KENT'S
MECHANICAL ENGINEERS' HANDBOOK

IN TWO VOLUMES

Power Volume

Prepared by a Staff of Specialists

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ABSTRACT FROM PREFACE TO THE FIRST EDITION, 1895

More than twenty years ago the author began to follow the advice given by Nystrom: "Every engineer should make his own pocket-book, as he proceeds in study and practice, to suit his particular business." The manuscript pocket-book thus begun, however, soon gave place to more modern means for disposing of the accumulation of engineering facts and figures, viz., the index rerum, the scrap-book, the collection of indexed envelopes, portfolios and boxes, the card catalogue, etc. Four years ago, at the request of the publishers, the labor was begun of selecting from this accumulated mass such matter as pertained to mechanical engineering, and of condensing, digesting, and arranging it in form for publication. In addition to this, a careful examination was made of the transactions of engineering societies, and of the most important recent works on mechanical engineering in order to fill gaps that might be left in the original collection, and insure that no important facts had been overlooked.

Some ideas have been kept in mind during the preparation of the pocket-book that will, it is believed, cause it to differ from other works of its class. In the first place it was considered that the field of mechanical engineering was so great, and the literature of the subject so vast, that as little space as possible should be given to subjects which especially belong to civil engineering.

Another idea prominently kept in view by the author has been that he would not assume the position of an "authority" in giving rules and formulae for designing, but only that of compiler, giving not only the name of the originator of the rule, where it was known, but also the volume and page from which it was taken, so that its derivation may be traced when desired. When different formulae for the same problem have been found they have been given in contrast, and in many cases examples have been calculated by each to show the difference between them. In some cases these differences are quite remarkable. Occasionally the study of these differences has led to the author's devising a new formula, in which case the derivation of the formula is given.

Much attention has been paid to the abstracting of data of experiments from recent periodical literature, and numerous references to other data are given. In this respect the present work will be found to differ from other pocket-books.

William Kent

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AIR

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PROPERTIES OF AIR AND OF MIXTURES OF AIR AND WATER VAPOR

BY JOSEPH KAYE AND JOSEPH H. KEENAN

FLOW OF AIR AND GASES

BY A. H. EHLINGER

FLOW OF AIR IN PIPES

BY A. H. EHLINGER

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BY THEODORE BAUMEISTER

FANS AND BLOWERS

BY T. A. WALTERS

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PROPERTIES OF AIR AND OF MIXTURES OF AIR AND WATER VAPOR

By Joseph Kaye and Joseph H. Keenan

THE GENERAL DEVELOPMENT OF GAS TABLES. The equation of state for many gases over a range of states which includes most engineering applications can be represented accurately by the relation

$$pv = RT \tag{1}$$

where p denotes the pressure, v the specific volume, R the gas constant, and T the absolute thermodynamic temperature, all in consistent units. When the gas in question has a critical temperature that is low relative to the range of temperatures encountered in engineering work, the deviation of eq. 1 from the true equation of state may be quite small. Thus for air at 32 F the deviation is of the order of 1% at 300 psi, and 0.1% at atmospheric pressure; at higher temperatures the deviation is smaller. For a gas with a critical temperature greater than that of air, the deviation of eq. 1 from the true equation of state will in general be greater than the corresponding deviation for air at the same temperature and pressure.

Equation 1 leads to great simplification of the presentation of the properties useful in engineering as compared with the tables and charts necessary to an adequate statement of the properties of a vapor.

It may be shown from eq. 1 that the *internal energy u and enthalpy h are functions of temperature only* (Ref. 1). Complete presentation of these quantities will consist, therefore, of a table with a single argument, the temperature.

The entropy, on the other hand, proves to be a function of pressure as well as temperature, so that an equally simple presentation is not possible.

Tabulations of entropy serve primarily in identifying states along an isentropic, that is, in the selection of states of equal entropy. This selection, however, is not necessary for a substance conforming to eq. 1 because it may be shown that the ratio of the pressures corresponding to a given pair of temperatures is the same for all isentropics and the ratio of the volumes is the same for all (Ref. 2). These relations are given by:

$$\left(\frac{p_a}{p_b}\right)_{s=\text{constant}} = \frac{p_{ra}}{p_{rb}} \tag{2}$$

and

$$\left(\frac{v_a}{v_b}\right)_{s=\text{constant}} = \frac{v_{ra}}{v_{rb}} \tag{3}$$

where p is the absolute pressure, v the volume, p_r the relative pressure, v_r the relative volume, and subscripts a and b refer to two states along a given isentropic. It has been shown (Ref. 2) that the relative pressure and relative volume are functions of temperature only.

The entropy s at any state reckoned from an arbitrary zero at a temperature T_0 and unit pressure is given by

$$s = \int_{T_0}^{T} \frac{c_p}{T}\, dT - R \ln p = \phi - R \ln p \tag{4}$$

in which c_p is the specific heat at constant pressure and

$$= \int_{T_0}^{T} \frac{c_p}{T}\, dT \tag{5}$$

Since c_p is a function of temperature only, ϕ is likewise. The change in entropy between states 1 and 2 is then

$$s_2 - s_1 = \phi_2 - \phi_1 - R \ln \frac{p_2}{p_1} \tag{6}$$

Thus, for a gas whose behavior is given by eq. 1, a table in which the temperature is the independent argument serves to present values of h, u, p_r, v_r, and ϕ. Such tables are presented below for air.

1. DRY AIR

The composition of dry air at sea level is given in Table 1 with sufficient precision for engineering calculations. This composition is independent of locality at sea level, but it varies somewhat with altitude. The exceedingly small percentages of carbon dioxide and hydrogen in the atmosphere (Ref. 3) are combined with the rare gases and labeled as argon in Table 1.

Table 1. Composition of Dry Air

Gas	Molecular Weight	Percentage by Volume
Nitrogen	28.016	78.03
Oxygen	32.000	20.99
Argon	39.95	0.98

The molecular weight of dry air based on the composition in Table 1 is 28.97. The gas constant for air, R, is 53.34 ft-lb per lb-F or 0.06855 Btu per lb-F.

The properties of air at low pressures are given in Table 2 (Ref. 4).

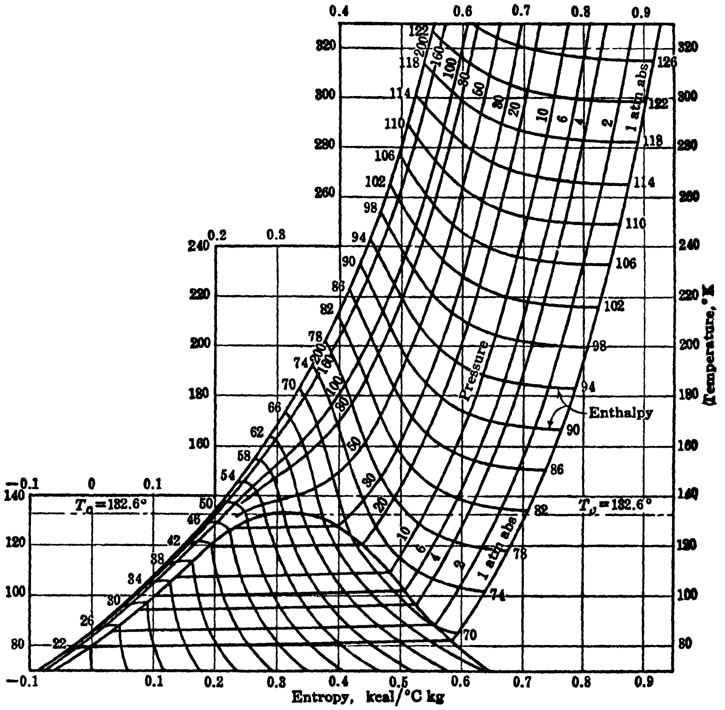

FIG. 1. Temperature-entropy diagram for dry air at low temperatures. (Ref. 6.)

Specific heats at constant pressure and volume are shown in Table 3, as well as the ratio of these specific heats, and the velocity of sound for dry air at low pressures. These values are condensed from Keenan and Kaye's *Gas Tables* (Ref. 4).

Table 2 gives the values of the properties of air with high precision provided that the pressure at the state in question is very low. A systematic error occurs whenever its values are applied to states of finite pressure. It has been shown (Ref. 4) that the systematic error is negligible for calculations of isentropic processes up to pressures of 200 psia.

The effect of pressure on the specific heat at constant pressure, c_p, and on the isothermal enthalpy-pressure derivative $(\partial H/\partial p)_T$, of air is shown in Table 4 (Ref. 5).

The properties of dry air at low temperatures are shown in Fig. 1 in the form of a temperature-entropy diagram, copied from Hausen (Ref. 6). The temperature is given in degrees Kelvin, the entropy in kilocalories per kilogram °C, the pressure in metric atmospheres (1 metric atm is 1 kg per cm^2), the enthalpy in kilocalories per kilogram. The critical temperature is 132.6 K. The two-phase region is also shown in Fig. 1. As the temperature increases, the lines of constant enthalpy become more nearly horizontal, indicating that the behavior of dry air approximates that of a perfect gas as the temperature increases.

Table 2. Air at Low Pressures

(For One Pound)

(The properties given here are condensed by permission of authors and publisher from *Gas Tables*, by J. H. Keenan and J. Kaye, published by John Wiley and Sons, 1948.)

T, °F abs	t, °F	h, Btu/lb	p_r	u, Btu/lb	v_r	ϕ, Btu/lb °F	T, °F abs	t, °F	h, Btu/lb	p_r	u, Btu/lb	v_r	ϕ, Btu/lb °F
100	−360	23.7	.00384	16.9	9640	.1971	1200	740	291.3	24.0	209.0	18.51	.7963
120	−340	28.5	.00726	20.3	6120	.2408	1220	760	296.4	25.5	212.8	17.70	.8005
140	−320	33.3	.01244	23.7	4170	.2777	1240	780	301.5	27.1	216.5	16.93	.8047
160	−300	38.1	.01982	27.1	2990	.3096	1260	800	306.6	28.8	220.3	16.20	.8088
180	−280	42.9	.0299	30.6	2230	.3378	1280	820	311.8	30.6	224.0	15.52	.8128
200	−260	47.7	.0432	34.0	1715	.3630	1300	840	316.9	32.4	227.8	14.87	.8168
220	−240	52.5	.0603	37.4	1352	.3858	1320	860	322.1	34.3	231.6	14.25	.8208
240	−220	57.2	.0816	40.8	1089	.4067	1340	880	327.3	36.3	235.4	13.67	.8246
260	−200	62.0	.1080	44.2	892	.4258	1360	900	332.5	38.4	239.2	13.12	.8285
280	−180	66.8	.1399	47.6	742	.4436	1380	920	337.7	40.6	243.1	12.59	.8323
300	−160	71.6	.1780	51.0	624	.4601	1400	940	342.9	42.9	246.9	12.10	.8360
320	−140	76.4	.2229	54.5	532	.4755	1420	960	348.1	45.3	250.8	11.62	.8398
340	−120	81.2	.2754	57.9	457	.4900	1440	980	353.4	47.8	254.7	11.17	.8434
360	−100	86.0	.336	61.3	397	.5037	1460	1000	358.6	50.3	258.5	10.74	.8470
380	−80	90.8	.406	64.7	347	.5166	1480	1020	363.9	53.0	262.4	10.34	.8506
400	−60	95.5	.486	68.1	305	.5289	1500	1040	369.2	55.9	266.3	9.95	.8542
420	−40	100.3	.576	71.5	270	.5406	1520	1060	374.5	58.8	270.3	9.58	.8568
440	−20	105.1	.678	74.9	241	.5517	1540	1080	379.8	61.8	274.2	9.23	.8611
460	0	109.9	.791	78.4	215.3	.5624	1560	1100	385.1	65.0	278.1	8.89	.8646
480	20	114.7	.918	81.8	193.6	.5726	1580	1120	390.4	68.3	282.1	8.57	.8679
500	40	119.5	1.059	85.2	174.9	.5823	1600	1140	395.7	71.7	286.1	8.26	.8713
520	60	124.3	1.215	88.6	158.6	.5917	1620	1160	401.1	75.3	290.0	7.97	.8746
540	80	129.1	1.386	92.0	144.3	.6008	1640	1180	406.4	79.0	294.0	7.69	.8779
560	100	133.9	1.574	95.5	131.8	.6095	1660	1200	411.8	82.8	298 0	7.42	.8812
580	120	138.7	1.780	98.9	120.7	.6179	1680	1220	417.2	86.8	302.0	7.17	.8844
600	140	143.5	2.00	102.3	110.9	.6261	1700	1240	422.6	91.0	306.1	6.92	.8876
620	160	148.3	2.25	105.8	102.1	.6340	1720	1260	428 0	95.2	310.1	6.69	.8907
640	180	153.1	2.51	109.2	94.3	.6416	1740	1280	433.4	99.7	314.1	6.46	.8939
660	200	157.9	2.80	112.7	87.3	.6490	1760	1300	438.8	104.3	318.2	6.25	.8970
680	220	162.7	3.11	116.1	81.0	.6562	1780	1320	444.3	109.1	322.2	6.04	.9000
700	240	167.6	3.45	119.6	75.2	.6632	1800	1340	449.7	114.0	326.3	5.85	.9031
720	260	172.4	3.81	123.0	70.1	.6700	1820	1360	455.2	119.2	330.4	5.66	.9061
740	280	177.2	4.19	126.5	65.4	.6766	1840	1380	460.6	124.5	334.5	5.48	.9091
760	300	182.1	4.61	130.0	61.1	.6831	1860	1400	466.1	130.0	338.6	5.30	.9120
780	320	186.9	5.05	133.5	57.2	.6894	1880	1420	471.6	135.6	342.7	5.13	.9150
800	340	191.8	5.53	137.0	53.6	.6956	1900	1440	477.1	141.5	346.8	4.97	.9179
820	360	196.7	6.03	140.5	50.4	.7016	1920	1460	482.6	147.6	351.0	4.82	.9208
840	380	201.6	6.57	144.0	47.3	.7075	1940	1480	488.1	153.9	355.1	4.67	.9236
860	400	206.5	7.15	147.5	44.6	.7132	1960	1500	493.6	160.4	359.3	4.53	.9264
880	420	211.4	7.76	151.0	42.0	.7189	1980	1520	499.1	167.1	363.4	4.39	.9293
900	440	216.3	8.41	154.6	39.6	.7244	2000	1540	504.7	174.0	367.6	4.26	.9320
920	460	221.2	9.10	158.1	37.4	.7298	2020	1560	510.3	181.2	371.8	4.13	.9348
940	480	226.1	9.83	161.7	35.4	.7351	2040	1580	515.8	188.5	376.0	4.01	.9376
960	500	231.1	10.61	165.3	33.5	.7403	2060	1600	521.4	196.2	380.2	3.89	.9403
980	520	236.0	11.43	168.8	31.8	.7454	2080	1620	527.0	204.0	384.4	3.78	.9430
1000	540	241.0	12.30	172.4	30.1	.7504	2100	1640	532.6	212	388.6	3.67	.9456
1020	560	246.0	13.22	176.0	28.6	.7554	2120	1660	538.2	220	392.8	3.56	.9483
1040	580	251.0	14.18	179.7	27.2	.7602	2140	1680	543.7	229	397.0	3.46	.9509
1060	600	256.0	15.20	183.3	25.8	.7650	2160	1700	549.4	238	401.3	3.36	.9535
1080	620	261.0	16.28	186.9	24.6	.7696	2180	1720	555.0	247	405.5	3.27	.9561
1100	640	266.0	17.41	190.6	23.4	.7743	2200	1740	560.6	257	409.8	3.18	.9587
1120	660	271.0	18.60	194.2	22.3	.7788	2220	1760	566.2	266	414.0	3.09	.9612
1140	680	276.1	19.86	197.9	21.3	.7833	2240	1780	571.9	276	418.3	3.00	.9638
1160	700	281.1	21.2	201.6	20.29	.7877	2260	1800	577.5	287	422.6	2.92	.9663
1180	720	286.2	22.6	205.3	19.38	.7920	2280	1820	583.2	297	426.9	2.84	.9688

Table 2. Air at Low Pressures—*Continued*

(For One Pound)

(The properties given here are condensed by permission of authors and publisher from *Gas Tables*, by J. H. Keenan and J. Kaye, published by John Wiley and Sons, 1948.)

T, °F abs	t, °F	h, Btu/lb	p_r	u, Btu/lb	v_r	ϕ, Btu/lb °F	T, °F abs	t, °F	h, Btu/lb	p_r	u, Btu/lb	v_r	ϕ, Btu/lb °F
2300	1840	588.8	308	431.2	2.76	.9712	3000	2540	790.7	941	585.0	1.180	1.0478
2320	1860	594.5	319	435.5	2.69	.9737	3020	2560	796.5	969	589.5	1.155	1.0497
2340	1880	600.2	331	439.8	2.62	.9761	3040	2580	802.4	996	594.0	1.130	1.0517
2360	1900	605.8	343	444.1	2.55	.9785	3060	2600	808.3	1025	598.5	1.106	1.0536
2380	1920	611.5	355	448.4	2.48	.9809	3080	2620	814.2	1054	603.0	1.083	1.0555
2400	1940	617.2	368	452.7	2.42	.9833	3100	2640	820.0	1083	607.5	1.060	1.0574
2420	1960	622.9	380	457.0	2.36	.9857	3120	2660	825.9	1114	612.0	1.038	1.0593
2440	1980	628.6	394	461.4	2.30	.9880	3140	2680	831.8	1145	616.6	1.016	1.0612
2460	2000	634.3	407	465.7	2.24	.9904	3160	2700	837.7	1176	621.1	.995	1.0630
2480	2020	640.0	421	470.0	2.18	.9927	3180	2720	843.6	1209	625.6	.975	1.0649
2500	2040	645.8	436	474.4	2.12	.9950	3200	2740	849.5	1242	630.1	.955	1.0668
2520	2060	651.5	450	478.8	2.07	.9972	3220	2760	855.4	1276	634.6	.935	1.0686
2540	2080	657.2	466	483.1	2.02	.9995	3240	2780	861.3	1310	639.2	.916	1.0704
2560	2100	663.0	481	487.5	1.971	1.0018	3260	2800	867.2	1345	643.7	.898	1.0722
2580	2120	668.7	497	491.9	1.922	1.0040	3280	2820	873.1	1381	648.3	.880	1.0740
2600	2140	674.5	514	496.3	1.876	1.0062	3300	2840	879.0	1418	652.8	.862	1.0758
2620	2160	680.2	530	500.6	1.830	1.0084	3320	2860	884.9	1455	657.4	.845	1.0776
2640	2180	686.0	548	505.0	1.786	1.0106	3440	2880	890.9	1494	661.9	.828	1.0794
2660	2200	691.8	565	509.4	1.743	1.0128	3360	2900	896.8	1533	666.5	.812	1.0812
2680	2220	697.6	583	513.8	1.702	1.0150	3380	2920	902.7	1573	671.0	.796	1.0830
2700	2240	703.4	602	518.3	1.662	1.0171	3400	2940	908.7	1613	675.6	.781	1.0847
2720	2260	709.1	621	522.7	1.623	1.0193	3420	2960	914.6	1655	680.2	.766	1.0864
2740	2280	714.9	640	527.1	1.585	1.0214	3440	2980	920.6	1697	684.8	.751	1.0882
2760	2300	720.7	660	531.5	1.548	1.0235	3460	3000	926.5	1740	689.3	.736	1.0899
2780	2320	726.5	681	536.0	1.512	1.0256	3480	3020	932.4	1784	693.9	.722	1.0916
2800	2340	732.3	702	540.4	1.478	1.0277	3500	3040	938.4	1829	698.5	.709	1.0933
2820	2360	738.2	724	544.8	1.444	1.0297	3520	3060	944.4	1875	703.1	.695	1.0950
2840	2380	744.0	746	549.3	1.411	1.0318	3540	3080	950.3	1922	707.6	.682	1.0967
2860	2400	749.8	768	553.7	1.379	1.0338	3560	3100	956.3	1970	712.2	.670	1.0984
2880	2420	755.6	791	558.2	1.348	1.0359	3580	3120	962.2	2018	716.8	.657	1.1000
2900	2440	761.4	815	562.7	1.318	1.0379	3600	3140	968.2	2068	721.4	.645	1.1017
2920	2460	767.3	839	567.1	1.289	1.0399	3620	3160	974.2	2118	726.0	.633	1.1034
2940	2480	773.1	864	571.6	1.261	1.0419	3640	3180	980.2	2170	730.6	.621	1.1050
2960	2500	779.0	889	576.1	1.233	1.0439	3660	3200	986.1	2222	735.3	.610	1.1066
2980	2520	784.8	915	580.6	1.206	1.0458	3680	3220	992.1	2276	739.9	.599	1.1083

EXAMPLE 1. COMPRESSION OF AIR IN STEADY FLOW. Air at a pressure of 1 atm abs and a temperature of 520 F abs is compressed in steady flow to a pressure of 6 atm abs. Find the work of compression and the temperature after compression for (1) 100% efficiency of compression and (2) 60% efficiency of compression. The efficiency of compression is here defined as the ratio of the isentropic work of compression to the actual work of compression.

Solution. (1) From Table 2 we get for $T_1 = 520$ F abs,

$$p_{r1} = 1.215, \qquad h_1 = 124.3 \text{ Btu/lb}$$

where subscript 1 refers to the state at the compressor inlet. To determine the properties at the compressor outlet for isentropic compression we compute the relative pressure there

$$p_{r2} = 6/1 \times 1.215 = 7.29$$

Interpolating in Table 2 with this value of p_r, we find, for h_{2s} and T_{2s}, the enthalpy and temperature at the compressor outlet for isentropic compression

$$h_{2s} = 207.6 \text{ Btu/lb}, \qquad T_{2s} = 864.6 \text{ F abs}$$

The work of compression for 100% efficiency is then

$$h_{2s} - h_1 = 83.3 \text{ Btu/lb}$$

(2) Since the efficiency of compression η is defined by the equation $\eta = (h_{2s} - h_1)$/work per pound, we have for 60% efficiency

$$\text{Work per pound} = \frac{83.3}{0.60} = 138.8 \text{ Btu/lb}$$

For the enthalpy at state 2, the state at the compressor outlet, we have

$$h_2 = 124.3 + 138.8 = 263.1 \text{ Btu/lb}$$

Interpolating in Table 2 with this value of the enthalpy, we get for the temperature at the compressor outlet

$$T_2 = 1088.4 \text{ F abs}$$

(The value of p_{r2} is irrelevant because the process is not isentropic.)

If in this problem the definition of the efficiency is altered to be the ratio of the reversible isothermal work of compression to the actual work of compression, Table 2 is not necessary to the solution, for it is readily shown that the work of reversible isothermal compression in steady flow is given by

$$RT \ln \frac{p_2}{p_1}$$

provided only that

$$pv = RT$$

Table 3. Air at Low Pressures

(These values are condensed by permission of authors and publisher from *Gas Tables*, by J. H. Keenan and J. Kaye, published by John Wiley and Sons, 1948.)

T, °F abs	t, °F	c_p, Btu/lb °F	c_v, Btu/lb °F	$k = \frac{c_p}{c_v}$	V_s, ft/sec	T, °F abs	t, °F	c_p, Btu/lb °F	c_v, Btu/lb °F	$k = \frac{c_p}{c_v}$	V_s, ft/sec
200	−260	.2392	.1707	1.402	694	3200	2740	.2950	.2264	1.303	2680
400	−60	.2393	.1707	1.402	981	3400	2940	.2969	.2283	1.300	2760
600	140	.2403	.1718	1.399	1200	3600	3140	.2986	.2300	1.298	2830
800	340	.2434	.1748	1.392	1382	3800	3340	.3001	.2316	1.296	2910
1000	540	.2486	.1800	1.381	1539	4000	3540	.3015	.2329	1.294	2980
1200	740	.2547	.1862	1.368	1679	4200	3740	.3029	.2343	1.292	3050
1400	940	.2611	.1926	1.356	1805	4400	3940	.3041	.2355	1.291	3120
1600	1140	.2671	.1985	1.345	1922	4600	4140	.3052	.2367	1.290	3190
1800	1340	.2725	.2039	1.336	2030	4800	4340	.3063	.2377	1.288	3260
2000	1540	.2773	.2088	1.328	2140	5000	4540	.3072	.2387	1.287	3320
2200	1740	.2813	.2128	1.322	2230	5200	4740	.3081	.2396	1.286	3390
2400	1940	.2848	.2162	1.317	2330	5400	4940	.3090	.2405	1.285	3450
2600	2140	.2878	.2192	1.313	2420	5600	5140	.3098	.2413	1.284	3510
2800	2340	.2905	.2219	1.309	2510	5800	5340	.3106	.2420	1.283	3570
3000	2540	.2929	.2243	1.306	2590	6000	5540	.3114	.2428	1.282	3630

Table 4. Effect of Pressure on Specific Heat and on Isothermal Enthalpy-Pressure Derivative of Air

(These values are abstracted from Thermodynamic Properties of Air, by R. V. Gerhart, F. C. Brunner, H. S. Mickley, B. H. Sage, and W. N. Lacey, *Mech. Eng.*, pp. 270–272, April 1942.)

Pressure, psia	32 F		130 F		250 F		370 F		550 F	
	c_p, Btu/lb	$\left(\frac{\partial H}{\partial p}\right)_T$ $\frac{\text{Btu}}{\text{lb}} \Big/ \frac{\text{lb}}{\text{in.}^2}$	c_p, Btu/lb	$\left(\frac{\partial H}{\partial p}\right)_T$ $\frac{\text{Btu}}{\text{lb}} \Big/ \frac{\text{lb}}{\text{in.}^2}$	c_p, Btu/lb	$\left(\frac{\partial H}{\partial p}\right)_T$ $\frac{\text{Btu}}{\text{lb}} \Big/ \frac{\text{lb}}{\text{in.}^2}$	c_p, Btu/lb	$\left(\frac{\partial H}{\partial p}\right)_T$ $\frac{\text{Btu}}{\text{lb}} \Big/ \frac{\text{lb}}{\text{in.}^2}$	c_p, Btu/lb	$\left(\frac{\partial H}{\partial p}\right)_T$ $\frac{\text{Btu}}{\text{lb}} \Big/ \frac{\text{lb}}{\text{in.}^2}$
0	.2397	−.00809	.2405	−.00560	.2420	−.00348	.2444	−.00216	.2491	−.000840
500	.2534	−.00760	.2514	−.00518	.2486	−.00312	.2490	−.00182	.2530	−.000607
1000	.2668	−.00702	.2617	−.00470	.2550	−.00276	.2530	−.00152	.2557	−.000330
1500	.2785	−.00629	.2712	−.00418	.2610	−.00241	.2572	−.00124	.2583	−.000137
2000	.2887	−.00539	.2796	−.00363	.2665	−.00205	.2612	−.000994	.2608	.000046
2500	.2970	−.00445	.2867	−.00305	.2714	−.00175	.2649	−.000827	.2633	.000184
3000	.3033	−.00342	.2921	−.00238	.2755	−.00140	.2682	−.000618	.2658	.000361
3500	.3088	−.00222	.2966	−.00170	.2791	−.00106	.2713	−.000388	.2683	.000539

2. MIXTURES OF AIR AND WATER VAPOR

Several terms, used often in dealing with mixtures of air and water vapor, are defined as in the paragraphs below.

Dry air or simply air denotes the usual mixture of atmospheric gases, exclusive of water vapor, the composition of which is given in Table 1. This differentiation is made between water vapor and the other atmospheric gases because the mass of water vapor in a given mass of air varies widely in contrast to the constant composition of the other gases.

Specific humidity ω is the ratio of the mass of water vapor to the mass of air in a given volume of mixture.

Relative humidity ϕ is the ratio of the mass of water vapor in unit volume of mixture to the density of saturated water vapor at the temperature of the mixture. When the relative humidity is unity, the mixture is saturated.

The dew point of a mixture is the temperature to which the mixture must be cooled at constant pressure before liquid water will form.

Dry-bulb temperature t_d is the temperature of the mixture recorded by a thermometer in thermal equilibrium with the mixture.

Wet-bulb temperature t_w is the temperature recorded under steady-state conditions by a thermometer whose surface is saturated with liquid water and is simultaneously exposed to the mixture of air and water vapor. The wet-bulb temperature does not represent a case of thermal equilibrium but rather one of simultaneous heat transfer and mass transfer. The upper limit of the wet-bulb temperature is the dry-bulb temperature and the lower limit is the dew point; for a saturated mixture of air and water vapor these three temperatures are identical.

3. SATURATED MIXTURES OF AIR AND WATER VAPOR

The thermodynamic properties of saturated mixtures of air and water vapor are given in Table 5 (Ref. 7). The first column gives the temperature in degrees Fahrenheit, the second the values of the vapor pressure of pure liquid water, the third the specific humidity, and the last three columns give the specific volume, enthalpy, and entropy of the mixture.

Table 5. Properties of Mixtures of Air Saturated with Water Vapor at a Pressure of One Atmosphere

(Condensed by permission of the authors and the editor of the journal from Thermodynamic Properties of Moist Air, by J. A. Goff and S. Gratch, *J. Am. Soc. Heating Ventilating Engrs.*, June 1945.)

t, °F	p_s, in. Hg	ω, lb of vapor/ lb of dry air	v_m, ft^3 of mixture/ lb of dry air	h_m, Btu/ lb of dry air	s_m, Btu/ °F lb of dry air
−160	0.101×10^{-6}	0.212×10^{-8}	7.52	−38.50	−0.1030
−140	0.100×10^{-5}	0.211×10^{-7}	8.03	−33.67	−0.0874
−120	0.765×10^{-5}	0.161×10^{-6}	8.54	−28.85	−0.0728
−100	0.467×10^{-4}	0.977×10^{-6}	9.05	−24.04	−0.0590
−80	0.236×10^{-3}	0.493×10^{-5}	9.55	−19.22	−0.0459
−60	0.101×10^{-2}	0.212×10^{-4}	10.1	−14.39	−0.0335
−40	0.379×10^{-2}	0.792×10^{-4}	10.6	−9.53	−0.0216
−20	0.126×10^{-1}	0.263×10^{-3}	11.1	−4.53	−0.0100
0	0.376×10^{-1}	0.787×10^{-3}	11.6	0.84	0.0019
20	0.103	0.215×10^{-2}	12.1	7.11	0.0153
40	0.248	0.521×10^{-2}	12.7	15.23	0.0319
60	0.522	0.111×10^{-1}	13.3	26.46	0.0539
80	1.03	0.223×10^{-1}	14.1	43.69	0.0864
100	1.93	0.432×10^{-1}	15.1	71.7	0.1374
120	3.45	0.815×10^{-1}	16.5	119.5	0.2216
140	5.88	0.153	18.8	205.7	0.3686
160	9.66	0.299	23.1	376.3	0.6511
180	15.3	0.658	33.0	791.8	1.319
200	23.5	2.30	77.1	2677	4.266

4. PSYCHROMETRIC CHARTS

It is convenient for many purposes to represent the composition of a mixture of air and water vapor in terms of wet-and-dry bulb temperatures on a psychrometric chart. The chart is valid only for a given value of the pressure of the mixture. Figures 2 and 3 represent two psychrometric charts for air and water vapor for a pressure of 1 atm (Ref. 8). The abscissa is the dry-bulb temperature and the ordinate is the specific humidity in pounds of water vapor per pound of dry air. On these charts are lines for constant values of the wet-bulb temperature and lines for constant values of the relative humidity. (See also Air Conditioning, Section 12, for psychrometric chart.)

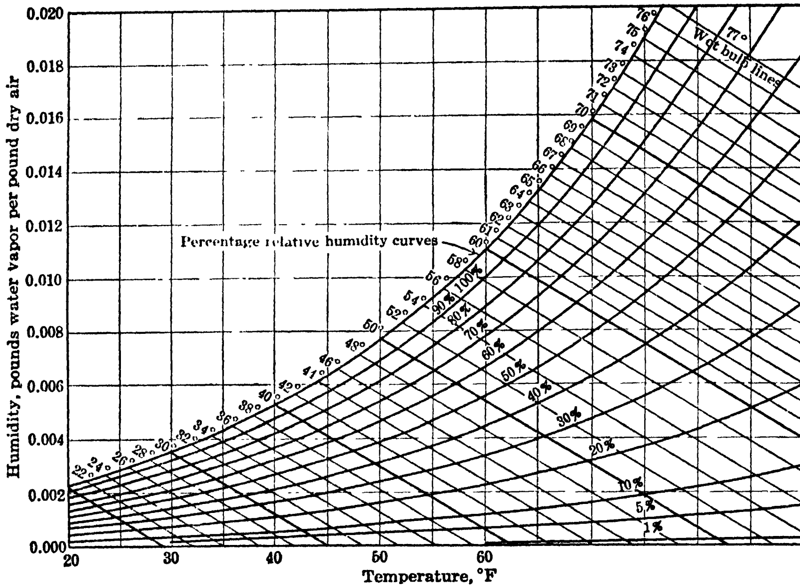

Fig 2. Psychrometric chart: temperature range, 20 to 90 F; pressure, 29.921 in. Hg. (Condensed by permission of the authors and publishers from *Psychrometric Tables and Charts*, by O. T. Zimmerman and I. Lavine, Industrial Research Service, Dover, N. H., 1945)

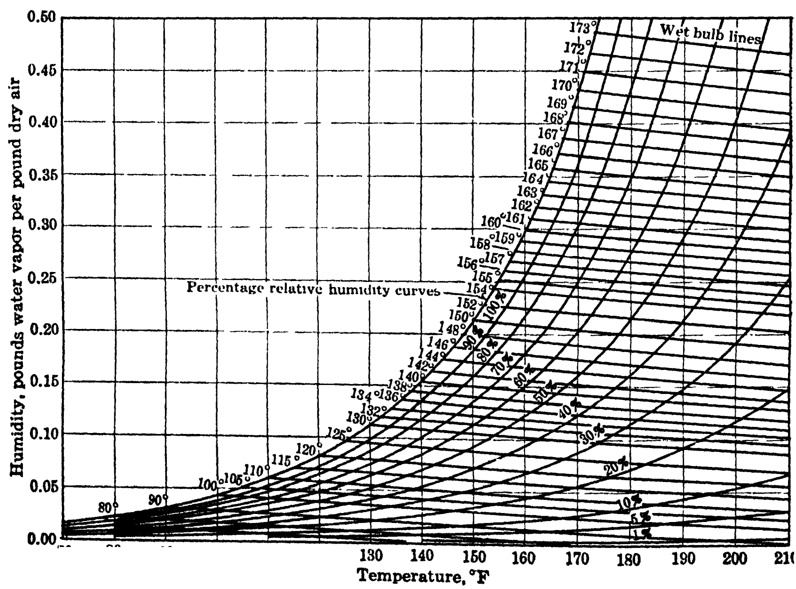

Fig. 3. Psychrometric chart: temperature range, 70 to 210 F; pressure, 29.921 in. Hg. (Condensed by permission of the authors and publishers from O. T. Zimmerman and I. Lavine, *op. cit.*)

5. THE STANDARD ATMOSPHERE

Properties of the Standard Atmosphere are presented in Table 6. They were condensed in 1925 from *NACA Report* 218 by W. S. Diehl, with the permission of the NACA.

Conversion data are shown in Table 7 by which feet of air may be determined for any pressure, as well as theoretical velocity of air for these pressures. These data are applicable at atmospheric pressure, as stated in the table.

Table 6. Standard Atmosphere

(Condensed by permission of the NACA from *NACA Report* 218, by W. S. Diehl, 1925.)

Altitude, ft	Temperature °F	Temperature °F abs	Pressure in. Hg	Pressure psia	Density, lb/ft³
0	59.0	518.7	29.92	14.696	.07651
2,000	51.9	511.6	27.82	13.664	.07213
4,000	44.7	504.4	25.84	12.692	.06794
6,000	37.6	497.3	23.98	11.778	.06395
8,000	30.5	490.2	22.22	10.914	.06013
10,000	23.3	483.0	20.58	10.108	.05649
12,000	16.2	475 9	19.03	9.347	.05303
14,000	9.1	468 8	17 57	8.630	.04973
16,000	1.9	461.6	16.21	7.962	.04658
18,000	−5.2	454.5	14.94	7.338	.04359
20,000	−12.3	477.4	13.75	6.753	.04075
22,000	−19.5	440 2	12.63	6.203	.03806
24,000	−26.6	433.1	11.59	5.693	.03550
26,000	−33.7	426.0	10.62	5.216	.03308
28,000	−40.9	418.8	9.72	4.774	.03078
30,000	−48.0	411.7	8.88	4.362	.02861
32,000	−55.1	404.6	8.10	3.978	.02656
34,000	−62 2	397.5	7.38	3.625	.02463
36,000	−67.0	392.7	6.71	3.296	.02265
38,000	−67.0	392.7	6.10	2.996	.02059
40,000	−67.0	392 7	5.54	2.721	.01872
42,000	−67.0	392.7	5.04	2.475	.01701
44,000	−67.0	392.7	4.58	2.250	.01546
46,000	−67.0	392.7	4.16	2.043	.01405
48,000	−67.0	392.7	3.78	1.857	.01277
50,000	−67.0	392.7	3.44	1.690	.011610
52,000	−67.0	392.7	3.12	1.532	.010550
54,000	−67.0	392.7	2.84	1.395	.009591
56,000	−67 0	392.7	2.58	1.267	.008718
58,000	−67.0	392.7	2.35	1.154	.007922
60,000	−67.0	392.7	2.13	1.046	.007201
62,000	−67.0	392.7	1.94	0.953	.006546
64,000	−67.0	392.7	1.76	0.864	.005949

Table 7. Conversion Table for Air Pressures

	Lb per sq ft	In. of Water	Oz per sq in.	Ft of Water	In. of Mercury	Psi	Ft of Air at 62 F *	$V = \sqrt{2gH}$ ft per sec †
1 lb per sq ft	1	0.19245	1/9	0.01604	0.01414	1/144	13.14	29.1
1 in. water at 62 F	5.196		0.5774	1/12	0.07347	0.036085	68.30	66.3
1 oz per sq in.	9	1.732	1	0.1443	0.1272	1/16	118.3	87.2
1 ft water at 62 F	62.355	12	6.928	1	0.8816	0.43302	819.6	230
1 in. mercury at 32 F	70.73	13.612	7.859	1.1343	1	0.49117	929.6	245
1	144	27.712	16	2.3094	2.036	1	1,893	349
1 atm	2116.3	407.27		33.94	29.921	14.6963	27,815	1338

* The figures in this column show the head in feet of air of uniform density at atmospheric pressure and 62 F corresponding to the pressure in the preceding columns.

† The figures in this column show the theoretical velocities corresponding to these heads, or the velocities of a jet flowing from a frictionless nozzle.

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FLOW OF AIR AND GASES

By A. H. Ehlinger

6. IDEAL FLOW FORMULAS FOR NOZZLES, ORIFICES, AND VENTURI TUBES

Symbol	Description	Units
a	Cross-sectional area of pipe (general)	sq in.
a_1	Cross-sectional area of pipe, initial point	sq in.
a_2	Throat area of nozzle or venturi tube; also area of orifice	sq in.
C	Discharge coefficient	dimensionless
c_p	Specific heat at constant pressure	Btu/lb/°F
c_v	Specific heat at constant volume	Btu/lb/°F
d_1	Internal diameter of pipe	in.
d_2	Throat diameter of nozzle or venturi tube; also diameter of orifice	in.
E	Area multiplier for thermal expansion of measuring element	dimensionless
g	Ratio of 1 slug to 1 lb = 32.17	dimensionless
h	Enthalpy of fluid (note units)	ft-lb/lb
U	Internal energy of fluid (note units)	ft-lb/lb
K	Flow coefficient	dimensionless
k	Ratio of specific heats = c_p/c_v	dimensionless
p	Absolute pressure of fluid (general)	psi
p_1	Absolute inlet static pressure	psi
p_{1t}	Absolute inlet total pressure	psi
p_2	Absolute static pressure at throat of nozzle or venturi tube; also absolute static pressure at downstream side of orifice plate	psi
p_3	Absolute pressure to which volumetric rate of flow is referred	psi
Δp	$p_1 - p_2$	psi
Δp_t	$p_{1t} - p_2$	psi
Q	Volumetric rate of flow	cu ft/min
q	Volumetric rate of flow	cu ft/sec
R	Gas constant = approx. 1545/mol.wt.	ft-lb/lb/°F
R_d	Reynolds' number	dimensionless
t	Observed temperature	°F
T	Absolute temperature = $t + 460$	°F
T_1	Absolute inlet temperature	°F
T_3	Absolute temperature to which volumetric rate of flow is referred	°F
V	Velocity of fluid	ft/sec
v	Specific volume of fluid = $1/\rho$	cu ft/lb
w	Gravimetric rate of flow	lb/sec
Y	Empirical expansion factor	dimensionless
z	Elevation of stream above datum	ft
μ	Absolute viscosity	lb/ft-sec
μ/ρ	Kinematic viscosity	sq ft/sec
ρ	Density of fluid	lb/cu ft
ϕ	Expansion factor for eq. 7	dimensionless
θ	Expansion factor for eq. 9	dimensionless
π	3.14159	dimensionless

Formulas developed herein cover flow of fluids through a restriction. During the flow process the velocity of the fluid is increased, and a pressure differential must exist across the restriction in order to sustain the flow. The velocity of the fluid continues to increase until the static pressure of the stream is reduced to the level of the static pressure on the downstream side of the restriction. The general method for developing a flow equation is to equate the energy of the fluid at one point to its energy at some other point farther downstream. By this procedure it is possible to determine gain in velocity of the fluid as it proceeds from the initial to the final point. If initial and final areas of the stream are known, it is possible to calculate the rate of flow through the restriction.

In the development of theoretical flow equations flow is considered to be frictionless and adiabatic, that is, isentropic. For flow with friction, see Flow of Air in Pipes, p. 1-22.

If properties of the fluid at the initial point are denoted by subscript 1 and at the final point by subscript 2, the general energy equation of the fluid for adiabatic flow is

$$h_1 + \frac{V_1^2}{2g} + z_1 = h_2 + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

For any fluid $h = U + pv$. Furthermore, if consideration is given only to horizontal flow, the Z terms cancel. Thus, the general energy equation for adiabatic horizontal flow may be written

$$U_1 + p_1v_1 + \frac{V_1^2}{2g} = U_2 + p_2v_2 + \frac{V_2^2}{2g} \qquad (2)$$

EQUATIONS FOR INCOMPRESSIBLE FLUIDS. For frictionless flow $U_1 = U_2$. Solving eq. 2 for the gain in kinetic energy:

$$\frac{V_2^2 - V_1^2}{2g} = 144(p_1 - p_2)v = \frac{144\,\Delta p}{\rho} \qquad (3)$$

Since $V = 144q/a = 144w/\rho a$, the foregoing equation may be written in terms of w, a, and ρ. Solving for w:

$$w = \sqrt{\frac{2g}{}}\sqrt{\frac{a_1^2}{a_1^2 - a_2^2}}\,a_2\sqrt{\rho\,\Delta p} = 0.668\frac{1}{\sqrt{1 - \left(\frac{d_2}{d_1}\right)^4}}\,a_2\sqrt{\rho\,\Delta p} \qquad (4)$$

FUNDAMENTAL EQUATION FOR COMPRESSIBLE FLOW. Theoretical equations for compressible flow usually are developed for perfect gases, that is, gases which obey the equation, $pv = RT$, wherein R is a constant of the gas. It can be shown that for the flow process for a perfect gas, $\Delta h = 778c_p\,\Delta t$. Therefore, the following equation can be derived from eq. 1:

$$\frac{V_2^2 - V_1^2}{2g} = h_1 - h_2 = 778c_p(t_1 - t_2) \qquad (5)$$

For a perfect gas undergoing an isentropic change of state, the following relationships exist (see also Engineering Thermodynamics, Section 3):

$$778c_p = R\frac{k}{k-1}$$

$$t_1 - t_2 = T_2\left[\left(\frac{p_1}{p_2}\right)^{(k-1)/k} - 1\right] = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k}\left[\left(\frac{p_1}{p_2}\right)^{(k-1)/k} - 1\right]$$

$$\rho = \frac{144p}{RT} \qquad V = \frac{144q}{a} = \frac{144w}{\rho a} = \frac{wRT}{pa}$$

$$pv^k = \text{constant}$$

By combining these basic relationships *the most important equation of all for compressible flow may be found.* This equation, based on isentropic flow, is fundamental. All others are variations, used for special purposes. In this equation the fluid is initially static, at a pressure p_1, and expands to a static pressure p_2.

$$w = A\sqrt{2g\frac{k}{k-1}\cdot\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{(k+1)/k}\right]} \quad \text{(in fps units)} \qquad (6)$$

where w is weight of gas discharged per second, pounds; A is area of cross section of jet, square feet; p_1 is initial pressure, p_2 is exhaust pressure, *pounds per square foot;* v_1 is initial specific volume, cubic feet per pound; k is ratio of specific heat at constant pressure to that at constant volume.

In practice flow is neither frictionless nor perfectly adiabatic, and the amount of heat entering or leaving the gas is not known. Hence the weight actually discharged is found by introducing a coefficient of discharge (generally less than unity), depending on the form of orifice employed. Equations given hereinafter may be compared with the basic equation. Much similarity will be found, the chief difference being inclusion of correction for velocity of approach, use of pressure drop instead of pressure ratio, inclusion of lumped constants, etc.

SPECIAL EQUATIONS FOR COMPRESSIBLE FLOW. Substituting the isentropic perfect gas relationships in eq. 5 and solving for w:

$$w = \sqrt{\frac{2g}{RT_1}} \frac{1}{\sqrt{1 - \left(\frac{d_2}{d_1}\right)^4}} a_2\phi\sqrt{p_1\,\Delta p} \tag{7}$$

where

$$\phi = \left[\frac{\frac{k}{k-1}\left(\frac{p_2}{p_1}\right)^{2/k}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]}{1 - \frac{p_2}{p_1}}\right]^{\frac{1}{2}} \left[\frac{1 - \left(\frac{d_2}{d_1}\right)^4}{1 - \left(\frac{d_2}{d_1}\right)^4\left(\frac{p_2}{p_1}\right)^{2/k}}\right]^{\frac{1}{2}} \tag{8}$$

This equation for ϕ was developed from purely theoretical considerations; hence it does not include allowance for practical deviations from theory, such as contraction of the jet (vena contracta), etc. In practice these factors are important. They are taken into account by use of a factor Y (see Fig. 2), values of which have been worked out by the ASME Power Test Codes Committee and published in *Flow Measurement*, 1940, by ASME. An approximate empirical equation for Y is given on p. 1-14, eq. 15.

Use of Pitot Tube for Upstream Pressure. In the use of nozzles initial pressure is sometimes measured by a pitot tube, which indicates total pressure of the fluid. In such cases initial (or total) pressure is indicated by the symbol p_{1t}, and the initial velocity need not be considered separately. Equation 7 then may be written

$$w = \sqrt{\frac{2g}{RT_1}}\, a_2\theta\sqrt{p_2\,\Delta p_t} \tag{9}$$

where $\Delta p_t = p_{1t} - p_2$, and in which

$$\theta = \sqrt{\frac{k}{k-1}\,\frac{\left(\frac{p_{1t}}{p_2}\right)^{(k-1)/k}\left[\left(\frac{p_{1t}}{p_2}\right)^{(k-1)/k} - 1\right]}{\frac{p_{1t}}{p_2} - 1}} \tag{10}$$

Values of θ are plotted in Fig. 1.

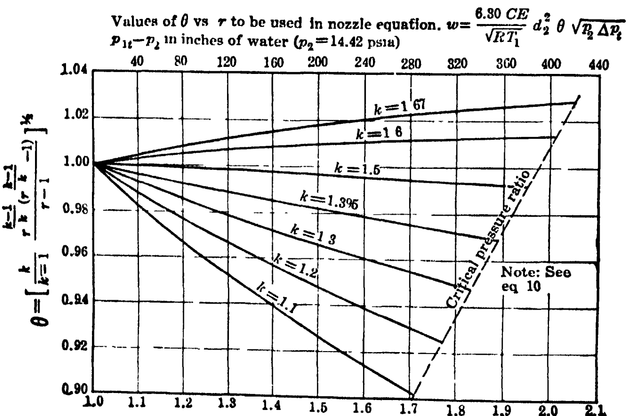


FIG. 1. Values of θ for use in eq. 9.

CRITICAL FLOW EQUATIONS FOR COMPRESSIBLE FLUID. From the fundamental flow equation for compressible fluids it is seen that the flow per unit of stream area is proportional to

$$\left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{(k+1)/k}\right]^{\frac{1}{2}}$$

in which p_1 is equivalent to p_{1t}, since the approach velocity is zero. By differentiating this function and setting the result equal to zero, we may find the pressure ratio for maximum flow per unit of stream area. This is found to be

$$\frac{p_2}{p_1} = \frac{p_2}{p_{1t}} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \tag{11}$$

The maximum flow per unit of stream area will occur where the stream area is smallest. For a nozzle or a venturi tube the minimum area is at the throat section. Therefore, the ratio of initial *total* pressure to static pressure at the throat cannot exceed the value $\left(\frac{k+1}{2}\right)^{k/(k-1)}$. This is known as the *critical pressure ratio*, and the rate of flow at this pressure ratio is known as the *critical rate of flow*. If the value $\left(\frac{k+1}{2}\right)^{k/(k-1)}$ is the following equation can be derived for critical rate of flow:

$$w = a_2 p_{1t} \sqrt{\frac{2g}{RT_1}} \left(\frac{2}{k+1}\right)^{1/(k-1)} \left(\frac{k}{k+1}\right)^{1/2} \tag{12}$$

where a_2 is equal to the throat area. (See also Section 3, Art. 25.) Substitution of $R = 53.3$, $k = 1.4$ in this equation yields Fliegner's formula for flow of air with greater than critical pressure drop across the nozzle.

Fliegner's Equation for Flow of Air.

$$w = C\,0.53a_2 \frac{p}{\sqrt{T}} \tag{13}$$

where w is flow, pounds per second; a_2 is throat area of the orifice, square inches; p is absolute pressure in the orifice chamber, pounds per square inch; T is absolute temperature, °F, of the air in the chamber; and C is flow coefficient. The formula applies only when the absolute pressure in the reservoir is greater than 1.89 times the exhaust pressure.

7. MEASUREMENT OF FLOW

To apply the theoretical flow equations for measurement purposes it is necessary to modify the calculated flow by a *discharge coefficient*, C (see Table 3), to compensate for friction losses, and by an *area multiplier*, E (see Fig. 3), to compensate for thermal expansion of the measuring element.

For applications where *static pressure* at the upstream side of the measuring element is used, a *flow coefficient* K is required. This coefficient simultaneously corrects for the velocity of approach and for the discharge coefficient.

$$K = \frac{C}{\sqrt{1 - \left(\frac{d_2}{d_1}\right)^4}} \tag{14}$$

The quantity $\sqrt{1 - (d_2/d_1)^4}$, known as the *velocity of approach factor*, is unity for flow from a large reservoir ($d_1 \cong \infty$). It is also equal to unity when *total pressure* is substituted for *initial static pressure*.

Measurement by Nozzles. For nozzles static pressure at the throat is used as the downstream pressure. When total pressure ahead of the nozzle is used, coefficient C is used as a factor of eq. 9. When static pressure ahead of the nozzle is recorded, coefficient C is used as a factor of eqs. 4 and 7. The denominator of coefficient K is already shown in eqs. 4 and 7, but C must be applied as a factor.

Measurement by Orifices. To determine the rate of flow through an orifice, both upstream and downstream static pressures are measured. Equations 4 and 7 are applied in conjunction with coefficient C. In application of these equations the area of the orifice is taken as a_2 and the diameter of the orifice as d_2. Since area of the orifice is greater than final area of the stream, it becomes necessary, in theory, to multiply rate of flow by the ratio of the final area of the stream to the area of the orifice. In practice, this ratio is included in the flow coefficient K. It has been found by experiment that geometrically similar measuring elements operating at the same Reynolds' number will have the same flow coefficient. Factors affecting similarity of an orifice arrangement are ratio of orifice diameter to pipe diameter and location of pressure taps. Values of K for orifices are plotted against Reynolds' number for various diameter ratios. For compressible fluids it is also necessary to modify the value of ϕ, because eqs. 7 and 8 were developed with d_2 equal to the final area of the stream. The modified form of ϕ, represented by Y, is given in terms of orifice diameter by the empirical equation

$$Y = 1 - \left[0.41 + 0.35\left(\frac{d_2}{d_1}\right)^4\right]\left[\frac{\Delta p}{p_1}/k\right] \tag{15}$$

Equation 15 may be used for all three types of pressure taps described below; it applies in particular when $\Delta p/p_1$ is less than 0.3. Values of Y for air computed from eq. 15 are given in Fig. 2.*

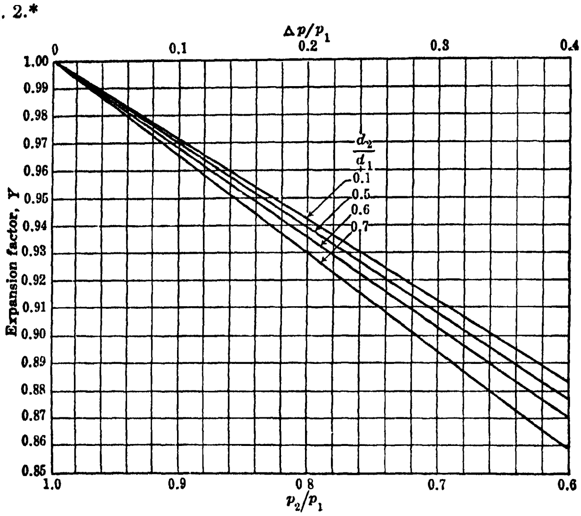


FIG. 2. Expansion factors for air, $k = 1.4$, computed from

$$Y = 1 - \left[0.41 + 0.35\left(\frac{d_2}{d_1}\right)^4\right]\left[\frac{\Delta p}{p_1}/k\right]$$

(*ASME Fluid Meters Report*, Part 1, 4th Ed., 1937, Fig. 72)

Reynolds' Number. Reynolds' number as used herein is defined by

$$R_d = \frac{V_2 d_2 \rho_2}{12\mu_2} = \frac{48w}{\pi d_2 \mu_2} \tag{16}$$

where d_2 is diameter of the orifice or throat of nozzle. The difference between μ_2 and μ_1 is usually small enough to permit using μ_1.

DATA FOR USE IN MEASURING FLOW. Figures 3 through 11 present curves abstracted, by permission, from *Flow Measurement*, 1940, prepared and published by the ASME Power Test Codes Committee. Figure 3 gives values of the expansion factor, E, as a function of temperature. Figures 4 and 5 give viscosity and density data for water. Figures 6 to 8 give viscosity data for various gases, air, and steam (see also Section 6, Art. 15). Figure 9 gives values of the flow coefficient, K, for square-edged orifices in 4-in. pipe, with flange taps; Fig. 10 gives similar data for vena contracta taps; and Fig. 11 gives the same data for radius taps, as defined below. For other pipe sizes, from 2 to 16 in., see *Flow Measurement*.

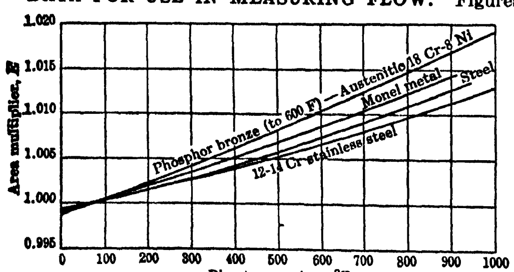


FIG. 3. Area multipliers for thermal expansion of primary elements. (Figures 3 through 11 are abstracted, by permission, from *Flow Measurement*, 1940, prepared and published by the ASME Power Test Codes Committee)

STATIC PRESSURE TAPS. The flow coefficients, K, given in Figs. 9 through 11, are for orifices located in thin flat plates held between the upstream and downstream sections

* Taken, by permission, from material prepared and published by the ASME Power Test Codes Committee, *Flow Measurement*, 1940. Other curves in this chapter are from same source.

of pipe. The cylindrical edge of the orifice must be square with the upstream surface of the plate, free from burrs, concentric with the pipe, and not longer than one-fiftieth of

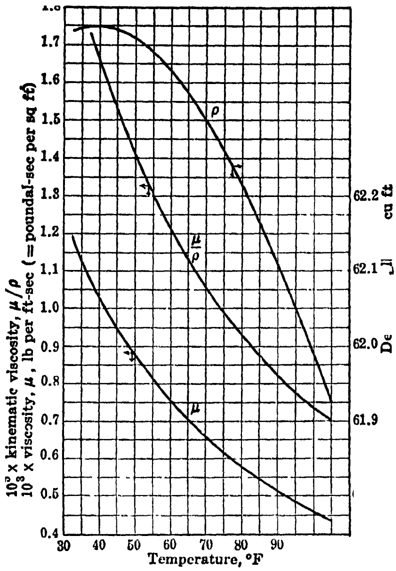


FIG. 4. Absolute viscosity, kinematic viscosity, and density of water below 105 F.

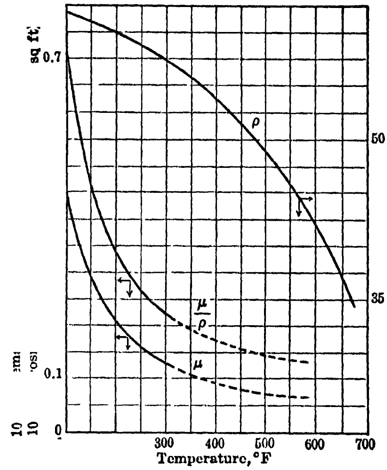


FIG. Absolute viscosity, kinematic viscosity, and density of water above 100 F.

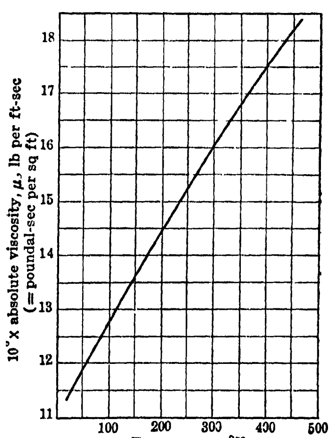


FIG. 6. Absolute viscosity of air.

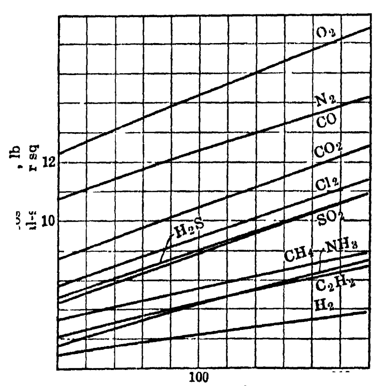


FIG. 7. Absolute viscosities of gases. (Data from Landolt-Bornstein Tables, 1923)

the pipe diameter. When the orifice plate is thicker than this dimension, the downstream edge of the orifice must be beveled at an angle of 45 degrees or less to the downstream face of the orifice plate. The length of straight pipe before and after the orifice plate must be in accordance with Fig. 12.

Locations of pressure taps used in conjunction with these orifices must be in accordance with Fig. 13 and the following rules.

Flange Taps. The centers of the holes shall be 1 in. from the respective faces of the orifice plate. (If $^1/_{16}$-in. gaskets are to be used this will require that the centers of the holes shall be $^{15}/_{16}$ in. from the bearing face of either flange.)

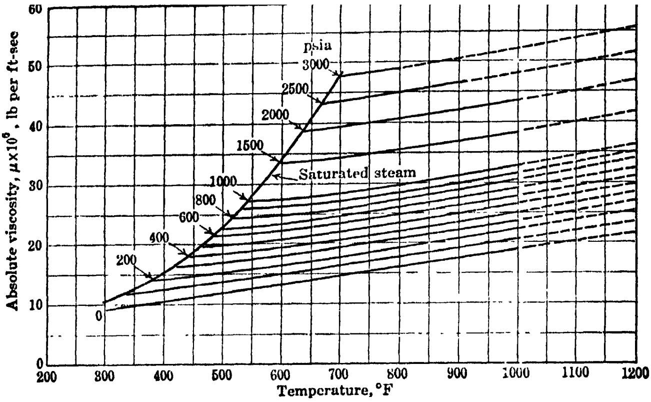


FIG. 8. Absolute viscosity of superheated steam.

Vena Contracta Taps. The center of the upstream hole shall be placed one pipe diameter from the upstream face of the orifice plate. The distance from the downstream face of the orifice to the center of the downstream hole shall be determined from Fig. 14 for the particular ratio of orifice to pipe diameter to be used.

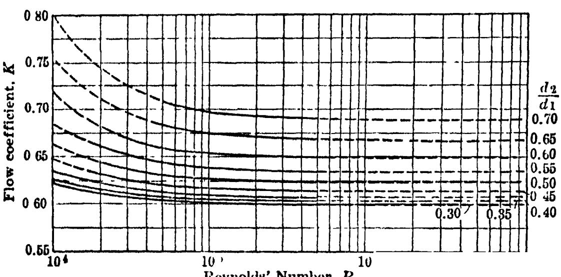


FIG. 9. Flow coefficient, K, for square-edged orifice plates, flange taps, 4-in pipe

Radius Taps. The center of the upstream hole shall be one pipe diameter from the upstream face of the orifice plate. The center of the downstream hole shall be one-half pipe diameter from the downstream face of the orifice plate.

The actual diameters of the pressure holes (i.e., drill sizes) at the inner surface of the pipe shall not exceed: $^1/_4$ in. for 3-in. pipe or smaller; $^3/_8$ in. for 3 $^1/_2$- to 6-in. pipe, inclusive; and $^1/_2$ in. for pipes larger than 6 in.

The corners of the holes at the inner surface of the pipe shall not only be free from burrs but shall also be smoothed off or slightly rounded (as with emery cloth).

LIMITATIONS OF COEFFICIENTS. The values of K and Y for square-edged orifice plates given herein apply particularly to conditions within the following limits:

$$2 \text{ in.} < d_1 < 24 \text{ in.}$$

$$d_2 \geq {}^3/_8 \text{ in.}$$

$$0.15 < \frac{d_2}{d_1} < 0.75$$

$$\frac{\Delta p}{p_1} \leq 0.2$$

Within these limits the values of K and Y (except for steam) may generally be relied upon to within ±0.5%. For steam, the "limit of error" of expansion factor, Y, is ±1%.

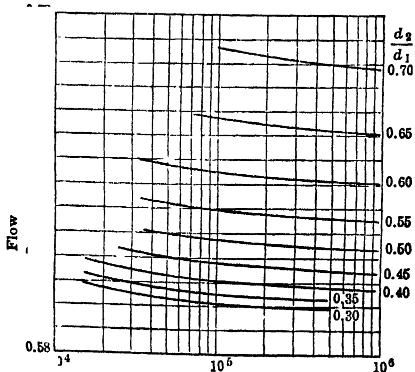


FIG. 10. Flow coefficient, K, for square-edged orifice plates, vena contracta taps, 4-in. pipe.

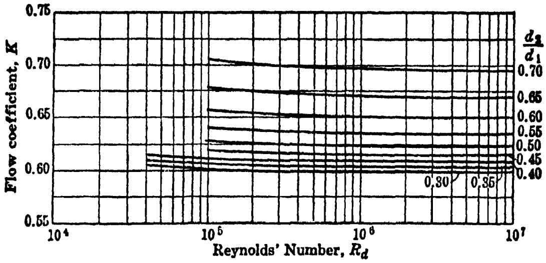


FIG. 11. Flow coefficient, K, for square-edged orifice plates, radius taps, 4-in. pipe.

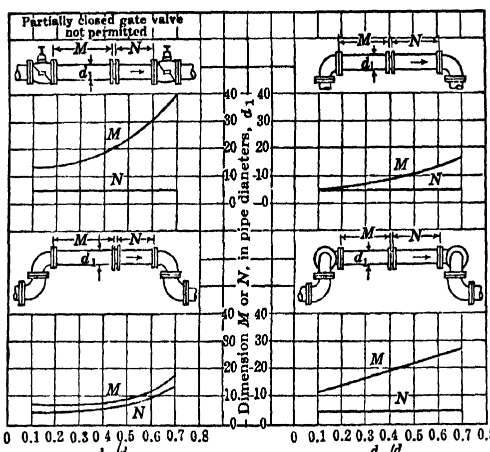


FIG. 12. Minimum pipe dimensions for undisturbed flow and no straighteners. (Based on Figs. 1-5, Report of the Joint AGA-ASME Committee on Orifice Coefficients, 1935, and Figs. 27, 27a, 28, and 29, Durchflusszahlen von Düsen und Staurandern, R. Witte, *Technische Mechanik und Thermodynamik*, Vol. 1, March 1930)

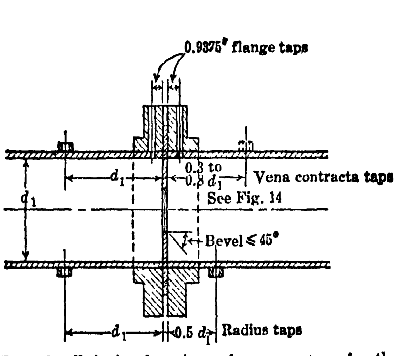


FIG. 13. Relative locations of pressure taps for the three types of taps.

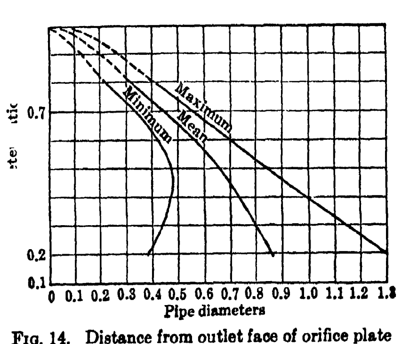


FIG. 14. Distance from outlet face of orifice plate to outlet pressure tap for vena contracta taps. (*ASME Fluid Meters Report*, Part 1, 4th Ed., 1937, Fig. 10)

8. FLOW OF AIR FROM A RECEIVER

In flow from a receiver, static air pressure in the receiver is equal to total pressure. For subcritical flow, where the pressure ratio across the restriction is less than 1.89, eq. 9, containing factor θ, applies. For critical flow, where the pressure ratio across the restriction is greater than 1.89, Fliegner's formula, eq. 13, must be used. In these equations p_{1t} is the static pressure of the receiver, p_2 is equal to the static pressure at the downstream side of the restriction, Δp_t is equal to $(p_{1t} - p_2)$ and d_2 is the diameter of the restriction.

Discharge coefficient, C, must compensate for contraction of the jet and will vary with rate of flow or pressure drop. According to Weisbach, the approximate range of values of C for various restrictions is

	Values of C
Square-edged circular orifices in thin plates	0.56 to 0.79
Short cylindrical mouthpieces	0.81 to 0.84
Short cylindrical mouthpieces, rounded at inner end	0.92 to 0.93
Conical converging mouthpieces	0.90 to 0.99

The Compressed Air and Gas Institute has adopted the value of $C = 0.65$ for sharp-edged orifices and $C = 0.97$ for an orifice with a well-rounded entrance. Table 1 gives theoretical rates of flow from receiver to atmosphere for various orifice sizes and receiver pressures. Values given are for a receiver temperature of 70 F and $C = 1.0$. Appropriate value of C must be chosen by the user and applied to the result as a factor.

Table 1. Discharge of Air through an Orifice

In cubic feet of free air per minute at standard atmospheric pressure of 14.7 psia and 70 F

(Table reprinted from *Compressed Air Handbook*, copyright 1947, by Compressed Air and Gas Institute, New York)

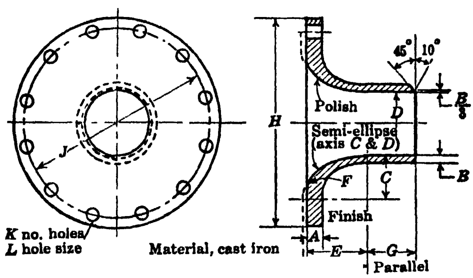
Gage Pressure before Orifice, psi	Diameter of Orifice, in.										
	1/64	1/32	1/16	1/8	1/4	3/8	1/2	5/8	3/4	7/8	1
1	.028	.112	.450	1.80	7.18	16.2	28.7	45.0	64.7	88.1	115
2	.040	.158	.633	2.53	10.1	22.8	40.5	63.3	91.2	124	162
3	.048	.194	.775	3.10	12.4	27.8	49.5	77.5	111	152	198
4	.056	.223	.892	3.56	14.3	32.1	57.0	89.2	128	175	228
5	.062	.248	.993	3.97	15.9	35.7	63.5	99.3	143	195	254
6	.068	.272	1.09	4.34	17.4	39.1	69.5	109	156	213	278
7	.073	.293	1.17	4.68	18.7	42.2	75.0	117	168	230	300
9	.083	.331	1.32	5.30	21.1	47.7	84.7	132	191	260	339
12	.095	.379	1.52	6.07	24.3	54.6	97.0	152	218	297	388
15	.105	.420	1.68	6.72	26.9	60.5	108	168	242	329	430
20	.123	.491	1.96	7.86	31.4	70.7	126	196	283	385	503
25	.140	.562	2.25	8.98	35.9	80.9	144	225	323	440	575
30	.158	.633	2.53	10.1	40.5	91.1	162	253	365	496	648
35	.176	.703	2.81	11.3	45.0	101	180	281	405	551	720
40	.194	.774	3.10	12.4	49.6	112	198	310	446	607	793
45	.211	.845	3.38	13.5	54.1	122	216	338	487	662	865
50	.229	.916	3.66	14.7	58.6	132	235	366	528	718	938
60	.264	1.06	4.23	16.9	67.6	152	271	423	609	828	1082
70	.300	1.20	4.79	19.2	76.7	173	307	479	690	939	1227
80	.335	1.34	5.36	21.4	85.7	193	343	536	771	1050	1371
90	.370	1.48	5.92	23.7	94.8	213	379	592	853	1161	1516
100	.406	1.62	6.49	26.0	104	234	415	649	934	1272	1661
110	.441	1.76	7.05	28.2	113	254	452	705	1016	1383	1806
120	.476	1.91	7.62	30.5	122	274	488	762	1097	1494	1951
125	.494	1.98	7.90	31.6	126	284	506	790	1138	1549	2023

Table is based on 100% flow coefficient. For well-rounded entrance multiply values by 0.97; for sharp-edged orifices use a multiplier of 0.65. Table assumes 70 F temperature in receiver. Weight flows were converted to volume flow in cfm by using a density of 0.0749 lb per cu ft (dry air at 14.7 psia, 70 F).

9. MEASUREMENT BY ASME STANDARD FLOW NOZZLES

The ASME long-radius low-ratio nozzle has been adopted by the Compressed Air and Gas Institute for measuring capacity of displacement or centrifugal type compressors or exhausters. The form of the nozzles and the related dimensions for nozzles having throat diameters from $^1/_8$ to 24 in. are given in Table 2. The approximate rates of flow in cubic feet per minute for a nozzle discharging into atmosphere is also given as a guide in selecting a nozzle size.

Table 2. Dimensions for Standard Long-radius Low-ratio Flange Type Nozzles



(Reprinted by permission from *Compressed Air Handbook*, Compressed Air and Gas Institute, 1947.)

D	*A*	*B*	*C*	*E*	*F*	*G*	*H*	*J*	*K*	*L*	Approx. Flow, cfm 10 H_2O	40 H_2O
0.125	0.437	0.250	0.09	0.121	0.01	0.437	4.25	3.125	4	0.562	1	2
0.1875	0.437	0.250	0.13	0.181	0.01	0.468	4.25	3.125	4	0.562	2	4
0.250	0.437	0.250	0.17	0.242	0.01	0.500	4.25	3.125	4	0.562	4	8
0.375	0.625	0.250	0.25	0.363	0.02	0.562	7.50	6.00	4	0.750	9	18
0.500	0.625	0.250	0.34	0.484	0.03	0.625	7.50	6.00	4	0.750	16	32
0.750	0.625	0.250	0.50	0.726	0.04	0.750	7.50	6.00	4	0.750	36	71
1.000	0.9375	0.250	0.67	0.969	0.05	0.875	9.00	7.50	8	0.750	62	127
1 375	1.000	0 250	0.92	1.332	0.07	1.063	11.00	9.50	8	0.875	119	239
2.000	1.000	0.313	1.33	1.938	0.10	1.500	11.00	9.50	8	0.875	253	506
2.500	1.000	0.375	1.67	2.422	0.13	1.875	11.00	9.50	8	0.875	397	790
3.000	1.000	0.375	2.00	2.906	0.15	2.250	11.00	9.50	8	0.875	565	1,127
4.000	1.125	0.438	2.67	3.875	0.20	3.000	13.50	11.75	8	0.875	1,010	2,020
5.000	1.188	0.500	3.33	4.844	0.25	3.750	16.00	14.25	12	1.000	1,590	3,160
6.000	1.250	0.500	4.00	5.812	0.30	4.500	19.00	17.00	12	1.000	2,260	4,510
8.000	1.438	0.625	5.33	7.750	0.40	6.000	23.50	21.25	16	1.125	4,050	8,100
10.000	1.688	0.625	6.67	9.688	0.50	7.500	27.50	25.00	20	1.250	6,350	12,600
12.000	1.875	0.750	8.00	11.625	0.60	9.000	32.00	29.50	20	1.375	9,100	18,200
18.000	2.375	1.000	12.00	17.438	0.90	13.500	46.00	42.75	32	1.625	18,000	36,000
24.000	2.750	1.125	16.00	23.250	1.20	18.000	59.50	56.00	44	1.625	39,500	78,000

Nozzle Coefficients. For small nozzles (below 8 in. in diameter) the nozzle flow coefficient, *C*, varies with flow conditions and size of the nozzle. Figure 15 in conjunction with Table 3 gives the nozzle coefficient for standard flow nozzles of the form given in Table 2. For nozzles 8 in. in diameter and larger a nozzle coefficient of 0.995 should be used. Note that the inner surface of the nozzle must be polished.

TESTING BY LOW-PRESSURE NOZZLES.* Measurement of air capacity in accordance with the ASME Code requires measurement of pressure drop across a long-radius low-ratio nozzle (Table 2) and measurement of total temperature at the upstream side of the nozzle. The nozzle may be arranged (*A*) at the end of a suitable length of straight pipe and discharging to atmosphere, (*B*) at the entrance to a suitable length of straight

* See also Fan Testing, p. 1-70.

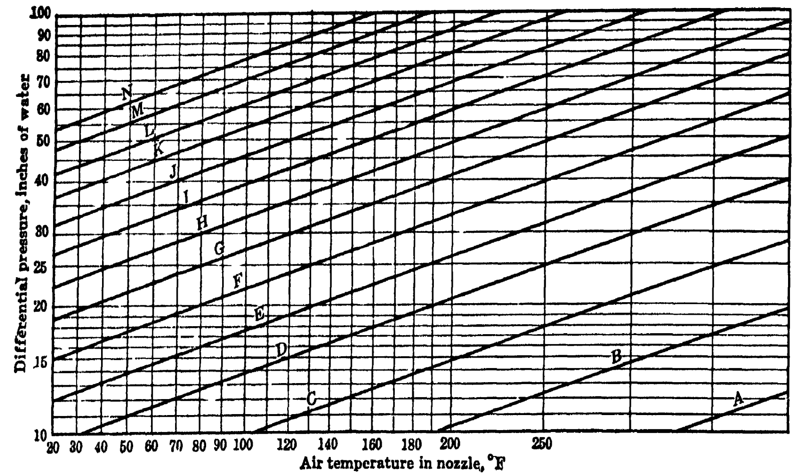


FIG. 15. Curve for selecting nozzle coefficient from Table 3. (Reprinted from *Compressed Air Handbook*, copyright 1947 by Compressed Air and Gas Institute, New York)

Table 3. Nozzle Coefficients for Air

(Applicable to arrangements *A* and *B*, Fig. 15)

(Reprinted by permission from *Compressed Air Handbook*, Compressed Air and Gas Institute, 1947.)

	Nozzle Diameter, in.														
Curve	1/8	3/16	1/4	3/8	1/2	3/4	1	1 3/8	2	2 1/2	3	4	5	6	8 *
A	.938	.946	.951	.957	963	.968	.973	.977	.982	.984	.986	.990	.993	.994	.995
B	.942	.948	.955	960	.965	.971	.975	.979	.984	.987	.989	.992	.994	.995	.995
C	.944	.952	.959	.964	.968	.974	.978	.981	.986	.990	.991	.994	.995	.995	.995
D	.947	.954	.961	.966	.970	.976	.980	.983	.988	.991	.993	.994	.995	.995	.995
E	.950	.957	.963	.968	.972	.977	.982	.985	.990	.992	.994	.995	.995	.995	.995
F	.953	.958	.964	969	.973	.978	.983	.986	.991	.993	.994	.995	.995	.995	.995
G	.956	.960	.966	.970	.974	.979	.984	.987	.922	.994	.995	.995	.995	.995	.995
H	.958	.962	.967	.972	.976	.980	.985	.988	.993	.995	.995	.995	.995	.995	.995
I	.959	.964	.968	.974	.978	.982	.986	.989	.994	.995	.995	.995	.995	.995	.995
J	.960	.965	.970	.975	.979	.983	.987	.990	.994	.995	.995	.995	.995	.995	.995
K	.961	.966	.971	.976	.980	.984	.988	.991	.994	.995	.995	.995	.995	.995	.995
L	.962	.967	.972	.977	.981	.985	.989	.992	.995	.995	.995	.995	.995	.995	.995
M	.963	.968	.973	.978	.982	.986	.990	.993	.995	.995	.995	.995	.995	.995	.995
N	.964	.969	.974	.979	.983	.987	.991	.994	.995	.995	.995	.995	.995	.995	.995

* For flow conditions specified in the ASME Code the coefficient for nozzles 8 in. in diameter and larger will be 0.995.

pipe and drawing air from the atmosphere, and (*C*) in either the intake or discharge pipe in a closed system (see Fig. 16). The nozzle pipe or reservoir is throttled (preferably by means of a butterfly valve) so that the nozzle drop is not less than 10 in. of water or greater than 100 in. of water.

For centrifugal compressors in which the flow is steady and uniform, the compressor can deliver directly into the nozzle pipe, as shown in Fig. 17 or draw air directly from the nozzle pipe when tested as an exhauster, as shown in Fig. 18.

For positive displacement compressors because of the intermittent and pulsating flow, a receiver must be installed between the compressor and the nozzle pipe.

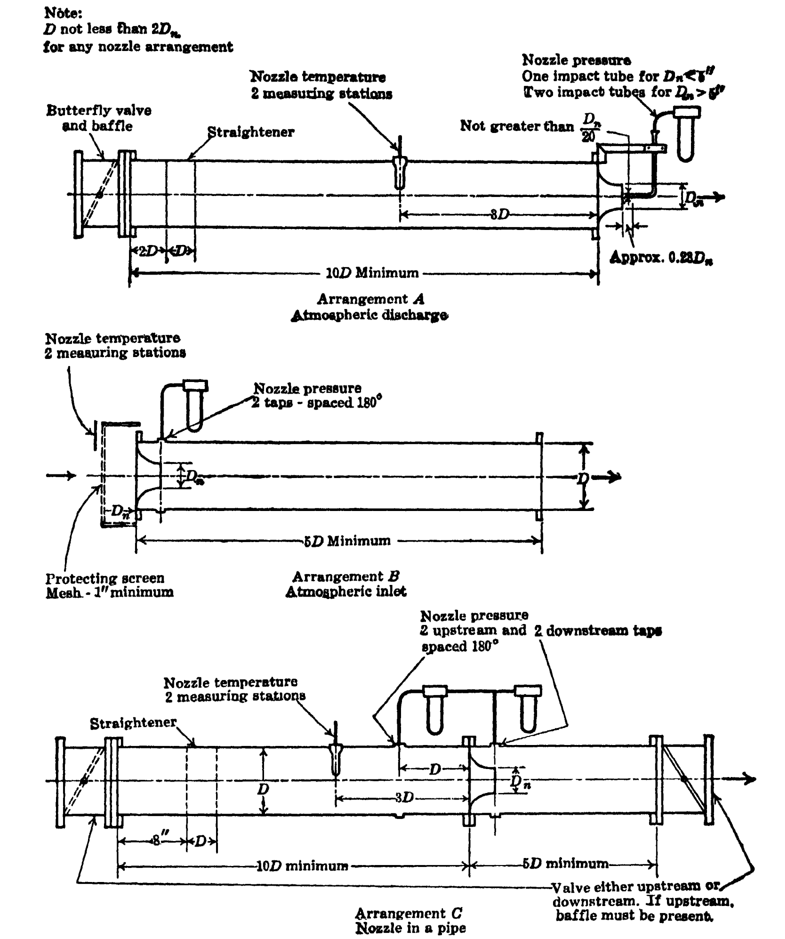


FIG. 16. Various arrangements of flow nozzles for compressor tests. (From *Compressed Air Handbook*, 1947)

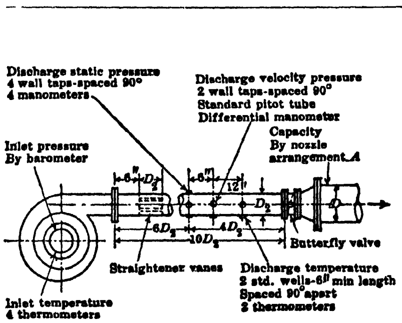


FIG. 17. Test setup No. 1. Volute type compressor, atmospheric inlet. (From *Compressed Air Handbook*, 1947)

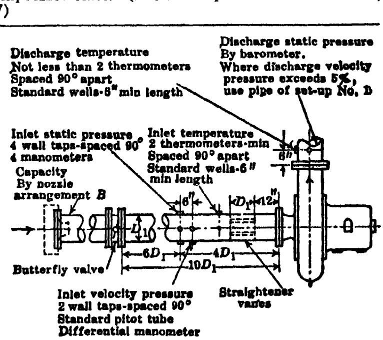


FIG. 18. Test setup No. 2. Single-stage compressor, atmospheric discharge. (From *Compressed Air Handbook*, 1947)

FLOW OF AIR IN PIPES

By A. H. Ehlinger

GENERAL. In all types of flow the analytical relations are based on the first law of thermodynamics, which states that energy cannot be destroyed. A mutual interchange between kinetic and potential energy takes place, but the total energy in the flowing fluid remains constant unless there is an interchange with the surroundings. Basic relationships include the Bernoulli equation

$$\frac{V^2}{2g} + \frac{p}{\rho} + z = \text{constant} \qquad \text{(in fps units)}$$

the equation of continuity

$$\rho A V = \text{constant} \qquad \text{(in fps units)}$$

and the Fanning equation, which permits taking pipe friction into account

$$p_1 - p_2 = 4f\rho\left(\frac{V^2}{2g}\right)\frac{l}{d} \qquad \text{(in fps units)}$$

In this equation the dimensionless proportionality constant, f, is defined by the equation itself. It has been found by many experimenters to be a function of (1) Reynolds' number $\left(\frac{\rho V d}{\mu}\right)$ and (2) the roughness of the pipe. (See Section 6, Art. 14, for friction factor values.)

EMPIRICAL EQUATIONS. Although the basic equation for flow in pipes is the Fanning equation, it is strictly valid only for *incompressible flow*, that is, flow in which the initial specific volume is a sufficiently close approximation to the true specific volume throughout the entire flow path. It is, however, the basic equation from which many special types are derived. When pressure drops become large (i.e., larger than 5 to 10% of the initial pressure) a correction must be applied to take account of the potential energy released to provide increase in kinetic energy necessitated by the increase in specific volume. Various forms of equation given herein accomplish this result.*

TURBULENT AND LAMINAR FLOW. Flow through a pipe may be either *laminar* or *turbulent*. In laminar flow pressure drop varies directly with *viscosity* of the fluid and its *velocity*; for turbulent flow pressure drop varies directly with *density* of the fluid and *square of its velocity*. A different flow equation must be used for each type of flow. However, for air and other gases laminar flow takes place only at extremely low velocities; hence all commonly used equations apply to turbulent flow.

NOMENCLATURE. Formulas given herein are for straight horizontal pipes of uniform diameter. The notation used is

p_1 = initial pressure, psia
p_2 = final pressure, psia
d = internal diameter of pipe, in.
a = area of pipe, sq. in.
l = length of pipe, in.
L = length of pipe, ft.
R = gas constant
R_d = Reynolds' number
ρ = density of fluid, lb per cu ft
v = specific volume of fluid, cu ft per lb
G = specific gravity of gas (air = 1.0)
g = acceleration of gravity, ft per sec^2
μ = absolute viscosity of fluid, lb per ft-sec
t = temperature of gas in pipe, °F
T = absolute temperature of gas in pipe $(460 + t)$
w = rate of flow, lb/sec
Q = rate of flow, cu ft per min
q = rate of flow, cu ft per sec
V = velocity of fluid, ft per sec

During the flow process the fluid moves against a resistance force, F_r, expending a quantity of work ($F_r \times$ distance traveled). Experiments indicate that, for turbulent flow, the resistance force, F_r, varies approximately: (1) with degree of roughness of pipe surface; (2) directly with area of rubbing surface; (3) directly with square of velocity; (4) directly with density of fluid; (5) slightly with viscosity; and (6) independently of pressure.

* For a graphical solution see Section 6, Art. 16.

The foregoing relationship may be stated in equation form:

$$F_r = C_1 \pi d l \rho V^2 \tag{1}$$

Where C_1 is a constant of proportionality whose value depends chiefly upon the ratio of the magnitude of surface irregularities to the diameter of the pipe. For pipes having the same degree of smoothness the value of C_1 is often expressed as a function of internal pipe diameter.

10. FLOW WITH FRICTION, SMALL PRESSURE DROP

In the development of this equation density of the fluid is considered constant. This is known as the *incompressible flow approximation.* When the equation is applied to flow of gases, initial density of the gas generally is used. With constant density, velocity of the fluid will also be constant, and pressure forces on the body of fluid in the pipe will be used only in overcoming the resistance force. These pressure forces do work in overcoming resistance to flow. During the interval dt the work done is

$$\frac{\pi}{4} d^2(p_1 - p_2) V\,dt = C_1 \pi\, dl\, \rho V^2 V\,dt$$

Simplifying,

$$\frac{p_1 - p_2}{\rho} = \frac{4C_1 l V^2}{d}$$

For constant density $[144(p_1 - p_2)/\rho]$ is loss in head, h_f, in feet of fluid. The term h_f, in terms of the kinetic energy of the fluid, is therefore

$$h_f = \frac{144(p_1 - p_2)}{\rho} = (4f)^* \frac{l}{d} \frac{V^2}{2g} \tag{2}$$

where $f = 144 \times 2gC_1$ (for f, see Section 6, Art. 14). The term l/d is equivalent to length of pipe, expressed in diameters. The rate of flow, in pounds per second, is

$$w = 0.1516 \sqrt{\frac{\rho(p_1 - p_2)d^5}{(4f)L}} \tag{3}$$

The pressure drop, in pounds per square inch, is

$$p_1 - p_2 = 43.5 \frac{(4f)Lw^2}{\rho d^5}$$

11. FLOW WITH FRICTION, LARGE PRESSURE DROP

The flow equation for a large pressure drop is given below for *constant temperature* flow of a perfect gas, i.e., the gas neither gains nor loses enthalpy while flowing. Such an assumption considerably simplifies the derivation, and often is approximated in practice. Nearly all the many equations for large pressure drops are derived with this assumption. Under this assumption, the flow is not entirely without heat interchange to the surroundings, because some heat must be added to the gas through the pipe wall to provide the increase in kinetic energy resulting from the decreasing density of the gas. This analysis applies, therefore, to constant temperature flow.

Heat is flowing into the gas to increase its kinetic energy; heat generated by friction also remains in the gas. Heat generated by friction is equal to the resistance force F_r times the distance traveled by the gas. During the time interval dt the heat produced is $F_r V dt$. Since the flow of gas during that same interval is equal to $w dt$, the heat of friction per pound of gas is equal to $F_r V/w$; from the continuity equation $V/w = [4(144)v]/\pi d^2$. Therefore, the heat per pound of gas *generated by friction* is equal to $[4(144)F_r v]/\pi d^2$. If gain in kinetic energy is neglected, heat added to a quantity of gas is equal to gain in internal energy of the gas plus the work done by expansion of the gas. Since gain in kinetic energy is supplied by the surroundings,

$$\frac{4(144)F_r v}{\pi d^2} = \Delta U + 144 \int p\,dv \tag{4}$$

where ΔU = gain in internal energy in foot-pounds per pound. From eqs. 1 and 2, and

* Much confusion exists as to whether the factor 4 is included in a given value of f. To avoid such confusion, the quantity $(4f)$ is carried in subsequent equations.

the continuity relationship

$$F_r = \frac{\pi(4f)LdV^2}{96gv} = \frac{144^2(4f)Lw^2v}{6\pi gd^3}$$

For an elemental length of pipe, dL, eq. 4, becomes

$$\frac{4(144)^2(4f)w^2v^2\,dL}{\text{[illegible]}} = \frac{dU}{144} + p\,dv$$

For constant enthalpy flow, $dU = 0$ and $d(pv) = p\,dv + v\,dp = 0$. Therefore $p\,dv = -v\,dp$. Likewise $v = RT/144p$. Substituting and solving for dp gives

$$-p\,dp = \frac{0.3021(4f)w^2RT\,dL}{d^5} \tag{5}$$

Integrating eq. 5,

$$\frac{p_1^2 - p_2^2}{2} = \frac{0.3021(4f)w^2RTL}{d^5} \tag{6}$$

Since $(p_1^2 - p_2^2)/2 = (p_1 - p_2)(p_1 + p_2)/2$, eq. 6 becomes

$$p_1 - p_2 = \frac{0.3021(4f)w^2RTL}{p_md^5} = \frac{43.5(4f)w^2v_mL}{d^5} \tag{7}$$

where $p_m = (p_1 + p_2)/2$ = mean pressure and v_m = mean specific volume. (The same result can be achieved by substituting the average value of density in eq. 3.) Solving eq. 6 for w, we obtain a basic expression for the flow of gas with large friction pressure drop, at constant temperature,

$$w = 1.289\sqrt{\frac{(p_1^2 - p_2^2)d^5}{(4f)RTL}} \tag{8}$$

$T = 460 +$ (temperature of gas in °F).

12. SPECIAL PIPE FLOW FORMULAS

Accuracy of special pipe flow formulas given herein depends upon the validity of the assumptions made and reliability of the values used for resistance coefficient, f. For turbulent flow eq. 2 is valid, but f is a function of viscosity, density, and pipe diameter, i.e., Reynolds' number. To keep eq. 2 dimensionally homogeneous, it is written in the form

$$h_f = (\text{constant})\left(\frac{\mu}{\rho d}\right)^n \frac{l}{d} V^{2-n}$$

Since Reynolds' number for pipe flow is defined as $R_d = \rho Vd/12\mu$,

$$h_f = \left(\frac{\text{constant}}{R_d^n}\right)\frac{l}{d}V^2$$

In application of eq. 2, friction factor, f, is a function of Reynolds' number for a set of geometrically similar pipes, that is, pipes having the same roughness ratio. Roughness ratio is defined as magnitude of surface irregularities divided by diameter of the pipe (see Section 6, Art. 14). For pipes having the same roughness but of different diameters, the value of f will therefore be a function of diameter. In pipe flow equations various simplified expressions are used for f:

Harris: $(4f)\ \dfrac{0.0313}{d^{0.31}}$

Weymouth: $(4f) = \dfrac{0.0321}{d^{1/3}}$

Unwin: $(4f) = 0.0112\left(1 + \dfrac{3.6}{d}\right)$

Except at small diameters, these values of f agree well with those commonly used for flow of water through smooth pipes. Values of f used herein are for clean smooth pipes. (See also Flow of Fluids in Pipes, Section 6.)

HARRIS EQUATION. One equation, by Professor E. G. Harris (University of Missouri *Bulletin*, Vol. 1, No. 4, 1912), is based on eq. 3. It is valid for $(p_1 - p_2) \leq 0.10p_1$. (For larger pressure drops see the Fritzsche and Weymouth equations below.) When f is ex-

Table 1. Loss of Air Pressure Due to Friction

In pounds per square inch in 1000 ft of pipe *

(Adapted from *Compressed Air Handbook*, Copyright 1947, Compressed Air and Gas Institute, New York)

Cubic Feet of Free Air per Minute	Equivalent Cubic Feet of Compressed Air per Minute	Nominal Diameter in Inches															
		1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	6	8	10	12
At 60 psig Initial Pressure																	
20	3.94	39.7	5.99	1.71	.39	.18											
40	7.86		24.7	6.85	1.59	.71	.19										
60	11.81		55.5	15.4	3.58	1.57	.43										
80	15.72			27.4	6.37	2.82	.75	.29									
100	19.60			42.8	9.95	4.40	1.18	.46									
150	29.45				22.4	9.90	2.64	1.02	.32	.15							
200	39.40				39.7	17.60	4.71	1.83	.57	.27							
250	49.20					27.5	7.37	2.85	.89	.42	.21						
300	58.90					39.6	10.55	4.11	1.30	.60	.31						
400	78.8						18.6	7.30	2.30	1.06	.53	.30					
500	98.4						29.7	11.4	3.60	1.67	.85	.46					
600	118.1						42.3	16.4	5.17	2.40	1.22	.67					
800	157.2							29.2	9.16	4.26	2.18	1.20					
1,000	196.0							45.7	14.3	6.65	3.40	1.87					
2,000	394.0								57.5	26.6	13.6	7.5	4.18	1.53	.36		
3,000	589									60.0	30.7	16.7	9.2	3.48	.81	.24	
4,000	788										54.5	29.7	16.5	6.17	1.44	.44	
6,000	1,181												37.0	13.9	3.25	.98	.38
8,000	1,572													24.7	5.80	1.73	.71
10,000	1,960													38.6	9.05	2.72	1.06
15,000	2,945														20.3	6.10	2.36
20,000	3,940														36.2	10.8	4.22
30,000	5,890															24.4	9.5
At 80 psig Initial Pressure																	
20	3.10	31.4	4.72	1.35	.31												
40	6.20		19.5	5.40	1.25	.56											
60	9.29		43.8	12.16	2.82	1.24	.34										
80	12.40		78.2	21.6	5.03	2.22	.59										
100	15.5			33.8	7.85	3.47	.93	.36									

* For longer or shorter lengths of pipe the friction loss is proportional to the length, i.e., for 500 ft, one-half of the above, for 4000 ft, four times the above, etc.

(*Table continued on p. 1-26*)

Table 1. Loss of Air Pressure Due to Friction—*Continued*

In pounds per square inch in 1000 ft of pipe

(Adapted from *Compressed Air Handbook*, Copyright 1947, Compressed Air and Gas Institute, New York)

Cubic Feet of Free Air per Minute	Equivalent Cubic Feet of Compressed Air per Minute	Nominal Diameter in Inches															
		1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	6	8	10	12
At 80 psig Initial Pressure—*Continued*																	
150	23.2			76.2	17.7	7.82	2.08	81									
200	31.0				31.4	13.9	3.72	1.44	.45								
250	38.7				49.0	21.7	5.82	2.25	.70	.33							
300	46.5				70.6	31.2	8.35	3.24	1.03	.47							
400	62.0					55.5	14.7	5.76	1.82	.84	.42						
500	77.4						23.3	9.0	2.84	1.32	.67	.30					
600	92.9						33.4	12.9	4.08	1.89	.96	.53					
800	124.0						59.3	23.1	7.15	3.36	1.72	.95					
1,000	155							36.1	11.3	5.27	2.68	1.48					
2,000	310								45.3	21.0	10.7	5.9	3.30	1.21	29		
3,000	465									47.4	24.2	13.2	7.3	2.74	.64	.19	
4,000	620										43.0	23.4	13.0	4.87	1.14	.34	
6,000	929											53.0	29.2	11.0	2.57	.77	.29
8,000	1,240												52.1	19.5	4.57	1.36	.56
10,000	1,550													30.5	7.15	2.14	.84
15,000	2,320													68.5	16.0	4.82	1.86
20,000	3,100														28.6	8.55	3.33
30,000	4,650														64.2	19.3	7.5
At 100 psig Initial Pressure																	
20	2.56	25.9	3.90	1.11	.25	.11											
40	5.12		16.0	4.45	1.03	.46											
60	7.68		36.2	10.0	2.32	1.02	28										
80	10.24		64.5	17.8	4.14	1.83	.49	.19									
100	12.81			27.9	6.47	2.86	.77	.30									
150	19.23			62.8	14.6	6.43	1.72	.66	.21								
200	25.62				25.9	11.4	3.06	1.19	.37	.17							
250	31.64				40.4	17.9	4.78	1.85	.58	.27							
300	38.44				58.2	25.8	6.85	2.67	.84	.39	.20						
400	51.24					45.8	12.1	4.75	1.50	.69	.35	.19					

500	63.28					71.6	19.2	7.42	2.34	1.09	.55	.30					
600	76.88						27.6	10.7	3.36	1.56	.79	.44					
800	102.5						49 0	19.0	5.89	2.77	1.42	.78					
1,000	126.6						76.9	29.8	9.3	4.35	2.21	1.22					
2,000	256.2								37.4	17.3	8.8	4.9	2.72	.99	.24		
3,000	384.6								84.1	39.1	20.0	10.9	6.0	2.26	.53		
4,000	512.4									69.4	35.5	19.4	10.7	4.01	.94	.28	
6,000	768.8										80.0	43.7	24.1	9.1	2.11	.64	.24
8,000	1,025											77.5	42.9	16.1	3.77	1.12	.46
10,000	1,280												67.1	25.1	5.88	1.77	.69
15,000	1,923													56.6	13.2	3.97	1.53
20,000	2,560														23.6	7.0	2.74
30,000	3,850														53.0	15.9	6.17
At 125 psig Initial Pressure																	
20	2.11	21.3	3.21	.92	.21												
40	4.21		13.2	3.67	.85	.38											
60	6.32		29.7	8.25	1.86	.84	.23										
80	8.42		53.0	14.7	3.41	1.51	.40										
100	10.50			22.9	5.32	2.36	.63	.25									
150	15.79			51.6	12.0	5.30	1.41	.55	.17								
200	21.05				21.3	9.4	2.52	.98	.31								
250	26.30				33 2	14.7	3.94	1.53	.48	.22							
300	31.60				47.3	21.2	5.62	2.20	.70	.32							
400	42.10					37.6	10.0	3.91	1.23	.57	.28						
500	52.60					58.8	15.7	6.10	1.93	.89	.46	.25					
600	63.20						22.6	8.8	2.76	1.28	.65	.36					
800	84.20						40 2	15.6	4.85	2.28	1.17	.64					
1,000	105.1						63.2	24.5	7.7	3.57	1.82	1.00					
2,000	210.5								30.7	14.2	7.3	4.0	2.24	.82	.19		
3,000	316								69.2	32.1	16.4	9.0	4.9	1.86	.43		
4,000	421									57.0	29.2	15.9	8.9	3.30	.77	.23	
6,000	632										65.7	35.8	19.8	7.5	1.74	.52	.20
8,000	842											63.7	35 2	13.2	3.10	.93	.38
10,000	1,051												55.2	20.6	4.85	1.46	.57
15,000	1,579													46.5	10.9	3.26	1.26
20,000	2,150														19.4	5.8	2.20
30,000	3,158														43.5	13.1	5.1

Table 2. Factors for Calculating Loss of Air Pressure Due to Pipe Friction Applicable for Any Initial Pressure *

(Adapted from *Compressed Air Handbook*, copyright 1947, Compressed Air and Gas Institute, New York)

Cubic Feet Free Air per Minute	Nominal Diameter in Inches																
	1/2	3/4	1	1 1/4	1 1/2	1 3/4	2	2 1/2	3	3 1/2	4	4 1/2	5	6	8	10	12
5	12.7	1.2	0.5														
10	50.7	7.8	2.2	0.5													
20	202	30.4	8.7	2.0	0.9												
30	456	70.4	19.6	4.5	2.0	1.1											
40	811	125.3	34.8	8.1	3.6	1.9											
60		282	78.3	18.2	8.0	4.2	2.2										
80		503	139.2	32.3	14.3	7.5	3.8	1.5									
100		785	217.4	50.5	22.3	11.7	6.0	2.3									
120			318	72.7	32.2	16.8	8.6	3.3									
140			426	98.9	43.8	22.9	11.7	4.6	1.4								
160			570	129.3	57.2	29.9	15.3	5.9	1.9								
180			705	163.3	72.6	37.9	19.4	7.5	2.4								
200			870	202	89.4	46.7	23.9	9.3	2.9								
240				291	128.7	67.3	34.4	13.4	4.2								
280				395	175	91.6	46.8	18.2	5.7								
320							61.1	23.8	7.5	3.5							
360							77.3	30.1	9.5	4.4	2.2						
400							94.7	37.1	11.7	5.4	2.7						

500							150.0	58.0	18.3	8.5	4.3	2.4					
600							215	83.5	26.3	12.2	6.2	3.4					
800							382	148.4	46.7	21.7	11.1	6.1	3.3				
1,000							600	232	73 0	33.8	17.3	9.5	5.2	1.9			
1,200							850	344.0	105.2	48.8	25.0	13.7	7.5	2.8			
1,400										66.3	33.9	18.6	10.2	3.8			
1,600										86.6	44.3	24.2	13.4	5.1			
1,800										110	56.1	30.7	16.9	6.4			
2,000										135	69.3	37.9	20.9	7.8	1.8		
2,400										195	99.8	54.6	30.1	11.3	2.6		
2,800										265	136	74.3	41.0	15.4	3.6		
3,200										347	177	97.1	53.5	20.1	4.7		
3,600										438	224	122.8	67.7	25.4	5 6	1.8	
4,000										542	277	151	83.6	31.4	7.3	2.2	
5,000											433	236	131	49.1	11.5	3.4	
6,000												341	188	70.7	16.5	5.0	1.9
8,000													335	125.7	29.4	8.8	3.6
10,000													523	196	45.9	13.8	5.4
15,000														442	103.2	31.0	12.0
20,000															184	55.0	21.4
30,000															413	123.9	48.2

* To determine the pressure drop in pounds per square inch, the factor listed in the table for a given capacity and pipe diameter should be divided by the ratio of compression (from free air) at entrance of pipe, multiplied by the actual length of the pipe in feet and divided by 1000.

pressed in terms of d this equation gives the following expression for flow in cubic feet per second:

$$q = rw = 0.856\sqrt{\frac{r(p_1 - p_2)d^{5.31}}{L}} \tag{9}$$

Solving eq. 9 for pressure drop,

$$p_1 - p_2 = \frac{1.365w^2vL}{d^{5.31}} = \frac{1.365q^2L}{vd^{5.31}} \tag{10}$$

If $v_s/v = q_s/q = r$, where v_s = specific volume of air at 68 F and 14.7 psia (13.3 cu ft per lb),

$$p_1 - p_2 = \frac{0.1025q_s^2L}{rd^{5.31}} \tag{11}$$

This equation is valid for pressure drops up to 10% of p_1, only. q_s = flow in cubic feet per second when referred to 68 F and 14.7 psia. Flow q_s often is referred to as cubic feet of *free air* per second. Term r is known as *ratio of compression* (from free air) at entrance to pipe.

Equation 11 has been adopted by the Compressed Air and Gas Institute for the compilation of pressure drop tables reproduced herein. See Table 1, which gives pressure drops for various flows and pipe sizes. Table 2 gives factors by which pressure drop can be calculated for initial conditions not covered by Table 1.

THE UNWIN EQUATION is a form of eq. 3 and is therefore valid for $(p_1 - p_2) \leqq 0.1p_1$.

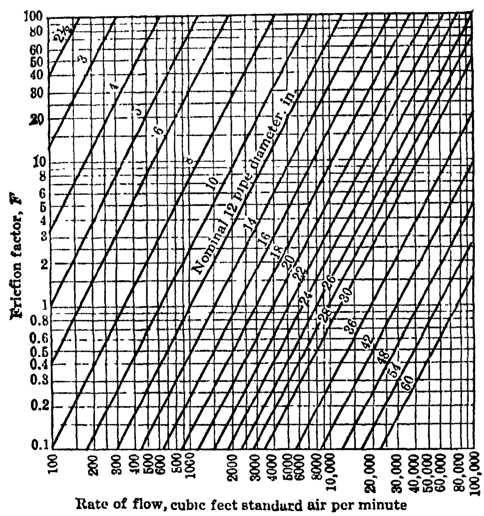


FIG. 1. Loss of air pressure due to pipe friction measured at standard conditions of 14.7 psia and 60 F. (From *Compressed Air Handbook*, 1947)

When expressed in pounds per second the flow given by the Unwin equation is

$$w = 1.43\sqrt{\frac{(p_1 - p_2)d^5}{vL\left(1 + \frac{3.6}{d}\right)}} \tag{12}$$

THE FRITZSCHE EQUATION is frequently used to compute pressure drops for the flow of air in relatively large pipes at moderate velocities. The equation is based upon the assumption that the resistance force varies with the 1.85 power of the velocity and that f = constant/$d^{0.3}$. Specific volume generally is taken as the average of initial and final values. The form of the equation is

$$w = \left[1.22\frac{(p_1 - p_2)d^5}{vL}\right]^{1/1.85} \tag{13}$$

Since $w = q/v = q_s/v_s$, the expression for pressure drop may be written

$$p_1 - p_2 = 0.82 \frac{q_s^{1.85} v L}{v_s^{1.85} d^5}$$

This equation is put in more convenient form by assuming that the temperature is 60 F. This makes specific volume a function of pressure only. When 60 F and 14.7 psia are standard conditions,

$$p_1 - p_2 = 0.7 \frac{Q_s^{1.85}}{p_m d^5} \frac{L}{1000} = \frac{F}{p_m} \frac{L}{1000} \qquad (14)$$

where $p_m = (p_1 + p_2)/2$; Q_s = flow, cubic feet per minute at standard conditions, 60 F and 14.7 psia; and $F = 0.7Q_s^{1.85}/d^5$. (See Fig. 1.)

Equation 14 applies with reasonable accuracy to flow of air through clean steel pipes at 60 F when $p_1 - p_2 \leq 0.2p_1$. When the temperature of the air is other than 60 F, the pressure drop may be obtained by multiplying the value obtained in eq. 14 by $T/520$, where T = 460 + actual temperature, °F. Values of F to be used in eq. 14 are given in Fig. 1 for various rates of flow and various pipe sizes.

THE WEYMOUTH EQUATION is a form of eq. 8 used for compressible flow, i.e., in applications where the pressure drop is 20% or more of the initial pressure. The specific gravity of a gas (referred to air as 1.0, as is common practice) is equal to $53.3/R$. This equation is usually written in terms of specific gravity, represented by G. Flow in pounds per second is

$$w = 0.985 d^{2.667} \sqrt{\frac{G(p_1^2 - p_2^2)}{TL}} \qquad (15)$$

The flow in cubic feet per minute of gas at standard conditions is

$$Q_s = 21.85 \frac{T_s}{p_s} d^{2.667} \sqrt{\frac{p_1^4}{GTL}} \qquad (16)$$

Where T_s and p_s are the absolute temperature (°F + 460) and absolute pressure (psia) used in defining standard conditions. Equation 16 is often used in computing the rate of flow through gas transmission lines, where pressure drops usually are appreciable.

Gas Transmission Lines. A commonly used form of the Weymouth equation gives the result in dimensions used by the industry:

$$Q_s = 871 d^{2.667} \left(\frac{p_1^2 - p_2^2}{L} \right)^{1/2}$$

valid for G = 0.60, $T = T_s$ = 60 F, atmospheric pressure = 14.65 psia, where Q_s = cubic

Table 3. Volume of Air Transmitted in Cubic Feet per Minute in Pipes of Various Diameters

Formula $Q = (0.7854/144)\, d^2 v \times 60$

Velocity, of Flow, ft per sec	Actual Diameter of Pipe, in. 1	2	3	4	5	6	8	10	12	16	20	24
1	0.327	1.31	2.95	5.24	8.18	11.78	20.94	32.73	47.12	83.77	130.9	188.5
2	0.655	2.62	5.89	10.47	16.36	23.56	41.89	65.45	94.25	167.5	261.8	377.0
3	0.982	3.93	8.84	15.7	24.5	35.3	62.8	98.2	141.4	251.3	392.7	565.5
4	1.31	5.24	11.78	20.9	32.7	47.1	83.8	131	188	335	523	754
5	1.64	6.54	14.7	26.2	41.0	59.0	104	163	235	419	654	942
6	1.96	7.85	17.7	31.4	49.1	70.7	125	196	283	502	785	1131
7	2.29	9.16	20.6	36.6	57.2	82.4	146	229	330	586	916	1319
8	2.62	10.50	23.5	41.9	65.4	94	167	262	377	670	1047	1508
9	2.95	11.78	26.5	47	73	106	188	294	424	754	1178	1696
10	3.27	13.1	29.4	52	82	118	209	327	471	838	1309	1885
12	3.93	15.7	35.3	63	98	141	251	393	565	1005	1571	2262
15	4.91	19.6	44.2	78	122	177	314	491	707	1256	1963	2827
18	5.89	23.5	53	94	147	212	377	589	848	1508	2356	3393
24	7.85	31.4	71	125	196	283	502	785	1131	2010	3141	4524
30	9.8	39.3	88	157	245	353	628	982	1414	2513	3927	5655

feet per 24 hr measured at 14.65 psia, 60 F; d = internal diameter of pipe, inches; p_1 = initial or upstream pressure, psia; p_2 = final or downstream pressure, psia; L = length of line, miles.

EQUALIZATION OF PIPES. It is frequently desired to know what number of pipes of a given size are equal in carrying capacity to one pipe of a larger size. At the same velocity of flow the volume delivered by two pipes of different sizes is proportional to the squares of their diameters; thus, one 4-in. pipe will deliver the same volume as four 2-in. pipes. With the same head, however, the velocity is less in the smaller pipe, and the volume delivered varies about as the square root of the fifth power (i.e., as the 2.5 power). Table 4 has been calculated on this basis. The figures opposite the intersection of any two sizes is the number of the smaller-size pipes required to equal one of the larger. Thus one 4-in. pipe is equal to 5.7 two-in. pipes.

Table 4. Equalization of Pipes

Diam., in.	1	2	3	4	5	6	7	8	10	12	14	16	18	20	24
2	5.7	1	...												
3	15.6	2.8	1												
4	32.0	5.7	2.1	1											
5	55.9	9.9	3 6	1.7	1										
6	88.2	15.6	5.7	2.8	1.6	1									
7	130	22.9	8 3	4 1	2.3	1.5	1								
8	181	32 0	11.7	5.7	3.2	2.1	1.4	1							
10	316	55 9	20 3	9.9	5.7	3.6	2.4	1.7	1						
12	499	88 2	32 0	15.6	8.9	5 7	3.8	2.8	1.6	1					
14	733	130	47.0	22.9	13.1	8.3	5.7	4.1	2.3	1.5	1				
16		181	65 7	32 0	18.3	11 7	7 9	5.7	3.2	2.1	1.4	1			
18		243	88 2	43 0	24.6	15 6	10 6	7.6	4.3	2.8	1.9	1.3	1		
20		316	115	55 9	32 0	20 3	13.8	9.9	5.7	3.6	2.4	1.7	1.3	1	
24		499	181	88 2	50 5	32 0	21.8	15.6	8.9	5.7	3.8	2.8	2.1	1.6	1
30		871	316	154	88 2	55.9	38.0	27.2	15.6	9.9	6.7	4.8	3.6	2.8	1.7
36			499	243	130	88.2	60.0	43.0	24.6	15.6	10.6	7.6	5.7	4.3	2.8
42			733	357	205	130	88 2	63.2	36.2	19 0	15.6	11 2	8 3	6.4	4.1
48				499	286	181	123	88.2	50.5	32.0	21.8	15.6	11.6	8.9	5.7
54				670	383	243	165	118	67.8	43.0	29.2	20.9	15.6	12.0	7.6
60				871	499	316	215	154	88.2	55.9	38.0	27.2	20 3	15.6	9.9

13. FLOW OF AIR THROUGH RECTANGULAR DUCTS *

When two pipes of equal length and roughness ratio carry fluid of the same density at the same velocity, the pressure drops are proportional to the wetted perimeter, that is, the perimeter of the sectional area of the pipe. One equation for rectangular ducts is a modified form of eq. 14. Considering a circular pipe and a rectangular duct, each having the same length and flow rate, the pressure drops are proportional to the wetted perimeters, provided density and velocity of the fluid are the same in each case.

With flow rate, density, and velocity equal, the duct area must be equal to the pipe area; hence, if a and b are assumed to be dimensions of the rectangle in inches,

$$d^5 \quad 32\left(\frac{ab}{\pi}\right)^{2.5}$$

The ratio of wetted perimeters is equal to

$$\frac{2a + 2b}{\pi d} = \frac{a + b}{(\pi ab)^{1/2}}$$

If these values are substituted in eq. 14, the pressure drop for the rectangular duct is

$$p_1 - p_2 = 0.216\,\frac{a + b}{(ab)^3}\,\frac{Q_s^{1.85}}{p_m}\left(\frac{L}{1000}\right) \tag{17}$$

* See also Section 12, Heating, Ventilating and Air Conditioning, Art. 9.

where Q_s = flow in standard cubic feet per minute at 60 F and 14.7 psia; p_m = average pressure, in pounds per square inch absolute; a and b are the dimensions of the rectangle in inches.

When the temperature is other than 60 F multiply the value obtained from eq. 17 by $T/520$, where $T = 460 +$ temperature in °F.

Compare eqs. 14 and 17. For the same duct length and the same flow rate the pressure drop for a rectangular duct will equal the drop for a circular duct when

$$d = \frac{2}{\pi^{0.4}} \frac{(ab)^{0.6}}{(a+b)^{0.2}} = \frac{1.265(ab)^{0.6}}{(a+b)^{0.2}} \tag{18}$$

This diameter is known as the *equivalent diameter* for the duct. Letting $a = nb$, the equivalent diameter is

$$d = 1.265 \frac{n^{0.6}}{(n+1)^{0.2}} b \tag{19}$$

Values of equivalent diameters for various rectangles are given in Fig. 2.

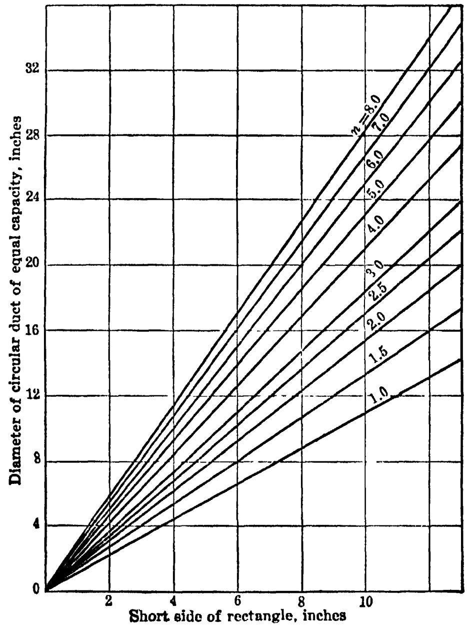


FIG. 2. Diameter of circular ducts of same flow capacity as rectangular ducts of various sizes; $n = a/b$, where a = long side of rectangle, b = short side of rectangle. (*Note:* Circular duct sizes corresponding to a given rectangular duct have same pressure drop per unit length when carrying same flow.)

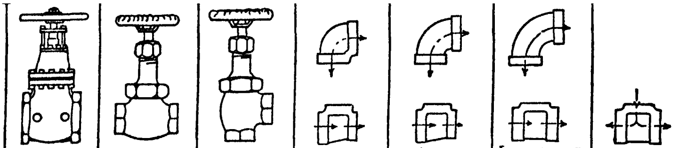
14. EQUIVALENT PIPE LENGTHS FOR VALVES AND FITTINGS *

Loss of head due to flow of fluids through valves and pipe fittings may best be expressed as a constant times the velocity head. To determine the equivalent length of straight pipe, the loss of head for the pipe is equated to the loss of head for the fitting:

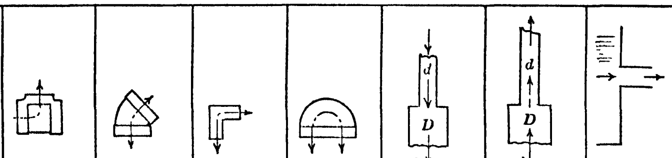
$$h_f = (4f) \frac{l}{d} \frac{V^2}{2g} = K \frac{V^2}{2g}$$

where K is a constant of the fitting. Since f is a function of d, the equivalent length of straight pipe is also a function of d. Figure 3 gives equivalent lengths in feet of standard pipe for valves and fittings of various types and sizes.

* See also Flow of Fluids in Pipes, Section 6, Art. 14.



Nominal diameter of standard pipe	Gate valve fully open	Globe-valve fully open	Angle valve fully open	Standard elbow—or run of tee reduced ½	Medium sweep elbow—or run of tee reduced ¼	Long sweep elbow—or run of standard tee	Standard tee
½"	0.4	17	8	1.5	1.4	1.0	3.3
¾"	0.5	22	11	2.0	1.8	1.4	4.5
1"	0.6	27	14	2.7	2.2	1.7	5.7
1¼"	0.8	36	18	3.5	3.0	2.3	7.6
1½"	1.0	43	22	4.3	3.6	2.7	9.0
2"	1.2	55	27	5.5	4.5	3.5	12.0
2½"	1.4	67	33	6.5	5.2	4.2	14.0
3"	1.7	82	41	8.0	6.8	5.2	17.0
4"	2.3	110	53	11.0	9.0	7.0	22.0
5"	2.9	140	70	14.0	12.0	9.0	27.0
6"	3.5	170	80	16.0	14.0	11.0	33.0
8"	4.5	225	120	20.0	17.0	14.0	43.0
10"	5.8	280	140	26.0	22.0	17.0	53.0
12"	6.8	330	160	32.0	25.0	20.0	68.0
14"	8.0	380	190	37.0	31.0	24.0	78.0
16"	9.0	440	220	42.0	35.0	27.0	88.0



Nominal diameter of standard pipe	Standard tee through side outlet	45° Elbow	Square elbow	Close return bend	$\frac{d}{D} = \frac{1}{2}$ Sudden enlargement	$\frac{d}{D} = \frac{1}{2}$ Sudden contraction	Ordinary entrance
½"	3.3	0.8	3.3	8.6	1.0	0.6	0.9
¾"	4.5	1.0	4.5	5.0	1.4	0.8	1.3
1"	5.7	1.3	5.7	6.0	1.7	1.0	1.5
1¼"	7.6	1.7	7.6	8.3	2.3	1.3	2.0
1½"	9.0	2.0	9.0	10.0	2.7	1.5	2.4
2"	12.0	2.5	12.0	13.0	3.5	1.9	3.0
2½"	14.0	3.0	14.0	15.0	4.2	2.2	3.6
3"	17.0	3.8	17.0	18.0	5.2	2.8	4.5
4"	22.0	5.0	22.0	24.0	7.0	3.8	6.0
5"	27.0	6.3	27.0	31.0	9.0	4.7	7.5
6"	33.0	7.5	33.0	37.0	11.0	5.8	9.0
8"	43.0	10.0	43.0	51.0	14.0	7.5	13.0
10"	53.0	13.0	53.0	61.0	17.0	10.0	15.0
12"	68.0	15.0	68.0	74.0	20.0	12.0	18.0
14"	78.0	17.0	78.0	85.0	24.0	13.0	20.0
16"	88.0	19.0	88.0	100.0	26.0	15.0	23.0

FIG. 3. Feet of standard pipe to give the same pressure drop as various fittings. (Adapted by permission from Natural Gasoline Supply Men's Association *Technical Manual*)

COMPRESSED AIR

By Theodore Baumeister

Compressors are built in the following types:

1. Positive displacement.
   1. Reciprocating (Figs. 1, 2, and 3).
   2. Reciprocating, wet.
   3. Rotary, without liquid seals (Fig. 4).
   4. Rotary, with liquid seals (Fig. 5).

2. Free compression, turbos.
    1. Centrifugal fans (see p. 1-72).
    2. Centrifugal compressors (Fig. 6).
    3. Axial-flow fans (see p. 1-93).
    4. Axial-flow compressors (Fig. 1, p. 1-96).
3. Free compression, jet.
    1. Hydraulic.
    2. Steam, air, or vapor (Fig. 7).

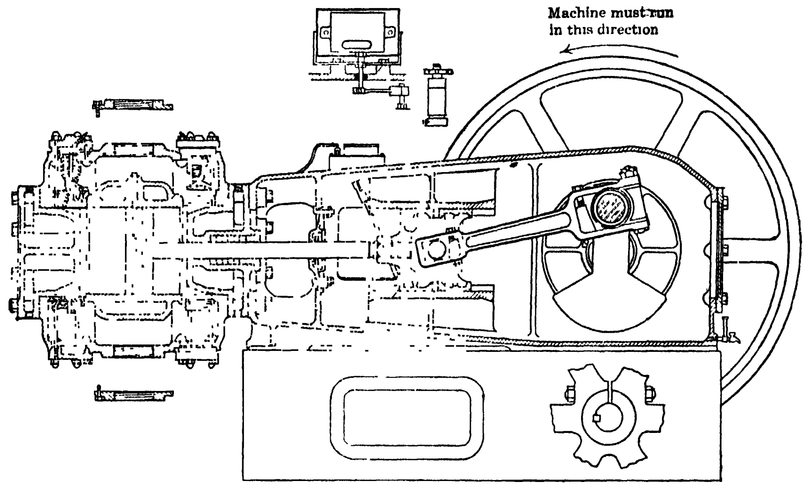


FIG. 1. Single-stage belt-driven compressor. (Chicago Pneumatic Tool Co.)

FIELDS OF APPLICATION AND RANGE OF PERFORMANCE. **Reciprocating compressors** are built in sizes as large as 5000 to 10,000 cu ft per min piston displacement. Pressures range up to 1000 atm and vacua down to 0.5 in. Hg abs. Vacuum pumps are

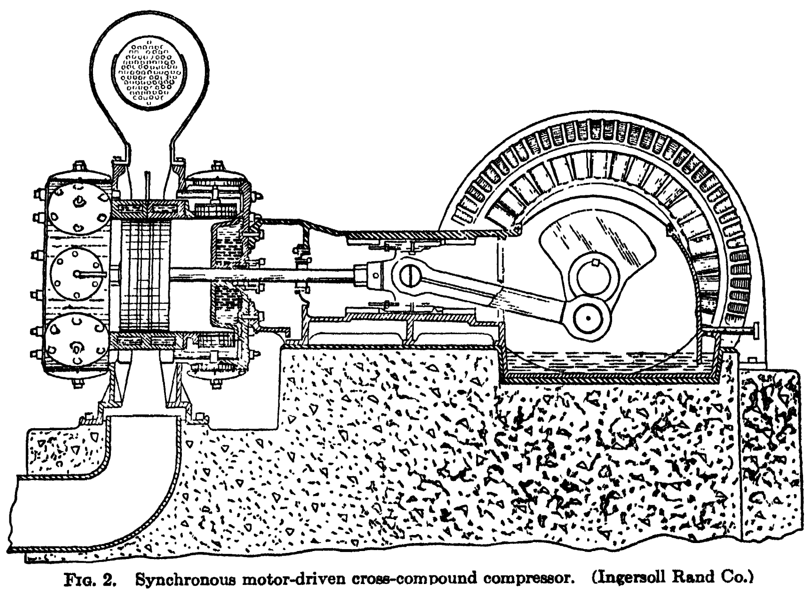


FIG. 2. Synchronous motor-driven cross-compound compressor. (Ingersoll Rand Co.)

usually single stage, seldom two stage, but positive pressure compressors for large capacities and for higher pressures require multistaging with 100 psi for single stage; 50 to 300 psi for two stage; 200 to 1000 psi for three stage; and 3000 to 4000 psi for four stage.

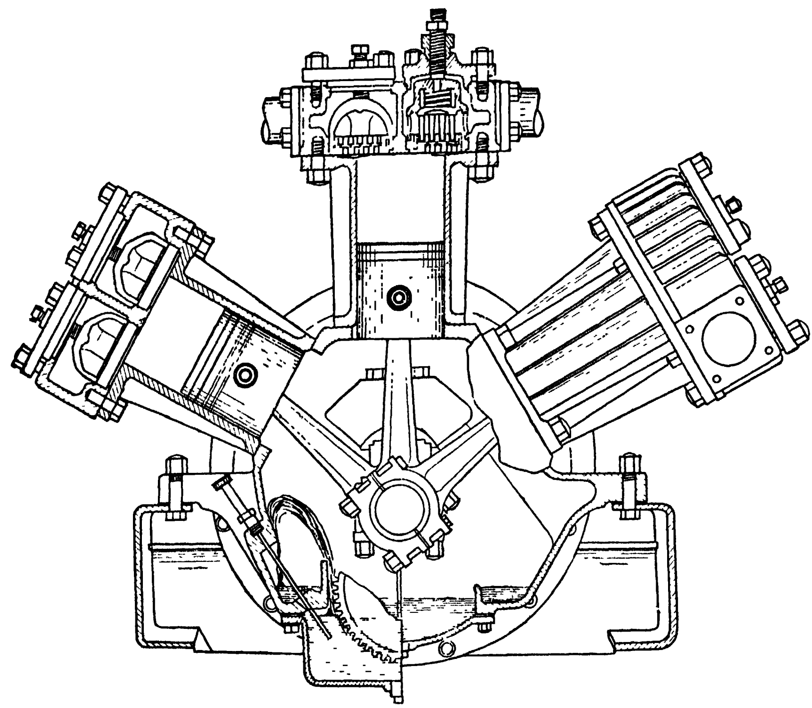


FIG. 3. Portable two-stage three-cylinder air compressor. (Ingersoll Rand Co.)

Reciprocating compressors running wet are an old form particularly adapted to vacuum service where both air and water are to be removed. Piston displacement ranges up to 500 cu ft per min and vacua are of the order of 22 to 28 in. Hg, referred to 30 in barometer.

Rotary compressors without liquid seals are available in sizes as large as 50,000 cu ft

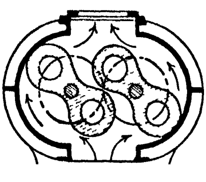


FIG. 4. Positive displacement rotary compressor. (Roots-Connersville Blower Corp.)

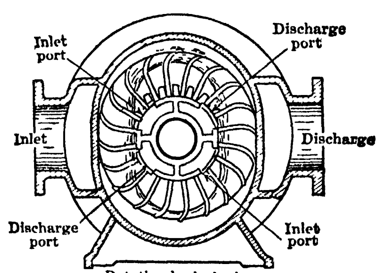


FIG. 5. Section through Nash Hytor vacuum pump. (Nash Engineering Co.)

per min. They are suitable for moderate vacua, not less than 15 in. Hg abs, and for moderate pressures, 5 to 20 psi. Some designs with sliding vanes can be carried to pressures of 100 psi and to higher vacua, particularly if multistage.

Rotary compressors with liquid seals are built in sizes up to 5000 cu ft per min and are suitable for ratios of compression reflected in vacuum pump service to 28 in. Hg and positive pressure service to 75 psig. If liquid is to be pumped simultaneously with the gas, this kind of machine is particularly favored, as in wet vacuum service where 20 to 25 in. Hg vacuum can be most satisfactorily met.

Centrifugal fans are built in sizes of 100 to 500,000 cu ft per min and with developed pressures ranging up to 50 or 60 in. of water, positive or negative (see p. 1-73).

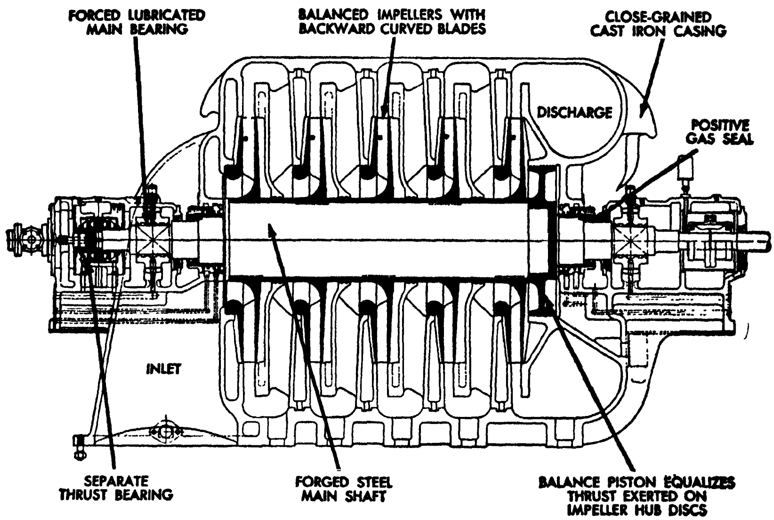


FIG. 6. Section of five-stage centrifugal compressor. (Allis-Chalmers Manufacturing Co.)

Centrifugal compressors are built in capacities of 500 to 100,000 cu ft per min and deliver pressures as high as 150 psi, where 10 to 15 stages would be required. Lower pressure units can deliver 1 to 20 psi in a single stage. Lightweight high-speed centrifugal compressors developed during the war for jet engines sometimes attain as much as 4 : 1 pressure ratio in a single stage. (See Sections 10 and 15.)

Axial-flow fans are primarily for delivery of the largest volumes under the lowest pressures. If operated without a casing the static pressure is zero and all energy is kinetic. If a casing is included, heads up to 24 in. water can be developed with the highest rotative speeds and with a single stage. Capacities are less than 100,000 cu ft per min in the commercial sizes.

Axial-flow compressors extend the field of the axial flow fans by the use of multistaging so that pressures of 75 psi can be developed with a 20 or 25 stage unit. Aircraft-type units use about one-half this number of stages. Capacities range from 2000 to 100,000 cu ft per min. See also p. 1-96 for a complete discussion.

Hydraulic jet compressors were formerly more popular than they are today. They were primarily employed for vacuum pump service requiring 1 to 4 in. Hg abs pressure and capacities of 1000 cu ft per min at suction conditions.

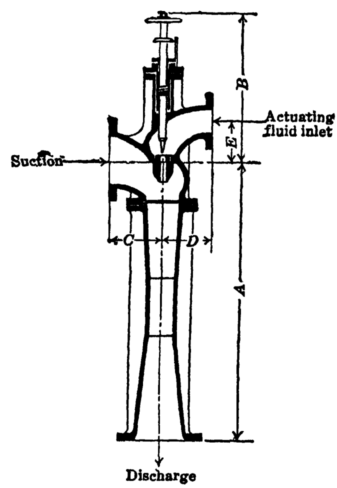


FIG. 7. Cross section of a manually controlled jet compressor. (Courtesy of Schutte and Koerting Co.)

Vapor jet compressors with steam as the usual actuating fluid are used for both exhauster and blower service. On the latter, delivery pressures run to 25 psi and, on the former, a suction pressure of 5 or 6 in. Hg abs can be maintained with single-stage constructions, 0.5 to 1 in. Hg abs with two stage, and 0.25 in. Hg with three stage. Suction volumes of 500 to 2000 cu ft per min are common on vacuum pump services, but, when developed for thermocompressor application, capacities range to 25,000 cu ft per min.

15. RECIPROCATING COMPRESSORS

THEORY OF COMPRESSOR PERFORMANCE. The basic thermodynamic cycle for a compressor is represented on the pressure-volume diagram, or indicator card (Fig. 8), as a suction phase (1–2) at constant supply pressure, p_s, compression phase (2–3) following the gas law $pv^n = C$, and a discharge phase (3–4) at constant delivery pressure, p_d. The net area of this cycle diagram, shown cross hatched, represents the ideal work required to drive the compressor and is evaluated graphically or by the relation

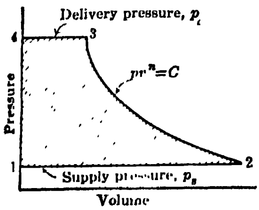


FIG. 8. Pressure-volume diagram for a compressor.

$$\Delta W_{\text{cycle}} = \int_2^3 v\,dp \tag{1}$$

Integration of this expression can be effected through the gas law

$$pv^n = C \tag{2}$$

If the exponent n equals unity (isothermal compression) the resultant equation is of a logarithmic form in which

$$\Delta W_{\text{cycle}} = 144 p_2 v_2 \log_e \frac{p_3}{p_2} \tag{3}$$

where pressures, p, are measured in pounds per square inch abs, volumes, v, in cubic feet, and work, ΔW, in foot-pounds.

If the exponent n in eq. 2 has a value other than unity, the resultant equation is of an exponential form in which

$$\Delta W_{\text{cycle}} = 144 \frac{n}{n-1} p_2 v_2 \left[\left(\frac{p_3}{p_2} \right)^{(n-1)/n} - 1 \right] \tag{4}$$

If the fluid follows the law $pv = C$, the compression is process carried out at constant temperature, and the work for the *isothermal* standard can be evaluated by eq. 3. This equation can be written in more generally useful form to give the power requirement, isothermal horsepower, for a given low pressure volume flow, v_s, in cubic feet per minute

$$\text{Isothermal horsepower} = \frac{p_s v_s}{229.2} \log_e \frac{p_d}{p_s} \tag{5}$$

where p_s = supply pressure, pounds per square inch abs; and p_d delivery pressure, pounds per square inch abs.

If, on the other hand, the gas is compressed rapidly or in a perfectly insulated cylinder, without internal or external friction losses, the process follows the equation $pv^k = C$, where k is the ratio of the specific heats at constant pressure and constant volume (c_p/c_v). Some values of k are given in Table 1; they range from 1.05 to 1.67. (For additional data, see Section 2, Art. 18). Such compression is referred to thermodynamically as a reversible adiabatic or *isentropic* process. For brevity in compressor practice, this is generally called the *adiabatic* standard even though this term, in its strictest sense, is a misnomer. The work can be evaluated by eq. 4, which can be rewritten in more useful form of horsepower for a given low-pressure volume flow, v_s cubic feet per minute:

$$\text{Isentropic or adiabatic horsepower} = \frac{k}{k-1} \times \frac{p_s v_s}{229.2} \left[\left(\frac{p_d}{p_s} \right)^{(k-1)/k} - 1 \right] \tag{6}$$

Equations 3 to 6 can be evaluated by Fig. 9.

For real gases, it may be necessary, for greater accuracy, to use tables of air properties, p. 1-04 for finding the theoretical work as the difference between the initial and the final enthalpy at constant entropy. (See also bibliography, end of chapter.)

Pressures p, volumes v, and absolute temperatures T at suction conditions (subscript s) and at discharge conditions (subscript d) are related by the equations

$$p_s v_s^k = p_d v_d^k \quad \text{or} \quad p_s v_s^n = p_d v_d^n \tag{7}$$

$$T_s v_s^{k-1} = T_d v_d^{k-1} \quad \text{or} \quad T_s v_s^{n-1} = T_d v_d^{n-1} \tag{8}$$

$$T_s p_d^{(k-1)/k} = T_d p_s^{(k-1)/k} \quad \text{or} \quad T_s p_d^{(n-1)/n} = T_d p_s^{(n-1)/n} \tag{9}$$

These equations are most conveniently solved by the use of ratios whose values are always greater than zero or

$$\text{Ratio of pressures, } R_p = \frac{p_d}{p_s} \tag{10}$$

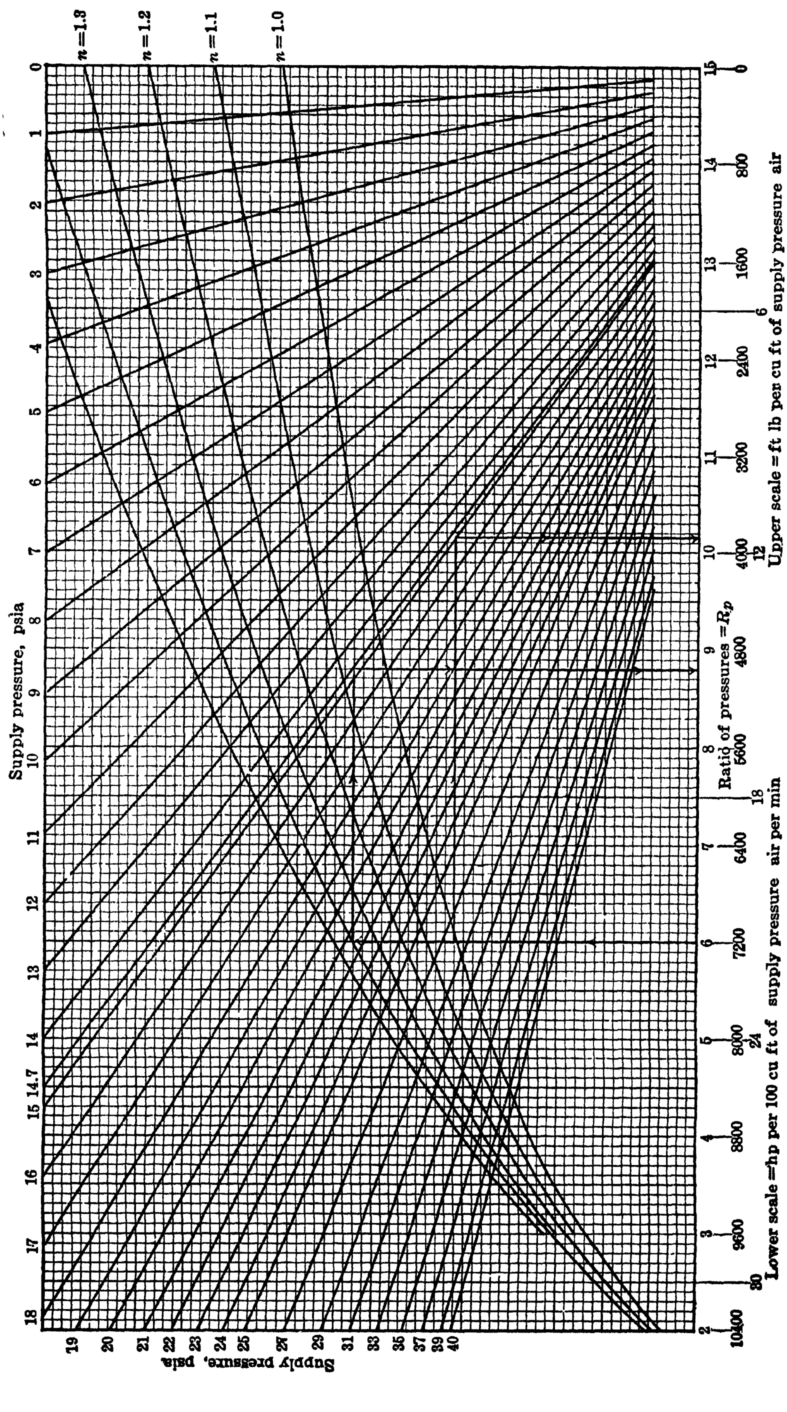


FIG. 9. Work per cubic foot and horsepower per 100 cu ft per minute of supply pressure gas for single-stage compressors. (After Lucke)

Table 1. Properties of Various Gases *

All figures are given on the basis of 60 F and 14.7 psia

(Adapted by permission, *Worthington Pump & Machinery Corp.*)

Name of Gas	Symbol	$c_p/c_v = k$	Specific Gravity Air = 1.00	Molecular Wt.	Pounds per Cubic Foot	Cubic Feet per Pound	Boiling Point at Atmos. Pressure, °F	Critical Temp., °F	Critical Pressure, psia
Acetylene	C_2H_2	1.3	0.9073	26.0156	.06880	14.534	−118	96	910
Air		1.406	1.000	28.9752	.07658	13.059	−317	221	546
Ammonia	NH_3	1.317	0.5963	17.0314	.04509	22.178	−28	270	1638
Argon	A	1.667	1.379	39.944	.10565	0.467	−302	−187	705
Benzene	C_6H_6	1.08	2.6953	78.0468	20640	4.845	176	551	700
Butane	C_4H_{10}	1.11	2.067	58.078	.15350	6.514	31	307	528
Butylene	C_4H_8		1.9353	56.0624	.14826	6.7452			
Carbon dioxide	CO_2	1.30	1.529	44.000	.11637	8.593	−109	88	1072
Carbon disulfide	CS_2	1.20	2.6298	76.120	.20139	4.965	115	523	1116
Carbon monoxide	CO	1.403	0.9672	28.000	.07407	13.503	−313	−218	514
Carbon tetrachloride	CCl_4	1.18	5.332	153.828	.40650	2.4601	170	541	661
Carburetted water gas		1.35	0.4090						
Chlorine	Cl_2	1.33	2.486	70.914	.18750	5.333	−30	291	1118
Dichloromethane	CH_2Cl_2	1.18	3.005	84.9296	.22450	4.458	105	421	1490
Ethane	C_2H_6	1.22	1.049	30.0468	.07940	12.594	−127	90	717
Ethyl chloride	C_2H_5Cl	1.13	2.365	64.4960	.17058	5.866	54	370	764
Ethylene	C_2H_4	1.22	0.9748	28.0312	.07410	13.495	−155	50	747
Flue gas		1.40							
"Freon" (F-12)	CCl_2F_2	1.13	4.520	120.9140	.31960	3.129	−21	233	580
Helium	He	1.66	0.1381	4.002	.01058	94.510	−452	−450	33
Hexane	C_6H_{14}	1.08	2.7395	86.1092	.22760	4.393	156	454	433
Hexylene	C_6H_{12}		2.9201	84.0936	22250	4.4951			
Hydrogen	H_2	1.41	0.06952	2.0156	.00530	188.62	−423	−400	188
Hydrogen chloride	HCl	1.48	1.268	36.4648	.09650	10.371	−121	124	1198
Hydrogen sulfide	H_2S	1.30	1.190	34.0756	.09012	11.096	−75	212	1306
Isobutane	C_4H_{10}	1.11	2.0176	58.078	.15365	6.5135	14	273	543
Isopentane	C_5H_{12}		2.5035	72.0936	.19063	5.2451			
Methane	CH_4	1.316	0.5544	16.0312	.04234	23.626	−258	−116	672
Methyl chloride	CH_3Cl	1.20	1.785	50.4804	.13365	7.491	−11	289	966
Naphthalene	$C_{10}H_8$		4.423	128.0624	.33870	2.952			
Natural gas † (app. av.)		1.269	0.6655	19.463	.05140	19.451			
Neon	Ne	1.642	0.6961	20.183	.05332	18.748	−410	−380	389
Nitric oxide	NO	1.40	1.037	30.008	.07935	12.605	−240	−137	954
Nitrogen	N_2	1.41	0.9672	28.016	.07429	13.460	−320	−232	492
Nitrous oxide	N_2O	1.311	1.530	44.016	.11632	8.595	−129	98	1053
Oxygen	O_2	1.398	1.105	32.000	.08463	11.816	−297	−182	730
Pentane	C_5H_{12}	1.06	2.471	72.0936	.19055	5.248	97	387	485
Phenol	C_6H_5OH		3.2655	94.0468	.24870	4.022			
Propane	C_3H_8	1.15	1.562	44.0624	.11645	8.587	−48	204	632
Propylene	C_3H_6		1.4505	42.0468	.11115	8.997	−52	198	661
Refinery gas † (app. av.)		1.20							
Sulfur dioxide	SO_2	1.256	2.264	64.060	.16945	5.901	14	315	1141
Water vapor (steam)	H_2O	1.33 ‡	0.6217	18.0156	.04761	21.004	212	706	3206

* See also Section 2, Art. 18

† To obtain exact characteristics of natural gas and refinery gas, the exact constituents must be known.

‡ This k value is given at 212 F. All others are at 60 F.

Since authorities differ slightly, foregoing data are average results.

$$\text{Ratio of volumes, } R_v = \frac{v_s}{v_d} \tag{11}$$

$$\text{Ratio of temperatures, } R_t = \frac{t_d}{t_s} \tag{12}$$

From eq. 7,

$$R_v = (R_p)^1 \qquad R_v = (R_p)^{1/n} \tag{13}$$

and from eq. 9,

$$R_t = (R_p)^{(k-1)/k} \qquad R_t = (R_p)^{(n-1)/n} \tag{14}$$

Figure 10 is helpful in solving these equations.

Figure 11 shows comparatively the isothermal ($n = 1$), isentropic ($n = k$), and polytropic ($1 < n < k$) compressor cyclic standards (eqs. 4 to 6). The higher the value of n, the steeper the compression curve and the greater the power needed to deliver a given quantity ($v_s = v_2 - v_1$) of gas. Actual compression tends to follow the isentropic standard so that isothermal compression, if attainable, would result in substantial power savings as evidenced by the cross-hatched area of Fig. 11.

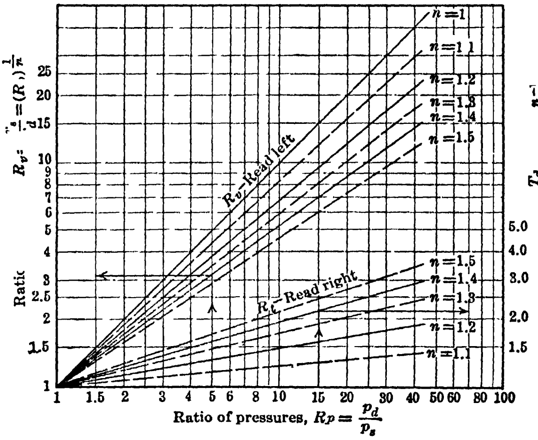


FIG. 10. Ratio of pressures, volumes, and temperatures.

Thermodynamic analyses indicate that the above equations for work and power are equally valid for cases involving cylinder clearance. If the same suction volume, v_s, is handled by the compressor, the power requirement is independent of clearance. The clearance, however, vitally influences the useful capacity delivered by a given cylinder. As the clearance is increased, the capacity is reduced. In Fig. 12, the phase (4–1) represents the clearance re-expansion loss and the net volume taken in by the cylinder is $v_2 - v_1 = v_s$. The total cylinder volume is the sum of the displacement, $D = v_2 - v_4$, and the clearance, v_4. The ratio of

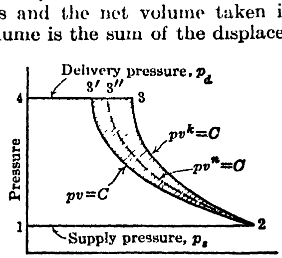


FIG. 11. Pressure-volume diagram showing isothermal, isentropic, and polytropic compression.

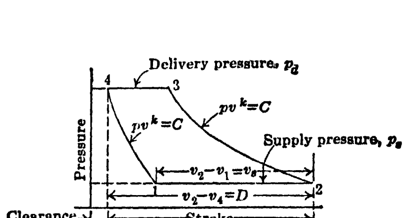


FIG. 12. Pressure-volume diagram, single-stage compressor with clearance.

the ideal useful capacity, v_s, to the piston displacement, D, is the conventional ***volumetric efficiency***, e_v, and is given by the equation

$$e_v = \frac{v_s}{D} = 1 + C - CR_p^{1/n} \tag{15}$$

where C = clearance, expressed as a fraction of the displacement, D.

The solution of this equation is facilitated by the use of Fig. 13. The limiting theoretical cases for volumetric efficiency are represented by the isothermal ($n = 1$) and the isentropic ($n = k$) standards.

Multistage compression, with intercooling between stages, results in (1) a saving in power and (2) improved volumetric efficiency of the low-pressure cylinder and, therefore,

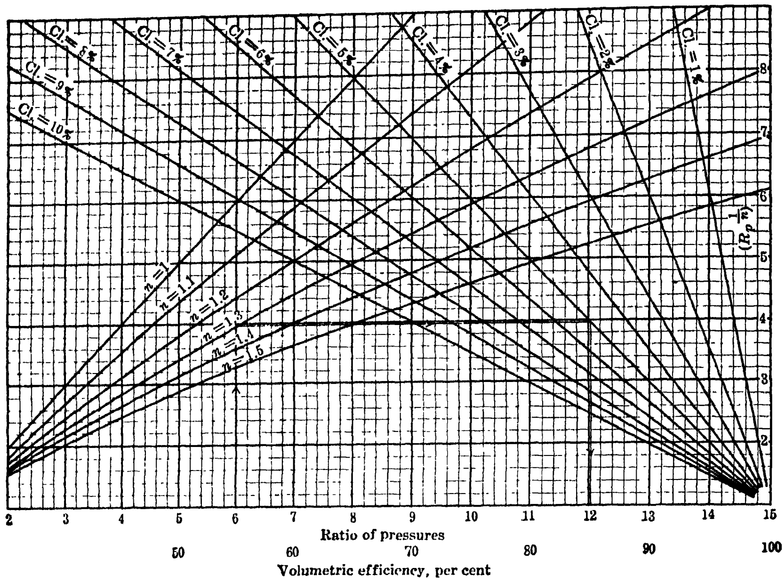


FIG. 13. Volumetric efficiency (conventional) for compressor cylinders. (After Lucke)

of the unit. Perfect intercooling, as illustrated in Fig. 14, for two-stage compression, requires that the gas be cooled back to the original temperature as represented by the phase (5–5′). Point 5′ is thus located on the isothermal compression line (2 3). The power saving, over single-stage isentropic compression, is shown by the cross-hatched area. The extent of the saving is determined not only by the degree of intercooling but also by the pressure at which the intercooling is effected. The maximum saving, with perfect intercooling, obtains when the work is allocated equally among the stages or where the ratios of compression in all stages are equal, or

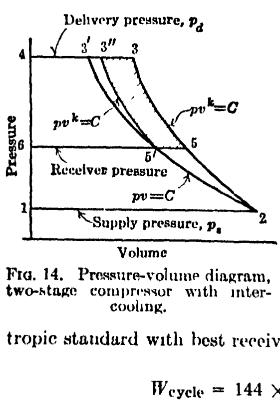


FIG. 14. Pressure-volume diagram, two-stage compressor with intercooling.

$$R_{p1} = R_{p2} = R_{p3}, \text{ etc.} = R_p^{1/N} \tag{16}$$

where R_{p1}, R_{p2}, R_{p3} = compression ratio in stages one, two, and three, respectively; R_p = overall compression ratio for the machine; and N = number of stages of compression.

The work and power required for the multistage isentropic standard with best receiver pressure and perfect intercooling are found by

$$W_{\text{cycle}} = 144 \times N\left(\frac{k}{k-1}\right)(p_2v_2)\left[\left(\frac{p_3}{p_2}\right)^{(k-1)/Nk} - 1\right] \tag{17}$$

and

$$\text{Isentropic horsepower} = N\left(\frac{k}{k-1}\right)\frac{p_sv_s}{229.2}\left[R_p^{(k-1)/Nk} - 1\right] \tag{18}$$

where p_s = supply pressure, pounds per inch² absolute; and v_s = capacity at supply pressure, p_s, cubic feet per minute.

These equations are readily solved for two-stage compression by use of Fig. 15. The isothermal standard does not change with staging and consequently can be evaluated by eqs. 3 and 5. Multistage compressors are in operation which utilize as many as seven stages, with intercooling between each stage. The power requirement is, again, theoretically independent of clearance. The volumetric efficiency of the low-pressure cylinder is the volumetric efficiency of the entire machine.

COMPRESSION EFFICIENCY. The performance of real compressors is referred to the ideal cyclic values so that compression efficiency is defined as

$$\text{Compression efficiency} = \frac{\text{Ideal horsepower}}{\text{Actual horsepower}} \tag{19}$$

The ideal values are computed by eqs. 5, 6, and 18 for the volume flow rate actually

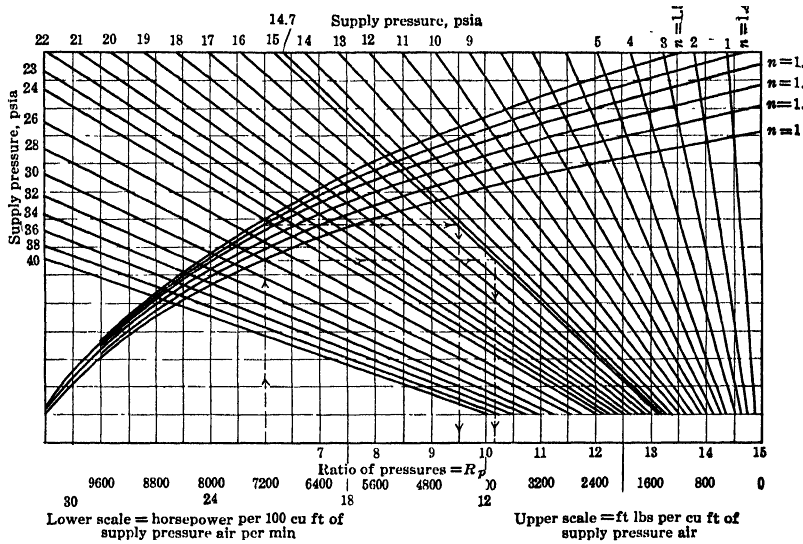


FIG. 15. Work per cubic foot and horsepower per 100 cu ft per minute of supply pressure gas for two-stage compressors, best receiver pressure, perfect intercooling. (After Lucke)

handled by the compressor. The ideal power may be any one of the hypothetical standards, and the reference base *should be clearly defined as either isothermal compression efficiency or adiabatic compression efficiency.* The actual horsepower used in the denominator of eq. 19 may be determined (1) from real indicator cards (Fig. 16) taken on the compressor cylinder, (2) from indicator cards taken on the engine cylinders if the compressor is engine driven, (3) as the shaft horsepower of the compressor, or (4) as the electric horsepower input to the motor terminals if the compressor is electrically driven. It is customary to use the air cylinder horsepower in the denominator of the equation. It is best, however, clearly to define the point at which the power is measured, in such terms as "air card adiabatic compression efficiency" or "overall isothermal compression efficiency."

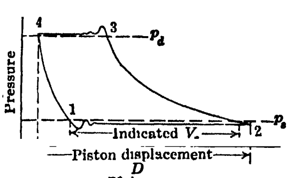


FIG. 16. Actual indicator card from a compressor cylinder.

ACTUAL CAPACITY. The capacity of a compressor is less than the piston displacement. Conventional volumetric efficiency is defined by eq. 15 for the ideal cases. The real indicator card (Fig. 16) will give a different value defined as

$$\text{Indicated volumetric efficiency} = \frac{\text{Indicated } v_s}{\text{Piston displacement}} \tag{20}$$

Indicated volumetric efficiency differs from the conventional because in the latter no allowance is made for suction throttling and suction heating effects of real compressors. Stated otherwise, the indicator card carries no temperature scale. Furthermore, if the actual capacity is separately metered and then referred to the piston displacement, the result is

$$\text{Actual volumetric efficiency} = \frac{\text{Actual metered capacity}}{\text{Piston displacement}} \tag{21}$$

Of the three volumetric efficiencies obtainable on the same data (eqs. 15, 20, and 21), the last, or actual, volumetric efficiency, while it is the smallest value numerically, is the true value and should always be chosen in preference to the others. Slippage efficiency is defined as

$$\text{Slippage efficiency} = \frac{\text{Actual volumetric efficiency}}{\text{Indicated volumetric efficiency}} \tag{22}$$

The actual volumetric efficiency may be estimated from indicator card values according to the equation

Estimated actual volumetric efficiency =

$$\frac{\text{Indicated volumetric efficiency}}{1 + (0.25 \text{ to } 0.50)\left[\dfrac{\text{Absolute temperature air delivered}}{\text{Absolute temperature air supplied}} - 1\right]} \tag{23}$$

where the lower constant is used with those machines offering the least suction heating effect.

POSITIVE DISPLACEMENT, RECIPROCATING COMPRESSORS. Older compressors followed steam-engine practice of large bore and stroke, few cylinders, low-speed, double-acting, horizontal constructions. The automotive-type internal-combustion engine has led to the development of small-bore, short-stroke, multicylinder, high-speed, vertical constructions. Figures 1 and 2 are representative of long-lived slower-speed units, and Fig. 3 is representative of high-speed units. (See p. **1-35.**)

Design data representative of air compressors are

Clearance, 3 to 12%
Piston speed, 500 to 1000 ft per min
Bore : stroke ratio = 0.75 to 1.25
Maximum cylinder size seldom exceeds 3 ft
Main bearing pressures = 150 to 250 lb per sq in. of projected area

Ratio of compression per stage seldom exceeds 7; usually averages 2 or 3 on multistage units.

Valves are of the automatic type because of better timing and lower cost. Mechanical valves are found on some older machines where minimum pressure drop is sought as with

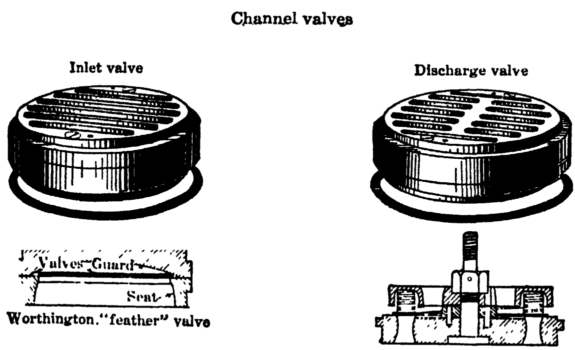


FIG. 17. Channel valves.

high vacuum. Automatic valves are of plate or disk type (Fig. 17) or mushroom or poppet type (Fig. 18). Each operates by pressure difference (1 to 5 psi) on the two sides of the moving member which has a limited travel (less than 1/4 to 1/2 in.), limited by a stop and opposed by a light spring-pressure. Air speeds approximate 5000 ft per min through the openings. Moving elements must be light to reduce inertia forces. Spring pressures are low to reduce frictional resistance but must be adequate to minimize chatter (see indicator card of Fig. 16). These requirements are best met by small valves used in multiplicity (poppets less than 2 or 3 in. diameter and plate valves less than 5 or 6 in. diameter). Poppet valves, because of superior strength as a circular beam, are preferred for high-pressure service. Many plate valve designs can be used interchangeably for inlet or discharge by the simple expedient of reversing the position of the cage assembly in the port. Valves are subject to heating effects and the carbonizing of lubricants on surfaces both of which are cumulative in their ill effects

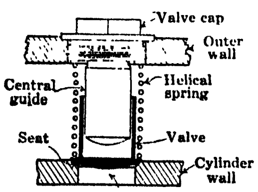


FIG. 18. Poppet-type discharge valve.

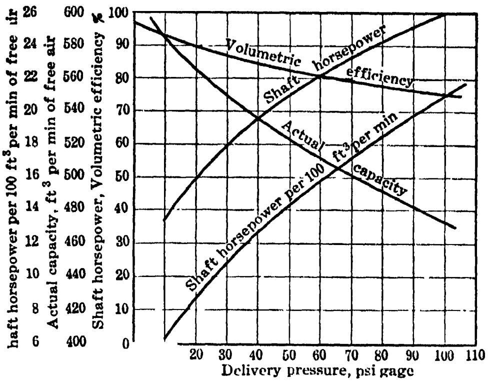


FIG. 19. Actual performance, horizontal water-cooled single-stage compressor. (Piston displacement = 630 cu ft per min, supply pressure = 14.7 psia)

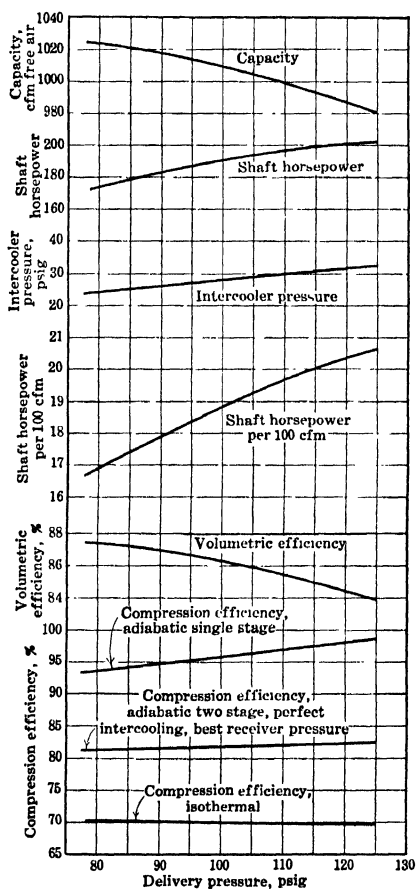


FIG. 20. Performance of two-stage air compressor. (18″ and 11″ x 10″ at 400 rpm; displacement = 1170 cfm)

on performance. The overall criteria for judging valve suitability are (1) adequacy of timing, (2) pressure drop, (3) reliability, and (4) replacement cost.

ACTUAL PERFORMANCE. The actual performance of a single-stage compressor is shown in Fig. 19 and of a two-stage compressor in Fig. 20. In each instance the independent variable is delivery pressure with the unit running at constant speed. The data of Fig. 21 are useful in estimating the power requirements of air and gas compressors. Compressors are greatly influenced as to capacity and horsepower by operation at altitude. Figures 22 and 23 illustrate this influence and give a means of correcting sea-level performance to the values obtaining at altitude.

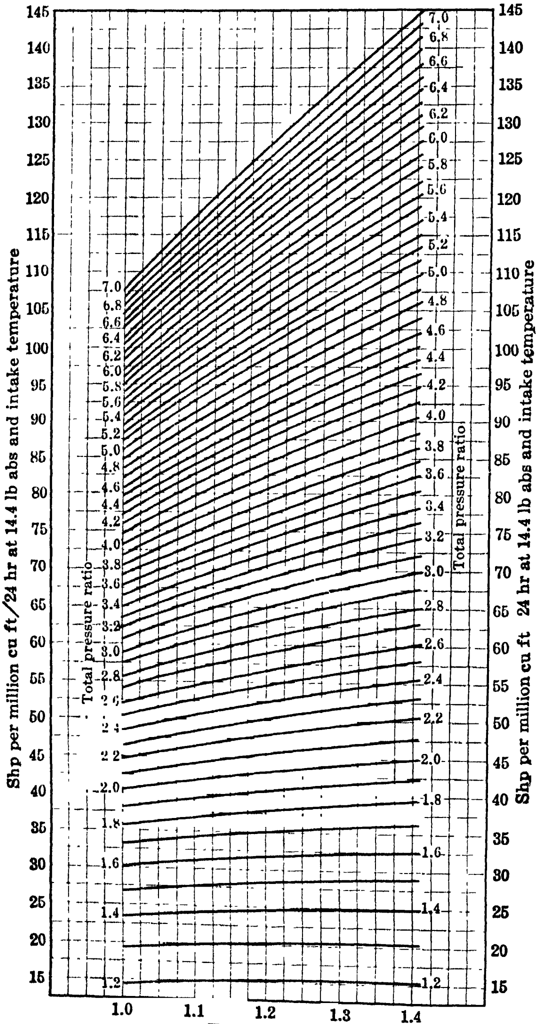


FIG. 21. Approximate horsepower to compress air or gas. Find exponent of gas from Table 1, p. 1-40. If single stage, multiply actual capacity (in cu ft free gas per min) by 0.00144 to obtain capacity in million cu ft per 24 hr. Multiply this value by the ordinate obtained from the chart to obtain the total horsepower. If two stage, take square root of pressure ratio. Read horsepower from chart for this ratio; multiply by 2 (for two stages), and add 3% for cooler loss. (Adapted by permission of Worthington Pump and Machinery Corp.)

Dry Vacuum Pump. The data of Fig. 24 are pertinent to the performance of a rotative dry vacuum pump.

DRIVES. Reciprocating compressors of the slow-speed type are adaptable to direct drive by steam and internal-combustion engines, and by synchronous motors. Gears and belts are otherwise needed unless the small bore, multicylinder, high-speed compressor is used which permits of direct connection to high-speed a-c and d-c motors, and to automotive-type gasoline or Diesel engines. The required turning effort is not uniform throughout the revolution of the crank shaft because of (1) the gas pressure loadings on the piston and (2) the inertia and centrifugal forces of the moving masses. With direct drive by steam or internal-combustion engines, the maximum driving pressure is available at the beginning of the stroke when the minimum resistance pressure is offered in the compressor cylinder. Heavy flywheels are commonly employed to equalize these effects. Crank effort diagrams must be carefully checked for synchronous motor drive to obviate hunting. Unloading devices are needed for starting in these applications.

GOVERNING AND REGULATION. Air compressors may be controlled for constant volume delivery, but the more usual requirement is for the maintenance of constant delivery pressure regardless of the capacity demand. Many basic methods of regulation have been used of which the following are of present commercial importance.

Speed Control. If the speed of the compressor is altered, the capacity will be changed in proportion to the speed. Variable speed requires a driver which can be readily and economically controlled such as a steam engine or an internal-combustion engine. Variable speed generally precludes the use of electric motors except for the smallest sizes and for some d-c services.

Throttled Suction. The suction of the compressor can be throttled, thus increasing the ratio of compression and decreasing the volumetric efficiency and the capacity as shown on the pressure volume diagram of Fig. 25*a*. Part-load operation for protracted periods may lead to ill effects from overheating and carbonization. The high-compression ratios at partial loads give accompanying high-temperature ratios between outlet from and inlet to the compressor cylinder.

Valve Control. The inlet valves can be held opened during the delivery stroke returning the supply air from the cylinder to the intake line as represented on the pressure volume diagram of Fig. 25*b*. This is usually a more expensive construction than the above two methods, but it gives advantages with constant-speed drive that include sustained efficiency and reliability, without danger of overheating.

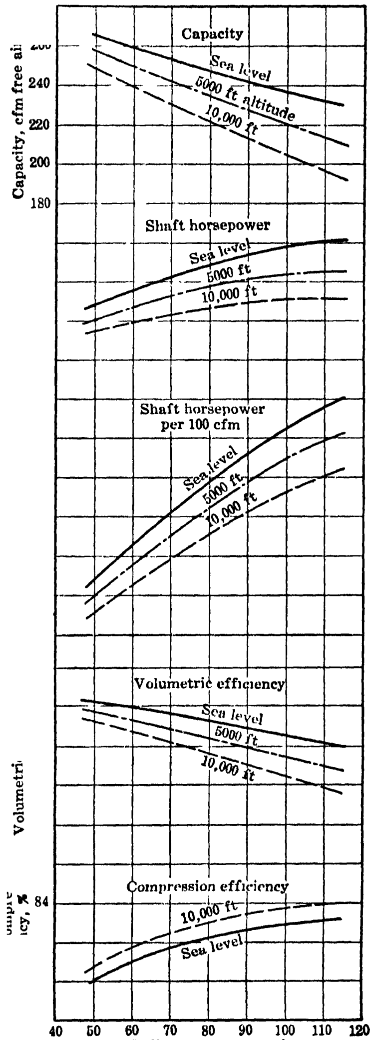


FIG. 22. Performance of a single-stage air compressor. (10″ x 12″ at 300 rpm; displacement = 327 cfm)

Clearance Volume Control. The clearance volume can be altered, usually in three or five steps, by opening or closing valves connected to clearance pockets in the cylinder heads. An increase in clearance gives an increase in re-expansion loss and a decrease in volumetric efficiency and capacity (Fig. 25*c*). Clearance control is a most effective method of governing as it requires minimum power despite the high initial cost of equipment. It

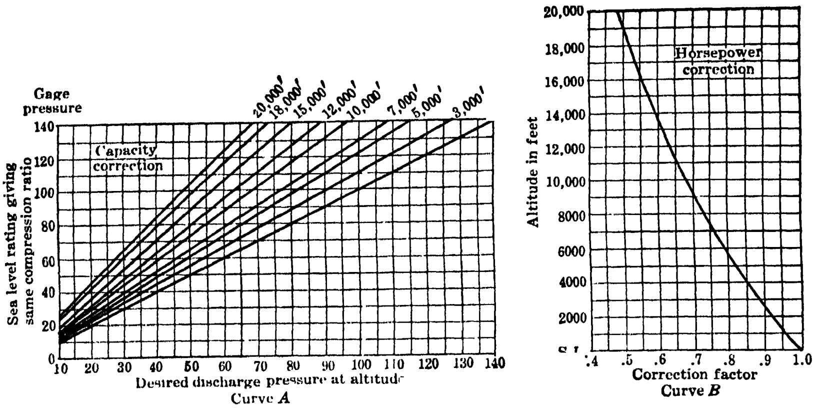


FIG. 23. Altitude correction curve for capacity and horsepower of compressors. (Courtesy of Ingersoll Rand Co.)

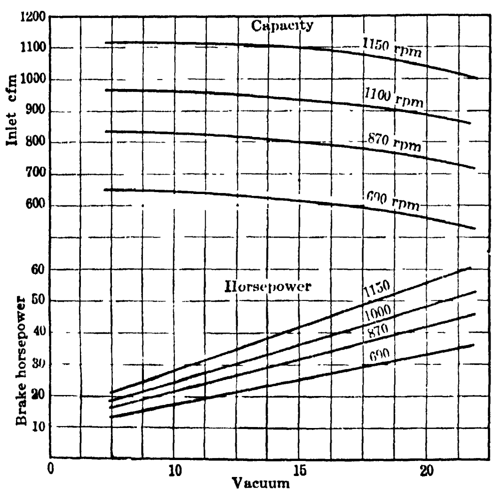


FIG. 24. Performance characteristics positive displacement rotary vacuum pump (8" x 11"). (Roots-Connersville Blower Corp.)

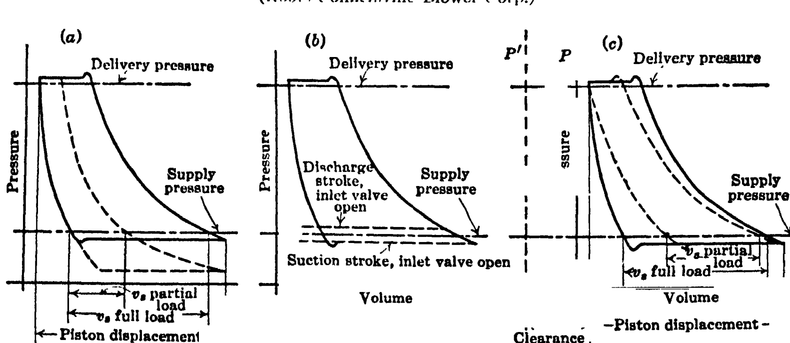


FIG. 25. Pressure-volume diagrams for different methods of regulating and governing compressors (a) suction throttling, (b) open inlet valves, (c) clearance control.

is particularly useful on synchronous motor-driven units where the results on two-stage (100 psi) service with five-step control would be summarized as:

Capacity, %	Power Required, %
100	100
75	77 to 78
50	53 to 55
25	29 to 32
0	5 to 10

16. ROTARY, HYDRAULIC, AND JET COMPRESSORS

A ROTARY COMPRESSOR WITH LIQUID SEALS is shown in Fig. 5. Intake and discharge ports give access to and from the cylinder without the use of valves. The circular impeller accelerates water or other liquid to the outer periphery of the elliptical casing. Gas from the intake port is trapped in the space between adjacent blades, and as this space is turned toward the discharge port the liquid in the partially filled cylinder acts as a piston to compress the gas. This type of compressor serves well as a vacuum pump or a positive pressure blower. Pressures are moderate (28 in vacuum or 75 psi delivery pressure on single-stage arrangements), and the units are particularly suited to service where liquids are to be handled simultaneously with gases, such as wet vacuum pump applications on steam-heating systems. The delivery of clean, oil-free air is assured by the scrubbing action of the liquid ring, and many other condensing, drying, or chemical effects may be obtained with the selection of suitable liquids for a given pumping condition. They are suitable for direct drive by electric motors; some representative performance data are given in Fig. 26.

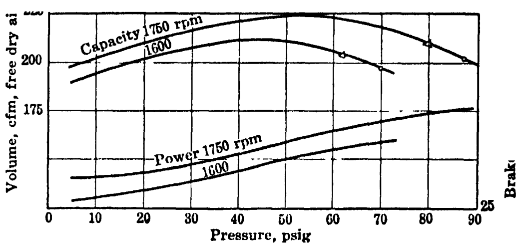


FIG. 26. Nash Hytor compressor performance curves. Δ = recommended maximum unloading pressure. o = recommended maximum operating pressure. (Nash Engineering Co.)

A ROTARY COMPRESSOR WITHOUT LIQUID SEALS and using a pair of multiple-lobe impellers is shown in Fig. 4. The impellers are held in correct alignment by gears external to the casing so that there is no rubbing or contact between the impellers themselves or between the impellers and the casing. The internal surfaces run dry. The external gears must be well constructed, well lubricated, and well maintained to hold the close operating clearances. These units are limited to ratios of compression of less than 2 to 1 for positive pressure or vacuum service with capacities as high as 50,000 cfm. Two units may be placed in series for a higher ratio of compression. Simplicity of construction and absence of intake

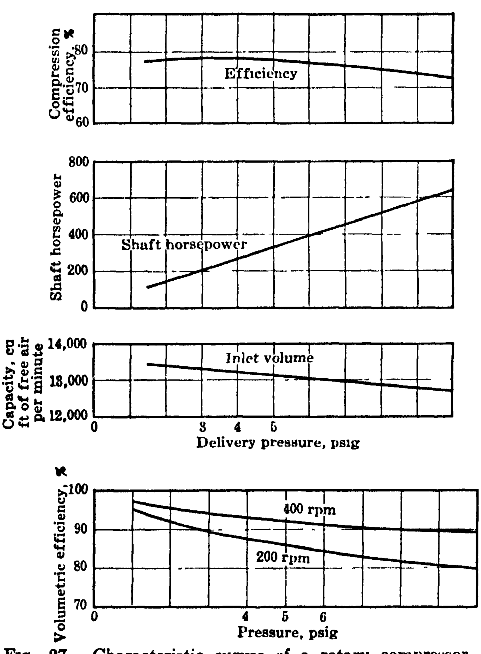


FIG. 27. Characteristic curves of a rotary compressor—28" x 32½" (Courtesy of Roots-Connersville Blower Corp.)

and discharge valve constitute real economic advantages. These blowers find common service in the gas industry, where they can readily handle tar-laden gases and operate simultaneously as a gas meter. They are also looked upon with favor for use in scavenging and supercharging of internal-combustion engines. Representative performance data for a Roots-Connersville rotary compressor (Fig. 4) are shown in Fig. 27.

A ROTARY COMPRESSOR which utilizes sliding vanes and lubricated internal surfaces is often substituted for other types. A floating ring may prevent metal-to-metal contact between the vanes and the cylinder wall. The lubricant film makes it possible to maintain tight seals so that these units will deliver air with compression ratios as high as 20 to 1 on a single stage. Three or four to one is the compression ratio prevailing for single-stage units. The possible variations in design detail on these compressors are almost limitless so that many features are covered by patents. These units are suitable for direct electric motor drive, if speeds are not too high, but they often suffer on the score of operating economy and maintenance when contrasted with other types of compressors.

THE HYDRAULIC AIR COMPRESSOR utilizes directly the effect of falling water to compress air. Water, from an elevated supply as in a hydroelectric plant, is allowed to compress the air in a vertical column. The advantages of this arrangement over the conventional motor-driven reciprocating compressor lie in the absence of moving machine parts and in the power saving attendant upon isothermal compression. The large mass of water involved, together with the high heat capacity of water, makes the compression proceed substantially at constant temperature. The installations are few, primarily because of the high investment costs.

THE JET COMPRESSOR (Fig. 7) consists essentially of a nozzle in which high-pressure gas or vapor is allowed to expand; the high-velocity jet serves as the actuating fluid to entrain air or gas and deliver the mixture to the diffuser tube in which kinetic energy is regained as pressure energy. The load may be imposed either on the suction or on the discharge with the jet compressor serving as an exhauster or a blower, respectively. Steam is frequently used as the actuating fluid. Such apparatus finds use for forced and induced draft service; air supply for process where the steam is not detrimental; air ejectors for vacuum applications; thermocompressors for heat pump and air-conditioning installations. The jet pump is a simple, rugged, static machine with no moving parts. It is consequently foolproof, low in maintenance costs, and low in investment. Its major disadvantage is its poor efficiency as a pump. Efficiency is defined as

$$\text{Ejector efficiency} = \frac{\left\{\begin{array}{l}\text{Work done on entrained gas between}\\ \text{suction and discharge pressure}\end{array}\right\}}{\left\{\begin{array}{l}\text{Work available from expansion of}\\ \text{actuating fluid from line pressure}\\ \text{to discharge pressure}\end{array}\right\}} \tag{24}$$

These values of work can be calculated either from the Mollier chart as enthalpy differences for isentropic expansion or compression or by the equivalent equations for work. (See eqs. 3 and 4.)

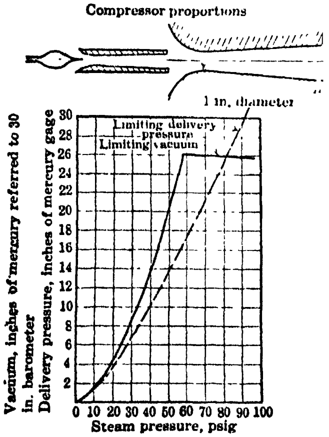


FIG. 28. Steam-jet compressor operating (a) as a blower and (b) as an exhauster.

A single-stage steam-jet air ejector will produce a compression ratio of 5 or 6 to 1 when serving as a vacuum pump. It is extremely sensitive to steam pressure, to physical properties of the actuating fluid, to velocity and surface of the actuating jet. Some representative performance curves of a simple ejector serving as a blower and as an exhauster are shown in Figs. 28 and 29.

In many applications of jet compressors it is necessary to substitute some other actuating gas or vapor for steam. In estimating the results under such conditions, it should be recognized that the entraining capacity of a jet is a function of (1) jet velocity; (2) densities of the fluid media; and (3) surface offered by the jet for contact with the second fluid. For a jet of circular cross section, the capacity is a function of a diameter, but the kinetic energy is a function of the area or the diameter squared. This will lead to the use of multiple nozzles with a single common combining tube, thus increasing capacity by offering more surface for entrainment with the same quantity of actuating fluid flowing. It follows, in turn, that the substitution of one actuating fluid for another will substantially alter the performance even without any change in the basic design. Thus, if compressed air is used instead of steam, the net result of the change in jet velocity

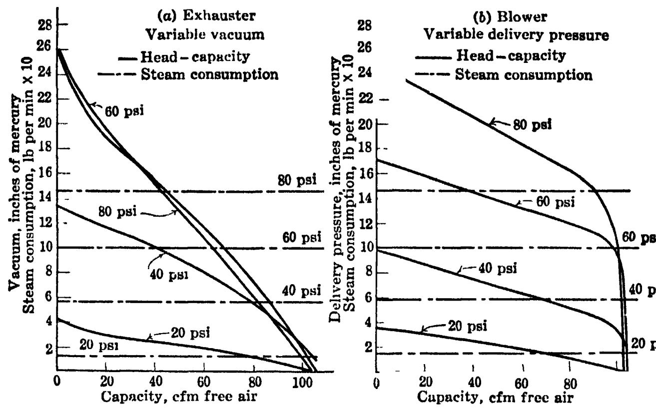


FIG. 29. Steam-jet compressor. Head-capacity, steam consumption, and thermal efficiency characteristics as (a) exhauster and (b) blower.

and in fluid density will be to lower the entraining capacity to such an extent that air consumption will be 1.5 to 2 times steam consumption. For additional data on the steam-jet air pump, see Section 9, Art. 3.

17. TURBO-, CENTRIFUGAL, AND AXIAL COMPRESSORS

TURBOCOMPRESSORS, of the centrifugal or axial-flow types, exemplify the pumping of gases or vapors under conditions of free compression, steady flow, in a jet and vane device. They are the modern counterpart of centrifugal and propeller fans as extended for high head service. The line of demarcation between fans and compressors can be drawn so as to separate them into those devices in which the fluid can be considered incompressible, like water, and those in which the compressibility effect is substantial. If the change in density, on passage through the unit, must be recognized in the calculations on performance, the unit is properly called a compressor rather than a fan. The laws of thermodynamics, instead of hydraulics, apply to the performance of compressors.

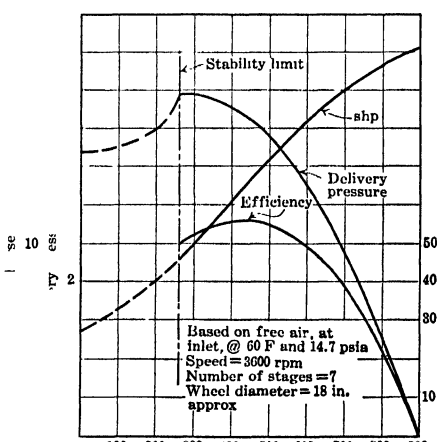


FIG. 30. Characteristics of a seven-stage turboblower. (Ingersoll Rand Co.)

A multistage centrifugal compressor is shown in Fig. 6 and a multistage axial flow compressor in Fig. 1, p. 1-96. The trend with modern axial and centrifugal compressors is to use air cooling exclusively, even with the highest ratios of compression. Multistage centrifugal units were formerly designed with water jackets and water-cooled diaphragms. Axial flow units have seldom, if ever, used water cooling in practice. The present reliance on air cooling recognizes the relative ineffectiveness of any jacket from a heat transfer viewpoint, and the increased investment and operating cost for a water-cooling system. Likewise, the potentialities of intercooling on multistage machines were formerly looked upon with greater favor than they are today. Both the increased cost and the pressure drop of the gas limit intercooling to those few cases in which a multiple-cylinder construction is substituted for the more usual single-barrel unit.

The overall performance of turbocompressors is best represented by characteristic curves which are definitive of their performance. In other words, the compressor must operate at some point on the characteristic. It may establish its own balance between system resistance and developed head to determine the capacity delivered. Figures 30 and 31 show, respectively, the characteristic performance curves for a centrifugal and an axial-flow compressor. Characteristics can also be plotted on a dimensionless basis, as in Fig. 32, for an axial compressor, to facilitate comparative analyses.

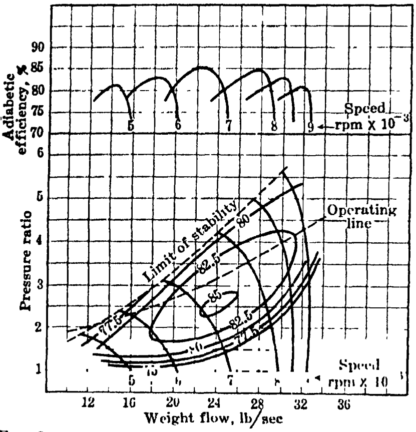


FIG. 31. Typical performance characteristic curves of an axial-flow compressor. (After Ponomareff)

The characteristics show the pressure, horsepower, and efficiency as functions of capacity for constant impeller speed. All turbocompressors are very sensitive to change in speed, and for a given point on the efficiency curve the basic relations are that capacity is proportional to speed, pressure proportional to the square of the speed, and horsepower to the cube of the speed. These relations are not rigorously exact as flow patterns must be based for similarity on Reynolds and Mach number criteria. The relations, nevertheless, are useful for the development of various dimensionless performance coefficients which enable the comparison of model and prototype performance and correction for scale effects.

The characteristics for all turboblowers show a region of unstable operation (see Figs. 30, 31, and 32). If an attempt is made to operate in this area the flow will pulsate so that there is blowback through the suction pipe with violent hunting, pumping, and surging. There is, accordingly, for any speed with a given unit, a lower limit of capacity below which instability prevails. On axial-flow compressors the stable operating range (Figs. 31 and 32) is small and approaches a minimum as the operating speed is increased. With centrifugal compressors the range of stable operating load is larger and is a function of the blade curvature. The widest stable operating range obtains with backward-curved blades, the narrowest range with forward-curved blades, and the intermediate range with the radial tip construction. Centrifugal blowers are accordingly designed either with backward or radial blades.

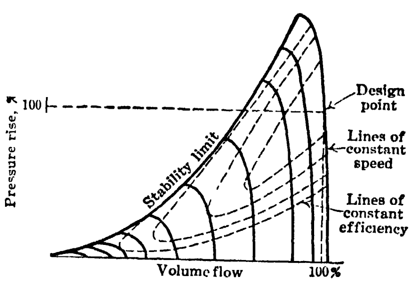


FIG. 32. Percentage characteristics for an axial-flow compressor. (From Salisbury, *Mechanical Engineering*, 1944)

Centrifugal and axial-flow compressors are designed with typical performance as shown in Table 2. The vector triangles representative of the several types of blade curvature for centrifugal blowers are shown in Fig. 33. The energy added to the fluid or the theoretical pressure developed, W, is given by Euler's equation

$$\Delta W,\ \text{ft-lb per lb} \quad u_2t_2 - u_1t_1 \tag{25}$$

where u is the absolute blade velocity, feet per second, and t is the tangential component (in the direction of u) of the absolute fluid velocity, v, in feet per second. If the spin, t_1, at the entrance to the blade is zero, the head developed is solely a function of u_2 and t_2, the velocities at the tip. In order to develop maximum head per stage both u_2 and t_2 should be made as large as possible. The former is dependent on structural design limitations. The latter is dependent on the tip curvature and is a maximum with forwardly inclined blades (Fig. 33*a*). Straight radial blades (Fig. 33*b*) can be built in heavy sections to utilize top tip speeds and thus, in turn, reduce the number of stages in multistage machines. The head developed per stage on real centrifugal compressors approximates

10,000 ft-lb per lb (13 Btu per lb). Tip speeds seldom exceed 1000 ft per sec while rotative speeds may run as high as 20,000 or 30,000 rpm.

For design method and data on axial flow compressors see p. 1-96.

Table 2. Representative Turboblowers and Compressors—Dimensions and Performance *

	Service Application					
	Aircraft Engine Super-charger	Multi-stage	Blast Furnace Blower	High-Pressure Air Com-pressor	Gas Turbine Com-pressor	Gas Works Blower
Compressor type	Centrifugal	Centrifugal	Centrifugal	Centrifugal	Axial	Centrifugal
Number of stages	1	7	5	13	20	1
Fluid	Air	Air	Air	Air	Air	Gas $w = 0.05$ $k = 1.33$
Capacity, cfm	2,000	1,000	30,000	15,000	40,000	15,000
Supply pressure, psia	15	15	15	15	15	14.7
Discharge pressure, psia	25	24	45	115	60	20.4
Ratio of compression, R_p	1.67	1.6	3.0	7.67	4	1.4
Speed, rpm	25,000	3,500	3,000	3,000	3,600	14,000
Shaft horsepower	104	60	3,130	3,600	5,500	470
Compression efficiency						
Single stage adiabatic	70%	55%	81%	75%	80%	72%
Isothermal	65%	51%	69%	55%	65%	68%
Energy added per stage, ft-lb/lb	15,000	1,400	7,500	6,000	2,400	15,000

* See also Section 10, Art. 14.

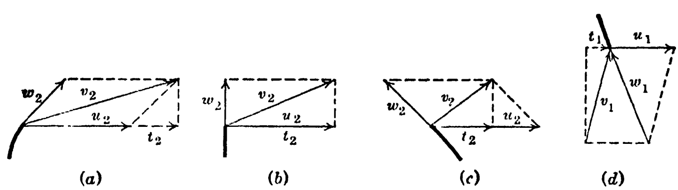


FIG. 33. Centrifugal compressor vector diagrams; (a), (b), and (c) tip conditions, (d) heel conditions. (Rotation toward right.)

18. INSTALLATION, OPERATION, AND USES

INTAKE CONDITIONS. The air supply to a compressor should be at the highest possible density in the interests of maximum volumetric efficiency and maximum useful capacity. A variation in the inlet temperature of 5 F will alter the useful capacity 1% or more. Pressure drop through the suction line should be held to a minimum (1) by limiting flow velocities to 1000 or 2000 ft per min; (2) by using the shortest, most direct (tapered if necessary) intake pipe line; and (3) by omitting valves, fittings, sharp bends, and abrupt changes of section wherever possible. Air intake screens and filters, of the wet or dry types, are recommended practice, but they should be sized liberally to balance pressure drop against maintenance costs from dirt in the air supply. Humidity at the intake should be as low as possible because water vapor at the compressor suction will be condensed to some extent in the intercooler, aftercooler, receiver, or transmission line. This condensation not only is a nuisance and a hazard but it is also a means of reducing the effective capacity of a compressor. Thus a compressor delivering air at 100 psi may have 2 to 5% lower useful delivered capacity when supplied with humid or saturated air than when supplied with dry air at the same temperature. The supply system on a positive displacement compressor should also be designed to meet the vibrational loads due to pulsating air flow.

LUBRICATION of air compressor cylinders requires the use of sufficient oil of proper grade. The leading oil companies are of great assistance in the solution of this problem. The Compressed Air and Gas Institute has published its recommended physical specification, as summarized in Table 3, and recommended quantities, as synopsized in Table 4. An excess, as well as a deficiency, of oil should be avoided. A deficiency will permit metal to metal contact: an excess will result in carbonizing, leaky discharge valves, and overheating. It should be noted that many compressors are designed and used with no lubri-

Table 3. Approximate Guide to Physical Characteristics of Air Compressor Cylinder Oils

(Reprinted from *Compressed Air Handbook*, copyright 1947, by Compressed Air and Gas Institute, New York)

Flash point, °F	350 min
Viscosity, seconds Saybolt Universal at 100 F	245 min
Viscosity, seconds Saybolt Universal at 210 F	45 min
Pour point, °F	35 max
Neutralization number	0.10 max
Conradson carbon residue, %	2.0 max

Table 4. Minimum Rate of Oil Feed for Air Compressor Cylinder Lubrication

(Reprinted from *Compressed Air Handbook*, copyright 1947, by Compressed Air and Gas Institute, New York)

Cylinder Diameter, in.	Piston Displacement per Cylinder, cu ft per min	Swept Surface per Cylinder, sq ft per min	Oil Feed per Cylinder Drops per min	Pints per 10 hrs *
Up to 6	Up to 65	Up to 500	0.67	.05
6 to 8	65 to 125	500 to 750	1.00	.075
8 to 10	125 to 225	750 to 1100	1.33	.10
10 to 12	225 to 350	1100 to 1500	1 to 2	.112
12 to 15	350 to 600	1500 to 2000	2 to 3	.188
15 to 18	600 to 1100	2000 to 2600	3 to 4	.262
18 to 24	1100 to 1800	2600 to 3600	4 to 5	.338
24 to 30	1800 to 3000	3600 to 4800	5 to 6	.412
30 to 36	3000 to 4500	4800 to 6000	6 to 8	.525
36 to 42	4500 to 6500	6000 to 7500	8 to 10	.675
42 to 48	6500 to 9000	7500 to 9000	10 to 12	.825

* Based on 8000 drops per pint at 75 F.

cating oil supplied to the cylinder because of the dangers of contamination. Such compressors are built with carbon rings on the pistons to prevent metal-to-metal contact and eliminate the need for oil, grease, water, glycerol, or the like. A thousand hours of operation without shifting or replacement of the carbon wearing rings is reasonable performance.

RECEIVERS, AFTERCOOLERS, AND INTERCOOLERS. Compressed air must be delivered to a receiver (1) to remove the pulsating effects of the compressor discharge; (2) to act as a storage vessel, keeping the fluid at a substantially constant service pressure; and (3) to remove moisture and excess oil. The volume of the receiver for good practice on 100 psi air supply should be such as to hold approximately one-half minute's discharge capacity of the connected compressor. Receivers, intercoolers, and aftercoolers are covered by legal limitations as to design and operation. The most generally accepted regulations are contained in the ASME Code for Unfired Pressure Vessels. The receiver should be placed outdoors for most effective cooling and must be equipped with safety valves, pressure gages, and blowdown valves. No shut-off valves should be installed between the compressor discharge and the receiver. Each compressor should have its own receiver. Sectionalization of the system, for maintenance and operation, should be effected by valving beyond the receiver. An aftercooler will greatly assist in bringing the air back to the compressor supply temperature, thus assuring removal of moisture at the compressing plant and reducing the hazards of deposition and freezing in the transmission and service piping. Aftercoolers and intercoolers will cool the air within 10 or 20 F of the available water supply temperature; water velocities are of the order of 5 to 10 ft per sec; heat transmission rates range from 5 to 20 Btu per hr per sq ft per °F mean temperature difference; pressure drops, on the air side, seldom exceed 2 to 5 psi; and on the water side they are 5 to 15 psi. Reasonable cooling water recommendations are contained in Table 5.

TRANSMISSION OF AIR. The piping system which transmits the air from the compressor to the point of use must be designed to comply with the standards for pressure piping (see Section 6). It should be sized to give the best economic balance between investment charges and loss in available energy due to friction. Pressure drop can be evaluated by the detail methods of p. 1-22. It should be borne in mind that compressed air energy is expensive so that allowable velocities do not ordinarily exceed 1500 or 2000 ft per min. Table 1, p. 1-25, gives pressure drops per 1000 ft of straight pipe and permits ready estimation of pipe sizes. Blowdown connections and traps should be installed at low points and adjacent to connected tools and equipment for the convenient removal of water, lubricant, and accumulated dirt. Connections to tools should be from the top of

Table 5. Cooling Water Requirements Recommended for Intercoolers, Cylinder Jackets, and Aftercoolers

Gallons of Water per 100 cu ft per min of free air

(Reprinted from *Compressed Air Handbook*, Copyright 1947, by Compressed Air and Gas Institute, New York)

Intercooler, separate	2.5 to 2.
Intercooler and jacket, in series	2.5 to 2.
Aftercoolers	
80 to 100 psi, two stage	1.25
80 to 100 psi, single stage	1.8
Two-stage jackets alone (both)	0.8
Single-stage jackets	
40 psi	0.6
60 psi	0.8
80 psi	1.1
100 psi	1.3

the line to avoid carryover of moisture. Joints may be screwed, flanged, or welded. Hose lines, armored and unarmored, serve as the standard connecting system between the distribution piping and the pneumatic tool. Compressed air hosing sizes and pressure drops are shown in Table 6.

Table 6. Friction of Compressed Air in Hose Lines

Correct for hose with smooth inside lining. Rough inside lining may cause 50% greater loss than the figures given

(After Peele)

Size of Hose	Gage Press at Pipe Line	Cu Ft Free Air per min													
		20	30	40	50	60	70	80	90	100	110	120	130	140	150
		Loss of pressure, psi, 50-ft Hose Length *													
1/2 in.	50	1.8	5.0	10.1	18.1										
with	60	1.3	4.0	8.4	14.8	23.4									
coup-	70	1.0	3.4	7.0	12.4	20 0	28.4								
lings	80	0.9	2.8	6.0	10.8	17.4	25.2	34.6							
at	90	0.8	2.4	5.4	9.5	14.8	22.0	30.5	41.0						
each	100	0.7	2.3	4.8	8.4	13.3	19.3	27.2	36.6						
end	110	0.6	2.0	4.3	7.6	12.0	17.6	24.6	33.3	44.5					
3/4 in.	50	0.4	0.8	1.5	2.4	3.5	4.4	6.5	8.5	11.4	14.2				
with	60	0.3	0.6	1.2	1.9	2.8	3.8	5.2	6.8	8.6	11.2				
coup-	70	0.2	0.5	0.9	1.5	2.3	3.2	4.2	5.5	7.0	8 8	11.0			
lings	80	0.2	0 5	0.8	1.3	1.9	2.8	3.6	4.7	5.8	7.2	8.8	10.6		
at	90	0.2	0.4	0.7	1.1	1.6	2.3	3.1	4 0	5.0	6.2	7.5	9.0		
each	100	0 2	0.4	0.6	1.0	1.4	2.0	2 7	3.5	4.4	5.4	6.6	7.9	9.4	11.1
end	110	0.1	0.3	0.5	0.9	1.3	1.8	2.4	3.1	3.9	4.9	5.9	7.1	8.4	9.9
1 in.	50	0.1	0.2	0.3	0.5	0.8	1.1	1.5	2.0	2.6	3.5	4.8	7.0		
with	60	0.1	0.2	0.3	0.4	0.6	0.8	1.2	1.5	2.0	2.6	3.3	4.2	5.5	7.2
coup-	70		0.1	0.2	0.4	0.5	0.7	1.0	1.3	1.6	2.0	2.5	3.1	3.8	4.7
lings	80		0.1	0.2	0.3	0.5	0.7	0.8	1.1	1.4	1.7	2.0	2.4	2.7	3.5
at	90		0.1	0.2	0 3	0.4	0.6	0.7	0.9	1.2	1.4	1.7	2.0	2.4	2.8
each	100		0.1	0.2	0.2	0.4	0.5	0.6	0.8	1.0	1.2	1.5	1.8	2.1	2.4
end	110		0.1	0.2	0.2	0.3	0.4	0.6	0.7	0.9	1.1	1.3	1.5	1.8	2.1
1 1/4 in.	50			0.1	0.2	0.2	0.3	0.4	0.5	0.7	1.1				
with	60				0.1	0.2	0.3	0.3	0.5	0.6	0.8	1.0	1.2	1.5	
coup-	70				0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.7	0.8	1.0	1.3
lings	80					0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0
at	90					0.1	0.2	0.2	0.3	0.3	0.4	0.5	0.6	0.7	0.8
each	100						0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.7
end	110						0.1	0.2	0.2	0.3	0.3	0.4	0.5	0.5	0.6

* For longer or shorter lengths, friction loss is proportional to the length.

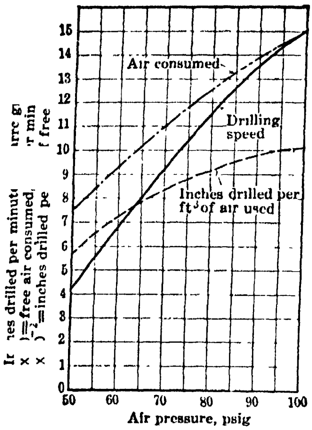


FIG. 34. Rock drill performance. (Drilling granite horizontally, wet operation, 1 5/8 in. diameter, 6 point bit, weight of drill = 150 lb.)

ROCK DRILLS AND PNEUMATIC TOOLS. Many pneumatic tools employ a reciprocating piston to deliver a rapid succession of hammer blows for operation of percussion rock drills, riveters, chippers, tampers, and impacters. In a second group rotary motion is delivered from a reciprocating engine or compressed air turbine for the operation of rotary drills, reamers, hoists, grinders, buffers, and polishers. These tools are essentially portable even though some, such as wagon drills, may be mounted on a large and heavy frame. The diversity of services from this group of tools is so great that their performance cannot be covered adequately in a handbook. To show some aspects of the great field of compressed air machinery, the data of Fig. 34 and Table 7 are included. Figure 34 covers the performance of a representative rock drill commonly employed in mining operations for drifting and on open-cut construction work or for sinking when mounted on a movable frame or wagon. Dimensions and air requirements on compressed air hoists and sinking pumps are contained in Tables 8 and 9.

Table 7. Design and Performance Data on Rock Drills

Carbon steel bits, lb	50 to 250
Weight of piston, oz	50 to 100
Cylinder bore, in.	2 to 3
Stroke of piston, in.	2 to 4
Drilling speed, in. per min in granite at 90 psi	10 to 16
Blows per min	1500 to 2500
Rotation ratio, blows per revolution	10 to 12
Air used, cu ft per min free air	50 to 200
Air pressure, psig	60 to 110

Table 8. Volume of Free Air Required for Operating Compressed Air Hoists

Air Pressure, 60 psi

(Courtesy of Ingersoll Rand Co., New York)

Diam of Cylinder,	Stroke, in	Rpm	Nominal Horsepower	Actual Horsepower	Wt. Lifted, Single Rope, lb	Cu Ft Free Air Required
SINGLE-CYLINDER HOISTING ENGINE						
5	6	200	3	5.9	600	75
5	8	160	4	6.3	1,000	80
6 1/4	8	160	6	9.9	1,500	125
7	10	125	10	12.1	2,000	151
8 1/4	10	125	15	16.8	3,000	170
8 1/2	12	110	20	18.9	5,000	238
10	12	110	25	26.2	6,000	330
DOUBLE-CYLINDER HOISTING ENGINE						
5	6	200	6	11.8	1,000	150
5	8	160	8	12.6	1,650	160
6 1/4	8	160	12	19.8	2,500	250
7	10	125	20	24.2	3,500	302
8 1/4	10	125	30	33.6	6,000	340
8 1/2	12	110	40	37.8	8,000	476
10	12	110	50	52.4	10,000	660
12 1/4	15	100	75	89.2	....	1125
14	18	90	100	125	....	1587

Table 9. Volume of Free Air Required for Operating Sinking Pumps

Sea Level

(Pumps working continuously at listed capacity, 10% allowed for slippage)

Size, in	Capacity, gal per min	Piston Speed, ft per min	Air Pressure at Pump, psi				
			50	60	70	80	100
6 x 3 x 7	27	75	71	81	93	104	126
6 x 6 and 4 x 7	48	67	63	73	83	92	112
8 x 4 x 12	83	128	213	246	278	312	378
10 x 5 x 13	130	128	333	384	435	486	590
12 x 5 x 13	130	128	480	554	628	702	850
12 x 7 x 13	206	103	385	444	503	563	682
14 x 7 x 13	206	103	525	605	686	768	930
18 x 9 x 16	247	75	635	732	830	930	1120
16 x 10 1/2 x 16	427	95	635	732	830	930	1120
18 x 10 1/2 x 16	427	95	800	922	1050	1170	1420

Multipliers for Altitude

Altitude, ft	1000	2000	3000	4000	5000	6000
Multiplier	1.04	1.08	1.12	1.165	1.21	1.255

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FANS AND BLOWERS

By T. A. Walters

(See also Axial-flow Compressors, p. 1-96.)

19. FAN TYPES

A fan is a machine for applying power to a gaseous fluid, to increase its energy content. This energy enables movement or flow of the gas against various degrees of resistance. The function of a fan is to move air or gases through distribution systems and apparatus required for conditioning of the gas medium, such as systems for heating, ventilating, and air conditioning of buildings; for drying and cooling of materials and products; for pneumatic conveying of materials; for dust collection, separation, and exhaust; for industrial process work; for mine and tunnel ventilation; for forced and induced draft of steam-generation plants.

The fan consists of a rotating member, called the wheel or impeller, and a stationary member called the housing. The housing is provided with an intake opening (inlet) and with a discharge opening (outlet). The flow of air or gas is caused by the pressure differential created by the energy transmitted to the gas by the rotating wheel. If no resistance to flow exists, as in the case of a fan in free space with no inlet and no outlet duct, the fan provides the gas with velocity energy only, and no compression or rarefaction occurs. When either inlet or outlet duct is added frictional resistance is imposed and partial compression occurs on the outlet side, whereas partial rarefaction occurs on the inlet side. The extent of the resistance imposed at the discharge governs the quantity of gas delivered by the fan. The greatest volume is delivered under zero resistance or "free delivery" conditions. As the resistance to flow is increased, the volume is decreased progressively until at infinite resistance the volumetric delivery is zero, corresponding to "blocked tight" or "static no delivery" condition.

Blowers are fans used to force air under pressure, that is, the resistance to gas flow is imposed primarily upon the discharge.

Exhausters are fans used to withdraw air under suction, that is, the resistance to gas flow is imposed primarily upon the inlet.

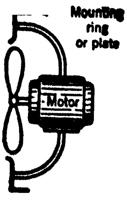
FAN CLASSES. The centrifugal fan has an impeller of the squirrel-cage type, operating within a scroll-type housing. Gas enters the inlet in a direction substantially parallel with the impeller axis and turns through an angle of approximately 90 degrees, flowing radially through the wheel and issuing from the wheel periphery. The scroll-type housing serves to convert part of the kinetic energy imparted to the gas into potential or static pressure energy.

The axial-flow fan has a wheel of the propeller or disk type, operating within a mounting ring or cylindrical housing. Gas flow is essentially parallel to the wheel axis at both inlet and outlet.

FAN STANDARDS. The principles and practices of fluid engineering applied by the fan and blower industry have undergone a steady refinement to the point of universal acceptance and standardization of fan types, designations, terminology, and engineering fundamentals. Portions of these standards, published in *Bulletin* 105 by the National Association of Fan Manufacturers, are here reproduced, by permission.

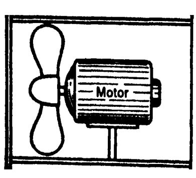
20. STANDARDS, DEFINITIONS, AND TERMS *

Table 1. Fan Types, Names, and Definitions



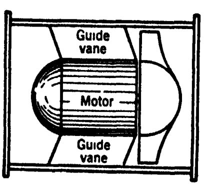
Propeller Fan

A propeller fan consists of a propeller or disk type wheel within a mounting ring or plate and including driving mechanism supports either for belt drive or direct connection.



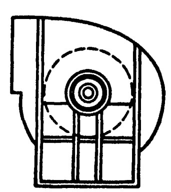
Tubeaxial Fan

A tubeaxial fan consists of a propeller or disk type wheel within a cylinder and including driving mechanism supports either for belt drive or direct connection.



Vaneaxial Fan

A vaneaxial fan consists of a disk type wheel within a cylinder, a set of air guide vanes located either before or after the wheel and including driving mechanism supports either for belt drive or direct connection.



Centrifugal Fan

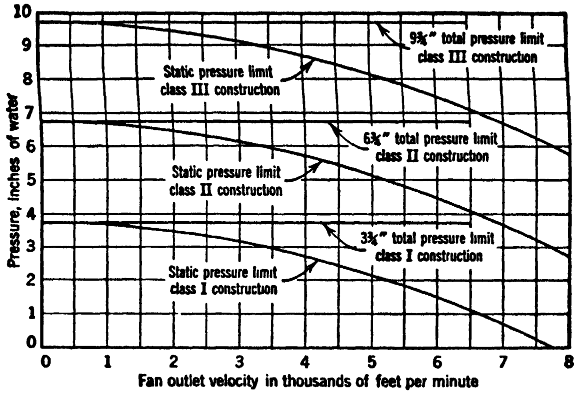
A centrifugal fan consists of a fan rotor or wheel within a scroll type of housing and including driving mechanism supports either for belt drive or direct connection.

* Published by The National Association of Fan Manufacturers. Detroit. Mich.

Operating Limits for Classes I, II, III, and IV Fans

A decade ago fan manufacturers designed a fan suitable for 5 to 6 in. static pressure and offered such a design in the heating and ventilating field as well as for industrial applications. During the interval there has been an increasing growth of various types of installations requiring fans designed more nearly for specific duties. In recognition of this situation the Association developed standards which would meet field conditions realistically. These standards given below in chart form include classes of fans designed for certain duties as follows:

Class I Fans—3 3/4 in. Maximum Total Pressure (Adopted Standards)
Class II Fans—6 3/4 in. Maximum Total Pressure (Adopted Standards)
Class III Fans—9 3/4 in. Maximum Total Pressure (Recommended Practice)
Class IV Fans—Greater than 9 3/4 in. Total Pressure (Recommended Practice)



Since fans are selected on the basis of outlet velocity and static pressure, by inspection of the chart it is possible to determine which class of fan is satisfactory for a specific duty. However, all values are based on air handled at 70 F and 29.92 in. barometer. For changes in temperatures and altitude a correction factor must be applied. These factors are covered in tabular form below. To apply them, divide the static pressure by the proper factor for temperature and altitude. The intersection of the resultant static pressure and outlet velocity on the chart will indicate the class of fan to be selected.

Table 2. Air Density Ratios at Various Altitudes and Air Temperatures

Unity basis = Standard air density of .075 lb per cu ft

(At sea level [29.92 in. Hg barometric pressure] this is equivalent to dry air at 70 F.)

Air Temperature	Altitude in Feet above Sea Level							
	0	2000	4000	6000	8000	10,000	15,000	20,000
	Barometric Pressure in Inches Hg							
	29.92	27.82	25.84	23.98	22.22	20.58	16.88	13.75
70°	1.000	.930	.864	.801	.743	.688	.564	.460
100°	.946	.880	.818	.758	.703	.651	.534	.435
150°	.869	.808	.751	.696	.646	.598	.490	.400
200°	.803	.747	.694	.643	.596	.552	.453	.369
250°	.747	.694	.645	.598	.555	.514	.421	.344
300°	.697	.648	.604	.558	.518	.480	.393	.321
400°	.616	.573	.532	.493	.458	.424	.347	.283
500°	.552	.513	.477	.442	.410	.380	.311	.254
600°	.500	.465	.432	.400	.372	.344	.282	.230
700°	.457	.425	.395	.366	.340	.315	.258	.210

Density directly proportional to Barometric Pressure established by the U. S. Standard Atmosphere-Altitude-Pressure relation. (Bureau of Standards Publication 82.) Density inversely proportional to absolute temperature.

Standard Sizes for Multiblade and Non-overloading Fans

The scope of standardization covers centrifugal fans designed for application in air conditioning, heating, ventilating, and general air-handling problems.

Standards are established for 25 sizes of fans by setting up a series of maximum wheel diameters and maximum outlet areas to correspond to these sizes. The series is based on geometric progression advocated by the American Standards Association.

Table 3. Outlet Areas for Wheel Diameters 12 in. to 132 in.

Index Number for Fans	Maximum Wheel Diameter		Maximum Outlet Area	
	Theoretical Diameter, in.	Practical Diameter,* in.	Single Width Single Inlet (square feet)	Double Width Double Inlet (square feet)
A	12.18	12 1/4	0.86	1.548
B	13.46	13 1/2	1.05	1.890
C	14.86	15	1.28	2.304
D	16.40	16 1/2	1.56	2.808
E	18.10	18 1/4	1.90	3.420
F	20.00	20	2.32	4.176
G	22.10	22 1/4	2.83	5.094
H	24.42	24 1/2	3.46	6.228
I	26.99	27	4.22	7.596
J	29.80	30	5.15	9.270
K	32.90	33	6.28	11.304
L	36.33	36 1/2	7.66	13.788
M	40.17	40 1/4	9.35	16 830
N	44.38	44 1/2	11.41	20.538
O	48.99	49	13.91	25.038
P	54.10	54 1/4	16.98	30.564
Q	59.76	60	20.71	37.278
R	66.00	66	25.27	45.486
S	72.90	73	30.82	55.476
T	80.58	80 3/4	37.61	67.698
U	88 91	89	45.89	82.602
V	98.20	98 1/4	55 98	100 764
W	108 60	108 3/4	68.30	122.940
X	119.80	120	83.33	149.994
Y	132 40	132 1/2	101 66	182.988

* For any index line the practical wheel diameters shall not be exceeded under these standards.

Methods of Testing Fans

For a complete description of the methods of testing fans for performance, the reader is referred to *Bulletin* 103 covering the Standard Test Code for Centrifugal and Axial Fans, Third Edition. The Code was first issued in 1923 by a joint committee of the American Society of Heating and Ventilating Engineers and the Engineering Committee of the National Association of Fan Manufacturers, with cooperation from Committee 10 of the Power Test Code Committee of the American Society of Mechanical Engineers. The Third Edition is issued under the same authorities.

The Standard Test Code is recognized and accepted not only by the Fan and Blower Industry, but also by architects, engineers, colleges of engineering, U. S. Governmental Bureaus, and industrial organizations.

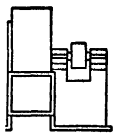
In recognition of the public demand for lower sound levels in the operation of machinery, a code for conducting and reporting sound measurement of fans was developed in 1942. The subject is covered in *Bulletin* 104, Sound Measurement Test Code for Centrifugal and Axial Fans, First Edition. It is issued by the Engineering Committee of the National Association of Fan Manufacturers and is based on definitions, standards, and specifications published by the American Standards Association as well as supplementary research by the U. S. Navy. The Code, therefore, provides a commonly used method of sound determination and gives designers and users a relative idea of sound conditions.

Fan Performance

Fan performance is a statement of volume, total pressures, static pressures, speed, power input, mechanical and static efficiency, at standard air density.

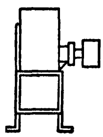
Fan performance curves are the graphical presentation of total pressure, static pressure, power input, mechanical and static efficiency as abscissa for the desired range of volumes all at constant speed and standard air density.

Table 4. Arrangements of Drive



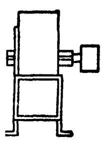
Arrangement 1.

For belt drive. Wheel overhung. Bearings on pedestal.



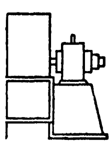
Arrangement 2.

For belt drive. Pulley and wheel overhung. Bearings in bracket on fan housing. Made only in smaller sizes for reversible discharge.



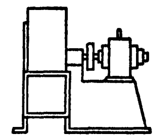
Arrangement 3.

For belt drive. Pulley overhung. Bearings supported on fan housing.



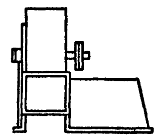
Arrangement 4.

For direct drive. Wheel overhung. No bearings on fan. Wheel mounted on motor or engine shaft. Pedestal for motor or engine.



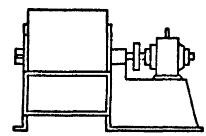
Arrangement 5.

For direct drive. Wheel overhung Includes housing, wheel, shaft, one intermediate bearing, flanged coupling and pedestal only for motor or engine.



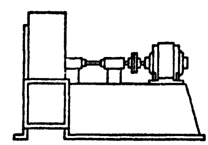
Arrangement 6.

For direct drive. Three-bearing arrangement with fan bearing at inlet side. Includes housing, wheel, shaft, one bearing (in inlet), rigid coupling, and pedestal only for motor or engine.



Arrangement 7.

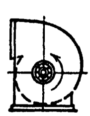
For direct drive. Similar to arrangement 6, but with two bearings on fan, and flexible instead of rigid coupling.



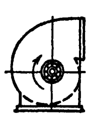
Arrangement 8.

Similar to arrangement 5, but with two bearings on pedestal with motor, and flexible instead of rigid coupling.

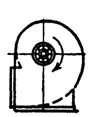
Table 5. Designation of Direction of Rotation and Discharge



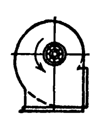
Counterclockwise Top Horizontal



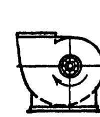
Clockwise Top Horizontal



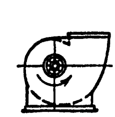
Clockwise Bottom Horizontal



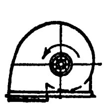
Counterclockwise Bottom Horizontal



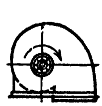
Clockwise Up Blast



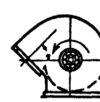
Counterclockwise Up Blast



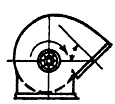
Counterclockwise Down Blast



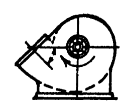
Clockwise Down Blast



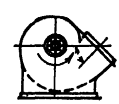
Counterclockwise Top Angular Down



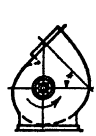
Clockwise Top Angular Down



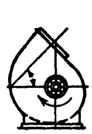
Clockwise Bottom Angular Up



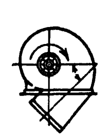
Counterclockwise Bottom Angular Up



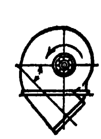
Counterclockwise Top Angular Up



Clockwise Top Angular Up



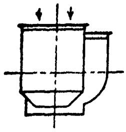
Clockwise Bottom Angular Down



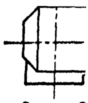
Counterclockwise Bottom Angular Down

Direction of rotation is determined from the drive side for either single or double width or single or double inlet fans. (The driving side of a single inlet fan is considered to be the side opposite the inlet regardless of the actual location of the drive.)

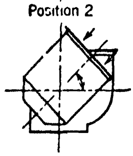
Table 6. Designation of Position of Inlet Boxes



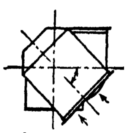
Position 1



Position 2



Position 3



Position 3*

Definitions

Reference line is a horizontal line through center of fan shaft.

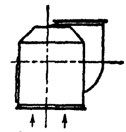
Air entry to inlet box is determined from drive side of fan.

On single-inlet fans the drive side is always considered as the side opposite the fan inlet.

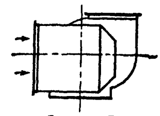
When drives are on both ends of fan shaft, the drive side is that side having the higher horsepower driving unit and is the same side from which the fan rotation is designated.

Air Entry Position Designation

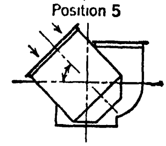
1. Top intake.
2. Horizontal right intake.
3. (Number of degrees) Above or below horizontal center line on the right.
4. Bottom intake.
5. Horizontal left intake.
6. (Number of degrees) Above or below horizontal center line on the left.



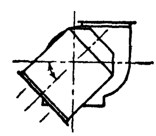
Position 4



Position 5



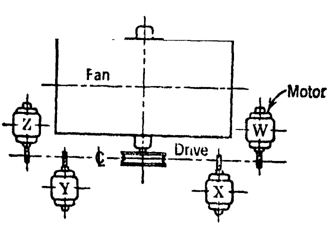
Position 6



Position 6*

* It will be found in some cases that this arrangement interferes seriously with the framing of the floor structure by the amount of floor space required.

Table 7. Motor Position, Belt, or Chain Drive



Location of motor is determined by facing the drive side of fan or blower and designating the motor position by letters W, X, Y or Z as the case may be.

Definitions

The volume handled by a fan is the number of cubic feet of air per minute expressed at fan outlet conditions.

The total pressure of a fan is the rise of pressure from fan inlet to fan outlet.

The velocity pressure of a fan is the pressure corresponding to the average velocity determination from the volume of air flow at the fan outlet area.

The static pressure of a fan is the total pressure diminished by the fan velocity pressure.

The power output of a fan is expressed in horsepower and is based on fan volume and the fan total pressure.

The power input to a fan is expressed in horsepower and is measured horsepower delivered to the fan shaft.

The mechanical efficiency of a fan is the ratio of power output to power input.

The static efficiency of a fan is the mechanical efficiency multiplied by the ratio of static pressure to the total pressure.

The fan outlet area is the inside area of the fan outlet.

The fan inlet area is the inside area of the inlet collar.

Abbreviations

Volume of air per minute	cfm	Power output (air horsepower)	ahp
Total pressure	TP	Power input (brake horsepower)	bhp
Velocity pressure	VP	Mechanical efficiency	ME
Static Pressure	SP	Static efficiency	SE
Speed	rpm		

Typical Specifications

The specifications following herewith are offered to architects and engineers in the hope that they will serve as a useful guide. They may be modified to meet the requirements for a specific installation.

General. The types of fans covered by the specifications are classified as "Multiblade" and "Non-overloading," or those types of fans designed for application in heating, ventilating, and air-conditioning fields.

The size of fans shall conform to the standards established by the National Association of Fan Manufacturers. The minimum gages for fan housings, i.e., sides and scroll, shall not be less than those recommended by these same standards.

Fan performance shall be based on laboratory tests conducted in accordance with the Standard Test Code for Centrifugal and Axial Fans, Third Edition, approved jointly by the American Society of Heating and Ventilating Engineers and the National Association of Fan Manufacturers.

Fan. Furnish ... . No. .... (give type) fan, single (or double) width, clockwise (or counterclockwise) rotation, ... discharge, and full (or 7/8) housing. The fan shall have a capacity of . . . . cubic feet of air per minute against a resistance of . . .. . inches static pressure with an outlet velocity of .... feet per minute. A tolerance of 2 1/2% shall be permitted for outlet velocity.

Standard Air, and Flue Gas Densities

Standard air density is 0.075 lb per cu ft at sea level (29.92 in. barometric pressure), dry air, and 70 F. Standard flue gas density is 0.078 lb per cu ft at sea level (29.92 in. barometric pressure) and 70 F.

Abrasion

Experience having shown on the basis of present-day knowledge of the art that erosion, corrosion, or results of excessive heat are speculative assumptions of maintenance, it is the policy of the Association to advise buyers of fan equipment that guarantees covering such conditions cannot be made in good faith.

Field Test of Fans

Research undertaken by the Members of the Association in cooperation with Test Code Committees of the Engineering Societies reveals thus far that no accurate or practical method of testing fans in the field has been developed by means of gas analysis or by use of the Pitot tube.

Since the values obtained from field tests vary so widely from actual results, guarantees of the performance of fans can be made only from laboratory tests conducted by the manufacturer in accordance with the Standard Test Code for Centrifugal and Axial Fans, approved jointly by the National Association of Fan Manufacturers and the American Society of Heating and Ventilating Engineers, with cooperation from Committee 10 of the Power Test Code Committee of the American Society of Mechanical Engineers.

In accordance with Paragraph 17, *Bulletin* 103 of the Standard Test Code for Centrifugal and Axial Fans, the performance of fans with wheels larger than 35 in. in diameter may be calculated from tests made on fans of the same design and similar proportions having a wheel not less than 35 in. in diameter. For fans having wheels less than 35 in. in diameter the performance may be calculated from tests made on fans of the same design and similar proportions and having a wheel diameter not greater than the rated size. The manufacturer reserves the right in all cases to test proportional fans as outlined in this paragraph.

21. FAN CHARACTERISTICS AND LAWS

FAN CHARACTERISTICS is the term for the variation in fan capacity or volume, pressure, power requirement, and fan efficiency, with degree of restriction or resistance to gas flow, at constant fan speed. *The characteristics of a fan are inherent in the type and design of the individual fan.*

Primary Characteristics. Five primary characteristic values are essential to describe completely the operation of any fan:

(1) Rate of volumetric flow	cfm
(2) Pressure differential maintained	TP, SP, VP
(3) Operating speed	rpm
(4) Driving power	bhp
(5) Gas density	lb per cu ft

A statement of fan performance is incomplete without inclusion of all five primary characteristics. Additional secondary characteristic values may be derived from the five fundamentals and are of value. The five basic characteristic functions are established by rating tests under standardized procedure; all other secondary characteristics are subsequently derived for convenience in selection and application of the fan.

Secondary characteristics include functions such as

(1)	Static efficiency	SE
(2)	Mechanical efficiency	ME
(3)	% Volume	% cfm
(4)	% Pressure	% SP
(5)	% Power	% bhp
(6)	Ratio of velocity to total pressure	VP ÷ TP
(7)	Ratio of velocity to static pressure	VP ÷ SP
(8)	Total pressure ratio	TP ÷ PVP
(9)	Velocity pressure ratio	VP ÷ PVP
(10)	Static pressure ratio	SP ÷ PVP
(11)	Peripheral velocity	PV
(12)	Outlet velocity	OV

PVP denotes peripheral velocity pressure, the velocity pressure ($V^2/2g$) of a gas stream when flowing at the rate of the peripheral velocity or tip speed of the fan wheel. Ratio of fan pressures to peripheral velocity pressure provides a convenient measure of the pressure-generating ability of the fan in relation to required operating speed.

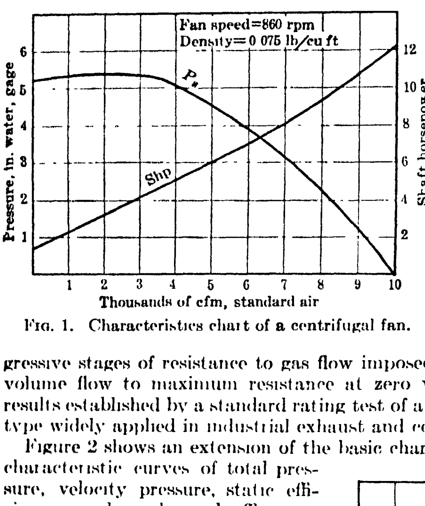


FIG. 1. Characteristics chart of a centrifugal fan.

Characteristic Curve Form. The conventional fan chart plots primary characteristics of pressure and horsepower input against volume for a fan of given size operating at fixed speed and handling gas or air of constant density. Figure 1 shows a typical characteristic or performance chart of a conventional centrifugal fan. It illustrates the basic form of static pressure and power input curves of a given fan, when operating at fixed speed and gas density, with progressive stages of resistance to gas flow imposed, from minimum resistance at maximum volume flow to maximum resistance at zero volume flow. The values are typical of results established by a standard rating test of a fan of the simple radial-bladed centrifugal type widely applied in industrial exhaust and conveying systems.

Figure 2 shows an extension of the basic characteristics of Fig. 1, to include additional characteristic curves of total pressure, velocity pressure, static efficiency, and mechanical efficiency. This type of chart is called a compound characteristic chart, combining the basic characteristics with the various secondary characteristics.

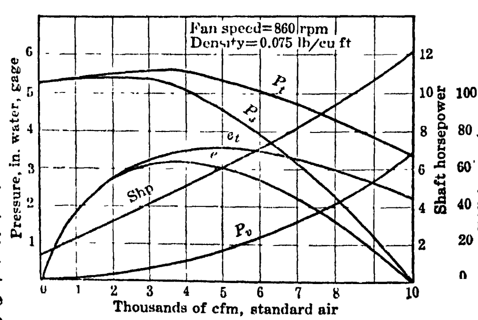


FIG. 2. Compound characteristics chart.

Fan characteristics may further be resolved by plotting fan volume, pressure, and power in percentage of maximum value. Figure 3 illustrates such nondimensional plotting for the fan shown by Figs. 1 and 2. Here the volume function is expressed as % cfm, in relation to the free delivery capacity, or the volume at zero static pressure. The maximum horsepower input to the fan shaft occurs at free delivery or 100% cfm for this type of fan. Such charts represent inherent characteristics common to fans of geometrically similar design. Curves of this type, established by rating test of a model fan, may be used to determine the performance of other fans of geometrically similar design.

FAN LAWS, which later will be discussed in detail, are the principles that govern the change in fan characteristic functions which occurs when (1) the fan speed is varied, but the external system to which the fan is connected remains unchanged in every respect;

(2) the fan size is changed in geometric proportion and the "point of rating" is maintained constant; (3) the gas density is changed. *Fan laws apply to all fans and are independent of the type and design of fan.*

A statement of fan performance specifies the characteristic functions for a specific condition or point of operation, including fan size, operating speed, volumetric delivery, pressure, power input, and density of gas. The fan laws apply for changes in fan speed, fan size, or gas density, singly or in combination, at a fixed point of rating, where the ratio of basic characteristic functions undergoes no change. The fan laws make possible calculations of the change in volume, pressure and power input with change in fan speed and in fan sizes of identical design. At a fixed "point of rating," "ratio of opening," or "point of operation" the fundamental functions of efficiency, proportionate volume, proportionate power, and pressure ratios are constant regardless of fan speed, size, or gas density. Thus the characteristic curves of a fan of a given series of geometrically similar design may be applied for any fan of the design throughout a wide range of application. The concept of constant fundamental characteristic functions of the fan is valid throughout normal fan application ranges. Modification in some degree becomes necessary when the principle of identical characteristics is extended into a range of compression ratios higher than encountered in conventional fan practice, or where wide range in size variation introduces "size effect." *Size effect* is descriptive of the tendency for small fans to exhibit lower efficiency and lower pressure characteristics than larger fans of geometrically similar design. Development tests establish that sizes larger than 33 to 36 in. in wheel diameter yield identical characteristics. Rating performance for any larger size may be derived, therefore, from the 33 to 36 in. diameter model tests. Sizes smaller than approximately 33 in. diameter exhibit a decrease in efficiency and developed pressure ratio to an extent which depends upon the size variation and design of fan; reliable ratings require additional rating tests to establish the extent of size effect.

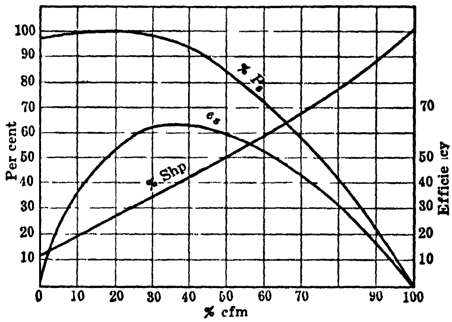


FIG. 3. Proportionate characteristic chart.

BASIC LAWS. Three basic fan laws encompass all fan functional principles. Numerous corollary laws may be formulated for specific conditions, but all originate from the basic three. (See also Pumps, Section 5.)

1. Fan Speed Variations (Constant Fan Size, Constant System, Constant Density).

(1) Capacity or cfm varies as fan speed.
(2) Pressure varies as square of fan speed.
(3) Power varies as cube of fan speed.

2. Fan Size Variation (Geometrically Similar Fans, Fixed Point of Rating, Constant Density).

(1) Capacity varies as square of wheel diameter.
(2) Pressure remains constant.
(3) Power varies as square of wheel diameter.
(4) Rpm varies inversely as wheel diameter.
(5) Tip speed remains constant.

3. Gas Density Variation (Constant Fan Size and Speed, Constant System or Point of Rating).

(1) Capacity remains constant.
(2) Pressure varies directly as gas density.
(3) Power varies directly as gas density.

COROLLARY LAWS. Useful corollary laws contributing to fan applications include

Fan Size Variation at Constant Rpm (Fixed Point of Rating, Constant Density).

(1) Capacity varies as cube of wheel diameter.
(2) Pressure varies as square of wheel diameter.
(3) Power varies as fifth power of wheel diameter.

Fan Size Variation at Varying Rpm (Fixed Point of Rating, Constant Density).

(1) Capacity varies as $(\text{diameter})^3 \times$ rpm ratio.
(2) Pressure varies as $(\text{diameter})^2 \times (\text{rpm ratio})^2$.
(3) Power varies as $(\text{diameter})^5 \times (\text{rpm ratio})^3$.

Gas Density Variation (to Maintain Constant Pressure) (Fixed Point of Rating, Fixed Fan Size).

(1) Capacity varies inversely as square of density.
(2) Power varies inversely as square of density.
(3) Rpm varies inversely as square of density.

Gas Density Variation (to Maintain Constant Volume) (Fixed Point of Rating, Fixed Fan Size).

(1) Capacity remains constant.
(2) Pressure varies directly as density.
(3) Power varies directly as density.
(4) Rpm remains constant.

Gas Density Variation (to Maintain Constant Gas Weight Flow) (Fixed Point of Rating, Fixed Fan Size).

(1) Capacity varies inversely as density.
(2) Pressure varies inversely as density.
(3) Power varies inversely as density.
(4) Rpm varies inversely as square of density.

SYSTEM CHARACTERISTICS. The term *system* refers to the external physical installation to which the fan is connected. It is any combination of gas flow passages and apparatus which imposes resistance to flow of the gas. It usually is comprised of duct work and connections, heaters, air washers, filters, economizers, preheaters, furnaces, etc. The pressure required to force air or other gas through a system is dependent upon the rate of volumetric flow. The system resistance is expressed in terms of the design volume, cfm, and that static pressure, P_s, required to maintain flow at the design rate. The system characteristic is independent of the type, size, or design of any fan used to serve the system.

In the common fixed-resistance system, the required pressure rise above atmospheric varies as the square of the air flow. The system characteristic may be graphically represented on the same coordinates as the fan pressure characteristics. The curve is of simple parabolic form, frequently referred to as the "curve of the square" or the "system square curve." Where complex overall system characteristics exist, because of the necessity for maintaining fixed pressure drop across certain components of the system, resolution into the square parabolic characteristic is necessary for each operating setting, to establish the relationship with the fan characteristic.

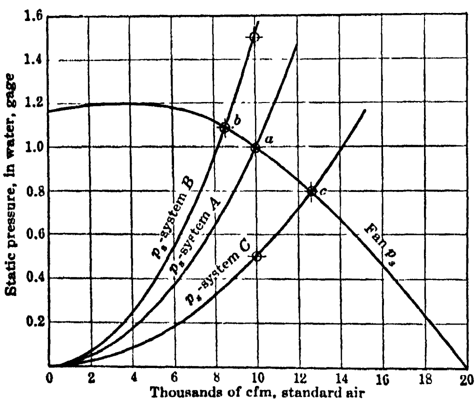


FIG. 4. System characteristic curves.

Correlation of system characteristics with fan characteristic pressure is accomplished by superimposing the system curve on the fan characteristic chart. The intersection of system characteristic curve and fan characteristic curve is the only point at which equilibrium exists. At this point the requirement of the system is satisfied by the available pressure from the fan, and continuous operation results. Figure 4 illustrates system characteristic curves, A, B, and C, which intersect the fan static pressure characteristic curve at points a, b, and c, respectively. The fan will operate to deliver 10,000 cfm at 1.00 in. p_s to system A, 8500 cfm at 1.08 in. p_s to system B, and 12,600 cfm at 0.8 in. p_s to system C. There can be no other point of operation so long as constant fan speed is maintained, and the system characteristic remains fixed. Figure 4 illustrates the effect upon volumetric delivery of error or variation in the system characteristic from the design condition. If the pressure of design system A (calculated to require 1.00 in. p_s for a flow of 10,000 cfm) were underestimated by $^1/_2$ in. water gage, the actual system would then have the characteristics of system B and would operate at point b with actual delivery of 8500 cfm, 15% under the design volume. If the pressure of system A were overestimated by $^1/_2$ in. water gage, the actual system would then correspond to system C and operation would occur at point c with a delivery of 12,600 cfm, 26% more than the design volume.

EXAMPLES OF FAN LAWS.

Effect of fan speed variation, utilizing Fan Law 1. Figure 5 illustrates the change in fan characteristics due solely to a change in speed. The basic fan speed is 860 rpm. Curves at 946 rpm show the effect of 10% increase in fan speed; curves at 774 rpm show the effect of 10% decrease in fan speed. The delivery volume varies in direct proportion to the speed, the developed pressure rise varies directly as the square of the speed, and power input to the fan varies as the cube of the speed. Application of these laws to several points of the basic curves at known fan speed establishes the performance at any other speed. The system characteristic variation of pressure with the square of the volume is identical with the relation of pressure to volume produced by the change in fan speed. The system characteristic curve intersects each fan pressure characteristic curve at a fixed *point of rating*, that is, at fixed proportionate volume, constant efficiency, and fixed pressure ratio. The fan is operating at 50% of free-delivery flow in the system shown, with a static efficiency of 59%; neither value is affected by the operating speed.

At 860 rpm: cfm = 5000

P_s = 4.5 in.

shp = 6.0

At 946 rpm: $\text{cfm} = 5000 \times \frac{946}{860} = 5500$

$$P_s = 4.5 \times \left(\frac{946}{860}\right)^2 = 5.45 \text{ in.}$$

$$\text{shp} = 6.0 \times \left(\frac{946}{860}\right)^3 = 7.97$$

At 774 rpm: $\text{cfm} = 5000 \times \frac{774}{860} = 4500$

$$P_s = 4.5 \times \left(\frac{774}{860}\right)^2 = 3.64 \text{ in.}$$

$$\text{shp} = 6.0 \times \left(\frac{774}{860}\right)^3 = 4.37$$

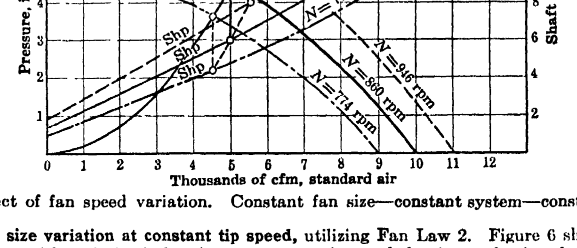


FIG. 5. Effect of fan speed variation. Constant fan size—constant system—constant density.

Effect of fan size variation at constant tip speed, utilizing Fan Law 2. Figure 6 shows change in fan characteristics with variation in fan size, at constant tip speed, density, and point of rating. Volume and shaft horsepower vary directly as the square of the wheel diameter, and the pressure remains constant. For fans operating at constant tip speed and at fixed point of rating, pressures remain fixed. Consequently, velocities through fan inlet, impeller, and fan outlet are identical for each size of fan. The effective area of each fan varies with the square of the wheel diameter; hence volumetric capacity, a function of velocity and area, is directly proportional to outlet area. Power input varies in proportion to delivered volume, since pressure and efficiency undergo no change. Figure 6 graphically illustrates the law for three sizes of fan, each operating at the same tip speed.

Fan *A* represents the basic size, with a relative wheel diameter of unity. Fan *B* has a wheel diameter 12 1/2% larger than fan *A*, and fan *C* has wheel diameter 12 1/2% smaller than fan *A*. At a selected point of rating, the change in volume takes place along the horizontal line traced upon the chart at 4 1/2 in. static pressure.

Fan A: cfm = 5000

P_s = 4.5 in.

shp = 6.0

Fan B: $\text{cfm} = 5000 \times \left(\frac{1.125}{1.000}\right)^2 = 6320$

$P_s = \text{constant} \qquad 4.5 \text{ in.}$

$\text{shp} = 6.0 \times \left(\frac{1.125}{1.000}\right)^2 = 7.6$

Fan C: $\text{cfm} = 5000 \times \left(\frac{0.875}{1.000}\right)^2 = 3830$

$P_s = \text{constant} = 4.5 \text{ in.}$

$\text{shp} = 6.0 \times \left(\frac{0.875}{1.000}\right)^2 = 4.6$

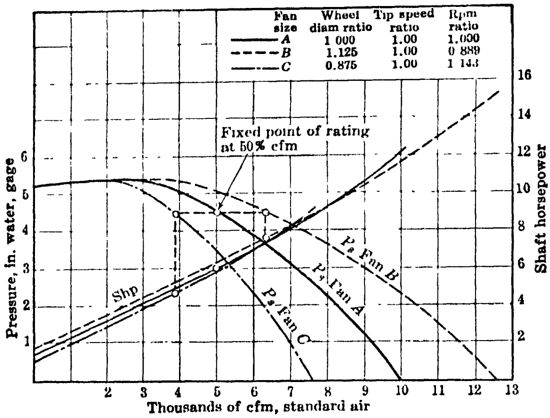


FIG. 6. Effect of fan size variation (geometrically similar fans). Fixed point of rating—constant density—constant tip speed.

Effect of gas density variation, utilizing Fan Law 3. Figure 7 is a plot of the characteristics of a fan operating in a system in which only the density of the air or gas changes. The fan is a constant-volume machine, maintaining constant gas velocity and constant volumetric flow through the fixed system. Both the pressure demand of the system and the fan developed pressure change in direct proportion to the density. The change in energy of the gas stream is reflected directly in the horsepower of the fan. In Fig 7, curves *A* show static pressure and shaft horsepower of a fan working with air or gas at standard density of 0.075 lb per cu ft. Curves *B* are characteristic curves of the same fan with gas density increased 25% to 0.0938 lb per cu ft. Curves *C* are for a 25% decrease in density to 0.0562 lb per cu ft.

At 0.075 lb per cu ft: $\text{cfm} = 5000$

$P_s = 4.5 \text{ in.}$

$\text{shp} = 6.0$

At 0.0938 lb per cu ft: $\text{cfm} = \text{constant} = 5000$

$P_s = 4.5 \times \frac{0.0938}{0.075} = 5.62 \text{ in.}$

$\text{shp} = 6.0 \times \frac{0.0938}{0.075} = 7.5$

At 0.0562 lb per cu ft: $\text{cfm} = \text{constant} \qquad 5000$

$P_s = 4.5 \times \frac{0.0562}{0.075} = 3.38 \text{ in.}$

$\text{shp} = 6.0 \times \frac{0.0562}{0.075}$

Gas density is inversely proportional to absolute temperature, directly proportional to barometric pressure, and proportional to the molecular weight of the gas.

Effect of fan size variation at constant rpm is illustrated by Fig. 8, showing the characteristics obtainable by changing fan size and maintaining rpm at a fixed value. This special condition is a combination of Laws 1 and 2. The end result can be obtained by applying these laws consecutively. Curves *B*

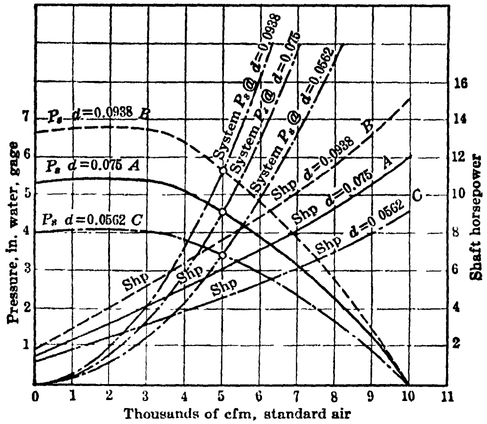


FIG. 7. Effect of gas density variation. Constant fan size and speed—constant system—or point of rating.

and curves C show the variation in characteristics for fans 12 1/2 % larger and 12 1/2 % smaller in wheel diameter than the basic fan, curves A. The trace line between points upon each pressure characteristic denotes the common "point of rating."

Fan A. cfm = 5000

P_s = 4.5 in.

shp = 6.0

Fan B:
$$\text{cfm} = 5000 \times \left(\frac{1.125}{1.000}\right)^3 = 7119$$

$$P_s = 4.5 \times \left(\frac{1.125}{1.000}\right)^2 = 5.70 \text{ in.}$$

$$\text{shp} = 6.0 \times \left(\frac{1.125}{1.000}\right)^5 = 10.8$$

Fan C:
$$\text{cfm} = 5000 \times \left(\frac{0.875}{1.000}\right)^3 = 3350$$

$$P_s = 4.5 \times \left(\frac{0.875}{1.000}\right)^2 = 3.45 \text{ in.}$$

$$\text{shp} = 6.0 \times \left(\frac{0.875}{1.000}\right)^5 = 3.08$$

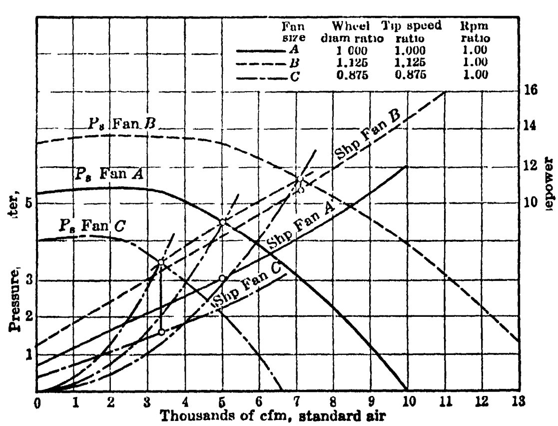


FIG. 8. Effect of fan size variation (geometrically similar fans). Fixed point of rating—constant density—constant rpm.

22. FAN TESTING

Testing of fans falls in one of these classifications: (1) research and development tests; (2) rating rests; (3) witness tests; (4) installation or field tests.

Research and development tests are (1) for the purpose of development of new equipment and new fields of application or (2) for improvement in existing equipment of standard design with respect to performance, suitability, and production economy. Such tests are conducted under controlled laboratory conditions in accordance with an accredited test code.

Rating tests are to establish data for fan capacity tables and characteristic charts, for the selection and application of production units; they are conducted under controlled laboratory conditions in strict conformance with the Standard Test Code. (See Article 23.)

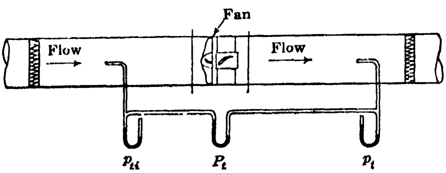


FIG. 9a. Fan pressures (universal). All tubes are facing impact tubes.

p_{ti} = inlet duct or region total pressure

p_{to} = outlet duct or region total pressure

P_t = fan total pressure = $p_{to} - p_{ti}$

P_v = fan velocity pressure = $\left(\frac{\text{outlet velocity}}{K}\right)$

P_s = fan static pressure = $P_t - P_v$

EXAMPLES:

p_{to}	p_{ti}	P_t	P_v	P_s
1″	−1″	2″	1″	1″
2″	−1″	3″	1″	2″
2″	−2″	4″	1″	3″
1″	0″	1″	1″	0″

Witness tests, for verification of specified performance, are formal laboratory tests under Standard Code provisions.

Installation or field tests are conducted at the site, with the fan installed and operating in conjunction with its system. Here little control can be exercised over conditions under which the fan operates insofar as adjustment of the operating point is concerned, since it is limited to that imposed by the characteristic of the system. The purposes of field tests are numerous. They may include verification of fan capacity, effect of inlet and outlet connections upon fan performance, verification of system characteristic or determination of its deviation from design, investigation of operation of component parts of system, establishment of data to predict corrective measures and modification of the system or adjustment in fan and driving equipment when required. In general, the results of field tests are approximate. The facilities available and conditions existing in most installations are not conducive to obtaining accurate and reliable test data, particularly volume and pressure determinations. Usually the fan is connected to the system in proximity to various elbows, transformations in duct section, or other variations in shape and size which eliminate the straight length of duct required to obtain

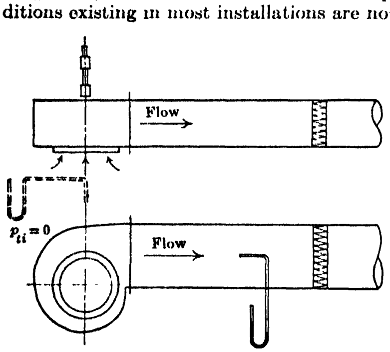


FIG. 9b. Fan pressures (with atmospheric inlet).

P_t = fan total pressure = $p_{to} - p_t$

P_v = fan velocity pressure = $\left(\frac{\text{outlet velocity}}{K}\right)^2$

P_s = fan static pressure = $P_t - P_v$

EXAMPLES:

p_{to}	p_{ti}	P_t	P_v	P_s
2″	0″	2″	1″	1″
4″	0″	4″	1″	3″
1″	0″	1″	1″	0″

p_{ti} $p_{to} = P_v$

FIG. 9c. Fan pressures (with atmospheric outlet).

P_t = fan total pressure = $p_{to} - p_{ti}$

P_v = fan velocity pressure

$= \left(\frac{\text{outlet velocity}}{K}\right)^2$

P_s = fan static pressure = $P_t - P_v$

EXAMPLES:

p_{to}	p_{ti}	P_t	P_v	P_s
1″	−1″	2″	1″	1″
1″	−3″	4″	1″	3″
1″	0″	1″	1″	0″

accurate pressure readings; when this is true, the accuracy of laboratory tests cannot be duplicated.

FAN TESTING is the procedure of subjecting the fan to certain prescribed conditions, then making precise measurement of the fan characteristics. The objective is to measure and establish volumetric delivery, pressure differential, and power requirements of the fan, when operating at controlled and measured operating speed and gas density.

Standard procedure involves a multi-point traverse of the fan inlet and outlet ducts with standard pitot tubes to measure pressure in the gas stream, with simultaneous measurement of fan speed, input, and gas density. The measured velocity pressure, converted to velocity of flow at the duct section area, is used to measure capacity or volumetric delivery. The elements of pressure determinations for any fan are shown by Fig. 9*a*. The following definitions of pressure are used:

Fan total pressure, P_t, is the rise of pressure from fan inlet to fan outlet measured by two *impact* tubes, one in the inlet duct and one in the outlet duct, corrected for friction losses not chargeable to the fan.

Fan velocity pressure, P_v, is the pressure corresponding to the average velocity at fan outlet area.

Fan static pressure, P_s, is the fan total pressure P_t minus the fan velocity pressure P_v.

Outlet duct total pressure, p_{to}, is the total pressure measured by an impact tube in the outlet duct.

Inlet duct total pressure, p_{ti}, is the total pressure measured by an impact tube in the fan inlet duct.

Fan total pressure, $P_t = p_{to} - p_{ti}$
Fan velocity pressure, P_v = velocity pressure at fan outlet
Fan static pressure, $P_s = P_t - P_v$

The foregoing relations completely define the pressure gradient credited to the fan. It credits the fan with energy required to maintain the pressure differential in the gas stream between fan inlet region and fan outlet region. No single pressure measurement is adequate to evaluate fan pressure, since fan pressure is a *differential* pressure, requiring measurement at two stations. When no inlet duct is used, the total pressure at fan inlet is zero, and the inlet total pressure reading may be omitted; the total pressure in the outlet duct is the fan total pressure. When no outlet duct is used, only the inlet total pressure need be measured, provided the fan is credited with the discharge total (velocity) pressure. Here the fan total pressure is the difference between outlet *velocity pressure* and inlet total pressure. See Fig. 9*b* for example of fan with atmospheric inlet pressure and Fig. 9*c* for fan with atmospheric outlet pressure.

23. FAN TEST CODES

Capacity Rating Tests. Formal rating tests for publication of fan capacities and performance, and witness and acceptance tests for verification of contractual specifications are conducted in conformance with accredited Standard Test Codes. The test codes embody specifications of a uniform method for conducting tests of fans and presentation of test results. Codes have been formulated by committees of leading engineering societies with personnel drawn from industry, technical societies, and educational institutions.

Two such codes are currently used in the United States, nationally recognized, and accepted by the Fan and Blower Industry, industrial organizations, technical societies, architects, engineers, colleges of engineering, and U. S. Government Bureaus.

ASHVE Code. The original Standard Test Code, issued in 1923, was prepared by a joint committee of the American Society of Heating and Ventilating Engineers and the National Association of Fan Manufacturers, in cooperation with PTC Committee 10 of ASME. It is generally referred to as the ASHVE Code or as the NAFM Code.

The first edition of the ASHVE and NAFM Code provided basic methods for testing and rating disk and propeller fans, centrifugal fans, and blowers. The second edition, issued in 1932, covered additional data relative to testing centrifugal fans with inlet boxes. The third edition, the currently governing code, issued in 1938, was extended to include all types of centrifugal and axial fans, clarification of definitions and nomenclature, additional procedure for specific types and arrangements of fans, addition of "egg crate" straighteners to test ducts, and modification of duct friction allowance.

ASME Code. The second code, entitled Test Code for Fans, was published in 1946 by ASME. The provisions of both the ASHVE and ASME codes for fans are identical in objective, fundamental theory, and instrumentation. Such differences as exist lie in terminology, nomenclature, and minor variations in procedure for test. The ASHVE Code provides extension of basic test methods to several types and arrangements of fans covered broadly but not specifically by the ASME Code. Conformance to either code results in identical performance.

Copies of Codes. The Standard Test Code for Centrifugal and Axial Fans, third edition, sponsored by the American Society of Heating and Ventilating Engineers and the National Association of Fan Manufacturers, may be obtained by writing to the National Association of Fan Manufacturers, 5-208 General Motors Building, Detroit, Michigan. It is given in *Bulletin* 103. This code, which contains much general information of value, should be in the library of any user of fans.

The ASME Test Code for Fans may be obtained by writing to the American Society of Mechanical Engineers, 29 West 39th St., New York, N. Y. It is of similar value and usefulness in fan work.

24. FAN NOISE

(For discussion of vibration and noise in general, see *Design and Production Volume*, Section 9, Kent's *Mechanical Engineers' Handbook*.)

SOUND MEASUREMENT TESTS. The need for sound control in the application of fans has become increasingly apparent, particularly in the heating, ventilating, and air-conditioning field. (See Section 12.) To a lesser degree the need exists in industrial applications. Recent developments in communications and acoustic industries have established sound-intensity measuring instruments and engineering principles which supply designers and fan users with additional guides for the selection and evaluation of equipment in keeping with acoustical requirements.

The technique, methods, and practices currently employed by the Fan and Blower Industry for the testing and reporting of fan sound levels are consolidated in The Sound Measurement Test Code for Centrifugal and Axial Fans, prepared by the National Association of Fan Manufacturers. This code, *Bulletin* 104, may be obtained by writing to the Association at the address given at the end of Art. 23, above.

25. CENTRIFUGAL FANS

Centrifugal fan types differ fundamentally by variation of blade inclination. These differences are reflected in distinctive characteristics inherent in each type. The three basic types are commonly designated (1) *forward curved blade fans*, (2) *radial blade fans*, (3) *backward curved blade fans*. (See Fig. 10.)

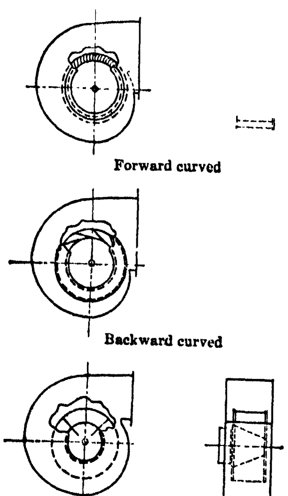


Fig. 10. Centrifugal fans—basic types.

Forward curved blade fans have blades inclined in the direction of wheel rotation. Number of blades varies from 24 to 64, depending upon size of fan and design. Wheels of 30-in. diameter and larger usually have the greater number of blades. Smaller wheels have progressively fewer blades to a minimum of about 24. Commercial sizes are available in wheel diameters of 3 to 132 in. This fan is frequently designated as the *multiblade fan*.

Radial blade fans have blades which are radial elements of the wheel. Many variations exist in the actual shape of blades of this type, such as radial tip with forward inclined entry portion, backward inclined tip with radial entry portion, and other combinations. Number of blades varies from 4 to 24; common usage is 6 to 16 blades. Commercial sizes are available in wheel diameters of 6 to 100 in.

Backward curved blade fans have blades inclined away from the direction of rotation. Number of blades varies from 8 to 16. Commercial sizes are available in wheel diameters of 12 to 132 in.

COMPARISON OF OPERATING CHARACTERISTICS. Volumetric capacity, pressure generation, power requirement, efficiency, and operating speed are primarily affected by the type of blading and secondarily by the proportions of associated fan housing, inlet, outlet, and structural members. Mechanical efficiencies over 80% have been attained for all three types. Commercial units with static efficiencies of 67 to 77% are readily available.

Pressure Developed. The most apparent difference between the three types lies in the pressure developed at a given tip speed. At a tip speed of 4000 ft per min, typical static pressures, in inches water, gage, will be: forward curved, 2.3 in.; radial bladed, 1.3 in.; backward curved, 0.75 in. Hence relative tip speed and rpm required for a given pressure are widely different for the three types. To maintain a static pressure of 1 in. water, gage, at standard air density of 0.075 lb per cu ft, the required tip speed will be: forward curved, $\sqrt{1/2.3} \times 4000 = 2640$ ft per min; radial, $\sqrt{1/1.3} \times 4000 = 3500$ ft per min; backward curved, $\sqrt{1/0.75} \times 4000 = 4620$ ft per min. These values indicate the relative differences in tip speeds of the three blade types. The radial-bladed type requires about 35% higher tip speed and the backward curved type about 75% higher tip speed than the forward curved type.

Speed Classification. This distinction leads to a further general classification of forward curved fans as the *low-speed* type, radial fans as the *medium-speed* type, and backward curved as the *high-speed* type. The volumetric capacity, that is, the relative delivery at a fixed static pressure of fans of the same wheel diameter, is highest with the forward curved type, intermediate with backward curved fans, and lowest with radial-bladed fans. Hence for a given duty of fixed volume and pressure, the forward curved type will have the smallest wheel diameter, the backward curved will be intermediate in wheel diameter, and the radial types will require the largest wheel diameter. In most radial fans, the necessary increase in wheel diameter requires that its rotative speed be lower than that of the forward curved type, despite the higher tip speed of the radial type. The volumetric capacity of the backward curve or high-speed type has been developed to approach the volumetric capacity of the forward curved type, and only small differences exist in the wheel diameters of the two types for a given duty. The higher tip speed of the backward curved fan makes it the highest rotative speed type. Conventional designs of the three basic types possess size and speed characteristics summarized as follows: forward curved, minimum size, lowest tip speed, medium rotative speed; radial-bladed, maximum size, medium tip speed, minimum rotative speed; backward curved, medium to minimum size, highest tip speed, highest rotative speed.

The pressure that can be produced by a fan is limited only by mechanical strength of the impeller in its ability to operate continuously and safely at high speeds. Conventional riveted and welded construction, utilizing standard materials, permits operation at tip speeds in the order of 20,000 ft per min. Special materials and design refinements extend the maximum operating tip speed to about 30,000 ft per min. At the normal limit of 20,000 ft per min tip speed, pressures that may be attained are: forward curved, 58 in. water, gage, = 2.1 psi; radial, 33 in. = 1.2 psi; backward curved, 20 in. = 0.72 psi. At the upper limit of 30,000 ft per min tip speed, the pressures that may be produced are: forward curved, 130 in. water, gage, = 4.7 psi; radial, 73 in. = 2.64 psi; backward curved, 45 in. = 1.62 psi.

FAN WHEELS. Centrifugal wheels consist of a hub and driving disk, called the *backplate* on single-inlet fans, or the *centerplate* on double-inlet fans; the blades, attached to the backplate or centerplate; and a blade rim or *shroud* member to encase the inlet side of the blades. Some designs utilize a driving spider member, with each blade mounted to an individual driving arm. The blade end shrouds or rims include simple radial disks, tapered or conical rings, and die-formed faired members of precise contour. Blade shapes vary from simple flat, rectangular, or trapezoidal plates to complex formed members of curved and warped surfaces. The preferred types utilize die-formed blade elements to obtain duplication in production and uniformity of operation in application.

Proportions of fan wheels vary widely, are established by extended development and research work. Fundamental factors include number of blades, type and shape of blading, internal or eye diameter of the wheel, and blast width of blades and wheel. Proportions in conventional design are listed in the following table; proportions are given as a ratio to tip diameter of the wheel. Proportions of blast width as listed apply to the maximum

Type Wheel	Forward Curved	Radial	Backward Curved
Number of blades	24–64	4–24	8–16
Internal eye diameter	0.87	0.50–0.75	0.65–0.80
Blast width	0.54	0.35–0.45	0.25–0.30

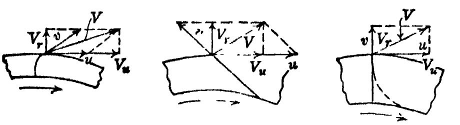
width found to be effective. Narrower wheels of partial width blades are effective in reducing volumetric capacity. The proportions shown are for single-inlet, single-width wheels. Double-inlet, double-width wheels are essentially two single-inlet, single-width wheels back to back, with blades fabricated integrally to one centerplate or driving spider.

FAN BLADE CHARACTERISTICS. The shape, contour, and angles of centrifugal fan blades exert the influences responsible for the distinctive characteristics of the various types of fan. Basic theory, embracing principles of fluid flow and vector analysis, has developed ideal equations which establish approximate quantitative design values. The ideal equations assume that the fluid flow is in conformance to the blade surfaces, and that the fluid is evenly distributed between the blades, flowing at equal velocity throughout a given section. Such ideal conditions are rarely approached in practice, and the theory must be modified by coefficients, derived from tests of similar fans.

The theoretical head available is the pressure increase transferred to the fluid by the impeller, consisting of the head created by centrifugal force of the rotating fluid, plus the velocity head conversion through the impeller, plus the increase in absolute velocity head between intake and exit blade sections.

$$H_t \quad \frac{(u_2 V_{u_2} - u_1 V_{u_1})}{g}$$

where H_t = ideal head in feet of fluid; u_2 = absolute velocity of impeller at exit; V_{u_2} = tangential component of fluid at exit; u_1 absolute velocity of impeller at intake; and V_{u_1} = tangential component of fluid at intake.



Forward curved   **Backward curved**   **Radial**
——Straight radial
---Radial tip

FIG. 11. Velocity diagrams for the three types of blade.

Velocity diagrams indicate the character of fluid flow at a selected blade section. Figure 11 shows the representation of velocity vectors for ideal conditions at the outlet from the basic types of blading. (See also Fig. 33, p. 1-53.) In the diagram, u represents absolute velocity of a point on the impeller, v is velocity of fluid relative to impeller and V represents absolute velocity of fluid. V_r is the radial component of absolute velocity V, and V_u is the tangential component of absolute velocity V.

FAN HOUSINGS encase the wheel and serve as directional collection space to conduct the gas from impeller to outlet. Sides of the housing or casing, called *side sheets*, are usually parallel plates separated by the transverse scroll sheet. The contour of the scroll may be of simple cylindrical or concentric shape, with tangential outlet or the more efficient spiral boundary. Actual practice modifies theoretical spiral contour in the interests of space saving and production economy. The scroll terminates at its inner extremity in a section designated as the *cutoff*, which diverts the flow from within the housing through the fan outlet to the connected system. The cutoff location varies, dependent upon the type of fan, and exerts an appreciable influence on fan capacity and efficiency. The nose of the cutoff is blunt or rounded to minimize noise generation. The radial distance between cutoff and fan wheel is seldom less than 7% of the wheel diameter, to avoid objectionable noise emission.

Dimensions of centrifugal fan housings have developed, through requirements for minimum space, to the point where only minor differences exist between available types. Overall dimensions of conventional models are shown in the following tabulation, given as a ratio to wheel diameter. Dimension h refers to height of top horizontal discharge fan illustrated in Fig. 10. Dimension L refers to length or dimension parallel to horizontal

	h	L	w
Forward curved	1.7 1.9	1.5–1.6	0.78–0.82
Radial	1.5-1.6	1.4–1.5	0.40-0.50
Backward curved	1.7 -2.0	1.5–1.7	0.78–0.82

centerline, whereas w indicates width of the housing. w as listed is for single-inlet, single-width fans. The width of double-inlet, double-width fans is generally 1.7 to 1.8 times the width of the corresponding single-inlet, single-width fans.

FAN INLETS are formed passages to permit entrance of the fluid to the fan impeller with minimum energy loss and turbulence. The preferable design is of the streamlined orifice shape which performs the dual function of reducing entry losses to a minimum and of guiding the fluid into the fan blades most rationally, reducing shock and eddy losses. The streamlined inlet is of particular merit where the inlet is open to atmosphere, or when not connected to a straight length of inlet duct that would eliminate the *vena contracta* and consequent entry losses. Wide fans of large volumetric capacity are materially benefited by a streamlined inlet which reduces the losses accompanying high inlet velocities.

Alignment and clearance of the inlet with respect to the fan wheel is of considerable importance in some types, minor in others. The backward curved blade type is most critical in this respect, requiring close radial clearance and concentric alignment between inlet lip and wheel rim. Proper axial alignment between inlet and wheel contributes to retention of the basic characteristics of the backward curved design. Radial-bladed fans are less critical, and conventional designs do not incorporate the close inlet-to-wheel clearances required by the backward curved type. Forward curved blade types are least critical, being relatively insensitive to moderate variations in clearance between inlet and wheel.

Inlet guide vanes are sometimes provided to guide the gas into the fan wheel with a spin or whirl in the direction of the wheel rotation. Such action modifies the fundamental characteristics of the fan, producing a reduction in capacity, pressure, and horsepower. The action produces a small increase in efficiency of some fans over a narrow range of initial spin intensity, due to improvement in entrance-to-blade flow pattern and reduction in initial shock losses. Higher angularity of vanes, giving greater intensity of spin, results in large reduction in capacity and appreciable reduction in overall fan efficiency. The major effect of fixed inlet vanes lies in reduction of fan capacity at a given fan speed or, conversely, increase in fan speed to attain a fixed duty requirement. The fixed vane is utilized as (1) an auxiliary device to obtain minor increase adjustment in operating speed of a standard design fan for a specific duty or (2) a standard design feature incorporated into a fan to obtain more desirable speed and power input characteristics than possessed by the fan without such vanes.

INLET CONNECTION. The design of the inlet connection between system and fan inlet can produce a marked effect, frequently detrimental, upon fan operation. Losses that arise from improper inlet connections and duct work cannot be evaluated as equivalent to the loss of an elbow or other variable gas passage. Such directional passages not only exert their inherent pressure losses but also create unbalanced, restricted, or whirling flow into the fan inlet, seriously reducing the basic pressure and efficiency. Precautions against losses of this nature are best taken in the original design and layout of the system, where some measure of control can be exercised. As a general guide, the inlet duct work and connection to fan should be designed to permit the gas flow to duplicate the action with the fan inlet open to atmosphere, or with a section of straight inlet duct. Unobstructed

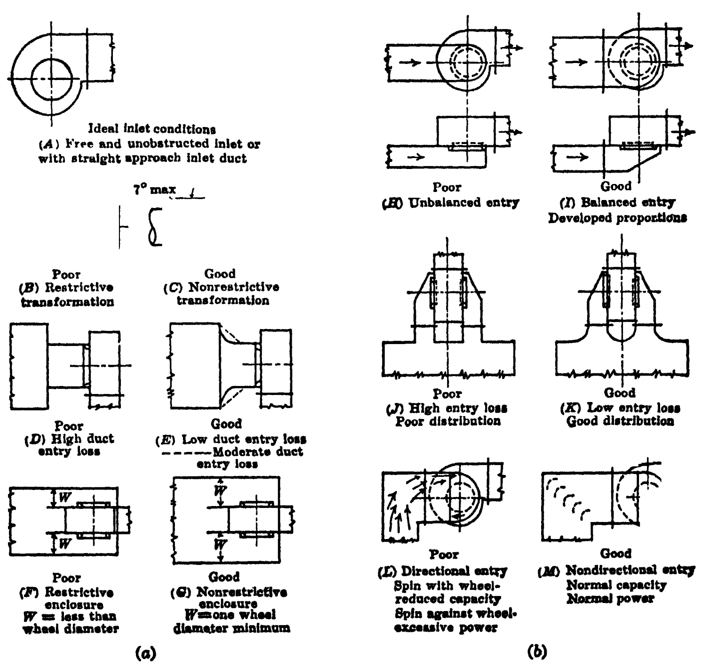


FIG. 12. Inlet connections.

inlets for blowers and straight inlet ducts for exhausters are the basis of Standard Code tests from which rating performance is established. When these normal inlet conditions cannot be substantially duplicated in application, allowances should be made in selection of the fan. Unusual conditions may warrant laboratory test to determine the extent of the connection effect upon fan performance.

Specific types of fan, used primarily for exhausting duty, such as induced draft units, have *inlet boxes* as a feature of the design. The inlet duct is connected to the inlet box in a manner to insure straight-line flow into the inlet box conforming to the axis of the box. Size, shape, and proportions of inlet boxes are established by laboratory development tests in conjunction with a particular type of fan. Properly developed, inlet boxes impose only slight penalty in fan performance. Rating information for fans with inlet boxes is customarily presented as the available capacity of fan with inlet boxes in place, tested in accordance with the Standard Test Code. Figure 12*a* and Fig. 12*b* show commonly encountered arrangements of inlet connections which adversely affect fan operation, together with corrective measures that eliminate or reduce the losses to a practical minimum.

FAN OUTLET. The outlet of the centrifugal fan is the termination of the scroll-shaped housing, that is, the point of connection of fan to external system. When a straight outlet duct is attached, the change from spiral flow in the housing is partially completed at the fan outlet, continued, and finally completed in the outlet duct. The pattern of gas flow at fan outlet is directional, influenced by the extent of the conversion and distributional action within the fan housing, and by the character of the discharge from periphery of impeller in the region adjacent to the fan outlet. The velocity of emission from fan outlet, commonly referred to as *outlet velocity*, varies appreciably in magnitude and direction, dependent upon the type and design of fan and the point of rating at which the fan is operated. Conservation of velocity energy at the fan outlet is of importance, since the velocity pressure ranges from 5 to 25% of the static pressure. When the fan outlet discharges directly to atmosphere or to a large plenum chamber, the complete velocity pressure is dissipated and wasted. Gradual reduction of velocity by properly proportioned expanding outlet connections reduces the final velocity pressure and converts an appreciable portion of the initial velocity pressure into useful static pressure. Such evasé outlet members can be used to advantage in many applications, providing reduction in operating power and speed of a given fan, or permitting reduction in size of fan for a given duty requirement and fixed operating speed. Reduction of operating power by as much as 10% and speed by as much as 5% is possible with properly proportioned outlet connections. The characteristic directional flow at the outlet requires that the proportions of the evasé outlet be established by laboratory test for each type of fan.

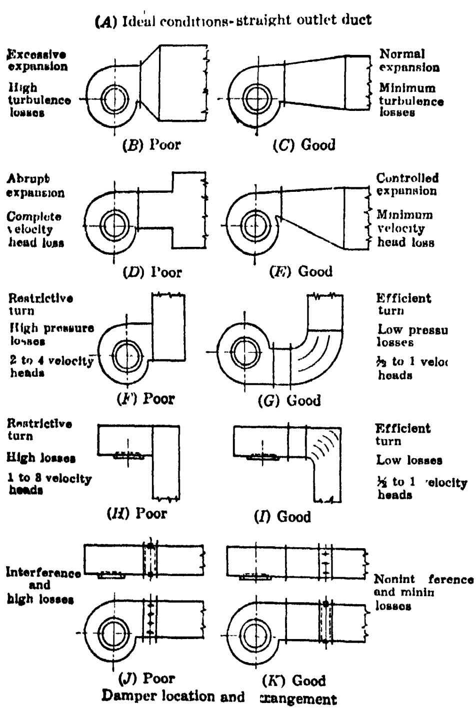


FIG. 13. Outlet connections.

The arrangement of the outlet connection can contribute to the elimination of losses, making more of the total input available to the system. Avoidance of abrupt or excessive expansion, restrictive sections, abrupt turns, or rapid change in direction is good practice. Figure 13 illustrates the more common precautions to be exercised in disposition of fan outlet connections.

CHARACTERISTICS OF CENTRIFUGAL FANS. Characteristics of volumetric capacity, pressure, driving power, and fan efficiency are determined by tests in accordance with the provisions of the Standard Test Codes. The quantities volume, pressure, power input, gas density, and operating speed are measured at various adjusted systems or points of rating from full-open or free delivery to full-closed or static no delivery. At the time of test, results are plotted in the general form of the basic characteristic chart of Fig. 1. Extension of the primary characteristics and conversion into percentages and fundamental pressure ratios permit the plotting of characteristics curves used to determine performance of fans of identical design under any circumstances of gas density or fan speed. Note that all pressures are rendered dimensionless by relating them to the peripheral velocity pressure, PVP, corresponding to tip velocity. Curves of this master rating type are shown in Figs. 14, 15, 16, and 17, which illustrate the effect of blade design on both trend of curves and developed pressure. Each chart is typical of fans of the particular blade shape classification shown. Individual design and construction introduce considerable variation in specific values of capacity, pressure, power input, and efficiency, demanding individual rating tests for selection and application.

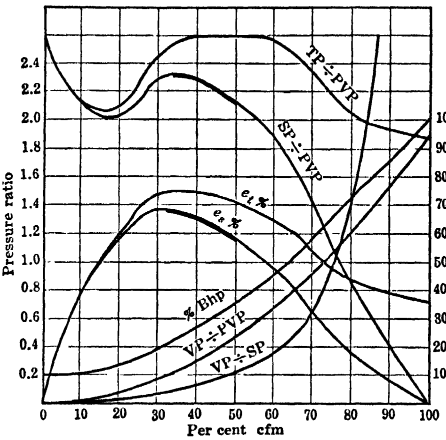


Fig. 14. Characteristic curves of forward-curved-blade fan.

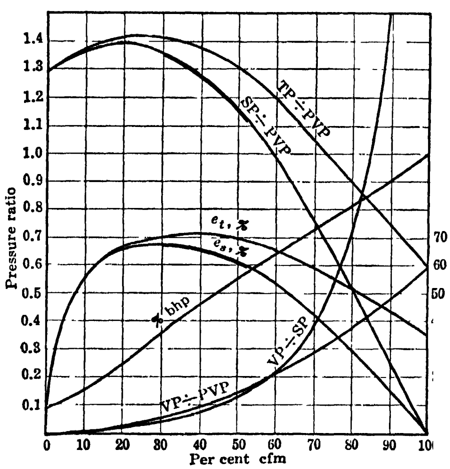


Fig. 15. Characteristic curves of radial-blade fan.

The relative distinguishing characteristics of the basic blade types are tabulated in Table 1.

For any given requirement of volume and pressure, a fan may be selected of a size to give operation at maximum efficiency and lowest sound emission. Selection of a larger size results in excessive operating and initial cost, with possibility of increased sound emission. Selection of a smaller size reduces initial cost at the expense of operating cost and increased sound emission.

Variation of requirements for a fan produces changes in the operation point, the effect of which is most clearly discernible by analysis of fan

Table 1. Basic Characteristics of Centrifugal Fans

Blade Type	Forward Curved	Radial	Backward Curved
Tip speed, for fixed pressure	Low	Medium	High
Pressure trend characteristics	Flat	Medium	Steep
Power trend characteristics	Rising	Rising	Nonoverloading limit load
Peak pressure ratio (SP ÷ PVP)	2.2 to 2.6	1.1 to 1.4	0.70 to 0.80
Tip speed for 1 in. P_s	2700 to 2500	3800 to 3400	4800 to 4500
Peak static efficiency, %	65 to 75	60 to 75	70 to 80
Wheel diameter	Minimum	Maximum	Intermediate
Space requirement	Minimum	Maximum	Intermediate
Operating rpm	Minimum to moderate	Moderate to minimum	Maximum

characteristics in relation to system characteristics. Figure 18 shows characteristic curves of a forward curved blade fan upon which are imposed system characteristic curves representing three different conditions of operation. Intersection of the system curve and the fan characteristic curve is the point of equilibrium, satisfying both system and fan pressure characteristics and resulting in steady-state flow.

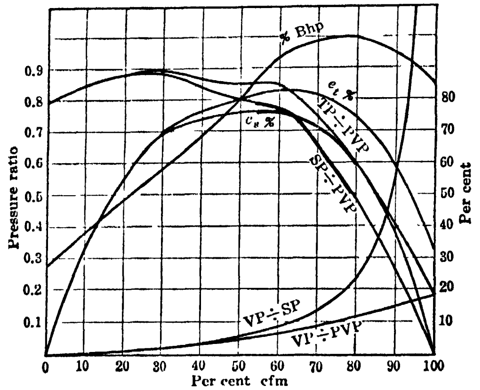


FIG. 16. Characteristic curves of full-backward-curved-blade fan.

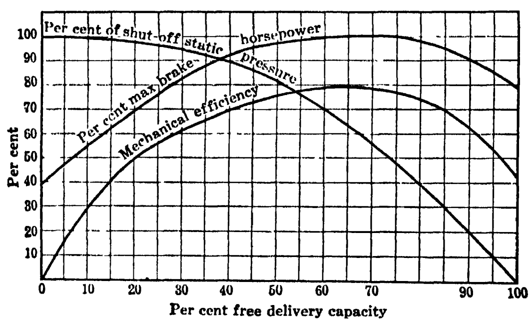


FIG. 17. Characteristic curves of double-curved multiblade fan.

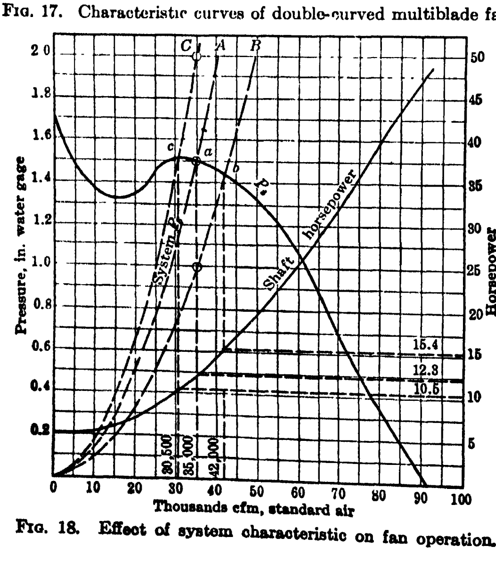


FIG. 18. Effect of system characteristic on fan operation.

Example. A No. 593 SI SW Sirocco Fan is selected for a duty of 35,000 CFM against a system resistance of 1 1/2 in. static pressure, operating at 208 rpm and requiring 12.3 shaft horsepower input, with a 15-hp driving motor. (See Table 5.) The system characteristic curve for this design volume and pressure is curve *A* (Fig. 18), operation resulting at point *a*. If the system actually imposed only 1.0 in. static pressure instead of 1 1/2 in. at 35,000 cfm, the system characteristic would be curve *B*. Actual operation would be at point *b*, 42,000 cfm at 1.44 in. static pressure, requiring 15.4 shp. This analysis shows that had the system pressure been overestimated by 50%, the fan selected would deliver 25% in excess of the design volume without imposing an overload upon the driving motor.

If the actual system pressure were greater than calculated, the system curve would be curve *C*. Here the actual system pressure is 2 in. static pressure. Operation would be at point *c*, 30,500 cfm against 1.51 in. static pressure, with a fan power requirement of 10.5 shp. This shows that when the actual system pressure requirement is underestimated operation will result with a reduction in the design volume and operating power, to a degree governed by the characteristics of the fan. These curves show the importance of carefully analyzing the system.

Selection of a fan for any application may be made by reference to the catalogs, bulletins, or performance charts issued by fan manufacturers and by comparison of the size, operating speed, power requirement, and other pertinent characteristics of the various types of fans available for the required purpose. (See tables, pp. 1-82 to 1-90.)

VELOCITY AND PRESSURE RELATIONS. The velocity pressure of a gas is that pressure which the gas stream possesses by virtue of its motion or rate of flow. The velocity head of a gas stream in motion may be determined by the formula $v = \sqrt{2gH}$, where v = velocity of gas in feet per second, H = head of gas in feet, and g = acceleration of gravity (32.17). In fan practice, unit of velocity is feet per minute and unit of pressure is inches water, gage. Conversion to these units may be made by substitution as follows: $v = V/60$, where V = velocity in feet per minute; $H = (P_v/12) \times (62.3/d)$, where P_v = velocity pressure, inches of water; d = density of gas, pounds per cubic foot. Substituting, the relation becomes $V/60 = \sqrt{2g(P_v/12) \times (62.3/d)}$, or $V = 1096.2\sqrt{P_v/d}$. At standard density (dry air at 70 F and 29.92 in. Hg barometric pressure), d = 0.075 lb per cu ft, and $V = 4005\sqrt{P_v}$. The velocity at any temperature and barometric pressure may be found by substituting the corresponding value of density in $V = 1096.2\sqrt{P_v/d}$.

At standard air density of 0.075 lb per cu ft, a velocity pressure of 1 in. water, gage, results from an air velocity of 4005 ft per min. Common usage accepts the value as 4000 ft per min; this value is called the *velocity constant*, corresponding to unit pressure in inches water, gage, for standard air density. The velocity constant is directly proportional to the square root of the absolute temperature and inversely proportional to the square root of the barometric pressure. Hence $V = K_v\sqrt{P_v}$, where K_v = velocity constant =

Table 2. Velocity of Standard Air, in ft per min, for Various Velocity Pressures

(d = 0.075 lb per cu ft, at 70 F and 29.92 in. barometer)

Velocity Pressure, in. water	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0	1266	1791	2193	2533	2832	3102	3351	3582	3800
1	4005	4200	4387	4566	4739	4905	5066	5221	5373	5523
2	5664	5803	5940	6074	6704	6332	6457	6581	6701	6820
3	6937	7052	7164	7274	7385	7492	7599	7704	7807	7910
4	8010	8109	8207	8305	8401	8496	8590	8683	8775	8866
5	8955	9043	9134	9220	9306	9392	9477	9562	9642	9728

Table 3. Velocity Constant, K_v, for Various Temperatures and Altitudes

(Gas = standard air of density = 0.075 lb/cu ft at 70 F and 29.92 in. barometer. Altitude, ft, and barometric pressure, in. Hg abs.)

Temp., °F	0 29.92	1000 28.86	2000 27.82	3000 26.81	4000 25.84	6000 23.98	8000 22.22	10,000 20.58
0	3729	3797	3867	39[illegible]	4013	4165	4327	4496
70	4005	4076	4151	4229	4307	4471	4645	4826
100	4115	4189	4267	4347	4427	4596	4775	4961
150	4294	4372	4453	4536	4621	4797	4983	5080
200	4467	4548	4632	4719	4807	4990	5183	5386
300	4793	4880	4971	5063	5158	5354	5562	5779
400	5099	5192	5288	5386	5487	5695	5917	6148
500	5387	5485	5587	5691	5797	6017	6251	6495
600	5661	5764	5781	5980	6091	6323	6569	6825
700	5922	6030	6141	6256	6372	6615	6872	7140

$4000 \times \sqrt{\frac{T}{530}} \times \frac{29.92}{P_a}$, T = absolute temperature, and P_a = barometric pressure under consideration. Tables 2 and 3 will save time in calculation of velocities and velocity constants. Note that values in Table 2 may be corrected for barometric pressure and temperature, using the ratio of constants in Table 3 to 4005 as a basis of proration.

HORSEPOWER OF FANS. The output of a fan can be derived from the power equation: Horsepower = foot-pounds per minute/33,000. The output is in terms of air quantity moved and pressure head. Thus

$$\text{Air horsepower} = \frac{\text{Weight of gas per minute} \times H}{33{,}000} = \frac{\text{cfm} \times d \times H}{33{,}000}$$

where d = gas density and H = head in feet of gas.

Related to the fan *total* pressure, P_t, inches water, gage, $H = P_t/12 \times 62.3/d$.

$$\text{Air horsepower (hp)}_t = \frac{\text{cfm} \times d \times P_t \times 62.3}{33{,}000 \times 12 \times d}$$

$$(\text{hp})_t = \frac{\text{cfm} \times P_t}{6356}$$

Related to the fan *static* pressure, P_s, inches water, gage,

$$\text{Air horsepower (hp)}_s = \frac{\text{cfm} \times P_s}{6356}$$

Horsepower input to fan shaft (shp) = output divided by fan efficiency,

$$\text{Shp} = \frac{\text{cfm} \times P_t}{6356 \times E_t} = \frac{\text{cfm} \times P_s}{6356 \times E_s}$$

where E_t = mechanical efficiency (based on total pressure) and E_s = static efficiency (based on static pressure).

FAN EFFICIENCY. The efficiency of a fan may be stated as *mechanical* efficiency or *static* efficiency. Current practice tends to evaluate the efficiency in terms of the air horsepower output, based upon *static* pressure.

$$\text{Static efficiency, } E_s = \frac{(\text{hp})_s}{\text{shp}} = \frac{\text{cfm} \times P_s}{6356 \times \text{shp}}$$

When required, the mechanical efficiency, based upon total pressure, may be determined from

$$E_t = \frac{(\text{hp})_t}{\text{shp}} = \frac{\text{cfm} \times P_t}{6356 \times \text{shp}}$$

SELECTION OF FANS. The essential data required to select the proper type and size of fan are (1) volume rate of flow in cubic feet per minute, (2) static pressure required to maintain the flow, (3) density of the gas, (4) type and speed of motive power available, (5) character of system, constant or variable flow or pressure, and (6) degree of permissible sound emission.

Theoretically a fan of any type may be selected to fulfill the bare requirements of a given duty of volume and pressure. The essential difference between the various types will be evidenced in (1) operating speed, (2) size of fan or space occupied, (3) efficiency or operating power cost, (4) initial cost, and (5) sound emission. The choice of the type lies in the relative importance of various operational characteristics. In some installations, initial cost or space occupied may be of major importance. In other installations, operating cost may overshadow all other considerations, dictating type and size to attain maximum possible efficiency. In many cases the necessity for driving the fan at a predetermined fixed speed will govern. Frequently, in the heating, ventilating, and air-conditioning field, quietness of operation becomes the primary factor, and those types are selected which have been developed for application to such exacting requirements. In general, *the intensity of sound emission of a fan of any type is lowest when the fan is operating at or near the peak of its static efficiency.* Hence selection of that fan which will attain maximum efficiency insures the quietest fan. Fans which must operate against high static pressures necessarily operate at higher speeds and produce higher sound intensity than when operating against lower pressures. Consequently, installations demanding extreme quietness of operation require design of the system to attain the lowest static pressure possible, in addition to proper selection of a fan for the system. Table 4 shows the relation of outlet velocity and tip speed to static pressure for fans operating in the peak efficiency range. Fans selected in accordance with these ranges will provide minimum power consumption and sound emission.

Table 4. Outlet Velocities and Tip Speeds for Multiblade Ventilating Fans

(Peak efficiency range, minimum power, and sound emission)

Static Pressure, in. water	Forward Curved Blade Fans		Backward Inclined and Double Curved Blade Fans	
	Outlet Velocity, ft per min	Tip Speed, ft per min	Outlet Velocity, ft per min	Tip Speed, ft per min
1/4	600- 900	1400–1600	500– 800	2400– 2700
1/2	800–1300	1900–2100	700–1100	3300– 3800
3/4	1000–1600	2400–2600	900–1400	4000– 4700
1	1200-1800	2700–3000	1000–1600	4600– 5400
1 1/2	1500 2200	3300–3700	1200–2000	5600– 6600
2	1700–2500	3800–4300	1400–2300	6500– 7600
2 1/2	1900-2800	4300–4800	1600-2500	7300- 8500
3	2100-3000	4700-5200	1800–2800	8000- 9400
4	2400–3600	5400- 6000	2000–3200	9200–11000
5	2700–4000	6000–6700	2200–3600	10000–12000

FAN RATINGS. Fan-capacity ratings are published in capacity table form (Tables 5, 7, 9, 11, 13, and 15) or graphically in curve form similar to Figs. 14, 15, 16, or 17. Manufacturers' catalogs may be consulted for data. The performance on which such ratings are

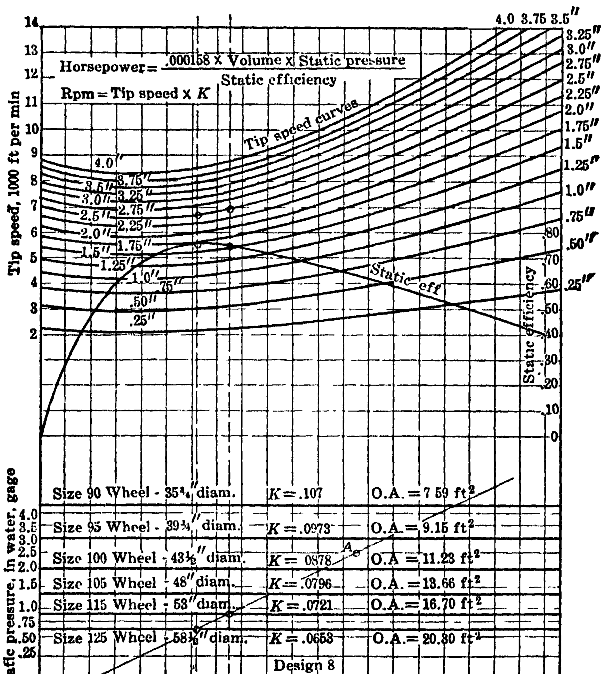


FIG. 19. Fan selection chart.

based is established by laboratory tests conforming to the provisions of the Standard Test Code. (See Article 23.) Practice is to present data for standard air density of 0.075 lb per cu ft. Variations in density modify the fan performance in accordance with the basic fan laws. (See Article 21.)

Table 5. Capacity Table of Forward Curved Blade Fan

No. 593 S.I. S.W. Sirocco Fan, Series 81

(Courtesy of American Blower Corp., Detroit, Mich.)

Cfm	Outlet Velocity	Outlet Velocity Pressure	1/2 in. SP		1 in. SP		1 1/2 in. SP		2 in. SP		2 1/2 in. SP		3 in. SP		4 in. SP		5 in. SP		6 in. SP	
			Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp
16568	800	.040																		
18639	900	.051	119	2.20																
20710	1000	.063	120	2.47																
22781	1100	.076	121	2.77																
24852	1200	.090	122	3.18	168	5.75														
26923	1300	.106	125	3.61	169	6.30														
28994	1400	.123	127	4.07	170	6.89														
31065	1500	.141	128	4.56	171	7.54	206	10.77												
33136	1600	.160	131	5.18	172	8.28	207	11.56												
35207	1700	.181	133	5.92	173	9.04	208	12.48	238	16.28										
37278	1800	.203	136	6.70	175	9.88	209	13.41	239	17.37										
39349	1900	.226	138	7.49	177	10.80	210	14.55	240	18.51	266	22.85								
41420	2000	.250	141	8.36	179	11.75	211	15.69	241	19.68	267	24.02								
43491	2100	.276	145	9.42	182	12.81	213	16.85	242	20.90	268	25.65	291	30.07						
45562	2200	.303	149	10.48	184	13.98	215	18.08	243	22.42	269	27.14	292	31.92						
47633	2300	.331	154	11.78	186	15.17	217	19.32	244	23.99	270	28.63	293	33.71						
49704	2400	.360	158	13.16	188	16.69	219	20.63	245	25.65	271	30.21	294	35.50	335	46.03				
51775	2500	.391	162	14.66	190	18.27	221	22.23	247	27.36	272	32.05	295	37.18	336	48.20				
53846	2600	.423	167	16.15	193	19.81	223	23.86	249	29.07	273	34.14	296	39.41	337	50.37				
55917	2700	.456	171	17.70	196	21.41	225	25.65	251	30.83	275	36.29	297	41.66	338	52.71	375	64.19		
57988	2800	.490	175	19.24	199	23.01	228	27.41	254	32.73	277	38.45	298	43.91	339	55.15	376	65.54		
60059	2900	.526	179	20.95	203	24.94	230	29.64	256	34.74	279	40.63	300	46.17	341	57.75	377	69.21		
62130	3000	.563	184	22.80	207	27.00	232	31.97	259	36.91	281	42.80	302	48.44	342	60.36	378	73.14	411	85.76
64201	3100	.601	188	24.83	211	29.18	235	34.25	261	39.19	283	44.97	304	50.97	343	63.13	379	75.72	412	88.48
66272	3200	.640	192	27.09	215	31.75	237	36.37	263	41.52	285	47.50	305	53.68	344	65.95	381	79.11	413	92.82
68343	3300	.681	196	29.28	219	34.47	240	38.54	265	43.97	288	50.21	308	56.45	345	68.94	382	82.23	415	96.08
70414	3400	.723	201	31.48	224	37.32	242	41.20	267	46.68	290	52.92	310	59.30	347	72.19	383	85.76	416	99.74
72485	3500	.766	204	33.65	228	40.03	246	44.10	269	49.94	291	55.77	313	62.56	349	75.58	384	89.29		
74556	3600	.810	207	36.10	232	43.02	250	47.28	271	53.19	294	58.70	315	65.68	351	79.03	385	93.09		
76627	3700	.856	210	38.95	236	46.14	254	50.48	273	56.45	296	62.42	317	68.94	353	82.51	387	96.89		
78698	3800	.903	214	41.80	241	49.12	258	53.79	276	59.71	298	66.22	319	72.60	355	86.41	388	100.69		
80769	3900	.951	218	44.86	245	52.24	262	56.99	280	62.96	301	70.16	321	76.29	358	90.32	390	104.62		
82840	4000	1.000	222	47.55	250	55.64	265	60.25	283	66.76	303	73.82	324	80.20	360	94.01	391	108.83		

Outlet { 64 3/4 in. x 46 7/8 in. outside. Area 20.710 sq ft inside.

Class I rating above heavy line.
Class II rating below heavy line.
For dimensions, see Table 6.

Wheel { 59 3/8 in. diameter. 15.54 ft circumference.

Wheel peripheral velocity
15.54 × rpm = fpm

Inlet { 65 1/4 in. diameter outside. Area 23.044 sq ft inside.

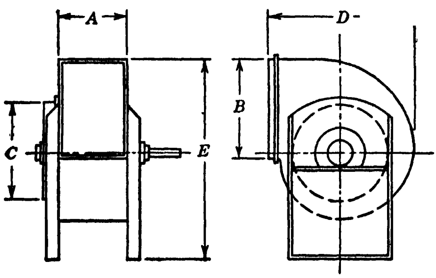
Capacity tables give fan performance over a wide range of volume and pressure. The capacity table usually gives volume, static pressure, outlet velocity, outlet velocity pressure, operating speed, and operating power. It also includes wheel diameter, wheel circumference, outlet dimensions and area, inlet dimensions and area, class designations, and operation limits. Tables 5, 7, 9, 11, 13, and 15 show typical capacity tables for several types of fan.

Manufacturers' catalogs and bulletins include separate capacity table ratings for fans of each size. The performance of other sizes may be calculated from a given fan-capacity table by use of the fundamental fan laws. (See Article 21.)

Characteristic Curves. The curves of a fan may be developed from the tabulated capacity table by reading various points from the table and converting the volume, pressure, and horsepower to that available at the required operating speed, on the basis that capacity varies directly as speed, pressure varies directly as the square of speed, and power

Table 6. Dimensions of American Sirocco Fans—Series 81, Single Inlet, Single Width

(Courtesy of American Blower Corp., Detroit, Mich.)



Fan Size	Dimensions, in.									
	Wheel Diam.	Width, A, S.I. S.W.	Outlet Depth, B	Inlet Diam., C	Length, D		Height, E			
					THD * BHD	UBD DBD	THD	BHD	UBD	DBD
122	12 1/4	9 3/4	13 1/16	13	22 1/8	23 5/8	28 5/8	25	26	26 1/8
135	13 1/2	10 3/4	14 3/8	14 3/8	24 1/4	25 7/8	31	27	28	28 1/4
150	15	11 15/16	15 13/16	15 7/8	26 1/2	28 5/8	34	29 5/8	30 1/2	31
165	16 1/2	13 1/16	17 9/16	17 3/8	28 3/4	31 1/4	37 1/8	32 1/4	33	33 3/4
182	18 1/4	14 7/16	18 5/16	19 3/8	31 1/4	34 1/2	41	35 1/2	36	37 1/4
200	20	15 13/16	21 5/16	21 1/4	34 1/8	37 5/8	44 3/4	38 7/8	39 1/2	40 5/8
222	22 1/4	17 9/16	23 5/8	23 5/8	37 5/8	41 3/4	49 5/8	43 1/8	43 1/2	45 1/8
245	24 1/2	19 3/8	26 1/8	26	41	45 3/4	54 1/2	47 1/4	47 1/2	49 1/2
270	27	21 3/8	28 5/8	28 5/8	44 5/8	50 1/4	59 5/8	51 5/8	51 1/2	54 1/8
300	30	23 5/8	31 7/8	31 3/4	49 1/8	55 5/8	65 3/4	56 7/8	56 1/2	59 5/8
330	33	26	35 1/8	35	53 5/8	61 1/8	72 1/2	62 5/8	62	65 5/8
360	36	28 3/8	39 1/4	37 7/8	58 1/4	66 1/2	78 5/8	68	67	71 1/4
397	39 3/4	31 5/8	43 5/8	45 3/8	62 1/2	73 1/4	75 1/8	74 1/8	64 3/4	62 1/2
440	44	34 7/8	48 1/4	50	70 3/8	81	83 1/8	81 7/8	72 1/2	70 3/8
486	48 5/8	38 1/2	53 1/4	55 3/8	77 1/2	89 1/4	91 1/2	90 3/8	79 3/4	77 1/2
537	53 3/4	42 1/2	58 3/4	59 1/4	85 1/2	98 1/2	100 7/8	99 3/4	88	85 1/2
593	59 3/8	46 7/8	64 3/4	65 1/4	93 3/4	108 3/4	110 3/4	110	96	93 3/4
657	65 3/4	52	71 1/4	72 7/8	103 1/4	120 1/2	122 3/4	121 1/4	105 1/2	103 1/4
726	72 5/8	57 3/8	78 5/8	79 3/4	114 1/2	132 7/8	135 3/8	134	117	114 1/2
803	80 3/8	63 5/8	87 3/8	88 1/2	129 1/8	148	151 5/8	149 1/4	133	129 1/8
887	88 3/4	70 1/4	96 1/4	97	142 5/8	163 1/4	167 7/8	165 5/8	147 1/2	142 5/8
980	98	77 1/2	106 1/4	106 1/2	157	179 3/4	184 5/8	182 1/8	162	157
1082	108 1/4	85 1/2	117 1/4	117	173 1/8	198 1/4	202 7/8	200 3/4	178	173 1/8
1195	119 1/2	94 1/4	129 3/8	128 1/2	191 1/8	218 5/8	223 3/4	221 1/4	196	191 1/8
1320	132	104	142 3/4	141 1/2	211 3/8	241	246 3/8	246 1/4	216 1/4	211 3/8

* THD = top horizontal discharge. BHD = bottom horizontal discharge. UBD = upward blowing discharge. DBD = downward blowing discharge.

Table 7. Capacity Table of Backward Curved Blade Fan
No. 593 S I S W. High Speed Fan, Series 81
(Courtesy of American Blower Corp., Detroit, Mich.)

Cfm	Outlet Velocity	Outlet Velocity Pressure	1/2 in SP		1 in SP		1 1/2 in SP		2 in. SP		2 1/2 in. SP		3 in. SP		4 in SP		5 in SP		6 in SP	
			Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp
16568	800	.040																		
18639	900	.051	215	1.98																
20710	1000	.063	223	2.25																
22781	1100	.076	231	2.61																
24852	1200	.090	242	2.99	297	5.18														
26923	1300	.106	253	3.45	306	5.70														
28994	1400	.123	265	3.91	315	6.24														
31065	1500	.141	279	4.42	324	6.92	367	9.72												
33136	1600	.160	293	4.99	333	7.60	375	10.50												
35207	1700	.181	308	5.67	345	8.41	382	11.37	420	14.66										
37278	1800	.203	324	6.43	358	9.23	393	12.38	428	15.74										
39349	1900	.226	336	7.22	370	10.18	405	13.35	437	16.88	469	20.46								
41420	2000	.250	349	8.03	382	11.13	416	14.47	445	18.05	479	21.87								
43491	2100	.276	362	9.04	395	12.27	427	15.71	454	19.19	488	23.26	518	27.25						
45562	2200	.303	375	10.18	409	13.43	438	16.96	463	20.73	498	24.75	525	28.80						
47633	2300	.331	390	11.34	422	14.71	449	18.21	476	22.25	508	26.27	532	30.34						
49704	2400	.360	405	12.48	436	16.01	461	19.70	488	23.80	518	27.76	541	32.19	594	41.47				
51775	2500	.391	420	13.65	450	17.51	471	21.20	501	25.32	527	29.37	551	34.14	604	43.42				
53846	2600	.423	435	14.79	464	19.00	482	22.93	514	26.87	536	31.10	562	36.12	612	45.60				
55917	2700	.456	448	16.34	477	20.76	494	24.67	527	28.39	546	33.25	572	38.10	621	47.77	664	57.94		
57988	2800	.490	463	18.27	491	22.53	508	26.43	540	30.70	556	35.39	582	40.22	630	49.94	671	60.69		
60059	2900	.526	478	20.17	506	24.43	524	28.42	553	33.00	565	37.53	593	42.34	639	52.54	678	63.72		
62130	3000	.563	493	22.09	521	26.33	538	30.42	561	35.39	580	39.68	605	45.16	648	55.37	634	66.76	727	75.80
64201	3100	.601	508	24.02	536	28.50	553	32.89	574	37.75	594	42.34	618	47.98	657	57.97	691	69.78	738	79.79
66272	3200	.640	523	26.22	551	30.67	568	35.39	587	40.14	608	45.03	631	50.81	668	60.79	705	72.82	748	83.78
68343	3300	.681	538	28.44	565	32.98	583	37.89	600	42.61	623	47.71	644	53.63	679	64.05	718	75.86	758	87.77
70414	3400	.723	553	30.64	579	35.28	598	40.38	613	45.30	637	50.91	656	56.45	691	67.31	731	78.90	769	91.76
72485	3500	.766	568	32.87	592	38.00	613	42.88	635	47.98	651	54.14	668	59.27	703	70.56	745	82.53		
74556	3600	.810	583	35.55	606	40.71	627	45.62	647	50.64	666	57.89	681	62.10	715	73.82	759	86.17		
76627	3700	.856	598	38.43	621	43.70	642	49.10	660	53.74	681	61.63	696	65.62	727	77.51	773	90.43		
78698	3800	.903	613	41.31	636	46.68	657	52.60	673	57.18	694	63.60	708	69.86	739	79.85	786	94.66		
80769	3900	.951	628	43.91	651	50.07	672	56.10	685	61.01	707	67.47	721	74.09	751	85.11	799	98.93		
82840	4000	1.000	643	46.60	667	53.47	687	59.57	702	64.70	721	71.90	734	78.33	764	89.02	813	103.16		

Outlet { 64 3/4 in. x 16 7/8 in. outside.
Area 20.710 sq ft inside.

Wheel { 59 3/8 in. diameter.
15.54 ft circumference.

Inlet { 65 1/4 in. diameter outside.
Area 23.044 sq ft inside.

Class I Rating above heavy line.
Class II Rating below heavy line.
For dimensions see Table 8.

Maximum bhp = $206 \left(\frac{rpm}{1000}\right)^3$

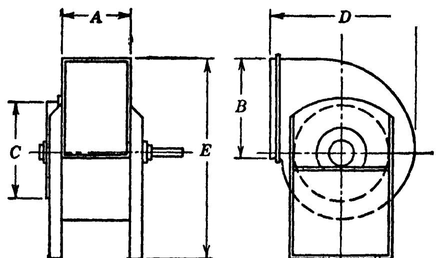
Wheel peripheral velocity
15.54 × rpm = fpm

directly as the cube of speed. For specific applications, involving special fan designs for unusual requirements, the manufacturer can supply design data and performance rating curves not readily available in conventional equipment catalogs.

Various forms of direct-reading fan charts are sometimes used for selection of fans, performing the same function as multirating capacity tables. One form of such a fan chart is illustrated in Fig. 19. To use the chart, locate point A, at the required volume and pressure. A line is drawn through this point and the origin of the chart as shown. Vertical lines drawn through the point of intersection of this line and lines denoting the various sizes of fan intersect the static efficiency curve and the tip speed curve (for the required static pressure) at the applicable static efficiency and required tip speed. See example for sizes 115 and 125. Rotative speed is then determined by rpm = tip speed ÷ wheel circumference or rpm = tip speed $\times$ K (K is listed on chart for each size of fan). Horsepower is calculated by the conventional formula shp $= \dfrac{\text{cfm} \times P_s}{6356 \times E_s}$

Table 8. Dimensions of American H.S. Fans, Series 81, Single Inlet, Single Width

(Courtesy of American Blower Corp., Detroit, Mich.)



Fan Size	Dimensions in Inches									
	Wheel Diam.	Width, A	Outlet Depth, B	Inlet Diam., C	Length, D		Height, E			
					THD* BHD	UBD DBD	THD	BHD	UBD	DBD
122	12 1/4	9 3/4	13 1/16	13	22 1/8	23 5/8	28 5/8	25	26	26 1/8
135	13 1/2	10 3/4	14 3/8	14 3/8	24 1/4	25 7/8	31	27	28	28 1/4
150	15	11 15/16	15 13/16	15 7/8	26 1/2	28 5/8	34	29 5/8	30 1/2	31
165	16 1/2	13 1/16	17 9/16	17 3/8	28 3/4	31 1/4	37 1/8	32 1/4	33	33 3/4
182	18 1/4	14 7/16	19 5/16	19 3/8	31 1/4	34 1/2	41	35 1/2	36	37 1/4
200	20	15 13/16	21 5/16	21 1/4	34 1/8	37 5/8	44 3/4	38 7/8	39 1/2	40 5/8
222	22 1/4	17 9/16	23 5/8	23 5/8	37 5/8	41 3/4	49 5/8	43 1/8	43 1/2	45 1/8
245	24 1/2	19 3/8	26 1/8	26	41	45 3/4	54 1/2	47 1/4	47 1/2	49 1/2
270	27	21 3/8	28 5/8	28 5/8	44 5/8	50 1/4	59 5/8	51 5/8	51 1/2	54 1/8
300	30	23 5/8	31 7/8	31 3/4	49 1/8	55 5/8	65 3/4	56 7/8	56 1/2	59 5/8
330	33	26	35 1/8	35	53 5/8	61 1/8	72 1/2	62 5/8	62	65 5/8
360	36	28 3/8	39 1/4	37 7/8	58 1/4	66 1/2	78 5/8	68	67	71 1/4
397	39 3/4	31 5/8	43 5/8	45 3/8	62 1/2	73 1/4	75 1/8	74 1/8	64 3/4	62 1/2
440	44	34 7/8	48 1/4	50	70 3/8	81	83 1/8	81 7/8	72 1/2	70 3/8
486	48 5/8	38 1/2	53 1/4	55 3/8	77 1/2	89 1/4	91 1/2	90 3/8	79 3/4	77 1/2
537	53 3/4	42 1/2	58 3/4	59 1/4	85 1/2	98 1/2	100 7/8	99 3/4	88	85 1/2
593	59 3/8	46 7/8	64 3/4	65 1/4	93 3/4	108 3/4	110 3/4	110	96	93 3/4
657	65 3/4	52	71 1/4	72 7/8	103 1/4	120 1/2	122 3/4	121 1/4	105 1/2	103 1/4
726	72 5/8	57 3/8	78 5/8	79 3/4	114 1/2	132 7/8	135 3/8	134	117	114 1/2
803	80 3/8	63 5/8	87 3/8	88 1/2	129 1/8	148	151 5/8	149 1/4	133	129 1/8
887	88 3/4	70 1/4	96 1/4	97	142 5/8	163 1/4	167 7/8	165 5/8	147 1/2	142 5/8
980	98	77 1/2	106 1/4	106 1/2	157	179 3/4	184 5/8	182 1/8	162	157
1082	108 1/4	85 1/2	117 1/4	117	173 1/8	198 1/4	202 7/8	200 3/4	178	173 1/8
1195	119 1/2	94 1/4	129 3/8	128 1/2	191 1/8	218 5/8	223 3/4	221 1/4	196	191 1/8
1320	132	104	142 3/4	141 1/2	211 3/8	241	246 3/8	246 1/4	216 1/4	211 3/8

* THD = top horizontal discharge. BHD = bottom horizontal discharge. UBD = upward blowing discharge. DBD = downward blowing discharge.

Table 9. Capacity Table of Radial-blade Fan

No. 600 Type E Fan with HE Wheel

(Courtesy of American Blower Corp., Detroit, Mich.)

Cfm	Outlet Velocity	1 in. SP		2 in. SP		3 in. SP		4 in. SP		6 in. SP		8 in. SP		10 in. SP		14 in. SP	
		Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp	Rpm	Bhp
3295	1000	331	0.75	457	1.52	550	2.27										
3954	1200	341	0.91	463	1.80	555	2.67	634	3.60								
4613	1400	356	1.10	470	2.06	562	2.95	642	4.24	772	6.22						
5272	1600	373	1.36	481	2.43	566	3.49	647	4.75	780	7.07	896	9.85				
5931	1800	389	1.64	492	2.78	579	4.02	661	5.50	787	7.98	903	10.88	1002	14.04		
6590	2000	407	1.94	504	3.16	589	4.49	668	5.90	793	8.63	909	11.96	1010	15.23	1189	21.39
7249	2200	423	2.27	522	3.67	602	5.10	674	6.58	801	9.57	916	13.22	1013	16.61	1196	23.47
7908	2400	442	2.69	540	4.19	617	5.71	683	7.27	811	10.62	923	14.33	1019	18.04	1198	25.39
8567	2600	461	3.16	556	4.77	631	6.34	700	8.10	818	11.51	926	15.35	1024	19.77	1202	27.17
9226	2800	480	3.67	573	5.36	649	7.14	712	7.70	831	12.54	938	16.57	1035	20.80	1205	29.13
9885	3000	500	4.26	591	6.08	662	7.86	729	9.83	842	13.69	946	17.74	1044	22.28	1211	31.15
10544	3200	519	4.91	607	6.81	680	8.80	746	10.88	855	14.79	956	19.16	1053	23.84	1216	32.97
11203	3400	538	5.62	626	7.65	698	9.69	765	12.03	870	16.05	967	20.59	1061	25.30	1227	34.10
11862	3600	558	6.22	644	8.47	714	10.72	778	13.03	886	17.62	982	22.16	1072	26.91	1233	37.18
12521	3800	577	7.23	663	9.52	732	11.79	795	14.16	904	19.05	994	23.54	1086	28.88	1243	39.22
13180	4000	599	8.19	685	10.65	749	13.01	812	15.51	922	20.62	1013	25.60	1099	30.91	1251	41.70
13839	4200	621	9.24	701	11.79	768	14.27	836	17.20	935	22.14	1025	27.24	1112	32.76	1261	44.25
14498	4400	641	10.37	721	13.06	788	15.68	850	18.28	952	23.87	1044	29.25	1125	34.63	1271	46.43
15157	4600	663	11.63	741	14.41	807	17.06	864	19.66	969	25.62	1060	31.38	1145	37.44		
15816	4800	685	12.99	760	15.94	825	18.72	883	21.55	988	27.50	1076	33.51	1158	39.66		
16475	5000	707	14.46	791	18.21	844	20.24	901	23.40	1003	29.27	1092	35.52	1172	42.45		
17134	5200									1021	31.38	1112	38.28	1192	44.79		
17793	5400									1041	33.81	1127	40.51	1205	47.50		
18452	5600									1056	35.99	1145	43.08	1222	49.73		
19111	5800									1075	38.47	1161	45.61	1243	53.24		
19770	6000									1095	41.21	1180	48.67	1258	55.93		
20429	6200									1113	44.02	1197	51.48	1275	59.41		
21088	6400									1132	46.82	1220	55.34				
21747	6600									1152	50.08	1233	57.80				

Outlet { 23 3/4 in. x 20 7/16 in. outside. Area 3.295 sq ft or 474 sq in.
For dimensions see Table 10.

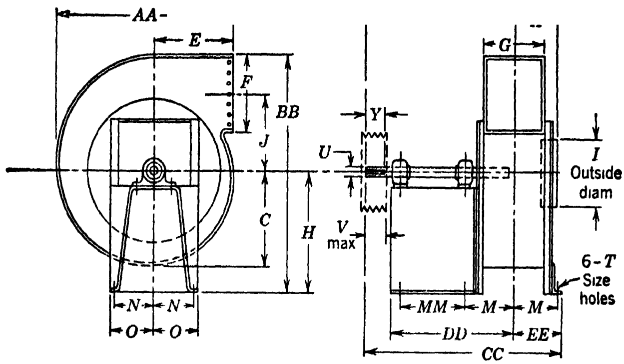
Standard wheel { 41 3/4 in. diameter. 10.93 ft circumference.

Inlet { 25 3/16 in. diameter outside. Area 3.319 sq ft or 478 sq in.

Table 10. Dimensions of Radial-blade Fans

Type E Fans with HE Wheels

(Courtesy of American Blower Corp., Detroit, Mich.)



Arrangement 1-*B* consists of housing, overhung fan wheel, two bearings located on drive side, and the running gear support is a tapered box type pedestal with straight top. The shaft is extended beyond the outer bearing and key-seated to overhang the sheave or pulley.

This is the most popular arrangement as it permits the application and removal of belts without dismantling the bearings and shaft.

Pulley or sheave shown in the above diagram is for illustration purposes only and is not included as regular equipment.

Fan Size	*C*	*E*	*F*	*G*	*H*	*I*	*J*	*M*	*N*	Keyway, HE Wheel	*U*, HE Wheel
250	12 7/16	12	9 7/8	8 9/16	17	10 1/2	10 3/16	7 5/8	6 7/16	3/8 × 3/16	1 7/16
300	15 11/16	14 1/2	12 7/16	10 3/4	21	13 3/16	12 3/4	8 11/16	7 7/8	3/8 × 3/16	1 11/16
350	17 1/2	16	13 7/8	11 15/16	23	14 5/8	14 5/16	9 5/16	8 7/8	1/2 × 1/4	1 15/16
400	19 3/4	18	15 5/8	13 1/2	26	16 1/2	16 3/16	10 1/16	9 5/8	1/2 × 1/4	1 15/16
450	22 3/16	20	17 1/2	15 1/8	29	18 1/2	18 3/16	11 1/8	10 7/8	1/2 × 1/4	2 3/16
500	24 15/16	22 1/2	19 11/16	16 15/16	32 1/2	20 3/4	20 1/2	12 1/8	12 1/8	5/8 × 5/16	2 7/16
550	27 3/8	24 1/2	21 5/8	18 5/8	35 1/2	22 13/16	22 1/2	12 15/16	13 5/8	5/8 × 5/16	2 7/16
600	30 1/8	27	23 3/4	20 7/16	38 1/2	25 3/16	24 3/4	13 7/8	14 5/8	5/8 × 5/16	2 11/16
700	33 3/16	30	26 1/4	22 5/8	42 1/2	27 11/16	27 1/4	16 3/16	16 1/2	5/8 × 5/16	2 11/16
750	36 9/16	33	28 7/8	24 7/8	47	30 3/8	30	17 5/16	19 1/4	3/4 × 3/8	2 15/16
800	40 3/16	36	31 11/16	27 5/16	51 1/2	33 3/8	33	19 9/16	19 7/8	7/8 × 7/16	3 7/16
900	44 5/16	40	34 7/8	30 1/16	56 1/2	36 3/4	36 3/8	20 15/16	21 3/4	7/8 × 7/16	3 7/16

Fan Size	*O*	*T*	*W*	*X*	*AA*	*BB*	*CC*	*DD*	*EE*	*MM*	Dimension *V*, HE Wheel *		*Y*
											SAO	D	
250	7 1/16	5/8	27 1/2	8 1/16	25 3/4	32 1/8	36	21 3/4	8 1/2	13 1/4	5 1/4	4	4
300	8 1/2	5/8	30 11/16	9 1/16	31 7/8	40	40 1/4	23 13/16	9 9/16	14 1/4	6 3/8	4 1/2	4 1/2
350	9 3/4	5/8	33 5/16	9 13/16	35 3/8	44 1/4	43 1/2	25 15/16	10 3/16	15 3/4	6 3/16	5	5
400	10 1/2	5/8	37 1/16	10 9/16	39 7/8	50	48	29 3/16	10 15/16	18 1/4	6 11/16	5 1/2	5 1/2
450	12	3/4	30 3/4	11 5/16	44 9/16	55 15/16	53	32 1/8	12 1/4	20	8 1/8	6	6
500	13 1/4	3/4	45 5/8	12 3/16	50 1/8	62 13/16	58 7/8	36 1/8	13 1/4	23	9	6 1/2	6 1/2
550	14 3/4	3/4	48 15/16	12 15/16	54 13/16	68 13/16	63	38 15/16	14 1/16	25	9 1/2	7	7
600	15 3/4	3/4	53 7/8	13 15/16	60 3/8	75 1/8	68 7/8	42 7/8	15	28	9 5/16	7 1/2	7 1/2
700	18	7/8	56	15 11/16	66 3/4	82 7/8	73 3/4	45 11/16	17 11/16	27 1/8	9 7/8	7 13/16	8
750	20 3/4	7/8	57 1/4	16 15/16	73 1/2	91 7/16	76	46 13/16	18 13/16	27 1/8	9 1/2	8 7/16	8
800	21 3/8	7/8	63	18 9/16	80 1/2	100 5/16	84	50 9/16	21 1/16	28 5/8	11 1/2	9 7/16	9
900	23 1/4	7/8	67 1/4	19 13/16	89	110 5/16	89 5/8	53 15/16	22 7/16	30 5/8	12 3/8	10 5/16	10

* SAO = ball bearings; D = sleeve bearings

Table 11. Capacity Table of Typical Vaneaxial Fan

No. 32 Type B Vaneaxial Fan

(Courtesy of Buffalo Forge Co., Buffalo, N.Y.)

Size 32

Limit Load Hp = $2.47 \left(\frac{\text{Rpm}}{1000}\right)^3$

Outlet Velocity *	Cfm	1/2 in. SP		1 in. SP		1 1/2 in. SP		2 in. SP		2 1/2 in. SP	
		Rpm	Hp	Rpm	Hp	Rpm	Hp	Rpm	Hp	Rpm	Hp
1600	9130	784	1.17	981	2.35						
1800	10260	828	1.39	1011	2.60						
2000	11400	880	1.64	1051	2.86	1205	4.37				
2200	12550	934	1.94	1091	3.20	1238	4.75	1378	6.50		
2400	13700	990	2.31	1135	3.59	1279	5.17	1410	6.96		
2600	14830	1053	2.68	1186	4.04	1316	5.64	1447	7.46	1560	9.44
2800	16000	1117	3.15	1237	4.57	1362	6.17	1482	8.03	1597	10.1
3000	17100	1180	3.64	1291	5.12	1406	6.74	1522	8.65	1635	10.7
3200	18250	1245	4.21	1350	5.75	1459	7.45	1565	9.36	1670	11.5
3400	19400	1311	4.84	1403	6.49	1506	8.21	1610	10.1	1710	12.3
3600	20550	1379	5.56	1466	7.27	1560	9.06	1658	11.0		

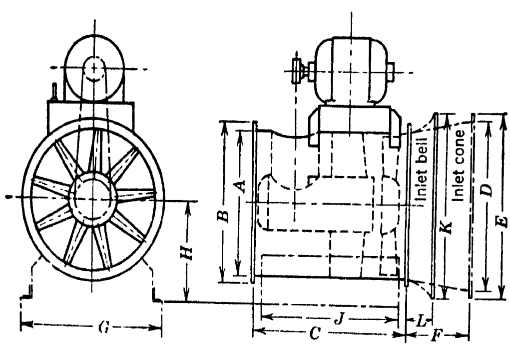
* Outlet velocity is through fan casing. Outlet velocity through large diam. cone = 0.725 times outlet velocity given. Ratings are based on tapered cone outlet. If cones are not used, deduct 4% from cfm.

See Table 12 for dimensions of size 32 fan.

Table 12. Dimensions of Vaneaxial Fans

Buffalo Type B Vaneaxial Fans

(Courtesy of Buffalo Forge Co., Buffalo, N.Y.)



Dimensions in Inches

Size	A	B	C	D	E	F	G	H	J	K	L
15	15 5/32	18 5/8	22	18	21 1/4	7 1/2	18 1/4	12	18	19 1/2	3
18	18 3/16	21 5/8	23 1/2	21 1/2	24 3/4	9	21 1/4	13 1/2	19 1/2	23 1/4	3 9/16
21	21 7/32	24 5/8	26 1/2	25	28 1/4	11	25 1/4	15 1/2	22 1/2	27 1/4	4 3/16
24	24 1/4	27 5/8	29 1/2	28 1/2	31 3/4	12	28 1/4	17	24 1/2	31	4 3/4
28	28 9/32	31 5/8	33 1/2	33 1/2	36 3/4	14	32 1/4	19 1/2	28 1/2	36 1/2	5 9/16
32 *	32 5/16	35 7/8	37 1/2	38	41 3/8	16	37 1/4	22	33	41 1/2	6 3/8
30	36 11/32	39 7/8	42 1/2	43	46 3/8	18	41 1/4	25	37 1/2	46 1/2	7 3/16
42	42 3/8	45 7/8	49	50	53 3/8	21	47 3/4	29	44	54 1/4	8 3/8
48	48 7/16	52 1/8	57 1/2	57	60 3/8	24	53 3/4	33	53	62 3/8	9 9/16

* Table 11 gives ratings for this size.

Table 13. Skeleton Capacity Table for Tubeaxial Fans *

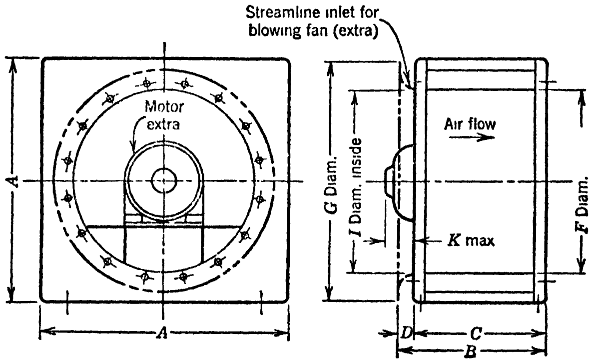
Tubeaxial Fans, Series 201
(Courtesy of American Blower Corp., Detroit, Mich.)

Fan Size	Rpm	Motor hp	Capacity in Cubic Feet per Minute			
			1/2 in. SP	1 in. SP	1 1/2 in. SP	2 in. SP
18-3	3450	5	7990	7130	6500	5815
	1750	1/2	3000			
22-3	1750	1 1/2	7025			
27-3	1750	5	12680	10850		
	1150	1	6680			
33-3	1750	10	23100	20900	18700	
	1150	3	13280			
40-3	1150	7 1/2	26500	22250		
	860	3	17350			
49-3	1150	20	50600	45750	40600	
	860	10	34800	27800		
	690	5	25200			
60-3	860	25	67000	57100		
	690	15	49800			
	570	7 1/2	36300			
73-3	860	60	129200	119700	109700	98700
	690	30	100400	87600	72500	
	570	20	78200	61800		

* Intermediate sizes are available. Consult catalog.
See Table 14 for dimensions and diagram.

Table 14. Dimensions of Tubeaxial Fans

Tubeaxial Fans, Series 201
(Courtesy of American Blower Corp., Detroit, Mich.)



Fan Size	A	B	C	D	F	G	I	K	Fan Size	A	B	C	D	F	G	I	K
18	25	17 5/8	15 1/4	2 3/8	18	22 1/2	18	2 3/8	40	53	29	24 1/2	4 1/2	39 7/8	47 5/16	39 7/8	5 1/8
22	30	19 1/8	16 3/4	2 3/8	22 1/8	26 1/2	22 1/8	1 1/4	49	65	33 5/8	28 1/4	5 3/8	48 11/16	57 3/16	48 11/16	9 3/8
27	37	22 5/8	19 1/2	3 1/8	27 1/16	33	27 1/16	3 3/8	60	80	41 7/8	35 1/4	6 5/8	59 9/16	69 1/2	59 9/16	3 3/4
33	45	26	22 1/4	3 3/4	32 9/16	39 1/8	32 9/16	5 1/2	73	96	50 1/8	42	8 1/8	72 3/4	92 7/8	72 3/4	1 5/8

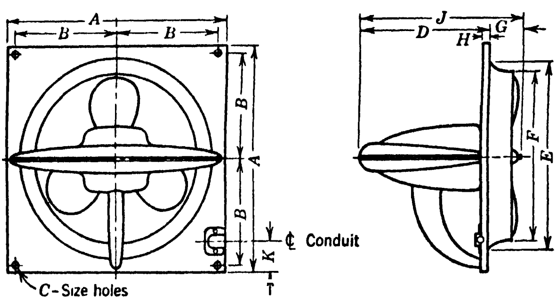
Motor supports designed for standard Nema frames.

Table 15. Capacity Table and Dimensions of Propeller Fans

(Courtesy of American Blower Corp., Detroit, Mich.)

(Model A Ventura fan Direct drive, constant speed 60 cycle, 115 or 230 volts, single pha 60 cycle, 220 or 440 volts, 3 phase.)

Catalog Number	Cfm Free Delivery	Fan rpm	Motor hp	Nominal Wheel Diameter, inches	Decibel Rating	Approx. Shipping Weight, pounds
10A	1000	1150	1/20	12	58	36
15A	1500	1725	1/12	12	65	40
20A	2000	1150	1/12	16	61	56
26A	2600	1150	1/8	18	62	68
30A	3000	1725	1/4	16	69	62
36A	3600	1150	1/6	20	63	72
40A	4000	1725	1/3	18	72	79
45A	4500	860	1/6	24	66	90
50A	5000	1150	1/4	24	73	90
60A	6000	1150	1/3	24	74	105
72A	7250	860	1/3	30	71	128
97A	9700	1150	3/4	30	77	143



Dimensions in Inches

Catalog Number	A	B	C	D	E	F	G	H	J	K
10A	16	7 5/8	7/16	11 3/8	14 1/2	12 1/2	2 1/4	3/4	13 5/8	2 1/4
15A	16	7 5/8	7/16	11 3/8	14 1/2	12 1/2	2 1/4	3/4	13 5/8	2 1/4
20A	22	10 1/2	7/16	12 3/4	19 1/2	16 5/8	3	3/4	15 3/4	3
26A	24	11 1/2	7/16	15	22 1/2	18 1/2	3 1/8	1	18 1/8	3 1/2
30A	22	10 1/2	7/16	12 3/4	19 1/2	16 5/8	3	3/4	15 3/4	3
36A	26 1/2	12 3/8	7/16	15	24	20 1/4	3 3/4	1	18 3/4	3 1/2
40A	24	11 1/2	7/16	15	22 1/2	18 1/2	3 1/8	1	18 1/8	3 1/2
45A	31	14 5/8	9/16	15 1/2	27 3/4	25 3/8	3 7/8	1 1/4	19 3/8	3 1/2
50A	31	14 5/8	9/16	15 1/2	27 3/4	25 3/8	3 7/8	1 1/4	19 3/8	3 1/2
60A	31	14 5/8	9/16	15 1/2	27 3/4	25 3/8	3 7/8	1 1/4	19 3/8	3 1/2
72A	36	17 1/8	9/16	18 1/8	33 3/8	30 3/8	4 5/8	1 1/4	22 3/4	3 1/2
97A	36	17 1/8	9/16	18 1/8	33 3/8	30 3/8	4 5/8	1 1/4	22 3/4	3 1/2

26. FAN CAPACITY CONTROL

Regulation of the capacity output of the fan or variation in the volume delivered by the fan to the system is an objective in many installations. A number of methods of capacity control are utilized in current practice. The relative importance of various factors of the particular application governs the choice of method. Fundamentally, maximum efficiency in control or maximum reduction in operating power to accompany the required reduction in volumetric capacity is a primary design function. Selection of the method of control will result from evaluation of such factors as initial cost, operating cost, range of capacity regulation available, speed of response to demand change, simplicity of operation, reliability, life, and maintenance.

The methods of fan control currently utilized include the following:

	Control Method	Fan Speed	Driver Speed
1.	Damper control	Constant	Constant
2.	Inlet-vane control	Constant	Constant
3.	Inlet-louver control	Constant	Constant
4.	Fluid drive (hydraulic coupling)	Adjustable	Constant
5.	Magnetic drive (electric coupling)	Adjustable	Constant
6.	D-c motor drive	Variable	Variable
7.	Slip ring a-c motor drive	Step-variable	Step-variable
8.	Two or three speed a-c motor drive	Semivariable	Semivariable
9.	Turbine drive	Variable	Variable
10.	Variable pitch sheave and belt	Variable	Constant

The control methods fall into three general classifications: (1) constant driver and fan speed, with auxiliary device to modify fan or system characteristic; (2) constant speed driver, with auxiliary intermediate device between driver and fan to provide adjustable speed of fan; and (3) variable speed driver direct-connected to fan, thereby providing variable fan speed to accomplish variation in fan capacity output.

DAMPER CONTROL. Dampering is a throttling action applied to the gas flow from or to the fan. At some convenient point between the fan and its external system, a damper or variable throttle is positioned so that additional resistance to gas flow may be imposed on the fan. The total resistance of the damper, in a partially closed position, and the system forces the fan to operate at a lower point of rating, with an attendant reduction in volumetric flow. The process is a pressure-dissipation action, forcing the fan to operate at a pressure in excess of the system pressure demand. The change in power input to the fan is coincident with the basic shaft horsepower characteristic curve, over the range of volumetric change effected. Fans with a steep horsepower characteristic damper better than fans with a flat horsepower characteristic. The regulating damper method of control provides minimum first cost, maximum operating cost, wide range regulation, simplicity of operation, and relatively long life. Figure 20 illustrates the variation in shaft horsepower with volume regulation under damper control for the forward curved blade fan and the backward curved blade fan.

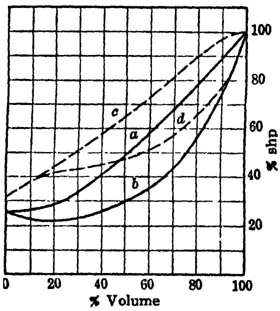


FIG. 20. Typical fan control characteristics, constant fan speed: (a) Forward-curved-blade fan, damper control. (b) Forward-curved-blade fan, inlet-vane or louver control. (c) Backward-curved-blade fan, damper control. (d) Backward-curved-blade fan, inlet-vane or louver control.

INLET-VANE CONTROL consists of a series of adjustable position vanes or blades located at the inlet to the fan. The vanes can be adjusted to various positions so that the entering air is given a change in direction or spin in the direction of rotation. The initial spin modifies the basic characteristics of pressure output and power input, resulting in a new and reduced pressure and horsepower characteristic, relative to the basic ones. Adjustment of the vanes to various positions, changing the extent of the initial spin, gives regulation to any required volumetric flow at only the pressure demanded by the system. The excess pressure dissipation of damper control is avoided, and the power demand of the fan is sensibly reduced from that of damper control. Inlet-vane control provides moderate first cost, good operating cost, wide range regulation, simplicity of operation, low maintenance, and relatively long life, except under conditions of exposure to corrosive or abrasive dust-laden gases. Figure 20 illustrates typical inlet-vane control characteristics of operating power versus volumetric flow for forward curved and backward curved blade fans.

INLET-LOUVER CONTROL, used primarily for induced draft fans, consists of a series of adjustable directional vanes located in the approach section of the inlet box, so disposed as to impart an initial spin in the gas stream adjacent to and within the inlet passage communicating with the fan wheel. The action is similar to that of inlet vanes, and the result is comparable in effecting modification of the fan pressure and horsepower characteristics. The control obtained with inlet-louver control is comparable to inlet-vane control in efficiency and operating cost and in wide range regulation. Long life and low maintenance are enhanced by location of the operating mechanism outside the gas stream. Typical characteristic variation of shaft horsepower with volume is indicated on Fig. 20, being substantially similar to inlet-vane control. Exact values of power versus volume reduction are a function of the design and construction of each type of control. The relation of power reduction to volume reduction is a function of the point of rating of the fan, varying materially between wide-open and shut-off systems. The variation is not large throughout chanical efficiencies attainable range from 65 to 75%, whereas static efficiencies range from 55 to 60%. Major application is in industrial ventilation and process work involving moderate static pressures, where simplicity of installation in ducts is required. Characteristic curves are similar in trend though lower in pressure level and efficiency than the curves shown on Fig. 23, for a Vaneaxial fan. Tables 13 and 14 show capacity table and dimensions of one model of commercial Tubeaxial fan.

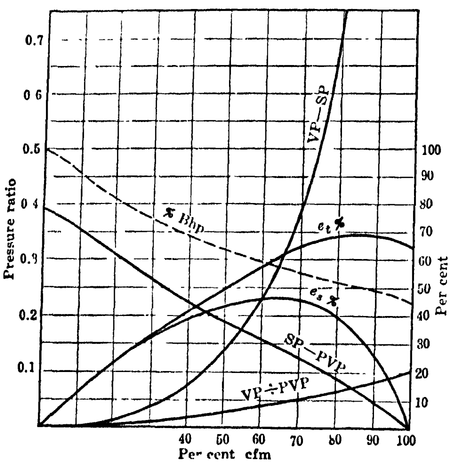


FIG. 22. Characteristic curves of propeller fan.

VANEAXIAL FANS. The Vaneaxial fan consists of an axial-flow wheel within a cylindrical housing, a set of air guide vanes located either before or after the wheel, and driving mechanism supports either for belt drive or direct connection. The Vaneaxial fan is a result of extension of the simple propeller fan into a precision machine, evolved by application of refined aerodynamic design principles and precise manufacturing procedures and control. Where such principles and technique are observed, excellent characteristics of efficiency, capacity, pressure availability, and sound emission are attainable. Development models are reported to show mechanical efficiencies in excess of 90%. Commercial units are currently offered with mechanical efficiencies in the range of 80% and static efficiencies of 75%. Sizes available as standardized units cataloged in manufacturers' bulletins range from 15 to 72 in. wheel diameter, capacities from 2000 to 150,000 cu ft per min, at static pressures of 1/4 in. water, gage, to 3 in. water, gage; a few models are rated as high as 9 in. water, gage. Standard models are available as belt-driven units or direct-driven by a-c motors of speeds up to 3450 rpm.

The major inherent virtue of the Vaneaxial fan, as a generic class, is its conservation of space and convenience of installation in connection with in-line ducts, with respect to the centrifugal fan. Conversely, where installation in ducts requires a change in direction, the centrifugal fan is better fitted with normal inlet and outlet connections than the Vaneaxial fan, which requires straight-line approach and discharge ducts to retain its normal characteristics of capacity and efficiency. Changes in direction or section of the ducts in close proximity to inlet or discharge of axial fans exert an adverse influence upon performance and should be avoided. In general, the Vaneaxial fan is somewhat smaller, and operates at materially higher speeds, than the conventional centrifugal fan, for comparable performance. The efficiency and sound emission of well-designed and constructed Vaneaxial fans compare favorably with centrifugal fans. The higher precision required in manufacture tends to offset any saving in cost that might accrue from its smaller size. The centrifugal fan is better adapted for applications where hot or contaminated gases are encountered.

AXIAL-FLOW FAN DESIGN ELEMENTS. The theory of airfoil performance and application of aerodynamic principles to axial-flow fan design has become reasonably well established only in recent years. The blade section transfers energy to the gas, and ideal conditions are realized when the transfer is accomplished uniformly along the radius of the blade from hub to tip, resulting in uniform pressure generation, minimum losses, and maximum efficiency and stability. Choice of blade section is dictated by aerodynamic characteristics, varying in practice from precise airfoil profiles of cast, molded or formed materials, to single-thickness plate materials. Attainment of uniform pressure rise along the blade at different radii involves variation of the angle of the blade from hub to tip. The number of blades is related to the pressure required. Low-pressure designs utilize fewer blades, three to five, whereas high-pressure designs may have as many as twenty-four blades.

The diameter of the wheel hub is a function of the pressure characteristics required. In general, low-pressure designs have small hub diameters in relation to tip diameters, ranging from one-third to one-half of the wheel diameter. High-pressure designs utilize larger hubs, 75 to 85% of the tip diameter.

Close clearance between blade tips and fan housing is a stringent requirement of the axial-flow fan. Clearances in excess of 1/32 in. will materially affect efficiency, pressure, and sound level, particularly in higher-pressure designs. Guide vanes reduce the whirl losses imparted to the gas in its passage through the wheel and convert energy of rotation into useful pressure. Commercial practice favors guide vanes on the downstream side of the wheel. Guide vanes may be of cast or formed air-foil shape or formed-plate members.

The cylindrical housing of the Vaneaxial fan may be cast or rolled. The section encasing the blades is formed accurately or subsequently machined to attain the required close clearance. Intake to housing should be fitted with well-formed bell-mouth entry where used without an inlet duct. Outlet diffuser or expanding-cone outlet is frequently utilized to convert a portion of discharge velocity pressure into useful static pressure.

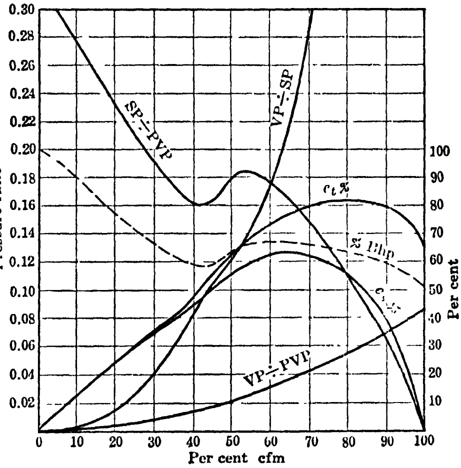


FIG. 23. Characteristic curves of Vaneaxial fan.

AXIAL-FLOW FAN CHARACTERISTICS. The pressure-volume characteristics of axial-flow fans cover wide ranges of absolute values of pressure and capacity, but virtually all possess a relatively steep pressure characteristic from free delivery to a peak pressure, then a flat or recession range beyond which occurs a steep rise again to shut-off or zero delivery. Throughout the low-capacity range from peak pressure back to shut-off the flow through the wheel is in "stall," with consequent low efficiency and high noise level. Operation of the fan should be limited to the range between peak pressure and free delivery to realize the efficient quiet-operating characteristics of a well-

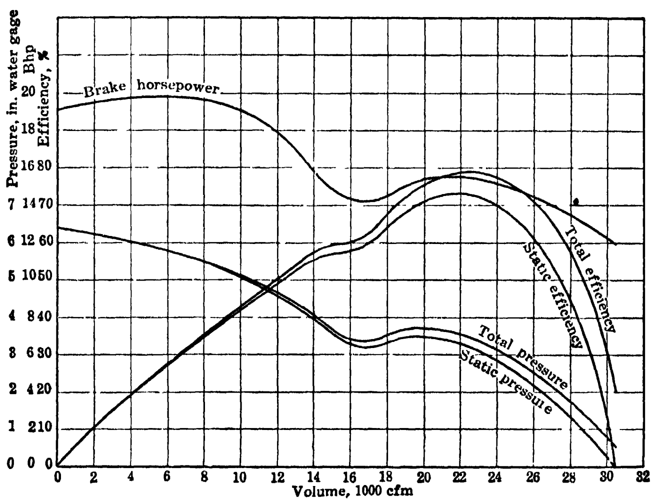


FIG. 24. Characteristic constant speed curves of Vaneaxial fan. (B. F. Sturtevant Co., Hyde Park, Boston)

designed Vaneaxial fan. Where system demand is variable selection of the proper fan size will insure that the fan is never forced to operate in the stall region.

The horsepower characteristic of the conventional Vaneaxial fan is similar in trend to its pressure curve. The power curve is relatively flat in the working range, falling in some degree as free delivery is approached and rising as shut-off conditions are approached. An essentially nonoverloading power characteristic is exhibited by most designs where operation is not shifted too widely from the normal application region. Typical characteristic curves of Vaneaxial fans are shown by Figs. 23 and 24. Tables 11 through 14 provide typical rating capacity tables and physical dimensions of typical Vaneaxial and Tubeaxial fans.

CAPACITY CONTROL OF AXIAL-FLOW FANS. Control and regulation of volumetric capacity may be accomplished by any of the several conventional methods of dampering, variable-inlet vanes, variable-speed or multispeed motors, and adjustable speed devices of the fluid or magnetic type. In addition, the axial fan may be built with adjustable blades to effect variation in volumetric capacity or available pressure. The blade is pivoted and locked in desired position. The method is best suited to service where infrequent changes in capacity are necessary.

28. DUCTS AND DISTRIBUTION SYSTEMS

(See Section 12 for a complete discussion.)

AXIAL-FLOW COMPRESSORS

By E. L. Hunsaker and W. A. Stoner

29. DESIGN CHARACTERISTICS

PHYSICAL DESCRIPTION. The term *axial-flow compressor* identifies a type of compressor that has greatly increased in importance with the advent of the gas turbine as an accepted prime mover. It is distinguished by the annular passage through which working fluid flows in a direction parallel to the rotor axis and by alternate rows of stationary and moving blades that increase the pressure level of the fluid. A moving-blade row and an adjacent stationary-blade row constitute a stage.

Factors favoring this type of compressor are (1) Fluid leaving one stage enters the next stage without long connecting ducts or change in direction, thus permitting staging for higher pressure ratios. (2) The arrangement is compact, lends itself to small size for a given flow capacity. (3) Moving blades lie along radial lines, permitting high rotative speed without excessive centrifugal stress.

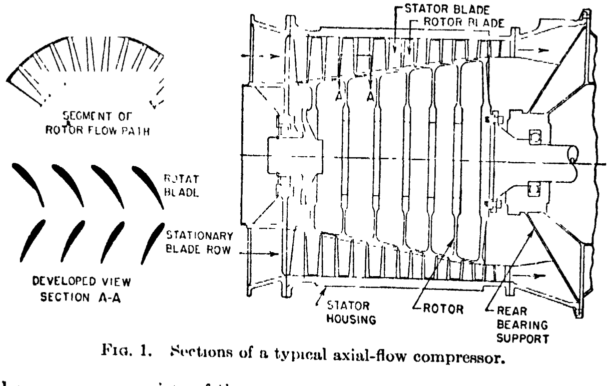


FIG. 1. Sections of a typical axial-flow compressor.

A typical compressor consists of these components.

The rotor supports the moving blade rows (rotor blades) and often forms the inner wall of the annular-shaped duct through which air passes. The rotor may be of hollow, drum-type construction; alternatively, the disk type of rotor may be utilized in which blade rows are mounted in disks shaped to minimize centrifugal stress.

Bearing supports (generally two) form sections of the entrance and exit annular ducting and provide spokes which transmit loads from bearings to the outer casing. A balance piston frequently is used to reduce thrust loads.

Stator housing supports stationary-blade rows (stator blades) and forms the outer wall of the annular duct. The stator housing, which is attached to the bearing supports, may be made in two semicircular sections to facilitate assembly and disassembly.

DISTINGUISHING CHARACTERISTICS. The application of aerodynamic principles to axial-flow compressor design has resulted in machines with high efficiency especially well adapted to certain fields of application. Advantages and limitations are discussed below.

Efficiency. Refined aerodynamic blade design theory makes possible compressor designs in which overall efficiencies exceeding 85% may be realized at design conditions. This high efficiency makes the axial-flow compressor especially attractive in such applications as gas turbine prime movers where compressor power absorption greatly affects shaft output. (See Section 10.) High efficiency is obtained over a relatively small range of operating conditions. The "air receiver" must be matched closely to the compressor characteristic to insure operation near peak efficiency.

Such factors as sensitivity to deposits of foreign matter on blades make the axial-flow compressor more susceptible than other types to efficiency decrease with use. Where space requirements prevent use of inlet-air filters, expedients such as spraying the compressor inlet with cleansing liquid frequently are used.

Airflow and Pressure Ratio. A wide range of design values of airflow and pressure ratio may be obtained with an axial-flow compressor. (See p. 1-111.) The airflow capacity varies as the square of the compressor diameter, and the pressure rise produced varies directly as the number of stages. The facility with which stages may be added to the axial-flow compressor makes it attractive for high compression ratios. However, factors such as off-design-point operation and starting power requirements limit the pressure ratio utilized in practice in a given compressor. A compression ratio of about 8 to 1 is the maximum currently used without resorting to special design features to minimize the above effects.

Because compressor aerodynamic design principles are largely independent of physical size, compressor designs may readily be scaled up or down in size to handle different airflows. The maximum practical size is dictated by manufacturing and economics rather than aerodynamic limitations. In general, more accurate blade profiles can be obtained on larger units so that compressor performance can be expected to improve as size is increased.

As compressor size is decreased, two factors adversely effect performance. Low Reynolds' numbers decrease the compressor efficiency and pressure rise. Manufacture of accurate blade profiles in small sizes further limits compressor performance. Such considerations reduce the attractiveness of axial-flow compressors for very small sizes and favor other compressor types which do not utilize airfoil section blading.

Range of Operation. At design conditions, pressure and airflow of the compressor should match the requirements of the "air receiver" near the peak-efficiency region.

The compressor characteristic (Fig. 11, p. 1-111) has two regions of performance: the *unstalled* or normal operating region in which air follows blade passages without separation, and the *stalled* region in which the air separates from the blades in the majority of the stages. Operation in the stalled region is characterized by low efficiency and possible *surging*, in which violent fluctuating exchanges of flow and pressure take place between compressor and air receiver. The stalled region is therefore to be avoided during normal operation.

During unstalled operation the volume of inlet air handled by the compressor at a given rotational speed is nearly constant regardless of outlet pressure. The region of high efficiency for a given rotational speed is relatively narrow. These considerations restrict the useful operating range of the compressor at a given speed.

A significant change in compressor airflow requires an alteration in rotational speed. To remain near the peak-efficiency region as speed is changed, the pressure airflow characteristic line of the air receiver must be roughly parallel to the stall line. A receiver in which pressure demand is proportional to airflow, such as a turbine nozzle with supercritical flow, usually can be made to match well a compressor characteristic.

Size, Weight, and Geometrical Shape. These factors are of major concern chiefly in mobile applications where space and weight are at a premium. In particular, much attention has been given to size, weight, and shape considerations of the compressor component of aircraft gas turbines. (See Section 15.) The axial-flow compressor has a smaller frontal area than other types, resulting in low external air drag.

Although the centrifugal-type compressor generally weighs slightly less than the axial-flow, the higher efficiency and better shape of the latter make it more attractive for aircraft propulsion units.

Cost. A major problem in the construction of axial-flow compressors is to produce blades economically in the precise and intricate shapes required for efficient performance. Important steps have already been made toward improving manufacturing technique and cutting blade costs. At the present stage of development, the manufacturing cost per horsepower of axial-flow types tends to be somewhat higher than for other compressors.

30. AERODYNAMIC CONSIDERATIONS

SYMBOLS

a	Sonic velocity at compressor inlet stagnation (impact) temperature, ft/sec
A	Annular area, ft^2
A_t	Cross-sectional area, ft^2
b	Distance from leading edge of blade to maximum camber point, ft
c	Absolute velocity, ft/sec
c_p	Specific heat at constant pressure, Btu/lb °F
c_v	Specific heat at constant volume, Btu/lb °F
C_D	Drag coefficient
C_L	Lift coefficient
D	Diameter, ft
E	Modulus of elasticity, lb/ft^2
F	Force, lb
g	Acceleration of gravity, ft/sec^2
I	Moment of inertia, ft^4
J	Mechanical equivalent of heat, $778 \frac{\text{ft-lb}}{\text{Btu}}$
k_1, k_2, k_3	Constants
l	Blade chord, ft
L	Blade length, ft
m	Bending moment due to rotor weight, lb-ft
M	Mach number $= \frac{\text{velocity}}{\text{local sonic velocity}}$
n	Number of blades
N	Revolutions per minute, rpm
p	Static pressure, lb/ft^2
P	Total pressure, lb/ft^2
PR	Total pressure ratio (always greater than unity)
Q	Volume flow, ft^3/sec
r	Radius, ft
R	Gas constant, ft/°R
s	Blade spacing or pitch, ft
S	Stress, lb/ft^2
t	Static temperature, °R
T	Total temperature, °R
u	Wheel speed, ft/sec
w	Velocity relative to blade, ft/sec
W	Weight flow, lb/sec, or weight, lb
x	Distance along rotor axis, ft
y	Deflection of rotor shaft due to its own weight, ft
β	Angle relative to blade measured from compressor axis (degrees)
γ	Ratio of specific heats
δ	$\frac{P_i\ (lb/ft^2)}{2116}$
Δ	Change in quantity
ϵ	Absolute air angle measured from compressor axis, degrees
η	Isentropic efficiency
η_b	Blade row static efficiency
η_{power}	Incompressible flow power efficiency
η_e	Expansion efficiency
θ	T_i °R/520
λ	Trailing edge deviation angle, degrees
μ	Viscosity $\left(\frac{\text{lb-sec}}{\text{ft}^2}\right)$
ν	Poisson's ratio
ξ	Radius ratio $\left(\frac{\text{inner radius}}{\text{outer radius}}\right)$
ρ	Static density, lb/ft^3
σ	Blade solidity, l/s

ϕ Flow coefficient, C_m/u
χ Airfoil camber angle (leading edge camber direction — trailing edge camber direction degrees)
ψ Pressure coefficient, $\Delta P/{}^1/_2\rho u^2_{tip}$
ω Radians/second

SUBSCRIPTS

1 Entering rotating blade row
2 Leaving rotating blade row, entering stationary blade row
3 Leaving stationary blade row
∞ Equivalent velocity at infinity, mean obtained as one-half the vector sum of velocity entering and leaving a blade row
C Compressible
cr Critical
i Inlet
I Incompressible
m Axial direction
OD Outer diameter
u Tangential direction
x Arbitrary radial position

Aerodynamic considerations, of controlling importance in axial-flow compressor design, largely determine the diameter, length, weight, rpm and performance of the compressor. Advances in airplane wing design have been primarily responsible for the excellent compressor performance realized in current machines, since compressor blades are essentially airfoils.

Figure 2 shows a developed view of a typical stage of an axial-flow compressor. When developed in two dimensions, the blade group or *grid* is considered to be infinite in extent.

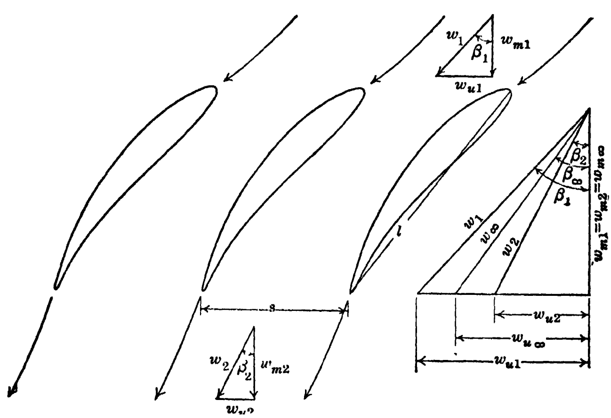


FIG. 2. Typical stage of an axial-flow compressor.

When the ratio of blade chord to blade pitch (solidity) is small, isolated airfoil data available from wing tests may be used directly. The solidity of most compressor grids, however, is sufficiently high so that the mutual interference effect of adjacent blades cannot be ignored. Correction curves are available (Refs. 1, 2) to take account of this effect for moderate blade solidities. For high-solidity grids, isolated airfoil data are not representative, and theoretical or experimental results of grid performance must be utilized.

Basic relationships between grid forces, velocities, and pressure rise will be developed herein. Although the local Mach number (see Section 3, Art. 26) on the airfoil shapes may reach unity, the compressibility effects upon stage velocity diagrams are relatively small for most compressor designs. Stage pressure ratio is generally less than 1.20. As a first approximation the airflow may be considered incompressible. Relationships developed below are for ideal incompressible flow in two-dimensional grids.

RELATION BETWEEN VELOCITY DIAGRAM AND PRESSURE RISE. To illustrate the process by which pressure rise is accomplished, consider a typical two-dimensional blade row acting upon an ideal incompressible fluid. By selecting an observation station on the blade row, equations developed will hold for both moving and stationary blades.

The airfoil-section blades are arranged to turn the air so that the *area* between streamlines leaving the grid is greater than between those entering, resulting in lower velocity and higher static pressure. The process involves converting part of the kinetic energy entering the grid to pressure energy. Bernoulli's equation relates these energies as

$$\frac{w_1^2}{2g} + \frac{p_1}{\rho} = \frac{w_2^2}{2g} + \frac{p_2}{\rho}$$

$$\frac{w_{m_1}^2}{2g} + \frac{w_{u_1}^2}{2g} + \frac{p_1}{\rho} = \frac{w_{m_2}^2}{2g} + \frac{w_{u_2}^2}{2g} + \frac{p_2}{\rho} \tag{1}$$

A second relationship, the continuity equation, equates the weight flow leaving the grid to that entering.

$$w_{m_1} = w_{m_2} \tag{2}$$

Combining eqs. 1 and 2, the change in static pressure through the grid is

$$\frac{p_2 - p_1}{\rho} = \frac{w_{u_1}^2 - w_{u_2}^2}{2g} \tag{3}$$

Substituting, for a rotating-blade row,

$$w_{u_1} = u - c_{u_1}$$

$$\frac{p_2 - p_1}{\rho} = \frac{c_{u_1}^2 - c_{u_2}^2}{2g} + \frac{u}{g}(c_{u_2} - c_{u_1}) \tag{4}$$

The last term represents energy added to the air by the moving blades, which in turn represents the rise in *total* pressure.

$$\Delta P = \frac{\rho u}{g}(c_{u_2} - c_{u_1}) \tag{5}$$

By using alternate rows of rotating blades to add energy to the fluid, and alternate stationary-blade rows properly to direct the fluid for succeeding rotor rows, each blade row contributes to the static pressure rise. Total pressure increases only through moving blade rows.

RELATION BETWEEN AIRFOIL CHARACTERISTICS AND VELOCITY DIAGRAMS. Momentum equations applied to circumferential and axial blade forces are given below. In the circumferential direction

$$F_u = \frac{W}{g}(c_{u_1} - c_{u_2}) \tag{6}$$

In the axial direction:

$$F_m = A(p_1 - p_2) \tag{7}$$

The resultant of these two forces equals the resultant of the lift and drag forces of all blades in the blade row.

For grid flow w_∞ is related to blade lift force and circulation in the same manner that undisturbed free-steam velocity is related to forces and circulation on an isolated airfoil (Refs. 1, 3). All lift and drag coefficients are therefore based upon this velocity.

Equations 3, 6, and 7 combine to relate airfoil characteristics to the velocity diagram:

$$F = \frac{\rho n s L}{g}(w_{u_1} - w_{u_2})w_\infty \tag{8}$$

By definition

$$F = \frac{\rho n l L C_L w_\infty^2}{2g} \tag{9}$$

Then

$$C_L\sigma = 2\frac{(w_{u_1} - w_{u_2})}{w_\infty} \quad \text{where } \sigma = \frac{c}{s} \tag{10}$$

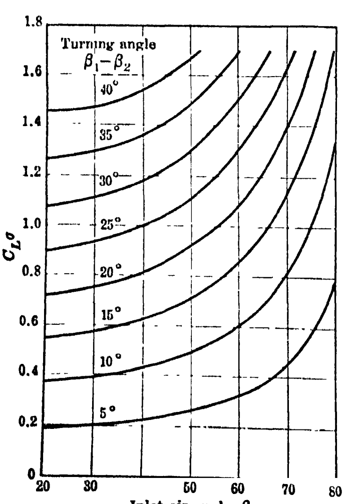


FIG. 3. The relation of $C_L\sigma$ to flow angles.

It is often more convenient to express the velocity diagram in terms of air angles:

$$C_L\sigma = 2(\tan\beta_1 - \tan\beta_2)\cos\beta_\infty \tag{11}$$

Figure 3 relates σC_L to flow angles.

Combining eq. 10 with the head input expression (developed in eq. 5) gives

$$\Delta P = \frac{\rho u}{g} \frac{\sigma C_L w_\infty}{2} \tag{12}$$

The total pressure rise obtained in the grid is a function of allowable wheel speed and relative velocity, limited in the actual compressor by Mach number considerations and the solidity times lift coefficient, which is limited by airflow separation effects in the blading.

LOSSES AND COMPRESSIBILITY. In a conservatively designed axial-flow compressor, the effect of losses and compressibility upon velocity diagram relationships is small. When losses and compressibility are considered, the energy equation replaces Bernoulli's equation for grid flow:

$$\frac{w_1^2}{2gJ} + c_p t_1 = \frac{w_2^2}{2gJ} + c_p t_2 \tag{13}$$

The static pressure ratio of the blade row is

$$\frac{p_2}{p_1} = \left[1 + \eta_b \left(\frac{w_1^2 - w_2^2}{2gJc_p t_1}\right)\right]^{\gamma/(\gamma-1)} \tag{14}$$

The equation relating airfoil characteristics to the velocity diagram is

$$C_L\sigma = \frac{2 \cos \beta_\infty (\tan \beta_1 - \tan \beta_2)}{1 + \dfrac{C_D}{C_L} \tan \beta_\infty} \tag{15}$$

when axial-velocity changes are small across the blade row.

The C_D/C_L ratio, combined with the velocity vector diagram, defines the efficiency of blade and stage.

The energy equation across the moving blade row is

$$\Delta T_{1-2} = \frac{u}{gJ} (w_{u_1} - w_{u_2}) \tag{16}$$

Total pressure ratio for the compressor stage is then

$$PR = \left(1 + \eta \frac{\Delta T_{1-2}}{T_i}\right)^{\gamma/(\gamma-1)} = \left(1 + \eta \frac{u(w_{u2} - w_{u1})}{gJc_pT_i}\right)^{\gamma} \tag{17}$$

where η is the isentropic efficiency of the stage.

The overall isentropic efficiency of a multistage compressor is defined by the ratio of temperature changes (isentropic and actual) in eq. 18, assuming constant specific heat of the fluid.

$$\eta = \frac{T_i(PR^{(\gamma-1)/\gamma} - 1)}{\Delta T_{\text{actual}}} \tag{18}$$

The use of isentropic efficiency in defining compressor performance results in an overall value lower than the isentropic efficiency of the individual stages, because of the existence of *reheat factor*.

AERODYNAMIC LIMITS. Many factors affect performance of axial-flow compressors. Some have such pronounced effects that they limit the practical design range. These limits are generally characterized by sharp breaks in the performance, usually brought about by sudden changes in flow pattern. The successful design does not exceed these limits, but approaches as closely as possible to them whenever this enhances the performance of the machine.

Equations 11 and 12 indicate that maximum $C_L\sigma$ or air-turning angle and maximum Mach number limit the pressure rise obtainable in a grid. They are discussed below.

Rate of Diffusion. The maximum $C_L\sigma$ or the turning angle which can be utilized in a compressor grid is determined by the lift coefficient or air deflection at which the airflow separates from the blading. This is a function of the allowable diffusion rate in the blade passage. The greatest rate of diffusion occurs on the convex surface of the airfoil shape, and it is here that separation first occurs. The blade row is then said to be stalled, and there is a sudden loss in pressure rise and efficiency. A limit to rate of diffusion may be set up by limiting either the lift coefficient or air-turning angle for a particular blade configuration.

The allowable value of $C_L\sigma$ air-turning angle must be determined from experimental data upon the particular blade shapes utilized. For low-solidity grids, $\sigma < 1.0$, NACA experimental data on isolated airfoils furnish a background for selecting maximum values of lift coefficient. Such factors as wall effects, mutual interference of velocity fields, nonuniform flow conditions, and considerations of off-design-point compressor operation limit the design C_L considerably below maximum values obtained in isolated airfoil wind tunnel tests. Typical current practice is a design C_L of 0.8 with $\sigma = 1.0$.

For high-solidity grids maximum turning angles utilized are currently about 30 degrees (Ref. 4).

Mach Number. Mach number, defined as the ratio of fluid velocity to speed of sound in the fluid (see Section 3, Art. 26), is a measure of compressibility. Mach numbers are less than 1.0 for subsonic flows, and greater than 1.0 for supersonic. The allowable value is determined by the velocity at which the *local* airfoil surface Mach number becomes sufficiently large so that compressibility effects give large shock losses and inefficient operation.

Conventional compressor blades are designed for subsonic flow, and they will not operate satisfactorily at local Mach numbers much in excess of 1.0. Shock waves appear when the critical Mach number is reached and increase in intensity with increasing entering Mach number. The loss in total pressure through a moderate shock wave is not excessive, but the change in air angle through an inclined shock will generally result in separation of the flow from the blade surface, unless the blade is explicitly designed for the condition. The resulting losses are always large.

Experimental or theoretical blade-surface velocity distributions are required to relate local surface Mach numbers to those of the velocity diagram. When the above data are available for the incompressible flow case, an approximate correction developed for isolated airfoils is frequently used to account for the change with entering Mach number in the ratio of local velocity to entering velocity. (See Ref. 7.)

$$\left(\frac{p_{\text{surface}} - p_{\text{stream}}}{\frac{\rho_C w_C^2}{2g}}\right)_C = \frac{\left(\dfrac{p_{\text{surface}} - p_{\text{stream}}}{\frac{\rho_I w_I^2}{2g}}\right)_I}{\sqrt{1 - M^2} + \dfrac{M^2}{2(1 + \sqrt{1 - M^2})}\left(\dfrac{p_{\text{surface}} - p_{\text{stream}}}{\frac{\rho_I w_I^2}{2g}}\right)} \tag{19}$$

where M = stream Mach number.

Mach number limitations are most important for the first stages of a compressor. Here the temperature, and accordingly the speed of sound, is low; on the other hand, the volume flow of air is highest. Thick, closely spaced, highly cambered blades may be limited to entering Mach numbers of 0.6; thin, widely spaced, low cambered blades may operate well with entering Mach numbers above 0.8.

REYNOLDS' NUMBER. Turbulence has long been known to correlate with Reynolds' number. Since any diffusion process is enhanced by turbulent interchange of energy between the center stream and the boundary layer, the efficiency of a compressor is affected by Reynolds' number. Reynolds' number of a blade row generally is based on the blade chord and entering air velocity.

$$\text{Reynolds' number} = \frac{w_1 l \rho}{g\mu} \tag{20}$$

Tests have indicated that compressor efficiency and maximum pressure rise increase with increasing Reynolds' number, though not at a pronounced rate in the normal range of operation. Although effects of Reynolds' number are normally small, an abrupt decline in efficiency occurs with decreasing Reynolds' number at values below 100,000. This limits the attractiveness of utilizing axial-flow compressors in small units.

VELOCITY DIAGRAMS. Selection of a velocity diagram is one of the first problems of compressor design. Numerous configurations will satisfy a given performance requirement; it is a matter of judgment and experience to select one that will do the job most efficiently. The discussion that follows is concerned with velocity diagrams for a *repeating* stage, as in the center of a compressor, when the same blading is used for preceding and following stages. One requirement for a repeating stage is that the whirl component of velocity leaving the stage equal that entering. This requires the same change in whirl velocity in stator and rotor

Then, in Fig. 4

$$c_{u\,\infty\text{rotor}} = c_{u\,\infty\text{stator}} \tag{21}$$

$$u = w_{u\,\infty\text{rotor}} + c_{u\,\infty\text{stator}}$$

$$\frac{u}{c_m} = \tan\beta_\infty + \tan\epsilon_\infty \tag{22}$$

Equations developed above are independent of the velocity diagram.

Types of Compressor Stage. Velocity diagrams may be classified according to the relative pressure rise in rotating and stationary elements. The terms *impulse* and *reaction* refer to the action of the rotor blades upon the working fluid. In an impulse stage (Fig. 5*a*) the rotor changes the fluid direction without changing the magnitude of the velocity relative to the moving blades. As a result, there is no change in static pressure through the rotor, but merely an increase in absolute velocity. Stator blades then convert part of this increased kinetic energy into pressure energy.

In a 50% reaction stage (Fig. 5*b*) half the static pressure rise occurs in the rotor and half in the stator. The velocity diagram is described as *symmetrical* because rotor and stator blades are set at the same angle, one being the mirror image of the other. In a 100% reaction stage (Fig. 5*c*) all the static pressure rise occurs in the rotor. The stator blades merely change direction of flow without changing magnitude of velocity or pressure.

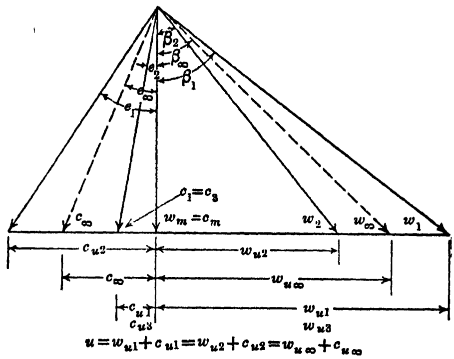


FIG. 4. Typical compressor stage velocity diagram for repeating velocity pattern.

These cases illustrate extremes. Any intermediate configuration, or even the more extreme cases for which there is a static pressure *drop* in rotor or stator, may be used.

SELECTION OF A VELOCITY DIAGRAM. Several factors must be considered in the selection of the velocity diagram.

1. For a given C_D/C_L the loss due to airfoil drag is a minimum with the blade at an angle of about 45 degrees. This favors a symmetrical velocity diagram with the axial component about half the wheel speed.

2. The above condition does not give the highest pressure rise for a given wheel speed when Mach number is not the limiting consideration. If the wheel speed cannot be increased because of other considerations, the pressure rise may be increased at some expense to efficiency by increasing the axial velocity or, in some cases, by adopting a nonsymmetrical diagram.

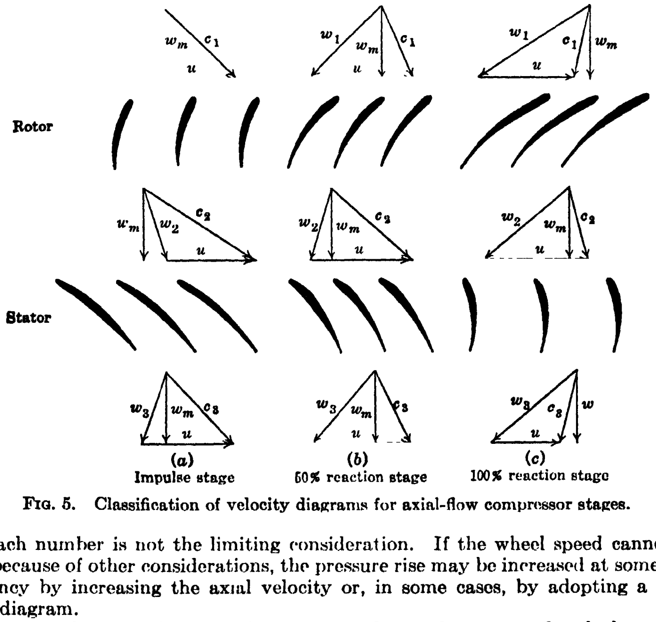


FIG. 5. Classification of velocity diagrams for axial-flow compressor stages.

3. If rotor and stator blade tip clearances are large, the reverse flow leakage at these points may result in important losses. These can be minimized on shrouded blades by keeping the static pressure rise across the blade row small near the blade tip. This is

done by biasing the design toward an impulse diagram near the outer radius, and toward a 100% reaction diagram near the inner radius.

4. Boundary layers forming on the blade surfaces tend to be thrown outward by centrifugal force on rotor blades, and be pushed inward by the pressure gradient in the surrounding air on stator blades. This effect means that more diffusion may be done at the inner radius by the rotor and at the outer radius by the stator before excessive boundary layer build-up causes separation and stall. This consideration also favors an impulse diagram near the outer radius, and a 100% reaction diagram near the inner radius.

5. Friction losses at the inner and outer walls can be important. They can be minimized by using blades of high aspect ratio and small axial clearance, and by selecting a velocity diagram with low velocity relative to the wall. This means low axial velocity, low whirl velocity at the outer wall, and whirl velocity close to the rotor speed at the inner walls. This consideration favors a 100% reaction diagram at the outer radius, and an impulse diagram at the inner radius.

6. If Mach numbers relative to the blades are near critical values, they become important considerations in selection of a velocity diagram. For structural reasons, it is generally true that the rotor blade may be made thinner at the outer radius than the stator blade. This implies higher permissible relative velocities near the rotor blade tips, favoring a 100% reaction diagram near the outer radius and an impulse diagram near the inner radius.

7. Since wheel speed varies with radius it is not possible to select a set of velocity diagrams that will give constant blade sections for both rotor and stator. If cost of the unit is important, the possibility should be considered of making one a constant section at all radii while the other is allowed to assume a twist. This is approximated by use of constant exit angles from one of the blade rows.

The conflicting requirements of the factors listed indicate that a wide range of velocity diagrams may be utilized, depending upon the relative importance assigned to the various considerations.

Theory is not adequate to predict the best design. The importance of carefully planned and executed tests of proposed designs cannot be overemphasized

BLADE ANGLE SETTING. When the grid velocity diagram, aerodynamic coefficients, and blade shape have been selected, the blade angle must be chosen to conform. For low-solidity grids, the blade setting is based upon attack angle required between w_∞ and the blade chord line to develop the chosen lift and drag coefficients. The attack angle may be taken from isolated airfoil data with suitable correction factors applied for mutual interference (Refs. 1, 2).

In high-solidity grids conventional practice is to base the blade angle upon the grid discharge flow angle desired. Reference 4 indicates that proper angle settings are obtained for a wide range of solidities and blade angles if the air deviation angle, or angle between trailing edge camber line direction and leaving air direction, is given by eq. 23.

$$\lambda = \left[0.23\left(\frac{2b}{l}\right)^2 + 0.1\left(\frac{\beta_2}{50}\right)\right](\chi)\left(\frac{s}{l}\right)^{1/2} \tag{23}$$

The leading edge camber line direction is generally set parallel to the direction of the entering air.

RADIAL EQUILIBRIUM CONSIDERATIONS (VORTEX DESIGN). The aerodynamic equations developed above refer to two-dimensional flow. In most compressors the ratio of blade height to diameter is sufficiently large so that the flow cannot be considered two dimensional. If, however, the flow stream lines have only a small radial motion, the flow may be considered as taking place on a series of thin coaxial cylinders. Two-dimensional flow theory may be applied to the developed grid contained within each thin cylinder.

For a typical stage in a multistage compressor, the flow pattern at each radius repeats after passing through each successive stage.

The pressure rise at each radial station on the rotor blade must be constant to comply with the repeating pattern. Thus

$$\Delta c_u = \frac{k_1}{r}$$

$$\Delta P = \rho k_2 r \Delta c_u = \rho k_1 k_2 \tag{24}$$

To establish the radial distribution of whirl velocities that will satisfy the condition of no radial flow, ideal incompressible flow across a blade row is examined at stations 1 and 2 which are at the same radial location of the blade row, upstream and downstream.

$$\iota + \frac{k_1}{r} \tag{25}$$

The energy equation is

$$p_1 + \frac{\rho c_{u_1}^2}{2g} + \frac{\rho c_{m_1}^2}{2g} + \rho k_1 k_2 = p_2 + \frac{\rho c_{u_2}^2}{2g} + \frac{\rho c_{m_2}^2}{2g} \tag{26}$$

For no radial acceleration at stations 1 and 2,

$$\frac{dp}{dr} = \frac{\rho c_u^2}{r} \tag{27}$$

If no radial displacement of streamlines occurs, the axial velocity distribution must be the same behind the blade row as ahead for incompressible flow.

$$c_{m_1} = c_{m_2}$$
$$\frac{dc_{m_1}}{dr} = \frac{dc_{m_2}}{dr} \tag{28}$$

Differentiating eq. 26, and combining with eqs. 25, 27, and 28, gives

$$c_{u_1} \cdot r = k_3 \tag{29}$$

Thus, *to satisfy the condition of no radial streamline acceleration and motion, the whirl velocity distribution must vary inversely as the radius.* This is the well known *free-vortex flow pattern.* It is interesting to note that the conclusions apply both to rotating and stationary blade rows.

If the additional condition of equal total streamline energies at all radial positions is imposed, the axial-velocity distribution is constant with radius. Conventional compressor designs follow this distribution. When the velocity diagram has been chosen at any radial position, the free-vortex pattern establishes the diagram at all other radial locations.

When the energy added by the blade row is constant, radially, the energy input is generally limited by conditions at the blade inner radius where large values of Δc_u are required.

TYPICAL SINGLE-STAGE PERFORMANCE. It is helpful to present single-stage compressor performance data in the form of dimensionless parameters. This is a suitable set of parameters for the incompressible case.

Flow Coefficient.

$$\phi = \frac{c_m}{u_{\text{tip}}} \tag{30}$$

A particular flow coefficient determines the angle of attack at which each blade section in a stage will operate.

Pressure Coefficient.

$$\psi = \frac{\Delta P}{\rho\left(\frac{u_{\text{tip}}^2}{2g}\right)} \tag{31}$$

This expresses total pressure rise in the stage as a ratio to the dynamic pressure the fluid would have moving at the rotor tip velocity.

Efficiency.

$$\eta_{\text{power}} = \frac{\Delta P \cdot Q}{\text{power input}} \tag{32}$$

These three parameters completely describe the performance of a compressor for a range of conditions in which Reynolds' number and compressibility effects are unimportant. ϕ is the independent variable which determines stage performance. Typical performance data for a single stage compressor are shown in Fig. 6, making use of the above parameters. These curves apply very closely for a wide range of rotor speeds. To know that blade angles of attack increase as flow coefficient decreases helps in visualizing the compressor performance. At higher than design flow coefficient the blades are at a low angle of attack and little work is being done on the air. Thus the low pressure coefficient. As the flow coefficient is decreased, the angle of attack increases until the point is reached where the lift to drag ratio is most favorable. This point corresponds to the peak in the efficiency curve; a further decrease in flow coefficient brings the angle of attack to the angle at which the blades stall. A sudden decrease in pressure coefficient and efficiency occurs. The performance continues to be poor for all lower flow coefficients. Thus the ϕ, ψ curve for a compressor is found to be

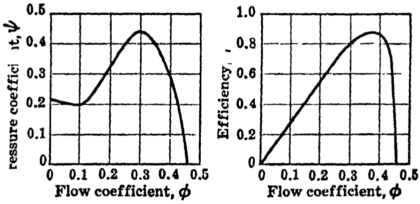


FIG. 6. Typical single-stage compressor performance characteristics in terms of dimensionless parameters. (See eqs. 30, 31, and 32.)

similar to a lift coefficient versus angle of attack curve for a single airfoil, whereas the ϕ, η curve for a compressor bears a similarity to a lift to drag ratio versus angle of attack curve for a single airfoil.

INLET GUIDE VANES. The first row of blades in a compressor may be either moving or stationary. If stationary, they are designed to impart an initial whirl to the working fluid either with or against the moving blades of the first rotor. Since the angle of attack entering the vanes will not change, a thin vane-like construction is often used with good results. Thus the term *guide vanes*. Costs may be minimized by making guide vanes of sheet metal stampings.

Since guide vanes add whirl to the working fluid, their use is accompanied by a drop in static pressure. They are justified only if they enhance the performance of the following blade rows. The usual reason for using them is to keep the Mach number on the first rotor blade below the critical value. In this case, whirl is added in the direction of rotor rotation. Where rotor speed is low and Mach number is not a problem, it may be found desirable to direct whirl *against* the rotor, thereby increasing the pressure rise in the first stage.

INLET AND DISCHARGE DUCTING. During its passage through a compressor, the working fluid has a significant portion of its energy in the kinetic form. This is true at the inlet to the first stage and at the discharge from the last one. It is the function of inlet ducting to convey air to the first stage inlet with minimum loss and with uniform velocity profile. It can best be done if the ducting is short with smooth contours, without sharp convex corners. If possible, the air is accelerated at all points along the duct by a smoothly converging section. This condition minimizes boundary layer and consequent losses, providing an ideal entrance condition. Deceleration of air in entrance ducting is sometimes unavoidable, as in a unit mounted in a very high speed aircraft, and where space limitations require restriction in duct cross-sectional area. In either, higher losses occur; use of a long, gradually diverging duct in these regions is indicated.

For most applications high static pressure at the compressor discharge is desired. Energy tied up in velocity leaving the last stage serves no purpose unless converted to pressure energy; hence the last stator stage usually is designed to discharge the air axially, leaving no whirl component. The discharge ducting then must diffuse the axial velocity, converting as much of the kinetic energy into pressure energy as practical. A smooth, gradually diverging duct is desirable.

31. MECHANICAL DESIGN FEATURES

BLADE STRESS AND VIBRATION. Stress and vibration considerations may determine the maximum blade length permissible with a given diameter and rpm. Compressor blade chord generally is fixed by allowable air bending stress. The blade chord, in turn, determines overall compressor length.

Blade stresses may be divided into three types: centrifugal stresses, independent of blade chord; steady gas bending stresses, which may be controlled by the blade chord selected; and vibratory gas bending stresses, generally assumed to be some function of the steady gas bending stress. Steady stresses are more readily calculated.

The centrifugal stress at any blade radius is given by

$$S_x = \frac{\displaystyle\int_{r_x}^{r_{OD}} \frac{\rho A_t r \omega^2 dr}{g}}{A_{t_x}} \tag{33}$$

For calculating bending stress the blade may be considered as a beam and usual methods of stress analysis applied. The load on any element of the blade is given by eqs. 6 and 7. Blade section properties may be found by graphical integration.

By inclining the center of gravity line of the rotor blades with respect to the radial direction, a centrifugal force bending moment may be introduced which *opposes* the design condition gas bending moment. Similar action may be obtained at all operating conditions by use of a flexible rotor-blade mount, permitting the blade to seek its own radial orientation.

Vibration. The problem of blade vibration does not yield readily to quantitative analysis. Theory, however, is helpful in predicting conditions at which vibration may be expected. There are several causes of vibration. Periodic pulsations in the air stream can cause blade vibration if they match one of the natural frequencies of the blade. This may occur when rotor blade wakes pass stator blades or when stator blade or strut wakes are passed by rotor blades. If the exciting pulse has a frequency which is an even multiple of a blade natural frequency, large vibration amplitudes may result. The amplitude of the vibration will increase until energy absorbed by damping forces equals energy fed into the vibrating system.

It is helpful to plot frequency of pulsations and frequency of blade vibration as a function of compressor rpm. The intersection of two lines represents an operating condition where vibration may be excited. Figure 7 illustrates such a plot for a typical single-stage machine.

The frequency of a twisted blade of varying section is difficult to calculate. Whenever possible, an actual blade mounted to simulate attachment in the machine is vibrated and natural frequencies measured. The blade usually is found to vibrate readily at several different frequencies, each representing a different nodal pattern. Thus several lines may appear on the frequency plot for a single blade. It is readily seen that in a multistage compressor, it is almost impossible to avoid all the resonant conditions between blades and periodic pulsating forces. Resonance with large periodic forces, such as inlet strut wakes, should be avoided.

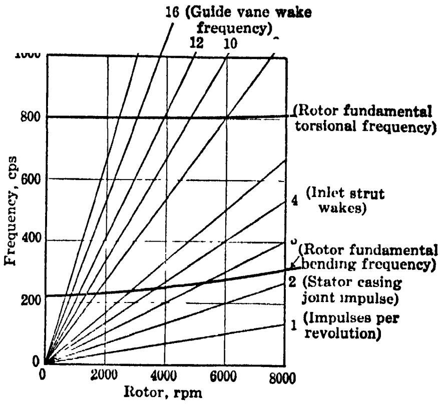
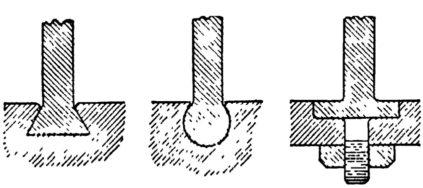


FIG. 7. Interference diagram for single-stage compressor.

Blade self-induced vibrations may result from several conditions. Classical *flutter* may occur if the blade torsional frequency is low and the relative air velocity high (Ref. 5). Stall flutter and Kármán vortices may also excite the blade row to large resonant amplitudes. Few data are available on these effects, although, in general, blades of high torsional frequency and low aspect ratio appear less susceptible to self-induced vibration difficulties. Because of insufficient data on vibratory loads, the value of vibratory stress is generally taken as several times the steady gas bending stress.

It is generally conceded that blade vibration cannot be entirely avoided but must be allowed for in compressor design. The effects of vibration are minimized by selection of blade materials with good damping properties, by use of a nonrigid blade attachment designed to dampen vibration, and by use of shrouds or wires connecting adjacent blades.

AXIAL AND RADIAL BLADE CLEARANCE. Mechanical considerations require that ample radial and axial blade clearances be maintained. Radial blade-tip clearance must accommodate eccentricity of the parts, differential thermal expansion, uncertainty of shaft location in bearings, and the strain of moving parts due to centrifugal force. Large blade-tip clearances, however, are detrimental to aerodynamic performance of the compressor; these losses generally limit the ratio of clearance to blade length. Current practice is to maintain this ratio less than about 3%. Nominal changes in axial clearance between moving and stationary blades do not affect compressor performance to a marked extent. The axial clearance is generally dictated by consideration of axial rotor shift, differential thermal expansion, and blade bending upon load.



Dovetail anchor Cylindrical anchor Bolted anchor

FIG. 8. Blade anchor attachments.

BLADE ATTACHMENT. The desired blade-attachment features are light weight, low cost, ability to withstand high blade loads, ease of blade replacement, and high vibratory-damping coefficient. The large number of methods of blade attachment now in use testify to the fact that no single design fully complies with all requirements. A few common methods of attachment are the dovetail anchor, cylindrical anchor, and bolted attachment, as shown in Fig. 8.

ROTOR CONSTRUCTION. Two commonly used forms of rotor construction are the hollow-drum type and the disk type. The hollow-drum type is simpler but limited to applications with moderate rotor speeds.

Centrifugal stress in a thin hollow-drum rotor, without blades, is given by:

$$S = \rho u_{rim}^2 \qquad (34)$$

while for a constant-thickness disk-type rotor, without blades, the maximum stress (at the center of rotation) is given by

$$S = \frac{3+\nu}{8} \cdot \frac{\rho u_{\mathrm{rim}}^2}{g} \tag{35}$$

For steel $\nu = 0.3$, so that the constant-thickness disk-type rotor has a centrifugal stress 41% of that of a drum-type rotor of the same material and peripheral speed. Furthermore, the maximum stress in a disk-type rotor may be reduced by thickening the disk near the center. Thickness distributions calculated to produce constant disk stress are commonly used.

It is of interest to compare the peripheral speeds obtainable with different types of rotors without rims or blades, assuming the same allowable tensile stress for each. For steel with a density of 0.282 lb per cu in. and an allowable stress of 35,000 psi, a peripheral speed of 576 ft per sec is permissible with the drum type and of 897 ft per sec with the constant-thickness disk type. A disk with tapered thickness is necessary if higher speeds are required. Stodola (Ref. 6) suggests a practical limit of 1300 ft per sec above which unreasonable disk shapes are encountered.

The method used to connect disks of a multistage disk-type rotor must provide enough rigidity to keep the critical speed at a safe value. If stress considerations permit, it is convenient to do this by joining the disks by rims near the periphery. If this is impractical, more than two rotor bearings may be required.

STATOR CONSTRUCTION. The stator structure connects the front and rear bearing supports, forms the outer wall of the annular duct, and supports stationary blading. It must withstand pressure forces, transmit blade loads to bearing supports, and provide sufficient rigidity to the machine. Rigidity of the stator assumes great importance because of its effect on rotor critical speed. The advantage in critical speed gained by use of center bearings is partially lost if the structure of the stator is not rigid.

A typical stator consists of an outer casing, split horizontally and held together by a bolted flange to provide ease of disassembly. Bolted flanges are also employed to attach the casing to bearing supports. Stator blades are attached to semicircular rings mounted in the outer casing, making any stator blade row readily replaceable.

CRITICAL SPEED OF ROTATING ELEMENTS. Critical speed considerations often are controlling in rotor design, particularly for multistage machines with a considerable bearing span. In general, rotor operating speeds are sufficiently high that design analysis should include calculation of critical speed.

The margin required between critical and operating speeds varies with the application and depends upon the accuracy with which the rotor is balanced and the degree of symmetry of parts which give the rotor rigidity. Nonsymmetry will result in different natural frequencies in different planes of vibration, and consequently a widened range of critical operating speeds. Careful dynamic balancing of the rotor will help to narrow the range of speeds at which operation is dangerous, but perfection is never attained. It is good practice to design for an operating rpm at least 20% below the critical speed.

If it is impractical to design with enough rigidity for operation below critical speed; operation above it may be necessary. This entails careful balance of the rotor and rapid acceleration through the dangerous region. Since the rotor will tend to rotate about its center of gravity rather than its mechanical axis, some flexibility in shaft and bearing mounts is desirable. If operation is intended at speeds considerably above the first critical, calculation of the second critical speed is desirable.

Most compressor rotors are of such complex form that standard critical-speed equations do not apply, and graphical integration must be used. A useful equation for this purpose relates critical speed to deflection of the shaft under its own weight and distribution of weight along the shaft:

$$N_{cr} = \frac{60}{2\pi}\sqrt{\frac{g\int y\,dW}{\int y^2\,dW}} \tag{36}$$

The two integrals are performed along the shaft length by graphical means. The deflection (y) may be found by a graphical integration using the expression

$$\frac{d^2y}{dx^2} = \frac{m}{EI} \tag{37}$$

The quantity m/EI is plotted as a function of x and the resulting curve integrated twice to obtain a plot of y versus x.

This method is not exact, in that it assumes that the deflection curve of the shaft under its own weight corresponds in shape to the deflection of a vibrating shaft; results, however, are very close. Sometimes gyroscopic effects are also important in determining rotor critical speed (Ref. 6)

32. PERFORMANCE CHARACTERISTICS

PEAK EFFICIENCY POINT. In designing a multistage compressor, the pressure ratio, weight flow, and rotor speed at which peak efficiency is desired must be selected. When this is done, each stage can be designed to give maximum efficiency at these conditions. This implies the proper choice of annular areas through the machine to give suitable axial velocities entering each stage and selection of blade angles to provide the optimum angle of attack at design conditions.

In some applications, it may be desirable to sacrifice efficiency at the design point to obtain smaller dimensions. In such cases the design values of Mach number and blade loading do not correspond to peak efficiency values. Peak compressor efficiency then generally occurs at lower rpm and pressure ratio than design value.

OFF-DESIGN PERFORMANCE. When the design of a multistage compressor is fixed, operation at an off-design point will always result in a change in attack angle on some of the compressor stages. Consequently, any large variation from the design condition usually results in a decrease in efficiency, as can be illustrated by examination of a few cases.

Design Rotor Speed, Small Change of Flow. With a small decrease in flow the first compressor stages are only slightly affected, their pressure ratio being slightly higher than design, since they operate at a somewhat larger angle of attack. The effect is cumulative, however, and in the latter stages, where increased pressure results in higher density and lower axial velocity, a small decrease in inlet flow to the compressor will result in a large attack-angle increase. Thus pressure ratio of the latter stages will increase greatly and result in an overall pressure ratio increase and an efficiency decrease. When the angle-of-attack change is sufficiently large so that a number of the latter stages are operating stalled, the overall pressure ratio may decrease with a flow decrease. This occurs in the stall or inefficient operating region of the compressor.

A small increase in inlet flow will increase the axial velocity in the last stages, thereby greatly decreasing the overall pressure ratio and the efficiency. Thus a machine designed for a large pressure ratio approaches the characteristic of constant flow at constant rotor speed.

Rotor Speed Too Low, Design Flow Coefficient at First Stage. In this case the angle of attack on the first-stage blading is correct, but the pressure rise is too low to maintain design flow coefficient on succeeding stages as the annular area decreases through the machine. As a result, the last stages operate at too high a flow coefficient and too low an angle of attack. In extreme cases the latter stages may perform as inefficient turbine stages taking a large pressure drop and returning some power to the shaft.

When necessary to operate at off-design rotor speed, best performance is obtained by operating with a flow corresponding to design flow coefficient somewhere near the middle stages.

If off-design performance is necessary a good part of the time, and high efficiency is important, some scheme for improving the off-design efficiency should be considered. Since poor angle of attack is the basic cause of the poor efficiency, one obvious remedy is to build a unit with variable blade angles. The blades in each row would be turned a different amount. A considerable improvement in off-design performance could be realized by adjustment of stator blade angles alone, which would somewhat simplify the difficult mechanical problem.

An alternative is to change angles of attack by changing rotor speed. The first and last sections of the compressor would be on different shafts driven at different speeds.

A third possible way of regulating angles of attack is to adjust the flow area. It is not feasible, mechanically, to change the annular area in a bladed compressor, but some other schemes do this, in effect. If a compressor is operating at a speed below design so that the first stages are stalled and the last stages are operating with too high a flow coefficient, it is often possible to obtain a substantial gain in performance by bleeding off part of the air at some intermediate stage. This action wastes energy in the bled air, but more than pays for itself by unstalling the early stages, allowing less air to pass through the latter stages so that they perform efficiently.

STALLING AND SURGING OF A COMPRESSOR. To analyze operation of a compressor, characteristics of both compressor and air receiver to which the compressor supplies air must be known. Such a system might consist of ducting with pressure losses, a combustion chamber and turbine, or any other system requiring air at increased pressure. In any event, the system generally requires increased pressure for increased weight flow, and therefore has a characteristic typically represented by line A in Fig. 9.

The compressor, on the other hand, produces greater pressure with decreasing flow and has a characteristic represented by line B. The two lines intersect at point P, and the compressor will operate at this point when connected to the system and operated at the same speed. To alter the point of operation, holding constant compressor speed, the characteristic curve of the system must be changed. This might be done by placing a throttle valve in the ducting or by firing the combustion chambers to a higher temperature in a gas turbine application. The resulting new characteristic is represented by the dotted line C, which is illustrated as the special case where the two lines no longer intersect at a point but are nearly tangent along a section of the curve. Slight changes in condition produce large unstable changes in flow and pressure. The resulting performance, known as *surging*, is characterized by violent fluctuation in the flow through the compressor and system. The stalled part of the compressor curve is characterized by inefficient operation. The stall or surge line of a compressor, therefore, is a limit line bounding the useful operating region (Fig. 11). Performance of a multistage compressor in the stalled region cannot be readily predicted from performance of single-stage components, because of the complex three-dimensional flow patterns in the multistage unit.

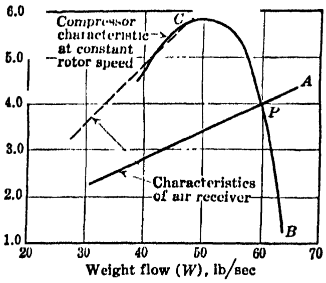


FIG 9. Characteristic curves of compressor and air receiver.

33. DESIGN PROCEDURE

For most applications airflow and pressure requirements of the axial-flow compressor are determined by external conditions. Many different designs will satisfy the requirements. Consideration of the aerodynamic and mechanical factors discussed in the previous sections and evaluation of their relative importance for the particular application aid in selecting a design.

The inlet compressor stage usually has the most severe aerodynamic and stress conditions and to a large extent determines compressor rpm and diameter. When the inlet stage design has been selected the compressor diameter is given by eq. 38.

$$Q \quad c_{m_i} \frac{\pi}{4} D_i^2(1 - \xi_i^2) \tag{38}$$

The inlet static air density, ρ, is lower than ambient, ρ_{amb}, because of the inlet pressure drop required to accelerate the air. Its value may be found by a simple calculation which takes into account efficiency of expansion, η_e, as in the following equation:

$$= \rho_{amb} \frac{\left(1 - \dfrac{c_i^2}{\eta_e 2gJc_pT_i}\right)^{\gamma/(\gamma-1)}}{\left(1 - \dfrac{c_i^2}{2gJc_pT_i}\right)} \tag{39}$$

where

$$\rho_{amb} \quad \frac{P_i}{RT_i}$$

The weight flow of air is found by combining with eq. 38:

$$W \quad \frac{P_i}{RT_i} \frac{\left(1 - \dfrac{c_i^2}{\eta_e 2gJc_pT_i}\right)^{\gamma/(\gamma-1)}}{\left(1 - \dfrac{c_i^2}{2gJc_pT_i}\right)} c_{m_i} \frac{\pi}{4} D_i^2(1 - \xi_i^2) \tag{40}$$

where the subscript i refers to values entering the first stage.

When compressor diameter is determined the operating rpm is given by

$$N \cdot \frac{60}{\pi} \frac{u_{tip}}{D_i} \tag{41}$$

As air passes through the compressor and density increases, the annulus flow area usually is decreased to maintain an approximately constant axial velocity. If the stage

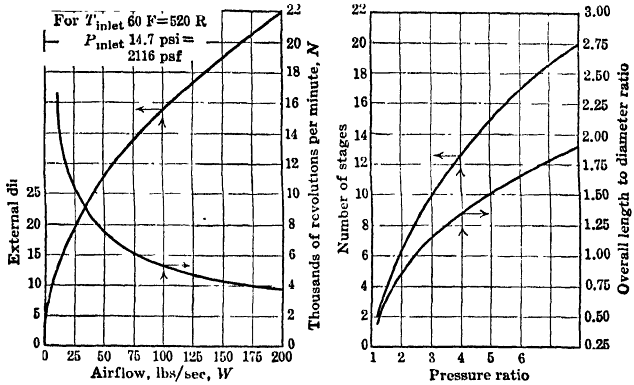


FIG. 10. Typical compressor dimensions and rpm as a function of airflow and pressure ratio.

velocity diagram is constant throughout the compressor, the air temperature rise at each stage is the same. The number of stages required to produce a given pressure ratio is expressed below, where η is overall isentropic efficiency.

$$\text{Stages} = \frac{(PR^{(\gamma-1)/\gamma} - 1)}{\eta \Delta T_{\text{stage}}} T_i \tag{42}$$

The overall compressor length is primarily a function of the blade chord selected from stress considerations, the blade angle, and the number of stages. Overall external dimensions and rpm of typical conservative compressor designs are given as a function of flow and rpm in Fig. 10.

MULTISTAGE COMPRESSOR DIMENSIONLESS PERFORMANCE PARAMETERS. When compressibility effects are not negligible an examination of velocity diagram and pressure relationships indicates that Mach number and flow coefficient determine stage performance. When stages are arranged in series, the flow coefficient and Mach number completely define the aerodynamic performance of the machine, assuming negligible Reynolds' number effect and constant specific heat of the working fluid. It is more convenient to use functions of these parameters than to use Mach number and flow coefficient directly to describe performance. The ratio of actual to maximum airflow or sonic airflow through the compressor entering annulus $\left(\frac{W}{W_{\text{sonic}}}\right)$ is a convenient measure of flow Mach number. For air

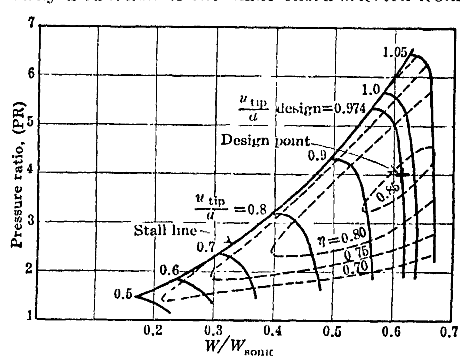


FIG. 11. Typical compressor performance in terms of dimensionless parameters.

$$W_{\text{sonic}} = 0.531 \frac{P_i}{\sqrt{T_i}} A_i \tag{43}$$

Flow coefficient may be expressed as the ratio of $\frac{v_{m_i}}{\sqrt{\gamma g R T_i}}$ to $\frac{u_{\text{tip}}}{\sqrt{\gamma g R T_i}}$. The value $\frac{}{\sqrt{\gamma g R T_i}}$ however, is a function of flow Mach number and may be determined from W/W_{sonic}. Therefore, W/W_{sonic} and $\frac{u_{\text{tip}}}{\sqrt{\gamma g R T_i}}$ $\frac{u_{\text{tip}}}{a}$ may be used as dimensionless parameters to identify a particular operating condition with the corresponding pressure ratio and efficiency. Performance expressed in this manner is independent of size of the compressor and inlet air pressure and temperature insofar as Reynolds' number effects and changes in specific heat may be neglected. Figure 11 shows performance of a typical multistage axial-flow compressor presented in dimensionless parameters. The figure represents a family of compressor designs of various sizes, but having the same aerodynamic performance coefficients.

In presenting such data for a *given* compressor the flow parameter $\frac{W\sqrt{\theta}}{\delta}$ and speed parameter $N/\sqrt{\theta}$ are more frequently used, where $\delta/\sqrt{\theta}$ is proportional to W_{sonic} and $N/\sqrt{\theta}$ is proportional to $\frac{u_{tip}}{\sqrt{\gamma g R T_t}}$.

Use of dimensionless parameters is illustrated by the following examples.

Example 1. The compressor with performance shown in Fig. 11 is operated at a rotor speed of 9400 rpm, and a pressure ratio of 3.5. The inlet total pressure is 1950 lb per ft² (abs) and the inlet total temperature is 40 F (500 R). If the wheel tip diameter (D) is 2.0 ft, and the inner diameter at entrance to the first stage is 1.2 ft, find the weight flow W and isentropic efficiency η. Using eq. 41:

$$u_{tip} = \frac{\pi ND}{60} = \frac{\pi}{60}(9400)2 = 984 \text{ ft/sec}$$

$$\frac{u_{tip}}{\sqrt{\gamma g R T_t}} = \frac{984}{\sqrt{(1.395)(32.2)(53.3)(500)}}$$

From Fig. 11:

$$\frac{W}{W_{sonic}} = 0.55,$$

Using eq. 43:

$$W_{sonic} = 0.531\frac{P_t}{\sqrt{T_t}}A_t = \frac{0.531 \times 1950}{\sqrt{500}} \cdot \frac{\pi}{4}[2^2]\left[1 - \frac{1.2^2}{2.0^2}\right]$$

$$= 93.1 \text{ lb/sec}$$

$$W_{actual} = 0.55(93.1) = 51.2 \text{ lb/sec}$$

Example 2. A design is required for an altitude condition where inlet total pressure is 1572 lb per ft² (abs) and inlet total temperature is 30 F (490 R). The weight flow is to be 30 lb per sec and the pressure ratio 5.

Using Fig. 10, find the dimensions of the compressor, the number of stages required, and the rotor speed.

Since Fig. 10 is for standard sea level conditions it can only be applied using the performance the compressor would have if operated at sea level. This may be calculated by making use of the fact that the same pressure ratio and efficiency will be obtained if the machine is operated at the same values of $\frac{u_{tip}}{\sqrt{\gamma g R T_t}}$ and W/W_{sonic}, i.e.,

$$\left(\frac{W}{W_{sonic}}\right)_{\text{sea level}} = \left(\frac{W}{W_{sonic}}\right)_{\text{altitude}}$$

$$\left(\frac{W}{0.531\frac{P_t}{\sqrt{T_t}}A_t}\right)_{\text{sea level}} = \left(\frac{W}{0.531\frac{P_t}{\sqrt{T_t}}A_t}\right)_{\text{altitude}}$$

$$W_{\text{sea level}} = W_{\text{altitude}}\frac{P_{t\,\text{sea level}}}{P_{t\,\text{altitude}}}\sqrt{\frac{T_{t\,\text{altitude}}}{T_{t\,\text{sea level}}}}$$

$$= 30 \cdot \frac{2116}{1572} \cdot \sqrt{\frac{490}{520}}$$

$$= 39.2 \text{ lb/sec}$$

Entering figure with $W = 39.2$ lb/sec and $PR = 5$ gives $N_{\text{sea level}} = 8600$, $D = 24$ in., $\frac{\text{length}}{\text{diameter}} = 1.5$, number of stages = 15.

The rotor speed at altitude is computed to give the same value of $\left(\frac{u_{tip}}{\sqrt{\gamma g R T_t}}\right)$ as at sea level.

$$\left(\frac{u_{tip}}{\sqrt{\gamma g R T_t}}\right)_{\text{altitude}} = \left(\frac{u_{tip}}{\sqrt{\gamma g R T_t}}\right)_{\text{sea level}}$$

$$\left(\frac{\frac{\pi}{60}ND_t}{\sqrt{\gamma g R T_t}}\right)_{\text{altitude}} = \left(\frac{\frac{\pi}{60}ND_t}{\sqrt{\gamma g R T_t}}\right)_{\text{sea level}}$$

$$N_{\text{altitude}} = N_{\text{sea level}}'$$

$$= 8600\sqrt{\frac{490}{520}}$$

$$= 8350 \text{ rpm}$$

Preliminary figures to be used for the design at altitude are therefore

$PR = 5, \quad W = 30, \quad$ number of stages $= 15$
$N = 8350, \quad D_i = 24$ in., $\quad$ length $= 36$ in.

Using the dimensionless parameter curves gives values of centrifugal stress proportional to inlet temperature. For high inlet air temperature an independent check of stress must be conducted.

SECTION 2

COMBUSTION AND FUELS

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COMBUSTION

By Richard C. Corey and Ernst G. Graf

1. PRINCIPLES OF COMBUSTION

DEFINITION OF COMBUSTION. Combustion is broadly defined as any chemical reaction accompanied by light and heat but is more commonly understood as the union of substances with oxygen. Substances of technical interest which undergo combustion are those which consist primarily of carbon and hydrogen or compounds of these elements.

MECHANISM OF COMBUSTION. The mechanism of combustion is concerned with the intermediate processes in the overall combustion process, whereby the atoms of the reacting substance (combustible) react with oxygen to form gaseous products.

The mechanisms of the combustion of solid fuels are complicated and not yet fully understood (Ref. 1); considerable controversy centers about the question of whether CO_2 or CO is the primary product of combustion on the surface of carbonaceous fuels. The mechanism of the combustion of gaseous and liquid fuels is better understood (Refs. 2 and 3). The combustion of fuels occurs in a number of simple or complex steps, depending on the fuel, resulting in intermediate products and proceeding at widely varying rates. The various steps respond differently to the chemical and physical conditions of the reactions, and these properties make it difficult to determine the mechanism. Knowledge of the mechanism of any combustion reaction is important to the design of fuel-burning equipment, since, if the heat-absorbing (endothermic) reactions of, say, a slagging gas producer are kept remote from regions where heat-evolving (exothermic) reactions occur, better control over operation of the unit will be achieved; conversely, clinkering difficulties in a fuel bed may be alleviated if the zones are made to overlap.

Certain physical factors also are of considerable significance in combustion, and in many technical processes they play a decisive part in the design of equipment. Of primary consideration is the fact that a gas cannot react at a solid surface at a rate greater than that at which the gas reaches the surface; nor can two gases react at a rate greater than that at which they are mixed, the controlling factor being the rate of diffusion under the specified conditions. Thus, under circumstances where the rates of chemical reaction are very rapid, as in most technical processes, the relative velocities of the gas and solid or the turbulence of gas streams are the limiting factors in the overall rate of reaction. The following facts attest the importance of physical factors in combustion processes: A particle of coal requires 40,000 to 60,000 times its volume of air, measured at the temperature of combustion, for complete combustion, and the rate of reaction of carbon with oxygen may be measured in milliseconds, whereas approximately 1 sec is required to burn a particle of pulverized coal.

2. STOICHIOMETRY AND ENERGETICS OF COMBUSTION

WEIGHT RELATIONSHIPS IN COMBUSTION. Simple numerical relationships exist between the reactants and products, and in the resulting energy changes, for the overall combustion reaction from its initial to final state. Thus, when graphitic carbon is burned with oxygen to form CO_2, the overall reaction is represented by the equation

$$C\ (gr) + O_2\ (g) = CO_2\ (g)$$
$$Q_c = 169{,}044 \text{ Btu} \tag{1}$$

This equation states that 1 *lb-mole* * of graphitic carbon (12.01 lb) reacts with 1 lb-mole of gaseous oxygen (32.00 lb) to form 1 lb-mole of gaseous CO_2 (44.01 lb), all at 25 C and 1 atm pressure, and, owing to the enthalpy change, releases 169,044 Btu per lb-mole of carbon.

Except in very special cases, industrial combustion processes do not use pure oxygen but depend upon air for the source of oxygen. For most engineering purposes (Ref. 4) dry air may be considered to consist of 21% by volume of oxygen of molecular weight

* The mass in pounds numerically equal to the molecular weight, sometimes shortened to simply *mole.*

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32.00 and 79% by volume of atmospheric nitrogen of molecular weight 28.16. On a *weight basis*, dry air contains 23.2% oxygen and 76.8% atmospheric nitrogen. The *molecular weight* of dry air is 28.97. Since equal volumes of perfect gases, measured under identical conditions, contain equal numbers of moles, each mole of oxygen in air has associated with it 79.0/21.0 = 3.762 moles of atmospheric nitrogen. Thus, for combustion of carbon with air, eq. 1 may be written

$$C\ (gr) + O_2\ (g) + 3.762N_2\ (g) = CO_2\ (g) + 3.762N_2\ (g) \qquad Q_c = 160{,}944 \text{ Btu} \tag{1a}$$

The *weight* of atmospheric nitrogen participating in the reaction remains unchanged—3.762 × 28.16 = 105.94 lb per lb-mole of carbon. The weight of dry air used is 32.00 + 105.94 = 137.94 lb, and the weight of the products of combustion is 44.01 + 105.94 = 149.95 lb per lb-mole of carbon.

VOLUME RELATIONSHIPS IN COMBUSTION. The volumes occupied by a pound-mole of all perfect gases are identical for any stated conditions of pressure and temperature. For example, at 1 atm pressure (29.92 in. Hg) and a temperature of 80 F, the *molar volume* of any perfect gas is 394.5 cu ft. At any other temperature or pressure, the molar volume of a perfect gas is given by

$$V_M = 394.5 \times \frac{t + 460}{540} \times \frac{29.92}{p} \tag{2}$$

where V_M = molar volume, cubic feet; t = temperature, °F; and p = absolute pressure, inches of Hg.

At ordinary pressures and temperatures, the properties of most combustion gases deviate slightly from the perfect gas laws but approach the properties of perfect gases as the temperature is increased. For engineering calculations, however, no serious errors are introduced by considering reactant and product gases as perfect gases. For more exact calculations at ordinary temperatures and pressures and for all conditions where deviations from the perfect gas laws are appreciable, special methods must be used (Ref. 5) or empirical correction factors may be employed (Ref. 6). To demonstrate volume relationships in combustion, eq. 1*a* becomes, at 1 atm and 80 F,

$$C\ (gr) + 394.5\ \text{ft}^3\ O_2 + 1384.1\ \text{ft}^3\ N_2 = 394.5\ \text{ft}^3\ CO_2 + 1384.1\ \text{ft}^3\ N_2 \tag{1b}$$

Density of Mixtures. Frequently, it is desired to calculate the *density* of gas mixtures; this is easily done in terms of the molecular weight of the mixture. The molecular weight is first obtained by multiplying the *mole fraction* of each constituent in the mixture by its respective molecular weight and adding the products. The mole fraction is obtained by dividing the number of moles of gas by the total number of moles present. For gases, the mole fraction is the percentage by volume divided by 100. Thus, for the product gases in eq. 1*a*,

	(1) Number of Moles	(2) Percentage by Volume	(3) Molecular Weight	(2) × (3) ÷ 100
CO_2	1.00	21.0	44.01	9.24
N_2	3.762	79.0	28.16	22.25
			Molecular weight of mixture	31.49

Therefore, the density of the gas mixture is, at 80 F and 1 atm pressure, 31.49/394.5 = 0.0798 lb per cu ft. The percentage of CO_2 shown in the example is the maximum that can be obtained by combustion with air. (See also Gas Density, p. 2-75.)

HEATING VALUE OF FUELS. The heating value of a solid fuel is determined by combustion with excess oxygen in a constant-volume bomb calorimeter (ASTM D271-46). The value reported is the heat of combustion at 20 C of the fuel at constant volume when the fuel has burned to ash, CO_2 (gas), SO_2 (gas), and H_2O (liquid). This value is termed the *gross calorific value* (ASTM) or the *high-heat value* (ASME). The *net calorific value* (ASTM) or *low-heat value* (ASME) is calculated from the high-heat value by deducting 1030 Btu for each pound of water derived from a unit quantity of fuel,* including both the water originally present as moisture and that formed by combustion (ASTM D407-44). The high and low heats of combustion of several fuels are given in Tables 1 and 2.

Dulong Formula. In many instances, the heating value of a fuel may be calculated from the ultimate analysis by assuming that the heat evolved is the sum of the heat of combustion of the carbon, hydrogen, and sulfur. Some error is introduced by the fact that the heat of formation of the compounds of these elements in the fuel, particularly if

* This value includes a correction to place the low-heat value on basis of constant pressure.

Table 1. Heating Value and the Products of Combustion

	Substance	Molecule or Atomic Weight	Overall Combustion Reaction	Heat of Combustion *			
				High-heat Value, Btu		Low-heat Value, Btu	
				Per lb	Per cu ft †	Per lb	Per cu ft
1	Graphite	12.01	$C + O_2 = CO_2$	14087		14087	.... ..
2	Carbon (coal)	(12.01)	$C + O_2 = CO_2$	14447	.	14447	......
3	Carbon (coal)	(12.01)	$C + 0.5O_2 = CO$	4341		4341	... .
4	Carbon monoxide	28.01	$CO + 0.5O_2 = CO_2$	4344	321	4344	321
5	Sulfur	32.06	$S + O_2 = SO_2$	3980	.	3980	....
6	Hydrogen	2.016	$H_2 + 0.5O_2 = H_2O$	60958	325	51571	275
7	Hydrogen sulfide	34.08	$H_2S + 1.5O_2 = SO_2 + H_2O$	7180	639	6620	589
8	Carbon disulfide	76.13	$CS_2 + 3O_2 = CO_2 + 2SO_2$	12050	620	12050	620
9	Ammonia	17.03	$2NH_3 + 1.5O_2 = N_2 + 3H_2O$	9668	435	8001	360
10	Methane	16.04	$CH_4 + 2O_2 = CO_2 + 2H_2O$	23861	1011	21502	911
11	Ethane	30.07	$C_2H_6 + 3.5O_2 = 2CO_2 + 3H_2O$	22304	1772	20416	1622
12	Propane	44.09	$C_3H_8 + 5O_2 = 3CO_2 + 4H_2O$	21646	2522	19929	2322
13	Butane	58.12	$C_4H_{10} + 6.5O_2 = 4CO_2 + 5H_2O$	21293	3270	19665	3020
14	Hexane (vapor)	86.17	$C_6H_{14} + 9.5O_2 = 6CO_2 + 7H_2O$	20928	4765	19391	4414
15	Octane (vapor)	114.23	$C_8H_{18} + 12.5O_2 = 8CO_2 + 9H_2O$	20747	6266	19256	5812
16	Ethylene	28.05	$C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$	21625	1603	20276	1503
17	Propylene	42.08	$C_3H_6 + 4.5O_2 = 3CO_2 + 3H_2O$	21032	2338	19683	2188
18	Butylene	56.10	$C_4H_8 + 6O_2 = 4CO_2 + 4H_2O$	20833	3076	19484	2877
19	Acetylene	26.04	$C_2H_2 + 2.5O_2 = 2CO_2 + H_2O$	21460	1473	20734	1423
20	Benzene (vapor)	78.11	$C_6H_6 + 7.5O_2 = 6CO_2 + 3H_2O$	18172	3745	17446	3595
21	Toluene (vapor)	92.13	$C_7H_8 + 9O_2 = 7CO_2 + 4H_2O$	18422	4490	17601	4285
22	Naphthalene (vapor)	128.16	$C_{10}H_8 + 12O_2 = 10CO_2 + 4H_2O$	17300	5854	16700	5654
23	Methylalcohol (vapor)	32.04	$CH_3OH + 1.5O_2 = CO_2 + 2H_2O$	10270	855	9080	755
24	Ethyl alcohol (vapor)	46.07	$C_2H_5OH + 3O_2 = 2CO_2 + 3H_2O$	13170	1575	11930	1425
25	Lignite ‡	.	$C_{24}H_{18}O_5 + 28.5O_2 = 24CO_2 + 9H_2O$	12055	.	11505	. .
26	Bituminous coal ‡		$C_{24}H_{20}O_2 + 29.0O_2 = 24CO_2 + 10H_2O$	14550	.	14055	..
27	Anthracite ‡		$C_{48}H_{18}O + 52O_2 = 48CO_2 + 9H_2O$	15230		14940	..

* Data for heat of combustion for items 1 to 4 and 10 to 21, inclusive, based on Selected Values of Properties of Hydrocarbons, F. D. Rossini et al., *Natl. Bur. Standards* (*U. S.*) *Circ.* C461, Nov. 1947. Reactants and products at 25 C (77 F).

† Based upon the standard cubic foot (scf) at 60 F and 30 in Hg of a perfect gas. One lb-mole = 379 scf.

it is coal, are neglected. The Dulong formula (modified) is useful for calculating the heating value of coal.

$$\text{Btu per pound} = 145.4C + 620\left(H - \frac{O}{8}\right) + 41S \tag{3}$$

where C, H, O, and S are, respectively, the weight percentages of carbon, total hydrogen, oxygen, and sulfur. The results with this formula are quite accurate for coke and have an average deviation of only 2% for coals containing up to 10% oxygen, on the moisture- and ash-free basis. (See also p. 2-26.)

The Vondracek formula is reasonably accurate for all solid and liquid fuels.

$$\text{Btu per pound} = (160.5 - 0.112C_1)C + 486\left(H - \frac{O}{10}\right) + 45S \tag{4}$$

where C, H, O, and S are the weight percentages in the fuel of carbon, hydrogen, oxygen, and sulfur, and C_1 is the weight percentage of carbon in solid fuels on the moisture- and ash-free basis.

AIR REQUIRED FOR COMBUSTION (THEORETICAL AIR). Of primary interest in engineering calculations is the theoretical weight of air required for combustion of a unit weight of a given fuel, or the weight of air required to obtain a unit quantity of heat from a given fuel. Table 1 presents a summary of the stoichiometric and energetic relationships for the combustion of several solid, liquid, and gaseous fuels, giving these quantities.

The oxygen required, in pounds per pound of fuel, is obtained by dividing the total molecular weight of the oxygen from the overall combustion reaction by the total molecular

of Various Solid, Liquid, and Gaseous Fuels

Combustion with Theoretical Amount of Air

Lb per lb							Cu ft per cu ft							Lb Air per 1000 Btu		CO_2, % by Volume (Dry Basis)	
Required		Products of Combustion					Required		Products of Combustion								
O_2	Air	N_2	CO_2	H_2O	SO_2	Total	O_2	Air	N_2	CO_2	H_2O	SO_2	Total	LHV	HHV		
2.67	11.50	8.83	3.67	....	.	12.50	...	..		.	.	...	.....	0.82	0.82	21.0	1
2.67	11.50	8.83	3.67	.....	...	12.50	.	.	.		....	....	....	0.80	0.80	21.0	2
1.33	5.73	4.40	2.33 (CO)	..	....	6.73	...	.	.		.	....	.....	.....	..	..	3
0.57	2.46	1.89	1.57	..		3.46	0.50	2.38	1.88	1.00		...	2.88	0.57	0.57	34.6	4
1.00	4.31	3.31	...	..	2.00	5.31				.		....		1.08	1.08	...	5
7.94	34.34	26.40	.	8.98		35.38	0.50	2.38	1.88		1.00		2.88	0.67	0.56	..	6
1.41	6.10	4.69		0.53	1.88	7.10	1.50	7.14	5.64		1.00	1.00	7.64	0.92	0.85		7
1.26	5.44	4.18	0.58		1.68	6.44	3.00	14.28	11.28	1.00		2.00	14.28	0.45	0.45	21.0 §	8
1.41	6.08	5.49		1.59	...	7.08	0.75	3.57	3 32		1.50	.	4.82	0.76	0.63	..	9
4.00	17.27	13.27	2.74	2.25	..	18.26	2.00	9.52	7.52	1.00	2.00	..	10.52	0.80	0.72	11.7	10
3.73	16.12	12.39	2.92	1.80	..	17.11	3.50	16.65	13.15	2.00	3.00	..	18.15	0.79	0.72	13.2	11
3.64	15.70	12.06	2.99	1.64	...	16.69	5.00	23.80	18.80	3.00	4.00	...	25.80	0.79	0.72	13.8	12
3.58	15.44	11.86	3 03	1.56	...	16.45	6.50	30.90	24.40	4.00	5.00	....	33.40	0.79	0.72	14.1	13
3.53	15.21	11.68	3.07	1.46	...	16.21	9.50	45.20	35.70	6 00	7.00	.	48.70	0.79	0.78	14.4	14
3.50	15.10	11.60	3.08	1.42	.	16.10	12.50	59.50	47.00	8.00	9.00	...	64.00	0.78	0.73	14.5	15
3.42	14.75	11.33	3.14	1.29	.	15.76	3.00	14.28	11.28	2.00	2.00	...	15.28	0.73	0.68	15.1	16
3.42	14.75	11.33	3.14	1.29	.	15.76	4.50	21.40	16.90	3.00	3.00	.	22.90	0.74	0.70	15.1	17
3.42	14.75	11.33	3.14	1.29		15.76	6.00	28.55	22.55	4.00	4.00	..	30.55	0.76	0.71	15.1	18
3.07	13.23	10.16	3.38	0.69		14.23	2.50	11.90	9.40	2.00	1.00	..	12.40	0.64	0.62	17.5	19
3.07	13.23	10.16	3.38	0.69	..	14.23	7.50	35.70	28.20	6.00	3.00		37.20	0.76	0.73	17.5	20
3.13	13.50	10.35	3.35	0.78	..	14.48	9.00	42.80	33.80	7.00	4.00	..	44.80	0.77	0.73	17.2	21
3.00	12.93	9.93	3.44	0.56	...	13.93	12.00	57.10	45.10	10.00	4.00	..	59.10	0.77	0.75	18.1	22
1.60	6.90	5.30	1.37	1.13	....	7.80	1.50	7.14	5.64	1.00	2.00	.	8.64	0.76	0.67	15.1	23
2.08	8.96	6.89	1.91	1.17	.	9.96	3.00	14.28	11.28	2.00	3.00	..	16.28	0.75	0.68	15.1	24
2.14	9.22	7.08	2.71	0.43	.	10.22	..	...	.....	....	..	...		0.80	0.76	19.5	25
2.60	11.21	8.60	3 08	0.52		12.21	.		.....	....	..	.	....	0.80	0.77	18.6	26
2.74	11.83	9.08	3.46	0.28		12.83			.			.	...	0.79	0.78	19.5	27

‡ Computed from average moisture- and ash-free ultimate analysis of 17 lignites, 27 medium and high-volatile coals, and 5 anthracite coals in the United States. The formulas for lignite, bituminous, and lignitic coals, given in column 3, do not represent the true constitution of the coal molecule, which is much more complex, but are adequate for stoichiometric calculations.

§ $CO_2 + SO_2$.

Table 2. Heating Value and Composition of Liquid Fuels *

	API Gravity, 60 F	Specific Gravity, 60 F	lb/gal	C	H	S	N	O	HHV	LHV	HHV
									Btu/lb	Btu/lb	Btu/gal
Commercial gasoline	55.5	.757	6.30	84.3	15.7			.	21,000	19,506	132,384
Commercial kerosene	41.8	.817	6.80	84.7	15.3			..	20,000	18,545	136,040
Gas oil	32.7	.863	7.18	85.5	†13.0	0.8			19,200	17,940	138,100
Fuel oil, mid-continent	27.2	.892	7.43	85.6	†12.0	0.4	0.5	0.6	19,376	18,236	144,000
Fuel oil, California	16.7	.955	7.96	84.7	12.4	1.1		.	18,835	17,755	150,000

* See also p. 2-45.

(or atomic) weight of the fuel in question. In eq. 1*a*, for example, the oxygen required is 32.00/12.01 = 2.67 lb of air per lb of graphitic carbon, and, since air consists of 23.2% oxygen and 76.8% atmospheric nitrogen, *by weight*, each pound of oxygen is equivalent to 100/23.2 = 4.31 lb of air. Table 1 gives these data for the various fuels.

Air Required per 1000 Btu. The weight of air required for *perfect* combustion to obtain 1000 Btu low-heat and high-heat value is given in Table 1. It is of interest that, with few exceptions, 0.65 to 0.80 lb is required. The amount of air in excess of the theoretical amount, to secure complete combustion, will depend on the process, furnace design, and operating practice.

EXCESS AIR. More than the theoretical amount of air is necessary in practice to attain complete combustion. The excess air is expressed either in percentage of the theoretical air or as the total air divided by the theoretical air. It is computed conveniently from the oxygen data.

$$\text{Percentage excess air} = \frac{\text{Total oxygen added} - \text{Theoretical oxygen}}{\text{Theoretical oxygen}} \times 100$$

$$= \frac{\text{Excess oxygen}}{\text{Theoretical oxygen}} \times 100$$

where excess oxygen may be determined from an Orsat analysis. If carbon monoxide is present, the oxygen required to burn it is deducted from the excess oxygen. Excess air may be calculated readily from the Orsat analysis. Assume flue-gas composition, per 100 moles of dry flue gas, to be as shown in the following table.

	Percentage by Volume (Orsat)	Equivalent Moles of O_2 in Flue Gas *
CO_2	10	10
CO	2	1
O_2	4	4
N_2	84 (by difference)	0
	100	15

* See chemical-reaction equations, Table 1.

Since N_2 in the fuel is assumed to be negligible, all of it came from air supplied for combustion. The moles of oxygen accompanying the N_2 in the combustion air were, therefore,

$$84.0 \times \frac{20.9}{79.1} = 22.19 \text{ moles per 100 moles of flue gas}$$

To burn the remaining CO to CO_2 (line 4, Table 1) would have required $2 \times 0.5 = 1$ mole of the excess oxygen in the flue gas. If complete combustion had occurred, the excess oxygen would be only $4.0 - 1.0 = 3.0$ moles per 100 moles of flue gas. Theoretical oxygen = (actual oxygen supplied − excess oxygen), all per 100 moles of dry flue gas. Therefore, the percentage of excess air was

$$\frac{\text{Excess oxygen}}{\text{Theoretical oxygen}} \times 100 = \frac{3.0}{22.19 - 3.0} \times 100 = 15.63\%$$

For the general case, where the nitrogen content of the fuel can be neglected and CO, hydrogen, and methane appear in the flue gases, the percentage of excess air may be calculated from the Orsat analysis as in eq. 5.

$$\text{Percentage excess air} = \frac{O_2 - (\tfrac{1}{2}CO + \tfrac{1}{2}H_2 + 2CH_4)}{\tfrac{21}{79}N_2 - O_2 + (\tfrac{1}{2}CO + \tfrac{1}{2}H_2 + 2CH_4)} \times 100 \tag{5}$$

When only the CO_2 in the flue gas and the maximum theoretical CO_2 are known for the given fuel, eq. 6 is useful.

$$\text{Percentage excess air} = 7900 \frac{CO_2\,(\text{max}) - CO_2}{CO_2\,[100 - CO_2\,(\text{max})]} \tag{6}$$

where CO_2 (max) is the maximum percentage of CO_2 obtainable (Table 1) and CO_2 is the percentage of CO_2 in the dry flue gas as determined by the Orsat.

It should be emphasized, particularly for solid fuels, that such calculations are based only on the *fuel burned* and do not include unburned combustibles in the ash or as soot.

Design Excess Air. Table 3 (Ref. 7) gives the range of values of excess air (at the point where the gases leave the furnace) commonly used by designers of industrial boilers and furnaces.

Table 3. Excess Air at Furnace Outlet for Various Fuels

Adapted by permission from *Combustion Engineering*, Combustion Engineering Co., Inc., New York, 1947.

Fuel	Excess Air, %	Fuel	Excess Air, %
Coal	10–40	Natural gas	5–10
Coke	20–40	Refinery gas	8–15
Wood	25–50	Blast-furnace gas	15–25
Bagasse	25–45	Coke-oven gas	5–10
Oil	8–15		

THE OSTWALD TRIANGULAR CHART in Fig. 1 is a useful means of checking flue-gas analyses and may be constructed for any given fuel, if it is assumed that all hydrogen in the fuel is burned completely to water and that carbon losses are negligible. The ordinate is drawn to represent, to some arbitrary scale, the maximum theoretical CO_2 of the fuel in question (Table 1). The maximum abscissa corresponds to the oxygen in air, 20.9% by volume. A CO line, originating on *AB* (at an oxygen concentration equal to the percentage excess over the theoretical oxygen required to burn the fuel only to CO) is drawn normal to the hypotenuse. For example, pure carbon burned to CO_2, gives a flue gas containing 21% CO_2 and requires 1 mole O_2; if the reaction goes only to CO, there will be 0.5 mole O_2 in excess. Consequently, the dry gas contains 0.5 mole O_2 + 1.0 mole CO + 3.76 moles N_2, and the percentage (by volume) of excess O_2 is 9.51. For pure carbon, this would be the origin of a line similar to *DE* on *AB*. Distance *AD* for more complex fuels is given in the table for various values of theoretical CO_2.

CO_2 (max), % by Volume	Origin of *DE* * on *AB*
21	9.51
19	8.60
17	7.70
15	6.79
13	5.89
11	4.98

* Note that these values are proportional to CO_2 (max), that is, the line *CD* has the same slope for all fuels.

DE then is drawn normal to the hypotenuse and is divided into equal-length units equal in number to twice that of the oxygen percentage at *D*, or 19.0 units in Fig. 1.

The hypotenuse *BC*, representing excess air, then is divided into ten equal parts, representing the reciprocal of the excess air number $\left(1 + \frac{\%\text{ excess air}}{100}\right)$; for example, 100% excess air = 2.0 excess air number, the reciprocal of which is 0.5, shown on the hypotenuse scale.

To use the chart, determine the intersection, as *P*, of the O_2 and CO_2 lines. By interpolation between lines parallel to *CD*, the excess air number is found, 0.565 in Fig. 1, which corresponds to 77% excess air. If a line is traced through *P* parallel to *CB*, the CO scale, *DE*, may be used to find the CO percentage, 0.3 per cent in Fig. 1.

Once this chart is constructed for a given fuel, it saves much calculation. Lines parallel to *CD* may be labeled directly in percentages of excess air, and lines may be drawn parallel to *CB* to show percentages of CO.

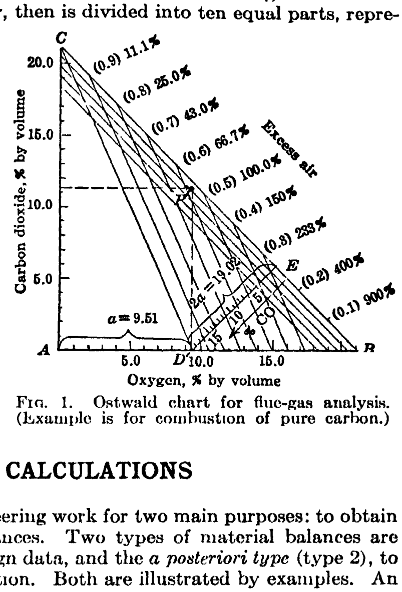

FIG. 1. Ostwald chart for flue-gas analysis. (Example is for combustion of pure carbon.)

3. COMBUSTION CALCULATIONS

Combustion calculations are made in engineering work for two main purposes: to obtain material balances and to obtain energy balances. Two types of material balances are used, the *a priori type* (type 1), to obtain design data, and the *a posteriori type* (type 2), to determine the performance of a unit in operation. Both are illustrated by examples. An energy balance is also made and is illustrated for the second type.

MATERIAL BALANCE—TYPE 1. Calculations for design purposes generally assume that the fuel will be burned with excess air.

EXAMPLE 1. A mixture of West Kentucky coals is to be burned in a pulverized-coal-fired furnace with 25% excess air (1.25 times the theoretical air). Assuming complete combustion of carbon and sulfur in the coal, calculate (1) weight of dry air required per 100 lb of coal, as fired, (2) weight of flue-gas components, CO_2, H_2O, SO_2, O_2, and N_2, per 100 lb of coal, as fired, (3) total weight of dry and wet flue gases per 100 lb of coal, as fired and analysis of the dry flue gas, and (4) weight of moisture in the flue gas resulting from moisture in the coal, net hydrogen in the coal, and moisture in the air.

Analysis of the Coal on As-fired Basis

Proximate Analysis	Percentage	Ultimate Analysis	Percentage
Moisture	8.8	Hydrogen	5.2
Volatile	35.4	Carbon	62.2
Fixed carbon	42.6	Nitrogen	1.3
Ash	13.2	Oxygen	14.3
		Sulfur	3.8

Air used for combustion has moisture, lb per lb of dry air = 0.0056.

(1) Weight of Dry Air Required for Combustion (per 100 lb coal fired).

Coal	Pounds	Moles *	Moles O_2 for Combustion †	Moles of Products †	
Carbon	62.2	5.18	5.18	CO_2	5.18
Sulfur	3.8	0.12	0.12	SO_2	0.12
Hydrogen	5.2	2.58	1.29	H_2O	2.58
Oxygen	14.3	0.45		N_2	0.05
Nitrogen	1.3	0.05			
			6.59		

* Weight in pounds divided by molecular weight.
† Mole relations between reactants and products are given in Table 1; e.g., 1 mole C forms 1 mole CO_2; 1 mole H_2 requires 1/2 mole O_2 and yields 1 mole H_2O, etc.

(Total oxygen required − oxygen in coal) = 6.59 − 0.45 = 6.14 moles = theoretical O_2
Moles oxygen to be used = 6.14 × 1.25 = 7.68
Moles oxygen in excess = 7.68 − 6.14 = 1.54
Moles nitrogen from combustion air

$$= \text{moles oxygen from combustion air} \times \left(\frac{\text{moles nitrogen}}{\text{mole oxygen}}\right) = 7.68 \times 3.762 = 28.89$$

Moles dry air to be used = 7.68 + 28.89 = 36.57
Weight dry air to be used = 36.57 × 28.97 = 1059 lb

$$\text{Moisture in combustion air} = 1059 \text{ lb dry air} \times 0.0056 \frac{\text{lb } H_2O}{\text{lb dry air}} = 5.9 \text{ lb}$$

(2) Weight of Flue-gas Components (Dry-gas Analysis) (per 100 lb coal fired).

Formed from Coal and Combustion Air

	Moles	Pounds (moles × *MW*)	
CO_2	5.18	228.0	
SO_2	0.12	7.7	
H_2O coal	2.58	46.5	52.4 total
H_2O air	0.33 *	5.9	
N_2 coal	0.05	1.3	814.8 total
N_2 air	28.89	813.5	
O_2 (excess)	1.54	49.3	

* 0.33 .0056 × 28.97/18 × 36.55.

(3) Total weight dry gas from (2) is 1100 lb per 100 lb coal fired.
Total weight wet gas from (2) is 1152 lb per 100 lb coal fired.
Total moles dry gas, adding dry components in first column of (2), is 35.76 moles per 100 lb coal fired.
Dry Flue-gas Analysis, Percentage.

$$CO_2 + SO_2 = 100\left(\frac{5.18 + 0.12}{35.76}\right) = 14.8$$

$$O_2 = 100\left(\frac{1.54}{35.76}\right) = 4.3$$

$$N_2 = 100 - (CO_2 + SO_2) - O_2 = \underline{80.9}$$

$$100.0$$

These computations may be checked by determining if the total weight of wet gas, calculated as weight of coal plus weight of air plus weight of moisture in air minus ash in coal, equals the weight of wet gas calculated above—1152 lb.

$$100 + 1059 + 5.9 - 13.2 = 1151.7 \text{ lb}$$

(4) Weight of Moisture in the Flue Gases (per 100 lb coal fired).

From moisture in coal	= 8.8 lb	(proximate analysis)
From net hydrogen in coal = 46.5 − 8.8	= 37.7 lb	
From moisture in air	= 5.9 lb	
Total	52.4 lb	

MATERIAL BALANCE—TYPE 2. The calculation of a material balance of the second type is based on representative analyses of flue gas, fuel fired, and residue, and weight of fuel fired.

EXAMPLE 2. With the same coal as in the first example, the analysis of the dry flue gas at the air-heater outlet was:

Orsat Analysis

(Dry gas, volume basis, percentage)

$CO_2 + SO_2 = 12.8$
$O_2 = 6.3$
$N_2 = 80.9$

The carbon loss in the residue was found to be 0.2 lb per 100 lb of coal fired.

The air for combustion contained 0.0056 lb moisture per lb dry air.

Assuming sulfur in the flue gas and residue to be negligible, determine (1) weight of the dry flue-gas components, CO_2, O_2, and N_2, (2) weight of dry air *supplied* for combustion, *theoretical* air needed for perfect combustion, and excess air based on coal as fired, (3) weight of moisture in flue gas due to moisture in the coal, moisture in the air, and moisture from combustion of net hydrogen in the coal, (4) total weight of dry and wet flue gases.

(1) Weight of Dry Flue-gas Components. This generally is calculated from the *carbon balance*, since carbon in both fuel and flue gas is determined with greater precision than other components.

Carbon Burned (per 100 lb coal fired).

$$\begin{aligned} &\text{Carbon in coal} &&= 62.2 \text{ lb} \\ &\text{Carbon in residue} &&= \underline{\ 0.2 \text{ lb}} \\ &\text{Carbon burned} &&= 62.0 \text{ lb} = 5.16 \text{ lb-moles} \end{aligned}$$

Carbon in Flue Gas (per 100 lb-moles dry flue gas).

$$\text{C in } CO_2{}^* = 12.80 \text{ lb-moles}$$

Moles of Dry Flue Gas (per 100 lb coal fired).

$$100 \times \frac{5.16}{12.8} = 40.3 \text{ lb-moles}$$

Total Dry-gas Components (per 100 lb coal fired).

$$CO_2 = \frac{40.3 \text{ lb-moles dry flue gas}}{100 \text{ lb coal}} \times \frac{12.8 \text{ lb-moles } CO_2}{100 \text{ lb-moles dry flue gas}} \times \frac{44 \text{ lb } CO_2}{\text{lb-mole } CO_2} = 227 \text{ lb}$$

$$O_2 = 40.3 \times \frac{6.3}{100} \times 32 = 81 \text{ lb}$$

$$N_2 = 40.3 \times \frac{80.9}{100} \times 28.2 = \underline{918 \text{ lb}}$$

$$\text{Total dry gas per 100 lb coal fired} = 1226 \text{ lb}$$

(2) Weight of Dry Air Supplied, Theoretical, and Excess (per 100 lb coal fired).

(*a*) *Dry air supplied.* This generally is calculated from a nitrogen balance.

$$N_2 \text{ from air} = \frac{40.3 \text{ lb-moles dry flue gas}}{100 \text{ lb coal fired}} \times \frac{80.9 \text{ lb-moles } N_2}{100 \text{ lb-moles dry flue gas}} = 32.6 \text{ lb-moles}$$

$$\text{Dry air supplied} = \frac{32.6 \text{ lb-moles } N_2}{100 \text{ lb coal fired}} \times \frac{100 \text{ lb-moles air}}{79 \text{ lb-moles } N_2} = 41.3 \text{ lb-moles}$$

$$41.3 \text{ lb-moles} \times \frac{28.97 \text{ lb air}}{\text{lb-mole}} = 1196 \text{ lb per 100 lb coal fired}$$

(*b*) *Theoretical dry air for combustion.* This is air supplied for combustion *plus* air needed to burn the unburned carbon, *minus* air in the flue gas, per 100 lb of fuel.

$$\text{Air supplied for combustion} = 41.30 \text{ lb-moles}$$

$$\text{Air needed to burn unburned carbon} = \frac{0.20}{12.01} \text{ lb-moles } O_2 \times \frac{4.76 \text{ lb-moles air}}{\text{lb-moles } O_2} = 0.08 \text{ lb-mole}$$

$$\text{Air in the flue gas} = 40.3 \frac{\text{lb-moles dry flue gas}}{100 \text{ lb coal fired}} \times 6.3 \frac{\text{lb-moles } O_2}{100 \text{ lb-moles gas}} \times \frac{4.76 \text{ lb-moles air}}{\text{lb-mole } O_2} = \underline{12.09 \text{ lb-moles}}$$

Subtracting the last two items from the first, dry air theoretically required for combustion = 29.13 lb-moles

$$29.13 \text{ lb-moles} \times 28.97 \frac{\text{lb}}{\text{lb-moles}} = 844 \text{ lb}$$

(*c*) *Excess air.* The excess air is 41.30 − 29.13 = 12.17 lb-moles

$$\text{Percentage of excess air} = \frac{12.17}{29.13} \times 100 = 41.8\%$$

(3) Moisture in the Flue Gas (per 100 lb coal fired).

$$\begin{aligned} &H_2O \text{ from air} = 1196 \times 0.0056 &&= 6.70 \text{ lb} \\ &H_2O \text{ from moisture in coal} &&= 8.80 \text{ lb} \\ &H_2O \text{ from total hydrogen in coal} = 2.58 \text{ lb-moles} \times 18.02 \frac{\text{lb } H_2O}{\text{lb-mole}} &&= 46.49 \text{ lb} \\ &H_2O \text{ from hydrogen in } \textit{dry} \text{ coal} = 46.49 - 8.80 &&= 37.69 \text{ lb} \\ &\text{Total } H_2O \text{ from air and coal} = 6.70 + 46.49 &&= 53.19 \text{ lb} \end{aligned}$$

* Since not all the SO_2 is absorbed in the Orsat, no serious error is introduced by assuming this to be the true CO_2.

(4) Total Weight of Dry and Wet Flue Gas (per 100 lb coal fired).

Dry gas, item (1) above	= 1226 lb
Moisture, item (3) above	= 53 lb
Total weight of gas	= 1279 lb

ENERGY OR HEAT BALANCE of a combustion process equates the enthalpy of all materials entering the process, plus the heat of combustion of the fuel at some standard state, to the heat absorbed by the process, plus the enthalpy of the materials leaving the process, plus the losses due to combustible losses and heat transfer to the surroundings. Where the heat absorbed by the process is not known, the method of heat losses is used, the heat absorbed being the heat input minus the heat losses, unaccounted-for losses usually being closely estimated for many operations. The following example, with the same data as in Example 2, above, illustrates a typical energy balance from test data for a coal-fired boiler.

EXAMPLE 3.

High-heat value of coal, Btu per pound, as fired = 11,200
Temperature of coal and air entering system, °F = 80
Temperature of flue gas leaving system, °F = 340
Coal analysis, flue-gas analysis, and combustible in ash, same as for example 2, p. 2-08.

Determine, per 100 lb of coal fired, (1) heat lost as enthalpy of dry flue gas, (2) heat lost in combustible in ash, and CO, H_2, and CH_4 in flue gas, (3) heat lost as enthalpy of moisture in fuel, of moisture from H_2 in fuel, and of moisture in air used for combustion, (4) heat lost as enthalpy of refuse, (5) heat-transfer losses, and (6) thermal efficiency.

All calculations that follow are on the basis of 100 lb of coal fired, reference temperature, 80 F.

Table 4. Mean Molar Specific Heat above 80 F.* †

(Btu per lb-mole, °F)

Temp, °F	Atm N_2 ‡ M.W. = 28.16	O_2 § M.W. = 32.00	CO_2 § M.W. = 44.01	H_2O § M.W. = 18 016	SO_2 ‖ M.W. = 64.07	Air ¶ M.W. = 28.97
80	6.94	7.02	8.96	7.98	9.53	6 96
140	6.942	7 050	9.117	8.017	9.680	6.963
240	6.950	7.100	9.369	8.069	9.920	6.981
340	6 958	7.154	9.623	8.115	10.149	7.000
440	6 975	7.211	9.861	8.167	10.364	7.025
540	6.998	7 272	10.080	8.226	10.566	7.054
640	7 027	7.377	10.284	8.291	10.757	7.091
740	7.059	7 401	10.474	8.360	10.923	7.132
840	7.093	7.463	10.652	8.432	11.077	
940	7.129	7.524	10.820	8.505	11.216	
1040	7.166	7.582	10 977	8.579	11.352	
1140	7.203	7.637	11.126	8.655	11.477	
1240	7.243	7.688	11.266	8.731	11 586	
1340	7.283	7 737	11.398	8.809	11.679	
1440	7.324	7.785	11.523	8.887	11.766	
1540	7.364	7 829	11.640	8.967	11.858	
1640	7 403	7.871	11.750	9.047	11.944	
1740	7.440	7.910	11.854	9.128	12.017	
1840	7.476	7.947	11.953	9.207	12.086	
1940	7.511	7.983	12.046	9.286	12.150	
2040	7.545	8.016	12.135	9.364	12.210	
2140	7.579	8.049	12.220	9.440	12.269	
2240	7.611	8.080	12.301	9.516		
2340	7.642	8.110	12.738	9.591		
2440	7.672	8.139	12.451	9.664		
2540	7.700	8.167	12.521	9.736		

* See also Tables 3, 4, and 5, p. 2-98, for Keenan and Kaye data for products of combustion.

† For chart showing heat content of gases found in flue products, see Gaseous Fuels, Fig. 8, p. 2-74.

‡ Calculated from data of Heck for N_2. Argon, C_p = 4.964, Btu per lb-mole, °F; atm N_2 = 98.76 mole % N_2, 1.24 mole % argon.

§ Calculated from data of R. C. H. Heck, The New Specific Heats, *Mech. Eng.*, Vol. 62, Jan. 1940, pp. 9-12.

‖ Interpolated from data of E. Justi, *Spezifische Wärme, Enthalpie, Entropie und Dissoziation technischer Gase*, Table 106, p. 148, Julius Springer, 1938.

¶ Calculated from data for atmospheric N_2 and O_2; air = 79.00 mole % atm N_2, 21.00 mole % O_2.

(1) Enthalpy of Dry Flue Gas. The components of the dry flue gas, per 100 lb of coal fired, are given in Example 2. The mean molar specific heats, above 80 F, are given in Table 4. The enthalpy is obtained from $Q = nC_{p_m}\,\Delta t$, where n is the number of moles of gas per 100 lb coal fired, C_{p_m} is the mean molar specific heat of the gas at a given temperature, and Δt is the temperature difference between the flue gas and the air used for combustion, i.e., 340 − 80 = 260 F.

Flue-gas Component	Lb-moles per 100 lb Coal	C_{p_m}	Enthalpy, Btu per lb-mole	Enthalpy, Btu per 100 lb Coal
CO_2	5.16	9.623	2,502	12,910
SO_2	0.12	10.149	2,639	317
O_2	2.60	7.154	1,860	4,836
N_2	33.41	6.958	1,809	60,439
		Total enthalpy of dry flue gas, Btu =		78,502

(2) Heating Value of Combustible in Ash and Flue Gas. If the heating value of the carbon in the refuse is assumed to be 14,500 Btu per lb of combustible, then 0.2 lb carbon per 100 lb coal fired gives a heating value of 0.2 × 14,500 Btu = 2900.

Since no unburned gases are present in the flue gas, there is no heat loss in this respect. However, when CO, H_2, or CH_4 are present, the weights present may be calculated as in Example 2, and the heating value may be obtained by multiplying by the low-heat value of the respective gases, given in Table 1, p. 2-04.

(3) Enthalpy of Water Vapor in Flue Gas.

Moisture in Flue Gas	Pounds	Pound-moles
From coal	8.8	0.488
From combustion of H_2	37.7	2.092
From air used for combustion	6.8	0.377
	Total moisture =	2.957

The mean molar heat capacity of water vapor is 8.115 Btu per lb-mole, °F at 340 F; therefore, the enthalpy is

$$2.957 \times 8.115 \times 260 = 6239 \text{ Btu}$$

(4) Enthalpy of Refuse. This quantity usually is quite small and is included merely to illustrate the calculation. In practice, the weight of refuse generally is somewhat smaller than the weight of the ash in the coal, but no appreciable error is introduced if it is assumed to be the same. Thus, the ash is 13.2 lb per 100 lb coal fired, and its specific heat is approximately 0.27 Btu per lb, °F. Assuming 20% of the ash discharged to the ash pit at 2000 F and 80% to the flue gas at 340 F, the total enthalpy would be

$$13.2 \times 0.27(0.80 \times 260 + 0.20 \times 1920) = 2110 \text{ Btu}$$

(5) Heat-transfer Losses. Radiation and convection heat losses from large boilers may be estimated from Plate 3 of the ASME *Code for Stationary Boilers, 1946*. In the present case, it was found to be 3800 Btu per 100 lb coal fired = 3800 Btu.

Total losses due to enthalpy of waste solids and gases and to heat transfer to surroundings = 93,551 Btu

Alternate (Short-cut) Method for Computing Total Heat Losses Found Above. Where limited data are available, the heat losses may be computed with fair accuracy from eq. 7.

$$Q_L = \left(a + \frac{b}{CO_2}\right)(t_f - t_a) \tag{7}$$

where Q_L = enthalpy of waste products, as percentage of low-heat value of fuel; a and b = constants for given fuel (see Table 5); t_f = temperature of flue gas, °F; t_a = temperature of air, °F; and CO_2 = percentage by volume in dry flue gas (Orsat).

When CO is present in the flue gas, ΔQ_L should be added to Q_L.

$$\Delta Q_L = c\frac{CO}{CO_2 + CO} \tag{8}$$

where c = constant for given fuel and CO = percentage by volume in flue gas.

Table 5. Values of Constants in Equations 7 and 8 for Various Fuels

	a	b	c
Solid fuels			
Anthracite	.0023	365	59
Bituminous coal	.0031	340	55
Lignite and sub-bituminous coal	.0048	375	61
Coke	.0018	395	64
Fuel oil	.00395	280	45
Gaseous fuels			
Natural gas	.0056	225	37
Blast-furnace gas	.00375	850	14
Coke-oven gas	.0063	170	28
Carburetted water gas	.0051	260	43
Producer gas (from coal)	.0046	.455	74

In the present example,

$$Q_L = \left(0.0031 + \frac{0.34}{12.8}\right)(340 - 80) = 7.71\%$$

On the basis of the low-heat value of the coal, the total heat losses would be

$$100 \times 10{,}721 ^* \times 0.0771 = 82{,}660 \text{ Btu per 100 lb coal fired}$$

Although the equation does not consider the enthalpy losses in the refuse and heat-transfer losses, agreement with the more precise procedure is fairly good, and the method serves as a good approximation, especially when these losses can be added separately.

(6) Thermal efficiency of a process is defined as the useful heat absorbed divided by the heat input. In many cases, however, the heat absorbed is determined more readily indirectly by evaluating the losses, as above, and subtracting them from the heat input.

On the basis of the low-heat value of the coal—10,721 Btu per lb in the present example—the heat input per 100 lb coal fired would be 1,072,000 Btu. The heat absorbed would be 1,072,000 − 93,550 = 978,450 Btu per 100 lb coal fired, or 1,072,000 − 82,660 = 989,340 Btu by eq. 7. The percentage thermal efficiency then would be

$$100 \times \frac{978{,}450}{1{,}072{,}000} = 91.3\% \text{ by method of losses}$$

or

$$100 \times \frac{989{,}340}{1{,}072{,}000} = 92.3\% \text{ by eq. 7}$$

Where the high-heat value of the fuel is used, the thermal efficiency would be

$$100 \times \frac{978{,}450}{1{,}120{,}000} = 87.4\%$$

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COMPARISON OF FUELS

CLASSIFICATION. There are three general types of fuel.

Solid, including coal, coke, peat, briquets, wood, charcoal, and waste products.

Liquid, including petroleum and its derivatives, synthetic liquid fuels manufactured from natural gas and coal, shale oil, coal by-products (including tars and light oil), and alcohols.

Gaseous, including natural gas, manufactured and industrial by-product gases, and the propane and butane or liquefied petroleum (LP) gases that are stored and delivered as liquids under pressure but used in the gaseous state.

4. RELATIVE ECONOMY OF VARIOUS FUELS

THE RELATIVE ECONOMY OF VARIOUS FUELS does not depend on the heating value of the fuel itself as much as on the conditions attending its use. Martin Frisch (*Trans. Am. Soc. Mech. Engrs.*, FSP-52-11, 1930) describes boiler tests with the nine fuels shown in Table 1, from which he derives Table 4 showing the relative price that can be paid for different fuels to generate steam at the same total cost with each fuel. The solid fuels were burned in pulverized form. The index of boiler performance was the sensible

* LHV of coal is HHV − (1030 × total water from coal), or $11{,}200 - \left(1030 \times 0.465 \frac{\text{lb}}{\text{lb}}\right) = 10{,}721$ Btu per lb.

Table 1. Comparative Analyses and Heating Values of Various Fuels

Analysis, %	Texas Lignite	Illinois Bituminous	Pittsburgh Bituminous	Pocahontas Semi-bituminous	River Anthracite	Oil	Blast-furnace Gas	Natural Gas	Coke-oven Gas
C	39.6	68.0	75.4	80.7	77.6	84.3	...	...	...
H_2	3.0	4.3	4.8	4.7	2.3	12.7	0.2	7.0	9.7
O_2	10.0	8.3	6.1	4.5	3.6	1.0	0.1	0.8	...
N_2	0.9	1.5	1.4	1.3	0.8	0.2	59.6	7.3	30.8
S	0.5	1.4	1.4	1.2	0.7	0.8	...	...	...
Ash	10.0	10.0	8.5	5.0	14.0	...	...	...	...
Moisture	36.0	6.5	2.4	2.6	1.0	1.0	...	...	...
CO	...	...	...	...	...	...	27.6	25.6	15.2
CH_4	...	...	...	...	...	...	0.4	30.3	41.3
CO_2	...	...	...	...	...	...	12.1	16.0	3.0
C_2H_4	...	...	...	...	...	...	...	5.6	...
C_2H_6	...	...	...	...	...	...	...	7.4	...
Volatile matter	32.0	37.0	34.5	18.2	8.2	...	...	...	...
Fixed carbon	22.0	46.5	54.6	74.2	76.8	...	...	...	...
Heating value, Btu per lb	7000	11,700	13,400	14,500	12,450	18,600	1426	15,530	16,354
Fusion point of ash, °F	2200	2,000	2,100	2,500	2,400	.		.	.....
Theoretical air required, lb per lb of fuel	5.21	9.04	10.14	10.77	9.63	14.12	0.82	10.3	10.86
Flue gas analysis, %									
CO_2	15.0	15.0	15.0	15.0	15.0	13.0	21.6	11.3	8.4
O_2	4.3	4.2	3.9	4.1	5.0	3.1	0.9	2.0	1.9
N_2	80.7	80.8	81.1	80.9	80.0	83.9	77.5	86.7	89.7

heat imparted to the products of combustion per pound passed through the boiler. This index depends on (1) heating value of the fuel, (2) weight of gas and vapor formed by its combustion, and (3) amount of heat made unavailable by incomplete combustion and evaporation of moisture and formation of water vapor by burning hydrogen. The effect of (2) is small whereas that of (3) may be large. Excess air influences the magnitude of the index as it affects the total quantity of gas formed per available heat unit. The author presents curves based on this index, which show that the fuels imparting the greatest amount of sensible heat to the flue gases have, in all cases, lower exit temperatures of flue gases and lower draft losses than fuels imparting less sensible heat. See Table 2 compiled from curves in the paper.

Table 2. Sensible Heat, Exit Temperatures, and Draft Losses with Various Fuels

(Based on 15% CO_2 in flue gas with solid fuels, 13% CO_2 with oil fuel, and 10% excess air with gaseous fuels. Fuels burned in straight-tube boiler.)

Fuel	Net Available Heat, Btu per lb of Flue Gas	Percentage of Rated Capacity of Boiler											
		100	200	300	400	500	600	100	200	300	400	500	600
		Exit Temperatures, °F						Draft Loss, in. of Water					
Blast-furnace gas	740	503	581	683	792	915	1042	.36	1.24	2.71	4.89	7.38	9.76
Texas lignite	890	489	544	616	700	789	889	.18	0.80	1.82	3.29	5.00	6.80
River anthracite	944	484	533	600	669	752	818	.13	0.65	1.58	2.87	4.40	6.07
Illinois bituminous	953	484	533	600	669	752	818	.13	0.65	1.56	2.84	4.36	6.00
Pittsburgh bituminous	986	482	531	587	658	731	818	.11	0.85	1.42	2.67	4.09	5.62
Pocahontas semi-bituminous	1001	482	526	580	650	722	800	.11	0.56	1.36	2.58	3.91	5.42
Oil	1001	482	526	580	650	722	800	.11	0.56	1.36	2.58	3.91	5.42
Coke-oven gas	1117	475	513	562	617	681	745	.03	0.44	1.09	2.13	3.22	4.36
Natural gas	1130	475	513	560	613	672	733	.00	0.38	1.00	2.00	2.96	4.00

The capacity developed with a given draft loss increased with the sensible heat per pound of gas passed through the boiler. Flame temperature has a greater effect on capacity than has exit temperature. For example, the combustion temperature of natural gas was about 1000 F higher than that of blast-furnace gas, but the exit temperatures were within 35 F of each other. Preheating the combustion air increased efficiency about 1% at 500% of rating and decreased draft loss over 3/4 in. It also lowered exit temperatures. The difference in efficiency attainable with the different fuels at a given percentage of boiler rating is due also to the completeness of combustion and latent heat losses, which vary with each fuel. Combustion losses with gaseous fuels and oil are negligible, but latent heat losses are high and may exceed 10% with coke-oven and natural gas. With solid fuels burned in pulverized condition, the losses due to incomplete combustion and unburned carbon increase with increasing fixed carbon content. Curves in the paper show that the losses due to unburned carbon range about as follows:

Btu liberated per cu ft of combustion space	8000	12,000	16,000	20,000	24,000	30,000
Loss due to unburned carbon, %						
Anthracite	7.0	7.10	7.25	7.4	7.6	8.1
Pocahontas semi-bituminous	1.30	1.40	1 55	1.70	1.95	2.70
Pittsburgh bituminous	0.60	0.72	0.82	0.90	1.00	2.05
Illinois bituminous	0.35	0.40	0.45	0.50	0.60	0.75
Lignite	0.20	0.20	0.25	0.25	0.25	0.40

The efficiency attainable with Pittsburgh coal is about 0.5% higher than with Pocahontas and about 0.75% higher than with Illinois coal. Maximum attainable efficiencies with anthracite and Texas lignite are 5 to 6% lower than with Pittsburgh coal, whereas they are but 2 to 2.75% lower with mechanically atomized oil. The attainable efficiencies with blast-furnace gas and with coke-oven or natural gas are, respectively, 2 to 20% and 3 to 5% lower than with Pittsburgh coal. See Table 3.

Table 3. Typical Efficiencies with Various Fuels at Different Rates of Driving

Fuel	Efficiency, %						Maximum Efficiency	
Percentage of Rated Capacity of Boiler	100	200	300	400	500	600	Percentage of Rated Capacity	Efficiency, %
Texas lignite	77.48	78.00	76 73	74 72	72.39	69 85	160	78.26
Illinois bituminous	82.41	83.40	82.51	80.86	78.62	76.38	180	83.35
Pittsburgh bituminous	83.14	84.00	83.00	81.41	79.81	77.28	190	84.10
Pocahontas semi-bituminous	82.70	83 71	82 55	81 60	79.00	76 69	200	83.71
River anthracite	78.29	79.14	77 86	76.15	73 85	71.48	160	79.31
Oil	80.41	81 21	80 46	80.48	77.52	75.62	175	81.38
Blast-furnace gas	80.90	80 00	77 30	73.55	68 76	64.19	115	81 00
Natural gas	77 00	78 48	78.00	77.07	75.93	74 66	200	78.48
Coke-oven gas	76.14	77.46	77.05	76.25	75.00	73 67	230	77 42

The final cost of steam which determines the most economical fuel depends on (1) fixed operating and maintenance costs of boiler plant, exclusive of fuel and fan equipment, (2) cost of fuel, (3) fixed operating and maintenance charges on fuel storage, handling, preparation, and burning equipment, (4) fixed and operating charges on fans and draft equipment, and (5) fixed charges on plant capacities reserved to provide for peak requirements of fuel and fan equipment. Item 1 is common to all fuels; items 2 to 5 vary with each fuel.

Table 4 was derived from a study of the cost of operating steam plants designed to burn the fuels shown in the first column. The several columns of the table show the maximum price that can be paid for different fuels other than that for which it was designed if steam is to be generated at the same cost. The figures are absolute only for the conditions stated in the paper, but they will serve as a guide to the relative value of the various fuels when a choice is to be made of fuel in a primary design or when it becomes necessary to change to a different fuel. In general, a plant designed for any solid fuel will be most economical when burning that fuel or any higher grade fuel obtainable at the same price per Btu without additional capital expenditure. Some of the higher-grade fuels may be more economical, even at a higher cost per Btu. A plant designed for a high-grade solid fuel will not be economical with a lower-grade fuel. Thus, a plant designed for lignite probably would prove more economical with Pocahontas or Pittsburgh coal, but if designed for these it could not develop its required capacity with anthracite or lignite without

Table 4. Maximum Relative Price to Be Paid for Fuels to Produce Steam at Equal Cost

(Prices: solid fuels, dollars per ton, oil, dollars per barrel; natural gas, cents per 1000 cu ft)

Fuel for Which Furnace Is Designed	Total Cost of Steam, 30¢ per 1,000,000 Btu							Total Cost of Steam, 40¢ per 1,000,000 Btu							Total Cost of Steam, 50¢ per 1,000,000 Btu						
	Maximum Price of Fuel Actually Used							Maximum Price of Fuel Actually Used							Maximum Price of Fuel Actually Used						
	Lignite	Illinois Bituminous	Pittsburgh Bituminous	Pocahontas Semi-bituminous	River Anthracite	Oil	Natural Gas	Lignite	Illinois Bituminous	Pittsburgh Bituminous	Pocahontas Semi-Bituminous	River Anthracite	Oil	Natural Gas	Lignite	Illinois Bituminous	Pittsburgh Bituminous	Pocahontas Semi-bituminous	River Anthracite	Oil	Natural Gas
Lignite only	1.34	2.66	3.15	3.42			..	2.38	4.52	5.30	5.74				3.41	6.38	7.45	8.06	...	...	...
Lignite and natural gas	1.31	2.59	3.07	3.33	.		12.1	2.35	4.45	5.22	5.64			19.8	3.38	6.31	7.37	7.96	...	..	27.2
Lignite and oil	1.18	2.35	2.79	3.04		.62	...	2.22	4.21	4.94	5.36		1.07		3.25	6.07	7.09	7.68	...	1.53	..
Lignite, natural gas, and oil	1.15	2.30	2.72	2.97		.61	10.7	2.19	4.16	4.87	5.29	.	1.06	18.3	3.22	6.02	7.02	7.61	...	1.52	25.8
Illinois bituminous only		3.02	3.55	3.90		...			4.88	5.70	6.22	..			.	6.74	7.85	8.54	...	...	.
Illinois bituminous and natural gas		3.00	3.51	3.85			13.5		4.86	5.66	6.17			21.0		6.72	7.81	8.49	...	...	28.5
Illinois bituminous and oil	..	2.72	3.20	3.51		.70			4.58	5.35	5.83		1.16			6.44	7.50	8.15	...	1.62	..
Illinois bituminous, gas, and oil	..	2.64	3.16	3.47		.69	12.3		4.50	5.31	5.79		1.15	19.8		6.36	7.46	8.11	...	1.61	27.3
Pittsburgh bituminous only	...	.	3.61	3.96		.				5.76	6.28				.		7.91	8.60	...	...	...
Pittsburgh bituminous and natural gas	...	.	3.57	3.92			13.7	.	.	5.72	6.24	.		21.2		..	7.87	8.56	...	...	28.7
Pittsburgh bituminous and oil	...	.	3.26	3.58	.	.72		..	.	5.41	5.90	.	1.17		...	..	7.56	8.22	...	1.63	..
Pittsburgh bituminous, gas, and oil	...	...	3.22	3.54	.	.71	12.5	.	...	5.37	5.86	..	1.16	20.0	.	...	7.52	8.18	...	1.62	27.5
Pocahontas semi-bituminous only	...	...	.	3.99		...	...	.	...	...	6.31	...	...		.	...	.	8.63	...	...	...
Pocahontas semi-bituminous and natural gas	...	...	...	3.94	...	.	13.8	..	...	...	6.26	.		21.3	...	...	...	8.58	...	..	28.9
Pocahontas semi-bituminous and oil	...	...	...	3.60		.72	...	...	...	...	5.92	.	1.18		.	...		8.24	.	1.63	...
Pocahontas semi-bituminous, gas, and oil	...	...		3.56		.71	12.6	...	...	...	5.88		1.17	20.1	..	...	..	8.20	...	1.62	27.6
River anthracite only	...	...	3.34	3.69	2.40		..	..	...	5.49	6.01	4.27			.	.	7.64	8.33	6.14	...	...
River anthracite and natural gas	...	...	3.31	3.66	2.38		12.7	.	...	5.46	5.98	4.25		20.3	.	...	7.61	8.30	6.12	...	27.8
River anthracite and oil	...	.	2.99	3.31	2.10	.72		..	...	5.14	5.63	3.97	1.18		..	..	7.29	7.95	5.84	1.63	...
River anthracite, gas, and oil	...	.	2.96	3.27	2.07	.71	11.5	.	.	5.11	5.59	3.94	1.17	19.1		.	7.26	7.91	5.81	1.62	26.6
Natural gas	...		..	..	.		15.2	.	...		...	..		22.7				...	...	...	30.2
Oil	...		...	...	.	.79	..		.			.	1.25				.	...	...	1.72	...
Natural gas and oil	...		...	...		.79	13.7	.	...	...	...		1.24	21.2	...	...	.	...	...	1.70	28.8

additional capital expenditure. Pulverized solid fuels can compete with oil or natural gas, except in certain localities where oil or gas prices are low or where gas is a waste product.

5. TYPICAL ECONOMIC STUDY *

By R. E. Morgan

In making an economic comparison of different types of fuel, relatively simple calculations are necessary, based on the fuel's delivered unit cost, its total heat, and its efficiency of use.

EXAMPLE. Coal is usually sold by the short ton (2000 lb), oil by the gallon, and gas by the thousand cubic feet. A comparison of the cost per million Btu for (1) a coal costing 10 dollars per short ton, 13,500 Btu per pound as received, (2) an oil costing 7 cents per gallon, 144,000 Btu per gallon as received, and (3) a natural gas costing 40 cents per 1000 cubic feet, 1000 Btu per cu ft as received, is calculated.

$$\text{Coal: } \frac{\$10.00}{13{,}500 \text{ Btu} \times 2000 \text{ lb}} = \$0.370 \text{ per } 1{,}000{,}000 \text{ Btu as received}$$

$$\text{Oil: } \frac{\$0.07}{144{,}000 \text{ Btu}} = \$0.486 \text{ per } 1{,}000{,}000 \text{ Btu as received}$$

$$\text{Gas: } \frac{\$0.40}{1000 \text{ Btu} \times 1000 \text{ cu ft}} = \$0.400 \text{ per } 1{,}000{,}000 \text{ Btu as received}$$

The Btu content of different fuels varies considerably. In the trade, Btu content frequently is expressed on a *dry* basis, whereas it should be expressed on the *as-received* basis to make correct comparisons, since some coals have 10% or more inherent moisture. The Btu value of oil usually varies with its heaviness; the heavier the oil, the greater the Btu content per gallon. The Btu value of gas depends on its composition. Table 5 gives general values, many of which are also given elsewhere in Section 2.

Table 5. Heating Value of Various Types of Fuel

Fuel	Btu "As Received"
Coal (per pound)	
Anthracite, Semi-anthracite	11,500– 13,300
Bituminous	10,500 – 14,400
Sub-bituminous	7,800– 11,800
Lignite	6,500– 8,000
Coke (from coal)	
High temperature	12,000 – 13,300
Low temperature	12,300 – 13,600
Coke (from oil)	14,500 – 15,400
Manufactured briquets	11,700– 14,400
Oil (per gallon)	
No. 1	133,000–138,000
No. 2	135,000–141,000
No. 3	138,000–144,000
No. 5	145,000–152,000
No. 6	150,000–155,000
Gas (per cubic foot)	
Natural gas	800– 1,150
Manufactured gas (as delivered)	450– 600

To determine the comparative cost of fuels under different efficiencies, the efficiency factor must be used. In the example given above, assume the efficiencies to be coal, 65%; oil, 70%; and gas, 75%. These efficiency percentages, divided into previously found costs, give a true index of fuel cost: coal, \$0.570 per 1,000,000 useful Btu; oil, \$0.694 per 1,000,000 useful Btu; and gas, \$0.533 per 1,000,000 useful Btu. Yearly fuel bills would be in the same ratios.

Efficiency for Various Fuels. Table 6 gives a general idea of the values of the overall efficiencies that can be anticipated when the different fuels are fired.

Efficiencies attained in the large utility-type plants, equipped with economizers and preheaters, are higher than the figures shown.

* Adapted from Bureau of Mines, *Questions and Answers for the Home Fireman*, J. F. Barkley, U. S. Bureau of Mines.

The cost of steam, which determines the most economical fuel in steam plants, depends on (1) charges for operation and maintenance, (2) cost of fuel, (3) charges for handling fuel, (4) cost of operation and maintenance of auxiliary equipment, and (5) fixed charges for stand-by capacity.

Table 6. Typical Efficiencies for Various Fuels

Fuel	Efficiency in Ordinary Equipment, % High	Low
Anthracite	75	55
Bituminous	80	50
Lignite	65	40
Coke	75	55
Oil	80	55
Gas	80	60

Table 7 shows items to be considered in working out a cost comparison for different fuels.

Table 7. Economic Analysis

Item	Coal	Oil	Gas
	Basic Data		
Personnel required			
Engineers	2	2	2
Firemen	4	4	4
Coal passers or helpers	4	0	0
Fuel required			
Tons of coal	5,200	....	....
Barrels of oil	....	18,190	.. ..
M cu ft of gas	....	....	112,500
Price in dollars			
Per ton of coal	4.10	... .	....
Per barrel of oil	....	2.48	....
Per M cu ft of gas	....	....	0.3056
Heating value, million Btu			
Per ton of coal	22.6	....	. ...
Per barrel of oil	....	6.03	....
Per M cu ft of gas	. . . .	....	0.975
Estimated efficiency	70	75	75
Useful heat, million Btu	15.82	4.523	0.7313
Cost of equipment, dollars	30,000	20,000	12,500
	Operating Cost (dollars)		
Labor	22,396	15,196	15,196
Fuel	21,320	45,110	34,380
Depreciation	900	600	375
Interest	1,500	1,000	625
Annual maintenance	500	300	200
Electric power	900	400	0
Atomizing steam	0	0	0
Pumping	0	325	0
Ash handling	440	0	0
Total annual operating	47,956	62,931	50,776
Differential	0	14,975	2,820

SOLID FUELS

By E. P. Carman, R. C. Corey, R. E. Morgan, and R. E. Brewer

6. COAL CLASSIFICATION

By E. P. Carman

Three methods of classifying coals have been adopted as standard in the United States as the result of a 10-year study begun in 1927 by a large group of specialists from the United States and Canada. These classifications are: by *rank* (Ref. 1) (degree of metamorphism, or progressive alteration, in the natural series from lignite to anthracite); by *grade* (Ref. 2) (quality determined by size designation, calorific value, ash, ash-softening temperature, and sulfur); and by *type* or *variety* (Ref. 3) (determined by the nature of the original plant material and subsequent alteration thereof). Other methods of coal classification are by use or suitability for specific purposes or types of combustion equipment, and by various trade systems set up to meet particular conditions in a given area or time. Examples of the use or special-purpose type of classification are given in two other standards that have been adopted in this country. One of these classifies coal by ash content (Ref. 4) and the other, a standard for gas and coking coals, classifies by use (Ref. 5).

CLASSIFICATION BY RANK. Probably the most universally applicable method of classification is by rank, in which coals are arranged according to fixed carbon content and calorific value, in Btu, calculated on the mineral-matter-free basis. The higher-rank coals are classified according to fixed carbon on a dry basis; the lower-rank coals, according to Btu

Table 1. Classification of Coals by Rank *

(From *Bulletin* 446 of the Bureau of Mines)

(FC = fixed carbon; VM = volatile matter; Btu = British thermal units)

Class	Group	Limits of Fixed Carbon or Btu (Mineral-matter-free Basis)	Requisite Physical Properties
I. Anthracitic	1. Meta-anthracite	Dry FC, 98% or more (dry VM, 2% or less)	....
	2. Anthracite	Dry FC, 92% or more, and less than 98% (dry VM, 8% or less, and more than 2%)	....
	3. Semi-anthracite	Dry FC, 86% or more, and less than 92% (dry VM, 14% or less, and more than 8%)	Nonagglomerating †
II. Bituminous ‡	1. Low-volatile bituminous coal	Dry FC, 78% or more, and less than 86% (dry VM, 22% or less, and more than 14%)	....
	2. Medium-volatile bituminous coal	Dry FC, 69% or more, and less than 78% (dry VM, 31% or less, and more than 22%)	....
	3. High-volatile A bituminous coal	Dry FC, less than 69% (dry VM, more than 31%), and moist § Btu, 14,000 ‖ or more	....
	4. High-volatile B bituminous coal	Moist § Btu, 13,000 or more, and less than 14,000 ‖	....
	5. High-volatile C bituminous coal	Moist Btu, 11,000 or more, and less than 13,000 ‖	....
III. Sub-bituminous	1. Sub-bituminous A coal	Moist Btu, 11,000 or more, and less than 13,000 ‖	Both weathering and nonagglomerating ¶
	2. Sub-bituminous B coal	Moist Btu, 9500 or more, and less than 11,000 ‖	....
	3. Sub-bituminous C coal	Moist Btu, 8300 or more, and less than 9500 ‖	....
IV. Lignitic	1. Lignite	Moist Btu, less than 8300	Consolidated
	2. Brown coal	Moist Btu, less than 8300	[illegible]

* This classification does not include a few coals that have unusual physical and chemical properties and which come within the limits of fixed carbon or Btu of the high-volatile bituminous and sub-bituminous ranks. All these coals either contain less than 48% dry, mineral-matter-free fixed carbon or have more than 15,500 moist, mineral-matter-free Btu.

† If agglomerating, classify in low-volatile group of the bituminous class.

‡ Moist Btu refers to coal containing its natural bed moisture but not including visible water on the surface of the coal.

§ It is recognized that there may be noncaking varieties in each group of the bituminous class.

‖ Coals having 69% or more fixed carbon on the dry, mineral-matter-free basis shall be classified according to fixed carbon, regardless of Btu.

¶ There are three varieties of coal in the high-volatile C bituminous group, namely, (1) agglomerating and nonweathering, (2) agglomerating and weathering, and (3) nonagglomerating and nonweathering.

on a moist basis (containing natural bed moisture). Agglomerating and weathering indexes are used to differentiate between certain adjacent groups. The classification is summarized in Table 1, but the complete standard (Ref. 1) contains details of classification, methods of sampling, analysis, and testing, and calculation of test results. In this standard, condensed symbols are provided to simplify the designation of the classification of any particular coal; numbers in parentheses signify properties on mineral-matter-free basis. The first number indicates fixed carbon on the dry basis to the nearest whole per cent; the second indicates Btu per pound, on the moist basis, to the nearest hundred. Thus, a coal with 62% fixed carbon and 14,580 Btu would be represented as (62–146). When classification of a coal by rank involves agglomerating or weathering properties, the following symbols are used: ag for agglomerating, na for nonagglomerating, we for weathering, and nw for nonweathering.

A graphic representation of the ASTM classification of coals by rank is shown in Fig. 1 (after Freeman, Ref. 6).

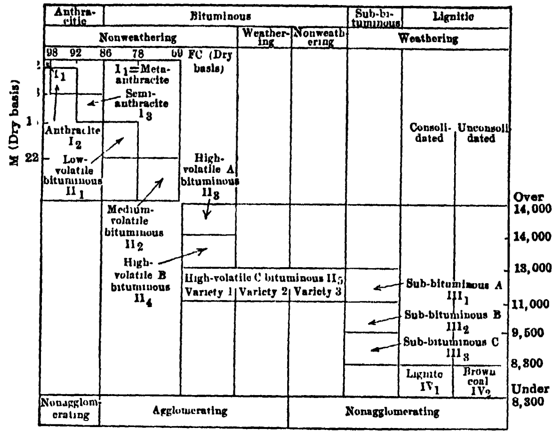


FIG. 1. ASTM classification of coals by rank. (After Freeman)

CLASSIFICATION BY GRADE. The standard for classification of coal by grade (Ref. 2) provides a symbol designation system indicating *size, Btu content, ash, ash-softening temperature*, and *sulfur content* of coals. The size designation is given first in accordance with the standard screen analysis method (Ref. 7),* followed by calorific value (expressed in hundreds of Btu per pound to the nearest hundred), and symbols representing ash, ash-softening temperature, and sulfur, in accordance with Table 2. For example, 4 x 2 in., 132–A8–F24–S1.6, indicates a coal 4 by 2 in. in size (through a 4-in. and on a

Table 2. Symbols for Grading Coal According to Ash, Ash-softening Temperature, and Sulfur *

Expressed on basis of coal as sampled

Ash †		Softening Temperature of Ash ‡		Sulfur †	
Symbol	%,§ inclusive	Symbol	°F, inclusive	Symbol	%, inclusive
A4	0.0– 4.0	F28	2800 and higher	S0.7	0.0–0.7
A6	4.1– 6.0	F26	2600–2790	S1.0	0.8–1.0
A8	6.1– 8.0	F24	2400–2590	S1.3	1.1–1.3
A10	8.1–10.0	F22	2200–2390	S1.6	1.4–1.6
A12	10.1–12.0	F20	2000–2190	S2.0	1.7–2.0
A14	12.1–14.0	F20 minus	Less than 2000	S3.0	2.1–3.0
A16	14.1–16.0			S5.0	3.1–5.0
A18	16.1–18.0			S5.0 plus	5.1 and higher
A20	18.1–20.0				
A20 plus	20.1 and higher				

* Reprinted by permission of the American Society for Testing Materials from "Standard specification for classification of coals by grade," ASTM D389-37, *ASTM Standards on Coal and Coke*, Committee D-5, 1948, p. 118.

† Ash and sulfur shall be reported to the nearest 0.1% by dropping the second decimal figure when it is 0.01 to 0.04, inclusive, and by increasing the percentage by 0.1% when the second decimal figure is 0.05 to 0.09, inclusive. For example, 4.85 to 4.94%, inclusive, shall be considered to be 4.9%.

‡ Ash-softening temperatures shall be reported to the nearest 10 F. For example, 2635 to 2644 F, inclusive, shall be considered to be 2640 F.

§ For commercial grading of coals, with ash less than 2%, ranges in the percentage ash smaller than 2% are commonly used.

* Data from this specification reprinted by permission of the American Society for Testing Materials.

2-in. round-hole screen); with approximately 13,200 Btu per lb (13,150 to 13,249 Btu, inclusive); an ash content of 6.1 to 8.0%, inclusive; ash-softening temperature of 2400 to 2590 F, inclusive; and sulfur content of 1.4 to 1.6%, inclusive. Analyses are expressed on the basis of the coal as sampled.

Screen Sizes. The standard for screen analysis (Ref. 7) lists the following series of screens.

Round-hole screens: 8, 6, 5, 4, 3, 2 1/2, 2, 1 1/2, 1 1/4, 1, 3/4, 1/2, and 3/8 in.

Wire-cloth sieves with square openings: No. 4 (4760 μ), No. 8 (2380 μ), No. 16 (1190 μ), No. 30 (590 μ), No. 50 (297 μ), No. 100 (149 μ), and No. 200 (74 μ).

The standard specifies that the upper limiting screen shall be the smallest screen of the series upon which less than 5% of the sample remains after screening, and the lower limiting screen shall be the largest on which at least 85% of the sample is retained. The size or number representing the upper screen is written first, and the lower screen size or number, second.

Anthracite Sizes. A separate standard (Ref. 8) covers size designations of anthracite as adopted by the Anthracite Committee of the Production Control Plan for the Anthracite Industry, Harrisburg, Pennsylvania, effective July 28, 1947 (see Table 5, p. 2-31).

When coal is classified by both rank and grade, the standards specify that grade designation shall follow rank designation, e.g., (62 146), 4 x 2 in., 132–A8 -F24–S1.6. This represents a coal with 62% fixed carbon on the dry, mineral-matter-free basis; approximately 14,600 Btu per lb heating value on the moist mineral-matter-free basis; more than 95% of the coal passing a 4-in. screen and over 85% retained on a 2-in. screen; an "as-sampled" heating value of approximately 13,200 Btu per lb; ash content between 6.1 and 8.0%, inclusive; ash-softening temperature within the range 2400–2590 F, inclusive; and sulfur content between 1.4 and 1.6%, inclusive.

CLASSIFICATION BY TYPE. In classifying coals by type or variety, consideration is given to the nature of the original plant ingredients and the biochemical alteration of these ingredients during the various steps in transforming living plant tissues into coal. Four commercial varieties of bituminous and sub-bituminous coal have been accepted as standard (Ref. 3).

Common banded coal is the common variety of bituminous and sub-bituminous coal. It consists of a sequence of irregularly alternating layers or lenses of (1) homogeneous black material with a brilliant vitreous luster, (2) grayish-black, less brilliant, striated material, usually of silky luster, and (3) generally thinner bands or lenses of soft, powdery, fibrous particles of mineral charcoal. The difference in luster of the bands is greater in bituminous than in sub-bituminous coal.

Splint coal is a variety of bituminous or sub-bituminous coal, commonly having a dull luster and grayish-black color, of compact structure, often containing a few thin irregular bands with vitreous luster. It is resonant when struck. It is hard and tough, breaks with an irregular, rough, sometimes splintery fracture, is free burning, and does not swell when heated.

Cannel coal is a variety of bituminous or sub-bituminous coal of uniform and compact fine-grained texture, with a general absence of banded structure. It is dark gray to black, has a greasy luster, is noticeably of conchoidal or shell-like fracture, is noncaking, yields a high percentage of volatile matter, ignites easily, and burns with a luminous, smoky flame.

Boghead coal is a variety of bituminous or sub-bituminous coal resembling cannel coal in appearance and behavior during combustion. It is characterized by a high percentage of algal remains and volatile matter. Upon distillation, it gives exceptionally high yields of tar and oil.

Many methods of coal classification were proposed, both in the United States and abroad, before the standard classifications by rank, grade, and variety were adopted. Ralston (Ref. 9) proposed a classification based on taking the sum of the carbon, hydrogen, and oxygen of the fuel as 100% and plotting the percentages of these three elements on trilinear coordinates. Rose (Ref. 10) continued the graphic study of coal classification from ultimate analyses and developed a "multibasic coal chart" which permits visual comparison of coal analyses graphed according to fixed carbon, volatile matter, Btu content, or ultimate analyses calculated on the basis of any of several different purity bases that had been proposed. The multibasic coal chart provides the most satisfactory graphic method for comparing coal classifications and is frequently desirable for presenting coal analyses for other purposes. Reference 10 shows the method of constructing and using this type of chart.

7. SAMPLING AND ANALYSIS OF COAL

By E. P. Carman

As a result of its mode of formation, coal is a very complex substance. Any two small lumps from the same bed are never absolutely the same in every respect. It is fundamental in coal sampling and analysis, therefore, that, although general characteristics may be determined for a given quantity of coal, not every small piece in that quantity of coal will have the same characteristics as the average characteristics for the lot as determined by the analysis. Because of this variation in the coal substance, it is extremely important to follow proper sampling procedures. The most accurate analysis possible means little if the sample is not representative of the lot of coal being sampled. The same considerations apply to the sampling and analysis of coke and, in varying degree, to all solid fuels.

METHODS OF SAMPLING COAL AND COKE. Mine samples are used by coal-mining companies and by the Bureau of Mines, U. S. Department of the Interior, for the Federal Government in determining the characteristics of coal produced in any given mine or district. Mine samples, however, may not be representative of commercial coal in ash content and heating value, and caution must be exercised in estimating the grade of commercial coal on the basis of mine samples. Frequently, the ash of the shipped coal is several per cent higher than the ash reported in mine samples, particularly with unwashed, mechanically mined coal.

The mine sample represents the quality of coal that *can* be obtained when the impurities are readily separated from the coal by washing or other coal-preparation methods and when extreme care is taken in the mine and in the preparation plant to obtain the cleanest possible coal. A mine sample is taken at the selected point in the mine by cutting away the face coal for a width of 1 ft and for the entire height of the bed to a depth of at least 1 in. The sample is then cut as a "channel" 2 in. deep and 6 in. wide in harder coals, or 3 in. deep and 4 in. wide in the softer coals, from the roof to the floor, down the middle of the foot-wide cut previously made in the coal face. The sample is collected on a sampling cloth spread on the mine floor; parting material more than $^3/_8$ in. and other impurities ordinarily rejected, such as sulfur lenses, are discarded. The sample is mixed and broken down by specified methods to a quantity small enough to fill a sample can, which then is carefully sealed before it is taken from the mine for shipment to the analytical laboratory. (For more complete detail on sampling coal in the mines, see Ref. 11.)

Tipple, Breaker, Truck, Railroad Car, and Ship Samples of Coal. Most coal samples are taken after the coal has been removed from the mine—at mine tipples; at breaker and preparation plants; from trucks, railroad cars, and ships; or upon delivery at the ultimate destination. "Increments," each consisting of the coal obtained by a single motion of the sampling instrument through or into the coal being sampled, are taken in accordance with standardized procedures. The sample is crushed, mixed, and reduced by riffling until a sample of not less than 1 $^3/_4$ lb is obtained. This sample is then sealed in a sample can or Mason jar for shipment to the laboratory. Special procedures are required in sampling for total moisture content. Complete details and specifications for sampling coal for shipment or delivery are given in Ref. 12.

Sampling Coke. Complete details of methods for sampling coke, including size of sample, crushing, mixing, and reduction of size, and size of final sample sealed in suitable containers for shipment to the laboratory are covered in Standard Method of Sampling Coke for Analysis (Ref. 13). The size of original samples ranges from 500 lb for large coke to 125 lb for coke breeze, and the number of increments specified ranges from the maximum of 50 to the minimum of 9. The standard illustrates methods of piling, mixing, crushing, and quartering samples for reduction to laboratory sample size.

ANALYSIS OF COAL AND COKE, INCLUDING PREPARATION OF LABORATORY SAMPLE. The most frequently used analyses of coal and coke are the proximate analysis, sulfur, and heating value. The proximate analysis involves determining moisture, volatile matter, fixed carbon, and ash. Determination of sulfur is employed to supplement the proximate-analysis data, and the heating or calorific value gives the heat produced by a unit quantity of the fuel burned under certain standard conditions. Complete details and instructions for preparing the small laboratory sample from the larger sample and standardized methods of performing the above analyses, as well as others, are given in Ref. 14.

The heating or calorific value of solid fuels is determined by burning a weighed quantity in a bomb calorimeter with oxygen and carefully determining the temperature rise in the calorimeter with thermometers, graduated to 0.01 to 0.02 C and certified for accuracy by a government testing bureau. Complete instructions and specifications for making the heating value determination are given in Ref. 14. The value determined in the bomb calorimeter is the total heat developed by complete combustion, with all products of combustion cooled down to the temperature of the calorimeter (about 20 C).

When data from the ultimate analysis (see below) of coal are available, the "as-received" heating value can be approximately computed by Dulong's formula

$$\text{Btu per pound} = 14{,}544C + 62{,}028\left(H - \frac{O}{8}\right) + 4{,}050S$$

where C, H, O, and S are the actual fractions by weight of these elements in the sample (see also p. 2-04). The formula assumes that the oxygen in the coal combines with some of the hydrogen to form water, while the remaining hydrogen and the carbon and sulfur are available to supply heat. With the higher-rank anthracites and bituminous coals, the formula gives values that seldom deviate more than 1 1/2% from those of the bomb calorimeter; for the lower-rank sub-bituminous and lignitic coals, variations up to 4 or 5% may be found. Formulas for calculating the heating value from proximate-analysis data cannot be recommended for general use.

Ultimate Analysis. From the ultimate analysis of coal are determined the percentages of carbon, hydrogen, nitrogen, and oxygen in the coal. The carbon given by the ultimate analysis is *all* the carbon in the coal, including that in the volatile matter as well as that in the fixed carbon of the proximate analysis.

The determinations, with specific directions for apparatus, preparation of solutions, etc., are described in Ref. 14.

Methods of Reporting Coal Analyses. Three methods of reporting coal analyses are in general use—"as received," "moisture free," and "moisture and ash free." The "as-received" basis is particularly useful where moisture may play an important part in determining whether coal is up to purchase specifications. Since surface moisture is a variable constituent, depending on weather, exposure, etc., the "moisture-free" determination more truly represents the true coal substance but includes the ash. The "moisture- and ash-free" analysis gives the percentages of the combustible matter only in the coal. Conversion from one type of reporting to another can readily be made from moisture and ash determinations of the proximate analysis.

SPECIAL TESTS, fully described in the *ASTM Standards on Coal and Coke,* Committee D-5, 1948, and in *Technical Paper* 8 of the Bureau of Mines, are briefly described below. The ASTM designation number is given in parentheses.

Fusibility of Ash (D271-48) is used to determine on a small scale the softening temperature of coke and coal ash. Initial deformation temperature and fluid temperature are frequently determined by this test.

Free-swelling index (D720-46) is a small-scale laboratory test of the free-swelling properties of coal to indicate its coking characteristics when burned as a fuel; it does not measure the expanding properties of coals in coke ovens. Indexes of 1 to 3 represent relatively free-burning (nonexpanding) coals in the fuel bed; 3 1/2 to 6 represent moderately expanding coals in fuel beds; and 6 1/2 to 9 represent strongly swelling coals in fuel beds.

Agglutinating-value test is a small-scale laboratory test that indicates the caking and coking properties of coal and gives an approximate measure of the material in coal that fuses and becomes plastic when heated. In this test, the average crushing strength of six specially prepared buttons, if within specified limitations, gives the agglutinating-value index, reported to the nearest tenth of a kilogram.

Plastic properties of coal may be determined by two methods proposed for standardization; the Davis-type plastometer and the Gieseler-type plastometer. These laboratory methods determine the plastic properties of coal when heated in the absence of air, as in coke and gas ovens. They are useful in predicting the coking behavior of bituminous coals and coal blends and in determining the extent of weathering a coking coal has undergone, when results are known for the fresh coals.

Expansion Pressure of Coal during Coking. At least four types of tests have been developed to determine expansion pressures and strains developed on coke-oven walls during coking.

Tests of weight in pounds per cubic foot of crushed bituminous coal (D291-29) **and of coke** (D292-29) are designed to provide reproducible results in determining bulk density and are not applicable to powdered fuels.

Determination of the true specific gravity of coal and coke is described in *Technical Paper* 8 of the Bureau of Mines, 1938 edition, pp. 37 and 38.

Grindability of coal (data in Section 7), an important factor in the pulverizing of coal for pulverized-coal-fired boilers, can be determined by the Hardgrove Machine method (Tentative D409-37T) or by the ball-mill method (Tentative D408-37T).

Size stability of coal and coke is determined by drop-shatter tests (Tentative D440-48T for coal; D141-48 for coke) and by tumbler tests (D441-45 for coal; D294-29 for coke). In the drop-shatter test, the fuel is dropped 6 ft from a hinged-bottom box onto a cast-iron or steel plate; coal is dropped twice, and coke is dropped four times. The product is screened, and results are reported as percentages of weight retained on and passing through specified sizes of screens.

Friability or abradibility of coal and coke in shipping, handling, and use are determined by tumbler tests (D441-45 for coal; D294-29 for coke).

The test for dustiness of coal and coke (D547-41) is designed to provide a relative index of dust produced when these fuels are handled.

The fineness test for powdered coal (D197-30), a procedure for determining fineness of powdered coal by sieve analysis, is of importance to pulverized-coal-furnace operators, since ease of ignition, length and stability of flame, and amount of combustible lost in the fly ash are directly affected by particle size of the powdered fuel.

Sizes of anthracite are standardized, and the method of determining such sizes are given in D310-34.

The volume of cell space in lump coke is determined by standard method D167-24.

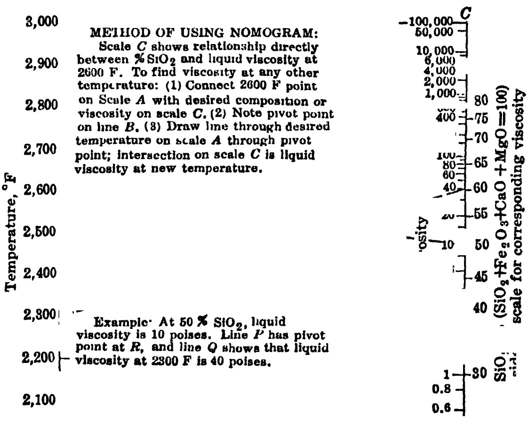
8. PROPERTIES OF COAL ASH

By Richard C. Corey

PROPERTIES OF COAL ASH. The most frequent causes of outage of coal-fired steam boilers are excessive draft losses and overheating and failure of heat-absorbing surfaces due to accumulations of ash and slag. Knowledge of the properties and the behavior of coal ash consists principally of the results of studies of relations between the chemical composition of coal ash and its viscosity, tendency to form clinkers, and fusion characteristics.

Composition. Coal-ash slags are composed essentially of silica, alumina, iron oxides, lime, and magnesia, with minor quantities of the oxides of phosphorus, titanium, and alkali metals. The composition of the original ash may vary over a wide range with respect to silica, alumina, and iron oxides, depending on the source of the coal. For any given coal, however, wide variations in the composition of the ash and slag may result from the classifying action that occurs in the furnace. For example, although the original coal ash may contain in the order of 0.001% of arsenic, expressed as As_2O_3, deposits on secondary heating surfaces have been found to contain as much as 15% As_2O_3.

Viscosity of Liquid Slags. Any coal ash, when heated to a sufficiently high temperature, undergoes chemical reactions that lead to the formation of a liquid slag. If the viscosity of a slag is known at one temperature, the entire viscosity-temperature relationship for the liquid slag can be ascertained from the nomogram shown in Fig. 2, which readily gives the viscosity of a liquid slag at any desired temperature if the viscosity is known at a particular temperature. The additional scale at the right side of the figure permits calculation of the viscosity at 2600 F, an arbitrarily selected base temperature, if the chemical composition of the slag is known. Therefrom, the viscosity of the same slag at any other temperature can be determined.



Nomogram

FIG. 2. Viscosity of coal-ash slags in the liquid state as a function of composition and temperature.

FLOW CHARACTERISTICS OF COAL-ASH SLAGS IN THE SOLIDIFICATION RANGE. When a slag is cooled from a temperature at which it is completely liquid, the viscosity increases. At a certain temperature, defined as the *liquidus temperature*, crystals of a composition usually different from that of the mother liquid begin to separate. The process is similar to that which occurs during cooling of ordinary noneutectic lead-tin solders from above the melting point, where the crystals that separate during cooling cause the mass to go through a "mushy" stage before the entire mass solidifies. The increase in viscosity when a liquid slag cools is the net result of a progressive change in viscosity of the liquid phase, due to the change in composition as crystals separate, the presence of crystals suspended in the liquid phase, and the increased viscosity of the liquid phase. Since the increased viscosity is the predominant effect and masks the other two, a continuous, smooth, viscosity-temperature curve is obtained for some temperature interval below the liquidus temperature. At a definite temperature for each slag, however, the accumulation of crystals causes an abrupt change in the flow properties of the slag, owing to the fact that at temperatures below this critical temperature the slag has plastic rather than viscous flow properties. This temperature, the *temperature of critical viscosity*, T_{cv}, is of the greatest significance because for low shear stresses the apparent viscosity at temperatures below T_{cv} will be quite high and will increase rapidly as the temperature decreases.

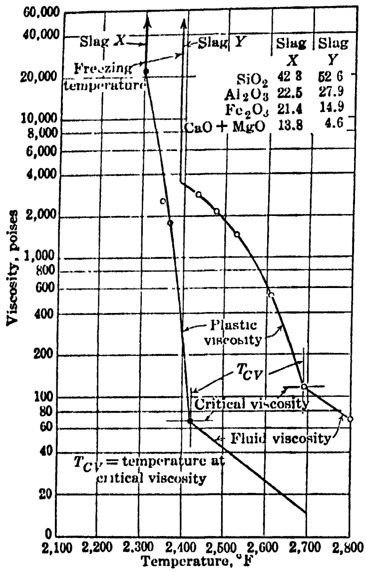


FIG. 3. Effect of cooling on solidification of two typical coal-ash slags, showing critical viscosity and region of plastic viscosity, for equilibrium conditions in air at each indicated temperature.

In Fig. 3 flow curves are shown for two typical slags over the entire temperature range of interest. The radical change in flow properties at T_{cv} is evident. The freezing temperature of slags, or the freezing interval, is also of considerable interest. The slags of Fig. 3 differ considerably in this respect, slag X freezing between 2420 and 2300 F, or a range of 120 F, and slag Y over a range of 300 F.

The temperature of critical viscosity and the freezing temperature not only are functions of the ultimate composition of the slag but also depend on the state of oxidation of the iron in the slag. The latter is conveniently expressed as the *ferric percentage*, which is defined as

$$\text{Ferric percentage} = \frac{100 \times Fe_2O_3\%}{Fe_2O_3\% + 1.11FeO\%}$$

PRACTICAL APPLICATIONS OF ASH DATA. Application of these data to the problem of tapping slag from pulverized-coal-fired furnaces is discussed by Cohen and Corey (Ref. 15), examples being given which show the feasibility of using typical American coals in slagging furnaces. It should be noted, however, that there is no simple quantitative relation between clinkering, bird-nesting, etc., and the ASTM cone-fusion test, or the flow properties as determined by viscosimetric methods; the processes of sintering, which lead to the formation of clinker, consist of complex chemical reactions of the mineralogical constituents in the ash and physical reactions among the individual particles.

In so far as burning coal on grates is concerned, a few rather general conclusions may be stated with regard to the relation of clinkering to ASTM cone-fusion results. (1) Clinkering usually increases as the fusion temperature decreases, for coals from the same bed; (2) clinkering usually increases as the ash content increases, for coals from the same bed; and (3) the relation between clinkering and ash-fusion temperatures, for coals from different beds, may vary widely or be lost entirely. Barrett (Ref. 16) has discussed the entire subject in a clear and comprehensive manner.

9. COAL SPECIFICATIONS, COMPOSITIONS, AND MISCELLANEOUS DATA

By R. E. Morgan and E. P. Carman

CONTRACTS. Wide variations exist in specifications for the purchase of coal adopted by different organizations. Specifications for special types of coal, as well as standards for coal and coke, have been adopted by the ASTM (*ASTM Standards on Coal and Coke*, Aug. 1947). The two general types of contract ordinarily used by purchasers of large amounts of coal are (1) the premium-penalty type of contract with an established Btu content as a base and (2) the minimum-standards type of agreement. Contracts also usually specify (1) name of mine or mines and location, (2) size of coal, (3) analytical constituents (within limits), (4) ash-softening temperatures, and (5) coal seam.

In most modern coal contracts, wage clauses are included to compensate for changes in wage rates instituted during the contract period. Also, clauses are included to permit the purchaser either to reject unsatisfactory coal or to receive an adjustment on the contract price and to terminate a contract if coal of an inferior quality is shipped repeatedly.

Coal may be sampled continuously or at intervals, as the purchaser decides is advisable for his particular case. An accepted method of collecting and preparing samples is described in *Technical Paper* 133 of the Bureau of Mines and by the ASME Solid Fuels Test Code.

Some stipulations in government coal contracts are quoted.

Unsatisfactory Coal. If coal of three or more consecutive deliveries, or of deliveries aggregating 10 percent or more of the contracted quantity, proves to be unsatisfactory in the use for which it may have been purchased, although all other conditions of the contract have been met by the contractor, the Government may, at its option:

(*a*) Decline to accept additional coal from the mine or mines named herein and authorize the contractor in writing to supply coal from other mines producing coal of a quality contemplated to meet the requirements of the contract, or

(*b*) Terminate the contract without liability to either party.

(The following paragraph shall apply only when purchases may be subject to analytical tests as will be set forth in the schedules.)

Sampling. All coal may be regularly and continuously sampled, or only part or parts thereof may be sampled as the Government may elect; in the latter event the analysis of a sample or samples shall be used as representing only the tonnage actually sampled. The collecting and preparing of samples shall be in accordance with *Technical Paper* No. 133 of the United States Bureau of Mines or any revision thereof. The contractor may be present at the taking of samples, but the government shall be under no obligation to notify the contractor to be present. Samples shall be analyzed by the United States Bureau of Mines, and the analysis shall be final and conclusive on the parties hereto. Coal not sampled and coal sampled and found by analysis not to show a percentage of ash 2 percent or more in excess of the ash content specified by the contractor shall be accepted and paid for at the contract price. If, however, the percentage of ash is shown by analysis to be 2 percent or more in excess of the ash content specified by the contractor, the Government may, at its option:

(*a*) Exercise its right to reject the coal, and require that all or any part thereof shall be removed by and at the expense of the contractor promptly after notification of rejection; or

(*b*) Retain the coal, in which event a reduction in price shall be computed by multiplying the value of the coal . . . by the difference between the percentage of ash by analysis and the percentage of ash specified by the contractor. The resultant product, computed to the nearest cent, shall then be deducted from the contract price.

Provided, however, that should the analyses of samples representing three or more deliveries, or of deliveries aggregating 10 percent or more of the contracted quantity, show the coal to be in any way inferior to contract requirements, the Government may, at its option:

(*a*) Decline to accept additional coal from the mine or mines named herein and authorize the contractor in writing to supply coal from other mines producing coal of a quality contemplated to meet the limits specified by the contractor, or

(*b*) Decline to accept additional coal from the mine or mines named herein or subsequently authorized and purchase coal in the open market, in which event the contractor and his sureties shall be liable to the Government for any excess costs occasioned thereby.

TYPICAL COALS OF THE UNITED STATES are shown in Table 3.*

MISCELLANEOUS DATA ON COAL. Anthracite slush or culm is the name usually given to material of a size $^1/_{32}$ in. and smaller, produced in the preparation of standard sizes of anthracite. This product has several applications. (1) It can be pulverized and burned under boilers (more widely used each year); it is particularly adaptable for large boiler units and economically feasible when they are located within a reasonable distance of the anthracite mining region. (2) It can be used as a sintering medium. (3) It can be mixed with bituminous coal in the manufacture of by-product coke. (4) It can be briquetted. (5) It can be fired on stokers of various types when mixed with larger sizes of anthracite or with bituminous coal (see Refs. 17–19).

Sizes of Anthracite and Bulk Density (Space Occupied). Table 5 gives the standard specifications for Pennsylvania anthracite approved by the Anthracite Committee of the Production Control Plan for the Anthracite Industry, Harrisburg, Pennsylvania, effective July 28, 1947.

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* Prepared by E. P. Carman.

Table 3. Analyses, Ash Fusibility, and Heating Value of Typical Coals of the United States and Alaska *

State and County	Town, Bed, and Mine	Description and Agglomerating Index †	Kind of Sample ‡	Proximate, % §: Volatile Matter	Proximate, % §: Fixed Carbon	Proximate, % §: Ash	Ultimate, %: Sulfur	Ultimate, %: Hydrogen	Ultimate, %: Carbon	Ultimate, %: Nitrogen	Ultimate, %: Oxygen	Fusibility of Ash, °F: Initial Deformation	Fusibility of Ash, °F: Softening Temperature	Fusibility of Ash, °F: Fluid Temperature	Calorific Value: As Received	Calorific Value: Mineral-matter Free Basis, Dry	Calorific Value: Mineral-matter Free Basis, Moist
I. Anthracite																	
Pa., Carbon	(1)	Chestnut	B	3.0	82.0	9.0	0.6	2.5	79.8	0.8	7.3	2910+	....	.	12,570	14,940	....
Pa., Carbon	(2)	..	M	4.3	81.4	11.0	0.5	2.2	81.5	0.7	4.1	2380	2570	2850	12,730	15,030	....
Pa., Lackawanna	(3)	..	M	5.2	82.6	9.5	0.5	2.7	81.6	0.7	5.0	2950+	.	.	13,190	15,170	.....
Pa., Luzerne	(4)	Chestnut	B	4.8	82.8	8.4	0.6	..	...	..	....	2820	2910+	...	13,170	15,170	....
Pa., Luzerne	(5)	.....	M	3.0	82.4	9.5	0.7	2.4	80.9	0.7	5.8	2840	2940	2970	12,720	15,060	....
Pa., Luzerne	(6)	Chestnut	B	3.9	82.7	9.2	0.8	2.8	80.9	0.9	5.4	2840	2880	2930–	13,070	15,250	....
Pa., Northumberland	(7)	Chestnut	B	7.5	78.8	10.0	0.9					2910+	...	...	13,120	15,380	.....
Pa., Schuylkill	(8)	Chestnut	B	4.1	80.8	11.6	0.5	2.5	79.4	0.8	5.2	2650	2730	2890	12,660	15,100	....
Pa., Schuylkill	(9)	Chestnut	B	3.1	85.2	7.6	0.5	2.2	83.7	0.7	5.3	2410	2740	2820	13,080	14,940	.....
II. Semi-anthracite																	
Pa., Sullivan	(10)	......	M	7.8	78.6	11.2	0.6	3.5	79.4	0.9	4.4	2940+		...	13,360	15,650	.
Va., Montgomery	(11)	Composite NAb	M	12.6	65.0	21.2	0.7	3.3	70.4	0.8	3.6	....		..	11,860	15,660	15,420
III. Low-volatile Bituminous																	
Ark., Sebastian	(12)	10″ lump	T	18.5	76.5	5.0	0.8	4.6	85.8	1.9	1.9	..	2450	....	14,570	15,630	....
Md., Garrett	(13)	Run-of-Mine	T	16.7	69.9	12.0	2.2	..	..						13,490		
Pa., Cambria	(14)	Composite	M	18.3	70.3	8.5	2.7	4.6	78.6	1.4	4.2	..	....	..	13,800	....	....
Pa., Huntingdon	(15)	Egg	D	16.1	71.1	11.6	2.0	...	.....			.....	.....	.....	13,570	....	......
Pa., Somerset	(16)	Run-of-Mine	T	17.6	74.4	8.0	1.0	.			.	.....		.....	13,570	....	.....
Pa., Tioga	(17)	Composite	M	19.1	68.9	8.6	2.7	4.7	77.5	1.1	5.4	.....	2800	.....	14,210	15,740	.....
W. Va., McDowell	(18)	Composite	M	18.1	74.6	4.7	0.5	4.5	84.3	1.2	4.8	....	....	..	13,740	....	......
W. Va., Mercer	(19)	Run-of-Mine	T	16.8	73.1	8.0	0.6	..	.		.	....	2620	....	14,560	15,800	15,430
W. Va., Raleigh	(20)	Composite Cg	M	16.5	76.2	5.7	0.9	4.6	83.8	1.3	3.7	......	......	......	14,520	15,780	15,510
IV. Medium-volatile Bituminous																	
Pa., Cambria	(21)	Composite	M	25.2	64.1	7.6	2.2	5.0	78.2	1.4	5.6	....	....	....	14,000	....	....
Pa., Centre	(22)	Composite	M	24.1	63.8	8.2	2.0	5.0	76.9	1.4	6.5	.....	....	....	13,630	....	....
Pa., Clearfield	(23)	Run-of-mine	D	22.4	66.5	9.7	1.1	...	.....	.....	..	.....	....	......	13,830	....	....

Va., Buchanan	(24)	Composite, Cg	M	21.7	68.0	8.2	1.5	4.8	79.8	1.3	4.4	...	...		14,060	15,850	15,490
Va., Tazewell	(25)	Composite	M	25.3	65.6	5.7	0.7	5.2	81.0	1.3	6.1	...	...	...	14,360	15,910	15,330
W. Va., Greenbrier	(26)	Run-of-mine	D	24.7	67.8	4.3	0.9	...	...	...	...	...	...	...	14,230	15,480	14,950
W. Va., McDowell	(27)	5" x 2 1/2"	T	23.0	72.0	3.2	0.9	...	...	...	...	...	2440	...	14,980	15,850	15,550
W. Va., Wyoming	(28)	Composite	M	21.7	72.6	3.0	0.6	5.0	84.7	1.4	5.3	...	...	...	14,780	15,740	15,300
V. High-volatile A Bituminous																	
Ala., Bibb	(29)	Composite	M	34.7	57.4	5.2	1.3	5.3	79.7	1.4	7.1	...	...	...	14,160	...	...
Ala., Jefferson	(30)	Composite	M	27.0	58.0	12.1	1.3	4.8	73.7	1.6	6.5	...	...	...	13,050	...	...
Ala., St. Clair	(31)	...	T	33.6	54.1	10.1	0.6	...	...	...	...	...	...	...	13,300	...	...
Ala., Walker	(32)	Composite	M	35.2	56.3	7.1	1.3	5.1	76.3	1.8	8.4	...	2440	...	13,800	...	...
Col., Las Animas	(33)	Composite	M	30.3	58.3	9.4	0.6	5.4	76.3	1.7	6.6	...	...	...	13,790	...	...
Ky., Bell	(34)	Composite	M	37.2	55.8	4.4	1.1	5.6	78.4	1.8	8.7	...	...	...	14,000	15,150	14,730
Ky., Floyd	(35)	4" lump, Cf	T	39.6	53.0	4.1	1.2	...	...	...	...	...	2430	...	13,850	15,050	14,530
Ky., Harlan	(36)	Composite	M	37.5	56.2	3.4	0.9	5.6	78.8	1.7	9.6	...	...	...	14,100	15,120	14,670
Ky., Harlan	(37)	...	M	35.5	55.0	6.1	0.7	...	...	...	...	...	2210	...	13,710	15,260	14,700
Ky., Perry	(38)	5" lump, Cf	T	37.7	55.2	3.8	1.2	...	...	...	...	...	2140	...	13,920	15,070	14,550
Ky., Pike	(39)	Composite, Cg	M	32.3	61.8	1.9	0.5	5.5	80.8	1.5	9.8	...	...	...	14,300	15,240	14,620
Ky., Whitley	(40)	5" lump, Cf	T	37.6	56.1	2.8	0.8	...	...	...	...	...	2360	...	13,960	14,960	14,420
Pa., Allegheny	(41)	6" x 4" egg	T	39.0	52.7	5.4	1.4	...	...	...	...	...	2550	...	13,800	...	...

* These analyses taken from the following *Bur. Mines Tech. Papers.* 682, Alaska; 347, Alabama; 416, Arkansas; 574, Colorado; 641, Illinois; 417, Indiana; 269, Iowa; 652 and 308, Kentucky; 465, Maryland; 529, Montana; 590 (and *Supplement* 645), Pennsylvania Bituminous; 659, Pennsylvania Anthracitic; 671 and 356, Tennessee; 345, Utah; 656 and 365, Virginia; 491 (and *Supplement* 618), Washington, 626 and 405, West Virginia, and 484, Wyoming; 700, North Dakota.

† See Table 4, p. 2-31, for key to agglomerating index. All coals in condition "as received."

‡ M, mine sample; T, tipple sample; D, delivered coal; and B, breaker sample.

§ % moisture = 100 − percent (volatile matter + fixed carbon + ash).

Key to Town, Bed, and Mine (see second column of table): (1) Lansford; Mammoth, Orchard, Primrose; Lansford. (2) Nesquehoning; Orchard; No. 3 Shaft. (3) Olyphant; Dunmore No. 3; Eddy Creek. (4) Ashley; Kidney, Hillman, Stanton, Five Foot, Cooper, Baltimore, Ross, Top and Bottom, Red Ash; Huber No. 20. (5) Jeddo; Wharton (Top Bench); Jeddo No. 7. (6) Wilkes-Barre; (not known); Dorrance. (7) Shamokin; (not known); Glen Burn. (8) Minersville; Ridge to Tracy; Oak Hill. (9) Tamaqua; Mammoth, Orchard, Primrose; Tamaqua. (10) Bernice; "B"; Connell. (11) McCoy; Merrimac; Superior. (12) Excelsior; Hartshorne; Boyd Excelsior. (13) Kitzmiller; Davis and Thomas; Hamill No. 1. (14) Beaverdale; Lower Freeport; Logan No. 6 1/2. (15) Robertsdale; Fulton; Rockhill No. 1. (16) Cairnbrook; Lower Kittanning; Loyal Hanna No. 6. (17) Morris Run; Bloss and Seymour; Morris Run. (18) Algoma; Pocahontas No. 3; Algoma (Piney). (19) Dott; Pocahontas No. 3; Modoc. (20) Affinity; Pocahontas No. 4; Affinity. (21) Gallitzin; Upper Freeport; Penn No. 10. (22) Moshannon; Lower Kittanning; Morgan No. 20. (23) Coalport; Lower Kittanning; Cambria Smokeless. (24) Keen Mountain; Cary (Lower Banner); Keen Mountain. (25) Jewell Ridge; Jewell (Raven); Jewell Ridge No. 1. (26) Anjean; Sewell; Anjean. (27) Bradshaw; Bradshaw; Bradshaw. (28) Wyoming; Sewell; Wyoming. (29) Belle Ellen; Woodstock; Belle Ellen. (30) Labuco; Mary Lee; Labuco. (31) Acmar; Henry Ellen; Acmar. (32) Gorgas; Pratt; Gorgas. (33) Valdez; Frederick; Frederick. (34) Balkan; Creech; Balkan. (35) Drift; Elkhorn No. 2; Beaver No. 1. (36) Coalgood; Harlan; Mary Helen. (37) Kenvir; Darby No. 5; Great Heart No. 30. (38) Allais; Hazard No. 4; Columbus No. 4. (39) Lookout; Pond Creek; Henry Clay. (40) Gatliff; Jellico; Gatliff Nos. 2, 3, 4. (41) Imperial; Pittsburgh; Champion No. 1.

Note. All values given in these tables are *gross* calorific values.

(Table continued on p. 2-28)

Table 3. Analyses, Ash Fusibility, and Heating Value of Typical Coals of the United States and Alaska *—*Continued*

State and County	Town, Bed, and Mine	Description and Agglomerating Index †	Kind of Sample ‡	Proximate, % §: Volatile Matter	Proximate: Fixed Carbon	Proximate: Ash	Ultimate, %: Sulfur	Ultimate: Hydrogen	Ultimate: Carbon	Ultimate: Nitrogen	Ultimate: Oxygen	Fusibility of Ash, °F: Initial Deformation	Fusibility: Softening Temperature	Fusibility: Fluid Temperature	Calorific Value: As Received	Calorific Value, Mineral-matter Free Basis: Dry	Calorific Value, Mineral-matter Free Basis: Moist
V. High-volatile A Bituminous—*Continued*																	
Pa., Allegheny	(42)	Composite	M	34.7	54.0	8.6	1.5	5.4	75.4	1.5	7.6	...	...	...	13,630	...	...
Pa., Armstrong	(43)	Composite	M	38.6	52.3	7.4	2.3	5.7	76.7	1.4	6.5	...	...	...	13,920	...	...
Pa., Butler	(44)	4" x 2" egg	T	35.2	53.9	7.7	1.3	...	...	...	...	...	2660	...	13,430	...	...
Pa., Clearfield	(45)	Run-of-mine	D	29.4	61.4	5.9	1.4	...	...	...	...	...	...	...	14,120	...	...
Pa., Fayette	(46)	Composite	M	35.7	54.8	7.1	2.0	5.3	76.8	1.6	7.2	...	...	...	13,810	...	...
Pa., Greene	(47)	3/4" lump	T	36.2	55.6	6.4	1.0	...	...	...	...	...	2660	...	13,920	...	...
Pa., Indiana	(48)	6" x 2" egg	T	30.4	59.8	8.5	1.4	...	...	...	...	...	2620	...	13,980	...	...
Pa., Jefferson	(49)	6" x 3" egg	T	31.9	59.8	6.3	2.4	...	...	...	...	...	...	...	14,140	...	...
Pa., Washington	(50)	Composite	M	37.7	52.4	6.0	1.6	5.9	75.0	1.4	10.1	...	...	...	13,460	...	...
Pa., Westmoreland	(51)	Composite	M	34.2	54.4	9.0	1.5	5.2	74.6	1.4	8.3	...	...	...	13,340	...	...
Tenn., Anderson	(52)	Composite	M	35.9	56.1	6.2	1.2	5.4	77.7	1.9	7.6	...	...		13,890	15,220	14,930
Tenn., Campbell	(53)	8" x 5", Cf	T	37.6	56.7	2.7	0.8	5.6	79.5	2.0	9.4	2500	2640	2760	14,200	15,120	14,650
Tenn., Claiborne	(54)	6" lump, Cf	T	38.3	56.3	2.7	0.9	5.5	78.7	1.9	10.3	2480	2650	2910+	14,170	15,040	14,620
Tenn., Scott	(55)	−6" resultant, Cg	T	37.5	49.4	10.9	3.7	5.1	70.0	1.6	8.7	1960	2140	2420	12,880	15,110	14,730
Utah, Carbon	(56)	Composite	M	38.0	51.3	6.7	1.2	5.6	72.8	1.6	12.1	...	...	...	13,120		...
Va., Buchanan	(57)	Composite	M	31.9	59.4	6.7	1.3	5.2	78.8	1.6	6.4	...	...	...	14,120	15,610	15,270
Va., Lee	(58)	2 1/2" lump, Cf	T	37.5	54.1	6.1	0.7	5.4	77.1	1.6	9.1	2450	2640	2910+	13,760	15,130	14,750
Va., Wise	(59)	Run-of-mine, Cg	T	35.5	56.2	5.8	0.8	5.4	77.8	1.5	8.7	2630	2760	2910+	13,870	15,230	14,820
W. Va., Barbour	(60)	Run-of-mine, Cg	T	35.2	55.0	7.2	1.9	...	...	...	...	...	2180	...	13,660	15,320	14,880
W. Va., Boone	(61)	Egg	D	34.0	56.4	5.2	0.9	...	...	...	...	...	...	...	13,740	...	...
W. Va., Boone	(62)	Composite	M	38.1	53.5	5.7	1.7	5.6	76.8	1.5	8.7	...		...	13,920	15,340	14,890
W. Va., Fayette	(63)	2" nut and slack	D	32.2	58.4	5.3	0.8	...	...	...	...	...	2910+	...	13,920	...	...
W. Va., Harrison	(64)	2" lump, Cg	T	39.4	51.5	7.0	2.2	...	...	...	...	...	2120	...	13,820	15,380	15,030
W. Va., Kanawha	(65)	Cg	M	34.3	55.8	5.2	0.7	5.6	76.5	1.4	10.6	...	2500	...	13,610	15,200	14,440
W. Va., Kanawha	(66)	6" x 2"	D	37.4	55.5	4.9	0.7	...	...	...	...	...	...	...	14,190	...	...
W. Va., Logan	(67)	Composite	M	36.1	55.6	5.5	0.9	5.4	78.5	1.5	8.2	...	...	...	14,000	...	...
W. Va., Marion	(68)	Composite	M	39.2	52.0	6.9	2.4	5.4	76.9	1.5	6.9	...	...	...	13,890	...	...

W. Va., Mingo	(69)	Composite	M	33.8	50.9	10.3	0.9	5.2	71.1	1.5	11.0	...	...	...	12,550	...	...
W. Va., Mingo	(70)	5" x 3", Cg	T	35.8	56.5	5.5	1.0	...	...	...	...	...	2810	...	14,100	15,390	15,030
W. Va., Monongalia	(71)	Composite, Cg	M	30.0	58.0	9.1	1 0	5 1	75.8	1.4	7.6	...	...	...	13,500	15,510	15,010
W. Va., Raleigh	(72)	5" x 2" Cf	T	35.4	56.8	4 8	0.6						2910+	. .	13,980	15,250	14,760
VI. High-volatile B Bituminous																	
Alaska, Matanuska	(73)	...	M	35.5	39.3	21.6	0.5	4 7	58.6	1.2	13.4	2050	2310	2470	10,440	14,310	13,630
Ky., Hopkins	(74)	Composite, Cf	M	37.6	46.0	9.9	3.7	5 5	67.5	1.3	12.1	.	..	. .	12,220	14,900	13,790
Ky., Muhlenberg	(75)	Composite, Cf	M	36 6	44.6	10.4	3.5	5.5	65.7	1.4	13.5	.	..	. .	11,850	14,880	13,440
Utah, Carbon	(76)	..	M	43.7	46.5	4.8	0.4	5 8	71.9	1.3	15.8	...	2130	...	12,820	...	...
Wash., Kittitas	(77)	3" x 1 5/8", egg	T	38.2	45.5	12.7	0.4	.	.	...	...	...	2280	...	12,370	...	...
VII. High-volatile C Bituminous																	
Col., Routt	(78)	. . .	M	38.4	45.9	5 1	0.4				..	. .	2690	.	11,620	.	...
Ill., Christian	(79)	Composite	M	36.5	39 3	10 3	4 3	5.8	58.6	1.1	19.9	.		..	10,690	14,430	12,110
Ill., Franklin	(80)	Composite	M	34.3	47.8	7.7	0 8	5.5	66.6	1.5	17.9	...	..	...	11,830	14,550	12,920
Ill., Franklin	(81)	. .	M	30.6	49.3	9 3	0.5		. .		.		.	. .	11,450	14,480	12,740
Ill., Fulton	(82)	6" x 3"	D	36.8	39.6	12.0	3.4	.	.		.	.	. .		11,030	...	...
Ill., Sangamon	(83)	6" x 3", Cf	T	34.0	40.6	11.8	3 9	.				.	1980		10,660	14,630	12,300
Ill., Washington	(84)	6" x 3", Cf	T	36.1	42.3	11.7	3.7		...	.	.	. .	2010	.	11,170	14,570	12,880
Ind., Greene	(85)	.. ....	M	36.8	43.0	6 5	1.0	.	....	.	.		2710		11,730	...	...
Iowa, Appanoose	(86)	... ...	M	35.4	40.4	7 1	4.0	.	.			.	.	.	10,931	...	...
Wyo., Sweetwater	(87)	Composite	M	36.2	46.5	5.9	0.8	5 6	63.9	1.4	22.4	.			11,210	. .	...

Key to Town, Bed, and Mine (see second column of table): (42) Renton; Thick Freeport; Renton No. 3. (43) Cadogan; Lower Kittanning; Cadogan No. 1. (44) Cosco; Middle Kittanning; Cosco Nos. 1, 2. (45) Helvetia; Lower Freeport; Helvetia. (46) Point Marion; Pittsburgh; Frederick No. 1. (47) Clarksville; Pittsburgh; Emerald. (48) Ernest; Upper Freeport; Ernest. (49) Punxsutawney; Lower Freeport; Lindsey No. 8. (50) McDonald; Pittsburgh; Montour No. 9. (51) Adamsburg; Redstone; Adamsburg. (52) Coal Creek; Coal Creek; Cross Mountain No. 1. (53) Morley; Jellico; Blue Rose. (54) Fork Ridge; Mason; Fork Ridge. (55) Dean; Dean; Straight Fork. (56) Sunnyside; Lower Sunnyside; Sunnyside No. 1. (57) Big Rock; Clintwood; Buchanan No. 1. (58) Calvin; Low Splint; Calvin. (59) Imboden; Imboden; Imboden. (60) Century; Redstone; Century No. 1. (61) Highcoal; Dorothy; Anchor No. 1. (62) Ramage; Alma; Spruce River No. 4. (63) Elkridge; Powellton; Powellton No. 2. (64) Clarksburg; Pittsburgh; Consolidation No. 25. (65) Cannelton; Lower Kittanning (No. 5 Block); Cannelton Coal and Coke No. 6. (66) Ward; Cedar Grove; Ward No. 4. (67) Omar; Island Creek (Cedar Grove); Omar No. 5. (68) Fairmont; Pittsburgh; Consolidation No. 38. (69) Chattaroy; Winifrede; Buffalo. (70) Red Jacket; Cedar Grove; Junior. (71) Dellslow; Upper Freeport; Ind. Col. Corp. No. 21. (72) Eunice; No. 5 Block; Princess Dorothy. (73) Jonesville; No. 00; Evan Jones. (74) Earlington; No. 9; North Diamond. (75) Graham; No. 9; Graham. (76) Kenilworth; Kenilworth; Kenilworth. (77) Roslyn; Roslyn; Roslyn No. 5. (78) Mount Harris; Wadge; Harris. (79) Tovey; No. 6, Hawthorn No. 8. (80) Buckner; No. 6; Old Ben No. 11. (81) Zeigler; No. 6; Zeigler No. 1. (82) Cuba; No. 5; Cuba No. 9. (83) Springfield; No. 5; Peerless No. 59. (84) Centralia; No. 6; Centralia No. 5. (85) Linton; No. 4; Vandalia No. 24. (86) Centerville; Lower Mystic; No. 3. (87) Rock Springs; No. 1; Rock Springs No. 4.

(Table continued on p. 2-30)

Table 3. Analyses, Ash Fusibility, and Heating Value of Typical Coals of the United States and Alaska *—*Continued*

State and County	Town, Bed, and Mine	Description and Agglomerating Index †	Kind of Sample ‡	Proximate, % §: Volatile Matter	Proximate, % §: Fixed Carbon	Proximate, % §: Ash	Ultimate, %: Sulfur	Ultimate, %: Hydrogen	Ultimate, %: Carbon	Ultimate, %: Nitrogen	Ultimate, %: Oxygen	Fusibility of Ash, °F: Initial Deformation	Fusibility of Ash, °F: Softening Temperature	Fusibility of Ash, °F: Fluid Temperature	Calorific Value: As Received	Calorific Value: Mineral-matter Free Basis: Dry	Calorific Value: Mineral-matter Free Basis: Moist
VIII. Sub-bituminous																	
Alaska, South-western	(88)	NAa	M	35.8	31.5	10.8	0.4	6.4	49.7	0.8	31.9	2250	2300	2500	8,710	13,120	9,860
Col., El Paso	(89)	.	M	31.4	38.0	6.0	0.4					.	2320		8,680	...	...
Col., Huerfano	(90)	Composite	M	36.3	47.0	8.8	0.5	5.7	65.8	1.1	18.1	...	...	...	11,630	...	...
Col., Weld	(91)	.	M	28.2	44.7	4.2	0.5						2090		9,810	...	...
Mont., Rosebud	(92)	Composite	M	31.5	36.3	8.3	0.9	5.5	47.9	0.9	36.5	.			7,910	...	...
Wash., King	(93)	+3 1/2" lump	T	39.4	47.1	3.9	0.5					.	2290		12,080	...	...
Wash., Whatcom	(94)	+3 1/2" lump	D	36.2	43.2	13.9	0.5					.		.	10,880	...	...
Wyo., Carbon	(95)	... ...	M	40.6	44.1	4.1	0.3	6.0	65.8	0.9	22.9	.	2410		11,460	...	...
Wyo., Sheridan	(96)	. .	M	34.3	38.4	3.4	0.4	6.3	54.1	1.1	34.7	.	2190		9,340	.	...
IX. Lignite																	
N. Dak., Burke	(97)	. . .	M	26.0	31.7	8.3	0.4	6.7	41.3	0.7	42.6	2080	2150	2180	7,050	12,370	7,740
N. Dak., Divide	(98)	NAa	M	26.8	30.5	7.8	0.7	6.8	42.6	0.7	41.4	2160	2200	2440	7,270	12,850	7,930
N. Dak., M'Lean	(99)	... . ..	M	29.6	32.7	4.6	0.4	6.7	43.6	0.7	44.0	2330	2400	2460	7,430	12,010	7,820
N. Dak., Mercer	(100)	Composite	M	26.2	31.7	5.8	0.7	6.8	42.2	0.6	43.9	.. .	. .	... .	7,140	12,450	7,610
N. Dak., Mercer	(101)	10" x 6" NAa	T	26.7	31.6	5.5	0.5					2330	2370	2490	7,140	12,360	7,590
N. Dak., Stark	(102)	NAa	M	25.4	26.9	6.5	0.5	7.3	37.6	0.5	47.6	2100	2140	2420	6,340	12,260	6,810
X. Cannel and Miscellaneous																	
Ind., Clay	(103)	....	M	35.2	49.9	1.1	1.5	...	..	..	....	....	... .	. ...	12,400	....	....
Ky., Floyd	(104)	Cannel	M	40.1	43.5	15.3	0.8	..		.	.	.. ...	2730	....	13,080	..	.. .. .

Key to Town, Bed, and Mine (see second column of table): (88) Suntrana; "E"; New Suntrana Hill. (89) Colorado Springs; Fox Hill; City. (90) Tioga; Kebler; Kebler No. 2. (91) Erie; Laramie; Imperial. (92) Colstrip; Rosebud; Colstrip. (93) Ravensdale; McKay; McKay. (94) Bellingham; Bellingham; Bellingham. (95) Hanna; Hanna No. 2; Hanna No. 4. (96) Monarch; Monarch; Monarch. (97) Columbus; Noonan; Kincaid. (98) Noonan; Noonan; Baukol-Noonan. (99) Garrison; (not known); Stevens. (100) Beulah; Beulah-Zap; Knife River (Beulah). (101) Zap; Beulah-Zap; Indian Head. (102) Dickinson; (not known); Lehigh. (103) Brazil; Lower Block; Brazil No. 1. (104) Drift, Elkhorn No. 1; Beaver No. 1.

Table 4. Agglomerating Properties of Coals Based upon Examination of Residue Incident to the Volatile-matter Determination

(Based upon Agglomerating and Agglutinating Tests for Classifying Weakly Caking Coals, by R. E. Gilmore, G. P. Connel, and J. H. H. Nicolls. *Trans. Am. Inst. Mining and Met. Engrs., Coal. Div.*, Vol. 108, 1934, pp. 255–265)

Designation: Class	Designation: Group	Appearance of Residue from Standard Method for Determination of Volatile Matter in Coal
Nonagglomerating * (button shows no swelling or cell structure and will not support a 500-g weight without pulverizing)	Na—nonagglomerate	NAa—noncoherent residue NAb—coke button shows no swelling or cell structure and after careful removal from the crucible will pulverize under a 500-g weight carefully lowered on button
Agglomerating * (button shows swelling or cell structure or will support a 500-g weight without pulverizing)	A—agglomerate (button dull black, sintered, shows no swelling or cell structure and will support a 500-g weight without pulverizing.	Aw—weak agglomerate (button comes out of crucible in more than one piece) Af—firm agglomerate (button comes out of crucible in one piece)
	C—caking (button shows swelling or cell structure)	Cp—poor caking (button shows slight swelling with small cells, has slight gray luster) Cf—fair caking (button shows medium swelling and good cell structure, has characteristic metallic luster) Cg—good caking (button shows strong swelling and pronounced cell structure, with numerous large cells and cavities, has characteristic metallic luster)

* Agglomerating index—coals that in the volatile-matter determination produce either an agglomerate button that will support a 500-g weight without pulverizing or a button that shows swelling or cell structure shall be classified as agglomerating.

Table 5. Standard Pennsylvania Anthracite Specifications Approved by Anthracite Committee, Effective July 28, 1947

Size	Size of Mesh (Round), in.		Oversize (Max), %	Undersize, %		Maximum Impurities, %		Ash, % †
	Through	Over		Max	Min	Slate *	Bone	
Broken	4 3/8	3 1/4–3		15	7 1/2	1 1/2	2	11
Egg	3 1/4–3	2 7/16	5	15	7 1/2	1 1/2	2	11
Stove	2 7/16	1 5/8	7 1/2	15	7 1/2	2	3	11
Chestnut	1 5/8	13/16	7 1/2	15	7 1/2	3	4	11
Pea	13/16	9/16	10	15	7 1/2	4	5	12
Buckwheat No. 1	9/16	5/16	10	15	7 1/2			13
Buckwheat No. 2 (Rice)	5/16	3/16	10	17	7 1/2			13
Buckwheat No. 3 (Barley)	3/16	3/32	10	20	10			15
Buckwheat No. 4 ‡	3/32	3/64	20	30	10			15
Buckwheat No. 5 ‡	3/64		30	No limit				16

* When slate content on broken to pea sizes, inclusive, is less than the above standards, bone content may be correspondingly increased, but slate content specified above shall not be exceeded in any event, and the total maximum impurities shall not exceed those specified.

† Ash determinations are on a dry basis.

‡ These specifications do not cover coal used for special purposes.

A tolerance of 1% is allowed on the maximum percentage of undersize and the maximum percentage of ash content.

The maximum percentage of undersize is applicable only to anthracite as it is produced at the preparation plant.

"Slate" is defined as any material which has less than 40% of fixed carbon.

"Bone" is defined as any material which has 40% or more but less than 75% of fixed carbon.

The space occupied by anthracite varies with the size of the coal. *Technical Paper* 184 of the Bureau of Mines gives a table for various coals, from which the following figures are taken: furnace or broken, 52 to 55 lb per cu ft; egg, 53 to 58 lb per cu ft; chestnut, 52.5 to 56.5 lb per cu ft; pea, 53.5 to 54.5 lb per cu ft; No. 1 buckwheat, 50.5 lb per cu ft.

Sizes of Bituminous Coal and Bulk Density (Space Occupied). Space occupied by bituminous coal varies with specific gravity of the coal, proportions of lump and slack, moisture content, and degree of settling. Table 6 shows a range of weights of bituminous coals given in *Technical Paper* 184 of the Bureau of Mines. Other things being equal, the sample with the higher moisture content will weigh more per cubic foot than will a sample with lower moisture content. The increase in volume of wet coal is not proportionately so great as the increase in weight per cubic foot. Slack, comprising a mixture up to and including nut size, weighs more than screened nut coal.

Table 6. Weight of Bituminous Coal

(From *Technical Paper* 184 of the Bureau of Mines)

Coal from	Size *	Lb/cu ft	Coal from	Size *	Lb/cu ft
Alabama	D	45.5	Oklahoma	40-20- 20	50.0
"	RM	51-54	"	35-45- 20	48.5
Arkansas	RM	49.5-59.0	Pennsylvania	90- 5- 5	47-49.5
Colorado	Lump	50.5-52.5	"	70-20- 10	50.5
"	D	49.0	"	60-25- 10	50.5
Georgia	60-10-30	54	"	20-30- 50	52
Illinois	D, Lump	49.5	"	10-15- 75	52
"	RM	54.5-55.5	"	0-10- 90	49.5-52.0
"	Lump	48.5	"	0- 0-100	52
Indiana	Lump	44.0	"	Lump	46.5
Iowa	60-25-15	46.5	Utah	95- 0- 5	44.5
Kansas	95- 5- 0	55.5	West Virginia	75-15- 10	55.5
Kentucky	95- 5- 0	43.0-54.5	" "	60-30- 10	47.0
"	Lump	45-47.5	" "	20-10- 70	55.0
Montana	90- 5- 5	52	" "	5-10- 85	55.5
Ohio	95- 5- 0	49	" "	4- 2- 94	54
"	70-15-15	47.5	" "	3- 5- 92	57.5
"	60-30-10	46.5	" "	0- 5- 95	56.5

* D = Domestic; RM = run-of-mine; the figures represent the respective percentages of lump, nut, and slack.

Angles of Repose and Sliding Angles. The *angle of repose* is the angle with the horizontal at which material will stand when piled. For anthracite it is about 30 degrees. This varies according to (1) size and shape of coal, (2) percentage of undersize, (3) moisture content, and (4) method of piling. The angle of repose for coke is about 41 degrees, with variations due to the factors mentioned above. For bituminous coal there is wide variation in the angle of repose; however, 45 degrees is often used.

The *sliding angle* is the angle at which material will move freely on an inclined surface. On bright steel, anthracite will slide at about 18 degrees, coke at about 24 degrees, and bituminous coal at about 22 degrees. These figures are affected by various factors, including (1) size and shape of material, (2) uniformity of size, and (3) moisture content. For coal of medium size and for similar materials, chutes are usually set at an angle of about 45 degrees.

Mixtures of Hard and Soft Coal. A hand-fired boiler plant, designed for bituminous coal and operating on natural draft, is capable of using 20% of steam sizes of anthracite mixed with the soft coal without considerable loss in efficiency and without change of equipment, provided sufficient grate surface is available; between 20 and 40% of small-size anthracite may be burned with the bituminous by installing forced-draft equipment (*Technical Paper* 220 of the Bureau of Mines).

Report of Investigations 3916 (1946) of the Bureau of Mines states that field tests of various mixtures of bituminous slack coal and anthracite barley on industrial-type stokers show that the use of mixtures is practical and that it is possible to change and usually to adjust to an improved condition the characteristics of a fuel bed of eastern bituminous slack coal by the addition of proper amounts of anthracite. The improved uniformity of the fuel bed of single-retort stokers makes possible less manual attention to the bed. Air flow through the fuel bed is affected. Smoke is decreased. In general, fly ash, unburned combustible in the fly ash, and refuse are increased. Clinkering in the fuel bed is generally lessened. The peak-load capacity of the stoker is affected, usually adversely, for high percentages of anthracite. Efficiencies obtained on single-retort stokers with lower percentages of anthracite in the mixtures are, in general, about the same as those obtained with bituminous coal only. With higher percentages of anthracite, over-all efficiencies are somewhat lower.

The best percentage of anthracite for a given installation depends on many factors, such as size range, volatile and ash content and ash fusion temperature, load conditions, smoke production, and type of equipment. It must be determined by trial for each installation.

Weathering or slacking of coal is a characteristic that results in disintegration when the coal is alternately dried and wetted by exposure to weather. Lower-rank coals, such as lignite, sub-bituminous coal, and high-volatile, high-inherent-moisture bituminous coals, disintegrate much more readily when exposed to the elements than do the higher-rank coals. Studies of Bureau of Mines tests show a wide variation in slacking tendency, with slacking indexes ranging from about zero for some bituminous coals to nearly 100 for a Texas lignite. For coals having a natural bed moisture up to about 7%, slacking is small; above this value, slacking is more erratic and, in general, increases appreciably with bed-moisture content.

Spontaneous combustion, caused by the union of coal with oxygen, is similar to the combustion process in the furnace but progresses at a considerably lower rate. *Technical Paper* 409 of the Bureau of Mines states: "The process of spontaneous heating is operative at room temperatures as soon as freshly broken coal is exposed to the air. It begins with the physical absorption of oxygen and is continued by the formation of a solid chemical compound of coal and oxygen, which is gradually decomposed as the temperature rises. The coal increases in weight by the amount of oxygen retained. There follows the breaking up of the solid compound of coal and oxygen and the formation of the final oxidation products—carbon dioxide, carbon monoxide, and water. This process generates heat."

Generally speaking, coals of lowest rank, such as sub-bituminous and lignite, are most susceptible to spontaneous heating. There are no known records of heating of high-grade screened anthracite stored under normal temperature conditions. The reaction between coal and oxygen is considered to double for about each 15 F rise.

Finely divided pyrite can increase the tendency of a coal to heat spontaneously. Although coals containing virtually no pyrite have fired spontaneously, fine pyrite does increase the rate of oxidation (*Technical Paper* 409 of the Bureau of Mines).

The total exposed surface area of the coal is an important factor in the heating process; the greater the surface, the more chance of oxidation. Table 7 shows the variation of surface area with size (*Information Circular* 7235 of the Bureau of Mines). Experience indicates that egg or lump bituminous coal of fairly uniform size ordinarily gives no trouble from spontaneous heating when stored, mainly because there is not enough surface area. Coals that slack readily in storage expose more surface area for oxidation.

Table 7. Variation of Surface Area of a Ton of Coal with Coal Size (Coal Considered to Be in the Form of Cubes or Spheres of the Size Designated)

Size	Sq ft/ton	Size, mesh	Sq ft/ton
2.83-ft cube	48	4	8,727
6-in. lump	272	8	17,416
3-in. nut	544	16	34,796
1 1/2 in.	1088	30	70,341
3/4 in.	2176	50	139,479
3/8 in.	4352	100	276,595

Methods of storing coal that will lessen or prevent spontaneous heating are: (1) Submerge the coal in water. (2) Layer and compact the pile with a bulldozer or similar device. (3) Store screened, large-size coal. (4) Keep storage area free of extraneous material. (5) Prevent segregation of sizes in the pile. (6) Store in shallow piles (not over 6 to 8 ft deep, unless extreme care is taken in making the pile). (7) Do not store coal near external source of heat. (8) Avoid draft of air through pile. (9) Use older storage first.

Arrangements should be made to check the temperature of stored coal frequently. If thermometers are not available, an iron rod forced into the pile and removed after a short time will roughly reveal the temperature within the pile. When temperatures above 120 F are discovered, it is advisable to move the hot coal. Application of water to stored coal for the purpose of reducing the temperature is not recommended because addition of moisture frequently accelerates oxidation. If a section of the storage pile has heated to the extent that some of the coal has coked, water applied to the top surface of the pile probably will not reach the affected area because of the shell of coke.

10. COMBUSTION OF COAL

By Richard C. Corey

GRATES AND STOKERS.* There are three principal methods of feeding and burning coal on fuel beds, characterized by the relative directions of the flow of fuel and the air used for combustion. These are designated as the overfeed, underfeed, and crossfeed (or frontfeed) methods. Stoker and grate design is based essentially on these methods, but analysis of commercial units shows that operation involves at least two of the basic methods. For example, a single-retort stoker, though designed primarily to secure the upward flow of coal and air, has some crossfeed action.

When fresh coal enters a hot furnace, moisture and volatile matter first are distilled off. Then the combustibles in the flue gas and the hydrogen and carbon monoxide produced by the reaction of hot carbon with CO_2 and water vapor burn in the furnace space above the fuel bed. Combustion occurs in two zones in the fuel bed—an *oxidation* zone, where high specific rates of combustion and high temperatures occur and the oxygen is reduced to zero; and a *reduction* zone, where the CO_2 from the first zone reacts with hot carbon and is reduced to CO. It has been indicated by research that CO also is produced to some extent in the oxidation zone. The main requirements for efficient combustion in fuel beds are (1) a uniform and thin layer of raw fuel, (2) a sufficient supply of secondary air over the fuel bed to burn the combustible gases, and (3) sufficient furnace volume to obtain mixing of the gases with oxygen, so that they are burned before they leave the furnace.

THE OVERFEED METHOD, shown schematically in Fig. 4(*a*), uses the countercurrent flow of fuel and air, fuel flowing downward and air upward, and is best exemplified by the domestic fireplace, the hand-fired domestic furnace, and the spreader stoker. The freshly charged fuel is heated by burning coal on the grate and hot combustion gases rising from the grate. As the combustible burns, the residue moves toward the grate, from which it is removed periodically. Anthracite and free-burning bituminous and sub-bituminous coals burn well by this method of firing, but strongly coking coals and those with a low ash-fusion temperature are not suitable.

Spreader Stoker. By far the most versatile of overfeed methods for burning solid fuels is the *spreader stoker*, which combines the features of suspension burning and the overfeed principle. The coal is projected through the front wall of the furnace by rotating blades or by steam or air jets, the mechanical type being most widely used in industrial and utility furnaces. The coal falls on a stationary, dumping, or continuous discharge (traveling-grate) type of grate. The coal fines burn largely in suspension and the heavier portions on the grates. Although this type of firing was devised primarily to burn low grades of coal, which it does very well, it is adaptable to a wide range of coals, from anthracite to lignite. Caking qualities of the coal have no appreciable effect on the performance of the spreader stoker. The size consist of the fuel is the most important factor for best performance, a range of sizes from slack to 3/4 in. giving good distribution on the grate and an appreciable amount of burning in suspension. Coal too closely sized tends to fall within a narrow portion of the grate. If the percentage of fines is too high, excessive carbon losses to the stack will result, although recirculation of cinders to the furnace is sometimes used to reduce the carbon losses. Air or steam jets directed across the fuel bed produce sufficient turbulence to reduce cinder losses and smoke and to prevent stratification of furnace gases. Best efficiencies are obtained when all combustion air is admitted through the grate and steam is used to obtain turbulence. Spreader stokers are characterized by rapid response to load changes. In refractory furnaces, the combustion rate should not exceed 35,000 Btu per hr-ft^3, and for water-cooled furnaces it should not exceed 45,000 Btu per hr-ft^3.

THE UNDERFEED METHOD, shown schematically in Fig. 4(*b*), is based on the upward, parallel flows of fuel and air. Since fresh coal is admitted beneath the burning fuel bed, volatile matter, moisture, and air pass *through* the active burning zone, and less soot and smoke are produced than with overfeed firing. Combined movements of the fuel tend to break coke structures near the top of the fuel bed. A disadvantage is that ash pushed to the hot zone on top of the fuel bed tends to clinker and impede normal movement of the fuel. This method of firing is best exemplified by the single- and multiple-retort types of stoker. In the single-retort stoker, the coal has a combination of upward and rearward motion through the retort and sidewise motion over the grate. Ash is discharged at the outer ends of the grates to dump plates, from which it is periodically

* See also Section 7, Art. 23, for additional data on coal burning on stokers.

discharged to the ash pit. The multiple-retort stoker has an inclined grate, and the underfeed principle is combined with the crossfeed. In addition to upward motion, there is rearward motion of the coal, normal to the flow of air and produced by auxiliary rams. These rams also break coke structures. Ash is discharged from the rear of the grate by dump grates.

CROSSFEED (FRONTFEED). No type of fuel-burning equipment yet developed has wider application for burning solid fuels: except for strongly caking bituminous coals, every type of coal and coke breeze may be burned successfully. The principle involved is shown diagrammatically in Fig. 4(c_1), the fuel moving essentially at a right angle to the flow of air. Pure crossfeed would be obtained if α, the angle of the plane of ignition with respect to the plane of the grate, were 90 degrees. However, in practice, ignition occurs across the top of the incoming fuel, either by radiant heat, conduction, and convection from hot gases, or by ignited fuel blown from the rear of the grate. The result is that the ignition plane travels in a direction nearly opposite to that of the air flow. Therefore, over the length of the ignition plane, the burning is essentially underfeed. Thereafter, the incandescent fuel burns crossfeed. Since the air requirements for the zones E, F, and G differ considerably, the windbox of forced-draft units usually is divided into sections with individual damper controls.

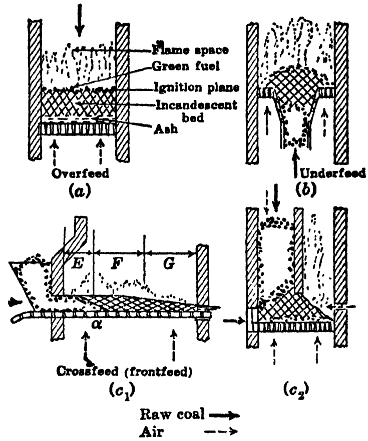

Fig. 4. Diagrams of methods of feeding and firing coal.

A modification of the crossfeed principle is illustrated in Fig. 4(c_2), representing a downdraft space heater. Heat is transferred to the raw fuel by conduction through the separating wall and by radiation from the incandescent fuel. This form of crossfeed operation is useful for domestic heating with sub-bituminous and lignite coals.

Table 8 gives the general characteristics and uses of the various types of stoker.

Table 8. Nominal Characteristics and Uses of Industrial Stokers

	Type		
	Overfeed	Underfeed	Crossfeed
Application	Inclined grate Spreader type	Single retort Multiple retort	Traveling grate Chain-grate
Fuel	Anthracite, high-volatile coking coals, lignite, coke and refuse fuels	High-volatile coking coals, slack or fines High ash fusion desirable	All except strongly coking bituminous coals Best operation with 10–20% ash coals
Fuel-bed thickness, in.	6–7 (bituminous)		5–6 (No. 3 buckwheat) 4–5 (No. 4 buckwheat)
Continuous combustion rate, max for best efficiency, lb per hr per ft^2 of grate area	30 (IG) *	30 (SR, 6–7′ wide) * 40 (SR, 7–10′ wide) * 40 (MR) *	50 (bituminous) 45 (No. 3 buckwheat) 35 (No. 4 buckwheat) 35 (Coke breeze)
Draft, inches of water, natural forced	 0.2–0.6 1–3	 2–4	 0.2–0.6 1–3

* IG = inclined grate. SR = single retort. MR = multiple retort.

Note. All stokers perform better with preheated and overfire air. Where forced draft is used, a windbox should be zoned to secure optimum amount of air for various sections. Preheat temperature of about 350 F is best for minimum stoker maintenance.

PULVERIZED-COAL FIRING (see Section 7, Art. 28). In contrast to grate and stoker operation, where ignition and burning occur in a bed of fuel, pulverized coal is burned in suspension in the furnace cavity. This type of firing is unequaled for high capacities, wide choice of fuels, and high boiler availability. Several boilers in this country operate continuously to generate 1,000,000 lb per hr of high-pressure steam. The economic factors that led to the extensive use in this country of pulverized-coal-fired steam generators were efficient utilization of the cheaper, low grades of fuel, exceptionally high boiler availability, high unit capacity, amenability to automatic control and flexible operation, low unit maintenance costs, and elimination of banking losses.

Pulverized-coal particles burn in suspension in the furnace atmosphere, going through the same stages as occur in fuel beds: heating to the ignition point, distillation and combustion of the volatile matter, and combustion of the coke residue. Ignition occurs in the order of a few hundredths of a second, and practically complete combustion occurs in approximately 1 sec, indicating that the combustion of the coke particles is the slowest part of the process. This is an important factor in the design of the burner and the furnace from the standpoint of carbon losses to the stack and the ashpit. The primary variables during the period of combustion of the devolatilized particles are particle size, type of fuel, furnace temperature, and excess air. The fraction of unburned carbon may be related empirically to the various pertinent factors (Ref. 20).

Burners for Pulverized Coal. Figure 5 shows schematically the basic methods of feeding pulverized coal and air to furnaces. The function of any burner is to supply coal and air

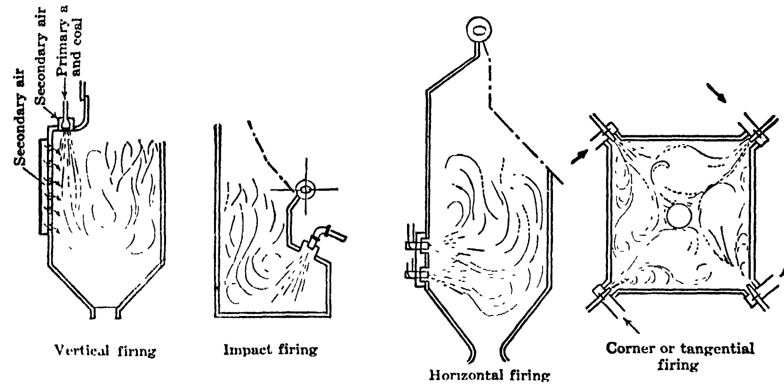

Fig. 5. Basic types of firing in pulverized-coal-fired furnaces.

in such a manner as to obtain (1) complete combustion within the furnace, thereby minimizing carbon losses and utilizing the heat-absorbing surface most effectively, (2) adequate mixing of the coal and air, (3) stable ignition to prevent furnace pulsations, (4) uniform distribution of temperature and composition of the gases leaving the furnace, (5) minimum slag and ash deposits on boiler or secondary heating surfaces, and (6) sufficient flexibility to burn a range of quality of coal.

Vertical firing, although an early method, still is used extensively, but with all the secondary air admitted around the burner nozzle so that it mixes quickly with the coal-primary air mixture from the burner nozzle. Occasionally the admission of secondary air along the front wall is used with considerable success, particularly in connection with very low-volatile coals, which require long flame travel, or in high, narrow furnaces.

Impact firing, a form of vertical firing, consists of burners located in an arch low in the furnace or in the side walls and directed toward the furnace door, with high velocities of both primary and secondary air. This type of firing is used exclusively in wet-bottom or slagging-type furnaces.

Horizontal firing employs a turbulent burner, which consists of a circular nozzle within a housing provided with adjustable valves, the unit being located in the front or rear wall. The primary air and coal are fed to the nozzle, in which the mixture is given a rotary motion by narrow, spiral vanes. The secondary air enters the outer housing through the adjustable vanes, which provide rotary motion at an angle different from that of the primary air and coal, and, meeting the primary-air-coal mixture at the periphery of the nozzle, creates a high degree of turbulence. This type of burner is suited to high-capacity and dry-bottom furnaces.

Corner or tangential firing is characterized by burners located in each corner of the furnace and directed tangent to a horizontal, imaginary circle in the middle of the furnace, thereby making the furnace the burner in effect, since turbulence and intensive mixing occur where the streams meet. The coal and primary air enter through rectangular or square coal nozzles; secondary air is supplied partly around the nozzles and partly through ports above and below them. Dampers proportion the secondary air to the various sections. The relative velocities of gas and fuel produce a scrubbing action that promotes the transport of oxygen to the fuel, through the film of combustion products around the particles. Further, the tangential motion of the gases produces a vortex, which effectively lengthens the time that the combustible is in the furnace. This type of firing is suited to either wet- or dry-bottom furnace operation and medium- or high-volatile coals, and it is capable of extremely high capacities.

FURNACE HEAT RELEASE AND HEAT AVAILABLE. Furnaces for pulverized-coal firing are designed either to remove the ash as molten slag intermittently or continuously (wet bottom), or as dry ash (dry bottom). Wet-bottom construction generally is chosen for low-grade coals that have low fusion characteristics, whereas dry-bottom construction often is selected for high-fusion coals. Experience has shown, however, that it is possible to design reliable dry-bottom units to burn any grade of coal available, at continuous maximum loads with low maintenance and high boiler availability. Pulverized-fuel firing is used for steam capacities ranging from 50,000 to 1,000,000 lb per hr, capacities above 150,000 lb per hr being almost exclusively fired with pulverized coal. The *furnace heat release* varies from 15,000 to 30,000 Btu per cu ft per hr—21,000 to 22,000 Btu for best performance of wet-bottom furnaces or for dry-bottom units burning coal with an ash-fusion temperature above 2100 F. The *available furnace heat* is defined as the heat in the coal as fired, plus the heat in the preheated air, minus the latent heat of water vapor in the flue gases, minus one-half the radiation and unaccounted-for losses, minus the heating value in the unburned carbon. This value, divided by the projected area, in square feet, of the furnace wall tubes plus the plane of the first row of boiler tubes, gives a useful factor for comparing furnaces. For round tubes, the projected area is taken as the diameter multiplied by the length; and, with finned tubes and studded tubes, the projected area, including fins and studs, is used. Most central station boilers in this country have values for the available furnace heat between 50,000 and 100,000 Btu per sq ft of heat-absorbing surface.

11. COKE

By R. E. Brewer

Coke is the solid, infusible, cellular residue left after fusible bituminous coals are heated, in the absence of air, above temperatures at which active thermal decomposition of the coal occurs. *Pitch coke* and *petroleum coke* of somewhat different characteristics are obtained by similar heating of coal-tar pitch and petroleum residues.

High-temperature coke is made from coal at temperatures ranging from 1500 to 2000 F (average practice, 1700 to 1900 F).

Low-temperature coke is formed at temperatures below 1300 F. The residue, if made from a noncoking coal, is known as char.

Typical analyses of various high-temperature cokes are given in Table 9. Such values depend on the coal or coal blend, its preparation, and the conditions of coking.

Production and disposal, by principal uses, of high-temperature cokes and coke breeze or screenings in the United States in 1945 are given in *Bureau of Mines Minerals Yearbook*, 1945, pp. 958–1016. By-product and beehive cokes are mainly metallurgical fuels, in blast furnaces and foundry cupolas, although considerable quantities are consumed for gas making and domestic heating. Gas-house or retort cokes are used principally by the producer for gas making and for bench fuels. Coke breeze or screenings are of value chiefly in making steam.

Typical analyses of low-temperature cokes and chars are given in Table 10. Of the processes listed for coal, only the Wisner, Curran-Knowles, and Lurgi methods have been operated commercially. The other processes using coal have furnished fundamental technical data from which new commercial processes eventually may be developed.

TYPES OF COKE. The quality of cokes is judged by physical, chemical, and thermal properties. Physical properties of cokes, such as size, shape, color, and cell structure, are appraised by visual examination. Strength, hardness, density, porosity, and weight per cubic foot are evaluated by special apparatus and test procedures. Chemical analyses for moisture, volatile matter, fixed carbon, ash, sulfur, phosphorus, and calorific value are important criteria in judging the quality of coke for special purposes. Ultimate analyses of the coke and the composition and fusion temperature of the ash are sometimes determined. Thermal properties, such as ignitability, combustibility, and reactivity of the coke, are important in evaluating cokes for metallurgical uses.

Table 9. Typical Analyses of Various High-temperature Cokes

Kind of Process	"As-received" Basis (or "Dry" Basis When No Moisture Is Reported)										Gross Calorific Value, Btu/lb
	Proximate, %				Ultimate, %						
	Moisture	Volatile Matter	Fixed Carbon	Ash	Hydrogen	Carbon	Nitrogen	Oxygen	Sulfur	Phosphorus	
By-product coke *											
Foundry		0.91	90.45	8.64					.62	.020	13,340 †
Other coke		0.98	88.48	10.54					.82	.018	13,060 †
Beehive coke ‡	1.1	1.80	85.20	11.90	0.8	83.4	1.2	1.9	.80	. .	12,370
Gas-house or retort coke §											
Horizontal retorts	0.8	1.40	88.00	9.80	0.7	86.8	1.1	0.9	.70		12,820
Vertical retorts	1.3	2.50	86.30	9.90	1.1	85.4	1.4	1.5	.70		12,770
Narrow coke ovens	0.7	2.00	85.30	12.00	0.6	84.6	1.2	0.9	.70		12,550
Pitch coke ‖	0.3	1.10	97.60	1.00	0.6	96.6	0.7	0.6	.50		14,097

* *Bur. Mines Minerals Yearbook*, 1945, p. 989. Averages for thirteen foundry cokes tested during 1945 1948 by Bureau of Mines for U. S. Navy were: volatile matter, 1.3; fixed carbon, 88.8; ash, 10.0, sulfur, 0.7, phosphorus (two samples), 0.022; and calorific value, 12,790 Btu per lb.

† Calculated from the equation: Btu per pound = [14,600 (100 − % ash)]/100.

‡ Unpublished analyses by Bureau of Mines, 1943.

§ J. D. Davis and J. W. Greene, Reactivity of Pulverized Cokes in Air, Carbon Dioxide, and Water Vapor, *Proc. Am. Gas Assoc.*, 1926, pp. 1160–1164.

‖ S. P. Kinney, and G. St. J. Perrott, The Shatter and Tumbler Tests for Metallurgical Coke, *Ind. Eng. Chem.*, Vol. 14, 1922, pp. 926–931.

Oven coke, made either in by-product or beehive ovens, may vary in color and luster from dull dark-gray to light silver-gray. It is hard, blocky, and fine-grained, with few cross-fracture and shrinkage cracks. The cell walls may be thick or thin, depending on the kind of coal carbonized and the rate of heating through the plastic temperature range during the coking process. High-quality foundry coke usually contains less volatile matter, ash, and sulfur, and more fixed carbon than do other cokes for metallurgical purposes.

Gas-house or **retort coke** contains more volatile matter than metallurgical cokes and is, therefore, generally more suitable as a domestic fuel. When used as fuel in the manufacture of water gas, retort cokes should be able to retain heat during the air blow and to decompose steam during the steam run.

Pitch coke is produced from coal-tar pitch in Alabama and in the Chicago district of Indiana and Illinois. Its high purity (high carbon and low ash) makes it desirable for manufacturing electrode carbon and for melting base-metal alloys in foundry cupolas. Limited production and high cost of manufacture have precluded its wide use.

Disco is produced at about 820 F in rounded, homogeneous, ball-shaped pieces by continuous heating and carbonizing of finely crushed coal in an inclined revolving mild-steel retort with additions of small quantities of air and steam at the discharge end. It is an excellent fuel for hand-fired furnaces and fireplaces.

Coke from Curran-Knowles sole-heated ovens is made by heating the oven mainly from the bottom. Thus it has the characteristics of high-temperature coke at the bottom and those of low-temperature coke at the top of the charge. The tar from this process resembles tars made by other low-temperature processes. The coke is widely distributed as a domestic fuel in the St. Louis area.

The four processes studied by the Utah Foundation produced low-temperature (1200 F) cokes whose analyses are given in Table 10. The internally heated retorts using superheated steam produced higher-volatile cokes than did the externally heated retorts.

Coke from the Grank Forks, N. Dak., pilot plant was produced at an average temperature of 1345 F in the combustion chamber, which accounts for its relatively low-volatile matter content.

Lurgi lignite-char briquets, made from lignite char and a petroleum or coal-tar-pitch binder, are used chiefly as domestic fuel and, to a limited extent, industrially as bakery and blacksmith fuels. The lignite char, produced at 930 to 1100 F, has also been used for the latter purposes.

Table 10. Typical Analyses of Various Low-temperature Cokes and Chars

Kind of Process	"As-received" Basis (or "Dry" Basis When No Moisture Is Reported)									Gross Calorific Value, Btu/lb
	Proximate, %				Ultimate, %					
	Moisture	Volatile Matter	Fixed Carbon	Ash	Hydrogen	Carbon	Nitrogen	Oxygen	Sulfur	
Externally heated, horizontal retorts										
Wisner (Disco), inclined rotary *	....	17.0	72.8	10.2	.....	.. ..	... .	.....	2.1	13,100
Utah Foundation, rotary†	....	8.3	83.9	7.8	2.3	85.7	1.5	2.1	0.6	13,214
Externally heated, vertical retorts										
Curran-Knowles, sole-heated ‡	....	4.2	80.3	15.5	.....	. .	... .	....	1.2	12,180
Utah Foundation †	....	8.1	83.8	8.1	2.2	84.6	1.5	3.1	0.5	12,969
Grand Forks pilot plant §	1.4	10.2	72.8	15.6	1.9	76.6	0.7	4.7	0.5	12,040
Internally heated, horizontal retorts										
Utah Foundation, rotary, superheated steam †	....	12.2	80.9	6.9	2.8	83.8	1.7	4.3	0.5	13,000
Internally heated, vertical retorts										
Utah Foundation, superheated steam †	. .. ..	15.0	76.5	8.5	3.1	80.0	1.7	6.2	0.5	12,856
Lurgi lignite-char briquets ‖	7.0	15.4	61.1	16.5	3.2	70.8	0.7	6.9	1.9	11,700
Petroleum coke, coking still ¶	0.6	2.1	95.8	1.5	. .	.....	..	...	0.5	14,480

* C. E. Lesher, Production of Low-temperature Coke by the Disco Process, *Am. Inst. Mining Met. Engrs., Tech. Pub.* 1176, 1940, 30 pp., p. 10.

† Utah Conservation and Research Foundation, *State of Utah Low-Temperature Carbonization of Utah Coals*, p. 92, Quality Press, Salt Lake City, Utah, 1939, 872 pp.

‡ G. Thiessen, Coke from Illinois Coals, *Ind. Eng. Chem.*, Vol. 29, 1937, pp. 506–513, p. 510.

§ V. F. Parry and others, Gasification of Lignite and Sub-bituminous Coal. Progress Report for 1945–46, *Bur. Mines Rept Invest.* 4128, 1947, 69 pp., p. 39.

‖ V. F. Parry and others, Gasification of Lignite and Sub-bituminous Coal. Progress Report for 1944, *Bur. Mines Rept. Invest* 3901, 1946, 59 pp., p. 14.

¶ J. C. Morrell and Gustav Egloff, Petroleum Coke, *Chemistry & Industry*, Vol. 51, 1932, pp. 467–469.

Petroleum coke is used as refinery and industrial fuel (often powdered) in the manufacture of carbon electrodes, brushes, plates, abrasives, artificial graphite, and calcium carbide, as metallurgical fuel, and in the ceramic industries.

12. WOOD AND HOGGED FUEL

By E. P. Carman

These data on wood fuel are taken largely from Basic Facts on Wood Burning, by L. E. Webber (published in *Power*, Vol. 85, March 1941, pp. 156–158) and the book *Combustion Engineering* by Otto de Lorenzi (published by Combustion Engineering Co., Inc., New York, 1947). Other references are given at the end of the chapter (Ref. 21).

Wood fuel may come to the boiler plant in the form of cordwood, slabs, edgings, bark, sawdust, or shavings, and frequently several forms are available together. From 30 to 50% of the lumber delivered to woodworking mills becomes waste available as fuel, the percentage depending on whether the mill is of the "rough" or "finishing" type. Waste from finishing mills usually runs 25 to 40% of lumber processed and is usually of smaller size consist, is drier, and contains less bark and foreign material. Technically, "hog" fuel, a term sometimes loosely applied to sawdust, shavings, and bark, is only that wood which has been chopped up in "hog" choppers, which may be (1) steel disks with attached knives, (2) two concentric cones bearing knives and revolving in a conical housing, (3) a cylinder with attached knives revolving in a cylindrical housing, or (4) "hammer hogs," in which wood is broken by impact of hammers against anvils. Dull-knived hogs may shred rather than cut wood; such shredded wood in long, stringy pieces may clog mechanical feeders.

PROPERTIES OF WOODS. The major variable in wood is moisture content; air-dried wood seldom contains less than 12% water, whereas kiln-dried usually contains from 1 to 7%. Moisture in wood from rough mills averages 30 to 50%; waste from logs floated to mills often contains up to 70%. Well-dried wood is hygroscopic; i e., it will absorb moisture from the air. The specific gravity of wood ranges from 0.3 to 1.2; the heating value of dry wood (except where resin increases heating value) is approximately proportional to the specific gravity. Moisture in newly felled wood varies with the species but averages 40%. Table 11 shows the chemical composition and heating value (dry) of typical woods common in this country.

Table 11. Chemical Composition and Heating Value of Dry Woods

Species	Constituents, % by Weight C	H	O	N	Ash	Heating Value, Btu/lb
Oak	50.16	6.02	43.26	.09	0.37	8316
Ash	49.18	6 27	43.19	.07	0.57	8480
Elm	48.99	6.20	44.25	.06	0.50	8510
Beech	49.06	6.11	44.17	.09	0.57	8591
Birch	48 88	6 06	44.67	.10	0.29	8586
Pine	50.31	6.20	43.08	.04	0.37	9153
Poplar	49.37	6.21	41.60	.96	1.86	7834
California redwood	53 50	5.90	40.30	.10	0.20	9220
Western hemlock	50.40	5.80	41.40	.10	2.20	8620
Douglas fir	52.30	6.30	40.50	.10	0.80	9050
Pine sawdust	51.80	6.30	41.30	.10	0.50	9130

A cord of wood is a pile 4 x 4 x 8 ft = 128 cu ft. B. E. Fernow (Ref. 22) gives the percentage of solid wood in a standard cord of various types of wood as follows: timber cords, 74.07%; firewood cords (diameter over 6 in.), 69.44%; billet cords (diameter over 3 in.), 55.55%; brushwood cords (diameter less than 3 in.), 18.52%; roots, 37%. The difference between the solid wood percentage and 100% is voids.

Hog fuel is usually sold on the basis of a *unit* of 200 cu ft, as measured in the containing transporation vehicle and without packing. The unit contains 1700 to 2300 lb of dry wood, depending on species, moisture content, and amount of shavings or sawdust present in the mixture.

The weight per cord of various woods, their heating value, and equivalent heating value expressed in pounds of 13,500 Btu per lb coal per pound of wood are given in Table 12.

Table 12. Heating Value of Woods

	Weight per Cord, lb		Heating Value, Btu/lb		Equivalent Pounds of Coal of 13,500 Btu/lb	
	Green	Air-dried	Green	Air-dried	Green	Air-dried
Ash, white	4300	3800	4628	5395	.343	.400
Beech	5000	3900	3940	5359	.292	.397
Birch, yellow	5100	4000	3804	5225	.282	.387
Chestnut	4900	2700	2633	5778	.195	.428
Cottonwood	4200	2500	3024	6000	.224	.444
Elm, white	4400	3100	3591	5710	.266	.423
Hickory	5700	4600	4053	5391	.300	.399
Maple, sugar	5000	3900	4080	5590	.302	.414
Maple, red	4700	3200	3745	5969	.277	.442
Oak, red	5800	3900	3379	5564	.250	.412
Oak, white	5600	4300	3972	5558	.294	.412
Pine, yellow	3100	2300	7097	9174	.526	.680
Pine, white	3300	2200	4226	5864	.313	.434
Walnut, black	5100	4000	4078	4650	.302	.344
Willow	4600	2300	2370	5870	.176	.435

COMBUSTION OF WOOD AND WOOD WASTE requires intelligent handling, knowledge of their composition and the important influence of moisture, and understanding of the three-stage wood combustion process. These three stages of combustion involve (1) evaporation of moisture, (2) distillation and burning of volatile matter, and (3) burning of fixed carbon, i.e., the residual charcoal. However, these steps usually overlap somewhat. The first and second stages *absorb* heat from the furnace, whereas the burning of volatile matter and fixed carbon *give up* heat to the furnace.

There are three general methods of burning wood fuels, though combinations may be used. Wood fuels may be burned (1) in a moving bed on an inclined grate, (2) in suspension, as in spreader stokers, or (3) in piles on flat grates. Method 3 is the slowest. Method 1 tends to segregate the three combustion stages (a not undesirable effect). It is necessary to supply excess air in burning wood fuels. Table 13 (from Webber) shows the flue-gas analysis for combustion of wood, both with perfect combustion and for varying percentages of excess air.

Table 13. Flue-gas Analysis for Complete Combustion of Wood

(Average Gottlieb analysis)

	Composition of Dry Products, % by Volume					
% Excess Air	0	20	40	60	80	100
CO_2	20.1	16.8	14.4	12.5	11.2	10.0
O_2	0.0	3.6	6.1	8.0	9.5	10.6
N_2	79.9	79.6	79.5	79.5	79.3	79.4

13. MISCELLANEOUS SOLID FUELS

By E. P. Carman

CHARCOAL. Charcoal provided the only carbon for steel making and other metal smelting from prehistoric times up to the eighteenth century in Europe and up to early in the nineteenth century in the United States, when coke gradually began to take the place of charcoal in steel making. The U. S. Department of Agriculture estimates (Ref. 23) that annual production of charcoal in the United States is still about 350,000 tons annually and lists several dozen market outlets in the domestic and specialized fuel, metallurgical, and chemical fields.

Production. Charcoal is produced by partial combustion of wood at about 400 C and with limited air. It may be made in kilns, ovens, buried pits, or any suitable type of enclosure in which wood can be piled and burning can be restricted through control of inlet air. The object is to char the wood without burning any more of it than is necessary to accomplish the charring operation. Kilns are frequently constructed of moundlike piles of wood covered with sod or turf and provided with a central flue and with air-inlet ports around the periphery. Kilns vary in capacity from 15 to 45 cords of wood. The time required to char a kiln of wood depends on the moisture content of the wood and the size of the kiln. It may take as long as two weeks. The process is complete when smoke from the kiln becomes thin and blue. Portable kilns that can be moved to new supplies of wood have received increasing attention; one of this type is the Black Rock Forest kiln (Ref. 23).

By-products. Both hardwoods and softwoods are now used in the production of charcoal; hardwood charcoal weighs about 20 lb per bu and softwood charcoal about 18 lb per bu (Ref. 24). When very resinous woods are processed in sloped clay-floor kilns, tar is formed from the resin in the wood. The tar collects on the floor and can be drained off and recovered. Other by-products are seldom recovered from small charcoal operations. With operations of sufficient size to make recovery and refining economical, large volumes of gas, a watery pyroligneous acid condensate, and tar can be recovered (Ref. 25). An average gas yield of approximately 8000 cu ft per cord of wood has been obtained from large commercial plants. Typical gas composition is: CO_2, 59%; CO, 33%; CH_4, 3.5%; H, 3.0%; Vapors, 1.5%. Considerable variation in gas yield and composition is reported; for example, volumes of 4600 to 8000 cu ft of gas per cord, and methane content of 3.5 to 18%. The watery pyroligneous acid contains a complex mixture of organic acids, alcohols, aldehydes, ketones, etc., and approximately 80 to 90% water. (See Ref. 25 for a list of products reported in this mixture.) Formic and acetic acids, methyl (wood) alcohol, formaldehyde, acetaldehyde, turpentine, and acetone are some of the more familiar by-products recovered. The tar is a complex mixture containing most of the products found in the pyroligneous acid and many others. It may be distilled to give "light" oils, "heavy" oils, and pitch. (See Ref. 25 for a list of identified products.)

Specifications. Charcoal is seldom sold on specification; the usual market guarantees relate only to weight per bushel and to volatile and moisture content. The maximum of 14% volatile and 2% moisture is customarily established. The heating value of charcoal ranges from 11,000 to 14,000 Btu per lb and can be approximately calculated from Dulong's formula (see p. 2-04). References cited previously and Ref. 26 give details of charcoal manufacture, kiln and oven descriptions and dimensions, etc.

PEAT. About 95% of the 224,785 tons of peat produced and imported in 1946 was sold for soil improvement and for the manufacture of mixed fertilizers (Ref. 27). Other uses include litter for barns and poultry yards, improvement of lawns and golf courses, mulching in nurseries and greenhouses, and packing material for plants, fruits, vegetables, eggs, and other fragile articles. No sales of peat for fuel were reported to the Bureau of Mines for 1946.

There are, however, extensive reserves of peat in this country, estimated by the U. S. Geological Survey (Ref. 28) to be 13,827,000,000 tons. Minnesota, Wisconsin, and Michigan, combined, contain about 75% of the reserves, Florida has 14%, and the remainder is scattered throughout about half the states, mainly in New England and the Pacific Coast states. Alaska (Ref. 29) has an estimated 110,000,000 acres of peat land or "muskeg" and grassy marshland.

BRIQUETS AND PACKAGED FUEL (PRESSED FUELS). Briquets and packaged fuel frequently are made of almost the same materials, but the methods of marketing differ. Both usually are made from fine coal or chars, with the addition of a binder, although some briquets are made without binder (Ref. 30). Briquets, which may be in the shape of "pillows," small barrels, cylinders, or cubes, range in weight from 1 1/2 to 20 oz. They are made with sufficient strength to stand handling and weathering and therefore constitute a closely sized fuel, easy to handle and to control during combustion. If made of anthracitic or low-volatile bituminous coals of reasonably low ash content, they are an excellent, nearly smokeless, domestic fuel that retains the shape of the individual briquets in the fire, does not cake, and burns to a fine ash.

Typical proximate analyses by the Bureau of Mines of various types of briquets made in the United States are given in Table 14.

Table 14. Proximate Analysis and Heating Value of Briquets Made in the United States

(Unpublished analyses from files of the Fuel Inspection Section, Courtesy of U. S. Bureau of Mines)

Type of Coal Used as Major or Only Coal Constituent	Proximate Analysis as Received, % Moisture	Volatile Matter	Fixed Carbon	Ash	Heating Value as Received, Btu/lb
Anthracite fines	3.7	13.3	74.7	12.0	12,750
Semi-anthracite	4.5	14.6	73.2	12.2	12,640
Petroleum coke	12.7	13.6	85.9	0.5	13,220
Medium-volatile bituminous coal	2.3	22.3	69.7	8.0	14,230
Pocahontas coal, cement binder	1.1	17.9	72.5	9.6	14,060
Pocahontas coal, starch binder	1.4	19.6	74.9	5.5	14,520
Illinois coal	6.0	34.1	50.6	15.3	11,440
Wyoming sub-bituminous	10.9	44.4	49.8	5.8	11,320

Petroleum asphalt binders, usually about 4 to 8% of the finished briquets, are favored in most briquetting operations in this country, although some starch (particularly in packaged-fuel operations), coal-tar pitch, and oil-gas pitch are used; cement, although feasible, adds to the ash content. Table 15 gives specifications of a binder used in Bureau of Mines tests on briquetting sub-bituminous coal.

Table 15. Properties of a Typical Briquetting Binder

(Adapted from V. F. Parry and John B. Goodman, Briquetting sub-bituminous coal, *Bur. Mines Rept. Invest.* 3707, June 1943, p. 7)

Property	Binders A	B	C	D
Viscosity, Saybolt Furol, at 275 F	74	330		
Penetration—100/5/77	18	18	21	4
Softening point—ring and ball, °F	133	141	154	176
Flash—Cleveland Open Cup, °F	470	610	650	
Ductility at 77 F/50 mm/min	196+	194+	5.8	
Solubility in CCl_4, %	98.99	99.68		
Specific gravity at 77 F	1.1030	1.047	1.0331	
Proximate analysis, %				
Moisture	0.0	0.0	0.0	0.0
Volatile matter	71.30	76.30	72.30	73.80
Fixed carbon	28.63	23.66	27.40	25.50
Ash	0.07	0.04	0.30	0.70
	100.00	100.00	100.00	100.00

Production of 3,171,596 tons of briquets in 1947 was 5.6% higher than in the previous year and continued the upward trend since 1938. Major production was in Wisconsin, where low-volatile bituminous-coal fines from lake docks and coal yards constituted the principal fuel used. There was substantial production of briquets in Pennsylvania from anthracite fines. Plants in 14 other states used various raw materials, including Arkansas and Oklahoma anthracitic coals, high-volatile bituminous coals, lignite char, residual chars from pyrolysis of natural gas and manufacture of oil gas, and petroleum coke.

Briquets are used primarily in domestic furnaces, stoves, and water heaters and in small industrial and commercial heating appliances, such as chicken brooders, greenhouses, store heaters, etc., where their uniform size and favorable combustion characteristics warrant paying the higher price that closely sized fuels command over slack and run-of-mine sizes. They cannot compete in larger fuel-burning installations where stokers permit efficient use of cheaper coals.

Packaged fuel, produced as 3- to 4-in. cubes and usually wrapped six to the package in sturdy paper sealed with gummed tape, is clean to deliver and handle. Packages are piled neatly in the basement or utility room and are fired as a unit, either with or without cracking the wrapper, which in any case soon burns off, allowing the cubes to spread over the fuel bed. Packaged fuel, because of its high price to cover the additional cost of wrapping and handling in relatively small units, is used more as a spring and fall fuel when moderate fires are desired (Ref. 31).

(For additional references to literature on briquetting, see Paul L. Fisher, A Selected Bibliography on Briquetting of Coal and Other Carbons, *Information Circular* 7469, Bureau of Mines, July 1948. Copies may be obtained by writing to the Bureau of Mines, Department of the Interior, Washington 25, D. C.)

BAGASSE. Bagasse is the fibrous refuse remaining after the juice has been extracted from sugar cane. According to one study (Ref. 32), bagasse leaving the last rolls of Cuban sugar mills contains about 40% fiber, 1 1/2 to 2 1/2% sugar, 45 to 55% moisture, and 1 1/2 to 2 1/2% ash. Typical analysis and heating value of dry bagasse is carbon, 45%; hydrogen, 6%; oxygen, 43%; nitrogen, negligible; ash, 2%; higher heating value, 8000 to 9100 Btu per lb.

In general, the steam requirements of sugar mills are easily supplied by the burning of the bagasse they turn out. Bagasse is burned in Dutch-oven-type furnaces on horseshoe-shaped grates built in multiple cell units, on inclined grates, and on spreader stokers. The ash from bagasse, a fine silt, is frequently fusible and produces a slag difficult to remove, but, with the cell-unit type of furnace, one cell can be cleaned while others maintain the load.

WET BARK. Wet bark in substantial quantities is available at most paper-mill operations, and, if not burned for heating and power, it creates a disposal problem. Since bark discharged from the barking drums contains as much as 80% moisture, it is, in that condition, of no value as a fuel. The heating value of the bark is insufficient to evaporate this much water (see Table 16). Mechanical presses are used to squeeze water out of the bark before it is burned. These presses reduce the moisture content to about 65%, raising the "as-fired" heating value to the point where the moisture can be evaporated and residual heat is available for steam generation. Table 16 gives the heating value for bark containing various percentages of moisture.

Table 16. Heating Value of Typical Bark Containing Various Percentages of Water

(Otto de Lorenzi, *Combustion Engineering*, Combustion Engineering Co., Inc., New York, 1947, pp. 12:16–23)

Moisture, % by weight	Btu per lb
0	8750
20	7000
40	5250
50	4375
60	3500
70	2625
80	1750
90	875

Furnace Types. Originally furnaces for burning bark and paper-mill refuse were considered as incinerators, and good fuel was used to fire them to get rid of the waste products. Newer furnaces, together with air heaters and bark driers, not only dispose of the waste, without using additional fuel, but provide steam for power and heating as well. The older Dutch-oven-type furnace, in which bark was burned in piles on flat grates, provided the incinerator type of operation. The hog-fuel type of furnace, with preheated air or with preliminary flue-gas drying in tandem with preheated air, is now used with satisfactory results to generate steam from wet bark and other paper-mill wood refuse.

Sloping-grate furnaces are very satisfactory for wet bark and are claimed to have distinct advantages over the conical-pile flat-grate method. Burning in suspension—i.e., with a spreader stoker—has been found quite satisfactory for burning bark in a Stirling boiler designed to operate at 550 psi and 700 F (Ref. 33).

Burning Rates. Disposal rates of 20 to 35 lb of dry solids per square foot of grate surface per hour are possible with bark of 65 to 70% moisture content if auxiliary fuel is used; if the moisture content can be reduced to 55 to 60% before the bark comes onto the grate, rates of 35 to 50 lb of dry solids are possible without auxiliary fuel (Ref. 34, *Combustion Engineering*, pp. 12:16–23). To heat up the furnace after shutdowns, to take up the load if the bark supply is low, or to increase the overall capacity to meet peak-load demands, most bark-burning furnaces are so designed that auxiliary fuels—coal, oil, or gas—can be burned when necessary.

STRAW, PAPER, AND MISCELLANEOUS WASTE FUELS. With properly designed equipment, almost any solid material having a heating value exceeding that required to evaporate the moisture in the material can be used to produce heat and power. The important consideration is that an adequate and assured supply of the material be available at a price, including transportation and handling, to make the installation economically sound. Table 17 (Ref. 35) gives the heat of combustion of various substances

Table 17. Heat of Combustion of Various Substances, on a Dry Basis

Substance	Heating Value, Btu per lb, dry	Substance	Heating Value, Btu per lb, dry
Petroleum coke	15,800	Rags (linen)	7,132
#1 Gilsonite selects *	17,699	Rags (cotton)	7,165
Asphalt	17,158	Cotton batting	7,114
Pitch	15,120	Corrugated fiber carton	5,970
Soot (from oil)	11,787	Newspaper	7,883
Soot (from smokeless coal)	7,049	Wrapping paper	7,106
Soot (Island Creek)	5,425	Oats	7,998
Soot (Red Jacket Thacker)	10,569	Wheat	7,532
Soot (Crystal Block Winifrede)	4,951	Oil (cottonseed)	17,100
Wood sawdust (oak)	8,493	Oil (lard)	16,740
Wood sawdust (pine)	9,347	Oil (olive)	16,803
Wood sawdust (pine)	9,676	Oil (paraffin)	17,640
Wood sawdust (hemlock)	7,797	Oil (rape)	17,080
Wood sawdust (fir)	8,249	Oil (sperm)	18,000
Wood sawdust (spruce)	8,449	Candy	8,096
Wood shavings	8,248	Butter	16,560
Wood shavings (hardwood auto bodies)	8,878	Casein	10,548
Wood bark (spruce)	8,817	Egg white	10,260
Wood bark (hemlock)	8,753	Egg yolk	14,580
Wood bark (fir)	9,496	Fats (animal)	17,100
Wood bark (fan)	7,999	Hemoglobin (blood)	10,620
Brown skins from peanuts	10,431	Waste hemp hurds	7,982
Corn on the cob	8,100	Cottonseed hulls (fusion 2342 F)	8,600
Rags (silk)	8,391	Cottonseed hull brans (fusion 2307 F)	8,675
Rags (wool)	8,876	Pecan shells	8,893
		Coffee ground	10,058
		Pecan shells (few meats left in them)	10,144

* Material used for cores in foundries.

on a dry basis. Methods of handling and firing vary, but, if the fuel material has a residual heating value higher than that required to evaporate its moisture content, furnaces and equipment can be designed to handle them satisfactorily.

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LIQUID FUELS

By Harry F. Tapp

14. CHARACTERISTICS OF FUEL OIL

FUEL OIL is defined (ASTM D288-47) as any liquid or liquefiable petroleum product burned for the generation of heat in a furnace or firebox, or for the generation of power in an engine,* exclusive of oils with a flash point below 100 F, by the Tag closed tester, and oils burned in cotton or wool-wick burners. Fuel oils in common use fall into four classes:

* For specifications of diesel engine fuels, see Section 13, Art. 11.

(1) residual oils, which are topped crude petroleums or viscous residuums obtained in refinery operations; (2) distillate fuel oils, which are distillates derived directly or indirectly from crude petroleum; (3) crude petroleums and weathered crude petroleums of relatively low commercial value; and (4) blended fuels, which are mixtures of two or more of the preceding classes.

COMMERCIAL FUEL OIL SPECIFICATIONS (ASTM D396-48T) cover five standard grades limited by the detailed requirements summarized in Table 1. The several grades are defined as: No. 1—a distillate oil intended for vaporizing pot-type burners and other burners requiring this grade of fuel; No. 2—a distillate oil for general-purpose domestic heating in burners not requiring No. 1 fuel oil; No. 4—an oil for burner installations not equipped with preheating facilities; No. 5—a residual-type oil for burner installations equipped with preheating facilities; No. 6—an oil for burners equipped with preheaters permitting a high viscosity fuel.

Flash point (ASTM D93-46) is the temperature to which oil must be heated to give off sufficient vapor to form an inflammable mixture with air. It varies with apparatus and procedure, and both must be specified when flash point is stated. The minimum flash point usually is controlled by law. If no legal requirements exist, minimum values of Table 1 are used.

Table 1. Detailed Requirements for Fuel Oil *

(ASTM D396-48T)

Grade†	Flash Point, °F, Min	Pour Point, °F, Max	Water and Sediment, %, Max.	Carbon Residue on 10% Bottoms, %, Max	Ash, %, Max	Distillation Temperatures °F: 10% Point, Max	Distillation Temperatures °F: 90% Point, Max	Distillation Temperatures °F: End Point, Max	Viscosity: Saybolt Universal at 100 F, Max	Viscosity: Saybolt Universal at 100 F, Min	Viscosity: Kinematic Centistokes at 100 F, Max	Viscosity: Kinematic Centistokes at 100 F, Min	Viscosity: Saybolt Furol at 122 F, Max	Viscosity: Saybolt Furol at 122 F, Min	Gravity, Degrees API, Min
1 ‡	100 or legal	0	Trace	0.15		420		625			2.2	1.4			35
2	100 or legal	20 §	0.10	0.35		‖	675		40						26
4	130 or legal	20	0.50		0.10				125	45					
5	130 or legal		1.00		0.10					150			40		
6	150		2.00 ¶										300	45	

* Because of the necessity for low-sulfur fuel oils used in connection with heat treating, nonferrous metal, glass, and ceramic furnaces, and other special uses, a sulfur requirement may be specified in accord with the following tables:

Grade of Fuel Oil	Sulfur, % Max
No. 1	0.5
No. 2	1.0
Nos. 4, 5, and 6	No limit

Other sulfur limits may be specified only by mutual agreement between the purchaser and the seller.

† It is the intent of these classifications that failure to meet any requirement of a given grade does not automatically place an oil in the next lower grade, unless in fact it meets all requirements of the lower grade.

‡ No. 1 fuel oil must pass copper strip corrosion test, 3 hr at 122 F. The exposed copper strip shall show no gray or black deposit.

§ Lower or higher pour points may be specified whenever required by conditions of storage or use. However, these specifications shall not require a pour point lower than 0 F under any conditions.

‖ The 10% point may be specified at 440 F maximum for use in other than atomizing burners.

¶ The amount of water by distillation plus the sediment by extraction shall not exceed 2.00%. The amount of sediment by extraction shall not exceed 0.50%. A deduction in quantity shall be made for all water and sediment in excess of 1.0%.

Pour point (ASTM D97-47) is the lowest temperature at which oil will flow under prescribed conditions.

Water and sediment (ASTM D96-47) are excluded almost entirely in No. 1 and 2 oils but are allowed to limited extent in No. 4, 5, and 6 oils. Water and sediment are determined together by the centrifuge, except that, in No. 6 oil water is determined by distillation (ASTM D95-46) and sediment is determined by extraction with benzol (ASTM D473-46T).

Carbon Residue (ASTM D524-42). The carbon residue test, in connection with other tests and the use for which the oil is intended, furnishes information and throws light on the relative carbon-forming qualities of an oil. For No. 1 and 2 oils, the Ramsbottom carbon residue test is made on 10% bottoms. For medium viscosity and blended oils, it is used to detect heavy residual products.

Ash (ASTM D482-46). The ash test determines the amount of noncombustible impurities, which come principally from the natural salts present in the crude oil, from chemicals used in refinery operations, or from sea-water contamination, as in the case of residual fuels transported by sea. They also may come from scale and dirt picked up from containers and pipes. Depending on its chemical composition, the ash in fuel oil may cause rapid deterioration of refractory materials in the combustion chamber, particularly at high temperatures. Some ash-producing impurities are abrasive and destructive to pumps, valves, control equipment, and other burner parts. Ash specifications are included to minimize these operating difficulties.

Distillation temperatures (ASTM D86-46 for No. 1 oil, ASTM D158-41 for No. 2 oil) of a sample under prescribed conditions are an index of volatility. The 10% and 90% points represent, respectively, temperatures at which 10% and 90% of the sample are distilled over. The end point is the maximum temperature recorded by the distillation thermometer at the end of distillation. The 10% point is an index of ease of ignition. The 90% point and the end point are specified to insure that the oil will burn completely and produce a minimum of carbon.

Viscosity * is a measure of the resistance of oil to flow (ASTM D88-44 for Saybolt viscosity). It is the time in seconds in which a definite volume of oil will pass through a tube of specified dimensions at a definite temperature. For oils having viscosities less than 32 sec. Saybolt Universal, such as No. 1 fuel oil, it is necessary to determine Kinematic viscosity in centistokes (ASTM D445-46T). Viscosity decreases as temperature increases. Preheating makes possible the use of oils of relatively high viscosities at normal temperatures. Maximum viscosity is limited because of its effect on oil flow in pipe lines and on the degree of atomization that can be had in given burner equipment. The Saybolt Universal viscosimeter is used for low-viscosity fuel oils, and the Saybolt Furol viscosimeter for heavier oils. Other types of viscosimeters for fuel oils are the Redwood and Engler. Kinematic viscosity in centistokes may be converted to Saybolt Universal by ASTM D446-39 and to Saybolt Furol by ASTM D666-44. See Table 2 for approximate viscosity equivalents at the same temperature.

Table 2. Approximate Viscosity Equivalents at the Same Temperature *

Kinematic, centistokes	Saybolt Universal seconds	Saybolt Furol, seconds	Redwood No. 1, seconds	Kinematic, centistokes	Saybolt Universal, seconds	Saybolt Furol, seconds	Redwood No. 1, seconds
2.0	32.6	. .	28.6	20.6	100	....	88.4
2.1	33.0	. ...	29.0	25.3	120	....	105.9
2.2	33.3	. .. .	29.2	29.8	140	....	123.6
2.3	33.7	... ...	29.6	34.3	160	....	141.1
2.4	34.0	....	29.8	38.8	180	....	158.8
2.5	34.4	....	30.2	43.2	200	23.0	176.4
2.7	35 0	.. ....	32.2	65.0	300	32.5	265.0
4.3	40	. ... .	36.2	108.2	500	51.0	441.0
7.4	50	.. ..	44.9	216.5	1,000	100.0	882.0
10.3	60	..	53.5	433.0	2,000	200	1,763.0
13.1	70	.	62.3	649.5	3,000	300	2,646
15.7	80	. .	71.0	1,082.5	5,000	500	4,408
18.2	90	. .	79.6	2,165.0	10,000	1,000	8,816

* Kinematic centistokes at 100 F = Saybolt Universal seconds at 100 F × 0.2165.
Redwood No. 1 seconds at 100 F = Saybolt Universal seconds at 100 F × 0.8773.
Engler degrees at 100 F = Saybolt Universal seconds at 100 F × 0.0285.

The viscosity-temperature chart (ASTM D341-43) is convenient for estimating Saybolt and Kinematic viscosities at temperatures other than standard test temperatures.

Specific gravity is the ratio between the weight of any volume of oil at 60 F and the weight of an equal volume of pure water at 60 F. It always is used for solid petroleum products and often for liquids. Except for exact laboratory work, gravity determinations on liquid petroleum are made by hydrometer (ASTM D287-39T), the depth to which it sinks in the liquid, as shown by the scale, determining specific gravity direct, or the gravity in degrees API. The API gravity of pure water at 60 F is 10°. The range for fuel oils is approximately 10° to 40° API.

To overcome the confusion due to the use of two so-called Baumé scales, for light liquids, the American Petroleum Institute, the U. S. Bureau of Mines and the National Bureau of Standards agreed, in 1921, to recommend that only the scale based on the modulus 141.5

* For additional information on viscosity, see Section 6, Art. 15.

be used in the petroleum-oil industry and that it be known as the API scale. The relation of degrees API to specific gravity is expressed by

$$\text{Degrees API} = [141.5/(\text{sp. gr. } 60\text{ F}/60\text{ F})] - 131.5$$

Liquid fuels are purchased by volume. All gravity readings and volume determinations should be corrected to the standard temperature, 60 F. For correction tables, see *Circular* 410, National Bureau of Standards. The unit of volume is the barrel (42 U. S. gal) = 5.6 cu ft approx. The weight of fuel oil ordinarily is taken as 60 lb per cu ft, whence 1 bbl = 336 lb at 60 F. The coefficient of volume expansion of the average fuel oil is approximately 0.0004 per °F.

The corrosion test (ASTM D130-30) is to detect free sulfur or corrosive sulfur compounds in No. 1 fuel oil. The test is the same as for gasoline except for interpretation of the exposed copper strip.

Heating Value. Exact determination of the heating value of fuel oil is made in a bomb calorimeter (ASTM D240-39). This calorimeter determination is unnecessary for most uses, as the heating value characteristic is associated with normally determined physical properties. For distillate fuel oils and residual fuel oils having gravities higher than 18° API, a close approximation of the gross heating value may be made as follows: Btu per lb = 18,250 + (40 × degrees API). For residual fuels having gravities less than 18° API, Btu per lb = 17,690 + (58 × degrees API). A correction should be made if water and sulfur are present in appreciable amount. See *Significance of Tests of Petroleum Products—A Report Prepared by ASTM Committee D-2 on Petroleum Products and Lubricants.* As fuel oils are generally purchased on a volume basis, it is important to keep in mind that the heating value of fuels on a volume basis is inversely related to the Btu per pound. For example, a fuel of 10° API having 18,270 Btu per lb will have 152,153 Btu per gal; a fuel of 15° API having 18,560 Btu per lb will have 149,297 Btu per gal. See *Publication* 97 of the National Bureau of Standards, for additional information on heating values. See Table 3 for analysis and heating value of typical oils.

Table 3. Analysis and Calorific Values for Various Oils

Oil	Ultimate Analysis, %					API Gravity, degrees	Flash Point, °F	Saybolt Viscosity		Higher Heating Value Gross Btu per lb, Calculated	Lower Heating Value, Btu per lb, Calculated	Higher Heating Value, Btu per gal, Calculated
	C	H	S	O and N	Ash			Universal, 100 F	Furol, 122 F			
Mid-continent distillate	86.1	13.2	0.6	0.1	Trace	34 7	150	39		19,638	18,484	139,430
Venezuelan cracked distillate	87.0	12 0	0.8	0.2	Trace	23.2	208	48	. . .	19,178	18,130	145,945
Texas residuum	84.6	10.9	1.6	2.9	. .	22 3	142	.	.	19,142	18,192	148,436
Mid-continent residuum	87.0	11.7	0.9	0.4	.01	15.9	360	...	179	18,612	17,590	148,890
Texas cracked residuum	86.3	10.5	2.1	1.1	.06	11.2	230	..	129	18.340	17,426	151,305
Texas cracked residuum	...	....	..	....	...	12.0	...	.....	. .	18,386	. .	150,765
Venezuelan residuum	86.0	11.2	2.1	0.8	.07	14.9	204	. .	140	18.554	17,579	149,360
Venezuelan cracked residuum	86.0	10.3	2.3	1.2	.08	11.3	198	......	102	18,345	17,445	151,163
Venezuelan cracked residuum	......	......	2.4	....	.08	13.5	230	......	145	18,473	... . .	150,000

SPECIFIC HEAT of fuel oils varies from 0.4 to 0.55 for the temperature range generally used. Specific heat increases with temperature, and decreases as the specific gravity of the oil increases. The Prime-movers Committee of the National Electric Light Association, 1930, gives the following mean specific heats of a California oil of 18° API.

Temp. °F	100	120	140	160	180	200	220	240	260	280
Mean specific heat	.450	.456	.463	.470	.479	.490	.502	.514	.527	.541

Specific heat of flue gas with oil fuel varies with temperature. F. G. Philo (*Trans. ASME* FSP-54-11, 1932) presents a curve which shows the specific heat to vary in a straight line from 0.245 at 300 F to 0.27 at 2000 F.

CHEMICAL COMPOSITION OF PETROLEUM. Petroleum is composed of carbon and hydrogen combined as hydrocarbons, and small quantities of oxygen, nitrogen, sulfur and ash. The range of ultimate analysis of fuel oils is C, 85–90%; H_2, 9–15%; S, up to 5%; O_2 and N_2, up to 3%; ash, 0 to 0.10%.

The ultimate analysis determines the theoretical air required for combustion (see p. 2-04) and the maximum CO_2 possible, which depends on the chemical analysis or on the carbon-hydrogen ratio. The carbon-hydrogen ratio is C/H, but for purposes of combustion calculations may be expressed as (C + 0.4S)/H. Table 4 gives the relation of C–H ratio, percentage of excess air, and CO_2. In practice, burning of fuel oil will give CO_2 as follows: high average, 12–14%; average, 10–12%; low average (poor), 5–8%.

Table 4. Relation of C-H Ratio, CO_2, and Excess Air

	Excess Air, %				
	0	10	30	50	100
			CO_2, %		
C-H ratio = 6 (light oil)	14.9	13.5	11.3	9.7	7.2
C-H ratio = 7 (medium oil)	15.6	14.1	11.8	10.1	7.5
C-H ratio = 8 (heavy oil)	16.1	14.5	12.2	10.5	7.8

OIL COMBUSTION THEORIES. Three theories of the burning of hydrocarbons are: (1) The hydrogen burns with oxygen before the carbon unites with oxygen. (2) The carbon burns in preference to the hydrogen. (3) A preliminary combination of oxygen with the hydrocarbon forms an intermediate hydroxylated compound, which, in turn, burns or is broken down thermally.

Investigations by W. A. Bone and others (*Flame and Combustion in Gases*, see General References) indicate a combination of hydrocarbons with oxygen, preliminary to final combustion (theory 3), i.e., hydrocarbons combine with oxygen to form alcohol and aldehydes as a preliminary to burning to CO, CO_2, and H_2O. This process is termed *hydroxylation*.

In ordinary combustion of hydrocarbons, no soot will form if conditions favor hydroxylation, viz., premixture with air and ample time for oxygen to enter into the hydrocarbon molecule. If conditions favor cracking, a smoky flame results. For example, if hydrocarbons and oxygen from the air are not thoroughly mixed, the heat due to burning part of the hydrocarbons decomposes or cracks the remainder. For the conditions of ordinary combustion, the hydrocarbon, plus a small amount of oxygen, may be assumed to become a mixture of $CO + H_2$, which burns as if the reactions were $2H_2 + O_2 = 2H_2O$, and $2CO + O_2 = 2CO_2$.

Unburned Residues from Oil Fuels. In burning distillate fuel oils, which normally contain only traces of ash, there are negligible deposits of scale or slag, although some soot may be formed, particularly with inadequately designed or improperly operated burners handling fuels high in carbon-hydrogen ratio. Residual fuels are usually slower-burning than distillates, and, although added precautions against smoke and soot are taken, some carbonaceous material is usually deposited. This residue should be blown out regularly to avoid possible corrosion difficulties and to preserve efficiency of heat transfer. The ash-forming constituents in residual fuels, though present in only small concentration, are often predominantly salts of the alkaline metals and, on passing through the combustion chamber, are converted largely to the sulfate form. These constituents have low fusion temperatures and may adhere tenaciously to the heating surfaces of high-capacity boilers, particularly high-temperature superheaters. When tubes are kept clean by regular blowing, this tendency is minimized, because the molten droplets of ash are chilled below their fusing point upon impact and are eliminated as dust up the stack. If the slag is allowed to accumulate, a water-washing technique may be required for complete removal.

COMPARATIVE FUEL COSTS are estimates of performance based on known and assumed factors, supposed to represent average conditions. Known factors are cost per unit and heat content per unit. The assumed factor is the efficiency of utilization of the heat content of the fuel (see p. 2-17). To compare fuel costs intelligently, conditions under which the fuels are used must be studied, and each problem must be considered separately. The accuracy of an estimate of the probable relative consumption of two fuels will be in direct proportion to the accuracy of information available.

The actual efficiency of a given heating operation can be determined only by test. If tests are not available, assumed efficiencies must be used, based on knowledge and experience. Factors to be considered are fuel used, air-fuel ratio, furnace volume, combustion temperatures, amount and condition of heating surface, relation between heating surface and flow of gases, heat-absorbing medium (water, air, metal, etc.), load factor, load fluctuation, and temperature of the escaping combustion gases.

Other factors than fuel cost that must be considered to determine real comparative operating cost, are (1) cost of installation, (2) labor, (3) results obtained, i.e., quality of finished product, (4) uniformity or flexibility of heat control, (5) reliability, (6) maintenance, (7) depreciation of equipment, and (8) operating power cost for equipment, including auxiliaries. Installation cost is important but should not be considered until study of the whole problem is complete. The advantages of a given fuel may so outweigh this factor as to make it negligible in the final analysis.

ADVANTAGES OF FUEL OIL. (Haslam and Russell, *Fuels and Their Combustion*, see General References.) (1) Weight 30% less and space occupied 50% less than coal of equivalent heat content. (2) No deterioration in storage. (3) Freedom from spontaneous combustion. (4) Storage may be distant from furnaces. (5) Fuel is immediately available and may be stored or removed with practically no labor. (6) High combustion rates per cubic foot of combustion space. (7) Great flexibility in furnaces to carry peak and valley loads readily and economically. (8) Low labor cost to handle oil at the furnace and to clean boiler tubes. (9) No labor for cleaning fires or removing ashes. (10) High efficiency and practically no smoke. (11) Absence of wear on machinery due to ash and dust. (12) Low pressure drop through the furnace. (13) Minimum of excess air required for complete combustion.

15. METHODS OF BURNING FUEL OIL*

OIL BURNER TYPES. Oil burners can be classified as (1) natural draft vaporizing, (2) natural draft atomizing, (3) mechanical draft vaporizing, (4) mechanical draft atomizing. They also can be classified according to most common use made of each class, as (1) domestic burners, using No. 1 and 2 oils, full- or semi-automatic or manually controlled, used in domestic heating systems, (2) commercial burners, using No. 4, 5, or 6 oils, full- or semi-automatic or manually controlled, used in heating boilers for apartment houses, office, manufacturing, and public buildings, etc., usually semi-automatic; (3) industrial burners using No. 6 oil, used for industrial or power steam generation, and using any grade of oil from No. 1 to 6, to supply heat for industrial processes.

Mechanical draft burners use a fan or blower to supply the air for combustion. Mechanical draft may be used with either vaporizing or atomizing burners. Its advantages are: (1) A more constant supply of air is provided under varying draft conditions. (2) Velocity of the air may be used to increase turbulence or mixing effect. (3) Sufficient air for clean combustion from a cold starting condition is supplied without dependance on chimney draft. (4) It can be used to develop high ratings when necessary. Its disadvantages are (1) increased first cost, (2) cost of power, and (3) mechanical wear and noise.

ATOMIZATION breaks the fuel into fine particles that readily mix with the air for combustion. The fuel then burns with a clean hot flame, being vaporized and oxidized by the resulting combustion before cracking takes place. In pressure atomizing burners, the fineness of spray increases as pressure increases and as viscosity decreases. When No. 6 oil is burned, a pulsating flame results if viscosity is reduced to a point where the preheat temperature tends to vaporize the fuel. Table 5, from *U. S. Navy Manual of Engineering Instruction*, shows the effect of pressure and viscosity on spray angles. This effect will vary with different burners and must be determined for each design size. If particles of oil or vaporized oil escape from the combustion zone because of improper atomization,

Table 5. Effect of Pressure and Viscosity on Spray Angle of Pressure Atomizing Burners

Size	340 Sec Saybolt Universal Viscosity			150 Sec Saybolt Universal Viscosity			50 Sec Saybolt Universal Viscosity		
	Pressure, pounds per square inch gage								
	125	200	300	125	200	300	125	200	300
	Spray Angle, degrees								
5520	35	35	35	41	39	37	46	42	38
5320	36	37	38	46	44	41	53	50	46
5220	37	38	39	47	44	41	55	51	47
5020	41	43	45	50	48	47	57	57	52
4430	47	48	50	57	55	53	61	59	56

* See also Section 7, Art. 25.

combustion chamber design, or air control, their heat content will escape with the flue gases. These losses are known as hydrocarbon losses and may be determined with the Burrell-Orsat gas analysis apparatus. Factors affecting hydrocarbon losses are (1) insufficient turbulence, (2) cracking of oil and vapor in hot inert gases of combustion, and (3) cooling effect of excess air.

The advantages of atomization of oil are: (1) Atomizing burners can be used with heavier grades of oil. (2) Atomization can be adapted to large applications because of larger capacity range. (3) Complete combustion is assured by the ability of the small particles to penetrate turbulent combustion. (4) Accurate metering of the fuel is possible, resulting in uniform combustion conditions. Disadvantages are necessity of power-driven units to effect atomization and higher installation cost.

Mechanical pressure atomizers are designed for capacities ranging from 6 to 5000 lb of oil per hour per burner. Oil is delivered to the burner, preheated if necessary for pumping and atomization, under pressures ranging from 40 to 250 lb per sq in. or more, depending on the quantity and grade of oil. Pressure is varied to increase or decrease the capacity of the tip. With low-capacity burners, 0.75 to 10 gal per hr (diameter less than 0.030 in.), the capacity range generally is determined at pressures of 65 to 150 lb per sq. in. for the lighter oils No. 1 and 2; for the heavier, more viscous grades, No. 4, 5, and 6, the orifice size is increased and the capacities are determined at pressures of 100 to 250 lb per sq in. The oil leaving the atomizer tip is broken into fine spray by centrifugal force and by the rapid expansion following a sudden reduction in pressure. A regulating valve, manual or automatic, maintains a uniform pressure at the atomizer. Automatic valves usually are controlled directly by steam pressure or indirectly by electric control devices.

Because of the high pressures used, the dimensions of the various parts of the atomizer must be held to limits of 0.001 to 0.005 in., if uniform results are to be expected. Even slight imperfections in the oil passages and orifice will cause faulty atomization. Care must be used in handling and cleaning burner tips. In shipping, storing, and handling, tips should be individually wrapped and protected against damage. The nose of the atomizer should have a shallow counterbore to protect the orifice.

The principal advantages of mechanical pressure atomization are (1) simplicity, (2) uniform atomization, (3) accurate and uniform metering of fuel, and (4) high efficiency at high ratings. The disadvantages are (1) clogging of small orifices and passages (can be reduced materially by providing suitable strainers), (2) narrow range capacity of individual tips, except with special *wide-range* burners controlled by regulating the return flow from the swirl chamber of the burner tip, and (3) high preheating temperature required with heavy grades of oil.

Steam-atomizing burners are classified as outside mixers or inside mixers. Both types can be designed to produce either round or flat flames. They are designed for capacities ranging from 5 to 1500 lb of oil per hour per burner. A burner using steam for atomizing also may be used with compressed air. The oil is delivered to the burner oil-regulating valve at pressures ranging from 5 to 50 lb per sq in., preheated if necessary for pumping and atomization. The preheating temperature for steam-atomizing burners usually is lower than that for mechanical pressure atomizers, as the viscosity is decreased by the heat of the steam used for atomization.

Steam usually is delivered to the burner steam-regulating valve at boiler pressure. It should be dry or superheated, as moisture causes the flame to sputter. The amount of steam used for atomization varies with the design of the burner, the skill of the operator, and the boiler capacity. Under average conditions it ranges from 2 to 4% of boiler output. With competent operators and well-designed burners, the steam consumed may be as low as 1 1/2 to 2%; with careless operators or poor burner design, it may be 4 to 6% or more.

The principal advantages of steam-atomizing burners are (1) simplicity of design, (2) low first cost of installation, (3) low preheating temperature, (4) low pumping pressures, (5) flexibility and high efficiencies at low and moderate rates of driving, and (6) ability to burn extremely heavy oils. The disadvantages are (1) steam consumption of burners, (2) limitation in boiler capacity, and (3) decreased efficiency at high rates of driving.

Low-pressure Air Atomizers. Atomization of light oils can be accomplished satisfactorily with low-pressure air, without depending on oil pressure, if a sufficient quantity of air is supplied. The energy in a large volume of low-pressure air equals the energy in smaller volumes of air at higher pressures. The quantity of air required depends on design of burner, degree of atomization required, grade of oil, its pressure and temperature. Table 6 gives the approximate minimum quantity of atomizing air required at different pressures.

Table 6. Percentage of Combustion Air Required for Atomization

Air pressure, psi	.25	.50	1	2	5	10	25	60	100
Air required, %	68	52	42	33	25	19	13	9	7

Mechanical Rotary Atomizers. Oil is fed by a pump or by gravity through a regulating valve to a rapidly rotating tapered cup. The oil enters at the rear or at the point of least diameter, is carried forward and spread out in a film on the inner surface, and is atomized by centrifugal force as it is thrown off the edge of the cup. The cup is rotated either by a motor or low-pressure air turbine. Rotary cup burners are designed for capacities ranging from 5 to 2000 lb of oil per hour per burner.

The principal advantages of rotary disk, or cup, burners, are (1) flexibility over large capacity ranges, (2) high efficiencies, and (3) ability to burn heavy oils at low preheating temperatures. The disadvantages are (1) high first cost, (2) vibration and noise from moving parts, and (3) inefficient combustion resulting from operating burner beyond its rated capacity.

COMBINATION OIL AND GAS BURNERS. The need for a reliable fuel supply in installations burning natural gas has led to the development of combination oil and gas burners. The necessity of thoroughly mixing natural gas and air for combustion makes the oil burner of any type ideal equipment for such combinations. The oil and gas ratings are such as to permit either fuel to be used for carrying the load.

TESTING OF OIL BURNER TIPS. To insure equal amounts of oil being fed to each burner in a battery, frequent tests of burner tips are advisable. Tips wear, and leakage between burner parts may occur. Both factors tend to increase flow, and considerable variation in capacity of individual burners thus may exist.

PREPARATION OF LIQUID FUEL. Many variations and combinations of the methods used to prepare liquid fuel for combustion, including domestic burners, will be found in *Handbook of Oil Burning*, by H. F. Tapp (see General References).

SOURCES OF INFORMATION. For information on various types of oil burners, the publications of the following manufacturers are suggested.

Pressure Atomizing Burners. (1) Domestic, automatic: Delco Appliance Corp., Rochester, N. Y.; Gilbert and Barker Manufacturing Co., West Springfield, Mass., Silent Glow Oil Burner Corp., Hartford, Conn. (2) Commercial, industrial, and marine: Anthony Co., Long Island City, N. Y.; Babcock and Wilcox Co., New York, N. Y.; Peabody Engineering Co., New York, N. Y.; Todd Shipyards Corp., Elmhurst, N. Y.

Air and Steam Atomizing Burners. (1) Domestic, automatic: Eureka-Williams Corp., Bloomington, Ill.; General Electric Co., Schenectady, N. Y.; Lammert and Mann Co., Chicago, Ill. (2) Commercial and industrial: Coen Co., San Francisco, Calif.; The Engineer Company, New York, N. Y.; Hauck Manufacturing Co., Brooklyn, N. Y.; National Airoil Co., Philadelphia, Pa.

Rotary Atomizing Burners. (1) Domestic, automatic: Anchor Post Products Co., Baltimore, Md.; Automatic Burner Corp., Chicago, Ill.; Timken Silent Automatic Co., Detroit, Mich. (2) Commercial, industrial, and marine: Gilbert and Barker Manufacturing Co., West Springfield, Mass.; Ray Burner Co., San Francisco, Calif.; Petroleum Heat & Power Co., Stamford, Conn.; Preferred Utilities Manufacturing Co., New York, N. Y.

Vaporizing Burners. (1) Domestic, automatic: Kresno-Stamm Manufacturing Co., Palisades Park, N. J.; H. C. Little Burner Co., San Rafael, Calif.; Quaker Manufacturing Co., Chicago, Ill.

OIL BURNER CONTROLS may be classified as (1) manual, (2) continuous operation (high-low), (3) continuous operation (graduated control), (4) intermittent operation (full automatic), and (5) combinations of 2 and 4, or of 3 and 4. For data on oil burner controls, the publications of the following manufacturers are suggested.

Electric controls for domestic, commercial, and industrial burners; Minneapolis-Honeywell Regulator Co., Minneapolis, Minn.; Mercoid Corp., Chicago, Ill.; Penn Electric Switch Co., Goshen, Ind.; Photoswitch Inc., Cambridge, Mass.

Mechanical controls for domestic, commercial, and industrial burners: Automatic Products Co., Milwaukee, Wis.; Bailey Meter Co., Cleveland, Ohio; Hagan Corp., Pittsburgh, Pa.; Hays Corp., Michigan City, Ind.

A full automatic control system incorporates a device responsive to changes in temperature or pressure, a sequence control device, and a device to ignite the oil or combustible mixture. If all operations except ignition are performed automatically, the control is *semi-automatic*. The method of control depends on design characteristics of the burner, grade and amount of oil burned, and character of the load. Manual control is used largely in the industries for metal melting, varnish cooking, forge furnaces, ceramic furnaces, etc., where temperature or pressure control is relatively unimportant or where the operation requires close attention and the temperatures or pressures are changed during critical reactions in the process. In general, however, automatic controls are advantageous in most applications.

Limit controls to protect boilers or furnaces from excessive operation of the burner respond to the amount of pressure or heat developed in the boiler or furnace. These controls usually are designed for electric connection to the control of the burner. They are so connected that, if the boiler or furnace control has broken the circuit, owing to excess pressure or heat, the burner is inoperative regardless of the demand from the thermostat or other normal control device.

BURNER AND FLAME APPLICATION. The design and type of burner governs the general shape of the flame and determines in part the method of installing the burner and the path of flame in the boiler or furnace. Pressure atomizing burners usually have a cone-shaped flame, the included angle varying between 45 and 120 degrees, depending on the atomizer tip design and control of air flow. By controlling air flow, it also is possible to obtain a flat flame. Steam- and air-atomizing burners in general can be designed to produce either flat or cone-shaped flames; rotary atomizers are limited to cone-shaped flames, whose included angle may be varied by control of air flow about the atomizer cup. Specific instructions for control of flame shape are given in manufacturers' instructions. Most burners are sufficiently flexible in this respect to be generally adaptable to boiler settings, but the type of boiler or furnace and also the shape and size of the combustion chamber should be considered in selecting the type and number of burners. No more burners should be used than are needed to utilize the available combustion volume with sufficient flexibility to meet existing load conditions. For example, for a relatively narrow and deep combustion chamber, one burner is preferable, provided the oil rate can be varied to suit both the minimum and maximum load.

Adequate flame distribution requires that the flame conform as nearly as possible to the shape of the combustion chamber in order to (1) utilize maximum combustion volume, (2) provide for thorough mixing of fuel and air during combustion, insuring efficient combustion, (3) permit using a minimum of excess air, (4) reduce hydrocarbon losses, (5) permit quiet combustion, (6) obtain maximum thickness of flame, providing maximum transfer of radiant heat, and (7) reduce flame impingement on refractory walls of combustion chamber.

DESIGN OF COMBUSTION CHAMBER. Refractory combustion chambers are used, principally (1) to permit maintenance of high temperature in the combustion chamber to vaporize quickly raw fuel injected by the burner and (2) to protect parts of the boiler or furnace that are not cooled properly. Additional benefit may be obtained by building brickwork to guide and increase flame travel in the combustion chamber.

The more important points to be considered in the design of combustion chambers are: (1) The refractory surfaces provide a radiant heating surface, which aids in combustion of the oil. Hence, it should be of minimum bulk so that it will heat rapidly and bring the combustion chamber to an efficient operating temperature. (2) Usually the combustion chamber must be so designed as to protect metal parts that are not cooled by water. (3) The shape of the combustion chamber should (*a*) provide maximum flame distribution, giving consideration to the size and shape of the flame and (*b*) provide maximum flame travel, taking into consideration location of flue passes to the indirect heating surface. (4) All the direct heating surface of the furnace should, if possible, be exposed to the radiant heat of the flame and should not be covered with refractory material, except to prevent direct flame impingement. (5) For uniformly satisfactory flame distribution, square corners must be avoided; surfaces of the combustion chamber should curve upward to increase the turbulence within the chamber without creating downward eddies from the flame. (6) The center line of the burner should be high enough to prevent flame impingement on the hearth and to allow air to circulate below the flame for complete combustion. (7) For light oils at firing rates under 10 gal per hr, the combustion chamber hearth should have a minimum area of 75 sq in per gal per hr. For heavy oils and for firing rates over 10 gal per hr, the minimum area should be 70 sq in per gal per hr. The maximum area should be 100 to 115 sq in per gal per hr.

COMBUSTION VOLUME. In any installation, two combustion volumes are to be considered: (1) available volume and (2) effective volume.

Available combustion volume is the total volume enclosed by the boiler and its foundation or setting which may be used as combustion space. This is space enclosed by the floor or hearth, side walls, or water legs, up to the crown sheet or equivalent, which is a plane tangent to the bottom of the lowest row of tubes or other water-backed surfaces. A bridge wall should be considered as a side wall.

Effective combustion volume is the space actually occupied by the flame and circulating gases. It is controlled by the design, application, and adjustment of the burner and should be as nearly equal to the required available volume as conditions will permit. The required combustion volume varies with different classes of work; volume rate usually is expressed as Btu released per cu ft per hr. There are no fixed rules except that, in general, higher temperatures are involved with higher Btu releases and must be considered in connection with the life and services of refractory materials and structural supports of boiler or furnace. The intensity of combustion noise also tends to increase with high heat releases. Unless the refractory walls of the combustion chamber are cooled, high Btu releases should not be used except in emergencies. The average range of Btu releases for various types of installation is given in Table 7.

Table 7. Average Range of Btu Releases for Various Classes of Oil-burning Installations

Type of Installation	Btu Release per cu ft per hr
Domestic and commercial heating boilers	40,000– 80,000
High-pressure steam boilers (medium)	40,000– 80,000
High-pressure steam boilers (large)	40,000–200,000
High-pressure steam boilers (marine)	40,000–200,000
High-pressure steam boilers (naval)	60,000–300,000
Industrial furnaces	30,000–200,000

FIREBRICK AND REFRACTORY CEMENT. Firebrick and refractory cement should be selected on the basis of the service in which they are used. A grade higher than absolutely necessary should be chosen because of abuse under extreme operating conditions. The life of refractory material in combustion chambers is shortened by sustained high temperature, by rapid changes in temperature, and by panting or vibration from combustion. High temperatures result from operation above normal rating, normal operation with insufficient combustion chamber volume, flame impingement, and combustion chambers designed for high Btu releases. Rapid temperature changes may be reduced to the minimum by the operating personnel. A cold boiler should be brought up to operating temperature and pressure as slowly as possible. When taking the boiler out of service, registers and dampers must be closed tightly to allow the boiler to cool slowly. Panting is usually due to improper drafts, faulty atomization, fluctuating oil pressure or high Btu releases. Sputtering results from water in the oil or wet steam supplied to steam-atomizing burners.

FURNACE FLOORS. The burner manufacturer usually specifies the furnace floor construction. The several layers are as follows: (1) insulating brick or material; (2) first course of brick, dry, laid $^1/_{16}$ in. apart to provide for expansion, joints broken between adjacent rows: (3) dry refractory cement, filling all cracks and covering bricks to depth of $^1/_8$ in.; (4) second course of brick similar to first, overlapping joints in first course; and (5) dry refractory cement as in (3). After firing, the bricks take a permanent set and the cement vitrifies to a hard surface. For air ports built into the floor, the bricks may be set in refractory cement mortar.

METAL COMBUSTION CHAMBERS. For wet-base domestic heating boilers and forced warm air furnaces, stainless steel combustion chambers are used extensively. Type 430 stainless steel (17% chromium) is representative of the lowest-grade material that may be used for this service.

16. STORAGE AND HANDLING OF FUEL OIL

FUEL OIL STORAGE TANKS generally are classified by material, as steel or concrete; by size, as gallons, etc.; by location, as exposed or inside, underground or buried; and by use, as light or heavy oil tanks. The essential requirements for tanks are tightness and durability. The specifications of Underwriters' Laboratories, Chicago, or of the National Board of Fire Underwriters, are generally accepted standards. See Tables 8 and 9. Some cities and states require special construction, and local regulations should be studied before installation. Tanks for heavy oil usually have a manhole and provision for a tank preheater, using either steam or hot water. Such tanks should be designed to heat the oil in the vicinity of the suction pipe to not over 100 F.

Table 8. Specifications for Underground Oil Storage Tanks *

(National Board of Fire Underwriters, Revised 1947)

Maximum capacity, gal	285	560	1100	4000	12,000	20,000	30,000
Gage of metal	16	14	12	7	$^1/_4$ in.	$^5/_{16}$ in.	$^3/_8$ in.
Weight of metal, lb per sq ft	2.50	3.125	4.375	7.50	10.00	12.50	15.00

* Top of underground tanks to be not less than 2 ft underground. Material to be galvanized steel, basic open-hearth, or wrought iron. Joints to be welded, or riveted and caulked. When the tank is installed inside buildings without enclosure, the maximum capacity is 275 gal and the minimum gage is 14.

Table 9. Specifications for Aboveground Oil Storage Tanks *

(National Board of Fire Underwriters, Revised 1947)

Maximum capacity, gal	60	350	560	1100	Over 1100
Gage of metal	18	16	14	12	

* Thickness of metal for outside aboveground tanks of over 1100 gal capacity to be calculated by the following formula: $t = H \times D/8450 \times E$, where t = thickness of metal in inches; H = height of tank in feet above bottom of ring under consideration; D = diameter of tank in feet; E = efficiency of vertical joint in ring under consideration where tensile strength of steel shall be considered to be 55,000 psi and shearing strength of rivets to be 40,000 psi. Minimum thickness of shell or bottom is $^3/_{16}$ in. and of roof $^1/_8$ in.

Capacity and Location of Tanks. The location of a tank with respect to distance from tank shell to line of adjoining property or nearest building depends on the construction, contents, equipment, and greatest dimension (diameter, length, or height) of the tank and should be in accordance with Table 10.

Table 10. Capacity and Location of All-steel Gastight Tanks

Class of Tanks and Contents	Approved Attached Extinguishing System or Approved Floating Roof	Distance between Shell and Property Line or Nearest Building
Group A for refined petroleum products not subject to boil-over	Yes	Not less than greatest dimension (diameter, length, or height); maximum distance required, 120 ft
Group B for refined petroleum products not subject to boil-over	No	Not less than 1 1/2 times the greatest dimension (diameter, length, or height); maximum distance required, 175 ft
Group C for crude petroleum and flammable liquid subject to boil-over	Yes	Not less than twice the greatest dimension (diameter, length, or height); minimum distance required, 20 ft; maximum distance required, 175 ft
Group D for crude petroleum and flammable liquid subject to boil-over	No	Not less than three times greatest dimension (diameter, length, or height); minimum distance required, 20 ft; maximum distance required, 350 ft

The minimum distance between shells of any two all-steel, gastight tanks should be not less than one-half the greatest dimension (diameter, length, or height) of the smaller tank except that such distance should not be less than 3 ft; for tanks of 18,000 gal or less, the distance need not exceed 3 ft.

Tanks should be so located as to avoid possible danger from high water.

When tanks are located on a stream without tide, they should, where possible, be downstream from burnable property.

For additional details, see latest revision of *Pamphlet* 31, National Board of Fire Underwriters.

Care of Tanks. Oil tanks should be cleaned at least once a year. Water and foreign material which settles out of the oil will accelerate corrosion if allowed to remain on the tank bottom. Large storage tanks should have a manhole for entrance for periodic examinations and cleaning. To insure that all oil vapors have escaped, before the tank is entered, the manhole should be left open for several hours and air circulation established by steam, compressed air, or a fan. The tank should be examined carefully, the bottom thoroughly cleansed, and all discolored or rusty spots scraped and painted with a composition paint, insoluble in oil or water, made especially for this purpose. Ordinary paint is soluble in oil and therefore unsuitable.

OIL PUMPS. The primary purpose of an oil pump in connection with oil burners is to draw oil from the storage tank by suction and deliver it to the burner. With pressure atomizing burners, the pump is also used to deliver oil under sufficient pressure to produce the atomization necessary for combustion.

PIPING SYSTEMS depend on design of burner, storage system, grade of oil, and requirements of local authorities.

Heavy Oil Systems. The principal difference between the piping for light and heavy oils is the use of preheaters. Consult burner manufacturers for detailed requirements for individual types of burners. The kind and amount of equipment necessary depend on design of burner, grade of oil, location of tank, character of load, and degree of automatic operation required. Figure 1 shows a typical heavy oil piping layout.

Fill Lines. Not less than 2-in. pipe should be used for light oils (No. 1); for heavy oils (No. 6), 6- or 8-in. pipe should be used. A pipe too large is better than one too small. The fill line for any storage tank should pitch from the fill box to the tank. A trap should be provided, either directly inside or outside of the tank, or the fill line sealed by ending it in the tank below the bottom of the suction line. The fill line always should be connected at the low end of the tank and never cross-connected to the vent pipe.

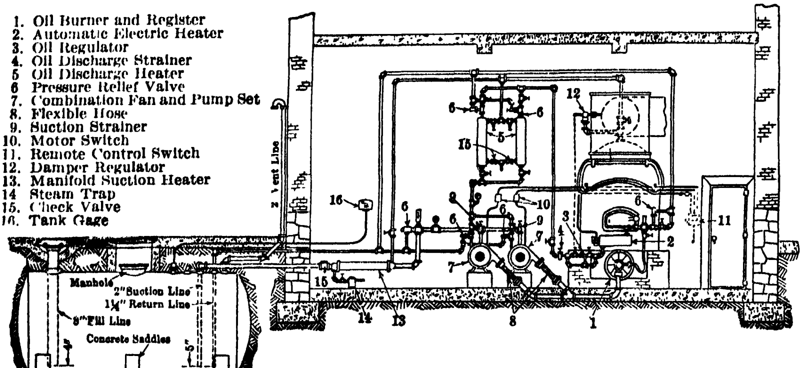

FIG. 1. Typical heavy oil piping layout.

Vent Pipe. All fuel oil storage tanks must be vented. The size of the vent pipe should be in proportion to the size of the fill line and should never be less than 1 1/4-in. pipe. Where tight fill-box connections are used for pressure filling, the vent must be of adequate size to prevent pressures being built up in the storage tank. The vent line should be without traps, pitched up from the tank, and connected near the high end. It should be installed with swing joints and should not be cross-connected with any line other than a vent. The vent pipe terminal should be visible from the fill box, weather-proof, and preferably without screens.

The suction line is a most important part of an installation. Between buried storage tanks and the building wall, it should pitch toward the tank and always should have swing joints. Usually a foot-valve is advisable on the suction line inlet in the tank, about 2 in. above the bottom of the tank for light oil, and 4 in. above for heavy oil. A testing tee should be in the suction line to permit testing at any time.

Strainers. Every oil line to the burner should have a suitable strainer, either suction or discharge. If the plant is shut down regularly, single strainers are satisfactory, but, for continuous operation, duplex strainers are advisable. The size of mesh depends on grade of oil, location of strainer, and design of burner. In general, the mesh should not be greater than 75% of the smallest port area protected and should not be finer than necessary to protect equipment in the portion of the line where the strainer is located. For light oils, suction strainers are 30 mesh or finer; discharge strainers 40 mesh or finer. For heavy oil, suction strainer meshes range from 8 to 30; discharge strainers, from 20 to 30 mesh. The area of strainer surface is of the same importance as the size of mesh. Suction strainers should be at least double the area of the suction line. Discharge strainers should have an area of at least 1/2 sq in. per gal of No. 1 or 2 oil burned per hour and at least 1 sq in. per gal of No. 4, 5, or 6 oil, but not less than 10 sq in. for burners having a capacity of not more than 10 gal per hr. For over 10 gal per hr, the area should be 10 sq in. plus 1/2 sq in. for each gallon per hour in excess of 10.

PREHEATERS may be classified according to (1) use, i.e., tank, suction, discharge, or auxiliary; (2) construction, i.e., manifold, coil, multipass, or film; and (3) heating medium, i.e., steam, hot water, or electric.

Tank heaters are used only where oil temperature in the tank is so low that oil cannot be pumped. Tank heaters sometimes are called suction heaters. However, for long suction lines a manifold heater often is used to insure a steady flow of oil. Steam or hot water furnishes heat, depending on grade of oil, temperature required, and size of heater. Steam generally is used, but some city ordinances require hot-water heaters to insure control of maximum temperature.

Discharge heaters reduce the viscosity of the oil to permit atomization. They always should be used for No. 5 and 6 oils, regardless of burner design. With pressure atomizing burners they often can be used to advantage with No. 4 oil. At least two discharge preheaters should be provided, each of sufficient capacity to carry the normal load, to permit cleaning and inspection without interrupting service. For wide variations in load, several heaters should be installed, to be cut in and out as needed. It is best to use one heater at maximum capacity rather than several in parallel at partial capacities. To avoid excessive heat losses, discharge heaters should be placed as close to the burner as structural conditions will permit.

Auxiliary Heaters. Auxiliary electric heaters, used to preheat heavy oils, permit starting when the system is cold. This type of heater also is incorporated in full automatic heavy-oil burners. It requires approximately 0.0146 kw-hr to raise 100 lb of oil 1 F. This type also is incorporated in full automatic heavy-oil burners.

Oil Temperature. The temperature required for the oil depends on design of the burner and viscosity of the oil. In general, the lower the viscosity, the better the atomization. For large capacity burners, the viscosity should not exceed 200 sec Saybolt Universal; and for small capacity (under 10 gal per hr) burners, 100 sec Saybolt Universal. The optimum viscosity varies with burner and furnace design and available draft. For satisfactory pumping, the viscosity should be held between 400 and 500 sec Saybolt Universal. Table 11 gives the range of temperatures commonly used. The table is based on gravity

Table 11. Approximate Preheating Temperatures for Fuel Oil

Gravity, API	Temperature, °F	Gravity, API	Temperature, °F	Gravity, API	Temperature, °F
10-12	275-325	16-18	150-200	22-24	70-100
12-14	220-275	18-20	140-160	24-26	70- 80
14-16	175-250	20-22	100-140	.....	....

and should be used only as an approximation. The correct temperature required to *reduce the viscosity* of the oil to 70-140 sec Saybolt Universal should be determined by test, or ASTM specification D341-431, and maintained. Gravity is not an accurate index to viscosity. Because of operating difficulties in pumping and atomizing, oil should not be heated above its flash point.

Preheaters designed for steam generally should have from 0.15 to 0.3 sq ft of heating surface for each gallon per hour capacity, depending on the grade of oil and design of heater. So many variables are involved, i.e., types of heaters (manifold, coil, multipass, and film), heating mediums (high- and low-pressure steam, hot water), temperature required by service conditions, grade of oil, types of burners, etc., that a preheater should be either selected on burner manufacturers' recommendations or designed as a heat exchanger on the basis of data applying to specific installation requirements. In all cases, best practice calls for automatic temperature regulation. Heaters should be examined regularly to detect leaks between the oil and steam passages. Such leaks usually can be detected by examining a sample of condensate from the heater. Heaters should be cleaned regularly, using steam in the oil passes and boiler compound in the steam passes. After disconnection or repair, heaters should have a hydrostatic pressure test before being put in service. Good practice provides discharge heaters with a by-pass relief valve to the suction or return line, to prevent damage by abnormal pressures if the oil lines of the heater are cut out and the steam left on.

17. GASOLINE AND KEROSENE

GASOLINE is defined (ASTM D288-47) as a refined petroleum naphtha which by its composition is suitable for use as a carburetant in internal combustion engines. Naphtha is a generic term applied to readily vaporizable hydrocarbon liquids. The term is often applied to hydrocarbon liquids used as solvents for specific purposes, such as cleaning, manufacture of rubber cement, or manufacture of paints and varnishes. For example, cleaners' naphtha or Stoddard solvent (ASTM D484-40) has a distillation range of about 300 to 400 F and a minimum flash point requirement of 100 F.

Although gasoline and kerosene are not invariable in composition and properties, they vary only within limits of quality requirements recognized by refiners and consumers of these products. The broad requirements are set forth in specifications such as those in the *Federal Standard Stock Catalog* (VV-L-791C, Section IV, Part 5) and the *ASTM Standards on Petroleum Products* (ASTM D439-40T).

Motor gasoline, for automotive use, is a mixture of hydrocarbons distilling in the range of 100 to 400 F by the standard method of test (ASTM D86-46). The hydrocarbons belong chemically to four principal classes: paraffins, olefins, naphthenes, and aromatics. A typical motor gasoline is a blend of (1) straight-run or prime-cut naphtha, i.e., the portion of natural crude oil boiling at temperatures up to 400 F; (2) reformed naphtha, i.e., the product of the same volatility obtained by thermal treatment or by catalytic dehydrogenation of the heavy straight-run naphtha; (3) cracked naphtha, i.e., the product of the same volatility obtained by thermally or catalytically decomposing gas oil and less volatile portions of the crude oil; and (4) casinghead gasoline and other light ends, i.e., the liquefiable hydrocarbons, including substantially none more volatile than isobutane, normally carried as vapor in natural gas or in stabilizer gases from cracking processes.

Compounds other than hydrocarbons occur in only very minor proportions in gasoline. Tetraethyl lead is often present, usually as an antiknock compound (see also Section 14, Art. 22), in concentrations not exceeding 3 cc per gal of motor gasoline and 4.6 cc per gal of aviation gasoline. Sulfur compounds of noncorrosive properties may be present, since sulfur compounds occur in crude oil, but their concentration in gasoline rarely represents a content of sulfur greater than 0.1% by weight. When stored for a long time, gasoline may form organic peroxides up to about 200 parts of active oxygen per million parts of gasoline, and resinous polymers, called *gum*, up to about 30 mg per 100 cc of gasoline. Many commercial gasolines contain minor concentrations of antioxidants, and some contain solvent oil to guard against the deposition of gum. Commercial gasolines on the market are usually free of suspended water, acid compounds, and other deleterious impurities.

The elementary composition of gasoline by weight is, in general, not far from 85% carbon and 15% hydrogen. The air-fuel ratio for stoichiometric requirements in the combustion of gasoline and kerosene varies between 14 and 15 lb of air per lb of fuel. Computations of the thermodynamic properties of the products of combustion of gasoline in air may be found in the *Transactions* SAE, Vol. 39, No. 4, 1936, pp. 411–424, and the attainable efficiencies for the Otto cycle may be found in the *Journal IME*, Vol. 143, 1940, pp. 289–298.

Gasoline ordinarily is graded by volatility and antiknock value, or octane number, into motor gasoline of regular and premium grades and into aviation gasoline of several antiknock grades, of which the most generally used are 91/98 and 100/130 (ASTM D910-47T). Typical characteristics of gasoline are listed in Table 12. The volatility of motor gasoline is varied seasonally, being made more volatile in winter than in summer. An ASTM pamphlet, entitled *The Significance of Tests of Petroleum Products*, includes a discussion of the characteristics of gasoline. Other typical physical properties of gasoline are: (1) volume coefficient of thermal expansion, per °F at 60 F, 0.0006 to 0.0007 (ASTM D206-36); (2) latent heat of vaporization, at 1 atm vapor pressure, 130 Btu per lb; (3) specific heat of vapor at 1 atm pressure and 100 F, 0.4 Btu per (lb × °F); (4) thermal conductivity of liquid for 1-ft depth, 0.08 Btu per (hr × sq ft × °F); (5) electrical restivity of water-free liquid, 2×10^{16} ohm per cu cm; (6) dielectric constant at 68 F, 2.2 referred to air as unity; (7) surface tension against air at 68 F, 21 dynes per cm for aviation grade, 25 dynes per cm for motor gasoline.

Table 12. Characteristics of Typical Gasolines

Use:	Summer Automotive		Aviation
Grade:	Regular	Premium	100/130
Distillation (ASTM, D86-46)			
Initial boiling point, °F	101	102	104
10% evaporated at °F	140	140	140
50% evaporated at °F	230	225	203
90% evaporated at °F	338	320	262
Final boiling point, °F	400	356	320
Vapor pressure, psi at 100 F	7.8	7.8	6.8
Motor octane number (ASTM D357-47)	74	78	100

EXPLOSIVE MIXTURES OF GASOLINE. Mixtures of air and gasoline vapor containing from 1.3 to 6.0% of gasoline vapor by volume are explosive (Properties of Flammable Liquids, Gases and Solids, *Ind. Eng. Chem.*, Vol. 32, No. 6, June 1940).

KEROSENE is defined (ASTM D288-47) as a refined petroleum distillate having a flash point not below 73 F as determined by the Abel tester (which is approximately equivalent to 73 F as determined by the Tag closed tester, ASTM standard method D56) and suitable as an illuminant when burned in a wick lamp.

Typical kerosenes have the following ranges of properties: distillation, 320 to 550 F; API gravity, 40 to 48 degrees; Tag flash 110 to 130 F; Kinematic viscosity at 100 F, 1.4 to 2.0. Other properties are listed in Table 13.

Even in areas where electrification has made kerosene lamps obsolete, kerosene has continued to be an important fuel for heating purposes, being consumed in wick-type and various vaporizing-type burners in stoves, heaters, and furnaces. In such cases, the product is frequently known as range oil. The specifications for No. 1 fuel oil also include produces of the kerosene type.

SPECIFIC VOLUMES of gasoline and kerosene completely vaporized at 1 atm pressure and at 60 F are listed in Table 13, together with some typical values of other properties which normally vary with the densities of these products.

Table 13. Specific Volume and Other Properties of Gasoline and Kerosene

	Gasoline				Kerosene		
For the Liquid							
Gravity, °API	55	60	65	70	40	45	50
Specific gravity, 60°/60 F	0.7587	0.7389	0.7201	0.7022	0.8251	0.8017	0.7796
Pounds per gallon	6.316	6.151	5.994	5.845	6.870	6.675	6.490
Specific heat at 100 F, Btu per (lb × °F)	0.500	0.515	0.530	0.545	0.475	0.495	0.505
Viscosity, centipoises * at 68 F	....	0.5	....	0.5	1.4	1.6	2.0
Net heating value, at constant pressure, Btu per lb	18,500	18,700	18,900	19,100	18,700	18,900	19,100
For the Vapor							
Specific vol., cu ft per lb, at 60 F	...	3.45	.	3.60	.....	3.05	...

* Centipoise is the cgs unit of viscosity and is equal to kinematic viscosity in centistokes × the density of the liquid.

18. OTHER LIQUID FUELS

ALCOHOL. The alcohol most frequently considered as fuel for internal combustion engines is ethyl alcohol, sometimes called grain alcohol. Its modern chemical name is ethanol. Two other alcohols that have been used as fuel are methanol and isopropanol, which are also called methyl alcohol and isopropyl alcohol, respectively.

Ethanol has the chemical formula, C_2H_5OH. When sold for industrial use, it is mixed with a minor proportion of a denaturant to make it unfit for human consumption, since alcohol for beverage is subject to special taxation. One denaturant is methanol, formerly called wood alcohol, which has the chemical formula, CH_3OH. The Department of Internal Revenue prescribes various approved formulas for denatured alcohol. These are changed from time to time, and the Department should therefore be consulted for an up-to-date list.

Table 14. Specific Gravity of Ethanol at 60 F Compared with Water at 60 F

(Based on U. S. Department of Agriculture Data and Smithsonian Tables)

Specific Gravity	% Alcohol Weight	% Alcohol Volume	Specific Gravity	% Alcohol Weight	% Alcohol Volume	Specific Gravity	% Alcohol Weight	% Alcohol Volume
.8339	85.8	90.0	.818	91.9	94.5	.8079	...	97.0
.828	88.1	91.8	.8161	92.6	95.0	.8035	...	98.0
.824	89.6	92.9	.814	93.3	95.5	.7989	...	99.0
.8199	91.1	94.0	.8121	94.0	96.0	.7939	100	100

The gross (higher) heating value of pure ethanol is 12,770 Btu per lb, and its net (lower) heating value at constant pressure is 11,585 Btu per lb. The products of its complete combustion in oxygen are carbon dioxide and water. For aqueous alcohol the net calorific value is lower, owing in part to the inertness of water and to the absorption of its latent heat of vaporization, which is 1050 Btu per lb at 77 F. For example, *Bulletin* 43, Bureau of Mines, reports a value of 10,440 Btu per lb for the net heating value of ethanol containing 8.9% water by weight. The latent heat of vaporization of pure ethanol is 367 Btu per lb. For specific gravity of ethanol compared with water, see Table 14. Owing to its relatively low heating value, ethanol is rarely used alone as fuel. It is sometimes used in about 20% concentration as a supplement in gasoline, particularly in countries lacking petroleum resources. Such blended gasolines generally contain about 15% benzol also, in order to make the blend less likely to be separated into two phases in the presence of water. Aqueous alcohols may be injected as auxiliary fuel in the intake manifold of Otto cycle engines being operated at full power output. The relatively high latent heat of vaporization of the alcohols, which serves to cool the fuel-air mixture, and their relatively high antiknock value, especially in rich fuel-air mixtures, permit higher power output than the knocking tendency of the main fuel, if used alone, would permit.

A comprehensive bibliography on alcohol as fuel in internal combustion engines is published by the Coordinating Research Council, Inc., 30 Rockefeller Plaza, New York 20, N. Y. The thermodynamic properties of the working fluid in the Otto cycle with ethanol as fuel are given in the *Ind. Eng. Chem.*, Vol. 34, 1942, pp. 575–580, 673.

SHALE OIL is obtained by the destructive distillation of oil shale. This is a compact rock of sedimentary origin, with an ash content of more than 33%. It contains organic matter yielding oil when destructively distilled, but not appreciably when extracted with the ordinary solvents for petroleum. Shale oil is not yet important as a commercial product.

COAL TAR AND TAR OIL. Coal tar is a product of the destructive distillation of bituminous coal carried out at high temperature. A typical composition of tar is: C, 86.7%; H, 6.0%; N, 0.1%; S, 0.8%; O, 3.1%; ash, 0.1%; water, 3.2%. The black color is due to free carbon in suspension (about 4%). The high heating value equals 16,340 Btu per lb. The viscosity is about 140 Saybolt sec at 140 F. Coal tar weighs 9.5 lb per gal. This analysis shows tar to have almost the same chemical composition as the combustible matter of the coal from which it is made. Tar is used principally in reheating furnaces and open-hearth furnaces of steel works. It is not easily obtainable in the open market. Since it is a by-product, its price is more or less arbitrary.

LIQUEFIED PETROLEUM GASES (LPG) are mixtures of hydrocarbons liquefied under pressure for efficient transportation, storage, and use. They are generally composed of ethylene, propane, propylene, butane, isobutane, and butylene. Commercially, they are classed as propane, propane-butane mixtures, and butane. They are odorless, colorless, and nontoxic. They should always be odorized so that leaks may be detected long before the lower explosion limit of the gas-air mixture is reached. These gases are heavier than air and seek ground level. Leaks will result if dangerous accumulations collect and are not dispersed by wind or other means. Liquefied petroleum gases are derived in most part from gases produced in petroleum refining operations and also in substantial quantities from natural gas. The sulfur content is generally low, particularly in gases produced from natural gas. Butane is not used as extensively as propane for two reasons: (1) Its relatively high boiling point makes it necessary to add external heat when the temperature drops below 32 F; and (2) butane has high economic value in the manufacture of synthetic rubber and for high octane gasolines. The physical properties of propane and butane are given in Table 15.

Table 15. Properties of Commercial Propane and Butane

Property	Propane	Butane
Chemical composition	C_3H_8	C_4H_{10}
Boiling point, °F	−43.8	+31.1
Specific gravity, liquid, at 60/60 F	0.508	0.584
Specific gravity, vapor, at 60 F, 14 psia (air = 1)	1.522	2.006
Specific heat, vapor, at 14 psia, Btu/lb, c_p	0.390	0.396
Specific heat, vapor, at 14 psia, Btu/lb, c_v	0.346	0.363
Heat of vaporization, at 14 psia, Btu/lb	183	166
Weight, lb/gal	4.23	4.86
Vapor produced, cu ft/gal	36.5	31.8
Heat content, gross Btu/lb	21,690	21,340
Explosion limits, % in air (lower)	2.0–2.4	1.5–1.9
Explosion limits, % in air (upper)	7.0–9.5	5.7–8.5
Air required for combustion, lb/lb of fuel	15.6	15.4

The distribution and uses of LP gases have expanded very rapidly. The uses include domestic water heating, cooking, refrigerating, and space heating. In small communities the gases are distributed from a central point in place of manufactured gas. They are also used in the gas industry for enriching manufactured gas and as a stand-by supply. Commercially and industrially, they are used as a fuel for internal combustion engines and for any of the various applications where manufactured or natural gas might be used.

LP gases are stored in portable and semiportable cylinders containing up to 100 lb of liquid, and in above- or below-ground storage tanks with capacities up to 30,000 gal. All storage installations should be made in accordance with the requirements of local authorities and the standards outlined in *Pamphlet* 58, National Board of Fire Underwriters. Cylinders should be constructed to meet the requirements of the Interstate Commerce Commission. Storage tanks should be constructed and tested in accordance with the requirements of the ASME and ASME-API codes for unfired pressure vessels.

Vegetable Oils as Internal Combustion Engine Fuel. Tests of internal combustion engines using palm oil or cotton seed oil showed the following fuel consumption: 265 grams per cheval-heure (1 cheval-heure = 0.9863 hp-hr) in a 4-cycle, 25-hp engine; 320 grams per cheval-heure in a 2-cycle, 168 hp engine; 285 grams per cheval-heure in a 2-cycle, 33-hp engine. The calorific value of palm oil and cotton seed oil averaged 9350 cal per kg. Thermal efficiencies were 25.9, 21.45, and 24.1%.

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GASEOUS FUELS

By L. L. Newman

19. CHARACTERISTICS AND PROPERTIES OF FUEL GASES

Advantages. Gaseous fuels commonly used in industry, whether distributed by public utilities or produced in isolated plants, are composed of one or more *simple* gases in varying proportions. They can be burned in furnaces or other appliances under conditions in which the supply can be varied almost instantaneously between wide limits by the manual or automatic manipulation of a valve. Because complete combustion is obtained with low excess air, flue losses are low and operation is smokeless. The atmosphere in the furnace may be maintained *oxidizing* or *reducing* with ease and with little reduction in efficiency. No storage facilities are needed on the premises of the consumer if the gas is furnished by a public utility.

Table 1 gives the combustion constants for various simple gases present in commercial gaseous fuels. (See also Table 1, p. 2-04.) The composition of typical commercial gases is given in Table 2.

GAS ANALYSIS. In ordinary methods of gas analysis, the gas passes through a series of absorbents, each of which removes a definite component or group of components. The remainder of the gas is subjected to combustion with oxygen or air. Measurements are made on a volume basis, and the results are expressed in percentages, on a dry basis, even though the actual sample may have been saturated with water vapor. Most of the equipment available for absorption methods of analysis provides for determining CO_2, illuminants, O_2, CO, H_2, CH_4, C_2H_6, and N_2 in the order listed. CO_2 is absorbed in a sodium or potassium hydroxide solution; illuminants in sulfuric acid, bromine water, or cuprous *beta*-naphthol; O_2 in alkaline pyrogallate or chromous chloride; CO in acid or alkaline cuprous chloride, cuprous sulfate *beta*-naphthol; H_2, CH_4, and C_2H_6 by combustion methods; and N_2 by difference (Ref. 1).

Other methods of analysis include distillation methods in which the sample of gas is liquefied and distilled or fractionated in suitable apparatus (Ref. 2), the use of the mass spectrometer (Ref. 3), and infrared spectroscopy (Ref. 4).

HEATING VALUE. The *total heating value* (or gross heating value, or higher heating value, hhv) of a gas is the number of Btu produced by combustion at constant pressure of 1 cu ft of the gas, measured at 60 F and 30 in. Hg, with air of the same pressure and temperature as the gas, when the products of combustion are cooled to the initial temperature of gas and air and *when the water formed by combustion is condensed to the liquid state.*

The *net heating value* (or lower heating value, lhv) is the number of Btu produced by combustion at constant pressure of 1 cu ft of the gas, measured at 60 F and 30 in. Hg, with air of the same pressure and temperature as the gas, when the products of combustion are cooled to the initial temperature of gas and air and *when the water formed in combustion remains in the vapor state.*

Table 1. Combus-

(Reprinted by permission from *Gaseous Fuels*, L. Schnidman (Editor),

No.	Substance	Formula	Molecular Weight*	Lb per Cu Ft †	Cu Ft per Lb †	Sp Gr, Air = 1.000 †	Heat of Combustion: Btu per Cu Ft ‡ Gross	Btu per Cu Ft ‡ Net §	Sat. Gas, Btu/cu ft Gross	Btu per Lb ‡ Gross	Btu per Lb ‡ Net §
1.	Carbon	C	12.01							14,093	14,093
2.	Hydrogen	H_2	2.016	.005327	187.723	0.06959	325.0	275.0	319.4	61,100	51,623
3.	Oxygen	O_2	32.000	.08461	11.819	1.1053			..	..	....
4.	Nitrogen (atm)	N_2	28.016	.07439¶	13.443¶	0.9718¶			.	. . .	....
5.	Carbon monoxide	CO	28.01	.07404	13.506	0.9672	321.8	321.8	316.2	4,347	4,347
6.	Carbon dioxide	CO_2	44.01	.1170	8.548	1.5282			...	....	....
	PARAFFIN SERIES, C_nH_{2n+2}										
7.	Methane	CH_4	16.041	.04243	23.565	0.5543	1013.2	913.1	995.7	23,879	21,520
8.	Ethane	C_2H_6	30.067	.08029¶	12.455¶	1.04882¶	1792	1641	1761	22,320	20,432
9.	Propane	C_3H_8	44.092	.1196¶	8.365¶	1.5617¶	2590	2385	2545.2	21,661	19,944
10.	*n*-Butane	C_4H_{10}	58.118	.1582¶	6.321¶	2.06654¶	3370	3113	3311.7	21,308	19,680
11.	Isobutane	C_4H_{10}	58.118	.1582¶	6.321¶	2.06654¶	3363	3105	3304.8	21,257	19,629
12.	*n*-Pentane	C_5H_{12}	72.144	.1904¶	5.252¶	2.4872¶	4016	3709	3946.5	21,091	19,517
13.	Isopentane	C_5H_{12}	72.144	.1904¶	5.252¶	2.4872¶	4008	3716	3938.7	21,052	19,478
14.	Neopentane	C_5H_{12}	72.144	.1904¶	5.252¶	2.4872¶	3993	3693	3923.9	20,970	19,396
15.	*n*-Hexane	C_6H_{14}	86.169	.2274¶	4.398¶	2.9704¶	4762	4412	4679.6	20,940	19,403
	OLEFIN SERIES, C_nH_{2n}										
16.	Ethylene	C_2H_4	28.051	.07456	13.412	0.9740	1613.8	1513.2	1585.9	21,644	20,295
17.	Propylene	C_3H_6	42.077	.1110¶	9.007¶	1.4504¶	2336	2186	2295.6	21,041	19,691
18.	*n*-Butene (Butylene)	C_4H_8	56.102	.1480¶	6.756¶	1.9336¶	3084	2885	3030.6	20,840	19,496
19.	Isobutene	C_4H_8	56.102	.1480¶	6.756¶	1.9336¶	3068	2869	3014.9	20,730	19,382
20.	*n*-Pentene	C_5H_{10}	70.128	.1852¶	5.400¶	2.4190¶	3836	3586	3769.6	20,712	19,363
	AROMATIC SERIES, C_nH_{2n-6}										
21.	Benzene	C_6H_6	78.107	.2060¶	4.852¶	2.6920¶	3751	3601	3686.1	18,210	17,480
22.	Toluene	C_7H_8	92.132	.2431¶	4.113¶	3.1760¶	4484	4284	4406.4	18,440	17,620
23.	Xylene	C_8H_{10}	106.158	.2803¶	3.567¶	3.6618¶	5230	4980	5139.5	18,650	17,760
	MISCELLANEOUS GASES										
24.	Acetylene	C_2H_2	26.036	.06971	14.344	0.9107	1499	1448	1473.1	21,500	20,776
25.	Naphthalene	$C_{10}H_8$	128.162	.3384¶	2.955¶	4.4208¶	5854**	5654**	5752.7	17,298**	16,708**
26.	Methyl alcohol	CH_3OH	32.041	.0846¶	11.820¶	1.1052¶	867.9	768.0	852.9	10,259	9,078
27.	Ethyl alcohol	C_2H_5OH	46.067	.1216¶	8.221¶	1.5890¶	1600.3	1450.5	1572.6	13,161	11,929
28.	Ammonia	NH_3	17.031	.0456¶	21.914¶	0.5961¶	441.1	365.1	433.5	9,668	8,001
29.	Sulfur	S	32.06		. . .	.			.	3,983	3,983
30.	Hydrogen sulfide	H_2S	34.076	.09109¶	10.979¶	1.1898¶	647	596	635.8	7,100	6,545
31.	Sulfur dioxide	SO_2	64.06	.1733	5.770	2.264		.		.	. .
32.	Water vapor	H_2O	18.016	.04758¶	21.017¶	0.6215¶			.	. . .	....
33.	Air	. . . .	28.9	.07655	13.063	1.0000				.	.

All gas volumes corrected to 60 F and 30 in. Hg, dry. For gases saturated with water at 60 F, 1.73% of the Btu value must be deducted.

* Calculated from atomic weights given in *J. Am. Chem. Soc.*, Feb. 1937.

† Densities calculated from values given in grams per liter at 0 C and 760 mm in the *International Critical Tables*, allowing for the known deviations from the gas laws. Where the coefficient of expansion was not available, the assumed value was taken as 0.0037 per °C. Compare this with 0.003662, which is the coefficient for a perfect gas. Where no densities were available, the volume of the mole was taken as 22.4115 liters.

‡ Converted to mean Btu per pound (1/180 of the heat per pound of water from 32 to 212 F) from data by Frederick D Rossini, National Bureau of Standards, letter of April 10, 1937, except as noted

tion Constants

American Gas Association, New York, 1948, Table 3, p. 118.)

Cu Ft per Cu Ft of Combustible						Lb per Lb of Combustible						Experimental Error in Heat of Combustion, %, + or −
Required for Combustion			Flue Products			Required for Combustion			Flue Products			
O_2	N_2	Air	CO_2	H_2O	N_2	O_2	N_2	Air	CO_2	H_2O	N_2	
.	.		. .			2.664	8.863	11.527	3.664		8.863	.012
0.5	1.882	2.382		1.0	1.882	7.937	26.407	34.344		8.937	26.407	.015
. . .	... .	. .	.	.	.			... .	..	.. ..	. . .	. .. .
0.5	1.882	2.382	1.0	.	1.882	0.571	1.900	2.471	1.571	. . ...	1.900	.045
.	.	. ...	..	.			..	..		.	.	. .
2.0	7.528	9.528	1.0	2.0	7.528	3.990	13.275	17.265	2.744	2.246	13.275	.033
3.5	13.175	16.675	2.0	3.0	13.175	3.725	12.394	16.119	2.927	1.798	12.394	.030
5.0	18.821	23.821	3.0	4.0	18.821	3.629	12.074	15.703	2.994	1.634	12.074	.023
6.5	24.467	30.967	4.0	5.0	24.467	3.579	11.908	15.487	3 029	1.550	11.908	.022
6.5	24.467	30.967	4.0	5.0	24.467	3.579	11 908	15.487	3.029	1.550	11.908	.019
8.0	30.114	38.114	5.0	6.0	30.114	3.548	11.805	15.353	3.050	1.498	11.805	.025
8.0	30.114	38.114	5.0	6.0	30.114	3.548	11.805	15.353	3.050	1.498	11.805	.071
8.0	30.114	38.114	5.0	6.0	30.114	3.548	11.805	15.353	3.050	1.498	11.805	.11
9.5	35.760	45.260	6.0	7.0	35.760	3.528	11.738	15.266	3.064	1.464	11.738	.05
3.0	11.293	14.293	2.0	2.0	11.293	3.422	11.385	14.807	3.138	1.285	11.385	.021
4.5	16.939	21.439	3.0	3.0	16.939	3.422	11.385	14.807	3.138	1.285	11.385	.031
6.0	22.585	28.585	4.0	4.0	22 585	3.422	11.385	14 807	3.138	1.285	11.385	.031
6.0	22.585	28.585	4.0	4.0	22.585	3 422	11.385	14.807	3.138	1.285	11.385	.031
7.5	28.232	35.732	5.0	5.0	28.232	3.422	11.385	14.807	3.138	1.285	11.385	.037
7.5	28.232	35.732	6.0	3.0	28.232	3.073	10.224	13.297	3.381	0.692	10.224	.12
9.0	33.878	42.878	7.0	4.0	33.878	3.126	10.401	13.527	3.344	0.782	10.401	.21
10.5	39.524	50.024	8.0	5.0	39.524	3.165	10.530	13.695	3.317	0.849	10.530	.36
2.5	9.411	11.911	2.0	1.0	9.411	3.073	10.224	13.297	3.381	0.692	10.224	.16
12.0	45.170	57.170	10.0	4.0	45.170	2.996	9.968	12.964	3.434	0.562	9.968	....
1.5	5.646	7.146	1.0	2.0	5.646	1.498	4.984	6.482	1.374	1.125	4.984	.027
3.0	11.293	14.293	2.0	3.0	11.293	2.084	6 934	9.018	1.922	1.170	6.934	.030
0.75	2.823	3.573	.	1.5	3.323	1.409	4.688	6.097	. . ..	1.587	5.511	.088
....	. ..		.	..	.	0.998	3.287	4.285	SO_2 1.998	......	3.287	.071
1.5	5.646	7.146	SO_2 1.0	1.0	5.646	1.409	4.688	6.097	SO_2 1.880	0.529	4.688	.30
...	. .. .	. .	.	. .	.	.	.	.	.. .	. .. ..	..	....
.....	...	. .			.			. .		.	.	.. ..
.. ..								.				... .

§ Deduction from gross to net heating value determined by deducting 18,919 Btu per pound-mole of water in the products of combustion. Osborne, Stimson, and Ginnings, *Mech. Eng.*, March 1935, p. 163, and Osborne, Stimson, and Fiock, National Bureau of Standards, *Research Paper* 209.

¶ Denotes that either the density or the coefficient of expansion has been assumed. Some of the materials cannot exist as gases at 60 F and 30 in. Hg pressure, in which case the values are theoretical ones given for ease of calculation of gas problems. Under the actual concentrations in which these materials are present, their partial pressure is low enough to keep them as gases.

** From *Combustion*, C. George Segeler (Editor), 3rd ed., American Gas Association, New York, 1932.

Table 2. Properties of Typical

(Reprinted by permission from *Combustion*, 3rd ed., C. George Segeler

No.	Gas	Constituents of Gas, % by Volume						
		CO_2	O_2	N_2	CO	H_2	CH_4	C_2H_6
1	Natural gas (Birmingham)			5.0	..	...	90.0	5.0
2	Natural gas (Pittsburgh)		..	0.8	.		83.4	15.8
3	Natural gas (Southern California)	0.7	.	0.5			84.0	14.8
4	Natural gas (Los Angeles)	6.5		.	.	..	77.5	16.0
5	Natural gas (Kansas City)	0.8	. ..	8.4		.	84.1	6.7
6	Reformed natural gas	1.4	0.2	2.9	9.7	46.6	37.1	. . .
7	Mixed, natural, and water gas	4.4	2.1	4.7	25.5	35.1	23.1	4.7
8	Coke oven gas	2.2	0.8	8.1	6.3	46.5	32.1	.... .
9	Coal gas (continuous verticals)	3.0	0.2	4.4	10.9	54.5	24.2	....
10	Coal gas (inclined retorts)	1.7	0.8	8.1	7.3	49.5	29.2	... .
11	Coal gas (intermittent verticals)	1.7	0.5	8.2	6.9	49.7	29.9	....
12	Coal gas (horizontal retorts)	2.4	0.75	11.35	7.35	47.95	27.15	. ...
13	Mixed coke oven and carburetted water gas	3.4	0.3	12.0	17.4	36.8	24.9	. ....
14	Mixed coal, coke-oven and carburetted water gas	1.8	1.6	13 6	9.0	42.6	28.0	......
15	Carburetted water gas	3.0	0.5	2.9	34.0	40.5	10.2	....
16	Carburetted water gas	4.3	0.7	6.5	32.0	34.0	15.5	....
17	Carburetted water gas (low gravity)	2.8	1.0	5.1	21.0	47.5	15.0	....
18	Water gas (coke)	5.4	0.7	8.3	37.0	47.3	1.3	. ...
19	Water gas (bituminous)	5.5	0.9	27.6	28.2	32.5	4.6	....
20	Oil gas (Pacific Coast)	4.7	0.3	3.6	12.7	48.6	26.3	......
21	Producer gas (buckwheat anthracite)	8.0	0.1	50.0	23.2	17.7	1.0	... . .
22	Producer gas (bituminous)	4.5	0.6	50.9	27.0	14.0	3.0	.. .
23	Producer gas (0.6 lb steam per lb of coke)	6.4		52.8	27.1	13.3	0.4	....
24	Blast-furnace gas	11.5	. .	60.0	27.5	1.0	. ..	... ...
25	Commercial butane	. .	(C_4H_{10}, 93.0)		. .	(C_3H_8, 7.0)		.. .
26	Commercial propane		(C_3H_8, 100.0)		. .	....		. . ..

This table shows the properties of typical commercial gases as served. The analyses were taken by averaging the gas composition as reported on the district in various cities. . . . The Btu value of the saturated gases would be 1.73% lower than the amounts given. Corresponding changes would have to be made all through the table to take care of saturated gases.

It might also be well to remember that analyses made by the ordinary chemical methods, even when made over water, are on the dry basis. . . .

The last two columns in the table require some additional explanation. Both relate to the theoretical flame temperature corrected for dissociation. They represent in a measure the relative usefulness of the various gases. They do not represent this measure accurately because the values hold true only for one condition, namely, 100% primary aeration.

The net heating value is less than the total heating value by the heat of vaporization, at the initial temperature of the gas and air, of the water formed in combustion of the gas.

The heating value of a gas may be determined directly by calorimetric methods or may be calculated from the analysis. For descriptions and directions for operating intermittent calorimeters, see *Circular* 48 of the National Bureau of Standards. The Thomas calorimeter, a typical recording instrument, makes a continuous record of the heating value of gases, accurate within 1% from half to maximum rating.

Table 3 may be used for rapid calculation of the heating value from the analysis. Since the illuminants are a complex mixture of several unsaturated hydrocarbons, a value is usually assigned to them on the basis of the composition of the gas obtained by analysis and the heating value obtained by calorimeter. The heat content of the simple gases is deducted from the total and the difference is assigned to the illuminants. Watson and Ceaglske (Ref. 5) described a procedure for determining the average values of the subscripts in complex mixtures of paraffin hydrocarbons (C_nH_{2n+2}) and unsaturated hydrocarbons (illuminants, C_aH_b). The heating values of these hypothetical hydrocarbon components may be calculated by means of the approximate formulas: Btu per cu ft of paraffin hydrocarbons, C_nH_{2n+2}, at 60 F and 30 in. Hg, saturated $= 745n + 258$; Btu per cu ft of unsaturated hydrocarbons, C_aH_b at 60 F and 30 in. Hg, saturated $= 459a + 132b + 135$. (See Ref. 6.)

Commercial Gases

(Editor), American Gas Association, New York, 1932, Table 53, p. 95.)

Constituents of Gas, % by Volume		Specific Gravity	Cu Ft Air Req. for Comb. of Cu Ft of Gas	Btu per Cu Ft		Products of Combustion per Cu Ft of Gas				Ultimate Percentage CO_2	Btu Net per Cu Ft of Prod. of Comb.	°F Flame Temp, No Excess Air
Illuminants												
C_2H_4	C_6H_6			Gross	Net	H_2O	CO_2	N_2	Total			
.....	....	0.60	9.41	1002	904	2.02	1.00	7.48	10.50	11.8	86.0	3565
.....	....	0.61	10.58	1129	1021	2.22	1.15	8.37	11.73	12.1	87.0	3562
....	......	0.64	10.47	1116	1009	2.20	1.14	8.28	11.62	12.1	87.0	3550
....	....	0.70	10.05	1073	971	2.10	1.16	7.94	11.20	12.7	86.7	3550
.....	..	0.63	9.13	974	879	1.95	0.98	7.30	10.23	11.9	86.0	3535
1.3	(C_3H_6, 0.8)	0.41	5.22	599	536	1.30	0.53	4.16	5.99	11.3	89.6	3615
0.2	0.2	0.61	4.43	525	477	1.01	0.64	3.55	5.20	15.3	91.7	3630
3.5	0.5	0.44	4.99	574	514	1.25	0.51	4.02	5.78	11.2	87.0	3610
1.5	1.3	0.42	4.53	532	477	1.15	0.49	3.62	5.26	11.9	90.7	3645
0.4	3.0	0.47	5.23	599	540	1.23	0.57	4.21	6.01	11.9	89.9	3660
3.0	0.1	0.41	4.64	540	482	1.21	0.45	3.75	5.41	10.7	89.0	3610
1.32	1.73	0.47	4.68	542	486	1.15	0.50	3.81	5.46	11.6	89.0	3600
3.7	1.5	0.58	4.71	545	495	1.04	0.62	3.85	5.51	13.9	90.0	3630
2.4	1.0	0.50	4.52	528	475	1.11	0.50	3.71	5.32	11.8	89.3	3640
6.1	2.8	0.63	4.60	550	508	0.87	0.76	3.66	5.29	17.2	96.2	3725
4.7	2.3	0.67	4.51	534	493	0.75	0.86	3.63	5.24	17.1	94.2	3700
5.2	2.4	0.54	4.61	549	501	0.98	0.64	3.70	5.31	14.7	94.3	3690
.	.	0.57	2.10	287	262	0.53	0.44	1.74	2.71	20.1	96.6	3670
0.4	0.3	0.70	2.01	261	239	0.47	0.41	1.86	2.74	18.0	87.2	3510
2.7	1.1	0.47	4.73	551	496	1.15	0.56	3.77	5.48	12.9	90.5	3630
.	.	0.86	1.06	143	133	0.22	0.32	1.34	1.88	19.4	70.5	3040
..	......	0.86	1.23	163	153	0.23	0.35	1.48	2.06	18.9	74.6	3175
....	...	0.88	1.00	135	128	0.17	0.34	1.32	1.82	20.5	70.3	3010
..	.....	1.02	0.68	92	92	0.02	0.39	1.14	1.54	25.5	59.5	2650
..	.....	1.95	30.47	3225	2977	4.93	3.93	24.07	32.93	14.0	90.5	3640
	..	1.52	23.82	2572	2371	4.17	3.00	18.82	25.99	13.7	91.2	3660

In practice two things are true: first, the maximum flame temperature does not occur at the point of 100% primary aeration, but usually on the gas-rich side of this figure; second, the point of maximum flame speed occurs slightly to the gas-rich side.

Even to this generalization there are certain exceptions. The presence of excess air or deficiency in the air supply will lower the maximum temperature attainable through the operation of several causes; first, the reduction in the rate of flame propagation removes the rate of energy release further from the ideal assumption of instantaneous effect; second, the presence of excess air introduces additional nitrogen which must be heated to the flame temperature without adding to the energy liberated. In the case of air deficiency, unburned combustible reduces the total amount of energy liberated.

RADIATION FROM FLAMES. In considering radiation from nonluminous flames, only the triatomic gases such as carbon dioxide and water vapor are of importance in normal industrial heating operations. Von Helmholtz (Ref. 7) found that in nonluminous flames 8.74% of the heat of combustion of carbon monoxide was radiated when this gas was burned. The corresponding figure for hydrogen was 3.63%. Thus the amount of radiant energy for the carbon dioxide resulting from the burning of carbon monoxide is approximately 2.4 times that from the water vapor resulting from the combustion of hydrogen. The wavelengths of the bands of carbon dioxide and steam in the infrared region of the spectrum do not overlap. However, the heats transmitted by radiation from carbon dioxide and steam when present together may not be added because each gas is somewhat opaque to the other. For a computation of heat transmission by radiation see Section 3, Art. 7. (See also Ref. 8.)

Luminous flames radiate 20 to 120% more energy than nonluminous flames, the radiation generally amounting to 10 to 40% of the heating value of the gas. It has been claimed that, for open-hearth steel furnaces, a heat transfer of 55 Btu per sq ft per degree difference per hour could be obtained from a luminous flame, in contrast to 10 Btu per sq ft per degree difference per hour from a nonluminous flame. The presence of tar in a producer-gas flame enables it to radiate more heat.

Table 3. Data for the Calculation of Heating Value in Btu per Cu Ft of a Gas from Its Analysis

Gross Heating Value	Percentage, by Volume									
	10	20	30	40	50	60	70	80	90	100
CO	32.2	64.4	96.5	128.7	160.9	193.1	225.3	257.4	289.6	321.8
H_2	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5	325.0
CH_4	101.3	202.6	304.0	405.3	506.6	607.9	709.2	810.6	911.9	1013
C_2H_4*	161.4	322.8	484.1	645.5	806.9	968.3	1130	1291	1452	1614
C_2H_6	179.2	358.4	537.6	716.8	896.0	1075	1254	1434	1613	1792
C_3H_6*	233.6	467.2	700.8	934.4	1168	1402	1635	1869	2102	2336
C_3H_8	259.0	518.0	777.0	1036	1295	1554	1813	2072	2331	2590
C_4H_{10}	337.0	674.0	1011	1348	1685	2022	2359	2696	3033	3370
Net Heating Value	**Percentage, by Volume**									
	10	20	30	40	50	60	70	80	90	100
CO	32.2	64.4	96.5	128.7	160.9	193.1	225 3	257.4	289.6	321.8
H_2	27.5	55 0	82 5	110.0	137 5	165 0	192 5	220 0	247.5	275.0
CH_4	91.3	182.6	273.9	365.2	456.6	547.9	639.2	730.5	821.8	913.1
C_2H_4*	151.3	302.6	454.0	605.3	756.6	907 9	1059	1211	1362	1513
C_2H_6	164.1	328.2	492.3	656.4	820.5	984.6	1149	1313	1477	1641
C_3H_6*	218.6	437.2	655.8	874.4	1093	1312	1530	1749	1967	2186
C_3H_8	238.5	477.0	715.5	954.0	1193	1431	1670	1908	2147	2385
C_4H_{10}	311.3	622.6	933.9	1245	1557	1868	2179	2490	2802	3113

Example of a Heating-value Calculation

Analysis, %		Gross †	Net †
CO	24.8	64.4	64.4
		12.87	12.87
		2 574	2.574
H_2	15.0	32.5	27.5
		16.25	13.75
CH_4	0.9	9.119	8.218
		137.713	129.312

* The values given for C_2H_4 may be used for the illuminants present in producer gas, and the values for C_3H_6 for illuminants in coke-oven gas, carburetted-water gas, and oil gas (see p. 2-04 for more precise method of calculation).

† Note that all values are found from table by moving the decimal point.

20. GAS INFLAMMABILITY

IGNITION TEMPERATURES. The ignition temperature must be reached at some point in a combustible mixture before the whole mixture will burn. The ignition temperature is not a fixed physical value that can be tested, like specific gravity. It represents the temperature at which the heat loss by conduction, radiation, etc., is more than counterbalanced by the rate at which it is developed by the combustion reaction, thus becoming self-sustaining. The ignition temperatures given in Table 4 accordingly are not definite absolute properties of the gases but useful approximations. They can be used only in a relative sense and may even be misleading unless complete details are given of the procedures by which the results were obtained. The results obtained depend upon and are affected by a number of variables, the most important of which are the percentage of combustible in the mixture, the oxygen concentration, the "lag" or time required at a given temperature to cause ignition, the size, composition, and dimensions in which the tests are made, the pressure at which the mixture is confined at the time of ignition, and the presence of catalysts and impurities in the mixture.

LIMITS OF INFLAMMABILITY. Gaseous mixtures are inflammable in air only between two extreme limits. Table 4 gives these limits for simple gases and a number of typical commercial gases. No differentiation can be made between explosive limits and inflammability limits. Industrial safety rules require that values for the limits of inflam-

Table 4. Limits of Inflammability of Gases and Vapors in Air and Minimum Ignition Temperatures and Flash Points of Combustible Liquids, Gases, and Vapors

(Adapted by permission from *Gaseous Fuels*, L. Schnidman (Editor), American Gas Association, New York, 1948, Appendix Tables 8 and 9, pp. 353–356, compiled by G. W. Jones, U. S. Bureau of Mines, 1946.)

1	2	3	4	5	6	7	8	9	10	11
Name	Formula	Limits of Inflammability, % by Volume		Combustibles in Air, % by Volume. Mixture for Theoretical Complete Combustion	Ratio of Lower Limit PCC,* Column 3 ÷ 5	Ratio of Upper Limit PCC,* Column 4 ÷ 5	Ignition Temperatures		Flash Point	
		Lower	Upper				°F	°C	°F	°C
Hydrogen	H_2	4.00	74.20	29.50	.14	2.52	1065	574	Gas	Gas
Carbon monoxide	CO	12.50	74.20	29.50	.42	2.52	1128	609	Gas	Gas
Methane	CH_4	5.00	15.00	9.47	.53	1.58	1170	632	Gas	Gas
Ethane	C_2H_6	3.10	12.45	5.64	.55	2.21	882	472	Gas	Gas
Propane	C_3H_8	2.10	10.10	4.02	.52	2.51	898	481	Gas	Gas
Butane	C_4H_{10}	1.86	8.41	3.12	.60	2.70	826	441	Gas	Gas
Isobutane	C_4H_{10}	1.80	8.44	3.12	.58	2.71	1010	543	Gas	Gas
Pentane	C_5H_{12}	1.40	7.80	2.55	.55	3.06	527	275	−40	−40
Isopentane	C_5H_{12}	1.32	...	2.55	.52	...	...	...	...	...
Hexane	C_6H_{14}	1.25	6.90	2.16	.58	3.19	478	248	−15	−26
Ethylene	C_2H_4	2.75	28.60	6.52	.42	4.39	914	490	Gas	Gas
Propylene	C_3H_6	2.00	11.10	4.44	.45	2.50	856	456	Gas	Gas
Butylene	C_4H_8	1.98	9.65	3.37	.59	2.86	829	443	Gas	Gas
Amylene	C_5H_{10}	1.65	7.70	2.72	.61	2.84	523	273	...	...
Benzene	C_6H_6	1.35	6.75	2.72	.50	2.49	1078	580	12	−11
Toluene	C_7H_8	1.27	6.75 †	2.27	.56	2.97	1026	552	40	4
o-Xylene	C_8H_{10}	1.00	6.00 †	1.95	.51	3.08	925	496	63	17
Naphthalene	$C_{10}H_8$	0.90 †		1.71	.53	...	1038	559	176	80
Acetylene	C_2H_2	2.50	80.00	7.72	.32	10.36	581	305	Gas	Gas
Methyl alcohol	CH_4O	6.72	36.50 †	12.24	.55	2.98	878	470	52	11
Ethyl alcohol	C_2H_6O	3.28	18.95 †	6.52	.50	2.91	738	392	54	12
Ammonia	NH_3	15.50	26.60	21.82	.71	1.22	1204	651	Gas	Gas
Cyanogen	C_2N_2	6.60	42.60	9.47	.70	4.50	1562	850	Gas	Gas
Hydrogen sulfide	H_2S	4.30	45.50	12.24	.35	3.72	558	292	Gas	Gas
Carbon disulfide	CS_2	1.25	50.00	6.52	.19	7.67	246	120	−22	−30
Carbon oxysulfide	COS	11.90	28.50	12.24	.97	2.33			...	...
Ethyl mercaptan	C_2H_6S	2.80	18.20	4.44	.63	4.10	570	299	...	...
Blast-furnace gas		35.00	73.50	...	...	...				...
Coal gas		6.50	36.00	...	...	...	...	...	...	...
Coal gas		5.30	33.00	...		...	...	...	...	...
Natural gas		4.30	13.50	...		...	...	...	...	...
Natural gas		4.90	15.00	...	...	...	...	...	...	...
Oil gas		4.75	32.50	...		...	...	...		...
Producer gas		20.70	73.70	...		...	...	...	...	...
Water gas		6.00	70.00	...	...		...	...	...	...
Gasoline, regular		1.40	7.50	...	...	...	536	280	−47	−44
Gasoline, 73 octane		1.50	7.40	...	...	...	570	299	...	...
Gasoline, 92 octane		1.50	7.60		...		734	390	...	...
Gasoline, 100 octane		1.45	7.50	...	...	...	804	429	...	...
Naphtha		1.10	6.00	...	...	...	450–531	232–277	+20–110	−7–+43
Kerosene		...	...	...	...	...	491	255	100–165	38–74

* PCC is percentage combustible in air. Mixture for complete combustion.

† Determinations made at elevated temperatures.

mability of gases and vapors in air, obtained in apparatus giving the widest limits only, be used.

Inflammability of Simple Gas Mixtures. The limits of inflammability of simple mixtures may be calculated from a knowledge of the limits of each constituent by means of LeChâtelier's mixture law, expressed by the equation

$$L = \frac{100}{\frac{P_1}{N_1} + \frac{P_2}{N_2} + \frac{P_3}{N_3} + \frac{P_4}{N_4} + \text{etc.}}$$

where P_1, P_2, P_3, P_4, etc., are the proportions of each combustible gas present in the mixture, free from air and inerts, so that $P_1 + P_2 + P_3 + P_4$, etc. = 100; N_1, N_2, N_3, N_4, etc., are the lower limits of inflammability of each combustible in air; and L is the lower limit of inflammability of the mixture. The same method can be applied for the upper limit.

This rule does not hold strictly for hydrogen-ethylene-air mixtures or for mixtures containing carbon disulfide. The rule does not hold for methane-dichlorethylene-air mixtures and is only approximately correct for mixtures of methyl and ethyl chlorides. It is therefore apparent that the mixture rule, useful when its application has been proved, cannot be applied indiscriminately but must first be proved for the gases being investigated.

EXAMPLE. As an example of the application of this law, we may take a natural gas of the following composition (based on the material in Ref. 9).

Hydrocarbon	Percentage by Volume	Lower Limit
Methane	80.0	5.00
Ethane	15.0	3.10
Propane	4.0	2.10
Butane	1.0	1.86

$$\text{Lower limit } L = \frac{100}{\frac{80.0}{5.00} + \frac{15.0}{3.10} + \frac{4.0}{2.10} + \frac{1.0}{1.86}} = 4.28$$

Limits of inflammability of complex mixtures, which may have a considerable inert content, may be calculated by the method given by G. W. Jones (Ref. 10), in which the inerts are combined with the combustible components and the inflammable limits of these combinations are used in the "mixture law" in place of the limits of the pure gases.

EXAMPLE. In a producer gas of the composition given in Table 5, the inert CO_2 and N_2 may be apportioned with the different combustibles in any one of several ways, one of which is shown in column 3, Table 5. Thus, the 6.2% CO_2 is combined with the 12.4% H_2, making a total of 18.6%; the 53.4% N_2 is combined with the 27.3% CO, making a total of 80.7%. There being no other inert gases, the 0.7% CH_4 is taken alone. Next, the ratio of inert to combustible is found for each of the

Table 5. The Calculation of Inflammable Limits

(Reprinted by permission from *Gaseous Fuels*, L. Schmidman (Editor), American Gas Association, New York, 1948, Table 2, p. 236.)

1	2	3	4	5	6	
Gas Analysis		Combinations Chosen	Total, %	Ratio Inert/ Combustible	Explosive Limits (Fig. 1)	
Gas	Composition, %				Lower, %	Upper, %
H_2	12.4	$12.4H_2 + 6.2CO_2$	18.6	0.50	6.0	70.5
CO	27.3	$27.3CO + 53.4N_2$	80.7	1.96	39.8	73.0
CH_4	0.7	$0.7CH_4$	0.7	0.00	5.0	14.0
CO_2	6.2					
O_2	0.0					
N_2	53.4					
Total	100.0		100.0			

$$\text{Lower limit} = \frac{100}{\frac{18.6}{6.0} + \frac{80.7}{39.8} + \frac{0.7}{5.0}} = 19.0$$

$$\text{Upper limit} = \frac{100}{\frac{18.6}{70.5} + \frac{80.7}{73.0} + \frac{0.7}{14.0}} = 70.5$$

groups, as shown in column 5, and the explosive limits for each one is obtained from Fig. 1 or from Table 4. Le Châtelier's mixture law is now applied, with the data of Table 5, and the calculated limits are found to be 19.0% and 70.5%, as compared to the figures of 20.7% and 73.7% determined experimentally.

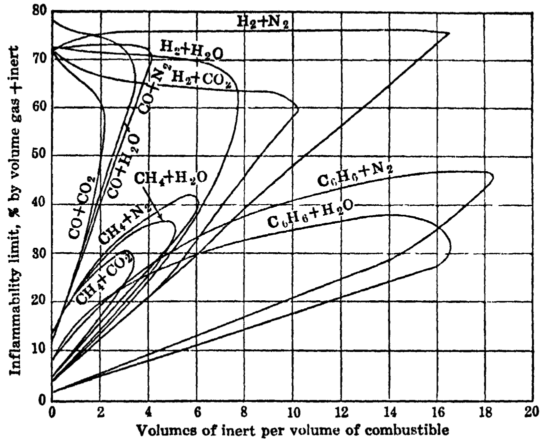


FIG. 1. Inflammable limits for combustible gases diluted with nitrogen, carbon dioxide, and water vapor. (Reprinted by permission from *Gaseous Fuels*, L. Schnidman, Editor, American Gas Association, New York, 1948, Chart 1, p. 236)

21. GAS-FLAME VELOCITY

FLAME VELOCITY, or the rate of flame propagation of gases, is an important factor in the design of gas burners. The flame velocities of individual gases vary from each other over a wide range. Each gas attains its maximum flame velocity with a certain percentage of air. Beyond this point, velocity decreases until, at the lower limit of inflammability, it reaches zero. Similarly, the flame velocity of a gas in too little air reaches zero theoretically at the upper limit of inflammability. The maximum velocity mixtures do not coincide with perfect air-gas mixtures and are generally on the gas-rich side. Flame velocity is not a characteristic of the gas itself but a property of the system as a whole, including the apparatus; and the identity of the gas can be considered as only one of several factors which, taken together, determine the numerical value in a given case.

The flame velocity of gas mixtures cannot be calculated indiscriminately by means of Le Châtelier's mixture law. The simplest way to approximate the results for gases in which the principal combustible constituents are carbon monoxide, hydrogen, and methane is by reference to a Gibbs triangular coordinate chart prepared by Fritz Schuster (Ref. 11) on the basis of measurements by Bunte and Litterscheidt. This chart is shown in Fig. 2 as presented by W. Gumz (Ref. 12). The broken lines in Fig. 2 give the percentage of the gas in the gas-air mixture. N_2 and CO_2 in the lower the value of the maximum flame velocity in accordance with the equation

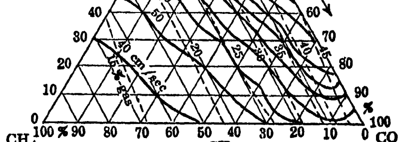


FIG. 2. Maximum flame velocity for gas mixtures containing hydrogen, carbon monoxide, and methane. (Reprinted by permission from *Kurzes Handbuch der Brennstoff und Feuerungstechnik*, by W. Gumz, p. 136, Springer-Verlag, Berlin, 1942)

$$\left[1 - \frac{N_2\,(\%) + 1.67CO_2\,(\%)}{100}\right]$$

where v_0 = velocity from Fig. 2 and v = velocity of the gas, including CO_2 and N_2.

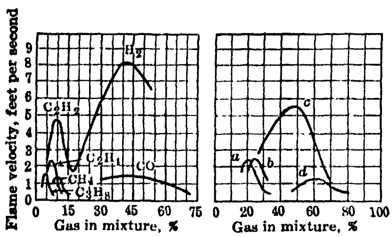
Example of Use of the Chart. Producer-gas analysis: CO_2, 7.4%; C_2H_4, 1.0%; CO, 22.5%; H_2, 12.5%; CH_4, 2.1%; N_2, 54.5%; the sum of C_2H_4, CO, H_2, and CH_4 is 38.1. Because of the relatively small amount of C_2H_4, consider $C_2H_4 + CH_4$ as CH_4 for calculating purposes; then 3.1/38.1 = 8.1% CH_4, 22.5/38.1 = 59.1% CO, and 12.5/38.1 = 32.8% H_2. These coordinates intersect at a point which is on the 100 cm/sec line.

$$v = 100\left[1 - \frac{54.5 + (1.67 \times 7.4)}{100}\right] \quad 33 \text{ cm/s}$$

$$1 \text{ cm} = 0.0328 \text{ ft}$$

$$33 \times 0.0328 = 1.1 \text{ ft/sec}$$

Figure 3 gives flame velocities of various gas-air mixtures. Figures 2 and 3 apply to streamline flow at room temperature (68 F) and atmospheric pressure. The effect of increased temperature at atmospheric pressure is shown in Figs. 4 and 5, drawn for the expression $V = BT^2$, where V = flame speed in feet per second, T = °F absolute, B = proportionality constant depending on the type of gas and having values of 27.6×10^{-6} for H_2, 4.66×10^{-6} for CO, 3.34×10^{-6} for CH_4, 5.98×10^{-6} for C_2H_4, 3.59×10^{-6} for natural gas, 7.94×10^{-6} for coke-oven gas, 7.75×10^{-6} for carburetted water gas, 14.5×10^{-6} for blue gas, and 3.08×10^{-6} for producer gas.



Gas	a	b	c	d
Kind	Coke Oven	Carb. Water Gas	Blue Gas	Producer Gas
CO_2	1 6	4.5	0.2	4.4
Ill	3.6	2.4		
O_2	1.0	0 2	0.4	
CO	5.5	20 8	47.0	29.1
H_2	54.5	51.8	50 5	10.2
CH_4	27.2	14.9		
N_2	6.6	5.4	1.9	56.3
Total	100 0	100.0	100.0	100.0

Fig. 3. Flame propagation velocities of various gas-air mixtures. (Data on simple gases in left-hand chart from Problems of Stationary Flames, by Francis A. Smith, *Chemical Reviews*, Vol. 20, p. 400, 1937. Data on industrial gases in right-hand chart from *Handbuch der Gasindustrie*, by H. Bruckner, Vol 6, p. 125)

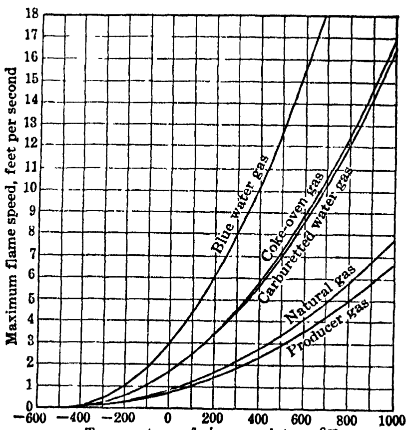


Fig. 4. Effect of temperature on flame velocity $V = BT^2$ where V = flame speed, feet per second, T = °F abs, B = proportionality constant. (Reprinted by permission from *Combustion*, 3rd Ed., C. George Segeler, Editor, American Gas Association, 1932, Chart 30, p. 94)

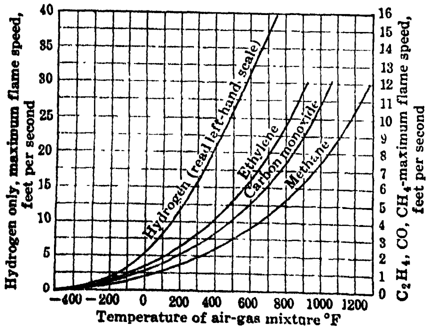


Fig. 5. Effect of temperature on flame velocity. (Reprinted by permission from *Combustion*, 3rd Ed., C. George Segeler, Editor, American Gas Association, 1932, Chart 31, p. 94)

22. GAS-FLAME TEMPERATURE CALCULATION

THEORETICAL FLAME TEMPERATURE. The following information is based in part on material in *Gaseous Fuels*, edited by L. Schnidman (American Gas Association, New York, 1948). A very useful measurement for explaining many of the results of gas burning is the flame temperature of the gas. This is a purely theoretical figure, inasmuch as a number of factors conspire, in practice, to prevent attaining the calculated temperatures. Calculated values can be made to approximate closely the measured flame temperatures obtained when short flames burning with nearly perfect mixtures are studied. Discrepancies are brought about by (1) radiation losses, (2) convection losses, and (3) conduction losses. In addition, (4) calculations of theoretical temperature assume that all energy in the fuel will be instantly available on combustion. Since the combustion process takes time, flame temperatures are actually lower than theoretical computations indicate. Two gases having equal heating value and specific gravity may burn with quite different flames. If the flames are to be identical, the gases must possess equal rates of flame propagation. The concept of *specific flame intensity* has been introduced to evaluate these differences between flames. Further discrepancies between calculated and measured temperatures are caused by (5) effect of excess air, (6) objects placed in the flame (tend to lower flame temperature), and (7) dissociation of CO_2 and H_2O.

Methods of calculating theoretical flame temperature involve the plotting and construction of a combustion chart. Then a method of trial using the following equation can readily be applied:

$$H = a[R_{CO_2}x + {}^3/_2 R_{O_2}(1 - x) + 321.8(1 - x)] + b[R_{H_2O}y + R_{H_2}(1 - y) + {}^1/_2 R_{O_2}(1 - y) + 275.0(1 - y)] + R_{O_2}(c + d)$$

where H the net Btu value.
R heat content of a cubic foot of the various gases above 60 F indicated by the subscript (R_{O_2} stands for nitrogen, carbon monoxide, or oxygen, since their heat contents are the same).
a cubic feet of CO_2 in the flue gases per cubic foot of gas burned.
b cubic feet of H_2O in the flue gases per cubic foot of gas burned.
c cubic feet of O_2 in the flue gases per cubic foot of gas burned.
d cubic feet of N_2 in the flue gases per cubic foot of gas burned.
$1 - x$ fraction of CO_2 dissociated.
$1 - y$ = fraction of H_2O dissociated.

Before a combustion chart can be plotted and constructed, it is necessary to determine the amount of moisture present as a result of incomplete decomposition of steam or scrubbing with water. Clean, cold gas may be assumed to be saturated and carries, for example, 1.73% moisture at 60 F. The moisture in hot, raw producer gas can be computed from Fig. 6. Data for the construction of the combustion chart are given in Table 1.

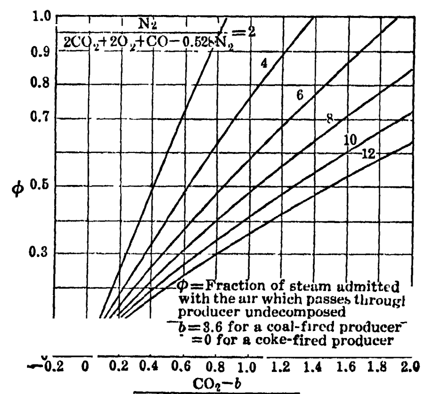


FIG. 6. Steam decomposition in gas producers. (Reprinted by permission from *Principles of Chemical Engineering*, by Walker, Lewis, McAdams, and Gilliland, McGraw-Hill, New York, 1937)

EXAMPLE. Calculation of the theoretical flame temperature of producer gas from coke. (See also Table 6 for clarification of calculation method.)

Composition of producer gas: CO_2, 3.56%; CO, 30.96%; H_2, 9.30%; CH_4, 0.70%; N_2, 55.43%; O_2, 0.05%. For Fig. 6, b = 0 for coke.

Evaluating the abscissa and parameter of Fig. 6:

$$\frac{CO_2 - b}{2CO_2 + 2O_2 + CO - 0.528N_2} = \frac{3.56 - 0}{2 \times 3.56 + 2 \times 0.05 + 30.96 - 0.528 \times 55.43} = \frac{3.56}{8.91} = 0.40$$

$$\frac{N_2}{2CO_2 + 2O_2 + CO - 0.528N_2} = \frac{55.43}{8.91} \quad 6.22$$

Table 6. Combustion Table—Producer Gas from Coke

Constituent	Mixture before Combustion: Per Mole or per Cu Ft	Mixture before Combustion: O_2 Required	Mixture before Combustion: Air and H_2O	Products of Combustion: H_2O	Products of Combustion: CO_2	Products of Combustion: O_2	Products of Combustion: N_2	Products of Combustion: Total	Btu Values: Gross per Cu Ft	Btu Values: Net per Cu Ft	Btu in Gas: Gross	Btu in Gas: Net
CO_2	0.0356				0.0356			0.0356				
O_2	0.0005	−0.0005										
CO	0.3096	0.1548			0.3096			0.3096	321.8	321.8	99.6	99.6
H_2	0.0930	0.0465		0.0930				0.0930	325.0	275.0	30.3	25.6
CH_4	0.0070	0.0140		0.0140	0.0070			0.0210	1013.2	913.1	7.1	6.4
N_2	0.5543						0.5543	0.5543				
Total (dry gas)	1.000										137.0	131.6
H_2O in gas	0.0646		0.0646	0.0646				0.0646				
Theoretical totals, using O_2		0.2148	0.0646	0.1716	0.3522		0.5543	1.0781				
Using dry air, add							0.8085	0.8085				
Totals, using dry air		0.2148	1.0233	0.1716	0.3522		1.3628	1.8866				
Using 10% excess air		0.0215	0.1023			0.0215	0.0809	0.1024				
Totals, using 10% excess air		0.2363	1.1256	0.1716	0.3522	0.0215	1.4437	1.9890				
Moisture in air			0.0079	0.0079				0.0079				
Total products of combustion				0.1795	0.3522	0.0215	1.4437	1.9969				

Dry air required: 1.1256 cu ft
Moisture in air at 40% relative humidity: 0.0070 cu ft per cu ft
$1.1256 \times 0.0070 = 0.0079$

$$\text{Partial pressure } CO_2 = \frac{0.3522}{1.9969} = 0.176. \quad \text{Partial pressure } H_2O = \frac{0.1795}{1.9969} = 0.090.$$

Constituent	Heat Content of Gas at 600 F: Cu Ft Entering	Heat Content of Gas at 600 F: Heat Content, Btu per Cu Ft	Heat Content of Gas at 600 F: Btu from Each Constituent	Heat Content of Gas at 1400 F: Cu Ft Entering	Heat Content of Gas at 1400 F: Heat Content, Btu per Cu Ft	Heat Content of Gas at 1400 F: Btu from Each Constituent
CO_2	0 0356	14	0.498			
O_2	0.0005	10	0.005	0.2363	25.6	6.049
CO	0.3096	10	3.096			
H_2	0.0930	10	0.930			
CH_4	0.0070	15	0 011			
N_2	0.5543	10	5.543	0.8894	25.6	22.769
Total, dry	1.0000	.....	10.083	1.1257	. .	28.818
H_2O	0.0646	12	0.775	0.0079	32	0.253
Grand total	1.0646	.	10.858	1.1336	. .	29.071

From Fig. 6, $\phi = 0.26$, fraction of steam undecomposed. $100(1 - \phi) = 100(1 - 0.26) = 74\%$ steam decomposed. The number 6.22 represents moles of nitrogen per mole of hydrogen from decomposed steam.

$\frac{1}{6.22} \times 100 = 16.08$ moles of H_2 formed from steam per 100 moles of N_2

$16.08 \times \frac{55.43}{100} = 8.91$ moles of H_2 in gas formed from steam

$9.30 - 8.91 = 0.39$ mole of H_2 in gas from volatile matter in the coke

$\frac{16.08}{1 - 0.26} \quad \frac{16.08}{0.74}$ 21.73 moles of steam injected per 100 moles of N_2

$21.73 \times \frac{55.43}{100}$ 12.04 moles of steam injected per 55.43 moles of N_2 or per 100 moles of dry gas

Since 8.91 moles of H_2 were formed from steam, 8.91 moles of steam were decomposed, leaving $12.04 - 8.91 = 3.13$ moles of undecomposed steam per 100 moles of dry gas. Assume 70 cu ft of gas per lb of dry coke with 10% moisture in the coke, 100 moles of dry gas = 37,800 cu ft.

$$\frac{37{,}800}{70} \cdot 540 \text{ lb of dry coke}$$

$$\frac{540}{0.90} - 540 = 60 \text{ lb of moisture per 100 moles of gas}$$

$$= 3.33 \text{ moles of moisture from coke per 100 moles of gas}$$

Total H_2O is $3.13 + 3.33 = 6.46$ moles per 100 moles of dry gas, which enter into the products of combustion in Table 6.

Assume that the gas is at a temperature of 600 F and that the air for combustion is at a temperature of 1400 F. H = the net Btu value of the gas + the heat content of the gas at 600 F + the heat content of the air at 1400 F = $131.6 + 10.9 + 29.1 = 171.6$.

From Fig. 7, dissociation of CO_2 and H_2O at various partial pressures:

Temp.	3400 F	3500 F	Temp.	3400 F	3500 F
$1 - x$	7.0%	8.9%	$1 - y$	2.9%	3.5%
	93.0%	91.1%	y	97.1%	96.5%

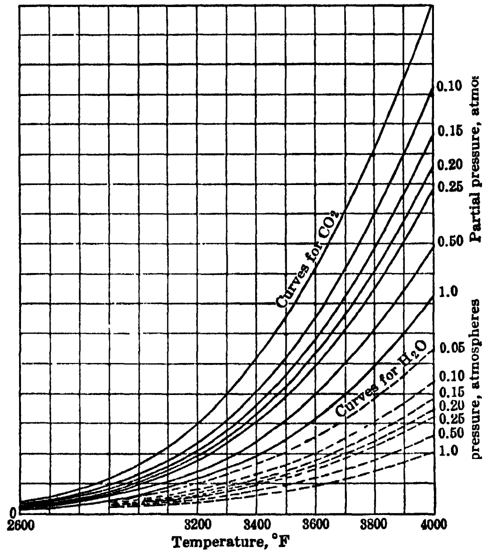


FIG. 7. Dissociation of CO_2 and H_2O at various partial pressures in mixtures containing no oxygen. (Reprinted by permission from *Gaseous Fuels*, L. Schnidman, Editor, American Gas Association, 1948, Chart 2, p. 121)

From Fig. 8, heat content of gases found in flue products:

Temp.	3400 F	3500 F	From Combustion Table Table 6
R_{H_2}	67	69	$a = 0.3522$
R_{O_2}	68	70.5	$b = 0.1795$
R_{CO_2}	112	116.5	$c = 0.0215$
R_{H_2O}	93	96.5	$d = 1.4437$

When these values are substituted in the equation: at 3400 F, $H = 164.93$, which is less than 171.6; at 3500 F, $H = 173.18$, which is greater than 171.6. By interpolation, $173.2 - 171.6 = 1.6$; $173.2 - 164.9 = 8.3$; $(8.3 - 1.6)/8.3 = 0.8$. Therefore, *the maximum theoretical flame temperature under the specified conditions is 3480* F. More precise calculations may be made by considering the dissociation of CO_2 and H_2O to yield H, OH, and O in addition to CO, H_2, and O_2, which become important in combustion with oxygen or oxygen enriched air.

The ratio of actual to ideal temperature rise during combustion is found in practice to be about 77% (Ref. 13). The equivalent temperature of gas plus air, if mixed before combustion, would be

$$\frac{(1.1366)(1400 + 460) + (1.0646)(600 + 460)}{1.1336 + 1.0646} = 1475 \text{ F absolute}$$

$$1475 - 460 = 1015 \text{ F}$$

$$1015 + (0.77)(3480 - 1015) = 1015 + (0.77)(2465) = 2913 \text{ F (actual flame temperature)}$$

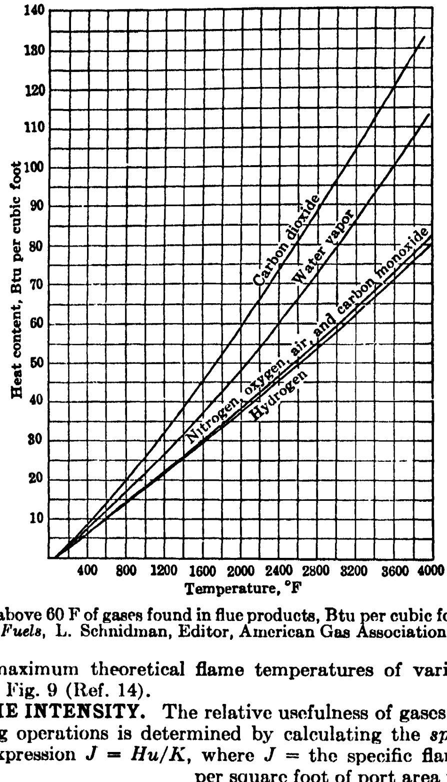


FIG. 8. Heat content above 60 F of gases found in flue products, Btu per cubic foot. (Reprinted by permission from *Gaseous Fuels*, L. Schnidman, Editor, American Gas Association, 1948, Chart 7, p. 130)

Curves for the maximum theoretical flame temperatures of various fuels burned in oxygen are given in Fig. 9 (Ref. 14).

SPECIFIC FLAME INTENSITY. The relative usefulness of gases for specialized high-temperature heating operations is determined by calculating the *specific flame intensity* by means of the expression $J = Hu/K$, where J = the specific flame intensity in Btu per square foot of port area per second, H = the net heating value in Btu per cubic foot of air-gas mixture issuing in 1 sec from a given burner, u = the rate of flame propagation of air-gas mixture in feet per second, and K = ratio of burner area to the inner flame cone area. Specific flame intensity expresses the *actual ability of the flame to deliver heat*, and the effectiveness of the flame is proportional to this factor (Ref. 15).

Specific flame intensity may be expressed as the *primary* flame intensity when it is based on the heat developed solely in the primary combustion, and as the *total* flame intensity when it is based on the heat developed both in the primary and in the secondary combustion. For the primary flame intensity, the value of H is determined from the relation $H = \dfrac{hx_t(1 - x)}{(1 - x_t)}$, and, for the total flame intensity, $H = hx$, where h = net heating value of the gas, Btu per cubic foot at 60 F, 30 in. Hg, dry; x_t = fraction of gas in the theoretical air-gas mixture required for complete combustion; and x = fraction of gas in the actual mixture.

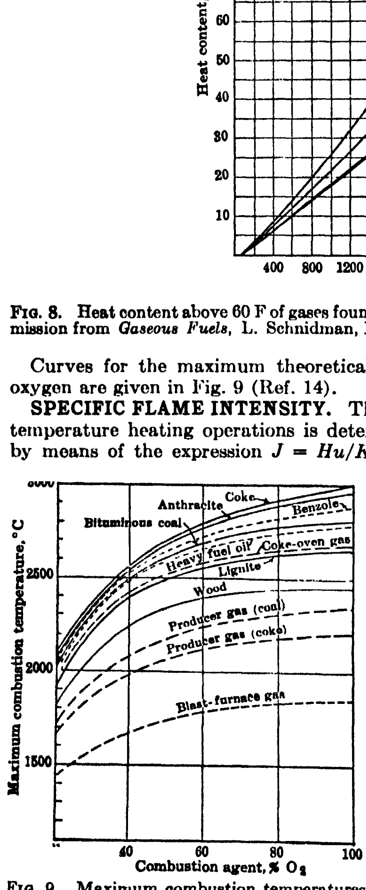


FIG. 9. Maximum combustion temperatures with theoretical O_2 fuel ratio. (Reprinted by permission from Combustion with Oxygen and Oxygen-enriched Air, by H. R. Fehling, *J. Inst. Fuel*, June 1948, Vol. XXI, No. 120, p. 231)

EXAMPLE. What are the primary and total specific flame intensities for hydrogen in a 51% hydrogen-air mixture? The net heating value of hydrogen, $h = 275$ Btu per cu ft at 60 F, 30 in. Hg, dry; $x_t = 0.295$, the fraction of hydrogen in the theoretical hydrogen-air mixture required for complete combustion; and $x = 0.51$ for the fraction of hydrogen in the actual mixture.

$$H = \frac{275 \times 0.295(1 - 0.51)}{1 - 0.295} = 56.4 \text{ for the primary flame intensity}$$

$$H = 275 \times 0.51 = 140.2 \text{ for the total flame intensity}$$

With a value of 0.5 for K and a flame velocity of 7.2 ft per sec (from Fig. 3)

$$J = \frac{7.2 \times 56.4}{0.5} = 815 \text{ Btu per sq ft per sec, primary flame intensity}$$

and

$$J = \frac{7.2 \times 140.2}{0.5} = 2020 \text{ Btu per sq ft per sec, total flame intensity}$$

23. GAS CALCULATIONS

GAS DENSITY is expressed in Table 1 in pounds per cubic foot, measured at 60 F and 30 in. Hg, "dry." The "wet" density is related to the dry-gas density as follows:

$$d_{gw} = \frac{d_{gd}(p - p_w) + d_w p_w}{p}$$

where d_{gw} = density of wet gas, d_{gd} = density of dry gas, d_w = density of water vapor, p = total absolute pressure of gas and water vapor, and p_w = partial pressure of water vapor in wet gas, all in consistent units.

The density of a gas mixture may be calculated from its analysis in terms of the *molecular weight of the mixture*, which is obtained by multiplying the *mole fraction* of each constituent in the mixture by its respective molecular weight and then adding. The mole fraction is obtained by dividing the number of moles of gas by the total number of moles present. For gases, *the mole fraction is the percentage by volume divided by 100.* Thus, for the product gases in the equation

$$C + O_2 + 3.76N_2 = CO_2 + 3.76N_2$$

	(1) No. Moles	(2) % by Vol.	(3) Mol. Wt.	(4) (2) × (3) ÷ 100
CO_2	1.00	21.0	44.01	9.24
N_2	3.76	79.0	28.16	22.24
			Molecular weight of mixture	31.48

The molar volumes vary slightly with the compressibility factors of the components, but for general purposes it is accurate enough to use a volume of 378 cu ft per lb-mole of the dry gas measured at 60 F and 30 in. Hg. The density of the gas mixture is, therefore, 31.48/378 = 0.0833 lb at 60 F and 30 in. Hg.

SPECIFIC GRAVITY of a dry gas is expressed as the ratio of its density to the density of dry air at the same conditions of temperature and pressure, $s = d_g/d_a$, where s = specific gravity on a dry basis and d_g and d_a = density of dry gas and dry air, respectively, at the same temperature and pressure.

The specific gravity of a gas mixture on a dry basis is

$$\text{Specific gravity (dry basis)} = \frac{\text{Molecular weight of gas mixture}}{\text{Molecular weight of air}}$$

Since the molecular weight of air is 28.97, a gas with a volume analysis, in percentage, of CO_2 = 21 and N_2 = 79 and with a molecular weight of 31.48 has a specific gravity of 31.48/28.97 = 1.09 (air = 1.00).

The specific gravity on a wet basis is

$$s_{\text{saturated}} = \frac{s(p - p_w) + s_w p p_w}{p}$$

where s = specific gravity on a dry basis; p = total pressure of gas and water vapor; p_w = partial pressure of water vapor as found by measurement or, if saturated, from the water vapor pressure table (steam table, Section 4) corresponding to the temperature; and s_w = specific gravity of water vapor (referred to dry air), generally taken as 0.622.

If the gas is saturated or contains determined amounts of water vapor, the specific gravity, s, on a dry basis is calculated from the formula

$$s = \left(\frac{\text{Density of gas and water} - d_w p_w}{\text{Density of air and water} - d_w p_w}\right)\left(\frac{\text{Pressure of air and water} - p_w}{\text{Pressure of gas and water} - p_w}\right)$$

where d_w = the density of water vapor, and p_w = the partial pressure of water vapor as determined or, if saturated, from the water vapor pressure table.

When the specific-gravity value is obtained by effusion apparatus, the air and gas are both saturated with water. The specific gravity, on a dry basis, may be converted to a saturated basis, as obtained by the effusion apparatus, by the formula $s_{\text{saturated}} = (s + k)/(1 + k)$, where s = specific gravity of the dry gas, $k = \dfrac{d_w p_w}{d_a(p - p_w)}$ d_w = density of water vapor, d_a = density of air, $\dfrac{d_w}{d_a}$ 0.622 (usual value), p = absolute pressure of gas and air, and p_w = partial pressure of water vapor at the temperature in question.

For descriptions and directions for using instruments for the determination of densities and specific gravities of gases, see Ref. 16.

GAS-VOLUME CORRECTION. To reduce the observed volume of a *dry gas* to the equivalent volume at 60 F and 30 in. Hg, dry, use the formula

$$V_s \text{ (dry)} = V \times \frac{459.6 + 60}{459.6 + t} \times \frac{H}{30}$$

For correcting observed volumes of *saturated gas* to the equivalent volume at 60 F and 30 in. Hg, saturated, use the formula

$$V_s \text{ (sat.)} \quad \frac{V \times (459.6 + 60)}{459.6 + t} \quad \frac{H - A}{30 - 0.522}$$

$$= V \times 17.626 \times \frac{H - A}{459.6 + t}$$

where V = observed volume, V_s = volume at standard conditions, 60 F, and 30 in Hg, H = absolute pressure of the gas, in. Hg (equals corrected barometric pressure plus gas pressure), A = water vapor pressure in. Hg for gas at t F (from steam tables, Section 4), and t = temperature of the gas, °F. Table 7 gives the correction factors for *dry gas* and Table 8 for *saturated gas*.

Table 7. Factors for Reducing Volumes of Dry Gas to Equivalent Volumes at 60 F and 30 In. Barometer *

(Multiply the observed volume by the factor to obtain the equivalent volume.)

Temp., °F	Absolute Pressure, Inches of Mercury										
	30.0	29.8	29.6	29.4	29.2	29.0	28.8	28.6	28.4	28.2	28.0
−30	1.2095	1.2014	1.1934	1.1853	1.1772	1.1692	1.1611	1.1530	1.1450	1.1369	1.1288
−20	1.1820	1.1741	1.1662	1.1583	1.1505	1.1426	1.1347	1.1268	1.1189	1.1111	1.1032
−10	1.1557	1.1480	1.1403	1.1326	1.1249	1.1172	1.1095	1.1018	1.0941	1.0863	1.0786
0	1.1306	1.1230	1.1155	1.1079	1.1004	1.0929	1.0853	1.0778	1.0703	1.0627	1.0552
10	1.1065	1.0991	1.0917	1.0843	1.0770	1.0696	1.0622	1.0548	1.0474	1.0401	1.0327
20	1.0834	1.0762	1.0689	1.0617	1.0545	1.0473	1.0401	1.0328	1.0256	1.0184	1.0112
30	1.0613	1.0542	1.0471	1.0401	1.0330	1.0259	1.0188	1.0118	1.0047	0.9976	0.9905
40	1.0400	1.0331	1.0261	1.0192	1.0123	1.0053	0.9984	0.9915	0.9845	.9776	.9707
50	1.0196	1.0128	1.0060	0.9992	0.9924	0.9856	.9788	.9720	.9652	.9584	.9516
60	**1.000**	0.9933	0.9867	.9800	.9733	.9667	.9600	.9533	.9467	.9400	.9333
70	0.9811	.9746	.9680	.9615	.9550	.9484	.9419	.9353	.9288	.9223	.9157
80	.9629	.9565	.9501	.9437	.9373	.9308	.9244	.9180	.9116	.9052	.8987
90	.9454	.9391	.9328	.9265	.9202	.9139	.9076	.9013	.8950	.8887	.8824
100	.9285	.9223	.9161	.9099	.9037	.8976	.8914	.8852	.8790	.8728	.8666
110	.9122	.9061	.9000	.8940	.8879	.8818	.8757	.8696	.8636	.8575	.8514
120	.8965	.8905	.8845	.8785	.8726	.8666	.8606	.8546	.8486	.8427	.8367

* Formula: Equivalent volume = Observed volume × $[519.6/(t + 459.6)] \times (B/30)$.

Table 8. Factors for Reducing Volumes of Gas Saturated with Aqueous Vapor to Equivalent Volumes at 60 F and 30 In. Barometer *

(Multiply the observed volume by the factor to obtain the equivalent volume.)

Temp., °F.	Absolute Pressure, Inches of Mercury										
	30.0	29.8	29.6	29.4	29.2	29.0	28.8	28.6	28.4	28.2	28.0
32	1.069	1.062	1.055	1.048	1.040	1.033	1.026	1.019	1.012	1.005	.997
35	1.062	1.055	1.048	1.041	1.033	1.026	1.019	1.012	1.005	0.998	.991
40	1.050	1.043	1.036	1.028	1.021	1.014	1.007	1.000	0.993	.986	.979
50	1.025	1.018	1.011	1.004	0.997	0.990	0.983	0.977	.970	.963	.956
60	**1.000**	0.993	0.986	0.980	.973	.966	.959	.953	.946	.939	.932
70	.974	.967	.961	.954	.947	.941	.934	.927	.921	.914	.907
80	.946	.940	.933	.927	.920	.914	.907	.901	.894	.887	.881
90	.917	.910	.904	.897	.891	.884	.878	.872	.865	.859	.852
100	.884	.878	.871	.865	.859	.853	.846	.840	.834	.827	.821
110	.848	.842	.836	.829	.823	.817	.811	.805	.799	.792	.786
120	.808	.801	.785	.789	.783	.777	.771	.765	.759	.753	.747

*Abridged by permission from *Combustion*, C. George Segeler (Editor), 3rd ed., American Gas Association, New York, 1932, Table 15, p. 199. (For formula, see text.)

24. INDUSTRIAL GASES

BLAST-FURNACE GAS is derived from partial combustion of coke in a blast furnace. Although the blast furnace is not operated primarily to produce gas for utilization outside the process, it may, nevertheless, be considered as a huge gas producer operated on preheated forced blast. Modern furnaces, producing 1000 tons of iron per day on a fuel consumption of 1800 lb of coke per ton of iron, have a daily output of about 127,000,000 cu ft of gas. A typical blast-furnace gas (Table 2) contains over 70% inert gases and less than 30% combustible gases, mainly CO. It has the lowest heating value of all commercial gases, between 90 and 110 Btu per cu ft, depending on quality of coke, speed of combustion, character of ore, and other factors; consequently, the gas cannot be transported economically over long distances. It is used to preheat air required for blowing the furnace, as fuel for supplying the motive power for the blowers, for heating by-product coke ovens, or for mixing with coke-oven gas to furnish fuel for miscellaneous plant uses.

PRODUCER GAS is derived from the reaction of a mixture of steam and air blown continuously through a deep bed of solid fuel, which may consist of coke, the entire range of ranks of coal, peat, or wood. The heating value of the gas is 120 to 180 Btu per cu ft, depending on the fuel; it is, therefore, uneconomical to transport the gas over long distances. In many applications, it is desirable to locate furnaces adjacent to producers to permit the gas to be burned in its hot, raw state, saving the sensible heat. The gas is widely used in ceramic kilns, in glass-melting furnaces, for underfiring of coke ovens, and for numerous other purposes. Typical analyses are given in Table 2, and detailed discussion of the process is given on p. 2-87.

BLUE GAS is produced by blowing air and steam *alternately* through a bed consisting of a good grade of coke, anthracite, or bituminous coal.

The "Blow." During the "blow," air is blown long enough to raise the temperature of the fuel bed high enough for rapid reaction of the steam with the incandescent carbon. With properly sized generator fuels that do not cake or decrepitate in the fuel bed, the rate of blow may be 175 to 250 cu ft of air per square foot of fuel-bed cross section per minute. At this rate, complete combustion takes place just above the ash and clinker zone, with the reaction,

$$C + O_2 + 3.76N_2 = CO_2 + 3.76N_2$$

taking place as long as free oxygen remains in the gas stream. The CO_2 is in part reduced to CO, according to the reaction $CO_2 + C = 2CO$, as the gases flow upward through the fuel bed, so that the blast gas leaving the generator contains considerable CO. To obtain maximum efficiency in blue-gas operation, it is desirable to produce the minimum of CO during the blow. This is accomplished by using a shallow fuel bed and as high a rate of air blast as the fuel will permit. The blast gas usually is burned with secondary air to produce steam in a waste-heat boiler and is finally discharged to the atmosphere. The reactions taking place during the blow are

$$C + O_2 + 3.76N_2 = CO_2 + 3.76N_2$$
$$C + CO_2 + 3.76N_2 = 2CO + 3.76N_2$$
$$2C + O_2 + 3.76N_2 = 2CO + 3.76N_2$$
$$2CO + O_2 + 3.76N_2 = 2CO_2 + 3.76N_2$$

The "Run." Steam is blown through the incandescent fuel bed during the "run," which is generally composed of an "up-run" and a "down-run." The terms refer to the direction in which steam flows through the generator. Principal reactions during the run are

$$C + H_2O = CO + H_2$$
$$C + 2H_2O = CO_2 + H_2$$
$$CO + H_2O = CO_2 + H_2$$
$$C + CO_2 = 2CO$$

To a minor degree, $CO + 3H_2 = CH_4 + H_2O$ also takes place.

Cycle. For the purpose of clinker and fuel-bed control, the rate and total time of admission of steam during the up-run are varied in relation to those of the down-run, and in relation to the rate and total time of admission of air during the blow. The cycle is adjusted so that, at the end of the blow, the fuel bed at the combustion zone will not be overheated to the point where an unmanageable clinker is produced and will not be cooled at the end of the run to the point where the rate of steam decomposition drops too low and the gas quality deteriorates. Charging of fuel and removal of clinker and ash generally have been manual in earlier units, requiring intermittent shutdowns. Automatic charging and clinkering equipment on modern blue-gas generators permits uninterrupted repetition of the blow-and-run cycle. The gas produced during the run of each cycle flows through a *wash box*, which acts as a hydraulic check valve and cooler, into a *relief holder* provided to equalize flow of gas from the generator to the condensing, scrubbing, and purification system. Ultimate use of the gas governs the extent of condensing, scrubbing, and purification to which the gas is subjected. Table 9 gives typical data on manufacture of blue gas.

Table 9. Typical Data in the Manufacture of Blue Gas

(Reprinted by permission from *Chemistry of Coal Utilization*, H. H. Lowry, Editor, John Wiley and Sons, New York, 1945, Vol. 2, Table VI, p. 1718.)

	merican	British	American
Material per 1000 cu ft			
Coke, dry, lb	34.7	38.8	33.8
Air for blast, cu ft	2230	1720	1610
Steam used, lb	51.9	34.6	49.6
Moisture in coke, lb	1.5	4.7	....
Steam decomposed, lb	23.85	....	....
Steam undecomposed, lb	29.55	.... .	....
Analysis of coke			
Moisture, %	4.20	10.8	... .
Volatile matter, %	2.69	....	0.8
Fixed carbon, %	89.80	.... .	92.2
Ash, %	7.51	12.3	12.3
Heating value, Btu per lb	2650	11260	....
Analysis of blue gas			
Carbon dioxide, %	5.4	4.5	5.3
Oxygen, %	0.7	0.1	0.2
Carbon monoxide, %	37.0	40.7	39.2
Hydrogen, %	47.3	49.2	48.6
Methane, %	1.3	0.6	0.8
Nitrogen, %	8.3	4.9	5.8
Total heating value, Btu per cu ft	287	296	285
Blast gases entering waste-heat boiler			
Carbon dioxide, %	19.9	....	....
Oxygen, %	1.1	....	....
Nitrogen, %	79.0	....	....
Temperature of blue and blast gases			
Entering the waste-heat boiler, °F	1300	....	1250–1300
Leaving the waste-heat boiler, °F	550	....	400–420
Steam from waste-heat boiler, lb per 1000 cu ft	57.0	. ....	33.8

Effect of Fuel. The character of the fuel has a marked influence on operation. Generator fuel should permit as high a blast rate as possible, should be closely sized and strong enough to resist excessive breakage and production of fines during handling, and should not appreciably decrepitate from thermal shock. The presence of fines lowers the blast rate which the fuel bed can resist without excessive fuel blow-over. (For decrepitation of anthracite, see Ref. 17.) Bituminous coal should have a minimum of caking under the conditions of generator operation.

The following sizes give good results: egg coke, 2 1/2 by 1 7/8 in.; broken anthracite, 4 3/8 by 3 1/4 in.; bituminous coal, 6 by 3 in. The ash-fusion temperature should exceed 2300 F to prevent excessive clinker formation. Table 10 gives comparative results obtained in blue gas operation using coke and bituminous coal. A heat balance of blue gas operation is given in Table 11. Typical analyses of blue gas (also known as water gas) are given in Table 2, p. 2-64.

Table 10. Comparative Results Obtained with Coke and Bituminous Coal in Blue Gas Operation

(Abridged by permission from *Gaseous Fuels*, L. Schnidman, Editor, American Gas Association, New York, 1948, p. 44.)

		Bituminous Coal with Blast Gas, %		
	Coke	0	10	30
Generator fuel				
Lb per M	40	48	44.5	37.5
Btu per lb	13,000	14,000	14,000	14,000
Steam, lb per M	45	50	45	35
Gas				
Btu per cu ft	300	335	316	277
CO_2, %	5.1	7.0	6.7	6.0
Illuminants, %	0.0	1.0	0.9	0.8
O_2, %	0.0	0.0	0.0	0.0
CO, %	40.2	34.4	33.5	31.7
H_2, %	50.0	48.8	44.5	35.8
CH_4, %	0.7	4.8	4.6	4.3
N_2, %	4.0	4.0	9.8	21.4
Relative capacity, %	100	70–90	80–100	100–120

Table 11. Heat Balance

(Reprinted by permission from "Water Gas," by J. J. Morgan, in *Chemistry of Coal Utilization*, H. H. Lowry, Editor, John Wiley and Sons, New York, 1945, Chapter 37, p. 1719.)

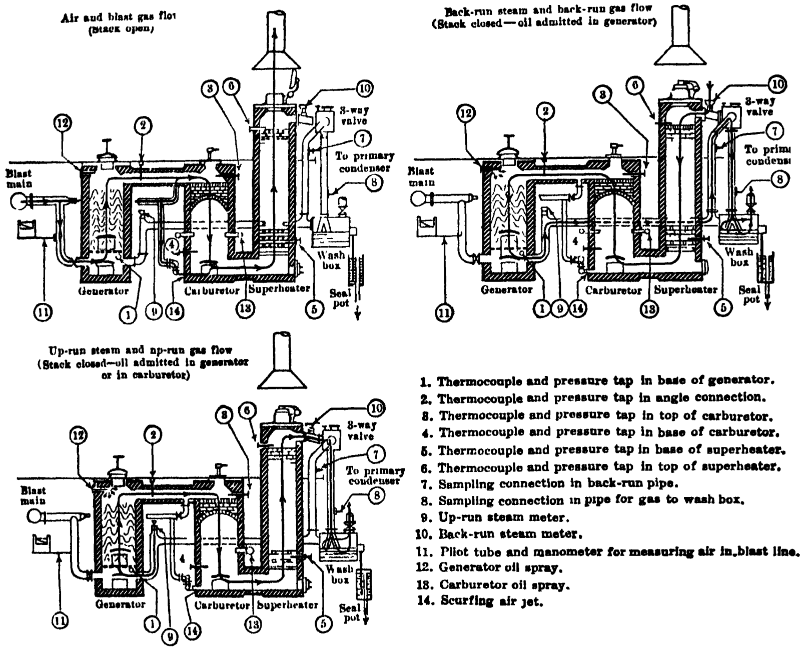
	American		British	
	Btu	%	Btu	%
Heat in				
Coke burned	458,040	100.0	490,000	100.0
Steam used	58,932	12.8	38,600	7.9
Blast		...	.....	
Total	516,972	112.8	528,600	107 9
Heat out, recovered				
Calorific value in gas	287,000	62.7	296,000	60.5
Steam made in waste-heat boiler *	64,700	14.1	... .	....
Heat losses				
Sensible heat in blue gas and blast gas to condenser and stack	31,130	6.8	63,300	12.8
Potential heat in blast gas to stack	....	.....	104,200	21.3
Sensible heat in undecomposed steam and moisture from coke to condenser	6,970	1.5	16,300	3.3
Latent heat lost in undecomposed steam and coke moisture	29,550	6.4		
Loss in unburned carbon in ashes	59,000	12.9	29,400	6.0
Radiation, boiler blow-down, and all other losses	38,622	8.4	19,400	4.0
Total	516,972	112.8	528,600	107.9

* Includes steam used under grate (91%) and excess steam (9%).

Blue gas has a heating value of 285 to 310 Btu per cu ft. It is usually carburetted or mixed with gases of higher heating value before distribution in city mains. A few large manufacturing establishments produce blue gas for plant use, as for forge welding operations. A large number of blue-gas machines are in use for the generation of synthesis gas for ammonia, methanol, and other chemical operations.

The production rates and fuel economy in blue gas or synthesis gas operation are greatly improved by converting the intermittent generators to continuous oxygen-blown generators or producers. (See Wright, Barclay, and Mitchell, The Production of Hydrogen and Synthesis Gas by the Oxygen Gasification of Solid Fuel, and L. L. Newman, Oxygen in the Production of Hydrogen or Synthesis Gas; both in *Ind. Eng. Chem.*, April 1948.)

CARBURETTED WATER GAS consists of blue gas mixed with hydrocarbon vapors, usually produced by thermal cracking of gas oil or heavy fuel oil in the same operation. A typical carburetted water gas machine consists of a *generator*, a *carburetor*, and a *superheater* lined with firebrick, all connected in series. Figure 10 illustrates the apparatus and



1. Thermocouple and pressure tap in base of generator.
2. Thermocouple and pressure tap in angle connection.
3. Thermocouple and pressure tap in top of carburetor.
4. Thermocouple and pressure tap in base of carburetor.
5. Thermocouple and pressure tap in base of superheater.
6. Thermocouple and pressure tap in top of superheater.
7. Sampling connection in back-run pipe.
8. Sampling connection in pipe for gas to wash box.
9. Up-run steam meter.
10. Back-run steam meter.
11. Pilot tube and manometer for measuring air in blast line.
12. Generator oil spray.
13. Carburetor oil spray.
14. Scurfing air jet.

FIG. 10. Phases of operation of a carburetted water-gas machine. (Reprinted from Test Results on the Use of Anthracite in Heavy-oil Water-gas Operation at the Pottsville Gas Works, L. L. Newman, C. C. Wright, and A. W. Gauger, *Penna. State Coll. Mineral Inds. Expt. Sta. Bull.* 32, 1941, pp. 8, 9, and 11)

three phases of the operating cycle, running on heavy oil. If gas oil is used for carburetion, both carburetor and superheater are filled with checkerbrick, but, when heavy oil is used, only the superheater is filled.

Cycle of Operation. Manufacture of blue gas is the first step. Blast gases leaving the generator during the blow are burned to heat the walls and checkerbrick of the carburetor and superheater. Products of combustion leaving the superheater may be used in a waste-heat boiler or discharged directly to atmosphere. During the run, gas oil is vaporized in the carburetor, and partly cracked in an atmosphere of blue gas flowing from the generator. The mixture leaving the carburetor flows through the superheater, in which the hydrocarbon gases are fixed, and finally through a wash box to the relief holder. In modern "back-run" water-gas machines, steam and gas flow in reverse along the path followed during the up-run.

The average temperature of the make gases entering the wash box in regular up- and down-run operation is 1400 F; in back-run operation, the temperature of the up-run gas is about 1100 to 1200 F and the temperature of the back-run gas is 400 to 600 F, resulting in saving of generator fuel. Further saving is effected by the lowered load on the cooling and condensing system and by reduced labor and material charges because of a less-frequent need for recheckering.

Heavy oil has replaced gas oil in many back-run water-gas machine operations. During the up-run, a portion of the heavy oil is admitted to the top of the generator fuel bed, to the carburetor, or to both, and the remainder is admitted to the top of the generator during the back-run. Because carbon deposits from heavy oil would quickly plug the checkerbrick in the carburetors, they are operated without checkerbrick, radiation from the walls supplying the heat necessary for vaporization and partial cracking of the oil.

Typical analyses of carburetted water gas are given in Table 2, p. 2-64. Operating data for a battery of eight carburetted water gas machines are given in Table 12.

Table 12. Operating Data for Typical Carburetted Water Gas Sets

(Reprinted by permission from Water Gas, by J. J. Morgan, in *Chemistry of Coal Utilization*, H. H. Lowry, Editor, John Wiley and Sons, New York, 1945, Chapter 37, p. 1745.)

Gas made, million cubic feet	20,606
Btu per cubic foot, average	530
Specific gravity, average	0.69
Per set per day, 1000 cu ft	5,847
Coke, pounds per 1000 cu ft, average	12.40
Boiler fuel, pounds per 1000 cu ft, average	5.10
Oil, gallons per 1000 cu ft, average	4.22
Percentage to generator	60
Percentage to carburetor	40
Percentage re-formed	25
Degrees API gravity	19.5
Coke residue, %	5.4
Steam, pounds per 1000 cu ft, average	19.5
Operating cycle, minutes	4.0
Blow, percentage of cycle	33
Blow-run, percentage of cycle	8
Up-run, percentage of cycle	37
Back-run, percentage of cycle	20
Air purge, percentage of cycle	2

Gas is pumped from the relief holder through a condensing, scrubbing, and purification system, before it is admitted to distribution holders from which it may flow into the mains under holder pressure or be compressed for distribution to distant points.

A heat balance of six carburetted water-gas plants is given in Table 13. For results of tests using anthracite as a water-gas generator fuel, see Ref. 18.

Carburetted water gas is distributed alone or in a mixture with other gases in city gas mains. Heating-value requirements vary from 500 to 600 Btu per cu ft, 530 Btu being most common. It is widely distributed by public utilities for general use.

COAL GAS AND COKE-OVEN GAS are produced by destructive distillation of bituminous coal, during which coal is exposed to heat from the retort or coke-oven walls in the absence of air. Coal distillation processes are referred to as carbonization. Commercial gaseous products available in the United States are principally products of high-temperature carbonization, with oven walls at 1800 F, or higher. For composition of these gases, see Table 2. In addition to gas, products of carbonization consist of coke, tar, ammonia, light oil, cyanogen, and naphthalene.

Retorts are made of fire clay or silica and may be horizontal, inclined, or vertical. Horizontal retorts are usually long, semicylindrical or dome-shaped chambers. They are either (1) "stop-end" retorts, with one end closed and the other fitted with a cast-iron mouthpiece and lid, or (2) "through" retorts, with both ends open except for mouthpieces and lids. The internal cross section varies from 14 by 24 in. to 16 by 28 in. Stop-end retorts are 10 to 12 ft long and hold 250 to 400 lb of coal per charge of 4- to 6-hour duration. Through retorts are 11 to 22 feet long and hold 400 to 1000 lb per charge of 4 to 12 hours. Retorts of either type are set in groups of six to twelve, called a bench. They are heated by a furnace or a gas producer, usually employing coke as fuel. Vertical retorts are rectangular or elliptical chambers, larger in size than the horizontal retorts. The length is usually 25 ft. Vertical retorts may be continuous or intermittent.

Continuous vertical retorts are often operated with the introduction of steam at the bottom, increasing the volume of gas produced but reducing the coke yield.

By-product coke ovens are long, narrow chambers of silica brick, 35 to 45 ft long, 12 to 18 ft high, and 14 to 18 in. wide, with a taper of about 3 in. They can carbonize 15 to 20 tons of coal in 14 to 18 hours. The ovens are built in batteries of 25 to 75, with vertical heating flues between them. Blast-furnace gas, producer gas, blue gas, or coke-oven gas is employed for heating the flues. Gas leaves the retorts or coke ovens through suitable standpipes or offtake pipes and hydraulic mains and passes through coolers, exhausters, tar extractors, saturators, light-oil scrubbers, purifiers, etc., in which the tar, ammonia, light oil, and sulfur are successively removed. Numerous variations in the procedures are followed for recovery of by-products.

Table 13. Heat Balance of Carburetted Water Gas Process

(Adapted by permission from "Water Gas," by J. J. Morgan, in *Chemistry of Coal Utilization*, H. H. Lowry (Editor), John Wiley and Sons, New York, 1945, Chapter 37, p. 1746.)

	A	B	C	D	E	F
External diameter of generator, feet	11	11	9	11	11	11
Average depth of fuel, feet	7.5	10.5	8.0	....	....	......
Duration of test	1 yr	1 mon.	24 hr	......	....	2 yr
Gas made, million cubic feet per set per day	2.4	3 6	1.8	4.5	5.7	5.8
Btu per cubic foot	600	530	520	535	535	530
Generator fuel	Coke and gas coal	Coke	Bituminous coal	Coke	Coke	Coke
Pounds per 1000 cu ft	30.6	27.7	28.7	26.2	15.8	12.4
Oil used, API gravity	32 0	24 0	36.0	24.2	15.3	19.5
Gallons per 1000 cu ft	3.68	2.98	2.59	2.90	4.18	4.22
Steam, pounds per 1000 cu ft	54.0	51.8	17.1	38.0	30.3	19.5
Steam from waste-heat boiler, pounds per 1000 cu ft	.	20.5	......	.....	. ....	13.0
Temperature, make gases leaving set, °F	1300	1100	. .	.. .	. .	.. .
Percentage of Total Heat Input						
Input						
Generator fuel	48.78	39.4	51.3	41.8	23.3	19.5
Enriching oil	52.94	53 1	44.6	52.3	72.8	77 8
Steam, total heat	6.28	7 1	4.1	5.9	3.9	2.7
Feed water	..... .	0.4	..	..	. .	
	100 00	100.00	100.00	100.00	100.00	100 00
Output						
Heating value						
Gas	61.88	64.7	67.0	65.8	61.1	63.8
Tar	7.89	12.3	10.6	13.5	15.3	17.0
Drip oil	0.65	1.1	..	1.0	1.4	0 9
Total or "efficiency" of set	70.48	78.1	77.6	80.3	77.8	81 7
Sensible heat:						
In gas	3.06	2.8	2.0	.. . ..	....	.. ..
In tar	0.36	0.6	0.3	....	....	... .
In drip oil	0 02	.		... . .	....	...
In dry blast products	5.53	1 8	4.3	.....	. . . ...	.. .
Combustible in stack gases	0.35	0.4	2.7	..	....	. .
Total heat in undecomposed steam and water vapor	5.41	5.8	6.4	.	. ....	.
Combustible in refuse	1.30	1.9	2.4	......	......	.
Steam in waste-heat boiler	.	2.9	.	... . ..	. .....	1.6
Radiation and unaccounted for loss	13.55	5.7	4.3	.. .	....	.. .
	100.00	100.00	100.00	....	......	....

Normal yields per ton of coal charged are 11,000 cu ft of gas, 1500 lb of coke and coke breeze, 10 gal of tar, 3.5 gal of light oil, and 28 lb of chemicals, principally ammonium sulfate. For composition of these gases, see Table 2.

Calorific Value. Coal or coke-oven gases have a calorific value of 520 to 575 Btu per cu ft, depending on the amount of steaming and light-oil removal. They generally are distributed by public utilities in mixtures with other gases because carbonization processes are not flexible enough to meet wide variations in gas load.

OIL GAS. Almost from the beginning of the business of gas manufacture, oil gas has been produced by distilling oil in iron or fire-clay retorts similar to those used for coal gas. ***Pintsch gas***, with a heating value of about 1300 Btu per cu ft, is made from oil gasified in the upper of two cast-iron retorts and fixed in the lower. Modern oil-gas machines (Jones) consist of two cylindrical steel shells of equal diameter but different heights, connected at the bottom, lined with firebrick, and partly filled with checkerbrick.

Table 14. Comparative Operating Data for Representative Oil-gas Plant Tests

(Data from *Efficiency of Manufacture, Distribution and Utilization of Oil Gas in California*, final report of investigation made by the Joint Committee on Efficiency and Economy of Gas of the Railroad Commission of the State of California, May 3, 1924.)

Item	Potrero	San Jose	Santa Barbara	Southern California Gas Co.	Los Angeles Gas and Electric Co.
Kind of Oil-gas Generator	Jones Improved Two-shell	Jones Improved Two-shell	Straight-shot	Straight-shot	Straight-shot
Generator dimensions					
Primary					
Height	49 ft 0 in.	30 ft 9 in.	28 ft 0 in.	36 ft 0 in.	35 ft 0 in.
Outside diameter	18 ft 9 in.	12 ft 0 in.	14 ft 0 in.	20 ft 0 in.	22 ft 0 in.
Inside diameter	14 ft 9 in.	8 ft 8 in.	.	16 ft 0 in.	. . .
Secondary					
Height	63 ft 0 in.	42 ft 6 in.	.. .	. . ..	... .
Outside diameter	18 ft 9 in.	12 ft 0 in.	....	. . ..	.. ..
Inside diameter	14 ft 9 in.	8 ft 8 in		. ...	... .
Operating cycle					
Dry blast, min	5	2	5	3	5
Heat, min	5	4 1/2	9	7	10
Make, min	8	6 1/2	20	15	22
Purge, min	2	2	6	5	8
Operating cycle, min	20	15	40	30	45
Number of runs per operating hour	3	4	1.5	2	1.33
Gas data					
Calorific value, Btu per cu ft	550	550	550	550	550
Average gas made per hour, M cu ft	200	88	55	100	116
CO_2, %	4.6	4.5	1.2	1.8	3.0
C_6H_6, %	1.2	0.8	0.8	0.9	0.9
C_nH_{2n}, %	2.5	3 0	3.9	1.9	3.4
O_2, %	0.3	0.1	0.2	0.3	0.5
CO, %	12.9	12.7	8.0	9.9	11 6
H_2, %	48 7	45.2	55.3	51.2	51.3
CH_4, %	26.4	28.1	24.8	28.5	25.4
N_2, %	3.4	5.6	5.8	5.5	3.9
Specific gravity of purified gas (air = 1)	0.467	0.484	0.389	0.415	0.434
Air supply data					
Blast period, cu ft per M cu ft gas	1,460	569	1,146	624	1,000
Heating period, cu ft per M cu ft gas	1,274	1.688	1,833	1,550	2,000
Steam data					
Pounds per M cu ft, dry blast	1.5	1.2	0.8	0.8	0.9
Pounds per M cu ft, heat period	5.9	5.0	4.0	2.7	3.0
Pounds per M cu ft, make period	24.8	23.5	14.2	12.2	10.5
Pounds per M cu ft, purge period	6.4	3.5	2.0	7.6	6.0
Total, lb per M cu ft	38.6	33.2	21.0	23.3	20.4
Oil data					
Heat oil, gal per M cu ft	0.87	0.92	1.02	0.65	1.01
Heating value, dry basis, Btu per M cu ft	130,376	137,750	151,904	96,755	150,505
Make oil, gal per M cu ft of gas	6.38	6.43	7.38	7.20	7.39
Heating value, dry basis, Btu per M cu ft	956,596	962,936	1,099,988	1,071,890	1,101,469
By-products					
Dry lampblack, lb per M cu ft	12.0	13.0	22.0	19.6	21.7
Tar (moisture free), lb per M cu ft	4.0	4.5	2.5	1.5	1.5
Percentage overall efficiency $= \frac{\text{Btu in gas}}{\text{Btu in heat + make oil}}$	50.8	50.0	43.8	47.1	43.8

Heat required to gasify the oil is obtained during the blow by burning carbon deposits; oil is then steam-sprayed into the apparatus through sprays or burners, entering the shorter (primary) generator near the top. The air blast enters the vessel through the top. A secondary air supply enters at the bottom of the taller or secondary generator, and the products of combustion escape through a stack valve opening at the top. When the apparatus has been heated to gas-making temperature—about 1800 to 2100 F at the top of the primary generator, 1800 F in the secondary generator—the blast is shut off, the stack valve closed, and the "run" begun by turning in steam or steam and oil through gas-making burners at the top of the primary generator and at top or bottom of the secondary one. The run is continued until the temperature of the checkerbrick has been reduced below that at which gas can be made economically. The oil is then shut off, and the apparatus is purged with steam in preparation for a repetition of the cycle. A typical operating cycle consists of: Blow (air only, heating with carbon), 5 min; blow (heating with oil), 5 min; total blow, 10 min. Run (with steam only), 1 min; run (with oil and steam), 7 min; purging with steam, 2 min; total run, 10 min. Total cycle, 20 min.

Many oil-gas machines are of the *straight-shot, heat-up, make-down* type. During the blast, air and oil are admitted at the base of the generator. Products of combustion travel upward and are discharged to the atmosphere through a stack valve. When the checkerbrick has reached the required temperature, the blast is discontinued and oil and steam are admitted at the top of the checkerbrick. The oil is gasified and deposits carbon which reacts with the steam to form blue gas. The mixture leaves the generator through an offtake at the bottom and passes through a wash box to the condensing system.

Recent trends have been to combine so-called *straight-shot units* by linking them in pairs by a tunnel at the base. Provisions are made for admitting air at the top of each shell. Air is admitted during the blow at the top, the blast traveling down one generator and up the other, with the direction of flow reversed during alternate cycles. After each blow, steam and oil are admitted at the top of each generator, and the gas leaves the bottom of each generator and passes through wash boxes to the condensing system. It has been claimed that no heating oil is required with this method of operation—the deposited carbon being sufficient to supply the heat—and that more uniform heats prevail throughout the checkerbrick.

Results of oil-gas operation in two-shell units (Jones) and straight-shot units are given in Table 14. For gas composition, see Table 2. Oil gas distributed on the Pacific coast closely resembles coal or coke-oven gas.

REFORMED NATURAL GAS is manufactured by decomposing natural gas into a gas of lower calorific value suitable for mixing with coke-oven gas, carburetted-water gas, and oil gas. Reforming processes may use *thermal cracking* alone or *catalytic cracking*. Thermal cracking may be done in the fuel bed of a water-gas generator or in the checkerbrick of an oil-gas generator. Catalytic cracking is done in externally heated tubes filled with suitable catalysts. For typical analysis of reformed gas, see Table 2, p. 2-64.

NATURAL GAS is obtained by tapping underground supplies by drilling. Advances in high-pressure distribution have made it possible to supply natural gas to large municipalities a thousand or more miles away from the well. In some cities, natural gas is mixed with manufactured gas before distribution. Natural gas has a calorific value of approximately 1000 to 1150 Btu per cu ft and is widely used for many industrial purposes and for cooking and space heating. For typical analysis of natural gas, see Table 2, p. 2-64.

Auxiliary Fuels. Natural gas is sold by public utilities to large industrial users on an interruptible basis. When demand for gas by domestic and institutional users exceeds a definite amount, it sometimes is necessary to interrupt the service to the industrial users. Although conditions and costs vary considerably in different parts of the country, a general report on auxiliary fuels for use in case of interruption of natural-gas service has been prepared by the Industrial Processing Committee of the American Gas Association (Ref. 19). This report, based on 1948 prices, gives data on the cost of stand-by plants using fuel oil, propane, butane, producer gas, casinghead (natural) gasoline, electricity, and coal.

Table 15 gives comparative costs of various auxiliary stand-by fuels. Table 16 compares the characteristics of these fuels.

PROPANE AND BUTANE are derived from natural gas and refinery oil gases. (See Liquid Fuels, p. 2-45.)

ACETYLENE as an industrial fuel is produced from calcium carbide and water; 64 lb CaC_2 and 36 lb H_2O produce 26 lb of acetylene. The chemical reaction is

$$CaC_2 + 2H_2O = C_2H_2 + Ca(OH)_2$$

Calcium carbide is produced in the electric arc furnace from a mixture of finely divided and intimately mixed calcium oxide, or quicklime, and coke. The chemical reaction is $CaO + 3C = CaC_2 + CO$. The principal use of acetylene as a fuel gas is in oxyacetylene welding and cutting operations. It is also used as an illuminating gas and in chemical operations, such as the production of synthetic rubber. In large-scale chemical operations, acetylene may be produced from other hydrocarbon gases by special processes.

Table 15. Comparative Costs of Auxiliary Fuels in Cents per Therm *

(Reprinted by permission from Fuels and Their Characteristics, by James E. Coleman, *Information Letter* 22, American Gas Association, Industrial Processing Committee, Aug. 2, 1948.)

Delivered Cost, cents/gallon, dollars/ton, cents/kw-hr	Light Fuel Oil	Heavy Fuel Oil	Propane	Butane	Natural Gasoline	Producer Gas	Coal	Electricity
0.5								14.7
1.0								29.4
2.0								58.8
3.0	2.2	2.0	3.2	2.9	2.7	2.9	1.1	
4.0	2.9	2.7	4.3	3.9	3.6	3.3	1.4	
6.0	4.3	4.1	6.5	5.8	5.4	4.3	2.1	
8.0	5.8	5.5	8.7	7.8	7.2	5.2	2.8	
10.0	7.2	6.8	10.9	9.7	9.0	6.2	3.6	
12.0	8.7	8.2	13.0	11.6	10.8	7.2	4.3	
14.0	10.1	9.6	15.2	13.6	12.5	8.1	5.0	

* One therm = 100,000 Btu.

Oil utilization cost adds at least 1/2 cent per gallon to above delivered costs.

Producer gas cost based on adding $3.00 per ton for gasification, amortization and interest charges, and 75% efficiency on conversion.

Electricity usually utilized at a higher efficiency than solid, liquid, or gaseous fuels.

Propane, butane, and natural gasoline costs do not include maintenance, amortization, or utilization cost.

Coal costs are the delivered cost of coal only.

Table 16. Comparative Characteristics of Auxiliary Fuels

(Reprinted by permission from Fuels and Their Characteristics, James E. Coleman, *Information Letter* 22, American Gas Association, Industrial Processing Committee, Aug. 2, 1948.)

Characteristics	Light Fuel Oil	Heavy Fuel Oil	Propane	Butane	Producer Gas	Vaporized Fuel Oil	Natural Gasoline	Electricity	Coal
Reasonably available	Yes	Yes	No	No	?	Yes	No	Yes	Yes
Low cost of installation	Yes	Yes	No	No	No	Fair	Fair	No	?
Readily started for intermittent use	Yes	No	Yes	Yes	No	Yes	Yes	Yes	No
Low sulfur content	Yes	No	Yes	Yes	?	Yes	Yes	Yes	No
Utilizes present gas piping, controls, and burners	No	No	Yes	Yes	No	Yes	?	No	No
Fuel can be changed without upsetting furnace conditions			Yes	Yes		Yes	Yes	No	No
Suitable for low-temperature operations as well as high-temperature	Yes	No		Yes	Yes	Yes	Yes		No

25. FURNACE ATMOSPHERES

Furnace atmospheres in direct-fired furnaces are called "reducing," "neutral," or "oxidizing," depending on their chemical analysis and not on their actual reaction with the material treated in the furnace. The American Gas Association Testing Laboratory defines atmospheres as: *reducing*, $(CO + H_2) > 0.05\%$, $O_2 < 0.05\%$; *neutral*, $(CO + H_2) < 0.05\%$, $O_2 < 0.05\%$; and *oxidizing*, $(CO + H_2) < 0.05\%$, $O_2 > 0.05\%$. At high temperatures, these atmospheres are actually oxidizing because of the release of oxygen from the dissociation of carbon dioxide and water vapor. In practice, waste of fuel is reduced by use of gases that lend themselves readily to precipitation of carbon while the fuel is burning, creating a protective atmosphere.

Methods of Creating Protective Atmospheres. The burners most commonly used to create the carbon atmosphere are (1) diffusion flame gas burners, which keep a rich gas blanket near the material being heated and, in addition, precipitate carbon, (2) premix gas burners in combination with carbon gas burners, in which the carbon gas burners precipitate the carbon and the premix burners provide the required heat, and (3) oil burners, which make a very good carbon flame, but, owing to atomizing limitations, are not so reliable as gas burners.

Gases of higher calorific value are desirable for producing a carbon flame. Liquefied petroleum gases, natural gas, coke-oven gas, or carburetted-water gas can be readily used. Mixed coke-oven gas, blast-furnace gas, blue gas, and producer gas below 300 Btu per cu ft will not precipitate carbon unless preheated.

Purification. Steam, oxygen, carbon dioxide, and sulfur must be eliminated or compensated for before any atmosphere can be considered truly reducing. Sulfur in the form of H_2S is removed by purification with iron oxide. Organic sulfur may be removed by adsorption or by conversion to H_2S before purification. Water vapor can be removed by cooling and refrigeration or by removal with silica gel or activated alumina, etc. Free oxygen can be eliminated by cracking or reforming the gas above the ignition temperature of one of its components. Nitrogen is inert, except in the nascent condition, in most applications of furnace atmospheres.

Recirculation. Reducing-furnace atmospheres should be recirculated to achieve uniform operation and to obtain a thoroughly mixed gas in chemical equilibrium. The inside atmosphere of a furnace may be recirculated by compressing the fresh gas to about 10 psi and inspirating it at a suitable location above the material treated in the furnace. Ten volumes of the old gas may thus be recirculated to one of fresh gas entering.

Safety Note. Since a great many furnace atmospheres use large quantities of toxic constituents, such as carbon monoxide, every possible precaution should be taken to prevent contaminating the air in the room. This may require ventilating hoods at loading and unloading ends, if not over the entire furnace. Extreme care is also necessary to avoid explosive mixtures. It is recommended that a continuous pilot flame be provided on controlled-atmosphere furnaces to serve as a source of ignition at the furnace openings for prepared atmosphere gases high in carbon monoxide or other combustible constituents, to prevent escape of combustible atmosphere constituents into surrounding work areas. (See also Safe Operating Procedures for Different Types of Special Atmosphere Furnaces, by C. George Segeler, *Information Letter* 20, Metals Committee, Industrial Commercial Gas Section, American Gas Association, New York, Aug. 2, 1948. For additional discussion of furnace atmospheres, see *Gaseous Fuels*, L. Schnidman (Editor), American Gas Association, New York, 1948, pp. 203–230; *Furnace Atmospheres*, A. H. Fisher, C. M. Kemp Manufacturing Co., Baltimore, Md., presented to the Association of Iron and Steel Engineers, Philadelphia Section, Feb. 6, 1937; *Industrial Furnaces*, W. Trinks, Vol. II, 2nd ed., John Wiley and Sons, New York, 1942, pp. 171–205.)

15. H. Brückner, *Handbuch der Gasindustrie*, Vol. 6, *Technische Gase und deren Eigenschaften*, R. Oldenbourg, Munich and Berlin, 1937, pp. 125–130.
16. F. A. Smith, J. H. Eiseman, and E. C. Creitz, Instruments for the determination, indication or recording of the specific gravity of gases, *Natl. Bur. Standards (U. S.) Misc. Pub.* M-177, Jan. 20, 1947.
17. C. C. Wright, L. L. Newman, and A. W. Gauger, The resistance of anthracite to mechanical and thermal shock, *Penna. State Coll. Mineral Inds. Expt. Sta. Bull.* 31, 1941.
18. L. L. Newman, C. C. Wright, and A. W. Gauger, Test results on the use of anthracite in heavy-oil water-gas operation at the Pottsville Gas Works, *Penna. State Coll. Mineral Inds. Expt. Sta. Bull.* 32, 1941; L. L. Newman, C. C. Wright, and A. W. Gauger, Test results on the use of anthracite in gas-oil water-gas operation at the Carlisle Gas Works, *Penna. State Coll. Mineral Inds. Expt. Sta. Bull.* 36, 1942.
19. James E. Coleman, Fuels and their characteristics, *Information Letter* 22, Industrial Processing Committee, Subcommittee on Ceramics, American Gas Association, New York, Aug. 2, 1948.

GAS PRODUCERS

By L. L. Newman

26. GAS-PRODUCER ZONES AND FUELS

A gas producer converts solid fuel into combustible gas by blowing air or steam and air through a deep bed of incandescent carbon. The simplest gas producer is a vertical brick-lined or water-jacketed cylindrical vessel with a grate to support the fuel bed, an inlet for the blast (through the grate), an inlet for fuel at the top, and an outlet for gas at the top. This is an *updraft* producer; hydrogen, methane, illuminants, tars, and water vapor in varying quantities are distilled from freshly charged fuel and added to CO_2, CO, hydrogen, methane, and nitrogen, which result from reaction of the air, or steam and air, with the incandescent carbonaceous fuel bed.

REACTION ZONES. Figure 1 shows zones and their temperatures in an updraft producer for coal or coke fuels. Although reactions in a gas producer depend on physical and chemical properties of the fuel and on operating conditions, reactions taking place in *oxidation* and *reduction* zones may be assumed to be the same for all solid fuels. Carbon reacts with air and steam flowing up through the fuel bed, some or all of the reactions reaching a balance below limiting equilibrium conditions of individual reactions. The degree of approach to equilibrium depends on temperature, time of contact, reactivity of fuel, and presence of catalytic material.

Ash Zone. The air or steam and air mixture is preheated in passing through the *ash and clinker zones.*

In the oxidation zone, oxygen molecules that reach the fuel surface react with carbon and leave the particle surface mainly as CO if the temperature exceeds 1500 F. As long as free oxygen remains in the gas stream, a large part of the CO leaving the surface of fuel particles will be burned in voids between them to CO_2, evolving considerable heat (Ref. 1).

Steam reacts at a much slower rate, if at all. Hydrogen formed in the steam reaction is burned in the presence of free oxygen about 2.9 times as fast as CO in the range 1500 to 3000 F (Ref. 2).

In the reduction zone, CO_2 and steam are reduced to form CO and hydrogen, absorbing considerable heat. Some of the hydrogen reacts with carbon or CO to produce methane with evolution of heat.

In the preheating zone, the sensible heat of the gases preheats the coke. If coal is charged, the sensible heat of the gases rising from the *preheating zone* is used to carbonize the coal in the *carbonization zone* and finally to dry and preheat the incoming coal above this zone. If coke is used, the carbonization stage is eliminated.

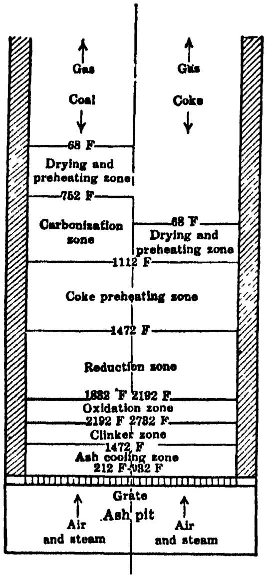


Fig. 1. Reaction zones in a gas producer. (Adapted from *Handbuch der Gasindustrie*, H. Bruckner, Vol. 2, R. Oldenbourg, Munich and Berlin, 1940)

LIMITS OF THE REACTION. All simple gas reactions are incomplete, though as a rule their progress can be observed only at high temperatures. Every chemical reaction will continue only until the reacting substances have each attained a definite relative concentration, corresponding to their temperature and total pressure. Thus, CO_2 and CO always will have the same concentration if left long enough together in the presence of carbon at the same temperature and pressure. When this equilibrium is disturbed by raising the temperature, some CO_2 will react with carbon to form CO, with the absorption of heat, which tends to diminish the temperature. In all chemical equilibria, when the temperature increases, the system will change so as to absorb heat, which tends to annul the temperature change (Le Châtelier principle). Thus, at high temperatures, since CO_2 combining with carbon to form CO absorbs heat, less CO_2 and more CO exist when the mixture is in equilibrium.

Table 1 gives the equilibrium constants that are of greatest importance in gas-producer operation.

It is evident from the equilibrium data that, as the temperature rises, the ratio of the concentration of CO to that of CO_2 and the ratio of the product of the concentrations of CO and steam to that of CO_2 and hydrogen will increase, and the ratio of the concentration of methane to that of hydrogen or the ratio of the product of the concentrations of methane and steam to that of CO and hydrogen will decrease. The ratio of the product of the concentration of CO and steam to that of CO_2 and hydrogen is unity at approximately 1500 F; that is, CO and hydrogen are equally strong reducing agents. At high temperatures hydrogen is the stronger reducing agent, and at low temperatures CO is the stronger.

Methods for calculating the composition of the gas that results from the reactions in a gas producer have been developed. The calculations are quite complicated, since in every real gasification process there are nine unknowns that require the solution of nine simultaneous equations. For a calculation method developed by Traustel based on the assumption that the Boudouard, water-gas, and methane equilibria are fully attained, see Ref. 3. For a somewhat less complicated method developed by John A. Goff, see Ref. 4.

FUEL FOR GAS PRODUCERS. Almost any kind and grade of fuel, including wood, peat, lignite, bituminous coal, anthracite, and coke, may be gasified in suitably designed producers. The choice is governed by local factors, such as cost and quality of gas required.

Bituminous Coal. If hot, raw gas can be used in large furnaces, bituminous coal is highly desirable. The coal must be crushed to pass through a 4-in. round test mesh screen, and may contain up to 35% fines below $^3/_4$-in. screen size, although mine-run or slack coals may be used, provided they can be handled by the mechanical charging equipment.

If hot, raw gas cannot be used, an elaborate condensing and scrubbing system is required for cooling, cleaning, and detarring gas made from bituminous coal. Anthracite and coke permit production of clean, cold gas with a minimum of condensing and scrubbing equipment.

Coke. In coke-fired gas producers, nut-size coke (passes through a 1 $^1/_4$-in. screen and remains on a $^3/_8$-in. screen) is preferred for high rates of gasification, but, because of lower cost, pea and breeze cokes containing less than 15% of material passing through a $^1/_8$-in. screen are frequently used.

Anthracite. The most widely used anthracite is buckwheat No. 2 (passes through a $^5/_{16}$-in. screen and remains on a $^3/_{16}$-in. round mesh screen). Buckwheat No. 1 (passes through a $^9/_{16}$-in. screen, remains on a $^5/_{16}$-in. screen) permits higher rates of gasification, is more expensive. Buckwheat No. 3 (passes through a $^3/_{16}$-in. screen, remains on a $^3/_{32}$-in.

Table 1. Values of Equilibrium Constants for Some Producer Gas Reactions

(Data from "Heats, free energies, and equilibrium constants of some reactions involving O_2, H_2, H_2O, C, CO, CO_2 and CH_4," Donald D. Wagman, John E. Kilpatrick, William J. Taylor, Kenneth S. Pitzer, and Frederick D. Rossini, *Research Paper* RP 1634, *J. of Research Natl. Bur. Standards*, Vol. 34, Feb. 1945, pp. 143–161.)

Temperatures—°K / °C / °F		900 / 627 / 1160	1000 / 727 / 1340	1100 / 827 / 1520	1200 / 927 / 1700	1300 / 1027 / 1880	1400 / 1127 / 2060	1500 / 1227 / 2240
Reaction	Equilibrium Constant	Values of Equilibrium Constants						
(3) $C + CO_2 = 2CO$	$K_3 = \frac{(CO)^2}{CO_2}$	.1926	1.900	12.20	57.09	208.3	628.6	1623
(6) $CO_2 + H_2 = CO + H_2O$	$K_6 = \frac{(CO)(H_2O)}{(CO_2)(H_2)}$	.4537	0.7278	1 059	1.436	1.840	2.270	2.670
(7) $C + 2H_2 = CH_4$	$K_7 = \frac{CH_4}{(H_2)^2}$	.3250	9.829×10^{-2}	3.677×10^{-2}	1.608×10^{-2}	7.932×10^{-3}	4.327×10^{-3}	2.554×10^{-3}
(8) $CO + 3H_2 = CH_4 + H_2O$	$K_8 = \frac{(CH_4)(H_2O)}{(CO)(H_2)^3}$	.7660	3.765×10^{-2}	3.192×10^{-3}	4.044×10^{-4}	7.003×10^{-6}	1.562×10^{-5}	4.248×10^{-6}

screen), without a suitable agitator, may be used only at very low gasification rates and requires considerable attention.

The ash-fusion temperature of bituminous coal, coke, or anthracite should exceed 2200 F.

27. DESIGN AND OPERATION OF GAS PRODUCERS

For efficient operation, the design must provide means for maintaining an *even fuel bed*, *uniform distribution of blast*, and *close control of steam* added to the blast under varying loads. An even fuel bed can be maintained by proper charging, spreading, and agitation of the fuel, and by correctly removing ash. Uniform blast distribution depends on design of the tuyères and homogeneity of fuel and ash beds. By regulation of the saturation temperature, the amount of steam added to the blast can be closely controlled. Accurate control is essential for maintaining the fire at the most suitable temperature for the ash-fusion characteristics of the fuel.

CHARGING THE FUEL. Fuel must be charged as continuously as possible and uniformly distributed over the cross section of the producer. It is more important that size distribution be uniform along each horizontal cross section than along vertical sections. With suitable fuel-charging equipment, uniform horizontal distribution may be obtained, even though segregation of the sizes received from storage bins may result in wide variations in average size distribution along vertical lines.

Bituminous-coal feeders are generally of the revolving-drum type, subdivided to admit small quantities at short intervals. Coke or anthracite generally is charged from hoppers with upper and lower valves which provide continuous feed from a magazine.

When closely sized coke or anthracite is used, fuel from the hopper may be distributed uniformly by (1) subdividing fuel charged from the hopper into feed pipes which supply several sections of the bed; (2) feeding fuel into a central magazine of adjustable height from which fuel flows continuously over the surface of the bed toward the circumference; (3) feeding fuel into a magazine of a ring-feed producer from which the fuel flows continuously over the surface of the bed from the circumference to the center; (4) charging the fuel through a double bell, which permits the operator to drop fuel either to the center or to the periphery, as required by the condition of the fire.

When the coke or anthracite varies in size, larger lumps roll away from the point of feed, while smaller lumps remain close to the point of feed; the result is uneven resistance to the blast. This may be overcome by increasing the number of feed pipes to subdivide the charge, or by using the "pants-leg" feed, which spreads fuel over the bed in three concentric circles by means of three spouts at different radii, attached to and rotating with the central magazine. Segregation of sizes in the ring-feed producer results in distribution of fines around the circumference. The coarse fuel remains in the center. This neutralizes the natural tendency of the gases to seek a path up the producer wall and distributes the gas flow more uniformly across the cross section of the fire.

AGITATION. Bituminous coal, if properly agitated, will descend through a producer so that all parts of the fuel are exposed to the blast. If caking coal is not agitated, the fuel particles will become plastic and agglomerate to a pasty mass of uneven resistance, producing blow holes, reduction of the effective area of the fuel bed, troublesome clinker formation due to excessive temperature in the blow holes, and poor gas quality.

Modern producers designed for bituminous coal provide for continuous mechanical agitation which reduces uneven temperatures, formation of clinker, and caking. The fuel can be prevented from caking by one of the following methods: (1) The shell or base of the producer may be rotated; (2) the entire depth of the fuel bed may be agitated by pokers; (3) the upper surface of the bed may be leveled; or (4) the bed may be leveled and agitated for a few inches below the surface.

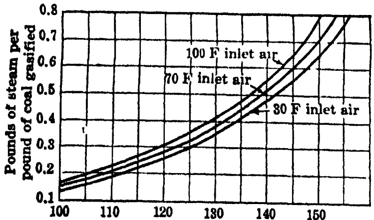


FIG. 2. Relation between steam used per pound of coal and blast saturation temperature. (Reprinted from Utilization of Producer Gas in Industrial Furnaces, D. B. Hendryx, *Trans. Am. Soc. Mech. Engrs.*, Vol. 68, No. 8, Nov. 1946)

CONTROL OF STEAM IN BLAST. The steam supplied to the blast may be obtained from a separate boiler, from the exhaust (steam) of the turboblower, from steam generated in the water jacket, or by the flow of the air supply over the surface of the hot water in fully water-jacketed producers. The amount of steam added to the air is controlled by maintaining the temperature of the air-steam mixture (blast saturation temperature) at a level suited to the character of the ash. Where the air is saturated over the surface of hot water in a fully jacketed producer, the saturation temperature is adjusted upward by decreasing the water circulation through the jacket to raise its temperature and vice versa. Figure 2 shows the relation between the amount of steam per pound of coal gasified and the blast saturation temperature.

DEPTH OF FUEL BED, determined by measurement or by observing the position of certain types of agitators, is adjusted to suit the character of the fuel and the load. With bituminous coal, the depth is adjusted to suit its coking characteristics. If a gummy condition develops, the depth of the bed is lowered, thus raising the temperature at the top enough to overcome the gummy condition. A free-burning coal provides more latitude in the choice of fuel-bed depth. Gas leaving a deep bed is cooler, has a less viscous tar, and has a higher calorific value than gas from a thin bed.

With coke or anthracite, the average depth of the fuel bed measured from the top of the grate may be limited only by the dimensions of the producer. Deeper fuel beds result in cooler gas with effectively lower velocities and smaller amounts of blown-over fuel. In producers with water-sealed ash pans, the blast pressure is limited by the depth of the water seal.

ASH REMOVAL. Homogeneity of the fire, uniformity of distribution, saturation of the blast, and condition of the ash are interdependent. A constant ash level is maintained by varying the rate of removal according to the gasification rate and ash content of the fuel. The depth of ash is governed by the requirement for keeping the grate cool and for distributing the blast.

A good mechanical grate must (Ref. 5) (1) maintain the lower part of the fuel bed in a steady, continuous, but *slow* movement, (2) not present sudden changes, as sudden projections near the fire zone are subject to heavy wear, (3) distribute the blast evenly over the fuel-bed surface, (4) maintain free and open air channels when the grate is revolved, and (5) crush large clinkers to small pieces before they reach the ash-removing appliance.

RATE OF GASIFICATION is limited by the amount of blast the fuel can stand before developing blow holes. Size and coking properties of the fuel, fusibility of the ash, and size and design of the producer have a marked effect on the gasification rate. When large, freeburning fuels are used, very high rates are obtainable (up to 90 lb of fuel per square foot of cross section). With smaller sizes, the gasification rate must be kept below the point where blow holes are formed and carry-over losses become excessive. If slack is used or if the fuel decrepitates when heated, slightly coking coal will yield a higher rate of gasification than noncoking coal, because the fines agglomerate into larger pieces and offer less resistance to the passage of the gas. With strongly coking coals, the gasification rate drops if agitation is not sufficient to prevent excessive coking and maintain an even fuel bed.

If the ash-fusion temperature is too low, gasification rate may be reduced by the need for lowering the combustion intensity to minimize the amount of clinker formed. In practice, the saturation temperature of the blast is increased to reduce clinker formation, but this increases the velocity of the gases, tends to form blow holes, and lowers the gasification rate. Moreover, too great an increase in the amount of steam in the blast lowers the gas quality. In automatic, high-capacity, gas-producer operation the reliability and continuity of operation is a prime consideration, and the steam rate is chosen to suit the fusion temperature of the ash, composition of the gas being fixed by this factor.

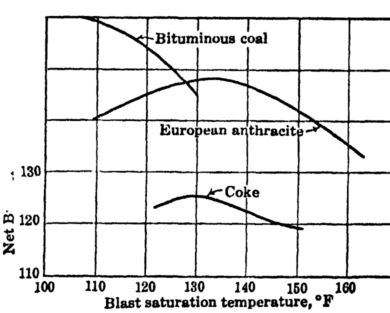
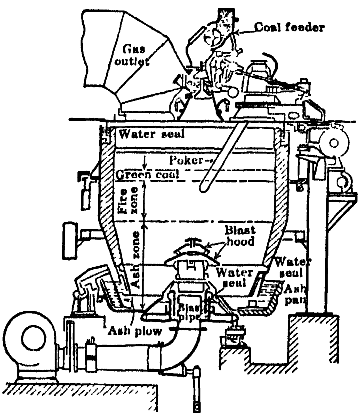


FIG. 3. Relation between heating value of gas and blast saturation temperature. (Bituminous coal, after D. B. Hendryx; European anthracite, through 5/8-in. screen, on 3/8-in. screen, 6.0 to 7.5% V.M. on dry basis, after H. R. Forman; coke, through 1-in. screen, on 1/2-in. screen, average 5% oversize, 6% undersize, after Chas. R. Locke)

Figure 3 shows the relation between the heating value of the gas and the blast saturation temperature for bituminous coal, anthracite, and coke. The curve for bituminous coal is based on a report by W. P. Chandler, Jr. (Ref. 6). The curve for anthracite was drawn from data in a paper by H. R. Forman (Ref. 7). The curve for coke is based on a curve in a report by C. R. Locke (Ref. 8).

TYPICAL PRODUCER. In the *Wellman* mechanical producer (Figure 4) the ash pan and shell revolve, and the top is stationary. The coal feed comprises two bells so timed that one always is closed gastight. The rate of feed is controlled by a variable-speed vane wheel operated by a ratchet wheel and an adjustable stroke crank. The water-cooled poker is set at an angle, with the point in advance, and swings in an arc from the center to the side of the wall. The semiplastic coal mass is thus gently lifted, making it porous for the free exit of the gas and closing holes and pipes. The blast hood is mounted on the ash pan and gives a uniform blast over the entire area of the producer. The ash pan is stopped three times in each revolution of the shell for approximately 21 degrees (total) to loosen up the ash bed. Scrapers and ejectors force the ashes to the outside of the pan where they are removed continuously by a stationary ash plow.



. Wellman gas producer. (Court The Wellman Engineering Co.)

28. GAS-PRODUCER AUXILIARY EQUIPMENT

A gas-producer plant, in addition to the generators, must include equipment for supplying fuel, air, steam, and cooling water; for removing ashes, and for cleaning and distributing the gas.

FUEL AND ASH HANDLING. Fuel may be delivered to overhead bunkers by a skip hoist or conveyor. Bins should be designed to minimize segregation of coarse and fine fuel. Ashes should be discharged to conveyors or sluice pits to eliminate hand labor. Where ashes accumulate in receivers beneath the grates, headroom should be provided to allow discharge directly into trucks or railroad cars.

AIR, STEAM, AND WATER SUPPLY. The air required for a producer ranges from 40 to 55 cu ft per lb of dry fuel, depending on the volatile content. In a 10-ft ID producer (standard), the air required at a gasification rate of 70 lb of fuel per square foot per hour ranges from 3700 to 5000 cfm. A blower 5000 to 5500 cfm capacity, operating at a pressure of 20 to 24 in. water, gage, and driven by a 40-hp motor or turbine, is used. Provisions for steam are made at the rate of 0.5 to 0.6 lb per lb of fuel gasified. The steam required by coke and anthracite producers generally is provided by evaporation of jacket water. In addition to jacket water, which is approximately 0.1 gal per lb of fuel gasified, approximately 200 gal of water per hour are needed in all bituminous-coal producers for cooling the poker, top, and coal feeder. This water may be recirculated. If a waste-heat boiler is used in coke operation, approximately 1 lb of steam is generated per pound of fuel gasified. Waste-heat boilers are not suitable for bituminous-coal operation, because of the sticky tar and dust deposited on the tubes.

CONTROLS. Each producer should be equipped with an air meter, a steam meter, a thermometer for saturation temperature, and a pyrometer for the gas offtake temperature. Gages and manometers should be provided for measuring pressures of air, steam, blast below the grate, and gas above the fuel bed. All instruments and controls should be on the operating floor close to the producer.

CLEANING AND DISTRIBUTION OF PRODUCER GAS. Hot, raw gas from bituminous coal leaving a producer contains tar in the form of vapor as well as dust and soot produced by decomposition of hydrocarbons. Weill (Ref. 9), referring to updraft producers, states that the gas contains 440 to 530 grains of tar per 100 cu ft, 130 to 220 grains of heavy dust per 100 cu ft, and 390 to 480 grains of soot per 100 cu ft. The tar, soot, and dust may remain in the gas if the furnace is near the producer; in fact, the tar adds to the calorific value of the gas and renders the flame luminous. Generally, a dust collector is installed near the producer outlet. The main principles involved in dry-dust separation are reduced gas velocity, impingement against baffles, and change of direction (Section 7, Art. 31). An efficient dry-dust collector removes 60 to 80% of the dust and soot in producer gas (Ref. 10). The gas offtake pipe, dust collector, and mains are brick-lined. The gas at 1200 F, including the steam, has about four times the volume of clean, cold gas. The actual velocities in a 3-ft ID offtake pipe are about 20 ft per sec when 30 lb of coal are gasified per square foot per hour and 50 ft per sec when the rate is 70 lb per hour. Even at these high velocities, much dust drops out and accumulates in horizontal mains, requiring frequent shut-downs for burning out. This is done by introducing air at intervals along the mains and removing the products of combustion through connections to a stack. Steeply inclined pipes with dust pockets at the lower ends, provided with a steam-jet vacuum system or sealed-in sluiceways, permit removal of the dust while the gas is flowing and eliminate frequent shut-downs for cleaning. The red-hot dust is dampened enough by the steam-jet vacuum system to extinguish the fire or is quenched by the stream of water in the sluice pit. Steam jets may be used to blow down any dust lying on the slopes. The dust and soot, ranging from 2 to 5% of the coal charged and containing 75 to 80% combustible material, may be reclaimed for boiler fuel. The dust catcher and the main must be sufficiently insulated to prevent the temperature of the gas from falling below the point where tars and other vapors begin to condense on cooling. The total temperature drop from producer to furnace ports must not exceed 300 F.

PROPERTIES OF COMBUSTION GASES

By Joseph Kaye and Joseph H. Keenan

29. PRODUCTS OF COMBUSTION

Properties of products of combustion are often required in engineering calculations. The following discussion and tables are limited to the products of combustion of hydrocarbon fuels with air.

It may be shown * that the number and scope of the required tables may be greatly simplified if each table is based on a *pound-mole* of mixture in place of a *pound* of mixture. In fact, only three major tables, for products of combustion with infinite, 400%, and 200% theoretical air, are sufficient to permit calculation of all processes encountered in design of a gas turbine over a wide range of hydrogen-carbon ratios of fuel and over the range of fuel-air ratios corresponding to lean mixtures. The first of these three major tables, that for infinite theoretical air, is the table for dry air converted from a basis of 1 lb to 1 lb-mole. Tables 1 and 2 for products of combustion with 400% and 200% theoretical air, respectively, based on a pound-mole of mixture, are condensed from *Gas Tables.* In these tables, the bar placed over a symbol denotes a molal quantity.

In Tables 1 and 2, T = temperature, degrees R (degrees F absolute); t = temperature, degrees F; $\bar{h}$ = enthalpy, Btu per pound-mole; p_r = relative pressure; $\bar{u}$ = internal energy, Btu per pound-mole; v_r = relative volume; ϕ = specific entropy, Btu per pound-mole degrees F.

* J. Kaye, ASME Journal of Applied Mechanics, Dec. 1948.

Table 1. Products—400% Theoretical Air

For 1 lb-mole

(Condensed from *Gas Tables*, by J. H. Keenan and J. Kaye, John Wiley and Sons, New York, 1948)

T, °F abs	t, °F	$\bar{h}$, Btu/lb-mole	p_r	$\bar{u}$, Btu/lb-mole	v_r	$\bar{\phi}$ Btu/lb-mole °F	T, °F abs	t, °F	$\bar{h}$, Btu/lb-mole	p_r	$\bar{u}$, Btu/lb-mole	v_r	$\bar{\phi}$ Btu/lb-mole °F
300	−160	2086	0.1726	1490	18650	42.24	1400	940	10095	45.6	7315	330	53.31
320	−140	2226	0.2166	1590	15850	42.69	1420	960	10251	48.2	7431	316	53.42
340	−120	2365	0.2681	1690	13610	43.11	1440	980	10406	50.9	7546	304	53.53
360	−100	2505	0.328	1790	11780	43.51	1460	1000	10562	53.8	7663	292	53.64
380	−80	2645	0.397	1890	10280	43.89	1480	1020	10719	56.7	7780	280	53.74
400	−60	2785	0.475	1990	9030	44.25	1500	1040	10876	59.8	7987	269	53.85
420	−40	2925	0.565	2091	7980	44.59	1520	1060	11033	63.0	8015	259	53.96
440	−20	3065	0.665	2191	7100	44.92	1540	1080	11191	66.4	8132	249	54.06
460	−0	3206	0.778	2292	6340	45.23	1560	1100	11349	69.9	8251	240	54.16
480	20	3346	0.905	2393	5690	45.53	1580	1120	11507	73.5	8369	231	54.26
500	40	3487	1.046	2494	5130	45.82	1600	1140	11666	77.3	8488	222	54.36
520	60	3627	1.202	2595	4640	46 09	1620	1160	11825	81.2	8608	214	54.46
540	80	3768	1.374	2696	4220	46.36	1640	1180	11984	85.4	8727	206	54.56
560	100	3909	1.563	2797	3840	46.61	1660	1200	12144	89.6	8847	198.8	54.65
580	120	4050	1.771	2899	3510	46.86	1680	1220	12304	94.0	8968	191.7	54.75
600	140	4192	2.00	3000	3220	47.10	1700	1240	12464	98.6	9088	184.9	54.84
620	160	4334	2.25	3102	2960	47.33	1720	1260	12625	103.4	9210	178.5	54.94
640	180	4475	2.52	3204	2730	47.56	1740	1280	12786	108.4	9331	172.2	55.03
660	200	4618	2.81	3307	2520	47.78	1760	1300	12948	113.6	9453	166.3	55.12
680	220	4759	3.12	3409	2340	47.99	1780	1320	13109	118.9	9574	160.7	55.22
700	240	4902	3.47	3512	2167	48.20	1800	1340	13272	124.4	9697	155.2	55.31
720	260	5044	3.84	3614	2014	48 40	1820	1360	13434	130.2	9820	150.0	55.40
740	280	5187	4.23	3718	1876	48 59	1840	1380	13597	136.2	9943	145.0	55.48
760	300	5330	4.66	3821	1750	48.78	1860	1400	13760	142.4	10067	140.2	55.57
780	320	5474	5.12	3925	1635	48.97	1880	1420	13924	148.8	10190	135.6	55.66
800	340	5618	5.61	4029	1531	49.15	1900	1440	14087	155.4	10314	131.2	55.75
820	360	5762	6.13	4133	1435	49.33	1920	1460	14251	162.3	10438	127.0	55.83
840	380	5906	6.70	4238	1346	49.50	1940	1480	14416	169.4	10563	122.9	55.92
860	400	6050	7.29	4343	1265	49.67	1960	1500	14580	176.7	10688	119.0	56.00
880	420	6195	7.93	4448	1191	49.84	1980	1520	14745	184.3	10813	115.3	56.09
900	440	6340	8.61	4553	1122	50.00	2000	1540	14910	192.2	10939	111.7	56.17
920	460	6486	9.33	4659	1058	50.16	2020	1560	15076	200.4	11064	108.2	56.25
940	480	6632	10.10	4765	999	50.32	2040	1580	15242	208.8	11190	104.9	56.33
960	500	6778	10.92	4872	944	50.47	2060	1600	15408	217.4	11317	101.7	56.41
980	520	6925	11.78	4979	893	50.62	2080	1620	15574	226.4	11443	98.6	56.49
1000	540	7072	12.69	5086	845	50.77	2100	1640	15740	236	11570	95.6	56.57
1020	560	7220	13.66	5194	801	50.92	2120	1660	15908	245	11698	92.7	56.65
1040	580	7367	14.68	5302	760	51.06	2140	1680	16074	255	11824	90.0	56.73
1060	600	7516	15.77	5411	721	51.20	2160	1700	16242	265	11952	87.4	56.81
1080	620	7664	16.91	5519	686	51.34	2180	1720	16409	276	12080	84.8	56.89
1100	640	7813	18.12	5628	652	51.48	2200	1740	16577	287	12208	82.3	56.96
1120	660	7962	19.38	5738	620	51.61	2220	1760	16745	298	12337	80.0	57.04
1140	680	8112	20.72	5848	590	51.75	2240	1780	16914	309	12465	77.7	57.12
1160	700	8262	22.1	5958	562	51.88	2260	1800	17082	321	12594	75.5	57.19
1180	720	8412	23.6	6069	536	52.01	2280	1820	17251	334	12723	73.3	57.26
1200	740	8563	25.2	6180	512	52.13	2300	1840	17420	346	12852	71.3	57.34
1220	760	8715	26.8	6292	488	52.26	2320	1860	17589	359	12982	69.3	57.41
1240	780	8867	28.5	6404	467	52.38	2340	1880	17758	373	13112	67.4	57.48
1260	800	9019	30.3	6517	446	52.50	2360	1900	17928	386	13241	65.5	57.56
1280	820	9171	32.2	6629	426	52.62	2380	1920	18098	401	13372	63.7	57.63
1300	840	9324	34.2	6742	408	52.74	2400	1940	18268	415	13502	62.0	57.70
1320	860	9478	36.3	6856	390	52.86	2420	1960	18438	430	13633	60.3	57.77
1340	880	9631	38.4	6970	374	52.97	2440	1980	18609	446	13763	58.7	57.84
1360	900	9786	40.7	7085	358	53.09	2460	2000	18780	462	13894	57.2	57.91
1380	920	9940	43.1	7199	344	53.20	2480	2020	18950	478	14025	55.7	57.98

Table 1. Products—400% Theoretical Air—*Continued*

T, °Fabs	t, °F	$\bar{h}$, Btu/lb-mole	p_r	$\bar{u}$, Btu lb-mole	v_r	$\bar{\phi}$ Btu/lb-mole °F	T, °Fabs	t, °F	$\bar{h}$, Btu/lb-mole	p_r	$\bar{u}$, Btu/lb-mole	v_r	$\bar{\phi}$, Btu/lb-mole °F
2500	2040	19121	495	14157	54.2	58.05	3000	2540	23457	1097	17499	29.4	59.63
2520	2060	19293	512	14288	52.8	58.12	3020	2560	23632	1130	17635	28.7	59.69
2540	2080	19464	530	14420	51.4	58.18	3040	2580	23808	1163	17771	28.0	59.74
2560	2100	19636	548	14552	50.1	58.25	3060	2600	23984	1197	17907	27.4	59.80
2580	2120	19808	567	14684	48.8	58.32	3080	2620	24160	1232	18043	26.8	59.86
2600	2140	19980	586	14816	47.6	58.38	3100	2640	24336	1268	18179	26.2	59.92
2620	2160	20152	606	14949	46.4	58.45	3120	2660	24512	1305	18316	25.7	59.97
2640	2180	20324	627	15082	45.2	58.52	3140	2680	24688	1342	18453	25.1	60.03
2660	2200	20497	647	15215	44.1	58.58	3160	2700	24865	1381	18589	24.6	60.08
2680	2220	20670	669	15348	43.0	58.65	3180	2720	25041	1420	18726	24.0	60.14
2700	2240	20843	691	15481	41.9	58.71	3200	2740	25218	1460	18863	23.5	60.20
2720	2260	21016	714	15614	40.9	58.77	3220	2760	25395	1502	19000	23.0	60.25
2740	2280	21189	737	15748	39.9	58.84	3240	2780	25572	1544	19137	22.5	60.31
2760	2300	21363	760	15882	39.0	58.90	3260	2800	25748	1586	19275	22.0	60.36
2780	2320	21536	785	16016	38.0	58.96	3280	2820	25926	1630	19412	21.6	60.42
2800	2340	21710	810	16149	37.1	59.03	3300	2840	26103	1675	19550	21.1	60.47
2820	2360	21884	836	16284	36.2	59.09	3320	2860	26280	1721	19687	20.7	60.52
2840	2380	22058	862	16418	35.4	59.15	3340	2880	26458	1768	19825	20.3	60.58
2860	2400	22232	889	16553	34.5	59.21	3360	2900	26636	1816	19963	19.9	60.63
2880	2420	22407	916	16688	33.7	59.27	3380	2920	26814	1865	20101	19.4	60.68
2900	2440	22581	945	16822	32.9	59.33							
2920	2460	22756	974	16958	32.2	59.39							
2940	2480	22931	1004	17093	31.4	59.45							
2960	2500	23106	1034	17228	30.7	59.51							
2980	2520	23281	1065	17363	30 0	59.57							

EXAMPLE 1. The products of combustion of benzene with 200% of theoretical air expand in a turbine from an initial temperature of 1500 F absolute and an initial pressure of 10 atm to an exit pressure of 1 atm. The efficiency of the turbine is 80%, based on the isentropic work of expansion. Calculate the turbine work per pound of products.

Solution. The composition of benzene is C_6H_6. A molal products table based on a fuel composition of $(CH_2)_n$ will yield precise results for the products of combustion of a hydrocarbon with the same composition as benzene. Using Table 2 for 200% theoretical air, one obtains for the isentropic expansion:

$$T_1 = 1500 \text{ Fabs}, \quad h_1 = 11050 \text{ Btu/lb-mole}, \quad p_{r1} = 63.9$$

$$p_{r2} = 63.9 \times 1/10 = 6.39$$

$$T_{2s} = 825.4 \text{ Fabs}, \quad h_{2s} = 5863 \text{ Btu/lb-mole},$$

where subscript 1 refers to the inlet of the turbine and subscript $2s$ refers to a state at the exit for isentropic expansion.

The work per pound of products is given by

$$W = \frac{0.8(h_1 - h_{2s})}{M} \quad \frac{0.8(11050 - 5863)}{29.445} = 140.9 \text{ Btu/lb}$$

where M is the molecular weight of the products of combustion for benzene with 200% theoretical air.

Table 2. Products—200% Theoretical Air

For 1 lb-mole

(Condensed from *Gas Tables*, by J. H. Keenan and J. Kaye, John Wiley and Sons, New York, 1948)

T, °F abs	t, °F	$\bar{h}$, Btu/lb-mole	p_r	$\bar{u}$, Btu/lb-mole	v_r	$\bar{\phi}$ Btu/lb-mole °F	T, °F abs	t, °F	$\bar{h}$, Btu/lb-mole	p_r	$\bar{u}$, Btu/lb-mole	v_r	$\bar{\phi}$, Btu/lb-mole °F
300	−160	2097	0.1677	1501	19200	42.18	1400	940	10251	48.4	7470	310	53.43
320	−140	2238	0.2107	1602	16300	42.63	1420	960	10410	51.2	7590	298	53.54
340	−120	2378	0.2612	1703	13970	43.06	1440	980	10569	54.2	7710	285	53.65
360	−100	2519	0.320	1804	12070	43.46	1460	1000	10729	57.3	7830	274	53.76
380	−80	2660	0.388	1906	10520	43.84	1480	1020	10890	60.5	7950	262	53.87
400	−60	2801	0.466	2007	9220	44.21	1500	1040	11050	63.9	8071	252	53.98
420	−40	2943	0.554	2109	8140	44.55	1520	1060	11211	67.4	8193	242	54.09
440	−20	3085	0.654	2211	7220	44.88	1540	1080	11373	71.1	8315	232	54.19
460	0	3227	0.766	2313	6440	45.20	1560	1100	11535	74.9	8437	224	54.30
480	20	3369	0.892	2416	5770	45.50	1580	1120	11697	78.9	8559	215	54.40
500	40	3511	1.033	2518	5190	45.79	1600	1140	11860	83.1	8682	206.6	54.50
520	60	3654	1.189	2621	4690	46.07	1620	1160	12023	87.4	8806	198.8	54.60
540	80	3796	1.362	2724	4260	46.34	1640	1180	12186	92.0	8929	191.4	54.70
560	100	3939	1.553	2827	3870	46.60	1660	1200	12350	96.7	9054	184.2	54.80
580	120	4083	1.762	2931	3530	46.85	1680	1220	12514	101.6	9178	177.5	54.90
600	140	4226	1.99	3035	3230	47.09	1700	1240	12679	106.7	9303	171.0	55.00
620	160	4370	2.24	3139	2970	47.33	1720	1260	12844	112.0	9428	164.8	55.10
640	180	4514	2.52	3243	2730	47.56	1740	1280	13009	117.5	9554	158.9	55.19
660	200	4659	2.81	3348	2520	47.78	1760	1300	13175	123.3	9680	153.2	55.29
680	220	4803	3.14	3452	2330	48.00	1780	1320	13340	129.2	9806	147.8	55.38
700	240	4948	3.49	3558	2154	48.21	1800	1340	13507	135.4	9932	142.6	55.47
720	260	5093	3.86	3663	1999	48.41	1820	1360	13674	141.9	10060	137.7	55.57
740	280	5238	4.27	3768	1859	48.61	1840	1380	13840	148.5	10186	132.9	55.66
760	300	5384	4.71	3874	1731	48.80	1860	1400	14008	155.5	10315	128.4	55.75
780	320	5530	5.18	3981	1615	48.99	1880	1420	14176	162.6	10443	124.0	55.84
800	340	5676	5.69	4088	1509	49.18	1900	1440	14344	170.1	10571	119.9	55.93
820	360	5823	6.23	4195	1412	49.36	1920	1460	14512	177.8	10700	115.9	56.02
840	380	5970	6.82	4302	1323	49.54	1940	1480	14681	185.8	10829	112.0	56.10
860	400	6118	7.44	4410	1241	49.71	1960	1500	14850	194.1	10958	108.4	56.19
880	420	6265	8.10	4517	1166	49.88	1980	1520	15020	202.7	11088	104.8	56.28
900	440	6413	8.81	4626	1096	50.05	2000	1540	15189	212	11218	101.4	56.36
920	460	6562	9.56	4734	1032	50.21	2020	1560	15359	221	11348	98.2	56.44
940	480	6710	10.37	4844	973	50.37	2040	1580	15530	230	11478	95.0	56.53
960	500	6860	11.22	4953	918	50.53	2060	1600	15700	240	11609	92.0	56.61
980	520	7010	12.13	5064	867	50.68	2080	1620	15871	250	11741	89.2	56.69
1000	540	7160	13.09	5174	820	50.83	2100	1640	16042	261	11872	86.4	56.78
1020	560	7310	14.11	5285	776	50.98	2120	1660	16214	272	12004	83.7	56.86
1040	580	7461	15.19	5396	735	51.13	2140	1680	16386	283	12136	81.1	56.94
1060	600	7613	16.33	5508	696	51.27	2160	1700	16558	295	12268	78.6	57.02
1080	620	7764	17.54	5620	661	51.42	2180	1720	16730	307	12401	76.2	57.10
1100	640	7916	18.82	5732	627	51.56	2200	1740	16902	319	12534	74.0	57.18
1120	660	8069	20.17	5845	596	51.69	2220	1760	17076	332	12667	71.7	57.26
1140	680	8222	21.60	5958	567	51.83	2240	1780	17249	345	12800	69.6	57.33
1160	700	8376	23.1	6072	539	51.96	2260	1800	17422	359	12934	67.6	57.41
1180	720	8529	24.7	6186	513	52.09	2280	1820	17595	373	13068	65.6	57.49
1200	740	8684	26.3	6301	489	52.22	2300	1840	17769	388	13202	63.7	57.56
1220	760	8839	28.1	6416	466	52.35	2320	1860	17943	403	13336	61.8	57.64
1240	780	8994	29.9	6531	444	52.48	2340	1880	18118	418	13471	60.1	57.71
1260	800	9149	31.9	6647	424	52.60	2360	1900	18292	434	13606	58.4	57.79
1280	820	9305	33.9	6763	405	52.72	2380	1920	18467	450	13741	56.7	57.86
1300	840	9462	36.0	6880	387	52.84	2400	1940	18642	467	13876	55.1	57.93
1320	860	9619	38.3	6997	370	52.96	2420	1960	18817	485	14012	53.6	58.01
1340	880	9776	40.6	7115	354	53.08	2440	1980	18993	503	14147	52.1	58.08
1360	900	9934	43.1	7233	339	53.20	2460	2000	19169	521	14283	50.6	58.15
1380	920	10092	45.7	7351	324	53.32	2480	2020	19344	540	14419	49.3	58.22

Table 2. Products—200% Theoretical Air—*Continued*

T, °Fabs	t, °F	$\bar{h}$, Btu/lb-mole	p_r	$\bar{u}$, Btu/lb-mole	v_r	$\bar{\phi}$ Btu/lb-mole °F	T, °Fabs	t, °F	$\bar{h}$, Btu/lb-mole	p_r	$\bar{u}$, Btu/lb-mole	v_r	$\bar{\phi}$, Btu/lb-mole °F
2500	2040	19521	560	14556	47.9	58.29	3200	2740	25806	1708	19451	20.10	60.51
2520	2060	19697	580	14693	46.6	58.36	3220	2760	25988	1758	19594	19.66	60.56
2540	2080	19874	601	14830	45.4	58.43	3240	2780	26171	1809	19736	19.22	60.62
2560	2100	20050	622	14967	44.2	58.50	3260	2800	26353	1861	19879	18.80	60.68
2580	2120	20228	644	15104	43.0	58.57	3280	2820	26536	1914	20023	18.39	60.73
2600	2140	20405	667	15241	41.8	58.64	3300	2840	26719	1968	20166	17.99	60.79
2620	2160	20582	690	15379	40.8	58.71	3320	2860	26902	2024	20309	17.60	60.84
2640	2180	20760	714	15517	39.7	58.78	3340	2880	27086	2081	20453	17.23	60.90
2660	2200	20938	738	15655	38.7	58.84	3360	2900	27269	2139	20597	16.86	60.95
2680	2220	21115	763	15793	37.7	58.91	3380	2920	27453	2198	20741	16.50	61.01
2700	2240	21294	789	15932	36.7	58.97	3400	2940	27636	2259	20884	16.15	61.06
2720	2260	21472	816	16070	35.8	59.04	3420	2960	27820	2321	21029	15.81	61.12
2740	2280	21651	843	16210	34.9	59.11	3440	2980	28004	2385	21173	15.48	61.17
2760	2300	21829	871	16348	34.0	59.17	3460	3000	28188	2450	21317	15.16	61.22
2780	2320	22009	900	16488	33.1	59.24	3480	3020	28373	2516	21462	14.84	61.28
2800	2340	22188	930	16627	32.3	59.30	3500	3040	28557	2584	21606	14.54	61.33
2820	2360	22367	960	16767	31.5	59.36	3520	3060	28741	2653	21751	14.24	61.38
2840	2380	22546	991	16906	30.7	59.43	3540	3080	28926	2724	21896	13.95	61.43
2860	2400	22726	1023	17046	30.0	59.49	3560	3100	29110	2797	22041	13.66	61.49
2880	2420	22906	1056	17186	29.3	59.55	3580	3120	29295	2870	22186	13.39	61.54
2900	2440	23086	1090	17327	28.6	59.62	3600	3140	29480	2946	22331	13.11	61.59
2920	2460	23266	1124	17468	27.9	59.68	3620	3160	29665	3023	22476	12.85	61.64
2940	2480	23447	1160	17608	27.2	59.74	3640	3180	29850	3102	22621	12.60	61.69
2960	2500	23627	1196	17749	26.6	59.80	3660	3200	30035	3182	22767	12.34	61.74
2980	2520	23808	1233	17890	25.9	59.86	3680	3220	30221	3264	22913	12.10	61.79
3000	2540	23988	1271	18031	25.32	59.92	3700	3240	30406	3347	23058	11.86	61.84
3020	2560	24170	1311	18172	24.73	59.98	3720	3260	30592	3433	23204	11.63	61.89
3040	2580	24351	1351	18314	24.16	60.04	3740	3280	30777	3520	23350	11.40	61.94
3060	2600	24532	1392	18455	23.60	60.10	3760	3300	30963	3609	23496	11.18	61.99
3080	2620	24714	1434	18597	23.05	60.16	3780	3320	31149	3700	23642	10.97	62.04
3100	2640	24895	1477	18739	22.53	60.22	3800	3340	31335	3792	23788	10.75	62.09
3120	2660	25077	1521	18881	22.02	60 28	3820	3360	31521	3887	23935	10.55	62.14
3140	2680	25259	1566	19023	21.52	60.34	3840	3380	31707	3983	24082	10.35	62.19
3160	2700	25441	1612	19166	21.03	60.39	3860	3400	31893	4081	24228	10.15	62.24
3180	2720	25623	1660	19308	20.56	60.45	3880	3420	32080	4182	24375	9.96	62.28

EXAMPLE 2. The products of combustion of benzene with 300% theoretical air flow through a heat exchanger and drop in temperature from 1200 to 800 F absolute at a constant pressure of 1 atm Calculate the heat transferred per pound of products.

Solution. Linear interpolation between the molal tables for 200 and 400% theoretical air is necessary to obtain the answer for 300% theoretical air. The table given below presents the values obtained from the two products tables.

Products for 400% air	Products for 200% air
$T_1 = 1200$ Fabs	$T_1 = 1200$ Fabs
$h_1 = 8563$ Btu/lb-mole	$h_1 = 8684$ Btu/lb-mole
$T_2 = 800$ Fabs	$T_2 = 800$ Fabs
$h_2 = 5618$ Btu/lb-mole	$h_2 = 5676$ Btu/lb-mole
$h_1 - h_2 = 2945$ Btu/lb-mole	$h_1 - h_2 = 3008$ Btu/lb-mole

Since 300% theoretical air is equivalent to 33.33% theoretical fuel, linear interpolation with respect to the percentage of fuel leads to Q, the heat transferred per pound of mixture,

$$Q = \frac{\left(\dfrac{33.33 - 25.0}{50.0 - 25.0}\right)(3008 - 2945) + 2945}{29.289} = 101.3 \text{ Btu/lb}$$

where 29.289 is the molecular weight of the mixture.

Molal specific heats at constant pressure and constant volume are shown in Tables 3, 4, and 5, as well as the ratio of these specific heats for products of combustion with 400, 200, and 100% theoretical air, respectively, and for three compositions of hydrocarbon fuels. The effect of variation of composition of the fuel is small so that for engineering calculations one need select only one fuel composition, such as $(CH_2)_n$, to cover a large range of fuel composition.

Table 3. Products of Combustion for 400% Theoretical Air

(Condensed from *Gas Tables*, by J. H. Keenan and J. Kaye, John Wiley and Sons, New York, 1948)

T, °F abs	$(CH_1)_n$			$(CH_2)_n$			$(CH_3)_n$		
	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k
400	6.996	5.010	1.396	7.003	5.017	1.396	7.007	5.021	1.395
800	7.195	5.210	1.381	7.189	5.203	1.382	7.185	5.199	1.382
1200	7.566	5.580	1.356	7.555	5.570	1.357	7.548	5.562	1.357
1600	7.954	5.968	1.333	7.943	5.957	1.333	7.936	5.950	1.334
2000	8.275	6.289	1.316	8.267	6.281	1.316	8.262	6.276	1.316
2400	8.512	6.526	1.304	8.508	6.522	1.304	8.505	6.519	1.305
2800	8.693	6.707	1.296	8 693	6.707	1.296	8.693	6.707	1.296
3200	8.834	6.848	1.290	8.838	6.852	1.290	8.841	6.855	1.290
3600	8.946	6.961	1.285	8 953	6.968	1.285	8.958	6.973	1.285
4000	9.040	7 054	1.282	9.050	7.064	1.281	9.057	7.071	1.281

Table 4. Products of Combustion for 200% Theoretical Air

(Condensed from *Gas Tables*, by J. H. Keenan and J. Kaye, John Wiley and Sons, New York, 1948)

T, °F abs	$(CH_1)_n$			$(CH_2)_n$			$(CH_3)_n$		
	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k
400	7.058	5.072	1.391	7.071	5.085	1.391	7 079	5.093	1.390
800	7.338	5.352	1.371	7.324	5.338	1.372	7.314	5.328	1.373
1200	7.749	5.763	1.345	7.726	5.740	1.346	7.710	5.724	1.347
1600	8.166	6.180	1.321	8.143	6.157	1.323	8.126	6.140	1.323
2000	8.512	6.526	1.304	8.493	6.508	1.305	8.481	6.495	1.306
2400	8.768	6.783	1.293	8.757	6.771	1.293	8.749	6.764	1.294
2800	8.965	6.979	1.285	8.962	6.976	1.285	8.959	6.973	1.285
3200	9.118	7.132	1.278	9.121	7 136	1.278	9.124	7.138	1.278
3600	9.238	7.252	1.274	9 248	7.262	1.273	9.255	7.269	1.273
4000	9.336	7.351	1 270	9 352	7.366	1.270	9.362	7.377	1.269

Table 5. Products of Combustion for 100% Theoretical Air

(J. Kaye, *J. Applied Mechanics* [ASME], Dec. 1948)

T, °F abs	$(CH_1)_n$			$(CH_2)_n$			$(CH_3)_n$		
	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k	$\bar{c}_p$, Btu/lb-mole °F	$\bar{c}_v$, Btu/lb-mole °F	k
400	7.180	5.195	1.382	7.200	5.215	1.381	7.214	5.228	1.380
800	7.614	5.628	1.353	7.580	5.594	1.355	7.557	5.571	1.356
1200	8.104	6.118	1.325	8.050	6.064	1.327	8.014	6.028	1.329
1600	8.578	6.592	1.391	8.522	6.536	1.304	8.483	6.497	1.306
2000	8.972	6.986	1.284	8.923	6.938	1.286	8.890	6.905	1.288
2400	9.266	7.280	1.273	9.231	7.245	1.274	9.208	7.222	1.275
2800	9.493	7.507	1.265	9.472	7.486	1.265	9.458	7.472	1.266
3200	9.667	7.682	1.259	9.660	7.674	1.259	9.655	7.669	1.259
3600	9.803	7.817	1.254	9.807	7.821	1.254	9.810	7.824	1.254
4000	9.912	7.926	1.251	9.926	7.941	1.250	9.936	7.950	1.250

SECTION 3

HEAT AND HEAT EXCHANGE

By

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THERMAL UNITS AND PROPERTIES

HEAT TRANSMISSION

By C. O. Mackey

HEAT INSULATION

By C. F. Kayan

ENGINEERING THERMODYNAMICS

By C. O. Mackey

THERMODYNAMICS OF GASES AT HIGH VELOCITY

By Neil P. Bailey

EVAPORATORS AND EVAPORATION

By W. L. Badger

DRYING AND DRYING MACHINES

By F. E. Finch

THERMAL UNITS AND PROPERTIES

1. UNITS OF HEAT MEASUREMENT

Several units for measuring the quantity of heat in a body are in use. The relations between them are given in Table 1.

Table 1. Relation of the Various Units of Heat

	Btu	Kg-cal	Gram-cal	Mean cal	Ostwald cal	Lb-cal
1 Btu =	1.0	0.252	252.0	251.93	2.5193	0.55555
1 kilogram-calorie =	3.968	1.0	1000.	999.76	9.9976	2.2044
1 gram-calorie =	0.003968	0.001	1.0	0.999658	0.00996	0.002204
1 mean calorie =	0.003969	0.00100024	1.00024	1.0	0.0099991	0.00220499
1 Ostwald calorie =	0.3969	0.100024	100.024	100.	1.0	0.220499
1 pound-calorie =	1.8	0.4536	453.6	453.474	4.53474	1.0

The British thermal unit (Btu) generally used in engineering work in the United States and Great Britain is $^1/_{180}$ of the heat required to raise the temperature of 1 lb of water from 32 to 212 F. Originally it was defined as the quantity of heat required to raise 1 lb of water from 62 to 63 F, but the former definition is now generally accepted.

The kilogram-calorie or large calorie (kg-cal) is the heat required to raise the temperature of 1 kg of water from 14.5 to 15.5 C, or $^1/_{100}$ of the heat required to raise 1 kg of water from 0 to 100 C.

The gram-calorie, small calorie, or 15° calorie (g-cal), generally used in scientific work, is the heat required to raise 1 gram of water from 14.5 to 15.5 C.

Table 2. Mechanical Equivalent of Heat

	Ft-lb	Kg-meter	Joules Int.	Joules Abs.	Btu	G-cal	Watt-hr, int.	Hp-hr
1 ft-lb =	1000 $\times 10^{-3}$	138.26 $\times 10^{-3}$	1355.4 $\times 10^{-3}$	1355.8 $\times 10^{-3}$	1.2849 $\times 10^{-3}$	323.79 $\times 10^{-3}$	0.3765 $\times 10^{-3}$	0.50505 $\times 10^{-6}$
1 Kg-meter =	723.30 $\times 10^{-2}$	100 $\times 10^{-2}$	980.37 $\times 10^{-2}$	980.67 $\times 10^{-2}$	0.92938 $\times 10^{-2}$	234.20 $\times 10^{-2}$	0.27233 $\times 10^{-2}$	0.36530 $\times 10^{-5}$
1 Joule, int. =	737.78 $\times 10^{-3}$	102.00 $\times 10^{-3}$	1000 $\times 10^{-3}$	1000.3 $\times 10^{-3}$	0.9480 $\times 10^{-3}$	238.89 $\times 10^{-3}$	0.27778 $\times 10^{-3}$	0.37262 $\times 10^{-6}$
1 Joule, abs. =	737.56 $\times 10^{-3}$	101.97 $\times 10^{-3}$	999.7 $\times 10^{-3}$	1000 $\times 10^{-3}$	0.9477 $\times 10^{-3}$	238.82 $\times 10^{-3}$	0.27769 $\times 10^{-3}$	0.37251 $\times 10^{-6}$
1 Btu =	778.26	107.60	1054.9	1055.2	1	252.00	0.29302	0.39307 $\times 10^{-3}$
1 g-cal =	308.84 $\times 10^{-2}$	42.70 $\times 10^{-2}$	418.61 $\times 10^{-2}$	417.73 $\times 10^{-2}$	0.3968 $\times 10^{-2}$	100 $\times 10^{-2}$	0.11628 $\times 10^{-2}$	0.15598 $\times 10^{-5}$
1 watt-hr, int. =	2656.0	367.21	3600 *	3601.1	3.41275	860 *	1	1.3414 $\times 10^{-3}$
1 Hp-hr =	1980.0 $\times 10^{3}$	273.75 $\times 10^{3}$	2683.7 $\times 10^{3}$	2684.5 $\times 10^{3}$	2.5441 $\times 10^{3}$	641.01 $\times 10^{3}$	0.74548 $\times 10^{3}$	1

* Exact value, by definition.

The mean calorie (mean cal) is $^1/_{100}$ of the heat required to raise the temperature of 1 gram of water from 0 to 100 C.

The Ostwald calorie (Ostwald cal), frequently used in electrochemistry, is the heat required to raise the temperature of 1 gram of water from 0 to 100 C.

The pound-calorie (lb-cal) is the heat required to raise the temperature of 1 lb of water 1 C.

The mechanical equivalent of heat is the number of foot-pounds of mechanical energy equivalent to one British thermal unit, heat and mechanical energy being mutually convertible. 1 Btu = 778.26 ft-lb, which is $^1/_{180}$ of the change in total heat along the saturated liquid water line from 32 to 212 F. Table 2 gives the mechanical equivalent of heat as expressed in various units.

ABSOLUTE TEMPERATURE. The absolute zero of a gas is a theoretical consequence of the law of expansion by heat, assuming that it is possible to continue the cooling of a perfect gas until its volume is diminished to zero.

The volume of a perfect gas increases 1/273.1 of its volume at 0 C for every increase of temperature of 1 C, and decreases 1/273.1 of its volume at 0 C for every decrease of temperature of 1 C. At −273.1 C the volume would then be reduced to zero. This point, −273.1 C = −459.6 F, or 491.6 F below the temperature of melting ice, is called the absolute zero; absolute temperatures are measured on either the centigrade or the Fahrenheit scale, from this zero. The freezing point, 32 F, corresponds to 491.6 F, absolute. If p_0 = pressure, and v_0 = volume of a perfect gas at 32 F = 491.6 F absolute, = T_0, and p = pressure and v = volume of the same weight of gas at any other absolute temperature T,

$$\frac{pv}{p_0v_0} = \frac{T}{T_0} = \frac{t + 459.6}{491.6}; \qquad \frac{pv}{T} = \frac{p_0v_0}{T_0} = R$$

A cubic foot of dry air at 32 F at the sea level (barometer = 29.921 in. Hg) weighs 0.080728 lb. The volume of one pound is 1/0.080728 = 12.387 cu ft. The pressure is 2116.3 lb per sq ft.

$$R = \frac{p_0v_0}{T_0} = \frac{2116.3 \times 12.387}{491.6} = \frac{26{,}214}{491.6} = 53.32$$

2. SPECIFIC HEAT

THERMAL CAPACITY. The thermal capacity of a body between two temperatures T_0 and T_1 is the quantity of heat required to raise the temperature from T_0 to T_1.

SPECIFIC HEAT. The specific heat of any substance is the ratio of the heat required to raise the temperature of a unit weight of that substance 1 degree and the quantity of heat required to raise the temperature of the same weight of water 1 degree, usually from 62 to 63 F.

DETERMINATION OF SPECIFIC HEAT. A hot body of known weight and temperature, immersed in a mass of liquid of known specific heat, weight, and temperature, will raise the temperature of the liquid until both liquid and body are at the same temperature. If c_1 and c_2 = specific heat, w_1 and w_2 = weight, t_1 and t_2 = temperature, of the hot body and the liquid respectively, and if T = final temperature of both, then $c_1 = c_2w_2(T - t_1) \div w_1(t_1 - T)$.

Another method consists of determining the amount of electrical energy required to raise the temperature of a unit weight of the substance 1° in 1 min, converting the result into heat units on the basis of 1 watt = 0.05685 Btu per min.

SPECIFIC HEATS OF VARIOUS SUBSTANCES. The specific heats of the chemical elements and of the more commonly used gases, liquids, and solids of engineering are given in Tables 3 to 7. Except where otherwise noted, these tables are based upon the physical tables published by the Smithsonian Institution.

The specific heat of many substances varies with temperature and probably is greater when the substance is liquid than when solid. In a number of cases the specific heat at several temperatures is given. Where a range of temperature is specified, the mean specific heat for that range is given.

Table 3. Specific Heat of the Chemical Elements

(For gases see Table 4; for liquids see Table 5)

Element	Temp., °F	Specific Heat	Element	Temp., °F	Specific Heat	Element	Temp., °F	Specific Heat
Aluminum	32	.2089	Indium	32–212	0.0570	Rubidium	32	0.0802
Aluminum	932	.2739	Iodine	48–208	0.0541	Ruthenium	32–212	.0611
Aluminum	61–579	.2250	Iridium	54–212	0.0323	Selenium	−306 to	
Antimony	59	.0489	Iron *	99	0.1092		+64	.068
Antimony	212	.0503	Iron	1832–2192	0.1989	Silicon	135	.1833
Antimony	392	.0520	Iron	32–1112	0.1396	Silicon	450	.2029
Arsenic (gray)	32–212	.0822	Iron	32–1472	0.1597	Silver	32–212	.0559
Arsenic (black)	32–212	.0861	Iron	32–1832	0.1557	Silver	212	.05663
Barium	−121		Iron (cast)	68–212	0.1189	Silver	63–945	.05987
	to +68	.068	Iron (wrought)	59–212	0.1152	Sodium	−301 to	.253
Bismuth	68–212	.0302	Lanthanum	32–212	0.0448		+68	
Boron	32–212	.307	Lead	61–493	0.0319	Sulfur	−306 to	.137
Bromine (solid)	−108		Lithium	212	1.0407		+64	
	to −4	.0843	Magnesium	68–212	0.2492	Sulfur (rhombic)	32–129	.1728
Barium (fluid)	55–113	.107	Manganese	68–212	0.1211	Sulfur (liquid)	246–297	.235
Cadmium	212	.0570	Mercury	−121 to	0.032	Tantalum	−301 to	.033
Cesium	32–79	.0482		+68			+68	
Calcium	32–358	.170	Mercury	212	0.03284	Tantalum	2552	.043
Carbon	52	.160	Molybdenum	68–212	0.0647	Tellurium	59–212	.0483
Carbon	1789	.467	Nickel	64–212	0.109	Thallium	68–212	.0326
Carbon	3146	.50	Nickel	212	0.1128	Thorium	32–212	.0276
Cerium	32–212	.0448	Nickel	932	0.1299	Tin (cast)	70–228	.0551
Chlorine (liquid)	32–75	.2262	Nickel	1832	0.1608	Tin (fluid)	482	.05799
Chromium	32	.1039	Osmium	66–208	0.0311	Tin (fluid)	2012	.0758
Chromium	212	.1121	Palladium	32–2309	0.0714	Titanium	32–212	.1125
Chromium	1112	.1872	Phosphorus			Tungsten	32–212	.0336
Cobalt	932	.1542	(red)	32–124	0.1829	Tungsten	1832	.0337
Cobalt	1832	.204	Phosphorus			Uranium	32–208	.028
Copper	59–450	.0951	(yellow)	55–97	0.202	Vanadium	32–212	.1153
Copper	912	.0942	Platinum	68–212	0.0319	Zinc	68–212	.0931
Copper	1652	.1259	Platinum	68–2372	0.0359	Zinc	212	.0951
Gallium	54–235	.080	Potassium	−301 to	0.170	Zinc	572	.1040
Germanium	32–212	.0737		+68		Zirconium	32–212	.0660
Gold	32–212	0.0316	Rhodium	50–207	0.0580			

* The following mean specific heats were obtained in experiments on the specific heat of nearly pure iron.

Temp., °F	500	600	800	1000	1200	1300
Specific heat	0.1228	0.1266	0.1324	0.1388	0.1462	0.1601
Temp., °F	1500	1800	2100	2400	2700	
Specific heat	0.1698	0.1682	0.1667	0.1662	0.1666	

The specific heat increases steadily between 500 and 1200 F. Then it increases rapidly to 1400 F, after which it remains nearly constant.

Table 4. Specific Heat of Gases and Vapors

Substance	Temperature Range, °F	Specific Heat at Constant Pressure	Mean Ratio of c_p/c_v*	Substance	Temperature Range, °F	Specific Heat at Constant Pressure	Mean Ratio of c_p/c_v*
Acetone, C_2H_6O	79–230	.3468	. . . . .	Chloroform, $CHCl_3$	72–172	0.1489	1.150
Air †	−22 to +50	.2394	1.398	Ether, $C_4H_{10}O$	156–435	0.4797	1.029
Air	68–824	.2469	1.384	Hydrochloric acid, HCl	55–212	0.1940	1.389
Air	68–1472	.2562	1.366	Hydrogen	70–212	3.4100	1.419
Alcohol, C_2H_5OH	226–428	.4534	1.133	Hydrogen sulfide, H_2S	68–403	0.2451	1.324
Alcohol, CH_3OH	214–433	.4580	1.256				
Ammonia	73–212	.5202	1.3172	Methane, CH_4	64–406	0.5929	1.316
Argon	68–194	.1233	1.667	Nitrogen	68–824	0.2419	1.405
Benzene, C_6H_6	95–356	.3325	1.403	Nitric oxide, NO	55–342	0.2317	1.394
Blast-furnace gas	. . . .	.2277	. . . .	Nitrous oxide, N_2O	61–405	0.2262	1.3111
Bromine	181–442	.0555	1.293	Oxygen	55–405	0.2175	1.3977
Carbon dioxide	52–417	.2169	1.3003	Sulfur dioxide, SO_2	61–396	0.1544	1.256
Carbon monoxide	79–388	.2426	1.395	Water vapor	32	0.4655	1.274
Carbon disulfide	187–374	.1596	1.205	Water vapor	212	0.421	1.33
Chlorine	61–649	.1125	1.336	Water vapor	356	0.51	1.305

* c_p and c_v = specific heat at constant pressure and constant volume, respectively.
† See also p. 1-06.

Table 5. Specific Heat of Liquids

Substance	Temperature, °F	Specific Heat	Substance	Temperature, °F	Specific Heat
Alcohol (ethyl)	104	.648	Oils: Turpentine	32	.411
Alcohol (methyl)	59–68	.601	Petroleum	69.8–136.4	.511
Aniline	59	.514	Potassium hydrate, $KOH + 30H_2O$	64.4	.876
Benzole, C_6H_6	104	.423	Sea water, sp. gr. 1.0043	63.5	.980
Calcium chloride, $CaCl_2$, sp. gr. 1.14	104	.787	Sea water, sp. gr. 1.0235	63.5	.938
Calcium chloride, $CaCl_2$, sp. gr. 1.26	104	.676	Sea water, sp. gr. 1.0463	63.5	.903
Copper sulfate, $CuSO_4 + 50H_2O$	53.6–59	.848	Sodium hydrate, $NaOH + 50H_2O$	64.4	.942
Ethyl ether	32	.529	Sodium chloride, $NaCl + 10H_2O$	64.4	.791
Glycerin	59–122	.576	Sodium chloride, $NaCl + 200H_2O$	64.4	.978
Naphthalene, $C_{10}H_8$	176–185	.396	Toluol, C_6H_8	149	.490
Nitrobenzole	57.2	.350	Water: See Section 4	. . . .	. . . .
Oils: Castor	. . . .	.434	Zinc sulfate, $ZnSO_4 + 50H_2O$	68–125.6	.842
Citron	42	.438			
Olive	44	.471			

Table 6. Specific Heat of Sodium Chloride Solutions

Temp, °F	Parts NaCl by Weight in 100 Parts of Solution							
	2	4	6	8	10	12	16	20
32	.966	.944	.923	.904	.885	.869	.840	.814
60	.971	.949	.929	.909	.891	.874	.844	.817
100	.980	.958	.936	.916	.899	.881	.851	.823
140	.986	.964	.942	.922	.903	.885	.855	.827

Table 7. Specific Heat of Various Solids

(For specific heat of solid chemical elements, see Table 3.)

Substance	Temperature, °F	Specific Heat	Substance	Temperature, °F	Specific Heat
Alloys:			Hematite, Fe_2O_3	59–210	.1645
Bell metal	59–208	.0858	Ice	0 to − 108	.463
Brass (red)	32	.0899	India rubber	?–212	.481
Brass (yellow)	32	.0883	Kaolin	68–208	.224
Cu 80, Sn 20	57–208	.0862	Lava	77–212	.197
Cu 88.7, Al 11.3	68–212	.1043	Limestone	59–212	.216
German silver	32–212	.0946	Magnetite	64–113	.156
Sb 37.1, Pb 62.9	50–208	.0388	Marble	32–212	.21
Bi 63.8, Sn 36.2	68–210	.0400	Mica	68	.10
Asbestos	68–208	.195	Paraffin	95–104	.622
Borax (fused)	51–208	.2382	Paraffin (fluid)	140-145	.712
Brick	32–212	.22	Porcelain	32–212	.22
Concrete	32–212	.156	Pyrites (copper)	59–210	.1291
Cork	32-212	.485	Quartz, SiO_2	54–212	.188
Corundum	42–208	.1976	Quartz, sand	68–208	.191
Dolomite	68–208	.222	Rock salt	55–113	.219
Earth	32-212	.44	Sandstone	32-212	.25
Galena, PbS	32–212	.0466	Talc	68–208	.2092
Glass (crown)	50–122	.161	Vulcanite	68–212	.3312
Glass (flint)	50–122	.117	Wood:		
Gneiss	63–210	.196	Fir	32–212	.65
Granite	54–212	.192	Oak	32 212	.57
Graphite	32–212	.201	Pine	32–212	.67

Table 8. Melting and Boiling Points, °F, of the Chemical Elements

Element	Melting Point	Boiling Point	Element	Melting Point	Boiling Point	Element	Melting Point	Boiling Point
Aluminum	1217.66	3272.	Indium	311.	...	Radium	1292.	......
Antimony	1166.0	2624.	Iodine	236.3	<392.	Rhodium	3542.	...
Argon	−306.4	−303.	Iridium †	4262.?	..	Rubidium	100.4	1284.8
Arsenic	1562.	680.*	Iron	2786.	4442.	Ruthenium †	4442.	......
Barium	1562.	......	Krypton	−272.2	−241.	Samarium	2372.–2552.	..
Beryllium	2336.	.	Lanthanum †	1490.?	.....	Selenium	422.6–428.0	1274
Bismuth	519.8	2606.	Lead	620.6 ± 4.5	2777.	Silicon	2588.	.
Boron †	3992.–4532.?	....	Lithium	366.8	2552.	Silver	1760.9	3551.
Bromine	−18.86	142.	Magnesium	1203.8	2048.	Sodium	207.5	1382.
Cadmium	609.62	1432.4	Manganese	2246.0	3452.		235.	
Cesium	78.8	1238.	Mercury	−37.98	674.6	Sulfur ‡	246.6	832.5
Calcium	1490.		Molybdenum	4595.	6548.		224.2	
Carbon	<6332.*	6512.	Neodymium †	1544.?	......	Tantalum	5252.	..
Cerium	1184.	....	Neon	−423.?	−398.2	Tellurium	845.6	2534.
Chlorine	−150.7	−28.5	Nickel	2645.	....	Thallium	575.6	2336.
Chromium	2939.	3992.	Niobium	3092.?	...	Thorium	<3092.	
Cobalt	2696.	..	Nitrogen	−347.8	−319.	Tin	449.4 ± .4	4118.
Copper	1981.4 ± 5.4	4190.	Osmium †	4892.?	..	Titanium	3263.	
Fluorine	−369.4	−304.6	Oxygen	−360.4	−296.9	Tungsten	6152.	10,526.
Gallium	86.18	......	Palladium	2820. ± 9	...	Uranium	>3362.	
Germanium	1756.4	.....	Phosphorus	111.6	550 4	Vanadium	3128.	....
Gold	1945.4	.	Platinum	3191. ± 9	7070.	Xenon	−220.	−164.4
Helium	>−455.8	−448.6	Potassium	144.1	1313.6	Yttrium	2714.	.
Hydrogen	−434.2	−422.6	Praseodium	1724.	....	Zinc	786.9	1706.
						Zirconium †	3092.?	....

* Sublimes.

† Value uncertain. Temperatures above the melting point of platinum may be 100 F in error.

‡ Melting-point temperatures are for various forms of sulfur.

3. LATENT HEAT, BOILING, AND FUSION

LATENT HEAT is the quantity of heat required to change the state, as solid or liquid, of a body without rise of temperature.

Latent heat of fusion is the quantity of heat required, at the fusion temperature, to change a body from the solid to the liquid state, without change of temperature. When the body changes from the liquid to the solid state this same amount of heat is rejected to the atmosphere or other surrounding bodies. See Tables 13 and 14 for latent heats of various materials.

Table 9. Melting Points of Alloys, °F

ALLOYS OF TIN, LEAD, AND BISMUTH

	Percentage									
Lead	32.0	25.8	25.0	43.0	33.3	10.7	50.0	35.8	20.0	70.9
Tin	15.5	19.8	15.0	14.0	33.3	23.1	33.0	52.1	60.0	9.1
Bismuth	52.5	54.4	60.0	43.0	33.3	66.2	17.0	12.1	20.0	20.0
Solidifies at	204.8	213.8	257.0	262.4	293.0	298.4	321.8	357.8	359.6	453.2

LOW-MELTING-POINT ALLOY

	Percentage						
Cadmium	10.8	10.2	14.8	13.1	6.2	7.1	6.7
Tin	14.2	14.3	7.0	13.8	9.4	....	....
Lead	24.9	25.1	26.0	24.3	34.4	39.7	43.4
Bismuth	50.1	50.4	52.2	48.8	50.0	53.2	49.9
Solidifies at	149.9	153.5	155.3	155.3	169.7	193.1	203.0

Table 10. Melting and Boiling Points, °F, of Inorganic Compounds

Compound	Chemical Formula	Melting Point	Boiling Point
Aluminum oxide	Al_2O_3	3668.	....
Ammonia	NH_3	−103.	28.3
Ammonium sulfate	$(NH_4)_2SO_4$	284.	....
Borax	$Na_2B_4O_7$	1041.8	....
Calcium chloride	$CaCl_2$	1425.2	....
Carbon tetrachloride	CCl_4	−11.2	159.2
Hydrochloric acid	HCl	−168.3	−117.
Hydrofluoric acid	HFl	−134.14	−33.8
Mercurous chloride	Hg_2Cl_2	842±	....
Mercuric chloride	$HgCl_2$	539.6	581.
Nitric acid	HNO_3	−43.6	186.8
Potassium chlorate	$KClO_3$	701.6	....
Sodium chloride	NaCl	1472.	....
Sodium carbonate	Na_2CO_3	1565.6	....
Sodium sulfate	Na_2SO_4	1623.2	....
Sulfur dioxide	SO_2	−104.8	14.
Sulfuric acid	H_2SO_4	40.7	640.4
Zinc chloride	$ZnCl_2$	689.	1310.
Zinc sulfate	$ZnSO_4 + 7H_2O$	122.	....

Latent heat of vaporization is the quantity of heat required to change a liquid, at the boiling point, to a vapor under constant pressure, without change of temperature. When the substance is changed from vapor to liquid, under the same conditions of temperature and pressure, this same quantity of heat will be rejected from the substance. See Table 13 for latent heats of vaporization of various substances.

Table 11. Melting and Boiling Points of Organic Compounds

Compound	Chemical Formula	Melting Point, °F	Boiling Point, °F
Acetylene	C_2H_4	−113.8	−118.8
Acetic acid	$C_2H_4O_2$	62.0	244.4
Alcohol, ethyl	C_2H_6O	−173.2	172.4
Alcohol, methyl	CH_4O	−142.6	150.8
Aniline	C_8H_5N	17.6	363.0
Beeswax	....	143.6	.. ....
Benzine	C_6H_6	41.8	176.3
Benzoic acid	$C_7H_6O_2$	249.8	480.2
Camphor	$C_{10}H_6O$	348.8	408.2
Carbolic acid	C_6H_6O	109.4	359.6
Carbon disulfide	CS_2	−166.	115.
Carbon tetrachloride	CCl_4	−22.	170.
Chloroform	$CHCl_3$	−85.	142.1
Ether, ethyl	$C_4H_{10}O$	−180.4	94.3
Gasoline	....	....	158–194
Glycerin	$C_3H_8O_3$	68.	554.
Naphthalene	$C_6H_4 \cdot C_4H_4$	176.	424.4
Nitrobenzine	$C_6H_5O_2N$	41.	411.8
Olive oil	....	68±	572±
Oxalic acid	$C_2H_2O_4 2H_2O$	374.	. . .
Paraffin wax (soft)	....	100-125	662-734
Paraffin wax (hard)	....	125–133	734–806
Spermaceti	....	113±	....
Sugar (cane)	$C_{12}H_{22}O_{11}$	320.	....
Tallow (beef)	....	80–100	....
Tallow (mutton)	....	90–142	....
Tartaric acid	$C_4H_6O_6$	338.	.. ...
Toluene	C_7H_8	−133.6	230.5

ENTHALPY OF EVAPORATION is the heat required to evaporate one pound of a liquid at its boiling point and change it into saturated vapor at the same temperature.

For the total heat, latent heat, etc., of steam at different pressures, see table of the Properties of Steam, Section 4. For tables of total heat, latent heat, and other properties of vapors of ether, alcohol, acetone, chloroform, chloride of carbon, and bisulfide of carbon, see Röntgen's *Thermodynamics* (Dubois's translation). Tables of the properties of ammonia, carbon dioxide, sulfur dioxide, and other refrigerants are given in Section 11, Refrigeration and Ice Making.

Table 12. Increase in Temperature of Boiling by Salts in Solution

Salt	Rise in Boiling Point, °C									
	1	3	5	7	10	20	40	60	80	100
	Number of Parts of Salt Added to 100 Parts of Water									
$CaCl_2$	6 0	16 5	25.0	32.0	41.5	69.0	137.5	222.0	314.0	
KOH	4.7	13.6	20.5	26.4	34.5	57.5	92.5	121.7	152.6	185.0
KCl	9.2	23.4	36.2	48.4	.	... ..	......	....	......	
$MgSO_4 + 7H_2O$	41.5	138.0	262.0				. .	..	.	
NaOH	4.3	11.3	17.0	22.4	30.0	51.0	93.5	150.8	230.0	345.0
NaCl	6.6	17.2	25.5	33.5		. ..	.. ...	....	... ..	. .
NH_4Cl	6.5	19.0	29.7	39 6	56.2	...	...	......	... ...	..
NH_4NO_3	10.0	30.0	52.0	74.0	108.0	248.0	682.0	1370.0	2400.0	4099 0
$C_4H_6O_6$	17.0	52.0	87.0	123 0	177.0	374.0	980.0	3774.0	.... . .	.... .

RISE OF BOILING POINT OF SALT SOLUTIONS. The boiling point of salt solutions is raised as the strength of the solution increases. Table 12 shows the number of parts of various salts that must be added to 100 parts of water, to raise the boiling point the number of degrees given.

Table 13. Latent Heats of Vaporization

Substance	Boiling Point, °F	Latent Heat, Btu per lb	Substance	Boiling Point, °F	Latent Heat, Btu per lb
Acetic acid	244.4	152.82	Ethyl iodide	159.8	84.6
Alcohol (ethyl)	172.4	369.0	Heptane	194.	140.0
Alcohol (methyl)	150.8	480.6	Hexane	158.	142.6
Aniline	363.0	198.0	Iodine	.....	42.3
Benzine	176.3	167.2	Mercury *	674.6	117.0
Bromine	141.8	82.1	Octane	266.0	126.0
Carbon disulfide	115.0	150.8	Pentane	86.	154.4
Chloroform	142.1	105.3	Sulfur	600.8	651.6
Ether (ethyl)	94.3	159.1	Toluol	230.5	154.8
Ethyl bromide	100.7	108.7	Turpentine	318.8	133.3

* For data on Mercury, see Section 4, Art. 1, Table 1.

Table 14. Latent Heats of Fusion

Substance		Melting Point, °F	Latent Heat, Btu per lb	Substance	Melting Point, °F	Latent Heat, Btu per lb
Alloys:				Ice	32	143.35
Pb	Sn			(from sea water)	17.6	97.2
1	9	456.8	50.4 *	Lead	620.6	9.65
30.5	69.5	361.4	30.6	Mercury	−38.2	5.07
36.9	63.1	354.2	27.9	Naphthalene	175.8	64.11
63.7	36.3	351.5	20.88	Nickel	2615	133.0
77.8	22.2	349.7	17.17	Palladium	2813	65 34
Aluminum		1216.4	170.3	Phosphorus	111.6	8.94
Benzol		41.7	55.08	Platinum	3191	48.96
Bromine		18.9	29.16	Potassium	143.6	28.26
Bismuth		514.4	22.75	Paraffin	126.3	63.18
Cadmium		609.3	24.59	Silver	1761.8	37.92
Calcium chloride + $6H_2O$		85.3	73.26	Sodium	206.6	49.5
Copper		1981.4	75.6	Spermaceti	111.0	66.56
Iron, gray cast		......	41.4	Sulfur	239	16.87
Iron, white		......	59.4	Tin	449.6	25.2
Iron, slag		......	90	Wax	143.2	76.14
Iodine		......	21.08	Zinc	786.2	50.63

* Total heat from 32 F.

Table 15. Freezing Mixtures

Ingredient		Mixed with		Temp. before Mixture, °F	Temp. of Mixture, °F	Lowering of Temp., °F
	Parts		Parts			
Calcium chloride (crystals)	250	Water	100	51.4	11.1	40.3
Sodium hyposulfite (crystals)	110	Water	100	51.2	11.0	40.2
Potassium sulfate	10	Snow	100	30.2	28.6	1.6
Sodium carbonate (crystals)	20	Snow	100	30.2	28.4	1.8
Calcium chloride	30	Snow	100	30.2	12.4	17.8
Ammonium chloride	25	Snow	100	30.2	4.3	25.9
Ammonium nitrate	45	Snow	100	30.2	1.9	28.3
Sodium chloride	33	Snow	100	30.2	− 6.3	36.5
Sulfuric acid + water (H_2SO_4, 66.1%)	1	Snow	1.097	30.2	−34.6	64.8
Sulfuric acid + water (H_2SO_4, 66.1%)	1	Snow	13.08	30.2	3.2	27.0
Alcohol at 39.2 F	77	Snow	73	32.	−22.	54.0

4. THERMAL EXPANSION

EXPANSION OF AIR. In the centigrade scale the coefficient of expansion of air per degree is 0.003665 = $^1/_{273}$; that is, the pressure being constant, the volume of a perfect gas increases $^1/_{273}$ of its volume at 0 C for every increase in temperature of 1°. In Fahrenheit units it increases $^1/_{491.6}$ = 0.002034 of its volume at 32 F for every increase of 1 F.

Table 16. Expansion of Solids

Substance	t° F *	Coefficient of Linear Expansion, per °F †
Aluminum	M	.00001233
Aluminum	1112	.00001750
Antimony	M	.00000587
Bismuth	M	.00000731
Brass, cast	M	.00001042
Brass, wire	M	.00001072
Bronze (3 Cu, 1 Sn)	M	.00001024
Bronze (86.3 Cu, 9.7 Sn, 4 Zn)	104	.00000990
Cadmium	M	.00001755
Carbon, diamond	104	.00000066
Carbon, gas carbon	104	.00000300
Carbon, graphite	104	.00000437
Carbon, anthracite	104	.00001154
Caoutchouc	M	.00003650
		.00003811
Cobalt	104	.00000687
Constantan	39-84	.00000846
Copper	M	.00000926
Ebonite	77-95	.00004677
Fluorspar, CaF_2	M	.00001083
German silver	M	.00001020
Gold	M	.00000817
Gold-platinum (2 Au, 1 Pt)	M	.00000846
Gold-copper (2 Au, 1 Cu)	M	.00000862
Glass, tube	M	.00000463
Glass, plate	M	.00000495
Glass, crown	M	.00000498
Glass, flint	M	.00000438
Glass, Jena 16^{III}	M	.00000450
Glass, Jena 59^{III}	M	.00000322
Glass, quartz	61-1832	.00000032
Gutta-percha	68	.00011016
Ice	-4 to +30	.00002833
Indium	104	.00002317
Iridium	M	.00000489
Iron, soft	104	.00000672
Iron, cast	104	.00000589
Iron, wrought	0-212	.00000633
Iron, steel	104	.00000734
Iron, steel annealed	M	.00000608
Lead	M	.00001516
Lead-tin (2 Pb, 1 Sn)	M	.00001393
Magnalium	54 102	.00001322
Magnesium	104	.00001450
Manganin	....	.00001005
Marble	59-212	.00000650
Nickel	104	.00000566
Osmium	104	.00000365
Palladium	104	.00000653
Paraffin	32-61	.00005923
Paraffin	61-100	.00007238
Paraffin	100 120	.00026501

Substance	t° F *	Coefficient of Linear Expansion, per °F †
Phosphorus	32-104	.00006961
Platinum	104	.00000499
Platinum-iridium (10 Pt + 1 Ir)	104	.00000491
Platinum-silver (1 Pt + 2 Ag)	M	.00000846
Porcelain	68-1454	.00000229
Porcelain Bayeux	1832-2552	.00000307
Potassium	32-122	.00004607
Quartz (‖ to axis)	32-176	.00000443
Quartz (⊥ to axis)	32-176	.00000743
Rhodium	104	.00000472
Rock salt	104	.00002244
Rubber, hard	32	.00003839
Ruthenium	104	.00000535
Selenium	M	.00003669
Silicon	104	.00000424
Silver	104	.00001067
Sodium	32-194	.00012554
Speculum metal	M	.00001074
Sulfur	M	.00006556
Tellurium	M	.00002048
Thallium	104	.00001678
Tin	M	.00001275
Topaz, ‖ to lesser horiz. axis	M	.00000462
Topaz, ‖ to greater horiz. axis	M	.00000464
Topaz, ‖ to vertical axis	M	.00000262
Type metal	62-489	.00001084
Vulcanite	32 64	.00003533
Wedgwood ware	M	.00000494
Wood, ash ‡	M	.00000528
Wood, ash §	35.6-93.2	
Wood, beech ‡	35.6-93.2	.00000143
Wood, beech §	35.6-93.2	.00003411
Wood, chestnut ‡	35.6-93.2	.00000361
Wood, chestnut §	35.6-93.2	.00001806
Wood, elm ‡	35.6-93.2	.00000314
Wood, elm §	35.6-93.2	.00002461
Wood, mahogany ‡	35.6-93.2	.00000201
Wood, mahogany §	35.6-93.2	.00002244
Wood, maple ‡	35.6-93.2	.00000354
Wood, maple §	35.6-93.2	.00002689
Wood, oak ‡	35.6 93.2	.00000273
Wood, oak §	35.6-93.2	.00003022
Wood, pine ‡	35.6-93.2	.00000301
Wood, pine §	35.6-93.2	.00001894
Wood, walnut ‡	35.6-93.2	.00000366
Wood, walnut ‡	35.6-93.2	.00002689
Wax, white	50-79	.00012778
Wood, white	79-88	.00017333
Wax, white	88 109	.00027000
Wax, white	109-135	.00084594
Zinc	M	.00001653

* M = Mean coefficient, 32-212 F.

† Cubical expansion may be taken as (3 × linear expansion). Coefficient of expansion per °C. = coefficient per °F × 9/5.

‡ Parallel to fiber.

§ Across fiber.

EXPANSION OF SOLIDS, LIQUIDS, AND GASES. The coefficients of expansion of various solids, liquids, and gases are given in Tables 16 to 18. These tables are based on the tables compiled by the Smithsonian Institution, Washington, from various sources.

Table 17. Coefficients of Cubical Expansion of Liquids, per °F

Liquid	$C \times 10^3$	Liquid	$C \times 10^3$
Acetic acid	.595	Glycerin	.281
Acetone	.826	Hydrochloric acid (33.2% solution)	.253
Alcohol, amyl	.501	Mercury	.101
Alcohol, ethyl	.622	Olive oil	.401
Alcohol, methyl	.666	Pentane	.893
Benzine	.687	Potassium chloride (24.3% solution)	.196
Bromine	.629	Phenol	.606
Calcium chloride (5.8% solution)	.139	Petroleum (density 0.8467)	.531
Calcium chloride (40.9% solution)	.254	Sodium chloride (20.6% solution)	.230
Carbon disulfide	.677	Sodium sulfate (24% solution)	.228
Carbon tetrachloride	.687	Sulfuric acid (100%)	.310
Chloroform	.707	Turpentine	.541
Ether	.920	Water	See Section 4

Table 18. Coefficients of Expansion of Gases, per °F

(Pressures given are in centimeters of mercury.)

	Constant Volume		Constant Pressure	
Substance	Pressure	Coefficient	Pressure	Coefficient
Air	75.2	.00203667	76.0	.00203944
Air, 32–212°	100.1	.00204133	100.1	.00204044
Air	200.0	.00205016	257.0	.00205167
Argon	51.7	.00203778	.....	....
Carbon dioxide	76.0	.00204756	76.0	.00206111
Carbon dioxide, 32–212°	51.8	.00205400	51.8	.00205961
Carbon dioxide, 32–212°	99.8	.00207011	99.8	.00207833
Carbon monoxide	76.0	.00203706	76.0	.00203833
Helium	56.7	.00203611	.....	....
Hydrogen, 32–212°	76.4	.00202800	100.0	.00203333
Nitrogen, 32–212°	100.2	.00204133	..	....
Nitrous oxide	76.0	.00204222	76.0	.00206611
Oxygen	75.9	.00203783	.....	....
Sulfur dioxide	76.0	.00213611	76.0	.00216833
Water vapor, 32–392°	.....	....	76.0	.00218778

EXPANSION OF STEEL AT HIGH TEMPERATURES. Coefficients of expansion (for 1 C) of annealed carbon and nickel steels at temperatures at which there is no transformation of the steel are given in Table 19.

Table 19. Coefficients of Expansion of Steel at High Temperatures, per °C

Composition of Steels				Mean Coefficients of Expansion from			Coefficients between	
C	Mn	Si	P	15 to 200°	200 to 500°	500 to 650°		
0.03	0.01	0.03	0.013	11.8×10^{-6}	14.3×10^{-6}	17.0×10^{-6}	880 and 950°	24.5×10^{-6}
0.25	0.04	0.05	0.010	11.5	14.5	17.5	800 and 950°	23.3
0.64	0.12	0.14	0.009	12.1	14.1	16.5	720 and 950°	23.3
0.93	0.10	0.05	0.005	11.6	14.9	16.0	720 and 950°	27.5
1.23	0.10	0.08	0.005	11.9	14.3	16.5	720 and 950°	33.8
1.50	0.04	0.09	0.010	11.5	14.9	16.5	720 and 950°	36.7
3.50	0.03	0.07	0.005	11.2	14.2	18.0	720 and 950°	33.3

Nickel Steels			Mean Coefficients of Expansion from				
Ni	C	Mn	15 to 100°	100 to 200°	200 to 400°	400 to 600°	600 to 900°
26.9	0.35	0.30	11.0×10^{-6}	18.0×10^{-6}	18.7×10^{-6}	22.0×10^{-6}	23.0×10^{-6}
28.9	0.35	0.36	10.0	21.5	19.0	20.0	22.7
30.1	0.35	0.34	9.5	14.0	19.5	19.0	21.3
34.7	0.36	0.36	2.0	2.5	11.75	19.5	20.7
36.1	0.39	0.39	1.5	1.5	11.75	17.0	20.3
32.8	0.29	0.66	8.0	14.0	18.0	21.5	22.3
35.8	0.31	0.69	2.5	2.5	12.5	18.75	19.3
37.4	0.30	0.69	2.5	1.5	8.5	19.75	18.3
25.4	1.01	0.79	12.5	18.5	19.75	21.0	35.0
29.4	0.99	0.89	11.0	12.5	19.0	20.5	31.7
34.5	0.97	0.84	3.0	3.5	13.0	18.75	26.7

HEAT TRANSMISSION

By C. O. Mackey

THE FUNDAMENTAL HEAT TRANSFER PROCESSES are conduction, convection, and radiation. Evaporation and condensation are special forms or combinations of conduction and convection.

Conduction is the transmission of heat by molecular vibration from one part of a body to another or from one body to another body in direct contact with it.

Convection is the transfer of heat between a fluid and a surface by the circulation or mixing of the fluid. In free or natural convection the fluid motion is caused by gravity forces due to difference in density between the hotter and cooler parts; in forced convection the motion is produced artificially, as by a pump, blower, or other external forces not connected with the temperature of the fluid.

Radiation is the transmission of heat in the form of radiant energy or wave motion from one body to another across an intervening space. This term sometimes is used loosely to denote dissipation of heat from the outer surface of a furnace or pipe, which usually includes both radiation and convection.

Heat may be transmitted by any one of the three processes acting alone or by combinations acting in series or in parallel, as in the case of most practical applications. Although it is sometimes convenient to deal only with the combined or overall heat transfer, as from one fluid to another across a dividing wall, this sort of treatment does not bring out the effects of the individual components, and the data are not of such general utility as when these components are treated separately. Whenever the total or overall heat transmission is desired, care should be taken to include all the processes acting in conjunction, as shown herein.

SYMBOLS AND UNITS *

q = rate of heat transfer, Btu per hr
t = temperature, °F
θ = temperature difference, °F
A = area, sq ft
L = length, ft
r = radius, ft
D = diameter, ft
h = film coefficient of heat transfer, Btu/(hr)(sq ft)(°F)
R = thermal resistance, (hr)(sq ft)(°F)/Btu
V = velocity, ft per hr
w = weight of fluid flowing in unit time, lb per hr
G = mass velocity = w/A = ρV, lb/(hr)(sq ft)
k = thermal conductivity, Btu/(hr)(ft)(°F)
U = overall coefficient of heat transfer, Btu/(hr)(sq ft)(°F)
ρ = density, lb per cu ft
c = specific heat, Btu/(lb)(°F)
μ = absolute viscosity, lb/(hr)(ft)
a = thermal diffusivity = $k/\rho c$, sq ft per hr
β = coefficient of cubical expansion $(\partial V/V\,\partial T)_p$, F^{-1}
τ = time, hr
N_{Gr} = Grashof number = $(L^3\rho^2\beta g\theta)/\mu^2$, dimensionless when acceleration of gravity is expressed in ft per (hr)^2
N_{Gz} = Graetz number = wc/kL, dimensionless
N_{Nu} = Nusselt number = hD/k, dimensionless
N_{Pr} = Prandtl number = $\mu c/k$, dimensionless
N_{Re} = Reynolds' number = $DV\rho/\mu$ = DG/μ, dimensionless

* Many different systems of units will be found in the literature on heat transfer. Units shown constitute one consistent system, but there are others. Engineers should use consistent units unless dimensional constants are introduced into equations.

5. CONDUCTION

STEADY STATE. Conduction is in the *steady state* when the temperature gradient at a given point in the material remains constant with respect to time. The rate of steady conduction of heat in one direction perpendicular to the area A is given by $q = -kA(dt/dx)$. The temperature change in the direction of heat conduction is $-(dt/dx)$; the thermal conductivity, k, is a property of the material.

STEADY CONDUCTION. For the steady conduction of heat through a homogeneous material of thickness L feet with plane surfaces at temperatures t_1 and t_2, the rate of conduction of heat is

$$q = \frac{kA}{L}(t_1 - t_2)$$

The ratio k/L is called the *thermal conductance* of the material.

For the steady conduction of heat through n plane materials in intimate contact (no thermal resistances at the bounding surfaces), the rate of conduction of heat is

$$q = \frac{A(t_1 - t_{n+1})}{\dfrac{L_1}{k_1} + \dfrac{L_2}{k_2} + \cdots \dfrac{L_n}{k_n}}$$

The total thermal resistance of the n plane materials is the sum of the several individual thermal resistances, $\Sigma(L/k)$.

For the steady conduction of heat in a radial direction through a homogeneous cylinder of inside radius, r_1, and length, L, outside radius r_2, the rate of conduction of heat is

$$q = \frac{2\pi kL(t_1 - t_2)}{\ln \dfrac{r_2}{r_1}}$$

where ln is the natural logarithm (to the base e).

For the steady conduction of heat in a radial direction through n concentric, hollow cylinders of the same length in intimate contact (no thermal resistances at the bounding surfaces), the rate of conduction of heat is

$$q = \frac{2\pi L(t_1 - t_{n+1})}{\dfrac{1}{k_1}\ln\dfrac{r_2}{r_1} + \dfrac{1}{k_2}\ln\dfrac{r_3}{r_2} + \cdots \dfrac{1}{k_n}\ln\dfrac{r_{n+1}}{r_n}}$$

For consistency in this section, the thermal conductivity k is expressed in the units (Btu)(ft) per (hr)(sq ft)(°F) or, more simply, as Btu/(hr)(ft)(°F). Thermal conductivities of insulating materials are often expressed in the units of (Btu)(in.) per (hr)(sq ft)(°F). The thermal conductivity of a thickness of one inch of a material is twelve times the thermal conductivity of a thickness of one foot of the same material. In the cgs system of units, the units of thermal conductivity are (cal)(cm) per (sec)(sq cm)(°C) or, more simply, cal/(sec)(cm)(°C). To convert one cal/(sec)(cm)(°C) to the units of Btu/(hr)(ft)(°F), multiply by 241.9.

CONDUCTIVITY OF METALS AND ALLOYS. The thermal conductivity of many pure metals and of some alloys is directly proportional to the electrical conductivity. As with nonmetallic crystals, the thermal conductivity of pure metals is inversely proportional to absolute temperature. For most alloys (except ferrous metals) as for amorphous materials, the thermal conductivity increases slightly as the temperature increases.

The addition of a small amount of another metal, or the presence of an impurity, usually will result in a sharp drop in thermal conductivity of a pure metal, particularly when a solid solution is formed. Practically all the alloy steels have lower conductivities than wrought iron. When the percentage of added constituents (C, Co, Cr, Mn, Mo, Ni, Si, V, W, etc.) is small, the value of k usually is about 24 Btu/(hr)(ft)(°F), with lower values as the percentage increases. Thermal conductivities of some metals are given in Table 1.

CONDUCTIVITY OF NONMETALLIC SOLIDS. The conductivities of nonmetallic solid materials generally increase with increases in density, temperature, and moisture content. For porous or fibrous insulating materials, there may be an optimum density for lowest conductivity. Conductivities given in Table 2 are representative values for conditions of normal use. When reliable values of the thermal conductivity of dry, nonmetallic solids have not been obtained by measurement, approximate values may be estimated from these empirical equations:

Density, ρ, between 5 and 20 lb per cu ft: $k = 0.0137\rho^{0.3}$
Density, ρ, between 20 and 60 lb per cu ft: $k = 0.00118\rho^{1.13}$
Density, ρ, of 60 lb per cu ft and higher: $k = \left(\dfrac{\rho}{143}\right)^{2.5}$

Table 1. Thermal Conductivity k of Metals, Btu/(hr)(ft)(°F)

Metal	Temp., °F	k	Metal	Temp., °F	k
Aluminum	32	123	Manganin	32	12
	400	130		212	15
Antimony	32	10.6	Mercury	32	4.8
	212	9.7		120	4.6
Bismuth	64	4.7	Molybdenum	32	83
	212	3.9		212	80
Brass, yellow	68	64	Monel metal	90	20
	400	84			
Brass, red	32	60	Nichrome	90	8
	212	69			
Cadmium	32	54	Nickel	32	34
	212	52		400	32
Constantan	68	13.5	Platinum	32	40
	212	15.5		212	42
Copper	32	223	Platinoid	64	14.5
	400	217			
Gold	32	172	Potassium	68	56
	212	169			
Iron, pure	64	39	Rhodium	63	51
	212	37			
Iron, cast	32	32	Silver	32	238
	400	28		400	217
Iron, wrought	32	35	Sodium	32	79
	400	30		212	70
Steel (1% C)	32	26	Tantalum	32	31
	400	25		3092	42
Steel, stainless (18-8)	400	12	Tin	32	38
				212	35
Lead	32–212	20	Tungsten	64	115
				2912	60
Magnesium	32	91	Zinc	32	65
	400	85		212	63

Table 2. Thermal Conductivity k of Various Nonmetallic Solids, Btu/(hr)(ft)(°F)

(For conductivity of heat insulators, see p. 3-38; of building materials, see p. 12-04.)

Material	k	Material	k	Material	k
Carbon, graphite	2.9	Glass, pyrex	0.62	Sand, moist	0.7
Cardboard	0.15	Granite	1.25	Sandstone	1.0
Celluloid	0.12	Ice at 32 F	1.25	Sawdust	0.03
Chalk	0.48	Lime	0.07	Silicon carbide, powdered	0.12
Coke, powdered	0.11	Linoleum	1.0	Silk	0.03
Concrete, cinder	0.2	Mica	0.33	Slate, ⊥ to cleavage	0.8
Concrete, stone	0.7	Paper	0.08	Slate, ‖ to cleavage	1.5
Cotton	0.03	Porcelain	0.83	Snow, fresh	0.06
Earth's crust	0.97	Quartz, ⊥ to axis	3.9	Snow, old	0.3
Fiber, sheet	0.17	Quartz, ‖ to axis	6.8	Soil, dry	0.08
Firebrick	0.75	Rock salt	4.0	Soil, moist	0.83
Glass, window	0.3–0.6	Rubber, solid	0.1	Sulfur	0.15
Glass, crown	0.50	Rubber, sponge	0.03	Vulcanite	0.21
Glass, flint	0.48	Sand, dry	0.02	Wool	0.02

CONDUCTIVITY OF LIQUIDS. Water has the highest thermal conductivity of all the nonmetallic liquids. The thermal conductivity of most organic liquids is about 0.1 Btu per (hr)(ft)(°F); in general, the thermal conductivity of organic liquids decreases slightly with increasing temperature. For water, k increases to a maximum at about 270 F and then decreases until its value at 570 F is about the same as at 40 F. The conductivity of aqueous solutions generally is lower than that of water by an amount roughly proportional to the concentration of the solute. Thermal conductivities of miscellaneous liquids are given in Table 3.

Table 3. Thermal Conductivity k of Various Liquids, Btu/(hr)(ft)(°F)

Liquid	Temperature, °F	k	Liquid		Temperature, °F	k
Acetic acid	68	0.100	Oil, petroleum		55	0.09
Acetone	68	0.103	Oil, turpentine		68	0.073
Alcohol, methyl	68	0.124	Pentane (n)		68	0.078
Alcohol, ethyl	68	0.105	Petrolatum		68	0.11
Alcohol, amyl	68	0.085	Sulfur dioxide		68	0.195
Ammonia	5–86	0.29	Toluene		68	0.088
Aniline	32	0.104	Water		32	0.321
Benzene	68	0.098	Water		140	0.378
Carbon dioxide	68	0.121	Water		270	0.397
Carbon disulfide	68	0.092	Water		420	0.379
Chloroform	54	0.070				
Ether	68	0.079				
"Freon-12"	0–150	0.06–0.04	Solution	Specific Gravity		
Gasoline	86	0.08				
Glycerin	68	0.165	$CuSO_4$	1.160	40	0.294
Kerosene	68	0.09	KCl	1.026	55	0.281
Oil, castor	68	0.10	NaCl	1.178	50	0.278
Oil, lubricating	68	0.10	H_2SO_4	1.054	69	0.306
Oil, olive	39	0.10	$ZnSO_4$	1.134	40	0.292

CONDUCTIVITY OF GASES AND VAPORS. According to kinetic theory, thermal conductivity of a gas is proportional to the product of absolute viscosity and specific heat at constant volume, proportional to the square root of absolute temperature, and independent of pressure. This equation may be used to estimate the thermal conductivity of a gas at moderate temperatures and pressures when more precise data are not available:

$$k = b\mu c_v$$

where k = thermal conductivity, Btu/(hr)(ft)(°F); μ = absolute viscosity, lb/(hr)(ft); c_v = specific heat at constant volume, Btu/(lb)(°F); and b = a constant that depends chiefly upon the number of atoms in the gas molecule with approximate values as follows: for monatomic gases, 2.44; diatomic gases, 1.90; triatomic gases, 1.70; complex gases, 1.31.

The influence of temperature upon the thermal conductivity of some gases and vapors may be expressed as

$$k_T = k_{492}\left(\frac{492 + C}{T + C}\right)\left(\frac{T}{492}\right)^{1.5}$$

where C is a constant, depending on the gas.

Values for several gases and vapors are given in Table 4.

UNSTEADY STATE. Conduction is in the unsteady state when the temperature at a fixed point in the material changes with time. The basic differential equation for the unsteady conduction of heat is based upon a combination of the Fourier equation for conduction and a heat balance upon an element of volume; for a material of uniform *thermal diffusivity*, $a = k/\rho c$, and in terms of the rectangular coordinates of a point (x, y, z) in the material,

$$\frac{\partial t}{\partial \tau} = a\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right)$$

Many solutions of this basic equation for the heating or cooling of various shapes under different boundary conditions will be found in the references given at the end of this chapter.

An approximate solution for the unsteady conduction of heat in one direction only, often called the Schmidt method, is given here. Assume conduction of heat in one direction normal to a plane wall; replace the partial derivatives by small finite changes; then

$$\left(\frac{\Delta t}{\Delta \tau}\right)_x = a\left(\frac{\Delta^2 t}{\Delta x^2}\right)_\tau$$

Table 4. Thermal Conductivity of Gases and Vapors

$$k_T = k_{492}\left(\frac{492 + C}{T + C}\right)\left(\frac{T}{492}\right)^{1.5} \text{ Btu/(hr)(ft)(°F)}$$

Gas or Vapor	Temperature, °F	Thermal Conductivity, k	k_{492}	C
Air	32	0.014	.014	225
Ammonia	32 212	.012 .017		
Carbon dioxide	32 212	.0081 .011		
Carbon monoxide	32	.013	.013	281
Chlorine	32	.0042		
Ethane	32	.010		
Ethylene	32	.0095		
"Freon-12"	32	.0047	.0047	480
Helium	32	.081	.081	59
Hydrogen	32	.094	.094	169
Methane	32	.0175		
Methyl chloride	32	.0048		
Nitrogen	32	.014	.014	205
Oxygen	32	.014	.014	259
Steam, saturated	212 400 600	.014 .034 .11		
Sulfur dioxide	32	.0047		

The temperature of the material depends upon both time and position. Let the time be divided into equal increments of $\Delta\tau$, and the thickness of the material into slabs with equal increments of Δx. Use a double subscript on temperature, the first to denote time and the second, position. For example, $t_{n,m}$ denotes the temperature after the passage of a time ($n\Delta\tau$) from the reference time at a position ($m\Delta x$) from the reference surface of the wall. Choose the increment in time so that

$$\Delta\tau = \frac{(\Delta x)^2}{2a}$$

Then

$$t_{n+1,m} = \tfrac{1}{2}(t_{n,m+1} + t_{n,m-1})$$

This equation will give the temperature of neither surface of the plane wall. To find a surface temperature, the rate of entry of heat into a surface may be set equal to the rate at which heat is conducted through the hypothetical surface slab of thickness, Δx, or

$$h_0(t_f - t_0) = \frac{k}{\Delta x}(t_0 - t_1)$$

where h_0 is the fluid film coefficient of heat transfer, t_f is the temperature of the fluid that surrounds the surface, t_0 is the surface temperature, and t_1 is the temperature of the material at the inner surface of the first hypothetical slab.

The accuracy of this approximate method of solving problems of heating and cooling of solids is increased by using small increments, Δx, of material thickness.

6. CONVECTION

Free or Natural Convection

Heat may be transferred through a fluid not only by conduction but also by *mixing* of portions of the fluid at different temperatures. It is not practicable to separate the individual thermal resistances along the path of heat flow from the solid surface to the main body of the fluid. Instead, the thermal resistances are combined, and the following equation may be written for the rate of heat transfer by either natural or forced convection:

$$q = hA\theta$$

The film coefficient, h, is expressed in Btu per (hr)(sq ft)(°F); θ is the difference between the temperature of the solid surface and the temperature of the main body of the fluid. The reciprocal of the film coefficient ($1/h$) represents the total resistance to the transfer of heat between the solid surface and main body of the fluid. Convection film coefficients do not include radiant heat transfer, which, in the case of some gases, should be computed separately as an additional component of the total transfer of heat.

Experimental results obtained for the film coefficients of heat transfer for natural convection have been correlated in terms of three dimensionless groups of the variables:

$$N_{\mathrm{Nu}} = \phi(N_{\mathrm{Gr}})\psi(N_{\mathrm{Pr}})$$

where ϕ is a function of the Grashof number and ψ is a function of the Prandtl number.

$$N_{\mathrm{Nu}} = \frac{hL}{k}; \qquad N_{\mathrm{Gr}} = \frac{L^3\rho^2\beta g\theta}{\mu^2}; \qquad N_{\mathrm{Pr}} = \frac{\mu c}{k}$$

where L is a characteristic linear dimension of the surface in feet. Although the ϕ and ψ functions are not precisely the same, some data have been correlated by plotting the Nusselt number against the product of the Grashof and Prandtl numbers; this latter product is a function of properties of the fluid and L and θ. If

$$M = \frac{\rho^2 g\beta c}{\mu k} \quad \text{in ft}^{-3}\,\text{F}^{-1}$$

$$(N_{\mathrm{Gr}})(N_{\mathrm{Pr}}) = ML^3\theta$$

M, which is a function of properties of the fluid, may be called the *free convection modulus;* a few values for M are given in Table 5.

$$M = \frac{\rho^2 g\beta c}{\mu k}\ \text{ft}^{-3}\,\text{F}^{-1}$$

Fluid	Temperature, °F	M ft^{-3} F^{-1}
Air, atmospheric pressure	32	2.3 (10)^6
	100	1.3 (10)^6
	200	0.62 (10)^6
Water	32	4.2 (10)^7
	100	59 (10)^7
	200	200 (10)^7

The following empirical equation correlates with fair accuracy experiments on free convection in various fluids—air, water, oil, and alcohol—with different shapes of the solid surface—horizontal and vertical cylinders, spheres, and vertical planes:

$$\log_{10}(N_{\mathrm{Nu}}) = 0.125(1 + 0.1\log_{10} ML^3\theta)\log_{10} ML^3\theta$$

This equation is approximate and holds for the film coefficient of heat transfer by free convection from a solid surface to any fluid except when the natural circulation of the fluid is restricted, as in the case of a warm horizontal surface facing downward. Increasing the characteristic dimension of the surface, L, beyond about 2 ft seems to have little effect on the average film coefficient. In the case of short cylinders or planes, where the horizontal and vertical dimensions may influence the rate of heat transfer to a like degree, the characteristic dimension of the surface may be found as follows:

$$L = \frac{1}{\dfrac{1}{L_{\mathrm{hor.}}} + \dfrac{1}{L_{\mathrm{vert.}}}}$$

More restricted forms of the general equation that apply to surfaces of special shapes or to particular fluids are described below.

Free Convection in Air. For warm horizontal surfaces in still air at atmospheric pressure,

$$h = 0.38\theta^{0.25} \quad \text{for surfaces facing up}$$
$$h = 0.20\theta^{0.25} \quad \text{for surfaces facing down}$$

For vertical surfaces,

$$h = 0.27\theta^{0.25}$$

For horizontal or vertical pipes in air, the film coefficient of heat transfer by natural convection depends upon the diameter of the pipe, expressed here in feet, as well as the temperature difference, and

$$h = 0.27\left(\frac{\theta}{D}\right)^{0.25}$$

Free Convection in Liquids. In this case the convection coefficient represents the total heat transfer, because radiation is not involved. Ordinarily, the effects of the shape or size of the body are not significant, except that for fine wires the coefficients will be considerably higher than those given below.

Free Convection in Water. For bodies of ordinary size in unagitated water,

$$h = 0.165(t_w + 30)\sqrt{\theta}$$

where t_w is the water temperature, °F. This formula does not apply when the surface temperature is above the boiling point of the water. Values of h for various values of t_w and θ are given in Table 6.

Table 6. Heat-Transfer Coefficients for Surfaces in Unagitated Water

Btu per hr per sq ft per °F

Water Temperature, t_w, °F	Temperature Difference θ, °F 10	20	30	40	50	60	70	80	90	100
40	37	52	63	73	82	90	96	103	110	116
60	47	66	81	94	105	115	124	133	141	149
80	57	81	100	115	128	140	152	162	172	182
100	68	96	117	136	152	166	179	192	203	214
120	78	110	136	156	175	192	207	221	235	...
140	89	125	154	177	198	217	235	...	...	...
160	99	140	172	198	222	...	...	...	...	...
180	110	155	190	...	...	...	...	...	...	...

Free Convection in Oils. If μ is the absolute viscosity in lb/(hr)(ft) at the average of the surface and oil temperatures, the free convection coefficient in oils is, approximately,

$$h = \frac{24\theta^{0.25}}{\mu^{0.4}}$$

Forced Convection

When the fluid is circulated artificially over the heat-transfer surface, the value of the coefficient h is governed by the velocity and physical properties of the fluid, and by the size, shape, arrangement, and nature of the surface. In general, roughening the surface, or anything that promotes turbulence in the fluid flow, will increase the heat-transfer coefficient. In liquids, the convection coefficient represents the total heat exchange. This also generally is true of gases, as regards the exchange of heat between the surface and the gas, but with exposed surfaces the additional effect of radiation to or from the surroundings may be relatively significant, particularly at low velocities and high temperatures. The effect of humidity generally is negligible, except when the temperature of the surface is below the dew point of the gas. Condensation then will occur as an additional process, governed by the vapor pressure difference.

GAS FILM COEFFICIENTS. The empirical formulas listed below represent the best available data from various sources: h = convection coefficient, Btu/(hr)(sq ft)(°F); v = gas velocity, ft/sec; G = mass velocity, lb/(hr)(sq ft of cross section); c_p = specific heat at constant pressure, Btu/(lb)(°F); μ = absolute viscosity, lb/(hr)(ft); k = thermal conductivity, Btu/(hr)(ft)(°F); t = average gas temperature, °F; d = pipe diameter, in.

Turbulent flow of gases inside straight tubes,

$$h = \frac{0.044\, c_p G^{0.8}\mu^{0.2}}{d^{0.2}}$$

Turbulent flow of gases inside helical coil, multiply h **for straight tubes by** $(1 + 3.5d/d_h)$, where d/d_h is the ratio of pipe diameter to helix diameter.

Gas flow at right angles to a single tube with Reynolds' number greater than 1000,

$$h = \frac{0.7c_p^{0.3}G^{0.6}k^{0.7}}{d^{0.4}\mu^{0.3}}$$

Air flow at right angles to a single tube with Reynolds' number greater than 1000,

$$h = 0.06(1 + 0.00047t)\frac{G^{0.6}}{d^{0.4}}$$

For gas flow normal to banks of tubes, the film coefficient of convective heat transfer is somewhat higher when the banks are staggered than when they are arranged in line with the flow. The mass velocity, G, should be found per square foot of *minimum free area* rather than per square foot of *face area.*

For gas flow at right angles to a bank of staggered tubes,

$$h = \frac{0.031c_p(t + 460)^{0.3}G^{0.67}}{d^{0.33}}$$

For air flow at right angles to a bank of staggered tubes,

$$h = \frac{0.0075(t + 460)^{0.3}G^{0.67}}{d^{0.33}}$$

For atmospheric air flowing over smooth, plane surfaces at velocities under 15 ft per sec,

$$h = 0.8 + 0.22v$$

At higher velocities of air flow,

$$h = 0.56v^{0.75}$$

For rough surfaces, such as brick, concrete, and stucco, film coefficients are 20 to 50% higher than for smooth surfaces.

The film coefficients of Fig. 1 are for air at or near atmospheric pressure and a temperature of 100 F. For other conditions use $v = 0.004G$. The formulas and curves for air may be used for other similar gases such as N_2, O_2, and CO. The value of h for combustion gases is usually about 25% higher than for air. Curves marked *cross flow* are representative of the data for finned tubing. Table 7 shows the effect of tube diameter upon the above coefficients.

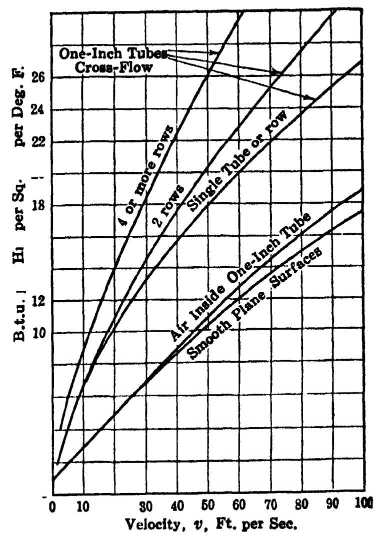


FIG. 1. Coefficient of heat transfer for air at atmospheric pressure and moderate temperature.

Table 7. Effect of Tube Diameter upon Forced Convection Coefficients

Diam. in.	0.1	0.25	0.5	0.75	1	1.5			
	Multiply coefficient for 1-in. tube by								
Flow inside tube	1.59	1.32	1.15	1.06	1.0	0.92	0.87	0.76	0.66
Cross flow, single tube	2.60	1.77	1.33	1.13	1.0	0.84	0.75	0.56	0.42
Tube banks	2.15	1.58	1.25	1.10	1.0	0.87	0.79	0.63	0.50

Coefficients for Flow in Channels or Annular Sections. For fluids, either liquids or gases, flowing in channels or annular spaces, an equivalent diameter should be used for d in the preceding equations. The equivalent diameter is four times the hydraulic radius or four times the area of cross section divided by the perimeter; for concentric pipes, d is the difference between the diameters.

LIQUID FILM COEFFICIENTS. The empirical formulas given below have constants based on the units given above for gas film coefficients. The properties of the liquid may be found at the temperature of the main body of the liquid.

Turbulent flow of liquids inside straight pipes,

$$h = \frac{0.038k^{0.6}G^{0.8}c^{0.4}}{d^{0.2}\mu^{0.4}}$$

The properties of any one liquid may be replaced with an appropriate function of temperature.

Free Convection in Air. For warm horizontal surfaces in still air at atmospheric pressure,

$$h = 0.38\theta^{0.25} \quad \text{for surfaces facing up}$$
$$h = 0.20\theta^{0.25} \quad \text{for surfaces facing down}$$

For vertical surfaces,

$$h = 0.27\theta^{0.25}$$

For horizontal or vertical pipes in air, the film coefficient of heat transfer by natural convection depends upon the diameter of the pipe, expressed here in feet, as well as the temperature difference, and

$$h = 0.27\left(\frac{\theta}{D}\right)^{0.25}$$

Free Convection in Liquids. In this case the convection coefficient represents the total heat transfer, because radiation is not involved. Ordinarily, the effects of the shape or size of the body are not significant, except that for fine wires the coefficients will be considerably higher than those given below.

Free Convection in Water. For bodies of ordinary size in unagitated water,

$$h = 0.165(t_w + 30)\sqrt{\theta}$$

where t_w is the water temperature, °F. This formula does not apply when the surface temperature is above the boiling point of the water. Values of h for various values of t_w and θ are given in Table 6.

Table 6. Heat-Transfer Coefficients for Surfaces in Unagitated Water

Btu per hr per sq ft per °F

Water Temperature, t_w, °F	Temperature Difference θ, °F: 10	20	30	40	50	60	70	80	90	100
40	37	52	63	73	82	90	96	103	110	116
60	47	66	81	94	105	115	124	133	141	149
80	57	81	100	115	128	140	152	162	172	182
100	68	96	117	136	152	166	179	192	203	214
120	78	110	136	156	175	192	207	221	235	...
140	89	125	154	177	198	217	235	...	...	...
160	99	140	172	198	222	...	...	...	...	...
180	110	155	190	...	...	...	...	...	...	...

Free Convection in Oils. If μ is the absolute viscosity in lb/(hr)(ft) at the average of the surface and oil temperatures, the free convection coefficient in oils is, approximately,

$$h = \frac{24\theta^{0.25}}{\mu^{0.4}}$$

Forced Convection

When the fluid is circulated artificially over the heat-transfer surface, the value of the coefficient h is governed by the velocity and physical properties of the fluid, and by the size, shape, arrangement, and nature of the surface. In general, roughening the surface, or anything that promotes turbulence in the fluid flow, will increase the heat-transfer coefficient. In liquids, the convection coefficient represents the total heat exchange. This also generally is true of gases, as regards the exchange of heat between the surface and the gas, but with exposed surfaces the additional effect of radiation to or from the surroundings may be relatively significant, particularly at low velocities and high temperatures. The effect of humidity generally is negligible, except when the temperature of the surface is below the dew point of the gas. Condensation then will occur as an additional process, governed by the vapor pressure difference.

GAS FILM COEFFICIENTS. The empirical formulas listed below represent the best available data from various sources: h = convection coefficient, Btu/(hr)(sq ft)(°F); v = gas velocity, ft/sec; G = mass velocity, lb/(hr)(sq ft of cross section); c_p = specific heat at constant pressure, Btu/(lb)(°F); μ = absolute viscosity, lb/(hr)(ft); k = thermal conductivity, Btu/(hr)(ft)(°F); t = average gas temperature, °F; d = pipe diameter, in.

Turbulent flow of gases inside straight tubes,

$$h = \frac{0.044\, c_p G^{0.8} \mu^{0.2}}{d^{0.2}}$$

Turbulent flow of gases inside helical coil, multiply h for straight tubes by $(1 + 3.5d/d_h)$, where d/d_h is the ratio of pipe diameter to helix diameter.

Gas flow at right angles to a single tube with Reynolds' number greater than 1000,

$$h = \frac{0.7c_p^{0.3}G^{0.6}k^{0.7}}{d^{0.4}\mu^{0.3}}$$

Air flow at right angles to a single tube with Reynolds' number greater than 1000,

$$h = 0.06(1 + 0.00047t)\frac{G^{0.6}}{d^{0.4}}$$

For gas flow normal to banks of tubes, the film coefficient of convective heat transfer is somewhat higher when the banks are staggered than when they are arranged in line with the flow. The mass velocity, G, should be found per square foot of *minimum free area* rather than per square foot of *face area.*

For gas flow at right angles to a bank of staggered tubes,

$$h = \frac{0.031c_p(t + 460)^{0.3}G^{0.67}}{d^{0.33}}$$

For air flow at right angles to a bank of staggered tubes,

$$h = \frac{0.0075(t + 460)^{0.3}G^{0.67}}{d^{0.33}}$$

For atmospheric air flowing over smooth, plane surfaces at velocities under 15 ft per sec,

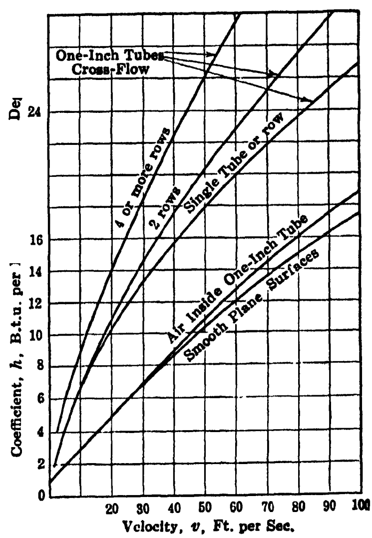
$$h = 0.8 + 0.22v$$

At higher velocities of air flow,

$$h = 0.56v^{0.75}$$

For rough surfaces, such as brick, concrete, and stucco, film coefficients are 20 to 50% higher than for smooth surfaces.

The film coefficients of Fig. 1 are for air at or near atmospheric pressure and a temperature of 100 F. For other conditions use $v = 0.004G$. The formulas and curves for air may be used for other similar gases such as N_2, O_2, and CO. The value of h for combustion gases is usually about 25% higher than for air. Curves marked *cross flow* are representative of the data for finned tubing. Table 7 shows the effect of tube diameter upon the above coefficients.



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Coefficients for Flow in Channels or Annular Sections. For fluids, either liquids or gases, flowing in channels or annular spaces, an equivalent diameter should be used for d in the preceding equations. The equivalent diameter is four times the hydraulic radius or four times the area of cross section divided by the perimeter; for concentric pipes, d is the difference between the diameters.

LIQUID FILM COEFFICIENTS. The empirical formulas given below have constants based on the units given above for gas film coefficients. The properties of the liquid may be found at the temperature of the main body of the liquid.

Turbulent flow of liquids inside straight pipes,

$$h = \frac{0.038k^{0.6}G^{0.8}c^{0.4}}{d^{0.2}\mu^{0.4}}$$

The properties of any one liquid may be replaced with an appropriate function of temperature.

Turbulent flow of water inside straight pipes,

$$h = \frac{0.008(1 + 0.011t)G^{0.8}}{d^{0.2}}$$

$$h = \frac{150(1 + 0.011t)v^{0.8}}{d^{0.2}}$$

Turbulent flow of liquids inside helical coils, multiply h for straight pipes by $[1 + (3.5d/d_h)]$, where d/d_h is the ratio of pipe diameter to helix diameter.

Corrosion, scale, or dirt on the pipe surface will reduce the coefficient considerably, in extreme cases by as much as 50%. The effect of pipe length is generally negligible when length exceeds about twenty diameters; for shorter lengths, the coefficient may be appreciably higher. Turbulence promoters, such as internal ribs or fins, swirlers or wire coils tightly fitting the inside surface of the pipe, are effective in increasing film coefficients of heat transfer. Such turbulence promoters will also generally increase the pressure drop.

Coefficients for Cooling. When the liquid properties do not vary widely over the temperature range, the preceding formulas will apply to cooling as well as heating. With petroleum oils, however, the variation of viscosity with temperature is so great that it is difficult to obtain a formula that is valid for all conditions. Most of the data on heating of oils in turbulent flow may be represented with fair accuracy by

$$h = \frac{122v}{\mu^{0.63}}$$

where μ is the absolute viscosity of the oil in lb/(hr)(ft) at the average of inlet and outlet temperatures. If the oil is being cooled, the film coefficient is usually about 25% lower.

Streamline or Viscous Flow. For the heating or cooling of liquids in streamline flow, the results of experiments on heat transfer may be correlated with fair accuracy in terms of the dimensionless Nusselt number (N_{Nu}) and the dimensionless Graetz number (N_{Gz}). The film coefficient of heat transfer in the Nusselt number is commonly based on the arithmetic mean of the terminal temperature differences between surface and liquid.

For the heating of liquids in streamline flow inside horizontal or vertical pipes,

$$N_{\mathrm{Nu}} = 2.5(N_{\mathrm{Gz}})^{0.333}$$

For the cooling of liquids in streamline flow inside horizontal or vertical pipes,

$$N_{\mathrm{Nu}} = 1.5(N_{\mathrm{Gz}})^{0.333}$$

If the liquid has a large change of viscosity with temperature, it may be necessary to introduce the ratio of the viscosity of the liquid at average main-body temperature (μ) to the viscosity of the liquid at the average temperature of the inside surface of the pipe (μ_s). Then, for the heating or cooling of liquids of high viscosity in streamline flow inside horizontal or vertical pipes, the equation becomes

$$N_{\mathrm{Nu}} = 2.0\left(\frac{\mu}{\mu_s}\right)^{0.14}(N_{\mathrm{Gz}})^{0.333}$$

7. RADIATION

Radiant energy may be regarded as a form of wave motion in free space, which is manifested in such various forms as radio waves, light, heat, and x-rays, depending on the wavelengths. In the range of short wavelengths, known as the visible spectrum, radiant heat and light are identical physically. In the longer wavelengths, which are associated with lower temperatures, the radiation is invisible but it still follows the general laws of optics as regards propagation and reflection; i.e., it travels in straight lines with the speed of light, the intensity at any point varies inversely as the square of the distance from the source, and for a polished surface the angle of reflection is equal to the angle of incidence. On the other hand, it is important to note that this long-wave radiation from sources at temperatures below incandescence may be emitted, absorbed, reflected, or transmitted to a very different degree from short-wave radiation from luminous sources. For example, ordinary window glass will transmit about 90% of the solar radiation falling upon it, but will almost completely absorb radiation from a source at a temperature below 1000 F. Also, at ordinary temperature, a white surface may be as good a radiator or absorber as a black one, whereas absorption of solar radiation increases with the darkness of the color.

DEFINITIONS. A black body is a body that absorbs all the radiant energy falling upon it. **Such a body also radiates energy at the maximum rate possible by virtue of its temperature.**

The emissivity of a body is the ratio of its radiating power to that of a black body at the same temperature.

The absorptivity of a body is the fraction of the radiant energy falling upon it that is absorbed.

The reflectivity of a body is the fraction of the radiant energy falling upon it that is reflected.

The transmissivity of a body is the fraction of the radiant energy falling upon it that is transmitted directly.

THE STEFAN-BOLTZMANN LAW, which has been verified experimentally, states that the total radiation from a black body is proportional to the fourth power of its absolute temperature. This may be expressed in the form

$$q = 0.173(10)^{-8}AT^4$$

where q = rate of emission of radiant energy, Btu per hour; A = area of black body, square feet; and T = absolute temperature of black body, °R.

For ordinary, nonblack surfaces, the rate of emission of radiant energy is

$$q = 0.173(10)^{-8}eAT^4$$

where e = the emissivity of the surface.

Kirchhoff's law states that the emissivity of a surface at the temperature T_1 is equal to the absorptivity of that surface for radiation emitted by a source at the temperature T_1. It is not necessarily true, however, that the emissivity of a surface at the temperature T_1 is the same as the absorptivity of that surface for radiant energy emitted from sources at temperatures greatly different from T_1.

EMISSIVITY OF METALLIC SURFACES. Values of emissivity are given in Tables 8 and 9. The emissivity of a metallic surface depends to a marked extent on the degree of oxidation. Values of e for bright metal surfaces are given in Table 8. The figures given in Table 9 show the normal variation of e for moderately to badly oxidized surfaces. For slightly oxidized or tarnished surfaces, values of e intermediate between those of Table 8 and Table 9 should be used.

Table 8. Emissivity *e* of Polished Metal Surfaces

	Temperature		
Metal	70 F	1000 F	3000 F *
Aluminum	0.05	0.075	....
Brass	0.05	0.06	....
Copper	0.04	0.08	0.15
Gold	0.03	0.05	....
Iron, cast or wrought	0.20	0.25	0.28
Lead	0.08	.....	
Monel metal	0.07	0.10	
Nickel	0.06	0.10	
Platinum	0.036	0.10	0.20
Silver	0.025	0.035	....
Steel	0.20	0.25	0.28
Tin	0.08	.....	....
Tungsten	0.03	0.09	0.25
Zinc	0.10		

* Or molten, if melting point is below 3000 F.

Table 9. Emissivity *e* of Oxidized Metals at Temperatures below 1500 F

Metal	e	Metal	e
Aluminum	0.10–0.20	Iron and steel	0.60–0.90
Brass	0.25–0.60	Monel metal	0.40–0.50
Copper	0.55–0.75	Nickel	0.40–0.60

The emissivity of aluminum or bronze paints varies from 0.3 to 0.6, depending on age and amount of lacquer.

EMISSIVITY OF NONMETALLIC MATERIALS. A careful study of the results of tests on several hundred different materials reported by about thirty investigators indicates that practically all nonmetallic materials, such as porcelain, glass, rubber, paper, cloth, refractories, building materials, enamels, and paints *of any finish or color*, have emissivities between 0.85 and 0.95. In view of the lack of agreement in many cases, a value of e = 0.9 is recommended for all such materials.

INTERCHANGE OF RADIANT ENERGY. Any radiating surface is usually surrounded by other radiating surfaces, and there is an interchange of energy by radiation when these surfaces are at different temperatures. The term *angle factor* is used to describe the geometry of the arrangement of these surfaces. Angle factor (F_{12}) is defined as the fraction of the energy emitted from one surface (1) which is in such direction as to be intercepted at the other surface (2). If two surfaces are arranged so that the normal to area 1 makes an angle ϕ_1 with the center-to-center line, while the normal to area 2 makes the angle ϕ_2 with the center-to-center line, and if the length of the center line connecting the two surfaces is r, the angle factors for special cases follow:

Two infinitesimal surfaces, dA_1 and dA_2:

$$F_{dA_1,dA_2} = \frac{\cos\phi_1 \cos\phi_2}{\pi r_2} dA_2$$

One infinitesimal surface dA_1 and one finite surface A_2:

$$F_{dA_1,A_2} = \int_{A_2} \frac{\cos\phi_1 \cos\phi_2}{\pi r^2} dA_2$$

Two finite surfaces A_1 and A_2:

$$F_{A_1,A_2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\phi_1 \cos\phi_2}{\pi r^2} dA_1 dA_2$$

Angle factors and areas are related as follows:

$$A_1 F_{A_1,A_2} = A_2 F_{A_2,A_1}$$

BLACK-BODY RADIATION. The rate of transfer of heat by radiation between two black surfaces when separated by a medium that does not asborb or emit radiant energy is

$$q_{12} = 0.173 A_1 F_{12} \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]$$

NONBLACK-BODY RADIATION. When the surfaces exchanging heat by radiation are not black, calculation of the rate of heat transfer becomes more complex (Ref. 1). The following limiting assumptions simplify the calculation: (1) the emissivity of each surface is the same as the absorptivity of that surface for radiant energy emitted by the other surface; (2) the fraction of radiant energy reflected from one surface and intercepted by the other is the same as the fraction of radiant energy emitted from that surface and intercepted by the other. With these assumptions, equations for the rate of transfer of heat by radiation between two surfaces separated by a medium that neither absorbs nor emits radiant energy are given below.

Common Form of Radiation Equation. The equations for rate of heat transfer by radiation between two surfaces have the following common form:

$$q_{12} \quad 0.173 F_{ea} A_1 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]$$

where F_{ea} is a factor that evaluates the geometrical arrangement and the nonblackness of the surfaces in any particular case. Its value is apparent in the cases which follow.

Case A. Surface 1 "sees" both surface 2 and surface 1 and no other reflecting surface that sees 2 or 1; surface 2 sees both surface 1 and surface 2 and no other reflecting surface that sees 1 or 2:

$$q_{12} = \frac{0.173 e_1 e_2 F_{12} A_1 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]}{1 - r_1 F_{11} - r_2 F_{22} - r_1 r_2 F_{12} F_{21}}$$

where r_1 is the reflectivity of surface 1 and r_2 is the reflectivity of surface 2. If the transmissivity of each surface is zero, $r = 1 - e$. If surface 1 sees only 2 and 1, and surface 2 sees only 1 and 2,

$$F_{11} = 1 - F_{12}; \qquad F_{21} = \frac{A_1 F_{12}}{A_2}; \qquad F_{22} = 1 - F_{21}$$

Case B. Surface 1 is totally enclosed by and sees only surface 2; surface 2 sees both surface 1 and surface 2 and no other reflecting surface that sees 1 or 2:

$$q_{12} = \frac{0.173 e_1 e_2 A_1 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]}{1 - r_2 F_{22} - r_1 r_2 F_{21}}$$

The angle factor $F_{12} = 1$; $F_{21} = A_1/A_2$; if surface 2 sees only 1 and 2, $F_{22} = 1 - A_1/A_2$. If neither surface transmits radiant energy that is incident upon it, $r_1 = 1 - e_1$ and $r_2 = 1 - e_2$; then

$$q_{12} = \frac{0.173 A_1 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]}{\frac{1}{e_1} + \frac{A_1}{A_2} \left(\frac{1}{e_2} - 1 \right)}$$

A further special case under B is where the enclosed area A_1 is very small relative to the area of the enclosure A_2; then A_1/A_2 approaches zero, and

$$q_{12} = 0.173e_1A_1\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]$$

Case C. Surface 1 sees surface 2 and no other reflecting surface that sees 2 or 1; surface 2 sees surface 1 and no other reflecting surface that sees 1 or 2:

$$q_{12} = \frac{0.173e_1e_2F_{12}A_1\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_1}{100}\right)^4\right]}{1 - r_1r_2F_{12}F_{21}}$$

A further special case under C occurs when A_1 is equal or nearly equal to A_2 and $F_{12} = F_{21} = 1$. This condition would be met by infinite parallel planes or by the total enclosure of one surface by another of nearly equal area; then

$$q_{12} = \frac{0.173A_1\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

EQUIVALENT FILM COEFFICIENT. It is often convenient to use an equivalent film coefficient of heat transfer (h_r) by radiation, where

$$= \frac{0.173F_{ea}\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{T_1 - T_2} = \frac{q_{12}}{A_1(T_1 - T_2)}$$

Let F_T then be the temperature factor, where

$$F_T = \frac{0.173\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{T_1 - T_2}$$

Then $h_r = F_{ea}F_T$. Values of F_T are given in Fig. 2.

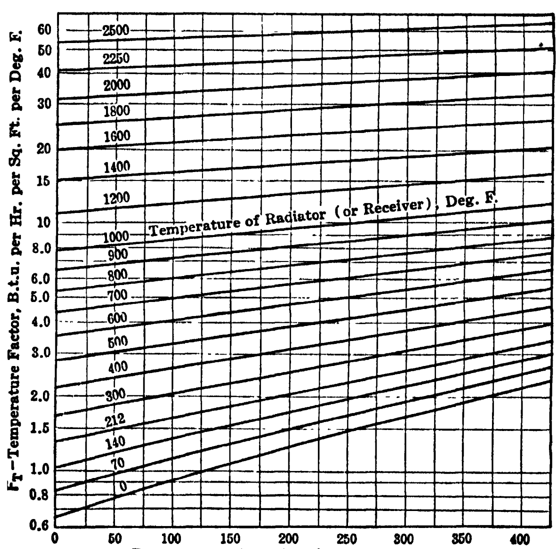

FIG. 2. Temperature factor for radiant heat transfer:

$$F_T = \frac{0.173\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{(T_1 - T_2)}$$

RADIATION IN GASES AND FLAMES. Radiation from combustion products may include radiant energy emitted by small particles of burning soot, larger particles of ash or coal, and gaseous products. Radiation from gaseous products of combustion does not follow the Stefan-Boltzmann law for radiation between solid surfaces. When radiant energy passes through columns of certain gases or vapors, it may be found that these gases or vapors absorb an appreciable portion of that radiant energy within certain bands of wavelengths whereas practically no radiant energy will be absorbed within other bands. Also, if these gases or vapors are heated, they will emit radiant energy at these same wavelengths. Water vapor, carbon dioxide, carbon monoxide, hydrocarbons, sulfur dioxide, ammonia, and hydrogen chloride emit and absorb radiant energy to an appreciable degree. Many common gases, such as nitrogen, oxygen, dry air, and hydrogen, emit and absorb negligible amounts of radiant energy for the conditions common in heat transfer in engineering apparatus.

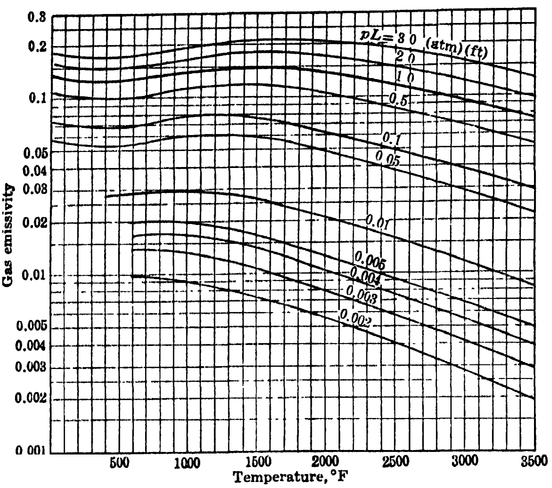

FIG. 3. Emissivity of carbon dioxide.

The following method of estimating the rate of heat transfer by radiation from carbon dioxide and water vapor to bounding solid surface is recommended by Hottel and Egbert (Ref. 2).

Radiation of Hemispherical Gas Mass. The rate of transfer of heat by radiation between a hemispherical mass of gas with radius L feet, partial pressure of gas emitting radiation of p atmospheres, at the uniform absolute temperature of T_g, and a small element

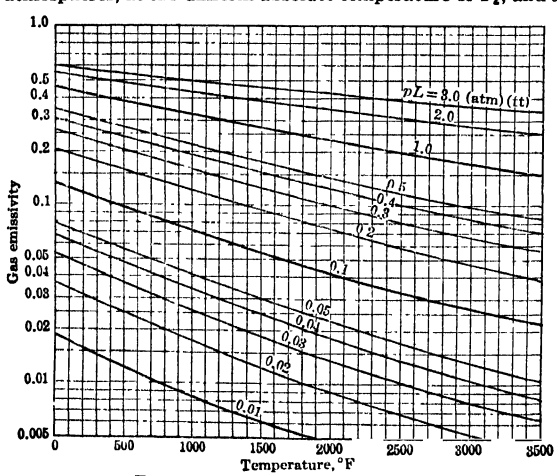

FIG. 4. Emissivity of water vapor.

of surface at absolute temperature T_s and surface emissivity e_s located on the base of the hemisphere at its center is

$$\frac{q_{gs}}{A} = 0.173e_s\left[e_{gg}\left(\frac{T_g}{100}\right)^4 - e_{gs}\left(\frac{T_s}{100}\right)^4\right]$$

where e_{gg} = emissivity of gas at T_g and e_{gs} = emissivity of gas at T_s when $T_g > T_s$.

The emissivity of the gas depends on the temperature of the gas and the product pL. Recommended values of the emissivity of CO_2 are given in Fig. 3; values for water vapor are given in Fig. 4.

Correction Factor. When the gas mixture contains both CO_2 and water vapor, the combined radiation from these two constituents is less than the sum of the separate effects. The combined radiation may be estimated by using a correction factor K:

$$\frac{q}{A} = (1 - K)\left[\left(\frac{q}{A}\right)_{CO_2} + \left(\frac{q}{A}\right)_{H_2O}\right]$$

where $(q/A)_{CO_2}$ = rate of heat transfer by radiation from CO_2 to the bounding surface; $(q/A)_{H_2O}$ = rate of heat transfer by radiation from H_2O to the bounding surface.

The value of the correction factor depends on the partial pressure of the $CO_2(p_c)$ and of the water vapor (p_w) in the mixture, the beam length (L), and the temperature. The temperature effect is ignored in Fig. 5, where K is given as a function of $p_c/(p_c + p_w)$ and $(p_cL + p_wL)$.

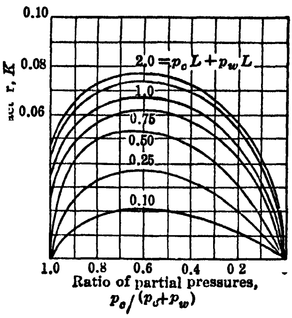

FIG. 5. Correction factor for combined CO_2 and H_2O.

For gas shapes other than hemispheres, an equivalent radius or beam length, L, may be estimated. As the product pL approaches zero, L approaches a value of four times the ratio of the volume of the gas to the area of the bounding surface. For larger values of pL, L is always less than this limiting value, and 85% of the limiting value is a satisfactory approximation. Equivalent beam lengths for various gas shapes are given in Table 10.

Table 10. Equivalent Beam Lengths for Gas Radiation

Shape	Characteristic Dimension, D	Equivalent Beam Length, L
Sphere	Diameter	$0.6D$
Infinite cylinder	Diameter	$0.9D$
Space between infinite parallel planes	Separating distance	$1.8D$
Cube	Edge	$0.6D$
Rectangular parallelopiped (1 x 2 x 6) radiating to any face	Shortest edge	$1.06D$
Space surrounding infinite bank of tubes with centers on equilateral triangles; clearance equal to tube diameter	Clearance	$2.8D$
Same as preceding, but with clearance twice tube diameter	Clearance	$3.8D$

EXAMPLE. This is an example of the calculation of rate of heat transfer by radiation from gases. A mixture of gases at atmospheric pressure and a temperature of 2500 F contains 10% by volume of CO_2 and 7% by volume of H_2O. This gas exchanges heat by radiation with a bounding cubical surface with an edge of 5 ft, a surface emissivity of 0.9, and a temperature of 900 F. For the CO_2, $p_cL = 0.1(0.6)(5) = 0.3$; at $t_g = 2500$ F, $e_{gg} = 0.075$; at $t_s = 900$ F, $e_{gs} = 0.10$ (these values are from Fig. 3). If the CO_2 were present alone, the rate of transfer of heat by radiation from CO_2 to bounding surface would be

$$q = 150(0.173)(0.9)[0.075(29.6)^4 - 0.10(13.6)^4]$$
$$= 1{,}265{,}000 \text{ Btu/hr}$$

For the water vapor, $p_wL = 0.07(0.6)(5) = 0.21$; at $t_g = 2500$ F, $e_{gg} = 0.062$; at $t_s = 900$ F, $e_{gs} = 0.135$ (these values are from Fig. 4). If the water vapor were present alone, the rate of transfer of heat by radiation from H_2O to bounding surface would be

$$q = 150(0.173)(0.9)[0.062(29.6)^4 - 0.135(13.6)^4]$$
$$= 1{,}004{,}000 \text{ Btu/hr}$$

In order to find the correction factor, K, use Fig. 5 with $p_cL + p_wL = 0.17(0.6)(5) = 0.51$ and with $p_c/(p_c + p_w) = 0.59$. Then $K = 0.053$, and the rate of heat transfer to the bounding surface due to the combined effects of carbon dioxide and water vapor is

$$q = 0.947(1{,}265{,}000 + 1{,}004{,}000) = 2{,}149{,}000 \text{ Btu/hr}$$

SOLAR RADIATION. Solar radiation is received on a plane outside the earth's atmosphere and perpendicular to the rays of the sun at a rate of about 420 Btu per (hr)(sq ft). Part of this radiant energy is absorbed and scattered by gases, vapors, and dust in the earth's atmosphere. Solar radiation reaching the earth's surface is part direct and part scattered or sky radiation. Standard values of the direct solar radiation incident upon a plane perpendicular to the sun's rays at the earth's surface which are suitable for many engineering calculations have been proposed (Ref. 3). These values are given in Table 11 for various altitudes of the sun.

Table 11. Solar-radiation Data

Solar Altitude, H, degrees	Direct Solar Radiation at Normal Incidence, I_n, Btu/(hr) (sq ft)	Ratio, Direct to Sky Radiation in Horizontal Surface (summer in eastern states), I_H/I_s
5	65	0.70
10	122	1.40
15	165	1.85
20	196	2.30
25	219	2.70
30	234	3.10
35	245	3.48
40	253	3.84
50	266	4.55
60	276	5.20
70	283	5.63
80	289	5.90
90	294	6.10

Sky Radiation. In addition to the direct solar radiation, sky radiation is incident upon all surfaces regardless of orientation. The ratio of direct solar radiation to sky radiation depends on orientation of the surface, solar altitude, cloudiness, time of year, and locality. Approximate values for the ratio of direct to sky radiation received on a *horizontal* surface in the summer in eastern states on clear days are also given in Table 11.

The solar altitude, H degrees, depends on the latitude, L degrees, the hour angle, θ degrees (1 hr = 15 degrees), and the declination of the sun, D, degrees, and

$$\sin H = \sin D \sin L + \cos D \cos L \cos (360 - \theta)$$

The direct solar radiation received on a horizontal plane is $I_n \sin H$ Btu per (hr)(sq ft).

EXAMPLE. Estimate the solar radiation incident upon a horizontal plane on a clear day at north latitude of 40°, sun time of 3 P.M. ($\theta = 45°$), when the declination of the sun is +20° (north).

$$H = \sin^{-1} [\sin 20 \sin 40 + \cos 20 \cos 40 \cos 315] = 46° 47'$$

From Table 11, $I_n = 262$ Btu/(hr)(sq ft) and $I_H/I_s = 4.32$.

The direct solar radiation incident upon the horizontal surface is 262(0.7288) or 191 Btu per (hr)(sq ft), and the sky radiation is 191/4.32 or 44 Btu per (hr)(sq ft). The total solar radiation incident upon this horizontal surface is (191 + 44) or 235 Btu per (hr)(sq ft).

Solar radiation incident upon a surface is either absorbed, reflected, or transmitted directly. The absorptivity of a surface for solar radiation of a surface that does not transmit solar radiation depends primarily upon color. In general, the darker the color, the greater is the solar absorptivity. For materials that transmit solar radiation, like glass, the absorptivity may also depend on the angle of incidence of that radiation. Approximate values of absorptivity for solar radiation are given in Table 12.

Table 12. Absorptivity for Solar Radiation of Various Surfaces

Surface	Solar Absorptivity	Surface	Solar Absorptivity
Aluminum paint	.35	Nickel, polished	.40
Copper, polished	.50	Red brick or tile	.65
Galvanized iron, new	.65	Silver, polished	.07
Glass, window	.08	Slate, gray	.90
Lampblack	.97	Steel, polished	.45
Magnesium carbonate	.02	Whitewash	.25

8. HEAT TRANSFER TO BOILING LIQUIDS

Film coefficients for the transfer of heat from metal surfaces to boiling liquids are subject to extremely wide variations and depend chiefly on temperature difference, nature of the surface, and nature and temperature of the liquid.

THE MECHANISM OF HEAT TRANSFER from a submerged heated surface to a boiling liquid is similar to free convection but complicated by the agitation that results from the formation of bubbles of vapor. Heat is transferred from the surface to the liquid and from the liquid to the vapor bubble; the temperature of the liquid is commonly higher than the temperature of the saturated vapor. As the temperature difference between the surface and the liquid is increased, the rate of heat transfer per unit of area of the submerged surface first increases, then passes through a maximum, and finally decreases. The decrease in the rate of heat transfer occurs when film boiling replaces nuclear boiling, that is, when a film of vapor of low thermal conductivity begins to collect on the submerged surface. For water boiling at 212 F, for example, the critical temperature difference for the maximum rate of heat transfer is about 45 or 50 F; the corresponding maximum value of the film coefficient of heat transfer is between 7000 and 9000 Btu per (hr) (sq ft) (°F). Conditions that tend to give high film coefficients include the critical temperature difference, freedom from surface scale, use of wetting agents, and high thermal conductivity, specific heat, and density combined with low viscosity of the liquid. Over a very limited range of temperature difference considerably below the critical value of the temperature difference, this empirical equation for film coefficient fits data for eleven liquids boiling at atmospheric pressure and heated by a brass cylinder of 1 in. diameter (Ref. 4).

$$h = \frac{\theta^{2.4}\rho^{2.4}k^{1.8}c^{0.4}}{222\mu^{3.2}} \text{ Btu/(hr)(sq ft)(°F)}$$

where θ = difference between temperature of submerged surface and liquid, °F; ρ = density of liquid, lb/ft³; k = thermal conductivity of liquid, Btu/(hr)(ft)(°F); c = specific heat of liquid, Btu/(lb)(°F); and μ = absolute viscosity of liquid, lb/(hr)(ft).

When the liquid boils inside tubes and when the circulation is forced, the mechanism of the heat transfer process changes. For the boiling of refrigerants, such as "Freon-12," sulfur dioxide, methyl chloride, and ammonia, in evaporators of the dry expansion types, values of the film coefficient will range from 100 to 700 Btu per (hr)(sq ft)(°F); the higher values occur at the higher loads.

9. HEAT TRANSFER FROM CONDENSING VAPORS

Two types of condensation of a saturated vapor on a surface may exist either alone or in combination. In *dropwise condensation*, the vapor condenses in a number of small drops which grow in size but leave the surface before a continuous film of liquid is formed. In *filmwise condensation*, a layer of condensed liquid covers the cooling surface. Film coefficients of heat transfer for dropwise condensation are many times as great as for filmwise condensation. Filmwise condensation occurs, for example, when clean steam condenses on clean surfaces, either smooth or rough. Dropwise condensation of steam may be promoted by use of a contaminant, such as benzyl mercaptan on copper or brass or oleic acid on copper, brass, nickel, or chromium. The function of the contaminant is to prevent wetting of the metal surface. The only safe design procedure is to use film coefficients based upon the assumption of filmwise condensation.

For filmwise condensation of pure saturated vapor outside horizontal tubes arranged in vertical tiers N rows in height, a conservative equation for film coefficient of the Nusselt type is

$$h = 0.725\left(\frac{k^3\rho^2 g h_{fg}}{ND\mu\theta}\right)^{0.25} \text{ Btu/(hr)(sq ft)(°F)}$$

where k = thermal conductivity of condensate, Btu/(hr)(ft)(°F); ρ = density of con-condensate, lb/(ft³); g = acceleration of gravity, ft/(hr)²; h_{fg} = latent heat of condensation, Btu/(lb); D = outside diameter of tube, ft; μ = absolute viscosity of condensate, lb/(hr)(ft); and θ = temperature difference between vapor and tube surface, °F.

Properties of the condensate may be separated from this equation to leave

$$h = \frac{B}{(ND\theta)^{0.25}}$$

where $B = 0.725\left(\frac{k^3\rho^2 g h_{fg}}{\mu}\right)^{0.25}$.

Values of B then depend upon the liquid and its temperature. **Approximate values of** B at 90 F are for water, 1850; for ammonia, 1600; for "Freon-12," 310.

Another form of the equation for the film coefficient of heat transfer for filmwise condensation of saturated vapor outside horizontal tubes is

$$h = 0.955\left(\frac{k^3\rho^2 g L}{\mu W}\right)^{0.333}$$

where L = length of horizontal tube, feet and W = rate of flow of condensate from lowest point on condensing surface, pounds per hour.

Properties of the condensate at the average film temperature may be separated as before to leave

$$= B'\left(\frac{L}{W}\right)^{0.3}$$

where $B' = 0.955\left(\frac{k^3\rho^2 g}{\mu}\right)^{0.33}$

Approximate values of B' at ordinary film temperatures are for water, 3300; for ammonia, 3500; for "Freon-12," 800. For many pure hydrocarbons or petroleum fractions, B' is between 650 and 800.

EFFECT OF NONCONDENSING GASES. The presence of even a small amount of noncondensing gas, as air, in a vapor may have a marked effect in reducing the heat transfer coefficient. Othmer (Ref. 5) found that the presence of 1.07% of air in steam reduced the coefficient to 55% of its value for pure steam. Under the conditions of good engineering practice, the average value of h is about 2000 for steam and 1000 for ammonia, bearing in mind that these are film coefficients on the vapor side only. Values of h for other vapors usually are lower; McAdams and Frost (Ref. 6) obtained coefficients of about 300 for carbon tetrachloride and 350 for benzine.

10. COMBINED CONDUCTION AND CONVECTION

A common example of heat transfer in engineering apparatus is the transfer of heat (1) by convection from a hot fluid to a solid surface, (2) by conduction through the solid material, and (3) by convection from another surface of the solid to a cold fluid. The overall coefficient of heat transfer U is the reciprocal of the sum of the individual thermal resistances encountered in series along the path of heat flow.

Heat Flow through Plane Surfaces. If the cross section of the path perpendicular to the direction of heat flow is constant, as for plane surfaces of the solid material, the overall coefficient of heat transfer is

$$U = \frac{1}{\Sigma\left(\frac{1}{h}\right) + \Sigma\left(\frac{L}{k}\right)} = \frac{q}{A(t_h - t_c)} \text{ Btu/(hr)(sq ft)(°F)}$$

where $\Sigma(1/h)$ = the summation of the reciprocals of the several individual film coefficients of heat transfer by convection, (hr)(sq ft)(°F)/Btu; $\Sigma(L/k)$ = the summation of the several individual thermal resistances to heat transfer by conduction, (hr)(sq ft)(°F)/Btu; q = the rate of heat transfer by the combined processes in series, Btu/hr; A = the area of the plane surface perpendicular to the path of heat flow, sq ft; t_h = the temperature of the hot fluid, °F; and t_c = the temperature of the cold fluid, °F.

For radial flow of heat through cylindrical pipes, the area of cross section perpendicular to the path of heat flow increases directly with the radius. The overall coefficient of heat transfer may be based either on the inner pipe area of radius r_1, in which case it will be called U_1, or on the outer pipe area of radius r_2, in which case it will be called U_2. Then

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1}{k}\ln\left(\frac{r_2}{r_1}\right) + \frac{r_1}{r_2 h_2}}$$

and

$$U_2 = \frac{r_1 U_1}{r_2}$$

where h_1 = film coefficient of heat transfer by convection from fluid inside pipe to inner pipe surface, Btu/(hr)(°F)(sq ft of inner pipe surface) and h_2 = film coefficient of heat transfer by convection from outer pipe surface to fluid outside pipe, Btu/(hr)(°F)(sq ft of outer pipe surface).

The rate of heat transfer by the combined processes in series is

$$q = 2\pi r_1 L U_1(t_h - t_c) = 2\pi r_2 L U_2(t_h - t_c)$$

where L = length of the pipe, feet.

FINNED SURFACE. (See also Economic Use of Secondary Surface, Art. 13, p. 3-33.) When one film coefficient of convective heat transfer is considerably smaller than the other, it may be desirable to use secondary, extended, or finned surface on the side of the greater thermal resistance. Assume a transfer of heat by convection from fluid 1 to the extended surface, by conduction through the extended surface to the primary surface, and by convection from the primary surface to fluid 2. Assume, also, that the thermal resistance of the primary material is negligible. This is commonly the case for metals, but, if it is not true in a particular case, it is possible to decrease the film coefficient of heat transfer from primary surface to fluid 2 in order to compensate for the thermal resistance of the metal. Assume that the ratio of the area of primary surface in contact with fluid 1 to that in contact with fluid 2 is R_p. Also assume that the entire primary surface is at a uniform temperature t_p; the base of the fin at its point of attachment to the primary surface is also assumed to be at the temperature t_p. If the temperature of fluid 1 (which contacts the extended surface) exceeds that of fluid 2, the average temperature of the secondary surface, t_f, will be greater than t_p. Let the *fin effectiveness* be defined as follows:

$$f = \frac{t_1 - t_f}{t_1 - t_p}$$

The area of primary surface in contact with fluid 2 is A_p sq ft; the area of secondary surface in contact with fluid 1 is A_f sq ft. With these assumptions, the rate of heat transfer from fluid 1 to fluid 2 is

$$q = h_1 A_f(t_1 - t_f) + h_1 R_p A_p(t_1 - t_p) = h_2 A_p(t_p - t_2)$$

where h_1 = the film coefficient of heat transfer from fluid 1 to primary and secondary surface, Btu(hr)(sq ft)(°F) and h_2 = the film coefficient of heat transfer from primary surface to fluid 2, Btu/(hr)(sq ft)(°F).

If the overall coefficient of heat transfer, U_p, is based on the area of primary surface in contact with fluid 2,

$$U_p = \frac{q}{A_p(t_1 - t_2)} = \frac{1}{\dfrac{1}{h_2} + \dfrac{1}{h_1(R_p + fR_a)}}$$

where $R_a = A_f/A_p$.

Fin Effectiveness. One form of fin is the *bar fin*. With conduction of heat perpendicular to the primary surface, the bar fin has a constant area normal to the path of heat conduction, A, and a constant perimeter of the surface, p. If there is negligible transfer of heat from fluid 1 to the fin tip, the effectiveness of a bar fin with a thermal conductivity of k is

$$f = \frac{\tanh (mL)}{mL}$$

where $m = \sqrt{h_1 p/kA}$, with the units of ft^{-1} and L = distance from primary surface to fin tip (fin length), ft.

Another type of fin is the *plate fin* with rectangular plates or circular disks of uniform thickness attached to primary tubes or pipes in perpendicular arrangement. The effectiveness of *annular* plate fins with an inner radius of r_a ft, an outer radius of r_b ft, and a thickness of b ft has been evaluated (Ref. 7) in Fig. 6. The effectiveness is plotted against the dimensionless quantity, P $r_a\sqrt{2h_1/bk}$, and lines of constant $R = r_b/r_a$ are shown.

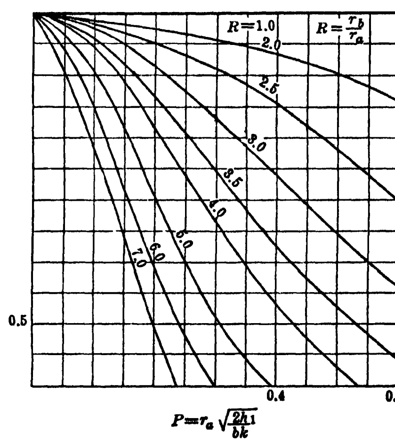

FIG. 6. Effectiveness of annular fins.

The effectiveness of *rectangular* plate fins may be estimated by using the effectiveness of an annular fin of the same area.

EXAMPLE. Find the effectiveness of a 3-in. square plate fin made of aluminum (k = 130), with a thickness of 1/16 in. when used on a 5/8-in. OD tube. On the fin side, the air film coefficient of heat transfer may be assumed to be 12 Btu per (hr)(sq ft)(°F).

$$P = \frac{\quad}{16(12)} = 0.155$$

The area of square fin and equivalent annular fin, in square inches, is

$$9 - \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = \pi\left[r_b^2 - \left(\frac{5}{16}\right)^2\right]$$

from which r_b = 1.7 in. and R = 1.7/0.3125 = 5.44.

From Fig. 6, the fin effectiveness f = 0.77.

11. COMBINED CONVECTION AND RADIATION

Many surfaces transfer heat by both convection and radiation. The total rate of heat transfer is found by *adding* the rates of heat transfer by convection and radiation. An example of this type of heat transfer is a surface at temperature t (absolute temperature T) transferring heat by natural convection to surrounding still air at temperature t_a and by radiation to totally enclosing surfaces of large area at a uniform (or average) temperature of t_s (absolute temperature T_s).

For cylindrical pipes of outside diameter D ft, length of L ft, and surface emissivity of e, the total rate of heat transfer is

$$q = 0.848LD^{0.75}(t - t_a)^{1.25} + 0.543DLe\left[\left(\frac{T}{100}\right)^4 - \left(\frac{T_s}{100}\right)\right] \text{ Btu/hr}$$

For large vertical plane surfaces of area A sq ft and emissivity e,

$$q = A\left\{0.27(t - t_a)^{1.25} + 0.173e\left[\left(\frac{T}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]\right\} \text{ Btu/hr}$$

The rate of heat transfer from the top surface of warm, horizontal planes of area A sq ft and emissivity e is

$$q = A\left\{0.38(t - t_a)^{1.25} + 0.173e\left[\left(\frac{T}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]\right\} \text{ Btu/hr}$$

The rate of heat transfer from the bottom surface of warm, horizontal planes of area A sq ft and emissivity e is

$$q = A\left\{0.2(t - t_a)^{1.25} + 0.173e\left[\left(\frac{T}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]\right\} \text{ Btu/hr}$$

An equivalent combined film coefficient of heat transfer for natural convection in parallel action with radiation may be calculated with sufficient accuracy for many estimates of heat loss by assuming in each of the preceding equations that $t_a = t_s = 70$ F and that $e = 0.9$. This combined film coefficient is $h_c = \dfrac{q}{A(t - 70)}$. Values are given in Table 13 for different values of t and different pipe sizes and plane surface arrangements.

Table 13. Equivalent Combined Film Coefficients of Heat Transfer for Warm Surfaces in Still Air (Natural Convection and Radiation)*

(Surrounding surface and air at 70 F)

Nominal Diameter of Standard Pipe, in.	Outside Diameter Pipe, in.	Equivalent Combined Film Coefficient, h_c, Btu/(hr)(sq ft)(°F)									
		Temperature of Warm Surface, °F									
		80	100	150	200	300	400	500	600	700	800
½	0.840	1.88	2.24	2.68	3.10	3.77	4.45	5.18	6.00	6.92	7.93
1	1.315	1.74	2.05	2.45	2.83	3.46	4.11	4.81	5.62	6.52	7.51
2	2.375	1.67	1.96	2.33	2.70	3.31	3.94	4.64	5.43	6.32	7.31
4	4.50	1.56	1.82	2.16	2.49	3.07	3.68	4.36	5.14	6.02	6.99
8	8.625	1.47	1.70	2.01	2.32	2.87	3.46	4.12	4.89	5.76	6.72
Vertical plane surface (walls)		1.43	1.64	1.94	2.24	2.78	3.36	4.02	4.78	5.64	6.61
Top of horizontal plane surface (floors)		1.63	1.90	2.26	2.61	3.21	3.83	4.52	5.30	6.20	7.18
Bottom of horizontal plane surface (ceilings)		1.31	1.48	1.74	2.00	2.50	3.06	3.70	4.44	5.29	6.24

* For heat transfer data in *panel heating* applications, see p. 12-57.

Example. Estimate the hourly loss of heat by convection and radiation per foot of length of a 4-in. (nominal) standard pipe surrounded by still air and room surfaces at 70 F, when the temperature of the outside surface of the pipe is 300 F. In Table 13, the equivalent combined film coefficient is $h_c = 3.07$; in a 1-ft length of this pipe there are 1.178 sq ft of surface. The hourly loss of heat from 1 ft of this pipe is 1.178(3.07)(300 − 70) or 832 Btu.

12. MEAN TEMPERATURE DIFFERENCE

Logarithmic Mean Temperature Difference. In the exchange of heat between two fluids where radiation is not great, the rate of heat transfer is proportional to the difference between the temperature of the two fluids; for this case,

$$dq = U(t_h - t_c)\, dA = U\theta\, dA \text{ Btu/hr}$$

If the only heat transfer involved is between the two fluids, the quantity of heat given up by the hot fluid (h) equals that absorbed by the cold fluid (c). Then

$$dq = W_cC_c\, dt_c = -W_hC_h\, dt_h$$

These equations may be integrated after making the following assumptions: (1) constant U; (2) constant specific heats; (3) constant flow rates; (4) either parallel or counterflow of the two fluids in contact with the element of surface dA (no cross flow). The mean difference between the temperatures of the two fluids is, therefore,

$$\theta_m = \frac{\theta_a - \theta_b}{\ln\left(\dfrac{\theta_a}{\theta_b}\right)}$$

where θ_a is the difference between the temperatures of the two fluids at one end of the heat exchanger and θ_b is the difference between the temperatures of the two fluids at the other end.

The rate of heat transfer is

$$q = UA\theta_m$$

Arithmetic Mean Temperature Difference. If the ratio, θ_a/θ_b, is less than 2, the arithmetic mean temperature difference, $0.5(\theta_a + \theta_b)$, may be used with little error.

CROSS-FLOW COEFFICIENT. In some cases, the logarithmic mean temperature difference is *not* the true mean temperature difference. These cases include those where there is considerable variation in U over the entire surface, cross flow of the fluids, and change of phase of either or both fluids.

Shell-and-tube heat exchangers consist of a bundle of tubes inside a shell. Fluid flowing through the inside of the tubes is designated as the *tube-side fluid*, and fluid flowing on the outside of the tubes is designated as the *shell-side fluid*. The heads of multipass heat exchangers may contain baffles arranged to cause the tube-side fluid to flow back and forth from one

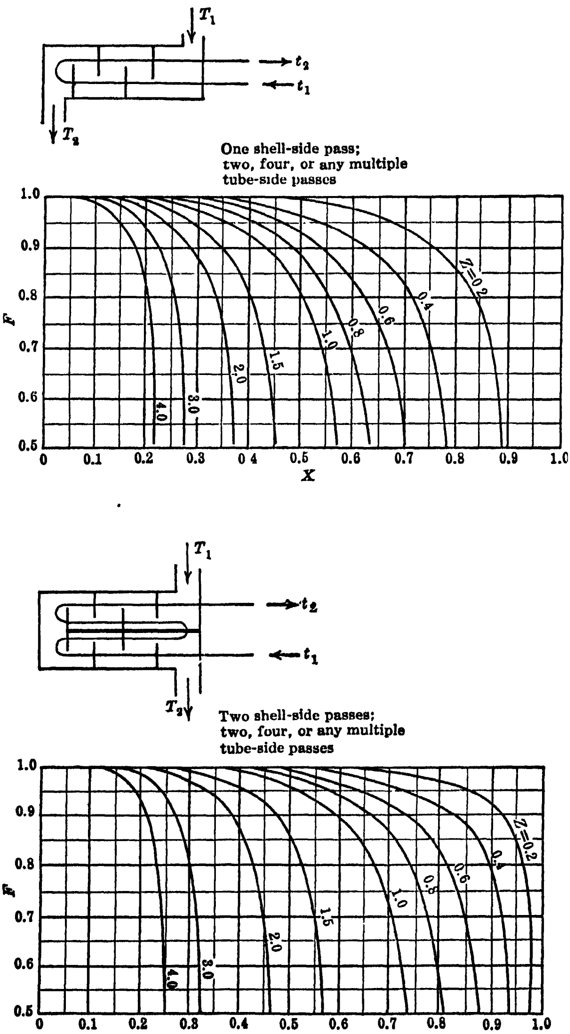

FIG. 7. Cross-flow correction factor, F, versus X for various values of Z.

end of the exchanger to the other a number of times. The shell-side fluid in a horizontal heat exchanger may be kept mixed by the use of vertical baffles which do not extend across the entire section. Also, horizontal or longitudinal baffles may be used in a horizontal exchanger to cause the shell-side fluid to pass from one end of the heat exchanger to the other a number of times. In some local sections of such exchangers, the principal flow of the shell-side fluid may be across the tubes; in other local sections, the principal flow of the shell-side fluid may be either parallel or counter to the flow of the tube-side fluid. The mean temperature difference for multipass exchangers may be found by multiplying the logarithmic mean temperature difference for counterflow by a suitable correction factor, or the mean temperature difference is

$$\theta_m = \frac{F[(T_1 - t_2) - (T_2 - t_1)]}{\ln \dfrac{T_1 - t_2}{T_2 - t_1}}$$

where T_1 = inlet temperature of shell-side fluid, °F; T_2 = outlet temperature of shell-side fluid, °F; t_1 = inlet temperature of tube-side fluid, °F; and t_2 = outlet temperature of tube-side fluid, °F.

Graphs of the correction factor F have been presented by Bowman, Mueller, and Nagle (Ref. 8). In these graphs, two of which are shown in Fig. 7, F is plotted versus X, and lines representing constant values of Z appear, where

$$X = \frac{t_2 - t_1}{T_1 - t_1}$$

and

$$Z = \frac{T_1 - T_2}{t_2 - t_1}$$

In deriving the values shown for F, assumptions are made that the area of heat transfer surface is the same in each pass and that the shell-side fluid is mixed at a given section of a shell-side pass.

Example. Find the mean temperature difference for a heat exchanger with one shell-side pass and four tube-side passes when the temperature of the entering shell-side fluid is 300 F, the temperature of the leaving shell-side fluid is 220 F, the temperature of the entering tube-side fluid is 130 F, and the temperature of the leaving tube-side fluid is 230 F. Assume constant U, constant specific heats, and constant flow rates. The logarithmic mean temperature difference for counterflow is

$$\theta_m = \frac{(300 - 230) - (220 - 130)}{\ln 70/90} = 79.6 \text{ F}$$

$$X = \frac{230 - 130}{300 - 130} = 0.588$$

$$Z = \frac{300 - 220}{230 - 130} = 0.8$$

From Fig. 7 read a correction factor $F = 0.71$, the effective mean temperature difference is

$$\theta_m = 0.71(79.6) = 56.5 \text{ F}$$

13. ECONOMICS OF HEAT TRANSFER

There are many important economic problems in the design and operation of heat-transfer apparatus, including economic extent of surface, economic thickness of insulation, and economic use of secondary surface.

Economic Extent of Surface. As the area of heat-transfer surface installed increases, the amount of heat transferred annually increases but not in direct proportion to the area because of the gradual decrease in mean temperature difference between the two fluids. The annual owning and operating expense for this surface increases more nearly in proportion to the area. There is, usually, an economic extent of surface. If more than this critical value of surface is installed, the annual value of the additional heat transferred is not so great as the increase in the annual owning and operating cost.

Economic Thickness of Insulation. The cost of each additional inch of thermal insulation applied to a pipe line or flat surface is nearly constant, but the heat saved and the value of the heat saved in one year does not increase in direct proportion to the thickness of the insulation. The economic thickness of insulation is that thickness which gives the minimum sum of the annual cost of insulation and the annual value of the heat transferred through the insulation.

For flat surfaces when the cost of one square foot of the insulation is directly proportional to its thickness and where the thickness of the insulation is small relative to the dimensions of the enclosure, the economic thickness of insulation, in *inches*, is

$$\qquad F \qquad - 12Rk$$

where $Y = BH\Delta t/1{,}000{,}000$; B = value of heat transferred, dollars per million Btu; H = hours of operation per year; Δt = temperature difference across insulation and other thermal resistances in series with the insulation, °F; k = thermal conductivity of insulation, Btu/(hr)(ft)(°F); F = annual charges for insulation expressed in dollars per square foot for a thickness of one inch (board foot); and R = thermal resistances in series with the insulation, (hr)(sq ft)(°F)/Btu.

Note. The thermal conductivity per foot of thickness is one-twelfth of the thermal conductivity per inch, so $12k$ may be replaced by the thermal conductivity in (Btu)(in.)/(hr)(sq ft)(°F).

EXAMPLE. An insulation is to be applied to a building wall. The annual charge for this insulation is \$0.006 per sq ft for 1 in. of thickness; k = 0.025 Btu/(hr)(ft)(°F); an average temperature difference of 25 F will exist across the insulation and the boundary thermal resistances for 6000 hours during the year. Total boundary thermal resistances are 0.78(hr)(sq ft)(°F)/Btu, and the value of the heat saved is \$1.10 per million Btu. Estimate the economic thickness of the insulating material.

$$Y = \frac{1.10(6000)(25)}{1{,}000{,}000} \quad 0.165$$

$$= \sqrt{\frac{12(0.165)(0.025)}{0.006}} - 12(0.78)(0.025)$$

$$= 2.87 - 0.23 = 2.64 \text{ in.}$$

ECONOMIC USE OF SECONDARY SURFACE. Heat transfer through primary surface and secondary (or extended) surface is treated in Article 10 of this section, Combined Conduction and Convection. If extended surface is added to primary surface on the side of the greater film resistance, there is an economic limit to the amount of secondary surface which should be used. The notation of the previous discussion of this subject is followed with some additions. Let the weight of primary surface in contact with fluid 2 be W_p lb per sq ft and the weight of the secondary surface per square foot of total surface area be W_f lb. There is one ratio of the area of secondary surface to area of primary surface (or one value of R_a) for which the *total weight of the heat exchanger will be a minimum* if the other variables are assigned fixed values. This optimum ratio is

$$R_a = \frac{1}{f}\left[\sqrt{R_h(fR_W - R_p)} - R_p\right]$$

where $R_h = h_2/h_1$ and $R_W = W_p/W_f$.

If the cost of the secondary surface per unit weight is not greatly different from the cost of the primary surface, the heat exchanger of minimum weight should be the one of nearly minimum cost.

If, however, the cost per pound of primary surface is C_p cents and the cost per pound of secondary surface is C_f cents, the ratio of area of secondary surface to primary surface *for minimum cost of the heat exchanger* is

$$R_a = \frac{1}{f}\left[\sqrt{R_h(fR_WR_c - R_p)} - R_p\right]$$

where $R_c = C_p/C_f$.

EXAMPLE. A refrigerant evaporator (air-to-boiling-refrigerant heat exchanger) is to be made with a primary surface of $^5/_8$ in. (OD) copper tubing. Aluminum plate fins 3 in. square with a thickness of $1/16$ in. are to be used for secondary surface; the aluminum weighs 160 lb per cu ft. A refrigerant boils on the inside of the tubing with a film coefficient of 300 Btu per (hr)(°F)(sq ft of inside tube surface), whereas the film coefficient of heat transfer on the air side is 12 Btu per (hr)(°F)(sq ft of fin surface). The spacing of the fins for minimum total weight of the evaporator is desired.

The weight of primary surface (copper) per square foot of inside surface of the tubing is W_p, 1.77 lb. The weight of the aluminum fin per square foot of total fin surface is 0.416 lb, and R_W = 1.77/0.416 = 4.25; $R_h = h_2/h_1$ = 300/12 = 25. The ratio of primary surface in contact with air to primary surface in contact with refrigerant depends upon the fin spacing, but will be slightly less than the ratio of outside diameter to inside diameter of the copper tubing; a value of R_p = 1.1 will be assumed. The effectiveness of these fins, f, was found (p. 3-29) to be 0.77.

The ratio of area of secondary surface to area of primary surface for minimum total weight of the evaporator is

$$R_a = \frac{1}{0.77}\left[\sqrt{25[0.77(4.25) - 1.1]} - 1.1\right] = 8.14$$

In each fin, there is a total face area (exclusive of fin edge) of 0.12 sq ft. In each foot length of tubing there are 0.145 sq ft of primary inside surface. The number of fins which should be used per foot of tube length for minimum weight is $\frac{8.14(0.145)}{0.12}$ or 10. A fin spacing of 1.2 in. will meet this requirement. With this spacing, the ratio of primary surface in contact with air to that in contact with refrigerant is 1.07 so that the value of 1.1 assumed for this ratio need not be changed.

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HEAT INSULATION

By C. F. Kayan

14. SCOPE OF THERMAL INSULATION PRACTICE

Heat insulation achieves the reduction of heat transfer between realms of different temperatures by presenting increased resistance to heat flow. Heat transfer involving insulation is characteristically of two types, steady-state and unsteady-state or transient heat transfer.

In steady-state transfer, temperatures at given internal locations, under fixed terminal temperatures, are constant with respect to time; in unsteady-state, they vary with time. This condition of varying temperature with time changes the problem of heat-insulation calculation materially. Since, however, the introduction of insulation is usually to safeguard against long-time heat losses, steady-state flow is more often encountered, although under special circumstances, unsteady-state conditions must be studied. Because unsteady-state transfer must provide for the heat stored in the insulation itself, specific heat and density of the material are important. Normally the actual weight of insulation material is not large so that the total heat quantity involved in temperature changes will not greatly enter into a heat-flow total, particularly if the period of operation is long.

The most common application of thermal insulation is as single or multiple layers of homogeneous material applied to flat walls. Next in importance is insulation applied to curved walls, such as pipes, and also as homogeneous material in single or multiple layers. These cases may be regarded as simple. Beyond the field of simple cases is a vast realm of complex cases, such as walls of special geometrical forms, such as regular and irregular, two- and three-dimensional corners, and walls of nonhomogeneous structure, as, for example, those embodying relatively highly conductive metal members. The calculation of such cases is truly complex. Special studies have been made of certain complex cases; the work of Van Dusen (Ref. 1) and of Kayan (Ref. 2) should be consulted as examples in this realm.

INSULATION FIELD. The utilization of heat insulation is in different fields, with division on the basis of temperature ranges which fix the type of material best suited to the working conditions. The prime requirement of suitability is met through different properties such as low-conducting qualities, structural stability and life under temperature and atmosphere conditions, and reasonable cost. (See Table 1.)

COMMERCIAL INSULATORS are:

1. **Loose vegetable or animal fibers,** felted or molded, are used only at low temperatures on refrigerating, cold-water, or hot-water pipes or surfaces. Such insulators are wool and

Table 1. General Division of Heat Insulation Fields

Field	Temperature Range	Applications	Materials Used
1	Below 32 F	Refrigeration Cold storage	(a) Organic materials, wood, cork, vegetable and animal fibers (b) Rock wool (c) Air spaces (d) Vacuum (e) Metallic sheets (f) Cellular glass
2	32 to 100 F	Cold-water pipes Building and room insulation	
3	100 to 230 F	Hot-water heating Low-pressure steam heating Hot-air heating	(a) Air spaces (b) Lower-grade asbestos goods (c) Molded materials (d) Rock wool (e) Metallic sheets (f) Cellular glass
4	230 to 800 F	High-pressure steam plants Industrial processes Heating ovens, etc.	(a) 85% magnesia (600 F) (b) High-grade asbestos goods (c) Diatomaceous earth
5	800 to 1800 F	Furnace settings Kilns High-temperature stills, etc.	(a) Diatomaceous earth (b) Clays
6	Above 1800 F	Firebrick and ceramic products field	Firebrick and clays of various types
7	3000 F up Fire protection only	Safes and vaults Walls and buildings Structural steel	Varies with the temperature likely to occur

hair felt, and cork, by themselves or combined with asbestos or roofing papers or containers.

2. Asbestos insulation is made from loose fibers, molded to shape, and the surface hardened with a binder; or formed into a mattress between woven asbestos cloth; also formed into paper and built-up with intervening air spaces. Available for all moderate temperatures up to the limit of steam temperatures, and, when properly supported, for fire protection.

3. Mineral wool is made by steam blasting blast-furnace slag or fusible rocks. It is used for stuffing spaces and for forming into blocks with other materials. It is used chiefly for low-temperature conditions. It does not rot, but is brittle, easily shaken down, and should be supported by wires or by tufting.

4. Molded powder with or without binders. The efficiency of such insulators depends largely on the proportion of entrained air in the molded product. The larger the percentage of voids, the greater will be its efficiency as an insulator. Those powders whose crystals have the smallest absolute conductivity and the best reflecting surface have the best efficiency.

Plaster of Paris, lime, gypsum, and other materials have been used. Plaster of Paris, being an acid salt, corrodes metals; lime and gypsum naturally have low efficiency because of their high density when molded.

Molded cork scrap is used for low-temperatures and refrigerating conditions.

Infusorial earth molds to a low apparent density, but unless used in block form, as cut from its bed, it has poor binding qualities and requires an artificial binder to give it cohesion. Being an oxide of silica, it can, in its natural form, be used on the highest temperatures.

Basic hydrated carbonate of magnesia molds into shapes with 90% voids, and, because of interlocking of the minute crystals, possesses considerable inherent strength. Commercially, mixed with 10 to 15% of asbestos fiber to give added strength, it is known as *85% Magnesia*. It can be machined accurately to shape. A limiting maximum temperature of 600 F generally is recommended.

Alumina and other refractory powders also are molded with binders. Lightweight refractories are made by mixing clay with carbonaceous materials which are burnt out during firing. Cellular glass slabs and shapes are available for low- and medium-temperature conditions.

5. Metallic sheets, usually with bright surfaces, depend on countering heat flow by screening and reflecting radiant energy. They also introduce an air-space resistance effect.

Table 4. Thermal Conductivities of High-temperature Insulation

k = Btu/(hr)(sq ft)(°F/in.)

Material *	Wt./cu ft, lb	Max. Temp., °F	Mean Temperature, °F										
			400	600	800	1000	1200	1400	1600	1800	2000	2200	2400
1. Sil-O-Cel, natural, good grade	29	1600	0.66	0.76	0.84	0.93	1.01	1.09	1.17		.	.	.
2. Sil-O-Cel, natural, laminated	29	1600	0.92	1.00	1.09	1.17	1.25	1.33	1.42				.
3. J. M. Co. C-22	36	2000	1.28	1.43	1.59	1.75	1.90	2.06	2.22	2.38	2.54	..	..
4. J. M. Co., Super Sil-O-Cel	44	2500	1.67	1.81	1.96	2.10	2.25	2.40	2.54	2.70	2.84	2.98	3.12
5. P. C. Co., Alumino	25	1850	0.59	0.65	0.72	0.78	0.85	0.92	0.98	1.05	...	..	.
6. B. and W. No. 80 insulating	34	2800	1.25	1.36	1.46	1.57	1.68	1.78	1.89	2.00	2.11	2.22	2.32
7. A. C. Co., Non-pariel	30	1600	0.90	0.97	1.07	1.16	1.25	1.34	1.42		.		
8. A. C. Co., Armstrong	36	2500	1.18	1.43	1.68	1.92	2.16	2.40	2.65	2.90	3.13	3.38	3.62
9. Corundite, L. W.-10	48	2600	1.64	1.79	1.94	2.09	2.25	2.40	2.75	2.70	2.85	3.00	3.16

* The materials listed may be described in the order listed: (1) Naturally compacted diatomaceous silica brick. (2) Naturally compacted diatomaceous silica, laminated. (3) Molded and calcined diatomaceous earth brick. (4) Calcined semi-refractory diatomaceous earth brick. (5) High-porosity alumina made from bauxite, chemically processed. (6) High-porosity kaolin insulating brick. (7) Molded diatomaceous earth brick. (8) Semi-refractory diatomaceous earth brick. (9) Bloated clay mixture.

building materials, such as brickwork and concrete. The true insulating value of a wall, particularly one exposed to wind or gas pressure, will depend on its permeability to gas flow as well as on its conductivity.

Apparent differences are sometimes due to a confusion of terms. In some cases the thermal resistance due to the surface effect is included with that of the material; that is, the overall conductivity from air to air may be given instead of the conductivity from surface to surface. The former *decreases* with the thickness of the test piece, whereas the conductivity is the same for *any* thickness of test sample.

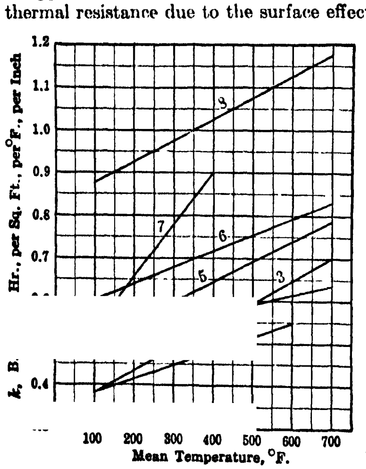

FIG. 1. Mean conductivity of insulation. 1. Asbestos paper and sponge (sponge felt, multi-ply); 2. 85% Magnesia; 3. Rock wool; 4. High-temperature compound (High-temp., super X); 5. Molded diatomaceous earth (Non-Pareil); 6. Asbestos with hard coal (Thermalite); 7. Asbestos air cell; 8. Asbestos molded with silicate of soda (Firefelt).

In view of the foregoing, values of conductivity to be used in computation are used in the expectation that they will actually be in effect on the installation, in service. The conductivities shown in this section are to be regarded as typical only.

Air spaces have value as insulators under given conditions, particularly where cross radiation through the space is not great, as in low-temperature applications. An overall conductance value for the air space, C_s, permits its inclusion in calculations. Air space is effective in the application of metallic insulation, in which the insulating result is partly due to entrapped air, with mitigation of radiation effect through the medium of multiple-radiation screens and the high reflectivity of the metallic surfaces.

Typical values for thermal conductivity are shown in Tables 2, 3, and 4. Table 2 covers materials used in atmospheric and low-temperature conditions; Table 3, refractories for high-temperature conditions; and Table 4, high-temperature insulation. Tables 3 and 4 show variation with temperature, necessary with wide extremes that may exist with the materials involved. For the medium temperature range, Fig. 1 shows values of conductivity and their variation with temperature for typical materials. Whereas general material descriptions have been used, many materials are sold under trade names.

HEAT TRANSFER AT EXPOSED SURFACES. The surface conductance h is comprised of two components, convection and radiation:

$$h = h_c + h_r$$

Convection conductance h_c may be due to natural or free convection, caused by temperature difference between the surface and air, or to forced convection, from wind velocity, for example.

Natural convection depends on size, shape, position of the surface, and temperature difference. For large vertical surfaces, an approximate formula for free convection is

$$h_c = 0.22\Delta t^{1/3}$$

For horizontal surfaces the convection is increased if a hot surface faces upwards or a cold surface faces downwards; it is decreased if a cold surface faces upwards or a hot surface faces downwards. The approximate increase or decrease may be taken as 30%. The presence of near-by surfaces may restrict natural air currents. Wind greatly increases the loss by convection from exposed surfaces. (See also p. 3-18.)

Radiation conductance h_r is

$$h_r = 0.173F_eF_a[(T_1/100)^4 - (T_2/100)^4]/(T_1 - T_2)$$

where T_1 = temperature, °F abs., of the surface considered; T_2 = temperature, °F abs., of the surrounding or confronting surfaces; F_e = emissivity factor representing the combined effect of both the source and the receiver surface emissivity e; and F_a = the configuration factor which for most practical problems here may be taken as 1.

The value of surface emissivity e relative to that of a black body (e = 1.00)—hence also F_e—is uncertain and subject to judgment, as it depends on material, polish, oxidation, and cleanliness of the surface. The following average values are sufficient for most insulation problems:

Surface	e	Surface	e
Aluminum or tin, polished	0.08	Steel and iron, commercial	0.80 to 0.90
Aluminum or tin, varnished	0.20	Nonmetallic materials	0.92 to 0.96
Aluminum paint	0.40	Lampblack	0.96
Oxide paints, all colors	0.94		

F_e depends on the configuration as well as on the individual emissivities. For a small body in a relatively large enclosure, F_e may be taken equal to the emissivity e of the small body surface. For large parallel surfaces, large as compared with the distance between them, for long concentric cylinders, and for large enclosed bodies,

$$F_e = \frac{e_1e_2}{e_1 + e_2 - e_1e_2}$$

where e_1 and e_2 = emissivities of the two surfaces.

For most purposes, labor in computation may be saved by expressing

$$h_r = F_eF_aF_t(t_1 - t_2)$$

where

$$F_t = 0.173[(T_1/100)^4 - (T_2/100)^4]/(T_1 - T_2)$$

See Heat Transmission, p. 3-23, Fig. 2, for values of F_t.

BARE VERTICAL SURFACES. Table 5 shows typical values for heat loss, combined surface conductance, and equivalent pounds of coal, for heat transfer by natural convection and radiation from heated vertical surfaces, such as metal casings and walls, for various temperature differences.

The total surface conductance value with wind is difficult to establish. Acceptable values may be employed using the relationship $h = A + BV$, comprising both convection and radiation components, based on studies by Rowley *et al.* (Ref. 3). V = wind velocity in miles per hour; A and B are constants.

Kind of Surface	A	B
Smooth (glass or paint)	1.50	0.24
Moderately rough	1.80	0.36
Very rough (stucco)	2.00	0.46

INSULATING EFFECT OF AIR SPACES. Air space, rightly used, is an effective insulator at atmospheric and low temperatures. The transfer of heat across it involves both direct radiation and air effects of convection and conduction. The radiation factor depends on the type of surface; is usually assumed to be independent of width of space. This is not strictly true because of the effect of the sides of the spacers. Conduction is inversely proportional to width of space, until it becomes wide enough (at least 3/8 in.) to permit movement of air, so that convection assists the transfer.

Up to about 3/8 in. width, the total transfer with a horizontal space is smaller than for a vertical space if the hot face is on top, and greater if it is below. Effectiveness of air spaces

Table 5. Heat Loss from Flat Bare Vertical Surfaces

(Air at 70 F. Coal taken as 13,000 Btu per lb.)

Temp. Difference, °F	Actual Surface Temp., °F	Heat Loss per sq ft: Surface Coefficient, h	Heat Loss per sq ft: Btu per hr	Heat Loss per sq ft: Pounds Coal per Year (300 days)
25	95	1.80	45	25
50	120	1.91	95	53
75	145	2.02	150	83
100	170	2.10	210	115
125	195	2.23	280	155
150	220	2.34	350	195
175	245	2 46	430	240
200	270	2.59	520	290
225	295	2.73	615	340
250	320	2.85	715	395
275	345	3.00	825	460
300	370	3.14	940	520
325	395	3 31	1075	595
350	420	3.48	1220	675
375	445	3.68	1380	765
400	470	3.87	1550	860
425	495	4 08	1735	960
450	520	4.28	1930	1060
475	545	4.51	2140	1180
500	570	4.72	2360	1300

depends on their being sealed so that no air can circulate through them. If circulation occurs the insulating effect of the spaces is lost; it may actually *increase* the transfer.

Table 6 gives typical values and typifies air space effectiveness as an insulator. It shows values of C_s determined by the National Bureau of Standards (Ref. 4) for mean tempera-

Table 6. Average Value of Conductance for Vertical Air Spaces 24 in. High

(Btu/hr)(sq ft)°F, surface to surface)

Width of Space, in.	Air Transfer Coefficient h_c. Conduction and Convection (without Radiation) for Given Width	Total Transmission Coefficient, C_s, for Given Width, Btu					
		Between Building Materials		Between Bright Tin Surfaces		Between Building Materials and Bright Tin	
		At 32 F	At 75 F	At 32 F	At 75 F	At 32 F	At 75 F
1/8	1.36	2.14	2.30	1.40	1.41	1.43	1.45
1/4	0.68	1.46	1.62	0.72	0.73	0.75	0.77
3/8	0.45	1.23	1.39	0.49	0.50	0.52	0.54
1/2	0.34	1.12	1.28	0.38	0.39	0.41	0.43
5/8	0.28	1.06	1.22	0.32	0.33	0.35	0.37
3/4	0.25	1.03	1.19	0.29	0.30	0.32	0.34
7/8	0.23	1.01	1.17	0.27	0.29	0.30	0.32
1	0.21	0.99	1.15	0.25	0.26	0.28	0.30
2	0.18	0.96	1.12	0.22	0.23	0.25	0.27
3	0.19	0.97	1.13	0.23	0.22	0.26	0.28

tures of 32 and 75 F and an assumed temperature difference of 18 F between faces; changes from 18 F appear to make little difference. Emissivities were assumed as follows: of building materials, 0.94; of bright tin and aluminum, 0.08. The resulting values of radiation conductance h_r are:

Combination	At 32 F	At 75 F
Building material to building material	0.78	0.94
Bright metal to bright metal	0.04	0.05
Building material to bright metal	0.07	0.09

16. INSULATION OF COLD SURFACES

(See also Refrigeration, p. 11-41.)

The insulation of cold surfaces covers applications where transfer of heat from warmer air must be prevented, and includes refrigerating and cold-storage plants, air-conditioning installations, low-temperature process installations, and pipes for refrigerants and cold water. Its distinguishing feature is that the insulation is at a lower temperature than that of the air, so that moisture may condense in it, and thus lower its insulating effectiveness. Air currents also tend to carry moisture from the hot side and condense it on the cold side.

Because of the small temperature range normally involved, except in unusual circumstances of deep temperature cases, variations with temperature of the conductivity k and surface conductance h values are normally neglected. These values have been determined at room temperature, and will be smaller for lower temperatures as long as the material is dry. It is often questionable whether in service it actually remains so. Practical problems usually involve compound walls of such thickness that they constitute the controlling resistance, and the surface thermal resistance has little effect on the result. The normal calculation of transfer through the walls does not include transfer of heat due to air leakage, infiltration, and thermal short circuits.

Air spaces find application in low-temperature insulation practice, as do metallic insulations such as aluminum foil and ferrous sheet metal types, embodying the principle of radiation screens. Where reflective characteristics underlie the value of metallic insulation, the effectiveness depends on the maintenance of the low emissivity over the period of service. Conductivity values for materials used in low-temperature and general building service are given in Table 2. For additional values and for more extensive information in the field of refrigeration, the *Refrigerating Data Book* (Ref. 5) should be consulted, and in the field of air conditioning, the ASHVE *Guide* (Ref. 6).

PREVENTION OF CONDENSATION ON COLD SURFACES. It is desirable to make insulation thick enough to maintain the air-side surface temperature higher than the dew point of the air. The following values are good general practice in using cork in cold-storage work:

Inside air temperature, °F	−20 to −5	−5 to +5	5 to 20	20 to 35	35 to 45	Over 45
Thickness, in.	8	6	5	4	3	2

The maintenance of a high surface temperature avoids condensation of moisture on the outer surface of the insulation. Such moisture, penetrating internally, would subsequently lower the insulation value and would set up a tendency to rot. The penetration of moisture is one of the special problems of low-temperature insulation in general, necessitating special provisions against vapor penetration by means of vapor barriers. Vapor tends to diffuse through porous material, and will precipitate out when its dew point is reached. Insulated walls have been seriously damaged by an accretion of frost from this source which builds up to heavy thicknesses over a period of time.

Moisture will condense on a surface whose temperature is below that of the dew point of the air in contact with it. In using insulation to bring up the surface temperature above that of the dew point, the first step is to fix the expected dry-bulb temperature of the warm air and its maximum dew-point temperature. Table 7 gives the temperature drop t_d to

Table 7. Temperature Difference between Dry Bulb and Wet Bulb for Various Relative Humidities, t_d °F

Relative Humidity, %	Air Temperature, Dry Bulb, °F			Relative Humidity, %	Air Temperature, Dry Bulb, °F		
	50	70	90		50	70	90
	Temperature Difference, °F				Temperature Difference, °F		
20	....	43	47	70	9.5	11	11.5
30	29	33.5	36.5	80	6.5	7	7
40	23	25.5	28	90	3.5	3.5	3.5
50	18.5	20	21.5	100	0	0	0
60	13.5	15	16				

the dew point for different atmospheric conditions. If t_0 is the air temperature, the surface temperature may not be less than $(t_0 - t_d)$.

If the cold surface temperature is assumed to be unchanged by the application of insulation, the following formulas may be used for determining the required thickness of insulation. Here t_0 = air temperature, t_1 = temperature of surface to be insulated, t_d = temperature drop to dew point, h_2 = air-side conductance, k = insulation conductivity, and L = thickness, inches. h_2 may be taken to be 1.4.

For flat surfaces:

$$L = \frac{k}{h_2} \times \frac{t_0 - t_1 - t_d}{t_d}$$

For pipes, the thickness is obtained from

$$r_2 \log_{10} \frac{r_2}{r_1} = \frac{k}{2.3h_2} \quad \frac{t_0 - t_1 - t_d}{t_d}$$

where r_1 = radius of pipe, inches; and r_2 = radius of outside insulation. A few trial values will give r_2.

For a wall exposed to air on both sides, let t_0 and t'_0 be temperatures of the air on the hot and cold sides, respectively; let the original wall without insulation have an overall coefficient from air to air of U, and a thickness conductance from surface to surface of C_w; let h_1 and h_2 be surface conductances on cold and hot sides respectively, k insulation conductivity, and L thickness in inches.

$$L = \frac{k\left[\dfrac{t_0 - t'_0}{h_2 t_d} - \dfrac{1}{U}\right]}{\left[\dfrac{t_0 - t_0' - t_d}{h_2 t_d} - \left(\dfrac{1}{C_w} - \dfrac{1}{h_1}\right)\right]}$$

17. INSULATION OF HOT SURFACES UP TO 800 F

A satisfactory insulation for this type of service should have good insulating properties, should be fireproof, easily molded, light in weight, impervious to moisture, insoluble, unaffected by steam, noncorrosive, structurally strong to resist handling and vibration, sanitary, and vermin-proof. Many types of satisfactory insulation are on the market under special trade names. Figure 1, Art. 15, shows values of conductivity and their variation with temperature for various materials suitable for this medium-temperature service.

STANDARD COMMERCIAL SIZES. Commercial insulation is made in block form and as pipe covering. Block insulation is made in various sizes according to the material, but molded materials are standardized for 6 by 36 in., or 3 by 18 in., and of thicknesses from 1/2 to 4 in.

Pipe covering is made in 36-in. lengths in sectional and segmental forms. Sectional forms, split longitudinally for ease of application, are used for all pipes up to 10 in. in diameter, and for larger sizes in laminated installations. Molded covering for pipes over 10 in. in diameter is supplied as segmental blocks about 6 in. wide.

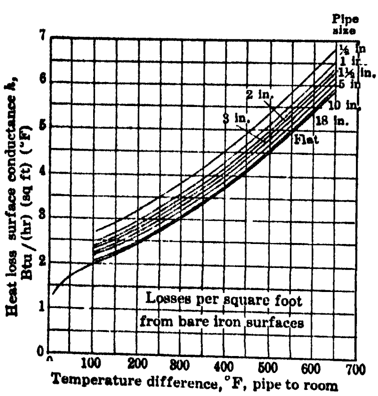

FIG. 2. Heat loss from bare iron pipe.

HEAT LOSS FROM BARE SURFACES. The value of heat losses is conveniently expressed in terms of a given quantity of fuel wasted. It may then be translated into money values by multiplying by the unit value of the fuel as fired. If surfaces are heated by steam, hot water, or hot air, values so found must be divided by the efficiency of the heat generator.

Table 5 may be used for calculating losses from furnace casings and similar heated surfaces, such as brick walls. The actual loss will vary with height, the convection loss per square foot increasing with decrease below 24-in. height. The loss from exposed horizontal surfaces facing upward will be greater because of increase in the convection component; with the hot surface facing downward, a decrease may be expected, because of the reduction in the convection component. Actual heat losses under these varying circumstances are difficult to establish definitely, since working conditions may defy accurate definition.

HEAT LOSSES FROM BARE IRON PIPES. Table 8, based on work by Heilman (Ref. 7), shows values of heat losses for horizontal iron pipes of different sizes at common steam temperatures, under natural circulation. Figure 2 shows the variation of the heat loss surface conductance h with temperature difference for different pipe sizes. Those

Table 8. Losses from Horizontal Bare Iron Steam Pipes per Lineal Foot per Hour

(Air temp. 70 F. Coal taken as 13,000 Btu per lb. Boiler efficiency 70%.)

Pressure	Hot Water		10 psig		80 psig		120 psig		160 psig		200 psig		600 psig	
Pipe Temp.	180 F		239 F		324 F		350 F		370 F		388 F		488 F	
Pipe Size, in.	Btu	Lb Coal	Btu	Lb Coal	Btu	Lb Coal	Btu	Lb Coal	Btu	Lb Coal	Btu	Lb Coal	Btu	Lb Coal
1/2	67	.008	113	.012	198	.022	228	.025	254	.028	278	.031	425	.047
3/4	80	.009	137	.015	239	.026	277	.030	310	034	336	.037	512	.056
1	97	.011	164	.018	289	.032	330	.037	372	.041	407	.045	620	.067
1 1/4	118	.013	203	.022	357	.039	411	.045	460	.051	502	.055	779	.085
1 1/2	134	.015	229	.025	403	.044	470	.052	525	.058	574	.063	885	.097
2	164	.018	271	.030	495	.055	575	.063	643	.071	675	.071	1110	.121
2 1/2	197	.022	336	.037	590	.065	690	.075	770	.084	810	.089	1340	.145
3	231	.025	416	.045	701	.077	815	.089	900	.099	1000	.110	1590	.174
3 1/2	262	.029	449	.049	800	.088	925	.101	1030	.113	1140	.125	1810	.198
4	292	.032	500	.055	895	.098	1040	.114	1150	.126	1260	.138	2020	.222
4 1/2	322	.035	551	.060	985	.108	1140	.125	1270	.139	1400	.153	2220	.243
5	352	.039	605	.067	1070	.118	1240	.136	1390	.152	1520	.167	2490	.273
6	414	.045	716	.079	1270	.139	1480	.162	1650	.181	1820	.200	2900	.319
7	476	.052	815	.089	1470	.161	1700	.186	1890	.207	2080	.228	3340	.365
8	555	.061	920	.101	1640	.180	1910	.208	2140	.235	2320	.255	3760	.412
9	590	.065	1030	.113	1830	.201	2130	.234	2380	.261	2610	.286	4130	.452
10	652	.072	1110	.121	2030	.223	2360	.260	2640	.290	2910	.320	4520	.495
12	765	.084	1330	.146	2390	.261	2800	.306	3120	.342	3530	.387	5400	.590
14	840	.092	1460	.160	2630	.288	3060	.336	3400	.372	3740	.410	5900	.648
16	952	.104	1650	.181	2970	.325	3460	.380	3840	.420	4300	.471	6650	.730
18	1060	.116	1820	.200	3300	.361	3860	.425	4310	.474	4730	.518	7450	.816

for vertical pipes would be somewhat smaller. The pipes are presumed to have their natural oxidized finish.

Losses from hot surfaces are dependent only on surface temperature. The surface temperature may, however, be lower than that of the contained fluid. For saturated steam the difference is very small if the pipe is well drained. With superheated steam, however, there may be an appreciable drop, the amount depending on size of the pipe, pressure and velocity of the steam, and rate of loss. The drop will increase with surface loss, and thus will be more for bare than for insulated pipe. It is to be noted, furthermore, that as loss takes place along a superheated steam line, temperature falls correspondingly.

HEAT TRANSFER COMPUTATIONS. The fundamental formulations previously cited are used, but because of large temperature ranges, k and h cannot be considered constant except for approximate values. Values of k at mean temperatures, customarily published by manufacturers, should be consulted for the material used. Computation for heat flow for a given case usually must be by trial, in which as one step an air-side surface temperature must be assumed. If conductivity at the proper mean temperature of the insulation is used, the rate of heat flow should equal computed surface-to-air loss; if it does not, another surface temperature must be tried.

For canvas covering with natural or painted finish (other than aluminum paint), the value of the surface conductance h may satisfactorily be taken as $(1.2 + 0.01\Delta t)$ for 4-in. outside diameter; $(1.3 + 0.01\Delta t)$ for 4 to 8 in., and $(1.4 + 0.01\Delta t)$ for larger sizes. Here Δt = difference in temperature between surface of covering and the air. A clean aluminum paint surface will have values lower by about 25%.

HEAT LOSS WITH INSULATION. Average calculated values for the heat lost through good insulating coverings for flat surfaces and for different pipe sizes are given in Tables 9 and 10 for various temperature differences. Intermediate temperatures may be interpolated. The values shown are for a typical insulation, and may be prorated for any specific material. Naturally, all commercial insulations show some variations for different samples, depending mainly on the density of the material. The tables have been calculated on the following assumed conductivity, varying with temperature as indicated:

Temperature, °F	100	200	300	400	500	600	700
Conductivity, k	0.50	0.52	0.55	0.58	0.62	0.66	0.71

The approximate value of h used: $h = 1.25 + 0.01\ \Delta t$.

It must again be pointed out that the different insulation materials available have limiting operating temperatures for satisfactory performance, specified by the manufacturers.

ECONOMIC THICKNESS OF INSULATION. Computations for economic thickness of insulation involve graphical or trial-and-error methods. Since they include obvious uncertainties of predictions for the value of heat and the life of the insulation, great accuracy usually is neither possible nor required. Approximate values may be obtained from Fig. 3, the work of McMillan (Ref. 8). The chief uncertainty in carrying out a determination is in the choice of value for mean conductivity, but temperature drop from the surface of the insulation to air as given in Table 9 may be used as a guide to the surface temperature of the insulation.

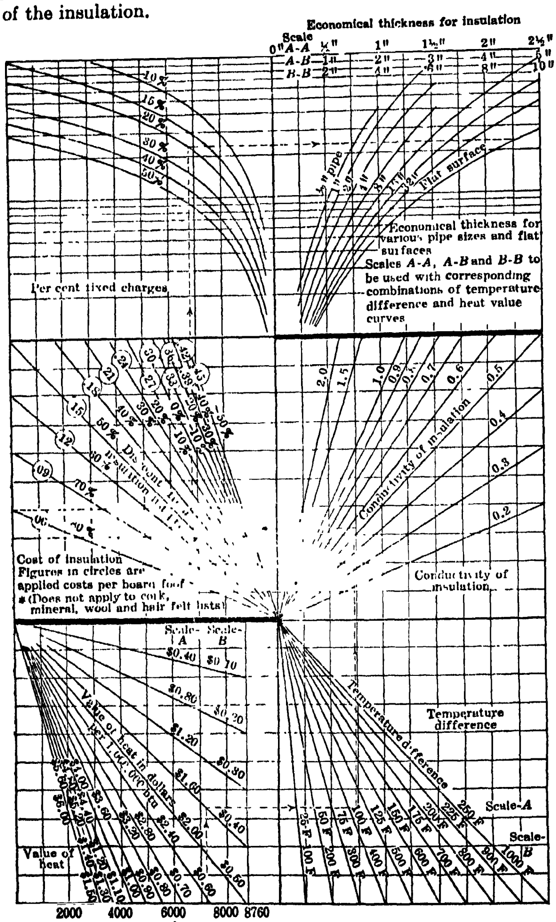

FIG. 3. Economic thickness of insulation. (McMillan, *Proc. Natl. Dist. Heat. Assoc.*, Vol. 18, p. 138)

INSULATION OF UNDERGROUND STEAM MAINS. Exact calculations of loss of heat from pipes buried in the ground should include the thermal absorptivity of the soil, its change in temperature, effect of seasons, rainfall, and intermittent operation. According to an early study by Allen (Ref. 9) based on the assumption that constant conditions have been reached, the following are significant: (1) Heat lost per square foot of pipe decreases with increasing diameter. (2) Since soil acts as an insulator, the advantages of an increase of thickness of insulation is not so great as for an exposed pipe. (3) Little increase in insulating effect is gained by burying the pipe more than 2 ft. (4) Character and dampness of soil will materially affect results.

Table 9. Losses and Surface-air Temperature Drops for Good Insulations on Flat Surfaces

Column *A*—Temperature Drop, °F, insulation surface to air
Column *B*—Losses through the insulation, Btu/(hr)(sq ft)

Temp. Diff. Surface to Air, °F	Actual Surface Temp. Air = 70 F	Steam Pressure, psig	Thickness of Insulation													
			1 in.		1 1/2 in.		2 in.		2 1/2 in.		3 in.		4 in.		5 in.	
			A	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*
			°F	Btu	°F	Btu	°F	Btu	°F	Btu	°F	Btu	°F	Btu	°F	Btu
50	120	....	12	18	10	15	8	12	6	9	5	7	4	6	3	5
100	170	......	24	38	18	27	14	21	11	17	10	15	8	12	6	10
120	200	..	31	50	23	36	19	30	15	23	13	19	10	15	9	13
169	239	10	40	67	30	48	24	38	20	30	17	25	14	21	11	17
200	270	27	46	79	34	56	28	44	23	35	20	30	16	24	13	20
254	324	80	58	105	43	75	34	58	29	46	25	39	20	31	16	25
280	350	120	63	116	47	82	38	64	32	51	28	44	22	34	18	28
300	370	160	67	126	50	88	40	68	34	56	30	48	23	37	20	30
317	387	200	70	134	53	94	42	72	36	60	31	51	25	39	21	32
350	420	300	76	147	58	105	46	79	39	65	34	56	27	43	23	35
400	470	500	86	173	65	121	52	93	44	76	39	65	31	50	26	40
418	488	600	89	182	68	128	55	99	46	79	40	67	32	52	27	42
450	520	800	95	200	73	142	60	109	50	88	43	75	35	58	29	47
500	570	1200	105	228	80	158	65	122	56	100	49	85	38	65	32	53
550	620	1800	114	255	87	177	71	135	62	113	53	95	42	72	35	59
600	670	2500	123	286	95	200	78	152	68	127	58	105	46	81	39	66

The general systems of underground steam pipes and the insulation associated with them are: (1) The insulated pipe is buried in the ground without additional construction other than loose stone drainage under the pipe if the soil is very damp. The pipe is insulated as for overhead construction, covered with overlapping layers of tar paper sealed with asphalt. Insulation must be able to sustain the weight of the soil. (2) Conduit systems, where pressure of the soil is taken by a conduit; such systems always have effective drainage. The pipe is laid in rubble, which, if necessary, contains drainage pipes. Some types of ducts are: (*a*) Impregnated wood ducts; pipes are insulated in the regular

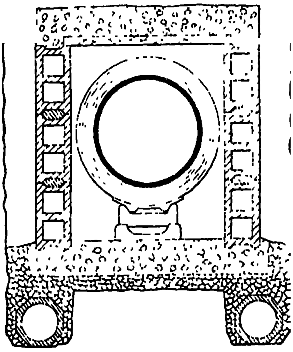

Fig. 4. Rectangular pipe duct.

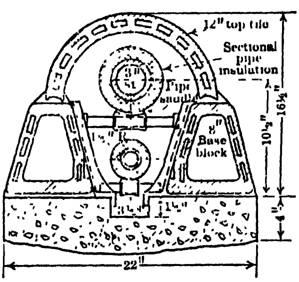

Fig. 5. Commercial pipe duct construction.

manner, supported in the center of the duct, and thus have the additional insulating value of the air space. (*b*) Built-up ducts within a trench; ducts may be all concrete, or concrete top and bottom with tile or brick sides. (*c*) Split molded tile with sealed joints. (3) Man-size tunnels; method of insulation is similar to that of overhead construction, but there may be some gain if the air temperature becomes higher than normal. Figures 4 and 5 show two types of duct.

How much the heat loss is reduced by burial of pipe or conduit depends on the dampness of the soil. The most important single factor is to keep the insulation dry, whether

Table 10. Heat Losses through Good Pipe Insulations

Nominal Pipe Size, in.	Thickness of Covering, in.	Temp. Difference, °F, between Air and Hot Surface													
		100	130	169	200	254	280	300	317	350	400	450	500	550	600
		Actual Hot Surface Temp., °F; Air = 70 F													
		170	200	239	270	324	350	370	387	420	470	520	570	620	670
		Equivalent Steam Gage Pressure, psi													
				10	27	80	120	160	200	300	500	800	1200	1800	2500
		Btu per hour per lineal foot													
3/4	S— 7/8	19	28	37	44	57	64	69	74	83	96	110	125	139	156
	1 1/2	16	23	29	35	46	52	55	58	65	75	86	98	109	122
	D—1 15/16	15	21	27	33	43	48	52	55	61	72	81	91	102	114
	2	14	20	26	31	40	45	49	53	57	67	76	86	96	107
	3	12	17	22	26	34	38	41	44	48	57	65	73	82	91
1	S— 7/8	23	32	42	50	65	72	78	83	93	107	124	141	157	175
	1 1/2	18	25	33	39	47	57	62	65	73	85	97	110	123	137
	D—1 15/16	17	24	31	37	48	53	57	61	70	79	90	102	114	128
	2	16	22	29	34	45	50	54	57	64	74	84	94	106	117
	3	14	19	24	29	38	42	45	48	54	62	71	80	90	100
1 1/4	S— 7/8	29	38	50	59	78	87	94	100	111	129	149	169	188	211
	1 1/2	21	29	38	46	59	67	71	76	84	98	112	127	142	158
	D—1 15/16	19	27	35	42	55	61	66	70	78	91	104	118	131	146
	2	18	25	32	39	51	56	61	64	71	83	95	108	120	134
	3	17	21	27	32	42	46	50	53	59	69	79	89	99	111
1 1/2	S— 7/8	31	42	55	65	85	95	103	109	122	141	163	185	207	237
	1 1/2	24	32	42	50	64	71	77	82	91	106	122	137	154	172
	D—1 15/16	22	29	38	47	60	66	71	76	84	98	112	127	141	158
	2	20	27	36	43	56	62	67	71	79	92	105	121	134	148
	3	17	23	30	35	46	51	55	58	65	75	86	97	109	121
2	S—1 1/32	33	45	58	70	90	101	109	115	129	150	172	196	218	244
	1 1/2	27	36	47	56	74	82	88	94	105	121	139	158	177	197
	D—2 5/32	23	30	40	47	62	69	74	79	88	101	116	132	147	164
	3	19	25	33	40	51	57	62	65	73	84	97	109	122	136
2 1/2	S—1 1/32	38	51	67	80	104	116	125	133	148	173	199	224	252	282
	1 1/2	31	41	54	64	84	94	101	107	119	139	159	180	202	224
	D—2 5/32	26	34	45	54	71	78	84	89	99	115	132	150	167	186
	3	21	28	37	44	58	64	69	73	81	94	108	122	136	151
3	S—1 1/32	44	60	77	92	120	134	144	153	172	199	230	260	291	325
	1 1/2	36	48	62	73	97	107	116	123	137	160	182	207	231	258
	D—2 5/32	30	39	51	61	80	88	96	102	113	132	151	170	190	212
	3	24	32	42	52	65	72	77	82	91	106	121	137	153	170
3 1/2	S—1 1/32	49	67	86	102	134	150	161	171	192	222	257	290	325	363
	1 1/2	39	52	69	81	106	118	127	135	151	176	202	238	255	285
	D—2 1/4	33	43	56	67	88	98	105	111	124	144	165	187	208	232
	3	26	35	46	57	71	78	84	89	100	106	132	150	167	186
4	S—1 1/8	32	69	89	114	140	155	167	178	198	231	266	288	336	376
	1 1/2	43	57	75	88	117	129	140	148	164	192	220	250	278	310
	2	37	48	62	74	97	108	116	123	137	159	183	207	230	257
	D—2 1/4	33	44	58	69	90	100	108	114	128	147	170	192	214	239
	3	28	37	49	58	76	84	90	96	107	124	142	161	179	200
4 1/2	S—1 1/8	35	74	97	116	152	169	183	194	216	252	289	328	366	410
	1 1/2	47	62	81	96	126	140	151	160	178	208	238	260	300	338
	2	39	52	67	81	105	117	126	133	149	173	198	224	250	279
	D—2 5/16	37	48	63	75	98	108	117	124	138	160	183	208	232	258
	3	31	41	53	63	82	91	98	104	115	134	153	174	194	215
5	S—1 1/8	61	81	106	128	165	185	199	211	235	274	315	358	400	438
	1 1/2	51	67	88	118	137	153	165	175	194	226	509	297	328	367
	2	42	57	73	87	114	126	136	144	161	187	214	242	270	302

S = standard thickness; *D* = double thickness.

Table 10. Heat Losses through Good Pipe Insulations—*Continued*

Nominal Pipe Size, in.	Thickness of Covering, in.	Temp. Difference, °F, between Air and Hot Surface													
		100	130	169	200	254	280	300	317	350	400	450	500	550	600
		Actual Hot Surface Temp., °F; Air = 70 F													
		170	200	239	270	324	350	370	387	420	470	520	570	620	670
		Equivalent Steam Gage Pressure, psi													
				10	27	80	120	160	200	300	500	800	1200	1800	2500
		Btu per hour per lineal foot													
5 (*cont.*)	*D*—2 5/16	39	52	67	81	105	117	126	134	149	172	198	223	250	278
	3	33	43	57	67	88	97	105	112	124	144	164	186	208	232
6	*S*—1 1/8	69	91	120	145	187	209	224	239	267	310	357	405	453	505
	1 1/2	57	76	100	119	156	173	187	199	220	256	294	341	373	415
	2	48	64	83	99	129	144	155	164	183	212	243	276	307	343
	D—2 5/16	45	59	76	91	119	132	142	151	168	195	224	253	282	315
	3	37	49	64	76	99	109	118	126	140	162	186	210	234	261
7	*S*—1 1/4	74	97	127	152	180	219	237	252	279	327	374	425	475	531
	1 1/2	65	86	113	133	174	194	211	222	248	286	331	374	418	466
	2	53	70	92	100	144	160	173	183	203	236	271	308	343	383
	D—2 1/2	45	61	79	94	124	137	147	157	175	202	232	262	294	327
	3	40	53	70	83	108	120	130	137	153	178	203	230	258	286
8	*S*—1 1/4	80	108	141	168	219	243	263	278	310	363	415	477	529	590
	1 1/2	73	96	125	149	195	217	236	248	276	322	369	418	465	520
	2	58	77	102	121	158	176	190	201	224	260	298	383	378	421
	D—2 1/2	50	67	80	104	136	151	163	173	193	224	256	290	324	361
	3	46	59	77	92	120	133	144	152	170	196	225	254	284	316
9	*S*—1 1/4	89	120	156	187	243	271	292	310	345	404	462	525	589	655
	1 1/2	78	105	137	159	212	236	256	270	302	349	401	456	509	567
	2	64	85	112	133	173	193	207	220	244	284	326	370	412	460
	D—2 1/2	54	73	88	113	148	165	178	189	211	244	279	316	353	394
	3	48	64	84	100	131	145	157	166	185	214	246	278	310	346
10	*S*—1 1/4	98	132	171	205	267	298	321	342	380	444	510	579	647	723
	1 1/2	86	115	151	179	234	261	283	298	332	385	442	504	561	626
	2	71	94	123	146	191	212	228	242	269	313	359	407	455	507
	D—2 1/2	60	80	96	124	161	179	194	206	230	266	301	345	384	429
	3	52	70	92	109	141	158	170	181	201	234	267	303	337	376
12	*S*—1 1/2	100	133	175	208	272	303	325	346	386	447	515	585	653	728
	2	82	108	142	169	219	244	263	279	310	361	413	470	525	585
	3	60	80	106	125	162	181	196	208	232	269	306	347	387	432
	4	49	65	85	101	132	146	158	168	186	216	247	280	312	348
14 in. OD	1 1/2	108	143	189	225	293	326	351	373	417	484	555	631	705	785
	2	89	116	153	182	238	264	284	302	335	390	447	508	567	632
	3	65	87	114	135	175	196	212	224	250	290	332	375	418	467
	4	53	70	92	110	143	158	170	181	201	234	268	303	338	376
16 in. OD	1 1/2	123	163	214	255	333	370	398	424	473	550	630	715	800	890
	2	99	131	170	203	265	295	318	338	375	436	500	566	634	706
	3	73	97	127	151	198	220	237	252	281	328	372	421	470	525
	4	59	78	102	122	158	175	188	200	223	258	298	326	374	416
20 in. OD	1 1/2	155	205	270	358	419	465	500	534	595	690	794	898	1010	1140
	2	125	164	214	254	333	370	398	423	470	547	626	710	795	895
	3	89	118	155	184	240	267	288	308	341	396	452	512	551	638
	4	72	95	124	149	177	213	230	244	271	316	361	408	455	507
24 in. OD	1 1/2	182	240	316	420	490	546	588	625	697	810	934	1060	1180	1320
	2	146	191	250	297	390	433	465	495	550	641	734	831	930	1035
	3	103	137	179	214	279	310	334	355	394	458	525	594	640	739
	4	83	109	143	170	221	245	264	281	311	363	415	470	525	584

moisture comes from the soil or from steam leaks. Insulation should not be damaged by wetting and subsequent drying; water vapor produced by drying should be able to escape. If the packing is loose, it should not sag if wetted, and it should be able to withstand vibration if buried below traffic lanes. Insulation with low water-absorbing properties is desirable. The effect on insulation of movement of the pipe, due to expansion, should be considered.

Computations for heat loss must depend on assumptions for the character of the soil and gradual heating-up of surrounding conduits and soil. The desirable thickness of insulation for overhead service is sometimes computed, and its thickness then decreased $^1/_2$ in.

18. HIGH-TEMPERATURE AND FURNACE-WALL INSULATION

The high-temperature field covers all conditions where the temperature is greater than 1000 F (as in furnaces, boiler walls, kilns, petroleum stills, and ovens) and deals with heat transfer through refractories with or without added thickness of more effective heat-insulation material. A good refractory is usually strong and dense, hence a good conductor of heat. Under these circumstances, to prevent excessive heat loss, the refractory either must be made very thick or a back-up insulating material must be built into the wall on the cooler side of the refractory. The total heat loss is great because of the large area of the outer surfaces of furnaces, with resultant waste of fuel, uneven temperatures, undue heating of surroundings, and conduction of heat to floors.

The high-temperature field of heat insulation has received much attention in recent years in design of structures and in development of materials to withstand higher temperatures without deterioration and of refractories with lower thermal conductivity. In addition, industrial process developments, typically in the oil and chemical industries, have made it profitable to prevent losses, either because of the higher value of heat or because of the advantages of more uniform operation, temperature-wise, resulting from decreased heat loss. One of the developments in this high-temperature range has been in insulating firebrick, an industrial product usable in numerous refractory services, with insulating properties considerably better than those of conventional refractories.

HEAT-TRANSFER COMPUTATIONS. These follow the usual formulations, but owing to the extreme temperature drop through the material, the variation in conductivity must be taken into account. Great accuracy in computation, however, is usually unwarranted for furnaces, since the temperatures on the hot side of the walls are not readily predictable, are nonuniform over the surface, and are not constant with time. However, since radiation is predominant on the hot side, particularly with flame, it is reasonable to take the hot-surface temperature as that of the furnace. Certain definite difficulties enter into a guarantee of predicted performance: (1) uncertainty of conductivity of refractories; (2) effect of slag erosions, slag accumulations, or contamination of the refractory by slag; (3) deterioration of wall structure. In many instances, on the other hand, rigorous computation is warranted.

THERMAL CONDUCTIVITY OF REFRACTORIES AND HIGH-TEMPERATURE INSULATION. Values of conductivities for refractories are not today satisfactorily defined because of variation in properties of materials, and because experimental work at elevated temperatures is difficult. Values obtained by some investigators indicate that the straight line relationship, $k = a + bt$, is not adequate, and therefore the mean conductivity will depend on the temperature range involved. Table 3 shows values for high-temperature refractories on a mean conductivity basis.

Table 4 presents typical values for insulation material capable of withstanding high temperatures, such as back-up material for furnace refractories. As for refractories, great discrepancies exist in reported conductivity values for high-temperature insulation, even with homogeneous materials.

The surface conductance value h for the air side may be taken from Table 5, values for still air. These values, which will give conservative heat-loss results, are subject to modification if the air motion is vigorous.

USE OF INSULATION. The decision on whether it will pay to apply insulation to a furnace cannot be made solely on an economic basis. It is an engineering problem for each class of furnace and for the conditions of its use, and it depends particularly on whether the service is continuous or intermittent.

These are some of the main considerations and necessary precautions. (1) Insulation must not be used at the risk of weakening the structure. (2) Thickness of insulation, and its location in the walls, must be such that the maximum temperature in the insulation will not exceed the limiting temperature specified for it; its highest working temperature must obtain under the most severe conditions of operation, not the average conditions. (3) Addition of insulation usually will not increase the temperature at the hot face of the refractory; it will, however, decrease the temperature drop through it, and thus increase its average temperature. If the furnace temperature is so high that the refractory is near its melting point, or where rapid erosion by slagging will occur, it is necessary to consider whether or not the life of the refractory may be shortened. (4) Insulation exposed to weather should be well waterproofed, particularly for furnaces which may be shut down for the winter, when insulation may be damaged by freezing of included moisture. (5) If a furnace operates intermittently, the effect of the heat stored in the walls may become important.

Table 11 shows heat-transfer and heat-storage values for a number of typical furnace walls of composite construction, operating at elevated temperatures.

Table 11. Heat Loss and Heat Capacity of Typical Composite Furnace Walls

Heat flow = Btu/(hr)(sq ft); heat capacity (H.C.) = 1000 Btu/(sq ft)

Thickness, in.			Hot Face Temperature, °F							
			1600		2000		2400		2800	
Total	Brick	Insulation	Flow	H.C.	Flow	H.C.	Flow	H.C.	Flow	H.C.
13 1/2	13 1/2	0	770	35	1060	42	1400	50	1800	58
13 1/2	9	4 1/2	400	37	570	47	750	57	940	66
18	18	0	600	43	830	53	1080	64	1340	75
18	13 1/2	4 1/2	340	52	500	65	660	78	830	91
18	9	9	260	45	380	57	510	68	650	80
22 1/2	22 1/2	0	440	52	610	65	780	78	970	90
22 1/2	18	4 1/2	340	64	490	80	660	97	830	113
22 1/2	13 1/2	9	230	57	330	72	440	86	550	100
27	18	9	200	73	290	91	390	110	490	130

19. UNSTEADY-STATE OPERATIONS

The calculation of transient heat flow, involving the element of time, cannot readily be generalized since it is dependent on details of wall structure and time schedule. However, in many cases it assumes importance. If, for example, a furnace operates intermittently, with extensive changes in the wall mean temperature, the effect of heat capacity of the walls is important. The magnitude of this is apparent in Table 11. If a furnace must be cooled between runs, it is desirable that insulation be as near to the hot side as otherwise allowable; if, on the other hand, temperature is to be maintained between runs, insulation should be placed on the outside.

Cyclic operation accentuates the importance of heat-storage effects in walls, particularly under elevated operating temperatures. For further information consult Bradley and Ernst (Ref. 10). In the realm of heat-storage losses for furnace linings, dealing with accumulated heat in walls above room temperature (see Table 11), the study by McCullough (Ref. 11) is of interest.

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ENGINEERING THERMODYNAMICS

By C. O. Mackey

Engineering thermodynamics is the science of the relation between the forms of energy that are important to the engineer.

SYMBOLS AND UNITS

m = mass, lb
M = molecular weight, lb per lb-mole
p = absolute pressure, lb per sq ft
t = temperature, °F
T = absolute temperature, °R
v = specific volume, cu ft per lb
V = total volume, cu ft, or velocity, ft per sec
u = specific internal (molecular) energy, Btu per lb
U = total internal energy, Btu ($U = mu$)
h = specific enthalpy, Btu per lb
H = total enthalpy, Btu ($H = mh$)
s = specific entropy, Btu per lb °F
S = total entropy, Btu per °F ($S = ms$)
R = gas constant, ft-lb per lb°F
c_p = specific heat at constant pressure, Btu per lb °F
c_v = specific heat at constant volume, Btu per lb °F
γ = c_p/c_v
Q = quantity of heat, Btu
w = weight flow, lb per sec, or weight, lb
W = work, ft-lb
J = mechanical equivalent of heat = 778.26 ft-lb per Btu

20. DEFINITIONS AND LAWS

ENERGY is the capacity for producing an effect. It is convenient to classify energy as stored or transient (in transition).

A thermodynamic system consists of a restricted region of space containing a finite portion of matter to which thermodynamic analysis is to be applied. Everything outside the system having a direct bearing upon its behavior is known as *surroundings*. Thermodynamic analysis should be limited to systems that are large relative to the individual particles that comprise these systems. Thermodynamic laws are valid regardless of the theories of the constitution of matter. Present theories postulate matter to consist of atoms and molecules, atoms to consist of electrons, positrons, neutrons, protons, and mesons.

Energy in transition across the boundaries of a thermodynamic system in the absence of electricity, magnetism, and capillarity consists of *work* or *heat*. Energy stored within the thermodynamic system is often given the broad name of *internal energy*, but for the sake of clarity it is here called *stored energy*.

Work is measured as the product of a force and the distance over which it is applied in the direction of the force. Work is commonly expressed in the units of foot-pound. In thermodynamic processes, the expansion of a fluid is often used to move the piston of an engine; in finding the work done, consideration must be given to the fact that the magnitude of the force may not be constant throughout the motion.

Heat is energy in transition between a system and its surroundings because of a difference in temperature, only.

Various forms of stored energy are:

(1) The **gravitational potential energy** stored in m lb of a substance at a vertical height of Z ft above a horizontal datum plane in the gravitational field of the earth is mZ ft-lb.

(2) The **kinetic energy** of mass motion stored in m lb of a substance with a uniform velocity of V ft per sec relative to the earth is $mV^2/2g$ ft-lb, where g is the acceleration of gravity in feet per second per second.

(3) **Internal (molecular) energy** is stored in molecular systems by virtue of the relative motions of, and the forces of interaction between, the individual molecules. The internal

energy of 1 lb of a substance is u Btu per lb; for m lb of a substance, the internal energy is $mu = U$ Btu.

(4) **Chemical energy** is stored energy that is released or absorbed during chemical reactions. Until the achievement of the first divergent nuclear chain reaction in 1942, the release of the largest quantities of stored energy resulted from chemical reactions like combustion.

Internal energy is stored in subatomic systems by virtue of the relative motions of, and the forces of attraction between, particles that constitute the atom. Fission of the nucleus of the atom, for example, releases large quantities of stored energy, but only a portion of the still larger quantities of energy that are stored in mass.

If 1 lb of a substance with a specific volume of v cu ft per lb flows into a system from surroundings where a constant pressure of p lb per sq ft abs is maintained, the surroundings do work on the substance and system in the amount of pv ft-lb per lb. For simplicity, m lb of a flowing fluid may be said to have energy stored in the form of *flow work* in the amount of mpv ft-lb.

Units of Energy: (1) The foot-pound (ft-lb) is the amount of work done by a constant force of 1 lb acting through a distance of 1 ft in the direction of the force.

(2) The British thermal unit (Btu), in the past, has generally been defined as $^1/_{180}$ of the quantity of heat required to raise the temperature of 1 lb of water from 32 F to 212 F at a pressure of 1 standard atmosphere (14.696 psi). In 1929, the First International Steam Tables Conference proposed the definition of 1 international steam-table calorie as $^1/_{860}$ of the international watthour. The equivalent of 1 Btu in foot-pounds is given the symbol J ($J = 778.26$).

(3) The horsepower-hour (hp-hr) is the transition of energy at the rate of 550 ft-lb per sec over a period of 1 hr, or 1 hp-hr = 2544.1 Btu.

(4) The absolute kilowatthour (kwhr) is the transition of energy at the rate of 1000 absolute watts per hour over a period of 1 hr, or 1 kwhr = 3412.75 Btu.

The *enthalpy* (formerly called total heat or heat content) of 1 lb of a substance is the sum of its internal energy and its pressure-specific volume product at the given state or $h = u + (pv/J)$ Btu per lb, where p = absolute pressure, pounds per square foot, and v = specific volume, cubic feet per pound. For m lb, the enthalpy is $H = mh$ Btu. Enthalpy, like internal energy and flow work, is a property of state.

THE FIRST LAW OF THERMODYNAMICS is a statement of the principle of conservation of energy. If any system is carried through a complete cycle (the end state and the initial state of the system being the same), the net quantity of heat Q Btu added to the system from the surroundings must be equal to the net work W ft-lb delivered by the system to the surroundings; or $Q = W/778.26$ Btu.

Non-flow Process. Let the state of any substance change from 1 to 2 in the absence of any motion, any change in the center of gravity of the substance, any chemical reaction, or any release of stored atomic energy. The work done by the substance is $W_{12} = m \int_{v_1}^{v_2} p\,dv$ ft-lb. The quantity of heat added to the substance is Q_{12} Btu. Then $Q_{12} = U_2 - U_1 + (W_{12}/J)$.

If the volume of the substance remains constant during the process, the heat added is used entirely in increasing the internal energy of the substance, or $Q_{12} = U_2 - U_1$, if $v_2 = v_1$.

If the pressure of the substance remains constant during the process, $Q_{12} = U_2 - U_1 + (mp/J)(v_2 - v_1)$ Btu, or $Q_{12} = H_2 - H_1$.

If the temperature of the substance remains constant, the process is an isothermal process.

If there is no supply or removal of heat during the process (adiabatic process), $W_{12} = J(U_1 - U_2)$ ft-lb.

Many actual expansions and compressions of fluids may have a pressure–specific volume relationship of the form pv^n = a constant, with a constant value of the exponent n; such processes are called *polytropic* processes.

Steady-flow Process. Let the state of a flowing substance change from 1 to 2 in passing through apparatus in which no chemical reaction or release of stored atomic energy occurs. If the mass of substance that passes section 1 in unit time is the same as that passing section 2 in unit time, if the properties, elevation, and velocity of the substance remain constant and uniform at each section, and if there is no storage or recovery of energy within the apparatus during the unit time,

$$Z_1 + \frac{V_1^2}{2g} + JU_1 + p_1v_1 + JQ = Z_2 + \frac{V_2^2}{2g} + JU_2 + p_2v_2 + W$$

where Q Btu is the net quantity of heat supplied per pound of substance from the surroundings between the reference sections, and W ft-lb is the net work delivered to the surroundings per pound of substance by the apparatus between the reference sections. Note that enthalpy may be substituted in this equation for the sum of internal energy and flow work.

Let A denote the area of cross section (square feet) of the pipe or conduit through which the substance is flowing, and let w represent the rate of flow (pounds per second); the *equation of continuity* for steady flow is

$$w = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

Temperature. The temperature of a system is a property that determines whether or not a system is in thermal equilibrium with other systems. On the Fahrenheit scale, the ice point is given the number 32 and the steam point (the temperature at which steam is in equilibrium with pure water under standard atmospheric pressure) is given the number 212. On the Rankine scale of absolute temperature, the ice point is 491.7 F. Absolute temperature, therefore, in degrees Rankine is Fahrenheit temperature (t) plus 459.7, or $T = t + 459.7$.

THE SECOND LAW OF THERMODYNAMICS. Many equivalent statements of this law exist. The Kelvin-Planck statement is that it is impossible to construct an engine that, operating in a cycle, will produce no effect other than the extraction of heat from a reservoir and the performance of an equivalent amount of work. The Clausius statement is that it is impossible to construct a device that, operating in a cycle, will produce no effect other than the transfer of heat from a cooler to a hotter body.

A limit to the thermal efficiency of heat engines and to the coefficient of performance of refrigerators or heat pumps is imposed by the second law. A heat engine is a device that receives heat at a high level of temperature, delivers work, and rejects heat at a low level of temperature.

The **thermal efficiency** of a heat engine is the ratio of the work delivered to the heat received. If a heat engine receives heat from a high-temperature reservoir at absolute temperature T_1 and rejects heat to a low-temperature reservoir at absolute temperature T_2, the maximum thermal efficiency that the engine may have is $\eta = (T_1 - T_2)/T_1$.

The **coefficient of performance** of a refrigerator is the ratio of heat removed from the low-temperature reservoir to the net work supplied to operate the machine. The maximum coefficient of performance that the refrigerating machine may have is $T_2/(T_1 - T_2)$. If the purpose of operation of the heat pump is to effect warming rather than refrigeration, the coefficient of performance is the ratio of heat delivered to the high-temperature reservoir to the net work required to operate the machine. In this latter case, the maximum coefficient of performance is $T_1/(T_1 - T_2)$.

REVERSIBILITY. A process is said to be *reversible* when the following conditions are fulfilled. (1) When the direction of the process is reversed, the system taking part in the process can assume in reverse order the states traversed in the direct process. (2) The external actions are the same for the direct and reversed processes. (3) Not only the system undergoing the change, but also all connected systems, can be restored to initial conditions. (Goodenough.)

Any process that fails to meet these requirements is an *irreversible process.* Three irreversible processes are of frequent occurrence: (1) The direct conversion of work into heat through the agency of friction. (2) The transfer of heat due to temperature difference. (3) The throttling of a fluid in flowing through an orifice from a region of high pressure to a region of low pressure.

ENTROPY. Entropy is a property of state, like pressure, specific volume, and temperature. Specific entropy, s, or the entropy of 1 lb of a substance has the units of Btu/lb °F; the total entropy, S, of m pounds is $S = ms$ Btu/°F.

The increase in entropy of w pounds of a substance during a change of state from 1 to 2 may be evaluated for *reversible* processes, *only,* by the following equation:

$$S_2 - S_1 \quad \int_{T_1}^{T_2} \frac{dQ}{T}$$

where dQ is the very small quantity of heat added to the substance at the instantaneous absolute temperature T.

The increase in entropy of a substance during an *irreversible* process may be found by evaluating the increase in entropy for any imaginary reversible process or processes that may connect the initial and final states of the substance.

As a coordinate, specific entropy is often used as the abscissa of a coordinate system where absolute temperature is the ordinate. On this T-s diagram, the area under the path of a *reversible* process $\int_{s_1}^{s_2} T\,ds$ represents the quantity of heat added to 1 lb of the working substance during the reversible process that changes the state of the substance from 1 to 2.

AVAILABILITY OF ENERGY. Assume that a very small quantity of heat, dQ Btu, is added to a substance at an instantaneous absolute temperature of T. Let T_0 be the absolute temperature of the coldest reservoir at hand. Only a portion of the small quantity of heat received by the substance may be transformed into work. The maximum amount of energy *available* for doing work is $dQ[1 - (T_0/T)]$ Btu. The minimum amount of energy that has become *unavailable* for doing work is $dQ(T_0/T)$ Btu.

During any reversible process, the entropy of the *entire system* (universe) affected by the process is not changed, although the entropy of the *substance* undergoing the process may increase or decrease. During any irreversible process, the entropy of the *entire system* affected by the process increases. Adiabatic processes may be *either* reversible or irreversible. During an irreversible adiabatic process, the entropy of the fluid undergoing the process always increases, whether the process is a compression or an expansion; there is an accompanying loss of available energy. The entropy of a fluid undergoing a reversible adiabatic (isentropic) process remains constant, and there is no loss of available energy.

Change in entropy during either a reversible or an irreversible process may be considered as an index of the unavailability of energy. When a very small quantity of heat, dQ, is added to a substance at an instantaneous absolute temperature T, the quotient obtained by dividing the minimum amount of energy that has become unavailable for doing work by the absolute temperature of the cold reservoir (with reference to which the unavailable energy has been computed) is $(T_0\,dQ)/T_0T$ or dQ/T. This quotient is independent of the temperature of the cold reservoir and is the increase in entropy of the substance resulting from the addition of heat.

A **reversible adiabatic process** is one of constant entropy (isentropic). The *maximum* amount of work that may be recovered from the *adiabatic* flow of 1 lb of a substance from a reservoir at steady high pressure p_1 through any device, in which there is no chemical reaction or no release of stored atomic energy, into a reservoir in which a steady low pressure p_2 is maintained with no change in the elevation of the substance is the isentropic drop in enthalpy of the substance, or $W_{max} = J(h_1 - h_2)$ ft-lb. The enthalpy of the substance h_2 must be evaluated at the pressure p_2 and for the entropy $s_2 = s_1$.

The *minimum* amount of work that must be supplied by a pump to transfer 1 lb of a substance, adiabatically, under conditions of steady flow from a reservoir maintained at steady low pressure p_a into a reservoir maintained at steady high pressure p_b with no change in the elevation of the substance is the isentropic rise in enthalpy of the substance, or $W_{min} = J(h_b - h_a)$ ft-lb. The enthalpy of the substance h_b must be evaluated at p_b for the entropy $s_b = s_a$.

21. PERFECT GASES

The **characteristic equation of state** of a perfect, or ideal, gas is $pv = RT$ or $pV = mRT$. The product of the molecular weight of the perfect gas and the gas constant R is a constant; $MR = 1545$ ft-lb per lb-mole °F, the universal gas constant.

The equation for the change of state for a fixed mass of a perfect gas between any two states 1 and 2 is $p_1V_1/T_1 = p_2V_2/T_2$.

The quantity of heat that must be added to unit mass of a substance to increase its temperature 1 F depends upon the process followed during the addition of heat. In general, the instantaneous value of the specific heat at the temperature t is $c = dQ/(m\,dt)$. For a constant-pressure process, the specific heat is given the symbol c_p, and for a constant volume process of addition of heat, c_v.

For a perfect gas, $R = J(c_p - c_v)$ (ft-lb) per (lb)(°F).

The characteristic equation of state of perfect gases is a limit approached by the equation of state of real gases as the pressure of the gas approaches zero. The internal energy of the perfect gas is a function of temperature, only. It is customary to specify that c_p and c_v are constant and independent of temperature for perfect gases; for real gases, c_p and c_v are functions of temperature and pressure. The model of the perfect gas according to the kinetic theory meets these conditions: (1) A chemically homogeneous gas is composed of identical molecules moving in random fashion. (2) The actual space occupied by the molecules is negligible when compared with the space between them. (3) The molecules exert no forces upon each other except during collision. (4) On the average, the impacts between molecules or between molecules and containing walls are perfectly elastic.

Values of gas constants at zero pressure and 70 F are given in Table 1.

Table 1. Gas Constants for Perfect Gases *

(At zero pressure and 70 F; also hold closely for real gases at moderate pressures and 70 F.)

Name of Gas and Formula	Molecular Weight	Gas Constant in Equation of State, ft-lb/lb °F	Specific Heat at Constant Pressure, Btu/lb °F	Specific Heat at Constant Volume, Btu/lb °F	Ratio of Specific Heats
	M	R	c_p	c_v	$\gamma = c_p/c_v$
Hydrogen, H_2	2.016	767	3.42	2.43	1.40
Methane, CH_4	16.032	96.4	0.530	0.405	1.31
Carbon monoxide, CO	28.000	55.2	0.249	0.178	1.40
Nitrogen, N_2	28.016	55.1	0.248	0.178	1.40
Ethylene, C_2H_4	28.032	55.1	0.364	0.293	1.24
Oxygen, O_2	32.000	48.3	0.219	0.157	1.39
Carbon dioxide, CO_2	44.000	35.1	0.201	0.156	1.29

* For air see Section 1 and text, below.

For the **change of state** of m pounds of a perfect gas from state 1 to state 2 along *any* path:

$$U_2 - U_1 = mcv(t_2 - t_1) \text{ Btu}$$

$$H_2 - H_1 = mc_p(t_2 - t_1) \text{ Btu}$$

$$S_2 - S_1 = m\left(c_p \ln\frac{v_2}{v_1} + c_v \ln\frac{p_2}{p_1}\right) \text{ Btu/°F}$$

MIXTURES OF PERFECT GASES. For mixtures of perfect gases that have no chemical action upon one another, the total pressure of the mixture is the sum of the partial pressures of the constituents:

$$p_m = p_1 + p_2 + p_3 + \cdots$$

The volume of each constituent under its partial pressure is the same as the volume of the mixture under the total pressure and at the same temperature:

$$m_m v_m = m_1 v_1 = m_2 v_2 = m_3 v_3 = \cdots$$

The gas constant of the mixture is

$$R_m = \frac{\Sigma m_i R_i}{\Sigma m_i}$$

The specific heats of the mixture are

$$c_{pm} = \frac{\Sigma m_i c_{pi}}{\Sigma m_i} \quad \text{and} \quad c_{vm} = \frac{\Sigma m_i c_{vi}}{\Sigma m_i}$$

where the subscript $_i$ refers to the respective components.

The mixture of perfect gases may be treated as if it were one perfect gas with the properties found as above.

Atmospheric air may be treated as a mixture of perfect gases and may be used as an illustration of the application of the laws of mixtures of perfect gas. The composition of the mixture of *dry* gases constituting atmospheric air changes with altitude. At or near sea level, the composition of dry air is as shown in the tabulation.

Constituent	Fraction by Volume, f	Fraction by Weight $g = \frac{Mf}{\Sigma Mf}$
Nitrogen	0.7803	0.7547
Oxygen	0.2099	0.2319
Monatomic gases (principally argon)	0.0094	0.0130
Carbon dioxide	0.0003	0.0004
Hydrogen	0.0001	0.0000

For dry air of this composition, the equivalent molecular weight of the mixture, or ΣMf, is 28.966. The gas constant of the mixture is $R_m = 53.35$ ft-lb per lb °F. If the monatomic gases of this mixture are assumed to have a specific heat at zero pressure of 0.124 Btu per lb °F, the specific heat at constant (zero) pressure and at 70 F is

$$c_p = 0.7547(0.248) + 0.2319(0.219) + 0.013(0.124) + 0.0004(0.201) = 0.240 \text{ Btu/lb °F}$$

The specific heat at constant volume for zero pressure and 70 F is

$$c_v = 0.240 - \frac{53.35}{778.26} \quad 0.171 \text{ Btu/lb °F}$$

The ratio of specific heats is $\gamma = 1.40$.

MAXWELL RELATIONS. Six properties of the state of any fluid have been introduced: T, p, v, s, u, and h. In addition, free energy, or the Gibbs' zeta function, is $Z = h - Ts$; also the Gibbs' psi function (Helmholtz function) is defined as $\psi = u - Ts$. Eight properties of the state of any fluid have now been defined. The total differential of each of these properties is an exact differential in accordance with the following criterion:

$$z = f(x, y)$$
$$dz = M\,dx + N\,dy$$

where M and N are also functions of x and y, and

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

A number of useful relations between partial derivatives of the properties of state have been written. Some of these relations, called the Maxwell relations, are listed, together with their sources, the various total differential equations. In these relations the symbol A has the special meaning of $1/J = 1/778.26$, or the heat equivalent of work in Btu per foot-pound.

Total Differential Equation	Corresponding Maxwell Relations
$du = T\,ds - Ap\,dv$	$\left(\frac{\partial T}{\partial v}\right)_s = -A\left(\frac{\partial p}{\partial s}\right)_v$
	$\left(\frac{\partial u}{\partial v}\right)_s = -Ap$
	$\left(\frac{\partial u}{\partial s}\right)_v = T$
$dh = T\,ds + Av\,dp$	$\left(\frac{\partial T}{\partial p}\right)_s = A\left(\frac{\partial v}{\partial s}\right)_p$
	$\left(\frac{\partial h}{\partial p}\right)_s = Av$
	$\left(\frac{\partial h}{\partial s}\right)_p = T$
$dZ = -s\,dT + Av\,dp$	$\left(\frac{\partial s}{\partial p}\right)_T = -A\left(\frac{\partial v}{\partial T}\right)_p$
$d\psi = -s\,dT - Ap\,dv$	$\left(\frac{\partial s}{\partial v}\right)_T = A\left(\frac{\partial p}{\partial T}\right)_v$

Useful differential equations that involve the specific heats of fluids are:

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p$$

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v$$

$$c_p - c_v = T\left[\left(\frac{\partial s}{\partial T}\right)_p - \left(\frac{\partial s}{\partial T}\right)_v\right] = AT\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial T}\right)_v$$

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -AT\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

and

$$\left(\frac{\partial c_v}{\partial v}\right)_T = AT\left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

Table 2. Processes of Perfect Gases

In every case: $\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2} = mR$; $U_2 - U_1 = mc_v(T_2 - T_1)$; $H_2 - H_1 = mc_p(T_2 - T_1)$

Process	pv Relation	Final Pressure, lb/ft² abs	Final Volume, cu ft	Final Absolute Temperature, degree Rankine	Work Done by Gas, ft-lb	Heat Added, Btu	Increase in Entropy, Btu/°F
		p_2	mv_2	T_2	W_{12}	Q_{12}	ΔS_{12}
Constant pressure	$p_1 = p_2$	p_1	$mv_1 \frac{T_2}{T_1}$	$T_1 \frac{v_2}{v_1}$	$mp_1(v_2 - v_1)$	$mc_p(T_2 - T_1)$	$mc_p \ln \frac{T_2}{T_1}$
Constant volume	$v_1 = v_2$	$p_1 \frac{T_2}{T_1}$	mv_1	$T_1 \frac{p_2}{p_1}$	0	$mc_v(T_2 - T_1)$	$mc_v \ln \frac{T_2}{T_1}$
Constant temperature (isothermal)	$p_1v_1 = p_2v_2$	$\frac{p_1v_1}{v_2}$	$\frac{mv_1p_1}{p_2}$	T_1	$mp_1v_1 \ln \frac{v_2}{v_1}$	$\frac{mRT_1}{J} \ln \frac{v_2}{v_1}$	$\frac{mR}{J} \ln \frac{v_2}{v_1}$
Constant entropy (isentropic)	$p_1v_1^{\gamma} = p_2v_2^{\gamma}$	$p_1 \left(\frac{v_1}{v_2}\right)^{\gamma}$	$mv_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$	$T_1 \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$ or $T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$	$\frac{m}{\gamma - 1}(p_1v_1 - p_2v_2)$	0	0
Polytropic	$p_1v_1^n = p_2v_2^n$	$p_1 \left(\frac{v_1}{v_2}\right)^n$	$mv_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$	$T_1 \left(\frac{v_1}{v_2}\right)^{n-1}$ or $T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$	$\frac{m}{n - 1}(p_1v_1 - p_2v_2)$	$\frac{mc_v(n - \gamma)(T_2 - T_1)}{n - 1}$	$\frac{mc_v(n - \gamma)}{n - 1} \ln \frac{T_2}{T_1}$
Adiabatic throttling	The adiabatic throttling process with no change in elevation or velocity of the flowing gas occurs with $h_2 = h_1$ and with $t_2 = t_1$.						

These partial differential equations have many uses in the study of thermodynamics. It may be shown, for example, that $c_p = c_v$ for any fluid when the density has a maximum value. Note that

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1$$

Then

$$c_p - c_v = AT\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_v = -AT\left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$$

For maximum density,

$$\left(\frac{\partial v}{\partial T}\right)_p = 0 \quad \text{and} \quad c_p = c_v$$

Certain characteristics of perfect gases may also be derived from these equations. For a perfect gas with the equation of state $pv = RT$,

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -AT\left(\frac{\partial^2 v}{\partial T^2}\right)_p = 0$$

The specific heat at constant pressure of a perfect gas does not vary with pressure at constant temperature.

PROCESSES OF PERFECT GASES. For several processes of m pounds of a perfect gas, Table 2 gives equations for the work done by the gas, the quantity of heat added to the gas, and the increase in entropy of the gas, as well as relations between the properties in the end states.

22. REAL GASES

Many empirical equations of state have been proposed for real gases where the simple characteristic equation of state for perfect gases does not have the desired accuracy.

THE BEATTIE-BRIDGEMAN EQUATION OF STATE is

$$pv = \frac{RT}{v}(1 - \epsilon)(v + B) - \frac{A}{v}$$

where $A = A_0\left(1 - \frac{a}{v}\right)$, $B = B_0\left(1 - \frac{b}{v}\right)$, and $\epsilon = \frac{C}{vT^3}$.

Experimental values for the constants in the Beattie-Bridgeman equation are given in Table 3 for several real gases.

Table 3. Constants in the Beattie-Bridgeman Equation of State

Gas	R, ft-lb/lb °F	A_0, ft^4/lb	a, ft^3/lb	B_0, ft^3/lb	b, ft^3/lb	C, ft^3(°R)^3/lb
Air	53.35	842	0.0107	0.0255	−0.00061	14(10)^4
Carbon dioxide	35.12	1,403	0.0260	0.0382	0.0263	140(10)^4
Carbon monoxide	55 19	932	0.0150	0.0288	−0.00396	14(10)^4
Ethylene	55.13	4,240	0.0283	0.0693	0.0205	75.6(10)^4
Hydrogen	766 6	26,400	−0.0402	0.167	−0.347	2.33(10)^4
Methane	96.39	4,810	0.0185	0.0558	−0.0158	74.8(10)^4
Nitrogen	55.16	930	0.0149	0.0288	−0.00395	14(10)^4
Oxygen	48.29	790	0.0128	0.0232	0.00211	14(10)^4

The Beattie-Bridgeman equation of state in the form previously given may not be solved directly for specific volume when the pressure and temperature of a given gas are known. Another form of the equation of state that permits direct solution for specific volume is

$$v = \frac{RT}{p} + \frac{\alpha_1}{RT} + \frac{\alpha_2 p}{R^2T^2} + \frac{\alpha_3 p^2}{R^3T^3}$$

In this equation, all coefficients of the terms containing pressure are functions of temperature alone, for any one gas. The previous form of the Beattie-Bridgeman equation may be approximated by this equation by using the following values of the coefficients:

$$\frac{\alpha_1}{RT} = B_0 - \frac{A_0}{RT} - \frac{C}{T^3}$$

$$\frac{\alpha_2}{R^2T^2} = \frac{-B_0 b}{RT} + \frac{A_0 a}{R^2T^2} - \frac{B_0 C}{RT^4}$$

$$\frac{\alpha_3}{R^3T} = \frac{B_0 bC}{R^2T^5}$$

As many terms may then be carried in this equation as are necessary for the accuracy desired. For example, an equation of state for dry air with all terms dropped beyond the second is

$$v = \frac{53.35T}{p} + 0.0255 - \frac{15.78}{T} - \frac{14(10)^4}{T^3}$$

ZERO PRESSURE PROPERTIES. The **specific heat at constant pressure** and the **specific heat at constant volume** of any real gas depend principally upon temperature, but also on pressure. The latest values for the specific heats of gases at zero pressure have been obtained by application of quantum theory to spectroscopic data. Values for specific heats in Table 4 have been found by interpolation from *Bulletin* 30 of the Cornell University Engineering Experiment Station; Table 4 gives the specific heats at constant *zero pressure* for several gases over a range in temperature.

Table 4. The Specific Heat at Constant Zero Pressure, Btu/lb°F

Temperature, °F	Gas							
	Air	CO_2	CO	C_2H_4	CH_4	H_2	N_2	O_2
−100	.240	.176	.249	0.301	0.500	3.24	.249	.218
0	.240	.189	.249	0.340	0.512	3.37	.249	.219
100	.240	.202	.249	0.376	0.540	3.43	.249	.220
200	.241	.216	.250	0.423	0.579	3.45	.249	.223
300	.243	.227	.251	0.472	0.626	3.46	.250	.227
400	.245	.237	.253	0.515	0.672	3.47	.251	.230
500	.248	.246	.256	0.552	0 720	3.47	.253	.234
600	.252	.254	.260	0.590	0.772	3.48	.256	.239
700	.256	.261	.263	0 627	0.819	3.48	.259	.243
800	.259	.268	.267	0.660	0.865	3.49	.263	.247
900	.262	.273	.270	0.692	0 910	3.50	.266	.250
1000	.265	.279	.273	0.720	0.954	3.53	.269	.253
1500	.275	.299	.288	0.830	1.14	3.62	.284	.264
2000	.283	.312	.298	0.907	1.25	3.76	.294	.271
2500	.289	.322	.304	0.960	1.33	3.90	.301	.276
3000	.295	.328	.309	1.00	1.39	4.02	.306	.281
3500	.300	.333	.312	1.03	1.44	4.13	.310	.286
4000	.304	.337	.314	1.04	1.47	4.22	.313	.291

The data of Table 4 may be fitted over a limited range in temperature by equations of the following form:

$$c_p^{(0)} = A + BT + \frac{C}{\sqrt{T}} + \frac{D}{T} \quad \text{Btu/lb °F}$$

The empirical constants A, B, C, and D are given in Table 5.

Table 5. Empirical Constants in the Equation for the Specific Heats of Gases at Constant Zero Pressure

Gas	Range of Absolute Temperatures Covered by Equation, °R	A	B	C	D	$1.986/M$
Air	800–4500	0.320	$4.47(10)^{-6}$	−2.34	0	0.0686
CO_2	800–5000	0.435	$-3.12(10)^{-6}$	−5.69	0	0.0451
CO	800–5000	0.342	$2.78(10)^{-6}$	−2.72	0	0.0709
C_2H_4	900–3000	1.29	$3.85(10)^{-5}$	−23.8	0	0.0708
CH_4	700–4000	4.30	$-1.63(10)^{-4}$	−173	2059	0.1239
H_2	550–4500	2.86	$2.87(10)^{-4}$	+9.98	0	0.9851
N_2	800–5000	0.323	$5.14(10)^{-6}$	−2.27	0	0.0709
O_2	800–4500	0.320	$2.53(10)^{-6}$	−2.70	0	0.0621

At zero pressure, the specific heat at constant volume may be found from the specific heat at constant pressure as follows: $c_v^{(\infty)} = c_p^{(0)} - 1.986/M$. For example, the specific heat at constant volume of oxygen at zero pressure and at a temperature of 2000 F is $\left(0.271 - \frac{1.986}{32}\right)$ or 0.209 Btu per lb °F.

PROPERTIES AT PRESSURES GREATER THAN ZERO. Specific Heat. The Beattie-Bridgeman equation of state and the specific heats of real gases at zero pressure may be used to find the specific heats of real gases at pressures greater than zero in the following steps. (1) Find the specific heat at constant volume for zero pressure as indicated above. (2) Find the specific heat at constant volume for the volume v and absolute temperature T from

$$c_v = c_v^{(\infty)} + \frac{6RC}{JT^3}\left(\frac{1}{v} + \frac{B_0}{2v^2} - \frac{B_0 b}{3v^3}\right)$$

(3) Find the difference between the specific heat at constant pressure and the specific heat at constant volume in the desired state from

$$c_p - c_v = \frac{\dfrac{T}{J}\left[\left(v + B_0 - \dfrac{B_0 b}{v}\right)\left(R + \dfrac{2RC}{vT^3}\right)\right]^2}{2pv^3 - \dfrac{RC}{T^2}\left(v + B_0 - \dfrac{B_0 b}{v}\right) - RT\left(1 - \dfrac{C}{vT^3}\right)(v^2 + B_0 b) + A_0 a}$$

At constant temperature, an increase of pressure will increase the instantaneous specific heats at constant pressure and constant volume. This effect is not large at high temperatures, but it is very significant at low temperatures. For example, air at a temperature of 2000 F has a constant-pressure specific heat of 0.2834 Btu per lb °F at zero pressure and 0.2840 at an absolute pressure of 1000 psia; at 0 F, the constant pressure specific heat is 0.240 at zero pressure and 0.278 at a pressure of 1000 psia. The effects of pressure and temperature upon the specific heat at constant pressure of dry air are shown in Fig. 1.

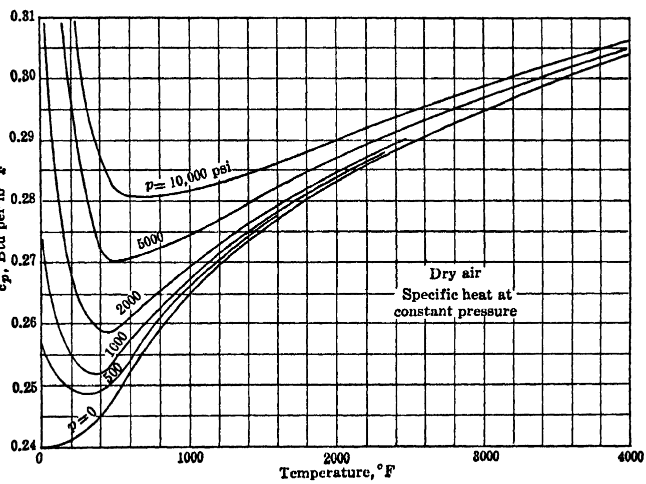

FIG. 1. Variation of the specific heat of air with temperature and pressure. (*Bulletin* 30, Cornell University Engineering Experiment Station)

The **enthalpy** of the real gas in any state may be found from the following equation:

$$h = \frac{1}{J}\left\{\int_0^p \left[\frac{\partial(v/T)}{\partial(1/T)}\right]_p dp\right\}_T + \int_{T_0}^{T} c_p^{(0)}\, dT + h_0$$

where h_0 is the arbitrary value of the enthalpy at the reference state of zero pressure and absolute temperature T_0.

The **entropy** of the real gas may be found from the following equation:

$$-R \ln p - \frac{1}{J}\left\{\int_0^p \left[\frac{\partial\left(v - \dfrac{RT}{p}\right)}{\partial T}\right]_p dp\right\}_T + \int_{T_0}^{T} {}_p^{(0)}\, dT + s_0$$

where s_0 is the arbitrary value of the entropy at the reference state of zero pressure and absolute temperature T_0.

The **internal energy** of the real gas in any state may be found from the equation that defines enthalpy, or $u = h - (pv/J)$.

23. VAPORS

Vapors are substances in conditions intermediate between the liquid and gaseous states that can completely fill a container, exerting a pressure as does a gas. As used in engineering apparatus, vapors may depart considerably from the perfect gas laws. Tables, like those of Keenan and Keyes, give the properties of saturated water, saturated steam, and superheated steam; charts of the properties of vapors, like the *Thermodynamic Charts* of Ellenwood and Mackey, are also useful in the solution of many problems.

A **saturated liquid** is in a state where addition of heat at constant pressure will start immediate vaporization at constant temperature, and where removal of heat at constant pressure will result in immediate drop of liquid temperature, or subcooling. This state is commonly designated by the subscript f.

A **saturated vapor** is in a state where addition of heat at constant pressure will result in immediate temperature rise or superheating, and where removal of heat at constant pressure will start immediate condensation at constant temperature. This state is commonly designated by the subscript g.

The temperature of a *superheated vapor* is higher than the saturation temperature corresponding to the pressure of the vapor.

A **mixture of saturated liquid and saturated vapor** is called a *wet mixture.* The ratio of the weight of saturated vapor to the total weight of the mixture is the *quality* of the mixture, designated by the symbol x. The increase in any property of the fluid during complete vaporization at constant pressure is designated by the subscript fg; for example, h_{fg} is the increase in enthalpy during complete vaporization at constant pressure (the latent heat of vaporization); $h_{fg} = h_g - h_f$.

Properties of 1 lb of a wet mixture may be found in terms of the quality of the mixture and of the properties of the saturated liquid and saturated vapor at the same pressure:

$$h = h_f + x(h_g - h_f) = h_f + xh_{fg}$$

$$v = v_f + x(v_g - v_f) = v_f + xv_{fg}$$

$$s = s_f + x(s_g - s_f) = s_f + xs_{fg}$$

The internal energy of a pound of vapor in any state may be found from the enthalpy, absolute pressure, and specific volume as $u = h - (pv/J)$.

At very low pressures and in states sufficiently far removed from the saturated states, many vapors will obey, closely, the laws of perfect gases. For example, water vapor mixed with dry air in the atmosphere may be assumed to obey the laws of perfect gases with sufficient accuracy for many calculations.

Table 6. Critical-State Properties

(*International Critical Tables* and Other Sources)

Fluid	Critical Temperature, °F	Critical Pressure, psia	Critical Specific Volume, cu ft/lb
Air	−220	547	.046
Alcohol (methyl)	464	1157	.059
Alcohol (ethyl)	470	927	.058
Ammonia	270	1639	.068
Argon	−188	705	.030
Butane	307	529	.071
Carbon dioxide	88	1073	.035
Carbon monoxide	−218	514	.051
Carbon tetrachloride	541	661	.029
Chlorine	291	1119	.028
Ethane	90	717	.076
Ethylene	49	848	.073
Helium	−450	33	.231
Hexane	455	434	.068
Hydrogen	−400	188	.516
Methane	−116	673	.099
Methyl chloride	290	967	.043
Neon	−380	391	.033
Nitric oxide	−137	955	.031
Nitrogen	−233	492	.052
Octane	565	362	.068
Oxygen	−182	730	.037
Propane	204	632	.071
Sulfur dioxide	315	1142	.031
Water	705	3206	.050

Properties of many vapors, such as steam, ammonia, carbon dioxide, and "Freon," are discussed in other sections of this book.

At a certain temperature, called the *critical temperature*, the latent heat of vaporization vanishes; the saturated liquid then becomes saturated vapor with no change in volume and with no addition of heat. The pressure that is just sufficient to liquefy the fluid at the critical temperature is called the *critical pressure*. Above the critical temperature, a separation of the liquid and gaseous states cannot be effected by the action of pressure, alone.

In Table 6 are given values of the temperature, absolute pressure, and specific volume for many different fluids in the critical state.

24. THE CARNOT CYCLE

(See Section 4.)

25. ISENTROPIC FLOW OF GASES AND VAPORS

(See also Section 1 and p. 3-65.)

For steady, frictionless, adiabatic flow of a gas in a horizontal passage, let the initial properties of the gas be designated by the subscript 1 at the section of area A_1 sq ft; let the final properties of the gas be designated by the subscript 2 at the section of area A_2 sq ft. At section 2, the velocity of the gas is

$$V_2 = 8.02 \sqrt{\frac{\gamma p_1 v_1 [1 - (p_2/p_1)^{\frac{\gamma-1}{\gamma}}]}{(\gamma - 1)[1 - (A_2/A_1)^2 (p_2/p_1)^{\frac{2}{\gamma}}]}} \text{ ft/sec}$$

The weight of gas flowing is

$$w = 8.02 A_2 \sqrt{\frac{\gamma p_1 [(p_2/p_1)^{\frac{2}{\gamma}} - (p_2/p_1)^{\frac{\gamma+1}{\gamma}}]}{v_1(\gamma - 1)[1 - (A_2/A_1)^2 (p_2/p_1)^{\frac{2}{\gamma}}]}} \text{ lb/sec}$$

If the initial velocity of the gas is negligible, corresponding equations are

$$V_2 = 8.02 \sqrt{\frac{\gamma p_1 v_1}{\gamma - 1} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]} \text{ ft/sec}$$

and

$$w = 8.02 A_2 \sqrt{\frac{\gamma p_1}{v_1(\gamma - 1)} \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right]} \text{ lb/sec}$$

The shape of a passage designed to give steady isentropic flow of a gas is convergent at first; if the ratio of the discharge pressure to the admission pressure is low enough, the passage becomes divergent. The section of least area is called the *throat*. If the initial velocity is negligible, the velocity at the throat is

$$V_t = 8.02 \sqrt{\frac{\gamma p_1 v_1}{\gamma + 1}} \text{ ft/sec}$$

The ratio of the pressure at the throat to the admission pressure is called the critical pressure ratio, r_c:

$$r_c = \frac{p_t}{p_1} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$$

The weight rate of discharge for steady isentropic flow of the gas is not changed by reducing the discharge pressure below the critical value if there is no change in the initial state of the gas and no change in the area of the minimum section. For any discharge pressure *less* than the critical and for an area of minimum section of A_t sq ft, the weight rate of discharge is

$$= 8.02 A_t \sqrt{\frac{p_1}{v_1} \left(\frac{\gamma}{\gamma - 1}\right) \left[\left(\frac{2}{\gamma + 1}\right)^{\frac{2}{\gamma - 1}} - \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma - 1}}\right]} \text{ lb/sec}$$

Let

$$B = 8.02\sqrt{\frac{\gamma}{\gamma - 1}\left[\left(\frac{2}{\gamma + 1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

then

$$w = \frac{BA_t p_1}{\sqrt{RT_1}} \text{ lb/sec}$$

Values of the critical pressure ratio and B are given in Table 7 for different values of γ.

Table 7

γ	Critical Pressure Ratio, r_c	B
1.00	.607	3.42
1.02	.602	3.46
1.04	.598	3.49
1.06	.593	3.51
1.08	.588	3.54
1.10	.584	3.56
1.15	.574	3.62
1.20	.564	3.68
1.25	.555	3.73
1.30	.546	3.78
1.35	.537	3.83
1.40	.528	3.88
1.45	.520	3.93
1.50	.512	3.98

EXAMPLE. Find the weight rate of isentropic flow of air ($\gamma = 1.4$) through an orifice with a diameter of 0.25 in. if the initial velocity of the air is negligible, the initial pressure is 90 psia; the initial temperature is 100 F, and the final pressure is 1. 60 psia, 2. 14.7 psia.

Solution. (1) Critical pressure ratio is 0.528. A discharge pressure of 60 is greater than the critical pressure; hence the true pressure ratio is used in the flow equation. The initial specific volume of the air is $v = 2.3$ cu ft per lb; $p_2/p_1 = 0.667$; $A_2 = 0.000341$ sq ft. The weight rate of isentropic flow is

$$w = 8.02(0.000341)\sqrt{\frac{1.4(90)(144)}{2.3(0.4)}(0.667^{1.429} - 0.667^{1.714})}$$

$$= 0.0951 \text{ lb / sec}$$

(2) The discharge pressure of 14.7 is *less than* the critical pressure; hence the critical pressure ratio of 0.528 must be used to calculate the weight rate of isentropic flow. From Table 7, $B = 3.88$, and then

$$w = \frac{BA_t p_1}{\sqrt{RT_1}} = \frac{3.88(0.000341)(90)(144)}{\sqrt{53.35(559.7)}}$$

$$= 0.0992 \text{ lb / sec}$$

For determining the **isentropic flow of vapors,** tables or charts may be used to find the properties of the vapor instead of using an approximate value of γ for the isentropic expansion. Enthalpy, h, is the property most frequently required from the charts. For the steady, isentropic flow of vapors in horizontal passages, the final velocity is

$$V_2 \quad 223.7\sqrt{\frac{h_1 - h_2}{[1 - (A_2/A_1)^2(v_1/v_2)^2]}} \text{ ft/sec}$$

For negligible initial velocity, the final velocity is

$$V_2 = 223.7\sqrt{h_1 - h_2} \text{ ft/sec}$$

The weight rate of flow may be found from the equation of continuity as

$$w = \frac{A_2V_2}{v_2} \text{ lb/sec}$$

The final enthalpy (h_2) and the final specific volume (v_2) of the vapor are found at the final pressure (p_2) at the same entropy as the initial state ($s_2 = s_1$).

EXAMPLE. Find the weight rate of isentropic flow of steam through a nozzle with a minimum area of cross section of 0.0015 sq ft if the initial absolute pressure of the steam is 100 psi, the initial temperature is 600 F, and the discharge pressure is 14.7 psia. The initial velocity is negligible. From steam tables or charts, $h_1 = 1329$ Btu per lb, $v_1 = 6.22$ cu ft per lb, $s_1 = s_2 = 1.758$. For the isentropic expansion of superheated steam, γ is approximately 1.3, and the critical pressure ratio is 0.546. At $p_2 = 54.6$ psia, $h_2 = 1264$ and $v_2 = 9.85$. The velocity of the steam at the section of minimum area is $V_2 = 223.9\sqrt{65} = 1800$ ft per sec, and the weight rate of isentropic flow is $w = 0.0015(1800)/9.85 = 0.274$ lb per sec.

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THERMODYNAMICS OF GASES AT HIGH VELOCITY

By Neil P. Bailey

SYMBOLS AND UNITS

A = area, sq ft
c_p = specific heat at constant pressure, Btu per lb per °F
c_v = specific heat at constant volume, Btu per lb per °F
d = total differential
e = diffuser pressure rise ratio
F = frictional force, lb
f = friction factor
g = acceleration of gravity, 32.2 ft per sec per sec
J = 778.26 ft-lb per Btu
L = mechanical work, Btu per lb
M = Mach number
m = hydraulic radius, area over perimeter, ft
P = pressure, lb per sq ft, abs
Q = heat added, Btu per lb
R = gas constant, 53.3 for air
T = temperature, degree Rankine (°R)
t = time, sec
V = velocity, ft per sec
v = specific volume, cu ft per lb
w = weight flow, lb per sec
x = distance, ft
ρ = mass density, slugs per cu ft
γ = ratio of specific heats, 1.395 for cold air

26. BASIC CONCEPTS

The basic equations for the flow of gases at high velocities are simple individually, but when they are solved simultaneously the resulting relationships are often complicated and tedious to use. This is because the state of a moving gas cannot be defined by fewer than three of the variables commonly used. A further complication is the fact that the general gas equation $Pv = RT$ was devised for a gas at rest. When it is used to define the condition of a moving gas both the temperature T and the pressure P must be *static* values measured relative to the stream; that is, they must be values that would be measured by instruments moving with the stream.

MACH NUMBER. Much of this complication can be avoided by replacing the velocity V and the temperature T in terms of the Mach number M of the gas. The simplest definition of Mach number is the ratio of flow velocity to local velocity of sound in the gas at that point.

$$M = \frac{V}{\sqrt{\gamma g R T}} \tag{1}$$

Translation energy of flow depends on the velocity V and the thermal energy upon T; hence it is not surprising that this ratio as measured by the Mach number proves to be very useful as a fundamental working variable. For later use it is convenient to express eq. 1 in the differential form:

$$\frac{dM}{M} = \frac{dV}{V} - \frac{1}{2}\frac{dT}{T} \tag{2}$$

Gas Equation. The statement that the medium involved is a gas is

$$Pv = RT \tag{3}$$

or

$$\rho = \frac{1}{vg} = \frac{P}{gRT} \tag{4}$$

which, in a differential form, is

$$\frac{d\rho}{\rho} = \frac{dP}{P} - \frac{dT}{T} \tag{5}$$

ENERGY EQUATION. As a pound of gas flows a distance dx if an amount of mechanical work dL is done on it and an amount of heat dQ is added to it, the sum must appear as increased kinetic energy $(1/Jg)V\,dV$, as flow work or as an internal or thermal energy change. The forcing of a fixed volume of gas v into a region of higher pressure involves an amount of work $(1/J)v\,dP$. If the gas is compressible and experiences a volume change dv, an amount of energy $(1/J)P\,dv$ is involved. Since the internal energy of a gas depends only on temperature, the energy equation is

$$dL + dQ = c_v\,dT + \frac{1}{J}\,v\,dP + \frac{1}{J}\,P\,dv + \frac{1}{Jg}\,V\,dV \tag{6}$$

but

$$c_v\,dT + \frac{1}{J}\,d(Pv) = \left(c_v + \frac{R}{J}\right)dT = c_p\,dT \tag{7}$$

and

$$c_p = \frac{\gamma}{(\gamma - 1)}\frac{R}{J} \tag{8}$$

$$dL + dQ = \frac{\gamma}{(\gamma - 1)}\frac{R}{J}\,dT + \frac{1}{Jg}\,V\,dV \tag{9}$$

For the special case of no mechanical work dL or heat transfer dQ,

$$\frac{1}{(\gamma - 1)}\frac{dT}{T} = -\frac{V^2}{\gamma gRT}\frac{dV}{V} = -M^2\frac{dV}{V} \tag{10}$$

FLOW EQUATIONS. If it is assumed that the flow is uniform across a channel of area A, the statement of steady flow is

$$\rho VA = \text{constant} \tag{11}$$

or, in the differential form,

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \tag{12}$$

FORCE EQUATION. **With no channel wall friction** forces or other losses, any pressure change dP acting over the area A must involve the acceleration of the mass of gas $\rho A\,dx$ or

$$A\,dP_a + \rho A\,dx\frac{dV}{dt} = A\,dP_a + \rho A\;V\,dV = 0 \tag{13}$$

With wall or pipe friction, the additional pressure change may be expressed as

$$dP_f = -\frac{\rho f\,dx\,V^2}{2m} \tag{14}$$

where m is the hydraulic radius which is defined as the channel cross sectional area divided by the wetted perimeter and f is the flow friction factor, which is a function of Reynolds' number. The general pressure equation is

$$dP = dP_a + dP_f = -\rho V\,dV - \frac{\rho f\,dx\,V^2}{2m} \tag{15}$$

Combining eqs. 1, 4, and 15 gives

$$\frac{dP}{P} = -\gamma M^2\left[\frac{f dx}{2m} + \frac{dV}{V}\right] \tag{16}$$

REVERSIBILITY EQUATION. Flow that is reversible or without friction, turbulence, or heat transfer is characterized by internal-energy changes that are entirely due to compression or expansion work $(1/J)P\,dv$.

$$c_v\,dT + \frac{1}{J}\,P\,dv = 0 \tag{17}$$

From the general gas equation,

$$P\,dv = R\,dT - v\,dP = R\,dT - \frac{RT}{P}\,dP \tag{18}$$

Using,

$$c_v = \frac{1}{(\gamma - 1)} \frac{R}{J} \tag{19}$$

the condition of reversibility is

$$\frac{dT}{T} = \frac{(\gamma - 1)}{\gamma} \frac{dP}{P} \tag{20}$$

SUMMARY OF BASIC EQUATIONS.

Mach number:
$$\frac{dM}{M} = \frac{dV}{V} - \frac{1}{2}\frac{dT}{T} \tag{21}$$

Gas equation:
$$\frac{d\rho}{\rho} = \frac{dP}{P} - \frac{dT}{T} \tag{22}$$

Energy:
$$\frac{1}{(\gamma - 1)} \frac{dT}{T} = -M^2 \frac{dV}{V} \tag{23}$$

Flow:
$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \tag{24}$$

Pressure:
$$\frac{dP}{P} = -\gamma M^2 \left[\frac{f\,dx}{2m} + \frac{dV}{V} \right] \tag{25}$$

Reversibility:
$$\frac{dT}{T} = \frac{(\gamma - 1)}{\gamma} \frac{dP}{P} \tag{26}$$

If eq. 23 is combined with eq. 25 for the case of no friction ($f = 0$), the result is eq. 26. This means that there are but five independent equations. When f is not zero, eq. 26 does not apply and when $f = 0$, eqs. 25 and 26 are identical.

The form of these basic equations immediately suggests that to solve them simultaneously, dV/V, dP/P, dT/T, and $d\rho/\rho$ should be considered as variables and not V, P, T, and ρ. With the exception of M all variables appear only in this form. This immediately suggests that each and every other variable is expressible in terms of Mach number M.

TOTAL AND STATIC TEMPERATURES. The general relationship between the static temperature T when the gas is moving at a Mach number M and the impact or total temperature T_0 indicated by an ideal stationary thermometer that brings the gas to rest without heat transfer may be obtained from eq. 9 when dL and dQ are zero.

$$\gamma g R \int_T^{T_0} dT + (\gamma - 1) \int_V^0 V\,dV = 0 \tag{27}$$

giving

$$T_0 - T = \frac{(\gamma - 1)}{2\gamma g R} V^2 \tag{28}$$

or

$$\frac{T_0}{T} = 1 + \frac{(\gamma - 1)}{2} \frac{V^2}{\gamma g R T} = 1 + \frac{(\gamma - 1)}{2} M^2 \tag{29}$$

The total temperature T_0 is measured when a gas is at rest and the static temperature T is the temperature that would be indicated by a thermometer moving with the gas stream. It is the temperature that determines the gas density, internal energy, and Mach number. It is not directly measurable by usual methods and so is ordinarily a calculated quantity. (See column *a*, Table 1, p. 3-66, for solutions of eq. 29 for low-temperature air.)

27. REVERSIBLE OR FRICTIONLESS FLOW

Truly reversible flow occurs in the free stream deceleration at the stagnation zone in front of an impact tube and is approximately realized in well-designed nozzles. It is also a useful first approximation for the solution to other problems such as decelerating flow in diffusers.

PRESSURE RATIO. The relationship between Mach number change and pressure change for constant total energy reversible processes may be found by eliminating dV/V and dT/T from eqs. 21, 23, and 26 to give

$$\frac{dP}{P} = -\frac{\gamma M\,dM}{1 + \left(\frac{\gamma - 1}{2}\right) M^2} \tag{30}$$

When a moving gas at any Mach number M and static pressure P is decelerated to rest ($M = 0$) reversibly, the final pressure is known as the total pressure P_0.

$$\int_P^{P_0} dP/P = -\int_M^0 \frac{\gamma M\, dM}{1 + \left(\frac{\gamma - 1}{2}\right) M^2} \tag{31}$$

This gives

$$\frac{P_0}{P} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\gamma/\gamma - 1} \tag{32}$$

This same equation holds equally well for the ratio of total to static pressure necessary to accelerate a gas from rest to a Mach number M.

Values for eq. 32 are listed in Table 1, column b, for low-temperature air. The pressure ratio involved in a change from any Mach number to any other value becomes only the ratio of the two pressure ratios of Table 1, column b.

AREA RATIO. Eliminating $d\rho/\rho$, dP/P, dV/V and dT/T from eqs. 21, 22, 23, 24, and 26 gives the relationship between Mach number and flow area for a reversible change, such as

$$\frac{dA}{A} = -\frac{dM}{M} + \left(\frac{\gamma + 1}{2}\right) \frac{M\, dM}{\left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]} \tag{33}$$

It is not feasible to integrate eq. 33 for the area change between a Mach number M and one of zero, since an infinite area would be required at zero velocity.

Integrating from any A_0 and M_0 to A and M in eq. 33,

$$\frac{A}{A_0} = \frac{M_0}{M} \left[\frac{1 + \left(\frac{\gamma - 1}{2}\right) M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M_0^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{34}$$

Equation 34 can be readily solved for the area ratio needed to make a given change in Mach number, but must be solved by trial and error for Mach number. To facilitate such calculations, Table 1, column c, gives values for eq. 34 for the case of $M_0 = 1.0$. As before, this table can be used between any two Mach numbers or area ratios merely by using the ratio of the two ratios from column c.

Table 1. Cold Air

$\gamma = 1.395$

M	(a) T_0/T	(b) P_0/P (ideal)	(c) $A/A(M = 1)$ (ideal)	(d) $w\sqrt{T_0}/AP$	(e) $\frac{f\gamma x}{2m}$ to $M = 1$
0.00	1.000	1.000	∞	0.0000	∞
0.10	1.002	1.007	5.825	0.0918	46.8500
0.20	1.008	1.028	2.965	0.1842	10.1800
0.30	1.018	1.064	2.036	0.2776	3.7112
0.40	1.032	1.116	1.591	0.3726	1.6170
0.50	1.049	1.186	1.340	0.4698	0.7490
0.60	1.071	1.274	1.188	0.5696	0.3440
0.70	1.097	1.386	1.094	0.6724	0.1459
0.80	1.126	1.522	1.038	0.7788	0.0507
0.90	1.160	1.689	1.009	0.8891	0.0102
1.00	1.197	1.890	1.000	1.0038	0.0000
1.10	1.239	2.131	1.008	1.1231	0.0070
1.20	1.284	2.420	1.030	1.2474	0.0236
1.30	1.334	2.765	1.067	1.3771	0.0455
1.40	1.387	3.176	1.115	1.5125	0.0700
1.50	1.444	3.664	1.177	1.6535	0.0955
1.60	1.507	4.242	1.251	1.8009	0.1210
1.70	1.571	4.928	1.339	1.9544	0.1460
1.80	1.640	5.737	1.441	2.1144	0.1699
1.90	1.713	6.693	1.558	2.2810	0.1928
2.00	1.790	7.816	1.691	2.4544	0.2144

MACH NUMBER DETERMINATION. There are, in general, two ways of determining the Mach number of a gas flowing in a passage. A value of static pressure from a wall pressure tap or a traverse tube and the value of the total pressure from an impact tube give the value of (P_0/P) to use with eq. 32 or Table 1, column b. When the flow is subsonic, the deceleration in front of a properly designed impact tube gives the true total pressure because the deceleration is reversible.

The other method applies to the case where the weight flow w, the total temperature T_0, the flow area A, and the static pressure P are known. The weight flow for a uniform velocity distribution is given by

$$w = \rho g V A = \frac{P}{RT} VA \tag{35}$$

The velocity V may be eliminated by using $\sqrt{\gamma g R T}\, M$ to give

$$w = \frac{P}{\sqrt{T}} \sqrt{\gamma g R}\, MA \tag{36}$$

If now T is evaluated from eq. 29, it gives

$$\frac{w\sqrt{T_0}}{AP} = M \sqrt{\left(\frac{\gamma g}{R}\right)\left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]} \tag{37}$$

Equation 37 uniquely determines the Mach number whenever w, T_0, A, and P are known and Table 1, column d, facilitates its use.

28. FLOW WITH FRICTION

When gases flow at high velocities, the changes in pressure caused by pipe or duct friction result in variations in density ρ which initiate additional velocity changes. These accelerations may produce pressure changes much greater than the original effects that produced them.

MACH NUMBER CHANGES. The pressure change when friction is present is defined by eq. 25. If the flow equation (eq. 24) for constant area is combined with the gas equation (eq. 22), the result is

$$\frac{d\rho}{\rho} = \frac{dP}{P} - \frac{dT}{T} = -\frac{dV}{V} \tag{38}$$

If this value of dP/P is used in the pressure equation,

$$\frac{dP}{P} = \frac{dT}{T} - \frac{dV}{V} = -\gamma M^2 \left(\frac{f\,dx}{2m} + \frac{dV}{V}\right) \tag{39}$$

and if the value of dT/T from the Mach number equation (eq. 21) is used in eq. 39,

$$\frac{dT}{T} = \frac{dV}{V} - \gamma M^2 \left(\frac{f\,dx}{2m} + \frac{dV}{V}\right) = \frac{2dV}{V} - \frac{2dM}{M} \tag{40}$$

Similarly, eliminating dT/T from eq. 21 and the energy equation (eq. 23)

$$\frac{dT}{T} = \frac{2dV}{V} - \frac{2dM}{M} \qquad -(\gamma - 1) M^2 \frac{dV}{V} \tag{41}$$

Eliminating dV/V from eqs. 40 and 41,

$$\frac{dV}{V} = \frac{(2dM)/M}{2 + (\gamma - 1)M^2} = \frac{(2dM)/M - (\gamma M^2 fx)/2m}{1 + \gamma M^2} \tag{42}$$

Equation 42 can now be solved for the frictional flow distance to produce a Mach number change in a constant area duct.

$$\frac{\gamma f\,dx}{2m} = \frac{dM}{M^3} \frac{(1 - M^2)}{1 + \frac{(\gamma - 1)}{} M^2} \tag{43}$$

Since dx can have only a positive sense, dM and $(1 - M^2)$ are the only terms that can become negative, and they must reverse at the same time.

This means that when M is less than unity, dM must be positive, and when M is greater than one, dM must be negative. Also as M approaches unity, dM/dx is infinite.

In words, this means that with flow below sound velocity, friction increases Mach number and, above the acoustic, friction decreases Mach number. Also, it means that flow cannot continue at unity Mach number in a constant area passage with friction once this velocity is reached.

For purposes of integration, eq. 43 may be expressed as

$$\frac{\gamma f}{2m}\int_0^x dx = \int_{M_1}^{M_2}\frac{dM}{M^3} - \left(\frac{\gamma+1}{2}\right)\int_{M_1}^{M_2}\frac{dM}{M} + \left(\frac{\gamma+1}{4}\right)\int_{M_1}^{M_2}\frac{(\gamma-1)M\,dM}{\left[1+\frac{(\gamma-1)}{2}M^2\right]} \tag{44}$$

or

$$\frac{\gamma f x}{2m} = -\frac{1}{2}\left(\frac{1}{M_2^2}-\frac{1}{M_1^2}\right) - \frac{(\gamma+1)}{2}\log_e\frac{M_2}{M_1} + \frac{(\gamma+1)}{4}\log_e\left[\frac{1+\left(\frac{\gamma-1}{2}\right)M_2^2}{1+(\gamma-1)M_1^2}\right] \tag{45}$$

To aid with the solution of eq. 45, Table 1, column *e*, gives values of $fx/2m$ necessary to go from M_1 to $M_2 = 1.0$ or acoustic flow by friction in a constant area duct. The friction factor f is a function of Reynolds' number, and values are given in NACA *Technical Memorandum* 844. For most channels of interest, f will range from 0.004 to 0.005 for both subsonic and supersonic flow.* See also Section 6 for discussion of f.

ACCELERATION AND FRICTION EFFECTS. Since the flow is at constant area A, constant weight flow w, and constant total energy or total temperature T_0, eq. 37 may be used for constant area friction to give

$$\frac{w\sqrt{T_0}}{A} = P_1M_1\sqrt{\frac{\gamma g}{R}\left[1+\frac{(\gamma-1)}{2}M_1^2\right]} = P_2M_2\sqrt{\frac{\gamma g}{R}\left[1+\frac{(\gamma-1)}{2}M_2^2\right]} \tag{46}$$

This gives the static pressure ratio for both friction and acceleration,

$$\frac{P_2}{P_1} = \frac{M_1\sqrt{\frac{\gamma g}{R}\left[1+\frac{(\gamma-1)}{2}M_1^2\right]}}{M_2\sqrt{\frac{\gamma g}{R}\left[1+\frac{(\gamma-1)}{2}M_2^2\right]}} \tag{47}$$

and Table 1, column *d*, may be used to an advantage to solve friction problems.

The portion of this pressure difference $(P_1 - P_2)$ that is the result of acceleration or deceleration may be formed from eq. 13:

$$(\Delta P)_a = -\int_1^2 \rho V\,dV \tag{48}$$

For constant area,

$$\rho V = \text{constant} = \rho_1 V_1 = \rho_2 V_2 \tag{49}$$

$$(\Delta P)_a = -(\rho_2 V_2^2 - \rho_1 V_1^2) = -(\gamma P_2 M_2^2 - \gamma P_1 M_1^2) \tag{50}$$

$$\frac{(\Delta P)_a}{P_1 - P_2} = \frac{\gamma(P_2/P_1)M_2^2 - \gamma M_1^2}{(1 - P_2/P_1)} \tag{51}$$

For subsonic flow both friction and acceleration produce pressure decreases, but with supersonic flow the deceleration resulting from friction may produce a larger increase in pressure than the decrease caused by friction itself. This results in the apparent contradiction of friction causing a pressure increase.

29. NOZZLES AND DIFFUSERS †

In the same way as in Article 28, the Mach number equation, gas equation, energy equation, and the pressure equation may be combined with the flow equation for variable area to give

$$m\frac{dM}{dx} = \frac{\left(-\frac{m}{A}\frac{dA}{dx} + \frac{f\gamma M^2}{2}\right)M\left[1+\frac{(\gamma-1)}{2}M^2\right]}{(1-M^2)} \tag{52}$$

It is convenient that for conical and square section passages that are tapered, the term $(m/A)(dA/dx)$ is the tangent of the wall angle.

It follows then that for converging passages $(m/A)(dA/dx)$ is negative and for diverging passages it is positive. This means that the divergence term $(m/A)(dA/dx)$ may be either plus or minus and greater or smaller numerically than the friction term $(f\gamma M^2)/2$.

SUBACOUSTIC NOZZLES. Since the walls of subacoustic nozzles converge, $(m/A)(dA/dx)$ is negative, and friction and convergence combine to produce acceleration (dM/dx is plus). Furthermore, as M approaches unity, the denominator of eq. 52 approaches zero, and the acceleration tends to become infinite. These factors explain

* Note that the *smaller* value of f is used in this chapter.

† See also Supersonics, Section 15.

why it is possible to construct very efficient subacoustic nozzles, and why a short parallel section at the end of the convergence is quite commonly used in subacoustic nozzle design.

SUBACOUSTIC DIFFUSERS. In diverging subacoustic passages that are known as diffusers, $(m/A)(dA/dx)$ is plus. This means in eq. 52 that friction is an accelerating influence contrary to the decelerating action of the divergence. For wall angles of one degree or slightly less, the two factors balance, and flow at constant Mach number $(dM/dx = 0)$ can result. Unfortunately, the friction and divergence terms are not independent variables. As a rule, if the wall angle exceeds five or six degrees, diffusion effectiveness actually decreases, indicating that the increased wall angle has the effect of increasing friction.

SUPERACOUSTIC NOZZLES. After M exceeds unity, the denominator of eq. 52 becomes negative, and the friction term in the numerator produces a negative dM/dx or a deceleration. Acceleration is accomplished only as long as the divergence $(m/A)(dA/dx)$, which is positive, exceeds the friction. Thus, in the supersonic portion of a nozzle, friction opposes the accelerating effect of divergence, and any nozzle design has a limiting maximum Mach number.

The ideal area ratio for any nozzle may be calculated from eq. 34 or from Table 1, column c. The actual area ratio may be found by integrating eq. 52 in the form

$$\frac{dx}{m} = \frac{(1 - M^2)\,dM}{\left(-\frac{m}{A}\frac{dA}{dx} + \frac{f\gamma M^2}{2}\right) M \left[1 + \frac{(\gamma - 1)}{2} M^2\right]} \tag{53}$$

This integration can be accomplished since m may be expressed in terms of x, and $(m/A)(dA/dx)$ is a constant for conical or square sections. There is little information as to the proper value of f to be used, but it can be evaluated from tests of existing successful nozzle designs.

SUPERACOUSTIC DIFFUSERS. In a supersonic diffuser eq. 52 indicates that a convergence or negative $(m/A)(dA/dx)$ will combine with the friction effect and together produce a negative dM/dx or deceleration. This will continue until M approaches one, at which time the deceleration becomes infinite. Very few actual data exist for such diffusers, for, as will be indicated in Article 30, their action is usually confused by shock phenomena.

DIFFUSER EFFICIENCY. The purpose of a nozzle is to change thermal energy into directed kinetic energy, and its efficiency is usually defined as the kinetic energy change produced divided by the kinetic energy increase available with a reversible expansion between the same pressures.

Since the purpose of a diffuser is to produce a pressure rise starting with a given initial kinetic energy, it is apparent that the efficiency concepts of nozzles cannot be applied to diffusers without modification. Equation 32 and Table 1, column b, give the ideal pressure ratio available when air at any Mach number is brought completely to rest reversibly.

Since it would be illogical to charge diffusers with bringing flow completely to rest, it is convenient to use the *pressure rise ratio*. It is defined as the ratio of the actual pressure rise of a diffuser to the ideal pressure rise available from a reversible deceleration between the same Mach numbers. This ratio compares the pressure rise $(P_2 - P_1)$ of an actual diffuser of area ratio (A_2/A_1) with the ideal pressure rise $(p_2 - p_1)$ of an ideal diffuser of area ratio (a_2/a_1) that changes the flow from the same M_1 to the same M_2.

Since the leaving Mach number is the same in the ideal and actual cases and the total temperatures are equal, it follows that the static temperatures are equal. Thus the necessary leaving flow area is inversely proportional to the static pressures or

$$\frac{A_2}{a_2} = \frac{p_2}{P_2} \tag{54}$$

For the same $A_1 = a_1$ and $P_1 = p_1$,

$$\frac{A_2/A_1}{a_2/a_1} = \frac{p_2/p_1}{P_2/P_1} \tag{55}$$

From the definition of pressure rise ratio or efficiency (e),

$$e = \frac{P_2 - P_1}{p_2 - p_1} = \frac{P_2/P_1 - 1}{p_2/p_1 - 1} \tag{56}$$

Using eq. 55 to eliminate P_2/P_1 from eq. 56 gives

$$\left(\frac{A_2}{A_1}\right)_{\text{actual}} = \frac{(a_2/a_1)_{\text{ideal}}\,(p_2/p_1)_{\text{ideal}}}{1 + [(p_2/p_1)_{\text{ideal}} - 1]e} \tag{57}$$

This equation gives a basis for designing actual diffusers from the ideal area ratio and pressure ratio, once the pressure rise efficiency e is known or estimated.

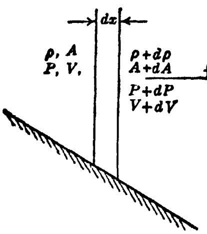


FIG. 1. Diagram of steady gas flow through a channel.

Diffuser pressure rise efficiencies range from very poor ones at $e = 0.60$ to quite good ones at $e = 0.85$.

Wall angles up to 4 or 5 degrees divergence per wall usually give good results, and the efficiency of a given diffuser is ordinarily best at low Mach numbers and in large sizes.

MOMENTUM. Many problems are solved best by force summations; others yield best to an energy analysis.

A third method, using momentum concepts, has quite a wide application in flow problems.

Momentum Concepts. When a gas flows steadily through a channel as shown in Fig. 1, the force of the wall in the direction of flow exerted on the element of gas in length dx is $P\,dA$. Any wall friction, obstruction, or anything opposing flow may be represented by a force dF. The gross force accelerating the flow is $-A\,dP$. This is minus because it is contrary to the force dP.

Summing up forces on an element gives

$$-A\,dP - dF - \rho A\,dx\,\frac{dV}{dt} = 0 \tag{58}$$

But

$$\frac{dx}{dt} = V \quad \text{and} \quad A\,dP = d(PA) - P\,dA \tag{59}$$

So

$$-d(PA) + P\,dA - F\,dx - \rho AV\,dV = 0 \tag{60}$$

However, for steady flow,

$$\rho AV = \text{constant} \tag{61}$$

giving

$$d[PA + (\rho AV)V] = P\,dA - dF \tag{62}$$

This can also be written

$$d(PA + \gamma PAM^2) = P\,dA - dF \tag{63}$$

The expression $(P\,dA - dF)$ represents the net wall force on the flowing gas in the direction of flow and it is equal to the change in $PA\,(1 + \gamma M^2)$, which is called the momentum per second (pounds) passing any point. Thus any change in the rate of flow of momentum represents a wall force on the moving gas.

THRUST CALCULATION. For any passage in a free stream as shown in Fig. 2, the net internal force of the stream on the object (acting to the left) is

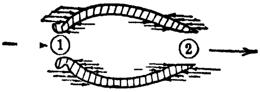


FIG. 2. Forces on a duct in a free stream.

Internal thrust

$$= P_2A_2(1+\gamma M_2^2) - P_1A_1(1 + \gamma M_1^2) \tag{64}$$

This holds regardless of what changes in pressure temperature or velocity occur between 1 and 2 and regardless of what causes the changes.

The internal force is measured in terms of absolute pressures, so to account for the fact that an external pressure P_0 acts over the area difference $A_2 - A_1$, eq. 64 must be corrected to

$$\text{Net thrust} = P_2A_2(1 + \gamma M_2^2) - P_1A_1(1 + \gamma M_1^2) - P_0(A_2 - A_1) \tag{65}$$

Equation 65, properly evaluated for entering and leaving gas conditions, gives the net thrust which needs only to be corrected for the external flow drag to give the effective thrust.

30. COMPRESSION SHOCKS

When flow is subsonic, it can adjust for whatever flow conditions are ahead, but once it is supersonic no down-stream effects can be felt up stream. The result is that flow adjustments, instead of being smooth and continuous, occur as sudden discontinuities or compression shocks.

PLANE SHOCK THEORY. When a gas flows steadily in a constant area channel, a change which takes place in a short distance may be thought of as occurring at constant total momentum (eq. 63) and at constant total energy or total temperature (eq. 37).

Combining these equations to eliminate P_1 and P_3 gives

$$\frac{(1 + \gamma M_2^2)}{M_2\sqrt{1 + \left(\frac{\gamma - 1}{2}\right) M_2^2}} = \frac{(1 + \gamma M_1^2)}{M_1\sqrt{1 + \left(\frac{\gamma - 1}{2}\right) M_1^2}} \tag{66}$$

By clearing fractions, rearranging terms, and factoring, this equation becomes

$$-\left(\frac{\gamma - 1}{2}\right)(M_2^2 - M_1^2)(M_1^2 + M_2^2) + \gamma M_1^2 M_2^2 (M_2^2 - M_1^2) = M_2^2 - M_1^2 \tag{67}$$

Since $(M_2^2 - M_1^2 = 0)$ or $(M_2^2 = M_1^2)$ is one obvious solution for constant area flow with no wall forces, it may be factored out to leave

$$-\left(\frac{\gamma - 1}{2}\right)(M_2^2 + M_1^2) + \gamma M_1^2 M_2^2 = 1 \tag{68}$$

which yields

$$M_2^2 = \frac{1 + \left(\frac{\gamma - 1}{2}\right) M_1^2}{\gamma M_1^2 - \left(\frac{\gamma - 1}{2}\right)} \tag{69}$$

This shows mathematically what is an observed physical fact, that, for constant-area supersonic flow at M_1, there is a subsonic flow at M_2 which satisfies flow, force, and energy conditions, and will occur as a sudden transition when the proper pressure conditions are imposed.

Since this change occurs at constant momentum per second, eq. 69 may be used in eq. 63 to give

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{(\gamma + 1)} \tag{70}$$

This shock is a nonreversible process which can occur in one direction only since the P_2/P_1 of eq. 70 is less than the reversible pressure ratio for the same Mach number change.

SHOCKS AND FRICTION. The distance that a superacoustic gas stream can flow at constant area with friction before reaching $M = 1$ is given by eq. 45 and Table 1, column *e*. If supersonic flow enters a duct with a greater friction length, a plane compression shock will occur. A shock can also be forced in a shorter length by impressing the correct overall pressure ratio.

The greatest length channel which a given supersonic flow can enter is one where a plane shock occurs at the entrance and the friction length given by Table 1, column *e*, is such as to bring the flow by friction from the shock M_2 back to $M = 1$, and this could occur only at one particular pressure ratio.

The most practical method of checking for the possibility of shock in a supersonic channel is trial and error.

Supersonic friction is assumed up to some arbitrarily chosen point, and the conditions found from Table 1, column *e*. Next, a plane shock is assumed, satisfying eqs. 69 and 70. Next, subsonic friction for the remaining length is solved from Table 1, column *e*. In this way, a final leaving Mach number and overall pressure ratio may be found. If this is done for all points from the entrance to the discharge of the duct, the range of operating conditions that can produce a shock may be found. In all cases, it must be remembered that flow with friction cannot continue once $M = 1$ is reached.

SUPERSONIC IMPACT PRESSURE. When an impact or total pressure tube is placed in a subsonic stream, a reversible compression occurs, and the true total pressure is recorded. However, when an impact tube is placed in a supersonic stream, it is preceded by a bow shock which approximates a plane shock in intensity. Back of this shock a reversible compression occurs from the shock to rest. The overall pressure ratio recorded by the tube is the product of the shock pressure ratio and this subsonic diffusion pressure ratio and is less than the reversible pressure ratio from the original supersonic Mach number.

EVAPORATORS AND EVAPORATION

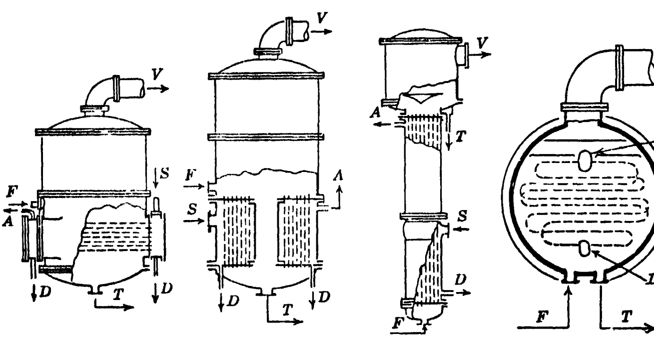
By W. L. Badger

This section is written for: (1) the engineer who must operate or maintain process evaporators, (2) the engineer who plans the energy balance of plants in which both power and process steam must be generated, and (3) the engineer who is concerned with an evaporator only as a means of producing boiler-feed make-up.

Evaporation may be carried out by the use of any suitable source of heat, but certain methods, because of their convenience and economy, are most practical. Evaporation by solar heat is practical in very few locations in the United States, and is confined entirely to the manufacture of common salt and similar compounds. For evaporation in sprays and cooling towers, see Section 9. Evaporation by direct fire is a province of the designer of steam boilers. (See Section 7.) Where liquids are evaporated by direct fire or by waste heat, the apparatus never has been standardized. In most operations in practice where a solution is to be concentrated or water is to be distilled, some type of steam-heated apparatus almost invariably is used. Many types of construction are found, but certain constructions are so common as to be almost standard.

31. EVAPORATOR CONSTRUCTION

Steam-heated evaporators may be classified in four general types: those with horizontal tubes, with inclined tubes, with vertical tubes, and with coils. Vertical tube evaporators may be subdivided into evaporators with natural circulation and evaporators with forced circulation. Figure 1 gives conventionalized illustrations of these types.



Round body horizontal A — Standard vertical B — Long tube vertical C — Horizontal D

FIG. 1. Types of evaporators. (*A*, noncondensed gas vent; *D*, condensate removal; *F*, feed connection; *S*, steam connection; *T*, concentrated solution discharge; *V*, vapor discharge.)

THE HORIZONTAL-TUBE EVAPORATOR (Fig. 1*A*) consists of a vertical cylindrical body that may be 3 to 15 ft in diameter and 4 to 15 ft high. Two rectangular steam chests in the lower section contain tube sheets, between which the tubes are fastened. The tubes, usually 7/8 in. in diameter in smaller evaporators, and 1 1/4 in. in larger ones, generally are fastened to the tube sheets by rubber packing rings and packing plates held down by studs. Connection for inlet and outlet of liquor may be at any convenient point. The vapor offtake always is in the center of the top. It may or may not be provided with internal or external foam catchers or entrainment separators. Steam for heating enters one steam chest, and condensate and noncondensible gases are removed from the other. This evaporator is used widely, although it is not the commonest type. It is primarily suitable for nonviscous liquids that do not deposit salt or scale during evaporation. It is being replaced by other types.

THE STANDARD VERTICAL-TUBE EVAPORATOR (Fig. 1*B*) consists of a vertical cylindrical shell which may be closed with a flat bottom, a deep dish, or a conical bottom. Vertical tubes are fastened between two horizontal tube sheets extending across the entire body near the bottom. The tubes, which always are held in place by rolling, are 1 1/2 to 4 in. in diameter (2 and 2 1/2 in. are most common) and 30 in. to 6 ft long (5 ft most common). A central downtake well generally is provided, of cross-sectional area of about 75% of the combined cross-sectional area of the tubes. The liquid is inside the tubes. Steam enters the space outside the tubes and between the tube sheets through suitable connections. Condensate is taken off the bottom tube sheet, and noncondensed gases usually are removed from the top tube sheet at a point opposite the steam inlet. During boiling, normal circulation is up through the tubes and back through the downtake. Connections for admitting feed and discharging thick liquor may be made where desired.

This evaporator is probably the most widely used of all types. It can be adapted to more different purposes and can be built in larger units than the horizontal tube evaporator. It can be used for liquids that deposit salt or scale during boiling. It can carry liquids to relatively high viscosities. The construction is so common that it often is referred to as the standard evaporator.

There may be many minor variations of the standard vertical tube evaporator. The downtake may be at one side. It may consist of a number of small downtake tubes, it may consist of an annular ring, or it may consist of downtake passages entirely external to the main body. The construction shown in Fig. 1*B* is the only important one.

LONG-TUBE NATURAL-CIRCULATION VERTICAL EVAPORATORS (Fig. 1*C*), have tubes 15 to 20 ft long. The liquor space is reduced to a very small chamber below the bottom tube sheet. There is a relatively small vapor head above the top tube sheet. In operation, the normal liquor level is relatively low, and the liquor being evaporated is carried to the top of the evaporator as a film along the tube wall. Immediately above the tubes, some type of entrainment separator deflects the liquid into a channel, whence it is drawn off. In this type the liquor passes through the evaporator but once. A return connection for recirculation from the vapor head back to the bottom of the tubes may be provided, but is less common.

This type has found wide acceptance and promises to be the most important type for nonsalting or nonscaling liquids. It can be used up to quite high viscosities, and it is one of the cheapest types (per unit of capacity).

FORCED-CIRCULATION EVAPORATORS (Fig. 1*D*) consist of a bundle of tubes, usually $^7/_8$ in. in diameter and 8 ft long. Steam is introduced around these tubes near the bottom of the heating element, passes up behind an inner cylindrical baffle, and flows downward along the tubes. Noncondensed gases and condensate are taken off near the bottom tube sheet. A portion of the heating element projects into the main body of the evaporator. Liquid is pumped through the tubes at velocities of 6 to 12 ft per sec, usually by a low-head centrifugal pump. The liquid issues from the tubes as a mixture of vapor and spray, striking a parabolic deflector that throws the liquid down into the lower part of the evaporator body, whence it returns to the circulating pump. Concentrated liquor usually is taken off from the body, and feed is introduced into the pump suction.

This evaporator is suitable for the concentration of liquids to extremely high viscosities, for liquids that tend to deposit salt or scale, for foamy liquids or cases where entrainment must be reduced to a minimum, or for cases where high-priced metals must be used for the heating surface. This evaporator, with nickel tubes, has become the standard in the United States for the evaporation of caustic soda. A modification using horizontal or vertical external heaters is also used.

Inclined tube evaporators have been popular at one time in both Europe and the United States, but they are not now offered by the well-known makers of evaporators. The long-tube natural-circulation type has all the advantages claimed for the inclined tubes, and it requires less floor space.

COIL EVAPORATORS. Many types of evaporators have been built with coils as the heating surface. These have been U-bends, flat pancake coils, vertical helical coils, and many others. One type, not necessarily the commonest, but widely used, is shown in Fig. 1*D*. Steam is introduced into a number of parallel coils through a header, and condensate is collected in another header. The body is usually a horizontal cylinder, nearly filled with coils. This leaves a small vapor space, and hence elaborate entrainment separators must be introduced. In some types of such evaporators, scale can be removed by heating the coils to steam temperature and then introducing cold water into the evaporator. This method of scale removal can be used only on coil evaporators.

Coil evaporators are used only for making distilled water for boiler-feed make-up.

The purpose for which evaporators may be used and the types of liquids to be evaporated are so varied that it is impossible to make definite statements as to the principal field of usefulness of each type. In many industries the type of evaporator used is dictated by custom rather than by sound engineering principles.

32. HEAT TRANSFER IN EVAPORATORS

The chapter, Heat Transmission (see p. 3-12), indicates that the only logical method of studying heat-transfer coefficients is to separate them into their separate film coefficients. Very little work of this type has yet been done on boiling liquids.

TEMPERATURE DIFFERENCES. Figure 2 shows diagrammatically the thermal conditions in the evaporator in the most general cases. Temperatures are plotted on the *Y* axis, and distance along the heating surface is plotted along the *X* axis. Steam enters somewhat superheated at temperature T_1 and is first cooled to saturation temperature T_2.

The heating surfaces should be so proportioned that there is no appreciable pressure drop between steam inlet and condensate outlet and, therefore, condensation takes place throughout the heating surface at T_2. Before leaving the evaporator the condensate may be cooled to T_3. It is obvious that in practice these operations do not take place in three distinct stages, but are more or less simultaneous. For discussion, however, it is convenient to separate them as shown.

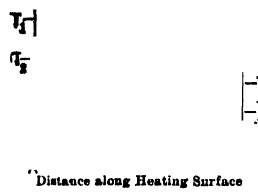


FIG. 2. Temperatures in evaporators.

The liquid enters the evaporator somewhat below its boiling point at T_4. It then is heated to boiling point, T_5, at which temperature it evaporates throughout the body. In practically every case T_5 is higher than the boiling point of pure water, T_6, under the pressure existing in the body of the evaporator.

The true temperature difference between steam and liquid would be the area $T_1T_2T_3T_5T_4T_1$ divided by distance AB. In most cases in practice, however, the superheat temperature, the condensate temperature, and the feed temperature are neglected, and the working temperature drop is considered to be the difference between the temperature of the steam and the temperature of the liquid.

It is much easier to measure the pressure in the vapor space of the evaporator than it is to measure temperature of the boiling liquid. It is customary, therefore, to calculate temperature T_6 from the pressure of the vapor space by means of steam tables, and the temperature difference $T_2 - T_6$ is termed the *apparent temperature difference.* Heat-transfer coefficients based on this apparent temperature difference are called *apparent heat-transfer coefficients.*

If elevation in boiling point of the solution in question is known, or can be determined, temperature T_5 is obtained by adding the known elevation in boiling points to the apparent boiling point T_6. When temperature T_5 thus obtained is subtracted from T_2, this gives a *temperature drop corrected for boiling-point elevation;* the heat-transfer coefficients based on this temperature drop are called *coefficients corrected for elevation in boiling points.*

In practice, evaporators installed solely for producing distilled water for boiler-feed make-up or similar purposes are usually operated on solutions so dilute that the elevation in boiling point $T_5 - T_6$ is a fraction of a degree and therefore negligible. In any case, in such evaporators the boiling-point elevation is too small compared to the apparent temperature difference to be of significance. Any corrections to the apparent temperature drop for steam superheat $T_1 - T_2$, condensate cooling $T_2 - T_3$, or feed heating $T_5 - T_4$ must be made by highly arbitrary methods and are of questionable significance.

The boiling point T_6 exists only at the surface of the liquid. The lower layers of liquid are under a pressure greater than that of the vapor space and, therefore, must have a higher boiling point than the surface layers. Consequently, the true mean working temperature drop is less than either of the temperature drops described above. A correction for this effect of hydrostat c head, although important, cannot be made with certainty (because of rate of circulation of the liquid). It, therefore, ordinarily is not included in evaporator calculations.

It should be noted that in no case can the elevation in boiling points of a solution be calculated from known rules, except for solutions so dilute that they are of no practical importance. Furthermore, the elevation in boiling point changes with changes in concentration and in pressure, and, therefore, must be determined by experiment. Many data on boiling points of pure substances are given in reference books, especially International Critical Tables, but for many commercial solutions these elevations are incorrect. Table 1 gives the boiling points of the NaCl solutions of various concentrations at various pressures.

Table 1. Boiling Points of Sodium Chloride Solutions

Pressure, mm	Grams NaCl per 100 grams of Water										
	0	4	8	12	16	20	24	28	32	36	Sat.
	Boiling Point, °F										
760	212.0	213.1	214.2	215.6	217.0	218.6	220.3	222.1	223.9	225.7	227.7
540	195.3	196.4	197.6	198.9	200.2	201.6	203.2	205.0	206.8	208.6	210.2
380	179.1	180.1	181.3	182.5	183.8	185.2	186.8	188.4	190.2	192.0	193.6
240	150.3	160.2	161.2	162.3	163.4	164.8	166.3	167.9	169.7	171.5	172.4
160	142.9	143.8	144.8	145.8	146.8	148.2	149.6	151.1	152.8	154.6	155.5
100	124.8	125.6	126.5	127.4	128.5	129.7	131.1	132.8	134.4	136.1	136.9

THE OVERALL HEAT-TRANSFER COEFFICIENT in an evaporator is obviously the resultant of the steam film coefficient, the resistance of the metal wall, together with any scale it may carry, and the resistance of the liquid film. Since in most cases the liquid film is in natural or free convection, it practically is impossible to calculate overall heat-transfer coefficients for any except forced-circulation evaporators. In these the liquids inside the tubes may be considered as nonboiling through a considerable part of their length.

While there are many data in the literature on heat-transfer coefficients in evaporators, they cover such a small portion of the entire field that the average engineer cannot predict heat-transfer coefficients. To give an idea of variations that may be expected, a set of such determinations is reproduced in Fig. 3. This represents overall apparent coefficients between steam and boiling distilled water, in a vertical-tube evaporator with tubes 2 in. in diameter and 30 in. long. The general shape of the curves is similar for other tube proportions. Note that the heat-transfer coefficient increases with increasing temperature drop, due to more vigorous boiling at the higher temperature drops, with correspondingly more rapid circulation. Also note that for a given temperature drop the heat-transfer coefficient increases as the boiling point increases. This is due largely to a decreased viscosity of water at higher temperatures, with consequent increase in rate of circulation. Changes of the same, or even greater, order of magnitude can be caused by a change in the type of liquid, depth of liquid, diameter of tubes, length of tubes, shape and size of the body, and many other factors. It is obvious that it is impossible to present in any summary a definite statement as to what heat-transfer coefficient may be expected in a given case. The author has tests of evaporators showing overall heat-transfer coefficients ranging from 4000 to 2 Btu per sq ft per hr per °F. In practice, unless data are available from plant tests on evaporators of the same type and size as the one under consideration, and operating under the same conditions on the same liquor, it is necessary to depend on the knowledge of the companies manufacturing commercial evaporators. In general, with ordinary nonviscous, nonscaling liquids, the horizontal-tube or the standard vertical-tube evaporator will have heat-transfer coefficients between 200 and 500 Btu per sq ft per hr per °F. The long-tube natural-circulation evaporator and the forced-circulation evaporators may reach 1000 to 1200 Btu in the same units.

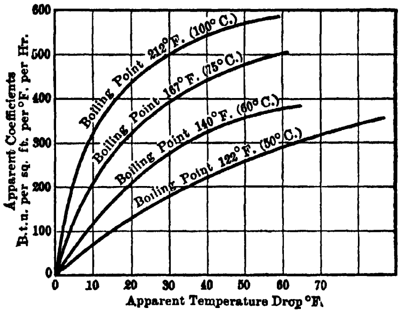


FIG. 3. Heat transfer coefficients.

33. MULTIPLE-EFFECT EVAPORATION

A multiple-effect evaporator is a series of evaporators so connected that the vapor from one body is used as the heating steam in the next. To provide a working temperature drop in each body or effect, the pressure in the vapor space of each body must be lower than the preceding one. The individual bodies of a multiple-effect evaporator are similar in all respects to the construction of single-effect evaporators.

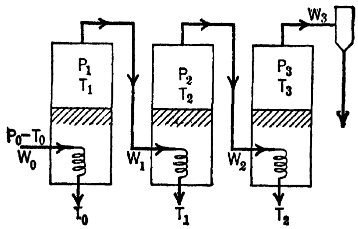


FIG. 4. Diagram of multiple effect evaporator.

HEAT RELATIONS. Imagine each body in Fig. 4 to be fed with liquid heated to the boiling point for the particular body to which it is fed. Let the liquid being evaporated be either pure water or a dilute solution whose thermal properties are not appreciably different from those of pure water. Consider radiation losses to be negligible. Then if W_0 pounds of steam at pressure P_0 are fed to the heating surface of the first effect, and if condensate is assumed to leave at T_0, the stauration temperature corresponding to the pressure P_0, then W_0L_0 Btu, are delivered to the heating surface. L_0 is latent heat of steam at pressure P_0. The liquid is at pressure P_1 and has a boiling point of T_1. T_1 will not be greatly different from T_0 and, therefore, L_1, the latent heat of the liquid in the first effect, will be nearly the same as L_0. W_0 pounds of steam entering the first effect will evaporate W_1 pounds of water in the first effect, and W_1 and W_0 will be nearly equal.

When the vapor produced by the boiling liquid in the first effect enters the second effect it will condense at the pressure P_1 and will, therefore, have latent heat L_1 very nearly the same as L_0. The liquid in the second effect will have a latent heat of L_2, not greatly different from L_1. Therefore, W_2 also will be nearly equal to W_0. The same line of reasoning may be continued, from which it may be concluded that in an N-effect evaporator, one pound of steam will evaporate N pounds of water. In practice, this statement must be modified to take into consideration such details as heat used for heating of feed, changes in latent heats when there are very large temperature drops across the evaporator, heat losses by radiation, and similar effects. The principle remains unchanged, that increasing the number of effects increases the *economy* of operation.

RELATIVE CAPACITY OF SINGLE- AND MULTIPLE-EFFECT EVAPORATORS. The pressure and temperature of steam for operating the evaporator and the pressure and temperature that may be produced in a condenser usually are fixed by conditions external to the evaporator. If T_0 is the saturation temperature of the steam available, and T_N is the saturation temperature corresponding to the pressure in the condenser, the total temperature drop available for the operation of the evaporator is $T_0 - T_V$. If a single-effect evaporator is used, of a heating surface of A square feet and an overall coefficient U, the heat transmitted by this evaporator will be $UA(T_0 - T_N)$.

Suppose that a double-effect evaporator is operated with steam to the first effect at T_0 and that the pressure in the vapor space of the second effect corresponds to temperature T_N. The boiling point in the first effect (and consequently the temperature of the heating steam in the second effect) is represented by T_1. If both evaporators have the same surface A per effect, and if their heat transfer coefficients are U_2 and U_1, respectively, the heat transferred through the first effect will be $U_1A(T_0 - T_1)$; the heat transferred through the second effect will be $U_2A(T_1 - T_N)$. If heating of feed and losses by radiation, etc., are neglected, it follows that $U_1A(T_0 - T_1)$ must be approximately equal to $U_2A(T_0 - T_1)$. That is, the evaporator will come to equilibrium with T_1 at such a value that the temperature drops in the two effects will be approximately inversely proportional to the heat transfer coefficients. Temperature T_1 cannot be set arbitrarily or controlled mechanically, as it is solely the result of thermal equilibrium between the effects. If any operating condition changes so that U_2 decreases, steam will not be condensed in the heating surface of the second effect as fast as it is generated in the first effect. As a result, pressure will build up in the first effect with consequent rise in temperature T_1. This will decrease the temperature drop across the first effect and increase it across the second effect until the evaporator has attained an equilibrium corresponding to the new conditions.

The above reasoning may be extended to any number of effects and the conclusion reached that, in a multiple-effect evaporator, the temperature distribution between effects represents a normal and automatically attained thermal equilibrium. This cannot be altered mechanically, and will be such that the temperature drops across the various effects will be approximately inversely proportional to the heat-transfer coefficients in those effects.

It will also appear from the above reasoning that, since $(T_0 - T_1)$ is only a part of the single-effect temperature drop, $(T_0 - T_N)$, A square feet of heating surface in the first effect of a double-effect evaporator will transmit much less heat than the same number of square feet in a single-effect evaporator working between the same terminal temperatures T_0 and T_N. Furthermore, since the heat transmitted in the first effect is approximately equal to the heat transmitted in the second effect, it follows that even if the heat-transfer coefficients of a double-effect evaporator were equal to the heat-transfer coefficient in a single-effect evaporator, there would be needed A square feet in *each* effect of the double-effect evaporator to transmit the same total quantity of heat with the same terminal temperature drop as would be transferred by A square feet in a single-effect evaporator. This same reasoning can be continued to any number of effects, and it leads to the conclusion that a multiple-effect evaporator, to have a given total evaporation, must have as much heating surface in *each* effect as the heating surface of a single-effect evaporator operating under the same terminal temperatures. Furthermore, Fig. 3 shows that as temperature drops decrease, heat-transfer coefficients decrease. Consequently, in practice it usually is necessary to have more surface per effect in a multiple-effect evaporator than would be required in a single-effect evaporator to do the same work.

The first cost of an evaporator, and consequently fixed charges and maintenance, thus will increase in proportion to the number of effects. Another statement of the same conclusion is that the capacity of the evaporator per square foot decreases in proportion to the number of effects. Consequently, although passing from single effect to multiple effect improves steam *economy*, it decreases *capacity*, and increases fixed charges. The economic number of effects for any particular case is obviously that number which shows the minimum total cost. The total cost is the result of adding steam costs and condenser water costs (which decrease with number of effects) to fixed charges and maintenance (which increase with number of effects). Such total cost curves usually show a marked minimum.

In many cases in present practice single- and double-effect evaporators are used. Triple and quadruple effects are very common and there are a few septuple effects. Evaporators with eight effects or more are rare.

34. CALCULATIONS FOR MULTIPLE-EFFECT EVAPORATORS

The most important results to be obtained from the preliminary calculations for multiple effect evaporators are (1) quantity of steam required, (2) heating surface required, (3) temperature in the various effects, (4) water consumption of the condenser. As heat-transfer coefficients must be determined by experience, and as they are not available to the average engineer, result 2 will be only approximate. The other results, however, will be quite accurate. These figures, especially item 1, are of the greatest importance in making preliminary decisions as to the number of effects, the arrangement of the evaporator, and its influence on the heat balance of the plant.

In Fig. 5, thin liquor is fed to the first effect, the liquor is passed from effect to effect in the direction of decreasing pressures, and thick liquor is removed from the last effect. This is the commonest method of feeding evaporators.

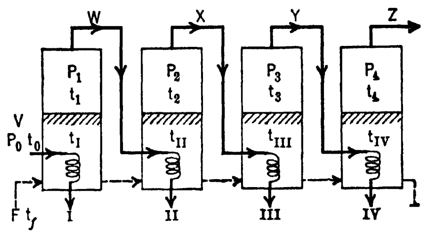


FIG. 5 Multiple-effect evaporator symbols.

In most evaporator installations, all the effects are of the same size and construction. This usually is a requisite condition in all such calculations. In order to make the calculations, approximate values of the heat-transfer coefficients in different effects must be available. The most obvious method of attack would be to write heat-balance equations across each effect. This method, however, would involve the temperature in each effect, but, as heretofore noted, temperatures in the various effects are the results of the evaporator coming to an equilibrium determined by the relation between the heat-transfer coefficients. This prevents writing a set of equations that can be solved directly, so that trial and error must be used.

EVAPORATOR CALCULATIONS. So many evaporator arrangements and so many complicating conditions are possible that no general formulation can be made. One case will be presented with certain simplifying assumptions, and the method followed through for this case. It will be necessary to develop similar equations by methods that should be obvious for whatever arrangement may occur in practice.

In practically all cases in practice the liquid being evaporated has a higher boiling point than pure water under the same conditions. Hence the vapor leaving the liquid will be superheated. The amount of superheat contained in the vapor under ordinary circumstances will be so small a part of the total heat available from the steam that its transfer is accomplished in a very small fraction of the apparatus, and most of the heating surface will be transmitting heat from saturated steam. Therefore, it is usual to disregard the effect of superheat on the temperature drop in evaporators, but not necessarily to disregard it as it may effect heat balances.

Assumptions. (1) Condensate leaves the heating surface at the saturation temperature of the steam. (2) Radiation is negligible. (3) Superheat in the vapor does not affect the temperature difference in the next effect. (4) Secondary thermal effects such as heat of concentration are negligible. (5) No appreciable amount of solids separate from the liquid during evaporation. (6) Coefficients of heat transfer have been corrected for elevation in boiling point. (7) Boiling-point elevations are known for the liquor in question at all concentrations and pressures. (8) Specific heat of the solution to be evaporated is 1.00 at all concentrations.

Notation. Let I, II, III, IV = first, second, third, and fourth effects respectively; F = pounds of thin liquor fed per hour; E = total evaporation, pounds per hour; V = pounds of steam used per hour; W, X, Y, Z = evaporation in I, II, III, and IV, respectively, pounds; A_1, A_2, A_3, A_4 = heating surface in I, II, III, and IV, respectively, sq ft;

t_0 = saturation temperature of steam to I, °F; t_{I}, t_{II}, t_{III}, t_{IV} = boiling point of the solution in the several effects, °F; t_1, t_2, t_3, t_4 = saturation temperatures of vapor from the several effects, °F; L_0, L_1, L_2, L_3, L_4 = latent heat of vaporization of steam from the solution at t_0, t_1, t_2, t_3, t_4, Btu; h_0, h_1, h_2, h_3, h_4 = heat present as superheat in vapor from the various effects, Btu; U_1, U_2, U_3, U_4 = overall heat transfer coefficients in the various effects, corrected for elevation of boiling point. t_f = temperature of feed, °F; Δt_1, Δt_2, Δt_3, Δt_4 = net working temperature drops in the various effects, °F.

Heat balance equations are

Across I $\quad V(L_0 + h_0) = F(t_{I} - t_f) + W(L_1 + h_1)$ (1)

Across II $\quad W(L_1 + h_1) + (F - W)(t_{I} - t_{II}) = X(L_2 + h_2)$ (2)

Across III $\quad X(L_2 + h_2) + (F - W - X)(t_{II} - t_{III}) = Y(L_3 + h_3)$ (3)

Across IV $\quad Y(L_3 + h_3) + (F - W - X - Y)(t_{III} - t_{IV}) = Z(L_4 + h_4)$ (4)

Material Balance. In addition to the foregoing, a material balance equation can be written:

$$E = W + X + Y + Z \tag{5}$$

Heating Surface and Heat Transfer. Next a set of equations connecting the heating surface and the heat transfer in each effect may be written:

$$A_1 = \frac{V(L_0 + h_0)}{U_1(t_0 - t_{I})} = \frac{W(L_1 + h_1) + F(t_{I} - t_f)}{U_1(t_0 - t_1)} \tag{6}$$

$$A_2 = \frac{W(L_1 + h_1)}{U_2(t_1 - t_{II})} \tag{7}$$

$$A_3 = \frac{X(L_2 + h_2)}{U_3(t_2 - t_{III})} \tag{8}$$

$$A_4 = \frac{Y(L_3 + h_3)}{U_4(t_3 - t_{IV})} \tag{9}$$

The steps in the solution are: (1) An approximation is made of the values of W, X, Y, and Z; from these the approximate concentrations in each effect is determined. (2) From this approximate concentration the elevation in boiling point in each effect is determined. (3) The pressure and temperature of steam to the first effect and the vacuum to be maintained in the last effect usually are available as fixed conditions in the problem. If not, values for them are assumed, and, from these, total available temperature drop for the whole evaporator is determined. (4) From the total available temperature drop, the sum of the elevations in boiling point is subtracted and the remaining net or effective temperature drop is divided between the effects, approximately inversely to the heat-transfer coefficients. This, with the approximate elevation in boiling point, gives the temperature of liquid and the saturation temperature of vapor for each effect. (5) On the basis of these assumed temperatures the heat-balance equations are solved for V, W, X, Y, and Z. (6) On the basis of these values for evaporation in each effect, the heat-transfer equations are solved for the surface in each effect. (7) If these surfaces are not sensibly equal (assuming that it is a condition of the problem that all evaporator bodies must be of the same size), the temperatures in the various bodies are readjusted and steps 4, 5, and 6 are repeated. This process is continued until a set of temperatures is found that will give results satisfying the condition that all heating surfaces are equal. Values for V and Z thus obtained ordinarily are carried to within a few per cent. If final values for W, X, Y, and Z result in concentrations so different from those assumed in step 1 that the preliminary elevations in boiling point determined in step 2 are incorrect by more than one or two degrees, the calculations must be repeated on the basis of the new values for elevations of boiling point. Although the method sounds tedious, it is usually possible with a little experience to make the second or third trial yield results with an accuracy ample for any preliminary calculations. This example will illustrate the application of the method.

Example. A quadruple-effect evaporator is to be fed with 50,000 lb per hr of 5% sodium chloride solution at 150 F. Steam to the first effect will be at 35 psig. Vacuum on the last effect will be 26 in. referred to a 30-in. barometer. The solution is to be concentrated to 25% solids. Required the approximate heating surface, the steam used, and the heat above 32 F going to the condenser.

Assumptions. (1) Feed will be forward. (2) Radiation losses will be negligible. (3) All specific heats may be taken as 1.00. (4) There will be no appreciable heat of concentration. (5) All condensate will leave steam chests at saturation temperature. (6) All effects are to have the same heating surface. (7) Superheat in vapor due to elevation in boiling point will not affect temperature drop in next effect. (8) Coefficients, in Btu per hr per sq ft per °F, corrected for elevation in boiling point will be first effect, 375; second effect, 350; third effect, 300; fourth effect, 200. (9) Elevation in boiling point of salt solution may be taken from Table 1. (10) No salt or scale will separate. (11) Steam to the first effect is dry and saturated.

Solution. Step 1. The total evaporation is determined as follows:

	H_2O, lb	NaCl, lb	Total, lb
Feed	47,500	2,500	50,000
Product	7,500	2,500	10,000
Evaporation	40,000		40,000

Assume that the evaporation will be approximately equal in all effects. Then the concentrations will be:

	H_2O, lb	NaCl, lb	Total, lb	Concentration, %
Feed to I	47,500	2,500	50,000	5.00
Evaporation in I	10,000		10,000	
Feed to II	37,500	2,500	40,000	6.25
Evaporation in II	10,000		10,000	
Feed to III	27,500	2,500	30,000	8.33
Evaporation in III	10,000		10,000	
Feed to IV	17,500	2,500	20,000	12.50
Evaporation in IV	10,000		10,000	
Product	7,500	2,500	10,000	25.00

Step 2. The elevations will be approximately.

Effect	First	Second	Third	Fourth
Concentration	6.25%	8.33%	12.50%	25.00%
Elevation, °F	1	2	3	10

Step 3. Steam to first effect—35 psig = 281 F
Vacuum on last effect—26 in. = 125 F

Total temperature drop = 156 F

Step 4. Total available temperature drop = 156 F
Sum of all boiling point elevations = 16 F

Net temperature drop = 140 F

After several trials it is found that the desired temperature drops are $\Delta t_1 = 35°$; $\Delta t_2 = 25°$; $\Delta t_3 = 31°$; $\Delta t_4 = 49°$. The conditions will be:

		Latent heat, Btu	Superheat, Btu	$L + h$
Steam to I	$t_0 = 281$ F	923.5	......	923.5
	$\Delta t_1 = 35$			
Boiling point in I	= 246			
Elevation in I	= 1			
	$t_1 = 245$	948.6	0.5	949.1
	$\Delta t_2 = 25$			
Boiling point in II	= 220			
Elevation in II	= 2			
	$t_2 = 218$	966.4	1.0	967.4
	$\Delta t_3 = 31$			
Boiling point in III	= 187			
Elevation in III	= 3			
	$t_3 = 184$	987.4	1.5	988.9
	$\Delta t_4 = 49$			
Boiling point in IV	= 135			
Elevation in IV	= 10			
	$t_4 = 125$	1022.2	5.0	1027.2

Step 5. Substituting in eqs. 2, 3, 4, and 5 gives

$$949.1W + (50{,}000 - W)(246 - 220) = 967.4$$
$$967.4 + (50{,}000 - W - X)(220 - 187) = 988.9Y$$
$$988.9Y + (50{,}000 - W - X - Y)(187 - 135) = 1027.2Z$$
$$W + X + Y + Z = 40{,}000$$

A simultaneous solution of these equations gives $W = 8690$ lb; $X = 9650$ lb; $Y = 10{,}500$ lb; $Z = 11{,}160$ lb.

Step 6. Substituting in eqs. 6, 7, 8, and 9, gives $A_1 = 1014$ sq ft; $A_2 = 972$ sq ft; $A_3 = 1007$ sq ft; $A_4 = 998$ sq ft; or an average of 1000 sq ft per effect.

S [illegible]

bet [illegible]

of a degree, which is not justified, this solution may be considered [illegible] not be repeated. A repetition of steps 1 and 2, with these values of evaporation in each effect, shows that the real concentration does not differ enough from that obtained by the first approximation to change the elevations in boiling point by more than a fraction of a degree. Therefore, this solution may be considered final. Substituting in eq. 1 gives $V = 14{,}410$ lb of steam. The heat above 32 F in the vapor going to the condenser is $11{,}160 \times (1027.2 + 125 - 32) = 12{,}389{,}800$ Btu.

EXTRA STEAM. The phrase "extra steam" has a special meaning in connection with evaporator flow sheets. It means vapor withdrawn from any body of an evaporator for use elsewhere in the plant.

In a plant having a complicated steam flow sheet, especially as regards the use of process steam, the possibilities of a multiple-effect evaporator as a producer of low-pressure steam rarely is given sufficient consideration. For instance, if vapor is taken from the second effect of a multiple-effect evaporator for use as process steam, provided that the temperature of the second effect is sufficiently high for the purpose desired, this vapor may be considered as steam that has already evaporated twice its weight of water and is therefore that much more economical than steam from the mains. The removal of such quantities of vapor from a multiple-effect evaporator alters the evaporator balance somewhat, and makes the evaporator, as an evaporator, somewhat less economical. However, the effect on the heat balance of the plant as a whole is always highly favorable. If large quantities are withdrawn from any one body in comparison to the amount generated in that body, the temperature distribution over the evaporator may be too much disturbed. It may then be more practical to put additional heating surface in the body from which such vapor is withdrawn and all bodies ahead of it. If the amount withdrawn is 25% or less of the total amount generated in any given body, it usually still will be possible to make all bodies alike without having abnormal operating conditions. The advantages of this method of operation are fully understood only in the beet sugar industry. They deserve much more consideration in other industries than they have had heretofore.

35. FITTING EVAPORATORS TO THE STEAM-FLOW DIAGRAM OF A PLANT

Where a given plant must generate both power and process steam, the balancing of conditions to secure maximum overall economy calls for the consideration of a wide range of operating conditions. Where the process steam is largely used in evaporators, a proper choice of conditions for evaporator operation is of great importance. Too often the conditions of operation of the power-generating units are arbitrarily chosen, without due consideration of the process equipment.

In the ideal scheme the terminal pressures on the power-generating equipment and the terminal temperatures on the evaporators are chosen so that the exhaust from the power units and the steam demands for the evaporators are just balanced. This scheme introduces also the consideration of how many effects are to be used, because there are usually a number of arrangements that will satisfy the conditions postulated, but differing widely in equipment costs and operating costs.

Evaporators are usually (but not necessarily) operated at the lower end of the temperature range available. Vacuum on the last effect is usually 26 to 28 in. Higher vacuums would seem to give a greater available temperature range, but usually a vacuum much in excess if 28 in. decreases the coefficient more than it increases the temperature drop and hence decreases capacity.

It is not necessary for the evaporators to be operated under vacuum. In the beet sugar industry, triple-effect evaporators are operated with the third effect under a pressure of 1 atm or more, and all the evaporator vapor is used as process steam. In one case, in another industry, a double-effect evaporator is operated with the last effect at 80 psig, to furnish process steam at that pressure. The evaporators may be operated in any part of the available temperature range that may be desired, unless there is some special consideration, such as sensitivity to high temperature of the solution being concentrated.

In general, the method of establishing a steam flow sheet for a plant requires (1) determining initial steam temperature and exhaust pressure for the power units so that the requisite amount of power is generated and (2) determining the final temperature and number of effects for the evaporators in order to use the steam available from (1). There is no mathematical method for solving such problems. They are attacked by setting up as many flow sheets as the ingenuity of the designer can provide, calculating each through, and then selecting the one where the sum of operating costs (mainly fuel and water) and fixed charges on the equipment is at a minimum.

No attempt should be made to establish a value of exhaust steam and to calculate the cost of operation of power generation and process operation separately. Such methods usually give false results. The only correct method is to calculate the entire flow sheet for each case.

36. POWER-PLANT MAKE-UP EVAPORATORS

If the plant contains a multiple-effect evaporator as the principal user of process steam, condensate from the first effect is returned as boiler feed. To supply additional feed to make up for blow-downs, losses, and condensate not returned from other process steam users, condensate from later effects may be used if free from entrained material that would be undesirable in the boiler.

Where large proportions of the boiler feed must be supplied, and the source of make-up is such that distillation (rather than softening) is the preferred method, an ordinary multiple-effect evaporator can be installed, whose operating conditions are determined by the methods that would be applied to any process evaporator. The type of evaporator to be chosen depends on the type of impurity separated by distillation.

In the case of power plants not complicated by large amounts of process steam, practice has become somewhat standardized. Two general systems are in use, the high-heat-level and the low-heat-level systems.

In low-heat-level systems, steam at some low pressure is diverted to an evaporator, of one or more effects, whose final vapor is sent to the main power-plant condensers or a condenser operating at similar pressures.

In high-heat-level systems, steam is withdrawn from the power-generating system at such a pressure that it can be used in the evaporator and evaporator vapor absorbed in feed heaters or other uses. Very complicated flow sheets have been employed in such systems. A common one is to extract steam from a bleeder turbine at any suitable pressure, operate a single-effect evaporator, preheat the evaporator feed with hot evaporator condensate, which is then added to the main stream of boiler feed. Evaporator vapors are then used in a feed heater, whose position in the feed heating system with respect to open heaters or closed high-pressure heaters depends on the temperature at which the evaporator operates.

37. EVAPORATION TO THE ATMOSPHERE

EVAPORATION OF WATER FROM TANKS AND RESERVOIRS (*Tech. Bull.* 271, U. S. Dept. of Agriculture, Dec. 1931). A series of experiments extending from 1923 to 1930 to determine the evaporation from tanks, reservoirs, open channels, etc., developed these formulas:

For Tanks, $E = (1.465 - 0.0186B)(0.44 + 0.118W)(e_s - e_d)$ (10)

For Reservoirs, $E = 0.771(1.465 - 0.0186B)(0.44 + 0.118W)(e_s - e_d)$ (11)

where E = evaporation, inches per 24 hr; B = mean barometer reading, inches of mercury at 30 F; W = mean velocity of ground wind or water surface wind, miles per hour; e_s = mean vapor pressure of saturated vapor at temperature of water surface, inches of mercury; e_d = mean vapor pressure of saturated air at temperature of dew point, inches of mercury. Values of the factor $(1.465 - 0.0186B)$ for various altitudes are given in Table 2.

Table 2. Value of Factor (1.465–0.0186B) for Various Altitudes

Altitude above Sea Level, ft	Barometer, in. of Hg	Factor	Altitude above Sea Level, ft	Barometer, in. of Hg	Factor	Altitude above Sea Level, ft	Barometer, in. of Hg	Factor	Altitude above Sea Level, ft	Barometer, in. of Hg	Factor
0	29.90	.91	4000	25.81	0.98	8,000	22.28	1.05	12,000	19.23	1.11
1000	28.82	.93	5000	24.88	1.00	9,000	21.47	1.07	13,000	18.53	1.12
2000	27.78	.95	6000	23.98	1.02	10,000	20.70	1.08	14,000	17.86	1.13
3000	26.78	.97	7000	23.11	1.04	11,000	19.95	1.09	15,000	17.22	1.14

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DRYING AND DRYING MACHINES

By F. E. Finch

38. CHARACTERISTICS OF DRYERS AND MATERIALS

THE DRYING PROCESS. Drying is the process of removing moisture in varying amounts from solid or semi-fluid materials. The drying process may be accomplished by pressure, suction, decantation, or evaporation. The common term for processes using only pressure, suction, or decantation is dewatering, whereas the process of drying has evaporation as its main principle. The difference between a dewaterer and a dryer is often very slight because of the various uses to which both are put. Primarily a dewaterer will remove or reduce only a portion of the surface moisture, but a dryer will not only reduce or remove the surface liquid but will also remove internal moisture and in many cases water in chemical combination.

Moisture Occurrence. Surface moisture clings to the outer surfaces of the material. Internal water is within the material. Inherent moisture and bed moisture are other terms used for describing the moisture in the mass that is not on the surface. Chemically combined water is that water occurring when a chemical component of the material changes its chemical composition by heat or other means.

It is necessary to divide all problems in continuous drying in three parts and to arrive at as close a balance of the three parts as practical. These parts are: (1) Application of heat to the material and/or to the air mixture which carries away the moisture. (2) Means for removing the water vapor, steam, or mixture. This includes a study of the vapor pressures involved. (3) Conveying the material in its wet, semi-dried, and dried condition into, through, and out of the apparatus, allowing the material the proper time for contact with the heating and moisture-removing elements. For batch or intermittent type dryers this third part is not as important. The main purpose of the heat is to raise the temperature of the material and the water it carries to a required level, to evaporate the liquid,

Table 1. Heat in Hundreds of Btu Required per Hundred Pounds of Dried Material, at 100% Thermal Efficiency

(Specific heat of material 0.21. Temperature of feed 60 F)

Original Moisture Content, % Wet Weight	Final Moisture Content, %										
	0	1	2	4	6	8	10	12	15	20	25
3	76.5	56.5	35.5								
5	90.5	80.5	69.5	48.5							
8	129	119	107	85.5	63.5						
10	156	145	134	111	88.5	66.0					
12	180	173	161	139	115	92.5	69.5				
15	229	218	206	182	158	134	110	85.5			
20	311	299	284	261	235	210	185	158	120		
25	405	388	379	351	329	296	269	241	199	130	
30	511	498	483	453	423	395	364	335	285	216	142
35	634	619	604	572	539	507	476	443	396	315	234
40	774	763	745	710	675	640	605	570	512	430	343
45	946	931	912	873	835	796	758	720	662	566	470
50	1150	1132	1110	1068	1026	983	941	898	835	729	623
55	1398	1379	1356	1308	1261	1214	1166	1119	1048	929	807
60	1709	1689	1662	1606	1554	1501	1447	1393	1312	1178	1044

to heat the drying equipment to its proper point, and to replace heat constantly being lost by radiation.

Types of Dryer. Dryers are made in three general types, the type determined by the method used in transferring heat to the material being dried. These types are direct, indirect, and steam-heated. Each type is subdivided into many more detailed classifications. The direct-heat type may have the flame from combustion impinging on the material being dried, or the gases of combustion may be mixed with additional air so that the mixture in contact with the material is reduced in temperature. In another division of the direct-heat type the hot combustion gases pass through spaces to heat the drying compartment by indirect means before the gases are taken into direct contact with the material being treated. In the indirect-heat types the gases of combustion pass through spaces surrounding, or in other ways heating, the drying chamber, but the gases are not allowed in contact with the material being dried at any time. In steam-heated types the material is in contact with steam pipes or the air is passed over steam heaters and then over or through the material being dried.

Table 2. Humidity Tables for Drying Calculations

Temp., °F	Vapor Tension, mm of Mercury	Water Vapor per lb of Air, lb	Humid Heat, Btu	Humid Volume, cu ft	Density, lb per cu ft at 760 mm		Volume, cu ft per lb of	
					Dry Air	Saturated Mixture	Dry Air	Saturated Mixture
32	4.569	0.003761	0.2391	12.462	.080726	.080556	12.388	12.414
35	5.152	0.0042435	0.2393	12.549	.080231	.080085	12.464	12.496
40	6.264	0.0050463	0.2398	12.695	.079420	.079181	12.590	12.629
45	7.582	0.0062670	0.2403	12.843	.078641	.078348	12.718	12.763
50	9.140	0.0075697	0.2409	12.999	.077867	.077511	12.842	12.901
55	10.980	0.0091163	0.2416	13.159	.077109	.076685	12.968	13.041
60	13.138	0.010939	0.2425	13.326	.076363	.075865	13.095	13.180
65	15.660	0.013081	0.2435	13.501	.075635	.075039	13.222	13.325
70	18.595	0.015597	0.2447	13.683	.074921	.074219	13.348	13.471
75	22.008	0.018545	0.2461	13.876	.074218	.073471	13.474	13.624
80	25.965	0.021998	0.2478	14.081	.073531	.072644	13.600	13.777
85	30.573	0.026026	0.2497	14.301	.072852	.071744	13.726	13.938
90	35.774	0.030718	0.2519	14.539	.072189	.070894	13.852	14.106
95	41.784	0.036174	0.2545	14.793	.071535	.070051	13.979	14.275
100	48.679	0.042116	0.2575	15.071	.070894	.069179	14.106	14.455
105	56.534	0.049973	0.2610	15.376	.070264	.068288	14.232	14.643
110	65.459	0.058613	0.2651	15.711	.069647	.067383	14.358	18.840
115	75.591	0.068662	0.2699	16.084	.069040	.066447	14.484	15.050
120	87.010	0.080402	0.2755	16.499	.068443	.065477	14.611	15.272
125	99.024	0.094147	0.2820	16.968	.067857	.064480	14.736	15.509
130	114.437	0.11022	0.2896	17.499	.067380	.063449	14.863	15.761
135	130.702	0.12927	0.2987	18.103	.066713	.062374	14.989	16.032
140	148.885	0.15150	0.3093	18.800	.066156	.061255	15.116	16.325
145	169.227	0.17816	0.3219	19.609	.065601	.060104	15.242	16.643
150	191.860	0.21005	0.3371	20.559	.065154	.058865	14.368	16.993
155	216.983	0.24534	0.3553	21.687	.064539	.057570	15.494	17.370
160	244.803	0.29553	0.3776	23.045	.064016	.056218	15.621	17.788
165	275.592	0.35286	0.4054	24.708	.063502	.054795	15.748	18.250
170	309.593	0.42756	0.4405	26.790	.062997	.053305	15.874	18.761
175	347.015	0.52285	0.4856	29.454	.062500	.051708	16.000	19.339
180	388.121	0.64942	0.5458	32.967	.062015	.050035	16.126	19.987
185	433.194	0.82430	0.6288	37.796	.061529	.048265	16.253	20.719
190	482.668	1.00805	0.7519	44.918	.061053	.046391	16.379	21.557
195	536.744	1.4994	0.9494	56.302	.060588	.044405	16.505	22.521
200	595.771	2.2680	1.3147	77.304	.060127	.042308	16.631	22.638
205	660.116	4.2272	2.1562	131.028	.059674	.040075	16.758	24.954
210	730.267	15.8174	15.9148	562.054	.059228	.037323	16.884	26.796

These three general types are made both for batch and intermittent operation and also for continuous feed and discharge.

The wetness of a material is expressed numerically as the percentage of the wet weight that is lost when a weighed sample of the wet material is heated to a specified temperature (usually a few degrees above the boiling point of water) for a specific time. Both the temperature that should be used as well as the time of heating will vary for different materials. The American Society for Testing Materials has set up standard procedures, including temperatures and time, for testing the water content of many materials.

GENERAL DESIGNS OF DRYERS. For direct- and indirect-heat dryers the hot furnace gases are generated from coal, wood, gas, oil, or other fuel. For steam-heated dryers either steam direct from a boiler or exhaust steam is used. Except for special cases superheated steam or that at high pressures is as efficient as steam at lower pressure.

Moisture is usually removed from the material and the drying chamber by using an exhaust fan although in a few cases a stack only is used. Exhaust fans should have few blades on the impeller for if dust is drawn off with the moisture it usually passes through the fan unless a dust collector is used between drying chamber and exhauster. Stacks are satisfactory only with fairly high and uniform temperatures and are difficult to regulate as their operation depends partially on atmospheric conditions. With either exhaust fan or stack the temperature of the exhaust gases should be such that the relative humidity of the gases does not approach the dew point so that condensation will not occur in the exhaust system.

Conveying the wet material into and through the dryer and the dried material out of it depends on the physical and chemical properties of the material and the product desired. Both the capacity and thermal efficiency of a machine depend greatly on selecting the proper type dryer for treating the specific material.

SELECTION OF A DRYER. As many hundred different designs of drying machines are built, the selection of a dryer for a given purpose involves the problems stated above and many other considerations. The type, detailed design, and size of a dryer selected for a specific problem must take into consideration the material to be handled, available sources of fuel or heat and of power, space occupied, operating labor required, costs of erection and maintenance, and most important, whether the type and size selected will give the desired product at the lowest cost. In considering this cost, thermal efficiency, materials used in construction, interest and depreciation on erected cost, and cost of maintenance are the most important items.

39. CLASSIFICATION OF DRYERS

MECHANICAL. These are used principally as a preliminary step in drying and are therefore sometimes called dewaterers. They are a centrifuge, either batch type or continuous feed type. They revolve at fairly high speeds and the batch type consists of a revolving basket into which the material has been placed. It is necessary to stop the machine to take out the dried product. This batch type is used in dry-cleaning plants and laundries and also for those chemicals and materials that are made in batch lots. The continuous type has the revolving basket built in the shape of a frustrum of a cone with the feed entering the smaller end, and as the material feeds through the basket towards the larger diameter it is subjected to greater and greater centrifugal force. The rotating basket is usually made of alloy steels, and to the inside of the basket screen plates of various size openings are attached. This type of dryer is suitable only for use on granular materials or in those places where the loss of the extreme fines with the effluent is of little or no account.

ROTARY DRYERS. Probably more rotary dryers are in use than any other type, and they are made in many and varied designs. The simplest and most common is the single-shell type; they may have the flow of material parallel or counter to the flow of hot gases or warm air through the shell.

Direct-heat single-shell types use heat generated in a combustion chamber with the hot gases passing directly into the shell and in contact with the material being dried. In other cases air is heated by means of steam heaters before entering the drying chamber. The cooled gases or air together with the moisture that has been given up by the material is exhausted at the end opposite the furnace or heater. Much thought should be given to the number of and design of the lifting flights or vanes inside of and attached to the revolving shell, because selection of the proper design will increase the capacity and lower the amount of fuel required. The shell of the dryer revolves at fairly slow speed, driven by a large ring gear around the shell, meshing with a pinion which is connected through speed reducers to a motor. An exhauster to induce the draft through the shell is placed at the end of shell opposite the furnace or heater. The discharge end of practically all rotary dryers is lower than the feed end. The louver-type rotary dryer has ducts and louvers attached to the inside of the revolving shell. The hot gases are taken from the furnace or heater by a fan and forced through a header into these louvers. The hot or warm air gives up some of its heat to the material while passing through the ducts before coming into direct contact with the material. See Figs. 1 and 2.

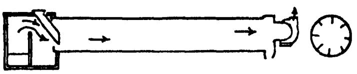

FIG. 1. Single-shell parallel-flow type dryer.

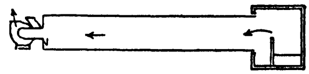

FIG. 2. Single-shell counterflow type dryer.

The double-shell semi-direct heat dryer is used for those materials that can stand a fairly high temperature but not as high as would be obtained in a single-shell direct-heat type. The gases from the combustion chamber pass down an internal tube in the shell and then back in the annular space between the two shells and over the material being dried. This type has the advantage of the highest heat at the feed end where the material carries the highest moisture and still has the effect of countercurrent flow of gases. The highest thermal efficiencies of any heat dryers are claimed for this double-shell design.

Indirect rotary dryers (see Fig. 3) are made in two general designs. In one design the gases of combustion pass down a center flue and then back in tubes or small flues attached to the inside of the outer shell which act as lifting or showering vanes for the material.

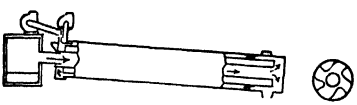

FIG. 3. Double-shell indirect type dryer.

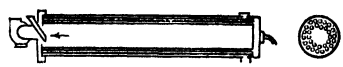

FIG. 4. Steam-tube type heater dryer.

In the other general design the outer shell is set in an insulated stationary compartment with the gases of combustion passing around the rotating shell and sometimes back through tubes attached to the inside of the shell. This type of dryer is used on china clay, kaolin, and other materials that can stand a fairly high temperature but cannot be contaminated by the products of combustion.

Steam-tube rotary dryers (see Fig. 4) have pipes attached to the rotating shell, and through a rotary valve attached to a header at one end of the shell, live or exhaust steam is introduced into these pipes, which act as the lifting vanes.

SCREEN TYPES consist of moving screens, usually operating within a compartment or chamber, over which the wet material passes while subject to drafts of heated air. In some designs one or more sections of the screen will be subject to downdraft while the adjacent section has an updraft forced through the screen and the material. Screen-type dryers are not suitable for materials having a particle size smaller than 10 mesh, and are used in practically all the reclaimed rubber plants and for other similar materials. The heat comes from banks of steam heaters or the compartments have a steam-heated wall. In a few cases, however, gases of combustion tempered with outside air have been used.

TOWER TYPES are vertical shafts connected at the bottom to some source of heat. The wet material is introduced at the top and falls downward over baffles installed in the shaft and at the same time comes into contact with the hot air or gases which rise and exhaust at the top of the stack. This type is usually made without any moving parts except any required by the source of heat. As the draft is fixed by the design of the tower or shaft and as the flow of material is by gravity, little regulation of this type can be obtained outside that in the heat-generating part.

RABBLE AND HEARTH TYPES, as made today, are improvements over one of the earliest dryers ever made. The material to be dried is supported on a floor or hearth made of brick, steel, or other materials and receives its heat through such floor or hearth. In the earliest form the material was moved over the hearth by means of hand rakes, shovels, etc., but now such dryers are designed with continuous conveyors or rabbles moving the material over the hearth. The hearth may be heated by direct contact with furnace gases, by waste heat, or by steam. In the most efficient type the hearth is enclosed by a hood, and an exhaust fan or stack connected to one or more places in the hood carries off the water vapor driven off. This exhaust decreases the time required for drying and increases the capacity of this type.

TRAY AND TUNNEL DRYERS are used when only a small quantity of material is to be handled or for those materials that require very careful handling and must not be agitated such as brick and sanitary ware, which also require slow drying. The tray type is a compartment into which trays carrying the material are placed whereas the tunnel type is a larger compartment having the material on a slowly moving conveyor or on cars

Performance Data

ROTARY DRYERS

Type of dryer	Single Shell, Counterflow	Single Shell, Parallel Flow	Single Shell, Parallel Flow	Single Shell, Counterflow	Single Shell, Parallel Flow	Single Shell, Counterflow	Double Shell, Semi-direct	Double Shell, Indirect Heat	Steam Tube, Counterflow
Material	Limestone	Phosphate Conc.	Lead Conc.	Silica Sand	Alfalfa	Starch	Bituminous Coal	Kaolin	Spent Brewer's Grain
Size, diameter and length, ft	8 × 70	7 × 70	5 × 35	4 × 40	6 × 33	3 × 30	7 1/2 × 55	6 × 45	6 × 40
Feed, tons per hr wet weight	65	44	7.25	18	2.62	0.57	25.8	4.5	0.65
Feed moisture, %	4.8	14.8	12.5	6.2	67	43.5	11.8	21.	67
Fuel, type	Coal	Oil	Oil	Oil	Hot-air oil-fired heater	Warm-air heater	Coal	Coal	Steam at 60 psig
Btu per fuel unit	13,800	145,000	141,000	140.000			13,900	12,200	
Fuel used per hr, lb or gal	1,250	180	24.7	43.9	39.5	....	715	340	1,450
Delivered weight, tons per hr	62.1	38.3	6.55	16.9	0.99	0.39	23	4.1	....
Moisture in discharge material, %	0.5	2.1	3.2	0.6	12.5	17.4	1.1	2.1	12
Thermal efficiency, %	65.3	60.4	58.7	57.2	....	...	77.7	53.1	....
Power used, hp	85	80	14	24	31	...	66	33	35

HEARTH DRYERS

Material	Copper-Iron Sulfide	Pyrite	Vanadium	Copper Concentrates
Hearth area, sq ft	180	252	424	528
Feed per hr, lb	4,750	2.920	3,570	5,676
Feed moisture, %	10.35	24	71	12
Delivered material, lb	4,470	2,413	1,679	5,342
Moisture in product, %	4.65	8	38	6.5
Fuel	Coal	Coal	Oil	Oil
Fuel per hr, lb or gal	77	79.2	21.5	4.1
Btu per fuel unit	14,000	12.000	138,000	135,000
Thermal efficiency	47.2	71.7	64.9	63.8
Hp required	3	5	9.5	....

STEAM-HEATED TRAY DRYER

Material, paint pigment; original moisture, 87%; final moisture, 0.16%; time required, 22 hr; weight of dry material, 459 lb; thickness of material bed in trays, 1.25 in.; steam pressure, 42 lb; temperature in drying chamber, 226 F; steam used per pound of water evaporated, 3.85 lb.

and allowed to remain in the compartment or tunnel a definite length of time. The choice between a tray or compartment and a tunnel depends on the size and amount of material and on the continuity of production desired. With both types various flows of heated air are employed. They may be updraft, downdraft, or a combination. In others the air is sometimes reheated after passing through one section, making the operation similar to a combination of single dryers placed in series. In some tunnel dryers provision is made to use the air over again, and occasionally the air is cooled after each passage to remove its moisture by condensation.

FLASH TYPE. This is a fairly new type that has recently been used at a large number of plants for drying of wheat and wood flours, chemical salts, spent grains, sewage sludge, and for secondary drying of small sizes of coal. It consists of an air heater, piping, cyclone collectors with air locks, exhaust fans, and in most cases a mixer to blend a certain portion of the dried product with the incoming wet feed. This blending is done so that the wet product may be more easily conveyed by air, for the main principle of this type is the conveying of the wet product by means of warm or hot air from the feed inlet to the cyclone collectors where the dried product is trapped. On very finely divided materials it has been found necessary to use a secondary dust collector or air scrubber to prevent dust nuisance in the air exhaust from the cyclones.

INFRARED RAY. This is the latest of the many types of drying. Heat comes from banks of infrared lamps with a reflector bowl directing the infrared rays towards the material to be dried. The usual practice has been to install banks of these lamps in chambers through which the material to be dried is carried by conveyors. Heating by means of these rays is almost instantaneous provided the bed of material is not too thick. When used for drying painted metal parts, the heat passes through the paint to the metal itself and starts drying the paint or material on the surface from the inside out. Although infrared-ray drying is very new, it is already used in drying glue, porcelain, enamel, grinding wheels, china ware, pigments, as well as the paint on metal parts.

SECTION 4

STEAM, WATER, AND ICE

By

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STEAM-POWER CYCLES

By J. K. Salisbury

1. CYCLE DIAGRAMS *

Power produced by steam represents the major part of the entire production of power in the United States. For this reason it is well for the engineer to have a firm concept of the thermodynamic properties of steam and the various cycles in which steam can be used in a power plant to produce power.

A cycle is defined as a series of *states* through which a working fluid repetitively passes, and as a result of which useful mechanical work is produced.

The thermodynamic properties of steam, water, and ice are covered in this section in considerable detail. Tables and charts are given from which may be determined all the important properties of steam. The following discussion, however, will help to clarify some of the definitions and characteristics of steam.

The temperature of steam in contact with water depends on the pressure under which it is generated. At atmospheric pressure (14.7 psia) its temperature is 212 F. As the pressure is increased, as when the steam is being generated in a closed vessel, its temperature, and that of the water in its presence, increases. See p. 4-36 for pressure-temperature relations.

Enthalpy, formerly called total heat, is arbitrarily taken as zero at 32 F. Enthalpy is defined as the sum of the two important kinds of energy which a gas may have. One of these is internal energy, which results from molecular motion of the fluid. As the temperature is increased the intensity of molecular motion also increases, so that the internal energy is a function only of the temperature, for a perfect gas. The second kind of energy is the pv energy of a gas, which is made up entirely of the work put into the gas to cause it to reach any given volume when opposed by the external pressure under which it is contained. For additional discussion of enthalpy, see Section 3.

ENTHALPY OF STEAM includes three elements. (1) The *total heat of saturated liquid*, h_f, which is the heat required to raise the temperature of the water from 32 F to the boiling temperature at the given constant pressure at which it boils. (2) The heat required to completely evaporate the water at that pressure and temperature. This heat is called the *internal latent heat*, h_i. (3) The *external work* done by the steam in making room for itself against the pressure under which it is generated, Apv_{fg}, where $A = 1/778.26$; p = absolute pressure, pounds per square foot; v_{fg} = change in volume of vapor, cubic feet per pound. The sum of the last two elements is the *latent heat* of steam, h_{fg}, or the *total heat of evaporation.*

EXAMPLE. The heat required to generate 1 lb of steam at 212 F, from water at 32 F and 14.696 psia, is

Total heat of saturated liquid at 212 F, h_f	=		180.07 Btu
Internal latent heat of steam, h_i	=	897.5	
External work, Apv_{fg} = [14.696 × 144(26.80 − 0.0167)]/778.26	=	72.8	
Total heat of evaporation, h_{fg}	=		970.3
Enthalpy or total heat of saturated vapor, h_g	=		1150.4 Btu

The enthalpy or total heat of 1 lb of wet steam at quality x is $h_x = h_f + xh_{fg}$.

The enthalpy or total heat of superheated steam is found by adding to the total heat of saturated vapor the energy added in the superheat. This is found by the calculus from $h_s = \int_{T_1}^{T_2} c_p\,dt = \bar{c}_p(T_2 - T_1)$, where h_s is the energy added; $\bar{c}_p$ = mean specific heat per pound of superheated steam between T_1 and T_2 at constant pressure; T_1 = absolute temperature of evaporation corresponding to the given pressure; and T_2 = absolute temperature of the superheated steam.

PROPERTIES OF STEAM. See p. 4-29.

TEMPERATURE-ENTROPY DIAGRAM OF WATER AND STEAM. Changes taking place in steam expansion or compression may conveniently be represented on a tempera-

* Revision of material originally prepared by A. G. Christie.

ture-entropy diagram. On this diagram the entropy of water at 32 F is arbitrarily taken to be zero. Thus in Fig. 1 the line OT becomes the y axis along which is laid off the temperature of the substance under consideration, in degrees Fahrenheit absolute. Entropies are laid off along the x axis beginning at zero at the point O.

The diagram represents changes in the state of 1 lb of water due to the addition or subtraction of heat or to changes in temperature. Any point on the diagram is called a *state point*. A is the state of 1 lb of water at 32 F or 491.6 F abs, B the state at 212 F, and C at 392 F, corresponding to about 226 psia pressure. K is the state point at the critical temperature 705.4 F. At 212 F the area $OABb$ is the heat added to the water, and Ob is the increase of entropy. At 392 F, $bBCc$ is the further addition of heat to the water, and the entropy at C, measured from OT, is Oc. The two quantities added are nearly the same, but the second increase of entropy is the smaller, since the mean temperature at which it is added is higher. If Q is the quantity of heat added and T_1 and T_2 are the lower and the higher temperatures, respectively, the addition of entropy from 32 F to 212 F, $s_f = \bar{c}_p \log_e (T_2/T_1) = 0.3120 = Ob$, where $\bar{c}_p$ is the mean specific heat of water over this temperature range. Accurate values of the entropy of water, taking into account the variation in specific heat, are given on p. 4-34.

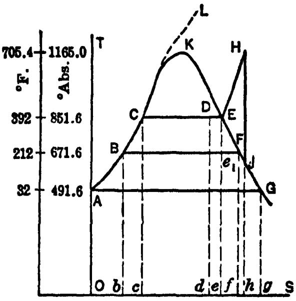


Fig. 1. Temperature-entropy diagram.

Let 1 lb of water at state B have heat added to it at the constant temperature of 212 F until it is evaporated. The quantity of heat added is the latent heat of evaporation at 212 F, or $h_{fg} = 970.3$ Btu, and it is represented on the diagram by the rectangle $bBFf$. Dividing by $T_1 = 671.6$, the absolute temperature, gives $s_{fg} = 1.4446 = BF$. Adding $s_f = 0.3120$ gives $s_g = 1.7566$, the entropy of 1 lb of steam at 212 F measured from water at 32 F $= Of$.

In like manner, if we take $h_{fg} = 833.6$ for steam at 392 F, $s_{fg} = 0.9787 = CE$, and s_f = entropy of water at 392 F $= 0.5564$, $= Oc$, the sum $s_g = 1.5351 = Oe$.

E is the state point of dry saturated steam at 392 F and F is the state point at 212 F. The line EFG is the *saturation line* and the line ABC the *liquid line*. The line CE represents the increase of entropy in the evaporation of water at 392 F. If entropy CD only is added, or $cCDd$ of heat, then part of the water will remain unevaporated, viz., the fraction DE/CE of 1 lb. The state point D thus represents wet steam having a dryness fraction of CD/CE.

K is the *critical state for water* at a temperature of 705.4 F and a pressure of 3206.2 psia. To the left of K the substance is water; above K, to the right, it becomes superheated steam. At pressures above 3206.2 psia, the water has no latent heat but passes directly to superheated steam as at K. Line AL represents (but not to scale) the states for a pressure of 6000 psia.

If steam having a state point E is expanded isentropically to 212 F, its state point is e_1, having the same entropy as at E, a lower total heat by the amount represented by the area $BCEe_1$, and a dryness fraction Be_1/BF. If it is expanded while remaining saturated, heat must be added equal to $eEFf$, and the entropy increases by ef.

If heat is added to the steam at E, both temperature and entropy increase, the line EH representing the *superheating* and the area $EHhe$ the head added. If from the state point H the steam is expanded isentropically at constant entropy, the state point follows the line HJ until it cuts the line EFG, when the steam is saturated; if it crosses this line the steam becomes wet. If the state point follows a horizontal line to the left of line EFG and to the water line ABC, it represents *condensation at constant temperature*, the amount of heat rejected being shown by the area under the horizontal line down to line Og.

In practical calculations with the temperature-entropy diagram it is necessary to have at hand tables or charts of entropy, enthalpy, and specific volume, such as are given in Keenan and Keyes' *Thermodynamic Properties of Steam*, extracts of which are on pp. 4-30 to 4-37.

An isentropic expansion on the temperature-entropy diagram is a vertical line at constant entropy. One difficulty with the temperature-entropy diagram is that enthalpies cannot be represented except by areas. This is highly desirable in discussing qualitatively power-plant cycles but is of no quantitative value. It is of value, however, to be able to picture the relationship of the various thermodynamic quantities, and entropy is useful when the diagram is used in conjunction with the steam tables, as shown by the example.

Example. Let state 1 be the initial condition and state 2 the final condition after isentropic expansion. With wet steam $s_{f1} + x_1 s_{fg1} = s_{f2} + x_2 s_{fg2}$ from which x_2 can be found and the enthalpy or total heat at state 2 calculated from $h_2 = h_{f2} + x_2 h_{fg2}$. The heat drop or *available energy* $(h_1 - h_2)$ can then be found. With superheated steam expanding into the wet region, $s_1 = s_{f2} + x_2 s_{fg2}$. Then x_2 can be found and h_2 calculated as above.

AVAILABLE ENERGY. If a hypothetical turbine (or other steam-driven prime mover) operates on the cycle in Fig. 2, designated as $ABCDEA$ and universally known as the *Rankine cycle*, the heat added is equal to the area under the broken line $ABCD$. The area of the heat supplied must always be taken down to the entropy axis, that is, it is the area $aABCDda$. The heat rejected is the area under the line EA (given in this order because this is the direction of the path of the state point) and is equal to the area $dEAad$. The difference between the heat supplied and the heat rejected in the Rankine cycle is the *available energy* of the cycle, which is the maximum possible quantity of heat that can be converted into useful work. This is true because the expansion path DE is an isentropic line, representing a completely reversible process; by the second law of thermodynamics such a process is the most efficient process possible. Thus the available energy of the cycle is represented by the area $ABCDEA$.

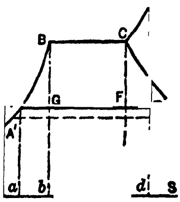


Fig. 2. Rankine cycle.

This cycle is known either as the *Rankine* or *Clausius* cycle. It is the yardstick by which all steam power cycles are measured. The available energy may be found from *theoretical steam rate tables* (see p. 4-42) by dividing the theoretical steam rate into 3412.75, the result being given in Btu per pound. The efficiency of a turbine (or engine) is defined as the ratio of the theoretical steam rate to the actual steam rate (see Section 8). It is readily seen that the efficiency is also given by the ratio of the actual heat converted into useful work by the turbine to the available energy of the Rankine cycle. The efficiency of the *Rankine cycle* is the ratio of the heat available for work to the heat supplied, that is, the area $ABCDEA/aABCDEda$. Frequently the efficiency of a turbine is described as a *Rankine efficiency*, whereas actually this is improper nomenclature. The intent is to state that the efficiency of the turbine is, for example, 82% "referred to the Rankine cycle." That is, the actual turbine converts into useful work only 82% of the energy that would be available if it were to operate on the Rankine cycle.

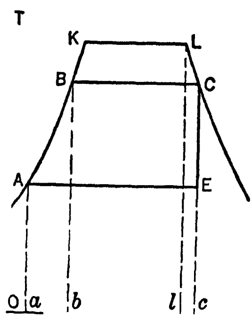


Fig. 3. Effect of increased pressure.

The efficiency of the Rankine cycle can be increased by increasing the heat available for work in a greater ratio than the increase of heat supplied. This may be done by lowering the exhaust temperature. Obviously from Fig. 2, when the exhaust temperature is lowered to $A'E'$, less heat is rejected to the condenser and more heat is available for work. Hence when low absolute pressure or vacuum can be utilized it is thermodynamically desirable to lower the vacuum temperature as close as possible to that of the coldest available cooling water. If the pressure is increased without superheat, the efficiency increases as the pressure rises from BC to KL (Fig. 3). The enthalpy or total heat of *saturated* steam increases up to 440 psia, as shown in the steam tables, and then decreases to a minimum at the critical pressure of 3206.2 psia. The thermal efficiency of the Rankine cycle with saturated steam increases to a maximum at 2000 psia, and then decreases slightly. Increased pressure leads to wetter steam at the lower stages of expansion, and in turbines this would cause internal moisture losses to offset the gain. In all modern plants the moisture is limited to 10 to 15%, regardless of initial pressure, by use of appropriate initial superheats.

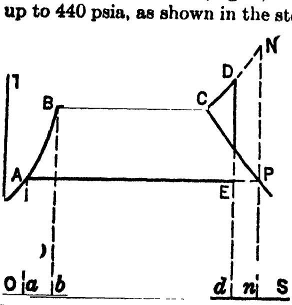


Fig. 4. Effect of increased steam temperature.

Increased steam temperature at moderate pressure, as shown in Fig. 4, by the Rankine cycle $ABCNPA$, gives only a small increase in cycle efficiency. In turbines, such an increase in temperature leads to a substantial increase in efficiency, as less of the expansion takes place in the saturated region and this causes a decrease in moisture losses in the turbine. An increase in both pressure and superheat improves cycle efficiency without detrimental moisture loss.

CARNOT CYCLE. Sadi Carnot proposed over a century ago that the most efficient cycle operation for a prime mover of any type is one in which all the processes are reversible. Such a cycle would be represented on the temperature-entropy diagram of Fig. 2 by *GBCFG*. If a prime mover were to operate on this cycle, since both *CF* and *GB* are isentropics (reversible) and both *BC* and *FG* are isothermals (also reversible), all the components of the cycle would be reversible. According to Carnot, a prime mover operating on this cycle attains the highest possible efficiency for the temperature limits imposed. This cycle is impossible of attainment, practically. It is, however, another yardstick by which the performance of other power cycles may be measured. Since it is impossible of attainment, the Rankine cycle has been accepted by engineers for many years as a better criterion of performance of steam-driven prime movers.

REHEAT CYCLE. The use of high pressures with low superheat causes high moisture loss in a turbine. This loss is reduced by *resuperheating*, as shown in Fig. 5, where pressure is increased from *BC* to *KL*, but inlet temperature remains constant at *ND*. After expansion from *N* to *C* the steam is resuperheated to the original temperature *D* and expanded to *E*; this is known as *reheating*. The advantages, as measured by cycle efficiencies, are small, but the practical gains due to decreased moisture loss in steam turbines and engines are considerable. Gains of 4 to 6% in turbine-cycle heat rate usually result.

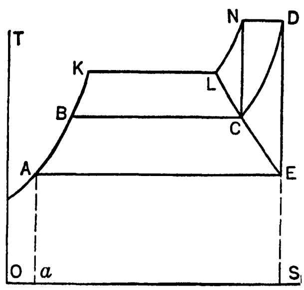


Fig. 5. Reheat cycle.

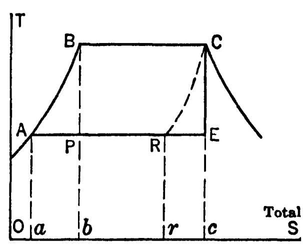


Fig. 6. Extraction or regenerative cycle.

EXTRACTION OR REGENERATIVE CYCLE. Probably the most popular method of improvement of the basic steam cycle is that in which feedwater heaters are used. They heat water leaving the condenser hotwell by using steam extracted from the turbine. The benefits of this cycle may most readily be explained by an illustration based on a turbine supplied with saturated steam, as shown in Fig. 6. Steam enters the turbine at point *C*. After expanding isentropically (ideally) through an infinitesimal portion of the pressure range, a small part of the steam is extracted from the turbine and supplied to a feedwater heater. This heats feedwater from point *A* (hotwell condition) through an infinitesimal temperature range. After further infinitesimal expansion within the turbine, an additional infinitesimal part of the steam is withdrawn, so that for successive extractions the steam proceeds along the path *CR*.

It will be noted that the abscissa in Fig. 6 is *total entropy* of the steam in the turbine, which decreases as steam is withdrawn. In other diagrams the abscissa is the *specific entropy* or entropy per pound. Because in Fig. 6 the abscissa is total entropy, removal of a portion of the steam from the turbine decreases the entropy, causing the state point of steam within the turbine casing to traverse a path as shown by the dash line *CR*.

The line *CR* is essentially "parallel" to *AB* because the heat added to the feedwater, *aABba*, must equal the total heat abstracted from the turbine, *rRCcr*. It is readily seen that the true cycle of this power plant is *ABCRA*, and that this cycle is equivalent to the Carnot *PBCEP*. Thus in this specific cycle, by use of an infinite number of heaters, we are able to equal the Carnot cycle efficiency. Although this is possible theoretically, it is impossible practically because an infinite number of heaters could not be used, if only because the turbine has a finite number of stages, thus offering only a finite number of extraction points. In addition, only the saturated steam cycle has even the theoretical possibility of approaching the Carnot. As soon as superheat is added to the steam, the top temperature of the cycle is higher than the line *BC*. Any superheated steam abstracted from the turbine would be forced to transfer its heat through a finite temperature difference (causing a loss of availability, by the second law of thermodynamics), even to transfer heat to feedwater at the temperature *B*.

The cycle has become so widely used in the power plant industry, that many references and analyses of it appear in the literature. (Refs. 1, 2, and 3.) A more extensive discussion of the regenerative cycle is given in Section 8 of this book.

Regenerative-reheat Cycle. Some power stations operate on the regenerative cycle, at the same time using a reheat cycle. In such applications it is of value to use as one extraction point the exhaust from the high-pressure turbine. For further data on the use of the regenerative reheat cycle, see Ref. 4.

BINARY-VAPOR CYCLES. The critical state point of steam is 3206.2 psia, at a temperature of 705.4 F. Several plants have been built to use a fluid with higher boiling temperature superimposed on the regular steam cycle, forming a *binary-vapor* cycle. The mercury vapor-steam cycle provides one of the most efficient means of generating power from fuel. (See Section 8.)

Mercury-vapor-steam Cycle. Figure 7 illustrates the mercury-vapor-steam cycle on a temperature-entropy diagram. At 180 psia pressure, mercury vapor boils at 999.5 F. It is then expanded isentropically to 28 in. vacuum, in a mercury turbine. The mercury condenses at 456.4 F in a condenser-boiler, which transfers the exhaust heat from mercury to steam. Saturated steam is generated at 360 psig, 438.3 F, by condensation of mercury vapor. For efficient use in the steam turbine, the steam is superheated by flue gases from the mercury boiler to 700 F and expanded to 29 in. vacuum, using three stages of regenerative feedwater heating. Because of the low latent heat of mercury, about 10 lb of mercury per 1 lb of steam are used in the mercury-vapor-steam cycle. With isentropic expansion of both mercury vapor and steam, and with three stages of extraction feedwater heating in the cycle shown in Fig. 7, an efficiency of 58.8% may be obtained. The Carnot (maximum possible) cycle efficiency between the temperature limits of 999.5 and 79 F is 63.4%. Higher efficiences can be developed on this combined cycle than on any of the preceding cycles.

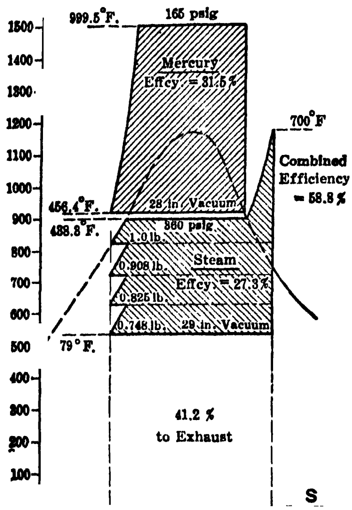


FIG. 7. Temperature-entropy diagram of mercury-vapor-steam cycle.

The properties of mercury vapor are given in Table 1. For additional discussion of mercury-vapor cycles, see Section 8.

SUPERSATURATED STEAM. The isentropic expansion of superheated steam follows closely the equation $pv^{1.3} = constant$, where p = absolute pressure and v = specific volume. If we assume that isentropic expansion of saturated steam takes place in thermal equilibrium, condensation proceeds in the wet region as indicated on a Mollier diagram. Such condensation takes considerable time, for droplets must form and grow in the mass of the steam. Consequently, sudden expansion of a saturated vapor usually produces a temporary unstable condition in which the mass continues to expand as superheated steam without any condensation. This phenomenon is called *supersaturation*. The density of the vapor in this state is abnormal, higher than the density of saturated vapor at the same pressure. The temperature at the end of expansion is lower than the temperature of saturation at that pressure, and the vapor is said to be *undercooled*. The supersaturated condition soon disappears through condensation of part of the vapor. The temperature of the remaining mass is raised by the latent heat given off during condensation until thermal equilibrium is restored. Supersaturation may be assumed to occur whenever steam is expanded through a nozzle or orifice and the saturation line is passed during expansion. Such expansion can be represented by the equation $pv^{1.3} = constant$.

Effect of Supersaturation. One effect of supersaturation in nozzles passing saturated steam is to increase the discharge of steam by about 5% for a given throat area as compared with that calculated from equilibrium data for saturated steam. A second effect is to cause an increase of entropy and volume after passing the throat when the steam in the nozzle becomes wet. It can be shown that there is less heat available to do work from that portion of the expansion below the saturation line when expansion continues in the

Table 1. Properties of Saturated Mercury Vapor

(Abstracted from *Properties of Mercury Vapor*, by Lucian A. Sheldon. Courtesy of General Electric Company, 1948.)

For calculations involving superheated mercury vapor, the mean specific heat at constant pressure, c_p, may be taken as 0.02474 Btu/lb-°F

Pressure, psia	Temperature, °F	Specific Volume, cu ft/lb	Density lb/cu ft	Enthalpy, Btu/lb			Entropy, Btu/lb-°F		
				Saturated Liquid	Evaporation	Saturated Vapor	Saturated Liquid	Evaporation	Saturated Vapor
0.49	413.76	94.065	0.01063	12.535	126.922	139.456	.0189	.1453	.1642
0.735	438.22	64.485	0.01550	13.329	126.816	140.141	.0198	.1412	.1610
0.980	456.42	49.33	0.02027	13.917	126.730	140.647	.0204	.1383	.1587
1.2	469.75	40.85	0.02448	14.346	126.670	141.016	.0209	.1363	.1572
1.4	480.10	35.42	0.02823	14.680	126.623	141.303	.0212	.1347	.1560
1.6	489.35	31.29	0.03196	14.973	126.582	141.555	.0215	.1334	.1549
1.8	497.50	28.08	0.03561	15.236	126.545	141.781	.0218	.1322	.1540
2.0	504.93	25.39	0.03939	15.476	126.512	141.988	.0221	.1312	.1532
3.0	535.25	17.50	0.05714	16.439	126.377	142.816	.0230	.1271	.1501
4.0	557.85	13.38	0.07474	17.161	126.275	143.436	.0237	.1241	.1479
5.0	575.7	10.90	0.09174	17.741	126.193	143.934	.0243	.1219	.1462
6.0	591.2	9.26	0.10799	18.233	126.124	144.357	.0248	.1200	.1448
7.0	604.7	8.040	0.12438	18.657	126.065	144.722	.0252	.1185	.1436
8.0	616.5	7.120	0.14045	19.035	126.011	145.046	.0255	.1171	.1426
9.0	627.3	6.390	0.15649	19.381	125.962	145.343	.0258	.1159	.1417
10.0	637.0	5.810	0.17212	19.685	125.919	145.604	.0261	.1148	.1409
15.0	676.05	4.020	0.24876	20.934	125.743	146.677	.0272	.1107	.1379
20.0	706.0	3.090	0.32362	21.864	125.609	147.473	.0280	.1078	.1358
25.0	730.05	2.525	0.39604	22.627	125.500	148.127	.0287	.1055	.1341
30.0	750.6	2.140	0.46729	23.277	125.407	148.684	.0292	.1036	.1328
35.0	768.45	1.860	0.53763	23.837	125.327	149.164	.0296	.1020	.1317
40.0	784.4	1.648	0.60680	24.345	125.255	149.600	.0300	.1007	.1307
45.0	798.85	1.482	0.67409	24.793	125.191	149.983	.0304	.1000	.1299
50.0	812.1	1.348	0.74184	25.203	125.131	150.334	.0307	.0984	.1291
55.0	824.3	1.239	0.80710	25.583	125.076	150.659	.0310	.0974	.1284
60.0	835.7	1.144	0.87413	25.940	125.024	150.964	.0313	.0965	.1278
65.0	846.35	1.066	0.93809	26.274	124.977	151.250	.0315	.0957	.1272
70.0	856.4	0.998	1.0020	26.585	124.931	151.516	.0318	.0949	.1267
75.0	865.85	0.961	1.0406	26.880	124.889	151.769	.0320	.0942	.1262
80.0	874.8	0.885	1.1299	27.159	124.849	152.008	.0322	.0936	.1257
85.0	883.4	0.838	1.1933	27.425	124.810	152.235	.0324	.0929	.1253
90.0	891.5	0.797	1.2547	27.680	124.774	152.454	.0326	.0923	.1249
100.0	906.8	0.725	1.3793	28.152	124.706	152.858	.0330	.0913	.1242
110.0	921.0	0.667	1.4993	28.596	124.641	153.237	.0332	.0903	.1235
120.0	934.3	0.617	1.6207	29.005	124.582	153.587	.0335	.0894	.1229
130.0	946.6	0.575	1.7391	29.390	124.526	153.916	.0338	.0886	.1223
140.0	958.3	0.538	1.8587	29.748	124.474	154.222	.0340	.0878	.1218
150.0	969.4	0.507	1.9724	30.090	124.424	154.514	.0343	.0871	.1213
160.0	979.9	0.478	2.0921	30.415	124.376	154.791	.0345	.0864	.1209
170.0	989.9	0.453	2.2075	30.724	124.331	155.055	.0347	.0858	.1205
180.0	999.5	0.431	2.3202	31.018	124.288	155.306	.0349	.0852	.1201

supersaturated condition than if it took place in thermal equilibrium. This decrease in available energy usually is considered in steam turbine design, and is an important factor in causing the lowered stage efficiencies experienced with saturated steam. For further information, see Refs. 5–11.

2. THE MOLLIER DIAGRAM

By J. K. Salisbury

CONSTRUCTION. A 20 by 32 in. Mollier diagram accompanies Keenan and Keyes' book, *Thermodynamic Properties of Steam*. (A large-scale Mollier chart is given on pp. 4-10 to 4-28 of this book.) (See p. 4-09.) On this chart the enthalpy of 1 lb of steam above 32 F is plotted as ordinate, and entropy above 32 F as abscissa. Lines of constant absolute pressure in pounds per square inch slope up from left to right. In the low-pressure region at the right-hand side, dotted lines represent absolute pressures in inches of mercury, and are convenient for exhaust steam calculations. Below the saturation line, curves of constant moisture content, *in percentage*, slope down from left to right. Above the saturation line are lines of constant temperature and lines of constant superheat, both in degrees Fahrenheit. Enthalpy may be found directly from the diagram. For instance, the enthalpy at 650 psia, 900 F, from the chart = 1460.8 Btu per pound at entropy 1.6665. The total heat at 1 in. Hg absolute and 90% quality (10% moisture) = 991.4 Btu per pound.

THROTTLING EFFECTS. In throttling processes the enthalpy of steam remains unchanged, provided no heat is lost by radiation, hence a line of constant enthalpy represents throttling on the Mollier diagram. Thus, if steam at 650 psia, 900 F, is throttled to 400 psia, its condition on the constant total heat line for 1460.8 Btu at 400 psia is 888 F.

The quality of wet steam can readily be determined on the Mollier diagram from throttling calorimeter readings. Given: steam line pressure, 125 psig (140 psia); atmospheric pressure, 14.696 psia, in the calorimeter; temperature in the calorimeter, 280 F. From the chart, enthalpy or total heat in calorimeter at 14.696 psia and 280 F = 1183.3 Btu. A constant enthalpy line, representing throttling, intersects the 140-lb absolute pressure line at 1.1% moisture giving 98.9% quality. (See also Section 7.)

Available energy on the *Rankine cycle* is the enthalpy change for an isentropic expansion, which follows a line of constant entropy.

Example. A turbine receives steam at 450 psia, 750 F, and exhausts at 1 in. Hg abs. Total heat at 450 psia, 750 F, h_1 = 1387.2 Btu; s = 1.6480. At 1 in. Hg abs, s = 1.6480, h_2 = 885.8 Btu. Heat available on the Rankine cycle or heat drop $(h_1 - h_2)$ = 1387.2 − 885.8 = 501.4 Btu per lb of steam.

Heat added as reheat in a reheating turbine is readily found on the Mollier chart.

Example. Steam leaves the high-pressure turbine at 120 psia, 370 F, and is reheated with a 10 psi pressure drop in the reheater to 700 F. Total heat at 120 psia, 370 F, = 1207.4 Btu. Total heat at 110 psia, 700 F, = 1378.3 Btu. Heat added by reheater = 1378.3 − 1207.4 = 170.9 Btu per lb.

The condition of exhaust steam can be found quickly on a Mollier diagram. In a steam turbine,

$$h_e = h_o - \frac{3413}{\text{S.R.} \times \text{M.E.}}$$

where h_e = enthalpy of exhaust steam, Btu per pound; h_o = enthalpy of steam at throttle, Btu per pound; S.R. = turbine steam rate, pounds per kilowatt-hour; and M.E. = mechanical and electrical efficiency.

The moisture content may be read from the chart at the intersection of exhaust pressure and enthalpy lines.

LARGE-SCALE MOLLIER CHART. The small-scale Mollier chart supplied with the steam tables sometimes is inadequate for accurate reading in the design of steam turbines. A large chart from which such readings may be made with considerable improvement in accuracy is given on pp. 4-09 to 4-28. Portions of the original ASME chart have been especially selected and prepared for use in this book, by permission of ASME.

To incorporate the most useful portions of the large chart in the format of this book it has been necessary to use several pages. Each page of the charts which follow is identified by a key number, indicating the portions of the chart covered, as shown on the key chart, Fig. 8. Key numbers *without* suffixes pertain to the *central* portion of the chart (normally used for state lines of modern steam turbines). The suffix (A) indicates portions of the chart to the *left* (lower entropies) of the central portion (normally used in determination of adiabatic enthalpy differences or available energy). The suffix (B) indicates portions of the chart to the *right* (higher entropies) of the central portion. Correspondence of the numerical portions of the key numbers indicates the same enthalpy coverage.

Skeleton pressure and temperature lines are shown in Fig. 8 to facilitate reference to the proper chart. For example, both charts 3 and 3B cover an enthalpy range from 1310 to 1380 Btu per lb.

The respective entropy ranges are:

3: 1.520 to 1.720
3B: 1.720 to 1.920

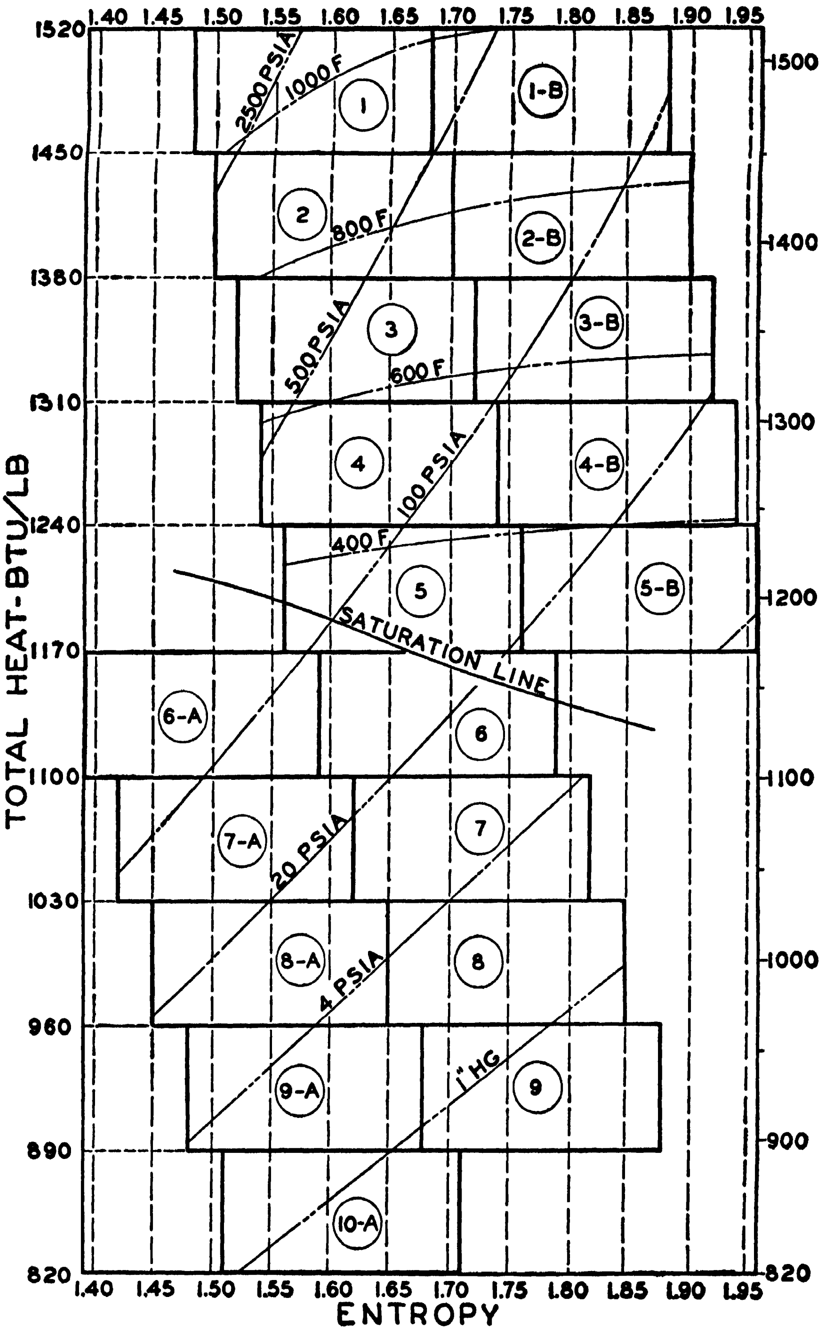
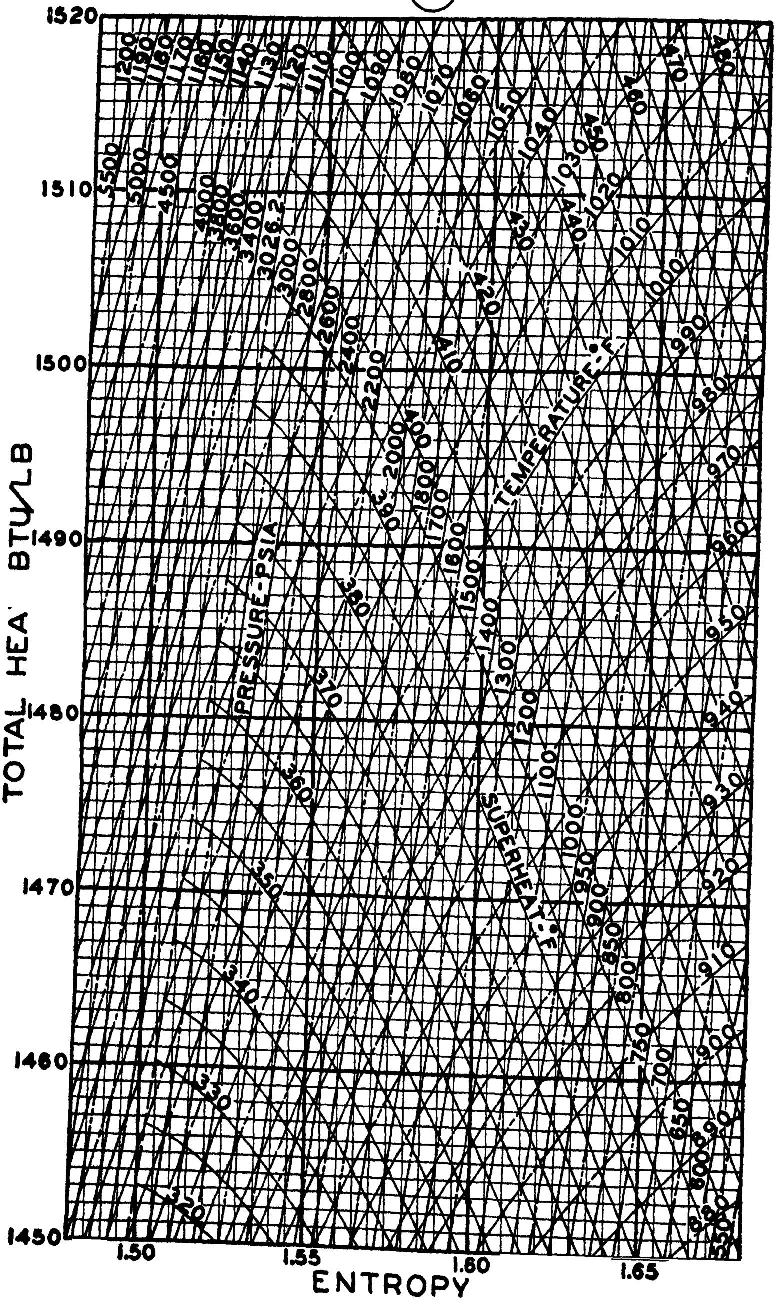
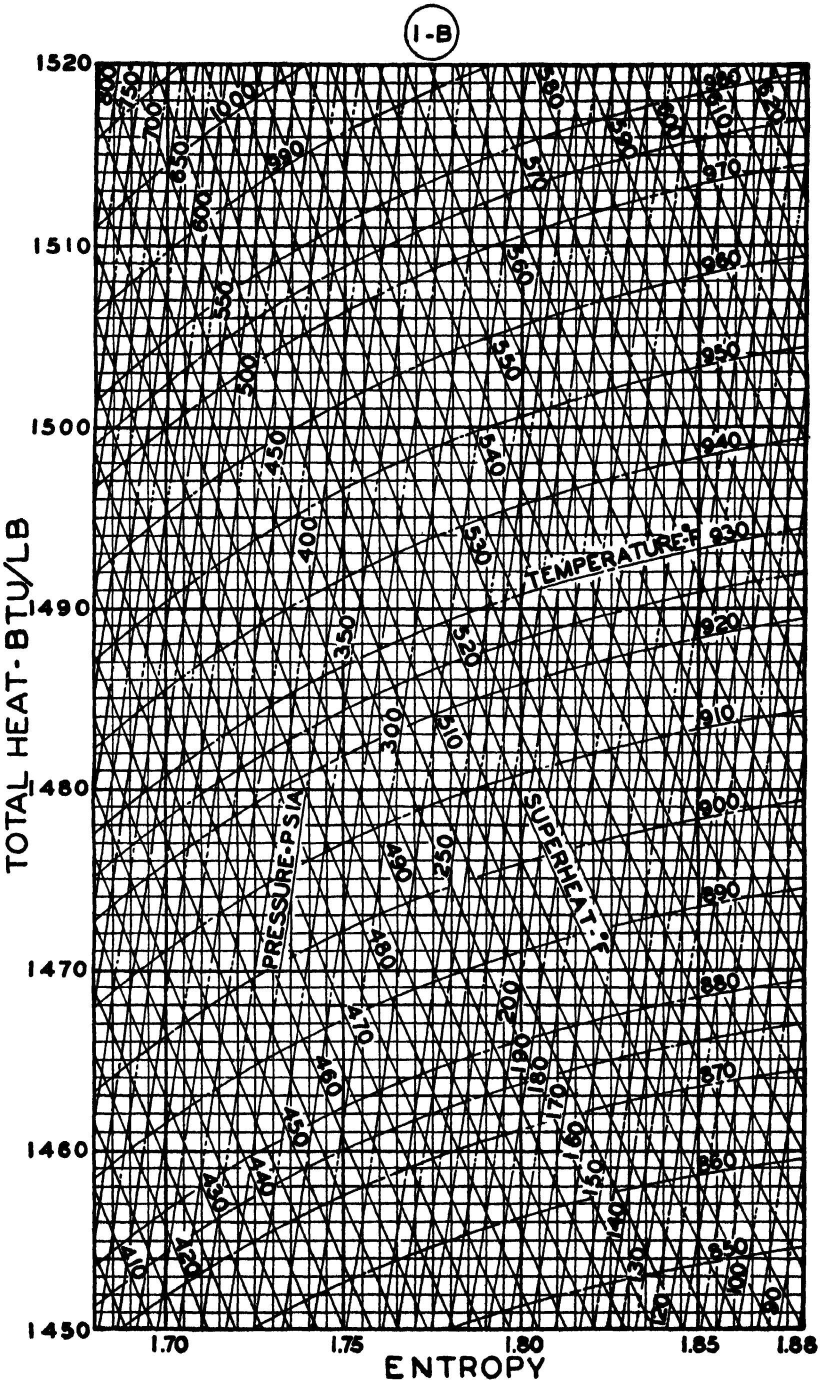
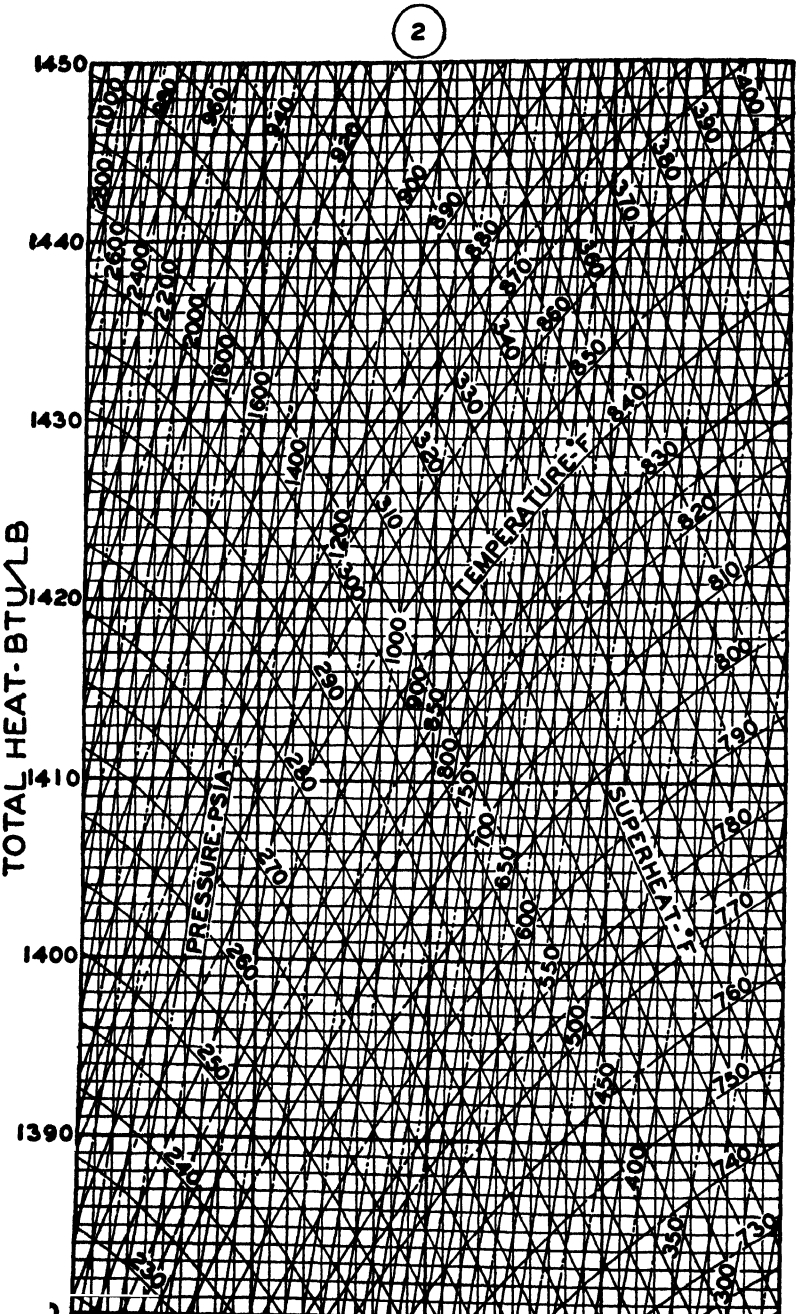


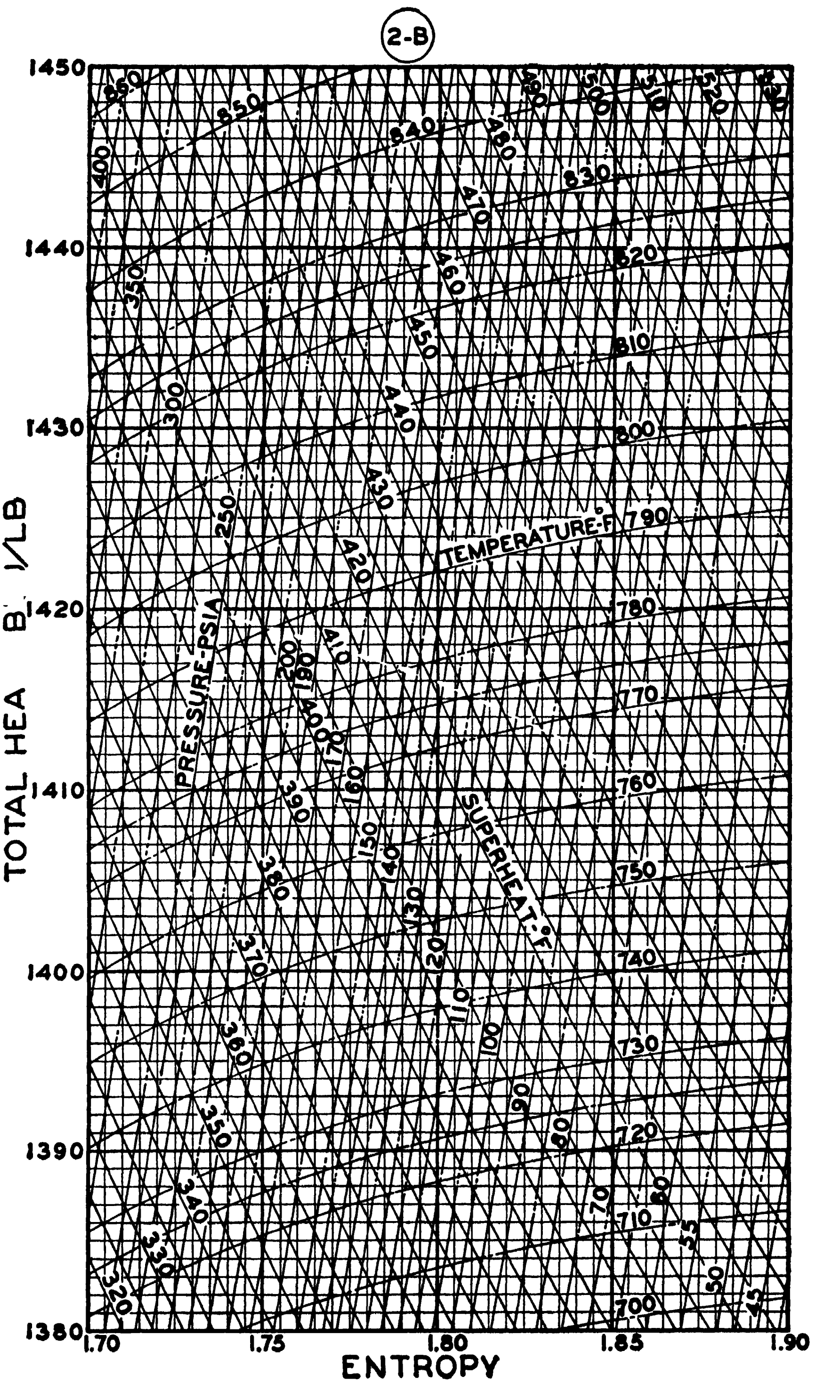
FIG. 8. Key diagram to be used in selection of proper chart from the following pages.

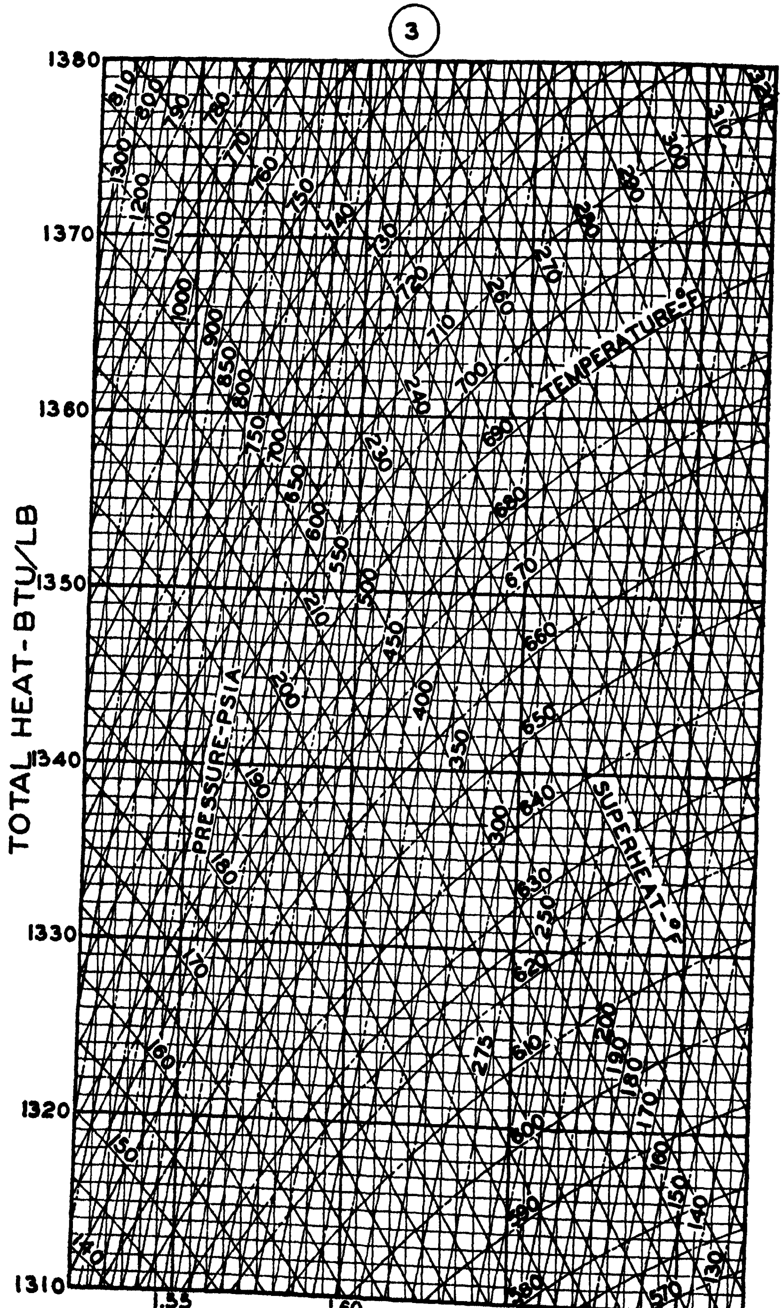
(1)

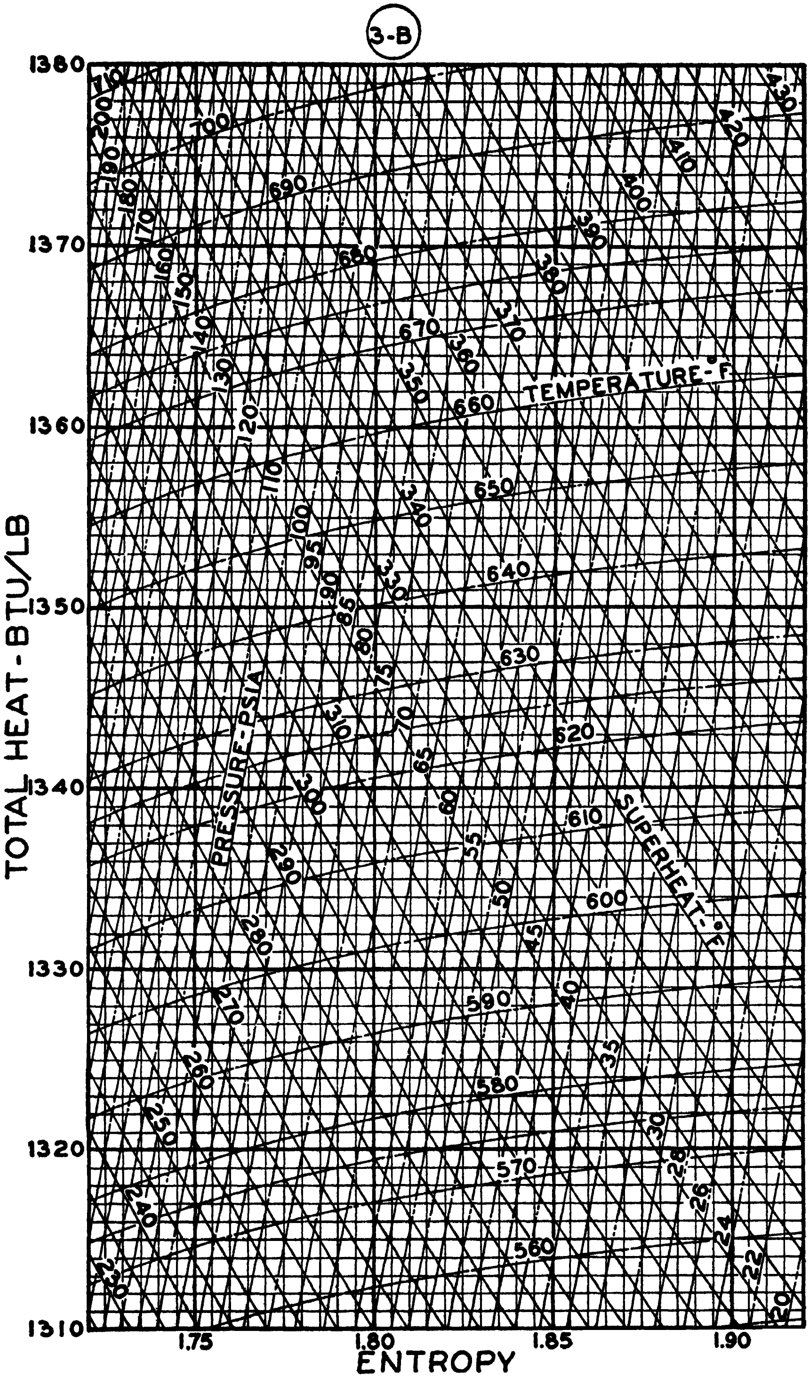

TOTAL HEAT BTU/LB
ENTROPY
PRESSURE-PSIA
TEMPERATURE-F
SUPERHEAT-F
1520
1510
1500
1490
1480
1470
1460
1450
1.50
1.55
1.60
1.65

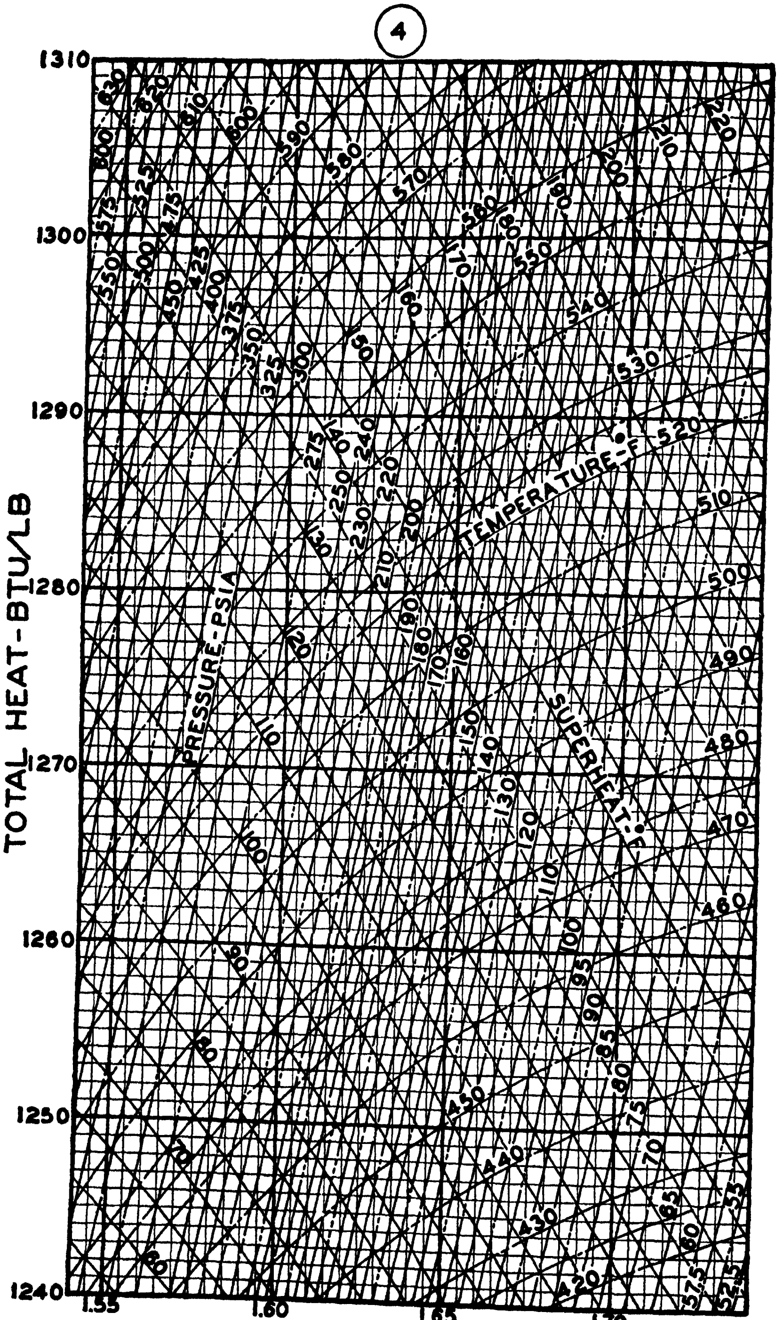

1-B
TOTAL HEAT-BTU/LB
ENTROPY
TEMPERATURE-°F
PRESSURE-PSIA
SUPERHEAT-°F
1520
1510
1500
1490
1480
1470
1460
1450
1.70
1.75
1.80
1.85
1.88

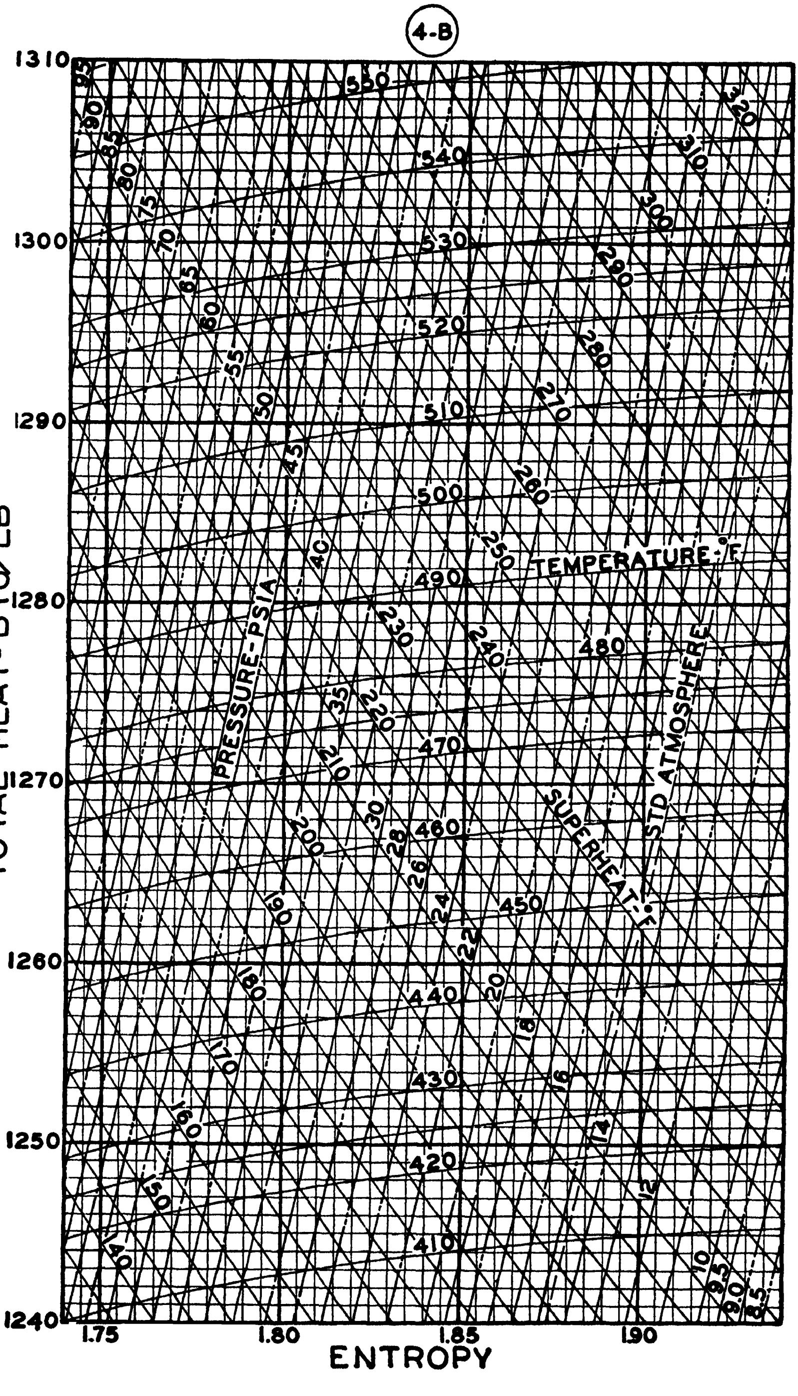

2
TOTAL HEAT-BTU/LB
ENTROPY
PRESSURE-PSIA
TEMPERATURE-F
SUPERHEAT-F
1450
1440
1430
1420
1410
1400
1390
1.50
1.55
1.60
1.65
1.70

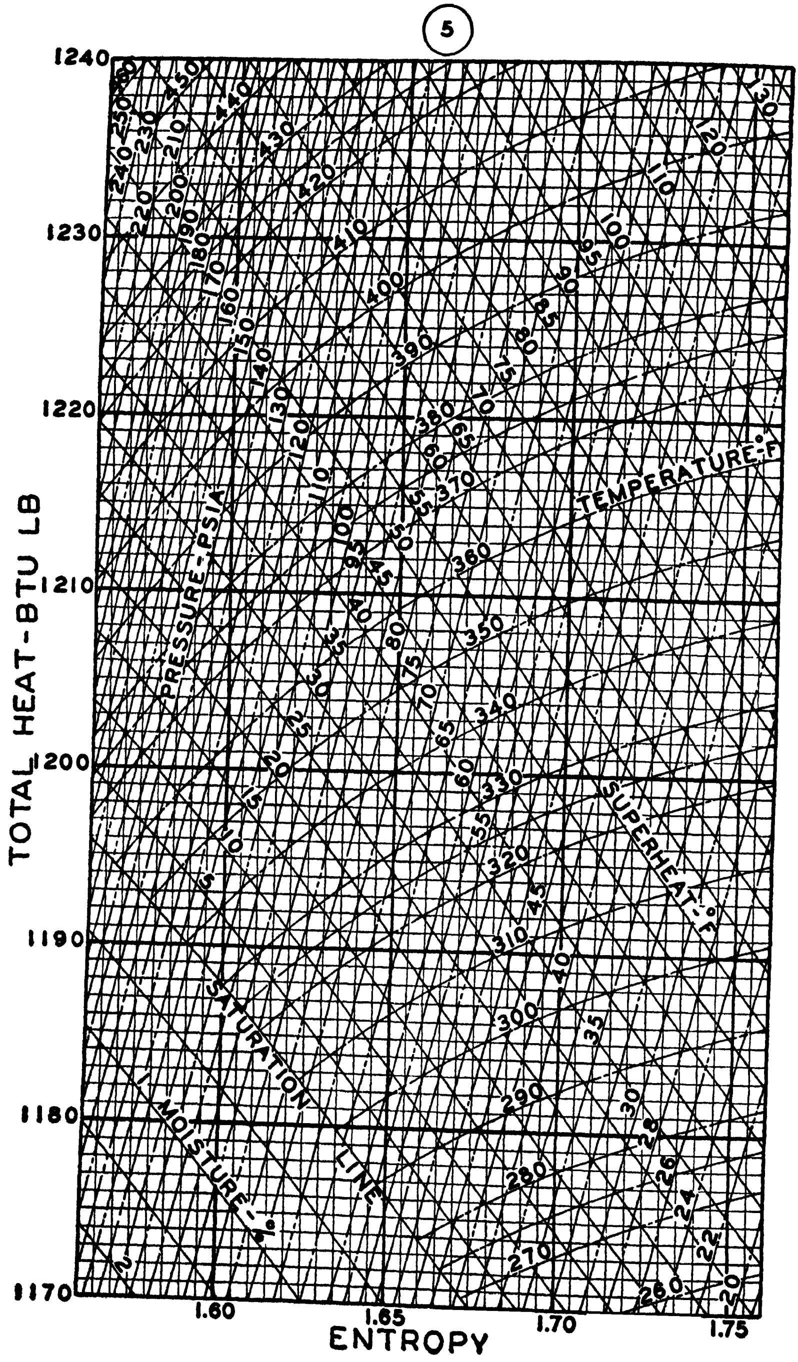

2-B
1450
1440
1430
1420
1410
1400
1390
1380
1.70
1.75
1.80
1.85
1.90
ENTROPY
TOTAL HEA B /LB
TEMPERATURE-F
PRESSURE-PSIA
SUPERHEAT-°F

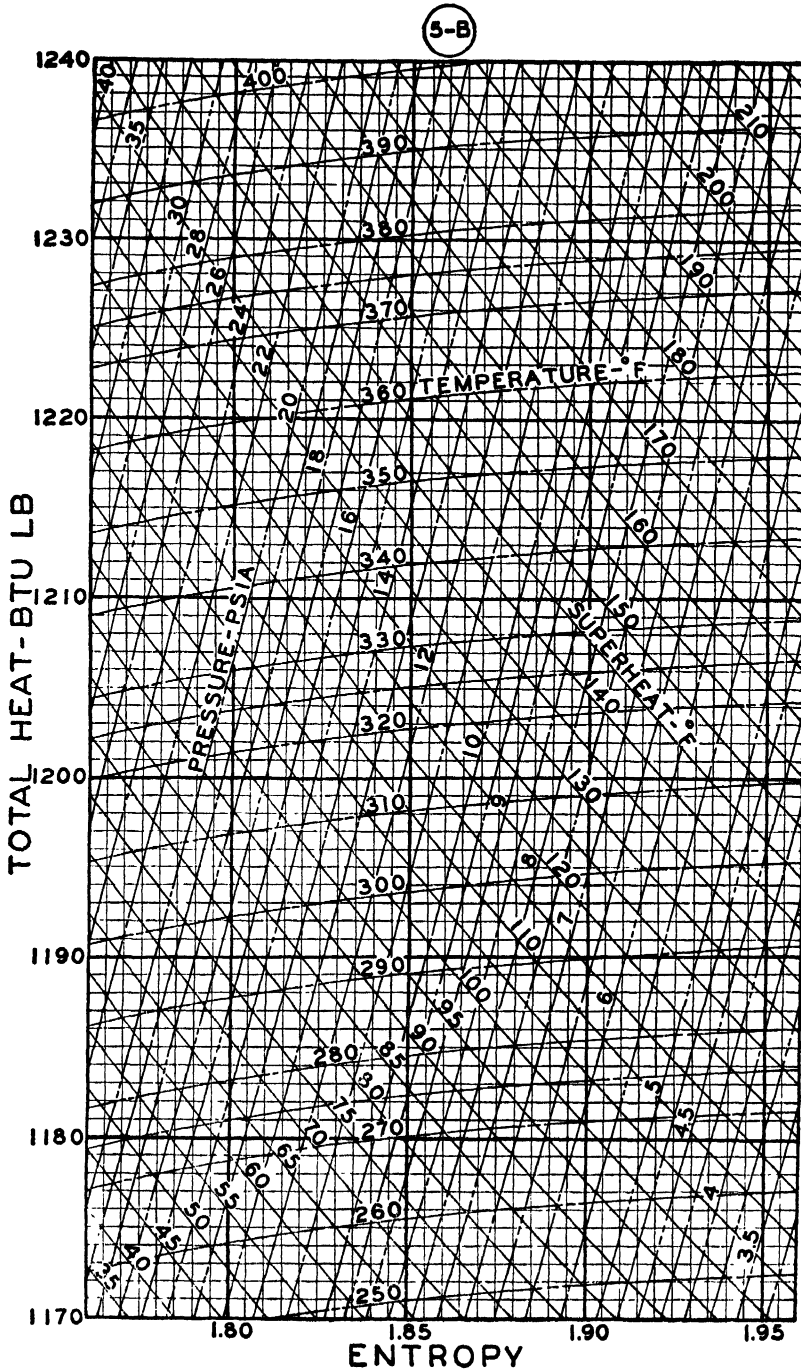

3
TOTAL HEAT - BTU/LB
ENTROPY
1380
1370
1360
1350
1340
1330
1320
1310
1.55
1.60
1.65
1.70
PRESSURE-PSIA
TEMPERATURE-°F
SUPERHEAT-°F


3-B
TOTAL HEAT - BTU/LB
ENTROPY
TEMPERATURE-°F
PRESSURE-PSIA
SUPERHEAT-°F
1380
1370
1360
1350
1340
1330
1320
1310
1.75
1.80
1.85
1.90

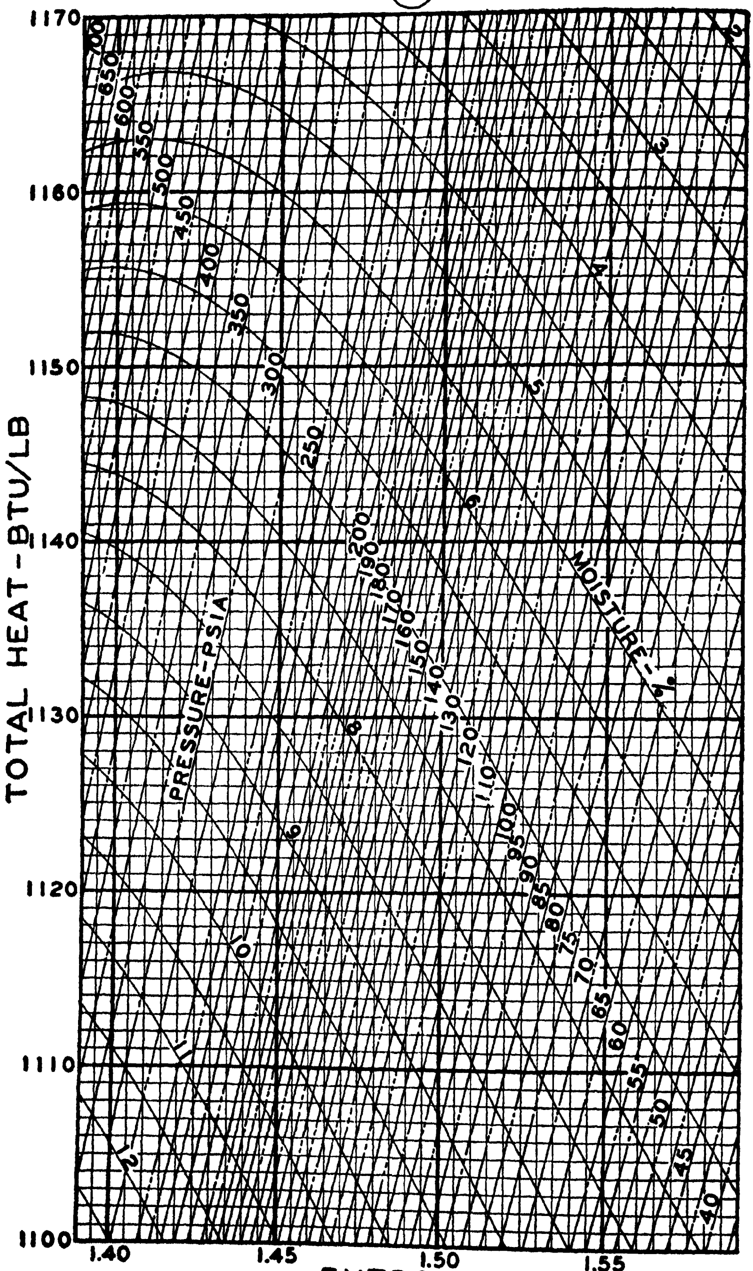

4
TOTAL HEAT-BTU/LB
ENTROPY
PRESSURE-PSIA
TEMPERATURE-F
SUPERHEAT-F
1310
1300
1290
1280
1270
1260
1250
1240
1.55
1.60
1.65
1.70

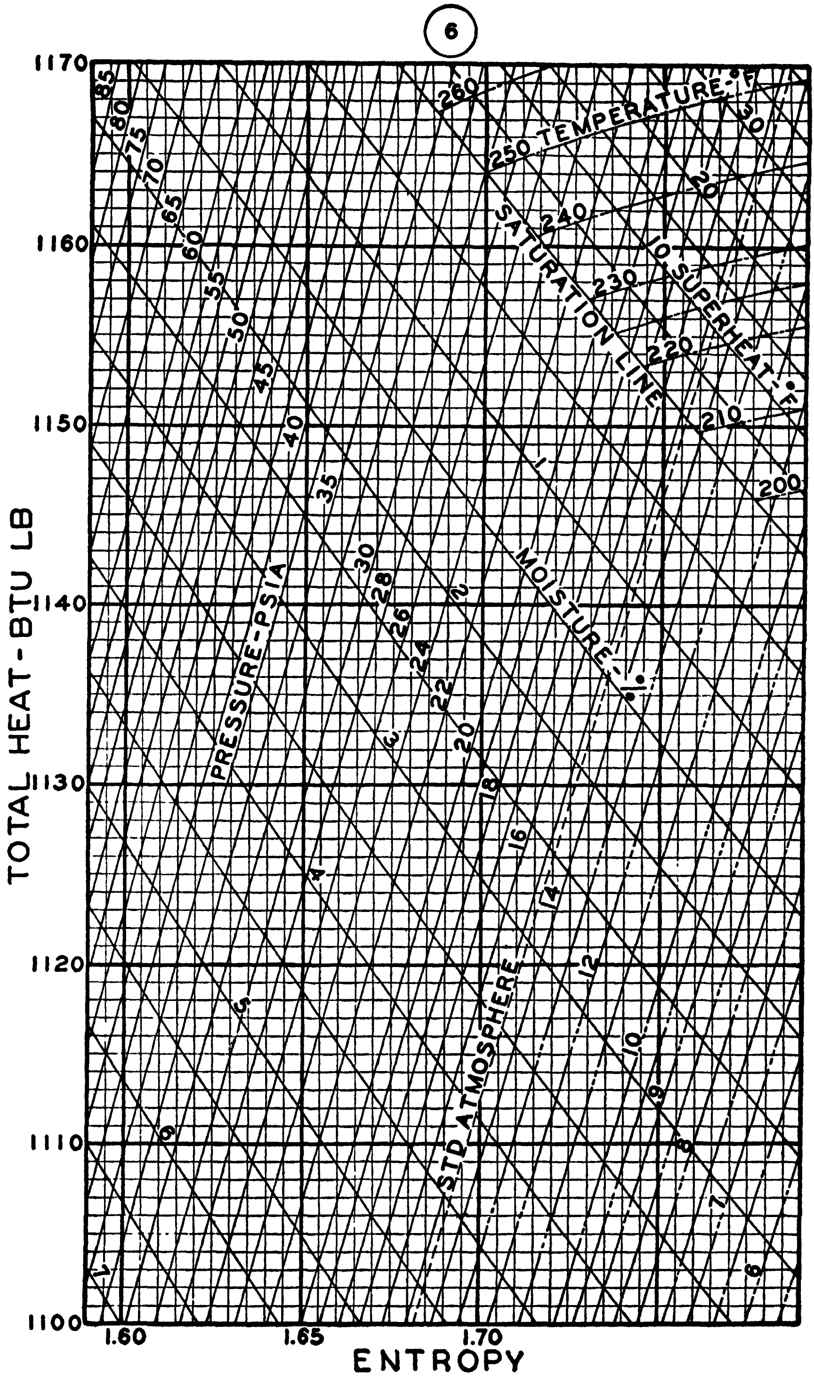

4-B
TOTAL HEAT-BTU/LB
ENTROPY
1310
1300
1290
1280
1270
1260
1250
1240
1.75
1.80
1.85
1.90
PRESSURE-PSIA
TEMPERATURE-°F
SUPERHEAT-°F
STD ATMOSPHERE

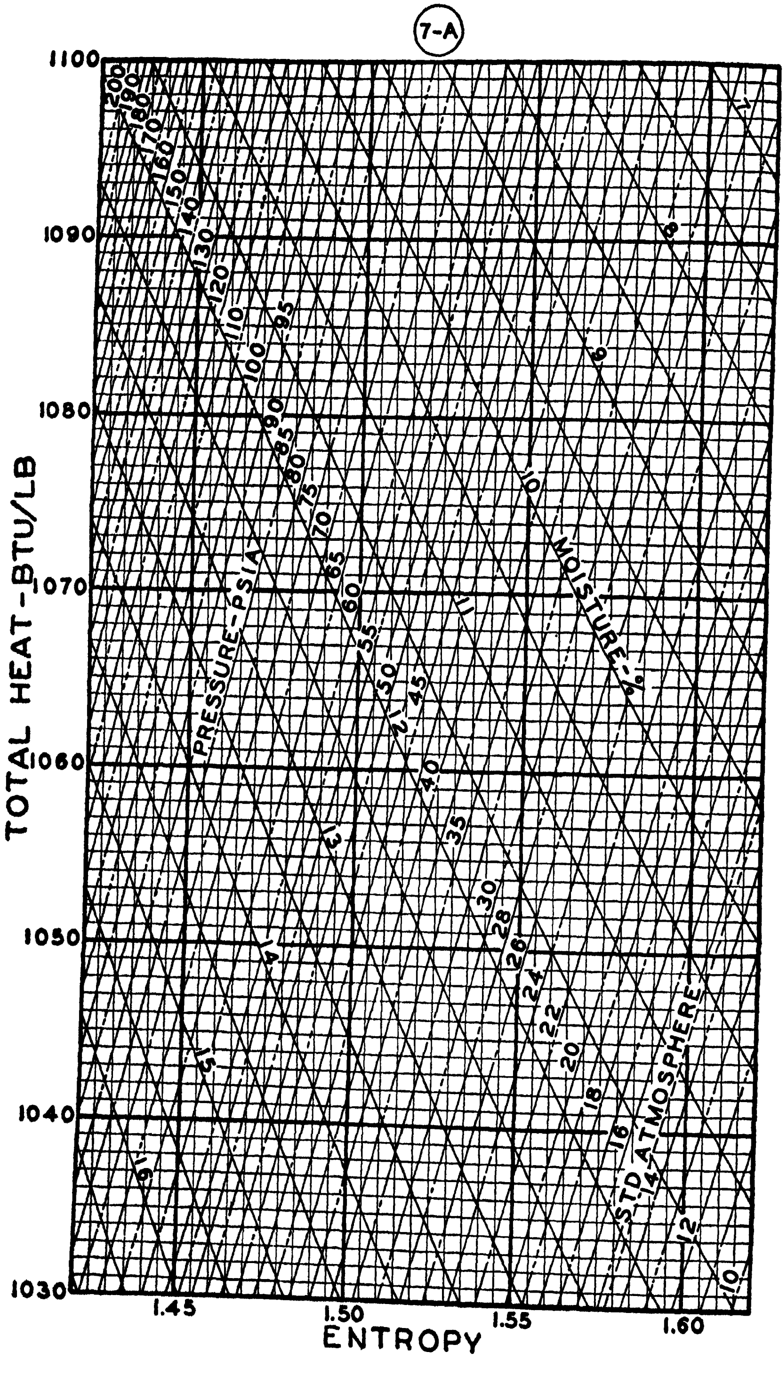

5
1240
1230
1220
1210
1200
1190
1180
1170
TOTAL HEAT-BTU LB
1.60
1.65
1.70
1.75
ENTROPY
PRESSURE-PSIA
TEMPERATURE-F
SUPERHEAT-°F
SATURATION LINE
MOISTURE-%

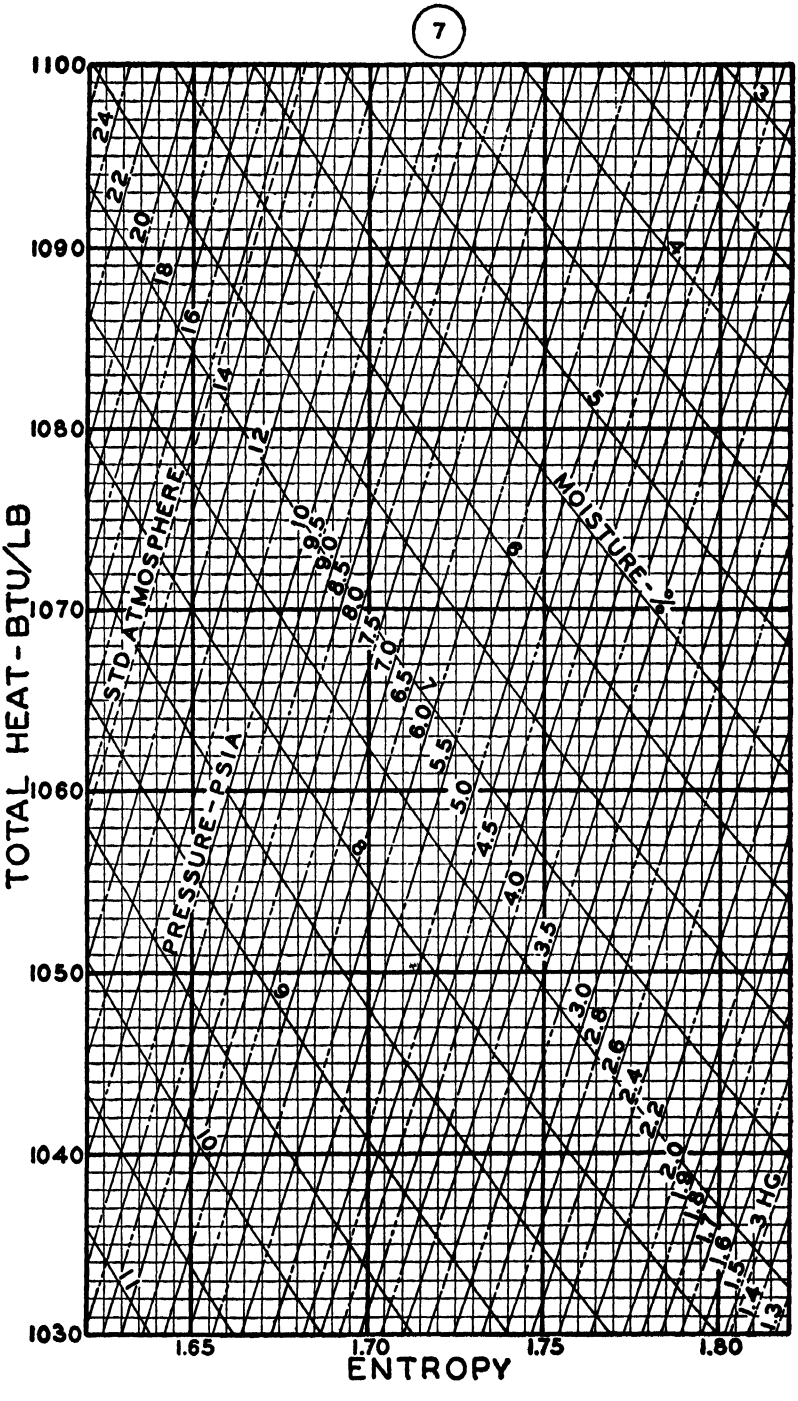

5-B
TOTAL HEAT-BTU LB
ENTROPY
TEMPERATURE-°F
PRESSURE-PSIA
SUPERHEAT-°F
1240
1230
1220
1210
1200
1190
1180
1170
1.80
1.85
1.90
1.95
400
390
380
370
360
350
340
330
320
310
300
290
280
270
260
250
210
200
190
180
170
160
150
140
130
120
110
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
28
26
24
22
20
18
16
14
12
10
9
8
7
6
5
4.5
4
3.5

6-A

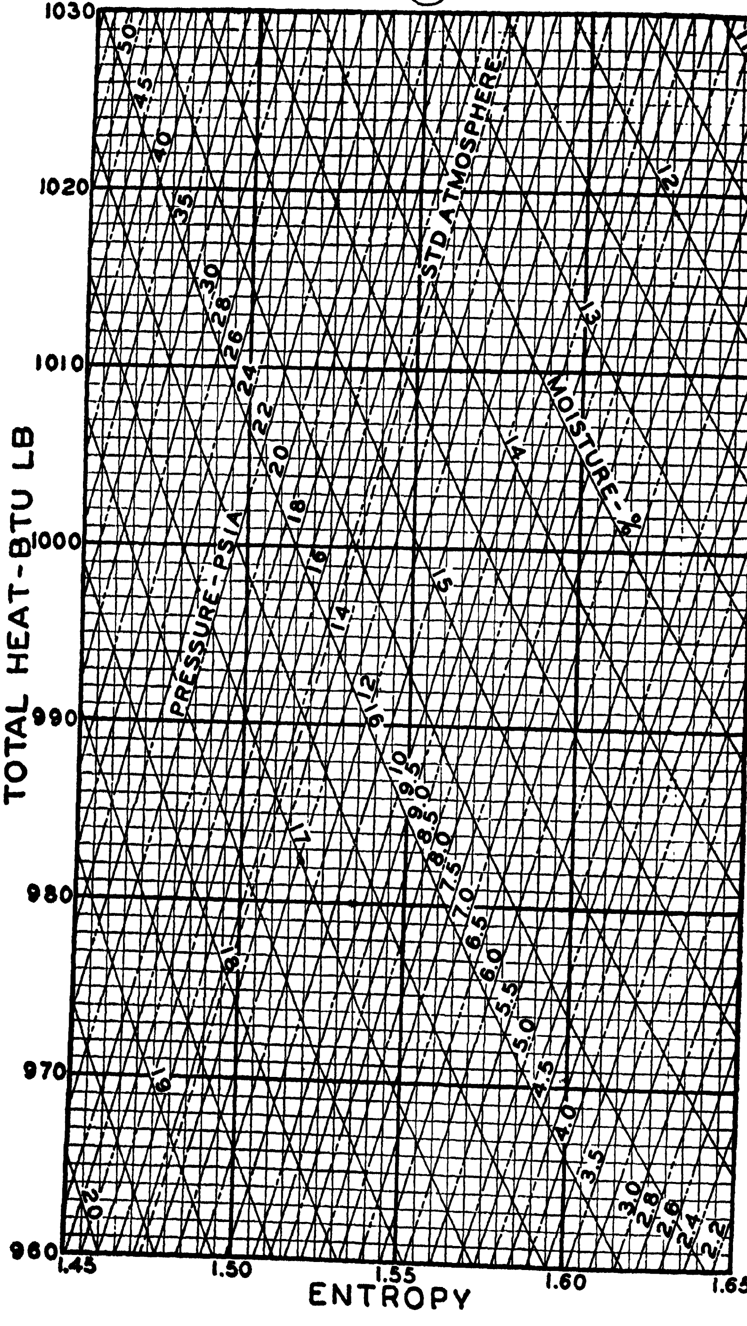

TOTAL HEAT - BTU/LB
ENTROPY
PRESSURE-PSIA
MOISTURE-%
1100
1110
1120
1130
1140
1150
1160
1170
1.40
1.45
1.50
1.55


6
TOTAL HEAT - BTU LB
ENTROPY
1170
1160
1150
1140
1130
1120
1110
1100
1.60
1.65
1.70
TEMPERATURE-°F
SUPERHEAT-°F
SATURATION LINE
MOISTURE-%
PRESSURE-PSIA
STD ATMOSPHERE


7-A
1100
1090
1080
1070
1060
1050
1040
1030
TOTAL HEAT-BTU/LB
1.45
1.50
1.55
1.60
ENTROPY
PRESSURE-PSIA
200
190
180
170
160
150
140
130
120
110
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
28
26
24
22
20
18
16
14
12
10
MOISTURE-%
STD ATMOSPHERE


7
TOTAL HEAT - BTU/LB
ENTROPY
1100
1090
1080
1070
1060
1050
1040
1030
1.65
1.70
1.75
1.80
STD ATMOSPHERE
PRESSURE - PSIA
MOISTURE - %
24
22
20
18
16
14
12
10
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.8
2.6
2.4
2.2
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
3 HG
3
4
5
6
7
8
9
10
11

8-A



TOTAL HEAT-BTU LB
ENTROPY
1030
1020
1010
1000
990
980
970
960
1.45
1.50
1.55
1.60
1.65
PRESSURE-PSIA
STD ATMOSPHERE
MOISTURE-%
50
45
40
35
30
28
26
24
22
20
18
16
14
12
10
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
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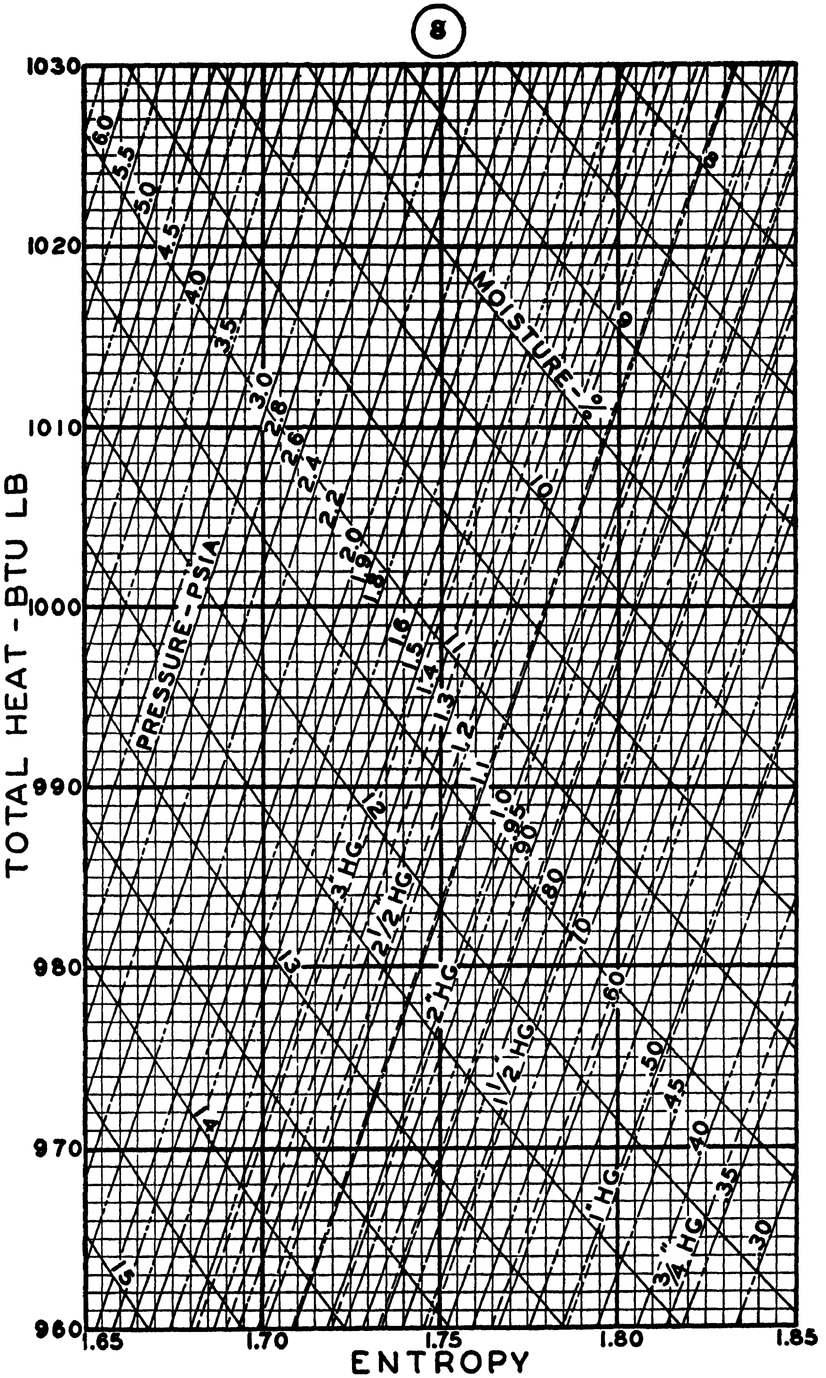
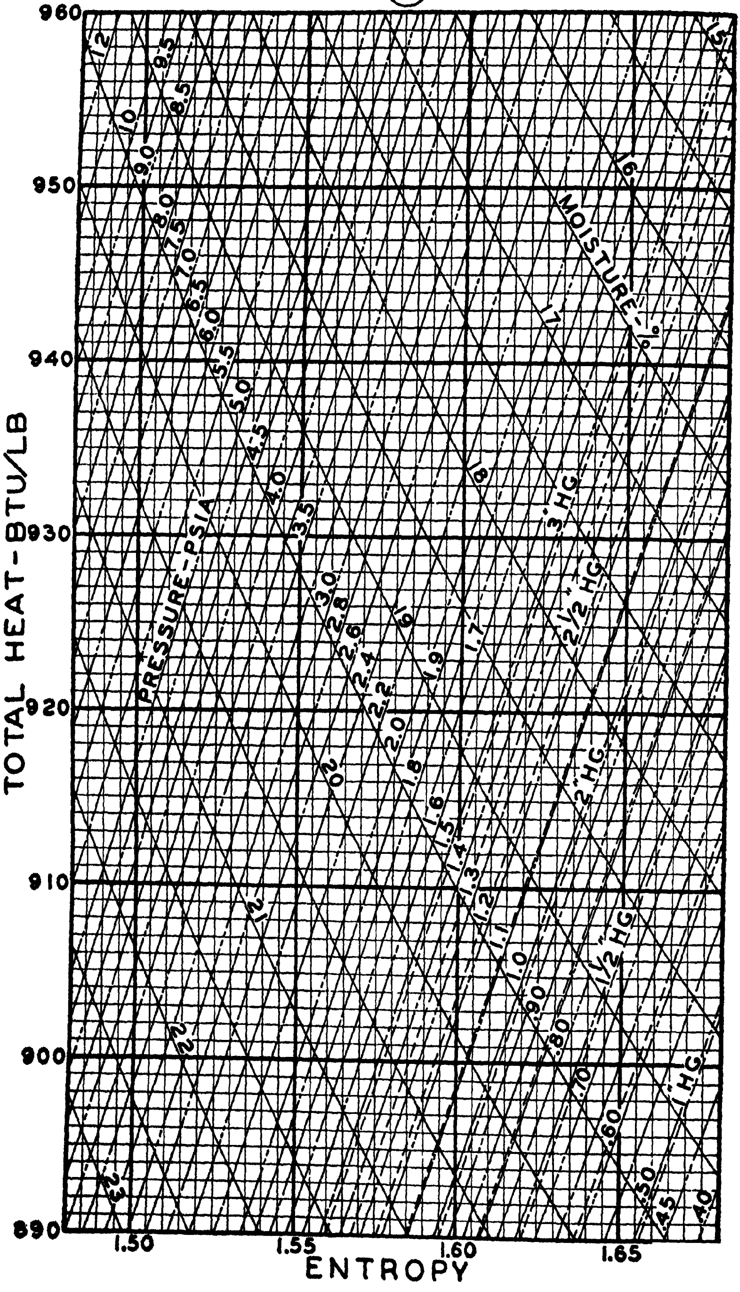
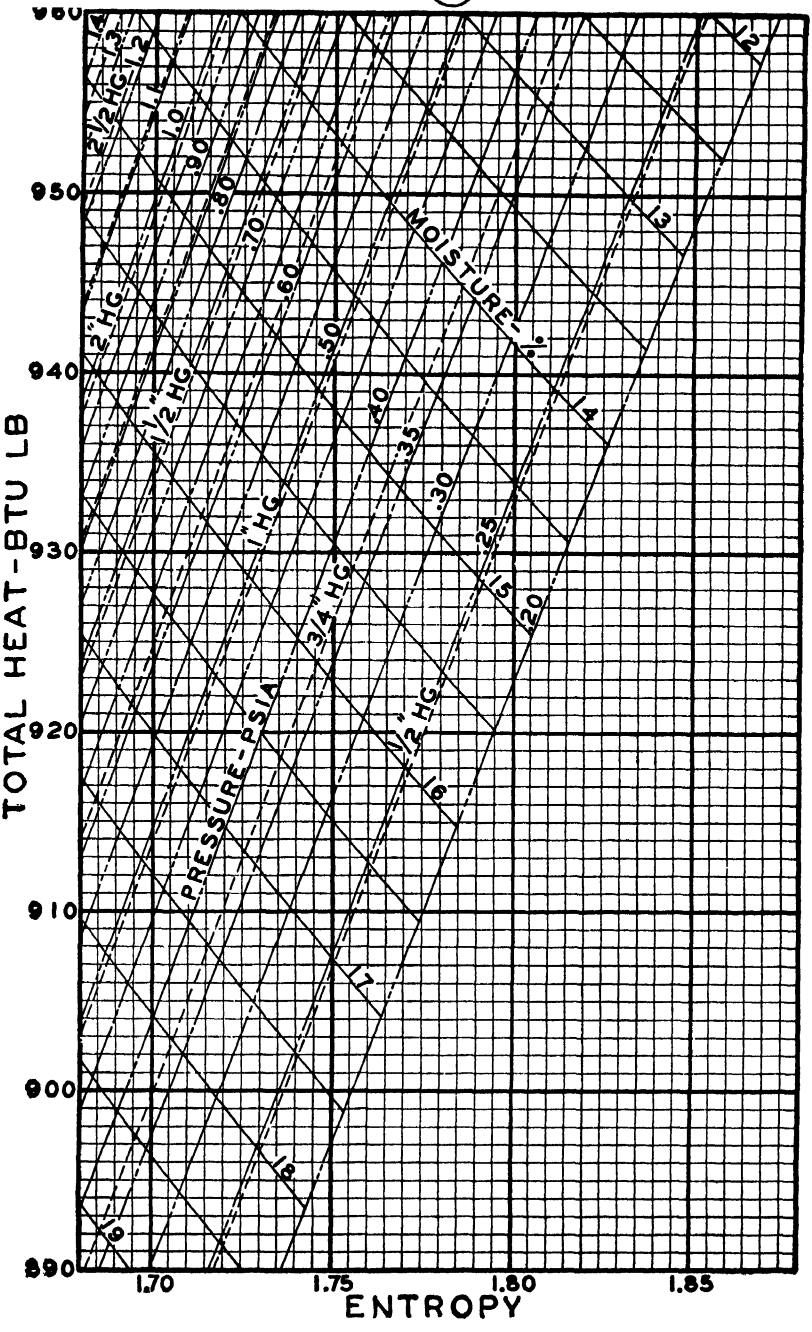
16
17
18
19
20


8
TOTAL HEAT - BTU LB
1030
1020
1010
1000
990
980
970
960
ENTROPY
1.65
1.70
1.75
1.80
1.85
PRESSURE - PSIA
MOISTURE - %
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.8
2.6
2.4
2.2
2.0
1.9
1.8
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.95
.90
.80
.70
.60
.50
.45
.40
.35
.30
3" HG
2 1/2" HG
2" HG
1 1/2" HG
1" HG
3/4" HG
15
14
13
12
11
10
9
8

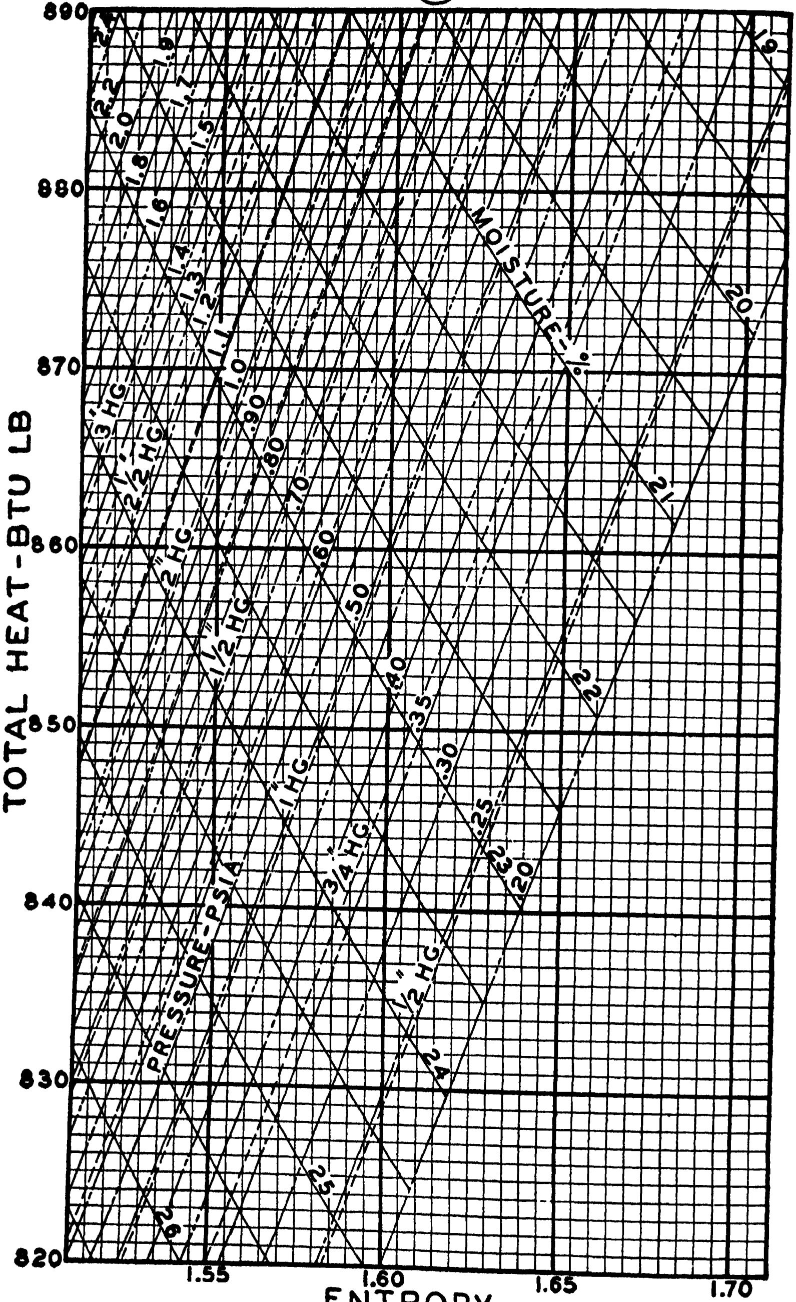
9-A


TOTAL HEAT-BTU/LB
ENTROPY
PRESSURE-PSIA
MOISTURE-%
960
950
940
930
920
910
900
890
1.50
1.55
1.60
1.65
1" HG
1/2" HG
2" HG
2 1/2" HG
3" HG

9


TOTAL HEAT-BTU LB
ENTROPY
950
940
930
920
910
900
890
1.70
1.75
1.80
1.85
MOISTURE-%
PRESSURE-PSIA
2 1/2" HG
2" HG
1 1/2" HG
1" HG
3/4" HG
1/2" HG
1.4
1.3
1.2
1.1
1.0
.90
.80
.70
.60
.50
.40
.35
.30
.25
.20
.15
12
13
14
15
16
17
18
19

10-A


TOTAL HEAT-BTU LB
ENTROPY
890
880
870
860
850
840
830
820
1.55
1.60
1.65
1.70
PRESSURE-PSIA
MOISTURE-%
3" HG
2 1/2" HG
2" HG
1 1/2" HG
1" HG
3/4" HG
1/2" HG
2.4
2.2
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.90
.80
.70
.60
.50
.40
.35
.30
.25
.23
.20
19
20
21
22
24
25
26

3. FLOW OF STEAM IN PIPES

(See p. 1-10 and p. 6-35.)

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THERMODYNAMIC PROPERTIES OF STEAM, WATER, AND ICE

By Joseph Kaye and Joseph H. Keenan

Water in its diverse forms is used in industry and in other human endeavor more frequently than any other single substance. For this reason the properties of water have been studied in greater detail and with greater precision than those of most other substances. A wealth of data on water is available in many books, publications, and reference works.

The objective of this section is to provide the engineer with a coherent set of useful data in tabular form. The tables give the important thermodynamic properties of the vapor phase, of the liquid phase, and of the regions comprising equilibrium between liquid and vapor, and vapor and solid.

4. STEAM

SUPERHEATED VAPOR. Table 1 presents properties of steam or superheated water vapor as a function of temperature and pressure. The data in this table are based on values established by international agreement and represent the best values for steam. The table is condensed from *Thermodynamic Properties of Steam*, by J. H. Keenan and F. G. Keyes, John Wiley and Sons, 1936, by permission of the authors and the publisher.

In Table 1, the following symbols and units are used: h = enthalpy, Btu per pound; p = pressure, pounds per square inch, absolute; s = entropy, Btu per pound-°F; t = temperature, °F; and v = specific volume, cubic feet per pound.

EQUILIBRIUM OF LIQUID AND VAPOR. Tables 2 and 3 give the thermodynamic properties of equilibrium states for liquid and vapor. Temperature is the independent argument in Table 2 and pressure in Table 3. They are condensed from *Thermodynamic Properties of Steam*, by J. H. Keenan and F. G. Keyes, John Wiley and Sons (1936), by permission of the authors and publisher.

In Tables 2 and 3 the subscripts f, g, and fg refer to saturated liquid, saturated vapor, and to the difference between the saturated vapor and saturated liquid, respectively.

Table 1. Properties of Superheated Steam

(Condensed by permission from Keenan and Keyes, *Thermodynamic Properties of Steam*, Wiley, 1936)

Pressure, psia (Sat. Temp., °F)		Sat. Liquid	Sat. Vapor	Temperature—Degrees Fahrenheit																	
				140	160	180	200	220	240	260	280	300	400	500	600	700	800	900	1000	1100	1200
1 (101.74)	v	0.02	333.6	356.6	368.6	380.6	392.6	404.5	416.5	428.4	440.4	452.3	512.0	571.6	631.2	690.8	750.4	809.9	869.5	929.1	988.7
	h	69.7	1106.0	1123.3	1132.4	1141.4	1150.4	1159.5	1168.5	1177.6	1186.7	1195.8	1241.7	1288.3	1335.7	1383.8	1432.8	1482.7	1533.5	1585.2	1637.7
	s	0.1326	1.9782	2.0081	2.0230	2.0373	2.0512	2.0647	2.0779	2.0907	2.1031	2.1153	2.1720	2.2233	2.2702	2.3137	2.3542	2.3923	2.4283	2.4625	2.4952
2 (126.08)	v	0.02	173.7	177.96	184.01	190.04	196.06	202.1	208.1	214.1	220.0	226.0	255.9	285.7	315.5	345.4	375.1	404.9	434.7	464.5	494.3
	h	94.0	1116.2	1122.6	1131.8	1140.9	1150.0	1159.1	1168.2	1177.3	1186.5	1195.6	1241.6	1288.2	1335.6	1383.8	1432.8	1482.7	1533.5	1585.1	1637.7
	s	0.1749	1.9200	1.9308	1.9458	1.9603	1.9743	1.9879	2.0011	2.0140	2.0265	2.0387	2.0955	2.1468	2.1938	2.2372	2.2778	2.3159	2.3519	2.3861	2.4188
4 (152.97)	v	0.02	90.63		91.71	94.77	97.81	100.84	103.86	106.87	109.88	112.88	127.86	142.80	157.73	172.64	187.55	202.4	217.4	232.3	247.1
	h	120.9	1127.3		1130.6	1139.9	1149.2	1158.5	1167.6	1176.8	1186.0	1195.2	1241.3	1288.1	1335.5	1383.7	1432.7	1482.6	1533.4	1585.1	1637.7
	s	0.2198	1.8625		1.8678	1.8827	1.8969	1.9107	1.9240	1.9370	1.9496	1.9618	2.0189	2.0703	2.1173	2.1608	2.2013	2.2394	2.2755	2.3097	2.3424
6 (170.06)	v	0.02	61.98			63.00	65.05	67.09	69.12	71.14	73.16	75.17	85.18	95.16	105.12	115.07	125.01	134.95	144.89	154.82	164.76
	h	138.0	1134.2			1138.9	1148.3	1157.7	1167.0	1176.3	1185.5	1194.7	1241.1	1287.9	1335.3	1383.6	1432.6	1482.6	1533.4	1585.0	1637.6
	s	0.2472	1.8292			1.8367	1.8512	1.8651	1.8786	1.8917	1.9044	1.9167	1.9740	2.0255	2.0725	2.1160	2.1566	2.1947	2.2307	2.2650	2.2977
8 (182.86)	v	0.02	47.34				48.67	50.22	51.75	53.28	54.80	56.31	63.84	71.34	78.82	86.28	93.74	101.20	108.66	116.11	123.56
	h	150.8	1139.3				1147.5	1157.0	1166.3	1175.7	1185.0	1194.3	1240.8	1287.7	1335.2	1383.5	1432.6	1482.5	1533.3	1585.0	1637.6
	s	0.2674	1.8057				1.8184	1.8325	1.8462	1.8594	1.8721	1.8845	1.9420	1.9936	2.0407	2.0842	2.1248	2.1630	2.1990	2.2333	2.2660
10 (193.21)	v	0.02	38.42				38.85	40.09	41.33	42.56	43.78	45.00	51.04	57.05	63.03	69.01	74.98	80.95	86.92	92.88	98.84
	h	161.2	1143.3				1146.6	1156.2	1165.7	1175.1	1184.5	1193.9	1240.6	1287.5	1335.1	1383.4	1432.5	1482.4	1533.2	1585.0	1637.6
	s	0.2835	1.7876				1.7927	1.8071	1.8208	1.8341	1.8470	1.8595	1.9172	1.9689	2.0160	2.0596	2.1002	2.1383	2.1744	2.2086	2.2413
12 (201.96)	v	0.02	32.40					33.34	34.38	35.41	36.43	37.45	42.51	47.52	52.51	57.50	62.48	67.45	72.42	77.40	82.36
	h	170.0	1146.6					1155.4	1165.0	1174.6	1184.0	1193.4	1240.3	1287.3	1335.0	1383.3	1432.4	1482.4	1533.2	1584.9	1637.5
	s	0.2967	1.7730					1.7860	1.8000	1.8134	1.8264	1.8389	1.8969	1.9486	1.9958	2.0394	2.0800	2.1182	2.1543	2.1885	2.2212
14.696 (212.00)	v	0.02	26.80					27.15	28.00	28.85	29.70	30.53	34.68	38.78	42.86	46.94	51.00	55.07	59.13	63.19	67.25
	h	180.1	1150.4					1154.4	1164.2	1173.8	1183.3	1192.8	1239.9	1287.1	1334.8	1383.2	1432.3	1482.3	1533.1	1584.8	1637.5
	s	0.3120	1.7566					1.7624	1.7766	1.7902	1.8033	1.8160	1.8743	1.9261	1.9734	2.0170	2.0576	2.0958	2.1319	2.1662	2.1989
20 (227.96)	v	0.02	20.09						20.48	21.11	21.74	22.36	25.43	28.46	31.47	34.47	37.46	40.45	43.44	46.42	49.41
	h	196.2	1156.3						1162.3	1172.2	1182.0	1191.6	1239.2	1286.6	1334.4	1382.9	1432.1	1482.1	1533.0	1584.7	1637.4
	s	0.3356	1.7319						1.7405	1.7545	1.7679	1.7808	1.8396	1.8918	1.9392	1.9829	2.0235	2.0618	2.0978	2.1321	2.1648
30 (250.33)	v	0.017	13.746							13.957	14.390	14.816	16.897	18.933	20.95	22.96	24.96	26.95	28.95	30.94	32.93
	h	218.8	1164.1							1169.1	1179.3	1189.3	1237.9	1285.7	1333.8	1382.4	1431.7	1481.8	1532.7	1584.5	1637.2
	s	0.3680	1.6993							1.7063	1.7203	1.7336	1.7937	1.8464	1.8940	1.9379	1.9786	2.0169	2.0530	2.0873	2.1201
40 (267.25)	v	0.017	10.498								10.711	11.040	12.628	14.168	15.688	17.198	18.702	20.20	21.70	23.20	24.69
	h	236.0	1169.7								1176.5	1186.8	1236.5	1284.8	1333.1	1381.9	1431.3	1481.4	1532.4	1584.3	1637.0
	s	0.3919	1.6763								1.6855	1.6994	1.7608	1.8140	1.8619	1.9058	1.9467	1.9850	2.0212	2.0555	2.0883
50 (281.01)	v	0.017	8.515									8.773	10.065	11.309	12.532	13.744	14.950	16.152	17.352	18.550	19.747
	h	250.1	1174.1									1184.3	1235.1	1283.9	1332.5	1381.4	1430.9	1481.1	1532.1	1584.0	1636.8
	s	0.4110	1.6585									1.6721	1.7349	1.7887	1.8368	1.8809	1.9219	1.9602	1.9964	2.0308	2.0636
60 (292.71)	v	0.017	7.175									7.259	8.357	9.403	10.427	11.441	12.449	13.452	14.454	15.453	16.451
	h	262.1	1177.6									1181.6	1233.6	1283.0	1331.8	1380.9	1430.5	1480.8	1531.9	1583.8	1636.6
	s	0.4270	1.6438									1.6492	1.7135	1.7678	1.8162	1.8605	1.9015	1.9400	1.9762	2.0106	2.0434

Pressure, psia (Sat. Temp., °F)		Sat. Liquid	Sat. Vapor	Temperature—Degrees Fahrenheit																	
				340	360	380	400	420	440	460	480	500	600	700	750	800	850	900	1000	1200	1400
70 (302.92)	v	0.017	6.206	6.571	6.762	6.950	7.136	7.320	7.502	7.683	7.863	8.041	8.924	9.796	10.230	10.662	11.093	11.524	12.383	14.097	15.808
	h	272.6	1180.6	1201.0	1211.5	1221.9	1232.1	1242.3	1252.3	1262.2	1272.2	1282.0	1331.1	1380.4	1405.2	1430.1	1455.2	1480.5	1531.6	1636.3	1744.6
	s	0.4409	1.6315	1.6576	1.6707	1.6832	1.6952	1.7068	1.7181	1.7291	1.7397	1.7501	1.7988	1.8432	1.8641	1.8843	1.9039	1.9228	1.9591	2.0263	2.0879
80 (312.03)	v	0.018	5.472	5.718	5.888	6.055	6.220	6.383	6.544	6.704	6.862	7.020	7.797	8.562	8.942	9.322	9.700	10.077	10.830	12.332	13.830
	h	282.0	1183.1	1198.8	1209.7	1220.3	1230.7	1240.9	1251.1	1261.1	1271.1	1281.1	1330.5	1379.9	1404.7	1429.7	1454.8	1480.1	1531.3	1636.2	1744.5
	s	0.4531	1.6207	1.6407	1.6541	1.6669	1.6791	1.6909	1.7023	1.7134	1.7242	1.7346	1.7836	1.8281	1.8491	1.8694	1.8889	1.9079	1.9442	2.0115	2.0731
90 (320.27)	v	0.018	4.896	5.053	5.208	5.359	5.508	5.654	5.799	5.942	6.084	6.225	6.920	7.603	7.942	8.279	8.616	8.952	9.623	10.959	12.291
	h	290.6	1185.3	1196.6	1207.7	1218.6	1229.1	1239.5	1249.8	1260.0	1270.1	1280.1	1329.8	1379.4	1404.3	1429.3	1454.5	1479.8	1531.0	1635.9	1744.3
	s	0.4641	1.6112	1.6256	1.6393	1.6523	1.6648	1.6767	1.6883	1.6995	1.7103	1.7209	1.7702	1.8149	1.8359	1.8562	1.8758	1.8948	1.9311	1.9984	2.0601
100 (327.81)	v	0.018	4.432	4.521	4.663	4.801	4.937	5.071	5.202	5.333	5.462	5.589	6.218	6.835	7.141	7.446	7.749	8.052	8.656	9.860	11.060
	h	298.4	1187.2	1194.3	1205.7	1216.8	1227.6	1238.1	1248.6	1258.8	1269.0	1279.1	1329.1	1378.9	1403.8	1428.9	1454.1	1479.5	1530.8	1635.7	1744.2
	s	0.4740	1.6026	1.6117	1.6258	1.6391	1.6518	1.6639	1.6756	1.6869	1.6979	1.7085	1.7581	1.8029	1.8240	1.8443	1.8639	1.8829	1.9193	1.9867	2.0484
110 (334.77)	v	0.018	4.049	4.085	4.216	4.345	4.470	4.593	4.714	4.834	4.952	5.069	5.644	6.207	6.486	6.763	7.040	7.316	7.866	8.961	10.053
	h	305.7	1188.9	1192.0	1203.7	1215.0	1226.0	1236.7	1247.3	1257.7	1268.0	1278.2	1328.4	1378.3	1403.4	1428.5	1453.7	1479.2	1530.5	1635.5	1744.0
	s	0.4832	1.5948	1.5988	1.6133	1.6269	1.6398	1.6521	1.6640	1.6754	1.6865	1.6972	1.7471	1.7921	1.8132	1.8336	1.8532	1.8723	1.9087	1.9761	2.0378
120 (341.25)	v	0.018	3.728	.....	3.844	3.964	4.081	4.195	4.307	4.418	4.527	4.636	5.165	5.683	5.940	6.195	6.449	6.702	7.207	8.212	9.214
	h	312.4	1190.4	.....	1201.6	1213.2	1224.4	1235.3	1246.0	1256.5	1266.9	1277.2	1327.7	1377.8	1402.9	1428.1	1453.4	1478.8	1530.2	1635.3	1743.9
	s	0.4916	1.5878	.....	1.6017	1.6156	1.6287	1.6413	1.6533	1.6649	1.6760	1.6869	1.7370	1.7822	1.8033	1.8237	1.8434	1.8625	1.8990	1.9664	2.0281
130 (347.32)	v	0.018	3.455	.....	3.529	3.641	3.751	3.858	3.963	4.066	4.168	4.269	4.761	5.240	5.478	5.714	5.949	6.183	6.650	7.579	8.503
	h	318.8	1191.7	.....	1199.5	1211.3	1222.7	1233.8	1244.6	1255.3	1265.8	1276.2	1327.0	1377.3	1402.5	1427.7	1453.0	1478.5	1529.9	1635.1	1743.7
	s	0.4995	1.5812	.....	1.5908	1.6050	1.6184	1.6312	1.6433	1.6550	1.6663	1.6773	1.7277	1.7730	1.7942	1.8147	1.8344	1.8535	1.8900	1.9575	2.0193
140 (353.02)	v	0.018	3.220	.....	3.258	3.365	3.468	3.569	3.667	3.764	3.860	3.954	4.413	4.861	5.082	5.301	5.520	5.738	6.172	7.035	7.895
	h	324.8	1193.0	.....	1197.3	1209.4	1221.1	1232.3	1243.3	1254.1	1264.7	1275.2	1326.4	1376.8	1402.0	1427.3	1452.6	1478.2	1529.7	1634.9	1743.5
	s	0.5069	1.5751	.....	1.5804	1.5950	1.6087	1.6217	1.6340	1.6458	1.6573	1.6683	1.7190	1.7645	1.7858	1.8063	1.8260	1.8451	1.8817	1.9493	2.0110
150 (358.42)	v	0.018	3.015	.....	3.023	3.124	3.223	3.318	3.411	3.502	3.592	3.681	4.113	4.532	4.738	4.944	5.148	5.352	5.758	6.564	7.367
	h	330.5	1194.1	.....	1195.1	1207.5	1219.4	1230.8	1242.0	1252.9	1263.6	1274.1	1325.7	1376.3	1401.5	1426.9	1452.3	1477.8	1529.4	1634.7	1743.4
	s	0.5138	1.5694	.....	1.5706	1.5856	1.5995	1.6127	1.6252	1.6372	1.6487	1.6599	1.7109	1.7566	1.7779	1.7984	1.8182	1.8374	1.8740	1.9416	2.0034
160 (363.53)	v	0.018	2.834	.....	.....	2.914	3.008	3.098	3.187	3.273	3.359	3.443	3.849	4.244	4.438	4.631	4.823	5.015	5.396	6.152	6.906
	h	335.9	1195.1	.....	.....	1205.5	1217.6	1229.3	1240.6	1251.6	1262.4	1273.1	1325.0	1375.7	1401.1	1426.4	1451.9	1477.5	1529.1	1634.5	1743.2
	s	0.5204	1.5640	.....	.....	1.5766	1.5908	1.6042	1.6169	1.6291	1.6407	1.6519	1.7033	1.7491	1.7705	1.7911	1.8109	1.8301	1.8667	1.9344	1.9962
170 (368.41)	v	0.018	2.675	.....	.....	2.728	2.818	2.904	2.989	3.071	3.152	3.232	3.617	3.990	4.173	4.355	4.536	4.717	5.076	5.789	6.498
	h	341.1	1196.0	.....	.....	1203.5	1215.8	1227.7	1239.2	1250.4	1261.3	1272.1	1324.2	1375.2	1400.6	1426.0	1451.5	1477.2	1528.8	1634.3	1743.1
	s	0.5266	1.5590	.....	.....	1.5679	1.5825	1.5961	1.6090	1.6213	1.6331	1.6444	1.6962	1.7421	1.7636	1.7842	1.8040	1.8232	1.8599	1.9276	1.9894
180 (373.06)	v	0.018	2.532	.....	.....	2.563	2.649	2.732	2.813	2.891	2.969	3.044	3.411	3.764	3.937	4.110	4.281	4.452	4.792	5.466	6.136
	h	346.0	1196.9	.....	.....	1201.4	1214.0	1226.1	1237.8	1249.1	1260.2	1271.0	1323.5	1374.7	1400.2	1425.6	1451.2	1476.8	1528.6	1634.1	1742.9
	s	0.5325	1.5542	.....	.....	1.5596	1.5745	1.5884	1.6015	1.6139	1.6258	1.6373	1.6894	1.7355	1.7570	1.7776	1.7975	1.8167	1.8534	1.9212	1.9831
190 (377.51)	v	0.018	2.404	.....	.....	2.414	2.498	2.577	2.655	2.730	2.804	2.877	3.226	3.562	3.727	3.891	4.053	4.215	4.537	5.177	5.812
	h	350.8	1197.6	.....	.....	1199.3	1212.2	1224.5	1236.3	1247.8	1259.0	1270.0	1322.8	1374.2	1399.7	1425.2	1450.8	1476.5	1528.3	1633.9	1742.8
	s	0.5381	1.5497	.....	.....	1.5517	1.5668	1.5810	1.5943	1.6069	1.6189	1.6305	1.6829	1.7292	1.7507	1.7714	1.7913	1.8106	1.8473	1.9151	1.9770
200 (381.79)	v	0.018	2.288	.....	.....	.....	2.361	2.438	2.513	2.585	2.656	2.726	3.060	3.380	3.537	3.693	3.848	4.002	4.309	4.917	5.521
	h	355.4	1198.4	.....	.....	.....	1210.3	1222.9	1234.9	1246.5	1257.8	1268.9	1322.1	1373.6	1399.2	1424.8	1450.4	1476.2	1528.0	1633.7	1742.6
	s	0.5435	1.5453	.....	.....	.....	1.5594	1.5738	1.5873	1.6001	1.6123	1.6240	1.6767	1.7232	1.7448	1.7655	1.7855	1.8048	1.8415	1.9094	1.9713

(Table continued on p. 4-32)

Table 1. Properties of Superheated Steam—*Continued*

Pressure, psia (Sat. Temp., °F)		Sat. Liquid	Sat. Vapor	400	420	440	460	480	500	520	540	560	580	600	700	800	900	1000	1200	1400	1600
				Temperature—Degrees Fahrenheit																	
220	v	0.019	2.087	2.125	2.198	2.267	2.335	2.400	2.465	2.528	2.590	2.652	2.712	2.772	3.066	3.352	3.634	3.913	4.467	5.017	5.565
(389.86)	h	364.0	1199.6	1206.5	1219.5	1231.9	1243.8	1255.4	1266.7	1277.8	1288.7	1299.5	1310.1	1320.7	1372.6	1424.0	1475.5	1527.5	1633.3	1742.3	1854.7
	s	0.5537	1.5372	1.5433	1.5603	1.5742	1.5874	1.5998	1.6117	1.6231	1.6342	1.6448	1.6551	1.6652	1.7120	1.7545	1.7939	1.8308	1.8987	1.9607	2.0181
240	v	0.0186	1.9183	1.9276	1.9964	2.062	2.126	2.187	2.247	2.306	2.364	2.421	2.477	2.533	2.804	3.068	3.327	3.584	4.093	4.597	5.100
(397.37)	h	372.1	1200.6	1202.5	1216.0	1228.8	1241.1	1253.0	1264.5	1275.8	1286.9	1297.8	1308.5	1319.2	1371.5	1423.2	1474.8	1526.9	1632.9	1742.0	1854.5
	s	0.5631	1.5298	1.5319	1.5475	1.5619	1.5754	1.5882	1.6003	1.6120	1.6232	1.6339	1.6444	1.6546	1.7017	1.7444	1.7839	1.8209	1.8889	1.9510	2.0084
260	v	0.0187	1.7748	......	1.8257	1.8882	1.9483	2.006	2.063	2.118	2.172	2.226	2.278	2.330	2.582	2.827	3.067	3.305	3.776	4.242	4.707
(404.42)	h	379.8	1201.5	...	1212.4	1225.7	1238.3	1250.5	1262.3	1273.8	1285.0	1296.0	1306.9	1317.7	1370.4	1422.3	1474.2	1526.3	1632.5	1741.7	1854.2
	s	0.5719	1.5229	...	1.5354	1.5503	1.5642	1.5773	1.5897	1.6015	1.6129	1.6238	1.6344	1.6447	1.6922	1.7352	1.7748	1.8118	1.8799	1.9420	1.9995
280	v	0.0188	1.6511	...	1.6789	1.7388	1.7960	1.8512	1.9047	1.9569	2.008	2.058	2.107	2.156	2.392	2.621	2.845	3.066	3.504	3.938	4.370
(411.05)	h	387.0	1202.3	...	1208.7	1222.4	1235.4	1247.9	1260.0	1271.7	1283.1	1294.3	1305.3	1316.2	1369.4	1421.5	1473.5	1525.8	1632.1	1741.4	1854.0
	s	0.5801	1.5164	...	1.5238	1.5392	1.5536	1.5670	1.5796	1.5917	1.6032	1.6143	1.6251	1.6354	1.6834	1.7265	1.7662	1.8033	1.8716	1.9337	1.9912
300	v	0.0189	1.5433	...	1.5513	1.6090	1.6638	1.7165	1.7675	1.8170	1.8654	1.9128	1.9594	2.005	2.227	2.442	2.652	2.859	3.269	3.674	4.078
(417.33)	h	393.8	1202.8	...	1204.8	1219.1	1232.5	1245.3	1257.6	1269.5	1281.1	1292.5	1303.7	1314.7	1368.3	1420.6	1472.8	1525.2	1631.7	1741.0	1853.7
	s	0.5879	1.5104	...	1.5126	1.5286	1.5434	1.5572	1.5701	1.5824	1.5941	1.6054	1.6163	1.6268	1.6751	1.7184	1.7582	1.7954	1.8638	1.9260	1.9835
320	v	0.0190	1.4485	...	...	1.4950	1.5479	1.5985	1.6472	1.6945	1.7406	1.7856	1.8298	1.8734	2.083	2.285	2.483	2.678	3.063	3.443	3.822
(423.29)	h	400.4	1203.4	...	...	1215.6	1229.5	1242.6	1255.2	1267.4	1279.2	1290.7	1302.0	1313.2	1367.2	1419.8	1472.1	1524.7	1631.3	1740.7	1853.5
	s	0.5952	1.5046	...	...	1.5184	1.5337	1.5478	1.5611	1.5736	1.5855	1.5969	1.6079	1.6186	1.6673	1.7108	1.7508	1.7880	1.8565	1.9187	1.9763
340	v	0.0191	1.3645	...	...	1.3941	1.4454	1.4941	1.5410	1.5863	1.6303	1.6733	1.7155	1.7569	1.9562	2.147	2.334	2.518	2.881	3.240	3.596
(428.97)	h	406.7	1203.7	...	...	1212.1	1226.4	1239.9	1252.8	1265.2	1277.2	1288.9	1300.4	1311.6	1366.1	1419.0	1471.5	1524.1	1630.9	1740.4	1853.2
	s	0.6022	1.4992	...	...	1.5085	1.5243	1.5388	1.5524	1.5652	1.5773	1.5889	1.6000	1.6108	1.6599	1.7036	1.7437	1.7811	1.8496	1.9119	1.9695
360	v	0.0192	1.2895	...	...	1.3041	1.3539	1.4012	1.4464	1.4900	1.5322	1.5734	1.6137	1.6533	1.8431	2.025	2.202	2.376	2.719	3.059	3.396
(434.40)	h	412.7	1204.1	...	...	1208.4	1223.2	1237.1	1250.3	1263.0	1275.2	1287.1	1298.7	1310.1	1365.0	1418.1	1470.8	1523.5	1630.5	1740.1	1853.0
	s	0.6090	1.4941	...	...	1.4989	1.5152	1.5301	1.5440	1.5571	1.5694	1.5812	1.5925	1.6033	1.6529	1.6968	1.7371	1.7745	1.8431	1.9055	1.9631
380	v	0.0192	1.2222	...	...	1.2232	1.2719	1.3178	1.3616	1.4037	1.4444	1.4840	1.5226	1.5605	1.7419	1.9149	2.083	2.249	2.575	2.897	3.217
(439.60)	h	418.5	1204.3	...	...	1204.6	1219.9	1234.2	1247.7	1260.7	1273.1	1285.2	1297.0	1308.5	1363.8	1417.3	1470.1	1523.0	1630.0	1739.8	1852.7
	s	0.6153	1.4891	...	...	1.4895	1.5063	1.5217	1.5359	1.5493	1.5618	1.5738	1.5852	1.5962	1.6462	1.6904	1.7307	1.7683	1.8370	1.8994	1.9571
400	v	0.0193	1.1613	...	...	...	1.1978	1.2426	1.2851	1.3259	1.3652	1.4034	1.4406	1.4770	1.6508	1.8161	1.9767	2.134	2.445	2.751	3.055
(444.59)	h	424.0	1204.5	...	...	...	1216.5	1231.3	1245.1	1258.3	1271.0	1283.3	1295.2	1306.9	1362.7	1416.4	1469.4	1522.4	1629.6	1739.5	1852.5
	s	0.6214	1.4844	...	...	...	1.4977	1.5135	1.5281	1.5417	1.5546	1.5667	1.5783	1.5894	1.6398	1.6842	1.7247	1.7623	1.8311	1.8936	1.9513
500	v	0.0197	0.9278	...	...	...	...	0.9543	0.9927	1.0289	1.0633	1.0963	1.1282	1.1591	1.3044	1.4405	1.5715	1.6996	1.9504	2.197	2.442
(467.01)	h	449.4	1204.4	...	...	...	...	1215.4	1231.3	1246.0	1260.0	1273.4	1286.2	1298.6	1357.0	1412.1	1466.0	1519.6	1627.6	1737.9	1851.3
	s	0.6487	1.4634	...	...	...	...	1.4752	1.4919	1.5072	1.5213	1.5345	1.5470	1.5588	1.6115	1.6571	1.6982	1.7363	1.8056	1.8683	1.9262
600	v	0.0201	0.7698	...	...	...	...	...	0.7947	0.8286	0.8602	0.8901	0.9187	0.9463	1.0732	1.1899	1.3013	1.4096	1.6208	1.8279	2.033
(486.21)	h	471.6	1203.2	...	...	...	...	...	1215.7	1232.5	1248.1	1262.7	1276.6	1289.9	1351.1	1407.7	1462.5	1516.7	1625.5	1736.3	1850.0
	s	0.6720	1.4454	...	...	...	...	...	1.4586	1.4759	1.4916	1.5061	1.5196	1.5323	1.5875	1.6343	1.6762	1.7147	1.7846	1.8476	1.9056
700	v	0.0205	0.6554	...	...	...	...	...	...	0.6832	0.7134	0.7416	0.7681	0.7934	0.9077	1.0108	1.1082	1.2024	1.3853	1.5641	1.7405
(503.10)	h	491.5	1201.2	...	...	...	...	...	...	1217.5	1235.0	1251.1	1266.3	1280.6	1345.0	1403.2	1459.0	1513.9	1623.5	1734.8	1848.8
	s	0.6925	1.4296	...	...	...	...	...	...	1.4463	1.4640	1.4800	1.4947	1.5084	1.5665	1.6147	1.6573	1.6963	1.7666	1.8299	1.8881
800	v	0.0209	0.5687	...	...	...	...	...	...	0.5715	0.6015	0.6288	0.6541	0.6779	0.7833	0.8763	0.9633	1.0470	1.2088	1.3662	1.5214
(518.23)	h	509.7	1198.6	...	...	...	...	...	...	1200.5	1220.5	1238.6	1255.2	1270.7	1338.6	1398.6	1455.4	1511.0	1621.4	1733.2	1847.5
	s	0.7108	1.4153	...	...	...	...	...	...	1.4172	1.4375	1.4553	1.4715	1.4863	1.5476	1.5972	1.6407	1.6801	1.7510	1.8146	1.8729

Pressure, psia (Sat. Temp., °F)		Sat. Liquid	Sat. Vapor	Temperature—Degrees Fahrenheit																	
				560	580	600	620	640	660	680	700	720	740	760	780	800	900	1000	1200	1400	1600
900 (531.98)	v	0.0212	0.5006	0.5396	0.5644	0.5873	0.6089	0.6294	0.6491	0.6680	0.6863	0.7041	0.7215	0.7385	0.7552	0.7716	0.8506	0.9262	1.0714	1.2124	1.3509
	h	526.6	1195.4	1224.9	1243.2	1260.1	1275.9	1290.9	1305.1	1318.8	1332.1	1345.0	1357.5	1369.9	1382.0	1393.9	1451.8	1508.1	1619.3	1731.6	1846.3
	s	0.7275	1.4020	1.4313	1.4492	1.4653	1.4800	1.4938	1.5066	1.5187	1.5303	1.5413	1.5519	1.5620	1.5719	1.5814	1.6257	1.6656	1.7371	1.8009	1.8595
1000 (544.61)	v	0.0216	0.4456	0.4668	0.4915	0.5140	0.5350	0.5546	0.5733	0.5912	0.6084	0.6251	0.6413	0.6571	0.6726	0.6878	0.7604	0.8294	0.9615	1.0893	1.2146
	h	542.4	1191.8	1209.7	1230.3	1248.8	1265.9	1281.9	1297.0	1311.4	1325.3	1338.7	1351.7	1364.4	1376.9	1389.2	1448.2	1505.1	1617.3	1730.0	1845.0
	s	0.7430	1.3897	1.4073	1.4273	1.4450	1.4610	1.4757	1.4893	1.5021	1.5141	1.5256	1.5365	1.5470	1.5572	1.5670	1.6121	1.6525	1.7245	1.7886	1.8474
1100 (556.31)	v	0.0220	0.4001	0.4053	0.4307	0.4532	0.4738	0.4929	0.5110	0.5281	0.5445	0.5602	0.5755	0.5904	0.6049	0.6191	0.6866	0.7503	0.8716	0.9885	1.1031
	h	557.4	1187.8	1192.6	1216.1	1236.7	1255.3	1272.4	1288.5	1303.7	1318.3	1332.2	1345.8	1358.9	1371.7	1384.3	1444.5	1502.2	1615.2	1728.4	1843.8
	s	0.7575	1.3780	1.3827	1.4055	1.4251	1.4425	1.4583	1.4728	1.4862	1.4989	1.5108	1.5222	1.5330	1.5435	1.5535	1.5995	1.6405	1.7130	1.7775	1.8363
1200 (567.22)	v	0.0223	0.3619	......	0.3786	0.4016	0.4222	0.4410	0.4586	0.4752	0.4909	0.5060	0.5206	0.5347	0.5484	0.5617	0.6250	0.6843	0.7967	0.9046	1.0101
	h	571.7	1183.4	......	1200.4	1223.5	1243.9	1262.4	1279.6	1295.7	1311.0	1325.6	1339.6	1353.2	1366.4	1379.3	1440.7	1499.2	1613.1	1726.9	1842.5
	s	0.7711	1.3667	......	1.3832	1.4052	1.4243	1.4413	1.4568	1.4710	1.4843	1.4968	1.5086	1.5198	1.5306	1.5409	1.5879	1.6293	1.7025	1.7672	1.8263
1300 (577.46)	v	0.0227	0.3293	......	0.3328	0.3569	0.3778	0.3966	0.4139	0.4301	0.4454	0.4600	0.4739	0.4874	0.5004	0.5131	0.5728	0.6284	0.7333	0.8336	0.9314
	h	585.4	1178.6	......	1182.4	1209.0	1231.6	1251.8	1270.2	1287.3	1303.4	1318.6	1333.3	1347.3	1361.0	1374.3	1437.0	1496.2	1611.0	1725.3	1841.3
	s	0.7840	1.3559	......	1.3596	1.3850	1.4061	1.4246	1.4412	1.4563	1.4703	1.4834	1.4957	1.5073	1.5184	1.5290	1.5769	1.6190	1.6927	1.7577	1.8169
1400 (587.10)	v	0.0231	0.3012	......	......	0.3174	0.3390	0.3580	0.3753	0.3912	0.4062	0.4203	0.4338	0.4468	0.4593	0.4714	0.5281	0.5805	0.6789	0.7727	0.8640
	h	598.7	1173.4	......	......	1193.0	1218.4	1240.4	1260.3	1278.5	1295.5	1311.5	1326.7	1341.3	1355.4	1369.1	1433.1	1493.2	1608.9	1723.7	1840.0
	s	0.7963	1.3454	......	......	1.3639	1.3877	1.4079	1.4258	1.4419	1.4567	1.4704	1.4832	1.4953	1.5068	1.5177	1.5666	1.6093	1.6836	1.7489	1.8083
1500 (596.23)	v	0.0235	0.2765	......	......	0.2815	0.3045	0.3240	0.3413	0.3572	0.3719	0.3857	0.3989	0.4114	0.4235	0.4352	0.4893	0.5390	0.6318	0.7199	0.8056
	h	611.6	1167.9	......	......	1174.5	1203.9	1228.3	1249.8	1269.3	1287.2	1304.1	1320.0	1335.2	1349.7	1363.8	1429.3	1490.1	1606.8	1722.1	1838.8
	s	0.8082	1.3351	......	......	1.3412	1.3688	1.3912	1.4105	1.4278	1.4434	1.4578	1.4712	1.4837	1.4956	1.5068	1.5569	1.6001	1.6750	1.7406	1.8002
2000 (635.82)	v	0.0257	0.1878	......	......	......	......	0.1936	0.2161	0.2337	0.2489	0.2624	0.2748	0.2863	0.2972	0.3074	0.3532	0.3935	0.4668	0.5352	0.6011
	h	671.7	1135.1	......	......	......	......	1145.6	1184.9	1214.8	1240.0	1262.3	1282.6	1301.4	1319.0	1335.5	1409.2	1474.5	1596.1	1714.1	1832.5
	s	0.8619	1.2849	......	......	......	......	1.2945	1.3300	1.3564	1.3783	1.3974	1.4145	1.4300	1.4443	1.4576	1.5139	1.5603	1.6384	1.7055	1.7660
2500 (668.13)	v	0.0287	0.1307	......	......	......	......	......	......	0.1484	0.1686	0.1841	0.1973	0.2090	0.2196	0.2294	0.2710	0.3061	0.3678	0.4244	0.4784
	h	730.6	1091.1	......	......	......	......	......	......	1132.3	1176.8	1210.0	1237.6	1261.8	1283.6	1303.6	1387.8	1458.4	1585.3	1706.1	1826.2
	s	0.9126	1.2322	......	......	......	......	......	......	1.2687	1.3073	1.3357	1.3589	1.3789	1.3967	1.4127	1.4772	1.5273	1.6088	1.6775	1.7389
3000 (695.36)	v	0.0346	0.0858	......	......	......	......	......	......	......	0.0984	0.1251	0.1416	0.1548	0.1660	0.1760	0.2159	0.2476	0.3018	0.3505	0.3966
	h	802.5	1020.3	......	......	......	......	......	......	......	1060.7	1136.1	1180.1	1213.8	1242.2	1267.2	1365.0	1441.8	1574.3	1698.0	1819.9
	s	0.9731	1.1615	......	......	......	......	......	......	......	1.1966	1.2611	1.2981	1.3259	1.3490	1.3690	1.4439	1.4984	1.5837	1.6540	1.7163
3206.2 (705.40)	v	0.0503	0.0503	......	......	......	......	......	......	......	......	0.1020	0.1220	0.1363	0.1480	0.1583	0.1981	0.2288	0.2806	0.3267	0.3703
	h	902.7	902.7	......	......	......	......	......	......	......	......	1090.7	1150.2	1190.6	1222.9	1250.5	1355.2	1434.7	1569.8	1694.6	1817.2
	s	1.0580	1.0580	......	......	......	......	......	......	......	......	1.2190	1.2690	1.3024	1.3287	1.3508	1.4309	1.4874	1.5742	1.6452	1.7080
4000	v	......	......	......	......	......	......	......	......	......	0.0287	0.0328	0.0478	0.0761	0.0927	0.1052	0.1462	0.1743	0.2192	0.2581	0.2943
	h	......	......	......	......	......	......	......	......	......	763.8	817.5	930.7	1064.2	1129.8	1174.8	1314.4	1406.8	1552.1	1681.7	1807.2
	s	......	......	......	......	......	......	......	......	......	0.9347	0.9806	1.0757	1.1862	1.2397	1.2757	1.3827	1.4482	1.5417	1.6154	1.6795
5000	v	......	......	......	......	......	......	......	......	......	0.0268	0.0287	0.0316	0.0368	0.0465	0.0593	0.1036	0.1303	0.1696	0.2027	0.2329
	h	......	......	......	......	......	......	......	......	......	746.4	783.9	828.4	887.8	966.2	1047.1	1256.5	1369.5	1529.5	1665.3	1794.5
	s	......	......	......	......	......	......	......	......	......	0.9152	0.9472	0.9847	1.0337	1.0975	1.1622	1.3231	1.4034	1.5066	1.5839	1.6499

Table 2. Saturation: Temperature Table

(Condensed by permission from Keenan and Keyes, *Thermodynamic Properties of Steam*, Wiley, 1936)

Temp., Fahr., t	Pressure		Specific Volume			Enthalpy			Entropy		
	psia, p	In. Hg	Sat. Liquid, v_f	Evap., v_{fg}	Sat. Vapor, v_g	Sat. Liquid, h_f	Evap., h_{fg}	Sat. Vapor, h_g	Sat., Liquid, s_f	Evap., s_{fg}	Sat. Vapor, s_g
32	0.08854	0.1803	0.01602	3306	3306	0.00	1075.8	1075.8	0.0000	2.1877	2.1877
35	0.09995	0.2035	0.01602	2947	2947	3.02	1074.1	1077.1	0.0061	2.1709	2.1770
40	0.12170	0.2478	0.01602	2444	2444	8.05	1071.3	1079.3	0.0162	2.1435	2.1597
45	0.14752	0.3004	0.01602	2036.4	2036.4	13.06	1068.4	1081.5	0.0262	2.1167	2.1429
50	0.17811	0.3626	0.01603	1703.2	1703.2	18.07	1065.6	1083.7	0.0361	2.0903	2.1264
55	0.2141	0.4359	0.01603	1430.7	1430.7	23.07	1062.7	1085.8	0.0459	2.0645	2.1104
60	0.2563	0.5218	0.01604	1206.6	1206.7	28.06	1059.9	1088.0	0.0555	2.0393	2.0948
65	0.3056	0.6222	0.01605	1021.4	1021.4	33.05	1057.1	1090.2	0.0651	2.0145	2.0796
70	0.3631	0.7392	0.01606	867.8	867.9	38.04	1054.3	1092.3	0.0745	1.9902	2.0647
75	0.4298	0.8750	0.01607	740.0	740.0	43.03	1051.5	1094.5	0.0839	1.9663	2.0502
80	0.5069	1.0321	0.01608	633.1	633.1	48.02	1048.6	1096.6	0.0932	1.9428	2.0360
85	0.5959	1.2133	0.01609	543.4	543.5	53.00	1045.8	1098.8	0.1024	1.9198	2.0222
90	0.6982	1.4215	0.01610	468.0	468.0	57.99	1042.9	1100.9	0.1115	1.8972	2.0087
95	0.8153	1.6600	0.01612	404.3	404.3	62.98	1040.1	1103.1	0.1205	1.8750	1.9955
100	0.9492	1.9325	0.01613	350.3	350.4	67.97	1037.2	1105.2	0.1295	1.8531	1.9826
105	1.1016	2.2429	0.01615	304.5	304.5	72.95	1034.3	1107.3	0.1383	1.8317	1.9700
110	1.2748	2.5955	0.01617	265.3	265.4	77.94	1031.6	1109.5	0.1471	1.8106	1.9577
115	1.4709	2.9948	0.01618	231.9	231.9	82.93	1028.7	1111.6	0.1559	1.7898	1.9457
120	1.6924	3.4458	0.01620	203.25	203.27	87.92	1025.8	1113.7	0.1645	1.7694	1.9339
125	1.9420	3.9539	0.01622	178.59	178.61	92.91	1022.9	1115.8	0.1731	1.7493	1.9224
130	2.2225	4.5251	0.01625	157.32	157.34	97.90	1020.0	1117.9	0.1816	1.7296	1.9112
135	2.5370	5.1653	0.01627	138.93	138.95	102.90	1017.0	1119.9	0.1900	1.7102	1.9002
140	2.8886	5.8812	0.01629	122.99	123.01	107.89	1014.1	1122.0	0.1984	1.6910	1.8894
145	3.281	6.680	0.01632	109.13	109.15	112.89	1011.2	1124.1	0.2066	1.6722	1.8788
150	3.718	7.569	0.01634	97.06	97.07	117.89	1008.2	1126.1	0.2149	1.6537	1.8685
155	4.203	8.557	0.01637	86.51	86.52	122.89	1005.2	1128.1	0.2230	1.6354	1.8584
160	4.741	9.652	0.01639	77.27	77.29	127.89	1002.3	1130.2	0.2311	1.6174	1.8485
165	5.335	10.863	0.01642	69.17	69.19	132.89	999.3	1132.2	0.2392	1.5997	1.8388
170	5.992	12.199	0.01645	62.04	62.06	137.90	996.3	1134.2	0.2472	1.5822	1.8293
175	6.715	13.671	0.01648	55.76	55.78	142.91	993.3	1136.2	0.2551	1.5649	1.8200
180	7.510	15.291	0.01651	50.21	50.23	147.92	990.2	1138.1	0.2630	1.5480	1.8109
185	8.383	17.068	0.01654	45.29	45.31	152.93	987.2	1140.1	0.2708	1.5312	1.8020
190	9.339	19.014	0.01657	40.94	40.96	157.95	984.1	1142.0	0.2785	1.5147	1.7932
195	10.385	21.144	0.01660	37.07	37.09	162.97	981.0	1144.0	0.2862	1.4984	1.7846
200	11.526	23.467	0.01663	33.62	33.64	167.99	977.9	1145.9	0.2938	1.4824	1.7762
210	14.123	28.755	0.01670	27.80	27.82	178.05	971.6	1149.7	0.3090	1.4508	1.7598
212	14.696	29.922	0.01672	26.78	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566
220	17.186	34.992	0.01677	23.13	23.15	188.13	965.2	1153.4	0.3239	1.4201	1.7440
230	20.780	42.308	0.01684	19.365	19.382	198.23	958.8	1157.0	0.3387	1.3901	1.7288
240	24.969	50.837	0.01692	16.306	16.323	208.34	952.2	1160.5	0.3531	1.3609	1.7140
250	29.825	60.725	0.01700	13.804	13.821	218.48	945.5	1164.0	0.3675	1.3323	1.6998
260	35.429	72.134	0.01709	11.746	11.763	228.64	938.7	1167.3	0.3817	1.3043	1.6860
270	41.858	85.225	0.01717	10.044	10.061	238.84	931.8	1170.6	0.3958	1.2769	1.6727
280	49.203	100.18	0.01726	8.628	8.645	249.06	924.7	1173.8	0.4096	1.2501	1.6597
290	57.556	117.19	0.01735	7.444	7.461	259.31	917.5	1176.8	0.4234	1.2238	1.6472
300	67.013	136.44	0.01745	6.449	6.466	269.59	910.1	1179.7	0.4369	1.1980	1.6350
310	77.68	....	0.01755	5.609	5.626	279.92	902.6	1182.5	0.4504	1.1727	1.6231
320	89.66	....	0.01765	4.896	4.914	290.28	894.9	1185.2	0.4637	1.1478	1.6115
330	103.06	....	0.01776	4.289	4.307	300.68	887.0	1187.7	0.4769	1.1233	1.6002
340	118.01	....	0.01787	3.770	3.788	311.13	879.0	1190.1	0.4900	1.0992	1.5891
350	134.63	....	0.01799	3.324	3.342	321.63	870.7	1192.3	0.5029	1.0754	1.5783
360	153.04	....	0.01811	2.939	2.957	332.18	862.2	1194.4	0.5158	1.0519	1.5677
370	173.37	....	0.01823	2.606	2.625	342.79	853.5	1196.3	0.5286	1.0287	1.5573
380	195.77	....	0.01836	2.317	2.335	353.45	844.6	1198.1	0.5413	1.0059	1.5471
390	220.37	....	0.01850	2.0651	2.0836	364.17	835.4	1199.6	0.5539	0.9832	1.5371

Table 2. Saturation: Temperature Table—*Continued*

(Condensed by permission from Keenan and Keyes, *Thermodynamic Properties of Steam*, Wiley, 1936)

Temp., Fahr., t	Pressure		Specific Volume			Enthalpy			Entropy		
	psia, p	In. Hg	Sat. Liquid, v_f	Evap., v_{fg}	Sat. Vapor, v_g	Sat. Liquid, h_f	Evap., h_{fg}	Sat. Vapor, h_g	Sat., Liquid, s_f	Evap., s_{fg}	Sat. Vapor, s_g
400	247.31	......	0.01864	1.8447	1.8633	374.97	826.0	1201.0	0.5664	0.9608	1.5272
410	276.75	......	0.01878	1.6512	1.6700	385.83	816.3	1202.1	0.5788	0.9386	1.5174
420	308.83	.....	0.01894	1.4811	1.5000	396.77	806.3	1203.1	0.5912	0.9166	1.5078
430	343.72	.....	0.01910	1.3308	1.3499	407.79	796.0	1203.8	0.6035	0.8947	1.4982
440	381.59	.....	0.01926	1.1979	1.2171	418.90	785.4	1204.3	0.6158	0.8730	1.4887
450	422.6	.....	0.0194	1.0799	1.0993	430.1	774.5	1204.6	0.6280	0.8513	1.4793
460	466.9	....	0.0196	0.9748	0.9944	441.4	763.2	1204.6	0.6402	0.8298	1.4700
470	514.7	.	0.0198	0.8811	0.9009	452.8	751.5	1204.3	0.6523	0.8083	1.4606
480	566.1	...	0.0200	0.7972	0.8172	464.4	739.4	1203.7	0.6645	0.7868	1.4513
490	621.4	.....	0.0202	0.7221	0.7423	476.0	726.8	1202.8	0.6766	0.7653	1.4419
500	680.8	...	0.0204	0.6545	0.6749	487.8	713.9	1201.7	0.6887	0.7438	1.4325
510	744.3	....	0.0207	0.5935	0.6142	499.8	700.3	1200.1	0.7008	0.7223	1.4231
520	812.4	..	0.0209	0.5385	0.5594	511.9	686.4	1198.2	0.7130	0.7006	1.4136
530	885.0	..	0.0212	0.4886	0.5098	524.1	671.8	1195.9	0.7252	0.6788	1.4040
540	962.5	.....	0.0215	0.4434	0.4649	536.6	656.6	1193.2	0.7374	0.6568	1.3942
550	1045.2	...	0.0218	0.4022	0.4240	549.3	640.8	1190.0	0.7497	0.6346	1.3843
560	1133.1	......	0.0221	0.3647	0.3868	562.2	624.2	1186.4	0.7621	0.6121	1.3742
570	1226.5	...	0.0224	0.3304	0.3528	575.4	606.7	1182.1	0.7746	0.5893	1.3638
580	1325.8	....	0.0228	0.2989	0.3217	588.9	588.4	1177.3	0.7872	0.5659	1.3532
590	1431.2	...	0.0232	0.2700	0.2931	602.8	569.0	1171.8	0.8001	0.5421	1.3422
600	1542.9	....	0.0236	0.2432	0.2668	617.0	548.5	1165.5	0.8131	0.5176	1.3307
610	1661.2	....	0.0241	0.2185	0.2426	631.6	526.7	1158.4	0.8264	0.4924	1.3188
620	1786.6	...	0.0247	0.1955	0.2201	646.7	503.6	1150.3	0.8398	0.4664	1.3062
630	1919.3	......	0.0253	0.1740	0.1992	662.3	478.8	1141.1	0.8536	0.4394	1.2930
640	2059.7	......	0.0260	0.1538	0.1798	678.6	452.0	1130.5	0.8679	0.4110	1.2789
650	2208.2	.....	0.0268	0.1348	0.1616	695.7	422.8	1118.5	0.8828	0.3809	1.2637
660	2365.4	.....	0.0278	0.1165	0.1442	714.2	390.2	1104.4	0.8987	0.3485	1.2472
670	2531.8	......	0.0290	0.0987	0.1277	734.4	353.2	1087.7	0.9159	0.3127	0.2285
680	2708.1	......	0.0305	0 0810	0.1115	757.3	309.9	1067.2	0.9351	0.2719	1.2071
690	2895.1	....	0.0328	0.0625	0.0953	784.4	256.0	1040.4	0.9578	0.2227	1.1805
700	3093.7	....	0.0369	0.0392	0.0761	823.3	172.1	995.4	0.9905	0.1484	1.1389
705.4	3206.2	..	0.0503	0	0.0503	902.7	0	902.7	1.0580	0	1.0580

Table 3. Saturation: Pressure Table

(Condensed by permission from Keenan and Keyes, *Thermodynamic Properties of Steam*, Wiley, 1936)

Abs. Pressure In. Hg, p	Temp. Fahr., t	Specific Volume		Enthalpy			Entropy			Internal Energy		
		Sat. Liquid, v_f	Sat. Vapor, v_g	Sat. Liquid, h_f	Evap., h_{fg}	Sat. Vapor, h_g	Sat. Liquid, s_f	Evap., s_{fg}	Sat. Vapor, s_g	Sat. Liquid, u_f	Evap., u_{fg}	Sat. Vapor, u_g
0.5	58.80	0.01604	1256.4	26.86	1060.6	1087.5	0.0532	2.0453	2.0985	26.86	1003.5	1030.4
1.0	79.03	0.01608	652.3	47.05	1049.2	1096.3	0.0914	1.9473	2.0387	47.05	990.0	1037.0
1.5	91.72	0 01611	444.9	59.71	1042.0	1101.7	0.1147	1.8894	2.0041	59.71	981.4	1041.1
2.0	101.14	0.01614	339.2	69.10	1036.6	1105.7	0.1316	1.8481	1.9797	69.10	974.9	1044.0
2.5	108.71	0.01616	274.9	76.65	1032.3	1108.9	0.1449	1.8160	1.9609	76.65	969.8	1046.4
5	133.76	0.01626	143.25	101.66	1017.7	1119.4	0.1879	1.7150	1.9028	101.65	952.6	1054.3
10	161.49	0.01640	74.76	129.38	1001.4	1130.8	0.2335	1.6121	1.8456	129.37	933.4	1062.8
15	179.14	0 01650	51.14	147.06	990.7	1137.8	0.2616	1.5508	1.8125	147.04	921.1	1068.1
20	192.37	0.01658	39.07	160.33	982.7	1143.0	0.2822	1.5069	1.7891	160.30	911.7	1072.0
25	203.08	0.01666	31.70	171.09	975.9	1147.0	0.2985	1.4726	1.7711	171.05	904.0	1075.0
Pressure, psia												
1	101.74	0.01614	333.6	69.70	1036.3	1106.0	0.1326	1.8456	1.9782	69.70	974.6	1044.3
2	126.08	0.01623	173.73	93.99	1022.2	1116.2	0.1749	1.7451	1.9200	93.98	957.9	1051.9
3	141.48	0.01630	118.71	109.37	1013.2	1122.6	0.2008	1.6855	1.8863	109.36	947.3	1056.7
4	152.97	0.01636	90.63	120.86	1006.4	1127.3	0.2198	1.6427	1.8625	120.85	939.3	1060.2
5	162.24	0.01640	73.52	130.13	1001.0	1131.1	0.2347	1.6094	1.8441	130.12	933.0	1063.1
6	170.06	0.01645	61.98	137.96	996.2	1134.2	0.2472	1.5820	1.8292	137.94	927.5	1065.4
7	176.85	0.01649	53.64	144.76	992.1	1136.9	0.2581	1.5586	1.8167	144.74	922.7	1067.4
8	182.86	0.01653	47.34	150.79	988.5	1139.3	0.2674	1.5383	1.8057	150.77	918.4	1069.2
9	188.28	0.01656	42.40	156.22	985.2	1141.4	0.2759	1.5203	1.7962	156.19	914.6	1070.8
10	193.21	0.01659	38.42	161.17	982.1	1143.3	0.2835	1.5041	1.7876	161.14	911.1	1072.2
12	201.96	0.01665	32.40	169.96	976.6	1146.6	0.2967	1.4763	1.7730	169.92	904.8	1074.7
14.696	212.00	0.01672	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	180.02	897.5	1077.5
15	213.03	0.01672	26.29	181.11	969.7	1150.8	0.3135	1.4415	1.7549	181.06	896.7	1077.8
20	227.96	0.01683	20.089	196.16	960.1	1156.3	0.3356	1.3962	1.7319	196.10	885.8	1081.9
25	240.07	0.01692	16.303	208.42	952.1	1160.6	0.3533	1.3606	1.7139	208.34	876.8	1085.1
30	250.33	0.01701	13.746	218.82	945.3	1164.1	0.3680	1.3313	1.6993	218.73	869.1	1087.8
35	259.28	0.01708	11.898	227.91	939.2	1167.1	0.3807	1.3063	1.6870	227.80	862.3	1090.1
40	267.25	0.01715	10.498	236.03	933.7	1169.7	0.3919	1.2844	1.6763	235.90	856.1	1092.0
45	274.44	0.01721	9.401	243.36	928.6	1172.0	0.4019	1.2650	1.6669	243.22	850.5	1093.7
50	281.01	0.01727	8.515	250.09	924.0	1174.1	0.4110	1.2474	1.6585	249.93	845.4	1095.3
55	287.07	0.01732	7.787	256.30	919.6	1175.9	0.4193	1.2316	1.6509	256.12	840.6	1096.7
60	292.71	0.01738	7.175	262.09	915.5	1177.6	0.4270	1.2168	1.6438	261.90	836.0	1097.9
65	297.97	0.01743	6.655	267.50	911.6	1179.1	0.4342	1.2032	1.6374	267 29	831.8	1099.1
70	302.92	0.01748	6.206	272.61	907.9	1180.6	0.4409	1.1906	1.6315	272.38	827.8	1100.2
80	312.03	0.01757	5.472	282.02	901.1	1183.1	0.4531	1.1676	1.6207	281.76	820.3	1102.1
90	320.27	0.01766	4.896	290.56	894.7	1185.3	0.4641	1.1471	1.6112	290.27	813.4	1103.7
100	327.81	0.01774	4.432	298.40	888.8	1187.2	0.4740	1.1286	1.6026	298.08	807.1	1105.2
110	334.77	0.01782	4.049	305.66	883.2	1188.9	0.4832	1.1117	1.5948	305.30	801.2	1106.5
120	341.25	0.01789	3.728	312.44	877.9	1190.4	0.4916	1.0962	1.5878	312.05	795.6	1107.6
130	347.32	0.01796	3.455	318.81	872.9	1191.7	0.4995	1.0817	1.5812	318.38	790.2	1108.6
140	353.02	0.01802	3.220	324.82	868.2	1193.0	0.5069	1.0682	1.5751	324.35	785.2	1109.6
150	358.42	0.01809	3.015	330.51	863.6	1194.1	0.5138	1.0556	1.5694	330.01	780.5	1110.5
160	363.53	0.01815	2.834	335.93	859.2	1195.1	0.5204	1.0436	1.5640	335.39	775.8	1111.2
170	368.41	0.01822	2.675	341.09	854.9	1196.0	0.5266	1.0324	1.5590	340.52	771.4	1111.9
180	373.06	0.01827	2.532	346.03	850.8	1196.9	0.5325	1.0217	1.5542	345.42	767.1	1112.5
190	377.51	0.01833	2.404	350.79	846.8	1197.6	0.5381	1.0116	1.5497	350.15	763.0	1113.1
200	381.79	0.01839	2.288	355.36	843.0	1198.4	0.5435	1.0018	1.5453	354.68	759.0	1113.7
220	389.86	0.01850	2.087	364.02	835.6	1199.6	0.5537	0.9835	1.5372	363.27	751.3	1114.6
240	397.37	0.01860	1.9183	372.12	828.5	1200.6	0.5631	0.9667	1.5298	371.29	744.1	1115.4
260	404.42	0.01870	1.7748	379.76	821.8	1201.5	0.5719	0.9510	1.5229	378.86	737.3	1116.1
280	411.05	0.01880	1.6511	386.98	815.3	1202.3	0.5801	0 9363	1.5164	386.01	730.7	1116.7
300	417.33	0.01890	1.5433	393.84	809.0	1202.8	0.5879	0.9225	1.5104	392.79	724.3	1117.1
320	423.29	0.01899	1.4485	400.39	803.0	1203.4	0.5952	0.9094	1.5046	399.26	718.3	1117.6
340	428.97	0.01908	1.3645	406.66	797.1	1203.7	0.6022	0.8970	1.4992	405.46	712.4	1117.9
360	434.40	0.01917	1.2895	412.67	791.4	1204.1	0.6090	0.8851	1.4941	411.39	706.8	1118.2
380	439.60	0.01925	1.2222	418.45	785.8	1204.3	0.6153	0.8738	1.4891	417.10	701.3	1118.4
400	444.59	0.0193	1.1613	424.0	780.5	1204.5	0.6214	0.8630	1.4844	422.6	695.9	1118.5
420	449.39	0.0194	1.1061	429.4	775.2	1204.6	0.6272	0.8527	1.4799	427.9	690.8	1118.7
440	454.02	0.0195	1.0556	434.6	770.0	1204.6	0.6329	0.8426	1.4755	433.0	685.7	1118.7
460	458.50	0.0196	1.0094	439.7	764.9	1204.6	0.6383	0.8330	1.4713	438.0	680.7	1118.7
480	462.82	0.0197	0.9670	444.6	759.9	1204.5	0.6436	0.8237	1.4673	442.9	675.7	1118.6

Table 3. Saturation: Pressure Table—*Continued*

(Condensed by permission from Keenan and Keyes, *Thermodynamic Properties of Steam*, Wiley, 1936)

Pressure, psia	Temp. Fahr., t	Specific Volume Evap., v_{fg}	Specific Volume Sat. Vapor, v_g	Enthalpy Sat. Liquid, h_f	Enthalpy Evap., h_{fg}	Enthalpy Sat. Vapor, h_g	Entropy Sat. Liquid, s_f	Entropy Evap., s_{fg}	Entropy Sat. Vapor, s_g	Internal Energy Sat. Liquid, u_f	Internal Energy Vapor, u_g
500	467.01	0.9081	0.9278	449.4	755.0	1204.4	0.6487	0.8147	.4634	447.6	1118.6
520	471.07	0.8717	0.8915	454.1	750.1	1204.2	0.6536	0.8060	.4596	452.2	1118.4
540	475.01	0.8379	0.8578	458.6	745.4	1204.0	0.6584	0.7976	.4560	456.6	1118.3
560	478.85	0.8065	0.8265	463.0	740.8	1203.8	0.6631	0.7893	.4524	460.9	1118.2
580	482.58	0.7772	0.7973	467.4	736.1	1203.5	0.6676	0.7813	.4489	465.2	1118.0
600	486.21	0.7497	0.7698	471.6	731.6	1203.2	0.6720	0.7734	.4454	469.4	1117.7
620	489.75	0.7238	0.7440	475.7	727.2	1202.9	0.6763	0.7658	.4421	473.4	1117.5
640	493.21	0.6995	0.7198	479.8	722.7	1202.5	0.6805	0.7584	.4389	477.4	1117.3
660	496.58	0.6767	0.6971	483.8	718.3	1202.1	0.6846	0.7512	.4358	481.3	1117.0
680	499.88	0.6553	0.6757	487.7	714.0	1201.7	0.6886	0.7441	.4327	485.1	1116.7
700	503.10	0.6349	0.6554	491.5	709.7	1201.2	0.6925	0.7371	.4296	488.8	1116.3
720	506.25	0.6156	0.6362	495.3	705.4	1200.7	0.6963	0.7303	.4266	492.5	1116.0
740	509.34	0.5973	0.6180	499.0	701.2	1200.2	0.7001	0.7237	.4237	496.2	1115.6
760	512.36	0.5800	0.6007	502.6	697.1	1199.7	0.7037	0.7172	.4209	499.7	1115.2
780	515.33	0.5635	0.5843	506.2	692.9	1199.1	0.7073	0.7108	.4181	503.2	1114.8
800	518.23	0.5478	0.5687	509.7	688.9	1198.6	0.7108	0.7045	.4153	506.6	1114.4
820	521.08	0.5329	0.5538	513.2	684.8	1198.0	0.7143	0.6983	.4126	510.0	1114.0
840	523.88	0.5186	0.5396	516.6	680.8	1197.4	0.7177	0.6922	.4099	513.3	1113.6
860	526.63	0.5049	0.5260	520.0	676.8	1196.8	0.7210	0.6862	.4072	516.6	1113.1
880	529.33	0.4918	0.5130	523.3	672.8	1196.1	0.7243	0.6803	.4046	519.9	1112.6
900	531.98	0.4794	0.5006	526.6	668.8	1195.4	0.7275	0.6744	.4020	523.1	1112.1
920	534.59	0.4673	0.4886	529.8	664.9	1194.7	0.7307	0.6687	.3995	526.2	1111.5
940	537.16	0.4558	0.4772	533.0	661.0	1194.0	0.7339	0.6631	.3970	529.3	1111.0
960	539.68	0.4449	0.4663	536.2	657.1	1193.3	0.7370	0.6576	.3945	532.4	1110.5
980	542.17	0.4342	0.4557	539.3	653.3	1192.6	0.7400	0.6521	.3921	535.4	1110.0
1000	544.61	0.4240	0.4456	542.4	649.4	1191.8	0.7430	0.6467	.3897	538.4	1109.4
1050	550.57	0.4000	0.4218	550.0	639.9	1189.9	0.7504	0.6334	.3838	545.8	1108.0
1100	556.31	0.3781	0.4001	557.4	630.4	1187.8	0.7575	0.6205	.3780	552.9	1106.4
1150	561.86	0.3581	0.3802	564.6	621.0	1185.6	0.7644	0.6079	.3723	559.9	1104.7
1200	567.22	0.3396	0.3619	571.7	611.7	1183.4	0.7711	0.5956	.3667	566.7	1103.0
1300	577.46	0.3066	0.3293	585.4	593.2	1178.6	0.7840	0.5719	.3559	580.0	1099.4
1400	587.10	0.2781	0.3012	598.7	574.7	1173.4	0.7963	0.5491	.3454	592.7	1095.4
1500	596.23	0.2530	0.2765	611.6	556.3	1167.9	0.8082	0.5269	.3351	605.1	1091.2
1600	604.90	0.2309	0.2548	624.1	538.0	1162.1	0.8196	0.5053	.3249	617.0	1086.7
1700	613.15	0.2111	0.2354	636.3	519.6	1155.9	0.8306	0.4843	.3149	628.7	1081.8
1800	621.03	0.1932	0.2179	648.3	501.1	1149.4	0.8412	0.4637	.3049	640.1	1076.8
1900	628.58	0.1769	0.2021	660.1	482.4	1142.4	0.8516	0.4433	.2949	651.2	1071.4
2000	635.82	0.1621	0.1878	671.7	463.4	1135.1	0.8619	0.4230	.2849	662.2	1065.6
2200	649.46	0.1358	0.1625	694.8	424.4	1119.2	0.8820	0.3826	.2646	683.9	1053.1
2400	662.12	0.1128	0.1407	718.4	382.7	1101.1	0.9023	0.3411	.2434	706.0	1038.6
2600	673.94	0.0918	0.1213	743.0	337.2	1080.2	0.9232	0.2973	.2205	728.8	1021.9
2800	684.99	0.0719	0.1035	770.1	284.7	1054.8	0.9459	0.2487	.1946	753.8	1001.2
3000	695.36	0.0512	0.0858	802.5	217.8	1020.3	0.9731	0.1885	.1615	783.4	972.7
3206.2	705.40	0	0 0503	902.7	0	902 7	1.0580	0	.0580	872.9	872.9

5. WATER

PROPERTIES OF COMPRESSED LIQUID. Table 4 is a table of properties of compressed liquid water. The independent variables are temperature and pressure, but the nature of the dependent variables has been altered in order to reduce greatly the size of the table. Instead of giving the enthalpy directly, Table 4 gives the excess of the enthalpy over the saturation enthalpy at the temperature in question, namely $(h - h_f)$. Corresponding constant-temperature differences are given for the specific volume and entropy, namely $(v - v_f)$ and $(s - s_f)$.

In order to obtain properties for a state intermediate between the states tabulated, it is necessary to interpolate linearly in Table 4 for the differences from the saturation values. The saturation values themselves can be obtained from Tables 2 and 3. For purposes of interpolation at low pressures, note that the value of each of the differences—$(v - v_f)$, $(h - h_f)$, and $(s - s_f)$—becomes zero at the saturation pressure corresponding to each temperature.

Values in Table 4 are taken from *Thermodynamic Properties of Steam*, by J. H. Keenan and F. G. Keyes, John Wiley and Sons (1936), by permission of the authors and publisher.

Figure 1, extracted by permission from the same source, graphically illustrates the properties of compressed liquids.

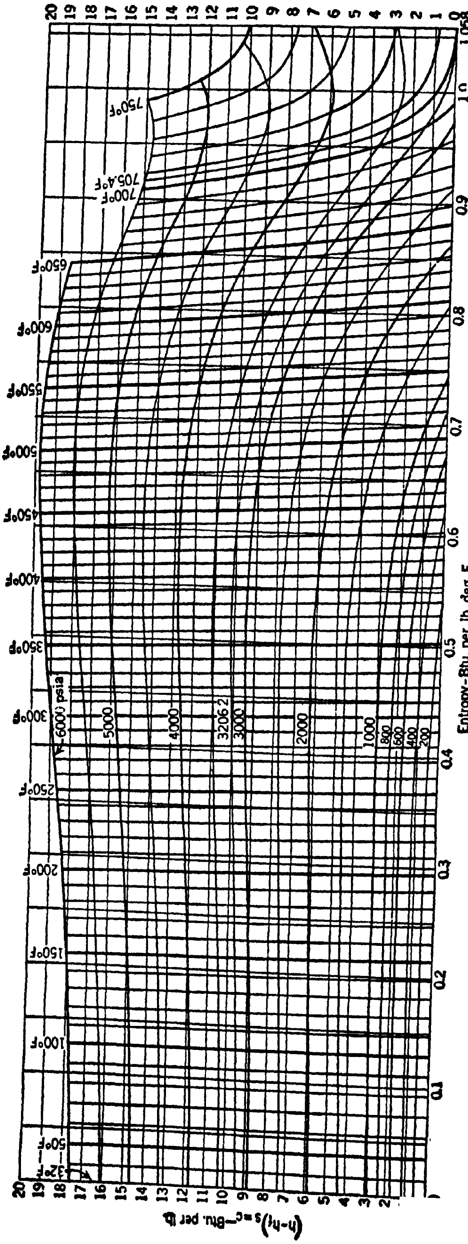

FIG. 1. Enthalpy-entropy chart for compressed water.

Example. Liquid water is at a temperature of 300 F and a pressure of 900 psia. Find the specific volume, enthalpy, and entropy.

Solution. The saturation values are given in Table 2 as follows: pressure = 67.013 pounds per square inch, absolute; specific volume = 0.017449 cubic feet per pound; enthalpy = 269.59 Btu per pound; and entropy = 0.43694 Btu per pound, ° F.

For 900 psia by linear interpolation in Table 4, we have

$$(v - v_f) = -6.3 \times 10^{-5}$$

$$v = 0.017449 - 0.000063 = 0.017386 \text{ ft}^3/\text{lb}$$

Likewise, we get

$$(h - h_f) = 1.55$$

$$h = 269.59 + 1.55 = 271.14 \text{ Btu/lb}$$

and

$$(s - s_f) = -1.46 \times 10^{-3}$$

$$s = 0.43694 - 0.00146 = 0.43548 \text{ Btu/lb F}$$

Table 4. Compressed Liquid Water

Pressure, psia (Sat. Temp., °F)		Temperature—Degrees Fahrenheit							
		32	100	200	300	400	500	600	700
	Saturated Liquid p	0.08854	0.9492	11.526	67.013	247.31	680.8	1542.9	3093.7
	v_f	0.016022	0.016132	0.016634	0.017449	0.018639	0.020432	0.023629	0.03692
	h_f	0	67.97	167.99	269.59	374.97	487.82	617.0	823.3
	s_f	0	0.12948	0.29382	0.43694	0.56638	0.68871	0.8131	0.9905
200 (381.79)	$(v - v_f)\cdot 10^5$	−1.1	−1.1	−1.1	−1.1				
	$(h - h_f)$	+0.61	+0.54	+0.41	+0.23				
	$(s - s_f)\cdot 10^3$	+0.03	−0.05	−0.21	−0.21				
400 (444.59)	$(v - v_f)\cdot 10^5$	−2.3	−2.1	−2.2	−2.8	−2.1			
	$(h - h_f)$	+1.21	+1.09	+0.88	+0.61	+0.16			
	$(s - s_f)\cdot 10^3$	+0.04	−0.16	−0.47	−0.56	−0.40			
600 (486.21)	$(v - v_f)\cdot 10^5$	−3.5	−3.2	−3.4	−4.3	−4.4			
	$(h - h_f)$	+1.80	+1.67	+1.31	+0.97	+0.39			
	$(s - s_f)\cdot 10^3$	+0.07	−0.27	−0.74	−0.94	−0.96			
800 (518.23)	$(v - v_f)\cdot 10^5$	−4.6	−4.0	−4.4	−5.6	−6.5	−1.7		
	$(h - h_f)$	+2.39	+2.17	+1.78	+1.35	+0.61	−0.05		
	$(s - s_f)\cdot 10^3$	+0.10	−0.40	−0.97	−1.27	−1.48	−0.53		
1000 (544.61)	$(v - v_f)\cdot 10^5$	−5.7	−5.1	−5.4	−6.9	−8.7	−6.4		
	$(h - h_f)$	+2.99	+2.70	+2.21	+1.75	+0.84	−0.14		
	$(s - s_f)\cdot 10^3$	+0.15	−0.53	−1.20	−1.64	−2.00	−1.41		
1500 (596.23)	$(v - v_f)\cdot 10^5$	−8.4	−7.5	−8.1	−10.4	−14.1	−17.3		
	$(h - h_f)$	+4.48	+3.99	+3.36	+2.70	+1.44	−0.29		
	$(s - s_f)\cdot 10^3$	+0.20	−0.86	−1.79	−2.53	−3.32	−3.56		
2000 (635.82)	$(v - v_f)\cdot 10^5$	−11.0	−9.9	−10.8	−13.8	−19.5	−27.8	−32.6	
	$(h - h_f)$	+5.97	+5.31	+4.51	+3.64	+2.03	−0.38	−2.5	
	$(s - s_f)\cdot 10^3$	+0.22	−1.18	−2.39	−3.42	−4.57	−5.58	−4.3	
2500 (668.13)	$(v - v_f)\cdot 10^5$	−13.7	−12.3	−13.4	−17.2	−24.8	−37.7	−61.9	
	$(h - h_f)$	+7.49	+6.58	+5.63	+4.55	+2.66	−0.41	−4.9	
	$(s - s_f)\cdot 10^3$	+0.25	−1.48	−2.97	−4.25	−5.79	−7.54	−8.5	
3000 (695.36)	$(v - v_f)\cdot 10^5$	−16.3	−14.7	−16.0	−20.7	−30.0	−47.1	−87.9	
	$(h - h_f)$	+9.00	+7.88	+6.76	+5.49	+3.33	−0.41	−6.9	
	$(s - s_f)\cdot 10^3$	+0.28	−1.79	−3.56	−5.12	−7.03	−9.42	−12.4	
4000	$(v - v_f)\cdot 10^5$	−21.5	−19.2	−21.0	−27.5	−40.0	−64.5	−132.2	−821
	$(h - h_f)$	+11.88	+10.49	+9.03	+7.41	+4.71	−0.16	−10.0	−59.5
	$(s - s_f)\cdot 10^3$	+0.29	−2.42	−4.74	−6.77	−9.40	−13.03	−19.3	−55.8
5000	$(v - v_f)\cdot 10^5$	−26.7	−23.6	−26.0	−34.0	−49.6	−80.5	−169.3	−1017
	$(h - h_f)$	+14.75	+13.08	+11.30	+9.36	+6.08	+0.25	−12.1	−76.9
	$(s - s_f)\cdot 10^3$	+0.22	−3.07	−5.92	−8.40	−11.74	−16.47	−25.3	−75.3
6000	$(v - v_f)\cdot 10^5$	−31.7	−27.8	−30.8	−40.5	−58.7	−96.1	−202.9	
	$(h - h_f)$	+17.60	+15.72	+13.62	+11.39	+7.50	+0.77	−14.0	
	$(s - s_f)\cdot 10^3$	+0.10	−3.72	−7.06	−10.00	−13.96	−19.57	−30.6	

DENSITY OF LIQUID WATER AT ONE ATMOSPHERE. The density of liquid water at a pressure of one standard atmosphere is given in Tables 5 and 6 over the temperature range of 0 to 100 C.

In Table 5 the densities are given in grams per milliliter. Since the density of liquid water is very nearly 1 gram per milliliter at 4 C (temperature of maximum density), the values given in Table 5 represent also the densities at various temperatures relative to the density at 4 C. The values in Table 5 are taken by permission from the *International Critical Tables*, Vol. III, by the National Research Council, copyrighted, 1928, published by the McGraw-Hill Book Company, Inc.

Table 5. Density of Water in Grams per Milliliter between 0 C and 100 C at One Atmosphere

°C	0	1	2	3	4	5	6	7	8	9
0	.99987	.99993	.99997	.99999	1.00000	.99999	.99997	.99993	.99988	.99981
10	.99973	.99963	.99952	.99940	0.99927	.99913	.99897	.99880	.99862	.99843
20	.99823	.99802	.99780	.99757	0.99733	.99707	.99681	.99654	.99626	.99597
30	.99568	.99537	.99505	.99473	0.99440	.99406	.99371	.99336	.99299	.99262
40	.99225	.99186	.99147	.99107	0.99066	.99024	.98982	.98940	.98896	.98852
50	.98807	.98762	.98715	.98669	0.98621	.98573	.98525	.98475	.98425	.98375
60	.98324	.98272	.98220	.98167	0.98113	.98059	.98005	.97950	.97894	.97838
70	.97781	.97723	.97666	.97607	0.97548	.97489	.97429	.97368	.97307	.97245
80	.97183	.97121	.97057	.96994	0.96930	.96865	.96800	.96734	.96668	.96601
90	.96534	.96467	.96399	.96330	0.96261	.96192	.96122	.96051	.95981	.95909
100	.95838									

The densities in Table 6, expressed in pounds per cubic foot for various Fahrenheit temperatures were obtained from the densities given in Table 5.

Table 6. Density of Water in Pounds Per Cubic Foot between 32 F and 212 F at One Atmosphere

°F										
30			62.418	62.420	62.422	62.423	62.425	62.425	62.426	62.426
40	62.426	62.426	62.425	62.424	62.423	62.421	62.419	62.417	62.415	62.412
50	62.409	62.406	62.403	62.399	62.395	62.391	62.386	62.382	62.377	62.372
60	62.366	62.361	62.355	62.349	62.343	62.336	62.330	62.323	62.316	62.309
70	62.301	62.294	62.286	62.278	62.269	62.261	62.252	62.244	62.235	62.225
80	62.216	62.207	62.197	62.187	62.177	62.167	62.156	62.146	62.135	62.124
90	62.113	62.102	62.090	62.079	62.067	62.055	62.043	62.031	62.019	62.006
100	61.994	61.981	61.968	61.955	61.942	61.929	61.915	61.902	61.888	61.874
110	61.860	61.846	61.831	61.817	61.802	61.788	61.773	61.758	61.743	61.728
120	61.713	61.697	61.681	61.666	61.650	61.634	61.618	61.602	61.585	61.569
130	61.552	61.535	61.519	61.502	61.485	61.467	61.450	61.433	61.415	61.398
140	61.380	61.362	61.344	61.326	61.308	61.289	61.271	61.252	61.233	61.215
150	61.196	61.177	61.158	61.139	61.119	61.100	61.080	61.061	61.041	61.021
160	61.001	60.981	60.961	60.941	60.920	60 900	60.879	60.859	60.838	60.817
170	60.796	60.775	60.754	60.732	60.711	60.689	60.668	60.646	60.625	60.602
180	60.580	60.558	60.536	60.514	60.492	60.469	60.447	60.424	60.401	60.378
190	60.355	60.332	60.309	60.286	60.263	60.239	60.216	60.192	60.169	60.145
200	60.121	60.097	60.073	60.049	60.025	60.000	59.976	59.951	59.927	59.902
210	59.877	59.853	59.828							

6. ICE

EQUILIBRIUM OF VAPOR AND ICE. Table 7 gives the properties of equilibrium states for vapor and ordinary ice. Temperature is the independent argument. The subscript i refers to saturated ice. The last column has values of the internal energy of the saturated vapor, u_g. This table is taken from *Thermodynamic Properties of Steam*, by J. H. Keenan and F. G. Keyes, John Wiley and Sons (1936), by permission of the authors and publisher.

Table 7. Equilibrium of Solid and Vapor

t, °F	p, psia	v_i	$v_g \times 10^{-3}$	h_i	h_{ig}	h_g	s_i	s_{ig}	s_g	u_g
32	.0885	.01747	3.306	−143.35	1219.1	1075.8	−.2916	2.4793	2.1877	1021.6
30	.0808	.01747	3.609	−144.35	1219.3	1074.9	−.2936	2.4897	2.1961	1021.0
25	.0640	.01746	4.508	−146.84	1219.6	1072.8	−.2987	2.5159	2.2172	1019.4
20	.0505	.01745	5.658	−149.31	1219.9	1070.6	−.3038	2.5425	2.2387	1017.7
15	.0396	.01745	7.14	−151.75	1220.2	1068.4	−.3089	2.5698	2.2609	1016.1
10	.0309	.01744	9.05	−154.17	1220.4	1066.2	−.3141	2.5977	2.2836	1014.4
5	.0240	.01743	11.53	−156.56	1220.6	1064.0	−.3192	2.6260	2.3068	1012.8
0	.0185	.01742	14.77	−158.93	1220.7	1061.8	−.3241	2.6546	2.3305	1011.1
−5	.0142	.01742	19.04	−161.27	1220.9	1059.6	−.3294	2.6842	2.3548	1009.5
−10	.0108	.01741	24.67	−163.59	1221.0	1057.4	−.3346	2.7143	2.3797	1007.8
−15	.0082	.01740	32.1	−165.89	1221.1	1055.2	−.3397	2.7449	2.4052	1006.2
−20	.0062	.01739	42.2	−168.16	1221.2	1053.0	−.3448	2.7764	2.4316	1004.5
−25	.0046	.01739	55.8	−170.40	1221.2	1050.8	−.3500	2.8085	2.4585	1002.9
−30	.0035	.01738	74.1	−172.63	1221.2	1048.6	−.3551	2.8411	2.4860	1001.2
−35	.0025	.01737	99.3	−174.82	1221.2	1046.4	−.3602	2.8745	2.5143	999.6
−40	.0019	.01737	133.9	−177.00	1221.2	1044.2	−.3654	2.9087	2.5433	997.9

THEORETICAL STEAM RATE TABLES *

By J. K. Salisbury

The theoretical steam rate of an engine or turbine in pounds per kilowatt hour is defined as

$$\text{Theoretical steam rate} = \frac{3412.75}{h_1 - h_2}$$

where h_1 is the enthalpy in Btu per pound of steam supplied to the engine or turbine, and h_2 is the enthalpy in Btu per pound of steam at the exhaust pressure, at the entropy of the steam supply. In other words, $(h_1 - h_2)$ represents the isentropic energy drop across the engine or turbine.

It is evident from its definition that the quantity tabulated below is distinctly different from the actual steam rate of an engine or turbine. The ratio of the two is the efficiency of the engine or turbine, i.e.,

$$\text{Efficiency} = \frac{\text{Theoretical steam rate}}{\text{Actual steam rate}}$$

This efficiency is called the "engine (or turbine) efficiency, referred to the Rankine cycle."

* Adapted by permission of ASME, from *Theoretical Steam Rate Tables* by Keenan and Keyes, published by ASME, 1938.

Table 1. Theoretical Steam Rates in Pounds per Kilowatt Hour

Exhaust Pressure, in. Hg abs	Initial Temperature, °F													
	600	650	700	750	800	850	900	600	650	700	750	800	850	900
	Initial Pressure, 400 psig							Initial Pressure, 450 psig						
	Initial Superheat							*Initial Superheat*						
	151.9	*201.9*	*251.9*	*301.9*	*351.9*	*401.9*	*451.9*	*140.5*	*190.5*	*240.5*	*290.5*	*340.5*	*390.5*	*440.5*
0.5	7.04	6.831	6.631	6.440	6.256	6.080	5.910	6.983	6.770	6.571	6.381	6.199	6.024	5.857
1.0	7.51	7.28	7.06	6.856	6.654	6.459	6.272	7.44	7.21	6.994	6.787	6.586	6.393	6.208
1.5	7.84	7.59	7.36	7.14	6.924	6.717	6.518	7.76	7.51	7.28	7.06	6.849	6.644	6.447
2.0	8.09	7.84	7.60	7.37	7.14	6.919	6.710	8.01	7.75	7.51	7.28	7.05	6.839	6.634
2.5	8.31	8.04	7.79	7.55	7.31	7.09	6.871	8.22	7.95	7.70	7.46	7.22	7.00	6.789
psig 0	12.59	12.11	11.65	11.20	10.77	10 35	9.96	12.31	11.83	11.38	10.95	10.53	10.13	9.75
10	14.42	13.83	13.27	12.72	12.19	11.68	11.20	14.03	13.45	12.90	12.38	11.87	11.39	10.92
20	16.02	15.33	14.66	14.02	13.41	12.82	12.29	15.52	14.85	14.21	13.60	13.01	12.44	11.92
100	28.08	26.49	25.09	23 86	22 76	21.77	20.87	26.29	24.81	23.49	22.31	21.26	20.31	19.46
150	37.5	35.4	33.6	31.9	30.5	29.14	27.96	34.2	32.2	30.5	28.94	27.59	26.36	25.27
200	51.1	48.3	45.8	43.5	41.5	39 7	38.1	44.5	42.0	39.7	37.8	36.0	34.4	33.0
250	... ..	69.1	65.5	62.3	59.4	56 8	54.6	60 0	56.3	53.3	50.6	48.2	46.0	44.2
	Initial Pressure, 500 psig							Initial Pressure, 600 psig						
	130	*180*	*230*	*280*	*330*	*380*	*430*	*111.2*	*161.2*	*211.2*	*261.2*	*311.2*	*361.2*	*411.2*
0.5	6.937	6.722	6.523	6.333	6.151	5.977	5.810	6.872	6.649	6.445	6.254	6.073	5.900	5.734
1.0	7.39	7.15	6.935	6.727	6.529	6.338	6.154	7.31	7.06	6.841	6.633	6.436	6.246	6.065
1.5	7.69	7.44	7.21	6.995	6.784	6.582	6.387	7.60	7.34	7.11	6.888	6.680	6.479	6.288
2.0	7.94	7.68	7.43	7.20	6.985	6.772	6.568	7.83	7.56	7.32	7.09	6.872	6.663	6.462
2.5	8.14	7.87	7.62	7.38	7.15	6.930	6.720	8.02	7.75	7.50	7.26	7.03	6.814	6.606
psig 0	12.07	11.60	11.16	10.74	10.33	9.94	9.57	11.71	11.24	10.80	10.40	10.01	9.64	9.29
10	13.70	13.13	12.60	12.10	11.61	11.14	10.69	13.20	12.65	12.14	11.64	11.19	10.74	10.32
20	15.09	14.44	13.83	13.24	12.68	12.14	11.63	14.46	13.83	13.24	12.68	12.15	11.65	11.17
100	24.89	23 55	22.28	21.12	20.11	19.20	18.40	22.88	21.67	20.52	19.44	18.49	17.63	16.87
150	31.7	29.81	28.19	26.75	25.48	24.33	23.32	28.32	26.65	25.15	23.83	22.66	21.62	20.70
200	40.2	37.7	35.7	33 9	32.3	30.8	29 50	34.7	32.4	30.6	28.99	27.59	26.32	25.17
250	51.8	48.6	45.9	43.5	41.4	39.5	37.9	42.4	39.6	37.3	35.3	33.6	32.1	30.7

Exhaust Pressure, in. Hg abs	Initial Temperature, °F													
	650	700	750	800	850	900	950	650	700	750	800	850	900	950
	Initial Pressure, 650 psig							Initial Pressure, 700 psig						
	152.6	*202.6*	*252.6*	*302.6*	*352.6*	*402.6*	*452.6*	*144.6*	*194.6*	*244.6*	*294.6*	*344.6*	*394.6*	*444.6*
0.5	6.622	6.417	6.226	6.043	5.869	5.703	5.545	6.600	6.392	6.196	6.015	5.841	5.676	5.518
1.0	7.03	6.806	6.596	6.397	6.208	6.029	5.856	7.00	6.774	6.563	6.365	6.176	5.997	5.824
1.5	7.30	7.07	6.846	6.636	6.437	6.247	6.065	7.27	7.03	6.813	6.602	6.401	6.211	6.030
2.0	7.52	7.27	7.04	6.826	6.617	6.418	6.228	7.48	7.24	7.01	6.787	6.578	6.379	6.190
2.5	7.70	7.45	7.21	6.982	6.765	6.560	6.363	7.66	7.41	7.17	6.939	6.725	6.519	6.321
psig 0	11.10	10.67	10.27	9.88	9 51	9.17	8.83	10.98	10.55	10.15	9.77	9.41	9.06	8.73
10	12.46	11.95	11.46	11.01	10.58	10.17	9.77	12.30	11.79	11.31	10.86	10.44	10.03	9.64
20	13.59	13.02	12.47	11.95	11.45	10.98	10.54	13.39	12.82	12.28	11.77	11.28	10.82	10.39
100	20.96	19.87	18.83	17.90	17.05	16.30	15.63	20.39	19.32	18.32	17.40	16.57	15.83	15.16
150	25.53	24.10	22.81	21.68	20.66	19.76	18.95	24.64	23.23	21.96	20.86	19.85	18.97	18.19
200	30.7	28.90	27.37	26.01	24.79	23.72	22.73	29.34	27.55	26.05	24.74	23.57	22.52	21.58
250	36.9	34.7	32.8	31.2	29.73	28.44	27.26	34.8	32.6	30.8	29.27	27.88	26.64	25.51
	Initial Pressure, 800 psig							Initial Pressure, 850 psig						
	129.7	*179.7*	*229.7*	*279.7*	*329.7*	*379.7*	*429.7*	*122.7*	*172.7*	*222.7*	*272.7*	*322.7*	*372.7*	*422.7*
0.5	6.568	6.352	6.154	5.970	5.795	5.631	5.473	6.560	6.337	6.136	5.950	5.776	5.611	5.454
1.0	6.961	6.723	6.510	6.309	6.120	5.941	5.771	6.947	6.705	6.488	6.286	6.097	5.917	5.747
1.5	7.22	6.975	6.750	6.539	6.340	6.150	5.970	7.21	6.954	6.725	6.512	6.312	6.122	5.944
2.0	7.43	7.17	6.937	6.718	6.510	6.313	6.125	7.41	7 15	6.910	6.688	6.480	6.284	6.097
2.5	7.60	7.34	7.09	6.866	6.652	6.448	6.253	7.58	7.31	7.07	6.837	6.622	6.419	6.223
psig 0	10.79	10.36	9.96	9.58	9.23	8.89	8.57	10.71	10.28	9.87	9.49	9.14	8.81	8.50
10	12.03	11.52	11.05	10.61	10.20	9.80	9.43	11.92	11.41	10.94	10.50	10.09	9.71	9.34
20	13.05	12.48	11.96	11.46	10.99	10.55	10.13	12.92	12.35	11.82	11.33	10.87	10.43	10.02
100	19.45	18.44	17.48	16.60	15.80	15.08	14.43	19.07	18.07	17.14	16.27	15.48	14.77	14.13
150	23.22	21.86	20.66	19.60	18.64	17.78	17.03	22.63	21.33	20.16	19.09	18.14	17.31	16.57
200	27.21	25.52	24.06	22.82	21.72	20.72	19.84	26.39	24.77	23.31	22.09	20.99	20.03	19.17
250	31.7	29.64	27.97	26.49	25.18	24.03	23.00	30.5	28.54	26.85	25.43	24.15	23.04	22.06
300	36.7	34.4	32.4	30.7	29.24	27.87	26.70	35.1	32.8	30.9	29.24	27.78	26.52	25.38

Table 1. Theoretical Steam Rates in Pounds per Kilowatt Hour—*Continued*

Exhaust Pressure, in. Hg abs	Initial Temperature, °F: 700	750	800	850	900	950	1000	700	750	800	850	900	950	1000
	Initial Pressure, 900 psig							Initial Pressure, 1000 psig						
	Initial Superheat							*Initial Superheat*						
	166.1	*216.1*	*266.1*	*316.1*	*366.1*	*416.1*	*466.1*	*153.6*	*203.6*	*253.6*	*303.6*	*353.6*	*403.6*	*453.6*
0.5	6.324	6.121	5.934	5.758	5.594	5.436	5.284	6.309	6.096	5.907	5.729	5.563	5.405	5.254
1.0	6.688	6.469	6.268	6.077	5.898	5.727	5.562	6.666	6.438	6.233	6.040	5.860	5.689	5.527
1.5	6.934	6.702	6.490	6.289	6.101	5.921	5.747	6.905	6.667	6.451	6.248	6.060	5.879	5.708
2.0	7.13	6.885	6.664	6.456	6.260	6.073	5.891	7.09	6.845	6.621	6.410	6.214	6.027	5.850
2.5	7.29	7.04	6.810	6.595	6.392	6.197	6.010	7.25	6.993	6.764	6.547	6.345	6.151	5.966
psig														
0	10.20	9.80	9.43	9.08	8.75	8.43	8.13	10.08	9.67	9.30	8.95	8.63	8.31	8.02
10	11.31	10.84	10.41	10.00	9.62	9.25	8.90	11.15	10.68	10.25	9.84	9.46	9.10	8.77
20	12.22	11.70	11.22	10.76	10.33	9.92	9.53	12.02	11.49	11.02	10.57	10.14	9.74	9.36
100	17.74	16.84	16.00	15.21	14.50	13.87	13.29	17.22	16.32	15.51	14.76	14.05	13.43	12.86
150	20.84	19.69	18.65	17.73	16.91	16.17	15.50	20.09	18.95	17.94	17.03	16.22	15.49	14.85
200	24.08	22.66	21.44	20.39	19.45	18.60	17.83	23.00	21.62	20.42	19.37	18.46	17.63	16.92
250	27.55	25.92	24.52	23.30	22.22	21.25	20.37	26.10	24.43	23.12	21.92	20.87	19.93	19.12
300	31.4	29.57	27.99	26 60	25.36	24.26	23.25	29.42	27.55	26.03	24.71	23.54	22.48	21.55

Exhaust Pressure, in. Hg abs	700	750	800	850	900	950	1000	700	750	800	850	900	950	1000
	Initial Pressure, 1100 psig							Initial Pressure, 1200 psig						
		192	*242*	*292*	*342*	*392*	*442*		*181.2*	*231.2*	*281.2*	*331.2*	*381.2*	*431.2*
0.5	...	6.081	5.886	5.705	5.538	5.380	5.227	...	6.071	5.873	5.688	5.519	5.358	5.206
1.0	...	6.421	6.211	6.013	5.830	5.660	5.495	...	6.403	6.187	5.990	5.807	5.633	5.469
1.5	...	6.644	6.423	6.216	6.025	5.846	5.672	...	6.624	6.398	6.190	5.999	5.817	5.645
2.0	...	6.818	6.589	6.376	6.178	5.991	5.811	...	6.797	6.562	6.347	6.148	5.958	5.780
2.5	...	6.965	6.729	6.510	6.304	6.111	5.924	...	6.942	6.700	6.479	6.274	6.078	5.894
psig														
0	...	9.58	9.20	8.85	8.52	8.22	7.93	...	9.50	9.12	8.76	8.44	8.14	7.85
10	...	10.55	10.11	9.71	9.33	8.98	8.64	...	10.44	10 00	9.60	9.23	8.87	8.55
20	...	11.33	10.85	10.41	9.99	9.59	9.22	...	11.20	10.71	10.27	9.86	9.46	9.10
150	...	18.37	17.38	16.48	15.68	14.97	14.32	...	17.89	16.92	16.04	15.24	14.53	13.90
200	...	20.80	19.63	18.60	17.69	16.89	16.17	...	20.16	19.01	17.97	17.09	16.30	15.59
250	...	23.36	22.01	20.84	19.83	18.94	18.11	...	22.49	21.19	20.01	19.03	18.13	17.34
300	...	26.07	24.57	23.27	22.15	21.14	20.22	...	24.94	23.45	22.16	21.08	20.10	19.23
400	...	32.4	30.5	28.92	27.50	26.27	25.11	...	30 4	28.58	27.06	25.74	24.52	23.46

Exhaust Pressure, in. Hg abs	700	750	800	850	900	950	1000	700	750	800	850	900	950	1000
	Initial Pressure, 1250 psig							Initial Pressure, 1400 psig						
		176.1	*226.1*	*276.1*	*326.1*	*376.1*	*426.1*			*211.5*	*261.5*	*311.5*	*361.5*	*411.5*
0.5	...	6.068	5.867	5.680	5.510	5.349	5.196	...	...	5.854	5.664	5.489	5.325	5.173
1.0	...	6.398	6.179	5.981	5.796	5.622	5.458	...	...	6.163	5.957	5.770	5.593	5.428
1.5	...	6.619	6.389	6.180	5.987	5.804	5.632	...	...	6.370	6.153	5.956	5.771	5.599
2.0	...	6.790	6.552	6.334	6.135	5.945	5.767	...	...	6.529	6.305	6.101	5.910	5.730
2.5	...	6.932	6.688	6.464	6.258	6.063	5.879	...	...	6.661	6.431	6.221	6.024	5.841
psig														
0	...	9.46	9.08	8.73	8.40	8.10	7.81	...	...	8.99	8.63	8.30	8.00	7.72
10	...	10.40	9.95	9.55	9.18	8.83	8.50	...	...	9.83	9.42	9.06	8.70	8.38
20	...	11.14	10.65	10.21	9.80	9.41	9.05	...	...	10.51	10.06	9.65	9.26	8.91
150	...	17.69	16.70	15.83	15.05	14.36	13.72	...	...	16.21	15.34	14.58	13.87	13.25
200	...	19.89	18.74	17.71	16.82	16.05	15.34	...	...	18.08	17.06	16.17	15.39	14.70
250	...	22.13	20.80	19.64	18.67	17.81	17.01	...	...	19.93	18.79	17.82	16.95	16.18
300	...	24.48	22.96	21.70	20.61	19.67	18.79	...	...	21.84	20.57	19.50	18.56	17.73
400	...	29 68	27.81	26.27	24 96	23.81	22.75	...	...	25.98	24.48	23.21	22.07	21.09

Exhaust Pressure, in. Hg abs	Initial Temperature, °F: 850	900	950	1000	850	900	925	950	975	1000	850	900	950	1000
	Initial Pressure, 1600 psig				Initial Pressure, 1800 psig						Initial Pressure, 2000 psig			
	243.9	*293.9*	*343.9*	*393.9*	*227.8*	*277.8*	*302.8*	*327.8*	*352.8*	*377.8*	*213.1*	*263.1*	*313.1*	*363.1*
0.5	5.652	5.472	5.304	5.149	5.651	5.463	5.375	5.290	5.209	5.132	5.657	5.461	5.284	5.122
1.0	5.941	5.747	5.566	5.400	5.935	5.734	5.639	5.548	5.462	5.379	5.940	5.729	5.537	5.365
1.5	6.133	5.929	5.741	5.566	6.124	5.913	5.813	5.718	5.628	5.541	6.125	5.906	5.706	5.525
2.0	6.280	6.070	5.875	5.695	6.270	6.052	5.950	5.851	5.757	5.667	6.270	6.042	5.836	5.648
2.5	6.406	6.188	5.987	5.802	6.392	6.168	6.062	5.960	5.864	5.772	6.390	6.156	5.944	5.752
psig														
0	8.54	8.21	7.90	7.62	8.48	8.14	7.98	7.82	7.68	7.54	8.44	8.08	7.77	7.48
10	9.30	8.93	8.57	8.25	9.22	8.83	8.65	8.48	8.31	8.15	9.16	8.76	8.40	8.08
20	9.90	9.49	9.11	8.75	9.79	9.38	9.18	8.99	8.81	8.64	9.72	9.29	8.90	8.54
150	14.86	14.10	13.41	12.79	14.51	13.74	13.38	13.06	12.74	12.44	14.22	13.47	12.78	12.17
200	16.42	15.53	14.75	14.07	15.96	15.08	14.67	14.29	13.93	13.60	15.59	14.72	13.94	13.24
250	17.95	16.97	16.12	15.37	17.36	16.38	15.92	15.50	15.12	14.77	16.89	15.93	15.05	14.31
300	19.51	18.44	17.50	16.70	18.77	17.68	17.19	16.74	16.32	15.93	18.20	17.12	16.17	15.36
400	22.80	21.54	20.46	19.51	21.65	20.39	19.82	19.30	18.82	18.37	20.83	19.54	18.46	17.54

(*Table continued on p. 4-44*)

Table 1. Theoretical Steam Rates in Pounds per Kilowatt Hour—*Continued*

Exhaust Pressure, in. Hg abs	Initial Temperature, °F													
	850	900	950	1000	850	900	925	950	975	1000	850	900	950	1000
	Initial Pressure, 2200 psig				Initial Pressure, 2400 psig						Initial Pressure, 3000 psig			
	Initial Superheat				*Initial Superheat*						*Initial Superheat*			
	199.6	*249.6*	*299.6*	*349.6*	*187*	*237*	*262*	*287*	*312*	*337*	*153.9*	*203.9*	*253.9*	*303.9*
0.5	5.673	5.467	5.284	5.116	5.693	5.477	5.378	5.286	5.197	5.114	5.792	5.533	5.317	5.128
1.0	5.952	5.733	5.536	5.356	5.972	5.741	5.635	5.535	5.441	5.352	6.073	5.796	5.562	5.361
1.5	6.136	5.907	5.702	5.514	6.154	5.912	5.803	5.700	5.602	5.509	6.256	5.966	5.722	5.513
2.0	6.279	6.041	5.829	5.636	6.296	6.046	5.934	5.828	5.725	5.628	6.397	6.098	5.847	5.631
2.5	6.399	6.155	5.937	5.738	6.415	6.158	6.042	5.933	5.828	5.729	6.515	6.208	5.950	5.729
psig														
0	8.41	8.05	7.73	7.43	8.41	8.03	7.86	7.70	7.55	7.40	8.48	8.04	7.66	7.34
10	9.12	8.71	8.35	8.02	9.10	8.68	8.49	8.30	8.13	7.97	9.15	8.66	8.24	7.88
20	9.67	9.22	8.82	8.47	9.64	9.18	8.97	8.77	8.58	8.41	9.68	9.15	8.69	8.30
150	14.02	13.25	12.57	11.95	13.87	13.08	12.73	12.39	12.08	11.79	13.69	12.80	12.06	11.43
200	15.32	14.46	13.67	12.96	15.10	14.22	13.82	13.45	13.09	12.76	14.82	13.84	13.01	12.30
250	16.56	15.58	14.73	13.95	16.30	15.30	14.86	14.44	14.05	13 68	15.91	14.81	13.89	13.11
300	17.79	16.69	15.76	14.92	17.45	16.36	15.87	15.41	14.99	14.59	16 93	15.75	14.75	13.89
350	19.00	17.79	16.79	15.90	18 61	17.39	16.85	16.36	15.91	15.49	17.94	16.67	15.57	14.64
400	20.23	18.92	17.85	16.89	19.75	18.44	17.86	17.32	16.84	16.40	18.97	17.57	16.39	15.39
450	21.47	20.05	18.92	17.90	20.91	19.47	18.87	18.31	17.80	17.33	19.98	18.48	17.19	16.14
600	25 43	23.79	22.40	21.22	24.49	22.81	22.08	21.42	20.83	20.30	23.04	21.19	19.68	18.47

SECTION 5

HYDRODYNAMICS, HYDRAULICS, AND PUMPS

By

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HYDRODYNAMICS

By George E. Barnes

HYDRAULICS

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HYDRAULIC TURBINES

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PUMPS

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HYDRAULIC COUPLINGS

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HYDRODYNAMICS

By George E. Barnes

1. FUNDAMENTALS

DEFINITION OF A FLUID. A fluid is a homogeneous substance that cannot remain motionless under shearing stress. Both liquids and gases have this characteristic and are therefore classified as fluids. Because of this characteristic, the behavior of fluids under stress and motion differs from the behavior of solids. A fluid is put in motion under shearing stress, and the motion is sustained as long as the stress is applied, however small it may be. Conversely, the motion of a fluid results in shearing stresses between fluid layers having unequal velocities. It follows that in a fluid at rest there can be no internal shear, and that the pressure at any point within the mass is exerted equally in all directions.

VISCOSITY. The shearing stress within a fluid is proportional to the velocity gradient across the sheared section. (See also p. 6-41.) The proportionality factor, inherent in the physical structure and composition of the fluid, is called the *absolute* or *dynamic viscosity* μ (or, more simply, the viscosity) and must be found by test. If τ is the unit shear and dV/dy the velocity gradient across a sheared section of the fluid of width dy,

$$\tau = \mu \frac{dV}{dy} \tag{1}$$

Viscosity is one of the controlling influences on fluid motion. For a given fluid, it varies greatly with temperature and, in general, very little with pressure. The viscosity of liquids decreases with an increase in temperature (see Fig. 5, p. 6-43); that of gases increases with increase in temperature (see Fig. 7, p. 1-15). With few exceptions, change in pressure has no measurable effect on viscosity except at extraordinarily high pressures.

FLUID MECHANICS. Many dependable formulas relating to the flow of water, air, and steam are in use. These formulas do not directly include viscosity (or certain other physical properties of the fluid) and are, therefore, not interchangeable or applicable to the flow of fluids in general. Many advances made through analytical and experimental methods of fluid mechanics have resulted from recognition of these properties as variables, and to development of basic and more rational formulas, applicable to all fluids when their physical properties are known.

PHYSICAL PROPERTIES OF FLUIDS of interest in fluid mechanics are (1) mass density, ρ; (2) viscosity, μ; (3) modulus of elasticity in compression, E; and (4) surface tension σ. Derived properties include (5) the unit weight $\delta = \rho g$; and (6) kinematic viscosity $\nu = \mu/\rho$; and others calculated from the foregoing. The density of gases may be calculated by means of the perfect gas law, or equation of state (see also p. 3-53):

$$pv = RT \tag{2}$$

or

$$p = \delta RT$$

where p = absolute pressure, pounds per square foot; v = unit volume = $1/\delta$, cubic feet per pound; R = the gas constant (53.3 for air); and T = absolute temperature, °F.

The gas constant may be determined approximately from Avogadro's law:

$$R = \frac{1544}{m} \tag{3}$$

where m = molecular weight and 1544 = average universal gas constant.

When the pressure or volume of a gas changes from one state to another, the process may be isothermal, polytropic, or isentropic. The equation of state may be rewritten in conformity with each of these processes as follows:

(*a*) Isothermal or constant temperature change:

$$pv = \text{constant} \tag{4}$$

(*b*) General or polytropic change:

$$pv^n = \text{constant} \tag{5}$$

(*c*) Isentropic change, no heat added or subtracted:

$$pv^k = \text{constant} \tag{6}$$

where

$$k = \frac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}}$$

Physical properties of common fluids are given in Table 1.

Table 1. Physical Properties of Common Fluids

Fluid Property and Symbol	Units of Measurement	Temperature, °F	Fluid: Water	Oil	Air	CO_2	NH_3
Density, ρ	slugs/ft^3	32	1.940	1.84	.00251	.003835	.001495
		60	1.938	1.64	.00237	.003620	.001412
		100	1.927	1.58	.00220	.003365	.001315
		200	1.868	1.40	.00187	.002850	.001111
Viscosity, μ	lb-sec/ft^2	32	3.746×10^{-5}	1.10×10^{-2}	3.67×10^{-7}	2.92×10^{-7}	1.92×10^{-7}
		60	2.359	0.23	3.78	3.05	2.02
		100	1.424	0.06	4.06	3.27	2.19
		200	0.637	0.0105	4.53	3.89	2.72
Modulus of elasticity, E	lb/ft^2	32	41.6×10^6	18.0×10^6	*Note for gases:* $E = p$ (absolute pressure) for isothermal change; $E = kp$ for isentropic change		
		60	44.9	27.0			
		100	47.6	26.9			
		200	44.5	38.9			
Surface tension, σ (in contact with air)	lb/ft	32	.00520	About .00220	Not a property of gases		
		60	.00500				
		100	.00485				
		200	.00413				
Unit weight, $\delta = \rho g$	lb/ft^3	32	62.42	59.0	.08071	.12341	.04813
		60	62.37	54.5	.07625	.11670	.04550
		100	62.00	52.5	.07095	.10830	.04230
		200	60.12	45.1	.06010	.09190	.03580
Kinematic viscosity, $\nu = \mu/\rho$	ft^2/sec	32	1.931×10^{-5}	6.0×10^{-3}	1.460×10^{-4}	$.762 \times 10^{-4}$	1.285×10^{-4}
		60	1.217	1.4	1.595	.842	1.430
		100	0.739	0.38	1.847	.971	1.663
		200	0.341	0.075	2.425	1.708	2.442

THE CONCEPT OF DYNAMIC SIMILARITY stems from Newton and is of fundamental importance in fluid mechanics. From the basic law (force = mass × acceleration), it may be deduced that a *system* of forces, however complex, acting upon a *system* of particles, however disposed, will produce a unique set of motions. In fluid flow, this is called the *flow pattern*. In two systems of particles similarly disposed as to internal and boundary positions, the particles of both systems can be made to traverse geometrically similar paths in proportional times, if the two related force systems are of a kind and if the corresponding forces in the two systems are, each to each, in fixed and equal ratio.

Forces operating in fluid flow are of the following kinds: (1) inertia forces, (2) gravitational forces, (3) viscosity forces, and (4) elasticity forces. In open channel flow, (1) and (2) have dominant influence on the flow pattern; the others may frequently be neglected without sensible error. In pipe flow, or in the motion of a solid through an enveloping fluid (such as an aircraft or submarine) at less than supersonic speeds, (1) and (3) are dominant.

Consider the problem of securing dynamic similarity in two fluid-flow systems, say a model and its prototype, so that the flow pattern for one is the same as for the other; that is, one is a kind of slow-motion picture of the other. The stipulation is that the ratio of inertia forces in model and prototype shall be fixed and equal to the ratio of x forces in model and prototype, where x means (a) gravitational forces, (b) viscosity forces, or (c) elasticity forces. This stipulation is met for (a) when Froude's number $N_F = V^2/gL$ is the same for model and prototype; for (b) when Reynolds' number $N_R = VD\rho/\mu$ is the same for each, and for (c) when Mach number $N_M = V/\sqrt{E/\rho}$ is the same for each. Here L = length, V = velocity, D = characteristic dimension in consistent units, and other symbols are as previously defined.

Using the subscript p for prototype quantities and m for model quantities, we may summarize by saying that dynamic similarity is secured (or that the flow pattern will be the same in model and prototype) when the two systems are geometrically similar, and when the physical quantities, such as velocities, length, or diameter parameters, and the pertinent fluid properties are equated as follows:

For gravitational flow

$$\frac{V_m^2}{gL_m} = \frac{V_p^2}{gL_p} \qquad (N_{F_m} = N_{F_p})$$

For subsonic flow in closed systems

$$\frac{V_m D_m \rho_m}{\mu_m} = \frac{V_p D_p \rho_p}{\mu_p} \qquad (N_{R_m} = N_{R_p})$$

For supersonic flow

$$\frac{V_m}{\sqrt{E_m/\rho_m}} = \frac{V_p}{\sqrt{E_p/\rho_p}} \qquad (N_{M_m} = N_{M_p})$$

The determination of such resistance coefficients as pipe friction factors (see p. 6-37) and lift and drag coefficients (see Section 15), as functions of N_R and N_M, is one important development of fluid mechanics and permits generalization never before secured by empirical means.

2. DIMENSIONAL ANALYSIS

DIMENSIONAL ANALYSIS is an algebraic method for examining physical equations to determine their correct form. The method, of prime importance in fluid mechanics, will be briefly treated for particular applications.

A physical equation is one containing physical quantities such as forces and velocities, all of which (in mechanics) are concepts of mass, length, and time. Thus any physical quantity may be abstractly described by not more than three dimensions (M, L, and T), whatever the units of measurement may be. For examples see Table 2, which lists only a few of the many physical quantities.

For validity, a physical equation must equate equivalent kinds of quantities. For example, in the simple equation $V = \sqrt{2gH}$, we have dimensionally

$$LT^{-1} = (M^0L^0T^0)\,(LT^{-2})^{1/2}(L)^{1/2} = LT^{-1}$$

so that the equation is *dimensionally correct*. It equates a velocity to the product of two quantities which are the dimensional equivalent of a velocity. Furthermore, it is complete, for it states not only that V is a function of $\sqrt{gH}$, but it also shows what the function is, i.e., V is invariably $\sqrt{2}$ times $\sqrt{gH}$. By dimensional analysis, we could find the exponents of g and H and thus discover the dimensionally correct *form* of the equation.

Table 2. Dimensions of Physical Quantities

Physical Quantity	Symbol	Dimensions	Number of Dimensions, k	English Units
Velocity	V	LT^{-1}	2	ft/sec
Acceleration	a (general)			
	g (gravitational)	LT^{-2}	2	ft/sec^2
Force	F	MLT^{-2}	3	lb
Mass density	ρ	ML^{-3}	2	slugs/ft^3
Work	W	ML^2T^{-2}	3	ft-lb
Unit pressure	p	$ML^{-1}T^{-2}$	3	lb/ft^2
Head	H	L	1	ft
Hydraulic radius	R	L	1	ft
Viscosity	μ	$ML^{-1}T^{-1}$	3	lb-sec/ft^2
Modulus of elasticity	E	$ML^{-1}T^{-2}$	3	lb/ft^2
Radians (ratio of angles)	Pure number	$M^0L^0T^0$	None	None
Slope (ratio of distances)	Pure number	$M^0L^0T^0$	None	None
N_F, N_R, N_M (ratio of forces)	Pure number	$M^0L^0T^0$	None	None

We could not find the *function*, however; that would have to be the result of physical experimentation.

EXAMPLE. Suppose $f(V, g, \rho, H) = 0$. That is, let us assume that there is some physical phenomenon that can be expressed as an equation involving V, g, ρ, and H. What are the exponents of these quantities, to form a dimensionally correct equation?

Solution. Assume that one would like to have the expression in terms of V, so that the exponent of V becomes unity. Then

$$V = f'(g, \rho, H) = f'(g)^x(\rho)^y(H)^z$$

or, dimensionally,

$$(LT^{-1}) = (LT^{-2})^x(ML^{-3})^y(L)^z$$

the sum of the exponents of L and of T and of M on one side of the equation must equal the sum on the other side. Equating the exponents for

$$M: \quad 0 = y$$

$$L: \quad 1 = x - 3y + z$$

$$T: \quad -1 = -2x$$

Solving, we get

$$x = 1/2, \quad y = 0, \quad \text{and} \quad z = 1/2$$

Therefore

$$V = f'(g)^{1/2}(\rho)^0(H)^{1/2}$$

or, finally,

$$V = f'\sqrt{gH}$$

The analysis shows (1) that there is no possible physical law relating *all* of the four quantities originally assumed and that ρ, the only term containing mass, must drop out; (2) that assuming the exponent for V = unity, the correct exponents for g and H are $1/2$; and (3) that there is the possibility of V varying as $\sqrt{gH}$. The proportionality constant must be evaluated by test. The relationship may not exist at all; or it may exist and prove to be either simple or complex. The equation may also be written $f(V^2/gH) = 0$ or, more significantly, $f(N_F) = 0$.

BUCKINGHAM'S π THEOREM. The method of dimensional analysis described above becomes cumbersome for any but simple equations; the more general method devised by Buckingham, called the π theorem, is preferred. The method is here outlined and applied to illustrate the problem of developing a dimensionally correct general equation for the flow of a fluid through a pipe.

Argument. The general form of a physical equation is

$$f(Q_1, Q_2, Q_3, \cdots Q_n) = 0$$

where the terms Q stand for the physical quantities involved. Let there be n such quantities, and let k dimensions be sufficient for describing all the quantities Q. k in mechanics does not exceed three (M, L, and T).

The equation sought will be of the form

$$f(\pi_1, \pi_2, \cdots \pi_{n-k}) = 0$$

where the terms π stand for a dimensionless ratio obtained by combining the Q terms to the number of $k + 1$, with suitable exponents. Of the $(k + 1)$ Q terms, k will be common to all π terms, and must be chosen so that they are not of a kind, i.e., they must be described by different sets of dimensions.

Method.

1. Determine n and k, and consequently the number of π terms to appear in the final equation $= n - k$.
2. Choose the Q terms to appear in all values of π, to the number of k, and to be incommensurate as to dimensions.
3. Assign in succession to each value of π a different Q term, not among those in (2).
4. Find the exponents in each π term, by the simple algebra of the previous example (p. 5-05).
5. Write the equation in terms of the π values, and perform such algebraic operations as may be necessary to rearrange the terms, for an equation in suitable form.

Example. Let it be required to find a dimensionally correct formula for the flow of a fluid through a pipe, on the assumption that the following quantities are interdependent in such an equation:

V	= mean velocity	LT^{-1}
L	= pipe length	L
D	= pipe diameter	L
ρ	= mass density	ML^{-3}
μ	= viscosity of the fluid	$ML^{-1}T^{-1}$
e	= pipe roughness, as arbitrarily described by the linear height of protuberances of the inner surface	L
p	= pressure drop as a measure of energy loss	$ML^{-1}T^{-2}$

Then

$$f(V, L, D, \rho, \mu, e, p) = 0$$

$$\left.\begin{matrix} n = 7 \\ k = 3 \end{matrix}\right\} n - k$$

Therefore the equation will be in the form:

$$f(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

Choose the k quantities common to all values of π, say V (described by L and T), D (described by L), and ρ (described by M and L). Then:

$$\pi_1 = D^x V^y \rho^z \mu$$
$$\pi_2 = D^a V^b \rho^c L$$
$$\pi_3 = D^m V^n \rho^k e$$
$$\pi_4 = D^r V^s \rho^t p$$

Solving dimensionally for $\pi_1 = (L)^x(LT^{-1})^y(ML^{-3})^z(ML^{-1}T^{-1})$,

$$\begin{matrix} M: & z + 1 = 0 \\ L: & x + y - 3z - 1 = 0 \\ T: & -y - 1 = 0 \end{matrix} \quad \left\{\begin{matrix} x = -1 \\ y = -1 \\ z = -1 \end{matrix}\right.$$

From which

$$\pi_1 = \frac{\mu}{VD\rho} = \frac{1}{N_R}$$

Solving dimensionally for $\pi_2 = (L)^a(LT^{-1})^b(ML^{-3})^c(L)$

$$\begin{matrix} M: & c = 0 \\ L: & a + b - 3c + 1 = 0 \\ T: & -b = 0 \end{matrix} \quad \left\{\begin{matrix} a = -1 \\ b = 0 \\ c = 0 \end{matrix}\right.$$

From which

$$\pi_2 = \frac{L}{D}$$

Solving dimensionally for $\pi_3 = (L)^m(LT^{-1})^n(ML^{-3})^k(L)$

$$\begin{matrix} M: & k = 0 \\ L: & m + n - 3k + 1 = 0 \\ T: & -n = 0 \end{matrix} \quad \left\{\begin{matrix} m = -1 \\ n = 0 \\ k = 0 \end{matrix}\right.$$

from which

$$\pi_3 = \frac{e}{D}$$

Solving dimensionally for $\pi_4 = (L)^r(LT^{-1})^s(ML^{-3})^t(ML^{-1}T^{-2})$

$$
\begin{array}{lrr}
M: & t + 1 = 0 & 0 \\
L: & r + s - 3t - 1 = 0 & -2 \\
T: & -s - 2 = 0 & -1
\end{array}
$$

From which

$$\pi_4 \quad \frac{p}{V^2\rho}$$

The equation sought is then of the form

$$f\left(\frac{1}{N_R}, \frac{L}{D}, \frac{e}{D}, \frac{p}{V^2\rho}\right) = 0$$

We are now at liberty to perform any convenient algebraic transformations on this equation, provided only that we do not disturb the dimensional relationships described. Thus a dimensionally equivalent equation would be

$$\frac{p}{\rho V^2} = f'\left(N_R, \frac{e}{D}\right)\frac{L}{D}$$

Again, dividing each side of the equation by g (the gravitational constant) and using the pure number "2" to express the right-hand side in terms of velocity head, we have finally:

$$\frac{p}{\delta} = \left[\phi\left(N_R, \frac{e}{D}\right)\right]\frac{L}{D}\frac{V^2}{2g}$$

which is the Darcy formula.

The importance of the equation in this form cannot be overestimated. The *friction factor* (term in brackets) is shown to depend on the *relative roughness* of the pipe surface, as given by e/D, and on the value of *Reynolds' number*. (See also p. 6-35). Once this relationship is determined by test, a *universal* friction factor is obtained for the flow of any fluid in any pipe of circular section.

3. APPLICATIONS

RESULTS OF THE METHOD. This article demonstrates one or two of the most important analytical procedures in fluid mechanics and brings out the significance of the results obtained in contradistinction to those reached by empirical methods. If the basic formula above, obtained by abstract reasoning and dimensional analysis, can be validated by experiment (as actually is the case), then the results of relatively few experiments will give useful coefficients. These coefficients are restricted neither to the kind of fluid used in the tests nor to the range of velocities measured nor even to the particular kind of pipe employed in the experiments: the solution is dimensionless and perfectly general.

THE FRICTION FACTOR f. The Darcy formula for flow in pipes is written:

$$\Delta H = \frac{fL}{D}\frac{V^2}{2g}$$

in which ΔH is the head loss due to pipe friction, and where, from the preceding material, $f = \phi(e/D, N_R)$. This is in the form of $x = \phi(y, z)$ so that it is possible to show the relationship between x (the friction factor) and z (Reynolds' number) as a two-dimensional plot, with a family of curves for each of which e/D is constant, as shown in Fig. 1, for subsonic velocities. The nature of the function (the manner in which f varies with N_R) must be found by experiment. If the relation is simple it may be expressed by a mathematical equation; for example, in the range of laminar flow (sometimes called streamline flow or viscous flow) $f = 64/N_R$. If the relation is complex, it may be possible to express it only approximately by an equation.

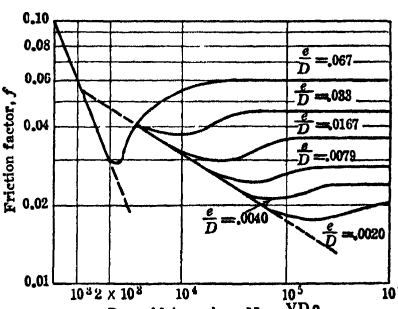

FIG. 1. Resistance coefficients for artificially roughened pipes. (Adapted from Rouse, *Fluid Mechanics for Hydraulics Engineers*, p. 250, McGraw-Hill Book Co., 1938)

The curves shown in Fig. 1 are after Nikuradse, and the values of e/D pertain to pipes with artificial lining of sands of selected grain size, to attain geometrical similarity throughout a range of pipe sizes for different degrees of roughness.

Although these curves prove the argument presented by the formula they cannot, unfortunately, be applied directly to the solution of engineering problems, because there is no commercial pipe of constant e/D. The surface texture of cast-iron pipe, for example, does not change much with size. The roughness e is approximately constant, so that e/D would diminish with increasing diameters. Therefore a single curve for cast-iron pipe would cut across curves of e/D = constant in Fig. 1.

Pigott and Kemler have related these curves to others for pipes of commercial nature, as shown in Fig. 2. Here the curves are numbered, and a listing is given with the chart to show what kind of pipe and what range of diameters apply to each. Such curves permit the use of universal friction factors for the more common types of commercial pipe, in terms of their effective roughness and the numerical value of N_R.

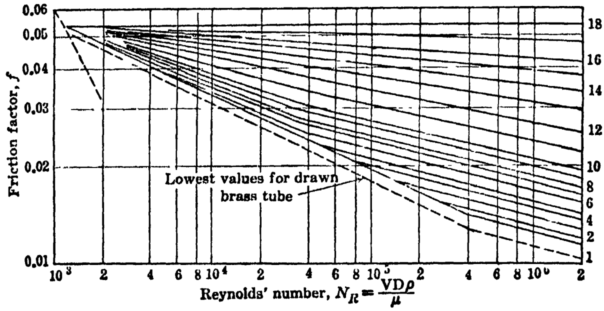

FIG. 2. Resistance coefficients for commercial pipe. (Adapted from Cox and Germano, *Fluid Mechanics*, D. Van Nostrand Co., 1941) See also Fig. 1, p. 6-37, for additional data.

OTHER RESISTANCE COEFFICIENTS may be evaluated in similar fashion:

Discharge Coefficient for an Orifice. Assume that the velocity through orifices of a given type, with geometrically similar approach conditions, is given by

$$V = f(D, H, \mu, \rho)$$

where D is the diameter and H the head. Dimensional analysis gives

$$V = [\phi N_R]\sqrt{2gH}$$

where the discharge coefficient $[\phi N_R]$ depends only on N_R.

Figure 3 illustrates curves found by experiment (after G. L. Tuve), each of which corresponds to a certain geometry of orifice and feed pipe.

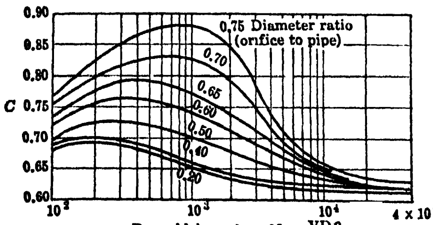

FIG. 3. Variation of orifice discharge coefficient with Reynolds' number. (After G. L. Tuve)

Drag Coefficient. Assume that the unit resistance (drag force divided by area normal to flow) of a fluid flowing past a solid of given shape is given by

$$\frac{F}{A} = f(D, V, \mu, \rho, E)$$

where E is the coefficient of compressibility of the fluid. Dimensional analysis leads to:

$$F = [(\phi N_R, N_M)]\rho V^2$$

where the drag coefficient depends on N_R (in the range of turbulent subsonic flow) and on Mach number (in the range of supersonic flow).

Figure 4 illustrates curves found by experiment, each of which corresponds to a certain geometry of shape for the object which obstructs the flow.

SHAPE FACTOR. The flow pattern depends on the geometry of the boundaries confining the flow and on the physical properties of the fluid. Included in the boundary geometry is the shape factor. In incompressible flow it is difficult to predict the influence of shape on the flow pattern without resorting to complex mathematical procedures. In general, predictions are based on experiment.

In pipe flow, for instance, the shape factor is that imposed by a circular conduit, with certain surface roughness. The friction factor for noncircular pipes is not known, in general, but satisfactory approximations can be made by assuming that a conduit (square, rectangular, etc.), with hydraulic radius R, will give the same resistance to flow as a circular pipe whose diameter is $4R$, since for a circular pipe $R = D/4$. However, the hydraulic radius is not a completely satisfactory parameter for the flow pattern, particularly where the conduit section is not reasonably concentric about a point.

Where it is not possible to conduct experiments to find the resistance coefficients for particular shapes, the engineer must carry out calculations using conformal transformations, and conformal mapping of source and sink distributions. The art of analysis by such methods has been developed to a high degree by aerodynamicists since the advent of the airplane in important commercial and military fields.

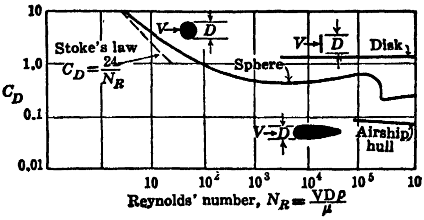

FIG. 4. Subsonic drag coefficients for sphere, disk, and streamlined body. Above $M = 0.7$, N_R has negligible effect on C_D. (Adapted from Vennard, *Elementary Fluid Mechanics*, p. 297, John Wiley and Sons, 1940)

HYDRAULICS

By William P. Creager

4. FLOW THROUGH ORIFICES AND SHORT TUBES

ORIFICES AND SHORT TUBES. Figure 1 shows typical examples of orifices and short tubes. A knowledge of the laws of the flow of water through them is necessary to determine the discharge through sluiceways and the entrances to conduits. If the entrance is not properly shaped, a contraction of the jet occurs as in a, c, and h (Fig. 1), and the area of the jet is not as great as that of the orifice or tube. For properly rounded approaches to orifices, as in b and e, and in the constant-diameter short tubes in d, f, and g, the diameter of the jet equals the area of the orifice or tube. In short tubes without rounded entrances, the contraction does occur; but the jet, with certain exceptions as explained below, re-expands as indicated, a partial vacuum occurring just inside the entrance.

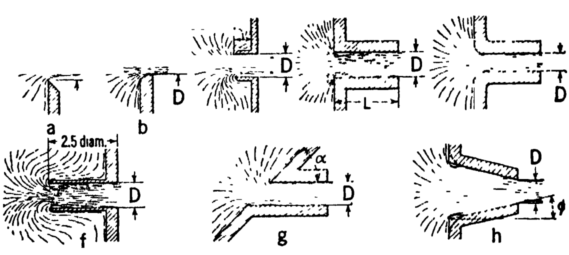

FIG. 1. Types of orifice.

Let H = head of water, feet, on the center line of a freely flowing orifice or tube, or the difference in water level for a submerged orifice or tube; a = area, square feet, of the orifice or tube; V = theoretical velocity, feet per second, corresponding to head H; g = acceleration of gravity = 32.2 ft per sec; Q = discharge, cubic feet per second; C_2 = coefficient of contraction, or ratio of area of jet to area of orifice or tube; C_1 = coefficient of friction; and C = coefficient of discharge.

The general equation for the velocity of spouting water is

$$V = \sqrt{2gH} \tag{1}$$

Considering friction, the actual velocity due to head H is $V = C_1\sqrt{2gH}$.

The discharge is equal to the product of the actual velocity and the area of the jet, or, since the area of the jet is C_2a, $Q = VC_2a$, or $Q = C_1C_2a\sqrt{2gH}$.

In experiments conducted to determine the discharge through orifices and tubes, the coefficient of friction, C_1, and the coefficient of contraction, C_2, are combined and the general equation for the discharge is

$$Q = Ca\sqrt{2gH} \tag{2}$$

The value of C_1 varies with the shape of the orifice or tube. Table 1 gives average experimental values of C, for use in eq. 2 for several types of orifices and tubes.*

These orifices and tubes are circular unless otherwise noted. According to Bovey and other authorities on orifices in thin plates (Fig. 1*a*), the value of C for circular orifices is about 2% less than for square orifices; 3 to 4% less than for rectangular orifices having a ratio of length to height of 4; and 5 to 7% less than for rectangular orifices having a ratio of length to height of 10. A similar relation probably exists for tubes having square-cornered entrances. The coefficient C is not greatly affected by submergence.

Coefficients for short tubes apply only to heads less than about 40 ft. For higher heads the expansion heretofore explained does not occur, and the coefficients C approach those for orifices of similar type.

The expansion of the jet within the short tubes may not occur if they are not submerged and if sufficient friction is lacking, i.e., the jet, after contracting, may pass through the tube without touching its sides, if under a high head. Even if the trajectory is such that the jet strikes the bottom of the tube, expansion will not occur if the friction along the bottom is insufficient.

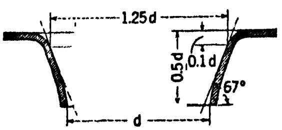

FIG. 2. Standard mouthpiece. (A. H. Gibson, after Weisbach)

The shape of the jet from a sharp-cornered round orifice (Fig. 1*a*) between the orifice and the vena contracta, or section at which the area of the jet becomes constant, is indicated in Fig. 2 and should be used for rounded entrances as in Fig. 1*e*.

Values of C for vertical orifices with full contraction (Fig. 1*a*) with a free discharge into the air, with the inner face of the plate, in which the orifice is pierced, plane, and with sharp inner corners so that the escaping vein only touches these inner edges are given in Table 2, an abridgment of tables compiled by Hamilton Smith, Jr., from many experimental data. The coefficients in Tables 1 and 2 have been obtained from experiments under ideal conditions.

Table 1. Coefficients of Discharge, C, through Orifices and Tubes for Equation 2

(Circular Except as Noted)

Fig. No.	Type of Orifice								
1*a*	In thin plate, C =								0.60
1*b*	Rounded, C =								0.97
1c	Inwardly projecting, C =								0.50
1*d*	Short tube, sharp-cornered entrance: *								
	L/D =	0	0.25	0.50	0.75	1.00	1.50	2.50	3.50
	C =	0.60	0.63	0.67	0.72	0.76	0.79	0.80	0.80
1*e*	Short tube, rounded entrance, C =								0.97
1*f*	Inwardly projecting, sharp-cornered entrance, C =								0.72–0.80
1*g*	Inclined short tube sharp-cornered entrance: †								
	α =	90°	80°	70°	60°	50°	40°	30°	
	C =	0.82	0.80	0.78	0.76	0.75	0.73	0.72	
1*h*	Convergent short tube: ‡								
	ϕ =	0°	5.75°	11.25°	22.5°	45°			
	C (sharp-cornered entrance) =	0.82	0.94	0.92	0.85	......			
	C (rounded entrance) =	0.97	0.95	0.92	0.88	0.75			

* From experiments by Rogers and Smith on submerged tubes (*Eng. News*, vol. lxxvi, p. 827). The coefficient for L/D = 2.5 and more has been found by other experimenters to be 0.82.

† According to Weisbach.

‡ H. W. King after Unwin.

Equation 2 corrected for velocity of approach, may be written

$$Q = Ca\sqrt{2g(H + \beta h_v)} \tag{3}$$

where h_v = head corresponding to average velocity of approach, and β a coefficient which must be determined experimentally. Unfortunately, β is not well known for many types of orifices, and may vary between 1.0 and 2.0, depending on the location and relative size of the orifice.

* Where the head is large in comparison with size of orifice or tube. For head on orifice equal to (1.5 × height of orifice), results are about 1% too large.

Table 2. Values of Coefficient C for Vertical Orifices with Sharp Edges, Full Contraction, and Free Discharge into Air

Head from Center of Orifice H	Square Orifices. Length of the Side of the Square in Feet							Circular Orifices. Diameters in Feet						
	.02	.05	.10	.20	.40	.80	1.0	.02	.05	.10	.20	.40	.80	1.0
0.4	....	.637	.621	..	....	. .	.....	.....	.637	.618		.	.	.....
0.6	.660	.630	.617	.605	.601	.596	.....	.655	.624	.613	.601	.596	.590	.....
1.0	.648	.622	.613	.605	.603	.600	.599	.644	.617	.608	.600	.598	.593	.591
6.0	.623	.609	.605	.604	.604	.602	.602	.618	.604	.600	.598	.598	.596	.596
10.	.616	.606	.604	.603	.603	.602	.601	.611	.601	.598	.597	.597	.596	.595
20.	.606	.603	.602	.602	.601	.601	.600	.601	.598	.596	.596	.596	.595	.594
100. (?)	.599	.598	.598	.598	.598	.598	.598	.593	.592	.592	.592	.592	.592	.592

RECTANGULAR ORIFICES AND TUBES UNDER LOW HEAD. If the area of the freely flowing orifice or tube is large in comparison with the head, eq. 1 should be written

$$Q = \frac{2LC}{3}\sqrt{2g}\,(H_1^{3/2} - H_2^{3/2}) \tag{4}$$

where H_1 and H_2 = heads on the bottom and top of the orifice, respectively. If the orifice is completely submerged, the head is the difference in level between the upper and lower water surfaces, and eq. 2 applies.

If, in eq. 4 the top of the orifice is at the water surface, $H_2 = 0$, and the equation reduces to

$$Q = \frac{2LC}{3}\sqrt{2g}\,H^{3/2} \tag{5}$$

This is the basic theoretical equation for discharge over weirs. Equation 2 can be used with an error not greater than 1% if the depth of water to the top of the freely flowing orifice is greater than twice the height of the orifice.

DISCHARGE THROUGH SLUICE GATES. Sluice gates are made in a variety of forms. Many types of sluices, controlled by sluice gates, are in reality short conduits in which skin friction is a large percentage of the total loss. A discussion of the losses through conduits is given in Article 5 of this section. If the sluice is short, the discharge may be considered as that through a short tube (Figs. 1*d* to 1*h*, inclusive), or, if the sluice gate is in a thin wall (Figs. 1*a* to 1*c*, inclusive), the discharge may be obtained from eqs. 2 or 3 with the proper coefficient of discharge selected from Table 1 according to the details of the sluice gate opening.

5. FLOW OF WATER IN CONDUITS

BERNOULLI'S THEOREM. The velocity head, or head required to produce a given velocity, is

$$\text{Velocity head} = h = \frac{V^2}{2g} \tag{5a}$$

According to Bernoulli's theorem, the general law governing steady flow of water in conduits is: For steady flow in a conduit, the sum of the velocity head, the pressure head, and the potential head at any point, A, is equal to the sum of the corresponding heads at any upstream point, B, less the frictional resistance between the points A and B (Fig. 3).

Expressed mathematically, we have

$$\frac{V^2}{2g} + h_p + h_e = \frac{V_1^2}{2g} + h_p' + h_e' - h_f, \tag{5b}$$

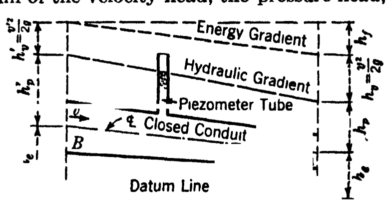

Fig. 3. Closed conduits.

where V and V_1 = the velocities, feet per second, at points A and B, respectively; h_p and h_p' = corresponding pressure heads, feet; h_e and h_e' = the corresponding potential heads above a common datum plane, feet; h_f = total frictional resistance or lost head, feet, between points A and B.

If a vertical or oblique tube (Fig. 3) is inserted in a pipe containing water under pressure, the water will rise in the tube, and the vertical height to which it rises will be the head, h_p, producing the pressure at the point where the tube is attached. Such a tube is called a piezometer. If water in the piezometer falls below its proper level, it shows that pressure in the main pipe has been reduced by an obstruction between the piezometer and the reservoir. If water rises above its proper level, it indicates that pressure has been increased by an obstruction beyond the piezometer.

If we imagine a pipe full of water to be provided with a number of piezometers, a line joining the tops of the columns of water in them is the hydraulic gradient, and $h_p + (V^2/2g)$ defines the corresponding elevation of the energy gradient.

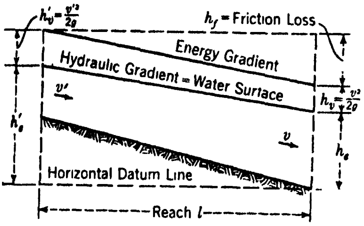

FIG. 4. Open conduits.

In an open conduit, h_e and h_e' are measured to the water surface, as in Fig. 4, and h_p and h_p', measured to the same place, are equal to zero. Therefore, the hydraulic gradient for open conduits is at water surface.

The head lost by friction and eddies between any two points in a closed or open conduit is equal to the drop in level of the energy gradient between the two points. This in turn is equal to the drop in level of the hydraulic gradient, as shown by piezometer readings (or water surface in open conduits), plus the velocity head at the upper point and less the velocity head at the lower point.

FLOW IN CLOSED CONDUITS. The quantity of water discharged through a pipe depends on the *head*, i.e., the vertical distance between the level surface of still water in the chamber at the entrance end of the pipe and the level of the center of the discharge end of the pipe; also on the length of the pipe, on the character of its interior surface as to smoothness, and on the number and sharpness of the bends. It is independent of the position of the pipe, as horizontal, or inclined upwards or downwards.

The head, instead of being an actual distance between levels, may be caused by pressure, as by a pump, in which case the head is calculated as a vertical distance corresponding to the pressure, 1 psi = 2.309 ft head, or 1 ft head = 0.433 psi.

The total head operating to cause flow is divided into four parts: (1) The *velocity head*, which is the height through which a body must fall to acquire the velocity with which the water flows into the pipe, is $V^2 \div 2g$, in which V is the velocity in feet per second and $2g = 64.32$. (2) The *entry head* is that required to overcome the resistance to entrance to the pipe. With sharp-edged entrance the entry head is about one-half the velocity head; with smooth rounded entrance the entry head is inappreciable. (3) The *friction head* is due to the frictional resistance to flow within the pipe. (4) The *eddy losses* are due to bends, valves, and sudden changes in area of the pipe. All these heads are lost except the velocity head, a large part of which may be regained at any point by gradual enlargement of the pipe.

FLOW IN OPEN CONDUITS. The theory of the flow of water in closed conduits applies directly to the flow of water in open conduits. In the case of open conduits, however, the elevation of the surface of the water is the piezometer readings.

LOSS OF HEAD IN CONDUITS. The loss of head in conduits may be divided into two general groups: (1) *Eddy losses*, caused by sudden changes in the direction of flow, as at bends, branches, etc., or by sudden changes in velocity due to sudden changes in area, as at the entrance, sudden enlargements, and valves. (2) *Skin friction* in straight, uniform conduits. These losses are described in detail in the material that follows.

EDDY LOSSES IN CONDUITS. It is convenient to measure eddy losses in terms of the velocity head of the flowing water. The velocity head, or head required to produce a given velocity, may be obtained by transposing eq. 1:

$$\text{Velocity head} = h_v = \frac{V^2}{2g} \tag{6}$$

Then h_f, the head lost at any point in the conduit due to eddies, is

$$\text{Eddy loss} = h_f = Kh_v = K\frac{V^2}{2g} \tag{7}$$

where K = coefficient of eddy loss and V = highest velocity at the point under consideration.

LOSSES AT CONDUIT ENTRANCES. A direct relation exists between the coefficient of discharge for short tubes and the coefficient of eddy loss K, eq. 7, which can be derived as follows. A drop in pressure or head at the entrance to a conduit is required for two purposes: (*a*) velocity head to provide the necessary velocity; (*b*) head to overcome friction due to eddies. Or,

$$H = h_v + h_f = \frac{V^2}{2g} + K\frac{V^2}{2g} = \frac{V^2}{2g}(1 + K)$$

As $Q = aV$, $H = (Q^2/2ga^2)(1 + K)$. Also from eq. 2,

$$H = \frac{Q^2}{2ga^2} \times \frac{1}{C^2} \quad \text{and} \quad K = \frac{1}{C^2} - 1 \tag{8}$$

The loss of head in short tubes where there is no residual contraction in the jet, as in Figs. 1*d*, 1*e*, 1*f*, and 1*g*, may be assumed to be the same for similar entrances to closed conduits. Values of K derived from the experimental values of C are given in Table 3.

Table 3. Coefficients for Eddy Loss for Equation 7

Fig. No.	Type of Conduit						
1d	Short tube with sharp-cornered entrance, K =						0.56
1e	Short tube with rounded entrance, K =						0.06
1f	Inwardly projecting tube with sharp-cornered entrance,* K =						0.56–0.93
	Inclined tube with sharp-cornered entrance:						
1g	α = 90°	80°	70°	60°	50°	40°	30°
	K = 0.49	0.56	0.65	0.73	0.78	0.88	0.93

* Depending on distance of projection.

EDDY LOSSES AT CONDUIT BENDS. (For closed conduits, see Section 6.) Few experiments have been made on the losses in bends of open conduits. The few experiments that have been made seem to indicate that the loss due to bends in open conduits is much less than for closed conduits, and values equal to one-half of those in closed conduits are recommended.

Recommended safe values for K for miscellaneous closed conduit fittings are given in Fig. 5.

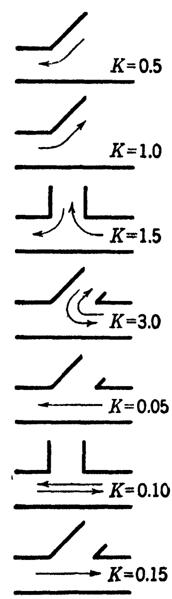

Fig. 5. Values of K for miscellaneous fittings.

EDDY LOSSES AT CONDUIT VALVES. The value of K for use in eq. 7 for wide-open gate valves is probably less than 0.1, although few experiments are available. E. A. Dow's experiments on the disk-arm type of butterfly valves indicate a value of

$$K = \frac{t}{d} \tag{9}$$

where t = thickness of valve disk and d = valve diameter.

The value of V for use in eq. 7 is that used for the normal section of the conduit.

On the basis of three needle-valve experiments, the following equation has been devised:

$$K = \frac{0.183}{\sqrt[3]{d}} \tag{10}$$

where d = diameter at small end, feet. The value of V for use in eq. 7 is that for the small end of the valve.

Equation 10 applies only to needle valves in closed conduits. For free outlets the loss due to the valve can be determined by eq. 2. In this case a is the smaller area of the valve in square feet and C is the coefficient of discharge, varying from 0.64 to 0.76, depending on the make of valve.

MISCELLANEOUS CONDUIT EDDY LOSSES. As all losses other than those due to skin friction and bends are due to sudden changes in section of the conduit, knowledge of the laws governing losses due to sudden contractions and enlargements will assist materially in determining losses caused by various irregularities in conduits. An equation giving losses correct within 10% for values of $(V_1 - V_2)$, between 1 and 13 ft per sec is

$$h_f = \frac{(V_1 - V_2)^2}{2g} \tag{11}$$

For losses due to gradual enlargements in a closed conduit Etcheverry gives

$$K \left(1 - \frac{a_1}{a_2}\right)^2 \sin\theta \tag{12}$$

where θ = angle formed by intersection of one side of the taper with the center line and a_1 and a_2 = areas of the smaller and larger sections respectively.

To determine the value of K for sudden enlargements in open conduits and for the junction of a closed and open conduit the value of K should be taken from the equation

$$K = \left(1 - \frac{a_1^2}{a_2^2}\right)C \tag{13}$$

The value for V for use in eq. 7 is that in the smaller section. The coefficient C has a value depending on the nature of the enlargement. Scobey (Ref. 1) recommends the following values for the coefficient C: for square enlargements, $C = 0.75$; for enlargements where the angle of flare of each side of the conduit makes an angle of 30 degrees with the center line, $C = 0.50$; and for perfectly designed (Ref. 2) enlargements, $C = 0.25$.

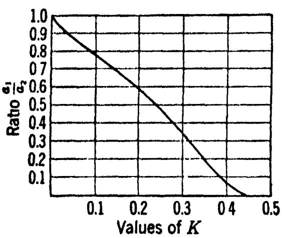

FIG. 6. Values of K for sudden contractions in closed conduits.

For sudden contractions in closed conduits, values of K are given in Fig. 6, using a coefficient of contraction for a sharp-edged orifice equal to 0.60. The value of V for use in eq. 7 is that in the smaller section. Losses due to gradual contraction are very small. The loss in gradual contractions in open conduits is negligible.

SKIN FRICTION CONDUIT LOSSES. Most of the many empirical equations for the uniform flow of water in conduits are based on the equation developed by Chezy in 1775.

$$V = CR^{1/2}S^{1/2} \quad \text{or, in its more general form,} \quad V = CR^xS^y \tag{14}$$

where V = average velocity, feet per second; R = hydraulic mean radius, feet, being the cross-sectional area divided by the wetted perimeter; S = sine of the angle of slope of the energy gradient; C, x and y = coefficients depending on the type of conduit and the condition of the wetted perimeter, all of which must be determined by experiment. A sufficient number of experiments have not been made to enable the engineer to adopt precise coefficients C (or its function n in Kutter's and Manning's equations, given below).

The character of the wetted perimeter is subject to considerable change during the life of the conduit, because of corrosion, tuberculation, growth of fungi, weeds, and other vegetation, silt deposits, scour, ice, etc. The chief difficulty in the use of existing experimental data is inability to define the condition of the wetted perimeter of the experimental conduit. For these reasons, refinement in flow equations seems unwarranted at present, and a considerable factor of safety must be adopted to compensate for possible errors in the choice of the coefficient C.

Tabulations given hereinafter are based on an extensive study by F. C. Scobey of available sources of information regarding experimental determination of the friction coefficient. (Ref. 3). The values corresponding to best and worst conditions embrace practically all variations in the data studied.

Unless otherwise stated, the friction coefficients in the tables are based on straight or slightly sinuous conduits free from the following influences: (1) curvature, other than slightly sinuous; (2) settlement of open conduits or other defects of construction; (3) sediments, rocks, or other deposits washed or fallen in; (4) plant growth, moss, etc.; (5) ice covering; (6) wind movement.

All experimental data were adjusted as closely as possible to correct for curvature, so that all coefficients are practically for straight conduits. Obviously, the results of defects in construction, settlement, and obstructions are indeterminate, although an attempt has been made in the tabulation to give approximate coefficients for certain conduits which should cover the range of reasonable maintenance.

Kutter's equation, or its approximation, Manning's equation, is used for flow in open channels, and all other skin friction equations herein given are used for flow in closed conduits.

OPEN CONDUITS.

Kutter's equation is

$$V = \left[\frac{\dfrac{1.811}{n} + 41.66 + \dfrac{0.00281}{S}}{1 + \left(41.66 + \dfrac{0.00281}{S}\right)\dfrac{n}{\sqrt{R}}}\right]\sqrt{RS} \tag{15}$$

where n = coefficient of roughness; R = hydraulic mean radius, feet; and S = sine of angle of slope of the energy gradient.

Manning's equation

$$V = \frac{1.486R^{2/3}S^{1/2}}{n} \tag{16}$$

is a simplified approximation of Kutter's equation, and the same values of n that would be adopted for Kutter's equation are to be used.

The Manning and Kutter equations agree fairly closely for values of n between 0.012 and 0.020.

Values of Kutter's Coefficient, n

In dry excavated earth canals the value of n increases with the life of the canal unless constantly maintained. Slightly silted waters will "slick" over an original rough surface so that the value of n becomes less. Heavily silted water will decrease n, but also will decrease the area of the water prism. Best conditions are found in tough silt or clay soils, with velocities below scouring limits. $n = 0.016$ may be acquired by silt deposit free from growths.

New canals in sandy loam to clay loam range from class above to one next below, and $n = 0.020$.

Medium to Large Canals. The accepted value for *medium to large canals* in firm earth or gravelly loam with silty water, with reasonable maintenance, is $n = 0.0225$.

Small ditches, easily influenced by slight roughness, and *larger canals poorly maintained,* $n = 0.025$.

Mountain power canals, with cobble bottoms but without finer materials for a graded bedding, $n = 0.028$.

Dredged earth canals are rougher than those excavated by hand or bulldozers. Likewise, a dipper dredge leaves a rougher bottom than a drag line. Differences in value of n are brought about largely by adaptability of soil types and silt in the water to smooth over the original roughness. For best conditions $n = 0.0225$; probable conditions, $n = 0.030$; worst conditions, without neglect of maintenance, $n = 0.040$.

For canals excavated in rock the net section, neglecting possible overbreak, should be used. It is possible for excavation to be done in horizontally stratified rock, resulting in a very smooth bottom. Such canals, if very wide, will have a low value of n as the rough sides have relatively little influence and, if the canal is of ordinary size and no attempt is made to smooth the sides, it is considered that the minimum value of $n = 0.023$.

For usual or probable conditions, with care in smoothing the rock cut by breaking off projections, $n = 0.033$. Under worst possible conditions, there is no limit, but it is seldom above $n = 0.040$. Silt and gravel deposits in rock canals may lower n by filling in the holes in the bottom.

Natural Channels. It is impossible to describe accurately the conditions of a natural channel that correspond to any given value of n. The best natural channels have a value of n seldom below $n = 0.025$. Average natural channels have a value of n probably in the neighborhood of $n = 0.030$. For the worst possible conditions, there is no limit. Judgment and experience are necessary to fix the value of n accurately for natural channels (Ref. 4).

Concrete Linings. Best possible, with neat cement, extremely well-troweled surface, $n = 0.010$. This value is seldom realized in practical construction.

The highest grade of practical concrete linings in best condition, with surface troweled as smooth as hand-troweled sidewalks, and expansion joints perfectly smooth: Best, $n = 0.011$; probable, $n = 0.012$; worst, $n = 0.013$.

Surface as left by smooth jointed forms, or roughly troweled, and expansion joints fair. The value usually adopted for concrete lining: Best, $n = 0.013$; probable, $n = 0.014$; worst, $n = 0.015$.

Concrete having prominent form marks, or previous types subject to deposits of gravel on the bottom: Best, $n = 0.015$; probable, $n = 0.016$; worst, or probable maximum value not subject to rejection because of bad workmanship, $n = 0.018$. If liable to a growth of moss, the foregoing values should be increased by adding 0.002.

Gunnite Linings. Concrete linings deposited by a cement gun, from the inside: best, if lightly troweled, $n = 0.014$; if scrubbed with wire brush, $n = 0.016$; if not scrubbed, $n = 0.019$; worst, for poor workmanship, $n = 0.021$.

MISCELLANEOUS MASONRY LININGS. Glazed brickwork: Best, $n = 0.011$; probable, $n = 0.013$; worst, $n = 0.015$.

Brick in cement mortar: Best, $n = 0.012$; probable, $n = 0.015$; worst, $n = 0.017$.

Dressed ashlar surface: Best, $n = 0.013$; probable, $n = 0.015$; worst, $n = 0.017$.

For bench flume, consisting of natural rock surface for the uphill side, a smooth concrete retaining wall on the downhill side, and with a floor between, lined with concrete and clean, the uphill side being without projecting points, $n = 0.020$.

For the same construction, but with the floor covered with sand or gravel, or left as excavated without projections, uphill side with a few projecting points, such as obtained with careful excavation in hard rock, $n = 0.025$.

Cement-rubble surface: Best, $n = 0.017$; probable, $n = 0.025$; worst, $n = 0.030$.

Dry-rubble surface: Best, $n = 0.025$; probable, $n = 0.033$; worst, $n = 0.035$.

WOODEN BOX FLUMES. Planed lumber, longitudinal boards sides and bottom: Best, $n = 0.011$; probable, $n = 0.014$; worst, after years of service, $n = 0.018$.

Unsurfaced lumber, longitudinal boards sides and bottom: Best, $n = 0.012$; probable, $n = 0.015$; worst, after years of service, $n = 0.018$.

Roofing paper lining varies with the type, generally from $n = 0.010$ to $n = 0.017$.

Wood-stave Flumes. Creosoted: Best, $n = 0.011$; probable, $n = 0.012$; worst, $n = 0.014$. Untreated: Best, $n = 0.010$; probable, $n = 0.012$; worst, $n = 0.014$.

SMOOTH INTERIOR STEEL FLUMES. For smooth-interior flumes, as manufactured and erected under various trade names, when unpainted: best, $n = 0.0105$; probable, $n = 0.012$; worst, $n = 0.014$. When painted: Best, $n = 0.012$; probable, $n = 0.013$; worst, $n = 0.017$.

Friction Coefficient in Closed Conduits

STEEL PIPE. (See also Section 6.) Scobey's equation for flow in steel pipe (Ref. 5) of 4 in. size and larger is

$$V = C_1 R^{0.58} S^{0.526} \tag{17}$$

in which C_1 ranges from 154 to 120 for new continuous-interior pipe to new full-riveted pipe, respectively, and decreases from this initial value about 8% for each 10 years in service.

MISCELLANEOUS CLOSED CONDUITS.

Williams and Hazen's equation as published in their *Hydraulic Tables* (Ref. 6) is

$$V = 1.32 C R^{0.63} S^{0.54} \tag{18}$$

It is used for closed conduits of the types given below.

Cast-iron Pipe. On account of the growth of tubercles on the inside of the pipe, which decreases its area as well as increases its roughness, the value of C for a given age decreases as the diameter, but variation of C with age depends largely upon the composition of the water flowing in the pipe. Values are, therefore, rough approximations. Williams and Hazen recommend the average values given in Table 4. Cleaning old pipe increases the coefficient materially.

Table 4. Average Value of C for Cast-iron Pipe

Diameter of Pipe, in.	Age in Years						
	0	5	10	20	30	40	50
4	130	118	107	89	75	64	55
8	130	119	109	93	83	73	65
12	130	120	111	96	86	77	70
16	130	120	112	98	87	80	72
24	130	120	113	100	89	81	74
60	130	120	113	100	90	83	77

Unlined Tunnels in Rock. Recommended values of C for unlined tunnels in rock, based on net sections and neglecting possible overbreak, are: Best, $C = 50$; probable, $C = 45$; worst, $C = 38$.

Small Smooth Pipe. For smooth pipe of brass, lead, tin, glass, and drawn copper, new and in good condition, $C = 140$; average, $C = 130$; bad, $C = 120$. The same values apply to new small wrought-iron and steel pipe. Falling off at C with age depends on indeterminate conditions which are accentuated in small pipe. Therefore, an ample factor of safety should be used. For additional data, see Section 6.

6. MISCELLANEOUS DATA ON CONDUITS

PERMISSIBLE VELOCITIES IN CANALS. Economy usually requires as small a section and hence as high a velocity as the material will stand. Therefore, if the character of the bed is such as to require low velocities, sedimentation and plant growth are necessary evils, unless the alternative of a smaller lined section is adopted.

A mean velocity of 2 or 3 ft per sec generally will be sufficient to prevent the deposit of silt. Sand and gravel entering the canal will not be deposited with velocities somewhat smaller than the maximum given hereinafter to prevent scour of beds of like materials.

Plant growth has seriously affected the capacity of some canals. A temperature below 65 F, turbid or deep water, or a velocity greater than 2.5 ft per sec usually prevents serious growth.

The Special Committee on Irrigation Structures of the ASCE recommends the values given in Table 5 for permissible canal mean velocities to prevent scour (Ref. 7). This table is for canals on tangents and for depths not exceeding 3 ft. For sinuous alignment, reduce velocities 25%. For greater depths, use velocities not exceeding 0.5 ft per sec greater. Column 3 recognizes that colloidal silts will precipitate and eventually form a plastic, highly cohesive mass, provided the canal is not fully loaded until seasoned. Column 4 recognizes that waters conveying abrasive sand or gravel will furnish a graded bedding and more resistance in some cases, but may assist scour in shales and clays.

Table 5. Permissible Canal Velocities

Original Material Excavated for Canal	Velocity, ft per sec, after Aging in Canals Carrying:		
	Clear Water, No Detritus	Water Transporting Colloidal Silts	Water Transporting Noncolloidal Silts, Sands, Gravels, or Rock Fragments
Fine sand (noncolloidal)	1.50	2.50	1.50
Sandy loam "	1.75	2.50	2.00
Silt loam "	2.00	3.00	2.00
Alluvial silts when noncolloidal	2.00	3.50	2.00
Ordinary firm loam	2.50	3.50	2.25
Volcanic ash	2.50	3.50	2.00
Fine gravel	2.50	5.00	3.75
Stiff clay (very colloidal)	3.75	5.00	3.00
Graded, loam to cobbles, when noncolloidal	3.75	5.00	5.00
Alluvial silts when colloidal	3.75	5.00	3.00
Graded, silt to cobbles, when colloidal	4.00	5.50	5.00
Coarse gravel (noncolloidal)	4.00	6.00	6.50
Cobbles and shingles	5.00	5.50	6.50
Shales and hardpan	6.00	6.00	5.00

WATER HAMMER. When selecting valves and fittings, the possibility of shock or strain due to water hammer, in excess of the average working pressure of the line or system, should be considered. Many valves and fittings, installed where the working pressure under normal conditions would be low, have failed because of pressure due to water hammer. This danger can be avoided by proper cushioning of the line by air chambers or by relief valves.

When a valve in a pipe is closed while the water is flowing, the velocity of the water behind the valve is retarded and a dynamic pressure is produced. When the valve is closed quickly this dynamic pressure may be very great. It is then called *water hammer* or *water ram*, and it sometimes causes fracture of the pipe. It is prevented by arrangements which prevent rapid closing of the valve.

The excess pressure in feet of water produced by the instantaneous closure of a valve in a pipe is

$$h = \frac{aV}{g} \tag{19}$$

where a = velocity, feet per second of wave propagation up the pipe; V = reduction of velocity, feet per second. The velocity of wave propagation is given by Joukowsky's equation

$$a = \frac{4660}{\sqrt{1 + KB}} \tag{20}$$

where K = ratio of the elastic moduli of water to the material of the pipe shell (0.01 for steel pipe) and B = ratio of pipe diameter to thickness.

Equation 19 applies to any time in seconds of valve closure less than

$$t = \frac{2L}{a} \tag{21}$$

where L = length of pipe, feet. The pressure produced by a closure in time greater than $2L/a$ is less than given by eq. 19, but its determination is quite complex. (See Ref. 8; also see p. 5-46.)

7. FLOW OVER DAMS

The basic theoretical expression for flow over weirs is given in eq. 5, which, if all constants are combined, may be written

$$Q = Clh^{3/2} \tag{22}$$

where Q = total discharge, cubic feet per second; C = coefficient of discharge, which depends on the shape of the crest and the head on the crest; l = net or effective length of crest, feet, i.e., the total length of crest corrected for end contractions due to piers and sharp-cornered abutments; and h = actual or measured head on the crest, feet, taken at a point sufficiently remote from the dam to avoid the surface curve.

Francis determined that, to allow for the effect of the velocity of approach, this equation should be written

$$Q = Cl(h + h_v)^{3/2} - h_v^{3/2} \tag{23}$$

where h_v = head corresponding to velocity of approach. An approximate form of Francis's equation is

$$Q = Cl(h + h_v)^{3/2} \tag{24}$$

Equation 24 gives values of Q in excess of that from eq. 23. The error for a depth of channel approach greater than twice the head on the crest is less than 2%.

Francis's equation for the necessary correction due to complete sharp-cornered end contractions is

$$l = l_t - 0.1nh \tag{25}$$

where l_t = total or gross length of crest between abutments and piers and n = number of complete contractions.

If the crest is obstructed by wide rectangular piers, n represents the number of corners that deflect the water, there being two for each pier and one for each abutment. However, if the piers are very thin or pointed upstream, or if the abutments are well rounded or continuous upstream, the effective reduction in crest will be much less.

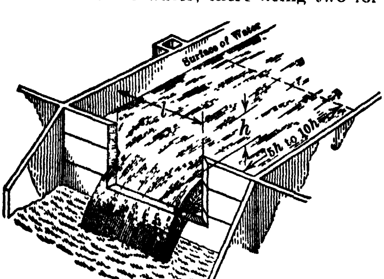

Fig. 7. Weir and end contractions.

When the length of the crest between complete end contractions becomes less than about three times the head on the crest, or when the length of the crest between piers of the usual type becomes less than about two times the head on the crest, the discharge should be found by hydraulic model experiment.

The value of the discharge coefficient C for use in eqs. 23 and 24 has been determined experimentally for spillways of many different types. These experiments have been carefully tabulated by R. E. Horton (Ref. 9). Coefficients adaptable to standard dam crests are given in Ref. 8. For sharp-crested weirs, as in Fig. 7, the coefficient C is approximately 3.33.

SUBMERGED SPILLWAYS. If the crest of the spillway is submerged, the discharge coefficient for use in eqs. 23 and 24 should be modified according to the degree of submergence, as indicated in Table 6. In this table C is the coefficient for *free* discharge over a similar crest under the same head, and C' is the modified coefficient due to submergence. The head h is the aforementioned head on the dam, and h_s is the corresponding superelevation of tailwater above the crest.

Table 6. Relative Coefficients, Submerged Crest and Free Crest

h_s/h =	0.0	0.2	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C'/C =	1.000	0.983	0.956	0.937	0.907	0.856	0.778	0.621	0.000

8. MEASUREMENT OF FLOWING WATER *

MEASURING WEIRS. The measuring weir consists of a dam which may extend the full width of the channel or may have a crest consisting of a notch, rectangular or otherwise, cut in the dam, in which case it is called a weir with end contractions.

RECTANGULAR WEIRS. Figure 7 shows a rectangular weir. The general formula for discharge of water over a rectangular weir is eq. 22.

Where l_t = total length of crest, feet, the usual expression for the effective length of crest, with two complete sharp-edged end contractions, is from eq. 25:

$$l = l_t - 0.2h \tag{26}$$

Experiments on many different types of crests have been made which indicate that C varies from about 2.6 for flat-crested dams to about 4.0 for rounded crests scientifically designed to fit the bottom contraction (see Ref. 8). These experiments, with the resulting values of C, are summarized by R. E. Horton in Ref. 9. Measurement of discharge over existing dams of similar types of crest can be made by comparison with these experiments within only a fair degree of accuracy.

Unless duplicating exactly a given set of accurate experiments, measuring weirs, constructed for that purpose, should have the following characteristics: (1) Straight uniform channel with uniform velocity, provided by stilling racks if required. (2) Suppressed end contractions, provided by making sides of channel form the end of the weir. This results in $l = l_t$ in eq. 26. (3) Weir to have free overfall with complete aeration of the nappe. Aeration usually requires air passages leading to the space between the nappe and the dam. (4) A metal crest free from rust, with sharp right-angle corner on the upstream edge, a crest width of $^1/_8$ in. and beveled to an angle of 45 degrees on the downstream face. The crest edge should be level.

A number of equations, based on experiments with sharp-crested rectangular weirs of the foregoing specifications, have been proposed. The following equations apply to such weirs with suppressed end contractions.

Francis Weir Equation. The best-known equation is based on the *Lowell Hydraulic Experiments*, by J. B. Francis (Ref. 10)

$$Q = 3.33l(h + h_v)^{3/2} - h_v^{3/2} \tag{27}$$

where h_v is the head due to the velocity of approach. With no velocity of approach, eq. 27 reduces to

$$Q = 3.33lh^{3/2} \tag{28}$$

Francis's equations are subject to considerable error, particularly for high velocities of approach and small values of h. The equation should not be used for accurate weir measurements and is given here only because eq. 28 is a convenient one to remember for rough approximations.

King Weir Equation. King (Ref. 11) proposes the following equation, which, after considerable study, he finds to agree more closely with various experimental data than any of the others:

$$Q = 3.34lh^{1.47}\left(1 + \frac{0.56h^2}{D^2}\right) \tag{29}$$

where D is the depth, in feet, of water upstream of the weir.

Rehbock's Weir Equation. Perhaps the most accurate of all weir equations is Rehbock's (see Ref. 12). If P = the height of weir in feet, h = head on weir in feet, this equation is

$$Q = \left(3.228 + 0.435\frac{h_e}{P}\right) lh_e^{3/2} = Clh_e^{3/2} \tag{30}$$

where

$$h_e = h + 0.0036 \tag{31}$$

The ASME Test Code specifies the use of eq. 30 for sharp-crested weirs without end contractions. Table 7 gives typical Code values of C in eq. (30).

Table 7. Test Code Values of C (Abridged)

Head h, feet *	Height of Crest P in Feet (above Bottom of Channel of Approach) 4	8	16
1.0	3.337	3.283	3.255
2.0	3.446	3.337	3.283

* Use linear interpolation between 1 and 2 ft.

* See also ASME Test Code for Hydraulic Prime Movers.

TRIANGULAR NOTCH WEIRS. Triangular or V-notch weirs, in which the apex is down, are adaptable to small discharges. In this type of weir, the head for extremely small discharges is proportionally greater due to the reduction of crest length near the apex. This results in greater accuracy.

From his own experiments and those of Barr, King (Ref. 11) gives the following equations for discharge:

For a sharp-edged right-angle notch cut in a large sheet of commercial steel plate,

$$Q = 2.52h^{2.47} \tag{32}$$

For a similar notch cut in a polished brass plate

$$Q = 2.48h^{2.48} \tag{33}$$

D. R. Yarnell (Ref. 13) gives the following coefficients for the equation

$$Q = Ch^{5/2} \tag{34}$$

Head, h	0.4	0.6	0.8	1.0	1.2
Coefficient, C	2.511	2.492	2.484	2.481	2.480

These coefficients apply to a right-angle notch cut in a smooth brass plate, with edges very carefully finished but not highly polished. An accuracy within one-half of 1% is claimed.

THE CIPPOLETTI OR TRAPEZOIDAL WEIR. Cippoletti found that by using a trapezoidal weir with the sides inclined 1 horizontal to 4 vertical, with end contraction, the discharge is equal to that of a rectangular weir without end contraction (that is, with the width of the weir equal to the width of the channel) and is represented by the simple formula $Q = 3.367lh^{3/2}$. In experiments with a trapezoidal weir, with values of l from 3 to 9 ft and of h from 0.24 to 1.40 ft it was found that the value of the coefficient averages 3.334, the water being measured by a rectangular weir and results being computed by Francis's formula, and 3.354 when Smith's formula was used. Cippoletti's formula, when applied to a properly constructed trapezoidal weir, will give the discharge with an error due to combined inaccuracies not greater than 1%.

CURRENT METERS. The current meter is a mechanism consisting of cups or vanes, on vertical or horizontal axes, held stationary in a stream of flowing water. The vanes are revolved by the motion of the water. The number of revolutions in a given time is proportional to the velocity of flow. Current meters are rated by drawing them through still water at several velocities.

Current meters are not extremely accurate means of testing, on account of extraneous components of flow in streams caused by eddies. Some meters overregister and others underregister, in turbulent flow, by an amount which can be estimated by holding the meter obliquely during rating. For greater accuracy, two types of meters having opposite characteristics with respect to turbulence are sometimes employed, and a weighted average of the readings is used.

The section of the stream chosen for the test should be as uniform in area and as smooth as possible for some distance up- and downstream. The minimum dimension of the measuring section should be not less than twenty times the diameter of the rotating part of the current meter. The minimum metered velocity in the measuring section should be not less than twice the stalling velocity of the current meter. The maximum velocity in the measuring section should not exceed 12 ft per sec.

Tests are made both by the point by point and the integrating traverse methods. In the point by point method, the number of metering points should be not less than four times the square root of the metering area in square feet. In the integrating traverse method, a reliable mechanical means should be provided for obtaining uniform traversing speeds.

For the point by point method, vertical velocity curves should be plotted and the areas planimetered to obtain the mean velocity for each vertical section. A horizontal velocity curve should then be plotted from the mean vertical values as obtained above or as given directly by the integrating traverse method, and the area planimetered to obtain the mean velocity for the entire section.

The ASME Test Code specifies that in drawing the velocity curve at the boundaries the seventh root law shall be used as follows:

$$V_x = VX^{1/7}$$

where X = distance from the boundary to the point being plotted, in fractional portion of the total distance from boundary to first metering point ($X = 1$ at first metering point); V = velocity at first metering point; and V_x = velocity at any point.

The methods recommended by the ASME Code should be strictly followed.

FLOAT MEASUREMENTS. The velocity of a stream can be found by laying off 100 ft of the bank and throwing a float into the middle, noting the time taken in passing over the 100 ft. By doing this a number of times and taking the average, the velocity at the surface is determined by dividing the average by the distance. As the top of a stream flows faster than the bottom or sides, the average velocity being about 83% of the surface velocity at the middle, it is convenient to measure a distance of 120 ft for the float and reckon it as 100.

PITOT TUBES. The Pitot tube is used for measuring the velocity of flowing fluids and gases. Its essential feature is a thin-edged orifice at the end of a bent tube facing the flow. The impact of the fluid causes an excess pressure in the tube equal to the velocity head. The orifice is made to traverse the pipe on at least two mutually perpendicular diameters. The pressure in the tube is

$$H = h_p + h_v = h_p + \frac{V^2}{2g} \tag{35}$$

where h_p and h_v = pressure and velocity heads respectively.

Since the pressure in the tube is the pressure head plus the velocity head, the Pitot tube reading must be compared with the average of at least four piezometers placed around the pipe to measure the static pressure head.

The Pitot and piezometer tubes usually are joined through a manometer in which the differential head is directly measured. The differential head is, from eq. 35,

$$h_v \quad H - h_p = \frac{V^2}{2g}$$

from which

$$V = \sqrt{2gh_v} \tag{36}$$

Piezometers should be installed in pairs diametrically opposite each other and equally spaced around the penstock in the plane of the Pitot tube orifices. The mean velocities so determined must be multiplied by the following coefficients, then used with the area to obtain the discharge.

Ratio of Mean Velocity to Velocity at Center of Pipe	Coefficient
0.95	0.9925
0.90	0.9850
0.85	0.9775
0.80	0.9700
0.75	0.9625

Instructions of the ASME Code should be strictly followed.

THE VENTURI METER. The venturi meter, as shown in Fig. 8, consists of a contraction in a pipe or other closed conduit for the purpose of accelerating the fluid and lowering its static pressure. Piezometers are placed at 1, the upstream end of the contraction and at 2, the lower end. The equation for the discharge past the meter is

$$Q = \frac{CAa\sqrt{2gh}}{\sqrt{A^2 - a^2}} \tag{37}$$

Fig. 8. Venturi meter.

where Q = discharge, cubic feet per second; A = area at 1, square feet; a = area at 2, square feet; h = difference between the pressure heads at 2 and 1, as shown by the piezometers; and C = a coefficient which varies between 0.97 and 1.0.

From data in Part 1 of the *ASME Fluid Meters Report*, 1927, the following empirical formula for the value of C for water has been devised:

$$1 - \frac{0.051}{(60Sd/t)^{0.284}} \tag{38}$$

where S = velocity at the throat, feet per second; t = temperature of the water °F; and d = diameter at the throat, inches.

King (Ref. 11) claims that experiments by Ledoux (Ref. 14) indicate values of the coefficients to be about 2% smaller for throat velocities of 5 ft per sec, and about 1% smaller for the higher velocities. However, the exact determination of the coefficient can be obtained only by test.

Equation 38 applies to meters having the following usual dimensions, where D is the normal diameter of the pipe: Diameter of the throat at 2 from $D/4$ to $D/2$; length of the throat at 2 from $D/4$ to $D/2$; entrance cone to have a total angle of about 21 degrees; exit cone to have a total angle of about 5 to 7 degrees; throat to be accurately machined to exact diameter; diameter at 1 to be smooth and to accurate dimension; angles in entrance cone at 2 to be rounded off to an easy tangential curve; length of straight pipe before meter at least $5D$, and preferably more.

The tubes from the piezometers 1 and 2 are joined through a manometer and the differential pressure h_v read directly. Special attachments may be obtained for indicating, recording, and integrating the flow.

The loss of head in passing through the meter can be calculated from eq. 12, practically the entire loss being confined to the enlargement below the throat.

There is no limit to the sizes of the meters or the quantities of water that may be measured. Three venturi tubes, approximately 17 ft in diameter, are installed in the Catskill Aqueduct supply to New York. While the venturi meter originally was applied to the measurement of water, it has since been used extensively for the measurement of sewage, gases, steam, and many other fluids. (See also Section 1.)

SALT VELOCITY METHOD OF MEASURING FLOW. The salt velocity method of measurement is based on the fact that salt in solution increases the electrical conductivity of water. Brine (salt solution) is injected through a system of piping and pop valves at any point in the conduit, usually at the upper end. The introduction of this brine and its passage past one or more pairs of electrodes at other points in the conduit, together with the elapsed time between points, are recorded graphically. The electrodes are connected in parallel to the electrical recording instrument. The time of passage of the salt solution is computed from the center of gravity of pop valve injection curve to the center of gravity of the electrode curve or between the centers of gravity of two or more electrode curves as determined by the physical conditions. The discharge $Q = V/t$ cu ft per sec, where V = volume of the conduit test section, cubic feet, t = time of passage of the salt solution, seconds.

In applying the method, the following rules should be observed:

(1) The test section should be straight, not converging, although bends of slight angle are permitted. There should be no reversed or idle flow in the test section.

(2) When the salt introduction station is at the intake or when two sets of electrodes are used, the test section should be not less than 50 ft long.

(3) When the salt-introduction station is located in the conduit, the test section should be not less than 80 ft long.

(4) The time of passage of the salt should be not less than 9 sec at rated full load of the system under test.

(5) In large conduits there should be at least one injection valve for each 20 sq ft of cross section at the salt-introduction station.

This salt velocity method was developed by C. M. Allen, professor of Hydraulic Engineering, Worcester Polytechnic Institute. For a complete description of the method, see Ref. 15. It has been used extensively in measuring water in both closed and open conduits, and particularly in connection with field efficiency tests of water wheels, and has been developed and improved so that it is now accepted as one of the standard methods. The instructions of the ASME Code should be strictly followed.

THE GIBSON METHOD OF MEASURING FLOW. The Gibson method is based on the equation of impulse and momentum applied to an enclosed column of water in motion. It is applicable in testing hydraulic power plants where the turbine is supplied with water through a closed conduit and means are available, such as turbine gates, for interrupting the flow of water. To apply the method it is necessary to measure the physical dimensions of the conduit and to obtain pressure-time diagrams, which show the changes of pressure with respect to time that occur in the conduit during and after the closing of the turbine gates. There are two kinds of such diagrams: (*a*) *Simple diagrams*, in which the changes of pressure at one point in the conduit are recorded; (*b*) *Differential diagrams*, in which the difference between the changes of pressure at two points in the conduit are recorded.

The length of conduit upstream from the piezometer section for simple diagrams or the length between the two piezometer sections for differential diagrams should be not less than 30 ft nor less than twice the maximum dimension of the conduit cross section. Also the product of L and V_a should be not less than 200, where V_a is the mean velocity in the conduit in feet per second when the turbine is carrying rated full load and L is the length in feet of the conduit used for the test. The conduit measurements should be made as precisely as possible.

The leakage past the turbine gates, or other device used in producing the pressure rise, must be added to the flow determined by the diagram to obtain the total flow in the conduit. This leakage is determined by a special leakage test.

The method may be used whether the conduit is of uniform or variable cross-sectional area but, for simplicity, a section of greatest regularity should be selected. For complete description of the Gibson method, see Ref. 16. This method is applicable only to closed conduits. The instructions of the A.S.M.E. Code should be strictly followed.

SALT-SOLUTION METHOD OF MEASURING FLOW. In cases where the flow is too turbulent for other methods, the salt-solution method has been used. In this method salt in solution is introduced at the inlet at a uniform known rate and its concentration measured at a point downstream, usually at the outlet. The determination of the relative salt content of the water should be made at a point far enough downstream from the inlet to insure a thorough mixture. Also the section where the solution is introduced should be free of reverse currents which would carry a portion of the solution upstream.

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HYDRAULIC TURBINES

Revised by William J. Rheingans *

9. GENERAL

A hydraulic turbine consists of a runner connected to a shaft for producing prime motive power, a mechanism for controlling water flow to the runner, and water passages leading to the control mechanism and away from the runner.

FUNDAMENTAL FORMULAS. Hydraulic turbines derive energy from water under pressure. Power that a turbine can develop is a function of pressure and quantity of water available. The available pressure is called *head* on the turbine, designated by H. It corresponds to the difference in feet between the elevation of water at the intake and the elevation of *tailwater* level, at the discharge from the power plant.

The quantity of water available, called *quantity* or *discharge*, is designated by Q and measured in cubic feet per second.

The theoretical horsepower of a hydraulic turbine can be expressed as

$$\mathrm{Hp}_T \quad \frac{H \times Q \times W}{550} \quad \frac{HQ}{8.82}$$

where Hp_T = theoretical horsepower and W = weight of a cubic foot of water (approximately 62.4 lb).

The actual horsepower of a hydraulic turbine is the theoretical horsepower multiplied by the turbine efficiency e:

$$\mathrm{Hp} = \mathrm{Hp}_T \times e = \frac{H \times Q \times e}{8.82} \tag{1}$$

* Based on material originally prepared by R. E. B. Sharp.

Assuming a turbine with fixed discharge orifice, the velocity V through the orifice varies as the square root of the head, since

$$V = \sqrt{2gH}$$

Therefore, the discharge varies as the square root of the head

$$Q \propto \sqrt{H} \tag{2}$$

and

$$\text{Hp} \propto H \times \sqrt{H}$$

$$\text{Hp} \propto H^{3/2} \tag{3}$$

Since the bucket angles of a runner are fixed, the runner has a definite peripheral speed for any linear velocity of the entering water. At this speed there is least disturbance to the entering water and the turbine, therefore, will develop its maximum efficiency. This speed, the *best efficiency speed* of the runner, has a constant ratio to the velocity of the entering water. The corresponding angular velocity of the runner, designated by n, is measured in revolutions per minute, rpm.

Since the velocity of water varies as the square root of the head,

$$n \propto \sqrt{H} \tag{4}$$

In this discussion it has been assumed that runner size remained constant while head varied. Size of a runner usually is designated either by the diameter measured at the bucket inlet or by discharge diameter. This diameter, designated by d, usually is measured in inches. When the size of a given runner is changed, the entire runner is changed homologously, i.e., all dimensions are changed in the same ratio but all angles remain constant.

Since with change in size, area of the orifices in the runner vary directly as the diameter squared,

$$Q \propto d^2 \tag{5}$$

hence

$$\text{Hp} \propto d^2 \tag{6}$$

Since bucket angles remain constant regardless of size, linear speed of the buckets for best efficiency at a given head remains constant. However, since the size of the runner has changed, *angular speed* of the runner must vary *inversely* as the diameter d, to keep linear bucket speed constant

$$n \propto \frac{1}{d} \tag{7}$$

The above are fundamental equations for hydraulic turbines and hold true regardless of the type of turbine. For quick reference these relations are repeated:

$$\text{Hp} = \frac{H \times Q \times e}{8.82} \tag{1}$$

$$Q \propto \sqrt{H} \tag{2}$$

$$\text{Hp} \propto H^{3/2} \tag{3}$$

$$n \propto \sqrt{H} \tag{4}$$

$$Q \propto d^2 \tag{5}$$

$$\text{Hp} \propto d^2 \tag{6}$$

$$n \propto \frac{1}{d} \tag{7}$$

SPECIFIC SPEED. Turbine runners of different types have widely varying characteristics of power, speed, dimensions, and operating head. In order to have a common basis for comparison of all turbines, a quantity called *specific speed* is used.

Any given turbine runner has a definite horsepower at one foot head. It also has a definite speed at which it develops maximum efficiency. If the size of the runner is changed homologously, the horsepower output varies as the square of the diameter and the best efficiency speed varies inversely as the diameter. Hence $d^2 = C\ \text{Hp}_1$, where C = a constant, Hp_1 = horsepower developed under one foot head; and $d = K/n_1$, where K = a constant and n_1 = best efficiency speed under one foot head. Thus $d = \sqrt{C\ \text{Hp}_1} = K/n_1$. Hence $K/\sqrt{C} = n_1\sqrt{\text{Hp}_1}$. Since K and C are constants, we can give them a symbol

$$N_s = n_1\sqrt{\text{Hp}_1} \tag{8}$$

where N_s is the *specific speed* (also called *characteristic speed*) and remains constant regardless of the physical size of runner for a given runner configuration.

If size of the runner remains constant while head varies

$$n = n_1\sqrt{H}$$

and

$$\text{Hp} = \text{Hp}_1 \times H^{3/2}$$

Therefore,

$$n_1 = \frac{n}{\sqrt{H}}$$

and

$$\text{Hp}_1 = \frac{\text{Hp}}{H^{3/2}}$$

Substituting these in eq. 8,

$$N_s = \frac{n}{\sqrt{H}} \times \sqrt{\frac{\text{Hp}}{H^{3/2}}} = \frac{n \times \sqrt{\text{Hp}}}{\sqrt{H} \times \sqrt{H^{3/2}}}$$

$$N_s = \frac{n \times \sqrt{\text{Hp}}}{H^{5/4}} \qquad (9)$$

(To raise a number to a fractional power, see Section 20.)

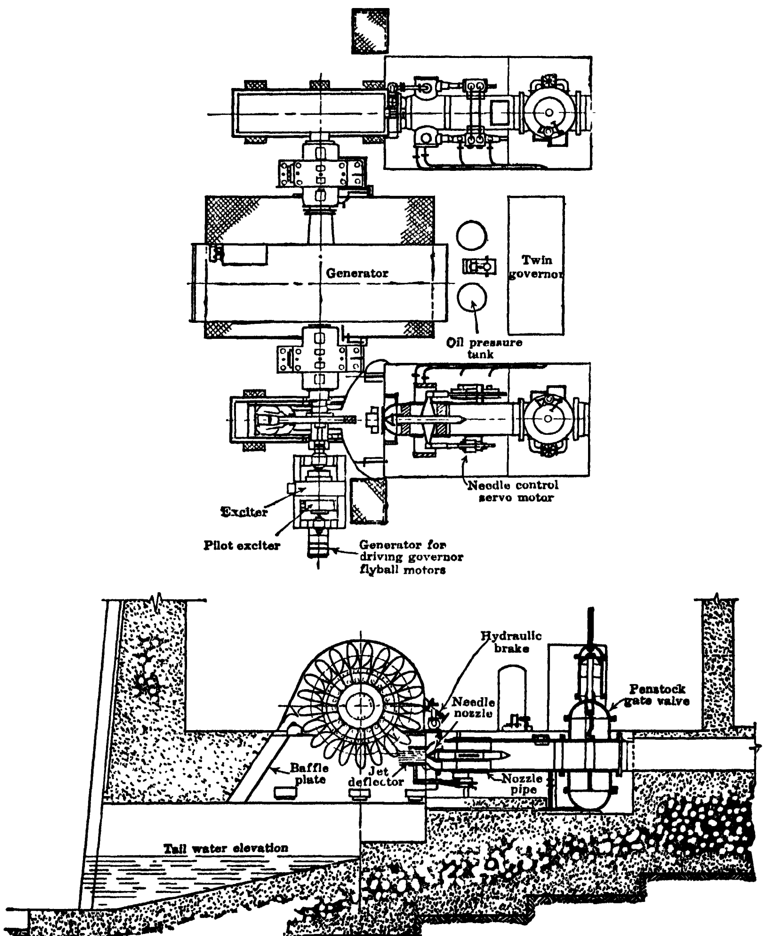

FIG. 1. Impulse turbine.

Each type and design of runner will have a given N_s, independent of size, but characteristic of the shape or geometry. This value is used in selecting a runner for a given set of conditions of head, power, and speed.

TYPES OF TURBINE. Modern hydraulic turbines are divided into two general groups, *impulse* and *reaction* (or pressure) turbines. A further classification of reaction turbines is (1) Francis turbines, (2) adjustable-blade propeller turbines, and (3) fixed-blade propeller turbines. The two general groups are not well named because both types involve both reaction and impulse. However, through common usage, the terms are well understood by hydraulic engineers.

Impulse. Figure 1 shows a typical cross-section and plan view of a double-overhung impulse turbine driving a generator. A free jet of water discharging into an aerated space impinges on the buckets of the runner, and is controlled by a needle-type nozzle. The power output is controlled either by actuating the needle in the center of the nozzle or by deflecting the stream between nozzle and runner by means of a jet deflector.

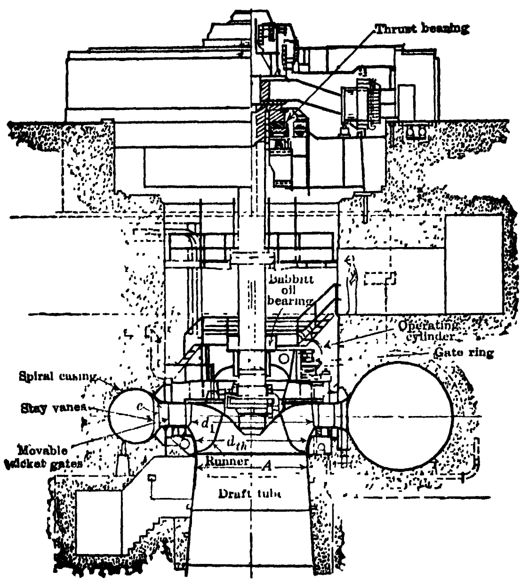

FIG. 2. Reaction turbine (Francis type) driving a generator.

Reaction. Figure 2 shows a Francis-type, inward-flow reaction turbine. Movable wicket gates with axes parallel to the turbine shaft control the flow. Water enters the spiral casing from the intake passages, or penstock, passes through the stationary stay vanes, movable wicket gates, and runner into the draft tube, through which it flows into the tail race below the powerhouse. Power is controlled by actuating the movable wicket gates either manually, by a motor, or by a governor.

Figure 3 shows an adjustable-blade propeller turbine with movable runner blades, adjusted during operation synchronously with the movable wicket gates to give high part-load efficiency.

EFFECT OF HEAD ON SELECTION OF TYPE. The type of turbine selected for a given set of conditions depends largely on the net head available. The following table is a general guide.

Net Head	Type of Turbine
Up to 70 ft	Propeller type (either fixed or adjustable)
70 ft to 110 ft	Propeller type or Francis
110 ft to 800 ft	Francis
800 to 1300 ft	Francis or impulse
1300 ft and above	Impulse

Special conditions at a power development site may require that a certain type of turbine be selected to operate at heads less than those shown above. It is possible to operate

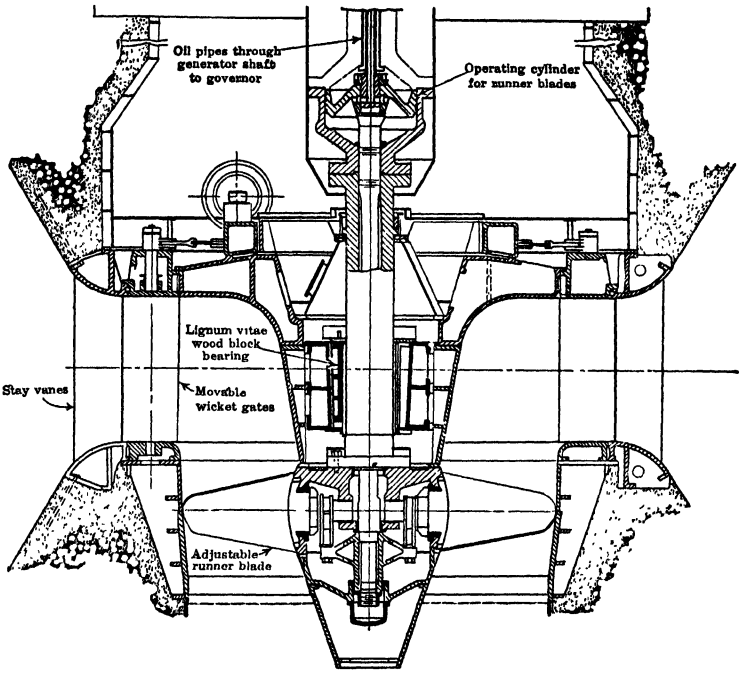

FIG. 3. Reaction turbine, Kaplan type.

a Francis or even an impulse turbine at heads below 70 ft, and special considerations sometimes warrant such use. On the other hand, propeller turbines are rarely used for heads over 110 ft, and Francis turbines are not used for heads over 1300 ft.

Each turbine type has a limited range of specific speed N_s inherent in its design. Figure 4 indicates the dividing line between the two types of turbine and typical efficiencies obtained for varying values of N_s. It is necessary to employ the impulse type in some instances when the head is lower than 900 ft, if the power developed is relatively small, because in such case a reaction turbine (having a higher N_s) would have to operate at a speed too high from a mechanical design standpoint.

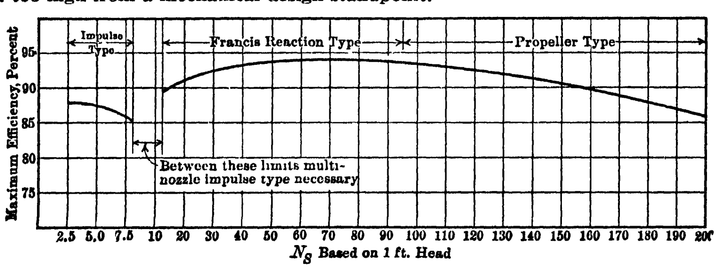

FIG. 4. Efficiency of various types of hydraulic turbines.

SYNCHRONOUS SPEEDS. Hydraulic turbines generally are direct-connected to a-c generators, hence must operate at some synchronous speed nearest the best speed from a hydraulic and mechanical standpoint.

The synchronous speed of a generator is

$$\text{Rpm} = \frac{\text{Frequency} \times 120}{\text{Number poles on generator field}}$$

where rpm = revolutions per minute. The number of poles on the generator field is always an even number.

10. REACTION TURBINES

SELECTION OF TYPE. When head conditions call for a reaction turbine, the particular type is determined by consideration of the conditions to be met. Heads between 70 and 800 ft generally indicate the Francis type. For capacity above 1000 Hp the vertical shaft arrangement (Fig. 2) should be used unless local conditions require a horizontal shaft. For heads above 100 ft, a metal casing of cast iron, cast steel, riveted or welded steel plate construction is used. If the power to be developed is small, the horizontal shaft type (Fig. 5) often is used for heads up to 800 ft because of greater accessibility.

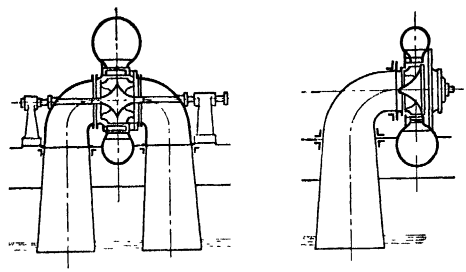

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Fig. 5. Horizontal-shaft reaction turbine.

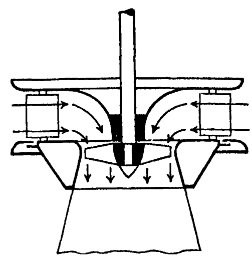

Fig. 6. Vertical-shaft reaction turbines, fixed-blade propeller type.

Heads below about 70 ft usually call for the propeller type, either with adjustable runner blades (Fig. 3) or fixed runner blades (Fig. 6). For run-of-river installations with varying available flow, the adjustable runner blade type offers decided advantages in maintaining high efficiency under reduced flow. Figure 7 compares performance of an adjustable blade with that of fixed blade type. The increase in efficiency for part loads is notable. Several types of adjustable-blade propeller turbines are available.

The manually adjustable propeller turbine has runner blades manually adjustable at the coupling between turbine and generator shaft. Although this type has the advantage of low cost, the unit has to be stopped whenever a change in runner blade pitch is made; hence it is practical only where infrequent load changes are required.

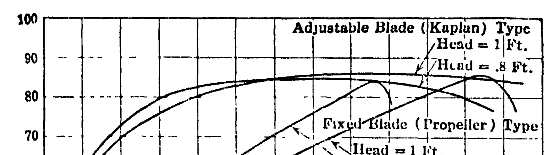

Fig. 7. Comparison of performance of Kaplan type and fixed-blade propeller type turbine.

Motor-operated adjustable propeller turbines have electric motors mounted in the turbine shaft coupling to vary the pitch of the runner blades under load. This type has the advantage of lower initial cost and lower maintenance cost than oil-operated Kaplan turbines. Pitch of the turbine blades must be adjusted manually by a switch at the control stand. Automatic control in synchronism with the turbine gates is possible but entails a complicated electrical arrangement.

Automatically balanced adjustable blades are pivoted slightly ahead of their centers of pressure so that they tend to follow movement of the wicket gates. Water forces on the blades create a moment which tends to open the blades, and is counterbalanced by water pressure acting on a balance piston in the runner hub that tends to close the blades. Water pressure on the balance piston is supplied from the penstock through a pressure-control valve, operated by a cam on the gate-operating mechanism. By this means the runner blades are forced to take the position for best efficiency for each wicket gate position but have freedom to assume other desirable positions, e.g., to open when starting and to open during runaway.

The Kaplan type is the most commonly used adjustable-blade propeller turbine. Runner blades are adjusted automatically in synchronism with turbine wicket gates by an oil servomotor in the turbine shaft. Oil is supplied to the servomotor from the turbine governor oil system, through the generator shaft, and through a control valve. The control valve is operated by a cam which is operated by the turbine gate-operating mechanism. By this means the runner blades are forced to take the position for best efficiency for each wicket-gate position.

SETTINGS. For heads below 20 to 25 ft, the *siphon setting* (Fig. 8) is advantageous for propeller turbines of all types. It reduces excavation costs, permits the generator floor to be set above headwater elevation. If the headwater is fairly constant, it may be possible to omit head gates and employ stop logs at the intake, for occasional inspections of the casing. Ejectors are used to exhaust air from the casing for starting.

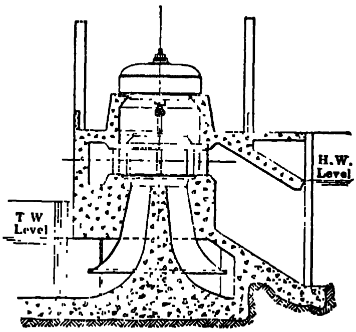

FIG. 8. Siphon setting.

For heads up to 35 ft, the *open flume setting* (Fig. 9) is sometimes used, where the power is small and where low first cost is important. The turbine is completely submerged in an open flume or pit. One disadvantage is the difficulty of lubricating the operating mechanism, with consequent relatively rapid wear. An important feature is prevention of air vortices. There is no known formula for determining the submergence required to prevent them, as the tendency varies with different types of turbine (greater with horizontal shaft installations) and even with different designs of the same type. Because of this, and because of poor lubrication, the open flume setting should be avoided.

DETERMINATION OF SPEED. Figure 10 shows values of N_s generally used for various values of head H, as the upper safe limit of N_s. Higher values should be avoided because of cavitation, rough operation, and poor performance. Although costs vary inversely with the selected N_s, careful balance should be made between good performance and low maintenance costs, versus low initial cost.

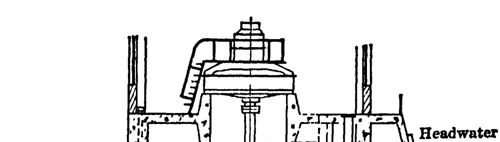

FIG. 9. Open flume setting.

Selection of N_s should also be governed by the draft head adopted. When the latter is great, N_s should be lowered to give a suitable σ. (See eq. 17, p. 5-38.)

NUMBER OF UNITS REQUIRED depends on cost, efficiency, flow characteristics, and maximum unit capacity available.

Cost. The cost of turbines per horsepower developed decreases with increasing capacity up to a runner discharge diameter of about 100 in. Above this diameter cost per horsepower for a single turbine increases with capacity, because weight and cost increase faster than horsepower developed. Electrical connections, maintenance, and operating costs, however, decrease with decreasing number of units, and in general, the smaller the number of units, the lower the overall cost.

Efficiency. With a given specific speed, the larger the unit, the higher the efficiency. (See eq. 10, p. 5-32.)

Flow Characteristics. If flow is widely variable, reasonably high efficiency at low flows is important. By adopting turbines with high part-load efficiency, minimum flow conditions may be efficiently met; alternatively, this may be done by selecting a small number of large units and one small unit of high part-load efficiency. The latter scheme has the disadvantage of requiring a more complex operating plan to achieve best efficiency. (See Ref. 1.)

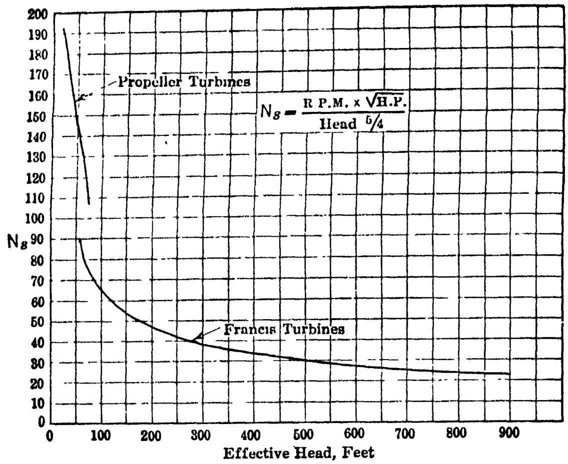

Fig. 10. Values of N_s generally used for various values of H.

Maximum Unit Capacity Available. With cost in mind, if a large amount of power is to be developed, the largest practical unit capacities should be adopted. The largest existing capacities are:

Tennessee Valley Authority, Pickwick Plant
Head 43 ft, 48,000 hp
U. S. Government, Bonneville Plant
Head 55 ft, 66,000 hp
Susquehanna Power Company, Conowingo Plant
Head 89 ft, 54,000 hp
U.S.S.R., Dneiperstroy Development
Head 116.5 ft, 84,000 hp
U. S. Government, Grand Coulee Plant
Head 330 ft, 165,000 hp
U. S. Government, Hoover Dam
Head 510 ft, 115,000 hp

VARIATION OF CHARACTERISTIC CURVES WITH N_s AND TYPE. Figure 11 shows typical efficiency curves plotted against output, for various values of N_s. These curves, starting with curve 1 ($N_s = 21$), show how increasing N_s above a value of about 35 has the effect of reducing part-load efficiency, this effect continuing to the fixed-blade propeller type, curve 6 ($N_s = 120$). The Kaplan-type propeller turbine, with runner blades adjustable during operation, has a very high efficiency, curve 7, over wide ranges of power. The part-load efficiencies are even higher than those of low N_s Francis-type turbines.

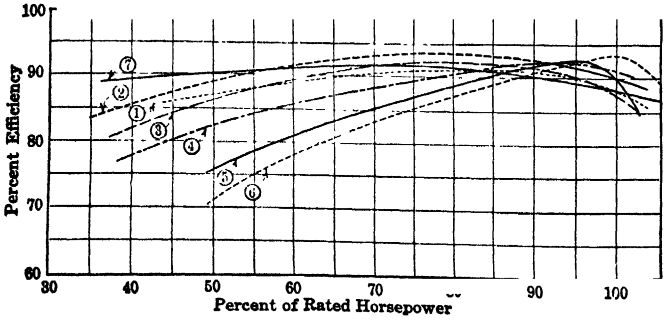

Fig. 11. Typical efficiency curves for turbines having specific speeds ranging from $N_s = 21$ (curve 1) to $N_s = 120$ (curve 6) and adjustable-blade Kaplan type (curve 7).

MODEL RUNNER TESTS, correctly interpreted, may be used as a reliable indication of the performance of large units. It is vitally important, however, that other considerations than that of runner alone be taken into account. If the large turbine is to be installed under restricted conditions affecting design of casing and draft tube, these conditions must be reproduced faithfully in the test of the model runner to obtain a prediction of performance of the large unit. This is particularly important for the draft tube, if N_s is high. To apply these tests to large installations, they should be provided with liberally designed casings and modern draft tubes.

Assume that a water-power site is to be developed where the effective head will be 100 ft and that the unit capacity is to be 10,000 hp. With N_s = 64, rpm = $(64 \times 100^{5/4})/\sqrt{10{,}000}$ = 202.5. With a frequency of 60 cycles, 200 rpm corresponds to a generator having 18 pairs of poles, and the corrected value of N_s = 63.3. Assume that the test curves shown in Fig. 12 represent performance of a model runner and draft tube that may be stepped up in dimensions for this project. It will be noted that this runner develops a value of N_s = 63.3 at ϕ = 0.891. The term ϕ is (peripheral velocity in feet per second $\div \sqrt{2gH_0}$), where g = acceleration due to gravity = 32.2 ft per sec per sec and H_0 = effective head on turbine. ϕ is measured at the throat of the runner (see Fig. 2). This value of N_s is not computed at maximum power but at 95% of maximum capacity, which is the *rated capacity*. The performance curve may now be drawn. For a value of ϕ = 0.891, the corresponding values of efficiency and horsepower should be tabulated. The throat diameter, d_{th} (in.) of the runner under consideration is

$$d_{th} = \frac{(0.891 \times 1836 \times \sqrt{100})}{200} \quad 81.75 \text{ in.}$$

in which 1836 is a constant: $60 \times 12\sqrt{2g}/\pi$.

Remembering that

$$\text{Hp} \propto \text{diam}^2 \times H^{3/2}$$

$$\text{Hp} = \text{Hp}_1 \times \left(\frac{81.75}{12}\right)^2 \times 100^{3/2} = \text{Hp}_1 \times 46{,}500$$

At rated capacity as taken from Fig. 12, Hp_1 = 0.215, and 0.215 × 46,500 = 10,000 hp. This agrees with the rated capacity of the turbine under consideration, thus indicating that the selected value of ϕ is correct. If the head is variable, Hp-efficiency curves may be drawn for the values desired by obtaining new values of ϕ and new values of $(d_{th}/12)^2 H^{3/2}$. If the minimum head is considerably less than the normal value, a frequent condition at low-level developments, it is desirable that the maximum power possible be developed at the low heads. This is equivalent to stating that the values of Hp_1 at higher than normal should be greater than at normal ϕ.

A complete group of curves, as shown in Fig. 12, is necessary to determine the characteristics of a Kaplan-type runner. Each curve represents a test at a fixed blade angle.

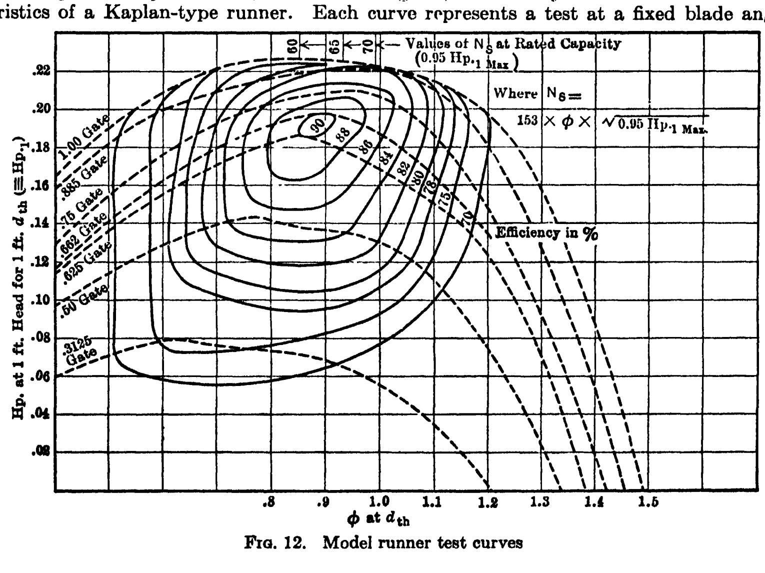

FIG. 12. Model runner test curves

The Hp-efficiency curve for any value of ϕ is obtained by plotting the Hp curves for all blade angles at that value of ϕ, and then drawing an envelope curve tangent to each blade angle curve.

Increase in Efficiency with Increase in Runner Diameter. The formula for stepping up efficiencies of model runners in order to predict that of the large prototype, as developed by Professor L. F. Moody, has been found to be quite accurate. This formula is

$$E_1 \qquad (1 - E)\left(\frac{D}{D_1}\right)^{1/4}\left(\frac{H}{H_1}\right)^{1/10} \tag{10}$$

where E and E_1 = efficiency of small and large runners, respectively; D and D_1 the diameters, and H and H_1 = the heads acting.

RUNNER PROPORTIONS. After determining the value of N_s and consequently the rpm of a runner, it is necessary (unless the runner to be constructed is to be stepped up from a small model runner as described above) to select arbitrarily a value of ϕ for the determination of the diameter at the throat and at the bucket tips. It is also necessary to select a ratio of e_1/d_{th} (see Fig. 2) in order to fix the former. The curves in Fig. 13 indicate values of ϕ_{th} (at throat), ϕ_1 (at bucket tips), and e_1/d_{th} = (distributor width ÷ diameter at throat) for varying values of N_s which have been found by test to give the most

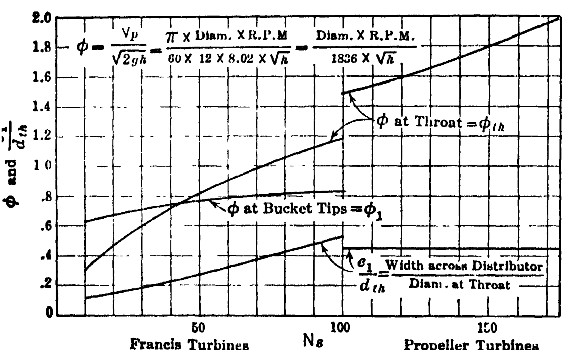

FIG. 13. Relation of ϕ_{th}, ϕ_1, e_1/d_{th} to N_s.

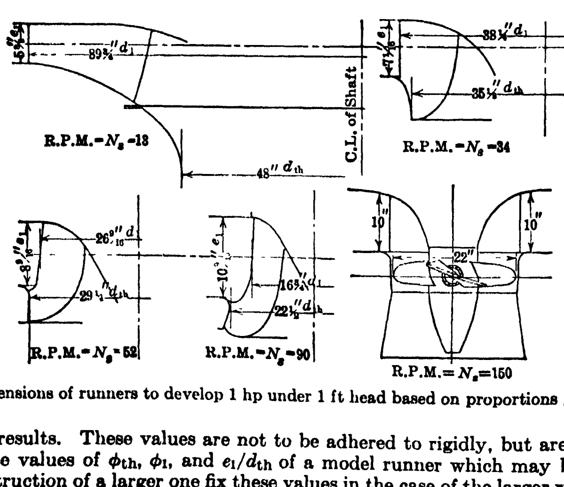

FIG. 14. Dimensions of runners to develop 1 hp under 1 ft head based on proportions given in Fig. 13.

satisfactory results. These values are not to be adhered to rigidly, but are submitted as a guide. The values of ϕ_{th}, ϕ_1, and e_1/d_{th} of a model runner which may be used as the basis of construction of a larger one fix these values in the case of the larger runner without reference to Fig. 13, except for purposes of comparison. Again, it may be necessary for draft head reasons (see p. 5-29) to increase the value of ϕ above that indicated in Fig. 13, thus reducing the throat velocity and increasing the allowable distance that the runner may be placed above tailwater. Figure 14 gives profiles of runners of varying values of

N_s, having proportions obtained from Fig. 13, and drawn to such size that each would develop 1 hp under 1 ft head. A propeller runner also is shown. The N_s of these runners is therefore synonymous with the rpm as noted. These profiles indicate clearly the desirability of increasing N_s as a means of reducing the cost of the unit.

METHODS OF INCREASING POWER UNDER REDUCED HEAD CONDITIONS. A characteristic of most low-head developments is the marked reduction in available head with increased flow conditions during flood seasons.

A device for suppressing the tailwater elevation during flood condition by causing waste water to flow over the exit of the draft tube has been used successfully in a hydraulic-plant installation. (See Ref. 2.)

11. THEORY OF REACTION TURBINE RUNNERS

The design of reaction turbine runners has as its basis the theorem, "The power of a turbine in steady motion equals the angular velocity multiplied by the change of angular momentum experienced by the mass of water flowing in a unit of time in its passage through the turbine." This principle, probably discovered by Leonhard Euler in 1754, is known as the Eulerian theorem.

Notation. All velocities, except where noted, are in feet per second. Let C_0 = absolute velocity of water leaving movable wicket gates; C_1 = absolute velocity of water at radius r_1, feet; C_2 = absolute velocity of water at radius r_2, feet; U_1 = absolute velocity of turbine runner at radius r_1, feet; U_2 = absolute velocity of turbine runner at radius r_2, feet; CU_1 = tangential component of $C_1 = C_1 \cos \alpha_1$; CU_2 = tangential component of $C_2 = C_2 \cos \alpha_2$; C_{m1} = radial component of $C_1 = C_1 \sin \alpha_1$; C_{m2} = radial component of $C_2 = C_2 \sin \alpha_2$; w_1 = velocity of water at r_1 relative to runner; w_2 = velocity of water at r_2 relative to runner; ω = angular velocity of turbine runner, radians per second; M = mass of water discharged by runner per second = W/g; Q = quantity discharged, cubic feet per second, W = weight of water flowing per second, pound; 62.4 = weight of 1 cu. ft of water, pound; g = acceleration due to gravity = 32.2 ft per sec per sec; H = head acting on turbine runner; e = hydraulic efficiency of turbine runner; K_1 = moment arm of C_1, feet; K_2 = moment arm of C_2, feet.

Figure 15, wherein this theory is applied to the design of a Francis-type turbine, shows angular moment at entrance to runner to be $K_1 \times C_1$ and at exit $K_2 \times C_2$.

$$\text{Power delivered by water to turbine} = P = \omega \times M \times (K_1C_1 - K_2C_2)$$

$$= \frac{\omega \times Q \times 62.4}{g} (K_1C_1 - K_2C_2) \tag{11}$$

By similar triangles, $K_1/r_1 = CU_1/C_1$ and $K_2/r_2 = CU_2/C_2$. Therefore,

$$\text{Power} = \frac{\omega Q \times 62.4}{g} (r_1CU_1 - r_2CU_2) \tag{12}$$

Since (angular velocity × radius) = linear velocity at outer end of radius, $\omega r_1 = U_1$ and $\omega r_2 = U_2$, and

$$P = \frac{Q \times 62.4}{g} (U_1CU_1 - U_2CU_2) = Q \times 62.4 \times H \times e \tag{13}$$

whence

$$gHe = U_1CU_1 - U_2CU_2 \tag{14}$$

Equation 14 may be used as the basis of turbine runner design, for determining the entrance and discharge angles of the runner buckets and also of the movable wicket gates at the entrance to the runner.

Referring to Fig. 16, it will be noted that, except at very low values of N_s, there is a whirl component CU_2 in the direction of rotation at maximum efficiency At rated capacity, however, there is a whirl component against the direction of rotation for values of $N_s < 50$. The curves shown in Fig. 16 are the results of observations of the flow for turbines of efficient design.

With the controlling data given, i.e., the horsepower to be developed, the effective head, and the rpm, the value of N_s may be calculated (see p. 5-25). For the calculated value of N_s, values of ϕ_1, ϕ_{th}, and e_1/d_{th} may be found from Fig. 13. For the specific speed under consideration, an inspection of the curves in Fig. 11 will show the approximate percentage of the rated turbine capacity at which maximum efficiency will occur. The maximum efficiency which may be attained may be estimated from a consideration

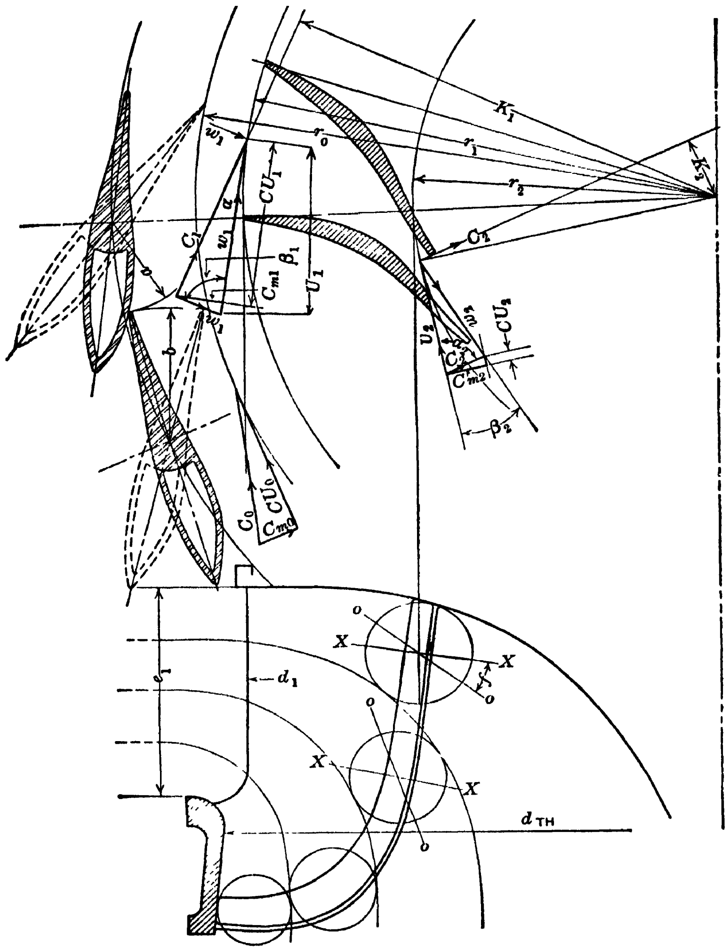

FIG. 15. Velocity diagrams of a reaction turbine.

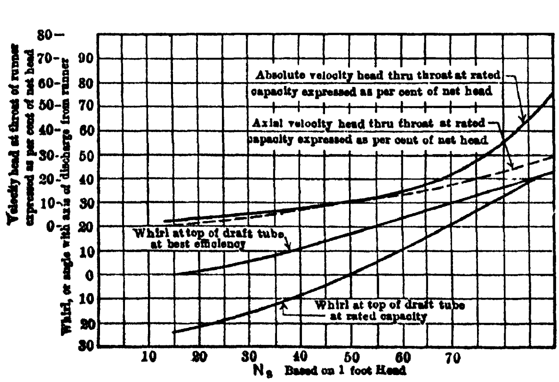

FIG. 16. Whirl in draft tubes

of the dimensions of the runner (see eq. 10) and from Fig. 4. The quantity discharged at maximum efficiency may be determined by the formula

$$Q = \frac{\text{Hp} \times 550}{H \times 62.4 \times E} = \frac{\text{Hp}}{0.1134 \times H \times E} \tag{15}$$

where H = head acting on turbine and E = efficiency.

PROFILE OF RUNNER BUCKETS. The profile of the runner buckets, including the runner band and runner crown, may be laid down as indicated in Fig. 15. The curvature from the lower distributor to the throat of the runner should have as large a radius as practicable, and the flow from the runner band into the top of the draft tube should be without any sudden break. The shape of the profile should be such as to give sufficient depth to the buckets for proper guidance and flow of the water, without resulting in high friction losses. In particular, with the higher speed runners, where the profile of the buckets at outflow is not at right angles to the direction of flow, the runner should be subdivided into several sections by means of equal quantity flow lines, and the design in each of these subdivisions should be treated separately. Table 1 may be used as an approximate guide for the number of runner buckets.

Table 1. Approximate Number of Runner Buckets or Blades

	Francis Type					Propeller Type		
N_s	12–18	18–30	30–45	45–70	70–100	100–130	130–175	175–200
No. of wheel vanes	19	18	17	16	15	6	5	4

For the specific speed under consideration the amount of whirl in the draft tube at best efficiency may be taken from Fig. 16. This angle of whirl has been determined by test to be approximately constant at all distances from the center line of the shaft. Considering the upper subdivision of the runner in Fig. 15, where $(90° - \alpha_2)$ is the angle of whirl, it is seen that $CU_2 = C_{m2} \tan (90° - \alpha_2)$. C_{m2} is the velocity of the water in a vertical plane passing through the axis of the runner, and is determined by the area between the flow lines at radius r_2, allowance being made for the area occupied by the runner buckets. It is then possible to obtain the term CU_1 from the general equation

$$gHe = U_1CU_1 - U_2CU_2 \tag{16}$$

since all other terms are known. C_{m1} may then be found from the area at the intake to the runner. The values w_2, C_2, and C_{m2} in the plan view of Fig. 15 should be considered as lying in the direction of flow, that is, in direction OO. In determining β_2 on the line XX, which is at right angles to the surface of the runner bucket at outflow, the outflow triangle should be constructed on the basis that C_{m2}' (in the direction XX) $= C_{m2} \times \cos f$.

WICKET GATES. After determining the number of wicket gates and the distance between the center lines of the gate shanks and of the turbine shaft, the radius r_0 may be determined as being the radius at which the water leaving the two sides of any gate converges, forming a solid mass. The number of wicket gates is generally between 12 and 24, the number being smaller for mechanical reasons for turbines of smaller proportions. There is no well-defined relation between the number of wicket gates and the value of N_s. From the relation $r_1CU_1 = r_0CU_0$, the value of CU_0 (Fig. 15) can be determined. The value of C_{m0} obviously can also be determined, and hence the value of C_0; the angle and opening of the gates necessary may be determined from the latter value. Additional gate opening must be provided for that portion of the performance curve (see Fig. 11) between maximum efficiency and maximum power. Maximum designed horsepower should be about 5 or 6% greater than the rated or guaranteed value. This margin is necessary to insure the attainment of the rated horsepower under test and to allow for variations between design and actual construction and for slight inaccuracies in calculations. To design for maximum efficiency at too low a percentage of the rated horsepower is to risk failing to attain the rated capacity. For the application of rational theory to the design of propeller type runners, see Ref. 3.

12. FEATURES OF TURBINE DESIGN

CASING. The spiral casing is universally accepted as most efficient. Figure 17 shows a concrete spiral casing used on turbines up to about 90 ft head. Figure 18 shows a metal casing either of cast steel or steel plate, for heads above 90 ft. Areas of casing passages surrounding the stay ring should be determined on the basis of the vortex law, $(CU_x)(r_x) = J$, where CU_x = tangential component of velocity of any filament, at radius r_x of filament (measured from the centerline of the shaft), and J = a constant. This gives increasing velocity as water passes around the casing toward the baffle vane of the stay ring. If casing areas are larger than given by the above law the excess area will be occupied by eddies. An attempt to prevent excessive friction loss thus may lead to excessive eddy loss. The velocity head at the intake to a concrete casing of the type shown in Fig. 17 should be about 2 to 3% of the effective head on the turbine. For the metal casing shown in Fig. 18, the velocity head can be as high as 4% of the effective head on the turbine, although absolute velocities for extremely high-head turbines usually are limited to about 32 ft per sec.

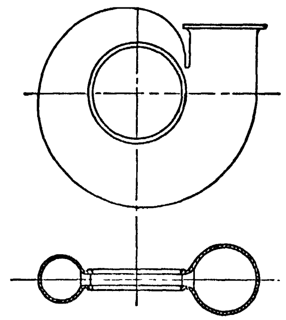

Fig. 18. High-head spiral steel casing.

THRUST BEARINGS. This part of the hydroelectric unit invariably is supplied with the generator. Two types are in general use, the Kingsbury and the General Electric spring type. Both operate in oil baths under atmospheric pressure, and depend on the viscosity of the oil carrying a wedge-shaped oil film between the rotary and stationary surfaces. Most recent practice, particularly with larger units, is to have the thrust bearing located below the generator and combined with a guide bearing. This arrangement results in the supporting beams being of reduced length and weight, and in general eliminates an upper generator guide bearing.

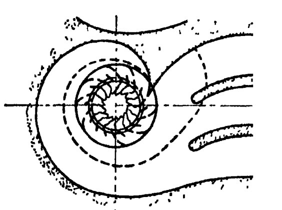

Fig. 17. Low-head spiral concrete casing.

RUNAWAY SPEED. Both generator and turbine parts should be designed safely to withstand the full runaway speed of the turbine, with maximum gate opening and no load on the generator. For impulse wheels the runaway speed is generally 80 to 90% above normal speed. For Francis turbines of low specific speed it is 65 to 80%; for high specific speed Francis turbines, 80 to 90%, and for the Kaplan type it may be as high as 180% above normal speed. Runaway speeds should be based on the maximum operating head rather than the normal value.

COMPUTATION OF LOADS ON THRUST BEARINGS. Allowance must be made for the weight of the turbine runner, turbine shaft, and the amount of hydraulic thrust on the turbine runner in the design of the thrust bearing of the vertical-shaft single-runner turbine. This thrust bearing also carries the weight of the generator revolving parts. The weight of the turbine shaft may be readily computed. A rough idea of the weight of any Francis runner may be obtained by multiplying the cube of the throat diameter in feet by the constant 35. This applies to cast-iron or cast-steel runners with the buckets and hubs cast integrally. For propeller-type runners with fixed blades, the constant may be taken as 12; for Kaplan-type runners, as 22. The largest factor dealt with generally is the hydraulic thrust; hence a comparatively large percentage of error in the weight of the runner has no great effect on the total estimated thrust-bearing load allowance.

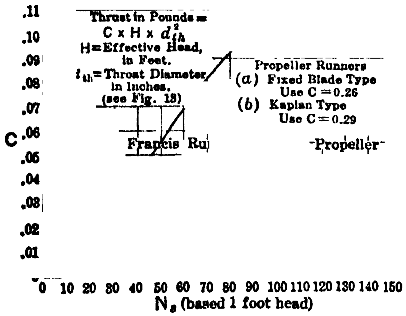

Fig. 19. Thrust on runners as a function of specific speed.

Hydraulic thrust on Francis runners, although complex, is subject to analysis. It is necessary to take into account the pressure between movable gates and runner; the seal design, and area; the method of venting, and proportions of the runner. Figure 19 gives a curve for computing the thrust on runners of varying specific speeds, where the runner is vented through a passage in the head cover leading to cored passages through the runner hub.

RUNNER LOSSES. Impact and eddy losses in the runner at low wicket gate openings are unavoidable, since the buckets of a Francis runner cannot be made movable. Friction losses can be reduced to a minimum by proper finish of the runner buckets and by proper selection of the number of runner buckets and the amount of bucket surface.

Impact and eddy losses in Kaplan runners are reduced effectively by the adjustment of the runner blades during operation to suit the wicket gate opening. The most serious loss in this type of runner is that due to friction on the blade surfaces caused by the necessarily high values of ϕ, and resulting high relative velocities. On this account, the proper finishing of the blade surfaces is extremely important, and recourse is sometimes made to machine finishing.

Leakage loss around a Francis runner is greatest for low values of N_s, and vice versa, since this varies with a function of ϕ/N_s on the basis of uniform seal clearances and seal design. This loss can be effectively reduced, theoretically, by using labyrinth seals. In practice, elaborate designs of this type of seal are not in wide use, because of the destructive effects of contact between the rotating and stationary seals caused by bearing wear or incorrect alignment. Present practice for large turbines of low specific speed embodies the use of rotating seal rings on the runner, preferably of stainless steel, in conjunction with stationary seals of a dissimilar softer metal. Very small clearances are used with this design, as indicated in Fig. 20, and if actual contact does occur, the softer metal wears away locally with no injurious generation of heat.

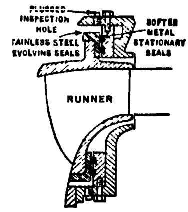

FIG. 20. Labyrinth seals.

The friction of water surrounding the runner against rotating external surfaces of the runner causes *disk loss*. This loss may be maintained at a minimum by finishing external runner surfaces smoothly and by reducing the space between runner and adjacent parts to the lowest practicable value. This loss, like loss from leakage, is greatest at low values of N_s. Experiments which have been made on machined brass disks give a value of K of 0.000,000,000,413 in the formula $\text{Hp}_{df} = KD^5N^3$, where Hp_{df} = horsepower loss due to disk friction, D = diameter of disk, or runner, feet; and N = rpm. This formula takes care of the loss on both sides of the disk. No experiments have been made in this connection on turbine runners, but the formula may be applied thereto for comparative purposes. This loss affects the efficiency at part load to a greater extent than at full load. The experiments on brass disks, cited above, indicated greater loss when the disk was in a large chamber than in a restricted chamber with smooth surfaces close to the disk. Therefore, less disk loss is encountered in a turbine when the stationary surfaces are smooth and are in close proximity to the external surfaces of the runner.

CAVITATION. In any water passage not occupied by steadily flowing water, eddies of rapidly whirling water are formed. When the head or pressure, acting on this water passage, is reduced to that of vapor pressure (about 1.25 ft absolute head at usual water temperature), *flashing* of water into vapor (steam) occurs, and voids or cavities form, causing what is known as *cavitation*. Under such conditions, slight changes in static pressure or velocity, with resultant changes in pressure, cause alternate formation and collapsing of these cavities, accompanied by intense local water hammer, with the formation of high local momentary pressure. If these cavities collapse on the surface of runner blades or draft tubes, the pressure generated tends to enter microscopic cracks, causing *pitting*.

It thus is important to design all water passages to avoid areas where eddies tend to form, as indicated on the back of the runner bucket in Fig. 21, and to place the runner sufficiently close to tailwater level. Pitting nearly always occurs on the back or underside of the buckets of vertical-shaft type turbines, as this is the low pressure side. The face of the buckets, which receives the reactive force in the form of higher pressure, is much less susceptible to pitting.

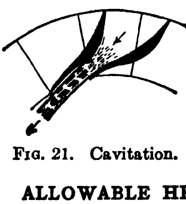
FIG. 21. Cavitation.

High-head, low specific speed turbine runners are, in general, less susceptible to cavitation than are high specific speed, low-head runners. Of all types, propeller runners are most susceptible, because of their high relative velocity and small blade area.

ALLOWABLE HEIGHT OF TURBINE ABOVE TAILWATER is one of the most important[7] dimensions in power-house design. Numerous installations exist which have been all but ruined by fixing the runner at an excessive distance above tailwater. As a result, in such cases, excessive pitting and vibration have occurred, with heavy maintenance costs and undue limitations in power output. Although a given turbine operating under high head must be placed closer to tailwater than when operating under a low head, it is the low-head plants, i.e., below about 60 ft, that are most often in difficulties, because of excessive draft head. This results from the use of higher specific speeds with higher relative velocities at the low heads.

CAVITATION COEFFICIENT. (See also Pumps, p. 5-67.) D. Thoma originated the cavitation coefficient through his work at Munich in the years 1920 to 1923 (Ref. 4).

He defined this coefficient as

$$\frac{H_b - H_v - H_s}{H} \tag{17}$$

where σ = cavitation coefficient, dimensionless; H_b = barometric pressure head at elevation of runner above sea level, feet; H_v = vapor pressure of water at the temperature existing, feet; H_s = static draft head or elevation of runner above tailwater, measured at the throat of a Francis runner and at the centerline of the blades of a propeller runner, feet; H = total effective head on turbine, feet.

The physical meaning of this coefficient can be readily visualized. Suppose a model turbine is operating without cavitation in a testing laboratory where forebay and tailrace elevations can be varied at will. The tailrace can be lowered while the total effective head is maintained constant. As the draft head is thus increased, the pressure on certain parts of the runner will decrease until it just reaches the vapor pressure H_v, at which point cavitation starts.

The static pressure on the bucket is represented by barometric pressure H_b minus the draft head H_s. However, during operation of the unit another factor lowers the absolute pressure on the runner bucket. This is the velocity head H_x of the water flowing in the runner passages.

Since we have assumed that cavitation starts when the absolute pressure on the runner bucket equals the vapor pressure,

$$H_b - H_s - H_x = H_v \tag{18}$$

or

$$H_x = H_b - H_s - H_v \tag{19}$$

The velocity head H_x is proportional to the square of the discharge of the turbine ($H^x = V^2/2g$). Since the square of the discharge is proportional to the head, H_x is proportional to the head; therefore, $H_b - H_s - H_v$ is also proportional to the head.

It is thus possible to use in place of H_x, which applies only to the conditions of test, the *ratio* H_x/H or $(H_b - H_s - H_v)/H$, a constant for a given design of runner, which applies to all heads and is identified as σ.

The velocity head H_x can be calculated. However, it can be only an approximate value because of the unknown effects that a variation in shape of the runner buckets and passageways has on the drop in pressure and distribution of the flow of water in these passageways. Therefore, for exact determination of σ, cavitation tests of a model are necessary. These tests are not required, however, where it is practicable to provide ample margin in σ.

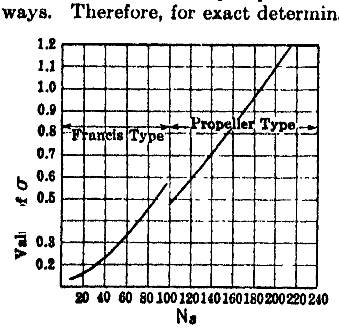

FIG. 22. Relation of and specific speed.

The σ–N_s curve of Fig. 22, based on usual practice, may be used as a general guide in the absence of cavitation tests. Tests, however, should be made for confirmation, if local conditions require the use of a low σ. Having selected σ either from Fig. 22 or from model tests, the runner setting above tailwater can be obtained by transposing eq. 17.

$$H_s = H_b - H_v - \sigma H \tag{20}$$

For positive values of H_s the runner can be set above tailwater, but for negative values the runner must be set below tailwater by the calculated amount, to avoid cavitation.

The σ at which cavitation actually occurs on a model or full-size turbine, or at which it is estimated cavitation will probably occur, is called the *critical* or *cavitation* σ. The value of σ at which a turbine operates is called the *operating* σ. To avoid excessive cavitation, the operating σ should exceed the critical σ. The greater this margin, the less the possibility of cavitation during operation.

SPECIAL MATERIALS TO RESIST CAVITATION. Materials commonly used for hydraulic turbine runners have various resistances to cavitation action. Table 2 gives relative pitting resistance of turbine runner materials.

The wide range illustrates the importance of material selection with respect to life expectancy under cavitation conditions. In large power-plant developments the cost of excavation is so great that it is uneconomical to set the runner centerline far enough below normal tailwater elevation to prevent all cavitation. In many of these installations the runners are prewelded with stainless steel at areas where greatest cavitation occurs. In

Table 2. Pitting Resistance of Various Turbine Runner Materials

Type of Material	Relative Rate of Loss of Metal Due to Cavitation
Welded or cast stainless, 18% Cr, 8% Ni steel	
Rolled stainless, 18% Cr, 8% Ni steel	1.5
Cast stainless, 14% Cr, 1% Ni steel	4
0.33% carbon cast steel	
Manganese bronze	23
Cast iron	50 to 75

some instances, cast stainless steel runners and guide vanes are provided, as well as stainless steel wearing rings and facing plates above and below the guide vanes. However, because of high cost, this is done only where excessive cavitation is anticipated or for very high-head Francis-type turbines (600 ft head and over).

DRAFT TUBES permit conservation and utilization of the head represented by the difference in elevation between the discharge of the runner and the tailrace when the value of σ permits the runner to be placed some distance above tailwater level. Draft tubes also regain a portion of the kinetic energy in water leaving the runner, making possible good efficiencies with high runner outflow velocities.

Two types of draft tubes are in use, the *symmetrical* and the *elbow*. The former may be a straight cone or a White hydraucone. The latter may have a curved or a square elbow. Figure 23 shows the forms mentioned. All these draft tubes are capable of developing

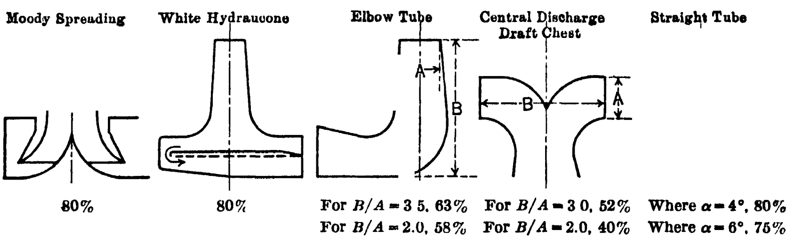

FIG. 23. Types of draft tube.

high efficiency if correctly designed. The value of α should not exceed 5 to 6 degrees for any of the tubes. The most important design feature, regardless of type, is the ratio B/A. The greater this value, the greater the regain and the higher the turbine efficiency. Good design usually requires a value of B/A between 2.5 and 3.0.

The higher the specific speed of a runner, the greater the loss in efficiency when a low value of B/A is used. Figure 24 shows the relation between the loss of efficiency of a runner and the values of B/A for various specific speeds. These curves are based on both theoretical considerations and a large number of tests of runners, using various lengths and types of draft tube. This loss in efficiency of the turbine holds true, regardless of the type or design of draft tube.

Tests on model draft tubes show that for a vertical setting an addition to the horizontal length of the draft tube is only about one-fourth to one-third as effective in regaining kinetic energy from the water discharged from the runner as the same addition applied to the vertical length.

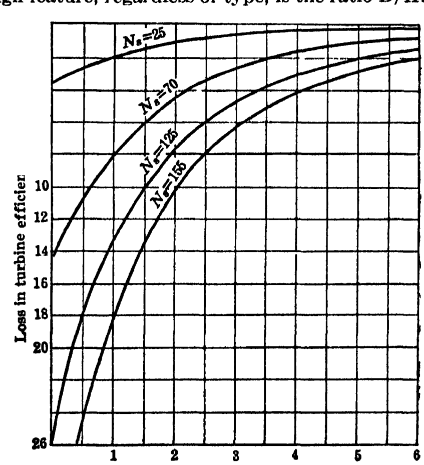

FIG. 24. Relation between loss of turbine efficiency and draft tube length.

13. IMPULSE TURBINES

The horizontal shaft type of impulse turbine (Fig. 1), is in more extensive use than the vertical type. A few vertical installations have been made with the usual design of bucket, and good efficiencies have been obtained by the provision of a suitable baffle to prevent loss of efficiency due to the discharge from the upper portion of the buckets. It is not considered practicable, unless efficiency is of small consideration, to adopt a value of N_s

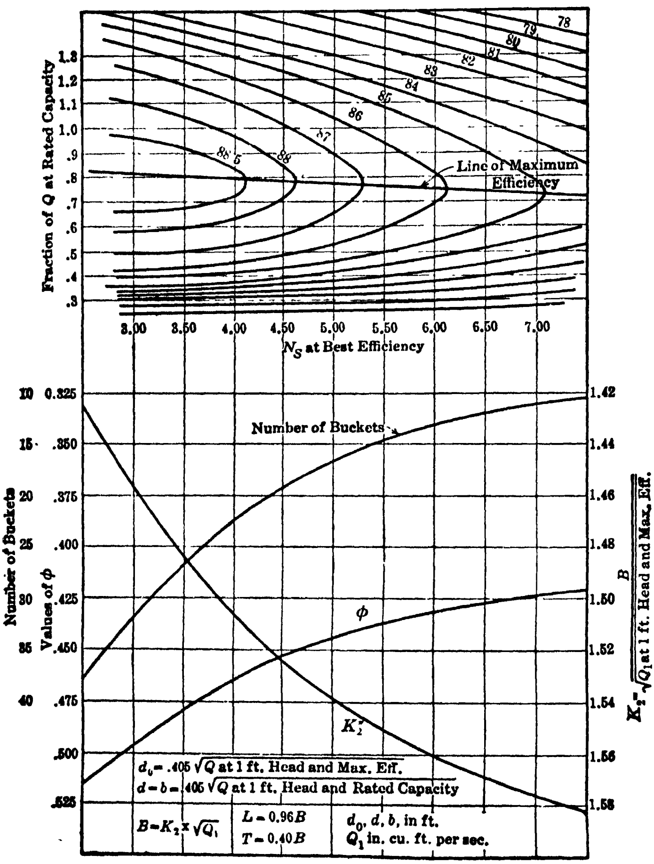

Fig. 25. Relation between ϕ and N_S.

greater than about 8.0. The interval between this value (see Fig. 4) and the lowest allowable value of N_s with a reaction turbine is covered by the use of multiple-runner units, or multi-nozzle units, or even multiple-runner multi-nozzle units. Units with more than one nozzle per runner offer complications in design of operating mechanism and may suffer loss of efficiency due to interference of the discharge from the multiple nozzles, unless very careful attention is given to their relative position. When the power to be developed with the head available results, with a single-runner single-nozzle unit in too low a value of rpm, a good arrangement may be adopted by locating a runner on each end of the generator shaft with the runners overhung. In addition to allowing, with a given value of N_s, a value of rpm higher by $\sqrt{2}$ than that of a single-runner unit, this arrangement permits shutting down one runner at part load and operation of the remaining runner at nearly maximum efficiency. For large units it is the best practice, when the foregoing arrangement is adopted, to provide two separate governing mechanisms—one for each runner.

The value of N_s for maximum efficiency varies with the head. For heads around 1000 ft,

N_s = 5.0 to 5.5 gives high efficiency. For heads around 2000 ft, maximum efficiency is attained near N_s = 3.8. Part of this variation is due to physical inability to use the proper number of buckets on the wheel disk at the higher values of N_s. Figure 25 gives an idea of the manner in which part-load efficiencies vary with N_s.

THEORY OF IMPULSE TURBINES. The force exerted by a stream upon a bucket (Fig. 26) is

$$P = MC(1 - \cos\theta) = (W/g) \times C(1 - \cos\theta) \tag{21}$$

for a stationary bucket, where M = mass per second = W/g; C = velocity of stream, feet per second; W = weight of water flowing per second, pound; and θ = angle through which water is turned relative to bucket, degrees. If the bucket is moving in the direction of the stream with a velocity U at the pitch diameter (see below)

$$P = (W/g)(C - U)(1 - \cos\theta) \tag{22}$$

$$\text{Work done by stream} = PU = (WU/g)(C - U)(1 - \cos\theta) \tag{23}$$

This is 0 when U = 0 and when U = C, and is a maximum when $U(C - U)$ is a maximum or when $U = {}^1/_2\, C$. When θ is greater than 90 degrees the cosine becomes negative. For instance, if θ = 174°, cos = − sin (174 − 90°) = − 0.9945. Then

$$PU = (WU/g) \times (C - U)(1 + 0.9945)$$

Actually the water discharged must have some velocity and the value of U consequently is made somewhat less than ${}^1/_2\, C$, except at very low values of N_s. Figure 25 indicates a relation between ϕ and N_s which has been established by test. For discussion of ϕ, see page 5-31. The diameter of the circle which falls tangent with the centerline of the stream is known as the pitch diameter of the runner, and it is at this point that ϕ is measured (Fig. 27). The value of ϕ at r (Fig. 27), it will be noted, decreases with increasing N_s. That, however, at r_a increases with increasing N_s because of increasing relative size of the buckets. The angle α at the center line of the bucket should be calculated from the velocity triangle which obtains as the bucket enters the stream, where C = velocity of stream, u_a = that of bucket at radius r_a, and w_a = relative velocity. The under side of the bucket should be finished at a greater angle than α to avoid pitting of the bucket. The number of buckets on a runner should be such that all portions of the stream will react on the bucket with the maximum attainable efficiency. Figure 25 indicates the most efficient number and proportions of buckets, as determined by tests. By the relation of stream and bucket velocities it

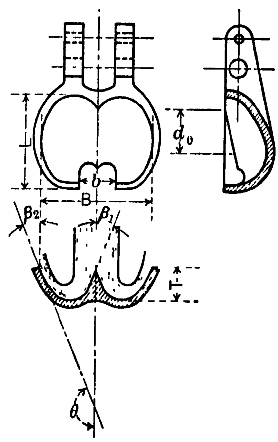

Fig. 26. Impulse turbine bucket.

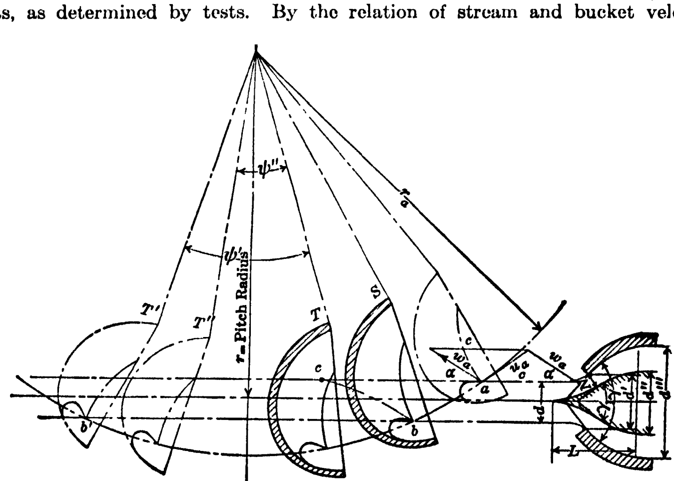

Fig. 27. Velocity diagrams of an impulse hydraulic turbine.

may be established that while the bucket travels from a to b (Fig. 27), the upper edge of the stream travels from a to c, and that the bucket cuts the stream along the line cb. Assume that when the bucket S is in the position shown, the next bucket is in the position T. In order for the particle of water at b to react on the bucket T the latter must reach the position T' before b reaches the position b'. As a matter of fact the stream, particularly the under surface, reacts only very imperfectly on the bucket when the latter is in the position T'. Therefore the bucket T' should be in some position T'', at the time b would reach b' if the bucket were removed. Good design dictates a value of ψ'' of $0.6\psi'$ to $0.7\psi'$. On this basis $\psi''/(6.0 \times \text{rpm})$ may be made equal to $(bb'/\sqrt{2gH_0})$. That is, bucket T will reach the position T'' in the time that a particle of water at b would reach b'. The angle between S and T represents the pitch, and for a fixed position for S, the position of T changes with the number of buckets selected.

It is important to maintain a small value of discharge angle β_2 (Fig. 26), but this should not be so small that the discharge comes in contact with the back of the bucket just ahead. Such contact will produce greater losses than the gain obtained with the smaller discharge angle. Also for turbines operating under high heads, water striking the backs of the buckets can produce excessive cavitation or even erosion. Attention should be given to finishing the internal or working surfaces, and the intake and discharge angles β_1 and β_2.

BUCKETS can be of bronze, cast iron, cast steel, stainless steel, or fabricated and welded plate steel. Cast-steel buckets are used for high heads or where bucket size is relatively large. Fabricated buckets of pressed plate steel, welded to lugs, have been used for a number of modern high-head large-capacity turbines. Stainless steel is used where buckets are subject to erosive action under high heads. Accepted practice is to bolt them to the cast-steel hub with two or more fitted bolts, or to cast them integrally with the hub, when N_s is high. The bolts should be of sufficient cross-section to resist the stresses at zero speed with maximum $P = (W/g) \times C(1 - \cos\theta)$, and at runaway speed with $P = 0$, also with centrifugal stresses, due to runaway speed of about 1.8 of normal speed. The dimensions B, L, T of the bucket, it will be noted (Figs. 26 and 27), are considered in terms of $\sqrt{Q_1}$ at best efficiency, although the dimension b is dependent on $\sqrt{Q_1}$ at rated capacity. Q_1, the discharge Q reduced to 1 ft head, is given by the formula $Q_1 = Q/\sqrt{H}$.

EXAMPLE. Determine the outline dimensions of the buckets of an impulse wheel to develop 30,000 hp when operating at a head of 2000 ft, to generate 60-cycle alternating current.

Assume N_s at rated capacity to be 4.2. Then from eq. 9, the speed will be $(N_s \times H^{5/4}) \div \sqrt{\text{Hp}} = (4.2 \times 2000^{5/4}) \div \sqrt{30{,}000} = 324$ rpm. By adopting a two-runner unit, one-half the horsepower will be developed in each runner, and the speed will be $(4.2 \times 2000^{5/4}) \div \sqrt{15{,}000} = 459$. A 16-pole generator at 450 rpm will give 60-cycle current, and the two-runner unit is selected. The value of N_s for this speed will be $N_s = (450 \times \sqrt{15{,}000}) \div 2000^{5/4} = 4.12$.

The efficiency at rated capacity E_r may be assumed as 87.9%; whence the quantity of water at rated capacity:

$$Q_r = (\text{Hp} \times 550) \div (W \times H \times E_r) = (15{,}000 \times 550) \div 62.4 \times 2000 \times 0.879$$
$$= 75.2 \text{ cu ft per sec}$$

In order to arrive at the value of N_s at best efficiency (Fig. 25) use the cut-and-try method. Assume as a trial that Q at best efficiency $= Q_{m.e} = 0.8 \times Q_r$; whence $Q_{m.e} = 0.8 \times 75.2 = 60.2$ cu ft per sec. Assuming again a maximum efficiency E_m of 89.0%, horsepower at this efficiency will be

$$\text{Hp}_{m.e.} = \frac{Q_{m.e} \times W \times E_m}{550} = \frac{60.2 \times 62.4 \times 2000 \times 0.89}{550} = 12{,}100$$

The corresponding value of N_s at best efficiency = 3.70. For $N_s = 3.70$ at maximum efficiency, the efficiency at rated capacity is about 87.9%, which corresponds with our original assumption. Also, the line of maximum efficiency crosses $N_s = 3.70$ at about 0.8 of Q at the rated capacity, which checks our original assumption.

The optimum number of buckets (Fig. 25) corresponding to $N_s = 3.70$ is 25. Also, from Fig. 25, $\phi = 0.476$; whence

$$r = (\phi\sqrt{2gH} \times 60) \div (2\pi \times \text{rpm})$$
$$= (0.476\sqrt{64.4 \times 2000} \times 60) \div (6.283 \times 450)$$
$$= 3.63 \text{ ft}$$

$$Q_1 \text{ at maximum efficiency} = \frac{Q}{\sqrt{H}} = \frac{60.2}{\sqrt{2000}} = 1.346 \text{ cu ft per sec}$$

$$Q_1 \text{ at rated capacity} = \frac{75.3}{\sqrt{2000}} = 1.684 \text{ cu ft per sec}$$

Diameter of jet. At maximum efficiency $d_0 = 0.405\sqrt{Q_1 \text{ at maximum efficiency}} = 0.470$ ft; at rated capacity $d_0 = 0.405\sqrt{Q_1 \text{ at rated capacity}} = 0.526$ ft.

Dimensions of buckets (from Fig. 25).

$$K_2 = 1.495 \text{ for } N_s = 3.70$$

$$\text{Then } B \text{ (Fig. 25)} = K_2 \times \sqrt{Q_1 \text{ at maximum efficiency}}$$

$$= 1.495 \times \sqrt{1.346} = 1.734 \text{ ft}$$

$$L = 0.87B = 1.51 \text{ ft}; \quad T = 0.40B = 0.694 \text{ ft}$$

$$b = 1.03d = 1.03 \times 0.526 = 0.542 \text{ ft}$$

THE NEEDLE NOZZLE should be placed as close to the buckets as possible, as the stream tends to lose its compactness of form shortly after emerging from the nozzle, owing partly to air friction, partly to the centrifugal effect caused by whirl components, and partly to expanding air in the water. The efficiency of a well-designed nozzle is usually between 95 and 97%, corresponding with a velocity in the free jet at its smallest point between $0.975\sqrt{2gH}$ and $0.985\sqrt{2gH}$, where H = pressure head at d''' (Fig. 27) + the velocity head. Where d is the diameter of the free jet at the rated capacity of the unit, the main dimensions of the nozzle (see Fig. 27) may be found as follows:

$$d' = 1.25d; \quad \gamma = 70 \text{ to } 80°; \quad d'' = 1.55d; \quad d''' = 3.2d; \quad L = 2.2d$$

The coefficient of contraction at exit from nozzle is about 0.75; that is, area of d divided by area N = 0.75. Care should be taken to have γ_1 smaller than γ(γ_1 being the angle of needle at the portion of greatest taper), otherwise this portion of the needle will be subject to pitting due to water not adhering to the needle surface. The needle as well as the nozzle near the outflow should be very smoothly finished.

The nozzle pipe leading to the needle should be as straight as possible (Fig. 1). Elbows or bends which change the direction of flow near the needle nozzle introduce whirl components, which tend to break up the jet before it reaches the buckets, with a consequent loss in efficiency. The higher the head, the greater the tendency for whirl components to break up the jet.

CASING DESIGN. The casing in the vicinity of the nozzle should be large enough to allow free discharge from the buckets. Baffles should be installed to prevent water from being carried around to the upper casing. This upper casing need be only large enough to clear the buckets, as windage is increased by large clearances. The discharge passage from the wheel should have its outlet above tailwater; otherwise, air confined in the casing will become entrained in the water and the vacuum created will cause the water level in the discharge passage to rise until it reaches the runner buckets. Venting of the upper casing also is beneficial.

Long penstocks usually necessary with impulse turbine installations prevent quick movement of the needle by the governor, especially in the closing direction. Relief valves and jet deflectors permit quick closing of the needle and prevent injurious water hammer.

14. SPEED REGULATION

TURBINE GOVERNORS. In a hydraulic turbine inertia, friction, and hydraulic load acting against the movement of the turbine gates necessitate the introduction of a force external to, but controlled by, the governor for overcoming these resistances. A pump generally is used to force oil under pressure into one or two operating cylinders which actuate the turbine gates.

Figure 28 is a diagrammatic sketch of a turbine governor. When an increase in speed takes place, the revolving flyballs T cause the right-hand end of the lever L to move upward, and open the ports of the governor valve, thus admitting oil under pressure from A to C and simultaneously connecting B with D. A gate-closing movement of the piston of the operating cylinder results, continuing until the restoring rod E has raised the left-hand end of the lever L sufficiently to close the ports of the governor valve by lowering the right end of L. During this operation the right end of L' may be considered as fixed. This leaves the unit at a higher speed than the normal value, but with the ports of the governor valve momentarily closed and the gates momentarily stationary.

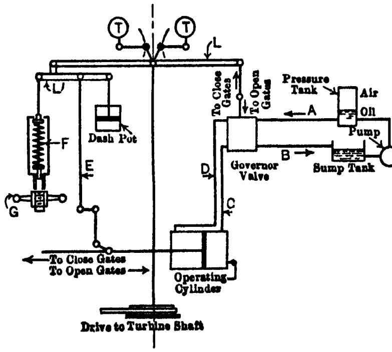

FIG. 28. Diagram of hydraulic turbine governor.

The spring F, however, is compressed since the left end of L' has moved upward from its original position. This compression causes the lever L' now to move downward slowly at the left end at a rate determined by the dash pot by-pass, L', for the time being, turning about a pivot formed by the upper end of E. This results in an upward movement of the right end of L with consequent closing movement of the turbine gates. The load on the turbine now being steady, the closing movement of the gates results in a reduction in speed until the spring F has reached its normal position, which is necessarily accompanied by normal speed and closed ports in the governor valve. Thus, following a change in load, the functioning of the governor is divided into two distinct processes. The first involves the movement of the gates to supply or cut off the necessary amount of hydraulic energy to suit the new load; the second involves the restoration of speed to the normal value necessitating an additional (but small) movement of the gates.

The hand wheel G permits the case containing spring F to be raised or lowered, and forms a means of controlling the normal speed of the unit.

FLYBALL DRIVES. The simplest method for driving governor flyballs is through a belt direct-connected to the shaft of the turbine. This drive has disadvantages of belt wear and slippage and the possibility of breaking, and it necessitates locating the governor near the turbine. In recent years belt drive has been discarded in favor of motor drive. Power for driving the motor is obtained either from transformers in the generator leads or from a separate direct-connected permanent-magnet generator mounted on the generator or exciter. The permanent-magnet generator eliminates an outside source of electricity and insures power for driving the flyball motor whenever the unit is rotating. At speeds as low as 5 rpm modern permanent-magnet generators will produce sufficient power to drive the flyball motors.

DETERMINATION OF GOVERNOR CAPACITY. Figure 29 serves as a very approximate means of determining the governor capacity required for the movement of the turbine gates. This curve is plotted between unit governor capacity and specific speed. There is indicated in this figure the formula for arriving at the approximate governor capacity for any given conditions of head and N_s. The values for very large turbines tend to be materially smaller than this curve indicates, because of relatively less friction and large number of wicket gates used.

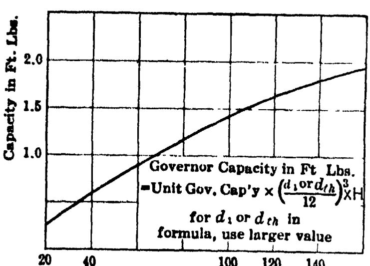

Fig. 29. Governor capacity.

The above curve is for use where the operating mechanism is lubricated, as on the outside type (Fig. 2). When the mechanism is submerged, and is lubricated by water only, the values from the above curve should be increased by about 1.25. This curve does not include allowances for relief valves or for capacity required for runner blades of Kaplan-type turbines.

REGULATION FOLLOWING SUDDEN LOAD CHANGES. The inertia of water in the turbine water passages, including penstock and draft tube, prevents prompt increase or decrease in energy supplied to the turbine as called for by the governor, thus aggravating the speed change which occurs. The change in speed as the result of a given sudden load change is a function of (1) the inertia effect, or WR^2 of the rotating element of the unit; (2) the inertia effect of the water passages; and (3) the relation between time and wicket gate movement.

With the above values and relations fixed, and neglecting the effect of lost motion, all governors give a uniform degree of speed regulation.

Let N = rpm of unit before gate movement. This is generally the synchronous value at full load, and 3 to 5% higher at no load. This difference in speed is known as the inherent speed change, also called the speed droop, and is necessary in order that the percentage of load on turbines operating in parallel may be maintained by the governors at about the same value. Therefore, N for loads going off is lower than for loads going on; however, since the term speed regulation generally relates to the percentage of momentary speed change succeeding a load change, the fact that the speed returns to a value of N slightly higher or lower than N before the load change may be neglected.

Let N_1 = maximum or minimum rpm succeeding the load change; WR^2 = product of weight of revolving parts of unit (including generator) and the square of the radius of gyration, feet; H_0 = head acting on turbine before load change; L = length, feet, of enclosed water passages of turbine = sum of L_P (penstock) + L_c (casing) + L_d (draft tube); h = average (as distinguished from maximum) change in head, feet, during time T, caused by inertia of water in passages; V = average velocity, feet per second, in enclosed water passages of turbine = $(L_P V_P + L_c V_c + L_d V_d)/L$; T = time, seconds, for gate movement; Hp_1 = load on turbine before gate movement; Hp_2 = load on turbine after gate movement; a = velocity of pressure wave in penstock, feet per second. In order to avoid danger of penstock collapse, T for load increases should be greater than $L_P V_P/12H_0$ and should also be greater than $2L_P/a$. (See page 5-46 for maximum pressure changes in penstocks.) With T in the above limits, h may be considered as LV/gT. Where the profile of the penstock departs appreciably from a straight line between intake and turbine, a greater value of T than otherwise is necessary, and a careful study should be made to avoid collapse.

EXAMPLE. Consider a sudden load increase, and neglect the time interval between the load change and the beginning of gate movement. The load demand in foot-pounds during T is $\mathrm{Hp}_2 \times 550 \times T$. This is supplied partly by hydraulic energy acting on the turbine runner and partly by energy given up by WR^2 of the revolving element in slowing down from N to N_1.

The energy of the rotating mass $= MV^2/2 = WV^2/2g = [W(2\pi RN/60)^2]/2g = WR^2N^2/5870$. For a reduction of speed from N to N_1, the energy given up by the revolving mass is $[WR^2(N^2 - N_1^2)]/5870$.

It is not possible to determine exactly the hydraulic energy supplied to the turbine during transition without analyzing the changes in quantity, head, efficiency, and gate opening, by subdividing T into smaller intervals. The method described by E. B. Strowger and S. L. Kerr in Ref. 5 gives an accurate determination of this energy.

The time of gate movement T employed should never be less than about 1.25 sec for complete travel, and should be slower than this with long penstocks in order that $LV/HT \leqq 12$, and also in order that $T > 2L/a$. If the latter requirements are fulfilled, T, seconds, for governors should be, roughly, not less than 1.25 + (ft-lb of governor/150,000).

The time of gate movement for part travel is less than for full travel, although the average rate is slower than for full travel. The following figures may be used as a guide:

Percentage load change	100	75	50	25	10
Time of wicket gate movement in percentage of time required for full travel	100	87	72	57	48

It is usually allowable to have the speed change 12 to 14% for half the rated turbine capacity rejected, although some loads require closer regulation. A usual value of WR^2N^2/Hp is 5,000,000 to 10,000,000. These values generally are exceeded where the value of H_0 is high.

INERTIA DUE TO PENSTOCK LENGTH. The problem of satisfactorily allowing for inertia due to excessive penstock length may be taken care of in four ways: (1) By providing a synchronous relief valve in conjunction with the turbine, which prevents velocity change in the penstock when load on the turbine is changed. (2) By increasing the time of governor operation, thus reducing pressure changes. (3) By providing a jet deflector between the nozzle and the buckets (on impulse turbines only) to regulate the amount of water reaching the buckets. (4) By introducing a surge tank as near the powerhouse as possible to prevent sudden changes in velocity in the pipe line between surge tank and intake.

Synchronous relief valves are employed frequently on reaction or impulse turbines in the West, particularly where irrigation requirements necessitate a constant discharge from the turbine. At low loads there is a corresponding large waste of water which is by-passed through the relief valve without doing work. The relief valve is connected directly to the gate operating ring of a reaction turbine or the needle control mechanism of an impulse turbine so that closing of the turbine gates or needle results in opposite movement of the relief valve; the sum of the discharge through relief valve and turbine is thus made constant.

Water-saving relief valves which close at a slow rate after having been opened by the governor often are provided. This type prevents excessive pressure rises for sudden load decreases, but does not avoid the danger of penstock collapse for sudden load increases. For this condition, the time of governor operation (opening) must be limited to a value such that LV/HT is less than about 12 and that $T > 2L/a$. It is extremely important that relief valve operation be positive. It is preferable that a rigid connection be used between valve and gate operating ring or needle stem, so that the turbine gates or needle also will be prevented from moving in the closing direction if the valve becomes stuck, for any reason. Otherwise, if the relief valve becomes deranged, the turbine gates might close with disastrous results.

Increasing Governor Time. The second solution, increasing governor time, requires that means be provided to prevent the governor from operating at a rate faster than that decided upon. It often is not practicable to employ this method, since the flywheel effect resulting from the comparatively long time of gate operation is too great to be taken care of in the generator rotor; the awkwardness of employing a separate flywheel frequently necessitates adoption of either the first, third, or fourth methods.

Jet deflectors applicable only to impulse turbines (Fig. 1), are installed between nozzle and buckets. The governor rapidly moves this deflector into the jet, cutting off the load. It is not unusual for a jet deflector to cut off the entire stream in 1 1/2 sec. Since the deflector acts on the stream after it leaves the nozzle, there is no change of flow in the penstock, hence no pressure rise. For a water-wasting by-pass arrangement, the needle remains in some predetermined position, and load on the unit is regulated entirely by the jet deflector. This has the same disadvantage as the synchronous by-pass relief valve in that there is a corresponding large waste at light load. For a water-saving arrangement, the needle slowly follows the closing of the jet deflector to cut off flow at a rate which does not cause serious pressure rise; at the same time the jet deflector slowly moves out of the stream. However, the needle must also move slowly in the opening direction for oncoming loads to avoid penstock collapse due to large pressure drops. This prevents use of the unit for close speed regulation.

Jet deflectors have practically eliminated use of relief valves for impulse turbines, because they are simple in construction and operation, have positive, quick action, and are low in cost.

Surge tanks, where practicable, are most desirable. Adoption of a surge tank permits quick gate or needle movements in both directions, without adding to maintenance costs of the turbine resulting from the additional moving parts embodied in a relief valve, and without requiring waste of water for sudden loss of load.

Two types of surge tanks are in use: (1) the simple tank, consisting of a cylindrical tank with a pipe connection to the conduit; (2) the Johnson differential tank, of much smaller size. This tank differs from the simple tank primarily by the addition of a riser in the center of the tank proper. At the base of the riser, an annular port communicates with the tank, the port area being proportioned to suit the conditions under which the tank is to operate.

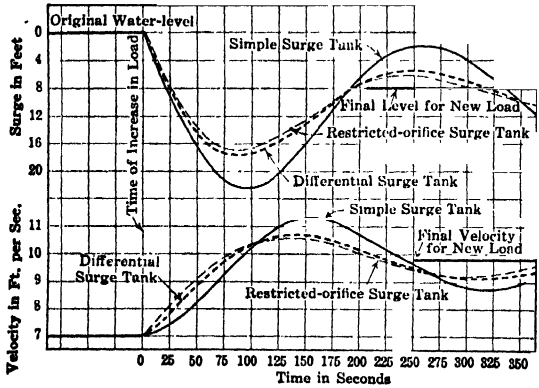

FIG. 30. Comparison of different types of surge tanks. (From *Hydroelectric Handbook*, by W. P. Creager and J. D. Justin, 1950, John Wiley & Sons)

Figure 30 shows a comparison of the action of different types of surge tanks, as affecting the water level in the tank and the velocity in the conduit. The upper curves indicate the change of level in the tank and the lower group of curves, the conduit velocity. The time in seconds is reckoned from the instant at which the load is thrown on the turbine, followed practically immediately by the opening of the turbine gates. The curves for the restricted orifice type of tank refer to a form of differential tank, having, instead of a riser, restricted ports at the base of the tank. On account of the restricted openings this type gives relatively great pressure changes at the turbine, and is infrequently used on account of poor governing qualities.

PRESSURE CHANGES IN PENSTOCKS. Notation. Let a = velocity of pressure wave along pipe, feet per second. (See Fig. 32.) A = cross-sectional area of penstock, square feet; g = acceleration due to gravity, feet per second = 32.2; h = pressure rise or excess head above normal, feet, also = pressure drop below normal, feet; h_{max} = pressure rise due to instantaneous closure = aV_0/g, feet; H_0 = initial steady head near turbine gates, corresponding to V_0, feet; K = pipe line constant = $h_{max}/2H_0 = aV_0/2gH_0$; L = length of penstock to forebay, or other point of relief, feet; N = time constant or number of $2L/a$ intervals in time of closure = $aT/2L$; P = pressure rise as a proportion of $h_{max} = h/h_{max} = h \div (aV_0/g) = gh/aV_0$; Q_0 = initial steady flow in pipe prior to start of gate closure, corresponding to H_0, cubic feet per second; T = time of gate movement for complete closure, if rate is uniform, seconds. If rate is faster in middle portion of stroke than at beginning and end of stroke, as is usually the case, consider that $T = 0.85 \times$ time for complete closure; V_0 = velocity in pipe near turbine gates, corresponding to H_0 and Q_0, feet per second; $Z = (2LV_0/gT\sqrt{H_0})^2$.

The maximum pressure rise theory originally was developed by Joukovsky (Ref. 6) and expanded by Allievi (Ref. 7), Gibson (Ref. 8), and others. Allievi prepared a chart for determination of maximum total pressure. R. S. Quick (Ref. 9) devised the chart shown in Fig. 31 for determining values of P. This chart may be used to obtain maximum

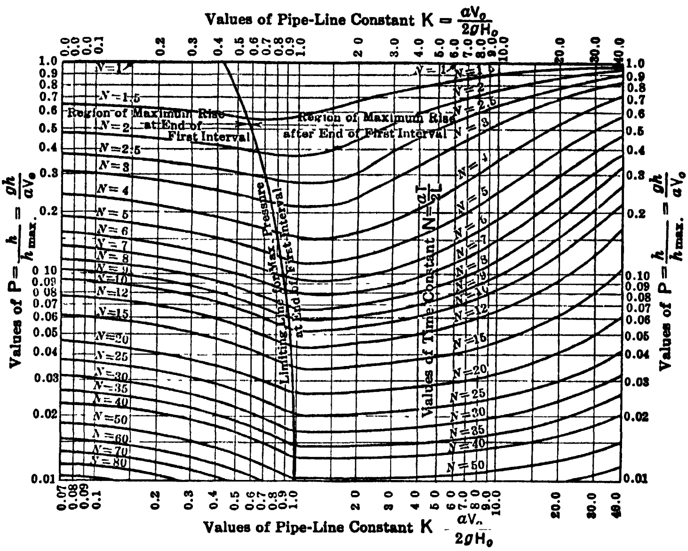

FIG. 31. Values of P. (After R. S. Quick)

pressure rise with uniform gate motion, and complete gate closure. For values of K and N falling to the left of the line marked Limiting Line for Maximum Pressure at End of First Interval, this chart may be used for partial gate closure.

Figure 32 is a chart prepared by Quick to assist in determination of value of a for various penstocks. In this chart $a = 4660/\sqrt{1 + (Kd/Eb)} = 4660/\sqrt{1 + (d/100b)}$, where K = bulk modulus of elasticity of water = 294,000; E = Young's modulus for pipe walls = 29,400,000 for steel; d = inside diameter of pipe, inches; b = thickness of pipe wall, inches.

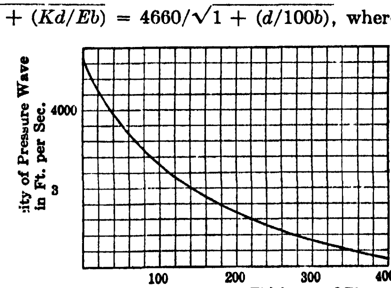

FIG. 32. Values of a.

For values of N and K falling to the right of the almost vertical line in Fig. 31, it is important, as demonstrated by S. L. Kerr (Ref. 10), to reduce rates of movement at the closing end of the gate stroke to prevent the pressure rise for partial gate movements to the closed gate position from being greater than for complete gate stroke.

Maximum Pressure Drop. Methods for determining pressure drop are similar to those used for pressure rise. See chart developed by Kerr (Ref. 11) for obtaining value of fall in pressure h, for various values of Z.

$$\text{Pressure } h = {}^1\!/_2(-Z + \sqrt{Z^2 + 4ZH_0}$$

where

$$= \left(\frac{2}{g} \times \frac{L}{T} \times \frac{V_0}{H_0}\right)^2$$

H_0 = normal head, ft

This formula holds for any pipe line where (1) rate of opening of gates is uniform and (2) T is not less than $2L/a$. T here is the total time of gate movement. Where the gate movement is not uniform, the formula holds for all practical purposes, but the maximum pressure drop in this case may occur near the end of the stroke instead of at the end of the first interval (= $2L/a$).

15. TURBINE TESTS AND CODES

Testing of hydraulic turbines to determine efficiency involves measurement of work available in the water supplied to the turbine (water horsepower) and turbine output (developed horsepower).

$$\text{Water horsepower (whp)} = (Q \times 62.4 \times H_0) \div 550 = 0.1134 \times Q \times H_0$$

where Q = cubic feet per second of water passing through the turbine and H_0 = effective head, feet, acting on the turbine. Q and H_0 are measured as discussed below.

In a few cases, turbine output may be measured by an absorption dynamometer, but in most installations the turbine is connected to a generator. Developed horsepower = generator output plus generator losses (supplied by the turbine) plus input to all auxiliary drives supplied by the turbine. Turbine efficiency = dhp/whp.

See Test Code for Hydraulic Prime Movers (1949).

MEASUREMENT OF POWER OUTPUT. When practicable, the generator is to be separately excited during turbine tests, and excitation loss is not included in computing turbine output. Turbine output in kilowatts then is (generator output) + (armature I^2R) + (generator windage and friction) + (stray load loss) + (open-circuit core loss) + (input to auxiliaries, when furnished by turbine), where all losses are expressed in kilowatts. (If the exciter is direct connected to the generator, the input to the exciter must be included.)

MEASUREMENT OF POWER INPUT OR WATER HORSEPOWER. The effective head on the turbine is the difference between the elevation corresponding to the pressure in the penstock near the entrance to the turbine casing and the elevation of the tailwater, corrected by adding the velocity head in the penstock at the point of measurement and subtracting the residual velocity head at the point of measurement in the tailrace. When turbines are set in an open flume, the head is measured by gages located immediately above the center of the turbine in the tailrace. The effective head on such a turbine is taken as the difference between the elevation of the free water surface immediately above the center of the turbine and the elevation of the tailwater, corrected by subtracting the residual velocity head at the point of measurement in the tailrace.

MEASUREMENT OF QUANTITY OF WATER. These accepted methods of water measurement are listed in ASME Test Code for Hydraulic Prime Movers; Allen Salt Velocity Method, Current Meters, Gibson Pressure-Time Method, Pitot Tube, Cole Pitometer, Venturi Meter, Salt Solution, and Weir. (See *Trans. Am. Soc. Mech. Engrs.* Vol. xlv, 1923, for description of Allen salt-velocity method and Gibson method.)

TURBINE COSTS. It is advisable for the owner to employ reliable consulting hydroelectric engineers to make surveys and submit a report on the advisability of proceeding with a development. Such a report involves a study of stream flow, head of water and power available, cost of the development, market, probable revenue, and estimated rate of return on the investment.

In obtaining prices and data from turbine manufacturers, the following information should be furnished: (1) Flow of water, cubic feet per second. (2) Normal, maximum, and minimum heads. (3) Maximum fluctuation of upper (head) and lower (tail) water levels. (4) Elevation of site above sea level. (5) Number of units under consideration. (6) Importance of efficiency at part loads. (7) Length and diameter of penstocks. (8) Frequency of electric current.

This information will enable the turbine manufacturer to make recommendations of type, unit capacity, and rpm, to estimate turbine and governor costs, and to prepare drawings of equipment.

The importance of purchasing reliable and efficient equipment is very great where hydraulic turbines are concerned. This equipment is generally embedded in the substructure of the power house and can be replaced only at very great expense.

STANDARD NOMENCLATURE. The Hydraulic Turbine Section of NEMA (National Electrical Manufacturers Association) * has adopted standard nomenclature and definitions for hydraulic turbines and governors which may be obtained from that body by users.

Manufacturers. Principal manufacturers of hydraulic turbines are: Allis-Chalmers Manufacturing Company, Milwaukee, Wisconsin; the Baldwin Locomotive Works, Philadelphia, Pennsylvania; James Leffel and Company, Springfield, Ohio; S. Morgan Smith Company, York, Pennsylvania; Newport News Shipbuilding and Dry Dock Com-

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* Member hydraulic turbine manufacturing companies of NEMA: Allis-Chalmers Manufacturing Company, the Baldwin Locomotive Works, James Leffel and Company, S. Morgan Smith Company, Pelton Water Wheel Company, and Woodward Governor Company.

pany, Newport News, Virginia; and the Pelton Water Wheel Company, San Francisco, California, a subsidiary of the Baldwin-Southwark Corporation. These firms manufacture reaction turbines of the Francis, propeller, Kaplan and impulse types. The Allis-Chalmers Manufacturing Company and the Woodward Governor Company of Rockford, Illinois, manufacture turbine governors.

PUMPS

By A. J. Stepanoff

16. CENTRIFUGAL PUMPS

INTRODUCTION. Centrifugal pumps comprise a class of pumping machinery in which pumping of liquids or generation of head is effected by rotary motion of one or more impellers. The great variety of centrifugal pumps may be reduced to a few fundamental types. Every pump consists of three principal parts: an *impeller* which forces the liquid into a rotary motion; the *pump casing* which directs the liquid to the impeller and leads it away under a higher pressure; and a *drive* to put the impeller into rotary motion. The latter includes pump shaft, supported by bearings, and driven through a flexible or rigid coupling by the drive.

Stuffing boxes are provided in places where the shaft extends outside the pump casing. Closely fitted wearing rings restrict leakage of high-pressure liquid back to the pump suction. Impeller vanes or blades and impeller side walls or shrouds form the impeller channels. In a double-suction impeller, liquid is introduced at both sides. Frequently impellers are built "open," i.e., with the front shrouds removed. Impeller vanes are always curved backwards and are called plain or radial (erroneously) if they are of single curvature. Wider impellers have vanes of double curvature, the suction ends being twisted (Francis type). An impeller is designated as radial if the shrouds are essentially normal to the shaft axis, being only slightly curved at the entrance. Pumps fitted with such impellers are called straight centrifugal. In an axial-flow pump, liquid approaches the impeller axially and the forward component of velocity, in passing through the impeller, is parallel to the shaft axis. Mixed-flow impellers occupy a position intermediate between radial and axial-flow types. Extreme mixed-flow and axial-flow pumps are also called propeller pumps. Both types are built with open impellers only.

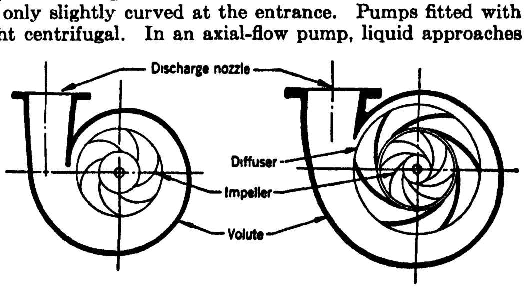


FIG. 1. Volute and diffusion casing pumps. (Reproduced by permission from Stepanoff, *Centrifugal and Axial Flow Pumps*, John Wiley and Sons, 1948)

As a result of the impeller action, liquid leaves the impeller at a higher pressure and velocity than exists at its entrance. The velocity is partly converted into pressure by the pump casing before the liquid leaves the pump through the discharge nozzle. This conversion is accomplished either in a volute casing or in a set of diffusion vanes, as shown in Fig. 1.

When the total pressure cannot be produced efficiently by one impeller, several impellers are arranged in series resulting in a *multistage pump.* In some designs, for hydraulic reasons, the total pump capacity is divided between two impellers operating in parallel. Any pump can be arranged with either a horizontal or a vertical shaft, depending on the application, type of driver, or other requirements.

Upper limits of head or capacity produced by centrifugal pumps are constantly increasing. Pressures as high as 5400 psi are being produced by centrifugal pumps (three in series) for operation of hydraulic presses. The largest pumps in the United States are those of the Grand Coulee project, each of which delivers 607,000 gpm against 310-ft head, driven by 65,000-hp motors at 200 rpm. In small sizes, centrifugal pumps are produced in excess of one million units a year for domestic water supply and industrial use.

PUMP CHARACTERISTICS. The volume of liquid pumped per unit time is referred to as *capacity*, generally measured in gallons per minute (gpm). Large capacities frequently are stated in cubic feet per second or million gallons per day. Petroleum products are measured in barrels (42 gal) per day. The following are conversion factors:

1 cubic foot per second = 448.8 gpm.
1,000,000 gallons per day = 694.4 gpm.
1000 barrels per day = 29.2 gpm.
1 liter per second = 15.85 gpm.

The height to which liquid can be raised by a centrifugal pump is called *head*, measured in feet. This *does not depend* on the liquid specific gravity. The head can also be expressed in pounds per square inch. For a horizontal pump the total dynamic head is

$$H = H_d - H_s + \frac{V_d^2}{2g} - \frac{V_s^2}{2g} \tag{1}$$

where H_d is the discharge head in feet, measured at the discharge nozzle, and H_s is the suction head in feet, both referred to the pump shaft centerline. Velocities at discharge and suction are V_d and V_s, respectively. If the suction head is negative (lift), H_s becomes additive in eq. 1. The last two terms represent difference in kinetic energy or velocity head between discharge and suction nozzles. For a vertical pump with the pumping element submerged the total dynamic head is $H = H_d + H_s + (V_d^2/2_g)$. In this case the loss in the suction bell and discharge column, up to the point where discharge head is measured, is charged against the pump. The Hydraulic Institute Test Code * gives a detailed procedure for head and capacity measurements for all practical cases.

Efficiency. The degree of hydraulic and mechanical perfection of a pump is judged by its gross efficiency, defined as

$$\text{Efficiency} = \frac{\text{Pump output}}{\text{bhp}} = \frac{Q\gamma H}{550 \times \text{bhp}} = \frac{\text{gpm} \times H}{3960 \times \text{bhp}} \tag{2}$$

where Q is capacity in cubic feet per second, γ is the specific weight of liquid (for cold water = 62.4 lb per cu ft); bhp is the brake horsepower of the driver or the pump input power. The pump output for water, expressed in horsepower, is referred to as *water horsepower* (whp). If a liquid other than cold water is used, the whp should be multiplied by the liquid specific gravity to obtain the pump output.

Performance Curves and Affinity Laws. The variation of head with capacity at a constant speed is one *pump characteristic* (Fig. 2). A complete set of pump characteristics also includes efficiency and bhp curves. Head, capacity, and bhp of a pump vary with speed in such a way that the performance curves retain their characteristic features. This variation follows laws known as *affinity laws.* Applicable to any point on the head-capacity curve, these laws state that (1) when speed is changed capacity varies directly as the speed; (2) the head varies directly as the square of the speed; and (3) the bhp varies directly as the cube of the speed or

$$\frac{Q_1}{Q_2} = \frac{n_1}{n_2}; \quad \frac{H_1}{H_2} = \frac{n_1^2}{n_2^2}; \quad \frac{(\text{bhp})_1}{(\text{bhp})_2} = \frac{n_1^3}{n_2^3} \tag{3}$$

For usual changes in speed the efficiency remains the same for each point. Figure 3 shows three head-capacity curves at speeds n_1, n_2, and n_3.

* 90 West Street, New York 6, N. Y.

Geometrically Similar Pumps. If two geometrically similar pumps are operated at the same rotational speed, capacity varies directly as cube of the ratio of some linear dimension such as a ratio of impeller outside diameters $(D_2/D_1)^3$. The head varies directly as the square of the same ratio, and the brake horsepower changes directly as the fifth power of the ratio:

$$\frac{Q_2}{Q_1} = \left(\frac{D_2}{D_1}\right)^3; \quad \frac{H_2}{H_1} = \left(\frac{D_2}{D_1}\right)^2; \quad \frac{(\text{bhp})_2}{(\text{bhp})_1} = \left(\frac{D_2}{D_1}\right)^5 \tag{4}$$

Points connected by affinity laws are called *corresponding points* and lie on a parabola. All have the same efficiency and are of the same specific speed.

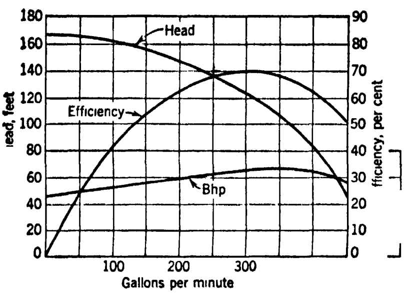


FIG. 2. Typical centrifugal pump characteristics. (Stepanoff, *op. cit.*)

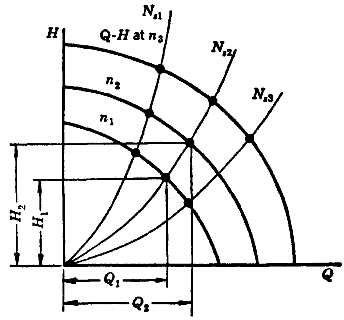


FIG. 3. Variation with speed of the head-capacity characteristic. (Stepanoff, *op. cit.*)

Reduction of Impeller Diameter. At constant rotational speed a reduction of head and capacity of a given centrifugal pump can be accomplished by reducing the impeller diameter. Rules for estimating the performance of a pump for a given reduction in impeller diameter are closely associated with the affinity laws, but are less accurate. These rules are: capacity is reduced directly as the impeller diameter ratio (D_2'/D_2); head is decreased as the square of the impeller diameter ratio $(D_2'/D_2)^2$, and consequently bhp is reduced as the cube of the diameter ratio $(D_2'/D_2)^3$. The efficiency usually decreases with appreciable reduction of impeller diameter. For example, on Fig. 4 it is required to reduce impeller diameter so that the head-capacity curve Q–H passes through the specified point A, below the curve. Take an arbitrary capacity Q_b, higher than the specified capacity Q_a, and calculate the head H_b by applying the affinity law $H_b = H_a(Q_b/Q_a)^2$. Connect points B and A to obtain point C on the curve Q–H. The required impeller diameter ratio is Q_a/Q_c. Axial-flow impellers cannot be cut to reduce the head capacity of the pump. A new casting with a smaller vane angle is required.

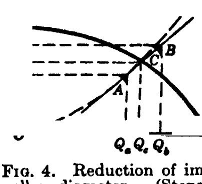


FIG. 4. Reduction of impeller diameter. (Stepanoff, *op. cit.*)

Specific Speed. (See also Hydraulic turbines, p. 5-24.) Centrifugal pumps are classified according to specific speed $N_s = n\sqrt{Q}/H^{3/4}$, where N_s is specific speed, n is speed in rpm, Q is capacity in gpm, and H is head in feet. As a *type number*, specific speed is applied to the best efficiency point only, does not change with speed, and is constant for all similar pumps. With each specific speed are associated definite proportions of major impeller dimensions. The specific speed of centrifugal pumps varies from 500 to 15,000. The lower numbers refer to radial centrifugal pumps of small capacity and high head, characterized by a very narrow impeller. The larger numbers apply to axial-flow propeller pumps. For multistage centrifugal pumps, specific speed is calculated on the basis of the head per stage. When the specific speed of a double suction impeller is compared with that of a single suction impeller, the capacity of the former should be divided by 2. Specific speed is used as a basis for classification of the performance and design constants of centrifugal pumps in the same manner that Reynolds' number is used as a criterion of pipe flow. For the same head-capacity requirements, higher-specific-speed pumps will run at a higher speed and will be of smaller physical dimensions. Limitations as to the maximum head for each specific speed are imposed by the available suction head, or the impeller submergence, known as net positive suction head, or NPSH.

Net Positive Suction Head (NPSH) is defined as the gage reading in feet, taken at the suction nozzle and referred to the pump centerline, minus the gage vapor pressure in feet corresponding to the temperature of the liquid, plus the velocity head at this point. When pumping boiling liquids from a closed vessel, NPSH is the static liquid head in the vessel above the pump centerline minus losses of head in the suction pipe. Hydraulic Institute charts B-24, B-25, B-26, and B-27 give NPSH values recommended for boiler-feed and condensate pumps (see p. 7-42).

THEORETICAL RELATIONSHIPS. The head produced by the impeller of a centrifugal pump (axial flow included) is given by Euler's equation

$$H_i = \frac{u_2 c_{u2} - u_1 c_{u1}}{g} \tag{5}$$

where H_i is the input head or actual theoretical head based on the power input to the pump, u_1 and u_2 are impeller peripheral velocities at inlet and discharge, and c_{u1} and c_{u2} are the tangential components of the absolute liquid velocity at impeller inlet and discharge (Fig. 5). If the entrance liquid velocity is without prerotation or is strictly meridional (radial for radial impellers and axial for axial impellers), eq. 5 reduces to $H_i = u_2 c_{u2}/g$. By trigonometric substitutions eq. 5 can be transformed to

$$H_i = \frac{u_2^2 - u_1^2}{2g} + \frac{c_2^2 - c_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} \tag{6}$$

where c_2 and c_1 are true absolute velocities at discharge and entrance and w_2 and w_1 are the true relative velocities at discharge and entrance. However, the true value of these velocities and their directions are never known. If entrance and discharge velocity diagrams are drawn on the impeller vane entrance and discharge angles and the average meridional velocity (c_{m1} and c_{m2}), and values of the velocities are substituted into eqs. 5 or 6, the value of the head obtained (H_e = Euler's head) is absurdly high and the equation loses its meaning. No actual impeller can transmit to the liquid the power to produce Euler's head H_e. The ratio H_t/H_e is the vane efficiency (effectiveness). The difference $H_e - H_i$ is not a loss as there is no power input corresponding to this difference in head. The difference between H_e and H_t is explained by (1) uneven velocity distribution through the impeller channel, resulting in a theoretical head lower than that based on an average velocity, (2) relative circulation of liquid within the channel due to inertia effect, (3) non-active part of impeller vane tips at discharge, and (4) prerotation of liquid at impeller entrance not taken into account.

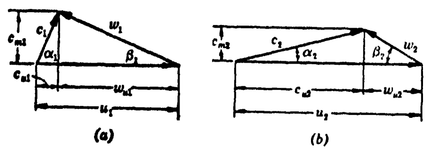


FIG. 5. (a) Entrance velocity triangle; (b) discharge velocity triangle. (Stepanoff, *op. cit.*)

Efficiency Definitions. All the head in a centrifugal pump is generated by the impeller. The rest of the parts contribute nothing to the head, but incur inevitable losses—hydraulic, mechanical, and leakage. All losses of head between the points where the suction and discharge pressures are measured constitute hydraulic losses. *Hydraulic efficiency* is defined

$$\frac{H}{H_i} = \frac{H_i - \text{hydraulic losses}}{H_i} \tag{7}$$

The capacity available at the pump discharge is smaller than that passed through the impeller by the amount of leakage through the clearances between the rotating element and the stationary parts of the pump. The ratio of the two is called the *volumetric efficiency*

$$\frac{Q}{Q_i} = \frac{Q}{Q + Q_L} = e_v \tag{8}$$

where Q_L is the leakage. Mechanical losses include loss of power in bearings and stuffing boxes, and disk friction. *Mechanical efficiency* is the ratio of power actually absorbed by the impeller and converted into head and power applied to the pump shaft, or

$$\frac{\text{bhp} - \text{mechanical losses}}{\text{bhp}} \tag{9}$$

The relationship between the efficiencies defined above and the gross pump efficiency is

$$e = e_h e_v e_m \tag{10}$$

The vane efficiency $e_{vo} = H_i/H_e$ represents the ratio of the true tangential component of the absolute velocity c_{u2}' to that taken from the Euler's velocity triangle, c_{u2}, based on vane angles. The product $e_{vo}e_h = e_{man}$, sometimes referred to as *manometric coefficient*, has no meaning and is frequently confused with the hydraulic efficiency.

THEORETICAL CHARACTERISTICS, AND DIMENSIONLESS HEAD AND CAPACITY. It is convenient to reduce pump head and capacity to dimensionless forms which apply to all similar pumps and do not depend on the actual rotative speed of the pump. The dimensionless head, called the *head coefficient,* is defined as $\psi_e = H_e/(u_2^2/g)$ for Euler's head; $\psi_i = H_i/(u_2^2/g)$ for the input head; $\psi = H/(u_2^2/g)$ for the pump total head. The dimensionless capacity, called *capacity coefficient,* is defined as $\phi = c_{m2}/u_2$ or is a ratio of the meridional velocity (proportional to capacity) to the peripheral velocity of the impeller.

The head coefficient often is plotted against the capacity coefficient, as in Fig. 6*a*. The expression for the Euler's head $H_e = u_2 c_{u2}/g$ (for meridional entrance velocity) can be transformed to

$$\psi_e = 1 - \phi/\tan \beta_2 \tag{11}$$

This is an equation of a straight line intersecting the axis of ψ at $\psi_e = 1$ and the axis of ϕ at $\phi = \tan \beta_2$ in Fig. 6*a*. This head-capacity characteristic applies to pumps of *all specific speeds* which use the *same discharge angle,* β_2. All points corresponding to the best efficiency points of different specific speeds (N_{s1}, N_{s2}, N_{s3}) are located on BA, the specific speeds increasing from B to A. By using velocity ratios, the velocity triangles also become dimensionless and apply to all similar pumps irrespective of rotational speed. It is convenient, for this purpose, to divide all velocities by u_2. On Fig. 6*b*, OBC, OBD, OBE are discharge velocity triangles for three specific speeds N_{s1}, N_{s2}, N_{s3}, all having the same discharge angle β_2. Since c_{m2}/u_2 is the dimensionless capacity ϕ and c_{u2}/u_2 is the dimensionless head ψ_e, the velocity diagram may also serve as a theoretical characteristic of the pump. Conversely, by connecting points C, D, and E to the point O on Fig. 6*a*, the discharge velocity triangles are obtained (OBC, OBD, OBE) which are identical with those on Fig. 6*b*.

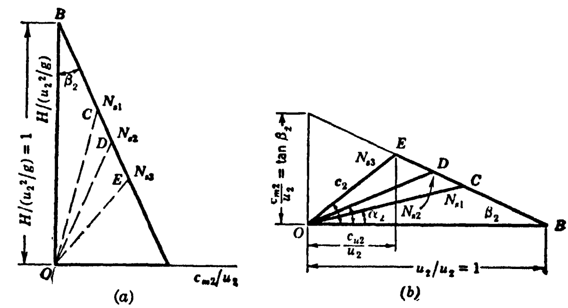


FIG. 6. (*a*) Euler's head-capacity characteristic. (*b*) Euler's discharge velocity triangle. (Stepanoff, *op. cit.*)

The input head-capacity line DA (Fig. 7*a*) is drawn so that

$$\frac{DO}{BO} = \frac{c_{u2}'}{c_{u2}} = e_{va} \text{ (vane efficiency)} \quad \text{and} \quad OA = \tan \beta_2$$

This applies to pumps of all specific speeds, using the same value of β_2. The location of line DA can be determined by locating one point on this line, estimating or calculating the hydraulic efficiency for the best efficiency point of an existing pump of any specific speed. On Fig. 7*b*, $OC'B$ is the input discharge velocity triangle, showing the actual velocities and β_2' is the true discharge angle of the flow from the impeller. OCB is the Euler's velocity triangle, β_2 being the vane angle. In practice β_2 varies between 15 and 35 degrees, the normal range being from 20 to 25 degrees.

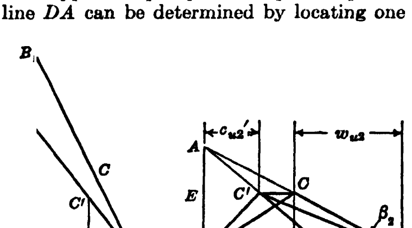


FIG. 7. (*a*) Euler's and input head-capacity curves; (*b*) Euler's and input discharge velocity triangles. (Stepanoff, *op. cit.*)

IMPELLER DESIGN. Selection of specific speed for a given set of head-capacity conditions is guided by the following conflicting considerations. (1) High specific speeds lead to smaller pumps. (2) Each specific speed has its limitations, depending on the cavitation characteristics (Fig. 31). (3) Driver speed selection has limitations, particularly when electric motors are used. (4) Optimum pump efficiency depends on the specific speed (Fig. 8). (5) Specific speed may be varied by changing the number of stages or dividing capacity between several pumps. (6) Sometimes it may be advantageous to place the operating point off peak and use a more efficient type.

Mean Effective Diameter. Centrifugal pump characteristics and design data for different specific speeds fall into continuous curves if they are compared on the basis of the mean effective diameter of the impeller, D_m, defined by (Fig. 9):

$$D_m^2 = \frac{D_{2o}^2 + D_{2i}^2}{2} \quad \text{(for mixed flow impellers)}$$
$$D_m^2 = \frac{D_o^2 + D_h^2}{2} \quad \textit{(for axial flow impellers)} \tag{12}$$

where D_{2o} is the impeller outside diameter at outer shroud; D_{2i} the diameter at inner shroud; D_o the outside diameter and D_h the hub diameter of an axial-flow impeller. The impeller vane angles and velocities are selected for the mean effective diameter. The shape of the vane at any other diameter is established from Euler's velocity triangles.

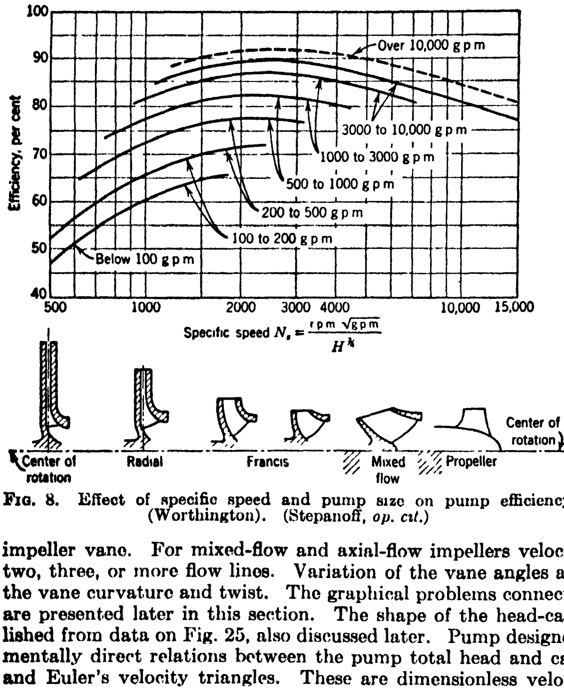


FIG. 8. Effect of specific speed and pump size on pump efficiency (Worthington). (Stepanoff, *op. cit.*)

Design Constants. Both the impeller profile and vane layout are possible if the following elements are known: (1) meridional velocities at inlet and outlet, (2) impeller outside diameter and (3) impeller vane inlet and outlet angles. These same quantities determine Euler's entrance and discharge triangles. For straight radial impellers only one entrance and one discharge triangle determine the design of the impeller vane. For mixed-flow and axial-flow impellers velocity triangles are drawn for two, three, or more flow lines. Variation of the vane angles along the radius determines the vane curvature and twist. The graphical problems connected with an impeller layout are presented later in this section. The shape of the head-capacity curve can be established from data on Fig. 25, also discussed later. Pump designers have established experimentally direct relations between the pump total head and capacity at the design point and Euler's velocity triangles. These are dimensionless velocity ratios, independent of pump size and speed, which are correlated on the basis of specific speed. In addition, a number of ratios of important linear dimensions, not directly related to velocities, are found helpful in perfecting the hydraulic design of impellers and pump casings.

Speed constant is defined as

$$K_u = \frac{u_2}{\sqrt{2gH}}; \qquad \text{hence } u_2 = K_u\sqrt{2gH} \quad \text{and} \quad H = \frac{1}{K_u^2}\cdot\frac{u_2^2}{2g}$$

The speed constant K_u is used for calculation of the impeller diameter when the head H is given and the speed is selected. Figure 9 gives values of K_u for different specific speeds based on the impeller outside diameter D_{2o} and the mean effective diameter D_m. The head coefficient $\psi = H/(u^2/g)$ can be used as a speed constant; it is connected with K_u as follows:

$$\psi = \frac{1}{2K_u^2}$$

Capacity constant is defined as $K_{m2} = c_{m2}/\sqrt{2gH}$, where c_{m2} is the meridional velocity at impeller discharge calculated (disregarding leakage, but allowing for the vane thickness) (see Fig. 10) from

$$c_{m2} = \frac{Q}{(D_{2\text{ave.}} - zs_u)d_2}$$

where z is the number of vanes. K_{m2} is connected to the capacity coefficient as follows: $\phi = K_{m2}/K_u$. This can also be used as a capacity constant.

A similar constant used for determination of the meridional velocity at entrance is $K_{m1} = c_{m1}/\sqrt{2gH}$, where c_{m1} is calculated for the area at the vane entrance tips, again omitting the leakage. The vane thickness can be disregarded as the vane tips are well tapered and c_{m1} can be assumed to be the velocity just ahead of the vanes. Referring to Fig. 10,

$$c_{m1} = \frac{Q}{\pi D_{1m} d_1}$$

The velocity through the impeller eye is either equal to c_{m1} or slightly lower. Neglecting the leakage introduces an error in vane angles β_1 and β_2, as determined from Euler's velocity triangle, of not over one degree and can be disregarded. The values of K_{m2} and K_{m1} are also given in Fig. 9. On the same figure are shown ratios of the impeller-eye diameter to the impeller outside diameter; and also the speed constant $K_{us} = u_2/\sqrt{2gH_s}$ based on the pump shut-off or zero capacity head H_s. This enables determination of the shut-off head H_s when the impeller diameter D_2 is established. In this way it is possible to estimate rather closely the shape of the head-capacity curve.

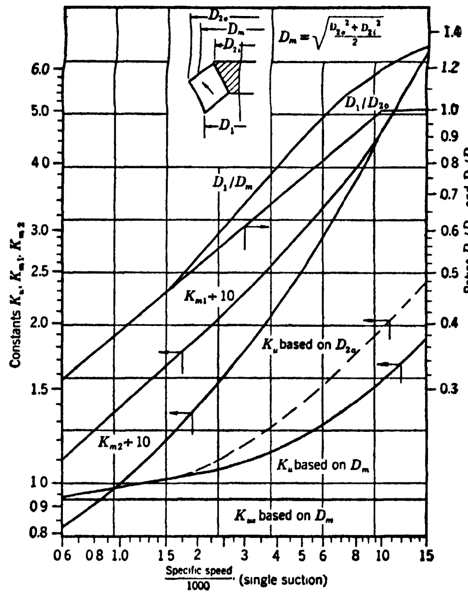


FIG. 9. Impeller constants. (Stepanoff, *op. cit.*)

The vane discharge angle, β_2, is the most important single design element. The theoretical characteristics are determined by the vane angle alone. The rest of the design constants depend on the value of β_2. An average value of $\beta_2 = 22\ 1/2$ degrees may be considered as normal for all specific speeds. Figure 9 is based on this value of β_2. The design constants for other values of β_2 can be selected from Fig. 25. Lower values of β_2 result in a steeper head-capacity curve (H_s remaining the same for all values of β_2) but may reduce the efficiency slightly on account of the larger impeller diameter necessary to produce the same head (disk friction loss is higher). Higher angles are resorted to when it is desired to increase the output of a given pump.

The vane entrance angle, β_1, is established from the following considerations. (1) Since "shockless" entrance occurs always at a capacity greater than the normal (best efficiency point) prerotation should be allowed on the entrance velocity triangle ($\alpha_1 \approx 75°$, Fig. 5a). (2) The ratio of relative velocities on Euler's triangles is maintained within the limits $w_1/w_2 = 1.15$ to 1.25, the higher ratio referring to higher specific speeds. With mixed-flow impellers, w_1 and w_2 are taken at the average or mean effective diameters. (3) For the average range of specific speeds (1500 to 4500) the entrance angle at the outer flow line becomes equal to the discharge angle, or $\beta_1 \approx \beta_2$.

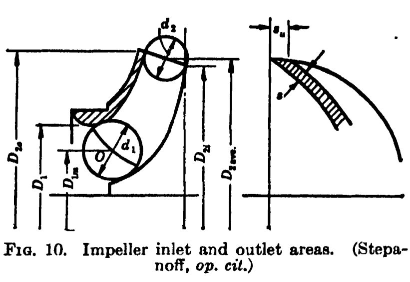


FIG. 10. Impeller inlet and outlet areas. (Stepanoff, *op. cit.*)

Vane angles for mixed-flow and axial-flow pumps for several flow lines are obtained from Euler's velocity triangles for the values of u_2, u_1, c_{m2}, c_{m1}, β_2, and β_1, selected for the mean effective diameter at discharge and average entrance diameter at inlet. The forced vortex pattern of absolute flow is assumed in the impeller approach, through and beyond the impeller. Forced vortex is defined as rotary motion of the liquid at a constant angular velocity ω' at all radii. This is satisfied if the tangential components of the absolute velocities vary directly as the radii or peripheral velocities.

$$\frac{c_{u2h}}{u_h} = \frac{c_{u2o}}{u_o} \qquad \frac{c_{u2h}}{r_h} = \frac{c_{u2o}}{r_o} = \omega' \tag{13}$$

Figures 11*a* and 11*b* show the entrance and discharge velocity triangles for an axial-flow pump for two flow lines, one at the hub and the other at the periphery of the impeller. On both, extensions of lines representing the vane angles intersect on the axis of ordinates at the same point E. This is a necessary and sufficient condition to produce a forced vortex and satisfy eq. 13. The head produced at the hub is lower than that at the periphery; both are equalized in the casing beyond the impeller without cross flows, by conduction. It can be shown that the pump total integrated head H is the arithmetical average of the head generated at the hub and at the periphery and equal also to the head produced at the mean effective diameter. Point E on Fig. 11*b* is determined by the intersection of the line representing the discharge angle at the mean effective diameter (not shown on figure).

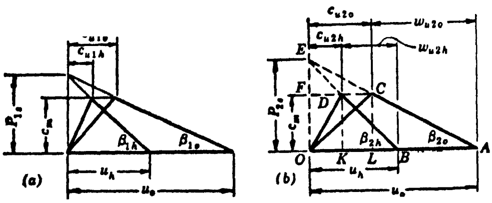


FIG. 11. (*a*) Axial flow pump entrance triangle with prerotation. (*b*) Discharge velocity triangle (Euler's), axial flow pump. (Stepanoff, *op. cit.*)

The ratio P_{2s}/c_{m2} (called the impelling ratio) is an important design factor and can be obtained from Fig. 25.

IMPELLER LAYOUT. The impeller profile is drawn for the known impeller eye and impeller outside diameter, and the meridional velocities at entrance and discharge. For the average range of specific speeds (1500 to 4500) three flow lines are sufficient, a_1a_2, b_1b_2, and c_1c_2 (Fig. 12*a*). The middle flow line can be drawn as a line dividing the flow into two equal parts or drawn as a median line between the two shroud profiles. The latter method is simpler and just as accurate as the first. Next, one of the flow lines (a_1a_2) is divided into a number of parts and then laid out the same distances along the remaining flow lines, points 1*a*, 2*a*· · ·8*a*; 1*b*, 2*b*· · ·10*b*; 1*c*, 2*c* · ·11*c*. Parallel lines spaced g_1, g_2, g_3 · · ·apart, equal to the spacing along the flow lines, are drawn on a plane for the vane development, Fig. 12*b*. The vane development of each flow line is drawn on this plane, using the vane angles β_1 and β_2, between the parallel lines limiting the flow lines on profile 1*a*, 8*a*; 1*b*, 9*b*; 1*c*, 10*c*. The true vane thicknesses s_u, s_{u1}, s_{u2}, etc., are drawn for each flow line (c_1c_2 is shown only on Fig. 12*b*). Now the flow lines are plotted on the plan view (Fig. 13*a*) point by point. The radial distances for each point are taken from the elevation view r_{1a}, r_{2a} · · · (Fig. 12*a*), while the advances of each point along the circles D_2, r_{1a}, r_{2a} · · · are h_1, h_2, h_3 · · · (from Fig. 12*b*). The construction of flow line a_1a_2 is shown; b_1b_2 and c_1c_2 are plotted in the same manner.

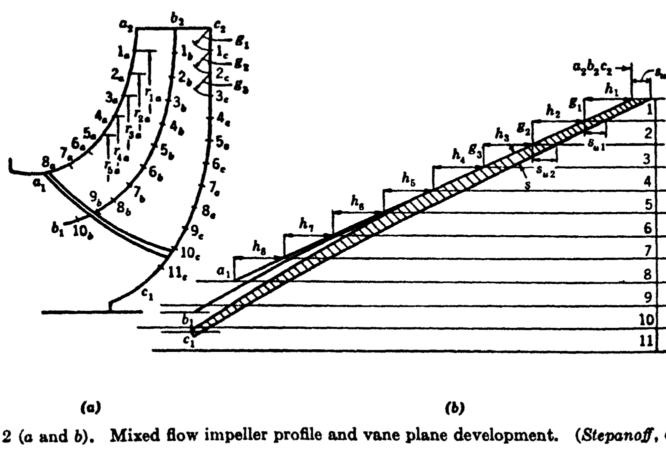
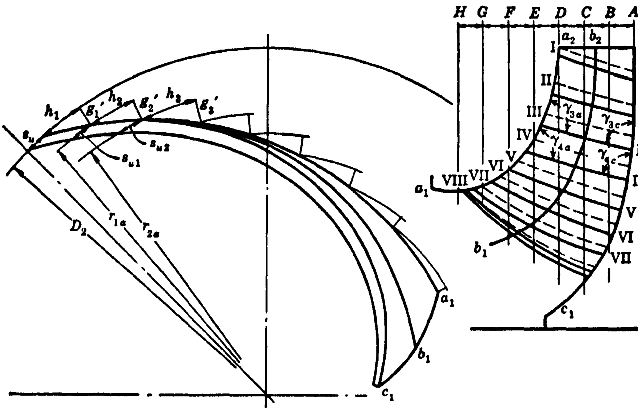


FIG. 12 (*a* and *b*). Mixed flow impeller profile and vane plane development. (*Stepanoff*, op. cit.)

The vane thickness s_u, s_{u1}, s_{u2}, etc., taken from the vane development, is laid off along the arcs of radii r_{1a}, r_{2a}, etc. The flow lines are the first set of construction lines used for plotting the vane pattern sections. For a second set of lines uniformly spaced radial sections are drawn on the plan view such as I, II, III (Fig. 14). The intersections of the flow lines with the radial sections, for both the front and back of the vanes, are plotted on the elevation view I, II, III (Fig. 13*b*). In the next step, vane pattern "board" sections (vane pattern is built up from wooden slices, i.e., boards) are drawn on the elevation view (*A*, *B*, *C*, Fig. 13*b*), which are then transferred to the plan view, using the points of intersection of the board sections with the radial lines I, II, III for plotting. To avoid confusion of



(*a*) (*b*)

FIG. 13. (*a*) Impeller vane plan view; (*b*) Radial sections on profile view. (Stepanoff, *op. cit.*)

lines it is better to separate the views of the front and back sides of the vane, showing them in their respective positions for two adjacent vanes. This will define the impeller channel and determine the shape of the core box if individual core boxes are made for each impeller channel. However, except for very large impellers, one core is made for the whole impeller. It is baked with metal vanes in place and then broken to remove the vanes, after which the parts of the core are reassembled. For a pattern of this type the wooden vane pattern is first made, from which metal vanes are cast.

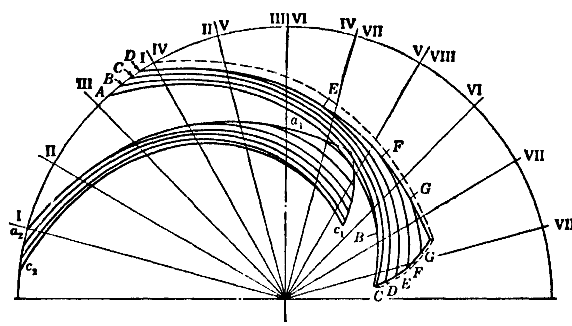


FIG. 14. Vane pattern sections. (Stepanoff, *op. cit.*)

To build the wooden vane, the vane sections are cut to proper shape and thickness and, when they are glued together in the proper order and their corners shaved off, they will give the vane shape. The vane section drawings are obtained by placing the two views of the front and back sides of the vane in their true relative position, one on top of the other. From this the vane sections can be picked out for each board.

Hydraulically, the best form of impeller channel is obtained when the true angles between the impeller vanes and shrouds are close to 90 degrees. For impellers having considerable curvature in their profiles, and for all extreme mixed-flow and axial-flow impellers, this becomes difficult to accomplish. The channel form may be improved by tilting the vane with respect to the shrouds. This is done by moving the flow lines on the plan view through a certain angle in respect to each other, thus changing the angle between the vane and the impeller shrouds without changing the vane angularity (compare Fig. 14 with Fig. 13*a*). The radial sections I, II ··· on the elevation view give approximately the value of the angle between the vane and the shrouds in space (γ_{3a}, γ_{3b}, γ_{4a}, γ_{4b}, Fig. 13b). The method of the vane layout presented here is known as the method of "error triangles." It is also recommended for plain radial vanes with single curvature, although quicker and simpler ways of drawing the plain vane are in use.

VOLUTE PUMP CASING. Except for vertical pumps of the turbine type, the majority of the centrifugal pumps built in the United States are of the volute type. There are several design elements of volute casings which determine their hydraulic characteristics: volute areas, volute angle α_v, volute width b_3, and volute base circle D_3. The best modern pumps are designed for a constant average volute velocity for all volute sections. This means that volute areas are increased in proportion to their angular advance from the cut-water (point A, Fig. 15). The average volute velocity c_3 is determined experimentally from $c_3 = K_3\sqrt{2gH}$. Figure 16 shows values of K_3 for different specific speeds for best efficiency points. In laying out volute areas it is immaterial whether they are measured from the base circle D_3 or from the inside edge of the cut-water at D_v. The variation of the volute angle α_v with the specific speed, as shown on Fig. 16, follows closely the general trend of variation of the angle α_2' between the absolute and the peripheral velocities at discharge on the input (true) velocity triangles as it appears on Fig. 25. Volute width b_3 varies from $b_3 = 1.6b_2$ to $b_3 = 2.0b_2$, where b_2 is the impeller width at discharge. The higher value applies to small pumps of low specific speed.

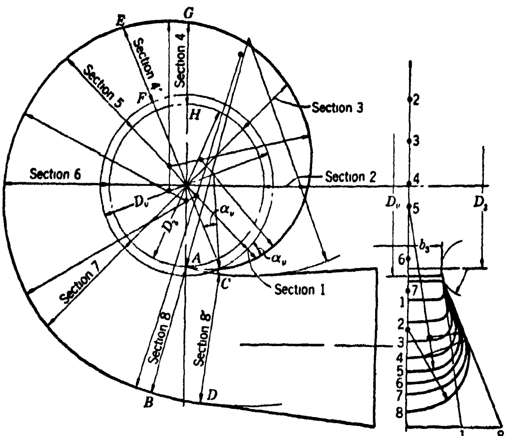


Fig. 15. Pump volute casing. (Stepanoff, *op. cit.*)

The following expression connecting the volute width b_3, base circle diameter D_3, and volute exit area A_8 (AB on Fig. 15) should be satisfied:

$$D_3\pi b_3 = \frac{A_8}{\sin \alpha_v}$$

The base circle D_3 is used for drawing the volute layout, while the cut-water diameter D_v determines the physical limitations of the maximum impeller diameter. Figure 16 gives the minimum diametral gap between the cut-water and the impeller expressed as a ratio to the impeller diameters.

The kinetic energy contained in the flow in the volute expressed as a fraction of the total head is equal to

$$K_3^2 = \frac{c_3^2}{2g}\cdot\frac{1}{H}$$

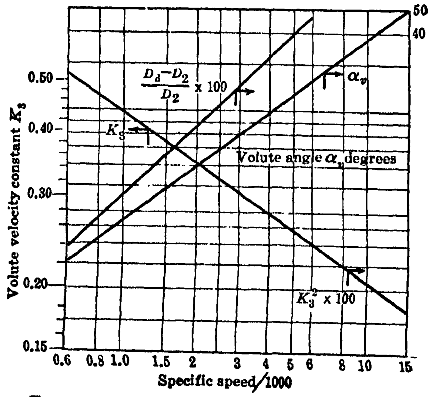


Fig. 16. Volute constants. (Stepanoff, *op. cit.*)

The average pressure in the volute casing, above suction pressure, is equal to $H_v = H(1 - K_3^2)$, disregarding the loss of head due to friction in the volute and the velocity head in the suction nozzle.

Radial Thrust in the Volute Casing. In a volute casing designed for a constant average velocity at the best efficiency point the pressure is the same in all volute sections around the impeller. However, at capacities lower than that of the best efficiency point the pressures in all volute sections are not the same, and radial forces develop which deflect the shaft toward the smaller volute sections. The direction of these forces is reversed at capacities beyond the best efficiency point. The radial thrust can be expressed by a formula

$$P = \frac{KHD_2B_2}{2.31} \quad \text{where} \quad K = 0.36\left[1 - \left(\frac{Q}{Q_n}\right)^2\right] \tag{14}$$

P = the radial resultant force, pounds; H = head, feet, D_2 = the impeller diameter, inches; B_2 = the impeller width including shrouds; Q is any capacity and Q_n is the normal or design capacity. A great many shafts have been broken because of fatigue resulting from shaft deflection caused by radial forces in double-suction pumps having large span between the bearings.

CROSSOVER. In multistage pumps, the channel leading from the discharge volute of one stage to the impeller eye of the next stage is called a crossover. This channel must perform several important functions: (1) convert high velocity in the volute into pressure by reducing the velocity sufficiently to perform the next functions with a minimum loss; (2) make a 180-degree turn; and (3) change the shape of the channel so that it will distribute the flow uniformly around the eye of the next impeller. Each of these functions should be completed independently, or one at a time. A circular section is best for the major portion of the crossover channel. A fully developed crossover may show an efficiency gain up to four points as compared with a short cast-on crossover.

DIFFUSION CASINGS are used chiefly in multistage pumps and combine the functions of the volute and the crossover. Selection of diffusion casing areas (AB, Fig. 17) is made in the same manner as for volute pumps; but since the outlet from the impeller is spread all around the periphery, the width of the diffusion ring b_3 is made only about $b_3 = 1 \cdot 1b_2$. Only a small gap is allowed between the diffusion vanes and the impeller, and tapering impeller and diffusion vane tips becomes more important. The diffusion vane angle α_3 is selected from the same data as the volute angle α_v, but the area of the diffusion ring is of much greater importance than the angle. The diffusion ring depth is maintained so that the ratio D_4/D_3 remains within the values 1.25 to 1.50. A greater number of vanes is used with the lower ratio D_4/D_3. The number of vanes, the vane angle, and diffusion ring depth are adjusted to obtain a square shape of the channel. The vane layout is prepared by using the *error triangles* method, putting part of the vane in the ring and the remainder into the return channel (Fig. 17).

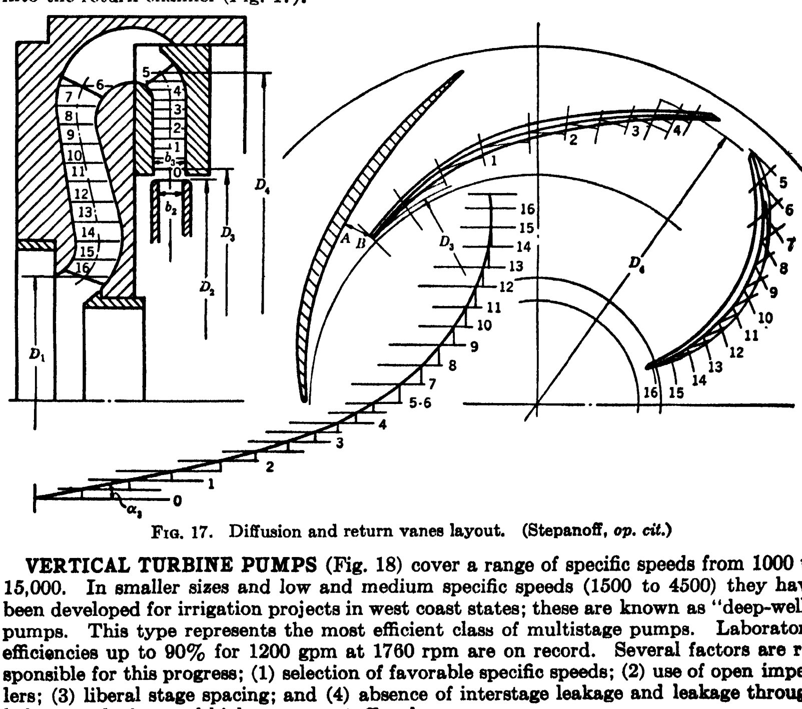


FIG. 17. Diffusion and return vanes layout. (Stepanoff, *op. cit.*)

VERTICAL TURBINE PUMPS (Fig. 18) cover a range of specific speeds from 1000 to 15,000. In smaller sizes and low and medium specific speeds (1500 to 4500) they have been developed for irrigation projects in west coast states; these are known as "deep-well" pumps. This type represents the most efficient class of multistage pumps. Laboratory efficiencies up to 90% for 1200 gpm at 1760 rpm are on record. Several factors are responsible for this progress; (1) selection of favorable specific speeds; (2) use of open impellers; (3) liberal stage spacing; and (4) absence of interstage leakage and leakage through balancing devices and high-pressure stuffing boxes.

Because of the vertical arrangement and limited outside diameter, both the diffusion casing and the impeller profile are extended in the axial direction. The diffusion vanes are developed in one piece without sharp turns (Fig. 18). With increasing specific speed the impeller profile gradually changes from a straight radial to a conical mixed flow and finally to a straight axial flow as represented by the angle δ on Fig. 19. Diffusion vane

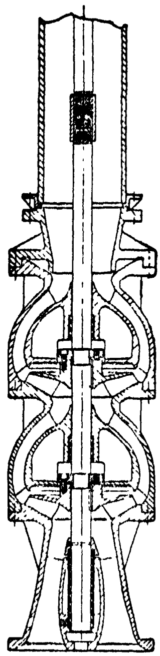


FIG. 18. Vertical turbine (Ingersoll-Rand). (Stepanoff, *op. cit.*)

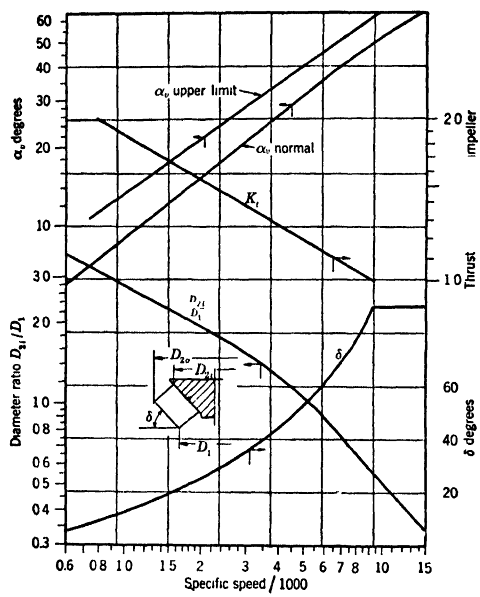


FIG. 19. Vertical turbine pump constants. (Stepanoff, *op. cit.*)

angles and velocities are selected according to the same guiding considerations. Much higher velocities through the diffuser are possible with vertical turbine pumps than with the volute pumps of the same specific speed, requiring higher diffusion vane angle α_v.

AXIAL-FLOW PUMPS cover a range of specific speeds from 10,000 to 15,000. Different specific speeds within this range are obtained by a suitable combination of several design elements. (1) The ratio of the impeller hub diameter to the impeller outside diameter. (2) Vane area or vane length. This is usually given as the ratio of the vane developed length (l, Fig. 20) to the vane spacing t. This ratio, sometimes called "solidity," varies along the radius, increasing toward the hub. (3) The number of vanes for a given solidity should be small for best efficiency. Most axial-flow impellers have three or four vanes. Figure 21 shows the three factors in terms of specific speed, compiled from a number of successful pumps. The vane hydraulic characteristics are completely described by the vane curvature $\beta_2 - \beta_1$, the angles β_2 and β_1 being referred to the vane mean or median line. The vane thickness is built up equally on both sides of the mean line. For thickness variation along the mean line, airfoil sections can be followed. All good airfoil sections have nearly the same thickness variations, the maximum thickness being different for different profiles.*

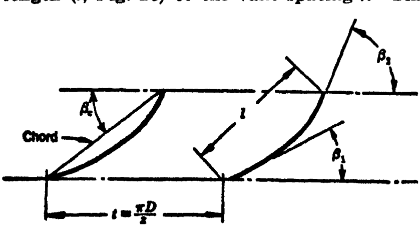


FIG. 20. Axial flow pump vane spacing and angles. (Stepanoff, *op. cit.*)

* National Advisory Committee for Aeronautics. *Report* 460, 1935, p. 300.

Effect of Number of Vanes. The thinnest vane consistent with mechanical strength and foundry practice gives best efficiency. It is essential to have the vane discharge edge tapered to a sharp edge. The effect of several design elements, determined experimentally, is given below. (1) Tests with two to five identical vanes show that (*a*) the normal capacity and capacity at zero head remain the same, because they are determined mainly by the vane entrance angle. (*b*) The normal head and head at zero capacity increase with number of vanes so that their ratio remains the same, due entirely to the increase of the l/t ratio. (2) The results would be the same if the l/t ratio were changed and the number of vanes remained the same. (3) If the impeller vanes are turned through the same angle, (*a*) the normal head and the shut-off head remain the same, because $\beta_2-\beta_1$ is not changed; (*b*) the capacity increases approximately as tan β_1.

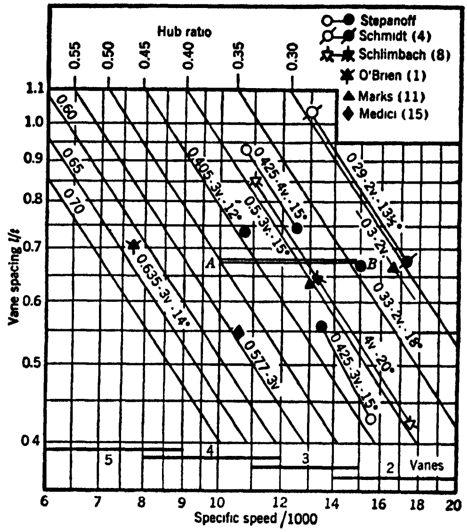


FIG. 21. Hub ratio, number of vanes, and l/t ratio for axial flow pumps. (Stepanoff, *op. cit.*)

Adjustable Vanes. Figure 22 shows the performance of an axial-flow pump with adjustable impeller vanes. Adjustable vanes permit regulation of capacity to suit the demand. Considerable power saving can be effected in this manner. In some services, such as condenser circulating pumps, capacity may follow the load on the condenser, and make it advisable to compensate for water temperature variations with the season of the year. By pumping through the condenser only the water quantity required by the steam load, the life of the condenser tubes is prolonged. Other applications, such as irrigation, drainage, storm water, and dry dock use, require a maximum capacity, irrespective of the suction level. Adjustable-vane axial-flow pumps are particularly suited for such services.

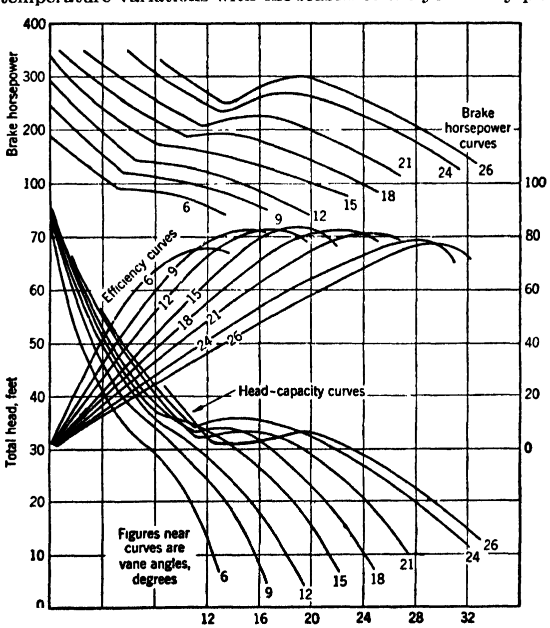


FIG. 22. Test curve of axial-flow pump with adjustable vanes. (Stepanoff, *op. cit.*)

HYDRAULIC LOSSES. Progress in centrifugal pump design has been accomplished largely in an experimental way, the pump gross efficiency being the only criterion of improvement in performance. Hydraulic losses are least known of all pump losses. In general, hydraulic losses are caused by (1) skin friction and (2) eddy and separation losses due to changes in direction and magnitude of velocity of flow. The latter group includes the so-called shock loss and diffusion loss. It is customary to assume that the hydraulic friction and diffusion losses vary as the square of the capacity and can thus be expressed as $h_{fd} = K_1Q^2$, where K_1, a constant for a given pump, includes all lengths, areas, area ratios, and friction coefficients. This can be represented by a parabola with its own axis on the h axis (Fig. 23). The shock losses take place at the impeller entrance and discharge because of a sudden change in direction or magnitude of velocity of flow. If these losses are zero at a certain capacity Q_s, they will increase on both sides of this capacity directly as $(Q - Q_s)^2$ or $h_s = K_2(Q - Q_s)^2$, where K_2 combines all constants for a given pump. This represents a parabola with its axis at Q_s (Fig. 23). The constants K_1 and K_2 vary from pump to pump and are inconsistent along the same head-capacity curve; for this reason there have been no serious attempts to establish values for these constants.

From Fig. 23 it is evident that if both friction and shock losses are combined, the minimum total loss occurs at a capacity lower than the "shockless" capacity. Therefore, for the normal or design capacity, some prerotation should be allowed at the impeller entrance. Since the theoretical head (input head) is represented by a straight line, the shape of the total head-capacity curve is determined by the hydraulic losses (Fig. 24).

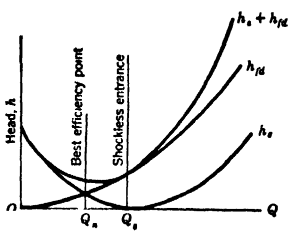


FIG. 23. Hydraulic losses. (Stepanoff, *op. cit.*)

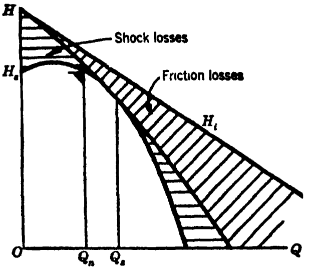


FIG. 24. Q-H curve is obtained by subtracting hydraulic losses from input head. (Stepanoff, *op. cit.*)

STEPANOFF'S DIAGRAM OF CENTRIFUGAL PUMP CHARACTERISTICS. For large pumps in which the scale effect is negligible (i.e., 12 in. and larger) it has been found that the head-capacity points at best efficiency of pumps of different specific speeds using the same discharge angle β_2 fall on a straight line somewhat below the input-head line. In Fig. 25, CE shows one such line for $\beta_2 = 35$ degrees, BE is the input head line, and AE is the Euler's head line for the same angle β_2. The dimensionless capacity ϕ and head ψ are used as coordinates for this diagram. Specific speeds N_s are marked along the line CE. It will be noted that CO/BO is equal to the hydraulic efficiency, and BO/AO is the vane efficiency. Both remain constant for all specific speeds and vane angles β_2. Any number of such lines can be drawn for different values of the angle β_2, determined on the axis of ϕ by $\tan \beta_2 = \phi$. Lines CM, CL, etc., are drawn for β_2 at five-degree intervals. The input head lines for these angles can be drawn by connecting point B with the same points on the ϕ axis. Similarly, lines of constant Euler's head can be established. On the same figure curves of constant specific speed in the dimensionless form $\omega_s = \phi^{1/2}/\psi^{3/4}$ are drawn converging at the origin. To convert the specific speed N_s into the dimensionless specific speed ω_s, this formula is used:

$$N_s = \frac{n\sqrt{\text{gpm}}}{H^{3/4}} = 9675\left(\frac{b_2}{D_m}\right)^{1/2}\left(\frac{D_{\text{ave}}}{D_m}\right)^{1/2} \times \frac{\phi^{1/2}}{\psi^{3/4}} \tag{15}$$

Figure 25 shows the essential elements of design and performance of all centrifugal pumps, including (1) Euler's, input and "actual" discharge velocity triangles for the best efficiency points; (2) design constants to establish the main physical dimensions of the pump; (3) actual performance data indicating the shape of the head-capacity curve. Any point on the diagram (for instance, N) represents the best efficiency point of a pump, the specific speed of which can be read on the scale ($N_s = 6500$; or $\omega_s = 1.25$). The capacity coefficient is $\phi = 0.24$, and the head coefficient is $\psi = 0.29$. The discharge angle $\beta_2 = 22\ 1/2$ degrees. Line CJ is the locus of the best efficiency points of all specific speeds using the same β_2. AJ is the Euler's head-capacity line. The input-head line BJ is not shown. Point D is the actual shut-off head for all specific speeds. From the diagram it follows immediately that steep head-capacity curves are obtained with low values of β_2 and high values of specific speeds. For point N, OPA is the Euler's discharge velocity triangle. ONA is an excellent approximation of the actual velocity triangle. (The error is introduced by disregarding the losses in the pump casing.) The head at the impeller discharge is slightly higher than that measured at the pump discharge. Although the triangle *ONA* gives the true value of the absolute velocity at impeller discharge, c_2'/u_2, the average volute velocity is considerably lower, and should be taken from Fig. 16. The value of the absolute velocity angle α_2' agrees very well with the normal volute angle α_v, shown in Fig. 19. To use the chart for selection of design data the diagram is entered

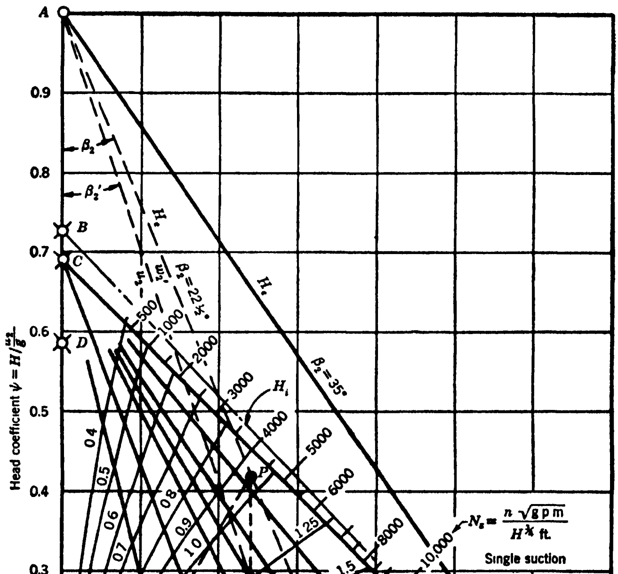


FIG. 25. Stepanoff's diagram of centrifugal pump characteristics. (Stepanoff, *op. cit.*)

through the specific speed scale; i.e., for a given head-capacity condition, the rpm should be chosen first. The best efficiency points ϕ and ψ are read for a desired vane angle β_2. The peripheral velocity u_2 and impeller mean effective diameter D_m are found from the head constant $\psi = 1/2K_u^2$; the meridional velocity c_{m2} is obtained from $\phi = c_{m2}/u_2$. For axial-flow impellers the outside diameter D_o is calculated from

$$D_o^2 = \frac{2D_m^2}{1 + \rho^2} \tag{16}$$

where $\rho = D_h/D_o$ is the assumed hub ratio.

The ratio $\tan \beta_2/\phi$ is the impelling ratio, an important factor for drawing the discharge velocity diagrams for several flow lines. The vane solidity for axial-flow impellers or vane number for mixed flow and centrifugal impellers is found from $l/t = 4.75\psi$, established experimentally. It is important to remember that all values of u_2, β_2, etc., on the diagram are given for the mean effective impeller diameter and not for the outside diameter.

LOSSES IN PUMPS. Leakage Loss. Leakage can take place in one or several of the following places: (1) between casing and impeller at the impeller eye; (2) between two adjacent stages in multistage pumps; (3) through stuffing boxes; (4) through axial thrust balancing devices; (5) through bleed-off bushings when used to reduce the pressure on the stuffing box; (6) past vanes in open impeller pumps; and (7) through bearings and stuffing box for cooling purposes. The volumetric efficiency $e_v = Q/(Q + Q_L)$ accounts only for leakage through the wearing rings and balancing devices. The rest of the items should be treated separately. In each case, to obtain the power loss, the amount of leakage should be multiplied by the pressure drop across the clearance. For a known pressure drop across the clearance the amount of leakage can be calculated by the formula

$$H_L = f\frac{L}{d}\cdot\frac{V^2}{2g} + 0.5\frac{V^2}{2g} + \frac{V^2}{2g} = \left(f\frac{L}{d} + 1.5\right)\frac{V^2}{2g} \qquad (17)$$

where H_L is the head across the clearance, feet; f is a friction coefficient (dimensionless) taken from Fig. 26; V is the velocity through the clearance, feet per second; L is the length of the throttling surface, feet; d is the diameter, in feet, of a circular pipe having the same hydraulic radius as the annular channel of the clearance. It so happens that $a = d$, where a is the diametral clearance, both in feet. The values of f are plotted against Reynolds' number, $R = vd/\nu$, where ν is the kinematic viscosity of liquid, feet square per second. The pressure across the wearing rings at the impeller eye is given by the expression

$$H_L = H(1 - K_3^2) - \frac{1}{4}\frac{u_2^2 - u_r^2}{2g} \qquad (18)$$

where u_r is the peripheral velocity of the impeller wearing ring. The leakage loss through the wearing rings expressed in percentage of the pump input decreases rapidly with increasing specific speed. A few examples calculated for double-suction pumps will serve for illustration.

Specific speed	500	1000	2000	3000	4000	5000	6000
Leakage loss, %	8.0	3.5	1.5	1.0	0.6	0.4	0.3

For similar pumps, the percentage leakage loss remains constant.

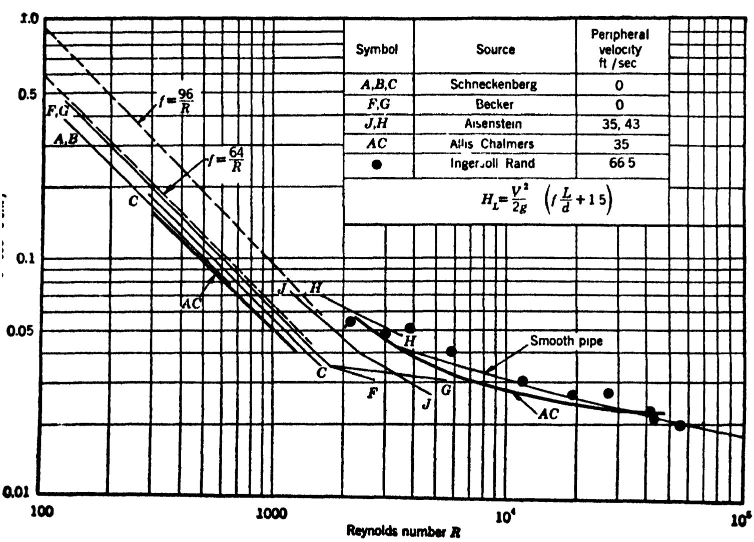


FIG. 26. Friction coefficient for flow through annular clearances. (Stepanoff, *op. cit.*)

Disk friction loss is by far the most important of all external mechanical losses. For water the following formula can be used for calculating disk friction horsepower loss; $(hp)_d = 0.375n^3D^5/10^9$, where D is the impeller diameter, feet. For determination of disk friction loss for viscous liquids a modified von Kármán formula is frequently used.

$$\text{For viscous flow, } M = \frac{1.30}{\sqrt{R}}\gamma r^5\frac{u^2}{2g}, \qquad \text{for turbulent flow, } M = \frac{0.0418}{\sqrt[5]{R}}\gamma r^3\frac{u^2}{2g} \qquad (19)$$

where M is the resisting moment, foot-pounds; R is Reynolds' number, dimensionless; r is the disk radius, feet; u is the peripheral velocity at radius r, feet per second; and γ is the liquid density, pounds per cubic foot.

The percentage of power lost in disk friction is constant for pumps of the same specific speed irrespective of the pump size and speed. However, disk friction loss increases rapidly as specific speed is reduced. Disk friction loss is reduced by polishing the impeller shrouds and cleaning and painting the pump casing walls. Increasing the clearance between the impeller and the casing walls increases the disk friction loss.

Mechanical losses include stuffing box and bearing friction losses. The stuffing box loss depends on the arrangement, size, and lubrication. The bearing losses depend on the type of thrust bearing (ball or Kingsbury) and on the axial load carried. Actual measurements show that both losses do not exceed 1% for multistage high-speed pumps. Also, in single-stage large pumps (over 8 in.) these losses are of the same order. For small pumps, the mechanical losses may be 2 or 3% or more of the pump power input.

PUMP THRUST. Axial Thrust. Single-suction impellers (Fig. 27) are subjected to an axial thrust because an area equal to the impeller eye is under discharge pressure on the back shroud side, while the same area at the impeller inlet is under suction pressure. The magnitude of the axial thrust can be calculated approximately from

$$T = (A_1 - A_s)\gamma H_L$$

where T is the axial thrust, pounds; A_1 is the area corresponding to the diameter of the impeller wearing ring, square feet; A_s is the area of the shaft sleeve through the stuffing box, square feet; H_L is the pressure at the wearing ring, feet, given by eq. 18. When the axial thrust can safely be carried by a thrust bearing, this is the most efficient way to take care of it.

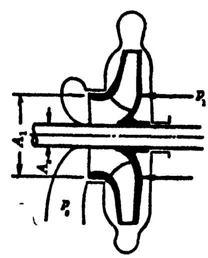


FIG. 27. Axial thrust on a single-suction impeller. (Stepanoff, *op. cit.*)

There are two methods in use for reduction or elimination of the axial thrust in single-stage pumps. In the first method a chamber on the back of the impeller is provided with a closely fitted set of wearing rings. Suction pressure is admitted to this chamber either by drilling holes through the impeller back shroud into the eye or by providing a special channel connecting the balancing chamber to the suction nozzle. For complete balance the diameter of the balancing chamber should be greater than that of the impeller-eye wearing rings. Evidently the leakage loss is double by this method of balancing the axial thrust.

In the second method, radial ribs are provided on the back shroud to reduce the pressure in the space between impeller and pump casing by forcing liquid in this space into rotation. The extra power absorbed by the ribs is less than the extra leakage loss of the first method. This power loss remains constant while the leakage loss increases as wearing clearances increase. In addition, the second method is cheaper and more effective. Figure 28 shows the pressure distribution between the impeller shrouds and the casing. All pressures are taken above the suction pressure. The axial forces on the impeller shrouds are represented by the weights of the liquid enclosed by the surfaces of revolution $ABCD$ on the front shroud and $ABEF$ on the back shroud. The volume $GKFG$ represents the reduction of the axial thrust by the radial ribs. The pressure distribution between shrouds and casing is based on the assumption that the angular velocity of rotation of the liquid in this space is equal to one-half that of the impeller for impellers without radial ribs, and full impeller angular velocity in the space swept by radial ribs. For a complete balance the following equation should be satisfied:

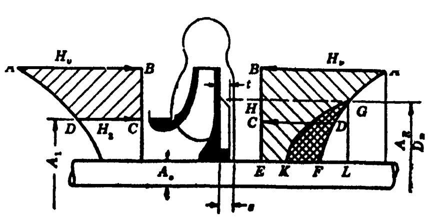


FIG. 28. Balancing axial thrust with radial ribs. (Stepanoff, *op. cit.*)

$$T = (A_1 - A_s)\gamma H_L = \frac{3}{8}(A_R - A_s)\left(\frac{u_R^2 - u_s^2}{2g}\right) \tag{20}$$

where A_R is the area of the circle corresponding to the diameter of the ribs D_R; u_R is the peripheral velocity at the diameter D_R; and u_s is the same at the diameter D_s of the shaft or the shaft sleeve. For overhung impellers there is additional axial thrust in a direction opposite to the impeller suction equal to $T_s = p_s A_s$ where p_s is the suction pressure. Thus for overhung impellers the diameter of the balancing ribs depends on the suction pressure.

Axial Thrust, Multistage Pumps. There are several ways in which the axial thrust of multistage pumps is balanced: (1) by using all double-suction impellers; (2) by arranging impellers in two opposing groups; and (3) by providing balancing devices such as an automatic balancing disk or drum. The automatic balancing disk operates as follows (Fig. 29). On the back of the last-stage impeller a balancing chamber, connected through a throttle A to the first-stage suction, is formed. The balancing disk C is larger in diameter than the impeller wearing rings. The rotating element is free to move axially. Axial thrust tends to move the disk to the left, thus closing the gap between the disk and stationary face B.

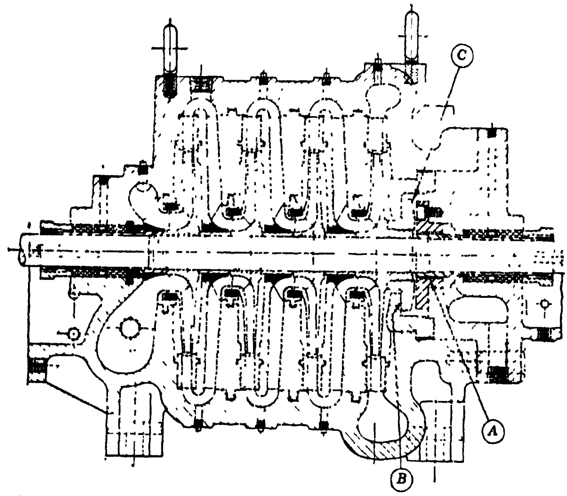


FIG. 29. Multistage centrifugal pump with automatic balancing disk. (Stepanoff, *op. cit.*)

This reduces the pressure in the balancing chamber behind disk C. At the same time the full pump pressure will move disk C to the right until a perfect balance is reached. The amount of leakage is controlled by throttle A. When this becomes worn the gap between disk C and face B increases in order to maintain the required pressure on the back of the disk. To protect disk C and face B from damage under any possible condition the travel of the rotating element is limited and a thrust bearing is usually provided.

Open impellers produce higher thrust than closed impellers. The thrust on the back shroud is only partly balanced by the pressure inside the impeller (Fig. 30). The net axial thrust is the difference between the two (double cross hatching).

$$T = (A_2 - A_s)\left[H_v - \frac{1}{8}\frac{u_2^2 - u_s^2}{2g}\right]\gamma - (A_2 - A_1)\frac{H_v}{2}\gamma \tag{21}$$

Multistage pumps with open impellers are always of the vertical turbine type with all impellers facing in the same direction and the axial thrust taken up by the driver's thrust bearing. Radial ribs are often used to reduce the axial thrust if it is beyond the bearing capacity, but there is a loss of approximately two points in efficiency. For practical purposes great accuracy in calculating thrust is seldom required since a thrust bearing is always used to take care of unbalanced thrust. For that reason simple experimental formulas are in use. One of them has the form $T = A_e p K_t$, where T is thrust, pounds; A_e is the impeller eye area, square inches; p is the pump total head, pounds per square inch; and K_t is an experimental factor which equals 1.0 for axial-flow pumps and increases for mixed-flow and radial impellers. The value of K_t for different specific speeds is given in Fig. 19.

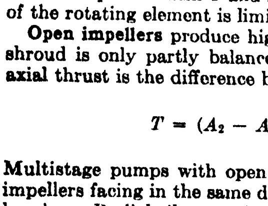


FIG. 30. Axial thrust on open-impeller centrifugal pumps. (Stepanoff, *op. cit.*)

CAVITATION. The term cavitation refers to conditions within the pump where, owing to a local pressure drop, cavities filled with vapor are formed. These cavities collapse as soon as the vapor bubbles reach regions of higher pressure on their way through the pump. Cavitation may appear along stationary parts of the pump casing or along moving vanes of the impeller. The reduction of the absolute pressure to that of vapor tension may be *general* (for the whole system) or merely *local*. The general reduction may be produced by: (1) an increase in the static lift; (2) a decrease in atmospheric pressure; (3) a decrease in the absolute pressure in the system when pumping from a vessel; and (4) an increase in the temperature of liquid. A local decrease in pressure may be caused by dynamic means: (1) an increase in velocity by speeding up the pump; (2) a result of separation and contraction of flow due to a sudden change in direction of flow. The signs of cavitation are: (1) noise and vibration, (2) drop in head-capacity and efficiency curves, and (3) impeller vane pitting. For pumps of low specific speed, the decrease with cavitation in head-capacity characteristics and efficiency is rapid; for medium specific speeds it is more gradual at first and then rapid; for propeller pumps the decrease exists over the whole range of capacity.

Different materials resist cavitation to varying degrees. The most common metals in order of their resistance are cast iron, bronze, carbon steel, and stainless steel. Cavitation conditions can also appear as a result of vibration of parts in contact with water, without an apparent local pressure drop or high velocity of flow. Water is unable to follow the frequency of the vibrating body, and at deflection vapor-filled cavities are formed which collapse on reversal. Theoretical determination of the minimum suction pressure to suppress cavitation is unreliable because the maximum local velocity through the impeller is never known. Therefore, the pump designer has to depend on experimental constants to evaluate the cavitation characteristics of impellers of different specific speeds.

Cavitation constant sigma (see also Hydraulic Turbines, p. 5-38) was introduced by Thoma and defined as

$$\sigma = \frac{NPSH}{H} = \frac{H_a + h_s - h_l - h_v}{H} \tag{22}$$

where H_a is the absolute pressure prevailing at the surface of the pump suction supply; h_s is the static level in the vessel above the pump centerline; h_l is the head loss in the suction pipe; and h_v is the vapor pressure. If the liquid is boiling, $H_a = h_v$ and $\sigma = (h_s - h_l)/H$. The constant σ is the same for all similar pumps irrespective of the pump size and speed. Figure 31 shows variation of σ with the specific speed for an average design. With special designs lower values of σ are possible. A reduction of the minimum required net positive suction head, NPSH, can be realized in low-specific-speed pumps by increase of impeller suction passages, even at the expense of reducing the number of vanes and by use of lower vane entrance angle. For high-specific-speed pumps of mixed-flow and axial-flow types an increase in the number of vanes may reduce the local dynamic depression and the required NPSH. Decreasing the velocity (absolute) reduces the NPSH for impellers of all specific speeds. A turn in the impeller approach affects adversely the cavitation characteristics of impellers, particularly those of higher specific speeds.

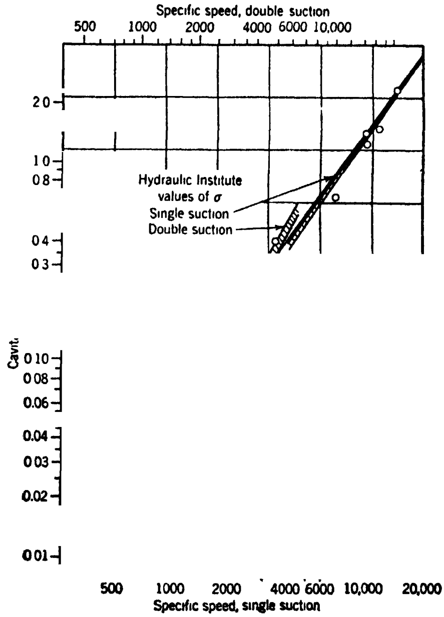


FIG. 31. Typical values of cavitation constant, σ. (Stepanoff, *op. cit.*)

REVERSE ROTATION. If a pump is suddenly stopped and reverse flow develops under the static head in the discharge pipe, the pump will operate as a turbine under no load, or at what is known as the "runaway" condition. It will rotate in the reverse direction, and under a head equal to its normal head it will attain a maximum speed of about 125% of normal speed. At heads lower than the normal head the speed will reduce as the square root of the head. This applies to pumps of all specific speeds. However, when a pump handles liquids at or near their boiling points, such as light hydrocarbons or boiler feed-water, and the check valve fails (mechanical obstruction) in a sudden power failure, the liquid flowing under full discharge pressure will flash into vapor. Under these conditions the pump will reverse under a head of gas several hundred times the pump normal head in feet, and the runaway speed may exceed the normal speed a great many times, if not prevented by mechanical damage to the pump or driver.

The screwed type of shaft coupling is universally used in this country for vertical-turbine and deep-well pumps. The hand of the threads is such that in normal operation the torque of the driver tightens the threads. When the flow and rotation are reversed, the pump becomes a driver and tends again to tighten the threads of the shaft couplings. With the introduction of open impeller construction into the deep-well pump field it was discovered that pressure waves developed in the discharge column of pumps with long settings when the flow was reversed. This caused impellers to hammer against their seats while rotating, resulting in the impeller grinding to such an extent that it was impossible to maintain normal impeller clearances. To eliminate this wear on the impeller vanes many deep-well pumps are equipped with motors having nonreversing ratchets which prevent rotation of the shaft in the reverse direction. This scheme, however, eliminates neither the pressure waves nor the mechanical vibration caused by them.

Starting Pumps Running Backwards. A centrifugal pump running in the reverse direction at a runaway speed does not generate any torque. However, if the motor is put on the line the backward speed of the pump will decrease to zero and then accelerate in the normal direction. The pump will develop an opposing torque, which, under full pump head, may exceed the pump normal torque. Prevention of reverse rotation by mechanical means does not change the torque requirements at zero speed, which still may be higher than the normal torque. A standard motor is able to accelerate the pump from 120% of reverse speed, but starting time is increased and starting current is higher.

On propeller pumps the brake horsepower at shut-off may be twice as much as normal power. Thus if the pump is started with the discharge valve closed the pull-in torque is twice as much as normal; a standard motor cannot provide the necessary torque. When possible, propeller pumps are started with discharge valve partially open, and pump running in the opposite direction. Another frequently used scheme is to provide a check valve in addition to a gate valve and start the pump with the gate valve open against static head only.

PUMPING VISCOUS LIQUIDS. In pumping viscous liquids, such as petroleum products, head and capacity at the best efficiency point are reduced and the brake horsepower increases primarily because of increased disk friction. This increase in brake horsepower is the same through a wide range of capacities. The performance of a pump handling viscous liquids is usually estimated by means of corrections applied to the water performance. Figure 32 gives correction factors for head and capacity at the best efficiency point. On the same figure is given the reduction of the peak efficiency with viscosity. The head on water H_w is not immediately known, and head on oil H_o can be substituted for the first approximation. The head-capacity curve is estimated by drawing a smooth curve through the corrected best efficiency point and the shut-off head point, which remains essentially constant at all viscosities. To draw the efficiency curve the brake horsepower is calculated for the best efficiency point; then the brake-horsepower curve is drawn by following the general slope of the brake-horsepower curve for water. The efficiency curve is calculated from the new brake-horsepower and head-capacity curve.

These experimental results are helpful in evaluating the performance of centrifugal pumps on viscous oil: (1) the affinity laws hold for all viscosities with less accuracy than for water; (2) at constant speed the head and capacity decrease as the viscosity increases in such a way that the specific speed at the best efficiency points remains constant; (3) for a constant viscosity and variable speed, the efficiency is higher at higher rotative speeds. For a given set of head-capacity and viscosity conditions the pump with higher rotative speed (higher specific speed) will show a lower drop in efficiency due to viscosity; and (4) the brake horsepower at best efficiency points increases with viscosity.

HEAD-CAPACITY SURGES. If the head-capacity curve has a peak (maximum head) at partial capacity, operation on the part of the curve from shut-off to maximum head may be unstable under certain conditions. These conditions are (1) The mass of water must be free to oscillate. This happens when the water mass is suspended between two free surfaces as in boiler-feed and condensate pumping cycles. (2) There must be a member in the system which can store and give back the pressure energy, or act as a spring. In a boiler-feed pump cycle the elastic steam cushion in the boiler serves this purpose. (3) There must be a member that will provide impulses at regular intervals to start the head and capacity swings. With an unstable head-capacity curve such conditions are present when the capacity is reduced to that corresponding to the maximum head. The

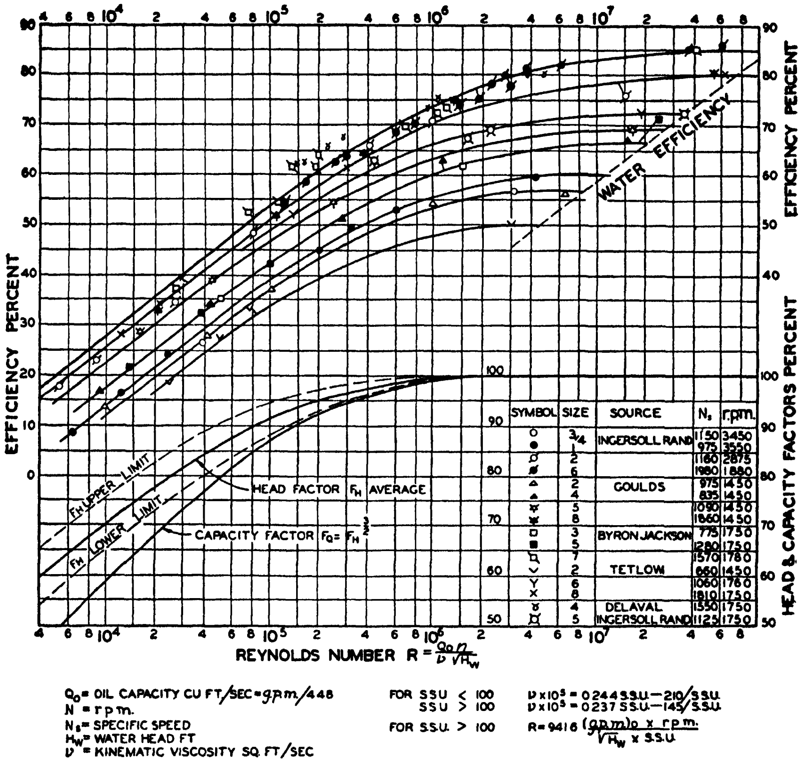


FIG. 32. Head and efficiency corrections for viscosity at best efficiency points. (Courtesy of Ingersoll-Rand Co.)

cause of the pressure pulsations lies in the fact that at certain times the pressure in the discharge line becomes higher than the pump head and a tendency to reverse flow appears. Even with a stable curve, surges may appear if excessive prerotation is allowed in the impeller approach (vane entrance angle is too high). The "droop" in the head-capacity curve can be avoided by using six impeller vanes and vane angles not exceeding 20 degrees. Surges can be suppressed by throttling the pump discharge slightly at a point near the discharge nozzle.

PIPE-LINE CHARACTERISTICS. When a pump has to overcome the pipe-line resistance in addition to a static head, the head varies with the capacity. The operating point is best determined graphically by plotting on the same sheet both the pump head-capacity curve and the pipe-line resistance curve. The following cases may be encountered. (1) When more than one pump in series is used to produce the total head, the combined head-capacity curve is obtained by adding heads for the same capacity. (2) When two or more pumps in parallel are used to deliver the total capacity, the combined head-capacity curve is obtained by adding capacities for the same heads. (3) When the pipe line consists of two or more sections of different diameters in series, the individual pipe characteristics are plotted first, and then a combined or total pipe line characteristic is plotted by adding the resistance of the individual sections for the same capacity.

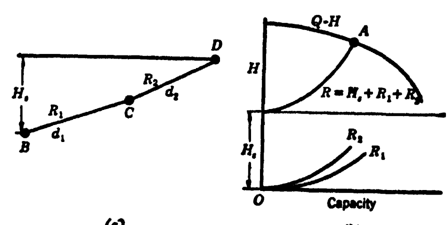


FIG. 33. Determination of pump operating point from pipe-line characteristics. (Stepanoff, *op. cit.*)

When the final point of destination is at a different elevation from the pumping station the static head is added (or subtracted) to the combined pipe-line characteristic. (4) When the pipe line consists of two or more sections in parallel the combined pipe-line characteristics are obtained by adding capacities of the individual pipe characteristics for the same head. Figure 33 shows an example where pumping takes place from the point B to D, the latter being higher than B by H_s feet. The pipe line BC is of diameter d_1 and has a characteristic R_1; R_2 is the characteristic of the line CD; R is the combined pipe-line characteristic. The operating point A is obtained as in intersection of R and the pump head-capacity curve Q–H.

CAPACITY REGULATION at constant speed can be accomplished in one of the following ways: (1) by providing a storage capacity and intermittent pump operation; (2) by dividing the maximum capacity demand among several units and operating one or more units at a time; (3) by throttling the pump discharge or by-passing part of the capacity; (4) by using adjustable-vane axial-flow pumps for low heads. Variable-speed drives permit capacity variation by changing speed. The following variable-speed drives are in use: (1) steam turbine or engine drive; (2) variable-speed electric motors; (3) constant-speed electric motors with variable-speed coupling. The first type of drive does not involve any waste of power. The d-c electric motor, the most efficient electric drive for variable-speed conditions, is widely used for marine service. Direct-current motors are not available at high speed owing to commutation difficulties. The wound-rotor induction motor efficiency varies approximately directly as the speed, but since the pump output varies directly as the cube of the speed, a considerable power saving can be effected with this type of drive. Variable-speed couplings can be of the magnetic or the hydraulic type. The latter is more widely used in practice. The economy of both is approximately the same.

MODEL TESTING is usually resorted to for one or more of the following reasons: (1) as part of the development of a new type of pump to be built in several sizes; (2) to determine pump behavior under special conditions by reproducing those existing in a new proposed plant, such as the cavitation characteristics, and the effect of different arrangements within the pumping station; (3) as an acceptance test where tests of the full-size pump are impossible owing to physical limitations or economic considerations. In general a model test is more accurate than a field test on a large pump because the accurate measurement of large quantities of water presents difficulties. To reproduce the cavitation conditions on the model it should be tested at the same head and submergence as the prototype. If this is impossible, the model may be operated at any head but the submergence must be adjusted to obtain the same value of the cavitation constant σ. Where a model is operated at the same head as the prototype the capacity varies as the square of the size ratio, and the speed varies inversely as the same ratio.

Experience has shown that the efficiency of large units is higher than that of their models. The Hydraulic Institute recommends that the efficiency guarantee should be made on the basis of a model acceptance test. Moody's formula, developed for water turbines, is used occasionally by pump engineers for estimating efficiency of a full-size pump from a model test

$$\frac{1-E}{1-e} = \left(\frac{d}{D}\right)^{1/4}\left(\frac{h}{H}\right)^{1/10} \qquad \text{if } h = H, \text{ then } \quad \frac{1-E}{1-e} = \left(\frac{d}{D}\right)^{1/4} \tag{23}$$

where E is the efficiency, D the impeller diameter, and H the head of the prototype; e, d, and h are the same variables for the model. The same formula can be used for estimating efficiency from a reduced-speed test of the full-size pump, in which case $d/D = 1$.

CRITICAL SPEEDS of centrifugal pump shafts are determined by following the same procedure as that used for shafts of other machines, such as steam turbines, blowers, or electric motors. However, the operation of centrifugal pumps differs from that of these machines in that (1) the pumped liquid exerts a definite damping effect on the shaft vibrations; (2) there are a number of closely fitted parts inside the pump which reduce, or limit, the amplitude of vibrations or serve as bearings; (3) stuffing boxes of modern pumps serve as perfect bearings, and thus reduce the span of the shaft between the supports. Tests on pumps with stuffing boxes packed have shown that the first critical speed takes place at a speed corresponding to that for a shaft with the shaft ends fixed at the middle of the stuffing boxes. On this basis the calculated critical speeds are usually found to be well above 3600 rpm. However, for higher speeds and when mechanical seals are used (no stuffing boxes) shafts should be checked for critical speeds. There are a great many pumps in successful operation running above the first critical speed.

BOILER FEED PUMPS. See Section 7 of this book.

17. RECIPROCATING PUMPS

RECIPROCATING PUMPS VERSUS CENTRIFUGAL PUMPS. Reciprocating pumps are particularly adapted for low capacities and high pressures. Direct-acting steam pumps have an advantage in certain applications because of their flexibility in capacity head and speed, and their uniform efficiency over a wide range of conditions. The advantages of centrifugal pumps lie in their lower initial cost, smaller floor space requirements, quiet operation, absence of pulsating flow, and adaptability for direct connection to high-speed electric motors and steam turbines. The field of application of centrifugal pumps is constantly increasing at the expense of the reciprocating pumps.

DRIVE CLASSIFICATION. All reciprocating pumps are divided into two groups, based on the drive used: (1) direct-acting steam-driven pumps and (2) power pumps. In the former a steam piston is connected directly to the liquid piston or plunger through a common piston rod. In the latter the pistons are actuated by a crank shaft driven through a suitable gear reduction by electric motor or belt. There is another "heavy-duty" crank-and-flywheel reciprocating type driven by compound, cross-compound, or triple-expansion steam engines. In large sizes such units are known as *pumping engines*. They were widely used in the past for waterworks service but are now almost entirely displaced by centrifugal pumps.

Power pumps driven by a constant-speed motor operate at practically constant capacity against widely varying heads, up to the limit permitted by the pull-out torque of the motor. If the discharge is closed the pressure may rise enough to cause mechanical damage to the pump. For this reason they should be equipped with a pressure-relief valve on the discharge. In direct-acting steam pumps an increase in discharge pressure slows down the pump, and a decrease in pressure causes an increase in speed. As soon as the total force acting on the water piston equals that on the steam piston the pump stalls. To maintain constant capacity a direct-acting steam pump can be provided with an automatic control, or governor. The speed of direct-acting pumps can be controlled by throttling the steam inlet pressure.

Pump ends can be divided into two types, piston or inside packed (Fig. 34) and plunger or outside packed. The piston types have packing rings carried by the piston; in the plunger types the plunger moves in stationary packing surrounding the plunger. Plunger

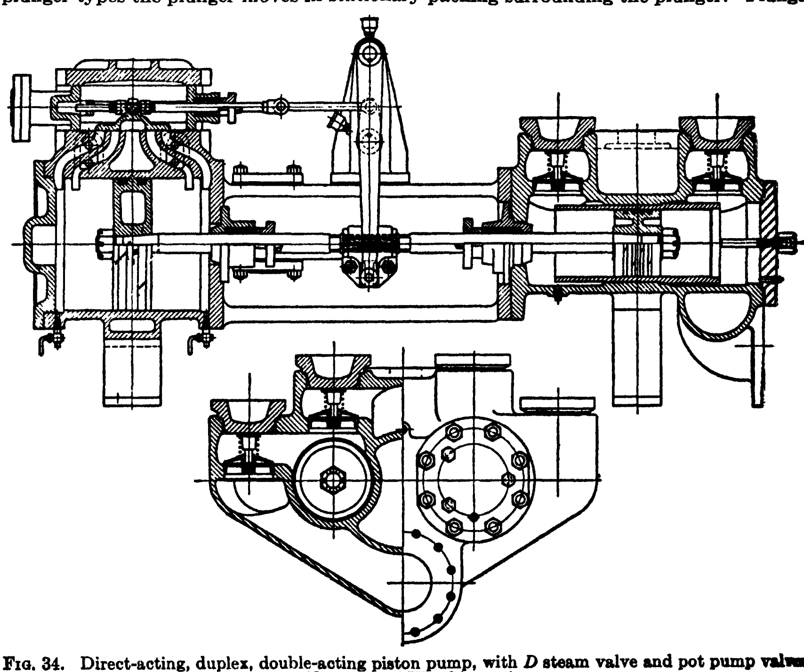


FIG. 34. Direct-acting, duplex, double-acting piston pump, with *D* steam valve and pot pump valves. (Courtesy of Worthington)

pumps are subdivided into "center-packed" or outside-packed types (Fig. 35), depending on the location of the plunger packing. The plunger pumps are used chiefly for high-pressure duty and are heavier and more expensive than the piston types.

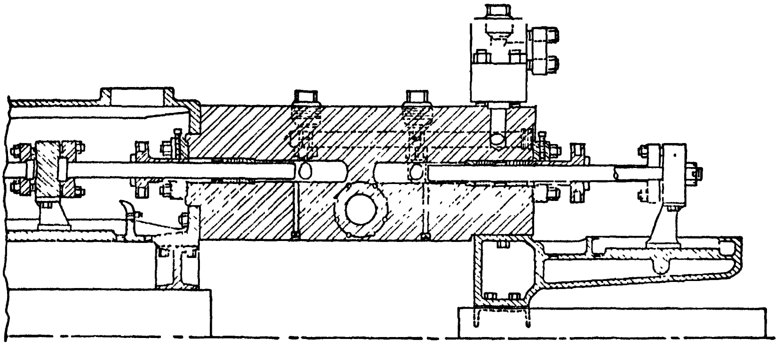


FIG. 35. Horizontal duplex plunger, high-pressure power pump. (Courtesy of Worthington)

According to the method of operation the pump ends may be either single acting or double acting. Single-acting pumps have one discharge and one suction stroke per cycle. In a double-acting pump there are two discharge and two suction strokes per cycle. Nearly all direct-acting pumps are double acting. If a pump has one pump cylinder and one steam cylinder, it is called a "simplex" pump. A "duplex" pump consists of two simplex pumps arranged to operate together as a single unit (Fig. 34). Since the duplex pump has twice the number of discharge strokes per cycle as a simplex pump, the resultant flow of liquid is more even. The duplex pump is the most widely used type of steam pump. The size of a direct-acting pump is designated by the diameter of its steam and liquid pistons and the length of stroke. Thus 6 × 4 × 6 designates a pump with a 6-in. diameter steam piston, a 4-in. diameter liquid piston, and a 6-in. stroke. Motor-driven power pumps usually are built as simplex, duplex, or triplex units, and occasionally with more cylinders. Both direct-acting and power pumps may be disposed either horizontally or vertically, to suit different service applications.

HEAD, CAPACITY, EFFICIENCIES. The total head as defined for centrifugal pumps also applies to reciprocating pumps. It is the general practice of manufacturers of reciprocating pumps to state capacities in terms of piston or plunger displacement without deduction for the piston rod area or slippage. Volumetric efficiency is defined as $e_v = \frac{Q}{Q + Q_L}$, where Q is the actual volume of liquid discharged and $Q_L + Q$ is the true piston or plunger displacement. Q_L includes all losses of capacity due to leakage past piston packing, stuffing boxes, and valves, and also that loss due to delayed closing of valves. All losses of capacity given in percentage of the displacement are referred to as slip ($= 1 - e_v$). In new pumps the slip is of the order of 2%. In packed low-head pumps the slip may be as low as 1/2% or may even be negative as a result of the inertia of the liquid column, which continues to move even after the plunger has stopped.

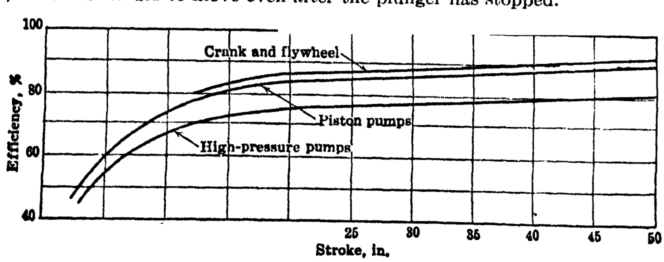


FIG. 36. Steam pump gross efficiencies.

When liquids are pumped at high temperatures and pressures (petroleum products) they cannot be considered incompressible. In such case e_v must include also the loss of capacity due to compression of the liquid on the discharge stroke and re-expansion of the liquid retained in the cylinder clearance on the suction stroke. For compressible liquids, at high pressures, it is essential to reduce the cylinder clearance to a minimum.

Hydraulic and mechanical efficiencies as defined for centrifugal pumps are rarely considered separately but are included in the gross pump efficiency $e = \text{whp}/(\text{hp})_i$, where whp is the pump output (water or liquid horsepower) and $(\text{hp})_i$ is the indicated steam horsepower of a steam pump or brake horsepower of the motor of a power pump. Figure 36 gives commercial efficiencies for direct-acting steam pumps, and Table 1 for power pumps.

Table 1. Approximate Efficiencies of Power Pumps

Water, hp	3	4	5	7 1/2	10	15	20	30	40	50	60	75	100	150	200	250
Efficiency, %	55	60	65	70	72	75	77	80	82	83	84	85	86	87	88	89

SUCTION LIFT. Reciprocating pumps are subject to suction limitations to at least the same degree as centrifugal pumps. The liquid cannot be "sucked" into the pump; on the contrary, flow to the pump is produced by external pressure (atmosphere), the pump producing a pressure reduction in the pumping cylinder. For a given pump (size of valve openings) and suction pipe the capacity or maximum speed is fixed by the available net positive suction head, NPSH. All factors leading to a reduction of NPSH for centrifugal pumps increase the maximum capacity (or speed) of reciprocating pumps. Figure 37 gives normal values of suction lift for water at different temperatures and elevations. For boiling water at corresponding pressure, 8 ft NPSH is considered as the minimum; normal NPSH is about 12 ft.

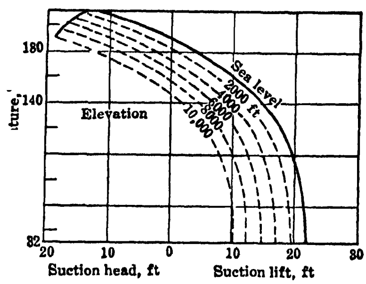


Fig. 37. Maximum suction lift recommended for reciprocating pumps for water at different temperatures.

VISCOSITY AND TEMPERATURE CORRECTIONS. In addition to suction lift, maximum pump speed and capacity are determined by the viscosity and temperature of the pumped liquid. Table 2, based on Chart D-25 of the Hydraulic Institute Standards, gives basic speeds for direct-acting and power pumps. Table 3, compiled from Chart D-26 of the Hydraulic Institute Standards, gives corrections for temperature and viscosity to be applied to basic speeds obtained from Table 2.

Table 2. Basic Speeds for Direct-acting and Power Pumps

Stroke, in.	Rpm of Simplex Power Pumps	Rpm of Duplex and Triplex Power Pumps	Strokes per min of Duplex Steam Pumps	Strokes per min of Simplex Steam Pumps
3	85	105	122	137
4	75	90	105	120
6	60	73	86	98
8	53	64	74	85
10	47	57	67	76
12	43	52	61	70
15	38	46	55	62
18	35	42	50	56
24	30	36	43	49

Table 3. Speed Corrections for Viscosity and Temperature

Viscosity, SSU	250	500	1000	1500	2000	3000	4000	5000
Reduction in speed, %	0	4.0	11	16	20	26	30	35
Temperature, °F	70	80	90	100	125	150	200	250
Reduction in speed, %	0	9	15	18	25	29	34	38

For handling semi-solid material such as acid sludge, very heavy residuum syrups, and molasses, a special design of the liquid end having no suction valve is used. The residuum gravitates into a hopper or funnel suction, entering the cylinder at midstroke (Fig. 38). The piston, of the solid plug type, shears off a slug of the material and forces it through large ball or disk discharge valves at each end of the cylinder.

AIR CHAMBERS are generally used to dampen discharge line pulsations. In simplex pumps and also in all power pumps air chambers are a necessity. The discharge of duplex pumps is sufficiently smooth so that no shock-absorbing devices are necessary. In most pumps air chambers are installed on the discharge lines. But with long suction pipes and

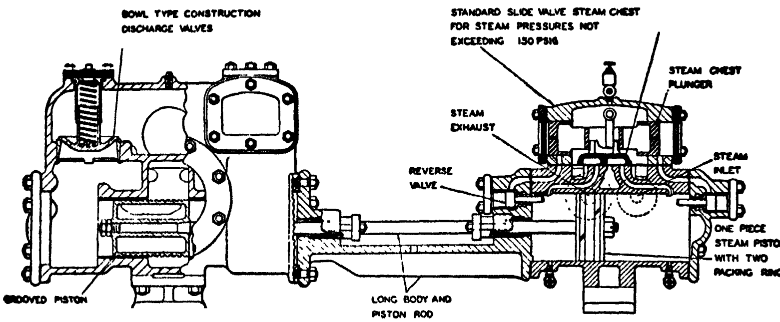


FIG. 38. Simplex double-acting pump for heavy substances. (Courtesy of Ingersoll-Rand)

high-speed large pumps an air chamber on the suction pipe contributes greatly to the smoothness of the pump operation and life of packing and valves. The size of the air chamber depends on the pump type, number of discharge strokes per cycle, and pump speed, and varies from two to six times the piston displacement per stroke.

If a crank of a radius r turns with a constant angular velocity ω, the crankpin velocity is $r\omega$. Assuming a connecting rod of infinite length, the piston position, measured from the dead end is $s = r(1 - \cos\phi)$, where ϕ is the crank angular position with respect to the axis of the cylinder. The piston velocity is $V = r\omega \sin\phi$. Hence the instantaneous pump discharge by each piston is $Q = FV = Fr\omega \sin\phi$, where F is the area of the piston.

Figure 39 shows diagrams of the pump discharge plotted against time for (a) a simplex double-acting pump, (b) a duplex double-acting pump, and (c) a triplex single-acting pump. The difference between the maximum discharge and the average is stored in the air chamber, to be returned to the discharge pipe when pump discharge falls below the average.

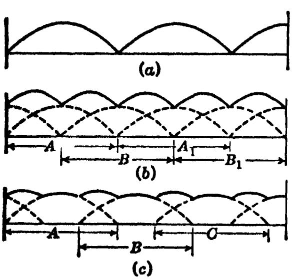


FIG. 39. (a) Simplex double-acting; maximum rate of flow above average is 60.0%, minimum below average is 100.0%. (b) Duplex double-acting; maximum rate of flow above average is 26.7%, minimum below is 21.6%. (c) Triplex single-acting; maximum rate of flow above average is 6.64%, minimum below is 18.4%.

PUMP VALVES. The cap and valve plate arrangement (Fig. 40) is the most frequently used valve in simplex and duplex pumps. The suction valves are in the lower chamber, and the discharge valves are in a removable valve deck; both are easily accessible. The valves are of either the disk type or disk valves faced with resilient materials such as rubber, leather, or composition. This type is used for general application and for pressures up to 350 psi. For higher pressures pot valve piston pumps are used. All cylinder surfaces exposed to the discharge pressure are of small area and reinforced by the pot valve construction. Each valve is in a separate chamber and easily accessible. This type originally was designed for oil refinery service, but a great many have been replaced by centrifugal pumps. For high pressures, wing-type valves with bevel seat are used, as shown in Fig. 35. For viscous material, valves with large port opening are required. Ball valves and mushroom-faced valves of the Rollo type (Fig. 38) present much less resistance to the liquid passage than disk- or wing-type valves. The ball type depends on its own weight to return it to its seat, and may lag behind the piston movement; the Rollo spring-loaded type, more positive in action, has all the advantages of the ball valve. Figure 41 shows a mushroom *dual-face valve* with wing guide for slush and cement service. A steel-to-steel bevel seat withstands closing impact and carries the hydrostatic load of the fluid; a rubber-to-steel seat provides a liquid-tight seal that prevents seat cutting. The wings are set at a small angle to the axis of the seat to give a slight rotary motion to the valve to insure uniform wear of the seat. For large capacities and viscous liquids two valves in parallel are frequently used. To handle mash, sewage, and other thick liquids, hinged-flap valves are used to a limited extent. The valves are flat, hinged at their upper ends, and are closed by flat springs on inclined seats.

Steam End, Direct Acting. A direct-acting pump has its steam piston connected to the pump piston or plunger by means of a rod without a crank. Since there is no moving mass to store energy to carry the piston over dead center, early cut-off of steam is not possible.

Therefore, the indicator diagram of the steam cylinder is a rectangle. The size of the steam end is selected so that the net steam pressure acting over the piston area provides the force necessary to move the liquid piston against the total liquid pressure (including losses through valves) and to overcome all mechanical losses of both steam and liquid ends.

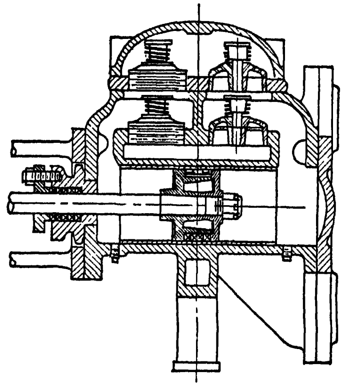


FIG. 40. Pump end with a cap and valve plate, or turret valve assembly. (Courtesy of Worthington)

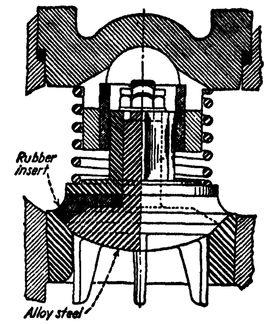


FIG. 41. Dual-face mushroom valve with wing guides for slush pump.

Simplex pumps have steam valves operated by pilot valves either moved by an external linkage similar to that used with duplex pumps (Fig. 42) or moved internally by the main steam pistons (Fig. 38). In the latter type the main steam valve, B-shaped, is moved by an auxiliary plunger located immediately above it, housed in the body of the steam chest and surrounded by high-pressure steam on all sides. When the steam piston approaches the end of its travel it opens the reversing valve housed in the steam cylinder heads. This releases the steam pressure on one end of the auxiliary plunger causing it to move to this end and move the main steam B valve. Both the steam piston and the auxiliary plunger are cushioned near the end of their travel by trapping portion of the steam between the piston and the cover. Figure 42 shows the steam end of a simplex pump with an external mechanism to actuate the pilot valve and a D main steam valve, moved by a plunger shuttle valve.

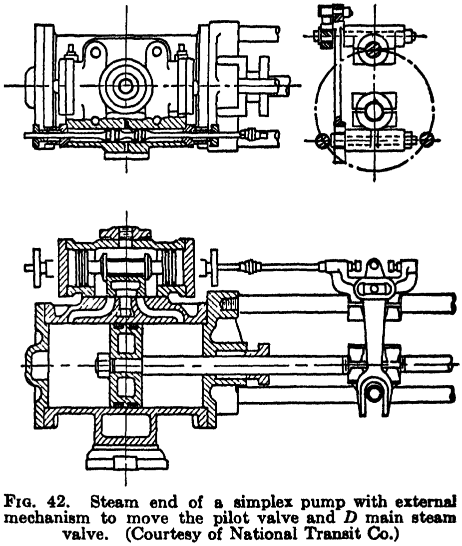


FIG. 42. Steam end of a simplex pump with external mechanism to move the pilot valve and *D* main steam valve. (Courtesy of National Transit Co.)

Duplex pumps have mechanically operated steam valves. The piston rod of one cylinder, while making its stroke, actuates the opposite steam valve, and thereby controls the admission or exhaust of steam of the other cylinder (Fig. 34). This arrangement avoids the "dead center" condition since one or the other steam cylinder port is always open. The pump is thus ready to start when steam is admitted to the steam chest. The valve is not rigidly connected to the crankpin; a small lost motion is allowed in order to delay the movement of the valve. This permits the steam piston to come to rest gradually. Increasing the lost motion lengthens the stroke, and if made excessive the piston will strike the cylinder head. Reducing the lost motion will shorten the stroke with a resultant loss in capacity. Most small pumps are fitted with a fixed amount of lost motion. On large pumps the lost motion setting is adjustable and can be made while the pump is in operation. The slide valve is of D type and is pressed against its seat by the full steam pressure. It will wear without proper lubrication, particularly with high steam pressure and temperature. Balanced piston valves are used for high pressures and temperature. Besides reducing the wear these valves also have the advantage that they give a large port opening area with a minimum of valve travel.

Cushion valves are provided on large pumps to control the amount of cushioning steam in the cylinder, depending on the pump speed and load. The steam end has five ports; the outside ones are for steam admission and the inside ones for steam exhaust (Fig. 34). As the steam piston approaches the end of the cylinder it covers the exhaust port, thereby trapping an amount of steam in the end of the cylinder. The cushion valve is a by-pass valve between the steam and exhaust ports, and by opening or closing this valve the amount of cushion steam can be controlled. When operating at high speeds the cushion valve is closed. Cushion valves also serve to regulate the length of stroke to prevent the piston from striking the cylinder head.

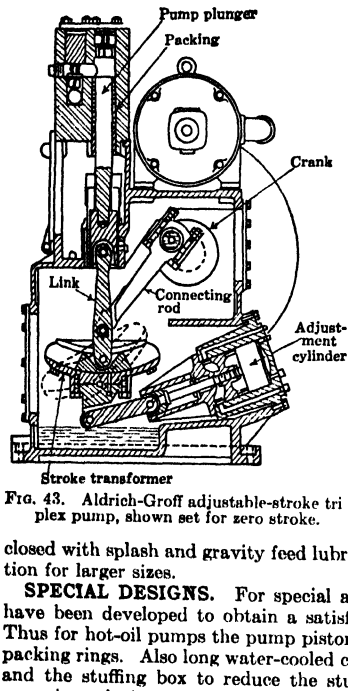


FIG. 43. Aldrich-Groff adjustable-stroke triplex pump, shown set for zero stroke.

VARIABLE-STROKE PUMPS. Small-capacity and high-pressure reciprocating pumps having an adjustable plunger stroke have been developed. They permit variation of capacity from zero to maximum capacity with a constant-speed driver. Variation of length of stroke is accomplished either by varying the position of the eccentric (Worthington) or by a movable slide which is a part of the linkage between the connecting rod and the crosshead (Fig. 43). Change in the rate of flow may be accomplished automatically by means of a remote hydraulic control. Variable stroke pumps are particularly suited for such services as boiler feed, stationary or marine; hydraulic presses; process charging in chemical plants or refineries; and repressuring of oil fields.

INVERTED POWER PUMPS are a later development in vertical triplex and quintuplex pumps (Fig. 43). In these the fluid end is on top of the frame, and the crankshaft extends through the crankcase, which forms the lower part of the frame. The whole unit is totally enclosed with splash and gravity feed lubrication for smaller sizes and full force-feed lubrication for larger sizes.

SPECIAL DESIGNS. For special and difficult services a number of special features have been developed to obtain a satisfactory operation and normal life of pump parts. Thus for hot-oil pumps the pump pistons are provided with hammered-iron self-adjusting packing rings. Also long water-cooled chambers are provided between the pump cylinders and the stuffing box to reduce the stuffing box temperature. For pumping acids and corrosive solutions, parts coming in contact with liquid are made of heat-resisting stoneware encased in cast iron, hard rubber, or rubber-covered metal. For pumping tar, asphalt, and heavy oils, the cylinders, heads, and discharge-valve chambers of the liquid end are steam jacketed to reduce the viscosity of the pumped liquid. Since exhaust steam from the steam end is used for these steam jackets, heating of the pump end adds little to the cost of pumping.

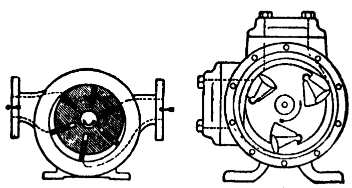
For materials for pumping various liquids, consult Hydraulic Institute Standards, Section G-41.

18. ROTARY PUMPS

Rotary pumps comprise an exceedingly great variety of types (the number of patents run into thousands), all of which have several common characteristics. They are of the positive-displacement type without valves and, except for leakage, can deliver a constant capacity against variable pressure. Pressures up to 3000 psi have been produced with rotary pumps at small capacities, although most of them are used for pressure not over 350 psi. Rotary pumps are particularly suited for handling viscous liquids up to 250,000 SSU (steam jacketed). For such services they are built in sizes up to 2500 gpm. To maintain volumetric efficiency rotary pumps require very close clearances between rubbing surfaces. Although used successfully for clean water and gasoline, their field of application lies primarily in pumping oils or other liquids having some lubricating qualities and sufficient viscosity to prevent excessive leakage. Several million rotary pumps are in service in domestic oil burners, refrigerators, lubricating oil circulators, and hydraulic controls for machinery.

The operation of the rotary pump depends on the formation of practically fluid-tight enclosed spaces filled with liquid on the suction side of the pump and the displacement of the liquid on the discharge side. Thus all of them are self-priming and work satisfactorily under suction lifts up to 20 ft. Higher lifts are possible under certain conditions, depending on the nature of the liquid, pump size, and speed. Although all rotary pumps can handle entrained air, gases, and vapors, detrainment or separation of gases affects adversely the pump volumetric efficiency, particularly at high suction lifts. Also they may become noisy and may vibrate and cause pulsating discharge pressure under such conditions. Entrainment of air and consequent foaming present a difficult problem in connection with circulation of lubricating oil for bearings, gears, and similar services. These factors reduce oil foaming: (1) low velocities in suction and discharge and low rate of circulation (large supply tank); (2) low suction lift and maximum submergence of suction and return pipe; (3) heating oil, where practical, to reduce viscosity, and use of special oils; (4) prevention of air leaks. Several leading types of rotary pumps are described below.

TYPES OF ROTARY PUMP. Sliding-vane type is shown in Fig. 44*a*. The rotor turns in an eccentric casing. The liquid trapped between the vanes is pressed out at the discharge. The vanes are pressed against the casing by centrifugal force, springs, or by pressure from behind the vanes. Leakage occurs across the tips and sides of the vanes. Increasing the number of vanes materially reduces leakage. High-pressure pumps of this type are built with oval-shaped casing having two intake and discharge strokes per revolution. Because of symmetry the radial load on the bearings is eliminated. Figure 44*b* shows a swinging-vane type of pump working on the same principle. Since the number of vanes is limited by space, this pump is essentially a low-pressure low-speed pump. Some of the pumps of this group have metal rollers in slots of the rotor which are thrown out by centrifugal force and act in the same manner as sliding vanes.



(*a*) (*b*)

FIG. (*a*) Sliding-vane pump. (*b*) Swinging-vane pump.

Gear pumps include spur gear, lobar (two and three teeth), internal gear, and helical and herringbone gear types. Figure 45 is a diagram of a spur or herringbone gear type. Liquid fills the spaces between the teeth on the suction side and is carried over to the discharge side, where it is pressed out by the engaging teeth. One gear is keyed to the shaft; the other rotates as an idler. Correct shape of teeth assures a tight seal where they mesh and at the same time will not trap liquid in the root of the teeth to build up high pressure there. Any oil entrapped in the roots of the teeth causes high radial forces on the shaft and results in noisy operation. A greater number of teeth reduces the leakage loss. Spur gear pumps operate at slow speeds up to about 600 rpm. Herringbone gear pumps operate at higher speeds, up to 1750 rpm in small sizes, and are free from axial thrust, but require accurate machining to keep the leakage at the meshing point low.

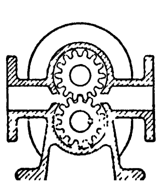


FIG. 45. Spur or herringbone gear pump.

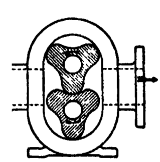


FIG. 46. Three-lobe pump.

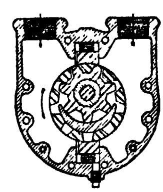


FIG. 47. Internal-gear pump.

The lobar type (Fig. 46) was used extensively in the past for low-pressure high-capacity pumps and blowers. They operate in the same manner as a two- or three-tooth gear pump, but require external pilot gears capable of transmitting half the total power of the pump. Lobar type of pumps and blowers gradually are being replaced by more efficient and quieter high-speed rotary or centrifugal machines.

Figure 47 shows an internal-gear pump with a difference of two teeth between the internal and the external gear. The overhung rotor revolves concentrically within the casing and engages the external gear rotating freely as an idler. A crescent piece attached to the casing cover is interposed in the clearance between the teeth of both gears, providing a seal between suction and discharge. The suction is separated from discharge by two spring-loaded shoes which serve as relief valves, and also compensate for wear.

Screw Pumps. Figure 48 shows a screw pump with one driving rotor and two driven idler screws closely meshed and running with a close clearance in the casing without timing gears. By intermeshing, the helical passages in the rotors are divided into compartments completely sealed, which, while rotating, progress from the suction to the discharge end. Multiple surfaces rather than line contacts between screws and casing reduce leakage and permit high pressures. Where right- and left-hand helices are used, axial thrust is eliminated. Because of the small rotor diameter and the shape of the rotor, high speeds (3500 rpm) are possible. The discharge is continuous and noiseless. Figure 49 shows a two-rotor screw pump with timing gears. For pumping liquids with low lubricating properties the rotors are supported on sleeve bearings. This reduces the rotor wear.

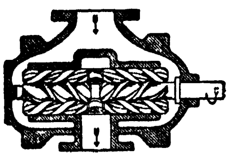


FIG. 48. Three-screw pump. (Courtesy of DeLaval Steam Turbine Co.)

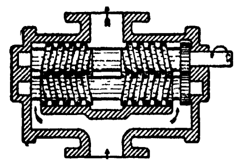


FIG. 49. Two-rotor screw pump.

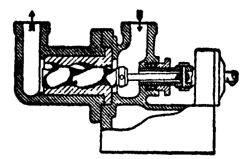


FIG. 50. Moyno single-screw pump.

The Moyno screw pump (Fig. 50) consists of one single-thread steel or bronze rotor running eccentrically in a double-threaded soft or hard rubber casing. The length of the rotor is such that liquid is trapped in the stationary helices between two successive threads of the rotor. The single screw pump is similar in operation to an internal gear pump with a one-tooth helical external gear engaging a two-tooth helical internal gear. Since the rotor has an eccentric motion as it rotates, it is connected to the drive shaft by a connecting rod and two universal joints.

Rotary plunger type of pumps are reciprocating pumps with suction and discharge valves. Their reciprocating motion is derived from a rotating shaft by means of eccentrics, cams, or wobble plate, built in the same casing. Nearly all such pumps have low capacity and high pressure (up to 10,000 psi). Higher capacity is obtained by increasing the number of plungers (up to seven), all driven by the same shaft.

19. IMPULSE TURBO-PUMPS

Pumps of this class are also known as "peripheral," "regenerative," or "turbine" pumps. Pumping is effected by a multiblade impeller (rotating in a concentric casing of constant passage area) by direct blade impulse on the liquid (Fig. 51). Liquid is thrown off at the periphery of the impeller and drawn in at a smaller radius, establishing a circulation from the impeller to the casing passage. The energy imparted by the impeller impact increases as liquid progresses along the casing channel. Partial conversion of the velocity into pressure takes place in the casing channel. High-pressure liquid is diverted into the discharge nozzle by a closely fitted stop, or cut-water, a new supply of the liquid being drawn from the suction nozzle. The pump becomes self-priming if the casing is arranged so that enough liquid is entrapped to separate the suction from the discharge chambers. Air or vapor is carried from the suction to the discharge nozzles by entrainment. Good efficiency (up to 50%) depends on close running clearances between rotor and casing side plates. The pump is free from axial thrust because of symmetry, but the rotor is subjected to a radial load caused by the uneven pressure distribution in the cas-

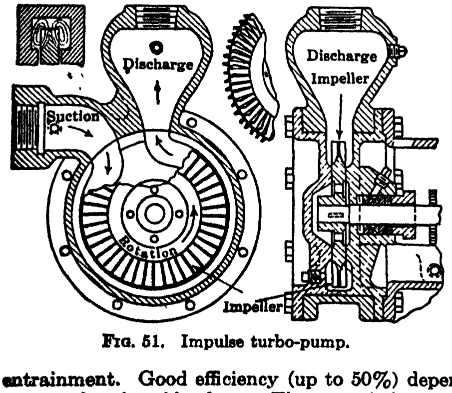


FIG. 51. Impulse turbo-pump.

ing. In small sizes impulse pumps operate at 3500 rpm. Built in sizes up to 2 1/2 in., impulse turbo-pumps are available for capacities up to 200 gpm and heads up to 350 ft at 1750 rpm. The head at best efficiency point is equal to approximately $H = 2u^2/g$, where u is the peripheral velocity at the impeller outside diameter. This is about four times the head produced by a centrifugal impeller of the same diameter. The shut-off head is 2.5 to 3.5 times the head at the best efficiency point. The liquid velocity c in the casing at the best efficiency point is approximately equal to $c = u/2$. The brake horsepower increases as the capacity decreases. The impulse pump follows the affinity laws of centrifugal pumps, i.e., capacity varies directly as the speed and head varies directly as the square of the speed.

20. CENTRIFUGAL JET-PUMP WATER SYSTEMS

GENERAL ARRANGEMENT. For small capacities and low lifts (up to 125 ft) a special type of pumping unit has been developed which consists of a combination of centrifugal pump and jet-pump or ejector. The centrifugal pump is motor-driven, at ground level, and furnishes the driving head and capacity for a jet pump placed in a well, for example, below the water surface (Fig. 52). For shallow wells, up to 25 ft, the jet pump can be placed on the surface of the ground or built into the centrifugal-pump casing. The mechanical advantage of this arrangement is evident as there are no moving parts in the well, and the centrifugal pump, with its motor, can be placed at some convenient point. The hydraulic advantages are steep head-capacity characteristics with operating head about 50% higher than that of the centrifugal pump alone, and a nonoverloading brake-horsepower curve. The peak efficiency of the combination is at least equal to that of the jet pump, but is lower than that of centrifugal or vertical turbine pumps. However, at the operating capacity, the efficiency is at least equal to that of the centrifugal pump, operating at the same capacity. In small sizes, this type of pumping unit is widely used for domestic water supply. According to the U. S. Department of Commerce Census, 351,905 jet-type water systems were sold in 1947, representing a value of $29,110,797. Most units are for capacities of 5 to 10 gpm.

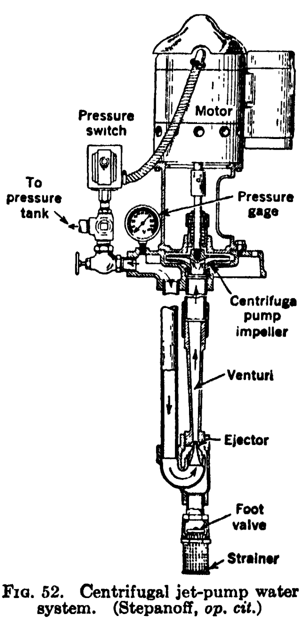


FIG. 52. Centrifugal jet-pump water system. (Stepanoff, *op. cit.*)

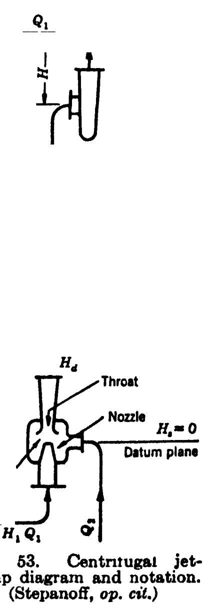


FIG. 53. Centrifugal jet-pump diagram and notation. (Stepanoff, *op. cit.*)

JET PUMPS. Figure 53 is a diagram of the jet-centrifugal combination, and Fig. 54 gives the performance of a jet pump under several driving heads, kept constant for each head-capacity curve. Note the resemblance of these curves to centrifugal-pump characto the casing cover is interposed in the clearance between the teeth of both gears, providing a seal between suction and discharge. The suction is separated from discharge by two spring-loaded shoes which serve as relief valves, and also compensate for wear.

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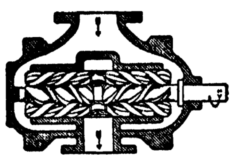

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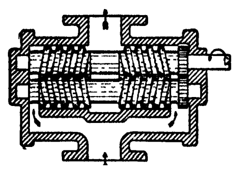

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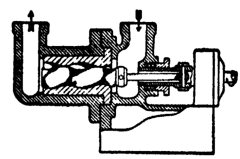

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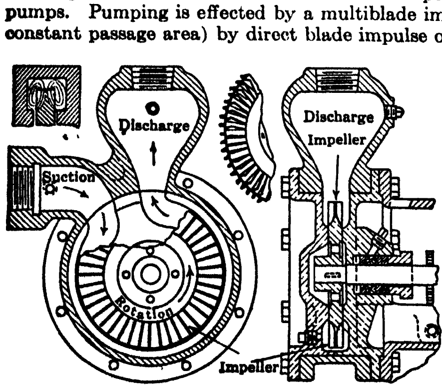

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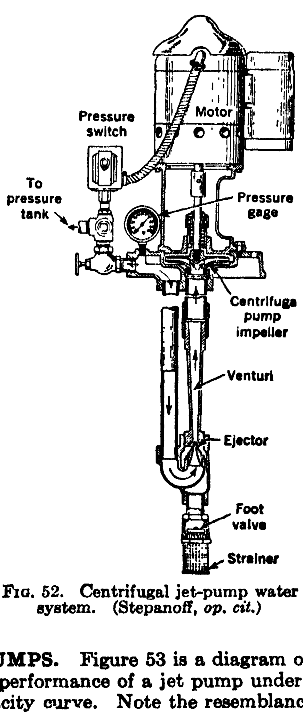

Fig. 52. Centrifugal jet-pump water system. (Stepanoff, *op. cit.*)

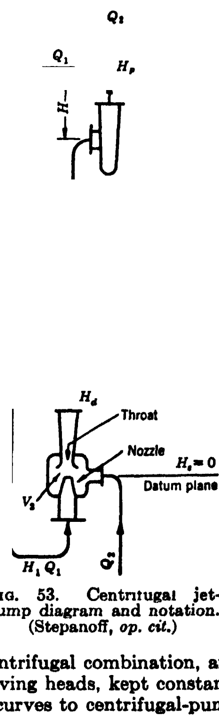

Fig. 53. Centrifugal jet-pump diagram and notation. (Stepanoff, *op. cit.*)

JET PUMPS. Figure 53 is a diagram of the jet-centrifugal combination, and Fig. 54 gives the performance of a jet pump under several driving heads, kept constant for each head-capacity curve. Note the resemblance of these curves to centrifugal-pump characteristics at several speeds. Characteristics of jet pumps can be described by three ratios:

$$R = \frac{A_1}{A_2} = \frac{\text{Nozzle area}}{\text{Throat area}} \tag{24}$$

(1)

$$M \frac{Q_2}{Q_1} = \frac{\text{Pumped capacity}}{\text{Driving capacity}} \tag{25}$$

(2)

$$N = \frac{H_d - H_s}{H_1 - H_d} \quad \frac{\text{Net jet-pump head}}{\text{Net driving head}} \tag{26}$$

(3)

The driving head H_1 and the driving capacity Q_1 are furnished from the outside source. Capacity Q_2 enters the jet-pump suction under the head H_s. The capacity leaving the jet-pump discharge equals the sum of the driving capacity and the jet-pump capacity: $Q = Q_1 + Q_2$. Figure 55 shows several typical characteristics of jet pumps in terms of M and N for four values of R. Except for extreme values of R the M-N curves are straight lines and apply to all similar jet pumps. The slope of the M-N lines is determined by the value of R. The position of the M-N lines is governed by the efficiency of the jet pumps, more efficient pumps giving higher values of M and N, as is evident from the definition of efficiency of jet pumps.

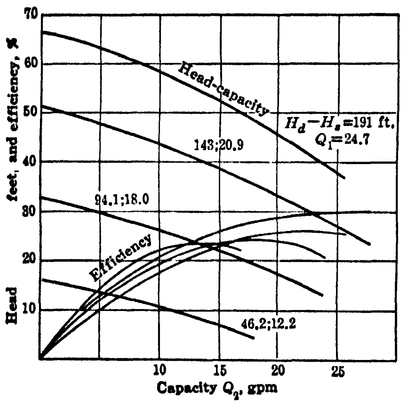

FIG. 54. Jet-pump performance. (After Gosline and O'Brien) (Stepanoff, *op. cit.*)

$$e_j = \frac{Q_2(H_d - H_s)}{Q_1(H_1 - H_d)} = MN \tag{27}$$

Since M-N characteristics are straight lines, they are completely defined by their intersections with the coordinate axes. Figure 56 gives values of M_0 for $N = 0$ and values of N_0 for $M = 0$ in terms of nozzle throat ratios. The fact that M-N curves are straight lines results in peak efficiency taking place at the values of M and N equal to half values of M_0 and N_0, respectively.

When the M-N curve is established, an efficiency curve can easily be plotted and is symmetrical about its peak. Since $MN = e_j$, $M = M_0/2$, and $N = N_0/2$ at the best efficiency point, $M_0N_0 = 4e_j$, where e_j is the jet-pump peak efficiency. Thus selection of M_0 and N_0 fixes the efficiency of the jet pump. For a given nozzle/throat ratio, efficiency increases with size of pump. The maximum efficiency attained on commercial jet pumps in sizes employed for jet-centrifugal combination systems ($0.25 < R < 0.625$) is about 35%. The performance of jet pumps is affected by the distance from the nozzle exit to the throat; optimum performance is obtained when this distance is equal to the nozzle diameter. Reduction of the distance below that value is impossible without obstructing the flow, resulting in head-capacity and efficiency reduction. Increase of the distance above one nozzle diameter has only a minor effect on pump performance. When pressure at the nozzle discharge reaches the vapor pressure, jet pumps show cavitation effects in the same manner as low-specific-speed

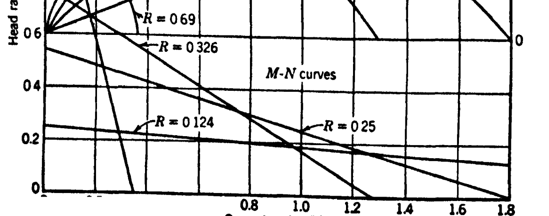

FIG. 55. Jet-pump characteristics for various values of the ratio R. (After Gosline and O'Brien) (Stepanoff, *op. cit.*)

centrifugal pumps; that is, the head-capacity curve drops abruptly. Actual measurements have been made which indicate a pressure at the throat within a fraction of 1 ft of the vapor pressure, at the water temperature.

PERFORMANCE OF A CENTRIFUGAL JET-PUMP COMBINATION. At the operating point, the following equation connecting the M and N values of the jet pump and head H and capacity Q of the centrifugal pump should be satisfied:

$$\frac{Q^2}{H_c} = \frac{2gC^2A_1^2(M+1)^2(N+1)}{1+B(N+1)} \qquad (28)$$

where $H_c = H - h_1$ is the net centrifugal head, considering the suction pipe as part of centrifugal pump; C is the coefficient of discharge for the nozzle of the jet pump; and $B = h_2/H_1$ is a fraction expressing loss in the driving pipe in terms of the driving head H_1. Equation 26 is best solved graphically. By substituting M and N values for an arbitrary point on the M-N curve (A in Fig. 57), the equation becomes $Q/\sqrt{H_c}$ = constant, which means that the equation can be satisfied by a given centrifugal pump at any speed or impeller diameter.

To find Q and H_c for a fixed rpm of the pump, an arbitrary value is assigned to Q, and H_c is obtained (point B). Points of the same unit capacity $Q/\sqrt{H_c}$ lie on a parabola with its apex at the origin. To determine its intersection with the Q-H_c curve (point D), another point C is located at an arbitrary capacity and connected with point B by the affinity relations. A straight line BC is drawn to intersect the Q-H_c curve at point D. When Q and H_c are known, the system head H_p and capacity Q_2 (point E) are found by using the following formulas:

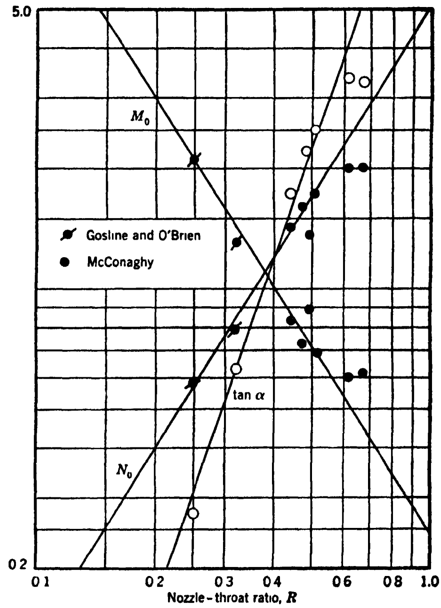

FIG. 56. M_0 and N_0 in terms of nozzle/throat ratio for 30% peak efficiency. (Stepanoff, *op. cit.*)

$$H_p = H_1(B+1); \qquad H_1 = H_c\frac{N+1}{1+B(N+1)} \qquad (29)$$

$$Q_2 = Q_1M; \qquad Q_1 = CA_1\sqrt{2gH_1} \qquad (30)$$

The efficiency of the jet-centrifugal combination e_c (point J) is obtained by dividing the output of the system by the brake horsepower of the centrifugal pump (points F and G). Any required number of points for the Q_2-H_p curve can be obtained by this method by selecting different points on the M-N curve.

AFFINITY RELATIONS. (1) When the centrifugal pump speed or impeller diameter is changed, M and N values remain constant, and the net system head H_p and the other heads H_1, H_d, h_1, and h_2, vary directly as the square of the speed or impeller diameter, or directly as the centrifugal head H_c. The capacity of the jet-centrifugal pump combination Q_2, and also Q_1 and Q, vary directly as the speed or impeller diameter of the centrifugal pump or directly as $\sqrt{H_c}$.

(2) If, for a given centrifugal pump, a larger jet pump of the same design (R constant) is used, M and N are constant except for the effect of change in peak efficiency. The ratio $Q/\sqrt{H_c}$ increases directly as the nozzle area A_1 (eq. 30). This means that the operating point of the centrifugal pump moves to a higher capacity and lower head. Since M is constant, Q is split in the same ratio, or Q_1 and Q_2 change directly as Q. All heads (H_p, H_d, H_1, h_1, and h_2) change directly as $\sqrt{H_c}$.

(3) If, for a given centrifugal pump, the nozzle-throat ratio R is changed, for instance, by *increasing the throat area* by reboring, R decreases. For the best-efficiency point, $MN = e$, remains essentially constant but M increases and N decreases. Thus, from eq. 28, the centrifugal-pump unit capacity $Q/\sqrt{H_c}$ increases (for simplicity, assume $B = 0$).

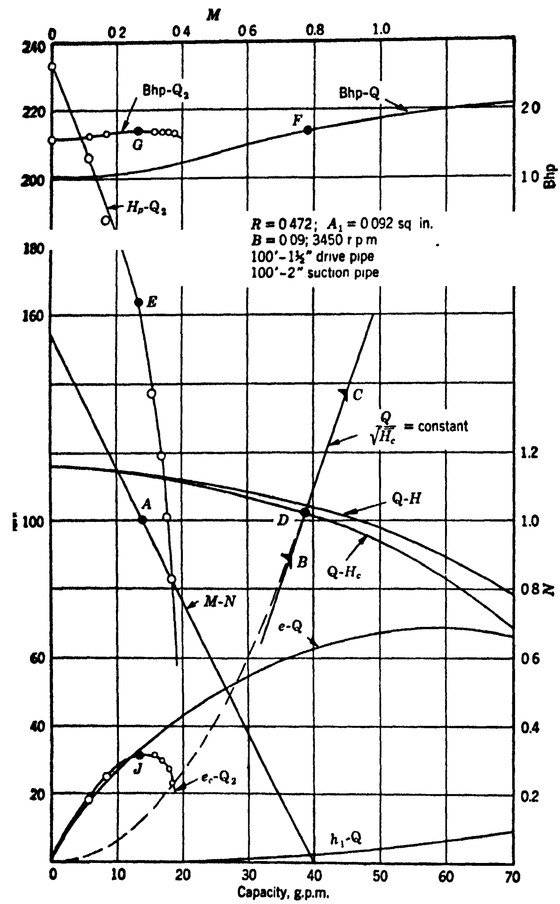

Fig. 57. Calculations of centrifugal jet-pump performance. (Stepanoff, *op. cit.*)

This means that the operating point of the centrifugal pump moves to a higher capacity Q, and lower head H_c; therefore, H_p and all heads decrease because of the decrease of both N and H_c. Capacity Q_2 increases, but Q_1 decreases because of the decrease of the driving head H_1.

(4) If the *nozzle ratio R is increased* by reboring the *nozzle*, variation of heads and capacities are in a direction opposite to that of case (3); that is, all heads increase, Q_1 increases, Q_2 decreases, and Q decreases.

(5) If the size or the length of the centrifugal-pump suction pipe is changed so that the hydraulic loss h_1 in this pipe is increased, H_c is lower (Fig. 57). The decrease in H_c is accompanied by a decrease in all heads and capacities. If the size or length of the drive capacity pipe is changed so as to increase its resistance h_2, the constant B increases, and both heads and capacities drop.

JET-PUMP APPLICATION includes the following services: pumping cesspools, draining large water turbine casings and drainage pits; pumping and washing sand. In addition, jet pumps are used for pumping large volumes of vapor and gas at low pressures in the chemical industry, using compressed air or steam as a driving fluid.

21. AIR LIFT

The air lift, as a means of lifting liquids, was developed for pumping water or oil from drilled wells. It consists of (1) a discharge pipe and (2) a smaller air pipe, both partially submerged in a well. (In oil fields natural gas is used in the same way for pumping crude oil from the wells.) Air from the air pipe is discharged into the water at the lower end, either through an open end or through perforations in the pipe; in both cases the air pipe is placed inside the discharge pipe. Sometimes special footpieces are used for the same purpose (Fig. 58) in connection with side air inlet. The design of the footpiece has little effect on the pump efficiency as long as it does not obstruct the flow of water, distributes air uniformly, and provides sufficient total opening area to introduce the required amount of air at about the same velocity as that of the water.

The operation of the air lift can be explained on the basis of the reduction of specific gravity of the mixture in the discharge pipe or by considering the buoyancy of air bubbles as a motive force. It is also possible to consider work done by air in the discharge pipe in the same manner as steam in a steam engine, first by displacement, and then by expansion. In every case the fundamental relationship remains the same, and frequently is expressed by the formula

$$Q_a = \frac{H_l}{C \log_{10} \dfrac{H_s + 34}{34}} \qquad (31)$$

where Q_a = cubic feet of free air required per gallon of water; H_l = total lift, feet; H_s = working submergence, feet; and C is a numerical constant incorporating the gross efficiency of the air lift unit. (See values below, for different submergences.) The efficiency of the air lift increases as submergence is increased. Higher submergence requires higher working air pressure, $P_w = H_s/2.31$; where P_w is the net air pressure, pounds per square inch. The air pressure should be high enough to start the air lift with the existing static submergence, which is much greater than the working submergence, on account of the water level "draw-down" when the flow is established. Besides, loss of air pressure in the air pipe should be provided for. Table 4 shows recommended submergences expressed in percentage of the total length of the discharge pipe.

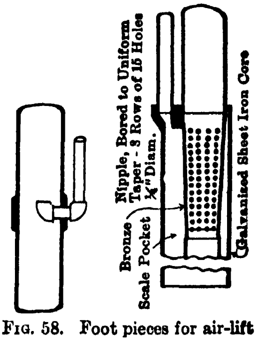

FIG. 58. Foot pieces for air-lift pumps.

Table 4. Recommended Submergences and Constant C in Eq. 31

Lift, ft	20 to 100	150	200	300	400	500	700
Submergence, %	70	65	60	55	50	45	40
C, outside air pipe	358	348	335	318	296	272	246
C, inside air pipe	322	306	285	262	238	214	185

Sizes of the discharge pipes, with a central air pipe, for various capacities are approximately:

Pipe size, in.	3	4	5	6	8	10	12
Average capacity, gpm	60	100	175	300	600	750	1000

For best efficiency the amount of air used must be kept at the minimum which produces a continuous discharge. Too little air results in intermittent discharge and surging. Excess of air increases friction losses in the pipes and waste of air due to incomplete expansion at discharge. As a rule increase of submergence improves the efficiency of the air lift, notwithstanding the increased friction in air and discharge piping, because entrance losses and the loss due to incomplete expansion of air at discharge are essentially constant per pound of air. Expressed as a percentage of total potential energy of air, these losses are smaller at higher submergence, i.e., at higher air pressures. For moderate lifts, up to 300 ft, air-lift efficiencies, based on the air horsepower at the air inlet, of over 70% have been obtained. To compare these with efficiencies of other type of pumps, the air-lift efficiency should be multiplied by the overall efficiency of the air compressor (75% is an average value).

COMPOUND AIR LIFT. If the difference between the static and pumping level is too great, high starting air pressure may overload the air compressor. In the compound air lift this difficulty is overcome by providing an auxiliary air inlet above the normal air inlet. This feature allows starting with rated compressor air pressure. Pumping is started with the auxiliary air inlet. When the level in the well drops sufficiently, the normal air inlet is opened and the auxiliary inlet is gradually turned off.

THE INJECTOR. See Section 7 of this book.

THE PULSOMETER AND HYDRAULIC RAM were used in the past to a limited extent, but have been entirely replaced by more modern and efficient pumping machinery. (See *Experimental Engineering*, by R. C. Carpenter and H. Diederichs, John Wiley, 1924, pp. 1038, 1080. Out of print.)

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HYDRAULIC COUPLINGS

By R. G. Olson

The fluid drive or hydraulic coupling was developed in Germany before World War I and was first used in that country for diesel ship propulsion. Its purpose was to prevent the transmission of torsional vibrations and to provide a convenient means of clutching and declutching where two or more engines were connected to a single propeller through gearing. Development of fluid drive for automotive and industrial applications started in England in 1927, and has since continued in that country as well as in the United States.

Two General Types. Of the two general types of fluid drives—constant-speed and variable-speed—the former has achieved the most popularity through its use on automobiles. Consisting of an impeller, a runner, an enclosing casing, and an oil seal, it is filled with an initial charge of oil and sealed. This type is called a constant-speed unit because in normal operation the output shaft runs constantly at 97 to 98% of the input speed. The variable-speed fluid drive, on the other hand, does not use an oil seal but has means of getting oil in and out of the rotating parts. A stationary scoop tube reaches inside the rotating parts and skims off fluid, circulating it outside where it can be cooled, the oil being returned by another passage.

22. DESIGN

PRINCIPLE OF OPERATION. Figure 1 illustrates fluid circulation from runner to impeller. The oil is pumped outward in the impeller, crosses the gap into the runner, and flows inward. Actually, the fluid follows a spiral path, as it has motion in the direction

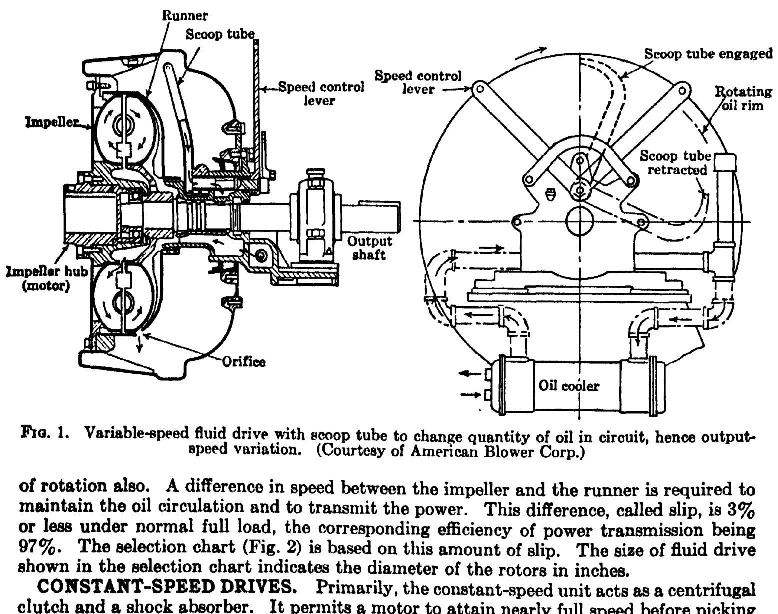

Fig. 1. Variable-speed fluid drive with scoop tube to change quantity of oil in circuit, hence output-speed variation. (Courtesy of American Blower Corp.)

of rotation also. A difference in speed between the impeller and the runner is required to maintain the oil circulation and to transmit the power. This difference, called slip, is 3% or less under normal full load, the corresponding efficiency of power transmission being 97%. The selection chart (Fig. 2) is based on this amount of slip. The size of fluid drive shown in the selection chart indicates the diameter of the rotors in inches.

CONSTANT-SPEED DRIVES. Primarily, the constant-speed unit acts as a centrifugal clutch and a shock absorber. It permits a motor to attain nearly full speed before picking up the load, greatly reducing motor current and simplifying the starting of heavy loads. A general-purpose motor with simple starter can be used where much more complicated starting equipment would otherwise be needed.

VARIABLE-SPEED DRIVES. Variable-speed fluid drives find their greatest use in connecting a constant-speed electric motor to a machine being driven at varying speeds (a fan, a pump, etc.). Change of speed is accomplished by changing the quantity of oil in the oil vortex. When the circuit is full of oil, the output shaft runs at 97 to 98% of motor

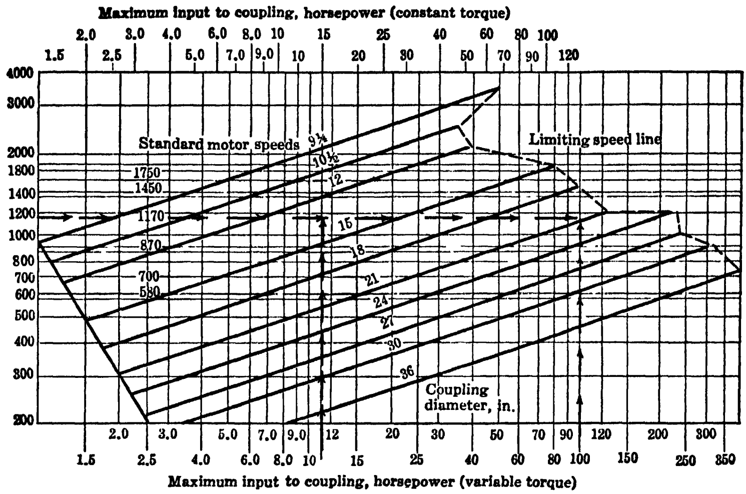

FIG. 2. Selection chart for fluid couplings (sizes for various combinations of motor speed and horsepower). Horsepower varies as cube of input speed and as fifth power of the coupling diameter. (Courtesy of American Blower Corp.)

speed. When oil is withdrawn, the output shaft slows down to some lower speed. It is thus possible to adjust the output speed accurately and without steps by gradually changing the fluid level in the rotors. The amount of oil in the vortex is adjusted in one of two ways: by a movable lever-controlled scoop tube or by a fixed scoop tube in conjunction with an auxiliary oil-control pump.

Lever Control. Illustrated in Fig. 1 is a lever-controlled fluid drive, available in a range of 1 to 300 hp, the hub of which mounts on the motor shaft. Idle oil not in the vortex between impeller and runner is stored centrifugally in the rotating outer casing in an annular ring. To fill the vortex, the movable scoop is immersed in the oil ring; it then picks up the oil and forces it outside through a cooler and back into the working circuit. A continuous stream of oil leaves the vortex through calibrated nozzles and enters the outer casing, where it is again picked up by the scoop tube and circulated. An end view of the scoop tube (Fig. 1) makes clear how it is possible to vary the position of the tip of the tube and thus adjust the oil quantity to any desired level by moving the external speed-control lever.

23. APPLICATION

CHARACTERISTICS OF VARIABLE-SPEED FLUID DRIVE. When oil is withdrawn from the rotors, the ability of a fluid drive to transmit power is reduced, and consequently the output shaft slows down. By this means, a continuous type of speed variation can be achieved.

Starting torque which this drive will transmit when the rotors are filled is equal to or greater than the peak torque of the motor normally selected for use with it. If the motor has a pull-out torque of 225%, then 225% torque is available through the fluid drive for starting the load. The fluid drive takes advantage of the "pull-out" torque of the motor to start the load, and, as a result, the starting torque is much higher than that of the motor alone.

Overload capacity is determined by the amount of oil in the coupling. With a full circuit, the oil vortex transmits the peak torque of the motor. By gradually withdrawing (or adding) oil, any degree of allowable overload can be preselected. It is possible, then, to limit output for such uses as tension control in winding, drawing tubes, and extruders.

AUTOMATIC AND REMOTE CONTROL. When remote or automatic control is used, (1) a motor must be energized to move the speed-control lever of the lever-controlled unit or (2) valves must be energized which admit oil to or release oil from the fluid drive to change speed of the pump-controlled unit, the control motor being actuated by any automatic control impulse that can make an electric contact. The automatic control may be measuring water pressure from a pump driven by the fluid drive, or air flow from a fan, or tension in steel strip being wound, or speed of a conveyor, or motor current on an agitator drive, etc. These are fairly common cases.

APPLICATIONS of fluid couplings to electric motor drives extend to every major industry. For easy analysis, the various applications are classified as follows for nature of load.

Variable-torque Load. (Fan or centrifugal pump.) The power demand of these units varies as the cube of rpm under average conditions, and speed range can be 5 to 1 or more.

Constant-torque Load. Horsepower varies directly with speed since friction is the load. These machines are further classified as: (1) smooth rolling load with rolling momentum; printing presses, paper slitters, and triplex pumps are typical examples. A range of 3 to 1 is quite practical for continuous operation, and lower speeds can be held during the starting period. (2) Fluctuating torque loads; vacuum filters, crushers, rotary kilns, and ball mills are typical examples. A speed range of 2 to 1 is the maximum recommended; lower speeds can be held, however, during a short starting period by means of manual control.

Machines on which variable speed fluid drives are used include textile spinning frames, paper coaters and slitters, agitators and mixers, ball mills, rotary kilns and dryers, drawbenches, extruders, cable and rope stranders, winches, and wire-drawing machines.

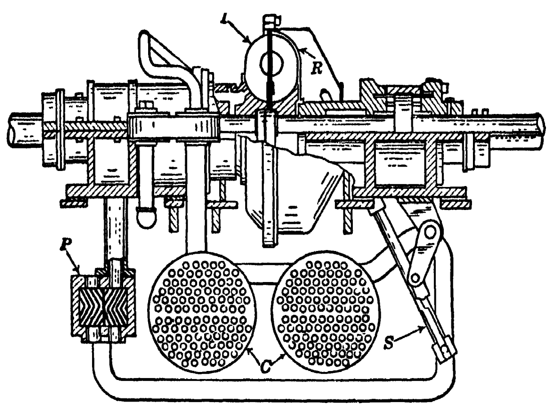

FIG. 3. Cross section through 3600 rpm adjustable-speed fluid drive for boiler feed pumps. *R* is runner; *I*, impeller; *P*, oil pump; *C*, oil coolers; *S*, speed controller. (Courtesy of American Blower Corp.)

A high-speed fluid drive used for the variable-speed drive of 3600 rpm boiler feed pumps and centrifugal compressors is shown in Fig. 3. It is a self-contained totally enclosed unit and is available in a horsepower range of 500 to 2500. The rotors, enclosed in an oil-tight welded steel housing, are connected by heavy input and output shafts to the driving motor and the load. The shafts are carried in oil-film type journal bearings. Similar units are available to cover a full range of heavy industrial drives. As in other types of fluid drive, this unit provides no-load starting, infinitely variable speed, and controlled acceleration to prevent motor overload when driving high inertia loads.

SECTION 6

PIPING

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STEAM POWER PLANT PIPING

By E. J. Wiseman

STRESSES IN PIPE LINES

By A. S. McCormick and W. A. Thomas

PIPE AND TUBING

VALVE AND FITTING DATA

FLOW OF FLUIDS IN PIPES

By R. J. S. Pigott, J. M. Cunningham, and J. B. Nichols

STEAM POWER PLANT PIPING

By E. J. Wiseman

1. CODES

Construction of steam and other pressure piping is covered by the Code for Pressure Piping, ASA B31.1-1942. Section 1 (Power Piping Systems), Section 6 (Fabrication Details), and Section 7 (Materials) contain requirements applicable to steam power plant piping. Other sections cover gas, air, oil, district heating, and refrigerating piping systems.

The Code sets forth minimum safety requirements for selection of suitable materials, designation of dimensional standards, design of component parts and the assembled unit, and tests of elements before erection and completed systems after erection. New materials and designs having safety characteristics equal or superior to those specified are allowed, provided minimum requirements of the Code are maintained.

Piping, valves, and fittings requirements prescribed in the ASME Code for Power Boilers (see Section 7) in general exceed those of the ASA Code for Pressure Piping. The following piping is governed by the Code for Power Boilers and should be designed in accordance with its rules: (1) steam piping to and including the outlet valve for a single boiler; (2) steam piping to and including the second valve where two or more boilers connect to a common header; (3) boiler feed piping to and including the second valve from the boiler for a single unit; (4) boiler feed piping to and including the second shut-off valve and the feedwater regulating valve in some piping arrangements. It is advisable to consult the applicable regulations for a specific geographic location before design and fabrication of boiler piping.

Much of the material which follows is based on the ASA Code for Pressure Piping. It is included chiefly to illustrate principal requirements for various types of service, since details of the Code cannot be presented in condensed form.

2. PIPE

PIPE MATERIALS and their uses are given below.

Nonferrous materials, including copper and brass, are generally used in sizes 3 in. and smaller for fluids corrosive to ferrous materials, salt or brackish waters, untreated fresh waters, and where reduction in pipe size due to corrosion is objectionable. Copper tubing is generally used for instrument and control connections. Copper is limited to 406 F, brass or bronze to 500 F. (See Table 2 for data.)

Cast iron was formerly used in sizes of 3 in. upward for steam and water lines at pressures below 250 psi, temperatures below 450 F. For these services it has been replaced by steel pipe, and at present is used chiefly for circulating water lines and underground piping.

Wrought iron offers better resistance to corrosion than steel, but less than cast iron. Present uses in power plant piping are rather limited.

Carbon-steel pipe, either welded or seamless, is the most widely used of all materials. Butt-welded piping is available in sizes 3 in. and smaller. In general, the manufacture of lap-welded pipe has been discontinued. Seamless pipe is available in a full range of sizes up to 24 in. OD, though as the wall thickness increases, the size limit is reduced. Where seamless material is not available, either electric fusion-welded or forged and bored pipe may be selected, depending on the service conditions. Although higher temperatures are allowable under the Code for Pressure Piping, carbon steel is seldom selected for service above 750 F.

Alloy-steel pipe containing molybdenum, or molybdenum and chromium, is made in seamless only. Sizes up to 16 in. OD are produced. The service temperature may be as high as 1100 F, depending on the alloy.

Stainless-steel pipe of the 18% chromium, 8 to 10% nickel analysis has been produced in seamless, for use in the process industry. The present temperature limit is 1200 F.

SPECIFICATIONS AND CHEMICAL AND PHYSICAL PROPERTIES of pipe materials are given in Table 1.

Table 1. Specifications and Properties of Piping Materials

(Adapted from ASA Code for Pressure Piping and ASME Boiler Code)

ASTM Specification	Description	Chemical							Physical			
									Tensile Strength, 1000 psi	Yield Point, 1000 psi	Elongation * in 8 in., %	Elongation * in 2 in., %
	Nonferrous	Cu	Pb, max	P, max	Sn	Fe, max						
B-42	Copper pipe	99.90		0.04								
B-43	Muntz metal	59–63	0.50			0.07	Zn = remainder					
	High brass	65–68	0.80			0.07						
	Admiralty metal	70–73	0.075		0.9–1.2	0.06						
	Red brass	83–86	0.06		0.15 †	0.05						
	Ferrous	C	Mn	P, max	S, max	Si	Cr	Mo				
A-72	Welded wrought iron		0.05 †						40	24	12	
A-120	Welded and seamless steel for ordinary service	No chemical or physical requirements specified. Hydrostatic test required.										
A-53	Lap welded acid Bessemer								50	30	18	
	Lap welded open hearth			0.06					45	25	20	
	Seamless or resistance welded—Grade A			0.045					48	30		35
	Seamless or resistance welded—Grade B			0.045					60	35		30
A-106	Grade A	0.25	0.30–0.90	0.04	0.06				48	30		35
	Grade B, silicon killed	0.35	0.35–1.00	0.04	0.06	0.10 §			60	35		30
	Grade B, acid Bessemer killed	0.25	0.35–1.00	0.11	0.06	0.10 §			60	35		30
A-134	Electric fusion welded, 30 in. and larger											
	Grade A	Composition varies with plate material. See ASTM A-283 and A-285.							45 §	0.5TS ‡		1,500,000/TS
	Grade B								50 §	0.5TS ‖		
A-135	Electric resistance welded—Grade A			0.045					48	30		35
	Electric resistance welded—Grade B			0.060					60	35		30
A-139	Electric fusion welded—Grade A			0.045					48	30		35
	Electric fusion welded—Grade B			0.060					60	35		30
A-155	Electric fusion welded, 18 in. and larger											
	Grade A — Plates $^3/_4$ in. and under in thickness	0.15 †	0.35–0.60	0.04 ¶	0.05	0.10 §			45 §	0.5TS ‡		1,500,000/TS
	Grade B — Plates $^3/_4$ in. and under in thickness	0.20 †	0.35–0.60	0.04 ¶	0.05	0.10 §			45 §	0.5TS ‖		
	Grade C — Plates $^3/_4$ in. and under in thickness	0.25 †	0.30–0.60	0.04 ¶	0.05	0.10 §			45 §	0.5TS		
	Grade A — Plates over $^3/_4$ in. thickness	0.17 †	0.35–0.60	0.04 ¶	0.05	0.10 §			45 §	0.5TS ‡		1,500,000/TS
	Grade B — Plates over $^3/_4$ in. thickness	0.22 †	0.35–0.60	0.04 ¶	0.05	0.10 §			45 §	0.5TS ‖		
	Grade C — Plates over $^3/_4$ in. thickness	0.30 †	0.30–0.60	0.04 ¶	0.05	0.10 §			45 §	0.5TS		
A-206	Carbon molybdenum—Symbol P 1	0.10–0.20	0.30–0.80	0.04	0.05	0.10–0.50		0.45–0.65	55	30		30
A-280	Chromium-molybdenum	0.10–0.20	0.30–0.60	0.04	0.05	0.10–0.30	0.40–0.60	0.45–0.65	55	30		30
A-158	Chromium-molybdenum, Grade P 11	0.15 †	0.30–0.60	0.03	0.03	0.50–1.00	1.00–1.50	0.45–0.65	60	30		30
A-158	Chromium-molybdenum, Grade P 3a	0.15 †	0.30–0.60	0.03	0.03	0.45–0.75	1.50–2.00	0.60–0.80	60	30		30
A-158	Chromium-molybdenum, Grade P 3b	0.15 †	0.30–0.60	0.03	0.03	0.50 †	1.75–2.25	0.45–0.65	60	30		30

* Elongations vary with type of specimen, are subject to correction for thickness and direction. Values shown are approximate. Consult specifications for exact figures.
† Maximum. ‡ Minimum, 24,000 psi. § Minimum. ‖ Minimum, 27,000 psi. ¶ Basic.

DIMENSIONAL STANDARDS. The former designations ***standard weight, extra strong,*** and *double extra strong* now are used only for ordinary grades of pipe; for the higher grades, these have been replaced by the American Standard for Wrought Iron and Wrought Steel Pipe, ASA B36.10–1939 (see Article 13, p. 6-24, Dimensions of Welded and Seamless Steel Pipe). These dimensions also apply to brass or copper pipe furnished in iron-pipe sizes.

Dimensions for cast-iron pipe are contained in ASTM A-44 or in the American Water Works Association standard.

3. VALVES AND FITTINGS

VALVE MATERIALS and their uses are given below.

Nonferrous valves and fittings are produced in screwed, flanged, and socket brazing types, although screwed are most widely used. Screwed valves have either screwed or union bonnets, and flanged valves have flanged bonnets. Seating and trim materials are bronze, nickel alloy, Monel metal, and hardened stainless steel.

Cast-iron valves and fittings are produced in screwed, flanged, and bell and spigot types. Screwed ends predominate in the smaller sizes, and flanged in the larger sizes. Bell and spigot type are usually used for underground service. Seating, trim, and stem materials of valves are generally bronze, although all-iron valves with steel stems are available.

Malleable-iron fittings, made in screwed type only, are less subject to breakage from mechanical strains than cast iron. Small-sized valves are also available in malleable iron.

Wrought-iron fittings of the butt-welding type are available, but seldom used in power plant piping.

Carbon and alloy steel valves and fittings are forged or cast; they are available in screwed, socket-welding, butt-welding, and flanged types. Screwed or socket-welding type is normally used in sizes 2 in. and smaller, and butt welding or flanged in sizes 2 1/2 in. and larger. Forged valves predominate in the small sizes, and cast valves in the larger sizes. Stem material is normally 12 to 14% chromium stainless steel. Seating materials are nickel alloy, Monel metal, 12 to 14% chromium stainless steel, 18% chromium and 8 to 10% nickel stainless steel, Stellite, and other hard facing alloys, and processed or surface-hardened stainless steel. Nickel alloy and Monel metal are not recommended for temperatures above 750 F. Either 12 to 14% chromium or 18% chromium and 8 to 10% nickel stainless steels are suitable for all water services and have been used for steam at 750 F and higher, although they have a tendency to gall or seize at the higher temperatures. For 750 F and above, Stellite and processed or surface-hardened stainless steel are the preferred materials.

Body materials are available in compositions comparable to the pipe materials with which they will be used. Some manufacturers have standardized on carbon-molybdenum steel bodies for 300 psi and higher, furnishing carbon steel bodies only if specifically ordered.

FITTING, VALVE, AND FLANGE MATERIAL SPECIFICATIONS are given for ready reference.

ASTM Specification No.	Description
Nonferrous	
B-61	Steam or Valve Bronze Castings
B-62	Composition Brass or Ounce Metal Castings
Cast Iron	
A-126	Gray Iron Castings for Valves, Flanges, and Pipe Fittings
Malleable Iron	
A-277	Malleable Iron Flanges, Pipe Fittings, and Valve Parts
Carbon Steel	
A-95	Carbon Steel Castings for Valves, Flanges, and Fittings for High Temperature Service
A-216	Carbon Steel Castings Suitable for Fusion Welding, for High Temperature Service
A-105	Forged or Rolled Steel Pipe Flanges, Forged Fittings, and Valves and Parts for High Temperature Service
A-181	Forged or Rolled Steel Pipe Flanges for General Service
A-234	Factory-made Wrought Carbon Steel and Carbon-molybdenum Steel Welding Fittings

(*Continued on p. 6-05*)

ASTM Specification No.	Description
Alloy Steel	
A-157	Alloy Steel Castings for Valves, Flanges, and Fittings for High Temperature Service
A-217	Alloy Steel Castings Suitable for Fusion Welding for High Temperature Service
A-182	Forged or Rolled Alloy Steel Pipe Flanges, Forged Fittings, and Valves and Parts for High Temperature Service
A-234	Factory-made Wrought Carbon Steel and Carbon-molybdenum Steel Welding Fittings

DIMENSIONAL STANDARDS *

Screwed fittings are covered by these standards:

ASA B16c—Malleable Iron Screwed Fittings for Maximum Working Saturated Steam Pressure of 150 psig
ASA B16d—Cast Iron Screwed Fittings for Maximum Working Saturated Steam Pressures of 125 and 250 psig
MSS † SP-31—Malleable Iron Screwed Fittings for Maximum Working Saturated Steam Pressure of 300 psig
ASA B16.15—Brass or Bronze Screwed Fittings for Maximum Working Saturated Steam Pressure of 125 psig
ASA B16.17—Brass or Bronze Screwed Fittings, for Maximum Working Saturated Steam Pressure of 250 psig

Flanges and flanged fittings are covered by these standards:

ASA B16b2—Cast Iron Pipe Flanges and Flanged Fittings for Maximum Working Steam Pressure of 25 psig
ASA B16a—Cast Iron Pipe Flanges and Flanged Fittings, Class 125
ASA B16b—Cast Iron Pipe Flanges and Flanged Fittings, Class 250
ASA B16e—Steel Pipe Flanges and Flanged Fittings

Welding fittings are covered by these standards:

ASA B16.11—Steel Socket Welding Fittings
ASA B16.9—Steel Butt Welding Fittings

Valves. Brass, bronze, and cast-iron valves vary in dimensions with the various manufacturers; catalogs should be consulted for exact values. Face-to-face dimensions of flanged and welding end steel valves are covered by ASA B16.10.

4. BOLTING

For low-pressure services or where cast iron or nonferrous flanges are used, headed carbon steel bolts with either square or hexagon heads and semifinished heavy series hexagon nuts are suitable, and can be readily obtained from jobbers' stocks. Although a higher grade of carbon steel bolting may be used at pressures up to 250 psi and temperatures up to 450 F, it is usually more satisfactory to use alloy steel bolting. Alloy material is supplied in stud form, threaded full length, with two nuts.

Studs and nuts are covered by the following ASTM specifications:

A-193—Alloy Steel Bolting Materials for High Temperature Service
A-194—Carbon and Alloy Steel Nuts for High Temperature Service

Not all the grades of material listed are readily available. For temperatures up to 750 F, grades BC and B-7 bolting with class 2 nuts are usually furnished. For temperatures above 750 F, grades B-13 and B-14 bolting with class 2-H nuts are generally specified.

* Although the 11th edition of this handbook carried extensive tables covering dimensions and other data for valves, fittings, and piping, frequent changes in such data make it advisable to substitute references to the applicable standards so that the user may consult the latest revision. EDITOR.

† MSS = Manufacturers Standardization Society of the Valve and Fittings Industry, 420 Lexington Ave., New York.

5. SPECIFIC REQUIREMENTS OF THE CODE FOR PRESSURE PIPING

(Adapted from ASA Code for Pressure Piping B31.1–1942)

STEAM PRESSURES 250 TO 1500 PSIG. TEMPERATURE 450 TO 1000 F.

Pipe. For pressures in excess of 400 psi, pipe shall be seamless steel in accordance with ASTM A-106, A-206, or A-158, or electric fusion-welded, ASTM A-155 (see Table 1).

For steam pressures 250 to 400 psi, pipe shall be lap-welded or seamless ASTM A-106, electric fusion-welded ASTM A-155, electric resistance-welded ASTM A-135, or seamless ASTM A-53.

Flanged openings or welding ends are required on all valves and fittings above the following pipe sizes in the given pressure ranges:

Pipe size, in.	3	2	1 1/2
Pressure range, psig	250–400	400–600	600–1500

Flanges and bolting of valves and fittings shall conform to ASA B16e.

Fittings shall be cast or forged steel, but nonferrous fittings may be used for temperatures under 500 F. Forged or cast steel screwed fittings may be used up to and including 3 in. for pressures 250 to 400 psi, 2 in. for 400 to 600 psi, and 1 1/2 in. for pressures 600 to 1500 psi if their design is suitable for the pressure and temperature. Malleable iron screwed fittings in accordance with the 300 lb MSS Standard Practice SP-31 may be used up to 300 psig and 500 F.

Valves shall be of cast or forged steel, or of forged or cast nonferrous material if temperature is under 500 F. Malleable iron valves may be used under the same limitations as for malleable fittings. Stem threads may be internal or external with reference to the valve bonnet, which may be joined to the body either by screwed or flanged connections in the following sizes: 3 in. and smaller for pressures of 250 to 400 psi; 2 in. and smaller for pressures of 400 to 600 psi; 1 1/2 in. and smaller for pressures of 600 to 1500 psi. Stem threads on valves larger than the above sizes shall be external to the valve body, and used in connection with a yoke and flanged bonnet. Steam valves 8 in. and larger shall have a 3/4-in. (minimum) by-pass of seamless steel pipe at least equal to ASTM specification A-106 with minimum thickness schedule 80 of ASA B36.10. (See Table 1, p. 6-25.)

Gaskets for service above 250 F shall be metallic, asbestos, or other nonburning material.

Unions shall be forged steel suitable for the service.

STEAM PRESSURES 125 TO 250 PSI. MAXIMUM TEMPERATURE 450 F.

Pipe to any of the specifications listed for 250 to 1500 psi service; and also electric fusion-welded ASTM A-134 or A-139 and welded wrought iron ASTM A-72 may be used. Brass pipe ASTM B-43 or copper pipe ASTM B-42 may be used up to 406 F.

Fittings. Steel flanged fittings shall conform to the 300 lb American Standard B16e; cast iron fittings to the 250 lb American Standard B16b for flanged and B16d for screwed fittings. Malleable iron fittings shall conform to the 300 lb MSS Standard Practice SP-31 except that the 150 lb American Standard B16c may be used in accordance with its table of adjusted pressure-temperature ratings. Fittings shall be of steel, cast iron, malleable iron, bronze (MSS-SP-11), or brass. Flanges of brass or bronze flanged fittings shall conform to MSS-SP-2 for 250 lb. The 150 lb American Standard B16e may be used in accordance with its table of adjusted pressure-temperature ratings.

Valves. Gate, angle, and globe valves 3 in. and smaller may have inside screw. Stop valves 8 in. and larger shall be by-passed. Pipe used in the by-pass shall be steel or wrought iron.

Unions shall be suitable for the pressure and service.

Gaskets subject to burning shall not be used for temperatures over 250 F.

STEAM PRESSURES 25 TO 125 PSI. MAXIMUM TEMPERATURE 450 F.

Pipe. All pipe listed for services over 125 psi may be used; and also welded steel pipe to ASTM specification A-120, and cast iron pipe.

Fittings shall be of the 125 lb American Standard, either screwed (ASA B16d) or flanged (ASA B16.1), or American Standard B16c malleable iron, or bronze (MSS-SP-10). Flanges of bronze fittings shall be in accordance with MSS-SP-2. Cast steel fittings shall conform to 150 American Standard B16e.

Valves shall be manufacturer's standard for the specified pressure, may have cast-iron, malleable-iron, steel, or brass bodies, bonnets, disks, and yokes. Drilling and facing of flanges in accordance with American Standard B16a is recommended.

Cast-iron pipe joints may be welded with bronze if the temperature is not over 353 F.

STEAM PRESSURES 25 PSI AND BELOW. MAXIMUM TEMPERATURE 450 F.

Pipe. All pipe listed for service at 25 to 125 psi is suitable.

Fittings. Flanged fittings shall conform to 25 lb American Standard B16b2. All screwed fittings listed for service at 25 to 125 psi are suitable.

Valves shall be the manufacturer's standard for 25 psi pressure, of materials as listed under 25 to 125 psi.

Cast-iron pipe joints may be welded with bronze if the temperature does not exceed 353 F.

EXPANSION of the more common piping materials shall be calculated on the basis of total expansion in inches per 100 ft as follows:

	Temperature, °F														
	32	100	150	200	250	300	350	400	450	500	550	600	650	700	750
	Expansion, in.														
Steel, Wrought iron	0	0.5	0.9	1.3	1.7	2.2	2.6	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
Cast iron	0	0.5	0.8	1.2	1.6	1.9	2.3	2.7	3.1	3.5					
Copper	0	0.8	1.4	2.0	2.5	3.1	3.7	4.3	4.9	5.6					
Brass, bronze	0	0.8	1.4	2.1	2.7	3.4	4.1	4.8	5.5	6.2					

Sabin Crocker * gives the following formula for the expansion of piping:

$$L_t = L_0\left[1 + a\left(\frac{t-32}{1000}\right) + b\left(\frac{t-32}{1000}\right)^2\right]$$

where L_t, L_0 = length at temperature t and at 32 F, respectively; t = final temperature of pipe, °F; a, b = constants whose values are:

	a	b
Cast iron	0.005441	0.001747
Steel	0.006212	0.001623
Wrought iron	0.006503	0.001622
Copper	0.009278	0.001244

The curves of Fig. 1 have been derived from this formula.

FLEXIBILITY OF PIPING SYSTEMS shall be sufficient to prevent thermal expansion from causing unsafe stresses in piping material, excessive bending moments at joints, or excessive thrusts on equipment or at anchorage points (see p. 6-15).

PIPING SUPPORTS shall be designed to allow free expansion and contraction of pipe without causing excessive strains in pipe, anchors, or supports. Supports shall permit side movement of pipe caused by normal expansion. Anchors and guides shall be located to confine and guide expansion in a direction permitting proper use of the flexibility of the system.

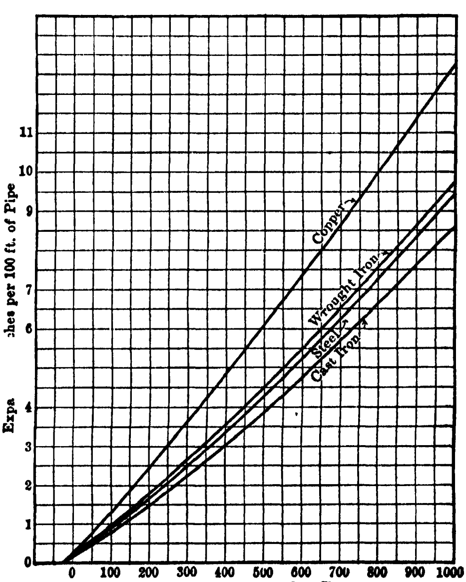


FIG. 1. Thermal expansion of piping.

6. PIPE WALL THICKNESS

CODE FOR PRESSURE PIPING. For inspection purposes, minimum thickness of pipe wall is

$$t_m = \frac{PD}{2S + 0.8P} + C \qquad (1)$$

where t_m = minimum pipe wall

* Sabin Crocker, *Piping Handbook*, McGraw-Hill Book Co., New York, 1945.

Table 2. Allowable Stress Values (S) for Pipe in Pressure Piping Systems *

Material	Specification	Values of S, psi, for Temperatures Not to Exceed †						
(From ASA Code for Pressure Piping)		406 F	450 F	500 F	600 F	700 F	750 F	950 F
Seamless copper pipe	ASTM B-42	4,000						
Seamless red brass pipe	ASTM B-43	4,500						
Seamless copper tubing	ASTM B-75	4,000						
Annealed	ASTM B-88	4,000						
Cast iron, pit cast	ASTM A-44		4,000					
Cast iron, centrifugally cast	FSB WWP-421		6,000					
Butt-welded wrought iron	ASTM A-72	5,350	5,250	5,150	4,900	4,600	4,150	
Lap-welded wrought iron	ASTM A-72	7,150	7,050	6,900	6,550	6,100	5,500	
Riveted steel or wrought iron	ASTM A-138	$(TS \times E)/5$						
Electric fusion-welded steel	ASTM A-134	$TS \times 0.16$						
Electric fusion-welded steel								
Grade A	ASTM A-139	8,600	8,400					
Grade B	ASTM A-139	10,800	10,560					
Electric fusion-welded steel								
Grade A	ASTM A-155	9,100	8,900	8,700	8,300	7,900	7,500	2,350
Grade B	ASTM A-155	10,100	9,900	9,650	9,200	8,650	8,100	2,350
Grade C	ASTM A-155	11,150	10,900	10,650	10,150	9,350	8,550	2,350
(From ASME Code for Power Boilers)		650 F	700 F	800 F	900 F	1000 F	1100 F	1200 F
Butt-welded steel	ASTM A-53	5,400	5,100	(not allowed over 750 F)				
Lap-welded steel	ASTM A-53	7,300	6,800	(not allowed over 750 F)				
Seamless steel, grade A	ASTM A-53	9,600	9,100	7,250	4,400	1,350		
Seamless steel, grade B	ASTM A-53	12,000	11,400	8,300	4,400	1,350		
Seamless steel, grade A	ASTM A-106	9,600	9,100	7,250	4,400	1,350		
Seamless steel, grade B, with 0.10% minimum Si	ASTM A-106	12,000	11,400	9,100	5,600	2,000		
Electric resistance welded steel, grade A	ASTM A-135	8,150	7,750	6,150	3,750	1,150		
Grade A, with 0.10% min Si	ASTM A-135	8,150	7,850	6,800	4,750	1,700		
Grade B	ASTM A-135	10,200	9,700	7,050	3,750	1,150		
Grade B, with 0.10% min Si	ASTM A-135	10,200	9,700	7,750	4,750	1,700		
Seamless alloy steel, grade P1	ASTM A-206	11,000	11,000	10,750	10,000	5,000		
Seamless alloy steel	ASTM A-280	11,000	11,000	10,750	10,000	5,000		
Seamless alloy steel	ASTM A-315	12,000	12,000	11,800	10,000	5,850	2,200	
Seamless alloy steel								
Grade P3a or P3b	ASTM A-158	12,000	12,000	11,800	10,000	5,850	2,200	
Grade P5a (molybdenum only)	ASTM A-158	12,000	12,000	11,800	10,000	5,850	2,200	
Grade P11	ASTM A-158	12,000	12,000	11,800	10,000	5,850	2,200	
Grade P8b or P8d	ASTM A-158	15,000	15,000	14,300	13,400	10,000	6,000	3,600

* To the minimum pipe wall thickness calculated from any of the above S values, the manufacturing tolerance, demanded for the pipe considered, must be added to obtain the nominal wall thickness. (See ASA B36, Table 2.)

† The several types and grades of pipe tabulated above shall not be used at temperatures in excess of the maximum temperatures for which S values are listed.

TS = Ultimate tensile strength of the material. E = Efficiency of joint.

thickness, inches; P = maximum internal service pressure, pounds per square inch, gage; D = actual outside diameter of pipe, inches; S = allowable stress in material at the operating metal temperature, pounds per square inch, plus water hammer allowance in case of cast iron pipe; C = allowance for threading, mechanical strength, and corrosion, inches. Values of S for various temperatures and materials are given in Table 2.

Values of C

Cast-iron pipe, cast in horizontal molds or centrifugally	0.14
Cast-iron pipe, pit cast	0.18
Threaded steel, wrought iron, or nonferrous pipe, where n = number of threads per inch	$0.8/n$
Grooved steel, wrought iron, or nonferrous pipe	depth of groove, inches
Plain end steel, wrought iron or nonferrous pipe or tube, 1 in. size and smaller	0.05
Over 1 in. size	0.065
Plain end nonferrous pipe or tube *	0.000

* Joined by flared compression couplings, lap joints, and welding.

Water hammer allowance to be added to P in formula is:

Pipe size, in.	4–10	12–18	20	24–30	36–48	54–84
Water hammer allowance, psi	120	110	100	95	90	85

ASME CODE FOR POWER BOILERS. For inspection purposes, the minimum thickness of pipe wall to be used for piping at different pressures and for temperatures not exceeding those allowed for the various materials shall be determined by eq. 1 for steel or wrought iron only. Consult the ASME Code for definitions of P for power boilers, factors to be applied to P, and special requirements for power boilers.

7. PIPE JOINTS

Pipe joints may be threaded, flanged, or welded. Special joints are permissible under certain conditions.

Nonferrous pipe or tubing may be joined by full-depth socket-brazed joints using solder in which copper plus silver is at least 60%. Fillet brazing is not acceptable. For service temperatures of 250 F and less, American Standard Soldered Joint Fittings (ASA A40.3) may be used for the pressure and temperature ratings specified therein. Flared- or compression-type connections may be used for pressures for which they are suitable.

Threaded joints shall conform to American Standard B2 for Taper Pipe Threads.

FLANGED JOINTS (see Fig. 2) are used in the following forms: (1) Flanges cast or forged integral with the pipe, fitting, or valve. (2) Screwed companion flanges, permitted in sizes and for maximum service ratings covered by ASA American standards. (3) Steel flanges grooved for rolling in the pipe with an expanding tool, permitted in the sizes and maximum service ratings given for screwed flanges. (4) Lapped (Van Stone) flanges, permitted in the sizes and service ratings for integral flanges. (5) Slip-on welding flanges, limited to service pressures of 300 psi. (6) Welding-neck flanges butt-welded to the pipe.

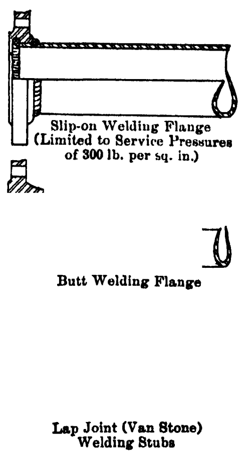


FIG. 2. Steel flange connections.

Flange Facings. Plain-face cast-iron and bronze flanges shall be faced smooth, except for the two concentric grooves for bronze flanges covered by MSS-SP-2. Flanges with 1/16 in. or 1/4 in. raised face described in the ASA American standards may be faced smooth, or finished with concentric or spiral grooves not over 1/32 in. deep, 16 per in. for cast iron and 32 per in. for steel. Steel flange facings include: (1) standard raised face; (2) large male and female with a relatively large contact area; (3) small male and female; (4) small tongue and groove with reduced contact area, faced smooth, with which metallic gaskets are recommended. Flanges with the larger contact areas are likely to give trouble with solid metallic gaskets unless the gasket width is reduced to increase the unit compression.

Reference to the Rules for Bolted Flange Connections contained in Section VIII of the ASME Boiler Code is recommended for a full treatment of the various types of gaskets.

Lapped (Van Stone) joints may be made with or without upsetting to increase the thickness of the lap. Where a female tongue and groove, ring joint, or spherical ground joint facing is to be machined in the lap, it must be upset so that the minimum thickness after machining is not less than the minimum pipe wall. The backs of laps and the faces of flanges should be machined true to provide an even bearing. Total clearance between the bore of a lapped flange and the outside diameter of pipe shall not exceed the following, except for pressures below 25 psi:

Pipe size, in.	6 and less	8 to 12	14 and over
Maximum total clearance, in.	1/8	3/16	1/4

Cast-iron flanges are not recommended.

Gaskets of paper or vegetable fiber shall not be used for temperatures over 250 F, and only where this type of material is required to resist the action of the fluid. Rubber inserted gaskets may be used with plain-face flanges for temperatures not over 250 F. Asbestos-composition gaskets may be used with any flange facing, except small male and female or narrow tongue-and-groove. Jacketed asbestos or metallic gaskets, either plain or corrugated, are not limited as to pressure or temperature.

8. PRESSURE-TEMPERATURE RATINGS

American Standard B16e, Steel Pipe Flanges and Flanged Fittings, contains seven pressure *series*, from 150 psi to 2500 psi pressure. Depending on the material and the type of facing, they may be used at pressures higher than their primary ratings, in accordance with Tables 3, 4, 5, and 6. Welded ends are rated the same as ring joints.

Table 3. Pressure-temperature Ratings for Steel Pipe Flanges and Flanged Fittings [1]

(Reprinted from Supplement No. 1 to ASA B16e–1939)

Material: Carbon Steels [2]
Facing: Other than Ring-joint [3]

Fluid	Primary Service Pressure Ratings	150	300	400	600	900	1500	2500
	Hydrostatic Shell Test Pressures [4]	350	900	1200	1800	2700	4500	7500
	Service Temperatures, °F	Maximum, Nonshock, Service Pressure Ratings at Temperatures from 100 to 1000 F						
Water, steam, oil	100	230	600	800	1200	1800	3000	5000
	150	220	590	785	1180	1770	2950	4915
	200	210	580	770	1160	1740	2900	4830
	250	200	570	760	1140	1710	2850	4750
	300	190	560	740	1120	1680	2800	4660
	350	180	550	725	1095	1645	2740	4565
	400	170	540	710	1075	1615	2690	4475
	450	160	525	700	1050	1580	2630	4380
	500	**150**	500	665	1000	1500	2500	4165
	550	140	475	630	950	1420	2370	3950
	600	130	445	590	890	1330	2220	3700
	650	120	415	550	830	1240	2070	3450
	700	110	380	500	760	1140	1900	3160
	750	100	340	450	680	1020	1700	2830
	800	92	**300**	**400**	**600**	**900**	**1500**	**2500**
	850	82	245	330	490	740	1230	2050
Oil	900	70	210	280	420	630	1050	1750
	950	55	165	220	330	495	825	1375
	1000	40	120	160	240	360	600	1000

Note. These ratings apply also to steel flanged valves designed for the same primary service pressure ratings and complying with the requirements of Section 1, General.

[1] All pressures are in pounds per square inch (gage). Temperatures and pressures listed are maximum internal fluid temperatures and pressures at flange.

[2] Carbon-steel castings: ASTM A95-44—"When required for fusion welding it is recommended that the maximum carbon content shall not exceed 0.35 per cent." ASTM A216-47T Grade WCB.
Carbon-steel forgings: ASTM A105-46 Grades I and II. ASTM A181-46 Grades I and II (for 150 and 300 primary service class only).

[3] Raised face, lapped, and large male and female facings, when used with flat solid metal gaskets, are permitted to be given the pressure-temperature ratings of this Standard only when the gasket contact area is not greater than the large tongue and groove gasket contact area and the gasket ID is not less than the large tongue and groove gasket ID.

[4] These are manufacturer's shell tests applicable to valves and fittings. Flanges may be tested after attachment to the pressure vessel in accordance with applicable code requirements, in which cases the test pressures may be higher than those given above. In such cases consideration should be given to proper gaskets for the blanking-off flanges. All tests shall be made with water at a temperature not to exceed 125 F or as required by applicable codes.

Table 4. Pressure-temperature Ratings for Steel Pipe Flanges and Flanged Fittings[1]

(Reprinted from Supplement No. 1 to ASA B16e–1939)

Material: Carbon Steels[2]

Facing: Ring-joint

Fluid	Primary Service Pressure Ratings	150	300	400	600	900	1500	2500
	Hydrostatic Shell Test Pressures[4]	425	1100	1450	2175	3250	5400	9000
	Service Temperatures, °F	Maximum, Nonshock, Service Pressure Ratings at Temperatures from 100 to 1000 F						
Water, steam, oil	100	275	720	960	1440	2160	3600	6000
	150	255	710	945	1420	2130	3550	5915
	200	240	700	930	1400	2100	3500	5830
	250	225	690	920	1380	2070	3450	5750
	300	210	680	910	1365	2050	3415	5690
	350	195	675	900	1350	2025	3375	5625
	400	180	665	890	1330	2000	3330	5550
	450	165	660	875	1320	1975	3295	5490
	500	**150**	625	835	1250	1875	3125	5210
	550	140	590	790	1180	1775	2955	4925
	600	130	555	740	1110	1660	2770	4620
	650	120	515	690	1030	1550	2580	4300
	700	110	470	635	940	1410	2350	3920
	750	100	425	575	850	1275	2125	3550
	800	92	365	490	730	1100	1830	3050
	850	82	**300**	**400**	**600**	**900**	**1500**	**2500**
Oil	900	70	210	280	420	630	1050	1750
	950	55	165	220	330	495	825	1375
	1000	40	120	160	240	360	600	1000

Note. These ratings apply also to steel flanged and welding-end valves designed for the same primary service pressure ratings and complying with the requirements of Section 1, General.

See Table 3 on opposite page for Notes 1, 2, and 4. Note 3 does not apply to this table.

Table 5. Pressure-temperature Ratings for Steel Pipe Flanges and Flanged Fittings[1]

(Reprinted from Supplement No. 1 to ASA B16e–1939)

Material: Carbon-molybdenum Steels[2] and Equivalent Alloy Steels
Facing: Other than Ring-joint[3]

Fluid	Primary Service Pressure Ratings	300	400	600	900	1500	2500
	Hydrostatic Shell Test Pressures[4]	900	1200	1800	2700	4500	7500
	Service Temperatures, °F	Maximum, Nonshock, Service Pressure Ratings at Temperatures from 100 to 1000 F					
Water, steam, oil	100	600	800	1200	1800	3000	5000
	150	590	785	1180	1770	2950	4915
	200	580	770	1160	1740	2900	4830
	250	570	760	1140	1710	2850	4750
	300	560	740	1120	1680	2800	4660
	350	550	725	1095	1645	2740	4565
	400	540	710	1075	1615	2690	4475
	450	525	700	1050	1580	2630	4380
	500	500	665	1000	1500	2500	4165
	550	475	630	950	1420	2370	3950
	600	445	590	890	1330	2220	3700
	650	415	550	830	1240	2070	3450
	700	380	500	760	1140	1900	3160
	750	360	475	720	1080	1800	2995
	800	340	450	680	1020	1700	2830
	850	320	425	640	960	1600	2665
	900	**300**	**400**	**600**	**900**	**1500**	**2500**
Oil	950	265	350	530	795	1325	2205
	1000	190	250	380	570	950	1580

Note. These ratings apply also to steel flanged valves designed for the same primary service pressure ratings and complying with the requirements of Section 1, General.

[1] All pressures are in pounds per square inch (gage). Temperatures and pressures listed are maximum internal fluid temperatures and pressures at flange.

[2] Carbon-molybdenum steel castings: ASTM A157-44 Grade C1; ASTM A217-47T Grade WC1. Carbon-molybdenum steel forgings: ASTM A182-46 Grade F1.

[3] Raised face, lapped, and large male and female facings, when used with flat solid metal gaskets, are permitted to be given the pressure-temperature ratings of this standard only when the gasket contact area is not greater than the large tongue and groove gasket contact areas and the gasket ID is not less than the large tongue and groove gasket ID.

[4] These are manufacturer's shell tests applicable to valves and fittings. Flanges may be tested after attachment to the pressure vessel in accordance with applicable code requirements, in which cases the test pressures may be higher than those given above. In such cases consideration should be given to proper gaskets for the blanking-off flanges. All tests shall be made with water at a temperature not to exceed 125 F or as required by applicable codes.

Table 6. Pressure-temperature Ratings for Steel Pipe Flanges and Flanged Fittings[1]

(Reprinted from Supplement No. 1 to ASA B16e–1939)

Material: Carbon-molybdenum Steels [2] and Equivalent Alloy Steels
Facing: Ring-joint

Fluid	Primary Service Pressure Ratings	300	400	600	900	1500	2500
	Hydrostatic Shell Test Pressures [4]	1100	1450	2175	3250	5400	9000
	Service Temperatures, °F	Maximum, Nonshock, Service Pressure Ratings at Temperatures from 100 to 1100 F					
Water, steam, oil	100	720	960	1440	2160	3600	6000
	150	710	945	1420	2130	3550	5915
	200	700	930	1400	2100	3500	5830
	250	690	920	1380	2070	3450	5750
	300	680	910	1365	2050	3415	5690
	350	675	900	1350	2025	3375	5625
	400	665	890	1330	2000	3330	5550
	450	660	875	1320	1975	3295	5490
	500	625	835	1250	1875	3125	5210
	550	590	790	1180	1775	2955	4925
	600	555	740	1110	1660	2770	4620
	650	515	690	1030	1550	2580	4300
	700	470	635	940	1410	2350	3920
	750	425	575	850	1275	2125	3550
	800	375	500	750	1125	1875	3125
	850	350	475	700	1050	1750	2925
	900	325	425	650	975	1625	2700
	950	**300**	**400**	**600**	**900**	**1500**	**2500**
Oil	1000	230	310	470	700	1170	1950
	1050	135	180	270	405	675	1125
	1100	90	120	180	270	450	750

Note. These ratings apply also to steel flanged and welding-end valves designed for the same primary service pressure ratings and complying with the requirements of Section 1, General.
See Table 5 on opposite page for Notes 1, 2, and 4. Note 3 does not apply to this table.

9. WELDING

Fusion welding of piping for power plant service is performed either under the ASA Code for Pressure Piping or the ASME Boiler Code. Both codes contain requirements intended to insure safe welds, and involve both procedure and operator qualifications.

Procedure qualification is to demonstrate that the methods and practices used by a manufacturer or contractor for a given material will result in satisfactory welds.

Operator qualification is a series of tests to demonstrate the ability of an individual operator to produce satisfactory welds under the specified conditions of his employer's procedure.

Joints made by the fusion process may be butt or fillet welds. (see Fig. 3.) The ASA Piping Code covers joints formed by pipe end to end, pipe branches, pipe to flanges, fittings and valves, and pipe, valves, or fitting to other equipment.

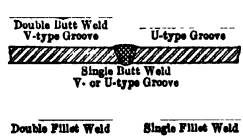


FIG. 3. Fusion welds in piping.

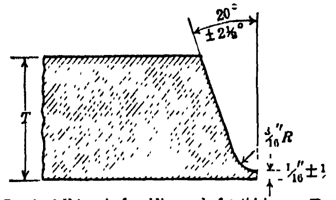
Materials for pipe, fittings, valves, and flanges must conform to the Code. They must be of good weldable quality, free from laminations, harmful ingredients, or defects. Filler metal, electrodes, welding wire, and welding rods shall be suitable for use with the base metals to be welded.

BUTT WELDS are those whose throat, i.e., the minimum thickness of the weld, not including reinforcement, along a straight line passing through the root, lies in a plane at approximately 90 degrees from the surface of at least one of the parts joined. In double-welded joints, filler metal is added to both sides; in single-welded joints it is added to one side only. Parts shall be prepared for fusion welding approximately as in Fig. 4. Welds may be single- or double-V type, or U-bevel. Welding procedure shall insure complete penetration of deposited metal to the bottom of the joint, and thorough fusion of deposited and base metal. Ferrules or backing strips inside the pipe may be used if properly secured and thoroughly fused to the weld. Minimum throat shall be the thickness of the thinner part joined. Welds shall be reinforced in excess of net throat not less than 1/16 in. for material up to and including 1/2 in. thick, and 1/8 in. for thicker material.

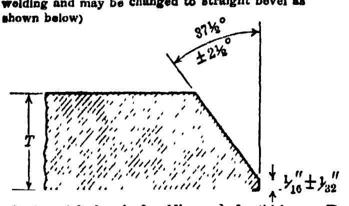
FILLET WELDS are of approximately triangular cross section, with the throat in a plane at approximately 45 degrees from the surface of the parts joined (see Fig. 3). They may be either single or double fillet. Welding procedure shall insure complete penetration and thorough fusion of deposited and base metal. Minimum throat shall be 0.75 × nominal size of weld, i.e., the width of the shortest of its adjacent fused legs.

SEAL WELDS are continuous arc or gas welds primarily intended to secure tightness. They may be either butt or fillet type, and should be made of as small cross section as practicable. They shall not be considered as contributing to the strength of the joint. Seal welding must be done so as to avoid undue straining of the joint by temperature changes.

CAST-IRON AND NONFERROUS MATERIALS shall be welded, when permitted, with bronze or other suitable filler metal. Copper pipe shall not be welded with copper welding rod, but may be welded with bronze rod; brass pipe also may be welded with bronze rod.



Standard U-bevel of welding ends for thickness (T) Greater than ¾ in.
(This form of bevel is not recommended for acetylene welding and may be changed to straight bevel as shown below)



Standard straight bevel of welding ends for thickness (T) 3/16 to ¾ in., incl.
(For electric arc or acetylene welding)

FIG. 4. Welding preparation for pipe. (Reproduced from ASA Code for Pressure Piping B31.1-1942)

FLANGES. Figure 2 shows designs of welds for steel flange connections. The slip-on flange is limited to service pressures not over 300 psi.

BACKING RINGS are used in butt-welded joints both to prevent weld metal entering the pipe at the joint and to assist in obtaining proper fusion in the initial bead. Commercial split backing rings are suitable for low and medium pressures; but for high pressures or where welds are to be x-rayed, one-piece machined backing rings should be used and pipe ends machined for a snug fit.

WELDING PROCEDURE. Beveling preferably shall be done by machine. Forms of welding bevels used are shown in Fig. 4. Torch beveling may be used if the surfaces subsequently are cleaned thoroughly from scale and oxidation. All surfaces must be free of paint, oil, rust, and scale; a light coat of linseed oil to prevent rust is permissible. No part may be offset from an adjacent part by more than 20% of the pipe thickness. Length of tack welds shall be approximately twice the thickness of the thinner material joined. Tack welds must be kept below the outside surface, and melted out during welding. No globules of metal may project within the pipe so as to seriously restrict its area or cause danger of loosening and falling into the pipe. The thickness of reinforcement of butt welds shall increase gradually from edge to center, and no depressions below the surface of the pipe are permissible. Nozzles, tees, and branches in materials of 3/4 in. or more wall thickness shall be welded under shop conditions. Precautions must be taken to insure that welding operations do not cause distortion by heat that would prevent any valve, sliding fixture, or other operating equipment from functioning properly.

Preheating is desirable if the carbon content exceeds 0.35%, for pipe wall thicknesses 3/4 in. or over, or for alloy material. Preheats of 400 to 450 F for carbon steel, or 500 F or higher for alloy material, are normally used.

Stress relief is mandatory for wall thicknesses of 3/4 in. or greater for ordinary carbon steels; and for thicknesses of 1/2 in. and greater if carbon exceeds 0.35%, or for alloy steels. It is accomplished by slowly heating to the required temperature, holding that temperature for one hour per inch of wall thickness, and cooling slowly. Stress-relief temperatures used are 1100 to 1250 F for carbon steel, 1250 to 1325 F for carbon-molybdenum steel, and 1300 to 1350 F for chromium-molybdenum steel.

10. HIGH PRESSURES AND TEMPERATURES

GRAPHITIZATION. An investigation of the failure of a welded carbon-molybdenum steel joint in a large central station in 1942 showed that it had resulted from graphitization. Under the combined influences of stress, elevated temperature, and time, a portion of the combined carbon in steel reverts to graphite. This condition is most likely to occur in the heat-affected zone adjacent to a weld. Graphite may be *nodular graphite*, occurring as flecks or spots, or *chain graphite*, in the form of long stringers. Nodular graphite in small amounts is not particularly serious, but the condition should be followed closely for further developments. Discovery of appreciable amounts of chain graphite usually calls for replacement of the joint.

It now has been established that the deoxidizing practice used in manufacture of the steel is pertinent, since graphitization occurs more readily in *aluminum-killed* steels than in *silicon-killed* steels. The use of more than 0.5 lb of aluminum per ton of steel appears to favor graphite formation. Under the same conditions, plain carbon steel is more likely to graphitize than carbon-molybdenum steel, and the latter is more susceptible to graphitization than chromium-molybdenum steel. Chromium makes the carbides more stable in both rolled and cast materials, though the latter require somewhat more chromium to accomplish a given degree of stabilization.

TEMPERATURE LIMITS FOR PIPE MATERIALS. Although it is not possible to fix exact temperature limits at which a given material will *not* be subject to graphitization, the following limits are consistent with current design principles:

ASTM Specification No.	Material	Temperature Limit, °F
A-106	Carbon steel	750
A-206	Carbon-molybdenum steel	825
A-280	Chromium-molybdenum steel (1/2% Cr)	900–950
A-315	Chromium-molybdenum steel (1% Cr)	950–1000
A-158 Grade P-11	Chromium-molybdenum steel (1 1/4% Cr)	950–1000
A-158 Grade P-3b	Chromium-molybdenum steel (2% Cr)	1000
A-213 Grade T-22	Chromium-molybdenum steel (2 1/4% Cr, 1% Mo)	1000

DESIGN CONSIDERATIONS. Above approximately 750 F and for the higher pressure range, it is desirable to use all-welded construction. Flanged joints should be fitted with the best grade of alloy bolting material and made up by micrometer measurements to insure uniformity. Initial (cold) bolt stresses should be limited to about 30,000 psi to avoid creep or relaxation.

Piping stresses will be fixed by the creep properties of the material used, and for temperatures above 750 to 800 F no reduction in stress should be assumed as a result of cold springing.

Hangers and supports become more important at elevated temperatures. The constant-support type of hanger is desirable as it tends to reduce the difference between hot and cold stress. Roller or sliding supports should be designed so that temperature does not impair their functioning.

Valves employing welded-in or seal-welded seat rings are better suited to high-temperature service than other designs. Seats should preferably be hard surfaced.

STRESSES IN PIPE LINES

11. GRAPHICAL SOLUTION FOR STRESSES

By A. S. McCormick

In the design of a piping structure the limiting features usually are the thrust that can be imposed on the members to which the piping is to be connected and the stress allowable in the pipe at the temperature at which it is to be used.

The first step, therefore, is to determine the thrust and stress on the proposed layout. Once these are determined it usually is possible to modify the design to meet the imposed conditions. If the design is too stiff and highly stressed it will be necessary to add length to some of the members, to add short lengths at right angles to each other on the plan of the U-bend, or to use some kind of expansion joint if the space is limited.

Stresses encountered come under several headings:

(1) Stress due to thermal expansion which can be determined by the graphical method.

(2) Bursting or hoop stress which is found from the equation

$$S = \frac{pd}{2t} \tag{1}$$

where p = pressure, psi; d = inside diameter of pipe, inches; t = wall thickness of pipe, inches.

(3) Longitudinal stress due to internal pressure, which is equal to $^1/_2 \times$ (hoop stress).

(4) Longitudinal bending stress due to the dead weight of unsupported lengths of piping. Stresses can be determined from beam formulas and hangers properly spaced to reduce the stresses.

(5) Stresses due to direct thrust or direct tension. These are equal to the direct forces divided by the net area of the pipe and usually are small.

(6) Torsion. If the piping between anchor points is in one plane no torsion will be present. However, if the piping lies in more than one plane the points where torsion will occur can be located by inspection and the torsional stress calculated by consideration of the forces applied.

In calculating the stresses due to thermal expansion the modulus of elasticity used should be selected for the temperature at which the pipe is to be used. The value of the modulus drops rapidly with rise in temperature. The proper value can be derived by the formula of G. A. Orrok (High Pressure Steam Boilers, *Trans. ASME*, FSP-50-28, 1928),

$$E_t = E_{32}\left(1 - \frac{t - 32}{1700}\right) \tag{2}$$

where E_{32} = modulus of elasticity of the material at 32 F; t = temperature at which the modulus is required; E_t = modulus of elasticity at temperature t.

The above formula is reasonably good for ordinary low-alloy materials, but the modulus of high-alloy steels of austenitic compositions falls off less markedly at high temperature. The practice of 100% cold springing enables designs to be based on the cold modulus. However, if less cold springing is employed, expansion stresses at temperature should be based on the hot modulus of the material.

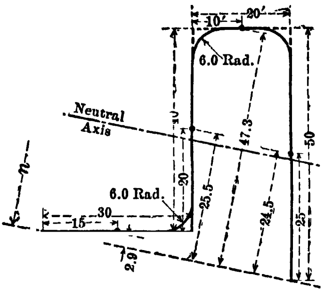
Present practice for cold springing of pipe varies from 50 to 100% of the thermal expansion, provided the elastic limit of the material is not exceeded. If 50% cold springing is used, there is a bending stress in the pipe in its hot condition which gradually will be reduced by temperature, due to creep of the material, and a corresponding stress will appear in the opposite direction in the piping when in the cold condition. However, if the pipe is cold sprung 100%, there will be little, if any, stress in the pipe in the hot condition, and it may therefore be designed for the bursting stress at operating temperature only. The full stress due to thermal expansion will then appear in the pipe when it is cold, and must be limited to some value below the elastic limit, to assure permanence of the arrangement.

A GRAPHIC METHOD FOR DETERMINING STRESSES IN PIPING is given by C. T. Mitchell in *Trans. ASME*, FSP-52-25, 1930. Being graphic, it is independent of tables and complicated formulas. It considers the pipe structure as a whole, and is applicable to any independent pipe structure in one or more planes.

Two assumptions are made in every problem. (1) The ends or anchor points are assumed to be 100% fixed. (2) The moment of inertia of the pipe is assumed to be constant, whether the section be taken at a bend or elsewhere. No attempt is made to determine the horizontal or vertical reactions at anchor points. Also, all bends are eliminated from the structure during the analysis, and square corners substituted in their place. A correction subsequently is made for the influence of bends.

Notation. A, B, C, F, G, H, J = moment areas, square feet; $a, b, c, f, g, h, j, l, m, n, o$ = distances on sketches, feet; D = outside diameter of pipe, inches; d = inside diameter of pipe, inches; Δ = expansion of pipe between anchors, inches; E = modulus of elasticity; I = moment of inertia, inches^4; $M = \Sigma$(moment areas $\times$ distance of center of gravity of each to neutral axis), i.e., $M = \Sigma(Aa + Bb + Cc + \cdots)$; P = thrust between anchors, pounds; P_c = thrust corrected for bends in pipe, pounds; R = shortest distance from neutral axis to point of greatest bending moment, feet; S = stress in outside fiber of pipe, psi; T = total length of pipe with square corner bends, feet; t = total length of pipe with full radius bends, feet.

The application of the method will be illustrated by a problem wherein the pipe is in a single plane, and one wherein it is in two planes.



$$n = \frac{30 \times 2.9 + 40 \times 25.5 + 20 \times 47.3 + 50 \times 24.5}{140} = 23.4$$

FIG. 1.

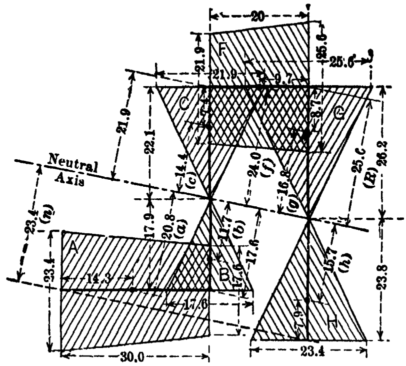


FIG. 2.

Typical Single-plane Problem. Arrangement of piping is shown in Fig. 1. Conditions: 12-in. pipe, wall thickness, 1.219 in.; D = 12 75 in.; I = 740.0; steam temperature, 750 F; room temperature, 70 F; expansion of steel pipe through 680 F, 7.20 in. per 100 ft.

The procedure is as follows. (1) Redraw the arrangement to a convenient scale, with square corners instead of bends (dotted lines, Fig. 1). (2) Locate neutral axis as shown in Fig. 1. (3) Draw moment areas A, B, C, F, G, H as in Fig. 2. (4) Compute the square feet in these areas, as below. The sum of the areas on each side of the neutral axis should be equal.

$$A = 1/2\,[(23.4 + 17.6) \times 30] = 615$$
$$B = 1/2\,(17.6 \times 17.9) = 158$$
$$C = 1/2\,(21.9 \times 22.1) = 242$$
$$F = 1/2\,[(21.9 + 25.6) \times 20] = 475$$
$$G = 1/2\,(25.6 \times 26.2) = 335$$
$$H = 1/2\,(23.4 \times 23.8) = 279$$

(5) Compute M. $M = Aa + Bb + Cc + Ff + Gg + Hh$ = (615 × 20.8) + (158 × 11.7) + (242 × 14.4) + (475 × 24 0) + (335 × 16.8) + (279 × 15.7) = 39,545. (6) Compute P. Distance between anchors = 51.0 ft. Consequently, Δ = (7.20 × 51)/100 = 3.68 in., and $P = \Delta EI/1728\,M$ = (3.68 × 29,000,000 × 740.0)/(1728 × 39,545) = 1160 lb. $P_c = PT/t$ = 1160 × 140/132.2 = 1230 lb. (7) Compute S. The greatest stress in the pipe line is located at a point at the greatest distance from the neutral axis. By scaling Fig. 2, this distance is found to be R = 25.6 ft. Then, by substitution in the equation,

$$S = \frac{6P_cRD}{I} \tag{3}$$

we find S = (6 × 1230 × 25.6 × 12.750)/740 = 3250 psi. The stress may be calculated at any point along the pipe by scaling the perpendicular distance from this point to the neutral axis, and substituting the value so found for the value of R in eq. 3.

Typical Two-plane Problem. Arrangement of the piping is shown in perspective in Fig. 3. Conditions: 6-in. pipe, wall thickness, 0.5 in.; D = 6.625 in.; I = 45.4; steam temperature, 750 F; room temperature, 70 F; expansion of steel pipe through 680 F, 7.20 in. per 100 ft.

Procedure is as follows: (1)(*a*) Draw to scale the plan view, Fig. 4, with line xx connecting anchors parallel to the upper edge of the paper, and all bends changed to square corners. (*b*) Draw elevation (Fig. 5) which shows line xx in its true length. (*c*) Project another plan (Fig. 6). This also shows line xx in its true length. (2) Locate neutral axis. (*a*) Draw neutral axis through the center of gravity of each view (Figs. 5 and 6, parallel to xx, taking care to use the true length of each pipe member in the calculations. Let U, V, W be the true length of the several pipe members, and u, v, w, respectively, the perpendicular distance from the midpoint of each member to the line x–x. Let n = distance of neutral axis from the line x–x. Then

$$n_1 = \frac{Uu_1 + Vv_1 + Ww_1}{U + V + W}$$

and

$$n_2 = \frac{Uu_2 + Vv_2 + Ww_2}{U + VW}$$

If the midpoints of the several sections lie on the same side of the line xx, the products Uu, Vv, Ww will be of like algebraic sign. If they lie on opposite sides of xx, the products will be of unlike sign.

$$n_1 = \frac{(10 \times 2.05) - (20 \times 2.00) - (15 \times 4.60)}{45} = -2.23 \text{ ft (Fig. 5)}$$

$$\frac{(10 \times 4.15) + (20 \times 8.35) + (15 \times 4.15)}{45} = +6.02 \text{ ft (Fig. 6)}$$

(3) Draw moment areas *A, B, C, F, G, H, J*, as shown in Fig. 7. (*a*) To determine moment areas, it is necessary to know the perpendicular distance from the neutral axis to all parts of the piping arrangement. The distance for any point is the hypotenuse of a right triangle, the legs of which are the ordinates to the neutral axis from the point in question, which may be scaled from Figs. 5 and 6. (*b*) Draw the moment areas about the line *YY*, Fig. 7. This line represents the total length of pipe, with each member drawn to scale. Above *YY*, and perpendicular to it, draw the ordinates for the end of each member, as scaled from Fig. 5. These are drawn *above YY*, irrespective of their position in Fig. 5. Connect the ends of the ordinates by straight lines. The irregular dotted line below *YY* is drawn so that at any point the vertical distance between it and the line above *YY* is the bending

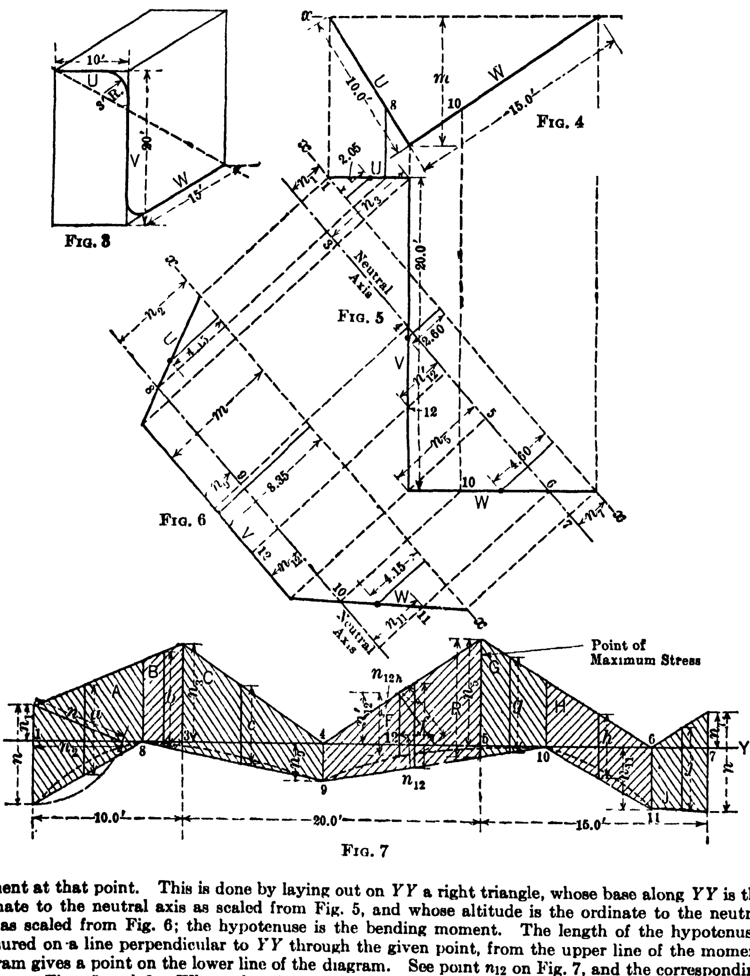


Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

moment at that point. This is done by laying out on *YY* a right triangle, whose base along *YY* is the ordinate to the neutral axis as scaled from Fig. 5, and whose altitude is the ordinate to the neutral axis as scaled from Fig. 6; the hypotenuse is the bending moment. The length of the hypotenuse, measured on a line perpendicular to *YY* through the given point, from the upper line of the moment diagram gives a point on the lower line of the diagram. See point n_{12} on Fig. 7, and the corresponding points on Figs. 5 and 6. Where the pipe intersects the neutral axis in Fig. 5, the ordinate at that point is, of course, zero, and the position of the point in the lower line of the moment diagram is determined by the length of the ordinate in Fig. 6. See points 9 and 11 in Figs. 5, 6, and 7. It is unnecessary to plot a large number of bending moments. The dotted line, forming the true boundary of the moment diagram, is a refinement that is not worth while. Using the trapezoidal areas *A, B, C*, etc., instead of the true moment area will give results that are sufficiently accurate. In locating points along *YY* in Fig. 7, the true distance along the pipe member under consideration should be taken, as determined by projecting the point to Fig. 4 or 5 as may be necessary. See point 8, which is projected from Fig. 6 to Fig. 4, to find its correct position in Fig. 7.

(4) The next step is to divide the total moment area into a convenient number of trapezoids as *A, B, C, F, G, H*, and *J*, and to compute the square feet in each area. Scaling the bases and altitudes of the several trapezoids in Fig. 7, we find the areas to be:

$$A = 1/2\,(6.4 + 5.2) \times 7.2 = 41.7$$
$$B = 1/2\,(5.2 + 6.9) \times 2.8 = 17.0$$
$$C = 1/2\,(6.9 + 2.3) \times 9.5 = 43.7$$
$$F = 1/2\,(2.3 + 7.7) \times 10.5 = 52.5$$
$$G = 1/2\,(7.7 + 4.5) \times 4.2 = 25.6$$
$$H = 1/2\,(4.5 + 4.0) \times 7.1 = 30.2$$
$$J = 1/2\,(4.0 + 6.4) \times 3.7 = 19.3$$

(5) Next compute M. The lengths of lines a, b, c, f, g, h, j are determined by locating the center of gravity of each trapezoid, as shown in Fig. 8, drawing a perpendicular to YY through each center of gravity, and scaling the intercept between the sides of the trapezoid. Then

$$M = Aa + Bb + Cc + Ff + Gg + Hh + Jj = (41.7 \times 5.8) + (17.0 \times 6.1) + (43.7 \times 5.0) + (52.5 \times 5.4) + (25.6 \times 6.2) + (30.2 \times 4.1) + (19.3 \times 5.2) = 1237$$

(6) Compute P. Distance between anchors = 26.9 ft, and $\Delta = (7.20 \times 26.9)/100 = 1.94$ in. Then $P = \Delta EI/1728\,M = (1.94 \times 29{,}000{,}000 \times 45.4)/(1728 \times 1237) = 1195$ lb.

$$P_c = \frac{PT}{t} = \frac{1195 \times 45}{42.6} = 1260 \text{ lb}$$

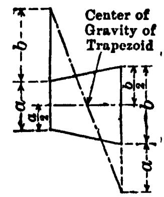


FIG. 8

(7) Compute S. The maximum stress occurs at the point of greatest bending moment. This point is shown in Fig. 7, and the distance R is found to be 7.7 ft. Substituting in eq. 3,

$$S = \frac{6 \times 1260 \times 7.7 \times 6.625}{45.4} = 8500 \text{ psi}$$

The stress may be calculated at any point along the pipe line by scaling the ordinate at this point in Fig. 7, and substituting the value so found for R in eq. 3.

Correction for square corners can be made by multiplying the values obtained for Δ in the above analysis by the factor

(Total length of actual structure) ÷ (Total length of square-corner structure)

Results will be 90 to 100% accurate, which is all that can reasonably be expected in problems of this sort, in view of the uncertainties always present. If allowance is made for flattened cross section of the pipe at bends, it should be remembered that the factor of safety of the structure thereby is lowered.

In two-plane problems the stress may be due to a combination of torsion and bending. Mr. Mitchell explains in the paper why the stress may be treated as a simple bending stress, and also explains the development of the correction factor for square corners. The theory underlying the graphic method is demonstrated.

12. METHOD OF MULTIPLE ANCHORS

By W. A. Thomas

Mitchell's method, described in the preceding article, is of much broader applicability than his original paper discloses. It may be extended to problems involving *any number of anchors, branch lines, differing pipe sizes, and to different types of end constraints.* Multiple-anchor problems are treated by first breaking down the given piping arrangement into its simple *two-anchor systems*, i.e., into all possible combinations of the n anchors taken two at a time. Thus there results $n(n - 1)/2$ such systems. Each system is considered to be temporarily detached from the others, and is treated by the Mitchell procedure to determine its force. From this point the procedure for determining stresses departs from Mitchell's. Stresses at each joint are determined for the two planes of each system moment diagram, then tabulated and combined vectorially to obtain resultant stresses.

This procedure will be illustrated by an example of a four-anchor piping system in which two different pipe sizes occur, shown in Fig. 9. For this piping arrangement there are six two-anchor systems ($4 \times 3 \div 2 = 6$). The example has been chosen so that three of these systems are identical in configuration with that of the example illustrated in Fig. 3, to avoid repeating Mitchell's method of constructing the moment diagrams. The remaining three systems are single-plane systems for which moment diagrams are readily constructed in the manner given for the example of Fig. 1.

THE PROCEDURE is outlined in the following seven steps:

(1) Sketch a perspective view (Fig. 9) of the entire piping arrangement, indicating pipe sizes and dimensions of the members. Number the anchors and joints. Compute the distances between *all*

pairs of anchors and the values of Δ along these distances. Indicate these on the sketch as shown.

(2) Draw a plan to scale and two projections of each anchor-pair piping system (Fig. 10), always forming the plan from the top side of the perspective (Fig. 9). Construct the moment diagrams and compute the values of M in the same manner as for the examples of Mitchell's method. These are *composite* moment diagrams for projections 1 and 2 and are to be used *only* for determining the forces, P_n. Detailed computations for locating the neutral axes and obtaining the values of M are omitted since they are obtained in the same manner as for the previous examples.

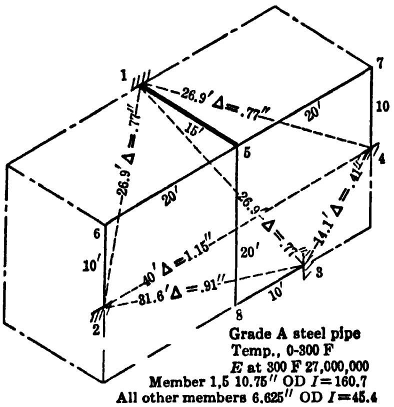


FIG. 9. Four-anchor system used as an example.

(3) Systems (1, 2), (1, 3), and (1, 4) each contain two different pipe sizes, so the values of M for these systems must be modified as follows:

$$M_{\text{mod}} = M + M_s\,[(I_l - I_s)/I_s]$$

$$= 1237 + 854\,[(160.7 - 45.4)/45.4] = 3407 \text{ ft}^3$$

where M = total for all members in a moment diagram; M_s = total for members having the smaller moment of inertia = 45.4; I_l = larger moment of inertia = 160.7; I_s = smaller moment of inertia = 45.4

(4) The forces P_n acting along the baselines connecting the pairs of anchors are determined by using the equation $P = \Delta EI/1728M$. For the forces $P_n = P_{12} = P_{13} = P_{14}$, I_l and M_{mod} are to be used in the equation.

$$P_n = \frac{0.77 \times 27{,}000{,}000 \times 160.7}{1728 \times 3407} \quad 570 \text{ lb}$$

For the forces P_{23}, P_{24}, and P_{34}, I_s and M are to be used.

$$P_{23} = \frac{0.92 \times 27{,}000{,}000 \times 45.4}{1728 \times 2212} \quad 300 \text{ lb}$$

$$P_{24} = \frac{1.15 \times 27{,}000{,}000 \times 45.4}{1728 \times 489} = 1670 \text{ lb}$$

$$P_{34} \quad \frac{0.41 \times 27{,}000{,}000 \times 45.4}{1728 \times 2620} = 110 \text{ lb}$$

These forces may be corrected for curvature of elbows if necessary by methods given in the preceding article.

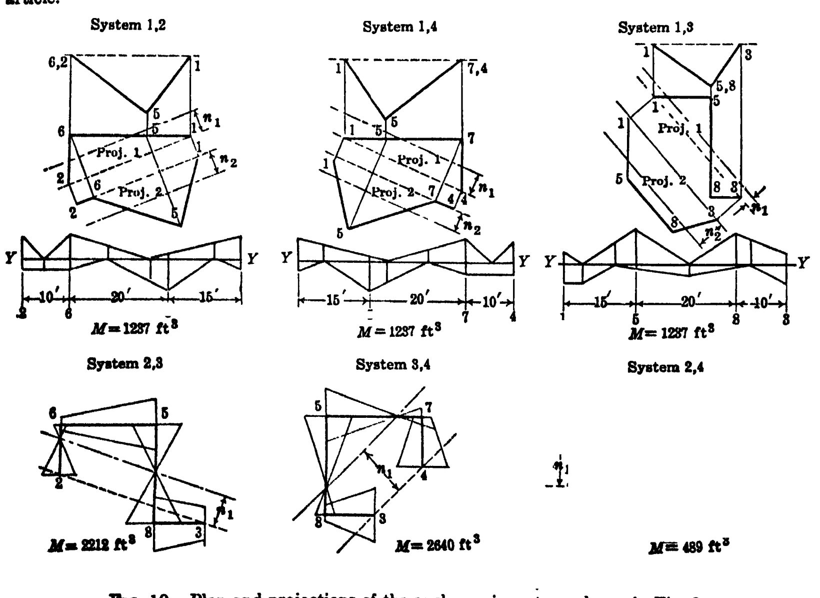


FIG. 10. Plan and projections of the anchor-pair systems shown in Fig. 9.

(5) Lay out a stress sheet, providing a table with columns headed *Joint, Plane, System and Stress*; and providing rows headed by the joint number and Plane I and II, as shown in Table 1. At the lower left corner of each cell enter the system number (1, 2), (1, 3) . . . in which the joint or anchor under consideration occurs. For this example three *System and Stress* columns are required because each joint or anchor occurs in three elemental systems. These entries having been made, the table is completely indexed, and the cells are ready to receive the stress entries. Note that stress at a joint means at a point immediately adjoining a square corner. For a junction of several pipes a point should be chosen as fixed upon a certain *member* so as to establish consistently to which anchor-pair systems the junction belongs in the analysis. For this example the point for joint 5 was chosen on member (5, 8) so that it occurs in systems (1, 3), (2, 3), and (3, 4). The stress for entry in each cell is found by the equation

$$S = P_n a(6D/I)$$

where P_n is the force for the system; n = (1, 2), (1, 3), etc.; a is the distance from the neutral axis to the point under consideration, scaled from Projection 1 for entry in a Plane I cell, or from Projection 2 for entry in a Plane II cell. The proper value of the force, P_n and of a is entered in each cell.

Only two pipe sizes occur in this example; hence there are only two values of $(6D/I)$, i.e., $(6 \times 10.75)/160.7 = 0.403$, and $(6 \times 6.625)/45.4 = 0.876$. The proper one of these two factors is entered in each cell, for use in the stress equation, given above.

After the foregoing three factors have been entered, their product is entered in the lower right corner of the cell. This product is the portion of the stress in that joint or anchor contributed by the anchor-pair system in the plane indicated by the column and row of the cell. Where an anchor-pair system is in a single plane there is of course but one plane in which stresses are entered in the table.

(6) The next step is to combine the contributions of Plane I stresses in each joint; they are obtained from the corresponding row of cells. In like manner the contributions of Plane II stresses are obtained from the corresponding row and combined. The combining is done vectorially following the usual principles of composition and resolution of vectors. The magnitudes of the stress vectors are obtained

Table 1. Stress Calculation Form

Joint	Plane	System and Stress, psi	System and Stress, psi	System and Stress, psi	Resultant S, psi
1	I	4.3 × 570 × .403 = (1, 2) 980	4.3 × 570 × .403 = (1, 4) 980	2.3 × 570 × .403 = (1, 3) 530	2,760
	II	4.5 × 570 × .403 = (1, 2) 1030	4.5 × 570 × .403 = (1, 4) 1030	6 × 570 × .403 = (1, 3) 1370	
2	I	4.3 × 570 × .876 = (1, 2) 2200	6.8 × 300 × .876 = (2, 3) 1780	8.5 × 1670 × .876 = (2, 4) 12,400	16,200
	II	4.5 × 570 × .876 = (1, 2) 2300	0	0	
3	I	2.3 × 570 × .876 = (1, 3) 1150	6.8 × 300 × .876 = (2, 3) 1780	10.5 × 110 × .876 = (3, 4) 1000	3,600
	II	6.2 × 570 × .876 = (1, 3) 3100	0	0	
4	I	4 × 570 × .876 = (1, 4) 2000	10.2 × 110 × .876 = (3, 4) 980	8.5 × 1670 × .876 = (2, 4) 12,400	15,800
	II	4.5 × 570 × .876 = (1, 4) 2300	0	0	
5	I	6.8 × 570 × .876 = (1, 3) 3390	8.8 × 300 × .876 = (2, 3) 2300	11 × 110 × .876 = (3, 4) 1060	4,300
	II	2 × 570 × .876 = (1, 3) 1000	0	0	
6	I	4.8 × 570 × .876 = (1, 2) 2400	2.6 × 300 × .876 = (2, 3) 680	1.5 × 1670 × .876 = (2, 4) 2200	5,500
	II	4.5 × 570 × .876 = (1, 2) 2300	0	0	
7	I	4.5 × 570 × .876 = (1, 4) 2300	3.2 × 110 × .876 = (3, 4) 310	1.6 × 1670 × .876 = (2, 4) 2400	4,700
	II	4.6 × 570 × .876 = (1, 4) 2300	0	0	
8	I	.8 × 570 × .876 = (1, 3) 400	9.8 × 300 × .876 = (2, 3) 2570	3.2 × 110 × .876 = (3, 4) 310	2,700
	II	2 × 570 × .876 = (1, 3) 1000	0	0	

from the table, but the angles are obtained from a key plan and elevation of the whole piping arrangement, drawn to scale, as in Fig. 11.

Mitchell's original method makes its analysis by projections of the piping arrangement upon two intersecting planes, the axis of which is coincident with the baseline connecting two anchors. This provides a definition of the expression *Planes I* and *Planes II*. In the plan of Fig. 11 the angles ϕ are the angles between the Planes I, which are perpendicular to the paper and pass through the line connecting anchors. In the elevation of Fig. 11 the angles θ are the angles between the Planes II, which are perpendicular to the paper and pass through the line connecting anchors.

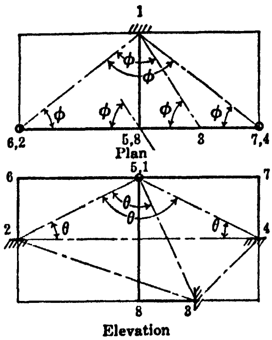


Fig. 11. Scale drawings of piping system used to obtain angles.

The tabulated contributing stresses are regarded as *stress disks* or vectors operating in Planes I and II. Their direction is found in Fig. 11. The resultant Plane I stress in a joint or anchor due to the contributing stresses of Planes I, and the resulting Plane II stress due to the contributing stresses of Planes II, are obtained graphically as shown in Fig. 12. Thus at joint 1, Plane I, the contributing stresses are, for system (1, 4) 980 psi; (1, 2) 980 psi; (1, 3) 530 psi. These are drawn to scale in both magnitude and direction. The resultant (shown dotted) of the three stresses is found by combining them in pairs. For other joints and planes the procedure is the same.

(7) Finally, for each joint the resultant Plane I stress, S_1, is combined with the resultant Plane II stress, S_2, to obtain the resultant combined stress, S_c, by the equation:

$$S_c = \sqrt{S_1^2 + S_2^2}$$

Thus at joint 1 the resultant combined stress of both planes is

$$S_c = \sqrt{1680^2 + 2200^2} = 2760 \text{ psi}$$

The results are entered in the last column of Table 1, and the maximum stress with its location becomes evident.

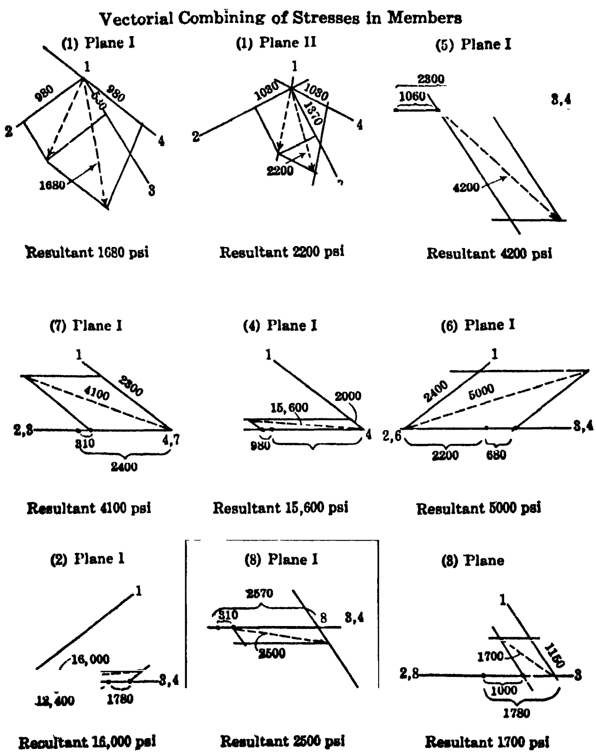


Fig. 12. Graphical combination of contributing stresses.

In this example magnitudes of the resultant combined stresses have been plotted to scale perpendicularly across the piping arrangement (Fig. 13) so that we may visualize the effect of thermal expansion. These stresses will remain proportional for any other temperature range. The reader is cautioned against concluding from this example that the maximum stress always occurs at an anchor. Probably more often it occurs at an intermediate joint. Note that stress at a joint means at a point immediately adjoining a square corner and for a junction of several pipes a point should be chosen as fixed upon a certain *member* so as to establish consistently to which anchor-pair systems the junction belongs in the analysis.

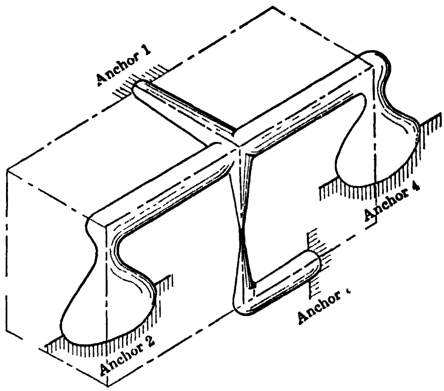


FIG. 13. Distribution of stress in the piping system.

PARTIAL END CONSTRAINTS. The Mitchell method can be extended to problems involving partial end constraints, where one or more degrees of freedom occur through the use of guides hinges instead of fixed anchors. Such problems arise when it becomes desirable to permit translation or rotation at the ends of a piping system.

Figure 14 shows the approximate position of the neutral axis for various end constraints

Position of Neutral Axis
For Different Constraints

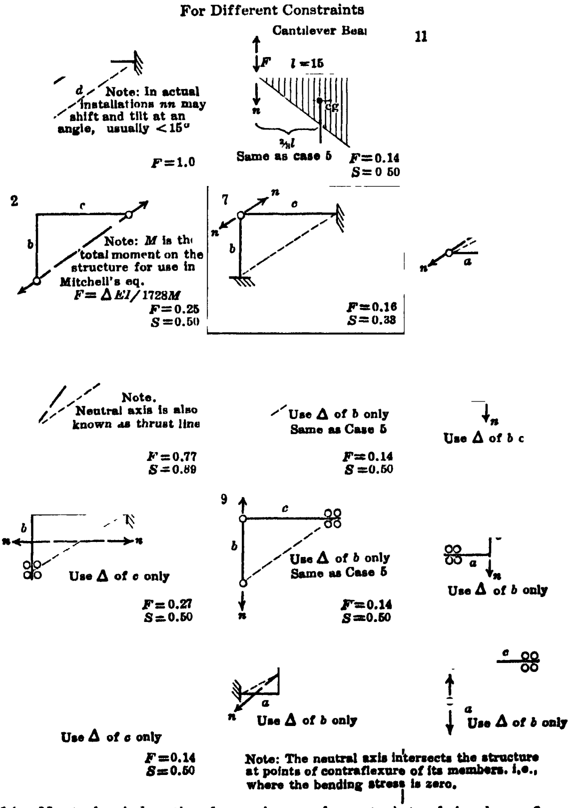


FIG. 14. Neutral axis location for various end constraints of simple configuration.

in two simple configurations that may be used as an aid to the construction of moment diagrams. Approximate values of force and stress multiplication factors (F and S, respectively) are also given, referred to Case 1, fixed anchors, as having unity force and maximum stress.

Charts. For the simpler piping configurations, time-saving charts are available such as for instance the Flex-Anal Charts (available from the Power Piping Division of Blaw-Knox Construction Company, Pittsburgh 22, Pa.)

REMEDIES FOR HIGH STRESSES. When calculated stresses or forces are too great, some of the available remedies are: (1) Add more pipe. (2) Reduce pipe size. (3) Change direction of members. (4) Prespring the system. (5) Relocate equipment. (6) Use corrugated pipe, five times as flexible. (7) Use material having smaller expansion coefficients. (8) Use material having lower modulus of elasticity. (9) Insert expansion joints. (10) Free one or more anchors. (11) Using external constraint, distribute stress to other portions of the system.

COMMERCIAL CONDITIONS. In commercial piping the anchors are seldom perfectly rigid and piping joints seldom of equal flexibility when installed. Moreover the actual displacements in a piping configuration are much greater than those permitted in such structures as buildings or bridges. These larger displacements cause a progressive shifting of the centroid from its initially calculated position, thus changing the location and slope of the neutral axis to a final position usually impossible to calculate precisely. Mill tolerances on pipe wall thicknesses and eccentricity are such that a departure of $\pm 10\%$ from calculated forces and stresses should be anticipated from these causes. It is probably best to avoid too detailed and costly calculations, especially when close agreement with test results is hardly likely, and any method of calculation necessarily is an approximation.

MODIFIED MITCHELL METHOD. In projections drawn for the Mitchell method, the neutral axis (which passes through the centroid) is assumed by Mitchell, for simplicity, to be parallel with the baseline x–x, connecting anchors, in all configurations. Actually this assumption holds only when the piping system comprises pairs of members of equal length, symmetrically disposed about a perpendicular through the midpoint of the baseline, x–x. For other configurations the neutral axis passes through the centroid but is inclined to it at a small angle, θ. It can be shown that for any configuration the neutral axis must cross at least two members and that θ varies from 0 and 12 degrees. For most piping systems θ is less than 8 degrees so that its influence upon moments does not usually affect theoretical reactions or maximum stress more than $\pm$ 10%.

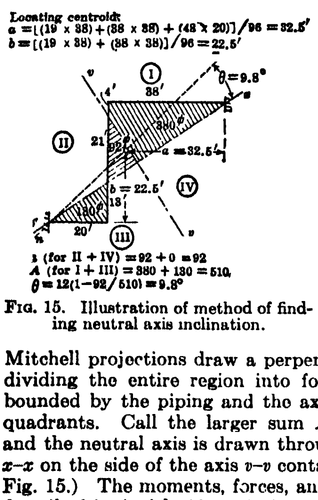


FIG. 15. Illustration of method of finding neutral axis inclination.

When more precise values of force and stress are required the Mitchell method may be modified to determine the angle θ, as follows. On each of the Mitchell projections draw a perpendicular v–v through the midpoint of the baseline x–x, dividing the entire region into four quadrants. Compute the area in each quadrant bounded by the piping and the axes v–v and x–x. Add the areas in *diagonally opposite* quadrants. Call the larger sum A and the smaller sum a. Then $\theta = 14[1 - (a/A)]$, and the neutral axis is drawn through the centroid of the projection, but sloping toward x–x on the side of the axis v–v containing the greatest total length of pipe. (See example, Fig. 15.) The moments, forces, and stresses are then found by the procedure of Mitchell, described in Article 11, p. 6-15.

PIPE AND TUBING

13. COMMERCIAL PIPE AND TUBING

Tables 1 to 11 give data on steel, wrought-iron, and nonferrous pipe commercially available for steam service, and also on seamless tubes and boiler tubes.

WELDED STEEL PIPE is furnished with threads and couplings and in random lengths unless otherwise ordered. Data in Tables 1 and 2 represent the latest American Standard. Table 3 covers the old "Double Extra Strong" classification.

Table 1. Dimensions of Welded and Seamless Steel Pipe

(ASA B36.10-1939)

Nominal Pipe Size	Outside Diameter	Nominal Wall Thicknesses for Schedule Numbers									
		Sched. 10	Sched. 20	Sched. 30	Sched. 40	Sched. 60	Sched. 80	Sched. 100	Sched. 120	Sched. 140	Sched. 160
1/8	0.405	......	......	......	**0.068**	......	**0.095**	......	......	......	......
1/4	0.540	......	......	......	**0.088**	......	**0.119**	......	......	......	......
3/8	0.675	......	......	......	**0.091**	......	**0.126**	......	......	......	......
1/2	0.840	......	......	......	**0.109**	......	**0.147**	......	......	......	0.187
3/4	1.050	......	......	......	**0.113**	......	**0.154**	......	......	......	0.218
1	1.315	......	......	......	**0.133**	......	**0.179**	......	......	......	0.250
1 1/4	1.660	......	......	......	**0.140**	......	**0.191**	......	......	......	0.250
1 1/2	1.900	......	......	......	**0.145**	......	**0.200**	......	......	......	0.281
2	2.375	......	......	......	**0.154**	......	**0.218**	......	......	......	0.343
2 1/2	2.875	......	......	......	**0.203**	......	**0.276**	......	......	......	0.375
3	3.5	......	......	......	**0.216**	......	**0.300**	......	......	......	0.437
3 1/2	4.0	......	......	......	**0.226**	......	**0.318**	......	......	......	......
4	4.5	......	......	......	**0.237**	......	**0.337**	......	0.437	......	0.531
5	5.563	......	......	......	**0.258**	......	**0.375**	......	0.500	......	0.625
6	6.625	......	......	......	**0.280**	......	**0.432**	......	0.562	......	0.718
8	8.625	......	0.250	**0.277**	**0.322**	0.406	**0.500**	0.593	0.718	0.812	0.906
10	10.75	......	0.250	**0.307**	**0.365**	**0.500**	0.593	0.718	0.843	1.000	1.125
12 *	12.75	......	0.250	**0.330**	0.406 *	0.562 *	0.687	0.843	1.000	1.125	1.312
14 OD	14.0	0 250	0.312	0.375	0.437	0.593	0.750	0.937	1.062	1.250	1.406
16 OD	16.0	0.250	0.312	0.375	0.500	0.656	0.843	1.031	1.218	1.437	1.562
18 OD	18.0	0.250	0.312	0.437	0.562	0.718	0.937	1.156	1.343	1.562	1.750
20 OD	20.0	0.250	0.375	0.500	0.593	0.812	1.031	1.250	1.500	1.750	1.937
24 OD	24.0	0.250	0.375	0.562	0.687	0.937	1.218	1.500	1.750	2.062	2.312
30 OD	30.0	0.312	0.500	0.625	......	......	......	......	......	......	......

Sizes below line in lower right corner not available in seamless pipe.

The decimal thicknesses listed for the respective pipe sizes represent their nominal or average wall dimensions. For tolerances on wall thickness, see appropriate material specification.

Thicknesses shown in bold-face type in Schedules 30 and 40 are identical with thicknesses for *standard weight* pipe in former lists; those in Schedules 60 and 80 are identical with thicknesses for *extra strong* pipe in former lists.

The schedule numbers indicate approximate values of the expression $1000 \times P/S$.

* Owing to a necessary departure from the old *standard weight* and *extra strong* thicknesses in the 12 in. size, Schedules 40 and 60, the new thicknesses are not as yet stocked by all manufacturers and jobbers. Hence, where agreeable to the purchaser and suitable for the service conditions, the old standard-weight 0.375 in. wall pipe corresponding to a $1000\ P/S$ value of 37.7 is still available and can be substituted for the 0.406 in. wall, and the old extra-strong 0.500 in. wall pipe corresponding to a $1000\ P/S$ value of 55 can be substituted for the 0.562 in. wall.

Table 2. Nominal Weights of Welded and Seamless Steel Pipe *

(ASA B36.10–1939)

Nominal Pipe Size, in.	Sched. 10	Sched. 20	Schedule 30		Schedule 40		Sched. 60	Sched. 80	Sched. 100	Sched. 120	Sched. 140	Sched. 160
	Plain Ends	Plain Ends	Plain Ends	Threads† and Couplings	Plain Ends	Threads† and Couplings	Plain Ends	Plain Ends	Plain Ends	Plain Ends	Plain Ends	Plain Ends
1/8					**0.25**	**0.25**		**0.32**				
1/4					**0.43**	**0.43**		**0.54**				
3/8					**0.57**	**0.57**		**0.74**				
1/2					**0.86**	**0.86**		**1.09**				1.31
3/4					**1.14**	**1.14**		**1.48**				1.94
1					**1.68**	**1.69**		**2.18**				2.85
1 1/4					**2.28**	**2.29**		**3.00**				3.77
1 1/2					**2.72**	**2.74**		**3.64**				4.86
2					**3.66**	**3.68**		**5.03**				7.45
2 1/2					**5.80**	**5.82**		**7.67**				10.0
3					**7.58**	**7.62**		**10.3**				14.3
3 1/2					**9.11**	**9.21**		**12.5**				
4					**10.8**	**10.9**		**15.0**		19.0		22.6
5					**14.7**	**14.9**		**20.8**		27.1		33.0
6					**19.0**	**19.2**		**28.6**		36.4		45.3
8		22.4	**24.7**	**25.0**	**28.6**	**28.8**	35 7	**43.4**	50.9	60.7	67.8	74.7
10		28.1	**34.3**	**35.0**	**40.5**	**41.2**	**54.8**	64 4	77 0	89.2	105	116
12		33.4	**43.8**	**45.0**	53.6	55.0	73 2	88.6	108	126	140	161
14 OD	36.8	45.7	54.6		63 3		85.0	107	131	147	171	190
16 OD	42.1	52.3	62.6		82.8		108	137	165	193	224	241
18 OD	47.4	59.0	82.0		105		133	171	208	239	275	304
20 OD	52.8	78.6	105		123		167	209	251	297	342	374
24 OD	63.5	94.7	141		171		231	297	361	416	484	536
30 OD	99.0	158	197									

* Weights are in pounds per linear foot and are for pipe with plain ends except for sizes which are commercially available with threads and couplings for which both weights are listed.

† The weights for line pipe with couplings are slightly greater than shown in Schedules 30 and 40 and may be found in API Specification 5-L.

Weights shown in bold-face type in Schedules 30 and 40 are identical with weights for *standard weight* pipe in former lists; those in Schedules 60 and 80 are identical with weights for *extra strong* pipe in former lists.

The schedule numbers indicate approximate values of the expression $1000 \times P/S$.

Table 3. Double Extra Strong Pipe *

Nominal Pipe Size	Outside Diameter	Nominal Wall Thickness		Weight per Foot, Plain Ends
		Wrought Iron	Steel	
1/2	0.840	0.307	0.294	1.714
3/4	1.050	0.318	0.308	2.440
1	1.315	0.369	0.358	3.659
1 1/4	1.660	0.393	0.382	5.214
1 1/2	1.900	0.411	0.400	6.408
2	2.375	0.447	0.436	9.029
2 1/2	2.875	0.565	0.552	13.695
3	3.500	0.615	0.600	18.583
4	4.500	0.690	0.674	27.541
5	5.563	0.768	0.750	38.552
6	6.625	0.884	0.864	53.160
8	8.625	0.895	0.875	72.424

All dimensions are in inches.

Weights are in pounds per linear foot.

*** Obsolete terminology. This table of *double extra strong* pipe sizes, although not a part of the American Standard, is included for the convenience of the user, inasmuch as this pipe is commercially available in both wrought iron and steel.**

WELDED WROUGHT-IRON PIPE has a thicker wall and smaller internal diameter than welded steel pipe. It is furnished with threads and couplings, and in random lengths. For dimensions and weights, see Tables 4 and 5.

Table 4. Dimensions of Welded Wrought-iron Pipe

(ASA B36.10–1939)

Nominal Pipe Size	Outside Diameter	Nominal Wall Thicknesses for Schedule Numbers					
		Schedule 10	Schedule 20	Schedule 30	Schedule 40	Schedule 60	Schedule 80
1/8	0.405	....	.	.....	**0.070**	....	**0.098**
1/4	0.540	...	. .	....	**0.090**	....	**0.122**
3/8	0.675	....	. . .	....	**0.093**	....	**0.129**
1/2	0.840	.. . .	..	.....	**0.111**	....	**0.151**
3/4	1.050	....	. .	....	**0.115**	....	**0.157**
1	1.315	....	....	....	**0.136**	....	**0.183**
1 1/4	1 660	.....	.. .	....	**0.143**	....	**0.195**
1 1/2	1.900	....	.. .	....	**0.148**	. ...	**0.204**
2	2.375	....	. . .	....	**0.158**	....	**0.223**
2 1/2	2.875	....	..... .	....	**0.208**	....	**0.282**
3	3.5	....	... . .	.....	**0.221**	....	**0.306**
3 1/2	4.0	....	......	....	**0.231**	. ...	**0.325**
4	4.5	....	.... . .	....	**0.242**	....	**0.344**
5	5.563	....	..... .	......	**0.263**	....	**0.383**
6	6.625	.... . .	....	....	**0.286**	....	**0.441**
8	8.625	....	... ...	. **0.283**	**0.329**	... .	**0.510**
10	10.75	... .. .	....	**0.313**	**0.372**	**0.510**	0.606
12 *	12.75	. .. .	....	**0.336**	0.414 *	0.574 *	0.702
14 OD	14.0	0.250	0.312	0.375	0.437	0.625	0.750
16 OD	16.0	0.250	0.312	0.375	0.500	0.687	....
18 OD	18.0	0.250	0.312	0.437	0.562	0.750	....
20 OD	20.0	. .....	0 375	0.500	0.562	....	....

All dimensions are in inches.

The decimal thicknesses listed for the respective pipe sizes represent their nominal or average wall dimensions. For tolerances on wall thickness, see appropriate material specification.

Thicknesses shown in bold-face type in Schedules 30 and 40 are identical with thicknesses for *standard weight* pipe in former lists; those in Schedules 60 and 80 are identical with thicknesses for *extra strong* pipe in former lists.

The schedule numbers indicate approximate values of the expression 1000 $\times$ P/S.

* Owing to a necessary departure from the old standard weight and extra strong thicknesses in the 12 in. size, Schedules 40 and 60, the new thicknesses are not as yet stocked by all manufacturers and jobbers. Hence, where agreeable for the purchaser and suitable for the service conditions, the old standard weight 0.382 in. wall pipe corresponding to a 1000 P/S value of 38.7 is still available and can be substituted for the 0.414 in. wall, and the old extra strong 0.510 in. wall pipe corresponding to a 1000 P/S value of 56.3 can be substituted for the 0.574 in. wall.

Table 5. Nominal Weights of Welded Wrought-iron Pipe *
(ASA B36.10–1939)

Nominal Pipe Size, in.	Schedule 10	Schedule 20	Schedule 30		Schedule 40		Schedule 60	Schedule 80
	Plain Ends	Plain Ends	Plain Ends	Threads and Couplings	Plain Ends	Threads and Couplings	Plain Ends	Plain Ends
1/8	...	...	...	...	**0.25**	**0.25**	...	**0.32**
1/4	...	...	...	...	**0.43**	**0.43**	...	**0.54**
3/8	...	...	...	...	**0.57**	**0.57**	...	**0.74**
1/2	...	...	...	...	**0.86**	**0.86**	...	**1.09**
3/4	...	...	...	...	**1.14**	**1.14**	...	**1.48**
1	...	...	...	...	**1.68**	**1.69**	...	**2.18**
1 1/4	...	...	...	...	**2.28**	**2.29**	...	**3.00**
1 1/2	...	...	...	...	**2.72**	**2.74**	...	**3.64**
2	...	...	...	...	**3.66**	**3.68**	...	**5.03**
2 1/2	...	...	...	...	**5.80**	**5.82**	...	**7.67**
3	...	...	...	...	**7.58**	**7.62**	...	**10.3**
3 1/2	...	...	...	...	**9.11**	**9.21**	...	**12.5**
4	...	...	...	...	**10.8**	**10.9**	...	**15.0**
5	...	...	...	...	**14.7**	**14.9**	...	**20.8**
6	...	...	...	...	**19.0**	**19.2**	...	**28.6**
8	...	...	**24.7**	**25.0**	**28.6**	**28.8**	...	**43.4**
10	...	...	**34.3**	**35.0**	**40.5**	**41.2**	**54.8**	64.4
12	...	...	**43.8**	**45.0**	53.6	55.0	73.2	88.6
14 OD	36.0	44.8	53.6	..	62.2	...	87.6	104
16 OD	41.3	51 4	61.4	..	81.2	...	111	...
18 OD	46.5	57.9	80.5	..	103	...	136	...
20 OD	...	77.0	103	..	115	...	.	...

* Weights are in pounds per linear foot and are for pipe with plain ends except for sizes which are commercially available with threads and couplings for which both weights are listed.

Weights shown in bold-face type in Schedules 30 and 40 are identical with weights for *standard weight* pipe in former lists; those in Schedules 60 and 80 are identical with weights for *extra strong* pipe in former lists.

The schedule numbers indicate approximate values of the expression $1000 \times P/S$.

SEAMLESS STEEL TUBES are both hot and cold drawn. Composition and physical properties of the various grades of steel used are given in Table 6. Weights given in Table 7 are based on 1 cu in. of steel = 0.2833 lb. This table was compiled from data supplied by the National Tube Co., Pittsburgh.

Table 6. Physical and Chemical Properties of Steels Used for Seamless Tubing
(Based on information from National Tube Co., Pittsburgh)

Grade or Designation	Percentage of C	Mn	P (max)	S (max)	Ni	Cr	Mo	Other	Condition	Yield Point, psi	Ultimate Strength, psi	Elongation in 2 in., %	Equivalent Rockwell	Equivalent Brinell
SAE 1015	.10–.20	.30–.60	.045	.055	...	...	...	...	Annealed *	30,000	48,000	40	...	...
									Hard Drawn	67,000	80,000	15	B-84	159
SAE X-1020	.15–.25	.70–1.00	.045	.055	...	...	...	...	Annealed	35,000	55,000	40	B-64	107
									Hard Drawn	70,000	85,000	10	B-87	170
SAE 1025	.20–.30	.30–.60	.045	.055	...	...	...	...	Annealed	35,000	55,000	35	B-64	107
									Hard Drawn	72,000	85,000	10	B-87	170
SAE 1035	.30–.40	.60–.90	.045	.055	...	...	...	...	Annealed	38,000	63,000	35	B-71	124
									Hard Drawn	80,000	90,000	10	B-89	179
SAE 1040	.35–.45	.60–.90	.045	.055	...	...	...	...	Annealed	40,000	70,000	30	B-77	137
									Hard Drawn	85,000	95,000	8	B-92	192
SAE 1045	.40–.50	.60–.90	.045	.055	...	...	...	...	Annealed	45,000	73,000	25	B-80	146
									Hard Drawn	90,000	100,000	7	B-95	207
SAE 1050	.45–.55	.60–.90	.045	.055	...	...	...	...	Annealed	60,000	78,000	20	B-83	156
									Hard Drawn	95,000	105,000	7	B-96	217
SAE 1115	.10–.20	.70–1.00	.045	.10–.20	...	...	...	...	Annealed	30,000	48,000	40	...	...
									Hard Drawn	60,000	75,000	10	B-81	149
SAE X-1315	.10–.20	1.30–1.60	.045	.10–.20	...	...	...	...	Annealed	40,000	58,000	35	B-66	112
									Hard Drawn	70,000	80,000	10	B-84	159
SAE 2315	.10–.20	.30–.60	.04	.05	3.25–3.75	...	...	...	Annealed	40,000	65,000	35	B-73	128
									Hard Drawn	85,000	95,000	10	B-92	192
SAE 2330	.25–.35	.50–.80	.04	.05	3.25–3.75	...	...	...	Annealed	55,000	75,000	30	B-81	149
									Hard Drawn	100,000	125,000	10	C-25	255

* Soft-annealed, represents softest possible condition.

(*Table continued on p. 6-30*)

Table 6. Physical and Chemical Properties of Steels Used for Seamless Tubing—*Continued*

(Based on information from National Tube Co., Pittsburgh)

Grade or Designation	Percentage of C	Mn	P (max)	S (max)	Ni	Cr	Mo	Other	Condition	Yield Point, psi	Ultimate Strength, psi	Elongation in 2 in., %	Equivalent Rockwell	Equivalent Brinell
SAE 2340	.35–.45	.60–.90	.04	.05	3.25–3.75	. .	. .	Si .15–.30	Annealed *	60,000	80,000	25	B-84	159
									Hard Drawn	110,000	130,000	10	C-26	262
SAE 3115	.10–.20	.30–.60	.04	.05	1.00–1.50	.45–.75			Hot Rolled	45,000	70,000	25	B-77	137
									Hard Drawn	90 000	100,000	15	B-94	202
SAE 3140	.35–.45	.60–.90	.04	.05	1.00–1.50	.45–.75	.	. .	Annealed	60,000	80,000	25	B-84	159
									Hard Drawn	110,000	130,000	10	C-28	269
SAE X-4130	.25–.30	.40–.60	.04	.05	.80–1.10	.	.15–.25	.	Annealed	50,000	75,000	30	B-81	149
									Hard Drawn	110,000	130,000	10	C-26	262
SAE 4140	.35–.45	.60–.90	.04	.05	.80–1.10	.	.15–.25		Annealed	50,000	80,000	25	B-84	159
									Hard Drawn	120,000	140,000	10	C-30	285
SAE 4615	.10–.20	.40–.70	.04	.05	1.65–2.00		.20–.30		Annealed	35,000	70,000	40	B-77	137
									Hard Drawn	95,000	105,000	20	B-96	212
5% Cr-Mo	.15 max	.50 max	.03	.03		4.0–6.0	.45–.65	Si .50 max	Annealed	25,000	60,000	30	B-87	163
									Hot Rolled	100,000	160,000	. .	C-35	331
USS 18-8	.07 max	.30–.70	.03	.03	8.0 min	17 min	.	Si .70 max	Annealed	35,000	80,000	50	B-80	146
									Hard Drawn	50–150,000	100–200,000	. .	B-100–C-35	241–331
USS 18-8Cb	.07 max	.30–.70	.03	.03	8.0 min	17.0 min		Si .40–.65	Annealed	35,000	80,000	50	B-80	146
								Cb .70	Hard Drawn	50–150,000	100–200,000	. . . . .	B-100–C-35	241–331
USS 18-8Ti	.07 max	.30–.70	.03	.03	8.0 min	17.0 min	. .	Si .75 max	Annealed	35,000	80,000	50	B-80	146
								Ti .35–.50	Hard Drawn	50–150,000	100–200,000	. . . .	B-100–C-35	241–331
USS Cor-Ten	.12 max	.10–.50	.07–.15	.05	.55 max	.50–1.25	.	Si .25–.75	Annealed	45,000	65,000	45	B-73	128
								Cu .30–.80	Hot Rolled	50,000	80,000	40	B-84	159

* Soft-annealed, represents softest possible condition

Table 7. Weight per Foot of Seamless Steel Tubing

(Condensed from table issued by National Tube Co., Pittsburgh)

Outside Diameter, In.	Thickness—B.W.G. and Decimals of an Inch								Thickness—Fractions and Decimals of an Inch												
	20	18	16	14	13	12	11	10	3/16	1/4	5/16	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
	0.035	0.049	0.065	0.083	0.095	0.109	0.120	0.134	0.188	0.250	0.313	0.375	0.500	0.625	0.750	0.875	1.00	1.125	1.250	1.375	1.500
1/2	0.17	0.24	0.30	0.37	0.41	0.46	0.49	0.52													
5/8	0.22	0.30	0.39	0.48	0.54	0.60	0.65	0.70													
3/4	0.27	0.37	0.48	0.59	0.66	0.75	0.81	0.88	1.13												
7/8	0.31	0.43	0.56	0.70	0.79	0.89	0.97	1.06	1.38												
1	0.36	0.50	0.65	0.81	0.92	1.04	1.13	1.24	1.63	2.00											
1 1/4	0.45	0.63	0.82	1.03	1.17	1.33	1.45	1.60	2.13	2.67	3.13	3.50									
1 1/2	0.55	0.76	1.00	1.26	1.43	1.62	1.77	1.95	2.63	3.34	3.96	4.51	5.34								
2		1.02	1.34	1.70	1.93	2.20	2.41	2.67	3.63	4.67	5.63	6.51	8.01	9.18							
2 1/2		1.28	1.69	2.14	2.44	2.78	3.05	3.39	4.63	6.01	7.30	8.51	10.68	12.52	14.02						
3				2.59	2.95	3.37	3.69	4.10	5.63	7.34	8.97	10.51	13.35	15.85	18.02	19.86	21.36				
3 1/2				3.03	3.45	3.95	4.33	4.82	6.63	8.68	10.64	12.52	16.02	19.19	22.03	24.53	26.70				
4					3.96	4.53	4.97	5.53	7.63	10.01	12.31	14.52	18.69	22.53	26.03	29.20	32.04				
4 1/2						5.11	5.61	6.25	8.64	11.35	13.98	16.52	21.36	25.87	30.04	33.88	37.38				
5						5.69	6.25	6.96	9.64	12.68	15.64	18.52	24.03	29.20	34.04	38.55	42.72				
5 1/2							6.90	7.68	10.64	14.02	17.31	20.53	26.70	32.54	38.05	43.22	48.06				
6								8.40	11.64	15.35	18.98	22.53	29.37	35.88	42.05	47.89	53.40				
6 1/2									12.64	16.69	20.65	24.53	32.04	39.22	46.06	52.57	58.74	64.58	70.09	75.26	80.10
7									13.64	18.02	22.32	26.53	34.71	42.55	50.06	57.24	64.08	70.59	76.76	82.60	88.11
7 1/2										19.36	23.99	28.54	37.38	45.89	54.07	61.91	69.42	76.60	83.44	89.95	96.12
8										20.69	25.66	30.54	40.05	49.23	58.07	66.58	74.76	82.60	90.11	97.29	104.13
8 1/2										22.03	27.33	32.54	42.72	52.57	62.08	71.26	80.10	88.61	96.79	104.63	112.14
9										23.36	28.99	34.54	45.39	55.90	66.08	75.93	85.44	94.62	103.46	111.97	120.15
9 1/2										24.70	30.66	36.55	48.06	59.24	70.09	80.60	90.78	100.63	110.14	119.32	128.16
10										26.03	32.33	38.55	50.73	62.58	74.09	85.27	96.12	106.63	116.81	126.66	136.17
10 1/2										27.37	34.00	40.55	53.40	65.92	78.10	89.95	101.46	112.64	123.49	134.00	144.18
11										28.70	35.67	42.55	56.07	69.25	82.10	94.62	106.80	118.65	130.16	141.35	152.19
11 1/2										30.04	37.34	44.56	58.74	72.59	86.11	99.29	112.14	124.66	136.84	148.69	160.20
12										31.37	39.01	46.56	61.41	75.93	90.11	103.96	117.48	130.67	143.51	156.03	168.21
13										34.04	42.35	50.56	66.75	82.60	98.12	113.31	128.16	142.68	156.86	170.72	184.23
14										36.71	45.68	54.57	72.09	89.28	106.13	122.65	138.84	154.70	170.22	185.40	200.25
15										39.38	49.02	58.57	77.43	95.95	114.14	132.00	149.52	166.71	183.57	200.09	216.27
16										42.05	52.36	62.58	82.77	102.63	122.15	141.35	160.20	178.73	196.92	214.77	232.29
17										44.72	55.70	66.58	88.11	109.30	130.16	150.69	170.88	190.74	210.27	229.46	248.31
18										47.39	59.03	70.59	93.45	115.98	138.17	160.04	181.56	202.76	223.62	244.14	264.33
19										50.06	62.37	74.59	98.79	122.65	146.18	169.38	192.24	214.77	236.97	258.83	280.35
20										52.73	65.71	78.60	104.13	129.33	154.19	178.73	202.92	226.79	250.32	273.51	296.37

Table 8. Dimensions and Weights in Pounds per Foot of Copper, Brass, and Everdur Pipe

(American Brass Co., Waterbury, Conn.)

Standard Pipe Size, in.	Outside Diameter, in.	Inside Diameter, in.	Wall Thickness, in.	67 Brass Density 0.307	85 Red Brass Everdur-1015 Density 0.316	Copper Density 0.323	Everdur-1010 Density 0.308
Regular							
1/8	0.405	0.281	.0620	0.2461	0.2533	0.2590	0.2469
1/4	0.540	0.375	.0825	0.4368	0.4496	0.4596	0.4383
3/8	0.675	0.494	.0905	0.6122	0.6302	0.6441	0.6142
1/2	0.840	0.625	.1075	0.9114	0.9381	0.9588	0.9143
3/4	1.050	0.822	.1140	1.235	1.271	1.299	1.239
1	1.315	1.062	.1265	1.740	1.791	1.831	1.746
1 1/4	1.660	1.368	.1460	2.558	2.633	2.692	2.567
1 1/2	1.900	1.600	.1500	3.038	3.127	3.196	3.048
2	2.375	2.062	.1565	4.018	4.136	4.228	4.031
2 1/2	2.875	2.500	.1875	5.832	6.003	6.136	5.851
3	3.500	3.062	.2190	8.316	8.560	8.750	8.343
3 1/2	4.000	3.500	.2500	10.85	11.17	11.42	10.89
4	4.500	4.000	.2500	12.30	12.66	12.94	12.34
5	5.563	5.062	.2505	15.40	15.85	16.20	15.45
6	6.625	6.125	.2500	18.45	18.99	19.41	18.51
8	8.625	8.000	.3125	30.06	30.95	31.63	30.16
10	10.750	10.019	.3655	43.93	45.22	46.22	44.07
11	11.750	11.000	.3750	49.37	50.82	51.94	49.53
12	12.750	12.000	.3750	53.71	55.28	56.51	53.88
Extra Strong							
1/8	0.405	0.205	.100	0.3530	0.3633	0.3714	0.3541
1/4	0.540	0.294	.123	0.5936	0.6110	0.6246	0.5956
3/8	0.675	0.421	.127	0.8055	0.8291	0.8475	0.8081
1/2	0.840	0.542	.149	1.192	1.227	1.254	1.195
3/4	1.050	0.736	.157	1.623	1.670	1.707	1.628
1	1.315	0.951	.182	2.387	2.457	2.511	2.394
1 1/4	1.660	1.272	.194	3.292	3.388	3.463	3.302
1 1/2	1.900	1.494	.203	3.987	4.104	4.195	4.000
2	2.375	1.933	.221	5.509	5.671	5.797	5.527
2 1/2	2.875	2.315	.280	8.409	8.656	8.848	8.437
3	3.500	2.892	.304	11.24	11.57	11.83	11.28
3 1/2	4.000	3.358	.321	13.67	14.07	14.38	13.71
4	4.500	3.818	.341	16.41	16.90	17.27	16.47
5	5.563	4.813	.375	22.52	23.18	23.69	22.59
6	6.625	5.751	.437	31.30	32.21	32.93	31.40
8	8.625	7.625	.500	47.02	48.40	49.47	47.17
10	10.750	9.750	.500	59.31	61.05	62.41	59.51
Double Extra Strong							
1/2	0.840	0.252	.294	1.858	1.912	1.955	
3/4	1.050	0.434	.308	2.645	2.723	2.783	
1	1.315	0.599	.358	3.965	4.081	4.172	
1 1/4	1.660	0.896	.382	5.650	5.816	5.945	
1 1/2	1.900	1.100	.400	6.944	7.148	7.306	
2	2.375	1.503	.436	9.784	10.07	10.29	
2 1/2	2.875	1.771	.552	14.84	15.28	15.61	
3	3.500	2.300	.600	20.14	20.73	21.19	
3 1/2	4.000	2.728	.636	24.76	25.49	26.05	
4	4.500	3.152	.674	29.85	30.72	31.40	
5	5.563	4.063	.750	41.78	43.00	43.96	
6	6.625	4.897	.864	57.61	59.30	60.61	
8	8.625	6.875	.875	78.48	80.78	82.57	

Variations from these weights must be expected in practice.

COPPER, BRASS, AND EVERDUR PIPE are furnished in regular, extra strong, and double extra strong weights, as shown in Table 8.

Table 9. Length of Pipe Nipples in Inches

Nominal Pipe Size	Wrought Iron Close	Wrought Iron Short	Wrought Iron Long* Min	Wrought Iron Long* Max	Bronze Close	Bronze Long* Min	Bronze Long* Max
1/8	3/4	1 1/2	2	3 1/2	3/4	1 1/2	6
1/4	7/8	1 1/2	2	3 1/2	7/8	1 1/2	6
3/8	1	1 1/2	2	3 1/2	1	1 1/2	6
1/2	1 1/8	1 1/2	2	3 1/2	1 1/8	1 1/2	6
3/4	1 3/8	2	2 1/2	4	1 3/8	2	6
1	1 1/2	2	2 1/2	4	1 1/2	2	6
1 1/4	1 5/8	2 1/2	3	4 1/2	1 5/8	2 1/2	6
1 1/2	1 3/4	2 1/2	3	4 1/2	1 3/4	2 1/2	6
2	2	2 1/2	3	4 1/2	2	3	6
2 1/2	2 1/2	3	3 1/2	5	2 1/2	3	6
3	2 5/8	3	3 1/2	5	2 5/8	3	6
3 1/2	2 3/4	4	4 1/2	6	2 3/4	4	6
4	2 7/8	4	4 1/2	6	2 7/8	4	6
5	3	4 1/2	4 1/2	6	3	4 1/2	6
6	3 1/8	4 1/2	4 1/2	6	3 1/8	4 1/2	6
8	3 1/2	5	6	8	....	......	.....
10	3 7/8	5	6	8	....	......	.....
12	4 1/2	6		8		...	.....

* Lengths advance by ½ in. up to and including 6 in.; 8-in. and 10-in. sizes advance by 1 in.

STANDARD PIPE THREADS (ASA B2.1–1945). The dimensions of standard pipe threads are given in Fig. 1 and Table 10. The formulas for these dimensions are:

Pitch Diameters. $E_0 = D - (0.05D - 1.1)\frac{1}{n}$; $\qquad E_0 + 0.0625L_1 \qquad (1)$

Length of Thread. $L_2 \quad (0.80D + 6.8)\frac{1}{n} \qquad (2)$

where E_0 = pitch diameter at end of pipe; E_1 = pitch diameter at gaging notch; D = outside diameter of pipe; L_2 = length of effective thread; L_1 = normal engagement by hand between male and female threads; $1/n$ = pitch of thread = distance axis will advance in one revolution, where n is expressed in threads per inch. All dimensions are in inches.

Taper of thread, 1 in 16, measured on the diameter.

Manufacturing Tolerance. The maximum allowable variation in the commercial product is one turn plus or minus from the gaging notch when working gages are used.

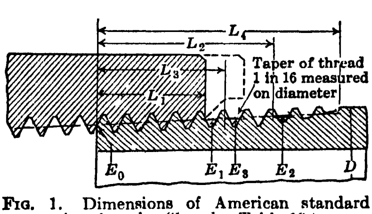


Fig. 1. Dimensions of American standard pipe thread. (See also Table 10.)

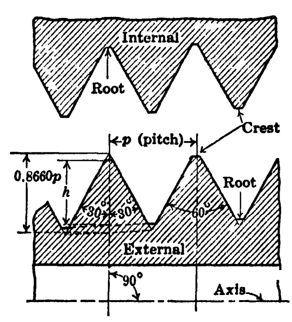


Fig. 2. Form of standard pipe threads.

Owing to an allowance of $1/2$ turn on the working gages, this is a maximum allowance of $1\ 1/2$ turns from the basic dimensions.

Form of Thread. The angle between the sides of the thread is 60 degrees when measured in an axial plane, and the line bisecting this angle is perpendicular to the axis. The depth of the sharp V thread, H, is

$$H = 0.8660p$$

The basic maximum depth of the truncated thread, h (see Fig. 2), is

$$h = 0.800p$$

The crest and root of pipe threads are truncated a minimum of $0.033p$.

Length of thread, as given by eq. 2, is the effective length, and includes two threads that are imperfect on the crest.

Drill sizes for tapped holes are given in Table 11.

Table 10. Dimensions of American Standard Taper Pipe Threads

(ASA B2.1-1945)

Nominal Size, in.	Outside Diameter, in.	Threads per in.	Pitch Diameter at Beginning of External Thread	Hand-tight Engagement		Effective Thread, External		Overall Length, External Thread, in.	Nominal Perfect External Threads		Depth of Thread, in. (Fig. 2)
				Length, in.	Diameter, in.	Length, in.	Diameter, in.		Length, in.	Diameter, in.	
	D	n	E_0	L_1	E_1	L_2	E_2	L_4	L_3	E_3	h
1/16	0.3125	27	0.27118	0.160	0.28118	0.2611	0.28750	0.3896	0.1870	0.28287	.02963
1/8	0.405	27	0.36351	0.180	0.37476	0.2639	0.38000	0.3924	0.1898	0.37537	.02963
1/4	0.540	18	0.47739	0.200	0.48989	0.4018	0.50250	0.5946	0.2907	0.49556	.04444
3/8	0.675	18	0.61201	0.240	0.62701	0.4078	0.63750	0.6006	0.2967	0.63056	.04444
1/2	0.840	14	0.75843	0.320	0.77843	0.5337	0.79179	0.7815	0.3909	0.78286	.05714
3/4	1.050	14	0.96768	0.339	0.98887	0.5457	1.00179	0.7935	0.4029	0.99286	.05714
1	1.315	11 1/2	1.21363	0.400	1.23863	0.6828	1.25630	0.9845	0.5088	1.24543	.06957
1 1/4	1.660	11 1/2	1.55713	0.420	1.58338	0.7068	1.60130	1.0085	0.5329	1.59043	.06957
1 1/2	1.900	11 1/2	1.79609	0.420	1.82234	0.7235	1.84130	1.0252	0.5496	1.83043	.06957
2	2.375	11 1/2	2.26902	0.436	2.29627	0.7565	2.31630	1.0582	0.5826	2.30543	.06957
2 1/2	2.875	8	2.71953	0.682	2.76216	1.1375	2.79062	1.5712	0.8875	2.77500	.100000
3	3.500	8	3.34062	0.766	3.38850	1.2000	3.41562	1.6337	0.9500	3.40000	.100000
3 1/2	4.000	8	3.83750	0.821	3.88881	1.2500	3.91562	1.6837	1.0000	3.90000	.100000
4	4.500	8	4.33438	0.844	4.38712	1.3000	4.41562	1.7337	1.0500	4.40000	.100000
5	5.563	8	5.39073	0.937	5.44929	1.4063	5.47862	1.8400	1.1563	5.46300	.100000
6	6.625	8	6.44609	0.958	6.50597	1.5125	6.54062	1.9462	1.2625	6.52500	.100000
8	8.625	8	8.43359	1.063	8.50003	1.7125	8.54062	2.1462	1.4625	8.52500	.100000
10	10.750	8	10.54531	1.210	10.62094	1.9250	10.66562	2.3587	1.6750	10.65000	.100000
12	12.750	8	12.53281	1.360	12.61781	2.1250	12.66562	2.5587	1.8750	12.65000	.100000
14 OD	14.000	8	13.77500	1.562	13.87262	2.2500	13.91562	2.6837	2.0000	13.90000	.100000
16 OD	16.000	8	15.76250	1.812	15.87575	2.4500	15.91562	2.8837	2.2000	15.90000	.100000
18 OD	18.000	8	17.75000	2.000	17.87500	2.6500	17.91562	3.0837	2.4000	17.90000	.100000
20 OD	20.000	8	19.73750	2.125	19.87031	2.8500	19.91562	3.2837	2.6000	19.90000	.100000
24 OD	24.000	8	23.71250	2.375	23.86094	3.2500	23.91562	3.6837	3.0000	23.90000	.100000

Wrench make-up: for internal thread pipes 1/16 to 2 in., 3 threads; for 2 1/2 in. to 24 OD, 2 threads. Number of vanished threads, 3.47 for all sizes of pipe. Increase in diameter per thread, all sizes, equals $0.0625/n$.

Table 11. Diameters of Twist Drills for Tapped Holes for Pipe Threads

(ASA B2.1-1945. All dimensions in inches.)

Nominal Pipe Size	Taper Thread: With Use of Reamer		Taper Thread: Without Use of Reamer		Straight Pipe Thread		Nominal Pipe Size	Taper Thread: With Use of Reamer		Taper Thread: Without Use of Reamer		Straight Pipe Thread	
1/16	.....	.240 *	.....	.246 *	1/4	.250 *	1	1 1/8	1.125	1 9/64	1.141	1 5/32	1.156
1/8	21/64	.328 *	...	.332 *	11/32	.344 *	1 1/4	1 15/32	1.469	1 31/64	1.484	1 1/2	1.500
1/4	27/64	.422	7/16	.438 *	7/16	.438 *	1 1/2	1 23/32	1.719	1 47/64	1.734	1 3/4	1.750
3/8	9/16	.562	9/16	.562	37/64	.578	2	2 3/16	2.188	2 13/64	2.203	2 7/32	2.219
1/2	11/16	.688	45/64	.703	23/32	.719	2 1/2	2 19/32	2.594	2 5/8	2.625	2 21/32	2.656
3/4	57/64	.891	29/32	.906	59/64	.922							

* American Standard twist drill sizes.

VALVE AND FITTING DATA

Editorial Note. Because of rapidly changing data on details of valves and fittings it is considered inadvisable to supply exhaustive data in this handbook. To replace these data it is recommended that the user obtain the applicable standard, as listed on p. 6-05, as well as current catalogs from manufacturers. Since such data are always kept up-to-date, the user may avoid costly design errors.

FLOW OF FLUIDS IN PIPES

14. PRESSURE LOSS IN TUBING, PIPE, AND FITTINGS

By R. J. S. Pigott

Roughness Effect. Since the work of Osborne Reynolds in 1884, relatively little rational use of the Reynolds' number $dv\rho/\mu$ was made as this criterion alone did not cover roughness effects, known to affect friction loss. In 1931 and 1932 an analysis of all the published tests on water, oil, air, and steam was made by Dr. Emory Kemler (Ref. 1). This study was the basis for a hypothesis that roughness had the same effect as reduction of diameter. This assumption was reduced to a series of curves of relative roughness showing friction factor against Reynolds' number (Ref. 2).

In the fifteen years since this solution for roughness effect was published, the curves of Ref. 2 have been widely adopted, and there is now ample proof of their reliability. W. B. Heltzel and others carried out very large-scale tests on oils in commercial pipe lines, 6 to 12 in. in diameter, and O. C. Bridgeman, Bureau of Standards, carried out for the Coordinating Research Council extensive tests on gasoline in drawn aluminum tubing; these extreme ranges have given excellent correlation with the curves.

In 1944 Professor Lewis F. Moody made a further improvement (Ref. 3). Using the Colebrook equations, he established smooth curves of f versus Rn for each relative roughness, and also confirmed the important fact that at some Reynolds' number for each relative roughness the friction factor becomes constant and does not decrease with further increase in Reynolds' number.

Symbols used herein are

d = pipe diameter, ft

v = average velocity, ft per sec $\left(\dfrac{Q}{\frac{\pi}{4}d^2}\right)$

Q = volume flow, cu ft per sec

ρ = density of fluid, lb per cu ft

μ = absolute viscosity, lb per ft-sec units (= 0.000672 × centipoises)

f = friction factor, a dimensionless number (see footnote p. 6-37).

l = length of pipe, ft

L_e = equivalent length of pipe for ζp_v

M = mean hydraulic radius $\left(\dfrac{\text{Area}}{\text{Wetted perimeter}}\right) = \left(\dfrac{d}{4}\text{ for round pipe}\right)$

ϵ = absolute roughness, ft
ϵ/d = relative roughness, a number
ζ = bend loss factor, a number; does not include pipe friction; fraction of velocity head
p = pressure, psi
p_v = velocity head, psi
Δp = pressure loss, psi, for length l
Rn = Reynolds' number $\frac{dv\rho}{\mu}$ or $\frac{4Mv\rho}{\mu}$
r = radius of bend, ft

PRESSURE LOSS. Viscous Flow. The formulas for pipe flow given below apply both to liquids and to gases, provided that for compressible fluids the pressure drop is not in excess of 10% of the initial pressure.

Viscous flow, $Rn < 1200$:

$$\text{Round pipes, } \Delta p = \frac{0.00691\mu l v}{d^2} \tag{1}$$

$$\text{Other shapes, } \Delta p = \frac{0.000432\mu l v}{M^2} \tag{2}$$
(Approximately circular)

Viscous flow will usually be found with heavy crude in pipe lines, in lubricating oil lines from tank to aviation engine, in internal lubrication conduits in practically all internal-combustion engines (drilled crankshafts, oil galleries, connecting rods), and in all bearings.

Turbulent Flow. Turbulent flow, $Rn > 3000$; safest also for $Rn = 1200$ to 3000:

$$\text{Round pipes, } \Delta p = \frac{0.000108 f l \rho v^2}{d} \tag{3}$$

$$\text{Other shapes, } \Delta p = \frac{0.000027 f l \rho v^2}{M} \tag{4}$$

Turbulent flow is usually found with water, air, and steam flowing in pipes.

Friction Factor. Figure 1 gives curves (for varying relative roughness) of f versus Rn. Relative roughness is given in Table 1. Values given are for clean pipe; rusted or carbunculated pipe may have several times the roughness of clean pipe. Under brass pipe and tubing is included all drawn material, such as lead, block tin, aluminum, copper, steel, and glass. It should be remembered that metal tubing 1/4 in. OD or less is often finished "no plug" draw, in which case it will not be smooth inside, but wrinkled, and must be considered as rough as cast iron.

Vapor-liquid Mixtures. Where vapor or gas is present with liquid, the presence of gas or vapor makes no difference in viscous flow; pressure drop is calculated from the liquid flow alone (eqs. 1 and 2), as if no gas or vapor were present.

In turbulent flow, eqs. 3 and 4 apply, but multiplied by the factor $\left(1 + \frac{G}{L}\right)$, where G is the vapor or gas volume and L the liquid volume in the mixture (Ref. 4). This condition occurs where vapor is released from the liquid in transit, such as gasoline, or water near the boiling point, or where the liquid is aerated. These equations can also be used for non-Newtonian materials, such as mud, cement slurries, greases, or any liquids having thixotropic properties. These are all materials having a shear strength when stationary (jelling), and an apparent viscosity that decreases with rate of shear. It is necessary to have the apparent viscosity at several flow rates in the viscous region and one in the turbulent region (Ref. 5). Such applications occur in oil-well drilling, in pumping mud down drill pipe and up the hole to carry out chips, in handling cement slurries, heavy sewage, mixed concrete delivery by pipe in tunnel construction, some paper pulp mixtures, and many less ordinary liquids, such as tomato ketchup and glue. In the viscous region the apparent viscosity varies with velocity and diameter; in the turbulent region, the viscosity becomes constant; the material behaves as a true liquid.

RESISTANCE OF ELBOWS, TEES, VALVES, AND PIPE BENDS has not up to now been rationalized to any extent. In the first place, there are few reliable test data; in the second, the loss must be measured as a relatively small difference between two much larger quantities. Under these circumstances, it is not surprising to find that test data on the loss in an elbow or bend vary 100 to 300%. However, the careful work of Freeman (Ref. 6) and Hofmann (Ref. 7) gives support to an empirical formulation by the writer.

Analysis of tests of several investigators leads to the hypothesis that there is a true bend loss ζ_1, which is a function only of r/d, and an additional loss, ζ_2, which is a function of ϵ/d, relative roughness and Reynolds' number Rn.

Table 1. Relative Roughness, ϵ/d

Diameter, in.	Brass and Tubing	Galv. Air Duct	Steel Pipe	Asphalted Cast Iron	Galv. Steel	Cast Iron	Concrete Smooth	Concrete Average	Riveted Light	Riveted Heavy; Brick
1/2	.000107	...	.0032	...	.0120	.0204	...	...	...	...
3/4	.000076	...	.0022	...	.0080	.0136	...	...	...	...
1	.000057	...	.0017	...	.0060	.0102	...	...	...	...
1 1/4	.000048	...	.0014	...	.0050	.0081	...	...	...	...
1 1/2	.000040	...	.0012	...	.0040	.0068	...	...	...	...
2	.000029	.00015	.00090	...	.0030	.0051	...	...	...	...
2 1/2	.000023	.00012	.00070	...	.0024	.0041	...	...	...	...
3	.000020	.00010	.00060	.00160	.0020	.0034	.0040	.016	.0120	...
4	.000015	.000075	.00045	.00120	.0015	.0026	.0030	.012	.0090	...
6	.000010	.000050	.00030	.00080	.0010	.0017	.0020	.0080	.0060	...
8	...	.000038	.00023	.00060	.00075	.0013	.0015	.0060	.0045	.045
10	...	...	.00018	.00048	.00060	.0010	.0012	.0050	.0036	...
12 *	.000005	.000025	.00015	.00040	.00050	.00085	.0010	.0040	.0030	.030
16	...	...	.00013	.00030	.00029	.00064	.00075	.0030	.0023	.023
20	...	...	.000090	.00024	...	.00051	.00060	.0024	.0018	.018
24	...	.000013	.000075	.00020	...	.00042	.00050	.0020	.0015	.015
36	...	...	.000050	.00013	...	.00028	.00033	.0013	.0010	.010
72	...	...	...	...	...	...	...	...	...	.0050
144	...	...	...	...	...	...	...	...	...	.0025

* The 12 in.-diameter values are also the absolute roughness, feet.

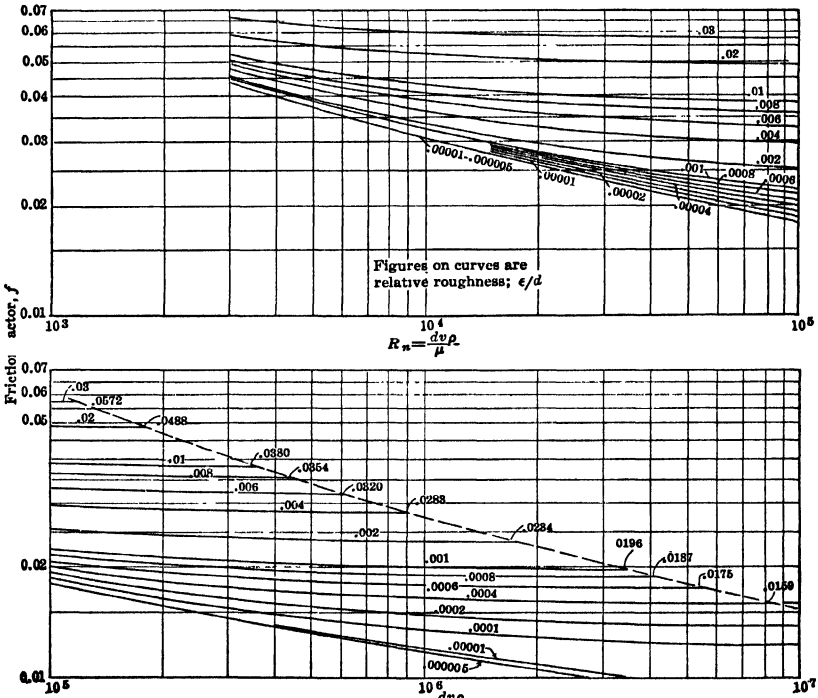


FIG. 1. Friction factor * as a function of Reynolds' number.

* *Editor's Note.* Friction factors have long caused engineers difficulty because of variation in usage among authors. The two most common friction factors differ in the ratio of four to one. Difference in nomenclature is inadvisable because the symbol f is used for both in the literature. In the laminar flow region they may be identified by the facts that

$$\text{Smaller } f = 16/Rn$$

$$\text{Larger } f = 64/Rn$$

Friction factors and formulas used in Arts. 14, 15, and 16 of this section consistently employ the *larger* value of f. Data in Fig. 1 are the larger values. Authors in Section 1 used formulas requiring the smaller value. This was called to the reader's attention by use of the combination $(4f)$ in formulas of Section 1. The value of this parenthetical product is read directly from Fig. 1.

The correlation fitting best is

$$\zeta = 0.106\left(\frac{r}{d}\right) + 2000f^2 \tag{5}$$

ζ is the fractional velocity head lost. Velocity head is

$$p_v = 0.000108\rho v^2, \text{ in pounds per square inch} \tag{6}$$

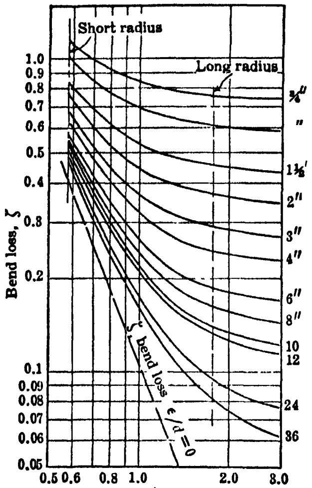


FIG. 2. Bend loss in cast iron or brass elbows (Rn = 83,300 per inch of diameter)

The bend loss in a fitting is ζp_v. As customarily employed ζ does *not* include the pipe friction of the fitting or bend; this is conveniently included with the pipe by using center-line distances for all runs.

Values of ζ for cast 90-degree elbows, both screwed and flanged, are given in Fig. 2. The average r/d for short radius ells is 0.58, and for long radius 1.78. Any other values can be taken from the curves. Figure 2 is also applicable to cast brass fittings, since the sand core determines the roughness and not the material in the elbow. It might be expected that ζ would be different for screwed and flanged elbows, but the enlargement of diameter used in screwed fittings cancels out the loss from enlargement and contraction of section at the inlet and outlet, so that values are the same for both types.

ζ values for drawn tubing and brass IPS pipe, together with commercial steel pipe, are shown in Fig. 3, upper. These curves can also be used for tube turns made of steel pipe, commonly used in welded lines. The lower part of Fig. 3 gives the ζ values for galvanized sheet steel duct used for ventilation and heating air; these are made of light gage and long sections, few joints, flat rivets, and have very low resistance. It has been found that turns do not cause as much loss with gases as with liquids, but no very clear quantitative data are available; for the present it is safer to use all values as for liquids.

For short-radius tees, ζ is 1.50 times that for 90-degree short-radius ells when the flow is entering the run and leaving the branch; 2.00 times, for flow entering the branch, leaving the run. For long-radius tees, flow either way, the value is 2.50 times that for 90-degree long-radius ells. Since only the inside radius of the tee can have a radius, the *outside* of the stream is still unguided, and the reduction in the loss is much less than for the corresponding increase in radius for an elbow. In small-diameter cast fittings (2 in. or less, not often used) the advantage of long-radius tees is negligible.

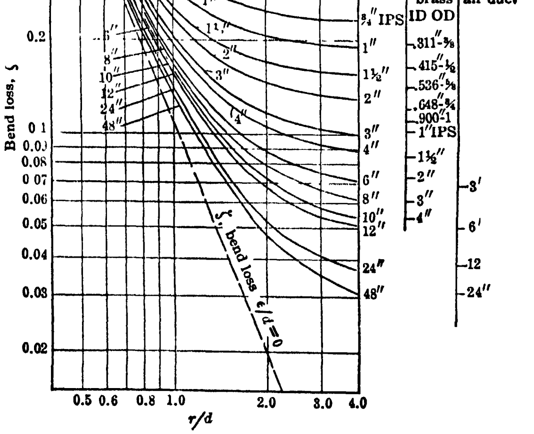


FIG. 3. Bend loss in drawn tubing, steel pipe, and galvanized air ducts (Rn = 83,300 per inch of diameter.)

Miter or sharp-angle turns occur in drilled or cored passages for oil in engines and in the drilled elbows and tees used for small tubing with compression or flared couplings, in automotive and aviation service. As a rule, these fittings do not extend beyond 1 1/4 in. OD for these services. The earlier fittings were drilled with the usual 60-degree point drill, and consequently there was an irregular hump in the elbow which rendered it somewhat worse than a miter bend. Later, round point drills were used that give a smooth bend, rounded at the corner, having a value of $r/d = 0.25$. This small rounding does not improve the loss over a miter bend, as no improvement shows before $r/d = 0.35$.

Calculated from eq. 5, using 1.00 for the first member, the values check tests by Kirschbach, Schubart, and Gibson very closely.

Die-cast elbows are nearly as smooth as drawn tubing; drilled elbows about the same as steel pipe (whether brass or steel, the drill does not leave a dead-smooth surface).

Values for the size range likely to be used are:

Size	ζ, Die Cast	ζ, Drilled
3/8 OD	1.19	1.43
1 OD	1.10	1.19
3/4 IPS	....	1.22
3 IPS	....	1.10
6 IPS	....	1.07

VALVES. There are few authoritative data on the resistance of valves, but values of ζ usually employed are

Gate valves	$0.02 \times \zeta$ or L_e for short-radius elbows
Globe valves	$11.0 \times \zeta$ or L_e for short-radius elbows
Angle valves	$4.0 \times \zeta$ or L_e for short-radius elbows

Differences in design, body space, diaphragms, etc., can materially change such values. For example, boiler nonreturn valves of two designs, one the ordinary flat disk and angular seat, gave a ζ value close to that given above; another design, with carefully faired disk and seat contours to reduce vena contracta and turn losses, had 40% of these values.

SCREWED COUPLINGS AND UNIONS. The resistance of these fittings is due chiefly to a small gap between the ends of the pipe, offering a sudden enlargement of section. The loss of this enlargement and subsequent contraction does not have the full value ordinarily assigned for 6 to 10 diameters length, because the stream leaving one pipe end has not diffused materially by the time the contraction occurs. The loss usually does not exceed $\zeta = 0.04$.

45-DEGREE ELBOWS AND RETURN BENDS. In smooth miter bends, ζ varies nearly as θ^2, where θ is the bend angle; but in actual fittings, the ζ_2 effect of roughness greatly affects the value. For cast-iron fittings, 45-degree elbows are 60% of the value for the same-radius 90-degree elbow; return bends, 140%. For steel pipe and drawn tubing, the loss appears to be nearly proportional to θ, the angle of bend.

45-DEGREE Y BRANCHES resemble the 45-degree miter elbow, except that with the flow entering the run and out of the branch, the first portion of the bend is unguided because of the run of the fitting going through. The loss is in general a little higher than a 45-degree miter of the same roughness. The value of the loss is 0.8 that of a short-radius 90-degree elbow of the same roughness.

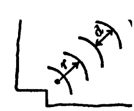


FIG. 4. Vane or blade turn.

VANE OR BLADE TURNS. For large ducts, space is often not available for long radius bends, and the vane or blade turn was devised and first used in wind tunnels. In 1913 and later the writer employed them in 16-in. water pipes and air ducts. About 1928 they were tried out in forced and induced draft ducts with excellent results. The usual construction uses one or two splitters to a much larger number of small vanes or blades (see Fig. 4).

The loss in these blade turns can be predicted by considering the passages between vanes as the controlling configuration, instead of the full duct. The loss will correspond to a 90-degree elbow of the same r/d and roughness.

LOSS IN TERMS OF EQUIVALENT PIPE. It is evident from eqs. 1 to 5 that calculation of ζ for various rates of flow, pipe size, and roughness is both tedious and cumbersome. The method of assigning equivalent lengths of pipe, long used, offers both a convenient and accurate solution. (See also p. 1-33.) The equivalent pipe length, as shown by eqs. 4 and 6, is

$$L_e = \frac{\zeta d}{f} \tag{7}$$

Since ζ is largely controlled by f, it is found that L_e is nearly constant for all values of Rn; hence calculation of pressure drop for any given pipe line becomes simple. Center line distances give the length of pipe plus fittings (correct this, if necessary, for the reduction of total length due to long radius bends and for the equivalent lengths for ζ of fittings).

Selection of f for any flow rate gives the total loss. Furthermore, for most ranges of flow in a single line of pipe, the lines in Fig. 1 are nearly straight; hence the slope can be used to calculate the pressure drop for any other flow by the relationship

$$\frac{\Delta p_2}{\Delta p_1} = \left(\frac{Q_2}{Q_1}\right)^{2-} \tag{8}$$

where f need be selected only for Q_1, p_1; n is the absolute value of the actual slope of the f line in the double log plot. For example, if $n = -0.2$, the exponent of the right-hand member of eq. 8 is 1.8.

Equivalent straight pipe for cast elbows is given in Table 2. Since cast-iron fittings are used with cast-iron and steel pipe, and cast-brass fittings are used with brass or copper pipe, the table gives equivalent lengths in terms of all three.

Table 2. Equivalent Feet of Straight Pipe

Cast 90-Degree Elbows

Nominal Size, in.	Cast-iron Pipe		Steel Pipe		Brass Pipe	
	$\frac{r}{d} = 0.58$	$\frac{r}{d} = 1.78$	$\frac{r}{d} = 0.58$	$\frac{r}{d} = 1.78$	$\frac{r}{d} = 0.58$	$\frac{r}{d} = 1.78$
3/4	1.8	1.2	2.9	1.9	3.9	2.5
1	2.2	1.4	3.5	2.1	4.7	2.7
1 1/4	2.7	1.5	3.6	2.3	4.3	3.1
1 1/2	3.2	1.7	4.9	2.6	6.2	3.3
2	4.2	2.0	6.2	2.9	7.8	3.7
2 1/2	5.1	2.2	7.6	3.3	9.4	4.1
3	6.1	2.5	8.8	3.5	11.0	4.4
4	8.1	2.9	11.5	4.2	14	5.1
6	12.4	3.8	17	5.3	20	6.3
8	17	4.7	23	6.5		
10	22	5.6	30	7.7		
12	27	6.4	37	8.9		
24	58	11.1				
36	93	16				

Equivalent lengths for bends in tubing, brass pipe, steel pipe, and sheet steel air ducts are shown in Table 3.

Table 3. Equivalent Feet of Straight Pipe

90-Degree Bends

OD	ID	Brass Tubing and Pipe; Sheet Steel Ducts				Steel Pipe			
		$\frac{r}{d} = 0.5$	$\frac{r}{d} = 1$	$\frac{r}{d} = 2$	$\frac{r}{d} = 3$	$\frac{r}{d} = 0.5$	$\frac{r}{d} = 1$	$\frac{r}{d} = 2$	$\frac{r}{d} = 3$
3/8	0.311	0.82	0.31	0.22	0.20				
1/2	0.415	1.1	0.39	0.27	0.24				
5/8	0.536	1.5	0.49	0.32	0.28				
3/4	0.648	1.9	0.59	0.36	0.32				
1	0.900	2.7	0.81	0.47	0.41				
3/4	IPS	2.3	0.72	0.43	0.38	2.0	0.81	0.59	0.55
1	IPS	3.2	0.93	0.53	0.45	2.8	1.0	0.71	0.65
1 1/4	IPS	4.1	1.2	0.62	0.52	3.6	1.2	0.83	0.76
1 1/2	IPS	5.1	1.4	0.71	0.59	4.3	1.4	0.94	0.86
2	IPS	6.9	1.8	0.88	0.71	6.0	1.9	1.2	1.0
2 1/2	IPS	9.1	2.2	1.1	0.84	7.6	2.3	1.3	1.2
3	IPS	10.9	2.7	1.2	1.0	9.3	2.7	1.5	1.3
4	IPS	15	3.6	1.6	1.2	13	3.5	1.9	1.6
6	IPS	24	5.4	2.2	1.9	20	5.2	2.6	2.1
8	IPS					29	7.1	3.3	2.6
10	IPS					37	8.9	3.9	3.0
12	IPS	146	11	4.2	2.9	46	11	4.6	3.4
16	IPS					64	15	5.7	4.2
20	IPS					82	18	6.9	5.0
24	IPS	117	24	8.0	4.8	102	22	8.2	5.7

LARGE PRESSURE DROP WITH GASES. Where pressure drop, Δp, is a considerable fraction of initial pressure, as in transmitting natural gas over long lines, the simple hydraulic formulas are no longer accurate enough.

For turbulent flow,

$$p_1 = \sqrt{p_2^2 + 2p_2\left(\frac{0.000108 f l \rho_2 v_2^2}{d}\right)} \tag{9}$$

$$p_2 = \sqrt{p_1^2 - 2p_1\left(\frac{0.000108 f l \rho_1 v_1^2}{d}\right)} \tag{10}$$

These formulas, based on initial pressure p_1 and final pressure p_2 for length l, do not lend themselves readily to solution for pressure drop Δp. It is simplest, therefore, to compute flow for two values of $p_2 - p_1$, and plot a straight line on double-log paper. This gives Δp for all flows quite as accurately as the roughness and friction factor are determinable. (See also p. 1-23.)

15. VISCOSITY

By J. M. Cunningham

VISCOSITY is defined as the tangential force on a unit area of either of two planes of indefinite extent, at unit distance apart, one moving relative to the other at unit velocity, the space between filled with the viscous fluid. It is that property by which a fluid resists any deforming force. The dimensions of viscosity are given by the ratio of *shear intensity* to *velocity gradient* as stated by Sir Isaac Newton. The numerical value of this ratio is called the *coefficient of absolute* (*or dynamic*) *viscosity*, represented by the symbol μ.

In the absolute system of units, M, L, and T (mass, length, and time), the dimensions of the coefficient of absolute viscosity are

$$\mu = \frac{M/LT^2}{(L/T)(1/L)} = \frac{M}{LT}$$

The cgs metric unit of coefficient of absolute viscosity in the absolute system of units is named a poise, after Poiseuille.

$$1 \text{ poise} = \frac{1 \text{ gram (mass)}}{1 \text{ cm} \times 1 \text{ sec}}$$

which is numerically equal to

$$\frac{1 \text{ dyne} \times 1 \text{ sec}}{1 \text{ cm}^2}$$

Because the poise is too large a unit for ordinary use, the centipoise (0.01 poise) is generally used. It so happens that the centipoise is almost exactly the viscosity of water at 20 C (actually 1.005 centipoises).

The English ft lb sec (abs) unit of coefficient of absolute viscosity in the absolute system of units has no name assigned. This unit is

$$\mu = \frac{1 \text{ lb (mass)}}{1 \text{ ft} \times 1 \text{ sec}}$$

which is numerically equal to $\dfrac{1 \text{ poundal} \times 1 \text{ sec}}{1 \text{ ft}^2}$.

In the gravitational system of units, F, L, and T (force, length, and time), the dimensions of the coefficient of absolute viscosity are:

$$\mu = \frac{F/L^2}{(L/T)(1/L)} = \frac{FT}{L^2}$$

In the English in. lb (force) sec gravitational system of units, the coefficient of absolute viscosity frequently is called a Reyn, after Osborne Reynolds.

$$1 \text{ Reyn} = \frac{1 \text{ lb (force)} \times 1 \text{ sec}}{1 \text{ in.}^2}$$

The metric cm gram (force) sec unit of the coefficient of absolute viscosity in the gravitational system of units has no name assigned. This unit is

$$\frac{1 \text{ gram} \times 1 \text{ sec}}{1 \text{ cm}^2} = 980.7 \text{ poises}$$

The conversion for coefficient of absolute viscosity from centipoises to a few values of other units is

$$\text{Centipoise} \times 6.72 \times 10^{-4} = \frac{\text{lb (mass)}}{\text{ft sec}} = \frac{\text{Poundal-sec}}{\text{ft}^2}$$

$$\text{Centipoise} \times 5.6 \times 10^{-5} \quad \frac{\text{lb (mass)}}{\text{in. sec}}$$

$$\text{Centipoise} \times 2.09 \times 10^{-5} \quad \frac{\text{lb sec}}{\text{ft}^2}$$

$$\text{Centipoise} \times 1.45 \times 10^{-7} = \frac{\text{lb sec}}{\text{in.}^2}$$

$$\text{Centipoise} \times 2.42 \times 10^{-9} = \text{lb min}$$

$$\text{Centipoise} \times 1.02 \times 10^{-4} = \frac{\text{kg sec}}{\text{m}^2}$$

VISCOSIMETRY. A number of viscosimeters are in use, such as the McMichael, the Kingsbury taper plug, and others of the rotational type which measure absolute viscosity directly; the instrument most commonly in use is the Saybolt Universal, which measures the time in seconds for a definite quantity of liquid to flow through a short tube of small diameter. The time in seconds for the definite quantity of liquid to flow in the short tube is known as the *Saybolt Universal Viscosity* (SUV) of the fluid, also called *Seconds Saybolt Universal* (SSU).

The Saybolt Universal viscosimeter is used for fluids that have an efflux time greater than 32 sec. Although there is no maximum limit of flow time, for very viscous fluids, as heavy oils above 250 SUV, the viscosity is more conveniently measured by the Furol viscosimeter, similar to the Saybolt Universal, except that it has a larger diameter tube resulting in an efflux time approximately one-tenth that of the Saybolt Universal viscosimeter. The time in seconds for the efflux of fluid in the Furol viscosimeter is known as the *Saybolt Furol Viscosity* (SFV).

This type of instrument indirectly measures the ratio of the coefficient of absolute viscosity to the fluid density and is limited to a Newtonian fluid. This ratio is named the *kinematic viscosity*, represented by the symbol ν. The dimensions of kinematic viscosity are given by the ratios of coefficient of absolute viscosity to density. In the absolute system of units, the dimensions are

$$\nu = \frac{\mu}{\rho} = \frac{M/LT}{M/L^3} = \frac{L^2}{T}$$

The dimensions of kinematic viscosity in the gravitational system of units are

$$\nu = \frac{\mu}{\rho} = \frac{\mu}{\delta/g} = \frac{FT/L^2}{(F/L^3)(T^2/L)} = \frac{L^2}{T}$$

Thus kinematic viscosity has the dimensions of area divided by time.

The cgs metric unit of kinematic viscosity is named the *stoke* after Sir George Stokes.

$$1.0 \text{ stoke} = \frac{1 \text{ cm}^2}{1 \text{ sec}}$$

and also

$$\nu \text{ (stokes)} = \frac{\mu \text{ (poises)}}{\text{specific gravity}}$$

Inasmuch as the poise is too large a unit for ordinary use, the stoke is also too large a unit for ordinary use and the *centistoke* (0.01 stoke) is generally used.

No name has been assigned to any other unit of kinematic viscosity, either English or metric units.

The theoretical relation between centistokes and the Saybolt Universal Viscosity is $\nu_{\text{centistokes}} = 0.220t_{\text{S.U.V.}}$. This formula must be corrected for entrance losses and other secondary effects which makes it necessary to refer to a standard conversion table or chart such as ASTM D446, Standard Method for Conversion of Kinematic Viscosity to Saybolt Universal Viscosity.

Conversion may be obtained with sufficient precision for solution of most engineering problems by:

$$\text{Centistokes} = 0.220t - \frac{180}{t} \qquad \text{For } t < 50$$

$$\text{Centistokes} = 0.220t - \frac{195}{t} \qquad \text{For } 50 < t < 100$$

$$\text{Centistokes} = 0.220t - \frac{135}{t} \qquad \text{For } t > 100$$

where t = Saybolt seconds.

The modified Ostwald and modified Ubbelohde viscosimeters are short glass capillary types which measure the time for a fluid meniscus to fall between two etched lines. Different ranges of viscosity use different sizes of capillaries, and all require calibration with standard fluids of known viscosities. Some authorities consider these two viscosimeters more accurate than the Saybolt instrument.

Other forms of viscosimeters include a long capillary tube, rolling ball in an inclined tube, falling ball in tube, and falling cylinder type.

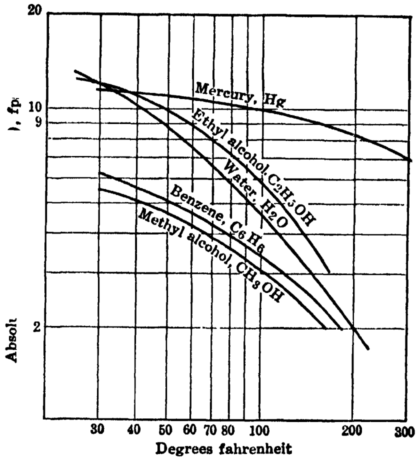


FIG. 5. Viscosity of common liquids.

The Engler viscosimeter is used on the continent, and in England the Redwood is used. These are of the type which measures the time of efflux of a definite quantity of fluid from a tube similar to the Saybolt Universal viscosimeter.

The viscosity from the Engler is expressed in degrees, which is the relative time of efflux of the fluid to the time of efflux of a standard fluid. The viscosity from the Redwood is expressed in seconds.

Conversion to centistokes is

$$\text{Centistokes} = 0.0147t_1 - \frac{3.74}{t_1} \quad \text{for the Engler}$$

$$\text{Centistokes} \quad 0.26t - \frac{172}{t} \quad \text{for the Redwood}$$

$$t_1 = 51.3 \times \text{Engler degrees}$$

$$t = \text{seconds Redwood}$$

The conversion for kinematic viscosity from centistokes to a few values of other units is

$$\text{Centistokes} \times 0.00001076 = \frac{\text{ft}^2}{[illegible]}$$

$$\text{Centistokes} \times 0.001550 = \frac{[illegible]}{\text{sec}}$$

Viscosity of a fluid varies with temperature and pressure. The viscosity of liquids in general decreases with an increase in temperature, and the viscosity of gases in general increases. For an increase in pressure the viscosity of liquids and gases increases, but the variation with pressure is much less rapid than the variation with temperature.

ASTM charts are available for plotting viscosity-temperature relations of petroleum products as straight-line functions. The plot on standard log-log paper is not quite straight.

Viscosities of a number of common gases and steam are given in Section 1, p. 1-15, and viscosities of liquids in Figs. 5 and 6 of this section.

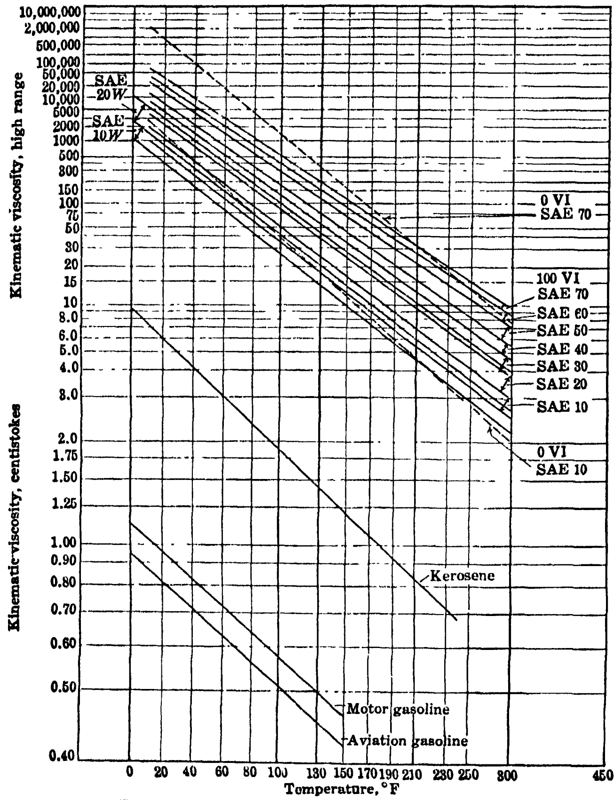


FIG. 6. Viscosity of common petroleum products.

16. COMPRESSIBLE FLOW OF AIR IN PIPES

By J. B. Nichols

EFFECT OF HIGH PRESSURE DROP. In most applications requiring piping of air, a pipe that will keep the pressure loss to some reasonably low value is chosen. The savings resulting from lower pressure drop in a larger pipe are usually balanced against increase in duct cost, yielding an optimum size.

Friction pressure drops greater than about 10% make the usual equation for pressure drop of little value, if accuracy is required. The incompressible flow equation

$$\Delta p = f \frac{\rho V^2}{2} \frac{x}{m}$$

assumes that density ρ and velocity V remain constant along the pipe, whereas in an insulated pipe velocity actually increases as pressure and density decrease. The total temperature remains constant, but the static temperature decreases. The combined result of increasing velocity and decreasing static pressure is a rapid increase in Mach number, defined by

$$M = \frac{V}{\sqrt{gkRT}}$$

Instead of equal pressure drops for equal lengths of pipe as in incompressible flow, each section of pipe has a higher pressure drop and a higher Mach number than the preceding section.

The magnitude of this effect is a function of the Mach number. The greater the Mach number, the greater the acceleration in that section. If the pressure in the duct is high enough and the pipe long enough, the Mach number will continue to increase until the velocity of sound is reached ($M = 1$). At this point, no further acceleration can occur as friction cannot accelerate subsonic flow through the sonic velocity, because this state is the condition of maximum entropy, hence the condition of greatest stability. (By the same token, friction alone cannot decelerate supersonic flow to subsonic velocities.)

Nomenclature

A = duct area, sq ft
f = friction factor, dimensionless
g = acceleration of gravity, 32.2 ft/sec^2
x = length of duct, ft
m = hydraulic radius $\left(\dfrac{\text{area}}{\text{wetted perimeter}}\right)$ (for round pipes $m = D/4$), ft
M = Mach number = $\dfrac{\text{velocity}}{\sqrt{gkRT}}$
p = static pressure, lb/sq ft
p_0 = impact pressure lb/sq ft
k = c_p/c_v (1.4 for air), dimensionless
R = gas constant (53.3 for air), ft-lb/lb-°F
T = static temperature, °F absolute
T_0 = impact temperature, °F absolute
V = velocity, ft/sec
w = air weight flow, lb/sec
ρ = mass density of air, slugs/ft^3

COMPRESSIBLE FLOW EQUATIONS. The solution for compressible pressure drop along a pipe normally requires the solution of the following equations.

Variation of Mach number

$$\frac{1}{2M_2^2} - \frac{k+1}{2}\log\frac{\sqrt{1+\frac{k-1}{2}M_2^2}}{M_2} = \frac{1}{2M_1^2} - \frac{k+1}{2}\log\frac{\sqrt{1+\frac{k-1}{2}M_1^2}}{M_1} - \frac{fkx}{8m}$$

tatic pressure ratio

$$\frac{p_2}{p_1} = \frac{M_1\sqrt{1+\frac{k-1}{2}M_1^2}}{M_2\sqrt{1+\frac{k-1}{2}M_2^2}}$$

Ratio of total to static pressure

$$\frac{p_0}{p} = \left(1+\frac{k-1}{2}M^2\right)^{k/(k-1)} = \left(\frac{T_0}{T}\right)^{k(k-1)}$$

Total pressure ratio

$$\frac{p_{02}}{p_{01}} = \frac{M_1\left(1+\frac{k-1}{2}M_2^2\right)^{\frac{1}{2}(k+1)/(k-1)}}{M_2\left(1+\frac{k-1}{2}M_1^2\right)^{\frac{1}{2}(k+1)/(k-1)}}$$

The initial Mach number, M_1, is determined by the continuity equation from the values of the air weight flow, pressure, temperature, and pipe cross-sectional area at the inlet.

GRAPHICAL SOLUTION OF COMPRESSIBLE FLOW WITH FRICTION. Figures 7 and 8 are charts of static pressure ratio and total pressure ratio, respectively, each presented as functions of entrance Mach number and $fkx/8m$. Static pressure ratio is more commonly used, but total pressure ratio is a more accurate indication of the available energy lost by friction.

Entrance Mach number may be determined from Fig. 9 by entering with $w\sqrt{T}/PA$, where T or T_0 and p or p_0 may be used with their respective curves and the Mach number read immediately.

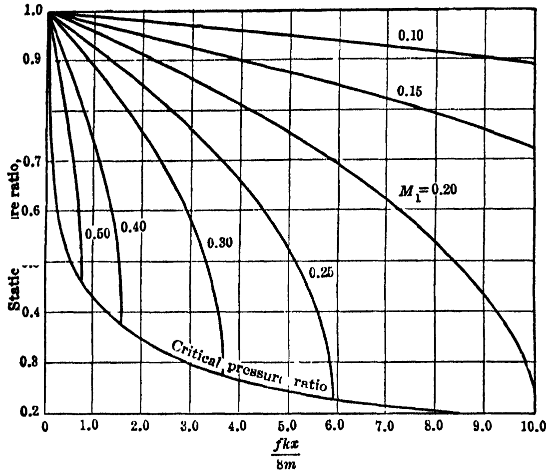


Fig. 7. Static pressure ratio across a pipe with compressible flow.

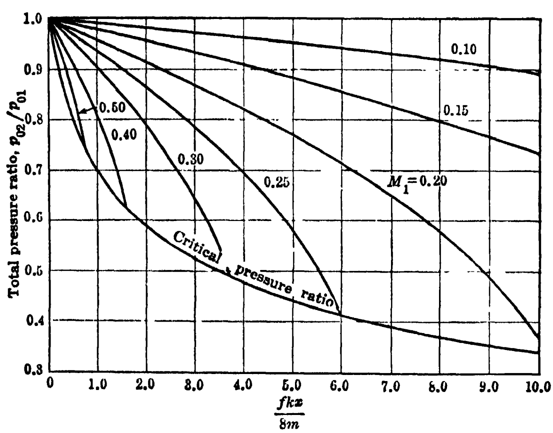


Fig. 8. Total pressure ratio across a pipe with compressible flow.

Example. Tank air at 2 atmospheres pressure and 100 F is to be transported 50 ft in a 4 in. x 6 in. rectangular duct of smooth steel ($f = 0.02$). What will be the pressure ratio across this duct when carrying 9 lb per sec?

$$\frac{w\sqrt{T_0}}{p_0A} = \frac{9\sqrt{560}}{2(14.7)(4 \times 6)} = 0.3015$$

From Fig. 9 $M_1 = 0.353$.

$$\frac{fkx}{8m} = \frac{0.02 \times 1.4 \times 50 \times 12}{8 \times 24/20} = 1.75$$

From Fig. 7,

$$\frac{p_2}{p_1} = 0.648$$

From Fig. 8,

$$\frac{p_{02}}{p_{01}} = 0.73$$

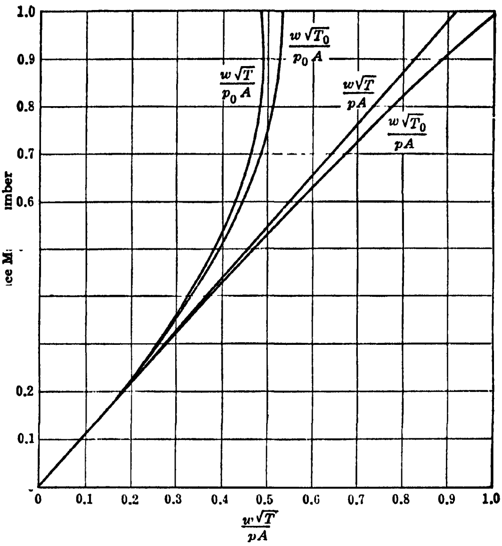


FIG. 9. Entrance Mach number versus various weight flow functions.

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SECTION 7

STEAM-GENERATING UNITS

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CHEMISTRY OF BOILER FEEDWATER

Frederick G. Straub and Others

BOILER FURNACES

By W. A. Carter

PULVERIZERS AND PULVERIZED COAL

By V. Z. Caracristi

FLY-ASH COLLECTION

By R. B. Foley

THE STEAM BOILER

By E. M. Powell

1. SELECTION OF EQUIPMENT

The modern steam boiler is only one part of the carefully integrated steam-generating unit, defined as a unit to which water, fuel, and air are supplied and in which steam is generated. It consists of a boiler with fuel-burning equipment and, typically, water-cooled furnace, superheater, reheater, economizer, and air heater, or combinations of these. The component parts are so interrelated that it is impracticable to discuss the steam boiler alone. This article will deal, therefore, with selection and performance of a steam-generating unit, with particular emphasis on the *boiler itself.*

For maximum overall reliability and operating economy each component must be correctly proportioned and related to the others. To produce a coordinated unit requires, first, a background of experience covering each element of the design and its functional relation to other elements; second, a wide variety of designs to choose from in order that the exact type and size best suited to the particular needs of any plant may be recommended and furnished.

Many pitfalls in the selection of steam-plant equipment can be avoided through the employment of competent and reliable consulting-engineering organizations with broad experience in power-plant work and the ability to integrate manufacturers' designs with the remainder of the plant.

FACTORS INFLUENCING SELECTION. Factors which exert the greatest influence on selection of fuel-burning and steam-generating equipment are fuel characteristics, capacity and steam conditions, space conditions, cost, and individual preference.

FUEL CHARACTERISTICS. Before attempting even a preliminary selection of equipment, complete information should be available as to fuels on which designs and predicted performance are to be based. This information should be established by a comprehensive survey of the market, to determine which fuel, available in quantities sufficient to guarantee a reliable source of supply, offers the greatest economic value over a long-range program. It usually is desirable to establish a *secondary* fuel supply for emergency use when the supply of primary fuel is interrupted or changes in price make the secondary fuel economically more attractive.

If possible, the equipment should be selected so that the performance with the secondary fuel will be equivalent to that with the primary fuel. However, if the reserve of primary fuel is ample to insure that any interruption in supply would be only temporary, and if the price differential between primary and secondary fuels is fairly stable, it may be better economy to design for maximum efficiency with the primary fuel and accept some compromise in performance and maintenance costs with the secondary fuel.

Fuels available to the average plant, in order of importance, are coal, fuel oil, and natural gas. (See Section 2.) Neither cost nor heating value is a true index of their economic value. These are only two of many factors in the overall cost per thousand pounds of steam generated.

Other factors to be considered are the efficiencies and operating costs obtained with different fuels. Natural gas, piped directly from the supply mains to the burners, incurs practically no operating costs; no refuse results from its use. The cost of burning fuel oil usually is low, consisting of storage, pumping, heating, atomization, and maintenance of equipment. Although the ash content is low, heating surfaces must be cleaned. Depending on the sulfur content, there may also be corrosion in air heater, economizer, fan and breeching connections. The cost of cleaning and maintaining such equipment must be charged to burning oil.

The cost of burning coal should include maintenance of equipment, handling charges from car to storage to steam-generating unit, cost of cleaning heating surfaces, and ash handling. The average cost of burning coal in the United States, exclusive of fixed charges, is about 5% of its cost, for fuel oil approximately 1.5% of the equivalent coal cost, and for natural gas 0.5%.

The type of fuel-burning equipment depends on burning characteristics of the fuel and the capacity for which the unit is designed. For example, stokers usually are the more

Table 1. Fuel-burning Equipment and Furnace Check List

(Adapted by permission from *Combustion Engineering*, Combustion Engineering-Superheater, Inc., New York, 1947)

Type of Fuel-burning Equipment	Approximate Continuous Combustion Rate, pounds per sq ft per hr (dry basis)								Range in Maximum Continuous Furnace Liberation Rates, Btu per cu ft per hr			
	Anthracite	Bituminous Coal		Sub-bituminous Coal	Lignite	Coke Breeze	Wood					
		High Quality	Low Quality				Dry Refuse	Wet Hogged	Coal	Oil	Gas	Wood
Pulverized coal	...	...	...	...	...	...	...	...	15,000 to 22,000	...	...	...
Single-retort stoker	20	30	25	30	30	...	...	...	30,000 to 40,000	...	...	...
Multiple-retort stoker	...	35	30	...	...	...	...	...	30,000 to 35,000	...	...	...
Spreader stoker	...	40	35	50	50	...		...	25,000 to 35,000	...	...	...
Chain-grate stoker	30	45	35	50	50	30	...	...	40,000 to 55,000	...	...	...
Traveling-grate stoker	30	45	35	50	50	30	...	...	40,000 to 55,000	...	...	...
Stationary grate	20	20	15	15	18	...	...	...	...	...	...	...
Special furnaces	...	...	...	...	...	...	100 to 125	100 to 125	...	25,000 to 30,000	20,000 to 30,000	25,000 to 30,000

economical selection for comparatively low capacity units and pulverized coal for high capacities. A more complete discussion of fuel-burning equipment is found on p. 7-64. Table 1 illustrates the adaptability of different firing methods to various commonly used fuels and required stoker and furnace sizes. All coals, including anthracite, have been burned successfully in pulverized form, except coke breeze.

The range of capacities for which each type of fuel-burning equipment is particularly suitable and most commonly used is shown in Table 2.

Table 2. Fuel-burning Equipment Selection Check List—Capacity Basis

(Combustion Engineering)

Fuel-burning Equipment	Continuous Capacity Range, pounds of steam per hour			
	1000 to 15,000	15,000 to 35,000	35,000 to 200,000	150,000 to 1,000,000
Pulverized coal	.....	✓ *	✓	✓
Single-retort stoker	✓	✓	✓ †	....
Multiple-retort stoker	....	✓	✓	✓ ‡
Spreader stoker, dumping-grate	✓	✓	✓ §	....
Spreader stoker, continuous-discharge	....	....	✓	✓ ‡
Chain-grate stoker	....	✓	✓	✓ ‡
Traveling-grate stoker	....	✓	✓	✓ ‡
Stationary grates	✓	....	....	....
Oil burners	✓	✓	✓	✓
Gas burners	✓	✓	✓	✓

* Occasionally.
† Rarely exceeding 40,000 lb per hr.
‡ Occasionally for capacities exceeding 200,000 lb per hr.
§ Up to approximately 125,000 lb per hr.

Special refuse or by-product fuels are available from industrial processes such as blast-furnace gas, other gases, paper-mill liquors, bagasse, bark, and wood refuse. Burning characteristics of these fuels and other design considerations given to their use are discussed in Section 2.

CAPACITY AND STEAM CONDITIONS. Capacity is one of the most important factors in determining the type of unit to be selected. Table 3 illustrates the capacity ranges for which each of the general types of boilers has been found most adaptable, the

Table 3. Steam-generating Equipment Selection Check List—Capacity Basis

(Combustion Engineering)

Type	Continuous Output, pounds of steam per hour				Maximum Design Pressure, psig	Maximum Temperature, °F
	1000 to 15,000	15,000 to 35,000	35,000 to 150,000	150,000 to 1,000,000		
Firetube boiler	✓	....	....	....	250	150 F superheat
Three-drum low-head boiler	✓	✓	....	....	400	150 F superheat
Two-drum vertical unit-type boiler	✓	✓	✓	✓ *	1000	900
Three- or four-drum vertical boiler	....	✓	✓	✓	1500	925
Sectional-header boiler	....	✓	✓	....	2000	900
Special utility-type boiler	....	....	....	✓	2500	1050 †
Controlled forced-circulation boiler ‡	✓	✓	✓	✓	(approx.) 3000	1050 †

* Up to approximately 225,000 lb per hr on coal, and 300,000 lb per hr on oil and gas.
† Present maximum used. Design and materials available for higher temperature.
‡ Controlled forced-circulation boilers are designed in various types and sizes for the full range in capacities.

maximum steam pressures for which they are usually designed, and the corresponding maximum steam temperatures. There is also a partial correlation between steam conditions and capacity. Maximum pressures and temperatures seldom prove economical except in the higher capacity ranges. The flow ranges have been selected so that 1000 to

15,000 represents the small heating plant, 15,000 to 35,000 the small industrial, 35,000 to 150,000 most of the industrial plants and some of the smaller utilities, and 150,000 to 1,000,000 nearly all the central-station types. Included in this latter group are some of the large units for industrial plants.

Limitations imposed by steam pressure and temperature are predominantly structural. They affect the weight of steel required, hence the cost; temperature affects the space required by the superheater and adaptability of the boiler to provide that space. Another condition affecting design is feedwater temperature. Boilers without economizers are not affected by variation in feedwater temperature, except as it affects the heat input to a pound of steam. However, in boilers equipped with economizers, boiler and economizer surface must be proportioned to suit the variations in feedwater temperature.

Heat-recovery Equipment. The selection of heat-recovery equipment is primarily an economic study based on capitalization of the fuel saving credited to its use. The choice between an air heater and an economizer (see p. 7-30) depends on (1) allowable maximum air temperatures (depends on type of fuel-burning equipment used); (2) boiler pressure and feedwater temperature as affecting allowable absorption in the economizer; (3) arrangement and amount of heating surface required; and (4) temperature of gas at boiler outlet.

Conditions other than economic may make the use of air heaters desirable. With pulverized coal firing, preheated air is necessary for drying coal in the pulverizer, and as an aid to combustion in the furnace. Use of air heaters with stoker-fired units will not yield as much increase in efficiency as with pulverized coal-fired units, because the preheated-air temperature is normally limited to 300 F, to avoid excessive maintenance of grates. Here economizers provide a better means of increasing the efficiency than air heaters.

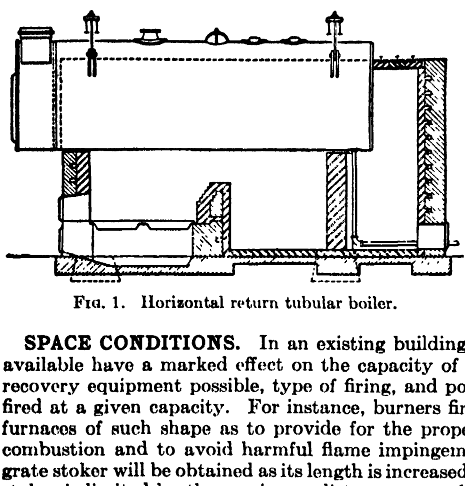


Fig. 1. Horizontal return tubular boiler.

In large utility-type designs, heat-recovery equipment almost always includes economizers *and* air heaters to obtain the highest efficiency which can be justified economically: 88 to 90%. Most of these units are designed for high-pressure operation, and the cost of boiler and economizer surface is high. Since thickness of air-heater parts is not affected by steam pressures, it is usually economical to use a large amount of air-heater surface.

SPACE CONDITIONS. In an existing building both shape and volume of the space available have a marked effect on the capacity of the unit which can be installed, heat-recovery equipment possible, type of firing, and possibly the range of fuels which can be fired at a given capacity. For instance, burners firing pulverized coal, oil, or gas require furnaces of such shape as to provide for the proper length of flame travel for complete combustion and to avoid harmful flame impingement. Best operation with a traveling grate stoker will be obtained as its length is increased, whereas the grate length of a spreader stoker is limited by the maximum distance over which it is possible to distribute the fuel uniformly. Coal having low ash-fusion temperatures frequently dictate the use of larger furnaces than are actually required for satisfactory combustion so that sufficient water-cooled surface can be provided. This cools the gases below the temperature at which excessive slag and ash accumulations would occur at the entrance to the first gas passage of the boiler.

COST. Extreme care should be exercised in the extent to which first cost is allowed to influence the equipment selected. A complete economic study should be made (see Section 16) considering the load factor of the plant, the cost of fuel, and the efficiency of the plant as a whole, rather than the steam-generating equipment alone.

For instance, a small plant located near an ample supply of low-priced fuel and having a seasonal load of a few months of each year can justify a standard boiler, a stoker, no water-cooling in the furnace or heat-recovery equipment, and will operate with natural draft. On the other hand, a base-load plant, whether industrial or utility, with a load factor approaching 100% and burning a high-priced fuel, can easily justify an efficient fuel-burning system, water-cooled furnace, high steam pressure and temperature, as well as heat-recovery equipment, forced- and induced-draft fans, and control equipment. The value of the fuel burned during the life of such a unit may represent forty times the initial investment. Even a small advantage in reliability, efficiency, or flexibility gives economic justification for the relatively small additional first cost necessary to provide the better unit.

Too many variables are involved to prepare any accurate cost comparisons that may be applied to specific cases. However, for a given set of steam conditions and firing methods, one large boiler can be constructed at a cost of 15 to 25% less than two boilers having the same combined capacity. A unit designed to operate at steam conditions of 600 psig—750 F will cost between 40 and 50% more than one for 200 psig—500 F.

INDIVIDUAL PREFERENCE. Well-founded individual preference, not personal prejudice, should be considered if it is based on the familiarity of the plant personnel with the operation of a given type of equipment or if the plant itself was designed for a specific type and not suited to others without expensive changes. However, improvements in design and the higher efficiency or capacity that may be obtained within the same space at reduced cost for labor and maintenance should not be overlooked. The types of boilers listed in Table 3 are described briefly below with some of their chief advantages and limitations.

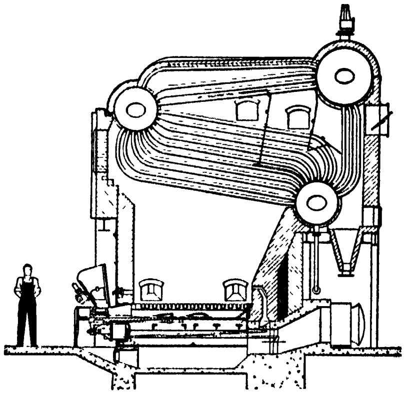


FIG. 2. Three-drum low-head boiler.

TYPES OF BOILERS. Fire tube boilers are relatively inexpensive compared to the corresponding water tube boilers. Their design and construction, however, are such that there is a definite limitation in the size and pressure for which they can be built. They are seldom used above 150 psig design pressure. Overload is limited, and exit gas temperature rises rapidly with increased output. One advantage is the large water-storage capacity. Because of this feature, wide and sudden fluctuations in steam demand are met with little change in pressure. There are many types of fire tube boilers in use today with internal or external furnaces, but the most widely used is the horizontal return tubular type illustrated in Fig. 1.

The three-drum low-head boiler, as its name implies, was designed for limited space, especially when headroom is low. Like other water tube boilers, they are seldom designed for pressures below 150 psig. Because of its low overall height, its capacity is limited by the hydraulic head available for maintaining circulation of water and steam through the tubes. Baffles are arranged for natural draft operation with a normal stack height. A typical application of this boiler is shown in Fig. 2.

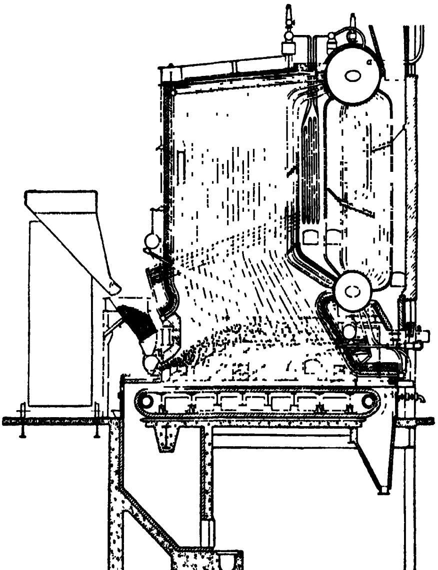


FIG. 3. Two-drum boiler with spreader stoker.

Two-drum boilers are the simplest of the bent tube types and are available in many designs. Because of the flexibility in their design and increased hydraulic head, they can be used for a wide range of capacities and pressures and are adaptable to any method of firing. A design for a large industrial plant is shown in Fig. 3, fired by a spreader stoker and using high-pressure overfire air to create turbulence in the furnace and shorten the length of flame travel.

Three- or four-drum vertical boilers in industrial and small utility plants have been replaced to a large extent by the less expensive two-drum boiler. Their chief application today is found in the special utility-type class, where they are designed to fit the conditions

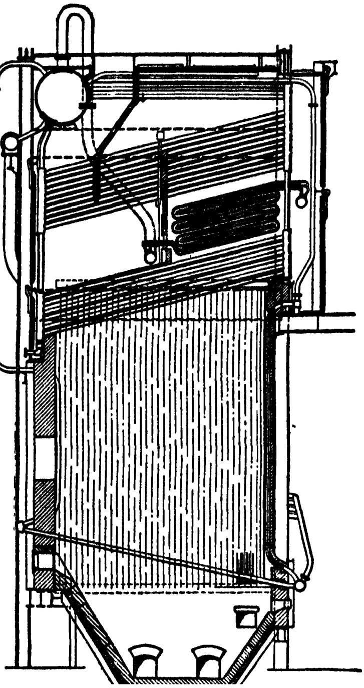


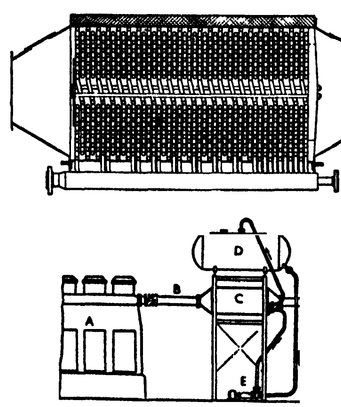
FIG. 4. Sectional header boiler.

of each particular installation. They are described and illustrated at length later in this discussion.

Sectional-header boilers were also designed to meet conditions of low headroom, and for that reason are still widely used in the marine field. Their chief advantages are simplicity, excellent performance, and adaptability to construction over a wide range in performance. The desire for higher capacities per foot of furnace width which exceeded the limit of satisfactory water and steam circulation in this type of boiler, together with the development of welding technique, served to accelerate the use of bent-tube designs particularly in the higher pressure and capacity field. A typical unit of this design is shown in Fig. 4.

WASTE-HEAT BOILERS. Waste heat, for our purpose, is defined as the sensible heat in noncombustible gas, such as gas leaving furnaces used for processing metals, ores, and other materials. Boilers are usually installed where the waste gases are continuously discharged from a process at a temperature well above that of saturated steam at the required pressure. Gas temperatures above 1000 F usually justify application of waste-heat boilers. High gas temperatures permit efficient heat transfer with low velocities, resulting in low draft losses by permitting natural draft operation. Low gas temperatures (1) require high velocities to obtain efficient heat transfer and (2) necessitate the use of induced-draft fans. One notable low-temperature application is where steam is required in a plant equipped with diesel engines. The exit gas temperatures range from 500 to 800 F, depending on whether the engine is designed for a two-stroke or four-stroke cycle. (See Section 13.)

Forced-circulation boilers are particularly suited to duty with diesel engines, since small tubes and high gas velocities can be used for most efficient heat transfer with the low temperatures involved. They can be of compact design and provide more flexibility of tube arrangement and location since they do not depend on hydraulic head for circulation, as do natural-circulation boilers. The low gas temperature limits the operating pressure to about 200 psig. Such a boiler is illustrated in Fig. 5. This boiler consists of a cylindrical-shaped shell containing heating surface as a number of horizontal pancake coils stacked one above the other. Each coil is a continuous circuit connecting the inlet and outlet headers. An orifice is used in the inlet header for each coil to insure uniform distribution of water. The boiler may be installed either vertically or horizontally and the headers arranged to suit installation requirements.



A = *Diesel Engine* *C* = *Waste Heat Boiler*
B = *Exhaust Pipe* *D* = *Steam and Water Drum*
E = *Circulating Pump Drawing Water from D, Delivering It to C and Returning It to D.*

FIG. 5. Forced circulation waste-heat boiler used with diesel engine.

Boilers used in *steel mills* containing open-hearth and continuous-heat furnaces can be either water tube or fire tube type. Fire tube boilers are sometimes preferred because of the low infiltration into the setting, thereby minimizing the possibility of the explosion of combustible gases.

Water tube boilers are usually preferred with *cement and lime kilns*, *copper and zinc* *furnaces*, because they are available in large sizes and permit baffling to recover solids suspended in the gas stream if desired. They are more accessible for cleaning and less susceptible to damage by sudden temperature changes than fire tube boilers. A water-cooled furnace can be readily applied to reduce the temperature of gases containing nearly plastic solid matter, thereby reducing slagging difficulties in the convection passes of the boiler.

Vertical bent-tube boilers are preferred to straight-tube boilers when *dust-laden gases* are present, since they are more flexible in tube arrangement, require less space for tube removal, and are more positive in water and steam circulation. Also, they are more adaptable to the application of superheaters, and the vertical tubes minimize dust accumulation.

Figure 6 shows one of four boilers serving two large reverberatory furnaces in a copper refinery. This design, illustrating all the points enumerated, was developed from experience with heat and chemical recovery from paper-mill liquor where gases contain large quantities of luminous solid matter at a temperature near the plastic point. Boiler tubes are widely spaced, vertical to minimize ash deposit. Soot blowers are provided in the furnace roof and in the boiler as indicated by double circles. The large hopper at the bottom of the furance and the hopper at the bottom of the boiler passes collect a large percentage of the dust from the gas stream for the recovery value of that dust. Numerous doors are provided for observation and manual cleaning if necessary. This particular boiler was designed for 45,000 lb of steam per hr at 750 psig and 661 F.

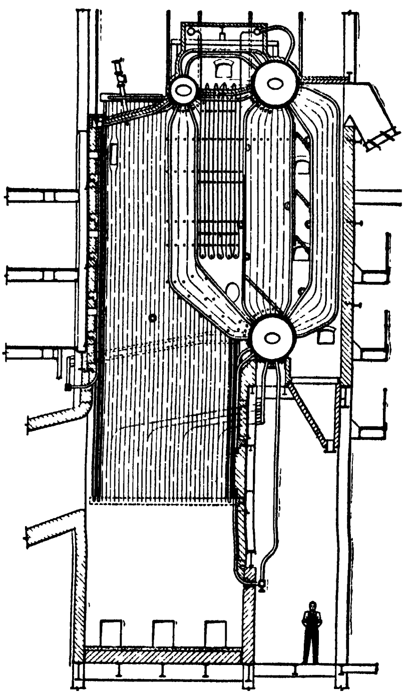


FIG. 6. One of eight waste-heat boilers installed at Phelps Dodge Corporation, Morenci, Arizona. Maximum continuous capacity is 45,500 lb per hr at 750 psig–661 F.

FORCED-CIRCULATION BOILERS. Natural circulation in a boiler may be defined as the movement of water and steam through boiler tubes in conformity with the available head resulting from the difference in density of the circulating fluid in the downcomer and riser circuits. Although natural circulating head may be present also in a forced-circulation boiler, the primary circulating force is supplied by a pump. There have been many different designs employing the forced-circulation principle, most of which can be grouped in four basic types.

(1) *The "once-through" type* does not require a separating drum. Water is fed into the boiler by the feedwater pump and passes progressively through the various sections of the unit for water heating, evaporating, and superheating. Since all the water is evaporated in one pass through the unit, all the solid matter in the feedwater must be deposited on the heating surface. This presents the serious problem of arranging the zone of deposit in a cool region to avoid overheating the tube metal, and of providing for washing the affected surfaces. Second, the low water-storage capacity makes necessary a sensitive control system for regulating the flow of fuel, air, and water.

(2) *The "recirculation" type* discharges water and steam into a drum in which the steam is separated from the water. The circulated water and feedwater are taken from the drum and fed to the evaporating circuits through a circulating pump. The concentration of solids in the boiler water may be controlled by blowdown from the drum, in the conventional manner.

(3) In *the third type*, evaporation takes place entirely in a drum, avoiding the deposit of solids on heating surface. The rate of evaporation is considerably greater than the output of the unit, and a pump is used to force this steam through the superheater. The excess steam is returned to the drum to evaporate feedwater by direct contact. The large volume of steam handled by the pump results in such high power requirements that this type of unit is not economical except at very high pressures.

(4) *The fourth basic type* avoids one limitation of the once-through type by discharging a mixture of water and steam into a *separator*, from which the water is removed and passed through heat exchangers for conservation of heat. This type requires no special circulating pump but does require sensitive control equipment.

The recirculation type is most widely used in this country because the methods of control and operation most nearly approach those of the natural-circulation boiler. Its chief advantages are (1) the ability to proportion the water flow to the various circuits in accordance with predetermined requirements; (2) the ability to make use of small-diameter, thin-walled tubes; and (3) the facility with which tubes may be arranged for most efficient utilization of the space available, without regard to natural circulating head.

TYPICAL CASE OF EQUIPMENT SELECTION. Additional power generation is required in a large public utility plant. The unit shown in Fig. 7 was designed to meet the steam conditions tabulated below and to burn a good grade of West Virginia bituminous coal with provision for other coals of lower heating value, grindability, and ash-fusion temperature.

Specifications

Pressure at superheater outlet, psig	1,285
Superheated steam temperature, °F	950
Continuous capacity, lb per hr	620,000
Control is to be provided so that a constant steam temperature of 950 F is maintained from 475,000 lb per hr to 620,000 lb per hr	
Approximate load factor, %	80
Feedwater temperature, °F	451

Selection of Equipment. The first design step in selecting such a unit is the determination of furnace dimensions. The width is set by the maximum evaporation per unit of drum length at which it is possible to operate and yet secure satisfactory steam separation and purification in the drum. Velocity of gas flow at entrance to boiler and superheater as it may affect slag accumulation is also a factor. With this unit, tangential burners are used. Furnace depth and height are proportioned to provide the required volume, as well as necessary height and cross-sectional area for satisfactory burner arrangement for the secondary low-quality fuel.

The furnace walls are fully water cooled to secure a gas temperature at the superheater inlet which is low enough to avoid slag accumulations. This gas temperature, however, must be sufficiently high to provide the required steam temperature at the minimum load for which superheat control is desired. The use of vertically adjustable burners which may direct the fuel and air either upward or downward in the furnace provides a relatively wide range in control of furnace outlet temperature, and thus increases the capacity range over which constant steam temperature is obtainable. In addition, dampers for by-passing a portion of the gas around the superheater surface are used for control of steam temperature beyond the range of burner regulation. In some cases, desuperheaters are also included to increase flexibility in control of steam temperature.

A large amount of superheating surface having a high percentage of alloying materials is required for a steam temperature of 950 F. To produce this steam temperature, the gas-temperature drop through the superheater is approximately 925 F. As a result, the temperature differential between gas and steam is relatively small. This condition requires use of a counterflow arrangement between steam and gas. Close spacing of the surface and high velocity will produce high heat-transfer rates, but will result in excessive draft loss, tube erosion, and slag accumulations. Operating experience on many installations with high steam temperature at high capacity provides the designer with many data from which he can determine correct spacing of superheater elements and optimum gas velocity.

After establishing superheater design, the next step is to select the air heater and economizer combination, and then fill in with boiler convection surface to secure the desired gas outlet temperature from the unit. On the basis of specified load factor and fuel cost, it is possible to justify an outlet-gas temperature of less than 300 F. However, 300 F is usually set as the allowable minimum to avoid corrosion in the air heater. Improvements in air heater cleaning methods, materials, arrangement of surface, and methods for increasing metal temperatures during low load operation, such as recirculating preheated air to the inlet of the air heater, indicate that lower outlet-gas temperatures may be practicable. The outlet-gas temperature and calculated gas and air quantities, having been determined, the maximum practical size of air preheater is selected. Performance calculations then establish gas temperature at air heater inlet, i.e., economizer outlet. The economizer performance is established by limiting the water temperature rise to avoid generating steam under any expected operating condition, and then determining the

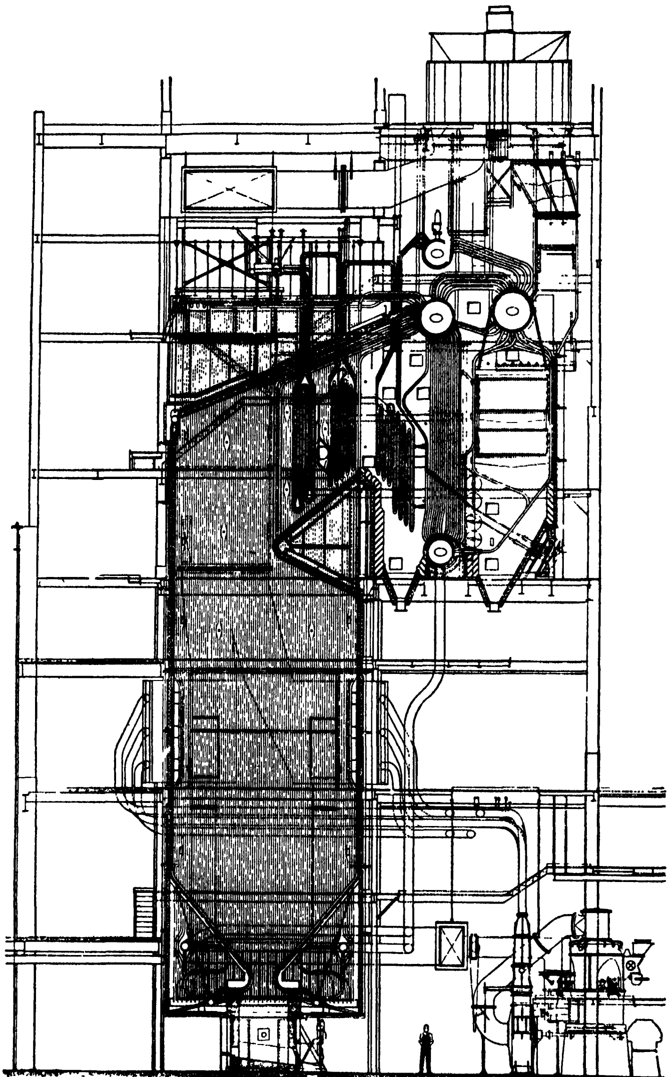


FIG. 7. Steam generating units for Cliffside Steam Station, Duke Power Company. Maximum continuous capacity is 620,000 lb per hr at 1285 psig–950 F.

corresponding drop in gas temperature. Length of economizer elements is usually equal to the furnace width. The number of tube rows in height and width is adjusted to provide this performance and thus sets the gas temperature at the economizer inlet. This temperature immediately indicates whether additional convection surface is needed between superheater outlet and economizer inlet. Surface of this type is part of the boiler and is generally arranged as a vertical tube bank between upper and lower drum. A minimum amount of this convection surface should be installed, to avoid increasing cost of drums.

The foregoing discussion outlines the fundamental approach to arranging and proportioning the several types of heat-absorbing surfaces of a steam-generating unit. There are a number of other factors which also require careful analysis to provide a well-balanced unit: the number and size of drums required to produce steam of acceptable purity must be established (for the specified capacity two upper drums are generally provided); arrangement of equipment must fit into space conditions existing at the plant; design of water walls and convection surface must provide for adequate steam and water circulation; and overall draft loss through the unit should be reasonable, to avoid excessive fan power.

The number of pulverizers (see p. 7-82) recommended for these larger units may be two, three, or four, depending on character of operating load and quality of coal. The pulverizers are selected with ample capacity to handle the lowest quality of coal, considering moisture, grindability, and heating value, that may be used.

2. BOILER DESIGN AND HEAT BALANCE

FACTORS OF PERFORMANCE. The term *performance* refers to the rate of output, efficiency of heat transfer, draft and pressure requirements of the steam-generating unit or any of its component parts.

Output or capacity of steam boilers is expressed in many ways. The term most commonly used is *actual evaporation*, the pounds of steam generated per hour at the given steam temperature and pressure. This term does not offer an accurate comparison between one unit and another since the heat transferred per pound of steam generated may vary widely, depending on steam pressure, temperature, and feedwater temperature. A more accurate method for comparison is to report output in terms of total heat transferred per hour to the water and steam as it passes through the unit. Turbine steam rates are usually expressed in pounds of steam required per kilowatt-hour generated, which probably accounts for the popularity of the term actual evaporation for rating the boilers supplying the steam.

When the heat-transfer equipment of a steam-generating unit consisted solely of a boiler and was used primarily in conjunction with a steam engine, it was common practice to rate the unit in terms of *boiler horsepower*. A boiler horsepower was defined as the evaporation of 34.5 lb of water per hour from a temperature of 212 F into dry saturated steam at the same temperature. This is equivalent to 33,475 Btu per hr. It was common practice to rate a boiler on the basis of 10 sq ft of heating surface per boiler horsepower. With improved firing methods it was found that a boiler could develop considerably more than its "rated capacity." The ratio of actual to rated capacity expressed in percentage came to be known as *per cent of rating*. With the advent of water-cooled furnaces and heat-recovery equipment, less work was required of the boilers for a given total output so that higher capacities could be developed per rated horsepower. Many steam-generating units today, particularly in central stations, contain *no boiler heating surface* as such. Boiler horsepower and per cent of rating would be meaningless in such cases, and, for that reason, these terms have become obsolete although they are still used occasionally with reference to standardized boilers of low capacity.

The *factor of evaporation* is the ratio of the heat actually required to heat one pound of feedwater to the final steam conditions, to the heat required to evaporate one pound of water from a temperature of 212 F to dry saturated steam at 212 F. *Equivalent evaporation* is the product of *actual evaporation* and the *factor of evaporation*. This term is seldom used today, although it does represent a true measure of the total heat output.

Efficiency of a steam-generating unit is the ratio of the heat absorbed by water and steam to the heat in the fuel fired. It is a measure of the potential heat energy in the fuel which has been converted and transferred to the steam, in which form it can do useful work.

There are two accepted methods of testing for efficiency, described in detail in the ASME Power Test Code for Stationary Steam Generating Units. (See p. 19-12.) The preferred method involves the direct measurement of input and output. The measurements needed are the quantity and heat value of the fuel, the quantity of steam generated, and the heat absorbed per pound. It is also necessary to obtain sufficient heat loss data to construct a heat balance to permit checking the results. The difficulties in accurately determining fuel and steam quantities and a representative heat content of each is so great with larger units that an alternate method is generally used. In this method, the efficiency is determined by calculating the losses in per cent and subtracting from 100. Minor losses such as sensible heat in the refuse, loss due to moisture in the air, and loss due to unburned hydrocarbons can be calculated from the Test Code or a nominal mutually agreed "unaccounted for" loss assigned to cover these and other unmeasured losses.

In this country the higher heating value is used in determining the heat input from the fuel, whereas in Europe the lower heating value is commonly used. The lower heating value equals the higher heating value minus 1020 Btu for each pound of water present as moisture in the fuel or formed by burning the hydrogen in the fuel.

When the boiler and fuel-burning equipment are of different manufacture, the boiler manufacturer can guarantee the overall efficiency based on an assumed CO_2 leaving the unit and an assumed loss due to unburned combustible. The manufacturer of fuel-burning equipment will guarantee to burn the required quantity of fuel so that the combined effect on the efficiency of the steam-generating unit will not be less favorable than that resulting with a given CO_2 in the exit gas and a given heat loss due to unburned carbon.

BOILER HEAT BALANCE (see also Combustion, Section 2). The losses usually included in a heat balance are these.

1. Heat loss due to dry gas represents the sensible heat in the dry flue gases and is equal to pounds of dry gas per pound as-fired fuel $\times c_p(t_{exit} - t_{air})$, where c_p is the specific heat of the gases, taken as 0.24 for approximate calculations. For accurate values see p. 2-10. The heat loss thus calculated is expressed as Btu per pound of "as-fired" fuel. If a complete analysis of the flue gas is available, the weight of dry gas per pound as-fired fuel may be calculated from

$$\frac{11CO_2 + 8O_2 + 7(N_2 + CO)}{3(CO_2 + CO)} \times (\text{lb carbon burned per lb as-fired fuel} + {}^3/_8 S) + {}^5/_8 S$$

where CO_2, O_2, N_2, and CO are the constituents of the dry flue gas expressed in per cent by volume, and S is the pounds of sulfur in the fuel per pound, as-fired.

2. Heat loss due to moisture in fuel is due to evaporating moisture in the fuel and superheating it to the temperature of the flue gas. Temperature at which evaporation begins may be quite low, owing to the low partial pressure of the water vapor in the gaseous products of combustion. The heat lost is the difference between the total heat of water vapor at exit temperature and of liquid water at fuel temperature. When the gas temperature leaving the unit is less than 575 F,

$$\text{Btu loss per lb as-fired fuel} = \text{lb } H_2O \text{ per lb as-fired fuel}(1089 - t_{fuel} + 0.46t_{exit})$$

When t_{exit} is greater than 575

$$\text{Heat loss} = \text{lb } H_2O \text{ per lb as-fired fuel}(1066 - t_{fuel} + 0.5t_{exit})$$

3. Heat loss due to water from combustion of hydrogen is determined in the same manner as outlined in (2), by calculating the water formed as 9 lb per lb H_2 in the as-fired fuel. If free moisture is included as hydrogen and oxygen, in the ultimate analysis of fuel as fired, the formulas will give the total moisture loss, and calculation of loss due to moisture in fuel outlined under (2) may be omitted.

Moisture in gaseous fuels exists in two separate forms which require different treatment in calculating the heat loss. Washed gas contains entrained water in the form of suspended globules of liquid. In this form it can be treated as described under (2). There is no entrained water in an unwashed gas. Nearly all gaseous fuels contain some water vapor. In natural gases the water vapor is present because of water that has been in contact with the gas in the ground or because of rehydration. In refinery gas, blast-furnace gas, or coke-oven gas, it is present owing to the nature of the process of which they are a product or because of subsequent cleaning operations. This item assumes a real importance with unwashed blast-furnace gas supplied to the burners at high temperature. The moisture in vapor form does not require the heat of vaporization. The heat loss, therefore, represents the difference in sensible heat between the fuel temperature and that of the flue gas according to the following equation:

$$\text{Btu loss per lb as-fired fuel} = \text{lb } H_2O \text{ per lb as-fired fuel} \times 0.47(t_{exit} - t_{fuel})$$

4. Heat loss due to moisture in air is treated in exactly the same manner as the moisture in fuel in vapor form as described under (3).

5. Heat Loss Due to Carbon Monoxide. The presence of CO as detected by flue-gas analysis indicates incomplete combustion. It is seldom found with properly operated modern fuel-burning equipment. A small amount represents an appreciable heat loss.

$$\text{Btu loss per lb as-fired fuel} = \frac{CO}{CO_2 + CO} \times 10{,}160 \times \text{carbon burned per lb as-fired fuel}$$

where CO_2 and CO represent percentages by volume of carbon dioxide and carbon monoxide in the dry flue gases and 10,160 is the difference in Btu evolved in burning 1 lb carbon to CO rather than to CO_2.

6. Heat Loss Due to Combustible in Refuse. To determine this loss accurately involves measuring separately the weight of refuse in the ash pit, boiler hoppers, and gas stream leaving the unit as well as the heat value of that refuse by calorimeter. The heat loss then becomes the product of pounds of refuse per pound of as-fired fuel and the heat value per pound of refuse. This loss is sometimes approximated by burning out a sample of refuse, determining the loss in weight, and considering that to be carbon with a heat value of 14,600 Btu per lb. This method should not be used in checking guarantees since the heat value of the combustible may vary as much as 9000 to 14,600 Btu per lb, depending on its composition.

7. Heat loss due to radiation includes all the heat lost to the surroundings by either radiation, convection, or conduction, through the setting or the casing of the unit. No satisfactory method has been developed for measuring the loss. However, it is known to vary with the type of setting and particularly the extent of water cooling of the furnace walls and capacity. Accordingly, the American Boiler Manufacturers Association issued Fig. 8, showing the variation for different wall constructions and capacities. It has been adopted by the industry and ASME Test Code.

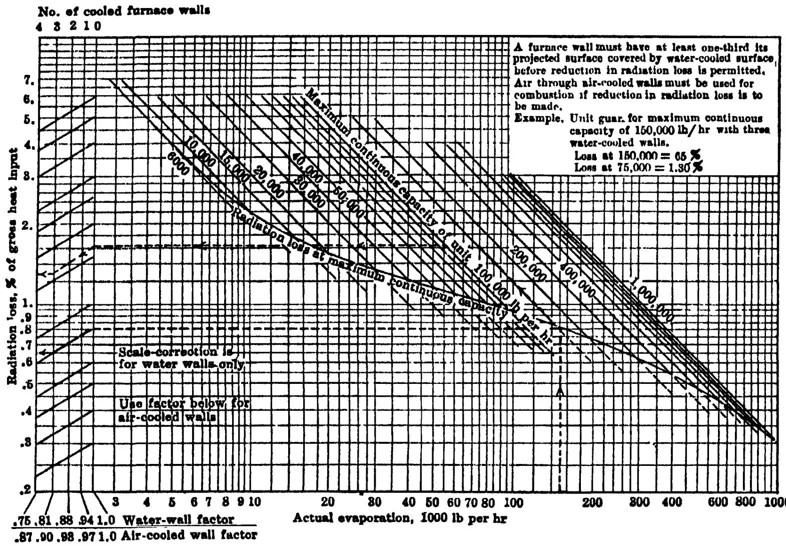


FIG. 8. Variation in heat loss due to radiation for different wall constructions and capacities. (American Boiler Manufacturers Assoc.)

HEAT TRANSFER. The theory of heat transmission is described in detail in Section 3 of this handbook. However, there are so many departures in the operation of a steam-generating unit from the controlled conditions of the laboratory that considerable modification is necessary before laboratory data can be used for design. All three methods of heat transfer—radiation, convection, and conduction—are present in each part of the steam-generating unit. For example, the heat transferred from the furnace gases to the water and steam within a water-wall tube must pass through a series of resistances interposed by the gas film adjacent to the tube, slag on the tube, tube wall, scale deposited on the inside by the water evaporated, and the film of water and steam. Additional resistance may be present between the slag and tube wall or the scale and tube wall, depending on the bond between them.

Heat transfer through the gas and evaporating films do not follow the simple equation for conduction. The temperature drop through the other resistances is inversely proportional to the thermal conductivity. For radial heat flow through curved surfaces, the temperature drop can be calculated from this equation:

$$\Delta t = \frac{Q}{12k} \times r_2 \times \left(\log_e \frac{r_2}{r_1}\right)$$

where Δt = temperature drop across any single resistance, °F; Q = heat absorption, Btu per hour per square foot of outside tube surface; k = thermal conductivity, Btu per square foot per hour per °F for 1 foot of thickness; r_2 = outside radius, inches; r_1 = inside radius, inches.

The conductivity of scale varies from 0.04 to 2.12, depending on its composition and temperature; steel varies from 21.5 to 30, and steam from 0.036 to 1.009.

The gas film is the dominating resistance to heat transfer with clean surfaces. However, accumulations of ash on the outside surface, or scale on the inside, may become the controlling resistance if allowed to continue unchecked, and will result in loss of efficiency. High metal temperatures and possibly failures will result from scale.

The rate of heat absorption in a given furnace, corresponding to a given rate of heat release, will vary over a wide range within the limits shown approximately in Fig. 9. It is impracticable to present in compact form curves more accurate than these, since the efficiency of heat absorption varies widely with the type of firing, fuel burned, amount and disposition of heating surface, and the cleanliness of that surface. Generally, the rapid combustion of pulverized coal and liquid or gaseous fuels occurs with a relatively short flame, and with a correspondingly high rate of heat absorption, particularly when combustion takes place near the bottom of the furance. This is found to be especially true with tangential firing.

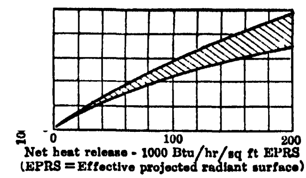


FIG. 9. Range of radiant heat absorption in water-cooled furnaces.

Heat transfer in tube banks, whether they form a part of the boiler, superheater, economizer, or air heater, differs in two ways from that in furnaces. The rate follows the laws of convection rather than radiation except for the relatively small quantity of nonluminous radiation from the CO_2, SO_2, and water vapor in the gas. The hourly quantity of heat transferred will be in direct proportion to the temperature difference between the hot and cold fluids. One or both of these temperatures will vary as the gas passes over the tube bank.

Fundamental heat-transfer equations (see also Section 3) which apply to most problems of boiler design are:

$$Q = R \times S \times \Delta t$$

where Q = heat absorbed, Btu per hour; R = overall rate of heat transfer, Btu per square foot per hour per °F; S = heating surface, square feet; and Δt = logarithmic mean temperature difference fluid to fluid, °F.

Also, neglecting leakages and radiation losses, the heat absorbed is equal to the heat given up by the gas or

$$Q = W \times c_p \times (t_1 - t_2)$$

where W = weight of gas, pounds per hour; c_p = mean specific heat of gas at constant pressure, Btu per pound per °F; and $(t_1 - t_2)$ = gas temperature drop, °F.

The overall rate of heat transfer in tube banks of boilers, superheaters, and economizers is dependent on the series of resistances, as in a furnace tube. The resistances through steam, water films, and tube wall are so small, however, that only the resistance of the gas film need be considered, with clean surfaces. In air heaters, the resistance through the air film is of the same order of magnitude as that through the gas film, so that the overall rate is $1/R = 1/R_g + 1/R_a$, where R = overall rate, Btu per square foot per hour per °F; R_g = thermal conductance through gas film, Btu per square foot per hour per °F; and R_a = thermal conductance through air film, Btu per square foot per hour per °F.

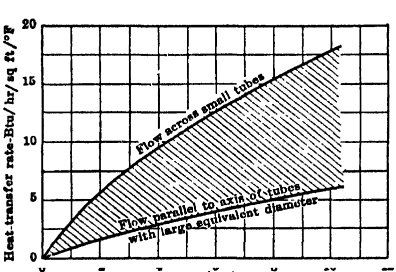


FIG. 10. Range of convection rate of heat transfer.

Whether the gas flows parallel or transverse to the axis of the tubes or through the tubes has a great effect on the rate of heat transfer. The approximate variation is shown in Fig. 10, where the upper limit represents flow across tubes of small diameter and the lower limit represents flow parallel to tubes on relatively wide centers. Pure cross flow or parallel flow are seldom found in boilers. This fact, along with stratification of gases and ineffective surfaces due to structural exigencies and ash accumulations, makes a background of practical and operating experience most important.

The temperature difference which produces the flow of heat depends on whether the temperature of one or both fluids vary, and the relative direction of both fluids passing through or over the heating surface, i.e., whether they flow counter to, parallel to, across one another, or some combination thereof. In any case, the mean temperature difference equals

$$\Delta t = \frac{\text{Greatest difference} - \text{least difference}}{\log_e \left(\dfrac{\text{greatest difference}}{\text{least difference}}\right)}$$

When the temperatures of both fluids vary, the greatest heat transfer with a given heating surface is obtained where they are arranged in counterflow. With the parallel flow arrangement, the highest temperature of the heated fluid can approach, but never reach, the lowest temperature of the heating fluid.

DRAFT LOSS. The various items included in the pressure differential across the convection surface of a steam-generating unit are friction due to flow across tubes; loss in head due to turns; friction due to flow through, or parallel to, tubes; and stack effect.

These equations show the mechanics of friction losses:

$$\text{Flow across tubes: } PD = fNH_v$$

$$\text{Turn loss: } PD = KH_v$$

$$\text{Flow along tubes: } PD = f\left(\frac{L}{D}\right)H_v$$

where PD = pressure drop, inches of water; f = friction factor, dimensionless; N = number of restrictions; K = constant, depending on type of turn; L = length of tube, feet; D = inside or equivalent diameter, feet; G = the mass velocity, pounds per hour per square foot furnace area; d = density, pounds per cubic foot; and H_v = velocity head, inches of water = $0.0002307[(G/1000)^2/d]$.

Data on the variation of friction factor with tube arrangement are available in the literature on the subject. For flow across tubes, a close approximation is 0.24 when the tubes are in line, and 0.36 for staggered tubes.

Turns in boilers and air heaters are usually of the severest type in that they are generally 180 degrees and very abrupt. The factor K may be taken as 1.5 when the velocity head corresponds to the average velocity at the turn.

The friction factor for flow along tubes varies, depending on Reynolds' number, tube diameter, and degree of roughness of the surface. A conservative approximation for this type of equipment is 0.02. The diameter to be used for flow through tubes is the actual inside diameter; for flow *along* a bank of tubes it is the *equivalent diameter*, four times the free gas-passage area divided by the gas-touched perimeter of the tubes.

Because of the great difference in coefficient of heat transfer between cross and parallel flow, lower velocities are usually used for the former in order to obtain reasonable draft losses. The overall rate of heat transfer Btu per hour per square foot, in the various parts of a typical high-capacity, high-temperature, steam-generating unit, normally falls within the following range: Waterwalls—45,000 to 80,000. Superheater—8000 to 12,000. Boiler—2000 to 5000. Economizer—3000 to 4500. Air heater—500 to 700.

BOILER CONSTRUCTION

By J. R. Kruse

3. SCOPE OF CODES FOR CONSTRUCTION

There are at least five sets of rules covering construction of steam boilers. Four of them are mainly for marine boilers. The code most widely used for construction of land-plant boilers is the Boiler Construction Code of the American Society of Mechanical Engineers, referred to herein as the Code. Codes for construction of boilers for marine use are U. S. Coast Guard Marine Engineering Regulations and Material Specifications; the American Bureau of Shipping Rules for Building and Classifying Steel Vessels; Boiler Construction Rules and Regulations by Lloyd's Register of Shipping; and Boiler Construction Rules for United States Navy Vessels.

All these rules are, in reality, "Safety Rules," written to provide maximum safety, and yet permit a construction well within practical methods. In the United States, most states have adopted the ASME Code as their safety code. A few have added several minor rules to the Code, for boilers operated in that state. A few others have their own rules, based to a great degree on the ASME Code. There are also a few states that have no rules at all, but it is common practice of boiler manufacturers to build boilers to meet the ASME Code.

All code committees are continually revising their rules to achieve uniformity. The ASME Code Committee is probably the most active; therefore, the other code bodies use ASME as a basis for revisions wherever practical. Many foreign countries are now accepting land-type boilers built to the ASME Code. Some, however, have rules that differ considerably from those of ASME.

Since all codes are continually being revised, it is important that the latest revised rules be followed in the design and construction of boilers. The latest editions or the addenda can be obtained from the following sources:

ASME Codes, from The American Society of Mechanical Engineers, 29 West 39th St., New York, N. Y.

Coast Guard Rules, from The United States Coast Guard, 1300 E St., Washington, D. C.

ABS Rules, from The American Bureau of Shipping, 45 Broad St., New York, N. Y.

Lloyd's Rules, from Lloyd's Register of Shipping, 17 Battery Place, New York, N. Y.

Navy Specifications, from the United States Navy, Boiler Section, Washington, D. C.

BOILERS AND UNFIRED PRESSURE VESSELS. Since construction requirements for unfired pressure vessels of the many different types are similar to those for boilers, the construction rules for both are similar in many phases.

The Boiler Code Committee of the American Society of Mechanical Engineers, in cooperation with other association committees, such as the American Society of Testing Materials, the American Welding Society, and the American Petroleum Institute, has prepared a number of different sections of the Code, listed below:

Section I. Rules for Construction of Power Boilers (includes Section VI, Rules for Inspection of Material and Steam Boiler and the Appendix).

Section II. Material Specifications.

Section III. Boilers of Locomotives.

Section IV. Low-pressure Heating Boilers.

Section V. Suggested Rules for Care of Power Boilers.

Section VII. Miniature Boilers.

Section VIII. Unfired Pressure Vessels.

Section IX. Qualification for Welding Procedure and Welding Operator.

An additional Unfired Pressure Vessel Code for Petroleum Liquids and Gases was prepared by a joint committee drawn from the ASME Committee and the American Petroleum Institute Committee. This code is designated as the API-ASME Code.

ENFORCEMENT OF CODE RULES. The enforcement of any code is carried out by duly authorized inspectors of the originating organization. This enforcement applies to construction, erection, and sometimes operation of the boilers. Enforcement of rules for boilers operating in the United States comes under the jurisdiction of the National Board of Boiler and Pressure Vessel Inspectors. This organization works closely with the ASME Boiler Code Committee, on which it has representation. The many boiler insurance carriers' inspectors hold National Board licenses, obtained only by taking written examinations given by one of the member states. These authorized inspectors are required to make shop and field inspections before certificates necessary for insurance coverage and operation can be issued to the owners.

FACTORS OF SAFETY USED IN VARIOUS CODES. No term has been misused so widely as the so-called factor of safety. Consider three types of vessel, designated as types A, B, and C.

Type A. Steam Boilers or Vessels Pierced with Unreinforced Holes. In these vessels, stress at the edge of holes may be over twice average hoop stress. In applying an initial hydrostatic test of ($1\ ^1/_2 \times$ working pressure) to a drum built with a so-called factor of safety of 5, the steel at the most highly stressed part is so strained as to produce a permanent set and redistribution of forces in the stressed area. The factor is based on ultimate strength, and with a steel in which yield strength is one-half the ultimate strength, yield strength is $2\ ^1/_2$ times average hoop stress at the working pressure. If stress at the sides of tube holes is ($2 \times$ average hoop stress), and the hydrostatic test is made at ($1\ ^1/_2 \times$ working pressure), stresses at the sides of tube holes are ($3 \times$ average hoop stress) at the working pressure. This would exceed yield strength of the material. If the so-called factor of safety is made less than 5, as is often done, there is a greater yielding of the material on application of the hydrostatic test and a greater redistribution of stresses.

Type B. Vessels Having No Holes or Other Stress Raisers in Their Shells. In these vessels, yield strength is not exceeded until hoop stress, which is uniform from end to end of the cylinder, is exceeded. Such vessels could be operated safely with a much lower so-called factor of safety than those with holes or other stress raisers in the shell. Application of a hydrostatic test of ($1\ ^1/_2 \times$ working pressure) to a vessel of this type, built with a factor of safety of 5, does not exceed the yield strength, as occurs with Type A vessels.

Type C. Penstocks for Boulder Dam. This construction is described by C. M. Day and Peter Bier of the U. S. Bureau of Reclamation in *Mechanical Engineering*, Aug. 1934. The yield strength of the special steel used in the penstocks is 38,000 psi; hoop stress was limited to 18,000 psi. The article describes tests made by elastic analysis, and by subjecting $^{1}/_{6}$ scale models of the penstocks to hydrostatic pressure. The parts were built so that maximum stress at any point would not exceed about 19,000 psi.

Each of the three types of vessels has its own field of application. Type A includes all pressure vessels having unreinforced holes, or in which unreinforced holes, as telltale holes for determining shell thickness, may be drilled after the vessels are in operation. This type is covered by ASME and similar construction codes.

Type B includes vessels constructed under rules of the Interstate Commerce Commission for transporting liquids and gases under pressure. For such vessels, a so-called factor of safety, of, say, 3 $^{1}/_{2}$ where there is no corrosion, gives as great a degree of safety as some Type A vessels where the factor is 5.

Type C represents the highest type of construction for pressure vessels where walls cannot be made of uniform thickness with no stress raisers. By limiting both hoop and maximum stresses, due to departing from uniform shell thickness, as well as at special branch connections and reinforced openings, no question can arise as to the effect of exceeding yield strength in any part of the structure.

THE ASME CODE FOR LOCOMOTIVE BOILERS specifies:

Factor of safety used in design and construction of new boilers shall be not less than 4.5.

Factor of safety used in determining maximum allowable working pressure calculated on conditions actually obtained in service shall be not less than 4.0.

Maximum allowable working pressure determined by conditions obtained in service shall not exceed that for which boiler was designed.

THE JOINT API-ASME CODE contains provisions for removing vessels from service if the factor of safety becomes lower than certain values. At first glance it would appear that factors of safety given in the API-ASME Code are lower than those given in the Unfired Pressure Vessel Section of the ASME Boiler Code in the ratio of 4 to 5. However, the factor of 4 is a figure below which vessels cannot be operated, whereas the ASME Code refers to initial factor of safety when boilers are built. Another feature in the two codes brings the two factors for stress-relieved unfired pressure vessels nearer together. In the API-ASME Code, tensile strength used in applying the formulas is that for coupons which are tested at the steel mill, after being stress-relieved in the same manner as the vessel will be stress-relieved; whereas in the ASME Code coupons for such vessels are not stress-relieved at the steel mill. In stress relieving the coupons, by holding them at a temperature of 1100 to 1200 F, for 1 hr per inch of thickness of plates, tensile strength may be lowered about 10%. The factor 5 in the ASME Code was considered ample to cover this lowering of the tensile strength; whereas in the API-ASME Code it was embodied in the Code on account of the lower operating factor.

The 1946 ASME Code for boilers and pressure vessels omits the term "Factor of Safety," in the formula for determining the maximum allowable working pressure. Instead, certain working stresses are allowed on various materials, at various temperatures. In effect this change is equivalent to reducing the so-called factor of safety to nearly 4 for some constructions.

RULES FOR CONSTRUCTION. In designing and fabricating boilers, it is essential that the proper rules be followed. These rules depend on type of service and locality, among other things. The correct rules may be ASME or state rules for a stationary boiler, operating in the United States. For merchant vessels under the United States flag, the rules of the U. S. Coast Guard or the Rules of the American Bureau of Shipping, or both, are applicable. If the boiler is for a Navy vessel of the United States, it may be built to the special Navy Specifications, or, if not a combat vessel, it may be built to either the ASME Rules or the U. S. Coast Guard Rules. If the boiler is for a vessel operating under a foreign flag, the Rules of Lloyd's Register of Shipping may apply.

These states require all stationary power boilers to be built according to the ASME Code rules: Arkansas, California, Delaware, District of Columbia, Indiana, Iowa, Louisiana, Maine, Maryland, Michigan, Minnesota, Nebraska, New Hampshire, New Jersey, New York, North Carolina, Ohio, Oklahoma, Oregon, Panama Canal Zone, Pennsylvania (a few minor exceptions or additions are required), Rhode Island, Tennessee, Texas, Utah, Vermont, Washington, West Virginia, and Wisconsin. The above states may have their own codes, which are the same as ASME. These states have boiler laws, but do not require that the boilers be built to any specific rules: Colorado, Connecticut, Mississippi, and Montana. These states will accept standard ASME boilers, or boilers stamped by the National Board. Massachusetts has her own boiler codes similar to the ASME Code. Slight differences are found in a few sections. The remaining states do not have boiler laws, but many of the Canadian provinces accept boilers built to the ASME Code rules.

The foregoing data on States Boiler Rules are in accordance with the Sept. 1, 1949, Synopsis of Boiler Laws as published by the National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y. This synopsis is brought up to date each year.

All ASME codes are kept up-to-date by issuance of annual code "pink sheet addenda," available about August of each year. "Interpretations" of the codes are issued whenever necessary, to clarify the intent of the code.

PREAMBLE TO THE ASME POWER BOILER CODE. To have better understanding of just what the Power Boiler Code scope is, the preamble to this code section is quoted.

Preamble

This code covers rules for construction of power boilers to be used in stationary service. Stationary boilers as herein considered include portable and tractor boilers.

The Code does not contain rules to cover all details of design and construction. Where complete details are not given, it is intended that the manufacturer, subject to the approval of the authorized inspector, shall provide details of design and construction which will be as safe as otherwise provided by the rules in the Code.

A pressure vessel in which steam is generated by the application of heat resulting from the combustion of fuel (solid, liquid, or gaseous) shall be classed as a fired steam boiler.

An unfired pressure vessel which generates steam for power or heat to be used externally to itself shall be classed as an unfired steam boiler. Such vessels may be constructed under the appropriate classification of Section VIII of the Code and shall be equipped with the safety devices required by Section I of the Code in so far as they are applicable to the service of the particular installation.

The material for forced-circulation boilers and boilers with no fixed steam and water line shall conform to the requirements of the Code. All other requirements shall also be met except where they relate to special features of construction made necessary in boilers of these types, and to accessories that are manifestly not needed or used in connection with such boilers, such as water gages, water columns, and gage cocks.

Separately fired steam superheaters which are not integral with the boiler or are separated from the boiler by stop valves are considered fired pressure vessels and their construction shall comply with Code requirements, including all piping, valves, and required safety devices, from the inlet flange to the outlet flange. If welding ends are used at the inlet or outlet of the superheater, Code requirements shall begin or end at the weld where flanges, if used, would have been placed. Such attachment welds to external connecting pipe are not within the scope of the Code, if they are not exposed to high-temperature gases.

These rules apply to the boiler proper and pipe connections up to and including the valve or valves as required by the Code. Superheaters, reheaters, economizers, and other pressure parts connected directly to the boiler without intervening valves shall be considered as parts of the boiler and their construction shall conform to the Code rules.

The eleventh edition of this Handbook carried voluminous extracts from the ASME Boiler Construction Code. Since it was published, increase in pressure and temperature has necessitated expansion of the Code's size so that abstraction of its requirements in useful form has become impossible. The reader, to be certain he has the latest information, should consult the latest revision. This is available (*ASME Boiler Construction Code* [Combined Edition], 1949 Edition) through ASME, 29 W. 39th St., New York, N. Y., for $12.50.

MOISTURE, SUPERHEATERS, AND REHEATERS

By P. B. Place and F. I. Epley

4. MOISTURE IN STEAM

By P. B. Place

CARRYOVER. Steam may carry various amounts of water and impurities out of the boiler. This contamination of the steam is called carryover.

Steam quality is the *liquid* contamination in the steam, expressed in percentage, by weight, of the mixture. Thus 99.8% quality steam contains 0.2% moisture. Steam quality is determined by calorimetric methods on saturated steam samples.

Steam purity is the *solids* impurity in the steam, expressed in parts per million (ppm) of impurity. Thus a 1 ppm steam contains 1 part by weight of solids contamination per million parts of steam. Steam purity is determined by evaporation, or by conductivity determination, of condensed steam samples of saturated or superheated steam.

If the moisture in the steam is boiler water of known concentration, the following equality exists between steam quality and steam purity:

$$\text{Ppm impurity} = \frac{\%\ \text{moisture} \times \text{ppm in boiler water}}{100}$$

$$\%\ \text{moisture} = \frac{\text{ppm impurity} \times 100}{\text{ppm in boiler water}}$$

STEAM SAMPLING. The objective in steam sampling is to obtain a sample of steam representative of the total steam flow. Usually some form of perforated nozzle is used in the saturated header of the superheater, in the steam circulators between the boiler and superheater, or in a main steam line. Figure 1 shows types of sampling nozzles suitable for use in headers, tubes, and pipes. In pipe and tube nozzles, the sampling holes face upstream and are spaced to sample from equal areas of cross section of the line.

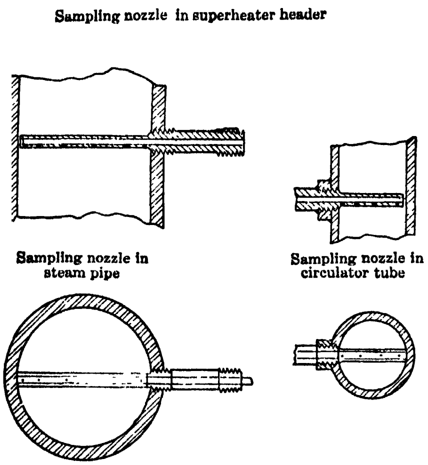


FIG. 1. Types of steam sampling nozzles.

Sampling rates are determined by the relation

$$f = \frac{a}{A} \times F$$

where f and F are sample and steam flow rates and a/A is the ratio of total sample hole area to pipe area. The total area of sample holes should be less than the cross-section area of the nozzle to insure a small pressure drop and distribution of flow. The number of holes may vary from 4 for pipe diameters up to 6 in., to 8 for pipe diameters over 12 in.

Sampling nozzles should be located after a length of straight pipe equal to at least ten diameters. Locations in order of preference are (1) vertical pipe, downward flow; (2) vertical pipe, upward flow; (3) horizontal pipe, vertical insertion; and (4) horizontal pipe, horizontal insertion. Steam containing less than 1% moisture is more easily and accurately sampled than steam containing higher amounts of moisture.

TYPES OF CARRYOVER. There are three types of carryover: (1) slugs of water, (2) foam, and (3) spray.

Priming. Carryover of type 1, known as priming, may occur under conditions of high water level or severe surging in the boiler drum. Priming is essentially mechanical and usually due to too high water level or spouting of submerged risers.

Foamover. Carryover of type 2, known as foamover, occurs when foam accumulates in the boiler drum and is carried out by the steam. Foamover is the most common type of excessive carryover. It is due to foaming of the boiler water and is caused by stabilization of bubble films by impurities in the boiler water. High concentrations, organic matter, oil, and suspended matter are often causes of foaming.

The American Boiler Manufacturers' Association recommends the following boiler-water concentration limits for various operating pressures:

Pressure, psi	Concentration, ppm
0 to 300	Not over 3500
301 to 450	Not over 3000
451 to 600	Not over 2500
601 to 750	Not over 2000
751 to 900	Not over 1500
901 to 1000	Not over 1250
1001 to 1500	Not over 1000

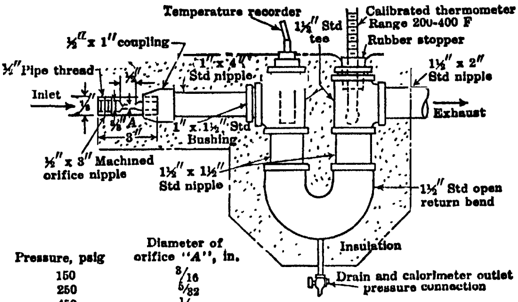
These limits are arbitrarily based on average experience. In some cases, excessive foaming may not develop at higher concentrations; in other cases excessive foaming may develop at lower concentrations.

Spray. Carryover of type 3 occurs when spray, mist, or fog, which are degrees of atomization of the boiler water, are steam borne from the drum in much the same manner as dust is carried by air currents. Spray carryover is due to incomplete purification of the steam, leakage in drum baffles, or operation beyond velocity limitations of the purification equipment. The normally small contamination in commercial steam is usually of this type.

STEAM-QUALITY DETERMINATIONS. Steam calorimeters are used to determine the quality of saturated steam, i.e., the amount of moisture in the steam. There are several types of calorimeters, but the most common is the throttling calorimeter.

The throttling calorimeter is a device in which a flowing sample of saturated steam of known temperature and pressure is expanded through an orifice to a lower pressure, usually atmospheric. The passage through the orifice and calorimeter involves no heat loss other than radiation, and since the total heat in the sample is greater than that of saturated steam at lower pressure, the excess heat in the sample is used in superheating the expanded sample and/or evaporating any moisture in the sample. If the temperature and pressure of the steam entering and leaving the calorimeter are known, the amount of moisture in the sample can be determined.

Figure 2 shows a design that can be made of standard pipe fittings and in which both a temperature recording bulb and calibrating thermometer may be used.



Pressure, psig	Diameter of orifice "A", in.
150	3/16
250	5/32
450	1/8
650	3/32
850	1/16

FIG. 2. Home-made throttling calorimeter.

The apparatus required for throttling calorimeter test is a suitable means of sampling the steam, the calorimeter, and a means of measuring the pressure and temperature of the inlet and outlet steam. Pressure of inlet steam is usually measured by calibrated steam pressure gage, and inlet-steam temperature may be obtained from steam tables or by thermometer in a suitable well, located upstream of the calorimeter orifice. Outlet-steam pressure is usually checked with a mercury manometer attached to the exhaust chamber of the calorimeter. In most cases, the orifice size and steam flow are adjusted to give

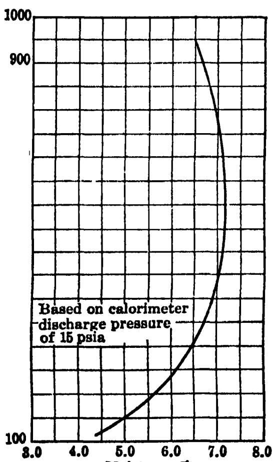


FIG. 3. Maximum moisture capacity of throttling calorimeters.

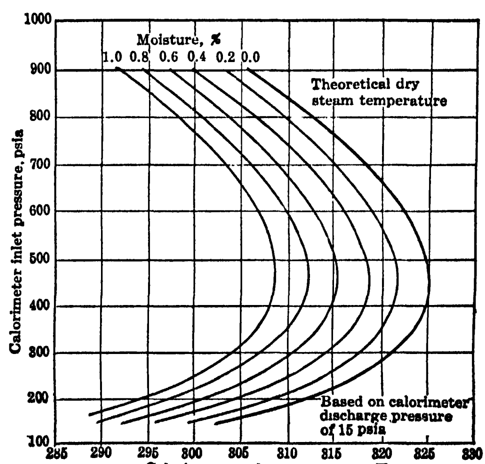


FIG. 4. Throttling calorimeter outlet temperature for moisture up to 1%. Values shown by 0% moisture curve are theoretical temperatures resulting from throttling dry and saturated steam from the pressures shown by the ordinate to 15 psia. They are useful in establishing the temperature correction. See example.

atmospheric discharge pressure. Outlet-steam temperature must be carefully measured with a calibrated thermometer or thermocouple. Radiation loss can be reduced by inserting the thermometer or couple in the exhaust steam flow without shielding in a thermometer well.

A minimum steam flow of 250 to 300 lb per hr through the calorimeter is desirable to minimize radiation effects. Different-sized orifices are required for different operating pressures to obtain suitable sampling rates.

Above and below 450 psig, the total heat in saturated steam decreases, and the available heat- and moisture-determining capacity of the calorimeter decreases. The practical range of operation for throttling calorimeters is from 150 to 900 psig, with most satisfactory performance between 350 and 550 psig. Figure 3 shows the maximum moisture content determinable by throttling calorimeter at various operating pressures, and Fig. 4 shows the maximum temperatures at calorimeter outlet for different pressures and moisture contents up to 1%. Steam containing more than 1% moisture is difficult to sample, and determinations of high moisture are subject to errors traceable to sampling difficulties.

Moisture in the steam is calculated from this equation:

$$m = \frac{H_1 - H_2}{L}$$

where m = pounds of moisture per pound of steam sample; H_1 = total heat in saturated steam at inlet temperature and pressure; H_2 = total heat in superheated steam at exhaust pressure and exhaust temperature (corrected for radiation); and L = latent heat of evaporation at inlet pressure.

Example.

Calorimeter inlet pressure	440 psia
Calorimeter back pressure	0.6 in. Hg
Calorimeter normal temperature	323 F
Calorimeter outlet temperature	318 F

Pressure correction:

0.6 in. Hg = 0.3 psi
14.7 + 0.3 = 15.0 psia

Temperature correction:

Theoretical dry steam temperature	= 325 F
Calorimeter normal temperature	= 323 F
Temperature correction	= + 2 F
Corrected outlet temperature	= 320 F
Total heat in sample at 440 psia	= 1204.6 Btu/lb
Total heat at 15 psia and 320 F	= 1202.2 Btu/lb
Latent heat at 440 psia	= 770.0 Btu/lb
Moisture in steam	= (1204.6 − 1202.2)/770.0
	= 0.003 lb per lb steam
	= 0.3%
Steam quality	= 99.7%

The separating calorimeter is a separation device consisting essentially of a perforated cup. Moisture is thrown out and deposited in a separating chamber while the dry steam passes up and out of the separating device through an annular steam jacket around the separating chamber. The dry steam discharges through an orifice. The steam flow may be determined by condensing and weighing, or by calculating the flow by Napier's equation provided the size of the orifice is known (see Sections 1 and 3).

The amount of moisture deposited in the separating chamber can be read directly from a gage glass graduated in $^1/_{100}$ lb units. Although the accuracy of this type calorimeter is less than that of the throttling type, it has a much wider range. If well insulated, the radiation loss is less than 0.05%.

The universal calorimeter consists of a separating and throttling calorimeter, of high and low range, respectively, in series. If Y_1 = percentage of moisture, by weight, in steam as determined by the combination calorimeter; w_1 = weight of moisture collected in separating calorimeter in a given time, pounds; w_2 = weight of dry steam condensed after passing through the throttling calorimeter, pounds; Y_2 = proportion, by weight, of moisture in steam discharged from separating portion as determined by throttling calorimeter; then, without radiation losses,

$$Y_1 = \frac{w_1 + w_2Y_2}{w_1 + w_2} \times 100$$

The electric calorimeter is one form of superheating calorimeter. Steam enters the bottom of the calorimeter, passes upwards over heating coils, and thence to atmosphere. To determine moisture content, it is necessary to know the electrical input, temperature of the exhaust steam, and the formulas and constants furnished by the manufacturers of the apparatus.

STEAM-PURITY DETERMINATIONS. Evaporation Method. The solids impurity in steam may be determined by evaporation of a known amount of sample under carefully controlled conditions. A relatively large sample of condensate is evaporated in a lightweight dish by radiant heat. The evaporation is done within a dustproof cabinet through which purified air passes to remove vapors. Automatic adjustment of sample feed to the evaporating dish is usually provided, and, when all the condensate is evaporated, the residual solids are determined by the difference between the final and original weight of the evaporating dish.

Conductivity Method (Ref. 1). Carryover of dissolved, ionizing, boiler water salts is usually determined by measuring the *electrical conductivity* of the condensate. The method does not measure suspended matter, organic material, or unionized solids such as silica. The usual unit of measurement is the micromho, which is the reciprocal of the resistance, in millions of ohms, corrected to standard temperature of 77 F.

$$\text{Equivalent ppm impurity in steam} = \text{micromho} \times 0.6.$$

Dissolved gases that ionize in solution, such as NH_3 and CO_2, must be eliminated from the sample by a degasification process, or suitable correction made for their presence. Continuous degasification and conductivity recording equipment is available which makes this method practical for continuous measurement of steam impurity of less than 0.5 ppm. Portable test equipment consists of a conductivity dip cell (usually having a cell constant of 0.1), a sample degasifier or chemical means of determining the amount of NH_3 and CO_2 in the sample, and a thermometer for checking condensate temperature. An approximate correction for dissolved gases may be obtained by determining the difference between the conductivity of the condensate before and after boiling (and recooling).

EXAMPLE.

Conductivity of condensate sample	= 4.5 micromhos
Conductivity after boiling and recooling	= 1.2 micromhos
Conductivity correction for gases	= 3.3 micromhos
Ppm impurity in steam = 1.2 × 0.6	= 0.7 ppm

STEAM PURIFICATION is obtained in boiler drums by means of baffles and devices known collectively as *drum internals*. The process of purification is a stage process that involves (1) primary separation, (2) steam washing, and (3) steam drying.

Primary separation of the bulk of the circulating water from the steam is the first step. This stage of purification, always necessary in water tube boilers and when generated steam and circulating water are delivered to a drum as a mixture, involves separation of a relatively large amount of water from the steam; such principles as gravity separation, change in direction, and centrifugal action are utilized. Separation of bulk water is not difficult, and failure of primary separation is usually due to failure to separate that portion of the water that is present in the form of excessive spray and/or foam films. Over 98% of the circulating water should be separated in this primary separation step to avoid possible overloading of the subsequent steps of washing and drying. Primary separation in fire tube boilers occurs below the water level in the drum, and separation equipment is not involved.

Steam washing is an optional step in purification, in which the separated steam is washed or rinsed in low concentration feedwater to reduce the concentration of impurities in the final moisture in the steam. Washing is not usually applied to single-drum boilers unless the drum is large enough to accommodate this extra step. Washing is a process of mixing the feedwater with the steam, and the steam is usually passed through the water dispersed as a spray. Current opinion is that steam washing with condensate feedwater reduces volatilized silica carryover in the steam.

Steam drying is the final stage of purification in which the residual moisture in separated or washed steam is removed. Drying, a process of removing small amounts of moisture from a relatively large volume of steam, is essentially a filtration process. Dryers have large surface areas on which moisture is deposited as it passes through at relatively low velocity. At low operating pressure, steam velocity through the dryer may be 8 to 10 ft per sec, but at high operating pressure this velocity should be 2 to 3 ft per sec or less. Dryers are designed to separate relatively small amounts of moisture and become overloaded when steam entering the dryer contains more than 5 to 10% moisture. Dry pipes and dynamic dryers, utilizing centrifugal action, reduce moisture in steam to tenths of 1%, but filter-type dryers, utilizing closely spaced screens or bent plates, are required to obtain steam having high purity. (See also Refs. 2 and 3.)

PERFORMANCE OF DRUM INTERNALS. Failure to obtain satisfactory steam quality and purity is usually due to one or more of these causes.

1. Excessive foaming of the boiler water, which is a chemical phenomenon and over which drum internals may have little control.
2. Improper installation of internals which may result in leakage of water or impure steam through assembly joints and contamination of the outlet steam.

3. Improper design of internals for the condition involved.
4. Exceeding the design capacity of internals which may be limited by such factors as pressure drop, drainage capacity, or flow velocity.
5. Plugging of internals with chemical sludges, poor distribution of water or steam flow, or chemical or mechanical damage to internals which may result in leakage.

5. SUPERHEATERS

By F. I. Epley

THE SUPERHEATING PROCESS. When water is heated to the boiling point in a closed vessel, the vapor released causes the pressure to rise. As the pressure increases, the boiling temperature also rises. Steam is usually generated in a boiler at constant pressure. During the change of state from a liquid to a vapor at constant pressure, the vapor in contact with the liquid remains at constant temperature until vaporization is complete. When all the water has been transformed into steam, further addition of heat will raise the temperature, causing the steam to be superheated.

Water enters the boiler at some specified pressure and temperature, and heat is added to bring it to the boiling point. Further heating causes evaporation of the water at the saturation temperature corresponding to boiler pressure. The saturated steam is then removed from the saturated liquid, and heat is again added at substantially constant pressure, the only pressure differences being those due to friction in various sections of the boiler.

The properties of steam accepted as a commercial standard are given in *Thermodynamic Properties of Steam*, by Keenan and Keyes, John Wiley and Sons, 1936. (See also Section 4.) A working knowledge of these tables is necessary for quick and accurate solution of thermodynamic problems of the steam boiler.

Saturated steam is delivered to the superheater in an almost-dry state. Steam containing water in any form, either as minute droplets, mist, or fog, or due to entrainment of water in boiling or to partial condensation is called *wet steam*. The enthalpy of the mixture is less than that of dry saturated steam at the same pressure, because vaporization is incomplete. If the steam contains 3% moisture, the quality is said to be 97%. For modern steam-generating units, the quality of the saturated steam delivered to the superheater is very high, in many boilers over 99.75%.

With saturated steam, the heat available for doing useful work depends entirely on the pressure, whereas with superheated steam additional heat is available as the degree of superheat increases. This additional available energy, obtained through increased expenditure of fuel, yields economic benefits and a net efficiency gain of considerable magnitude.

Because of the increase in thermal efficiency of the heat cycle and the reduced erosion of turbine buckets under the lower moisture conditions, steam superheaters are always used in modern steam power stations. In fact, the ability to obtain relatively high steam temperatures has made it economically possible to design and build high-pressure steam power generating units.

Practical limits of steam temperature and pressure are set by the materials available for superheater construction. Considerable development in the metallurgy of alloy steels has taken place during the last few years, as well as in manufacture of both tubing and finished sections of the superheater. These developments have made possible the design of satisfactory superheaters for high-temperature and high-pressure boiler installations. Improvements in welding and other methods of fabrication have also contributed greatly to the development of such installations.

THE INTEGRAL SUPERHEATER is a bundle of tubes located within the boiler setting, and receiving heat from the same gases that generate steam in the boiler. It may be either of the *radiant* or *convection* design, or a combination of both, depending on the manner in which heat is transferred from furnace gases to the steam.

Radiant superheaters absorb heat by direct radiation from the furnace gases, and may be located in one or more furnace walls. Since the furnace temperature and, therefore, the amount of heat available from radiation, does not rise as rapidly as the rate of steam flow, a radiant superheater has a *falling characteristic*, i.e., the steam temperature drops as the steam output rises. Tubes located in the furnace walls absorb heat at a high rate, and in order to minimize tube failures, high mass flows of steam through the tubes are necessary; this can be achieved only at the expense of pressure drop. The steam-temperature characteristic of a straight radiant superheater is shown as curve *A* in Fig. 5.

Convection superheaters absorb heat by impingement and flow of hot gases around the tubes. A true convection superheater has a *rising characteristic.* Mass flow and temperature of the gas entering the superheater zone, as well as the steam flow from the boiler, increase with an increase in firing rate. These changes in temperature produce a greater mean temperature difference between the gas and steam. This, together with the higher gas mass flow, causes an increased rate of heat absorption, resulting in an increased steam temperature. A convention superheater which is not entirely shielded from the furnace combines to some extent the effects of both radiant and convection heat absorption, resulting in a more nearly constant degree of superheat over the range of outputs. The steam-temperature characteristic of a straight convection superheater is shown as curve *B* in Fig. 5.

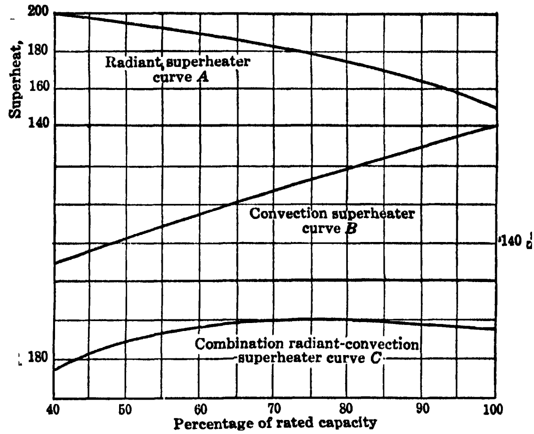


FIG. 5. Typical superheater performance, illustrating characteristic variation in superheat with load of radiant, convection, and combined types.

A combination of the rising steam-temperature characteristic of the convection superheater with the falling characteristic of the convection superheater has been used on a few installations to maintain nearly constant steam temperature over a greater range of ratings. The steam-temperature characteristics of such a combination of superheaters is shown as curve *C* in Fig. 5.

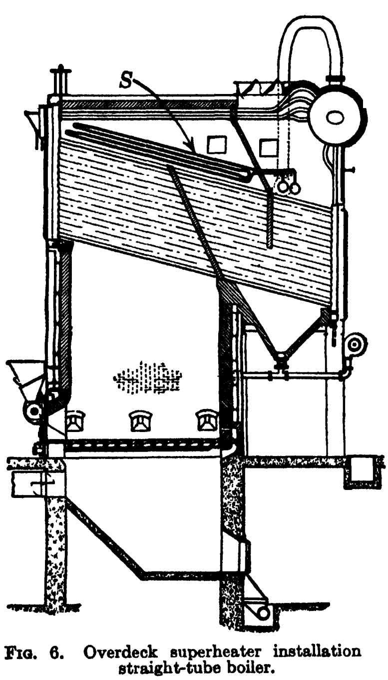


FIG. 6. Overdeck superheater installation straight-tube boiler.

SUPERHEATERS FOR STRAIGHT-TUBE BOILERS. For moderate steam temperatures, the superheater can be located above the boiler tube bank. The superheat which can be obtained in such a location is limited principally by the gas temperatures and the available space. Operating conditions and the design of the furnace and boiler also have a great effect.

In general 200 F superheat can be obtained with economical sizes of superheaters in coal-fired boilers without appreciable water-cooled surface in the furnace. With oil and natural gas fuels, the superheat will be about 150 F. A typical "overdeck" superheater installation is shown in Fig. 6.

When furnace walls are covered with generous amounts of water-cooled surface more heat is absorbed from the furnace gases, resulting in lower furnace-gas temperatures and lower gas temperatures throughout the boiler. The gas temperature entering the superheater is reduced to such a degree that considerably more surface is required for the same superheat. Approximately twice as much superheater surface is required in a boiler with full water-cooled surface, as in the same boiler with a refractory furnace.

When the superheat becomes too great, it is necessary to split the boiler tube bank to provide space to install an "interdeck" superheater over a small bank of screen tubes. In this way the superheater surface can be considerably reduced, owing to the higher gas temperature at this point. The steam temperature obtained is greatly increased with such an arrangement, and is sufficient to cover most needs of the present-day power plant.

SUPERHEATER FOR SEMIVERTICAL BOILERS. Superheaters can be installed in bent-tube boilers in an intertube position, as in Fig. 7. With this arrangement a large

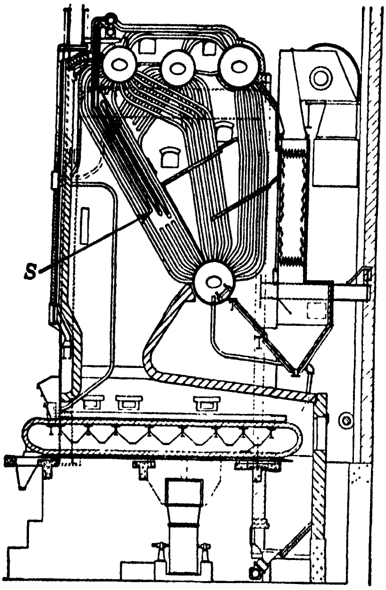


FIG. 7. Intertube superheater installation in bent-tube boiler.

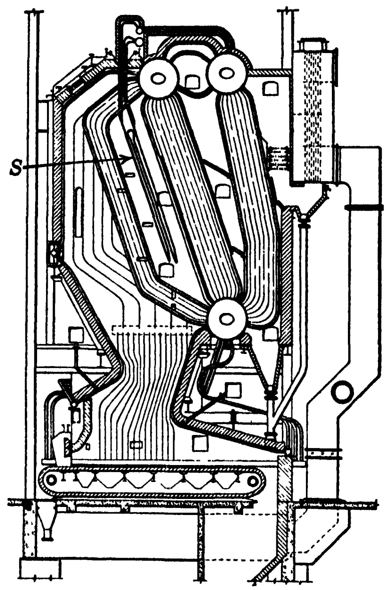


FIG. 8. Interbank superheater installation in bent-tube boiler.

part of the heat is absorbed by direct radiation from the furnace, resulting in a relatively flat steam temperature characteristic. This type can be installed in most semivertical boilers, although construction details in some boilers make such a design impractical.

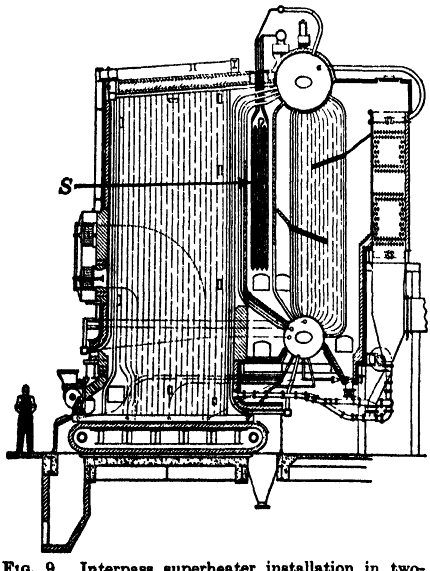


FIG. 9. Interpass superheater installation in two-drum vertical bent-tube boiler.

Most superheaters for semivertical boilers are installed in an interbank location, as shown in Fig. 8, or in an interpass location as shown in Fig. 9. The superheat obtained with either arrangement is ample for most requirements except where the available space is limited.

SUPERHEATER SURFACE REQUIRED. The relation between heat absorbed by steam in the superheater and that given up by gases of combustion, radiation included, is

$$AUT = Wc(t_1 - t_2)$$
$$AUT = W_1c_1(T_1 - T_2)$$

where A = area of superheater surface, square feet; U = conductance, Btu per hour per square foot per degree mean temperature difference; T = logarithmic mean temperature difference between steam and gases of combustion; W, W_1 = respectively, weight of gases of combustion and weight of steam passing through superheater per hour; c, c_1 = respectively, mean specific heat of gases of combustion and of steam; t_1, t_2 = respectively, gas temperature entering and leaving superheater, °F; and T_1, T_2 = respectively, temperature of superheated steam and saturated steam.

Surface required is $A = H/UT$, where H = heat to be absorbed by superheater, Btu per hour = $(h_2 - h_1)$; h_1, h_2 = respectively, total heat of superheated and saturated steam, Btu (see Section 4).

$$T = \frac{(\text{Maximum temperature difference}) - (\text{minimum temperature difference})}{\log_e \left(\dfrac{\text{Maximum temperature difference}}{\text{Minimum temperature difference}}\right)}$$

Conductance U varies with temperature of gases, gas velocity, steam velocity, tube size and spacing, surface cleanliness, and other variables of minor importance. Approximate values for 2-in. tube elements are

Mass flow	2000	3000	4000
U, interdeck superheater	7–7.5	8.5–9.5	10–11.5
U, overdeck superheater	6–6.5	7.5–8.5	9–10

Mass flow is defined as pounds of gas or steam per hour per square foot of minimum free flow area.

Temperature drop through the steam film in superheater tubes for various rates of heat absorption and steam mass flows is given in Fig. 10. Temperature drop $t_d = H/U_f$;

Table 1. Superheater Surface Required for Various Superheats

Conditions: Capacity, 300,000 lb steam per hr; pressure, 410 psig at superheat outlet; gas weight, 350,000 lb per hr; gas mass flow, 4000 lb per hr per sq ft of minimum free flow area through superheater.

Interdeck. Entering Gas, 1950 F			Overdeck. Entering Gas, 1300 F	
Superheat, °F	Sq ft		Superheat, °F	Sq ft
	Parallel Flow	Counterflow		
100	1400	1396	100	3220
200	3032	2962	200	8200
300	5275	4940	. . . .	. . . .
400	8835	7550		.

$U_f = 0.95G/1000$, where U_f = steam film conductance, Btu per hour per square foot of inside tube surface; G = mass steam flow; other notation as above. The curves are based on a specific heat of steam of 0.55 and a friction factor of 0.004. For any other specific heat or friction factor, temperature drop will vary inversely as the square root.

Table 1 illustrates typical data on superheater design.

CONTROL OF STEAM TEMPERATURE. For steam temperatures of 900 F and higher, it is necessary to use some means of control to hold the steam temperature within safe limits. Occasionally specifications require the designer to incorporate steam-temperature control for installations involving steam temperatures as low as 750 F.

Various means are employed to control steam temperatures. Some of these are described below.

Control by Firing. It is practical to control firing of fuel in the furnace so that the heat absorption in the furnace walls can be controlled over a broad range, thereby creating a change in the gas temperature entering the superheater. The normal furnace temperature characteristic falls off more at partial loads than the gas temperature required to maintain constant steam temperature, but with certain types of burners and fuels it is possible to raise the furnace temperatures at partial loads to compensate for this. In this way constant steam temperatures can be maintained over a wide range of output.

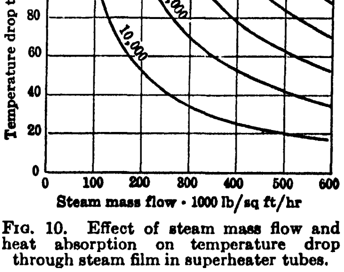


FIG. 10. Effect of steam mass flow and heat absorption on temperature drop through steam film in superheater tubes.

Damper Control. By-pass dampers for controlling steam temperatures have been widely used in this country. The general design is practically the same in nearly all cases, differing only in details. The principle involved is by-pass of a larger or smaller portion of the gases, only a part of the gases going over the superheater surface. In general the design has given satisfactory operation provided the damper mechanism is constructed to withstand the gas temperatures.

Desuperheating Control. Desuperheaters, both of the spray and noncontact type, have been used for steam-temperature control. The superheater is designed with excess surface so that at partial loads there is sufficient surface to give the required steam temperature. The desuperheater then is used to remove the heat represented by the excess steam temperature.

Desuperheaters can be located either at the superheater outlet or between sections of the superheater. In the former arrangement it is necessary to design the superheater materials to withstand steam temperatures in excess of the final superheated steam temperature. The excess depends on operating conditions but frequently exceeds 100 F for a high-temperature installation.

The desuperheater can be placed between the primary and secondary sections of the superheater. In this way it is possible to design the high-temperature section of the superheater for actual operating conditions existing at the outlet of the superheater, making allowances for only nominal excesses. The low-temperature section would then be uncontrolled, and the materials would be adequate to withstand the highest expected steam temperature.

For noncontact desuperheaters, the degree of control is less for those located interstage because of the lower temperature difference. However, the use of an interstage desuperheater gives a safer boiler unit than one in which the desuperheater is located beyond the superheater outlet.

When condensate or excellent feedwater is available, spray-type desuperheaters offer a satisfactory and economical means of control of steam temperature. Here too the interstage location of the desuperheater is preferable to a location beyond the superheater outlet. The degree of control is at least equal to any other means used in modern power boilers. With the spray desuperheater impurities may be injected into the superheated steam, and carried over into the turbine. With condensate such carryover usually is negligible. For the usual high-pressure high-temperature installation, each 10 F reduction in steam temperature requires about 0.5% of condensate or total feedwater flow to the boiler or economizer.

Control by Superheater Design. By combining radiant and convection surfaces in proper proportions, it is possible to maintain a substantially constant steam temperature over a range of loads. To make full and effective use of this device, it may be necessary to adjust burners or fuel-burning conditions in the most favorable manner.

EFFECT OF FEEDWATER TEMPERATURE. For the same fuel-burning rate, superheat increases with decrease in temperature of feedwater. Gas weights and temperatures entering superheater will not change, but steam weight through superheater will be less, since more heat is required to evaporate each pound of water. For the same capacity, the superheat will vary approximately in direct proportion to the heat absorbed per pound of steam in boiler and superheater (and economizer, if any), for a change of not more than 150 F in feed temperature. Excessive superheat may result if it is necessary to supply the boiler with cold feedwater.

SEPARATELY FIRED SUPERHEATERS. Where steam is used for process, close temperature control may be imperative. A separately set and separately fired superheater is sometimes employed. Such a unit may be directly fired with coal, oil, gas, or other fuels, and designed for any practical capacity or range of operating conditions. Automatic combustion or fuel control may be applied. Where the installation of integral superheaters is impractical, separately fired superheaters have been used in conjunction with existing power boilers. Separately fired superheaters are also designed for use in testing, to cover a wide range of operating conditions. In special cases, such superheaters have been designed for 1400 F low-pressure operation, but the majority of such installations are designed for 700 F and less.

THICKNESS OF SUPERHEATER TUBES. The ASME Code for power boilers prescribes methods of determining the allowable working pressure for superheater tubes. Stresses for the more common materials used in superheaters are given in Fig. 11. Since the maximum allowable stress is dependent on the *metal* temperature of the superheater tubes, it is necessary to have a means of predicting it. The principal factors entering into calculation of tube-wall temperatures are (1) quantity of heat being transmitted through the tubing, (2) steam velocity on inside tubes, and (3) degree of uniformity of steam temperature delivered from the multiple steam paths of the superheater. For the normal convection designs the difference between average steam temperature leaving the superheater and maximum tube temperature is less than 125 F, for high-temperature, high-pressure installations. For lower temperature installations, this difference may be as low as 75 F. An average value of 100 F may be used for the usual convection design of superheater. The effect of mass flow and heat absorption on the temperature drop through the steam film is illustrated in Fig. 10.

Convection superheaters located in moderate gas temperature zones receive a relatively small amount of heat directly from the furnace, hence heat absorption per square foot is low or moderate. On the other hand, radiant superheater surface located in the furnace where temperatures are high has a rate of heat absorption comparable to that obtained in furnace water-cooled surface, hence may have tubing temperatures considerably higher than in convection superheaters. Reduction in tubing temperature results from using high steam velocities to afford the maximum possible cooling effect of the steam, but even with high steam velocities it is not uncommon to have tubing temperatures which exceed steam temperatures by 300 F or more. For this reason, low steam velocities and low pressure drops are inadvisable for radiant superheaters.

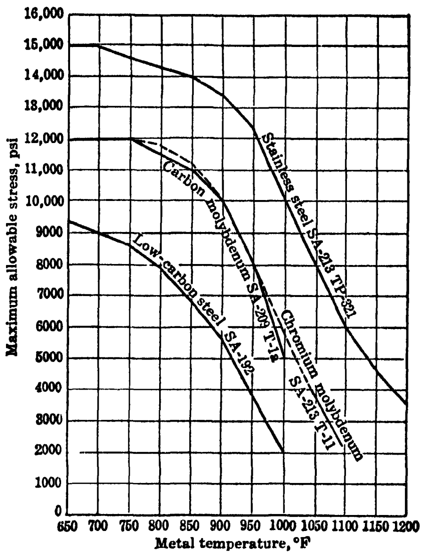


FIG. 11. Maximum allowable stresses for superheater materials. (Adapted from ASME Boiler Code, Table P-5)

USE OF ALLOY STEEL IN SUPERHEATERS. Superheaters designed for 800 F and higher may require use of several grades of alloy steel at various locations in the superheater because of temperature variation. Design pressure has a greater effect on the *thickness* of tubing required and a lesser effect on the *grade* of material required. General practice indicates that these materials are satisfactory for the stated *metal temperature* limits.

Material	ASME Specification No.	Temperature Limit, °F
Low carbon	SA-17 SA-192	850
Carbon molybdenum	SA-209	950
1 to 1/4% Cr, 1/2% Mo	SA-213 T-11	1050
5% Cr, 1/2% Mo	SA-213 T-16	1100
18% Cr, 8% Ni	SA-213 TP-321	1300

These data are to be used only as guides to practice. Because of economic and other considerations, the designer may find it satisfactory to deviate from these values.

PRESSURE DROP IN SUPERHEATERS. Ample steam-pressure drop in superheaters is desirable from the standpoint of the superheater designer. The greater the pressure drop and the higher the steam velocity, the more adequate is the protection to the superheater tubes; excessive pressure drop in the superheater, however, results in higher boiler design pressure. Experience indicates that a steam pressure drop through the superheater of 3 to 5% of the boiler design pressure is satisfactory. On large, high-capacity units it may be advantageous to use a higher pressure drop to simplify design.

Pressure drop in superheater tubes can be expressed by

$$p = \frac{400fV}{D} \times \left(\frac{G}{100{,}000}\right)^2$$

where p = pressure drop, pounds per square inch; f = friction factor; V = specific volume of steam, cubic feet per pound; D = inside diameter of tube, inches; G = steam mass flow, pounds per hour per square foot of free flow area. Bends will increase p from 50 to 100%, depending on their number per 100 ft of tube and their radius. Additional pressure drop for each 90-degree bend, expressed in equivalent feet of straight pipe is, approximately,

Radius of bend, tube diameters	1	2	3
Equivalent straight pipe, ft	6.2	4.3	3.3

Pressure drops of 20 to 25 psi per 100 ft of tube are not excessive.

SETTING OF SUPERHEATER SAFETY VALVES. In all superheaters, a certain steam velocity through the tubes is necessary for protection against burning. Superheater safety valves, therefore, should be set to operate at a pressure below that of the boiler (saturated) safety valves, to insure that superheater valves blow first. If safety valves on boiler and superheater are set for the same pressure, boiler safety valves will blow first, and the superheater will have little or no flow of steam, with consequent danger of tube burning.

6. REHEATERS

Within the last few years, attention has again been given to the *reheat cycle* in steam electric power stations (Refs. 4 and 5). The increase in efficiency of such a cycle has resulted in considerable saving in fuel, even when offset by additional fixed charges. The rise in the fuel price has made use of the reheat cycle attractive, in some applications.

A reheater or resuperheater is simply a second steam superheater located within the boiler setting or separately set and separately fired. The integral design of reheater has been more popular and perhaps more practical. The reheater surface may be either of the convection or radiant design, and important installations have been using both arrangements.

The reheater frequently is used in connection with compound or topping turbines, where it receives steam from the high-pressure section of the turbine at a lower pressure and temperature than the steam entering the high-pressure turbine. In the reheater, this steam is heated again to some specified temperature, usually near the original temperature. However, it does experience a friction pressure loss in the reheater tubes and piping, partially offsetting the intrinsic gain of the thermodynamic cycle. After the reheater, the steam passes to the low or intermediate pressure section of the turbine. (See also Sections 4 and 8.)

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ECONOMIZERS, AIR PREHEATERS, AND WASTE-HEAT UTILIZATION

By W. S. Patterson and H. Karlsson

7. ECONOMIZERS

By W. S. Patterson

Economizers, first used about 1860, antedate air preheaters. At that time fuel was very inefficiently burned, and even though low pressures and low capacities prevailed, a tremendous amount of waste heat was carried away in the flue gases. It was possible to recover economically much of this waste with a relatively inefficient economizer employing large, thick-wall tubes, widely spaced, so as to cause little added draft loss. Later, when use of mechanical stokers made it possible to operate boilers at higher capacities, the flue-gas temperature was high because of the increased capacity, even though combustion of the fuel was more efficiently accomplished. Thus there was a need for economizers even with low-pressure boilers. When both high pressures and high ratings became prevalent, the use of heat-recovery equipment was even more necessary to prevent excessive loss of heat in the flue gas.

In the early days it was not uncommon practice to install one economizer to serve several boilers. Modern practice is to provide an economizer to serve each steam-generating unit.

In their earliest applications, economizers were called upon to heat relatively cold water which was not deaerated and thus carried considerable oxygen. To reduce oxygen corrosion, the economizers were constructed of cast-iron tubes arranged vertically with the water entering all tubes at the bottom and leaving at the top. The gases were passed through at right angles to the tubes. With this arrangement, it was not possible to take advantage of the countercurrent principle, and the extent of heat recovery was limited. Tubes 4 in. in diameter were frequently used. The generally accepted maximum pressure for cast-iron economizers was 250 psi, and it was necessary to adopt other tube materials when higher pressures were encountered.

STEEL-TUBE ECONOMIZERS. Steel is the most suitable material for high-pressure economizers, except for corrosion resistance. The principal reasons are thinner tubes; smaller diameter; closer spacing; more heating surface in a given space; better heat transmission for a given weight or surface; lower cost; and less radiation loss.

A steel-tube economizer must be supplied with water that has been properly deaerated and heated to above 200 F. The water should first be raised to the boiling point in a well-vented open-type deaerating heater. With careful operation this method of treatment reduces the oxygen content to 0.05 ppm or less. In moderate pressure plants, about 400 psig, it is usually economical to heat the feedwater to a temperature between 180 and 220 F by means of bled steam, even when flue-gas economizers are used. With high pressures, such as 1400 psig, bleed heating of the feedwater is carried as high as 400 F. (See Section 8.)

ARRANGEMENTS OF ECONOMIZERS. Steel-tube economizers are arranged in vertical or horizontal banks. The vertical type is known as an *integral economizer* when it is arranged similar to a bank of boiler tubes and located within the boiler setting.

Integral economizers frequently are provided with two drums. The upper one is divided into two compartments, with feedwater introduced on one side of the partition and discharged from the other side. The water thus makes two passes through the economizer in flowing from one compartment through part of the tube bank to the lower drum and then back through the remainder of the tube bank to the other compartment of the upper drum. If the economizer is vertically baffled for two gas passes, these may be arranged to give countercurrent flow. An efficient arrangement of this type is shown in Fig. 1*A*. The design shown in Fig. 1*B* eliminates one of the economizer drums. In this one the feedwater is introduced into the lower drum, makes only one pass through the tube bank, and then is discharged directly to the boiler drum. Cross-flow baffles have been employed in some integral economizers as illustrated in Fig. 1*C*. Integral economizers have a low water velocity, and hence a low water-side pressure drop. This characteristic is not conducive to good distribution and positive circulation, and accelerates corrosion.

Horizontal-tube Type. Today the most widely used type of economizer employs horizontal steel tubes generally arranged in staggered, closely spaced rows with the gases flowing transverse to the axis of the tubes. The water flows progressively through the tubes, upward or downward from row to row. Upward flow of water is preferred by some engineers, particularly if there is any possibility of steam being generated in the economizer. However, many down-flow applications are in successful use.

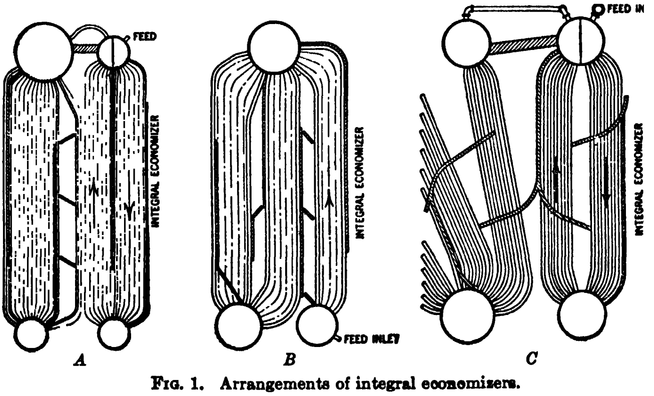


FIG. 1. Arrangements of integral economizers.

Cleaning. At one time it was considered necessary to make all economizer tubes accessible for cleaning. Small junction boxes were used, each serving as a return bend between a pair of straight tubes. The tubes were fastened into the boxes by rolling. A hand-hole fitting was provided for rolling, inspection, and tube cleaning. The pressure drop in these junction boxes was great, and their use has been discarded in favor of flanged return bends, each end of which is bolted to a similar flange attached to a straight tube.

Each straight tube, therefore, is accessible at one end. This type requires doors at one or both ends of the economizer. Illustrated in Fig. 2, this construction is recommended where poor feedwater makes frequent routine inspection or cleaning necessary. The tubes are accessible at one end only. Such construction permits the necessary tube cleaning, and has the advantage of reducing the number of bolted joints.

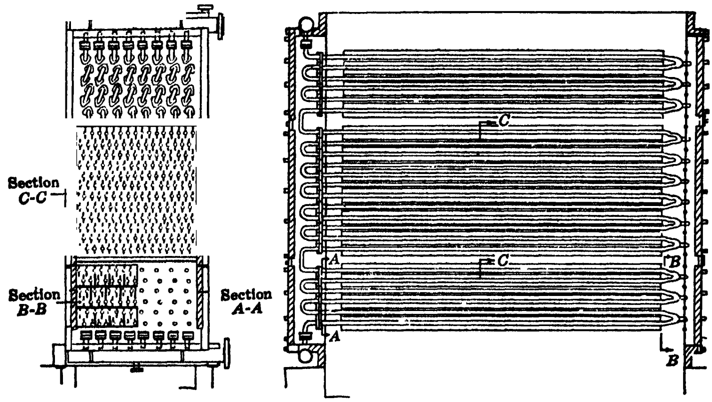


FIG. 2. Economizer with flanged return bends.

When feedwater is of intermediate quality requiring infrequent inspection and cleaning, the connection between pairs of tubes forming a water-flow circuit is made at one end by the use of a plugged bifurcate, actually an accessible return bend. By removing the plug, a quick access is possible, and the tube cleaner may be inserted in either tube. The opposite end is provided with bends of the same type as that shown in Fig. 2.

The recent adoption of inhibited hydrochloric acid for cleaning internal tube surfaces may lead to the omission of provisions for mechanical tube cleaning where it would otherwise be necessary.

The continuous-tube type, illustrated in Fig. 3, affords no access to the tube interiors, and is recommended only for installations where good feedwater conditions make

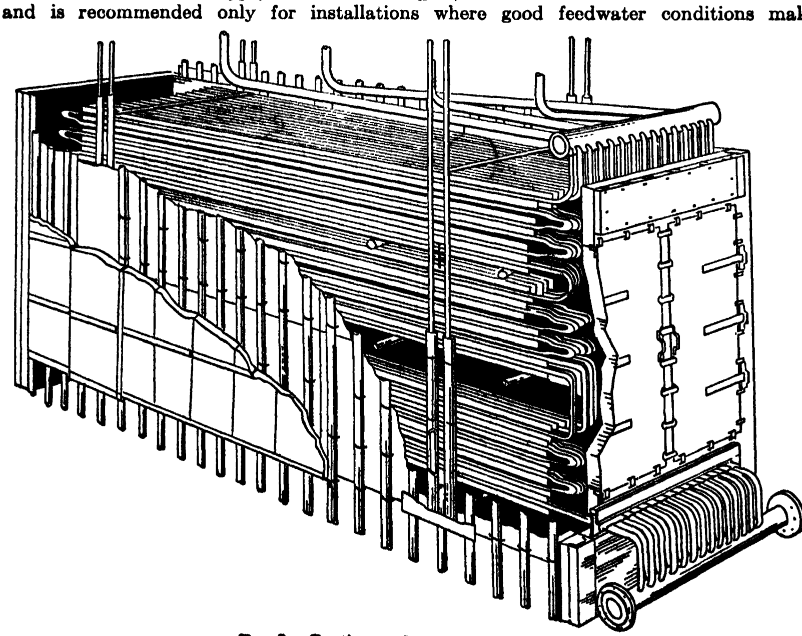


FIG. 3. Continuous-loop type economizer.

internal cleaning unnecessary. The inlet header is shown at the bottom, outside of the setting. The outlet header and the tubes for delivering the feedwater to the boiler are shown at the top, inside of the setting. Also illustrated are vertical support rods, located at both sides of the economizer, and used when an economizer is located inside the boiler setting.

The foregoing steel tube economizers employ 2-in. OD tubes spaced on $3^1/_2$-in. horizontal centers and use $5\ ^1/_2$-in. staggered vertical spacing. Longitudinal continuous fins, 2 in. wide and $^1/_4$ in. thick, are welded to top and bottom of tubes to increase the amount of effective heating surface provided in a given space. The fins not only increase the heating surface but also strengthen the tubes against bending and lessen the accumulation of ash on top of the tubes.

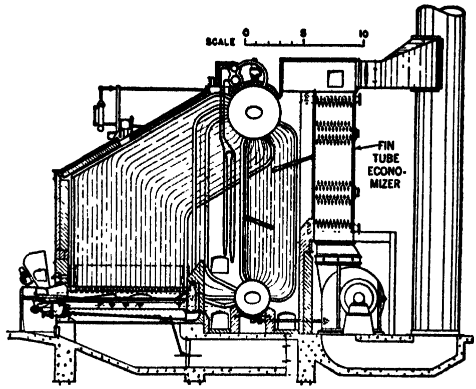


FIG. 4. External arrangement of economizer.

External Arrangement. Economizers are sometimes located outside the boiler setting as a separately supported and encased unit. Figure 4 illustrates an arrangement supported on outside steel but not entirely separate from the boiler in that one side of the economizer enclosure forms part of the rear boiler wall.

Illustrated in Fig. 5 is an effective arrangement in which the economizer is located within the boiler setting and directly below one of the drums. In this position, it may be supported from the boiler drum by means of alloy beams and hanger rods. Air-cooled and water-cooled hangers have also been used. Water-cooled headers are sometimes used for support beams. Feedwater at the same temperature and pressure as at the economizer inlet may be passed through the headers to cool these support members. The water flow rate is proportioned by the use of orifices in the various parallel circuits. An alternate arrangement is to make the support headers a part of the boiler circulation system.

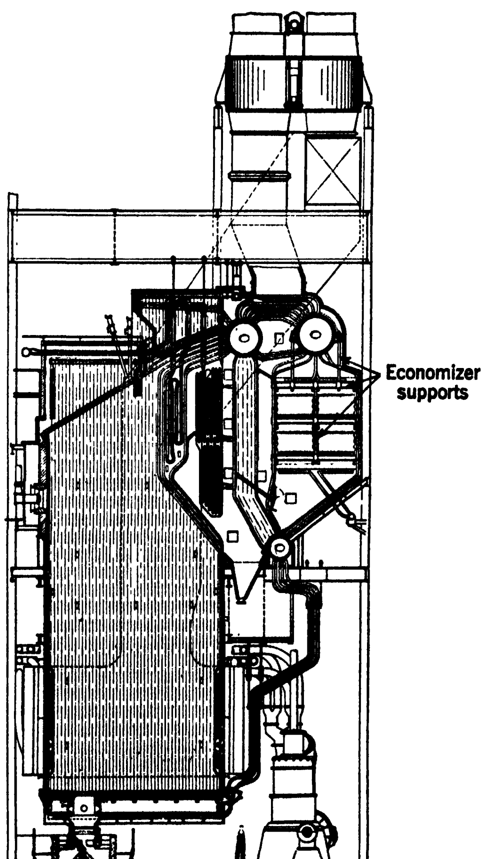


FIG. 5. Economizer mounted beneath drum.

Soot Blowers. For maximum effectiveness, economizer surface must be kept clean externally as well as internally. Therefore, provision frequently is made for the installation of revolving soot blowers in horizontal rows. Blowers located between supports and end panels are sometimes of the stationary type. Others are of the revolving type, designed for a 360-degree blowing arc. Eight to ten rows of tubes is the maximum number that can be used, with effective cleaning. Economizers, therefore, are divided into banks of tubes with soot blowers located in the space between banks.

ECONOMIZER SELECTION. Draft Loss. The amount of heating surface installed in an economizer depends on gas inlet and outlet temperatures, inlet water temperature, whether or not counterflow of fluids is employed, and allowable draft loss. Other things being equal, a low draft-loss economizer requires more surface, but in designing a high draft-loss economizer, the gas velocity must be kept below a limit of 6000 to 8000 lb per hr per sq ft of flow area if tube erosion is to be avoided with high-ash coal, depending on the abrasive qualities of the ash. High draft-loss economizers frequently also have a high pressure drop on the water side, generally limited to about 40 psi.

Economizer versus Air-heater Surface. Economizer surface being more expensive than air-heater surface, it is generally more economical to use an air heater of maximum size and an economizer of minimum size when both are required to obtain the desired heat recovery from the flue gases. The exception to this rule is when the unit is stoker-fired, in which case the air-heater size is limited by the air temperature permitted at the stoker grate, and a larger economizer must be used. Stoker-fired installations of moderate efficiency may work out more economically if the air heater is omitted entirely because the gas and air ducts associated with a small air heater have to be charged against the heater and may represent a large percentage of the cost of the air-heater installation.

The proportion of the heat required to generate steam *increases* for the economizer and *decreases* for the boiler as critical pressure is approached. In fact many higher pressure units generate 100% of the steam produced in the water walls surrounding the furnace and employ practically no boiler surface beyond the furnace. This practice has not, however, resulted in the use of enormous economizers because high-pressure systems employ several stages of feedwater heating by steam bled from the turbine so that water temperature entering the economizer may be 400 to 500 F. It is customary to heat the water in the economizer only to within 35 to 40 F of saturation temperature unless the economizer is designed for steaming under certain conditions of operation.

EXTERNAL CORROSION. The overall heat transfer rate obtained with economizers is of the order of 3000 to 4500 Btu per hr per sq ft. The temperature gradient through the tube wall is therefore very small, and since the conductance at the inside film is very high, the outside surface temperature of the tubes is just a few degrees higher than the water temperature. The gas in contact with the tubes is therefore cooled practically to water temperature. The dew point of flue gas is difficult to determine if sulfur is present in the fuel. With no sulfur present, it would normally be about 120 F, but with a small amount of SO_3 and water vapor both present, the dew point may be increased to more than 300 F. Consequently some external corrosion may be expected when the entering temperature of the water is lower than the dew point of the gas.

8. AIR PREHEATERS

By Hilmer Karlsson

Air preheaters in the power plant add heat to the combustion air by extraction of heat from the flue gases leaving the boiler. Their use permits high heat recovery, elimination of economizers, and use of turbine stage heating of feedwater. Preheated air increases furnace capacities, tends to stabilize the flame, and improves combustion. Air preheaters have been more widely used in the last decade because of the advancement in pulverized fuel firing and the successful application of water-cooled furnace construction.

Air preheaters on the boiler permit operation at low gas-exit temperatures. Owing to the relatively low cost of air-preheater surface, the present trend is toward higher recoveries in the air heater, thus reducing the amount of pressurized surface required in the boiler.

Air preheaters are of two basic types, *recuperative* and *regenerative*. The recuperative type is the conventional heat exchanger, with steady, continuous flow of both air and flue gases, each always on the same side of the heat exchange surface. In the regenerative type the two fluids also are separated by the heat-transfer surface, one fluid flowing on one side and the other on the opposite side. However, in the regenerative type (which can be divided into *intermittent* and *continuous* types) the heating surface is *alternately* exposed to one fluid and then the other. The checker work used in the steel industry with open-hearth furnaces is an example of the intermittent regenerative type. The best-known of the continuous regenerative types is the Ljungstrom.

TYPES OF AIR HEATERS. Plate-type Heaters. A typical unit of the plate type, illustrated in Fig. 6. is composed of welded envelopes, each envelope being complete with air inlet and outlet, air-side spacers, and gas-side spacers. The air passes through the envelopes. When stacked side by side in the casing, the gas passages are formed by the spaces between the adjacent envelopes. The width between envelopes or the gas passages varies from 1/2 to 1 1/4 in., closer spacing being used for clean gases and wider for gases containing solids in various degrees and of different types. Air spacing is usually smaller than gas spacing, but ordinarily neither is less than 1/2 in. This type of unit is generally of the same width as the economizer or boiler preceding it.

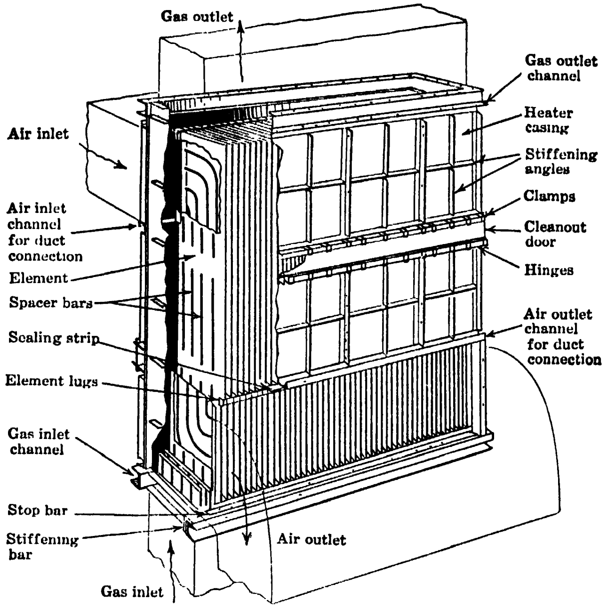


FIG. 6. Plate-type air preheater.

Tubular Air Preheater. A tubular air preheater (Fig. 7) consists of a tube bank with the tubes fastened into a stationary tube sheet at the top of the unit and a floating tube sheet at the bottom to provide for difference in expansion due to temperature differences between tubes and casing. The unit is further provided with baffles to direct the fluid over the outside of the tube surface. In the illustration shown, the flue gases are passed through the tubes while the air is passed outside. This is the arrangement most commonly used although there are several applications where the gases are taken outside the tubes and the air inside. The tubes can be either vertical as illustrated or horizontal. Furthermore, the unit may be built in one section as shown or in two sections. The latter type of construction is now advocated where low exit-gas temperatures prevail with fuels high in ash, sulfur, and moisture. In some instances, the tubular air heater is also separated in two units with a relatively small unit located ahead of the economizer, where an economizer is used, for preheating the primary air on pulverized coal-fired units. The large section is located after the economizer for preheating of the secondary air. The unit illustrated shows a cold-air by-pass for the purpose of controlling the temperature of the tubes at the cold end of the unit. In other makes, a certain amount of metal-temperature control is accomplished by reduction of air mass flow in relationship to gas mass flow at the cold end of the unit, thus maintaining metal temperature of the tubes at this point closer to the gas temperature. Tubes are usually 1 1/2 to 2 1/2 in. OD. The 2 in. and 2 1/2 in. OD tubes are the most popular sizes.

Regenerative-type Air Preheaters (Ljungstrom). This type of air preheater, illustrated in Fig. 8, has a slowly moving rotor containing the heating surface. Each revolution produces a complete cycle of exchange in which heat from the hot gases is absorbed by the heating surface in the rotor and given up as rotation moves it into the path of the combustion air.

As shown in Fig. 8, the heating surface, made up of specially formed sheets, provides a multiplicity of small channels for gas and air flow, respectively, usually made in two layers. With the shallow layer at the cold end, replacement cost is low for the part of the surface that is sometimes subjected to corrosive attack. The casing is divided into three sections; the middle section encloses the rotor, and the two end sections contain partitions for separation of gas and air flow, respectively, as well as connections for gas and air entering and leaving the unit. This type of air preheater is built for vertical flow as illustrated and also for horizontal flow of gas and air, respectively.

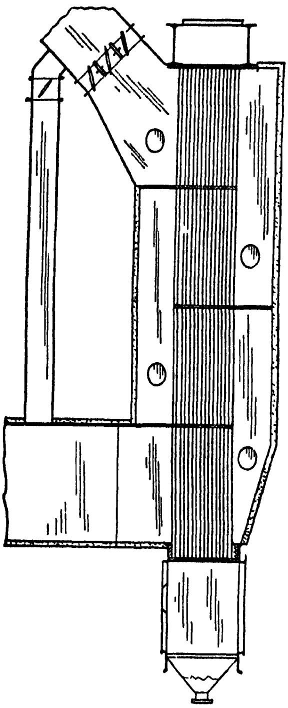


FIG. 7. Tubular air preheater. (Courtesy of Babcock and Wilcox Company)

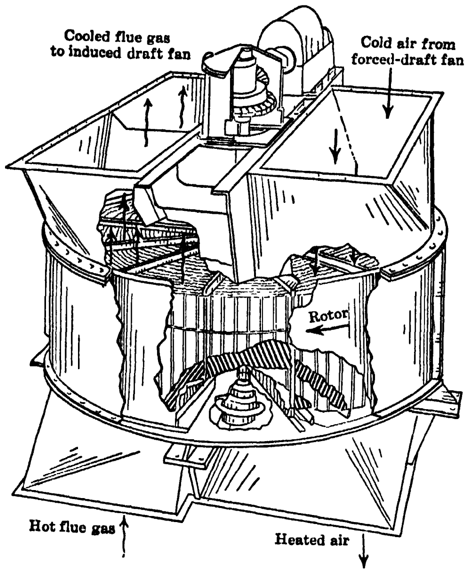


FIG. 8. Regenerative-type air preheater. (Courtesy of Air Preheater Corp.)

SELECTION OF AIR PREHEATER. Many factors must be considered in the selection of an air preheater. The major economic factors are fuel cost, fan power, maintenance expense, and cost of installation. The engineering factors are space available, type of fuel, and desired temperature of the exit gas and preheated air, respectively. Generally it will be found that for low capacities and low recoveries the recuperative type of unit is most economical whereas for large capacities and high recoveries, the regenerative type usually shows economic advantages. A detailed study is required to determine the type of unit to use.

The recuperative type of air preheater is stationary and considered airtight; the regenerative type of unit employs moving parts and has a certain amount of air leakage. Experience, however, has proved that the maintenance cost of the moving parts is within acceptable limits and that the loss due to air leakage is compensated for by the feasibility of operating this type of unit at lower exit-gas temperatures.

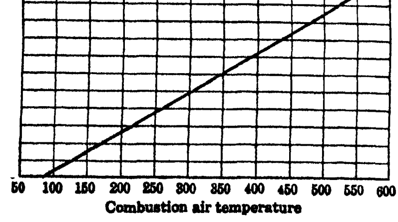


FIG. 9. Fuel saving with preheated air. (Courtesy of Babcock and Wilcox Co.)

PRACTICE. Fuel Saving. Preheated air gives a substantial fuel saving. For each 100 F drop in temperature of the stack gases, the efficiency of the steam-generating unit is increased 2.25 to 2.6%, as contrasted with 2% increase in efficiency for each 100 F rise in temperature of the air used for combustion. This gain in fuel saving is shown graphically in Fig. 9.

Air Temperatures. These temperatures of combustion air indicate present-day practice: pulverized coal firing, pulverized lignite firing, and oil and gas firing, 700 F; underfeed chain grate and spreader stoker coal firing, 350 F.

Although higher air temperatures are occasionally used for combustion air, especially in industrial applications where temperatures up to 1000 and 1200 F are not uncommon, these values have been found economical for most applications in power-plant practice. In burning coals of high moisture content and lignite fuels in pulverized form, it is necessary, for proper drying of the fuel, to use high primary-air temperatures; these requirements are illustrated in Fig. 10.

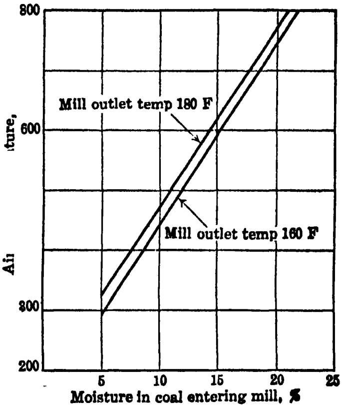


FIG. 10. Pulverizer air inlet temperature for various moisture contents of coal. (Based on 15% primary air.)

Gas-inlet Temperatures. Air preheaters used with steam-generating boilers are usually designed for maximum gas-inlet temperatures of 850 to 900 F, permitting open-hearth steel construction to be used throughout except for the cold end of the unit, where special corrosion-resistant materials may be employed. Air preheaters for industrial application are operated with gas-inlet temperatures up to 1600 F. It is necessary to construct such units from heat-resistant alloys. It is customary to vary the type of alloy used, in accordance with the requirements (temperature) existing in various parts of the unit, so as to obtain the most economical design.

Gas-outlet Temperatures. The permissible gas-outlet temperature depends on the characteristics of the fuel used as well as on the type of fuel-burning equipment. In order to avoid excessive maintenance cost of the cold-end portion of the heating surface in an air preheater, operating conditions should be such that corrosive attack by acid-forming constituents of the gases and by the entrained solids is held within reasonable limits. Up to the present time, it has been practice to limit the temperature of the gases leaving air

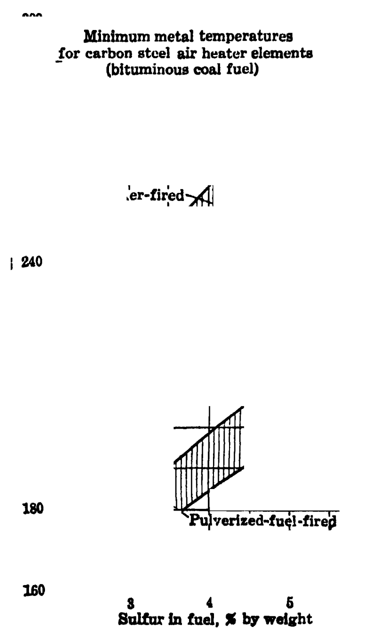


FIG. 11. Effect of sulfur content in fuel on minimum metal temperature of air heaters.

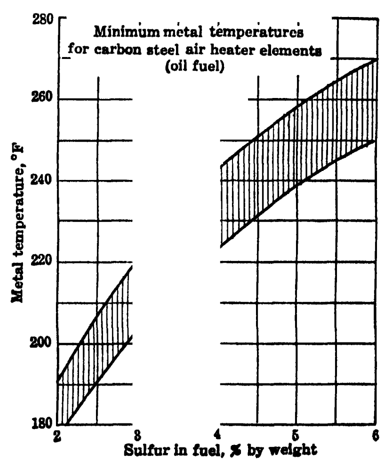


FIG. 12. Effect of sulfur content in fuel on minimum metal temperature of air heaters.

preheaters to values arrived at by experience, which vary with the sulfur content of the fuel and the firing method. Minimum metal temperatures are illustrated by Figs. 11 and 12. The curves of Fig. 11 show the limitation in metal temperature usually applied to bituminous coal of various sulfur contents when fired on stokers and in pulverized form; the curves of Fig. 12 show corresponding values for fuel oil.

Usually an air preheater is selected to operate with the gas-outlet temperatures given in Figs. 11 and 12 at normal load, to avoid undue maintenance from corrosion and deposits. This practice necessitates providing means at part-load operating conditions (where cold gas temperatures are encountered) for maintaining the gas outlet temperature at or above its permissible minimum value. One of the most common methods for both the recuperative and the regenerative types of air preheater is illustrated in Fig. 7. It by-passes part of the cold air around the air preheater during part-load operation so as to maintain the desired gas-exit temperature. This method of operation permits the metal temperature of the surface to approach more nearly the temperature of the gases leaving the unit. The second method increases the temperature of the air entering the air preheater above the ambient, either by passing the air from the forced draft fan over heating coils or by recirculating preheated air to the inlet of the forced draft fan. In the recuperative-type air preheater, this method is purely a means of increasing the metal temperature at the cold end of the unit. In the regenerative-type air preheaters, these methods, in addition, serve to reduce the relative humidity of the air entering the preheater, and thus to obtain a drying action on the air side of the unit and to elevate metal temperatures.

Heat-transfer Rates. Overall heat-transmission coefficients for air preheaters usually range from 2 $1/2$ to 5 $1/2$-Btu per hr per square foot per degree Fahrenheit mean temperature difference. The lower values apply to low mass velocities, and the higher values to preheaters where the allowable pressure drop permits operating at higher mass velocities.

Pressure Drop. Air preheaters are usually selected for an air-side pressure drop up to 6 in. water, gage, and a gas-side pressure drop up to 4 in. water, gage. Where the cost of fan power is high, it is economical to select the air preheater for lower resistances. An air heater is usually selected with higher pressure drop on the air side because the forced draft fan handles air at atmospheric temperatures whereas the induced draft fan used to overcome the pressure drop on the gas side handles gases of 250 to 400 F, thus requiring more power per pound of air handled, for the same pressure drop.

Corrosion and Deposits. Knowledge of the factors causing corrosion of air preheater materials is too limited to permit exact evaluation of their effects. It is known, however, that the humidity of the gas stream, sulfur content of the fuel, composition of the ash, and firing methods used, all have a definite bearing on the problem. Empirical curves, based on experience, have been formulated for the minimum preheater metal temperature permissible with various fuels and methods of firing, as shown by Figs. 11 and 12. These curves are useful for guidance only; it still is necessary to determine the safe or economical temperature at which to operate for any given condition on the basis of actual experience.

Sulfur in the fuel is first oxidized to sulfur dioxide and then partially oxidized to sulfur trioxide. Sulfur trioxide in the presence of water vapor produces sulfuric acid. Sulfuric acid of low concentration, in the presence of ferric sulfate, rapidly attacks steel. Ferric and aluminum sulfates, usually present in the deposits on the heating surface, have a high affinity for moisture; thus increased moisture content of the gas stream results in increased rate of corrosion, as does increased sulfur content of the fuel. Moisture in the gas stream may originate from the fuel, soot blower, economizer leaks, quenching-water in ash pits, or from steam lances used in the furnace.

A comprehensive study of corrosion has been published in *Bulletin* 228, of the University of Illinois Engineering Experiment Station.

Cleaning. Deposits forming on the surface at the cold end of the air preheater may be either of the bonded type or of a more powdery structure. They vary in nature with the type of fuel, firing conditions, etc.

Since air preheaters depend on clean heating surface for maximum efficiency, and since removal of deposits retards corrosion, surfaces of air preheaters working under conditions where deposits occur must be cleaned periodically. Deposits on the heating surface in recuperative-type units affect both heat transmission and pressure drop; in regenerative types, pressure drop only is affected.

In recuperative-type units, deposits are removed by air or steam, or by water washing, cleaning being performed either during operation or during outages, depending on the equipment used. With the Ljungstrom air preheater, steam, air, or water may be used for cleaning during operation, because this type of unit is equipped with manually or power-operated cleaning devices using any of these cleaning mediums. Where the deposits are of a powdery structure, blowing with steam or air is sufficient; where the deposits are bonded, water is necessary for cleaning.

In some cases, alkalized water has been used successfully for washing down the heating surface of air heaters, increasing its life, even under severe corrosive conditions. Coating the surface with lime water after removal of the deposit by washing has also been reported to be successful.

9. WASTE-HEAT UTILIZATION

By W. S. Patterson

GENERAL. All fuel-fired furnaces used in industrial processes, including steel-heating furnaces, cement kilns, lime kilns, and zinc and copper furnaces, can justify heat-recovery equipment if they are designed for continuous operation. In all these furnaces solid materials are introduced into the furnace and heated by burning of fuel. Therefore, several methods of recovery of waste heat are available: (1) preheating cold material with the flue gas; (2) preheating combustion air; (3) steam generation in waste-heat boilers; (4) steam superheating; and (5) water heating.

In oil stills, sulfur-burning furnaces, incinerators, diesel engines, and gas turbines, gas leaves at such a high temperature that heat-recovery equipment can be justified economically for heating combustion air or feedwater or for generation of steam. The first gas turbine put into service by a public utility in this country exhausts to a recuperator for heating feedwater for the boilers. Other gas-turbine applications have made use of air preheaters to recover exhaust heat, and in some instances waste-heat boilers may prove economical.

Paper Pulp Industry. Here the black liquor from the digesters contains not only a large amount of combustible matter but also valuable chemicals. In the sulfate process the make-up chemical is sodium sulfate; in the soda process the make-up is sodium carbonate. These are known as alkaline processes. Within the last twenty years great improvements have been made in equipment for recovering waste heat and chemicals from the liquor. In the sulfite process the liquor is acid. In a few sulfite mills by-products are obtained from the waste liquids, but generally they are discharged into streams, causing pollution. However, within the next few years several commercial waste-heat and chemical-recovery systems may be placed in operation in sulfite mills.

Successful, completely integrated chemical-recovery units comprising water-cooled furnace, liquor sprays, boiler, superheater, evaporators, chemical feeding and mixing equipment, fans, air, and gas systems, dissolving tanks, liquor pumps, controls, and instruments have been applied in sulfate mills. Units of this design have also been used in soda mills; they may also soon be applied in sulfite mills, with the omission or modification of the direct-contact cascade evaporators. In these units recovery of chemicals is the primary object; resulting heat recovery and steam generation are secondary objectives. However, the boiler is no longer referred to as a "waste-heat" type because steam is generated and superheated in an efficient manner, and the quantity is as important to the mill operation as that obtained from coal or oil-fired boilers, particularly since the steam generated by the recovery unit represents a large saving in expensive fuel that would otherwise have to be burned in other boilers.

Sewage-treatment plants are sometimes designed to utilize the heating value of sewage to supply heat for processing the sewage and for generation of steam. The primary object is to do it in an economical manner, which may involve burning the dried product if its value as fertilizer is less than its value as fuel. With such an arrangement the dried sewage

Table 1. Temperatures of Waste Gas from Industrial Furnaces

Type of Primary Furnace	Temperature, °F
Nickel-refining furnace	2500–3000
Beehive coke ovens	1950–2300
Zinc-refining furnace	1400–2000
Heating furnace	1700–1900 *
Copper reverberatory furnace	1650–2000
Copper-refining furnace	1450 †
Cement kiln (dry process)	1150–1350
Cement kiln (wet process)	800–1100
Open-hearth steel furnace (producer-gas-fired)	1200–1300
Open-hearth steel furnace (oil, tar, or natural gas)	800–1100
Gas benches	1050–1150
Oil stills	900–1000
Glass tanks	800–1000

* During operating periods. With furnace kept hot but heating no material, average temperature 1000–1100 F.

† Average over 36-hr cycle; range 500 to 2100 F.

is delivered to a storage bin and sacked as fertilizer or discharged to the furnace, burned in suspension, and the generated heat used in the drying process. Units having a water-cooled furnace and integral boiler for steam generation have been in successful operation for many years. Flash drying and incineration systems have been widely accepted and serve communities ranging in size from 6000 to 3,600,000 population. They are not limited to sewage sludge but have also been used in drying and burning the residue from the production of furfural (cotton-seed hulls, rice hulls, and corn cobs) and, at the same time, in producing all the steam necessary to support operation of the process.

STEAM SUPERHEATERS AND WATER HEATERS. Superheaters are sometimes used in conjunction with waste-heat boilers, even when the boiler is of the fire tube type, in which case the superheater is generally located in the gas inlet duct. Economizers are also sometimes used with waste-heat boilers. However, when the temperature of the waste gas is too low for steam generation, it may prove economical to recover the heat by means of water heaters which supply feedwater to other boilers, or hot water for space heating.

AIR HEATERS (RECUPERATORS AND REGENERATORS). **Air heaters for industrial furnaces and kilns** are referred to as *recuperators* or *regenerators*, depending on the principle of operation. Sometimes both the air and the gas fuel are preheated. The installation of a recuperator on a continuous steel heating furnace will return between 100 and 200% on the investment, even though the furnace is assumed to be in use during only one-third of all the available time, and the calculation is predicated on a recuperator life of only three years.

Recuperators for high temperatures are of two types: metallic and refractory. Refractory types may be so large that excavation and water proofing of the recuperator pit may cost more than the recuperator itself. For many applications, the gas inlet end of metallic recuperators is made of high-temperature heat-resisting metal. There must be a positive flow of the cooler fluid at all times, and the entering gas must be below the maximum temperature for which the apparatus was designed. A refractory chamber, enclosing a metallic recuperator in contact with hot gas flowing through it, will be heated to the gas temperature, but the air flow will keep the metal surfaces at a lower temperature. If the flow of gas and air are stopped simultaneously, the metallic surface may be damaged by heat radiated to it by hot refractories.

In a metallic recuperator handling very hot gas, the metal separating the air and gas streams will generally have a temperature closer to that of the gas. Cast iron and mild steel are unsuitable if the temperature at any time exceeds 900 to 1000 F. For higher temperatures calorized steel or nickel-chrome alloy steel must be used. Plate- and tubular-type recuperators of these materials are employed in the steel industry. Silicon carbide tubes have also been used. One comparatively new design of recuperator has cast sections of chrome-nickel alloy steel with extended surface on both the gas and the air sides.

Plate-type air heaters have been used as recuperators in conjunction with kilns for the manufacture of bricks, pottery, porcelain, plate glass, malleable castings, and cement. In most installations the gas temperature is 750 to 850 F, and the preheated air is used for combustion of the fuel. Special designs have been developed for entering gas temperatures exceeding 1000 F.

Regenerators used with steel-heating furnaces are of the refractory type. They handle gas up to 2300 F and preheat air up to about 1900 F. The hot-furnace gas is first passed over the refractory checkerwork in a recuperator unit, for a time, to heat it. Then the air is passed through to absorb stored heat. Two or more recuperator units are used to provide continuous cooling of gases and heating of air. Fluid flow is alternated on a schedule to maintain optimum operating economy.

The Ljungstrom regenerative preheater (see p. 7-35) is available in several standard designs for use with installations where gas-inlet temperatures are as high as 1800 F. Typical industrial-furnace applications are oil stills, sewage-disposal plants, metallurgical furnaces, car-thawing sheds, separately-fired superheaters, and numerous other processes.

Preheaters in the Oil-refining Industry. The advantages of preheaters designed for high temperatures with oil stills are covered by Mr. O. F. Campbell and T. B. Kimball in *Regenerative-type Air-preheaters for Refinery Use*, presented at the American Petroleum Institute meeting, 1941. The economy of eliminating the convection bank of a cracking-still furnace and substituting for it an air heater depends on the cost of the convection bank, the cost of air heater and fans, the cost of power to drive the fans, and fuel price. In a conventional cracking still, wherein flue-gas temperatures may exceed 1400 F when the gas leaves the radiant section, the installation of a convection bank, supplemented by an air heater, may prove more economical than the installation of a high-temperature air heater requiring costly alloys in its fabrication. Many cracking stills installed at the present time can justify, from a fuel-saving standpoint, the installation of an air heater.

PUMPING AND HEATING OF FEEDWATER

By A. J. Stepanoff, J. S. Daugherty, and G. D. Dodd

10. THE INJECTOR

Revised and rewritten by A. J. Stepanoff

Principle of Operation of the Injector. The simplest form of single-tube injector is shown in Fig. 1. Entering steam, in passing through the nozzle, acquires high velocity and is condensed by water in the combining tube, creating a vacuum into which water flows through the water-supply pipe. The high-velocity steam entering through the nozzle, comprising a mixture of condensed steam and water, flows into the delivery tube. There the energy of steam expanding from boiler pressure to a partial vacuum produced by condensation is sufficient to create pressure in the water as much as 50 to 80 psi in excess of the boiler pressure, for the range of pressures in which the injector is used. This excess pressure forces the water into the boiler.

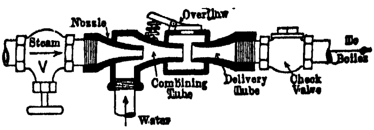


FIG. 1. Diagram of injector parts.

Positive and Automatic Injectors. Positive-type injectors have hand-controlled overflow valves, which are closed after operation has started and water appears in the overflow. The advantages of this type of injector are its ability to lift water to a greater height, to start with a lower steam temperature, and to discharge against a higher back pressure. In automatic injectors, opening and closing of the overflow are entirely automatic. This type is preferred for stationary work because of its restarting features.

THE INJECTOR AS A BOILER FEEDER is efficient and convenient. It has no moving parts, is compact, delivers hot water to the boiler without preheating, and has no exhaust steam to be disposed of. When used to feed water to a boiler, its thermal efficiency is 100%, since all heat rejected passes into the water and is carried into the boiler. The loss of work in the injector due to friction reappears as heat which is carried into the boiler. Heat converted into useful work in the injector reappears in the boiler as heat. Although the injector has perfect efficiency as a boiler feeder, it is not the most economical means of feeding because of its inability to handle hot water, thereby excluding utilization of other sources of waste heat for boiler-feed heating. It also is difficult to maintain continuous flow with the injector at low capacity because of the necessity of starting and stopping under such conditions.

The injector has been widely used on locomotives but has been displaced in certain cases by direct-acting feed pumps, especially when feedwater heaters are used. It is limited in stationary work to small or single boilers or as a reserve feeder. The injector used as a pump has an efficiency of approximately 1 to 2%. The weight of feedwater handled per pound of steam usually decreases as steam pressure increases, and varies between approximately 21 lb at 20 psig pressure and 10 lb at 100 psig pressure. The maximum temperature of feedwater which can be handled averages 120 to 140 F at sea level, and lower at higher altitudes.

Table 1. Test of Sellers Injector

(From *Practice and Theory of the Injector*, by S. L. Kneass)

Mean steam pressure, psig	30	60	121	150	200
Temperature of supply water, °F	67	67	54	54	50
Maximum capacity:					
Gallons water handled per hour	1912	2535	3517	3765	4005
Temperature of delivered water, °F	113	125	134	135	154
Weight of delivered water per pound of steam used, lb	25.9	19.1	13.6	12.6	10.3
Minimum capacity:					
Gallons water handled per hour	765	937	1290	1432	1732
Temperature of delivered water, °F	171	212	238	250	263
Ratio of minimum to maximum capacity	0.4	0.37	0.37	0.38	0.43

11. BOILER-FEED PUMPS

Except for very small capacities, centrifugal pumps are used for boiler-feed service almost universally. Their advantages—small size, high speed, and continuous steady flow—make them particularly suited for this service. To save space and weight in marine service, design speeds were gradually brought up to 7500 rpm. For stationary plants 3600 rpm is more common, but a trend to higher speeds is evident. Application of centrifugal pumps to boiler-feed service presented a number of problems not encountered in any other field of application involving high-pressure and high-temperature pumps.

REQUIREMENT OF A STABLE HEAD-CAPACITY CHARACTERISTIC See Section 5.

THE NET POSITIVE SUCTION HEAD (NPSH) REQUIREMENTS are in excess of those given by the cavitation constant (see Section 5) to prevent vapor binding resulting

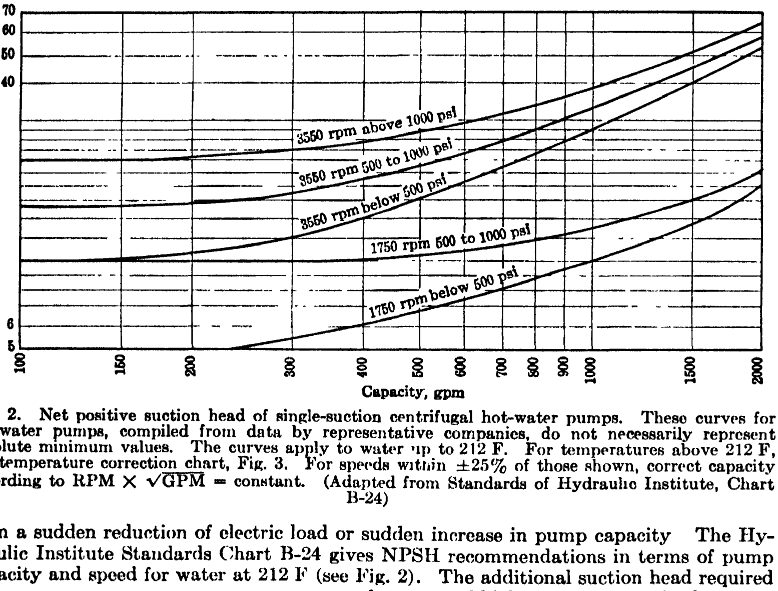


FIG. 2. Net positive suction head of single-suction centrifugal hot-water pumps. These curves for hot-water pumps, compiled from data by representative companies, do not necessarily represent absolute minimum values. The curves apply to water up to 212 F. For temperatures above 212 F, use temperature correction chart, Fig. 3. For speeds within ±25% of those shown, correct capacity according to RPM × $\sqrt{GPM}$ = constant. (Adapted from Standards of Hydraulic Institute, Chart B-24)

from a sudden reduction of electric load or sudden increase in pump capacity The Hydraulic Institute Standards Chart B-24 gives NPSH recommendations in terms of pump capacity and speed for water at 212 F (see Fig. 2). The additional suction head required for water of higher temperature is also given in the Hydraulic Institute Standards Chart B-26 (see Fig. 3).

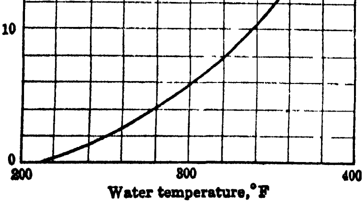


FIG. 3. Temperature correction chart for centrifugal hot-water pumps—single and double suction. (Adapted from Standards of Hydraulic Institute, Chart B-26)

MINIMUM FLOW. To protect boiler-feed pumps from overheating (vapor binding and scoring may follow) when the capacity is reduced below a safe limit, provision is made to by-pass 5 to 10% of the normal capacity back to the feedwater heater. Such by-pases may be operated manually or automatically, or left open continuously. The leak-off from the balancing devices can be used as a portion of the by-passed capacity. In every case the leak-off is piped to the heater storage space rather than to the pump suction.

SELECTION OF MATERIALS is governed by pressure, temperature, and water treatment. For discharge pressures below 1000 psi cast-iron casings are generally used. Rotors are bronze-fitted for temperatures up to 250 F, and stainless-fitted above 250 F. For pressures above 1000 psi, pump casings are made of steel, stainless-fitted. Figure 4 shows material selection based on the *p*H value of the boiler feedwater. When feedwater is pure condensate, the *p*H value does not accurately describe the corrosiveness of water, and corrosion-resisting materials (stainless steel) should be used.

RECIPROCATING PUMPS of the direct-acting duplex type are used for small capacities and moderate pressures. They use approximately 5% of the boiler steam, but if the exhaust is used to heat the feedwater the net heat consumption is less than 0.1%. For small capacities and high pressures, triplex power pumps are used. Special types with adjustable plunger stroke have been developed for boiler-feed service, permitting capacity regulation from zero to the rated capacity (see Section 5).

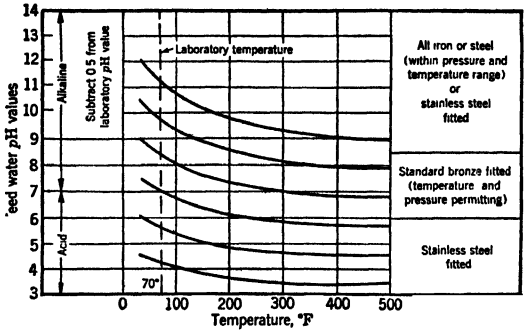


FIG. 4. Boiler feed pump materials used for various pH values of feedwater. (Reprinted by permission from A. J. Stepanoff, *Centrifugal and Axial Flow Pumps*, John Wiley and Sons, 1948)

12. OPEN FEEDWATER HEATERS

Revised by J. S. Daugherty

TYPES OF HEATERS. While any device used to heat feedwater, prior to admitting this water to the boiler, may be called a feedwater heater, the term generally is applied to equipment using steam for heating. This equipment comprises two general classes: (1) open, or direct-contact, heaters, in which the steam comes directly in contact with the water. Tray-type heaters and jet heaters form the two main subdivisions of this class. (2) Closed heaters, in which the heat from the steam is transmitted through tubular metallic walls to the feedwater. Either open or closed heaters can utilize the exhaust from engines or pumps, or be used as stage heaters supplied with steam extracted from bleeder turbines.

SAVINGS ACCOMPLISHED BY FEEDWATER HEATERS. Feedwater heaters, either open or closed, conserve the heat in pump or engine exhaust, high-pressure trap discharges, etc., which otherwise would be wasted. There is, roughly, a saving of 1% for every 10 F that the feedwater is heated by waste heat. The saving effected by a heater using waste heat may be determined from the formula $(h_2 - h_1)/(H - h_1)$, where h_1, h_2 = Btu per pound of feedwater entering and leaving heater, respectively; H = Btu per pound of steam at boiler pressure.

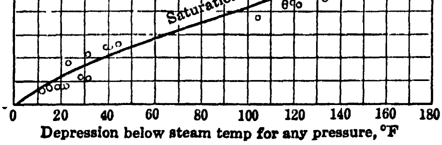


FIG. 5. Oxygen content of deaerated water leaving open heaters.

REMOVAL OF GASES FROM FEEDWATER. Of equal importance with the fuel saving effected by the open heater is its ability to liberate and remove dissolved gases from feedwater. Gases cannot stay in solution when the water is heated to the boiling point. The extent to which oxygen can be expelled by heating water in standard open heaters at atmospheric pressure is shown by Fig. 5.

THE TRAY-TYPE DEAERATING FEEDWATER HEATER (Fig. 6) has been developed from the older open heater. It is designed to accomplish practically complete removal of dissolved gases, of which oxygen is the most objectionable. This is effected by heating the water exactly to the saturated steam temperature, spreading it in thin sheets over successive layers of air-separating trays, agitating it thoroughly so that the gases may be brought to the surface and liberated, and sweeping the liberated gases away with the steam vented to the vent condenser.

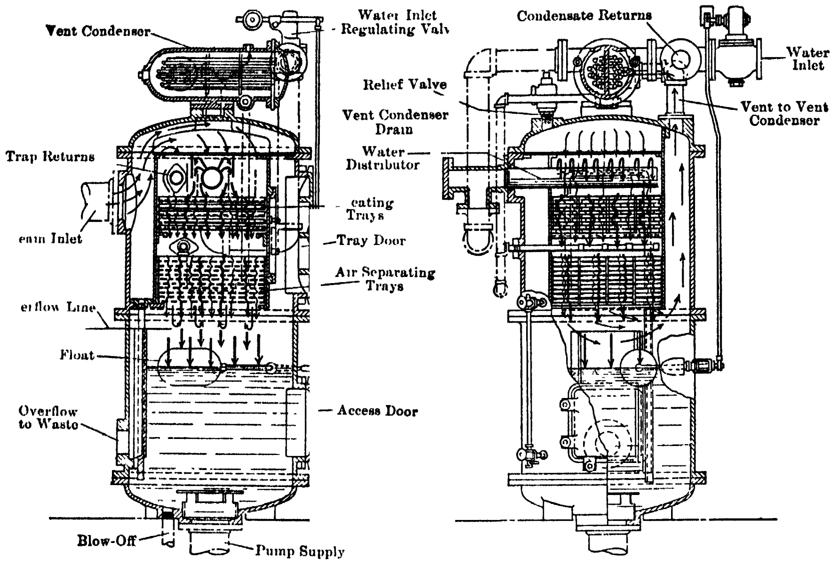


Fig. 6. Cochrane deaerating heater.

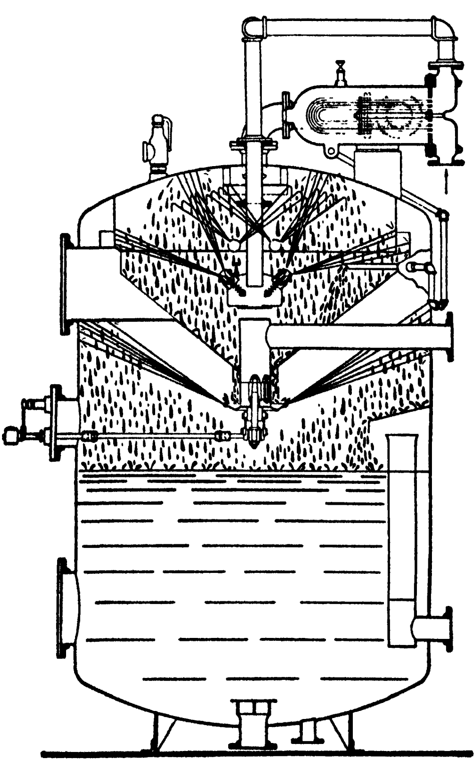


Fig. 7. Atomizing-type open feedwater heater. (Courtesy of Cochrane Corp.)

ATOMIZING-TYPE DEAERATOR. In the atomizing-type deaerator the incoming water is first heated to practically saturated steam temperature by being sprayed in direct contact with steam. After this initial heating, the water contacts a high-velocity steam jet which finely divides or atomizes it and scrubs away the noncondensable gases. The high-velocity steam jet is established by causing the incoming steam to pass through an opening or orifice. This process divides the water so finely that it provides tremendous contact surface between the steam and water permitting the oxygen to be removed in a fraction of a second.

Water Flow. Figure 7 illustrates the flow of water and steam. The water first passes through the vent condenser and then enters the preheating chamber, where it is sprayed into an atmosphere of steam. This spraying is accomplished by spring-loaded valves which distribute the water, evenly divided into small particles, causing it to be heated within 2 to 3 degrees of the steam temperature, and separating over 95% of the oxygen initially present in the raw water. This preheated water is then directed to the atomizer, and falls on the high-velocity steam jet, which divides it into infinitesimal particles or a heavy mist. The division of the water is so thorough that it is atomized, and from this process the deaerator obtains its name. Leaving the atomizer, the deaerated water falls to the storage section, and is available for use in boiler feed or process

Steam Flow. The steam first enters the equipment through the atomizer at a high velocity. After thoroughly atomizing the preheated water, the steam flows to the preheating section, where practically all of it is condensed. A small amount (1 to 2%) passes to the vent condenser with entrained oxygen and carbon dioxide. There the steam is condensed, and the noncondensable gases are liberated to the atmosphere. The condensed steam contains some oxygen and some carbon dioxide that go back into solution in the vent condenser. This condensate is drained to the preheating section so that it can be thoroughly deaerated by by-passing over the atomizer.

CONSTRUCTION OF OPEN HEATERS. When used for low pressures, open heaters usually are constructed of cast iron. When supplied with superheated steam or used with steam extracted from bleeder turbines, the shells of tray-type or jet-type heaters are rolled plate, either riveted or welded.

PROPORTIONS OF OPEN-TYPE FEEDWATER HEATERS. The importance of the jet-type open feedwater heater is due to its ability to heat large quantities of water in a relatively small space. One large manufacturer offers a line of standard jet heaters with outlet capacities ranging from 100,000 lb per hr to 1,000,000 lb per hr in which the internal volume may be approximated from the formula,

$$V = W/10{,}000$$

where V = internal volume, cubic feet; and W = outlet capacity, pounds per hour. The proportions of tray-type open heaters are governed primarily by the particular conditions of operation, and no general rule for proportions is available. An approximation of the size of the heater may be made by allowing at least 1 sq ft, in plan, of tray stock for each 15,000 lb per hr capacity. Vertical units vary in height from about 4 ft for small capacities to 10 ft for larger capacities. About half the height is used for water distribution and the tray stack.

Water storage capacity may be combined with open heaters. Where the feedwater is primarily all make-up and the load fluctuations are not severe, approximately 2 min boiler supply has been found sufficient. When the feedwater is condensate, with but a small amount of make-up, it often is the practice to incorporate condensate surge space in the heater storage compartment. The capacity for condensate surge varies from 5 to 30 min supply for the boiler.

The location of an open feedwater heater in relation to the boiler-feed pump is important. It must be at such an elevation above the pump inlet that the pump will receive only vapor-free liquid. The elevation will depend on temperature and pressure of water leaving the heater. See Figs. 2 and 3, and Section 5.

13. CLOSED FEEDWATER HEATERS

By G. D. Dodd

GENERAL. In closed or tubular-type feedwater heaters water flows *through* the tubes while the heating medium, generally steam, surrounds the tubes, the whole being enclosed by an outer shell. The water ordinarily passes through several tubes in parallel, and sometimes through groups of parallel tubes in alternate directions. This requires a header to introduce the water into the first group of tubes, called the first pass; the water flows back to the header through a group known as the second pass, and so on, for any desired number of passes.

Some designs use a header, known as the *return header*, at the end remote from the inlet header. Others dispense with the return header, bending the tubes into a U shape to return the flow. When a return header is enclosed in the outer shell surrounded by steam, providing for expansion independently of the shell, it is known as a floating header. Other designs have the return header fixed to the shell, providing for no expansion. Where both headers are fixed an unequal number of passes may be used, and water may enter the tube bundle and be withdrawn from opposite ends.

Common applications of closed heaters include the heating of water for domestic, industrial, or power-plant purposes. Only types used primarily with the regenerative feedwater heating cycle are discussed herein. Heaters in this classification may be installed on either or both sides of the boiler-feed pump. Those on the suction side of the pump are built for relatively low pressures in the tube and header assembly; those on the discharge of the pump are built for relatively high pressures.

Low-pressure heaters are built for tube-side pressures up to 400 psig; high-pressure heaters are built for pressures up to 3500 psig. In present-day practice shells are built for pressures of 50 psig to 900 psig.

HEATER PROPORTIONS. The required area or surface of tubes in a feedwater heater is determined by the equation

$$S = \frac{Q \times R \times C}{U \times LMTD} \tag{1}$$

where Q = pounds of feedwater per hour; R = water temperature rise, °F; C = average specific heat of the feedwater; U = conductivity in Btu per square foot of heating surface per °F per hour; and $LMTD$ = logarithmic mean temperature difference.

Two phrases frequently are encountered in discussion of normal heater calculations.

Initial terminal difference is the difference between inlet water temperature and the saturation temperature of steam at the heater shell pressure.

(Final) terminal difference is the difference in temperature between the outlet water and the saturation temperature of steam at the heater shell pressure. This is sometimes called the *approach.*

Equations for logarithmic mean temperature differences are found in Section 3.

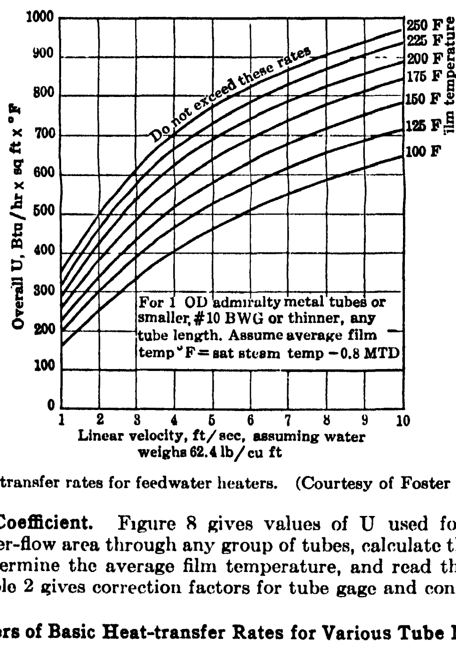


FIG. 8. Heat transfer rates for feedwater heaters. (Courtesy of Foster Wheeler Corp.)

Heat Transfer Coefficient. Figure 8 gives values of U used for commercial work. Determine the water-flow area through any group of tubes, calculate the velocity (in terms of cold water), determine the average film temperature, and read the U value from the proper curve. Table 2 gives correction factors for tube gage and conductivity.

Table 2. Multipliers of Basic Heat-transfer Rates for Various Tube Materials and Gages

For tube OD 5/8 to 1 in. inc.

(Courtesy of Foster Wheeler Corporation)

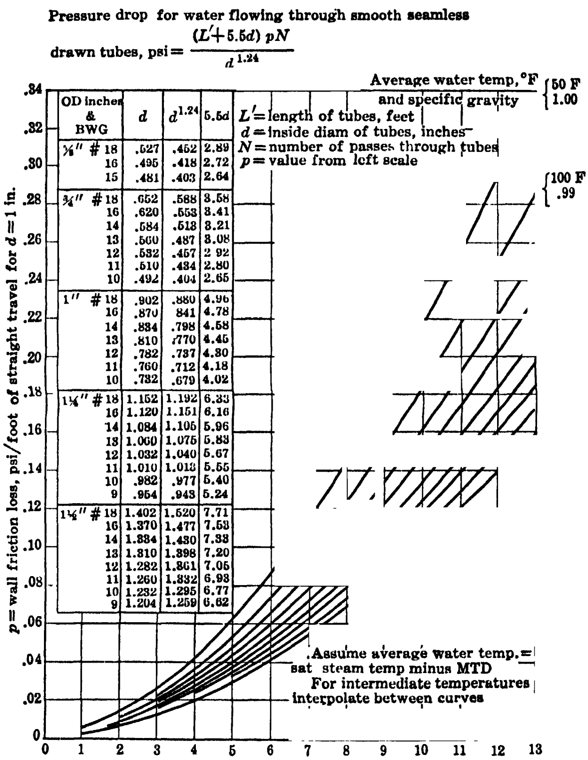
Gage	Arsenical copper	Admiralty	80–20 Cu-Ni	70–30 Cu-Ni	Monel	18-8 Stainless Steel	Low-carbon Steel *
18	1.00	1.00	.95	.92	.89	.85	.52
17	1.00	1.00	.91	.87	.85	.80	.51
16	1.00	1.00	.88	.84	.82	.77	.50
15	1.00	0.99	.86	.82	.79	.74	.49
14	1.00	0.96	.82	.77	.75	.70	.48
13	0.98	0.93	.78	.73	.70	.65	.47
12	0.95	0.90	.73	.68	.65	.60	.45
11	0.92	0.87	.70	.65	.62	.57	.44
10	0.89	0.83	.66	.60	.58	.52	.42
9	0.85	0.80	.62	.56	.54	.48	.41

* Factors for low-carbon steel include additional fouling resistance of 0.001.

Friction loss may be found from Fig. 9. The pressure drop per foot found from the chart is inserted in eq. 2, to find the total pressure drop.

$$\Delta p = \frac{(L' + 5.5d)pN}{d^{1.24}} \tag{2}$$

where L' = length of tubes, feet; d = inside diameter of tubes, inches; N = number of passes through tubes; p = wall friction loss, pounds per square inch per foot of travel in 1 in. diameter tubes; and Δp = pressure drop (total) in pounds per square inch.



OD inches & BWG	d	$d^{1.24}$	$5.5d$
⅝″ # 18	.527	.452	2.89
16	.495	.418	2.72
15	.481	.403	2.64
¾″ # 18	.652	.588	3.58
16	.620	.553	3.41
14	.584	.518	3.21
13	.560	.487	3.08
12	.532	.457	2.92
11	.510	.434	2.80
10	.492	.404	2.65
1″ # 18	.902	.880	4.96
16	.870	.841	4.78
14	.834	.798	4.58
13	.810	.770	4.45
12	.782	.737	4.30
11	.760	.712	4.18
10	.732	.679	4.02
1¼″ # 18	1.152	1.192	6.33
16	1.120	1.151	6.16
14	1.084	1.105	5.96
13	1.060	1.075	5.83
12	1.032	1.040	5.67
11	1.010	1.013	5.55
10	.982	.977	5.40
9	.954	.943	5.24
1½″ # 18	1.402	1.520	7.71
16	1.370	1.477	7.53
14	1.334	1.430	7.33
13	1.310	1.398	7.20
12	1.282	1.361	7.05
11	1.260	1.332	6.93
10	1.232	1.295	6.77
9	1.204	1.259	6.62

FIG. 9. Pressure drop in tubes. (Courtesy of Foster Wheeler Corp.)

HEATER CONSTRUCTION. Typical construction of low- and high-pressure heaters is shown in Figs. 10 to 13 inclusive.

Figure 10 illustrates shell constructions. The lower one, with removable cover, is adaptable for use with floating header heaters; the upper one, with integrally welded head, is used with U-bend types of heater. These shells are useful for both high and low pressure. Figure 11 is an exploded view of a low-pressure header assembly. Figure 12 illustrates the shear-block type *Lockhead* high-pressure closure in exploded view. A modified type is shown in Fig. 13.

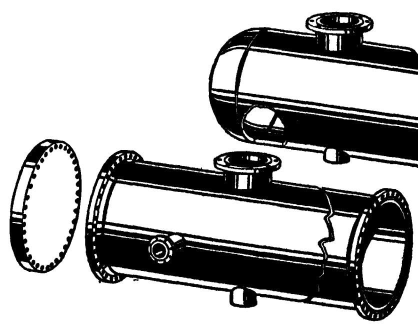


FIG. 10. Closed feedwater heater shell constructions. (Courtesy of Foster Wheeler Corp.)

Lockhead Design. Headers, also known as waterboxes or channels, are generally of forged carbon or low-alloy steel, machined as shown, with integral tube sheets. The Lockhead header provides a separate means for supporting head-pressure load and gasket-compression load. The isolated gasket load is carried by a set of bolts compressing the diaphragm gasket between the waterbox channel and a flexible diaphragm. The diaphragm is a disk of heavy flange rim section with a thin flexible center portion. The pressure load is transmitted through the flexible center of the diaphragm to a heavy pressure cover, held in position by a set of shear blocks or a separate set of bolts, as in Figs. 12 and 13. The initial application of pressure or variations thereof will cause movement or change of shape of pressure parts. The flexible section of the diaphragm compensates for this with negligible effect on the row of bolts clamping the diaphragm gasket to its seat. This insures an independently secured tight seal against the pressure, since the seal is entirely separate from the strength member supporting the head load.

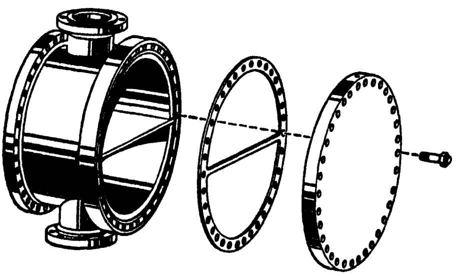


FIG. 11. Water-box construction of low pressure tubular-type feedwater heaters. (Courtesy of Foster Wheeler Corp.)

DRAIN-COOLER SECTIONS. The method of determination of the heating surface described above applies to a simple condensing type of feedwater heater. Use of a drain-cooler section improves the economy of the power plant. In a simple condensing heater the condensed steam is withdrawn at the saturated steam temperature and normally passes through a regulated orifice to the shell of the next-lower heater. This results in flashing of condensate into steam at the lower pressure. This steam is then condensed in the heater shell and reduces the extraction from this (lower) bleed point of the turbine, thus improving the economy.

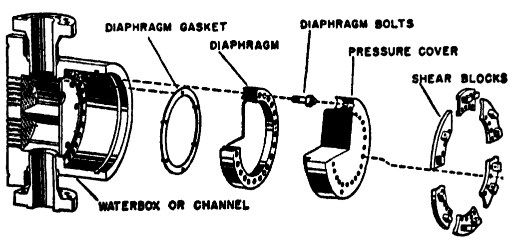


FIG. 12. High-pressure, shear-block type Lockhead closure for tubular-type feedwater heaters. (Courtesy of Foster Wheeler Corp.)

By using surface segregated from the *condensing* surface in the higher-stage heater, keeping this surface flooded at all times, and passing the condensed steam over it, the condensed steam can be cooled to a temperature approaching that of the inlet water to the heater. When these drains are rejected to the next-lower heater relatively little flashing occurs, and more steam is extracted from the next-lower stage of the turbine, resulting in additional power. This additional surface, known as a *drain-cooler section*, is shown in Fig. 14. Depending on the size of the turbine, the load factor, the price of coal, and other considerations it is possible to justify economically the value of drain-cooler sections in one or more heaters. (See *Steam Turbines and Their Cycles*, by J. Kenneth Salisbury, John Wiley and Sons, 1950.)

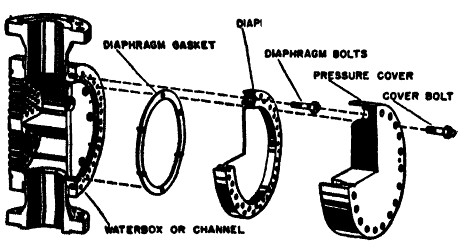
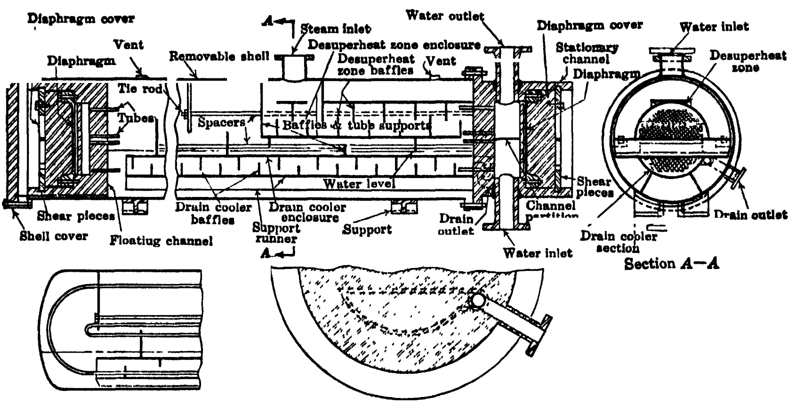


FIG. 13. High-pressure, modified Lockhead closure for tubular-type feedwater heaters. (Courtesy of Foster Wheeler Corp.)

DESUPERHEATING ZONE. Another refinement, in use for many years, is the desuperheating zone. It involves introducing (superheated) steam close to the outlet of the tube bundle system, where the water is at maximum temperature, leaving the condensing zone. This surface, *enclosed* so that the steam passes over the surface before delivery into the condensing section of the heater, is used where bled steam from the turbine contains a relatively large amount of superheat. The usable heat contained in the superheat depends on the temperature rise through the heater and the heat



U-tube construction Section through drain cooler outlet

FIG. 14. Cross section of a closed heater incorporating desuperheater and drain cooler sections. (Courtesy of Foster Wheeler Corp.)

of superheat contained in the bled steam. Superheated steam is cooled in passing through this zone, which adds heat to the water after it has left the condensing zone and provides an additional 3 to 15 degrees rise in the feedwater temperature leaving the heater. In a normal condensing heater, after approaching within 4 or 5 F of the saturation temperature, the cost of surface for each additional degree rise becomes uneconomical. By the introduction of a desuperheat zone considerably greater rise can be secured economically. The cost of surface to achieve this rise may be balanced against the value of the fuel saved, and thus proportioned to give maximum return on the investment.

Figure 14 illustrates a high-pressure heater incorporating a desuperheating zone, a drain-cooling section, and the normal condensing section.

PHYSICAL ARRANGEMENT AND OPERATION. The heaters described may be built either for horizontal or for vertical mounting. Where large high-pressure heaters are encountered the vertical arrangement is better because of the very heavy tube bundles. Heaters may be arranged with the inlet and outlet header at the top or the bottom, as desired. This freedom of arrangement applies to floating-header heaters as well as U-bend types. When a U-bend type is used with a cooler section, however, a very expensive construction is required to seal the wrapper plate of the cooler section to the tubes when the U section is at the top and the water header is at the bottom.

When horizontal installations with very heavy tube bundles are used it is desirable to withdraw the *shell* from the *bundle*, providing rollers for the shell. The bundle may be withdrawn from a fixed shell if internal rollers are used, but this requires a larger and more expensive shell.

Priming of Drain-cooler Sections. When drain-cooler sections are used with low-pressure heaters and vertical installations the pressure differential between the two stages may not be sufficient to prime the drain-cooler section. Thus unless the turbine is to be run constantly at very high loads, low-pressure heaters with integral drain-cooler sections should be of the horizontal type.

Cascading. When drains are cascaded consecutively from a high-pressure heater through a group of four or five others it often will be found that the drain-cooler section in the lowest heater is larger than the condensing portion of that heater. In such a situation a separate drain cooler is indicated.

Strength of Wrapper Plate. Drain-cooler sections and their wrapper plates must be structurally well built and, particularly with horizontal heaters, spaced from the condensing sections so that level control is possible, keeping the wrapper plate of the cooler section submerged at all times. Intermittent uncovering of the wrapper plate and pocketing of steam within it may result in condensation of the pocketed steam and collapse of the wrapper-plate assembly. Care should be taken in introducing drains from a heater containing no drain-cooler section into a heater containing one. These drains should never be introduced into the drain-cooler section of the next-lower heater since a pressure may be developed which will rupture the wrapper plate.

Floating-head versus U-bend Type. Selection of a floating-head heater versus a U-bend heater is entirely a matter of preference. The modern regenerative feed cycle contains very pure, clean water. U tubes can be cleaned chemically when used with clean water. In the past U tubes were seldom used because of the cleaning problem and because tube metal quality had not been developed as it is today. A U-bend type of heater consists of bends of varying radii nested together. If a considerable number of tube failures occurs, it is necessary to retube the entire heater since the inner tubes cannot be removed without removing most of the outer tubes.

Proper venting of closed heaters is particularly important since closed heaters use different relative locations of steam inlets and vents. The steam should be led across baffle plates from the steam inlet in a continuous unidirectional flow to the vents. The vents should be placed as close to the tube sheets as possible, to scavenge properly stagnant gases from points adjacent to baffle or support plates and tube sheets, where they come in contact with tubes. Otherwise condensation in the presence of stagnant gases at these points will corrode the tubes at their outer periphery.

CHEMISTRY OF BOILER FEEDWATER

By Frederick G. Straub and Others

14. COMPOSITION AND ANALYSIS OF FEEDWATER

IMPURITIES IN FEEDWATER. Natural feedwater supplies contain solids and dissolved gases which may promote these conditions in boilers: (1) incrustation or scale; (2) foaming, priming, and solids in steam; (3) corrosion; and (4) caustic embrittlement. To avoid these troubles, it is necessary to study each water supply individually and determine its individual characteristics and how it best may be treated.

Because of the high solids content, sea water and certain other bodies of water are unfit for use in boilers. Rainwater becomes contaminated in falling through the atmosphere, and always contains dissolved gases, including oxygen and carbon dioxide. Carbon dioxide forms a mild acid which greatly increases the solvent action of the water. Thus, with carbonic acid present, it can dissolve considerable amounts of materials such as calcium and magnesium salts from the ground through which the water passes.

CLASSES OF IMPURITIES. Table 1 is a partial list of impurities found in boiler feedwater, their effect in the boiler, and the usual method of treatment. The solubilities are listed to show constituents which may be present in water, and whether they can be expected to precipitate under boiler operating conditions. Increase of temperature increases the solubility of some solids and precipitates others. Regardless of whether solubility increases or decreases with temperature, concentration of solids in the boiler water increases with continued evaporation. Table 2 gives the solubility of substances listed in Table 1. As solubilities vary with temperature and authorities differ on the values for some constituents, it is not possible to estimate all solubilities under boiler conditions. The important feature is the probable effect of these constituents in the boiler. (See next to last column, Table 1.)

The impurities may be roughly classified under these headings.

Dissolved Gases. Inert gases, as nitrogen and the hydrocarbons. Corrosive or active gases, as oxygen, carbon dioxide, and hydrogen sulfide.

Dissolved Solids. *Slightly soluble solids* include most calcium and magnesium compounds. Also oil and silica.

Highly soluble solids include all soluble salts, as sodium chloride, sodium sulfate, sodium carbonate, sodium nitrate, and certain sodium silicates. Also sodium hydroxide, sodium phosphate, the acids, and certain organic compounds.

Suspended solids include the common clays and silts, organic and inorganic matter, found principally in rivers and streams, and all other insoluble matter.

Insoluble liquids, oils, greases, soaps, etc., have a deleterious effect on boiler water.

SPECIFICATIONS FOR BOILER WATER AND FEEDWATER. The following specifications are practicable with present feedwater treatment methods and equipment.

Feedwater. *Dissolved Oxygen.* Preferably zero and not over 0.05 cc per liter for boilers; zero where steel tube economizers are used. *pH Value.* Not less than 7. Excess alkalinity other than required for treatment or protection of feed lines, or to neutralize acids, should be reduced to a minimum. *Hardness.* Preferably zero. Not over 26 parts per million in terms of calcium carbonate. *Chloride.* Lowest practical minimum is desired. When due to condenser or other leakage not over 6 parts per million in terms of chlorine. *Oil.* None. *Total Solids.* Reduce to minimum. *Suspended Solids.* None. *Organic Matter.* Not more than 5 parts per million.

(*Continued on p. 7-52*)

Table 1. Usual Impurities of Boiler Feedwater

Impurity	Formula	Molecular Weight	Equivalent Weight	Solubility *	Probable Effect in Boiler	Methods of Treatment and Removal
Calcium bicarbonate	$Ca(HCO_3)_2$	162.10	81.05	Moderate	Scale and sludge. Liberates CO_2	In external treatment of calcium and magnesium compounds, lime and soda softeners plus coagulation and filtration give partial removal. Zeolite softeners and evaporators give more complete removal, the former replacing calcium and magnesium with sodium. Corrosive compounds require alkali treatment. In internal treatment, calcium and magnesium are precipitated as hydroxide and carbonates by sodium hydroxide and sodium carbonate. Calcium and sometimes part of the magnesium are changed to calcium and magnesium phosphates by treatment with sodium phosphates. Sodium hydroxide is preferred reagent for internal treatment of magnesium compounds. Calcium hydroxide is preferred for external treatment.
Calcium carbonate	$CaCO_3$	100.08	50.04	Slight	Scale and sludge. Liberates CO_2	
Calcium hydroxide	$Ca(OH)_2$	74.10	37.05	Slight	Scale and sludge	
Calcium sulfate	$CaSO_4$	136.14	68.07	Moderate	Hard Scale	
Calcium silicate	Variable	. .	.	Slight	Hard Scale	
Calcium chloride	$CaCl_2$	110.99	55.50	Very soluble	Corrosive. Scale and sludge	
Calcium nitrate	$Ca(NO_3)_2$	164.10	82.05	Very soluble	Corrosive. Scale and sludge	
Magnesium bicarbonate	$Mg(HCO_3)_2$	146.34	73.17	Moderate	Deposits. Liberates CO_2	
Magnesium carbonate	$MgCO_3$	84.32	42.16	Slight	Deposits. Liberates CO_2	
Magnesium hydroxide	$Mg(OH)_2$	58.34	29.17	Very slight	Deposits	
Magnesium sulfate	$MgSO_4$	120.38	60.17	Very soluble	Corrosive, deposits	
Magnesium silicate	Variable		. .	Slight	Hard Scale	
Magnesium chloride	$MgCl_2$	95.23	47.62	Very soluble	Corrosive, deposits	
Magnesium nitrate	$Mg(NO_3)_2$	148.34	74.17	Very soluble	Corrosive, deposits	
Sodium bicarbonate	$NaHCO_3$	84.00	42.00	Very soluble	Increases alkalinity and soluble solids. Liberates CO_2	Excess sodium alkalinity may be reduced by boiler blowdown. It sometimes is neutralized with sulfuric acid externally. Phosphoric acid and acid phosphates also are used. Evaporation is best practical means of removing sodium compounds from feedwater. Boiler blowdown used for internal reduction of soluble solids.
Sodium carbonate	Na_2CO_3	106.00	53.00	Very soluble	Increases alkalinity and soluble solids. Liberates CO_2	
Sodium hydroxide	NaOH	40.00	40.00	Very soluble	Increases alkalinity and soluble solids	
Sodium sulfate	Na_2SO_4	142.05	71.03	Very soluble	Inhibitor for caustic embrittlement. Increases soluble solids	
Sodium silicate	Variable		.	Very soluble	Increases alkalinity. May form silica scale	
Sodium chloride	NaCl	58.45	58.45	Very soluble	Increases soluble solids. Encourages corrosion	
Sodium nitrate	$NaNO_3$	85.01	85.01	Very soluble	Increases soluble solids	
Iron oxide	Fe_2O_3	159.68	26.61	Slight	Deposits. Encourages corrosion	Coagulation and filtration, evaporation, blowdown.
Alumina	Al_2O_3	101.94	16.99	Slight	May add to deposits	Coagulation and filtration, evaporation, blowdown.
Silica	SiO_2	60.06	30.03	Slight	Hard scale, acts as binder for deposits	Precipitation with aluminates, coagulation and filtration, evaporation, blowdown.
Dissolved oxygen	O_2	32.00	16.00	Slight	Corrosive	Deaeration preferred.
Carbonic acid or dissolved CO_2	H_2CO_3	62.02	31.01	Very soluble	Retards hydrolysis of carbonates. Reduces alkalinity	Deaeration and alkali treatment.
Hydrogen sulfide	H_2S	34.08	17.04	Very soluble	Corrosive	Deaeration and alkali treatment.
Acids, organic and mineral	. . . .	. . .	. . . .	Very soluble	Corrosive	Neutralization by alkali treatment.
Oil and grease	. . . . . .	. . . . . .	. .	Slight	Corrosive, deposits, foaming and priming	Coagulation and filtration, skimming.
Organic matter	. . . .	. . . .	. . . . . .	Very soluble	Corrosive, deposits, foaming and priming	Coagulation and filtration evaporation.

* See Table 2.

Table 2. Solubility of Impurities in Boiler Feedwater

Solubility in grams of substance in 100 grams of water

Substance	Formula	0 C	100 C	Authority
Calcium bicarbonate	$Ca(HCO_3)_2$	Soluble	Decomposes	
Calcium carbonate	$CaCO_3$	0.0013 (16 C)	0.002	Landolt Börnstein
Calcium hydroxide	$Ca(OH)_2$	0.1771	0.0667	*Inter. Crit. Tables*
Calcium sulfate	$CaSO_4$	0.1759	0.1688	*Inter. Crit. Tables*
Calcium silicate	$CaSiO_3$	0.0095 (17 C) *†		Seidell
Calcium chloride	$CaCl_2$	59.378	157.600	*Inter. Crit. Tables*
Calcium nitrate	$Ca(NO_3)_2$	102.061	362.630	*Inter. Crit. Tables*
Magnesium bicarbonate	$Mg(HCO_3)_2$	Soluble	Decomposes	....
Magnesium carbonate	$MgCO_3$	0.0106 (cold)		*Handbook of Chem. and Physics*—Hodgman-Lange
Magnesium hydroxide	$Mg(OH)_2$	0.0008		Seidell
Magnesium sulfate	$MgSO_4$	26.725 (1.8 C)	71.027	*Inter. Crit. Tables*
Magnesium silicate	$MgSiO_3$	*		
Magnesium chloride	$MgCl_2$	52.380	72.284	*Inter. Crit. Tables*
Magnesium nitrate	$Mg(NO_3)_2$	66.455	137.211 (90 C)	*Inter. Crit. Tables*
Sodium bicarbonate	$NaHCO_3$	6.888	16.465 (60 C)	*Inter. Crit. Tables*
Sodium carbonate	Na_2CO_3	6.996	45.153	*Inter. Crit. Tables*
Sodium hydroxide	$NaOH$	42.005	338.642	*Inter. Crit. Tables*
Sodium sulfate	Na_2SO_4	4.858	42.192	*Inter. Crit. Tables*
Sodium silicate	Na_2SiO_3	*		
Sodium chloride	$NaCl$	35.658	39.165	*Inter. Crit. Tables*
Sodium nitrate	$NaNO_3$	73.274	175.450	*Inter. Crit. Tables*
Iron oxide	Fe_2O_3	Insoluble		
Alumina	Al_2O_3	Insoluble		*Handbook of Chem. and Physics*—Hodgman-Lange
Silica	SiO_2	Insoluble		
Oxygen	O_2	0.0069 ‡	0.0	
Carbon dioxide	CO_2	0.3346 ‡	0.0576 (60 C)	
Hydrogen sulfide	H_2S	0 7066 ‡	0.0	

* The formulas and solubilities of the silicates are extremely variable, ranging from very high solubilities (especially sodium) to slight solubility.

† Per 100 cc solution.

‡ Pressure, 760 mm.

Boiler Water. *Sodium Phosphate.* With residual hardness in the make-up, 50 to 100 parts per million expressed as disodium phosphate. *Alkalinity.* Between 100 to 250 parts per million depending on silicates which also are present. The higher alkalinity is preferred when silicate concentration is 100 to 200 parts per million. *Chlorides.* Not over 500 parts per million expressed as chlorine. Preferably as low as possible. *pH Value.* Not less than 10.5, preferably 11.0. *Oil.* None. *Total Solids.* Not over 1700 parts per million.

ANALYSES. In examining raw water supplies to determine their suitability for feedwater and proper methods for purification, a complete analysis is preferred. When the purification plant has been standardized, control tests may be applied to feed and boiler water to maintain desired conditions. These usually consist of tests for alkalinity or acidity; *p*H value; hardness; chloride; sodium sulfate; dissolved oxygen; dissolved solids; turbidity.

Alkalinity or acidity is measured quantitatively by a titration method, using a standard acid or alkali in a burette and flask containing sample and color indicator. If indicator shows an alkaline reaction, sample is titrated with the standard acid until a certain color end point is reached. If indicator shows an acid reaction, it is titrated with the standard alkali to a predetermined end point. Alkalinity or acidity is then calculated in parts per million or grains per gallon of the predominating alkali or acid.

***p*H value (hydrogen-ion concentration)** is determined to measure the degree of acidity or alkalinity of a sample. In the colorimetric method (generally used) a measured amount of a chosen indicator is added to a measured volume of sample in a test tube or small cell. The color of the tube is compared to sets of color standards which represent the result for different *p*H values. A useful universal indicator that can be used for both titration work and approximate *p*H value can be obtained from such laboratory supply houses as Palo-Myers, Inc., New York.

EXPLANATION OF *p*H VALUE. All aqueous solutions contain hydrogen (H) and hydroxyl (OH) ions. The product of their concentrations is equal to a constant value which at room temperature is approximately 1×10^{-14}. Neutral water contains an equal number of hydrogen and hydroxyl ions. The hydrogen-ion concentration is, therefore, 1×10^{-7} gram of ionized hydrogen per liter.

When acid is added, the hydrogen-ion concentration increases with corresponding decrease in hydroxyl-ion concentration. When an alkali is added, the hydroxyl-ion concentration increases, and the hydrogen-ion concentration decreases. Since all acids and alkalies do not ionize alike, the quantity of acid or alkali does not give a direct measure of hydrogen-ion concentration. Strong acids, as hydrochloric, and strong alkalies, as sodium hydroxide, are much more effective in changing hydrogen-ion concentration than relatively weak materials, as carbonic acid and sodium carbonate.

For convenience, only the hydrogen-ion concentration is recorded, whether the solution is acid or alkaline. Hydroxyl-ion concentration may be found by dividing hydrogen-ion concentration into 1×10^{-14}. Thus, if hydrogen ion concentration is 1×10^{-8}, the hydroxyl ion is 1×10^{-6}. Hydrogen-ion concentration is expressed in terms of pH value, equivalent to log (1/H-ion concentration), that is, to the negative exponent. Thus if hydrogen ion concentration is 1×10^{-9}, pH value is 9. The lower the hydrogen-ion concentration, the higher the pH value. In neutral water $p\text{H} = 7$; in water that is relatively ten times as alkaline, $p\text{H} = 8$. If $p\text{H} = 6$, the water is relatively ten times as acid as at $p\text{H} = 7$. Table 3 shows hydrogen-ion concentration, its equivalent pH value, and corresponding color of the universal indicator. Table 4 lists several indicators, their solution concentrations, and the color change for the pH range to which they apply.

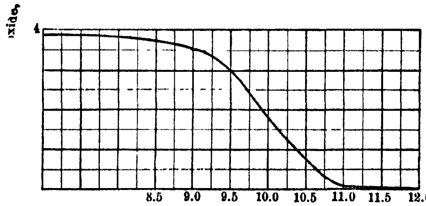


FIG. Relation between pH and solubility of iron in deaerated water at room temperature.

Table 3. Hydrogen-ion Concentration as Shown by Color Indicators

Hydrogen-ion Concentration, Gram-moles per Liter *		pH		Color of Universal Indicator
1.0	10^{-0}	0		
0.1	10^{-1}	1		
0.01	10^{-2}	2	Acid	
0.001	10^{-3}	3	range	
0.000,1	10^{-4}	4		Red
0.000,01	10^{-5}	5		Pink
0.000,001	10^{-6}	6		Yellow
0.000,000,1	10^{-7}	7	Neutral	Greenish yellow
				Green
0.000,000,01	10^{-8}	8		Blue
0.000,000,001	10^{-9}	9		Blue-violet
0.000,000,000,1	10^{-10}	10	Alkaline	Purple
0.000,000,000,01	10^{-11}	11	range	
0.000,000,000,001	10^{-12}	12		
0.000,000,000,000,1	10^{-13}	13		
0.000,000,000,000,01	10^{-14}	14		

* For hydrogen ion 1 gram-mole = 1 gram, but for hydroxyl ion 1 gram-mole = 17 grams.

Table 4. Colorimetric Indicator Solutions

Indicator	Concentration	pH Range	Color Change
Meta cresol purple	.04%	1.2– 2.8	Red–yellow
Bromophenol blue	.04	3.0– 4.6	Yellow–blue
Methyl red	.02	4.4– 6.0	Red–yellow
Bromocresol green	.04	4.0– 5.6	Yellow–blue
Bromocresol purple	.04	5.2– 6.8	Yellow–purple
Bromothymol blue	.04	6.0– 7.6	Yellow–blue
Phenol red	.02	6.8– 8.4	Yellow–red
Cresol red	.02	7.2– 8.8	Yellow–red
Thymol blue	.04	8.0– 9.6	Yellow–blue
Phthalein red	...	8.6–10.2	Yellow–red
Tolyl red	...	10.0–11.6	Red–yellow
Paraso orange	...	11.0–12.6	Yellow–orange
Acyl blue	...	12.0–13.6	Red–blue

This method of measuring acidity or alkalinity is useful in controlling corrosion and certain chemical reactions in treatment of feed and boiler water. For feedwater *p*H should be at least 7, and for boiler water at least 10.5. Figure 1 shows the relation between *p*H and solubility of iron in deaerated water.

HARDNESS. For control purposes, total hardness is determined by adding standard soap solution to a bottle containing a measured amount of sample, shaking the bottle vigorously between additions of soap solution, the bottle lying on its side, until an unbroken lather is maintained for 5 min on the water surface. Volume of soap solution used is referred to a chart or multiplied by a factor. The result is expressed in parts per million, grains per gallon, or equivalent calcium carbonate.

Actually, hardness consists of such materials as calcium and magnesium carbonate and bicarbonates, calcium and magnesium sulfates, and calcium and magnesium chlorides. These materials can be precipitated by boiling, and are known as temporary hardness. For example, the bicarbonates of calcium and magnesium are changed to carbonates, which are much less soluble. The remaining hardness is known as permanent hardness.

Chloride concentration is determined by titrating a measured volume of sample with standard silver nitrate solution, potassium chromate being used as an indicator. The end point is indicated by a red coloration. The result is expressed in parts per million, grains per gallon of chlorine, or equivalent sodium chloride.

Equivalent sodium sulfate determination is useful in boiler-water analyses. In control work, benzidine sulfate titration or the turbidity method is used. The titration consists of adding an excess of benzidine sulfate to a measured sample of water. After standing, to allow complete precipitation of sulfate as benzidine sulfate, filter and wash precipitate. Titrate the precipitate with a standard sodium hydroxide solution, with phenolphthalein as the indicator.

The turbidity method consists of adding hydrochloric acid and barium chloride to a measured sample of water, causing a white precipitate of barium sulfate to form. The sample is stirred to keep precipitate in suspension, and the mixture slowly poured into a graduated tube with a small light below it. When sufficient mixture has been added to just obscure the light filament, when looking down the tube, height of liquid is read, and equivalent sodium sulfate in parts per million or grains per gallon is estimated or read from the graduated tube.

Dissolved oxygen is an important test in controlling deaeration of feedwater. It involves sampling water through a cooling coil to reduce temperature to below 70 F, flowing water from the coil through a glass-stoppered sample bottle to wash out any air not in the sample. The sample is then fixed with three reagents, usually manganous sulfate, alkaline potassium iodide, and sulfuric acid. A measured volume of the sample is titrated with a standard sodium thiosulfate solution, with starch as an indicator. If dissolved oxygen is absent there will be no blue coloration when the indicator is added. Result is expressed in cc per liter or parts per million of dissolved oxygen.

Dissolved solids may be estimated in several ways. In the laboratory they are determined by evaporating a measured volume of sample and weighing the dried residue. For boiler water, hydrometer, densimeter, and conductivity tests are used, suitable calibrations being made for the type of water.

TURBIDITY tests are made by several methods, depending on the amount of suspended solids. For certain boiler waters containing considerable suspended matter, some type of turbidimeter may be used to regulate blowdown for suspended solids. The sulfate meter is operated by pouring liquid containing suspended matter into a tall glass cylinder until a light filament under the cylinder is no longer visible. Height of liquid in the cylinder is then read. Another method, for waters containing less suspended matter, involves the immersion of a graduated rod holding a wire at the end into the sample until the wire no longer can be seen. (See Refs. 1, 2, and 3.)

15. FEEDWATER TREATMENT

CAUSES OF SCALE FORMATION. Hard scale and incrustations of softer deposits result from the presence or formation of insoluble solids in feed and boiler water. Certain slightly soluble solids, when treated with water-softening chemicals, or heated and concentrated in the boiler water, become less soluble and precipitate. The most objectionable are calcium sulfate and silica, which have a strong tendency to crystallize and precipitate, forming hard scale which is difficult to remove. Both may act as a cement for other insoluble matter and hasten the formation of a heavy scale, which, because of its poor heat conductivity, will cause overheating and failures of boiler tubes. Calcium sulfate, and, to a slightly less extent, silica, tend to form scale on the hottest tubes. Calcium carbonate is more likely to precipitate in the boiler water than on the tubes. Its deposits tend to be greater in the cooler parts of the boiler. With external heating of feedwater containing calcium bicarbonate, the less soluble calcium carbonate often is formed and deposited in heaters and pipe lines. With this chemical present, the same result may be obtained by continuous addition of caustic soda to feedwater. Calcium phosphate tends to deposit in feed lines when sodium phosphate is used as a treating agent. Tannates have been used to delay precipitation of calcium compounds in the feed system. In general, calcium phosphate does not give serious trouble in boiler water, but periodic cleaning is advisable.

For explanation of the process of scale formation, see Ref. 4.

COAGULATION, SEDIMENTATION, AND FILTRATION may be used alone or in conjunction with lime-soda or other treatments. The removal of suspended matter is an important part of the treating system. Coagulation and sedimentation are carried on in large basins, or in tanks, depending on quantity of water to be handled. Usual coagulants are iron sulfate (copperas), aluminum sulfate, sodium aluminate, and lime. Cold-water filtration is carried on usually with gravity or pressure-type sand filters. In hot-water filtration, less soluble materials, as calcite or magnetite, should be used to avoid formation of calcium silicate, which may result in hard, dense scale. Sand filters usually are designed for a capacity of 2 to 4 gal of water per minute per square foot of cross-sectional area.

LIME-SODA TREATMENT is applied in several ways, with considerable variation in design of equipment. The principal differences are in the temperature of water, cycle of operation (whether continuous or intermittent flow), and method of applying and agitating chemicals. Hot-process equipment usually consists of chemical mixing tanks, chemical proportioner (which introduces chemicals in proportion to flow of water), a deaerating-type heater, placed above a reaction and settling tank, and a filter to remove unsettled suspended matter. The cycle usually is continuous. Water flows through the heater into the top of the reaction and settling tank, where chemicals are introduced and thence to the bottom of the tank, where suspended matter settles and is blown out. The water then rises through a central duct and is discharged through the side of the tank, at a point below the water level. Treated water finally passes through a closed or pressure-type filter. Retention time of water in the tank is preferably not less than one hour. Figs. 2 and 3 show two types of hot lime-soda water softeners.

In the cold process, equipment may be the same as in the hot process without the heater. Usually, it is desirable to have a longer reaction and settling time, and several treating tanks are provided. In these, water is treated, agitated, settled, and finally drawn off from the top to the filters. Each tank is treated in rotation. The cycle is so timed that treated water always flows from one tank to the filters while water in the others is being treated or settling. Filters frequently are of the open gravity type. Depending on analysis of raw water and excess of treating chemicals used, effluent water from a cold-process softener may have a hardness of 2 to 5 grains per gal, or from the hot-process softener a hardness of 1 to 3 grains per gal.

The chemicals used in these treatments are one or more coagulants (iron sulfate, aluminum sulfate, sodium aluminate), calcium hydroxide, and sodium carbonate. Coagulants are added either before or after lime and sodium carbonate, depending on the ease of coagulating suspended matter. Both iron sulfate and aluminum sulfate create acidity when added to neutral water, which must be corrected by alkaline chemicals. Sodium aluminate gives an alkaline reaction and, besides being a coagulant for suspended matter, it has water-softening properties similar to that of sodium carbonate.

Calcium hydroxide or hydrated lime combines with excess carbon dioxide, and reacts with calcium and magnesium bicarbonates to form less soluble calcium carbonate and magnesium hydroxide. Also when sodium carbonate is present, sodium hydroxide is formed. These reactions for lime are:

$$\underset{\text{Calcium hydroxide}}{2Ca(OH)_2} + \underset{\text{Magnesium bicarbonate}}{Mg(HCO_3)_2} = \underset{\text{Calcium carbonate}}{2CaCO_3} + \underset{\text{Magnesium hydroxide}}{Mg(OH)_2 + 2H_2O}$$

$$\underset{\text{Calcium hydroxide}}{Ca(OH)_2} + \underset{\text{Calcium bicarbonate}}{Ca(HCO_3)_2} = \underset{\text{Calcium carbonate}}{2CaCO_3} + 2H_2O$$

$$\underset{\text{Calcium hydroxide}}{Ca(OH)_2} + \underset{\text{Carbonic acid}}{H_2CO_3} = \underset{\text{Calcium carbonate}}{CaCO_3} + 2H_2O$$

$$\underset{\text{Calcium hydroxide}}{Ca(OH)_2} + \underset{\text{Sodium carbonate}}{Na_2CO_3} = \underset{\text{Calcium carbonate}}{CaCO_3} + \underset{\text{Sodium hydroxide}}{2NaOH}$$

The principal use of sodium carbonate is to react with calcium sulfate to form a less objectionable scale-forming compound. The treatment is also effective in changing acid-

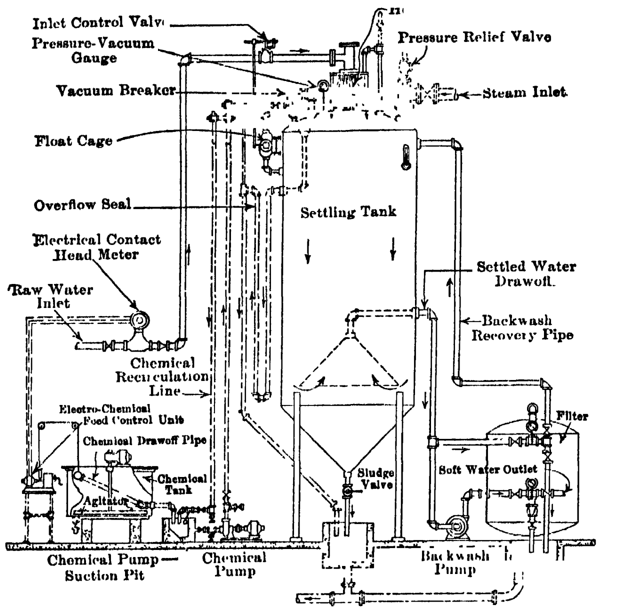


Fig. 2. Hot lime soda water softener. (Permutit Co.)

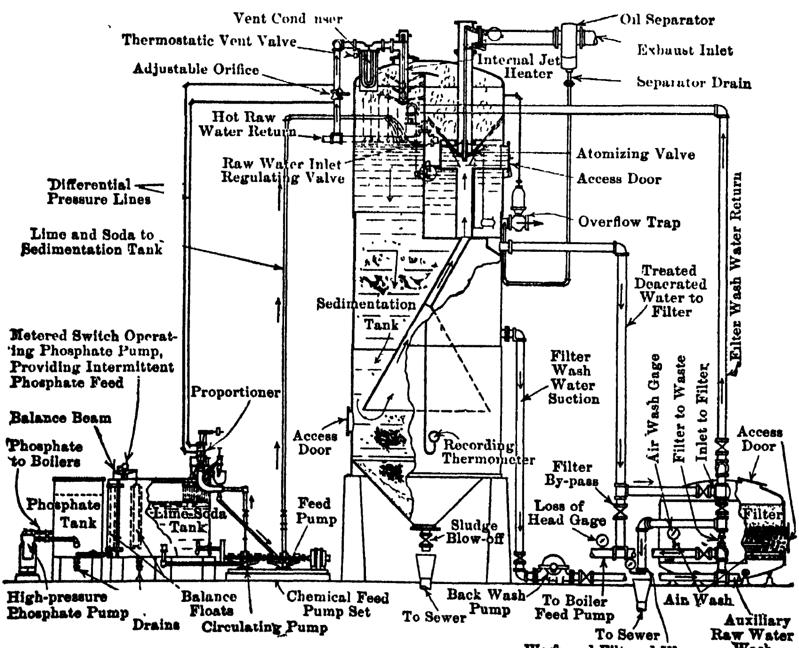


Fig. 3. Hot lime-soda deaerating feedwater heater and softener (Cochrane Corp.)

forming salts to neutral salts. The resulting magnesium hydroxide and calcium carbonate are largely precipitated in the softener. The principal reactions are:

$$\underset{\text{Calcium sulfate}}{CaSO_4} + \underset{\text{Sodium carbonate}}{Na_2CO_3} = \underset{\text{Sodium sulfate}}{Na_2SO_4} + \underset{\text{Calcium carbonate}}{CaCO_3}$$

$$\underset{\text{Calcium chloride}}{CaCl_2} + \underset{\text{Sodium carbonate}}{Na_2CO_3} = \underset{\text{Sodium chloride}}{2NaCl} + \underset{\text{Calcium carbonate}}{CaCO_3}$$

$$\underset{\text{Magnesium sulfate}}{MgSO_4} + \underset{\text{Sodium carbonate}}{Na_2CO_3} + \underset{\text{Calcium hydroxide}}{Ca(OH)_2} = \underset{\text{Magnesium hydroxide}}{Mg(OH)_2} + \underset{\text{Calcium carbonate}}{CaCO_3} + \underset{\text{Sodium sulfate}}{Na_2SO_4}$$

$$\underset{\text{Magnesium chloride}}{MgCl_2} + \underset{\text{Sodium carbonate}}{Na_2CO_3} + \underset{\text{Calcium hydroxide}}{Ca(OH)_2} = \underset{\text{Magnesium hydroxide}}{Mg(OH)_2} + \underset{\text{Calcium carbonate}}{CaCO_3} + \underset{\text{Sodium chloride}}{2NaCl}$$

Note: Sodium hydroxide may be substituted for sodium carbonate and calcium hydroxide in the last two equations.

The amounts and kinds of coagulants added are regulated mainly by coagulation tests. Lime and sodium carbonate additions are regulated by chemical analysis and control tests. The control tests usually made are soap hardness and alkalinity.

After a complete analysis of raw water, theoretically required amounts of lime (calcium hydroxide) and soda (sodium carbonate) may be calculated from the reacting molecular weights of the compounds shown in the foregoing equations. Actually some excess of treating agent is desirable and, owing to impurities, an allowance of 5 to 10% should be made for lime and of 1 to 2% for sodium carbonate.

EVAPORATION of make-up water is especially useful in power plants where the percentage make-up is small and a minimum quantity of boiler deposit is desirable. This process produces purer water than any other process now available. The amount of make-up water which can be so prepared is limited only by economic considerations. See Section 3 for a discussion of evaporators. Condensate from low-pressure boilers sometimes is used to supply make-up water for high-pressure boilers in which no appreciable quantity of scale can be tolerated. To assist continuous operation, feedwater to evaporators may be pretreated in the same manner as for boilers. In other cases it is preferred to crack scale from the evaporator tubes by the introduction of cold water when the scale is brittle. As with boilers, total solids in the water in the evaporator must be regulated to inhibit carrying undesirable quantities of solids into the steam.

INTERNAL WATER TREATMENT consists mainly of maintaining desirable concentrations of chemicals in the boiler water. Chemicals to be added and concentrations to be maintained vary with the nature of feedwater and results of operation. The usual agents for inhibiting scale formation are sodium carbonate, sodium phosphate, and sodium aluminate. Sodium hydroxide, tannates, and various prepared boiler compounds are used on occasion, or for some specific need. In general, prepared compounds are viewed with disfavor from both practical and economic standpoints.

Sodium carbonate is used to promote a desirable alkalinity and to inhibit formation of calcium sulfate scale. It also may retard formation of silica scale. In boilers, it hydrolyzes to form sodium hydroxide and CO_2 gas, the latter passing off with the steam The extent of this reaction depends mainly on the amount of carbonate in the feedwater but, in general, 70 to 90% of the sodium carbonate becomes sodium hydroxide.

Experiments indicate that under favorable conditions 2 to 3 grains of sodium carbonate in boiler water will inhibit calcium sulfate scale. The final concentration should be regulated by results of practical experience.

Sodium phosphate is used principally to precipitate (as tricalcium phosphate) calcium entering with the feedwater. This finely divided material has considerably less tendency to form objectionable deposits than the calcium compounds that otherwise would be present. A similar reaction may occur with magnesium, but in practice sufficient alkalinity usually is present to precipitate magnesium as magnesium hydroxide. The common forms of sodium phosphate are trisodium phosphate, Na_3PO_4; disodium phosphate, Na_2HPO_4; monosodium phosphate, NaH_2PO_4; sodium metaphosphate, $(NaPO_3)x$.

The less-alkaline phosphates, as monosodium phosphate, are used when there is excess alkalinity in feedwater or when calcium is present largely as calcium carbonate. Total al-

kalinity and total solids in boiler water thus are more easily controlled. Typical reactions of trisodium phosphate are:

Soluble Impurity		Treating Agent		Insoluble Product		Soluble Product
$3CaCO_3$	+	$2Na_3PO_4$	=	$Ca_3(PO_4)_2$	+	$3Na_2CO_3$
Calcium carbonate		Trisodium phosphate		Calcium phosphate		Sodium carbonate
$3CaSO_4$	+	$2Na_3PO_4$	=	$Ca_3(PO_4)_2$	+	$3Na_2SO_4$
Calcium sulfate		Trisodium phosphate		Calcium phosphate		Sodium sulfate

Calcium phosphate thus formed has considerable tendency to adhere to feed lines. It generally is safer to add the phosphates direct to the boiler, or in intermittent doses, so that a minimum of precipitate is formed external to the boiler.

Provided there is sufficient alkalinity in the boiler water, the less-alkaline phosphates give the same type of reaction as above. The following is an example:

$3CaCO_3$ +	$2NaH_2PO_4$ +	$4NaOH$ =	$Ca_3(PO_4)_2$ +	$3Na_2CO_3$	+ $4H_2O$
Calcium carbonate	Monosodium phosphate	Sodium hydroxide	Calcium phosphate	Sodium carbonate	

Only a small amount of phosphate need be maintained in boiler water to inhibit scale. Phosphates, unlike sodium carbonate, do not lose efficiency by hydrolysis in boiler water. As with other forms of internal treatment it is important that boilers and other equipment in feedwater and steam systems be inspected and cleaned periodically.

SODIUM ALUMINATE usually is given the formula $Na_2Al_2O_4$, but in liquid form may contain a higher ratio of sodium to alumina. While generally recommended as a coagulant for external treatment, it also is used as a substitute or aid to sodium carbonate and sodium phosphate in internal treatments. It has many of the qualities of sodium carbonate. In addition it tends to make calcium and magnesium precipitates less adherent than if precipitated alone. Under certain conditions it will reduce silica concentration of boiler water, and form calcium or magnesium aluminum silicates, which are not generally adherent. However, under certain adverse conditions adherent silicates have formed, and the manufacturer of the material should be consulted.

For further information, see publications of the National Aluminate Corporation, Chicago.

16. ION EXCHANGE SOFTENERS *

SODIUM CYCLE EXCHANGERS. An ion exchange softener operates without precipitation, but exchanges the ions in the water for ions that are usually more desirable than the original ions. Early ion exchangers were called zeolites. They occurred in natural deposits such as the New Jersey Greensands, and some were made synthetically. They are compounds of alumina and silica, called *aluminate silicates.* These silicates have low solubility in water and possess the property of so-called base exchange. The sodium of the zeolite is replaceable by other metal (positive) ions when in contact with their solutions. The reaction becomes reversible when the concentration of sodium ion is increased. Thus a solution of calcium or magnesium salt, when passed through a sodium zeolite bed, will exchange the calcium and magnesium for the sodium in the zeolite and the water will be soft until the sodium in the zeolite is used up. The sodium is replaced by so-called regeneration with a soluble sodium salt, usually sodium chloride, and rinsing out of the released calcium and magnesium chloride. The operation of this process is illustrated as follows:

$$\begin{matrix}\text{Calcium}\\ \text{or}\\ \text{magnesium}\end{matrix}\left\{\begin{matrix}\text{Bicarbonate,}\\ \text{chloride}\\ \text{or}\\ \text{sulfate}\end{matrix}\right. + \text{sodium zeolite}$$

$$\text{Sodium}\ \begin{matrix}\text{bicarbonate,}\\ \text{chloride,}\\ \text{or}\\ \text{sulfate}\end{matrix} + \begin{matrix}\text{calcium}\\ \text{or}\\ \text{magnesium}\end{matrix}\ \text{zeolite}$$

$$\begin{matrix}\text{Calcium}\\ \text{or}\\ \text{magnesium}\end{matrix}\left\{\ \text{zeolite} + \text{NaCl} = \text{Sodium zeolite} + \begin{matrix}\text{calcium}\\ \text{or}\\ \text{magnesium}\end{matrix}\right\}\text{chloride}$$

The calcium and magnesium content of the softened water is usually of a low value commonly referred to as zero hardness. The degree of residual hardness is governed by many factors and, if properly controlled, may be reduced to an extremely low value. In normal operation, the residual hardness runs less than three or four parts per million.

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* Articles 16–19 inclusive contributed by Frederick G. Straub, Research Professor of Chemical Engineering, University of Illinois.

The hardness which zeolite removes from a water depends on many factors, such as type of zeolite, rate of water flow, amount of salt used for regeneration, and chemical content of the water being treated. There are two main types of zeolites available, *natural* and *synthetic*. The natural zeolites, of the nonporous type, usually have a lower capacity for hardness removal. This capacity is expressed in terms of equivalent calcium carbonate removed per cubic foot of zeolite. It is about 2500 to 5000 grains of calcium carbonate per cubic foot for the nonporous types and increases to about 10,000 grains per cubic foot in the porous zeolites.

The zeolites may be used either in *gravity* or *pressure-flow* units. The water to be softened is passed at a suitable velocity through the unit holding the zeolite material until the bed becomes exhausted (the available sodium used). The bed is then backwashed with hard water to remove any suspended material that may have collected on the bed and to retard packing of the bed. This is accomplished by reversing the flow through the zeolite bed. A solution of the desired amount of salt or brine then is passed through the bed in the normal direction. Hard water then is passed through the bed to rinse out the calcium and magnesium chloride released from the zeolite and any excess sodium chloride. When the hardness reaches the desired value the unit is available for use.

Synthetic zeolites in general are not as stable as natural zeolites with waters of low hardness, low silica, or a *p*H value less than about 6.8. However, even natural zeolites should not be used for low or high *p*H waters. When waters have a *p*H below 6.8 or above 8.5, the *p*H should be adjusted before passing through the zeolite. Likewise, high temperature cannot be tolerated; consequently only waters with normal temperature may be softened by zeolites.

Waters of high hardness, around 30 grains per gallon (5000 ppm) are difficult to soften as are waters having high chlorides.

The exchange of two sodium ions (mol. wt. 2×23 or 46) for one calcium (mol. wt. 40) or one magnesium (mol. wt. 24) causes an increase in the total solids of the softened water. Thus a zeolite-softened water has an increase in total solids as compared to a decrease with the normal lime-softening process. In addition to a slight increase in total solids, the conversion to the sodium salts may bring about high alkalinities in evaporators or boilers. If the bicarbonate hardness predominates, the softened water will have the sodium bicarbonate predominating over the sodium chloride and sulfate.

The presence of high sodium alkalinity in many zeolite-softened waters is one of the disadvantages of this type of softening for boiler or evaporator make-up waters. To eliminate this difficulty, the waters often have been *pretreated* in cold-process lime-softeners or *after-treated* with sulfuric acid.

Pretreatment with cold-process lime-softening reduces the bicarbonates to a lower value so that the subsequent treatment with the zeolite softener gives a water lower in sodium alkalinity and lower total solids than if it is zeolite softened without pretreatment. However, the higher *p*H value of the lime-softened water will act to disintegrate the zeolite material and will also precipitate calcium carbonate in the softener due to after-precipitation in the cold lime-softened water. To prevent this trouble, the lime-softened water is acidified by means of sulfuric, phosphoric, or carbonic acid to about *p*H 8 before it is passed through the zeolite bed.

After-treatment with sulfuric acid of zeolite-softened water is simpler than pretreatment with lime; however, it does not reduce the total solids, but increases them. This increase is due to the reaction between the sulfuric acid and the equivalent amount of sodium carbonate; the sulfate ion (mol. wt. 96) replaces a carbonate ion (mol. wt. 60) with an increase in dissolved solids. In many instances this increase in dissolved solids is not detrimental to the operation, and its disadvantage is more than equalized by lowering of the alkalinity. Care must be taken in handling the acid so as to prevent acidity in the treated water, and to allow for release of the carbon dioxide formed. The carbon dioxide or carbonic acid tends to lower the *p*H of the treated water and may cause corrosion. Usually this carbon dioxide is removed by blowing air through the acid-treated water. The remaining sodium bicarbonate is slightly broken down to some sodium carbonate by passage of air through the solution with a subsequent increase in *p*H value.

This process is illustrated by the following reactions:

$$2NaHCO_3 + \text{air} = Na_2CO_3 + H_2O + CO_2 \tag{1}$$

$$2NaHCO_3 + H_2SO_4 = Na_2SO_4 + 2H_2CO_3 \tag{2}$$

$$H_2CO_3 = CO_2 + H_2O \tag{3}$$

If zeolite-softened water (having a *p*H of about 7) is treated by bubbling air through it or spraying the water through air, reaction 1 proceeds, to a small extent, to the right. The solution of sodium bicarbonate has a *p*H of about 7. As sodium carbonate is formed by removal of carbon dioxide, however, the *p*H increases because sodium carbonate is a salt of a strong base (NaOH) and a weak acid (H_2CO_3), so that the resulting solution is alkaline in nature (high pH). The amount of carbonate formed and the resulting pH depend on the amount of air used, the carbon dioxide in the air, the amount of bicarbonate present, and the temperature.

When sulfuric acid is added to a zeolite-softened water, the solution becomes acid (pH below 7) before a sufficient amount of sulfuric acid has been added to neutralize completely the sodium bicarbonate. The acidity is caused by the formation of carbonic acid, which tends to decrease the pH value of the solution. If air is blown through the solution, the carbonic acid breaks down to water and carbon dioxide, and the carbon dioxide is removed. If sufficient air is passed through the residual, bicarbonate forms some carbonate (reaction 1) with a slight increase in pH. This increase in the pH of a zeolite-softened water takes place during deaeration in the regular plant deaerator. The bicarbonate tends to form the carbonate as the carbon dioxide is released during deaeration.

Newer materials have been developed which operate on the same principle as the zeolite softener but have eliminated many of the undesirable features of this type of softening. These materials are *nonsiliceous* and are not attacked by acids or mild alkali. They will normally operate in a pH range between 2 and 9. Since they are nonsiliceous, they do not pick up silica when low-silica waters are softened.

These newer organic materials are of two types; one is the *sulfonated-coal* type, the other a *resinous* type. The sulfonated-coal type is often referred to as a *carbonaceous cation* exchanger; the other is called the *resinous cation* exchanger. The term *cation* is used since the positive ions or cations are exchanged.

When operating on the sodium cycle (regeneration with salt) these units operate in a manner similar to the older siliceous zeolites. They usually have a much higher exchange capacity than the natural zeolites, and are of about the same capacity as the older synthetic zeolites. However, some of the newer products may have a capacity as great as 35,000 grains of calcium carbonate per cubic foot. Their ability to work over a wider pH range, coupled with their freedom from silica contamination, makes them quite desirable for boiler make-up water softening. However, when operating with salt as a regenerating material, they still deliver the same type of softened water as the older zeolite softener. Since they are stable at low pH values, they are suitable for removal of all the positive cations, if properly regenerated. This type of operation is discussed in the next article.

17. HYDROGEN-CYCLE CATION EXCHANGERS

Since the nonsiliceous type of exchange material is stable at low pH values, it is possible to regenerate it with an acid such as sulfuric or hydrochloric. When so regenerated, it is free from sodium, calcium, or magnesium with the hydrogen ion replacing the sodium ion in the material. When acid-regenerated, it is said to be operating on the *hydrogen* or *acid cycle*. If regenerated with acid, and water is passed through, the sodium, calcium, and magnesium are all retained on the bed with the release of the corresponding acid. Thus the bicarbonates will form carbonic acid; the sulfates, sulfuric acid; and the chlorides, hydrochloric acid. All the cations are exchanged for hydrogen as shown

$$\begin{array}{llllll}
Ca(HCO_3)_2 & & & H_2CO_3 & & Ca \\
Mg(Cl)_2 & + & H_2 \text{ exchange bed} = & HCl & + & Mg \text{ exchange bed} \\
Mg(SO_4) & & & H_2SO_4 & & Na \\
NH_4 & & & & & NH_4
\end{array}$$

When all the hydrogen in the bed is used up, the exchange reaction stops. It may then be regenerated with sulfuric or hydrochloric acid, usually the former, and the softening cycle repeated.

Since the calcium, magnesium, sodium, and ammonium are all converted to the hydrogen equivalent, there is a reduction of solids. If all these positive ions were present as the bicarbonate, the resulting product would contain nothing but carbonic acid. If air were passed through the effluent, the carbon dioxide could be reduced to a low value and the resulting product would be almost pure water.

18. COMBINATION HYDROGEN- AND SODIUM-CYCLE CATION EXCHANGERS

In normal water supplies both chlorides and sulfates are present along with the bicarbonates. In order to control the residual alkalinity, it is possible to split the water flow so that part goes through the hydrogen unit and the rest goes through a sodium unit.

The effluents from the two units are then mixed in such proportions that the alkalinity of that from the sodium unit neutralizes the acidity of that from the acid unit, and the mixed water is degassed to remove the carbon dioxide. Since the chloride, sulfate, and silica pass through both units, the reduction in solids is only the amount of the calcium, magnesium, sodium, and ammonium removed in the hydrogen unit. The amount of excess sodium-softened water added above that necessary to neutralize the acidity determines the residual alkalinity of the treated water. This may be controlled at any desired value by varying the ratio of the waters passed through the hydrogen and sodium units. When a low residual alkalinity is desired, the presence of the small amount (3 to 5 ppm) of free carbon dioxide left after air degassing is sufficient to give a low *p*H (below 7) which makes the water quite corrosive, since it is saturated with air. If this water is to be heated before being deaerated, care must be taken to use acid-resistant materials. Of course the hydrogen unit and all its equipment must be of acid-resistant material. In some cases, the effluent from the softener is passed through an acid-resistant vacuum degasser to remove the carbon dioxide and then neutralized with sodium hydroxide. In others, the softened water is passed through a regular deaerating heater.

It is possible to control the amount of acid used in the unit so that it is sufficient to react with a definite portion of the water to be softened, and then to add a brine (salt solution) to regenerate the rest of the bed. Under proper control, water of the desired alkalinity may be obtained from one unit.

In general, the unit working on the hydrogen cycle has less capacity than the sodium unit. Usually the hydrogen cycle has an exchange capacity around 60 to 75% of that of a unit operated on the sodium cycle. The unit is backwashed and then regenerated, usually with dilute sulfuric acid. Care must be taken to keep the sulfuric acid concentration below 2%. If the acid strength is high and a high-hardness water is being softened, it is possible to precipitate calcium sulfate on the mineral bed and retard softening. Hydrochloric acid can be used to prevent precipitation, but sulfuric acid is more desirable from first cost and handling standpoints.

These types of softener may be operated to give residual hardness below 0.20 ppm if all operating conditions are properly controlled.

19. DEMINERALIZATION TREATMENT

It is possible to pass water through a combination of exchange materials and anion-adsorbent materials and, with suitable additional treatment, to obtain a water practically free of dissolved salts. This is commonly referred to as *demineralization treatment.* The first stage of the softening involves passing the water through a cation exchanger operating on the hydrogen cycle. This converts all negative ions to their acid form and removes all the positive ions (cations). The water is then passed through a second unit operating as an anion (negative ion) adsorber. This unit is regenerated with an alkaline material such as sodium hydroxide or sodium carbonate. The acids are adsorbed on the alkaline bed. The carbon dioxide passes through, along with the inactive silica, and the carbon dioxide is removed by air degassing, leaving a water practically free of all the original dissolved materials except silica. The anion adsorbers have capacities of 10 to 20 thousand grains of equivalent calcium carbonate per cubic foot of material. The alkaline bed cannot be backwashed or rinsed with raw hard water. Usually the effluent from the cation exchanger is used to prevent the precipitation of hardness in the anion adsorber. An excess of the theoretical amount of acid, as well as soda-ash to react with the dissolved materials in the water to be treated, is necessary. Since sulfuric acid and soda-ash are relatively expensive as compared to lime or salt, this method of treatment is more expensive than the older methods of softening. However, often, as in boiler-feed make-up for high-pressure boilers, the reduction in blowdown by the reduction of dissolved solids in the softened water may more than balance this increased cost. If the silica in the raw water is appreciable, it soon builds up in the boiler until the blowdown is regulated by the silica content, and the advantage of the demineralization is no longer evident. In order to adapt this method to treatment, several modifications have been tried. The first method involves changing the silica from an almost chemically inert form to an active or more highly ionized form so that it will be retained on the adsorbing bed. This is accomplished by having an excess of fluoride in the water above that necessary to form a fluosilicate. Thus, if hydrofluoric acid is present, it reacts as follows to form fluosilicic acid:

$$6HF + SiO_2 = H_2SiF_6$$

Thus six molecules of fluoride react with one of silica. The fluosilicic acid is adsorbed in the anion-removal bed.

There are several methods of introducing the hydrofluoric acid into the water. Hydrofluoric acid could be added to the effluent from the hydrogen cation exchanger, but the difficulty in handling hydrofluoric acid restricts the use of this method. In a second method, fluoride (as sodium fluoride) is added to the water ahead of the hydrogen cation exchanger. The sodium fluoride is converted to hydrofluoric acid in the cation exchanger. The hydrofluoric acid reacts with the silica to form the fluosilicic acid, which in turn is adsorbed on the anion removal bed. Calcium fluoride may also be added to the hydrogen cation exchanger to form hydrofluoric acid. It is necessary to modify the operation when the fluoride treatment is used so as to rinse the anion unit with excess acid from the cation unit to flush out the fluosilicic acid before regenerating with soda-ash. If the unit is regenerated directly with soda-ash, the silica precipitates in the anion adsorbing bed.

It is claimed that this process reduces the silica below 0.5 ppm and keeps the fluoride in the effluent below 0.5 ppm. Since the cost of the fluoride is rather high, it may be advantageous to use a silica reduction process in connection with lime treatment ahead of the demineralization treatment.

Another method being used is to demineralize and de-gas the water and then to pass it through a second anion alkaline-type adsorber that removes the silica in the water, which at that stage is free of all minerals except silica. This treatment gives a treated water with silica below 0.5 ppm.

20. EFFECTS OF IMPURE FEEDWATER

CORROSION. Corrosion in boilers or feedwater equipment may be explained by the electrochemical theory. (See Ref. 5.)

The principal accelerators of corrosion in boilers are (1) dissolved oxygen; (2) acids; (3) surface deposits, especially those that are electronegative to steel; (4) unlike metal couples, as brass and steel; and (5) electrolytes, as strong salt solutions. There are four common methods of preventing corrosion. (1) Removal of dissolved gases in feedwater, especially dissolved oxygen and carbon dioxide. This can be done by deaerating processes. (2) Neutralization of acids and maintenance of desirable alkalinity and hydrogen-ion concentration (pH value) in feed and boiler water. (See p. 7-52.) (3) Periodic mechanical cleaning. Protective paints may be applied if desired. (4) Avoidance of excessive salt concentrations.

HIGH-TEMPERATURE CORROSION. At higher temperatures, especially above 950 F, steam may react with iron according to the equation

$$\underset{\text{Iron}}{3Fe} + \underset{\text{Steam}}{4H_2O} = \underset{\text{Black magnetic oxide}}{Fe_3O_4} + \underset{\text{Hydrogen}}{4H_2}$$

At temperatures up to 950 F this corrosion reaction is slow. Above this temperature the rate is accelerated, and alloys like the Cr-Ni alloy KA2 are recommended. These materials form a protective oxide film on the metal surface, whereas steel does not.

CORROSION FATIGUE. Under certain conditions of corrosion and stress, metals may corrode in the form of deep sharp pits, which later develop into fissures or cracks. If the action is not stopped, the parts attacked eventually will fail. The action may be produced by subjecting metal to ordinary corrosive conditions while under stress, which may be intermittent in nature. Microscopic examination shows the action to occur in a characteristic manner, producing cracks that are transcrystalline and in a nearly straight line. The products of corrosion fill the crack, and the metal shows little or no elongation or distortion.

CAUSTIC EMBRITTLEMENT. With certain types of feedwater, cracks of a peculiar form may appear in the boiler plates, particularly at the riveted seams below the water line. These cracks are intercrystalline and do not follow the lines of the maximum stress, as they sometimes run past each other and around parts or islands of the plate. This action has been termed caustic embrittlement, as it occurs where concentrated sodium hydroxide (caustic soda) is present in the water in the boiler in the absence of certain other elements.

Embrittlement is inhibited when a certain ratio of sodium sulfate to sodium carbonate is maintained in the feedwater. This has led to ratios (given in Table 5) of sodium sulfate

Table 5. Recommended Ratios of Sodium Sulfate to Total Sodium Hydroxide

Working pressure of boiler, psig	0 to 150	150 to 250	250 and over
Sodium sulfate	1	2	3
Total sodium hydroxide and carbonate alkalinity as equivalent sodium carbonate	1	1	1

to total sodium hydroxide and sodium carbonate alkalinity, calculated to equivalent sodium carbonate, being recommended for different working pressures.

Embrittlement is caused by concentration of caustic soda at joints and through the effect of the stress in the metal at the joints. The trouble experienced with caustic embrittlement was a factor that led to the use of fusion-welded drums. (See Ref. 6.)

FOAMING AND PRIMING. Foaming may be described as the formation of a large amount of foam in the boiler, due to failure of steam bubbles to coalesce and break. It is accompanied by considerable increase in moisture content of the steam generated by the boiler.

Priming is characterized by large amounts of water passing out of the boiler with the steam, usually in intermittent slugs which endanger steam lines, turbines, and engines. It may occur simultaneously with foaming. High water levels in boilers promote priming.

Foaming and priming generally are caused by high concentration of dissolved and suspended solids, possibly accompanied by oil and soaps in boiler water, and sudden changes in boiler capacity. These conditions may be prevented by reducing boiler water concentrations by blowdown, elimination of sources of feedwater contamination, periodic cleaning of boiler, and proper regulation of water levels. Since operating conditions and the boiler equipment influence the amount and kind of solids that may be permitted in boiler water, no general concentration limits can be given.

REDUCTION OF CONCENTRATION BY BLOWDOWN. Maintenance of reasonably low concentrations in high-capacity boilers is facilitated by economical continuous blowdown, with or without flash tanks, and with one or more heat exchangers. Figure 4 indicates percentage blowdown required for any given percentage of make-up and ratio of concentration in make-up to that in boiler water.

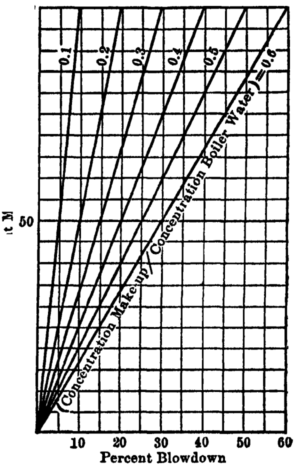

FIG. 4. Blowdown as affected by make-up and ratios of concentrations.

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BOILER FURNACES *

Revised and rewritten by W. A. Carter

21. BURNING OF COAL

See Section 2, Combustion and Fuels.

22. HAND-FIRED GRATES

See Refs. 1, 2, and 3.

In hand-fired boilers, ideal uniform combustion cannot be maintained because firing is intermittent. The best condition of fuel bed is obtained when the coal is fired frequently, in small amounts, and with proper distribution; when caked masses of coal (if any) are broken up as rapidly as formed; and when ashes and clinker are not allowed to clog the fuel bed.

* The author has drawn freely, by permission, on *Heat Power Engineering,* Part II, by Barnard, Ellenwood, and Hirshfeld (John Wiley and Sons) for many of the data in this chapter.

METHODS OF FIRING. Three methods in general use for hand-firing boilers are:

Spread Firing. A small amount of fresh coal is distributed evenly over the entire surface of the fire. It is commonly used with anthracite and other low-volatile coals.

Alternate Firing. Fresh coal is fired on but one-half of the grate at a time. The freshly liberated volatile matter absorbs the necessary heat for combustion from the brighter parts of the fire. It is particularly suitable for noncaking coals.

Coking-firing. Especially suited to caking coals. Fresh coal is placed on the front edge of the fire and allowed to coke. After distillation is complete, the coke is spread over the fire. Lower rates of combustion are obtained with this method than with the other two.

COMBUSTION CHAMBERS in hand-fired boilers usually have firebrick walls. These, when hot, help to maintain the high temperature required for combustion. The combustion chamber often is of special form to compel the volatile gases to mix with secondary air. With anthracite and other low-volatile coals a firebrick arch sometimes is sprung over the grate to assist ignition. The more volatile coals produce longer flames, and consequently the furnace must be made longer by setting it in front of the boiler as a Dutch oven, or using a deflecting arch under the boiler to postpone contact of burning gases with the relatively cold boiler surface. The Dutch-oven furnace helps to attain smokeless combustion, except when the burning rate is high. Arrangements to mix air and volatile gases include multiple arches, piers, or wing walls, and jets of air or steam injected through the front, sides, or bridge wall of the furnace.

COMBUSTION RATES depend on the characteristics of the coal and ash, thickness of fuel bed, total grate surface, air passage area of the grates, and the draft. Average rates, in pounds of fuel per square foot of grate surface per hour are:

Type of Draft	Anthracite	Semi-anthracite	Semi-bituminous	Bituminous		Lignite	Coke Breeze
				Eastern	Western		
Natural	15	16	18	20	30	25	..
Forced	20	25	35	30	35	35	20

23. STOKERS

Principal advantages of stokers over hand firing are continuous delivery of coal; progressive and gradual distillation of volatile matter; ability to obtain better performance and smokeless combustion because of the ease with which the operations can be regulated at all times; greater combustion capacity obtainable in a furnace; ability to meet varying load demands quickly; ability to burn poorer and cheaper grades of coal with less smoke and higher efficiency; relief of operators from strenuous duties, thus permitting more time for adjusting operating conditions; decreased labor costs in large boiler plants where the number of firemen can be reduced.

CLASSIFICATION. Stokers are classified as overfeed or underfeed or combination overfeed and underfeed. Certain stokers of each type require forced draft; others operate with natural draft.

Overfeed stokers include (1) Front-feed, inclined-grate stokers; coal enters at the front and is fed down an incline to the ash dump or clinker grinder at the bottom. (2) Double-inclined, side-feed stokers; coal is fed from both sides, down inclined grates to a refuse pocket at the center. (3) Chain-grate or traveling stokers; the entire coal bed moves

Table 1. Application of Stokers for Various Fuels

Data from Ref. 4.

Fuel	Preferable Stoker *
Anthracite	Traveling grate,† forced draft
Coke breeze	Traveling grate,† forced draft
Semi-anthracite	Traveling grate,† forced draft
Semi-bituminous (coking)	Underfeed and inclined overfeed
Bituminous (coking)	Underfeed and inclined overfeed
Bituminous (free-burning, ash > 10 or 12%)	Traveling grate,† forced or natural draft
Bituminous (free-burning, ash < 10 or 12%)	Traveling grate,† underfeed or side inclined ‡
Sub-bituminous	Traveling grate,† forced or natural draft
	Traveling grate,† forced or natural draft

* The spreader stoker may also be used for any of the fuels shown except anthracite.

† *Traveling grate* includes chain grates as well as traveling carrier-bar stokers.

‡ If ash fuses at a temperature below 2400 F, traveling grates are preferable. If the percentage of ash is less than 7, underfeed stokers are preferable.

horizontally from front to rear. (4) Spreader stokers; coal is fed over the entire grate area by revolving paddles; grates are stationary dumping or continuous ash discharge (chain grate).

Underfeed stokers include (1) Single-retort stokers, usually horizontal, with lateral ash dumps. (2) Multiple-retort stokers, usually inclined with refuse discharge at the rear.

Table 1 shows types of stokers generally most suitable for the various fuels.

OVERFEED STOKERS. Inclined overfeed stokers usually operate with natural draft. A coking arch at the front of the furnace, maintained at a high temperature, reflects heat to and distills volatile gases from the entering coal. Air, heated or otherwise, usually is admitted with coal under the arch. As a rule, these stokers require more attention than other types and seldom are used on boilers larger than 600 hp.

Practically all kinds of coal, sawdust, tan bark, and hog fuel can be burned in these stokers, but they are used principally with high-volatile, high-ash Midwestern coal. Average combustion rate with free-burning coals is 15 to 25 lb per sq ft of horizontal projected grate surface per hour, with a maximum of 35 lb. With caking coal, the maximum combustion rate is 25 lb.

Inclined front-feed stokers include a hopper, coal-pusher feeding device, dead plate, coking arch, and inlet for secondary air under the arch. The action of the pushers and grate bars can be regulated so that when the coal arrives at the ash table, it has been completely burned.

Double-inclined side-feed stokers have coal magazines at each side of the furnace. They feed coal to a coking plate, where it meets heated secondary air brought through a refractory arch that covers the entire stoker. The grate bars are inclined, each alternate bar being in constant motion to feed coal down to the clinker grinder. Exhaust steam from the stoker engine sometimes is admitted to the grinder to assist in breaking the refuse of clinkering coal.

Stokers of this type have large coking spaces, ample coking arches, and large combustion chambers. Ordinarily they are satisfactory for both uniform and varying loads, but at high combustion rates and with certain types of coal, the fuel may avalanche.

CHAIN- AND TRAVELING-GRATE STOKERS comprise series of small links, forming a broad endless belt conveyor carried on rolls or skids. In the traveling-grate type, crossbars, extending from endless chains on either side of the furnace, support short interlocking grate bars. Both types are driven by sprockets at variable speeds, in conformity with the load on the boiler. Raw fuel is fed at one end and discharged as burned-out refuse at the other. The fuel bed is undisturbed while passing through the furnace. Natural or forced draft may be used, depending on the design. A minimum ash content of 7% is necessary to protect the back end of the stoker from heat. With a properly designed furnace, this type of stoker can burn high-volatile coals without smoke, with either natural or forced draft, and also noncaking, clinkering coal, high in ash. Special designs can burn small-size anthracite and breeze, using forced draft.

Stokers of this type are relatively costly, but require little attention, and maintenance is low. They are not so well adapted as the underfeed stokers to meet sudden, heavy variations in demands for steam, unless forced draft is used.

Arches over the fire are necessary to cause mixing of gases from the rear of the grate, which are deficient in air, with excess air from the front. Another function is to maintain sufficient temperature to support combustion and to radiate heat to the front of the fuel bed to ignite entering fuel and distill volatile matter from it. Arches also prevent carrying away by a strong draft much of the fly fuel, which otherwise would be lost. Figure 1 shows typical installations.

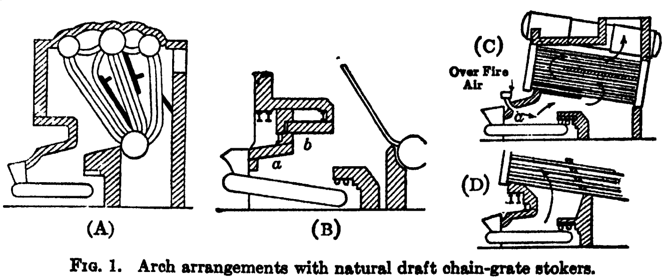


Fig. 1. Arch arrangements with natural draft chain-grate stokers.

Secondary air introduced under the front arch prolongs its life. A fan is preferable to a steam jet or induced draft.

Combustion Space Requirements. Adequate combustion chamber volume ordinarily should be provided by suitably locating boiler surface with respect to the grate according to recommended dimensions in Table 2.

Table 2. Height of Settings for Various Types of Stoker-equipped Boilers *

(Adapted from *Heat Power Engineering* Part II, by Barnard, Ellenwood, and Hirshfeld)

Type of Boiler	Underfeed Stoker				Chain Grate Stoker				Overfeed Stoker			
	Multiple Retort		Single Retort		Natural Draft		Forced Draft †		Side Feed		Front Feed	
	Types ‡ 4, 6, 11, 13, 15, 16		Types ‡ 2, 4, 5, 11, 16		Types ‡ 1, 2, 7, 8, 10		Types ‡ 1, 2, 3, 7, 8, 10		Types ‡ 4, 12		Types ‡ 9, 14	
	Min §	P.M. §	Min §	P.M. §	Min §	P.M. §	Min §	P.M. §	Min §	P.M. §	Min §	P.M. §
Water Tube												
Horizontal, all sizes	11'0"	13'0"	9'0"	11'0"	10'0"	12'0"	12'0"	14'0"	9'0"	11'0"	9'0"	11'0"
Inclined (H.M.D.), all sizes §	7'6"	8'6"	6'6"	8'6"	6'0"	8'0"	7'0"	8'0"	5'0"	7'0"	6'6"	8'0"
Inclined (V.M.D.), all sizes §	6'0"	7'0"	5'0"	7'0"	4'0"	5'0"	6'0"	8'0"	3'6"	5'0"	4'0"	5'6"
Vertical (H.M.D.), all sizes §	3'6"	5'0"	3'6"	5'0"	3'6"	4'6"	4'0"	5'0"	3'0"	4'0"	3'6"	5'0"
Vertical (V.M.D.), 1500 sq ft §	4'6"	5'0"	4'6"	5'0"	4'6"	5'0"	5'0"	5'6"	3'3"	....	3'6"	4'6"
Vertical (V.M.D.), 2500 sq ft §	5'6"	6'0"	5'6"	6'0"	4'6"	5'0"	5'0"	5'6"	3'3"	.....	3'6"	4'6"
Vertical (V.M.D.), 5000 sq ft §	6'0"	6'6"	6'0"	6'6"	4'6"	5'0"	6'0"	6'6"	3'3"	......	3'6"	4'6"
Horizontal Return Tubular												
72 in.	8'0"	10'0"	7'6"	8'6"	7'0"	8'0"	8'0"	10'0"	7'0"	8'0"	6'0"	8'0"
84 in.	8'0"	10'0"	7'6"	8'6"	7'0"	8'0"	8'0"	10'0"	7'0"	9'0"	6'0"	8'0"

* Setting heights are defined as follows. *Water-tube Boilers:* Horizontal tubes, floor line to bottom of header above stoker; inclined tubes (H.M.D.), vertical tubes (H.M.D.) floor line to center of mud drum; inclined tubes (V.M.D.), vertical tubes (V.M.D.), floor line to top of mud drum. *Horizontal Return Tubular Boilers:* Floor line to under side of shell.

† When burning coke breeze and anthracite fines, the setting heights indicated should be materially increased to provide for proper arch and furnace design.

‡ Types: 1. Babcock & Wilcox; 2. Burke; 3. Coxe; 4. Detroit; 5. Type E; 6. Frederick; 7. Green; 8. Harrington; 9. Huber; 10. Illinois; 11. Jones; 12. Murphy; 13. Riley; 14. Roney; 15. Taylor; 16. Westinghouse.

§ H.M.D. = horizontal mud drum; V.M.D. = vertical mud drum; Min = absolute minimum; P.M. = preferred minimum.

Air leakage around the grate may be minimized by: 1. Adjustable ledge plates to seal gaps between sides of stoker and furnace wall. 2. A tight ashpit to reduce infiltration at rear of stoker. 3. A well-fitted damper at the rear, between upper and lower runs. 4. A seal below lower run. A water back, connected in the boiler circuit, set into an overhanging bridge wall, close to the grate, will compress the back of the fuel bed and increase its density, thus decreasing air infiltration at this point. It also will reduce the amount of unburned fuel discharged to the ashpit, protect the bridge wall, and prevent adherence of clinker. Sidewall water boxes may be necessary to prevent clinker building up on the furnace walls, which then may cause increased air leakage.

Operation. The coal hopper outlet gate should be adjusted for proper thickness of fuel bed for the grade of coal burned, i.e., 2 1/2 to 5 in. for fine anthracite, 4 to 6 in. for Midwestern bituminous and lignite coals. The bed should be as thin as possible consistent with ignition and burning out at the proper point. Stoker speed and draft should be varied for load variations. For best results with the more volatile coals, coal that will pass through a 1-in. ring should be used, although screenings up to 2 in. will burn satsifactorily. A fuel bed of uniform density, offering correct resistance to air flow, is obtained with coal containing 50% fines. The addition of 3% moisture to the coal before firing will reduce sifting and blowholes in the fire, and thereby reduce excess air and improve ignition. This extra moisture reduces overall efficiency only a few tenths of 1%.

NATURAL-DRAFT CHAIN-GRATE STOKERS. Free-burning, high-volatile bituminous and sub-bituminous coals and lignite can be burned with high efficiency on natural-draft chain-grate stokers. The design of the arch depends on the percentage of volatile matter and heating value of the fuel, the combustion rate, and the stoker length. With natural draft, an arch height of about 36 in. at the front has been found to give good results. Figure 1 shows several arrangements of furnaces for natural-draft chain-grate stokers. Turbulence of the gases is insured by the arches *a* and *b* in furnace *B*, and by increased velocity of gases in the narrow throats of the other furnaces.

Combustion rates for most efficient operation with free-burning coal range from 20 to 30 lb per sq ft of grate surface per hour, with a maximum rate of 40 lb and a minimum

of 5 lb. Draft loss, up to combustion rates of 35 lb, is approximately 0.1 in. of water per 10 lb of coal per hour per square foot of grate, the loss increasing at higher combustion rates.

Operating results possible with proper operation, without an economizer or preheater, are monthly efficiencies of boiler and furnace of over 70%, with CO_2 at the boiler outlet of 12 to 13%. Combustible in refuse should range from 15 to 25% at combustion rates of 25 to 40 lb per sq ft per hr.

FORCED-DRAFT TRAVELING-GRATE STOKERS differ from natural-draft stokers in that a series of transverse independent forced-draft compartments, under the upper run of the traveling grate, are supplied, by a fan, with air under pressure from an air duct along the side of the boiler. Connections between the duct and compartments have dampers to control under-fire pressures in the various compartments. If the furnace has but a single arch, maximum under-fire pressure is carried only in the front compartment, the pressure tapering off to nearly zero in the rear compartment, as shown in Fig. 2. In furnaces with front and rear arches, maximum under-fire pressure is at about two-thirds of the distance to the rear, pressures of 1/4 to 1/2 in. of water being carried in front and rear compartments.

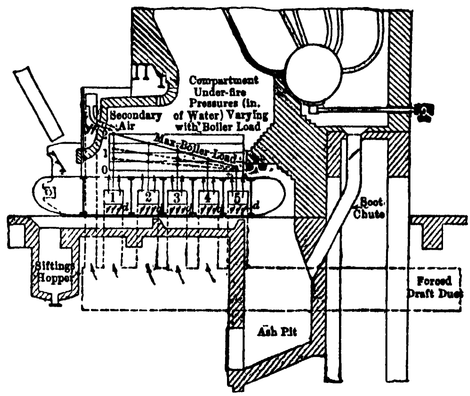


FIG. 2. Forced-draft traveling-grate stoker with independent air compartments.

Used with Low-volatile Coals. For small-sized anthracite, air openings in the grate must be small enough to prevent sifting. Special arrangements are necessary for primary kindling of the coal before it reaches the first forced-draft air compartment, utilizing heat radiated from the hot refractory surface of the arch. Entering fuel must "see" the arch through a greater angle, as A in Fig. 3 than that at B, through which it sees any relatively cold surface. In one stoker design, a small suction compartment at the front of the stoker draws some hot furnace gas down through the fresh fuel to ignite quickly moist or low-volatile fuels.

Stratification of gases and carrying of fly ash by the furnace gases can be overcome by introducing air over the fire or placing the arch at the rear. (See Fig. 4.)

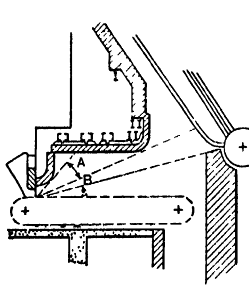


FIG. 3. Arch arrangement with traveling-grate stoker for anthracite.

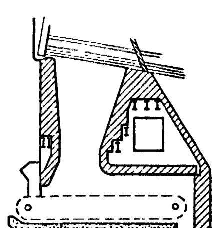


FIG. 4. Arch arrangement to reduce stratification and fly ash.

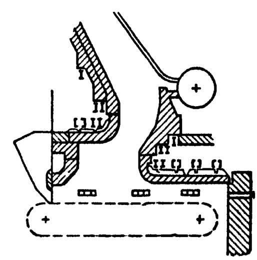


FIG. 5. Traveling-grate stoker with front and rear arches.

Front and rear arches often are used to form a narrow throat in which gases from the front and rear portions of the grate are mixed. Combustion is completed in the upper combustion chamber. (See Fig. 5.)

Used with Bituminous Coals and Lignite. Forced draft under traveling-grate stokers permits higher combustion rates of free-burning coals than natural draft. Efficiency curves are higher (5 to 6%), and flatter. Arches are smaller than with natural-draft stokers, but they must be set higher to prevent erosion. Side-wall water boxes and water backs are necessary. Caking and coking coals that could not be burned on these stokers with natural draft are burned successfully with forced draft by reason of the air pressure breaking up the fuel bed. Water-cooled arches and side walls avoid rapid destruction of brickwork by heat. An additional arch over the rear of the stoker is desirable when the coal varies in quality.

Combustion rates for best results with bituminous coal should range from 30 to 40 lb. per sq ft per hr, with a maximum of 60 lb when ash content ranges from 10 to 25%. A survey by NELA showed an average combustion rate of 43.5 lb and average stoker maintenance cost of 4 cents per ton of coal burned. With anthracite or coke breeze, combustion rates range from 30 to 38 lb per sq ft of grate per hour with forced draft of 1.5 to 2 in. of water, with maximum and minimum rates of 55 and 10 lb respectively.

Operating results with bituminous coal should show combined boiler and furnace monthly efficiencies of 70 to 77%; CO_2 of 12 to 15%; combustible in refuse, 10 to 20%. With anthracite and coke breese, monthly efficiencies of boiler and furnace should range from 72 to 76%.

UNDERFEED STOKERS operate at combustion rates as high as 110 lb per sq ft of grate per hour if ash fusion temperature is not below 2400 F. The field of the underfeed stoker comprises bituminous and semibituminous caking or free-burning coals, and to a lesser extent other grades of coal, including culm, coke breeze, and small-sized anthracite mixed with bituminous coal.

The essential principle of the underfeed stoker is a reciprocating ram or rams which feed coal from hoppers at the front of the furnace into the bottom of horizontal or slightly inclined retorts. The raw coal is underneath burning coal at the top of the fuel bed, which distills the volatile matter from the fresh coal. The liberated gases pass upwards through the burning coal, and are burned with air entering through tuyeres at the upper edges of the retorts. The coke which remains after distillation of the gases gradually is pushed upwards by entering fresh fuel and burns on the surface of the fuel bed. The entire fuel bed is worked toward the rear of the stoker or on to dead plates at the sides of the retort, ash and refuse being discharged into an ash hopper or removed by hand.

Forced draft always is necessary. Rams and pushers, and sometimes also the ash-disposal equipment, are driven by a motor or engine. Fuel and air supply can be regulated automatically by variations in steam pressure. Arches are unnecessary, and considerable heat is transmitted to the boiler by radiation. This results in a relatively low temperature of gases passing through the boiler, even at high combustion rates.

Simple single-retort stokers (Fig. 6) use a steam-driven ram or a screw feed, together with supplementary adjustable-stroke pushers, to distribute coal properly in the retort. From the surface of the fuel bed, refuse is deposited on dead plates, whence it is removed by hand through doors in the front. In some designs the dead plates may be dropped to dump to the ashpit. Access doors on the sides of the furnace are unnecessary.

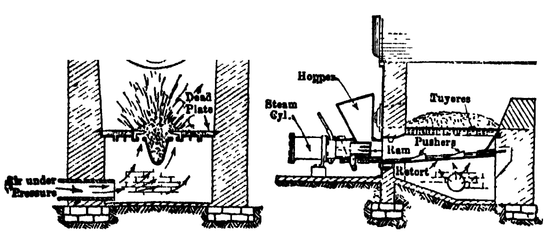


FIG. 6. Simple horizontal single-retort stoker.

At moderate combustion rates, even with high-volatile coals, combustion is complete within a short distance of the surface of the fuel bed. The capacity of these stokers is 800 to 1200 lb of coal per hour. For greater capacities, two or three stokers may be set in a single furnace.

Single-retort stokers with lateral grates resemble simple single-retort stokers, except that stationary or movable overfeed grates are interposed between the retort and the dead plates or dump plates. Air to the overfeed grates should be suitably regulated. The capacity of these stokers ranges from 1200 to 9000 lb of coal per hour.

Multiple-retort underfeed stokers occupy the full width of the furnace. The fuel bed constantly moves from front to rear, and refuse is fed continuously to an ash dump. (See Fig. 7.) These stokers are 6 to 28 ft or more wide, with 3 to 16 retorts, and 8 to 27 ft or more long. Each retort may have 13 to 69 or more replaceable tuyeres. Underfeed stokers can operate at higher combustion rates than other stokers, and in large units occupy a greater proportion of the area under the boiler. For a given rate of steam generation, they require less heat-absorbing surface and permit individual units to have high steam-generating capacity. These stokers can be brought quickly from bank to full capacity and can meet wide and rapid changes in load. Some furnaces have a stoker at each end discharging to a common ashpit.

Control of the shape of the fuel bed to give proper air distribution is by adjustment of the length of pusher strokes and speeds of the various groups of rams. This also keeps the fuel bed open and free of clinkers. The active area of the fuel bed may be zoned, with independent regulation of air supply to each zone.

Refuse discharging equipment comprises simple dump plates, double dumping grates, rocker plates, and clinker grinders. With clinker grinders, the final combustible in the refuse can be reduced to 5% if the grinder pocket is large enough to hold ash for 12 hr, and air is forced through its walls. Shortening the time of burning out refuse to 6 hr will raise the combustible to 15%. With dump grates, combustible in refuse may be 15 to 25%.

Furnace walls for high combustion rates must withstand high furnace temperatures and erosive and slagging action of molten fly ash. Materials used are special grades of firebrick, silicon carbide blocks (if ash is not high in iron oxide), hollow perforated blocks through which secondary air is discharged, or water-cooled refractory or metallic blocks. The walls may be made hollow, and primary combustion air circulated through them. Protection from erosion and adhesion of molten clinkers may be obtained by the use of high air-cooled side-wall tuyeres (see Fig. 7) or water-cooled metallic surfaces (see Fig. 8). Boilers fitted with underfeed stokers must be set in batteries of not more than two, as access doors for inspection and cleaning of side walls are necessary in at least one side of the furnace.

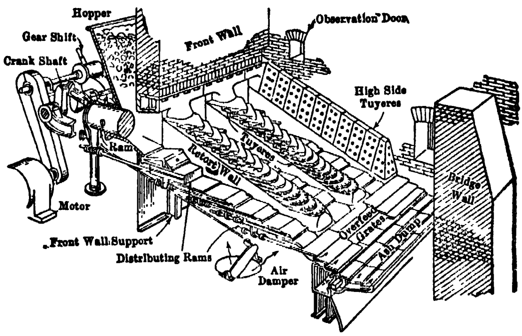


FIG. 7. Multiple-retort underfeed stoker.

Water-cooled underfeed stokers have been developed to burn low-grade Midwestern semibituminous coal, with ash fusion temperature as low as 1900 F, at a rate of 48 lb per sq ft of grate surface per hour (Ref. 5). Stoker tuyeres are cooled by forced circulation of water through groups of three tubes laid lengthwise of each tuyere stack, extending downward over stationary extension grates to a header near the clinker grinders. Groups of shorter tubes protect the remainder of the extension grates that register with the lower ends of the retorts. Side and rear furnace walls of such installations should be water-cooled to withstand the action of ash with such low fusion temperature.

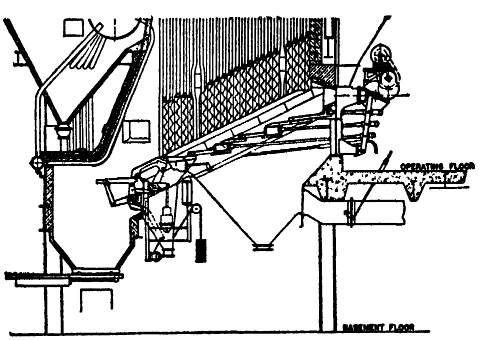


FIG. 8. Underfeed stoker with water-cooled walls and ash-dump plate.

Combustion rates range from bank to 60 lb of coal per square foot of projected grate area per hour for coal having an ash content in the neighborhood of 10%. They depend on furnace design and available draft. With zoned-air control, combustion rates as high as 90 to 100 lb have been carried successfully. Without zoned-air control, best operation is at combustion rates of 35 to 45 lb, although rates as high as 90 lb have been carried satisfactorily under favorable conditions and close attention to stoker operation and fuel bed condition.

Operating Conditions. Excess air required with underfeed stokers is relatively low, but it should not be reduced to a point where boiler exit gases contain CO, or furnace temperatures are greater than furnace walls can withstand. Forced-draft pressures range from $^3/_4$ to 1 in. of water per 10 lb of coal burned per square foot of projected grate surface per hour. Air preheated to 300 to 500 F sometimes is used. The closure of stoker air passages by expansion and growth of metals must be avoided by proper design and material. Prohibitive stresses and distortion also must be avoided.

Power required to operate underfeed stokers may be, under extreme conditions, as much as $^3/_4$ to 1 hp per retort, burning from 700 to 1100 lb of coal per hour.

GROSS EFFICIENCY of large steam-generating units with economizers, but without air preheaters, and equipped with multiple-retort stokers ranges from 90% at low loads to 75% at high loads. Under such conditions, excess air varies from 20 to 10%.

COMBINATION OVERFEED-UNDERFEED MULTIPLE-RETORT INCLINED STOKERS consist of two sections, an upper, underfeed section, and a lower, overfeed section, the link-grate section. The link-grate movement is an up-and-down undulating movement. The up movement breaks open the fuel bed and permeates it with low-pressure air flowing up through the grates. When the grates move down, the movement crumbles and conveys the burning fuel, ejecting the ash continuously off the ash-discharge plate. The ash pit may be equipped with a clinker grinder. This type of stoker is particularly well adapted for burning low-grade bituminous or semibituminous coals.

Figure 9 shows this stoker, and an enlarged section indicates the up-and-down movement of the link-grate section. Coal-burning rates are the same as for underfeed stokers.

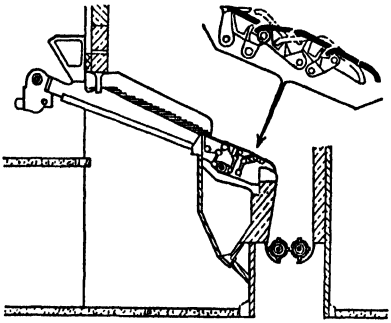


Fig. 9. Combination underfeed-overfeed stoker.

SPREADER STOKERS. In this type of stoker, coal is fed from a hopper into the path of rapidly revolving paddles, either *overthrow* or *underthrow*, which throw the coal into the furnace, where the finer portions are burned in suspension and the coarser particles on a grate. Figure 10, *a*, *b*, and *c*, shows three different types of coal feeders in common use. Spreader stokers are built up to 16 ft in depth and in varying width, single or multiple feeders.

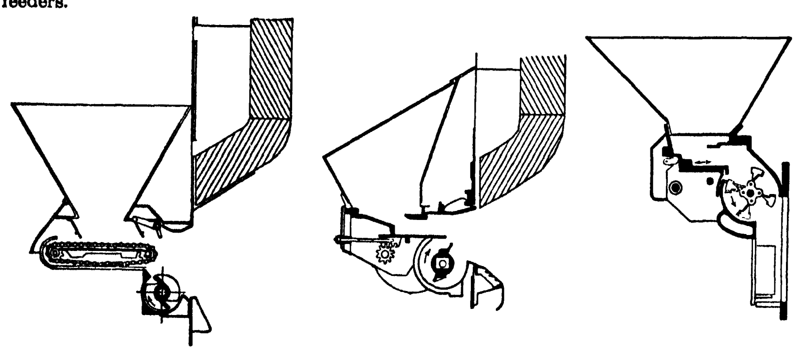


Fig. 10. (*a*) American Engineering spreader stoker. (*b*) Detroit spreader stoker. (*c*) Hoffman spreader stoker.

The grates may be of three different types, (1) stationary grates, (2) dumping grates, and (3) continuous ash discharge.

Stationary grates are used on small installations where low first cost is of importance. The fires must be cleaned by hoeing ashes out through the furnace doors, limiting this type of grates to smaller boilers, 100 to 250 hp rating.

Dumping grates eliminate hand cleaning of fires, are operated either by hand levers or mechanically by pressure cylinders. The efficiency is somewhat improved in comparison with stationary grates because the cleaning period is reduced to a minimum. However, disturbance of combustion conditions does result, and the CO_2 drops until the fuel bed again reaches equilibrium. These grates may be used over a wide size range. There should be at least two feeder units on a boiler equipped with dumping grates.

Continuous ash discharge grates are of the traveling- or chain-grate type. Ashes may be dumped either at the front or at the rear. This type of grate assures a minimum of ash handling and greater efficiencies by allowing an undisturbed CO_2 curve. They may be used for units as small as 25,000 to 30,000 lb per hr steam output.

The spreader stoker burns a wide variety of coals, particularly those grades that are

marketed at a low price because either poor quality or size makes them unsuitable for other types of stoker. The spreader stoker is well suited for these coals.

1. Midwestern bituminous coal from Illinois, Indiana, Ohio, western Kentucky, and adjacent fields; high-volatile, high-ash, free-burning, clinkering coals, difficult to burn on underfeed stokers.
2. Sub-bituminous and lignite, high-moisture, free-burning, clinkering coals.
3. Pacific Coast bituminous and sub-bituminous coals, high in ash and volatility.
4. Eastern bituminous coal: Coking, low-ash, high ash-fusion temperature. These coals are also successfully burned on underfeed stokers.
5. Coke breeze, at somewhat reduced combustion rates and efficiency as compared with bituminous coals.

Combustion Rates. It is desirable to limit the heat liberation for continuous rating to 500,000 to 600,000 Btu per sq ft of grate area per hour.

Furnace Design. Furnace-wall cooling is desirable where high rates of heat release are to be maintained. The heat-release rate should not exceed 35,000 Btu per cu ft of furnace volume per hour for all-refractory furnaces. Since much of the coal is burned in suspension, sufficient length of flame travel (14 ft or more) should be provided.

Cinder Losses. Because of relatively high rates of cinder emission with this type of stoker, it should not be used for plants in residential areas unless adequate dust collectors are installed. Cinders collected in hoppers and dust collectors may be reinjected into the furnace, thereby reducing cinder losses.

Air Requirements. Spreader stokers require forced draft, at a wind-box pressure of 2 to 3 in. of water. Air temperatures as high as 300 to 350 F may be employed.

OVERFIRE JETS. Overfire jets have been used for many years to reduce smoke and improve combustion efficiency. If the air deficiency is uniform over the fuel bed, a row of air jets strong enough to cover the entire fuel bed may be required. Air may be introduced by moderate- or high-pressure blowers, by steam jets, or by openings in the stoker front wall by induction only. (See Ref. 6.)

MANUFACTURERS of representative stokers are:

Overfeed Stokers. Detroit Stoker Company, Detroit. Riley Stoker Corporation, Worcester, Mass.

Chain- and Traveling-grate Stokers. Babcock & Wilcox Company, New York. Combustion Engineering-Superheater, Inc., New York. Riley Stoker Corporation, Worcester, Mass.

Underfeed Stokers. American Engineering Company, Philadelphia. Combustion Engineering-Superheater, Inc., New York. Detroit Stoker Company, Detroit. Riley Stoker Corporation, Worcester, Mass. Westinghouse Electric Corporation, Pittsburgh, Pa.

Spreader Stoker. Detroit Stoker Company, Detroit. American Engineering Company, Philadelphia. Hoffman Combustion Engineering Company, Detroit.

Combination Underfeed-overfeed Stokers. Westinghouse Electric Corporation, Pittsburgh, Pa.

24. GAS BURNERS

Gas burners used in boiler furnaces differ in the degree of mixing fuel and air that occurs in the burner. Long, luminous flames result from burners in which mixing is slight; short, nonluminous flames come from burners that mix gas with all the combustion air.

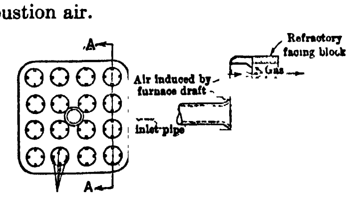


Fig. 11. Gas burner giving moderate mixing.

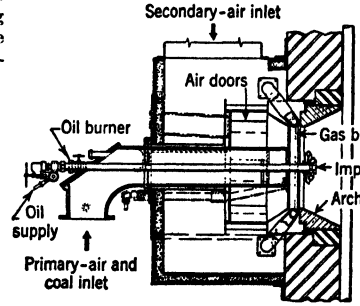


Fig. 12. Combination burner for coal, oil or gas.

Aspirating burners generally are used in boiler furnaces. Figure 11 shows a type that produces a moderate amount of mixing. Gas is introduced in various ways. In Fig. 12,

which shows a combination burner for gas, pulverized coal, and oil, a film of gas flowing around the circumference of the burner throat replaces the numerous small jets of Fig. 11. Either natural or forced draft may be used.

The venturi-type burner, with a central nozzle for gas injection, is used for rapid mixing. Primary air is induced by the reduced pressure in the venturi throat. (See Fig. 13.) A modified venturi-type burner, in which mixing is done in two stages, is shown in Fig. 14.

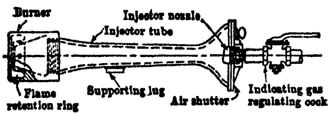


FIG. 13. Venturi-type gas burner.

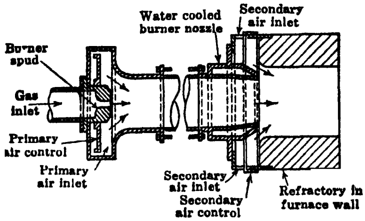


FIG. 14. Modified venturi-type gas burner.

MANUFACTURERS of representative gas burners are Bethlehem Steel Company, Bethlehem, Pa.; Hauck Manufacturing Company, Brooklyn, N. Y.; Peabody Engineering Corporation, New York; Todd Combustion Equipment, Inc., New York.

25. OIL BURNERS

Principal requisites of an oil burner are (1) it must completely atomize or vaporize oil; (2) it must not clog or drool; (3) the jet must be so shaped that it will completely mix with the air necessary for combustion; (4) combustion must be complete, and excess air at a minimum over the entire operating range; (5) the burner must be accessible for cleaning, and require a minimum of attention. An improperly shaped flame may cause flame impingement upon furnace walls or boiler tubes with resultant unburned oil droplets and, eventually, tarry residue on the relatively cool boiler tubes.

Two classes of atomizing burners are used: (1) those that effect atomization by spraying, usually by steam jets, although jets of compressed air may be used; (2) those that atomize mechanically, without any atomizing fluid. Vaporizing burners are not used in large boiler furnaces.

STEAM-ATOMIZING BURNERS use the atomizing fluid to break the oil into minute particles and carry them into the furnace. These burners are either external mixing or internal mixing; those of the latter type employ the premixing principle. Steam for atomization should be at a pressure of 75 to 150 psig. The amount of steam required for atomizing, pumping, and heating the oil ranges from 2 to 7% of the total steam generated. The temperature of oil delivered to the burner at a pressure of 10 to 15 psig is 150 to 190 F. This type of burner seldom is designed to pass more than 1200 lb per hr of oil when using natural draft, but some designs can burn up to 5500 lb per hr when using air at a pressure of 5 in. of water in the air register around the burner.

External-mixing burners usually are confined to boilers operating at steady, moderate rates. Figure 15 shows a simple form, giving a flat flame. With this type of burner, the combustion air usually enters through checker-work forming part of the furnace hearth. The combination of flame shape and method of supplying air limits the furnace to a single row of burners. Forcing the burner causes incomplete atomization, resulting in slower burning, smoking, fouling of boiler surfaces, and decreased efficiency. Figure 16*a* shows another burner of this type that produces a flat flame, and in which wearable parts are readily replaced. Figure 16*b* shows an external-mixing burner that employs compressed air at pressures up to 1 1/2 psig as the atomizing fluid. Its application is rather limited because of excessive operating cost.

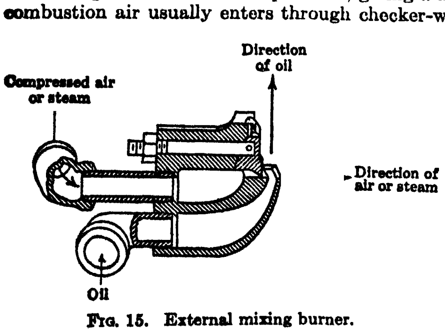


FIG. 15. External mixing burner.

Premixing burners usually deliver atomized oil and steam in the shape of a hollow cone, although they can be furnished to produce a flat flame. Figure 16*c* shows one of these burners and the method of installation. The air doors control the amount of air induced

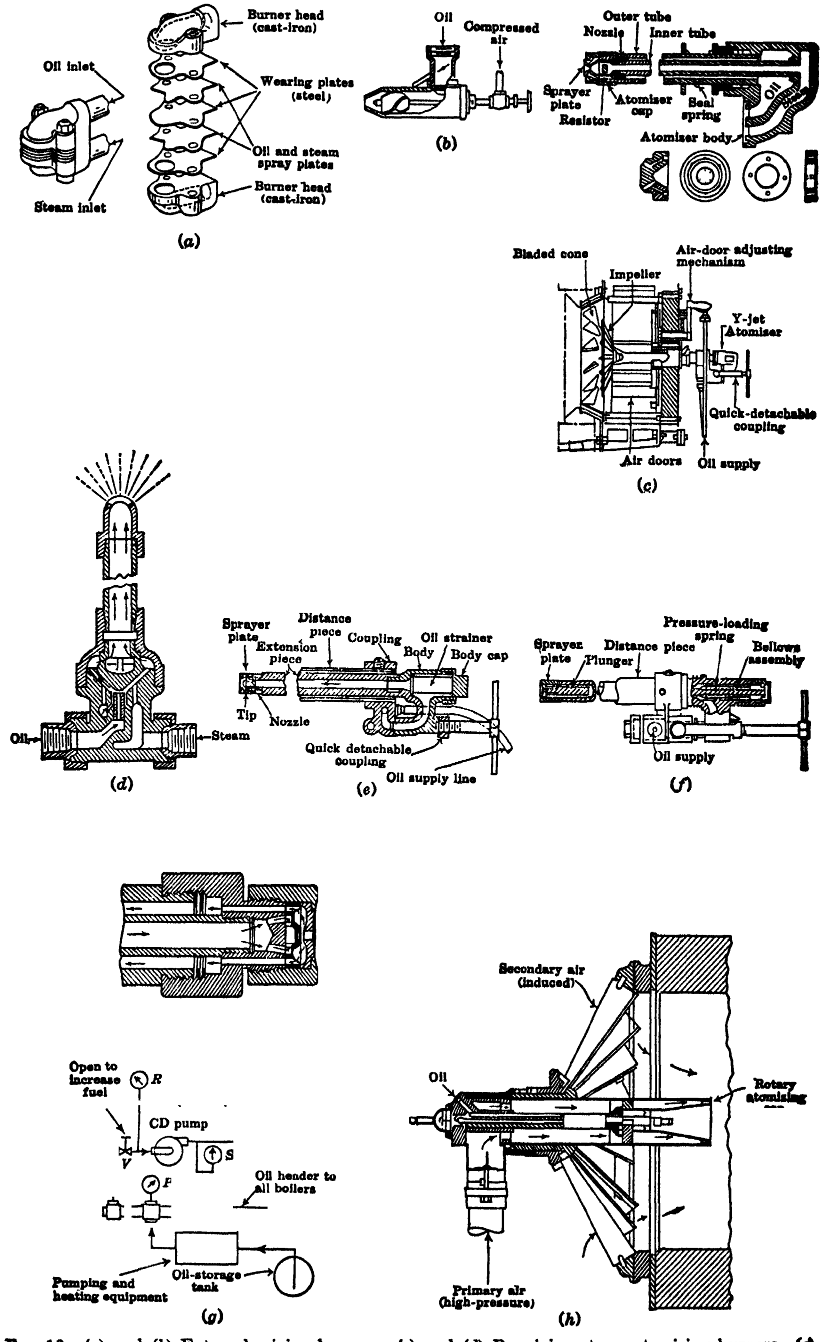


FIG. 16. (a) and (b) External mixing burners. (c) and (d) Premixing steam-atomizing burners. (e) Spray nozzle burner. (f) Pressure-plunger controlled spray nozzle burner. (g) Recirculating burner. (h) Rotary burner.

by the furnace draft and by aspiration of the steam jet. Air pressure, at high combustion rates, is as high as 5 in. of water. These burners can be set in multiple rows, providing large range of boiler output, limited only by furnace volume. Oil-burning rate ranges of 10 to 1 can be obtained with the best burners of this type by varying oil pressure and steam pressure within certain prescribed limits. The burner shown in Fig. 16*d* cannot operate over such a large range of burning rates.

Under certain operating conditions, steam-atomizing burners may be noisy. The blowpipe action may injure the walls of improperly constructed furnaces. Other objections are additional moisture produced in flue gases and the cost of atomizing steam. Nevertheless, they are widely used in small plants because of their simplicity and low initial cost.

MECHANICAL-ATOMIZING BURNERS comprise spray-nozzle burners and rotary burners, the latter being used generally only under low-pressure boilers.

Spray-nozzle burners are practically the only ones used in power-plant boiler furnaces. Oil under a pressure of 50 to 300 psig and at temperatures of 100 to 250 F issues in a hollow cone from a small orifice in the burner nozzle. Suitable passages in the nozzle cause a whirling motion of the oil as it is liberated. Combustion air enters, under furnace draft or forced draft, through a register around the burner. Sometimes it is given a relative spinning motion with respect to the flame. Figure 16*e* shows a typical burner tube of this type. The size of the openings in the tips depends on the quantity of oil to be burned. These burners are made with oil-burning capacities up to 5000 lb of oil per hour. The minimum operating rate on any single tip is about 40% of design flow. Several rows of burners can be installed in a furnace wall to obtain higher capacities. The steam equivalent of the power required to spray the oil seldom is more than 1% of the total steam generated.

Variation of oil pressure does not permit a large range of regulation of the oil-burning rate, as proper atomization is not obtained below 50 psig. Regulation by changing burner tips is objectionable because it interrupts operation. Regulation by varying the number of burners in operation is undesirable because it causes poor air distribution.

One method used to increase the range of oil-burning rate is to incorporate a pressure-loaded plunger in the burner tube that opens additional tangential holes in the nozzle as the oil pressure is increased. Such a design is shown in Fig. 16*f*. The range of this burner is 4 to 1.

Another method of increasing the capacity range, to as much as 14 to 1, is indicated in Fig. 16*g*. The burner line is supplied with a constant flow of oil by primary pumping to the burner line. The additional constant-differential (CD) pump is controlled by steam demand on the boiler being fired by the oil burners, and varies the oil pressure to atomize the required amount of oil through the burner tips, while the remainder of the oil recirculates.

Still another method for increasing the range of operation (to about 7 to 1) provides two independent sets of oil feed to the burner tips, either or both being used in accordance with steam demand.

Rotary burners are sometimes used to burn oils of higher viscosity than can be used in spray-nozzle burners. The oil needs little preheating and only pressure sufficient to deliver it to the revolving cup, in which centrifugal force atomizes it as it is discharged into the furnace. Figure 16*h* shows one of these burners, of the horizontal type, in which the stream of high-pressure primary air passes through the blades of a small turbine and rotates the atomizing cup at 3400 rpm. Secondary air is induced through the adjustable louvers by furnace draft. Other designs of this burner incorporate electrically driven blower for supplying the primary air, cup drive, automatic gas ignition, and controls suitable for burning up to 2000 lb of oil per hour.

MANUFACTURERS of representative oil burners are The Anthony Company, Long Island City, N. Y.; Babcock & Wilcox Company, New York, N. Y.; Combustion Equipment Division, Todd Shipyard Corporation, Elmhurst, Queens, N. Y.; The Engineer Company, New York, N. Y.; Hauck Manufacturing Company, Brooklyn, N. Y.; Peabody Engineering Corporation, New York, N. Y.; Ray Oil Burner Company, San Francisco, Calif.; and Schutte & Koerting Company, Philadelphia, Pa.

26. BOILER-FURNACE DETAILS

FACTORS INFLUENCING FURNACE DESIGN. Fuel and character of load variation are the most important items to consider in furnace design. The kind and characteristics of the fuel, including the properties of its ash, determine the method of burning it. For instance, solid fuels may be burned on grates, on stokers, or in pulverized form; the method of firing and type of burner are factors. Other items that will need to be considered in connection with the fuel are the amount of excess air, which influences boiler capacity and efficiency, and allowable carbon in fly ash and in refuse. The load characteristics include the minimum, normal, and maximum loads, and the duration of each. Heat release rates also are important, in that an increase in rate tends to decrease the size of the boiler for a given output of steam. This, in turn, affects the material and construction of furnace walls. Maximum temperatures for a given type of wall construction also must be determined. The number of variables involved require, for the most economical arrangement and construction, that each furnace be considered as a special case.

TYPES OF FURNACE WALLS, in the decreasing order of furnace volume per unit of steam output, and in the increasing order of heat release rates and furnace temperatures, are solid refractory walls, hollow air-cooled refractory walls, bare water-cooled metallic walls, and covered water-cooled metallic walls. The water-cooled walls are necessary for long-continued operation at high combustion rates and high temperatures. Solid refractory walls are suitable and economical for moderate rates and temperatures. For intermediate conditions, the hollow air-cooled wall or a combination of refractory and water-cooled walls may be satisfactory. Superheater or reheater surface may be substituted for some refractory or water-cooled surfaces.

Increasing excess air, to reduce furnace temperatures and decrease wall failures, is inadvisable in ordinary operation, as it also reduces efficiency; it may be justified at peak loads. The use of preheated air usually causes higher furnace temperatures than the use of room air. For long-continued, high-temperature operation, furnace walls should be designed with these conditions in mind.

MAXIMUM ALLOWABLE FURNACE TEMPERATURE depends on the behavior of the particular combination of fuel, ash, and material in the hot faces of the furnace walls. Depending on the composition of the ash, its fusion temperature, and the furnace-wall temperature, a refractory wall may be affected by slag penetration, chemical reaction, or erosion by molten slag running down the wall. If the temperature of a coal-fired furnace is not quite high enough to cause any of these effects on a solid refractory wall, solidified fly ash may deposit on it until the combined thickness becomes so great that the temperature at the surface equals the ash fusion temperature. Variation in furnace temperature causes the fly ash to melt or build up until equilibrium is established. The same is true of air-cooled or water-cooled refractory walls. Metallic walls give the least difficulty from adhering fuel ash, although fused ash flowing over them will, in time, be destructive.

FURNACE VOLUME depends on the total amount of heat required in a given time and on the permissible Btu release per hour per cubic foot of furnace volume. This heat-release rate depends on type of furnace construction, flame length, ash fusion temperature, method of firing, amount of excess air, and amount of turbulence in furnace. Table 3 gives permissible heat-release rates.

Table 3. Average Heat-release Rates

Method of Firing	Solid Refractory Walls		Water-cooled Metallic Walls	
	Continuous Operation	Peak Operation	Continuous Operation	Peak Operation
	Btu per hr per cu ft of Furnace Volume			
Chain- or traveling-grate stoker	15,000	25,000	30,000	45,000
Underfeed stoker	25,000	40,000	30,000	45,000
Spreader stoker	30,000	40,000	35,000	45,000
Pulverized coal firing	15,000	20,000	25,000	35,000
Oil firing	20,000	40,000	30,000	60,000
Gas firing	20,000	40,000	30,000	60,000

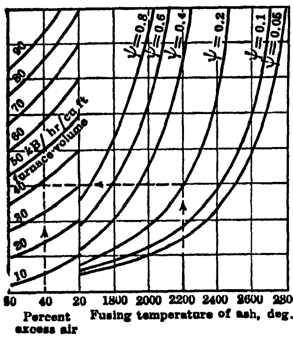
Since the heat-release rate in hand-fired furnaces is limited, the grate in horizontal return tubular boilers can be set within 3 or 4 ft of the boiler surface, and within 4 to 4 $^1/_2$ ft in horizontal water-tube boilers fired with volatile coal. With anthracite the distance can be slightly less. With stoker-fired furnaces, the distance should be made greater. (See Table 2.) For very high rates of combustion, even greater distances are required.

Pulverized-coal furnace volume usually is larger or the heat-release rates lower than in furnaces of equal capacity burning any other fuel. In general, under identical conditions, higher heat-release rates are allowable in small units than in large, since the ratio of wall surface to volume is greater. Figure 17 (Ref. 7) shows approximate relations

between heat-release rates, amounts of excess air, fusing temperatures of ash, and *fractions cold* (ψ), defined by

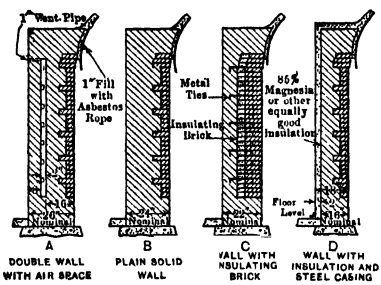
$$\psi = \frac{\text{(Actual extent of cold surface in furnace)}}{\text{(Maximum possible extent of cold surface in furnace)}}$$

Furnace design should consider the conditions to be met by the various elements of volume and wall surface. The use in design of average heat-release rates and average temperatures (which have been used in the above discussion) may lead to trouble because localized temperatures may be much higher than average temperatures.



ψ = fraction cold
kB = 1000 Btu

FIG. 17. Furnace heat-release relations.



Sections of side walls at rear of bridge wall

FIG. 18. Typical horizontal return tubular furnace settings.

SOLID REFRACTORY WALLS are usual in the furnaces of externally fired boilers, with low heat-release rates. The walls usually are integral with the boiler setting and are built of high-grade firebrick, second-grade firebrick, insulation, or some combination thereof. Some typical furnaces are shown below.

HORIZONTAL RETURN TUBULAR BOILERS. Figure 18 shows approved constructions. Type B is the least costly but is more liable to air leaks than type A. The hollow space in type A should be filled with sand or ashes to retard air infiltration in the event of cracks in the inner wall. Type C is more costly than type B. The insulation reduces heat loss through the walls, and raises the furnace temperature. Higher grade lining, therefore, is necessary. Metal ties are advisable to bond the lining to the outer walls, as the insulating brick has little mechanical strength. Type D is the most costly setting. It is similar to Type B, except that 85% magnesia replaces the hollow space, and a steel casing of No. 8 gage steel plate encloses the entire setting.

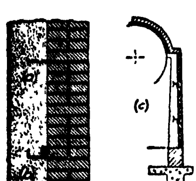


FIG. 19. Monolithic wall lining.

Jointless monolithic wall linings are made of plastic fireclay rammed into position and tied to the outer walls; three methods are shown in Fig. 19.

WATER-TUBE BOILERS. Stoker-fired furnaces may have solid refractory walls, whose arrangement depends on the type of stoker and boiler. The boiler should be so suspended from overhead beams that it cannot at any time come in contact with the furnace walls. Bridge walls and furnace linings should be high-grade firebrick. Cheaper grades of brick can be used behind the lining. Relieving arches, amply buttressed to carry thrust, may be built into the walls to relieve the load on the lower brick, in high settings, or to assist in wall repairs; expansion space should be provided below them. The upper part of high furnace walls sometimes is anchored to external steel work to prevent the wall falling inward as a result of alternate heating and cooling.

The common wall of furnaces grouped in batteries of two should be entirely of high-grade firebrick and much thicker than the side walls. Figure 20 gives typical sections through one type of wall construction for horizontal water-tube boilers. To avoid overheating, steel work supporting the boiler should not be enclosed in the brickwork.

Clinker belts, i.e., the lower parts of side walls of the furnace, adjacent to and just above the fuel bed, usually require special construction, as they are subject both to intense heat and to adhesion of clinker if the wall is of ordinary firebrick. Several solutions are available to meet these severe conditions. The wall at this point may be built of special slag-resisting blocks which may be solid or hollow, or air- or water-cooled metallic wall sections may be used.

Joints in refractory walls are vulnerable points for slag attack. The brick should be laid with a fireclay mortar with refractory properties equal to those of the brick itself. Finely ground raw fireclay and as much finely ground calcined fireclay or ground firebrick, free from slag, as will stay in suspension in a batter should be used in laying the brick.

Backing-up brick is bonded to the inner lining by header and stretcher courses. If the lining is only 4 1/2 in. thick, every fourth or fifth course should be a stretcher course as *d* in Fig. 20. A 9-in. wall can be laid as header courses with every fourth or fifth course a stringer course, i.e., a header course behind a stretcher course (*a*, Fig. 20).

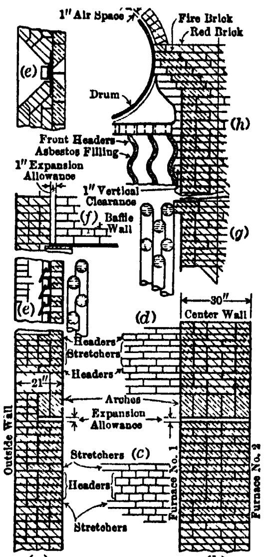


Fig. 20. Wall construction for horizontal water-tube boilers.

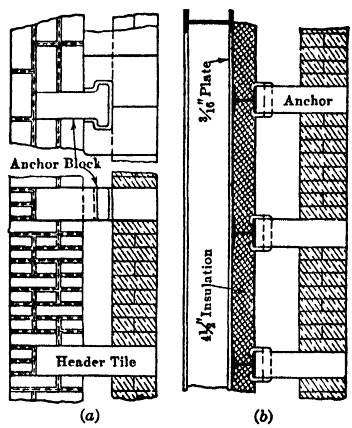


Fig. 21. Self-supporting air-cooled refractory wall.

AIR-COOLED REFRACTORY WALLS are either entirely self-supporting or sectionally self-supporting. Cooling air flows through ducts in the walls and into the furnace. Self-supporting air-cooled refractory walls may be built entirely of standard-size brick, or with special refractory tile in the furnace lining, bonded to the outer wall. Figure 21 shows two forms of this type of wall. In each, the inner wall is flexibly bonded to the outer wall, to provide for differences in expansion. Sometimes larger blocks are used, instead of the standard brick, to reduce the number of inner wall joints.

Special forms of air-cooled blocks sometimes are used in the clinker belt, and sometimes for lining the entire wall. Some blocks have openings that permit flow of air through the block and into the furnace in order to cool the surface next to the fire, thereby reducing adherence of clinker.

Self-supported hollow walls cannot be used in extremely high furnaces because of the inability of the lower part of the walls, when hot, to carry the load of the upper part.

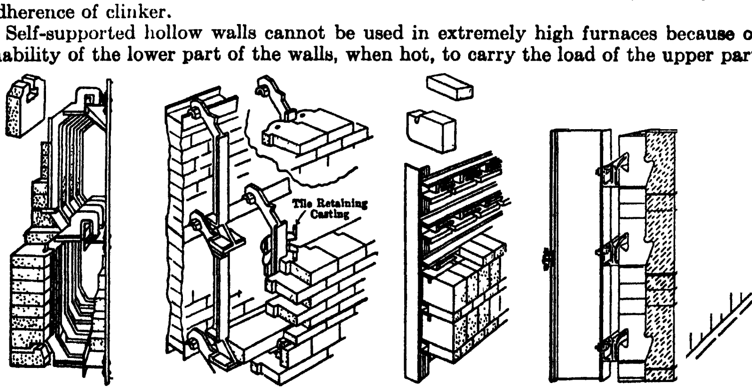


Fig. 22. Sectional self-supporting air-cooled refractory walls.

Sectionally supported air-cooled refractory walls usually are built in horizontal belts 2 to 3 ft high, attached to an outside steel structure. Static load on the brickwork is thus reduced and a means is provided to support the wall when refractory replacements are made. Figure 22 shows typical forms. Different makes vary in shape of the refractories, number of special shapes, brackets and type of supporting steel, methods of providing for expansion and for sealing joints.

Arches over the fuel beds are seldom curved or sprung. Flat suspended arches (see Fig. 23) are more desirable. They require less skill in erection, exert no end thrusts, do not distort when heated, and if necessary can be repaired while the furnace is in operation. The refractory tile are air cooled on the back side, flexibly supported, can expand or contract freely, and have no additional weight to support. One make, not shown, incorporates a veneer of silicon carbide, enabling the arch to withstand very high temperatures, rapid temperature changes, and slagging action.

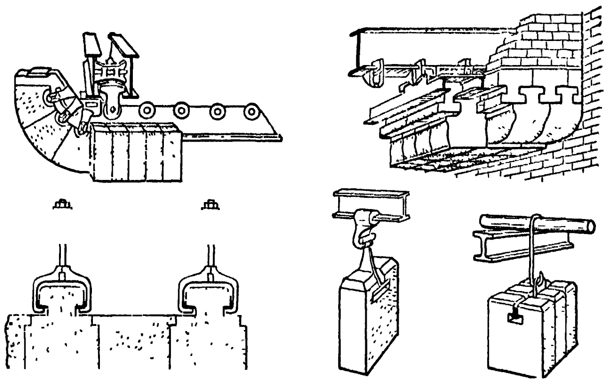


FIG. 23. Types of flat suspended arches.

Properties desired in a good refractory are relative infusibility, relatively low thermal conductivity, flexibility of structure, low thermal expansion, impermeability toward gases and liquids, chemical inertness, and resistance to abrasion. Failure of a refractory in a boiler furnace may be due to one or more of the following: fusion; subsidence under load; spalling; slag action; changes in dimension.

WATER-COOLED METAL FURNACE WALLS are of three types: bare-plate, bare-tube, and covered-tube walls. They are more costly than refractory walls, but can withstand more severe conditions. In general, they are used only in locations where the refractory wall would deteriorate rapidly, for instance, in the bridge wall of underfeed stoker furnaces, or that part of the side wall immediately adjacent to the fuel bed of traveling-grate stokers operating at moderate rates. If higher rates of combustion are maintained with either type of stoker, the entire wall surface and arches may require water cooling.

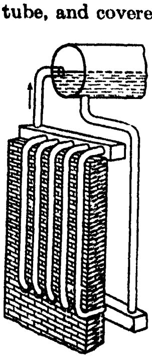


FIG. 24. Typical arrangement of bare-tube furnace wall.

Bare-plate wall furnaces are those in all internally fired boilers, as Scotch marine boilers and locomotive boilers.

Bare-tube walls are connected into the boiler circulation system, as shown diagrammatically in Fig. 24. They may be constructed of plain tubes (Fig. 25), fin tubes (Fig. 26), or studded tubes (Fig. 32). The plain-tube walls usually comprise tubes fairly closely spaced, the distance between tube centerlines ranging from approximately 6 in. to tube diameter (tubes touching each other). Other arrangements stagger the tubes in two rows or use special bifurcated tubes, usually 3-in. OD, on 3 $^1/_8$-in. centers.

In Fig. 25*a* the back sides of the tubes receive heat by radiation from the firebrick backing, the effectiveness of this radiation depending on the extent to which the space between the tubes is filled with sintered fly ash or molten slag. Figure 25*b* shows a modified form of firebrick backing. Arrangements shown in Figs. 25*c* and 25*d* are usually employed in larger furnaces fired by pulverized coal, oil, or gas, or a combination of all three fuels. These tangent tube walls require only a minimum of firebrick backing, the principal backing consisting of block

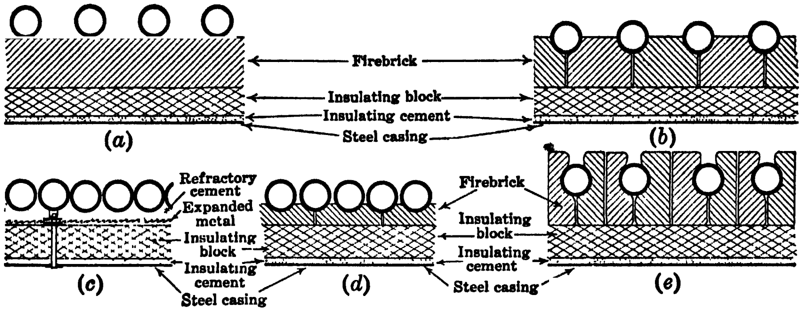


FIG. 25. Types of furnace water-wall construction.

insulation to reduce the heat loss. The wall is usually enclosed by a welded steel panel construction. Figure 25*e* shows an arrangement used in stoker arches.

In the fin-tube construction shown in Fig. 26, the longitudinal fins are welded to each tube usually at opposite ends of a diameter. Fins are ordinarily limited to 1 1/4 in. in width, and broken every 3 in. or so, longitudinally to prevent cracking. The backing is similar to that used behind plain tube walls.

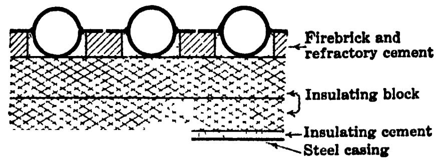


FIG. 26. Fin-tube water wall.

COVERED-TUBE WALLS usually consist of tubes protected either by integral blocks or attached blocks; the latter may be all metal, all refractory, or metal coated with refractory. The blocks generally are rectangular, with flat faces, and form a substantially continuous flat surface when placed close together.

Integral block construction is obtained by casting iron blocks on boiler tubes. (See Fig. 27.) Thermal contact between block and tube is good. Space must be left between blocks to permit growth of the cast iron. A close approximation to the good thermal contact of this construction is obtained by shrinking internally machined cast-iron blocks on accurately sized tubes. (See Fig. 28.)

Attached block construction comprises metallic blocks bolted to the water-wall tubes. Various types are shown in Figs. 29, 30, and 31. With such construction, furnace temperatures are higher than with bare-tube walls under identical conditions, because of the lower heat transmission of the block-tube walls. This may be important at light loads.

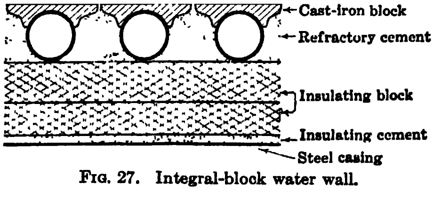


FIG. 27. Integral-block water wall.

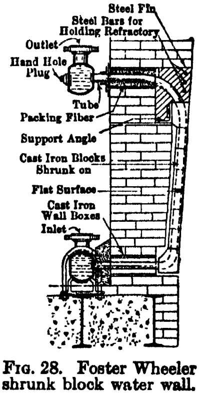


FIG. 28. Foster Wheeler shrunk block water wall.

Either cast iron or steel or alloy steel may be used for the blocks shown in Figs. 29 and 30. Depending on furnace conditions, the face exposed to the fire may be bare or coated with refractory; the bare face may be plain or ribbed. Refractory-faced blocks are used where high-temperature walls are necessary to assist combustion, and bare blocks where cooling surface is desirable. The blocks span the space between the tubes to which they are attached, and make good thermal contact by reason of ground joints and a suitable heat-conducting plastic filler.

In Fig. 31, the bare or refractory-faced blocks and tubes are brought in close contact by channels and toggle joints.

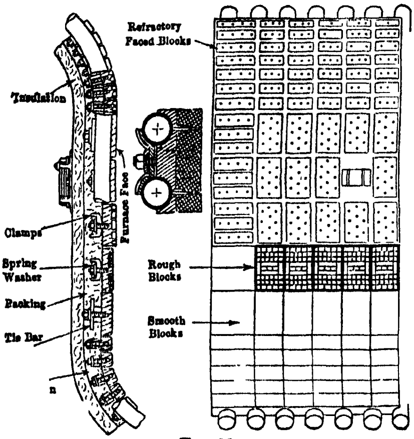


Fig. 29

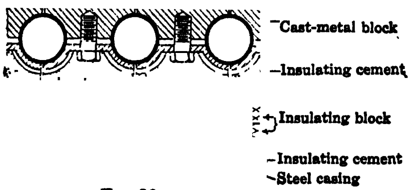


Fig. 30

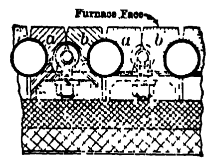


Fig. 31

Types of bolted-on block water walls.

Refractory-protected water tubes are shown in Figs. 32, 33 and 34. Walls of this type usually transmit heat less rapidly than walls of all-metal blocks. Refractory-protected water-tube walls materially assist in maintaining high furnace temperatures at low fuel-burning rates. In the stud-wall construction (Fig. 32), short iron studs are welded on the tube surface where plastic refractory is to be installed. The entire wall-tube surface in the hot parts of a furnace can be completely covered with a thickness of plastic refractory that will give the desired rate of heat absorption, while tubes in the cooler parts of the wall can be bare except for the refractory-covered studs between tubes. The studs support the refractory and cool it by providing a good heat conductor to the water in the tubes. In Fig. 33, small fireclay blocks are slipped around ordinary boiler tubes. The rate of heat transfer to the tubes can be increased by using silicon carbide blocks. Such walls may be backed with refractories, block insulation, or a combination of both. The outer surface of the wall should be coated with a sealing cement or steel casing to prevent infiltration of air. In Fig. 34, interlocking fireclay, silicon carbide, or cast-iron blocks maintain intimate contact with the tubes without the use of clamping devices. Horizontal structural-steel channels so support belts of the blocks that, by removal of key blocks, any block can be removed without disturbing any of those above it.

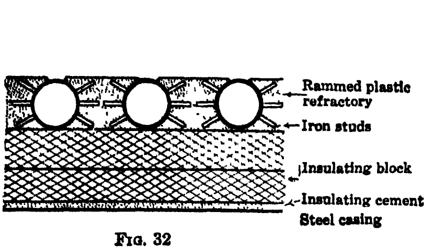


Fig. 32

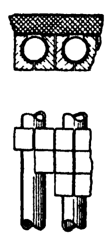


Fig. 33

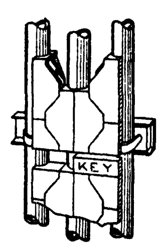


Fig. 34

Types of refractory-protected water-tube walls.

FURNACE BOTTOMS. The type of furnace bottom used depends on the fuel, characteristics, and methods of removal of the ash, method of firing, initial cost, and maintenance costs. Hand-fired or stoker-fired furnaces, operated at moderate rates, usually have ashpits, cleaned by hand. (See Fig. 2.)

Stoker-fired furnaces, operating at higher rates, have ash hoppers of large capacity.

The steel hopper is lined with second-grade, hard-burned firebrick, paving brick, or cast-iron air-cooled plates. The hot ashes usually are quenched by water sprays. In some installations, the ashes are carried away by hydraulic sluices, and in others by conveyor cars. (See Fig. 35.)

Oil- or gas-fired furnaces have solid bottoms, or bottoms with air-cooled passages. Air-cooled bottoms may be of refractory hollow tile, or of several layers of flat interlocking tile carried on standard brick on edge, but not in contact with each other.

Pulverized-coal-fired furnaces have either dry or wet bottoms.

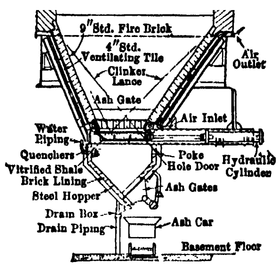


FIG. 35. Ash hopper for stoker-fired furnace.

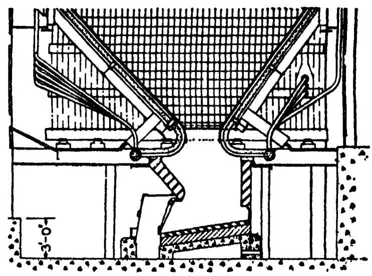


FIG. 36. Dry-bottom pulverized coal-fired furnace.

Dry bottoms are the more common, and may be of two types, the water-tube-screened refractory-hopper or the water-cooled hopper-bottom type. In the latter type the hopper bottom is formed by bending the front and rear wall tubes at the lower end. The tube slope is greater than the angle of repose for ash, and in this way forms a self-cleaning, water-cooled floor.

Figure 36 shows a typical hopper bottom of this type.

Wet bottoms, used in slag-tap or slagging boiler furnaces, form a hearth in which the molten ash collects in a pool. It remains molten and is tapped off either continuously or periodically, similar to the tapping of a foundry cupola. The molten ash, as it flows out, is granulated by a high-velocity water jet driving it against a plate, or by falling through a spray of multiple water jets. Wet-bottom furnaces originally were built to handle ash of fusion temperatures of 1900 to 2000 F, but inasmuch as the flat incandescent furnace bottom aids combustion, an additional advantage accrues from the saving in space requirements. As a result, furnaces have been developed to burn coals with ash fusion temperatures as high as 2600 F. In these furnaces, flames from the burners must bathe the hearth. Operation at high combustion rates only may be necessary, as the ash may solidify at low rates. Fluidity of the ash can be increased by adding limestone or other flux. (See Ref. 8.)

Preheated air can be used to full advantage to aid combustion of pulverized coal in wet-bottom furnaces without the troubles of ash removal from the furnace bottom that occur in dry-bottom furnaces.

A type of all-refractory wet bottom, usually installed in furnaces with water-cooled walls, is built on steel plates carried on an air-cooled structure of piers and I beams. Three or four courses of 2 $^1/_2$-in. firebrick are laid on the plates, and covered with 7 $^1/_2$ to 9 in. of burned dolomite or plastic chrome refractory. Extra courses of firebrick, laid near the furnace walls, form a saucer-shaped bottom. The tap hole at the side of the furnace is plugged by a ball of fireclay.

Furnaces with all-refractory wet bottoms are fairly satisfactory only when used with coals of low ash-fusion temperatures, at uniform, high combustion rates. Whenever the bottom cools, cracks may develop which will fill with slag. With frequent cooling the size of the bottom continually increases, ruining the seal at the furnace walls and displacing the water-cooled side walls. Iron sulfide, formed from iron pyrites in the coal, has a particularly bad erosive effect on the refractory, especially in cracks. The amount of iron sulfide formed can be reduced by pulverizing the coal until 80 to 90% passes through a 200-mesh sieve, as compared with the usual 65 to 70%.

MANUFACTURERS of representative water-cooled walls are: American Engineering Company, Philadelphia; Babcock & Wilcox Company, New York; Combustion Engineering-Superheater, Inc., New York; Foster Wheeler Corporation, New York; Riley Stoker Corporation, Worcester, Mass.; Springfield Boiler Company, Springfield, Ill.; Union Iron Works, Erie, Pa.; and The Wickes Boiler Company, Saginaw, Mich.

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PULVERIZERS AND PULVERIZED COAL

By V. Z. Caracristi

27. PULVERIZERS

PULVERIZED COAL utilized for the firing of steam-generating equipment is dried, ground, and classified to a fineness which may be transported and burned in suspension with air. The degree of fineness generally utilized ranges from 60 to 90% through a 200-mesh sieve. A typical sizing is represented by this screen analysis: 99.5% through 50 mesh; 96.5% through 100 mesh; and 80.0% through 200 mesh. The surface area of the particles of a pound of coal classified to this fineness is approximately 105,000 sq in. This area represents an increase of approximately 2300 times that of a single one-pound lump of coal.

Energy required for preparing, grinding, and transporting coal ranges from 10 to 20 kw-hr per ton of coal. The factors which materially influence the actual power required for a particular installation are type of equipment, grindability of coal, and degree of fineness. In general, the power is also influenced by the rating at which the equipment is operated with respect to the design capacity.

Cost of preparation varies from 3 cents to 15 cents per ton.

PREPARATION OF COAL FOR PULVERIZING. The necessity for supplying raw coal of uniform quality to the pulverizer at a metered rate requires cleaning, rough sizing, and bunkering of the coal. Cleaning necessitates the removal of foreign material, such as large pieces of wood, straw, rags, and iron. This foreign material, often present in the coal as delivered to the plant, is removed to facilitate uniform feeding and to protect equipment from damage. Iron may be removed by magnetic separators in the conveyor system; other foreign materials may be removed by screening, by manual removal, or by the equipment utilized for rough sizing or crushing.

Rough sizing or crushing eliminates oversize pieces of material which would jam the feeder or cause an irregular feed rate, and better distributes the moisture in the raw coal. The preferred sizing of the coal produced by the crusher is generally limited to a maximum of all through $^3/_4$ in. round screen. For small-capacity mills and to obtain a more uniform moisture distribution, the coal may be crushed down to all through a $^1/_2$ in. round screen. The bunker must be designed to give storage space for this prepared raw coal, and to provide a uniform supply of coal to the feeder. The method of filling the bunker must be considered as this is found to influence the degree of segregation and packing.

FEEDERS. An uninterrupted uniform feed to the pulverizer is essential to successful operation of a pulverized fuel system. In many cases the feeder is used as a source of metering the fuel supply to the system. Performance of the feeder is thus an important design consideration. Two types of feeders are generally used, the **table type** and the **roll type.** The table-type feeder feeds coal from a spout onto the table, the coal rotating with the table. The rate of feed is controlled by a cut-off arm, which scrapes off coal from the table, or by the speed of the table, or by a combination of the two. This type of feeder is subject to stoppages from foreign material and requires a relatively close sizing of raw coal to perform with any degree of uniformity. Simplicity, however, makes it well adapted for feeding where accurate metering is not necessary.

The roll-type feeder consists of a rotating spider with pockets which fill from the coal spout and discharge to the mill. The Raymond roll-type feeder is an example of this type.

PULVERIZERS. The function of a pulverizer is to grind, dry, and classify coal to a state in which it can be successfully transported and burned in mechanical suspension.

The principles of grinding are *impact*, *attrition*, and *crushing*. The application of one or more of these principles is employed in the various types of mill. The energy required for grinding is a function of the fineness produced and the hardness of the coal.

Energy Required. There is no generally accepted theory of the relation of energy to fineness of crushing. However, Rittinger's law, "The work to produce material of a given size from a large size is proportioned to the new surface produced," is a closer approximation than Kick's law, which states, "The energy required to effect crushing or pulverizing is proportional to the volume reduction of the particle." The hardness of the coal, or grindability, is a relative measure of the energy required for crushing. For lack of an exact law governing the energy required for crushing and a simple method for determining new surface, it has been desirable to predict mill-grinding performance on the basis of grindability and 200-mesh sieve fineness rather than on the surface area produced.

The Hardgrove method of grindability determination has been generally used because it is a direct measure of the 200-mesh fineness produced by a standard unit of energy input. (See also Section 2.) Principal features of the Hardgrove apparatus for fineness determination are shown in Fig. 1. Grindability is determined by placing a 50-gram sample of air-dried coal, sized to minus 16 and plus 30 mesh, in the mortar of the test machine. After turning the machine through 60 revolutions the sample is removed and screened. The quantity passing a 200-mesh sieve is used to determine the Hardgrove grindability index by the following empirical formula:

$$G = 6.93W + 13$$

where W is the weight in grams of the sample that passes a 200-mesh sieve.

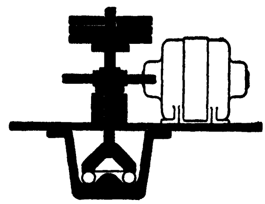

Fig. 1. Hardgrove apparatus for determining the fineness of pulverized coal.

The grindability index of various coals permit the manufacturer to predict the performance of a particular type mill with the coals to be used. Table 1 shows the grindability of several typical coals and Fig. 2 shows the effect of grindability on the grinding capacity of a particular mill. For additional data on coals, see p. 2-26.

The ability of a particular mill to dry coal depends on the heat added in the form of hot air or gas, and the energy that is converted to heat due to grinding. To utilize properly the heat added by hot air or gas there must be intimate mixing of the air and coal, and the incoming feed should be rapidly mixed with the dryer coal being pulverized or contained in the circulating load. Different types of pulverizers show considerable variation in drying performance. The ability to use high gas or air temperatures entering the mill rather than large quantities of gas or air also varies considerably with mills of different types. To enable high inlet drying temperatures there must be high velocities of gas or air. Dead pockets within the mill are a fire hazard. High inlet temperatures and consequent low gas or air quantities are desirable because of lower power requirements for handling a smaller weight of air, lower tempering air requirements (thus higher efficiency on direct-fired units supplied with gas-air heaters), and lower primary air quantities (thus better burner performance on direct-fired units).

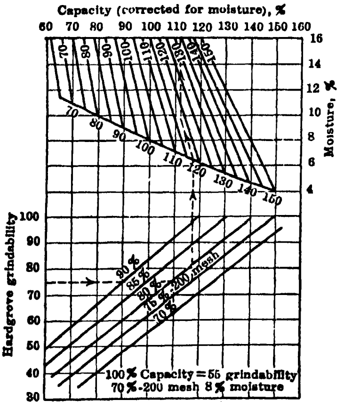

Fig. 2. Effect of grindability on capacity.

The extent to which drying must be accomplished in a mill depends primarily on the type of coal. In general, it is necessary to remove all surface moisture, leaving only the inherent moisture in the pulverized coal. Failure to remove the surface moisture limits the capacity of the mill considerably beyond the reduction of capacity that occurs due to moisture (see Fig. 2) in the raw coal, even when it is removed in the milling system.

The classification of coal size is determined by screening. The screens generally used and their corresponding sizes are shown in Table 2.

Classification of coal in mills generally is accomplished by means of air separation. Oversize particles are separated by a change in direction and returned to the grinding chamber. Means for application and control of this air separation principle differ for various mills. The classifier performance has a direct bearing on the grinding power required to obtain satisfactory combustion results. Satisfactory combustion results require

Table 1. Grindability of Typical Coals

State and County	Mining District or Seam	Grindability (Hardgrove)
Alabama, Jefferson	Mary Lee	62– 87
Walker	Mary Lee	51– 65
Arkansas, Franklin	Denning	99–102
Colorado, El Paso	Colorado Springs	38– 39
Las Animas	Trinidad	44– 54
Illinois, Franklin	Franklin	53– 63
Williamson	Williamson	52– 59
Sangamon	Springfield	54– 68
St. Clair	Belleville-Saunton	57– 62
Peoria	Peoria	65– 67
Fulton	Fulton	51– 68
Indiana, Clay, Greene, Vigo	No. 3	62– 66
Greene, Sullivan	No. 4	53– 60
Greene, Sullivan, Gibson	No. 5	60
Greene, Sullivan, Knox	No. 6	60– 65
Iowa, Appanoose, Wayne	Mystic	60– 70
Polk	....	62– 66
Boone	....	61
Kansas, Cherokee	Cherokee	61
Leavenworth	Leavenworth	70
East Kentucky, Floyd, Letcher, Pike, Perry, Breathitt	Elkhorn	50– 60
Knott, Letcher	Hazard No. 4	45– 55
Harlan	Harlan	47– 58
West Kentucky, Union, Webster Hopkins, Muhlenburg	Eastern Interior Seam No. 9	60– 65
Maryland, Allegany	Georges Creek	95–100
Michigan, Saginaw	Saginaw	50– 67
Missouri, Adair	Bevier	72– 75
Montana, Carbon	Red Lodge	50– 55
Carbon	Bear Creek	47– 56
New Mexico, McKinley	San Juan	29– 41
Santa Fe	Cerillos	65
North Dakota, Most Middle & Western Counties	(General)	50
Ohio, Morgan, Noble, Washington, Harrison	Meigs Creek	67
Belmont	Pittsburgh No. 8	50– 60
Oklahoma, Pittsburgh	McAlester	47– 67
Pennsylvania, Luzerne and Lackawanna	Northern Coal Field	25– 30
Dauphin, Schuylkill, Carbon	Southern Coal Field	35– 45
Cambria	Upper Kittaning	85– 87
	Lower Kittaning	107
Cambria	Upper Freeport	87
	Lower Freeport	99
Clearfield	Lower Kittaning	106
	Lower Freeport	87
Somerset	Upper Kittaning	95–100
	Lower Kittaning	115
Westmoreland	Redstone	60– 70
Allegheny	Upper Freeport	55– 60
Tennessee, Campbell	Jellico	45– 55
Bledsoe	Swanee	50– 60
Texas, Bowie S.W. to LaSalle	Lignite Fields	53– 79
Utah, Carbon	Castlegate	43– 49
Summit	Wasatch	47– 50
Virginia, Tazewell	Pocahontas	99–105
Wise	Norton	62
Washington, Kittitas	Clealum (Cle Elum)	49– 52
Kittitas	Roslyn	52
Pierce	High-volume Carbonado	69
Pierce	Medium-volume Carbonado	55
West Virginia, Monongalia, Marion, Harrison	Fairmont	50– 70
Fayette	New River	90–100
Mercer	Pocahontas	105
Kanawha, Fayette	Kanawha	40– 60
Mingo	Thacker	56

a minimum quantity of plus 50-mesh material; a large quantity of minus 200-mesh and superfine material is not necessary, however. Good classification is thus measured by the retention of a minimum quantity of 50-mesh material with a given quantity of minus 200-mesh material. Figure 2 illustrates the grinding capacity variation with 200-mesh fineness on a particular mill.

Table 2. Standard Screen Sizes

U. S. Standard Sieve			W. S. Tyler Sieve		
Mesh	Inches	Millimeters	Mesh	Inches	Millimeters
20	.033	.84	20	.033	.83
30	.023	.59	28	.023	.59
40	.0165	.42	35	.016	.42
50	.0117	.30	48	.0116	.30
60	.0098	.25	60	.0097	.25
100	.0058	.149	100	.0058	.15
140	.0041	.105	150	.0041	.10
200	.0029	.074	200	.0029	.074
325	.0017	.044	325	.0017	.043

TYPES OF PULVERIZER generally used are the ball, impact, ring roll, and ball race.

Ball mills consist of a horizontally rotating cylinder less than half full of balls of various diameters. The speed of rotation is approximately 20 rpm. Balls carried up the periphery in the direction of rotation continually cascade toward the center. Coal mixed with the ball charge is pulverized by impact, attrition, and crushing. Hot air passed through the mill dries the coal and removes the fines. A classifier is used in some designs to regulate the fineness of the finished product by returning the coarser particles. Low maintenance and quick response to change in output rate are characteristics of this mill. Power requirements, particularly at reduced capacity, are relatively high. Space requirements are relatively large for a given capacity, and with wet coal there is an extreme falling off in capacity. The large storage and heat capacity makes this type of mill poorly adapted for intermittent operation or quick starting.

Impact mills consist of a series of hammers or lugs revolving at high speed in an enclosed chamber. Grinding is by impact and attrition. Air passing through the mill dries the coal and carries away the fines. A means of classification is generally provided for returning oversize particles. This type is compact, low in cost, and may be built in very small sizes. It is well adapted to drying, because there is intimate mixing of air and coal. Maintenance and power consumption are relatively high, and it is difficult to maintain uniform fineness over the life of the wearing parts. The small storage capacity makes this type well adapted to quick starting and intermittent operation.

The ring-roll and ball-race mill principle is shown in Figs. 3 and 4. These mills pulverize by passing the coal between two surfaces, one rolling over the other. Grinding is accomplished by crushing and attrition. These types have low power consumption, are compact, maintain fineness over the life of the wearing parts, and handle wet coal with only a small reduction in capacity.

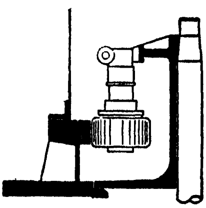

FIG. 3. Ring-roll mill.

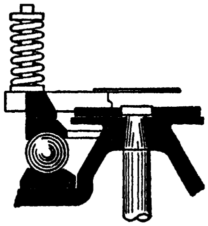

FIG. 4. Ball-race mill.

A unique application of this grinding principle is the Raymond Bowl Mill, shown in Fig. 5. The roll is restrained from coming in direct contact with the ring, so that there is no metal-to-metal contact. The grinding ring is rotated with the bowl at approximately 1200 ft per min. Stationary spring-loaded rolls are set with a travel-limit bar so that they do not come in contact with the grinding ring. A centrifugal-type classifier with adjustable inlet vanes is mounted directly over the center of the bowl. Raw coal fed to the bowl is thrown by centrifugal force to the face of the grinding ring, where it is passed under the spring-loaded rolls, which are free to rotate, thus grinding the coal as it is passed between the ring and the roll. As the coal is discharged over the rim of the grinding ring, it is thrown into an annular air passage where oversize particles are deflected back into the bowl. Pyrites and tramp iron are dropped to the bottom of the air inlet chamber, and the fines mixed with them are carried up by the hot air stream to the classifier inlet vanes. The coal deflected back into the bowl is again passed under the rolls for further grinding. Pyrites and tramp iron dropped to the bottom of the air chamber are discharged through a discharge spout by sweeps. Fines entering the classifier are separated by the centrifugal motion imparted by the inlet vanes, the oversize particles being returned to the center of the bowl to be mixed with the raw coal. The final ground, dried, and classified product is taken from the classifier outlet to the exhauster inlet.

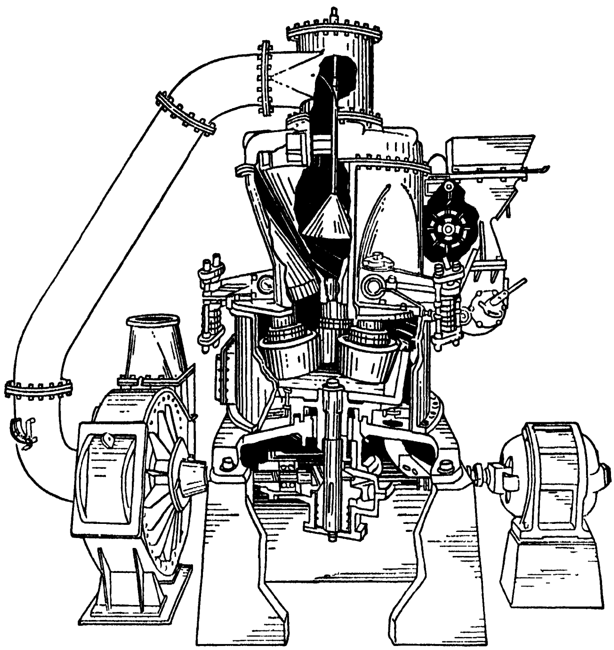

FIG. 5. Raymond Bowl Mill.

Evaluation of the mill best suited to a particular pulverized coal installation should consider the following factors: (1) Initial cost. (2) Power requirements at full and at partial load. (3) Maintenance cost and frequency. (4) Drying ability, limitation of drying temperatures. (5) Fineness and classification control. (6) Response to load changes. (7) Space requirements. (8) Quietness. (9) Ability to start and stop rapidly. (10) Physical limitations, consisting of (*a*) ease and reliability of raw coal supply to grinding elements, (*b*) lubrication ease and reliability, and (*c*) starting after trip-off. (11) Reliability. (12) Controllability—drying, fineness, output. (13) Accessibility and simplicity. (14) Rejection of pyrites. (15) Ability to handle foreign material.

28. PULVERIZED COAL SYSTEMS

Pulverized coal may be utilized in a *storage system* or in a *direct-fired system*. The latter system is characterized by immediate supply of coal to the burners and furnace as it is ground, with no part of it diverted to storage bins.

The bin or storage system has the advantage of operating flexibility, and permissible arrangement and location of equipment. This system permits preparation of coal during off-peak load hours at a constant (maximum) mill output rate. A few large-capacity pulverizers may be used without sacrificing flexibility. The ability to keep to a minimum the quantity of primary or carrier air used to convey the pulverized coal to the furnace is of distinct advantage in securing stability and range of operation of the fuel-burning equipment. These advantages are more than offset, in most cases, by the considerably higher equipment cost, the complication of venting the drying air or gas, transporting and storing the pulverized fuel, feeding the pulverized coal, and maintaining and operating the additional equipment. Justification of the storage system is difficult, except where low-volatile, hard-to-burn fuels, such as anthracite, must be utilized. A typical equipment arrangement for a storage system is illustrated in Fig. 6.

The direct-fired system, for which a typical equipment arrangement is illustrated in Fig. 7, is by far the most common arrangement. The advantages of this system are lower initial cost, simplicity of operation, and compactness of equipment. This system requires intelligent coordination in selecting milling, burning, and steam-generating equipment to obtain a reasonable degree of flexibility. The coordination of equipment must represent a satisfactory compromise between simplicity, operating range, types of coals (i.e., moisture, grindability, and volatile matter) to be burned, reliability, overall cost, efficiency, and excess milling and drying capacity.

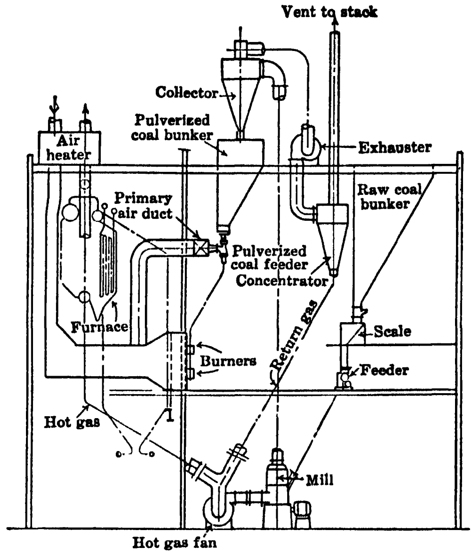

FIG. 6. Plant layout for storage-type pulverized coal system.

FUEL-BURNING EQUIPMENT. The function of fuel-burning equipment is to introduce fuel and air into a furnace in such a way that stability of ignition and substantially complete combustion with minimum excess air are obtained. For successful performance fuel-burning equipment must be designed in such a way that the following conditions can be obtained: (1) Uniform distribution of excess air and temperature at furnace outlet. (2) A means of ignition point and flame-shape control. (3) Freedom from localized slag deposition. (4) Protection against overheating and excessive wear of burner parts. (5) Accessibility for maintanance and adjustment.

Stability and Range. Design factors which control the stability of ignition are those which promote rapid supply of heat to fuel particles as they enter the furnace. Heat to evaporate moisture, distill off volatile matter, and raise the temperature to the kindling point must be supplied to each coal particle by the combustion process of the preceding fuel supply. Thus, to promote rapid ignition, stability, and wide range of operation, all essentially synonymous, the following design conditions can be utilized: (1) A furnace gas flow pattern to promote supply of heat to the incoming coal as it leaves the fuel nozzle. (2) A flow pattern of the fuel leaving the fuel nozzle which promotes low velocity eddies of some of the coal particles, allowing them to absorb heat before dilution by a large mass of combustion air. (3) A low relative quantity and high temperature of the air used to transport the fuel. (4) A high degree of fineness. (5) A high concentration of heat release close to the fuel nozzles. (6) A low heat absorption of the furnace surface in the vicinity of the fuel nozzles to permit a higher temperature for heating the incoming fuel.

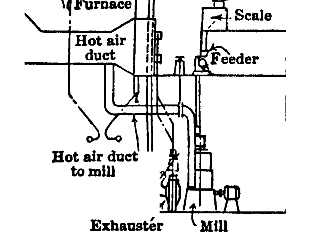

FIG. 7. Plant layout for direct-fired system.

In practice it is not usually considered necessary or desirable to obtain all these conditions; in some designs they may even be detrimental to the overall performance, because of deposition of an excessive amount of slag, production of a nonuniform furnace heat distribution, the necessity for utilizing uneconomically high fuel-air pressures, or high burner maintenance.

Completeness of Combustion. Design factors which control the completeness of combustion are those which contribute to the intimate mixing of burning fuel particles with the available oxygen in the combustion air. The importance of mixing is progressively greater as the combustion process nears completion and the oxygen concentration becomes low, so that inert gas shields the oxygen from the fuel.

Design conditions that promote intimate mixing of fuel and oxygen are: (1) Utilization of high air velocities penetrating the burning fuel streams. (2) Impingement of burning fuel streams upon each other. (3) Utilization of numerous individual equi-quantity fuel and air streams. (4) Utilization of cyclonic gas movement within the furnace chamber.

Mixing. Most types of firing utilize several burners in a furnace to obtain better mixing and more effective utilization of furnace volume and heating surface. The effectiveness of mixing with multiple burners is dependent on the accuracy of metering fuel and air to each nozzle, unless there is a high degree of mixing of the various streams from each nozzle with each other. The use of riffle-type distributors is one commonly used method of subdividing a stream of primary air-coal mixture for more than one burner nozzle. Secondary-combustion air is subdivided by maintaining equal pressures across ports of equal area.

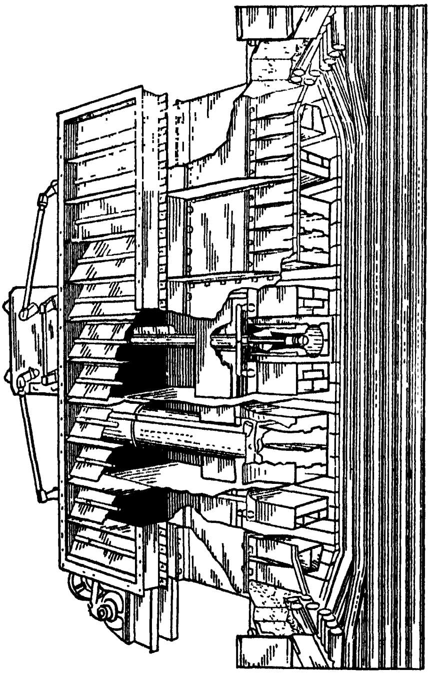

FIG. 8. Tangential burner.

Types of Firing. Fundamentally there are two basic types of firing. In one, the furnace volume is used as a mixing chamber for the air and fuel from all burners. In the other, individual burner nozzles have individual air-supply systems, necessitating an accurate air- and fuel-metering means for each burner assembly.

The tangential burner (Fig. 8) is representative of the first type, whereas horizontal (Fig. 9) and vertical burners (Fig. 10) are examples of the second type. Horizontal burners, where installed in opposite walls, more nearly resemble the first type of firing. However, if more than one burner is installed in each wall, accurate control of the air and fuel to each opposed-burner couplet becomes as important as in the second type of firing.

The tangential-type burner, Fig. 8, is located in each of the four corners of a furnace, and its air and coal streams are directed tangent to a circle in the center of the furnace. The velocity of these streams produces a rotary motion of the gas within the furnace. Thus the gas-flow pattern from each burner nozzle aids ignition of fuel from succeeding burner nozzles. Scrubbing and impinging action of the gas, air, and fuel streams assures maximum turbulence, so that the entire furnace cross section is uniformly and effectively utilized. Because this type of burner is essentially a series of straight-shot nozzles, furnace outlet-temperature control can be accomplished by vertically adjusting the angle of nozzle discharge, thereby utilizing more or less of the furnace heating surface. The ability to control the furnace outlet gas temperature permits a more conservative furnace design for steam-temperature control at partial ratings. Figure 11 illustrates the application of this principle.

The horizontal-type burner, Fig. 9, consists of a central coal nozzle concentric with a throat and a series of adjustable air-admission vanes. Primary air and coal are admitted tangentially to the coal nozzle, and peripheral fuel distribution is obtained at the nozzle discharge by internal ribs and vanes. The secondary air, admitted through adjustable vanes, is given a whirling motion in the same direction as the primary air-coal mixture. Turbulent mixing of the primary and secondary air streams is obtained in the burner throat after partial ignition of the coal. Flame shape and ignition point are readily controlled by the adjustable-vane position.

The vertical type burner, Fig. 10, has a straight-shot burner nozzle firing vertically downward in the furnace, with the main supply of combustion air being furnished through wall ports along the path of the flame travel. The flow pattern in the furnace requires the gas to pass back up the furnace adjacent to the burner nozzle locations. Ignition point and vertical flame travel are controlled by primary air pressure, quantity and velocity (of air admitted around the coal nozzles) and by selective elevation of the air admitted through the front wall ports.

Completeness of combustion for any burner is influenced by the fineness of pulverization and by the quantity of excess air. In practice the loss due to unburned fuel ranges from

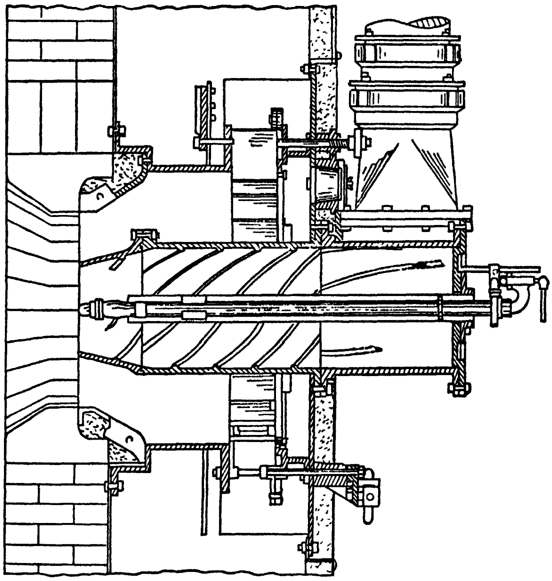

Fig. 9. Horizontal burner.

0.1 to 1.5% of the fuel supplied to the furnace. The approximate rate at which this loss varies with excess air and fineness of pulverization is shown in Fig. 12.

The range of operation of pulverized fuel burners varies considerably with many design factors, as well as with the type of coal. The range generally obtained may be from 2 : 1 to 6 : 1, with all burners in service in a particular furnace. By shutting off some of the burners, the actual range of heat input into a furnace may be controlled successfully in some cases over a range as great as 20 : 1.

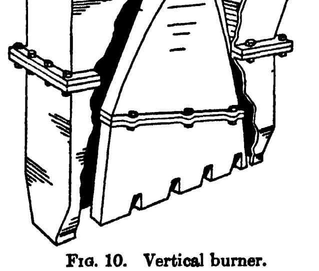

Fig. 10. Vertical burner.

Fly-ash Removal (see also p. 7-94). The removal of fly ash from the gas before it is discharged into the atmosphere is sometimes necessary. In many cases, to meet local ordinances, fly-ash removal has paid for itself on the basis of the increased life and availability of the induced draft fan through elimination of abrasive material before the gas enters the fan. The quantity and the character of the ash discharge vary with the quantity of ash in the coal, the completeness of combustion, type of furnace, and fineness of pulverization. In general, approximately 50% of the ash initially in the coal will be collected in a wet- or slagging-bottom furnace, whereas approximately 20% of the ash is caught in a dry-bottom furnace. The boiler-pass hoppers collect approximately 5% of the total ash. The remaining ash is entrained in the gas with the unburned carbon left after incomplete combustion. This ash plus unburned carbon generally represents a dust loading of approximately 2 to 5 grains per cubic foot in the gas leaving the boiler.

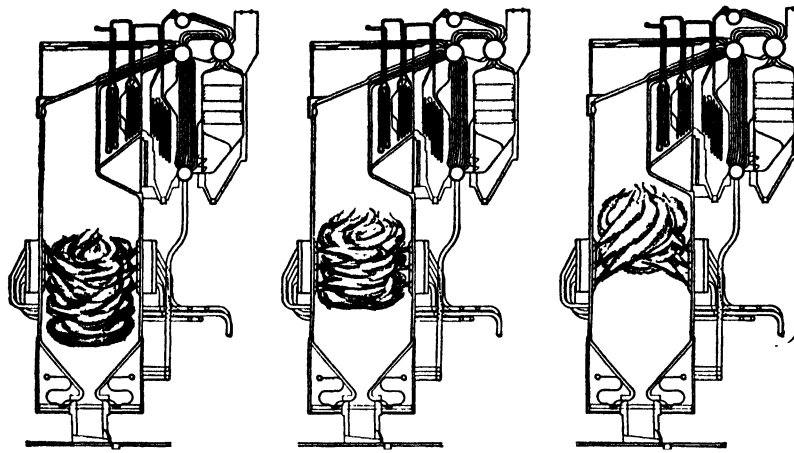

FIG. 11. Tangential burner with various angles of nozzle discharge.

A typical screen analysis of ash leaving the boiler would be:

+100 microns	0.25%
+74	0.8
+44	5
+30	13
+20	29
+10	66

Mechanical or electrical methods may be used for removing this ash. The mechanical method utilizes the change-of-direction principle as obtained in a cyclone separator; the electrical method uses the attraction and repulsion of ionized particles.

The limitation of the mechanical collector is the inability to separate the small and submicron sizes. Thus a collection efficiency of 80 to 90% is about the upper limit that can be anticipated for typical fly ash, using a gas pressure drop of approximately 4 in. of water. One limitation of the electrical precipitator is its inability to ionize carbon in the fly ash. The necessity for low velocities results in large sizes. Collection efficiencies up to 96% are obtainable with extremely low velocities and frequent rapping of plates.

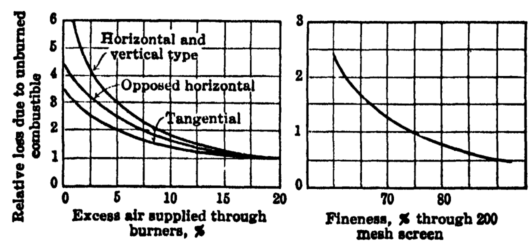

FIG. 12. Combustion loss as a function of excess air and fuel finene

In some instances it has been desirable to install both electrical and mechanical collectors in series. The larger particles, with high carbon content, are caught in the mechanical collector with a relatively low pressure drop and the smaller particles are caught in the electrical precipitator.

EXPLOSION AND FIRE HAZARDS OF PULVERIZED FUEL. In the handling, pulverization, storage, and burning of pulverized fuel there are recognized hazards of fire and explosion that must be considered in design and operation of the equipment. These hazards have been acknowledged by the National Board of Fire Underwriters in a pamphlet which the manufacturers of pulverized fuel equipment use in determining design strength and arrangement of equipment.

FLY-ASH COLLECTION

By R. B. Foley

29. FLY ASH

SOLID PARTICLES IN COMBUSTION GASES are classified as *smoke, fumes,* and *dusts.*

Smoke is unburned carbon particles of extremely small size, 0.001 to 0.25 micron. (*Note:* 0.25 micron = 0.00001 in., approximately.)

Fumes are condensed dispersoids 0.1 to 1.0 micron in size.

Dusts are solid particles larger than 1 micron in diameter. The dust resulting from combustion of solid fuels, consisting of ash and unburned carbon particles carried in the flue gases, is usually called *fly ash.* Although the larger carbon particles are sometimes called cinders, the term fly ash is commonly used to include *all* the solid particles larger than 1 micron.

COMPOSITION OF FLY ASH. The chemical components found most frequently in fly ash are listed in Table 1. The amount of unburned carbon varies from as low as 1% to more than 80%, depending on type of firing, furnace design, boiler load, and operating conditions. The percentages of various noncombustible oxides depend on the composition of the ash in the coal burned, but usually nearly half is silicon dioxide which, with ferric and aluminum oxide, accounts for more than 90% of the ash. In addition to the components listed, there are sometimes small quantities of alkalies and other metallic oxides (Ref. 1).

PHYSICAL CHARACTERISTICS of fly-ash particles, such as size, shape, and weight, are the properties which influence their behavior in the flue gases and the atmosphere. The carbon may be present as fairly large coke particles or as fine soot particles. The ash consists of both fused and crystalline particles. In the fly ash from pulverized-coal-fired boilers a large percentage of the ash particles is fused spheres, many of them hollow shell-like particles.

Some of the methods used to determine physical characteristics measure only the size of the particles. Standard sieves or screens are most commonly used to determine the size analysis of fly ash. Pulverized fuel fly ash, however, usually contains 60 to 90% of particles that pass through the finest standard sieve, which has 400 meshes per linear inch.

The two methods most commonly used for analyzing fly ash in the subsieve range, elutriation and sedimentation, depend on the aerodynamic characteristics of the particles, and thus measure the combined effect of all three of their physical properties on their behavior in flue gas or the atmosphere. In both these methods the fly-ash particles are separated into fractionations dependent on their terminal velocity. The terminal velocity of a dust particle is the ultimate velocity it will obtain in free fall through a given quiescent fluid; it is generally expressed in terms of the particle velocity in standard air.

Although it has been common practice to express the results of analyses made by the elutriation and sedimentation methods in terms of particle diameters in microns, the transition from the terminal velocities as measured to particle size is based on the assumptions that the particles are solid spheres of uniform density. Since fly ash is a heterogeneous mixture of materials of widely different densities (see Table 1) and since only a small percentage of the particles is solid spheres, these assumptions are not accurate. Moreover, it is terminal velocity of the particles, and not their size, which governs their movement in the flue gases, in the fly-ash collector, and in the atmosphere after they leave the stack.

Table 1. Chemical Components of Fly Ash

Compound	Symbol	Specific Gravity
Carbon	C	1.3 to 2.0
Silicon dioxide	SiO_2	2.20
Iron oxide	Fe_3O_4	5.20
Ferric oxide	Fe_2O_3	5.12
Aluminum oxide	Al_2O_3	3.99
Calcium oxide	CaO	3.32
Magnesium oxide	MgO	3.65
Sulfur trioxide	SO_3	1.923
Phosphorous pentoxide	P_2O_5	2.387

It must be remembered, therefore, that when fly-ash analyses, made by these methods, are expressed in terms of micron size, the values are not the true size of the particles but the equivalent size of solid spherical particles of a specified density. Although the various particles have different densities the average specific gravity of most composite fly-ash samples is approximately 2.0; this value is usually used in computing the equivalent micron size. Figure 1 shows the relationship between terminal velocity and equivalent micron size for various specific gravities.

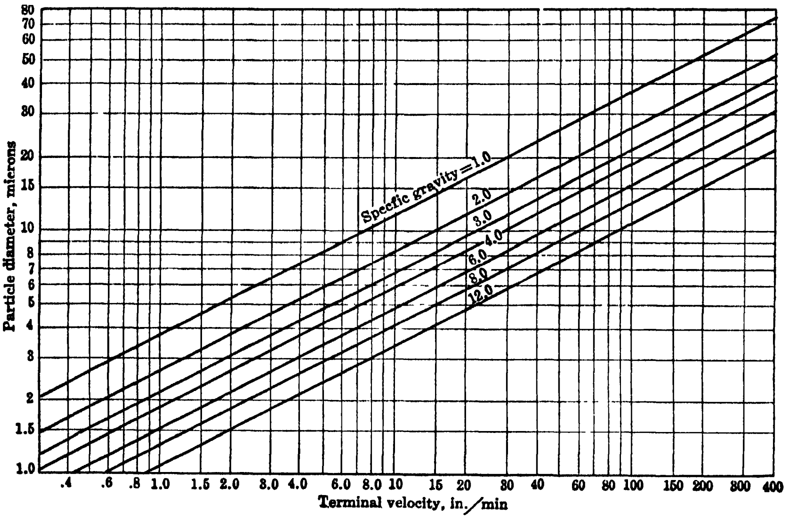

FIG. 1. Relationship between particle size and terminal velocity in standard air for solid spherical particles of various specific gravities. (Courtesy of American Blower Corp.)

INFLUENCE OF TYPE OF FIRING. Although many factors affect both quality and quantity of fly ash, the one of most influence is the method of firing the coal.

Pulverized-coal-fired boilers usually produce more and finer fly ash than any other type of firing. Since the coal is fine and burned in suspension in the furnace, a much higher percentage of the ash is carried out with the flue gases. At design rating, 70 to 85% of the ash may leave the furnace with the gases from the dry-bottom type of furnace; in wet-bottom or slag-tap furnaces 50 to 65% of the total ash is contained in the fly ash. The fly ash may contain up to 30 or 40% of unburned carbon, but generally the carbon content is much lower than that from stoker-fired furnaces. Although the degree of pulverization may materially affect the fineness of the fly ash, the total quantity is not affected as much as might be expected since the finer pulverization usually results in more complete burning and less unburned carbon. Depending on the percentage of ash in coal, furnace design, and operating conditions, the concentration of fly ash in the flue gases from a pulverized fuel-fired boiler may vary from 1 to 5 grains per cubic foot at the normal maximum boiler load. The analysis of the fly ash varies considerably with the boiler load and is also dependent on other factors such as fineness of the coal, type of coal, furnace velocities, and percentage of unburned carbon. Generally the unburned carbon particles are the largest; the finest particles are nearly all ash. Figure 2 shows graphically the usual ranges of analyses of pulverized fuel fly ash for maximum load operation. The center curve *B* represents an average analysis; curves *A* and *C* represent more extreme analyses.

Stoker-fired Boilers. Although the percentage of ash carried with the flue gas is usually considerably lower for stoker-fired boilers, there is generally more unburned carbon in the fly ash. The wide range of variables which affect the fly ash, such as type and size of coal, grate area, furnace volume and design, and operating conditions, causes such great differences in the fly ash that statements of analysis and concentration can be only very general.

With underfeed stokers, up to one-third of the ash may be carried out with flue gases at normal ratings. The fly ash concentration may vary from a few tenths of a grain per cubic foot when grate areas and furnace volumes are generous to two grains or more when the coal burned per square foot of grate area and furnace heat releases are high.

Since finer coals are often burned on chain-grate stokers the concentration may be somewhat higher, but the amount of fly ash varies considerably with the size of coal burned and the amount of unburned carbon.

With the spreader stoker, as much as 50% of the ash may be carried out of the furnace, because the fines are burned in suspension. The percentage of unburned carbon also materially affects the total fly-ash concentration. Owing to the wide variety of coals that may be burned on this type of stoker, and its flexibility in adaptation to furnace and boiler design, the concentration in the gases at the furnace exit under maximum operating loads may vary from about half a grain to as much as four grains per cubic foot.

In many stoker-fired installations, hoppers are provided under the rear boiler passes, and a large percentage of the coarser, heavier particles is trapped out of the gases at this point. The material caught in these hoppers is usually high in carbon content since 70 to 90% of the unburned carbon in the fly ash leaving the furnace is usually in the portion that stays on a 200-mesh screen.

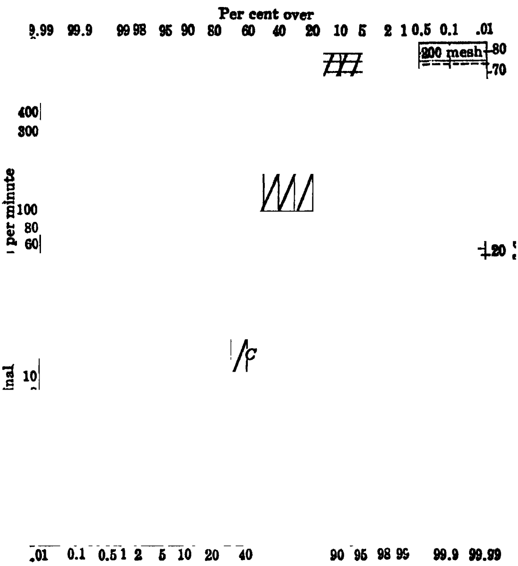

FIG. 2. Analysis of fly ash from pulverized coal fired boilers.

In many stoker-fired installations, especially when spreader stokers are used, fly ash is reinjected into the furnace. Since the carbon content in the material caught in the hoppers under the boiler passes is high, much of it can be reinjected without materially affecting the total concentration of solids in the flue gases. Fly-ash collectors, however, catch a much higher percentage of the fine ash particles, and when this ash is also reinjected the build-up of ash particles in the flue gases often results in doubling fly-ash concentrations.

Since it is difficult to give a typical analysis of the fly ash from stoker-fired boilers, those shown in Fig. 3 illustrate the wide range of analyses obtained.

FIG. 3. Analysis of fly ash from stoker-fired boilers. *A* and *B*—underfeed stokers; *C* and *D*—spreader stokers.

INFLUENCE OF TYPE OF COAL. The type of coal burned, its sizing, and its ash content materially affect the fly ash produced. Other factors being equal, the finer the coal, the greater the quantity of fly ash and the finer its analysis. In a given furnace under similar operating conditions the amount of fly ash in gases varies directly as the percentage of ash in the coal. As an illustration of the effect of the type of coal, tests on a spreader stoker installation showed a variation on the fly ash leaving the boiler of 12 to 40% of the total unburned solids when burning two different grades of coal. Fusion temperature of the ash affects the quantity retained in the furnace when the furnace temperature exceeds the lowest fusion point.

INFLUENCE OF FURNACE AND BOILER DESIGN. Design burning rates, furnace volumes, gas velocities in the furnace and through the boiler, and design and location of baffling are factors that influence the fly ash leaving the boiler. High furnace velocities not only increase the percentage of ash carried out but also result in a higher unburned carbon content in the fly ash. Under such conditions concentration is high and the analysis relatively coarse.

INFLUENCE OF OPERATING CONDITIONS. Smoke elimination is primarily a problem of proper operating conditions. The fly-ash problem can also be controlled to some extent by careful operation. By maintaining adequate furnace temperatures and properly controlling the fuel-air ratio, the quantity of unburned carbon and the fly-ash concentration (particularly in the nuisance range of coarser particles) are minimized.

Load variations have a pronounced effect on both the analysis and the concentration of the fly ash. Maximum loads result in highest concentration, coarsest analysis, and highest carbon content. Figure 4 illustrates the effect of load variations on the amount of fly ash produced with various types of firing. The effect of load variations is usually greater at the higher loads with underfeed stokers. The increase at low loads with spreader stokers is due to the lower furnace temperatures.

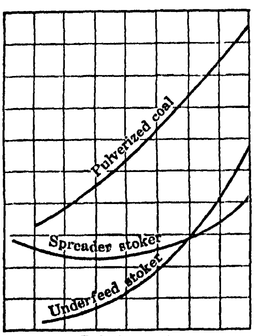

Fig. 4. Variation of fly-ash quantity with boiler load.

Fly-ash concentrations during periods of soot blowing reach many times their values during normal operation. More frequent blowing periods and more care in the proper operation of the blowers often materially reduce the amount of fly ash produced during the soot-blowing periods.

ORDINANCES GOVERNING FLY-ASH EMISSION. Many communities have smoke ordinances which also limit the fly-ash emission; others are now considering such ordinances. In some of them the emission of fly ash, in such a manner as to cause a "nuisance," is prohibited. In the majority of cases, however, definite limitations are set on the allowable concentration of fly ash that may be emitted from the stack. In addition to the limitation on the total fly-ash concentration, some ordinances also set a maximum allowable concentration of material retained on a 325-mesh sieve. This limitation on the coarser fly ash is desirable from the nuisance standpoint since these coarser particles are the greatest nuisance. The various ordinances now in effect limit the total fly-ash emission to 0.30 to 0.75 grain per cubic foot of flue gas, and most of them state that the flue-gas volume shall be determined at a temperature of 500 F with not to exceed 50% excess air. Where a limitation is also placed on the coarser material the allowable concentration of fly ash retained on a 325-mesh sieve is usually 0.20 grain per cubic foot.

A committee of The American Society of Mechanical Engineers has prepared a Model Ordinance which is being followed by many communities drawing up new laws or revising their present ones (Ref. 2). This model ordinance states that fly ash in the flue gas shall not exceed 0.85 lb per 1000 lb of gas adjusted to 50% excess air, except that it shall not be required that dust emitted to the atmosphere be less than 15% of the total dust entering the separating equipment. This limitation is equivalent to a concentration of 0.257 grain per cubic foot of gas at 500 F. It would be a difficult ordinance to meet in communities using high ash coal, were it not for the clause which waives this limitation as long as the dust collector is collecting at least 85% of the fly ash entering.

30. FLY-ASH COLLECTORS

MECHANICAL COLLECTORS. Baffle-type fly-ash collectors separate the fly ash from the flue gases by projecting the particles out of the gas stream when the gases make an abrupt change in direction. Usually the baffles are arranged in rows so that the gas stream is divided into a series of narrow ribbons. The draft loss is low, and, when necessary, collectors of this type can be designed to operate on natural draft installations or with existing induced draft fans when the available draft is limited. The collection efficiency is best on the larger, heavier particles. Collectors of the baffle type are also used to concentrate the ash into a small portion of the total gases, from which the ash is separated in a centrifugal-type separator.

Centrifugal separators, used for collection of all types of fly ash, usually consist of a number of high-velocity-type cyclones arranged in a common casing with one or more common hoppers. Draft loss is generally 2 to 4 in. of water at design load. Higher collection efficiencies are obtained with higher pressure loss, and maximum efficiency is obtained with the higher boiler loads. The space required for their installation varies considerably, depending on the size of the individual units. The location of the gas inlets and outlets can be adapted to most boiler and fan arrangements. Figure 5 illustrates a collector of this type in which the cyclonic action is produced by a series of vanes.

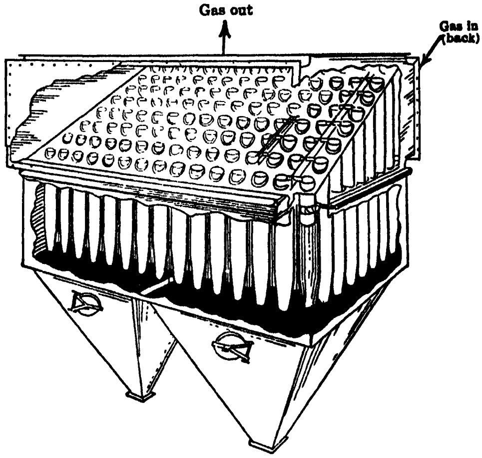

FIG. 5. Centrifugal fly-ash separator. (Courtesy of Western Precipitation Corp.)

Centrifugal concentrators concentrate fly ash into a small portion of the flue gas from which final separation is made in a secondary or auxiliary centrifugal separator. Fly-ash collectors of this type usually require the minimum amount of space. Several designs operating on this principle combine the fly-ash collector with the fan by replacing the fan inlet boxes with a scroll-shaped chamber in which the ash is thrown to the outer part of the scroll and drawn out with a small percentage of the gases through a scoop or shave-off located near the cut-off of the scroll. Figure 6 illustrates the principle of operation of one unit of this type. In some designs an auxiliary fan is used to draw off the secondary gases; in others the exhaust from the secondary separator is returned to center of the scroll opposite the fan inlet. The differential pressure between this point and the shave-off point produces the flow through the secondary circuit.

The principle of operation of another type of centrifugal concentrator is shown in Fig. 7. The centrifugal

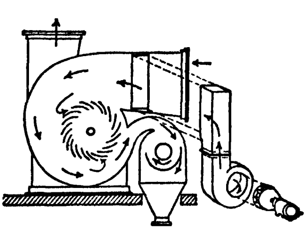
FIG. 6. Combination fly-ash collector and induced draft fan. (Courtesy Sturtevant Division, Westinghouse Electric Corp.)

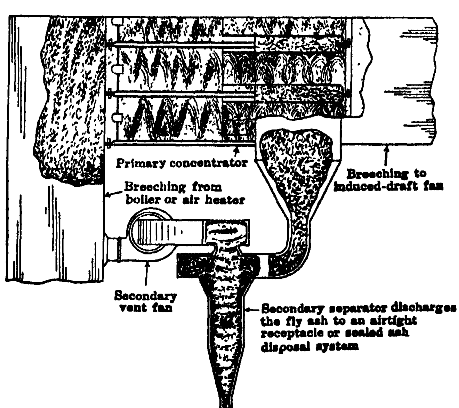

FIG. 7. Principle of operation of tubular-type centrifugal concentrator (American Blower Series 361 ST Fly-Ash Precipitator). (Courtesy American Blower Corp.)

concentrator consists of a number of tubes assembled in a common casing. The gases are started swirling by means of a spinner element located at the entrance of each tube. Fly ash is concentrated along the outer wall of the tube while clean gases pass through the center outlet tube. The outer portion of the gas stream into which the fly ash has been concentrated is drawn from the dust chamber through the high-velocity-type centrifugal separator by a secondary vent fan which discharges the cleaned gas back into the inlet of the unit. Since the secondary vent fan operates at constant speed and handles a nearly constant volume of gas, a higher percentage of the gases is handled at the lower loads, resulting in better efficiency characteristics at partial boiler loads. With fine pulverized fly ash, the design efficiency of this type of collector increases as the ratio of secondary gas volume to the total gases is increased. The tubes are assembled in a rectangular casing, and, within limits, the size of the unit for a given gas volume is governed by the available pressure drop, usually 2 to 3 1/2 in. water at design conditions.

ELECTRICAL PRECIPITATORS. Cottrell electrical precipitators used for collecting fly ash are usually of the horizontal-flow, plate type shown diagrammatically in Fig. 8. Flue gases are passed through an electrostatic field produced by a high-voltage d-c discharge between two sets of electrodes. The discharge electrodes, which must have a small radius of curvature, are usually wires supported from insulators and spaced between the vertical plates which form the collecting electrodes. The discharge electrodes are generally connected to the negative side of the high-voltage circuit and the collecting plates to the positive or grounded side. In passing through the corona discharge set up between the two electrodes, the fly-ash particles become ionized and are attracted to the collecting plates. Fly ash which adheres to the collecting electrodes is periodically removed by mechanically vibrating or rapping the plates.

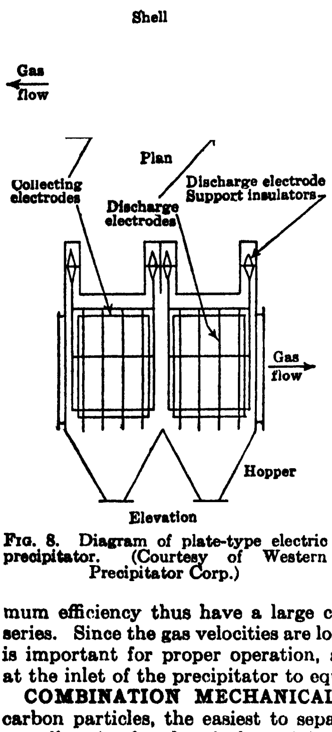

FIG. 8. Diagram of plate-type electric precipitator. (Courtesy of Western Precipitator Corp.)

The electrical equipment necessary to produce the high-voltage direct current consists of a transformer and either a mechanical or an electronic rectifier. The mechanical rectifier is usually driven by a synchronous motor, whereas the electronic type consists of a group of tubes arrranged to give full-wave rectification. The power consumed is about the same for either type. Although the voltage is high, usually 50,000 to 75,000 volts, the current is low so that the power consumption is low.

Electrical precipitators can be designed for high collection efficiency, especially with the fine fly ash from pulverized coal-fired boilers. The collection efficiency increases as the gas velocity decreases and is a function of the length of time the gas remains in the active field. Precipitators designed for maximum efficiency thus have a large cross-sectional area and consist of several sections in series. Since the gas velocities are low, the draft loss is low. Uniform velocity distribution is important for proper operation, and guide vanes or perforated plates are often used at the inlet of the precipitator to equalize the gas distribution.

COMBINATION MECHANICAL AND ELECTRICAL PRECIPITATORS. Coarse carbon particles, the easiest to separate by mechanical collectors, are the most difficult to collect in the electrical precipitator. Maximum efficiency of the mechanical type is obtained with higher boiler loads whereas the efficiency of the electrical precipitators is best at lower loads (gas velocities are lower). A combination of the two is therefore ideal for obtaining maximum collection efficiency. When a mechanical collector is placed in series with the electrical precipitator, the lower concentration of fly ash gives a slower build-up of dust on the collecting plates, resulting in better operation with less frequent rapping of the plates. There is a trend toward the use of such combination collectors on large pulverized coal-fired installation where continuous collection efficiencies higher than 90% are desired.

COST OF FLY-ASH COLLECTORS. The cost of fly-ash collection equipment is somewhat unstable. For estimating purposes, however, cost of the mechanical type of fly-ash precipitator varies from approximately 8 to 14 cents per cubic foot per minute of flue gas at design conditions, for the equipment alone. The installed price ranges between 20 and 25 cents per cubic foot per minute. The cost of electric precipitators on an installed basis varies from 30 to 60 cents per cubic foot per minute of flue gases, depending on design conditions and desired efficiency of precipitation.

DIMENSIONS OF FLY-ASH COLLECTORS. In any specific instance the manufacturer of the equipment should be consulted for dimensions. The following data are useful, however, for estimating. The base or plan area of the multiple-centrifugal separator type of unit is approximately 1 $^1/_4$ sq ft per 1000 cu ft per min of gas at design conditions. The height of the unit depends to a great extent on the storage capacity required in the hoppers. The centrifugal-concentrator type of unit illustrated in Fig. 7 is approximately 4 ft long in the direction of gas flow, and has a face area of approximately 1 sq ft per 1000 cu ft per min of gas flow.

SECTION 8

STEAM TURBINES AND ENGINES

By

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THE STEAM TURBINE

By A. G. Christie

THE STEAM ENGINE

By W. Trinks

THE STEAM TURBINE

By A. G. Christie

A steam turbine is a form of heat engine in which two distinct changes of energy take place. The available heat energy of the steam first is converted into kinetic energy by the expansion of the steam in a suitably shaped passage, or nozzle, from which it issues as a jet. A portion of this kinetic energy then is converted into mechanical energy by directing the jet, at a proper angle, against curved blades mounted on a revolving disk or cylinder and by the reaction of the jet itself as it leaves the curved passage.

The pressure on the blades, causing rotary motion, is due solely to the change of momentum of the steam jet during its passage through these blades.

Radiation and condensation losses in turbines are small. Leakage losses occur through clearances over the ends of reaction buckets and through labyrinths and glands. Friction of high-velocity steam jets through nozzle passages and across buckets, together with friction losses of high-speed revolving disks and idle buckets in steam-filled chambers, has considerable effect on the efficiency of the turbine.

1. TYPES OF TURBINE

THE SIMPLE IMPULSE TURBINE consists essentially of one or more nozzles supplied with high-pressure steam, with the discharge jet impinging, at a suitable angle, on a single row of buckets on a revolving disk. The steam expands in the nozzle to exhaust pressure, its velocity increasing during expansion. The resulting kinetic energy is partly converted into mechanical energy during the passage of steam across the buckets. The commonest type of simple impulse turbine is shown diagrammatically in Fig. 1.

For best efficiency of the simple impulse turbine the ratio ρ of wheel speed to steam speed of the jet issuing from the nozzle ought to be about 0.45 where the blading is short in radial length, and about 0.53 or higher for large radial lengths of blade, where pure impulse is seldom used. Available energy with these ratios cannot exceed 70 Btu and 55 Btu, respectively, with usual bucket speeds. Frequently the available energy exceeds these amounts, leading to a decrease in speed ratio ρ, with resultant lower efficiency. Simple single-stage impulse turbines usually are built for small output, although they have been used in units up to 3000 hp.

VELOCITY-COMPOUNDED TURBINES utilize a very high-velocity jet from a nozzle more efficiently than the simple impulse turbine; for moderate velocities the efficiency of the simple-impulse stage is higher. The steam, after passing through the first row of *moving* blades, flows through a set of *stationary* curved buckets or passages. These reverse the direction of the jet and redirect it against a second row of *moving* blades. This reversal and reimpingement may occur several times before the steam finally escapes to the outlet. When velocity compounding consists of two rows of revolving blades mounted on the same or parallel wheels, with intermediate stationary reversing blades, the form results in the well-known Curtis stage (Fig. 2), frequently called the two-row wheel, or "impulse" stage, even though the latter is not descriptive. This also is called a *velocity stage*. In another form, the reversing passages redirect the steam back against the same row of blades that the jet first crossed. This is known as the *re-entry* type of turbine (Fig. 3).

Helical flow Terry turbines (Fig. 4) employ a forged steel wheel in which semicircular buckets are milled in the rim at an angle of about 30 degrees with the tangent. The steam expands to exhaust pressure in the nozzle, which directs the steam into one side of the semicircular bucket. The steam gives up part of its energy in its first reversal through 180 degrees in the moving bucket but still retains considerable velocity. It then passes to a *reversing chamber*, which redirects the steam into the wheel buckets. This is repeated several times in additional reversing chambers until the kinetic energy of the steam is reduced to a low value. The whole operation occurs in a single wheel, frequently provided with several groups of nozzles and reversing chambers.

In all these velocity-compounded turbines, part of the kinetic energy is absorbed each time the steam passes through a revolving blade or bucket passage. More work per pound of steam is thus obtained than in a simple impulse turbine of the same bucket speed. Friction losses, however, increase with addition of the reversing blades or chambers. Although the various forms of velocity compounding have their widest application in small steam turbines, particularly for noncondensing auxiliary services, such as driving pumps, blowers, exciters, and stokers, the two-row wheel stage frequently is used as the first stage of very large compound or multistage units.

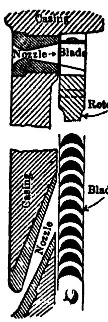


FIG. 1. Impulse.

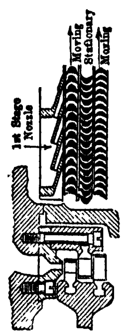


FIG. 2. Curtis or 2-row wheel.

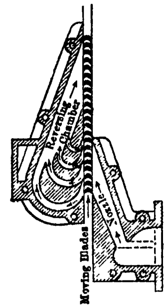


FIG. 3. Re-entry.

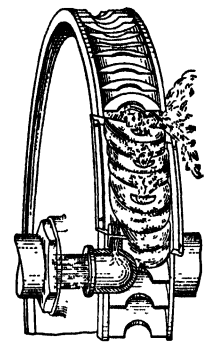


FIG. 4. Helical flow.

Types of Impulse Steam Turbine.

Stage, in an impulse type of turbine, is a term which signifies that part of a machine in which a decrease of pressure occurs with the accompanying generation of kinetic energy (nozzle), together with succeeding passages where no further drop of pressure occurs (bucket). A stage, therefore, includes (1) nozzles, (2) moving buckets, and (3) reversing elements, or chambers, when used.

THE MULTISTAGE IMPULSE TURBINE consists of a series of simple impulse turbines on the same shaft; each of these forms a stage. It is so designed that the steam expands through only a portion of the total pressure range in the nozzles of the first stage. On leaving the buckets of the first wheel, steam enters the second-stage nozzles (which are carried in a diaphragm forming the wall of the stage) and expands through a further pressure drop. The jet impinges on a second row of revolving blades. The operation is repeated in every stage until the steam is fully expanded in the final stage to exhaust pressure.

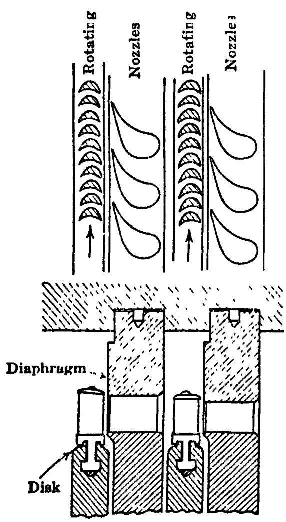


FIG. 5. Cross section and steam path of two typical stages of a multistage impulse turbine. Steam flows from right to left. Buckets shown have inside dovetail; outside dovetail, frequently used on large machines, as illustrated by last stage of turbine in Fig. 8, p. 8-05.

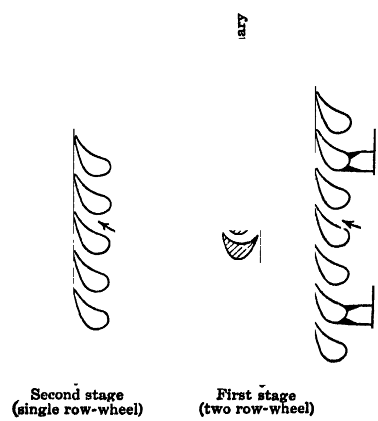


FIG. 6. Steam path of first two stages of a compound impulse turbine.

With this construction (Fig. 5) it is possible to maintain the most efficient ratio of wheel speed to steam speed by properly apportioning the total available energy, from initial conditions to final pressure, between a suitable number of stages. This is accomplished by proper choice of the nozzle areas, which determine the stage pressures. High efficiencies are possible with this type of turbine.

The total available energy from initial conditions to final pressure is divided in a particular manner, fixed by the manufacturer's construction, between the various stages. Steam speeds and wheel speeds are selected to give values of ρ (the ratio of wheel to steam speed) as near to the peak-efficiency value as commercially possible. Low-cost units with few stages have low values of ρ and consequent low efficiency. High-efficiency machines have high values of ρ (near 0.5) and many stages. Multistage impulse turbines can be built for the largest commercial ratings.

Another early form of turbine, chiefly of historical interest, comprised a *series* of velocity-compounded or two-row wheel stages. This construction was used on many early central-station turbines from the turn of the century throughout nearly two decades, but has been superseded by more efficient types, particularly the single-row stage.

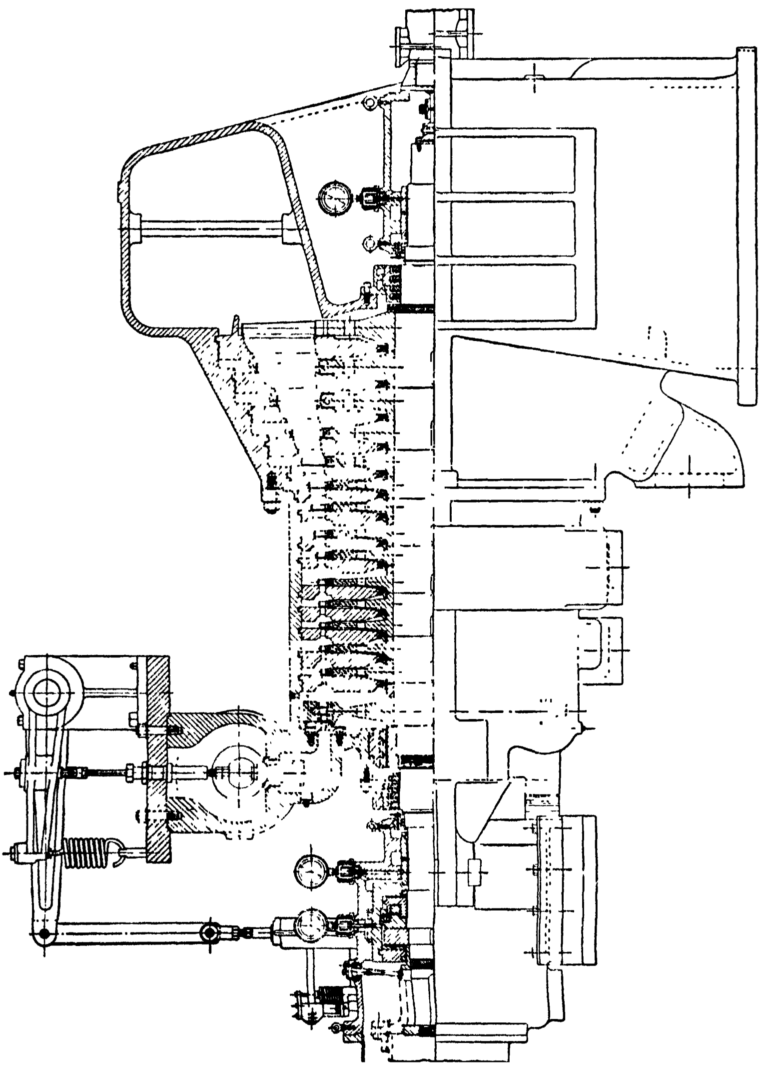


FIG. 7. Multistage turbine. (Courtesy DeLaval Steam Turbine Co.)

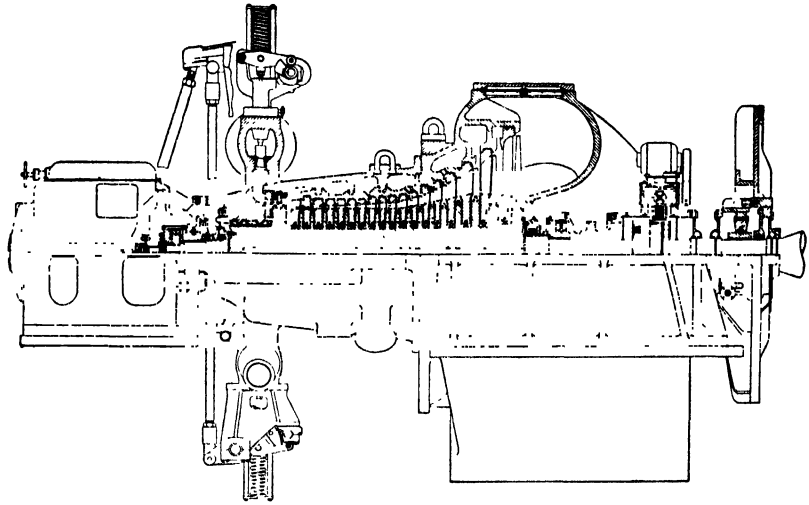


FIG. 8. Multistage turbine. (Courtesy General Electric Co.)

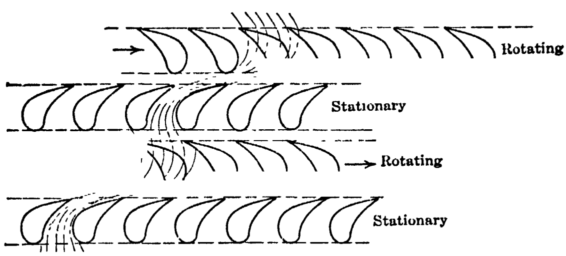


FIG. 9. Steam path of an impulse-and-reaction turbine (frequently called simply a reaction turbine) illustrating similarity of moving and stationary rows.

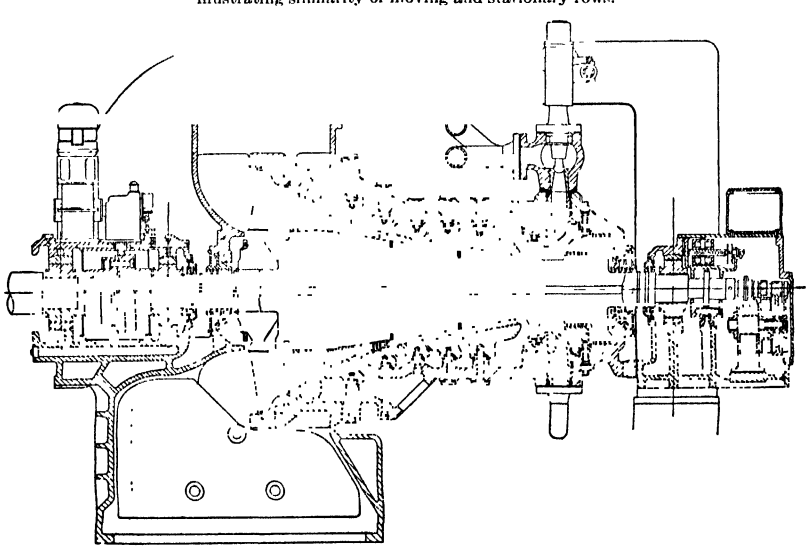


FIG. 10. Reaction turbine. (Courtesy of Allis-Chalmers Manufacturing Co.)

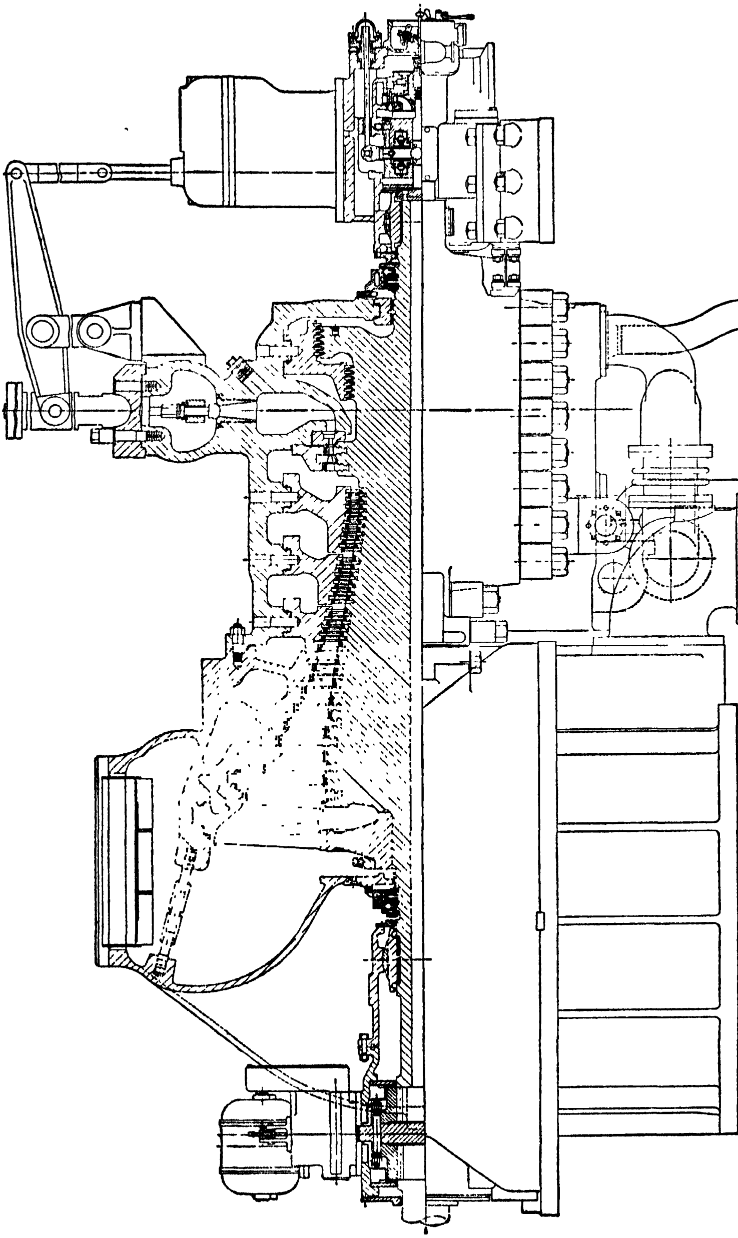


Fig. 11. Reaction turbine, 30,000 kw-1800 rpm, single-cylinder condensing turbine; 850 psig-900 F-1.5 in. Hg; AIEE-ASME Preferred Standard. (Courtesy of Westinghouse)

The theoretically high efficiency of a small multistage impulse turbine with few stages is partially offset by the large windage losses of the disks and buckets of the higher-pressure stages, which revolve in a dense atmosphere of high-pressure steam, and which usually have nozzles delivering steam over only a portion of the periphery (partial-arc admission).

A velocity-compounded or two-row stage often is substituted for several of the early simple stages in multistage impulse turbines. The resultant compound impulse turbine, two stages of which are shown in Fig. 6, is shorter, more compact, costs less, and is nearly as efficient as the pure multistage impulse turbine of the same capacity. Figure 7 shows such a unit built by the DeLaval Steam Turbine Company, and Fig. 8 a unit built by General Electric Company. Although units of this type are truly "compound," because they have two types of turbine, the term is not accepted in practice. Rather it is reserved to mean groups of stages contained in separate casings, but using the same steam, e.g., a *tandem-compound unit.*

The pressure drop in early impulse turbines occurred wholly in the nozzles between stages. Radial clearances over the bucket tips were large, as no pressure difference existed across the buckets. Increased efficiency sometimes may be achieved by assigning a portion of the stage energy to the moving buckets, if clearances over the blade ends are small to reduce leakage due to pressure difference. Suitable labyrinth packings must be provided in all stages where the shaft passes through the diaphragms between stages.

No distinct line exists between impulse and reaction turbines, as the majority of so-called impulse turbines have more or less reaction. The term *impulse* applies to stages with no reaction, or with appreciably less than 50% reaction. In *reaction* turbines as much as 50% of the total heat drop per stage is expended, with a corresponding pressure drop, in passing through the moving blade, thereby increasing the relative outlet velocity.

THE IMPULSE-AND-REACTION TURBINE, typified by the steam path shown in Fig. 9, consists of a barrel- or conical-shaped drum placed inside a cylinder, with rows of blades attached alternately to the stationary cylinder and to the revolving spindle. The passages between blades of *all* rows form contracting orifices, hence there is a drop in pressure of the steam through every row of blades. A row of stationary blades and its following row of revolving blades are together known as a *stage.* Figure 10 typifies this turbine as built by Allis-Chalmers Manufacturing Company.

In this section reaction blading indicates approximately 50% reaction, i.e., half of the available energy for the stage is released in the stationary blade and half in the moving blade. For best efficiency in a reaction turbine, blade speed should be about 90% of steam speed at the nozzle exit. Theoretically twice as many stages are required as in the impulse turbine. To obtain this efficiency, low steam speeds and many moving rows of blades, and consequently a long spindle, must be used. In general, lower ratios of blade to steam speed prove more desirable in practice, giving fewer rows of blades and shorter turbines. Clearances in reaction turbines must be kept small to prevent excessive leakage. With standard reaction blading this necessitates small radial clearances over the ends of both stationary and moving blades.

Many reaction turbines have blading with shrouds for strength and for reduction of tip leakage. Such blades usually have radial clearance seals. Some reaction turbines are fitted with shrouds to provide an axial (rather than radial) clearance which can be accurately controlled by axial adjustment of the thrust bearing (end tightening).

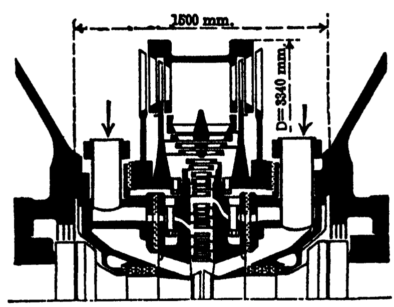


FIG. 12. Section of blading of 50,000-kw Ljungstrom radial-flow turbine.

On small reaction turbines using high steam pressure, full peripheral admission with the small steam volumes handled requires short blades, which are impractical and inefficient, so that a two-row impulse stage often replaces several reaction stages at the high-pressure end, as in the impulse type. The resulting machine (Fig. 11), sometimes known simply as the reaction turbine, is shorter and cheaper to build than the standard all-single-row unit. If a large pressure drop is allowed in the first-stage nozzles, casing temperatures are decreased and distortion troubles are lessened. This combination also permits reasonably long blades to be used on the first reaction row, which must have full peripheral admission.

A series of inlet valves on the first stage, with separate arcs for steam admission, provides better efficiency at partial loads than "throttling governing," which today is obsolete in

(*Continued on p. 8-10*)

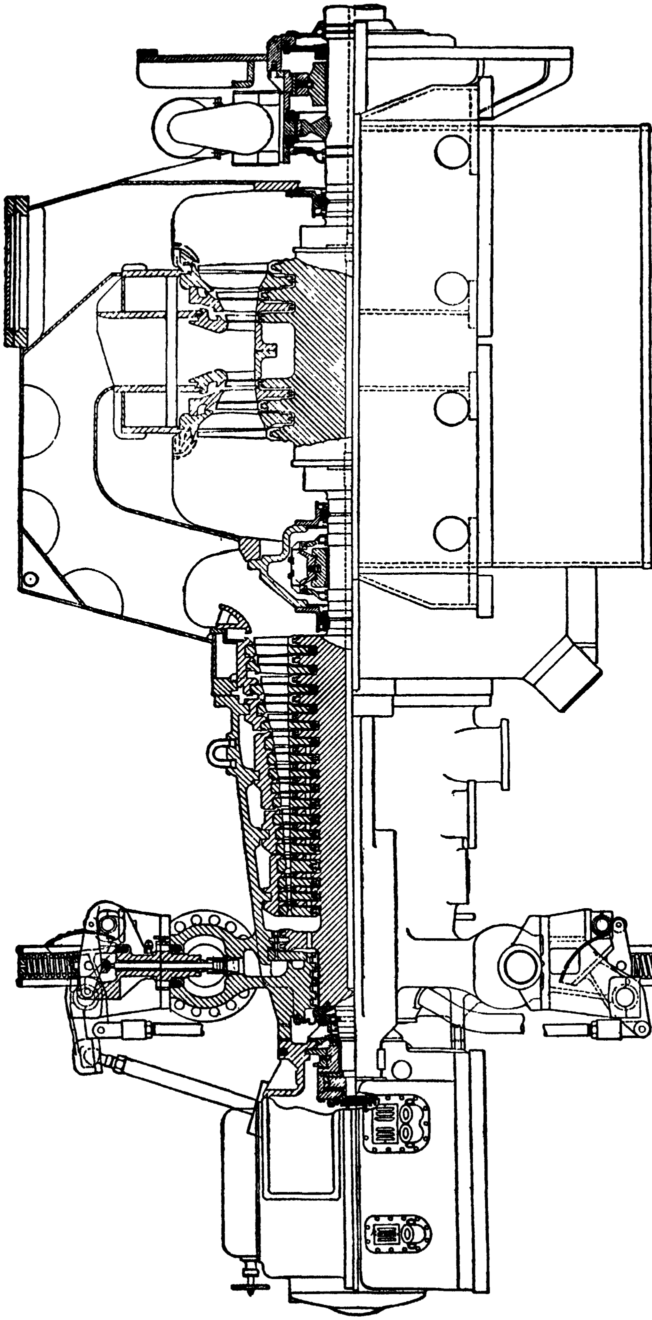


FIG. 13. 40,000-kw Preferred Standard tandem-compound double-flow impulse steam turbine. (Courtesy of General Electric Co.)

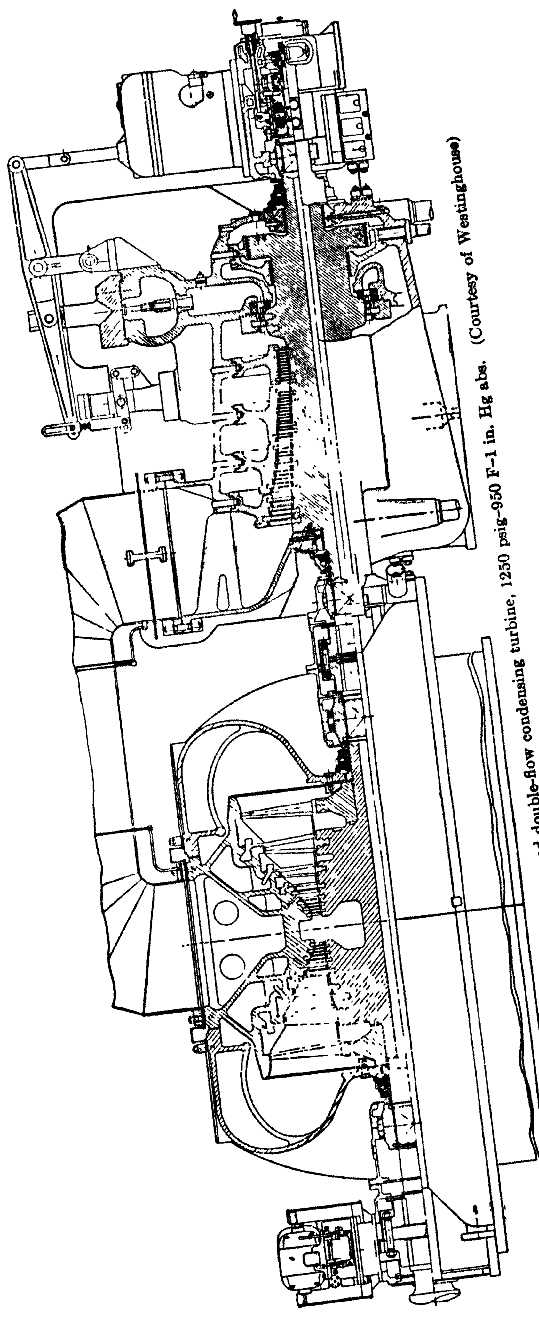


Fig. 14. 80,000-kw tandem-compound double-flow condensing turbine, 1250 psig–950 F–1 in. Hg abs. (Courtesy of Westinghouse)

large-turbine practice. Smaller and cheaper units use it because of its simplicity and low cost.

The Ljungstrom double-rotation turbine (Fig. 12), a radial-flow unit of the reaction type, consists of intermeshed sets of blading, each rotating in the opposite direction. Two generators, tied together electrically, are required. The relative velocity of the two sets of blades is twice that obtainable with a fixed casing and a single revolving spindle. This construction leads to high capacity and high efficiency for a given diameter of blade ring. The unit is compact and usually is placed above, and supported by, its surface condenser. Its construction permits the use of high-temperature steam and quick starting. The simple radial-flow design is applicable to back pressure, noncondensing, and the smaller sizes of condensing units. Large condensing units have double-flow axial blading in the exhaust end, as in Fig. 12.

APPLICATIONS OF STEAM TURBINES. The principal application of large turbines is to drive a-c generators through a solid or flexible couple. Turbines of various sizes, also direct connected, drive centrifugal pumps, small a-c generators, fans, blowers, etc.

For a given blade speed, turbines with small diameters, operating at high rpm, are most efficient. Such units may be connected through double helical reduction gearing to moderate-speed machinery such as fans, propeller shafts, compressors, stokers, pumps, and d-c and a-c generators of low ratings. Both driving and driven units then may operate under best conditions, and first cost of equipment is lower. Some geared turbines have operated rope and belt drives for factory machinery, as cotton mills, rolling mills, etc. Geared turbines also have been used to drive locomotives (see Section 14).

Single-cylinder condensing turbines are built in sizes up to 30,000 kw at 3600 rpm and 100,000 kw at 1800 rpm. Single-cylinder noncondensing (topping) units have been built for 65,000-kw output at 3600 rpm. Figure 13 shows a 40,000-kw Preferred Standard unit built by General Electric Company. Figure 14 shows a 80,000-kw unit built by Westinghouse Electric Corporation. Unidirectional, double-flow turbines in one cylinder provide large areas for the last blade rows. Larger units of either the tandem or cross-compound types often have double-flow low-pressure cylinders to obtain the desired low-pressure blade area. Compound units also are used with reduction gearing in marine and other service.

Impulse versus Reaction Turbines. The impulse type is best suited for use in the high-pressure region and for small steam quantities. The reaction type has advantages for the lower-pressure region, where large volumes of steam must be handled. Practice is tending toward the use of disks for low-pressure reaction blading at high blade speeds, rather than the "drum" construction. In commercial practice, there is no consistent difference in efficiency between the two types, as evidenced by test results and continuing sale of both types in large numbers to utilities. (See Ref. 1.)

Self-contained sets have a turbine mounted directly on or beside its condenser, and are generally connected to the generator through gearing. Exciter and condenser auxiliaries may be driven directly from the turbine through gearing, by connection to the main shaft, or by motors directly connected to the generator. Erection costs are low. Such sets are sometimes known as "packaged units."

STEAM CONDITIONS. Turbines can be built to operate at any steam pressure from a few inches of vacuum up to the highest steam pressures available. Central station pressures and temperatures generally are in the range shown in Table 1, although pressures up to 2400 psig are in use. Industrial turbines operate at all pressures from 100 to 1500 psig. The trend in initial temperature is distinctly upward, with 1050 F adopted for certain units. Temperature limits are fixed by available materials and required life. Increasing coal and labor costs will cause the adoption of even higher pressures and temperatures as suitable metals become available. (See Gas Turbines, Section 10.) Reheat cycles also become more attractive on certain systems with these conditions. Vacuum of 29 in. can be maintained with 57 F cooling water. With 70 F cooling water a vacuum of 28.5 in. can be obtained, using commercial condensers.

OPERATING CHARACTERISTICS. In the *straight condensing turbine* all the steam that enters the throttle, except some gland leakage in certain types, passes completely through the turbine to the condenser, in which a vacuum is maintained. Figure 15 shows a straight condensing turbine built by the Elliott Co.

Extraction or Regenerative Turbines. Steam is withdrawn from the turbine at intermediate stages and used to heat the feedwater in open or closed feedwater heaters. Because the extraction turbines give higher station economy, they are universally used in central station practice. (See Ref. 64.) Since steam is extracted at various points, the quantity of exhaust steam decreases, the size of condenser is smaller than otherwise, and an increase in the rating of a given casing is possible.

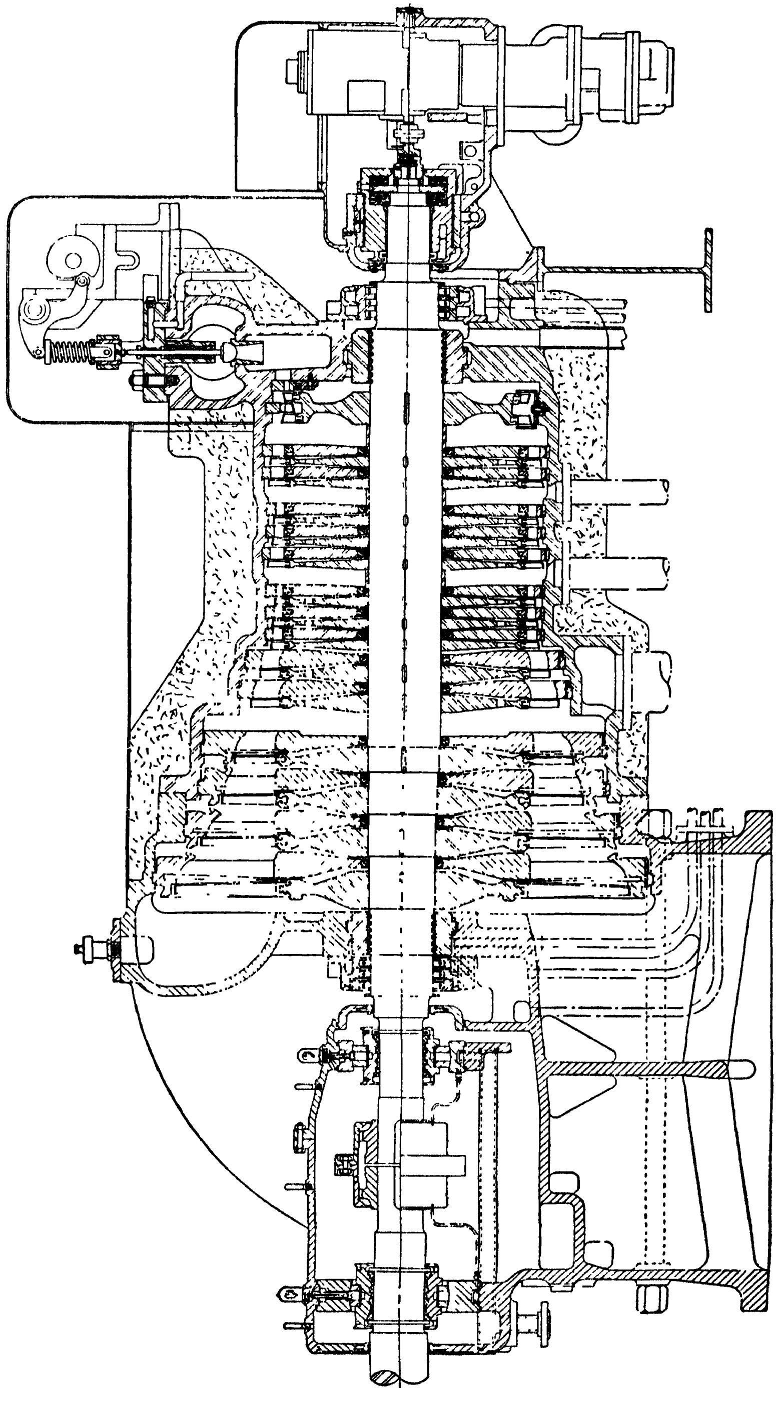


FIG. 15. Straight condensing turbine. (Courtesy of Elliott Co.)

High initial pressures, with moderate initial steam temperature, results in excessive moisture in the low-pressure stages. To avoid this, and to improve station economy, steam can be withdrawn from the turbine at an intermediate stage, reheated by flue gases, live steam, or other hot fluid, and then returned to the turbine. This cycle requires a *reheat turbine* as shown in Fig. 16. (See also Article 17.) Reheat turbines invariably have extraction heaters to heat the feedwater. This type is known as a *regenerative-reheat turbine.*

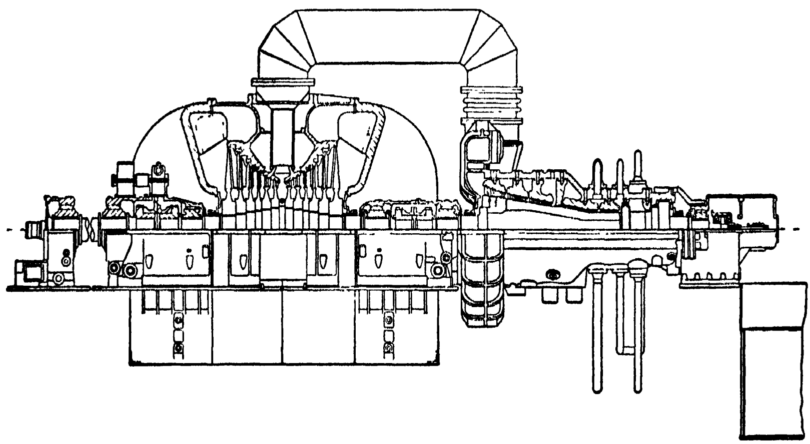


FIG. 16. 80,000 kw–1800 rpm reheat turbine. (Courtesy Allis-Chalmers Manufacturing Co.)

TOPPING TURBINE-GENERATORS have been installed with new high-pressure, high-temperature steam generators in old stations. These turbines exhaust at relatively high back pressure directly into the low-pressure steam main, furnishing steam to older lower-pressure units. This combination, which appreciably improves the station heat rate, is widely used.

Noncondensing turbines exhaust at atmospheric pressure or above. They form the high-pressure units of reheat turbines. They also exhaust to heating systems, to industrial processes, or simply to atmosphere in a few cases. While many small noncondensing turbines for auxiliary use have comparatively low efficiencies, those used for power generation can be designed for higher engine efficiencies than condensing units of the same size.

In the *bleeder* or *extraction turbine*, steam is extracted at one or more intermediate stages, often at comparatively high pressures, for industrial use. Frequently the pressures at these bleeder stages must be maintained constant by a special regulating device forming a part of the turbine; it is then called an *automatic extraction turbine*. The steam not withdrawn at the bleeder points expands through the remainder of the turbine to the exhaust. This type of turbine may operate at a given load with nearly all the steam that enters at the throttle flowing out of the extraction openings or with all throttle steam passing to the condenser when no steam is bled, or with any condition intermediate between these extremes. Such turbines are widely used in industrial plants. Figure 17 shows an extraction turbine built by the Worthington Pump and Machinery Corporation.

TURBINE SPEEDS. For 25 cycles, 1500 rpm; for 60 cycles, 3600, 1800, and 1200 rpm. European turbines for 50 cycles, 3000 and 1500 rpm. High-speed turbines are preferred for their lower weight and smaller floor space, although the efficiency is substantially equal to that of slow-speed units of equal ratings. Turbines direct-connected to pumps and blowers usually operate at the speed of the driven unit. Turbines for geared sets may run at any desired speed, and have been built for speeds of 6000 to 7200 rpm in the smaller sizes. This leads to low first cost and increased efficiency. Some marine turbines are built in speeds of 10,000 rpm and higher.

NEMA STANDARDS FOR TURBINE-GENERATOR SETS. See Section 16.

ASME-AIEE JOINT COMMITTEE PREFERRED STANDARDS are given in Table 1 for 3600 rpm central-station turbines.

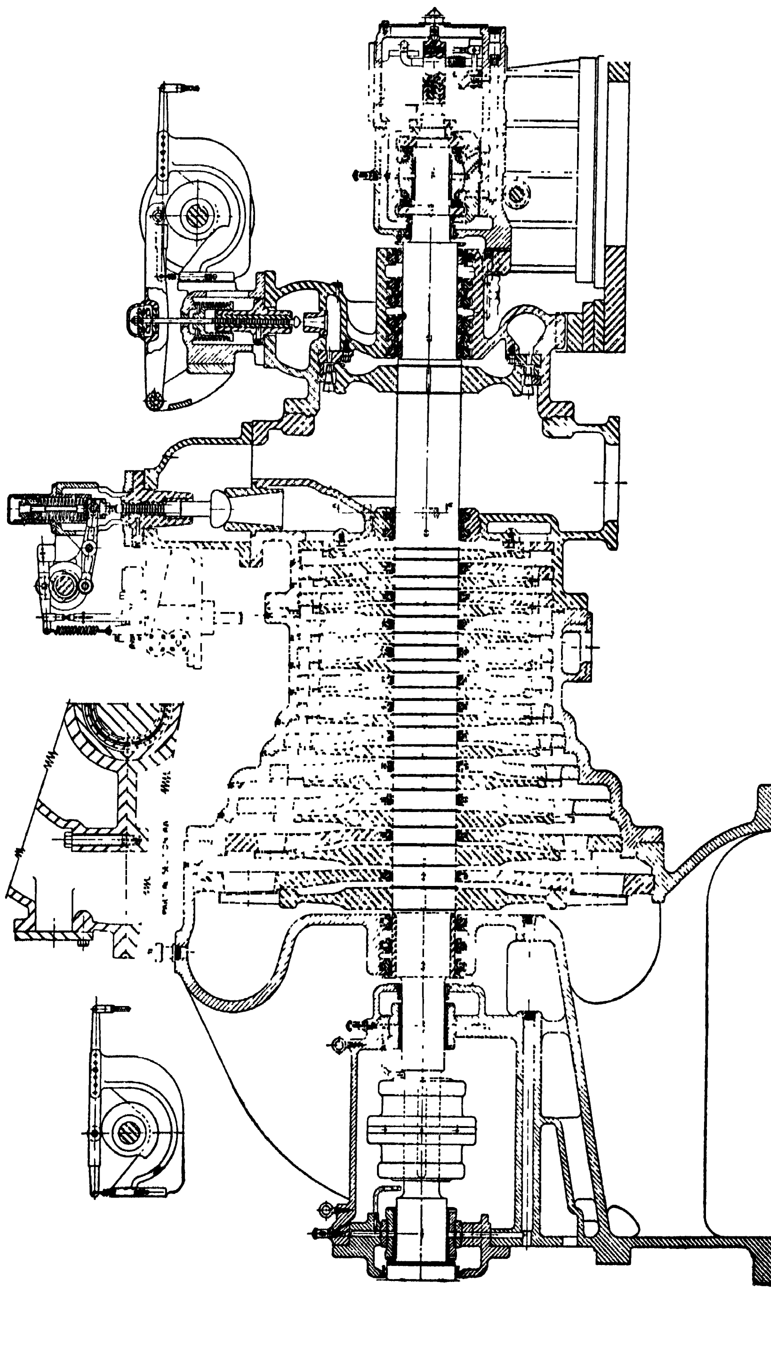


FIG. 17. Section through Worthington controlled-extraction condensing steam turbine.

Table 1. AIEE-ASME Preferred Standard Large 3600-Rpm, 3-Phase 60-Cycle Condensing Steam-turbine Generators

	Air-cooled Generator	Hydrogen-cooled Generator Rated at 0.5 psig Hydrogen Pressure						
Turbine generator rating, kw	11,500 *	15,000	20,000	30,000	40,000		60,000	
Turbine capability, kw †	12,650	16,500	22,000	33,000	44,000		66,000	
Generator rating, kva	13,529	17,647	23,529	35,294	47,058		70,588	
power factor	0.85	0.85	0.85	0.85	0.85		0.85	
short circuit ratio	0.8	0.8	0.8	0.8	0.8		0.8	
Throttle pressure, psig	600	850	850	850	850 or	1250	850 or	1250
Throttle temperature, °F	825	900	900	900	900	950	900	950
Number of extraction openings	4	4	4	5	5		5	
Saturation temperatures, °F, at openings at "turbine generator rating" with all extraction openings in service	(1st) 175	175	175	175	175		175	
	(2nd) 235	235	235	235	235		235	
	(3rd) 285	285	285	285	285		285	
	(4th) 350	350	350	350	350		350	
	(5th).....	....	....	410	410		410	
Exhaust pressure, in. Hg, abs	1.5	1.5	1.5	1.5	1.5		1.5	
Steam rates, lb per kwhr								
Per cent load / Power factor					850 psig 900 F	1250 psig 950 F	850 psig 900 F	1250 psig 950 F
25 / 0.85	10.20	8.94	8.80	8.70	8.60	8.25	8.45	8.15
50 / 0.85	9.00	8.08	8.00	7.93	7.83	7.48	7.75	7.43
75 / 0.85	8.60	7.82	7.78	7.71	7.59	7.25	7.56	7.21
100 / 0.85	8.46	7.78	7.73	7.68	7.57	7.22	7.55	7.19
110 / 0 935	8.46	7.81	7.76	7.73	7.60	7.24	7.60	7.23
Steam flow capacity of turbine, lb per hr	123,000	147,000	193,000	302,000	397,000	375,000	595,000	561,000
		With Hydrogen-cooled Generator Operated at 15 psig Hydrogen Pressure						
Generator capability at 0.85 pf, kva	......	20,294	27,058	40,588	54,117		81,176	

Notes:

1. A tolerance of plus or minus 10 F shall apply to above saturation temperatures. (Tolerances shall be unilateral so as not to reduce the spread in temperature between adjacent extraction openings.)

2. The *turbine capability* is guaranteed continuous output at generator terminals when the turbine is clean and operating under specified throttle steam pressure and temperature and 2.5 in. Hg abs, exhaust pressure, with full extraction from all extraction openings.

* For data on efficiency of turbines of lower rating, see Table 8, p. 8-61.

† For additional data on capability, see p. 8-64.

2. STEAM-TURBINE CYCLES

If steam could be expanded in a turbine with no friction or other losses, expansion would be isentropic. Theoretically, steam turbines operate on the Rankine cycle or its modifications, such as the regenerative, reheating, or regenerative-reheating cycles. Turbine problems involving energy transformations in the steam are based on isentropic adiabatic expansion with the necessary modifying factors. These problems can be readily solved on a Mollier diagram. (See Art. 2, Section 4.)

In the *Rankine cycle*, the throttle steam is expanded isentropically from the initial steam condition to the exhaust pressure. The available energy measures the heat thus theoretically available for work (see p. 4-04). The *engine efficiency* expresses the ratio by which the actual turbine approaches the Rankine cycle in converting into work the energy available with isentropic expansion. The numerator may be the heat equivalent of either internal, coupling, or generator output in kilowatts. The denominator is the product of available energy in Btu per pound and total pounds of steam per hour. It is necessary, therefore, to state whether the calculated ratio is engine efficiency based on internal kilowatts, engine efficiency based on coupling kilowatts, or engine efficiency based on generator output. 1 kwhr = 3413 Btu. Let I kw = internal kilowatts; h_1 = total heat per pound of steam at initial conditions before the throttle; h_2 = total heat per pound of steam after isentropic expansion to exhaust pressure; W = total pounds of steam per hour. Engine efficiency, based on internal kilowatts, = $(3413 \times I \text{ kw})/W(h_1 - h_2)$. Similar expressions can be written using coupling kilowatts or generator output. Where turbines are sold to drive pumps, fans, or other equipment, the rating frequently is expressed as brake horsepower at the coupling. The numerator in the above equation then becomes 2544 × bhp. Straight-condensing high-pressure and low-pressure turbines, with no extraction or reheating, follow the Rankine cycle, ideally.

The regenerative cycle, the reheating cycle, and the regenerative-reheating cycle (see Section 4) are used extensively for steam turbines. See ASME Power Test Code for Steam Turbines for efficiency calculation for the different cycles. (See Art. 5, Section 19.)

3. NOZZLES

Nozzle efficiency is of major importance in efficient turbine design. In general 1% gain in nozzle efficiency has about four times as much effect on stage performance as 1% gain in bucket efficiency. Nozzle efficiency should be tested experimentally before extensive application of a given design.

CRITICAL PRESSURE. The simplest form of nozzle consists of a circular hole with a well-rounded mouth. If installed in a chamber containing high-pressure steam at absolute pressure p_1, the flow of steam increases as the pressure p_2 on the discharge side of the nozzle becomes less than p_1 until the *critical pressure* is reached. The flow will not increase with further decrease in the pressure on the discharge side of the nozzle. The *critical pressure ratio* of throat pressure to initial pressure p_1 varies slightly with superheat. It can be used as 0.55 throughout the superheat range with no appreciable error. Its value can be found from the equation

$$\text{Critical pressure, } p_c = p_1 \left(\frac{2}{k+1}\right)^k$$

where p_1 = initial pressure, psia, and k = exponent for isentropic expansion at constant entropy (pv^k = constant) of the steam at the stated conditions. The value of k varies but for superheated steam, an average value is $k = 1.3$; for wet steam, k is variable, usually about 1.13, and critical pressure ratio is about 0.58. The velocity at critical pressure is that of sound in the gas or vapor at the pressure and density existing at the throat. (See Sections 1 and 3.)

The steam jet leaves the nozzle in practically straight lines as long as p_2 equals or is greater than the critical pressure. Hence, for these expansions, nothing more is needed in a turbine than a convergent passageway, with the discharge directed at the desired angle toward the buckets. The orifice must be convergent, because the rate of increase of velocity exceeds the rate of increase of volume, until the critical pressure is reached. The most efficient convergent nozzle in turbines is short, curved, and has a thin trailing edge.

If p_2 is less than the critical pressure, the pressure in the throat, or narrowest part of the orifice, remains at the critical pressure, and further expansion of the steam occurs after leaving the orifice. This causes the jet to expand in all directions. Walls are necessary to confine this further expansion with its accompanying increase in velocity and volume. To do useful work, the jet must be projected toward the buckets in a fixed direction, usually at an angle of 12 to 20 degrees to the plane of the wheel. Below the critical pressure the volume increases at a greater rate than the velocity. A diverging section, therefore, is added to the throat, forming a convergent-divergent nozzle, with mouth dimensions suitable for the range of expansion. The total angle of the divergent walls varies from 6 to 12 degrees. If the nozzles are of rectangular cross section, the sides continue to diverge for their full length. This also is done in some circular nozzles, and the result is a *reamed nozzle.*

Shapiro (Ref. 2) has shown that shock fronts occur in the diverging section of nozzles with straight diverging sides, because of the convergence of neighboring Mach lines, and appreciable losses result. A design of straight nozzle with curved diverging sides is proposed which obviates shock fronts. Analytical methods (e.g., Prandtl-Buseman) are available for design of such nozzles.

THEORETICAL NOZZLE VELOCITY. The available energy ($h_1 - h_2$) between any initial conditions and a final pressure is represented by A–B, in the Mollier diagram of Fig. 18.

As the steam expands in a nozzle, a portion of this available heat is transformed into kinetic energy and increases the velocity of the jet. The theoretical velocity V, feet per second, resulting from complete transformation of this available energy is $V = 223.9\sqrt{(h_1 - h_2)}$. This velocity is not obtained in an actual nozzle because of friction, eddy, and other losses.

Let ϕ = velocity coefficient for the nozzle, the fraction of the theoretical velocity actually generated at the nozzle exit.

The actual velocity V_1 of the steam jet leaving the nozzle is

$$V_1 = \phi V = 223.9\phi\sqrt{h_1 - h_2}$$

Nozzle efficiency, η_n, is the fraction of the available energy converted to useful kinetic energy. Then V_1 can be expressed as

$$V_1 = 223.9\sqrt{\eta_n(h_1 - h_2)}$$

The relation between ϕ and η_n is

$$\phi = \frac{V_1}{V} = \sqrt{\eta_n} \quad \text{or} \quad \eta_n = \phi^2$$

Nozzle losses appear as reheat. The total heat at the nozzle mouth is

$$h_3 = h_2 + (1 - \phi^2)(h_1 - h_2)$$

This is represented by point C on Fig. 18. The specific volume of the steam can be determined from conditions at point C.

In multistage turbines steam may enter the nozzles of all but the first stage, with an appreciable velocity V_c, the carryover from the preceding stage, and with a kinetic energy $(V_c/223.9)^2$. The velocity of steam leaving the nozzle becomes

$$V_0' = 223.9\phi\sqrt{(h_1 - h_2) + (V_c/223.9)^2}$$

The total heat at the nozzle exit is

$$h_3' = h_2 + (1 - \phi^2)[(h_1 - h_2) + (V_c/223.9)^2]$$

Carryover is the kinetic energy represented by the absolute velocity leaving the preceding stage. Improvements in design of nozzle entrances permit recovery of about 85 to 95% of the carryover energy at most efficient load. Even higher recoveries are possible with further improvement in designs.

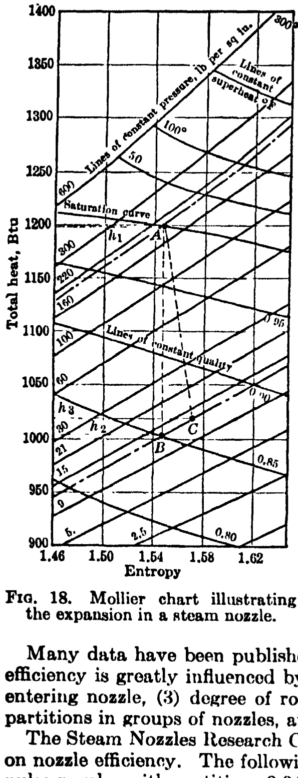


FIG. 18. Mollier chart illustrating the expansion in a steam nozzle.

Many data have been published on nozzle experiments. (See Refs. 3 and 4.) Nozzle efficiency is greatly influenced by (1) form of approach to nozzle, (2) condition of steam entering nozzle, (3) degree of roughness of nozzle, (4) length of nozzle, (5) thickness of partitions in groups of nozzles, and (6) condition of outlet edges of partitions.

The Steam Nozzles Research Committee of the IME has published many valuable data on nozzle efficiency. The following data show the performance of a set of converging impulse nozzles with partitions 0.04 in. thick, with 20-degree angle, and with a parallel portion beyond the throat equal in length to twice the width at the throat.

Theoretical velocity, V ft per sec	400	600	800	1000	1200	1400	1600	1800
Corresponding Btu per lb	3.2	7.2	12.8	20.1	28.8	39.2	51.2	64.8
Velocity coefficient, ϕ	0.976	0.955	0.947	0.946	0.944	0.944	0.945	0.946
Nozzle efficiency, η_n	0.953	0.915	0.897	0.895	0.892	0.892	0.894	0.895

The Nozzle Research Committee also noted that better coefficients were obtained when the nozzle partitions were well chamfered at the outlet edge. The angle of discharge is decreased by this chamfer and may be less than the nominal angle. These coefficients include any influences of supersaturation.

Tests indicate that nozzle performance is improved by omitting any parallel portion beyond the throat of converging nozzles.

The formula for theoretical nozzle discharge is

$$W = 3600\frac{A}{144}\sqrt{2g \times \frac{k}{k-1}\,\frac{144\,p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{(k+1)/k}\right]}$$

where W = discharge, pounds per hour; g = 32.2 ft per sec²; p_1 and p_2 = pressure before and after the nozzle, psi; v_1 = specific volume at p_1, cubic feet per pound; k = ratio of specific heats; and A = area of orifice in square inches. For pressure ratios less than critical (see p. 8-15) use the critical pressure ratio.

Horstman has a simplified but quite accurate approximation of this formula for superheated steam, particularly applicable to small pressure drops:

$$W = 2580A\sqrt{p_1/v_1 - [0.13 + (1.13p_2/p_1) - (p_2/p_1)^2]}$$

Wirt (Ref. 5) gives the results of tests of convergent nozzles by means of impact tubes using air. Warren and Keenan (Ref. 6), Keenan (Ref. 7), and Kraft (Ref. 8) give further data on tests of steam nozzles. The coefficients disclosed in Refs. 5 and 6 are:

Mach. Number (Ratio of Theoretical Velocity to Sound Velocity)	ϕ, Reaction Test Steam (Warren and Keenan)	ϕ, Impact Test Air (Wirt)
0.86	98.3%	98.4%
0.67	98.3%	98.0%
0.50	98.6%	98 65%

Modern converging nozzles of curved airfoil section with blunt entrance edges show high velocity coefficients comparable to those found by Warren and Keenan.

NOZZLE EFFICIENCY. Many factors influence nozzle efficiency. The end effects or secondary flows in nozzles of small radial height require correction coefficients varying from 0.95 for nozzles 0.5 in. high to 1.00 for nozzles 3 in. or more high. The total angle of turning in the nozzle influences efficiency. The greater the turning, the lower the velocity coefficient. Smoothness of all nozzle surfaces affects wall losses. The contour of the nozzle partitions is an important factor in nozzle efficiency, as shown by Kraft and Berry (Ref. 9). Also see New (Ref. 10). These authors outline modern methods of nozzle and blade testing and their application.

Shock frequently occurs in improperly designed converging-diverging nozzles, the result of overexpansion. It leads to nozzle losses when the pressure at the mouth is greater than that for which the nozzle is designed. The steam expands as though conditions at the mouth were those for which the nozzle was designed, until at a certain point in the nozzle the pressure becomes less than that at the mouth. Recompression to the pressure at the mouth then begins, and the volume decreases. This causes the jet to detach itself from the wall, and often results in setting up pressure pulsations. The detached jet may be no longer in the direction of the nozzle axis, and so will have an unfavorable angle of discharge. (See Ref. 3, p. 88, Steam Shock.)

When steam expands rapidly from a slightly superheated condition, it does not begin to condense when the saturated condition is reached but continues to expand as in the superheated region, thus becoming supersaturated. *Supersaturation* has been shown to exist in simple nozzles (Refs. 11, 12, and 13). Supersaturation, with its lesser steam volumes, causes greater nozzle discharges than saturated steam. Supersaturation also tends to lessen the efficiency of nozzles because of energy loss when drops start to form and the steam mass seeks equilibrium.

Nozzle coefficients ϕ for convergent-divergent designs are less than those for convergent nozzles because of losses in the divergent section. The coefficient is affected by nozzle form, dryness of steam, and final steam velocity. Values of $\phi = 0.96$ may be expected with superheated steam in good designs with proper expansion ratios, but ϕ may decrease to 0.93 with wet steam. Supersaturation increases the flow coefficient. Commercial nozzles may not have efficiencies as high as given above, because of poor entry conditions, too wide flare beyond the throat, or too thick and unchamfered partitions at the mouth. Gains of 5% in nozzle efficiency have resulted from the redesign of nozzles in certain cases.

FLOW OF STEAM IN NOZZLES. The design of all nozzles is based on the continuity equation:

$$w = A_t V_t \div 144 v_t = A_m V_m \div 144 v_m$$

where w = weight of steam flowing, pounds per second; A_t = area at throat, square inches; V_t = velocity at throat, feet per second; v_t = specific volume of the steam at throat conditions, cubic feet per pound; A_m, V_m, and v_m are similar conditions at the mouth, or exit.

The pressure at the throat of a convergent-divergent nozzle is always the critical pressure, if the discharge pressure is lower than critical. Hence, the flow of steam through such a nozzle is constant, regardless of the value of p_2, the discharge pressure, provided it is less than the critical pressure. The converging portion of such nozzles generally is short, particularly when the nozzle axis is straight. Expansion in the converging portion can be assumed to take place with a velocity coefficient $\phi = 0.99$ for straight nozzles and 0.98 if the approach to the throat is curved. The following equation applies to the flow of steam through a convergent-divergent nozzle if the back pressure is less than critical:

$$\text{Steam initially superheated: } w_s = 0.3155A_t\sqrt{p_1 \div v_1}$$

where w_s = pounds of superheated steam flowing per second; A_t = area of throat, square inches; p_1 = initial absolute steam pressure, psia; v_1 = specific volume of steam at pressure p_1 and the stated superheat, cubic feet per pound. Velocity in the throat, feet per second = $V_t = 72.24\sqrt{p_1v_1}$. There is evidence that supersaturation of initially saturated steam persists through a pressure ratio of about 4 (Wilson limit) with steam initially dry. The discharge w of a nozzle, pounds of steam per second, is given by the same equation as for superheat, but *supersaturation coefficients*, ranging from 1.0 for dry steam with low pressure drop to 1.06 for wet steam at critical pressure ratio, must be applied.

Converging nozzles formed of bent plates sometimes are used in low-pressure stages requiring long blades. Shorter nozzles are of the built-up type with curved airfoil partitions. This construction is being extended even to the lower-pressure stages.

When the mouth (exit) of the nozzle is too large the pressure in the nozzle falls below p_2 and the nozzle has "overexpansion." Serious eddies, shocks and other losses resulting from recompression are set up, and the loss from this cause may be serious. If the mouth is too small, the steam will not be fully expanded until after leaving the mouth of the nozzle, and is thus *underexpanded*. Stoney states that the loss in jet velocity M in percentage for various ratios R of actual nozzle mouth areas to theoretical mouth area is:

R	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.26
M	6.8	4.2	2.4	1.1	0.3	0	1.2	4.0	7.0

Note that the losses resulting from underexpansion are only about one-third of the losses from overexpansion, and not of large magnitude for usual values of R.

Flügel (Ref. 14, p. 72) states that, if in converging-diverging nozzles set at an angle to the axis of the blade row the actual back pressure exceeds the design pressure, compression shock takes place, so that the jet leaves the nozzle mouth at an angle *smaller* than the nominal nozzle angle. On the other hand, if the back pressure is less than design pressure, the jet tends to "turn the corner" so that it leaves the nozzle at an angle *greater* than the nominal nozzle angle; at the same time there is a tendency to start pulsation waves in the jet as it continues to expand beyond the nozzle mouth.

The lower efficiencies of converging-diverging nozzles have led many designers to use only converging nozzles in turbines designed for high efficiency, using divergent sections only for very large pressure ranges.

AREA OF NOZZLES. To direct the jet, a series of moderately small nozzles, spaced around the arc of the wheel, is used instead of one large nozzle. The total nozzle discharge area of a full circle of convergent nozzles is given by

$$A = \pi Dl\epsilon \sin \alpha$$

where A = nozzle flow area, square inches; D = pitch diameter of nozzle ring, inches; l = nozzle radial height (range: 0.5 to 40.0 in.); ϵ = edge thickness factor (range: 0.88 to 0.95); α = nozzle discharge angle (range: 12° to 20°).

CROSS SECTION OF NOZZLES. Converging-diverging nozzles may be of circular cross section at throat and mouth, or they may have rectangular cross section throughout. The former are used on single-stage turbines and sometimes in the first stage of multistage turbines of moderate efficiency. Frequently, these are pitched slightly towards the center of the shaft to cause the jet to enter the blades with less spilling at the shroud. Simple converging orifices in diaphragms are of rectangular cross section throughout, normal to the axis of flow.

The discharge angle α of convergent orifices must frequently be increased in the last stage of impulse turbines to provide passageway for the large volume of steam present, without excessive nozzle length. The pitch of nozzles is fixed arbitrarily. Usually it equals 1 $^1/_3$ to 3 times the blade pitch. Another rule is not to exceed 1 in. in width at the throat when measured at right angles to the axis of the jet at the mean diameter.

NOZZLE MATERIALS. Converging-diverging nozzles on some of the smallest turbines are made of brass or bronze, for low-pressure, low-temperature steam. Nozzles are formed of alloy steels (usually stainless, 12% chromium) when steam temperatures exceed 400 F or where the steam is wet and may corrode the metal.

Convergent nozzles for high pressures and temperatures usually are made of 12% chromium steel, machined and welded into the diaphragms. Large low-pressure nozzles are sometimes formed of bent plates of 12% chromium steel cast into diaphragms, or formed partitions may be used.

Variable entrance and exit angle nozzles and buckets are used to an increasing extent in low-pressure stages, to conform to the so-called vortex theory.

First-stage nozzles, of 12% chromium steel, are sometimes built up in sections. They are bolted or welded to the inlet steam belt of the casing.

4. BUCKETS

The steam leaving the nozzle in impulse turbines is directed against the revolving buckets at an angle of 12 to 16 degrees with the plane of the wheel for the early stages, increasing to 30 degrees in some low-pressure stages. The relative entrance velocity of the jet and its entering angle can be found from the velocity diagram, as in Fig. 19. The exit angle of the buckets is made the same as the entering angle in some small units, particularly those of the re-entry type. On larger turbines the exit angle is almost always less than the inlet angle.

LOSSES IN BUCKETS are due to secondary flows, to leakage over the tips, and to friction in the passages. Secondary flows are caused by compression and re-expansion of the steam jet on the curved face of the passage, and by the bottom wall of the passage on the disk and the top wall formed by the bucket cover. These wall losses are known as *end effects;* the longer the bucket, the less the influence of end effects upon efficiency. Some designers use the aerodynamicist's term *aspect ratio*, expressed as bucket height/bucket width. With a given value of ρ, efficiency increases with an increase of aspect ratio.

Other losses include an aspirating loss (sometimes called *nozzle-end loss*) due to mixing of the jet and dead steam in buckets leaving an idle arc or in the clearance between nozzle and bucket, and shock losses when the jet strikes the bucket at the wrong angle.

Because of these losses the relative velocity V_{ro} leaving the bucket is less than the entering relative velocity V_{ri}. The ratio of leaving to entering relative velocity is called the *bucket velocity coefficient*, i.e., $V_{ro}/V_{ri} = \psi$. This coefficient is influenced by the width of bucket, the form of its rear flank, the total angle through which the steam is turned, the relative velocity of the steam, and the smoothness of the passage.

COMPRESSION IN BUCKET PASSAGES. Stodola remarks that the greater part of bucket losses results from compression and re-expansion in the curved passages. Belluzo (Ref. 15, pp. 74–78) develops the principles of shock and recompression in curved passages. In a curved passage, recompression results from centrifugal force on steam molecules traveling at high relative velocity through the curved passage. If the curve of the passage is assumed to be an arc of a circle of radius R feet, and of width, h, feet (h is usually small); d_1 = density of steam, pounds per cubic foot; $g = 32.2$; V_r = mean velocity of steam crossing blade, feet per second; p_a = absolute pressure of steam at inlet and outlet of bucket (stage pressure), psia; p_x = absolute pressure that the steam assumes as a result of recompression due to centrifugal force, psia; then

$$144(p_x - p_a) = \frac{d_1}{g} \times \frac{h}{R} \times V_r^2 \quad \text{or} \quad p_x = p_a + \frac{d_1}{g} \times \frac{h}{R} \times \frac{V_r^2}{144}$$

If recompression were the only consideration, wide passages with wide buckets having curves of large radius would be best. Such buckets have large losses due to increased end effects or, in other words, to decreased aspect ratios. Hence widths have been a compromise, determined largely as a result of test and experience.

Velocity Coefficients. Zietemann (Ref. 16, p. 72) concludes that bucket velocity coefficients are principally influenced by the total angle through which the steam is turned. He presents the following values for commercial blading where β_1 and β_2 are the entering and leaving angles of the blade.

$(\beta_1 + \beta_2)/2$	10	20	30	40	50	60	70	80
ψ	0.77	0.85	0.89	0.918	0.938	0.953	0.962	0.966

Zietemann's values depend only on the total angle of deflection. Stodola (Ref. 3, p. 179) presents data from Brown Boveri experiments which indicate that the coefficient ψ, with buckets having 30-degree inlet and outlet angles, varies with relative entering steam velocity V_{ri}. For a given bucket, the coefficient curve is hyperbolic. The mean of the maximum values of ψ is:

V_{ri}	400	800	1200	1600	2000	2400	2800	3200
ψ	91.2	91	90.8	90.4	89.9	89	87.8	86.2

A correction factor for end effects in the bucket passage should be applied to the above values. The following correction factors have been used:

Bucket height, in.	0.5	0.75	1.00	1.50	2.00	2.50	3.00 and longer
Correction factor	0.95	0.97	0.98	0.99	0.995	0.999	1.00

The above data on the coefficient ψ may be used to calculate velocity diagram efficiencies that agree fairly well with practice.

Aerodynamic test methods provide correct values for ψ for various buckets. In some cases lift and drag coefficients are found. Sometimes the blade coefficient is related to Reynolds' and Mach numbers. Dollin presents values for reaction blade efficiencies as a function of Reynolds' numbers (Ref. 17).

VELOCITY DIAGRAMS. Figure 19*A* is the velocity diagram for one stage of a multistage turbine, which operates under the following conditions: pressure and temperature before nozzle (no carryover), 65 psia; 380 F; expands to 38 psia; nozzle angle, $\alpha = 14°$; nozzle coefficient, $\phi = 95.9\%$; wheel speed, $u = 650$ ft per sec; available energy, $h_1 - h_2 = 45.4$ Btu. Velocity leaving nozzle, $V_1 = 223.9 \times 0.959\sqrt{45.4} = 1446$ ft per sec.

From the diagram, the relative entering velocity $V_{ri} = 830$ ft per sec, and blade entrance angle $\beta_1 = 25°$. Assume $\beta_2 = 25°$ and $\psi = 0.87$. Relative leaving velocity $V_{ro} = 0.87 \times 830 = 722$ ft per sec. Absolute velocity $V_o = 304$ ft per sec.

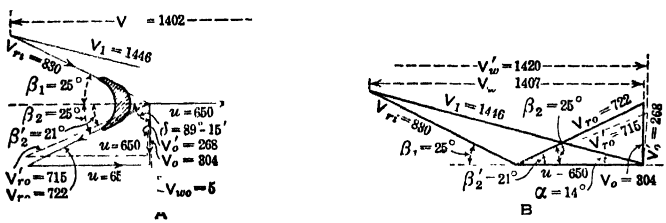


Fig. 19. Velocity diagram for impulse stage. Primed values illustrate the effect of using a smaller bucket exit angle.

The work done, in foot-pounds, is the product of the wheel speed u and the total (algebraic) change in tangential velocity, V_w, divided by gravity g. This can be readily determined from Fig. 19*B*, the usual form of combined velocity diagram. The lower triangle of Fig. 19*A* is placed on the same base u as the upper triangle, and $V_w = (V_{wi} + V_{wo})$ equals 1407.

$$\text{Work done in Btu} = \frac{uV_w}{778 \times g} = \frac{650 \times 1407}{778 \times 32.2} = 36.5 \text{ Btu}$$

Combined nozzle-bucket efficiency = 36.5/45.4 = 80.4%.

The volume of steam can be assumed to remain constant during its passage through the buckets. The length on the inlet side is usually about $^1/_{16}$ in. longer than the height of the nozzle exit.

Velocity diagrams for a 2-row wheel are shown in Fig. 20, *A* and *B*, based on the following assumptions: initial conditions, 140 psia; 450 F; back pressure, 20 psia; nozzle coefficient $\phi = 93.8\%$; nozzle angle, $\alpha = 14$ degrees; wheel speed, 525 ft per sec; available energy

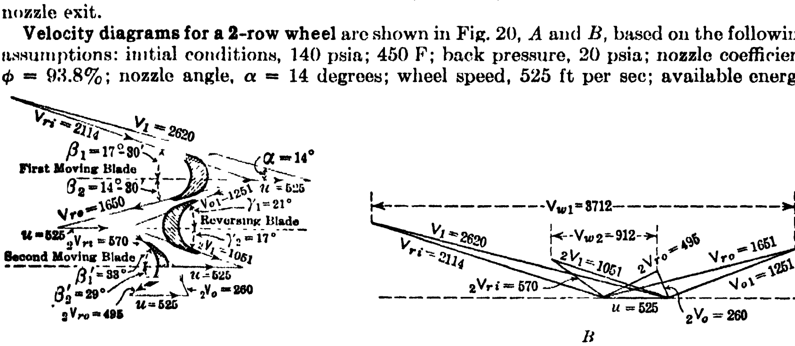


Fig. 20. Velocity diagram for two-row wheel.

$(h_1 - h_2) = 155.7$ Btu per lb. ψ_{b1}, ψ_{br}, and ψ_{b2} for first moving, reversing, and second moving rows respectively, are 0.83, 0.84, and 0.87. Usually only the diagram Fig. 20*B* is drawn. The velocities of whirl from the diagram are 3712 ft per sec for the first row, and 912 ft per sec for the second row, a total of 4624 ft per sec.

$$\text{Work done} = \frac{uV_w}{778g} = \frac{525 \times 4624}{778 \times 32.2} = 96.9 \text{ Btu per lb}$$

The combined nozzle-bucket efficiency = 62.2%. Three rows of moving blades are used on some wheels which operate at low wheel speeds, but these designs are rare.

REACTION. Stages are said to have reaction when the relative velocity across the bucket is increased by the expenditure in the blade of a portion of the total available energy of the stage. This portion may vary from 5% in high-pressure stages to 50% in the last stage. This last row then becomes a full-reaction stage. Since the available energy released results from a pressure difference across the bucket row, provision must be made to reduce steam leakage over the tip. Radial sealing strips extending from the casing, small clearances over the tips, if unshrouded, or axial sealing strips (end tightening) are used to reduce leakage. Axial seals are used on some high-efficiency impulse units and on two-row stages of some large turbines.

When reaction is added to an impulse stage the optimum value of the ratio of wheel speed to jet velocity at the nozzle exit, ρ, changes. The following values (practical compromises) may be used on impulse designs when reaction is expressed as a percentage of stage available energy:

% Reaction	ρ	% Reaction	ρ	% Reaction	ρ
0	0.430	20	0.530	40	0.685
5	0.450	25	0.565	45	0.735
10	0.475	30	0.600	50	0.800
15	0.500	35	0.640		

All values of ρ given tend to increase as the nozzle height increases to large values. The values quoted are for small to medium heights.

Zietemann's data show that impulse stage efficiency is influenced by the turning angle of the steam in the bucket. For a given total stage available energy, use of a portion of this available energy to give reaction in the buckets decreases the angle of turning, thus decreasing losses and increasing stage efficiency. Hodgkinson (Ref. 18) gives data on blade width-curvature relationships.

The inlet angles of commercial blades usually are increased several degrees over that found on the velocity diagram. This allows the steam to enter without striking the back of the bucket. Less loss occurs if steam strikes the face of the bucket at a slight angle than if it strikes the back. The former condition is *splashing*, the latter is *blunting;* both are undesirable.

BUCKET VIBRATION. Buckets are subject to bending stresses because of the tangential forces and because of the pressure difference across them. If there is any irregularity in the driving force due to interruption of steam flow from nozzles as at the horizontal flange, from buckets passing through the steam jet at partial load, or from nozzle partitions, vibrations are set up in the buckets which may lead to early fatigue failure. The amplitude of vibration is lessened by the damping properties of the bucket material. Studies have been made on relative damping properties of available materials, and one of the best, a 13% chromium steel, is frequently used. It not only has high internal damping, but also good stainless properties. (See Refs. 19 and 20.)

Various means are provided to lessen bucket vibration. Lashing or tie wires or other fastenings at intermediate positions on the bucket length have been used. Riveted covers on the bucket tips, tying several buckets together as a unit, also decrease vibration. Double bucket covers with overlapping joints are also used, but only in unusual designs. The longer buckets should be tuned away from their resonant frequencies by careful design, to avoid breakage by fatigue.

BUCKET WIDTH varies from $^1/_2$ to $^3/_4$ in. in small single-stage impulse turbines, and from $^3/_4$ to 2 in., or even larger, in multistage units. The wider buckets are used in the larger machines. The maximum length should not exceed 8 to 12 times the width which is basically fixed by the stresses.

In multivalve machines the maximum bending stress in the first-stage buckets occurs at light load, when the first open nozzles expand with a large pressure drop and high velocity. This condition gives the largest number of kilowatts per bucket, and therefore must be the basis of design. This condition may require a relatively wide bucket to withstand the tangential force, as well as vibrations that may be set up.

BUCKET DESIGN. When inlet and outlet angles and width have been chosen, the buckets can be designed. The curve forming the face of the bucket is drawn tangent to the lines forming inlet and exit angles. The pitch usually is chosen as 0.5 to 0.6 of the width, but seldom should exceed 1 in. With equiangular blades Kearton gives the pitch $P = b/2 \sin 2\beta_1$, where b = width, inches, and β_1 = entrance angle. The rear flanks are made parallel to the inlet and exit angles, and the curve of the back of the bucket is fixed by experience. The outlet edges are made as thin as manufacturing considerations will permit. The inlet edges of high-pressure buckets are made thin. In low-pressure blading, however, inlet edges are now thick and rounded, to reduce erosion and to improve efficiency.

VELOCITY RATIO. An important criterion of turbine performance is the velocity ratio ρ, the ratio of the mean wheel speed to the steam speed leaving the nozzle. Turbine efficiency depends on this ratio alone, and not on the individual values of the velocities. For maximum theoretical efficiency of impulse wheels with no bucket losses and no carryover, $\rho = \cos \alpha/2$, and for velocity compounded wheels, $\rho = \cos \alpha/2n$, where α = angle that the nozzle axis makes with the plane of the blades and n = number of moving rows in the stage.

Stodola indicates that highest efficiencies in a stage of a multistage impulse unit occurs when α = 12 to 15 degrees. Assuming α = 12 degrees, the maximum theoretical efficiency occurs at ρ = 0.489 for a single-row impulse stage and approximately 0.25 for a two-row stage. Consideration of stage losses and carryover lead to the use of higher values of ρ for maximum efficiency.

Experience has shown that a two-row stage is subject to less decrease in efficiency than a single-row stage when the available energy across the stage is increased as in the first stage of large units at light load. Hence, two-row stages are more frequently used than single-row stages for the first stage of large highly efficient turbines. When reaction is added to either two-row or single-row stages, the optimum value of ρ increases. Depending on the degree of reaction, ρ may increase to 0.3 for two-row and 0.7 for single-row stages.

The carryover energy of the absolute exit velocity of the previous moving row is added to the available energy in the nozzle. When carryover is large, larger values of ρ than $\cos \alpha/2$ may give higher stage efficiencies when ρ is calculated from the static isentropic available energy nozzle.

REACTION BLADING consists essentially of a series of converging nozzles. Figure 21 is a typical velocity diagram for a reaction stage of one stationary and one moving row.

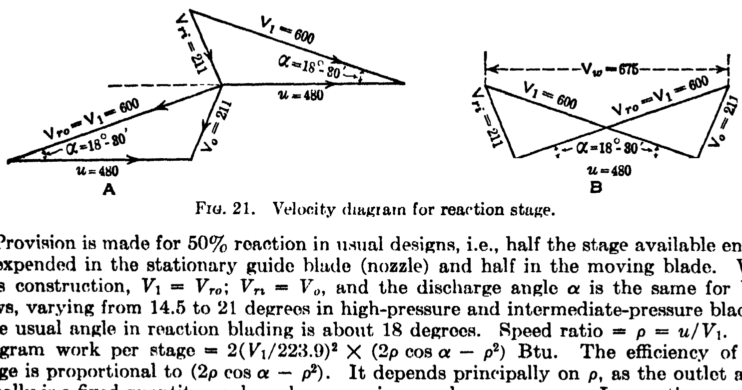


FIG. 21. Velocity diagram for reaction stage.

Provision is made for 50% reaction in usual designs, i.e., half the stage available energy is expended in the stationary guide blade (nozzle) and half in the moving blade. With this construction, $V_1 = V_{ro}$; $V_{r1} = V_o$, and the discharge angle α is the same for both rows, varying from 14.5 to 21 degrees in high-pressure and intermediate-pressure blading. The usual angle in reaction blading is about 18 degrees. Speed ratio = $\rho = u/V_1$. The diagram work per stage = $2(V_1/223.9)^2 \times (2\rho \cos \alpha - \rho^2)$ Btu. The efficiency of this stage is proportional to $(2\rho \cos \alpha - \rho^2)$. It depends principally on ρ, as the outlet angle usually is a fixed quantity, and reaches a maximum when $\rho = \cos \alpha$. In practice, ρ varies from 0.6 to 0.95 in land turbines and seldom is less than 0.85 in large units. The higher values of ρ give more expensive but more economical turbines. Lower values of ρ give smaller, and less efficient machines. The work done = $uV_w/778g$ Btu per lb, where V_w is found as indicated in Fig. 21*B*.

Losses in Reaction Blading. One of the principal losses in reaction blading is leakage over the blade tips, due to the difference in pressure across the blade row. The leakage passes directly across the clearance, while the working steam in the blades is turned through the angle α. The percentage loss from leakage through the clearance is

$$L = 100C \div (lM \sin \alpha + C)$$

where L = loss, percentage; C = clearance, inches; l = length of the blade, inches; α = exit angle of blade; and M = thickness coefficient (unity for most reaction blading).

Shrouded blades with radial seal strips inserted in the casing may have only 60 to 75% of the leakage calculated by the above equation.

The steam passing through the blades $W_1 = W/C_1$ where W = total steam per hour in pounds flowing through the turbine, and the clearance factor $C_1 = (1 + C/l \sin \alpha)$. The bucket efficiency $\eta_b = \eta_d/C_1$, where η_d = the diagram efficiency as calculated from a diagram similar to Fig. 21, using appropriate velocity coefficients.

Leakage losses are greatest with short blades and large clearances, and smallest for the long blades of the last rows. A two-row stage frequently replaces the high-pressure section of reaction turbines, where leakage and other losses are high. In such stages the clearance over the tip of the shortest blade is never less than 0.025 in. With no leakage the efficiency of reaction blades would be substantially that of nozzles, from 92 to 97%. Leakage, interstage, and rotation losses must be considered, however, in calculating the stage efficiency.

The face and rear flanks of reaction blading are made up of a series of curves. One effective form is made from the intersection of two ellipses. The entrance edge of reaction blades usually is quite blunt, to reduce erosion.

Widths of reaction blades vary from 1/2 in. for blades 4 in. long to 1 1/2 in. for blades 30 in. long. The pitch of blade rows is fixed somewhat by the type of shroud used to secure end packing, as it must be possible to shift the spindle toward the exhaust to clear these shrouds in lifting the rotor.

AREA THROUGH LAST ROW OF BUCKETS. A major consideration in turbine design is the large volume of steam to be passed through the last row of buckets, particularly at high vacuum. If the area of the passage through the last row is too small, the steam will have a high absolute leaving velocity, involving a considerable energy loss. This area, therefore, should be as large as possible, while still retaining sufficiently small blade angles to insure good diagram efficiency with safe bucket lengths.

Kearton (Ref. 21, p. 303) gives the centrifugal stress, psi, in steel blades of uniform cross section as

$$f_c = 4.09l\left(\frac{d}{100}\right) \times \left(\frac{N}{100}\right)^2 = 0.215u^2m$$

where d = mean diameter, inches; N = rpm; l = length of blades, inches; u = wheel speed, feet per second; and m = ratio of blade length to mean diameter. It is also shown that $f_c = 1.88A(N/100)^2$, where A = area of annulus of last blade row, square feet. From this it is evident that the stress in the blades of the last row is *directly proportional to the annular area of the bucket ring for a given speed.* These stresses should be multiplied by 1.074 for brass or bronze blades.

LEAVING LOSS. The absolute velocity from the last row of buckets represents kinetic energy generated from the available energy. This kinetic energy $(V/223.9)^2$, known as the "leaving loss," is one of the principal losses of the turbine. It cannot contribute useful work after leaving the last blade row. Blades as long as commercially justified are used to increase the exhaust annulus and to reduce this loss.

HOOD LOSS. Pressure drop through the exhaust hood due to friction and eddies causes a decrease in available energy known as *hood loss.* When added to the *leaving loss,* the total is known as *exhaust loss.*

VORTEX THEORY AND BUCKET DESIGN. It is general practice to put some degree of reaction in almost all stages, particularly in the low-pressure section. Many low-pressure stages have as much as 50% average reaction. This division of stage energy between nozzle and bucket lessens the velocity in the nozzle, but even so the velocity frequently exceeds the critical. In some cases limited last-stage area due to stress limits on blade length forces the use of reaction.

Radial pressure differences exist in the space between the stationary and moving elements; this difference is more marked in the greater bucket lengths. *Free vortex theory* is therefore used in design. Under the precepts of this theory the flow across the nozzle-blade passage is constrained to have a vortex velocity distribution by proper variation in angle (and passage area) of the nozzle and blade. Such distribution requires that the tangential components of the absolute velocities vary along the blade height in inverse proportion to the radii at which they occur. As a consequence of this vortex flow, motion of the steam particles exerts a centrifugal force which results in an increase in pressure from the inner to the outer radii of the blade row, maintaining equilibrium in the passage, and preventing establishment of radial velocity components.

As a result of this vortex action, varying degrees of expansion occur in the stationary nozzle and in the following bucket. The greater part of the pressure change of the stage takes place in the stationary element at the inner radius, and less pressure change occurs at this radius in the revolving bucket. As a result of this energy distribution, the bucket contour of long blades at the inner radius approaches that of an impulse bucket with its discharge angle slightly closed, whereas near the tip the configuration is more like that of a reaction stage. Thus the net result of vortex design is a stage which has radial variation of the degree of reaction.

The build-up of pressure at the outer radius of a long bucket leads to a smaller pressure drop in the nozzle and a larger pressure drop at the moving bucket tip. The buckets are warped from base to tip, with varying inlet angles, increasing from base to tip while discharge angles decrease. Some turbines are designed with variable-angle warped-surface nozzles in the stationary element of the stage to provide for the varying degree of expansion throughout its height. Other designers adhere to a constant discharge angle on the stationary element, depending on the radial spread at increasing diameters to give the increased nozzle flow area required by the increase in reaction.

Vortex design reduces the probability of radial flow in the space between stationary and moving elements in a stage. This theory has been applied to the last stage of many turbines, to all stages of some high-efficiency units built abroad and to at least one unit in this country. The stage efficiency is increased only a small amount on short buckets but may be increased by a measurable quantity on the last stages. No test evidence of an overall improvement has yet been published.

Various methods have been proposed to permit calculation of vortex stages, but they have not been published. (See also Axial Flow Compressors, Section 1.) One method is given by Pochobradsky (Ref. 22). In general all vortex designs are based on the relation $V_w u$ = constant, where V_w = (tangential) velocity of whirl at any radius, feet per second, and u = wheel speed at the same radius, feet per second.

BUCKET FABRICATION AND DESIGN. Buckets may be precision-cast, rolled, drop-forged, or machined from rectangular bars. Alloy steels generally are used for blading, consisting of 12 to 14% chromium and 0.10 to 0.15% carbon.

Some buckets are drop-forged with an axial bulb-shaped end on a straight shank at the blade base. After machining, they are driven axially into similar-shaped slots in the outer rim of the disk. The inverted T base with one or more sets of shoulders is a common type of fastening. Such buckets are inserted into rotor grooves which are closed by special devices when the last bucket has been inserted. Frequently T-base buckets are held tight against their shoulders by a small wedging strip driven into a shallow slot below the T base. Many long buckets have a straddle base which fits over a T- or tooth-shaped rim on the disk.

Spacing of most buckets is secured by machining the integral base to the proper width, thus also determining the discharge angle. One large manufacturer cuts the shorter buckets in low-temperature turbines from a rolled strip of copper-nickel alloy. They are set in a jig to insure uniform spacing and correct angle, and an alloy foundation ring is cast around their bases. A shroud strip is silver-soldered to the outer ends and the foundation ring is then machined, with one or more projections to match grooves in the spindle. These segments are balanced, inserted in grooves, and held in place by a caulking strip at one side. Serrated grooves are used with some forms of bucket root.

No sharp corners or edges are permissible on any part of the bucket, particularly at the base, since fatigue failures may start at sharp edges or corners Side-entry buckets are sometimes used on large low-pressure stages with each bucket row on a separate disk. In this design serrated bucket roots are driven into grooves milled either axially or in curved form across the rim of disks integral with or fastened to the shaft. The roots are locked in place by locking pins after being driven in place.

Buckets are examined carefully at each overhaul for signs of fatigue cracks, either by visual examination with a magnifying glass or by magnaflux.

Recent designs are curved airfoil sections, with well-rounded entrances. This construction is less sensitive to the angle of the entering steam jet, has a greater cross section, and is more rugged than buckets with finer entrances. A larger tenon through the shroud can be provided, which is highly desirable when it is riveted over the shroud.

Taper. Long low-pressure revolving buckets may have a tapering form to provide greater strength at the base, without excessive weight. C. A. Parsons and Company in England makes a hollow blade for low-pressure use, with increasing metal cross section from tip to base.

Lacing wires are silver-soldered to some of the larger buckets to reduce vibration. Long alloy steel buckets may have reinforcing projections, either welded to each individual bucket or forged integral with it. If welded, the buckets are heat treated before finishing. These projections on adjacent buckets can then be welded together after assembly without undue heating of the bucket. Thus a reinforcement is provided against vibration, at a midpoint in the blade length.

Gaging of reaction blades is the ratio of the net area for steam flow on the mean diameter measured at the blade outlet at 90 degrees to the direction of the jet, to the annular area occupied by the blade ring. Thus 25% gaging means a net steam flow area of 25% of the blade-ring annulus.

Shrouds. Some buckets, particularly small ones, are formed with projections at their tips to form their own shroud rings. Shroud strips may be held by riveting over the tenons machined at the ends of the buckets. Shrouds may consist of flat strips in impulse turbines with a clearance of 0.03 in. between adjacent groups, each containing six or eight buckets.

Sometimes flat shrouds, which project on one side toward the base of an adjacent row, are used for end tightening. The clearance between shroud and blade base can be adjusted to a much lesser amount than the permissible end clearance of similar blades. The reduced leakage with such shrouds results in better turbine economy.

Methods of Attaching Shrouds. Sometimes large cylinder blades are assembled in jigs, and the shroud is riveted on. The group is then inserted as a unit in the cylinder grooves. Cylinder blades are often inserted before the shroud is added and riveted. Flat shrouds may be riveted or silver-soldered to the blades and clearances maintained by radial sealing strips fastened to the casing. Channel and angle-shaped shrouds are also used, with the projecting strip serving as a seal.

Strength of Roots. All bucket roots must withstand the stresses due to centrifugal force. Methods of calculating such stresses can be found in Refs. 4 or 23. The centrifugal force on a bucket is

$$F = 0.00002842N^2Wr$$

where N = rpm; W = weight, pounds, of bucket and shroud ring beyond the section of the base carrying the load; r = radius, inches, from the center of the shaft to the center of gravity of the bucket and shroud beyond the section under stress. At normal speeds, buckets should not be stressed higher than 0.5 of their elastic limit. This allows the turbine a certain overspeed without risk of bucket trouble.

Bucket roots must be designed to withstand stresses due to vibration. Ryan and Rettaliata (Ref. 24) analyzed stresses in bucket roots and corners by means of plastic models and polarized light. Calibration of these models permitted calculations to be made of stresses in actual buckets.

Corrosion. Certain bucket materials corrode badly if the feedwater is not deaerated to free it of oxygen and carbon dioxide. Corrosion often is rapid in idle turbines into which steam leaks through the throttle valve. Aluminum-bronze blades have corroded badly in turbines receiving wet steam carrying magnesium and calcium chlorides.

Bucket failures have sometimes been attributed to corrosion fatigue due to the presence of salt deposits which accelerate cracking of the metal. Stages near the dew point in the turbine steam path are strengthened for greater corrosion fatigue strength.

Erosion of the inlet edges of low-pressure buckets in turbines with high tip speeds has sometimes required the renewal of buckets in 3 to 7 years. This is due to impact of water drops formed as a result of expansion. F. W. Gardner (Ref. 25) shows that the drops of water must be extremely fine, that they tend to concentrate on the outer ends, that the force of impact of the drops on buckets moving 1000 ft per sec is about 90,000 psi, that it is impossible to remove by any separating device all the water that condenses, and that hard bucket material and hard sheaths on blades (e.g., stellite) are necessary to withstand this erosion.

Expansion in most condensing turbines is carried to conditions where the moisture content in the exhaust steam ranges from 10 to 12%. This limit is fixed by energy losses due to mechanical interference between moisture drops and steam, and to the rapid erosion of the inlet edges of unprotected buckets caused by impingement of moisture drops. See Ref. 26 for curves showing braking losses caused by moisture in buckets, given in percentage of the useful output at different pressures and temperatures. Curves are shown of improvements resulting from drainage grooves.

Christie and Colburn (Ref. 27) found that erosion was most serious in turbine buckets with tip speeds over 1000 ft per sec, that it is more pronounced in reaction than in impulse buckets, and that erosion is most severe about 1 to 1 1/2 in. from the tip and around lacing wires. They also show how steam conditions at the entrance to the last row may be estimated.

Hardened strips fastened or brazed to the inlet edges of the buckets, coatings of hard alloys as stellite, tungsten tool steel, tantalum, etc., fused onto the flank, drainage systems to remove interstage moisture, alloy steel material, and combinations of these, reduce erosion.

Stainless steel buckets with 11 to 13% chromium are widely used in low-pressure buckets. In addition, they are protected on the inlet edge by a hard (stellite) shield one-half to two-thirds of the outer bucket length.

Water-drainage grooves are provided around the casing of most units to draw off water formed by condensation. These water-drainage devices remove 25 to 30% or more of the moisture present. The low-pressure bleeder opening also serves to draw off such water. Caldwell has proposed that hollow partitions be used for nozzle or bucket passages in low-pressure stages so that steam from a higher pressure stage can be provided to reheat the passing steam and to keep it relatively dry.

Fabrication. It is becoming general practice to mill buckets individually from stress-relieved bars or forgings. The roots are milled to serve as spacers. Necessary shoulders or serrated holding teeth are milled accurately on the sides of the base.

Some hollow low-pressure stationary blades for reaction turbines have been made by folding an alloy steel sheet over a form and welding the trailing edge, thus decreasing machine weight. Precision cast blades for complex shapes are under trial.

Material of many kinds has been tried for turbine buckets. Present practice favors an alloy steel with 11.5 to 13% chromium, 0.10 to 0.15% carbon, ultimate strength 90,000 to 100,000 psi, yield strength of 80,000 psi, and proof strength of 70,000 psi. A steel of 19% chromium, 9% nickel, and 0.5% tungsten has been used for 1000 F. Austenitic steels are used for higher temperatures. Monel metal (ultimate strength 85,000 psi and elastic limit 50,000 psi) is used in small units with moderate steam temperatures and stresses because it does not readily corrode. Special steel alloys and pure nickel are used in industrial turbines where corrosion may be serious.

Clearance. When the same pressure exists on both sides of impulse buckets, the clearance over their ends may be large. The axial clearance between nozzle exit and bucket entrance on small impulse turbines varies from $^1/_{32}$ to $^1/_{16}$ in. Large machines have axial clearances up to $^1/_2$ in.

The radial clearances over the ends of reaction or impulse buckets with moderate reaction must be kept small. Consideration also must be given to the rigidity of the casing. Various formulas are used by builders of reaction turbines for determining this clearance. Some formulas are based on distance between bearings and others on blade length and mean diameter of row, with consideration of the taper on the ends of the blades.

One formula for radial clearance of buckets without taper at the top is

$$C = 0.015 + 0.003D + 0.005\ l_b$$

where C = clearance, inches, D = mean diameter of blade ring, feet, and l_b = length of blades, inches.

This radial clearance, even when sealing strips are used, is seldom less than 0.020 in.

Stodola (Ref. 28) quotes one builder of reaction turbines as allowing on large single-cylinder turbines a clearance of 0.001 × mean diameter, and on short two-cylinder units 0.0008 × mean diameter.

The clearance of end-tightened blading can be set at substantially that of the dummy pistons. This usually will be less than half the end clearance given by the above formula, particularly when axial adjustments of the turbine rotor relative to the cylinder can be made with the unit under load. When measuring devices are installed, the operating clearance may be as close as 0.005 in.

Bucket length usually is limited to about 35% of the mean diameter of the bucket row. Maximum bucket lengths on 3600 rpm turbines are: 23 in. on a 42.5 in. disk, giving mean diameter of 65.5 in., mean blade speed of 1029 ft per sec, and tip speed of 1390 ft per sec; on 1800-rpm turbines, 40 in. long buckets on an 80-in. disk, mean diameter 120 in., mean blade speed 942 ft per sec, and tip speed 1257 ft per sec.

Minimum bucket length is seldom less than 2% of the mean diameter. This minimum is $^1/_2$ in. on small impulse turbines and generally 1 in. on reaction turbines.

EFFICIENCY AT THE WHEEL PERIPHERY OF IMPULSE UNITS is found by combining the nozzle efficiency η_n with the diagram efficiency η_d. Stodola (Ref. 3, p. 222) makes these statements on the efficiency at the wheel periphery: (1) Efficiency depends only on ρ, the ratio of wheel speed to steam speed, and not on the individual values of the velocities. (2) Efficiency varies with the peripheral velocity according to a parabolic law. (3) With a small nozzle angle α the maximum value of efficiency at the wheel periphery is attained when the peripheral velocity is nearly half the steam velocity. (4) The best efficiency for constant blade coefficient ψ is higher the smaller the nozzle angle α. (5) Energy loss in the nozzle is nearly four times as detrimental as energy loss in the buckets.

NOZZLE AND BUCKET EFFICIENCY. Data on nozzle and bucket losses are not complete. Many designers start with internal efficiencies based on tests of similar units, from which stage efficiencies can be calculated. Then nozzle and bucket efficiencies can be deduced with considerable reliability. Such deductive design based on test performance is preferable to synthetic design using assumed efficiency factors. Detailed methods of proportioning nozzles and blading will be found in Refs. 3, 4, 14, and 16.

DEPOSITS IN TURBINES. The steam supplied to turbines from boilers should be free from moisture, dust, acid, and corroding chemicals and as nearly pure as possible. Any corrosive element in the steam will rapidly destroy turbine blading, starting at the dew point. The surfaces are continually swept clear by the high-velocity steam, thereby accelerating corrosive attack.

Although feedwater in modern plants consists of condensate and evaporated make-up water, impurities may be carried into boilers from condenser leakage. Chemicals must be added to maintain the desired sulfate-alkalinity ratio so that the average concentration in the boiler drums may range from 1000 to 3500 parts per million. Because of the evaporation of any moisture in the superheater, some impurities are carried over as dust or vapor and deposited on the governor valves, nozzles, and buckets of the turbine, closing up the passages and decreasing the capacity. The decreased capacity may range from 15 to 50% in a few weeks, in some cases. (See Refs. 62 and 63.)

5. ROTORS

Rotors of small high-speed impulse turbines usually consist of a disk or wheel, carrying the buckets, pressed on a shaft and held against a shoulder by a lock nut. Some velocity compound turbines with low wheel speeds use two or more disks, each with a single row of buckets, instead of one disk with several rows. Small turbines generally are designed with shafts that operate well below their critical speed.

Multistage impulse turbines have a series of disks mounted on the shaft, with intermediate diaphragms between to carry the nozzles and the labyrinth packing. Surfaces of the disks should be smooth, and preferably polished.

Impulse-turbine rotors for high-pressure and topping units are frequently made of a single forging with disks formed by removing the steel between the disks; such construction is known as the "solid-rotor" type. The short rigid rotor permits operation below the calculated critical speed. Because of its extended surface the rotor is readily heated. Closeness of diaphragms prevents the rotor's cooling off faster than the shell, on shutting down. All rotors are now heat-stabilized before final machining, by heating in a furnace to about 900 F. This removes residual stresses, and has been found to give more smoothly operating machines.

Disks of two-row wheels have a rapidly tapering section and a heavy hub. Bucket speed at the mean diameter usually varies from 200 to 700 ft per sec.

Disks for multistage impulse turbines are made of uniform thickness, or of a hyperbolic tapering section. With only a relatively light rim, the stresses are less than in two-row wheels. They operate at rim speeds varying from 400 to 650 ft per sec.

DISK MATERIAL. The maximum working stress in disks at 25% overspeed should not exceed the elastic limit. At normal speed, the working stress of low-pressure, low-temperature disks should not exceed 16,000 to 20,000 psi. The working stress of disks used at high temperature are even lower, depending on the elastic limit at working temperature. Stresses at normal speed are designed to give a factor of safety of 2 or higher.

Disks of small turbines with low wheel stresses and low steam temperatures are of medium carbon steels with ultimate strength of 65,000 to 75,000 psi; elastic limit 30,000 to 40,000 psi. Larger disks at moderate stresses and temperatures are of carbon steel forgings with about 0.45% carbon, normalized and drawn, with ultimate strength 75,000 psi; yield point at room temperature, 40,000 psi. Where stresses are high, carbon-molybdenum disks, forged and heat treated, ultimate strength 100,000 psi, yield point 70,000 psi, are used.

A wide variety of alloy steels has been used for highly stressed disks operating at high temperatures. Among them are nickel steels, chromium-nickel, chromium-molybdenum, nickel-molybdenum, and molybdenum-vanadium. In general their ultimate strength varies from 85,000 to 105,000 psi, with yield points of 55,000 to 80,000 psi. (See ASTM A294 for details of composition.)

In general, chromium is added for greater strength and molybdenum for higher temperature resistance. Cobalt may be required with temperatures above 1000 F. Vanadium and columbium are stabilizing elements.

Certain manufacturers overstress the disks by operating them at overspeeds. The disks then are allowed to age, resulting in an increase in the elastic limit of the material.

Care must be taken in forging the disks to work all the metal thoroughly. This can be assured in forging by heavy blows which penetrate the whole metal and thus reduce grain size. Light forging or rolling work only the outside portions of the metal. No subsequent heat treatment can make up for lack of proper working.

DISK DESIGN. Disks usually are designed by assuming a thickness of disk under the rim, such that it will not buckle or bend during machining and erection. This thickness may be $^3/_8$ in. on small wheels, increasing to 1 in. on some two-row wheels and large-diameter impulse disks. The thickness at other points in a disk of hyperbolic profile is found from the equation $t = cr^a$, where t = thickness, inches; r = radius, inches; and c = a constant found for the conditions under the rim. The exponent a varies from (-0.4) to (-0.8) in multistage impulse wheels and is taken as (-1) for two-row wheels.

Wide variations exist in the disk designs of various builders. Some disks have been made too thin and have given trouble in service because of nodal vibration.

The first step in disk design is to assume the thickness under the rim. A value of exponent a is then chosen, depending on the type of disk desired. Since r is known, the value for the constant c can be found when the thickness is chosen. The thickness at any other radius can be readily calculated from the equation. Present practice on large turbines is to make the disks rather heavy, hence less liable to vibration.

The theoretical hyperbolic curve is modified near the hub to one or more arcs of a circle, with much greater curvature than the hyperbola, to provide a heavy hub in which a keyway may be cut without weakening the disk section. The proportions of the rim are determined by the size, number of rows of blades, and the methods of fastening the blades to the rim. The rim is connected to the narrowest part of the disk by a section of curved profile, frequently consisting of arcs of circles of short radius. The flanks of the disks between the curved sections at hub and rim are sometimes made straight taper or wedge shaped for easier machining. This form is slightly stronger than the hyperbolic profile, and is more easily made. Departures from true hyperbolic form make the solution of disk stresses complicated and tedious.

DETERMINATION OF DISK STRESSES. After preliminary designs are finished, calculations are made to determine stresses. Three stresses, radial, tangential, and axial, may act at any given point. The axial stress is of negligible value if there is no sudden change in axial thickness, as at a hub. Radial and tangential stresses can be calculated by neglecting axial stress. Stodola (Ref. 3) developed formulas for determining these stresses, but they are complicated in the form presented.

S. H. Weaver (Ref. 29) describes a method of calculation which is simpler and more readily applied. For machining purposes radial sections of disks usually consist of straight lines and arcs of circles. The equations of these lines present mathematical difficulties in calculating stresses. Hence the section is assumed to consist of one or more hyperbolas of the equation, $t = cr^a$. In this equation the exponent a, the shape constant of the profile, has a negative value when the thickness decreases with a larger radius, a zero value for a constant or uniform thickness, and a positive value when the thickness increases with radius. For a given disk profile, Fig. 22, the value of a may be found from

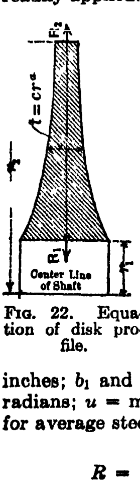


FIG. 22. Equation of disk profile.

$$a = \log_{10}\frac{t_2}{t_1} \div \log_{10}\frac{r_2}{r_1}$$

Stodola's equations for tangential and radial stress are given below.

Notation. m_1, m_2, and p are algebraic quantities as given in the equations of Group II, below; a = shape constant of profile of the particular portion of the disk section; V = Poisson's ratio of deformation = 0.3 for steel; E_1 = Young's modulus of elasticity; R = radial stress at radius r, psi; T = tangential stress at radius r, psi; r = any radius in disk section, inches; b_1 and b_{11} = boundary condition constants; ω = angular velocity of rotation in radians; u = mass of disk material per unit of volume = 0.2815 pounds per cubic inch for average steel; and y = radial elongation, inches.

$$R = \frac{\quad}{1 - V^2}[(3 + V)pr^2 + (m_1 + V)b_1r^{m_1-1} + (m_2 + V)b_{11}r^{m_2-1}]$$

$$\frac{\quad}{1 - V^2}[(1 + 3V)pr^2 + (1 + m_1V)b_1r^{m_1-1} + (1 + m_2V)b_{11}r^{m_2-1}]$$

It will be necessary to know two stresses in order to determine the values of the condition constants, b_1 and b_{11} and to transform these equations. Known radial stresses R_1 at radius r_1 and R_2 at radius r_2 are assumed. The tangential stresses at r_1 and r_2 are

$$\left.\begin{aligned} T_1 &= Ar_2^2 - BR_1 + CR_2, \\ T_2 &= Dr_2^2 - ER_1 + FR_2, \end{aligned}\right\} \qquad \text{(I)}$$

where A, B, C, D, E, and F have the values given in equations of Group II.

The stresses due to the external centrifugal load and the weight of the disk itself vary as the square of the speed. If all stress values are calculated for 1000 rpm, the stresses at any other speed can be found by multiplying by the square of the speed ratio.

The following formulas (Group II) based on 1000 rpm assist in solving for the various stresses:

$$
\left.
\begin{aligned}
K &= \frac{r_1}{r_2}, \qquad a = \frac{\log_{10}(t_2/t_1)}{\log_{10}(1/K)} \quad \text{or} \quad = -\frac{\log_{10}(t_1/t_2)}{\log_{10}(1/K)} \\
m_1 &= -(a/2) - \sqrt{a^2/4 - 0.3a + 1} \\
m_2 &= -(a/2) + \sqrt{a^2/4 - 0.3a + 1} \\
B &= \frac{m_1 K^{m_2-1} - m_2 K^{m_1-1}}{K^{m_2-1} - K^{m_1-1}} \\
E &= \frac{m_1 - m_2}{K^{m_2-1} - K^{m_1-1}} \qquad C = \frac{E}{K^{a+2}} \\
F &= B + a \\
A &= \frac{3.3(C - K^2B) - 1.9K^2}{1 + 0.4125a} \\
D &= \frac{3.3(F - K^2E) - 1.9}{1 + 0.4125a}
\end{aligned}
\right\} \qquad \text{(II)}
$$

The factors A, B, C, D, E, and F are functions only of the shape constant a and the ratio of radii K.

These factors become relatively simple for the portions of the disk where the wheel section is of constant thickness, as at the hub and sometimes at the rim. At 1000 rpm the functions reduce to these equations (Group III):

$$
\begin{aligned}
a &= 0 \qquad K = \frac{r_1}{r_2} \qquad C = \frac{2}{1 - K^2} \\
B &= F = C - 1 \qquad E = C - 2 \\
A &= 6.6 + 1.4K^2 \qquad D = 6.6K^2 + 1.4
\end{aligned}
\qquad \text{(III)}
$$

Since determination of the values of the functions takes time in making a stress calculation, Weaver has prepared the following approximate equations (Group IV) for the more rapid determination of these functions by the use of common logarithms. The error in these equations is about 0.7% as a maximum.

$$B = \frac{\log_{10}(1/K)}{5.43}(a^2 - 1.2a) - (a/2) + \left(\frac{2}{1 - K^2} - 1\right)$$

$$F = B + a$$

$$E = \frac{\log_{10}(1/K)}{10}(a^2 + 10a) - (a/2) + \left(\frac{2}{1 - K^2} - 2\right)$$

between the limits 0.8 and 0.1 for K and (-5) and 0 for a.

$$E = \frac{\log_{10}(1/K)}{7}(a^2 + 10a) - (a/2) + \left(\frac{2}{1 - K^2} - 2\right)$$

between the limits 0.97 and 0.8 for K and 40 and 0 for a. (IV)

$$C = \frac{\log_{10}(1/K)}{3.33}(a^2 + 4.8a) + (a/2) + \frac{2}{1 - K^2}$$

between the limits 0.8 and 0.4 for K and (-5) and 0 for a.

$$C = \frac{\log_{10}(1/K)}{4.65}(a^2 + 6a) + (a/2) + \frac{2}{1 - K^2}$$

between the limits 0.97 and 0.8 for K and 40 and 0 for a.

$$A = 3.1(1 - K)^{2.09}a + (6.6 + 1.4K^2)$$

$$D = 1.25(1 - K)^{1.75}a + (6.6K^2 + 1.4)$$

These functions can be plotted in simple alignment charts with negligible errors (see Refs. 30 and 31).

EXAMPLE. This example shows the method of applying these formulas. Figure 23 is a half section of a two-row wheel disk designed to run at 3600 rpm. The disk is divided into five sections, 1, 2, 3, 4, and 5. The curved profiles are assumed to be portions of hyperbolas whose a value can be found from equation of Group II for each section. All sections are assumed to have the same thickness where each joins the adjacent section. Rings 1 and 5 are of uniform thickness, hence $a = 0$. Ring 4 has a positive value of a, since thickness increases rapidly with the radius; a is negative in sections 2 and 3, since the thickness decreases as the radius increases.

Two of the radial stresses must be known. The radial stress at the bore may be taken as zero, as the shrink fit is supposed to be almost neutralized at normal speed by the centrifugal expansion of the bore, and at some overspeed the radial stress is zero. The outer radial stress of the blade load equals the centrifugal force of blades, shrouds, etc. The centrifugal force is

$$CF = 0.00001421N^2wd_1$$

where N = rpm; d_1 = diameter, inches, to center of gravity of blades; w = total weight of blades, shrouds, etc., pounds. The centrifugal force per inch of circumference at diameter d_2 at the edge of the disk = CF $\div \pi d_2$. This is assumed in the problem at 172 lb per in., at radius r = 15 3/4 in.

The unknown radial stresses at the lines dividing the imaginary rings are taken as e between rings 1 and 2, as f between rings 2 and 3, as g between rings 3 and 4, and as h between rings 4 and 5. The data may now be collected in Table 2. Constants A to F may be calculated from equations of Groups II and III; approximate results may be calculated from equations of Groups III and IV or may be read from the alignment charts referred to above. The values given in Table 2 are calculated from equations of Groups III and IV.

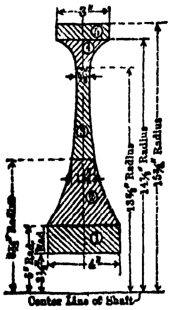


FIG. 23. Profile of a 3600-rpm two-row wheel disk. See also Table 2.

Table 2. Data on Stresses at Lines Dividing Imaginary Rings in Fig. 23

Ring No.	1	2	3	4	5
r_1	3.50	5.00	8.5	13.375	14.875
r_2	5.00	8.5	13.375	14.875	15.75
t_1	4.00	4.00	1.25	0.625	3.00
t_2	4.00	1.25	0.625	3.00	3.00
$K = r_1/r_2$	0.70	0.588	0.636	0.899	0.944
a	0	−2.19	−1.53	14.78	0
R_1	0	e	f	g	h
R_2	e	f	g	h	172
A	7.29	6.02	6.59	8.11	7.85
B	2.92	3.47	3.27	3.73	17.35
C	3.92	1.57	2.30	20.86	18.35
D	4.64	3.11	3.74	7.06	7.28
E	1.92	1.74	1.87	3.43	16.35
F	2.92	1.28	1.74	18.51	17.35

There is only one thickness at any one radius, hence, at the dividing line between any two imaginary rings there can be only one radial stress and one tangential stress. Take the line between rings 1 and 2, at radius 5 in. The outer tangential stress of ring 1 must equal the inner tangential stress of ring 2. Hence,

$$T_2 \text{ (ring 1)} = T_1 \text{ (ring 2)}$$

or

$$(Dr_2^2 - ER_1 + FR_2)_{\text{(ring 1)}} = (Ar_2^2 - BR_1 + CR_2)_{\text{(ring 2)}}$$

Similar equations can be written for each imaginary line at the given radius separating rings, substituting the values of radial stresses assumed above. These four equations can next be solved for the unknown radial stresses e, f, g, and h. The equations for the equal tangential stress at the various radii are:

At radius 5 in.:

$$4.64 \times (5.0)^2 - 1.92 \times 0 + 2.92 \times e = 6.02 \times (8.5)^2 - 3.47 \times e + 1.57 \times f$$

At radius 8.5 in.:

$$3.11 \times (8.5)^2 - 1.74 \times e + 1.28 \times f = 6.59 \times (13.375)^2 - 3.27 \times f + 2.3 \times g$$

At radius 13.375 in.:

$$3.74 \times (13.375)^2 - 1.87 \times f + 1.74 \times g = 8.11 \times (14.875)^2 - 3.73 \times g + 20.86 \times h$$

At radius 14.875 in..

$$7.06 \times (14.875)^2 - 3.43 \times g + 18.51 \times h = 7.85 \times (15.75)^2 - 17.35 \times h + 18.35 \times 172$$

These equations can be solved easily by a substitution method as follows.

From equation for 5 in. radius,

$$e = 0.246f + 49.9$$

Substituting this value of e in the equation for 8.5 in. radius and solving,

$$f = 0.558g + 252.5$$

Substituting this value of f in the equation for 13.375 in. radius and solving,

$$g = 4.71h + 360.8$$

When this value of g is substituted in the last equation it is found that

$$h = 243$$

Substituting in the three preceding equations, the following values are found:

$$g = 1503 \qquad f = 1091 \qquad e = 318$$

The tangential stresses at the various radii can be found by substituting in either side of the foregoing equations the values of e, f, g, and h:

At radius 5 in.:

$$T_5 = 4.64 \times (5.0)^2 + 2.92 \times 318 = 1045$$

At radius 8.5 in.:

$$T_{8.5} = 3.11 \times (8.5)^2 - 1.74 \times 318 + 1.28 \times 1091 = 1068$$

At radius 13.375 in.:

$$T_{13.375} = 3.74 \times (13.375)^2 - 1.87 \times 1091 + 1.74 \times 1503 = 1244$$

At radius 14.875 in.:

$$T_{14.875} = 7.85 \times (14.875)^2 - 17.35 \times 243 + 18.35 \times 172 = 894$$

(NOTE. Some decimals have been dropped to simplify the solutions.)

The tangential stress at the bore is found to be:

$$T_{1_b} = Ar_2^2 - BR_1 + CR_2 = 7.29 \times (5)^2 - 2.92 \times 0 + 3.92 \times 318 = 1430$$

At the rim:

$$T_{2_b} = Dr_2^2 - ER_1 + FR_2 = 7.28 \times (15.75)^2 - 16.35 \times 243 + 17.35 \times 172 = 817$$

The stresses at 1000 rpm are:

At radius	3.5	5	8.5	13.375	14.875	15.75
$R =$	0	318	1091	1503	243	172
$T =$	1430	1045	1068	1244	894	817

The stresses at intermediate points in any ring can be computed from the known values of radii and thickness at these points by equations of Group I.

At 3600 rpm the stresses above should be multiplied by the square of the ratio of the speeds, i.e., by $(3600/1000)^2 = 12.96$.

The resultant stresses are as follows:

At radius	3.5	5	8.5	13.375	14.875	15.75
$R =$	0	4,121	14,139	19,479	3,149	2,229
$T =$	18,533	13,543	13,840	16,122	11,586	10,588

Disks should be able to withstand an overspeed of 20% without exceeding the elastic limit. The stresses at this speed would be $(1.2)^2 = 1.44 \times$ stresses at normal load.

The maximum stress in the above table is 19,479 psi at radius 13.375 at 3600 rpm and at 4320 rpm (20% overspeed) it would be 28,050 psi, which is within the usual elastic limit of steel used in disks. If this stress is considered excessive, the design may be modified by thickening the metal at this point and allowing a smaller taper on the disk. The radial elongation in inches at any radius r in inches is

$$y = (T - 0.3R)r \div E_1$$

where E_1 = modulus of elasticity for steel = 29,000,000 for usual disk material. The radial elongation at 3600 rpm is:

Radius	3.25	5	8.5	13.375	14.875	15.75
Elongation, y	0.00212	0 00212	0.00281	0.00474	0.00546	0.00539

Weaver states that the approximate method generally gives stresses about 1% too high. The great advantage of this method is the reduction of the time required for computation.

H. Haerle (Ref. 32) presents a simple method for the determination of disk stresses from a single diagram, shown in Fig. 24, which can be applied to any disk profile, and yield results sufficiently accurate for all practical purposes. The general formulas for disk stresses given by Stodola, and stated above in discussing Weaver's methods, form the basis of Haerle's method. These formulas reduce to much simpler expressions when applied to a disk of constant thickness, in which case $a = 0$ in the expression $t = Cr^a$, where t = thickness of disk, inches, at radius r, inches, C = a constant, a an exponent governing the curvature and found by means of equations of Group II on page 8–29.

Let T = tangential stress, psi, and R = radial stress, psi.

Haerle's methods are based on the assumption that

$$S = T + R = \text{sum of principal stresses}$$

and

$$D = T - R = \text{difference of principal stresses}$$

The following equations, derived from those of Stodola, apply to a disk of constant thickness:

$$S = (1 + V)\frac{u}{2g}(-U^2 + K_1)$$

$$D = (1 - V)\frac{u}{4g}(U^2 + K_2U^{-2})$$

$$K_1 = \frac{4gEb_1}{(1 - V^2)u} \quad \text{and} \quad K_2 = \frac{8gE\omega^2 b_2}{(1 - V^2)u}$$

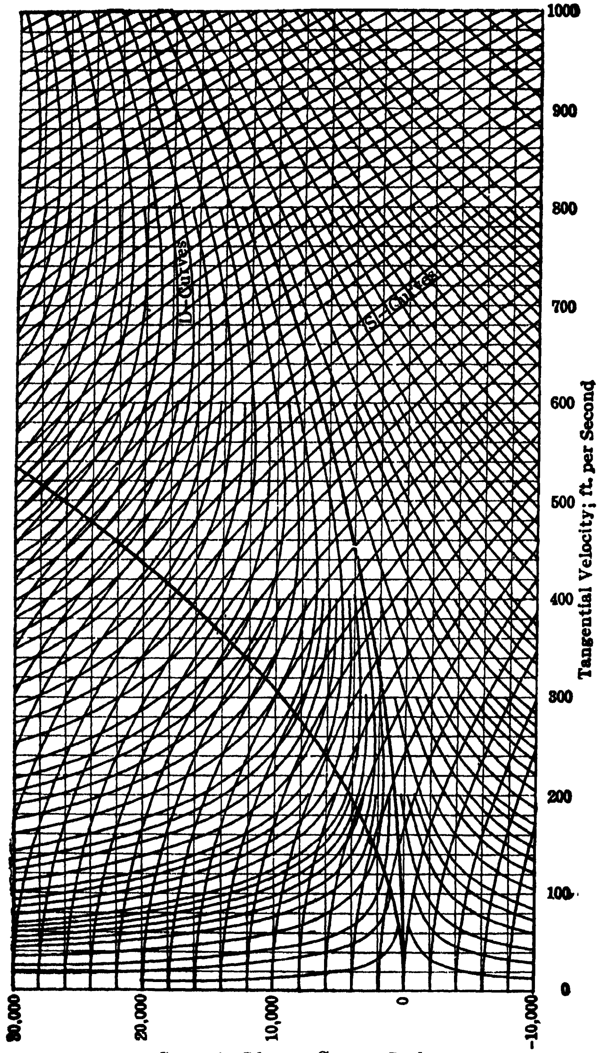


FIG. 24. Disk Stress Diagram.

where V = Poisson's ratio (= 0.3 for steel); u = weight of disk material, pounds per cubic inch; g = 32.2 × 12 (in.); ω = angular velocity, radians per second; U = tangential velocity of the disk, inches per second; E = Young's modulus (= 29,000,000 for steel); and b_1 and b_2 are constants depending on stress conditions at bore and rim as in Stodola's formulas.

The only variables at a given radius in the equations for S and D are K_1 and K_2. Series of curves as represented by the above equations are plotted in Fig. 24, each curve being based on a different value of K_1 and K_2, respectively. The problem is now simplified to selecting the proper curve or set of curves according to the specific details of the disk under consideration and finding S and D from the curves.

If S and D can be determined from the diagram, then

$$T = \frac{S + D}{} \quad \text{and} \quad R = \frac{S - D}{}$$

Figure 24 is plotted with stresses as abscissae and tangential velocities as ordinates. The heavy line on the diagram curving toward the left is the stress curve for a thin ring.

EXAMPLES. These examples show the application of this diagram. In the case of a disk of uniform thickness with a concentric bore and no load on the rim, the radial stress R at both bore and rim must be zero. Then

$$S = T + R = T \quad \text{and} \quad D = T - R = T \quad \text{or} \quad S = D = T$$

at both rim and at bore. That is, the S and D curves must intersect at both bore and rim, and these must be the same curves on the diagram at both places, for, since the boundary conditions are fixed, the values of K_1 or K_2 are the same throughout the disk. For example, let the rim speed be 500 ft per sec, and the bore speed 100 ft per sec. The same S and D curves must intersect at the 500-ft and 100-ft ordinates, as shown in Fig. 25, at 5500 lb. and at 22,000 lb., respectively, which must be

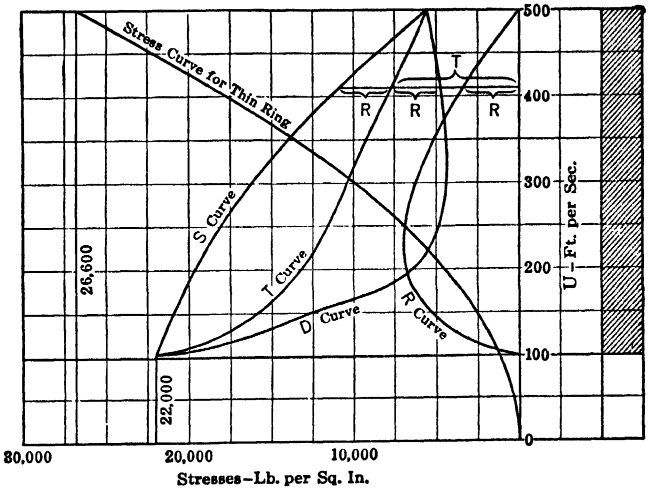


FIG. 25

the tangential stress at these points, since the radial stress at both points is zero. Tangential stresses at other speeds may be plotted by bisecting the distance between the S and D curves at each speed. Radial stresses can be found by halving the difference between the S and D curves and plotting from the zero ordinate.

The ease and rapidity with which problems relating to disks of uniform thickness can be solved by this S and D chart suggest the application of this method to disks of other than parallel profile. If the hyperbolic or other profile is replaced by a stepped disk consisting of a number of concentric rings, each of constant thickness as in Fig. 26, this method can be applied.

The assumption is made that, provided the steps are comparatively small, the stresses in adjacent concentric layers on either side of the step are inversely proportional to the axial dimensions or thickness t' and t, as in Fig. 26. Hence

$$\frac{t}{t'} = \frac{R'}{R} = \frac{T'}{T}$$

Let ΔR denote the increment (positive or negative) of the radial stress at the step. Then

$$\Delta R = R - R' = R\,[1 - (t/t')]$$

The following expression is derived for ΔT:

$$\Delta T = V\Delta R = VR\,[1 - (t/t')]$$

where V = Poisson's ratio = 0.3 for steel.

Combining these,

$$\Delta S = S - S' = 1.3\Delta R$$

and

$$\Delta D = D - D' = -0.7\Delta R$$

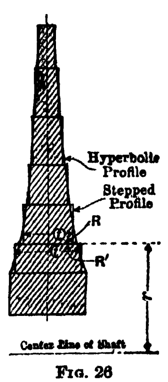


FIG. 26

for the change in the S and D curves at the step. It is now quite easy to determine the S and D values at all points on the irregular disk profile, as shown in the example (Fig. 27), when applied to a disk with tapered sides, for a two-row wheel. The mean diameter is 48 in., and the normal speed, 3000 rpm. The stepped disk, in substitution for the actual profile, is shown by the fine lines. Assumed peripheral velocity of the disk proper, 615 ft per sec. The net weight of blades, shrouds, etc., is assumed as 65 lb at normal speed, exerting a centrifugal pull of 2860 lb per in. of circumference. The net width of the disk at the periphery, after deducting for dovetail grooves for blades is $t' = 2.2$ in. Hence, radial stress due to blade load R (at periphery) $= 2860/2.2 = 1300$ psi. The tangential stress at the periphery may be assumed; let $T = 9000$ psi. Hence, at the periphery,

$$S = 9000 + 1300 = 10{,}300 \text{ psi}$$

$$D = 9000 - 1300 = 7{,}700 \text{ psi}$$

These values constitute the point of origin of the S and D curves across the outermost step. It is assumed that $R = 0$ at the bore. Hence $S = D = T$ at the bore, i.e., the curves intersect at the ordinate of the bore. If forced fit must be allowed for, R at the bore may be chosen with either a positive or negative value to suit the particular conditions.

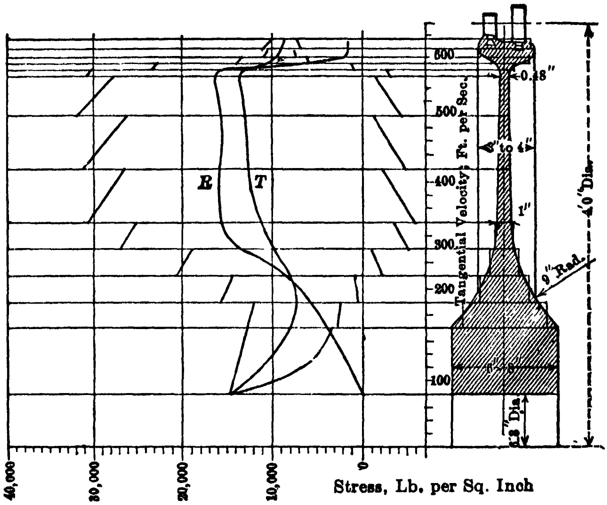


FIG. 27

Table 3. Stresses and Dimensions of Turbine Disk, Fig. 27

U	t	t'	$1 - \frac{t}{t'}$	R *	ΔR	ΔS	ΔD	T Actual	R Actual
615	....	2.2	....	1,300	0	0	0	9,000	1,300
600	2.2	3.26	0.325	2,120	690	900	− 484	9,400	1,300
590	3.26	1.80	−0.81	1,960	−1580	−2060	1150	9,800	2,500
580	1.80	0.68	−1.65	4,100	−6760	−8800	4750	10,900	6,800
570	0.68	0.47	−0.44	11,450	−5030	−6580	3550	13,600	16,300
560	0.47	0.55	0.145	17,050	2480	3220	−1740	14,100	16,600
500	0.55	0.73	0.246	18,250	4500	5850	−3150	13,500	16,200
420	0.73	0.92	0.206	18,170	3740	4860	−2620	13,100	16,200
340	0.92	1.18	0.220	18,440	4060	5300	−2850	11,700	16,000
300	1.18	1.90	0.380	16,280	6200	8060	−4350	10,000	13,600
260	1.90	3.20	0.406	11,500	4670	6100	−3270	8,300	8,800
220	3.20	5.11	0.375	7,800	2920	3800	−2050	7,700	5,900
185	5.11	6.30	0.189	5,150	970	126	− 680	8,300	4,200
82	6.30	6.30	0.0	0	0	0	0	14,800	0

* Values at inner diameter of steps.

Beginning at the tangential velocity of the periphery, follow the S and D curves across the radial extension of the first step, i.e., from tangential velocity, 615 ft per sec, to tangential velocity, 600 ft per sec. At step 600, read $R = (S - D)/2$, find ratio t/t' from the substituted profile, and calculate ΔR, ΔS, and ΔD. Plot the new S' and D' for the second step and continue the S and D curves across it. The procedure is the same for all other steps. If tangential stress T at the periphery has been chosen correctly, the S and D curves for the innermost step must intersect at the bore of the disk. If they do not, another value of T at the periphery may be chosen and the process repeated until intersection occurs at the bore.

The axial thickness of the several concentric rings which form the substituted disk section correspond to the thickness of the true profile at points midway between the steps, and the stresses as determined by the S and D curves at these points coincide with the actual stresses in the original profile. Hence, if the horizontal distance between the S and D curves in the center of the stepped rings is bisected, points on the true tangential stress curve are obtained. A smooth curve through this series of points represents graphically the distribution and magnitude of the tangential stress T throughout the disk. The radial stress R at the middle of each step can be determined by measuring the distance of the tangential stress curve T from the S or D curves, and plotting this distance from the zero axis at the various radii. Another smooth curve through these points indicates the magnitude and distribution of the radial stress R. Both these curves are clearly shown in Fig. 27. Values of R and T have been scaled off from these curves at radii corresponding to the various peripheral speeds U in the first column of Table 3, and are tabulated in the last two columns for general information. They could be scaled at any other point if desired. The maximum stress in this disk is a radial stress of 16,600 psi and it exists just below the rim.

M. G. Driessen (Ref. 33) discusses Haerle's method as outlined above and presents a larger chart for S and D curves than Fig. 24. The purpose of this paper is (1) to shorten the steps necessary to pass from one element of the disk to the next; (2) to provide an alternative to this cut-and-try method; and (3) to indicate a manner in which the results once obtained can be used for all other conditions under which this disk is used, i.e., for different loads at the circumference and for different speeds. The suggestions simplify Haerle's method.

As in Haerle's solution, the change of stress at a change of section is assumed inversely proportional to the thickness. Driessen proposes to find the new S and D curves for the new section as follows: ΔR for the new section $= R - R' = R(1 - t/t')$. Haerle shows that $\Delta S = 1.3\Delta R$, and $\Delta T = -0.7\Delta R$. For the first section $S - D = 2R$ and for the second section $S' - D' = 2R'$. The change is $(S' - D') - (S - D) = 2(R' - R) = 2\Delta R$. Hence $\Delta S - (1.3/2)\Delta R$ and $-\Delta D = (0.7/2)\Delta R$. If $(S - D)$ and $(S' - D'$ are measured in inches on the chart, the distances ΔS and $-\Delta D$ in inches can be readily found, and the new points for starting the S and D curves for the changed sections easily are located on the chart and the quantities in psi can be read from the diagram.

In Haerle's method the radial and tangential stresses at one diameter, usually the hub, were assumed, and the rim stresses determined. If the radial rim stress did not fit actual conditions, the disk was recomputed for other assumed tangential stresses at the hub until the desired radial rim stress results from the calculations. Driessen shows that if the tangential stress at the hub, T_h, is plotted against the radial stress at the rim, R_r, it follows a straight line. Hence when two points in this relationship are found, a straight line can be drawn through these points and any other T_h can be determined from this line for the actual R_r that prevails. This greatly shortens the computations.

Driessen points out that with large shrink fits, the stresses no longer are proportional to the squares of the speeds. He then outlines a method for computing and recording the limiting stresses of any disk, so that, if an earlier design of disk is considered for a new turbine, reference to these limiting stress values will determine its suitability.

Other methods for calculating disk stresses will be found in Refs. 3, 4, 21, and 34.

SHRINK FITS FOR DISKS. Experience has shown that disks tend to work loose on shafts if shrinkage allowance in the bore is insufficient. This may be due to enlargement of the bore from centrifugal stress at speed, or from relaxation due to high steam temperatures. Robinson (Ref. 35) and others discuss the first cause. Disk materials should be tested for relaxation at working temperature. Disks are usually designed for 100,000-hr operation without exceeding 80% of ultimate relaxation. The bore is proportioned to produce a stress in the cold state of 80 to 100% of the yield strength.

Disks frequently are secured to the shaft by a conical bushing. When a keyway is used on a disk, the key should serve only as a safety member, and exert neither radial nor lateral pressure. Sometimes two keyways are cut at opposite ends of a diameter to equalize stresses and to maintain balance. Disks of some turbines are held on the shaft by four fitted keys 90 degrees apart, by means of which expansion can take place equally in all directions and shrink fits are of less moment.

Often disks are bored with an allowance for pressing on the shaft of 0.0015 in. per in. of bore diameter, requiring 1 1/4 to 2 tons pressure per inch of shaft diameter to force each disk in place. Such press fit compensates for the increase in bore of the hub of the disk when under stress and prevents it from creeping along the shaft. Many builders shrink disks in place by heating the disk and chilling the shaft. This has advantages over forcing the disks into place by means of a hydraulic press.

In turbines having separate wheels (as opposed to the solid-rotor construction) it has been found necessary to be sure that the wheels stay central and drive the shaft under all temperature and stress conditions. This is accomplished by a pin bushing, keyed to the shaft with the wheel hub attached to the bushing by a number of radial pins. A separate key and bushing is provided for each wheel. When packing sleeves are placed between wheels, they are undercut to reduce heat transfer to the shaft should a rub occur. A lock or locating ring for each disk may be fixed on the shaft to prevent axial movement of the disk along the shaft.

DISK VIBRATION. (See Ref. 36.) Failures of some of the disks and buckets of early turbines led to intensive studies of vibration. These developed the fact that stationary disks and buckets vibrated harmonically in an even number of segments, between which were radial lines of quiet, called nodes. They appear as 4, 6, 8, 10, 12, or even higher numbers of nodes. Between these given frequencies the wheel was comparatively quiet. The higher the number of nodes, the higher the frequency of vibration and the less easily is vibration started. Both disk wheel and buckets vibrate as a continuous disk and must be considered as a unit in this type of vibration. The frequency of vibration is determined by two factors: *a*, the stiffness, and *b*, the mass of the vibrating body. The stiffer the body, the faster it vibrates, and the more massive it is, the slower it will vibrate. Centrifugal force exerts a stiffening effect on a disk and increases the frequency at which the nodes appear.

The combined frequency of a particle, f_r, due to the combined effects of stiffness and centrifugal force is $f_r = \sqrt{f_s^2 + BN_s^2}$, where f_s = natural frequency of a particle of mass m, with an elastic support of such stiffness that a force R_s is required to produce unit deformation; $f_s = (1/2\pi)\sqrt{R_s/m}$; N_s = revolutions per second; B = a speed coefficient which varies with the design of the wheel and the type of vibration. B has a low value when vibrating sectors extend well into the wheel, and higher values as the number of nodes increases. B usually varies from 2 to 3.

The critical speed $N_s = f_s/\sqrt{(n/2)^2 - B}$, where n = number of nodes. Minor resonant speeds occur, the equation for the first of which is $N_{s1} = f_s/\sqrt{(1 + n/2)^2 - B}$. (See Ref. 36 for other values.)

When disks are revolved, their circumferences develop, at certain speeds, a form of wave motion which travels around the wheel circumference in the opposite direction to rotation. This wave motion has an even number of nodes which move around the wheel with the wave. Every part of the wheel rim thus vibrates over a period of time during each revolution. If the number of nodes is the same, the frequency of vibration of every particle along the edge of a disk wheel is the same either for standing vibration or for traveling waves. The speed of the traveling wave per second equals the number of complete vibrations of the corresponding standing wave per second multiplied by the length of a complete wave. For a traveling wave all particles vibrate through the same amplitude, but their time phases vary successively along the wheel edge.

When these backward traveling waves have the same speed backward that the wheel has forward, a standing wave results. This condition has been most conducive to disk and blade failure. The speeds at which the standing wave forms are called the *wheel critical speeds*. Wave trains in disks may be started by the application of a small extra force at a given point due to uneven nozzle dimensions, thick partitions, or other lack of symmetry. These waves persist after formation if the speed is suitable for the wave.

The effects of vibration are: (1) The wheel may burst. (2) The wheel may rub. (3) Buckets may fail from fatigue. Mathematics has been developed to predict disk stresses, and disks are made heavier than formerly, but one can be sure of a wheel only after a direct test. For this purpose, machines have been developed in which to test disks for vibration under working temperatures and speeds. Safe limits between the operating speed and the critical speed are, for 4 nodes, 15% above or below the operating speed, and for 6 nodes, 10% above or below.

When disk wheel critical speeds fall within these limits, the wheel is *tuned*, that is, metal is removed either from the disk itself or from the buckets until the critical speed has been shifted beyond the specified limits. Disks are smooth finished throughout so that no tool marks may form starting points for fatigue cracks. All sharp edges are rounded off for the same reason.

BUCKET VIBRATION. In addition to disk vibrations, the buckets themselves may be excited into vibration. These vibrations may be in the direction of minimum or maximum stiffness, or they may be torsional vibrations of a bucket group. They are started by irregularities in the steam flow path which result from obstructions such as nozzle partitions and struts in the exhaust chamber. The vibrations set up stresses in the buckets, bucket fastenings, shrouds, and lacing wires which add to the stresses already imposed by centrifugal force and steam load.

Because the magnitude and wave form of the exciting force are unknown, the absolute value of vibration stresses cannot be calculated. Therefore, blades must be designed by allowing a margin above centrifugal and steam stress, in which vibration may increase the stress but not exceed the endurance limit. A method of making this calculation is given by Timoshenko (Ref. 37).

The amplification factor (margin by which vibration may increase the stress) is determined largely from experience, but in buckets whose lowest natural frequencies are relatively high harmonics of running speed (12 or more), vibration is difficult to excite, and vibratory stresses usually are small. For natural frequencies between the sixth and twelfth harmonic of running speed, a larger margin of stress must be provided for vibration. At harmonics lower than 6, large enough margin of stress cannot be provided, and it is necessary to design the bucket so that none of its lower natural frequencies coincides with a harmonic of running speed. To do this the first mode frequencies of the blades in the direction of minimum stiffness and of maximum stiffness may be calculated by the Stodola method (Ref. 38, p. 188). Calculations of the torsional mode and of the higher modes of bending vibration can be made but they are complicated, and tests are usually made for their determination. To adjust values of the various frequencies so that resonance is avoided at all harmonics below 6 requires adroit handling of bucket mass distribution, stiffness, root strength, number, position, and size of lashing wires and shroud, and selection of number of buckets in a group.

Centrifugal Stiffening Effect. In making frequency calculations, account must be taken of the increase in natural frequency due to the stiffening effect of centrifugal force. An approximation of this may be calculated (Ref. 38, p. 309) but most reliable information is obtained by test. The frequency at running speed may be obtained from

$$f^2_{\text{rotating}} = f^2_{\text{stationary}} + K\left(\frac{\text{rpm}}{60}\right)^2$$

where f is frequency and K is a coefficient which depends on bucket design and the type of the vibration. Values of K obtained from rotating frequency tests vary from 1 1/2 to 15, depending on the frequency and mode of vibration.

Tangential vibration may be set up in the buckets themselves at certain frequencies. Long slender blades must be so designed and tuned that the critical speed falls outside the limits above noted. Blades can be tuned by affecting their stiffness by: (1) soldering or welding the ends to a shroud; (2) adding an intermediate stiffening member between tip and base (lashing wire); and (3) decreasing the mass of the buckets. The position of the lashing wire has a very great effect on the resistance of blades to vibration.

The frequency of vibration of a reed is $f = tC/l^2$, where f = frequency, cycles per second; l = length, inches; t = thickness, inches; and C = constant of proportionality. For turbine buckets a factor S represents the scale of equivalent thickness. Comparing two buckets $S_2/S_1 = f_2l_2^2/f_1l_1^2$. Thus a test bucket is 14 in. long, with a frequency of 62. The scale of thickness for a similar bucket 24 in. long with a frequency of 37 is $S_2/S_1 = 37 \times 24^2/62 \times 14^2 = 1.755$. That is, the required thickness of the new bucket is 1.755 that of the one tested.

Buckets that may be subject to resonant vibration, which leads to fatigue failure, are heavier and somewhat wider than earlier forms and have both inlet and outlet edges rounded. Where possible, long buckets are made sufficiently stiff without the use of lashing wires.

For further details and the mathematics of the subject, see Refs. 36 and 39–44.

Compound Vibration. While the critical speed can be calculated by the preceding formula, and the expected vibration will occur at that speed, it often is noted in practice that the turbine goes through several critical frequencies before reaching the calculated critical speed, owing to the elastic scale of the foundations and the masses of the turbine parts. The foundation is subject to periodic forces induced by the turbine speed, to the influence of the superimposed mass, and to the elasticity of the supports. If deflection in the supports is large, vibration is increased. On the other hand, steel columns supporting the unit may be too stiff. The completed unit, therefore, frequently is studied by a vibrometer, and adjustment made both in machine balance and in foundations to secure the desired quiet operation at normal speed.

DISK LOSSES. Formulas for disk losses give conflicting values. Research has not definitely fixed these data. "Idle blades" are those revolving blades that are not receiving steam at the moment.

The following formulas may be used for disk and idle blade losses:

$$L_D = \text{disk loss, kilowatts} = 0.042D^2\,\frac{(u/100)^{2.9}}{v}$$

$$L_B = \text{idle blade loss, kilowatts} = \frac{0.19Dl^{1.25}(u/100)^{2.9}s}{v}$$

where u = wheel speed at mean diameter, feet per second; D = mean diameter, feet; l = blade length, inches; v = specific volume of steam, cubic feet per pound; and s = fraction of mean circumference *not* receiving steam from nozzles. For a two-row wheel, a correction factor of 1.23 is applied to L_B.

Another formula for disk loss is

$$L_D \text{ (in kilowatts)} = 0.03D_d^2(u/100)^3/v$$

where D_d diameter of disk at base of bucket feet; u and v are as above defined.

The idle bucket loss L_B can be diminished to 0.25–0.50 of the above value by enclosing the idle section by a channel-shaped ring or *shield* fastened to the casing.

The loss L_B given above for rotation in the normal direction is approximate. For backward rotation, as in the reverse elements of marine turbines, L_B must be multiplied by factors ranging from 10 for short blades to 30 for blades 8 in. long.

Data and other formulas on disk and blade losses may be found in Refs. 3, p. 201; Ref. 4, p. 529; Ref. 16, pp. 77-78; Ref. 28; and Ref. 45.

Displacement loss or nozzle-end loss is the energy required to sweep out inert steam from idle bucket passages when they come into the active arc of steam admission. This loss L, expressed as a fraction of the total steam flow, is

$$L = nK_d \frac{b}{AD}$$

where n = number of separate groups of nozzles interrupting continuous flow; b = bucket width of first rotating row, inches; A = percentage of circumference covered by nozzles; D = mean diameter of stage, inches; and values of K_d are:

Velocity ratio	0.05	0.10	0.15	0.25	0.30	0.40	0.50	0.60
K_d (two-row)	0.02	0.04	0.07	0.12	0.14	0.20	....	...
K_d (single-row)	....	0.03	0.05	0.09	0.11	0.15	0.19	0.235

SHAFT DIAMETERS for impulse turbines are a compromise between small diameters, which give low diaphragm-packing leakage, and the need for a reasonably stiff shaft, to insure operation above critical speed. Shaft sizes tend to larger diameters, favoring safe operation at some sacrifice of efficiency. With spherically seated bearings, the maximum deflection of shafts carrying disks varies from 0.005 to 0.030 in. Shafts with solid bearings are stiffer, and have about one half of these deflections.

Kearton suggests as a first approximation of shaft diameter: $d = l\sqrt{ND/K}$, where d = mean diameter of middle portion of spindle, inches; l = bearing span, inches; N = rpm; D = average diameter of disks, inches; K = a constant; for land turbines K = 6,000,000 or higher.

Rotors of reaction turbines consist of solid shafts throughout, of solid shafts in the high-pressure section with rings or disks on the low-pressure end, of a hollow cylinder fastened rigidly to the spindle ends, or of other modified constructions. In reaction turbines with small radial clearances of blades, larger and stiffer shafts are used than with impulse turbines. With spherically seated, self-adjusting bearings, the maximum deflection varies from 0.001 to 0.005 in. When high mean blade speeds are necessary in the low-pressure section of a reaction turbine, disks of hyperbolic or conical cross section, either integral with the spindle end or placed as rings on the shaft, are used. These disks carry one to seven rows of blading.

A bore hole concentric with the finished rotor, with a maximum eccentricity of about 0.02 in., is drilled through each large shaft for complete periscopic inspection of its interior metallurgical structure. This hole assists in attaining uniform heating and in relieving stresses.

In one design, solid disks are welded together at their peripheries, and annealed to eliminate welding stresses, no through shaft being provided. The advantages claimed are that the stiff rotor runs far below its critical speed; the distribution of material is excellent; the spindle heats quickly and uniformly; the wheels cannot work loose; no keys or keyway are needed; the stresses are small; and the weight is low.

STRESSES IN DRUM AND OTHER SOLID ROTORS. Solid rotors are used in many central station and main propulsion marine turbines. For the reaction-type machine, these rotors are grooved only for the rotating blade fastenings. For the impulse type machine a deep groove is cut between the wheels to provide space for the nozzle diaphragm, thus giving a rotor that is essentially a shaft with integral disks.

Mean tangential stress is the principal design criterion in either construction. Stresses computed on the basis of elastic theory are of secondary importance. This is particularly true of the tangential stress at the inspection bore of such rotors. Plastic flow redistributes the stress in the relatively ductile material which is used, and the rotor approaches a condition where the tangential stress is uniform from periphery to bore. This redistribution takes place rapidly when a rotor is overspeeded. It takes place more slowly, but just as surely, when a rotor is operated at design speed and elevated temperature. The calculated mean tangential stress then becomes the basis for the prediction of the overspeed bursting strength and the creep of the rotor at normal operating speed.

The mean tangential stress T_m, psi, is computed from the formula:

$$T_m = \frac{KI}{A} + \frac{P}{2\pi A}$$

where $K = 8 \times \left(\frac{\text{rpm}}{1000}\right)^2$ (for steel, where density = 0.28 lb per cu in.); I = rectangular moment of inertia about axis of rotation of cross-sectional area of solid of revolution (one side of axis only); A = net cross-sectional area capable of carrying tangential stress on one side of axis of rotation only, square inches; and P = total centrifugal force of buckets and any other parts not included in the section from which I is determined, pounds.

The limit for this mean tangential stress is then decided on two considerations. First, from the standpoint of a reasonable factor of safety for overspeeding, it is limited to one-third the yield strength of the material used at the normal operating speed. Second, it is limited to a value which will give an acceptable creep during the expected life of the machine at the temperature for which the machine is to be used.

STEEL FOR ROTORS. Carbon steel is still employed where stress and temperatures permit its use. Its service record has been good. Its heat treatment is known. However, increased stress and temperatures of 825 F and above require the use of alloy steels.

Mochel (Ref. 46) points out that the rotor must have physical characteristics to enable it to withstand rotation stress and to transmit load; at the same time it must operate smoothly for many years. The latter requirement can be met only by such heat treatment as will relieve all internal stress from the forging. Otherwise slow relaxation at operating temperatures may cause deformations which lead to rough operation. These careful heat treatments are specified in Mochel's paper and attention is called to factors influencing stress relief.

Mochel offers specifications for rotor steels, some of which are given in Table 4.

Table 4. Rotor Steels

Type of Steel	Carbon	Alloy 1	Alloy 2
Carbon	0.48 max	0.43 max	0.43 max
Manganese	0.40-0.80	0.40–0.80	0.40–0.80
Phosphorus	0.04 max	0.05 max	0.04 max
Sulfur	0.045 max	0.05 max	0.045 max
Nickel	....	2.50 min	2.50 min
Chromium	... .	0.30–0.60	0.60 max
Molybdenum	... .	0.20 min	0.30 min
Vanadium	.. .	....	0.25 max
Tensile strength, psi	75,000	90,000	95,000
Yield strength, psi	40,000	65,000	65,000
Elongation in 2 in., %	20.0	18.0	18.0
Reduction in area, %	35.0	40.0	35.0

See also ASTM A293 covering Carbon-steel and Alloy-steel Forgings for Turbine Rotors and Shafts and ASTM A294 covering Carbon-steel and Alloy-steel Forgings for Turbine Bucket Wheels.

DYNAMIC BALANCE. All rotors must be in dynamic balance to avoid excessive vibration at high speed. Lack of balance may be due to nonhomogeneous disk or drum material, to slight eccentricity of the rotating masses, or to errors in workmanship. Static balance first is obtained by mounting the shaft on carefully leveled knife edges and applying counterweights until it remains at rest in any position. Disk wheels are balanced similarly by mounting them on true arbors.

Static balance of the assembled shaft and its disk or drum is no assurance that it will be in good dynamic balance, as two heavy masses in the completed rotor may be placed so as to form a static couple. This will cause severe vibration at high speed, because of the dynamically unbalanced centrifugal forces resulting from these masses. They may be balanced by providing additional masses to set up an equal and opposite couple.

Dynamic balancing of high-speed rotors is required where the center of weight does not coincide with the axis of rotation. Balancing usually is accomplished by adding weights in tapped holes in a disk or in the shaft periphery in a position opposite the heavy point. For rotors that operate below the first critical speed, a balancing plane at each end of the rotor is usually sufficient to correct unbalance. Rotors that operate above the first critical speed may require additional planes if accurate balancing is necessary. The use of additional planes permits balance weights to be placed more nearly in the same axial plane as the unbalance, thus eliminating the introduction of internal bending moments which tend to bow the shaft at high speed. Balancing machines indicate the resultant of unbalances within the rotor but cannot show the distribution of the individual unbalances. When such unbalance distribution occurs in long rotors, the distribution of the balance weights may be estimated by analysis based on observation of the vibration at several speeds. In a well-balanced rotor, the mass center should be within 0.00025 in. of the true axis of rotation.

All manufacturing plants now have dynamic balancing machines in their plants where rotors, partly or completely assembled, can be quickly and accurately balanced. When it is necessary to balance a rotor in the field, it is run up to the operating speed, the high spot is marked, and counterweights are added by a cut-and-try method until balance is obtained. This operation is tedious and difficult, and final balance depends on the skill and experience of the operator.

CRITICAL SPEED. As every horizontal rotor deflects under its own weight, it is never possible to have the center of mass and the true center line of the shaft coincide. As the rotor speeds up, this eccentricity of mass results in an increasing centrifugal force tending to bend the shaft. At a certain speed this unbalanced centrifugal force *neutralizes* the elasticity of the shaft which resists deflection. The shaft deflection increases progressively, and, if unrestrained, failure would result. In an actual turbine the shaft will rub before this happens, causing considerable damage. The speed causing indefinitely large deflection of the rotor for a small initial eccentricity is the *critical speed* of the shaft. If the speed is increased above the critical speed, the shaft begins to straighten and tends to revolve about its true center of mass. Operation may be very smooth under these conditions, although other critical speeds may be encountered at still higher speeds.

The calculation of critical speed is difficult, except in the simplest forms of shafts and wheels. Methods of finding critical speeds will be found in Refs. 4, 21, and 47. Some engineers use this formula for a rough approximation:

$$\text{Critical speed, rpm} = 188/\sqrt{y}$$

where y = maximum shaft deflection, inches. The deflection y depends on bearing and coupling conditions, being less with solid bearings and couplings than with flexible ones.

Some small turbines run above their critical speed. Hence, in starting, they pass through this speed. To avoid serious deflections they are brought to speed quickly, passing through the critical speed so fast that no extreme vibration can occur.

Running speed should be 25% below the calculated critical speed, for "stiff-shaft" machines. For flexible shaft machines (which normally run above the critical speed) the running speed should be 15% above the calculated critical speed. In general the actual critical speed is less than the calculated and tends to equalize these margins. Balance also affects the actual critical speed.

Rotor shafts often are bent if a rub occurs from any cause. This is due to local overheating on the side that rubs. The resulting expansion makes the rub worse, and finally the overheating causes a permanent set in the shaft. Danger from rubs of such character is being reduced by the use of thin metal labyrinth packings in diaphragm glands and on shrouds and the use of thin-tipped blades in reaction blading when shrouds are not used.

6. TURBINE DETAILS

THRUST BEARINGS. There is little end thrust in impulse turbines. In small units, thrust bearings frequently consist of shaft collars on each side of one of the bearings. Ball bearings are used as thrust bearings in other types of small turbine.

Large turbines of early design frequently had a marine-type collar thrust bearing. Most modern impulse turbines have a single runner on the shaft which revolves between babbitt-faced bearing rings. Kingsbury thrust bearings are also widely used on large turbines.

The direction of thrust in operation is under control of the designer. End play of 0.010 to 0.015 in. may be permitted on the shaft with Kingsbury bearings. Thrust toward the exhaust tends to increase the clearances of end-tightened blades. Some builders, therefore, provide for thrust towards the steam end, where the thrust bearing is located, and provide position indicators to check shaft location. This insures minimum leakage, with end tightening.

Kingsbury Thrust Bearings. Reynolds' theory of lubrication has been applied to thrust bearings by Albert Kingsbury in America, and A. G. M. Michell in England, and their names designate forms of bearings widely used by turbine builders.

The Kingsbury bearing for steam turbines usually is designed to carry a load of 250 to 425 psi to allow for dirty or worn oil and to provide a wide margin of safety. Under ideal conditions it could carry 3000 psi. In the fixed type of Kingsbury thrust bearing, liners are placed between the thrust bearing cage and the pedestal to fix the spindle position in the turbine. The bearing sometimes is mounted in a cage which can be adjusted axially from the outside to locate the moving blades relative to the casing. Thrust collars of large turbines are integral with the shaft.

The mean speed on Kingsbury thrust blocks may be over 200 ft per sec. However, to keep the diameter small, lower speeds generally are used. Ample quantities of oil at low velocities must be supplied. Thermocouples are sometimes placed in thrust shoes to check incipient failure.

COUPLINGS. The claw-type coupling, which has a certain amount of flexibility, consists of two halves, each in two parts. The inner sleeve, keyed to the shaft, has jaws on the outer flange. The outer sleeve has a plain flange to bolt to the other outer sleeve on the other shaft. A set of claws, cut on the other end of the outer sleeve, fits into the jaws of the fixed sleeve. The two halves of the coupling are held together by fitted bolts. Lubrication of the bearing surfaces of the jaws is usually insured by an oil catcher and holes through the jaws to the bearing faces. Hardened steel plates are used on the jaw wearing surfaces. Various forms of pin-type and other couplings are used on small turbines. Falk couplings are also in service.

Flexible couplings of the Fast, Waldron, and Poole types comprise two hubs, each keyed to its respective shaft. Each hub has external spur teeth cut on it, at the maximum distance possible from the shaft end of the hub. A sleeve surrounding these hubs is flanged and split vertically at its center for disconnecting the two shafts. The two halves are bolted together through the flanges. Each half of the sleeve has internal spur teeth cut on its bore at its outside end, which engage the external teeth of the hub. The sleeve is carried at each end by an oiltight supporting ring. The error in alignment of the two shafts can be about ten times the clearance between the external and internal teeth, which are in an oil bath when in operation. These couplings have proved very satisfactory where expansion from heat, as on turbo drives for auxiliary equipment, makes it difficult to maintain correct alignment.

Solid couplings are extensively used on large turbines. They stiffen the shafts of both turbine and generator but require careful alignment. The coupling flanges may be integral with the shaft or shrunk on.

Various plans for securing true alignment include leveling pads on the bedplates, squaring and leveling coupling faces, stretching piano wire over the centers and checking up, and the use of surface gages. Allowance must be made in noncondensing units for expansion above the bedplate on heating up.

Turbines may be thrown out of alignment by pipe strains. Piping must have bends so arranged that no strains are transmitted to the turbine casing.

If the condenser is bolted to the turbine exhaust, consideration must be given to moments due to cooling water piping and to any eccentricity due to condenser water loading. If the condenser is not bolted to the exhaust, loadings due to the expansion joint may affect alignment.

DUMMY PISTONS. Balance, or dummy, pistons are used on reaction turbines to equalize the thrust toward the exhaust due to difference of pressure between the inlet and outlet of each row of moving blades and also to unbalanced pressure on annular surfaces when the drum is stepped-up in size. Two-step dummy pistons are used on large turbines, and single-step pistons on small units. Pipes fitted outside the casing equalize the pressures on the pistons with corresponding pressures on the bladed sections. Any unbalanced thrust is taken by the Kingsbury bearing.

Dummy packing in many designs consists of several axial knife edges projecting from the stationary elements which extend toward radial lands on the revolving pistons, thereby increasing the number of throttlings in a given axial length and reducing the distance between shaft bearings. Experimental data are used as a basis for calculating losses from such dummies. Dummies must provide space between throttling points to act as expansion chambers, must dissipate heat readily should contact occur, and one material must wear away rapidly with little heat generation. With radial clearances, dummies frequently consist of plain radial strips alternately deep and shallow, sealing against alternately low and high lands on the spindle dummy piston. Radial dummies depend on throttling through the small radial clearance at the tips of the projecting teeth for reducing the leakage. Both radial and axial sealing strips have been used. Sealing strips of nickel ribbon, chromium stainless steel, and other alloys are used.

Area of Dummy Pistons. Goudie (Ref. 4) says that the dynamic thrust on the blades in an axial direction is usually less than 1% and never exceeds 2% of the total thrust in reaction turbines, and may be neglected. For the annular area A_d of the balance piston for each cylinder of reaction blading, he gives on p. 424, the formula

$$A_d = \frac{P_2 A_1}{P_1 - P_c} + \frac{\Sigma(P_i - P_o)a}{2(P_1 - P_c)}$$

where P_1 = pressure on front of dummy, psia; P_2 = difference of pressure, psia, on any annular drum area A_1; A_1 = annular area of any step-up in drum at entrance; P_c = condenser or back pressure, psia; $(P_i - P_o)$ = drop in pressure in a group of blades of constant mean diameter; a = annular area between drum and casing at a group of blades; pressures are psia; areas are in square inches.

Leakage of Steam through Dummy Pistons. H. M. Martin (Ref. 48) discusses the leakage of steam through dummy pistons and submits a formula which is claimed to check within 1% of the actual loss. From this formula the following equation is derived:

$$w = 0.4722A\sqrt{\frac{P_1}{v_s}} \times \frac{1 - (1/r^2)}{N + \log_e r}$$

where w = steam leakage, pounds per second, through whole dummy; A = area available for flow of steam at any dummy constriction, square inches; P_1 = initial pressure, psia; v_s = specific volume of steam at pressure P_1, cubic feet per pound; N = number of throttlings in the dummy; r = ratio of initial to absolute final pressure over the dummy = P_1/P_2, where P_2 is the pressure on the rear side of the piston, psia. This is the formula generally used by American turbine builders. When the high-low type is used with sealing strips, the coefficient is reduced from 0.4722 to 0.40. Other values, based on tests, are used by some manufacturers.

Clearance of Dummies. Radial dummies must be used where the balance piston is distant from the thrust and considerable expansion can occur. Radial clearances on such seals are about 0.001 in. per in. of diameter. Side-contact dummies are ground to fit by revolving the spindle slowly and drawing up on the thrust until contact occurs. They are afterwards set, when thoroughly heated, by drawing up on the thrust, the spindle revolving very slowly, until first contact is heard by listening on the casing. The thrust then is moved to obtain the desired running clearance, which varies from 0.004 in. on small reaction turbines up to 0.012 to 0.015 on large turbines. Balance pistons near the middle of the spindle require somewhat larger clearance. When finally set, all thrust blocks are locked after allowing sufficient end play for lubrication. This adjustment should be checked periodically to detect wear in the thrust collars due to clogged oil supply or dirty oil.

LABYRINTH SEALS. Glands must be provided in all turbines where the shaft leaves the casing. Impulse turbines also require glands where the diaphragms between stages encircle the shaft.

Carbon ring glands comprise several carbon rings, each in its own compartment of a cast-iron or steel case. The several segments of the ring are pressed together by a garter spring or an arched flat spring. The rings usually are divided into three or four segments. Clearances on the shaft diameter are from zero to approximately 0.006 in., depending on the size of shaft.

Dummy piston glands are used in certain reaction turbines.

Labyrinth glands for diaphragms consist of cut or inserted teeth projecting from the shaft toward a smooth stationary casing, from the casing toward the shaft, or from both with the teeth alternating, similar to radial clearance dummy pistons. The ends of these teeth are knife-edged, and usually clearance of 0.002 in. per foot of bearing span is allowed in design. These constrictions throttle the steam into a larger space, where it forms eddies and restricts flow. Alternate high and low lands break up steam flow and reduce leakage.

The high-pressure labyrinth gland, where the shaft leaves the casing, can be made in two sections. The longer inner part seals against the internal pressure, the outer part against atmosphere. Steam is withdrawn from or supplied to the gland at this middle point. Steam above atmospheric pressure must be supplied at the intermediate point in the exhaust-end gland to seal it against air leakage, which would destroy the vacuum. An alternative common arrangement uses a water seal impeller capable of sealing up to 5 to 10 psig pressure to avoid the necessity for sealing steam at the outer end. Marine turbines (variable speed) must, however, use the steam sealing arrangement, because the water seal is not effective at reduced speed.

High-pressure gland leakage either is used in a low-pressure gland, or led from an intermediate pressure take-off to a feedwater heater. Provision usually is made to admit live steam to both low-pressure and high-pressure glands to seal them on starting. The clearance between the teeth of these glands and the shaft or casing is usually 0.005 to 0.020 in. Materials used for labyrinth glands are babbitt, aluminum, brass, and bronze, and for high temperatures, stainless or other alloy steels. An eductor is provided on these glands to prevent steam leakage into the room.

Some designs of labyrinth glands are planned so that any rubbing causes the parts automatically to separate. Glands made of packing rings of anti-friction metal are used on some small turbines which hardly warrant the expense of a more elaborate type. Relief grooves cut in the shaft on each side of labyrinth packings prevent bowing of shafts by localizing the heating if rubbing occurs.

WATER GLANDS, where the spindle leaves the casing, consist of a small impeller or paddle wheel, fastened to a long sleeve or hub on the shaft, which revolves in a gland casing. This gland is supplied with water under a pressure of 10 to 15 psig. The water is unable to leak along the shaft, as the action of the impeller holds it in a solid ring against the outer casing. On the other hand, air cannot leak into the turbine because of this solid ring of water under pressure much greater than atmosphere. Several forms of combined water and labyrinth glands can be steam sealed when the turbines are run at speeds too low to maintain the water-gland seal. The governor may regulate the supply of water or steam to the glands. Proper leak-off passages are provided for steam and water when needed.

Water glands generally are used outside the labyrinth glands on high-pressure ends of impulse turbines, to prevent steam leakage into the turbine room. On account of the high temperature at this gland, condensate must be circulated through it, discharging into the feed system.

Power Required by Water Gland. Guy and Jones (Ref. 49) state that experiments indicate that the power required by a water gland with the paddle completely immersed in water is $P = 6u^3D^2/10^6$, where P = horsepower required; u = peripheral velocity of paddle wheel, feet per second; and D = diameter of paddle wheel, feet. Under actual operating conditions the paddle is not fully immersed on both sides, and tests indicate that the power required is about 50% of that given by the formula.

The water required by water glands varies from 0.5 to 2% of the condensate, but little of this is lost, since at the low-pressure end the vapor from the gland enters the exhaust and is recovered in the condenser. At high-pressure glands, the water must circulate, and the heat it absorbs can be fully recovered. Water glands usually seal at any speed above one-half of normal speed.

Steam required by casing glands can be estimated by a chart to solve problems in labyrinth packing presented in *Engineering*, Vol. 128, p. 65, 1929. Martin's formula is used by many builders to compute diaphragm gland leakage. (See also Ref. 50.) Some builders use coefficients which depend on clearances, arrangement of lands, eddy chambers, etc., and which vary from 0.35 to 0.472 in the Martin formula.

BEARINGS. Turbine bearings may be divided into two classes, self-oiling and forced lubrication. Self-oiling bearings, used only on small turbines, generally consist of babbitt-lined cast-iron or cast-steel shells, with oil supplied by oil rings revolved by the shaft. Bearings for large units and for reduction gears always have forced lubrication from the main oiling system.

Large turbines may have spherically seated, self-aligning bearings. In reaction turbines, the spherical seats take the form of three or four pads under which are placed steel-adjusting shims of varying thickness. Clearances at the ends of the blades can be equalized by changing the shims and thus shifting the position of the shaft relative to the casing. A clearance of 0.008 to 0.012 in. is provided above the top pad to prevent the bearing being pinched, and to allow the shaft to be self-aligning.

Because impulse turbines do not require close clearances over the ends of the buckets, their bearings have plain spherical seats. When light shafts are used in some forms of impulse turbines, it is desirable to decrease the deflection and increase the critical speed, by using solid parallel bearings. Such bearings also are used for reduction gearing where accurate alignment is essential.

Large bearings consist of cast-iron or cast-steel shells split in half horizontally and lined with babbitt. They are relieved for 20 to 30 degrees at the sides above and below the joint, except for $^3/_4$ in. at each end, by reboring slightly oversize with a separating plate between the halves. The lower halves are scraped to fit the journal for an arc of 120 degrees at the bottom. Bearings usually are bored 0.001 to 0.003 in. large per inch diameter of journal. With forced lubrication, oil usually is supplied both at the top and sides of the bearings. Oil throwers either are turned on the shaft, or attached to it at the outer end of the bearing, to prevent escape of oil. Oil guards are provided on the bearing cover for the same purpose. Provision is made for the escape of entrained air from the bearing pedestals.

The design of the spindle usually fixes the size of the bearing. The rubbing velocity of the journal should not exceed 150 ft per sec. The bearing pressure is found by dividing the total load on the bearing by the product of its length and diameter. A safe limit ot this pressure is up to 200 psi. The ratio of bearing length to diameter varies from 0.75 to 1.5. Bearings are made shorter than formerly, as this reduces the total length of the turbine. The design should be such as to reduce spattering and splashing of oil, which lead to oxidation troubles and acid formation.

Pressures may reach 1000 psig at contact surfaces of gears driving the main oil pump and governor, because of minute errors in tooth pitch and profile that result in momentary increases in tooth loads. Ample lubrication of such gears is a necessity.

Oil whip has been experienced on turbine bearings at high rpm. While this is still under investigation, it appears desirable to maintain bearing loads on such turbines above 100 psig, to prevent oil whip. Special *pressure bearings* also are used, in some instances.

Bearings on some small turbines are heated by conduction through the casings and through the shaft. Such bearings may heat to 250 F, and a heavy oil is required. Usual bearing temperatures range from 125 F to 160 F; occasionally the oil leaves at 175 F.

Bearings depend for their proper functioning on the supply of a thick wedge-shaped oil film on the side of the bearing where the shaft turns downward. This film spreads and maintains a separation of the two metal surfaces. There is no true coefficient of friction, but the shearing action of the oil offers a resistance to motion which is the so-called coefficient of friction. Kraft (Ref. 1) states that this factor is 0.008 for a bearing of good workmanship. The heat, Btu per minute, generated in a bearing, $H = (\pi d N \mu W) \div (12 \times 778)$, where d = bearing diameter, inches; N = rpm; μ = so-called mean coefficient of friction between journal and bearing; W = total load on the bearing, pounds.

The oil required per bearing can be estimated from these formulas:

$$\text{Kilowatt loss per bearing} = \frac{0.26zN^2LD^2}{M \times 10^{12}}$$

where z = absolute viscosity of oil in centipoises (usually 14); M = clearance ratio of bearing expressed as clearance in inches per inch of diameter (M is usually 0.001 to 0.003 in.); N = rpm; L = bearing length, inches; and D = diameter, inches.

$$\text{Oil required in gallons per minute} = 0.8064 \times \text{kilowatt loss}$$

This quantity is based on assumptions of specific heat of oil = 0.5; temperature rise in bearing = 30 F; specific gravity of oil = 0.9; and an arbitrary multiplier 1.6 to allow for journal heating, etc.

Hot turbine bearings may be caused by insufficient oil, when oil pipes or oil grooves are plugged up or the supply fails. Sometimes bearing clearances are insufficient to admit the proper amount of oil, particularly at the sides of the bearings. Heating also may be due to too heavy an oil, to emulsified oil, or to old contaminated oil.

7. REDUCTION GEARING

See *Design and Production Volume* of Kent's *Mechanical Engineers' Handbook* for design of reduction gearing. See also Marine Engineering, Section 15 of this book, for additional data on marine turbine gearing.

The efficiency of reduction gears is difficult to determine by mechanical methods. A common method of calculating gearing efficiency is to measure the friction heat carried away by the lubricating oil and to allow for radiation from the gear casing. Carefully made laboratory tests by both input and output measurements, and by heat measurement on single-reduction gears, show practically the same losses. Large single-reduction gears have shown efficiencies of 98 to 99%. The efficiency of small single-reduction gear sets ranges from 96 to 98%. Double-reduction gearing has given efficiencies varying from 88 to 97% on test. During World War II nearly all high-power combatant vessels were propelled by cross-compound steam turbines driving double reduction geared propellers. Such units had both light weight and small space as prime advantages, and were highly reliable in service.

8. TURBINE LUBRICATION

OILING SYSTEMS. Small turbines provided with ring-oiled bearings require only the maintenance of a suitable supply of pure mineral oil, changed at frequent intervals, in the reservoir below the bearings. The outer surfaces of the pedestals dissipate the heat generated by friction.

Large turbines have a completely self-contained oiling system, including fine wire screens to remove particles, an oil pump, an oil cooler, suitable piping systems, and an oil reservoir.

Some turbines have a centrifugal pump on the main shaft of the unit. It delivers oil at high pressure to the governor system and also to an ejector which induces additional oil flow at lower pressures for shaft bearings and thrust bearing.

Sometimes the oil pump is driven by gearing from the main turbine shaft or, if reduction gearing is used, sometimes from the slower speed shaft. Separate motor-driven oil pumps are in use with some turbines. The pump, usually of the rotary gear type, supplies oil at pressures between 50 and 200 psig pressure. The volumetric efficiency of gear-type pumps ranges from 70 to 80% with hot oil. Oil pressures to the bearings range from 5 psig to 25 psig, usually obtained through a reducing valve. Relief valves discharge any surplus oil to the reservoir.

An auxiliary turbine- or motor-driven centrifugal oil pump is placed on medium-size and large turbines to circulate oil through the bearings before starting the unit or when the main unit is on the turning gear. Such pumps are equipped with regulators which start them automatically when the oil pressure drops below a minimum safe value. Low oil pressure alarms are often installed. Many turbines are automatically stopped by a tripping device on the throttle valve when oil pressure fails.

Oil coolers, built in many forms with brass, admiralty, or copper cooling coils, should be readily accessible for cleaning if raw water is used. Frequently, condensate is used as the cooling medium so that less cleaning is necessary.

Oil may circulate through the tubes or outside of them with shell baffles. The heat transfer coefficient in oil coolers is low, varying from 10 to 20 Btu per sq ft per hr per °F temperature difference. Coolers lower the oil temperature about 20 to 30 F. Oil should pass to the bearings at temperatures between 105 and 140 F. Temperatures leaving the bearings range from 130 to 160 F. It is frequently specified that sufficient oil-cooler capacity shall be installed to keep the maximum temperature of the oil below 150 F when using cooling water at the maximum temperature specified by the purchaser for summer conditions. Only enough water should be circulated through the oil cooler to maintain the desired minimum bearing oil temperature. Cooling water pressure should be lower than the pressure of the oil passing through the cooler.

Oil-pump capacity is fixed by the total amount of oil required by the bearings and governing system, together with a liberal margin to provide for pump slip, air vents, etc. Oil-pump capacities furnished by one builder for 3600 rpm units varies from 50 gal per min on 1000 kw units to 1200 gal per min on 100,000 kw turbines. On 1800 rpm units, capacities range from 290 gal per min on 50,000 kw units to 1500 gal per min on 165,000 kw turbines. (See Ref. 51.)

Oil reservoirs vary in size with the type of turbine and with the several manufacturers. A frequent requirement for units of 5000 kw and over is that the capacity of the oil reservoir at the turbine shall be such that it will take 5 to 10 min to circulate a quantity of oil equal to the tank capacity. Reservoir capacities range from 100 gal on a 500 kw unit to 3000 gal on a 100,000 kw, 3600 rpm turbine, and 4000 gal on an 1800 rpm, 165,000 kw unit. With smaller turbines the period for complete circulation shall be not less than 5 min. Special precautions must be taken to minimize the danger of fires in oil reservoirs. Emergency drain lines to reservoirs, CO_2 and other noninflammable flooding equipment, and location of the reservoir in a fireproof room below the turbine are used. Oil temperatures in reservoirs should exceed ambient to prevent condensation of moisture on the inside of the reservoir.

Oil Piping. Seamless copper tubing, with brazed flanges and cast-iron or brass fittings, is sometimes used for oil piping. Steel piping and steel tubing are extensively used, with welded nipples for branches and joints. This material should be well pickled before assembly to remove mill scale and rust. Threads are made tight against oil pressure by shellac or similar material, or by seal welding.

Oil velocities through pressure oil piping vary from 5 to 15 ft per sec. All drain lines should be sloped to drain back to the oil reservoir by gravity. Drain pipes are proportioned to run only half full, to allow for natural venting. All oil piping is placed on the opposite side of the turbine from the steam piping and stop valve, to lessen fire risk in case of pipe failure.

These recommendations for the construction of oil piping lessen dangers from fire.

Cleanliness and good housekeeping are of primary importance. Avoid accumulations of combustible material. Wipe up spilled or dripping oil. Prevent oil collection in pits or other depressions. Arrange piping for ready inspection, but enclose sufficiently so that it is not used as a ladder or for hanging things on. Design with good supports and as nearly free from stress as possible.

Main oil lines should be of standard-weight pipe, or seamless tubing of equal wall thickness, with extra heavy steel flanges and steel fittings. Use welds as extensively as possible. Erect barriers around all oil joints when they must be used. Weld all threaded joints. Provide lock washers on all flange bolts. Male and female flange joints are preferred. Use metal-to-metal unions.

No connections should be made with less than 1/2-in. pipe size, with shut-off valves as close as possible to the main pipe. Screen reservoir vents and locate them at a distance from the steam lines. Reservoir covers should be self-closing.

Avoid gage glasses and use a mechanical indicator for oil level. Nipples for pressure gages must have restricted openings.

Keep oil system, particularly oil storage or reservoir, as remote as possible from high-temperature steam connections.

TURNING GEAR. A motor-driven turning gear to rotate the shaft at slow speed when the turbine is taken out of service is frequently furnished for units of 10,000 kw rating and higher. Rotation prevents shaft distortion due to uneven cooling, keeps the rotor and casing at more uniform temperatures, and permits more rapid starts. The turning gear is also used when warming up a cold machine. Turning-gear speeds vary from 1 to 40 rpm on the main shaft with some preference for 25 to 30 rpm. The higher speeds maintain a better oil film in the bearings. Full oil circulation is desirable, even with the turning gear in operation, to prevent journals and bearings from overheating by conduction from the turbine or generator rotor when shutting down, since bearing metal tends to become plastic around 300 F. Oil to bearings should be at nearly normal operating temperature before the unit is brought up to speed.

LUBRICATING OIL FOR TURBINES must be a properly refined, highly filtered, pure mineral oil, free from alkali or acid, and inhibited against rust, corrosion, and oxidation. While organic acids form from oxidation of unsaturated compounds or those containing sulfur, oxygen, or nitrogen, in the presence of water, that oil should be chosen which has the lowest organic acidity, not only when fresh but also after continued use. Organic acidity, expressed as the amount of potassium hydroxide required to neutralize 1 gram of oil, should not exceed 0.10 mg, preferably 0.05 mg. This is known as the *neutralization number*. (See Ref. 52.)

Water may enter the oil system from steam leaks at glands, from condensation of atmospheric moisture on the walls of reservoir, pedestals, or oil piping, or by other means. Oil should be able to separate rapidly from water and not form emulsions. If, however, emulsions are formed, they should be quickly separated on heating. Oil must be free from components of low boiling point, to maintain constant viscosity. It should have little tendency to break down and form sludge, when agitated at the actual operating temperature and mixed with air and water. The ideal lubricating oil should have maximum adhesion and minimum cohesion.

Foaming oil may be caused by water in the system or it may be due to air entering through leaks in the oil-pump suction piping or strainer. Air becomes mixed with oil in the bearings and in return oil piping. Foaming can be corrected by removing the water and preventing the entrance of the air. Turbine oils should have nonfoaming characteristics and should permit the entrapped air to separate quickly from the oil in the reservoir before it is recirculated.

Oils oxidize at various rates. Oxidation increases rapidly above 150 F. Water in oil causes oxidation of iron surfaces. Some oils have natural oxidation inhibitors added. The addition of about 10% of old oil to a new batch of oil has been found effective in inhibiting oxidation, and is commonly practiced.

Oils which decompose and age rapidly in service deposit a hydrocarbon sludge in bearings, piping, and oil cooler. Besides plugging passages in bearings, thrusts, and couplings, it forms an insulation on the cooling coils. Higher oil temperatures result, causing still more rapid aging. Such oils should be removed and filtered, the system thoroughly cleaned, and new, higher-grade oil purchased. Cotton waste must not be used to clean the oil system, as lint causes trouble later by stopping up the oil passages. Helpful experiences with lubricating systems are presented by Lowe (Ref. 53).

Reduction gears have higher tooth pressures per square inch than the usual bearing pressures and require a heavier oil than bearings. In some cases, these gears have their own oiling systems, using a heavy oil. When both gears and turbine bearings are on the same system, either a medium or a heavy oil may be used. Hot oil may be supplied to turbine bearings and only the gear supply circulated through the oil cooler.

FILTRATION AND PURIFICATION OF OILS are necessary with moderate and large-size turbines, usually accomplished in one of these ways: (1) continuous by-pass system; (2) batch system; and (3) continuous by-pass batch system.

The continuous by-pass system continuously takes out a certain fraction of the oil for treatment. The batch system takes out a large amount at intervals. The continuous by-pass batch system is a combination of the other two methods. Separation of water and sludge may be secured by: (1) Passing the oil through a centrifugal separator or filter such as the DeLaval and Sharples, which separate the substances on the basis of their specific gravities. This removes water and dirt, but not alkalies and acids or soluble sludge. (2) Bag filters, which remove insoluble sludge but will not remove acids or soluble sludge. (3) Separating tanks, in which the oil is cooled and where water and sludge can settle out and be withdrawn. Usually oil reservoirs are provided with piping which allows water that gets into the oil to overflow automatically when it settles in the reservoir.

Turbine builders furnish purchasers with general specifications for turbine oils. Non-inflammable oils for lubricating systems have been used but are not popular.

CHARACTERISTICS OF OIL. Oil must be free from inorganic acid, asphaltum pitch, resinous substances, animal and vegetable oils, and soap or other substances added to give body to the oil. It should contain no dirt, grit, or other suspended matter. The specific gravity should be between 0.86 and 0.88 at 60 F.; flash point, not below 334 F.; fire test, 375 F. The higher the values of flash and fire point, the better, particularly in plants using highly superheated steam. Acids can be detected by blue litmus, which turns red in the presence of acids. Animal and vegetable oils, used as adulterants, can be detected by a milk-white emulsion which forms when the oil is shaken with a strong solution of borax. After standing in a cool place, the clear mineral oil will be at the top, the borax solution at the bottom, and the emulsion in between.

Sludge in oil plugs piping, coats oil cooler tubes, and causes hot bearings by reducing the lubricating value of the oil. Sludge forms as a result of agitation of impure oil in the presence of water and air. To test the sludging properties of oils, a sludge accelerator has been developed which produces sludge in 1% of the time in actual service. For a description of this equipment and information on its use, see Ref. 54. This report also contains detailed instructions for making flash, fire, viscosity, pour, total acidity, alkalinity, corrosion, carbonization, and demulsibility tests on lubricating oils.

Table 5 gives representative oil specifications for several services.

Table 5. Oil Recommendations for Various Types of Service

Properties	Forced Circulation			Ring Oiled Bearings		ASTM Standard Method for Test
	Direct-connected Units, Land and Marine	(1) Small Geared Units, Single Reduction Gears up to 500 kw, Land and Marine	Large Geared Units, Including Marine Propulsion Units, Land and Marine	Ring Oiled for Starting with Separate Oil Cooler and Forced Circulation	(2) Maximum Bearing Temperature less than 180 F	
Saybolt Viscosity, Seconds						
at 100 F	140 200	250-350	400-500	140-200	200-500	
at 130 F	76-105	120-165	175-220	76-105	175-220	D-88
at 210 F	40-46	48 55	54-64	40 46	54-64	
Flash point, °F	330 F	350 F	360 F	330 F	360 F	D-92
Neutralization number	0.10 max	0.10 max	0.10 max	0.10 max	0.10 max	D-663
Steam emulsion number	120 max	150 max	180 max	120 max	180 max	D-157
Corrosion resistance test	Shall pass	Shall pass	Shall pass	Shall pass	Shall pass	D-665
Normal temperature of oil to bearings	100 F-120 F	100 F-120 F	100 F-120 F	100 F-120 F	....	....
Minimum oil temperature when starting	50 F	70 F	80 F	50 F	....	....

Notes:

1. For marine service, where it is sometimes desirable to have only one grade of oil for both main propulsion and auxiliary units, an oil having a viscosity of 375 to 425 seconds Saybolt at 100 F can be used with the understanding that power losses in bearings will be increased when higher viscosity oils are used.

2. In some installations, normal bearing temperatures may exceed 180 F because of high ambient temperature, restricted ventilation, etc. For bearing temperatures higher than 180 F, consult turbine oil supplier for recommendation.

9. GOVERNORS AND CONTROL

Turbines may be governed by throttling or by nozzle control. In *throttling governing*, the main stream is throttled to some pressure lower than line pressure before it enters the first-stage nozzles, thus reducing the flow to a value just sufficient to maintain the load. In *nozzle governing*, a series of valves is provided to admit steam under control of the governor to each of a number of separate first-stage nozzle groups. The only valve under throttling action is the last one being opened; the others are wide open, with only a small pressure drop across them. Nozzle governing gives a better steam rate than throttling at partial loads and is widely used on large units.

Throttle governing is used on turbines having full peripheral admission, as on reaction turbines. The steam-rate curve with throttle governing can be made to approximate that of nozzle governing by selecting a *most efficient load* at a relatively low rating percentage and by providing additional capacity by secondary, tertiary, and even quaternary valves admitting steam to lower stages of the turbine.

Governors for Small Turbines. The speed of most small turbines is controlled by a powerful centrifugal governor mounted directly on the end of the rotor shaft. Movements of the governor spindle are transmitted through a single lever to a regulating throttle valve, usually a balanced valve. The governor weights are carried on levers by tool-steel knife edges in tool-steel bearings requiring no lubrication. In some types, ball bearings are used instead of knife edges. One type has weights which roll on flat tempered steel springs with no pins, bushings, bearings, or knife edges. Sometimes the lever connecting the governor spindle to the regulating valve also is provided with knife edges. These governors must be examined frequently to see that no parts are loose or worn. Speed regulation of these governors is about $\pm 2\%$.

MECHANICAL OIL RELAY GOVERNORS are used on most moderate-sized and large turbines both condensing and noncondensing. They include a centrifugal governor geared to the turbine shaft, and an oil pump which may also be geared to the shaft, mounted on the shaft itself, or motor driven. The governor operates a relay or pilot valve which admits oil at pressures between 100 and 200 psig to a servomotor that controls the steam-governing valves. A restoring mechanism provides *feedback* to reset the relay control after each servomotor movement. The servomotor acts directly on a single balanced steam-control valve or a series of valves through a cam and lever mechanism. Overload valves also are opened by the servomotor.

When a series of steam-inlet valves are used, they may be lifted by a bar to which the valves are loosely attached. The valve stems are of various lengths to assure opening in sequence. This is known as the bar-lift system.

Some industrial turbines, where a range of speed is desired and close regulation over the load range is not necessary, have hydraulic orifice governors, using oil as the operating medium.

Oil governors are used which depend for their functioning on the variation in oil pressure supplied by a centrifugal pump mounted on the turbine shaft. This oil pressure varies as the square of the turbine speed. A pressure transformer changes the small pressure variations due to speed changes into relatively large pressure changes on the operating relay. The basic impeller pressure is balanced by a spring adjustment which can bias the turbine speed. If the speed changes the change in oil pressure moves the relay, operating ports which control the flow of oil to the operating piston of the steam-valve servomotor. A compensating follow-up linkage insures stability. Various additional control devices can be incorporated readily in this type of governing system, and they frequently are added on special request.

Pressure-regulating governors are frequently applied to turbines driving pumps, etc., for controlling extraction pressures on single- or double-extraction turbines and sometimes for exhaust pressure control. These are usually rather simple regulators that admit sufficient steam to the turbine to maintain a given pressure or pressure differential at the desired point, such as at the discharge of a pump, the extraction line, or in the exhaust. Pressure-regulating systems must be arranged to coordinate with the regular speed and overspeed control governors.

Governors on turboalternators have their regulation fitted to the service for which the unit is designed. In general, 4% variation between no load and rated load is allowed for electric lighting service. Greater variation may be desirable where the load changes are large. Too close regulation is not necessary or desirable, as it may cause the load to surge between machines that are in electrical parallel. Surging often can be reduced by increasing the regulation of the governors. Turbines operating in parallel with hydro units usually do not require close regulation. (See Refs. 55 and 56.)

FREQUENCY CONTROL. Since the introduction of electric clocks, constant frequency must be maintained on the power system of public utilities, regardless of load. With major generating systems interconnected in *power pools*, one station fixes frequency for the pool. Frequency, system, and tie line loads can be maintained by hand operation of the governor auxiliary speed control, but automatic frequency regulators are used on many generating systems.

REGULATING VALVES. Balanced valves are used in nearly every case for regulating valves of the throttling type, particularly on small units with direct-connected governors. These were formerly of the double-seated poppet type. The difficulty of keeping the two seats tight when using superheat, led some builders to adopt the balanced single-seated valve. Valves and seats are generally of hardened steel. Such valves must be kept absolutely steamtight when closed. Stainless or other alloy steel, with stellite trimmings, is generally used for high-temperature steam valves.

CONTROL VALVES. In large units, a multiplicity of control valves is provided to give better efficiency over a wide range of load. Single-seated valves, streamlined to reduce eddy losses, are used on nozzle-controlled turbines. They are bar- or cam-operated by single or multiple servomotors. Control valves usually are streamlined to reduce losses. Some are of inverted mushroom form. The seat is generally well rounded and of venturi shape to regain a substantial portion of the velocity head of the flow through the valve. The bearing portion of the valve on its seat is frequently spherical in shape to insure line contact on the seat.

STEAM STRAINERS in front of the throttle valve prevent grit, pipe scale, or other foreign substances from reaching the turbine. They are made of stainless steel wire mesh for small units and drilled stainless steel plate for large units, with holes $^1/_{16}$ to $^1/_8$ in. in diameter, and generally in basket form, although flat strainer cages are used on some units. The combined area of these openings should be two to three times the valve area to reduce throttling losses. The steam chest may be integral with the turbine casing or may be separate. When separate it is bolted rigidly to the foundation and connected to the casing by several flexible seamless steel pipes. The latter construction, sometimes adopted for high temperature units, isolates the hot valve chest from the cooler casing. In general, the main steam line should be flexible enough to impose only negligible stress on the turbine casing.

OVERSPEED GOVERNOR. All turbines have some form of automatic overspeed governor. A small centrifugal governor sometimes is provided in the end of the shaft. At 10% above normal speed, it automatically trips either the stop valve, the governor valve, a separate overspeed valve, or combinations of these. The usual form of overspeed governor consists of a bolt-headed pin in the shaft end, at right angles to its axis. The centrifugal force on the unbalanced bolt head is opposed, up to 10% overspeed, by a spring. At that speed, the bolt flies out, striking a trigger, which releases the spring-loaded stop or other valve and permits it to close. On some large machines, the trigger operates an oil relay valve which causes a second small piston to unlatch the main stop or other valve and allows it to close. A modification of this form of governor is an eccentric ring on the governor spindle or main shaft, the centrifugal force of the eccentric part being balanced by a spring. At a given overspeed, this spring is compressed, the eccentric ring strikes a trigger, and the control valve is closed either by a spring or by an oil relay.

LOAD RELEASE. An anticipator device, sometimes provided on large turbines, causes the main stop valve or the governor valves to close on complete and sudden loss of load, before the rotor has speeded up sufficiently to operate the overspeed governor. See Ref. 57 for a description of supervisory instruments and their operation.

AUTOMATIC EXTRACTION PRESSURE CONTROL. Present governing specifications for automatic extraction turbines generally call for compensated control systems with interconnections between the control mechanisms of both the speed-governing and pressure-regulating systems. Simultaneous action of both systems takes place in response to a change in either speed or controlled steam pressure. With an increase in load, the controls open the control valves to supply additional steam, and simultaneously adjust the pressure-regulating valve to maintain extraction pressure. An increased demand for extraction steam causes the valves controlling steam to the portion of the turbine beyond the extraction point to close partially, thus tending to reduce the power. The slight decrease in power is simultaneously compensated for by an approximate correction of the main control valves, immediately followed by an accurate correction by the speed governor. Thus there is no change in total load.

THROTTLE VALVES, now going out of general use (see Stop Valves, below), often were opened against a heavy spring. The handle for opening the valve is fastened to the valve stem by a latch which may be released, usually by an oil relay, when the overspeed governor acts, thus forming a combined throttle and trip valve. It serves as a throttling valve only when coming up to speed. The unit always should be shut down by tripping this throttle valve. Some throttle valves are controlled by an oil-operated piston. Overspeed governors on most large units can be reset when the turbine speed drops to about 2% above rated speed. On smaller units the speed may drop considerably below rated speed before the trip mechanism can be set.

Other overspeed governors on small turbines release a flap valve, which closes of its own weight. Still others control a butterfly valve in the steam supply.

STOP VALVES. The old-fashioned throttle valve, applied to hundreds of central station turbines, now is largely superseded by an oil-operated stop valve. This valve, installed at the steam inlet to the turbine, is normally held open by a spring-loaded hydraulic cylinder. Upon overspeed, the oil is vented to a drain line, and the valve closes almost instantaneously. The stop valve has no intermediate position; it is always wide open or closed. Usually the control valves must be closed, and a small by-pass line around the stop valve opened to bleed steam into the steam chest, before the hydraulic cylinder has enough power to open the stop valve. This is a safety precaution which prevents accidental overspeeding in starting up the unit. The prime function of the stop valve is to prevent overspeeding and to isolate the turbine from the main steam line.

SPECIAL CONTROLS. Central station turbines sometimes are supplied with one or more of these controls, among others: *Steam flow limit device* to control the maximum output fixed for the turbine; *low-steam-pressure limit device* to reduce the load in order to permit the boiler to restore pressure; *low-frequency block load device* to provide for the rapid increase of steam flow should frequency drop.

10. CASINGS, DIAPHRAGMS, AND EXHAUST

CASING MATERIALS. Casings of turbines for low steam pressures and temperatures frequently are made of cast iron, split horizontally, with all steam and exhaust connections in the lower half. The steam chest sometimes is carried around one end in the form of a cored passage, with hand-controlled valves to the nozzles. When high-temperature steam is used, casings must be made of alloy steel. Casings for pressures of 1200 psig and higher have been made of forgings.

Casings for large turbines usually are split horizontally, so that the upper half can be removed to permit examination and repairs. Many single-cylinder casings have a bolted vertical joint between the high- and low-pressure sections. High-pressure sections of turbines carrying temperatures over 425 F are made of cast steel.

For steam temperatures above 825 F, casings are cast of chromium-molybdenum steel. Above 1000 F the casings are of cast austenitic stainless or other alloy steel. Many industrial turbines, marine turbines, and the low-pressure casings of central station turbines are fabricated from rolled flat steel plate.

CASING CONSTRUCTION. While strength is an important consideration in casing design, rigidity under varying temperature conditions is imperative. Hence, they usually are cast without ribs on the outside and with the smallest flanges consistent with strength requirements. They approach the smooth barrel form, and all abrupt changes in diameter are avoided. If circumferential flanges or ribs are provided, they must not be too deep or distortion will occur because of slow adjustment to changes in temperature. Cored passages are avoided.

Stationary dummy rings are generally cast separately and fastened to the inside of the casings. Stationary blade rings of reaction turbines are cast separately and are registered in a barrel-shaped casing in different ways to provide free expansion under rapid temperature changes. Double-shell construction frequently is used in high-pressure elements to lessen the pressure and temperature differences across the flanged joints, by confining the high pressure and high temperature to the inner casing.

Horizontal flanges of casings are lapped or scraped to make them steamtight in operation. Sometimes the joint is coated with thin linseed oil before closing.

Flange joints of high-pressure high-temperature turbines are difficult to keep tight. Bolts creep and relax. Flange faces sometimes are undercut at the bolt diameter to increase the moment effect of the outer section. Narrow flange faces are used to increase unit compressive stress on the joint and to increase responsiveness to temperature changes. These flanges are thick, usually about 2 1/2 times the bolt diameter. Sometimes slots are cut through the outer flanges to permit adjustment of temperature after sudden changes of load. (See Refs. 58 and 59.)

The high-pressure end of casings often is supported on sliding feet, located near the horizontal center line, to allow free expansion. Transverse alignment is obtained by vertical keys. In some designs, a flexible support, in the form of a steel I beam or channel under the high-pressure end, holds the unit in sidewise and vertical alignment but permits longitudinal expansion through flexure of its web. With such construction, the cylinder is bolted solidly to the pedestal which rests on the I section.

Casings must be designed to withstand bursting pressures 50% above normal working load. For low-pressure cylinders, normal pressure may be taken as 30 to 50 psig. The deflection between supports should be a minimum. The pull of the vacuum on the exhaust outlet, when an expansion joint is used, must be considered in calculating this deflection. The lifting force of a spring-supported condenser also must be considered.

Cylinders should be blanked off and subjected to about 25 psig steam pressure for about 24 hr before final boring, to season the metal and relieve casting stresses. Cylinder supports should be as near to the center line of the machine as possible, to avoid vertical changes in shaft alignment from temperature changes. Usually the generator end of the casing is fastened rigidly to the bedplate with provision for expansion at the opposite end.

The thickness of the metal casing is usually calculated by the thin cylinder formula, $t = pd/2f$, where t = thickness, inches; p = internal pressure, psig; d = internal diameter, inches; and f = allowable stress in the material, psig. See Stodola (Ref. 3) for further details.

Drainage grooves to remove moisture are provided in the last stages of large turbines. They are capable of removing about 25% of the moisture present.

BOLTS. Flange bolts are put as close to the inner diameter of the casing as possible, leaving sufficient metal (at least 1 in.) between the inner wall and the bolt hole. This requires a deep flange and long through bolts, closely spaced. Sometimes these bolts are turned down to a diameter slightly less than the bottom of the thread to allow uniform extension. Usually, such bolts are hollow. They are heated electrically to a predetermined extension, the nut is set up snug, and the bolt is allowed to cool. The nuts are usually cylindrical with a small hex on top to permit closer spacing of bolts. Some manufacturers mill the threads on these bolts with a larger clearance between nut and bolt threads than standard practice. Threads are sometimes tapered to give an even bearing on all threads, when stretch occurs under load. (See Ref. 60.)

Various alloys have been used for bolts. ASTM B-14 specifies a chrome-molybdenum-vanadium steel, tensile strength 100,000 psi, yield point 80,000 psi. The bolts are cold stressed to about 55,000 psi, equivalent to a stretch of 0.0018 in. per inch of bolt length. Some use similar alloys heat treated to 125,000 psi, ultimate strength. Hardened steel washers are used under the nuts. Threads are lubricated by various materials, such as mixtures of graphite, white lead, and penetrating oils.

Bolts are proportioned on their *relaxation* stress at the steam temperature. After operation for a period at high temperature, creep and relaxation lower the bolt stress to its relaxation stress, where it remains indefinitely. (See Ref. 59.)

DIAPHRAGMS, when cast in halves, must be made sufficiently strong and rigid to avoid undue deformations due to temperature and pressure differences. They may be dished towards the high-pressure side to increase strength. Diaphragms in large machines are of cast steel, fitting on centralizing supports in turned circumferential slots in the casing. These diaphragms sometimes carry a shoulder projecting almost to the adjoining diaphragms and just outside the blade ring.

Diaphragms for high-pressure high-temperature units are built up by welding. Nozzle partitions are usually of stainless material. Their inner and outer bounding walls, whether integral or separate, are highly finished. They are welded to an inner half disk and to an outer half ring; this results in robust rigid nozzles, of high efficiency, in strong diaphragms. The welded parts are then finish turned to size.

Diaphragms for low-pressure stages with long radial nozzles are made with steel plate nozzle partitions of suitable form, cast in position in the diaphragm, or of formed partitions. Other methods of constructing both high- and low-pressure diaphragms are used by the various manufacturers.

Design constants for diaphragms are largely based on experience, since split diaphragms are not easily analyzed mathematically. They must be designed to have a limited deflection, to avoid gland rubs, and frequently are pressure-tested under design conditions to confirm their suitability.

CASING CORROSION, often in the form of grooves in the interior casing walls between rows of blading, has occurred in certain units. This grooving often leads to a considerable loss of metal which adversely affects the structural integrity of the machine. Corrosion is worst in the neighborhood of the dew point of the turbine. The cause of this corrosion has been traced to corrosive gases which attack the metal in the presence of moisture. The attack first occurs at the dew point. Oxygen, carbon dioxide, sulfur dioxide, and hydrogen sulfide may contribute to corrosive action. These gases may enter the boiler with the feedwater or may result from the action of boiler compounds. Corrosion can be prevented by proper deaeration and chemical treatment of feedwater.

EXHAUST HOODS are made of cast iron or of welded steel plate. Exhaust outlets of large turbines are stiffened and reinforced against vacuum by cast-in or bolted partitions, or by heavy staybolts with pipe spacers. These partitions or struts must not be placed close to the last blade row since they may cause undue blade vibration. Exhaust outlets usually are designed to have about twice the annulus area of the last stage blading. Where possible, these outlets should diffuse the leaving steam to produce the lowest absolute pressure at the discharge of the last blade row.

Kraft (Ref. 1) states that the design of the exhaust should fulfill the following conditions: (1) The transformation into pressure of the kinetic energy of the steam leaving the blades must begin as soon as possible after the last wheel; it must take place as quickly as possible yet it must be gradual. (2) The curvature must be gentle; the area of the steam passage in a direction perpendicular to the flow must increase continuously, causing the steam velocity to decrease gradually. (3) The jets of steam escaping from the last wheel must not interfere with each other and cause eddies. (4) As few guiding surfaces as possible should be provided in the exhaust casing, to keep down the friction losses against the walls. (See Ref. 61 for data on exhaust hood losses.)

ATMOSPHERIC EXHAUST. Provision may be made for an atmospheric exhaust on condensing turbines, to discharge exhaust steam to atmosphere through a relief valve. These valves are difficult to keep tight. The valve and piping are expensive and large. In modern operation, where machines are started and shut down under vacuum, it is inadvisable to operate noncondensing, particularly when high superheat is used; hence relief valves are seldom, if ever, used. Many modern turbines are installed with a lead blow-out or explosion diaphragm set at 2 to 5 psig. These turbines have a special vacuum-control device which trips the stop valve if the vacuum drops below a certain value, thus avoiding rupture of the blow-out diaphragm in all but the most extreme situations.

11. ERECTION AND OPERATION

TURBINE FOUNDATIONS. Satisfactory turbine operation can be assured only when proper foundations are provided, particularly for large units. Turbine builders provide customers with detailed instructions covering the design and construction of satisfactory foundations.

The foundation should have sufficient weight and mass to hold the turbine rigid against vibration. The subfoundations are determined by the character of underlying material, but must be designed so that the concentrated weights of the turbogenerator are spread over an ample area to prevent springing or settling. A reinforced solid mat 2 to 5 ft thick is desirable except when the foundation is on rock.

The turbine foundation and its base should be independent of main building or other foundation structures. Floor beams should not rest on turbine foundations. Overhanging cantilever supports should be avoided. Large mass is desirable to insure smooth operation. Space must be provided at the generator end for removal of the field. The floor for this section must be strong enough to carry the field. Space for dismantling should be provided on floors of sufficient strength.

Preferences for foundation materials are, in the order named (1) steel heavily encased in concrete; (2) reinforced concrete; and (3) bare steel.

The foundation generally consists of heavy columns tied together by beams. Necessary openings for condenser connections, extraction and main steam lines, water and oil pipes, electrical conduits, etc., should be provided, with necessary reinforcing around these openings.

Manufacturers differ in their methods for calculation of loadings. In general, the vertical load includes the weight of the turbine and generator plus the reaction of the condenser or the pull of vacuum if a vertical expansion joint is used, plus 25 to 50% allowance for auxiliary parts and impact. Turbine outline drawings show the distribution of these weights.

To design bracings, horizontal loads at the shaft center line are assumed equal to 50% of the rotating weights on four-pole units and 100% on two-pole units. A longitudinal load of 10% of the total weight of the turbine generator is also assumed to act at the center line. Provision must be made for sliding forces when one end of the turbine slides on pads, and for thrust forces when thermal expansion is taken by flexing plates. In areas subject to earthquakes, an additional horizontal thrust must be assumed in the design.

Resonance deflections must be avoided. They are given by the various builders, and can be obtained before foundations are designed. In general, deflections of beams and columns should not exceed 0.020 in. for 1800 rpm units and 0.010 in. for 3600 rpm machines.

A concrete mixture is recommended that has a compressive strength of not less than 3000 psi when 28 days old. This corresponds roughly to a 1-2-3 mixture. Solid walls are preferred to columns. Care must be taken in pouring to get a monolithic structure. Temperature effects from hot exhaust pipes of superposed units must receive special consideration.

Structural steel foundations are sometimes used where subsoil conditions are bad, and loadings must be the minimum. Generally, the space between beams is filled with concrete; steel columns should be adequately braced; beams must have ample gusset plate stiffening; and main foundation members must be protected from radiant heat. Cantilever beams should be avoided where possible. Deflections should not exceed those given for reinforced concrete.

THE DESIGN AND CONSTRUCTION OF TURBINE FOUNDATIONS usually is carried out by the purchaser. While the turbine builder will assume no responsibility for such plans, it is desirable to submit them to the builder for his approval to avoid mistakes, omissions of openings for drains, ducts, etc., and for suggestions as to resonance, rigidity, etc.

Some small turbines are leveled by steel shims on the steel work, and bolted in place. In other cases, the steel is designed so that a concrete top is provided and 1 to 1 grout is poured around the bedplates in the usual way, after leveling with wedges.

Floors between adjacent turbines and between foundations and walls should be supported on steel work independent of the turbine foundations, to prevent vibration being carried to other structures. In many large stations only an operating platform about 6 ft wide is built around the turbine bedplate, leaving the remainder of the engine room open to the basement floor. This provides better light around the condenser auxiliaries, and the condenser equipment is more accessible to the crane. This platform should be carried on independent supports as cantilevered walkways may be subject to vibration. Dismantling and repairs can be carried out on the basement floor. If carried out on a floor at the turbine elevation, heavy steel or concrete flooring and floor supports would be required.

Structure Vibration. In case of vibration trouble, a study of the structure by means of a vibrometer may locate the cause of the difficulty, provided the revolving elements are in dynamic balance.

ERECTION OF STEAM TURBINES. In erection, if turbine and generator are on one shaft, it is necessary only to level the unit carefully. If the set consists of a turbine with a separately driven unit, as a generator or pump, or if gears are used, care must be taken to insure not only correct levels but also accurate alignment, particularly at the couplings. Leveling pads are furnished on many bedplates. Levels should be checked with the turbine heated to operating temperature, as the alignment of some designs is affected by expansion due to the heating of the exhaust end. Leveling is done by steel wedges at frequent intervals under the edge of the bedplate, allowing a space of 2 in. under the bedplate for grouting.

There should be sufficient wedges to insure that no deformation occurs between them. After the grout has set for a day, they can be withdrawn. Some engineers raise the outboard bearings of both turbine and generator slightly to obtain more accurate alignment at the coupling by allowing for the deflection due to weight of the turbine rotor and revolving field. Leveling always should be done with all weights in place. Detailed information on erection will be found in the instruction books of manufacturers. Kearton's *Steam Turbine Operation* also contains many data and suggestions for alignment methods, erection, etc.

When leveling is completed, the bedplate is grouted in place, using one part of high-grade portland cement and one part of sharp sand, mixed in a thin grout and well rammed to prevent air bubbles under the base. That should cover the foot of the bedplate at least 3 in. Provision usually is made to grout in bedplates supported on steel. Occasionally lead, $^1/_2$ to 1 in. thick, is used in place of grout on structural steel foundations. Grouting should be done before any steam, exhaust, or extraction piping is connected. Foundation bolts are drawn up tight after the grout has set.

Oil piping generally is blown out with steam to remove dirt that may have got in during shipment, and is allowed to dry thoroughly before assembly. Steam lines should be blown out with full steam pressure, to remove any pipe scale, welding shot, or dirt.

STEAM PIPING. Piping to turbines must provide for expansion and contraction of the turbine and of the pipe line, so that the least possible stress will be imposed on the turbine. Pipe vibrations also must be avoided.

The steam chest containing the control valves is either (1) bolted directly to the turbine casing, (2) cast integral with the cylinder, or (3) fastened firmly to the foundation with flexible pipe connections from it to the turbine casing. The stop valve, usually of the automatic spring-closing type operated by the overspeed governor, is usually separately supported.

It is desirable that there should be minimum stress on the stop valve from piping. Allowance should be made for the maximum turbine and piping expansion from cold to working temperature. This expansion may be halved by cold-springing the piping into position for half the total expansion. Torsional stresses should be avoided. See Section 6 for data on piping.

Welded pipe joints are generally used on high-pressure piping. Several forms of weld are used, but all are stress relieved by electrical heating on completion. Welding icicles must not be allowed in steam lines as they eventually break loose. High-temperature steam piping of alloy steel has heavy walls, and consequently provision for expansion is difficult. Special care is necessary to avoid excessive thrust on the turbine. (See Arts. 11 and 12, Section 6.)

Drains must be provided for all low points in piping, steam chests, bends, etc. The manufacturer's drawings show the location of these drains.

Exhaust piping must be absolutely airtight. When an expansion joint is used, consideration must be given to the collapsing effect of the vacuum, and provision made by suitable brackets, anchor bolts, or other devices to prevent distortion.

PLANT DESIGN. Turbogenerator plants are arranged in either of two ways. (1) The turbogenerators are set with their axes at right angles to the boiler-room wall. Condensers are placed directly under the turbines. The units are spaced to give ample room between them for operating and for dismantling. This distance often is fixed by the space required by auxiliaries, usually located in the basement. Space must be provided at the ends of the units to permit the field to be withdrawn from the generator. With large units, this results in a wide turbine room and long crane span. (2) Large turbines often are placed with their axes parallel to the boiler-room wall. The distance between turbines is fixed by the space necessary to withdraw the generator field. The condenser usually is set at right angles to the turbine axis. Space must be allowed for withdrawing condenser tubes, and also at the side of the turbine for dismantling.

Unit sets have been developed in the smaller outputs where turbine, generator, exciter, condenser, and pumps form a simple unit.

The condenser should be placed as close to the turbine exhaust as possible. In small sizes, a bellows-type copper expansion joint may be placed between turbine exhaust and condenser inlet. Large units often are built with no expansion joint, the condenser being bolted directly to the exhaust nozzle. The supporting pads on the condenser shell rest on springs. They are adjusted to carry all, or a major portion of, the weight of the condenser when full of water. They provide a certain amount of flexibility to care for expansion from temperature changes. All water, air pump, condensate pump, and atmospheric exhaust connections should have expansion joints when the condenser is bolted firmly to the turbine in this manner. Condensers on some large units are fastened to their foundations with a flexible connection to the turbine exhaust outlet.

Headroom under the crane above engine-room floor must be sufficient to permit lifting off the turbine upper-half casing and removing the rotor for repairs or inspection.

Piping. The purchaser furnishes piping for cooling water to and from the oil cooler, to and from the generator air cooler, for water glands, for steam to and from auxiliary oil pump, and for drain connections. Manufacturers' drawings show where these connections can be made. The manufacturer usually supplies all oil piping, piping to steam-sealed glands, and water-cooling piping to bearings, when used.

Cooling air for small generators may be taken from below the unit and, after passing through the generator, may be discharged into the engine room. In this method, much dirt enters the generator and adheres to the windings. In case of a short circuit, a fire in the windings usually results from the continued supply of fresh air. Larger generators, up to 10,000 kw capacity, now are furnished with a closed system of air circulation and with air coolers supplied with condensate or with cooling water from other sources. These eliminate the necessity of cleaning generators. In case of a short circuit the fire is smothered by exhaustion of the limited oxygen supply in the closed system. Small breathers with viscous air filters provide for changes in volume in the closed circuit.

HYDROGEN COOLING. Standard units of 15,000 kw and above, as shown in Table 1 (p. 8-14), are furnished with hydrogen-cooled generators built into the closed casing of the generator. This casing is explosion-proof, being designed for an internal pressure of about 80 psig. Normal hydrogen pressure is 0.5 psig. When hydrogen pressure is increased to 15 psig the generator load may be increased about 15%, with the same temperature rise as at normal load and 0.5 psig hydrogen pressure. Special seals prevent hydrogen leakage where the shaft passes out of the casing. These seals employ oil as a sealing medium, and this oil absorbs hydrogen. Separate equipment dehydrogenates the oil and returns it to the main system. Provision must be made for purging the generator of air before admitting hydrogen. This purging is usually done by carbon dioxide which displaces the air and, in turn, is displaced by hydrogen.

CARE AND OPERATION OF STEAM TURBINES. Manufacturers issue complete instructions for the care and operation of their respective units. The following notes direct attention to a few operating considerations:

The logical steps in starting a turbine are: First, the condenser circulating pump and the other condenser auxiliaries should be started; next, the auxiliary oil pump on the turbine, and a full oil supply furnished to all bearings. Cooling water should be turned on the oil cooler. Any new oil added to the system should be poured through several thicknesses of cheese-cloth to remove any chips, cuttings, etc.

The most difficult part of starting is the proper warming-up of spindle and casing. If a stationary turbine rotor is allowed to cool down from high temperature, cooling takes place unevenly, and the rotor may become bent. It cannot be operated in this condition without extreme vibration and rubs on sealing strips, thus increasing clearances and leakage areas. Careful warming-up is necessary to straighten the rotor. When the turbine has no turning gear, with steam-sealed glands, the vacuum pumps are started and steam turned into the glands. A small amount of steam allowed to pass the throttle valve will, on account of its density, rise to the top of the casing, causing it to heat and expand. The lower half remains filled with cold air and does not change in temperature or form. The result is a distorted cylinder. Disks also distort if one-half only is heated. It is evident that warming with a small amount of steam may cause undesirable distortions, which may result in blade rubs. A better method is suddenly to admit enough steam through the throttle valve to revolve the spindle, and to then close the throttle until only sufficient steam enters to keep the spindle turning over slowly. The rotating blades carry the steam around the casing, causing it to heat more evenly and rapidly than in any other way. This should continue until the turbine is evenly heated, before allowing the machine to speed up. All drains must be kept open during the warming process. A blade rub developed in starting probably is due to local distortion, and often may be relieved by allowing the turbine to stand for a short time while the heat in the casing diffuses through the whole body. Another careful start then may indicate that the blades are clear. It is unwise to bring a turbine up to speed without warming, as severe stresses undoubtedly are produced in certain parts.

Starting up and loading are important considerations on large turbines. The time required to bring turbines of 20,000 kw and larger from cold up to speed varies from 1/2 to 4 hr, and load may be added at the rate of 1000 to 3000 kw per min. If possible, large turbines ought not to be operated at less than 20% of rating unless designed for such light load operation.

When turbines are connected to generators in parallel with water-power plants, and must operate for considerable periods at no load, sufficient cooling steam must be admitted to cool the rotor by removing heat generated by windage.

As the turbine speeds up, the glands begin to seal and vacuum builds up. It is well to ascertain if the valves are sufficiently tight to prevent overspeeding with no load and full vacuum. The turbine next can be synchronized and load applied.

Shutting Down. The turbine is shut down by reversing the above processes in the regular order. The machine usually is stopped by tripping the valve, to see that it is free and acting properly, or by speeding up the turbine by holding the governor lever until the emergency governor acts, at 10% above normal speed.

When a unit with turning gear is shut down, it should be kept rolling by the turning gear until all parts are at room temperature to prevent bowing of the spindle. This may take 72 hr on large units. No steam should be allowed to leak into any turbine during a shutdown.

Inspection. Every turbine should be opened and inspected periodically, usually once a year, to note any wear of parts or other troubles. Corrosion of blades in rows at the dew point generally is due to impure feedwater or to poor deaeration. Wet steam may cause erosion of the last rows of blades. Oil reservoirs require cleaning. Water glands scale up, unless condensate only is used. Clearances require checking. Wear on thrust blocks should be noted and corrected. Leakage in stop and control valve seats should be stopped. Care must be used in reassembling to avoid damage to the blading.

The highest vacuum should be maintained at all times in the condenser exhaust. Frequent use of heavy asphaltum paint on all exhaust joints is a good preventive against air leaks.

REMOVAL OF TURBINE DEPOSITS. Continued carry-over from the boiler results in accumulated deposits on strainer, valves, steam passages, and blading. Increased deposition is indicated by increased stage pressures at a given load, or by reduction in maximum capacity. These deposits consist of soluble and insoluble material. The cause of carry-over should be investigated and the difficulty overcome at the boiler.

Soluble deposits may be removed by washing the affected parts with water when shut down, or by use of saturated steam when operating at light load or low speed. In the latter case, superheat is slowly reduced by water injection in the steam header. Reduction of steam temperature at the boiler also helps. Wet steam sometimes dissolves the deposits when steam washing is done at no load and at low rpm. Removal may be checked by the conductivity of the condensate. After removal, superheat can be slowly restored.

Insoluble deposits, often principally silica, are difficult to remove. Sometimes the turbine is opened and the deposit removed by an air blast, with powdered coal ash used as an abrasive. Or the turbine can be washed with caustic soda solution while the rotor is turned at a slow speed. All traces of soda must be removed when washing is completed. Turbine builders issue instructions for use of these processes on their turbines. (See Refs. 62 and 63.)

ACCIDENTS to steam turbines generally are due to one of these causes: failure of the oil supply; overspeeding due to failure of the overspeed governor to act; failure of buckets due to fatigue of material from vibration; fouling of blades with foreign parts in the casing; failure of the disks or drum from internal defects; or starting quickly an unevenly heated and distorted turbine. Many of these conditions can be foreseen and prevented by proper vigilance on the part of the operating engineer.

TURBINE OPERATING DATA for a period of years will be found in *Turbines*, NELA and EEI. The following terms have been adopted as standards in stating turbine performance (see also Art. 32, Section 16):

Period Hours. Total hours per year (8760 in a normal year).

Service Demand Factor. The ratio of demand hours to period hours.

Service Demand Availability Factor. The ratio of service hours to demand hours.

Unit Capacity Factor. The ratio of kilowatt-hours generated to the product of the unit rating and period hours.

Unit Output Factor. The ratio of kilowatt-hours generated to the product of the unit rating and service hours.

Unit Operation Factor. The ratio of service hours to period hours.

Maximum Possible Unit Operation Factor. The ratio of the sum of the service hours and the reserve hours to the period hours.

Turbine design and reliability have been improved until the service demand availability factor has reached values of 95%.

Availability and reliability are important factors in turbine construction and operation, since outages not only cost money but also may lead to shutdowns of the entire electrical system. Gains in economy sometimes have been sacrificed to obtain more rugged and reliable operating units.

SYNCHRONOUS CONDENSER OPERATION of steam turbogenerator sets is often practiced. Generally the manufacturer refuses to accept responsibility for such an operating condition, although few failures have been assigned to this cause. If the turbine is coupled to the generator during such operation, particular care must be taken to ventilate the unit with steam to remove heat generated by windage. This is done by admitting steam through an orifice into the first stage. This steam, while supplying some of the energy to overcome the mechanical losses of the set, is less than the normal no-load steam flow. The heat generated and the location of the hottest stage depend on the size, type, number of stages, and density. The condenser usually is kept in service to maintain a high vacuum. Thermometers are placed on the turbine casing to indicate the rise in temperature when operated as a synchronous condenser. Limiting temperatures are about 500 F for a steel casing and 400 F for a cast-iron casing for small units. In large turbines, manufacturers prescribe even lower limits.

When the turbine is uncoupled and the generator operated alone as a synchronous condenser, it is started as a motor by being tied in electrically with another operating unit.

12. CORRECTION FACTORS FOR TURBINE DATA

An increase in steam pressure increases the available energy, which tends to reduce steam consumption at a given load and leads to lower leaving losses. If initial temperature is constant, moisture increases in the low-pressure stages, tending to decrease efficiency. ρ, the ratio of wheel to steam speeds, decreases on the average, but R, the reheat factor, increases. The net result is a slight decrease in turbine efficiency, together with lower steam consumption, and better heat rate

An increase in vacuum increases the available energy and decreases steam consumption and heat rate. ρ decreases, tending to lower efficiency. The leaving loss increases. The net result is lowered efficiency, but also decreased steam consumption and heat rate.

An increase in initial steam temperature increases efficiency. It increases the available energy and decreases ρ, which tends to decrease efficiency. This often is partly offset by the decreased moisture in the low-pressure section, although leaving losses tend to increase. The net effect is an increase in efficiency, accompanied by a decrease in steam consumption and heat rate.

CORRECTION FACTORS. As it is impossible to reproduce on a plant test exactly the standard conditions specified in the contract, every steam turbine guarantee should contain in the contract corrections for such variations from standard conditions as may occur on test. The corrections will vary with the various types and sizes of turbine, and with certain assumptions in their design. They should cover variations in initial pressure, vacuum, and load.

Only the manufacturer can state, with any degree of assurance, reasonable correction factors for his particular design. The purchaser, however, can check these by noting whether there is any appreciable change in the engine efficiency when the corrections are applied. (See Fig. 60 for correction data on a 60,000-kw turbine.) Correction factors furnished by builders are generally based on tests of units of the size and type in question. For extraction turbines, the exhaust pressure correction is expressed most conveniently as gain or loss in output, expressed as a function of the steam flow to the condenser.

RULES FOR STEAM TURBINE TESTS are given in the ASME power test code on steam turbines (see Section 19). This code covers instruments, methods of measurement, and computation of results, and in an appendix gives examples of the various computations necessary to obtain the actual performance.

13. TURBINE PERFORMANCE

Steam-turbine performance usually is stated in tables giving size, speed, initial steam and exhaust conditions, pounds of steam per kilowatt-hour, and net Btu per kilowatt-hour. These tables, while of interest to engineers for reference, have three limitations. (1) It is impossible in a small space to quote tests for every possible steam condition under which turbines of many sizes may operate. (2) Builders seldom allow any tests to be published except the best records made by their equipment, and such tests obviously do not represent average conditions. (3) Turbine builders are prepared to furnish different types of the same size of unit, and frequently of different efficiencies, built to meet quite different conditions. For instance, a simple, low-cost unit, with a low ratio of wheel speed to steam speed and high leaving losses, may be best suited for certain commercial conditions. For other conditions, a more expensive turbine, of more refined construction, with a high ratio of wheel speed to steam speed, and with low leaving loss, may be most desirable. The latter will have a much lower heat consumption and higher cost than the first unit.

The following data will serve as a guide in estimating turbine performance or in checking tests.

ENGINE EFFICIENCY. Operating conditions, particularly on small turbines, vary over such a wide range that tables covering every condition are beyond space limitations. The engine efficiency is used in the following paragraphs as a measure of performance of the smaller units.

The steam consumption of these units can be readily determined by dividing 3413 (the heat equivalent of 1 kwhr) by the product of the engine efficiency, η_e, at the generator terminals and the available energy from initial steam conditions to exhaust pressure. Thus pounds per kilowatt-hour $= 3413/[\eta_e(h_1 - h_2)]$.

The available energy $(h_1 - h_2)$ can be found from a Mollier diagram or from the Theoretical Steam Rate Tables (see Section 4). The steam consumption in terms of brake-horsepower can be calculated by substituting 2544 for the horsepower and η_e for the engine efficiency at the coupling. Thus pounds per brake-horsepower-hour $= 2544/[\eta_e(h_1 - h_2)]$.

Leaving loss also may be considered at the same time as engine efficiency, as it is a measure of the unutilized velocity leaving the last row of blades.

Since turbines operate best under the conditions for which they are designed, the efficiency ratio under any other set of operating conditions will vary from the ratio at the specified conditions. Because of the leaving loss at the last row of blades of high-vacuum turbines, it usually happens that a higher efficiency ratio may be obtained at a lower vacuum than specified, even though the steam consumption per kilowatt-hour may be higher.

HEAT CONSUMPTION of a turbine is expressed in Btu per kilowatt-hour.

British practice expresses turbine performance in terms of thermal efficiency. This is the ratio of 3413 (the heat equivalent of 1 kwhr) to the heat consumption of the unit, expressed in Btu per kilowatt-hour.

LAMBDA OF A TURBINE. Another British practice is to state the lambda (λ) value for a unit where

$$= \left(\frac{\Sigma d^2}{100}\right) \times \left(\frac{\text{rpm}}{100}\right)^2$$

where Σd^2 is the sum of the squares of the mean diameters, inches, of all rows. The higher the value of λ, the greater the engine efficiency as a rule. Another similar factor, called by Kraft the *Parsons coefficient* or *quality factor* $q = \Sigma u^2/h_0$, where Σu^2 = the sum of the squares of the various wheel speeds, feet per second, and h_0 = isentropic available energy, Btu per lb, from initial conditions to final pressure. Σu^2 gives an idea of the bulk of a turbine and consequently of the price of the unit. Kraft gives the following values of engine efficiency at the coupling for various values of q:

q =	5000	7500	10,000	12,500	15,000	17,500
η_e =	69.8%	77.2%	81.5%	84.2%	85.5%	86%

He points out that an increase in q means an increased number of stages, and that a large increase in q gives only a comparatively small increase in efficiency in the higher ranges. Also, the quality factor q must be higher for reaction turbines than for impulse units for the same efficiency.

PERFORMANCE OF MECHANICAL-DRIVE UNITS.*

In setting up a preliminary heat balance it is necessary that the designer have some source of information with respect to the approximate efficiency of auxiliary drives. Where mechanical-drive turbines are used for driving auxiliaries they may be either

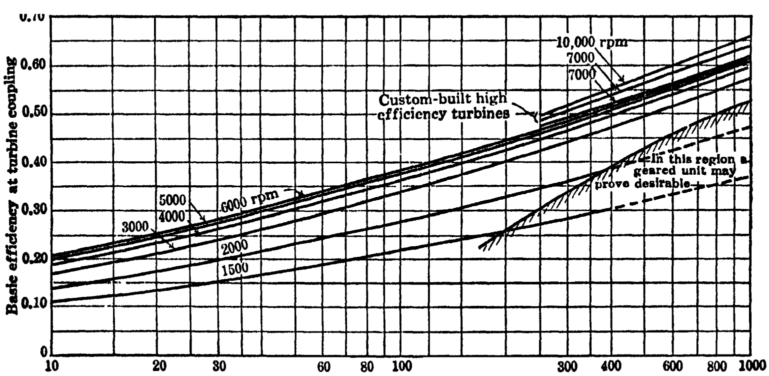


FIG. 28. Coupling efficiencies of mechanical-drive turbines at 400 psig–0 F superheat–15 psig exhaust pressure. Use correction factors from Figs. 29 and 30 for other steam conditions.

"direct-connected" or geared. In the larger sizes of turbine used for driving moderately slow-speed auxiliaries considerable advantage frequently can be gained by using geared sets rather than direct drive. Mechanical-drive turbines may be subdivided further into single-stage and multistage units. Although it is impossible to generalize with respect to the suitability of either type without consideration of the specific application, a broad general dividing line will fall in the neighborhood of 100 hp. Below this rating, single-stage turbines are commonly used; above this rating the multistage turbine is more popular. Occasionally, where space is at a premium and simplicity essential, the single-stage unit will be used for the larger ratings.

In accord with this general philosophy of mechanical drive of auxiliaries by steam turbines, Fig. 28 presents approximate turbine coupling efficiencies for estimating purposes. The efficiencies in Fig. 28 indicate the approximate level of currently quoted efficiencies on auxiliary-drive turbines. In using this curve, it must be realized that considerable latitude

* Contributed by the Editor and used, by permission, from Chapter I, "Heat Engineering and Thermodynamics," by J. Kenneth Salisbury, of Volume II, *Marine Engineering*, edited by H. L. Seward; published by the Society of Naval Architects and *Marine Engineers*, *1944*.

is available to the purchaser in the direction of either higher costs and better efficiencies or lower costs and poorer efficiencies. If any departure from the ordinary line of equipment is specified, it usually will be necessary that special designs be made, which may increase the cost.

In Fig. 28 turbine efficiencies based on the output at the turbine coupling have been plotted against rating in horsepower for various rotational speeds. All these efficiencies are based on steam conditions of 400 psig, 0 F superheat, and 15 psig exhaust pressure. For other steam conditions correction factors are provided to take care of inherent variation in efficiency with steam conditions. These corrections are shown in Figs. 29 and 30. Figure 29 presents the correction for initial pressures other than 400 psig. This correction is primarily a function of the rating of the unit; hence the parameter shown in the correction curves is horsepower rating. In Fig. 29 is shown an additional curve from which the superheat correction may be obtained. Large turbines are insensitive to moderate changes in available energy because they operate at or near their peak-efficiency speeds; superheat correction is intended to take care of the changes in efficiency resulting from change in moisture and supersaturation loss in the exhaust end

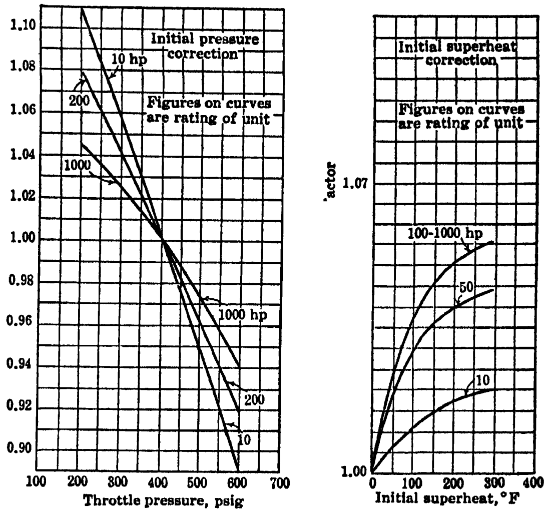


FIG. 29. Correction factors for initial pressure and superheat. These correction factors are to be applied to the efficiency obtained from Fig. 28.

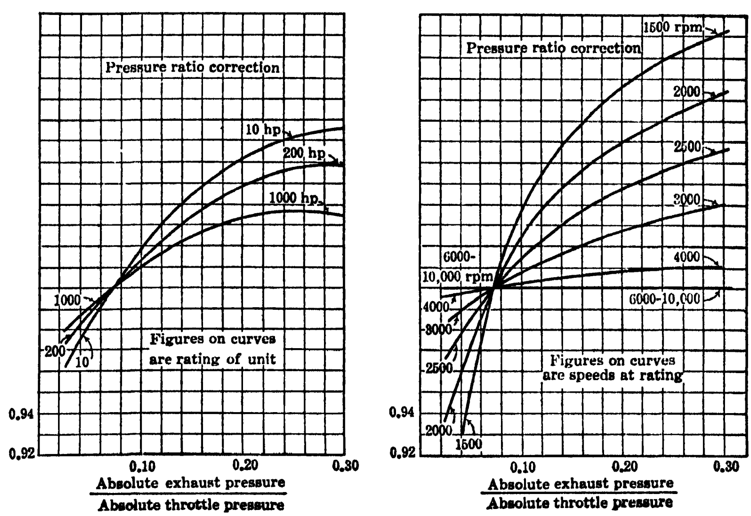


FIG. 30. Correction factors for exhaust pressure as a function of rating and rpm. These correction factors are to be applied to the efficiency obtained from Fig. 28.

of the turbine. Ordinarily mechanical-drive turbines, because of their somewhat poorer efficiency and higher exhaust pressures, have less moisture loss. They are sometimes "underspeeded," especially in the smaller sizes, for economic reasons; hence the superheat correction in this case takes care of the effect on efficiency of changes in velocity ratio that result from change in the initial superheat, at a given pressure.

Figure 30 provides correction factors resulting from changes in ratio of absolute exhaust pressure to absolute initial pressure. To cover this correction adequately it is necessary that two correction factors be used, one of them depending on the rating of the unit and the other on the design speed. These corrections, valid for exhaust pressures up to 50 psig, should not be used for units having an exhaust pressure higher than this value.

To obtain the coupling efficiency of an auxiliary-drive turbine the basic efficiency is read from Fig. 28 at the proper rating and design rotational speed. This basic efficiency is then multiplied by an initial pressure-correction factor taken from Fig. 29. The resulting efficiency is then corrected for turbine pressure ratio, using the data given in Fig. 30. The product represents the full-load efficiency of the turbine based on coupling output and referred to the Rankine cycle.

EFFICIENCY OF HIGH-SPEED AUXILIARY TURBINE-GENERATOR SETS.* The trend in auxiliary turbine-generator sets is toward efficient high-speed geared turbines. These turbines generally operate at about 10,000 rpm and are connected through gearing to low-speed (about 1200 rpm) generators. When built in this pattern the auxiliary turbine-generator sets have a level of efficiencies which is approximately 10 to 15% higher than the slower-speed sets. In some cases the improved efficiency will not be warranted, and the slower speed set may be chosen. On the other hand, since the high-speed geared set is more modern in design and shows promise of being the type of equipment that will be most widely used in the future, the curves of auxiliary turbine-generator set efficiencies shown in Fig. 31 are for this type of design.

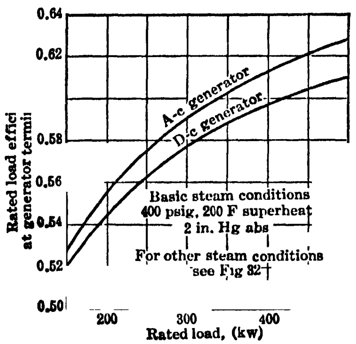


FIG. 31. Efficiency of geared auxiliary turbine-generator sets at 400 psig–200 F superheat–2 in. Hg abs. For other steam conditions, apply the correction factors of Fig. 32.

In Fig. 31 are shown the efficiencies referred to the Rankine cycle of a-c and d-c turbine generator sets in a range of powers from 150 to 500 kw. These efficiencies, based on the output at the auxiliary-generator terminals, are predicated on initial steam conditions of 400 psig, 200 F superheat, and 2 in. Hg abs exhaust pressure. While the basic steam conditions for which the curves are drawn are representative, correction factors are provided in Fig. 32 by which these efficiencies may be adjusted for the inherent variation in efficiency which results from changed steam conditions. In Fig. 32 three correction factor curves are shown. These factors correct for initial pressure, initial superheat, and exhaust pressure. Because the initial pressure-correction factor depends on the size of the unit under consideration, two curves are drawn, for 150 kw and 500 kw. For intermediate ratings the pressure-correction factor may be linearly interpolated. Superheat and exhaust pressure-correction factors are independent of the rating.

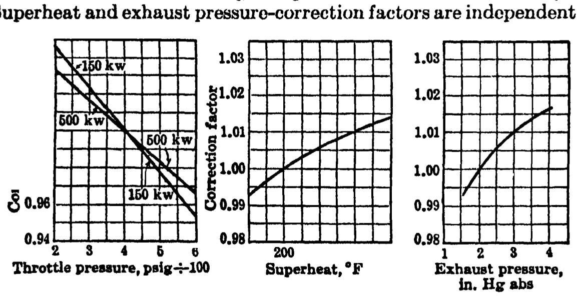


FIG. 32. Correction factors to be applied to the efficiency obtained from Fig. 31.

To determine the full-load efficiency of an auxiliary turbine-generator set the efficiency is read from Fig. 31 at the proper rating and then multiplied successively by the correction factors for initial pressure, superheat, and exhaust pressure.

* Contributed by the Editor and used, by permission, from Chapter I, "Heat Engineering and Thermodynamics," by J. Kenneth Salisbury, of Volume II, *Marine Engineering*, edited by H. L. Seward; published by the Society of Naval Architects and Marine Engineers, 1944.

EXAMPLE. Find the efficiency of an auxiliary a-c turbine-generator unit rated 250 kw, having steam conditions of 300 psig, 100 F superheat, and 3 in. Hg abs exhaust pressure.

Rated load efficiency (from Fig. 31) = 57.5%
Initial pressure-correction factor (from Fig. 32) = 1.02
Initial superheat-correction factor (from Fig. 32) = 0.982
Exhaust pressure-correction factor (from Fig. 32) = 1.010
Rated load efficiency = 57.5 (1.02) (0.982) (1.01) = 58.2%

EFFICIENCIES OF NONCONDENSING TURBINES, NEMA SIZES, at the generator terminals, are given in Table 6. Most efficient load is generally full rating.

Table 6. Engine Efficiencies of Noncondensing Turbines in Per Cent

(Various initial pressures; 200 F superheat; atmospheric exhaust pressure)

Rating, kw	Initial Pressure, psig			Rating, kw	Initial Pressure, psig		
	200	400	600		200	400	600
500	59.6	55.1	50.9	3000	74 0	71.4	68.7
700	62 9	58.5	54.4	3500	74.8	72 4	70.0
1000	66.0	62 1	58 4	4000	75.3	73 1	71.1
1500	69.3	65.8	62.2	5000	76.1	74.1	72.1
2000	71.4	68.2	65.1	6000	76.5	75 0	73.3
2500	72 9	70.1	67.1	7500	77.1	75.9	74.4

Note. In Tables 6, 8, and 11 the 500- to 1500-kw values are for high-speed turbines geared to 1200 rpm a-c generators. Turbine speeds are chosen to suit the steam conditions. Data for units 2000 kw and above are for 3600-rpm direct-connected turbine alternators.

Correction factors to noncondensing engine efficiencies for partial loads vary with initial conditions. Table 7 gives typical correction factors to engine efficiencies of the noncondensing units of Table 6 for one set of initial conditions.

Table 7. Correction to Engine Efficiencies of Noncondensing Turbines for Partial Load Operation

(400 psig, 200 F, superheat; atmospheric exhaust pressure)

Rating, kw	Load, %				
	25	50	75	100	125
500	.698	.883	.960	1.00	.985
700	.710	.889	.963	1.00	.985
1000	.718	.893	.965	1.00	.985
1500	.726	.897	.996	1.00	.984
2000	.730	.898	.966	1.00	.984
2500	.733	.899	.966	1.00	.984
3000	.735	.900	.966	1.00	.983
3500	.736	.900	.967	1.00	.983
4000	.738	.901	.967	1.00	.982
5000	.739	.902	.967	1.00	.982
6000	.740	.903	.968	1.00	.981
7500	.741	.904	.968	1.00	.980

EFFICIENCIES OF CONDENSING TURBINES, NEMA SIZES, at the generator terminals, at rated load are given in Table 8. (See note above regarding unit speeds.)

Table 8. Engine Efficiencies of Condensing Turbines in Per Cent

(Various initial pressures; 200 F superheat; 2 in. Hg abs exhaust)

Rating, kw	Initial Pressure, psig			Rating, kw	Initial Pressure, psig		
	200	400	600		200	400	600
500	61.2	58.5	56.3	3000	72.6	70.8	69.3
700	63.4	61.7	59.1	3500	73.2	71.5	70.2
1000	66.0	63.8	61.8	4000	73.7	72.1	70.9
1500	68.6	66.3	64.7	5000	74.6	73.1	71.9
2000	70.4	68.4	66.7	6000	75.2	73.8	72.6
2500	71.6	69.8	68.1	7500	75.9	74.5	73.5

Correction factors to condensing efficiencies for partial loads vary with initial conditions. Table 9 gives correction factors for the units of Table 8 when operating at the specified initial conditions.

Table 9. Correction to Engine Efficiencies of Condensing Turbines for Partial Load Operation

(400 psig; 200 F superheat, 2 in. Hg abs)

Rating, kw	Load, %			
	25	50	75	100 and 125
500	.754	.907	.970	1.00
700	.769	.914	.973	1.00
1000	.783	.920	.975	1.00
1500	.796	.925	.977	1.00
2000	.803	.929	.978	1.00
2500	.809	.932	.978	1.00
3000	.812	.933	.979	1.00
3500	.814	.934	.979	1.00
4000	.816	.935	.979	1.00
5000	.819	.936	.979	1.00
6000	.821	.936	.980	1.00
7500	.822	.937	.980	1.00

Superheat corrections for smaller turbines are given for the noncondensing units (Table 6) and the condensing units (Table 8) in Table 10.

Table 10. Superheat Correction to Engine Efficiencies

Superheat, °F	Noncondensing	Condensing
100	0.988	0.980
125	0.992	0.985
150	0.996	0.990
175	0.998	0.995
200	1.000	1.000
225	1.001	1.004
250	1.002	1.009
275	1.003	1.013
300	1.003	1.017

GEARED TURBINES are sometimes furnished for industrial use. The gear losses lower the efficiencies of these sets. Expected engine efficiencies of certain larger multistage geared condensing turbines operating at 400 psig–200 F superheat–2 in. Hg abs back pressure, 3600 rpm, and above are shown in Table 11.

Table 11. Engine Efficiencies of Geared Turbines

Brake Horsepower	Engine Efficiency, Including Gears, %	Brake Horsepower	Engine Efficiency, Including Gears, %
2000	65.6	7,500	73.4
3000	68.6	10,000	74.3
5000	71.5		

AIEE-ASME PREFERRED STANDARD TURBINES. The performance data in Table 1, p. 8-14, applies to large condensing turbine generator units (3600 rpm, 3 phase, 60 cycle) built according to the AIEE-ASME preferred standard ratings.

Maximum Capacity. AIEE-ASME preferred standard turbines have a guaranteed capability of 110% of rated kilowatts when operated with the initial pressures and temperatures shown in Table 1 (p. 8-14) and an exhaust pressure of 2 1/2 in. Hg abs. The steam-flow capacities stated in Table 1 correspond to guaranteed kilowatt capabilities of the turbines when operated with the standard number of feedwater heaters.

Effect of Vacuum. Capability of turbines when operated with rated initial steam pressure and temperature and with extraction for feedwater heating in the standard number of heaters, but with exhaust pressure other than 2.5 in. Hg abs, will change by the percentage given in Table 12.

Table 12. Effect of Vacuum on Capability

Initial Steam Pressure, psig	Initial Steam Temperature, °F	Per Cent Change in Kilowatt Capability					
		Exhaust Pressure, in. Hg abs					
		1.0	1.5	2.0	2.5	3.0	3.5
600	825	+3.9	+3.1	+1.8	0	−1.9	−4.0
850	900	+3.3	+2.6	+1.4	0	−1.7	−3.5
1250	950	+2.6	+2.0	+1.2	0	−1.5	−3.0

Effect of Initial Pressure. The kilowatt capability of turbines when operated at initial temperature, and with extraction for feedwater heating in the standard number of heaters, will be reduced by a 50-psi drop in initial pressure, as shown:

Initial Steam Pressure, psig	Initial Steam Temperature, °F	Reduction in Kilowatt Capability, %
600	825	9
850	900	6
1250	950	4

Effect of Initial Temperature. The kilowatt capability will be reduced 2% for 50 F reduction in initial steam temperature at rated initial pressure, and with full extraction for feedwater heating.

Effect of Bypassing Top Heater. When operated with rated initial steam conditions and an exhaust pressure of 2.5 in. Hg abs or lower, and with extraction for feedwater heating but with no steam extracted from the highest pressure extraction outlet provided in the standard turbine, the kilowatt capability of the turbine will be increased by 4%.

Extraction Openings. Table 1, p. 8-14, shows saturation temperatures at the extraction openings, with all openings in service.

14. PERFORMANCE CALCULATIONS

Steam-turbine calculations are made for the purposes of design, of estimating turbine performance with extraction heaters or bleeder connections, and of checking test results. Methods of making calculations are varied, since the procedure is not standardized. Many assumptions are made to simplify the work. The following methods yield results satisfactory for estimating and checking purposes. For an extensive discussion, see Ref. 64.

CHARACTERISTIC CURVES. Three characteristic curves are needed to analyze turbine performance: the *Willans line*, the *shell pressure curves*, and the *state-line*, or condition curve. All three are discussed in this article. The first of them, the *Willans line*, indicates the relation between total steam per hour and load. This line may be assumed to be straight between no load and *most efficient load.* Above most efficient load, the form of the line depends on the overload valve arrangement. The slope of the line may differ from that below most efficient load. With nozzle governing, the true Willans line consists of a series of scallops, concave down, each having a height of only about 0.5 to 3% of the steam flow. For all practical purposes a straight line up to most efficient load may be substituted.

When guarantees of steam consumption are not available, the Willans line may be readily estimated by methods given herein. The available energy from initial conditions to final pressure can be found using the Mollier chart, or the theoretical steam rate may be found (see Section 4). Warren and Knowlton (Ref. 65) present efficiency data representative of present practice in large steam turbines.

Figure 33 from Ref. 65 presents engine efficiencies for large turbines of various capacities, with 300 F initial superheat, at various initial steam pressures, and given generator efficiencies, with assumed mechanical and exhaust losses. The engine efficiency at other superheats can be found by *dividing* the engine efficiency found from Fig. 33 by the values given in Table 13.

Table 13. Superheat Corrections to Engine Efficiency

Superheat at throttle, °F	0	100	200	300	400	500	600
Correction factor	.080	1.044	1.018	1.000	0.987	0.978	0.970

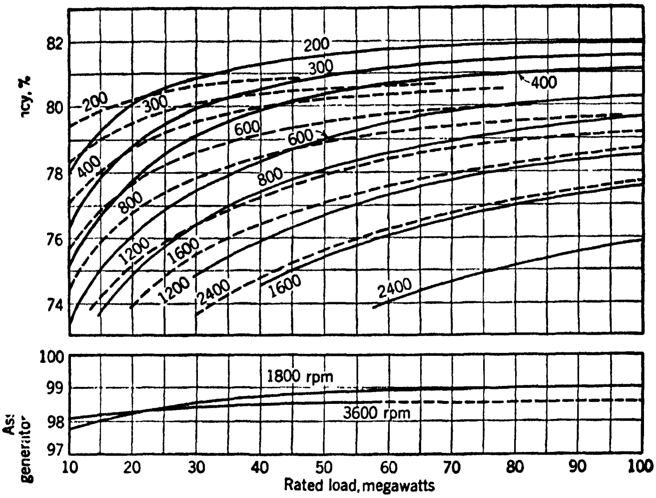


FIG. 33. Overall engine efficiencies of large General Electric condensing-turbine-generator units versus rated load, megawatts. Drawn for 300 F initial superheat, with 4% exhaust loss and 1.25% mechanical loss assumed at all ratings. Full lines, 1800 rpm; dash lines, 3600 rpm. Generator efficiencies assumed as shown; hydrogen cooling. Figures on curves are throttle pressure, psig. (Used by permission from Warren and Knowlton, Relative Engine Efficiencies Realizable from Large Modern Steam Turbine Generator Units, *Trans. ASME*, 1941)

Having an engine efficiency from Fig. 33 and the theoretical steam rate, the actual steam rate may be found:

$$\text{Actual steam rate} = \frac{\text{Theoretical steam rate}}{\text{Engine efficiency}}$$

Steam flow at rating is found by multiplying steam rate by the rated kilowatts. This flow is plotted as one point on the Willans line. For condensing units at normal exhaust pressures the no-load steam flow is approximately 0.5 lb per kw of rated capacity. This yields a second point. Connecting the two by a straight line gives a good first approximation of the Willans line. An approximate steam rate curve may be found by dividing various flows by corresponding loads. More accurate methods are given on p. 8-67.

CAPABILITY AND RATED LOAD. Some large units still are nominally rated at 80% power factor. Such units, however, are required to develop full kva at 100% power factor. The nominal rating thus is only 80% of the capability of the turbine-generator set.

When standardization of turbines was undertaken, the difference between nominal rating and capability was reduced. Capability is now 110% of nominal rating on standard units, a practice that may later be extended to all large units. Since capability is a true measure of the turbine's ability to deliver power, it is considered by some authorities as the true rating of the unit for power-generating purposes.

Capability of turbines rated less than 10,000 kw is generally 25% greater than the nominal rated capacity. The generator kva rating is at 80% power factor for nominal kilowatt capacity.

The maximum total steam required, in pounds per hour, is found by multiplying maximum output by the steam rate. The steam rate at maximum load may be larger than that at most efficient load by 0 to 4%.

NO-LOAD TOTAL STEAM FLOW with generator at full voltage depends on the design of turbine. Data collected from tests indicate that its value may be approximated

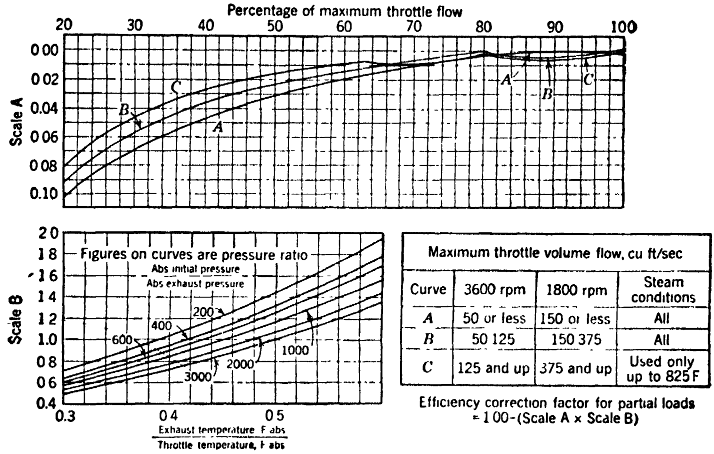
Table 14. No-load Steam Consumption of Condensing Turbines, Per Cent of Total Steam

Name plate rating, kw	1000	2000	3000	4000	5000	10,000	15,000	20,000	25,000	30,000	40,000	50,000 and over
% of total steam at most efficient load	13.8	12.5	11.8	11.3	10.9	9.9	9.2	8.8	8.6	8.4	8.2	8.0

from Table 14, which expresses no-load steam in percentage of total steam at most efficient load for various name plate nominal ratings. These figures apply to single-cylinder turbines with most efficient load at 80% of the name plate ratings for units rated 1000 kw and above. The values of the no-load factor apply only to straight condensing turbines and are somewhat higher than the average for modern high-efficiency units with hydrogen cooling.

Some designers use tables of steam-rate factors which, when multiplied by the steam rate at rated load, give the approximate steam rates at partial loads. Such factors are given in Tables 7 and 9 for NEMA sizes. See Ref. 66 for additional data.

Noncondensing no-load steam flow depends largely on steam conditions. For units of 500 kw and less, assume no-load steam = 20 to 25% of total steam at the most efficient load. Most efficient load in small noncondensing turbines is frequently full load.



Maximum throttle volume flow, cu ft/sec			
Curve	3600 rpm	1800 rpm	Steam conditions
A	50 or less	150 or less	All
B	50 125	150 375	All
C	125 and up	375 and up	Used only up to 825 F

Fig. 34. Partial-load correction factor for condensing turbines. (Warren and Knowlton, *op. cit.*)

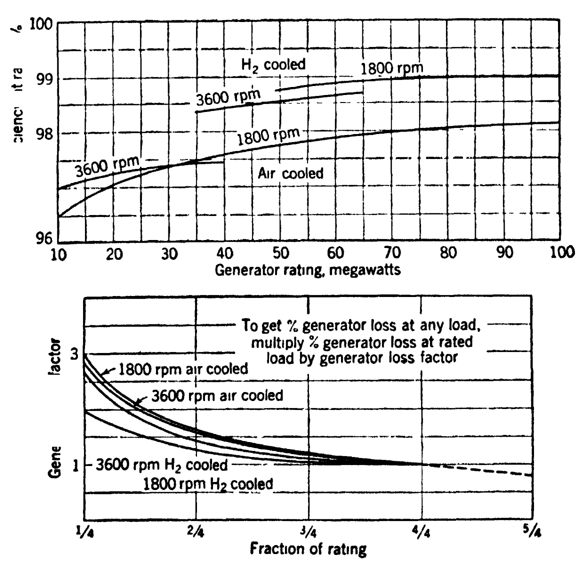


Fig. 35. Approximate efficiencies of General Electric turbine generator, 3600 and 1800 rpm, air- and hydrogen-cooled. Curves drawn for 0.80 power factor; for 0.90 power factor add 0.15% to rated-load generator efficiency; for 0.70 power factor subtract 0.25% from rated-load generator efficiency. (Warren and Knowlton, *op. cit.*)

ESTIMATING STEAM RATES FOR LARGE TURBINES. A more complete method of estimating turbine performance is presented by Warren and Knowlton in Ref. 65. Corrections for fractional load are shown in Fig. 34, for generator efficiency at other than rated load in Fig. 35, for mechanical losses in Fig. 36, and for exhaust losses in Fig. 37.

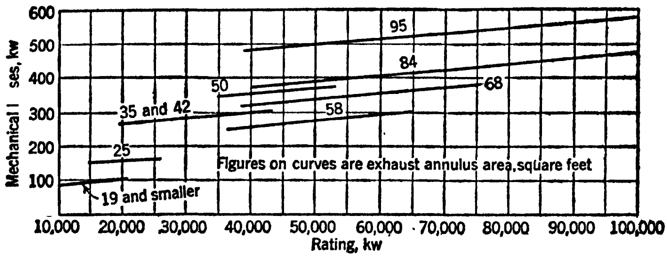


FIG. 36. Approximate mechanical losses of condensing turbines versus rated load, in kilowatts. Mechanical losses are constant at all loads for any given unit; therefore, the per cent mechanical loss varies inversely with load. (Warren and Knowlton, *op. cit.*)

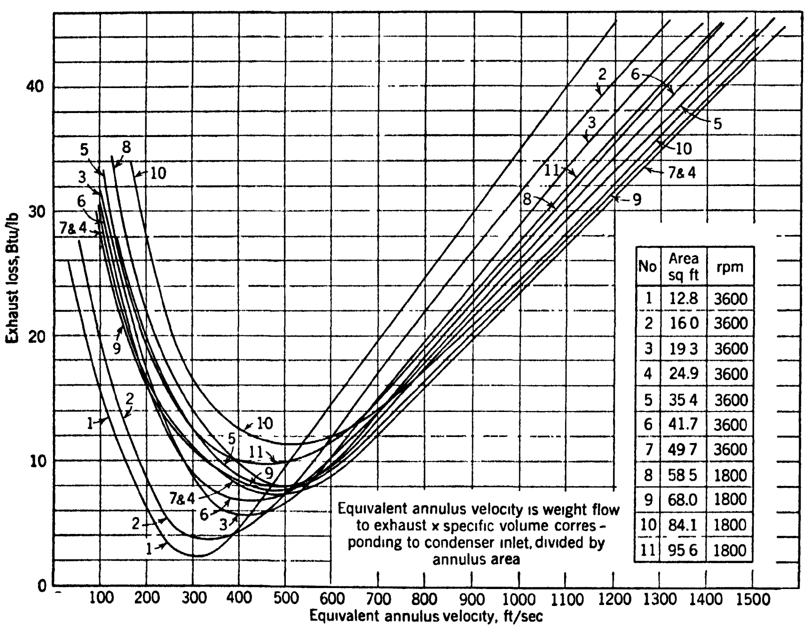


FIG. 37. Exhaust-loss curves for condensing turbines. (Warren and Knowlton, *op. cit.*)

The internal efficiency of a turbine is the ratio of the heat converted into work at the turbine shaft to the available energy. Figure 34 gives corrections to the basic internal efficiency due to change of load, but does not include change in exhaust loss, mechanical losses, or generator losses. The maximum throttle flow volume must be estimated before Fig. 34 can be used.

Exhaust losses vary with load. The magnitude of the exhaust loss depends largely on the area of the exhaust annulus. Various areas may be used for a turbine of a given rating, depending on the average vacuum and the desired efficiency. Because a large annulus decreases exhaust loss but increases first cost it is necessary for the designer to arrive at an economic balance, considering all factors. Exhaust annuli vary from 1 to 2 sq ft per 1000 kw of rated load.

The exhaust loss in Fig. 37 is expressed in Btu per pound of exhaust steam. At the lower velocities, the curves include internal losses in the last few stages, inherent in low steam flows in these sections. The exhaust loss in per cent equals (100 × exhaust loss in Btu per lb, Fig. 37) ÷ available energy, for nonextraction turbines. This percentage loss is determined

for each load. The final nonextraction engine efficiency, corrected to the true exhaust loss, equals efficiency at 4% exhaust loss from Fig. 33 $\times \dfrac{100 - \text{actual \% exhaust loss}}{96}$

EXAMPLE. Assume a condensing nonextraction turbine rated at 30,000 kw; 0.8 power factor; 850 psig–900 F–1 1/2 in. Hg abs; hydrogen-cooled generator; capability 33,000 kw.

From Fig. 33 the overall engine efficiency (with 4% exhaust loss, 1.25% mechanical losses, and 98.4% generator efficiency) at a capability of 33,000 kw with 850 psig, 300 F superheat = 77.63%. Engine efficiency corrected for superheat $= \dfrac{77.63}{0.99} = 78.4\%$. Available energy = 1453.1 − 896 = 557.1 Btu per lb. Therefore, approximate nonextraction steam rate $= \dfrac{3413}{0.784 \times 557.1} = 7.81$ lb per kwhr. Approximate maximum throttle flow = 33,000 × 7.81 = 257,730 lb per hr.

To find exhaust loss: Approximate heat to kw output $= \dfrac{3413}{7.81} = 436.8$ Btu per lb. Therefore, approximate exhaust condition = 1453.1 − 436.8 = 1016.3 Btu per lb. At 1 1/2 in. Hg abs and 1016.3 Btu, the approximate quality is 91.8%, and the specific volume is 0.918 × 444.9 = 408.4 cu ft per lb. Assume an exhaust of 35.4 sq ft area (No. 5 in Fig. 37). Then exhaust velocity $= \dfrac{257{,}730 \times 408.4}{3600 \times 35.4} = 826$ ft per sec. This corresponds on Fig. 37 to an exhaust loss of 18.3 Btu per lb. Hence exhaust loss = 18.3/557.1 = 3.28% of the available energy.

Partial load corrections. Specific volume of steam at throttle conditions of 864.7 psia, 900 F = 0.887 cu ft per lb. Maximum volume flow at throttle $= \dfrac{257{,}730 \times 0.887}{3600} = 63.5$ cu ft per sec. This places turbines in Class B (Fig. 34).

$$\frac{\text{Exhaust temperature, °F abs}}{\text{Throttle temperature, °F abs}} \quad \frac{459.6 + 91.7}{459.6 + 900}$$

$$\frac{\text{Absolute initial pressure}}{\text{Absolute exhaust pressure}} \quad \frac{850 + 14.7}{0.734} \quad 1178$$

Factor, scale B = 0.85

Conditions at partial loads can be computed and overall nonextraction steam rate found as in Table 15. The Willans line for this unit is shown in Fig. 38.

Table 15. Nonextraction Performance of 30,000-kw Turbine

1. Per cent of maximum throttle flow	20	40	60	80	100
2. Throttle flow, lb per hr	51,546	103,092	154,638	206,184	257,730
3. Basic overall efficiency of turbine = 78.4% with 4% exhaust loss, 1.25% mechanical loss, and 98.4% generator efficiency					
4. Basic internal efficiency, % $\dfrac{78.4}{0.96 \times 0.9875 \times 0.984}$ 84.0%					
Efficiency correction (Fig. 33)	0.922	0.969	0.987	1.00	1.00
Actual internal turbine efficiency (4 × 5), %	77.45	81.40	82.91	84.00	84.0
Equivalent annulus velocity, ft per sec	165	330	496	661	826
Exhaust loss (Fig. 36), Btu/lb	23	11	8	12	18.3
Exhaust loss % (8 ÷ available energy)	4.1	1.97	1.43	2.15	3.29
10. Turbine wheel efficiency $\left[6 - \left(\frac{9 \times 6}{100}\right)\right]$, %	74.27	79.80	81.72	82.19	81.2
11. Turbine wheel steam rate, lb per hr [100 × 3413] ÷ [(10) × available energy]	8.24	7.67	7.49	7.45	7.54
12. Heat to internal work, Btu per lb (10) × available energy	413.7	444.5	455.2	457.9	452.4
13. Heat to exhaust, Btu per lb	1039.4	1008.6	997.9	995.2	1000.3
14. Turbine wheel load, internal, kw (2 ÷ 11)	6255	13441	20645	27675	34181
15. Mech. losses, kw (Fig. 35)	285	285	285	285	285
16. Generator input (14 − 15), kw	5970	13156	20360	27390	33896
17. Generator loss, % (Fig. 34)	5.1	3.00	2.21	1.86	1.7
18. Generator loss $\left(\frac{16 \times 15}{100}\right)$, kw	304	394	450	509	576
19. Generator output (16 − 18), kw	5666	12762	19910	26881	33310
20. Overall nonextraction steam rate (2 ÷ 19), lb per kwhr	9.1	8.07	7.76	7.67	7.74

SHELL PRESSURE CURVES represent the second set of data required in analyzing turbine performance. In the older throttle-governed types of turbine the inlet pressure and all stage pressures are nearly proportional to the flow of steam through the turbine up to the most efficient load. The steam flow is related to load, by the Willans line, hence inlet pressure can be plotted against load. Inlet pressure at the most efficient load is less than throttle pressure by the pressure losses through throttle valve, strainer, and governor valve. This loss ranges from 3 to 5% of the absolute initial pressure. Below most efficient load, inlet pressure at any load is proportional to total steam flow. Above most efficient load, added steam passes through the throttle valve with added losses. These throttling losses above most efficient load can be assumed to vary as the square of total steam, since velocity through these areas is proportional to the total steam flow.

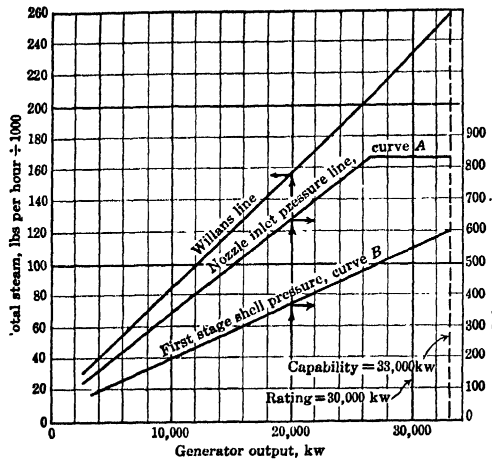


FIG. 38. Willans line and other data for turbine calculation given in the example, p. 8-67.

EXAMPLE. Assume that the 30,000-kw turbine above is throttle governed with most efficient load (full primary valve opening) at 26,881 kw when a 3% pressure drop occurs at the throttle. An overload valve is provided for higher loads. Inlet pressure at first-stage nozzle = 864.7 (1 − 0.03) = 838.7 psia. Inlet pressure at 5666 kw is proportional to steam flow and equals 838.7 (51,546/206,184) = 209.7 psia. Throttle pressure drop at 26,881 kw = 0.03 × 864.7 = 26 psi. Pressure drop at capability of 33,310 kw = 26 × $(257{,}730/206{,}184)^2$ = 40.6 psia. Inlet pressure at capability = 864.7 − 40.6 = 824.1 psia. An inlet pressure line *A* on Fig. 38 shows these conditions. Although many throttle-governed turbines of the type considered above are operating, few if any new turbines are of this design.

Assume that this turbine is nozzle governed, with all steam passing the first-stage nozzles, each group of nozzles being under the control of a separate valve, as in most modern units. The inlet pressure before the *first-stage* nozzles remains essentially constant at all loads. But the pressure before the nozzles of *each succeeding stage* can be assumed *approximately* (see Ref. 64) proportional to the steam flow, as shown on Fig. 56. If it is assumed that the pressure before the second-stage nozzles (commonly called the first-stage shell pressure) is 480 psia at 26,881-kw load, the pressure at 33,000 kw will be 480 × (257,730/206,184) = 600 psia; at 5666-kw output it will be, approximately, 480 (51,546/206,184) = 120 psia. These pressures are plotted as curve *B*, Fig. 38.

Frequently stage pressures in other stages are known at one particular flow or load. If they are, and if the variation in first-stage shell pressure (curve *B*, Fig. 38) is known, a complete set of shell pressure curves may be made up by proportioning each stage shell pressure to the first stage shell pressure in accordance with the ratio that exists at the known point. Such curves are illustrated in Fig. 56. For additional details, see Ref. 64.

THE STATE-LINE CURVE, also called the *condition curve* or the *expansion line,* is the third characteristic curve. It is the locus of points on a Mollier diagram representing the steam condition at each stage and the final end point. The exact condition curve can be plotted only from a step-by-step calculation of the nozzle, blade, disk, and leakage losses of each stage. For estimating purposes, such an accurately drawn curve is not necessary. Approximate curves serve ordinary needs, such as for estimates of station performance with regenerative heating.

Manufacturers usually furnish condition curves for given loads as in Fig. 54, or data on the condition of the steam at various extraction and reheat points, and at the exhaust, as well as data on the leaving losses. This information is part of the performance data on the unit. These condition points can readily be plotted on a Mollier diagram, and condition curves drawn through them. Similarly, if test data are at hand indicating steam conditions at various points, condition curves can be drawn.

An estimate of the state line of a turbine, needed for preliminary studies, can be made by using certain assumptions. The probable engine efficiency of a unit at most efficient load, if operated straight condensing, can be found from engine efficiencies previously given. The steam rate based on generator output can be computed from the engine efficiency.

The steam in expanding must furnish energy to supply the net output at the generator terminals, plus all generator and mechanical losses. This sum, the net energy being delivered to the turbine shaft, is known as the *wheel output*. It corresponds, in a way, to the indicated horsepower of a reciprocating engine. Since radiation from large turbines is small, this internal work is the *only* energy taken from the steam. All remaining energy therefore goes to the exhaust. The enthalpy of steam going to the condenser is equal to the initial enthalpy minus the heat that goes to wheel work on all straight condensing and noncondensing turbines. This enthalpy, known as the *exhaust point*, is used in estimating the state line.

Part of the heat to the exhaust is in the form of kinetic energy leaving the last row of buckets. This energy has been generated from a portion of the available energy, but cannot be converted into useful work. When this energy is subtracted from the exhaust point the state-line end point is obtained. A nearly straight line drawn on a Mollier diagram between the state-line end point and the initial conditions is a suitable first approximation to the condition curve, valuable for many estimating purposes. (See p. 8-83.)

Generator Losses. The manufacturer usually furnishes data on generator efficiency, or Fig. 35 may be used. If such data are not at hand, the following approximate formulas express generator efficiency at nominal or name plate rating at 80% power factor. Generator efficiency in per cent at nominal rating of unit, in kilowatts, is

$$\text{For 1800 rpm turbines } \eta_g = 98.85 - \frac{9}{(\sqrt{\text{rating}/1000})}$$

$$\text{For 3600 rpm turbines } \eta_g = 96.0 - \frac{2.5}{(\sqrt[3]{\text{rating}/1000})}$$

Correction factors for partial loads, in per cent of full load efficiencies, are approximately:

Generator load Per cent of rating	100	80	75	50	25
1800 rpm	100%	99.6%	99.5%	98.3%	95%
3600 rpm	100%	99.3%	99.0%	96.5%	92.5%

At turbine capability, the power factor of the generator is usually greater than 80%, with an increase in generator efficiency which would require a correction factor greater than 1.00. For estimating purposes a factor of 1.01 can be used for turbines under 10,000-kw rating.

The *coupling kilowatts* can be found by dividing the output by the generator efficiency.

Example. Assume a turbine with nominal rating of 7500 kw, 3600 rpm with most efficient load at 80% of rating, 80% power factor on the generator. The generator efficiency at rated load $\eta_g = 96.0 - (2.5^3\sqrt{7500/1000}) = 94.72\%$. At most efficient load of 6000 kw (80% of rating) generator efficiency, $\eta_g' = 0.993 \times 94.72 = 94.06\%$. Coupling kilowatts at most efficient load = 6000/0.9406 = 6379 kw.

The mechanical losses of a turbine consist of friction in journal and thrust bearings, power to drive governor, oil pumps, and water glands, and any other mechanical losses. Radiation is small, except in small auxiliary turbines where, for purposes of computation, it can be included in the mechanical losses.

Mechanical losses depend almost entirely on the speed of the unit, and may be considered constant at all loads, since the speed is maintained substantially constant. They are expressed as a percentage of the nominal rating of the generator, and can be supplied by the manufacturer or taken from Fig. 36. When not supplied they may be estimated by the following approximate formulas for single-cylinder units up to 100,000 kw:

1800 rpm turbines,

$$\text{Loss in per cent of nominal rating} = 1.5 - 1.15 \log_{10} (\text{rating}/10{,}000)$$

3600 rpm turbines,

$$\text{Loss in per cent of nominal rating} = \frac{4}{(\sqrt{\text{rating}/1000})}$$

Mechanical losses added to coupling kilowatts give *wheel kilowatts*, that is, the net power that must be developed on the shaft.

Example. The mechanical losses in the 7500-kw, 3600-rpm unit of the previous example would be $7500 \times 4/(100\sqrt{7500/1000}) = 110$ kw. Therefore, wheel kilowatts at most efficient load = 6379 + 110 = 6489 kw. Wheel kilowatts at other loads can be found by adding 110 kw to the coupling kilowatts at that load.

The heat-to-wheel work h_e in Btu per pound of steam at a given load now can be found.

$$h_e = \frac{3412 \times w\ \text{kw}}{Q}$$

where w kw = total wheel kilowatts and Q = total pounds of steam per hour passing through the turbine.

Of the total heat h_l that enters the throttle, h_e is all that passes out through the shaft as work; the remainder goes to the condenser with the exhaust steam. The heat to exhaust per pound of steam is $h_o = h_l - h_e$. h_o is known as the *exhaust point*.

EXAMPLE. Assume that the 7500-kw, 3600-rpm turbine operates at 450 psig, 750 F, 2 in. Hg abs with a steam rate at 6000 kw of 9.8 lb per kwhr. Total steam per hour = 6000 × 9.8 = 58,800 lb. Heat to internal work at most efficient load = (3412 × 6489)/58,800 = 376.5 Btu per lb. Enthalpy at 450 psig, 750 F = 1386.6 Btu per lb. Therefore, heat to exhaust = 1386.6 − 376.5 = 1010.1 Btu per lb. This is the exhaust point from which the leaving loss is subtracted to obtain the state-line end point on a Mollier diagram.

WETNESS AT THE EXHAUST is an important factor in leaving losses, erosion, resuperheating, and condenser design. It can readily be found from the Mollier diagram when the total heat h_o is determined. Moisture at the exhaust should not exceed 14% if excessive bucket erosion is to be avoided; moisture is usually limited to 11 to 12%. Turbine designers consider the stage efficiencies of those sections subject to wet steam to be decreased by 1 to 1.2% for each per cent of moisture present. If moisture content at the exhaust could be limited to 4 to 5% there would be no sacrifice in stage efficiency, since the steam would be supersaturated leaving the last blade row. This condition of 4 to 5% moisture at the exhaust can be secured with turbines operating on the reheat cycle, giving high efficiencies for such units.

LEAVING LOSS AND HOOD LOSS occurs in the exhaust hood between the last wheel exit and the exhaust flange to the condenser. (See Ref. 61.) The loss consists of both kinetic energy loss and pressure loss through the hood, since, according to the Power Test Code, vacuum is measured at the exhaust flange. This loss increases rapidly with load. The heat equivalent to the loss produces no useful work and passes to the condenser as part of h_o. The energy equivalent to this loss comes from the expansion of the steam from initial conditions to exhaust pressure.

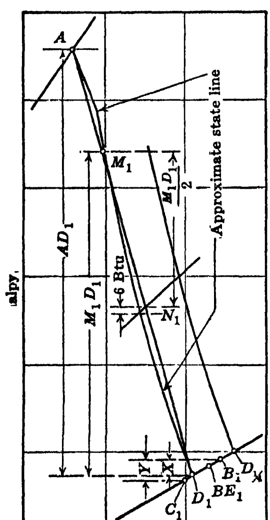


FIG. 39. Construction of the state line (see text).

The leaving loss and exhaust hood loss, sometimes called simply the *exhaust loss*, may be expressed in Btu per pound flow to the condenser, as a percentage of isentropic available energy.

The manufacturer may furnish an estimate of exhaust loss, as this can be determined from the turbine design. Robinson (Ref. 61) states that with a particular exhaust operating at fixed steam conditions, the exhaust loss increases roughly as the square of the quantity of steam flowing to the condenser. With a particular exhaust passing a fixed flow, increasing the total available energy by improved steam conditions, correspondingly reduces the percentage exhaust loss.

For heat-balance calculations the end point of the condition or state-line curve is taken as h_{D1}, Fig. 39. The condition curve represents the loci of steam states at each stage. Each stage available energy is multiplied by the applicable efficiency, to find a point on the condition curve. It is assumed that if it *were* possible to utilize the leaving loss, it would be utilized at the turbine internal efficiency, hence exhaust loss is always multiplied by the internal efficiency. Warren and Knowlton do this, as shown in Fig. 39. **B_1 is the exhaust point and D_1 corresponds to the state-line end point, h_{D1}. Y = exhaust loss and X/Y = internal efficiency of the turbine.**

Warren and Knowlton state that once the end point (such as D_1) of a condition curve is determined from data such as that in Table 15, the condition curve can be drawn. If a straight line is drawn from initial conditions A (Fig. 39) to the end point D_1, the condition curve at most efficient load will generally lie above this in the upper portion. Generally, it will cross the straight line about 25% of the way down. From the standpoint of feedwater heating and other calculations, the section above this crossing point may be neglected because, as a rule, steam is not extracted for feedwater heating much above this point. The true state line lies, on an average, about 6 Btu below this straight line midway between the

crossing point marked M_1, and D_1, as shown at N_1. For all practical purposes the condition curve or state line can be drawn $AM_1N_1D_1$. The exact shape is of no great importance, once the end points are determined.

From computations as in Table 15 it becomes possible to determine exhaust points and end points for partial loads. Most of the condition curves through these end points will be substantially parallel to the condition curve $M_1N_1D_1$ and can be so drawn.

THE INTERNAL EFFICIENCY, η_i, of a turbine with no extraction or reheat is the ratio of the utilized heat, as shown by the condition curve, to the total available energy. On Fig. 39, the condition curve efficiency $\eta_i = (h_A - h_{D1})$/available energy.

THE WHEEL EFFICIENCY, η_w of such a turbine is the ratio of the heat to work on the shaft to the total available energy. On Fig. 39, the wheel efficiency $\eta_w = (h_A - h_{B1})$ /available energy.

STAGE EFFICIENCY is the ratio of the heat to work in a stage of a turbine, measured on the shaft, to the isentropic available energy for that particular stage. Nozzle and blade losses together with disk and idle blade loss and leakage must be considered. In general, $\eta_s = \eta_n\eta_b\gamma$, where η_s = stage efficiency; η_n = nozzle efficiency; η_b = diagram efficiency; γ = a factor to cover disk, idle blade, leakage, and radiation losses. In large turbines γ is large, about 98%, for units with full peripheral admission and moderate blade speeds when labyrinth packings are tight. On small machines with partial admission, high wheel speeds and large leakage, γ is smaller.

REHEAT FACTOR. It is well known from thermodynamics that the heat returned as reheat in a stage increases the available heat to produce work in succeeding stages. Hence the total heat available to produce work in the several stages Σh_r will exceed the total available energy $(h_1 - h_2)$. The ratio $\Sigma h_r/(h_1 - h_2)$ is known as the *reheat factor*, R.

Let h_r = work per stage and $(h_1 - h_2')$ = stage available energy. Thus stage efficiency $\eta_s = h_r/(h_1 - h_2')$. *Let* η_s = average stage efficiency. It can be shown that $\eta_i = \eta_s R$. Hence, with the condition curve found, it is possible to estimate the average stage efficiency, provided the reheat factor R is known.

Reheat factors are discussed in Refs. 67–69.

Table 16 presents reheat factors taken from Thatcher's results, for an infinite number of stages. If R_∞ = reheat factor for an infinite number of stages and R_n = reheat factor for n stages, $R_n = 1 + (R_\infty - 1)[(n - 1)/n]$.

Table 16. Reheat Factors *

(Reprinted by permission from Thatcher, Reheat Factors for Expansion of Superheated and Wet Steam, *Trans. ASME*, 1939)

FOR SUPERHEAT REGION

PRESSURE RATIO, P_1/P_{sat}

η_s	5	10	20	30	40	50
.90	1.018	1.024	1.031	1.035	1.038	1.039
85	1.026	1.038	1.050	1.054	1.057	1.059
.80	1.034	1.051	1.066	1.072	1.077	1.081
.75	1.043	1.065	1.081	1.090	1.097	1.101
.70	1.053	1.078	1.100	1.111	1.119	1.123
.65	1.066	1.092	1.116	1.130	1.141	1.146
.60	1.073	1.105	1.132	1.151	1.165	1.170
.55	1.082	1.119	1.154	1.173	1.187	1.195
.50	1.092	1.134	1.173	1.195	1.211	1.222
.45	1.105	1.149	1.195	1.222	1.235	1.246

FOR SATURATION REGION

ISENTROPIC AVAILABLE ENERGY, BTU

η_s	80	120	160	200	240	280
.85	1.006	1.009	1.012	1.015	1.019	1.022
.80	1.008	1.012	1.017	1.021	1.025	1.029
.75	1.011	1.015	1.021	1.026	1.031	1.036
.70	1.013	1.019	1.025	1.031	1.038	1.044
.65	1.015	1.022	1.029	1.036	1.045	1.051
.60	1.017	1.025	1.034	1.042	1.052	1.058
.55	1.019	1.028	1.038	1.047	1.058	1.066
.50	1.021	1.031	1.042	1.053	1.064	1.074

* Values are for an infinite number of stages.

15. EXTRACTION CALCULATIONS

Turbines larger than 500 kw are generally provided with extraction openings from which steam may be bled for regenerative feedwater heating. Table 1 (p. 8-14) shows temperatures at extraction points of the AIEE-ASME Preferred Standard turbines. They may be considered representative of desirable extraction points on other turbines. The number of regenerative feedwater heaters varies from one to six in usual installations. If economizers are used, the highest temperature heater may be omitted and the extraction opening blanked off, resulting in a small increase in turbine capability. When air preheaters alone are installed all extraction points should be used to obtain optimum feedwater temperature to the boiler.

Effect of Regeneration. More throttle steam is required when steam is extracted for regenerative feedwater heating and the generator output is kept constant. A considerable portion of the original heat in the throttle steam is returned to the boiler in the hot feedwater leaving the last extraction heater, hence the heat per pound of steam to be supplied by fuel is reduced. There is also an increase in turbine efficiency. The net result of these effects is a reduction in turbine cycle heat rate, and a consequent improvement in station economy.

Complete heat-balance calculations under extraction conditions involve lengthy calculations, a knowledge of the particular turbine characteristics, and careful accounting for every heat unit entering the cycle from any source. Excellent approximations can be made by determining the engine efficiency and drawing condition curves for various loads by methods already described. (See pp. 8-68 and 8-70.)

THE HEAT BALANCE. A flow diagram is prepared including auxiliary services, such as steam jet ejectors, evaporators, boiler feed and other pumps, gland leak-offs, drain coolers, and make-up water. Assumptions are made of the pressure drops from turbine to heaters, of terminal differences in heaters and drain coolers, of heat losses by radiation, and of heat added in pumps. Heat-balance calculations are made for each heater, starting at the highest pressure heater. The power generated in each section of the turbine (between extraction points) is calculated, and the mechanical and electrical losses are deducted from the total internal power generated. The heat rate of the turbine in Btu per kilowatt-hour $= TS(H_1 - h_c) \div kw$, where TS = total pounds of steam per hour entering the throttle; H_1 = enthalpy at throttle; h_c = enthalpy of final feedwater, and kw = output at generator terminals. Methods for making such calculations are in various textbooks on steam turbines and power plants. (See Ref. 64.)

SHORT-CUT METHODS for quickly estimating heat rates have been developed. Some take the form of mathematical equations like those in Hendrickson and Vesselowsky's paper (Ref. 70) and others. Curves are presented in other papers which show the effects of the several variables on heat rates. (See Refs. 64, 72, and 73.)

The reduction in heat rate depends on the number of heaters, and increases with the number of heaters. The maximum gain theoretically occurs when an infinite number of heaters increases feedwater enthalpy to that of saturated steam at throttle pressure. This is not practically feasible, so that usually one to six or more heaters are used. There is also an optimum temperature to which feedwater can be heated in an ideal cycle to secure the maximum reduction in heat rate, for any specified number of heaters, as shown in Fig. 40, taken from Ref. 73. Theoretical reduction in the non-extraction heat rate for ideal cycles with an infinite number of contact heaters and no pressure drop in extraction lines is given in Table 18. Theoretical non-extraction heat rates are given in Table 17.

To obtain an estimate of heat rate divide the value obtained from Table 17 by the overall turbine-generator efficiency to obtain the actual nonextraction heat rate. Multiply the applicable value from Table 18, interpolating if necessary, by the applicable ordinate from Fig. 40 to obtain the percentage true reduction in nonextraction heat rate caused by regenerative feedwater heating. Decrease the actual nonextraction heat rate by the true reduction percentage to obtain the approximate final heat rate. Usually the final heat rate must be decreased by 1.0 to 2.5%, to allow for other gains which simultaneously occur.

Optimum feedwater temperature and reduction in heat rate with a given number of heaters vary with throttle pressure, temperature, and exhaust pressure. Data similar to Fig. 40 are shown in Ref. 71 and other technical literature.

In Fig. 40 the reduction in heat rate with any specified number of heaters decreases if the final feedwater temperature is either lower or higher than the optimum value, but it does not vary much over a considerable range of temperature on either side of the optimum. It is not always possible to extract steam at the pressure required for optimum feedwater

(*Continued on p. 8-75*)

Table 17. Theoretical Nonextraction Heat Rates, Btu per kwhr

(Taken by permission from Salisbury, The Steam Turbine Regenerative Cycle—An Analytical Approach, *Trans. ASME*, 1942)

Initial Pressure, psig	Initial Temperature, °F	Exhaust Pressures 1 in. Hg	1.5 in. Hg	2 in. Hg	2.5 in. Hg
200	650	10140	10528	10840	11094
	700	10031	10405	10704	10953
	750	9919	10280	10568	10812
	800	9804	10153	10432	10670
	850	9686	10023	10296	10528
	900	9565	9891	10160	10386
300	650	9681	10009	10279	10502
	700	9581	9907	10167	10383
	750	9481	9801	10053	10261
	800	9380	9692	9937	10137
	850	9280	9580	9819	10011
	900	9179	9466	9699	9883
400	650	9371	9677	9923	10110
	700	9284	9583	9822	10006
	750	9195	9487	9719	9900
	800	9105	9389	9613	9792
	850	9013	9289	9505	9682
	900	8919	9186	9396	9570
	950	8823	9081	9284	9456
600	650	8995	9261	9465	9646
	700	8914	9175	9377	9549
	750	8832	9087	9287	9451
	800	8750	8998	9194	9352
	850	8667	8907	9098	9252
	900	8583	8816	9001	9151
	950	8498	8725	8902	9050
	1000	8411	8633	8801	8949
800	650	8757	8992	9184	9341
	700	8679	8913	9099	9251
	750	8601	8832	9013	9161
	800	8522	8749	8925	9070
	850	8443	8665	8837	8978
	900	8364	8580	8749	8886
	950	8284	8495	8658	8792
	1000	8204	8409	8566	8697
1000	700	8513	8730	8898	9043
	750	8436	8651	8817	8951
	800	8360	8571	8735	8871
	850	8284	8491	8651	8784
	900	8208	8411	8567	8697
	950	8132	8330	8483	8609
	1000	8056	8248	8398	8521
1200	750	8314	8516	8675	8808
	800	8240	8438	8593	8723
	850	8165	8360	8511	8638
	900	8090	8282	8429	8553
	950	8015	8203	8347	8468
	1000	7939	8124	8265	8383
1800	850	7929	8102	8238	8349
	900	7855	8025	8157	8266
	950	7781	7948	8076	8183
	1000	7707	7871	7995	8100
2400	850	7797	7952	8076	8180
	900	7720	7875	7996	8098
	950	7646	7799	7917	8017
	1000	7565	7724	7839	7937

Notes. Table derived from *Theoretical Steam Rate Tables*, by J. H. Keenan and F. G. Keyes, ASME, New York, N. Y., 1938.

The values obtained from the tables have been altered slightly in a few cases to give smooth curves. Divide by the overall turbine-generator efficiency to obtain the non-extraction heat rate.

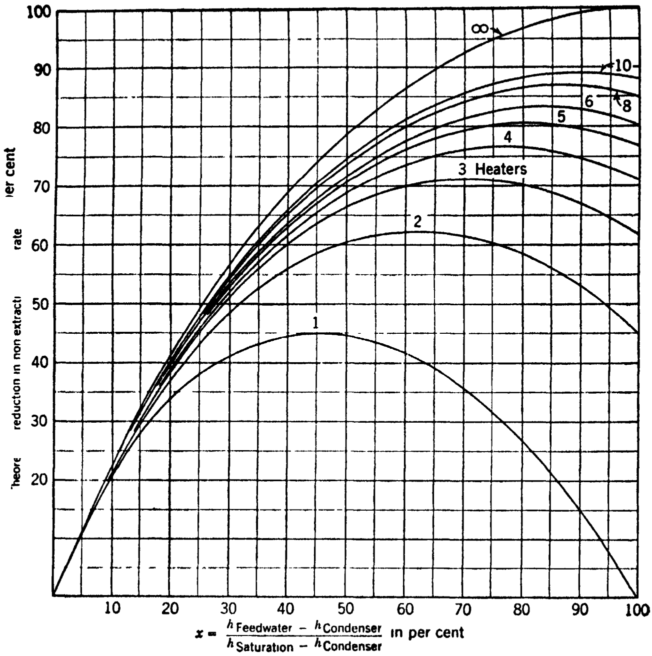


FIG. 40. Curve correlating fraction of "theoretical" gain against percentage of "possible" rise for various numbers of heaters. (Taken by permission from Salisbury, The Steam Turbine Regenerative Cycle—An Analytical Approach, *Trans. ASME*, 1942)

Table 18. Theoretical Percentage Reductions in Nonextraction Heat Rates for Various Initial Pressures, Temperatures, and Back Pressures in the Infinite-heaters Cycle *

(Taken by permission from Salisbury, The Steam Turbine Regenerative Cycle—An Analytical Approach, *Trans. ASME*, 1942)

Initial Pressure, psig	Initial Temperature, °F	Reduction in Non-extraction Heat Rate, %			
		1 in. Hg	1 1/2 in. Hg	2 in. Hg	2 1/2 in. Hg
200	700	10.57	10.33	10.15	10.01
	800	10.16	9.90	9.70	9.55
	900	9.75	9.46	9.25	9.08
400	700	12.34	12.17	12.04	11.94
	800	11.89	11.70	11.55	11.44
	900	11.44	11.22	11.06	10.93
	1000	10.99	10.75	10.57	10.43
600	700	13.57	13.44	13.34	13.26
	800	13.08	12.93	12.81	12.72
	900	12.59	12.41	12.28	12.18
	1000	12.10	11.90	11.75	11.63
800	700	14.64	14.54	14.47	14.41
	800	14.12	13.99	13.90	13.83
	900	13.60	13.45	13.33	13.24
	1000	13.08	12.90	12.77	12.67
1200	700	16.27	16.21	16.16	16.12
	800	15.67	15.58	15.52	15.47
	900	15.07	14.96	14.88	14.82
	1000	14.47	14.34	14.24	14.16
1800	700	18.35	18.32	18.30	18.28
	800	17.63	17.58	17.55	17.52
	900	16.91	16.85	16.80	16.76
	1000	16.19	16.11	16.05	16.00
2400	800	19.30	19.28	19.27	19.26
	900	18.47	18.44	18.41	18.39
	1000	17.64	17.59	17.55	17.52

*** Based on 80% turbine efficiency. For each 5 points increase in turbine efficiency the reduction in heat rate given increases 0.1, on the average.**

temperature. Furthermore the pressure at each extraction point varies with load. Figure 41 from Salisbury's paper shows the variation in the peak gain in heat-rate for various fractions of the optimum rise in feedwater enthalpy.

The extraction points on the AIEE-ASME Preferred Standard units were fixed after careful study of design, manufacturing, and operational considerations, and complete

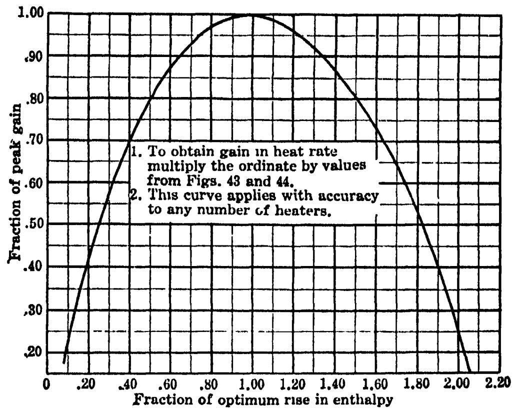


FIG. 41. Fraction of the peak gain in heat rate as a function of the fraction of the optimum rise in feedwater enthalpy. (Salisbury, *op. cit.*)

heat-balance data on such machines are available from manufacturers. Table 20, p. 8-82, shows how final feedwater temperature varies with load on a standard 60,000-kw unit. The optimum feedwater temperature for various pressures and number of extraction stages can be approximated from the curves in Figs. 40 and 42. (See also Ref. 64.)

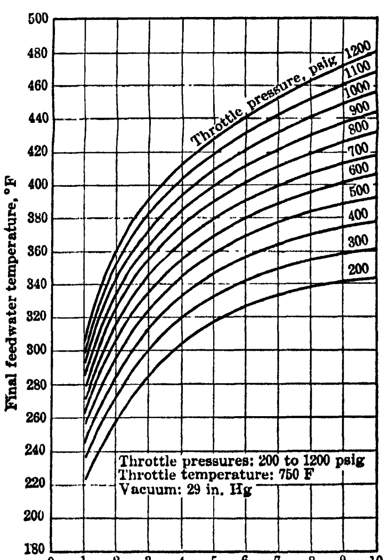


FIG. 42. Optimum final feedwater temperature.

While the data in Fig. 42 apply to 750 F throttle temperature, the final feedwater temperatures vary only a few degrees with change in throttle temperature from 750 F.

The percentage of reduction in nonextraction heat rate at optimum feedwater enthalpy and for various steam conditions is given in Figs. 43 and 44 from Salisbury's paper.

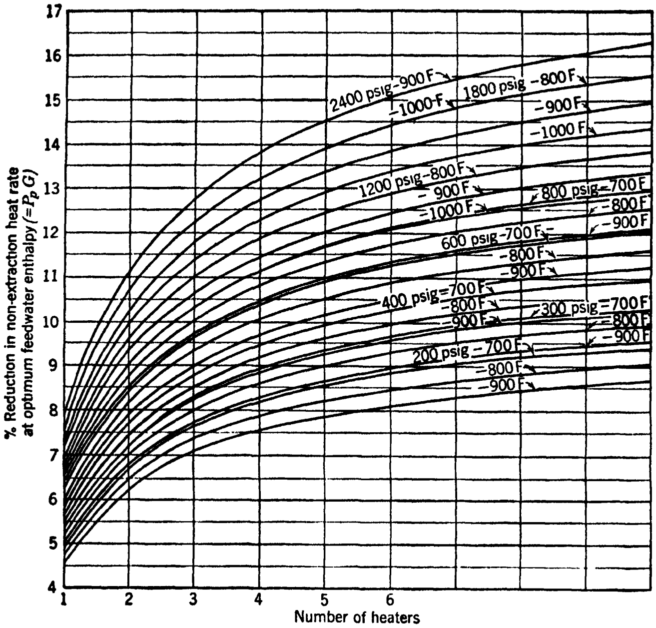


FIG. 43. Specific curves showing actual fractional gain over nonextraction heat rate for various steam conditions and number of heaters at 1 in. Hg abs. *Note:* For further gain due to reduction in exhaust loss, see text. For gain at reduced feedwater temperature, use Fig. 41. (Taken by permission from Salisbury, *op. cit.*)

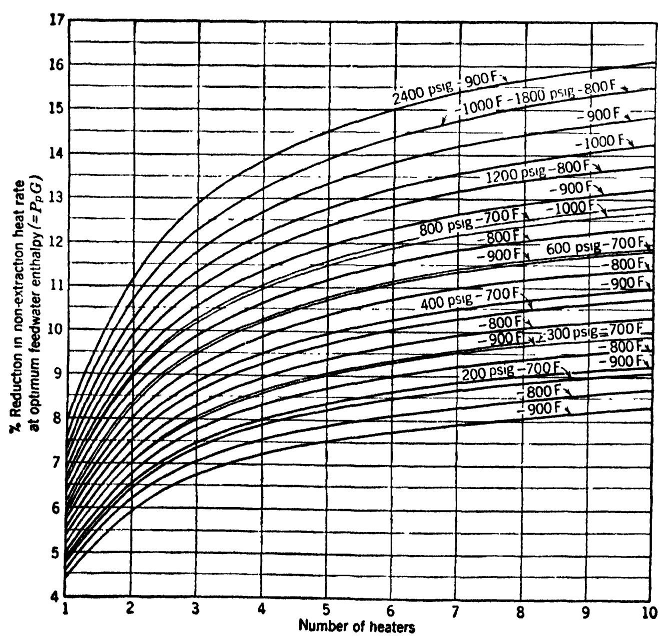


FIG. 44. Specific curves showing actual fractional gain over nonextraction heat rate for various steam conditions and numbers of heaters at 2 in. Hg abs. *Note:* For further gain due to reduction in exhaust loss, see text. For gain at reduced feedwater temperature, use Fig. 41. (Salisbury, *op. cit.*)

These curves indicate that (1) the gain obtainable by feedwater heating increases rapidly as initial pressures increase; and (2) the gain in heat rate decreases as initial steam temperature increases, at constant turbine inlet pressure, and that this decrease is greatest at the highest pressures.

ESTIMATING DATA FOR THE REGENERATIVE CYCLE.* Regenerative feedwater heating, or the extraction of steam from a turbine to heat boiler feedwater, results in a reduction in heat supplied to the cycle per unit of output, i.e., a better heat rate. The magnitude of the reduction depends on the operating steam conditions, the number of points at which steam is extracted, and the temperature to which the feedwater is heated.

Figures 45 to 48 permit a quick estimate to be made of the gain due to extraction for feed heating for typical steam conditions. These curves and the others which follow have been calculated from Salisbury's work, for steam conditions of

400 psig— 750 F—1 in. Hg abs
600 psig— 825 F—1 in. Hg abs
1250 psig— 950 F—1 in. Hg abs
1500 psig—1050 F—1 in. Hg abs

with 1, 2, 4, and 10 stages of feedwater heating.

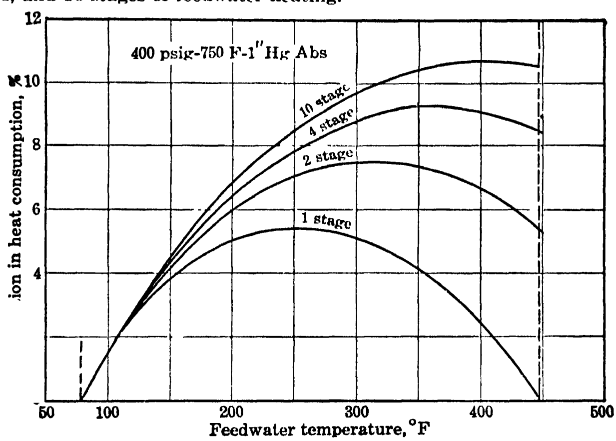


Fig. 45

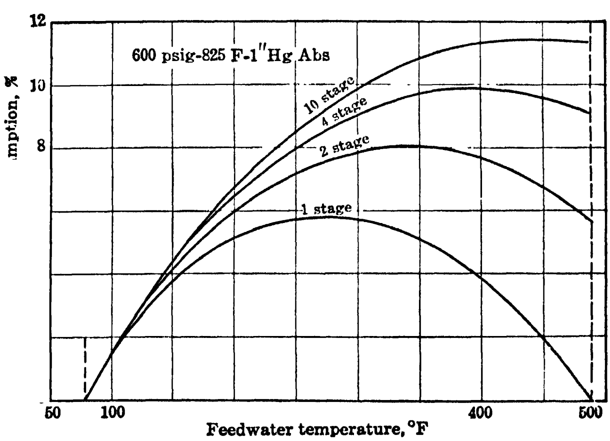


Fig. 46

Reduction in Nonextraction Heat Rate Due to Feedwater Heating in an Ideal Regenerative Cycle.

* This description, including Figs. 45 to 52, and Table 19, were contributed by A. O. White, Steam Turbine Engineering Division, General Electric Co.

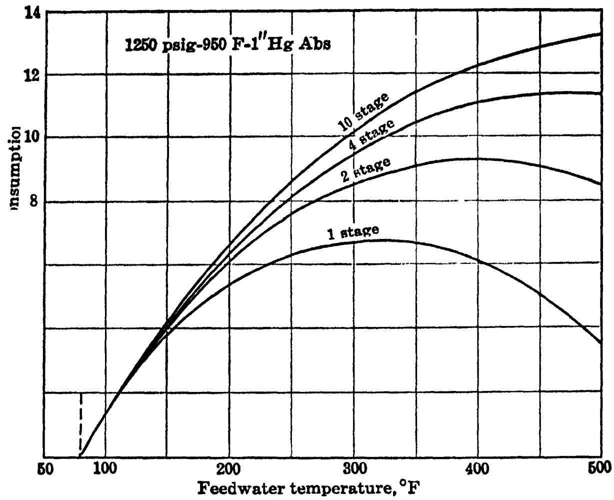


FIG. 47

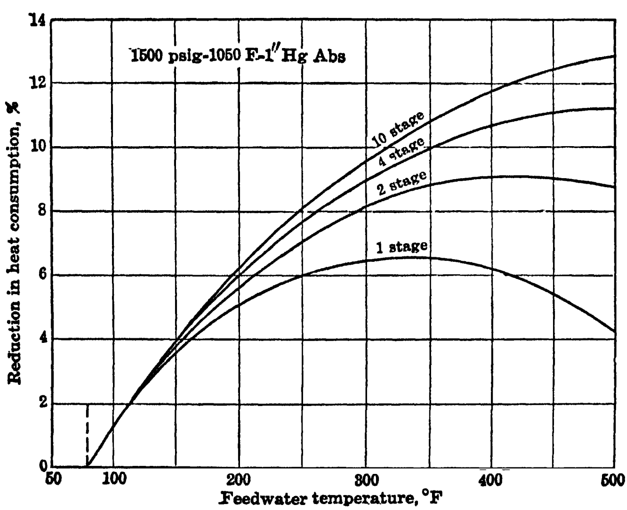


FIG. 48

Reduction in Nonextraction Heat Rate Due to Feedwater Heating in an Ideal Regenerative Cycle.

These figures show reduction in heat consumption, in per cent, with respect to the nonextraction heat rate, versus feedwater temperature, °F, for the four specified heater combinations.

Nonextraction heat rate is

$$HR_{nx} = (H_1 - h_c) \times SR$$

where H_1 = throttle enthalpy; h_c = condenser liquid enthalpy; and SR = steam rate, pounds per kilowatt-hour.

The steam rate may be either the *theoretical* steam rate (3412.75/available energy) or the *actual* steam rate for the turbine, yielding either *theoretical* or *actual* nonextraction heat rate, respectively. The theoretical nonextraction heat rate may also be found from

Table 17, taken from Ref. 73. The percentage reductions shown by the ordinate in Figs. 45 to 48 apply to the actual nonextraction heat rate. The resulting heat rate

$$\text{Extraction heat rate} = \text{Nonextraction heat rate}\left(1 - \frac{\%\ \text{reduction}}{100}\right)$$

is for a contact heater cycle as defined by Salisbury, with no terminal differences or pressure drops, and no credit for reduction in leaving loss. The heat rate found in this manner should be reduced approximately 1.0 to 2.5% to allow for reduced exhaust loss when steam

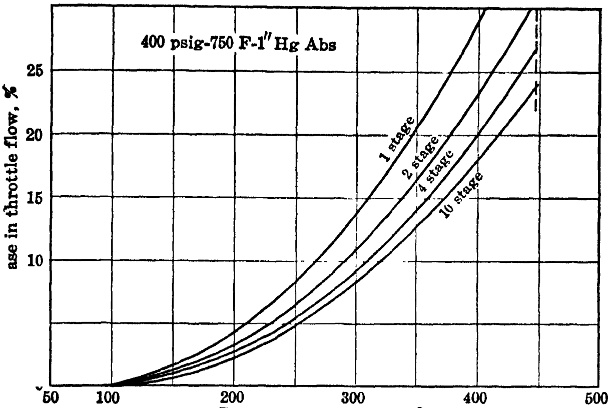


Fig. 49

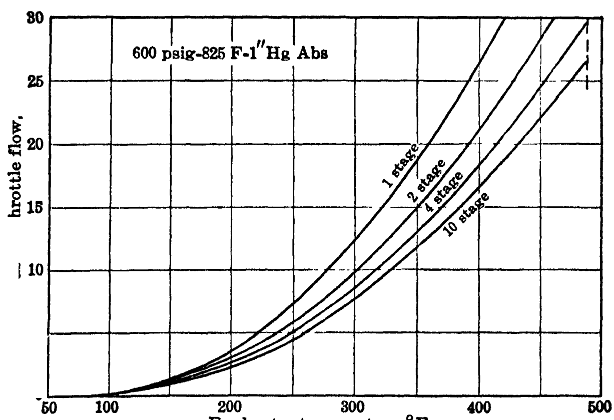


Fig. 50

Increase in Throttle Flow at Constant Load Due to Feedwater Heating in an Ideal Regenerative Cycle.

is extracted. It also should be increased about 0.5 to 1.0% to allow for cycle losses in the actual cycle, as compared with the ideal cycle. Thus

$$\text{Heat rate, extracting} = (\text{Nonextraction heat rate})\left(1 - \frac{\%\ \text{reduction}}{100}\right)(F)$$

where F = 0.98 to 1.00, the lower values applying to machines with large non-extraction exhaust loss and small cycle loss, and the higher values to machines with small non-extraction exhaust loss and poor heater arrangements. (See Refs. 64, 72, and 73.)

Comparisons between various steam conditions, number of heaters, and feedwater temperatures may be quickly made by use of these figures and the nonextraction heat rates in Table 17, provided engine efficiencies or the corresponding nonextraction steam rates are known for the specified steam conditions.

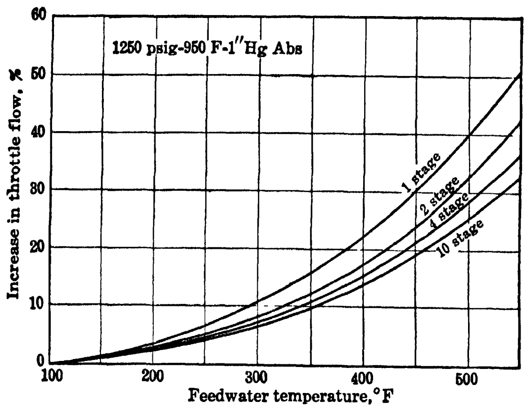


Fig. 51

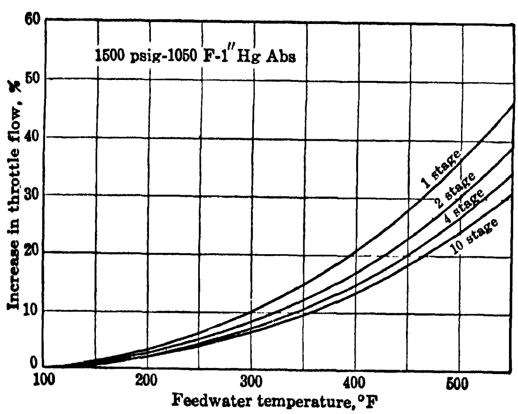


Fig. 52

Increase in Throttle Flow at Constant Load Due to Feedwater Heating in an Ideal Regenerative Cycle

Table 19. Total Steam Extracted, % of Throttle Flow

Final Feedwater Temperature, °F	Steam Pressure, psig							
	400		600		1250		1500	
	Stages of Feedwater Heating							
	2	10	2	10	2	10	2	10
150	7.0	7.1	6.9	7.0	......	......	......	......
200	11.4	11 8	11.3	11.6	11.2	11.5	10.7	11.0
250	15.6	16.2	15.5	16.0	15.5	15.9	12.6	13.1
300	19.6	20.6	19.5	20.4	19.5	20.3	18.8	19.5
350	23.6	24.8	23.5	24.6	23.6	24.6	20.7	21.6
400	27.1	29.0	27.1	28.8	27.4	28.9	26.4	27.8
450	......	......	30.2	32.5	31.1	33.2	30.0	32.0
500	......	......	......	......	35.0	37.4	33.9	36.1

Extraction for feedwater heating results in an increase in throttle flow for a given output, that is, an increase in throttle steam rate. Figures 49 to 52, inclusive, readily permit this increase to be determined. For the same steam conditions, number of extractions, and feedwater temperatures as shown in Figs. 45 to 48, the percentage increase in throttle flow for a given output is plotted against feedwater temperature for the four specified heater combinations. If the nonextraction steam rate is known, the extraction steam rate can be found from the relation

$$\text{Extraction SR} = \text{Nonextraction SR}\left(1 + \frac{\%\ \text{increase}}{100}\right)$$

When extracting for feedwater heating, the condenser flow is less than the throttle flow by the amount extracted. Table 19 permits the total steam bled (or extracted) to be determined as a percentage of the (extraction) throttle flow for four steam pressures and various feedwater temperatures. Values for other steam conditions, feedwater temperatures, or number of heaters can be obtained by interpolation. Condenser steam rate (pounds per kilowatt-hour) may be found from the increase in throttle steam rate determined from Figs. 49 to 52, and the percentage extraction found from Table 19.

Condenser steam rate (extracting)

$$= \text{Nonextraction steam rate}\left(1 + \frac{\%\ \text{increase}}{100}\right)\left(1 - \frac{\%\ \text{extraction}}{100}\right)$$

The condenser steam rate, together with an estimated exhaust enthalpy, enable the condenser size to be determined. Throttle flow and heat consumption determine the boiler size. Thus these curves and tables enable a quick estimate to be made of the specifications of major units in a power plant, for various conditions of design.

USE OF SHORT-CUT METHOD.

EXAMPLE. Estimate the heat rate for a turbine-generator operating at 800 psig–900 F–1 in. Hg abs exhaust pressure with six extraction heaters heating the feedwater to the optimum temperature. The nonextraction engine efficiency is 81% with nonextraction exhaust loss of 5%.

H_1 at 815 psia, 900 F = 1454.9 Btu/lb; S = 1.6382; H_2 at 1 in. abs = 880.6 Btu/lb

Available energy = 1454.9 − 880.6 = 574.3 Btu/lb
Steam rate = 3413 ÷ (574.3 × 0.81) = 7.334 lb/kwhr
Nonextraction heat rate = 7.334 (1454.9 − 47) = 10,326 Btu/kwhr
With six heaters gain from Fig. 43 = 11.38%
Decreased for cycle losses = 0.50
Net gain from six heaters = 10.88%
Heat rate uncorrected for exhaust loss = 10,326 × (1 − 0.1088)
= 9203 Btu/kwhr

Allow 2.5% for decrease in exhaust loss.

Final heat rate of turbine-generator for extraction at full load with six heaters = 9203 × (1 − 0.025)
= 8973 Btu/kwhr

PARTIAL LOAD CORRECTIONS. Salisbury points out that as the turbine load decreases there is a rapid drop in temperature of the feedwater, and hence in the duty of the lowest pressure heater. Occasionally, where generator air coolers and steam jets are used in the condensate circuit, the extraction for this heater ceases to be required. He suggests that the number of heaters may be considered to be reduced by one, and that the heat rate at partial load then be calculated under extraction conditions in the usual way. For additional discussion, see Ref. 64.

PERFORMANCE DATA ON EXTRACTION UNITS OF THE AIEE-ASME PREFERRED STANDARDS are available from manufacturers. Table 20, for a 60,000-kw unit, is representative of data on such units.

The heat rates given in Table 20 are based on the following:

1. 5% pressure drop from turbine extraction flange to heater.
2. 5 F terminal difference on all closed heaters; 0 F terminal difference on No. 2 contact heater; 10 F terminal difference on drain cooler.
3. Condensate leaves hotwell at saturation temperature corresponding to condenser pressure.
4. The heat rate is defined by:

$$\text{Heat rate (Btu/kwhr)} = \frac{\text{Throttle flow (throttle enthalpy} - \text{final feedwater enthalpy)}}{\text{Net generator terminal output *}}$$

* After deducting power required by motor-driven auxiliaries such as oil pumps and hydrogen equipment, etc.

Table 20. Turbine Performance Data—60,000-kw Rating, 1250 psig–950 F

(AIEE-ASME Preferred Standard)

Exhaust Pressure, in. Hg abs	Per Cent of Rated Load	Straight Condensing Steam Rate, lb/kwhr	Extraction Performance All 5 Heaters in Service				Extraction Performance Lower 4 Heaters in Service			
			Heat Rate, Btu/kwhr	Feedwater Temp., °F	Throttle Steam Rate, lb/kwhr	Condenser Steam Rate, lb/kwhr	Heat Rate, Btu/kwhr	Feedwater Temp., °F	Throttle Steam Rate, lb/kwhr	Condenser Steam Rate, lb/kwhr
1	25	7.85	10,097	291	8.36	6.74	10,242	252	8.21	6.91
	50	7.27	9,181	337	7.90	6.08	9,308	291	7.71	6.21
	75	7.13	8,874	369	7.87	5.84	8,984	319	7.62	5.95
	100	7.15	8,880	396	8.08	5.83	8,971	343	7.77	5.92
	110	7.20	8,915	407	8.20	5.85	8,999	352	7.85	5.94
1 1/2	25	8.15	10,436	293	8.65	7.07	10,586	254	8.49	7.22
	50	7.43	9,375	338	8.09	6.28	9,505	293	7.88	6.42
	75	7.21	8,987	370	7.98	5.99	9,098	320	7.72	6.10
	100	7.19	8,938	397	8.14	5.92	9,030	343	7.82	6.04
	110	7.23	8,956	408	8.24	5.95	9,040	352	7.89	6.04
2	25	8.43	10,796	296	8.97	7.38	10,951	256	8.80	7.54
	50	7.61	9,591	340	8.29	6.49	9,724	294	8.07	6.62
	75	7.33	9,127	372	8.11	6.14	9,240	321	7.85	6.25
	100	7.27	9,032	398	8.23	6.05	9,125	344	7.91	6.16
	110	7.29	9,035	409	8.32	6.05	9,120	353	7.97	6.15
2 1/2	25	8.72	11,113	298	9.25	7.66	11,273	257	9.07	7.82
	50	7.80	9,795	342	8.47	6.67	9,931	296	8.25	6.82
	75	7.46	9,283	373	8.26	6.29	9,398	323	7.99	6.41
	100	7.36	9,152	400	8.36	6.18	9,246	345	8.02	6.28
	110	7.38	9,140	410	8.43	6.17	9,226	354	8.07	6.26

Figure 53 shows the five-stage feedwater heating cycle used in computing the values given in Table 20.

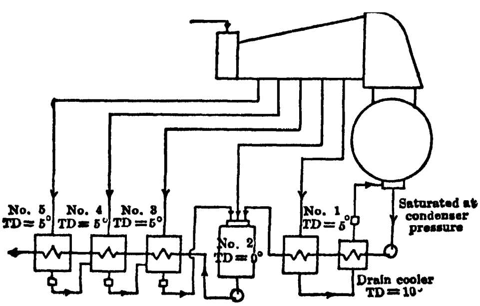


FIG. 53. Feedwater heating cycle for Preferred Standard unit of Table 20.

Cutting Out Top Heater. The purchaser may elect not to use the highest pressure extraction point for feedwater heating. It is then blanked off, and the turbine operates with the remaining lower pressure heaters. Performances with the lower four heaters in service are also given in Table 20.

Figures 54 to 60 present the usual technical data furnished by the General Electric Company for an AIEE-ASME Preferred Standard 60,000-kw turbine-generator set to operate at 850 psig–950 F–1.5 in. Hg. abs. These data supplement data such as Table 20.

SELECTION OF HEATERS. The economic selection of the proper number of heaters requires evaluation of the savings, with consideration given to average use factor and average load throughout the useful life of the turbine, the cost of fuel, and whether or not economizers are used. Against the fuel saving are charged the fixed charges on the investment in the added heater, valves and piping, repairs and maintenance on this equipment, and the cost of added power for pumping due to the friction head added by the heater

In many cases this balance will indicate that the savings from adding one or more additional heaters are not warranted by the returns on this investment.

When the final feed temperature is decided, temperatures leaving other heaters can be selected. It appears desirable to make the enthalpy rise of the feedwater in each of the heaters equal to the average rise per heater, i.e., to provide for equal enthalpy rise per heater. (See Refs. 64 and 73.) In actual cases this basic rule is slightly modified by drainage from higher-pressure heaters, by evaporators and their condensers, and by heat from boiler feed pump losses. Furthermore, the turbine manufacturer must locate extraction openings in the casing so as not to weaken the structure and to permit extraction piping to be attached. With a given final feedwater temperature, these small adjustments of the other heaters have little effect on the heat rate of the unit.

Terminal differences in feedwater heaters are 0 F in open heaters (deaerators) and usually about 5 F in closed heaters. Some engineers allow 10 F difference in the highest pressure heater to reduce the cost of the heater. Increased fuel prices favor lower terminal differences and more extensive use of drain coolers. For evaluation of the gains with drain coolers see Ref. 64. With the terminal difference fixed, the saturated steam temperature can be determined, hence the steam pressure in the heater. Some feedwater heaters for high-pressure bleeder points, where superheated steam is bled from the turbine, have been furnished with a countercurrent section to utilize the superheat, which in some cases has heated the feedwater to temperatures higher than the saturation temperature of the steam.

Pressure drop occurs in piping, nonreturn valves, and gate valves between turbine casing and heater, and in the turbine between the extraction stage and the extraction opening. The proportions of these connections vary, and the total pressure drop ranges from 5 to 10% of the absolute stage pressure. In older designs the major portion of this pressure drop was caused by the nonreturn valve. No serious error occurs in assuming the same percentage pressure drop from extraction point to heater at all turbine loads.

The pressure of the steam in the turbine casing now can be computed, and, when plotted on the condition curve, the total heat in the steam at the extraction point can be found.

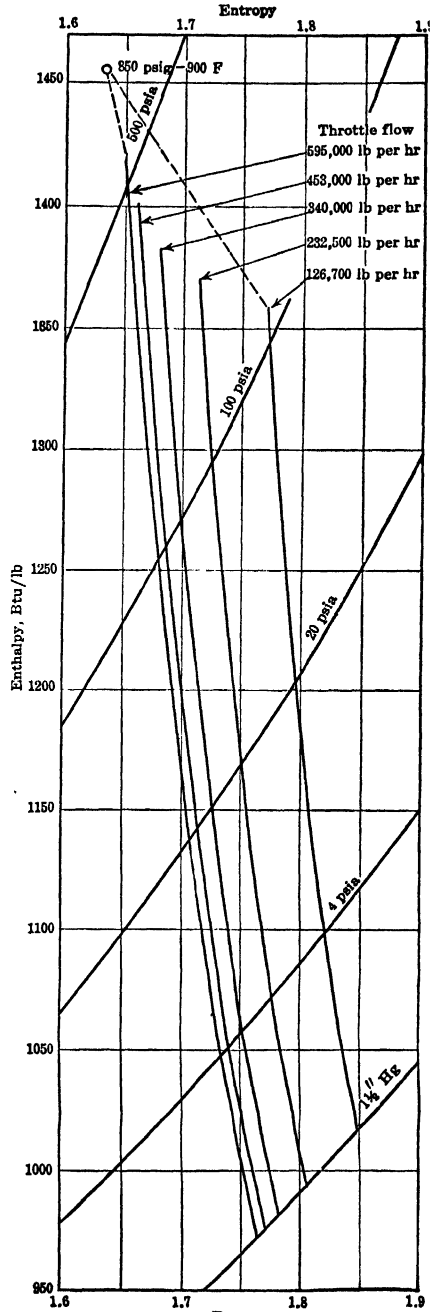


FIG. 54. State lines for Preferred Standard unit: 60,000 kw-850 psig-900 F-1.5 in. Hg. (Courtesy of General Electric Co.)

The flow of steam through a given area such as a stage in a steam turbine is theoretically proportional to $\sqrt{p/v}$, where p = psia and v = specific volume, cubic feet per pound, both taken at the stage inlet. Tests indicate that the relation between absolute pressure

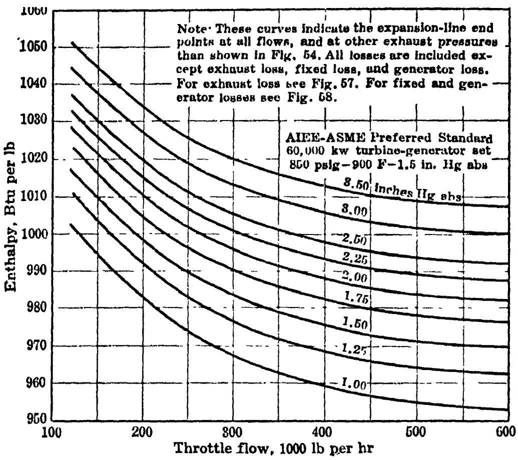


FIG. 55. State line end points. (Courtesy of General Electric Co.)

and steam flow at a given stage is approximately a straight line, as shown in Fig. 56. The pressure at each extraction point can be assumed proportional to the flow through the stage following the extraction point. Slight errors in extraction-point pressures have comparatively little effect on final unit performance.

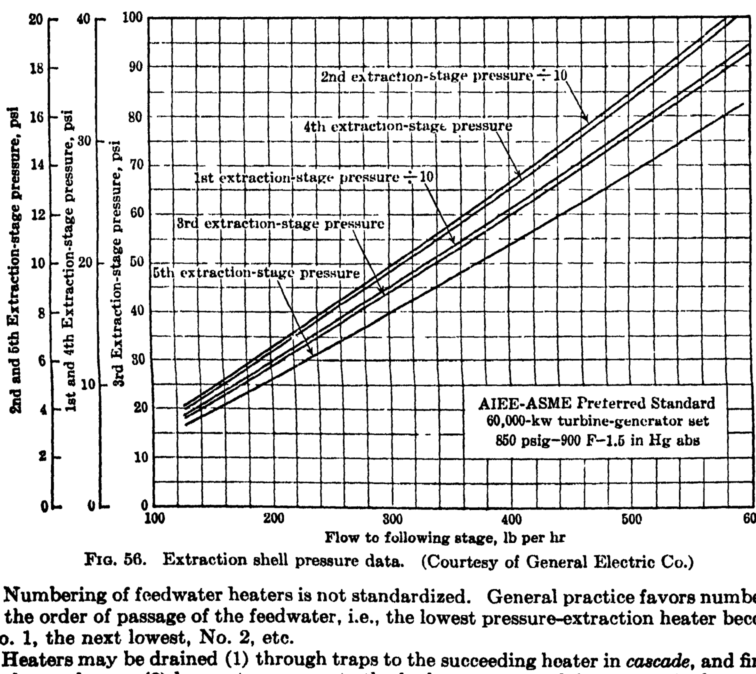


FIG. 56. Extraction shell pressure data. (Courtesy of General Electric Co.)

Numbering of feedwater heaters is not standardized. General practice favors numbering in the order of passage of the feedwater, i.e., the lowest pressure-extraction heater becomes No. 1, the next lowest, No. 2, etc.

Heaters may be drained (1) through traps to the succeeding heater in *cascade*, and finally to the condenser; (2) by a return pump to the feed system at each heater; or (3) by cascading to No. 2 heater, often a deaerating heater, and mixing with the feed. The drains from

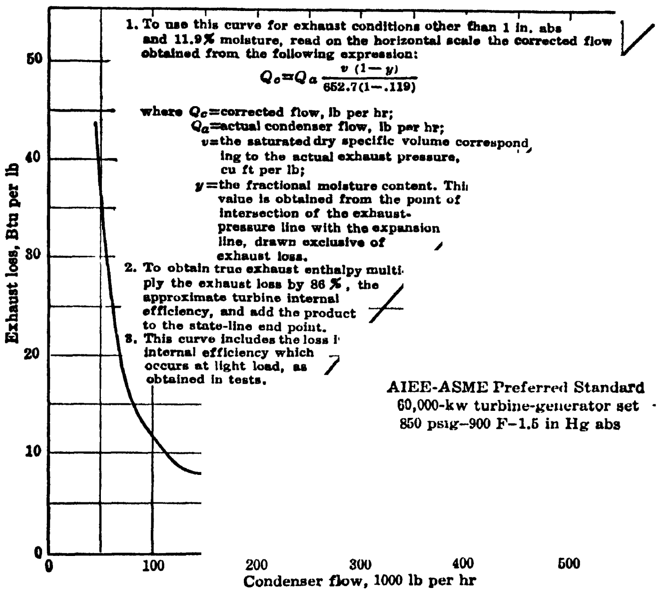


FIG. 57. Exhaust loss. (Courtesy of General Electric Co.)

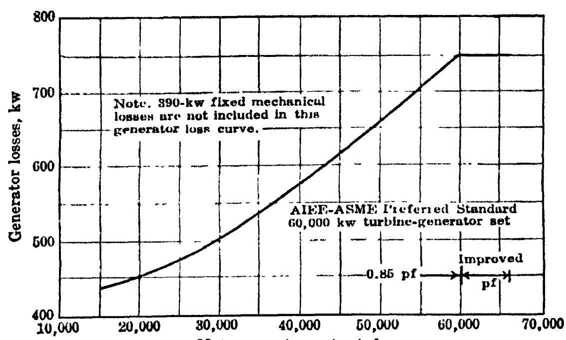


FIG. 58. Generator and mechanical losses. (Courtesy of General Electric Co.)

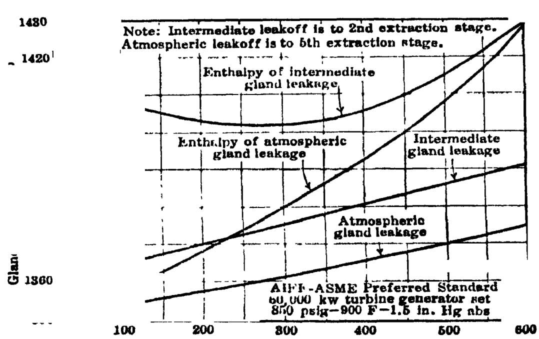
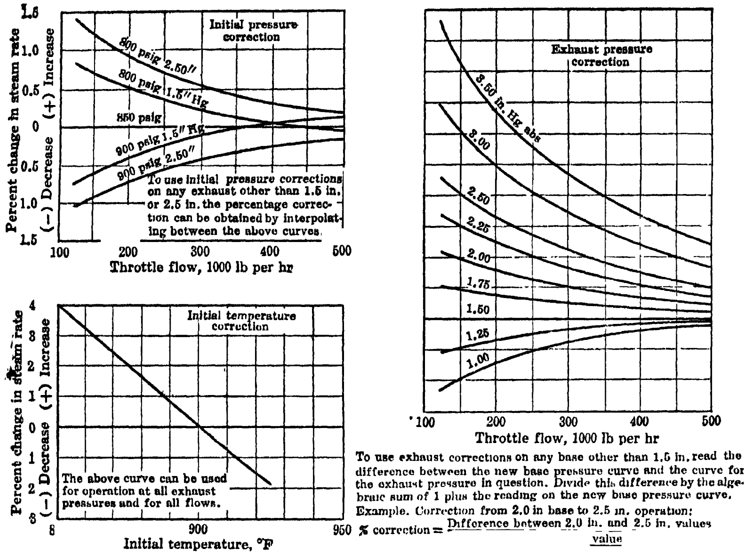


FIG. 59. Gland leakage data. (Courtesy of General Electric Co.)



METHOD OF USING CURVES

These correction factors assume constant control-valve opening, and are to be applied to guarantee non-extraction steam rates.

(1) For other than guarantee initial conditions the throttle flow can be found from the following expression;

where Q = throttle flow, lb per hr;
P = initial pressure, psia;
V = initial specific volume, cu ft per lb

The subscript G denotes guarantee conditions. Values without the subscript represent desired conditions.

(2) The steam rate at the desired condition can be found by multiplying the guarantee steam rate by the following:

$$1 + \frac{\text{Correction factor}}{100}$$

(3) The c rected generator output is equal to the following:

$$\frac{\text{Corrected flow (fro Item 1)}}{\text{Corrected steam rate (ite}}$$

Note: These correction factors are drawn from normal operating conditions of 850 psig—900 F—1.5 in. Hg abs. They can be used for other guarantee exhaust pressures as indicated in the note below each correction curve.

AIEE-ASME Preferred Standard
60,000-kw turbine-generator set
850 psig—900 F—1.5″ Hg abs

FIG. 60. Correction factors. (Courtesy of General Electric Co.)

No. 1 heater may either be pumped or may pass through a heat exchanger to the condenser. H. L. Guy (Ref. 74) shows that (2) will decrease heat consumption over (1), with 600 lb, 700 F steam conditions by about 0.87%; while (3) betters (2) by 0.7%. With 350 lb, 700 F steam conditions, the gains of (2) and (3) over (1) are 0.97 and 0.69%, respectively, for the conditions assumed. System (2) involves many small drip pumps, often of low capacity, and at the high-pressure heaters these must pump against full boiler feed pressure. These added auxiliaries decrease the reliability and availability factors of the turbine. Scheme (3) or a modification of this arrangement is generally preferred. For a complete discussion of losses in heater cycles, see Ref. 64.

FEEDWATER HEATING SYSTEMS. **An open heater** of the deaerating type is incorporated in the feedwater heating system of most turbines. Smaller turbines and those in industrial plants usually have only one or two points of extraction for feedwater heating. One of these heaters is generally an open heater from which the boiler feed pump takes its supply.

The function of the deaerator is to remove practically all the oxygen in the feedwater. Experience has indicated that it is undesirable to locate the deaerator in the cycle where the pressure in the deaerator may fall below atmospheric at low load. It is recent practice to locate the deaerator at that extraction point where the pressure in the deaerator remains above atmospheric pressure over the normal load range. The water storage of the system often is combined with the deaerator. The capacity of this storage is generally a 10-minute supply at maximum feedwater demand. To prevent oxygen from being absorbed in this storage water during short shutdown periods of the turbine, closed tubes are sometimes installed in the bottom of the storage tank, and live steam admitted to keep the water boiling during shutdown. Condensation from the heating tubes is removed by a trap.

Boiler-feed Pump. Boiler-feed pump suction usually is taken from the deaerator. Tube passes of heaters beyond the boiler feed pump must be designed for full pump discharge pressure. Some plants depend on deaeration in the condenser hotwell, and all heaters are of the closed type. The boiler feed pump is generally connected between the second and third heaters. When high pressures are used, a *booster* pump may be installed after the second heater and the main boiler feed pump after one of the higher pressure heaters.

Allowance may be made especially in high-pressure plants (800 psig and higher), for the heat added to the feedwater by losses in booster and boiler-feed pumps. It may be assumed that 10% of the losses disappear as radiation and 90% are carried away by the feedwater. On this assumption, the temperature rise in degrees Fahrenheit due to pump losses is

$$t_p = 0.9(1 - \eta_p)\frac{144Pd}{778\eta_p} = 0.1665Pd\left(\frac{1}{\eta_p} - 1\right)$$

where η_p = pump efficiency, as a decimal; P = total pressure added by pump, psig; d = specific volume of feedwater entering pump, cubic feet per pound.

Centrifugal condensate pump efficiency may vary from 25 to 60%, depending on working conditions. Centrifugal boiler-feed and booster pumps have efficiencies, at full load, of 50 to 75% depending on pressure and temperatures. These efficiencies decrease at partial loads.

The evaporator to provide distilled water for boiler-feed make-up usually forms an element in the extraction system. A single-stage evaporator generally is used, taking

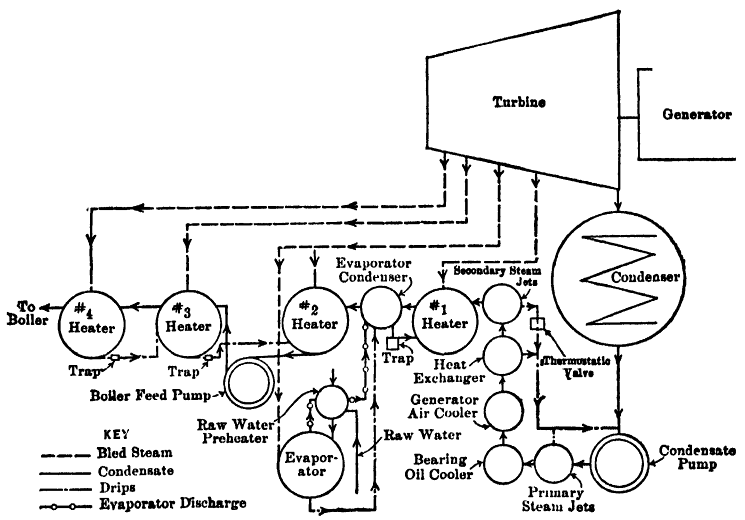


FIG. 61. Typical turbine-heater cycle.

steam from No. 2 or No. 3 extraction point, and exhausting into an evaporator condenser placed in the condensate line between the preceding heater and the heater at the extraction point from which steam is taken. The evaporator and its condenser serve as the first stage of feedwater heating at the extraction point from which steam is taken to the evaporator. Make-up in a modern station varies from 0.5 to 2% of the total steam to the turbine. In industrial plants, it may be much higher. See also Evaporators, Section 3.

Heat transfer in evaporator coils depends both on temperature head and on the vapor temperature and degree of purity of the water evaporated. Thus with 25 F temperature head, heat transfer rates of 425 Btu at 100 F vapor temperature, 500 Btu at 200 F and 550 Btu at 300 F might be expected. Generally the total heating surfaces in all stages of multiple-effect evaporators are made the same.

Condensate Circuit. Feed leaving the condensate pump usually passes first through the condenser for the primary steam jets of the air pump. Knowing the total steam required by the jets and the total heat in this steam, the temperature rise of the feedwater can be computed.

The feedwater next passes through the *generator air* or *hydrogen cooler* where use of clean condensate is desirable, as it eliminates the need of cleaning and will not corrode tubes. Few data have been published to show the relation which the heat recovered in the generator air cooler bears to the heat equivalent of the total electrical losses. Schoenherr (Ref. 75) indicates that at 29.5 in. vacuum, 92.5% of the electrical losses are recovered by the feedwater; at 29 in. vacuum, 84.5%; at 28.5 in. vacuum, 76%. Passing condensate through the generator air or hydrogen cooler reduces the steam bled at the first bleeder point, and results in a final saving in heat rate of 0.25 to 0.5%. See Ref. 64.

The feedwater next passes through the *oil cooler*. Data indicate that heat equivalent to the total mechanical losses is absorbed by the feedwater in this heater. Undoubtedly some of the heat in the generator losses and heat from the turbine itself passes by conduction to the bearings and is removed by the oil.

If a *heat exchanger* is used on drains, this can be calculated by assuming a terminal difference of 10 F between the cooled drains and entering condensate, and calculating the resultant temperature rise of the condensate.

The feedwater finally passes through the condenser of the secondary air jets of the vacuum pump, where temperature rise can be calculated, and thence to the remaining heaters of the cycle.

At light load, there may be insufficient condensate properly to cool the generator air or hydrogen. In such a case a thermostatically controlled valve may by-pass feedwater beyond the oil cooler to the condenser, where it flashes and thereby increases the flow to the generator air cooler to obtain the desired generator air temperature. Figure 61 shows a typical extraction layout. Further information on the computation of the heat balance on extraction turbines can be found in Refs. 64, 70, 72, 76–79.

16. AUTOMATIC EXTRACTION TURBINES

Extraction turbines are being used to an increasing extent in industry. Steam at a fixed pressure can be automatically extracted for process or other use in varying amounts. Regulators are provided to control extraction pressure and to maintain speed and load. The extraction-pressure regulator consists of an external or internal valve to control steam flow to the low-pressure section. Figure 17 shows a turbine of this type.

These turbines frequently exhaust to a condenser, but occasionally they are designed to exhaust at atmospheric pressure or higher. Some *double-automatic-extraction* units supply steam at two process pressures. However, most units are of the single-automatic-extraction condensing type.

Newman (Ref. 80) presents the approximate method for estimating performance of a single-automatic-extraction turbine upon which the following discussion is based.

Table 21 presents theoretical steam rates useful in this method.

Table 21. Condensed Table of Theoretical Steam Rates, TSR *

(Adapted by permission from *Theoretical Steam Rate Tables*, Keenan and Keyes, 1938, ASME)

Main Pressure, psig Initial Temp., °F Initial Superheat, °F	150 450 (84)	200 500 (112)	250 550 (144)	300 600 (178)	400 700 (252)	600 750 (261)	850 825 (298)
Exhaust Pressure	Theoretical Steam Rates, lb/kwhr						
1 in. Hg abs	9.09	8.54	8.10	7.71	7.06	6.63	6.19
2 in. Hg abs	9.98	9.32	8.79	8.34	7.60	7.09	6.58
3 in. Hg abs	10.62	9.87	9.28	8.78	7.96	7.40	6.85
0 psig	18.2	16.1	14.6	13.4	11.7	10.4	9.31
10 psig	22.4	19.3	17.2	15.6	13.3	11.6	10.30
20 psig	26.7	22.3	19.5	17.5	14.7	12.7	11.10
50 psig	42.4	32.3	26.8	23.1	18.5	15.4	13.10
100 psig	....	58.1	42.5	34.1	25.1	19.4	15.90
150 psig	....	....	....	51.1	33.6	23.8	18.60
200 psig	....	....	....	....	45.8	29.0	21.50
250 psig	....	....	....	....	....	35.3	24.80

* For more extensive table of Theoretical Steam Rates, see Section 4.

Table 22 gives the full-load nonextraction efficiency η of single-automatic-extraction condensing turbines.

Table 22. Full-load Nonextraction Efficiencies for Condensing Single-automatic-extraction Steam Turbines

Rating, kw, (0.8 pf)	Main Pressure, psig						
	150	200	250	300	400	600	850
	Efficiency, η						
500	.600	.595	.585	.580	.565	.545	
625	.615	.610	.605	.600	.580	.560	
750	.630	.625	.620	.610	.595	.575	
1000	.650	.645	.640	.630	.620	.600	
1250	.665	.660	.650	.645	.635	.615	
1500	.675	.670	.665	.660	.645	.630	
2000	.690	.685	.680	.675	.665	.645	
2500	.700	.695	.690	.685	.675	.660	
3000	.710	.705	.700	.695	.685	.670	
3500	.715	.710	.705	.700	.690	.680	
4000	.720	.715	.710	.705	.700	.685	
5000	.725	.720	.715	.710	.705	.695	.685
6000	.735	.730	.725	.720	.715	.705	.695
7500	.740	.735	.730	.725	.720	.715	.705

Half-load flow factors, H, for such units are given in Table 23.

Table 23. Half-load Factors

Rating, kw (0.80 pf)	Factor, H
500	0.590
625	
750	
1000	0.585
1250	
1500	
2000	0.580
2500	
3000	
3500	0.575
4000	
5000	
6000	0.570
7000	

ESTIMATING METHOD FOR AUTOMATIC EXTRACTION TURBINES. To estimate turbine performance, first find the value of the theoretical steam rate, TSR_1, from throttle conditions to exhaust; the theoretical steam rate, TSR_2, from throttle conditions to extraction pressure, both from Table 21; and the engine efficiency η for the turbine rating from Table 22. The rated-load nonextraction throttle flow is ($TSR_1 \times$ rating in kw) $\div\ \eta$. This throttle flow is represented by point A, Fig. 62. The half-load flow factor, Table 23, multiplied by the rated-load throttle flow A gives the throttle flow at half-load, point B, Fig. 62. AB is part of the Willans line for condensing operation.

The ratio TSR_1/TSR_2 is next determined, and the extraction factor, E, found from Fig. 63. The maximum desired extraction flow, F, must be known. The extraction factor, E, multiplied by the maximum desired extraction flow, F, equals the amount by which the condensing throttle flow must be *increased* at all loads to permit the desired extraction and still maintain the specified load on the generator. Hence, at rating, the throttle flow at full extraction = $A + (E \times F)$ and is represented by C, Fig. 62. Since $(E \times F)$ is an addition to condenser flow at all loads, the line for constant extraction flow can be drawn through C parallel to AB. Lines for extraction flows less than maximum can be drawn parallel to AB at distances proportional to the extraction quantity. Thus AC is divided into equal increments for the equal portions of the maximum extraction flow and lines drawn parallel to AB as shown in Fig. 62. A minimum flow to exhaust S, given in Fig. 64, is necessary to prevent overheating the low-pressure section.

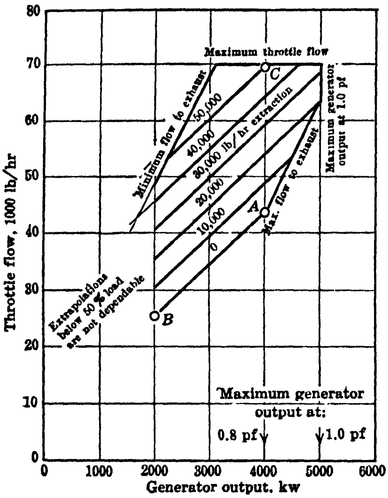


FIG. 62. Performance chart of a 4000-kw single-automatic extraction turbine.

Points are then plotted on the extraction lines where the throttle flow equals extraction flow plus minimum flow to exhaust. A straight line through such points forms the left boundary of the diagram (Fig. 62). The assumption is made that the turbine has an exhaust section capable of passing the flow required for rated nonextraction load without pressure rise at the extraction stage. If flow to the condenser is maintained at this value and progressively greater quantities of steam are extracted at the same time, both the throttle flow and generator output increase without rise in extraction pressure. This can continue up to maximum generator output. If, however, the throttle flow is increased without increase in extraction, the pressure will rise. The line of maximum flow to the condenser, as on Fig. 62, is found by plotting points on each extraction line where the throttle flow equals nonextraction rated-load throttle flow, as at A, plus the extracted flow.

The top line on Fig. 62 for maximum throttle flow may be fixed by the designer at any desired point, but there is little to be gained in extending it much beyond the throttle flow at C, especially since any such extension adversely affects the turbine efficiency.

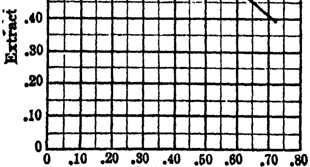


FIG. 63. Extraction (or replacement) factor for extraction calculations.

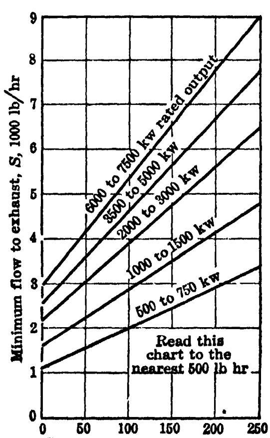


FIG. 64. Minimum or cooling steam flow to exhaust for extraction turbines.

EXAMPLE. Given a 4000-kw, 0.8-pf, 5000-kva, 60-cycle, 3600-rpm condensing single-automatic-extraction turbine generator, to operate at 400 psig–700 F–2 in. Hg abs, with an extraction pressure of 10 psig, and capable of a maximum extraction flow of 50,000 lb per hr.

From Table 21, $TSR_1 = 7.60$ lb per kwhr; $TSR_2 = 13.3$ lb per kwhr. From Table 22, engine efficiency $\eta = 0.70$; $A = (7.60 \times 4000) \div 0.70 = 43{,}400$ lb per hr. $B = 43{,}400 \times 0.575$ (from Table 23) $= 25{,}000$ lb per hr. $TSR_1/TSR_2 = 0.57$. E (from Fig. 63) $= 0.52$. Let $F = 50{,}000$ lb per hr. $C = 43{,}400 + (0.52 \times 50{,}000) = 69{,}400$ lb per hr. S (from Fig. 64) $= 3{,}000$ lb per hr. These values, together with extraction lines for every 10,000 lb per hr, are shown in Fig. 62, which represents the performance of this 4000-kw turbine from half rating to maximum generator output. The lines might be extended to less than half rating, with the understanding that the lower portion of the chart may be approximate.

Similar methods are given by Newman for single-automatic extraction noncondensing and for double-automatic extraction condensing turbines. (See Ref. 80.)

17. REHEATING TURBINES

Reheating steam after partial expansion in the turbine was originally considered when low initial steam pressures were in use as a means of reducing moisture at the exhaust, to minimize bucket erosion. The increased steam temperatures now in use have lessened this problem. Because of higher fuel prices and station costs, the reheat cycle with high initial and reheat temperatures is still used as a means of improving overall thermal efficiency. (See Ref. 81.)

When several boilers were installed per turbine, reheating introduced constructional and operational complications. The unit system of one boiler per turbine simplifies both construction and operation, and lowers the cost of reheating. Reheat turbines on the unit system can be taken on and off the line as required by load conditions with no greater difficulty than with non-reheat machines. However, reheating is best suited for base-load units.

To prevent overspeeding on the large volume of steam entrained in the system, intercepting valves, under governor control, are installed which stop steam flow to the low pressure cylinder if speed increases to over 105% of normal. In some turbines a dump valve is also provided which discharges this intercepted steam directly to atmosphere or to the condenser. These arrangements introduce control problems and complicate station layout to some extent.

Reheating is accomplished in some older plants by means of high-pressure-steam reheaters. The gains accomplished by this means are limited. Station economy is highest when reheating is done in the boiler, as in most current practice.

All reheat plants combine regenerative feedwater heating with reheating. Gains of 4 to 6% have been estimated for reheating. For example, in one plant a 100,000-kw reheating turbine operating with regenerative feedwater heating at 1250 psig–950 F–1 $^{1}/_{2}$ in. Hg abs, with reheat to 950 F, showed thermal gains over a similar unit with the same initial conditions and the same number of heaters of 6.3% at 50,000 kw load, 5% at 75,000 kw, 4.6% at 100,000 kw, and 4.2% at maximum capability of 110,000 kw.

Large cross-compound units have been considered with a 3600-rpm high-pressure turbine, a reheater, and an 1800-rpm low-pressure turbine.

Early writers have pointed out that turbine efficiency has little effect on the gain to be expected from reheating. The thermal gain from reheating decreases slightly as the number of stages of extraction for feedwater heating is increased. Also, the gain from reheating appears to decrease slightly with an increase in pressure. For a complete investigation of reheat see Ref. 82.

Pressure drop across the reheater of 5% of the absolute pressure leaving the high-pressure turbine was used in early designs. Consequently, piping and reheater were large. A total pressure drop of about 10% is now considered more satisfactory. Excessive pressure drops nullify thermal gains. (See Ref. 82.)

Studies are necessary in each application to determine the optimum reheat pressure, as it varies with turbine design, throttle pressure, and superheat, and with the number of stages of bleeding for feedwater heating. The reheat pressure falls generally in the range of 20 to 30% of the initial absolute pressure and is roughly proportional to load. On recent large units with high reheat temperatures there is only a small variation in overall heat rate for a considerable range in reheat pressures.

The gain from reheat increases with the temperature to which steam is reheated. When the final exhaust steam is in the 4 to 5% moisture range, the exhaust steam is in a supersaturated state leaving the last wheel, and there is little practical advantage in additional reheat to secure drier exhaust. Additional reheat would also increase the heat rejected to the condenser.

Since more heat per pound of steam is converted into work with reheating, less steam (only about 85%) enters the throttle for a given load and less passes to the condenser. But even though reheating involves rejection of more heat per pound of steam to the condenser, the net result is a substantial reduction in total heat rejected to the condenser and in quantity of circulating water needed. Smaller condensers, circulating water pumps, and boiler-feed pumps can be installed in reheat plants.

In general, reheat leads to increases in station economy when high pressures and temperatures are used. The justification for reheat must be established by full consideration of initial cost, fuel prices, load factors, reliability, and operating experiences. Some utilities have found it to be economically unjustifiable for their particular operating conditions.

18. SELECTION OF ECONOMIC OPERATING CONDITIONS

PRESSURE AND TEMPERATURES. Steam pressures and temperatures for turbines are still being increased. No serious difficulty has been experienced with pressures now in use up to 2300 psig. Turbines are in service with temperatures of 1000 F, and several plants are either planned or operating at 1050 F. Temperatures are limited by available materials for superheater, piping, and first stages of the turbine. New alloys are under development which may permit a further increase in temperature and a further improvement in turbine performance.

Stage efficiencies decrease with increased pressure particularly in the smaller sizes. Higher temperatures increase specific volumes and reduce moisture losses, thus improving turbine efficiency. The net result of these effects when taken together with the improved Rankine cycle efficiency is appreciable improvement in the heat rate. (See Ref. 83.)

EXHAUST CONDITIONS. The lowest exhaust pressure gives the greatest available energy and largest potential power per pound of steam. It also entails long low-pressure buckets, or in a given casing either limits output or increases exhaust loss. In any specific case the weighted average cooling water temperature should be determined from an analysis of water temperatures and loads throughout the year. This weighted average temperature may vary from the mean temperature by several degrees. When the weighted temperature is found, the corresponding vacuum may be estimated by allowing for a temperature rise (usually 10 to 15 F) and a terminal difference leaving the condenser (usually 9 to 12 F). When this vacuum is found the economic rating of a given casing can be chosen.

METALS FOR HIGH TEMPERATURE. Steel and other metals decrease rapidly in tensile strength at temperatures above 750 F. Much research work is being carried out on new alloys, particularly for gas-turbine work where temperatures up to 1800 F are considered. Such alloys will find eventual application in steam stations and in steam turbines. This will further extend the temperature range. Many data on these alloys are currently appearing in the technical press.

A further factor is *creep*, the name given to elongation that occurs at higher temperatures under constant load at stresses much below the elastic limit of the material. This rate of flow is stated in terms of hours. S. H. Weaver (Ref. 84) shows that the limiting creep stress on a 0.23% carbon steel at 900 F is 14,200 psi. A load of 10,300 psi produces a flow of 1% in 10,000 hr; a 1% flow results from 8000 psi load in 100,000 hr while a 6000 psi load produces an elongation of 0.1% in 100,000 hr. Certain designers have chosen the allowable rate of creep as 0.01% per year. Particular attention must be given to the matter of creep in bolting material, in disk and diaphragm construction, and in piping connections.

Baumann (Ref. 85) gives permissible creep rates per hour as follows: (1) turbine disks pressed on shafts 10^{-9}; (2) bolted flanges of turbine cylinders 10^{-8}; (3) steam piping, welded joints, boiler tubes 10^{-7}; and (4) superheater tubes, 10^{-6}. These stresses must be well below the yield point. He suggests a factor of safety of 3 based on the yield point at the working temperature. Similar stress limitations are used by turbine builders in designing for creep.

PROBABLE STATION CONDITIONS. Higher fuel and equipment costs with rising labor rates make use of high pressures and temperatures an economic advantage both for central stations and industrial plants. The choice of conditions requires a study of operating costs and fixed charges to obtain the lowest total cost per unit of net output.

The AIEE-ASME Preferred Standard Units are being adopted in many new plants. It is expected that this standardization will make performance and layout data immediately available, that less time will be spent in preliminary negotiations and plant design, and that costs will be less than with nonstandard units.

Single-cylinder turbines are available up to 30,000 kw at 3600 rpm and up to 100,000 kw at 1800 rpm. Most units of 15,000 kw and above are being purchased with hydrogen-cooled generators. Hydrogen cooling undoubtedly will be extended to still smaller units.

G. A. Gaffert (Ref. 86) considers the possibilities of various cycles. His results, given in Table 24, are based on the following assumptions: (1) An overall efficiency ratio of 82% for turbines of 30,000 to 50,000 kw capacity; (2) a maximum of 11% moisture at exhaust at full load; (3) terminal differences on feedwater heaters of 5 to 20 F for feed temperatures of 100 to 525 F; (4) steam generator efficiencies of 85%, including air preheater (if used); (5) pressure drops of 10% between boiler and turbine, bleed points and their respective heaters, reheater piping and reheater; (6) radiation loss of 2% from bleed point to heater and 3% for reheating lines; (7) normal auxiliary power allowances, with 20 kwhr per ton as power for pulverizing coal; (8) feedwater heated in equal temperature steps to a maximum of 75 to 80% of saturation temperature corresponding to throttle pressure when most economical number of feedwater heaters is employed; (9) an overall efficiency ratio of 75% for the mercury vapor turbine; (10) a terminal difference of 30 F across the mercury condenser boiler. With diphenyloxide, a difference of 30 F was assumed for the condenser boiler at 25 psig exhaust pressure for the diphenyloxide.

Table 24. Plant Performances for Steam and Binary Cycles

Cycle Conditions * Final vacuum = 29 in. in all cases	Number of Points of Steam Extraction			
	2	3	4	5
	Btu per kwhr of Station Output			
400 lb 800 F steam	12,800	12,600	12,460	12,400
400 lb 900 F steam	12,460	12,280	12,160	12,100
400 lb 1000 F steam	12,180	11,960	11,840	11,760
600 lb 800 F steam	12,180	11,930	11,830	11,760
600 lb 900 F steam	11,900	11,700	11,620	11,590
600 lb 1000 F steam	11,720	11,500	11,430	11,390
900 lb 1000 F steam	11,300	11,100	11,000	10,930
1200 lb 1000 F steam	11,050	10,800	10,680	10,580
1200 lb 800 F. R 200 lb 800 F †	11,300	11,060	10,880	10,800
1200 lb 1000 F. R 200 lb 1000 F †	10,630	10,400	10,260	10,220
2500 lb 800 F. R 500 lb 800 F †	11,080	10,850	10,680	10,580
2500 lb 1000 F. R 500 lb 1000 F †	10,300	10,080	9,930	9,860
3226 lb 800 F. R 900 lb 800 F. 2nd R 200 lb 800 F †	10,760	10,550	10,440	10,360
3226 lb 1000 F. R 900 lb 1000 F. 2nd R 200 lb 1000 F †	9,880	9,730	9,620	9,550
DPO ‡ 146 lb 750 F. 25 lb Exh.§ Steam 730 lb 800 F. R 100 lb 800 F †	10,920	10,780	10,700	10,680
DPO ‡ 210 lb 800 F 25 lb. Exh.§ Steam 730 lb 1000 F	10,630	10,550	10,480	10,460
Mercury 46 lb 800 F 4 in. Exh.§ Steam 500 lb 800 F	9,720	9,640	9,600	....
Mercury 95 lb 900 F 4 in. Exh.§ Steam 500 lb 800 F	9,300	9,210	9,170	....
Mercury 180 lb 1000 F 4 in. Exh.§ Steam 500 lb 800 F	8,900	8,850	8,830	....
Mercury 200 lb 1020 F 4 in. Exh.§ Steam-mercury Superheated at 500 lb 800 F	8,700	8,630	8,600	....

* Pressures, in psia. † R = Reheat conditions. ‡ DPO = Diphenyloxide. § Exh. = Exhaust pressure, psia or inches of mercury, absolute.

Gaffert concludes that higher steam pressures are justified economically and that the steam cycle has not reached its limit; that when metals become available for temperatures over 1000 F, the mercury-steam binary cycle is the only feasible one; and that when thermal advantages and capital costs are considered, there is little choice between mercury-steam, diphenyloxide-steam, and high-pressure steam cycles, assuming low fuel costs and 800 F initial temperature.

VARIABLE-PRESSURE OPERATION is possible with boilers operating at or above the critical pressure at constant temperature, such as the Benson. (See Ref. 87 for a description of this boiler.) The specific volume at approximately constant temperature varies inversely as the pressure. A turbine for 2250 psig pressure will pass about four times as much steam as one for 550 psig pressure and the same temperature, with some loss in vacuum. The available energy and heat utilized remain nearly constant in both cases. Hence, it appears possible to govern the load on the turbine by varying the boiler pressure, and thus the inlet pressure. This would eliminate the use of throttling or nozzle governing valves.

Many steam central stations are built as stand-bys to hydroelectric developments. In some cases the turbines operate normally as spinning reserves, and must pick up load instantaneously should there be a power failure on the hydro system. The Harbor Steam Plant of the City of Los Angeles has such turbines, which are supplied on a unit system by steam generators with very large water capacities. Pressure control is on the main boiler drum rather than located on the superheater outlet, as is usual practice. Hence the throttle pressure at the turbine varies with load due to the pressure drop through the superheater. When an outage occurs on the Boulder Dam transmission line, the turbine picks up full load almost instantly. The steam necessary to carry this load is supplied at decreasing throttle pressure by the accumulator effect of the large hot-water volume in the boiler. Sufficient steam is made available to maintain the turbine at full capacity while fans and fuel supply automatically go to full operating conditions.

THE LIMITING FACTOR IN TURBINE CAPACITY is the permissible area of the annulus of the last stage. The amount of steam that can be discharged to the condenser is determined by the allowable leaving loss, hence the allowable leaving velocity from this last stage. The greater the allowable leaving loss, the greater is the turbine capacity. For instance, if the leaving loss can be doubled, the turbine capacity is increased about 40%. Doubling the leaving loss does not signify an important decrease in turbine efficiency, because increased steam flow decreases disk, gland, and leakage losses, and increases the high-pressure section efficiency, while the mechanical efficiency increases. The turbine itself may decrease in efficiency by only a fraction of the amount of the added leaving losses.

The selection of the economic leaving loss for a given turbine, and hence its economic rating, depends on certain factors extraneous to the turbine, such as load factor, daily and annual load curves, coal cost, station cost, and cooling water temperatures. The load factor should be the average throughout the useful life of the turbine. Records show that this is comparatively low. Daily and annual load curves determine the nature of the loading, the point of best economy, and the permissible sacrifice of efficiency at occasional full load. Coal costs are highly important, as they fix the value of increased efficiency. Station cost influences the amount of money that may be spent on the turbine. Water temperatures influence turbine capacity. For instance, if only 28-in. vacuum can be obtained, practically twice the weight of steam can be passed through the last blade row as with 29-in. vacuum, at the same leaving velocity.

A given turbine casing can have a low rating for low leaving loss and maximum efficiency or a high rating with high leaving loss and decreased efficiency. Foreign practice favors low leaving loss, not exceeding 15 Btu per lb. American practice tends to restrict leaving losses to 4% of the available energy, or less. American practice in last-row annulus varies from 1.0 to 2.0 sq ft per 1000 kw rated capacity, with the trend towards the lower figure. Bleeding for regenerative heating of the feedwater decreases the steam to exhaust, and thereby increases the possible rating of a given casing. Reheating has a similar effect.

FLOOR SPACE AND WEIGHT REQUIREMENTS. The following tabulation of approximate dimensions and weights (without oil) for various units, at 3600 rpm, can be used for preliminary studies.

Size, kw	Overall Length	Width	Height	Total Weight, lb
1,500	22 ft 6 in.	6 ft 0 in.	6 ft 6 in.	64,000
3,000	25 6	7 6	7 0	85,000
5,000	28 6	8 0	7 0	103,000
10,000	33 0	13 0	7 6	237,000
11,500	41 6	13 0	8 0	270,000
15,000	42 6	13 0	8 9	300,000
20,000	43 6	13 0	9 0	330,000
30,000	45 6	14 6	10 6	400,000
40,000	57 6	14 6	11 0	550,000
60,000	64 0	15 6	12 6	710,000

THE SELECTION OF INDUSTRIAL TURBINES depends on the power demand, the efficiency warranted, use factor of unit, and cost. If the plant has large low-pressure steam requirements, a high-pressure turbine may serve as a reducing valve from high-pressure boilers. When all steam is utilized in an industrial plant, the electric energy is generated at all loads under average conditions at about 4600 Btu per kwhr or about $^1/_3$ lb of coal per kwhr.

Limited cooling water at an industrial plant may warrant high pressure to reduce the heat to condenser per kilowatt of plant output. These, and other economic influences, sometimes indicate use of higher pressures and temperatures in industrial plants. Many old industrial turbines, of uneconomical design, could be replaced by modern efficient units with the old alternator. Savings up to 15 to 20% are possible with no change in steam conditions. Additional savings may be effected by regenerative feedwater heating with new units.

Selection of an extraction turbine involves a careful study of the conditions in the plant for which it is intended. Performance diagrams like Fig. 62 can be prepared by

Newman's methods for preliminary use in estimating steam requirements. The principal difficulties in choosing extraction turbine requirements lie in uncertainties in regard to both extraction and electrical requirements. Many industrial operators can furnish only rough estimates of these quantities, and generally such data require careful checking to prevent expensive mistakes in turbine selection.

19. MERCURY POWER PLANTS *

Mercury as a power-generating fluid has high boiling temperatures for moderate pressures (see Table 1, p. 4-07). It does not decompose at any temperature employed in power generation. Mercury may be vaporized under pressure in a suitable mercury boiler, and can be utilized in a properly designed turbine, where it is expanded in stages to exhaust pressure. The condensing temperatures of mercury vapor at usual vacua (see Table 1, p. 4-07) are high enough to permit the heat of the condensing vapor to be transferred to water in a suitable boiler, producing steam at a pressure suitable for steam turbines. This combination leads to high plant economy, as shown in Table 24.

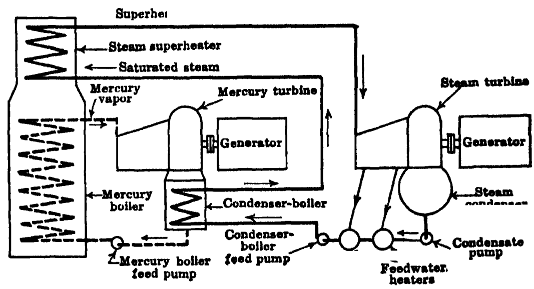


FIG. 65. Mercury-steam power plant flow diagram (Courtesy of General Electric Co.)

CYCLE EFFICIENCY. The efficiency of a vapor cycle is largely determined by the saturated temperature range through which it operates—the greater the range, the higher the efficiency of the cycle. The mercury cycle superimposed on a steam cycle raises the efficiency of the overall cycle because of the high boiling temperature of the mercury at moderate pressure; for example, mercury boils at 975 F under 140 psig, and with suitable steam turbine equipment can develop thermal efficiencies of 34 to 38%. With 1100 F mercury vapor at 300 psig the thermal efficiency may exceed 40%. Figure 65 shows the arrangement of the mercury-steam cycle when used for power generation. Figure 66 compares the performance of mercury-steam plants with normal steam-power stations.

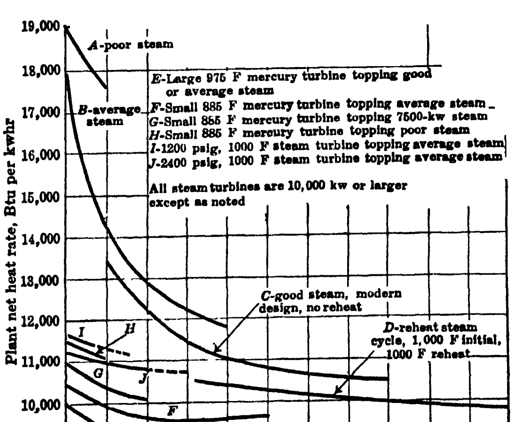


FIG. 66. Comparison of mercury-steam plant with steam plant heat rates. Used by permission, from *Power*, January, 1947.

Mercury condenser boilers can also be used to generate either high- or low-pressure steam for industrial purposes; in this case the power from the mercury turbine is a by-product of steam generation. The mercury condenser usually is arranged for a temperature head of about 30 F between condensing mercury and boiling water.

Standardization of Mercury Equipment. The principal mercury vapor equipment used in a mercury-steam power plant may be designed as standardized equipment. The cycle lends itself to standardized design in covering a wide range of steam pressures.

* The assistance of W. N. Oberly of the General Electric Company in supplying data for this article is gratefully acknowledged by the Editor.

This is possible because there is a large change in mercury-turbine exhaust temperature, which controls the pressure of steam generated in the condenser boiler, for a relatively small change in available energy in the mercury vapor used in the mercury turbine.

The mercury boiler may be designed so that with a minimum of changes it will be suitable for burning oil, gas, or coal of varying grades.

Mercury-turbine generators are particularly susceptible to standardization, and may be designed to cover a wide range of back pressures which determine the steam pressure generated in the condenser boiler.

Condenser boilers, being simple heat exchangers, may be designed as a standard piece of equipment to cover the maximum or minimum steam pressure available from a unit of given size.

The General Electric Company has built mercury-steam power plants both for process-steam production and for power production. Designs of complete mercury-steam power plants, including mercury boiler, turbine generator, condenser boiler, and all associated equipment, have been standardized and are offered as complete units. For example, a typical standardized plant, when used for power generation only, could consist of a 40,000-kw combination including a 15,000-kw mercury-turbine generator and a 25,000-kw condensing steam-turbine generator. Such a combination is installed at the Schiller Station of the Public Service Company of New Hampshire. It consists of two 7500-kw mercury-turbine generators operating at 113 psig, supplying steam to a 25,000 kw, 600 psig–825 F–1 in. Hg abs. turbine generator. The design plant net heat rate is below 9500 Btu per kwhr.

A vertical section of a 5000-kw mercury turbine and condenser boiler is shown in Fig. 67. The mercury turbine is a five-stage impulse unit overhung from the end of the generator

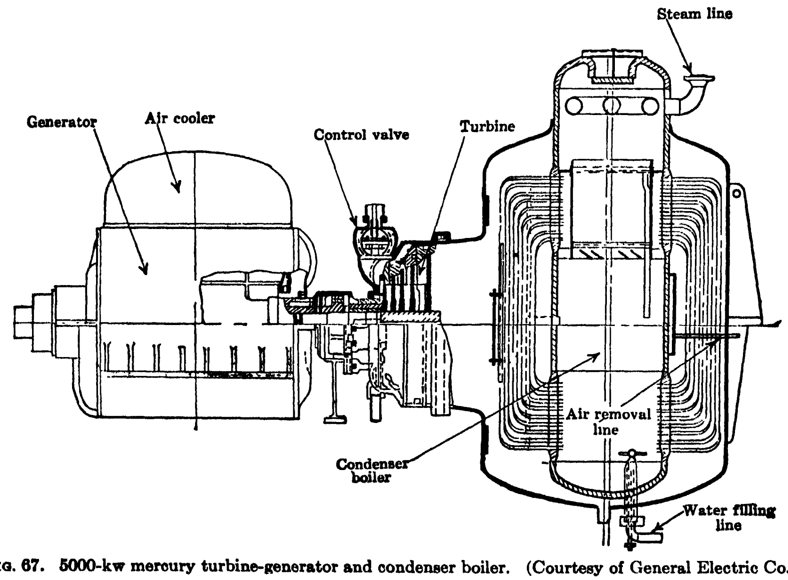


FIG. 67. 5000-kw mercury turbine-generator and condenser boiler. (Courtesy of General Electric Co.)

shaft to save one gland. Wheel speeds are comparatively low. Few data have been published on design and operation of mercury turbines, but their operation differs little from that of steam turbines. Special glands sealed by liquid mercury are used to prevent escape of mercury vapor.

Several mercury-steam plants have operated in the United States. A 10,000-kw mercury turbine was installed more than twenty years ago by the Hartford Electric Light Company to deliver steam to the main plant header. This turbine was a five-stage impulse unit with 70.7 psig mercury pressure and 28.5 in. vacuum. The combination developed power at the switchboard at an average fuel consumption of 10,250 Btu per kwhr.

After 120,000 hours of operation, the 10,000 kw unit at Hartford has been replaced with a 15,000-kw mercury-steam plant of modern design. The general plant arrangement is the same, and the existing building and steel were used. The new plant operates at a vapor pressure at the turbine throttle of 115 psig. Steam at 385 psig is delivered to the steam header for use in existing turbines.

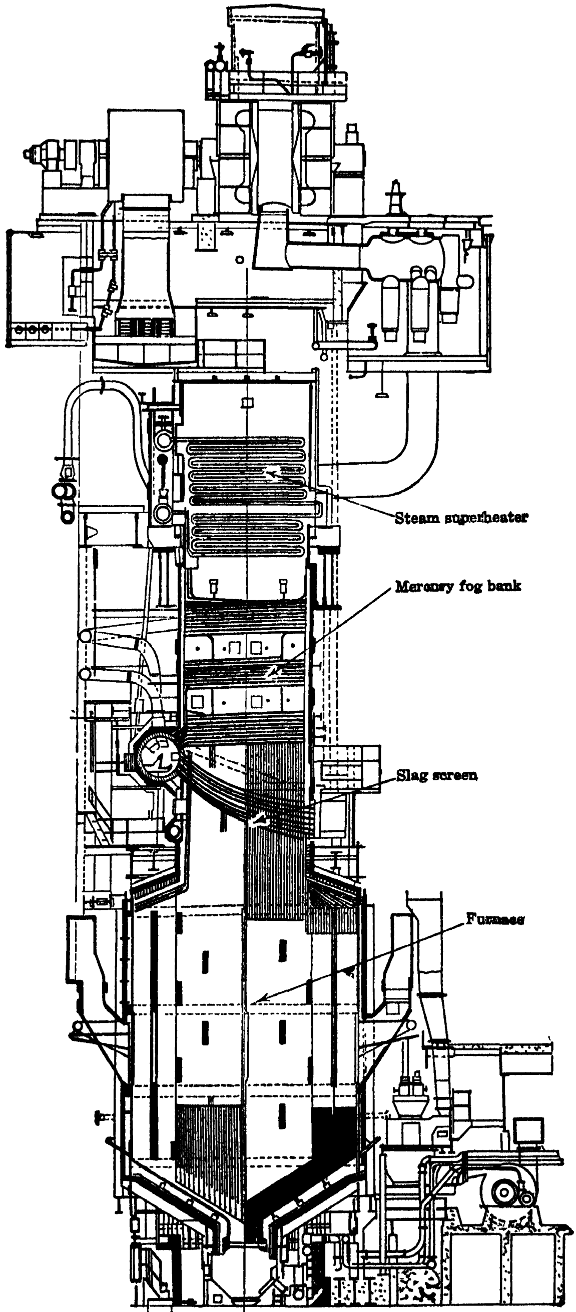


FIG. 68. Cross section of 20,000-kw mercury turbine-generator and boiler, Kearny station. (Courtesy of Public Service Electric and Gas Corporation, N. J.)

In 1932 the General Electric Company installed a 20,000-kw mercury turbine at its Schenectady plant which delivers 325,000 lb of 400 psig steam per hr to the company's steam system.

Another 20,000-kw unit, installed at the Kearny Station of the Public Service Electric and Gas Corporation, New Jersey, delivers steam to turbines in which 33,000 kw is generated, making a total of 53,000 kw for the combined mercury-steam unit. The heat rate for the combination is approximately 9175 Btu per kwhr. A cross section of this installation is shown in Fig. 68. Table 25 gives comparative operating conditions for these stations.

Table 25. Operating Conditions of Mercury-steam Stations

	Old Hartford	New Hartford	Kearny	Sch'dy	Schiller
Rating of mercury turbine, kw	10,000	15,000	20,000	20,000	15,000
Speed of mercury turbine, rpm	720	720	900	900	1,200
Total steam from unit, lb per hr	129,000	211,000	325,000	325,000	248,700
Steam pressure, psig	275	385	365	400	600
Steam temperature, °F	680	700	750	760	825
Mercury vapor pressure at turbine, psig	70.7	115	125	125	113
Mercury vapor temperature at turbine, °F	884.5	947	958	958	944
Vacuum of mercury condenser, in. Hg abs	1.5	3.0	3.0	3.0	5.2
Temperature of mercury vacuum, °F	440	485	485	485	523

THE STEAM ENGINE

By W. Trinks

History. The steam engine of the displacement type, made practical by James Watt between the years 1768 and 1790 is regarded by many as the most important and far-reaching invention of all time. It revolutionized manufacturing and, for over 100 years, was the principal prime mover. Steam engines have been supplanted for the most part by large steam turbines and internal-combustion engines, working in conjunction with electrical transmission of power to electric motors. However, the steam engine continues to be built for special uses. In sizes up to 500 or 600 kw, the noncondensing engine is more economical than the noncondensing steam turbine. If exhaust steam can be used for space heating or process work, the steam engine is preferred for even larger units. Reciprocating steam engines are frequently used for driving air compressors and gas compressors because the efficiency of a medium-sized reciprocating compressor exceeds the efficiency of rotating compressors.

Most railroad locomotives still are operated by steam engines. During World War II, many ocean-going ships were driven by triple-expansion steam engines because they could be produced quickly from existing drawings in almost any foundry and machine shop.

Characteristic Properties. The steam engine is a displacement machine in which work is done by pressure acting on a moving piston. The whole cycle is carried on in a given space, the walls of which are exposed alternately to high and to low temperature. These conditions result in: (1) Condensation of part of the entering steam on the walls of the cylinder and piston, and re-evaporation of the film of water near the end of the expansion and during exhaust, involving a transfer of heat energy into the exhaust without conversion into mechanical energy. (2) Variable torque and cyclical speed fluctuations, necessitating either a flywheel or else a multicylinder engine with several cranks. (3) Packing between cylinder and moving parts, requiring lubrication which contaminates the exhaust steam with lubricating oil. (4) Valve gearing to admit and exhaust steam alternately.

20. CLASSIFICATION OF ENGINES

CLASSIFICATION BY CONSTRUCTION. Horizontal Engine. For an explanation of running "over" or "under," see Fig. 1. An engine is *right hand*, if the flywheel is on the right-hand side of a person standing back of the cylinder and looking towards the shaft.

Vertical Engine. It may have the crankshaft either below (Fig. 2) or above the cylinder (Fig. 3).

Angle-type, or Horizontal-vertical Engine (Fig. 4). Usually, the two pistons act on the same crank. In multicrank engines, the steam cylinders are often vertical while the compressor cylinders are horizontal.

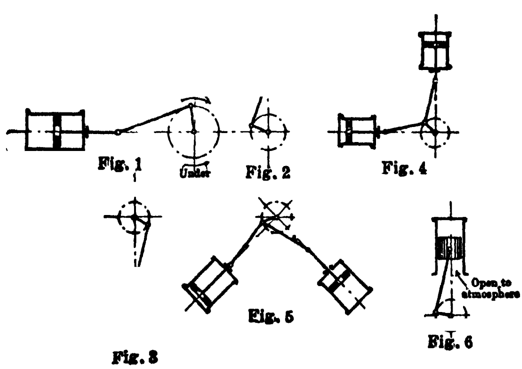
Hoisting engines sometimes have cylinders at 45 degrees (inverted V, Fig. 5).

Single-acting Engine (Fig. 6). The steam acts against only one side of the piston and does work during only one stroke, or half a revolution. It is usually of the vertical type.

Double-acting Engine (Figs. 1 to 5). Steam acts alternately on opposite sides of the piston.

Reciprocating Engine (Figs. 1 to 6). The piston moves in a straight line, but alternately in opposite directions.

Rotary Engine (Fig. 7). The pistons move continuously in a circular or other curved, closed path, never reversing their direction of motion.



FIGS. 1–6. Simple engines.

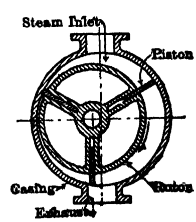


FIG. 7. Rotary engine.

CLASSIFICATION BY CONDITIONS OF OPERATION. A condensing engine discharges its exhaust steam into a vessel in which a pressure lower than atmospheric is maintained by condensation of the steam, and from which atmospheric pressure is excluded.

A noncondensing engine discharges its steam either into the atmosphere or against a pressure higher than atmospheric.

A bleeder-type or extraction-type engine is one from whose cylinder a portion of the steam is extracted, during either expansion or compression, at a pressure that is higher than the back pressure which acts during the exhaust stroke of the engine.

Simple Engine or Single-expansion Engine (Figs. 1 to 6). The complete expansion of the steam from boiler pressure to exhaust pressure is carried out in one cylinder or in each of several cylinders.

Multistage engine is one in which the expansion of the steam is divided up into stages. The steam expands in a high-pressure cylinder from boiler pressure to an intermediate pressure. It then flows into another cylinder, where it expands still further, and so on. Depending on whether the expansion is divided into two, three, or four stages, the engine is classified, respectively, as compound (Fig. 8), triple expansion (Fig. 9), or quadruple expansion (Fig. 10). A compound engine is *tandem compound* when the cylinders are arranged in line, one behind the other, and both act on the same piston rod. (See Fig. 11.) If the cylinders are side by side and act on cranks at right angles to each other, the engine is a *cross compound.* (See Fig. 8.) If the high-pressure cylinder discharges into two low-pressure cylinders, the engine is called a *three-cylinder compound.*

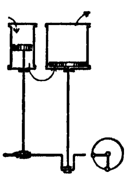


FIG. 8 Compound.

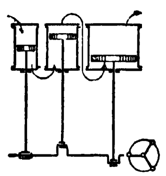


FIG. 9. Triple expansion.

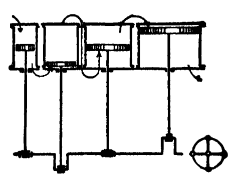


FIG. 10. Quadruple expansion.

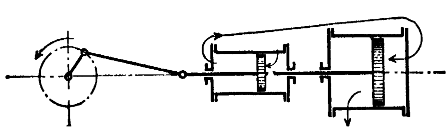


FIG. 11. Tandem compound.

Twin engines (Fig. 12) are simple engines having two cylinders of the same size side by side, with cranks at 90 degrees, as reversing mill engines, locomotive engines, and hoisting engines.

Uniflow engine (Fig. 13) is the "one-way" engine. In each cylinder end, steam flows in one direction only, from the cylinder head to the center of the cylinder. Admission valves are in or near the cylinder heads; exhaust ports are uncovered by the piston near the center of the cylinder. Many modern engines have auxiliary exhaust valves and are only partly of the uniflow type.

Duoflow or Counterflow Engine (Figs. 1 to 6, 14 and 15). Steam flows alternately in opposite directions in each of the cylinder ends, being first admitted and, later on, exhausted near the ends. The term covers all engines other than the uniflow type.

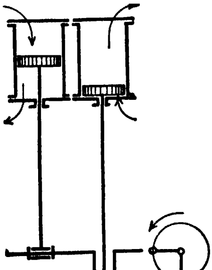


FIG. 12. Twin engine.

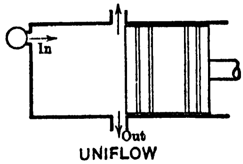


FIG. 13

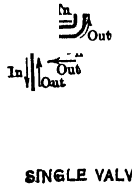


FIG. 14

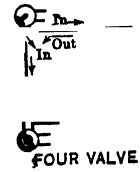


FIG. 15

CLASSIFICATION BY TYPE OF VALVE AND VALVE GEAR. (1) *Slide-valve engines:* the valve seat lies in a plane. (2) *Piston-valve engines:* the valve is a cylinder. (3) *Poppet-valve engines:* the valve lifts off the seat. (4) *Rocking-valve engines,* such as the "Corliss" engine: the valves move peripherally in a cylindrical seat. (5) *Automatic engines:* the valves are always mechanically connected to a driving crank or eccentric; cut-off is varied automatically by a governor, which is usually of the shaft-governor type. (6) *Releasing-gear engines:* governor causes steam valve to be periodically disconnected from the eccentric or crank. Valve is closed by an independent force.

CLASSIFICATION BY USE. This classification is rather loose. *Power engines* drive electric generators or deliver power to machinery through a belt, rope, or shaft drive. There are also pumping engines, blowing engines, locomotive engines, marine engines, hoisting engines, and many other types.

21. CAPACITY OF STEAM ENGINES

HORSEPOWER OF STEAM ENGINES. The rate at which steam does work on the engine piston is called *indicated horsepower* (ihp). It may be expressed in any of these four ways:

$$\text{Ihp} = \frac{\text{Average effective force on piston (lb)} \times \text{piston speed (ft per min)}}{33{,}000}$$

$$\text{Ihp} = \frac{\text{Average effective pressure on piston (psi)} \times \text{piston displacement (cu in. per min)}}{396{,}000}$$

$$\text{Ihp} = \frac{PLAN}{33{,}000}$$

where P = mean indicated pressure, psi; L = length of stroke, ft; A = effective area of piston, sq in., after deducting area of piston rod or tail rod; and N = number of power strokes per minute.

$$\text{Ihp} = \frac{\text{Mean effective pressure (psi)} \times \text{piston area (sq in.)} \times \text{piston speed (ft per min)}}{33{,}000}$$

The mean effective pressure is the average pressure shown on the indicator card (see Fig. 16) to be acting on the piston. Because of friction losses in the engine, the brake horsepower (power available at the shaft of the engine) is less than the indicated horsepower. The ratio of brake horsepower to indicated horsepower is the mechanical efficiency of the engine. For values, see Table 6.

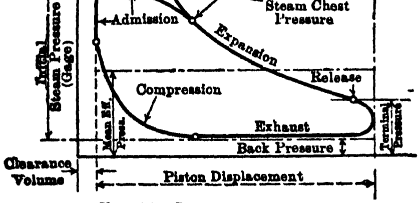


FIG. 16. Indicator diagram.

Greatest available mean effective pressure depends on initial steam pressure, back pressure, compression, clearance volume, area of ports, piston speed, type of valve gear, tightness of valves and pistons, superheat, and greatest possible length of steam admission (cut-off). The apparent cut-off on the indicator card is always shorter than the cut-off that is determined by the closing of the steam valve (see Fig. 16).

Initial Steam Pressure. For about one-half the steam engines currently being built, the initial steam pressure does not exceed 150 psig. Steam pressures up to 300 psi are in use if the engine is used in connection with process work (high back pressure) or as an auxiliary in a power plant with high steam pressure or in marine service. A few engines operated by steam of still higher pressure are in use.

TO FIND THE MEAN EFFECTIVE PRESSURE. Mean effective pressure (mep) from indicator cards of an existing engine is found either by planimetering or by calculating the mean value of equally spaced ordinates of the indicator card.

$$\text{Mep} = \frac{\text{Area of card, sq in.}}{\text{Length of card, in.}} \times \text{spring scale}$$

Spring scale is the number of pounds per square inch which produce an ordinate of one inch on the indicator card.

For predicting the greatest possible mean effective pressure of a new engine, three methods exist: (1) comparison with indicator cards of existing, similar engines; (2) design of the probable indicator card and measurement of its area; and (3) estimation from the ideal indicator card by means of a diagram factor.

The first method is used whenever information from other engines is available. (See Table 1.) The second method involves a large amount of numerical work and is, for that

Table 1. Commercial Mean Indicated Pressures on Which Engine Ratings Are Based

Initial Steam Pressure, psig	100	125	150	175	200
Engine Type	Rated Mean Effective Pressure, psi				
SIMPLE ENGINES					
Single-valve engine, condensing	45	49	53	57	62
Single-valve engine, noncondensing	51	58	64	71	77
Four-valve engine, condensing	42	46	50	54	58
Four-valve engine, noncondensing	48	54	60	66	72
Uniflow engine, condensing	35	42	48	52	55
Uniflow, noncondensing large clearance type	32	38	44	49	53
Uniflow, noncondensing, small clearance with auxiliary exhaust valves	37	44	51	56	59
MULTISTAGE ENGINES					
Compound condensing	25	28	30	32	34
Compound noncondensing	30	35	40	45	50
Triple-expansion, condensing	. . .	20	21	22	24

reason, seldom used. Although the third method is very convenient, it has led to great errors in the calculation of the greatest possible mean effective pressure, because of the assumption that the apparent cut-off on the indicator card will equal the actual cut-off of the valve gear. This coincides with assuming a *diagram factor* of 100%. For average diagram factors, see Table 2. The expansion line and the compression line are usually

Table 2. Diagram Factors

Average values for usual operating conditions, referred to ideal unmodified indicator diagram without clearance or compression (Fig. 17).

Engine Type	Diagram Factor at Rated Load	Diagram Factor at Maximum Overload
POWER ENGINES AND MILL ENGINES: HIGH SPEED		
Single-valve engine, small size, simple	.80	.70
Piston-valve engine, simple	.82	.74
Piston-valve engine, compound	.70	....
Automatic four-valve engine, nonreleasing valve gear, simple	.86	.82
Automatic four-valve engine, nonreleasing valve gear, compound	.74	....
Releasing gear engine, simple	.90	.88
Releasing gear engine, compound	.76	.74
Releasing gear engine, triple-expansion	.70	.68
Uniflow poppet-valve engine, condensing	.78	.75
Uniflow, noncondensing, with large clearance	.62	.60
Uniflow, noncondensing, with small clearance and auxiliary exhaust valves	.76	.75
PUMPING ENGINES: SLOW SPEED		
Releasing gear compound, without jackets	.82	.81
Releasing gear compound, with jackets	.93	.92
Releasing gear triple, without jackets	.73	.72
Releasing gear triple, with jackets and reheaters	.85	.84

constructed as equilateral hyperbolas. Because of the uncertainty of the diagram factor, a closer approach to the actual shape of the curves is not warranted.

Mean Effective Pressure from Ideal Card and Diagram Factor. The ideal hyperbolic diagram without clearance or compression is represented by 1–2–3–4–5–1 in Fig. 17. The average ordinate or mean effective pressure, pounds per square inch, of this diagram is

$$\text{Mep} = \frac{P(1 - \log_e R)}{R} - p$$

The meaning of the letters is clear from Fig. 17.

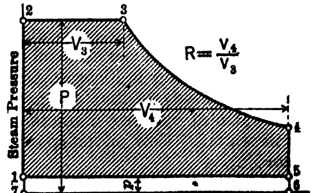


FIG. 17. Ideal indicator card.

Figure 18 shows the ideal mean effective pressure directly, for various expansion ratios and initial pressures, for the two conditions of back pressure of 15 psia, and for perfect vacuum. The actual mean effective pressure is less than that calculated by the formula or chart because the area of the card is reduced by release, by compression, and by throttling of steam through admission valve and exhaust valve. This throttling shortens apparent steam admission and makes compression begin earlier. The factor by which the ideal mean effective pressure is multiplied to obtain the actual mean effective pressure is called the *diagram factor*. Values which commonly apply (referring to the ideal card without clearance or compression) are shown in Table 2. This method of finding the

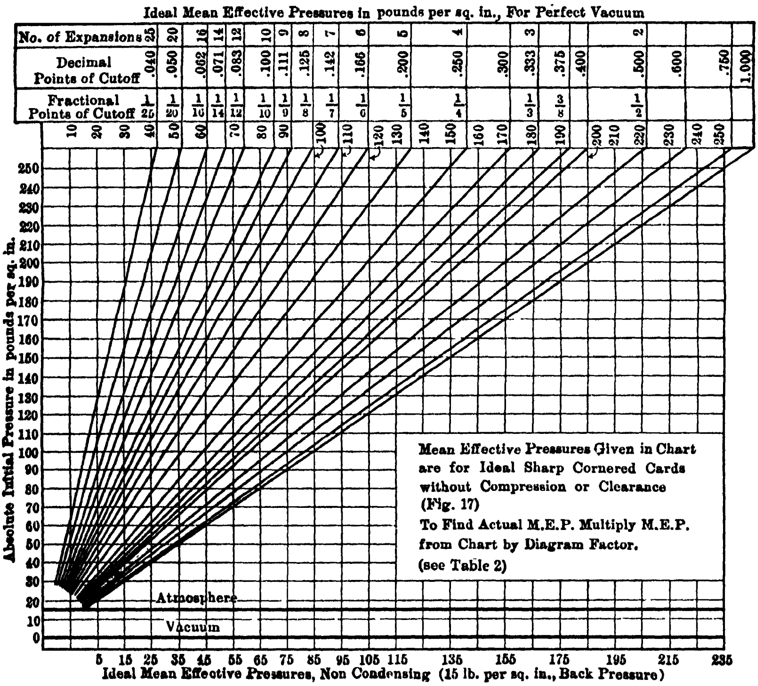


FIG. 18. Ideal mean effective pressure chart.

greatest possible mean effective pressure is used for approximate calculations. Commercial mean effective pressures and overload capacities for engines of different types are shown in Tables 1 and 3. Values of Table 1 represent mean effective pressures at which engines operate most economically, including consideration of investment costs. The diagram factor is reduced by restricted openings of valves and ports and by sluggish cut-off; it is also reduced by early compression, too early or too late release, high piston speed, and use of highly superheated steam.

Mean effective pressures in multistage engines are the sum of the mean effective pressures of all stages, referred to the low-pressure cylinder. Most economical mean effective pressure is usually 65 to 75% of rated mean effective pressure.

Clearance space includes all volume enclosed between piston and valves at one end of the cylinder, when piston is at dead center of that end. Clearance is kept as small as possible because of the harmful effect of its volume and surface on steam economy, except in noncondensing uniflow engines, in which it must be large (unless auxiliary exhaust valves are used) so that compression pressure will not rise above initial steam pressure. High ratio of cylinder diameter to stroke means large clearance in percentage of piston displacement. Low piston speed permits use of small ports, and clearance can be reduced accordingly.

Modified Hyperbolic Diagram. Figure 19 shows the sharp-cornered diagram 1–2–3–4–5–6–1, which includes the influence of clearance and of compression. The lines 3–4 and

Table 3. Overload Factors

Rated mean effective pressures of Table 1 are multiplied by these factors to find the *maximum* mean effective pressure which can be obtained.

Simple Engines	Initial Steam Pressure, psi	Overload Factors: Condensing Operation	Overload Factors: Noncondensing Operation
Automatic engine; or single-eccentric releasing gear engine	100	1.86	1.45
	125	2.05	1.60
	150	2.25	1.75
	175	2.46	1.90
	200	2.67	2.06
Double-eccentric releasing gear engine	100	2.18	1.70
	125	2.43	1.87
	150	2.65	2.02
	175	2.84	2.15
Uniflow engine	100	.63	1.30
	125	70	1.36 Large
	150	76	1.41 clearance
	175	81	1.44 type
	200	.87	1.48

Compound Engines

Cylinder Ratio	. . . .	3	3 1/2	4	4 1/2	5	2	2 1/2	3	3 1/2
Automatic engine	100	1.48	1.34	1.20	1.12	1.02	1.25	1.07	. . . . .	. . . . .
	125	1.59	1.47	1.32	1.22	1.12	1.39	1.20	1.02	. . . . .
	150	1.75	1.62	1.45	1.35	1.25	1.46	1.27	1.10	1.0
	200	2.21	1.97	1.79	1.66	1.55	1.60	1.42	1.27	1.17
Double-eccentric releasing gear engine	100	1.88	1.70	1.58	1.46	1.34	1.46	1.27	1.10	1.0
	125	2.05	1.86	1.71	1.59	1.48	1.81	1.54	1.37	1.21
	150	2.26	2.06	1.88	1.75	1.63	1.91	1.63	1.46	1.31
	175	2.49	2.28	2.07	1.92	1.79	2.00	1.71	1.54	1.40

Triple Expansion

Releasing gear engine, with cylinder ratios shown in Table 4	150	1.30
	175	1.44
	200	1.60

6–1 are usually constructed as hyperbolas (pv-constant) with point O as origin. The mean effective pressure is best determined with the aid of a planimeter.

Final Pressure of Compression. Pressure 8–1 of Fig. 19 is, in some engines, as low as 15% of 8–2. In high-speed engines it is as high as 75%, for quiet operation. In single-valve engines, the compression pressure varies with the load.

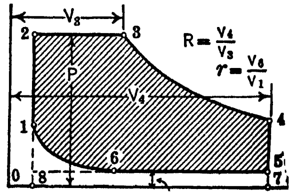


Fig. 19. Ideal indicator card with clearance and compression.

Piston Speed and Rotative Speed. The term *piston speed* is commonly understood to mean average piston speed, or 2 × stroke (ft) × rpm. High rotative speeds are desirable in engines direct-coupled to electric generators because size, weight, and cost are thereby reduced. High piston speed, obtained by lengthening the stroke rather than by increasing the rpm, cheapens neither engine nor generator. Economical piston speeds range from 500 ft per min for small engines to 1000 ft per min for large engines. Piston speeds up to 2000 ft per min have been attained under exceptional circumstances. Piston speeds below 500 ft per min result in poor steam economy because the time for cylinder condensation is increased. Some valve gears limit the rotative speed. Although releasing gears can be operated at speeds up to 150 rpm, they offer no advantages above 80 rpm and introduce difficulties above 120 rpm.

Figures 20 and 21 contain useful information on weight and space requirements of steam engines.

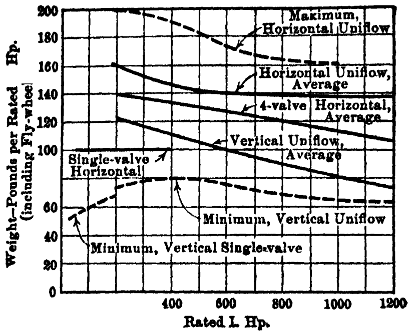


FIG. 20. Weight of engines per horsepower, including flywheel but not generator.

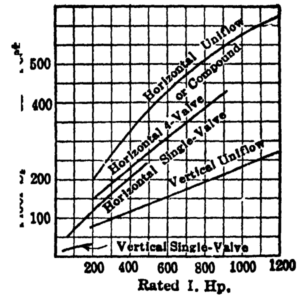


FIG. 21. Floor space required by engines.

22. CAPACITY OF COMPOUND ENGINES

Compound engines reduce the loss caused by cylinder condensation. Use of superheated steam and of uniflow engines has greatly reduced the value of compound engines.

The mean effective pressure of compound engines is referred to the low-pressure cylinder, and called the *equivalent mean effective pressure.* It equals Mep of low-pressure cylinder + (Mep of high-pressure cylinder × ratio of high-pressure piston displacement to low-pressure piston displacement). Commercial cylinder displacement ratios are given in Table 4. Receiver or cross-over pressure can be varied by changing the low-pressure cut-off.

Table 4. Commercial Cylinder Displacement Ratios of Multistage Engines

Ratios represent compromise between steam economy and overload capacity.

	Condensing Engines					Noncondensing Engines				
Initial Steam Pressure, psig	100	125	150	175	200	100	125	150	175	200
Automatic compound for electric power generation, or blowing engines	3.5	3.9	4.3	4.7	5.1	2.3	2.6	2.9	3.3	3.7
Automatic compound for large overload capacity (mill engines)	3.0	3.3	3.6	4.0	4.4	2.0	2.2	2.5	2.8	3.1
Compound engine with releasing valve gear (rocking or poppet valves)	4.0	4.4	4.8	5.2	.....	2.6	2.9	3.2	3.6	.....
Triple-expansion engine { h.p. / int. / l.p.	.... / ..... /	1: / 3.2: / 7.0	1: / 3.3: / 7.7	1: / 3.5: / 8.4	1: / 3.7: / 9.0					

Figure 22 shows the effect of low-pressure cut-off on ideal indicator diagrams for a compound engine with constant receiver pressure and without clearance or compression. Pressure drop through valves, unequal cylinder condensation, condensation in the receiver,

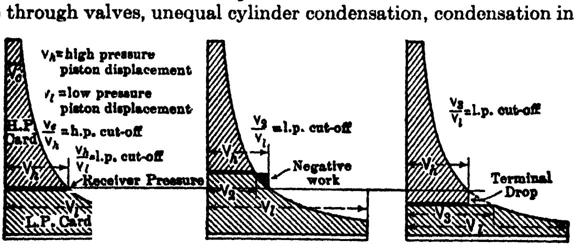


FIG. 22. Effect of low-pressure cylinder cut-off on indicator diagrams.

and other conditions cause the receiver pressure to be lower than that determined from the ideal diagram. Actual receiver pressures may be taken at about 22 psig for condensing and 40 psig for noncondensing engines, with 150 psig initial pressure.

23. TRIPLE EXPANSION ENGINES

Use of triple and quadruple expansion engines has become very limited. These engines are essentially constant-load machines because the small size of the high-pressure cylinder does not allow any appreciable overload capacity.

In multistage-expansion engines, superheated steam does not yield as much benefit as in simple or compound engines. In consequence, many triple-expansion engines in marine service are operated with saturated steam, so that cylinder lubrication becomes unnecessary.

Very few multistage steam engines are being built today. The number of such engines still in operation, however, is quite large.

EXTRACTION-TYPE ENGINES. In plants with condensing engines where a moderate quantity of 5 to 15 psig steam is needed for space heating or chemical processes, the steam may be obtained from the receiver of a compound engine, or tapped from the cylinder of a simple engine, through a check valve and port uncovered by the piston before the end of its stroke.

24. STEAM-ENGINE ECONOMY

The performance of steam engines is expressed in: pounds of steam per indicated bhp-hr; pounds of steam per bhp-hr; Btu per indicated hp-hr; Btu per bhp-hr (or, in each case, per kilowatthour); or by the Rankine efficiency ratio. If kilowatthours are the basis of reference, it must be stated whether or not generator efficiency is included.

The performance of pumping engines often is expressed as their "duty," the foot-pounds of work done by 1000 lb of dry steam, or per million Btu furnished by the boiler. Duty includes the performance of both engine and pump.

STEAM CONSUMPTION GUARANTEES. It is customary to base a steam-consumption guarantee (sometimes called a water-rate or steam-rate guarantee) on the performance of a similar engine, working under similar conditions of steam pressure, superheat, and back pressure. If this information is not available, steam consumption is estimated from data like those in Table 5. This table also contains values of the Rankine cycle ratio, η_R.

$$\eta_R = \frac{\text{Btu per brake horsepower-hour consumed by a perfect Rankine cycle engine}}{\text{Btu per brake horsepower-hour consumed by actual engine}}$$

Often the Btu values are replaced by equivalent steam consumptions. The Rankine-cycle-steam-consumption is (2544/Btu available per pound of steam). The divisor of this fraction is read from a Mollier chart. (See Section 4.)

EFFECT OF STEAM CONDITIONS ON STEAM ECONOMY. The effect of initial steam pressure is shown in Fig. 23. The superheat is also important, because superheat

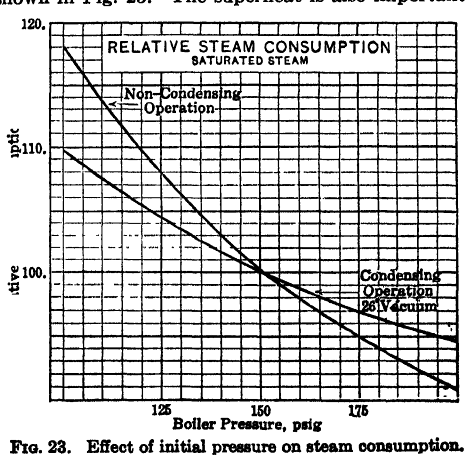


Fig. 23. Effect of initial pressure on steam consumption.

Table 5. Rankine Cycle Efficiency Ratio and Steam Consumption

Average values under operating conditions.

Type and Rating of Engine	Pressure, psig	Superheat	Vacuum	Rankine Cycle Ratio, %, Based on Ihp	Pounds of Steam Used per Ihp-hr	Thermal Efficiency, % Based on Bhp
Slide valve, simple, 50 hp	125	Saturated	N.C.	39.0	40.0	5.5
Piston valve, simple (same port for inlet and exhaust)	125	Saturated	N.C.	58.0	27.0	8.5
	125	150° superheat	N.C.	62.0	22.5	9.3
Piston valve, compound, 300 hp	150	Saturated	N.C.	66.0	22.0	10.3
	150	150° superheat	N.C.	70.0	18.5	11.2
	150	Saturated	26 in.	50.0	17.5	11.8
	150	150° superheat	26 in.	58.0	13.9	14.0
Four-valve, nonreleasing compound, 300 to 500 hp	150	Saturated	N.C.	74.0	19.5	11.6
	200	Saturated	N.C.	75.0	17.2	12.9
	150	Saturated	26 in.	61.0	14.5	14.3
	200	Saturated	26 in.	63.0	13.1	15.8
Corliss, simple, 300 hp	125	Saturated	N.C.	69.0	22.5	10.0
	125	Saturated	26 in.	50.0	18.5	11.3
Corliss, compound, 500 hp	150	Saturated	N.C.	76 0	19.0	11.9
	150	Saturated	26 in.	64.0	13.7	15.1
	150	50° superheat	26 in.	66.0	13.2	15.5
Triple-expansion marine engine	200	Saturated	26 in.	64.0	13.2	15.1
Triple-expansion power engine	175	150° superheat	26 in.	75.0	10.4	19.1
Triple-expansion pumping engine, 500 hp	175	150° superheat	26 in.	81.0	9.7	20.5
Uniflow (simple), 500 to 1000 hp	125	Saturated	N.C.	70	21.7	10.2
	150	Saturated	N.C.	71	20.3	11.1
	200	Saturated	N.C.	74	17 5	12.7
	125	150° superheat	N.C.	74	18.9	11.2
	150	150° superheat	N.C.	75	17.5	11.9
	200	150° superheat	N.C.	77	16.0	13.2
	125	Saturated	26 in.	63	14.5	14.3
	150	Saturated	26 in.	64	13.7	15.0
	200	Saturated	27 in.	64	12.7	16.2
	125	150° superheat	26 in.	67	12.5	15.6
	150	150° superheat	26 in.	68	11.8	16.4
	200	150° superheat	27 in.	68	11.0	17.7
	250	150° superheat	26 in.	69	10.5	18.2
	150	Saturated	5 lb *	87		

N.C. = Noncondensing. * Back pressure (gage).

Note. For larger sizes and higher superheats, values of the Rankine cycle efficiency ratio become greater than those given in the table. Variation of Rankine cycle ratio with size between limits of 100 and 1500 hp is shown with moderate accuracy by the empirical formulas:

$$E = A + 11\sqrt[6]{\text{ihp}}, \text{ for condensing engines, and } E = A + 8\sqrt[6]{\text{ihp}}$$

for noncondensing engines; factor A varies with type of engine.

reduces both cylinder condensation and steam density. The worse the cylinder condensation of an engine is, the more beneficial is superheat. Figure 24 shows the percentage saving in steam consumption due to superheat in engines of various types. Packings and lubrication problems limit initial steam temperature to 750–800 F.

Increasing *back pressure* rapidly increases the steam consumption per horsepower-hour. This effect is not harmful, if all the exhaust steam is required at the high back pressure for space heating or chemical processes. The engine then acts as a reducing valve which furnishes power as a by-product. The effect of reduced back pressure is illustrated in Fig. 25.

EFFECT OF OTHER FACTORS ON ECONOMY. Increase of *clearance volume* is harmful to steam economy because it increases both the surface for condensation and the free-expansion loss. The influence of clearance on steam economy cannot be simply expressed. The effect of *compression* varies with the type of engine. In engines with small

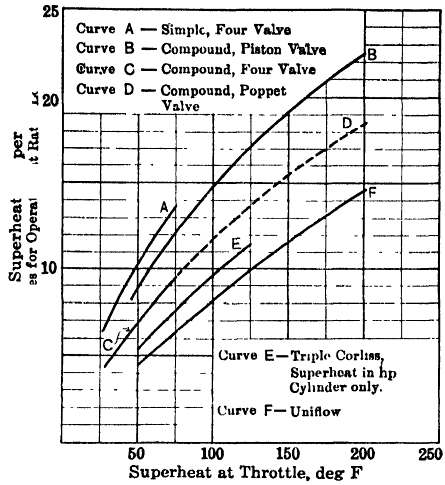


FIG. 24. Reduction in steam consumption due to superheat, for various types of engine.

clearance volume, long compression raises the steam temperature beyond that of the incoming live steam, and excessive cylinder condensation occurs during part of the compression. In engines with greater clearance, variation of length of compression has little influence on steam economy.

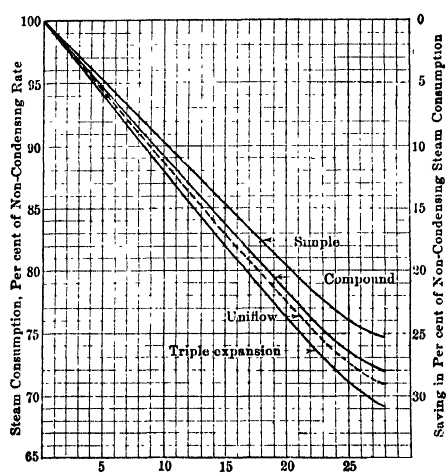


FIG. 25. Reduction in steam consumption by increase of vacuum.

Leaky pistons and valves waste more steam than the best valve gear can save, often 10% or more. For determination of engine leakage, see the ASME Steam Engine Power Test Code. The effect of leakage is greatest in single-valve engines because steam leaks past the valve directly into the exhaust, without entering the cylinder. Rocking valves (Corliss) and their seats are warped by high temperatures and sometimes leak badly.

The influence of cylinder condensation, leakage, and free expansion losses results in variation of specific steam consumption with load, as shown in Fig. 26. The type of valve gear affects specific steam consumption most seriously in single-valve engines, less in four-valve engines, and still less in uniflow engines.

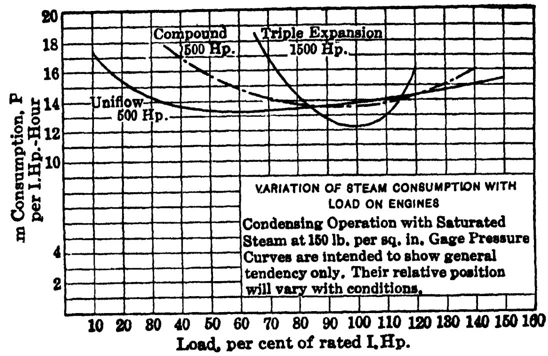


FIG. 26. Variation of steam consumption with load on engine.

Moisture increases steam consumption only by the weight of the moisture.

Steam jackets effect steam saving in slow-speed engines only, hence they have been abandoned, except in the heads of uniflow engines, where they save steam even at high speeds.

Engine friction affects both steam-engine economy and steam-engine capacity. The ratio, brake horsepower divided by indicated horsepower, is called mechanical efficiency. Figure 27 shows variation of mechanical efficiency with engine load. Table 6 gives additional data on average mechanical efficiencies.

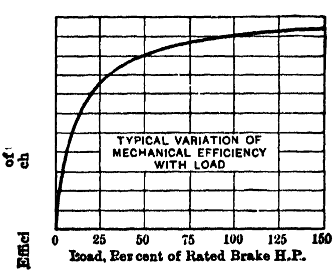


FIG. 27. Typical variation of mechanical efficiency with load.

Table 6. Mechanical Efficiency of Steam Engines, %

Average values at rated load for engines in good condition.

SIMPLE ENGINES:	Portable engine, 50 hp	83
	Horizontal automatic engine, 150 hp	90
	Horizontal Corliss engine, 300 hp	91
	Horizontal uniflow engine, 400 hp	90
	Locomotive	85
COMPOUND ENGINES:	Horizontal automatic engine, 300 hp	90
	Horizontal Corliss engine, 500 hp	89
	Horizontal blowing engine, automatic, 2000 hp	88
TRIPLE-EXPANSION ENGINES:	Vertical power engine	90
	Vertical pumping engine, slow speed, 1500 hp	94

25. OPERATING DATA

HOT BEARINGS result from causes such as insufficient supply of oil; dirty oil (even a powder so fine as not to cause any gritty feeling between fingers is sufficient to heat a heavily loaded bearing); water in oil; wrong viscosity of oil (too light or too heavy); undue tightness of bearing; wrong shape or location of oil grooves; improper bedding of shaft or pin in bearing, whereby metallic contact is caused in spots; shaft or pin too small for heavy alternating load, with deflection causing metallic contact in spots; binding due to lack of alignment; unsuitable bearing material; inaccurate finish of journal or box; insufficient opportunity to dissipate frictional bearing heat: an occasional overload on a hot day may have wiped the oil grooves shut.

STARTING. Warming up the engine before starting is advisable. Most four-valve engines and some single-valve engines are equipped for hand operation of valves so that engine may be warmed by steam coming through a by-pass or through the throttle valve. In engines which have to be started suddenly against full load, such as hoisting engines or locomotives, relief valves are made extra large and are set very loose. If the steam-supply line is not properly drained by a trap, a large hand-operated drain valve is provided.

VALVE SETTING should be checked regularly, at least once a month, preferably with an indicator. Nuts come loose, valve stems are twisted, eccentrics slip. Common defects and their causes are illustrated in Fig. 28.

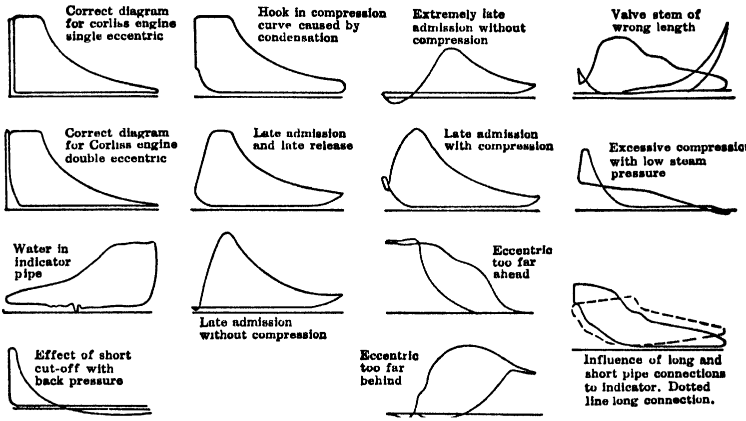


FIG. 28. Indicator card shape for various engine defects.

WEAR OF CYLINDER AND OF PISTON RINGS. Under average conditions, a set of piston rings travels over 300 million feet before renewal becomes necessary. Steam cylinders last for about 1500 million feet of piston travel before reboring becomes necessary. Cylinder walls are usually made thick enough for two reborings. For 24-hour-per-day operation, this means that a set of rings lasts approximately 14 months, that the cylinder must be rebored every 8 years, and that it must be replaced after 25 years. Both figures are influenced by the quality of rings and cylinder, by solids carried in the steam, and by effectiveness of lubrication. Piston rings of incorrect shape exert concentrated localized pressure, producing local wear and even glass-hard spots in the cylinder. These spots can be removed by grinding only. A set of rings is sometimes worn out in 2 months: another set will last 2 years. If cylinders are made of ordinary soft cast iron, the wear may be five times as rapid as indicated by the above figures, which apply to castings of hard cylinder iron. After a short period of use, cylinders made of the right kind of iron acquire a semiglazed and reasonably hard surface. From then on, the wear is slight. Wide pistons of horizontal engines, bearing on the cylinder surface and not supported by a rigid tail rod, cause very rapid wear, unless wide arc-shaped grooves encircle the piston.

BALANCING OF ENGINES. The balancing of steam engines differs in no detail from the balancing of other machinery with reciprocating parts. (See Steam Locomotives, Section 14.)

26. LUBRICATION

Lubrication of out-of-the-cylinder parts of steam engines is identical with that of the corresponding parts of internal-combustion engines and air compressors. In engines with multijointed valve gears, such as Corliss engines, automatic lubrication of the moving valve gear parts is not practical.

CYLINDER LUBRICATION. In engines working with high pressure or high superheat, lubrication of cylinders and of sliding valves is difficult because the rubbing surfaces cannot be completely separated by a film of oil. The hydrostatic lubricator, which serves well with low temperatures, is replaced by a mechanically operated (force pump) lubricator whenever high temperatures are encountered. Mechanical lubricators inject oil into the steam chamber close to the inlet valves or even into the steam cylinder. They are set to inject the lubricant at the proper time.

In operation with saturated steam the film of water on the walls washes off any straight mineral oil. For that reason, it is blended with tallow (usually 5%). For operation with superheated steam, high-grade uncompounded mineral oils are available.

The consumption of cylinder oil ranges from 0.1 pint to 4 pints per million square feet of surface swept over by the piston (perimeter of piston times distance traveled), with an average of 0.8 pint per million square feet. In engines using steam of 700 to 800 F, an excess of oil forms carbon, which produces wear.

Separation of Oil from Exhaust Steam. Where exhaust steam from reciprocating engines is to be used for heating feedwater or in process heating, the oil carried in the steam must be removed. The exhaust steam is passed through separators, usually of the baffle type, but occasionally of the centrifugal type. Two separators in series are sometimes provided. Oil drops only are removed; oil vapors pass through a separator without being removed. In Europe, charcoal oil separators have been used successfully.

27. SELECTION OF TYPE OF ENGINE

The most important question in the selection of the engine is whether to install a reciprocating steam engine or some other source of power, such as a steam turbine, gas engine, gasoline engine, or oil engine, or a secondary source of power, such as an electric motor. Steam engines have the advantage of easy reversibility. The steam boiler can be fired with any available fuel, including wood, wood refuse, bagasse, and peat. These two facts make the steam engine suitable for certain fields, such as hoisting, where no electrical energy is available, oil-field work, sawmills, and marine service. For generation of electrical energy, steam engines are frequently preferred in sizes of 800 hp or less, particularly if direct current is to be generated and if exhaust steam can be used for space heating or for process work. Since little, if any, waste heat is required for space heating during the summer, the engine should be of an economical type, four-valve or uniflow. In some cases, a summer engine driven by oil or natural gas is installed.

For sawmills (using sawdust for boiler fuel), for hoisting, and for oil-field work, steam engines of the simplest type are preferred. As a medium-size marine engine, the vertical tandem uniflow compound is growing in favor. The current trend is toward uniflow engines with auxiliary exhaust valves.

Vertical engines save floorspace and are frequently used in small sizes, particularly with horizontal compressors acting on common crankpins. They are not popular for large land units because they are inaccessible for repair.

Engines with rocking valves, Corliss engines, for instance, must not be used with high pressures or high superheat because distortion of valves and seats causes wear and breakage.

Steam for reciprocating engines used as auxiliaries in power plants is usually desuperheated and reduced in pressure.

28. TESTING OF STEAM ENGINES

All steam engine tests should be made in accordance with the ASME test code for reciprocating steam engines.

General References. Ripper, *Steam Engine Theory and Practice.* Ninde, *Design and Construction of Heat Engines.* Stumpf, *Uniflow Steam Engines*, 1922. Fernald and Orrok, *Engineering of Power Plants*, 1916. Furman, *Valves, Valve Gears, and Valve Diagrams.* Auchincloss, *Link and Valve Motions.* Dalby, *Valves and Valve Gear Mechanisms.* Hiscox, *Modern Steam Engineering.*

SECTION 9

CONDENSERS AND COOLING EQUIPMENT

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CONDENSERS

BY K. S. BRUNDIGE

COOLING EQUIPMENT

BY JAMES G. DEFLON

CONDENSERS

By K. S. Brundige

Classification. The two general types of condensers are: 1. *Direct contact condensers*, in which the steam to be condensed comes in direct contact with the condensing water. They include jet, barometric, and ejector condensers. 2. *Surface condensers*, in which steam and condensing water circulate on opposite sides of a metallic condensing surface.

1. DIRECT-CONTACT CONDENSERS

THE JET CONDENSER (Fig. 1) because of its low cost finds frequent application in plants of moderate size, particularly where cooling water is available in large quantity, and especially if this water is suitable for use in boilers. Since the application of a jet-type condenser is limited, development effort has been small in recent years. The operating cost of jet condensers is usually higher than that of surface-type condensers.

The jet condenser comprises a head into which exhaust steam is delivered and condensed by water sprays, a pump for the removal of condensate and condensing water,

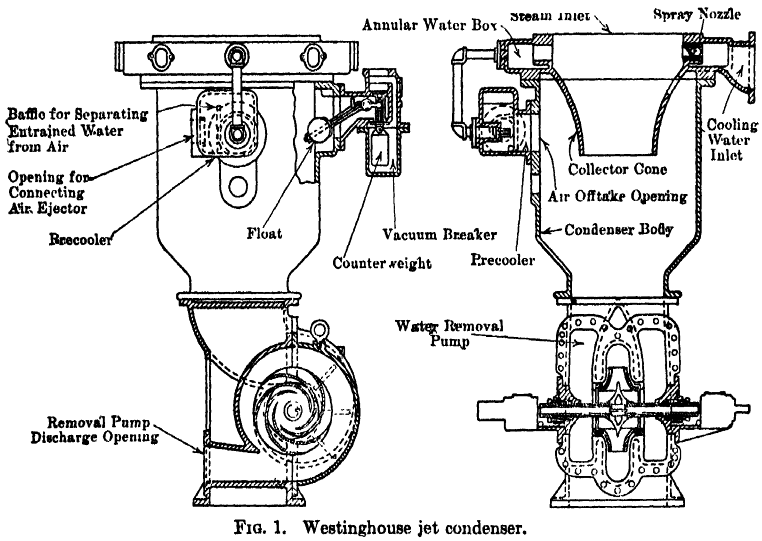

FIG. 1. Westinghouse jet condenser.

and a vacuum pump or steam jet air ejector (see p. 9-16) for removing air and non-condensable gases. Sometimes the same pump is used for removing both air and water. Condensing water is drawn into the head by the partial vacuum therein, and no circulating water pump is required.

Vacuum breakers are required on jet condensers to admit air to the condenser body if water rises in it to a predetermined level. If permitted to rise above this level, the water might flood and damage the prime mover. The vacuum breaker consists of a float that opens an air valve, destroying the vacuum and stopping the flow of condensing water.

Maximum suction lifts for the condensing water are 15 to 18 ft. If greater, an overload on the condenser would tend to decrease the vacuum so that insufficient condensing water would be drawn for operation. Figure 2 illustrates permissible condenser overloads at various suction lifts. The chart is based on condensing water at 70 F. At lower temperatures the permissible overload is greater.

The chart may be used in this way: Let L = load on condenser, pounds of water per hour, at suction lift m and vacuum v; l = relative load at lift m and vacuum v; X and X_a = actual load on condenser at incipient and absolute instability, respectively; x and x_a = relative load at point of intersection of curve of suction lift m, with lines of incipient and absolute instability respectively. Then

$$X = L\left(\frac{x}{l}\right) \quad \text{and} \quad X_a = L\left(\frac{x_a}{l}\right)$$

EXAMPLE. Load L = 50,000 lb per hr; vacuum, 27 in.; suction lift 18 ft. Relative load (v = 27; m = 18) = 140; relative load of incipient instability (m = 18) = 199. Load of incipient instability X = 50,000 (199/140) = 71,071 lb.

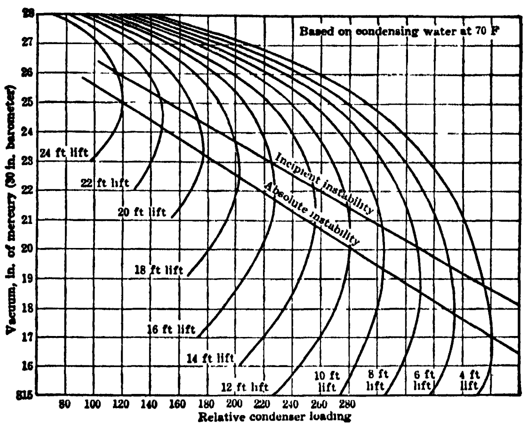

FIG. 2. Stability chart for jet condensers.

Dimensions of Jet Condensers. Shown in Fig. 3 and Table 1 are dimensions of typical vertical jet condensers.

THE BAROMETRIC CONDENSER consists of a condenser head similar to that of a jet condenser, mounted at the top of a discharge pipe whose length is at least 34 ft above

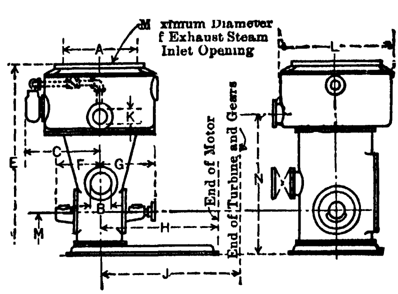

FIG. 3. Dimensions of Wheeler low-level jet condensers.

Table 1. Dimensions of Low Level Vertical Cylindrical Jet Condensers

(Courtesy of C. H. Wheeler Manufacturing Co., Philadelphia, 1949)

Size	Dimensions, in. (See Fig. 3)											
	A	*B*	*C*	*E*	*F*	*G*	*H*	*J*	*K*	*L*	*M*	*N*
JC	16	4	25	82	19	23	51	100	4	28	14	66
JD	20	5	28	88	20	24	58	102	4	34	17	70
JE	24	6	31	92 1/2	24	28	64	105	5	42	18 1/2	72
JF	28	7	33	97 1/2	24	28	67	111	6	48	20 1/2	74
JG	30	8	38	107	25 1/2	30	76	117	7	54	21	82
JH	36	9	47	112 1/2	27	32	78	120	8	62	22 1/2	84
JK	42	10	52	118 1/2	29	34	80	129	9	72	24 1/2	87
JL	48	12	56	127	32	37	90	132	10	80	29	92
JM	54	14	58	139 1/2	33	38	91	135	12	84	31 1/2	102
JN	60	16	62	146	34	39	97	138	14	92	33	104
JP	72	18	67	162	35	40	108	138	16	102	35	116

the level of the hotwell. Condensate and condensing water falling into the tail pipe draw out entrained air. No vacuum pump is required with this type of condenser although sometimes a dry air pump is used to remove air not removed by the descending water column. If the condensing water inlet is not over 20 ft above the source of supply, condensing water is drawn into the condenser by the partial vacuum, as in a jet condenser, and no circulating water pump is necessary.

Figures 4 and 5 show the Schutte & Koerting multijet barometric condenser. In Fig. 5 the condensing water is discharged through a series of small nozzles *A* into a combining tube *B* and an extension *C*, consisting of several sets of tapered rings. The water jets are directed into the throat *D*, where they unite to form a single jet. The vapor flows through the annular passes of the combining tube and there is condensed. The water jets entrain air and noncondensable gases and discharge them into the barometric *tail pipe*. This type of condenser has had wide application in connection with evaporators, vacuum pans, dryers, stills, etc. Table 2 gives dimensions and capacities.

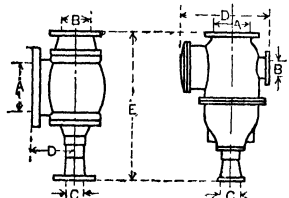

FIG. 4. Dimensions of Schutte and Koerting barometric jet condensers (for data, see Table 2).

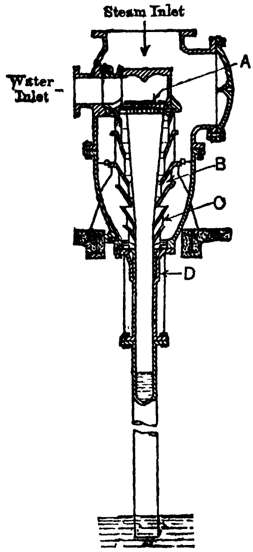

FIG. 5. Schutte and Koerting barometric jet condenser.

Table 2. Capacities and Dimensions of Barometric Multijet Condensers

(Courtesy of Schutte & Koerting Co., Philadelphia, 1949)

Size No.	Maximum Water Capacity, gpm	Connections, inches			Overall Dimensions		Approx. Shipping Wt., lb
		Vapor Inlet, *A*	Water Inlet, *B*	Discharge, *C*	*D*	*E*	
2	50	5	2 1/2	2	7″	23″	500
3	85	6	3	3	8″	30″	550
4	130	8	4	3	9″	32″	600
5	210	10	4	4	10 3/4″	36 7/8″	675
26	350	14	5	5	2′ 4 1/2″	4′ 5 3/4″	1,300
27	400	18	5	5	2′ 4 1/2″	4′ 5 3/4″	1,500
28	450	18	5	5	2′ 4 1/2″	4′ 5 3/4″	1,600
29	525	20	5	6	2′ 6 1/2″	4′ 6 1/4″	1,600
30	600	20	5	6	2′ 6 1/2″	4′ 6 1/4″	1,800
31	700	24	6	6	3′ 0″	5′ 2″	1,900
32	900	24	6	6	3′ 0″	5′ 2″	2,600
33	1,050	30	8	8	3′ 9 1/4″	5′ 6″	3,300
34	1,150	30	8	8	4′ 0″	6′ 0″	4,500
35	1,500	30	8	10	4′ 3″	6′ 8 1/2″	6,000
36	1,900	36	8	10	4′ 3 1/2″	7′ 1 1/4″	8,000
37	2,600	42	10	12	5′ 5″	8′ 0″	9,000
38	3,500	48	12	12	5′ 10″	8′ 8″	9,500
39	5,000	54	14	14	6′ 8 1/2″	9′ 8 1/4″	11,000
40	6,000	60	18	18	7′ 2″	11′ 8″	14,000
41	8,000	60	18	18	7′ 8″	12′ 6″	16,000
42	9,000	66	20	20	8′ 4″	13′ 4″	20,000
43	12,000	72	24	24	9′ 0″	14′ 2″	22,000

Weight

Pump Suction Temp. °F	29-in. Vacuum		28.5-in. Vacuum		28-in. Vacuum		27.5-in. Vacuum		27-in. Vacuum		26.5-in. Vacuum		26-in. Vacuum	
	Lb	Cu Ft	Lb	Cu Ft	Lb	Cu Ft	Lb	Cu Ft	Lb	Cu Ft	Lb	Cu Ft	Lb	Cu Ft
40	0.201	491.8	0.119	291.1	0.085	208.0	0.066	161.8	0.054	132.4	0.046	112.1	0.036	97.1
45	0.268	544.7	0.156	317.7	0.110	223.9	0.085	173.2	0.070	141.2	0.059	119.1	0.051	103.0
50	0.343	603.7	0.192	338.3	0.134	235.0	0.102	180.0	0.083	145.9	0.070	122.7	0.060	105.8
55	0.482	688.8	0.255	365.2	0.174	248.5	0.132	188.3	0.106	151.6	0.089	126.8	0.076	109.0
60	0.680	820.2	0.333	401.0	0.220	265.4	0.164	198.3	0.131	158.3	0.109	131.7	0.094	112.8
65	1.026	1048.0	0.442	451.2	0.282	287.5	0.207	211.0	0.160	163.2	0.135	137.7	0.115	117.3
70	1.764	1531.7	0.606	525.5	0.366	317.1	0.262	227.1	0.204	176.9	0.167	144.8	0.141	122.6
75	...	...	0.873	645.7	0.485	358.8	0.336	248.4	0.257	190.0	0.208	153.8	0.175	129.2
80	...	...	1.376	870.6	0.665	420.9	0.437	277.6	0.327	207.1	0.261	165.1	0.217	137.3
85	...	...	2.646	1437.2	0.963	523.1	0.589	319.8	0.424	230.2	0.331	179.9	0.272	147.6
90	...	...	...	...	1.533	717.5	0.822	384.8	0.562	262.9	0.427	199.7	0.344	161.0
95	...	...	...	...	3.044	1230.6	1.233	498.4	0.773	312.5	0.563	227.6	0.443	179.0
100	...	...	...	...	...	...	2.125	744.2	1.130	395.7	0.770	269.5	0.583	204.4
105	...	...	...	...	...	...	5.432	1653.5	1.848	562.6	1.114	339.0	0.797	242.5
110	...	...	...	...	...	...	...	...	3.921	1040.3	1.775	470.9	1.147	304.3
115	...	...	...	...	...	...	...	...	...	...	3.697	857.3	1.860	431.3
120	...	...	...	...	...	...	...	...	...	...	...	...	3.887	790.0

Table 4. Specific Volume and Partial Pressure of Saturated Air

Pump Suction Temperature, °F	Pressure in psia. Volume in cubic feet													
	29-in. Vacuum. 0.4910 psia Pressure		28.5-in. Vacuum. 0.7365 psia Pressure		28-in. Vacuum. 0.9820 psia Pressure		27.5-in. Vacuum. 1.2275 psia Pressure		27-in. Vacuum. 1.4730 psia Pressure		26.5-in. Vacuum. 1.7185 psia Pressure		26-in. Vacuum. 1.9640 psia Pressure	
	Pressure	Volume	Pressure	Volume	Pressure	Volume	Pressure	Volume	Pressure	Volume	Pressure	Volume	Pressure	Volume
40	.3639	491.8	.6148	291.1	.8603	208.0	1.1058	161.8	1.3513	132.4	1.5968	112.1	1.8423	97.1
45	.3435	544.7	.5890	317.7	.8345	223.9	1.0800	173.2	1.3255	141.2	1.5710	119.1	1.8165	103.0
50	.3130	603.7	.5585	338.3	.8040	235.0	1.0495	180.0	1.2950	145.9	1.5405	122.7	1.7860	105.8
55	.2770	688.8	.5225	365.2	.7680	248.5	1.0135	188.3	1.2590	151.6	1.5045	126.8	1.7500	109.0
60	.2349	820.2	.4804	401.0	.7259	265.4	0.9714	198.3	1.2169	158.3	1.4624	131.7	1.7079	112.8
65	.1856	1048.0	.4311	451.2	.6766	287.5	0.9221	211.0	1.1676	163.2	1.4131	137.7	1.6586	117.3
70	.1282	1531.7	.3737	525.5	.6192	317.1	0.8647	227.1	1.1102	176.9	1.3557	144.8	1.6012	122.6
75	...	...	.3070	645.7	.5525	358.8	0.7980	248.4	1.0435	190.0	1.2890	153.8	1.5345	129.2
80	...	...	.2298	870.6	.4753	420.9	0.7208	277.6	0.9663	207.1	1.2118	165.1	1.4573	137.3
85	...	...	.1405	1437.2	.3860	523.1	0.6315	319.8	0.8770	230.2	1.1225	179.9	1.3680	147.6
90	...	...	...	...	.2840	717.5	0.5295	384.8	0.7750	262.9	1.0205	199.7	1.2660	161.0
95	...	...	...	...	.1671	1230.6	0.4126	498.4	0.6581	312.5	0.9036	227.6	1.1491	179.0
100	...	...	...	...	...	...	0.2788	744.2	0.5243	395.7	0.7698	269.5	1.0153	204.4
105	...	...	...	...	...	...	0.1266	1653.5	0.3721	562.6	0.6176	339.0	0.8631	242.5
110	...	...		...	...	...		...	0.2030	1040.3	0.4485	470.9	0.6940	304.3
115	...	...		...	...	...	...	...	...	...	0.2485	857.3	0.4940	431.3
120	...			...					...	...	...	...	0.2720	790.0

THE EJECTOR CONDENSER operates in a manner similar to a steam ejector. In the eductor condenser, built by Schutte & Koerting Co., condensing water enters through a series of nozzles at a pressure of 5 to 10 psi and condenses the steam. The condensate and steam enter the discharge pipe at high velocity and discharge at atmospheric pressure, air and noncondensable gases being entrained and discharged with the jet. Water capacities of these condensers range from 15 to 2400 gal per min. Steam capacities, at 26-in. vacuum, range from 150 to 24,000 lb per hr. Condensing water is supplied by a circulating pump, no vacuum pump being required.

QUANTITY OF CONDENSING WATER REQUIRED. Let H = Btu per lb in exhaust steam at temperature t_s; h = Btu per lb in condensate at temperature t_c; W = weight of exhaust steam, lb per hr; Q = quantity of condensing water required, lb per hr; t_i = initial temperature of condensing water, °F; t_c = temperature of condensate, °F; t_f = final temperature of condensing water. Heat given up by the steam = $W(H - h)$. Heat absorbed by the condensing water = $Q(t_f - t_i)$, and $Q = W(H - h)/(t_f - t_i)$. Theoretically $t_c = t_s$, but owing to the presence of air and to imperfect mixing t_c should be taken from 10 to 15 F lower than t_s. $t_f = t_c$, whence

$$Q = \frac{W(H - h)}{t_c - t_i} \qquad (1)$$

The value of $(H - h)$ may be taken as 1000 Btu per lb for most purposes.

STEAM DATA of value in condenser calculations are presented in Tables 3 and 4. (See also Section 4.)

2. SURFACE CONDENSERS

Most modern steam plants employ the surface-type condenser rather than other types such as the barometric condenser. The surface-type condenser lends itself well to power plant construction, occupies less space than some of the other types, and may be mounted directly below the turbine. In most modern practice the condenser is independently supported, to avoid putting excessive loads on the turbine exhaust. The condenser may be connected to the turbine either through vacuum-tight flanges, or by direct welding of the condenser neck to the turbine exhaust. When the condenser is separately supported, springs are desirable to take care of the expansions which occur when the parts are hot.

Surface condensers usually are built of heavy rolled plate, with tube sheets into which are inserted a large number of tubes, usually of brass or other similar material. Modern practice dictates the use of tube sizes of $^5/_8$ to 1 $^1/_4$ in., with the intermediate sizes most common. Surface condensers may be either one-pass or two-pass. In the former, water passes through the condenser tubes in one direction only; in the latter the condensing water passes through half the tubes in one direction, is reversed in the water box, and returns in the opposite direction through the other half of the tubes. Occasionally condensers are built of more than two passes, sometimes as many as four. Water velocities in condenser tubes usually are limited to approximately 8 ft per sec. Steam condenses on the outside of the tubes, which are arranged in modern condensers in "lanes" that permit the steam to find its way readily to the bottom of the condenser and improve the vacuum obtainable at the turbine exhaust.

Because the heat transfer coefficient on the outside of the tube is greatly reduced by the presence of noncondensable gases, it is common practice to design the flow path of the steam so that after passing over a *cooler section* it enters a steam-jet air ejector (see p. 9-16) and is removed. The condensate, which drains over the tubes and enters the hotwell at the bottom of the condenser, is removed by a hotwell pump. This provides nearly complete recovery of the valuable pure condensate, so that the latter may be reused in the boiler. Frequently steam or hot drains are introduced into the hotwell, causing the water therein to boil and providing deaeration. This practice is recommended by some condenser builders.

CONDENSING SURFACE. Let S = area of outside surface of condenser tubes, square feet; Q = quantity of condensing water, pounds per hour; T_1 and T_2 = respectively, initial and final temperature of circulating water, °F; U = overall coefficient of heat transfer in Btu per square foot per hour per °F logarithmic mean temperature difference; t_m = logarithmic mean temperature difference between t_s, the steam temperature, and the temperature of the circulating water. Then

$$S = \frac{Q(T_2 - T_1)}{Ut_m} \qquad (1)$$

where

$$t_m = \frac{(T_2 - T_1)}{\log_e \frac{t_s - T_1}{t_s - T_2}} \tag{2}$$

From these basic relations is developed the equation used in predicting or checking operation of surface condensers.

$$T_2 = t_s - \frac{t_s - T_1}{\text{antiln}\left(\frac{SU}{Q}\right)} \tag{3}$$

$$t_s = \frac{T_2 - T_1}{\left[\text{antiln}\left(\frac{SU}{Q}\right)\right] -} + T_2 \tag{4}$$

Values of the heat transfer coefficient, U, are shown by Fig. 6.* These curves, developed from calculations, laboratory experiments, and operating test reports, have been adopted by the Heat Exchange Institute and are generally accepted by engineers and manufacturers. Values given are for commercially clean tubes made of materials shown in the notes, and with circulating water at 70 F.

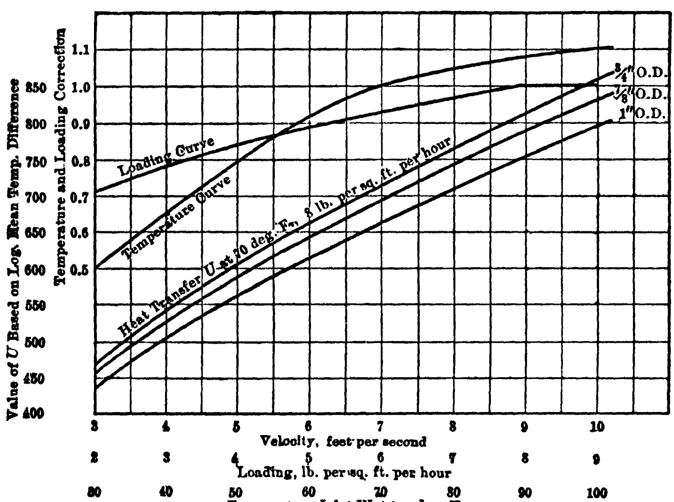

FIG. 6. Curves of values of U for surface condensers (with clean tubes) serving turbines.

Notes: 1. Heat transfer curves based on $C\sqrt{\text{velocity}}$ for velocities 3 to 8 ft per sec, inclusive. They are applicable to tube materials of the following alloys: Muntz Metal, Admiralty, Red Brass, Aluminum Brass, Copper, Arsenical Copper.

2. Loading curve based on $\frac{\sqrt[4]{\text{lb/sq ft/hr}}}{8}$

3. For condensers serving engines use 65% of U values given by Fig. 6.

4. For copper-nickel tubes use 90% of U values.

Figure 6 requires that at least two modifications be applied to curve values of U: (1) the temperature correction, which gives a factor for higher inlet water temperatures—greater than unity at temperatures higher than 70 F; (2) the empirical loading curve factor, which is based on the assumption that with a "loading" of less than 8 lb of steam per hr per sq ft of condenser surface it will be impossible to achieve adequate distribution of steam to all the condensing surface. This factor, less than unity, reduces the transfer coefficient, U.

A third factor must also be applied, depending on tube cleanliness. This factor varies from 75% with badly polluted water to 95% with very clean lake or river water. Many designers find that condenser selection on the basis of an 85% cleanliness factor results in best economics considering first cost and plant operation.

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Values of *U* are shown for three tube sizes used in modern condensers. Tubes are usually No. 18 BWG, but heat transfer values may be considered constant for all gages from 16 through 20.

The velocity of circulating water usually is between 6.5 and 8.0 ft per sec. The lower velocity is used with water carrying erosive material, but lower velocities than 6.5 ft per sec are generally economically unsound because of increased size and cost of equipment. The higher limit is set both by tube erosion and by pumping costs. Curves of pressure drop through tubes and water boxes, shown on Figs. 7 and 8, respectively, have also been adopted by the Heat Exchange Institute and accepted by manufacturers and users.

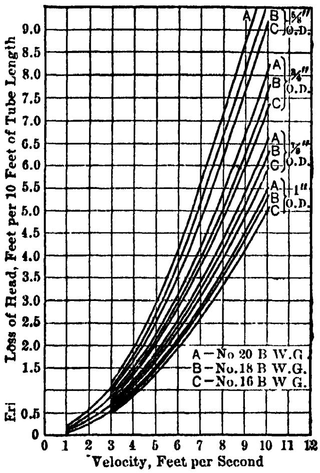


FIG. 7. Loss of head in tubes of surface condensers.

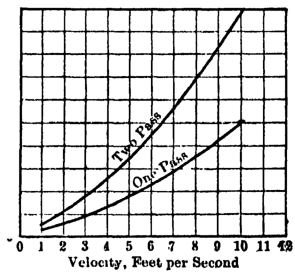


FIG. 8. Water box losses in surface condensers.

Tube diameter must be selected on the basis of thermal and space requirements. Although the smaller tubes have higher heat transfer coefficients, the larger ones carry more circulating water at a given velocity with less pressure drop. In general, $^3/_4$-in. tubes are suitable for units with surface up to about 10,000 sq ft, but larger tubes are advantageous in the larger units.

Tube length may be determined either by space limitations in the power plant (space must be provided for pulling tubes) or by first cost and pumping cost. A longer condenser is usually less expensive but requires more surface. The pumping power may not change materially, since the water quantity will be reduced. Since multipass units have increased length of water travel, they are particularly useful where the water supply is limited. Only exceptional situations require more than two passes, such as when available space limits tube length.

In determining the characteristics of circulating-water flow through a condenser, the following equations are used:

$$V = \frac{KGLN}{S}; \quad \frac{G}{S} = \frac{Q}{500S} = \frac{V}{KLN} \tag{5}$$

where V = water velocity through the tubes, feet per second; G = circulating water flow, gallons per minute; Q = circulating water flow, pounds per hour; L = tube length, feet; S = condenser surface, square feet; N = number of passes; and K = tube constant from Table 5.

Table 5. Condenser Tube Data

Tube, OD, in.	BWG	Tube Wall Thickness, in	K	Surface, sq ft/ft of Length	Tube Outside Cross sectional Area, sq in.
3/4	18	.049	.188	.1963	.442
	17	.058	.199		
	16	.065	.208		
7/8	18	.049	.155	.2291	.6013
	17	.058	.162		
	16	.065	.168		
1	18	.049	.131	.2618	.7854
	17	.058	.137		
	16	.065	.141		

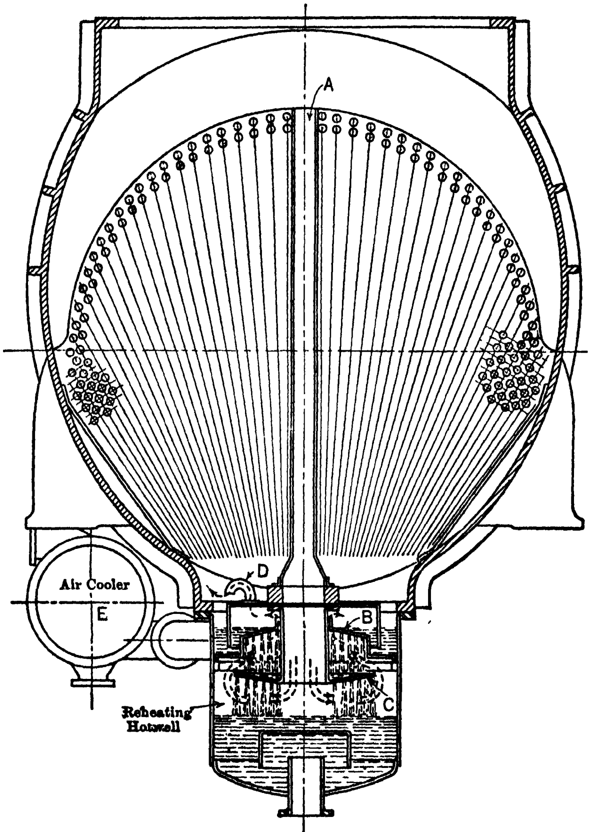


FIG. 9. Cross section of Foster-Wheeler single pass surface condenser with reheating hotwell and external air cooler.

Most condenser manufacturers have adopted combinations of linear and triangular tube spacing to get the desired steam distribution. Thus there is no direct method of calculating the required tube sheet area, but for preliminary studies it may be assumed that the tubes will occupy between 20 and 25% of the cross-sectional area of the condenser.

EXAMPLE. Determine the space required by a condenser to maintain an exhaust pressure of 1.5 in. Hg abs, when receiving 236,000 lb of exhaust steam per hr at an enthalpy of 1009.7 Btu per lb, with 65 F circulating water, and 85% clean tubes. The condenser is to use 7/8 in. OD No. 18 BWG tubes having an effective length of 24 ft arranged for two-pass circulating water flow. Velocity of circulating water in tubes must not exceed 7.0 ft per sec.

Solution.

$$\text{At 1.5 in. Hg abs, } t_s = 91.7 \qquad h = 59.7 \text{ Btu per lb}$$

$$\text{Heat rejection to condenser} = 1009.7 - 59.7 = 950 \text{ Btu per lb steam}$$

$$S \quad \frac{7.0}{0.155 \times 24 \times 2} = 0.94 \qquad \frac{w}{S} = 0.94 \times 500$$

From Fig. 6,

$$U = 695 \times 0.965 \times 0.85 = 570 \text{ Btu per hr per sq ft per °F}$$

From eq. 3,

$$T_2 = 91.7 - \frac{91.7 - 65}{\text{antiln} \dfrac{570}{0.94 \times 500}} = 91.7 - \frac{26.7}{3.36} = 91.7 - 7.95 = 83.75 \text{ F}$$

$$\frac{236{,}000 \times 950}{83.75 - 65} = 11{,}950{,}000 \text{ lb per hr} = 23{,}900 \text{ gal per min}$$

$$\text{Surface} = \frac{23{,}900}{0.94} = 25{,}400 \text{ sq ft}$$

$$\text{No. tubes} = \frac{12 \times 25{,}400}{0.875\pi \times 24} = 4620$$

Assuming the tubes to occupy 23% of the total condenser cross-sectional area,

$$\text{Condenser cross-sectional area} = \frac{4620 \times 0.6013}{0.23} = 12{,}080 \text{ sq in.} = 83.9 \text{ sq ft}$$

This would indicate a cylindrical condenser having approximately 10 ft diameter or alternatively, 10.5 ft wide by 8 ft high.

CONDENSER DESIGN. For all the surface to be useful, some means of controlling longitudinal distribution must be used. Condensers with surface up to approximately 20,000 sq ft can be made successfully in cylindrical shells with a distributing dome over the tube bundle. The tube sheet must allow adequate area in the top of the tube bundle, insuring a full supply of steam into the bottom, as illustrated in Fig. 9.

Figure 9 shows sections of a Foster-Wheeler condenser with a reheating hotwell and external air cooler. The tubes are arranged in straight lines diverging from the bottom. Steam enters at the top into a steam belt surrounding the upper part of the tube bank and flows through the straight lanes of diminishing area to the air offtakes at the bottom. A pipe *A* between the two tube banks conveys a portion of the steam to the hotwell, where it flows through two curtains of condensate falling from perforated plates *B* and *C*, being condensed thereby and raising the temperature of the condensate to within about 1 F of vacuum temperature. Entrained air is vented through pipe *D*. The air offtakes are connected to the external air cooler *E*, the cooling surface of which ranges from 5 to 15% of the cooling surface of the main condenser. The water velocity in the air cooler is less than 1 ft per sec.

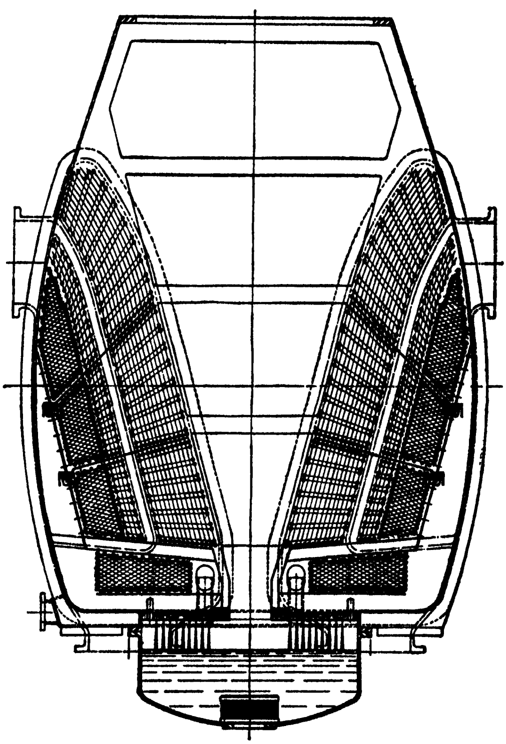


FIG. 10. Condenser with divided tube bundle. (Courtesy of Foster-Wheeler Corp.)

For larger condensers, the spacing between tube rows becomes large, making the condenser shell size excessive relative to the turbine foundation. One solution is illustrated in Fig. 10, where the tube bundle is divided into two parts, reducing the depth of steam travel to half that in a cylindrical condenser. This design eliminates the necessity of a distributing dome and reduces headroom required for installation.

In either design, pressure drop must be introduced in the steam flow path to prevent overpenetration in one section and starving of surface at the ends. In the cylindrical condenser this is accomplished by closely packed tubes in the bottom of the tube bank and restrictive venting into the air-cooler section. In the divided-tube-bank construction closely packed tubes at the end of the steam flow path and orifice plates installed along the outside of the tube banks control the steam flow into any zone of the condenser.

Condensers for both stationary and marine power plants are now normally built with internal air-cooler sections, having about 10% of the total condenser surface.

Hotwells, normally supplied to reheat condensate to the saturation temperature corresponding to exhaust pressure, also serve as deaerators; most manufacturers design deaerating sections to reduce oxygen content to less than 0.03 cc per liter. By addition of trays or other devices located in the steam path it now is possible to produce condensate with 0.01 cc of oxygen per liter.

CONSTRUCTION DETAILS. **Condenser shells** are made of copper-bearing steel plate (see Table 6) ribbed on the outside and designed with internal support plates to resist collapsing from atmospheric pressure. Large condensers which cannot be shipped

Table 6. Materials of Construction for Surface Condensers

Shells, Waterboxes	ASTM 126, Grade A Cast Iron
Shell plates	ASTM A-70, Flange Quality Copper-bearing Steel
Waterbox covers	ASTM A-70, Flange Quality Steel
Tube sheets	Rolled Muntz Metal
	USN 46-B-6, Rolled Naval Brass
Tube support plates	ASTM 126, Grade A Cast Iron
	ASTM A-70, Flange Quality Copper-bearing Steel
Tubes	ASTM B-111 (see Table 7)

assembled may be welded in the field or supplied with flanges at each shell section to permit bolting sections together in the field, with seal welding to prevent air leakage.

The exhaust connection may be either welded to the turbine or flanged for bolting, depending on construction of the turbine exhaust.

Tubes usually are Admiralty metal or arsenical copper (see Table 7) for fresh-water service. The latter is preferable where appreciable quantities of organic pollution are present in the circulating water to cause dezincification of the Admiralty metal. For salt-water service aluminum-brass usually is most satisfactory for stationary installations. Where service is unusually severe one of the copper-nickel alloys may be required. Tube sizes usually are limited to $^3/_4$, $^7/_8$, or 1 in. for stationary installations, although $^5/_8$-in. tubes are commonly used for marine work. Tube walls are between No. 16 and No. 20 BWG, with No. 18 most popular. Selection of tube size depends on the water quantity and allowable pressure drop, although large tubes are sometimes used to prevent clogging by extremely dirty water.

Tube Installation. Tubes are installed in condensers by expanding into tube sheets at both ends in most modern installations. They may be flared at the inlet ends to induce smooth flow at the entrance; as an alternative, they may be extended beyond the tube sheet, with a flow nozzle inserted in each tube. Differential expansion between tubes and shell is normally absorbed by an expansion joint between shell and one tube sheet. Tubes may be installed with an upward bow to provide drainage, or sloped in a straight line over the entire length of the tube. Some manufacturers recommend use of expanded tubes at both ends with an unusually large bow to absorb expansion, although this is not prevailing practice. Few condensers are currently built with ferrules to hold tubes in the tube sheets.

Tube spacing is determined both by steam-flow area requirements through the steam lanes and by the size of ligament required to hold the tube rigidly in place. Tube spacing should provide lanes which will allow an entrance velocity into the tube bank of 100 to 150 ft per sec. Most manufacturers believe that there should be a minimum ligament of $^3/_{16}$ in. along the rows to give adequate strength to the tube sheet and support plates.

Tube sheets usually are made of rolled Muntz or Admiralty metal. Thickness depends on the area of the tube sheet, but usually is between $^7/_8$ and 1 $^1/_2$ in.

Support plates currently are made almost entirely of copper-bearing steel welded to the shell. The thickness is determined by tube diameter and the internal support required to prevent collapse. Good practice makes the support plate thickness equal to the tube diameter. The longitudinal spacing is designed to prevent vibration of tubes at frequencies of near-by rotating equipment, and vibration caused by flow of steam into the tube bundle. One large builder recommends use of Fig. 11 to determine the natural frequency of tubes of various diameters and spans between support plates. From Fig. 11 the spacing to prevent tube vibration can be determined. Tube holes in support plates should be drilled and chamfered to a diameter of $^1/_{32}$ in. larger than the tube diameter.

Water boxes are either fabricated steel or cast iron, depending on circulating water characteristics, and are designed to promote smooth flow at the tube entrance. Inlet nozzles should allow normal flow of water at a velocity of 8 ft per sec. Outlet nozzles, located at the top of the outlet water box, to promote free removal of entrained air, are designed to provide a water velocity of about 5 ft per sec under any operating conditions.

The hotwell condensate outlet should be designed for a maximum velocity of 4 ft per sec, although low net positive suction head on the hotwell pumps may require velocities as low as 2 ft per sec (see Section 5).

Table 7. Chemical Composition of Tube Materials

(Reproduced by permission from ASTM B-111-40-T)

Alloy	Copper,* %	Tin, %	Aluminum, %	Nickel,† %	Lead, max, %	Iron, max, %	Zinc, %	Manganese, max, %	Arsenic, %	Antimony, max, %	Phosphorus, max, %
Muntz metal	59.0 to 63.0	...	...	...	0.3	0.07	Remainder	...	...	...	...
Admiralty metal:											
Type A	70.0 to 73.0	0.90 to 1.20	...	...	0.075	0.06	Remainder	...	...	...	...
Type B	70.0 to 73.0	0.90 to 1.20	...	...	0.075	0.06	Remainder	...	0.10 max	...	...
Type C	70.0 to 73.0	0.90 to 1.20	...	...	0.075	0.06	Remainder	...	...	0.10	...
Type D	70.0 to 73.0	0.90 to 1.20	...	...	0.075	0.06	Remainder	...	...	...	0.10
Red brass	84.0 min	...	...	...	0.075	0.06	Remainder	...	...	...	...
Aluminum brass: ‡											
Type A	76.0 min	...	1.75 to 2.50	...	0.075	0.06	Remainder	...	...	...	...
Type B	76.0 min	...	1.75 to 2.50	...	0.075	0.06	Remainder	...	0.10 max	...	...
Type C	76.0 min	...	1.75 to 2.50	...	0.075	0.06	Remainder	...	...	0.10	...
Type D	76.0 min	...	1.75 to 2.50	...	0.075	0.06	Remainder	...	...	...	0.10
Aluminum bronze	93.5 min	...	5.00 min	...	...	...	...	...	...	...	...
70-30 copper-nickel	Remainder	1.50 max	...	29.0 to 33.0	0.05	0.5	1.0 max	1.0	...	...	...
80-20 copper-nickel:											
Type A	Remainder	1.0 max	...	19.0 to 23.0	0.05	0.5	1.0 max	1.0	...	...	...
Type B	Remainder	1.0 max	...	19.0 to 23.0	0.95	0.5	3.0 to 6.0	1.0	...	...	...
Copper	99.90 min	...	...	...	...	...	...	...	...	...	0.035
Arsenical copper	99.45 min	...	...	...	...	...	...	...	0.15 to 0.50	...	...

* Silver counting as copper.
† Cobalt counting as nickel.
‡ Total other impurities, 0.30% max.
Note. Individual manufacturers of condenser tubes offer modifications of the types of alloys covered by these specifications. Some of these are proprietary alloys, sold under a trade mark.

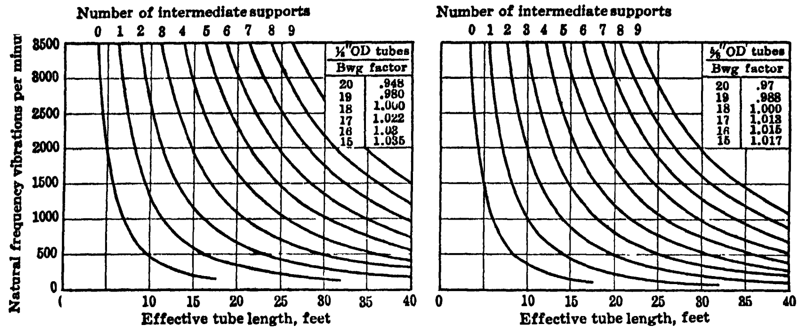


FIG. 11. Natural frequency of condenser tubes.

Since tube drainage is normally provided in the tube bundle the condenser shell is set in a horizontal position. Normal practice is to support the condenser on springs, with adjusting screws to permit the proper downward loading on the turbine exhaust. These springs may be supplied with either screw jacks or hydraulic jacks to relieve the spring loading and prevent overstressing of the turbine exhaust during operation or hydraulic testing. Very high circulating water pressure causes a moment on the turbine exhaust which precludes use of spring supports. In such installations a flexible expansion joint is installed between condenser and turbine. This expansion joint may be made of corrugated copper, rubber reinforced with fabric, or, in some instances, a slip-type joint sealed with condensate.

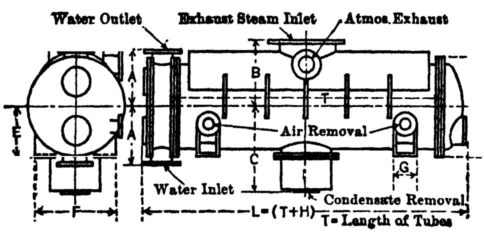


FIG. 12. Two-pass surface condenser (see Table 8).

DIMENSIONS OF CONDENSERS supplied by one manufacturer for small and medium-sized units are given in Fig. 12 and Table 8. The large units, such as are required by central station practice, usually are designed to meet the customer's specification for the particular application.

AIR LEAKAGE. In addition to air entrained in the feedwater, surface condensers contain air due to leakage through joints subject to vacuum. This leakage may range

Table 8. Dimensions of Two-pass Surface Condensers, Steel Plate Shell

(C. H. Wheeler Manufacturing Co., Philadelphia, 1949)

Size	Dimensions, in. (See Fig. 12.)							
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
20	15	18	22	25	11	24	6	13
22	17	20	24	28	12	24	6	15
25	19	21	26	30 1/4	13	24	6	16
28	20	24	27	33 1/2	16	26	6	20
30	21	24	34	36	18	30	8	23
33	23	23	36	39	18	32	8	25
36	24 1/2	30	40	42	20	36	10	28
39	27	29	48	46	21	36	10	28
42	28	33	54	49	22	42	10	30
45	31	34	54	52	24	42	12	31
48	33	36	54	55 1/2	26	48	12	35
54	36	40	58	62	28	50	12	38
60	40	48	66	68	32	54	15	42
66	42	48	64	74	35	60	18	46
72	45	54	69	80	38	66	18	50
76	48	58	73	84	40	66	18	53
80	48	60	75	88	42	72	18	58
86	54	66	83	96	44	84	18	60

from 1 to 25% of the volume of the feedwater, but average leakage is best estimated by consideration of the leakage quantities established as standard by the Heat Exchange Institute for use in design of steam-jet air ejectors. These standards recommend that the capacity of the air-removal equipment provided shall be not less than the values given in Table 9. The values in Table 9 are greater than the maximum leakage expected in a normally tight system and will provide a safe basis for design.

Table 9. Air Leakage Used for Design

(Reprinted from Standards of Heat Exchange Institute, Condenser Section, Copyright 1939, New York)

Maximum Pounds of Steam Condensed per Hour	Free Dry-air Leakage, 70 F, cu ft/min Surface Condensers Serving Turbines *
5,000 or less	2.20
5,001 to 10,000	2.50
10,001 to 15,000	2.65
15,001 to 20,000	2.80
20,001 to 25,000	3.00
25,001 to 30,000	3.20
30,001 to 35,000	3.35
35,001 to 40,000	3.50
40,001 to 45,000	3.65
45,001 to 50,000	3.80
50,001 to 75,000	4.50
75,001 to 100,000	5.00
100,001 to 150,000	6.50
150,001 to 250,000	8.50
250,001 to 350,000	10.00
350,001 to 450,000	11.50
450,001 to 600,000	13.50
600,001 and up	16.00

* Double the tabular values for condensers serving engines.

Effect of Leakage on Vacuum. G. A. Orrok has shown the effect of leakage on the vacuum of three units as follows.

Leakage, cu ft of free air per min	10	20	30	40	50
Size of unit			Vacuum, in. Hg		
8700 kw	28.25	27.85	27.4	27.0	26.6
8500 kw	28.4	28.1	27.8	27.5	27.2
4000 kw	28.7	28.35	28.0	27.65	....

CLEANLINESS FACTOR IN CONDENSER PERFORMANCE. Because deposits on the surface of condenser tubes greatly affect the performance of the condenser, a determination of the degree of cleanliness of the condenser is essential in evaluating its performance under given conditions. This is particularly important in tests to determine conformity with guarantees. Hardie and Cooper define cleanliness factor as the ratio of heat transmission of a fouled tube to that of a new tube through which water has passed only during the time necessary to start the test, the tubes being supplied with water at the same inlet temperature and subjected to the same conditions on the steam side. (See Section 19, Art. 6.)

ATMOSPHERIC RELIEF VALVES. Because of their large size neither condenser shells nor turbine exhaust is designed to withstand appreciable pressures. For this reason it is necessary to provide insurance that pressures exceeding approximately 5 psig do not occur within these shells. It has long been common practice to provide an atmospheric relief valve arranged to vent the condenser shell to atmosphere in the event pressures begin to build up due to malfunction of some part of the associated apparatus. Sizes of this type of relief valve have been standardized by the Heat Exchange Institute as shown in Table 10.

Under certain conditions some plants are required to operate turbines for short periods at atmospheric back pressure, exhausting the turbines to atmosphere, although this practice is discouraged, and sometimes forbidden by turbine manufacturers. This sometimes happens during flood conditions, or when auxiliary power for operation of station equipment is not available. The first column in Table 10 illustrates the size of valve required for normal service, that of venting the condenser in an emergency. The second column

Table 10. Atmospheric Relief Valve Sizes

(Reprinted from Standards of Heat Exchange Institute, Condenser Section, Copyright 1939, New York)

Pounds Steam per Hour	Size of Valve, in.	
	For Protection	For Maximum Noncondensing Operation
Up to 7,500	6	8
7,501 to 11,800	8	10
11,801 to 17,000	8	12
17,001 to 20,000	8	14
20,001 to 23,100	10	14
23,101 to 30,200	10	16
30,201 to 38,200	12	18
38,201 to 45,000	12	20
45,001 to 47,200	14	20
47,201 to 62,000	14	24
62,001 to 68,000	16	24
68,001 to 82,000	16	30
82,001 to 106,000	18	30
106,001 to 120,000	18	
120,001 to 170,000	20	
170,001 to 250,000	24	
250,001 to 380,000	30	
380,001 to 550,000	36	

is the size of valve required to operate the turbine for short periods at its maximum noncondensing capacity.

Blow-out Diaphragms. Because atmospheric relief valves and the associated piping are large and costly, particularly in the larger sizes, turbine manufacturers have developed a device which serves the same purpose, at lower cost. This arrangement, usually installed in the exhaust hood of the turbine and arranged for automatic operation, consists of a thin metallic diaphragm mounted on a circular knife edge of the same diameter as the diaphragm. The entire assembly is enclosed within a safety cage to prevent injury to personnel in the event of rupture of the diaphragm. When pressure rises to approximately 5 psig the diaphragm ruptures, and relieves all the steam flow to atmosphere.

Since it would be highly undesirable to have the blow-out diaphragm discharge large quantities of steam into the station, even at infrequent intervals, manufacturers have also developed a vacuum trip device which is installed in the governing mechanism of the turbine. This equipment is arranged to reduce or to shut off steam flow to the turbine in the event the pressure in the condenser rises above some predetermined (and adjustable) level. Usually this level is in the neighborhood of 10 in. Hg vacuum. This device is arranged so that it may be locked out of service under certain conditions, such as for starting with poor vacuum. Use of the turbine blow-out diaphragm has superseded to a considerable extent the use of atmospheric relief valves, particularly in large central station

3. AIR-REMOVAL EQUIPMENT

Removal of air from a steam condenser may be accomplished either by a steam-jet air ejector or by a mechanical pump. Although the steam jet is used in most new steam power plant installations, occasionally certain conditions make a mechanical pump desirable.

Steam-jet ejectors may be either *single stage* or *two stage*. Single-stage units are applicable to maximum vacua of 26.5 in. Hg. Two-stage units are used for 26.5 to 29.5 in. Hg. Since design conditions for a steam condenser in power plants rarely exceed 29.5 in. Hg there is practically no application for the three-stage unit. For many industrial applications where extremely high vacua are required, steam-jet ejectors are used with three or four stages; in a few installations requiring back pressures as low as 100 microns, five-stage units have been used.

In steam power plants the single-stage ejector is used as the main ejector (within the vacuum limitations stated) either with or without an *after condenser*, although an after condenser appreciably improves the economy. The single-stage noncondensing unit is normally used for rapid evacuation of large quanties of air, to prime the circulating water system or to establish a vacuum in the condenser shell and turbine casing before starting the turbine. Figure 13 shows a single-stage noncondensing ejector, with single steam nozzle, and Fig. 14 illustrates the multiple steam nozzle design.

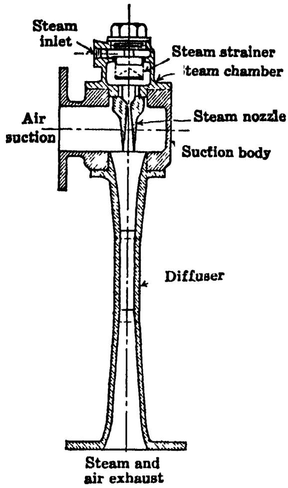


FIG. 13. Single-stage ejector with single steam nozzle.

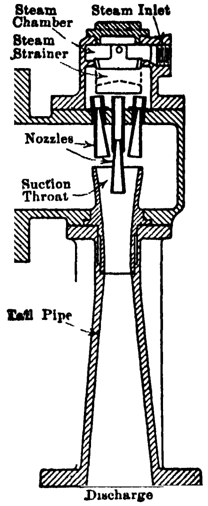


FIG. 14. Multiple-nozzle ejector.

Multistage ejectors usually have an intercondenser between stages and an aftercondenser after the final stage. Condensate is used as cooling water. All the heat in the steam jet thus is recovered, when the inter- and aftercondensers are of the surface type. Figure 15

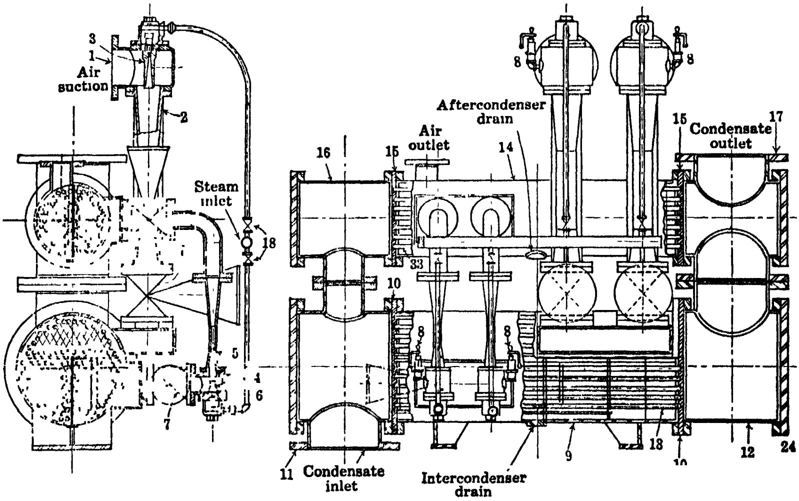


FIG. 15. Two-stage steam-jet air ejector, twin element type with separate inter- and aftercondensers and isolating valves (single-pass condensate design). (Courtesy of Foster-Wheeler Corp.)

1. Primary suction body
2. Primary diffuser
3. Primary steam nozzle
4. Secondary suction body
5. Secondary diffuser
6. Secondary steam nozzle
7. Isolating gate valves
8. Relief valves
9. Intercondenser shell
10. Intercondenser tube sheets
11. Intercondenser inlet water box
12. Intercondenser outlet water box
13. Intercondenser tubes
14. Aftercondenser shell
15. Aftercondenser tube sheets
16. Aftercondenser inlet water box
17. Aftercondenser outlet water box
18. Steam valves

shows a two-stage steam-jet air pump with surface inter- and aftercondensers in separate shells. Figure 16 shows a two-stage ejector with jet intercondenser.

CAPACITY OF STEAM-JET EJECTORS depends on the area of jet in contact with the air. A pump with multiple nozzles presents a much greater steam-jet surface than a single nozzle passing the same amount of steam. A single-stage pump operates economically within a compression ratio of not more than 8 to 1. Beyond this range, high vacuum can be obtained only at the cost of excessive steam consumption, and a two- or three-stage pump is indicated. Figure 17 shows the relative capacity of a two-stage and a three-stage Foster-Wheeler steam-jet ejector using the same amount of steam.

Since the usual central-station condenser is designed to operate at vacua higher than 26.5 in. Hg, the two-stage ejector is used almost exclusively for such service. In the interest of economy, these units are supplied with inter- and aftercondensers, arranged to

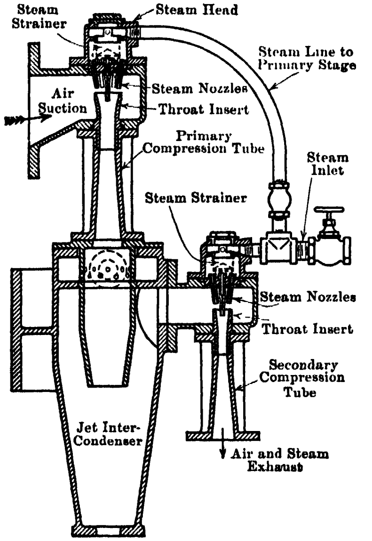


FIG. 16. Two-stage steam jet air pump with jet intercondenser.

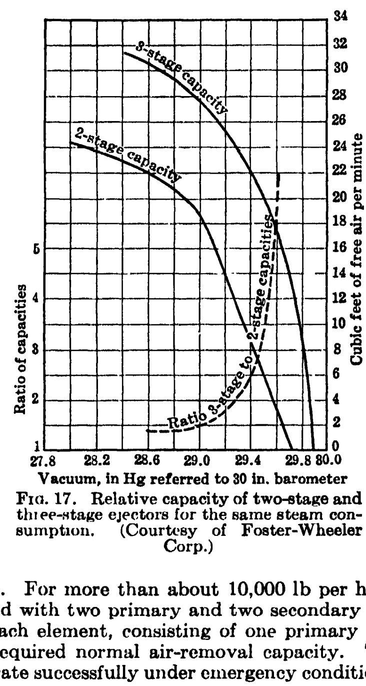


FIG. 17. Relative capacity of two-stage and three-stage ejectors for the same steam consumption. (Courtesy of Foster-Wheeler Corp.)

return the condensate to the main condenser. For more than about 10,000 lb per hr of steam the two-stage ejector usually is supplied with two primary and two secondary jets mounted on the inter- and aftercondenser; each element, consisting of one primary and one secondary jet, is designed to have the required normal air-removal capacity. The inter- and aftercondensers are designed to operate successfully under emergency conditions, using all four jets.

The steam consumption of two-stage ejectors is approximately 13.3 lb per hr of steam per pound of dry air, at 29 in. Hg vacuum and with a mixture temperature 7.5 F lower than saturation temperature of the steam entering the main condenser. Under these conditions, the mixture of air and vapor in the condenser comprises about 2.2 lb of vapor per 1 lb of dry air. The air to be removed is that which leaks into the system. The quantity may cover a wide range, depending on the construction, tightness of joints, and character of maintenance. Figure 18 gives data to be assumed for surface condensers, based on 29 in. vacuum and 7.5 F temperature depression. In general, the actual leakage in well-maintained systems will be lower than the figures given.

For conditions of vacuum and temperature other than those given, the necessary pump capacity can be estimated as follows. Let V_a = volume atmospheric air leaking into condenser, per hour, cubic feet; V_c = volume of air to be removed from condenser, per hour, cubic feet; v_a = specific volume of air at temperature t_a, cubic feet per pound; v_s = specific volume of steam at temperature t_m, cubic feet per pound; t_a = temperature of atmosphere, °F; t_m = temperature of mixture of air and vapor in condenser at pump suction, °F; p_a = pressure of atmosphere; p_c = total pressure in condenser; p_s = pressure of vapor in condenser; all pressures in inches of mercury; W_a = weight of air to be removed

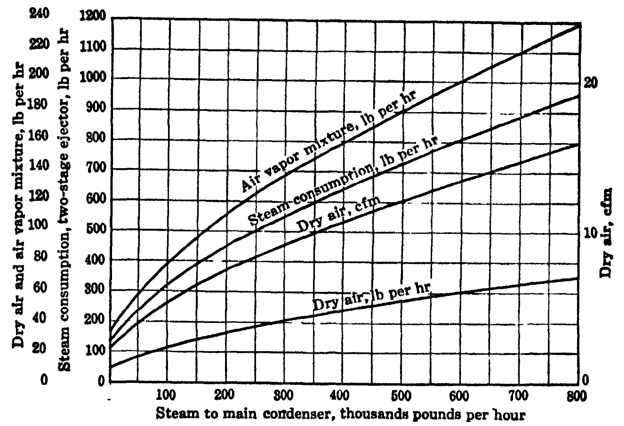


FIG. 18. Approximate data for design of air-removal systems for surface condensers. Curves are based on 29 in. Hg vacuum and 7.5 F temperature depression.

per hour, pounds; W_s = weight of vapor to be removed per hour, pounds; W_m = weight of mixture of vapor and air to be removed per hour, pound. Then

$$V_c = \frac{V_a p_a (t_m + 460)}{(p_c - p_s)(t_a + 460)} \tag{6}$$

$$W_a = \frac{V_a}{v_a} \tag{7}$$

$$W_s = \frac{V_c}{v_s} \tag{8}$$

$$W_m = W_a + W_s \tag{9}$$

Capacity of air pump for jet condensers may be estimated for preliminary calculations by assuming double the values given in Fig. 18. If the actual volume of air in the injection water is known, this figure should be used instead of the value from Fig. 18.

EXAMPLE. Required: The capacity of pump to remove air from a surface condenser condensing 250,000 lb of steam per hr. Temperature of atmosphere, 65 F; barometer, 29.6 in.; vacuum 28.5 in. (referred to 30 in. barometer); temperature of condensate, 87 F.

Solution. From Fig. 18, W_a = 38.0; the specific volume v_a of air at 65 F is 13.22, whence V_a = 13.22 × 38.0 = 502.5 cu ft. p_a = 29.6; p_c = 1.5; t_m = 87; t_a = 65. Interpolating in the temperature table for saturated steam (p. 4-34) for 87 F, the partial pressure of the steam is 1.293 in. Hg, or $(p_c - p_s)$ = 0.207.

Then from eq. 6,

$$V_c = \frac{502.5 \times 29.6 \times (87 + 460)}{0.207 \times (65 + 460)} = 74{,}800 \text{ cu ft/hr}$$

One lb of saturated air at 28.5 in. vacuum and at a temperature of 87.0 F will have a volume of 1990 cu ft and contain 3.88 lb of vapor. The weight of vapor to be removed, therefore, will be

$$W_s = \frac{74{,}800}{1990} \times 3.88 = 145.8 \text{ lb}$$

$$W_m = 38.0 + 145.8 = 183.8 \text{ lb/hr}$$

MECHANICAL AIR-REMOVAL EQUIPMENT may be desirable when it is preferred not to bring high-pressure steam into the plant for operation of auxiliary equipment. In some plants it is desirable to start the ejector equipment before boiler steam pressure is high enough to operate the steam jet properly. Since mechanical air pumps are normally constant-speed constant-volume flow units, they have a relatively large capacity at low vacuum and may therefore eliminate need for a special priming jet. This is an advantage, since only one piece of equipment may be used during the starting period of the turbine.

The disadvantage of the mechanical air pump is that, since it is a constant-speed constant-volume flow unit, the free-air capacity is low at high vacua, requiring a multiplicity of mechanical pumps to take care of actual needs with little margin for emergency conditions.

COOLING EQUIPMENT

By James G. DeFlon

4. COOLING-TOWER PRINCIPLES

In most industrial processes involving heating and cooling, a dependable and economical source of cold water is essential. One of the simplest means of assuring such a supply is a cooling tower. The selection of a tower of optimum size is important. Too small a tower is a continuous liability and will limit plant production. On the other hand, too large a tower, though it does the required cooling, will be uneconomical to purchase and operate. Between these two extremes lies the economical choice. In choosing a tower, the engineer must consider: (1) the type; (2) the size; (3) the design; (4) the performance; and (5) the cost.

PRINCIPLES OF EVAPORATIVE COOLING. The process of cooling water in a cooling tower is simple in principle. The wet-bulb temperature represents the minimum

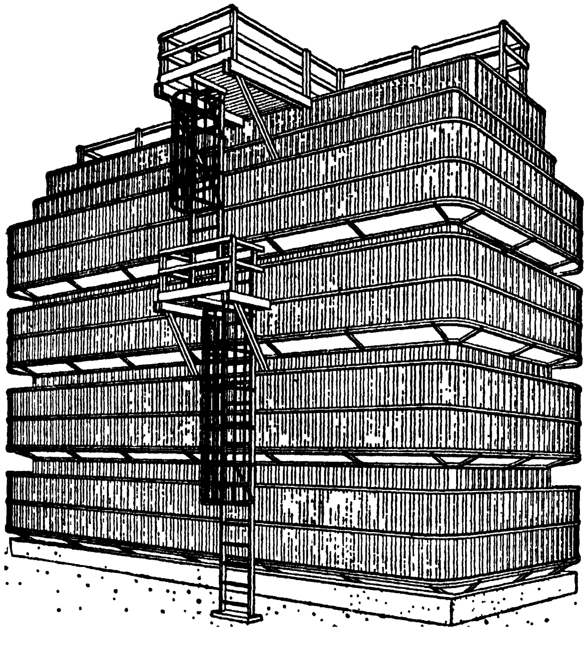


FIG. 1. Atmospheric cooling tower. (Courtesy of The Fluor Corp., Ltd.)

temperature to which it is possible to cool water by evaporation. This temperature varies with the time of day, the weather, and the season. In a cooling tower, finely divided water falls through an air stream whose wet-bulb temperature is below that of the water. The evaporation of a small portion of the water causes the air to absorb heat from the remaining water, due to the latent heat of the evaporating water. The evaporation of 1 lb of water absorbs about 1000 Btu. Thus, the evaporation of 1 lb of water cools 100 lb of water through a 10-degree cooling range. In addition, a small amount of sensible heat is transferred because of the temperature differential between the water and air (dry-bulb temperature). This may cause either a positive or a negative flow of heat from the water to the air, depending on the temperatures involved. The performance of a cooling tower is *independent of the dry-bulb temperature* of the air and dependent only on the *wet-bulb temperature* of the air in contact with the water. This means that the performance of a cooling tower is the same for a given wet-bulb temperature, regardless of the dry-bulb temperature, but the higher the relative humidity of the entering air, the more cooling has to be done by sensible-heat transfer and the less by latent-heat transfer. This is because the air's capacity for absorbing heat at a given wet-bulb temperature is always equal to the sum of the latent heat and the sensible heat.

In a commercial tower it is impossible for the wet-bulb temperature of the exhaust air to be equal to that of the hot water. Furthermore, it is impossible for the cold water to be reduced to the wet-bulb temperature of the incoming air. However, this latter condition can be approached in very high counterflow induced-draft cooling towers, where the coldest water meets the coldest air and the hottest air contacts the hottest water.

Types. Two distinct types of cooling towers are on the market today: (1) those dependent exclusively on *outside air movement*, like the atmospheric-type cooling tower, whether it contains decks or not; and (2) those assured of a constant air supply by means of power-driven fans, such as the *forced-draft type*, in which air is forced into the tower at the bottom; and the *induced-draft type*, in which the air is drawn out at the top. These are illustrated

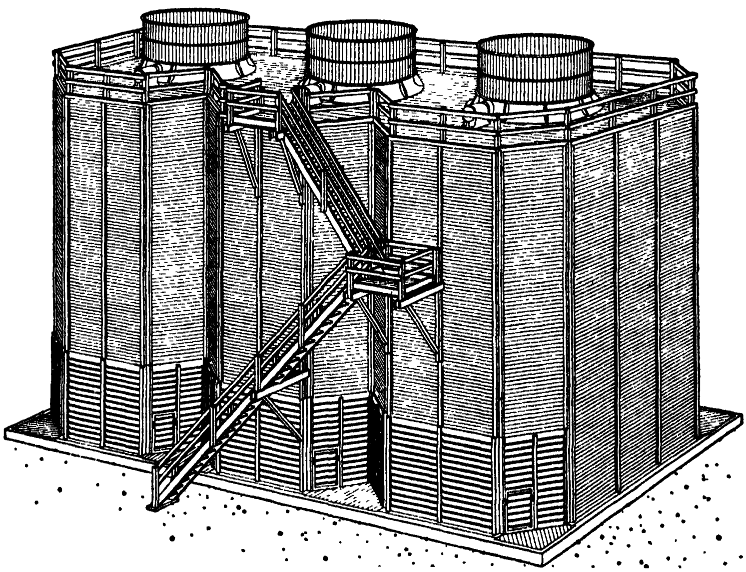


FIG. 2. Induced draft cooling tower. (Courtesy of The Fluor Corp., Ltd.)

in Figs. 1 and 2. The older chimney-type tower is now considered obsolete because of high initial cost and the limited performance available during hot weather.

Both atmospheric and mechanical-draft towers have their merits. The type chosen is determined by the climate, the industrial process involved, and the space and location available.

ATMOSPHERIC TOWERS. The commonest atmospheric tower consists of a series of decks over which water cascades and through which wind circulates. This *deck-type* tower operates satisfactorily through the same wide range of temperatures as the mechanical-draft tower and can be used in any operation where absolute temperature control is not essential for plant operation. A cross section of such a tower is shown in Fig. 3. This type of tower has these **advantages.**

1. It is by far the most dependable, since there are no mechanical parts to fail. When necessary, it can be designed to operate with zero wind; although it is not entirely dependent on wind for performance, wind will appreciably improve its performance.
2. It requires practically no expenditure for maintenance.
3. It has a long, trouble-free life.

4. It is not subject to recirculation of humid air, since the air enters the tower only from the windward side.
5. The average cold-water temperature is lower than for a mechanical-draft tower designed for the same conditions, since the design wind velocity is always less than the average wind velocity.

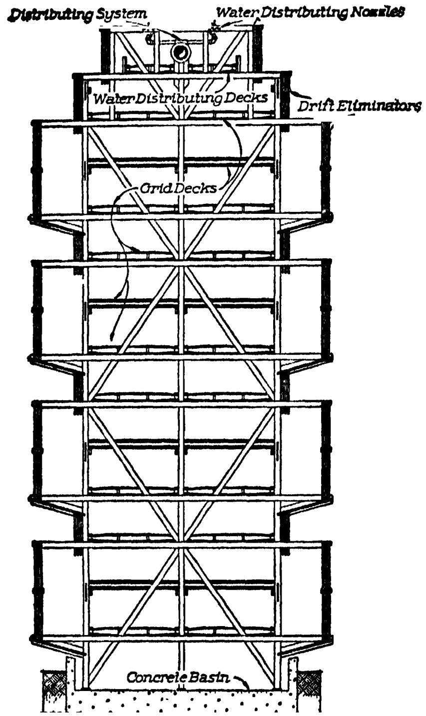


FIG. 3. Cross section of deck-type atmospheric cooling tower. (Courtesy of The Fluor Corp., Ltd.)

Some **disadvantages** of this type of tower are these.

1. A great length of tower is required for large installations.
2. It must be located in an exposed area.
3. The cold-water temperature fluctuates with change in wind intensity.
4. It usually requires a greater pumping head.

The former disadvantage of spray loss has been corrected in new atmospheric towers by addition of spray eliminators, so that certain atmospheric towers now compare very favorably with the best mechanical-draft towers in this respect. The atmospheric tower is usually comparable in price to a mechanical-draft tower, when its design is based on a 3-mile wind.

A somewhat different principle is employed in the spray cooler. In this type, there are no decks. The water is jetted downward from nozzles arranged in the top of the tower in such a way that a draft of air is induced; thus it is not dependent on outside air currents for its air supply. Because of the high air velocity at the bottom of the spray cooler created by the jetting action of the water spray itself, this type is subject to a high water loss. Some manufacturers build a spray cooler with a drift eliminator at this point to reduce spray loss.

Only limited performance is available with a spray cooler. The approach to the wet-bulb temperature is always equal to or greater than the cooling range. Its use is limited, therefore, primarily to small refrigeration and engine jacket-water cooling installations. It is particularly suited to such installations, because of its inherent trouble-free operation.

MECHANICAL-DRAFT TOWERS. The mechanical-draft tower creates its own air current by means of a motor-driven fan. It is of particular value in any operation in which a *close control* of the cold-water temperature is desired.

Advantages of this type of tower are:

1. Close control of the cold-water temperature.
2. Small space requirements.
3. The usually lower water-pumping head.

Disadvantages of a tower of this type are:

1. Operation of the fans requires a considerable expenditure of horsepower.
2. Subject to mechanical failures.
3. Subject to recirculation of hot, humid exhaust air vapors into the air intakes, thus increasing the average wet-bulb temperature of the entering air.
4. It has a higher maintenance cost than an atmospheric tower.

The principal difference in the two types of mechanical-draft towers lies in the location of the fan and mechanical assembly.

The induced-draft tower has the fans mounted on top of the tower to draw the air out. This location in the hot, humid exhaust air stream increases maintenance charges on the fan assembly and gear box. In spite of all precautions, the induced-draft gear box will breathe, causing water to condense in the oil, and necessitating frequent oil changes to prevent undue wear of gears and bearings. It is difficult to anchor the fan stack rigidly enough to prevent vibration. Vibration of the fan stack, however, usually has little or no effect on the life of the rotary mechanical assembly. A cross-sectional view of an induced-draft cooling tower is shown in Fig. 4.

The forced-draft tower has the fan, gear, and motor mounted on the lower side of the tower, where the air is blown in, permitting a more firm anchorage, thus reducing vibration.

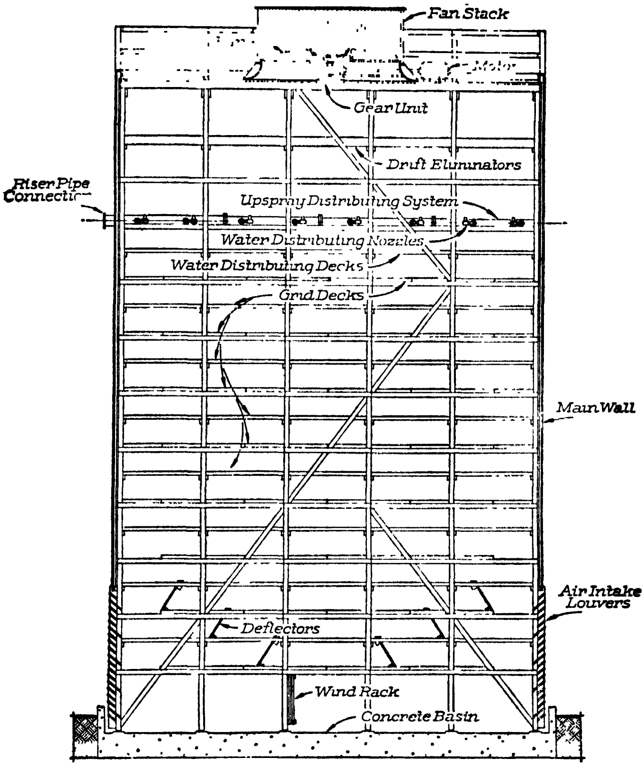


FIG. 4. Cross section of induced-draft cooling tower. (Courtesy of The Fluor Corp., Ltd.)

Location of the mechanical assembly in a comparatively dry air stream reduces moisture condensation in the gear box. The forced draft fan is more efficient than the induced draft by about 3%, because some of the fan velocity pressure is converted to static pressure in the tower. The forced-draft type is rapidly losing favor, because it is more likely to have recirculation of hot, humid exhaust vapors back into the air intake than the induced-draft type. Exhaust vapors leave the forced-draft tower at such a low velocity that a cross wind causes them to be drawn back into the fan suction on the leeward side, thus artificially raising the wet-bulb temperature of the entering air, and raising cold-water temperature. In induced-draft towers, the exhaust air leaves at a comparatively high velocity, and the tendency to recirculate is reduced. Induced-draft towers are always used when it is desirable to maintain the *lowest possible water temperature.*

SIZE OF TOWER. Before the size of a tower can be determined, these figures must be obtained: the **design wet-bulb temperature,* the approach,† the cooling range,‡ and the quantity of water to be cooled.**

* Wet-bulb temperature or adiabatic saturation temperature is that temperature at which the air would normally saturate without any change in its heat content. Design wet-bulb temperature is the accepted wet-bulb temperature for which the tower is to be designed.

† Approach is defined as the difference between the cold-water temperature and the wet-bulb temperature.

‡ Cooling range is the difference between the hot-water and cold-water temperatures.

Table 1. Meteorological Conditions throughout North America

Thirty-year average based on records of U. S. Weather Bureau for July

(Copyright by Cooling Tower Co., New York)

State	City	Dry Bulb, °F	Wet Bulb, °F	Relative Humidity, %	Wind, mph	State	City	Dry Bulb, °F	Wet Bulb, °F	Relative Humidity, %	Wind, mph
Alabama	Mobile	80.5	73.5	79.0	6.0	Montana	Helena	66.9	53.4	43.0	. . .
	Birmingham	79.8	73.3	73.0	5.0	Nebraska	Omaha	76.5	66.5	59.0	7.0
Arizona	Yuma	91.0	71.0	36.0	.....	New Jersey	Atlantic City	72.5	69.0	84.0	8.0
	Phoenix	90.5	68.5	31.0		New York	Albany	72.0	65.5	71.0	7.7
Arkansas	Little Rock	80.0	73.0	72.0	5.0		Buffalo	70.2	64.0	71.0	10.9
	Ft. Smith	80.5	72.5	68.0	5.0		New York	73.5	67.0	71.0	9.1
California	San Francisco	57.3	54.3	82.0	14.0		Rochester	70.4	63.2	67.0	7.1
	Sacramento	72.4	60.6	51.0	9.7	N. Carolina	Charlotte	77.8	70.3	69.0	5.0
	Los Angeles	67.4	61.6	72.0	4.5		Wilmington	78.7	73.7	79.0	.....
Colorado	Denver	71.8	57.8	45.0	7.5	N. Dakota	Bismarck	70.2	61.2	61.0	9.0
	Pueblo	72.6	58.0	44.0	7.0	New Mex.	Santa Fe	68.7	55.2	46.0	6.1
Conn.	New Haven	71.9	65.9	73.0	8.0	Ohio	Cincinnati	77.7	68.7	63.0	6.6
Dist. Col.	Washington	76.8	69.3	69.0	5.3		Cleveland	72.5	65.8	70.0	11.7
Florida	Jacksonville	80.9	75.0	75.0	8.0		Columbus	75.0	66.5	64.0	8.7
	Tampa	79.9	75.0	79.0	6.0	Oklahoma	Okla. City	79.0	70.0	64.0	9.0
Georgia	Atlanta	77.6	69.6	67.0	8.6	Oregon	Portland	66.3	58.0	60.0	7.9
	Savannah	80.5	75.5	79.0	6.4	Pa.	Erie	71.8	64.8	69.0	9.0
Illinois	Cairo	78.6	72.4	75.0	6.2		Philadelphia	75.8	67.8	66.0	9.4
	Chicago	72.3	65.8	71.0	15.1		Pittsburgh	74.6	67.1	68.0	5.2
	Springfield	76.1	68.0	64.0	6.6	S. Carolina	Charleston	81.3	75.3	76.0	9.8
Indiana	Indianapolis	76.4	68.0	65.0	8.2	S. Dakota	Yankton	74.6	66.8	66.0	6.5
Iowa	Davenport	75.4	67.2	65.0	7.4	Tennessee	Chattanooga	77.8	70.8	71.0	5.2
	Des Moines	75.0	67.0	66.0	7.1		Memphis	80.7	72.7	68.0	7.4
Kansas	Wichita	78.3	69.5	65.0	. . .	Texas	Galveston	83.0	77.0	76.0	10.0
Kentucky	Louisville	78.6	69.6	64.0	6.1		Ft. Worth	82.5	69.5	51.0	10.0
Louisiana	New Orleans	81.3	75.3	76.0	6.5	Utah	Salt Lake City	76.2	58.2	34.0	6.3
	Shreveport	82.1	75.0	72.0	5.0	Vermont	Burlington	68.2	63.2	76.0	......
Maine	Eastport	59.8	56.3	81.0		Virginia	Norfolk	78.4	72.6	74.0	....
	Portland	68.0	62.0	71.0			Richmond	79.2	72.0	70.0	....
Maryland	Baltimore	77.3	69.6	70.0	6.6	Wash.	Seattle	63.3	56.3	64.0	5.5
Mass.	Boston	71.3	64.8	70.0	9.3		Spokane	68.8	55.0	41.0	.
Michigan	Detroit	72.1	65.1	69.0	9.0	W. Virginia	Parkersburg	74.9	67.5	68.0	4.0
	Grand Rapids	72.6	65.4	68.0	9.0	Wisconsin	Milwaukee	69.7	64.7	77.0	9.8
Mississippi	Vicksburg	80.4	74.4	75.0	6.0	Wyoming	Cheyenne	67.4	54.6	46.0	8.0
Minnesota	St. Paul	72.1	64.5	67.0	7.2	Canada	Montreal	69.5	65.0	79.0	11.3
	Duluth	66.0	60.0	71.0	11.0		Toronto	68.5	62.3	70.5	7.9
Missouri	Kansas City	76.9	68.9	67.0	7.5		Winnipeg	66.0	61.0	77.5	11.3
	St. Louis	79.1	70.6	66.0	8.2						

Design Wet-bulb Temperature and Wind Velocity. The design wet-bulb temperature should be chosen high enough to include 95% of the maximum wet-bulb temperatures incurred during the three summer months.

Table 1 gives average air temperatures, humidity, and air velocities for July throughout the United States.* The wet-bulb temperature of Table 1 should be increased by from 5 to 7 F to cover 95% of the operating period during the summer months. The wind velocity shown should be reduced about 1 to 2 mph for design, if below 5 mph, and under no circumstances should a design wind velocity of more than 5 mph be selected, because design performance will not be obtained when the wind velocity drops to 2 or 3 mph.

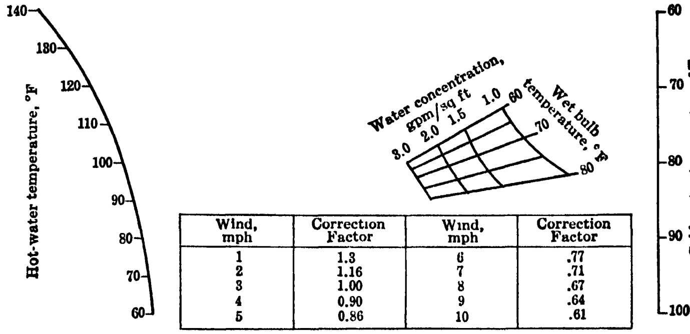
In selecting the design wet-bulb temperature, a correction must be made for mechanical-draft towers which recirculate, thereby increasing the wet-bulb temperature of the incoming air. Since all cooling towers, unless otherwise specified, are rated on the wet-bulb temperature of the entering air, these corrections are made for mechanical-draft towers: (1) Forced draft, add 2 to 4 F. (2) Induced draft, add 0 to 2 F.

No wet-bulb temperature correction is necessary for an atmospheric tower.

The recirculation of an induced-draft tower is primarily a function of the length of the

* Local summer atmospheric information can be obtained from local weather bureaus or from cooling-tower manufacturers.

tower and the vertical distance between the top of the air intake louvers and the top of the fan stack. Recirculation is of little consequence on an induced-draft tower of two or three cells, but it may become a serious problem on a tower of 15 or 20 cells, because a turbulence is created on the leeward side, causing the upper air to mix with the ground air. Air intakes on the leeward side of the center cells must draw their air from this mixture of ground air and exhaust air. The greater the height of the fan stack, the less the recirculation; in fact little or none would be expected from a tower of 5 or 6 cells, having 25 to 30 ft vertical distance between the top air intake louver and the fan deck and a fan stack about 16 ft high.



Wind, mph	Correction Factor	Wind, mph	Correction Factor
1	1.3	6	.77
2	1.16	7	.71
3	1.00	8	.67
4	0.90	9	.64
5	0.86	10	.61

FIG. 5. Performance curve for 12-deck atmospheric tower, 35 ft high, and designed for a wind velocity of 3 mph. (Courtesy of The Fluor Corp., Ltd.)

Note: To find required area of cooling tower, place straightedge on a line running through hot-water temperature and cold-water temperature. Read water concentration where straightedge crosses design wet-bulb temperature curve. Divide gallons per minute of water circulation by water concentration thus obtained to find area required. To find length of cooling tower, divide area by 12 (tower width). If wind velocity is other than 3 mph, multiply tower area by wind factor shown above. Use for preliminary estimates only. Do not extrapolate beyond limits of curve.

If a mechanical-draft tower is selected, its location must be considered. If the tower is located in a confined area, the wet-bulb temperature must be further corrected, for, if the vapors exhausting from a cooling tower are not carried away by the wind, the air in the surrounding area tends to saturate, and the average wet-bulb temperature of air entering the tower is increased. Then, too, if the temperature of the exhaust air is above that of the atmosphere, condensation takes place and a light precipitation occurs in the general area of the tower, increasing the average wet-bulb temperature of the air.

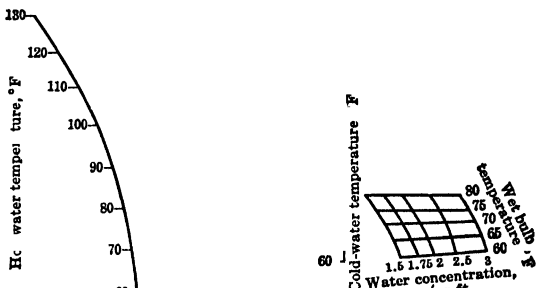


FIG. 6. Mechanical-draft cooling-tower sizing chart. (Courtesy of The Fluor Corp., Ltd.)

Note: To obtain the required water concentration in gallons per minute per square foot of tower area, draw a line through the hot-water temperature and the desired cold-water temperature, and read at the point where it intersects the wet-bulb temperature line. The required cross-sectional area of the cooling tower in square feet is then obtained by dividing the total water quantity to be cooled by the water concentration obtained. The curve is for a tower containing 22 ft of effective filling depth and does not apply when the difference between the wet-bulb and the cold-water temperature is less than 5 F.

Wind usually has a detrimental effect on mechanical-draft cooling performance. When wind blows across a fan, one side is overloaded and the other side is underloaded. The result is that less total air is handled than with zero wind. If the fan is installed in a stack the height of which is equal to the fan diameter, the effect of wind will be negligible. The wind also affects adversely the air distribution in the bottom of the tower, thus lowering performance.

Approach. Naturally, most plant operators are interested in obtaining as low a cold-water temperature as possible. However, the closer the approach the greater will be the size and the cost of the tower. Also, the higher the design wet-bulb temperature selected, the easier it is to obtain a close approach. For example, if two towers have the same wet-bulb temperature and heat load, one having a 10 F approach, and the other a 3 F approach, the latter will be more than twice as large as the former.

Take, for another example, two towers with the same heat load, water quantities, and approach, but rated on different wet-bulb temperatures. One rated on a 52 F wet-bulb temperature will be about twice as large as the one rated on an 80 F wet-bulb temperature, because the heat-absorbing capacity for a given air temperature rise is so much greater at the higher wet-bulb temperatures.

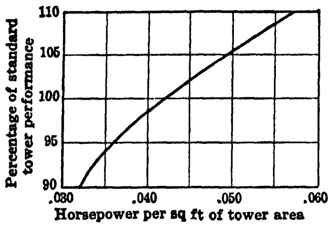


FIG. 7. Fan horsepower for mechanical draft towers. (Courtesy of The Fluor Corp., Ltd.)

Note: The area of the commercial mechanical-draft tower is rarely exactly equal to the required theoretical area. The performance is adjusted by the manufacturer to match the design performance by varying the air volume. The above curve shows the effect on the fan horsepower of varying the tower performance (by varying the air volume). To obtain total fan horsepower, multiply tower area by the factor read from the curve.

Cooling Range and Water Quantity. The product of the water quantity in pounds per hour by the cooling range (in °F) represents the *heat load* to be handled by the cooling tower. In determining cooling range and water quantity, it is more economical to purchase a cooling tower having a greater cooling range and reduced quantity of water than one having a smaller cooling range and greater quantity of water. This is because cooling-tower performance is not as sensitive to increased cooling range as it is to increased water quantity (water concentration). It is important to emphasize that increasing the flow of water to a cooling tower will *not* result in a lower cold-water temperature. To illustrate, consider a tower to cool 1000 gal per min through a 10 F range with a 5 F approach, as compared with a tower with an identical heat load in which only 250 gal per min are to be cooled through a 40 F range to a 5 F approach. The former will be about twice as large as the latter, even though the same heat load is handled by both.

The problem of determining wet-bulb temperature, approach, range, and gallons per minute must be solved for each particular plant. A method for determining relative size of the two types of cooling towers is shown in Figs. 5, 6, and 7.

5. APPRAISAL OF TOWER DESIGN

Before a cooling tower is purchased, it is wise to evaluate: (*a*) the *quality* of the structure; (*b*) the *accessories and services* included.

STRUCTURE. General. These questions should be considered:

1. *What wind load is the tower designed to withstand?* The usual design wind load is 30 lb per sq ft of vertical exposed surface based on the following formula:

$$P = 0.003V^2$$

where P = wind load, lb per sq ft; and V = air velocity, mph. This amounts to a 100-mph wind. It is well to specify a 30-lb wind load even though the tower may never be subjected to it, because a more rugged and durable tower is thereby obtained.

2. *What type of brace to post connection is used?*

3. *Is the tower structure designed in accordance with the new rigid standards* of the *Uniform Code* published by the Pacific Coast Building Officials Conference and *Technical Bulletin* 865, March 1944, of the United States Forest Products Laboratory, entitled Timber-connector Joints, Their Strength and Design, by John A. Scholten, as well as the requirements of the codes of major cities?

The two references mentioned above have, in general, reduced the allowable values for shear rings. The major city codes are also in the process of reducing the allowable values of shear rings as the result of failures in timber structures using shear rings.

4. *Are eccentric loads placed in the posts by the bracing and horizontal ties?*

An eccentric load of only a few inches often reduces the allowable wind load by one-half. This point should be thoroughly checked since many towers have been designed without considering the eccentricity in the joints.

5. *Are the braces designed to take both tension and compression,* or compression alone? In the most modern construction the braces and connectors are designed to take both compression and tension loads.

6. *Are the numbers and sizes of posts and braces sufficient* to insure long life for the tower?

7. *Are the transverse members of sufficient size* to carry the wind load from the exterior

panels to the braces? The minimum size of any member taking a horizontal load should be 2 in.

8. *What type of filling is used?* Is it 1/2-in. or 1-in. battens or merely 1/4-in. lath?

9. *Is the filling adequately supported* or will it sag in a few months, causing water channeling, thus seriously reducing performance? The filling should be of sufficient strength to allow a man to walk around freely on the decks.

10. *Will the filling remain in place* over a period of years? Is each deck supported by a bolted transchord or is the filling loosely stacked in place? The filling should be made up in panels so that it will withstand the force of a fire hose in cleaning.

11. *What type of hardware is used?* Are the bolts and other hardware to be black iron, galvanized iron, brass, or some other material? Is the hardware other than bolts to be cast iron, bronze, brass, or galvanized fabricated steel? Brass has usually proved the most serviceable material for both bolts and fittings.

Mechanical-draft Towers. In addition, these questions should be asked for mechanical-draft towers:

1. *What type of fan* is used? Of what is it made?

All cooling-tower fans are of the axial-flow type. The number of blades varies from two to six. In general, the greater the number of blades, the smoother the operation; however, increasing the number of blades tends to reduce the efficiency of the fan because of interference between blades. For instance, a two-bladed fan causes pulsation in air flow, vibrating the entire structure, whereas, with a six-blade fan, the pulsation disappears entirely. This is because the time which elapses between blade passings in a two-bladed fan running, say, at 200 to 250 rpm is so great that the air flow is actually reversed, causing a pulsation.

Are the blades made of aluminum, Bakelite, steel, or stainless steel? Are the blades cast, fabricated of steel plate, or built up of struts covered with some thin sheet material similar to that used on an airplane wing? The objection to the last type is that, if a nail or rock or hailstone falls in the fan, it will probably puncture the fan blade covering, causing the material to tear off and damaging the fan permanently.

2. *What is the static efficiency* of the fan (not total efficiency)? Most cooling-tower fans are 30 to 55% efficient. In cooling-tower work, a fan of more than 60% static efficiency should be seriously questioned. See also Section 1.

3. *What type of ring housing* is used? The efficiency of a fan depends on the type of fan ring orifice employed. All cooling-tower fan orifice rings fall between the best or bell-mouthed type and the poorest or straight-ring type. A fan in a straight-ring orifice requires about 15% more horsepower than a fan in a bell-mouthed ring.

4. *What type of gear unit* is employed? Is it a general-purpose gear, or a gear designed specifically for cooling-tower operation? Is the gear built and designed by a reputable gear manufacturer, or is it an assembly of gears and parts from automotive equipment adapted to a gear case? Are the horsepower ratings for the gears according to the American Gear Manufacturers Association for Class II service greater than the horsepower requirements of the fan? This standard allows a service factor of 1.25, which means that the unit is designed for an electric motor drive, continuous-duty, nonshock load. Is a generous oil sump supplied well below the lower main bearing for condensed moisture to collect, or will the lower main bearing run in water when condensation occurs?

5. *How large a cooling-tower basin* will be required? A low, wide tower requires a relatively large basin; a tall tower for the same capacity requires a relatively small basin. Cost of the basin should be considered in evaluating the total cost of the cooling-tower installation.

In an atmospheric tower it is well to consider what provision is made to eliminate spray loss, and what type of distributing system is employed. The wooden-flume-type distributing system is not acceptable since the wood will warp, sag, and open up at the seams. After such a system has operated for a year or so, the water distribution becomes poor.

Accessories. Before a tower is purchased, the buyer should know what accessories are included in the purchase. Does the purchase price include latter, stairway, walkway, distributing system, anchor bolts, foundation, riser pipes, and erection of the tower?

6. EVALUATING PERFORMANCE

Once the type of tower is chosen, it is well for the plant engineer to decide which of the towers offered will give the best performance. Considered in evaluating the performance of a cooling tower should be:

Effective Tower Volume. The product of the effective cross-sectional area by the effective height gives the effective tower volume. The effective tower height is the distance from the basin curb to the center line of the splash plates in a gravity-type system. In an upspray distributing system the effective height is the distance from the basin curb to the center line of the distributing system, plus 8 ft. *The greater the effective volume, the better the performance, other things being equal.*

Type of Distributing System. The two general types of distributing systems are the *gravity* system and the *pressure* piping system. The gravity system consists of troughs or flumes with holes drilled at intervals through which the water descends. The water then falls upon splash plates, which are placed 2 or 3 ft below. There is relatively little flexibility in this type of system. At a reduced water rate, the water is not uniformly distributed; at a high water rate, the flumes overflow.

The upspray system consists of pressure piping with upspray nozzles similar to those used in a spray pond. This upspray system is placed 6 to 8 ft below the drift eliminators to allow room for a spray chamber. The nozzles break the water into fine droplets, intimately mixing the water with the air. The upspray system increases the time of contact between water and air, since the water travels up to the drift eliminators and then back to the nozzles. As much as 30% of the heat can be "flashed off" before the water reaches the first deck. In effect, the upspray system is a spray pond on top of a cooling tower. The performance of an upspray distributing system represents the equivalent of adding 8 or 9 ft to the height of the cooling tower over that of the gravity-type system and, therefore, must be evaluated in determining the effective height of the tower and the time of contact. The maintenance cost of the upspray system is less than that of the gravity system, since there is sufficient pressure drop across the upspray nozzles (5 to 6 lb) to prevent algae from collecting and eventually plugging them.

Time of Contact. The time of contact between water and air is largely determined by the time required for the water to travel from the nozzle to the basin. The time of contact is therefore regulated largely by changing the height of the tower. Should the time of contact be insufficient, no amount of increase in the volume of air will accomplish the desired cooling. Therefore, it is essential that a minimum height of tower be maintained. When it is desirable to cool water through 25 to 35 F cooling range with a relatively large approach (15 to 20 F), a low cooling tower will suffice: one in which the water travels 15 to 20 ft from nozzles to basin. When the water is to be cooled 25 to 35 F with a moderate approach (10 to 15 F) a tower in which the water travels 25 to 30 ft is sufficient. When water is to be cooled 25 to 35 F with a small approach (4 to 8 F), 35 to 40 ft travel is required.

Counterflow cooling is the most effective of all types used in design. In true counterflow cooling, the coldest water contacts the coldest air, and the highest-temperature air contacts the warmest water. The height of tower required to give the necessary time of contact in the examples of the preceding paragraph was predicated on counterflow cooling. If the counterflow cooling is not employed, these figures must be increased by 20 to 30% for *cross-flow cooling* and 40 to 50% for *parallel-flow cooling.*

Wetted Surface. The wetted surface is the sum of the exposed deck surface and the drop surface in the tower. *The greater the wetted surface, the greater the cooling,* other things being equal.

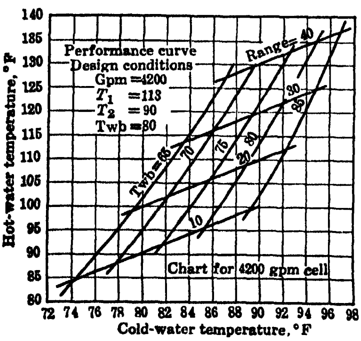


FIG. 8. Typical cooling tower performance curves. (Courtesy of The Fluor Corp., Ltd.)

Splash surface is that surface area on which water drops. Splash surface is more effective than wetted surface, provided, of course, that the water is in intimate contact with air when it splashes.

COOLING-TOWER GUARANTEES. Cooling towers are usually guaranteed on the basis of cooling a specified quantity of water through a given cooling range with a given approach to design wet-bulb temperature. Unless the stated water quantity, temperatures, and design wet-bulb temperature occur simultaneously (which may not occur during the year), the guarantee cannot be verified. It is well for the purchaser to obtain from the manufacturer at the time of purchase a series of performance curves similar to Fig. 8. This permits the tower to be tested under a wide range of temperatures.

JACKET WATER COOLING FOR DIESEL ENGINES. A cooling tower may be employed to cool jacket water by the *open system* or by the *closed system.* The open system consists of running cooling-tower water through the engine jacket. The closed system consists of passing raw cooling-tower water over coils which contain treated jacket water.

In the open system there is a tendency for salts to become concentrated during the evaporative cooling process. This concentrated salt solution (cooling-tower water) is then pumped directly to the engine water jacket, where salts are precipitated, necessitating frequent and expensive cleaning and overhauling of the engine. The open system is used only on very small engines, usually for temporary installations only.

COOLING WATER FOR REFRIGERATION LOAD. Cooling towers are used not only for refrigerating plants for ice making and food cooling but also for air-conditioning equipment. The heat discharged per ton of refrigeration (which must be eliminated by the cooling tower) is 250 Btu per min for compression plants on straight refrigeration and 550 Btu per min for absorption plants on straight refrigeration.

Mechanical refrigeration requires 30 gal-degrees of water per minute per ton of refrigeration; i.e., if but 1 gal of water per minute per ton of refrigeration is circulated through the tower, the cooling range will be 30 F; if 3 gal per min is circulated, the cooling range will be 10 F. Steam refrigeration requires circulation of at least three times these quantities.

7. COOLING PONDS

EVAPORATION FROM PONDS. In ponds, cooling is mainly by evaporation and is independent of the depth. Box gives for the rate of evaporation from a pond, in still air,

$$W = \frac{(240 + 3.7t)(p_s - p)}{7000} \tag{1}$$

where W = moisture evaporated per square foot per hour, pounds; t = average temperature of water, °F; p_s = saturation pressure of vapor at temperature t, inches of mercury; p = actual vapor pressure of air, inches of mercury.

Area of Pond Required. If the water is not sprayed into the pond, the theoretical area A st ft, required to cool Q lb of water to a final temperature of t_2, °F, is

$$A = \frac{Q(t_1 - t_2)}{H} \tag{2}$$

where t_1 = initial temperature of water entering pond; t_2 = final temperature of water in pond; H = heat dissipated per sq ft per hr, Btu = Wh; h = latent heat of water at temperature t_1; W = weight of water evaporated as given by eq. 1. The value of A so found may be smaller than is usual in practice. Heat dissipation is modified by variations in wind velocity, air temperature and relative humidity. Actual values of H range from 4 Btu per hr per sq ft per deg temperature difference in summer, to 2 Btu in winter. A good practical average is 3.5 Btu.

SPRAY PONDS. The heat-dissipating capacity of ponds may be increased greatly by spraying the water into them through nozzles which break the water into fine spray, increasing the evaporation and the cooling effect. Final temperatures depend on the cooling range, atmospheric conditions, arrangement of nozzles, and storage capacity of pond. A lower final temperature is obtainable with moderate cooling ranges (10 to 20 F) than with ranges of 30 to 40 F. At the same relative humidity, final temperatures will be lower in warm weather than in cold, due to the greater moisture-absorbing power of warm air. Table 2 compiled by the Cooling Tower Company gives average final temperatures that may be expected under various conditions.

Table 2. Average Final Temperatures of Spray Cooling Ponds

Temperature of Water Entering Nozzles, °F	Ammonia Condenser Service					Steam Condensing Service				
	Wet-bulb Temperature, °F					Wet-bulb Temperature, °F				
	60	65	70	75	80	60	65	70	75	80
	Temperature of Pond, °F					Temperature of Pond, °F				
60	60	....	....	....	.....	60.0	....	....	....	....
70	65	67.75	70.0	....	.....	66.0	68.25	70.0	.....	....
80	68.75	71.75	74.5	77.25	80.0	71.0	73.25	76.0	78.25	80
90	72.25	75.25	78.0	80.75	83.75	75.5	77.75	80.25	82.75	85.5
100	75.25	78.25	81.0	83.75	87.0	80.0	82.25	84.50	87.0	89.75
110	78.25	81.0	84.0	86.75	89.75	84.25	86.5	88.75	91.0	93.75
120	81.0	83.75	86.75	89.25	92.25	88.25	90.75	92.75	95.0	97.50
130	83.75	86.5	89.25	91.75	94.50	92.5	94.5	96.5	98.75	101.0

The quantity of water stored greatly exceeds the quantity sprayed per minute. Its average temperature is lower than that of the sprayed water, decreasing as the quantity stored increases. Figure 9 is a layout of a typical spray pond.

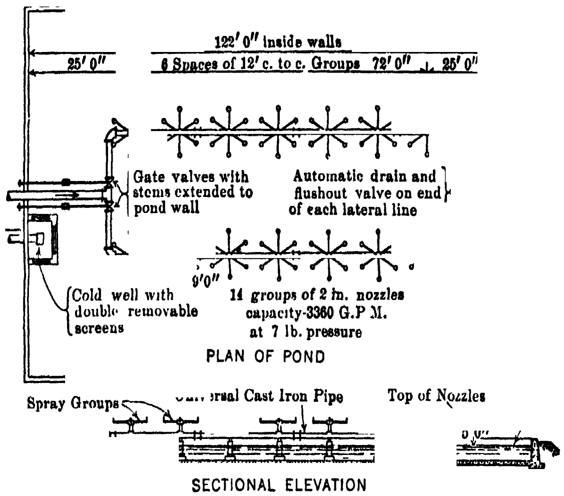


Fig. 9. Typical spray cooling pond.

Nozzle pressures of 7 to 10 psi are usual. Higher pressures produce a finer spray that may be blown away by the wind; lower pressures produce larger drops which retard cooling. Nozzles should be spaced 8 to 12 ft. The pond area should be 1 sq ft per 250 lb of water sprayed per hour for plants of 1000 hp or over, and 1 sq ft per 150 lb of water per hour for plants under 1000 hp. Depth need not exceed 3 ft. Power required to circulate the water ranges from 1 to 2% of the power developed by the prime mover under high vacuum.

SECTION 10

COMBUSTION GAS TURBINES

By

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COMBUSTION GAS TURBINES

The term *gas turbine* * applies to a variety of thermodynamically and mechanically related apparatus. The single feature common to all gas turbine applications is a turbine element in which the motive fluid is a heated permanent gas. There are several main categories of gas turbine application.

Supercharged and Compound Cycles. Turbo-superchargers are applied to four-cycle *Otto* or *diesel* engines. Compressed air is delivered to the engine intake manifold from a rotary compressor driven by a direct-connected turbine. The engine exhaust supplies the motive fluid for the turbine. In compound cycles, the turbine output exceeds compressor demands and may augment engine power in varying degrees.

Pressure Firing. In the *Velox boiler* and similar applications, furnace sizes are reduced and heat transfer rates greatly increased by firing the furnace under pressures higher than atmospheric. A turbine powered by the flue gases drives the charging compressor.

Compression of Process Gas. In chemical and refining operations typified by the *Houdry process*, a turbocompressor unit furnishes compressed air for use in the process. The air is heated in passing through the chemical system and is then expanded through the turbine. The latter may or may not develop power in excess of that required for compression.

Jet Propulsion (see Section 15). *Jet engines*, as applied to aircraft, are compressor-combustor-turbine combinations. Atmospheric air is aspirated, compressed by the centrifugal or axial flow compressor (see Section 1), heated by internal burning of fuel in the combustor(s), and partially expanded in the turbine. The pressure drop in the turbine is just sufficient to develop the power necessary to drive the compressor. The turbine exhausts into a jet duct in which the residual pressure drop to atmosphere develops a high-velocity jet. The reaction of this jet is the useful output of the engine; it is usually expressed in pounds of thrust. The power developed depends on the speed of the airplane.

Power Generation. A gas turbine power plant is usually comprised of one or more compressors, combustors (or heaters), and turbines. Certain additional items of heat-transfer apparatus are sometimes used to improve fuel economy. The power plant is characterized by the facts that the principal prime mover is of the turbine type and that the working medium is a permanent gas.

BASIC THERMODYNAMICS

1. IDEAL GAS TURBINE CYCLE

The usefulness of a gas turbine in practically any application is dependent on the fact that the work of reversible adiabatic compression or expansion of a perfect gas is directly proportional to the absolute temperature of the gas.

Isentropic work of compression (or expansion)

$$= \frac{k}{k-1} WRT_2 \left[1 - \left(\frac{p_1}{p_2} \right)^{(k-1)/k} \right] \text{ ft-lb} \quad (1)$$

where W = mass of gas, pounds; k = ratio of constant pressure and constant volume specific heats, c_p/c_v; R = gas constant, ft-lb/lb-°F abs; T_2 = temperature corresponding to the higher pressure p_2 (before expansion—after compression), °F abs; and p_2, p_1 = the higher and lower absolute pressure levels, respectively, lb/sq ft abs.

If we assume a combination of an ideal (100% efficiency) compressor and an ideal turbine, each handling the same mass of gas through the same pressure ratio, and that heat is added after compression, eq. 1 yields a simple expression for the useful work of a gas turbine cycle:

$$\text{Net useful work (ideal)} = \frac{k}{k-1} WR(T_{t2} - T_{c2}) \left[1 - \left(\frac{p_1}{p_2} \right)^{(k-1)/k} \right] \text{ ft-lb} \quad (2)$$

where

T_{t2}, T_{c2} = absolute temperatures before expansion and after compression, respectively

* For jet propulsion, see Section 15.

Since a temperature difference $T_{t2} - T_{c2}$ can be caused to exist by adding heat to the gas after compression, the turbine-compressor-heater (or combustor) combination constitutes a heat engine. The mechanical equivalent of the heat consumption is

$$W(T_{t2} - T_{c2})c_pJ \text{ ft-lb} \tag{3}$$

where

$$J = \text{mechanical equivalent of heat} = 778 \text{ ft-lb/Btu}$$

Hence the ideal thermal efficiency, η, of such an engine is (dividing 2 by 3)

$$\eta = \frac{k}{k-1}\frac{R}{Jc_p}\left[1 - \left(\frac{p_1}{p_2}\right)^{(k-1)/k}\right]$$

Or, since $R/Jc_p = (k-1)/k$,

$$\eta = 1 - \left(\frac{p_1}{p_2}\right)^{(k-1)/k} \tag{4}$$

This is the efficiency of the simple *Brayton cycle* illustrated in Fig. 1. The useful output, measured in Btu per pound, is the difference between the enthalpy changes represented by the isentropics (AB and CD) between the isobars p_1 and p_2. The net heat input is the difference in enthalpy between the compressor discharge, T_{c2}, and the turbine inlet, T_{t2}.

If p_1 is atmospheric pressure, the expansion ends at this pressure, and when gases are exhausted at this point there is a rejection of heat to atmosphere DA, which characterizes *open-cycle* operation. Heat abstraction after the turbine exhaust returns the working gas to its original state, A. This completes the cycle. *Closed cycles* are typified by rejection of this heat in an indirect-type heat exchanger.

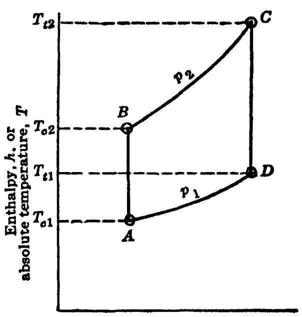

FIG. 1. Brayton cycle.

The thermal efficiency of the Brayton or gas turbine cycle approaches unity at infinite pressure ratio, as shown by eq. 4. Note that for this *ideal* cycle the efficiency is independent of the heat added and consequently of $T_{t2} - T_{c2}$. The theoretical useful work per pound of fluid also increases with increasing pressure ratio. However, it is proportional to the difference between compressor discharge and turbine inlet temperatures. Figure 8 (p. 10-14) includes a plot of simple cycle theoretical performance as related to pressure ratio and turbine inlet temperature. It is clear that theoretical performance is vastly altered by substitution of real machines for ideal compressors and turbines. In fact, for the actual cycle, with nonisentropic elements, the thermal efficiency is not independent of the heat added, $Wc_p\ (T_{t2} - T_{c2})$.

Use of a turbine as the expansion element in the cycle allows the realization of *complete expansion*, not attainable in piston engines having limited volume or expansion ratios, where the volume of the working fluid at the end of the expansion stroke is essentially the same as at the beginning of compression. The work available in the working fluid of a piston engine at the moment the exhaust valves or ports open is lost unless applied to an auxiliary such as the *turbocharger*. A turbine exhausting to atmosphere avoids this loss. (See Fig. 2.)

Another attribute of the gas turbine power plant, as opposed to the piston engine, is the *continuous execution of thermodynamic processes* in physically separated mechanical or thermal components. This makes possible the interposition of various heat-transfer devices (intercoolers, reheaters, regenerators) between compression and expansion elements. In fact, by the addition of these elements to the simple cycle any degree of approximation to the *Ericsson full-expansion 100% regenerative cycle* may be obtained. This cycle (isothermal compression, constant pressure heat addition from regenerator, isothermal expansion, constant pressure cooling in regenerator) has the same theoretical efficiency as the *Carnot cycle* and develops appreciably more useful work per pound of working gas.

These generalizations of gas-turbine characteristics predicated on ideal gas properties are not invalidated by deviations of real gases from these properties. However, the actual design of gas turbines and predictions of performance must be based on true gas characteristics. The thermodynamic properties of dry air (see Section 1) have been tabulated in much the same manner as for steam (Ref. 1). For properties of combustion products, see Section 2. For the moderate pressures of usual gas turbine power plants, enthalpy can be represented as a function of temperature alone. This permits a more concise tabulation than is possible for steam. On the other hand, variations in composition due to moisture content or the introduction of fuel into the working fluid make corrections necessary. Appropriate correction factors, together with other air properties, have been assembled in a U. S. Navy publication (Ref. 2).

GAS TURBINE APPLICATIONS

2. SUPERCHARGING AND COMPOUNDING

Turbo-superchargers in effect add turbo stages to both the compression and the expansion processes of reciprocating engine cycles. The combination of supercharger and engine thereby becomes a compound unit in which the complementary characteristics of turbo and reciprocating machines are utilized to secure performance not otherwise obtainable.

Full expansion of the working fluid in the engine cylinder is not practical. Therefore, a turbine applied to the exhaust develops additional power with virtually no change in engine operation. Because the turbine produces power at high rotative speeds, it is ideally suited to driving a centrifugal or axial-flow compressor, used to increase the density of air in the intake manifold. The engine output increases more rapidly than the mass flow, and even with relatively poor turbine and compressor efficiencies, overall economy equals or exceeds that of the unsupercharged engine. If turbine and compressor efficiencies are substantially improved (usually accomplished at the expense of weight, size, and loss of simplicity) and if manifold pressures are increased, a substantial excess of power may be obtained from the turbine for external use. In fact, any desired distribution of work between turbine and engine may be obtained. Figure 2 compares *pv* diagrams for several idealized supercharged and compound cycles. Test performances of machines executing these and other cycles in the same category are given in Table 1.

AIRCRAFT SUPERCHARGERS. One of the largest-scale applications of gas turbines is in the field of military aircraft supercharging. Here the principal objective is maintenance of sea-level performance of the engine at high altitudes. Fundamental requirements are simplicity and light weight, coupled with reliability for relatively short life, under all flight conditions.

Figure 3 illustrates application of a typical aircraft turbo-supercharger to an engine. The turbine is a *full-admission impulse stage* directly mounted on the exhaust manifold. A waste gate with automatic control is provided to insure that the compressor functions at all times within its range of stable operation and that the unit does not overspeed. The compressor delivers air to the carburetor through an intercooler. The fuel-air mixture is further compressed by a geared supercharger before delivery to the intake manifold. All cylinders exhaust into a common manifold, effectively throttling the exhaust gas from cylinder pressure at the end of the expansion stroke to a lower mean manifold pressure before expansion in the turbine. Turbo-supercharged engines maintain sea-level ratings at altitudes of 30,000 ft and above. Turbo-supercharger weights are as little as 6% of those of the engines to which they are applied.

BUCHI TURBOCHARGERS. The supercharging of four-cycle Diesel engines by the *Buchi method* constitutes the most extensive use of gas turbines outside the field of aviation. Applied to a particular engine, a Buchi turbocharger increases output by 50% or more and usually improves fuel consumption, particularly for overload operation. Thus four-cycle engines are made competitive in size and weight with two-cycle engines while retaining their inherent advantages in other respects. Long life and reliable performance with minimum maintenance are attained in the diesel turbocharger by the free use of cooling and more conservative use of high-temperature materials than in aircraft applications. See Fig. 11, Art. 3, Sec. 13, for cross section of a typical turbosupercharger for Buchi application.

Figure 4 illustrates application of the supercharger to an engine and indicates the principal thermodynamic distinction between constant pressure and Buchi supercharging. Divided exhaust manifolds connect separate groups of cylinders to corresponding segments of the turbine nozzle ring. The turbine thus operates at variable-pressure with multiple-admission and with maximum conversion to useful power of the exhaust pressure peaks. The additional power developed is applied in part to furnishing scavenging air. Because of the divided exhaust manifold scavenging air can be forced through the cylinder with moderate charging pressure. It serves to cool engine parts and clear the combustion space. In addition, it dilutes and cools the exhaust gases before they enter the turbine.

(Continued on p. 10-07)

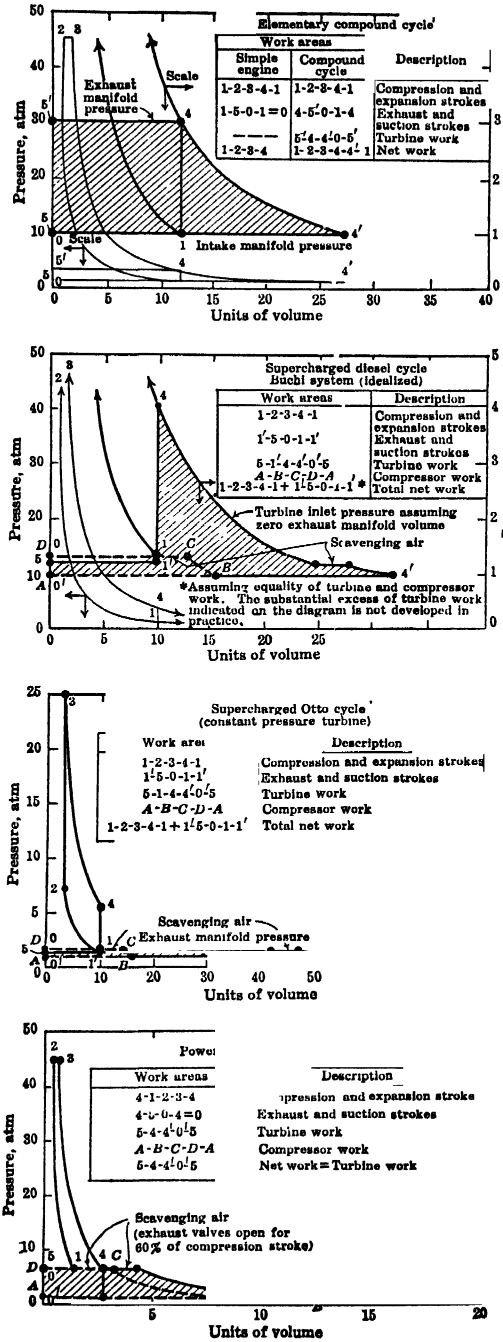

FIG. 2. Cycle *pv* diagrams for typical supercharged and compound engines.

Table 1. Typical Engine and Compound Cycle Performance

Power Plant Description	Rating, bhp	Fuel Rate, lb/bhp-hr	Supercharger Data	Turbine Data	Power Plant, Weight, lb	Remarks
Otto aircraft engine (with geared supercharger only)	1700 * at 7000 ft	.470	11.35 psia carburetor pressure, 11.35 psia ambient pressure	......	3,470	P & WA R-4360 (Wasp Major)
Otto aircraft engine (with geared and turbo superchargers)	1700 * at 25,000 ft	.445	13.25 psia carburetor pressure, 5.45 psia ambient pressure	1500 F (typical) inlet temperature	3,670	P & WA R-4360 (Wasp Major), GE-BH4 turbo-supercharger
4-Cycle heavy-duty diesel (unsupercharged)	425 (80 psi bmep) at 720 rpm	.380	......	......	16,000	8 Cylinders, 9-in. bore, 11 1/2-in. stroke
4-Cycle heavy-duty diesel (turbo-charged)	635 (120 psi bmep) at 720 rpm	.377	4.2 psig	$-1/2$ to 35 psig inlet pressure(3.3 psig av.), 970 F inlet temperature	17,000	8 Cylinders, 9-in. bore, 11 1/2-in. stroke, Elliott-Buchi BF-26. turbocharger
2-Cycle marine diesel (unsuper-charged)	5500 at 127 rpm	.333	......	......	......	Sulzer Brothers, 8 cylinders, 28.3-in. bore, 49.2-in. stroke
2-Cycle marine diesel (with direct-connected exhaust-gas turbine and reciprocating supercharger)	4000 at 445 rpm	.357	30 psia	27.5 psia inlet pressure 850 F inlet temperature	......	Sulzer Brothers development engine, 6 cylinders with opposed pistons, 12.6-in. bore, 2 x 15.75 in. stroke
2-Cycle diesel (with exhaust-gas turbine and axial-flow super-charger geared to engine)	2500 (175 psi bmep) at 1000 rpm	......	35.5 psia	......	20,000	Sulzer Brothers development engine, 8 cylinders with opposed pistons, 7.08-in. bore, 2 x 8.85 in. stroke

* Cruise power at critical altitude (48.5% max sea level power).

Buchi turbochargers have been successfully applied to engines ranging from 250 to 3000 bhp. In the United States more than 6,000,000 bhp of turbocharged engines have been produced since 1940. (See Diesel Engines, Section 13.)

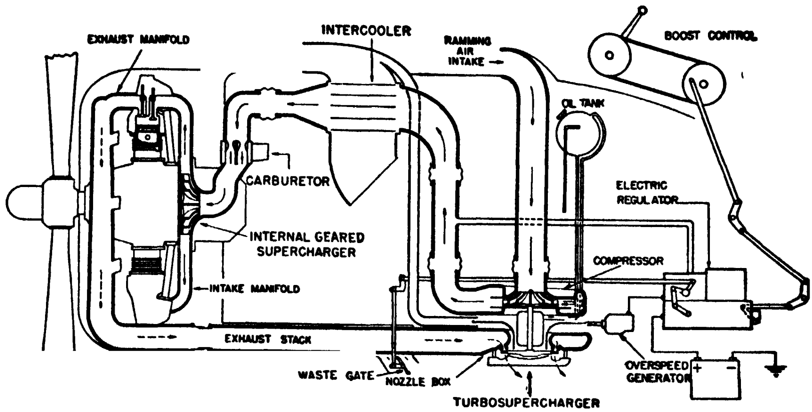

FIG. 3. Schematic diagram of turbo-supercharged power-plant unit. (Courtesy of General Electric Co.)

COMPOUND CYCLES (see also Ref. 3). Innumerable variations are possible in the compounding of Otto or diesel engines and turbines. These combinations usually include rotary compressors, but reciprocating compression also may be used.

If turbine and compressor are coupled mechanically to the engine crankshaft, a power balance is no longer required. This is a necessary condition for two-cycle supercharging of diesel engines because at low speeds the percentage of scavenging air is greatly increased. With the resultant lowering of turbine temperature, the turbine power is insufficient to drive the compressor. On the other hand, the higher intake and exhaust manifold pressures at normal speeds develop turbine power in excess of compressor requirements. With high supercharging pressure the turbine power may become equivalent to the entire net output of the turbine-engine combination.

Sulzer Brothers of Switzerland have developed several compound cycles. One such development involves the use of the engine solely to drive a reciprocating compressor and to deliver pressurized and heated exhaust gases to a power turbine.

GAS GENERATOR-TURBINE SYSTEM. From the point of view of fuel consumption, the most promising heat engine under present development is the free-piston gas generator–gas turbine combination (Pescara system), and its variants. Thermal efficiencies in excess of 40% have been predicted for this cycle. This system constitutes a limiting case of compounding, no useful power output being delivered by the reciprocating engine. Absence of crankshaft and connecting rods ("timing" linkages are required) and tandem arrangement of engine and compressor cylinders distinguish the free piston engine. The self-regulating variable stroke of the opposed pistons is another variation from conventional engine operation.

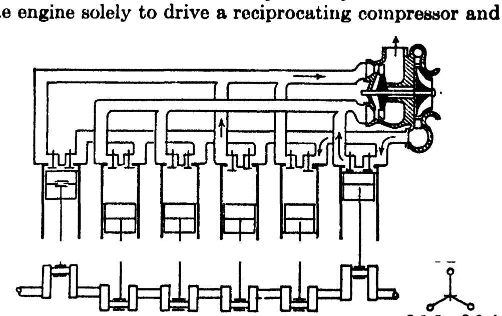

FIG. 4. Application of Buchi turbocharger to 6-cylinder diesel. (Courtesy of Elliott Co.)

As shown in Fig. 2, the overall pressure ratio of the cycle in the highly efficient reciprocating compression process greatly exceeds values obtainable with present types of rotary compressor. Effective cooling of the engine cylinders takes place because the combustion is intermittent, even though temperatures at the beginning of the expansion process are much higher than are permissible in present-day turbines. These characteristics of the diesel engine, coupled with the complete expansion feature of the gas turbine, result in a theoretical cycle of unequaled efficiency. In the Sulzer Brothers 7000-hp experimental version of this cycle, part of the exhaust gases is diverted to a second turbine which drives an axial-flow charging compressor. This unit supercharges the reciprocating compressor, increasing the capacity of the set. Figure 5 is a schematic arrangement of the Sulzer version of this power plant.

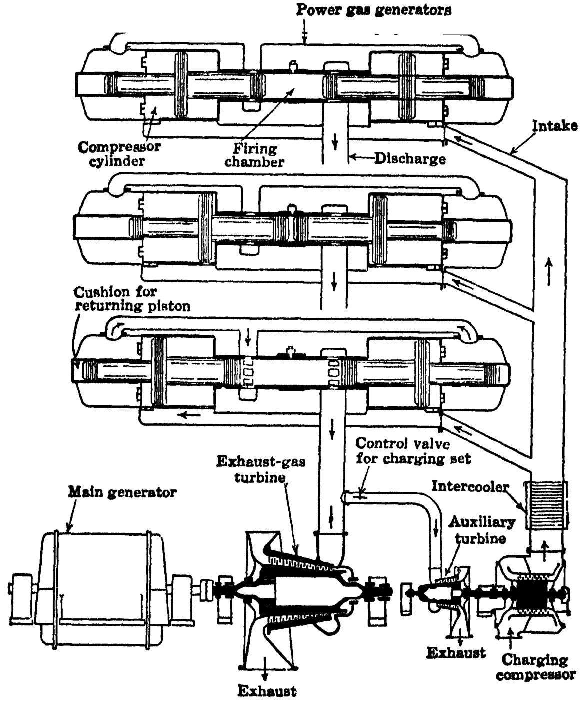

FIG. 5. Diagram of arrangement of 6000 bhp free piston power gas turbine cycle with exhast turbo-driven precompressor. (Courtesy of Sulzer Bros.)

PISTON-JET POWER PLANT. Compounding is particularly attractive where low fuel consumption is desirable. The Wasp Major-VDT (variable-discharge turbine), a piston-jet propulsion combination, has been applied for this reason to the long-range bomber. It delivers more than 4000 hp to a propeller, and at the same time provides several hundred pounds of jet thrust. A two-stage supercharger driven by a variable-discharge-exhaust gas turbine furnishes air through an intercooler to the intake of a 28-cylinder Wasp Major engine. Automatic control of the turbine discharge area determines the degree of supercharging, hence engine output. Thus the conventional throttle, with its attendant losses, is largely supplanted. The controlled exhaust jet adds appreciably to the power plant thrust.

3. PRESSURE FIRING OF BOILERS

Gas turbine pressure firing of steam generators is an important process in its own right. It is probably even more important as the forerunner of the modern power gas turbine. The Velox boiler, as manufactured by Brown Boveri, incorporates the principal elements of a gas turbine power plant. In particular, highly efficient compressors and turbines are essential to the process. The beneficial effect of high gas density on heat-transfer rates, as utilized in the Velox boiler, is of fundamental importance in the closed-cycle gas turbine.

In the Velox arrangement a charging compressor aspirates atmospheric air, raising its pressure to approximately 35 psig. High velocities are used in conjunction with this high density in the combustion, evaporation, and superheating sections of the boiler. Heat release and transfer rates are roughly ten times those developed in conventional furnaces and boilers. The high-pressure flue gases pass through a gas turbine, which powers the charging compressor, and are further cooled in an economizer. The boiler and charging unit occupy about the same floor space as the main turbo-alternator unit. In fact, the unit occupies one-fourth the space and weighs one-seventh as much as a conventional boiler. Concomitants of these advantages are quick starting, rapid response to load changes, and adaptability to automatic control. Velox boilers are reputed to handle almost any liquid or gaseous fuel.

4. PROCESS GAS COMPRESSION

Gas turbine-compressor units, for application to the Houdry catalytic cracking process for production of high octane gasoline, occupy a position relative to gas turbine development similar to that of the Velox boiler. Indeed, they are nearly identical with the charging units of the latter apparatus. Typical units deliver between 23,000 and 60,000 cu ft per min of atmospheric air compressed to 50 psig. This is passed over beds of catalyst which have been contaminated in the oil cracking process. The carbon contaminant is oxidized, thus heating the air which is returned to the turbine inlet at about 40 psig and 875 to 950 F. The turbine generally develops somewhat more than enough power to drive the compressor; the excess is absorbed in a direct-connected generator.

The necessary additions of a starting motor or turbine and a combustion chamber, for use during warm-up, make the complete Houdry charging unit an elementary power gas turbine. For this reason, and because a substantial amount of successful operating time has been logged on these units, they have created considerable confidence in the future of gas turbine power plants. (See Ref. 4.)

Other applications have been proposed in which the gas turbine utilizes process heat. Compression of air for blast furnaces is such an application in which both the heat and heating value of the blast furnace gas can be used to power the turbocompressor unit.

5. JET PROPULSION

(See Section 15.)

6. POWER GENERATION

DEFINITIONS. **A gas turbine power plant** may be defined as one "in which the principal prime mover is of the turbine type and the working medium is primarily a permanent gas." *

In the simplest form such a power plant includes a compressor, a combustor, and a turbine arranged as in Fig. 6A. Such a combination is the physical embodiment of the simple Brayton cycle. To achieve higher efficiencies or to fulfill the operating requirements, the three basic components may be used in multiple and in various arrangements. They are often augmented by other apparatus, generally for the purpose of approximating the regenerative Brayton or Ericsson cycles. The following are the principal additive components.

A regenerator is a heat exchanger which recovers waste heat from the cycle and transfers it to the fluid at a point where the pressure is higher and the temperature is lower. (See Fig. 6, *B*, *C*, and *F*.)

An intercooler is a heat exchanger which removes heat from the working medium between stages of compression, thereby giving an approximation to isothermal compression. Intercooler application is illustrated in Fig. 6, *C* and *F*.

A precooler is a heat exchanger which removes heat from the working medium prior to the first stage of compression, as in Fig. 6*D*. It may be called an *aftercooler* when used to cool the exhaust gases in a closed or semiclosed cycle (Fig. 6, *E* and *G*).

Heater is the general term for a thermal component in which heat is added to the working medium. It may be a *combustor* or a *heat exchanger* with or without a *combustion chamber*. The term is usually applied to the apparatus in which the principal heat addition is made in a closed cycle (Fig. 6*E*).

Reheater. When a heater is used to increase the temperature between expansion stages to approximate isothermal expansion (see Fig. 6*F*) it is called a *reheater*.

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* ASME PTC No. 22 (draft), A Test Code for Gas Turbine Power Plants. Other definitions in this subsection are adapted by permission from NEMA Standard, Definitions for Marine Propulsion Gas Turbines.

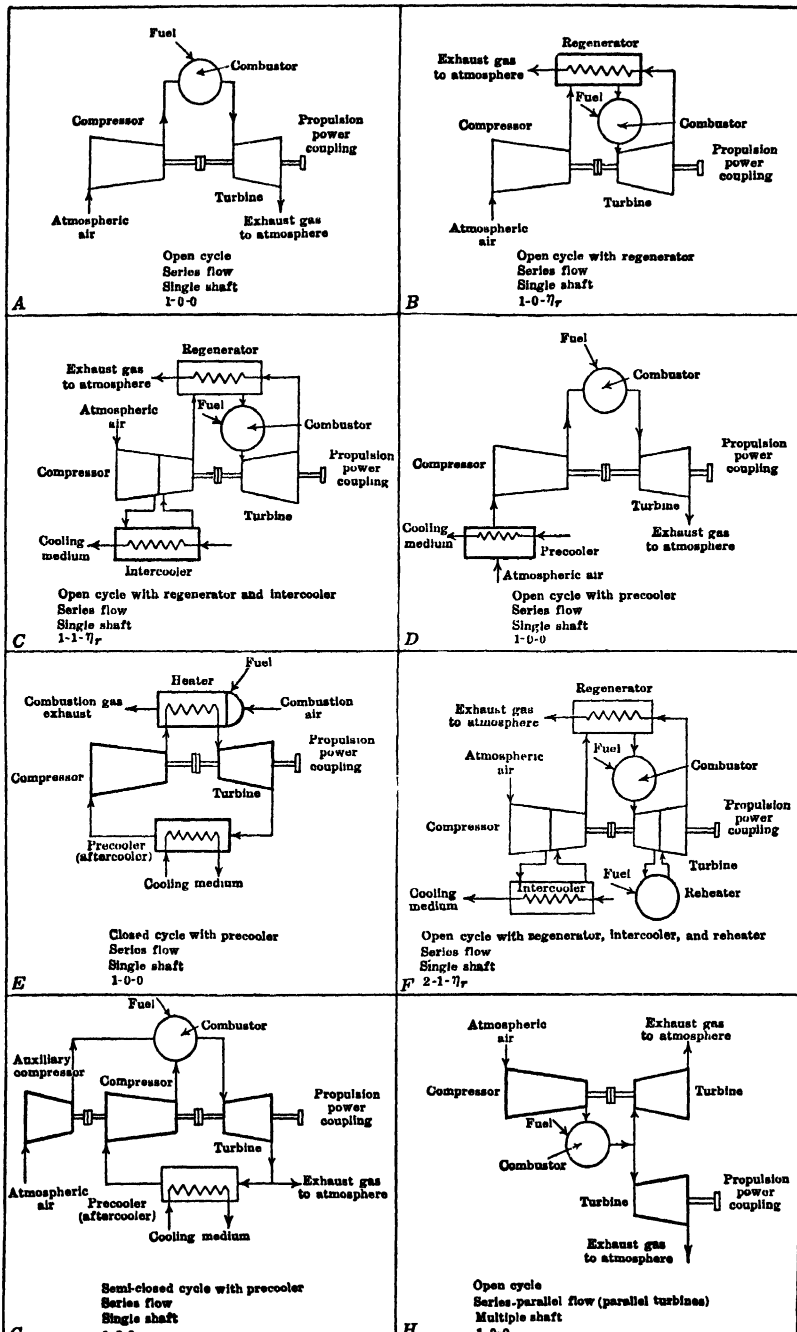


FIG. 6. Typical arrangements of gas turbine cycle components.

Comprex is a patented device combining expansion and compression functions in a single rotor permitting higher cycle temperatures.

Considered as a sequence of thermodynamic processes, any particular cycle is characterized by various stages of compression, heating, and expansion as well as by variations in the use of regeneration, intercooling, and reheating. The many possible gas turbine cycles, of which Fig. 6 illustrates a few typical examples, are classified as *open* if there is no recirculation of the working medium (Fig. 6, *A*, *B*, *C*, *D*, *F*, and *H*); *semiclosed* if part of the working medium is recirculated (Fig. 6*G*); or *closed* if there is total recirculation (Fig. 6*E*).

Any plant employing a given cycle may have various flow arrangements and physical disposition of components.

A series-flow arrangement is one in which all the working medium follows a single path through the entire cycle (Fig. 6, *A*, *B*, *C*, *D*, *E*, *F*, and *G*).

In a series-parallel-flow arrangement (Fig. 6*H*) the medium follows a divided path through some of the cycle.

Similarly, the basic physical arrangements are:

Single shaft when all rotors are coupled together on a common axis and rotate with fixed speed relationships (Fig. 6, *A*, *B*, *C*, *D*, *E*, *F*, and *G*).

Multiple shaft when the rotors lie on more than one rotational axis.

If no fixed speed relationship exists between the several shafts, all except the power coupling shaft are known as *floating shafts*. Figure 6*H* illustrates a multiple-shaft arrangement with floating shaft.

PERFORMANCE

7. CHARACTERISTICS

Characteristics of gas turbine power plants are expressed in terms of several parameters and ratios:

Thermal efficiency, η, is the ratio of the heat equivalent of the net output power to the total heat rate supplied by a specified fuel based on its higher heating value. (Lower heating value or heating value at temperature are sometimes used—the former particularly by European manufacturers—yielding *apparently* higher efficiencies.)

Air rate, w, in pounds per horsepower-hour, is the quantity of working medium entering the first stage(s) of compression per unit of output energy.

Fuel rate, in pounds per horsepower-hour, is the quantity of a specified fuel consumed per unit of output energy.

Cycle pressure ratio, ρ, is the ratio of the highest main compressor discharge pressure to the lowest main compressor inlet pressure.

Cycle pressure level is the ratio of maximum pressure in the cycle to the atmospheric pressure.

Work ratio, α, is the ratio of output power to the total installed turbine power.

Back-work ratio, $1 - \alpha$, is the ratio of compressor power to gross turbine power.

Regenerator effectiveness, η_r, is the ratio of the actual heat transfer in a regenerator to that theoretically possible if the heated fluid were to reach the temperature of the entering hot gas.

Thermal efficiency and *fuel rate* have obvious significance relative to operating economy. *Air rate* is a rough measure of the physical size of the equipment which must handle the specified quantity of fluid. As will be seen, each thermodynamic cycle has an optimum *cycle pressure ratio*. *Cycle pressure level* determines the reduction in physical dimensions of plant components resulting from closed-cycle operation. The *work ratios* are measures of the productive output of a cycle in relation to the amount of equipment required to implement it, being in this respect closely associated with air rate. Since work ratio also indicates the fraction of turbine work left for useful application after compressor power requirements are satisfied, it reveals the sensitivity of the cycle to variations in component efficiencies. *Regenerator effectiveness* is the fraction of recoverable "waste" heat actually restored to the cycle.

These performance ratios are primarily dependent on the cycle and working medium chosen, component efficiencies, parasitic losses, and similar items controllable by design. Of almost equal importance are various *operating variables*, e.g., compressor inlet temperature and pressure (altitude) and turbine inlet temperature. Thus the number of variables which influence gas turbine performance is so large that complete expression of their simultaneous effects is virtually impossible.

THERMAL EFFICIENCY DATA. The curves presented on Figs. 7, 8, 9, and 10 may be regarded as an incomplete set of typical sections through a multidimensional representation of generalized gas turbine performance. For convenience in discussing gas turbine performance, Lysholm has proposed a *short-hand designation of thermodynamic cycles.* Three numerals separated by hyphens are used to express (1) the *number of combustors* or heaters, which is also the number of expansions, (2) the *number of intercoolers,* which equals the number of compression stage groups less one, and (3) the *regenerator effectiveness.* Thus 2-1-0.75 represents a cycle with a single reheat, one intercooler, and 75% regeneration. This notation is used in Figs. 7, 8, 9, and 10, which are based on the following assumptions except as specifically noted:

(Working fluid is pure air throughout the cycle, no account being taken of change of mass due to fuel injection [the equivalent of indirect firing] or leakage.)

Compressor inlet temperature		= 70 F
Compressor inlet pressure		= 14.7 psia
Temperature after intercooling		= 90 F
Turbine efficiency (overall)	η_t	= 89%
Compressor efficiency (overall)	η_c	= 83%
Mechanical efficiencies		= 98%
Combustor efficiency		= 98%
Parasitic losses (pressure drop, radiation, etc.)		
Combustors		0.54 Btu/lb each
Intercoolers		0.37 Btu/lb each
Regenerators	$\eta_r = 0.50$	0.55 Btu/lb
	$\eta_r = 0.75$	1.10 Btu/lb
	$\eta_r = 0.90$	2.75 Btu/lb
Piping	No reheating	0.60 Btu/lb
	One reheat	0.80 Btu/lb
	Two reheats	1.00 Btu/lb

Thermal efficiency, η, is calculated as follows:

$$\eta = \frac{\left\{\begin{array}{l}\text{Enthalpy change in turbine} \times 0.98 - \text{Enthalpy change} \\ \quad \text{in compressor}/0.98 - \text{Parasitic losses}\end{array}\right\}}{\text{Enthalpy increase in heater}/0.98}$$

Note that this definition in effect credits the fuel with its "heating value at temperature" rather than with either its higher or lower heating value. This makes the results independent of fuel properties but gives a more optimistic picture of efficiency than the conventional HHV basis. The discrepancy increases for fuels with high hydrogen content and for high cycle temperatures.

The larger part of the data for curves on Figs. 7, 8, 9, and 10 has been taken from the paper, *The Gas Turbine as a Possible Marine Prime Mover* (Ref. 5). The assumptions for component efficiencies and parasitic losses are those of Messrs. Smith, Soderberg, and Lysholm. Although any such general and simple loss formulas are open to criticism—no account is taken, for example, of the effect of power plant size or pressure level—they appear on the whole to have been reasonably selected. For convenience of calculation, those cycles including intercooling or reheat have been assumed to be divided into equal pressure ratio steps. This does not necessarily lead to optimum performance (see the paper, *The Universal Optimum Power Cycle for Elastic Fluid Turbine Power Plants,* Ref. 6); but the relative merits of the various cycles are not grossly distorted by this simplification.

Figures 7*a*, 7*b*, and 7*c* are plots of the ideal performance of the three basic gas turbine cycles and the predicted performance of corresponding practical plants. Figure 7*a* deals with the simple Brayton cycle. As indicated above (eq. 4), the theoretical efficiency is a function of pressure ratio alone. When the theoretical processes are carried out in real machines, the efficiency becomes dependent on turbine inlet temperature; it no longer increases indefinitely with pressure ratio. The air rate, second only to efficiency in importance as a performance parameter, is even more sensitive to turbine inlet temperature. For example, a 1500 F simple gas turbine develops 60% more power than the same plant working at 1200 F.

As indicated on Fig. 7*b*, the maximum efficiency of the ideal regenerative Brayton cycle is developed at unity pressure ratio. This maximum is the Carnot efficiency corresponding to the upper and lower limits of cycle temperature. That is, it is a sensitive function of turbine inlet temperature. Although an actual regenerative cycle necessarily has zero efficiency at unity pressure ratio (zero output), it reflects the downward trend of the ideal cycle efficiency as pressure ratio increases. As the degree of regeneration is increased, the

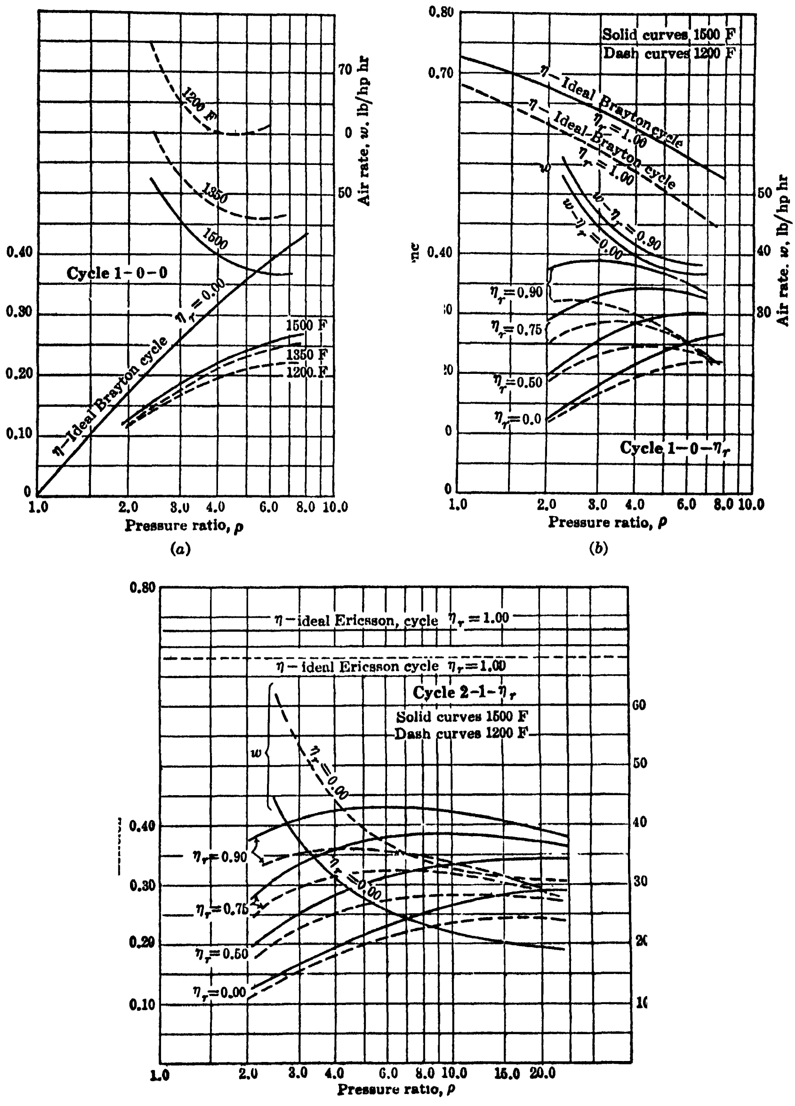


FIG. 7. Effect of pressure ratio, turbine inlet temperature, and regenerator effectiveness on cycle efficiency and air rate of various gas turbine cycle arrangements. (See p. 10-12 for cycle notation.)

optimum pressure ratio decreases. It is also clear that the addition of regeneration is particularly effective with high turbine inlet temperature. Since the addition of a regenerator scarcely affects the work developed by the turbine or that absorbed by the compressor (unless appreciable pressure losses occur), the net output, as measured by the air rate, is virtually unaffected.

The full regenerative Ericsson cycle represents the ultimate in power gas turbine processes. Figure 7*c* indicates that ideal cycle efficiency is independent of pressure ratio. For all values of *p* it equals the Carnot efficiency. The 2–1–η_r cycle introduces a single intercooler and a reheater in addition to a regenerator. It constitutes a first approximation to

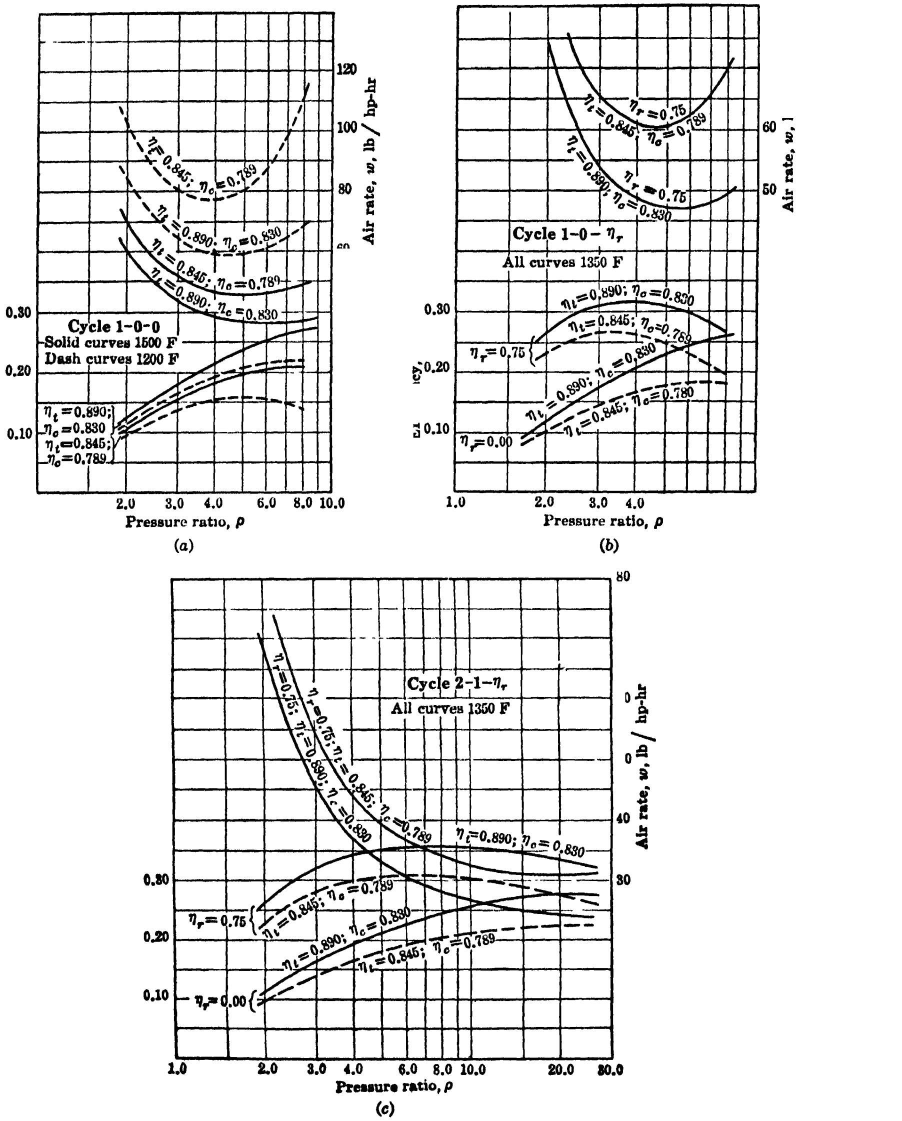


Fig. 8. Effect of pressure ratio and machine efficiency on cycle efficiency and air rate of various gas turbine cycle arrangements. (See p. 10-12 for cycle notation.)

isothermal compression and expansion and the full heat recovery of the ideal Ericsson cycle. This cycle, when it incorporates reasonably complete regeneration, reflects to a marked degree the independence of efficiency with respect to pressure ratio observed in the ideal cycle. As compared to the regenerative Brayton cycle, appreciable improvement in efficiency is indicated. However, the most striking gain is made in air rate. Almost twice as much useful output per pound of air is obtainable with this cycle as can be developed in the simple cycle. Thus the greater complexity of the equipment is offset by a corresponding reduction in its size.

Figure 8 illustrates the extreme sensitivity of the gas turbine power plant to individual

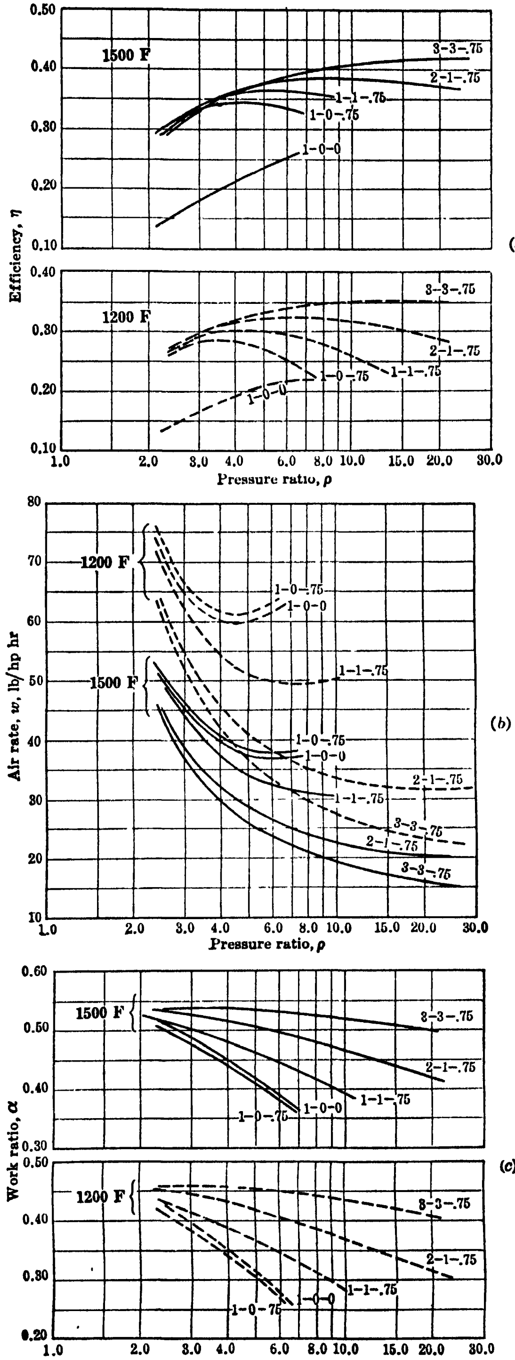


FIG. 9. Effect of pressure ratio and turbine inlet temperature on the efficiency, air rate and work ratio for various gas turbine cycle arrangements. (See p. 10-12 for cycle notation.)

component efficiencies. For the three principal types of cycle a comparison has been made between the performances indicated on Fig. 7 and those that would be developed using turbines and compressors which are each 5% inferior to those initially assumed.

Figure 8*a* provides an insight into the reasons for the long deferment of the development of a practical gas turbine. At temperatures below 1200 F, until relatively recently considered extreme, outstanding component efficiencies must be achieved if the simple cycle is to develop any net output. Figures 8*b* and 8*c* demonstrate that the additions of regenerator, intercooler, and reheater mitigate to some extent the unfavorable effect of reduced component efficiencies. However, it is clear that turbine and compressor performance is of paramount importance for all classes of gas turbine power plants.

PERFORMANCE RATIOS. Figure 9 constitutes a summary of the three most significant performance ratios, efficiency, air rate, and work ratio, for a variety of cycles. Combinations varying in complexity from the simple 1–0–0.75 cycle to the elaborate 3–3–0.75 have been studied at temperatures of 1200 and 1500 F. These temperatures approximately bracket current gas turbine practice. The assumed 75% regeneration lies at the approximate upper limit of practicality for low-pressure ratio open-cycle applications. It is somewhat low for closed-cycle designs.

The law of diminishing returns applies to the addition of elements to a gas turbine cycle. The advantages of added stages of intercooling and reheat are realized in full measure only by increasing cycle pressure ratio and meeting the attendant mechanical design problems. However, the favorable air rates associated with these optimum pressure ratios provide added incentive to suffer the added complications. This applies particularly to large capacity plants which would otherwise involve turbines of such size that disk forgings and similar high temperature parts would not be readily procurable in the alloys required.

Improvement in work ratio insures relative insensitivity to deterioration of component efficiency in service. This may be expressed with reasonable accuracy as follows:

$$\frac{\eta'}{\eta} = 1 + \epsilon_t \frac{\eta_m - (\eta/\eta_b)}{\alpha} \tag{5}$$

and

$$\frac{\eta'}{\eta} \quad 1 + \epsilon_c \frac{\eta_m - \alpha}{-} \tag{6}$$

where η and η' are cycle efficiencies before and after changes in turbine or compressor efficiency, respectively; ϵ_t and ϵ_c are changes in turbine and compressor efficiencies respectively, in percentage; η_m is mechanical efficiency (assumed equal for compressor and turbine); η_b is combustor efficiency and α is work ratio.

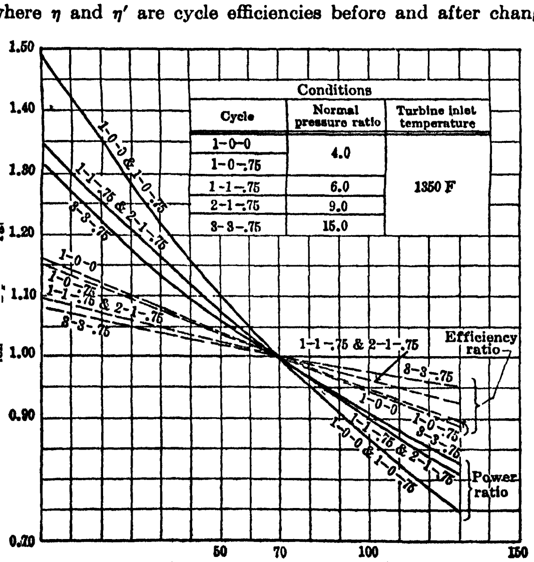


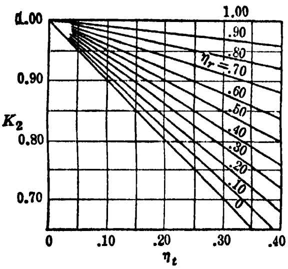
FIG. 10. Influence of compressor inlet temperature on output and efficiency of typical cycles. (See p. 10-12 for cycle notation.)

EFFECT OF DENSITY. Except for the minor influence of mechanical losses, the effect of altitude (inlet pressure) is to decrease the useful output of a given physical plant in proportion to the decrease in density of the air entering the first stage of compression. Conversely, a closed-cycle plant of given size develops power in direct proportion to the cycle pressure level.

EFFECT OF TEMPERATURE. Inlet temperature is of more fundamental significance. Since, for a given turbine design, the top working temperature is fixed, the theoretical efficiency of any cycle is a function of compressor inlet temperature. Moreover, actual machines are even more sensitive—both with respect to efficiency and air rate—to differences in relative temperature levels of compressor and turbine than the ideal cycles. Thus the low temperatures often associated with high altitudes may largely compensate, in aircraft, for the adverse effect of the latter on output and, at the same time, substantially improve efficiency. Figure 10 gives quantitative expression to the relationships between plant performance and intake temperature.

In reality the effect of compressor inlet temperature is dependent on compressor characteristics, intercooling temperatures, methods of control, etc. Figure 10 is based on the following representative conditions: (1) The mass of air inducted into the cycle is proportional to intake density. (2) The overall pressure ratio is proportional to intake density. (3) Intercooling temperature is independent of inlet temperature. Under these conditions it is seen that the sensitivity of a gas turbine power plant to compressor inlet temperature depends primarily on the number of stages of intercooling. As might be expected, the larger the number of compression stages, the less sensitive is the cycle to compressor inlet temperature.

EFFECT OF PRESSURE LOSS. Not indicated on the curves is the importance of minimizing parasitic pressure losses. Because the work of compression is such a large



Notes:

1. Losses found from these curves are closely approximate for other cycles.
2. 1% pressure drop exists across any restriction in the plant having 0.99 pressure ratio.
3. Various pressure drops may be added to get overall effect, to a first approximation.
4. Nomenclature:

K_1 = loss in output per 1% pressure drop

K_2 = recovery factor

K_1K_2 = % increase in fuel rate per 1% pressure drop

$K_1K_2 = \frac{x}{x-1}\,\frac{1}{3.5}\,(R-1)\left[1-\eta_t\,(1-\eta_r)\right]$

$R = \frac{\text{Gross turbine output}}{\text{Net plant output}} = \frac{1}{\alpha}$

η_t = plant thermal efficiency

η_r = regenerator effectiveness

*For increased accuracy divide the result by $1-(1+K_1)\Delta$ where

$$\Delta \equiv \frac{\Delta p}{p}$$

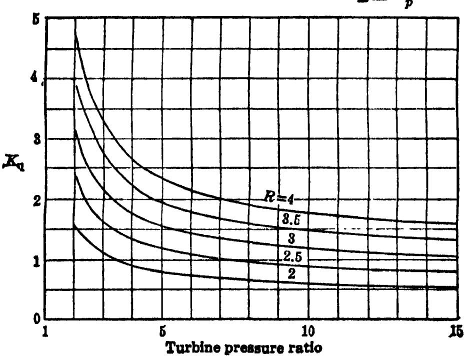


Fig. 11. Approximate effect of pressure drop on net output and fuel rate of simple gas turbine cycle, $1-0-\eta_r$.

fraction of the total work developed by the turbine, the pressure developed must be maintained at highest possible level for application to the turbine. An increase in parasitic pressure drop may be regarded as extra compression work or a decrease in the efficiency with which the compressor develops turbine inlet pressure. If we adopt the latter viewpoint, we may apply eq. 6 to evaluate the effect of a 1% pressure drop between compressor discharge and turbine inlet on each of several cycles. Table 2 shows the results of such a calculation. Increased cycle temperature reduces sensitivity to pressure losses; the addition of regeneration has little effect in this respect but intercooling and reheat increase the cycle pressure ratio at which sensitivity is a minimum.

Similar tables could be constructed showing the effects of parasitic losses ahead of the compressors and after the turbines. Likewise, the effects on cycle output may be determined. J. K. Salisbury, in his discussion of the paper *A Marine Gas Turbine Plant* (Ref. 7) presented the curves reproduced in Fig. 11. These curves make possible quick evaluation of the effects of pressure losses on fuel rate and plant output.

Table 2

Cycle	Turbine Inlet Temperature, °F	p	h_c Compression Work, Btu/lb	Δp Increment of Parasitic Pressure Drop, psi	$p' = 1.01p$	h_c' Compression Work, Btu/lb	Δh_c	$\epsilon_c = \frac{-\Delta h_c}{h_c}$	α	$\frac{\eta'}{\eta} - 1$
1-0-0	1200	2.0	32.75	0.293	2.02	33.26	0.51	−.0156	.464	−.0173
		4.0	72.85	0.587	4.04	73.48	0.63	−.0087	.363	−.0148
		6.0	100.1	0.880	6.06	100.83	0.73	−.0073	.290	−.0174
1-0-0	1500	2.0	32.75	0.293	2.02	33.26	0.51	−.0156	.546	−.0124
		4.0	72.85	0.587	4.04	73.48	0.63	−.0087	.460	−.0098
		6.0	100.1	0.880	6.06	100.83	0.73	−.0073	.399	−.0106
1-0-0.75	1350	2.0	32.75	0.293	2.02	33.26	0.51	−.0156	.491	−.0155
		4.0	72.85	0.587	4.04	73.48	0.63	−.0087	.407	−.0123
		6.0	100.1	0.880	6.06	100.83	0.73	−.0073	.342	−.0136
2-1-0.75	1350	4.0	66.85	0.587	4.04	67.87	1.02	−.0153	.491	−.0152
		9.0	112.72	1.32	9.09	113.89	1.17	−.0104	.441	−.0127
		16.0	148.50	2.35	16.16	149.78	1.28	−.0086	.403	−.0123

8. PARTIAL-LOAD PERFORMANCE

Figures 7 through 10 are constructed primarily to reveal the relationship of *design point* performance to the many variables at the designer's disposal. They provide, in addition, a certain insight into the *part-load* performance of various gas turbine cycles. Clearly the most significant factors affecting plant output are (1) air flow, (2) pressure ratio, and (3) turbine inlet temperature. Depending on the nature of the connected load, the cycle arrangement employed, the specific components used, these factors are controlled in many ways to vary load. Figures 3 and 4 give illustrative data on several types of plant.

PART-LOAD OPERATION. (a) If flow alone is varied (that is, pressure level), efficiency remains virtually independent of output down to loads where mechanical losses become important. The data presented in Table 4 for the Escher Wyss closed cycle illustrate this point.

(b) If both flow and pressure ratio are varied while turbine inlet temperature is held constant, similar favorable part-load characteristics are maintained. Figures 7, 8, and 9 indicate that the more complex cycles have efficiencies nearly independent of pressure ratio over considerable range. The Elliott marine turbine referred to in Table 4 demonstrates this mode of part-load operation.

(c) Diminished pressure ratio and turbine inlet temperature, at essentially constant air flow. For simple nonregenerative cycles this leads to very low efficiencies. However, the addition of a heat exchanger leads to considerable improvement in this respect, as may be seen on Fig. 7*b*. Unfortunately, off-design operation usually involves reduced component efficiencies and the losses in cycle performance indicated on Fig. 8. Table 4 includes data on the guaranteed performance of a Brown Boveri simple nonregenerative cycle for electric power generation.

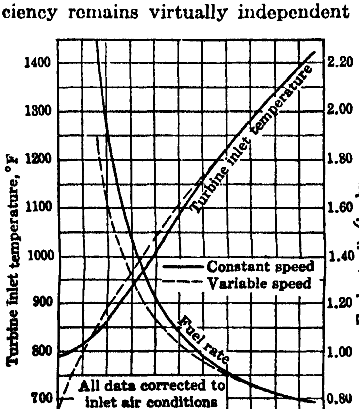


FIG. 12. Part-load performance of 2000-hp locomotive gas turbine unit. (Courtesy of Westinghouse Electric Corp.)

(d) Simultaneous reduction of flow, temperature, and pressure ratio (e.g., locomotive drives). Part-load performance in these instances falls between that for cases (b) and (c). Figure 12, which presents actual test data for the Westinghouse 2000-hp experimental gas turbine indicates the practical difference between constant- and variable-speed operation with respect to part-load performance.

Table 3. A Qualitative Comparison of Several Types of Gas Turbine Power Plants

	Classification of Plant	Applications	Probable Physical Characteristics: Size	Weight	Relative Capital Investment	Practical Capacity Range	Probable Fuel Consumption Characteristics: Full Load	Part Load	Operating Characteristics	Remarks
Open Cycles	Simple, nonregenerative 1-0-0	Mobile power units, locomotives, standby power	Very compact	Light	Lowest	100 to 5000 hp	Poor for low turbine temperatures, mediocre for high turbine temperatures	Very poor for constant-speed operation, poor for variable-speed operation	Requires no water, auxiliary equipment and controls simple and at a minimum	Favored for its extreme simplicity
Open Cycles	Simple, regenerative $1\text{-}0\text{-}\eta_r$	Mobile power units, locomotives, stationary	Compact	Moderate	Low	500 to 8000 hp	Fair	Poor for constant-speed operation, mediocre for variable-speed operation	Requires no water, auxiliary equipment and controls simple and at a minimum	Considerable improvement in economy gained at a reasonable sacrifice of simplicity
Open Cycles	Simple, with waste-heat boiler 1-0-0	Power plus process heat, power plus steam for power	Compact	Moderate	Low	500 to 8000 hp	Good provided $\frac{\text{power}}{\text{heat}}$ ratio is favorable	Good provided $\frac{\text{power}}{\text{heat}}$ ratio is favorable	Complications added to maintain proper $\frac{\text{power}}{\text{heat}}$ ratio	Good Btu economy possible for wide range of $\frac{\text{power}}{\text{heat}}$ ratios
Open Cycles	Compound, reheating, intercooling, and regenerative $n\text{-}m\text{-}\eta_r$	Marine, stationary	Large	Moderate	High	2000 to 40,000 hp	Good to excellent, depending on degree of compounding and maximum temperatures	Fair to very good, depending on load vs speed relationship and/or floating shaft arrangement	Requires some water and considerable amount of auxiliary equipment, control more complicated	Retains simplicity of internal combustion, regeneration limited owing to atmospheric pressure on gas side
Semiclosed Cycles	Heat addition by internal combustion only, compound, regenerative	Marine, stationary	Large	Moderate	High	5000 to 20,000 hp	Fair to very good, depending on degree of compounding and maximum temperatures	Good to very good, depending on load vs. speed relationship and/or floating shaft arrangement	Uses substantial quantity of water and considerable auxiliary equipment, control extensive	Combustor must operate with very little excess air, combustion products pass through turbines, aftercooler required
Semiclosed Cycles	Heat addition by combination of internal combustion and heaters, compound, regenerative	Marine, stationary	Large	Moderate	High	5000 to 30,000 hp	Fair to very good, depending on degree of compounding and maximum temperatures	Good to very good, depending on load vs. speed relationship and/or floating shaft arrangement	Uses substantial quantity of water and considerable auxiliary equipment, control extensive	Heat added to recirculating fluid through tube walls, combustion products pass through low pressure turbine(s), aftercooler required
Closed Cycles	Compound, regenerative	Marine, stationary	Large	Heavy	High	5000 to 35,000 hp	Good to very good, depending on degree of compounding and maximum temperatures	Excellent	Has largest water consumption and requires considerable auxiliary equipment, control very extensive	Use of heaters reduces permissible maximum operating temperatures and involves combustion loss but adds flexibility with respect to choice of fuel. Because of high pressure levels high regenerator effectiveness is possible

Table 4. Principal Features and Test Perform-

Type of Plant Cycle	Manufacturer and Application	Components				
		Turbines	Compressors	Combustors or Heaters	Regenerator	Coolers
Simple, no regeneration, single shaft 1-0-0	Brown Boveri (underground standby at Neuchatel)	One unit, reaction type, 7 stages, 1100 F inlet	One unit, axial type, 24 stages 4.4 : 1 pressure ratio	One unit, single burner. Fuel, gas or oil	None	None
Simple, no regeneration, single shaft 1-0-0	Westinghouse (locomotive or general purpose)	One unit, reaction type, 8 stages, 1350 F inlet	One unit, axial type, 20 stages, 5 : 1 pressure ratio	12 units, basket type. Fuel, no 2 oil (Bunker C burned experimentally)	None	None
Simple, no regeneration, single shaft 1-0-0	General Electric (locomotive or general purpose)	Two-stage, 1400 F inlet	Axial type, 15 stages, 6 : 1 pressure ratio	6 units, basket type. Fuel, Bunker C oil	None	None
Simple, regeneration, single shaft 1-0-η_r	Brown Boveri (locomotive on Swiss Federal Railways)	One unit, reaction type, 5 stages, 1100 F inlet	One unit, axial type, 18 stages, 4 : 1 pressure ratio	One unit, single burner. Fuel, no. 2 and heavy fuel oil	Tubular, cross flow. Air in tubes	None
Open with regeneration, reheat and intercooling 2-1-η_r	Elliott (experimental USN development)	Two units, high and low pressure, reaction type, 15 and 12 stages, 1200 F inlet	Two units, high and low pressure, positive displacement (Lysholm) type, l.p.—2 pair rotors in one casing, 2.9 : 1 pressure ratio, h.p.—1 pair rotors in casing, 2.3 : 1 pressure ratio	Two units, high and low pressure, elbow type, double burners. Fuel, no. 2 oil	Tubular, counterflow, air in tubes, η_r = 0.75	One compressor intercooler, flat finned tubes
Open with no regeneration, but with reheat and intercooling 2-1-0	Brown Boveri (stationary power plant at Filaret, Rumania)	Two units, high and low pressure, reaction type, about 7 stages each, 1100 F inlet	Two units, high and low pressure, axial type, l.p.—3.5 : 1 pressure ratio, h.p.—3.4 : 1 pressure ratio	Two units, high and low pressure, single burner in each. Fuel, gas	None	One compressor intercooler, plain tubes
Closed with regeneration and intercooling 1-2-η_r	Escher Wyss (experimental, stationary or marine)	Two units, high and low pressure, Rateau type, 6 stages each, h.p.—1280 F inlet, 8000 rpm, l.p.—1000 F inlet, 3000 rpm	Three units, low, intermediate and high pressure, axial type, 3.8 : 1 overall pressure ratio (92 to 350 psia), 8000 rpm	One oil-fired heater with air preheater, four burners	Tubular, counterflow, high pressure air in tubes	Two intercoolers, one aftercooler, plain tubes

ances of Several Gas Turbine Power Plants

Nominal Rating		Control	Performance (Horsepower at Turbine Coupling) (Fuel Rate, lb/hp-hr)				Date of Test (Approximate)	Remarks
Power (hp)	Speed (rpm)							
5360	3000	Temperature (constant speed)	5620	4275	500	.	July 1939	This unit is generally recognized as the first practical constant pressure gas turbine power plant
			0.77	0.85	4.1	.		
2000	8750	Temperature and flow (variable speed)	2000	1500	1000	500	Aug. 1947	The installation of one of these units as a compressor drive on a natural gas pipe line will be the first such application in the United States
			0.79	0.88	1.03	1.45		
4800	6700	Temperature and flow (variable speed)	6000	5000	2500	1000	Aug. 1947 to July 1948	One unit with waste-heat boiler installed by Oklahoma Gas & Electric Co. This is first gas turbine power-generating plant in the United States
			0 79	0.855	1.19	2.12		
2200	5200	Fuel flow and load (variable load)	2000	1500	1000	500	Fall 1941	This unit has had extensive actual railway service
			0.82	0.82	0.98	1.57		
2500	3000	Flow (variable speed) floating h-p turbine–l-p compressor shaft	2350	1750	1150	500	Dec. 1944	This was the first high-efficiency gas turbine plant to be tested in the United States
			0.47	0.49	0.55	0.72		
13,400	3000	Temperature, (constant speed on l-p compressor–l-p turbine power shaft)	16,100	11,840	8440	4560	Fall 1946	Standby installation does not justify installation of regenerator
			0.42	0.65	0.79	1.21		
2680	3000	Flow (variable density), speeds, temperatures and pressure ratios held constant	2730	2140	1300	575	Dec. 1944	This was the first closed cycle gas turbine plant
			0.44	0.45	0.47	0.57		

GAS TURBINE POWER PLANTS

9. COMPARISON WITH OTHER PRIME MOVERS

On purely technical grounds, several forms of gas turbine power plants have already demonstrated—at least on an experimental basis—certain superiorities with respect to Otto, diesel, and steam cycles. As opposed to the Otto aircraft engine the jet engine is far superior in specific weight, adaptability to ultrahigh speeds, and upper limits of thrust capacity in a single unit. It is considerably inferior in fuel consumption. It is not adapted to low-speed, light planes. "Prop-jet" units extend the range of gas turbines to lower speed applications and have improved fuel consumption. (See Section 15.)

Compared to the diesel as applied, for example, to road locomotives, simple gas turbines have advantages in space and weight and, presumably, maintenance. Their chief disadvantage is a less favorable fuel economy. This disadvantage can be expected to disappear on successful conclusion of current developments in coal combustion for gas turbines. The output of the gas turbine is more seriously affected by adverse ambient conditions (temperature and altitude) than is that of a corresponding diesel.

In heavy-duty stationary and marine applications, space-consuming heat-recovery devices, etc., partially wipe out the space and simplicity advantages in order to approach diesel efficiencies.

Process Heat. Gas turbine cycles may be adapted to supply process heat in addition to generating power. Thus application of a waste-heat boiler for the utilization of exhaust gas heat corresponds to back pressure or extraction operation of a steam turbine. By controlling the distribution of exhaust flow between regenerator and boiler a wide range of the heat generation to power development ratio is practicable.

Steam Plants. Although further development may permit favorable comparison with steam plants there are at this writing several unfavorable comparisons. The long-time reliability of the steam plant has not yet been demonstrated by the gas turbine. The installed turbine capacity for a given net output is more than twice as great. The compressors are far larger than the corresponding feedwater pumps. The plant capacity is more sensitive to deterioration of turbine and compressor efficiencies. Finally, the single-unit capacity of the gas turbine is, so far, considerably short of large steam turbine installations. However, in the relatively short development period of the gas turbine, enormous strides have been taken in the direction of increased unit outputs.

If we except aircraft applications, development of gas turbine power plants has been along the following lines and in this approximate order:

(1) Use in conjunction with *processes employing compressed or heated air.*
(2) *Stationary power applications* where light weight, compactness, and simplicity are of prime importance.
(3) *Transportation applications*—first locomotives, second ship propulsion.
(4) *Stand-by and emergency power stations.*
(5) *Moderate-capacity generating stations.*
(6) *Central station base load plants.*

Table 3 presented a qualitative comparison of various classes of gas turbine power plants. Table 4 set forth the salient features and test performances of representative pioneering units. The sections which follow describe these and other plants.

10. STATIONARY POWER PLANTS

Stationary plants for alternator drive are characterized by the necessity for a constant-speed output shaft determined by the a-c frequency. Since, for a particular output, design considerations determine the apparatus size and rpm, gearing is often necessary between turbine and alternator. The following units developed for this application are representative of commercially important gas turbine power plants.

BROWN BOVERI GAS TURBINE POWER PLANTS. This Swiss company was the first (1944) to offer a relatively complete line of gas turbines. This series of plants, as subsequently supplemented, ranges in capacity from 1000 to 27,000 kw. All plants incorporate axial-flow compressors, reaction turbines, and combustors capable of burning Bunker C oil. The basic arrangements offered are shown in Table 5.

Table 5. Brown Boveri Gas Turbine Power Plants

(Data courtesy of Brown Boveri)

Capacity, kw	Cycle	Arrangement	Full Load Efficiency,* %	Remarks
1000–5000	1–0–η_r	Single shaft	23.0–26.0	Regenerator surface from 0 to 13 sq ft/kw
5000–12000	2–1–0	Double shaft, h-p turbine and h-p compressor floating	22.0	Intended for standby service
5000–12000	2–2–η_r	Double shaft, l-p turbine and l-p and i-p compressor floating	27.7	Regenerator surface from 7.5 to 13 sq ft/kw; for base load service
27000	2–2–η_r	Similar to preceding plant except l-p turbine and compressor are double flow	34.0	High capacity associated with 40 F air inlet when used as wintertime supplement to hydro system

* Guarantee based on lower heating value of fuel.

ESCHER WYSS CLOSED-CYCLE TURBINE. The most notable performance thus far reported for a gas turbine power plant (excluding reciprocating engine-turbine combinations) is associated with this plant. A 2000-kw 1–2–η_r(η_r = 0.85 to 0.90) experimental version of this cycle, completed in 1939, has been thoroughly tested. (See Table 4.) Design studies have been made for commercial applications of this cycle in capacities up to 25,000 kw. A 12,500-kw plant is in an advanced stage of construction.

The Escher Wyss closed-cycle gas turbine (designed by Ackeret and Keller) takes advantage of the favorable effect of increased pressure level on cycle output. Instead of inducting fresh air at the low-pressure compressor inlet and heating it by internal combustion, the turbine exhaust gases are still at several atmospheres pressure; these gases are cooled in a regenerator and precooler, and then returned to the cycle. Heating is carried out *indirectly* in a tubular air heater. High thermal efficiencies are attained by approximating the Ericsson regenerative cycle by use of intercoolers, reheaters, and regenerators of high effectiveness. This scheme has several advantages in addition to the reduction in physical size of the turbine and compressor components. (1) Only clean air passes through compressors, turbines, and heat exchangers. (2) Part-load operation can be effected by reducing pressure level only. Speed remains constant, eliminating inertia effects; similarity of aerodynamic conditions throughout turbines and compressors over the load range insures essentially constant component efficiencies. (3) Indirect heating may eventually make possible the use of almost any fuel. (4) High density on both sides of the heat exchanger (regenerator) and the absence of contaminants make possible nearly complete regeneration (over 90%) with apparatus of reasonable dimensions. (5) High Reynolds' numbers, associated with high densities, and the fixed velocity ratios existing over the entire load range make the design of highly efficient turbines and compressors possible. (6) Since pressure *ratios* are constant in the cycle, temperature drops in the turbines remain close to their full load values over the entire load range, thus reducing the maximum temperature to which the heat exchanger is subjected.

The prices that must be paid for these advantages may be partly summarized as follows. (1) The air heater is an elaborate and technically difficult piece of equipment with a number of important auxiliaries. Tube temperatures necessarily exceed the peak cycle temperature even though they approach the latter more closely than the combustion temperatures. The design is feasible only on this account, and because the cleanliness of the gases inside the tubes permits the use of very small tubes (0.15 to 0.25 in. in diameter). (2) Combustion losses are inherent in the externally fired heater as opposed to the internally fired combustor. (3) The plant is complicated by the addition of a precooler and its attendant water consumption (air cooling is possible if more extensive heat-transfer surface is employed—in any case, considerably less water is required than for steam plant operation, because much larger temperature rises in the coolant are allowable). (4) Compressor inlet temperatures will generally be higher than for the open cycle, and pressure drops in the aftercooler and heater increase the back-work ratio. (5) Means must be provided for changing the cycle pressure level for load control (see below). (6) High pressures and high temperatures present design difficulties, e.g., shaft seals, expansion joints, turbine casings. For further details on this plant see Refs. 8 and 9.

SULZER HIGH-PRESSURE GAS TURBINE. Sulzer Brothers, another Swiss manufacturer, has developed a semiclosed cycle suitable for large capacities. A 20,000-kw plant for generation of supplemental winter energy at Weinfelden, Switzerland, has an application similar to that of Brown Boveri's 27,000-kw open-cycle unit. Figure 13 is a diagrammatic representation of this cycle. The basic plant is a 2–3–η_r open cycle from which part of the air is abstracted and recirculated through a 1–1–η_r closed cycle. The third and fourth compression stage groups of the open cycle also handle the recirculated air for the closed cycle. The function of the closed portion of the cycle is to power these compression stages. The two stages of open-cycle heating are by internal combustion. The recirculating air is heated indirectly by the hot gases of the first combustor before entering the turbine. Combustion air is preheated by the l-p turbine exhaust in the open-cycle regenerator. Similarly, the closed-cycle heat exchanger functions to preheat the recirculating air ahead of the heater.

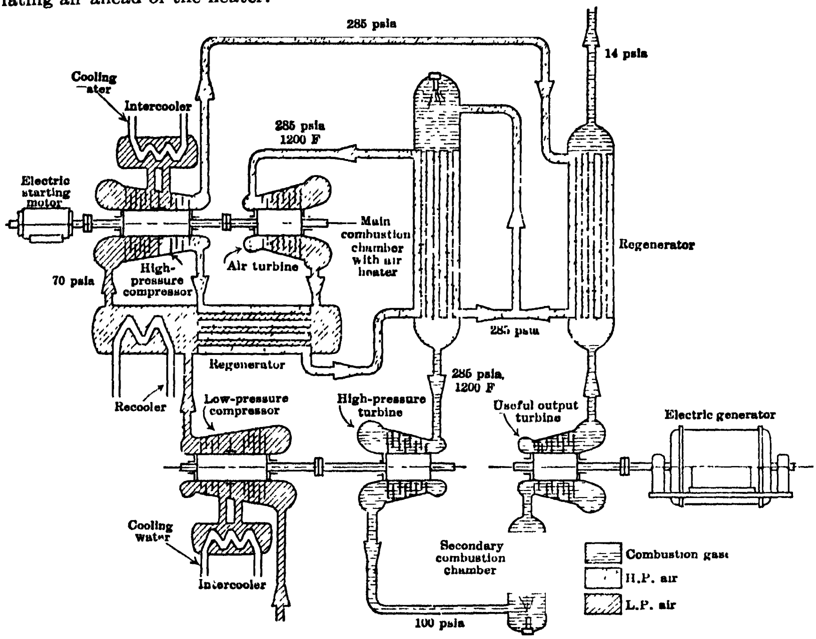


FIG. 13. Sulzer high-pressure semiclosed gas turbine cycle. (Courtesy of Sulzer Bros.)

The arrangement illustrated involves three shafts, two of which are floating. The turbine and compressor of the high-pressure circuit are coupled together and to the starting motor; the high-pressure combustion gas turbine powers the l-p compressor; and the l-p turbine following the reheater in the circuit is connected to the load. In common with other Swiss gas turbine plants, the Sulzer cycle incorporates reaction turbines and axial-flow compressors developing excellent efficiencies.

Since this development has apparently been directed at the same objectives as the Escher Wyss closed cycle, it is perhaps best evaluated by comparison with that plant. (1) Both plants make use of high-pressure levels to reduce the size of components. In addition, the main open cycle of the Sulzer plant operates at high-pressure ratio (20 : 1). So although all freshly inducted air is expanded to atmosphere, the final 7 : 1 expansion in the l-p turbine is devoted entirely to useful output. (2) Basic cycle efficiencies in each case are dependent on the degree of approximation of the Ericsson cycle. (3) Combustion products pass through two of the three Sulzer turbines and one of the two regenerators. (4) Part-load operation is less straightforward for the Sulzer plant involving changes in speeds and pressure ratios. On the other hand, control may actually be somewhat easier to effect through simple fuel regulation. (5) Perfect combustion and a virtually ashless fuel are essential for successful operation. (6) The high-pressure regenerator enjoys the same advantages as in the Escher Wyss cycle; the existence of atmospheric pressure on the gas side of the open-cycle exchanger reduces the practical limit of its effectiveness (probably to about 0.75 to 0.80). (7) The range of operating pressure ratios and speeds can be expected to make achievement of optimum component design somewhat more difficult in the Sulzer plant. (8) Under reduced load conditions regenerator average temperatures will increase while the pressures to which they are subjected decrease. (9) The

Sulzer air heater transmits only about half the input heat through the tubes. (10) Combustion losses in the Sulzer cycle are negligible, as in any internally fired plant. (11) Considerably less cooling is involved, because first-stage compression starts from the ambient temperature.

GENERAL ELECTRIC 5000-KW COMPOUND CYCLE. General Electric has projected a central station plant for delivery during 1949 (Ref. 10). Operating on a 1–1–0.75 cycle with 1500 F turbine inlet temperature, this plant is expected to have a fuel rate (Bunker C) of 0.49 lb per shp-hr or 0.70 lb per kw-hr generator output. Although reheat is not employed, the three-stage turbine is divided into two coaxial rotors, operating at independent speeds, to improve part-load performance. The first two stages are direct-connected to an 11-stage axial flow h-p compressor and through gearing to the load. The third turbine stage is coupled to a 9-stage l-p compressor. The mechanical features of the rotating components are similar to those of the 4800-hp locomotive plant described below.

Weighing approximately 300,000 lb, including the alternator, and occupying 1370 sq ft of floor space, this plant should be attractive as a standby, peak load, or end-of-line unit. Other favorable features are relatively good part-load performance and quick-starting characteristics.

11. LOCOMOTIVE POWER PLANTS

Gas turbine power plants for railroad application are attractive when compared to steam locomotives because of (1) no water consumption; (2) absence of reciprocating parts and the accompanying inertia forces; (3) lower fuel consumption; (4) improved starting traction; (5) better availability and lower maintenance; and (6) smokeless operation. Features 1 and 3, combined with better high-speed and starting characteristics, mean improved operating schedules. Gas turbines share with diesels the disadvantage of high initial cost. However, it appears that the diesel, which possesses several of the merits of the gas turbine, still justifies a larger capital expenditure. Because of size and weight considerations, larger single-unit locomotives are possible with gas turbine power. Higher fuel consumption of the gas turbine is offset by its ability to burn lower grades of oil. Successful completion of coal combustion developments would, of course, settle the item of fuel costs decisively in favor of the turbine. The greater simplicity and pure rotation of the gas turbine are expected to give maintenance and availability advantage relative to diesel installations. The possibility of using the gas turbine compressor to absorb power during dynamic braking represents a potential operational advantage.

Requirements for locomotive gas turbines are compactness, light weight, simplicity, reliability, quick starting, and reasonable fuel consumption. Gas turbines in general and simple cycles in particular lack flexibility with respect to speed-load relationships. An infinitely variable transmission (including a reversing feature) is essential, as it is for the diesel. Direct-current electric drives are generally favored. Hydromechanical transmissions have been studied by Allis-Chalmers, who have concluded they may ultimately be worthy of development. A flexible transmission system allows complete freedom in the choice of speed-power relationships for the turbine set. Generally speaking, it is found advantageous to reduce speed (and flow) at part load. This results in more economical part-load operation than is possible for constant-speed operation. (See Fig. 12.)

BROWN BOVERI LOCOMOTIVES. A 2200-hp unit developed by this manufacturer is in operation. Put in service on the Swiss Federal Railways in 1941, it has been used, when fuel oil was available, for local service. Some 60,000 miles of operation were reported in 1946. An exceptional number of train stops and gas turbine start-ups were included in this service. The power plant of this locomotive served as a model for a basic 2500-hp gas turbine which can be incorporated in multiple-unit locomotives to deliver up to 7500 hp. The simple cycle with regeneration (1–0–η_r) employed promises a peak thermal efficiency of 20% for new units. The turbine inlet temperature has been set at 1110 F. Suitable coordination of controls of multiple-unit plants will result in a broader range of total output with near-peak efficiency. The basic components, reaction turbine and axial-flow compressor, typify this manufacturer's design of similar apparatus. The following material is adapted from *Gas Turbine Locomotives*, by Walter Giger, delivered before ASME Metropolitan Section, May 23, 1946, and published in *Railway Mechanical Engineer*, Aug. 1946.

In 1939 [Brown Boveri] started construction on a 2200-hp gas turbine locomotive for the Swiss Federal Railways which was placed in service in 1941. This locomotive was described and its performance was given in an article in the *Railway Mechanical Engineer*, Feb. 1943, page 69. This article brings the record up to date and proposes designs of gas-turbine locomotives up to 7500 hp.

The original locomotive has covered approximately 60,000 miles since it was finished in 1941. It had to be taken out of service after its test runs due to lack of fuel oil. In May 1943, it was possible to obtain oil for operation on a small scale on a nonelectrified line of the Swiss Federal Railways. Only about 92 miles per day could be made and 22 stations had to be served on a local line. The average loading of the locomotive was about one-third of normal.

Considering that the gas turbine locomotive is designed for long-distance travel with relatively high average loads and few stops, this was about the worst service that could have been selected. However, there was no other line available, since all the heavy traffic main lines of the Swiss Railways are electrically operated and fuel oil had to be conserved. This service was a severe strain on the gas turbine plant, since the continuous starting and stopping meant continually varying temperatures in the turbine and the combustion chamber. The number of control functions thus performed would normally only be reached in long distance runs amounting to approximately 10 to 15 times the miles covered.

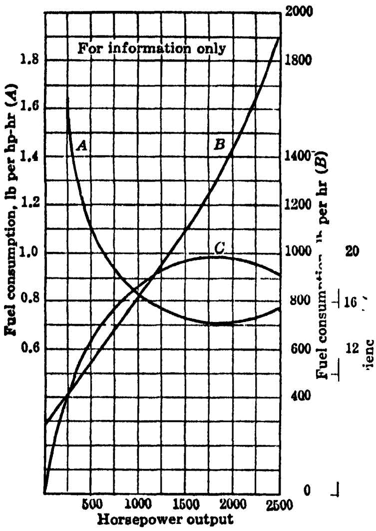


FIG. 14. Fuel consumption and efficiency of 2200-hp locomotive gas turbine at various output ratings. (Courtesy of Brown Boveri)

During this service in Switzerland, which had to be interrupted again after nine months of operation due to fuel oil shortage, the turbine was in operation approximately 1620 hr and it was started with the diesel-engine group not fewer than 1560 times. In Oct. 1945, the Swiss Federal Railways loaned the locomotive to the French National Railways and it is now [1946] operating daily on the line from Basle to Chaumont, making one round-trip of about 350 miles per day. On these runs the fuel consumption is about 45 lb per 1000 ton-miles. This fuel consumption is entirely in line with the results of many operating cost studies.

These studies included the operation of gas turbine locomotives on American railroads where the average load which the turbine has to develop is higher than on the runs from Basle to Chaumont, or for that matter on most European runs. For such trains our calculations with gas temperatures of 1110 F at the turbine intake show a fuel consumption of approximately 30 lb per 1000 ton-miles.

Considering the factors that enter into the economy of train operation, such as first cost, interest on investment, depreciation, maintenance, fuel and crew wage we estimated that the following savings per year would be reached for gas turbine locomotives as compared with diesel-electric units (with 1110 F gas temperature at turbine inlet):

(a) For a passenger train of 7 cars and 750 tons, with a 2500-hp locomotive, covering 187,000 miles per year in high-speed operations, approximately $10,000.

(b) For a passenger train of 14 cars and 1450 tons, with a 5000-hp locomotive, covering about 240,000 miles per year in high-speed operations, approximately $20,000.

These figures were estimated with fuel oil costs of approximately 2.7 cents per gallon for Bunker oil and about 4.5 cents per gallon for diesel oil, or a ratio of about 3.5.

The 2200-hp gas turbine locomotive built by Brown Boveri is the first of its kind. That company has built electrical equipment for over 2000 locomotives. The electrical parts of this locomotive are similar to the traction equipment previously built and did not involve any special problems. A different situation was found with the gas turbine plant and special designs had to be developed to be able to arrange the necessary machinery within the profile of the locomotive.

Table 6. Efficiency at the Rail of Various Locomotive Types *

Locomotive Type	Per Cent
Saturated steam, single expansion	5
Superheated steam, single expansion	7
Superheated steam, compound	8
Condensing steam turbine with electric transmission	8.5
850 ihp high-pressure steam, single expansion	10
Condensing steam turbine with gear transmission	10.2
850 ihp high-pressure condensing steam	12
Gas turbine with electric transmission	16
Diesel-electric locomotive	30

* For additional data on locomotives, see Section 14.

The locomotive is built for a maximum speed of 65 mph. During test runs speeds of 80 mph were reached and the locomotive proved to be one of the smoothest running Swiss locomotives. The gas turbine and the compressor unit also lived up to expectations and no disturbances were encountered.

Performance data for the 2200-hp locomotive are given in Fig. 14.

BROWN BOVERI 4000-HP TEST PLANT. A proposed modification of the 2500-hp basic locomotive plant incorporates a *comprex pressure-exchanger.* This device combines compression and expansion functions in a single rotor, making possible higher peak gas temperatures (1800 F) while maintaining considerably lower metal temperatures (1000 F). The cycle pressure ratio is increased from 4 to 10, accounting for an augmented output. Principal objectives of this development are reduction of space and weight.

L.D.C. COAL-BURNING GAS TURBINE LOCOMOTIVES. The Locomotive Development Committee of Bituminous Coal Research, Inc., has undertaken to produce two coal-burning gas turbine locomotives. The power plants have been built by the Elliott Company and Allis-Chalmers for installation in Baldwin and American Locomotive Company cabs, respectively. L.D.C is furnishing combustion equipment in each instance. To make the locomotives independent of fueling facilities, equipment for crushing, drying, and pulverizing run-of-mine coal is carried on the locomotives. All but the finest particles of fly ash are removed from the combustion products in a fly-ash separator. The disposition of this equipment in the Alco-Allis-Chalmers locomotive is indicated on the installation drawing (Fig. 15).

The A-C and Elliott power plants operate on similar cycles, essentially the same as in the Brown Boveri 2200-hp plant. In the A-C plant a 20-stage axial-flow compressor develops 4.8 : 1 pressure ratio. The turbine operates at 1300 F, using six reaction stages. The Elliott plant employs a two-stage centrifugal compressor to produce a 4 : 1 pressure ratio. Gas initially at 1275 F expands through the 4-stage reaction turbine. Using regenerators of about 50 to 60% effectiveness, both plants are expected to develop 4000 hp. Thermal efficiencies of 22 and 23.5% are predicted for the Elliott and A-C plants, respectively.

ELLIOTT OIL-BURNING LOCOMOTIVE PLANT. The Elliott Company and Baldwin Locomotive Works are building a 4000-hp locomotive for the Santa Fe railroad, using the same turbo-compressor design as for the L.D.C. locomotive. Figure 16, a sectional view of

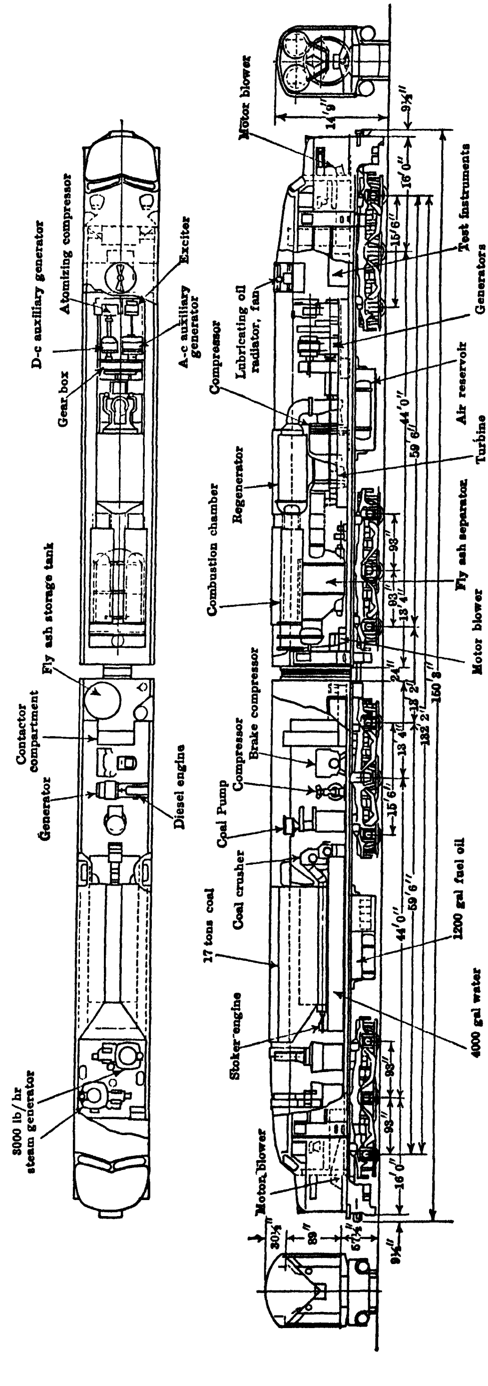


FIG. 15. Alco-Allis-Chalmers gas turbine locomotive. (Courtesy of the Locomotive Development Committee and Allis-Chalmers)

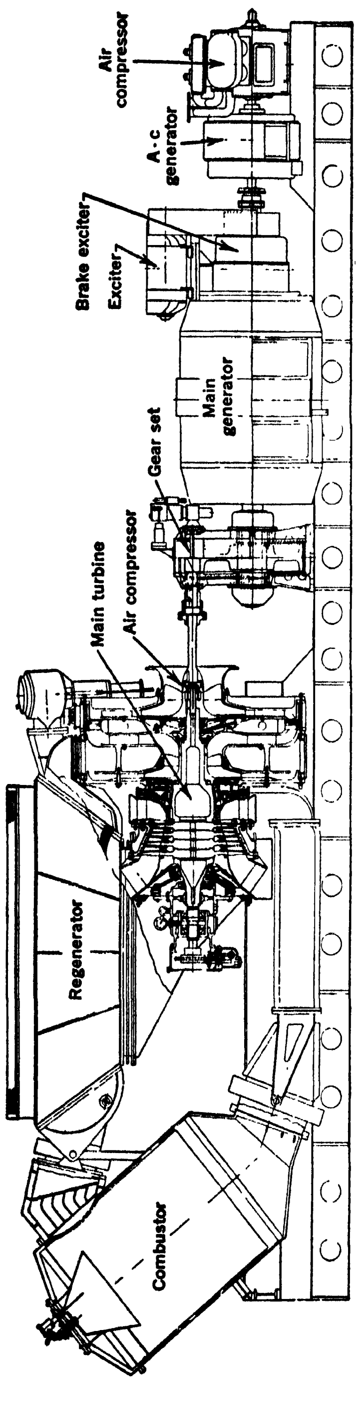


Fig. 16. Oil-burning gas turbine locomotive power plant. (Courtesy of The Elliott Co.)

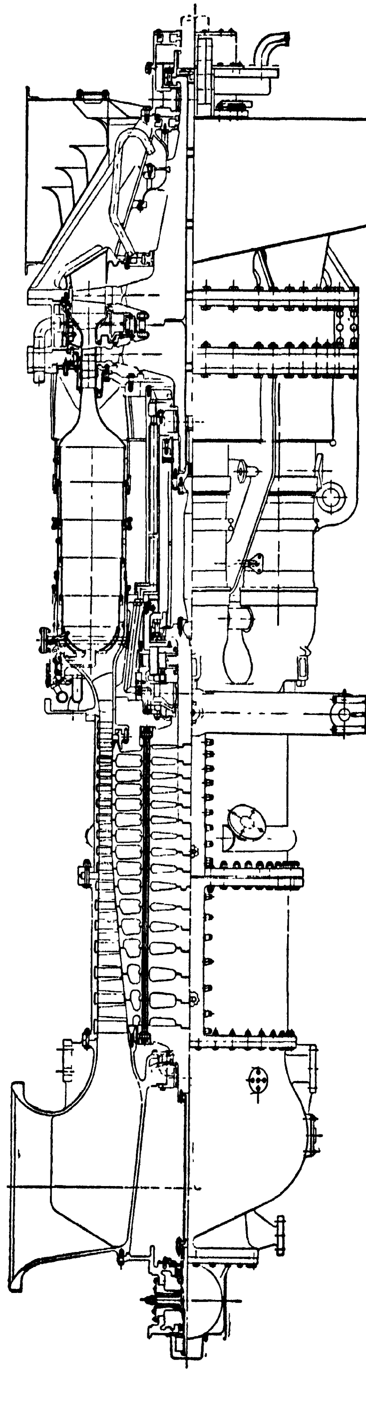


Fig. 17. Longitudinal semisection of a 4800-hp gas turbine for locomotive drive. (Courtesy of General Electric Co.)

this design, shows the relationship of the prime mover to the generating equipment. Two unusual features, of interest to railroads operating in mountainous regions and/or in hot climates, are being introduced. (1) Dynamic braking is to be provided by dumping a fraction of the cycle air after compression so that the net output of the plant becomes negative. (2) Power augmentation to offset altitude and unfavorable inlet temperature conditions is to be secured by humidifying the inlet air. By passing the inlet air over wetted filter panels its water content is increased to near saturation with an accompanying reduction in temperature. Thus mass flow is augmented, and compressor work per pound is reduced.

WESTINGHOUSE 2000-HP GAS TURBINE. This experimental plant, aimed primarily at locomotive application, has undergone considerable successful testing. (See Fig. 12.) The design will allow arrangement of two 2000-hp units side by side in a cab of standard width. Such a locomotive, developing 4000 hp, is approximately half the length of a diesel locomotive of the same output.

The 20-stage axial-flow compressor develops 5 : 1 pressure ratio. The compressed air is discharged through 12 basket- or cell-type combustors suitable for burning Bunker C fuel oil. Expansion takes place from an inlet temperature of 1350 F in eight reaction stages. A thermal efficiency of 16.7%, based on the HHV of the fuel, has been attained for full-load operation without the use of regeneration.

GENERAL ELECTRIC 4800-HP GAS TURBINE. This plant reflects the manufacturer's aircraft gas turbine experience. The sectional drawing (Fig. 17) illustrates the straight-through, direct-coupled, multiple combustion chamber arrangement which results in a compact, lightweight power plant. The gas turbine unit is self-supporting and can be mounted directly on the locomotive frame. Air cooling is used for the combustor, and for turbine stationary and rotating parts. Advantage is taken in the design of the temperature reduction thereby effected, and the use of austenitic alloys is reduced to a minimum. This economy with respect to material and adherence to standardized design is expected to solve the problem of costs.

Both compressor and turbine are high-specific-output machines. Approximately 78,000 cu ft of air per minute is compressed to 6 atmospheres in 15 axial-flow stages. The six combustion chambers are of the aircraft type, but of larger capacity and heavier construction, to increase expected life. They heat the air to 1400 F, after which expansion in the two-stage turbine develops almost 17,000 gross hp. The thermal efficiency, based on the LHV of Bunker C oil and the net power output, is 17%.

Assembled with a reduction gear and 3600-rpm alternator, this plant is rated at 3500 kw for stationary power generation. The first such installation was in the Arthur S. Huey Station of the Oklahoma Gas and Electric Company. It incorporates a waste-heat boiler to supplement the steam-generating capacity of the station.

12. MARINE POWER PLANTS

The gas turbine has been widely considered for ship propulsion, both for military and commercial service. It offers lighter weight and reduced space requirements, as compared with the steam turbine and its boiler. If extra fuel can be carried the consequent increase in cruising radius will be of military importance, and greater carrying capacity is obviously significant for merchant service. In comparison with the diesel, it is anticipated that maintenance will be reduced and more powerful units are possible. Also the higher fuel consumption can probably be offset by using cheaper heavy fuel oils.

A transmission system to provide for astern operation is an essential adjunct to the marine gas turbine. Either an electric-drive or a variable-pitch propeller (similar in construction and operation to a Kaplan turbine) may be employed. Both systems have the advantage of providing infinitely variable transmission, in addition to reversibility. However, the cube law variation of power with speed is well suited to most gas turbine cycles, so that complete flexibility of the transmission is superfluous. Either transmission must be supplemented by gearing to reduce the relatively high turbine speeds to suitable generator or propeller speeds. Electric drives, of course, are fully developed and easily controlled; however, they compare unfavorably with the variable pitch propeller with respect to weight, space requirements, cost, and efficiency.

BRITISH GUNBOAT GAS TURBINE. The significance of this plant is principally that it is actually installed in a vessel. It is a specialized military application in which the gas turbine is used to supplement the normal gasoline engines on attack runs. The power plant is a direct adaptation by the Metropolitan Vickers Electrical Company, Ltd., of a jet engine. A power turbine coaxial with the turbocompressor gas-generating unit, but not mechanically connected to it, replaces the normal jet-producing exhaust. The installation has a 9-stage axial-flow compressor, annular combustion chamber with multiple burners, two-stage compressor turbine, 4-stage power turbine, and gear. Maximum output is approximately 2600 hp with a fuel consumption of slightly over 1 lb per bhp-hr.

EXPERIMENTAL UNITS. In this country several manufacturers have developed gas turbine plants under the sponsorship of the Navy Department, Bureau of Ships. The first of these to reach completion (1944) was an Elliott Company plant designed to operate at 1200 F on a 2–1–0.75 cycle utilizing positive displacement (Lysholm) compressors. On test this plant delivered 2300 hp at a thermal efficiency (LHV) of 29.4%. Because reduction of load is accomplished by reducing flow and, consequently, pressure ratio while turbine temperatures remain fixed, exceptional part-load efficiencies were attained. (See Table 4.)

Allis-Chalmers has delivered to the Engineering Experiment Station in Annapolis an experimental 3500-hp unit designed for operation at 1500 F. This unit is undergoing exhaustive tests to develop fundamental data with respect to component performance. De Laval has likewise worked on experimental turbine plants for the Navy, but little is available on the results of these developments. Figure 18 includes a previously published sectional assembly and a view of the rotor of the high-pressure unit of a DeLaval power plant. It is of particular interest because of the use of mixed-flow (Birmann) compressor and turbine stages. Inlet temperatures up to 1500 F are possible through the use of unheated compressor air for turbine cooling.

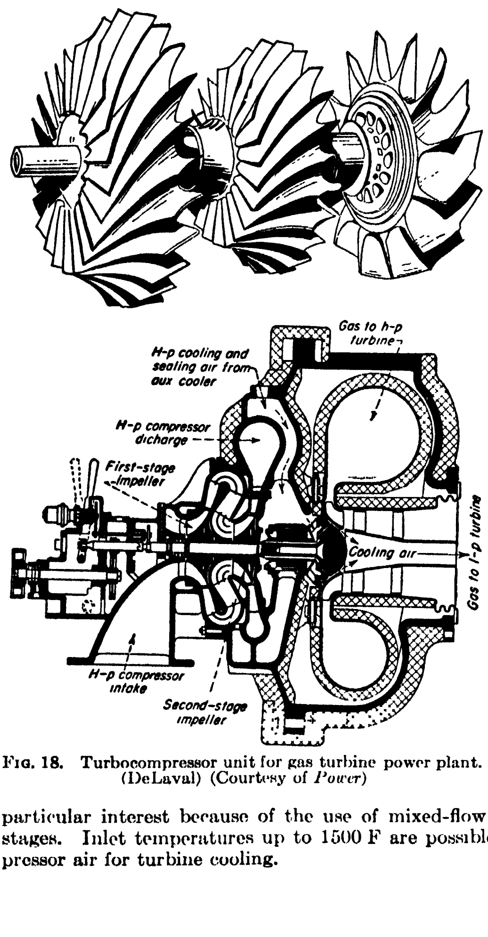


Fig. 18. Turbocompressor unit for gas turbine power plant. (DeLaval) (Courtesy of *Power*)

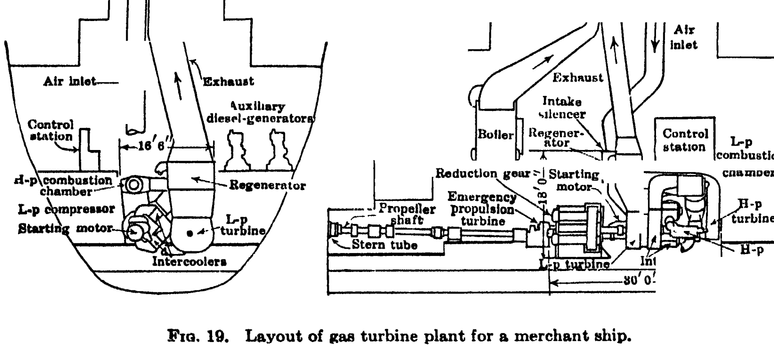


Fig. 19. Layout of gas turbine plant for a merchant ship.

ELLIOTT COMPANY 3000-HP MARINE GAS TURBINES. Three substantially identical plants are in manufacture. Two are for installation in destroyer escort vessels and one for a Maritime Commission collier. Diagrammatic installation plans are reproduced in Fig. 19. These plants are patterned closely after the earlier Navy experimental unit. The only significant thermodynamic change is the advancement of turbine inlet temperature to 1400 F to secure added output and improved efficiency. These plants embody a two-shaft arrangement in which the high-pressure turbine and low-pressure compressor comprise a floating shaft. The useful power is delivered by the low-pressure turbine, which also drives the high-pressure positive-displacement compressor.

GAS TURBINE COMPONENTS

13. TURBINES

High efficiency and suitability for high-temperature operation are prime requisites for the turbine elements of gas turbine power plants. As compared with typical steam turbines, the pressures to be handled are moderate and the inlet volume flows are high, but the expansion ratios are very much less with correspondingly lower exhaust volume flows. The absence of valve gear makes the typical constant pressure gas turbine an essentially simpler machine than its steam counterpart.

CLASSIFICATIONS OF TURBINES. (See also Section 8.) **Energy distribution** is one method of classification. Turbines may be classified as *impulse, reaction,* or *mixed staging.*

Impulse Stage. Following the established nomenclature of steam practice, a stage in which the principal pressure drop takes place in the stationary or nozzle element is called an impulse stage. For a given peripheral speed, the energy released in the impulse stage nozzle is substantially larger than in the stationary element of the reaction stage. The temperature drop is correspondingly greater, so that the rotating blading of an impulse stage runs cooler for a given stage inlet temperature. If the kinetic energy developed in the nozzle is converted to mechanical work in a single rotating row it is sometimes called a *Rateau stage.* Stages having two or more rotating rows with intermediate reversing buckets which redirect the flow, substantially without pressure drop, are called *Curtis stages.*

Curtis stages are relatively inefficient, hence not advisable where high turbine efficiency is of great importance. Rateau stages have been used in several classes of gas turbines: (1) in exhaust-gas-driven superchargers, (2) in jet engines, (3) in turbines utilizing a relatively large heat drop in the first stage to reduce the bucket temperature. (Allis-Chalmers employed this type for the experimental marine turbine.) (4) Multistage impulse turbines in which the advantages of high specific output and higher inlet temperature are balanced against somewhat lower turbine efficiencies (the General Electric locomotive turbine is an example of this arrangement).

Reaction Stage. If a large fraction of the stage pressure drop is taken across the rotating element, the arrangement is called a reaction stage. If the stage is *symmetrical* (equal pressure drops in rotating and stationary elements) it may be referred to as a Parsons stage. Such a flow arrangement has an optimum velocity ratio (ratio of tangential velocity of rotating blading to nozzle jet velocity) about equal to unity. This relatively high velocity ratio results in use of more stages than in an impulse turbine.

Mixed Stage. Because of centrifugal action and radially varying peripheral speeds, the velocity ratio and pressure distribution vary from blade base to tip in an axial-flow turbine. When the ratio of blade length to mean diameter is small (say $L/D < 0.10$) a two-dimensional basis of nozzle and blade design is adequate. Constant cross-section impulse or reaction blading is employed in such stages. For longer blading the three-dimensional aspects of flow become important, leading to the use of mixed stages using *vortex* or *constant circulation* blade design. (See Sections 1, 8, and 15.) This principle of design rests on the premise that free vortex flow should exist between all rows, suppressing radial components of flow, and giving equal aerodynamic force per unit of blade length, constant axial components of velocity over the entire blade height, and probably improved efficiency.

Constant-circulation blading has long been used in the low-pressure stages of condensing steam turbines. In gas turbines, even first-stage blades are often of such length that vortex design is applicable. Constant circulation is effected by the use of twisted blades. A constant circulation stage which is symmetrical at midheight has more than 50% reaction at the OD and approaches impulse design at the blading ID. In fact, if blade length is excessive, the base section of the stage becomes a compressing rather than expanding element. On this account $L/D = 0.3$ represents a design limit. Because of its superior efficiency, the mixed stage utilizing constant-circulation blading has been selected by many designers for power gas turbine applications.

Flow Pattern. Most gas turbines are of the axial-flow type. That is, radial components of gas velocity are negligible and the blading is arranged in radial array. Mixed-flow turbines have also been built. In the DeLaval mixed-flow stage utilizing a *Birmann wheel* the nozzles direct the flow tangentially with a radially inward component. The flow through the turbine wheel has high reaction due to the opposing centrifugal head developed. While flowing inwardly the gases are turned and discharged axially through the eye of the wheel, as shown in Fig. 18.

Cooling. Turbines are not easily classified mechanically. Two main approaches are used in dealing with high gas temperatures. These approaches result in *hot* or *cooled* turbines.

Hot turbines rely on symmetrical construction and various mechanical devices such as blade rings floating on radial pins or keys to maintain correct relationships between stationary and rotating parts. Because working parts attain gas temperature, this temperature must be moderate, and rotating speeds are necessarily conservative.

Cooled turbines make use of strategically directed auxiliary air, usually taken from an intermediate compressor stage, to reduce the temperatures of the hottest sections of the turbine. This makes possible utilization of higher inlet temperatures and higher speeds. Casings are sometimes kept cool by the use of internal insulation separating hot gases from the pressure-tight shell.

Actual turbines usually make use of one or more of these design expedients. The choice in each case depends on consideration of the following factors associated with extreme temperature design: (1) Material strength at temperature. (2) Distortions and relative expansions due to differences in temperature or coefficients of expansion. (3) Availability of high-temperature materials in required forms. (4) Cost of "superalloys" and of their fabrication.

These mechanical considerations are inextricably inter-related to the aerodynamic or blade path design. Thus the selection of high-energy stages for the G.E. 4800-hp locomotive turbine (Fig. 17) makes high peripheral speeds inevitable. High inlet temperature and the stresses associated with these speeds make disk cooling necessary. On the other hand, when cooled, the disk centers and shaft may be low-alloy forgings which develop greater low-temperature strength than the superalloys used in disk rims and buckets.

The larger number of reaction stages of the Westinghouse locomotive turbine permits lower speeds. Because the high aerodynamic efficiency attained makes very high inlet temperatures avoidable, the entire turbine is allowed to run "hot."

The Swiss, who carried on many of their developments during the shortages of World War II, minimized the use of high-alloy material. Thus their turbines have many stages, operate at conservative speeds, and rely for efficiency on aerodynamic excellence rather than on high inlet temperatures. They also maintain moderate casing temperatures by the use of internal insulation. This is a particularly logical solution for the problem of sustaining high pressures in the Escher Wyss cycle.

GAS TURBINE CONSTRUCTION AND MATERIALS. (For further information on materials, see *Design and Production* volume.) **Mechanical design** of gas turbines parallels that of steam turbines in many respects. Because of the thermodynamic importance of high temperatures, more attention is paid to the high-temperature resistance of materials and the stresses and strains resulting from thermal expansions. No large body of published gas turbine design practice has yet come into existence; it is important to note that individual practices lack the confirmation of prolonged operation. In the absence of service failures, the designer concerns himself with problems developed by experience with steam turbines. It can be expected that unanticipated problems peculiar to gas turbines will reveal themselves as more commercial gas turbine plants acquire service records. The time factor cannot be overemphasized. High-temperature material properties cannot be expressed except in terms of time. Furthermore, thermal stresses which follow a cyclic pattern matched to the operational cycle may reveal themselves as troublemakers only after substantial operating periods.

High-temperature Metallurgy. Because high temperatures radically reduce the strengths of even the best alloys, it becomes necessary to use these materials close to the limit if practical gas turbines are to be built. This limit is some "failing" stress of the material corresponding to service temperature and desired life. As in all design practice, the acceptability of a structure's calculated stress is judged by the margin existing between this stress and the failing stress. Depending on the method by which load is applied, e.g., pressure, centrifugal force, differential expansion, vibration, and initial set-up, and further depending on what constitutes failure, e.g., rupture, yielding, distortion, and loosening, any of several material properties may determine the failing stress. The following is a list of *significant material properties:* (1) Room-temperature yield point. (2) Short-time high-temperature yield. (3) Short-time ultimate strength (high temperature). (4) Rupture strength versus time (high temperature). (5) Creep strength (high temperature). (6) Relaxation strength. (7) High-temperature endurance limit. (8) Thermal expansion characteristics. (9) Corrosion resistance.

It is to be noted that establishment of the properties 4, 5, 6, and 7, of greatest interest to gas turbine designers, involves a large number of test determinations if *temperature* and *time* variables are to be well established. In the nature of things, the time variable cannot be fully explored. Most of the so-called 10,000-hour or 100,000-hour characteristics are obtained by extrapolating data obtained in much shorter time. For this and allied reasons, gas turbine life is somewhat speculative. Of necessity, tests of materials are made almost entirely on simple specimens simply stressed. The states of stress in a practical gas turbine structure are generally more complicated, and the relationship of the actual state of stress to the tested material properties is not always clear.

Design Criteria. To make the best use of the limited available data on high-temperature properties of gas turbine materials, it is necessary to abandon or modify the usual criteria for rotating machinery design. For example, under creep conditions, the classic elastic distribution of stresses in a rotating disk either is not developed or will not persist. It then becomes reasonable to suppose that the *average tangential stress* is a more logical, although still inadequate, measure of severity of service. This average stress may be compared to the rupture stress for the anticipated service life at operating temperature. On the other hand, this property of the material cannot be properly applied to the evaluation of the consequences of *thermal stresses* in disks. These stresses may arise from the use of composite construction (e.g., G.E. locomotive turbine rotor with austenitic rim and martensitic center) or from unfavorable temperature gradients resulting from such operational procedures as fast starting. Here distress might arise as a result of repeated plastic deformations. For some classes of thermal cycles initial yielding at high temperatures may give reversal of stress with subsequently reduced temperature; and if the cold stress does not exceed the cold yield point, reheating will not develop further permanent distortions.

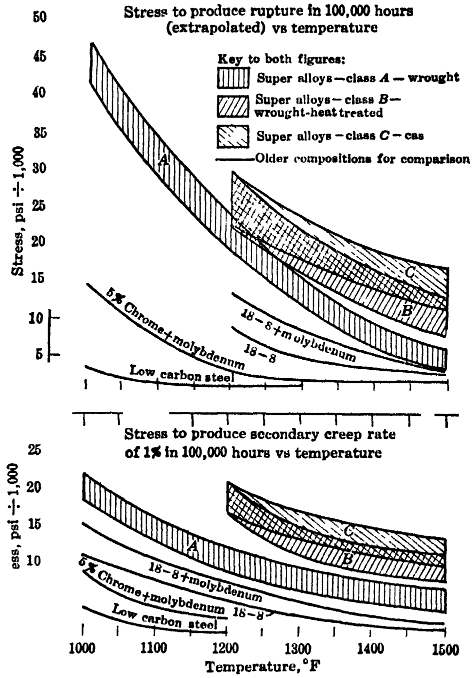


FIG. 20. Rupture and creep data for high-temperature gas turbine materials.

Since short-time high-temperature yield points are very large in relation to the long-time properties which are the basis for design, short periods of appreciable overspeeding are generally considered permissible for gas turbines. However, quantitative information on this point is lacking. The ultimate strength at temperature, as measured by short-time tests, is seldom used in design but is significant as a measure of material quality.

Gas turbine structures cannot be expected to maintain an absolutely fixed geometry. Frequently, the criterion of failure is the degree of permanent distortion anticipated after some particular period of service. This is related to the creep properties of the material. Thus, in time, rotating blades lengthen and move to a larger diameter, while stationary diaphragms may move downstream under the action of pressure forces. Similarly, bolting must be designed on the basis of relaxation properties. The elastic loading of a high-temperature bolt under constant total strain reduces continually in an approach to an asymptotic *relaxation stress.*

Blading Materials. One of the plagues of steam turbine design is fatigue failure of blading. It is clear from jet propulsion engine operating experience that gas turbines will not be free from this problem. The austenitic alloys used in gas turbine blades have very poor internal damping properties; hence large vibration amplitudes and possible failure can be anticipated when resonance exists and exciting forces have even moderate magnitude. Fortunately, gas turbine blade heights are usually comparatively short and gas forces moderate. Absence of valve gear and partial admission (except on superchargers) results in symmetry of flow and small excitations. Few data are available on the effect of alternating stresses superposed on steady loads. Indications are that even when the variable loading is a sizable fraction of the constant load, the stress-to-rupture properties are not affected appreciably.

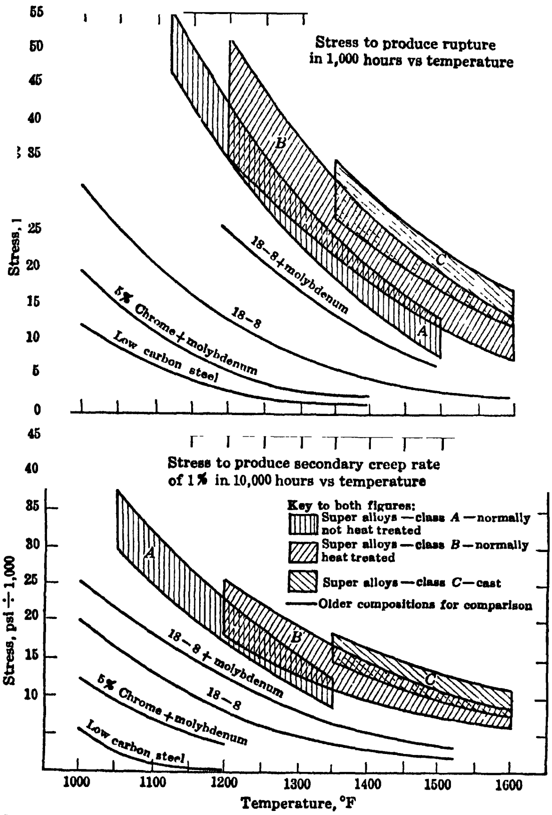


FIG. 21. Rupture and creep data for high-temperature gas turbine materials.

Superalloys. Major contributions to the gas turbine art have been made in recent years by metallurgists. This is forcefully illustrated by Figs. 20 and 21, where the properties of three classes of modern superalloys are compared with the standard stainless steels, representative high-temperature materials only a few years ago. In Table 7 a representative group of alloys are classified and their chemical compositions tabulated. It is seen that the gas turbine designer now has available 100,000 hour strengths as high as 20,000 psi at 1200 F. Even at 1500 F practical, long-life engineering structures now are possible.

Steam and gas turbines differ in design because of the large difference in costs of materials involved and limitations on the sizes of superalloy forgings obtainable. The rotor forging for the Westinghouse 2000-hp locomotive turbine represents the approximate present-day limit in size of austenitic forgings, thus accounting for the extensive use of built-up and disk-type rotors. Likewise, difficulties in producing sound stainless-steel castings in large sizes adds incentive to design fabricated casings.

Blading manufacture is a major problem for the gas turbine builder, both from the standpoint of functional adequacy and of cost. Techniques for blade fabrication are: (1) Machine from bar stock. (2) Cast to size by the lost wax precision casting method. Finish roots by milling, broaching, or crush grinding. (3) Forge to size. Finish roots as in (2). (4) Rough-forge and machine all over. Considerable work has been done in this country and abroad, particularly for aircraft application, on the fabrication of hollow blades from tubing and either flat or tapered sheet.

Blades are attached to disks by steam turbine methods, but axial or helical entry and multiple-land (Christmas tree) roots are more common in gas turbine practice. Welded

Table 7. Chemical Analyses of Some Old and New (Super) Heat-resisting Alloys Popular for Gas Turbine and Jet Engine Service in the United States

(Prepared by C. T. Evans, Jr.)

	Old Alloys				New (Super) Alloys									
					Class A		Class B						Class C	
	18-8 Columbium	18-8 Molybdenum	25-20	Inconel	19-9DL	Timken 16-25-6	N-155	S-588	S-590	S-816	Inconel X	Refractaloy 26	Vitallium (Cast)	X-40 (Cast)
Carbon	0.07	0.07	0.35	0.09	0.30	0.08	0.15	0.40	0.40	0.40	0.04	0.03	0.30	0.50
Manganese	1.00	1.00	0.75	0.75	0 75	1.50	1.00	0.75	0.75	0.75	0.75	0.70	0.30	...
Silicon	0.50	0.50	1.00	0.30	0.50	0.80	0.50	0.65	0 65	0.65	0.50	0.65	0.25	...
Chromium	18.50	17.00	25.00	13.50	19.00	16.50	21.00	19 00	20.00	20.00	15.00	18.00	28.50	25.00
Nickel	11.00	12.00	20.50	78.00	9.00	25.00	20.00	20.00	20.00	20.00	73.00	37.00	2.00	10.00
Cobalt	...	...	...	...	.	.	20.00		20.00	44.00	...	20.00	62.00	55.00
Molybdenum	...	2.50	...	...	1.25	6 25	3 00	4.00	4.00	4.00	...	3.00	5.50	...
Tungsten	...	...	...	...	1.25		2.00	4.00	4 00	4.00	..	.		7.00
Columbium	10 × C min	...		..	0.40		1.00	4.00	4.00	4.00	1.00	..	.	...
Titanium	...	.		..	0.35		.	...	.	.	2.50	3.00	.	...
Nitrogen	...			...	...	0 10	0.12	...	...	...			...	...
Aluminum	...	..		...	...	..	...	...	...	...	0.60	0.30	...	...
Iron (remainder)										3.00	7.00		1.00	0.60

attachments have been widely used for supercharger turbines and jet engines, particularly in connection with precision cast blades. The extension, in time, of inherent radial interblade cracks is a major problem in this connection.

The turbine designer must also deal with bearing and seal problems. In general bearings follow established turbine practice. Proximity of bearings to hot sections of the turbine causes heat dissipation through the lube oil. The amount of this heat rejection is seldom larger than the normal bearing losses. Therefore provision for reasonable increase of the usual bearing oil flow is adequate to carry away this additional heat. On the other hand, the existence of a temperature drop from gas inlet temperature to lube oil temperature in a short distance presents mechanical problems. If a continuous structure connects the hot portion of the unit to its bearing, an effort is made to secure continuity of temperature gradient. Another device used is interposition of an artificial temperature break or heat dam, with radially sliding keys or pins to maintain alignment. In either case, symmetry of structure is important.

Hot-gas seals are necessarily of metallic construction and may be of more or less elaborate labyrinth design. Secondary seals separating hot gases from lube oil may make use of cold sealing air which leaks into both the hot gas and the bearing spaces. In at least one instance—the Escher Wyss cycle—high-pressure oil is introduced into a bearing-like seal that prevents escape of secondary sealing air, which is then returned to the system at substantial pressure.

14. COMPRESSORS *

The output of a gas turbine power plant is the difference between turbine power and power required to drive the compressors. For this reason, the efficiency with which the compressors deliver air (or other working fluid) at specified conditions is of paramount

Table 8. Qualitative Comparison of Compressor Types

	Axial	Centrifugal	Lysholm
1. Relation to cycle characteristics			
A. Stability	Poorest—requires reduction in turbine temperature at part load	Fair—requires reduction in turbine temperature at part load	Excellent—no unstable range
B. Optimum compressor rpm compared to turbine rpm	May be made the same	May be made the same	Sometimes too low for turbine design
C. Peak efficiency	Highest	High	High
2. Physical attributes			
A. Size (See Table 9)	. . . .	. . . .	. . . .
B. Adaptability for intercooler or regenerator installation	Poorest	Fair	Excellent
C. Best type of combustion arrangement	Annular	Annular	In-duct
D. First cost	Greatest	Least	Intermediate
3. Operating characteristics			
A. Acceleration characteristics	Intermediate	Poorest	Best
B. Noise and vibration	Least	Intermediate	Silencers necessary
C. Effect of dirt in air flow	Loss in performance; filters needed	Slight loss in performance	No loss in performance, but mating surfaces may wear
4. Best applications	(1) Locomotive power plants (2) Large stationary plants (3) Closed-cycle plants (4) Large marine power plants (5) Power plants with little load variation	(1) Locomotive power plants (2) Medium stationary plants (3) Medium marine power plants	(1) Small stationary plants (2) Small marine power plants (3) Power plants with large load variation

* This article was prepared by A. H. Davis, Aerodynamics Division Engineer, Elliott Company.

importance. (See Fig. 8 and eq. 6.) The development of efficient air compressors has been a major factor in making the gas turbine power plant feasible.

Turbine-compressor Relation. The pressure-flow-temperature-efficiency curves for the turbine and the pressure-flow-speed-efficiency curves for the compressor are combined to calculate the temperature-speed-power curves of a gas turbine plant. The use of intercooling, regeneration, and reheating in the plant complicates the calculations; but the necessity for "matching" compressors and turbines is always present.

Figure 22*a* illustrates the relationship between compressor and turbine characteristics. Compressor pressure-flow characteristics are shown for several constant speeds. Turbine characteristics are shown by the dash pressure-flow curves for constant temperatures. A turbine-compressor unit is usually directly coupled so that turbine and compressor speeds are the same.

The part-load performance of the power plant is dependent not only on the characteristics of turbine and compressor but also on the speed-power relationship of the connected load. Thus if the plant powers a constant-speed alternator, more drastic reductions in temperature (hence efficiency) are necessary to reduce load than for load characteristics that permit speed reductions. Reduction of load by reduction of temperature alone does not introduce instability problems; however, the extent to which speed may be reduced at constant temperature is limited by the compressor surging limit, as shown in Fig. 22*a*. (See p. 10-39 for discussion of stability.) An optimum speed-temperature-load relationship exists for any given gas turbine power plant. An infinitely variable transmission, e.g., an electric drive, makes it possible to operate at optimum conditions.

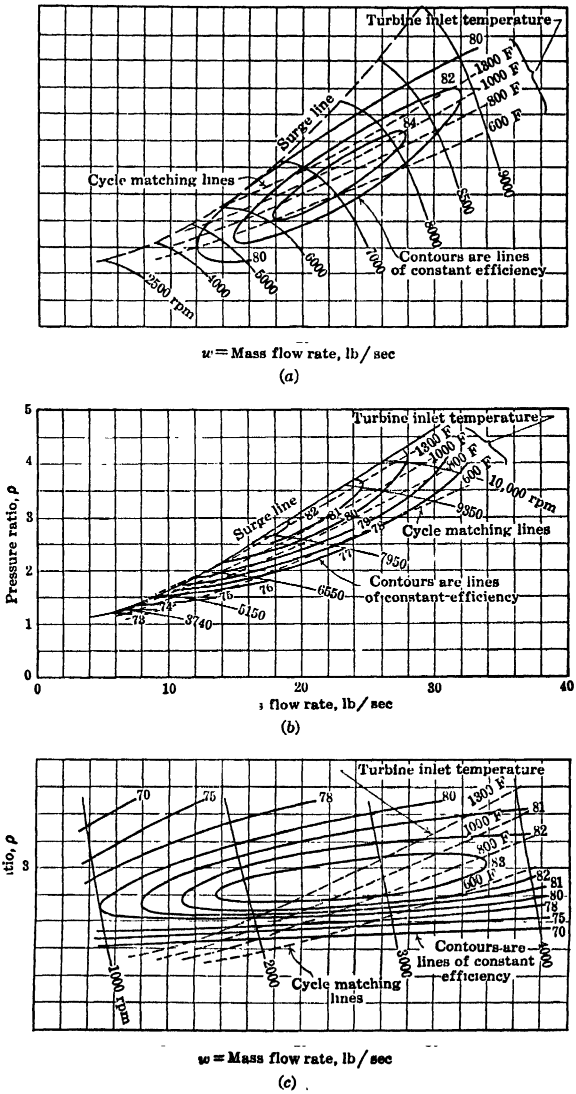


FIG. 22. Characteristics of (*a*) turbine and 20-stage axial-flow compressor; (*b*) turbine and 2-stage centrifugal compressor; (*c*) turbine and Lysholm single-stage compressor. Charts illustrate superposition of turbine flow-pressure characteristics on compressor performance map to determine operating point for any turbine inlet temperature and compressor speed.

The extent of the high-efficiency range for one compressor speed is important. It must

Table 9. Tabular Comparison of Estimated Compressor Designs

(80 F Inlet Temperature)

	20,000 cu ft/min			50,000 cu ft/min			100,000 cu ft/min		
	Axial	Centrifugal	Lysholm	Axial	Centrifugal	Lysholm	Axial	Centrifugal	Lysholm
Pressure Ratio = 4 (no intercooling)									
Number of stages	20	3	1 *	20	3	Not suitable	20	3	Not suitable
Tip speed for 80 F inlet, ft/sec †	685	1000	285	685	1000		685	1000	
Rpm for 80 F inlet	10,400	11,200	3450	6500	7050		4600	4950	
Adiabatic efficiency	0.84	0.80	0.80	0.86	0.82		0.88	0.84	
Stability, %	13	20	100	13	20		13	20	
Rotor diameter, in. †	15	20	19	24	33		34	46	
Overall diameter, in.	22	34	56	31	56		42	78	
Overall length, in.	58	35	72	90	58		125	81	
Weight, lb	4300 ‡	3400	14,000	13,000 ‡	11,000		27,000 ‡	25,000	
Pressure Ratio = 8 (with intercooling to 100 F, 2 stage groups)									
Number of stages, both stage groups	25	4	2 §	25	4	Not suitable	25	4	Not suitable
Tip speed for 80 F inlet, ft/sec ‖	755	1100	285	755	1100		755	1100	
Rpm for 80 F inlet ¶	11,500	12,000	3450	7150	8300		5050	6300	
Stage group efficiency	0.85	0.82	0.82	0.87	0.83		0.885	0.85	
Stability, %	12	17	100	12	20		12	25	
Rotor diameter, in.‖¶	15	21	19	24	30		34	40	
Overall diameter, in.¶	22	36	56	31	51		42	68	
Weight, both stage groups, lb	4300 ‡	4200	20,000	13,000 ‡	14,000		27,000 ‡	31,000	

* Two pairs of rotors in one casing, with parallel flow.
† Tip speed and diameter are those for largest rotor.
‡ Depends largely on type of rotor construction.
§ Low-pressure stage has two pairs of rotors in one casing, with parallel flow. High-pressure stage has one pair of rotors in casing.
‖ Tip speed and diameter are those for largest rotor in first stage group.
¶ Values given are for first stage group. Second stage group will usually be smaller and will run at higher speed.

be wide enough so that the compressor operates at favorable efficiency at all power-plant service conditions, including various atmospheric pressures and temperatures. Weight and size of compressors are frequently important, particularly in mobile power plants.

The three types of compressors successfully used in gas turbine power plants are discussed below briefly. Tables 8 and 9 summarize their relative merits and physical and thermodynamic characteristics.

AXIAL-FLOW COMPRESSORS (see also Section 1). **General.** The axial-flow compressor is well suited for gas turbine power plants requiring air flow greater than 20,000 cu ft per min. Alternate rotating and stationary blade rows compress air by changing its momentum. This classifies the axial-flow compressor as an *aerodynamic* or *turbo* machine, in contrast to the *displacement* type of compressor (see Section 1).

Each pair of rotating and stationary blade rows is called a *stage*. The combination of all the stages in one casing is called a *stage group*. The blades of a stage are frequently reaction blades; they are only slightly cambered, as compared to reaction blades for an axial-flow turbine. The direction of camber and air deflection in a compressor blade row is such that the air stream is decelerated relative to the row. This deceleration is accompanied by a pressure rise, the desired effect. However, because the air flow is in the direction of increasing average pressure, the tendency of the flow to "separate" from the blade and wall surfaces is much greater than in a turbine. This limits the stage work to a low value compared to that of a turbine stage. Hence, the number of axial-flow compressor stages necessary to effect a given pressure ratio may be three to five times as great as the number of turbine stages required to expand the flow through the same pressure ratio. The large number of stages and the critical aerodynamic design of the blading account for the relatively high cost of the axial-flow compressor. Recent developments in the field of *supersonic compressors*, in which each stage may be capable of as high as 4 : 1 pressure ratio, may drastically reduce the number of stages, hence the cost of this type.

Performance. The performance curves shown in Fig. 22*a* are typical. Pressure-flow-speed-efficiency relationships are shown. There is an area covering a range of pressure and flow in which the compressor operation is *unstable* and entirely unsuitable for application to gas turbine power plants. The limiting line of stable performance is variously called the *surge line*, *stability limit*, *pumping limit*, etc. Performance of the compressor in the unstable region is characterized by rapid pressure fluctuations and a surging of flow. The range of stable operation at one compressor speed defines a quantity known as *stability* (%). For any speed, the stability is

$$\text{Stability (\%)} = 100\left(1 - \frac{\text{flow at surge line for a given speed}}{\text{design flow for the same speed}}\right)$$

Axial-flow compressors generally have steep pressure-volume characteristics and relatively low stability (about 10 to 13%).

The axial-flow compressor is most suitable for large gas turbine power plants, where it is both more efficient and smaller in size than either the centrifugal or Lysholm types. For closed-cycle plants, the axial compressor has a decided advantage, for it can be made to operate at its peak efficiency for all loads, thus avoiding the disadvantages of its poor stability characteristics.

Construction. The mechanical problems associated with axial-flow compressors are similar to those familiar to turbine designers: blade attachment and vibration, bearings, seals, casing tightness, etc. Because of the lower working temperatures, these problems are less severe in a compressor. The *rotor* carrying the rotating blades is usually of the *drum* rather than the *disk* type, although solid rotors are sometimes used. The *running clearances* between free ends of rotating and stationary blades and their respective adjacent stationary and rotating wall surfaces must be small to minimize leakage. Rotating blades usually are not shrouded; the sealing line is the tip of the blade. Stationary blades, attached directly to the outer-casing shell, may be either shrouded or not shrouded. If shrouded, sometimes there are several labyrinth-forming seal strips running around the inner periphery of the shroud. Shrouds have the additional function of minimizing vibration of blades which, owing to their lack of camber, are particularly susceptible to fatigue.

The rotor weight usually is supported on conventional journal bearings, and the axial loads arising from pressure differences and aerodynamic reactions are carried on floating-shoe thrust bearings. Solidly coupled turbine-compressor combinations may be arranged to have their rotor thrusts partially balanced. A multiple labyrinth seal is provided around the shaft at the discharge end of the compressor to reduce leakage to atmosphere.

The air may be discharged from the compressor in an annular stream going to an annular heat exchanger or combustor; or it may be collected in a casing and discharged through a side opening into a duct.

CENTRIFUGAL COMPRESSORS. General. The centrifugal compressor is well suited for gas turbine power plants having an air flow less than 70,000 cu ft per min. It is an aerodynamic or turbo compressor since compression is produced by change in momentum of air passing through the rotor. A cross section of a multistage centrifugal compressor is shown in Fig. 16. Air enters the machine at the right in an axial direction. As it passes through the first impeller, the flow is turned from the axial to the radial direction. In this process the air acquires tangential momentum from the impeller blades and is discharged at an elevated pressure with high velocity. The impeller blades may have radial tips or they may be inclined backward at the impeller outside diameter. Backward-directed vanes in an impeller reduce the pressure obtainable for a given speed and size, but usually increase the stability and efficiency, compared to an impeller with radial blade tips. A *mixed-flow* impeller is one in which the velocity leaving the impeller has an axial component. The *Birmann wheels* of Fig. 18 typify the mixed-flow concept. Impellers frequently have an *inducer section.* This term applies to the first portion of the blading which accelerates the air tangentially while the velocity is essentially axial and before it has an appreciable radial component.

The *diffuser section* receives the high-velocity stream and decelerates it gradually, producing additional pressure. The diffuser usually has a set of guide vanes to aid in deceleration of the air. Proper design of these vanes is most important in obtaining good overall performance. The combination of one impeller and diffuser is called a *stage.* The combination of all the stages in one casing is called a *stage group.* As the air leaves the diffuser in a multistage machine, it is guided into the next impeller inlet by the *interstage passage.* After leaving the last diffuser, the air is received by the collector casing and is delivered into the discharge duct. The interstage passages frequently have one or more sets of vanes to complete the deceleration of the gas before entering the following impeller. The *collector chamber* may be either a plenum chamber built around the last diffuser discharge or a spiral volute.

Performance. A typical performance chart for a multistage centrifugal compressor is shown in Fig. 22*b*. There is a region of instability similar to that described for the axial-flow compressor. The definition of stability is the same as for axial-flow compressors. It will be seen from Fig. 22*b* that the performance curves are flatter than those of the axial-flow compressor, that is, the stability is greater.

Construction. The construction of centrifugal compressors is similar to that of other turbo machinery. Impellers may be fabricated by welding, or made from castings or forgings. They are usually made from steel or an aluminum alloy. Radial loads are usually supported by journal bearings and axial loads by floating-shoe thrust bearings. Leakage from high-pressure regions to low-pressure regions is reduced by labyrinth seals.

DISPLACEMENT COMPRESSORS. Lysholm Compressors. The Lysholm compressor, a displacement type, is the only rotary machine in this category that has been applied to gas turbines. It compresses air between closely fitting lobed rotors and their casing before discharging into the region of higher pressure. This differs from the action of other rotary-lobe blowers which merely transport air from a low-pressure to a high-pressure region, where compression is accomplished by the back flow of high-pressure gas.

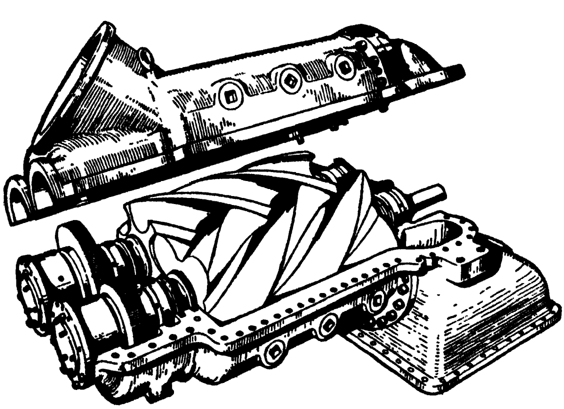


Fig. 23. A typical Lysholm compressor. (The Elliott Co.) (Courtesy of *Power*)

A typical Lysholm compressor is shown in Fig. 23. The rotors have an unequal number of conjugate helical lobes. The drive is directly coupled to one rotor shaft, and the other shaft is driven through timing gears. The lobes on the rotors mesh closely and have close radial and axial clearances in the casing. The intake flange is shown at the lower right of Fig. 23. The intake port is located so that air enters cavities formed between rotor lobes and the casing. As the rotors turn, each cavity increases to a maximum and is then sealed off by the end of the casing. The volume entrapped is then compressed by the further meshing of the lobes. When the meshing is partially completed, the exhaust port diagonally opposite the inlet is uncovered by the ends of the lobes. Further meshing delivers air to the discharge duct (upper left of Fig. 23).

Performance. Figure 22*c* is a typical performance plot for a Lysholm compressor. As would be expected of a displacement compressor, the flow depends almost entirely on the speed, being practically independent of pressure ratio. The outstanding characteristics are complete absence of regions of instability, and broad ranges of speed (flow) and pressure ratio over which high efficiency is obtained. The Lysholm compressor generally is not suited for flow in excess of 25,000 cu ft per min or single-stage pressure ratios greater than 4.0.

Construction. In addition to large axial forces on the rotors, characteristic of all classes of compressors, the Lysholm rotors must sustain large radial-pressure loads as well. Bearing pressures up to 600 psi are carried by 360-degree automotive-type journal bearings. Thrust loads are carried by floating-shoe thrust bearings. Hardened and ground gears running at pitch speeds up to 7000 ft per min are required to maintain *timing* of the rotors. Shaft seals may be of the segmented carbon-ring type or end-nose refrigerator type, of special design.

The close clearances between moving parts and unsymmetrical temperature distribution usually require that the casing be water jacketed to reduce thermal distortions. Discharge and inlet-flow conditions give pressure pulsations of considerable amplitude, producing a high noise level. This is particularly true of the large sizes, so that silencers are required. This noise is one of the principal disadvantages of all vane- and lobe-type compressors.

15. COMPREX

This recent Brown Boveri development is so-named because gases are *compr*essed and *ex*panded in a single casing utilizing the same rotor. Because of the low temperature level of the compression process, very high temperature may be associated with the expansion without overheating the rotor. It is applied as a second stage of compression, to increase the pressure ratio of a gas turbine power plant.

The rotor consists of a number of cells arranged around a shaft, constituting gas passages parallel to the shaft. These cells alternately carry air to be compressed and gas to be expanded. The rotor is simply a transporting device, externally powered. By proper choice of rotational speed and arrangement of gas and air inlet and exhaust ports, compression is achieved through a pneumatic-ram action. The compressed air is circulated by an auxiliary fan through an external combustion chamber where its temperature is raised. Then, returning to the comprex, part of the compressed and heated air is partially re-expanded, simultaneously making an energy contribution to the compression process. The remainder of the hot air (gases) proceeds directly to a high-pressure stage of the gas turbine. Gases that have been partially expanded in the comprex are delivered to a low-pressure portion of the gas turbine.

Combined compression and expansion efficiency of a developmental model was 69%; pressure ratios up to 3 : 1 were obtained; and a cycle temperature of 1800 to 2000 F was found admissible.

16. COMBUSTORS AND HEATERS

COMBUSTORS. Combustion of fuel in the working medium in a separate apparatus is peculiar to the open or semiclosed gas turbine power plant. Basic requirements are complete combustion, low pressure drop, mechanical suitability, and wide range of operation.

Complete combustion is desirable to insure full utilization of the fuel and to avoid fouling of turbines and heat exchangers. Accumulations of soot not only reduce the effectiveness of these components but also constitute fire or explosion hazards. To obtain complete and smokeless burning, the fuel must be finely divided (atomized) and mixed with the correct proportion of primary air to enable the combustion reaction to be completed in the combustion zone of the apparatus. The remainder of the combustor is used for mixing combustion products and secondary air. Temperature stratification is the consequence of inadequate mixing. This stratification may give as much as several hundred degrees variation in temperature across the profile of the gas stream; if it persists into the turbine, the stationary portions may be severely overheated, locally.

Low pressure drop is obviously essential because of the extreme sensitivity of the gas turbine cycle to parasitic losses. Both the combustion and mixing processes are made more difficult by economic limitations placed on pressure drop. One-third to 2 psi is the approximate desirable range of pressure drops in open-cycle combustors.

Mechanical suitability includes considerations of service life, geometric adaptability to plant arrangement, and size. Localized hot spots develop severe thermal distortions and ultimate failure. Therefore, metal temperatures must generally be held below combustion temperatures, with the result that carbon is deposited if unburned fuel is allowed to contact any part of the combustor. Parts exposed to combustion temperatures are usually cooled with secondary air. Because of increased radiation effects, this cooling is more difficult to accomplish as the size of the combustor, combustion temperature, and fuel density are increased. Considerations of plant arrangement influence the choice of combustor design. Compactness is almost always highly desirable and militates against reduction of pressure drop. Heat release rates up to 40×10^6 Btu per cu ft per hr are not uncommon in small combustors, at moderate pressures.

Range of operation is important not only with respect to variations in total pounds of fuel burned but also with respect to fuel-air ratios. For a typical simple-cycle plant the total fuel burned varies over a 6.5 : 1 range, and the fuel-air ratio over a 2 : 1 range. Thus the burner must be capable of handling a wide range of flows; and the combustor must correctly proportion the primary and secondary air flows at all loads.

Typical combustor designs directed toward solution of these problems are: (1) multiple *basket type* employed on many jet units and on the Westinghouse 2000-hp and G. E. 4800-hp locomotive turbines (see Fig. 17); (2) multiple-injection *annular type;* (3) *straight-through type* used by Brown Boveri; and (4) *elbow type* applied to marine plants by the Elliott Company.

In the basket-type combustor, primary air and fuel are burned in a flame tube, and secondary air is added from an outer annular space through slots or holes in the tube. Very high heat-release rates are achieved, but pressure drops are fairly high. The annular combustor is similar in principle, but a single flame space serves all burners. Performance characteristics are similar, but flame stability is a problem. In the straight-through type, a single burner serves a single-flame tube. Secondary air flowing outside the tube, thereby cooling it, mixes with combustion gases downstream of the tube. This combustor is essentially a larger version of the basket-type unit and usually has a lower heat release rate per unit of volume. In the elbow chamber, air entering the chamber at right angles to its axis is divided into primary and secondary air by a flame cone. The double vortex set up by turning the air stream through right angles mixes the combustion gases and the secondary air. Pressure drop is very low for the moderately high heat release effected; stratification is relatively severe.

Burner designs, as compared with diesel and boiler practice, generally accommodate wider ranges of operation and higher heat-release rates. Mechanical or air atomization may be employed with or without return-flow features. Control is effected either by varying the size of a regulating orifice or by varying the pressure applied to the atomizing orifice. Injection pressures vary from several thousand pounds per square inch to a few pounds above combustor pressure. Variation with load introduces additional control problems for low-pressure fuel systems. High-pressure levels in combustors make difficult or preclude the removal and replacement of burners during operation. For this reason a single burner of sufficient range to cover all operating conditions is desirable. If range must be extended by employing multiple burners, idle burners must be either removed or cooled.

Ignitors and flame protectors are the more important accessories of the combustor. Ignition is almost always initiated by a high voltage spark. The spark may ignite an acetylene- or propane-starting burner from a location out of the main combustion zone, or it may ignite fuel issuing from the main burner. In the latter case a retraction mechanism may be associated with the spark plug to remove it from the flame after ignition. To guard against accumulation of combustible vapors in the gas turbine plant in the event of flame failure, a flame protection system usually is provided. This system comprises a flame-sensitive element and a warning device and/or an automatic fuel cut-out. Flame radiation may be used to actuate a photoelectric cell, or the ionized gases in the flame zone may complete an electric circuit from a *flame rod* to the combustor walls. In either case the response of the sensitive element to flame failure is amplified electronically and applied to appropriate relays to give warning or shut off the fuel supply.

The ash problem is encountered when residual oils are burned or in combustion of coal. Little is known about the mechanical or chemical effects of finely divided ash on gas turbine blading. Presumably some limit of particle size exists below which erosion will not occur; but deposition and chemical corrosion may remain as problems. Vanadium-bearing ash is thought to be seriously damaging in this respect. Erosion difficulties with

Houdry process turbines due to catalyst dust have been overcome largely by using *Aerotec* vortex-type separators.

HEATERS FOR CLOSED-CYCLE GAS TURBINE POWER PLANTS have a certain similarity to superheaters in conventional steam boilers. In a proposed Escher Wyss heater for coal combustion the flue gases are recirculated to reduce the combustion-gas temperatures to about 1850 F before they enter the heat-transfer sections. By using tubes $^3/_4$ to 1 $^1/_2$ in. in diameter, with wall thicknesses 0.1 to 0.15 in., a heater of dimensions no greater than for a corresponding steam boiler is possible. Oil- or gas-fired heaters may be pressure fired, using an auxiliary gas turbine charging set similar to that for a Velox boiler. Here the heater structure is of cylindrical shape to withstand the charging pressure. Flue-gas losses are, of course, comparable to those for a steam boiler.

The Sulzer high-pressure cycle (see Fig. 13) employs a combination *heater-combustor*, in which the combustion products, at the same pressure as the gases being heated, are the working fluid for the power turbine. This eliminates flue-gas loss, but retains the requirement for ash-free combustion inherent in open-cycle combustors.

17. HEAT EXCHANGERS

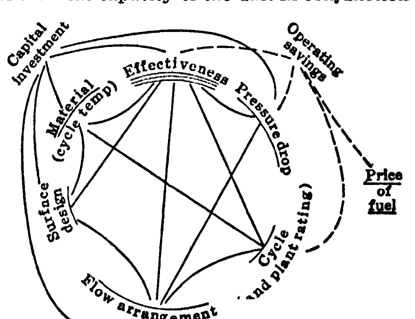
(See also Air Preheaters, Section 7.)

INTERCOOLERS for gas turbine power plant application are similar to those used in other compressor applications. However, because of the large air volumes handled and the necessity for low pressure losses, paramount considerations are low bulk and pressure drop. Except in closed-cycle applications, the resistance to heat transfer is far greater on the air side than on the water side. For this reason, extended surface on the air side is desirable to conserve space and weight. Finned tubes, similar to those used for generator air coolers, are one version of extended surface. Even greater space economy can be effected through use of built-up cores similar to tubular-type automobile radiator construction. If fouling from dirty water is anticipated, such construction may incorporate circular tubes for ease of cleaning instead of flattened or streamlined tubes, which are more effective as heat transfer elements.

Heat transfer in an intercooler does not take place at constant temperature of the working fluid as in a steam condenser. Therefore, through the use of counterflow or cross-counterflow arrangements, effective cooling is possible with substantial temperature rise of the cooling water, and water-consumption rates are comparatively low.

PRECOOLERS OR AFTERCOOLERS for closed cycles are functionally indistinguishable from compressor intercoolers.

REGENERATORS are thought by some to be misnamed because they generally involve *continuous* heat transfer rather than the use of the heat capacity of the unit in conjunction with an *intermittent* flow and transfer processes. There is also considerable disagreement on the economics of regeneration. In Fig. 24 the principal variables at the gas turbine designer's disposal are arranged in a circle and their interdependences indicated by connecting lines. The relationships of these design factors to capital investment, operating costs, and to plant size and weight are likewise indicated. This tangled skein of relationships makes it clear that every application must be considered on its own merits. It should be noted that developments in regenerator designs and in fabrication techniques may make necessary periodic re-evaluations of conclusions.



Note: All items in circle affect space and weight

FIG. 24. The interrelationship of factors affecting the economics of gas turbine regenerators.

Effectiveness is the thermodynamic quantity which best characterizes a regenerator. Size, weight, and cost increase rapidly as effectiveness is increased above 50%. One hundred per cent is unattainable (an infinite regenerator would be required), and considerably lower values are the ceilings for other than pure counterflow arrangements. In open-cycle applications, gas side heat-transfer coefficients are relatively unfavorable. This places a practical limit of $\eta_r \cong 0.80$ on regenerators for simple open cycles. More complicated open cycles require less heat-transfer surface to achieve a given effectiveness, as has been pointed out by Alf Lysholm in Table 10 (covering three cycles).

Table 10

(Reprinted by permission from a Contribution to the Solution of the Gas Turbine Problem, by Alf Lysholm, *Proc. Inst. Mech. Engrs.* (London), March 1947)

Cycle	1-0-0.75	3-2-0.90	5-4-0.90
Pressure ratio	5	15.7	100
Turbine inlet temperature, °F	1200	1200	1200
Power, hp/lb/sec	62	148	250
Thermal efficiency, %	28	39	40
Relative heat transmission coefficient	1	1.3	1.7
Regenerator surface, sq ft/hp	4.4	4.3	1.9

Better transfer associated with the high density of closed cycles makes an effectiveness of over 90% practicable. Figure 25 shows the relationship of effectiveness to the *number of transfer units* (NTU's) for various flow arrangements.

$$\text{NTU} = AU/c_pW$$

A = heat transfer surface, sq ft
U = overall coefficient of heat transfer, Btu/(hr)(sq ft)(°F)
c_p = specific heat at constant pressure, Btu/(lb)(°F)
W = mass flow, lb/hr

Thus NTU is a dimensionless expression of regenerator size. Actual physical dimensions and weight depend on the design of heat-transfer surface and on flow density and velocity. The area of transfer surface per cubic foot of exchanger volume may vary from 40 to 250 sq ft per cu ft. Regenerator cores weigh 60 to 100 lb per cu ft. A typical overall coefficient of heat transfer for a tubular regenerator for open-cycle application is 17 Btu per (hr)(sq ft)(°F). Heat transfer increases as the 0.5 to 0.8 power of the density at constant velocity. Thus more than 100 Btu per (hr)(sq ft)(°F) is attainable in closed-cycle applications.

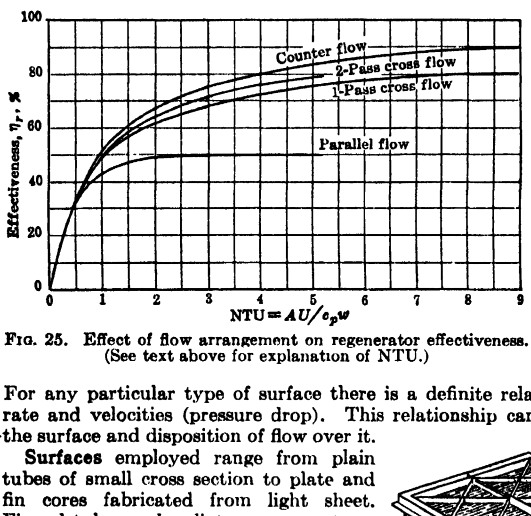


FIG. 25. Effect of flow arrangement on regenerator effectiveness. (See text above for explanation of NTU.)

Pressure drop is unavoidable. Because it increases as surface is added, practical limits are imposed on the useful size of the regenerator. For any particular type of surface there is a definite relationship between heat-transfer rate and velocities (pressure drop). This relationship can be modified by the design of the surface and disposition of flow over it.

Surfaces employed range from plain tubes of small cross section to plate and fin cores fabricated from light sheet. Finned tubes and radiator core constructions represent intermediate designs. Tubular regenerators represent the least departure from conventional pressurized heat-exchanger practice. The plate and fin units, similar in construction to aircraft intercoolers, promise the ultimate in effectiveness for a given weight and space and (presumably) cost. On the other hand, pin fins or staggered tubes may be used in the absence of space limitations, to achieve highest heat-transfer coefficients for a given pressure drop. Figure 26 is an isometric view of a plate and fin regenerator developed by the Harrison Radiator Division of General Motors for the Elliott Company's 3000-hp marine gas turbine plants. This is an essentially

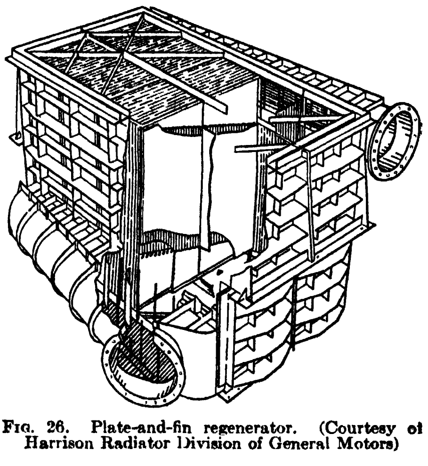


FIG. 26. Plate-and-fin regenerator. (Courtesy of Harrison Radiator Division of General Motors)

counterflow unit in which furnace brazed "tubes" are welded together to form their own "tube sheets" and air manifolds. The fabrication and thermal expansion problems are far less straightforward than for the tubular unit.

Materials used for regenerators depend on turbine inlet temperature, pressure ratio, and regenerator effectiveness. Thus the 90% regenerator for the Escher Wyss closed cycle, which always operates under full pressure *ratio* conditions and with moderate turbine inlet temperature, employs carbon steel tubes and shell. Martensitic alloys such as SAE 4130 are used where conditions are somewhat more severe. If high exhaust temperatures are anticipated, the corrosion resistance and high temperature strength of an austenitic material, e.g., inconel, may be required.

18. DUCT WORK

Because of the large air volumes handled at low velocities (75 to 150 ft per sec for reasonable pressure drops), ducts become major items in a gas turbine power plant. Also, because of the operating temperatures and high coefficients of expansion of austenitic materials, the accommodation of expansions becomes a technical problem of considerable magnitude. Both bulk and expansion problems are more serious in complex cycles than in simple-cycle arrangements.

Differential expansions are minimized (1) by judicious selection of anchor points on the machinery components, making expansions of parallel elements partially compensating; (2) by the use of internal insulation or by double wall construction with a sheet of cooling air flowing between the walls; and (3) by making all hot connections as short as possible.

The large cross-sectional areas of ducts mean that pressure thrusts are high; hence unrestrained expansion joints usually are not permissible. Consequently, linear expansions are usually absorbed by hinge-type joints acting in pairs. Thus a section of duct work between two joints becomes a link. The chain of which the link is a part must be given the necessary degrees of freedom to accommodate the differential expansions imposed.

The problem of maintaining a packed joint at gas turbine temperatures is so severe that slip joints are impractical. Most expansion joints are made of bellows-type elements. The linkage to sustain the pressure thrust may be of the conventional pin-and-knuckle type or of strictly elastic construction. If absorption of an axial expansion in a straight length of duct is unavoidable, the thrust either must be taken by the components between which the pipes run or by special compensating devices, e.g., balancing cylinders.

Other problems associated with duct work design are the staying and ribbing of flat surfaces; accommodation of temperature gradients associated with combustion processes or transition from internal to external insulation; and maintaining tightness of flanged joints. Because high-temperature gaskets are not truly "soft" and because relatively low relaxation strength limits effective bolt loading, flanges must be heavy and bolt spacing small in relation to the moderate pressures to be carried.

Insulation practice varies widely. In general the materials and techniques of boiler, process equipment, or aircraft manufacture are adapted. Refractories may be used between metal walls in internal insulation. Block-type insulation is applied externally where relative permanence is assured and the bulk can be tolerated. Glass and slag wool blankets are used for reduced bulk and weight and ease of removal. In locations of extreme temperature either refractory or leached glass wool materials are required. Where space and personal comfort are more important than the heat loss, dual layer construction with intermediate ventilation is appropriate (Ref. 11).

OPERATION OF GAS TURBINES

19. STARTING AND SHUTDOWN

STARTING of a gas turbine power plant requires an auxiliary power source. The plant's own compressor inducts air and compresses it to a pressure such that expansion from reasonable temperature will develop enough power to sustain operation. The *starter* may be an internal-combustion engine, a steam turbine, an auxiliary electric motor, or another gas turbine. It must be coupled to the turbocompressor shaft with a disengaging or over-running clutch. A main generator or its direct-connected exciter may be pressed into temporary service as a motor; in this case, electrical controls supplant the clutch.

Starting procedure is to roll the unit and induct air, actuate the combustion ignition system, and inject fuel. The fuel flow is controlled to obtain the necessary warm-up. Brown Boveri, General Electric, and Westinghouse all have reported starting operations which permit carrying full load within very few minutes after initiating the starting procedure. Enough fast starts have been made to indicate that no severe damage results from the necessarily rapid temperature changes. This permits the use of initial-combustion temperature in excess of rated turbine-inlet temperature to produce acceleration to *self-sustaining speed* from a lower starting speed. This transient condition is so brief that even turbine blades probably do not reach abnormally high temperatures. A more conservative procedure involves gradual increase in combustion temperatures with continuing, though diminishing, input of starting power. The gradual warm-up thus effected is designed to minimize thermal stresses calculated to exist during periods of rapid temperature change. This procedure will be employed where no severe penalty is attached to a relatively long starting period. The length of this period depends on the mass of the turbine and many other design features. In general, the gradual warm-up requires heavier duty starting auxiliaries because of the longer operation and because there is little or no distinction between starting and self-sustaining speed.

Starting procedures for multiple-shaft plants are more complicated than for single-shaft units. It is usually necessary to supply auxiliary power to each shaft in the correct speed ratio. The Elliott marine turbines have a novel starting arrangement which involves by-passing the high-pressure compressor and low-pressure turbine which are coupled together. The high-pressure turbine and low-pressure compressor combination is then started as a simple plant. The plant is "synchronized" by closing first the turbine by-pass valve and then the compressor by-pass. This operation may take place either before or after self-sustaining operation is achieved. No auxiliary power is required to start the low-pressure turbine shaft.

SHUTDOWN of a gas turbine plant occurs very quickly after fuel is cut off from the combustor(s). A rapid shutdown is desirable since it minimizes the chilling effect resulting from passing cold air through the hot turbine. Turning gears are usually employed, or alternatively the starting device may be operated at reduced speed to insure symmetrical cooling of the rotor. This avoids thermal "kinking" of the shaft.

20. CONTROL

The output of a gas turbine power plant is a sensitive function of the mass of gas flowing, cycle pressure ratio, and turbine inlet temperature. These variables are employed singly or in combination, directly and indirectly, in turbine plant control. The relative thermodynamic merits of these control means have been discussed under Performance. This article deals briefly with the physical means for effecting these controls.

A simple-cycle plant connected to a constant-speed load, e.g., an alternator synchronized with a large system, may be controlled by a simple manual fuel regulator. Since air flow and pressure ratio are nearly constant, this amounts to straight temperature control. If the burden of regulating speed is placed upon the power plant, the fuel regulator must be governor controlled. The open-cycle gas turbine plant has no reservoir of energy comparable to the boiler of a steam turbine to draw upon. Rapid governor action is essential to minimize adverse effects of reduced mass flow and pressure ratio corresponding to the drop in speed reflecting increased load.

If the power-speed characteristic of the load can be varied at will, as for a locomotive drive, the control may include regulation of this variable. One such arrangement utilizes a governor-actuated fuel control and a temperature-responsive generator field regulator. The plant thus operates at a constant temperature, and variable output is obtained by changing speed (flow and pressure ratio) only.

The complex cycles afford more possibilities for control. A closed-cycle plant, for example, is regulated by changing the density of the circulating medium. In the Escher Wyss cycle this is accomplished through the use of auxiliary high- and low-pressure accumulators. Flow from or to these reservoirs is regulated by governor-actuated valves while temperature is held constant by thermostatic control of fuel flow. Plants which incorporate floating shafts and multiple combustors (the Sulzer high-pressure cycle and Elliott marine plants are examples) may use separate controlling devices on the fuel flows to the several burners. Thus mass flow and pressure ratio may be indirectly regulated by governor control of fuel flow to the combustor ahead of the compressor turbine, while constant temperature is maintained at the power turbine inlet.

Use of throttling devices in gas turbine control introduces prohibitive losses. Variable turbine inlet area, however, has been built into a Birmann type turbine by DeLaval.

Another use of flow control has been employed by Escher Wyss to accommodate transient minor load fluctuations. In this arrangement high-pressure air is by-passed to a low-pressure part of the circuit when small decreases in load occur. Similarly, overspeed protection may include a dump valve to spill high-pressure air in addition to a fuel cut-out.

SECTION 11

REFRIGERATION AND ICE MAKING

By

B. H. JENNINGS, *Chairman, Department of Mechanical Engineering, Northwestern University*

REFRIGERATION

ICE MAKING

REFRIGERATION

1. REFRIGERATION EFFECT AND THE TON

Refrigeration means the production and maintenance in a given space of a temperature lower than that of the atmosphere or lower than the temperature in adjacent space. Before the advent of mechanical refrigeration, ice, formed by natural freezing and stored until used, was the only source of refrigeration. As ice, under atmospheric pressure, always melts at 32 F, it produces refrigeration as it absorbs heat in melting. So-called freezing mixtures of salt and ice produce temperatures lower than 32 F. When salt and finely divided ice (snow) are brought into contact, the melting (fusion) temperature is depressed, and heat is absorbed at this lower temperature, while the ice melts and the salt goes into solution. For example, ordinary table salt (NaCl) mixed with crushed ice causes the temperature of melting to fall from 32 to -6.3 F. Certain acids and alcohols have a similar effect in depressing the melting temperature of ice. (See p. 11-51 for freezing mixture data.) Another refrigerating material is solid carbon dioxide (dry ice), which at atmospheric pressure sublimes at -109.3 F and absorbs 246 Btu per lb of dry ice.

To obtain fully flexible ranges of temperature or to produce refrigeration in quantity, mechanical (artificial) means must be employed. The quantity of heat abstracted or absorbed is expressed in British thermal units (Btu) or in calories in countries where the metric system is used. The rate of heat absorption is expressed in Btu per hour, Btu per minute, or in tons of refrigeration. The ton of refrigeration is the absorption of heat at the rate of 12,000 Btu per hr or 200 Btu per min. Historically, the ton of refrigeration represented refrigeration equivalent to 1 ton weight of ice melting in 24 hr. The latent heat of fusion of ice was considered at 144 Btu per lb (although it is, more precisely, 143.3); and (2000 lb per ton) $\times$ (144 Btu per lb) = 288,000 Btu per 24 hr, or its equivalent 12,000 Btu per hr is 1 ton of refrigeration. This unit was formerly called the *standard ton.*

RATING OF REFRIGERATING MACHINES. The rating or capacity of a refrigerating machine or refrigerating unit is expressed in Btu per hour or in tons with a statement of the temperature (or temperature range) at which the machine or unit operates in producing its rating. Formerly all vapor refrigeration machines were rated in terms of the tons of refrigeration they could produce, when the evaporator operated at the pressures corresponding to boiling of the refrigerant at 5 F and to the condensation of the refrigerant at 86 F. Because of the broader present-day uses of refrigeration, as in air-conditioning, quick-freezing, low-temperature, and chemical-process refrigeration, the $+5$ F, $+86$ F rating is inadequate, and a larger number of rating temperatures are used. In addition to the rating of 5 F for the evaporator, -10 F, $+20$ F and $+40$ F are used, and condensation temperatures of 95 F, 100 F, 105 F, and 110 F allow for the more extreme conditions met when condensing with cooling tower water or with air. Among the more important rating and testing codes are Standard Methods of Rating and Testing Mechanical Condensing Units, ASRE *Circular* 14; Rating and Testing Water-cooled Refrigerant Condensers, *ASRE Circular* 22; Rating and Testing Forced Circulation and Natural Convection Air Coolers for Refrigeration, *ASRE Circular* 25; Rating and Testing Evaporative Condensers, *ASRE Circular* 20; Rating and Testing Refrigerant Compressors, *ASRE Circular* 23; American Standard Safety Code for Mechanical Refrigeration, ASA B-9, 1939, and *ASRE Circular* 15; and Equipment Standards of the Air Conditioning and Refrigerating Machinery Association, Inc. (ACRMA), published in five parts.

2. ARTIFICIAL REFRIGERATION METHODS

Low temperatures may be produced when appropriate substances absorb heat in changing state. This may be purely a physical phenomenon as when a liquid changes into a vapor, or may be a chemical-physical phenomenon as when ice melts in the presence of a salt which goes into solution at the same time. Freezing mixtures of the latter type have already been mentioned. Quantity refrigeration by this method is not feasible because it is not commercially possible to bring the constituents back to their original state by simple means.

SYSTEMS. 1. Vapor compression refrigeration, which is most commonly used, is associated with methods that involve physical changes of state which can be brought about by simple heat-transfer processes and mechanical work. This system involves the use of suitable refrigerants, such as ammonia (NH_3), "Freon-12" (CCl_2F_2), and sulfur dioxide (SO_2).

2. Steam-jet vacuum machines do not use a mechanical compressor and employ water vapor as the refrigerating medium.

3. Absorption machines customarily use ammonia as a refrigerant and employ direct heating as the main or only source of energy.

4. Cold-air machines, now largely obsolete, by expanding air that has been previously compressed and cooled, produce refrigeration without a change in phase.

VAPOR COMPRESSION MACHINES (Mechanical Refrigeration). The refrigerating mediums generally used in vapor compression machines exist normally only as gases or vapors at atmospheric pressure and ordinary temperatures, but they can be condensed if, after they are compressed to a sufficiently high pressure, heat is withdrawn from them. If the pressure on the resulting liquid is lowered, a portion of the liquid evaporates immediately as the temperature drops, while the remaining liquid in vaporizing absorbs heat from its surroundings, thereby creating refrigerating effect.

The essential parts of a refrigerating system are shown in Fig. 1.

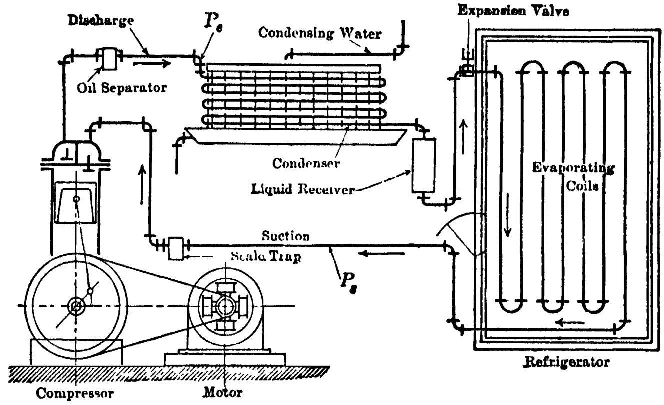


FIG. 1. Diagram of vapor compression refrigerating system.

1. In the evaporating coils the liquid refrigerant, in vaporizing, absorbs heat from brine or water or directly from the space being cooled.

2. The low-temperature vapor from the evaporator is drawn in by the compressor and is raised in pressure and temperature.

3. In the condenser the vapor from the compressor at the condenser pressure or head pressure of the system is condensed by the available circulating water or by ambient air temperature in the case of small air-cooled installations. The refrigerant must be sufficiently compressed so that its saturation temperature is higher than the temperature of the available cooling medium. After heat removal has caused condensation, the liquid refrigerant may be stored in a receiver or storage tank. However, this is not an absolutely essential part of a refrigeration system.

4. The high-pressure liquid passes through the expansion valve where the refrigerant throttles (drops) to the evaporator pressure of the system. In passing through the expansion valve, the liquid refrigerant cools itself at the expense of evaporating a portion of the liquid.

In the process of mechanical refrigeration the heat, absorbed at a low temperature, through the medium of mechanical work is raised to a sufficiently high temperature and pressure level to permit rejection of heat at this higher level. The work input to the compressor may be supplied by any convenient means, such as an electric motor, steam engine, or internal-combustion engine.

Items affecting the choice of a refrigerant and characteristics of refrigerants are given later, but computational methods are similar for any refrigerant. The low pressure of the evaporator for any given refrigerant is set by the temperature that it is desired to maintain in the cooled space while the high pressure is set by the temperature of the available cooling means, such as the cooling water or the atmosphere itself. For example, if it is desired to keep a storage space at 10 F, the refrigerant in the coils must be still cooler, say 0 F, in order to absorb the heat load from the storage space. This heat absorbed evaporates the liquid refrigerant into vapor, which in turn enters the compressor. Similarly, if the available cooling water supplied is 76 F and warms to 82 F in passing through the condenser, the condensing refrigerant must be warmer, say 86 F in the coils.

CYCLE REPRESENTATION ON THE *P-h* CHART. To illustrate the method of computing a refrigerant cycle, a computation will be carried through using "Freon-12" as the refrigerant. A typical cycle has been drawn on the pressure-enthalpy (p-h) chart (Fig. 2) which depicts the properties of this refrigerant. In refrigeration work, because of its convenience, the p-h chart is used almost to the exclusion of the T-s and h-s charts so common in power-cycle analysis. The line f-1-G of Fig. 2 is the relation between pressure and enthalpy of saturated liquid and D-8-5 is the similar line for saturated vapor. The region to the right of D-8-5 is superheated vapor. For the ideal cycle drawn, slightly superheated but cold vapor enters the compressor suction at 6 and is compressed isentropically to the head (discharge) pressure of the system p_c at point 7. This vapor, highly superheated, enters the condenser where it de-superheats to point 8 and then condenses to saturated liquid at point 1. The liquid before entering the expansion valve may become subcooled and arrive at point 2. The process in the expansion valve is one of throttling and takes place with enthalpy constant, i.e., from 2 to 3 (or 1 to 4). In passing through the expansion valve as the pressure falls the temperature of the refrigerant also falls to the low temperature of the evaporator, and this lowering in temperature occurs at the expense of vaporizing a portion of the original liquid refrigerant. Thus the refrigerant entering the evaporator is a wet vapor and has a quality x. Graphically the amount of flash vapor formed for subcooled liquid is indicated by the ratio of length $\overline{f\text{-}3}$ to length $\overline{f\text{-}5}$, or $x = \overline{f\text{-}3}/\overline{f\text{-}5}$. (If not subcooled, the corresponding lengths are $\overline{f\text{-}4}$ and $\overline{f\text{-}5}$.) The useful refrigerating effect is shown by the length $\overline{3\text{-}6}$ representing the enthalpy change as the refrigerant completes its vaporizing and slightly superheats in the evaporator.

RELATIONSHIPS FOR THE IDEAL CYCLE in terms of enthalpy values are given here for subcooled liquid.

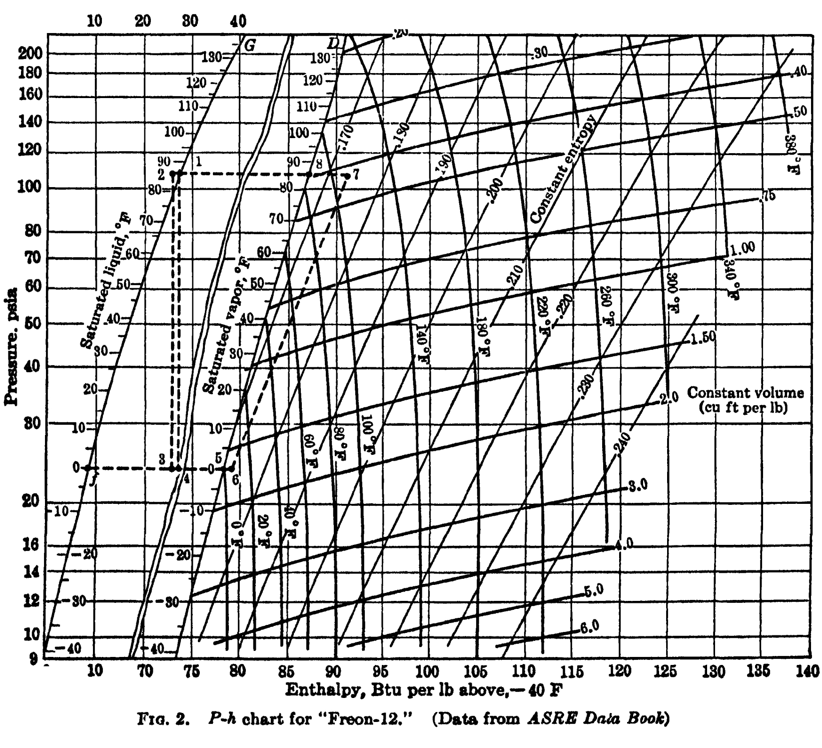


FIG. 2. *P-h* chart for "Freon-12." (Data from *ASRE Data Book*)

For the expansion valve,

$$h_2 = h_{fe} + xh_{fge} \tag{1}$$

Evaporator useful refrigerating effect (q_r) in Btu per pound of refrigerant,

$$q_r = h_6 - h_2 \tag{2}$$

or

$$q_r = h_5 - h_2 \tag{3}$$

if the vapor leaving the evaporator is not superheated and not wet.

Work of compression (W_i) in Btu per pound of refrigerant under ideal isentropic conditions,

$$W_i = h_7 - h_6 = h_c - h_s \tag{4}$$

If there is no superheat on entering the compressor h_6 coincides with h_5. The symbols h_c and h_s also indicate condenser and suction conditions.

Heat removal in condenser (q_c) consists of de-superheating ($h_7 - h_8$), condensing ($h_8 - h_1$), and subcooling ($h_1 - h_2$); or the total in Btu per pound is

$$q_c = h_7 - h_2 \tag{5}$$

Heat Balance. For the ideal case under adiabatic conditions

$$q_r + W_i = q_c \tag{6}$$

Refrigerant Flow Rate. The useful refrigeration per pound of refrigerant flowing is q_r as indicated by eq. 2. It is customary to express the rate of refrigerant flow in pounds per ton minute (M_r). As 200 Btu per min equals 1 ton of refrigeration,

$$M_r = \frac{200}{q_r} \tag{7}$$

For T tons of refrigeration the flow of refrigerant in pounds per minute is M, where

$$M = \frac{200T}{q_r} \tag{8}$$

Horsepower. The horsepower required by the compressor per ton can be found from the value of W_i of eq. 4:

$$\text{Hp} = \frac{M_r W_i 778}{33{,}000} = \frac{M_r W_i}{42.42} \tag{9}$$

For T tons the horsepower from eqs. 4 and 8 is

$$\text{Hp} = \frac{200TW_i}{q_r 42.42} = 4.717\frac{TW_i}{q_r} \tag{10}$$

This value is the minimum horsepower required under isentropic conditions for given evaporator and condenser temperatures.

The letter h in the previous equations represents the enthalpy in Btu per pound of refrigerant in its appropriate state.

EXAMPLE OF CYCLE CALCULATION FOR "FREON-12" SYSTEM. A "Freon-12" system has an evaporator at 0 F, and condensation takes place at 86 F. The liquid is subcooled in the lower part of the condenser to 80 F, and vapor entering the compressor is superheated 6 degrees. Find the significant items associated with the ideal cycle.

The cycle is pictured on the pressure-enthalpy chart of Fig. 2.

Subcooling and Expansion Valve. Start at 1 with saturated liquid after condensation at 86 F. Table 4 (p. 11-13) shows that the enthalpy of such liquid, h_f, is 27.72 Btu per lb and the pressure is 107.9 psia. As the condensed liquid is subcooled to 80 F, Table 4 shows that for this the h_f is 26.28 Btu. The subcooling is not affected by pressure, which is still at 107.9 psia. The subcooled liquid now enters the expansion valve and drops to 23.87 psia, the low pressure of the system corresponding to 0 F. The accompanying drop in temperature when the pressure falls takes place at the expense of vaporizing a portion of the original liquid "Freon." This amount can be computed by eq. 1, where h_2 = enthalpy of liquid before expansion valve, h_{fe} = enthalpy of liquid in evaporator, h_{fge} = enthalpy increase during vaporization in evaporator, and x = weight fraction of vapor formed. Selecting values at 0 F in Table 4,

$$26.28 = 8.25 + x(78.21 - 8.25)$$

and $x = 0.258$, or 0.258 lb of vapor forms from the expansion valve process for each pound of "Freon" fed to the evaporator. This point is shown as 3 on Fig. 2.

Evaporator. In the evaporator, the remainder of the liquid evaporates in absorbing heat from the space or medium being cooled and leaves at point 6 slightly superheated above the saturated condition. If saturated, its enthalpy is 78.21 Btu per lb (Table 4), and the useful refrigeration (q_r) by eq. 3 is

$$q_r = 78.21 - 26.28 = 51.93 \text{ Btu/lb}$$

The vapor, however, leaves the evaporator superheated a few degrees. As this is 6 degrees, using a specific of 0.144 for low-pressure "Freon-12" near saturation, the leaving enthalpy is $78.21 + 0.144 \times 6 = 79.07$ Btu per lb. Frequently the refrigerant superheating before entering the compressor may occur outside the refrigerated area and does not contribute a useful effect. For this case it does, and by eq. 2

$$q_r = 79.07 - 26.28 = 52.79 \text{ Btu/lb}$$

Compressor. The compressor must remove the saturated or slightly superheated vapor from the evaporator as fast as formed if the pressure (and temperature) is to be kept from rising in the refrigerated area. If, on the other hand, the capacity of the compressor (speed and displacement) is too great, the compressor removes vapor so fast that the evaporator pressure falls, and a lower temperature than desired may result. Ideal compression of the vapor is at constant entropy, and following in the direction of constant entropy lines locates point 7 at condenser pressure of 107.9 psia with an enthalpy of 91.0 Btu per lb at 109 F. Read from the chart, the compressor work supplied by eq. 4 is

$$W_t = 91.0 - 79.07 \text{ or } 11.93 \text{ Btu/lb of "Freon-12"}$$

Condenser. The vapor leaving the compressor is superheated gas. In the condenser, superheat is removed from the vapor, after which condensation takes place to saturated liquid followed by possible subcooling as well.

$$q_c = 91.0 - 26.28 = 64.72 \text{ Btu/lb}$$

Heat Balance. A heat balance of the cycle shows by eq. 6 that

$$52.79 + 11.93 = 91.0 - 26.28$$

$$64.72 = 64.72$$

Refrigerant Flow and Horsepower. By eqs. 7 and 9,

$$M_r = \frac{200}{52.79} = 3.79 \text{ lb/ton min}$$

$$\text{Hp} = \frac{(3.79)(11.93)}{42.42} = 1.07 \text{ hp/ton}$$

COEFFICIENT OF PERFORMANCE (CP). In reference to power-generation equipment, *efficiency* is defined as the work or useful output divided by the thermal energy required to produce that output. In a refrigeration system, the useful output is the heat absorbed at refrigeration temperature, and the input is the work required to produce that refrigeration. The ratio of these two values in appropriate energy or power units is called the *coefficient of performance* (CP), and is a real measure of the effectiveness of the system.

$$\text{CP} = \frac{\text{Refrigeration effect}}{\text{Work input}} = \frac{q_r}{W} \tag{11}$$

In power units using the ton of refrigeration and horsepower

$$\text{CP} = \frac{200}{42.42 \text{ hp}} = \frac{4.715}{\text{hp}} \tag{12}$$

Here hp is the horsepower required to produce one ton of refrigeration, and the equation is valid for the ideal or for the actual system or cycle. Note also that eq. 12 can be rewritten

$$\text{Hp} = \frac{4.715}{\text{CP}} \tag{13}$$

For the numerical example previously considered and using eq. 12,

$$\text{CP} = \frac{4.715}{1.07} = 4.4$$

COMPRESSOR DATA. Brake Horsepower (Bhp). The *p-h* chart, as has been shown, can give the minimum work required under isentropic compression conditons. The actual

Bhp required runs some 25 to 30% higher than the horsepower computed using this minimum theoretical value. By eq. 4 the ideal work, $W_i = h_c - h_s$, and by eq. 8 for a flow of M lb per min,

$$\text{Hp} = \frac{M(h_c - h_s)778}{33{,}000} = \frac{200T(h_c - h_s)}{q_r(42.42)} = 4.715\frac{(h_c - h_s)T}{q_r} \tag{14}$$

$$\text{Bhp} = \frac{4.715}{\eta_o}\frac{(h_c - h_s)T}{q_r} \tag{15}$$

where η_o is overall efficiency of compressor in terms of ideal to actual input work and ranges from some 0.75 to 0.80 with 0.77 as a representative value.

Where tabulations of refrigerant properties are not available or are incomplete, it is possible to compute approximately the ideal work of compression in Btu per pound of refrigerant from

$$W_i = \left(\frac{144k}{k-1}\right)\left(\frac{p_s v_s}{778}\right)\left[\left(\frac{p_c}{p_s}\right)^{(k-1)/k} - 1\right] \tag{16}$$

where p_s and v_s are the suction pressure in pounds per square inch, absolute, v_s is the specific volume of the refrigerant in cubic feet per pound at entry to the compressor, and p_c is condenser pressure in pounds per square inch, absolute. Values of k are given in Table 2. Equation 4 is always preferable to eq. 16.

Mean Effective Pressure. It was formerly customary to take indicator diagrams of compressor cylinders, and this is still done on some of the large slow-speed reciprocating machines. With the newer designs of high-speed machines, indicator cards are very seldom taken. However, indicated *mean effective pressure* (mep) is still an important factor in design. The mean effective pressure can be found by calculation from the compression work, as computed from the pressure-enthalpy diagram (eq. 4) or by eq. 16 after the application of a proper factor. Thus mep in pounds per square inch is

$$\text{mep} = \frac{W_i(778)}{(144)v_s\eta_i} \tag{17}$$

where η_i is the ratio of theoretical to indicated work of compression and ranges between 0.83 to 0.87 for many compressors. Also, by use of eq. 16,

$$\text{mep} = \frac{p_s}{\eta_i}\left(\frac{k}{k-1}\right)\left[\left(\frac{p_c}{p_s}\right)^{(k-1)/k} - 1\right] \tag{18}$$

The ratio of η_o/η_i ranges between 0.88 and 0.94, a representative value being 0.91.

Indicated horsepower (Ihp) can be found from mep or from Bhp.

$$\text{Ihp} = (\text{mep})(M)\frac{v_s}{33{,}000} \tag{19}$$

where mep is from eq. 17 or eq. 18, M is pounds per minute refrigerant flow, and v_s is cubic feet per pound specific volume of refrigerant at suction conditions.

$$\text{Ihp} = (\text{Bhp})\frac{\eta_o}{\eta_i} \tag{20}$$

Piston Displacement. The theoretical piston displacement of a compressor is set by specific volume of refrigerant at suction conditions and by the rate of refrigerant flow.

$$\text{Theoretical displacement (TD)} = Mv_s \text{ cu ft/min} \tag{21}$$

The actual displacement (D) is necessarily greater because the volumetric efficiency (η_v) is less than unity.

$$D = \frac{Mv_s}{\eta_v} \text{ cu ft/min} \tag{22}$$

where D is cubic feet per minute if M is in pounds per minute, v_s is always specific volume in cubic feet per pound of refrigerant at suction conditions, and η_v is the volumetric efficiency expressed as a decimal. Factors controlling volumetric efficiency are discussed later, and Table 1 and Fig. 4 give typical volumetric efficiency data.

The actual effective displacement (D) of a reciprocating machine depends on the bore (d inches), the stroke (l inches), the rpm (N), and the number of cylinders (C if single acting and $2C$ if double acting), as well as volumetric efficiency (η_v).

$$D = \eta_v C\frac{\pi d^2 l N}{(4)(1728)} = \eta_v C\frac{d^2 l N}{2200} \tag{23}$$

EXAMPLE. A "Freon-12" compressor is operating under the conditions of the example on p. 11-05. Compute its capacity and Bhp if it is a 2-cylinder single-acting 4 1/4 by 4 1/4-in. running at 850 rpm.

Figure 4 indicates a volumetric efficiency of 0.70 for a smaller faster rpm machine at a compression ratio of 4.5. This larger slower machine should be closer to $\eta_v = 0.80$. According to eq. 23,

$$D = (0.80)(2)\frac{(4.25)^2(4.25)(850)}{2200} = 47.5 \text{ cu ft/min}$$

Table 3 or Fig. 2 indicates the specific volume v_s of the suction vapor at 0 F and 23.87 psia as 1.637 cu ft per lb. The refrigerant flow is

$$M = \frac{D}{v_s} = \frac{47.5}{1.637} = 29.0 \text{ lb/min}$$

Using eq. 8 and $q_r = 52.79$ from the example on p. 11-05,

$$T = \frac{Mq_r}{200} = \frac{(29.0)(52.79)}{200} = 7.65 \text{ tons}$$

Using eq. 15 and η_o taken as 0.77,

$$\text{Bhp} = \frac{4.715}{\eta_o}\frac{TW_i}{q_r} = \frac{4.715}{0.77}\frac{(7.65)(11.93)}{52.79} = 10.6$$

The Bhp per ton is thus 10.6/7.65 = 1.39.

Volumetric Efficiency (η_v). This term is of significance with positive displacement machinery such as reciprocating and rotary compressors. It is defined as the ratio of actual weight of vapor handled per unit time by the compressor compared to the weight which could be handled by the piston displacement of the compressor computed at the temperature and pressure conditions the vapor possesses in the suction line of the compressor. Volumetric efficiency is affected by many factors, of which clearance is perhaps the most significant.

Clearance is conventionally expressed as a percentage or as a decimal fraction of the piston displacement. The following equation gives an expression for volumetric efficiency η_{vt} in terms of clearance alone. The effect of clearance is to reduce volumetric efficiency when the gases after compression and delivery at high pressure re-expand down to suction pressure and occupy a volume in the cylinder which prevents the complete charge of fresh gas entering the cylinder.

$$1 - m\left[\left(\frac{p_c}{p_s}\right)^{1/k} - 1\right] \tag{24}$$

Values of k in this equation are given in Table 2. p_c/p_s, the discharge to suction pressure ratio, is measured in any convenient units of absolute pressure, and m, the clearance, is expressed as a decimal. Figure 3 is an idealized pressure-volume plot of conditions in the compressor cylinder. The normal diagram, without clearance, indicates compression of the refrigerant charge from A to B with discharge of the gas taking place from B to C. With no clearance, all the gas is delivered from the cylinder. When the piston starts on its return stroke, the pressure suddenly drops from C to D and a full charge is drawn into the cylinder from D to A, represented by the length of the piston displacement, V_{PD}. With a small clearance indicated by C_1, the gas trapped in the compressor cylinder at the end of the discharge stroke re-expands along the line C_{S1}, and compression for this condition is AB_1. If the clearance in the cylinder is still larger, as indicated by C_2, the re-expansion of the gases follows the curve C_{S2} and compression follows the curve AB_2. The net result of clearance is to reduce the capacity of the cylinder by varying amounts, as indicated, from V_{PD} to V_{C1} and finally to V_{C2} for the conditions indicated. As these lengths are proportional to the weight of new charge drawn into the cylinder, it can be seen that increased clearance decreases the capacity of a machine for a given fixed amount of piston displacement. At the same time the work of compression per cylinder cycle is also decreased. For no clearance, the work area is proportional to $ABCD$, reducing to AB_2C_{S2} for the final condition shown of large clearance.

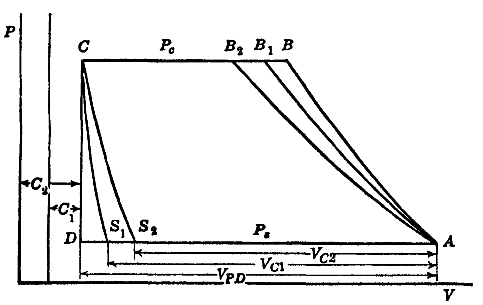


FIG. 3. Pressure-volume diagram for a compressor cylinder.

Actual volumetric efficiency, in addition to the effect of clearance, is also influenced by the superheating of the gases as they enter the cylinder that has recently been in contact with the hot discharge gases. This increases the specific volume of the gas, and it, in turn, prevents the full weight or charge from entering. Wire drawing or throttling always occurs when gas flow takes place through passages of limited cross section and around bends. The resulting pressure drop which develops also increases the specific volume of the gases entering the cylinder.

Various methods of increasing volumetric efficiency have been used. In some cases cylinder jackets with a cooling medium keep the cylinder walls at lower temperature. However, in most cases, the temperature of the inlet gases is so low that an ordinary water-cooled jacket would be subjected to freezing and would also add heat instead of dissipating it from the cylinder. *Wet compression* involves bringing into the cylinder a mixture for compression which has unevaporated liquid in it. This liquid, frequently in the form of fog suspension, keeps the cylinder temperature low and permits a larger charge to be taken in. However, inability to control wet compression and the possibility of slugs of liquid entering the cylinder and perhaps causing serious damage, as well as interfering with lubrication, have discredited this method of compression to a point where current practice calls for superheated gas entering the compressor.

Table 1 gives test results obtained by Reed and Ambrosius on a single-acting ammonia compressor with a clearance volume of 5.56% of the piston displacement. Figure 4 is a plot of theoretical volumetric efficiency computed by eq. 24 for a clearance of 4.5% on a small-bore four-cylinder high-speed (1200-rpm) "Freon-12" compressor. Also on the same chart are test data giving the actual volumetric efficiency for this compressor.

Table 1. Volumetric Efficiency of Single-acting Ammonia Compressor

Suction Pressure, psia	Discharge Pressure, psia			
	100	150	200	250
	Test Volumetric Efficiency, η_{VA}			
10	.826	.780	.735	.677
20	.882	.840	.790	.743
30	.925	.875	.823	.776
40	.970	910	.865	.817
50	.990	.945	.892	.848

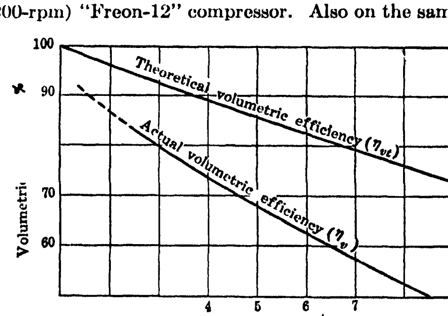


FIG. 4. Typical volumetric efficiency of small "Freon-12" compressor, 4.5% clearance, 1200 rpm, 4 cylinders, 2.25-in. bore x 1.687-in. stroke, single-acting.

COMPOUND OR MULTISTAGE COMPRESSION. It has been shown that the liquid entering the expansion valve must be cooled down to the evaporator temperature which can occur only at the expense of vaporizing a portion of the refrigerant in the evaporator. The specific volume and the pressure ratio through which the vapor has to be compressed increase as the evaporator temperature lowers. Where there is a large spread between evaporator and condenser pressures (and temperature) it is frequently desirable to multistage the machine and to use intercoolers between each stage. By this arrangement a portion of the liquid vaporizes in dropping to the interstage pressure. Here it is desirable to draw this vapor off because in so doing it will have only to be recompressed from interstage pressure up to condenser pressure. On the other hand, if the vapor expands completely to evaporator pressure, it occupies a much greater volume and must be lifted through the total pressure range of the system with a greater requirement of work. The intercooler, between stages, is usually arranged in such manner that the partly expanded liquid refrigerant comes in contact with the compressed vapor from the lower stage, and from the intercooler liquid refrigerant at lower pressure and temperature continues the expansion down to evaporator pressure, whereas the flash vapor and the compressed vapor from the lower stage are together drawn into the high-pressure compressor of the system. An appreciable saving in power input can be accomplished by multi-stage operation if the evaporator temperature is low. For temperatures below −20 F, multistaging is usually desirable.

Table 2. Physical Properties of Various Refrigerants

	Ammonia, NH_3	Carbon Dioxide, CO_2	Sulfur Dioxide, SO_2	Propane, C_3H_8	Methyl Chloride, CH_3Cl	Dichloro-ethylene, $C_2H_2Cl_2$ (Dielene)	Trichloro-ethylene, C_2HCl_3	Dichloro-methane, CH_2Cl_2 (Carrene)	Dichlorodi-fluoro-methane, CCl_2F_2 ("Freon-12")	Trichloro-monofluoro-methane, CCl_3F ("Freon-11")
	At 86 F									
Absolute pressure, psi	169.2	1043.0	66.45	156.0	94.7	6.9	1.72	10.6	107.9	18.28
Volume of vapor, cu ft per lb	1.772	0.047	1.185	0.68	1.081	8.5	25.2	6.68	0.389	2.24
Density of liquid, lb per cu ft	37.16	37.4	84.44	30.37	56.3	...	...	...	80.6	91.3
Enthalpy, Btu per lb										
Of liquid	138.9	83.3	42.12	72.7	56.2	23.2 *	20.1 *	...	27.7	25.3
Of vaporization	492.6	27.1	142.80	141.7	159.1	113.0	109.5	143	59.7	77.3
Of vapor	631.5	110.4	184.92	214.4	205.8	156.2	129.6	...	87.4	102.6
Specific heat, vapor, c_p	0.79	0.202	0.154	0.365	0.24	0.1625	0.12	0.154	0.16	0.14
Specific heat, vapor, c_v	0.4011	0.156	0.123	0.316	0.20	0.1425	0.105	0.128	...	0.124
	At 5 F									
Absolute pressure, psi	34.27	339.	11.81	41 9	21.15	0.82	0.16	1.17	26.5	2.93
Volume of vapor, cu ft per lb	8.150	0.267	6.421	2 5	4.47	63.0	240.0	49.9	1.49	12.27
Density of liquid, lb per cu ft	41.11	62.6	92.0	34.33	61.7	79.0	91.6	...	90.0	97.8
Enthalpy, Btu per lb										
Of liquid	48.35	21.3	14.11	23.8	21.15	1.35	1.17	...	9.3	8.88
Of vaporization	565.0	117.5	169.38	170.2	180.7	136.0	112.5	162	69.5	84.0
Of vapor	613.35	138.8	183.49	194 0	196.9	37.25	113.67	...	78.8	92.8
Specific heat of vapor, c_p	0.91	0.20	0.154	.47	0.24	0.1625	0.12	0.154	0.144	0.13
Specific heat of vapor, c_v	0.4011	0.16	0.123	.41	0.20	0.1425	0.105	0.128	0.126	...
$k = c_p/c_v$	1.3	1.31	1.25	1.16	1.20	1.14	1.14	1.21	1.14	1.13
Critical pressure, psi	1657	1066 2	1141.5	618	969.2	800	...	6.40	582	635
Critical temperature, °F	271 4	87.8	314.8	206.3	289.6	470	...	4.21	232.7	388.4
Molecular weight	17.03	44.0	64.06	44.09	50 48	96.9	131.35	84.9	120.9	137.38
Boiling point (1 atm), °F	−28.0	−109.3	14.00	−44 18	−10.66	122	187	105	−21.6	74.7

Except as indicated enthalpy is above 40 F.
* Above 0° F.

COMPOUND, COMBINED, OR BINARY VAPOR REFRIGERATION SYSTEMS. Sometimes, instead of operating with a compound system using a single refrigerant, binary systems have been arranged wherein a given refrigerant, such as carbon dioxide, may operate with an evaporator temperature of −60 F and deliver its discharge gas to a condenser operating at 0 F. The carbon dioxide condenser in this case is arranged to be the evaporator of a second refrigeration system using ammonia. It, of course, operates with an evaporator somewhat lower than 0 F, say −10 F, and delivers its gases to an ordinary condenser operating with cooling water or ambient air at perhaps 75 F.

Endeavoring to carry refrigeration through this range of 125 degrees in a single stage would present a difficult design problem; and, if ammonia were used alone, would require that the evaporator of the ammonia system work high in the vacuum region, a difficult problem for ordinary seals. Present design would envisage the use of "Freon-22" or even "Freon-12" as preferable refrigerants to carbon dioxide.

CAPACITY CONTROL FOR RECIPROCATING COMPRESSORS. A varying refrigerating load requires pumping a varying weight and volume of gas. Steam-driven compressors can do this by varying engine speed. With compressors driven by constant-speed motors or diesel engines, another means of varying the weight of vapor pumped is required. Throttling the suction decreases capacity, but is wasteful of power.

One method varies and reduces compressor capacity by clearance pockets around the compressor cylinder, which communicate through auxiliary valves with the clearance space. The pockets increase the clearance volume by fixed percentages of cylinder volume and vary the weight of vapor in the clearance space, and consequently the piston displacement volume available for drawing in vapor from the evaporator on the suction stroke. Some machines obtain 100%, 50%, and zero capacity. The clearance pockets provide means of varying compressor capacity with decreasing power requirements as capacity decreases.

3. REFRIGERANT TABLES AND PROPERTIES

The pressure temperature characteristics of a large number of common refrigerants are indicated in Table 2 with extensive tabulations of the very common refrigerants given in Tables 3–9 and on the *p-h* charts of Figs. 2, 5, 6, and 7.

There is no perfect or ideal refrigerant. Desirable properties of a refrigerant are low condensing pressures, low boiling temperature at atmospheric pressure, high critical

Table 3. Thermal Properties of Liquid and Saturated Ammonia

Figures in bold face are for *standard ton* conditions. This table and Table 5 were condensed from *Natl. Bur. Standards Tables for Ammonia*, Circular 142

Temp., °F t	Pressure, psia p	Specific Volume of Vapor, cu ft/lb v_g	Enthalpy above −40 F, Btu per lb			Entropy of Vapor s
			Liquid, h_f	Latent, h_{fg}	Total, h_g	
−60	5.55	44.73	−21.2	610.8	589.6	1.4769
−50	7.67	33.08	−10.6	604.3	593.7	1.4497
−40	10.41	24.86	0.0	597.6	597.6	1.4242
−30	13.90	18.97	10.7	590.7	601.4	1.4401
−28	14.71	18.00	12.8	589.3	602.1	1.3955
−20	18.30	14.68	21.4	583.6	605.0	1.3774
−10	23.74	11.50	32.1	576.4	608.5	1.3558
0	30.42	9.116	42.9	568.9	611.8	1.3552
5	**34.27**	**8.150**	**48.3**	**565.0**	**613.3**	**1.3253**
10	38.51	7.304	53.8	561.1	614.9	1.3157
15	43.14	6.562	59.2	557.1	616.3	1.3062
20	48.21	5.910	64.7	553.1	617.8	1.2969
25	53.73	5.334	70.2	548.9	619.1	1.2879
30	59.74	4.825	75.7	544.8	620.5	1.2790
40	73.32	3.971	86.8	536.2	623.0	1.2618
50	89.19	3.294	97.9	527.3	625.2	1.2453
60	107.6	2.751	109.2	518.1	627.3	1.2294
70	128.8	2.312	120.5	508.6	629.1	1.2140
77	145.4	2.055	128.5	501.7	630.2	1.2035
80	153.0	1.955	132.0	498.7	630.7	1.1991
86	**169.2**	**1.772**	**138.9**	**492.6**	**631.5**	**1.1904**
90	180.6	1.661	143.5	488.5	632.0	1.1846
95	195.8	1.534	149.4	483.2	632.6	1.1775
100	211.9	1.419	155.2	477.8	633.0	1.1705

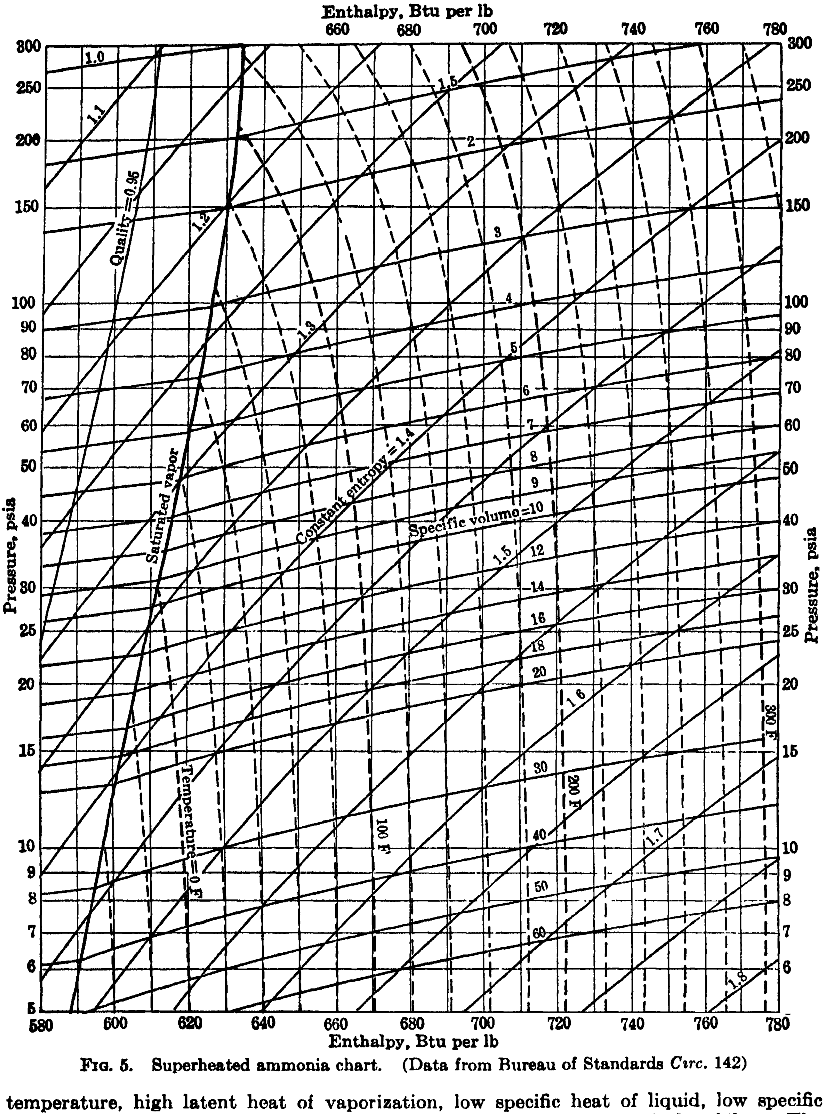


FIG. 5. Superheated ammonia chart. (Data from Bureau of Standards *Circ.* 142)

temperature, high latent heat of vaporization, low specific heat of liquid, low specific volume of vapor, absence of corrosive action on metals, and good chemical stability. The refrigerant should also be noninflammable and nonexplosive, and it should be nontoxic also, particularly when it is to be used in air-conditioning installations. A refrigerant should also have characteristics which do not harm the lubricants in the system and have satisfactory heat-transfer and viscosity characteristics. The refrigerant selected for a given job will satisfy many of these requirements, but no known refrigerant satisfies all of them.

The two most-used refrigerants are ammonia (NH_3) and "Freon-12" (CCl_2F_2). In addition there are many other refrigerants used in special installations or for special reasons. These refrigerants are described below in some detail.

AMMONIA is the most extensively used refrigerant, particularly in industrial and commercial refrigeration. When water-free it is known as anhydrous ammonia, and when mixed with water it is called aqua ammonia. As shown in Tables 3 and 5 and Fig. 5, its operating pressure range is moderate, not dropping below atmospheric pressure unless the temperature is less than −28 F. Recommended working pressures for design of the high side are 250 psig and for the low side 150 psig. Ammonia does not harm mineral oil for lubrication, and water mixed with ammonia does not freeze at expansion valves, which is a disadvantage with most other refrigerants. Ammonia is noncorrosive to iron and steel materials, but rapidly corrodes copper and copper alloys, such as brass and bronze. Ammonia gas is irritating to eyes and mucous membranes, and in quantities approaching 0.5% by volume in air it may cause serious injury if exposure is prolonged. Ammonia does not burn readily, but can form explosive mixtures with air when the ratio lies between 16 and 25% of gas by volume. No harmful decomposition products are formed.

"FREON-12," chemically known as **dichlorodifluoromethane,** is one of a large number of refrigerants made by synthesizing a basic hydrocarbon. The chemical formula of "Freon-12" can be written from its chemical name by starting with the formula for methane (CH_4) and replacing the hydrogen atoms with the proper number of chlorine and fluorine atoms, arriving at (CCl_2F_2). The whole "Freon" family of refrigerants has low

Table 4. Saturated Liquid and Vapor Table of Dichlorodifluoromethane ("Freon-12")

(Reprinted by permission from *ASRE Data Book*)

Temperature, °F t	Pressure, psia p	Specific Volume		Enthalpy, above −40 F			Entropy	
		Liquid, cu ft/lb v_f	Vapor, cu ft/lb v_g	Liquid, Btu/lb h_f	Vaporization Btu/lb h_{fg}	Vapor, Btu/lb h_g	Liquid s_f	Vapor s_g
−40	9.32	.0106	3.911	0	73.50	73.50	0	.17517
−30	12.02	.0107	3.088	2.03	72.67	74.70	.00471	.17387
−20	15.28	.0108	2.474	4.07	71.80	75.87	.00940	.17275
−10	19.20	.0109	2.003	6.14	70.91	77.05	.01403	.17175
0	23.87	.0110	1.637	8.25	69.96	78.21	.01869	.17091
2	24.89	.0110	1.574	8.67	69.77	78.44	.01961	.17075
4	25.96	.0111	1.514	9.10	69 57	78.67	.02052	.17060
5	26.51	.0111	1.485	9.32	69.47	78.79	.02097	.17052
6	27.05	.0111	1.457	9.53	69.37	78.90	.02143	.17045
8	28.18	.0111	1.403	9.96	69.17	79.13	.02235	.17030
10	29.35	.0112	1.351	10.39	68.97	79.36	.02328	.17015
12	30.56	.0112	1.301	10.82	68.77	79.59	.02419	.17001
14	31.80	.0112	1.253	11.26	68.56	79.82	.02510	.16987
16	33.08	.0112	1.207	11.70	68.35	80.05	.02601	.16974
18	34.40	.0113	1.163	12.12	68.15	80.27	.02692	.16961
20	35.75	.0113	1.121	12.55	67.94	80.49	.02783	.16949
22	37.15	.0113	1.081	13.00	67.72	80.72	.02873	.16938
24	38.58	.0113	1.043	13.44	67.51	80.95	.02963	.16926
26	40.07	.0114	1.007	13.88	67.29	81.17	.03053	.16913
28	41.59	.0114	0.973	14.32	67.07	81.39	.03143	.16900
30	43.16	.0115	0.939	14.76	66.85	81.61	.03233	.16887
32	44.77	.0115	0.908	15.21	66.62	81.83	.03323	.16876
34	46.42	.0115	0.877	15.65	66.40	82.05	.03413	.16865
36	48.13	.0116	0.848	16.10	66.17	82.27	.03502	.16854
38	49.88	.0116	0.819	16.55	65.94	82.49	.03591	.16843
40	51.68	.0116	0.792	17.00	65.71	82.71	.03680	.16833
50	61.39	.0118	0.673	19.27	64.51	83.78	.04126	.16785
60	72.41	.0119	0.575	21.57	63.25	84.82	.04568	.16741
70	84.82	.0121	0.493	23.90	61.92	85.82	.05009	.16701
80	98.76	.0123	0.425	26.28	60.52	86.80	.05446	.16662
86	107.9	.0124	0.389	27.72	59.65	87.37	.05708	.16640
90	114.3	.0125	0.368	28.70	59.04	87.74	.05882	.16624
100	131.6	.0127	0.319	31.16	57.46	88.62	.06316	.16584
110	150.7	.0129	0.277	33.65	55.78	89.43	.06749	.16542
120	171.8	.0132	0.240	36.16	53.99	90.15	.07180	.16495
130	194.9	.0134	0.208	38.69	52.07	90.76	.07607	.16438
140	220.2	.0138	0.180	41.24	50.00	91.24	.08024	.16363

toxicity, "Freon-12" of the group having about the lowest toxicity of all. This refrigerant has no odor and can be breathed with no apparent harm when its concentration in air is as high as 20% by volume. As for physiological action, it apparently acts as an inert material which is breathed in and exhaled and merely acts as an additional diluent for the oxygen in the air. For this reason, it is the most common refrigerant used in air-conditioning installations. Observation of its formula will indicate that it is actually a halogenated hydrocarbon, and such chemicals, in the presence of open flames or extremely hot surfaces, may break down and cause objectionable or toxic decomposition products. Caution must thus be exercised in using this gas where there is fire hazard or open flame. Tables 2 and 4 and Fig. 2 indicate that the pressure range is moderate. The latent heat is low, 50 to 85 Btu per lb, so that the weight of "Freon-12" circulated per minute per ton of refrigeration is much greater than that required for ammonia, but the volume handled

Table 5. Thermal Properties of Superheated Ammonia

v = specific volume, cu ft per lb; h = enthalpy, Btu per lb; s = entropy superheated vapor

Temp., °F	Pressures, psia, 170, psig, 155.3 Sat. Temp., 86.29 F			Pressures, psia, 180, psig, 165.3 Sat. Temp., 89.78 F			Pressures, psia, 190, psig, 175.3 Sat. Temp., 93.13 F		
t	v	h	s	v	h	s	v	h	s
Sat.	1.764	631.6	1.1900	1.667	632.0	1.1850	1.581	632.4	1.1802
100	1.837	641.9	1.2087	1.720	639.9	1.1992	1.615	637.8	1.1899
110	1.889	649.1	1.2215	1.770	647.3	1.2123	1.663	645.4	1.2034
120	1.939	656.1	1.2336	1.818	654.4	1.2247	1.710	652.6	1.2160
130	1.988	662.8	1.2452	1.865	661.3	1.2364	1.755	659.7	1.2281
140	2.035	669.4	1.2563	1.910	668.0	1.2477	1.799	666.5	1.2396
150	2.081	675.9	1.2669	1.955	674.6	1.2586	1.842	673.2	1.2506
160	2.127	682.3	1.2733	1.999	681.0	1.2691	1.884	679.7	1.2612
170	2.172	688.5	1.2873	2.042	687.3	1.2792	1.925	686.1	1.2715
180	2.216	694.7	1.2971	2.084	693.6	1.2891	1.966	692.5	1.2815
190	2.260	700.8	1.3066	2.126	699.8	1.2987	2.005	698.7	1.2912
200	2.303	706.9	1.3159	2.167	705.9	1.3081	2.045	704.9	1.3007
210	2.346	713.0	1.3249	2.208	712.0	1.3172	2.084	711.1	1.3099
220	2.389	719.0	1.3338	2.248	718.1	1.3262	2.123	717.2	1.3189
230	2.431	724.9	1.3426	2.288	724.1	1.3350	2.161	723.2	1.3278
240	2.473	730.9	1.3512	2.328	730.1	1.3436	2.199	729.3	1.3365
250	2.514	736.8	1.3596	2.367	736.1	1.3521	2.236	735.3	1.3450
260	2.555	742.8	1.3679	2.407	742.0	1.3605	2.274	741.3	1.3534
270	2.596	748.7	1.3761	2.446	748.0	1.3687	2.311	747.3	1.3617
280	2.637	754.6	1.3841	2.484	753.9	1.3768	2.348	753.2	1.3698
300	2.718	766.4	1.3999	2.561	765.8	1.3926	2.421	765.2	1.3857
320	2.798	778.3	1.4153	2.637	777.7	1.4081	2.493	777.1	1.4012
340	2.878	790.1	1.4303	2.713	789.6	1.4231	2.565	789.0	1.4163
360	2.957	802.0	1.4450	2.788	801.5	1.4379	2.637	801.0	1.4311
380	3.036	814.0	1.4594	2.863	813.5	1.4523	2.707	813.0	1.4456

is somewhat less. Pipe joints for the halogenated hydrocarbons must be carefully made to prevent gas leakage. Copper tubing, with solder-sweated or brazed joints, is customarily used in small installations, and even in some of the larger ones. For lubrication, mineral oil, free of water, is required. Oil and the "Freon" family are mutually soluble, and an excess of "Freon" may dilute the lubricating oil to a point where it loses its lubricating effectiveness. It is necessary to observe caution in operation to see that not too much oil from the compressor is carried over into the evaporator, and proper provision must be made for returning carry-over oil back to the compressor. Rubber gasket material is rapidly weakened by "Freon," but a group of synthetic gasket compositions, such as neoprene, are satisfactory. Water must be completely removed from a "Freon" system to prevent freezing up of the expansion valve. Corrosion of metals by "Freon" is trivial or nonexistent except in the presence of water.

"FREON-11," also known as Carrene 2, and chemically as trichloromonofluoromethane (CCl_3F), is one of the so-called *vacuum* refrigerants, that is, it exists as a liquid at normal atmospheric temperature and pressure and, in order to obtain refrigeration temperatures in the evaporator, it is necessary to vaporize the refrigerant at pressures below atmospheric. Table 6 shows its properties. This refrigerant is practically odorless and relatively

Table 6. Saturated Liquid and Vapor Tables for Various Refrigerants

Temperature, °F t	Pressure, psia p	Specific Volume of Liquid, cu ft/lb v_f	Specific Volume of Vapor, cu ft/lb v_g	Enthalpy Btu/lb above −40 F			Entropy	
				Liquid h_f	Vaporization, h_{fg}	Vapor h_g	Liquid s_f	Vapor s_g
Carbon Dioxide, CO_2 (Reprinted by permission from *ASRE Data Book*)								
−40	145.8	.01437	0.6113	0.00	137.8	137.8	1.0000	1.3285
−30	177.8	.01466	0.5029	4.5	133.7	138.2	1.0107	1.3218
−20	214.9	.01498	0.4168	9.1	129.4	138.5	1.0212	1.3154
−10	257.3	.01532	0.3472	13.9	124.8	138.7	1.0314	1.3091
0	305.5	.01570	0.2904	18.8	120.1	138.9	1.0418	1.3029
4	326.5	.01588	0.2707	20.8	118.0	138.8	1.0460	1.3006
6	337.4	.01596	0.2614	21.8	116.9	138.7	1.0481	1.2994
10	360.2	.01614	0.2437	24.0	114.7	138.7	1.0536	1.2980
20	421.8	.01663	0.2049	29.4	108.9	138.3	1.0648	1.2919
30	490.8	.01719	0.1722	35.4	102.4	137.8	1.0768	1.2859
40	567.8	.01787	0.1444	41.7	95.0	136.7	1.0884	1.2786
50	653.6	.01868	0.1205	48.4	86.6	135.0	1.1010	1.2709
60	748.6	.01970	0.0994	55.5	76.6	132.1	1.1145	1.2618
70	853.4	.02112	0.08040	63.7	63.8	127.5	1.1292	1.2497
80	968.7	.02370	0.06064	73.9	44.8	118.7	1.1486	1.2314
86	1043.0	.02686	0.04789	83.3	27.1	110.4	1.1646	1.2143
87.8	1066.2	.03454	0.03454	97.0	0.0	97.0	1.1890	1.1890
Trichloromonofluoromethane ("Freon-11" or Carrene 2) (Reprinted by permission of Kinetic Chemicals, Inc.)								
−80	0.157	.00961	189	−7.89	90.68	82.79	−0.0197	0.1995
−60	0.356	.00974	87.5	−3.94	89.06	85.12	−0.0096	0.2037
−40	0.739	.00988	44.2	0.00	87.48	87.48	0.0000	0.2085
−20	1.420	.01002	24.06	3.94	85.93	89.87	0.0091	0.2046
−10	1.920	.01010	18.17	5.91	85.16	91.07	0.0136	0.2030
0	2.555	.01018	13.94	7.89	84.38	92.27	0.0179	0.2015
5	2.931	.01022	12.27	8.88	84.00	92.88	0.0201	0.2009
10	3.352	.01026	10.83	9.88	83.60	93.48	0.0222	0.2003
20	4.342	.01034	8.519	11.87	82.82	94.69	0.0264	0.1991
30	5.557	.01042	6.776	13.88	82.03	95.91	0.0306	0.1981
40	7.032	.01051	5.447	15.89	81.22	97.11	0.0346	0.1972
50	8.804	.01060	4.421	17.92	80.40	98.32	0.0386	0.1964
60	10.90	.01069	3.626	19.96	79.57	99.53	0.0426	0.1958
70	13.40	.01079	2.993	22.02	78.71	100.73	0.0465	0.1951
80	16.31	.01088	2.492	24.09	77.84	101.93	0.0504	0.1947
86	18.28	.01094	2.242	25.34	77.31	102.65	0.0527	0.1944
90	19.69	.01098	2.091	26.18	76.95	103.12	0.0542	0.1942
100	23.60	.01109	1.765	28.27	76.03	104.30	0.0580	0.1938
110	28.09	.01119	1.499	30.40	75.08	105.47	0.0617	0.1935
120	33.20	.01130	1.281	32.53	74.10	106.63	0.0654	0.1933
Sulfur Dioxide, SO_2 (Reprinted by permission from *ASRE Data Book*)								
−40	3.136	.01044	22.42	0.00	178.61	178.61	0.00000	0.42562
−20	5.883	.01063	12.42	5.98	175.09	181.07	0.01366	0.41192
−10	7.863	.01072	9.44	9.16	172.97	182.13	0.02075	0.40544
0	10.35	.01082	7.280	12.44	170.63	183.07	0.02795	0.39917
5	11.81	.01087	6.421	14.11	169.38	183.49	0.03155	0.39609
10	13.42	.01092	5.682	15.80	168.07	183.87	0.03519	0.39306
20	17.18	.01102	4.487	19.20	165.32	184.52	0.04241	0.38707
30	21.70	.01114	3.581	22.64	162.38	185.02	0.04956	0.38119
40	27.10	.01126	2.887	26.12	159.25	185.37	0.05668	0.37541
50	33.45	.01138	2.348	29.61	155.95	185.56	0.06370	0.36969
60	40.93	.01150	1.926	33.10	152.49	185.59	0.07060	0.36405
70	49.62	.01163	1.590	36.58	148.88	185.46	0.07736	0.35846
80	59.68	.01176	1.321	40.05	145.12	185.17	0.08399	0.35291
86	66.45	.01184	1.185	42.12	142.80	184.92	0.08783	0.34954
90	71.25	.01190	1.104	43.50	141.22	184.72	0.09038	0.34731
100	84.52	.01204	0.9262	46.90	137.20	184.10	0.09657	0.34173
110	99.76	.01219	0.7804	50.26	133.05	183.31	0.10254	0.33611
120	120.93	.01236	0.6598	53.58	128.78	182.36	0.10829	0.33046

nontoxic. It is nonexplosive and practically noninflammable except for decomposition. It has no corrosive action on common metals of construction.

The specific volume of the so-called "vacuum" refrigerants is usually so great that large volumes of gas must be handled. Reciprocating machines of reasonable size cannot perform effectively in this case, and such refrigerants are normally used with centrifugal machines. These machines, in turn, require a small overall pressure range, usually less than 30 psi.

"FREON-22," known chemically as monochlorodifluoromethane ($CHClF_2$), is a higher-pressure member of the "Freon" group which is suitable for operation at lower temperatures. It is relatively nontoxic. Because of its higher pressure, it can work in a low-temperature region without running into the necessity of extremely high specific volumes. At −75 F the specific volume is only 8.3 cu ft per lb, and the pressure is 5.6 psi. This is appreciably lower than corresponding values for either "Freon-12" or ammonia. Its suggested design pressures for the high side are 300 psig and for the low side 225 psig. Data on this refrigerant are given in Table 7 and in Fig. 6.

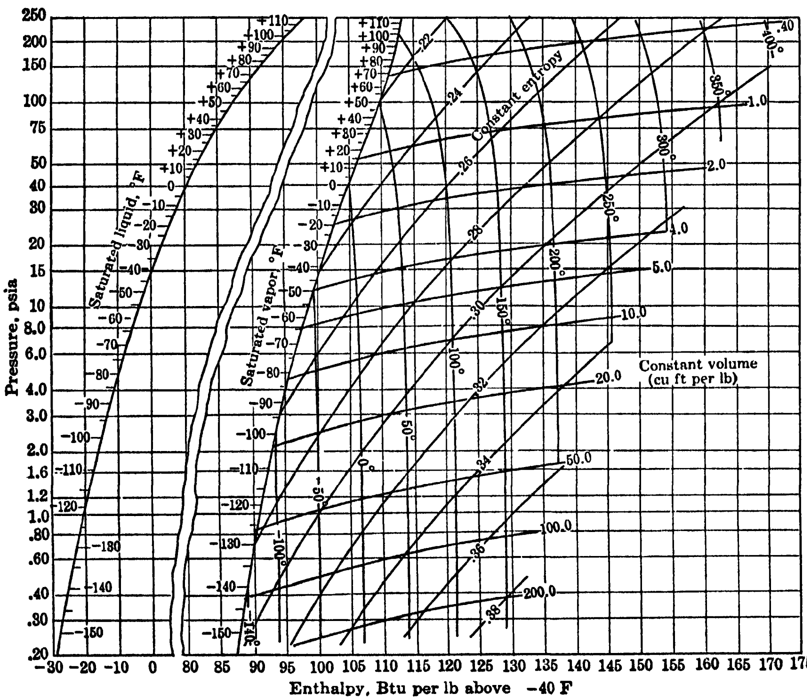


FIG. 6. P-h chart for "Freon-22." (Drawn from data of Kinetic Chemicals, Inc., by permission)

METHYL CHLORIDE (CH_3Cl) has been used rather extensively in small commercial refrigerating units and in small air-conditioning units. The pressure range for this refrigerant is moderate, the recommended design pressures being 215 psig on the high side, 125 psig on the low side. As a refrigerant its performance is good, with a latent heat of evaporation running in excess of 150 Btu per lb over the common operating range. The refrigerant is stable and relatively noncorrosive to common metals of construction, with the exception of aluminum. With aluminum, rapid and dangerous decomposition occurs if water is present, and care must be taken to see that this metal is never employed. Because methyl chloride has a sweet-smelling odor that is not seriously offensive it is desirable to add to it warning agents to indicate the presence of leaking gas. Concentrations of 2% by volume in air may be dangerous if the exposure lasts for a long period. The gas is also mildly anesthetic. It should never be used in direct-expansion air-conditioning systems. Methyl chloride is moderately inflammable and can be explosive between limits of 8.1% and 17.1% by volume in air. Properties are given in Tables 2 and 8 and in Fig. 7.

Table 7. Properties of "Freon-22" ($CHClF_2$)

Liquid and Saturated Vapor

(Condensed and reproduced by permission of Kinetic Chemicals Co.)

Temperature, °F	Pressure, psia, p	Volume, cu ft/lb		Enthalpy, Btu/lb above −40 F			Entropy		Temperature, °F t
		Liquid v_f	Vapor v_g	Liquid h_f	Vaporization h_{fg}	Vapor h_g	Liquid s_f	Vapor s_g	
−150	0.2605	.0103	146.1	−27.79	115.15	87.36	−0.0767	.2952	−150
−140	0.4332	.0103	90.61	−25.25	113.78	88.53	−0.0687	.2874	−140
−130	0.6949	.0104	58.21	−22.73	112.43	89.70	−0.0609	.2803	−130
−120	1.079	.0105	38.60	−20.22	111.10	90.88	−0.0534	.2738	−120
−110	1.626	.0106	26.33	−17.73	109.80	92.07	−0.0461	.2680	−110
−100	2.386	.0107	18.43	−15.23	108.50	93.27	−0.0390	.2627	−100
−90	3.417	.0108	13.20	−12.73	107.20	94.47	−0.0322	.2579	−90
−80	4.787	.01090	9.650	−10.22	105.90	95.68	−0.0255	.2535	−80
−70	6.57	.01100	7.192	−7.69	104.57	96.88	−0.0253	.2494	−70
−60	8.86	.01111	5.452	−5.16	103.24	98.08	−0.0126	.2458	−60
−50	11.74	.01123	4.192	−2.58	101.86	99.28	−0.0062	.2425	−50
−40	15.31	.01135	3.279	0.00	100.46	100.46	0.0000	.2394	−40
−30	19.72	.01148	2.590	2.62	99.01	101.63	0.0062	.2367	−30
−20	25.01	.01162	2.074	5.28	97.51	102.79	0.0123	.2341	−20
−10	31.29	.01177	1.681	7.96	95.96	103.92	0.0182	.2316	−10
0	38.79	.01192	1.373	10.63	94.39	105.02	0.0240	.2293	0
5	43.02	.01200	1.246	11.97	93.59	105.56	0.0268	.2283	5
10	47.63	.01208	1.130	13.29	92.79	106.08	0.0296	.2272	10
20	57.98	.01225	0.9369	15.98	91.15	107.13	0.0352	.2253	20
30	69.93	.01243	0.7816	18.74	89.39	108.13	0.0409	.2235	30
40	83.72	.01262	0.6559	21.70	87.39	109.09	0.0469	.2218	40
50	99.40	.01282	0.5537	24.73	85.25	109.98	0.0528	.2201	50
60	117.2	.01303	0.4695	27.83	82.95	110.78	0.0588	.2185	60
70	137.2	.01325	0.4000	30.99	80.50	111.49	0.0648	.2168	70
80	159.7	.01349	0.3417	34.27	77.86	112.13	0.0708	.2151	80
86	174.5	.01363	0.3113	36.28	76.19	112.47	0.0744	.2140	86
90	184.8	.01374	0.2928	37.61	75.06	112.67	0.0768	.2133	90
100	212.6	.01402	0.2517	40.98	72.08	113.06	0.0827	.2115	100
110	243.4	.01433	0.2167	44.35	68.94	113.29	0.0886	.2096	110
120	277.3	.01469	0.1871	47.85	65.67	113.52	0.0945	.2078	120

CARBON DIOXIDE (CO_2), before the development of the synthetic group of "Freon" refrigerants, was an extremely important refrigerant because of its low toxicity. CO_2 is noncorrosive and inert, has no odor, is nonirritating and essentially nontoxic, in fact, it is exhaled in breathing. However, as the concentration of this gas increases to values in excess of 6 to 8% by volume in air, it upsets the regulatory mechanism of the lungs and may cause breathing difficulties which can result in loss of consciousness and suffocation. Its main disadvantage as a refrigerant lies in the fact that it is an extremely high-pressure refrigerant (Tables 2 and 6) with a low critical temperature, 87.8 F at 1066 psia. At 20 F its evaporating pressure is 422 psia, and its triple point is −69.88 F at 75.1 psia. Thus liquid CO_2 cannot exist at atmospheric pressure, and the solid sublimes at −109.3 F at 14.7 psia.

Numerous other chemicals have been used as refrigerants. Some, used during the period of development of the industry, are now obsolete; others are required for special purposes. In particular, for very low temperature refrigeration, it has been necessary to use hydrocarbon gases or even some of the permanent gases. For example, *ethylene* (C_2H_4) has been used for certain low-temperature work in gas liquefaction and separation. This hydrocarbon boils at −154.7 F at 14.7 psia and has a critical temperature of 49.3 F at 743 psia. In very-low-temperature work of this type it is necessary to operate cascade or compound systems in which a low-temperature refrigerant dissipates its heat into a condenser, which, in turn, is the evaporator of a second refrigeration system operating in a higher range. In fact, sometimes as many as four systems are superimposed in cascade on each other to obtain the low temperatures desired. Space does not permit a discussion of all the refrigerants, but some mention of certain common refrigerants will be made.

DICHLOROMONOFLUOROMETHANE ($CHCl_2F$), also called **"Freon-21"**; **dichlorotetrafluoroethane** ($C_2Cl_2F_4$), also known as **"Freon-114"**; and **trichlorotrifluoroethane** ($C_2Cl_3F_3$), also known as **"Freon-113,"** are all vacuum-type refrigerants.

METHYLENE CHLORIDE (CH_2Cl) also known as Carrene 1, was one of the first vacuum-type refrigerants used in the early centrifugal compressors.

DICHLOROETHYLENE ($C_2H_2Cl_2$), also known as dielene, is a vacuum refrigerant formerly used in centrifugal refrigeration machines.

SULFUR DIOXIDE (SO_2) and **methyl formate** ($C_2H_4O_2$) are both relatively low-pressure refrigerants and have had some use in domestic refrigerators particularly those of the hermetically sealed type.

Table 9 lists toxic properties of certain refrigerants.

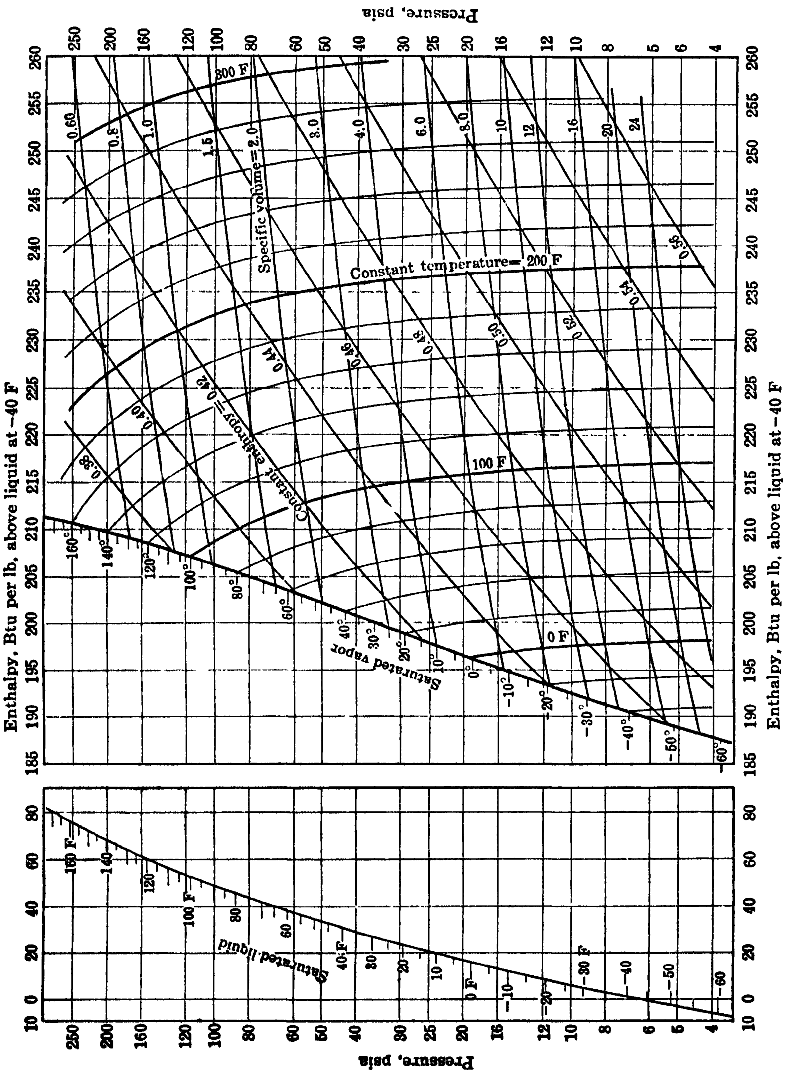


FIG. 7. Pressure-enthalpy diagram for methyl chloride.

Table 8. Methyl Chloride

Properties of Saturated Vapor

(Data from E. I. Du Pont de Nemours & Co.)

Temperature, °F t	Pressure, psia p	Specific Volume, cu ft/lb		Enthalpy above −40 F			Entropy		Temperature, °F t
		Liquid v_f	Vapor v_g	Liquid, Btu/lb h_f	Vaporization, Btu/lb h	Vapor, Btu/lb h_g	Liquid, Btu/lb F s_f	Vapor, Btu/lb F s_g	
−80	1.953	.01493	41.08	−13.888	198.64	184.75	−0 0351	.4882	−80
−60	3.799	.01523	22.09	−7.039	194.78	187.74	−0.0172	.4703	−60
−40	6.878	.01553	12.72	0.000	190.66	190 66	0.0000	.4544	−40
−30	9.036	.01568	9.873	3.562	188.52	192.08	0.0084	.4472	−30
−20	11.71	.01583	7.761	7.146	186.34	193.49	0.0166	.4405	−20
−10	14.96	.01598	6.176	10.75	184.11	194.87	0.0247	.4343	−10
0	18.90	.01613	4.969	14.39	181.85	196.23	0.0327	.4284	0
5	21.15	.01622	4.471	16.21	180.70	196.92	0.0367	.4257	5
10	23.60	.01631	4 038	18 04	179 53	197.58	0.0406	.4229	10
20	29.16	.01647	3.312	21.73	177.11	198.84	0.0484	.4177	20
30	35.68	.01665	2.739	25.44	174.59	200.03	0.0560	.4126	30
40	43.25	.01684	2.286	29.17	172.00	201.17	0.0636	.4079	40
50	51.99	.01704	1.920	32.93	169.35	202.28	0.0710	.4034	50
60	62.00	.01724	1.624	36.71	166.62	203.33	0.0784	.3991	60
70	73.41	.01744	1.382	40.52	163.82	204.34	0.0856	.3950	70
80	86.26	.01764	1.183	44.36	160.91	205 27	0.0928	.3910	80
86	94.70	.01778	1.081	46.67	159.13	205 80	0.0970	.3887	86
90	100.6	.01786	1.018	48 21	157.92	206.13	0.0998	.3872	90
100	116.7	.01808	0.8814	52 09	154.85	206 94	0.1069	.3836	100
110	134.5	.01833	0.7672	56.00	151.70	207.70	0.1138	.3801	110
120	154.2	.01859	0.6710	59.93	148.46	208.39	0.1206	.3768	120
130	175.9	.01887	0.5889	63.89	145.13	209.02	0.1274	.3736	130
140	199.6	.01915	0.5189	67.87	141.71	209.58	0.1341	.3705	140
150	225.4	.01945	0.4586	71.87	138.23	210 10	0.1407	.3674	150
160	253.5	.01978	0.4070	75.90	134 66	210 56	0.1473	.3646	160

Table 9. Toxic Properties of Refrigerants

(Reprinted by permission from *Refrigerating Eng.*, Vol. 39, 1940. Prepared by R. J. Thompson.)

Underwriters' Laboratory Group No.	Refrigerant	Formula	Cu ft/lb of Refrigerant, at 68 F and 14.7, lb/in.²	Boiling Point, °F	Kills or Seriously Injures		
					Duration of Exposure	Concentration in Air	
						% by Volume	lb/1000 cu ft
1	Sulfur dioxide	SO_2	6.01	14.0	5 min	0.7	1.165
2	Ammonia	NH_3	22.6	−28.0	1/2 hr	0.5– 0.6	0.221– 0.256
3	Methyl formate	$C_2H_4O_2$	6.41	89.2	1 hr	2 – 2.5	3.12 – 3.9
4	Methyl chloride	CH_3Cl	7.62	−10.6	2 hr	2 – 2.5	2.62 – 3.28
4	Dichloroethylene	$C_2H_2Cl_2$	3.97	118	2 hr	2 – 2.5	5.04 – 6.3
4	Ethyl chloride	C_2H_5Cl	5.96	54.5	1 hr	4.0	6.72
4	Methylene chloride	CH_2Cl_2	4.53	103.6	1/2 hr	5.1– 5.3	11.25 –11.70
4	"Freon-113"	$C_2Cl_3F_3$	2.1	117.6	.	....	... .
4	"Freon-21"	$CHCl_2F$	3.76	48.0	1/2 hr	10.2	27.1
5	Carbon dioxide	CO_2	8.75	−108.4	1/2 to 1 hr	29.0–30.0	33.2 –34.3
5	"Freon-11"	CCl_3F	2.8	74.7	2 hr	10	35.7
6	"Freon-114"	$C_2Cl_2F_4$	2.25	38.4	2 hr	*20.1–21.5	89.6 –95.7
6	"Freon-12"	CCl_2F_2	3.18	−21.7	2 hr	*28.5-30.4	89.6 –95.7

* Oxygen deficiency.

4. REFRIGERATION COMPRESSORS

RECIPROCATING COMPRESSORS. Both reciprocating and centrifugal compressors are used. Centrifugal compressors are applied to larger capacity units, 75 tons of refrigeration or more. Reciprocating machines that are more numerous and widely distributed run from small fractions of a ton to more than 100 tons capacity per unit. The horizontal, double-acting type (Fig. 8) is infrequently built with the present-day emphasis being on multicylinder, single-acting, *vertical* or *radial* or *W*- or *V-block* units (Fig. 9). Frequently these have the suction valve built integrally into the top of the piston although the use of separate ring-plate and poppet-spring-loaded valves on both suction and discharge is common. The compressors are built with small clearance (2 to 7% with 3 to 5% most common).

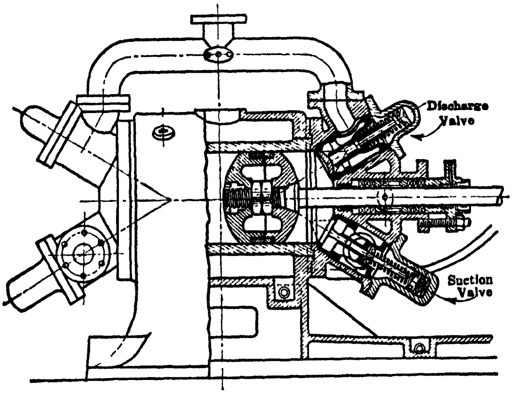


Fig. 8. Double-acting compressor cylinder.

To protect the compressor from excessive pressure rise in the event of liquid carry-over being trapped between the piston top and the cylinder head, it is customary to make a movable safety head inside the main cylinder. The safety head is normally held in fixed position by powerful springs but can move to relieve the pressure when required.

Packing around the shaft, of soft type, is still used on a few ammonia-type compressors. More common are revolving shaft seals in which a lapped and hardened seal ring rotates tightly against a mating lapped surface. With slight lubrication between the surfaces a sufficiently high pressure is exerted to prevent refrigerant leakage through the seal. Some form of tight bellows, hermetically fixed to the shaft, holds the spring and seal ring in position. Any packing or seal is subject to leakage, and a preferable construction that has been adopted for small compressors (in nearly all domestic sizes) is the so-called hermetic arrangement. In this the motor is housed in the casing, which constitutes part of the compressor or is a compressor extension. There is no shaft seal to outside, and the motor may come in contact with both the refrigerant gas and the lubricant. Extension of this design to large-size units is difficult, but domestic ($^1/_8$ to $^1/_2$ ton) and small units to 2-ton capacity have had very successful designs of this type. It should be realized that such compressors run at motor speed, and 1150 and 1750 rpm units are common. Some direct-connected units as large as 75 hp have been made in semihermetic construction with flanged and gasketed cover plates permitting access to the inside of the unit. Most units (except for the domestics) use shaft seals with some form of Vee belt drive, but more and more units are being manufactured using direct motor drive.

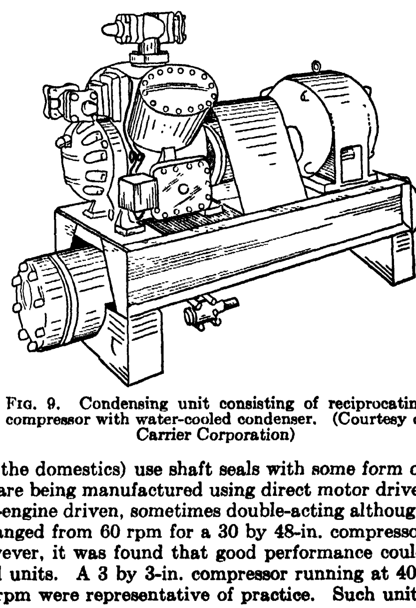


Fig. 9. Condensing unit consisting of reciprocating compressor with water-cooled condenser. (Courtesy of Carrier Corporation)

Earlier ammonia machines were steam-engine driven, sometimes double-acting although more frequently single-acting. Speeds ranged from 60 rpm for a 30 by 48-in. compressor to 125 rpm for a 9 by 12-in. unit. However, it was found that good performance could also be obtained from smaller high-speed units. A 3 by 3-in. compressor running at 400 rpm or a 6 by 6-in. one running at 360 rpm were representative of practice. Such units

Table 10. Ammonia Compressor Performance * at Various Operating Conditions for Vertical Single-acting Ammonia Compressors and Single-stage Compression

(Reprinted by permission from *ACRMA Equipment Standards* 413, Part II)

Pressure and Corresponding Saturated Temperature of Suction Gas		Discharge Pressure, Psig															
		135		145		155		165		175		185		195		205	
		Corresponding Saturation Temperature, °F															
		78.7		82.5		86.2		89.7		93.0		96.2		99.3		102.3	
Pressure, Psig	Temperature, °F	Cfm/Ton	Bhp/Ton	Cfm/Ton	Bhp/Ton	Cfm/Ton	Bhp/Ton	Cfm/Ton	Bhp/Ton	Cfm/Ton	Bhp/Ton	Cfm/Ton	Bhp/Ton	Cfm/Ton	Bhp/Ton	Cfm/Ton	Bhp/Ton
0	−28.0	12.34	2.51														
5	−17.2	8.42	2.02	8.64	2.12	8.89	2.22	9.16	2.32	9.40	2.41	9.66	2.50				
10	−8.4	6.36	1.694	6.53	1.782	6.70	1.869	6.84	1.955	7.01	2.04	7.16	2.12	7.35	2.20	7.53	2.28
15	−1.0	5.09	1.449	5.21	1.531	5.34	1.611	5.45	1.690	5.56	1.765	5.67	1.839	5.79	1.911	5.93	1.980
20	5.5	4.28	1.270	4.39	1.341	4.46	1.412	4.55	1.482	4.64	1.552	4.72	1.621	4.80	1.688	4.90	1.755
25	11.3	3.69	1.127	3.75	1.193	3.84	1.259	3.91	1.324	3.98	1.387	4.06	1.449	4.12	1.509	4.20	1.568
30	16.6	3.24	1.011	3.31	1.073	3.38	1.133	3.44	1.193	3.50	1.251	3.56	1.307	3.63	1.363	3.69	1.419
35	21.4	2.92	0.910	2.96	0.971	3.00	1.029	3.06	1.085	3.11	1.140	3.16	1.193	3.22	1.244	3.27	1.293
40	25.8	2.62	0.825	2.66	0.883	2.71	0.938	2.75	0.992	2.81	1.044	2.85	1.094	2.90	1.144	2.94	1.193
45	30.0	2.39	0.755	2.43	0.807	2.47	0.859	2.50	0.910	2.54	0.961	2.58	1.009	2.62	1.057	2.65	1.103
50	33.8	2.18	0.689	2.21	0.739	2.25	0.789	2.28	0.838	2.33	0.886	2.36	0.934	2.39	0.980	2.42	1.026

*Based on two-cylinder compressors with 6 in. bore and 6 in. stroke. For other compressor sizes, multiply by machine factors of Table 11.

The values above the heavy line in this table are for ratios of compression above 8.0, and for these conditions it is recommended that multistage compression be considered.

$\frac{\text{Cfm}}{\text{Ton}}$ = average values of swept volume, in cubic feet per minute, required per ton refrigeration based on a suction superheat of 10 F and on liquid ammonia entering the refrigerant control valve at the saturated temperature corresponding to the discharge pressure.

$\frac{\text{Bhp}}{\text{Ton}}$ = average values of actual compressor brake horsepower per ton refrigeration.

could also be motor driven with belt or Vee belt drive, and many such machines with bore stroke ratio of unity are in use. Table 10 gives performance data on a large family of such machines based on a 6 by 6-in. cylinder, with factors in Table 11 to adapt the table to other cylinder sizes. Data are given both on cubic feet per minute per ton and brake horsepower per ton.

Table 11. Machine Factors for Various Compressor Sizes

(Reprinted by permission from *ACRMA Equipment Standards* 413, Part II)

	Compressor Bore and Stroke, In.							
	3	4	5	6	7	8	9	10
$\frac{\text{Cfm}}{\text{Ton}}$	1.030	1.022	1.012	1.000	0.986	0.972	0.960	0.948
$\frac{\text{Bhp}}{\text{Ton}}$	1.106	1.063	1.027	1.000	0.977	0.960	0.944	0.930

The desirability of units of smaller physical size has led progressively to the development of small-cylinder compressors operating at motor speed. Such units are particularly common with the "Freon" refrigerants. To reduce piston speeds and maintain valve area with these high-speed units the bore-stroke ratio is usually in excess of unity. For example, one such unit has seven 3-in. bore by 2.75-in. stroke cylinders arranged radially and runs at 1150 rpm. Table 12 gives typical cylinder sizes for the units of one manufacturer.

Table 12. Physical Data—Carrier Reciprocating Compressors

(Courtesy of Carrier Corporation)

Model	Horse-power	Rpm	Capacity, 1000 Btu/hr *	No. of Cylinders	Bore × Stroke, in.
7J1-609	1/4	700	2.1	2	1 3/8 × 1
7J1-A189	1/3	900	2.7	2	1 3/8 × 1
7J1-789	1/2	1300	4.1	2	1 3/8 × 1
7L1-A189	3/4	1100	6.7	2	1 11/16 × 1 1/4
7L1-A229	1	1300	8.1	2	1 11/16 × 1 1/4
7G2-A189	1 1/2	1390	11.7	2	1 15/16 × 1 7/16
7G2-609	2	1690	15.2	2	1 15/16 × 1 7/16
7K3-549	2	1100	16 0	2	2 1/4 × 1 11/16
7K3-579	3	1500	20.4	2	2 1/4 × 1 11/16
5F20-889	5	1750	64.2	2	2 1/2 × 2
5F30-889	7 1/2	1750	96.3	3	2 1/2 × 2
5F40-899	10	1750	128.4	4	2 1/2 × 2
5F60-899	15	1750	192.6	6	2 1/2 × 2
5H40-899	25	1750	291.5	4	3 1/4 × 2 3/4
5H60-899	40	1750	436.0	6	3 1/4 × 2 3/4
5H80-899	50	1750	581.5	8	3 1/4 × 2 3/4
5H40-919	60	1750	726.5	10	3 1/4 × 2 3/4
5H60-919	75	1750	872.0	12	3 1/4 × 2 3/4
5H80-919	100	1750	1163.0	16	3 1/4 × 2 3/4

* Models 7J1-609 through 7G2-609 rated under ASRE group IV standards; 7K3-549 through 5H80-919 under ASRE group III.

Many manufacturers are building so-called condensing units which consist of the high side of a refrigerating system, namely the compressor, condenser and receiver. By shipping these as pre-assembled units, installation costs in the field can be reduced. Even units reaching 100 tons capacity are supplied in this manner.

CENTRIFUGAL COMPRESSORS are most commonly employed with the so-called vacuum refrigerants such as "Freon-11" and methylene chloride (Carrene 1). It is possible, however, to use Freon-12 and hydrocarbon refrigerants for special applications. Although the pressure range with centrifugal machines must be kept relatively low, it is possible to make the range as high as desired by increasing the number of working stages in the compressor. Capacities built into such machines are large, seldom less than 75 tons, and exceed 1000 tons per unit.

Figure 10 shows a centrifugal compressor. Vaporized refrigerant from the evaporator passes through eliminator plates and enters the suction side of the first stage of the compressor. In passing through the rotating impeller, the velocity of the vapor is greatly increased. In the flared diffusion passages of the casing, the velocity of the moving vapor decreases with a resulting increase in pressure, and the compressed vapor then passes to a succeeding stage of the machine. One or more stages can be employed, depending on the pressure range desired for the machine. The compressed vapor finally leaves the last stage of the machine and passes to the condenser. If very high pressure lifts are required, it is also possible to arrange to have two or more machines in series.

As an economy measure, the expansion valve of centrifugal machines can be made to operate in a series of steps so that the flash gas from intermediate steps can be led back to the appropriate suction point of a stage in the centrifugal machine. This prevents the

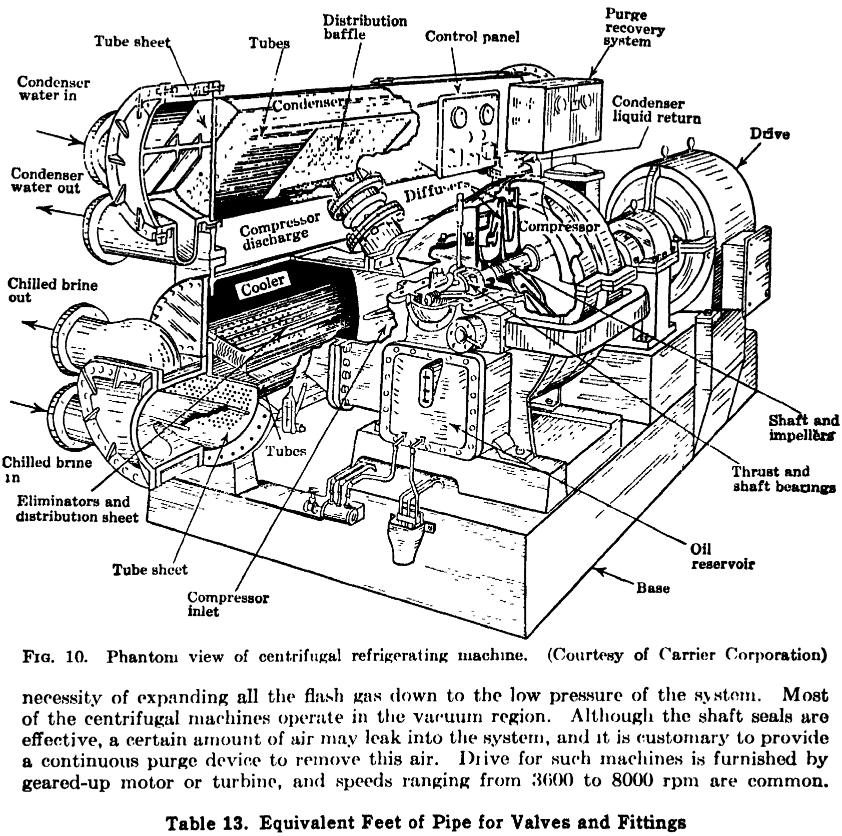


FIG. 10. Phantom view of centrifugal refrigerating machine. (Courtesy of Carrier Corporation)

necessity of expanding all the flash gas down to the low pressure of the system. Most of the centrifugal machines operate in the vacuum region. Although the shaft seals are effective, a certain amount of air may leak into the system, and it is customary to provide a continuous purge device to remove this air. Drive for such machines is furnished by geared-up motor or turbine, and speeds ranging from 3600 to 8000 rpm are common.

Table 13. Equivalent Feet of Pipe for Valves and Fittings

(Reprinted by permission from *ACRMA Equipment Standards*)

Line Size, in.	IPS	3/8	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	5	6	8	10	12
	OD	1/2	5/8	7/8	1 1/8	1 3/8	1 5/8	2 1/8	2 5/8	3 1/8	3 5/8	4 1/8	5 1/8	6 1/8	8 1/8	10 1/8	12 1/8
Globe valve (open)		14	16	22	28	36	42	57	69	83	99	118	138	168	225	280	335
Angle valve (open)		7	9	12	15	18	21	28	34	42	49	57	70	83	117	140	165
Standard elbow		1	2	2	3	4	4	5	7	8	10	12	14	16	20	26	31
Standard tee (through side outlet)		3	4	5	6	8	9	12	14	17	20	22	28	34	44	56	65

IPS—iron pipe size. OD—outside diameter of tubing.

Values shown are average values based on standard-weight pipe and type L tubing.

Note. Because right-angle forged fittings give a high pressure drop their use in suction lines should, in general, be discouraged.

Table 14. Suction Lines, Tons Capacity per 100 Ft Equivalent Length—Based on 105 F Condensing Temperature with "Freon-12"

(Reprinted by permission from *ACRMA Equipment Standards* 520)

Line Size, in. IPS	Evaporator Saturation Temperature, °F														
	−20			0				20				40			
	Pressure Drop per 100 ft Equivalent Length of Suction Line, psi														
	1/2	1	2	1/2	1	3	5	1/2	1	3	5	1/2	1	3	5
3/8	0.10	0.14	0.19	0.13	0.18	0.32	0.40	0.15	0.21	0.37	0.46	0.17	0.24	0.42	0.54
1/2	0.19	0.26	0.37	0.24	0.34	0.60	0.76	0.28	0.39	0.69	0.89	0.35	0.45	0.79	1.03
3/4	0.38	0.54	0.77	0.50	0.71	1.25	1.56	0.58	0.82	1.44	1.83	0.68	0.94	1.65	2.12
1	0.73	1.03	1.47	1.06	1.50	2.64	3.37	1.23	1.77	3.12	3.97	1.43	2.01	3.54	4.60
1 1/4	1.52	2.15	3.04	1.98	2.82	4.97	6.25	2.32	3.27	5.76	7.28	2.70	3.82	6.72	8.48
1 1/2	2.32	3.25	4.63	3.02	4.25	7.48	9.50	3.54	4.95	8.72	11.0	4.05	5.75	10.12	12.8
2	4.41	6.29	8.90	5.72	8.14	14.3	17.9	6.74	9.37	16.5	21.0	7.66	10.9	19.2	24.5
2 1/2	6.85	9.67	13.8	8.93	12.7	22.4	27.9	10.5	14.6	25.7	32.9	12.0	17.1	30.1	38.2
3	11.9	16.9	23.9	15.6	22.0	38.7	48.5	18.3	25.7	45.3	57.2	20.9	29.4	51.8	66.2
3 1/2	17.2	24.3	34.3	22.5	32.1	56.5	70.5	26.0	37.0	65.2	82.6	30.2	43.2	76.1	96.0
4	23.5	33.0	46.5	30.4	43.5	76.5	96.2	35.6	50.0	88.0	112	40.7	58.6	103	130
5	39.5	56.0	79.6	52.7	74.5	131	165	61.3	85.5	151	192	71.3	100	176	224
6	71.8	101	144	94	133	234	298	110	155	273	346	126	183	322	403
8	119	168	239	156	220	387	490	180	257	453	576	211	297	523	664
10	202	285	408	261	371	653	823	306	433	763	968	352	503	887	1130
12	308	438	621	405	573	1010	1290	474	674	1187	1500	550	780	1373	1748

Table 15. Discharge Lines, Maximum Tons of Compressor Capacity with "Freon-12" for Various Sizes

(Reprinted by permission from *ACRMA Equipment Standards* 520)

Condensing Temperature, °F	1/2 in. IPS	3/4 in. IPS	1 in. IPS	1 1/4 in. IPS	1 1/2 in. IPS	2 in. IPS	2 1/2 in. IPS	3 in. IPS	3 1/2 in. IPS	4 in. IPS	5 in. IPS	6 in. IPS	8 in. IPS	10 in. IPS	12 in. IPS
115	1.87	3.26	5.29	9.16	12.5	20.6	32.2	54.5	78.8	101.6	171.5	266	461	725	1041
90	1.50	2.62	4.25	7.35	10.0	16.5	25.9	43.8	63.3	81.6	137.8	214	370	582	836

Evaporator temperatures in the air-conditioning range, +30 to 45 F, as well as temperatures for industrial purposes ranging as low as −150 F to −200 F, are possible with such machines when they are properly designed.

In air-conditioning work, centrifugal compressors are frequently used with water-steam as the refrigerant operating under such vacuums that the water boils in the evaporator in the range of 35 to 45 F.

PIPE SIZE. To reduce pressure loss to the compressor, good practice calls for making the shortest possible piping runs from the evaporator and using pipe of adequate size. With ammonia vapor, velocities of 4000 to 5000 ft per min are used on suction and up to 6000 on discharge. "Freon-12" and methyl chloride are designed with lower velocities of 3000 to 4000 ft per min on suction and may reach 5000 on discharge. It is customary with ammonia to size suction mains with pressure drops per 100 ft equivalent length of run, of 0.25 psi for 5 psig suction pressure, 0.5 psi for 20 psig, and 1.0 psi for 45 psig. Discharge pipes are usually sized for 1.0 psi per 100 ft of equivalent length (see Table 13 for equivalent length of valves and fittings). For example, at 20 psig suction pressure and 0.5 psi loss per 100 ft: 5 lb ammonia vapor per min requires 1 1/2-in. steel pipe; 10 lb ammonia vapor per min requires 2-in. steel pipe; 25 lb ammonia vapor per min requires 3-in. steel pipe; and 50 lb ammonia vapor per min requires 4-in. steel pipe.

Ammonia pipes are always made of steel or wrought iron of not less than "standard weight" wall thickness (Schedule 40, see Section 6). Aluminum tubing is employed, however, in some ammonia heat-transfer surfaces, particularly of ammonia to air.

"Freon-12" and other "Freons" and methyl chloride use not only steel but also copper tubing with brazed or sweated fittings. Tables 14 and 15 give pressure drops per hundred feet of equivalent length of pipe for "Freon-12" systems.

5. STEAM-EJECTOR VACUUM REFRIGERATING SYSTEMS

The refrigerating effect in steam-jet-vacuum refrigeration is produced by evaporating part of the water circulated at a low absolute pressure (partial vacuum). Final temperature of the water corresponds to temperature of evaporation for the partial vacuum (Fig. 11). The relatively large volumes of water vapor to be removed and compressed, for refrigerating effect, have made reciprocating vapor compressors in this connection impracticable, although centrifugal machines can give good performance. Water is sprayed into the evaporator to present a surface from which evaporation may occur in a reasonable space. The unit is compact and simple and has few parts. It is used to cool water for industrial purposes, and is used with air-conditioning apparatus when steam is available. Usual final temperature, t_e, of the chilled water is 40 to 60 F. Considerably more condensing water is required than for power-driven vapor-compression machines, as not only must the evaporated water producing refrigerating effect be condensed but also the steam supplied to the jet.

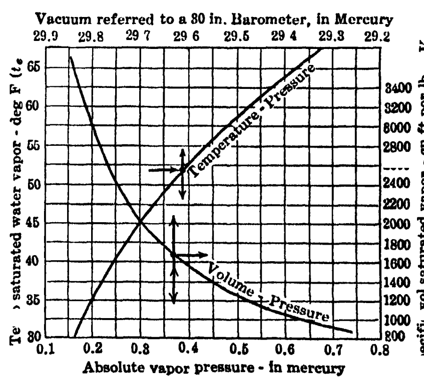


FIG. 11. Pressure-temperature-volume curves of saturated water vapor.

PRINCIPLE OF OPERATION. Figure 12 is a diagram of the apparatus. Steam expands in a single or multiple set of steam nozzles to a pressure corresponding to temperature t_e maintained in the evaporator. The velocity, feet per second, of steam leaving nozzle exit for adiabatic expansion is $w = 223.9\sqrt{h_1 - h_e}$, where h_1 and h_e = enthalpy of steam supplied to nozzle and after expansion to evaporator pressure, respectively. For an initial pressure p_1 = 125 psia, w = 4000 ft per sec (approximately) for usual chilled water temperatures.

The high-velocity steam mixes in the injector chamber with saturated vapor from the evaporator and passes into the thermocompressor. Mixture is compressed adiabatically to the pressure in the condenser. Part of the kinetic energy of the nozzle steam, S lb, is used to accelerate the evaporated vapor, W lb, to the velocity at which the mixture enters the compressor section. Part of the kinetic energy of mixture ($S + W$) is used to compress the mixture from evaporator pressure p_e to condenser pressure p_c. Condenser temperature t_c is approximately 5 to 10 degrees higher than initial temperature of condensing water. The usual partial vacuum for the condenser is 27.5 to 28.5 in. Hg. The vapor mixture enters the diverging portion of the unit, where it is compressed and its velocity

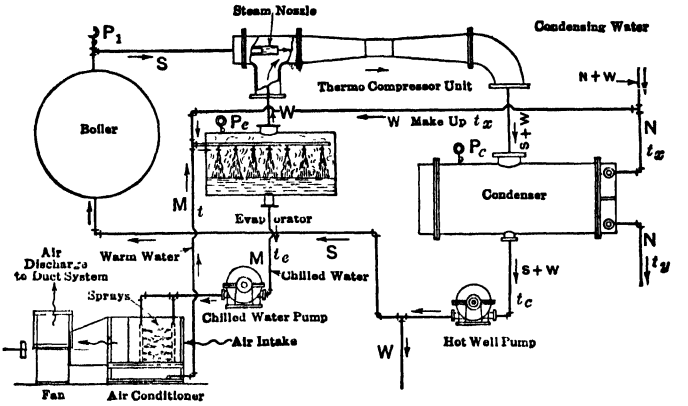


FIG. 12. Diagram of steam-jet refrigerating apparatus.

reduced to approximately 500 ft per sec, at which velocity it passes into the condenser. The compression is from p_e to p_c.

Cycle Relations. Let M = water to be cooled, pounds per hour; t, t_e = initial and final temperature of water to be cooled, °F; W = lb of water to be evaporated at evaporator temperature, t_e °F, per hour per ton of refrigeration; and c = specific heat of water equal to 1.0 for range in question. Then the refrigeration required in tons of capacity is

$$\text{Tons} \quad \frac{M(c)(t - t_e)}{12{,}000} = \frac{M(t - t_e)}{12{,}000} \qquad (25)$$

The water here performs the same function as the refrigerant in the compression system but usually leaves as a wet vapor. With an assumed quality of $x = 0.92$,

$$h_e = h_{fe} + xh_{fge} = h_{fe} + 0.92h_{fge} \qquad (26)$$

where h_{fe} and h_{fg} are read from steam tables for the evaporator temperature. The enthalpy of the water entering the sprays, h_{fm}, corresponds to the temperature after mixing of the return water at t and the make-up at t_x. Water evaporated per ton-hour (W) is

$$W = \frac{12{,}000}{h_e - h_{fm}} \qquad (27)$$

The weight of steam required per hour per ton of refrigeration varies with the evaporator (chilled water) temperature, the condenser temperature (partial vacuum), and the initial steam pressure. Some 40% of the isentropic available work from expansion of the line steam to evaporator pressure is realized in useful entrainment and compression. For a representative steam-supply pressure of 125 psig, Fig. 13 gives data on expected steam consumption and circulating water required by the condenser.

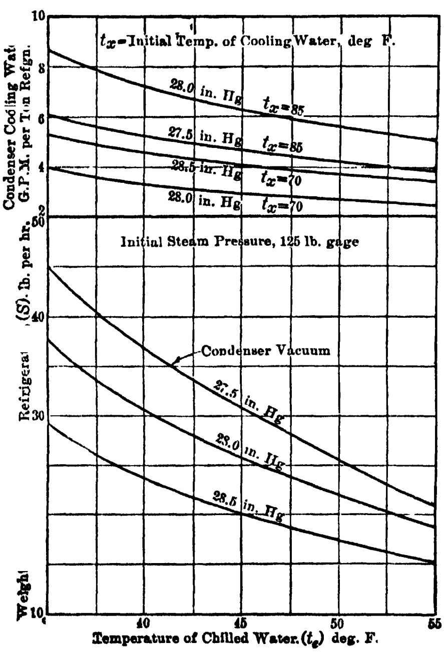


FIG. 13. Performance curve, steam-jet refrigerating apparatus.

EXAMPLE. Assume chilled water at 40 F is supplied for a cooling process. It warms a few degrees in the process, returns to the evaporator, mixes with the make-up water, and is rechilled to 40 F. For steam supply at 125 psig estimate the steam consumption and circulating water for a 28.0-in. Hg vacuum in condenser. According to Fig. 13, for 40 F chilled water read S = 31 lb of steam required per ton hour. Assume a 78 F circulating water supply temperature. Interpolation between the two 28-in. Hg curves shows 5.5 gal per min of circulating (condenser) water required per ton of refrigeration.

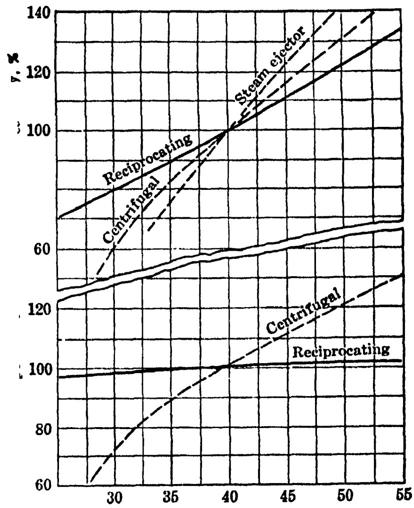


FIG. 14. Variation of performance with evaporator temperature.

Circulating Water. The circulating water must condense not only the steam supply (S) but also the water evaporated (W). The temperature rise in the water above its initial temperature, t_r, is most significant in setting the condenser pressure (partial vacuum). The circulating water usually leaves 5 to 8 degrees below the condensation temperature. A small temperature rise gives a lower condenser pressure (higher partial vacuum) with lower steam consumption but increases the amount of water required. An economic balance must be struck between cost of steam and cost of circulating water.

PRACTICE. A number of American manufacturers build steam-jet refrigerating machines; the majority use multiple nozzles in the ejector chamber. A surface condenser ordinarily is used, although barometric condensers may be employed. Either type is equipped with a two-stage steam-jet air eliminator having inter- and after-condensers. Maximum efficiency is obtained only when the machine operates at the designed rate of flow through the steam nozzles and other portions of the apparatus. It is customary to provide two or more separate steam-jet thermocompressor units to obtain the best overall efficiency under variable loads. The units are connected to the same evaporator and condenser.

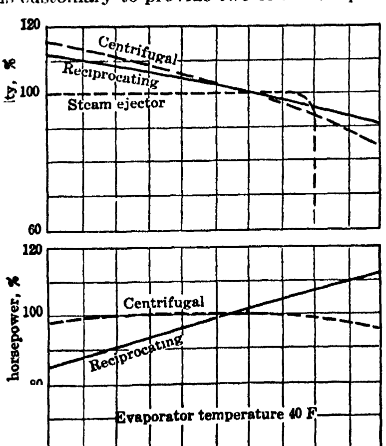


FIG. 15. Performance characteristics of compression refrigeration machines at constant speed and evaporator pressure.

PERFORMANCE VARIATION. The performance and capacity of a refrigeration system vary with evaporator temperature and also with the condenser temperature (pressure). Typical performance variation for changing evaporator temperatures (pressures) is illustrated in Fig. 14. For a given datum or reference temperature, in this case 40 F, the variation in machine capacity can be seen. As the evaporator temperature falls, capacity decreases least rapidly with a reciprocating machine and more rapidly with centrifugal and steam-jet equipment. The centrifugal-machine capacity reduces because of the inherent difficulties arising with increased pressure lift. Also, as the evaporator pressure falls, the specific volume of the refrigerant increases and tends to choke the suction system. At higher evaporator temperatures (pressures) it will be noticed that the centrifugal-machine capacity increases quite rapidly, and this occurs even more so with the steam ejector.

The effect on performance of variations in condenser temperature (pressure) is shown in Fig. 15. Here the centrifugal machine again decreases in capacity at a more rapid rate than the reciprocating, whereas the steam ejector has relatively constant capacity up to a break-off temperature. In the case of the centrifugal machine this falling characteristic can be used to some advantage to reduce the capacity of a constant-speed centrifugal machine. To reduce the capacity it is merely necessary to increase the temperature of the circulating water, which can be done easily by reducing its quantity. The horsepower characteristics of the centrifugal machine are also relatively constant over a wide range of either evaporator or condenser temperatures.

6. COLD-AIR MACHINES

Cold-air machines owe their importance today mainly to the historical part they played in the development of refrigeration. Their use requires the circulation of large volumes of air and consequent bulky equipment. In addition, they are less efficient than vapor-compression machines. They were formerly used extensively on shipboard because air represented a safe refrigerant and could always be replenished.

To reduce the size of the cold-air equipment, it was customary to use a closed cycle under pressure. The air was recirculated to keep the working temperature range low and also to prevent operating difficulties which could arise from freezing of moisture taken in with the make-up air added to the system. To reduce this possibility, the make-up air was chilled below its dew point before being added to the system. The compressor cylinder capacity was approximately sixteen times that required for equal capacity with ammonia as the refrigerant. The thermodynamic principle for development of refrigeration rests on the fact that when air is expanded adiabatically and does work, the temperature of the air is decreased. Air chilled in this way, in passing through the space to be cooled, can absorb heat and perform useful refrigeration.

Table 16. Tests on Cold-air Machines

(Linde, *Trans. ASME*, xiv, p. 1416)

	System		
	Bell-Coleman	Lightfoot	Haslam
Air pressure in receiver, psia	61.0	65.0	64.0
Temp. of air entering compression cylinders, °F	65.5	62.0	
Temp. of air after expansion, °F	−52.6	−82.0	−85.0
Ihp in compression cylinder	124.5	43.1	346.4
Ihp in expansion cylinder	58.5	28.0	176.2
Ihp in steam cylinder	84.4	24.6	332.7
Btu abstracted per hour per ihp of steam cylinder, at 20 F	668.0	1554.0	954.0

The **ALLEN DENSE-AIR MACHINE** was once used extensively, and a diagram of its elements appears in Fig. 16. Air is drawn into the compressor B, from the refrigerator coil at a pressure of 60 to 70 psig. It is compressed (single stage) to 210 to 240 psig and passes through a cooling coil E surrounded by water, which removes the heat of compression. The air leaves the coil at about 10 F higher than the final temperature of the cooling water. The reduced volume of cooled air expands in the cylinder D, which has an adjustable cut-off gear. The air, in expanding, assists in driving the compressor, thereby recovering a portion of the external work of compression. The expansion of the air causes its temperature to fall, and it then passes to the coils of the room to be refrigerated, returning thence to the compressor cylinder. An oil extractor F in the discharge line permits the frozen oil to be collected and melted. Make-up air required to keep the system fully charged is compressed in a small cylinder following which its temperature is lowered and moisture precipitated before it is introduced into the system.

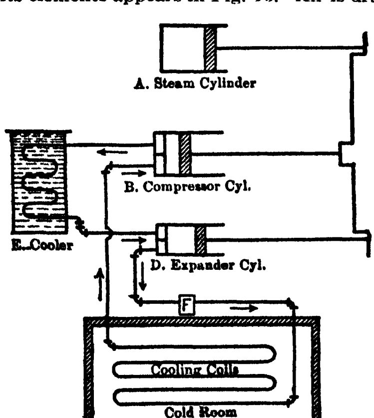


FIG. 16. Diagram of cold-air refrigerating machine.

The operation of the cold-air system is explained below. Compression of the air and its re-expansion are assumed to take place adiabatically (Fig. 17). Note that the cycle of operation is identical with the gas turbine cycle (see Section 10) operating in the reverse direction.

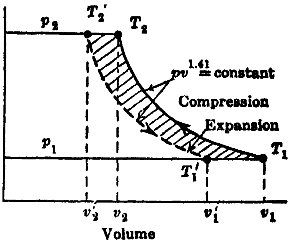


FIG. 17. Cycle diagram, cold-air machine.

Compression. Let v_1 = volume of air drawn into compressor cylinder per min, T_1 = absolute initial temperature of this air, and p_1 = its initial pressure. T_1 is about 10 degrees below the temperature to be maintained in the refrigerator. Let v_2, p_2, and T_2 be the volume, pressure, and absolute temperature, respectively, of the air after compression. All pressures are in pounds per square inch, absolute. Then

$$= T_1\left(\frac{p_2}{p_1}\right)^{0.29} \qquad v^{1.41} = \frac{p_1 v_1^{1.}}{p_2} \tag{28}$$

Work of compression, foot-pounds per minute, is $w = 3.45(p_2v_2 - p_1v_1) \times 144$ (29)

Cooling. Volume v_2 is reduced to volume v_2' in the cooler at constant pressure

$$v_2' \quad v_2\left(\frac{T_2'}{T_2}\right) \tag{30}$$

The Btu imparted to the cooling water per pound of air circulated is

$$H = c_{pa}(T_2 - T_2') \tag{31}$$

where c_{pa} = 0.24 = specific heat of air at constant pressure.

Expansion Cylinder. The reduced volume v_2' expands adiabatically to v_1' and pressure p_1'. Then

$$(v_2')^{1.41} = \frac{p_1(v_1')^{1.41}}{p_2'} \tag{32}$$

Work recovered by expansion, foot-pounds, is w' $3.45(p_2v_2' - p_1v_1') \times 144$ (33)

$$\text{Final absolute temperature} = T_1' = T_2'\left(\frac{p_1}{p_2}\right)^{0.29} \tag{34}$$

Expansion in Refrigerator Coils. Volume v_1' expands in the refrigerator coil at constant pressure p_1, returning to original volume v_1 and temperature T_1.

$$v_1 = v_1'\left(\frac{T_1}{T_1'}\right) \tag{35}$$

The refrigerating effect (heat removed = H) per cubic foot of piston displacement is:

$$H = c_{pa}d(T_1 - T_1')\ \text{Btu} \tag{36}$$

where d = density, pounds per cubic foot of air at temperature T_1 and pressure p_1. The density for any pressure is given by the characteristic equation of gases $PV = MRT$, where P = absolute pressure, pounds per square foot, T = absolute temperature, and R = 53.35 for air. If M is 1 lb and v is the volume, cubic feet, $Pv = RT$ or $1/v = P/RT$. But $1/v = d = P/RT$, which gives the density in terms P, R, and T.

Compressor Displacement Required per Ton of Refrigeration. Let D = displacement of compressor per 24 hr per ton of refrigeration, cubic feet, and E = volumetric efficiency of compressor (80% approximately). Then

$$D = 288{,}000 \div [d \times c_{pa}(T_1 - T_1') \times E] \tag{37}$$

and the net horsepower required is

$$W = (w - w') \div 33{,}000 = \frac{[3.45(p_2v_2 - p_1v_1 - p_2v_2' + p_1v_1') \times 144]}{33{,}000} \tag{38}$$

7. ABSORPTION REFRIGERATION

PRINCIPLE OF OPERATION. Absorption refrigeration resembles vapor-compression refrigeration in that the low side of the system is unchanged for either type. For the high side the compressor is replaced in the absorption system by a collection of heat-transfer vessels and a pump. These vessels make use of variations in the solubility of the refrigerant in an absorbing solution when the temperatures and pressures are changed. One absorbent is water, which will absorb large quantities of ammonia gas. If the ammonia delivered by an evaporator enters a tank supplied with a continuous stream of water, the pressure in the tank will remain lower than the evaporator pressure, constituting a crude form of absorber. This absorber serves the same purpose as the suction stroke of a compressor.

Most systems use ammonia as a refrigerant and solutions of ammonia-water as the absorbing medium. Figure 18 is a diagram showing the elements of a conventional absorption system. The amount of ammonia vapor which can be absorbed by water increases with increase in external pressure and decreases with rising temperature. The absorber of the system operates at a pressure just slightly lower than evaporator pressure and is supplied with a cool solution of water-ammonia that is not saturated with ammonia gas. Liquid in this condition is called "weak liquor," and in the absorber it takes up ammonia gas until it becomes saturated (holds as much gas as it can at the temperature and pressure existing in the vessel). Heat is generated during the process and must be removed by

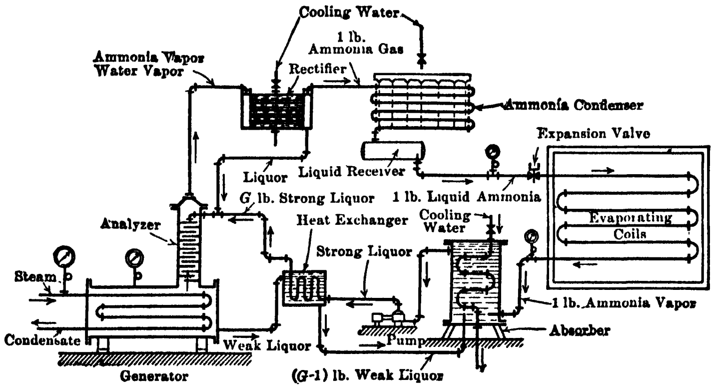


FIG. 18. Arrangement of ammonia-absorption plant.

cooling water. The liquid, saturated with ammonia, is called "strong liquor" and enters the pump of the system which raises its pressure and sends it through a heat exchanger into the generator. The generator operates at condenser pressure and is supplied with steam or other source of heat which drives off ammonia from the liquid, decreasing the concentration of the liquid until it becomes "weak liquor." The ammonia vapor from the generator passes through an analyzer which acts as a heat exchanger and tends to throw back a portion of the water vapor that was distilled off along with the ammonia.

The process of preferentially condensing out the water vapor is completed in the next vessel, called the rectifier, so that the vapor leaving the rectifier is essentially all ammonia. This ammonia enters the condenser where it changes into liquid as in the conventional refrigeration system, passes to the expansion valve of the system, and thence to the evaporator. The hot weak liquor from the generator passes through the heat exchanger, where it is cooled at the expense of warming the strong liquor and then enters the absorber to pick up a fresh charge of ammonia vapor. The strong liquor, by being warmed in the heat exchanger, requires less heating from the steam coils in the generator. Essentially two pressures exist in the absorption system: the head or condenser pressure which maintains in the generator, rectifier, condenser, and parts of the heat exchanger and the evaporator pressure which exists in the evaporator and absorber. The weak liquor from the generator, after leaving the heat exchanger, passes through an expansion valve in which its pressure is dropped from the high to the low pressure of the system.

The absorption system resembles the compression system in many ways, as the condenser, expansion valve, and evaporator are completely interchangeable. The energy input to the system consists of a small amount of power supplied to the aqua-ammonia pump and a large amount of thermal energy supplied by the heating medium to the generator. Notice that there are three circuits of fluid through the system: the ammonia circuit from the generator, finally arriving at the absorber, the strong-liquor circuit from the absorber to the generator, and the weak-liquor circuit from the generator back to the absorber. In addition, steam and circulating water are required.

The vapor distilled from the solution in the generator consists of ammonia along with small quantities of steam. When this vapor is cooled, the steam saturated with ammonia condenses out first. The analyzer first serves in this connection by bringing the vapor into contact with the strong liquor, which is richest in ammonia, and cools the vapor a certain amount by this means. In the rectifier, water cooling of the vapor completes the process. Traces of moisture in the ammonia leaving the rectifier and condenser are not serious, although this moisture may collect in the evaporator (unless the latter is of the straight through flow type) and must periodically be purged back to the absorber.

The properties of aqua-ammonia solutions have been tabulated by Jennings and Shannon and published as a miscellaneous bulletin of the American Society of Refrigerating Engineers. The same society has also published tables compiled by A. B. Stickney (*Refrigerating Engineering*, Oct. 1935). The Jennings-Shannon data are conveniently summarized in Fig. 19.

EXAMPLE OF COMPUTATION OF A TYPICAL AMMONIA-ABSORPTION REFRIGERATION SYSTEM. For representative conditions in an absorption system, consider a condenser pressure of 180.0 psia corresponding to 89.8 F, an evaporator pressure of 30 psia corresponding to −0.6 F, temperature of weak liquor leaving generator of 256.1 F with 24% concentration, and strong liquor leaving absorber at 90.9 F with 38% concentration. The vapor leaving the rectifier and entering the condenser will be considered anhydrous ammonia at 89.8 F. This is not quite true as some water vapor is carried with the ammonia, and the mixture will be at a higher temperature than saturation.

For each pound of ammonia absorbed from the evaporator the amount of ammonia liquor circulated in terms of the weight concentrations of strong liquor (x_s) and of weak liquor (x_w) is

$$G = \frac{1 - x_w}{x_s - x_w} \tag{39}$$

$$G - 1 = \frac{1 - x_s}{x_s - x_w} \tag{40}$$

where G = pounds of strong liquor circulated per pound of ammonia (NH_3), $G - 1$ = pounds of weak liquor circulated per pound of NH_3, x_s = weight concentration of strong liquor, and x_w = weight concentration of weak liquor.

For the condition given

$$G = \frac{1 - 0.24}{0.38 - 0.24} = 5.4 \text{ lb strong liquor per lb NH}_3$$

$$G - 1 = 5.4 - 1 = 4.4 \text{ lb weak liquor per lb NH}_3$$

Heat Exchanger. The strong liquor is warmed in the heat exchanger usually to its boiling point as the weak liquor is cooled in counterflow. Figure 19 shows that 38% liquor (x_f of Fig. 19) has a boiling temperature of about 200 F and an enthalpy h_{fSL} of 80 Btu per lb (precisely 201.1 F and h_{fSL} = 80.0). The strong liquor as it leaves the absorber is always a few degrees higher than circulating water temperature, and, if saturated strong liquor is assumed for the 38% concentration at 30 psia, Fig. 19 shows the temperature as 91 F, h_{fSL} = −44 Btu per lb (precisely 90.9 F and −44.3). The weak liquor leaving the generator at 256.1 F, x_f = 0.24, and 180 psia has an h_f = 160 Btu per lb from Fig. 19 (precisely 160.8 Btu per lb). Making a heat balance for the exchanger shows

$$G(h_{fSL\ \text{out}} - h_{fSL\ \text{in}}) = (G - 1)(h_{fWL\ \text{in}} - h_{fWL\ \text{out}}) \tag{41}$$

Substituting values,

$$(5.4)[80.0 - (-44.3)] = 4.4(160.8 - h_{fWL\ \text{out}})$$

$$h_{fWL\ \text{out}} = 8.25 \text{ Btu/lb}$$

Figure 19 shows that h_f = 8.25 Btu per lb at x_f = 0.24 has a temperature of 114 F. Notice that for subcooled liquid (below saturation temperature) the enthalpy is independent of the pressure.

Absorber. The vapor from the evaporator in coming into contact with the weak liquor from the heat exchanger is absorbed, and the resultant strong liquor leaves the absorber to enter the strong liquor pump. The process of absorption entails the removal of a quantity of heat (Q_A) equivalent to the heat of condensation, the heat of solution, and the heat removal associated with any temperature lowering of the final product (the strong liquor). Q_A can be found from a simple enthalpy balance of the components

$$(G - 1)(h_{fWL}) + h_{V\text{NH}_3} = Gh_{fSL} + Q_A \tag{42}$$

For the example, h_{fWL} = 8.25 Btu per lb for the weak liquor leaving the heat exchanger, h_{fSL} = −44.3 Btu per lb for the strong liquor leaving the absorber. The ammonia vapor entering the absorber is taken at 30 psia, although it is slightly less than evaporator pressure and will be considered as saturated vapor, $h_{V\text{NH}_3}$ = 533.7 Btu per lb from the tables or read approximately from Fig. 19. If the ammonia were superheated it would be necessary to use the anhydrous-ammonia tables or Fig. 5 to obtain the enthalpy. To put anhydrous-ammonia table values, which have a −40 F reference datum, to the +32 F reference datum of the aqua-ammonia tables, 77.9 Btu per lb must be subtracted from the anhydrous values. Substituting in eq. 42,

$$(4.4)(8.25) + 533.7 = (5.4)(-44.3) + Q_A$$

Q_A = 809.2 Btu must be removed in the absorber for each pound of ammonia vapor entering solution.

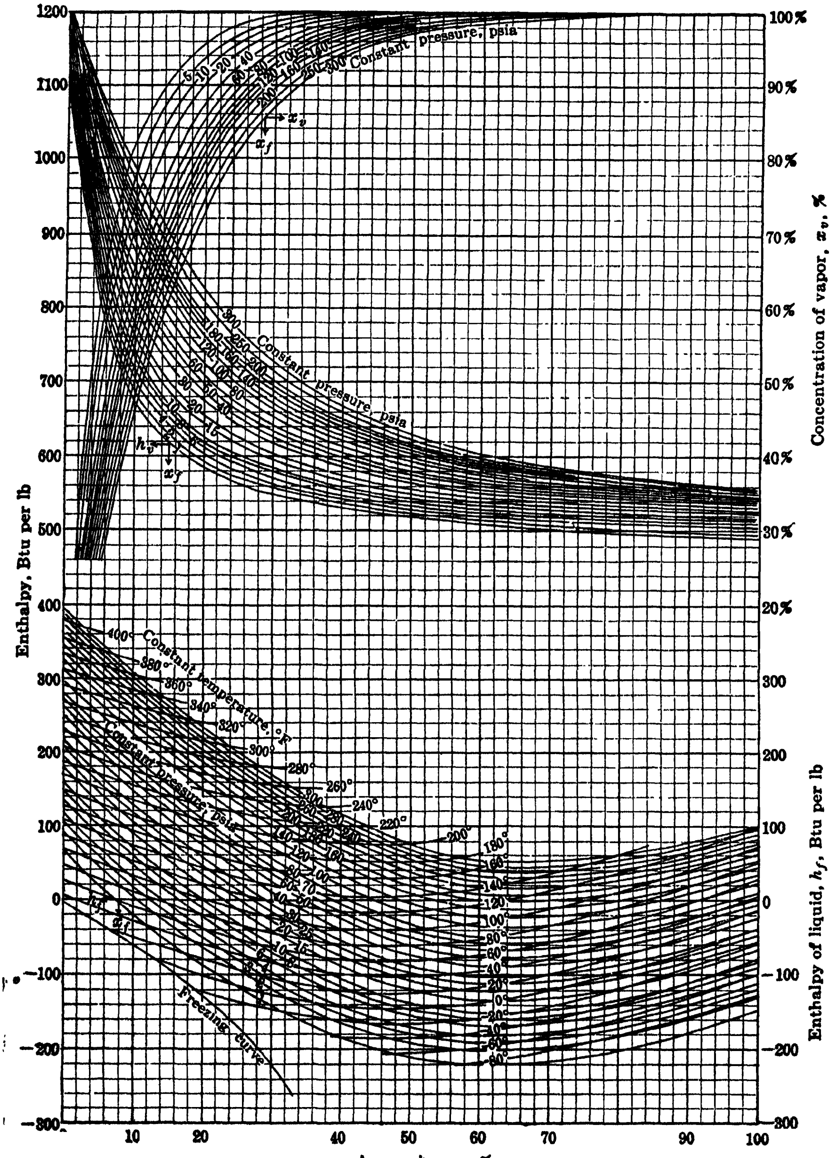


FIG. 19. Aqua-ammonia chart. (Reproduced by permission from *Thermodynamics*, by Emswiler and Schwartz, McGraw-Hill Book Co.)

Rectifier. In this vessel water vapor is condensed from the vapor entering until almost anhydrous ammonia is delivered to the condenser. The water vapor in condensing reabsorbs or condenses some ammonia so that the drips running back to the generator (analyzer) are very rich in ammonia. A computation for rectifier heat load sets the leaving conditions (usually 9 to 25 degrees higher than the condenser temperature) and sets the entering vapor conditions usually taken for equilibrium with the strong liquor supplied to the top of the generator (analyzer). Rectifier cooling if excessive can be very wasteful as drips, far in excess of the minimum for equilibrium, can be sent back in the generator for useless reheating. Rectifier drips are most effective if they are trapped and made to run counterflow to and in direct contact with the vapors rising to the rectifier. A mass balance to determine equilibrium concentrations appears below for 1 lb of vapor mixture leaving, having $x_{v\text{ out}}$ parts of ammonia by weight and $(1 - x_{v\text{ out}})$ parts of water vapor by weight. The liquid drips, which in returning to the generator are assumed to run counterflow and in equilibrium with the strong liquor, have a concentration of x_{fSL} for the ammonia and $(1 - x_{fSL})$ for the water while the vapor entering the rectifier in equilibrium with these drips and the strong liquor is x_{vSL} for the ammonia vapor and $(1 - x_{vSL})$ for the water vapor. Call the drips D and the vapor V, both expressed in pounds per pound of vapor delivered from the rectifier.

$$\text{Ammonia balance } Vx_{vSL} = (1)x_{v\text{ out}} + Dx_{fSL} \tag{43}$$

$$\text{Water balance } V(1 - x_{vSL}) = (1)(1 - x_{v\text{ out}}) + D(1 - x_{fSL}) \tag{44}$$

Solving these equations gives D and V, and then by using proper enthalpies the rectifier heat balance appears

$$(1)(h_{v\text{ out}}) + Dh_{fSL} + Q_R = Vh_{vSL} \tag{45}$$

At the 180 psia of the example, Fig. 19 or the aqua table shows for 98.9 F leaving rectifier (i.e., 9.1 degrees above condensation) $x_{v\text{ out}} = 0.9993$, $h_{v\text{ out}} = 561.6$; and for 0.38 strong liquor at 201.1 F, $x_{vSL} = 0.9516$, $h_{fSL} = 80.0$, and $h_{vSL} = 655.1$. Substituting in eqs. 43 and 44, and solving, shows $D = 0.083$, $V = 1.083$.

Substituting in eq. 45,

$$(1)(561.6) + (0.083)(80) + Q_R = (1.083)(655.1)$$

$Q_R = 141.2$ Btu removed from rectifier per lb of vapor sent to condenser.

Condenser. The condenser delivers the almost anhydrous ammonia as a liquid.

$$Q_C = h_{V\text{ out of rectifier}} - h_{fC} \tag{46}$$

Here $h_{V\text{out of rectifier}} = 561.6$ Btu per lb and $h_{fC} = 65.4$ Btu per lb for saturated ammonia at 180 psia leaving condenser; $Q_C = 561.6 - 65.4 = 496.2$ Btu, per lb of vapor condensed, to be removed.

Generator. The generator is supplied with strong liquor at h_{fSL} from the heat exchanger with drips from the rectifier h_{fD} (which may or may not be at the same condition as the strong liquor) and delivers weak liquor into the heat exchanger at h_{fWL} and vapor at an enthalpy leaving of h_{vSL}. The heating steam supplies Q_g Btu based on 1 lb of ammonia delivered by the condenser to the evaporator. As an equation

$$G(h_{fSL}) + Dh_{fD} + Q_g = Vh_{vSL} + (G - 1)h_{fWL} \tag{47}$$

$$(5.4)(80) + (0.083)(80) + Q_g = (1.083)(655.1) + (4.4)(160.8)$$

$Q_g = 978.4$ Btu per lb of ammonia passing through the evaporator.

Evaporator. Here the liquid from the condenser changes into the kind of vapor supplied the absorber. The heat added per pound, Q_E, is

$$Q_E = h_{vE} - h_{fC}$$

$$Q_E = 533.7 - 65.4 = 468.3 \text{ Btu/lb}$$

Pump Work. This can be closely computed by the usual equation for pump work, where Q_P is expressed in Btu per pound of ammonia circulated through the evaporator.

$$Q_P = \frac{Gv_{SL}(144)(P_G - P_E)}{778} = G\frac{P_G - P_E}{5.4}v_{SL} \tag{48}$$

where G = pounds of strong liquor per pound of ammonia; P_G and P_E = generator and evaporator pressures in pounds per square inch absolute; and v_{SL} = specific volume, cubic feet per pound of strong liquor as it leaves the absorber.

For the 0.38 strong liquor at 90.9 F, $v_{SL} = 0.0186$ cu ft per lb from Table 17.

$$Q_P = (5.4)\frac{(180 - 30)}{5.4}(0.0186) = 2.8 \text{ Btu}$$

Table 17. Specific Volume of Aqua-Ammonia Solutions

Cubic Feet per Pound of Liquid

(Prepared by Jennings and Shannon)

Temperature, °F	Weight Concentration, Ammonia in Solution										
	0.00	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.00
20	.0160	.0166	.0172	.0177	.0182	.0188	.0196	.0206	.0218	.0232	.0247
40	.0160	.0167	.0172	.0178	.0183	.0190	.0199	.0209	.0222	.0236	.0253
60	.0160	.0167	.0173	.0179	.0185	.0192	.0201	.0213	.0226	.0242	.0260
80	.0161	.0168	.0174	.0180	.0186	.0195	.0204	.0216	.0231	.0248	.0267
100	.0161	.0169	.0175	.0182	.0189	.0197	.0208	.0221	.0236	.0254	.0275
120	.0162	.0170	.0176	.0184	.0191	.0200	.0212	.0226	.0242	.0262	.0284
140	.0163	.0171	.0178	.0186	.0194	.0204	.0216	.0230	.0249	.0269	.0294
160	.0164	.0172	.0180	.0188	.0197	.0207	.0223	.0237	.0256	.0279	.0306
180	.0165	.0174	.0182	.0191	.0200	.0212	.0226	.0244	.0265	.0289	.0320
200	.0166	.0175	.0184	.0193	.0204	.0216	.0232	.0252	.0275	.0304	.0338
220	.0168	.0177	.0186	.0197	.0210	.0225	.0242	.0264	.0288	.0320	.0361

Thus the contribution of pump-work energy for this condition of no losses (100% hydraulic pump efficiency) is trivial compared to the other values in the heat balance. This is also true when real pump efficiencies of 55 to 85% are used.

Overall Heat Balance. For a heat balance all the energy added must equal the energy dissipated to circulating water. Summarizing in equational form,

$$Q_E + Q_g + Q_P = Q_A + Q_R + Q_C \tag{49}$$

where the subscripts refer to the different elements of the system. Substituting values

$$468.3 + 978.4 + 2.8 = 809.2 + 141.2 + 496.2$$

$$1449.5 = 1446.6, \text{ approximately}$$

The trivial discrepancy occurs from two sources; the pump work energy was purposely not added to the heat exchanger calculation to simplify the computations, and the weak and strong liquid values were not carried to a second decimal place.

Performance. The performance ratio (PR) of the system, the useful refrigeration effect compared to the energy input to produce it, can be expressed as

$$\text{PR} \quad \frac{Q_E}{Q_S + Q_P} \tag{50}$$

For the example,

$$\text{PR} \quad \frac{468.3}{978.4 + 2.8} = 0.48 \text{ or } 48\%$$

Various modifications of the cycle as given can be made which could improve economy and the performance ratios, reaching towards a limit of 100% or a performance ratio value of one. For example, the strong liquor can pick up heat by being used as a coolant in the rectification process before being led to the heat exchanger. Thus the weak liquor may warm the strong liquor to the point where some ammonia is already separated before it enters the generator, and this would decrease the heat consumption there.

The preceding theoretical example closely follows the actual cycle as most systems are essentially adiabatic because of effective insulation. Deviations occur through heat loss, inadequate heat-transfer surfaces, incomplete saturation in the absorber, and poor rectifier operation. Water not removed in the rectifier eventually passes to the evaporator and must periodically or (even continuously at a slow rate) be purged back to the absorber.

Steam Consumption. The refrigerant circulated per ton is obviously $12{,}000/Q_E$ lb per hr. Thus the heat consumption per hour per ton is

$$Q = Q_g \times \frac{12{,}000}{Q_E} \tag{51}$$

and the steam consumption per hour per ton is

$$M_S = \frac{Q_g}{Q_E}\frac{12{,}000}{h_{fg}} \tag{52}$$

where h_{fg} is the latent heat available for heating from each pound of steam. For the example,

$$Q = 978.4\,\frac{12{,}000}{468.3} = 25{,}071 \text{ Btu/hr ton}$$

With steam at 40 psia, $h_{fg} = 933.7$ and $M_S = \frac{978.4}{468.3}\frac{12,000}{933.7}$ 27 lb steam per hr per ton.

Table 18 gives values, based on tests, of steam consumption for ammonia-absorption refrigeration plants without analyzers. When these are used approximately 3 lb per hr per ton should be deducted from the tabular values. Moreover, modern designs with analyzers, rectifier refluxing, and better arrangement of heat transfer surface reduce the tabular values to about 75% of the values given.

Table 18. Steam Consumption, Pounds per Hour per Ton of Refrigeration for Various Condensing Pressures

(Evaporator pressure = 15.7 psig)

Steam Pressure, psig	Condensing Pressure						
	125	135	145	155	165	175	185
1	34 3	35.5	....	.....	......	....	.....
3	34 1	35.3	36.7	...	...	....	....
5	33.9	35.1	36 4	37.8	39.0	...	....
10	33.3	34.5	35.9	37.2	38 4	40.1	41.3
15	32.8	34.0	35.3	36 7	37.9	39.6	40.7
20	32.2	33.4	34.8	36.1	37.3	39.0	40.2
25	31.7	32.9	34.2	35.6	36.8	38.5	39.6
30	31.1	32 3	33.7	35.0	36.2	37.9	39.0
35	30.6	31.8	33.1	34.5	35 7	37.3	38.5
40	30.0	31.2	32.5	33.9	35.1	36.8	37.9
45	29.5	30.7	32 0	33.4	34.6	36.2	37.4
50	29 0	30.1	31.4	32.8	34.0	35.7	36.8

APPLICATIONS OF ABSORPTION SYSTEM. Fuel and Investment Cost Considerations. Although absorption refrigeration machines are not designed as conventional production units, nevertheless the absorption refrigeration machine has a definite place in the picture of industrial refrigeration. Where thermal energy in the form of waste heat or low-cost fuel is available, the absorption machine has economic possibilities. Because of the appreciably larger amounts of heat-transfer surface, the investment cost of an absorption unit is generally higher than for a compression machine of corresponding capacity.

Other Refrigerants. By far the greatest number of absorption machines utilize the ammonia, ammonia-water system for refrigerant, and solvent. Among other combinations which have been used are methylene chloride as a refrigerant, and the dimethyl ether of triethylene glycol as a solvent. In air-conditioning work, hygroscopic brines have been used with water as the refrigerant. Lithium chloride (LiCl) and lithium bromide (LiBr) have been most generally employed. Where water is the refrigerant, it must evaporate at very low pressure, so that brine systems are always vacuum systems.

Domestic. In the field of domestic refrigerators, the Electrolux refrigerator employs ammonia-water for the absorbent and ammonia for the refrigerant. These refrigerators, energized entirely by heat (a gas flame), use no pump but employ percolator and siphon action for moving the fluids throughout the system along with an ingenious application of Dalton's law of partial pressures. In order for the refrigerant in the evaporator to boil at a low pressure, an inert gas is supplied which circulates back and forth between the absorber and the evaporator. For example, if the condensing pressure of the system is 200 psia, this pressure must apply throughout the system, but to obtain the proper temperature in the evaporator the ammonia must boil at approximately 40 psia. Thus in the evaporator and absorber, the inert gas, which is usually hydrogen, operates under a partial pressure of 160 psi, and the ammonia vapor vaporizes at its partial pressure of 40. This ingenious arrangement often goes under the name of the original co-inventors, whence it is called the Platen-Munters system.

Silica gel (the common name for silicon dioxide [SiO_2]) is a hard glassy material resembling quartz sand. It is chemically inert toward most refrigerants with the possible exception of ammonia. Its structure consists of numerous ultramicroscopic pores in which it can adsorb large quantities of vapor. At one time it was used in an intermittent refrigeration system that employed SO_2 as the refrigerant, the SO_2 being adsorbed in the pores of the gel. When the gel was heated after it had adsorbed its charge, the SO_2 was driven off and condensed in an ordinary refrigerant condenser. One pound of silica gel can adsorb 0.25 to 0.35 lb of SO_2. During the process of adsorption, an amount of heat must be removed equivalent to the latent heat of the vapor itself plus an additional amount of heat associated with the bonding which takes place in the process. Roughly, this heat of bonding amounts to 0.2 to 0.4 of the latent heat. This system using SO_2 is now obsolete.

However, silica gel is extensively used as an agent for drying (dehumidifying) air and liquid refrigerants. In the air systems, the air to be dried is passed over beds or trays of silica gel and, in the process, the moisture content of the air is greatly reduced. The gel, during the process, rises in temperature from the heat of condensation as well as from the heat of bonding.

For every pound of vapor adsorbed, about 200 Btu in excess of the latent heat are generated because of the bonding effect. This bonding effect, in the case of water, frequently goes under the name of the heat of wetting. For each grain of moisture adsorbed per pound of air, these heat effects cause a rise in temperature of the air passing through the bed of approximately 0.7 F. The gel is reactivated after becoming charged with moisture, by heating to a temperature of 250 to 400 F. During the heating the excess moisture is driven off, and on cooling the gel is ready again to readsorb moisture. In drying refrigerants to remove the moisture that entered the system during construction or came into the system under vacuum operation or entered with the refrigerant supplied to the system, it is customary to place in parallel with the liquid refrigerant line or the vapor refrigerant line a canister containing silica gel. The affinity of the silica gel for water is such that as the system operates the gel progressively picks up a portion of the water in the system. Water in the system is objectionable not only because it can promote corrosion by hydrolysis but the moisture can freeze in the expansion valves, thereby stopping further flow.

As refrigerant dehydrators, other chemicals are sometimes used, such as activated alumina, Drierite, and various proprietary combinations.

8. REFRIGERATION LOAD AND HEAT TRANSMISSION

LOAD COMPONENTS. The load in a refrigeration system is the name applied to the quantity of heat that must be removed per unit of time, expressed usually in Btu per hour or in tons of refrigeration. The items contributing to refrigeration load consist of:

Conduction, heat which is transmitted through walls, partitions, floors, and ceilings surrounding the space being held at reduced temperature.

Infiltration, the energy which must be removed from warmer air entering the refrigerated space. This air may be the air supplied under controlled conditions for ventilation or it may be merely leakage air which enters because of door opening or because of infiltration through cracks, etc.

Product heat, which occurs from cooling the product at its entering temperature down to the temperature held in a refrigerated space. This may be merely lowering the temperature of the substance, but in some cases it may also mean freezing or solidifying the liquid parts contained in the substance, and then further cooling of the solidified product down to the storage temperature. An additional problem occurs in the case of leafy foodstuffs and fruits still undergoing living processes, as certain amounts of heat resulting from growth chemical change, enzyme action, etc., contribute additional heat load. This load is sometimes called product breathing.

Heat sources, miscellaneous sources inside the space. Lights, motors, people working, fans, and pumps within the space, all contribute to heat load. Such additional items as wash-down or cleaning the space with water, and the problem of defrosting also are included.

Defrosting. Whenever moist products are stored, there is a tendency for moisture to leave the products to enter the air and, along with the normal humidity in the air, to deposit on the refrigeration heat-transfer surfaces. When these surfaces are below the freezing point, deposition occurs in the form of ice, eventually becoming a severe retardent for transferring heat to the refrigerant or refrigerant brine. In refrigerators held above freezing, defrosting is accomplished by circulating warm air over the coils which melts off the ice. In low-temperature refrigeration, it is usually necessary to employ a warm-water or brine wash directed over the coils to melt off the ice on the surfaces. The hot gas leaving the condenser may be redirected by proper control of valves to pass through the evaporator coils which brings the surfaces of the coils to a sufficiently high temperature to melt off the ice. In cabinet-type units it is also possible to use electric heaters to warm up the inside of the cabinet space to cause defrosting. Whatever the method used, defrosting entails additional refrigeration load on the system.

HEAT TRANSMISSION OF BUILDING CONSTRUCTION. Calculation of Conduction. The walls, floors, ceilings, and partitions in refrigerated structures must be built to have the structural strength and life requisite for the type of building under consideration, and must be furnished with sufficient insulating material to keep the heat gain within reasonable economic limits. Heat is transferred through a wall (partition, etc.) whenever a temperature difference exists between the two sides of the wall. The heat (q) flowing through each square foot of area per hour can be computed from

$$q = UA\Delta t \tag{53}$$

where U = overall coefficient of heat transfer for the wall in Btu per (hr)(sq ft)(°F), A = wall surface area in square feet, Δt = temperature difference between the spaces adjacent to each side of the wall.

The value of U can be read from Table 20 for typical walls or computed for a composite wall not listed in the table by use of Eq. 54 and suitable coefficients of heat transfer from Table 19.

$$U = \frac{1}{\frac{1}{f_o} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \quad \frac{x_n}{k_n} + \frac{1}{a} + \frac{1}{C_n} + \frac{1}{f_i}} \tag{54}$$

Here f_o and f_i = surface coefficients of heat transfer on the outside and inside of a partition respectively. They may be taken as f_i = 1.6 Btu per (hr)(sq ft)(°F) for the still air such as usually exists on an inside wall and up to 6 if on an outside wall with a moderate wind blowing (15 mph); x_1, x_2, x_n = thickness in inches of the insulation or structural material of each particular type; k_1, k_2, k_n = coefficients of heat transfer in Btu per (hr)(sq ft)(°F) per inch of heat path; a = conductance of an air space if present; C = conductance of a construction unit (such as a 4-in. hollow tile) where conductance has units of Btu per (hr)(sq ft)(°F), for its actual thickness (not per inch).

Table 19. Conductivity (k) or Conductance (C) for Insulating and Cold-storage Materials

k is in Btu per (hr)(sq ft)(°F) per inch of thickness

C is in Btu per (hr)(sq ft)(°F) for thickness of block

Material	k	C
AIR SPACES		
Vertical, over 3/4 in.		1.10
Vertical, faced with aluminum foil		0.46
BRICK		
Common	4.8	
Face (4 in. thick)		2.30
Face brick and common 4″ + 4″ = 8 in.		0.77
CONCRETE		
Cinder aggregate	4.9	
Sand and gravel	12.0	
CORK		
Board (dense)	0.32	
Board (low density)	0.27	
Regranulated	0.30	
INSULATING BOARDS		
Cement and asbestos	2.70	
Cane fiber boards	0.33	
Gypsum board	1.40	
Wood fiber boards	0.33	
INSULATION LOOSE		
Diatomaceous earth	0.31	
Glass wool	0.27	
Hair felt	0.25	
Rock wool	0.27	
Sawdust	0.41	
Tree bark	0.31	
Vermiculite	0.32	
PLASTER		
Cement	8.00	
Gypsum (3/8 in.)		8.80
Metal lath (3/4 in.)		4.40
Wood lath		2.50
ROCK CORK		
Mineral in blocks	0.32	
SURFACE CONDUCTANCE		
Fifteen mph wind		6.00
Still air		1.65
TILE		
Hollow clay 4 in.		1.00
Hollow clay 6 in.		0.64
Hollow clay 8 in.		0.60
Hollow clay 12 in.		0.40
WOOD		
Oak or maple	1.15	
Yellow pine	0.80	

EXAMPLE. An outside wall of a cold-storage warehouse consists of 8-in. concrete, an asphalt binder against which is laid 4 in. of dense composition corkboard and 1/2 in. of cement plaster. Outside is 85 F and inside is −5 F. Find heat gain through each square foot of wall surface. If the inside walls, floor, and ceiling are adjacent to similar storage space, compute the gain through the 30 ft by 12 ft outside wall area.

By eq. 54 and using values from Table 19,

$$U = \frac{1}{1/6 + 8/12 + 4/0.32 + 0.5/8.0 + 1/1.65} = \frac{1}{14.0} = 0.071 \text{ Btu per (hr)(sq ft)(°F)}$$

$$q_w = U(1)\Delta t = (0.071)(1)[85 - (-5)] = 6.39 \text{ Btu/(sq ft) (hr)}$$

$$Q_w = UA\Delta t = (0.071)(30 \times 12)[85 - (-5)] = 2300 \text{ Btu per hour total gain}$$

Table 20. Overall Heat-transfer Coefficients for Composite Walls

Still Air on Inside. U in Btu/(hr)(sq ft)(°F)

Concrete	Corkboard or Rock wool Avg. k = 0.32	Plaster	Vertical Air Space 3/4 in. or more	U
6 in.	0	0	0	.78
6 in.	2	3/4	3/4	.116
6 in.	4	3/4	0	.071
10 in.	4	3/4	0	.070
Hollow Tile				
6 in.	4 in.	0	3/4	.063
6 in.	4 in.	3/4	0	.066
12 in.	4 in.	3/4	0	.062
Brick				
Face + Common				
4 in. + 4 in.	0	0	0	.65
4 in. + 4 in.	4 in.	3/4	3/4	.066
4 in. + 4 in.	4 in.	3/4	0	.070
4 in. + 4 in.	8 in.	3/4	0	.037

COOLING OF VENTILATING AND LEAKAGE AIR. Calculation of Infiltration. Such air must be cooled from supply conditions, and the greater part of the humidity must be condensed and frequently frozen as well. This heat removal, in Btu per hour, can be computed.

$$Q_V = (V)\left(\frac{1}{v}\right)[(h_o - h_s) + (W_o - W_s)h_{fg}] \tag{55}$$

where V = cubic feet per hour of ventilation air measured at conditions of v; v = cubic feet per pound specific volume of air specified either as inside of space or as supplied to space; h_o = enthalpy of air supplied, Btu per pound of dry air; h_s = enthalpy of air in cold-storage space, Btu per pound of dry air; W_o and W_s = specific humidity of air supplied to and inside of space, respectively, in pounds of water vapor per pound of dry air; and h_{fg} = enthalpy of condensation if condensed water does not freeze (use h_{ig}, the enthalpy of sublimation, if water freezes on coils).

Typical values for use are h_{fg} = 1074 and h_{ig} = 1220 Btu per lb. The additional small load of cooling the liquid or frozen water is disregarded.

EXAMPLE. A 20 by 30 by 12 ft high cold-storage room kept at 0 F, and 60% relative humidity, has an average of two air changes per hour, measured at inside conditions. The air enters from another part of the warehouse at 45 F and 90% relative humidity. Find the cooling load required for this air.

From the psychrometric chart (Fig. 20) read h_o at 45 F and 90% as 17.0 and h_s at 0 F and 60% as 0.4 Btu per lb, and v of cold-storage air as 11.59 cu ft per lb. Also W_o = 0.0057 and W_s = 0.0004 lb per lb of dry air.

By eq. 55,

$$Q_v = (2)(20 \times 30 \times 12)\left(\frac{1}{11.59}\right)[17.0 - 0.4 + (0.0057 - 0.0004)(1220)]$$

$$= (1242)[16.6 + 6.5] = 28{,}690 \text{ Btu/hour}$$

PRODUCT LOAD can be computed by the equation:

$$Q_p = M[c(t_o - t_f) + h_{fi} + c_f(t_f - t_s)] \tag{56}$$

where Q_p = Btu per hour; M = pounds of product supplied to storage space per hour (averaged over a period of 16 to 24 hr); c = specific heat of product above freezing temperature; t_o = temperature of product supplied, °F; t_f = temperature of final stored product or freezing (fusion) temperature of product, whichever is higher, °F; h_{fi} = heat of fusion, for products which are frozen, Btu per pound (see Table 21 or it can be computed approximately thus, h_{fi} = $(m/100)(144)$ = $1.44m$, where m is the percentage of water in the product); c_f = specific heat of product in frozen condition (see Table 21 or use approximately 0.5 to 0.7 of the specific heat before freezing); t_s = storage temperature if product is frozen.

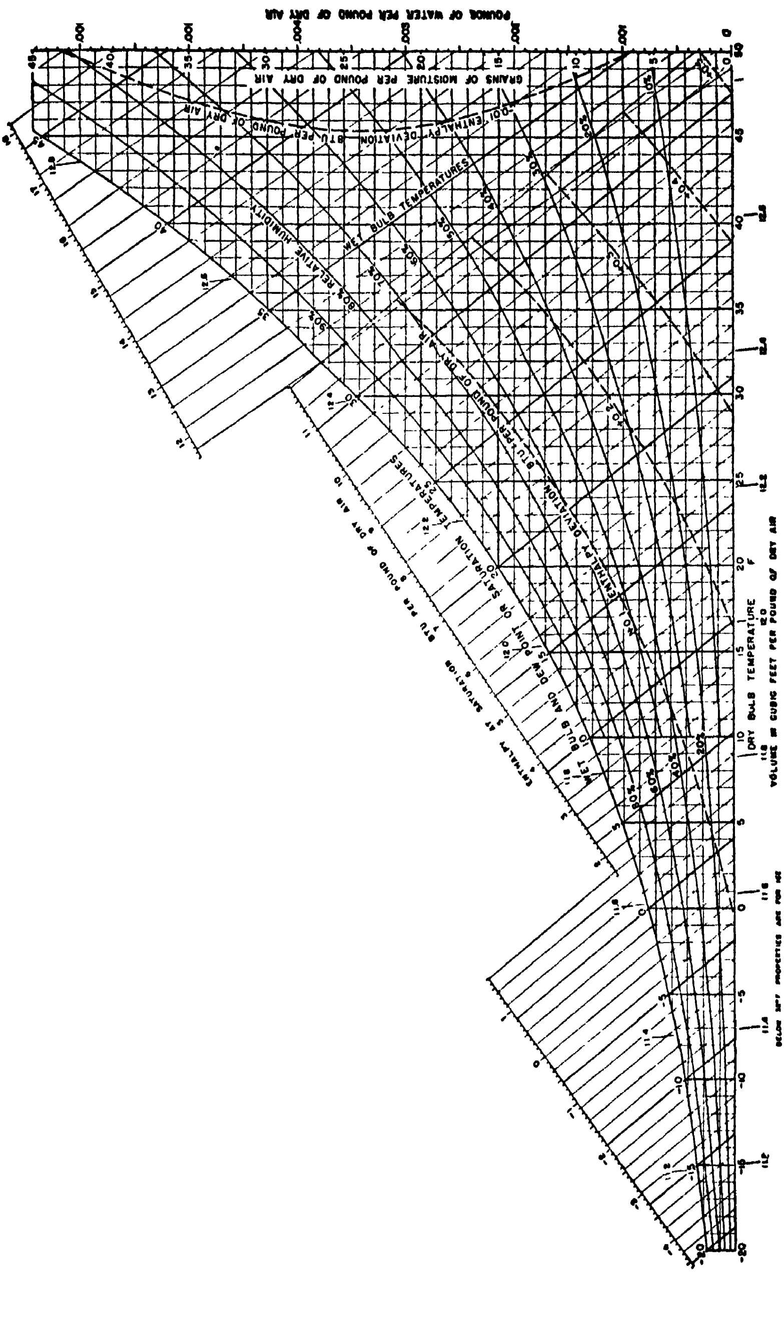


FIG. 20. Low-temperature psychrometric chart, barometric pressure 29.92 in. Hg. (Courtesy of Carrier C

EXAMPLE. Three tons of meat (beef) enter a freezing room each 24 hr, and 3 tons of meat are removed in the same period. The incoming meat is at 60 F, and the all-frozen meat is removed at 0 F. Compute the refrigeration load.

By eq. 56 and Table 21,

$$Q_p = (6000)[0.75(60 - 27) + 98 + 0.40(27 - 0)]$$
$$= 801{,}300 \text{ Btu per hr}$$

$$\frac{801{,}300}{12{,}000} = 66.8 \text{ tons of refrigeration}$$

Table 21. Storage Characteristics of Perishable Products

(Reprinted by permission from *Air Conditioning and Refrigeration*, by Jennings and Lewis, International Textbook Co.)

Products	Range of Storage, °F		Optimum Relative Humidity	Freezing Point, °F	Water in Composition, %	Latent Heat of Fusion	Specific Heat	
	Short Time	Warehouse					Above Freezing	Below Freezing
FRUITS								
Apples	35–40	30–32	85	28.5	85	122	0.90	.49
Bananas	55–56	55–56	80	26–30	75	.....	0.90	....
Oranges	40–45	32–34	80	28	86	124	0.90	.47
Peaches	35–40	31–33	80	29.5	88	128	0.92	.48
Strawberries	35–40	31–33	80	30	90.5	131	0.92	.48
VEGETABLES								
Bean (string)	40–45	32–34	85	30	68.5	98.5	0.80	.46
Beets	40–45	32–34	85	27	88.5	128	0.86	.48
Cabbage	35–40	32–34	90	31	91.5	132	0.93	.47
Lettuce	35–40	31–32	95	31	94.5	136	0.90	.46
Potatoes	36–50	38–42	85	29	78.5	113	0.86	.47
Tomatoes	50–55	50–55	80	30.5	94.5	132	0.92	.46
MEAT AND FISH								
Beef (fresh)	35–40	30–32	84	27	68	98	0.75	.40
Fish (frozen)	15–20	5–10	80	28	70	101	0.76	.41
Hams and loins	34–38	28–30	80	27	60	86.5	0.68	.38
Lamb	34–38	28–30	85	29	58	83.5	0.67	.30
Poultry (fresh)	28–30	28–30	84	27	74	106	0.79	.37
Poultry (frozen)	15–20	0– 5	85	27	74	106	0.79	.37
MISCELLANEOUS PRODUCTS								
Beer	35–40	34–38	83	28	92	.....	1.0	......
Butter	45–40	...	80	70–75	15	......	0.64	..
Cheese (American)	40–45	32–34	80	17	55	79	0.64	.36
Eggs (crated)	40–45	30–31	85	27	73	100	0.76	.40
Eggs (frozen)	15–20	0– 5	60	27	.....	100	.....	.41
Milk	35–40	35–40	70	31	87.5	124	0.93	.49

HEAT SOURCES IN A SPACE. Men working in cold-storage space contribute between 600 and 1000 Btu per hr per person. For motors and their driven equipment located in the space, consider as heat generation: 4250 Btu per hr per hp for $^1/_8$- to $^1/_2$-hp motors; 3700 Btu per hr per hp for $^1/_2$- to 3-hp motors; 2950 Btu per hr per hp for above 3-hp motors. For lights, multiply the total wattage by 3.41 to get the Btu generated for each hour of operation.

TOTAL REFRIGERATION LOAD. The sum of all the heat gains in a space expressed in Btu per hour, when divided by 12,000, gives the minimum tons of refrigeration required for the space. However, it is necessary to allow for contingencies such as excessive door opening, defrosting, unusual loading conditions, and cleaning. For such additional capacity it is customary to design the system so that if it operated steadily for less than 24 hr per day it could carry the total load. Thus

$$\text{Tons} = \frac{\text{Heat load}}{12{,}000} \times \frac{24 \text{ hr}}{x \text{ hr}} \tag{57}$$

where tons = actual installed capacity required, in tons of refrigeration; and x = hours of operation of the plant under which it could carry the base load.

Take $x = 16$ for a system operating with storage above 32 F and $x = 20$ for a system operating with storage below 32 F.

VAPOR BARRIERS. The problem of moisture migration or moisture travel through insulation is extremely important. This travel should be reduced as much as possible by use of vapor seals. The seal may take the form of a closely bonded layer of hot asphalt, or it may be an aluminum foil sheet bonded at the edges or moisture-resistant tar paper or the like. The seal should be placed on the outer surface of the cork (rock wool or major insulation) and adjacent to the outside masonry (brick or concrete wall). The vapor pressure of water vapor in air is greater at higher temperatures, and thus the direction of water-vapor migration is from the warm to the cold side. When moisture-laden air permeates a refrigerator wall, it eventually reaches a point of temperature low enough to cause condensation to take place, with the deposition of water inside the insulation. Condensation does not take place, however, if the water vapor is vented from the condensation zone at a fast enough rate to keep the moisture concentration above the equilibrium dew point. However, with improper venting, further progress of water vapor through insulation may carry it to a zone below 32 F, and freezing can occur. The effectiveness of insulation is seriously reduced as it becomes wet, and physical deterioration also takes place under the resultant expansion which occurs with freezing. Good construction should consider the following rules: (1) Install an effective vapor barrier. (2) Place the barrier on the warm side of the insulation at a location above the dew-point temperature. (3) Do not attempt to place additional impervious barriers past the first one if there is a possibility of the location of the second barrier being at a temperature zone lower than the dew-point temperature as any moisture which passes through the barriers must eventually vent itself into the refrigerated space, and not be stored in the wall itself. Figure 21 shows arrangements of cold-storage building construction. Note the pitch-cement vapor barrier next to the masonry.

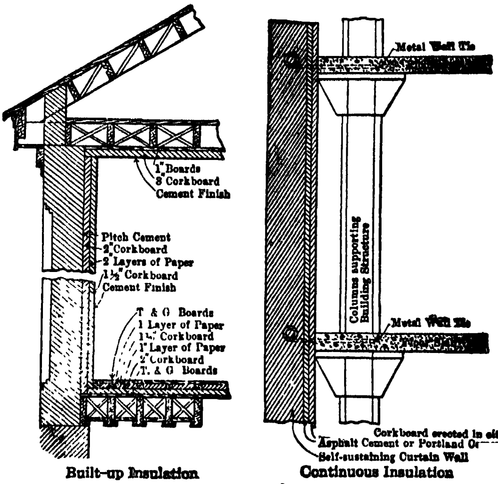


Fig. 21. Methods of insulating walls.

9. COLD STORAGE

PRACTICE. Insulation. A cold-storage structure should have sufficient insulation effectiveness to keep the refrigerator system load from being excessive because of heat gain through the walls. Table 22 indicates accepted practice in this respect.

Table 22. Insulation Thickness Practice, Referred to Corkboard (k 0.30)

Storage Temperature, °F	Thickness, in. Northern U. S.	Thickness, in. Southern U. S.
−40 to −10	9	10
−10 to 0	7	8
0 to 15	6	7
15 to 30	5	6
30 to 40	4	5
40 to 50	3	4
50 to 60	2	3

Pipe Covering. For refrigerated pipe carrying chilled water brine or refrigerant, it is customary to use molded (shaped) pipe covering of cork or rock wool composition. Covering, 1 1/2 to 2 in. thick, called "ice-water thickness" is used for temperatures above 25 F; "brine thickness" is 2 to 3 in., used for the range 0 to 25 F, and "heavy-brine thickness" 3 to 4 in. is used below 0 F.

Relative Humidity for Cold Storage. Data on optimum relative humidities are given in Tables 21 and 23. If air in a cold-storage room is recirculated, it rapidly decreases in moisture content as the moisture condenses or freezes on the refrigerator coil surfaces.

Table 23. Operating Conditions for Meat-storage Rooms

	Precooling Rooms	Cold-storage Room	Frozen-meat Room
Temperature, °F	41–46.4	35.6–39.2	21.2–15.8
Relative humidity, %	70–80	70–80	70–85
Meat stored, lb per sq ft	41	31	31
Initial meat temp., °F	82.4	59	32 *
Final meat temp., °F	59	37.4	21.2–17.6
Hours of cooling; freezing and cooling	20	30	72
Water evaporated, % of weight of meat:			
First day	0.65	......	....
Second and third day	....	0.35	......
In three days	....	......	0.345

* Meat already cooled to 32 F.

The ultimate result is evaporation of moisture from the stored product with loss of weight and quality of the product.

Condensation or freezing on the pipes or other heat-transfer surface is greatest when the surface is much colder than the temperature desired in the refrigerator space. A temperature difference of not more than 10 to 12 degrees between the room temperature and the coil temperature is desirable.

Control of Humidity. When precise control of humidity is required, a spray-type system, using water (or brine if below 32 F) can be employed. This will deliver air essentially saturated with moisture at the contact spray conditions, and, by reheating, the air can be brought to the precise relative humidity desired in the space. With brine sprays, consideration must be given to the equilibrium conditions of water vapor over brine, which is different from the equilibrium conditions of water vapor over pure water.

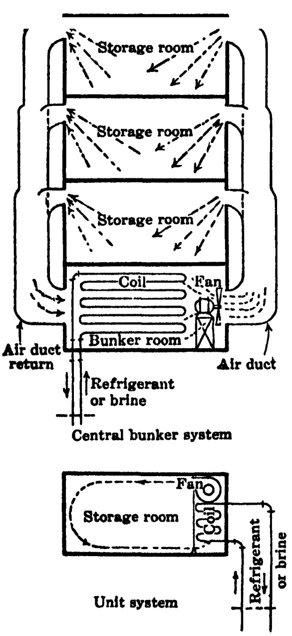


FIG. 22. Arrangement of two types of forced-circulation systems.

Pipe Coils. Older cold-storage room designs frequently used pipe coils placed along the ceiling or in elevated offset compartments called bunkers. Bunkers give better circulation of the air by forming a passage section. Warm air rising in the room to the top of the bunker, and then passing down over the bunker coils, falls as cooled air over the stored products. Natural circulation bunkers require more space than is required with forced circulation.

The pipe coils or other heat-transfer surfaces carry brine which has been cooled by the refrigerant in an evaporator, or can carry the refrigerant directly. When the refrigerant is fed to the coils and vaporizes in them as it absorbs heat from the air in the storage space the coils are called direct-expansion coils.

SYSTEMS. **Forced-circulation System.** In the forced-circulation, central-bunker system the coils are in a separate room, and a fan sends air over the coils through a duct system to the storage room (Fig. 22). This same arrangement can be used with a single room and a bunker for that particular room. Carried to the limit, this becomes the so-called air-unit or unit-conditioner, which is a heat-transfer coil and fan, built up as a factory assembly in a common housing. It can be placed where desired in a cold-storage space and connected up to the source of refrigeration.

The brine bunker system of room cooling is used principally in hog or beef chill rooms or meat-packing plants. It consists of spraying cold brine in spray bunkers or lofts near the ceiling. The brine spray induces rapid circulation of air and maintains a fairly high relative humidity. The meat chills rapidly without "case hardening" and without appreciable loss of weight or shrinkage. Pressure of 8 to 20 psi is maintained at the spray nozzles, spaced 1 to 5 ft centers. At each end of the bunker is a large gravity, air-circulation

Table 24. Space Required for Refrigerated Goods

Material	Average Weight, lb	Floor Space, sq ft	Space Occupied, cu ft	Clear Height of Room
1 barrel apples or potatoes	160	2.5	5.9	
1 tub butter	60	2.5	2.5	
1 cheese	60	2.0	2.0	
1 case eggs (30 doz)	70		3.0	
1 beef	700	9.0	108.0	12 ft 0 in.
1 sheep	75	2.0	16.0	8 ft 0 in.
1 hog	250			

Table 25. Approximate Refrigeration Required for Large Boxes

(Based on 24-hr continuous operation)

Cu Ft Space in Box or Room	Temperature = 20 F							Temperature = 10 F						
	Cu Ft per Ton of Refrigeration	Cu Ft per 1 ft of Pipe						Cu Ft per Ton of Refrigeration	Cu Ft per 1 ft of Pipe					
		Pipe Size, in., Direct Expansion			Pipe Size, in., Brine				Pipe Size, in., Direct Expansion			Pipe Size, in., Brine		
		1	1 1/4	2	1	1 1/4	2		1	1 1/4	2	1	1 1/4	2
12	113	1.6	2.2		1.4	2.0	.	93	1.0	1.2	...	0.6	1.1	....
20	137	1.7	2.3	.	1.4	2.0	.	112	1.0	1.2	.	0.6	1.1	...
50	160	1.8	2.4		1.5	2.1		130	1.1	1.2	....	0.6	1.1	....
100	205	2.0	2.6		1.6	2.2	.	168	1.2	1.3	.....	0.6	1.2	...
250	348	.	2.8			2.4	..	280	.	1.4	..	....	1.3	....
1,000	580		3.2			2.7	.	470	..	1.6	2.5		1.5	....
3,000	820		3.8	5.5		3.0	4.0	650	....	2.2	3.0	..	1.7	2.3
5,000	1100		4.5	6.5		3.4	4.5	840	..	2.5	3.6	..	1.9	2.6
10,000	1600		6.0	8.0		4.0	5.5	1140	.	3.2	4.6	.	2.2	3.3
20,000	2100	.	7.0	10.0		4.7	6.5	1600	..	4.0	5.7	.	2.6	4.0
40,000	2600	.	8.0	12.0	..	5.5	7.5	2100	.	4.8	6.8	..	3.0	4.7
70,000	3200	...	9.0	14.0	.	6.5	8.5	2600	...	5.5	8.0	..	3.5	5.5
100,000	4000	..	11.0	17.0		7.5	10.0	3100		6.5	10.0	....	4.2	6.7
150,000	4900		14.0	20.0		9.0	12.0	3800	...	8.0	12.0	...	5.0	8.0
	Mean temperature, ammonia expansion						0 F	Mean temperature, ammonia expansion						0 F
	Mean temperature, brine in coils						10 F	Mean temperature, brine in coils						5 F

duct. Air circulation is in direction of the spray. This system is cheaper than a cold-air fan circulating system.

REFRIGERATION REQUIREMENTS. Miscellaneous. Table 26 illustrates requirements for several types of establishment.

Table 26. Approximate Refrigeration Requirements for Various Purposes

General Cold Storage

Temperature desired, °F	0	5	10	20	32	36
Cu ft per ton, rooms below 1000 cu ft	500	1000	2000	3000	4000	5000
Cu ft per ton, rooms above 1000 cu ft	1000	2000	3000	5000	7000	8000

Apartment buildings: 16 to 20 apartment refrigerators per ton
Creameries: 3000 gal of milk cooled per day per ton; 1000 cu ft of storage per ton
Drinking water: 1000 gal per day from 75 F to 40 F per ton
Fur storage: 1500 cu ft of vault space per ton
Hotels: One ton will serve 1000 cu ft of kitchen and storage refrigerators, cooling drinking water for 60 rooms; ice-making for 125 rooms; preparation of ice cubes in a central freezing plant
Ice cream factory: 120 to 150 gal of ice cream made and hardened per day per ton
Ice plants: 1000 lb of ice per day per ton
Meat markets: 1000 cu ft of storage space or 50 lineal ft of display counter per ton
Restaurant: 1000 cu ft refrigerator space per ton
Skating rinks (ice:) 170 sq ft of ice surface maintained per ton; cut to half this figure if frequent replacement thawing and freezing is required

Small Boxes and Rooms. In large rooms the undesired heat gains may be analyzed with some degree of certainty when conditions of operation are known. For small refrigerators, as in hotels, kitchens, and private homes, the following data are recommended as giving better results than a more elaborate analysis. Heat gain, Btu per cubic foot per 24 hr: pantry refrigerator, 300; kitchen refrigerator, 600 to 900; butchers' display refrigerators, 200 to 250; long storage, 150 to 200. An allowance of 200 to 225 Btu is made per lb of ice. In applying the data, assume a refrigerator temperature of approximately 45 F and an average summer temperature of 72 F. For pantry and kitchen refrigerators, outside dimensions are used in figuring volumes.

Domestic refrigerators, in general, are built as hermetically sealed units, that is, with the motor and compressor enclosed in a shell, and with welded connections leading to the condenser and from the evaporator. The original charge of lubricating oil should last throughout the life of the equipment, and, unless leakage occurs through porosity of the metal welds, the refrigerant charge should be sufficient for the life of the unit. Various types of reciprocating or rotary positive-displacement compressors are used, and the condensers are fin-tube units. A forced-circulation fan, driven by a separate motor, is usually provided to improve the heat-transfer characteristics of the condenser.

One widely distributed domestic refrigerator, which works on the absorption system, uses a gas flame as its only source of energy. Domestic units of all types usually have capacities of less than one-quarter ton of refrigeration.

HEAT TRANSFER. Coefficients. Values of the overall coefficient of heat transfer (U) from refrigeration coils or surface are given in Table 27, expressed in Btu per (hr) (sq ft)(°F) of mean temperature difference.

Table 27. Coefficients of Heat Transfer for Refrigeration Surface

U in Btu per (hr)(sq ft)(°F)

Brine, or refrigerant pipe coils, air outside, quiet air (natural circulation)

Δt	5	10	20	30
U	1.5	2.3	3.1	3.4

Brine, or refrigerant pipe coils, air outside, air moved by fan

Velocity of air, ft/min	200	300	400	500
U	2.9	4.2	5.0	5.5

Condensers, ammonia

Atmospheric, gas in at top	U = 75–150
gas in at bottom	U = 125–250
Double pipe	U = 150–400
Shell and tube	U = 150–350
Submerged coil	U = 30– 40

Coolers

Baudelot with cream	U = 55
Baudelot with milk or water	U = 70
Baudelot with oil	U = 10

Shell and tube multipass brine cooler, brine velocity 100–400 ft/min

U = 40 – 120

Direct expansion coils

Finned air coils, air surface heat-transfer coefficient, h_s

Velocity, ft/min	300	400	600	800
h_s	4.3	5.1	6.5	7.7

"Freon-12," film surface coefficient, h = 150–250

Mean temperature difference (Δt) can be computed from Table 28, where D_s = smallest temperature difference and D_g = greatest temperature difference, and D_s/D_g = ratio of the temperature differences. Then $\Delta t = M\,D_g$, where M is given in Table 28.

Table 28. Mean Temperature Difference, $\Delta t = M\,D_g$

D_s/D_g	M	D_s/D_g	M	D_s/D_g	M	D_s/D_g	M
0.0025	0.166	0.08	0.368	0.20	0.500	0.50	0.724
.005	.189	.10	.391	.25	.544	.60	.786
.01	.215	.12	.418	.30	.583	.70	.843
.02	.251	.14	.440	.35	.624	.80	.897
.04	.298	.16	.461	.40	.658	.90	.953
.06	.335	.18	.478	.45	.693	1.00	1.000

EXAMPLE. Brine enters room coils at 15 F and leaves at 20 F, and air in the room passes in counter (and cross)-flow over the coils, entering at 40 F and cooling to 25 F.

The 40 F air and 20 F brine give $D_g = 40 - 20 = 20$, and $D_s = 25 - 15 = 10$. Thus $D_s/D_g = 10/20 = 0.5$, and M from Table 28 is 0.724.

$$\Delta t = M\,D_g = (0.724)(20) = 14.48$$

If there are 500 lineal feet of 2-in. pipe (2.375 in. outside diameter) in this coil and the air velocity is 300 ft per min, find the heat absorbed per hour by the brine. Select $U = 4.2$ from Table 27, and use

$$Q = UA\,\Delta t = (4.2)\left(500 \times \pi \times \frac{2.375}{12}\right)(14.48)$$

$$18{,}907 \text{ Btu per hr or } \frac{18{,}907}{12{,}000} = 1.58 \text{ tons of refrigeration}$$

10. REFRIGERATION ACCESSORY EQUIPMENT

CONDENSERS. Heat is absorbed in the condenser for desuperheating, condensing, and subcooling the refrigerant after this has been compressed. Very small compressors, such as in domestic units, use air-cooled condensers, but larger units use water almost exclusively.

The shell-and-tube condenser is most extensively used at the present time. It resembles the steam condenser of the power plant with the vapor passing inside the shell and the water passing through the tubes. Figure 23 is typical of modern design with low fins on the tubes, inside the shell, to increase the heat-transfer surface on the refrigerant side, particularly for "Freon" group refrigerants which carry oil. The water heads are made for multi-pass water flow, with sometimes as many as ten passes, but two to four passes are most common. Refrigerant vapor enters at the top of the horizontal shell and drains out at the bottom. Capacities to more than 100 tons are built in single shells.

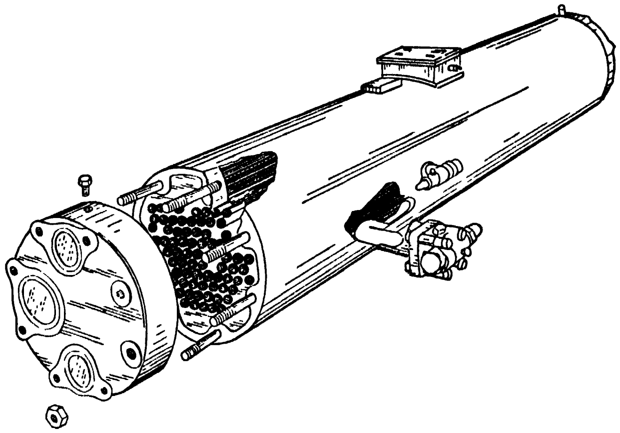


FIG. 23. Shell-and-tube condenser. (Courtesy of Carrier Corporation)

Double-pipe condensers were formerly common but are little used for new installations. The inside pipe carries the water, and the refrigerant condenses in the annulus between the inside and outside pipes. Counterflow between the refrigerant and water is possible, permitting subcooling, with the coldest water, and desuperheating, with the hottest water leaving. However, these disadvantages should be mentioned; the complexity of the tube end connections, the greater space required, and difficulty in cleaning.

Atmospheric condensers are used for outside locations such as on roofs or in open courts. They are relatively economical in the use of water and are easy to clean. The surface required with them is greater than that needed for other types. In such condensers the water is pumped to the top of the coil and washes or trickles down over the coils, permitting evaporative cooling to take place.

The Flooded Condenser. In the flooded condenser sufficient liquid ammonia is injected into a mixing chamber in direct contact with the ammonia gas to absorb the superheat and insure full saturation. Condensation is effected by cooling water applied externally. The advantages of the flooded type of condenser are lower first cost, reduced space, and reduced upkeep. The flooded condenser is built in the atmospheric, double-pipe and shell-and-tube types. One section of a flooded atmospheric condenser, 12 pipes high, 20 ft long, made of 2-in. pipe, or a total of 150 sq ft of pipe surface, is rated as follows when operating at a condensing pressure of 185 psi and various temperatures of water:

Temperature of water, °F	55	60	65	70	75	80	85	90
Tonnage per coil	54.6	49.9	44.4	39.3	33.5	26.5	17.1	11.8
Water per ton of refrigeration, gallons per minute	1.1	1.19	1.36	1.53	1.79	2.26	3.86	5.68

Similar data for a double-pipe coil, 8 pipes high, 18 ft 2 in. long, made of 2- and 3-in. pipe are:

Temperature of water, °F	55	60	65	70	75	80	85	90
Tonnage per coil	62.5	55.9	49.3	42.5	35.8	28.8	21.3	13.0
Water per ton of refrigeration, gallons per minute	1.44	1.61	1.83	2.12	2.51	3.13	4.22	6.9

Table 29. Dimensions of Atmospheric Ammonia Condensers

No. of Sections	Capacity, Tons of Refrigerator	Space Required, ft			Pipe Size, in.	Pipe Length, ft	No. of Pipes	Total Pipe, ft	Shipping Weight, lb
		Length	Width	Height					
1	7 1/2	19 1/2	2	8	1 1/4	18	20	360	2,300
2	15	19 1/2	3	8	1 1/4	18	40	720	4,600
1	12 1/2	21 1/2	2	11 3/4	2	20	24	480	3,200
2	25	21 1/2	3 1/2	11 3/4	2	20	48	960	6,400
3	37 1/2	21 1/2	5 1/2	11 3/4	2	20	72	1440	9,600
4	50	21 1/2	7 1/4	11 3/4	2	20	96	1920	12,800
6	75	21 1/2	12	11 3/4	2	20	144	2880	19,200
8	100	21 1/2	16	11 3/4	2	20	192	3840	25,600
12	150	21 1/2	24	11 3/4	2	20	288	5760	38,400
16	200	21 1/2	32	11 3/4	2	20	384	7680	51,200

Table 30. Dimensions of Double-pipe Ammonia Condensers

Capacity, Tons	No. of Sections	No. of Pipes	Length of Pipe, ft	Total Feet of Pipe	Space Required			Shipping Weight, lb
					Length, ft in.	Width, ft in.	Height, ft in.	
1	1	6	8	48	10 6	1 8	6 0	900
2	1	6	12	72	14 6	1 8	6 0	1,100
3	1	8	12	96	14 6	1 8	6 9	1,500
4	1	8	14	112	16 6	1 8	6 9	1,600
5	1	8	16	128	18 6	1 8	6 9	1,700
6	1	10	15	150	17 6	1 8	7 6	1,950
7	1	10	16	160	18 6	1 8	7 6	2,000
8	1	10	17.5	175	20 0	1 8	7 6	2,100
9	1	10	16.5	198	19 0	1 8	8 3	2,400
10	1	12	17.5	210	20 0	3 8	8 3	2,500
12.5	1	12	19	228	21 6	1 8	8 3	2,600
15	1	14	19	266	21 6	1 8	9 0	2,900
15	2	8	17.5	280	20 0	3 4	6 9	3,900
18	2	10	17.5	350	20 0	3 4	7 6	4,500
20	2	10	19	380	21 6	3 4	7 6	4,800
25	2	12	19	456	21 6	3 4	8 3	5,400
30	3	10	18	540	20 6	5 0	7 6	6,800
35	3	12	18	648	20 6	5 0	8 3	7,900
40	4	10	19	760	21 6	6 8	7 6	9,400
45	4	12	17.5	840	20 0	6 8	8 3	10,400
50	4	12	19	912	21 6	6 8	8 3	10,800
55	5	12	17.5	1050	20 0	8 4	8 3	12,700
60	5	12	18	1080	20 6	8 4	8 3	12,900

Table 31. Dimensions of Shell-and-Tube Type Ammonia Condensers

Diameter of shell, in.	16	20	24	34	38	42	46
No. of vertical 2-in. tubes	21	23	51	115	144	191	227
No. of horizontal 2-in. tubes	16	26	36	96	136	176	204
Shell thickness, in.	1/4	5/16	3/8	1/2	5/8	5/8	3/4

Water Circulation of Atmospheric Condensers. The water required per minute per ton of refrigeration in atmospheric condensers is as follows:

Temperature, °F	50	55	60	65	70	75	80	85
Gal per min	1/2	5/8	3/4	7/8	1	1 1/5	1 1/2	2

These quantities are based on water leaving the condenser at 95 F and are the smallest quantities that should be allowed.

Evaporative condensers are increasingly important for small and moderate-size installations, where water must be conserved, as they combine in one unit condenser and a cooling tower. They are built with a water sump at the bottom with a pump to deliver this water to the top of the closed cabinet, whence the water trickles down over the refrigerant coils that rest in the lower part of the cabinet above the water sump. A fan mounted at the top of the cabinet draws in air at the bottom; the air passes over the wetted coils, evaporating moisture as it rises, and is delivered to waste from the top of the unit. Heat is absorbed largely in evaporating a portion of the water. The water in evaporating approaches the wet-bulb temperature of the entering air. Thus condensation of the refrigerant is not fixed by the upper temperature to which water in passing through a condenser is raised. The water saving is approximately 95% as compared to use of a continuous supply of water such as from city mains. A similar result could be obtained by the use of a cooling tower built separate from the evaporative condenser.

Water quantity for condensers depends on the temperature rise permitted in the water and the plant capacity. In the example on p. 11-05 it was shown that a "Freon-12" system with 86 F condensation required the removal of 64.7 Btu per lb, and for a 100-ton plant 379 lb of refrigerant would flow per min. Thus the condenser load would be

$$64.7 \times 379 = 24{,}521 \text{ Btu/min}$$

For a 5-degree temperature rise

$$\frac{24{,}521}{5 \times 1 \times 8.3} \quad 591 \text{ gal/min of water are required}$$

For a 10-degree temperature rise it is obvious that only half as much water, 295 gal per min, is required. If water is available at 75 F and condensation takes place at 86 F, a 5-degree rise would be a conservative design as the fluid leaving is 6 degrees lower than condensation. A 10-degree rise, however, would give a leaving temperature only 1 degree lower, and unless the condenser had very ample surface the condensation temperature might rise. The problem is one of balancing water cost against the power cost of compression which increases as the condensing temperature rises.

EVAPORATORS. The shell and tube evaporator resembles the shell and tube condenser in its general design features. However, the evaporators are frequently operated flooded with boiling liquid occupying about half the total volume and the vapor passing off at the top. This design is very common for brine coolers and water chillers. Figure 10 illustrates such an evaporator.

Pipe coil evaporators and once-through tubing assemblies are also common as evaporators. In these the refrigerant enters at one end of the pipe or tubing coil and progressively vaporizes as it absorbs heat in passing through the coil. In small coils the refrigerant feed is restricted so that all the refrigerant evaporates in passing through the coil, and it leaves as a slightly superheated vapor. In the larger coils there may be some carry-over of liquid that can be separated out in an attached surge drum and returned to the coils for completion of vaporization.

EXPANSION VALVES. Expansion valves are usually automatic, although hand-operated valves can be used and are installed for emergency operation. In all expansion valves the objective is to feed refrigerant to the evaporator at a controlled or desired rate.

Thermal-expansion valves employ a thermostatic bulb attached to or inserted in the suction pipe of the evaporator. This bulb reacts to temperature changes in the suction gas leaving the evaporator. When the rate of refrigerant flow to the evaporator is inadequate the leaving refrigerant temperature will be higher than that corresponding to saturation in the evaporator. This elevated temperature reacts on the fluid in the thermostatic bulb, and thereby increases its pressure, and this, in turn, is transmitted back to a spring-controlled bellows or diaphragm which, through suitable mechanism, opens the feed valve, permitting more refrigerant to enter the evaporator. The valves are adjusted to give varying rates of flow and are operated with 3 to 20 degrees of superheat. Figure 24 shows a thermal-expansion valve.

Automatic high-side float valves operate in similar manner to a steam trap, in that they deliver all liquid supplied to them, without permitting the vapor to by-pass from the high to the low side of the system.

Automatic low-side float valves are used with flooded evaporators and operate to maintain a definite level in the evaporator at all times.

Automatic diaphragm expansion valves are responsive to the pressure in the evaporator and feed refrigerant in accordance with pressure variations in the evaporator. They are also known as constant-pressure valves.

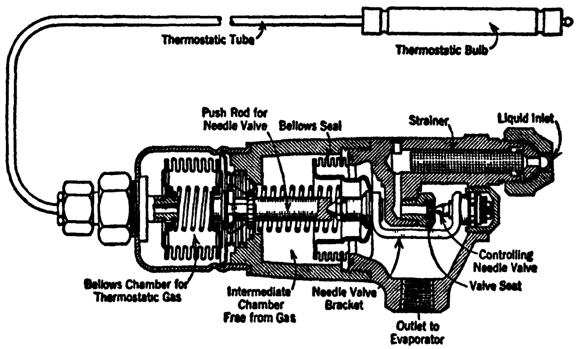


FIG. 24. Thermal-expansion valve for refrigerating system.

Note: The magazine *Refrigerating Engineering*, published by the ASRE, carries the most significant literature of the field, but numerous papers have also appeared in the *Transactions* of the ASHVE and the *Transactions* of ASME. Significant material also appears in the magazines *Ice and Refrigeration; Heating, Piping and Air Conditioning, Heating and Ventilating*, and *Zeitschrift für die Gesämte Kalte Industrie*. The *ASRE Refrigerating Data Book*, Vol. 1, covers properties of refrigerants, refrigeration theory, and equipment, and the *ASRE Data Book*, Vol. 2, covers refrigeration applications. Among the more general text and reference books are Jennings and Lewis, *Air Conditioning and Refrigeration;* H. J. McIntire, *Refrigeration Engineers' Handbook;* Raber and Hutchinson, *Refrigeration and Air Conditioning Engineering;* the *Guide* of the ASHVE; and H. R. Sparks, *Theory of Mechanical Refrigeration*. In addition to the very complete refrigerant tables in the *ASRE Data Book*, Vol. 1, separate tables of refrigerant properties appear in the National Bureau of Standards, *Circular* 142, on ammonia; the ACRMA publication, *Properties of Commonly Used Refrigerants*, and *Tables of Properties of the "Freon" Refrigerants* by Kinetic Chemicals, Inc.

ICE MAKING

11. ICE MANUFACTURE

PRACTICE. Formerly, nearly all artificial ice was made by the can or plate system, but at the present time a large amount of ice is being made in unit ice machines.

The Can System. In the can system of making ice, standard size ice cans (Table 1) are immersed in a brine solution which flows around the cans and causes the water to

Table 1. Size of Standard Ice Cans and Freezing Time

Size of Can, in.	Weight of Ice Block, lb		Gage of Metal		Time of Freezing, hr	
	Normal	Actual	Sides	Bottom	15° Brine	18° Brine
6 x 12 x 26	50	56	20	20	15	20
8 x 16 x 32	100	110	18	16	30	36
8 x 16 x 42	150	165	18	16	30	36
11 x 22 x 32	200	220	18	14	50	60
11 x 22 x 44	300	315	16	14	50	60
11 x 22 x 57	400	415	16	14	50	60

freeze. Under the older system it was necessary to use distilled water as the material supplied to the cans in order to produce a clear cake of ice. Distilled water was easy to obtain with the steam engine driven systems formerly so extensively used, as the exhaust steam could be used in the process. Exhaust steam from the main engine and auxiliaries is purified by reboiling and filtering and then is frozen in galvanized sheet steel cans. In the brine tank, the brine is agitated by a propeller wheel and cooled by direct expansion piping in the tank. The product is known as distilled-water ice. The ice grows from all sides of the can, and any mechanically suspended impurities in the water will appear in the ice at the center of the block. It is, therefore, essential that water free from impurities be used. A standard-size block is 11 by 22 by 44 in. and weighs approximately 300 lb. A typical plant of this type is shown in Fig. 1.

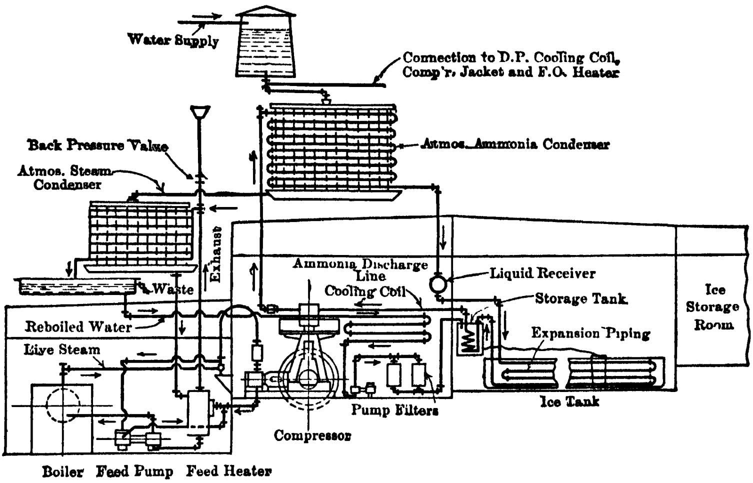


FIG. 1. Arrangement of distilled-water ice-making plant.

The raw-water system, using air agitation, is by far the most important method of ice manufacture at the present time. In this process the water in the cans is agitated during the entire freezing period by air discharged under a pressure of 3 to 6 psi at or near the bottom of the cans through a $^1/_{16}$-in. opening. The unfrozen water, heavily charged with impurities, is pumped out of the center of the block before this part is entirely closed and frozen; and the space is filled with raw water containing only the initial amount of impurities. As this system does not require distilling apparatus, the most economical form of drive may be used, the majority of plants being motor driven. The size of the motor should be ample to provide for a higher terminal pressure than ordinarily is considered for 70 F condensing water, to prevent possible overloading of motor.

Plate System. A tank approximately 10 ft deep by 12 ft wide, divided by $^1/_2$-in. plates, bolted to direct expansion piping, into compartments 30 in. wide is used. The plates form the freezing surface. Ice grows from the plates outwards, and mechanically suspended impurities in the water are separated and fall to the bottom of the tank. The ice cake usually is thawed from the plate by passing hot gas through the coils. Distilled water is not required for the manufacture of clear ice, and electric drive may, therefore, be used. The ice gradually forms in 8 to 10 days to a thickness of 12 to 14 in.

The freezing tank area required by the plate system is about twelve times that required by the can system, and the cubical contents are about four times as great. The advantage of the plate system that clear ice is produced without special apparatus is offset by the fact that the building up of the ice is slow and expensive; also, for continual operation several tanks are required, so that one or more may be frozen while the others are being emptied. The cost of the plate system is about one-third more than that of the can system. In the can system, ice is drawn throughout the day while in the plate system the entire product is harvested, cut, and stored in a few hours. The plate system of making ice has become practically obsolete.

ICE-MAKING CAPACITY. The capacity of a refrigerating machine for making ice is approximately 61% of its refrigerating capacity, based on rating conditions of a 5-degree evaporator and an 86-degree condenser, using ammonia which is most commonly employed in ice-making plants. Table 2 gives the suction pressures required for maintaining various brine temperatures with an ammonia system.

Table 2. Suction Pressures Required for Various Brine Temperatures

Back pressure, psig	5	10	15	20	25	30
Brine temperature, °F	−5	0	10	15	20	25

Size of Freezing Tank. Let W = weight of ice, pounds, to be pulled every 24 hr = tons rating $\times$ 2000; H = freezing time, hours, N = number of cans required; C = weight of one block of ice, pounds. Then $N = WH/(C \times 24)$. For 300-lb blocks, H = 50 and $N = W/144$, and the number of 300-lb cans per ton capacity rating of plant is 14. Some manufacturers recommend 16 cans per ton of capacity. Dimensions of the freezing tank may be approximated from data given in Table 3. Water consumption data for typical plants are given in Table 4.

Table 3. Dimensions of Freezing Tanks

Number Cans, Wide or Long	Width, ft	Width, in.	Length, ft	Length, in.	Number Cans, Wide or Long	Width, ft	Width, in.	Length, ft	Length, in
6	9	0	16	6	22	28	3	49	9
8	11	3	20	9	24	30	9	53	9
10	13	9	24	9	26	33	0	58	0
12	16	3	29	0	28	35	6	62	3
14	18	6	33	0	30	38	0	66	3
16	21	0	37	3	32	..	.	70	6
18	23	6	41	3	34	..	.	74	6
20	25	9	45	6					

Table 4. Water Consumption per Ton of Ice in Compression Plants

Initial temp. water over ammonia condenser, °F	55	60	70	80
Water temp. entering steam condenser, °F	80	85	90	95
Water temp. leaving steam condenser, °F	125	125	125	125
Gallons per minute	4	4.5	5.15	6

Expansion Pipe. Approximately 80 to 100 sq ft of pipe surface per ton of ice-making capacity is required with good brine agitation. The maximum length of pipe for one expansion is 1200 ft. See Table 5.

Table 5. Lineal Feet of Pipe per Ton of Ice

Pipe size, in.	1	1 1/4	1 1/2	2
15° brine, ft of pipe	400	320	270	210
18° brine, ft of pipe	450	360	310	240

The time of freezing a cake of ice can be approximated from this equation:

$$x = \frac{7a^2}{32 - t} \qquad (1)$$

where x = freezing time for cake, hours; a = thickness of cake, inches; t = temperature of brine, °F.

In a freezing tank, brine velocities of 16 to 30 ft per min are used, and the brine temperatures are held between 12 and 18 F. Although ice can be made at a more rapid rate by lowering the brine temperature, there is a tendency for ice made rapidly at very low temperatures to be brittle.

UNIT ICE MACHINES. Unit ice machines are made in a variety of designs. In one such machine, a refrigerated drum rotates, and onto this a stream of chilled water plays. On the surface of the drum a thin film of ice freezes, which is scraped off by a rigid bar. This ice product in small pieces is useful for many purposes.

Other machines produce a similar material and then compress the product into briquettes of proper size. In still another arrangement, ice is frozen in a series of vertical tubes of 1 to 2 in. in diameter, and this ice frozen in these small cylinders can be broken into whatever lengths are desired.

12. BRINE CIRCULATING SYSTEM

In the brine circulating system (Fig. 2) brine circulates through coils in the storage room. The evaporating coils of the refrigerating system are located in the brine tank, and the brine acts simply as a heat-transfer medium. The principal advantage of this system is that the refrigeration can be stored and the refrigerating machine shut down for part of the time. The length of refrigerant piping and number of joints also are reduced to a minimum.

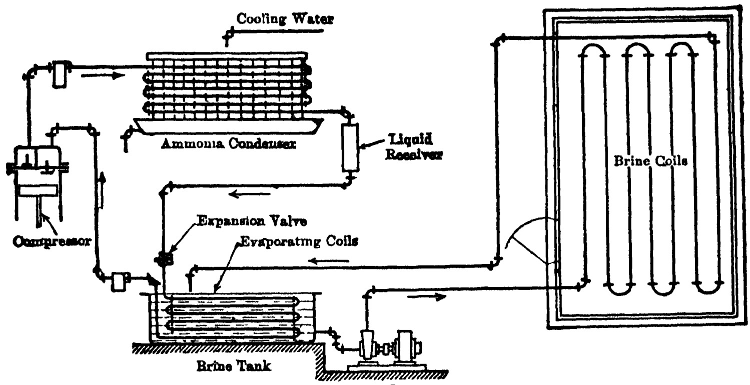


FIG. 2. Arrangement of brine circulating system.

Brine Tank. For continuous operation of the plant, about 60 cu ft of brine per ton of refrigeration is required. For noncontinuous plant operation, the weight of the brine required is

$$W = \frac{12{,}000R(24 - h)}{c_b(t_1 - t_2)} \tag{2}$$

where R = refrigeration, tons; h = hours of operation per day; c_b = specific heat of brine solution; t_1 = initial temperature of brine when machine is starting; and t_2 = final temperature of brine when machine is shut down. The specific gravity of the brine solution depends on the lowest temperature to be carried. This fixes the percentage of salt used in making up the solution. If x = specific gravity of solution at 60 F, the weight d, pounds per cubic foot = $62.5x$, approximately, and the contents of the brine tank = W/d cu ft. The size of the brine circulating pump is calculated in the same manner as that of a water pump.

Brine Solutions. The most common brine solutions employ common salt (NaCl) and calcium chloride ($CaCl_2$). Sodium chloride is cheaper but is not suitable for as low temperatures as calcium chloride. The eutectic or complete solidification point for sodium chloride occurs with a 23.3% solution, and solidification takes place at −6.0 F. The corresponding point for calcium chloride is −59.8 F. Both these brines must be treated to prevent corrosion of the system. The brine tank solution should be kept on the alkaline side, with a pH ranging of not less than 7.2 to not more than 8.5. The most effective treatment for brines is with sodium dichromate in proportions of approximately 125 lb of the dichromate per 8000 gal of calcium chloride solution. Except where the brine may come into contact with food materials, by far the greatest amount of commercial brine is made with calcium chloride. Tables 6 and 7 give properties of the two brines.

Table 6. Properties of Common Salt Brine at 60 F

Degrees Baumé	Degrees Salinometer	Specific Gravity	Salt, %	Wt. of 1 gal, lb	Wt. of 1 cu ft, lb	Freezing Point, °F	Specific Heat
5	20	1.037	5	8.7	64.7	27.0	.938
10	40	1.073	10	9.0	67.0	20.4	.892
15	60	1 115	15	9.3	69.6	12.0	.855
19	80	1.150	20	9.6	71.8	1.8	.829
23	100	1.191	25	9.9	74.3	+16.1	.783

Table 7. Properties of Calcium Chloride Brine

Specific Gravity at 68 F	Lb 75% $CaCl_2$ per gal	Lb 75% $CaCl_2$ per cu ft	Freezing Point, °F	Specific Heat
1.100	1.46	10.9	18.0	.88
1.125	1.83	13.7	12.5	.87
1.150	2.20	16.5	+6.5	.85
1.175	2.59	19.4	−2.0	.835
1.200	2.99	22.4	−12.5	.81
1.225	3.38	25.3	−23.5	.80
1.250	3.75	28.3	−36.5	.77

Brine Piping. Steel or wrought-iron pipe is normally used in constructing the brine system, and, with proper control of the brine, corrosion is not a severe problem. Factors entering into design of brine piping are (1) heat absorbed by each coil, basing calculations upon refrigeration required for the various rooms or refrigerators; (2) amount of brine to be circulated to absorb the calculated amount of heat with a temperature rise of 2 to 5 F; and (3) pipe friction of the system. Quantity of brine to be circulated per °F rise of temperature per ton of refrigeration, when the specific heat of brine is 0.829, is 242.4 lb per min or 25.2 gal per min. Regulation of flow is facilitated if the system is so arranged that it always will be full of brine while in operation and also if it is kept under pressure. To insure the latter, it may be advisable to place a reducing valve on the discharge.

13. QUICK FREEZING

If food materials, meat or vegetables, are frozen at a sufficiently rapid rate it is possible to control the size of crystals formed in the material to reduce the amount of cell tissue breakdown and reduce the amount of fluid migration between cells in the material. The tendency with slow freezing is for large crystals to form, and a greater amount of cell breakdown takes place. Quick freezing is characterized by low temperatures and a means of accelerating the heat-transfer process.

Before the term *quick freezing* became popular, a similar kind of freezing had been done in rooms in cold-storage plants maintained at low temperatures, usually below −10 F, called *sharp freezers*. They employed essentially still air but were quite effective, and are still employed for certain types of quick freezing. A more effective method uses an air blast in a room or tunnel kept at low temperature. The air blast may range from −10 to −40 F, with velocity of 500 to 1500 ft per min. It is frequently employed for freezing meats.

With small packages, an effective method uses refrigerated plates, between which the packages are placed. The plates coming into contact with the food material can rapidly remove heat by direct conduction. This arrangement is sometimes supplemented by an air blast in addition to the plates themselves.

Another method uses a refrigerant spray. A low temperature brine (NaCl) is allowed to wash over the product to be frozen, poultry, meat, or fish, and by direct contact with the product, or with the product wrapped in some type of parchment, rapid freezing takes place. Small fruits are frequently frozen by immersing the fruit in a refrigerated sugar solution of the proper temperature.

Facilities for commercial distribution and marketing of frozen food are increasing to such an extent that it has become a large industry.

SECTION 12

HEATING, VENTILATING, AND AIR CONDITIONING

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PANEL HEATING

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HEAT PUMPS

By E. R. Ambrose and Theodore Baumeister

VENTILATING

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AIR CONDITIONING

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HEATING

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1. HEAT LOSS FROM BUILDINGS

Heat loss from a building occurs whenever the inside temperature is higher than the prevailing outside temperature. The principal loss of heat from a building is through the walls, roof, and glass areas. Heat flows through these materials at varying rates, depending upon their conductivity and thickness, and is dissipated from the surfaces exposed to air by *convection* and *radiation*. For surfaces buried in the ground, such as basement walls and floors, heat is lost by conduction to the earth.

A large amount of heat is lost by the *infiltration* of air. Few buildings erected by the usual method are absolutely airtight. Cracks occur at doors and windows, and in masonry and frame construction. Temperature difference alone creates a difference in air pressure, and, if outdoor temperature is lower than indoor, the pressure difference or chimney effect will cause air leakage inward in the lower parts of the structure and outward in the upper parts. Air leakage is further increased by an increase in wind velocity. Wind pressure causes leakage on the windward side, the displaced heated air moving out on the lee side. Air introduced for ventilation purposes also adds load on the heating system.

Heat-loss calculations are made for the purpose of determining the size of the heat-emitting apparatus or, more specifically, the size of each component of the system to maintain a condition of heat equilibrium for an assumed temperature differential.

TEMPERATURES. Inside temperatures ordinarily assumed for different types of buildings are given in Table 1. Outside design temperatures are dependent on geographical location. It is customary to select as the outside design temperature not the lowest temperature on record, but a figure at least 15 F above the lowest temperature ever reported.

Table 1. Inside Temperatures

Building	°F	Building	°F
Churches	65-68	Offices	68-72
Factories	50-60	Residences	72-75
Garages	50-55	Restaurants	68-70
Hospitals	70-80	Schools	65-72
Hotels	68-72	Theatres	68-72

Customary design temperatures are given in Table 2. Wind velocity and direction used for infiltration calculations is the average velocity measured by the Weather Bureau for the months of December, January, and February.

Table 2. Temperatures and Wind in U.S. Cities

(Data compiled from U. S. Weather Bureau Records)

City	Average Tempera-ture, Oct. 1–May 1, °F	Lowest Tempera-ture Ever Reported, °F	Design Tempera-ture Usually Assumed, °F	Average Wind Velocity, Dec., Jan., Feb., mph	Direction of Prevail-ing Wind, Dec., Jan., Feb.
Atlanta	51.5	−8	+10	11.4	NW
Boston	38.1	−18	0	11.2	W
Chicago	36.4	−23	−10	12.5	W
Cleveland	37.2	−17	−5	15.0	SW
Denver	38.9	−29	−15	7.5	S
Detroit	35.8	−24	−10	12.7	SW
Minneapolis	29.4	−34	−20	11.3	NW
New York	40.7	−14	0	16.7	NW
Philadelphia	42.7	−11	0	11.0	NW
San Francisco	54.2	27	+30	7.5	N
St. Louis	43.6	−22	−5	11.7	S
Washington, D. C.	43.4	−15	0	7.9	NW

Moisture content and heat capacity of soil affect ground temperature at various depths, but for heat loss determinations it can be assumed that the ground temperature under an excavated basement floor will average around 50 F in most localities. Ground temperature surrounding a basement wall below grade and below a concrete slab poured on the ground may be assumed as 32 F.

Frequently it happens that some rooms in a building are to be unheated or that some rooms are under unheated attics or over unheated crawl spaces. A common method of handling this condition is to assign an arbitrary temperature to the unheated space, such as the mean of the inside and outside design temperatures, and proceed in the same manner as for boundaries exposed to outside air. However, if the boundary surfaces are insulated, the previously suggested procedure may lead to incorrect results; it is then preferable to use the data listed in Table 3.

Table 3. Temperatures of Unheated Spaces

Value of U for Partition or Ceiling	Temperature of Unheated Space to Be Assumed, °F — Design Outside Temperature, °F			
	+10	0	−10	−20
0.6	41	35	30	25
0.5	38	32	26	21
0.4	34	28	22	15
0.3	30	23	16	9
0.2	25	17	10	2
0.1	18	10	1	−8

In a high ceilinged room the temperature at the upper levels may be above the breathing-line temperature, resulting in a greater heat loss for those portions of the room. One satisfactory rule for computing the air temperature at the ceilings of rooms as high as 15 to 18 ft is to add 1% per ft of height above the breathing line (5-ft) level.

THERMAL PROPERTIES BUILDING MATERIALS. Walls, floors, or roofs which separate heated spaces from unheated spaces seldom consist of a single homogeneous material. Usually they are constructed of several materials, each with its characteristic resistance to the flow of heat.

A device known as a "hot box" (see Ref. 1) has been developed to measure the heat flow through sections of *built-up walls* of various types. However, this test procedure is very expensive to construct for separate tests on the variety of walls that the architect or engineer may devise. A simpler device known as a "hot plate" can be used economically for measuring the conductivity of the various materials which constitute a built-up wall section. In Table 4 are the conductivities of materials commonly used in building construction. Before the equation is developed for computing the unit heat flow through a wall section, commonly used heat transfer terms must be defined.

HEAT TRANSFER TERMS. Conductivity (symbol, k) refers to heat transfer by conduction through homogeneous materials. It is expressed as the rate of heat flow per unit area, per unit time, per degree temperature difference from surface to surface *for unit thickness.*

Conductance (symbol, C) refers to heat transfer by conduction through nonhomogeneous materials. It is expressed as the rate of heat flow per unit area, per unit time, per degree temperature difference from surface to surface for the thickness of the material *as it is used commercially.*

Surface conductance (symbol, f) refers to heat transfer to or from a surface; it includes heat transferred by radiation and convection. It is expressed as the rate of heat flow per unit area, per unit time, per degree temperature difference between surface and fluid.

Overall coefficient of heat transmission (symbol, U) in computing heat losses refers to heat transfer from air on one side of a wall to air on the other. (For floors in ground, it refers to heat flow from air to the ground.) It is expressed as the rate of heat transfer per unit area, per unit time, per degree temperature difference between air on one side and air on the other.

Thermal resistance (symbol, R), as the name implies, refers to the resistance offered to the flow of heat. It is the reciprocal of conductivity, conductance, etc.

HEAT FLOW THROUGH WALLS. The various materials which comprise the construction of wall, floor, or ceiling offer resistance to heat flow. Surfaces in contact with air also offer resistance. If surface areas are large, such as building walls, it is both convenient and safe to assume that all heat flow is perpendicular to the wall surface. For

Table 4. Conductivities *k* and Conductances *C* of Building and Insulating Materials

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These constants are expressed in Btu per hr per sq ft per °F temperature difference. Conductivities *k* are per inch thickness, and conductances *C* are for thickness or construction stated, *not* per inch thickness.

Material	Description	Density, lb per cu ft	Mean Temp., °F	Conductivity or Conductance		Resistance	Authority
				k	*C*	(1/*k*) or (1/*C*)	
Building boards (non-insulating)	Compressed cement and asbestos sheets	123	86	2.70	.	0.37	1
	Corrugated asbestos board	20.4	110	0.48	...	2.08	2
	Pressed asbestos millboard	60.5	86	0.84	.	1.19	3
	Gypsum board—gypsum between layers of heavy paper	62.8	70	1.41	...	0.71	3
	3/8-in. gypsum board	. .		.	3.73	0.27	
	1/2-in. gypsum board	.		. . .	2.82	0.35	
	1/2-in. gypsum board	53.5	90	. .	2.60	0.38	1
Frame construction combinations	1-in. fir sheathing and building paper	. . .	30	......	0.86	1.16	4
	1-in. fir sheathing, building paper, and yellow pine lap siding	....	20	....	0.50	2.00	4
	1-in. fir sheathing, building paper, and stucco	.. .	20	. ....	0.82	1.22	4
	Pine lap siding and building paper, siding 4-in. wide	. ...	16	...	0.85	1.18	4
	Yellow pine lap siding	. .	.. ...	. .....	1.28	0.78	4
Masonry materials:							
Brick	Damp or wet	.... ..	.. .	5.0 †	. ...	0.20	2
	Common yellow clay brick *	. ... .	.... ..	4.8	. .	0.21	4
	One tier yellow common clay brick, one tier face brick, approx. 8 in. thick *	... .	. . .	.. . ..	0.77	1.30	4
Clay tile, hollow	2-in. tile, 1/2-in. plaster both sides	120.0	110		1.00	1.00	2
	4-in. tile, 1/2-in. plaster both sides	127.0	100	.... .	0.60	1.67	2
	6-in. tile, 1/2-in. plaster both sides	124.3	105	. .	0.47	2.13	2
	8-in. tile, average of 8 types (walls 59, 63, 64, 66, 67, 90, 91, 92) *	....	....	. .	0.52	1.92	4
	12-in. clay tile wall: 8 in. × 5 in. × 12 in. and 4 in. × 5 in. × 12 in.*	. ... ..	... .	. . ...	0.26	3.84	4
Concrete	Sand and gravel aggregate, various ages and mixes ‡	.. ....	. .	11.35 to 16.36		0.09 to 0.06	5
	Sand and gravel aggregate	142	75	12.6	.	0.08	4
	Limestone aggregate	132	75	10.8	..	0.09	4
	Cinder aggregate	97	75	4.9	...	0.22	4
	Steam-treated limestone slag aggregate *	74.6	75	2.27	. .	0.44	4
	Pumice (mined in California) aggregate *	65.0	75	2.42	.....	0.41	4
	Expanded burned-clay aggregate *	59.9	75	2.28	...	0.44	4
	Burned-clay aggregate *	67.1	75	2.86	.....	0.35	4
	Blast-furnace slag aggregate	76.0	70	1.6	...	0.63	3
	Expanded vermiculite aggregate	20	90	0.68	.....	1.47	3
	Expanded vermiculite aggregate	26.7	90	0.76	.....	1.32	3
	Expanded vermiculite aggregate	35	90	0.86	.....	1.16	3
	Expanded vermiculite aggregate	50	90	1.10	.. ...	0.91	3
	Concrete plank	76	75	2.5	.. ..	0.40	3
	Cellular concrete	40.0	75	1.06	.. ...	0.94	3
	Cellular concrete	50.0	75	1.44	... .	0.69	3
	Cellular concrete	60.0	75	1.80	......	0.56	3
	Cellular concrete	70.0	75	2.18	. . .	0.46	3

See notes, p. 12-07.
See also Heat Insulation, p. 3-34.

Table 4. Conductivities k and Conductances C of Building and Insulating Material
Continued

Material	Description	Density, lb per cu ft	Mean Temp., °F	Conductivity or Conductance k	Conductivity or Conductance C	Resistance $(1/k)$ or $(1/C)$	Authority
Masonry materials—*Continued*							
8 in. concrete blocks 8 x 8 x 16 3-oval core concrete blocks	8-in. three-oval core, sand and gravel aggregate *	126.4	40	.	0.90	1.11	4
	8-in. three-oval core, crushed limestone aggregate *	134.3	40	. . .	0.86	1.16	4
	8-in. three-oval core, cinder aggregate *	86.2	40	.	0.58	1.73	4
	8-in. three-oval core, burned-clay aggregate *	67.7	40		0.50	2.00	4
	8-in. three-oval core, expanded blast-furnace slag aggregate *	. .	40	. .	0.49	2.04	4
12 in. concrete blocks 8 x 12 x 16 3-oval core concrete blocks	12-in. three-oval core, sand and gravel aggregate *	124.9	40	.	0.78	1.28	4
	12-in. three-oval core, cinder aggregate *	86.2	40		0.53	1.88	4
	12-in. three-oval core, burned-clay aggregate *	76.7	40	.	0.47	2.13	4
Gypsum	3-in. solid gypsum partition tile *	. .	.	2.41		0.42	4
	3-in. three-cell gypsum partition tile *			. .	0.74	1.35	4
	4-in. three-cell gypsum partition tile *		.	. .	0.60	1.67	4
	87 1/2 per cent gypsum, 12 1/2 per cent wood chips	51.2	74	1.66		0.60	4
	Gypsum plaster			3.30		0.30	
Plastering materials	Gypsum plaster, 3/8 in. thick	.	73		8.80	0.11	4
	Cement plaster	.	.	8.00	. .	0.13	2
	Wood lath and plaster, total thickness 3/4 in.	. .	70	.	2.50	0.40	4
	Gypsum plaster and expanded vermiculite, 4 to 1 mix	39.9	75	0.85	. .	1.18	3
	Insulating plaster 0.9 in. thick applied to 3/8-in. gypsum board	54.0	75	. .	1.07	0.93	3
Roofing	Asbestos shingles	65.0	75	.	6.0	0.17	3
	Asphalt, composition or prepared	70.0	75		6.5	0.15	3
	Asphalt shingles	70.0	75	. .	6.5	0.15	3
	Built-up roofing, bitumen or felt, gravel or slag surfaced §	. . .	. .	1.33	. .	0.75	2
	Slate	. .	. .	10.00		0.10	
	Wood shingles	. . . .	. . . .	.	1.28	0.78	
Woods	Balsa	20.0	90	0.58	.	1.72	1
	Balsa	8.8	90	0.38	. .	2.63	1
	Balsa	7.3	90	0.33	. . . .	3.03	1
	California redwood, 0% moisture *	28.0	75	0.70	. .	1.43	4
	Cypress	28.7	86	0.67	. . .	1.49	1
	Douglas fir, 0% moisture *	34.0	75	0.67	. . .	1.49	4
	Eastern hemlock, 0% moisture *	30.0	75	0.76	. . . . .	1.32	4
	Longleaf yellow pine, 0% moisture *	40.0	75	0.86	. . . .	1.16	4
	Mahogany	34.3	86	0.90	. . . . . .	1.11	1
	Hard maple, 0% moisture *	46.0	75	1.05	. . . . .	0.95	4
	Maple	44.3	86	1.10	. . . . .	0.91	1
	Maple, across grain	40.0	75	1.20	. .	0.83	3
	Norway pine, 0% moisture *	32.0	75	0.74	. . . . . .	1.35	4
	Red cypress, 0% moisture *	32.0	75	0.79	. . . . .	1.27	4
	Red oak, 0% moisture *	48.0	75	1.18	. . . . . .	0.85	4
	Shortleaf yellow pine, 0% moisture *	36.0	75	0.91	. . . . . .	1.10	4
	Soft elm, 0% moisture *	34.0	75	0.88	. . . . .	1.14	4
	Soft maple, 0% moisture *	42.0	75	0.95		1.05	4

(Table continued on p. 12-06)

Table 4. Conductivities *k* and Conductances *C* of Building and Insulating Materials—*Continued*

Material	Description	Density, lb per cu ft	Mean Temp., °F	Conductivity or Conductance k	Conductivity or Conductance C	Resistance $(1/k)$ or $(1/C)$	Authority
Woods—*Continued*							
	Sugar pine, 0% moisture *	28.0	75	0.64	....	1.56	4
	Virginia pine	34.3	86	0.96	...	1.04	1
	West coast hemlock, 0% moisture *	30.0	75	0.79	. .	1.27	4
	White pine	31.2	86	0.78		1.28	1
	Yellow pine	.		1.00		1.00	3
	Sawdust, various	12.0	90	0.41	..	2.44	1
	Shavings, various from planer	8.8	90	0.41		2.44	1
	Shavings, from maple beech and birch (coarse)	13.2	90	0.36	.	2.78	1
Insulating materials Blanket and batt insulations							
	Chemically treated wood fibers held between layers of strong paper	3.62	70	0.25		4.00	3
	Eel grass between strong paper	4.60	90	0.26		3.85	1
	Eel grass between strong paper	3.40	90	0.25		4.00	1
	Flax fibers between strong paper	4.90	90	0.28		3.57	1
	Chemically treated hog hair between kraft paper	5.76	71	0.26		3.85	3
	Chemically treated hog hair between kraft paper and asbestos paper	7.70	71	0.28		3.57	3
	Hair felt between layers of paper	11.00	75	0.25		4.00	3
	Kapok between burlap or paper	1.00	90	0.24		4.17	1
	Stitched and creped expanding fibrous blanket	1.50	70	0.27		3.70	3
	Paper and asbestos fiber with emulsified asphalt binder	4.2	94	0.28	.	3.57	1
	Cotton insulating batt	0.875	72	0.24		4.17	3
	Felted cattle hair	13.00	90	0.26	. .	3.84	1
	Felted cattle hair	11.00	90	0.26		3.84	1
	Felted hair and asbestos	7.80	90	0.28	.	3.57	1
	Ground paper between two layers, each 3/8 in. thick, made up of two layers of kraft paper (sample 3/4 in. thick)	12.1	75		0.40	2.50	4
	3-in. Mineral-wool batts, barrier lapped on warm side; horizontal position ‖	3.67	...	0.30	.	3.33	4
	3-in. Mineral-wool batts, barrier laid on warm side, horizontal position ‖	2.24	..	0.26		3.84	4
	3-in. Mineral-wool batts, barrier laid on warm side; vertical position ‖	2.24	..	0.25	.	4.00	4
	4-in. Mineral-wool batts, barrier lapped on warm side; horizontal position ‖	3.0	...	0.31	.	3.22	4
	4-in. Mineral-wool batts, barrier lapped on warm side; vertical position ‖	3.0	..	0.33	.	3.03	4
	4-in. Mineral-wool batts, no barriers; horizontal position ‖	1.77	....	0.30	...	3.33	4
Insulating board	Made from sugar-cane fiber	13.5	70	0.33	.	3.03	3
	Made from cornstalks	15.00	71	0.33	.	3.03	3
	Made from exploded wood fibers	17.90	78	0.32	..	3.12	4
	Made from hard wood fibers	15.20	70	0.32	..	3.12	3
	Made from wood fiber	15.90	72	0.33	....	3.03	3
	Made from wood fiber	15.00	70	0.33	....	3.03	3
	Made from wood fiber	......	52	0.33	.....	3.03	6
	Made from wood fiber	8.50	72	0.29	..	3.45	3
	Made from wood fiber	15.20	.....	0.33	.....	3.03	3
	Made from wood fiber	16.90	90	0.34	....	2.94	1
	Made from licorice root	16.1	81	0.34	.	2.94	3

Table 4. Conductivities *k* and Conductances *C* of Building and Insulating Materials— *Continued*

Material	Description	Density, lb per cu ft	Mean Temp., °F	Conductivity or Conductance *k*	*C*	Resistance (1/*k*) or (1/*C*)	Authority
Insulating materials—*Continued*							
Insulating board—*Continued*	1/2-in. insulating boards without special finish ¶ (11 samples)	16.5 to 21.8	90	0.33 to 0.40	. .	3.03 to 2.40	1
	1-in. insulating board *	13.2	.	0.34	. . .	2.94	4
Loose-fill type	Made from ceiba fibers	1.90	75	0.23	.	4.35	3
	Made from ceiba fibers	1.60	75	0.24	.	4.17	3
	Fibrous material made from dolomite and silica	1.50	75	0.27	.	3.70	3
	Fibrous material made from slag	9.40	103	0.27	..	3.70	1
	Redwood bark	3 00	90	0.31	. .	3.22	1
	Redwood bark	5.00	75	0.26		3.84	3
	Glass wool fibers 0.0003 to 0.006 in. in diameter	1.50	75	0.27	.	3.70	3
	Granular insulation made from combined silicate of lime and alumina	4.20	72	0.24		4.17	3
	Expanded vermiculite			0.48	.	2.08	1
	Expanded vermiculite, particle size −3 + 14	6.2		0.32	.	3.12	3
	Regranulated cork, about 3/16-in. particles	8.10	90	0 31	..	3.22	1
	Hand-applied granular mineral wool 2 to 6 in. thick; horizontal position.** No covering	6 05 to 7.13		0.30 to 0.33	. . .	3.33 to 3.03	4
	4-in. machine-blown granular mineral wool, horizontal position ** No covering	5.74		0.30	....	3.33	4
	Rock wool	10.0	90	0.27	. .	3.70	1
Slab insulations	Corkboard, no added binder	14.0	90	0.34	.	2.94	1
	Corkboard, no added binder	10.6	90	0.30		3.33	1
	Corkboard, no added binder	7.0	90	0.27		3.70	1
	Corkboard, no added binder	5.4	90	0.25	.	4.00	1
	Corkboard *	8 7		0.29		3 45	4
	Corkboard, asphaltic binder	14.5	90	0.32	...	3.12	1
	Chemically treated hog hair with film of asphalt	10.0	75	0.28	.	3.57	3
	Sugar-cane fiber insulation blocks encased in asphalt membrane	13.8	70	0.30	....	3.33	3
	Made from 85% magnesia and 15% asbestos	19.3	86	0.51	...	1.96	1
	Made from shredded wood and cement	24.2	72	0.46	..	2.17	3
	Made from shredded wood and cement *	29.8	..	0.77		1.30	4

Authorities:
1. National Bureau of Standards, tests based on samples submitted by manufacturers.
2. A. C. Willard, L. C. Lichty, and L. A. Harding, tests conducted at the University of Illinois.
3. J. C. Peebles, tests conducted at Armour Institute of Technology, based on samples submitted by manufacturers.
4. F. B. Rowley and others, tests conducted at the University of Minnesota.
5. American Society of Heating and Ventilating Engineers, Research Laboratory.
6. E. A. Allcut, tests conducted at the University of Toronto.

* F. B. Rowley and A. B. Algren, Thermal conductivity of building materials, *Univ. Minn. Eng. Exp. Sta. Bull.* 12.

† Harding and Willard, *Heating, Ventilating and Air Conditioning,* revised ed., John Wiley and Sons, 1932.

‡ F. C. Houghten and Carl Gutberlet, Conductivity of concrete, *Trans. Am. Soc. Heating, Ventilating Engrs.,* 1932.

§ Roofing 0.15 in. thick (1.34 lb per sq ft), covered with gravel (0.83 lb per sq ft), combined thickness assumed 0.25 in.

‖ F. B. Rowley and C. E. Lund, Heat transmission through insulation as affected by orientation of walls, *Trans. Am. Soc. Heating, Ventilating Engrs.,* 1943.

¶ National Bureau of Standards, Building Materials and Structures, *Report* 13.

** G. B. Wilkes and L. R. Vianey, The effect of convection in ceiling insulation, *Trans. Am. Soc. Heating, Ventilating Engrs.,* 1943

Table 5. Conductivities *k* and Conductances *C* Used in Calculating Heat-loss Coefficients *U*

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These constants are expressed in Btu per hr per sq ft per °F temperature difference. Conductivities *k* are per inch thickness and conductances *C* are for thickness or construction stated, *not* per inch thickness.

Material	Description	Conductivity or Conductance		Resistance
		k	*C*	(1/*k*) or (1/*C*)
Air Spaces				
Bounded by ordinary materials	Vertical,* 3/4 in. or more in width	...	1.10	0.91
Bounded by aluminum foil	Vertical,* 3/4 in. or more in width	...	0.46	2.17
Exterior Finishes (frame walls)				
Brick veneer	4 in. thick (nominal)	...	...	0.44
Stucco (1 in.)	...	12.50	...	0.08
Wood shingles	...	...	1.28	0.78
Yellow pine lap siding	...	...	1.28	0.78
Insulating Materials				
Aluminum foil	See air spaces	...	...	...
Batts	Enclosed both sides	0.27	...	3.70
Blankets	Made from mineral or vegetable fibers or animal hair	0.27		3.70
Corkboard	Pure, no added binder	0.30	...	3.33
Insulating board	...	0.33	...	3.03
Mineral wool	...	0.27	...	3.70
Vermiculite	...	0.48	...	2.08
Interior Finishes				
Composition wallboard	3/16 to 3/8 in. thick	0.50	...	2.00
Gypsum plaster	...	3.30	...	0.30
Gypsum board (3/8 in.)	Plain or decorated	...	3.70	0.27
Gypsum lath (3/8 in.) and plaster	Plaster thickness assumed 1/2 in.	...	2.4	0.42
Insulating board (1/2 in.)	Plain or decorated	...	...	1.52
Insulating-board lath (1/2 in.) and plaster	Plaster thickness assumed 1/2 in.	...	...	1.67
Insulating-board lath (1 in.) and plaster	Plaster thickness assumed 1/2 in.	...	...	3.18
Metal lath and plaster	Plaster thickness assumed 3/4 in.	...	4.40	0.23
Plywood (3/8 in.)	Plain or decorated	...	...	0.47
Wood lath and plaster	...	...	2.50	0.40
Masonry Materials				
Brick	Adobe	3.56	...	0.28
Brick	Common	5.00	...	0.20
Brick	Face	9.20	...	0.11
Cement mortar	...	12.00	...	0.08
3-in. clay tile (hollow)	...	...	1.28	0.78
4-in. clay tile (hollow)	...	...	1.00	1.00
6-in. clay tile (hollow)	...	...	0.64	1.57
8-in. clay tile (hollow)	...	...	0.60	1.67
10-in. clay tile (hollow)	...	...	0.58	1.72
12-in. clay tile (hollow)	...	...	0.40	2.50
16-in. clay tile (hollow)	...	...	0.31	3.23
Concrete	Lightweight aggregate †	2.50	...	0.40
Concrete	Sand and gravel aggregate	12.00	...	0.08
3-in. concrete blocks	Hollow, cinder aggregate	...	1.28	0.78
4-in. concrete blocks	Hollow, cinder aggregate	...	1.00	1.00
8-in. concrete blocks	Hollow, gravel aggregate	...	1.00	1.00
12-in. concrete blocks	Hollow, gravel aggregate	...	0.80	1.25
8-in. concrete blocks	Hollow, cinder aggregate	...	0.60	1.66
12-in. concrete blocks	Hollow, cinder aggregate	...	0.53	1.88
8-in. concrete blocks	Hollow, lightweight aggregate †	...	0.50	2.00
12-in. concrete blocks	Hollow, lightweight aggregate †	...	0.47	2.13
Gypsum fiber concrete	87 1/2 per cent gypsum and 12 1/2 per cent wood chips	1.66	...	0.60

Table 5. Conductivities k and Conductances C Used in Calculating Heat-loss Coefficients U—Continued

Material	Description	Conductivity or Conductance k	Conductivity or Conductance C	Resistance $(1/k)$ or $(1/C)$
Masonry Materials—*Continued*				
3-in. gypsum tile	Hollow	....	0.61	1.64
4-in. gypsum tile	Hollow		0.46	2.18
Stucco	....	12.50	......	0.08
Tile and terrazzo	For flooring	12.00	......	0.08
Roofing Materials				
Asbestos shingles	....	......	6.00	0.17
Asphalt shingles	....	......	6.50	0.15
Built-up roofing	Assumed thickness 3/8 in.	.....	3.53	0.28
Heavy roll roofing	....	...	6.50	0.15
Slate	....	10.00	.....	0.10
Wood shingles	....	......	1.28	0.78
Sheathing				
Gypsum (1/2 in.)	....	......	2.82	0.35
Insulating board (25/32 in.)	....	......	...	2.37
Plywood (5/16 in.)	....	......	......	0.39
Fir or yellow pine (1 in.)	Actual thickness 25/32 in.	......	..	0.98
Fir, plus building paper	Actual thickness 25/32 in.	......	0.86	1.16
Surfaces				
Still air	Ordinary nonreflective materials, vertical	......	1.65	0.61
15 mph wind velocity	Ordinary nonreflective materials, vertical	......	6.00	0.17
Woods				
Fir sheathing (1 in.) building paper and yellow pine lap siding	....	......	0.50	2.00
Maple or oak	....	1.15	.....	0.87
Yellow pine or fir	..	0.80	.....	1.25

* Conductance values for horizontal air spaces depend on whether the heat flow is upward or downward, but in most cases it is sufficiently accurate to use the same values for horizontal as for vertical air spaces.

† Expanded slag, burned clay, or pumice.

this assumption, essentially true except at corners, the various resistances in the wall are in series. Therefore, reasoning by analogy from Kirchhoff's law for series electrical circuits, the total resistance for the wall is equal to the sum of the individual resistances or

$$R_U = R_1 + R_2 + R_3 + R_4 + \cdots R_N \tag{1}$$

Overall Coefficients, U. Usually it is not necessary in computing heat loss to make use of eq. 1 because the overall transmission coefficients (U) have been calculated for many of the common constructions and are given in the *ASHVE Guide*. Some of the data used in calculation are given in Table 5. However, if the type of construction is different, it is important to be able to calculate the value of U from the various components of the wall section.

Example. To apply eq. 1, an uninsulated frame wall with lap siding is used as an example. Beginning on the inside, the resistances to heat flow are the inside surface, plaster and metal lath, an air space, sheathing and siding, and the outside surface.

Plaster on metal lath is considered a nonhomogeneous material; hence its conductance has been measured, $C = 4.4$, from Table 5. A conductance has also been assigned to air spaces as found in building construction, $C = 1.10$, from Table 5. Wood used for sheathing may be treated as a homogeneous material; but because it is used in a standard thickness of 25/32 inch throughout the United States and because siding is also of standard thickness, the heat transfer rate for the combination has been expressed as a conductance, $C = 0.5$, from Table 5.

The essential difference between inside and outside surfaces is that the outside surface is subjected to higher wind velocities, which increase rate of heat flow. It is standard practice to assume still air inside, $f_i = 1.65$, and 15 mph wind outside, $f_o = 6.00$, for determining surface conductances.

Thermal resistances are considered to be reciprocal functions of conductivities or conductances. Thus for the example the resistances are

Inside surface	$R_1 = \frac{1}{f_i}$
Plaster and lath	$R_2 = \frac{1}{C_1}$
Air space	$R_3 = \frac{1}{a}$
Sheathing and siding	$R_4 = \frac{1}{C_2}$
Outside surface	$R_5 = \frac{1}{f_o}$

Applying eq. 1, the total resistance

$$R = \frac{1}{U} = \frac{1}{f_i} + \frac{1}{C_1} + \frac{1}{a} + \frac{1}{C_2} + \frac{1}{f_o} \tag{2}$$

or, solving for U,

$$U = \frac{1}{\frac{1}{f_i} + \frac{1}{C_1} + \frac{1}{a} + \frac{1}{C_2} + \frac{1}{f_o}} \tag{3}$$

If typical values are used from Table 5, the solution becomes

$$U = \frac{1}{\frac{1}{1.65} + \frac{1}{4.4} + \frac{1}{1.1} + \frac{1}{0.5} + \frac{1}{6.00}} = 0.26 \text{ Btu/hr/sq ft/°F}$$

Equation 1 may be applied similarly to other constructions, by inserting a factor for each separate resistance. Tests by Houghten (Ref. 2) have indicated that a coefficient of $U = 0.10$ may be used for all types of concrete floors on the ground and for basement walls below grade, with or without insulation.

INFILTRATION. Two recognized methods are available for calculating infiltration losses. One is the crackage method, and the other the air-change procedure. The first method requires that the width of crack (at windows and doors) and the length of crack in each heated space be determined. An air-leakage rate is selected for the type of construction from Table 6. The product of length of crack and leakage rate is the rate of infiltration for the space.

Table 6. Infiltration Due to Windows of Various Types

(Reprinted by permission from *Heating, Ventilating, Air Conditioning Guide*, 1948, Chapter 8)

Type of Window	Infiltration, cubic feet of air per hour per foot of crack: Wind Velocity, 5 mph	Wind Velocity, 15 mph	Wind Velocity, 30 mph
Double-hung, wood-sash windows:			
Per foot of crack around sash and meeting rail of 1/16 in. width, approx. Window unlocked, without weather strips	6.6	39.3	103.7
With weather strips	4.3	23.6	63.4
Double-hung, metal-sash windows:			
Per foot of crack around sash and meeting rail. No weather strips, window locked	20	70	154
No weather strips, window unlocked	20	74	170
Weather strips, window unlocked	6	32	76
Rolled-section steel windows:			
Per foot of crack around ventilating sash			
Industrial type, 1/16 in. crack	52	176	372
Architectural type, 3/64 in. crack	20	88	208
Residential type, 1/32 in. crack	14	52	128
Heavy casement type, 1/32 in. crack	8	38	96

Crackage Method. As air does not leak into a building on all sides at the same time, some discrimination is required in determining the total length of crack to be used. The following rules have been developed:

Room with	Use
1 exposed wall	Total length of crack
2 exposed walls	Wall with greater length crack
3 exposed walls	Crack on 2 walls
4 exposed walls	Crack on 2 walls

The crackage method may be applied without great difficulty to existing buildings. The chief disadvantage is in the application to proposed construction. It is difficult for the designer to determine accurately beforehand the width of crack, nor is there any assurance that mechanics will install windows and doors and not exceed the assumed width of crack.

The air-change method assumes that the volume of air in the room or building is renewed a certain number of times per hour. The number of air changes usually assumed is given in Table 7. The heat required to supply the infiltration losses must be sufficient

Table 7. Air Changes for Various Rooms

Kind of Room or Building	Air Changes per Hour
Rooms with windows on one side	1
Rooms with windows on two sides	1 1/2
Rooms with windows on three sides	2
Entrance halls	2
Sun rooms with many windows on three sides	3
Factory buildings	1/2 to 1
Churches, public assembly rooms	1/2 to 3

to warm the air from the temperature of the outside air to that of the room and may be computed according to eq. 4:

$$H_l = Q \times d \times C_p(t - t_o) \tag{4}$$

where H_l = heat loss due to infiltration, Btu per hour; Q = volume air entering building, cubic feet per hour determined either from crackage or air-change method; d = density of air at temperature t, = 0.075 for 70 F air; c_p = specific heat of air at constant pressure = 0.24; t = inside temperature, °F; and t_o = outside temperature, °F.

AUXILIARY HEAT SOURCES. The heat supplied by persons, lights, motors, and machinery should generally be considered in a heat-loss computation. Heat emitted from electric lights is equal to the wattage of the lamps × 3.413 Btu per hour. Heat released from adults at rest is 400 Btu per hour, but this amount may be increased to 1200 Btu per hour under some working conditions.

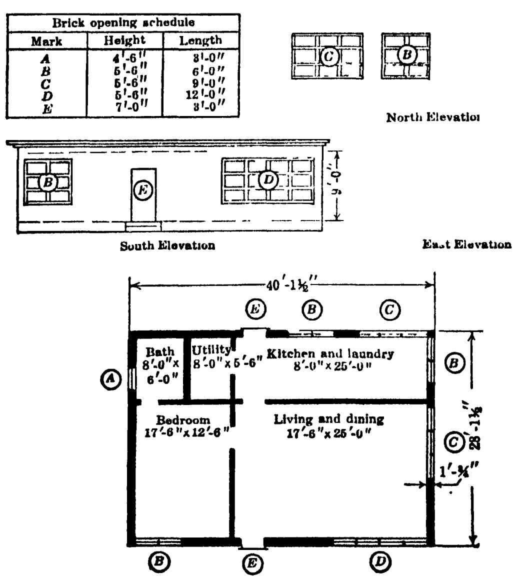


FIG. 1. Building dimensions.

HEAT-LOSS CALCULATIONS. The total heat loss from a room is obtained by taking a summation of the losses for each wall, roof or ceiling, and glass obtained by multiplying the area of each surface by the proper overall coefficient of heat transmission and by the design temperature difference. Add to the above, the infiltration losses and deduct any allowances for auxiliary heat sources. The net result will be the total amount of heat required.

The losses from the separate spaces are used to determine radiator sizes or, for a warm-air system, the quantity of air required for heating. The total for the building, together with other loads such as piping loss, water heating, ventilation, and warming-up allowance, is used to determine the size of boiler or furnace.

EXAMPLE. Calculate the heat loss for the building shown in Fig. 1, which is constructed of 4-in. brick and 8-in. hollow tile with furred metal lath and plaster. Floor is a 4-in. concrete slab on the ground with asphalt tile finish. The roof is 2 3/8-in. thick gypsum plank, with 1-in. rigid insulation and built-up roofing and a suspended plaster ceiling in metal lath. Windows are single glass, and both windows and doors are weather stripped. Assume that the inside temperature is 72 F and the building is located where the outside design temperature is −5 F and the average wind velocity 15 mph.

Solution. The overall transmission coefficient for the exposed walls, by the method of the example, p. 12-09, is found to be $U = 0.25$; roof, $U = 0.15$; windows and doors, $U = 1.13$; and floor, $U = 0.10$. From Table 6 the infiltration rate is found to be 52 cu ft per hr per lineal foot crack for casement windows with 1/32 in. crack.

Room	Exposure	Areas, sq ft			Temp. Diff.	U		Infiltration		Heat Losses, Btu per hour			
		Gross	Glass or Door	Net		Wall Roof Floor	Glass Door	Lin. ft crack	Cu ft/hr	Wall Roof Floor	Glass	Infiltration	Total
Living	Wall	383	137	246	77	0.25	1.13	88	52	4,720	12,000	6,350	
	Floor	437		437	40	0.10				175			
	Roof	437		437	80 *	0.15				5,250			28,495
Bed	Wall	271	33	238	77	0.25	1.13	34	52	4,600	2,870	2,450	
	Floor	219		219	40	0.10				84			
	Roof	219		219	80	0.15				2,640			12,644
Bath	Wall	126	14	112	77	0.25	1.13	18	52	2,160	1,220	1,300	
	Floor	48		48	40	0.10				19			
	Roof	48		48	80	0.15				576			5,275
Kitchen	Wall	297	137	160	77	0.25	1.13	105	52	3,080	12,000	7,550	
	Floor	200		200	40	0.10				80			
	Roof	200		200	80	0.15				2,400			25,110
Totals										25,784	28,090	17,650	71,524

* Temperature at ceiling $= 72[1.00 + (4 \times 0.01)] = 75 - (-5) = 80$ F.

2. HEATING SYSTEMS

HEATING SYSTEM is generally understood to mean the kind of heating medium and the type of apparatus used to release or transfer heat from the medium to the enclosure to be warmed.

All heating systems comprise at least two principal parts, or necessary apparatus to effect the desired result. (*a*) *Heat generator,* where the heat released from burning fuel is transferred to the heating medium. This apparatus is called the boiler in steam and hot-water systems, or simply a furnace in warm-air furnace systems. (*b*) *Distribution system* used to convey the medium from the generator to the heat emitting apparatus; for example, steam or hot-water piping and air ducts. (*c*) *Heat emitting* or releasing apparatus to which the distributing system is connected, called the radiator. In a simple warm-air furnace system, only *a* and *b* are required; in a steam or hot-water system, all three parts are essential.

Radiator commonly means a heat-emitting unit located within the enclosure to be warmed. If no provision is made for introducing outside air for ventilation, to be circulated over the radiator, it is called a direct radiator. Radiators often are installed in recesses in the walls of the building. The most common type of direct radiator is made of hollow cast-iron sections joined by malleable push or threaded nipples. Practically, only two types of direct cast-iron radiators are being made in recent years; (*a*) tubular (as used in many residences); (*b*) wall type (see Fig. 2). Direct radiators built of steel or iron pipe and standard fittings are still employed to some extent for industrial work (see Fig. 3 for a typical design).

Convectors of the built-in or concealed design (see Fig. 4) and of the cabinet type are constructed of brass, copper, or cast iron, with a large percentage of thin fin surface. The heating element which is shallow, is placed low in the enclosure to produce maximum chimney effect.

Radiant baseboard is a hollow cast-iron radiating element designed to take the place of a conventional wood baseboard. The type shown in Fig. 5 is available in 1- and 2-ft lengths, 7 in. high and 1 3/4 in. thick. The several sections are joined together by push nipples and short tie bolts. This design has a heat output of 275 Btu per hr per lineal foot of section with water at 200 F. This heat emission value may be increased 55% by using a variation of the above design, consisting of a hollow cast-iron section with fins cast integrally on the wall side of the unit. It is arranged much like a convector with air openings on the top and bottom of each section.

Panel heating is an arrangement whereby a fluid such as hot water, steam, air, or electricity is circulated through distribution units embedded in the building construction. A complete description of this system, together with a design procedure, is outlined in this section in the chapter Panel Heating.

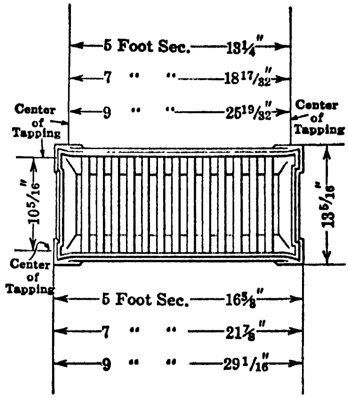
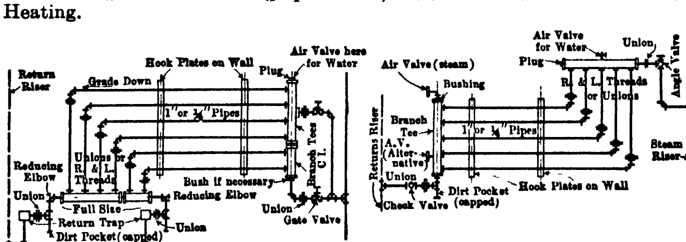


FIG. 2. Cast-iron radiators, wall type.



RADIATOR FOR 2-PIPE VACUUM SYSTEM, DIVIDED SURFACE

RADIATOR FOR 2-PIPE LOW PRESSURE STEAM SYSTEM

FIG. 3. Pipe coil radiators.

A UNIT HEATER is a combination of radiator and fan located within a common enclosure or casing and placed within or adjacent to the space to be warmed. The unit heater largely used for industrial heating is made in two general types: (*a*) floor type; (*b*) ceiling or hung type. The air passed through the casing is warmed by convection. Comparatively little heat is emitted by radiation.

Air for industrial installations almost invariably

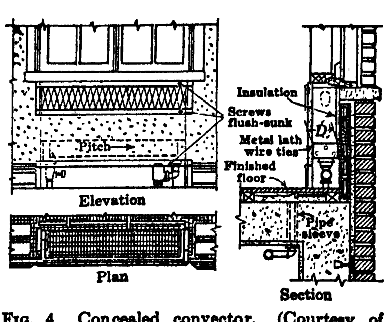


FIG. 4. Concealed convector. (Courtesy of Modine Manufacturing Co.)

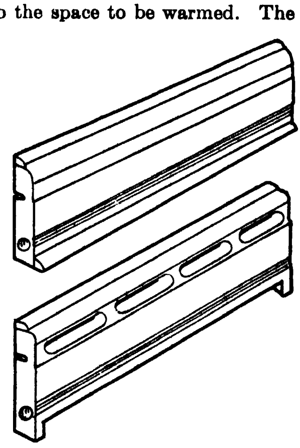


FIG. 5. Cast-iron baseboard radiator units. (Courtesy of Burnham Corp.)

is recirculated, with no provision for introduction of outside air to be passed through the unit. Ordinarily, sufficient air for adequate ventilation is provided by infiltration in single-story industrial buildings having monitors or saw-tooth roofs. Unit-type heaters are now, in practice, the standard heating equipment for industrial establishments. Steam ordinarily is used as the heating medium, although hot water occasionally is used. The circulation of air at a velocity of 1000 ft per min or more over a heating surface greatly increases the convection coefficient of the heat-emitting surface. This results in a large reduction in installed heating surface, as compared with direct radiation, to obtain the same heating results. The ratio is often as low as 1 to 5.

UNIT VENTILATOR is defined as a floor-type unit heater, designed to circulate all or part outdoor air, with or without recirculation. It is designed primarily for school and office buildings, where a positive ventilation is required. (See Fig. 32.) Provision often is made to introduce moisture into the air circulated through the ventilator, and thermostatic control ordinarily is provided. These units may supply both heating and ventilating requirements or ventilation alone in conjunction with direct radiation.

FAN OR BLAST HEATING is understood to mean a combination of heat-emitting surfaces, enclosed by a casing through which air is blown or drawn by a fan. As usually installed, the blast heater is located on the suction side and the duct system is connected to the fan outlet. (See Figs. 26 to 28, inclusive, and Figs. 30 and 31.)

RATING OF RADIATORS AND CONVECTORS: EQUIVALENT DIRECT RADIATION. All types of direct radiators are tested to determine the heat emission, in Btu per hour, of the assembled radiator in still air at 70 F, using dry steam at a temperature of 215 F, corresponding to a pressure of 0.50 psig at the radiator inlet. Manufacturers publish the rating in terms of square feet of *equivalent direct radiation* (edr), 1 sq ft of equivalent direct steam radiation being assumed equal to a heat emission of 240 Btu per hr. No fixed relation exists between the actual measured external heating surface of the many and varied types of radiators, particularly when they are of the extended surface type, and their heat emission; hence the necessity of either publishing radiator ratings in Btu or in terms of edr.

Cast-iron direct radiators, being constructed of sections, the edr per section is stated. (See Tables 8 and 9.) When a number of these sections are assembled, more or less heat

Table 8. Dimensions and Ratings of Small-tube Cast-iron Radiators

Section Spacing—1 3/4 in. Center to Center

Height, in.	Catalog Ratings per Section, sq ft edr			
	Three Tube	Four Tube	Five Tube	Six Tube
14				1.6
19		1.6		2.3
22		1.8	2.1	
25	1.6	2.0	2.4	3.0
32				3.7
	Nominal Width of Sections, in.			
	3 1/4–3 1/2	4 7/16–4 13/16	5 5/8–6 5/16	6 13/16–8

Table 9. Dimensions, Catalog Ratings, and Heat Output of Cast-iron Wall Radiators

Catalog Rating per Section, sq ft	Heat Output per Section, Btu per hr	Equivalent Rating per Section, sq ft, edr	Section Dimensions		
			Height, in.	Width, in.	Thickness, in.
5	1390	5.8	13 5/16–14 1/8	16 1/2 –16 5/8	3
7	1950	8.1	13 5/16–14 1/8	21 7/8 –22 7/8	3
9	2580	10.7	13 5/16–14 1/8	29 1/16–29 1/4	3
7	1730	7.2	21 7/8 –22 7/8	13 5/16–14 1/8	3
9	2310	10.4	29 1/16–29 1/4	13 5/16–14 1/8	3

actually may be emitted, depending on height and number of sections. If the same radiator is to be used in conjunction with a hot-water system, based on supplying the radiator with water at 180 F in still air at 70 F with a 20 degree drop in temperature passing through the radiator, the heat emission is assumed to be 150 Btu per sq ft of edr per hr. The temperatures stated for the steam or water and air surrounding the radiator are known as *standard conditions*. Thus, if the published rating of any type of direct radiator, unit heater, or blast heater is stated as R sq ft of edr, the expected heat emission, when the heat-emitting unit is supplied with steam at a temperature of 215 F, will be $R \times 240$ Btu per hr. The latent heat at this pressure is 967 Btu per lb. The weight of steam condensed per square foot of edr per hour is, therefore, 240/967, or approximately 0.25 lb per hr. The wide variation in convector design makes it desirable to refer to manufacturers' ratings for capacities. However, ratings may be approximated from Table 10, which gives heat

Table 10. Heat Output of Nonferrous Convectors

Convector Height, in.	Convector Depth			
	4 in.	6 in.	8 in.	10 in.
	Heat Output per Inch of Finned Length, sq ft edr			
18	0 52	0.75	0.93	1.01
20	0.63	0.94	1.10	1.24
24	0.79	1.10	1.29	1.43
32	0.86	1.26	1.47	1.68

outputs per inch of finned length for various depths and heights. The finned length may be assumed to be 4 in. less than the cabinet length. Convector ratings are given at standard steam and entering air conditions of 215 F and 65 F, respectively.

AMOUNT OF EDR TO BE INSTALLED. If H is the calculated heat loss, Btu per hour, the square foot R of edr to be installed is $R = H/240$ sq ft for steam at 0.50 psi gage in 70 F air. For a hot-water installation, $R = H/150$ sq ft with initial temperature of water of 180 F, and a 20-degree drop through the radiator.

Conditions Other Than Standard. Cases arise where both steam temperature and temperature of the room air are other than standard. The output of steam radiators and convectors at other than standard conditions may be determined by means of the correction factors given in Table 11. The output of hot water convectors must be determined from manufacturers' catalogs. Correction values in Table 11 are not applicable to hot-water convectors.

Table 11. Correction Factors for Direct Cast-iron Radiators and Convector Heaters

(Reprinted by permission from *Heating, Ventilating and Air Conditioning Guide*, 1948, Chapter 25)

Steam Pressure, Approximate		Steam or Water Temperature, °F	Factors for Direct Cast-iron Radiators							Factors for Convectors						
			Room Temperature, °F							Inlet-air Temperature, °F						
Vacuum in. Hg	Lb abs.		80	75	70	65	60	55	50	80	75	70	65	60	55	50
22.4	3.7	150	2.58	2 36	2.17	2.00	1.86	1.73	1.62	3.14	2.83	2.57	2.35	2.15	1.98	1.84
20.3	4.7	160	2.17	2.00	1.86	1.73	1.62	1.52	1.44	2.57	2.35	2.15	1.98	1.84	1.71	1.59
17.7	6.0	170	1.86	1.73	1.62	1.52	1.44	1.35	1.28	2.15	1.98	1.84	1.71	1.59	1.49	1.40
14.6	7.5	180	1 62	1.52	1.44	1.35	1.28	1 21	1.15	1.84	1.71	1.59	1.49	1.40	1.32	1.24
10.9	9.3	190	1.44	1.35	1 28	1.21	1.15	1.10	1.05	1.59	1.49	1.40	1.32	1.24	1.17	1.11
6.5	11.5	200	1.28	1.21	1.15	1.10	1.05	1 00	0.96	1.40	1.32	1.24	1.17	1.11	1.05	1.00
Psig																
1	15.6	215	1.10	1.05	1.00	0.96	0.92	0 88	0.85	1.17	1.11	1.05	1.00	0.95	0.91	0.87
6	21	230	0.96	0 92	0.88	0.85	0.81	0 78	0.76	1.00	0.95	0.91	0.87	0.83	0.79	0.76
15	30	250	0.81	0.78	0.76	0.73	0.70	0.68	0.66	0.83	0.79	0.76	0.73	0.70	0.68	0.65
27	42	270	0.70	0 68	0.66	0.64	0.62	0 60	0.58	0.70	0.68	0.65	0.63	0.60	0.58	0.56
52	67	300	0.58	0.57	0.55	0.53	0.52	0.51	0.49	0.56	0.54	0.53	0.51	0.49	0.48	0.47

22 i

installation.

Solution. From the values in Table 8, the rated surface of the radiators in edr may be determined as follows:

235 sections × 1.6	376 sq ft
260 sections × 2.1	546
167 sections × 3.0	501
	1423 sq ft

Heat emission at standard conditions = 1423 × 240 = 341,500 Btu per hr

The correction factor for 6 psig steam pressure (230 F) and 60 F room temperature from Table 11 is 0.81. The equivalent radiation, edr = 1423/0.81 = 1760 sq ft, and the heat output at these conditions = 341,500/0.81 = 422,000 Btu per hr.

EXAMPLE. Required the dimensions of a 6-tube, 32-in. high, tubular-type, cast-iron direct radiator to supply 14,400 Btu per hr. Standard conditions: steam 0.50 psig room temperature, 70 F. The edr rating required is 14,400/240 = 60 sq ft. The edr surface for a 6-tube, 32-in. high section is 3.7 (Table 8). The number of sections required is therefore 60/3.7, or (approx.) 17. The length of the assembled radiator will be 17 × 1 3/4, or 30 in., and the width will be 8 in. The actual installed edr is 17 × 3.7 = 63 sq ft. The expected weight of steam to be supplied (or resulting condensate) per hour will be 63 × 0.25 or 15 3/4 lb.

HEAT EMISSION OF DIRECT CAST-IRON RADIATORS. The coefficient of heat emission K of direct radiation in still air, or the Btu emitted per hour by the radiator per measured square foot of external surface per degree difference in temperature between the heating medium and the surrounding air, is found by tests to be a variable. Its value depends on the length, width, number of sections in the radiator, and the temperature difference. The value of K varies from 1.45 to 1.95, the average value being 1.70. The values may be applied to either steam or hot water.

The coefficient K is increased or decreased at the approximate rate of 0.2% per degree of temperature difference above or below the standard temperature difference of 145 F.

Effect on Heat Emission of Location of Direct Radiators. The maximum heat emission is obtained when the radiator is placed in the center of the room. According to Ref. 3, the following values of K were obtained by test: 1.76 with radiator placed in center of room and 1.588 with radiator placed under a window in the outside wall.

The heat emission of direct radiation should be reduced by the following percentages when inclosures are provided (see Ref. 4): radiator set in recess of wall without front grille, 8%; radiator with shelf over top, 10%; radiator with shelf over top and front grille, 20%.

Effect of Painting on Heat Emission. Tests by J. R. Allen indicate that the effect of painting the surface of radiators is to influence the loss of heat from the surface only and depends largely on the radiation factor for the surface in question. The following are the relative transmissions of various surfaces: bare iron, 1.00; aluminum and copper bronze, 0.75; snow-white enamel, 1.01; white-lead paint, 0.987; white-zinc paint, 1.01.

3. BOILER RATINGS AND SELECTION

Ratings for heating boilers are expressed in Btu per hour or equivalent direct radiation. Since there are several different ratings applied to heating boilers, it is essential to know the basis on which the published boiler rating has been established in order to make the correct selection of a boiler for a given load. The different methods of rating heating boilers and the loads involved in these ratings are given below.

NET LOADS. In selection of boilers net loads can be used except where the heating system contains an unusual amount of bare pipe or the nature of the connected load is such that the normal allowance for pipe loss and pickup do not apply. The net load consists of two items: radiation load and hot-water supply load.

Radiation load consists of the total heat output of all the connected radiation (direct, indirect, or forced convection coils). The different types of radiation and the heat outputs of each are given in Article 2. As practically all boilers are now rated on a Btu basis, it is usually unnecessary to convert the radiation load to equivalent square feet of radiation. If there are special loads attached, such as sterilizers, steam tables, and snow-melting equipment, the heat output of these special loads should be included in estimating the radiation load.

Hot-water supply load is the estimated maximum heat required in Btu per hour to heat water for domestic use. Recent tests have shown that the house-heating and water-heating loads are not strictly additive, because a large portion of the so-called water-heating load is actually utilized in supplying heat to the house. Thus, for smaller installations, it is not evident that it is necessary to make any water-heating allowance for the purpose of selecting a boiler. The usual allowance made for pickup and piping losses seems more than adequate to provide for the small additional load normally imposed by heating the domestic hot water. However, if there are more than two bathrooms to be served or if the use of domestic hot water exceeds 75 gal in 24 hr, the following allowance should be made:

Storage-type heater: 120 Btu per gal of storage tank capacity.
Tankless heater: 12,000 Btu for each bathroom in excess of two.

In larger installations the water-heating load may be quite large and it should, therefore, be carefully computed. Where the requirements for hot water are reasonably uniform, as in apartment buildings, hotels, and the like, smaller storage capacity is required than in factories, schools, and office buildings, where practically the entire day's usage of hot water occurs during a very short period. Correspondingly, the heating capacity must be proportionately greater with uniform usage of hot water than with intermittent usage, where there may be several hours between peak demands during which the water in the storage tank can be brought up to temperature. As a general rule it is desirable to have a large storage capacity in order that the heating capacity and consequently the size of the heater, or the load on the heating boiler, may be as small as possible; however in estimating the quantity of hot water which can be drawn from a storage tank only about 75% of the volume of the tank is available. By the time this quantity has been drawn off incoming cold water has cooled the remainder to a point where it can no longer be considered hot water.

Where steam from the heating boiler is used to heat domestic hot water the computed load on the boiler should be increased by 4 sq ft edr for every gallon of water per hour heated through a 100 F rise. The actual water requirement is 3.47 sq ft per gal $[(100 \times 8.33)/240]$ heated 100 F. The value 4 allows for transmission losses, etc.

ESTIMATING HOT WATER FOR A BUILDING. Two ways of estimating the hot-water requirements of a building are in common use: first, by the number of people and, second, by the number of plumbing fixtures installed. Where the number of people to be served is known or can be estimated, the data in Table 12 may be used.

EXAMPLE. According to Table 12, a residence housing one family of four persons would have a daily requirement of 50 gal per day (1×50). The heater should have a storage capacity of 30 gal $(50 \times 3/5)$ and a heating capacity of 7.1 gal per hr $(50 \times 1/7)$.

Table 12. Estimated Hot-water Demand per Person for Various Types of Buildings

(Adapted from *Heating, Ventilating and Air Conditioning Guide*, 1948, Chapter 50)

Type of Building	Hot Water Required at 140 F	Minimum Storage Capacity in Relation to Day's Use	Hourly Heating Capacity in Relation to Day's Use
Residence	50 gal per family per day *	3/5	1/7
Apartments, hotels, etc.	40 gal per person per day	3/5	1/7
Office buildings	2 gal per person per day	1/5	1/6
Factory buildings	5 gal per person per day	2/5	1/8
Restaurants:			
$0.50 meals	1.5 gal per meal	1/10	1/10
$1.00 meals	2.5 gal per meal		
$1.50 meals	4.5 gal per meal		
Restaurants			
3 meals per day		1/5	1/10
1 meal per day		2/5	1/6

* Assumed that a family consists of 4 persons.

Table 13 may be used to determine the size of water-heating equipment from the number of fixtures. To obtain the probable maximum demand, multiply the total quantity for

Table 13. Hot-water Demand per Fixture for Various Types of Buildings

(Adapted from *Heating, Ventilating and Air Conditioning Guide*, 1948, Chapter 50)

	Apartment House	Club	Gymnasium	Hospital	Hotel	Industrial Plant	Office Building	Private Residence	School	Y.M.C.A.
1. Basins, private lavatory	2	2	2	2	2	2	2	2	2	2
2. Basins, public lavatory	4	6	8	6	8	12	6	....	15	8
3. Bathtubs	20	20	30	20	20	30	.....	20	.....	30
4. Dishwashers	15	50–150	..	50–150	50–200	20–100	.....	15	20–100	20–100
5. Foot basins	3	3	12	3	3	12	......	3	3	12
6. Kitchen sink	10	20	...	20	20	20	......	10	10	20
7. Laundry, stationary tubs	20	28	......	28	28	....	.	20	......	28
8. Pantry sink	5	10	......	10	10	....	....	5	10	10
9. Showers	75	150	225	75	75	225	......	75	225	225
10. Slop sink	20	20	..	20	30	20	15	15	20	20
11. Demand factor *	0.30	0.30	0.40	0.25	0.25	0.40	0.30	0.30	0.40	0.40
12. Storage-capacity factor †	1.25	0.90	1.00	0.60	0.80	1.00	2.00	0.70	1.00	1.00

* Multiply by fixture quantity to obtain probable maximum demand.

† Ratio of storage-tank capacity to probable maximum demand per hour

Table 14. IBR Boiler Rating Table

Net IBR Rating			Hand Fired					Automatic Fired			
Steam, sq ft	1000 Btu	Piping Factor	Piping and Pick-up * Factor	Gross IBR Output, 1000 Btu	Time Available Fuel Will Last, hr	Maximum Stack Height † ft	Minimum Stack Area † sq in.	Piping and Pickup * Factor	Gross IBR Output, 1000 Btu	Minimum Stack Area † sq in.	Maximum Allowable Draft Loss
1	2	3	4	5	6	7	8	9	10	11	12
100	24.0	1.300	2.360	56.6	7.50	29.0	50.0	1.560	37.4	50.0	0.044
700	168.0	1.248	2.139	359.4	5.73	42.0	54.0	1.492	250.7	50.0	0.072
1,300	312	1.205	1.967	614	4.99	49.0	110.0	1.444	451	73.5	0.096
1,900	456	1.177	1.872	854	4.49	54.5	160.0	1.408	642	115.0	0.118
2,500	600	1.152	1.805	1083	4.24	59.0	207.5	1.382	829	155.0	0.141
3,100	744	1.136	1.744	1298	4.07	63.0	253.0	1.359	1011	192.5	0.162
3,700	888	1.122	1.691	1502	4.00	66.5	296.0	1.339	1189	230.0	0.183
4,000	960	1.120	1.666	1599	4.00	68.0	315.0	1.331	1278	249.0	0.192
5,000	1200	1.120	1.590	1908	4.00	72.5	367.0	1.305	1566	308.0	....
6,000	1440	1.120	1.526	2197	4.00	76.0	409.0	1.290	1858	359.0	....
7,000	1680	1.120	1.470	2470	4.00	79.5	446	1.288	2164	405	....
8,000	1920	1.120	1.426	2738	4.00	83.0	481	1.288	2473	446	....
9,000	2160	1.120	1.400	3024	4.00	87.0	515	1.288	2782	486	....
10,000	2400	1.120	1.400	3360	4.00	91.5	555	1.288	3091	522	.....
12,000	2880	1.120	1.400	4032	4.00	99.0	634	1.288	3709	596	
14,000	3360	1.120	1.400	4704	4.00	106.5	712	1.288	4328	668	
16,000	3840	1.120	1.400	5376	4.00	112.5	789	1.288	4946	740	
18,000	4320	1.120	1.400	6048	4.00	118.0	863	1.288	5564	810	
20,000	4800	1.120	1.400	6720	4.00	120.0	900	1.288	6182	877	. .

* Includes pickup allowance and correction for difference between test and operating conditions.
† To be specified in catalog.

Table 15. SBI Net Rating Data for Residential Steel Boilers—Oil Fired *

SBI Net Rating			Minimum Furnace Volume, cu ft	Heating Surface, sq ft
Steam, sq ft	Water, sq ft	Btu		
275	440	66,000	2.5	16
320	510	77,000	2.9	19
400	640	96,000	3.6	24
550	880	132,000	5.0	32
700	1120	168,000	6.4	41
900	1440	216,000	8.2	53
1100	1760	264,000	10.0	65
1300	2080	312,000	11.8	77
1500	2400	360,000	13.6	88
1800	2880	432,000	16.4	106
2200	3520	528,000	20.0	129
2600	4160	624,000	23.6	153
3000	4800	720,000	27.3	177

* Stoker-fired and gas-fired SBI net rating not greater than oil-fired. Hand-fired, SBI net rating (steam) not greater than fourteen times the square feet of heating surface.

Table 16. SBI Ratings for Commercial Steel Boilers

Mechanically Fired									Heating Surface, sq ft	Hand Fired						Minimum Grate Area, sq ft	Connections	
SBI Rating			SBI Net Rating			Minimum Furnace Volume, cu ft	Minimum Furnace Height,* in.			SBI Rating			SBI Net Rating				Steam Outlet IPS, in.	Steam Return IPS, in.
Steam, sq ft	Water, sq ft	Btu	Steam, sq ft	Water, sq ft	Btu					Steam, sq ft	Water, sq ft	Btu	Steam, sq ft	Water, sq ft	Btu			
2,190	3,500	526,000	1,800	2,880	432,000	15.7	26	129		1,800	2,880	432,000	1,500	2,400	360,000	7.9	6	3
2,680	4,280	643,000	2,200	3,520	528,000	19.2	28	158		2,200	3,520	528,000	1,830	2,930	439,000	8.9	6	3
3,160	5,050	758,000	2,600	4,160	624,000	22.6	29 1/4	186		2,600	4,160	624,000	2,170	3,470	521,000	9.7	6	3
3,650	5,840	876,000	3,000	4,800	720,000	26.1	29 1/2	215		3,000	4,800	720,000	2,500	4,000	600,000	10.5	6	3
4,250	6,800	1,020,000	3,500	5,600	840,000	30.4	30	250		3,500	5,600	840,000	2,920	4,670	700,000	11.4	6	3
4,860	7,770	1,166,000	4,000	6,400	960,000	34.8	30 1/2	286		4,000	6,400	960,000	3,330	5,330	800,000	12.2	6	3
5,470	8,750	1,313,000	4,500	7,200	1,080,000	39.1	31 1/4	322		4,500	7,200	1,080,000	3,750	6,000	900,000	13.4	6	3
6,080	9,720	1,459,000	5,000	8,000	1,200,000	43.5	31 3/4	358		5,000	8,000	1,200,000	4,170	6,670	1,000,000	14.5	6	3
7,290	11,660	1,750,000	6,000	9,600	1,440,000	52.1	32 3/4	429		6,000	9,600	1,440,000	5,000	8,000	1,200,000	16.4	8	4
8,500	13,600	2,040,000	7,000	11,200	1,680,000	60.8	34	500		7,000	11,200	1,680,000	5,830	9,330	1,400,000	18.1	8	4
10,330	16,520	2,479,000	8,500	13,600	2,040,000	73.8	35 1/2	608		8,500	13,600	2,040,000	7,080	11,330	1,700,000	20.5	8	4
12,150	19,440	2,916,000	10,000	16,000	2,400,000	86.8	37 1/2	715		10,000	16,000	2,400,000	8,330	13,330	2,000,000	22.5	8	4
15,180	24,280	3,643,000	12,500	20,000	3,000,000	108.5	40 1/4	893		12,500	20,000	3,000,000	10,420	16,700	2,500,000	25.6	8	4
18,220	29,150	4,373,000	15,000	24,000	3,600,000	130.2	43	1,072		15,000	24,000	3,600,000	12,500	20,000	3,000,000	28.4	8	4
21,250	34,000	5,100,000	17,500	28,000	4,200,000	151.8	46	1,250		17,500	28,000	4,200,000	14,580	23,330	3,500,000	30.9	8	4
24,290	38,860	5,830,000	20,000	32,000	4,800,000	173.5	48 3/4	1,429		20,000	32,000	4,800,000	16,670	26,670	4,000,000	33.2	8	4
30,360	48,570	7,286,000	25,000	40,000	6,000,000	216.9	54 1/2	1,786		25,000	40,000	6,000,000	20,830	33,330	5,000,000	37.4	10	6
36,430	58,280	8,743,000	30,000	48,000	7,200,000	260.3	60 1/2	2,143		30,000	48,000	7,200,000	25,000	40,000	6,000,000	41.2	10	6
42,500	68,000	10,200,000	35,000	56,000	8,400,000	303.6	66	2,500		35,000	56,000	8,400,000	29,170	46,670	7,000,000	44.7	10	6

* Bituminous stoker-fired.

the fixtures by the demand factor in line 11. The heater or coil should have a water-heating capacity equal to this probable maximum demand. The storage tank should have a capacity equal to the probable demand multiplied by the storage capacity factor in line 12.

EXAMPLE. *Determination of heater and storage-tank size for an apartment building from number of fixtures.*

60 lavatories × 2	= 120 gal per hr
30 bath tubs × 20	= 600 gal per hr
30 showers × 75	= 2250 gal per hr
60 kitchen sinks × 10	= 600 gal per hr
15 laundry tubs × 20	= 300 gal per hr
Possible maximum demand	= 3870 gal per hr
Probable maximum demand = 3870 × 0.30	= 1161 gal per hr
Heater or coil capacity	= 1161 gal per hr
Storage-tank capacity = 1161 × 1.25	= 1450 gal

DESIGN LOAD consists of the radiation load and hot-water supply load, as discussed under Net Loads, p. 12-16, plus an allowance for piping tax.

Piping Tax. Piping tax is the estimated heat emission in Btu per hour of the piping connecting the radiation and other apparatus to the boiler. There is lack of agreement on the allowance to be made for piping losses for installations of different size. The allowances for piping tax recommended by the Institute of Boiler and Radiator Manufacturers and the Steel Boiler Institute are indicated in Tables 14, 15, and 16, p. 12-18. For large installations it is usually advisable to compute the actual heat emission from all pipe surfaces rather than assume an arbitrary piping allowance.

Heat losses from the piping system, which comprises all connections not considered as direct radiation and includes all mains, branches and risers, may be estimated as follows. Let H = heat loss, Btu per hour per square foot of external surface of uncovered piping; t_1 = temperature of surrounding air; t = temperature of steam or hot water; $t_2 = (t - t_1)$; K = a constant = 2.0 for steam and 1.8 for hot water. Then $H = t_2 \times K$. Based on steam and hot-water temperatures of 219.4 F and 180 F, respectively, the following are values of H for various values of t_1:

t_1	40	45	50	55	60	65	70	75
H (steam)	358.8	348.8	338.8	328.8	318.8	308.8	298.8	288.8
H (hot water)	252	243	234	225	216	207	198	189

If pipe covering $^3/_4$ in., or more, thick is used, the above figures may be reduced 75%. If the covering is less than $^3/_4$ in. thick, the piping should be considered as bare.

GROSS OR MAXIMUM LOAD. The gross load is the estimated maximum load the boiler will be required to carry. It is equal to the design load plus a warming-up or pickup allowance.

The warming-up or pickup allowance is the estimated increase in the normal load caused by the heating up of the cold system. It represents the load due to heating the boiler and contents to operating temperature and heating up cold radiation and piping. The factors to be used for determining the allowance to be made may be selected from Table 17 and should be applied to the estimated design load. Although in every case the estimated maximum load will exceed the design load if adequate heating response is to be achieved, there is no object in overestimating the allowance, as the only effect would be to reduce the time of warming-up by a few minutes. Otherwise, it might result in firing the boiler unduly and increasing the cost of operation.

Table 17. Warming-up Allowances for Hand-fired Low-pressure Steam and Hot-water Heating Boilers

Design Load,* Btu per Hour	Equivalent Square Feet of Radiation †	Percentage Capacity to Add for Warming-up ‡
Up to 100,000	Up to 420	65
100,000 to 200,000	420 to 840	60
200,000 to 600,000	840 to 2500	55
600,000 to 1,200,000	2500 to 5000	50
1,200,000 to 1,800,000	5000 to 7500	45
Above 1,800,000	Above 7500	40

* Represents summation radiation load, hot-water supply load, and piping tax.
† 240 Btu per sq ft.
‡ This table refers to hand-fired, solid fuel boilers. A factor of 20% over design load is adequate when automatically fired fuels are used.

EQUIVALENT BOILER HORSEPOWER RATING OF HEATING BOILERS. The capacities of heating boilers may be stated in boiler horsepower, and its equivalent in square feet of radiation may be determined as follows: 1 boiler hp = 34.5 lb water evaporated from and at 212 F = 34.5 × 970.2 (latent heat at 212 F) = 33,472 Btu per hr. One sq ft of standard cast-iron steam radiation is assumed to transmit 240 Btu per hr, and 1 boiler hp = 33,472 ÷ 240 = 139.5 sq ft of equivalent direct radiation.

The equivalent boiler horsepower rating of a hot-water boiler is 33,472 ÷ 150 = 223.1 sq ft of direct cast-iron hot-water radiation.

BOILER SELECTION. Both gross outputs and net ratings of cast-iron boilers are usually available from manufacturers' catalogs. They may also be obtained from published tables of IBR ratings. Net ratings may also be obtained from recommendations of the Heating, Piping, and Air Conditioning Contractors National Association.

Net ratings can be used in selection boilers, unless the heating system contains an unusual amount of bare pipe, or the nature of the connected load is such that the normal allowances for pipe loss and pickup do not apply. In such a case the selection must be based on the gross output.

If a boiler bears an IBR rating it signifies that the boiler has been tested and rated in accordance with the IBR Testing and Rating Code. This code gives the detailed test procedure to be followed in order to determine the output of the boiler and the heating load which the boiler can conservatively handle.

Catalog ratings for steel boilers in accordance with the previously mentioned Steel Boiler Institute, Inc., code are intended to correspond to the estimated design load. When the heat emission of the piping is not known, the net load to be considered for the boiler may be determined from Tables 15 and 16.

Boilers with less than 177 sq ft of heating surface and having SBI net ratings (steam) of not more than 3000 sq ft, if mechanically fired, and 2480 sq ft if hand fired, are classified as residence size. An insulated residence boiler for oil, gas, or stoker firing may carry a net load expressed in square feet of steam radiation of not more than seventeen times the square feet of heating surface in the boiler, provided the boiler has been tested in accordance with the SBI Code for Testing Oil-Fired Steel Boilers at output rates of 125, 150, and 175% of the SBI net rating. The SBI net rating (square feet steam) for hand-fired residence boilers is not greater than fourteen times the heating surface. If the heat loss from the piping system exceeds 20% of the installed radiation, the excess is to be considered as a part of the net load.

Selection Based on Heating Surface and Grate Area. Where neither net load nor gross output ratings based upon performance tests are available, a good general rule for conventionally designed boilers is to provide 1 sq ft of boiler heating surface for each 14 sq ft of equivalent radiation (240 Btu per sq ft) represented by the design load. This is equivalent to allowing 10 sq ft of boiler heating surface per boiler horsepower. In this case it is assumed that the maximum load, including the warming-up allowance, will be provided by operating the boiler in excess of the design load, that is, in excess of the 100% rating on a boiler-horsepower basis.

Because manufacturers' ratings for boilers of approximately the same capacity vary widely it is advisable to check the grate area required for heating boilers burning solid fuel by the following formula:

$$G = \frac{H}{C \times F \times E} \tag{5}$$

where G = grate area, square feet; H = required gross output of the boiler, Btu per hr; C = desirable combustion rate for fuel selected, pounds of dry coal per square foot of grate per hour; F = calorific value of fuel, Btu per pound; and E = efficiency of boiler, usually taken as 0.60.

EXAMPLE. Determine the grate area for a required gross output of the boiler of 500,000 Btu per hr, a combustion rate of 6 lb per hr, a calorific value of 13,000 Btu per lb, and an efficiency of 60%.

$$G = \frac{500{,}000}{6 \times 13{,}000 \times 0.60} = 10.7 \text{ sq ft}$$

The boiler selected should have a grate area not less than that determined by eq. 5. With small boilers, where it is desired to provide sufficient coal capacity for approximately an 8-hr firing period plus a 20% reserve for igniting a new charge, more grate area may be required, depending upon the depth of the fuel pot.

Net ratings for coal-fired boilers are usually based on coal having a calorific value of 13,000 Btu per lb. For other calorific values the estimated net load should be corrected in the ratio of the calorific values.

EXAMPLE. Estimated net load = 50,000 Btu per hr. Calorific value of coal to be used = 11,500 Btu per lb. Select boiler having a net rating of 50,000 × (13,000/11,500) = 56,500 Btu per hr.

Practical combustion rates for different grades of coal used in heating boilers are given in Table 18.

Table 18. Practical Combustion Rates for Coal-fired Heating Boilers Operating at Maximum Load on Natural Draft of 1/8 in. to 1/2 in. Water *

Kind of Coal	Grate, sq ft	Coal, lb per sq ft Grate per hour
No. 1 Buckwheat anthracite	Up to 5	3
	5 to 9	3 1/2
	10 to 14	4
	15 to 19	4 1/2
	20 to 25	5
Anthracite pea	Up to 9	5
	10 to 19	5 1/2
	20 to 25	6
Anthracite nut and larger	Up to 4	
	5 to 9	9
	10 to 14	10
	15 to 19	11
	20 to 25	13
Bituminous	Up to 4	9.5
	5 to 14	12
	15 and above	15.5

* Steel boilers usually have higher combustion rates than those indicated, for grate areas exceeding 15 sq ft.

FUEL CONSUMPTION. The estimated fuel consumption of heating boilers per heating season may be based on grate area, square feet of radiation installed, or cubic contents of the building to be heated. The U. S. Treasury Department allows for government buildings 5 tons of coal per sq ft of grate area per season of 240 days, or 1 lb of coal per cu ft of building content. District steam-heating companies estimate 500 lb of steam per sq ft of direct radiation per season or about 70 lb of good coal. This is approximately equivalent to assuming that one-third of the radiation installed is operated continuously for 240 days.

Seasonal fuel consumption may be calculated utilizing a unit known as the *degree-day*. For any one day there are as many degree-days as there are degrees Fahrenheit difference in temperature between the mean temperature for the day and 65 F. The Weather Bureau has analyzed daily mean temperatures over a long period and has established average yearly degree-day figures, as shown in Table 19. If the maximum calculated hourly heat loss is known the yearly fuel consumption can be estimated from eq. 6.

Table 19. Average Degree-days for Various Cities

City	Average Yearly Degree-days	City	Average Yearly Degree-days
Atlanta	3002	New Orleans	1208
Boston	5943	New York	5306
Chicago	6287	Portland, Ore.	4379
Minneapolis	7989	Washington, D. C.	4598

$$F = \frac{H \times 24 \times D}{(65 - t_o) \times C \times E} \tag{6}$$

where F = yearly fuel consumption in units in which C is expressed; H = calculated heat loss for a given outside design temperature, Btu per hour; D = average yearly degree-days (Table 19); t_o = outside design temperature, °F; C = heating value one unit of fuel, Btu per pound coal, per gallon oil, per cubic foot gas; and E = efficiency of fuel utilization, %.

EXAMPLE. Estimate the yearly oil consumption for a residence in Chicago having a calculated heat loss of 132,500 Btu per hr. Assume oil to have a heating value of 140,000 Btu per gal and to be utilized at an efficiency of 70%.

Solution. According to Table 19 the average degree-days for Chicago are 6287, the outside design temperature from Table 2 = −10 F. From eq. 6:

$$F = \frac{132{,}500 \times 24 \times 6287}{(65 - [-10]) \times 140{,}000 \times 0.70} = 2700 \text{ gal}$$

CHIMNEYS FOR HEATING BOILERS. The minimum height of chimneys for low-pressure heating boilers, hot-water boilers, and hot-air furnaces is 35 ft measured from the grate. No flue should be less than 8 by 8 in. Many heating installation failures may be traced to insufficient draft to burn the fuel at the rate required for the rated capacity of boiler or furnace. Flue-gas temperatures should range between 400 and 500 F when the apparatus is worked at its rated capacity. The chimneys should be so located with reference to near-by higher buildings that wind currents will not form eddies and force air downward (see Fig. 6). The flue should be as straight as possible from base to top outlet and should have no opening except the boiler smoke pipe. The outlet must not be so capped that its area is less than the flue area. Sharp bends and off-sets in the flue may reduce the area and choke the draft. The flue must have no feature which reduces the area. In tile flues, joints must be well cemented, and all space between tile and brickwork tightly filled in. If crevices open into the flue where tile sections meet, the draft will be checked. With brick flues, the stacks should have outside walls at least 8 in. thick. Exposed bricks at the top should be laid in cement mortar to prevent acid fumes and rains from cutting out the joints, as will occur with lime mortar. The best location for a chimney is near the center of the building, as all walls then are kept warm. If there is a soot pocket in the flue below the smoke-pipe opening, the clean-out door should always be tightly closed. Other openings into it, from fireplaces, etc., check the draft and prevent best results. The smoke pipe should not extend into the flue beyond the inside surface of the latter, as its end cuts down the area of the flue. Joints where the smoke pipe fits the smoke hood of the boiler, or where the pipe enters the chimney, should be made tight with boiler putty or asbestos cement. The best practice uses fire-clay linings for small and medium-sized flues. Rectangular flue linings are rated by outside dimensions and round linings by inside dimensions.

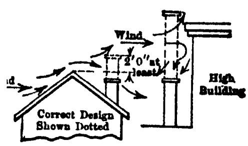


FIG. 6. Correct chimney location.

Table 20. Dimensions of Chimneys for Low-pressure Steam- and Hot-water Boilers and Hot-air Furnaces

(From ASHVE Code of Minimum Requirements for the Heating and Ventilation of Buildings, American Society of Heating and Ventilating Engineers)

Warm-air Furnace Capacity, sq in. of Leader Pipe	Steam Boiler Capacity, sq ft of Radiation	Hot-water Heater Capacity, sq ft of Radiation	Rectangular Flues			Round Flues		Height of Chimney from Grate, ft
			Nominal Dimensions of Fire Clay Lining, in.	Actual Inside Dimensions of Fire Clay Linings, in.	Actual Area, sq in.	Inside Diameter of Lining, in.	Actual and Effective Area, sq in.	
590	590	973	8 1/2 x 13	7 x 11 1/2	81	...	...	35
1000	690	1,140	...	...	...	10	79	35
	900	1,490	13 x 13	11 1/4 x 11 1/4	127	...	...	35
	900	1,490	8 1/2 x 18	6 3/4 x 16 1/4	110	...	...	35
	1,100	1,820	...	...	...	12	113	40
	1,700	2,800	13 x 18	11 1/4 x 16 1/4	183	...	...	40
	1,940	3,200	...	...	...	15	177	40
	2,130	3,520	18 x 18	15 3/4 x 15 3/4	248	...	...	40
	2,480	4,090	20 x 20	17 1/4 x 17 1/4	298	...	...	45
	3,150	5,200	...	...	...	18	254	50
	4,300	7,100	...	...	...	20	314	50
	4,600	7,590	24 x 24	21 x 21	441	...	...	55
	5,000	8,250	...	20 x 24	480	...	...	55
	5,570	9,190	...	24 x 24 *	576	...	...	60
	5,580	9,200	...	...	...	22	380	60
	6,980	11,500	...	...	...	24	452	65
	7,270	12,000	...	24 x 28	672	...	...	65
	8,700	14,400	...	28 x 28	784	...	...	65
	9,380	15,500	...	...	...	27	573	65
	10,150	16,750	...	30 x 30	900	...	...	65
	10,470	17,250	...	28 x 32	896	...	...	65
	11,800	19,500	...	...	...	30	707	70
	14,700	24,300	...	...	...	33	855	70
	17,900	29,500	...	...	...	36	1018	70

* Dimensions below are larger than those in which rectangular fire-clay flue linings are commercially available, and hence are for unlined rectangular flues—requiring thicker walls than when lined.

Table 20, giving dimensions and heights of chimneys, has been used with success in many heating installations. For large installations and for power boilers, draft losses should be estimated, and a height of chimney chosen to give sufficient intensity of draft to balance the sum of the losses. (See Fig. 7.)

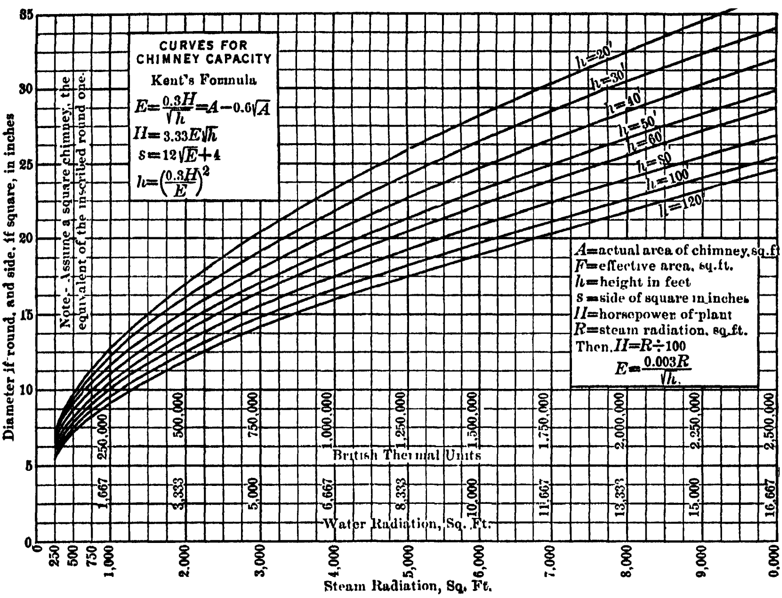


Fig. 7. Capacity of chimneys.

The loss of draft through a cast-iron sectional boiler, normal rating, is approximately 0.15 in. water column, and 0.07 in. for loss through fuel bed, with a combustion rate of 8.6 lb of egg coal per hr.

Stacks for tall buildings are special cases and should be designed by the methods used in the design of chimneys for power boilers.

4. DIRECT STEAM HEATING

Systems using direct steam radiators are (1) gravity circulating and (2) mechanical circulating. The distinguishing characteristic is the manner in which the condensate from the radiators returns to the boiler. In type 1 the condensate returns by gravity, because the static head exists in the returns, and the system is a closed circuit. The steam pressure is the same in boiler, mains, and radiator, except for friction-pressure losses due to the flow of steam. In type 2, the condensate returns to a receiver or feedwater heater, and then is forced into the boiler by a pump or return traps, or both. The system is not closed, and boiler pressure may be higher than that in mains and radiators. The receiver usually is vented to atmosphere, and in vacuum systems an additional pump, attached directly to the returns, discharges the condensate into the receiver or heater. Gravity circulating systems are also divided into one-pipe and two-pipe systems, with basement mains supplying risers to the various floors, or with overhead mains supplying drop risers to the floors below. In the latter arrangement, steam and the water of condensation in the risers flow in the same direction. As there are no counter-currents, less friction is produced, and somewhat smaller pipes may be used. The overhead system is commonly known as the Mills system. (See below.)

The following types of steam-heating systems are in common use: one-pipe circuit systems, Fig. 8; one-pipe relief systems, Figs. 9 and 10; two-pipe systems, Fig. 11; air-line systems, Fig. 12; vapor or air return systems (two-pipe), Fig. 13; vacuum systems, Fig. 14. **In all systems provision must be made to maintain the water in the boiler at the normal water-line level. A most prolific cause of cracking of sections in a cast-iron boiler is** the lowering of the water line, thereby uncovering heating surface which is practically in contact with the fire. Because of loss of pressure in a gravity return system caused by frictional resistance in piping, valves, etc., a static head of water must exist in the return piping, above the boiler water line, equivalent to this pressure loss (30 in. per lb loss in pressure). When the system is started with cold radiation, a greater volume of steam is moved through the piping. Consequently, greater loss in pressure results, and more water is drawn from the boiler than is necessary during the normal heating period to create the necessary static head in the return piping. It is during this starting period that cracking of cast-iron boiler sections sometimes occurs.

ONE-PIPE GRAVITY SYSTEM. The one-pipe *circuit system* (Fig. 8) with basement mains is commonly used for small residence heating. The main rises close to the basement ceiling, just above the boiler, grading down from this high point with a fall of $^3/_4$ or 1 in. per 10 ft to the last radiator riser. The main then drops below the boiler water line and, being required to carry only condensation, is reduced in size. This construction, called *wet return*, is the most satisfactory arrangement whenever its use is possible. If the return main is above the boiler water line, it is a *dry return*. Return mains slope to the boiler 1 in. per 30 ft. An automatic air valve should be placed on the main, at the drop, to remove air from the pipe system.

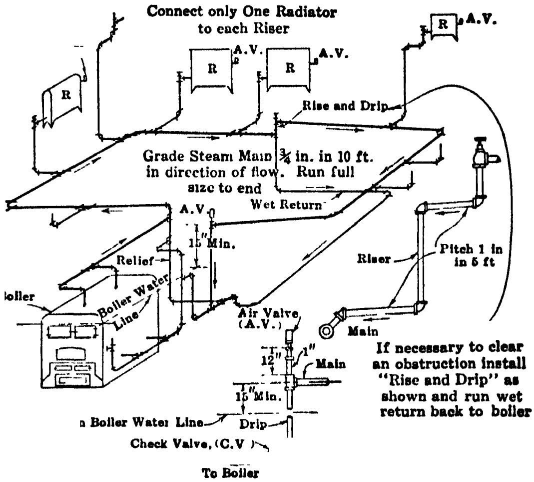


FIG. 8. One-pipe circuit system.

In mains of unusual length the height of the end of the main above the boiler water line must be carefully determined to prevent water backing up from the boiler and flooding the main, air valve, and branches. For steam mains up to 80 ft long, there should be at least 20 in. between the under side of the steam main at the low point and the normal water level in the boiler. This height should be increased 2 in. for every 10 ft of run above 80 ft in all types of gravity systems. In operation, steam and water flow in the same direction in the steam main, and in opposite directions in basement branches, risers, and radiator branches. This necessitates larger piping and valves than in any other steam system. The main, especially, must be full size from boiler to drop, unless dripped at intervals.

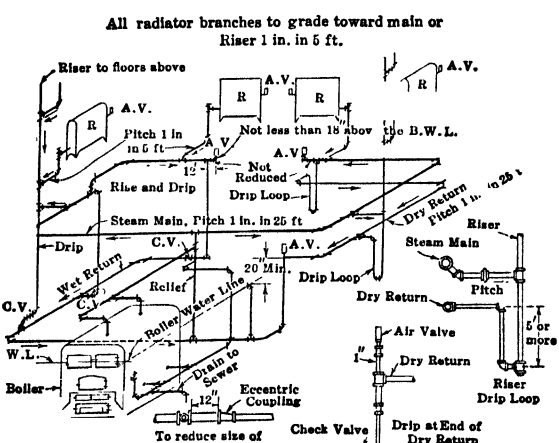
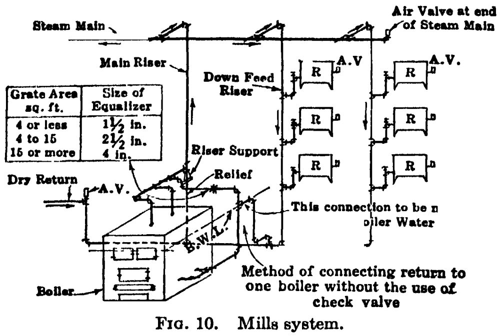


FIG. 9. One-pipe relief system.

THE ONE-PIPE RELIEF SYSTEM (Fig. 9) resembles the one-pipe circuit system, except that the individual risers drip to the return main, which may be either wet or dry. A wet return is preferred. The steam main carries no condensate, and also drips at intervals to the return. A *rise and drip*, as shown, is used when the head room under the steam main would be too much reduced. In this system it is possible to reduce the size of the main at each branch, and to run the main closer to the basement ceiling, which is important where basement space is valuable. This is the most common system in large installations.



Grate Area sq. ft.	Size of Equalizer
4 or less	1½ in.
4 to 15	2½ in.
15 or more	4 in.

FIG. 10. Mills system.

For tall buildings, the one-pipe system with basement mains and gravity circulation is frequently used. It is satisfactory if the piping is properly designed for the circulation of steam and return of condensate. In long, narrow buildings, using a gravity system, a deep boiler pit is necessary; otherwise the elevation of water in the return connections may flood the far end of the steam lines.

MILLS SYSTEM. A more satisfactory arrangement (Fig. 10) for tall buildings and factories is to run the steam main near the ceiling of the top floor or the roof and install down-feed risers to the radiation. This arrangement of the piping is known as the Mills system. In it, the steam and condensate flow in the risers in the same direction and with higher velocities. Consequently, smaller pipe may be used than in an up-feed system. The risers drip at the bottom to the return as previously indicated. These systems ordinarily operate at 2 to 5 lb boiler pressure at normal load. The steam piping usually is designed for a loss in pressure of approximately 1 oz per 100 ft of run, including allowances for ells and other fittings.

THE TWO-PIPE GRAVITY SYSTEM (Fig. 11) with basement mains often is used in large buildings, and *always* with indirect radiation. A thermostatic valve on each radiator will adapt it to vapor and mechanical vacuum systems. When used as a gravity system, the return from each radiator is separately sealed either by dropping below the boiler water line to a wet return or by using drip loops, before connection to a dry return. Even in one-pipe systems, all drips or reliefs should be sealed as in Fig. 9. If this is not done, steam may enter a drip or return from the outlet and cause water hammer, due to counter-currents of steam and condensate. All drips, reliefs, return risers, and connections from the steam to the return side of the system must be sealed, either by connection below the water line or by using a running or return trap on the connecting line. Failure so to seal will result in unsatisfactory operation.

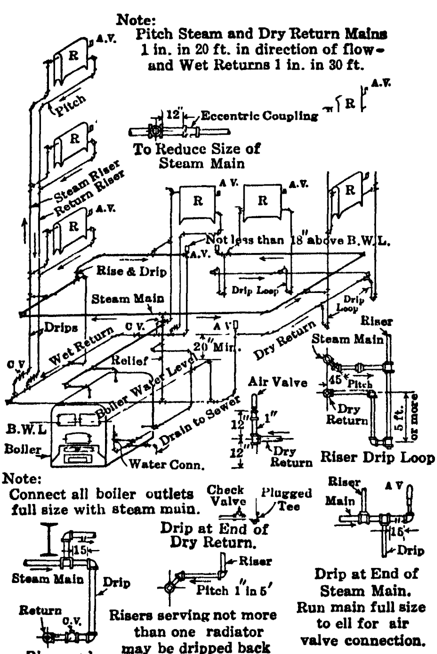
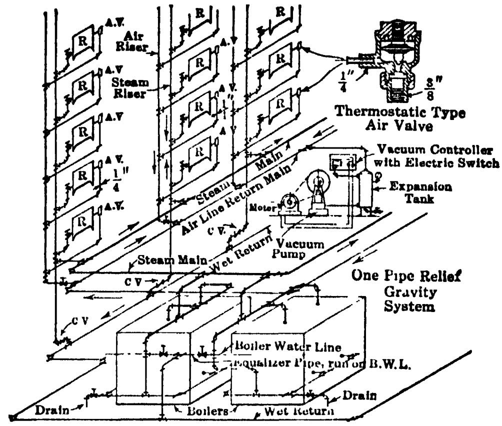


FIG. 11. Two-pipe gravity system.

SPECIAL GRAVITY SYSTEMS. Many special steam-heating systems, known as air-line and vapor systems, also operate with gravity return of the condensate. The air-line system may be applied to any one- or two-pipe gravity system, by connecting the automatic air valve of each radiator, by small size piping, to an exhauster maintaining a slight vacuum in the air piping and removing accumulated air from the radiators. The application of this scheme to the ordinary one-pipe or two-pipe gravity system will improve its operation. The exhauster for less than



Note:

All Connections to radiator air Valves $\frac{1}{4}$ in.

Air risers are $\frac{1}{2}$ in., up to 8 stories high.

Air mains in basement are: 1 in. for $\frac{1}{2}$ in. Risers and $1\frac{1}{4}-1\frac{1}{2}$ in. for $\frac{3}{4}$-in. Risers.

Air Valves are of the thermostatic type.

This illustration shows the recommended method of connecting piping for a two-boiler installation without the use of check valves.

FIG. 12. Air-line vacuum system.

Note: Pitch Steam and Dry Return Mains 1 in. in 20 ft. in direction of flow. All Radiator Branches to grade toward Main or Riser 1 in. in 5 ft.

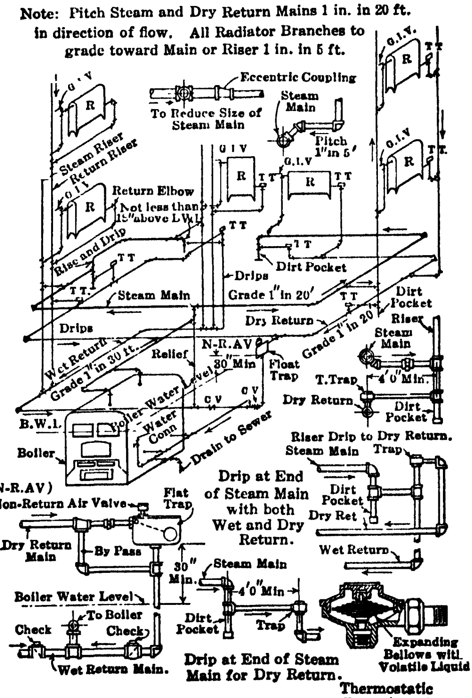


FIG. 13. Vapor system.

2500 sq ft of radiation is a water-driven vacuum pump with a pressure of at least 20 psi. Larger systems use a high-pressure steam jet or, if steam is not available, a motor-driven vacuum pump of about $^1/_4$ hp; 1-in. air mains in the basement are used, with a gate valve on each riser. (See Fig. 12.) The steam used varies from 1 to 5% of the total condensation.

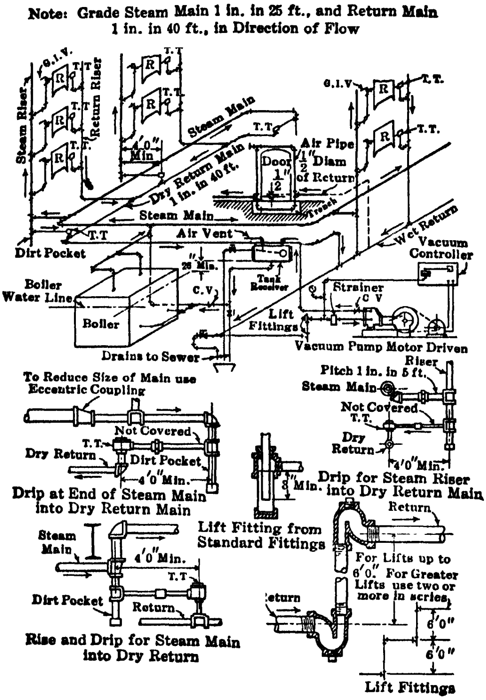


FIG. 14. Mechanical vacuum system.

THE VAPOR SYSTEMS, so-called (Fig. 13) are two-pipe gravity systems in which the accumulated air in the radiators is removed through the return, the air valve on the radiator being omitted. The return on each radiator has a check valve or thermostatic trap, and the dry main return in the basement terminates in a small receiver, having an automatic air valve of sufficient capacity to remove all accumulated air. Each radiator ordinarily is fitted with a graduated fractional valve on the steam connection, permitting partial heating of the radiator when desired.

MECHANICAL VACUUM SYSTEMS are of the two-pipe type and have a vacuum pump attached directly to the returns. (See Fig. 14.) This pump must be capable of handling both air and water, as no air valves can be used on the radiators in this system. The return end of each radiator has a radiator trap, usually of the thermostatic type. A volatile liquid in the thermostatic bellows vaporizes when steam comes in contact with the bellows, causing the bellows to expand and close the valve. The temperature of the condensate from the radiator is not sufficiently high to vaporize the liquid, and the valve, therefore, remains open to pass condensate and air until the steam starts to flow. The valves are very sensitive, and when in proper adjustment and repair will not blow steam. It is good practice to connect a $^1/_2$-in. cold-water line to the main return at the pump, to condense steam that may leak past the vacuum valves, because of dirt getting under the seat.

Figures 15*A* and 15*B* show the application of vacuum traps to the two-pipe system. The vacuum valve or trap is placed 4 ft from the riser or main which it drains. Otherwise, conduction of heat through the connection to the trap will keep the valve open. Return connections for a vacuum system are smaller than for the ordinary two-pipe system.

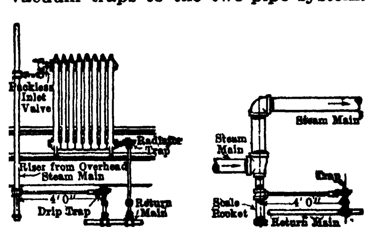


FIG. 15. Application of vacuum traps to 2-pipe system. *A*. Riser drip. *B*. Riser and drip connection.

Vacuum systems are used with exhaust steam heating, where the back pressure from engines or turbines ordinarily should not exceed 5 psi. A by-pass with a reducing valve cross-connects the live-steam main with the heating system, allowing live steam at reduced pressure (usually 2 to 5 lb) automatically to enter the heating system whenever the demand is greater than the supply from the engines, or when they are not in operating condition. The pump on the main return line ordinarily maintains a vacuum of about 10 in. of mercury. It is under automatic control, the negative pressure in the return line operating the controller.

RADIATOR VALVES. Table 21, giving radiator valve ratings, is based on average cast-iron radiation for a 20-min heating-up period. If the 20-min quick heating-up feature

Table 21. Rating of Radiator Valves When Full Open

Square Feet Equivalent Direct Radiation

(Warren Webster Co., Camden, N. J.)

Type of Valve	Size, in.	Pressure at Inlet Valve			
		2 oz	4 oz	8 oz	1 lb
Standard angle	1/2	30	42	60	84
" "	3/4	62	87	124	175
" "	1	102	147	204	294
" "	1 1/4	180	252	360	504
" "	1 1/2	258	364	516	728
Modulating *	1/2	27	38	54	76
"	3/4	57	80	113	160
"	1	94	132	187	265
"	1 1/4	160	225	319	450

* Fractional or graduated inlet type. Use 2-oz. rating for vapor and 8-oz. rating for vacuum systems.

is disregarded and ratings are desired for normal requirements after the radiator is fully heated, multiply values in Table 21 by 2. The capacity and rating of valves manufactured by various concerns vary somewhat.

PRESSURE LOSSES IN PIPING. The pressure loss due to friction in steam mains, valves, and traps in low-pressure gravity systems ordinarily should not exceed, approximately, 1 oz or 0.062 psi per 100 ft of run. Figure 16 shows the reason for limiting the pressure loss. As the steam flowing through the main loses pressure, the pressure at the last riser will be lower than in the boiler. The difference in pressure or pressure loss, p, causes the water line in the return main to be higher than in the boiler. The added height of water in the return is that of a column of water which pressure p will support. Thus, if boiler pressure is 2 psi, and the pressure at the far end of the main is 1 1/2 lb, with water weighing 61 lb per cu ft (0.035 lb per cu in.), the water in the return will stand $(2 - 1.50) \div 0.035$, or 14 in. above the water line of the boiler. In this instance, unless the water line of the boiler is 18 in. or more below the last riser or radiator connection, water is likely to flood the steam main and cause hammering and poor circulation in the radiators near the end of the run.

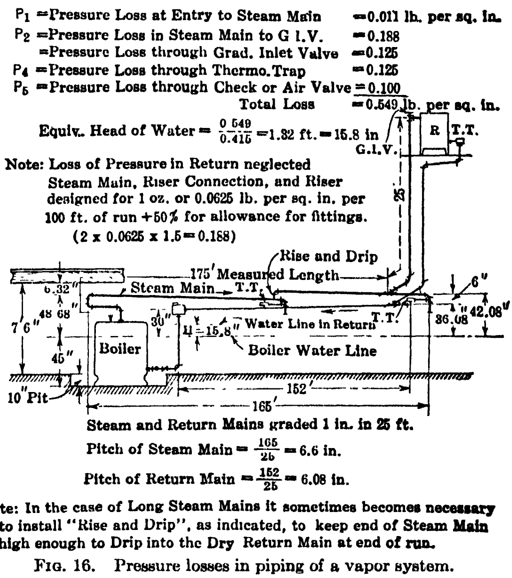


FIG. 16. Pressure losses in piping of a vapor system.

Figure 16, showing the various pressure losses existing in a vapor system, illustrates the method of approximately estimating pressure losses in any low-pressure heating system. The height of the water line in the return piping above the boiler water line may then be determined. The example is based on normal or designed load conditions of operation.

EXAMPLE. Let P_1 = pressure loss, or effective pressure, required to impart initial velocity in the steam main and overcome friction at entrance to main, pounds per square inch; H = head of water column, feet, $= P/0.415$; then $P_1 = 1.65 \times$ velocity pressure $= 1.65v^2d/9262$; v = velocity of steam, feet per second; d = density of steam. The velocity of the steam in the main at the boiler, as ordinarily designed, usually does not exceed 40 to 50 ft per sec at rated load. Assuming 1 lb gage pressure at the boiler, $d = 0.04$, and

$$P_1 = 1.65d\left(\frac{40^2 \times 0.04}{9262}\right) = 0.011 \text{ psi}$$

P_2 = pressure loss due to friction in steam main, horizontal connection to riser, and riser to inlet valve of radiator farthest from the boiler, pounds per square inch, taken as 1 oz or 0.0625 psi per 100 ft of run, including all allowances for loss through ells and tees. (See Figs. 17 and 18.) If no actual estimate for ells or tees is made, 1.5 L is usually sufficiently accurate, L being the measured length, feet, from boiler outlet to last radiator valve. Then

$$P_2 \text{ (approx.)} = 0.0625 \times \frac{1.5L}{100} = \frac{0.094L}{100} \text{ psi}$$

P_3 = pressure differential required to overcome pressure loss through a graduated inlet valve wide open. With proper valve size, P_3 should not exceed 0.125 psi at normal rated operation. Exact data should be obtained from the maker of valve used.

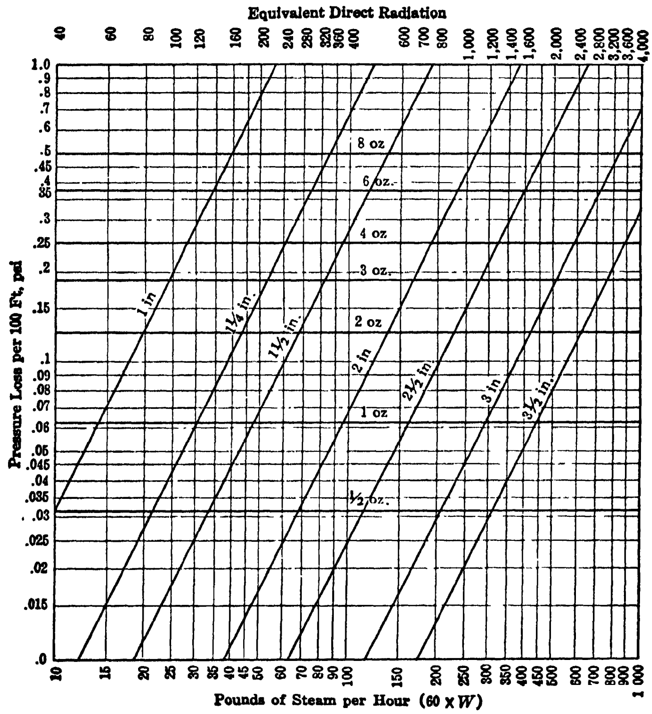


FIG. 17. Pressure losses in steam mains, 40 to 4000 sq ft edr.

P_4 = pressure differential required to overcome pressure loss through the radiator return valve as applied to a vapor system. In thermostatic return valves for vapor systems in normal operation, P_4 (approx.) = 0.125 psi. With thermostatic valves for pressure systems (1 to 2 lb), P_4 = (approx.) 0.25 lb.

P_x = pressure loss in radiator and return lines; it is negligible if returns are properly graded.

P_5 = effective pressure causing air-vent check valve to open at end of return line, or the effective pressure to operate check valve on return line at boiler, whichever is greater. Air-vent check valves usually are rated on a basis of completely venting the system of air in 40 min at starting, with a pressure differential of 0.0625 psi. In normal operation perhaps 0.05 lb or less is required. A horizontal swing check may require approximately 0.10 psi in normal operation. A swing check in the return main adjacent to the boiler, or on a drip line, requires an effective head of approximately 3 in. to 4 in. of water (0.103 to 0.138 psi). The cubic content of the radiation and connect piping is approximately 3 cu ft per 100 sq ft of installed direct radiation.

The sum of the losses P as indicated above is

$$P = P_1 + P_2 + P_3 + P_4 + P_5 = 0.011 + \frac{0.094L}{100} + 0.125 + 0.125 + 0.10 = 0.361 + \frac{0.094L}{100}$$

Assuming L = 200 ft, P = 0.361 + 0.188 = 0.549 psi; then $H = P/0.415$ = 1.32 ft = 15.8 in.

During the starting period (cold radiation and maximum pressure condition) the condensation rate and, consequently, the weight of steam flow and the pressure loss in the main, inlet valve, and radiator trap may approximate twice the amounts stated. For the example given, H then approximately equals 30 in. Vapor systems using thermostatic traps place the air trap on the return main 24 to 30 in. above normal water line of boiler to prevent flooding of the dry return main.

PRESSURE LOSS IN STEAM MAINS. The pressure loss in a steam pipe may be approximated by the Unwin or Babcock formula

$$W = 87 \sqrt{\frac{ypd^5}{L \times \left(1 + \frac{3.6}{d}\right)}} \tag{7}$$

where W = weight of steam flow, pounds per minute; L = length of pipe, feet;

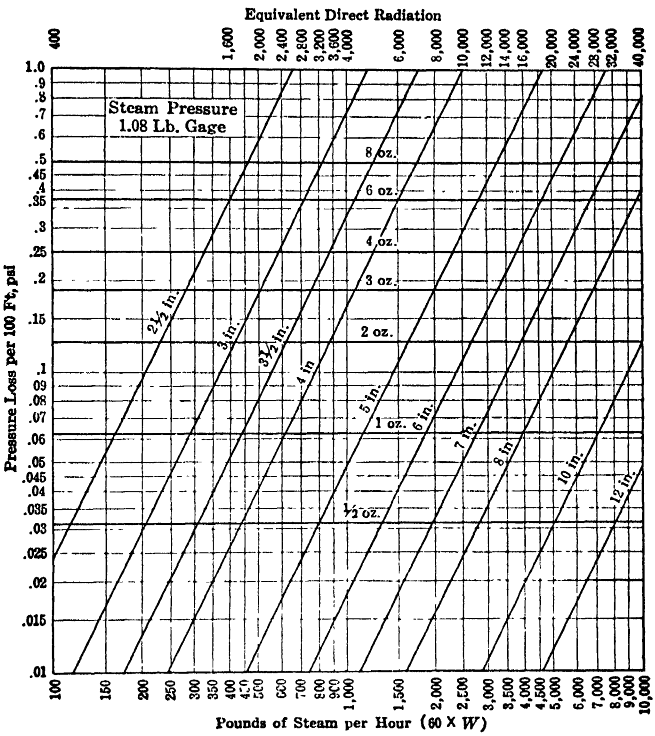


FIG. 18. Pressure losses in steam mains, 400 to 40,000 sq ft edr.

d = diameter of pipe, inches; y = density of steam, pounds per cubic foot; p = loss of pressure, pounds per square inch. Figures 17 and 18 are based on this formula. Under normal conditions of operation, 1 sq ft of direct radiation will condense 0.25 to 0.30 lb per hr. The density of steam, y, is 0.04 for 2.3 psig pressure. The capacities of steam mains in Table 24 were calculated by the above formula, the pressure loss p being limited to 1 oz or 0.062 psi per 100 ft of straight pipe. To allow for fittings, use the data in Table 22. The steam main should not be smaller than the riser connected to it.

Table 22. Resistance of Valves and Fittings

(Length of pipe to be added to measured length of run to obtain equivalent length of run, in feet)

Pipe Size, in.	2	2 1/2	3	3 1/2	4	5	6	7	8	9	10	12	14
Standard elbow	5	7	10	12	14	18	22	26	31	35	39	47	53
Side outlet tee	16	20	26	31	35	44	50	55	63	69	76	90	105
Gate valve	2	3	3	4	5	7	9	10	12	13	15	18	20
Globe valve	18	25	33	39	45	57	70	82	94	105	118	140	160
Angle valve	9	12	16	19	22	28	32	37	42	47	52	63	72

ALLOWABLE PRESSURE DROP IN LOW-PRESSURE, VAPOR, AND VACUUM STEAM-HEATING MAINS. Usual practice has been to design steam-heating mains in buildings on a basis of 1 oz pressure drop per 100 ft of run regardless of length. Good practice limits the total drop in pressure from boiler to the farthest radiator to approximately the values given in Table 23.

Table 23. Pressure Drop in Steam Mains

Type of System	Total Drop, oz
One-pipe low-pressure gravity systems, equivalent length of run 200 ft or less	2
Two-pipe low-pressure gravity systems, equivalent length of run 200 ft or less	2
Two-pipe vapor systems, equivalent length of run 200 ft or less	2
One-pipe low-pressure gravity systems, equivalent length of run 200 to 600 ft	4
Two-pipe low-pressure gravity systems, equivalent length of run 200 to 600 ft	4
Two-pipe vapor systems, equivalent length of run 200 to 400 ft	2
Two-pipe vapor systems, equivalent length of run 200 to 600 ft	4
Vacuum pump systems, equivalent length of run 200 to 600 ft	4
Vacuum pump systems, equivalent length of run 200 to 1200 ft	8

Table 24. Capacities of Steam Mains, Branches, and Risers

(Capacities stated in equivalent square feet of direct radiation. One sq ft of equivalent direct radiation assumed to condense 0.25 lb of steam per hr.)

Nominal Pipe Size, in.	Steam Mains and Down-feed Risers, Dripped; Branches to Risers, Dripped (Steam and Condensate Flowing in Same Direction)			Branches to Risers Not Dripped *		Up-feed Supply Risers	
	Pressure Loss, oz per 100 ft			One-pipe Gravity Systems	Two-pipe Gravity, Vapor, and Vacuum Systems	One-pipe Gravity Systems †	Two-pipe Gravity, Vapor, and Vacuum Systems ‡
	1	2	4				
3/4	§	§	§	......	....	25	30
1	55	80	110	20	26	45	55
1 1/4	120	175	245	55	58	98	120
1 1/2	190	270	380	80	95	152	190
2	385	550	770	165	195	288	385
2 1/2	635	900	1,270	260	395	464	635
3	1,165	1,645	2,325	475	700	799	1165
3 1/2	1,735	2,460	3,475	745	1150	1144	1735
4	2,460	3,475	4,915	1110	1700	1520	2460
5	4,545	6,430	9,090	2180	3150	.....	....
6	7,460	10,555	14,925	....	....	...	....
8	15,335	21,970	31,070	.....	....	..	....
10	28,345	40,085	56,690	....	....	....	....
12	45,490	64,335	90,990	.....	....	......	....

* Radiator branches more than 8 ft long to be one pipe size larger than table.
† Based on tests by ASHVE Research Laboratory.
‡ Based on 1 oz pressure loss per 100 ft run.
§ See Table 26 for size of radiator valves.

The length of run includes allowances for elbows, side outlet tees, and valves.

Example. In a vapor system the measured distance from the boiler to the farthest radiator, including allowances for ells, is 350 ft. Total allowable drop is 2 oz. Allowable drop per 100 ft, $2 + (350/100) = 0.57$ oz $= 0.0356$ psi. A horizontal line through this pressure loss per 100 ft in Fig. 17 intersects the diagonal pipe size lines. The equivalent direct radiation for a 2-in. pipe is read at the top of the chart as 285 sq ft.

PIPE SIZES FOR LOW-PRESSURE STEAM, VAPOR, AND VACUUM SYSTEMS. Tables 24 and 25 may be used to determine pipe sizes in buildings for all types of low-pressure steam and vapor systems. The rating of the steam mains is based on pressure losses of 1 oz, 2 oz, and 3 oz per 100 ft of run. To design the steam main for a fixed total pressure loss, P, for a length, L, determine the pressure loss per 100 ft of run, equal to $P + L/100$; locate this pressure loss on the chart; from the intersection of the horizontal pressure loss line with the vertical line corresponding to the weight of steam to be carried by the pipe per hour or the equivalent direct radiation, determine the nearest size of pipe required. It is advisable in any large gravity steam system to check the total pressure loss in the system.

Table 25. Capacities of Dry and Wet Return Mains

(Capacities stated in equivalent square feet of direct radiation.)

Nominal Pipe Size, in.	Dry Return Mains: 1- and 2-pipe Gravity and Vapor Systems up to 200 ft *	1- and 2-pipe Gravity Systems Exceeding 200 ft Length *, Length, L, ft: 300	400	600	2-pipe Vapor Systems Exceeding 200 ft Length *, Length, L, ft: 300	400
3/4	......	......	...	...	...	......
1	320	370	320	275	285	250
1 1/4	670	770	670	480	595	520
1 1/2	1,058	1,210	1,058	757	945	820
2	2,300	2,640	2,300	1,630	2,140	1,880
2 1/2	3,800	4,380	3,800	2,770	3,470	3,040
3	7,000	8,000	7,000	5,000	6,250	5,480
3 1/2	10,000	11,500	10,000	7,200	8,800	7,800
4	....	......	15,000	10,700	13,400	11,700

Nominal Pipe Size, in.	Vacuum System: Return Mains and Return Risers *, Length, L, ft: 100	300	600	Wet Return Mains: Gravity and Vapor Systems. Pressure Loss, 1/2 in. Water per 100 ft Run, Length, L, ft: 100	200	400	600
3/4	800	462	326	......	......	......	......
1	1,400	810	570	1,525	1,083	762	625
1 1/4	2,400	1,387	976	3,255	2,311	1,627	1,335
1 1/2	3,800	2,195	1,547	4,541	3,224	2,270	1,862
2	8,000	4,622	3,256	8,450	6,000	4,425	3,465
2 1/2	13,400	7,745	5,453	13,176	9,355	6,588	5,402
3	21,400	12,360	8,710	21,122	15,000	10,511	8,660
3 1/2	32,000	18,490	13,020	32,500	23,075	16,250	13,325
4	44,000	25,430	17,910	45,077	32,000	22,538	18,482

* Recommendations of Joint Committee, ASHVE and HPCNA, also ASHVE Minimum Requirements Code.

Note. For capacities for any length of run L_1, multiply capacities given in table in the column under length, L, by $\sqrt{L/L_1}$. Minimum grade for steam and dry return mains 1 in. per 40 ft. Minimum grade for horizontal branches to radiators 1 in. per 20 ft. Above table applies to pipes properly reamed and first-class workmanship.

Table 26. Radiator Valve Capacities and Vertical Connections

(Square feet, equivalent direct radiation.)

Size, in.	Single Pipe Gravity Systems	Two-pipe Gravity Systems: Radiator Supply Valve	Return Trap	Vapor and Vacuum Systems
3/4	...	30	120	Use manufacturers
1	20	55	190	listed capacities
1 1/4	55	120	385	for valves and
1 1/2	81	190	...	return traps
2	165	385	...	

5. EXHAUST STEAM HEATING

The economy of using exhaust steam for heating is apparent, since approximately only 10 to 20% of the heat above 32 F supplied to the average noncondensing engine or turbine appears as work in the steam cylinder. However, nearly all the heat in the exhaust may be utilized for heating, drying, etc. The steam consumption of *noncondensing* automatic high-speed engines and turbines in first-class condition with atmospheric exhaust is given in Table 27, when operating at normal load.

Table 27. Approximate Steam Consumption of High-speed Engines and Turbines with Atmospheric Exhaust Pressure

Size of Unit		Steam Consumption	
Ihp	Kw	Per Ihp-hr	Per Kw-hr
ENGINES 100 PSIG INITIAL PRESSURE			
10–25	. ..	45	.. ..
50	30	33	55
100	65	29	51
300	200	28	43
TURBINES 150 PSIG INITIAL PRESSURE			
....	50	. ...	56
... .	100	...	47
... .	200	..	41
....	500	.	36
. . .	1000	. .	34

1 kw = 1.34 electrical hp. Allowing for efficiency of engine and generator, 1 kw at the generator terminals requires approximately 1.55 ihp. The above steam rates will be increased approximately 3% for 2 psig back pressure and 10.5% for 5 psig back pressure. (See also Section 8.)

Direct-acting feed pumps consume approximately 4% of the total steam generated by boilers; forced draft equipment approximately 2 to 3%. A feedwater heater will condense approximately 17% of the total weight of exhaust steam when heating feedwater from 50 to 210 F, and 6% when heating the water from 150 to 210 F. The latter assumption may be used when all the exhaust is utilized and the heating returns are piped back to the feedwater heater. Allow 20% loss by radiation in piping and heater in determining the net direct radiation which the power equipment will supply.

EXAMPLE. Required the amount of direct radiation (0.25 lb condensation per sq ft per hr) which a 200-kw noncondensing engine-driven unit will supply; hand-fired natural-draft boiler plant and 5 psig back pressure on the engine.

Solution. Boilers must evaporate: $200 \times 28 = 5600$ lb of water per hr for engine; $(5600 \times 0.04)/(1.00 - 0.04) = 233$ lb per hr for feed pump, or a total of about 5825 lb per hr.

A vacuum system should be used in conjunction with exhaust steam heating in order to obtain good steam circulation with a minimum of back pressure on the engine. For additional information, see L. A. Harding, Power from process and space heating steam, *Trans. ASHVE*, 1930.

6. DIRECT HOT-WATER HEATING

SYSTEMS IN USE. Direct hot-water radiator heating systems may be divided into two general classes: (1) all systems operating by gravity only, depending on the difference in density of the water columns in the flow and return lines to cause circulation; (2) systems in which a forced circulation is maintained by a pump.

GRAVITY HOT-WATER HEATING SYSTEM. The gravity systems are (1) up-feed systems, using basement mains; (2) down-feed systems, using overhead or attic mains. Up-feed systems may have either a one-pipe or two-pipe basement main; and the latter type may have either a direct or a reversed return main. (See Fig. 20 for reversed return.) The down-feed systems may have either single or double risers. Either system may operate with an open or a closed expansion tank, as shown in Fig. 19. In general, the down-feed or overhead systems are more positive, permit the use of smaller mains and risers, and provide for the automatic removal of air from radiators and piping. For proper installation of overhead mains and branches, the headroom in the attic must be at least 4 ft. If the overhead mains can be run at the ceiling of the top floor, this restriction does not apply. Mains in attics must be well insulated to prevent freezing.

Under-feed systems are used where basement space of little or no value is available, and the radiation is located on two or more floors; or where attic space is so limited that overhead mains and branches cannot be installed. Underfeed systems are likely to be unsatisfactory in buildings less than two stories high, as the motive head, with radiators on the first floor only, is so slight that faulty or deficient circulation is probable.

The only rational method for designing gravity flow hot-water piping is to balance the friction head against the head available. The head available is calculated from the difference in weight of the water in the flow and return lines. The friction head formulas for pipe of American manufacture, valves, and fittings were determined by Dr. F. E. Giesecke, in 1924. (For further information on determining pipe sizes, see Refs. 5, 6, 7, 8. See also Section 6.)

Up-feed, One-pipe Systems. The up-feed, one-pipe system consists of a supply main in the basement, sloping down 3/4 in. per 10 ft, from a point close to the ceiling above the boiler to beyond the last supply branch, after which it drops and returns to the boiler, together with flow risers and radiator branches taken off at the top, and return branches entering at the side or bottom of the main. The main is of the same diameter throughout the circuit. In the case of branches near the boiler, or branches supplying only upper-floor radiators, flow connections may be made to the supply main at 45 degrees instead of at the top. Radiators on upper floors will assist the circulation in radiators on the first floor, if the upper-floor risers are taken from the side of the branches supplying the first floor radiators. (See Fig. 19*A* and 19*B*.) First-floor branches usually are all full size.

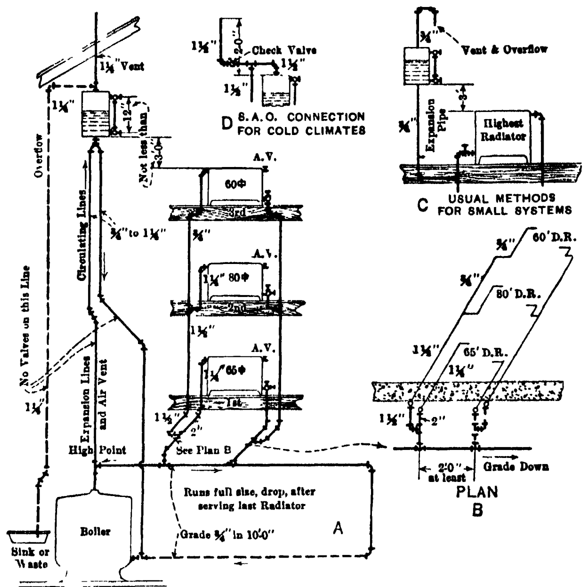


FIG. 19. Expansion tank connections.

Radiators should be connected at the top to the supply by a union elbow, and at the bottom to the return, by a quick-opening hot-water radiator valve. Only one valve is required to control a radiator. The area of the last radiators on a main should be increased from 5 to 10%, as the temperature of the water is gradually decreased in passing through the preceding radiators. It is advisable to increase the size of branch and riser connections at the end of a main by one pipe size.

Tables 28 and 29 give data for proportioning piping. In using the tables, all mains must be measured back to the boiler. For mains over 100 ft long, reduce the capacity in the ratio of $\sqrt{100 \div L}$. Risers to a given floor must be made large enough to supply not only the radiators on that floor, but also on all the floors above it.

Table 28. Sizes of Basement Mains for Gravity Hot-water, Up-feed, Open-tank Heating Systems

(Mains up to 100 ft long)

Pipe Size, in.	Direct Radiation, sq ft	Indirect Radiation, sq ft	Pipe Size, in.	Direct Radiation, sq ft	Indirect Radiation, sq ft	Pipe Size, in.	Direct Radiation, sq ft	Indirect Radiation, sq ft
1 1/4	135	100	3 1/2	850	650	7	4,800	3,900
1 1/2	220	135	4	1100	850	8	6,200	5,000
2	350	225	4 1/2	1350	1050	9	7,700	6,300
2 1/2	460	320	5	1700	1350	10	9,800	7,900
3	675	500	6	3600	2900	12	14,000	11,400

Table 29. Sizes of Branches and Risers for Gravity Hot-water, Up-feed, Open-tank Heating Systems with Basement Mains

Pipe Size, in.	Direct Radiation, sq ft				Pipe Size, in.	Direct Radiation, sq ft			
	1st Floor	2d Floor	3d Floor	4th Floor		1st Floor	2d Floor	3d Floor	4th Floor
3/4	30	45	55	70	2 1/2	400	490	525	550
1	60	75	85	95	3	620	650	690	730
1 1/4	110	120	135	150	3 1/2	820	870	920	970
1 1/2	180	195	210	230	4	1050	1120	1185	1250
2	290	320	350	370	4 1/2	1325	1400	1485	1560

Up-feed, Two-pipe System. The up-feed, two-pipe system (Fig. 20) comprises two mains in the basement, a feed and return main, connected, respectively, to the inlet and return risers of the radiators. This system will prove satisfactory if it has a "reversed return," that is, the return main begins at the first radiator connected to the feed main, and parallels the latter to the last radiator, whence it returns to the boiler. It is the same size throughout as the feed main. With a "direct return," that is, with the return main

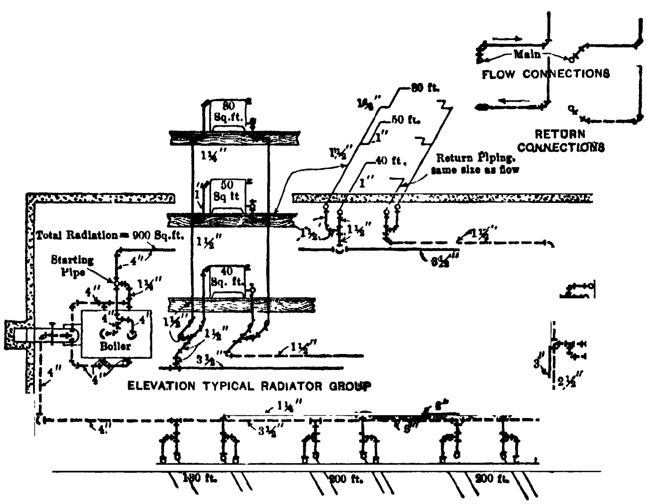


Fig. 20. Up-feed 2-pipe system with reversed return.

beginning at last radiator connected to the feed main, water will circulate first through radiators nearest the boiler, having heat abstracted from it, and then through succeeding radiators in turn. The last radiators thus will be slow in warming up, and the system may prove unsatisfactory. With the reversed return, each radiator offers the same resistance to the flow of water and all become warm at the same time. With the reversed return, the flow is in the same direction as in the flow main. The return increases progressively in size while the flow main decreases. Flow mains should slope up from, and return mains down to, the boiler 3/4 in. per 10 ft. Pipe sizes in Tables 28 and 29 will apply to two-pipe systems, and the size of main should be reduced or increased as rapidly as the change in radiation supply will permit. A "starting pipe" of 1 1/4 to 2 1/2 in. diameter is, in government work, installed between the flow main and the return at the boiler in under-feed systems, to assist establishing initial circulation between flow and return headers.

EQUALIZATION OF PIPES. The relative capacities of different sizes of pipe for the same friction loss per 1000 ft of run are:

Pipe size, in.	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	5	6	7	8
Relative capacity	2	5	10	20	30	60	110	175	260	380	650	1050	1600	2250

The equivalent numbers are proportional to the $^5/_2$ powers of the diameters and the quantity of water W flowing equals $Kd^{5/2}$, where K = a constant and d = pipe diameter, inches.

EXAMPLE. Find a pipe of equivalent capacity to a 1 1/4-, 1 1/2- and 2-in. pipe.

Solution. The equivalent capacity numbers are 1 1/4 = 20; 1 1/2 = 30; 2-in. = 60. 20 + 30 + 60 = 110, which is the number equivalent to a 2 1/2-in. pipe.

DETAILS OF PIPING SYSTEMS FOR GRAVITY HOT-WATER HEATING. Mains and branches must be uniformly graded with provision for expansion and contraction by means of flexible double elbow joints or otherwise. Air traps and pockets must be avoided, and automatic air outlets provided at the top of all points where such pockets may occur. Eccentric fittings must be used wherever the mains are reduced in size to keep the tops of the pipe in the same plane and avoid air pockets. The piping must be arranged so that the system will completely drain of water when the blow-off cock at the boiler is open. All branch mains from a header at the boiler must rise to the same elevation; the tops of all branches must lie in the same plane as they start away from the boiler.

Long sweep fittings must be used on all main piping and branches. Risers supplying radiators on two or more floors should be connected through special tees, known as O.S. fittings, to the branches. (See Fig. 21.) The deflector to divert the current of flow into the outlet of the tee will favor the radiators on the intermediate or lower floors. The maximum length of branch employed above the floor for connecting either steam or water radiators is 9 in. The branch must run in the floor or under the ceiling if longer than 9 in. Risers to upper floors should be not over 2 in. from finished walls.

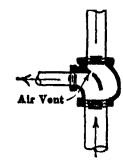


FIG. 21. O. S. fitting.

Radiators should be connected to the flow at the top and to the return at the bottom. A single valve on the return thus will control the radiator. With both connections made at the bottom, two valves are necessary.

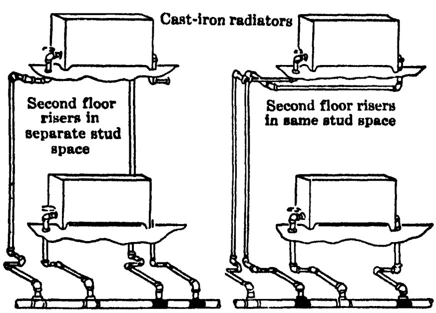


FIG. 22. Alternate piping connections for small one-pipe forced hot-water system.

Air Removal. Small air cocks or automatic air relief valves should be attached to the high point of each radiator on all up-feed systems and opened periodically to relieve air accumulations. Automatic air valves heretofore have been liable to derangement and to passing water as well as air. Unless air accumulations are removed, faulty circulation and failure to heat the radiator will result.

FORCED HOT-WATER HEATING SYSTEM. Concurrently with the development of a number of small circulating pumps in recent years, the one-pipe system of distribution has been studied and improved. In this system, special tees have been developed for diverting various percentages of flow to the radiators, thereby resulting in more uniform water temperature throughout the system. For heating loads not in excess of 150,000 Btu, a simplified design procedure has been developed (*IBR Installation Guide* No. 1, One-Pipe Forced Circulation Hot Water Heating System, June 1948) from which Tables 30 and 31 have been abstracted. Diagrammed in Fig. 22 are alternate procedures for installing the special diverting fittings which may be placed on either the supply or return connection from the radiator, depending on the recommendation of the manufacturer.

EXAMPLE. Select all pipe sizes for a single-circuit one-pipe forced hot-water heating system, similar to the arrangement shown in Figs. 19 and 23, having a total load of 72,000 Btu per hr. The measured length of the pipe circuit is 114 ft, and there are six radiators above the main having an output of 11,000 Btu each, and one radiator below the main with an output of 6000 Btu per hr.

Solution. By moving down the measured length column in Table 30 to 120 ft, the size of the trunk main throughout its entire length is found to be 1 1/4 in. For these small-capacity systems it is customary to select the pump the same size as the largest pipe used in the main. The size of closed expansion tank used for these systems should be not less than one gallon per 30 sq ft of installed radiation. If the heat-emission rate for an average radiator temperature of 195 F is assumed to be 200 Btu per hr, the amount of installed radiation is 360 sq ft, and the minimum expansion tank capacity

Table 30. Main Sizes for Small One-pipe Forced Hot-water, Closed Systems

(Abstracted from *IBR Installation Guide* No. 1)

Measured Length	Capacity of Main, Btu per Hour: 3/4 in. Main	1 in. Main	1 1/4 in. Main	1 1/2 in. Main
50	44,000	80,000	150,000	230,000
60	41,000	76,000	145,000	220,000
70	38,000	73,000	140,000	210,000
80	36,000	70,000	135,000	200,000
90	35,000	67,000	130,000	191,000
100	34,000	64,000	125,000	183,000
120	32,000	60,000	116,000	170,000
140	31,000	56,000	111,000	160,000
160	29,000	54,000	107,000	153,000
180	27,000	51,000	103,000	148,000
200	25,000	49,000	99,000	146,000
220	24,000	47,000	96,000	144,000
240	23,000	45,000	92,000	142,000

Table 31. Radiator Branch and Riser Sizes for Small One-pipe Forced Hot-water, Closed Systems

(Abstracted from *IBR Installation Guide* No. 1)

Total Load on Circuit, Btu/hr	Circuit Size, in.	Capacity: Radiators below Main, 1/2-in. Pipe	Radiators below Main, 3/4-in. Pipe	Radiators above Main, 1/2-in. Pipe	Radiators above Main, 3/4-in. Pipe
20,000	3/4	5,800	.	6,800	.. ..
30,000	3/4	8,700	..	10,200	. ...
	1	3,800	6,400	4,500	7,500
40,000	3/4	11,600	..	13,600	..
	1	5,100	8,500	6,000	10,000
50,000	1	6,400	10,600	7,500	12,500
	1 1/4	4,300	6,800	5,000	8,000
60,000	1	7,700	12,800	9,000	15,000
	1 1/4	5,100	8,200	6,000	9,600
80,000	1	10,200	17,000	12,000	20,000
	1 1/4	6,800	10,900	8,000	12,800
100,000	1 1/4	8,500	13,600	10,000	16,000
	1 1/2	6,800	9,400	8,000	11,000
120,000	1 1/4	10,200	16,300	12,000	19,200
	1 1/2	8,200	11,200	9,600	13,200
140,000	1 1/4	11,900	19,100	14,000	22,400
	1 1/2	9,500	13,100	11,200	15,400

Note. Branch and riser capacities listed in this table are based on a standard radiator branch circuit containing 15 ft of horizontal pipe, one cast-iron radiator having a resistance of 3 elbow equivalents, radiator valve and elbows totaling 12 elbow equivalents. If a heating unit having a resistance in excess of 3 elbow equivalents, an unusual number of elbows, or more than 15 ft of horizontal pipe are used, decrease listed capacity by 1% for each additional elbow equivalent or foot of horizontal pipe in excess of that included in standard branch circuit.

is 360/30 = 12 gal. From Table 31, for 80,000 Btu per hr total load on the circuit and 1 1/4-in. circuit size, the branch and radiator pipe size for the six radiators above the main is 3/4 in., and for the one radiator below the main is 1/2 in.

EXPANSION TANKS. A suitable tank to take care of the expansion and contraction of water in the system must be provided on all low-pressure hot-water heating systems. This tank should be opened and connected to the nearest return riser or to a separate expansion line at an elevation at least 3 ft above the highest radiator. The capacity of the tank depends on the amount of water in the system and on its temperature range. Water heated from 32 to 212 F increases in volume approximately 4.33%. Hence, for every 23 gal of water in the system at 32 F, a tank capacity of 1 gal must be provided, when water is heated to 212 F. Cast-iron radiators have an internal volume of about 1 1/2 pint per sq ft; steel radiators and 1-in. pipe contain about 1 pint per sq ft of surface. It generally is assumed that the internal volume of radiators is 50% of the volume of the

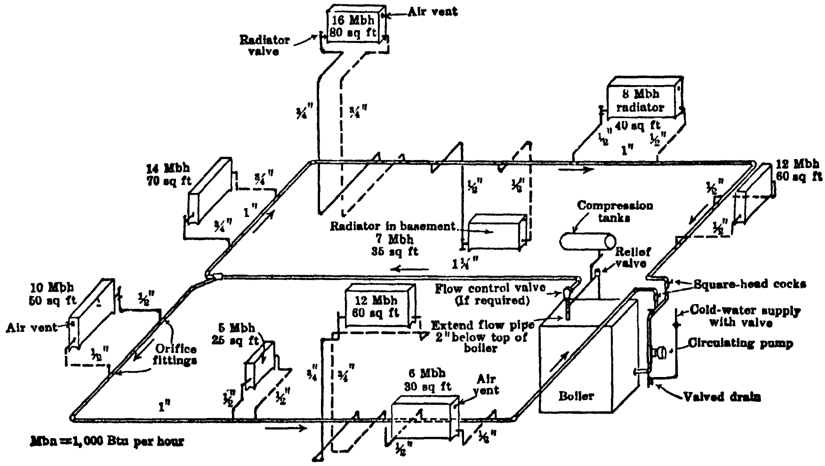


FIG. 23. Typical installation of a one-pipe forced hot-water heating system.

entire system, and the approximate capacity C of the tanks on this basis, therefore, is $C = (R \times 2) \times 0.01$, where C = capacity, gallons, and R = square feet of radiation in radiators.

Table 32 shows the sizes and capacities of commercial expansion tanks. They are slightly smaller in the larger sizes than given by the above formula, but will prove satisfactory with all systems employing up to 6000 sq ft of radiation. For larger systems, the size of the tank should be determined separately.

Table 32. Sizes and Capacities of Commercial Expansion Tanks

Size, in.	Capacity, gal	Radiation, sq ft
12 x 36	18	500
12 x 48	24	1000
12 x 60	30	2000
16 x 48	35	2500
16 x 54	40	3000
18 x 62	60	4000
24 x 45	80	4500

Note. Above sizes based on radiator heat emission 200 Btu per hr; if 240 Btu emission is used add 15% to tank size. For old systems with large pipe sizes, add 30% to tank size.

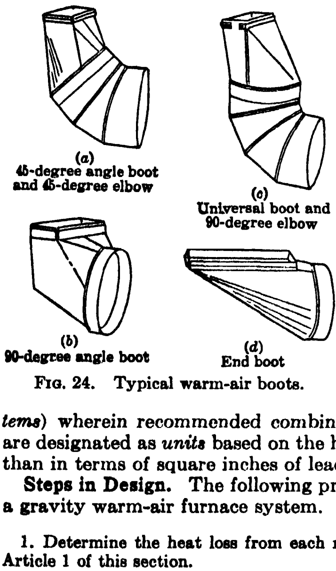
It is considered preferable to locate expansion tank at highest point of system whenever practicable, and it should be well protected from freezing. It is recommended that a three-way valve be installed for replacing the air in basement tanks.

Expansion tanks on one-pipe systems preferably are connected through an expansion line from the high point of the main, just above the boiler connection, to a return bend just below the tank, with a return circulating line connected through the side of the bend with the return main at the boiler. (See Fig. 19.) A 1 1/4-in. vent from the top of the tank should lead through the roof with a 1 1/4-in. overflow connected in the vent line just above the tank. The vent should discharge into an open sink or a drain near the boiler. In small installations, the expansion line may be connected to a radiator return riser.

CLOSED TANK SYSTEMS. In open-tank systems, the highest temperature practically possible is 212 F. At this temperature, the water, rising into the open tank, will boil and empty the system of water. The remedy is to raise the pressure on the system, which, by increasing the hydrostatic head, will raise the boiling point.

7. FURNACE HEATING

Furnace systems are either of the *gravity* or *forced warm-air* type. Both heating plants consist of a fuel-burning furnace or heater, enclosed in a sheet metal casing. In the gravity furnace system the motive head producing flow depends upon the difference in weight between the heated air leaving the top of the casing and the cooled air entering the bottom of the casing; in the forced furnace system a fan furnishes all the motive head. In a gravity system, the horizontal pipes are called leaders and the vertical pipes are referred to as stacks or risers.



(a) 45-degree angle boot and 45-degree elbow

(c) Universal boot and 90-degree elbow

(b) 90-degree angle boot

(d) End boot

FIG. 24. Typical warm-air boots.

DESIGN PROCEDURE. The proper determination of pipe sizes and furnace capacity for gravity systems is based largely on the recommendation of the National Warm Air Heating and Air Conditioning Association. For a long time a physical unit *square inches of leader pipe* was used for expressing the heat capacities of leader pipes, stacks, furnaces, and return ducts. By utilizing this term it was found that 1 sq in. of basement pipe or leader area would deliver 111 Btu per hr to a first-floor room, 167 Btu to a second-floor room, or 200 Btu to a third-floor room. In 1941 the Association adopted a design procedure (*Standard Code Application Manual for Gravity Warm Air Heating Systems*) wherein recommended combinations of leader pipes, fittings, stacks, and registers are designated as *units* based on the heat loss of the room expressed in Btu per hour rather than in terms of square inches of leader pipe.

Steps in Design. The following procedure is recommended in the *Manual* for designing a gravity warm-air furnace system.

1. Determine the heat loss from each room to be heated in terms of Btu per hour as outlined in Article 1 of this section.
2. Make a tentative layout of the basement plan to scale showing proposed location of (*a*) furnace, (*b*) smoke pipe connection to chimney, (*c*) warm-air registers (whether floor, baseboard, or wall), and (*d*) return-air grilles.
3. Indicate on each warm-air run (*a*) whether the room to be heated is on the first or second story, (*b*) the approximate length of leader pipe in basement, (*c*) the number of right-angle elbows as illustrated in Fig. 24. Capacity tables which follow are based on the use of the boots designated as the 90-degree angle boot, universal boot and 90-degree elbow, and 45-degree angle boot and 45-degree elbow. (If the one designated as the end boot in Fig. 24 is used, two additional elbows should be included in counting the total number of elbows. Consider each 45-degree elbow as equal to one-half of 90-degree elbow; and consider the two sharp 90-degree elbows in a crossover connection as equivalent to three 90-degree elbows.)
4. Show the number and possible location of return-air grilles and the type of return-air combination as illustrated in Fig. 25.
5. Recommended combinations of warm-air and return-air pipes, ducts, fittings, registers, and grilles are designated as *units*. From Table 33, for the first story, or from Table 34 for the second story, select the unit number which will supply the heat required to each room with the number of elbows and length of leader pipe previously determined. Using the unit number as found, read directly in Table 35 the leader, stack, and register sizes required. Double-wall stacks are sometimes used as listed in the last section of Table 35. These stacks have a double metal wall with an air space about 5/16 in. wide between the inner and outer walls.
6. From Table 36 select the unit number for the return-air system to correspond with the heat loss requirements and the type of system shown in Fig. 25. Then from Table 37 select the duct and grille size corresponding to the same unit number.
7. Select a furnace having a capacity rating, in Btu per hour, equal to the total heat loss from the structure.

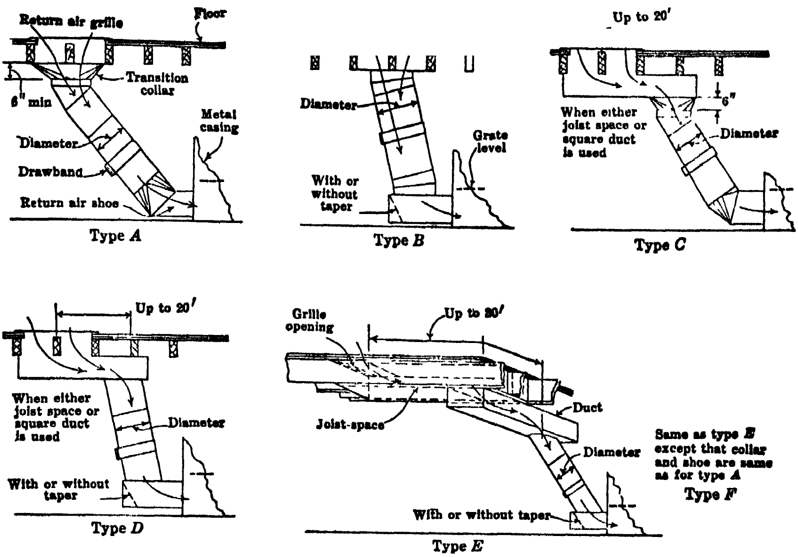


FIG. 25. Types of return-air ducts.

Table 33. Warm-air Carrying Capacity, Btu Delivered, First-story Registers

(Based on *Standard Code Application Manual for Gravity Warm Air Heating Systems*, National Warm Air Heating and Air Conditioning Association)

Unit	Length of Leader Pipe, ft									
							18	20	22	24
					Btu Delivery with One Elbow					
1	6020	5850	5680	5510	5340	5170	5000	4830	4660	4490	4320
2	7620	7400	7180	6970	6760	6540	6320	6110	5890	5680	5460
3	9400	9140	8870	8600	8340	8070	7810	7540	7270	7010	6740
4	13350	12970	12590	12210	11830	11450	11080	10700	10320	9950	9560
5	17520	17020	16530	16040	15550	15050	14550	14050	13560	13060	12560
					Btu Delivery with Three Elbows					
1	5620	5460	5310	5150	4990	4830	4670	4510	4350	4190	4030
2	7120	6910	6710	6510	6310	6110	5900	5700	5500	5300	5100
3	8780	8530	8280	8030	7780	7530	7290	7040	6800	6550	6300
4	12450	12100	11750	11400	11050	10700	10350	10000	9650	9300	8950
5	16360	15900	15440	14970	14510	14050	13600	13130	12660	12200	11750
					Btu Delivery with Five Elbows					
1	5240	5090	4940	4790	4640	4500	4350	4200	4050	3910	3760
2	6630	6440	6250	6060	5880	5690	5500	5320	5130	4940	4750
3	8180	7950	7720	7490	7260	7030	6800	6560	6330	6100	5860
4	11610	11290	10950	10620	10300	9970	9640	9320	8990	8660	8320
5	15250	14800	14380	13950	13520	13090	12650	12230	11800	11370	10940

Notes. (1) Table 33 applies to baseboard and floor locations of warm-air registers. (2) Consider each 45-degree elbow as equal to one-half of a 90-degree elbow. (3) Consider each 90-degree offset between boot and stack head equal to two 90-degree round elbows. (4) With an end boot, include two additional elbows. (5) Interpolate for two and four elbows.

Table 34. Warm-air Carrying Capacity, Btu Delivered, Second-story Registers

(Based on *Standard Code Application Manual for Gravity Warm Air Heating Systems*, National Warm Air Heating and Air Conditioning Association)

Unit	Length of Leader Pipe, ft										
	4	6	8	10	12	14	16	18	20	22	24
	Btu Delivery with One Elbow										
11–22	8370	8140	7900	7670	7430	7190	6950	6710	6470	6240	6000
12–24	10040	9760	9470	9190	8900	8620	8330	8050	7770	7480	7200
13	10880	10570	10260	9960	9650	9340	9030	8730	8420	8110	7800
14	11710	11380	11050	10720	10390	10060	9720	9390	9060	8730	8400
15	16200	15750	15300	14840	14380	13920	13460	13000	12550	12100	11640
16	18920	18390	17850	17310	16780	16240	15710	15180	14640	14100	13570
	Btu Delivery with Three Elbows										
11–22	7530	7320	7110	6900	6680	6470	6250	6040	5830	5620	5400
12–24	9030	8780	8520	8270	8010	7750	7500	7240	6990	6730	6470
13	9800	9520	9240	8960	8690	8410	8130	7850	7580	7300	7020
14	10530	10240	9940	9650	9350	9050	8750	8450	8160	7860	7560
15	14580	14180	13780	13370	12950	12530	12120	11710	11300	10890	10480
16	17040	16550	16070	15580	15110	14620	14140	13660	13180	12700	12210
	Btu Delivery with Five Elbows										
11–22	6700	6510	6320	6130	5940	5750	5560	5370	5180	4990	4800
12–24	8040	7810	7580	7350	7130	6900	6670	6440	6220	5990	5760
13	8710	8460	8210	7970	7720	7470	7230	6980	6730	6490	6240
14	9370	9110	8850	8580	8310	8050	7780	7510	7250	6980	6720
15	12970	12600	12240	11870	11500	11140	10770	10400	10040	9680	9310
16	15140	14710	14280	13850	13420	13000	12570	12140	11710	11280	10850

For Unit 21: for Btu values multiply 11–22 values by 0.83.
For Unit 23: for Btu values multiply 12–24 values by 0.83.
Notes. (1) Btu deliveries shown in Table 34 apply only to baseboard locations of warm-air registers. When floor registers are used deduct 15% from the Btu deliveries. (2) Consider each 45-degree elbow as equal to one-half of a 90-degree elbow. (3) Consider the two sharp elbows in a crossover connection as equivalent to three 90-degree round elbows. (4) Consider each 90-degree offset between boot and stack head equal to two 90-degree round elbows. (5) With an end boot, include two additional elbows. (6) Interpolate for two and four elbows.

EXAMPLE. For a living room having a heat loss of 16,500 Btu per hr, select the proper size of first-story warm-air system required. There are three 90-degree elbows, and the leader is approximately 8 ft long.

Solution. Since 16,500 Btu is beyond capacity of Table 33, it is necessary to choose two leaders of 8250 each. In the 8-ft leader column of Table 33 and in section for three elbows find 8280 as nearest capacity corresponding to unit 3 in the first column. Note that leaders in unit 3 of Table 35 should each be 10 in. in diameter and should be used with 10 by 12 in. floor registers or 12 by 9 in. baseboard registers with 3 1/4-in. extension.

EXAMPLE. Select a return-air system of type *D* to service 31,000 Btu per hr.

Solution. In Table 36 find unit 36, which will serve 31,300 Btu per hr. By referring to Table 37 it will be found that unit 36 will require a 20-in. diameter duct, a shoe area of 340 sq in., a metal grille of 14 by 30 in. or 18 by 24 in., and a duct 36 by 10 in. or 30 by 12 in. If joist lining is used, the minimum depth should be 12.5 in. for two joist spaces 14 in. wide.

FURNACE RATINGS. Gravity warm-air furnaces of conventional design, having ratios of heating surface to grate area of 15 to 1 or greater, are rated as follows:
For hand-fired furnaces converted to stoker, gas or oil firing:

$$\text{Bonnet capacity, Btu per hr} = 1785 \times S \times 1.33 \qquad (8)$$

For hand-fired furnaces with ratios heating surface to grate area greater than 15 to 1 and less than 25 to 1:

$$\text{Bonnet capacity, Btu per hr} = 1785 \times S \times 1.33 \qquad (9)$$

For hand-fired furnaces with ratios of heating surface to grate area in excess of 25 to 1:

$$\text{Bonnet capacity, Btu per hr} = 1785 \times 25 \times G \times 1.33 \qquad (10)$$

where S = heating surface, square feet, and G = grate area, square feet.

The register delivery rating (also considered to be same as calculated heat loss of room) is equal to 0.75 × bonnet capacity. The leader pipe rating in square inches, may be found by dividing the register delivery rating by 136. This latter value is the average of the first- and second-floor leader-pipe carrying capacities previously mentioned.

Table 35. Recommended Standard Commercial Sizes of Leaders, Stacks, Fittings, and Registers

(Based on *Standard Code Application Manual for Gravity Warm Air Heating Systems*, National Warm Air Heating and Air Conditioning Association)

First-story Warm-air Ducts

Unit	Leader-pipe Diameter, in.	Register Size, in.		
		Floor	Baseboard	
			Size	Extension *
1	8	8 x 10	10 x 8	2 1/4
2	9	9 x 12	12 x 8	2 1/4
3	10	10 x 12	12 x 9	3 1/4
4	12	12 x 14	13 x 11	5 1/4
5 †	14	14 x 16		

Second-story Warm-air Ducts, Single-wall Stacks and Fittings

Unit	Leader-pipe Diameter, in.	Stack ‡ Size, in.	Register Size, in.			
			Floor	Baseboard		Side Wall
				Size	Extension *	
11	8	10 x 3 1/4	8 x 10	10 x 8	2 1/4	10 x 8
12	9	12 x 3 1/4	9 x 12	12 x 8	2 1/4	12 x 8
14	10	14 x 3 1/4	....	12 x 8	2 1/4	12 x 8
15	12	12 x 5 1/4	.. . .	12 x 10	3 1/4	
16	12	14 x 5 1/4	. .	13 x 11	3 1/4	

Second-story Warm-air Ducts, Double-wall Stacks and Fittings

Unit	Leader-pipe Diameter, in.	Stack Size, in.		Register Size, in.			
		Internal	External	Floor	Baseboard		Side Wall
					Size	Extension *	
21	8	2 1/2 § x 10	3 1/8 § x 10 5/8	8 x 10	10 x 8	2 1/4	10 x 8
22	8	3 x 10	3 5/8 x 10 5/8	8 x 10	10 x 8	2 1/4	10 x 8
23	9	2 1/2 § x 12	3 1/8 § x 12 5/8	8 x 12	12 x 8	2 1/4	12 x 8
24	9	3 x 12	3 5/8 x 12 5/8	8 x 12	12 x 8	2 1/4	12 x 8

* Distance from room side of plaster line to outside of register base at floor level. This allows for an increase in floor-area opening to take care of large leader-pipe diameters required and thus permits the free flow of heated air to the room with a minimum amount of restriction.

† When the calculations indicate a requirement for a given room greater than unit number 4, two or more smaller units totaling the required capacity are recommended.

‡ Recommended stack sizes. Tables may also be applied to 3-in. and 3 1/2-in. stack depths.

§ Commercial sizes vary 1/8 in. from values shown.

Table 36. Return Air—Carrying Capacity—Btu Serviced

Return Air Combination	Duct Diam., in.	Type *A* Btu per hr	Types *B* and *C* Btu per hr	Type *D* Btu per hr	Type *E* Btu per hr	Type *F* Btu per hr
31	10	11,300	9,500	7,800	5,000	7,800
32	12	16,300	13,700	11,300	7,200	11,300
33	14	22,200	18,700	15,300	9,800	15,300
34	16	29,000	24,400	20,000	12,800	20,000
35	18	36,700	30,800	25,300	16,200	25,300
36	20	45,300	38,000	31,300	20,000	31,300
37	22	54,800	46,000	37,800	24,100	37,800
38	24	65,200	54,800	45,000	28,700	45,000

Table 37. Return Air Ducts

Combination	Duct Diam., in.	Area at Shoe Connection, sq in.	Metal Grille Sizes			When Joist Lining Is Used *		When Duct is Used	
			Choose One			No. of Joists Lined	Minimum † Depth, in.	Choose One	
			A	*B*	*C*				
31	10			8 x 14	10 x 12	1	7	14 x 6	12 x 8
32	12		6 x 30	8 x 24	12 x 14	1	9	22 x 6	16 x 8
33	14	170	8 x 30	10 x 24	14 x 16	1	12	28 x 6	22 x 8
34	16	220	10 x 30	12 x 24		2	8	28 x 8	22 x 10
35	18	280	12 x 30	14 x 24		2	10	36 x 8	28 x 10
36	20	340	14 x 30	18 x 24		2	12.5	36 x 10	30 x 12
37	22	420	18 x 30			2	15.0	42 x 10	36 x 12
38	24	500	20 x 30			2	18.0	42 x 12	36 x 14

* Based on 14-in. space between joists.

† Use full depth of joist except when joist depth is less than minimum depth required, *when pan must be used.*

FORCED WARM-AIR SYSTEMS. A similar simplified design and furnace-rating procedure is available for forced warm-air furnace systems (*Code and Manual for the Design and Installation of Warm-air Winter Air-conditioning Systems*, 1945, National Warm Air Heating and Air Conditioning Association), but the basic principles outlined in Article 8, below, are also applicable to residential systems.

8. FAN OR BLAST HEATING SYSTEMS

The mechanical indirect system of heating, known as the fan system or blast system, particularly adapted to the warming and ventilating of large structures, consists of three units: (1) a heater of pipes, tubes, or cast-iron sections through which steam, hot water, or hot gas is passed; (2) a fan or blower to circulate air over the heater surfaces, the air being the medium of heat transfer; (3) a system of ducts or pipes to convey the heated air from the heater. If the heater is located between the fan and main duct the system is called *blow-through.* If the fan is installed between the heater and the duct, it is called *draw-through.* (See Fig. 28.) The draw-through system is used for shops where compactness is desirable. The blow-through apparatus is used principally for hot and cold systems installed in schools and public buildings.

If ventilation is not required, or is relatively unimportant, the air simply may be recirculated, sufficient fresh air for ventilation being obtained by infiltration. The heat to be supplied to the heater in this case is the same as in a direct-radiation installation. When ventilation is required, a cold-air intake is provided, and sufficient fresh air is drawn into the system from outdoors to meet the ventilation requirement. The remainder of the air necessary for heating is recirculated. This is effected by an arrangement of ducts and dampers on the suction side of the fan. If the fresh air is to be washed or conditioned, the washer or humidifier and tempering coil is placed between the inlet for the recirculated air and the fresh-air intake. In factory work, the unit heater has largely supplanted the fan or blast system when heating is the only requirement. See Unit Heaters, p. 12-55.

ADVANTAGES OF THE FAN SYSTEM. The advantages of the fan system over direct radiation, briefly, are: (1) It affords positive ventilation, entirely independent of changing climatic conditions. (2) When a standard humidity of air is to be maintained, as is desirable in heating and ventilating, and essential to the manufacture of some materials, the air-conditioning apparatus may be made integral with the system. (3) Less radiating surface is required for equal heating duty, with a consequent reduction in the number of steamtight joints to keep in repair. (4) Air leakage being mostly outward, the building will be freer from drafts and more uniformly heated. If air is recirculated and no outside air is taken into the heating system from outside, this statement does not apply. The pressure of air in the building, even if all air enters the system from outside, is comparatively feeble, and some air will infilter around windows and doors on the windward side of the building, despite the often-made statement that the outward leakage prevents all infiltration of cold air. (5) The fan system is more easily regulated than direct radiation, and readily responds to changing outside temperature. (6) Air entering for ventilation may be cooled in summer by circulating cold water or brine, previously cooled by mechanical refrigeration, through the heater. (7) Running the fan will, in itself, relieve oppressiveness in sultry weather, and, if cold water is circulated through the coils, the difference is noticeable.

HEATER CALCULATIONS. Let H = heat loss; H_1 = heat to be supplied by heater; H_2 = heat to be supplied by tempering coil; H_3 = heat to be supplied to humidifier, all in Btu per hour. Let M_o = outside air introduced; M_r = recirculated air; $M = M_o + M_r$ = total air circulated, all in pounds per hour. Let T_o = temperature of outside air; T_1 = temperature of air entering heater; t = temperature to be maintained in room; t_1 = temperature of air leaving heater; t_2 = temperature loss in ducts; t_3 = temperature of air leaving ducts; t_4 = temperature of air leaving tempering coil; t_m = temperature of air entering ducts, all in °F; n = number of occupants of room; Q = ventilation requirements, cubic feet of air per hour per occupant = 900 to 1800 (usual school requirements). Specific heat of air at constant pressure = 0.24. Weight of air per cubic foot at 70 F = 0.075 lb.

Amount of Air to Be Circulated.

$$\text{For heating, } M = H \div 0.24(t_3 - t) \tag{11}$$

$$\text{For ventilation, } M_o = 0.075 \times n \times Q \tag{12}$$

Heat to Be Supplied by Heater.

$$H_1 = 0.24(t_1 - T_1)M \tag{13}$$

Temperature of Air Leaving Ducts.

$$t_3 = \frac{H}{0.24M} + t \tag{14}$$

Temperature of Air Entering Ducts.

$$t_m = t_3 + t_2$$

Temperature of Air Entering Heater. (*a*) When all air circulated is outside air, $T_1 = T_o$. (*b*) When all air is recirculated, $T_1 = t$. (*c*) When part of air is recirculated,

$$T_1 = (M_oT_o + M_rt) \div (M_o + M_r) \tag{15}$$

Temperature of air leaving the ducts, $t_3 = t_1 - t_2$. It depends on temperature T_1 of air entering heater, its velocity through clear area of heater, area of heating surface, steam temperature, and temperature loss in ducts. The usual range of t_3 is from 125 to 150 F. Tables 40 and 41 (p. 12-48) give t_1 for specific conditions. Values of t_2 depend on the location of the ducts. Heat loss from ducts in inside walls helps to heat the building, and t_3 may be taken as equal to t_1; that is, $t_2 = 0$, and $t_m = t_1$. Heat losses from ducts in outside walls or underground must be compensated by increasing t_1 by an amount equal to t_2.

A constant value of t cannot be maintained in several different rooms with different heat losses by controlling the temperature t_1 of air leaving the heater. Temperature t_3 usually will be different for each room. It is controlled by means of the double plenum chamber system described below.

Air that is recirculated and outside air are passed through a tempering coil that raises it to a temperature t_4, usually from 64 to 70 F. Part of the tempered air passes through the heater. The remainder is by-passed and is mixed with the hot air leaving the heater at temperature t_1. Calculations for the tempering coil are the same as for the heater. The required proportions of heated and tempered air are found by the *method of mixtures* as follows. Let x and $(1 - x)$ = respectively the proportions by weight of heated and tempered air. Then

$$xt_1 + (1 - x)t_4 = t_m \tag{16}$$

By assuming values for t_1 and t_4, usually 120 and 64 F, respectively, the equation can be solved for x and $(1 - x)$.

EXAMPLES. Determine quantity of air to be circulated, and the heat to be supplied to it to offset heat loss from a room maintained at t = 70 F, and whose heat loss is 120,000 Btu per hr. Room contains 60 persons. Outside temperature T_o assumed as 0 F.

Solutions. Several arrangements are possible.

I. All circulated air drawn from outside. Assume $t_2 = 5$; $t_3 = 120$; $t_1 = t_2 + t_3 = 125$.

$$M = M_o = H \div 0.24(t_3 - t) = \frac{120,000}{0.24}(120 - 70) = 10,000 \text{ lb}$$

$$H_1 = 0.24(t_1 - T_1) \times M = 0.24(125 - 0) \times 10,000 = 300,000 \text{ Btu}$$

II. All air to be recirculated. Here $T_1 = t = 70$.

$$M = M_r = H + 0.24(t_3 - t) = \frac{120,000}{0.24}(120 - 70) = 10,000 \text{ lb}$$

$$H_1 = 0.24(t_1 - T_1) \times M = 0.24(125 - 70) \times 10,000 = 132,000 \text{ Btu}$$

III. Part of air to be recirculated, and 1800 cu ft of outside air per hr per person to be supplied for ventilation.

$$M_o = n \times Q \times 0.075 = 60 \times 1800 \times 0.075 = 8100 \text{ lb per hr}$$

$$M = M_o + M_r = H \div 0.24(t_3 - t) = \frac{120{,}000}{0.24}(120 - 70) = 10{,}000 \text{ lb}$$

$$M_r = M - M_o = 10{,}000 - 8100 = 1900 \text{ lb}$$

$$T_1 = (M_oT_o + M_rt) \div (M_o + M_r) = [(8100 \times 0) + (1900 \times 70)] \div 10{,}000 = 13.3 \text{ F}$$

T_1 is the temperature of the mixture of outside and recirculated air entering the heater.

$$H_1 = 0.24(t_1 - T_1)M = 0.24(125 - 13.3) \times 10{,}000 = 268{,}000 \text{ Btu}$$

IV. Room heated by direct radiation. Air for ventilation supplied by a fan system. This combination is called a split system. Air conditioned to maintain a constant relative humidity of 35% in room. Outside air is passed through a tempering coil to bring its temperature to 35 F before it enters washer. With $t = 70$ F, a relative humidity of 35% corresponds to a dew point temperature of 41 F. Air passing through spray chamber of air washer is saturated and leaves at a temperature of $T_1 = 41$ F, at which temperature it enters the heater. Assume temperature t_3 of air entering the room to be 80 F. Then

$$M = M_o = n \times Q \times 0.075 = 60 \times 1800 \times 0.075 = 8100 \text{ lb}$$

For heater, $H_1 = 0.24(t_3 - T_1)M = 0.24(80 - 41) \times 8100 = 75{,}816$ Btu
For tempering coil, $H_2 = 0.24(t_4 - t_o)M = 0.24(35 - 0) \times 8100 = 68{,}040$ Btu
For washer, $H_3 = 7.3M = 59{,}130$ Btu

It is assumed here that a washer with a water heater is used that will fully saturate the air. The heat to be supplied to the washer is the difference between the heat content of dry air entering the washer at 35 F and of saturated air leaving it at 41 F, or 7.3 Btu per lb.

$$\text{Total heat} = H_1 + H_2 + H_3 = 202{,}986 \text{ Btu}$$

V. Constant temperature $t = 70$ F to be maintained in each of several rooms which have different heat losses, as given in Table 38. Ventilation to be provided at rate of 1800 cu ft per hr per occupant. Determine temperature t_3 of entering air. With room A-2 as an example,

$$M_o = Q \times n \times 0.075 = 1800 \times 53 \times 0.075 = 7125 \text{ lb}$$

$$t_3 = \frac{H}{0.24M_o} + t = \frac{21{,}000}{0.24 \times 7125} + 70 = 82.2 \text{ F}$$

Table 38. Data for Example

Room Number	Ventilation: Number of Occupants, n	Ventilation: Cubic Feet per Hour at 70 F $1800 \times n$	Ventilation: Weight per Hour, lb, M_o	Heat Loss, H	Temperature, T, of Air Leaving Ducts, by Formula
A-1	50	90,000	6750	32,000	89.5
A-2	53	95,400	7125	21,000	82.2
Hall		30,000	2250	4,000	77.4

AIR SUPPLIED FOR VENTILATING PURPOSES ONLY. A combination of direct radiation to offset the heat loss H and a fan system to supply fresh air needed for ventilation is considered best practice for schools and public buildings. The heater capacity usually is made sufficient to warm the air for ventilation to about 80 F.

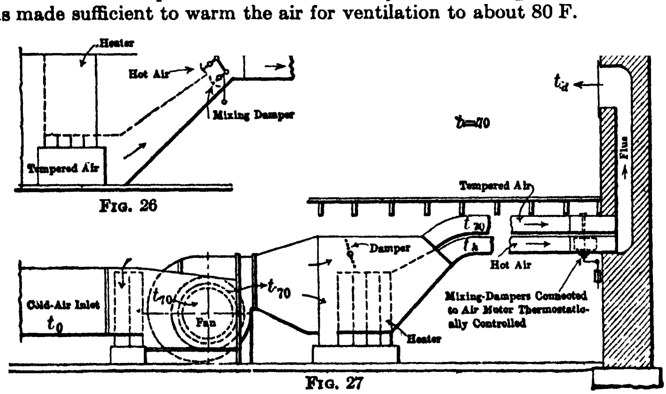


Fig. 26

Fig. 27

Figs. 26 and 27. Hot-blast plenum chambers.

HOT-AND-COLD OR DOUBLE-PLENUM-CHAMBER SYSTEMS. In these systems all air drawn from outdoors first passes through a tempering coil, designed to heat air to approximately 70 F. Part of the tempered air then passes through a heater and is raised to from 125 to 150 F. If varying proportions of the hot and tempered air are correctly mixed, the resulting temperature T is controlled without varying the quantity of air discharged, which must remain constant on account of the ventilation requirement.

The two methods of distribution used are shown in Figs. 26 and 27. In the single-duct system (Fig. 26) the hot and tempered air meet at the entrance to the ducts, at the end of the plenum chamber. The temperature of the mixture is controlled by mixing dampers, which may be operated by hand or automatic thermostatic control. The plenum chamber is divided, and each duct serving a room has an independent set of mixing dampers. In the double-duct system (Fig. 27) two ducts run from the plenum chamber to the base of each vertical flue, carrying hot air and tempered air, respectively, which are mixed at the base of the flue. The mixing dampers may be controlled by hand or by automatic thermostatic control through a compressed-air-operated damper.

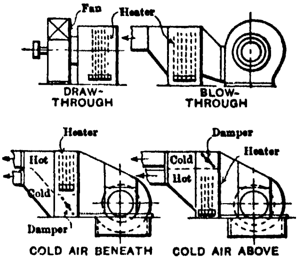


FIG. 28. Types of heater jackets.

BLAST HEATERS. Iron Pipe-coil Heaters. The standard pipe-coil heater for hot-blast work comprises four vertical rows of 1-in. pipe, spaced 2 1/8 to 2 3/4 in. centers, screwed into a cast-iron base. Nipples and ells cross-connect at the top the pipes in each row. The heater comprises a number of sections, each consisting of a base and its pipes, enclosed in a sheet-steel jacket, usually 22 gage. This type of heater has been supplanted largely by the cast-iron sectional and copper tube types. Figure 28 shows types of heater jackets.

Cast-iron Indirect Heaters. Special cast-iron sections for indirect heaters, such as the Vento, made by American Radiator Co., have been used extensively in this class of work. A stack of several sections has fewer joints than a pipe-coil section of equal heating surface. There is practically no deterioration of the cast-iron sectional type of heater, except for the right- and left-hand hexagonal nipples connecting the units of a stack. The four standard lengths of Vento heater-sections are indicated in Table 39, which includes other data needed in design. The Vento heater is designed for pressures not over 10 psig.

Table 39. Vento Hot-blast Heater Data

Length of Section, in.	Heating Surface, Regular Units, sq ft	Center Distance, in. 4 5/8	5	5 3/8
		Free Area of Unit Section, sq ft		
40	10.75	0.52	0.62	0.72
50	13.50	0.65	0.77	0.91
60	16.00	0.78	0.92	1.08
72	19.00	0.94	1.10	1.30

BRASS OR COPPER TUBE EXTENDED SURFACE HEATERS. Commercial heaters are now built of copper tubing on which is wound a spiral copper ribbon or pressed or bonded copper fin forming the extended surface. They are known under such trade names as Aerofin, Arco Blast, and Superfin. This construction, because of its compactness and light weight, has largely supplanted the cast-iron sectional heater. See manufacturers' catalogs for physical data, temperature rise of air passing over various number of rows of pipe, and resulting condensation for various steam pressures. These heaters also may be obtained for high-pressure work up to 350 psig.

TEMPERATURE RISE IN BLAST HEATERS. The temperature rise of air passing through blast heaters of various types has been well established by experiment. Manufacturers have published the results in the form of bulletins and catalogs. (See Tables 40 and 41.)

SELECTION OF BLAST HEATERS. The selection of a heater for a given service is based on the final temperature desired and on the net free area of pipe coil or Vento heaters, and gross face area of copper tube heaters, required for a certain allowable velocity. That is, for specified initial and final temperatures and a given number of net sections, a certain final temperature results when the velocity has been fixed in advance. Good practice limits velocities to those given in Table 42. High velocities are objectionable in public buildings on account of the resulting noise. Resistance through the heater increases as the square of the velocity, with an increase in the power required.

Rating of Blast Heaters. The rating of an assembled heater of several sections (pipe coil or copper tube type) or stacks (Vento type) is based on the temperature rise of the air

Table 40. Final Temperatures and Condensations, Vento Heaters

Regular section, standard spacing, 5-in. center to center, of loops. Steam, 5-lb, gage. C = condensation, lb per hr per sq ft of heating surface. FT = final temperature of air leaving heater. Velocity measured through free area.

Number of Stacks Deep	Temperature of Entering Air,* °F	Velocity through Heater, ft per min, Measured at 70 F							
		1000		1200		1400		1600	
		FT	*C*	*FT*	*C*	*FT*	*C*	*FT*	*C*
1	0	35	2.24	32	2.46	....	....	....	....
	20	51	1.99	49	2.23	47	2.42	45	2.56
	40	68	1.80	66	2.00	64	2.16	62	2.26
	60	84	1.54	82	1.69	81	1.89	80	2.05
2	−20	49	2.22	44	2.46	40	2.69	37	2.92
	0	62	1.99	58	2.23	54	2.42	51	2.62
	20	76	1.80	72	2.00	69	2.20	66	2.36
	40	90	1.60	86	1.77	83	1.93	81	2.10
	60	103	1.38	100	1.54	98	1.71	96	1.85
3	−20	75	2.03	69	2.28	64	2.51	59	2.70
	0	86	1.84	81	2.08	76	2.27	72	2.46
	20	97	1.65	92	1.85	88	2.06	85	2.22
	40	109	1.47	104	1.64	100	1.79	97	1.95
	60	120	1.28	116	1.44	113	1.58	110	1.71
4	−20	96	1.86	90	2.12	84	2.34	78	2.51
	0	106	1.70	100	1.92	95	2.13	90	2.31
	20	115	1.52	110	1.73	105	1.91	101	2.08
	40	124	1.35	119	1.52	115	1.68	111	1.82
	60	134	1.19	129	1.33	125	1.46	122	1.59
5	−20	114	1.72	107	1.95	100	2.15	94	2.34
	0	122	1.56	115	1.77	109	1.96	104	2.14
	20	130	1.41	124	1.60	119	1.78	114	1.93
	40	138	1.26	132	1.42	127	1.56	123	1.70
	60	145	1.09	140	1.23	136	1.36	133	1.50
6	−20	129	1.59	121	1.81	115	2.02	110	2.22
	0	135	1.44	129	1.65	123	1.84	118	2.02
	20	142	1.30	136	1.49	130	1.65	126	1.81
	40	148	1.15	143	1.32	138	1.47	134	1.60
	60	155	1.02	150	1.15	146	1.29	142	1.40
7	−20	141	1.47	134	1.69	128	1.90	122	2.08
	0	147	1.35	140	1.54	135	1.73	130	1.90
	20	152	1.21	146	1.39	141	1.55	136	1.70
	40	158	1.08	153	1.24	148	1.39	143	1.51

* Interpolate for other temperatures.

EXAMPLE. Determine size of Vento hot-blast heater necessary to heat a public building, whose calculated heat loss H = 1,420,000 Btu per hr for 70 F inside and 0 F outside temperature; temperature T of air entering rooms to be approximately 120 F, and of air entering heater 0 F; steam pressure, 5 lb, gage.

Solution. From Table 40, the number of stacks deep required for final temperature 120 F and entering air 0 F, using a velocity of 1000 ft per min, is 5. Weight of air to be circulated per minute is

$$M = \frac{H}{0.24(T - t) \times 60} = \frac{1{,}420{,}000}{0.24(120 - 70) \times 60} = 1972$$

Free area required is A = 1972/(0.075 × 1000) = 26.3 sq ft.

From Table 39, with a 60-in. length of unit, 5 in. on centers, free area per section is 0.92 ft. Number of sections, N, required across face of heater is $N = A/0.92$, or 26.3/0.92 = 28.

Heating surface per section is 16 sq ft. Total heating surface, therefore, is $S = 5 \times 28 \times 16$ = 2240 sq ft, and the total condensation C to be supplied by the boiler, or exhaust-steam, at 5 psig pressure is (Table 40)

$$C = 2240 \times 1.56 = 3494 \text{ lb per hr}$$

passing over the heating surface for certain velocities through the free or unobstructed area. For convenience in rating, velocity is based on the volume of air at an assumed temperature of 70 F.

Table 41. Final Temperatures and Condensations of Aerofin Heaters

(Steam at 5 psi and 227 F temperature)

Temp. of Entering Air, °F	Rows of Tubes Deep	Velocity of Air through Net Face Area,* ft per min, Measured at 70 F and 29.92 in. Barometer							
		400 ft Face Velocity. Friction per Row = 0.0337 in.		500 ft Face Velocity. Friction per Row = 0.052 in.		600 ft Face Velocity. Friction per Row = 0.074 in.		700 ft Face Velocity. Friction per Row = 0.100 in.	
		Final Temp. of Air, °F	Condensation, lb per hr, per Lineal ft of Tube	Final Temp. of Air, °F	Condensation, lb per hr, per Lineal ft of Tube	Final Temp. of Air, °F	Condensation, lb per hr, per Lineal ft of Tube	Final Temp. of Air, °F	Condensation, lb per hr, per Lineal ft of Tube
0	6	144.0	1.22	138.2	1.47	133.0	1.69	127.4	1.89
	7	155.0	1.13	149.5	1.37	144.0	1.58	139.0	1.78
	8	163.8	1.04	158.7	1.27	153.6	1.47	149.0	1.66
	9	170.5	0.96	166.0	1.17	161.5	1.37	157.2	1.56
	10	175.8	0.90	171.8	1.10	168.0	1.29	164.0	1.47
+20	1	54.2	1.71	52.0	2 00	49.0	2.17	48.0	2.45
	2	82.0	1.59	78.0	1.86	74.3	2.09	71.5	2.32
	3	104.6	1.44	99.5	1.69	95.5	1.92	91.3	2.12
	4	123.0	1.32	117.8	1.57	113.2	1.80	108.5	1.99
	5	138.0	1.20	132.4	1.44	127.6	1.65	123.0	1.84
	6	150.0	1.10	144.6	1.32	139.7	1.52	135.0	1.71
	7	159.8	1.02	154.7	1.23	150.0	1.42	145.5	1.61
	8	167.4	0.94	163.0	1.14	158.5	1.32	154.3	1.50
	9	173.4	0.87	169.4	1 06	165.5	1.23	161.6	1.40
	10	178.0	0.81	174.7	0.99	171.0	1.16	167.4	1.32
+40	1	70.4	1.52	68.3	1.77	66.5	1.99	64.5	2.14
	2	95.0	1.41	91.5	1.65	88.6	1.87	85.5	2.04
	3	115.3	1.28	111.3	1.51	107.6	1.72	103.6	1.89
	4	131.5	1.18	127.0	1.40	123.0	1.60	119.0	1.78
	5	144.7	1.07	140.0	1.28	135.8	1.47	131.4	1.63
	6	155.5	0.98	151.0	1.18	146.6	1.35	142.5	1.52
	7	164.2	0.91	160.0	1.10	155.7	1.27	151.6	1.43
	8	171.0	0.83	167.0	1.01	163.3	1.18	159.5	1.33
	9	176.0	0.77	172.6	0.94	169.2	1.09	165.7	1.24
	10	180.3	0.72	177.5	0.88	174.2	1.03	171.0	1.17
+60	1	86.8	1.34	84.5	1.53	83.4	1.75	81.5	1.88
	2	108.3	1.24	105.3	1.46	103.0	1.66	100.0	1.80
	3	126.0	1.12	122.4	1.32	119.5	1.51	115.8	1.66
	4	140.5	1.03	136.3	1.23	133.0	1.41	129.0	1.55
	5	152.0	0.94	148.0	1.12	144.0	1.29	140.4	1.44
	6	161.4	0.86	157.4	1.03	153.6	1.19	150.0	1.34
	7	168.6	0.79	165.0	0.96	161.5	1.11	158.0	1.25
	8	174.3	0.73	171.0	0.89	168.0	1.03	164.6	1.17
	9	179.0	0.67	176.0	0.82	173.0	0.96	170.0	1.09
	10	182.6	0.63	180.3	0.77	177.3	0.90	174.6	1.03
+70	1	94.7	1.23	93.0	1.44	91.5	1.61	90.0	1.75
	2	115.0	1.16	112.3	1.36	110.0	1.54	107.4	1.68
	3	131.5	1.04	128.0	1.23	125.0	1.40	122.0	1.54
	4	144.6	0.96	141.0	1.14	137.6	1.30	134.0	1.44
	5	155.6	0.87	151.8	1.05	148.2	1.20	144.8	1.34

* Net face area means only that area facing the tubes and does not include the headers or casings.

Table 42. Allowable Velocities, Feet per Minute of Air through Free Area of Blast Heaters *

No. of stacks deep	4	5	6	7	8
Velocity, public buildings	1000–1500	1000–1300	1000–1200	900–1100	800–1000
Velocity, factories	1200–1600	1200–1600	1200–1600	1200–1500	1200–1400

* Referred to a temperature of 70 F. Allowable velocities for gross or face area of heater is approximately one-half tabular values.

Let M = weight of air to be circulated through heater, pounds per minute; A = free area of heater, square feet; V = velocity of air, feet per minute, through free area based on 70° temperature; and 0.075 = density of air at 70 F. Then $A = M \div 0.075V$.

9. DESIGN OF AIR DUCTS *

ALLOWABLE VELOCITY OF AIR IN DUCTS AND FLUES. To limit the resistance or pressure loss in the duct system, velocities should be kept within the limits of Table 43. In public buildings, air should be delivered to the room at a velocity that will insure its movement to the desired points in the room without objectionable draft or noise when passing the register grilles.

The velocity through the fan outlet, under the ordinary conditions that obtain in heating work, varies from 1500 to 2500 ft per min.

Table 43. Allowable Velocities in Fan Systems

(Feet per minute)

Location	Residences	Schools and Public Buildings	Industrial Buildings
Outside air intakes	700–800	800–900	1000–1200
Filters	250–300	300–350	350
Heating coils	450–500	500–600	600–700
Air washers	500	500	500
Fan suction connection	700–900	800–1000	1000–1400
Fan outlets	1000–1700	1300–2200	1600–2800
Main ducts	700–1000	1000–1400	1200–2000
Branch ducts	600–700	600–1000	800–1200
Branch risers	500–650	600–900	800–1000
Baseboard registers	300–400	350–500	600–700
Wall registers	350–450	450–600	700–800
Ceiling registers	400–500	500–1400	800–2000
Return grilles	350–500	500–1000	800–1500

SHEET-METAL PIPES AND DUCTS. The recommended gage (U. S. Standard sheet-metal gage) for various sizes of galvanized sheet-steel pipes for heating and ventilating work, blowpiping and exhaust work is given in Table 44.

Table 44. Metal Gages for Ducts

Heating and Ventilating		Thickness and Weight		Blowpiping and Exhaust Work	
Diameter, in.	U. S. Standard Gage	Thickness, in.	Weight, lb per sq ft	Diameter, in.	U. S. Standard Gage
6–19	26	0.018	0.91	3–5	26
20–29	24	0.024	1.16	6–8	24
30–39	22	0.032	1.41	9–15	22
40–49	20	0.036	1.66	16–24	20
50 and over	18	0.048	2.16	26–30	18

PRESSURE LOSS. The pressure loss of air flowing through smooth sheet-metal ducts, measured in inches of water, for 70 F air, and for a length of duct of 100 ft, may be estimated by the formula

$$h = 0.000136 \times \frac{R}{A} \times v^2 \qquad (17)$$

where R = perimeter of duct, feet; A = area of duct, square feet; v = velocity of air, feet per second; and h = pressure loss, inches of water. For round ducts, the above formula reduces to $h = 0.00055v^2/D$, where D = diameter of duct, feet.

Figure 29, which is based on this formula, gives the diameter of a round duct for various velocities, and the pressure loss or resistance for various quantities of air flowing.

* See also Section 1, Flow of Air in Pipes.

EXAMPLE. To find the size of a round duct to convey 1500 cu ft of air per min at a velocity of 1800 ft per min, and also the pressure loss per 100 ft of duct: locate 1500 on the right-hand side of the diagram, pass horizontally to the left to the intersection of the diagonal 1800-ft velocity line. The duct nearest the required size is 12 in. diameter. At this intersection pass vertically down to the base line and read the pressure loss of 0.48 in. of water.

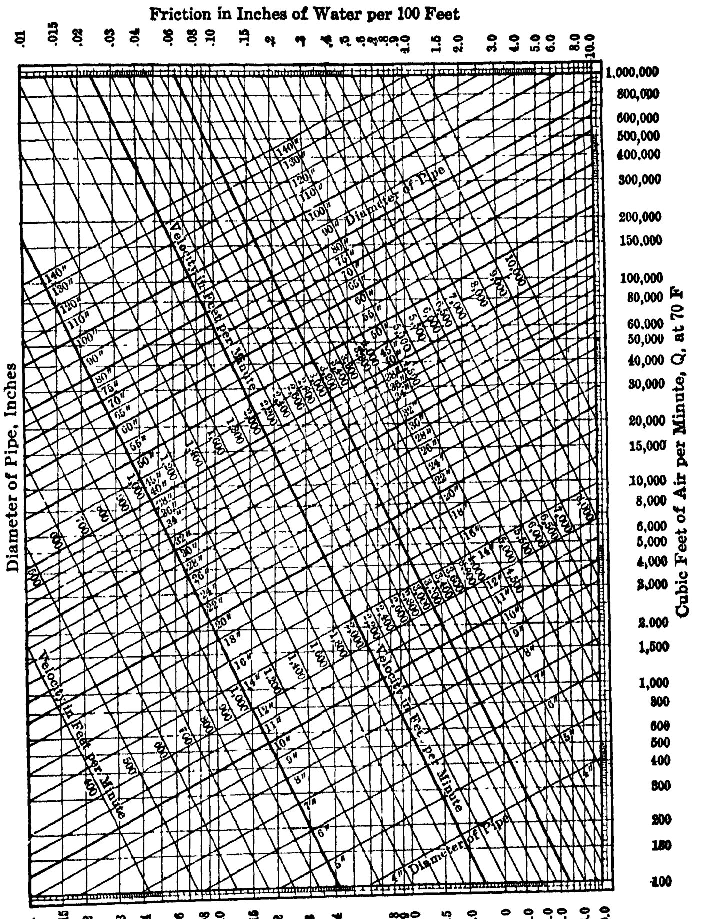


FIG. 29. Pressure drop chart for air ducts.

PRESSURE LOSS OF RECTANGULAR DUCTS. The simplest method of determining pressure loss for rectangular ducts is to proportion the system for round ducts throughout, and transfer to rectangular sizes giving equal pressure losses (not equal areas) using Table 45.

EFFECT OF TEMPERATURE ON PRESSURE LOSSES. The preceding data and eq. 17 on pressure losses in ducts, registers, and heaters are based on an air temperature of 70 F. For other temperatures, multiply results found by using eq. 17 by the ratio $530/(t + 460)$, where t is the temperature of the air, and v in eq. 17 is its true velocity at that temperature. For heaters use the average temperature of the air passing through the heater.

Table 45. Round and Rectangular Ducts of Equal Pressure Losses

Side of Rectangular Duct, in.	4	6	8	10	12	14	15	16	18	20	22	24
	Equivalent Diameters, in.											
5	4.9											
6	5.4	6.6										
7	5.8	7.0										
8	6.1	7.6	8.8									
10	6.8	8.4	9.8	11.0								
12	7.4	9.2	10.7	12.0	13.2							
14	7.9	9.9	11.5	12.9	14.3	15.4						
16	8.4	10.5	12.3	13.8	15.2	16.5	17.1	17.6				
18	8.9	11.1	13.0	14.6	16.1	17.4	18.1	18.7	19.8			
20	9.3	11.6	13.6	15.4	17.0	18.4	19.0	19.7	20.9	22.0		
22	9.7	12.1	14.2	16.1	17.8	19.2	19.9	20.6	21.9	23.1	24.2	
24	10.0	12.6	14.8	16.8	18.5	20.0	20.8	21.5	22.8	24.0	25.2	26.4
26	10.4	13.1	15.4	17.3	19.2	20.8	21.6	22.3	23.8	25.1	26.3	27.5
30	11.0	13.9	16.4	18.5	20.5	22.2	23.1	23.9	25.4	26.8	28.2	29.5
36	11.9	15.1	17.7	20.1	22.2	24.2	25.1	26.0	27.7	29.3	30.8	32.2
42	12.7	16.1	19.0	21.6	23.8	25.9	26.9	27.9	29.8	31.4	33.0	34.5
48	13.5	17.0	20.1	22.8	25.2	27.5	28.6	29.6	31.6	33.4	35.2	37.0
54	14.1	17.9	21.1	24.0	26.6	29.0	30.1	31.2	33.4	35.3	37.2	38.9
60	14.7	18.7	22.1	25.1	27.8	30.5	31.6	32.7	34.9	37.1	39.1	40.9
66	15.3	19.5	23.0	26.2	29.0	31.7	33.0	34.2	36.4	38.7	40.8	42.8

Example. A duct 12 in. in diameter and 120 ft long contains two 90-degree elbows, whose ratio of radius of throat to pipe diameter is 3; air flowing, 1500 cu ft per min at a velocity of 1800 ft per min. The total equivalent length of duct is 120 + (2 × 4.8 [Table 46]) = 129.6 ft. The pressure loss, from Fig. 29, is 0.53 in. per 100 ft. The loss is, therefore, 0.53 × (129.6)/100) = 0.69 in. of water. The pressure loss for square elbows is $0.85v^2/2g$ in. of water for round pipes, and $1.25v^2/2g$ for square pipes; v = velocity, ft per sec. The pressure loss through register grilles may be taken at 0.023 in. for a velocity of 400 ft per min through free area. The gross area of registers is about twice the free area. The pressure loss in air washers and humidifiers for a velocity of 500 ft per min through free area is 0.25 in. of water. The pressure loss through Vento heaters is given in Table 47, and through Aerofin heaters in Table 41.

DESIGN OF DUCT SYSTEMS. Two schemes are used to proportion air ducts: (1) the velocity method; (2) the method of equal friction pressure loss per foot of length. Method 1 involves fixing the velocities (see Table 43) in the various sections, and the gradual reduction of the velocity from beginning of the duct to point of discharge. Pressure loss is computed separately for each section having a different velocity, and the various pressure losses are added together to obtain total pressure loss. Method 2 is used principally in the design of duct systems for factory heating. The velocity in the outlet farthest from the fan is fixed and the area of this branch is determined by volume of air to be delivered. Determine friction pressure loss per 100 ft of duct of this size by Fig. 29 and proportion remainder of the main for this same pressure loss per 100 ft.

Example. Method 1. In the single-duct system (Fig. 30) the risers are based on a velocity of 600 ft per min or 10 ft per sec and 400 ft per min or 6.6 ft per sec, through free area of register grille. Velocity in the longest main, B, is 900 ft per min; volume of air to be delivered, 2000 cu ft per min, at 120 F. Area of riser required is 2000/600 = 3 1/3 sq ft = 480 sq in. An 18 by 27-in. riser (486 sq in. area) is used. Area of main, B, is 2000/900 = 2.22 sq ft = 320 sq in. Size of duct is 12 by 27 in. The diameters of round ducts of equivalent friction losses (Table 45) are: for riser, 24 in.; main, B 19.5 in.

Referring to Fig. 29, pressure losses are: riser, 0.032 in. per 100 ft; main, 0.09 in. per 100 ft. The main has one elbow and the riser one elbow, with ratio of radius at throat to diameter assumed to be 3, in both cases. Equivalent length of main B = 200 + (4.8 × 20/12) = 208 ft; pressure loss = 0.09 × (208/100) = 0.187 in. Equivalent length of riser = 34 + (4.8 × 24/12) = 44 ft; pressure loss = 0.032 × (44/100) = 0.014 in. Pressure loss through register grille is assumed to be 0.023 in.

Total resistance of the duct system is 0.187 + 0.014 + 0.023 = 0.224 in. Assuming five-section Vento heater to be used, with a velocity (based on free area) of 1200 ft per min, pressure loss through heater is 0.376 in. (Table 47). Total resistance against which fan must operate is 0.224 in. + 0.376 in. = 0.60 in., based on 70° air. Assuming temperature of air to be 120 F, resistance is: 0.60 × (530/580) = 0.55 in.

Method 2 is illustrated by example given under Application of Fan-blast Heating Data, below.

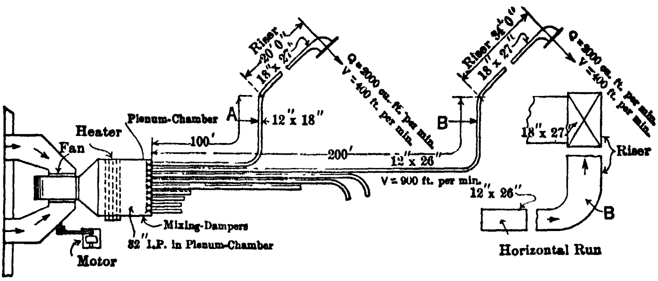


FIG. 30. Single duct system.

Table 46. Friction Pressure Loss of 90-degree Elbows

(Quantities in table expressed in diameters of pipe)

Radius of throat *	1/4	1/2	3/4	1	1 1/4	1 1/2	2	3	4	5
Straight pipe of equivalent pressure loss	67	30	16	10	7.5	6	4.3	4.8	5.2	5.8

* Ratio of throat radius to diameter of pipe.

Table 47. Friction of Air through Vento Heaters

(Friction loss, inches, of water. Vento stacks of regular section and standard 5-in. spacing of loops. Air temperature, 70 F.)

Velocity, ft per min	One Stack	Two Stacks	Three Stacks	Four Stacks	Five Stacks	Six Stacks	Seven Stacks
800	0.037	0.070	0.103	0.135	0.167	0.200	0.232
900	0.047	0.088	0.129	0.170	0.211	0.252	0.293
1000	0.059	0.109	0.160	0.211	0.262	0.313	0.364
1100	0.071	0 132	0.193	0.255	0.316	0.377	0.438
1200	0.084	0.157	0.230	0.303	0.376	0.449	0.522
1300	0.099	0.185	0.271	0.356	0.442	0.528	0.614
1400	0.115	0.214	0.314	0.414	0.513	0.612	0.712
1500	0.132	0.246	0.360	0.474	0.588	0.702	0.816
1600	0.150	0.280	0.410	0.540	0.670	0.800	0.930
1700	0.169	0.316	0.463	0.609	0.756	0.903	1.049
1800	0.190	0.354	0.518	0.683	0.848	1.012	1.177

APPLICATION OF FAN-BLAST HEATING DATA. The application of the preceding data may be illustrated by the design of a fan-blast draw-through heating system for the factory building (Fig. 31), steam pressure to be 5 psig. The calculated heat loss (average inside temperature of 70 F in zero weather) is:

	Square Feet	Btu Transmitted per sq ft		Total Btu per hr
1 1/2-in. roof	22,000	31.*	=	682,000
9-in. brick walls	1,830	25.	=	45,760
13-in. end walls	2,100	19.7	=	41,370
Side wall monitor	880	30.	=	26,400
Glass surface	7,300	79.	=	576,100
7/8 air-change per hr, cu ft	354,800	1.26	=	390,287
		Total	=	1,761,917 = H

* Temperature of air, under side of roof assumed to be $[70(1 + 18 \text{ ft} \times 0.01)] = 82.6$ F. U for roof $= 0.37 \times 82.6 = 30.6$ or 31 Btu per sq ft per hr.

Heater Calculations. A heater made up of five 60-in. Vento stacks will be used. Air will be recirculated and will enter heater at an assumed temperature 10 F lower than the average inside temperature, or $(70 - 10) = 60$ F. It will pass through free area of heater at a velocity of 1200 ft per min, measured at 70 F. From Table 40, final temperature of air leaving heater will be 140 F, and condensation per square foot of heating surface will be 1.23 lb per hr.

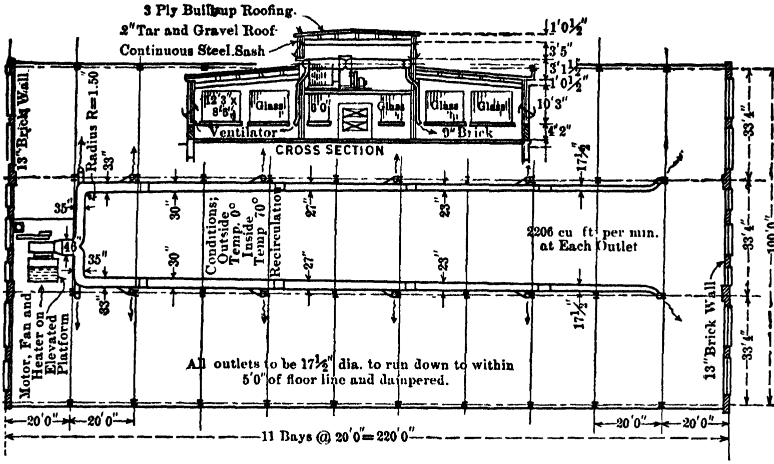


FIG. 31. Draw-through blast system in a factory.

$M = 1{,}761{,}917 + [60 \times 0.24(140 - 70)] = 1747$ lb of air to be circulated per min, or since specific volume = 13.3 at 70 F, $1747 \times 13.3 = 23{,}215$ cu ft of air per min, measured at 70 F.

$A = 23{,}215 \div 1200 = 19.34$ sq ft of free area through heater (Table 42)
$N = 19.34 \div 0.92 = 21$ sections, width of heater (Table 39)
$S = 21 \times 5 \times 16 = 1680$ sq ft, total heating surface (Table 39)
$C = 1680 \times 1.23 = 2066$ steam condensed per hour (Table 40)

Design of Duct System. The duct system in factory work ordinarily is designed on a basis of equal pressure loss per foot of length. Total air volume, measured at 140 F, is $1747 \times 15.1 = 26{,}379$ cu ft per min, and volume of air to be delivered by each of the 12 outlets is $26{,}379 \div 12 = 2198$ cu ft per min. With an assumed discharge velocity at outlets of 1350 ft per min (Table 43), required area per outlet is $2198/1350 = 1.6$ sq ft, equivalent to 17 1/2 in. diameter.

From Fig. 29, pressure loss per 100 ft of length of the last section of the 17 1/2-in. duct, discharging 2198 cu ft per min, is 0.20 in. of water column. Since the duct system is to be designed for equal pressure loss per foot of length, the remaining sizes of duct are determined by the intersection of the vertical 0.20-in. pressure loss line with the horizontal cubic foot lines for the quantities of air to be handled by the various sections of the duct. Each outlet should have a damper to equalize and adjust the flow of air. Total pressure loss of system is estimated as follows:

Loss through heater	0.376 in. of water
" " 200 ft pipe, 2 × 0.20	0.400 " " "
" " two 35-in. elbows	0.072 " " "
" " one 17 1/2-in. elbow	0.018 " " "
Total	0.866 " " "

The actual pressure loss will be somewhat less than the above total, as the figures are based on a density corresponding to an air temperature of 70 F. Usually no correction is made in practice for this difference.

The rated capacity of the fan required (see Section 1) for this installation is $26{,}379 \div 1.06 = 24{,}886$ cu ft per min (Table 48) at 7/8-in. static pressure or maintained resistance. The speed and horsepower as given by the fan tables should be multiplied by the factor 1.06.

RATING OF FANS. The volume of air, cubic feet per minute, at 70 F, which a fan will deliver, varies with the resistance against which it operates. To correctly choose a fan from manufacturers' tables (see Section 1), the resistance (static pressure) must be determined by the duct design, after the size of heater and its resistance have been determined. The speed and brake horsepower required to drive the fan also are stated in the tables. The temperature of air handled by the fan with draw-through apparatus is higher than 70°, except for a fan which is connected ahead of a tempering coil, usually a heater two sections deep. The tabulated speed, volume, and brake horsepower to maintain the pressure must be multiplied by the factors given in Table 48, for temperatures other than 70°. The factors in this table are the square roots of the ratios of the density of the air at 70° F to its density at the temperature stated.

Table 48. Factors for Speed, Volume, and Brake Horsepower of Fans

Temperature, °F	0	100	120	130	140	150
Factor	0.932	1.028	1.046	1.055	1.064	1.073

SELECTION OF MOTOR FOR FAN DRIVING. It is good practice to add 15% to the brake horsepower determined from the fan tables, for the rating of the motor. This allows for overload, when fan is operated under different conditions of pressure and speed than those under which it was originally rated.

Fan Engine. When high-pressure steam is available, an automatic high-speed engine often is used to drive the fan. Exhaust from the engine is used in the first section of the heater.

ADDITIONAL HEATING REQUIREMENT. It may be desirable to so proportion heating apparatus that the fan may be stopped at night and started about two hours before the shop is opened in the morning. It may be safely assumed that the temperature of the air in the building will not be below 30 F, when the fan is started, and that the air is all recirculated. Fan and heater must be of sufficient capacity to make up the heat loss from the building, including infiltration, and also to heat the contained air from 30 to 60 F in 2 hours. Assuming the same data as given in the preceding examples, the additional heat required, if the cubic contents of the building are 388,650 cu ft, will be

$$(388{,}650 \times 0.08 \times 0.24 \times 30) \div 2 = 119{,}931 \text{ Btu per hr}$$

or approximately 7% greater heating requirement than previously calculated. This may be obtained by increasing the steam pressure in the heater to approximately 10 psig. Catalogs, bulletins, etc., on the subject of fan-blast heating may be obtained from the American Blower Corporation, Detroit, Mich., the B. F. Sturtevant Company, Hyde Park, Mass., the Buffalo Forge Company, Buffalo, N. Y., and other manufacturers.

UNIT HEATERS, INDUSTRIAL TYPE. Propeller-fan type unit heaters have outlet velocities ranging from 300 to 600 ft per min. They ordinarily are suspended from the ceiling or structural framework of the building, at intervals of 60 to 100 ft. The floor-type unit heater discharges warmed air through one or several outlets at velocities of 1000 to 1700 ft per min. Some types will project and distribute the heating effect over a distance of approximately 200 ft. Most unit heaters are built with extended-surface copper-tube heating surface. Unit heaters may be distributed through the central portion of a room, discharging toward the exposed wall surfaces, or around the walls, discharging at an angle with the walls toward the interior of the room. The discharge from the heaters should be so directed that rotational circulation of all the air in the room is set up by the discharge from the heaters.

The usual capacity rating of unit heaters is Btu per hour heating effect, based on recirculation of air in the room. Some makers state air volume delivered and temperature rise. The number of uniform-size unit heaters required in a room depends primarily on the spacing necessary to obtain uniform heating. The required Btu capacity of each heater is the calculated heat loss of the room divided by number of heaters to be installed. Capacities of unit heaters as made by one manufacturer are given in Table 49.

Table 49. Unit-heater Capacities

(Data from Buffalo Forge Company, 1948)

Size	Rpm	Motor Hp	CFM *	Capacity † Btu per hr	Final Temp., °F	Overall Dimensions, in. High	Wide	Deep
			Ceiling Type					
121	1725	1/20	975	44,000	101	18	21	18
122	1725	1/20	900	60,000	121	18	21	18
181	1725	1/3	2,960	130,000	100	26	27	22
182	1725	1/3	2,750	190,000	123	26	27	22
241	1140	1/2	4,600	204,000	100	33	39	25
242	1140	1/2	4,370	312,000	125	33	39	25
			Floor or Wall Type					
F-122	1105	3/4	4,180	125,000	89	94	56	23
	985	1/2	3,640	120,000	92	94	56	23
F-153	945	2	10,850	515,000	92	102	92	32
	860	1 1/2	9,860	489,000	93	102	92	32
F-302B	755	7 1/2	29,400	2,740,000	126	132	92	78
	660	5	25,700	2,510,000	129	132	92	78

* Volume of air at 70 and 60 F entering air temperature.

† Capacity Btu per hour with 60 F entering air and 2 psig steam pressure.

EXAMPLE. A suspended-type unit-heater installation for the factory building shown in Fig. 31 conveniently could consist of ten units, in two groups of five in each side bay near the outside walls. Required capacity of each unit = 1,761,917 ÷ 10 = 176,192 Btu per hr. Air is to be recirculated at 70 F. Steam pressure, 5 psig. The nearest size of unit (Table 49) is a 182 unit, 2750 cu ft per min delivery, 190,000 Btu per hr, 1/3-hp motor; multiplying by factor 0.97 from Table 50 for 5-psig steam and 70 F air, 190,000 × 0.97 = 184,300 Btu per hr.

Table 50. Constants for Determining the Capacity of Unit Heaters for Various Steam Pressures and Temperatures of Entering Air

(Based on steam pressure of 2 psig and entering air temperature of 60 F.)

Steam Pressure, psi	Temperature of Entering Air, °F											
	−10		10	20	30	40	50	60	70	80	90	100
						Blow-through Type						
0	1.54	1.45	1.37	1.27	1.19	1.11	1.03	0	0.88	0.81	0.74	0.67
2	1.59	1.50	1.41	1.32	1.24	1.16	1.08	1	0.93	0.85	0.78	0.71
5	1.64	1.55	1.46	1.37	1.29	1.21	1.13	1.05	0.97	0.90	0.83	0.76
10	1.73	1.64	1.55	1.46	1.38	1.29	1.21	1.13	1.06	0.98	0.91	0.84
20	1.86	1.77	1.68	1.58	1.50	1.42	1.33	1.25	1.17	1.10	1.02	0.95
40	2.06	1.96	1.86	1.77	1.68	1.60	1.51	1.43	1.35	1.27	1.19	1.12
60	2.20	2.09	2.00	1.90	1.81	1.73	1.64	1.56	1.47	1.39	1.31	1.24
80	2.31	2.21	2.11	2.02	1.93	1.84	1.75	1.66	1.58	1.50	1.42	1.34
100	2.41	2.31	2.20	2.11	2.02	1.93	1.84	1.75	1.66	1.58	1.50	1.42
						Draw-through Type						
0	1.48	1.41	1.33	1.25	1.18	1.11	1.03	0.96	0.89	0.82	0.75	0.69
2	1.52	1.44	1.36	1.29	1.22	1.14	1.07	1.00	0.93	0.86	0.79	0.73
5	1.57	1.49	1.41	1.33	1.26	1.19	1.11	1.05	0.98	0.91	0.84	0.77
10	1.64	1.56	1.48	1.40	1.33	1.25	1.18	1.11	1.04	0.97	0.90	0.84
20	1.73	1.65	1.57	1.50	1.42	1.35	1.28	1.21	1.14	1.07	.00	0.94
40	1.86	1.79	1.71	1.64	1.56	1.49	1.42	1.35	1.28	1.22	.15	1.08
60	1.97	1.90	1.82	1.75	1.67	1.60	1.53	1.46	1.40	1.33	.26	1.19
80	2.06	1.99	1.91	1.84	1.77	1.70	1.63	1.56	1.49	1.42	.35	1.29
100	2.15	2.07	1.99	1.92	1.85	1.77	1.70	1.63	1.56	1.49	.43	1.36

Note. To determine capacity at any steam pressure and entering temperature, multiply constant from table by rated capacity at 60 F entering air and 2 psi.

When increasing steam pressure it is important to determine whether the heater is suitable for the increased pressure application and whether the resulting increased outlet temperature is satisfactory.

UNIT VENTILATORS. A considerable number of unit heaters, suitable for use in schools and public buildings ordinarily are designed for the ventilating requirement only, with sufficient direct radiation to provide for the calculated heat loss of the walls. The combination is termed a split system. The unit ordinarily is designed to heat to approximately 80 F outside air drawn into the apparatus. (See Fig. 32.)

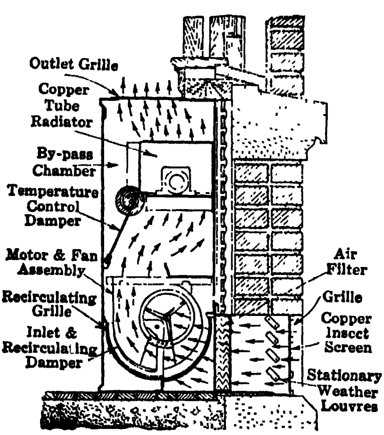


Fig. 32. Unit ventilator.

Unit ventilators ordinarily are placed under a window. They comprise a small ventilating fan, fan motor, and indirect heating surface of either cast-iron or extended-surface copper coils, all enclosed in a finished sheet steel casing, with an outside and recirculating intake. Each intake has a damper, and all dampers usually are so connected that the full opening of one closes the other. The fan draws in either all outside air, all recirculated air, or a mixture of the two as may be desired by the manipulation of the dampers. The fan discharge may be by-passed around the heater, all passed through the heater, or a portion only by-passed by means of a damper ordinarily under thermostatic control. The temperature of the air leaving the unit is controlled by the by-pass damper, the volume of air leaving the unit remaining constant.

10. HEATING BY ELECTRICITY

The relations involved and the underlying principles of electrical heating by resistance are expressed by these equations:

$$I = \frac{E}{R}; \quad R = \frac{kl}{a}; \quad W = I \times E = I^2R; \quad H = 3.413W = 3.413I^2R \text{ per hr}$$

where I = current flowing, amperes; E = electromotive force or potential difference, volts; R = resistance of conductor, ohms; k = coefficient of specific resistance, ohms per square inch per foot of conductor; l = length of conductor, feet; a = area of cross section, square inches; W = watts = volts × amperes; H = heating effect, Btu per hour.

It is evident that the heating value of a conductor varies as the square of the current flowing and directly with the resistances. For given potential differences, the current varies inversely as the resistance; hence, increasing the resistance only, in such a case, actually would cut down the heat supplied. The resistance varies with the material, with the length of conductor, and inversely as the area of the conductor. For a conductor of given material and length, subjected to a definite voltage, there is a certain diameter and corresponding resistance that will give the maximum heating effect, which can be determined from the above equations.

The economy of electrical heating depends entirely on the cost of generating and distributing the electricity, as all the energy supplied, if the heater is of the direct type, is converted into heat at the radiator. In addition to this, account must be taken of the simplicity, additional convenience, reduced attendance and repairs, and small space occupied by the electrical equipment. The actual money value of these factors is hard to determine. Any comparison with other forms of heating usually is based on heating effect obtained per pound of coal burned, or per dollar paid for fuel and electricity. Only when electricity can be generated at a very low cost, as in certain hydroelectric plants, can it be used for heating on a basis of equal cost compared with the ordinary steam or hot-water system of heating.

A new form of electrical heating, however, which currently is rapidly growing in importance, is the *heat pump*. By this means electricity is used to abstract heat from "cold" bodies, such as the earth, or even the atmosphere, and make it available in normal heating applications. For a complete discussion, see Arts. 12 to 16, inclusive.

EXAMPLE. A house-heating boiler burns coal of 12,000 Btu per lb with 60% efficiency. The coal required to give the heating effect of 1 kw hr is 3413/(0.60 × 12,000) = 0.47 lb. With a generating set thermal efficiency of 15%, the coal required at 60% efficiency to produce 1 kw-hr for electrical heating is

$$\frac{3413}{0.15 \times 0.60 \times 12{,}000} = 3.16 \text{ lb}$$

The relative cost of fuel at the same price per ton is, therefore, 3.16/0.47 = 6.7 times greater for electrical heating than for direct heating.

PANEL HEATING

By B. F. Raber and F. W. Hutchinson

11. DESIGN OF PANEL HEATING SYSTEMS

Panel heating for comfort is a method of controlling the rate of body heat loss by utilization of large, moderate temperature (80 to 160 F), flat surfaces located within thermal "sight" of the occupant. In most installations the surfaces used are ceiling, walls, or floor of the room. Various methods of bringing energy to the heating surface are in use. **Warm-air systems** provide plenum spaces above the ceiling or under the floor, the air acting merely as an energy-bearing working substance and not being admitted to the occupied enclosure. **Electrical systems** utilize resistance wiring embedded (after insulation) in the structure, or built into some type of fabric or prefabricated section for direct application to the walls or ceiling. Steam is rarely used in panel heating because of undesirably high temperature when at positive gage pressure. The essence of panel design is maintenance of a surface temperature in the range of 80 to 120 F (under unusual circumstances wall panels may be designed to operate at temperatures up to 160 F); hence, hot water is the most suitable working fluid.

Hot-water systems consist of coils or grids of ferrous pipe or nonferrous tube embedded in the structure a short distance (rarely over 1 in.) back of the heating surface. For ceiling installations pipe or tube in the diameter range of $^3/_8$ to $^3/_4$ in. is commonly attached either above or below metal lath and then plastered in. When coils are below the lath a heavy, more expensive plaster, is required, whereas location of coils above the lath will usually prevent complete embedment in the plaster.

ADVANTAGES AND DISADVANTAGES. From the standpoint of comfort, panel heating has been advocated as a means of offsetting the cold-wall effect commonly present during low-temperature weather in rooms heated by 100% convection systems. For residences and other nonmechanically ventilated structures there is no appreciable opportunity, with panel heating, of establishing a mean radiant temperature (taken approximately as the average surface temperature) more than two or three degrees greater than the room air temperature; hence, for such structures, optimum room air temperature will be in the range of 67 to 70 F. In larger structures where untempered air is introduced mechanically at air change rates in the range of 2 to 8 changes per hour the depression of optimum inside air temperature below 70 F may be of the order of 5 to 10 F; in such cases panel heating provides a definite and substantial reduction in total heat requirements.

The most important advantages of panel systems over usual methods of convection heating are: (1) Complete invisibility of heating elements. (2) Cleanliness (no inaccessible dust-collecting spaces). (3) Higher relative humidity because of lower dry bulb temperature; hence reduced drying rate on furniture, etc. (4) Warmer floors; hence greater comfort for small children.

The most important disadvantages of the systems are: (1) Sluggishness in responding to load changes. (2) Lack of sufficient available area to permit floor panels in average structures located in cold (−20 F) climates. (3) Difficulty of repair or alteration.

FLOOR VERSUS WALL VERSUS CEILING PANEL. The problem of selecting a location for heating panels is complex and controversial. From the thermal standpoint the ceiling is definitely better than the floor since, with operation at a surface temperature of 120 F, unit area of ceiling panel will dissipate 222% as much heat as unit area of floor at 85 F panel; the required area of ceiling panel is therefore less than one-half as great as that of equivalent floor panel. Reduced area is advantageous from the standpoint of first cost and also in that it permits use of panel heating in climates where insufficient total surface would be available if the dissipation rate of a floor at 85 F could not be exceeded.

Another advantage of the ceiling as a panel location is greater freedom from interference effects such as occur with floor panels because of furniture location and carpeting. By careful distribution of total panel area in a periphery ceiling pattern excellent uniformity of heating effect can be readily achieved. The surface temperature of unheated floors in a room with ceiling panels will be slightly greater than room air temperature; hence the advantage of warm floors is realized irrespective of panel location.

Advantages of floor over ceiling are greater simplicity of installation and reduced hazard to room furnishings in event of leakage. The first cost, per unit area, of floor panels is often less than that of ceiling units. However, since the required floor panel area is more than double that of ceiling units, it follows that first cost becomes favorable to floor installations only when the cost of unit area drops to less than 50% that of a ceiling panel.

BASIS OF DESIGN. Irrespective of the means used to bring energy to the heating surface, the major problem of panel heating design is to determine the area of panel—operating at design surface temperature—which is necessary to maintenance of a comfortable environment. An alternative design problem is determining the required surface temperature of a panel of arbitrarily selected area. For small structures—as residences—the latter design method is commonly used with panel area taken as 17, 20, or 23% of the total inside room surface area, whereas, for larger buildings, or for mechanically ventilated homes, the former method is most widely used.

Once the area and design surface temperature have been determined the remaining part of the design involves bringing energy—in warm air, hot water, or as electricity—to the panel surface.

GENERALIZED DESIGN PROCEDURE. The rational design of a panel heating system differs considerably from that for a convection system. The major point of distinction is that irradiation of unheated exterior surfaces sufficiently alters the usual combined convection-radiation inside film coefficient so that computation of room heat load by the customary equation, $Q = UA\Delta t$, is no longer accurate. A preferable procedure is to write a series of heat balances on the various room surfaces and solve the resultant equations simultaneously. To simplify this procedure an equivalent overall coefficient of heat transfer is defined by the equation

$$U_e = \frac{U_f A_f + U_c A_c + U_g A_g + U_i A_i + U_w A_w}{A_f + A_c + A_g + A_i + A_w} \tag{1}$$

where the U and A terms are overall heat transfer coefficients and areas, respectively, whereas the subscripts f, c, g, i, and w refer to the floor, ceiling, glass (windows), interior partitions, and exterior walls of the actual room. An equivalent conductance is then defined as

$$C_e = \frac{1}{\dfrac{1}{U_e} - \dfrac{1}{h_i}} \tag{2}$$

where h_i is the customary combined convection-radiation inside film coefficient [usually taken as 1.65 Btu/(hr)(sq ft)(°F)].

An overall heat balance can be written (Ref. 1) on the unheated room surface area, A_e, as

$$C_e A_e(t_e - t_o) = 0.8A_e(t_a - t_e) + 1.08A_p(t_p - t_e) \tag{3}$$

which simplifies to

$$t_e = \frac{1.08ut_p + 0.8vt_a + C_e vt_o}{1.08u + 0.8v + C_e v} \tag{4}$$

where t_e, t_p, t_a, t_o are temperatures of unheated inside surface, panel surface, inside air, and outside air, respectively. The terms u and v are, respectively, the heated and unheated fractions of total room surface.

A similar heat balance on ventilation air gives

$$0.24W(t_a - t_o) = h_p(t_p - t_a)A_p + 0.8A_e(t_e - t_a) \tag{5}$$

which simplifies to

$$t_a = \frac{h_p ut_p + 0.8vt_e + 0.018V_c t_o}{h_p u + 0.8v + 0.018V_c} \tag{6}$$

where W and V_c are the outside air ventilation rate expressed in pounds per hour and in cubic feet per hour per square foot of total surface, respectively. The term h_p is the film coefficient for convective transfer from the panel and can be taken as 0.4, 0.7, and 1.1 for ceiling, wall, and floor panels, respectively.

A heat balance on the occupant is written in the form of the so-called comfort equation (Ref. 1),

$$t_a = 140 - ut_p - vt_e \tag{7}$$

The simultaneous solution of eqs. 4, 6, and 7 will permit elimination of t_e and t_a as unknowns and the determination of panel area (for assumed panel surface temperature) or of panel temperature (for assumed panel area). Graphical solutions of these three equations are available in the literature (Ref. 2) for all common design conditions.

SPECIALIZED DESIGN PROCEDURE. For residences and other naturally ventilated structures a simplification can be achieved by fixing V_c as 2, which corresponds to one-half to one and a half air changes (depending on the geometry of the particular room) and is of adequate accuracy for most design applications. With panel area (u) fixed, solution of the three equations for panel temperature leads to a simple equation of the form

$$t_p = \frac{K_1 + K_2 C_e}{K_3 + K_4 C_e} \tag{8}$$

where the K terms are numerical coefficients whose value depends on panel location, design, outside temperature, and panel area. Table 1 evaluates these coefficients for the range of conditions that occur in practice.

Thus, for residential design a simplified procedure is to calculate V_c and C_e (eqs. 1 and 2), select the outside design temperature, and select the panel area (taking $u = 0.20$ as a first, and usually acceptable, value). Then go to Table 1 for values of K_1, K_2, K_3, K_4 and, using these values, solve eq. 8 for the panel temperature.

An approximate evaluation of total room load (*including* ventilation load) can then be obtained from the equations:

For ceiling panels $q = 1.48A_p(t_p - 70)$ (9)

For wall panels $q = 1.78A_p(t_p - 70)$ (10)

For floor panels $q = 1.18A_p(t_p - 70)$ (11)

Table 1. Design Coefficients

CEILING PANEL

t_o	$u = 0.17$		$u = 0.20$		$u = 0.23$	
	$K_3 = 0.3270$; $K_4 = 0.1530$		$K_3 = 0.3783$; $K_4 = 0.1722$		$K_3 = 0.4256$; $K_4 = 0.1889$	
	K_1	K_2	K_1	K_2	K_1	K_2
−20	27.16	98.72	30.40	93.90	33.73	88.76
−10	26.71	89.15	29.96	84.81	33.30	80.37
0	26.26	79.58	29.52	75.72	32.87	71.98
10	25.81	70.01	29.08	66.61	32.44	63.59
20	25.36	60.44	28.64	57.51	32.01	55.20
30	24.91	50.90	28.20	48.53	31.57	46.81

WALL PANEL

t_o	$u = 0.17$		$u = 0.20$		$u = 0.23$	
	$K_3 = 0.4050$; $K_4 = 0.2026$		$K_3 = 0.4708$; $K_4 = 0.2300$		$K_3 = 0.5344$; $K_4 = 0.2544$	
	K_1	K_2	K_1	K_2	K_1	K_2
−20	32.46	103.32	37.04	101.36	41.33	97.01
−10	32.01	94.40	36.60	91.91	40.90	88.19
0	31.56	85.48	36.16	82.42	40.47	79.38
10	31.11	76.56	35.72	72.95	40.04	70.57
20	30.67	67.64	35.28	63.47	39.61	61.76
30	30.21	58.72	34.84	53.99	39.17	52.95

FLOOR PANEL

t_o	$u = 0.17$		$u = 0.20$		$u = 0.23$	
	$K_3 = 0.5095$; $K_4 = 0.2686$		$K_3 = 0.5948$; $K_4 = 0.3066$		$K_3 = 0.6790$, $K_4 = 0.3414$	
	K_1	K_2	K_1	K_2	K_1	K_2
−20	39.76	114.16	45.64	111.38	51.43	107.98
−10	39.31	103.77	45.20	101.39	51.00	98.63
0	38.86	93.38	44.76	91.40	50.57	89.28
10	38.41	82.99	44.32	81.40	50.14	79.93
20	37.96	72.60	43.88	71.41	49.71	70.58
30	37.51	62.21	43.43	61.42	49.27	61.23

REQUIRED WATER TEMPERATURE AND SPACING. Experiment has shown (Ref. 3) that pipe or tube diameter, within the center-to-center range of 4 through 9 in., has very little effect on energy dissipation rate from embedded coils or grids. With accuracy to 90% the transfer rate can be taken as 0.77 Btu per hr per *lineal* foot of pipe or tube per 1 F temperature difference between water and panel surface. Thus, with panel-surface temperature, t_p, known as a result of the design calculations, the mean temperature of water in the panel—for a selected tube spacing—can be readily calculated by the equation

$$t_w = t_p + \frac{q}{0.77A_pL}$$

where q is calculated from eqs. 9, 10, or 11 and L is the selected lineal feet of pipe or tube per square foot of panel ($L = 1$ for 12-in. spacing; $L = 1\,{}^1/_2$ for 9-in. spacing; $L = 2$ for 6-in. spacing; $L = 3$ for 4-in. spacing).

The flow rate through the panel can be determined by dividing q by 500 and again by the arbitrarily selected temperature drop (often taken as 10 F) of the water passing through the panel.

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1. B. F. Raber and F. W. Hutchinson, *Panel Heating and Cooling Analysis*, John Wiley and Sons, New York, 1947.
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HEAT PUMPS

By E. R. Ambrose and Theodore Baumeister

HEAT PUMP is the name applied in present commercial practice to a year-round air-conditioning system employing refrigeration equipment in a manner which enables a surface to deliver usable heat to a space during the winter period and to abstract heat from the same space during the summer period. When operating as a *heating* system, one surface absorbs heat from an outside medium and delivers it at a higher temperature level, together with the heat equivalent of the work of compression, to a second surface, which in turn gives it up to the space to be heated. When operating on the *cooling* cycle, one of the two surfaces absorbs heat from the conditioned space and delivers it, together with the heat equivalent of the work of compression, to the other surface, which in turn rejects it to the outside medium.

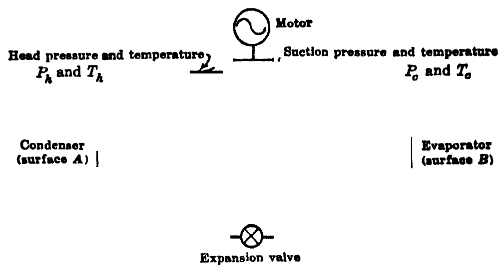


FIG. 1. Schematic arrangement of the vapor compression heat pump cycle.

A simple vapor-compression heat-pump system is illustrated in Fig. 1. The high-pressure vapor discharged from the compressor at the head pressure, P_h, is condensed to a liquid in the condenser A. From the condenser, the liquid refrigerant passes through the expansion valve to the evaporator B at the suction pressure, P_c, where it changes into a vapor by absorption of the latent heat of vaporization from an external source. From the evaporator, the refrigerant vapor returns to the compressor to complete the cycle. The heat pump uses the heat rejected from the condenser to heat the conditioned space during one period, and the refrigerating effect created in the evaporator to cool the space during the other period.

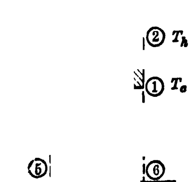


FIG. 2a. Carnot cycle temperature-entropy diagram of the heat pump cycle.

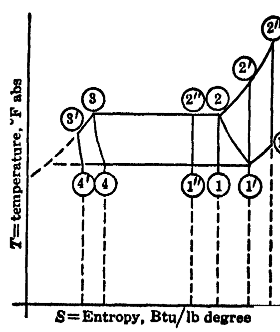


FIG. 2b. Rankine cycle temperature-entropy diagram of the heat pump cycle. 1–2 wet compression; 1′–2′ dry compression; 1″–2″ wet compression; 1‴–2‴ superheated compression; 3–4 constant enthalpy expansion, no subcooling; 3′–4′ constant enthalpy expansion, with subcooling.

The amount of heat rejected, Q_h, and the refrigerating effect produced, Q_c, together with the changes in the state of the refrigerant during the cycle, are best shown by the conventional temperature-entropy diagram. The Carnot cycle, which represents the highest possible limit of performance between two temperatures, T_h and T_c, is shown in Fig. 2a, where phase 1–2 is isentropic compression, 2–3 is isothermal compression or condensation, 3–4 is isentropic expansion, 4–1 is isothermal expansion or evaporation. The Rankine cycle, which is representative of the vapor-compression system of Fig. 1, is shown in Fig. 2b and substitutes a constant enthalpy phase in the expansion valve for phase 3–4 of the Carnot cycle. Wet, dry, and superheated compression, as well as liquid subcooling, are all shown in Fig. 2b. Work of the compressor is most conveniently evaluated by use of the Mollier chart for the refrigerant (see Section 11).

Coefficient of performance, C_p, is defined as

$$C_p = \frac{\text{Useful heat delivered, } Q_h}{\text{Work}} \qquad \text{(Heating cycle)} \qquad (1)$$

$$C_p = \frac{\text{Refrigeration, } Q_c}{\text{Work}} \qquad \text{(Cooling cycle)} \qquad (2)$$

For the Carnot cycle, these values are measured by the absolute temperatures, T_c and T_h, so that

$$C_p \text{ (Carnot) } \quad \frac{T_h}{T_h - T_c} \quad \text{(Heating cycle)} \tag{3}$$

$$C_p \text{ (Carnot) } = \frac{}{T_h - T_c} \quad \text{(Cooling cycle)} \tag{4}$$

Between any two given temperatures, the Carnot cycle gives the highest coefficients. Actual machines and systems run at lower values and may be referred to the Carnot or Rankine cycle as a standard. Actual performance of heat pumps is usually of the order of 40 to 75% of the theoretical.

12. BASIC HEAT-PUMP DESIGNS

The four basic types of heat-pump systems for comfort heating and cooling are: (1) air to air (refrigerant flow reversed), (2) air to air (fixed refrigerant circuit), (3) liquid to air (refrigerant flow reversed), and (4) liquid to air (fixed refrigerant circuit).

AIR-TO-AIR DESIGN (REFRIGERANT FLOW REVERSED)—FIG. 3a. During the cooling cycle, outdoor air is used to take rejected heat from refrigerant and air to transfer the cooling effect from refrigerant to conditioned space. During the heating cycle, outdoor air is used as the source of heat for refrigerant and air to transfer the heat from refrigerant to conditioned space.

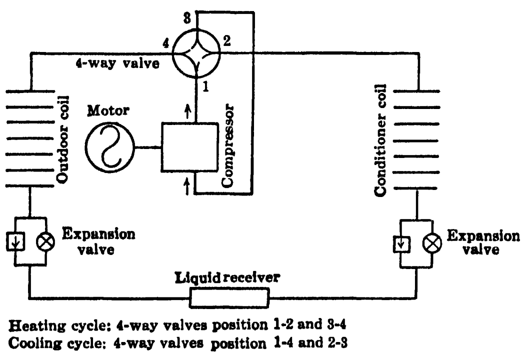


FIG. 3a. Basic heat pump, air-to-air design with reversible refrigerant flow.

To cool the conditioned space, the compressor delivers the hot compressed refrigerant gas through 4-way valve path 1–4 to the outdoor coil, where it is condensed, giving up the latent heat of condensation to the outside air. From the outside air coil, the liquid refrigerant flows through a check valve to the liquid receiver, then through the expansion valve to the conditioner coil. In the conditioner coil, the liquid refrigerant is changed into a gas absorbing the heat of vaporization from the air going to the space to be conditioned. From the conditioner coil, the refrigerant gas returns through the 4-way valve path 2–3 to the compressor to repeat the cycle.

To heat the conditioned space, the compressor delivers the hot compressed refrigerant gas through the 4-way valve path 1–2 to the conditioner coil where it is condensed, giving up the latent heat of condensation to the air going to the conditioned space. From the conditioner coil, the liquid refrigerant flows through the check valve, the liquid receiver, and the expansion valve to the outdoor air coil. In the outdoor air coil, the liquid refrigerant changes into a gas, absorbing the heat of vaporization from the outdoor air. From the outdoor air coil, the refrigerant gas returns through the 4-way valve path 4–3 to the compressor to repeat the cycle.

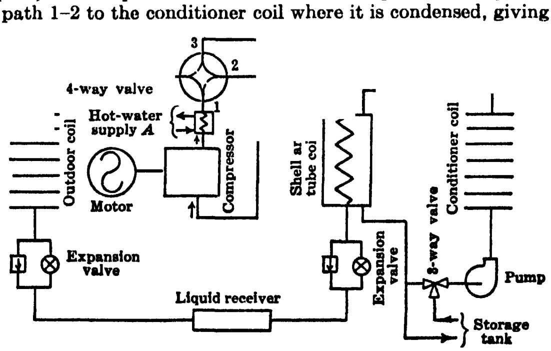
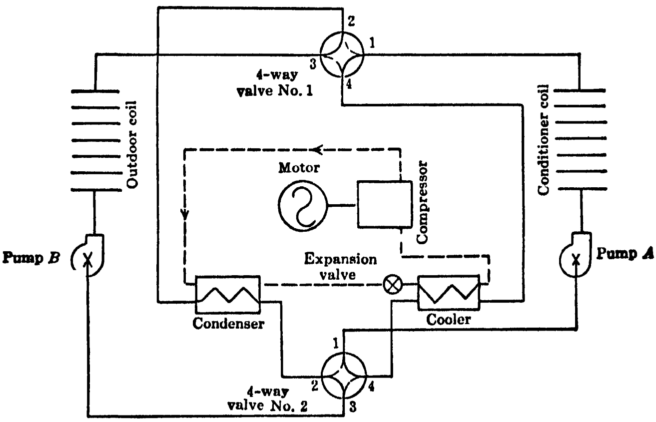


FIG. 3b. Basic heat pump, air-to-liquid design.

The air-to-liquid design, using a storage tank (Fig. 3b), is similar to the air-to-air design (Fig. 3a) except that a shell and tube surface is added to the circuit. A storage tank is employed in the design as indicated by Fig. 3*b*. In such a design, when using the storage tank, it is possible, when on the heating cycle, to store hot water during mild weather for use when lower outdoor temperatures are experienced, and when on the cooling cycle to store cold water during mild weather for use when higher outdoor temperatures are experienced. It is possible to store either hot or cold water, depending on the cycle of operation, during the night or offpeak periods for use during the day or when the demand is greatest. During both the cooling and heating cycles, the pump circulates water in a closed loop from the water coil and/or the storage tank, depending on the position of the 3-way valve, through the conditioner coil.

AIR-TO-AIR DESIGN (FIXED REFRIGERANT CIRCUIT)—FIG. 4. In this design the refrigerant circuit is fixed, going from the compressor to the condenser, through the expansion valve and cooler, back to the compressor. The heat rejected by the refrigerant



Heating cycle: 4-way valves position 1-2 and 3-4
Cooling cycle: 4-way valves position 1-4 and 2-3

FIG. 4. Basic heat pump, air-to-air design with fixed refrigerant circuit.

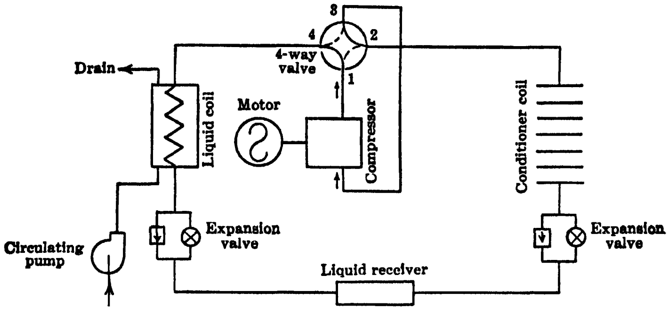
in the condenser or the cooling effect produced in the cooler is transferred to the air by means of an intermediate circulating liquid.

During the cooling cycle, pump *A* circulates the colder liquid through the conditioner coil, valve 1 path 1–4, the cooler, and valve 2 path 1–4 in a closed circuit. The circulating liquid transfers the heat absorbed from the air passing over the conditioner coil to the refrigerant in the cooler. Pump *B* circulates the warmer fluid through the outdoor coil, valve 1 path 3–2, condenser, and valve 2 path 2–3 in a closed circuit. The liquid in passing through the condenser absorbs heat from the refrigerant and gives it up to the outside air when passing through the outdoor coil.

During the heating cycle, pump *A* circulates the warmer fluid through the conditioner coil, valve 1 path 1–2, the condenser, and valve 2 path 2–1 in a closed circuit. In passing through the condenser, the liquid absorbs heat from the refrigerant which is transferred to the conditioned air passing over the conditioner coil. Pump *B* circulates the colder fluid through the outdoor coil, valve 1 path 3–4, the cooler, and valve 2 path 4–3 in a closed circuit. The liquid in passing through the outdoor coil absorbs heat from the outdoor air which is transferred to the refrigerant in the cooler.

LIQUID-TO-AIR DESIGN (REFRIGERANT FLOW REVERSED)—FIG. 5. In this design the earth, water, or any other suitable liquid is used as the heat source or for the heat rejected from the system, and air is used as the heating and cooling medium for the conditioned space.

During the cooling cycle, water from a well or the liquid from the ground coils is circulated by a pump through the liquid coil where it liquefies the refrigerant. The liquid is discharged to an exhaust well, or the drain if water is being used as a heat source, or is returned to the ground coils if earth is being used. The refrigerant path is from the compressor, through 4-way valve path 1–4, the liquid coil, the check valve, the liquid receiver, the expansion valve, the conditioner coil, and the 4-way valve path 2–3 back to the compressor to repeat the cycle. The refrigerant gas is liquefied in the liquid coil by giving up its latent heat of condensation to the water and is changed back into a gas in the conditioner coil by absorbing its heat of vaporization from the air going to the conditioned space.

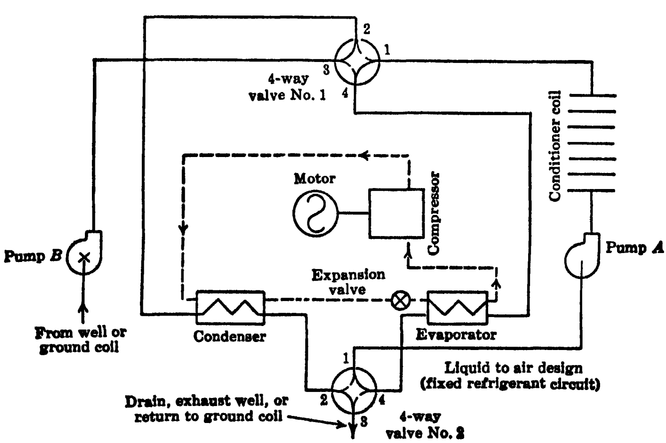


Heating cycle: 4-way valves position 1-2 and 3-4
Cooling cycle: 4-way valves position 1-4 and 2-3

FIG. 5. Basic heat pump, liquid-to-air design with reversible refrigerant flow.

During the heating cycle, the water from the well or the liquid from the ground coil is circulated through the liquid coil in a manner similar to that for the cooling cycle, except that heat is removed from the circulating liquid by the refrigerant. The refrigerant path is from the compressor, through 4-way valve path 1–2, the conditioner coil, the check valve, the liquid receiver, the expansion valve, the liquid coil, and the 4-way valve path 4–3 back to the compressor to repeat the cycle. Conversely to that for the cooling cycle, the hot compressed refrigerant gas from the compressor is liquefied in the conditioner coil by giving up its heat of condensation to the air going to the conditioned space and is changed back into a gas in the water coil by absorbing the heat of vaporization from the circulating liquid.

LIQUID-TO-AIR (FIXED REFRIGERANT CIRCUIT)—FIG. 6. This design is similar to the air-to-air design with a fixed refrigerant circuit (Fig. 4) except that earth



Heating cycle: 4-way valves position 1-2 and 3-4
Cooling cycle: 4-way valves position 1-4 and 2-3

FIG. 6. Basic heat pump, liquid-to-air design with fixed refrigerant circuit.

or water, or any suitable liquid other than air, is used as the heat source during the heating cycle and for the heat rejected from the system during the cooling cycle. The cycle of operation is the same as for the air-to-air design.

13. COMPARISON OF DESIGN FEATURES

The air-to-air design shown in Fig. 3a and the liquid-to-air design shown in Fig. 5 offer the simplest and most economical designs, from an equipment standpoint, when air, water, or the earth is used as the heat source. There are no intermediate heat-transfer surfaces so that the equipment and power requirements are the minimum and the coefficient of performance the maximum. However, the requirements of a highly reliable installation with minimum maintenance may favor one of the other designs. In the air-to-air and liquid-to-air designs, each of the two main surfaces serves the dual purpose of a condenser and evaporator. Care must be exercised to prevent trapping of refrigerant and oil. Since the reversing valve is located in the refrigerant lines a well-constructed, leakproof, reliable design is essential. Both the air-to-air design (Fig. 4) and the liquid-to-air design (Fig. 6), on the other hand, have fixed refrigerant circuits which reduce the difficulties with oil trapping, the amount of refrigerant piping, and the possibilities of refrigerant leaks. Equipment design is thus simplified, and the presence of a large quantity of liquid in the connecting piping acts as a thermal flywheel. The designs of Figs. 4 and 6 are particularly adaptable to installations where the outdoor coil must be located at a distance from the compressor or where multiple conditioner coils are required for zoning. The advantages of these indirect systems are somewhat offset by the lower suction and higher head pressures entailed by the necessary temperature gradients for the heat-transfer surfaces. Also, the circulating pumps increase the electrical input, thus reducing the coefficient of performance. Four 3-way valves or eight 2-way valves can be substituted for the 4-way valves shown in the diagrams in order to change the direction of the refrigerant or liquid flow. However, regardless of the type of valve employed, care must be exercised in valve design and selection to eliminate any possible leakage across the valve while maintaining the minimum flow resistance.

The usual precautions of a conventional refrigeration system must be followed in a heat-pump design. It is advisable to use oil traps in the high-pressure refrigerant line ahead of the compressor, to install the refrigerant piping and coils to prevent trapping of the oil and refrigerant in the system, and to proportion the refrigerant and liquid piping for most economical pressure drop.

14. HEAT SOURCES

The heat pump can use any source of low-grade heat. The coefficient of performance depends upon the temperature of the source. Air, water, or the earth are all in use, and at least one is conceivable in any location. An ideal source for comfort heating would be abundant, inexpensive, and at an average temperature of 50 to 80 F the year round.

WATER. Natural water from wells, lakes, and rivers, discharged water from manufacturing processes, or any other liquid whose temperature is too low for direct utilization will qualify, provided that it is chemically suited and does not require special metals or extensive treatment. The disposal of the water after heat abstraction constitutes a practical problem in many localities.

AIR. Outdoor air offers a universal heat source for the heat pump in locations where the minimum temperatures are not too low or of too long duration and where means can be found to offset the resulting loss of capacity caused by extremely low and wide fluctuations in daily temperatures. The main disadvantages of air as the heat source are that as the outdoor air temperature drops the heating demand of the structure increases and the output of the heat pump decreases and that as the outdoor temperature falls below 32 F frosting of the heat-absorbing coils is threatened. These disadvantages may be offset by having an auxiliary heat source or a storage tank and by providing a method of defrosting.

GROUND. The liquid-to-air designs (Figs. 5 and 6) are readily adaptable to the use of the ground as the heat source wherein a buried coil is substituted for the water source. Uniform temperature and universal availability of the ground make it attractive, but further study and research are needed to meet the problems on transfer of heat between the refrigerant and the ground during both heating and cooling cycles.

The few studies that have been made indicate that heat transfer is decidedly variable and depends on many factors including soil type, soil form, soil condition, moisture content, climatic conditions, and geological formation. The following data, from various sources, show that the *thermal conductivity* for buried pipe lines ranges from a minimum of 0.14 to a maximum of 2.62 Btu per sq ft per hr per °F per ft of thickness.

Thermal Conductivity. Preliminary tests indicate that a direct-expansion $^3/_4$-in. copper tube buried 3 ft or more below the earth's surface in soil with a thermal conductivity of approximately 1.0 Btu per sq ft per hr per °F per ft absorbs an average for the season of 30 Btu per linear ft per hr when the surrounding soil temperature is 50 F and the refrigerant temperature is 20 F.

Table 1. Thermal Conductivity of Soil

	Btu/hr/ft²/deg/ft Thickness
Clean dry white sand	0.14
Clean dry yellow sand	0.17
Clean yellow sand, 4% moisture	0.28
Clean yellow sand, 80% moisture	0.56
Ground, marshy, or constantly soaked	1.00
Soil, wet maximum	2.62
Soil, wet minimum	0.85
Soil, wet average	1.69
Soil, very light to dry	0.21
Soil, damp clay, or sand	0.92
Subsoil, wet	1.33

Data for table computed from Maker, *Special Problems in the Flow of Viscous Fluids*, Extension of the University of California, Feb. 3, 1938, and from *Oil and Gas Journal*, Sept. 15, 1945.

Water Heater. In all heat-pump designs it is possible to install a heat exchanger type of water heater in the hot-gas refrigerant line just ahead of the compressor to heat water for the humidifier or for general use. This arrangement can serve as a supplementary method to another independent hot-water heating system whenever the compressor is in operation during either the heating or the cooling cycle. Since the hot-water demand is not related to the space heating or cooling requirements, a water heater incorporated in the heat-pump design is not dependable within itself.

A self-contained unit, designed specifically for domestic hot-water supply, is shown in Fig. 7. With delivered hot-water temperatures of 120 to 140 F average, coefficients of performance of 2 1/2 to 4 are obtainable. Ambient air or any other heat source that is suitable for the year-round heat pump unit may be used.

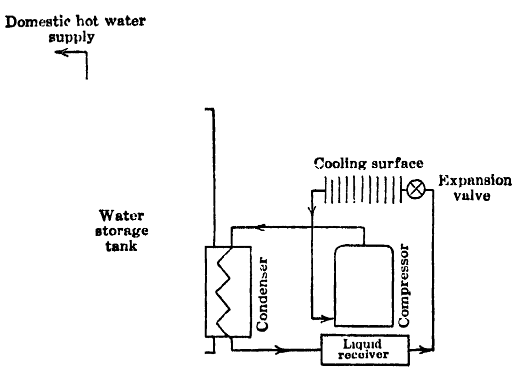


FIG. 7. Heat-pump water heater.

The equipment arrangement for a hot-water heat pump (Fig. 7) consists of the refrigerating compressor, cooling surface, expansion valve, condenser, water-storage tank, and connected piping assembled on a common base to give a compact, neat-appearing, self-contained unit. The expansion valve and all other working parts should be easily accessible for repair or removal. It is important to have a noncorrosive drain pan with a suitable drain connection for the evaporator surface. A gas-liquid heat exchanger can be incorporated in the design if it proves to be economically justifiable from a capacity and operation standpoint. The evaporator surface can be of either forced or natural circulation type.

15. ADVANTAGES AND DISADVANTAGES OF A HEAT-PUMP SYSTEM

The heat pump has all the inherent advantages of electric resistance heating plus the overall operating economy reflected in performance coefficients of 3 to 5. Instead of thermal efficiencies of 30%, this means values of 90 to 150% when referred to the fuel burned originally in the electric-generating plant. Since the same physical equipment is used both for cooling in summer and for heating in winter, the heat pump offers additional advantages over the conventional heating and cooling system:

Economy in Cost of Installation. By combining heating and cooling in the same equipment, the cost of installation is lower.

Compactness of Installation. The combination of heating and cooling in a single system results in minimum use of space.

Flexibility in Arrangement. The equipment can be located in the building, on the roof, in the basement, or elsewhere.

Concentration of Utility Service. By combining heating and cooling requirements into an all-year-round single system, the building can be operated on a single electric utility service without need for additional coal, oil, or gas.

The ordinary operating and maintenance difficulties of standard refrigeration systems are encountered. There are problems due to expansion and refrigerant valve failures, breakdown of compressor parts, refrigerant leaks, and defrosting.

COMPARATIVE HEATING COSTS. The cost of heating by alternative methods is best compared on the basis of the cost per million Btu delivered. Thus, the cost for fuel or electric energy is given by

$$\text{Cost per million Btu} = \frac{1{,}000{,}000 \times \text{Cost per unit}}{\text{Btu per unit} \times \text{Efficiency}} \tag{5}$$

$$1 \text{ kwhr} = 3413 \text{ Btu}$$

$$\text{Heat pump efficiency} = \text{Coefficient of performance}$$

$$\text{Resistance heating efficiency} = 1.0 \text{ (theoretical)}$$

Table 2 gives representative comparative heating costs on several alternative systems.

Table 2. Comparative Heating Costs

Method of Heating	Cost per Unit	Cost per Million Btu for Fuel or Equivalent
Electric resistance	1 1/2 cents per kwh	\$4.41
Electric resistance	2 cents per kwh	5.87
Heat pump, $C_p = 4$	1 1/2 cents per kwh	1.10
Heat pump, $C_p = 4$	2 cents per kwh	1.47
Heat pump, $C_p = 5$	1 1/2 cents per kwh	0.88
Heat pump, $C_p = 5$	2 cents per kwh	1.17
Heat pump, $C_p = 6$	1 1/2 cents per kwh	0.735
Heat pump, $C_p = 6$	2 cents per kwh	0.98
Coal, 12,000 Btu/lb	10 dollars per ton	0.69
Coal, 12,000 Btu/lb	15 dollars per ton	1.04
Oil, 140,000 Btu/gal	8 cents per gal	0.952
Oil, 140,000 Btu/gal	10 cents per gal	1.19
Natural gas, 900 Btu/cu ft	40 cents per M cu ft	0.741
Artificial gas, 550 Btu/cu ft	70 cents per M cu ft	2.12

Note. Fuel burning efficiencies taken as 0.60.

The *equipment and installation cost* of a heat pump cannot be directly compared with a conventional heating system because the heat pump is a year-round air-conditioning plant supplying both heating and cooling, whereas the conventional system supplies winter heating only. The heat-pump cost should be compared with the cost of a conventional heating system plus the cost of a conventional cooling system.

Several hundred heat pumps now are installed in the United States, ranging in size from 1/2 to 300 hp and furnishing heating and cooling to office buildings, commercial establishments, private homes, and dwellings. Climatic conditions and geographic location largely influence the selection of the heat source. Design and performance data are given in Table 3 for ten heat-pump systems installed in office buildings in the United States.

16. INDUSTRIAL APPLICATIONS

The industrial field offers opportunities for the economic utilization of the heat pump equal to if not more diverse than the field of comfort heating and cooling. Manufacturing plants, where cleanliness, temperature, humidity, and circulation of the atmosphere are controlled to improve product quality and to maintain the most favorable working conditions, offer real potentialities. The heat pump utilizes the same equipment for both the heating and cooling cycles, and these services may be provided simultaneously if desired. There are many industrial applications in which large quantities of cold liquid or hot liquid must be provided. Both hot and cold liquids may be required simultaneously in a plant where, by the use of the heat pump, the necessary refrigeration and the necessary heating can be accomplished in the same equipment with each service viewed as a mutual byproduct of the other.

Table 3. Design and Performance Data on Representative Office-building Heat-pump Installations

	Atlantic City Electric Co., Salem, N. J.	Southern California Edison Co., Whittier, Calif.	Ohio Power Co., Steubenville, Ohio	Southern California Edison Co., Montebello, Calif.	Southern California Edison Co., Santa Ana, Calif.	Southern California Edison Co., San Bernardino, Calif.	Westinghouse Electric Corp., Emeryville, Calif.	Ohio Power Co., Portsmouth, Ohio	Ohio Power Co., Coshocton, Ohio	Ohio Power Co., Brilliant, Ohio
1. Approximate date installed	1934	1937	1936	1938	1940	1937	1939	1940	1940	1946
2. Calculated:										
Heat loss, (Btu/hr) ÷ 1000	285	160	406.4	245.12	54.38	187.86	230	354.7	409.7	200
Heat gain, (Btu/hr) ÷ 1000	108	...	359	...	240.8	254.31	196.5	450.	404.7	164.85
3. Design temperature:										
Heating, inside dry bulb, °F	70	73	70	73	73	73	70	72	72	70
outside dry bulb, °F	0	30	0	30	35	30	35	0	−5	0
Cooling: inside, dry bulb, °F	80	78	80	78	78	78	75	78	78	78
inside relative humidity, %	50	50	50	50	50	50	62	50	50	50
outside dry bulb, °F	95	100	93	100	95	100	85	95	95	92
outside wet bulb, °F	75	70	75	70	67	70	68	75	75	72
4. Compressor, hp	4–5	1–10	2–20	1–10	2–7 1/2	1–10	2–7 1/2	2–25	1–10 and 1–15	1–15
5. Heat source:										
Well water	*	...	...	...	...	...	...	...	*	...
Outside air	...	*	*	*	*	*	*	*	...	*
6. Heat source, temperature, °F	57	Var.	Var.	Var.	Var.	Var.	Var.	Var.	55	Var.
7. Supplementary heating equipment	None	None						None	None	
Water	...	...	...	*	*	*	*	...	...	...
Electric resistance heater	...	...	15 kw	...	...	...	...	...	...	15 kw
Storage tank	...	...	*	...	...	...	...	...	...	*
8. Building volume, cu ft ÷ 1000	76.8	75.3	170	50.1	116.7	73.8	90	213	170	52
9. Average C_p †	3.5	...	3.5	3.81	...	4.28	3.5	3.0	3.6	3.5
10. Estimated annual energy consumption, kwh ÷ 1000										
Heating	...	7.996	...	8.556	21.8	11.440	...	156	70.72	...
Cooling	...	16.1	...	17	30.526	27.460	...	44	20.9	...
11. Estimated average annual consumption: kwh/1000 deg-days/1000 cu ft										
Heating	...	129.0	...	...	125.0	92.3	...	156.3	79.5	...
12. Reference	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

† Coefficients of performance, including all auxiliaries.

References: (1) *Trans. Am. Soc. Heating Ventilating Engrs.*, 1935. (2) Southern California Edison Co., *Bull.* 54. (3) *Heating and Ventilating*, Jan. 1944. (4) Southern California Edison Co., *Bull.* 54. (5) Southern California Edison Co., *Bull.* 54. (6) Southern California Edison Co., *Bull.* 54. (7) F. W. Jordon, Practical Aspects of the Heat Pump, *Electrical West*, Vol. 94, No. 4, April 1945, pp. 65–69. (8) *Trans. Am. Soc. Heating Ventilating Engrs.*, 1944. (9) *Trans. Am. Soc. Heating Ventilating Engrs.*, 1944. (10) *Trans. Am. Soc. Mech. Engrs.*, 1946.

EVAPORATION, DISTILLATION, CONCENTRATION, DRYING, AND DESICCATION. In industry a wide variety of applications of this type ideally suit the heat pump. The heat pump frequently will be competitive in this field with the conventional evaporator using steam from an external source. It offers the advantage of maintaining evaporating temperatures at low levels and within narrow ranges, thus safeguarding the delicate structure of many organic compounds and preventing loss of taste, aroma, flavor, or vitamin content. The goal is to evaporate a solution for recovery of distillate, for concentration of mass, or for extraction of solid content. The vapor issuing from the evaporator is led directly to the compressor suction (Figs. 8 and 9). It is then compressed to a suitable pressure for liquefaction and recovery in the condenser. Raw feed must be introduced into the evaporator shell for replenishment of the supply, and a blowdown system is necessary for removal of concentrate and accumulated solids. This cycle makes it possible to recover the solvent and thus conserve pure water or other valuable chemical reagent.

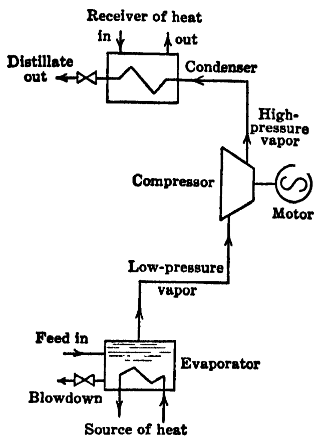


FIG. 8. Basic heat-pump flow diagram for industrial applications wherein the evaporated vapor is passed directly to the compressor and subsequently discharged as distillate from the condenser.

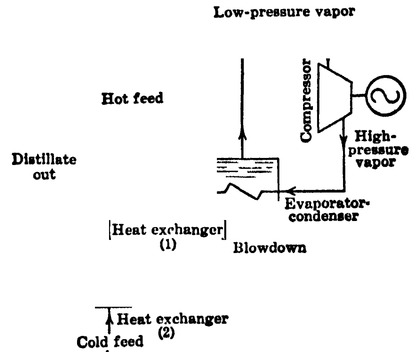


FIG. 9. Basic heat-pump or thermocompressor flow diagram for industrial applications with a combined evaporator-condenser and heat exchangers on blowdown and distillate for maximum thermal economy.

A rudimentary system like Fig. 8 can do this, but the ultimate development of the evaporative process is the *thermocompressor* as shown in Fig. 9, where the external source of heat is superseded by combining the condenser and evaporator into a single piece of equipment. In this cycle the low-pressure vapor leaves the evaporator shell, flows to the thermocompressor, and is discharged at a higher pressure to the condenser coil which is mounted in a submerged position within the evaporator shell. The saturation temperature, which is thus maintained on the condenser coil, is sufficient to support ebullition and evaporation in the shell. The condenser thus acts as the heat source for the evaporator.

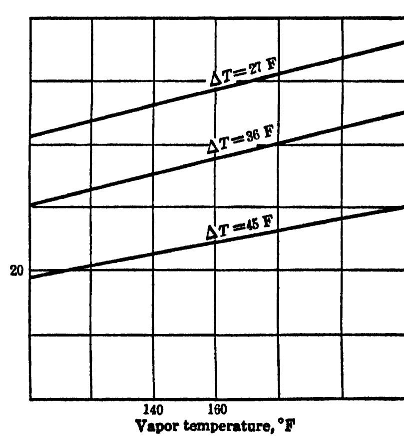


FIG. 10. Performance data for thermocompressor installation. ΔT is temperature difference between condensing and evaporating sides of heat-transfer surface.

The performance of a thermocompressor installation can be estimated by the data in Fig. 10. These curves are typical for operations involving aqueous solutions and are expressed on the basis of the weight of water evaporated per kilowatt-hour of energy input to the heat pump. The *evaporative economy* is expressed as a function of the saturation temperature (pressure) maintained in the evaporator shell for three representative values of temperature difference (ΔT) between the condensing and evaporating sides of the heat-transfer surface.

The curves show the improvement in economy which results from the use of high vapor temperatures and low mean temperature differences. It should be recalled that the mechanical equivalent of 1 kwh is 3413 Btu, and, with a latent heat of vaporization of approximately 1000 Btu per lb of water, the direct utilization of electric energy for evaporative purposes would result in the delivery of 3 to 4 lb of steam per kwh. The thermocompressor by the data of Fig. 10, on the other hand, makes it possible to raise this performance economy to a value of 20 to 60 lb of water per kwh for the practical range of operating conditions.

Industrial applications include the concentration of dyes and other chemicals in solution; the preparation of foodstuffs; the concentration of unfermented fruit juices, condensed milk, or sugar syrups; the evaporative process for preparing powdered milk, table salt, or sugar; the recovery of valuable solvents used in manufacturing operations; the distillation of water to remove impurities and foreign matter which would otherwise affect the potability or the utility of the water for manufacturing processes; and the distillation of make-up water in steam-power plants. The essential operating performance of several representative industrial installations is shown in Table 4. The compression distillation plant was brought to a high degree of perfection during World War II where the Army and Navy had need for the production of drinking water from sea water in many outlying areas and aboardship. Portable compression distilling plants were developed with a range

Table 4. Operating Results in Thermocompressor Plants for Evaporation and Distillation

Process	Input, Kwh	Evaporative Capacity, lb/hr	Water Evaporated, lb per kwh	Approximate C_p	Evaporator Temperature, °F	Suction Pressure, psia
1. Evaporating plant handling milk products	73	2,200	30.1	8.9	120	.....
2. Evaporating plant handling milk products and unfermented fruit juices	240	6,600	27.1	8.0	120	.....
3. Evaporating plant in chemical works	94	1,540	16.3	4.8	...	0.86
4. Water-evaporating plant for distillation of drinking water	75	2,750	36.6	10.6	212	14.65
5. Water-evaporating plant for distillation of drinking water from sea water	0.02	1	50.0	14.7	213	14.75
6. Estimated performance of make-up evaporator for a power plant	315	20,000	63	17	293	60

Data for table computed from Sporn, Ambrose, and Baumeister, *Heat Pumps*, John Wiley and Sons, 1947.

capacity up to 6000 gal per day. (See Latham, Compression Distillation, *Mech. Eng.*, March 1946.) These units were generally driven by automotive-type gasoline or Diesel engines which made them completely self-contained. Their servicing, maintenance, and operation required the minimum of supplementary training.

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VENTILATING

Revised and rewritten by John W. James

VENTILATION is the displacement of vitiated air from an inhabited room and its replacement with fresh air. It usually is expressed in the number of changes of air per hour. This is not strictly correct, as there is no positive change; the incoming air dilutes the foul air until it is suitable for respiration.

VENTILATION LAWS. Many states have laws stating the amount of ventilation to be supplied to public and semipublic buildings, temperature limits, etc. These codes and those of the various cities involved should be consulted before completing the design of any heating and ventilating system.

Table 1. Outside Air Ventilation Requirements

Type of Application	Cubic Feet per Minute per Person		Ceiling Height, feet				
			8	10	12	16	20
	Minimum	Preferred	Air Changes per Hour				
Apartment	10	15	3	2	1 1/2	1	3/4
Banking space	7 1/2	10	3	2	1 1/2	1	3/4
Barber shop	7 1/2	10	3	2	1 1/2	1	3/4
Beauty parlor	7 1/2	10	3	2	1 1/2	1	3/4
Brokers' board room	20	30	8	6	4 1/2	3	2 1/4
Cafeteria	15	20	6	4 1/2	3 1/2	2 1/2	1 3/4
Cocktail bar	20	30	8	6	4 1/2	3	2 1/4
Churches	5	7 1/2	3	2	1 1/2	1	3/4
Department store	7 1/2	15	3	2	1 1/2	1	3/4
Directors' room	30	40	8	6	4 1/2	3	2 1/4
Drugstore (no counter)	7 1/2	10	3	2	1 1/2	1	3/4
Drugstore (with counter)	10	15	5	3 3/4	3	2	1 1/2
Funeral parlor	5	7 1/2	3	2	1 1/2	1	3/4
Gambling rooms	20	30	8	6	4 1/2	3	2 1/4
Hospital room	10	15	3	2	1 1/2	1	3/4
Hotel room	10	15	3	2	1 1/2	1	3/4
Office, general	10	15	4	3	2 1/4	1 1/2	1
Office, private	10	15	5	3 3/4	3	2	1 1/2
Restaurant	12	15	5	3 3/4	3	2	1 1/2
Shop, retail	7 1/2	10	3	2	1 1/2	1	3/4
Theater	5	7 1/2					
Residence, or any room in a residence			3	2	1 1/2	1	3/4
Note: For general applications:							
Each person not smoking	5	7 1/2					
Each person smoking	20	30					

SYSTEMS OF VENTILATION. Ventilation systems are classified as: (1) Natural ventilation in which the movement of air in flues, ducts, etc., is induced by the thermal head produced by the difference in density due to the difference in temperature between the column of air in the ducts and that of the outside atmosphere. (2) Mechanical ventilation in which the movement of air is maintained by a positively driven fan. With the mechanical system, air at any desired temperature may be positively circulated to all parts of the building, giving practically uniform ventilation, regardless of outside weather conditions. For data regarding fan capacities in mechanical ventilating systems, see Blast Heating, p. 12-44, and Fans and Blowers, Section 1. For data on capacity of ventilators in natural ventilation, see Table 2.

Ventilation systems also are divided into (1) upward systems; (2) downward systems. Upward systems frequently are used for audience rooms. Air is supplied near the floor line through mushroom ventilators in the floor, through hollow pedestals in the chairs, or through floor registers. Vitiated air outlets are in the wall near the ceiling. The downward system generally is used in schoolrooms, hospitals, institutions, etc., where the occupants are not closely spaced. Air enters at 8 ft or more above the floor; the vitiated air outlet usually is at or near the floor line. Inlets and outlets should be placed if possible in the same inside wall, the former being at least 7 or 8 ft above the latter. Incoming air flows out across the room to the cold outside wall before it cools and drops to floor level. Practically uniform distribution of fresh air throughout the room is thus obtained. Theaters often are ventilated by a downward system, with the fresh air outlets located in the ceiling. (See p. 12-86.) Neither upward nor downward system can be characterized as superior; each has its proper place and limitations.

REQUIREMENTS FOR GOOD VENTILATION. Table 1 represents good ventilation practice with respect to amount of outside or new clean air to be introduced into rooms used for various purposes.

AUTOMATIC VENTILATORS. The chief differences in the various types of automatic ventilators for natural ventilation lie in the degree to which the wind is utilized to produce additional ventilation. Four general classes of such ventilators are: (1) stationary nonsiphoning type; (2) stationary siphoning type; (3) rotary plain head type; (4) rotary siphoning type.

Type 1 does not contemplate utilizing the wind to produce additional ventilating effect. Type 2 uses the wind to produce additional ventilating effect, by an ejector action. When there is no wind, ventilation is due to natural circulation.

The rotary or revolving head types 3 and 4 have a protecting cowl revolving on a central bearing. An opening in one side is always directed in the path of the wind through the movement of the cowl. The velocity of the wind creates a region of decreased pressure at the cowl opening, which in turn creates additional ventilation. Average results of a series of test conducted in 1920 by the Engineering Experiment Station of the Kansas State Agricultural College on different types of 10-in. automatic ventilators are given in Table 2.

Table 2. Air Velocities through Automatic Ventilators Produced by Various Wind Velocities

Wind Velocity, miles per hr	Velocity of Air in Ventilator, ft per min				Wind Velocity, miles per hr	Velocity of Air in Ventilator, ft per min			
	Stationary Nonsiphoning Type	Stationary Siphoning Type	Rotary Plain Head Type	Rotary Siphoning Type		Stationary Nonsiphoning Type	Stationary Siphoning Type	Rotary Plain Head Type	Rotary Siphoning Type
1	27	31	46	52	7	192	215	321	361
2	55	62	92	104	8	220	246	367	413
3	83	93	137	155	9	248	277	413	465
4	110	123	183	207	10	276	307	459	516
5	137	154	229	258	11	304	338	505	570
6	165	185	275	310	12	332	368	552	621

CAUSES OF AIR VITIATION. Air is vitiated by: (1) Generation of heat by occupants, lights, etc., above that required for warming. (2) Water vapor in excess of or below that required for desired relative humidity. (3) Dust, either brought in by the air or carried by persons within the building. (4) Bacteria. (5) Odors due to occupants or to manufacturing processes. The old theory that increased carbon dioxide and decreased oxygen content of the air are responsible for physical discomfort has been disproved. However, lack of ventilation may result in air conditions which are offensive and conducive to a feeling of lassitude. To alleviate these conditions, outside fresh air must be introduced in sufficient quantity to reduce the concentration of objectionable substances to a satisfactory level.

RELATION BETWEEN HUMIDITY AND TEMPERATURE. Air, on being heated, has its capacity for absorbing moisture greatly increased, giving the sensation due to so-called dry heat. This causes excessive and unnatural evaporation of moisture from the skin and respiratory organs, which lowers the surface temperature of the body and causes a temperature sensation several degrees lower than the actual temperature of the room. The remedy for too low or too high humidity in a room is the introduction or decrease of moisture present in the air. The relation of temperature and relative humidity to human comfort has been the subject of considerable research by the ASHVE. The results of this research are published in the *Transactions* of the Society. See *ASHVE Guide*, 1948, for comfort charts.

AIR MOTION. A certain amount of air motion is considered desirable. In still air the body is enveloped by a layer of warm and humid air, which a moderate air movement helps to blow away. An air movement that is just perceptible has a stimulating effect, but too great a velocity may cause extreme discomfort. The maximum air velocity comfortable to human beings at rest is approximately 50 ft per min.

DUST. The air contains a large amount of dust of various degrees of fineness and consisting of many different substances. Atmospheric impurities may be classified as: (1) *dust*—particles of 150 to 1 μ in diameter (micron, μ, is 1/25,000 in.; (2) *fumes*—particles of 0.2 to 1 μ, resulting from reactions such as distillation, oxidation of metal fumes, and purely chemical reactions; (3) *smokes*—particles less than 0.3 μ, resulting from incomplete combustion of materials such as coal, oil, tar, and tobacco. The hay-fever sufferer finds pollen from flowers and weeds a serious impurity in the air. Adequate filters are reasonably effective in reducing the pollen count.

BACTERIA. No rigid limits have been established for permissible bacteria content, but it is recognized that the degree of air contamination may be reduced by proper ventilation. The possibility of sterilizing air with ultraviolet light by destroying bacteria at their point of admission to the air stream shows some promise. Also the idea of employing bactericidal mists (propylene glycol and triethylene glycol) for controlling air-borne infection is being studied.

AIR FILTERS. The use of some form of air filter is a recognized necessity for the removal of dirt and dust from the air in a heating and ventilating system, or for the air supply to air compressors, internal-combustion engines, etc. Air filters often are used in conjunction with air-washer equipment when the removal of fine dust particles is considered important. They are distinguished from air washers in that they remove dirt and dust without a water spray, and hence do not change temperature or relative humidity of air passing through the filter.

All types of filters should (1) Be efficient in dirt removal. (2) Interpose low resistance to air flow. (3) Possess large dust-holding capacity. (4) Be easy to clean and handle. Air filters are classified as (1) dry filters; (2) viscous filters.

Dry filters were the first type to be used. They comprise a fine-mesh cloth, felt, or paper screen through which the air is filtered or strained. Several types of dry air filters are available. They are cleaned by rapping, by reversing the air flow, or by supplying new filter areas.

Viscous filters have largely superseded the dry type. Their cleaning action depends on the dirt impinging on a surface coated with a viscous fluid. All filters of this type operate on the principle of adhesive impingement. The viscous material employed should be odorless, fireproof, and germicidal in its action. This type of filter is constructed in units of various standard sizes so that practically any area of filtering surface may be obtained by the use of one or more units.

Electrostatic Cleaners. The inability of air filters effectively to remove fine dust particles and tobacco smoke has led to the development of the electrical precipitator. This method of precipitation consists of imparting an electrical charge to each dust particle by passing the air between electrodes of small wires and then collecting the dust on parallel plates as the air flows between them. (See Fly Ash Collection, Section 7.)

ODOR REMOVAL. When conditions make it necessary to remove odors, it is possible to absorb them in small perforated cylindrical canisters of activated carbon. Air washers should not be expected to deodorize air effectively, as few particles causing odor are soluble in water. Some odor problems may be solved by the dilution method, in which increased percentages of fresh outdoor air are introduced to the space being ventilated.

AIR CONDITIONING

Revised and rewritten by John W. James

AIR CONDITIONING generally is understood to mean the simultaneous control of temperature, relative humidity, air motion, air distribution, and ventilation within an enclosure. Air-conditioning systems are used in theaters, churches, auditoriums, schools, restaurants, offices, homes, etc., to produce or effect comfort for occupants by maintaining a temperature and relative humidity which will lie in the so-called comfort zone. The comfort zone for both winter and summer has been established by the ASHVE Research Laboratory. A system embodying many of the principles used in modern air conditioning was proposed for the U. S. Army hospitals by G. H. Knight in 1864.

Air-conditioning systems are used in industry to maintain the temperature and relative humidity most desirable for the process involved. For many industrial processes the most desirable temperature and relative humidity have been fairly well established. (See *ASHVE Guide*, 1948.)

The *addition* of moisture to air being circulated is humidification; the *removal* of moisture is dehumidification.

HUMIDIFICATION usually is effected by passing air through a finely divided water spray, in an air washer. This consists of a sheet-metal housing, enclosing one or more banks of spray nozzles. The air passes through the spray and then over bent plates, called eliminators, which abruptly change the direction of flow and remove free water held in suspension. Larger dust particles also are removed in the washer, but for complete dust elimination an air filter must precede the washer.

In some industrial plants, a steam jet is used for humidification. When both humidification and dehumidification are required for year-round operation an air washer is frequently used. Recirculated spray water is heated for humidification in winter and cooled for dehumidification in summer.

Surface-type coolers are used quite generally for summer comfort cooling. Cold water is circulated through the coils of the cooler or a refrigerant, such as the "Freon" group or methyl chloride, is evaporated in the coils. When temperature of cooling medium is lower than temperature of air entering cooler, dehumidification results. Artesian well water, if available, may be used as a cooling agent. Artificial refrigeration, however, is usual for comfort cooling.

17. HUMIDITY

Humidity is the water vapor (steam or moisture) mixed with air. The maximum weight of vapor which a given enclosure will contain depends only on the temperature and may be determined from steam tables, regardless of the presence or absence of any other vapor or gas. That is, the weight of vapor is exactly the same whether air is present or not. The pressure of the vapor is in accordance with Dalton's law.

Saturated Air. Air is saturated when it has mixed with it the maximum possible amount of vapor, which amount varies with temperature. The vapor itself under this condition also is saturated (quality $x = 1$). If the air is not saturated, the contained vapor is superheated. The *actual humidity* of the air, in meteorological work, is the number of grains (1 lb = 7000 grains) or pounds of water vapor contained in 1 cu ft of a mixture of air and vapor at the observed temperature.

Relative humidity, or degree of humidity, is the actual amount of moisture (grains or pounds) contained in 1 cu ft of the mixture divided by the amount which 1 cu ft of the mixture would hold at the same temperature if saturated. This condition is stated as per cent relative humidity.

Wet- and dry-bulb temperatures are the observed temperatures as indicated by wet- and dry-bulb thermometers respectively.

Dew-point temperature is the temperature corresponding to saturation (100% relative humidity) for a given weight of vapor. Any lowering of temperature produces a contraction of volume and partial condensation. Air with any amount of vapor or relative humidity has a dew point, as temperature may be lowered to a temperature where condensation begins.

EXAMPLE. Required the weight of vapor carried by 1 lb of air in a saturated air-vapor mixture at 60 F and atmospheric pressure (14.7 psia, or 29.92 in. Hg, absolute, at sea level).

Solution. Let p and p_1 = respectively, atmospheric and absolute partial pressures of air; p_2 = absolute partial pressure of the vapor corresponding to the temperature; all pressures in pounds per square inch. Then $p = p_1 + p_2 = 14.7$ at sea level.

From steam tables, at 60 F, $p_2 = 0.26$; density = 0.000828 lb per cu ft; $p_1 = 14.70 - 0.26 = 14.44$. From the relation $PV = MRT$, where $R = 53.35$ for air, $T = 459.6 + 60 = 519.6$, $P = 144 \times 14.44$, $M = 1$,

$$V = \frac{53.35 \times 519.6}{144 \times 14.44} = 13.33 \text{ cu ft} = \text{volume of air}$$

As the air and vapor occupy the same space, the volume of vapor also is 13.33 cu ft, whence the weight of saturated vapor in the mixture is $13.33 \times 0.000828 = 0.01104$ lb. Weight of vapor per cubic foot of mixture = $0.01104/13.33 = 0.000828$ lb = $0.000828 \times 7000 = 5.796$ grains.

TOTAL HEAT OF DRY AND SATURATED AIR. The total heat of dry air is $h = c_p t$, where h = total heat, Btu per pound; c_p = specific heat of air = 0.24 (usual figure); t = temperature of air, °F. In saturated air the "heat of the liquid" usually is neglected, the error introduced thereby being negligible in air-conditioning calculations. The total heat (sometimes referred to as the sigma function [Σ] or Carrier total heat) per 1 lb of dry air in the saturated air-vapor mixture is the heat required to raise the temperature of 1 lb of dry air from 0 F to saturation temperature t, and to evaporate the weight W of vapor in the mixture. Thus

$$h_m = c_p t + rW \tag{1}$$

where h_m = total heat of mixture, Btu per pound; r = latent heat of vapor at temperature t. For saturated air at 56 F, r = 1060.3, W = 0.00954 lb. Then

$$h_m = (0.24 \times 56) + (1060.3 \times 0.00954) = 23.555 \text{ Btu}$$

TOTAL HEAT OF PARTIALLY SATURATED AIR. A nonsaturated air-vapor mixture may become saturated by an adiabatic process (see below) at the wet-bulb temperature. The total heat of the mixture is the same as the total heat of saturated air at the wet-bulb temperature, as given by the psychrometric chart (Fig. 1). This relation is used in determining the total heat of partially saturated air.

WEIGHT OF VAPOR IN MIXTURE. Assume a saturated mixture of 1 lb of dry air, containing W lb of vapor at temperature t, to be heated at constant pressure to a dry-bulb temperature of t_d, with a corresponding wet-bulb temperature of t_w. If the mixture is cooled at constant pressure back to t, t is the dew-point temperature for a combination of dry- and wet-bulb temperature t_d and t_w, and weight W lb of vapor in the mixture is the same amount as contained in saturated air at the dew-point temperature. Thus if dry- and wet-bulb temperatures are known, the dew-point temperature can be determined from the psychrometric chart, and consequently the value of W.

Method of Mixtures. If x lb of air is mixed with y lb of air at dry-bulb temperatures t_1 and t_2, respectively, the resultant dry-bulb temperature is

$$t_3 = \frac{xt_1 + yt_2}{x + y} \tag{2}$$

If W_1 and W_2 = weight of moisture per 1 lb of air at dry-bulb temperatures t_1 and t_2, respectively, the weight of moisture per 1 lb of air in the mixture is

$$W_3 = \frac{xW_1 + yW_2}{x + y} \tag{3}$$

From the psychrometric chart (Fig. 1) the dew point corresponding to W_3 can be found. These relations often are used in air-conditioning work.

ADIABATIC SATURATION OF AIR. Air not completely saturated, when passed through a spray or over the surface of water housed in a completely insulated enclosure, will pick up vapor evaporated from the water. Dry-bulb temperature t_d of the leaving air will be lower than that of the entering air. If leaving air is saturated, its temperature will be the same as the wet-bulb temperature t_w of the entering air, which remains constant during the process. Since no external heat is supplied, and as heat is necessary to evaporate water, evidently the heat exchange has been between the air passing through the apparatus and the water. Such an exchange without transfer of external heat is called an adiabatic process. Let W_1 = initial vapor content, pound per pound of dry air, corresponding to dry- and wet-bulb temperatures t_d and t_w, respectively; W_2 = final vapor content corresponding to temperature t_w; r_w = latent heat corresponding to t_w, Btu per pound of vapor; c_p = specific heat of dry air = 0.24; c_s = specific heat of vapor = 0.4423 + $0.00018t_d$. Then heat to evaporate weight of vapor added to mixture = $r_w(W_2 - W_1)$ Btu per 1 lb of dry air, and $r_w(W_2 - W_1) = (c_p + c_sW_1)(t_d - t_w)$;

$$W_1 = \frac{r_wW_2 - c_p(t_d - t_w)}{r_w + c_s(t_d - t_w)} \tag{4}$$

AIR WASHERS. The ability of a washer to *cool* air depends on fineness of spray, water pressure at the nozzles, arrangement of nozzles, number of banks of nozzles through which air passes, and direction of spray discharge from the banks.

Humidifying efficiency of an air washer, considered as a cooling device, is

$$E = 1 - \frac{t_{dl} - t_w}{t_{de} - t_w} = \frac{t_{de} - t_{dl}}{t_{de} - t_w} \tag{5}$$

where E = efficiency, per cent; t_{de} and t_{dl} = dry-bulb temperatures of entering and leaving air, respectively; t_w = constant wet-bulb temperature. Values of E attained in practice with various arrangement of nozzles are: 2 banks upstream, 1 bank downstream, E = 1.00; 2 banks upstream, E = 0.95; 1 bank upstream, 1 bank downstream, E = 0.85; 1 bank upstream, E = 0.80; 1 bank downstream, E = 0.65.

Air washers are rated at 400 to 500 cu ft of air per min per sq ft of cross-sectional area through the spray chamber. The pump and spray nozzles should supply 5 to 6 gal of water per 1000 cu ft of air per min, the individual spray nozzles being rated at 1 1/2 gal per

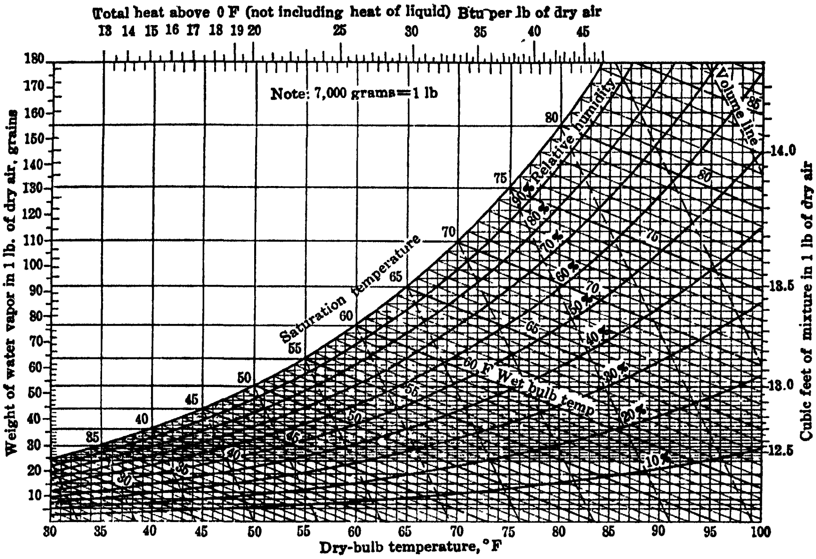

FIG. 1. Psychrometric chart (barometric pressure 29.92 in. Hg). (Courtesy of Buffalo Forge Co.)

min. The friction loss of air through a washer varies with the type. For 500 ft per min velocity it is approximately 0.25 in.

Psychrometric Chart (Fig. 1).

EXAMPLES IN THE USE OF CHART AND DIAGRAM. Required the relative humidity for a dry-bulb reading of 84 F and a wet-bulb reading of 66 F

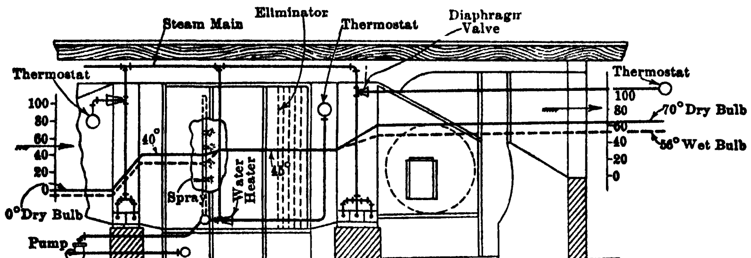

FIG. 2. Temperature relations in humidifying apparatus, measured in degrees Fahrenheit.

Solution. The intersection of a diagonal line through 66 F on the *saturation curve*, and the vertical through 84 F, dry bulb on the base line gives, approximately, 38% for the relative humidity. The *dew-point* temperature is found by tracing the corresponding *constant weight vapor line* to its intersection with the *saturation curve*, giving 56 F for the above condition. The actual weight of vapor per pound of dry air may be read directly from the saturation curve for the *dew-point* temperature of 56 F and is 66.8 grains or 0.00955 lb.

Humidifying. Assume a room temperature of 70 F and 40% relative humidity to be maintained when outside temperature is 0 F. Locate intersection of vertical 70 F dry-bulb temperature with 40% relative humidity curve, follow the *constant weight vapor line*, passing through the intersection, to its intersection with the saturation curve or 45 F corresponding to 0.0063 lb or 44.2 grains of vapor per lb of dry air. This is then the temperature at which saturated air must leave washer, and is the temperature for which thermostat controlling spray water heater must be set. (See Fig. 2.)

The heat per pound of dry air required for the tempering coil and water heater is read at the top of the diagram, and is 17.5 Btu to raise temperature of incoming air from 0 to 45 F and saturate it at this temperature. The additional heat required for the reheater will depend upon the final temperature desired for air entering the room.

Air Cooling. Entering air 89 F, dry bulb, and a relative humidity of 50% corresponding to a wet-bulb temperature of 74 F, wet-bulb depression (89 − 74) or 15 F. If the *humidifying efficiency* of a washer is 60%, temperature drop will be 15 × 0.60 or 9.0 F. Temperature of leaving air = (89 − 9.0) or 80 F. Wet-bulb temperature remains constant at 74 F. (See Fig. 3.)

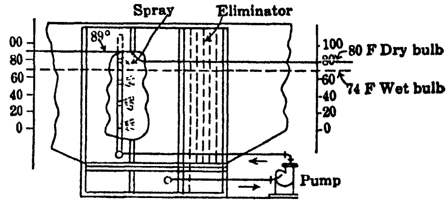

Fig. 3. Temperature relations in air cooler.

Drying. Outside air temperature 75 F and 50% relative humidity; heater to raise its temperature to 100 F, at which temperature it is introduced into drier. Air to leave drier, 70% saturated. From intersection of vertical 75 F dry-bulb temperature line and 40% relative humidity curve, follow the (horizontal) *equal weight of vapor line* until it intersects the vertical 100 F line corresponding to 70.4° wet bulb. Go diagonally to the left along a line of constant wet-bulb temperature to the intersection with the 70% curve. The corresponding dry-bulb temperature is 77.7 F, which is the required temperature of leaving air.

The weight of moisture evaporated per pound of dry air circulated is the difference between the weight of vapor per pound of dry air for 77.7 F and 70% humidity and 75 F and 50% humidity or 99.5 − 64.3 = 35.2 grains or 0.00503 lb.

18. COMFORT COOLING

Air is recirculated, and only enough outside air is introduced to provide for ventilation requirements, approximately 10 to 15 cu ft per min per occupant. Inside dry-bulb temperature to be maintained at breathing line should not exceed 10 to 15 F below outside design dry-bulb temperature. Another frequently used rule states that inside dry-bulb temperature should be midway between 70 F and outside temperature. Both rules are commonly disregarded, to the considerable discomfort of those concerned. The minimum dry-bulb temperature of air leaving cooler or washer depends largely on the distribution, which should be such as to prevent cold drafts over the occupants. Normally this temperature should be not less than dew-point temperature to be maintained in the room. A rule-of-thumb method for limiting minimum room inlet air temperature for horizontal outlets is: 2 F per foot of height of bottom of outlet above floor, below the room temperature.

Weight of air to be circulated depends on sensible heat to be removed. The sensible head load to be removed from the room is made up of: (1) Motors, electric lights, and electrical heating appliances = total watts × 3.41 Btu per hr. (2) Occupants at rest in 80 F air, 225 Btu per hr per person. (3) Heat gain from windows and glass blocks based on data given in Tables 1 and 2. (4) Heat gain from roofs (Table 3) and walls (Table 4). (5) Heat loss from insulated steam pipes (approximately 90 Btu per sq ft of pipe surface per hr). (6) Infiltration of air = pounds of air per hr × 0.24 × temperature difference between outside and inside air.

The so-called latent heat load is the heat to be removed by the washer or cooler to condense moisture that is to be removed from the air circulated through the cooler or washer. (See eq. 13.)

Example using Tables 3 and 4. Find the rate of heat flow into a room at 2 P.M. in July for an outdoor design temperature of 105 F and indoor conditions of 78 F for a 4-in. stone concrete roof covered with an average depth of 4-in. cinder concrete on which is placed a 3/8-in. thick felt roof with 1/2-in. pitch and slag surface exposed to sun. Daily range of temperature 30 F, that is, outdoor temperature 75 F at 4 or 5 A.M. Location is central part of United States.

Solution. Make correction in equivalent temperature differential according to notes in Table 3. The correction for 27 F design temperature difference is (27 − 15) = +12. The correction for 30 F daily range is −(30 − 20)/2 = −5. Net correction is +12 − 5 = +7 F.

To select the equivalent temperature differential the construction is assumed to be equal approximately to an uninsulated 6-in. concrete roof for which temperature is found from Table 3 to be 38 F. With above correction (38 + 7) temperature is equal to 45 F.

The overall transmission coefficient for roofs in summer should be selected using inside surface conductance of 1.2 and for outside 4.0. No distinction need be made between winter and summer coefficients for walls (Table 4) because there is little difference in the values.

U for roof is calculated as follows:

$$R = \frac{1}{1.2} + \frac{4}{12} + \frac{4}{4.9} + \frac{0.375}{1.33} + \frac{0.50}{1.00} + \frac{1}{4.0}$$

$$R = 3.03 \quad \text{or} \quad U = 0.33$$

Heat flow rate for roof is then 45 × 0.33 = 14.85 Btu per hr per sq ft.

Table 1. Instantaneous Rates of Heat Gain Due to Solar and Sky Radiation for Single Sheets of Unshaded Common Window Glass

(Computed for Solar Declination of 18 Degrees—August 1)

Note. To determine the total instantaneous rate of heat gain add the term 1.04 ($t_o - t_i$) to the values shown in the table.

Sun Time	Solar Altitude, degrees	Instantaneous Rate of Heat Gain, Btu per Hour for Each Square Foot of Unshaded Glass								
		N	NE	E	SE	S	SW	W	NW	Horizontal
		40 Degrees North Latitude								
5 A.M.	1.5	7	18	17	6	1	1	1	1	2
6	11.5	23	106	120	62	5	5	5	5	24
7	23.0	15	141	181	118	11	11	11	11	82
8	34.5	14	122	194	147	19	14	14	14	145
9	45.5	15	76	172	156	42	15	15	15	196
10	56.0	16	30	125	144	66	16	16	16	235
11	64.5	16	16	53	110	85	22	16	16	261
12	68.0	16	16	16	62	94	62	16	16	269
1 P.M.	64.5	16	16	16	22	85	110	52	16	261
2	56.0	16	16	16	16	66	144	125	30	235
3	45.5	15	15	15	15	42	156	172	76	196
4	34.5	14	14	14	14	19	147	194	122	145
5	23.0	15	11	11	11	11	118	181	141	82
6	11.5	23	5	5	5	5	62	120	106	24
7	1.5	7	1	1	1	1	6	17	18	2

Table compiled from solar radiation transmission data developed by ASHVE Research Laboratory. Values are for relatively clear atmosphere at sea level. For hazy atmosphere values may be reduced 10%. Above sea level add 1% per 1000 ft elevation.

If outside canvas awnings are used, multiply above values by 0.30; inside roller shade (aluminum finish) fully drawn by 0.45; inside venetian blind (aluminum finish) slats at 45 degrees fully covering window by 0.75.

Table 2. Total Instantaneous Rates of Heat Gain for Glass Blocks on Design Day of August 1

(Values tabulated include solar and sky radiation, convection, and conduction averaged for four types of blocks)

Mean Sun Time	Total Instantaneous Rate of Heat Gain, Btu per Hour for Each Square Foot of Sunlit Glass Block					
	Vertical Surface Facing					
	East	West	South			
North Latitude, degrees	30 to 45	30 to 45	30	35	40	45
7 A.M.	61	....	−4.5	−2.0	−0.5	1.0
8	78	......	0.0	2.0	4.0	5.0
9	74	5.0	5.0	7.0	10	12
10	58	6.5	11	15	18	21
11	45	7.5	17	22	26	32
12 noon	37	11	22	28	34	41
1 P.M.	30	22	25	32	39	46
2	24	35	26	32	39	47
3	20	55	24	30	37	45
4	16	77	20	26	32	41
5	13	86	15	20	25	34
6	11	55	9.5	14	18	26
7	8	19	3.5	7.0	11	18

From Heat Gain through Glass Blocks by Solar Radiation and Transmittance, by F. C. Houghten, et al., *ASHVE Transactions*, 1940.

Table 3. Total Equivalent Temperature Differentials for Calculating Heat Gain through Sunlit and Shaded Roofs

Description of Roof Construction *	8 A.M.	10 A.M.	12 N.	2 P.M.	4 P.M.	6 P.M.	8 P.M.	10 P.M.	12 M.
	Sun Time								
	Local Time								
Light Construction Roofs—Exposed to Sun									
1 in. Wood † or 1 in. Wood † + 1 in. or 2 in. insulation	12	38	54	62	50	26	10	4	0
Medium Construction Roofs—Exposed to Sun									
2 in. Concrete or 2 in. Concrete + 1 or 2 in. insulation or 2 in. Wood †	6	30	48	58	50	32	14	6	2
2 in. Gypsum or 2 in. Gypsum + 1 in. Insulation; 1 in. Wood † or 2 in. Wood † or 2 in. Concrete or 2 in. Gypsum + 4 in. rock wool in furred ceiling	0	20	40	52	54	42	20	10	6
4 in. Concrete or 4 in. Concrete with 2 in. insulation	0	20	38	50	52	40	22	12	6
Heavy Construction Roofs - Exposed to Sun									
6 in. Concrete	4	6	24	38	46	44	32	18	12
6 in. Concrete + 2 in. insulation	6	6	20	34	42	44	34	20	14
Roofs Covered with Water—Exposed to Sun									
Light construction roof with 1 in. water	0	4	16	22	18	14	10	2	0
Heavy construction roof with 1 in. water	−2	−2	−4	10	14	16	14	10	6
Any roof with 6 in. water	−2	0	0	6	10	10	8	4	0
Roofs with Roof Sprays—Exposed to Sun									
Light construction	0	4	12	18	16	14	10	2	0
Heavy construction	−2	−2	2	8	12	14	12	10	6
Roofs in Shade									
Light construction	−4	0	6	12	14	12	8	2	0
Medium construction	−4	−2	2	8	12	12	10	6	2
Heavy construction	−2	−2	0	4	8	10	10	8	4

* Includes 3/8 in. felt roofing with or without slag. May also be used for shingle roof.
† Nominal thickness of the wood.

Equation: $\left\{\begin{array}{l}\text{Total heat transmission from solar}\\ \text{transmission and temperature dif-}\\ \text{ference between outside and room}\\ \text{air. Btu per (hr) (sq ft) of roof area}\end{array}\right\} = \left\{\begin{array}{l}\text{Temperature differ-}\\ \text{ential from above}\\ \text{table}\end{array}\right\} \times \left\{\begin{array}{l}\text{Heat transmission co-}\\ \text{efficient for summer}\\ \text{Btu per (hr) (sq ft)}\\ \text{°F}\end{array}\right\}$

Notes.

1. *Source.* From paper, Solar heat gain through walls and roofs for cooling load calculations, by J. P. Stewart, *Journal ASHVE*, 1948. Estimated for July in 40° north latitude. For typical design day where the maximum outdoor temperature is 95 F and minimum temperature at night is 75 F (daily range of temperature, 20 F) mean 24-hr temperature 84 F for a room temperature of 80 F. All roofs have been assumed a dark color which absorb 90% of solar radiation, and reflect only 10%.
2. *Application.* These values may be used for all normal air-conditioning estimates; usually without correction, in latitude 0 to 50° north or south when the load is calculated for the hottest weather.

(*Table notes continued on p. 12-80*)

Note 5 explains how to adjust the temperature differential for other room and outdoor temperatures as well as for intermediate seasons when solar radiation is different.

3. *Peaked Roofs.* If the roof is peaked and the heat gain is primarily due to solar radiation, use for the area of the roof, the area projected on a horizontal plane.

4. *Attics.* If the ceiling is insulated and if a fan is used in the attic for positive ventilation, the total temperature differential for a roof exposed to the sun may be decreased 25%.

5. *Corrections. For temperature difference when outdoor maximum design temperature minus Room is different from 15 F.* If the outdoor design temperature minus room temperature is different from the base of 15 F, correct as follows: When the difference is greater (or less) than 15 F add the excess to (or subtract the deficiency from) the above differentials.

For outdoor daily range of temperature other than 20 F. If the daily range of temperature is less than 20 F, add 1° for every 2° lower daily range; if the daily range is greater than 20 F, subtract 1° for every 2° higher daily range. For example, the daily range in Miami, Florida, is 12 F or 8° less than 20 F; therefore, the correction is + 4 F at all hours of the day.

Light Colors. Credit should not be taken for light-colored roofs except where the permanence of the light color is established by experience, as in rural areas or where there is little smoke. When the exterior surface of roof exposed to the sun is a light color, such as white or aluminum (which absorb approximately 50% and reflect 50% of the solar radiation) add to the temperature differential for roof in shade 55% of the difference between the roof in sun and roof in shade. When the roof exposed to the sun is a medium color such as light gray, blue, or green, or bright red, add 80% of this difference.

Table 4. Total Equivalent Temperature Differentials for Calculating Heat Gain through Sunlit and Shaded Walls

North Latitude Wall Facing	Sun Time																	
	8 A.M.		10 A.M.		12 N.		2 P.M.		4 P.M.		6 P.M.		8 P.M.		10 P.M.		12 M.	
	Local Time																	
	Exterior color of Wall—D = dark, L = light																	
	D	L	D	L	D	L	D	L	D	L	D	L	D	L	D	L	D	L
Frame																		
NE	22	10	24	12	14	10	12	10	14	14	14	14	10	10	6	4	2	2
E	30	14	36	18	32	16	12	12	14	14	14	14	10	10	6	6	2	2
SE	13	6	26	16	28	18	24	16	16	14	14	14	10	10	6	4	2	2
S	−4	−4	4	0	22	12	30	20	26	20	16	14	10	10	6	6	2	2
SW	−4	−4	0	−2	6	4	26	22	40	28	42	28	24	20	6	4	2	2
W	−4	−4	0	0	6	6	20	12	40	28	48	34	22	22	8	8	2	2
NW	−4	−4	0	−2	6	4	12	10	24	20	40	26	34	24	6	4	2	2
N (Shade)	−4	−4	−2	−2	4	4	10	10	14	14	12	12	8	8	4	4	0	0
4-in. Brick or Stone Veneer + Frame																		
NE	−2	−4	24	12	20	10	10	6	12	10	14	14	12	12	10	10	6	4
E	2	0	30	14	31	17	14	14	12	12	14	14	12	12	10	8	6	6
SE	2	−2	20	10	28	16	26	16	18	14	14	14	12	12	10	8	6	6
S	−4	−4	−2	−2	12	6	24	16	26	18	20	16	12	12	8	8	4	4
SW	0	−2	0	−2	2	2	12	8	32	22	36	26	34	24	10	8	6	6
W	0	−2	0	0	4	2	10	8	26	18	40	28	42	28	16	14	6	6
NW	−4	−4	−2	−2	2	2	8	6	12	12	30	22	34	24	12	10	6	6
N (Shade)	−4	−4	−2	−2	0	0	6	6	10	10	12	12	12	12	8	8	4	4
8-in. Hollow Tile or 8-in. Cinder Block																		
NE	0	0	0	0	20	10	16	10	10	6	12	10	14	12	12	10	8	8
E	4	2	12	4	24	12	26	14	20	12	12	10	14	12	14	10	10	8
SE	2	0	2	0	16	8	20	12	20	14	14	12	14	12	12	10	8	6
S	0	0	0	0	2	0	12	6	24	14	26	16	20	14	12	10	8	6
SW	2	0	2	0	2	0	6	4	12	10	26	18	30	20	26	18	8	6
W	4	2	4	2	4	2	6	4	10	8	18	14	30	22	32	22	18	14
NW	0	0	0	0	2	0	4	2	8	6	12	10	22	18	30	22	10	8
N (Shade)	−2	−2	−2	−2	−2	−2	0	0	6	6	10	10	10	10	10	10	6	6

Table 4. Total Equivalent Temperature Differentials for Calculating Heat Gain Through Sunlit and Shaded Walls—*Continued*

North Latitude Wall Facing	Sun Time																	
	8 A.M.		10 A.M.		12 N.		2 P.M.		4 P.M.		6 P.M.		8 P.M.		10 P.M.		12 M.	
	Local Time																	
	Exterior color of Wall—D = dark, L = light																	
	D	*L*	*D*	*L*	*D*	*L*	*D*	*L*	*D*	*L*	*D*	*L*	*D*	*L*	*D*	*L*	*D*	*L*
8-in. Brick or 12-in. Hollow Tile or 12-in. Cinder Block																		
NE	2	2	2	2	10	2	16	8	14	8	10	6	10	8	10	10	10	8
E	8	6	8	6	14	8	18	10	18	10	14	8	14	10	14	10	12	10
SE	8	4	6	4	6	4	14	10	18	12	16	12	12	10	12	10	12	10
S	4	2	4	2	4	2	4	2	10	6	16	10	16	12	12	10	10	8
SW	8	4	6	4	6	4	8	4	10	6	12	8	20	12	24	16	20	14
W	8	4	6	4	6	6	8	6	10	6	14	8	20	16	24	16	24	16
NW	2	2	2	2	2	2	4	2	6	4	8	6	10	8	16	14	18	14
N (Shade)	0	0	0	0	0	0	0	0	2	2	6	6	8	8	8	8	6	6
12-in. Brick																		
NE	8	6	8	6	8	4	8	4	10	4	12	6	12	6	10	6	10	6
E	12	8	12	8	12	8	10	6	12	8	14	10	14	10	14	8	14	8
SE	10	6	10	6	10	6	10	6	10	6	12	8	14	10	14	10	12	8
S	8	6	8	6	6	4	6	4	6	4	8	4	10	6	12	8	12	8
SW	10	6	10	6	10	6	10	6	10	6	10	8	10	8	12	8	14	10
W	12	8	12	8	12	8	10	6	10	6	10	6	10	6	12	8	16	10
NW	8	6	8	6	8	4	8	4	8	4	8	4	8	6	10	6	10	6
N (Shade)	4	4	2	2	2	2	2	2	2	2	2	2	2	2	4	4	6	6
8-in. Concrete or Stone or 6-in. or 8-in. Concrete Block																		
NE	4	2	4	0	16	8	14	8	10	6	12	8	12	10	10	8	8	6
E	6	4	14	8	24	12	24	12	18	10	14	10	14	10	12	10	10	8
SE	6	2	6	4	16	10	18	12	18	12	14	12	12	10	12	10	10	8
S	2	1	2	1	4	1	12	6	16	12	18	12	14	12	10	8	8	6
SW	6	2	4	2	6	2	8	4	14	10	22	16	24	16	22	16	10	8
W	6	4	6	4	6	4	8	6	12	8	20	14	28	18	26	18	14	10
NW	4	2	4	0	4	2	4	4	6	6	12	10	20	14	22	16	8	6
N (Shade)	0	0	0	0	0	0	2	2	4	4	6	6	8	8	6	6	4	4
12-in. Concrete or Stone																		
NE	6	4	6	2	6	2	14	8	14	8	10	8	10	8	12	10	10	8
E	10	6	8	6	10	6	18	10	18	12	16	10	12	10	14	10	14	10
SE	8	4	8	4	6	4	14	8	16	10	16	10	14	10	12	10	12	10
S	6	4	4	2	4	2	4	2	10	6	14	10	16	12	14	10	10	8
SW	8	4	8	4	6	4	6	4	8	6	10	8	18	14	20	14	18	12
W	10	6	8	6	8	6	10	6	10	6	12	8	16	10	24	14	22	14
NW	6	4	6	2	6	2	6	4	6	4	8	6	10	8	18	12	20	14
N (Shade)	0	0	0	0	0	0	0	0	2	2	4	4	6	6	8	8	6	6

Equation: $\left\{\begin{array}{l}\text{Total heat transmission from solar}\\ \text{transmission and temperature dif-}\\ \text{ference between outside and room}\\ \text{air Btu per (hr) (sq ft wall area)}\end{array}\right\} = \left\{\begin{array}{l}\text{Temperature differ-}\\ \text{ential from above}\\ \text{table}\end{array}\right\} \times \left\{\begin{array}{l}\text{Heat transmission co-}\\ \text{efficient for wall Btu}\\ \text{per (hr) (sq ft) °F}\end{array}\right\}$

Notes.

1. *Source.* From paper, Solar heat gain through walls and roofs for cooling load calculations, by J. P. Stewart, *Journal ASHVE*, 1948. A north wall has been assumed to be a wall in the shade; this is

(*Table notes continued on p. 12-82*)

practically true. Dark colors on exterior surface of walls have been assumed to absorb 90% of solar radiation and reflect 10%; white colors absorb 50% and reflect 50%. This includes some allowance for dust and dirt since clean, fresh white paint normally absorbs only 40% of solar radiation.

2. *Application.* These values may be used for all normal air-conditioning estimates, usually without corrections, when the load is calculated for the hottest weather. Correction for latitude (Note 3) is necessary only where extreme accuracy is required. There may be jobs where the indoor room temperature is considerably above or below 80 F or where the outdoor design temperature is considerably above 95 F, in which case it may be desirable to make correction to the temperature differentials shown. The solar intensity on all walls other than east and west varies considerably with time of year.

3. *Corrections. Outdoor minus room temperature.* If the outdoor maximum design temperature minus room temperature is different from the base of 15 F, correct as follows. When the difference is greater (or less) than 15 F add the excess to (or subtract the deficiency from) the above differentials.

Outdoor daily range temperature. If the daily range of temperature is less than 20 F add 1° to every 2° lower daily range; if the daily range is greater than 20 F, subtract 1° for every 2° higher daily range. For example, the daily range in Miami, Florida, is 12 F, or 8° less than 20 F; therefore, the correction is +4 F.

Color of exterior surface of wall. Use temperature differentials for light walls only where the permanence of the light wall is established by experience. For cream colors use the values for light walls. For medium colors interpolate half way between the dark and light values. Medium colors are medium blue, medium green, bright red, light brown, unpainted wood, natural color concrete, etc. Dark blue, red, brown, green, etc., are considered dark colors.

19. AIR-CONDITIONING CALCULATIONS

Notation. t_{od}, t_{op}, t_{ow} = respectively, dry-bulb, dew-point, and wet-bulb design temperatures of outside air; t_{id}, t_{ip}, t_{iw} = respectively, dry-bulb, dew-point, and wet-bulb design temperatures of inside air; t_{ed}, t_{ep}, t_{ew} = respectively, dry-bulb, dew-point, and wet-bulb temperatures of air entering cooling apparatus; t_{ld}, t_{lp}, t_{lw} = respectively, dry-bulb. dew-point, and wet-bulb temperatures of air leaving cooling apparatus; t_{rd}, t_{rp}, t_{rw} = respectively, dry-bulb, dew-point, and wet-bulb temperatures of air entering room; w_o, w_i, w_e, w_l, w_r = weight of vapor per 1 lb of dry air, corresponding to dew-point temperatures with the same subscripts; h_1, h_2 = respectively, total heat, Btu per 1 lb of air-vapor mixture entering and leaving cooling apparatus, corresponding to wet-bulb temperatures t_{ew} and t_{lw}; H_s = computed estimate of sensible heat to be removed, Btu per hour, due to occupants, lights, electrical machinery, heat transmission, sun effect, infiltration, etc.; H_v = sensible heat to be removed, Btu per hour due to air introduced for ventilation; H_l = latent heat load, Btu per hour; W_1 = weight of vapor added to room air by occupants, infiltration, and other sources, pounds per hour; each occupant at rest in 80 F air gives off 0.17 lb of vapor per hr; W_2 = weight of vapor added to room return air by ventilation requirements, pounds per hour; M = weight of air to be circulated through room and cooling apparatus, pounds per hour; M_o = infiltration, pounds per hour; M_v = ventilation requirements, pounds per hour = $60n \times$ cubic feet of outside air per person $\times$ 0.075; n = number of occupants; c = cubic feet per minute of standard air, 70 F; 0.24 = specific heat of air at constant pressure. Then

$$W_1 = 0.17n + M_o(w_o - w_i) \tag{6}$$

$$W_2 = M_v(w_o - w_i) \tag{7}$$

$$M = \frac{H_s}{0.24}(t_{id} - t_{rd}) \tag{8}$$

The weight of air M is based on sensible heat to be removed from room. It does not include sensible heat to be removed by cooler from outside air drawn in at the cooler for ventilation. This latter must be included in the sensible heat to be removed by the cooler.

The air leaving cooler and entering room must contain a weight of vapor of

$$w_r = w_i - \frac{W_1}{M} \tag{9}$$

The corresponding dew-point temperature t_{rp} is found from Fig. 1.

DEHUMIDIFYING AIR WASHER (Fig. 4*A*). The air leaves a dehumidifying air washer in a saturated condition at temperature t_{ld}, which is the dew-point temperature of air entering the room, or $t_{ld} = t_{rp}$. The saturated air must be warmed by a heater, or by mixing room air by-passed around the washer, to a final dry-bulb temperature of t_{rd}. Dry-bulb temperature of air entering cooling apparatus is, by method of mixtures,

$$t_{ed} = \left(1 - \frac{M_v}{M}\right) t_{id} + \frac{M_v}{M} t_{od} \tag{10}$$

Weight of moisture per 1 lb of air entering cooling apparatus is, by method of mixtures,

$$w_e = \left(1 - \frac{M_v}{M}\right) w_i + \frac{M_v}{M} w_o \tag{11}$$

corresponding to a dew-point temperature of t_{ep}. Dry-bulb temperature of t_{ed} being known, wet-bulb temperature t_{ew} is obtained from the psychrometric chart, and h_1 thus is determined. The heat to be extracted from air circulated by the cooling apparatus is $M(h_1 - h_2)$ Btu per hour, or

$$\text{Tons of refrigeration} = \frac{M(h_1 - h_2)}{12{,}000} \tag{12}$$

The cooler must condense the weight of vapor $(W_1 + W_2)$, and also must remove the sensible heat load $(H_s + H_v)$.

$$H_v = 0.24(t_{od} - t_{id})M_v \tag{13}$$

Latent heat load may be approximated as

$$H_l = 1060(W_1 + W_2) \tag{14}$$

in which 1060 is the assumed latent heat of vapor at the temperature at which it is condensed. This assumption is not strictly correct. For most practical purposes

$$\text{Tons of refrigeration} = (H_s + H_v)\quad \frac{H_l}{12{,}000} \tag{15}$$

This approximation may be used for a by-pass dehumidifying washer or a surface-cooling unit, but not for a dehumidifying washer without by-pass. In the latter, the heat to be removed by the washer is somewhat greater.

Dehumidifying Washer with By-pass (Fig. 4*B*). To avoid necessity of reheating to dry-bulb temperature, t_{rd}, the saturated air leaving the washer at dew-point temperature t_{lp}, and also to reduce materially the refrigeration required, a return air by-pass around the washer is essential. Let x, y, and z represent the fractional part of M that is outside air for ventilation, recirculated air through washer, and by-passed air around washer, respectively. Then $x + y + z = 1$.

By the method of mixtures we may write the equations

$$zt_{id} + (x + y)t_{lp} = t_{rd} \tag{16}$$

$$zt_{ip} + (x + y)t_{lp} = t_{rp} \tag{17}$$

Equation 17 is based on the fact that, for comparatively small temperature ranges, dew-point temperatures may be substituted, without appreciable error, for the weight of vapor per pound of dry air.

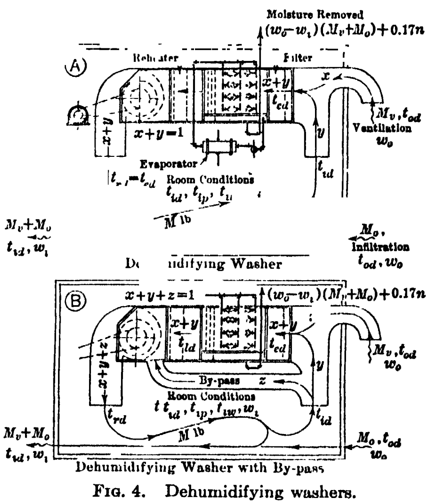

FIG. 4. Dehumidifying washers.

Subtracting eq. 17 from eq. 16, $z = (t_{rd} - t_{rp})/(t_{id} - t_{ip})$; substituting value of z in eq. 17, $t_{lp} = (t_{rd} - zt_{id})/(x + y)$. By method of mixtures, $t_{ed} = (xt_{od} + yt_{id})/(x + y)$, and $w_e = (xw_o + yw_i)/(x + y)$, which is the weight of vapor per pound of air entering washer. The corresponding dew point, t_{ep}, of air entering washer thus is ascertained. Corresponding wet-bulb temperature t_{ew} is given by the psychrometric chart, and h_1 is found in the saturated air table, p. 1-07. The value of h_2, from the saturated air table, corresponds to temperature t_{lp}. Tons of refrigeration required is found by eq. 12.

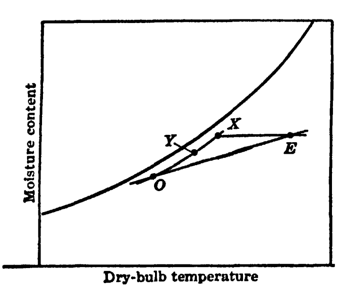

FIG. 5. Diagram illustrating load-ratio line.

Evaporator-type Cooling Surface. The heat transfer rate of any type of air cooling is a function of the velocity of air through free area of coils, and of the temperature of coil surface. All the foregoing analysis applies to spray washer-type equipment. With a coil the exit condition is not at saturation but at some relative humidity depending on the coil depth. If inlet and outlet conditions are located on a nonlogarithmic chart such as Fig. 1, the angle which a straight line connecting the two points makes with the horizontal is a direct measure of the relative amount of latent to sensible heat removal.

Assume that air enters a coil at conditions corresponding to point E in Fig. 5. If the coil surface temperature remains above the air dew point, dry cooling takes place until X is reached, which is the point where the coil surface temperature and air dew point are equal. Dehumidification occurs when the coil temperature is reduced below the dew point, and the air condition will be somewhere along line X, Y, O, which is on a curve at a constant horizontal distance from the saturation curve. This relationship indicates that the minimum dry bulb for no dehumidification (point X) and the exit conditions for any *load ratio* should have the same relative humidity. A line drawn through EY or EO represents the *load-ratio* line, the ratio of sensible heat removal to total heat removal.

Because of the many complex variables involved in predicting coil performance, numerous empirical formulas have been developed for selecting coil surface. Tuve and Siegel (Air Cooling Coil Problems and Their Solutions, *ASHVE Transactions*, 1945) have developed a simplified procedure for calculating the size and depth of coil, the actual condition of the air leaving the coil, and refrigerant temperature.

The relationship between number of coil rows, leaving air condition, and outside-surface coefficient may be expressed in eqs. 18, 19, and 20.

$$\frac{h_o A_s N}{0.24W} = \log_e \left(\frac{t_{ed} - t_{ep}}{t_{ld} - t_{lp}}\right) \tag{18}$$

$$t_s = \frac{\dfrac{(t_{ed} - t_{ep})t_{ld}}{t_{ld} - t_{lp}} - t_{ed}}{\dfrac{t_{ed} - t_{ep}}{t_{ld} - t_{lp}} - 1} \tag{19}$$

$$t_r = t_s - \frac{H_t R}{N A_s h_r} \tag{20}$$

where h_o = outside-surface coefficient (air side), Btu per hour per square foot per °F. For many conventional coils $h_o = 1.1\ (dV)^{0.6}$, where d = density of air, pounds per cubic foot, and V = air velocity, feet per minute.

A_s = outside surface area, square feet per square foot of coil face area per row of coil depth.

N = number of rows of coil depth.

W = weight of air-vapor mixture, pounds per hour per square foot coil face area.

t_s = coil-surface temperature, °F.

t_r = refrigerant temperature, °F.

H_t = total coil load, Btu per hour per square foot coil-face area.

h_r = inside-surface coefficient (refrigerant side), Btu per square foot per °F. A value of h_r = 325 is satisfactory for the "Freon" group.

R = ratio air-side to refrigerant-side surface area.

EXAMPLE. Determine the coil depth, face area, refrigerant temperature, and the actual air conditions leaving a coil to maintain room conditions of 80 F dry bulb and 67 F wet bulb for a calculated load of

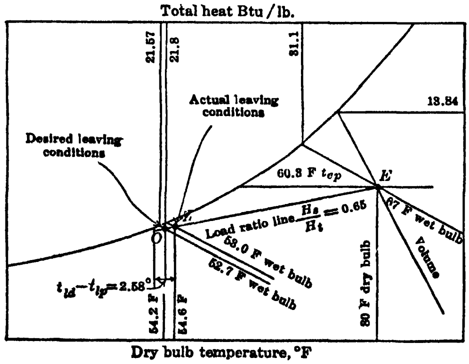

FIG. 6. Cooling-coil selection diagram.

60,000 Btu per hour with a *load ratio* of 0.65. The coil is to cool and dehumidify 1450 cu ft per min with a face-area velocity through the coil of 600 ft per min. Assume that the coil has 22 sq ft external surface, including fins and tubes, per sq ft face area per row of coil depth, and a ratio of air-side to refrigerant-side surface area of 15.

Solution. With a *load ratio* of 0.65, the sensible heat load is 0.65 × 60,000 = 39,000 Btu per hr, and by difference the latent heat load amounts to 21,000 Btu. Volume of room-air mixture from psychrometric chart (Figs. 1 and 6) is 13.84 cu ft per lb, or the density is 1/13.84 = 0.0723 lb per cu ft. The dew point t_{ep} of the entering air is found to be 60.3 F.

The amount of sensible and total heat that must be extracted per pound of air circulated in order that the actual leaving conditions may be estimated is

$$\frac{H_s}{60Qd} = \frac{39{,}000}{60 \times 1450 \times 0.0723} = 6.2 \text{ Btu per lb air}$$

$$H_s = c_p W(t_{ed} - t_{ld})$$

$$(t_{ed} - t_{ld}) = \frac{6.2}{0.24} = 25.8°$$

with

$$t_{ed} = 80°, \text{ then } (80 - 25.8) = t_{ld} = 54.2°$$

$$\frac{H_t}{60Qd} = \frac{60{,}000}{60 \times 1450 \times 0.0723} = 9.54 \text{ Btu per lb air}$$

The total heat from Fig. 1 corresponding to entering-air conditions E of 67 F wet bulb is 31.1 Btu per lb air. Subtracting, 31.1 − 9.54 = 21.56 Btu is the total heat of the leaving air. This corresponds to a wet-bulb temperature t_{lw} = 52.7° which intersects the dry-bulb temperature t_{ld} = 54.2°, at a dew point t_{lp} = 51.8°, thus establishing the desired air conditions O leaving the coil. Connecting points E and O in Fig. 6 gives the *load-ratio* line.

Before substituting in eq. 18 to determine coil depth, it is necessary to calculate the weight of air circulated, W.

The coil face area is obtained by dividing the air quantity by the velocity or 1450/600 = 2.42 sq ft. Then

$$W = \frac{60 \times 1450 \times 0.0723}{2.42} = 2600 \text{ lb per hr per sq ft of coil face area}$$

Also it is necessary to calculate the outside surface coefficient

$$h_o = 1.1(0.0723 \times 600)^{0.6} = 9.6 \text{ Btu per hr per sq ft}$$

Substituting in eq. 18

$$\frac{9.6 \times 22 \times N}{0.24 \times 2600} = \log_e \left(\frac{80 - 60.3}{54.2 - 51.8}\right)$$

$$N = 6.21$$

It is now possible to determine the *actual* location of the leaving conditions from eq. 18 by solving for the actual value $t_{ld} - t_{lp}$ for a 6-row coil.

$$\frac{9.6 \times 22 \times 6}{0.24 \times 2600} = \log_e \frac{(80 - 60.3)}{(t_{ld} - t_{lp})}$$

$$\text{antilog}_e\, 2.03 = 7.61 = \frac{(80 - 60.3)}{(t_{ld} - t_{lp})}$$

and

$$t_{ld} - t_{lp} = 2.58°$$

Next, locate point Z on the load ratio line at a horizontal distance of 2.58° dry-bulb degrees from the saturation curve. This is the only part of the procedure which requires a graphical determination. The actual leaving conditions are thereby established as t_{ld} = 54.6°; t_{lw} = 53.0°; t_{lp} = 52.0°; and total heat = 21.8 Btu per lb.

The coil surface temperature may now be found from eq. 19,

$$t_s = \frac{(7.61 \times 54.6) - 80}{(7.61 - 1)} = 50.8°$$

Total coil load may be calculated from the total heat difference between points E and Z.

$H_t = W(h_e - h_z) = 2600(31.1 - 21.8) = 24{,}200$ Btu per hr per sq ft of face area. Equivalent tons of refrigeration 24,200 × 2.42 sq ft face area = 58,500 Btu per hr or 4.875 tons of refrigeration.

Refrigerant temperature can be calculated from eq. 20,

$$t_r = 50.8 - \frac{24{,}200 \times 15}{6 \times 22 \times 325} = 42.3 \text{ F}$$

In summary, the coil characteristics are: depth of coil = 6 rows; face area = 2.4 sq ft; refrigerant temperature = 42.3 F; leaving conditions, dry-bulb = 54.6 F and wet-bulb = 53.0 F. By using these data and referring to any manufacturer's catalog information, a suitable coil can be selected.

The refrigerant temperature corresponds to the suction pressure of the refrigerant in direct expansion systems. The refrigerant temperature is automatically maintained at approximately the same temperature in all sections of the coil, except that a moderate amount of superheating (about 10°) is desirable to avoid the possibility of liquid refrigerant entering the suction of the compressor, with consequent danger (in a reciprocating machine) of damage to the compressor.

When chilled water is used in coils the condition is somewhat different. Since the water frequently flows through the coils in series, there is an appreciable temperature rise (10 to 15 F). The heat transfer depends largely upon the logarithmic mean temperature difference between air and water in the tubes, and an appropriate correction must be made for the "cross-flow" effect in such designs (see p. 3-31). The quantity of water circulated depends upon the total amount of heat removed from the air (sensible heat plus latent heat) and the design temperature rise of the water.

Other items to be calculated are the face area of the units, the air-friction drop, and the water-friction drop. The face area is usually based on a velocity of about 500 ft per min.

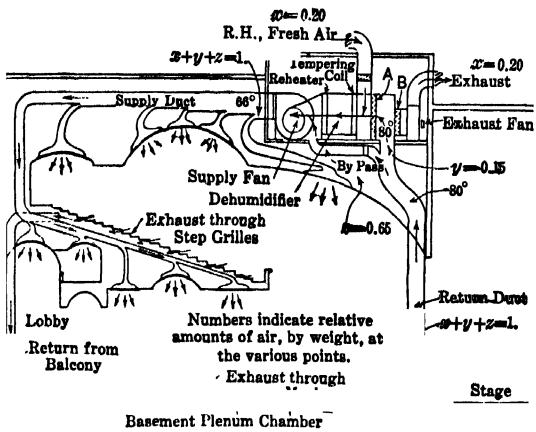

Fig. 7. Air conditioning of theater.

Friction drops are best taken from the manufacturer's data. Provision must be made for draining off the water of condensation from the coil casing.

UNIT COOLERS consisting of an assembly of fan, rows of coils, and a dry-type air filter are obtainable from several manufacturers. These assembled units are used largely for comfort cooling of restaurants, stores, offices, etc. Unit coolers, consisting of a direct expansion cooler and fan, similar to unit heaters, are used in cold-storage rooms in which the air is recirculated.

AIR CONDITIONING A THEATER. The relative weights of fresh outside air, recirculated air, and by-passed air are about as indicated in Fig. 7 if the formulas and data given above for the sensible and latent heat loads and usual outside design dry-bulb temperatures (90 to 95 F) and 60% relative humidity are applied. That is, $x = 0.20$, $z = 0.65$, $y = 0.15$. Usually the refrigeration requirement is 13.5 seats per ton or 74.3 tons per 1000 seats.

SECTION 13

INTERNAL-COMBUSTION ENGINES

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DIESEL ENGINES

BY JOHN W. ANDERSON

AIRCRAFT PISTON ENGINES

BY ROBERT INSLEY

AUTOMOBILE ENGINES

See Automotive Engineering, Section 14

GAS ENGINE COMPRESSORS

BY J. N. MACKENDRICK

DIESEL ENGINES

By John W. Anderson

A diesel engine is a prime mover actuated by the gases resulting from the combustion of a liquid or pulverized fuel, injected in a fine state of subdivision into the engine cylinder at or about the conclusion of the compression stroke. Gaseous fuels are also used but need a pilot charge of fuel oil, injected as stated, for ignition. The full charge of combustion air is taken by the engine during the intake period, and the heat generated by compression of this air within the cylinder is the sole means of igniting the liquid or pulverized fuel charge. After ignition, the fuel burns, and the gases expand as the piston recedes on the working stroke, converting the heat energy of the fuel into work.

Steam, gas, and oil engines were known and used prior to the invention of the diesel engine. The Hornsby-Akroyd (England) was the most successful engine to use liquid petroleum fuel. Ignition was obtained partly from the heat of compression but principally from contact of the fuel with the uncooled hot surface of the combustion chamber. Low-pressure pump injection of the fuel gave a coarse atomization.

In 1892 Rudolf Diesel patented (in Germany) his engine to operate on the Carnot cycle. The objective was a "rational heat engine" with the highest attainable efficiency. Although the original patent showed the use of powdered coal for fuel, this was of only incidental importance. All experimental work was with liquid fuel, and a second patent described this. The final experimental engine deviated from the Carnot cycle ideal to have nearly constant pressure rather than isothermal combustion. The first commercial engines were built in 1898 in Germany and in the United States. These were of the four-cycle type and used compressed air for injection and atomization of the fuel oil. (See Fig. 1.)

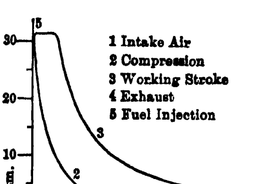


FIG. 1. Diesel engine four-stroke cycle.

The processes of evolution and development have eliminated all hot-surface-ignition engines as well as the air-injection diesel engine. The advent of a satisfactory commercial form of pump-injection system for the fuel made small-cylinder higher-speed diesel engines feasible. Today the generic name *diesel* is applied to all internal-combustion engines in which the fuel is ignited entirely by the heat resulting from compression of air supplied for combustion. The usual engine employs pump injection for the fuel oil. Natural gas and sewage gas are being used increasingly when the economics favor them. In Europe, a few coal-dust engines have been built.

At the time of its invention and development, the diesel was the most efficient prime mover known, and it still is. Its brake thermal efficiency is about 30% in the smaller engines, but 35% is a representative figure for medium sizes, and as high as 41% has been recorded for a large slow-speed engine.

Diesel engines are available in this country in units of 3 to over 10,000 bhp, and they are used for nearly all power purposes except airplanes and passenger automobiles. They are especially suitable where an independent source of power is required as in ships, locomotives, mobile equipment of all sorts, and isolated power plants.

1. CLASSIFICATIONS OF DIESEL ENGINES

DEFINITIONS. The Diesel Engine Manufacturers Association has established the following definitions.

An **oil-diesel engine** is one which operates on fuel oil injected after compression is practically completed.

The **gas-diesel** is an engine which operates on a combustible gas as primary fuel and in which the ignition of the gas is accomplished or aided by pilot-oil fuel injected after compression is practically completed. The gas fuel may be compressed in the engine cylinder with the air or it may be compressed separately and injected into the combustion chamber near the end of the compression stroke.

A **dual-fuel diesel engine** is one which may be operated as an oil-diesel or a gas-diesel or a combination of both, and is equipped with controls or parts to permit operating as one or the other.

A **single-acting engine** utilizes the working medium on only one side of a single piston.

A **double-acting engine** utilizes the working medium on both sides of a single piston.

An **opposed-piston engine** utilizes the working medium simultaneously on two pistons in the same cylinder.

A **trunk-piston engine** has the connecting rod connected directly to the wristpin in the piston. The side thrust caused by the angularity of the connecting rod is taken by the piston bearing against the cylinder wall.

A **crosshead engine** has the connecting rod connected to a crosshead traveling in guides, and the crosshead in turn is connected to the corresponding piston. The side thrust caused by the angularity of the connecting rod is taken by the crosshead and guides.

Fuel injection. The term *mechanical injection* refers to a method of introducing the fuel charge into the power cylinder of an engine. The injection system is completely filled with liquid fuel, and the fuel charge is injected into the power cylinder under pressure built up by a fuel pump. Mechanical injection systems have five subdivisions.

(1) *Pump-timed injection system,* in which the fuel is injected into the engine cylinder directly by the action of the fuel-injection pump plunger. The action of the pump both meters and times the injection of the fuel.

(2) *Pump-timed injection system, with distributor,* in which the fuel pump both times the injection and meters the fuel charge. The distributor selects the particular cylinder to which the fuel charge is delivered.

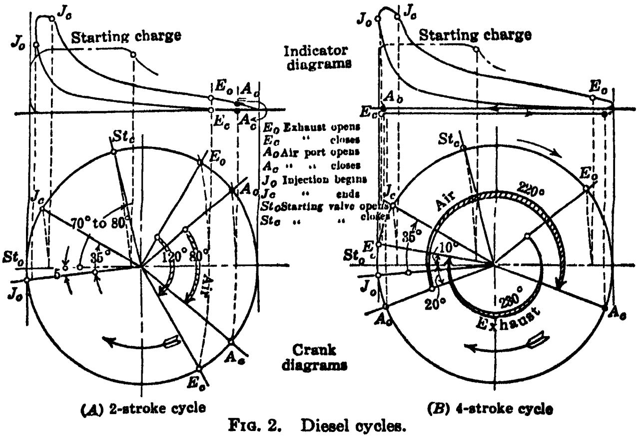
(3) *Common-rail system,* in which a fuel pump supplies fuel to a header, called the common rail, at a pressure above the cylinder pressure of the engine, the fuel being passed from this common rail to each cylinder in turn at the proper time through mechanically operated valves.

(4) *Controlled-pressure injection system,* in which a fuel pump supplies fuel to a header at varying pressures, the fuel being metered to each power cylinder through injectors by mechanically operated valves which reduce the line pressure to a low value after each injection.

(5) *Low-pressure distributor injection system,* in which a single pump plunger delivers the fuel, by means of a distributor, to multiple injectors, the timing and injection being accomplished by action of the injector, and the pump and distributor serving only as a metering device at relatively low pressure.

Scavenging air refers to air at low pressure used to force burnt gases out of the power cylinder during the exhaust period and, by this displacement, to furnish a supply of fresh air for the following cycle. There are several methods of compressing this air and of introducing it into the cylinder.

COMBUSTION CYCLES. Spark-ignition gas and gasoline engines operate on the explosion cycle; the combustion takes place at substantially constant volume. In early



(*A*) 2-stroke cycle (*B*) 4-stroke cycle

FIG. 2. Diesel cycles.

diesel engines combustion took place at substantially constant pressure. Modern diesel engines operate on the dual cycle; some of the combustion occurs at constant volume, and combustion is concluded at substantially constant pressure. The maximum combustion pressure may be from 25 to 100% above the compression pressure.

The Hesselman engine burns fuel oil injected during the latter part of the compression stroke and ignited by a spark. The engine operates on the explosion cycle with cylinder pressures similar to those obtaining in gas engines.

CYCLES OF OPERATION. Diesel engines operate as four-cycle or two-cycle. The four-cycle engine (four-stroke cycle, Otto cycle, Beau de Rochas cycle) uses four piston strokes per complete cycle: (1) air inlet, intake or suction; (2) compression; (3) working or expansion; (4) exhaust. (See Fig. 2*B*.) The two-cycle engine uses two piston strokes per complete cycle: (1) working or expansion; (2) compression. During the last part of the working stroke and the early part of the compression stroke, scavenging of the cylinder occurs. (See Fig. 2*A*.)

COMBUSTION CHAMBERS. Combustion chambers provide space for combustion of the fuel and, in conjunction with the fuel injection system, provide means for the process of mixing fuel and air before and during combustion. These chambers take many forms according to size of cylinder and the method and source of energy for mixing fuel and air. The mixing problem is especially acute in small cylinders because of small physical dimensions and time element.

The **open combustion chamber** is formed between the cylinder head and piston, or between the two pistons in an opposed piston engine, and is used on all sizes of cylinders; it is nearly always used in the medium and larger sizes. For mixing, there is the energy of expansion of the air in air injection engines, and there are the fuel jets themselves in mechanical injection types. In the latter case, moderate air swirl may also be used. The open chamber gives highest efficiency and the easiest cold starting, with the lowest compression pressure. In small cylinders, it is the most sensitive, with respect to obtaining smooth combustion. It is the easiest to arrange for good supercharging in four-cycle engines.

The **turbulent chamber** takes many forms. It may be included entirely in the piston top, entirely in the cylinder head, or at the side of the cylinder. As much of the combustion air as possible usually is concentrated in the chamber. Turbulence may be induced by air velocity at the entrance into the cylinder or by motion of the piston during compression. Used principally on smaller cylinders, some turbulent chambers have been applied to the larger sizes. Fuel is injected into the chamber.

The **precombustion chamber** is used only in small cylinders. It is located in the cylinder head, contains 30% or less of the total combustion air, and all the fuel is injected into it. The combustion of a portion of the fuel in the prechamber expels the flaming fuel charge out into the main combustion space in the cylinder, creating turbulence, mixing of fuel and air, and vigorous burning.

The **energy-cell type** is also used only in small cylinders. The energy cell contains 10% or less of the total combustion air. It is located in the cylinder head, and fuel is injected across the main combustion chamber into the energy cell. Not all the fuel enters the cell. Combustion of a portion of the fuel in the cell expels all of it into the main chamber and creates the proper turbulence and mixing of fuel and air.

Precombustion chambers and energy cells confine the peak combustion pressures and avoid sudden pressure rises in the main cylinder.

Turbulent chambers, precombustion chambers, and energy cells are used to obtain better and smoother combustion. However, the cooling effect of their extra surface tends toward a higher fuel consumption, and usually a higher compression pressure is needed for fuel ignition and starting. However, a lower fuel injection pressure is satisfactory.

DUTY. Diesel engines are rated according to service conditions or duty. If the duty is light, owing to intermittent use or low load factor, engines may be applied at high power and speed ratings. If duty is heavy or severe, because of continuous use or high load factor, moderate speed and power ratings are essential; overload capacity is usual. Every engine should be rated so that maximum load and speed requirements are within the ability of the engine to meet them in average everyday operating condition. Neither the piston speed nor the rotative speed of an engine is alone a suitable criterion for judging the ability of an engine to meet specific duty requirements. Engines with small cylinders are inherently more suitable for higher speeds than engines with large cylinders. Performance records form the soundest basis for judging application ratings.

2. THERMODYNAMICS OF DIESEL ENGINES

For the general subject of thermodynamics, see Section 3. The diesel engine is a heat engine whose purpose is to convert the combustion energy of its fuel into mechanical energy. The air which unites with the fuel always is precompressed to raise its temperature so that it will ignite the fuel injected near the end of the compression stroke, and to increase efficiency by subsequent expansion of the burned charge. In any internal-combustion engine, compression before ignition extends the range of effective expansion.

Figure 3 from *Diesel Power*, Dec. 1933, shows the heat balance throughout the range of load. Engines of different makes show considerable variation.

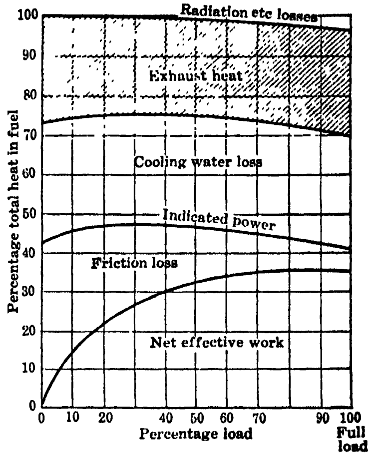


FIG. 3. Heat balance for a typical diesel engine.

Power and Efficiency Formulas

DISPLACEMENT of an engine is the volume, cubic feet per minute, swept by the piston or pistons during the power strokes. It is equal to: number of cylinders × area of each piston (sq ft) × stroke (ft) × number of power strokes per minute.

VOLUMETRIC EFFICIENCY is the ratio V_a/D, where V_a = volume of air, cubic feet per minute, at intake temperature and pressure, induced and compressed, and D = displacement.

INDICATED HORSEPOWER of an engine cylinder is the horsepower developed in the cylinder. The formula is

$$\text{Ihp} = (\text{Mip} \times L \times A \times N) \div 33{,}000 \tag{1}$$

where mip = mean indicated pressure, pounds per square inch; L = stroke of piston, feet; A = net piston area, square inches; and N = number of power strokes per minute.

MEAN INDICATED PRESSURE (mip) of an engine cylinder is average net pressure, pounds per square inch, acting on piston throughout one cycle.

BRAKE HORSEPOWER (bhp) is the horsepower delivered by the shaft at the output end. The formula is

$$\text{Bhp} = \frac{2\pi l n W}{33{,}000} \tag{2}$$

where l = distance between shaft center and bearing point of brake arm, feet; n = revolutions per minute of brake shaft; and W = net weight on brake arm, pounds.

BRAKE MEAN EFFECTIVE PRESSURE (bmep) is

$$\text{Bmep} = \frac{\text{bhp} \times 33{,}000}{L \times A \times N} \tag{3}$$

PISTON SPEED *is the total feet of travel made by each piston in one minute.* The formula is

$$\text{Piston speed} = 2 \times \text{stroke in feet} \times \text{rpm} \tag{4}$$

INDICATED THERMAL EFFICIENCY is the ratio of the heat equivalent of 1 hp-hr (2544 Btu) to the heat units actually supplied per ihp-hr, based on the higher heating value of the fuel.

BRAKE THERMAL EFFICIENCY is the ratio of the heat equivalent of 1 hp-hr to the heat units actually supplied per bhp-hr, based on the higher heating value of the fuel.

MECHANICAL EFFICIENCY is the ratio of brake horsepower to indicated horsepower.

LOSSES IN INTERNAL-COMBUSTION ENGINES are: (1) loss through externally cooled walls, when gases are at maximum temperature and during compression; (2) throttling of air inlet and back pressure during exhaust; and (3) incomplete combustion at maximum temperature and loss through exhaust. The distribution of heat energy in a typical diesel engine at full load is

	Btu per bhp-hr	Heat in Fuel, %
Brake work	2544	33
Friction	680	9
Heating of jacket water	2100	27
Heat in exhaust gases	2200	28
Radiation, etc.	255	
Total	7779	100

THERMODYNAMIC ANALYSIS OF INTERNAL-COMBUSTION ENGINE CYCLES. In a theoretical analysis of the cycle of an internal-combustion engine three degrees of approximation may be observed.

1. The simplest system of analysis gives the so-called air standard, used to estimate engine efficiencies. This analysis assumes that the working medium throughout the cycle

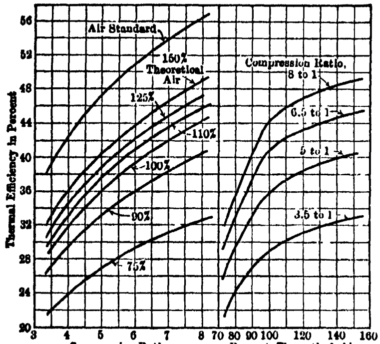


FIG. 4. Variation of efficiency with compression ratio and with mixture strength for Otto cycle.

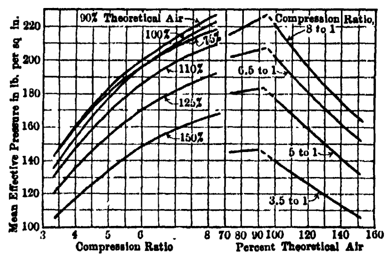


FIG. 5. Variation of mep with compression ratio and mixture strength for Otto cycle.

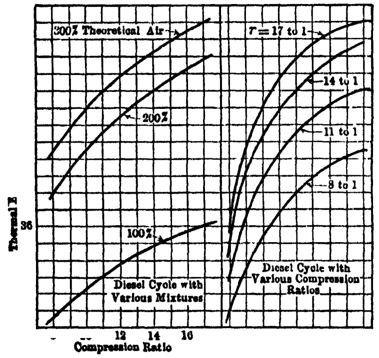


FIG. 6. Variation of efficiency with compression ratio and mixture strength for diesel cycle.

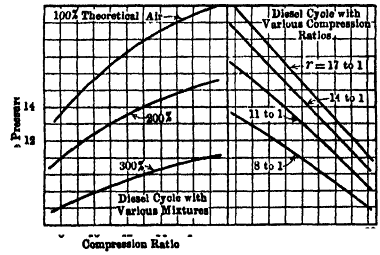


FIG. 7. Variation of mep with compression ratio and mixture strength for diesel cycle.

is air. During the combustion phase the air is supposed to receive an amount of heat equal to the heat of combustion of the fuel. The specific heat of air is taken as constant. The air standard efficiency deduced from this analysis always is 10 to 25% higher than the efficiency obtained from more accurate analyses.

2. The properties of the actual gas mixtures are used. The medium compressed is a mixture of fuel and air; the medium expanding adiabatically after combustion is an entirely different mixture, of different properties. In this analysis, it is assumed that combustion is complete before adiabatic expansion begins.

3. It is well known that at the maximum pressure and temperature attained in the cycle, combustion is incomplete, and that, owing to dissociation of CO_2 and H_2O at tem-

peratures above 2500 F, the mixture will contain unburned CO and H_2 at the beginning of adiabatic expansion. As the temperature falls during expansion, combustion continues,

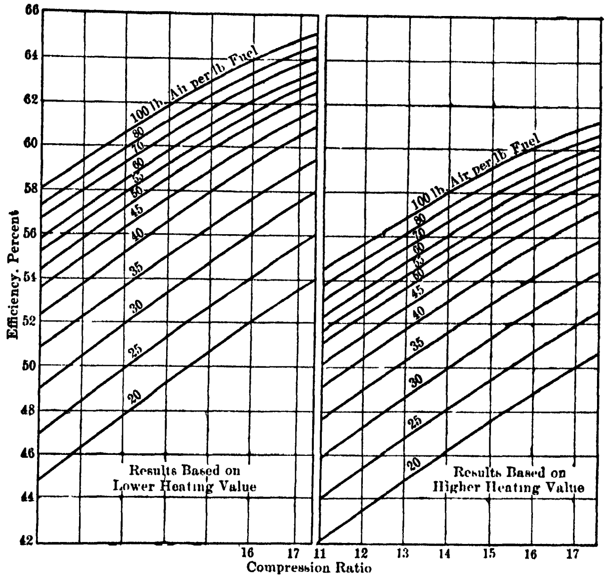


FIG. 8. Efficiency of ideal diesel engine using petroleum oil. (Ellenwood, Evans, and Chwang, *Trans. ASME*, OGP-50-5, 1928)

and at the end of expansion it is practically complete. The third system of analysis takes account of these phenomena.

For the Otto cycle the effect of compression ratio and mixture strength on thermal efficiency is shown in Fig. 4. The effect of these factors on mean effective pressure is shown in Fig. 5. Corresponding relations for the diesel cycle are shown in Figs. 6 and 7.

As a result of studies based on the third system of analysis, Goodenough and Baker concluded: (1) Efficiency increases with compression ratio, i.e., the higher the compression the higher the efficiency, other conditions being unchanged. (2) For the same compression, efficiency increases with amount of air used. A lean mixture gives higher efficiency than a rich mixture. (3) Mean effective pressure is a maximum when air supply is somewhat less than 100% of the theoretical amount (Fig. 5). The mixture for maximum power is a mixture of relatively low efficiency. (4) Ideal efficiencies obtained from various liquid fuels are practically the same. (5) Efficiencies of the diesel cycle, as a group, range higher than efficiencies of the Otto cycle. However, a comparison of the two efficiencies at the same compression ratio ($r = 8$) shows the Otto cycle to be inherently more efficient than the diesel cycle. The superior efficiency of the diesel cycle is due to the high compression ratio permitted by the system of operation.

Ellenwood, Evans, and Chwang in Fig. 8 show in convenient form the efficiency of an ideal diesel engine using petroleum.

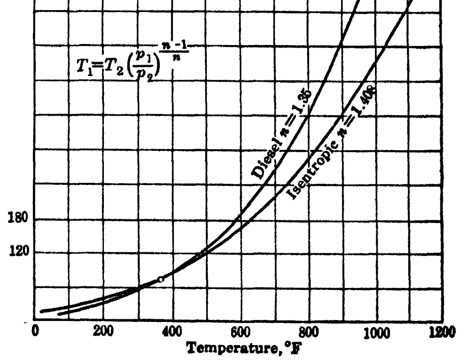


FIG. 9. Relation of pressure and temperature of air during compression, showing effect of heat loss.

COMPRESSION PRESSURES AND TEMPERATURES. The temperature of the air charge at the end of compression must be high enough to ignite the fuel. The relations of pressures and temperatures at beginning and end of the compression stroke are expressed by $T_1 = T_2(p_1/p_2)^{(n-1)/n}$, where T_1, T_2 = respectively, absolute temperatures at beginning and end of compression; p_1, p_2 = respectively, absolute initial and final pressures. For adiabatic compression, $n = 1.408$. In an actual engine, however, loss of heat occurs, and the value of n is about 1.35. These relations are shown graphically in Fig. 9.

Small cylinders which have relatively large cooling areas for their volume have still lower values of n. Also, under cold starting conditions, lower values of n prevail and reduce the compression temperature for fuel ignition.

THERMODYNAMIC LOSSES. The ideal diesel cycle calls for adiabatic compression and expansion, and complete combustion during the early part of the working stroke. All heat losses to the jacket water during compression, combustion, and expansion are thermodynamic losses, because otherwise that heat could do useful work. The heat loss to jacket water during the exhaust stroke, and from the exhaust passages outside the cylinder proper, are merely transfer of heat from the waste exhaust gases. Any leakage of gases past the piston rings during any part of the cycle is a thermodynamic loss; it is a loss of heat which otherwise could do useful work. All heat must be added (during combustion) at the highest temperature for full thermodynamic efficiency, and unless combustion is completed during the early part of the working stroke, full expansion of the gases and full conversion into work are not obtained. Thus while the normal heat rejection in the exhaust gases and to the jacket water are thermodynamic losses, they contribute to reliable engine operation. But the thermodynamic ideals of gas-tight pistons and complete combustion at the proper time must be sustained in everyday practice for good economy and reliable mechanical engine operation.

3. SUPERCHARGING

GENERAL. *Supercharging* refers to the practice of supplying the intake of an engine with air at a density greater than that of the surrounding atmosphere, this air being retained in the cylinders at the start of the compression stroke. Note that while scavenging air is supplied to two-cycle engines at a density greater than atmospheric, this does not supercharge the engine unless arrangements are made to retain a measure of the increased density in the power cylinders.

The purpose of supercharging is either to make up for the loss in power due to altitude, or to increase the power that can be obtained from an engine of a given piston displacement, thus reducing the cost, weight, or space occupied by an engine for a given power. A supercharger consists of an air compressor and a means of driving it. The air compressor may be of any type—reciprocating, rotary, positive-displacement, centrifugal, or axial-flow.

In all supercharging systems there is scavenging as well as supercharging. Two-cycle engines scavenge in the usual manner, and means are provided for extra charging of the cylinder, usually after the exhaust ports have closed. In four-cycle engines, scavenging occurs during the inlet- and exhaust-valve overlap period. Large gas flow areas (frequently dual inlet and exhaust valves in four-cycle engines) are important in all supercharged engines, except when supercharging only for elimination of loss of power at altitude.

METHODS OF SUPERCHARGING. (See also Section 10.) **Compressor Driven from Engine Crankshaft.** Such compressors are usually of the reciprocating, rotary, or positive displacement type. The power for compressing the air and for losses in the drive mechanism must be subtracted from the power developed by the engine, reducing the overall efficiency and increasing the specific fuel consumption. Volume of air supplied varies with engine speed and not in proportion to engine load. Higher relative compressor power loss occurs, therefore, at partial loads. This method is suitable for moderate increases in inlet pressure, up to, for instance, 35%. Figure 10 shows a method suitable for four-cycle engines of 300 hp or less. Two-cycle engines with oversize scavenging pumps have a slight supercharging at sea level and can deliver full power at altitude by loss of overload capacity.

Compressor Driven from External Source. The most efficient method is to obtain the driving power from an auxiliary diesel engine. In all cases, the main engine must be charged with the gross compressor driving power, including electrical losses when that form of drive is used. It is possible to vary air quantity supplied according to load, but extra controls are required. Any type of compressor can be used on any engine of any size. This method is useful for large two-cycle engines and for altitude supercharging.

Compressor Driven by Exhaust Gas Turbine. Engine and turbine-compressor unit are connected only by the fluids in the air and exhaust gas passages. The sole source of power for the turbine is the energy in the exhaust gases which otherwise would be wasted. The turbine-compressor set has no net power output but increases the output of the engine by maintaining both air and exhaust manifolds at pressures higher than atmospheric. Thus the engine operates under a dense atmosphere created by the turbine compressor, and more fuel can be burned. Speed of the turbine compressor is determined by engine speed (volume of exhaust gases), temperature of exhaust gases, barometric pressure and temperature, and the load imposed by the compressor. Thus speed changes both with engine load and with engine speed, maintaining a nearly constant ratio of air supply to load and fuel flow. As barometric pressure decreases, e.g., at higher altitudes, the turbine-compressor speed increases, making up for some or for all the barometric pressure loss.

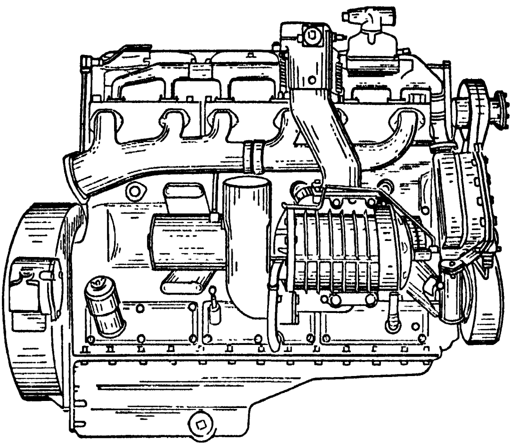


FIG. 10. 275-hp series NHS Cummins engine with positive-displacement type compressor. (Courtesy of Cummins Engine Co., Inc.)

The Buchi system is applied to four-cycle engines. Here the turbocharger (turbine compressor), Fig. 11, operates at about 4 to 6 psig supercharge pressure. Advantage is taken of *troughs* between the exhaust gas pressure waves to scavenge the clearance space. Inlet-exhaust valve overlap is extra long for this purpose. The exhaust header is divided into branches, with three cylinders or less per branch, to avoid overlap of exhaust periods from cylinders on a single branch.

High efficiencies of both turbine and compressor are important in producing a large weight of air at high supercharging pressure for a given temperature of exhaust gases.

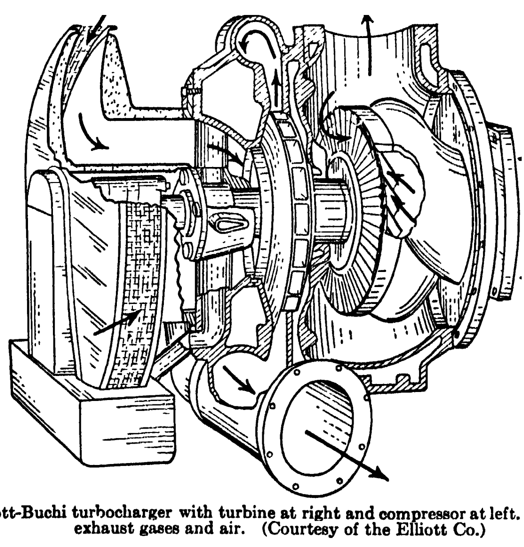


FIG. 11. **Elliott-Buchi turbocharger with turbine at right and compressor at left.** Arrows show flow of **exhaust gases and air. (Courtesy of the Elliott Co.)**

The turbine uses the same weight of gas as the compressor supplies, except for the addition of fuel in the cylinder. The usual supercharge pressure ratio is about 1.35 to 1.4, but higher readings occasionally are obtained.

Buchi turbocharged engines (see Fig. 12) have specific fuel consumptions at least as low as unsupercharged engines, over a wide range of load. Turbocharged engines usually are rated at brake mean effective pressures 50 to 60% above the corresponding atmospheric engines—as high as 115 to 125 psi. Test readings of 150 psi and higher have been obtained. Heat balance as compared with naturally aspirated engines is about as shown by the following typical figures:

	Naturally aspirated, %	Turbocharged, %
Brake work	33	34
Friction	9	7
Heat to jacket water	27	23
Heat in exhaust gases	28	33
Radiation, etc.	3	3
	100	100

These are relative figures. The increased flow of gases through the turbocharged engine raises the relative exhaust heat rejection, but a portion of it is utilized by the turbine of the turbocharger. Maximum combustion pressure of the turbocharged engine is higher. Exhaust temperatures are equal to or slightly higher than those of naturally aspirated engines.

The Buchi system can also be applied to the two-cycle engine.

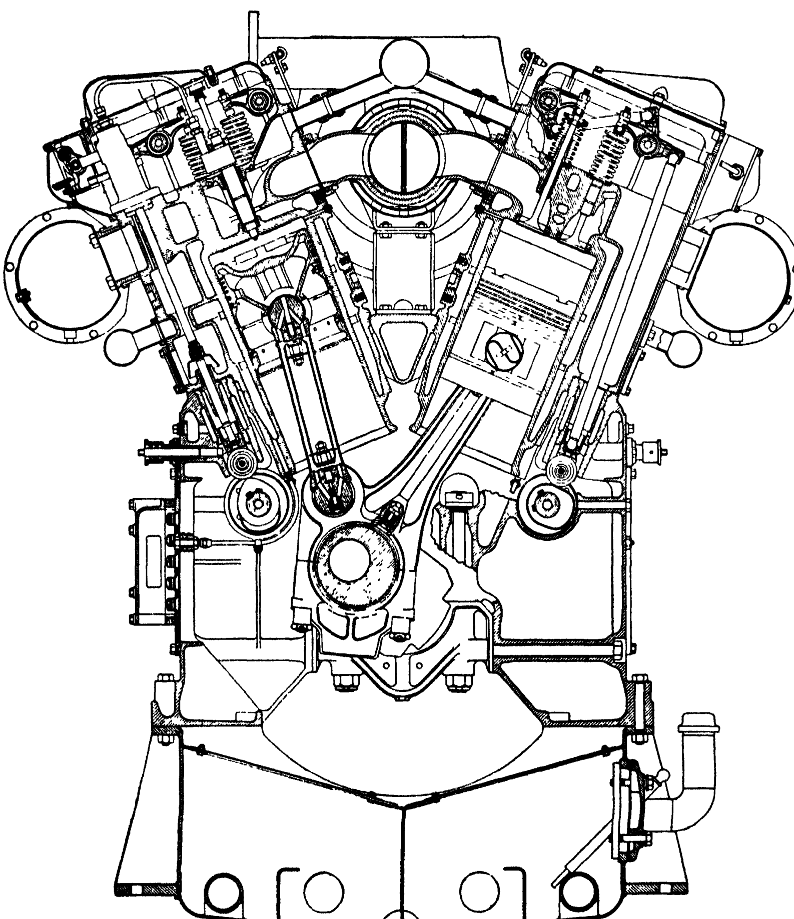


FIG. 12. Transverse section through a 12-cylinder Cooper-Bessemer type FV-T diesel engine. Operated at 900 to 1000 rpm, this Buchi-turbocharged engine is rated at 1200 hp at 1000 rpm. (Courtesy of Cooper-Bessemer)

The General Electric turbo-supercharger (turbine compressor) provides scavenging and a higher supercharging pressure. Pressure ratios (p_2/p_1) of 2.1 to 2.35 have been used on four-cycle locomotive engines to obtain brake mean effective pressures of 165 psi at normal rating. This is twice the mean effective pressure of the nonsupercharged engine; rate of air flow was about 2.8 times that through the nonsupercharged engine. Such engines produce rated sea-level power up to 10,000-ft elevation by speeding up of the turbo-supercharger. Still higher ratios of p_2/p_1 are possible with correspondingly higher brake mean effective pressures, but turbine and compressor efficiencies become increasingly important, and cooling of air after compression becomes essential.

Compressor Geared to Engine Shaft and Driven by Directly Coupled Exhaust-gas Turbine. Because of the inherently high speed at which the turbine-compressor unit operates, there must be a flexible connection, usually a hydraulic clutch, between engine and gearing to absorb shocks and sudden changes of speed of either engine or turbine-compressor unit without damage to either. With fixed ambient conditions there is only one engine load at each engine speed for which turbine and compressor are in balance. At lighter loads, turbine power is deficient and the engine makes up the difference. At higher loads, the turbine supplies excess power to the engine crankshaft. At light loads considerable power is used to compress air not needed by the engine.

This method has been used to supercharge opposed-piston two-cycle engines up to 2.5 atm gage ($p_2/p_1 = 3.5$); 190 lb bmep and higher was obtained. The power required to compress the air under such conditions is about one-third the engine power, but the power delivered by the turbine exceeds this, giving a net gain in total output.

Higher degrees of supercharging accentuate two problems. (1) *Turbine and compressor efficiencies.* The power required to compress air to higher pressures increases considerably, so that the efficiencies of both turbine and the compressor have an ever-increasing influence on power output and fuel consumption. (2) *Starting and maximum combustion pressure.* Since even a supercharged engine always *starts* unsupercharged, the compression ratio must provide a satisfactory ignition temperature under cold conditions. The higher the degree of supercharging, the higher the compression at full load, and the higher the maximum combustion pressure is likely to be. Such very high supercharging pressures may lead to combustion pressures that are untenable, unless suitable compensation is made by decreasing the engine compression ratio.

A supercharging pressure of 5 to 6 atm gage (p_2/p_1 = 6 to 7) requires the full power output of the engine to drive the supercharger, the turbine becomes the sole source of power, and there is no purpose in gearing the turbine to the engine. The engine then becomes a hot-gas generator, and the turbine becomes the power unit. To avoid starting and combustion-pressure problems, a free piston engine may be used. Because this is an opposed-piston engine *without* crankshafts, the stroke is variable. The pistons are connected by a suitable mechanism so that they work in phase with each other. The air compressor pistons may be on the outer ends of the working pistons. The air compressors supply air solely for combustion in the working cylinder, where combustion and expansion of the gases occur. The exhaust goes to the turbine, which delivers all the net-power output. There is no direct relationship between engine speed and turbine speed. Turbine speed may be constant or variable according to load requirements. Engine speed is a function of the natural frequency of the elastic system, and the stroke varies. Maximum speed and load bring the longest engine piston strokes, the greatest quantity of air compressed, the highest supercharging pressures, the largest quantity of fuel burned, and the maximum supply of hot gas to the turbine.

The transfer of the hot exhaust gases from engine to turbine is a potential source of loss, but the processes within the engine and turbine are inherently highly efficient. Work has been done in this country by at least three companies and abroad by several. No unit is yet in commercial service.

Kadenacy System. When the exhaust ports of a two-cycle engine open, there is a sudden outflow of exhaust gases, tending to create a vacuum in the cylinder, and to increase the flow of scavenge air through the inlet ports into the cylinder. Proper proportioning supercharges the cylinder and permits higher power outputs. This system may be used alone or in conjunction with a scavenging pump. When used with a scavenging pump the work of the pump is reduced, yielding a better engine mechanical efficiency and a lower fuel consumption. The degree of supercharging accomplished by this method is limited.

4. HEAT RECOVERY

Projects for utilizing jacket water and exhaust heat should be scrutinized carefully for their potential return on the initial investment and for their influence on the service cost and reliability of the whole plant. In Article 2 of this section, a typical heat balance was given which shows the heat available in the jacket water and exhaust. The available heat will vary with classes of engines and even between identical engines. Test figures should be used in any given case.

JACKET-WATER HEAT is completely recoverable, but consideration must be given to quality and temperature of the water. The jacket water may be recirculated and the heat used either directly or through a heat exchanger, the water may be used in process work, or the heat may be used and the water run to waste. In the last two cases the make-up supply must be dependable in both quantity and quality. The temperature of the water should be that recommended by the manufacturer. Usual outlet temperatures range from 140 to 180 F. Most modern engines are equipped with circulating-water systems that limit the temperature rise through the engine to about 10 to 15 F by vigorous circulation of water. Some engines have been equipped to operate with jacket-water temperatures of 212 to 220 F, and the water is circulated rapidly to wash away steam bubbles formed on the engine cooling surfaces.

EXHAUST HEAT RECOVERY is limited to about 60 to 70% of the total sensible heat, because of the necessity of maintaining the gas temperature at 250 F or higher to avoid condensation of water vapor with resulting corrosion. More heat can be recovered in heating water than in generating steam, and more heat can be recovered in generating low-pressure steam than high-pressure steam because of the permissible temperature differences. Figure 13 shows representative exhaust gas temperatures for different types of engines at various loadings.

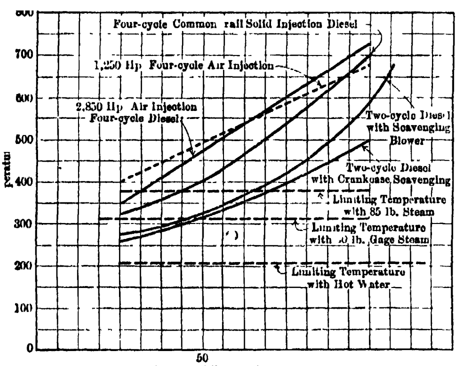


FIG. 13. Representative exhaust temperatures of various types of diesel engine.

The quantity of heat recoverable from the exhaust gas at any load may be estimated roughly from the formula

$$Q = \text{bhp} \times C \times D \qquad (5)$$

where Q = recoverable heat, Btu; bhp = brake horsepower at the load in question; C = a constant (approximately 12 for 4-cycle engines, 20 for 2-cycle engines); D = temperature drop of exhaust gases through heater, °F.

At full load, this formula indicates a heat recovery of up to 1500 Btu per bhp-hr. With a jacket-water recovery of 2100 Btu, there could be a total heat recovery of 3600 Btu or about 46% of the heat available, but individual installation circumstances would need to be very favorable.

Exhaust heaters may be fire-tube, water-tube, or thimble-type boilers, or have finned-wall or finned-tube heating surfaces. All such heaters are also fairly good mufflers. Some of these heaters or mufflers are arranged for dry operation when steam or hot water is not desired. Otherwise, a by-pass in the exhaust piping must be provided. The heating surfaces are easy to keep clean if the engine exhaust is kept clean. Individual boilers or heaters should be installed for each engine.

Table 1 gives the results of tests of a Foster-Wheeler water-tube muffler-boiler, with 1400 sq ft of extended heating surface, connected to a 4000-hp Hooven-Owens-Rentschler double-acting, 2-cycle, 4-cylinder, air-injection diesel engine.

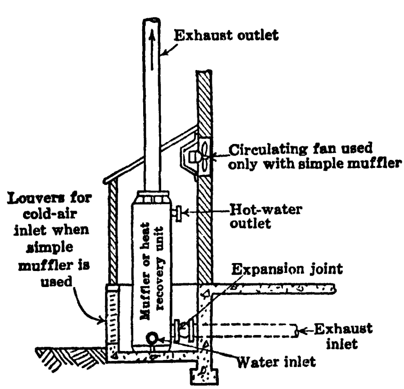


FIG. 14. Hot-air heating system, a heat recovery unit in the engine exhaust system. (Reprinted by permission of Diesel Engine Manufacturers Association)

In moderate climates, a very satisfactory way of heating engine room or building space is to utilize exhaust heat by air-jacketing the exhaust muffler and piping uptake

In the summer, it may be used as a *chimney* to get rid of heat; in the winter, a fan is required to circulate the air. The Diesel Engine Manufacturers Association suggests the arrangement shown in Fig. 14 as a convenient way of placing an exhaust-gas heat-recovery unit for circulating warm air.

Table 1. Test of Boiler Utilizing Diesel Engine Exhaust Gases

Steam pressure, psig	43.5
Back pressure on engine exhaust, in. of water	6.00
Temperature gases entering boiler, °F	538
Temperature of gases leaving boiler, °F	346
Temperature of water entering boiler economizer, °F	168
Indicated horsepower	4,812
Brake horsepower	4,019.5
Fuel consumption, lb per hr	1,669.5
" " lb per ihp-hr	0.346
" " lb per bhp-hr	0.438
Main engine speed, rpm.	110
Water fed to boiler, lb per hr	2,820
Exhaust gases, lb per hr	85,000
Stroke of main engine, in.	47.25
Diameter of cylinders, main engine, in.	27.5
Thermal value of fuel, Btu per lb	19,000

5. COMBINED DIESEL-STEAM POWER PLANTS

Diesel engines have been used in conjunction with noncondensing steam turbines to obtain greater overall economy in production of both power and heat. When all the exhaust of a steam power unit can be used for processes requiring heat, power is produced as a by-product. The boiler plant would be practically as large and burn as much fuel if it produced steam solely for heating. If the demand for heat is not large enough or uniform enough to utilize all the exhaust from engines or turbines, some exhaust heat is wasted. This waste may be reduced by the addition of diesel engine capacity to carry the load when there is no demand for heat, or to carry that portion of the load that exceeds the demand for exhaust steam.

The advantages of such a combination plant are: (1) The diesel engine is compact and self-contained. It can be installed easily in an existing plant. (2) It need not interfere with operation of the steam plant. (3) Investment is moderate. (4) The same staff can operate and maintain steam and diesel equipment. (5) The diesel equipment is an emergency reserve for the steam plant. (6) Heat in diesel jacket water generally can be used to advantage. If desired, an exhaust-gas boiler may be used to recover the diesel exhaust heat. (7) The foregoing combine with the fuel efficiency of the diesel engine to give low operating costs.

Exhaust-use Factor

The economy of a steam plant supplying power and heat depends on the exhaust-use factor, i.e., a ratio representing the proportion of the total steam power that is by-product. Before adding a diesel engine to an existing steam power plant, the exhaust-use factor must be determined accurately.

In determining the amount of by-product power, figures for power and heat requirement should be measured over short periods for each different operating condition. Daily averages may lead to large errors; longer periods, as a week or a month, are practically worthless, as long periods disregard the diversity of demands for power and heat. Even if daily average process or heating steam demand exceeds average power demand, there always are periods when conditions are reversed. When power demand exceeds heating demand, some or all of the exhaust steam is wasted. Hence the exhaust-use factor should be determined by readings for periods of one hour or less, taken at various times of the day, week, and year.

The addition of a diesel engine to a steam-power plant may improve overall economy by (1) increasing exhaust-use factor of steam plant and (2) replacing expensive *prime steam power* (steam power corresponding to wasted exhaust steam) with less expensive diesel power. In such case, steam units are loaded to the point of best overall economy for the entire plant, and the diesel generates the remainder of the power.

In a study of the joint use of the diesel and steam power in a definite project, it is best to select arbitrarily several different combinations of commercial sizes of diesel and steam turbines. With existing steam plant, only the amount of diesel power to be added is determined. For each combination a series of charts is made for all seasons of the year, based on curves of hourly demands for power and heat. These charts should show (1) amount of steam needed to supply heating load; (2) amount of by-product power that can be produced from that steam; (3) amount of additional power that diesel equipment can generate; (4) amount of prime steam power, if any, that must be produced if diesel plant cannot generate all power required in excess of by-product steam power. From these charts may be computed the quantities of coal and the diesel fuel that will be used, thus determining for each combination the annual cost of boiler and diesel fuel. To the fuel costs must be added cost of attendance, repairs, ash disposal, water, and supplies. Costs will vary with each combination, depending on relative capacity and power output of the steam and diesel parts of the combination. The sum of these items is total annual operating cost, to which are added fixed charges, as taxes, interest, amortization, and insurance. The resulting overall annual costs of the several combinations can then be compared.

6. DIESEL ENGINE STANDARDS

Standard Practices for Stationary Diesel Engines have been established by a book (1946) of that title published by the Diesel Engine Manufacturers Association (DEMA).

STANDARD SEA LEVEL RATING of a diesel engine is the net brake horsepower the engine will deliver continuously when in good operating condition and located at an altitude not over 1500 ft above sea level, with atmospheric temperature not over 90 F and barometric pressure not less than 28.25 in. Hg. The standard rating must be such that the engine will deliver an output of 10% in excess of the rating for 2 out of any 24 hours, with safe operating temperatures.

NET BRAKE HORSEPOWER is the horsepower delivered to the engine crankshaft coupling, *less* any power consumed by certain auxiliaries if used and separately driven, namely, injection-air compressor (for air-injection-type engines), scavenging-air pump or blower (for two-cycle engines), supercharging-air pump or blower (for supercharged engines), and pumps for circulating lubricating oil or piston coolant through the engine and the cooler. No deductions are made for the power to drive such auxiliaries if they are mechanically driven by the engine. No deductions are made for auxiliaries intermittently operated or governed in size or operation by special conditions of plant apart from the engine, such as oil-transfer pumps, circulating-water pumps, raw-water pumps, centrifuges, and compressors for starting air.

RATINGS AT HIGHER ALTITUDES. The power which any diesel engine is capable of delivering decreases as the altitude increases. The lower atmospheric density at higher altitudes causes this decrease in engine capacity because of the lower quantity of oxygen available for combustion in the engine cylinder. It is therefore standard practice to base the net brake horsepower ratings at altitudes on barometric pressures not more than those shown on the DEMA standard curve in Fig. 15 and on a standard maximum temperature of 90 F.

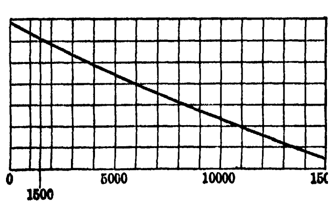


Fig. 15. Barometric pressure to be used for rating purposes at various altitudes. (Reprinted by permission of Diesel Engine Manufacturers Association)

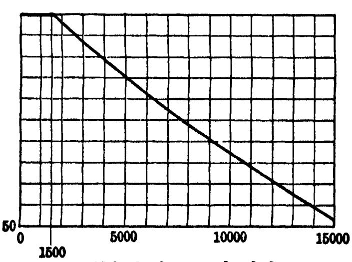


Fig. 16. Altitude rating for nonsupercharged diesel engines expressed in percentage of the sea-level brake horsepower rating. (Reprinted by permission of Diesel Engine Manufacturers Association)

The variation in net brake horsepower ratings of normal nonsupercharged engines at various altitudes is given in Fig. 16. The retention of sea-level rating up to 1500 feet is a purely practical provision to make it unnecessary to correct engine ratings for the average installation.

Reduction in rating due to increase in altitude for supercharged engines is basically the same as for nonsupercharged engines. Engine builders, however, use various means for supercharging, and proper allowance must be made for any additional power required by the superchargers and for any other factors affecting engine capacity in order to maintain the usual margin of overload capacity at the altitude without exceeding safe operating conditions.

BRAKE HORSEPOWER CAPACITY AND FUEL CONSUMPTION GUARANTEES are based on tests conducted by the engine builder on his shop test floor. Such tests are conducted on original models of a production series, and shop tests of subsequent engines are optional with the manufacturer. Test set-ups for the larger diesel engines which are usually dismantled for shipment are cumbersome and expensive; therefore, they are often tested in the field *after* installation.

Fuel-consumption guarantees are made in pounds per net brake horsepower-hour at one-half, three-quarter, and full load (sea-level rating or altitude rating) when operating at rated rpm. Gas-burning and dual-fuel diesel engine gas consumption guarantees are made in Btu per net brake horsepower-hour, based on the lower heating value (LHV). Fuel oil and pilot oil guarantees are based on a higher heating value (HHV) of 19,350 Btu per lb. All sea-level fuel consumption guarantees are contingent on the following conditions: (1) intake air temperature between 40 and 90 F, inclusive; (2) barometric pressure of intake air between 28.25 and 30 in. Hg, inclusive; (3) fuel to conform to the engine builder's specifications as set forth in his bid or tender.

Tolerances. In adopting these standard practices, the engine builder absorbs performance differences caused by three variables. Air density may vary up to 4.8% from the mean due to temperature; it may vary up to 2.9% from the mean due to pressure; the LHV of the fuel may vary as much as 300 Btu (1.6%) for the same HHV. These variables are independent of each other and, in any specific case, may accumulate adversely. Therefore, all fuel-consumption guarantees should be subject to a tolerance of 3% and a correction for any deficiency in the heating value of the fuel.

All heat engines can utilize only the lower heating value of the fuel used. However, owing to the greater ease and accuracy with which the higher heating value of fuel oil can be determined and checked, refiners and distributors sell oil on the higher heating-value basis exclusively. For this reason, engine builders base fuel-oil-consumption guarantees on the higher heating value.

Standard fuel consumption guarantees are not made for one-quarter load. Small differences in operating conditions, especially in new engines, make such tests inaccurate and not reproducible.

FIELD TESTS FOR HORSEPOWER CAPACITY AND FUEL CONSUMPTION. If the buyer so requests, a guarantee test for horsepower capacity and fuel consumption will be made after the engine is installed in its ultimate location. When the buyer requires more elaborate tests, they are conducted in accordance with the Test Code (see Section 19) and at the buyer's expense.

LUBRICATING OIL CONSUMPTION varies so widely because of the influences of plant conditions and operation that it is neither feasible nor logical for the engine builder to guarantee the lubricating oil consumption of his engines.

STANDARD EQUIPMENT FOR STATIONARY ENGINES. The following is a minimum list provided that the design of the engine requires these items. (1) Engine flywheel (except in the case of flywheel-type generator). (2) Extension shaft and outboard bearing if required. (3) Piping on engines to inlet and outlet connections. (4) Exhaust manifold or its equivalent. (5) Air-inlet manifold or its equivalent. (6) Blower and its driving equipment for supercharged engines. (7) Scavenging air equipment for two-cycle engines. (8) Lubricating-oil strainer or its equivalent. (9) Fuel-oil strainer or its equivalent. (10) Fuel-oil booster pump if required. (11) Lubricating-oil sump tank if required. (12) Force-feed lubricator if required. (13) Lubricating-oil pumps and coolers, if required. (14) Piston-cooling oil pump and cooler if required. (15) Pump for circulating lubricating oil before starting and after stopping oil-cooled-piston engine if required. (16) Lubricating-oil pressure gages for pressure system. (17) Cooling-water pressure gages if required on engine. (18) Thermometers for lubricating oil to and from engine and for piston cooling if required. (19) Thermometers for engine-cooling water supply and discharge. (20) Compression release valves and safety valves if required. (21) Suitable governing equipment. (22) Synchronizing device either hand-operated or electrically operated for generator units where necessary. (23) Flywheel barring device either hand-operated or power-operated. (24) A set of tools for each installation but not duplicated for more than one engine per engine room. (25) Engine builder's standard set of spare parts. (26) Engine platforms and steps if required but not interconnecting platforms. (27) Drilled and tapped holes for exhaust-temperature measuring devices but not the devices. (28) Drilled and tapped holes for the attachment of an indicator but not the indicator or the indicator valves or reducing motion. (29) Foundation bolts and nuts and anchor plates, including those for generator and exciter but not including sleeves nor casings. (30) Instruction book for operator. (31) Foundation prints for good soil. (32) Profile prints showing size and location of piping on engine to which buyer connects. (33) Diagrammatic prints showing recommended arrangement of typical station piping. (34) Engine part lists.

STANDARD GOVERNOR PERFORMANCE. For industrial electric drives involving a single generator where constant voltage is not important, for nonelectric drives, and where loading is relatively uniform, there is usually little need for governor regulation closer than 5 to 6% (variation between full-load and no-load speed in percentage of full-load speed). For multiple electric drives such as municipal plants, where time clocks and other constant-frequency equipment are involved, much closer regulation is required.

Recognized standards for nonisochronous governors of the centrifugal type are these. (1) Governor will control engine speed to 5 to 10% of rated speed upon gradual changes from no load to full load or vice versa. (2) For sudden changes in load within limits of engine rating and not exceeding one-half the rated load, the momentary speed change shall not exceed the normal no load–full load percentage regulation. This momentary speed change shall be in per cent of speed at instant of load change. (3) Under constant load there shall be no hunting; with changing load there shall be no sustained oscillations of speed or power output following a load change.

Recognized standards for nonisochronous relay type governors should be the same except that engine speed will be controlled within 2 to 8% upon gradual load changes.

Recognized standards for isochronous relay powered type governors with adjustable speed droop are these. (1) Governor will control the engine speed from 0 to 3% of the rated full-load speed on gradual load changes from no load to full load or vice versa, the percentage of regulation to be adjustable during engine operation. (2) Where engines are intended for parallel operation, the momentary speed change for sudden changes in load within the limits of engine rating and not exceeding one-half the rated load shall not exceed 5% of the speed at instant of load change. (3) Under constant load there shall be no hunting; with changing load there shall be no sustained oscillations of speed or power output following a load change.

Isochronous governors without speed droop adjustment may be furnished where such characteristics are suitable.

PARALLEL OPERATION. Engine builders usually will be able to furnish units that will operate in parallel with existing installations, provided such other generating units are capable of operating in parallel with each other, provided the other generators are equipped with damper windings, and provided that the station is equipped with voltage regulators with adequate cross-current compensations. Switchboard instruments should also be properly damped.

TORSIONAL VIBRATION AND CRITICAL SPEEDS. In the case of constant-speed units, it is standard practice to so coordinate the assembly of each engine and its driven equipment that there shall be no harmful torsional vibrations within 10% above or below the operating speed. When engine builder furnishes engine and all driven equipment, entire responsibility rests with him. When driven equipment is furnished by others, buyer must furnish engine builder with necessary information, and engine builder must coordinate the machinery arrangement.

In variable-speed units, the operating range is made as free from vibrational stresses as possible. Buyer is advised of any unsafe speed ranges which must be avoided. Torsional vibration damping devices are not usually furnished as standard equipment. They may be supplied as extra equipment if required by special operating conditions.

7. SELECTION AND INSTALLATION OF DIESEL ENGINES

The book *Standard Practices* (1946) published by the Diesel Engine Manufacturers Association has much additional information on this general subject, including suggestions for invitations for bids and specifications.

DIMENSIONS, OUTPUT, AND FUEL ECONOMY OF TYPICAL INTERNAL-COMBUSTION ENGINES. Tables 2, 3, and 4 give data on diesel engines, fuel-oil engines (low compression and spark ignition), gasoline engines, and a natural gas engine. The tables are separated according to cylinder size, but this also corresponds to other natural characteristics. Rated horsepowers are for continuous duty, according to builders recommendations, but design and rating objectives vary with service application conditions. The tables show typical engines and indicate the range of possibilities in engines of these sizes. Intermittent speed and output ratings are 10 to 25% higher than the figures given. Maximum figures are still higher.

Table 2. Data for Engines with Cylinders Less Than 4 Inches in Diameter

(All engines are of four-cycle type. Diesel engines have combustion chambers of either Lanova or air cell type.)

	Diesel Engines											Oil Engine	Gasoline Engines		
	Air-cooled														
	Fairbanks Morse Air Cell	Sheppard Air Cell	Hallett Air Cell	Witte Air Cell	Buda Lanova	Hallett Air Cell	Continental Lanova	International Harvester Air Cell	Waukesha Air Cell	Hercules Air Cell	Buda Lanova	Waukesha Hesselman	Continental	Buda	Hercules
No. cylinders	1	1	1	1	1	2	4	4	6	6	6	4	4	4	6
Bore, in.	3	3	3 1/2	3 1/4	3 7/16	3 7/8	3 3/8	3 7/8	3 5/16	3 5/8	3 5/8	3 3/4	3 3/8	3 7/16	3 1/2
Stroke, in.	4	4	3 5/8	4 1/2	4 1/8	4 1/4	4 3/8	5 1/4	3 3/4	4	5 1/8	5	4 3/8	4 1/8	4 1/4
Revolutions/min	1800	1800	1800	1200	1800	1500	1800	1500	1800	2400	1800	1500	2000	2000	2400
Brake horsepower	3.5	3.75	5	4	6.5	18	26.4	31	30	60	56	38	28.3	35	67
Total cyl. displacement, cu. in.	28	28	35	37	38	100	157	248	194	248	317	221	157	153	245
Engine weight, lb.	330	415	185	560	670	570	605	1215	800	850	1150	695	535	780	590
Mep, psi	54.5	58.4	63.0	70.7	74.8	94.8	74.2	66.1	76.5	80.1	77.8	91	71.5	90.8	90.2
Piston speed, ft/min	1200	1200	1088	900	1238	1063	1313	1313	1125	1600	1538	1250	1458	1375	1700
Weight/displacement, lb/cu in.	10.8	14.8	5.3	15.1	17.6	5.7	3.9	4.9	4.1	3.4	3.6	3.1	3.4	5.1	2.4
Specific weight, lb/bhp	94	111	37	140	103	31.7	22.9	39.2	26.7	14.2	20.6	18.3	18.9	22.3	8.8
Specific output, bhp/cu ft	216	232	248	187	296	311	290	216	267	418	305	297	332	395	473
Bhp/cylinder	3.5	3.75	5	4	6.5	9	6.6	7.8	5	10	9.3	9.5	7.1	8.75	11.2
Fuel at rated mep, lb/bhp-hr	.60	. . . .	.46	.50	.47	.42	.43	.44	.45	. . .	.47	.54	. . .	. . . .	.57

Table 3. Data for Engines with Cylinders 4 to 7 Inches in Diameter

(All engines are of four-cycle type, except as noted. Diesel engines have combustion chambers of Lanova, open, or air cell type.)

	Diesel Engines														Oil Engine	Gasoline Engine			Natural Gas
	Two-cycle Type																		
	G.M. Detroit Open Chamber	Harnischfeger Open Chamber	Fairbanks Morse Open Chamber	G.M. Cleveland Open Chamber	Fairbanks Morse Air Cell	Sheppard Air Cell	Buda Lanova	Continental Lanova	International Harvester Air Cell	Murphy, with Blower Open Chamber	Hercules Air Cell	Buda, with Blower Lanova	Cummins, with Blower Air Cell	Caterpillar, with Blower Air Cell	Waukesha Hesselman	Continental	Buda	Hercules	Buda, with Blower
No. cylinders	6	6	8	8	6	6	6	6	6	6	8	8	V-12	V-12	6	6	6	6	8
Bore, in.	4 1/4	4 1/2	6 1/4	6 1/2	4 1/4	4 1/4	4 1/4	4 3/4	5 3/4	6	6 1/4	6 3/4	5 1/8	5 3/4	5 1/4	4 3/4	5 5/8	5 3/4	6 3/4
Stroke, in.	5	5 1/2	9	7	5	5	5 1/2	5 3/8	7	6 1/2	6	8 3/4	6	8	6	5 3/8	6 1/2	6	8 3/4
Revolutions/min	1800	1200	720	1200	1200	1800	1600	1800	1375	1200	1700	1000	1800	12000	1400	1800	1400	1500	1000
Brake horsepower	140	120	280	450	56	67	75	96.5	144	165	325	308	350	40	130	108	160	169	332
Total cylinder displacement, cu in.	426	522	2210	1859	426	426	468	572	1091	1103	1474	2505	1486	2493	777	572	970	935	2505
Engine weights, lb	1700	1900	17,200	8400	1700	1790	1245	1785	3825	5400	4600	10,800	4550	11,000	1865	1670	3830	1830	13,240
Mep, psi	72.3	74.3	69.7	80	87.4	69.2	79.2	74.4	76.0	98.8	103	97.4	103.8	105.6	94.4	83.3	93.7	95.7	105
Piston speed, ft/min	1500	1100	1080	1200	1000	1500	1467	1612	1605	1300	1700	1458	1800	1600	1400	1612	1517	1500	1457
Weight/displacement, lb/cu in.	4.0	3.6	7.8	4.5	4.0	4.2	2.7	3.1	3.5	4.9	3.1	4.3	3.1	4.4	2.4	2.9	3.9	2.0	5.3
Specific weight, lb/bhp	12.1	15.8	61.5	18.7	30.4	26.7	16.6	18.5	26.6	32.7	14.2	35.1	13	27.5	14.3	15.5	23.9	10.8	39.9
Specific output, bhp/cu ft	567	397	219	419	227	272	277	292	228	259	381	213	407	278	289	326	285	313	229
Bhp/cylinder	23.4	20	35	56.3	9.3	11.2	12.5	16.1	24	27.5	40.6	38.5	29.2	33.3	21.7	18	26.7	28.2	41.5
Fuel at rated mep, lb/bhp-hr	.49	.45	.40	.....	.45	....	.47	.43	.44	.42	.	.47	.43	.44	. .	.	...	.56	......

Table 4. Data for Engines with Cylinders 7 to 10 Inches in Diameter

(All diesel engines have open combustion chambers except Waukesha with turbulent air cell.)

	Diesel Engines										Oil Engine
	Two-cycle			Four-cycle							
	G.M. Electromotive	G.M. Cleveland	Fairbanks Morse Opposed Piston	Waukesha	Cooper-Bessemer	Chicago Pneumatic Turbocharged	National Superior Turbocharged	Cooper Bessemer Turbocharged	American Locomotive Turbo-supercharged	Lima-Hamilton Turbocharged	Waukesha Hesselman
No. cylinders	V-16	V-16	12	6	8	8	8	V-16	V-16	8	6
Bore, in.	8 1/2	8 3/4	8 1/8	8 1/2	8	9	9	9	9	9	8 1/2
Stroke, in.	10	10 1/2	10	8 1/2	10 1/2	10 1/2	12	10 1/2	10 1/2	12	8 1/2
Revolutions/min	800	750	720	900	900	720	650	1000	1000	950	900
Brake horsepower	1500	1500	1920	240	410	568	625	1600	2000	1200	300
Total cyl. displacement, cu in.	9072	10,097	12,408	2894	4217	5342	6106	10,685	10,685	6106	2894
Engine weight, lb	29,000	29,500	40,840	11,100	15,000	21,200	24,500	34,000	37,000	16,600	10,750
Mep, psi	81.9	78.3	85	73	85.5	117	125	118.5	148.8	164	91
Piston speed, ft/min	1333	1312	1200	1273	1575	1260	1300	1750	1750	1900	1273
Weight/displacement, lb/cu in.	3.2	2.9	3.3	3.8	3.6	4.0	4.0	3.2	3.5	2.7	3.7
Specific weight, lb/bhp	19.3	19.7	21.3	46.2	36.6	37.3	39.2	21.2	18.5	13.8	35.8
Specific output, bhp/cu ft	286	257	268	143	168	184	177	259	324	340	179
Bhp/cylinder	93.8	93.8	160	40	51.2	71	78.2	100	125	150	50
Fuel at rated mep, lb/bhp-hr	....	....	.365	.46	.....	.....	.....	.37	.37	.39	.52

SELECTION OF SIZES. These are the important principles to be kept in mind in this connection. (1) Sufficient horsepower must be provided to satisfy the maximum demand for power. Engine overload capacity should not be depended on to handle regularly recurring peak loads. (2) Unit sizes should be selected to provide power at the lowest total cost for both operating expenses and fixed charges. (3) The most efficient load range, from the standpoint of fuel economy, is from half load to full load. (4) A small engine is nearly as efficient as a large one. (5) Additional engines can easily be added at any future date, and provision should be made for this.

LOAD VARIATION is presented most effectively in curve form, using successive power demands as ordinates and hours of the day as abscissae. Number and size of engines should be selected to conform with load conditions.

OVERLOAD CAPACITY. While diesel engines are rated to allow some overload capacity for short periods, this should not be utilized to handle routine peak loads. Any predictable load should be within the combined ratings of the engines.

POWER TRANSMISSION. Diesel engines may be direct-connected to driven machinery if starting torque can be limited to less than 50% of full-load torque.

CLUTCHES. Torque delivered by a diesel engine reaches full-load value at 25 to 35% of full-load speed. Machinery requiring a high starting torque, but requiring only normal torque immediately after starting, may be driven by diesel engines through clutches. A flour mill line shaft is a typical instance.

CLUTCH PULLEYS. For belt drives clutch pulleys should be on the driven, rather than on the driving, shaft. If driving shaft location is essential for the clutch, driving pulley should be on a quill.

ELECTRIC, HYDRAULIC, AND PNEUMATIC DRIVES. Where prolonged high starting torque is required, electrical, hydraulic, pneumatic, or a combination clutch and sliding gear drive is necessary. See Hydraulic Couplings, Section 5. Such transmissions are used in diesel locomotives, shovels, and drag lines.

MACHINERY LAYOUT. In designing a diesel plant, machinery should be laid out first and buildings designed around it. Good natural light contributes to lower operating costs. Ample space should be allowed around each engine. When engines and auxiliaries have been worked into the floor plan, a workroom and storeroom should be added. All floors should drain to one or more drains. The engine builder's specifications of minimum headroom from top of engine to bottom of crane hook should be observed.

SPACE FOR FUTURE UNITS. If more horsepower possibly will be needed at a future date, space should be allowed for it in the building, or the building designed so that it can be extended. Doorways should permit the introduction of future units and parts for present units.

CRANES. The best crane layout provides runways on either side of the building for a traveling crane that can serve the entire floor area. The crane may be hand operated, but with two or more engines, cost and time of erection can be reduced if the hoist is motor driven. Individual cranes may be mounted on I beam tracks over each engine center line, extending sufficiently beyond units to allow easy landing of parts.

ARRANGEMENT OF TWO OR MORE ENGINES. The most compact arrangement for two or more engines is side by side. This arrangement permits locating engine auxiliaries at one side of the room and driven apparatus with appurtenances at the opposite side.

TYPICAL PLANT LAYOUTS. One typical arrangement (Fig. 17) has a basement for many of the auxiliaries and provides an inside location for all air-intake filters and exhaust equipment, readily permitting servicing this equipment in severe weather. An alternative arrangement, without basement, places air filters and exhaust mufflers outside.

In all installations, provide adequate ventilation and heating for all conditions, provide adequate protection for flywheels and all other moving parts, provide for easy operation and maintenance of auxiliaries, provide for easily accessible valves and controls, and make the piping connections short and simple.

FOUNDATIONS. Manufacturers supply foundation drawings with each engine sent out. These drawings usually apply for a bearing of hard pan, confined gravel, hard clay, or rock. If such firm subsoil lies considerably below the minimum depth required, the foundation may be designed in the regular way, but supported on piling. An alternative construction is a reinforced slab supported by reinforced concrete pillars. An expert foundation engineer should be consulted where special conditions exist.

Minimum depth for foundations should be sufficient to prevent settling from frost, vibration, or influence of loads borne by adjacent ground, irrespective of firmness of subsoil. If solid rock exists at less than the recommended depth, excavation should be carried a foot or two into the rock to secure firm anchorage; in no event should absurdly shallow foundation be allowed.

Foundation Over Underground Water. An engine foundation never should be located over underground water without the advice of an experienced foundation expert, or without full details being given to the engine builder.

Foundation Bolt Template. The bolts should be supported by a template, carefully leveled, resting on the foundation forms. Each bolt should be surrounded with iron pipe or stove pipe, supported by the bottom washer, and stopped with waste at the upper end to prevent entrance of concrete.

Pouring the Concrete. The foundation should be of concrete, one part cement, two parts sand, and four parts broken stone or gravel (1 in. maximum). The entire foundation should be poured at one time, with no more interruptions than are required for proper spading and ramming. The top should be level, and left rough and clean for grouting. After pouring, the top should be covered and wet down twice daily until the forms are removed at the end of the third or fourth day. The engine should not be placed on the foundation until ten days have elapsed, or operated until another ten days have passed.

Leveling the Engine. The engine should be leveled by wedges resting on steel plates on top of the foundation, and grouted in with a thin mixture of one part cement and two parts sand. The grout should fill the pipes around the foundation bolts. The leveling wedges should be removed after grout has thoroughly set.

PIPING AND WIRING should be located in trenches with side walls and floors of concrete at least 4 in. thick, and covered by removable steel plates. Inlet, exhaust, fuel oil, lubricating oil, circulating water, and air piping trenches should be drained. Generator lead trenches should drain to their respective flywheel and generator pits through 3-in. drains All piping should be supported in trenches by racks. To avoid fire hazard, fuel and lubricating oil piping should not be located in the same trench with exhaust piping.

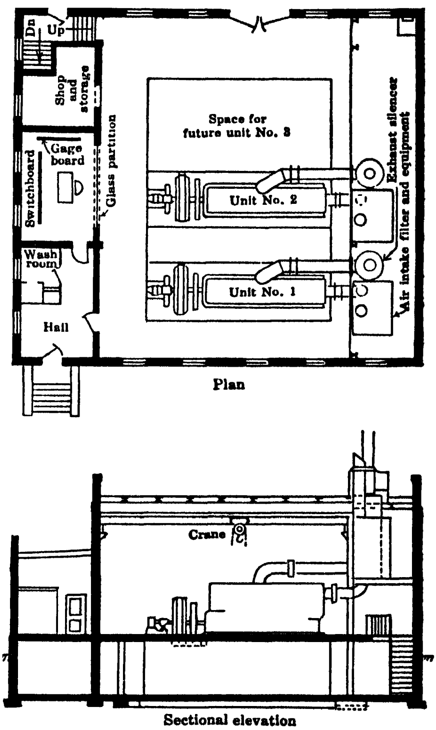


FIG. 17. Typical diesel power plant layout with basement. (Reprinted by permission of Diesel Engine Manufacturers Association)

INSULATING FOUNDATIONS AGAINST VIBRATION. Engines in residential districts, hotels, office buildings, or department stores should be set on cork-insulated foundations or spring dampers to minimize transmission of vibration. Industrial engines are also frequently mounted on spring dampers. All piping from engine to off-foundation locations should have flexible sections to prevent transmission of vibrations.

AIR INTAKE. Intake piping should be as short as possible and of adequate flow area. Filters should always be used, and where there is possibility of much dust this point requires special attention. If low noise levels are required, mufflers should also be used; two-cycle engines and turbocharged four-cycle engines always need mufflers. If the piping becomes long consult the engine manufacturer to avoid resonant air-column vibrations.

EXHAUST. Consult engine manufacturer and take special care to avoid back pressure and gas-column vibrations in exhaust piping and system. Mufflers are always needed, even on turbocharged engines, to reduce noise level to tolerable figures. Spark-arresting-type mufflers are needed in some locations. Exhaust system should be of steel piping; masonry, brick work, or tile is not usually satisfactory on account of poor resistance to thermal expansion and vibration. Allow for expansion due to 1000 F variation in temperature in normal installations.

FUEL STORAGE. Fuel-oil storage tanks may be of steel or concrete. Concrete tanks should be properly treated inside to prevent absorption of oil. Whether fuel is delivered by tank car, tank wagon, or barge, allow adequate storage capacity to operate the plant between deliveries, especially if deliveries may be irregular due to bad weather or other conditions. An unloading pump may be advisable if storage is some distance from delivery. If possible, divide main storage into two tanks to facilitate occasional cleaning without shutting down plant. In general, fire underwriters prefer tanks below ground, but if fire hazard is not serious, inspection and maintenance of tanks are easier for above-ground tanks. Heating coils around the suction pipe are recommended for storage tanks, especially those located in cold climates. They facilitate the flow of fuel in cold weather or when viscous fuels are used regularly. These heating coils should be without joints inside the tank to avoid possibility of steam or water leakage into fuel. Always provide means for gaging the tanks. A sounding pipe is essential, and one of the various types of liquid level indicators may be fitted in addition. Consult local ordinances and fire regulations in every case.

FUEL-HANDLING SYSTEMS. A common system of handling fuel oil is by means of individual day tanks for each engine. Oil is pumped from the main storage to the day tank, whence it flows by gravity to the engine pump. Engines sometimes are equipped with a supply pump which draws fuel directly from the main storage tanks.

Day Tanks. Day tanks above the floor line should have gage glasses to permit measurement of fuel consumed. Day tanks below the floor should have float or pressure-operated gages.

Fuel oil piping and connections should be made up tighter than is customary for water piping. Wrought-iron, steel, or preferably brass or copper pipe may be used for lines between fuel strainer and engine. Unions should be of brass without gaskets. Piping should be located so that oil cannot come in contact with hot exhaust piping.

Fuel-oil filtering or straining usually is sufficient purification for fuel oils; poorer grades often must be centrifuged. Filtering elements may be metal screens, cloth bags, or thin washers held tightly together by king bolts. Filters should be of duplex construction so that one half may be cleaned while the other half is in operation.

LUBRICATING-OIL SYSTEMS. Details of the lubricating-oil system vary with size and design of engine. In the smaller sizes, the system is entirely contained within the engine, and no attention is needed at installation except to provide easy access to fill and drain connections, oil level indicator, and filters. In larger engines, give attention to the same points, and provide for all oil transfers without slopping or waste of oil. Provide for draining entire system for cleaning oil as necessary and for refilling system. Nearly all engines have one pressure system with sump for all parts except additional mechanical lubricators for cylinder walls. Hand-operated or motor-driven pumps may be advisable in some cases for draining or filling purposes. On large engines, especially with piston cooling, an auxiliary pump is useful to circulate oil through the engine before starting, and after shutting down to carry away residual heat.

Piping may be black iron, but copper or brass is preferable.

Oil coolers are necessary when pistons are oil cooled, and are frequently needed for lubricating oil only. They should be of ample size because heat-transfer capacity is lost rapidly as the cooling surfaces become dirty. It is desirable to arrange a by-pass around the cooler on both the water and oil sides so that oil can be quickly warmed up when starting, and so that cooler can be isolated during operation if necessary.

Oil purification equipment is of prime importance. Various methods are used: full flow continuous, continuous by-pass of a portion of the circulating oil, the batch method where part or all of the oil is drained for replacement by clarified or new oil. Various methods are employed for clarification: settling tanks, reclaimers, centrifuges, and filters of cloth, cotton waste, cellulose, metal or paper disks with fine clearances between them, or fuller's earth. The choice depends on the type and size of engine, the conditions of operation, and the type of lubricating oil used. The engine manufacturer can well be consulted for his recommendations.

Automatic alarms or engine shut-down devices for low-lubricating oil pressures are frequently used.

COOLING WATER and water-cooling equipment must be clean to avoid deposits in water spaces. The water must be noncorrosive with a pH value of 7 to 7 $^1/_2$. The cooling system must be such that all engine parts are completely submerged, and proper venting should be arranged to accomplish this. The water must be soft to avoid deposits of scale on cooling surfaces. There must be sufficient circulation through the engine to hold the temperature rise within the limits recommended by the engine manufacturer. The larger engines should have a means of continuing the circulation through the engine after shutdown to carry away the residual heat.

Anti-freeze is satisfactory only in small engines; larger engines should be drained in cold weather or provision made for keeping the jacket-water temperature above freezing.

The cooling-water system should be arranged for easy draining in case of engine repairs and easy refilling.

Hard water may be softened with a Zeolite softener, which absorbs the calcium, magnesium, aluminum, and iron salts and gives soft water. Zeolite is regenerated by a brine solution (see Chemistry of Boiler Feedwater, Section 7).

Open and Closed Systems. The open type permits the cooling water to run to waste or cools it in a spray pond or cooling tower. Rarely is there a sufficient supply of satisfactory quality to permit running to waste unless the water is used for process work. Use of spray pond or tower is not usually recommended because of possible accumulations of scale and deposits in the engine jackets.

The closed system is preferable and usually used. The jacket water may be cooled by coils in the bottom of a cooling tower, by a shell and tube heat exchanger, by a radiator, or by an evaporative cooler. All except the radiator require raw water circulation which is cooled by evaporation in the tower, or in a spray pond, or in the evaporator. If the raw water is run to waste, a shell and tube heat exchanger is used. In the closed system there is little or no loss of jacket water. The choice of these methods depends on engine size, cost, and availability of raw water, space available, and the overall economics. For instance, a cooling tower may be best out of doors in a dry climate and an evaporative cooler in a basement installation in a city.

Quantity of Water. Recommendations of engine manufacturers should be followed for quantity of water circulated, temperature rise through engine, and outlet temperature. Usual figure for quantity is 0.4 gallon per minute per rated engine horsepower; this corresponds to 15 F rise.

Piping is usually of wrought iron or galvanized steel.

Alarms or shutdown devices for high water temperature are recommended and frequently used. Thermostatic controls over the water flow, or to supply cold water from the main source for make-up in case of excessive temperatures, are frequently used. In large multiple-unit plants, the individual engine-cooling systems are sometimes interconnected and a common set of circulating pumps used for the station, or at least available as standby.

STARTING AIR SYSTEM. Regardless of the total tank capacity, it is preferable to have a minimum of two tanks, so that there is one in reserve for contingencies. Tanks should be made in accordance with the Rules for Construction of Unfired Pressure Vessels, Section 8, American Society of Mechanical Engineers Boiler Construction Code; also in accordance with requirements of insurance companies, and any state or local codes. Install tanks with conveniently located drains for removal of condensed moisture. Each tank has its own safety valve.

Starting pressure is usually 350 psi or less, and standard black steel pipe is satisfactory with globe shutoff valves designed for the pressure employed. If pressures of 500 psi or higher are used, piping should be of seamless drawn steel tubing or copper tubing with needle valves.

8. DIESEL ENGINE APPLICATION DETAILS

There are many variations in engine details with type and make of engine, but any unit must be judged on its overall ability to meet the requirements. For a further discussion of this subject, see *Standard Practices* (1946) published by the Diesel Engine Manufacturers Association.

ATTACHED PUMPS. Smaller engines usually have built-in circulating water pumps. Larger engines usually require the installation of motor-driven pumps. Some 2-cycle engines have built-in scavenging pumps or blowers; others require motor-driven equipment.

FLYWHEELS. The dimensions and weight of flywheels for diesel engines depend on the equipment to be driven; its engineering data must be analyzed and evaluated with respect to parallel operation of a-c generators and torsional vibration. Flywheels may be of one- or two-piece construction, and must have suitable flywheel effect for the purpose intended. Some types of engines require only the flywheel effect normally incorporated in the generator rotor. Generators with weighted rotors are specified as flywheel type, or weighted-rotor type, as distinguished from standard engine type.

ENGINE EXTENSION SHAFTS. The length and diameter of extension shafts of diesel engines vary with type and speed of engine, type of generator, or other driven equipment, and critical speed or torsional vibration conditions. Flywheel-type generators usually are mounted on short extension shafts; standard engine-type generators require longer extension shafts. Exact dimensions cannot be determined until the driven equipment is selected and its engineering data analyzed.

FLYWHEEL-BARRING DEVICES are required for rotating the engine to its starting position or for adjustment and repair. The three types of barring devices used are manually, pneumatically, or electrically operated. The first two types usually are supplied by engine builders. Slow-speed engines up to about 750 hp generally have manually operated flywheel barring devices.

PYROMETERS, if used, may be attached to the engine, or may be mounted on the switchboard or engine-room wall.

OIL COOLERS for lubricating oil are standard equipment, where climatic conditions require them. Sizes and types are selected by the engine builder.

BEDPLATES. When engine-generator sets are mounted on concrete foundations, general practice is not to use an extension of the engine base or separate subbase to support the generator, exciter, and outboard bearing.

RIGHT- OR LEFT-HAND ENGINE indicates that the flywheels are on the right- or left-hand sides, respectively, when the engines are viewed from the operating side. Any advantages of using both right- and left-hand engines in the same installation are doubtful, and are lost when more than two engines are used. Using all engines of the same hand gives complete interchangeability of parts, a great advantage. If a central location for engine controls is desired, remote controls can be used.

ENGINE PLATFORMS. Differences in engine design (accessibility of controls and of parts for removal or adjustment) call for a variety of platform arrangements. When two or more engines are installed, the engine builder can supply interconnecting platforms and additional ladders as necessary.

WRENCHES AND TOOLS. All necessary special wrenches and tools are usually furnished, one set per installation.

ENGINE SPARES. Most engine builders supply standard sets of spare parts.

INDUSTRIAL POWER TAKEOFF. Engines designed for industrial power applications, when electric generators are not used, have clutches with extension shafts for easy application of pulleys or couplings. Consult with the engine builder about intended applications and details to make sure that design limitations are not exceeded.

9. ECONOMICS OF DIESEL POWER

GENERAL CONSIDERATIONS. Economics of diesel power includes the convenience of use of the diesel engine and its ability to meet the needs of a given power problem as well as the cost of operation. In fact, some economic advantages are not reducible to cost data. Costs themselves are relative and should be compared with competing sources of power suitable for the same application. After the plant has been placed in operation, good management, careful operation, and competent maintenance have important effects on costs.

Conditions and cost for a 5 to 10 hp plant are quite different from those for a 5000 to 10,000 hp plant. Smaller sizes have as competition the gas or gasoline engine, or electric power. Larger sizes compete with steam or electric power. If fuel gas of any sort is available, there is competition against fuel oil on a full, partial, or seasonal basis.

For any industrial power application where a mechanical drive is employed, the diesel has better pulling power throughout the speed range: the torque holds up better at reduced speeds. In many applications, the diesel uses less fuel than any other prime mover; hence in any application where the fuel is carried (e.g., transportation) or where fuel must be transported to an isolated location, the diesel usually has a distinct advantage on this point alone.

The diesel engine uses a safe fuel, free from ordinary fire hazards, and shows excellent economy over its whole load range—better than a gasoline engine, for example. Fuel oil is always lower in cost than gasoline. The diesel has the relative disadvantage of being higher in first cost than the gas or gasoline engine; hence hours of operation per year may be a determining point in some applications.

COSTS. Overall costs are made up of several items. Very small stationary engines never have regular attendants; operating costs include fuel, lubricating oil, supplies, and maintenance. Industrial-power applications may have operators for the machine the engine drives, but usually not for the engine itself. Hence only for regular power plants, independent of other near-by operations, are operators needed. Some of them are made entirely automatic in operation, with attendance for inspection and maintenance purposes only. In all cases, the number of operators on duty varies with the size of plant and the complexities of the operating conditions.

Municipal Electric Plant. Table 5 shows costs over a period of years for the municipal diesel-electric power plant at Algona, Iowa (William H. Gottlieb, *Diesel Progress*, April 1949). The plant consists of:

gines	Fuel Injection Method	Hp	Rpm	Year Installed	Kwh Produced per Gallon Fuel
1	Air	600	200	1928	10.75
1	Air	1000	200	1931	11.0
1	Mechanical	1000	225	1938	13.8
1	Mechanical	1500	225	1942	14.45
1	Mechanical	1500	225	1947	14.45

Table 5. Cost Data (in Dollars) for a Small Municipal Diesel Power Plant

Fiscal Year Ending March 31	Supervision and Labor	Fuel Charges	Lube-oil Charges	Supplies and Miscellaneous	Maintenance and Repairs	Total Generating Cost	Total Generating Cost per net kwh Distributed	Total Electric Department Expense	Depreciation	Total Revenue	Net Profit
1943	10,121	22,026	1087	2447	7255	42,939	.0083	57,590	25,263	127,920	45,066
1944	11,132	22,828	824	2022	6161	42,969	.0082	61,609	25,491	114,734	27,633
1945	13,840	26,705	958	2025	4387	47,917	.0075	68,790	25,616	132,851	38,444
1946	15,502	29,372	1352	2212	4918	53,359	.0078	73,779	25,529	132,198	32,889
1947	17,285	34,615	1604	2588	5806	61,900	.0081	83,756	25,720	141,856	32,380
1948	18,202	54,445	2560	3117	5659	83,984	.0095	112,193	26,284	184,890	46,413

Average load factor is 65%. Lighting rate is 6 cents per kwh for the first 50, 3 cents for the remainder; water heating rate is 1.5 cents per kwh. Ten per cent discount is allowed for prompt payment of bills. Population, about 7000. Fuel burned is a southwest Texas crude of about 24 gravity at a price of 11 cents per gallon. Cooling water is softened and cooled in a cooling tower.

Fuel costs vary with the method of delivery, whether by drum, tank wagon, tank car, or barge. There is usually an advantage in buying on yearly contract, whatever the delivery method. If the quantity required is large, the quality is important; some engines need a special quality for satisfactory operation. Engines with larger cylinders and slower speeds can frequently use the heavier fuels. Heating of such fuels may be necessary for easy flow through the piping and for good atomization. Where such fuels are used, the engine should always be started and stopped on lighter-grade, easy-flowing fuels. Cracked fuels or blended fuels may also be used to cut costs. For judging fuels, use as criterions clean combustion; lack of deposits on pistons, cylinder head, valves ports, and gas passages; lack of ill effect on the lubricating oil; no overbalancing increase in maintenance.

Fuels are not easily judged from the specifications; only the engine itself can evaluate any particular fuel. Comparisons with results in other installations is helpful but not conclusive because of possible unrecognized differences in operating conditions.

Small diesel engines have fuel consumptions of 6 to 7 gal per 100 hp-hr. Large engines burn 5 to 6 gal per 100 hp-hr. The larger the engine, the more important it is to determine this figure accurately from the engine test curves.

LUBRICATING OIL. The lubricating-oil cost depends on the type of engine, the grade of oil, the mechanical condition of the engine, operating temperatures, and other operating conditions; whether used oil is reclaimed or thrown away; and the quantity and kind purchased. Only the best quality of lubricating oil should be used.

MAINTENANCE COSTS are uncertain because of the influence of variables such as load factor, hours of operation per year, character of the load, and care of the engine. In most engines, wearing parts are replaceable; hence maintenance figures are significant only over a cycle covering complete replacement of all wearing parts or over the life of the engine.

ASME Report on Oil-engine Power Cost. The best figures available for operating costs for stationary diesel engines are those compiled annually by the ASME Subcommittee on Oil-engine Power Cost. These figures cover plants of various sizes, over a period of years. Although obtained from electric-generating stationary plants, the cost figures are applicable to other engines of similar type and size, running under similar load and operating conditions. When used for estimating purposes, it is best to select as a basis figures from several plants of the applicable size and type.

Figure 18 shows the lubricating-oil consumption of 93 diesel plants. Wide variations in performance are evident. Figure 19 shows the fuel consumption of 203 diesel plants. The symbols of the plotted points indicate the type of engine:

○ = four-stroke air-injection engines
○ = four-stroke mechanical-injection units
□ = two-stroke air-injection units
⊡ = two-stroke mechanical-injection units with separate scavenging pumps
+ = two-cycle mechanical-injection units of crankcase scavenging type
△ = mixed-type diesel engine units

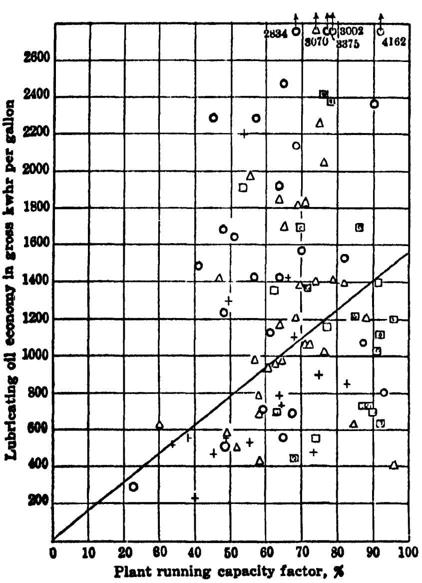


FIG. 18. Lubricating oil economy of 93 full-diesel plants. (Taken by permission from ASME Report on Oil-engine Power Cost)

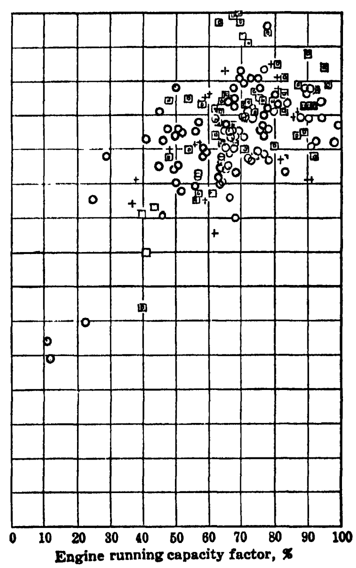


FIG. 19. Fuel economy of 203 full-diesel engines. (Taken by permission from ASME Report on Oil-engine Power Cost)

The following tabulation gives production costs for eight of the smallest plants ranging in size from 100 to 500 hp, and the twelve largest plants in the 1945 Report ranging in size from 5000 to 12,480 hp. Cost figures are in mills per net kilowatthour.

Costs in Mills per Kw hr for	Small Plants	Large Plants
Fuel	8.5	4.3
Lubricating oil	1.5	0.35
Attendance	7.0	1.9
Supplies	1.0	0.3
Engine repairs	4.5	0.65
Other repairs	0.5	0.2
Total production cost	23.0	7.70
Extremes for these plants	9.5 to 59.5	5.4 to 11.4

FIXED CHARGES depend on such factors as amount of investment (which depends on size and type of engines), completeness of accessory equipment, conditions surrounding plant location, and nature of building housing the plant; annual output; probable useful life; rate of amortization; rate of interest.

Adverse installation conditions which tend to increase the first cost should be avoided, such as poor soil conditions for foundations or neighborhoods requiring low noise levels. The plant site should be chosen to keep operating costs at a minimum. A plentiful supply of low-cost cooling water and easy transportation of supplies and fuel oil to the plant are important. All accessory equipment that helps to make safer, better, or lower-cost operation is essential; more than that is luxury. The actual life of most diesel engines is quite indefinite because wearing parts are replaceable. The useful life is limited only by the need of the engine in that service, its continued ability to handle the service loads, continued moderate cost of repairs and replacement of worn parts, and obsolescence of the general design. Many diesel engines have been in service for 25 years and more, but usual practice is to amortize the investment in 12 to 15 years. In general, permanent plants for primary power have a long useful life, but plants installed to supply power to a project which may prove temporary should have a correspondingly short amortization period. Industrial power engines should take the period of the machinery for which they are installed, even though the engine may be salvaged and reused on another machine or for other purposes. (See also Power Supply Economics, Section 16.)

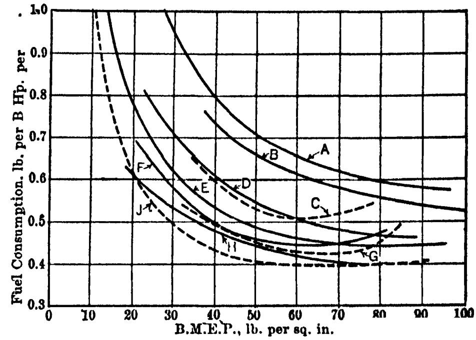


Fig. 20. Typical fuel consumption curves for small diesel engines.

A—6-cylinder, 4.33 x 6 in.; *B*—4-cylinder, 5.90 x 7.09 in.; *C*—4-cylinder, 4.33 x 7.09 in.; *D*—4-cylinder, 5.31 x 7.87 in.; *E*—2-cylinder, 3.15 x 11.00 in.; *F*—6-cylinder, 4.33 x 5.59 in.; *G*—4-cylinder, 4.72 x 7.09 in.; *H*—2-cylinder, 7.5 x 12.0 in.; *J*—4-cylinder, 4.5 x 5.5 in.

FUEL CONSUMPTION OF SMALL DIESELS typical of those used in the light transportation field, are shown in Fig. 20. The wide range of fuel rate is particularly apparent in small sizes, where similar sizes vary as much as 50%.

DIESELS VERSUS GASOLINE ENGINES IN CITY BUS SERVICE. In a paper presented at the SAE Annual Meeting in Detroit, Jan. 11, 1949, E. N. Hatch, Board of Transportation, New York City Transit System, gave some interesting data. In 1945, 250 electric-driven busses (60 diesel and 190 gasoline) operated in Brooklyn. Average data for the year were:

	Miles per Gallon	Fuel Cost, Cents per Gallon	Cost of Fuel, Cents per Mile
Gasoline type	1.85	9.31	5.03
Diesel type	2.97	7.5	2.53

(At 45,000 miles per year, the diesel type saved over $1100 in fuel.)

In 1946 and later, 1234 diesel and 510 gasoline engine busses were purchased. Average results for the entire fleet operating on various routes in New York City for the year ending June 1948 were:

	Miles per Gallon	Fuel Cost, Cents per Gallon	Cost of Fuel, Cents per Mile
Gasoline type	2.76	11.55	4.19
Diesel type	3.27	11.72	3.58

(At 45,000 miles per year, the diesel type saved about $275 in fuel.)

Comparative mileage for various hydraulic transmission busses were: gasoline engine averaged 3.33 to 3.53 miles per gallon, four-cycle diesel averaged 4.78 miles per gallon, and two-cycle diesel averaged 3.97 miles per gallon. On the average, for 40- to 45-passenger busses in this service, the diesel engine type cost $1000 more and saved $500 yearly, with fuel oil and gasoline at the same price per gallon.

Another SAE paper *Diesels in Transit Buses*, presented by M. C. Horine, Mack Manufacturing Corporation, points out that gasoline engines operate at a nearly constant air-fuel ratio of 12 to 15 to 1, but with sharply varying compression pressure, consequently widely variable thermal efficiency. Diesel engines have higher compression, full compression pressure at all loads, air-fuel ratio of 18 to 1 at full load, and well over 100 to 1 at light loads. Hence the diesel is more economical at full load, and much more economical at part load. Under drifting or decelerating conditions of the vehicle, the diesel governor usually shuts off all fuel above idling speed, while gasoline engines consume fuel at a high rate, though producing no useful power. The overall result is that the diesel gives savings up to 50% in fuel.

10. TESTS OF DIESEL ENGINES

Field tests of diesel engines are likely to be expensive. The engine should be in good mechanical condition and properly tuned if tests are expected to show normal or best results. Tests on new engines should be made promptly after a reasonable run-in period and after the engine builder's representative has had an opportunity properly to adjust the engine and its auxiliaries for the test.

The ASME Test Code for Internal-combustion Engines is applicable (see Section 19). In *Standard Practices* (1946) the Diesel Engine Manufacturers Association has suggested a procedure for testing slow and medium-speed stationary engine-generator units in the field which can be modified to suit other types, as necessary. The following discussion outlines some of the principles of the DEMA procedure. The object of tests is primarily determination of output and fuel consumption and, secondarily, collection of other operating data.

MEASUREMENTS. To attain the primary object, measure the following: (1) gross kilowatt-hour output as delivered to the switchboard; (2) net kilowatt-hour output, which is gross output minus essential auxiliaries as listed in Standard Equipment for Stationary Engines, Article 6; (3) weight or volume of fuel consumed; (4) speed in revolutions per minute.

To attain the secondary object, record the following, where applicable. (1) Pressures: barometric (may be obtained from the local Weather Bureau), injection air, scavenging air, lubricating-oil inlet, jacket-water inlet, jacket-water outlet, raw-water inlet, compression at stable operating temperature, and maximum combustion pressure. (2) Temperatures: engine room, intake air, scavenging air in crankcase or receiver, exhaust at manifold outlet, exhaust at cylinder outlets, lubricating-oil inlet, lubricating-oil outlet, piston cooling fluid from each piston, jacket-water inlet, jacket-water outlet, raw-water inlet, raw-water outlet. (3) Physical properties of fuel oil as determined by ASTM standard test methods.

INSTRUMENTS AND APPARATUS. The following are required for the measurements: (1) suitable tanks and scales, or meters, arranged for measuring fuel at each load during test; (2) pressure gages for required pressures; (3) thermometers and pyrometers for required temperatures; (4) diesel engine indicator for taking compression and combustion pressures; (5) tachometer or frequency indicator; (6) revolution counter (optional); (7) clock, watch, or electrical timing apparatus; (8) calibrated electrical instruments for gross kilowatt-hour output of unit and kilowatt-hour input of auxiliaries; (9) a water rheostat, unless steady load of accurately determinable power factor can otherwise be provided. Knowledge and understanding of the Instruments and Apparatus Sections of the ASME Power Test Codes will be found extremely helpful. (See Section 19).

If the engine drives equipment other than electric generator, some means must be provided to determine accurately the engine-coupling power. If power is not measured at the engine coupling, allowance must be made for losses of the driven equipment and losses in the transmission.

SCHEDULE OF PREPARATIONS. (1) Make sure engine is in proper condition and is functioning properly. (2) Check and install instruments and test apparatus. (3) Assure uninterrupted jacket-water supply. (4) Assure a sufficient fuel supply so that one grade will last through the tests. (5) Assure a sufficient lubricating-oil supply of one grade. (6) Record data called for in the test report form.

SCHEDULE OF TESTS. (1) Under steady load conditions, run engine 1 hour at half load, 1 hour at three-quarters load, 4 hours at full load. (2) Before each test is started, bring engine to stable state under conditions of the test; indicate attainment of this state by recording successive readings in report. (3) During each test period, record readings every 15 minutes. (4) Indicating instruments are read at least twice for each test period. Recording instruments are read and fuel measurements taken at the beginning and end of each test period. (5) Test for 10% overload, if run, must be considered as approximate only because generator efficiencies at overload are not normally given by generator builders. Also generator should be guaranteed for this load by the builder.

TABULATION OF DATA AND RESULTS. The following form is considered standard:

I. General information: date of test, owner, location, altitude, builder, name of test engineer, object of test.

II. Description of engine and equipment, including capacity, speed, make, type, and other pertinent data.

III. Test data and results: (1) Data required to obtain engine output and fuel consumption. (2) When additional operating data are desired, record pressures, temperatures and fuel properties as specified under Measurements. (3) Fuel quantities: (*a*) Calculation of net fuel per hour from measured consumption less recovered fuel. (*b*) Net fuel oil consumption in pounds per hour corrected to 19,350 Btu (higher heating value) per pound. For gas-diesel and dual-fuel diesel engines, gas consumption is expressed in Btu per hour lower heating value, and pilot oil in Btu per hour higher heating value. (4) Electrical data: (*a*) Meter readings including gross kilowatt output. (*b*) Standard generator efficiencies. (*c*) Corrected generator efficiencies, including losses listed under Calculation of Results, below. (*d*) Deductible kilowatt output to auxiliaries. (*e*) Net kilowatt output. (5) Power: (*a*) Brake output of engine, kilowatts. (*b*) Brake output of engine, brake horsepower. (6) Fuel consumption· (*a*) Pounds of fuel per kilowatt-hour, gross output. (*b*) Pounds of fuel oil per kilowatt-hour, net output, essential auxiliaries deducted. (*c*) Pounds of fuel oil per brake horsepower-hour. For gas-diesel and dual-fuel diesel engines, fuel consumption is expressed in Btu per kilowatt-hour or per brake horsepower-hour.

CALCULATION OF RESULTS. (1) Credit gross fuel consumption with collected uncontaminated leakage. Correct for deviation from higher heating value of 19,350 Btu per lb. Express in pounds of fuel oil per hour. Express gas consumption in Btu per hour lower heating value. Express pilot fuel oil consumption in Btu per hour higher heating value. (2) Gross kilowatt-hour output is obtained directly from calibrated instrument readings. (3) Net kilowatt-hour output is calculated by deducting input to essential auxiliaries from gross kilowatt output. (4) Shaft or brake horsepower output is calculated from gross kilowatt output after adding losses occurring between engine and test instruments. Such losses are: (*a*) I^2R losses of generator stator and coils. (*b*) Core losses. (*c*) Stray load losses. (*d*) Exciter losses. (*e*) Field rheostat losses. (*f*) Bearing friction and windage losses. (*g*) Friction losses in chains or belts. (*h*) Cable losses. If actual values are unavailable, use 0.3% of rated output of generator at rated load and vary as square of load.

With standard designs of engine or weighted-rotor types of generators with direct-connected, belted, or chain-driven exciters, subtract from generator efficiencies a figure to allow for exciter losses, cable losses, field rheostat losses, bearing friction and windage, none of which is included in standard generator efficiencies. When generator overall efficiencies are guaranteed, including above losses, use guaranteed values to determine net output from engine. Where overall efficiencies are not stated, exact determination of such losses is not practicable and deductions should be made according to the following tabulation. They are correct within limits of accuracy of field test.

	Deductions from Generator Efficiencies		
Full-load Generator Efficiency	Full Load	Three-quarter Load	Half Load
88.1 to 89	4.0	5.1	8.0
89.1 to 90	3.5	4.6	7.2
90.1 to 91	3.1	4.0	6.3
91.1 to 92	2.6	3.4	5.4
92.1 to 93	2.1	2.8	4.6
93.1 to 94	1.7	2.2	3.7
94.1 to 95	1.2	1.8	2.8
95.1 to 96	0.8	1.1	2.0

If excitation is external, obtain suitable correction figure from generator builder. Power factor should be maintained at or above that for which efficiencies are specified, throughout all tests.

FUEL SPECIFICATIONS FOR FIELD TEST. New pipe lines and tanks are likely to contain scale and other foreign matter. In any case, take special precautions to assure a supply of clean fuel to the engine. Fuel used during field test should be in accordance with engine builder's specifications.

TESTS FOR GOVERNING DIESEL ENGINE GENERATOR SETS. The object is to measure speed changes, to determine the performance of the governing equipment.

Apparatus Required. Either a tachograph having a sensitivity of one-quarter of 1% or an indicating tachometer of equal sensitivity is needed. Instruments should be calibrated through the range of speed to be measured, and a rating curve established 10% above and 10% below normal speed.

Governing Test Methods. Bring engine to normal speed without load and observe governor action. Without adjusting governor, gradually apply load and measure speed droop. Suddenly apply load and remove load, and note momentary speed changes.

Sensitiveness is not easily determined with accuracy in the field; laboratory instruments are required.

Stability is measured by tachograph equipment. At normal speed and any constant load, there should be no erratic movements of the fuel-control mechanism. On commercially variable load, departure from normal speed should be a minimum and return to

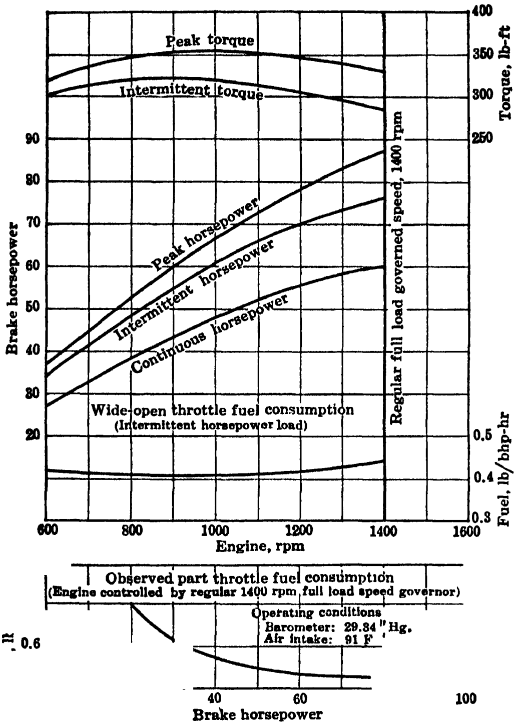


Fig. 21. Performance of a four-stroke-cycle four-cylinder diesel engine with precombustion chamber (Courtesy of International Harvester Co.)

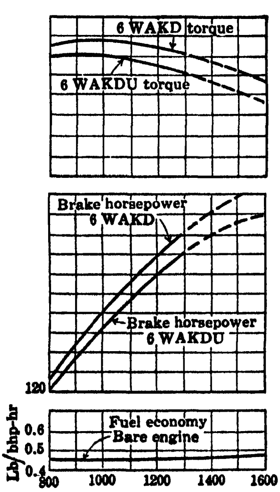


Fig. 22. Performance curves of Waukesha model 6WAKDU engine (6 1/4 x 6 1/2 in.; 6-cylinder). (Courtesy of Waukesha Motor Co.)

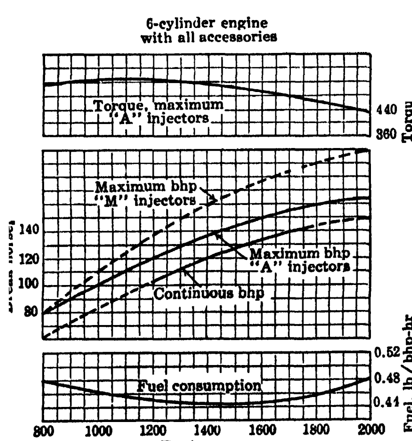


Fig. 23. Performance curves, General Motors-Detroit Diesel model 71 engine. (Courtesy of Detroit Diesel Division General Motors Corp.)

normal speed should be rapid and positive, without excessive hunting. It may be necessary to load engine through a water rheostat to obtain sufficiently steady load conditions to judge stability.

Speed drop is measured by adjusting the governor for rated engine speed with rated loading and then, without changing governor setting, slowly reducing load to no load, measuring the increase in engine speed.

Momentary speed changes are measured by applying or removing load suddenly. During these transient conditions the maximum percentage departure from the speed at the instant of sudden load variation is the applicable criterion. The stabilization period is the time from the initial load change until the speed has reached a steady condition. Except for single isochronous governing units, the new stabilized speed will be above or below the initial speed by a finite amount, according to the speed droop characteristic. These momentary speed changes are not easily handled in the field, except for generator units, where load can suddenly be dropped by tripping the circuit breaker.

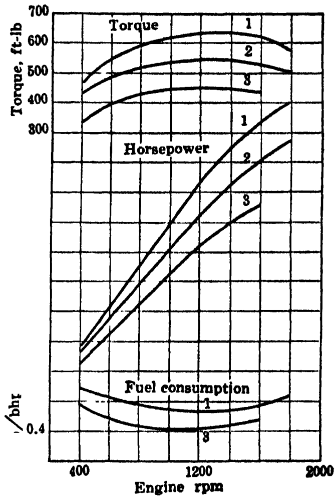


Fig. 24. Performance curves, Cummins engine corrected to standard conditions. Curves 1 represent maximum gross power applicable for automotive and similar applications. Curves 2 are for intermittent service. Curves 3 are for continuous service. (Courtesy of Cummins Engine Co.)

ACTUAL ENGINE TEST RESULTS. In many of the smaller engines in industrial-power applications, the engine power is taken off through a mechanical drive, and torque is important. Thus tests of smaller engines should include maximum power data at various speeds, from which torque curves are derived. Typical test results are given in Figs. 21 to 24. Figure 21 shows the performance of a four-cylinder engine with precombustion chamber. Figure 22 shows test

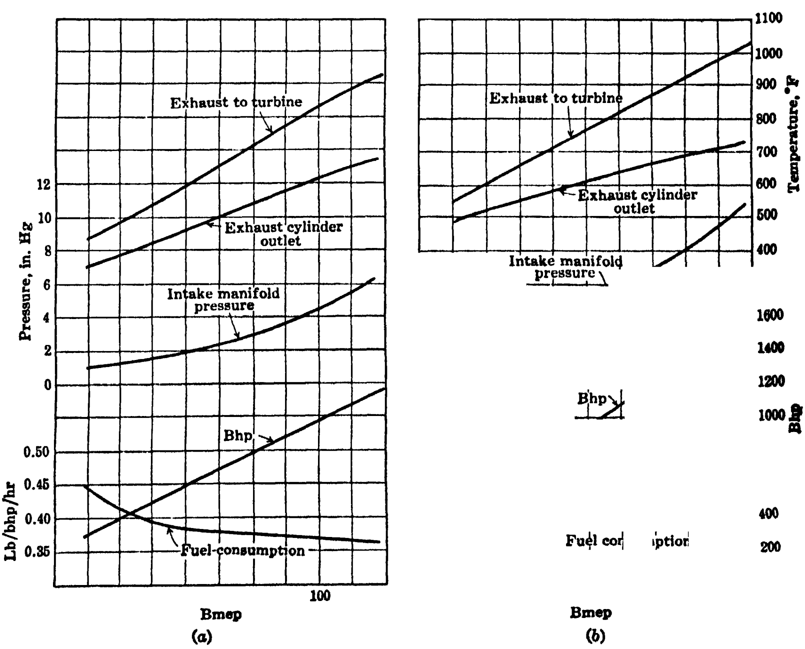


Fig. 25. Performance curves, Cooper-Bessemer model FV-T 16-cylinder engine. (a) at 720 rpm and (b) at 1000 rpm. (Based on 19,350 Btu/lb higher heating value.) (Courtesy of Cooper-Bessemer Co.)

results of a six-cylinder engine with turbulent combustion chamber entirely in the cylinder head. Both are four-cycle engines. Performance of a two-cycle engine with unit-type fuel injector and open combustion chamber is given in Fig. 23. Test results of a turbocharged four-cycle engine with open combustion chamber is as shown in Fig. 24.

Typical test results from a larger four-cycle engine, supercharged and nonsupercharged, are given in the following tabulation. The Cooper-Bessemer Type LS 8-cylinder engine has a rated speed of 327 rpm with two-valve cylinder heads for the nonsupercharged unit, and is rated at 360 rpm with four-valve cylinder heads for the supercharged unit.

	Nonsupercharged	Supercharged
Full load, bhp	1090	1800
Brake mep, psi	80	120
Fuel consumption, full load	0.386	0.384
three-quarter load	0.394	0.384
half load	0.43	0.406
one-quarter load (approximate)	0.57	0.472

		Cylinder Outlet	Turbine Inlet	Turbine Outlet
Exhaust temperature, °F, full load	855	760	925	735
three-quarter load	710	645	775	625
half load	570	545	620	515
one-quarter load	430	440	465	405

Performance curves for a sixteen-cylinder supercharged engine at two different speeds are shown in Fig. 25.

11. DIESEL FUEL OILS

(See also Liquid Fuels, Section 2)

Diesel engines in general are capable of burning a rather wide variety of liquid fuels; the large-cylinder slower-speed diesels will burn a wider range than the smaller engines. While vegetable and animal oils have been used to a limited extent, the most available and cheapest liquid fuels are mineral oils, usually derivatives of crude petroleum.

CLASSIFICATION. Refiners grade fuels broadly according to method of production: (1) Distillate fuels are produced by distillation of crudes. Various grades are distinguished according to choice of initial and final boiling points used in the process. (2) Residual fuels are those left after the distillation process. (3) Blended fuels are mixtures of straight distillate fuels with cracked fuel stocks. Cracked stocks are residuals of fuels which have been treated thermally or catalytically to obtain yields of lighter-grade fuels or gasoline.

Lightest grade distillates, classed as kerosene or No. 1 fuel oil, may have an initial boiling point of 350 F and end point of 500 F. Heaviest grades of distillates, classed as No. 3 or No. 4 fuel oil, may have initial boiling point of 450 to 500 F and end point of 650 to 700 F. Refiners may produce several grades of distillate fuel and usually try to produce fuels from the available crudes which will satisfy both domestic heating use and diesel engine requirements with least number of grades.

Residual fuels, No. 4 or No. 5, are suitable only for the slower-speed diesels. Residual and blended fuels have wide variations in characteristics and suitability for diesel use and each must be evaluated separately.

Fuel-oil price varies with many factors; one of them is quality. High-quality distillate fuels are more expensive than residual or blended fuels. High demands for gasoline and heating fuels indicate advisability, from the standpoint of fuel cost, of using the lowest suitable grade of diesel fuel available. However, lower fuel cost must be balanced against increased operating and maintenance costs. Always start from engine builder's specifications and depart from them only slowly and cautiously. Specifications are important in preliminary judgment of fuels, but the final criterion is acceptance of a fuel by the diesel engine itself.

SPECIFICATIONS. Terms usually employed in diesel-fuel specifications are:

Specific gravity, seldom used, compares the weight of the fuel with water; it is expressed as a decimal, with water taken as 1.0. The term API (American Petroleum Institute) gravity is frequently used. Water is taken as API 10 degrees. Oils lighter than water have higher degrees API gravity, according to the formula

$$\text{API degrees} = \frac{141.5}{\text{Specific gravity at 60 F}} - 131.5$$

Heat content is expressed in Btu per pound higher heating value. Fuel oil usually is purchased by the gallon, and the heavier fuels (low API gravity) have the lower cost per unit of heating value.

Flash point is the temperature at which the fuel gives off vapors ignited by an open flame and is significant only from handling and storage standpoints. Usually a flash point of 150 F meets all fire, insurance, and transportation regulations.

Pour point is important only for handling and storage reasons. Heating coils in storage tanks make possible the use of high pour point fuels in cold weather.

Viscosity is measure of resistance to flow. Important from the standpoint of handling through piping, especially in cold weather, and very important for injection characteristics. High-viscosity fuels do not atomize as freely and may upset combustion results in the engine. (See also Viscosity, Section 6.)

Volatility and Distillation Range. Volatility measures vaporizing tendencies and is usually expressed in distillation-range temperatures, sometimes as temperatures at which successive 10% increments of the fuel are distilled.

Cleanliness. Contamination such as water, abrasives, gummy constituents, pipe or tank scale, or sludge must be eliminated. The usual BS and W (bottom sediment and water) limit is 0.05% maximum when the engine is equipped with fuel filtration facilities. In large stations, with special filtration equipment, a limit of 1 to 2% is advisable.

Ash content includes uncombustible mineral materials, abrasive in action in engine cylinders, must be limited to less than 0.01% in higher-speed engines and 0.1% in slower-speed engines.

Corrosion. Fuel should be noncorrosive by test, but this may not correlate with corrosion of fuel-system parts. Acidity and alkalinity tests are not normally specified for distillate fuels because manufacturing process produces neutral fuels. All fuels should be neutral in reaction, and crudes or heavy fuels should be tested.

Sulfur may be present in many forms, some corrosive and some noncorrosive. Products of combustion of sulfur-containing fuels are likely to be corrosive or cause deposits in engines. In general, small engines have operated satisfactorily on fuels containing as much as 1% total sulfur, whereas large slow-speed units have operated on fuels with as much as 3% sulfur.

Carbon residue is sometimes called Conradson carbon. It is the carbonaceous residue remaining after destructive distillation, expressed in percentage by weight of the original sample. In light fuels, a test is run on the 10% remaining after the lightest 90% has been distilled off. This is called "carbon residue on 10% bottoms"; it gives values about ten times those obtained from the entire sample. Higher-speed engines function most satisfactorily on fuels having carbon residues on 10% bottoms of 0.25% or less, whereas some large low-speed engines have used fuels with much higher carbon residues. This test is believed to indicate the tendency of a fuel to form carbon deposits in an engine, but correlation between tests and actual engine results is not always good.

Ignition quality, in smaller engines, is one of the most important characteristics of a fuel. The term is used to express the speed at which combustion starts and continues under service conditions. When fuel is injected into a diesel engine cylinder, there is a delay until a portion of the fuel ignites. This burning increases the temperature and promotes general ignition and combustion. If ignition quality is low and engine speed is high, a large part of the fuel charge is injected before any appreciable ignition occurs. Hence, at the time of ignition, there is so much fuel present that combustion takes on the characteristics of an explosion, causing rough running.

Cetane Number. The usual method of expressing ignition quality is the determination of *delay angle* of the fuel in a standardized test engine. The delay angle is the angle of crankshaft revolution between the beginning of fuel injection and the first appreciable rise in pressure due to combustion. This is expressed by cetane number. Cetane is a hydrocarbon fluid of high ignition quality. It is mixed for trials with alpha-methylnaphthalene, which has a poor ignition quality. After determination of the delay angle of the fuel being tested, the performance is duplicated, using a mixture of these two pure compounds, and employing the trial-and-error method. The cetane number is the percentage of cetane used in the mixture.

Fuels with high cetane numbers give smooth combustion and provide easy starting. The slower the engine speed, the less the importance of the cetane number.

Diesel index is also used to express ignition quality of fuels:

$$\text{Diesel index} = \frac{\text{API gravity} \times \text{Aniline cloud point}}{100}$$

Both gravity and aniline cloud point are related to fuel composition, hence to ignition quality. Diesel index and cetane number can be fairly well correlated.

Another method of expressing ignition quality is by the empirical cetane number determined by a chart that takes into consideration a number of factors (including gravity, viscosity, and volatility) related to fuel composition.

In the range of 50 to 60, the diesel index is normally 5 to 10 points higher than the actual cetane number. In the range of 35 to 45, diesel index closely approaches cetane number. Below 30, diesel index is usually somewhat lower than cetane number. The correlation between empirical cetane number and actual cetane number is generally similar, but the results are more consistent than those obtained from the diesel index.

Additives (amyl nitrate, etc.) improve the ignition quality of fuels but add to the fuel cost. Engine tests alone are used for determining the ignition quality of additive-improved fuels.

FUEL SELECTION. Wide and numerous variations in engine design, such as size of cylinder, speed of revolution, form of combustion chamber, and injection system, affect fuel requirements. In selecting fuel oils, follow the engine builder's specifications but permit the fuel supplier as much latitude within them as possible. Restrictive specifications increase the fuel price. Increasing the cetane number above the minimum required for smooth running does not increase the operating efficiency, but may increase the fuel cost. The use of lighter fuels than actually required increases both the fuel cost per barrel and the fuel consumption.

GAS FUEL. The express purpose of the gas-diesel and dual-fuel engine is to take advantage of availability of low-cost gas fuel. Any gas suitable for fuel for gas engines, can be used, but natural gas and sewage gas are most common. The cost of pilot fuel oil is of lesser importance, and it is wise to use a good grade of fuel oil.

12. DIESEL LUBRICATING OILS

CLASSIFICATION. Crude oils are frequently described as "paraffinic," "naphthenic," or "mixed base," according to the physical characteristics of the crude. Many subclassifications of finished oils can be made, based on type of base stock, refining methods, and subsequent treatment, but these classifications do not describe the value of a lubricating oil in a diesel engine.

Two broad types of oil are in use, "straight" mineral oils and "additive" oils. "Straight" oils are produced entirely from the crudes chosen through elimination of undesired constituents by suitable refining processes. "Additive" oils are produced by adding to straight mineral oils certain oil-soluble compounds that enhance the lubricating-oil properties for use in a diesel engine. Additives are used principally to inhibit or slow down oxidation, to increase film strength, to keep solids in a finely divided state and to hold them in suspension (detergency), to improve the viscosity index, to lower the pour point, to decrease friction and wear under extreme pressure conditions, to reduce foaming, and as rust or corrosion inhibitors.

TYPES. The Society of Automotive Engineers and the American Petroleum Institute recognize three types of lubricating oil:

Regular type suitable for moderate operating conditions.

Premium type, having oxidation stability and bearing corrosion-preventive properties making it generally suitable for more severe service than regular-duty type. Operating circumstances which bring high load factor, or high temperatures from any cause, require premium oils. Elevated temperatures increase the rate of oxidation and tend toward harmful deposits in the engine. Oils having improved stability and oxidation resistance are required under such circumstances.

Heavy-duty type has oxidation stability, bearing corrosion-preventive properties, and detergent-dispersant characteristics for use under heavy-duty service conditions. Such oils resist deterioration under sustained heavy-duty operating conditions, carry carbon particles in suspension, and tend to eliminate deposits in the engine.

CHARACTERISTICS. Specifications for lubricating oil do not usually mention any physical characteristic except viscosity, but such characteristics should be considered.

Flash and fire points have the same meanings as for fuel oil, but the values are much higher. Carbon residue has similar significance, but the test figure is a doubtful indicator of the likelihood of engine deposits.

Viscosity must be high enough to provide an oil film under the load and temperature conditions prevailing between the sliding surfaces in the engine, and still flow freely through the passages and spread over sliding surfaces under the prevailing speed and clearance conditions. The latter is especially important when starting at low temperature.

Viscosity is usually expressed in seconds Saybolt or seconds SUS (Saybolt Universal seconds). It is determined by measuring the time in seconds required for a standard quantity of oil (60 cc) to flow through the orifice of the Saybolt viscosimeter at a standard temperature. Three standard temperatures are used, 100, 130, and 210 F.

SAE Grades. The viscosity of lubricating oils usually is expressed according to grades established by the Society of Automotive Engineers, given in the following table:

SAE Viscosity Number	Viscosity Range, SUS At 130 F Minimum	Maximum	At 210 F Minimum	Maximum
10	90	Less than 120		
20	120	Less than 185		
30	185	Less than 255		
40	255			Less than 80
50			80	Less than 105
60			105	Less than 125
70			125	Less than 150

Viscosity index indicates the relative change in viscosity of an oil for a given temperature change. The rate of change varies according to the type of base stock. Paraffin oils in general have a low rate of viscosity change with change in temperature; a particular paraffin oil was arbitrarily assigned an index number of 100. Naphthenic oils have a high rate of viscosity change, and a particular naphthenic oil was arbitrarily given an index number of zero. The viscosity index, or VI, of any oil compares its rate of viscosity change with temperature with these two standards.

Pour point is the lowest temperature at which oil will flow of its own accord. It is not the only factor influencing the ease of starting a cold engine, especially after it is exposed for some time to cold weather. Special low-point lubricating oils may be used under such circumstances, but the preferred method is to enclose suitably or protect the engine by heating so that the lubricating oil and jacket water are at reasonable temperatures *before* starting.

Engine Operation. Normal operation of a diesel engine subjects the lubricating oil to high pressures and temperatures. The result is oxidation and a tendency towards production of gums, resins, and acids. The products of oil deterioration are an almost infinite series of compounds, differing according to the oil used and the operating conditions. Furthermore, the oil may be contaminated by metal particles, raw fuel oil, or the products of combustion. Soot, ash, partially burned fuel, or raw fuel may mix with the cylinder wall lubricant and accelerate the deterioration of the lubricating oil.

Raw fuel oil will lower the flash and burning points of the lubricating oil, and lower the viscosity. Filtering alone will not eliminate fuel oil; a distillation process is necessary. The presence of fuel is detected by flash point and change in viscosity.

Oxidation of lubricating oil forms acidic products. Periodic tests for neutralization number will determine the rate of increase of acidity during service. Use the neutralization number of the original or new oil as a basis for comparison.

Small engines are likely to have filters of moderate capacity, and the lubricating oil is thrown away periodically after a certain number of hours of use. Medium and large engines have more elaborate systems of filtering the oil; the oil is clarified either by the batch system or continuously. In the batch system, oil is drawn from the engine periodically, clarified, and stored for future use. No clarified oil is ever thrown away; only residues or sludges are discarded. When the batch system is used, some operators have an analysis of the crankcase oil made periodically, and drain strictly according to the indications shown. The larger the establishment, the more essential that the correct grade of oil be used, that crankcase oil be kept in prime condition, and that losses and wastes be reduced to a minimum.

13. DIESEL ENGINE OPERATION AND MAINTENANCE

Successful operation and good maintenance are closely tied together. The primary purpose of good maintenance is to obtain safe, reliable, and economical operation. Competent operation requires close attention and good judgment of engine performance, to recognize what maintenance work should be done. Fixed maintenance schedules are a suitable guide but should be tempered by close observation of performance. Tearing down an engine to make sure everything is in order is seldom justified. Conversely, the engine should not be permitted to run until parts fail.

Performance records provide a benchmark in the form of past performance and, accordingly, provide a standard for judging current performance. Constant comparison provides a quick and convenient method of detecting early indications of operating deficiencies and difficulties. The larger the plant, the more elaborate these records need to be. The items given below should be regularly checked and recorded. Frequency of checking depends entirely on size of plant and service conditions. Records should provide a good indication of performance under everyday average conditions, and also under maximum or minimum or unusual conditions.

PRESSURES. Design compression and maximum combustion pressures must be maintained. All except the small-cylinder units have indicator connections for this purpose. Pressures are measured by one of the several forms of indicators available. Check compression with engine at operating temperature, running at rated speed with fuel cut-off from that cylinder. If a drum-type indicator is used, it is well to pull the drum by hand when taking the combustion pressure card and examine the shape of the combustion peak. It is usually sufficient to take these pressure readings once a month.

Causes of loss of compression are leaky cylinder head valves, low scavenging air pressure, obstructions in air intake lines or filters, leaky or sticky piston rings, and increased end clearance between piston top and cylinder head. It is best to limit the loss of compression pressure to within 20 psi of the recommended normal pressure, except that when it is due to normal wear of piston rings and cylinder liner, up to 50 psi may be tolerated. In the latter case, new piston rings, perhaps of the two-piece type, are needed for correction.

Combustion pressure is affected by compression pressure, fuel-injection timing, condition of fuel-injection system, and fuel-oil characteristics. Combustion pressure should be kept within 25 psi of the normal. Pressure conditions have a strong influence over the quality of combustion, which should be high at all times.

In small engines that have no indicator connections, reliance must be placed on good mechanical condition of the engine and correct adjustment of parts and fuel-injection system for maintenance of proper compression and combustion pressures. However, quality of combustion, which is a reflection of proper compression and combustion pressures, can be judged by the appearance and temperature of the exhaust.

EXHAUST TEMPERATURE is a sensitive and direct indication of proper combustion. The most efficient performance brings the lowest temperature; irregularities or deficiencies of combustion always bring temperatures *above* normal. Each load and speed combination has a normal temperature that should be established, recorded, and kept for reference and comparison. All except the smallest engines have permanent installations of pyrometers that indicate the exhaust temperature of each cylinder outlet and of the main exhaust-pipe outlets.

Fuel consumption is a simple and sure overall measure of engine performance. Meters or measuring tanks may be used for checking consumption. When engine-power output can readily be measured, as in generator installations, compare fuel consumption against kilowatt-hour output. Otherwise, record fuel consumption against hours of operation under similar load or service conditions. In comparing such records, allowance must be made for differences in the fuel used. In this connection also keep adequate records of engine speed, minimum and maximum load, and hours of operation.

SCAVENGING AIR PRESSURE in two-cycle engines is an important indicator of the continued satisfactory performance of the scavenging blower or pump, and of the condition of the scavenging and exhaust ports of the working cylinders. The most probable cause of low pressure is the scavenging pump. Higher than normal pressures usually indicate partial closing of scavenging and exhaust ports in the cylinders due to carbon deposits.

LUBRICATING-OIL PRESSURE. Pressure gages are needed to show oil pressure to engine and pressure loss through filter on larger engines; thermometers are needed to show temperature of oil entering and leaving the engine, and to show temperatures into and out of the cooler, when used. When piston cooling is used, there is usually a thermometer on each outlet, especially on larger engines.

Abnormally low lubricating oil pressure is dangerous; low-pressure shutdown devices or alarms are frequently used. Abnormally large pressure drop through the filter indicates clogging; low pressure drop indicates a break in filter medium. Rise in oil temperature indicates hot bearings; paradoxically, rise in temperature of the piston cooling oil indicates lack of circulation through piston.

Type and quantity of sediment in the filter are important. Watch for contamination from fuel oil, because this indicates poor combustion conditions. Water in the oil indicates leaks from jackets or head gaskets. Metal particles may be abnormal wear or breaking up of babbitt in the bearings. Abnormal quantity of sludge may indicate poor combustion or unsatisfactory type of lubricating oil for the engine and service conditions.

COOLING WATER SYSTEM. Inlet and outlet temperatures are especially important and recommendations of the engine builder must be followed. Large engines usually have a thermometer on the outlet of every cylinder. Pressure gages and open funnels help to indicate definite and adequate circulation of water.

GENERAL OBSERVATIONS. The result of any changes in the type or character of fuel and lubricating oil must be carefully watched. Any unusual sound or symptom must be promptly investigated. Record all pressure and temperature readings in tabulated or plotted form so that comparisons can be made readily and deviations quickly noted. It is these records that indicate what maintenance is required. Whenever any inspection work is done on an engine, extend the inspection to cover other parts that can be conveniently inspected at the same time. Similarly, when maintenance work is done on one part or assembly, recondition all parts in the assembly fully, as well as in any other assemblies or parts that can be conveniently handled at the same time.

Careful operation and maintenance procedures insure reliable operation. The only hazards then are those due to accidental breakage or failure of parts. They can be reduced to a minimum or eliminated by taking full advantage of inspection periods to check or test carefully each piece for hidden cracks or incipient failures. With reasonable precaution in operation, inspection, and maintenance, the diesel engine is a reliable source of power.

INSPECTION AND MAINTENANCE FORMS vary with the circumstances of each installation, but must list all essential items to serve as a proper reminder and record. Tables 6 and 7 given here are examples of forms that have proved satisfactory. Table 7 gives typical elapsed time hours as a guide for maintenance frequency but these figures will vary widely, depending on engine design and service.

Table 6. Inspection and Maintenance Schedule for Diesel Engines

Make.......... .Hp. Type .... Date ....

Enter in this schedule the maximum allowable operating hours between inspections of the listed parts. At the time of inspection all cleaning and adjusting consistent with good engine maintenance is to be done.

Part	Hours
CRANKSHAFT	
Main bearings	...
Outboard bearing	.....
Thrust bearing	......
CROSSHEADS, RODS, AND AUXILIARY SHAFTS	
Crankpins and bearings	.....
Piston pins and bearings	......
Crosshead pins and bearings	.....
Crosshead shoes and slides	. .
Compressor piston pin and bearing	. . .
Compressor crosshead pin and bearing	. ..
Compressor crosshead shoes and slides	....
Vertical shaft bearings	. ...
Vertical shaft thrust bearings	. . .
Camshaft bearing	.....
Camshaft thrust bearing	....
PISTONS AND CYLINDERS	
Power	.
Compressor, high stage	. . .
Compressor, intermediate and low stage	....
Scavenging pump	.
CYLINDER HEAD VALVES	
Fuel or spray	. .
Air inlet	....
Exhaust	......
Starting	. . .
Starting air check	....
Scavenging valves, mechanical	.....
Safety or relief	....
Fuel try or by-pass	....
GOVERNOR	
Links and bearings	......
Springs	......
COMPRESSOR VALVES	
Compressor suction and discharge	......
Scavenging pump suction and discharge	.....
SCAVENGING SYSTEM	
Ports and automatic valves	......
Exhaust gas flow regulators	......
Exhaust ducts and mufflers	......

Part	Hours
FUEL PUMPS	
Pistons	
Packing	
Valves	
FUEL OIL SYSTEM	
Filters or strainers	
Auxiliary storage tanks	
Supply lines	
Heaters	
GEARS OR CHAIN DRIVES	
Governor drive	
Camshaft drive	
Fuel pump drive	
Lubricating pump drive	
LUBRICATING SYSTEM	
Lubricating oil pump complete	
Lubricating oil supply lines	
Lubricating oil strainers or filters	
Lubricating oil tanks	
Lubricating oil coolers	
Engine crankcase	
Pressure feed lubricator and checks	
PISTON COOLING	
Cooling passages in piston	
Packing	
Bearings	
Ball and hinge joints	
SCALE AND SEDIMENT DEPOSIT	
Cylinder heads and jackets	
Air coolers	
Oil coolers	
AIR INTAKE SYSTEM	
Air filters	
Air suction ducts and mufflers	
PRESSURE GAGES	
Air, oil, and water gages	
RELIEF VALVES	
Air and water	
Lubricating oil	
Exhaust gas	

Table 7. Diesel Engine Maintenance Log

Engine No. .... Year.......... Month......................

Actual Running Time for Month, hours.. ....

A = Date work is scheduled
B = Date work was performed

No.	Operation	Maximum Elapsed Time Allowed, hours	No. of Operations Done at Same Time	Cyl. 1 A	Cyl. 1 B	Cyl. 2 A	Cyl. 2 B	Cyl. 3 A	Cyl. 3 B	Cyl. 4 A	Cyl. 4 B	Cyl. 5 A	Cyl. 5 B	Cyl. 6 A	Cyl. 6 B
1	Engine pistons taken out and cleaned	3600													
2	H.p. compressor pistons disassembled, cleaned and examined	1800													
3	I.p. and l.p. compressor pistons removed, cleaned and examined	1800													
4	Check clearance space between piston and head	300	36												
5	Main cylinder jackets and heads cleaned out for scale	3600	1-2 3-20												
6	Crankshaft and extended shaft alignment checked	3600													
7	Crankcase flushed out	1800													
8	Exhaust valves removed, cleaned, and ground	900													
9	Inlet valves removed, cleaned, and ground	1800													
10	Air intake valves removed, cleaned, and ground	1800													
11	Air starting check valves cleaned and ground	1800													
12	Safety valves removed, cleaned, and ground	1800													
13	H.p. compressor valves removed, cleaned and ground	600													
14	I.p. and l.p. compressor valves removed, cleaned, and ground	900													
15	Fuel valves removed, cleaned, and needles ground	900													
16	Fuel valves setting checked	300													
17	Fuel pump valves examined, cleaned, and ground	1800													
18	Test spray checks for tightness every day	10													
19	Camshaft gears or chains checked and adjusted	600													
20	Cam roller setting should be checked every time a valve cage is changed or cylinder head removed	3600	5												
21	Fuel pump driving mechanism should be examined	600													
22	Valve gears cleaned, examined, and corrected	300													
23	Lubricating oil filter cleaned	900													
24	Lubricating oil holes in crank pins cleaned	1800	36												
25	Lubricating oil piping cleaned	300													
26	Lubricating oil supply piping and tank cleaned	3600													
27	Lubricating oil pumps cleaned and examined	1800													
28	Examine lubricating oil piping for loose joints and leaks	300													
29	Examine lubricator driving gears	300	6-36 37												
30	Air injection piping burned out for carbon	3600													
31	Air cooler coils cleaned out for scale	3600													
32	Air receivers cleaned	3600													
33	Air tanks drained and cleaned	3600													
34	Check fastening of tube in bottle head	900													
35	Wristpin bearing checked and adjusted each time pistons are removed	1800	1-2-3												
36	Crankpin bearing checked and adjusted	1800													
37	Main bearings checked, adjusted for alignment, and clearance adjusted	3600													
38	Governor examined and adjusted	1800													

MAINTENANCE EQUIPMENT. The principal work involved in maintenance is disassembling and replacing of parts, especially on the larger engines. Maintenance processes may be classified as cleaning, adjusting, reconditioning, and renewal of parts. Renewal of worn parts or parts not operating satisfactorily is more common in smaller engines than in larger ones. Many engine builders provide a "unit exchange" parts service whereby a used part or assembly can be sent to the builder's factory or service branch for reconditioning or in exchange for a new or similar reconditioned part or assembly, reducing field work to a minimum. When reconditioning work is done in the power plant, special tools and equipment are justified, and skilled mechanics are needed. Engine builders supply all necessary special wrenches; special tools for specific overhauling operations, and for reconditioning and testing parts and assemblies, can be purchased from manufacturers who make a specialty of such items.

CRANKSHAFT AND BEARINGS. Many modern engines have precision bearing shells. The bearing clearance is never adjusted; when the wear becomes excessive, the worn shells are replaced. The limit for excessive wear can be obtained from the engine builder; in general it is when the bearing clearance becomes about 50% above the normal. Excessive bearing clearance means more oil leakage around the shaft and difficulty in maintaining the oil pressure. Excessive bearing wear may cause crankshaft misalignment and eventual breakage. In engines with adjustable bearing shells, the same observations apply, and in addition great care must be used in adjusting bearing clearances to compensate for wear, and in replacing shells to avoid misalignment of crankshaft.

ROCKER ARM ROLLER CLEARANCE. Too much rocker arm roller clearance is better than too little. Inadequate clearance may cause blocking of valves when engine is at operating temperature, resulting in loss of compression, overheating, and cutting of valve seats. Excessive clearance will change the timing and cause an undesirable increase in impact, leading to breakage or wear.

VALVES AND VALVE TIMING. It is poor practice to allow valves to wear so much that reseating is difficult. Leaking valves wear rapidly, reconditioning soon is impossible, and replacement is necessary. No corrections for exhaust and inlet valve timing should be made for variations of less than 5 degrees. Fuel or spray valve timing is extremely important. Much care should be used in its adjustment.

PISTON. The condition of piston and liner is a good indication of the quantity and quality of lubricating oil used. A bright surface on piston rings and liner, with rings free, indicates good oil supplied in proper quantities. Dull, grayish wearing surfaces, with rings stuck, indicates inferior oil. Too much oil, even of good quality, will cause a gummy deposit on rings and piston. If an engine operates with poor combustion, evidenced by smoky exhaust, a gummy residue will be found on the rings, even if a proper amount of good quality lubricating oil has been used.

PISTON DIAMETRAL CLEARANCE should be checked whenever a piston or liner is replaced, to insure that it compares closely with that recommended by the manufacturer. Aluminum pistons require greater clearance than cast-iron pistons.

PISTON RINGS must be kept in first-class condition. If they remain in the engine after blow-by has started, power will be lost, and piston and cylinder will be scored, as a result of hot gases destroying the lubrication. Ring condition should be investigated at the first sign of blow-by.

RING AND LINER WEAR. Piston and cylinder parts receive the greatest wear and require the most frequent renewal. Greatest wear in the liner invariably occurs near upper end of piston travel. Wear in other parts of the liner is inconsequential, even after several years. Wear at upper end of the liner usually necessitates renewal of the part, and replacement of piston rings. Replacement of liners is necessary when wear at top exceeds 0.005 in. per in. of cylinder diameter.

Ring and liner wear can be reduced by: (1) Using cylinder lubricant of suitable body, and in correct amounts. (2) Filtering air and fuel. (3) Maintaining a tight seal between rings and liner. If considerable wear has occurred, this may require use of two or more two-piece rings. (4) Keeping rings free in their grooves. (5) Keeping water jackets free of scale and water supply adequate.

THE FUEL-INJECTION SYSTEM must be kept clean throughout, and filters must be checked periodically. It is important to keep the quantity of fuel injected to each cylinder identical to insure even loading. Injection timing should be set exactly as specified by the engine builder. Spray-nozzle action should be watched carefully; faulty action usually can be detected by an increase in exhaust temperature and change in color of the exhaust. Equipment to test the spray action in the open is essential.

LUBRICATING-OIL SYSTEM. Utmost cleanliness is required in handling lubricating oil. Keep a record of the lubricating-oil consumption per engine operating hour. This not only is important from the standpoint of cost of oil used, but it also indicates whether the oil control (or wiper) rings are in satisfactory working order. Keep a record of the lubricating oil lost in the reclamation or clarifying process and of the sludge rejected. These are checks on the performance of both oil and engine.

OTHER ITEMS. Periodically check the quality of the cooling water and remove deposits from all parts of the system. Make sure that it is always full. Periodically drain the starting-air system of accumulated oil and water. Make sure of the constant availability of the auxiliary equipment. Where duplicate equipment is installed, operate the units alternately. Inspect and test all controlling devices, such as governors, periodically. Test all alarms and emergency shutdown devices regularly and frequently; usually they are arranged for such testing. Practice safety, cleanliness, and orderliness throughout the plant to attain lowest operating cost and most reliable service.

AIRCRAFT PISTON ENGINES

By Robert Insley

HISTORY. Within a seven-day period in December 1903, the first two American internal-combustion aircraft engines were flight tested. One airplane crashed and was totally wrecked; the other was damaged, but three days later it flew again to make history. The wrecked airplane was the Langley Aerodrome, powered by a five-cylinder radial, liquid-cooled engine designed by Charles Manly. The successful airplane was propelled by a four-cylinder horizontal in-line liquid-cooled engine that Wilbur and Orville Wright designed and built. Both engines were designed specifically for the purpose and both, even in the light of present-day engine experience, were highly creditable accomplishments. Of the two, the Manly engine, with 52 hp and a weight of less than 4 lb per hp, represented perhaps the more brilliant creation; but its unfortunate sponsorship precluded further test or exploitation. The Wright brothers' first engine, which produced 12 to 16 hp with a weight of approximately 170 lb, was a more modest effort but presaged a more distinguished career.

Pre-World War I Period. The history of aircraft engines from 1903 to World War I encompasses every conceivable mechanical arrangement and a wide variety of thermodynamic principles. Prominent among the types of engines in use during that period were the two-cylinder air-cooled "Vee," the three-cylinder air-cooled "W," four-cylinder and six-cylinder water-cooled in-line engines, the eight-cylinder water-cooled vee type, fixed air-cooled radial types, and the single- and two-row rotary radial engines, with stationary crankshaft and rotating crankcase and cylinders, which figured so prominently in the early years of World War I.

World War I provided a powerful stimulating and stabilizing influence on the development of aircraft engines. After work on the rotary engine and some tentative excursions into the air-cooled in-line and vee types, aircraft engine development was concentrated largely on two general types, the liquid-cooled engine (with both in-line and vee arrangements of cylinders) and the air-cooled radial type. The late and vigorous developments of World War I were directed almost wholly to the liquid-cooled in-line and vee types, the most prominent examples being the Siddeley and Beardmore in England and the BMW and Maybach in Germany (all six-cylinder in-line engines), the American Curtiss OX, the British Woolsley Viper, and the French Hispano-Suiza eight-cylinder vee engines, and the celebrated American Liberty 12 and the British Rolls Royce Eagle twelve-cylinder vee engines. The air-cooled fixed radial rose again to popularity shortly after World War I and received its greatest impetus in Lindbergh's famous flight with the Wright J-5 nine-cylinder radial engine in 1927. Until the emergence of the small two-, four-, and six-cylinder "opposed" air-cooled engines in the early 1930's, the fixed radial engines, with one or two rows of cylinders, and the liquid-cooled engines, principally of the twelve cylinder vee type, monopolized the development and production programs.

Despite the variety of engine types, the most noticeable feature of the overall aircraft engine development history has not been the evolution of improved models, but rather the extremely rapid increase in power ratings and corresponding weight reductions in engines of conventional form. The Wright brothers' first engine was intended to produce 8 hp and probably would have propelled the airplane satisfactorily at that rating, though its actual output was nearly twice the designed power. During the eleven years, to the beginning of World War I, normal ratings increased to about 100 hp. In four years the developments of World War I more than quadrupled the requirements, leading to the 450 hp rating of the Liberty 12. Progress moderated briefly in the early 1920's but, by 1930, the ceiling had been raised to 1000 hp, and at the beginning of World War II normal ratings were approaching 2000 hp.

World War II again provided a mighty impetus and doubled the output of piston engines. Astonishing progress has been made also in the enlargement of the power capabilities of individual engine models as exemplified by the Wright Cyclone, which, in a period of fifteen years, has quadrupled its output without enlargement of basic or overall dimensions.

Despite the increase in performance of aircraft engines, their endurance characteristics have improved similarly. All engines prior to World War I must be regarded as experimental types, and consequently of unspecified and variable endurance capabilities. During the latter portion of World War I and in immediately succeeding years, the standard measure of durability was the 50-hr type test, scheduled to reproduce as accurately as possible normal operation cycles. An engine capable of completing such a test without major failure was considered acceptable for military and civil service. Steadily that test schedule has been increased in both duration and in severity and now, for military or civil acceptance, an aircraft engine must successfully complete a much more rugged schedule of 150 hr without failure or excessive wear of any major part.

World War II, in addition to providing impetus to the development of piston engines, gave birth to the jet engine (turbo-jet, prop-jet, ram, and pulse jet, and rocket types, see Section 15) for aircraft propulsion. The advent of the jet power plant, however, and the likelihood of its early exclusive utilization in combat aircraft, dictates to some degree the scope of this section. Large liquid-cooled engines are used, almost without exception, in combat airplanes. Therefore, inasmuch as such engines seem now certainly on the verge of eclipse by jets, with no civil requirement in sight, they also will be given only passing mention in this chapter, and attention will be directed toward description and discussion of current and future aircraft engines for civil applications.

14. CLASSIFICATION OF AIRCRAFT ENGINES

CLASSIFICATION BY SIZE AND SERVICE. Modern aircraft engines of the piston type cover the power range from 50 to nearly 4000 hp. In this horsepower range fall two principal categories of service: (1) personal transportation (which includes flight training), (2) public transportation (which, in this classification, includes the certified and non-certified transport operators, but not the charter operators). Although no distinct line of demarcation exists, because of practical considerations a point of separation lies at about 500 hp. In general practice, engines of less than 500 hp are used in airplanes for personal transportation; aircraft for public transportation have engines of greater than 500 hp. Training planes fall in the smaller power category, but charter operators supply airplanes of size suited to the clients' requirements which fall into no specific power class. For economic reasons it seems unlikely that public transport operators in the future will be interested in smaller engines, and similarly for economic reasons there seems little likelihood that personal airplane purchasers will demand engines of higher power.

CLASSIFICATION BY THERMODYNAMIC CHARACTERISTICS. Thermodynamically, piston-type aircraft engines are impressively conventional. All, with a few isolated exceptions, are gasoline engines and operate on the four-stroke Otto cycle. Persistent efforts have been made to produce aircraft diesel engines, or more precisely, compression-ignition engines, with the three primary purposes of (1) extending cruising range by improved thermal efficiency, (2) reducing fire risk by use of lower volatility fuel, and (3) lowering fuel cost by virtue of the expected lower per gallon price of diesel fuel. Several reasonably satisfactory compression-ignition engines for aircraft have resulted, notably the German Junkers Jumo series of engines. In every case, however, the added complexity, weight, and cost of the compression-ignition engines seem to have outweighed all the benefits, with the possible exception of the expected reduction in fire risk. All important efforts in the direction of compression-ignition aircraft engines now appear to be dormant.

Similarly the two-stroke cycle gasoline engine for aircraft, though apparently offering distinct advantages in simplicity, low cost, and reduced weight, has failed to respond with tangible results after the expenditure of many thousands of hours of engineering effort and millions of dollars of public and private development funds.

Of all the thermodynamic variations which have appeared in aircraft engines in the forty-five years since the Wright brothers first flew, only two forms now seem likely to enjoy widespread use, (1) conventional four-stroke cycle piston engine (with spark ignition), and (2) the turbine. As will be noted later, a combination of the two, the compound engine, may be found valuable for special purposes.

CLASSIFICATION BY MECHANICAL ARRANGEMENT. The most discernible characteristic of the piston engine for aircraft is its type, or form, as determined by the arrangement of the cylinders. Automobile engines, with few exceptions, are made in two similar types, the in-line and the Vee types, nearly all liquid cooled. Aircraft engines, however, as has been noted previously, have been produced in every mechanically practicable cylinder arrangement, and nearly every type has appeared in both air-cooled and liquid-cooled versions. Moreover, the shape, size, cylinder arrangement, and method of cooling in the aircraft engine exert such a marked influence on the design and performance of the vehicle that classification by type has become a matter of some importance.

By the process of elimination over the years those engine types that have proved to be most costly to manufacture or have demonstrated high cooling drag, low inherent specific output (horsepower per cubic inch of piston displacement), high intrinsic weight, or other congenital defects have fallen by the way. As a result, three types of aircraft engines remain in general use, the air-cooled radial (with one, two, and four rows of cylinders), the air-cooled in-line and Vee engines, and the air-cooled "horizontally opposed" engines of two, four, six, and eight cylinders. Liquid-cooled engines are now used almost exclusively for military purposes, and consequently all the types which seem likely to be of interest for transport and private flying are air-cooled engines.

Radial engines are used almost exclusively for commercial transport service. They are built in single-row form with horsepower ratings up to approximately 1400 hp, in double-row models with 14 and 18 cylinders up to about 3000 hp, and in four-row form with 28 cylinders producing nearly 4000 hp.

Air-cooled in-line and Vee types are popular principally in England and in continental Europe. They are manufactured in 4- and 6-cylinder in-line and in 8- and 12-cylinder vee models, ranging in power from 60 to approximately 400 hp. However, they are intrinsically heavy and costly to build, and they seem likely to be supplanted eventually by the horizontally opposed forms which, up to the present time, have been almost exclusively American designs.

Horizontally opposed engines at present vary in power from 50-hp 4-cylinder models to 8-cylinder engines of approximately 300 hp. They are inherently lighter and somewhat less expensive than the in-line and Vee types, and frequently present much simpler installation and aircraft structural problems.

15. STRUCTURAL COMPONENTS AND MATERIALS

CRANKCASE CONSTRUCTION. Radial engine crankcases in low-power engines are ordinarily made in cast aluminum. In larger sizes, above 1000 hp, they are constructed of steel or aluminum forgings. Crankcases for in-line, Vee, and horizontally opposed engines are generally aluminum castings with a pronounced recent tendency toward magnesium castings.

CRANKSHAFTS. Forged steel crankshafts are used universally, the steel specification dictated by the hardening process to be used. Surface hardening of the bearing areas is general. Induction hardening is frequent, but the practice of nitriding all over for hardening of wearing areas and for improving physical properties is rapidly becoming accepted procedure. To suppress resonant vibration, the most frequent cause of shaft failure, tuned dynamic dampers (usually of the pendulum type) are made a part of nearly all radial engine crankshafts and of an increasing percentage of the crankshafts for in-line, Vee, and horizontally opposed engines. Radial engine crankshafts are usually separable at the crankpin, with clamped joint, although some multiple-row radial engines use integral shafts and separable master rods.

CONNECTING RODS. Steel forgings are used almost universally for connecting rods for both radial and in-line types of engines. Shot-peening all over or at points of stress concentration to improve physical properties is common practice, and nitriding for the same purpose has been employed in some instances. Connecting rods for in-line and opposed engines are conventional "split big-end" type, with as much as possible of the surface left in the "as forged" condition to reduce cost. Plain bearings are used almost universally. Master rods for single-row radial engines are ordinarily of one-piece construction, with crankshaft separable at the crankpin for assembly. Plain bearings are customary practice in radial engine master rods. The master rod serves one cylinder of the radial engine. Link rods for the remaining cylinders are attached to the master rod by link pins fixed to the master rod flanges. Although link rods in some radial engines have been made of aluminum forgings, common practice now is to use the same material as is used in the master rod, a steel forging machined all over.

BEARINGS. Roller main bearings and ball thrust bearings are common practice in radial engines, but, for practically all other applications, plain cylindrical, or sleeve, bearings are used. Materials are dictated by load requirements and permissible cost.

For maximum loads steel-backed silver bearings are used. For improved bearing properties the bores of these bearings are plated with lead which in turn is treated with indium to prohibit corrosion. Copper-lead lined steel-backed bearings are also used in highly loaded applications. Such bearings are commonly lined with indium-treated lead for improved bearing properties. Various proprietary arrangements of bearing materials, laminated, gridded, or otherwise prepared to combine the virtues of dissimilar metals, have been devised, and many of them are highly satisfactory. Bearings with very thin babbitt linings, to utilize the excellent bearing properties of babbitt and to avoid the crushing encountered with thick babbitt, are also in common use. Lightly loaded bushings used in accessory drives and similar applications are commonly made of bronze or aluminum. Not infrequently lightly loaded shafts are arranged to operate directly in the aluminum or magnesium castings which support them. For heavy but intermittent loads such as are encountered in piston pin and link pin bushings, bronze is a very satisfactory bearing material. Bushings for heavy, continuous loads are lined with silver, lead plated and indium treated, or with other materials used in high-duty connecting rod and main crankshaft bearings.

PISTONS AND RINGS. In engines of low specific output, cast aluminum pistons are universally used, but the temperatures and stresses encountered in high-output cylinders have made necessary the use of aluminum forgings. There is no general agreement concerning the need for an oil ring in the piston skirt. Highly successful engines have been produced with and without that feature, but the use of an oil ring immediately above the piston pin is universal practice. Taper face compression rings in the upper grooves are commonly used to control oil consumption, with one or two chromium-plated rings in the upper grooves to prevent feathering and scuffing.

CYLINDER CONSTRUCTION. Nearly all air-cooled cylinders are now of the "screwed and shrunk" type, consisting of a finned forged steel cylinder barrel to which is screwed and shrunk, by preheating, a finned aluminum cylinder head. The cylinder barrel is normally of alloy steel with a flange at the lower end for crankcase attachment, threads at the upper end to accommodate the head, and with integral machined steel fins for cooling. For improved heat dissipation, lower weight, and reduced cooling air requirement, aluminum fins are used on high-output cylinders. For this purpose three methods of construction are in common use, (1) a cast-on "muff," bonded to the steel barrel, with machined fins, (2) a forged "muff" shrunk over the barrel, with machined fins, and (3) formed sheet aluminum fins in "W" form, expanded at the roots into grooves machined in the outer surface of the barrel. There seems to be little difference in the cooling properties of the three types of construction though cost and convenience factors affect their acceptability in individual applications.

Cylinder heads are customarily aluminum alloy castings with integral cast fins. In recent years, however, for increased strength and improved cooling fin depth and spacing, cylinder heads for high-output air-cooled cylinders have been made from aluminum forgings with cooling fins produced by machining, usually milling or saw-cutting.

An important deviation from the screwed and shrunk cylinder construction, adopted by one engine manufacturer in the interest of cost reduction, is an integral cast-aluminum cylinder and head, with steel cylinder liner cast in place. Cooling and durability are considered satisfactory, and the cost is reported to be well below that of the screwed and shrunk type.

Cylinders. Two general types of cylinder construction are used in the principal liquid-cooled engines. In one, represented by the Allison engines, six individual steel cylinder barrels are shrunk into a cast cylinder head structure. The barrels are surrounded by a single cast light metal jacket which is attached to the head and sealed at the lower end of each barrel. The whole cylinder block unit is attached to the crankcase by studs extending from the crankcase through the cylinder head.

In the other basic construction, used in Rolls Royce engines, the cylinder barrels are assembled into the cylinder jacket casting to form an integral cylinder block. The cylinder head for each block is a removable assembly, the whole head and block unit attached to the crankcase by through-studs extending from the crankcase through the cylinder head.

VALVES AND VALVE MECHANISM. Nearly all modern air-cooled engines are built with one intake and one exhaust valve per cylinder, operated by push rods and rocker arms. Clearance adjustments are provided either in the push rod or on one end of the rocker arm. Some of the smaller air-cooled engines are equipped with automatic hydraulic clearance adjustments, but such equipment is not provided normally in the larger engines. Positive lubrication of the rocker arms and valve guides by forced circulation of oil is standard practice in all air-cooled engines. Solid stem intake and exhaust valves are commonly used in low output cylinders. Hollow stem exhaust valves without filling are sometimes used for increased guide area and reduced weight, but sodium-filled stems are almost universal practice for exhaust valves in high-output cylinders. Under extreme conditions of service sodium-cooled intake valves are also used.

Liquid-cooled engines commonly use overhead camshafts with cup followers or rocker arms.

SLEEVE VALVE ENGINES. Of all the unconventional valve types which have been proposed for use in internal-combustion engines, including rotating cylindrical valves, rotating cone valves, disk valves, cuff valves, and rotating and reciprocating sleeve valves, only one type at present seems to have established a place for itself in aircraft engine design. That is the single-sleeve valve, a ported cylinder liner given rotating and reciprocating motion by a driving crank to uncover at proper intervals exhaust and intake ports in the cylinder wall. This valve construction, invented by Burt and McCullom in 1909, was promoted in Britain by the Argyll and Vauxhall companies. A great amount of pioneer development work on this mechanism was carried on by H. R. Ricardo in England, but no commercial aircraft applications were made until the Bristol Aeroplane Company in 1927 began the development of a sleeve-valve air-cooled radial engine. From that start have come single- and double-row radial engines produced by Bristol in horsepower ratings of 1000 to 3000 hp. These engines served prominently in World War II and established creditable records in durability and economy. Using the same valve mechanism the Napier Sabre engine, a 24-cylinder "H" type liquid-cooled engine developed in England, has accomplished the highest specific power output published in standard military ratings.

AIRCRAFT ENGINE ACCESSORIES. In addition to such accessory components as are essential to the operation of the engine (such as carburetor, magnetos, and spark plugs) provision is normally made for starter, generator, fuel pump drive, tachometer drive, and one additional drive which may be used for vacuum pump, hydraulic pressure pump, propeller governor, or other purpose. In high-output engines, supplementary drives are frequently provided for additional hydraulic or vacuum pumps or other power requirements, but it has become common practice to incorporate in the design of large engines a high-capacity "power take-off" drive to which may be attached a supplementary unit with provision for driving as many additional accessories as may be required.

PROPELLER REDUCTION GEARS. Nearly all large aircraft engines and an increasing number of lower output models are equipped with propeller reduction gears to take advantage of the increased engine power available at high crankshaft speed without suffering from the noise and reduced propeller efficiency that attend high propeller speed. Two general types of reduction gears are used, (1) the offset spur gear, which is the simpler, lighter, and cheaper type, normally employed in in-line, Vee, and opposed engines, and (2) the epicyclic type (with either bevel or spur gears) commonly used to preserve the propeller concentricity in radial engines.

16. ENGINE PERFORMANCE CHARACTERISTICS

DEFINITIONS AND FORMULAS. Brake horsepower (bhp) of an aircraft engine is the power delivered by the engine to the propeller or to the transmission leading to the means of propulsion. The brake horsepower of an engine when measured by a dynamometer is determined by the formula:

$$\text{Bhp} = \frac{2\pi TN}{33{,}000}$$

(T is torque in pound-feet, and N is crankshaft revolutions per minute.)

Corrected brake horsepower (sometimes called "normal temperature and pressure" [ntp] power) is the power that would be produced by the engine under standard atmospheric conditions, 29.92 in. Hg barometric pressure and 60 F atmospheric temperature. Brake horsepower is converted to "corrected" brake horsepower by the following formula, which assumes that the power of an engine varies directly as the barometric pressure and inversely as the square root of the absolute temperature of the carburetor inlet air:

$$\text{Corrected bhp} = \text{bhp} \times \frac{29.92}{B} \times \sqrt{\frac{T}{520}}$$

(B is observed dry air barometric pressure, and T is observed absolute temperature.)

Brake mean effective pressure is the pressure which, if exerted in the cylinder through one stroke of the cycle, would produce the horsepower developed by the engine. It does not represent any condition actually occurring in the cylinder but is a figure of merit used for purposes of comparison. Brake mean effective pressure (bmep) is computed by the following formula:

$$\text{Bmep} = \frac{\text{bhp} \times 792{,}000}{D \times N}$$

(D is piston displacement in cubic inches, and N is crankshaft revolutions per minute.)

Indicated horsepower is the power developed in the cylinders. It is greater than the brake horsepower because of the pumping losses (the power required to pump air and exhaust gases through the manifolds and cylinders), the friction losses in bearings and other rubbing parts, and the power required to drive accessories. Ordinarily all such losses which combine to reduce indicated horsepower to brake horsepower are included in the designation *friction horsepower*. Indicated horsepower may be determined by means of cylinder pressure indicators which measure instantaneous pressures in the cylinder throughout the working cycle or by adding to the brake horsepower the power required to motor the engine. Both methods are of questionable accuracy, the first because of the difficulty of making precise determinations of cylinder pressure at high speed and the second because the friction and pumping losses when motoring the engine do not accurately reproduce the normal operating losses. The customary procedure for measuring friction horsepower is by motoring with a dynamometer; the formula for determining the power is the same as the formula for brake horsepower. Indicated horsepower is the sum of brake and friction horsepower, and the formulas for friction and indicated mean effective pressure (which are also simply figures of merit for purposes of comparison) are the same as the formula for brake mean effective pressure.

Mechanical efficiency is the ratio of net to gross power, or the ratio of brake to indicated horsepower. It is ordinarily expressed in percentage:

$$\text{Mechanical efficiency} = \frac{\text{bhp}}{\text{ihp}} \times 100$$

Volumetric efficiency is the degree of utilization of the piston displacement of the engine. It is the ratio, in percent, of the weight of air actually drawn into the cylinders to the weight of air equal to the full piston displacement of the engine.

Specific fuel consumption is the rate of fuel consumption expressed in pounds of fuel consumed per horsepower per hour of operation. It may be computed as *indicated specific fuel consumption* or *brake specific fuel consumption*. Similarly, *specific oil consumption* is the rate of consumption of oil in pounds per horsepower per hour, customarily computed only on the basis of brake horsepower.

The thermal efficiency of an engine is the ratio of output to input, or the ratio (ordinarily expressed in percentage) of the brake horsepower produced to the potential horsepower in heat units of the fuel consumed. Thermal efficiency may be computed as either brake or indicated thermal efficiency, using brake or indicated specific fuel consumption in the formula:

$$\text{Thermal efficiency} = \frac{2544 \times 100}{\text{Specific fuel consumption} \times h}$$

(h is heating value of fuel in Btu per pound.)

Air standard efficiency is a theoretical efficiency that assumes the working medium to be air of constant specific heat throughout the cycle. Efficiencies found on this basis are appreciably higher than the actual ones. (See also p. 13-06.)

Relative efficiency represents the ratio of the efficiency actually attained to that theoretically attainable on the air standard efficiency basis. It is the ratio, expressed in percentage, of the thermal efficiency determined by test to the air standard efficiency.

$$\text{Relative efficiency} = \frac{\text{Thermal efficiency}}{\text{Air standard efficiency}} \times 100$$

Compression ratio is the ratio of the total or maximum volume of the cylinder to the compressed or minimum volume: the piston displacement of the cylinder plus the clearance volume divided by the clearance volume.

FACTORS AFFECTING PERFORMANCE. Engine Speed. If all other factors were maintained constant, the power of an engine would increase in direct ratio to crankshaft speed. In practice, however, other factors do not remain constant. Such factors as mechanical and volumetric efficiency are normally adversely affected by speed increase, and consequently the power curve, instead of being a straight line (power proportional to speed) tends to droop or flatten out. This is illustrated in Fig. 1, which shows the effect of speed upon indicated mean effective pressure, brake mean effective pressure, indicated horsepower, brake horsepower, and brake specific fuel consumption at sea level in a non-

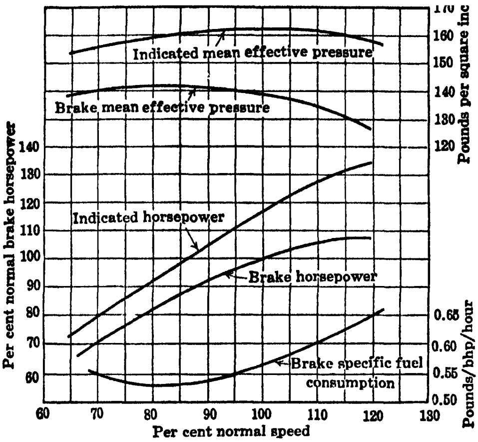


FIG. 1. Effect of speed on mean effective pressure, horsepower, and specific fuel consumption.

supercharged engine. The accompanying curves of Fig. 2 show corresponding values of mechanical efficiency, volumetric efficiency, and friction horsepower. The principal adverse factors illustrated by these curves are the friction horsepower, which increases at a rate greater than the speed increase rate (causing a droop in the mechanical efficiency curve), and the reduction in volumetric efficiency with speed caused by the increase of pressure losses in the manifolds accompanying the increase in velocity of the gases.

Mixture Ratio and Distribution. The internal-combustion engine is sensitive to the quality of fuel mixture it receives. The ratio of fuel to air, the *mixture ratio*, of the intake gas has a pronounced influence on both power and fuel consumption, as shown by the curves of Fig. 3 which illustrate the effect of variation of fuel-air ratio on engine performance and fuel consumption and emphasize the narrow range of mixture ratio that provides both maximum power and acceptable fuel consumption. To appreciate the importance and the difficulty of attaining satisfactory mixture distribution in a multicylinder engine, it must be realized that the condition illustrated by the curves of Fig. 3 exists in each cylinder and that an appreciable deviation from the desired mixture ratio in one or several cylinders produces an overall reduction in output, or increase in fuel consumption, or both.

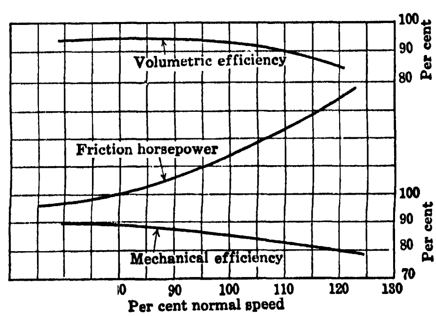


FIG. 2. Effect of speed on volumetric efficiency, friction horsepower, and mechanical efficiency.

Air Temperature and Pressure. Inasmuch as the power of an engine is directly dependent on the weight of gas entering the cylinder, the horsepower produced should vary directly with the density of the air entering the carburetor and inversely with the square root of the absolute temperature of the air. In actual operation the horsepower of a nonsupercharged engine has been found to vary in this manner. See formula for corrected brake horsepower, p. 13-44.

Spark advance (Fig. 4) influences engine power at various speeds, with full throttle operation.

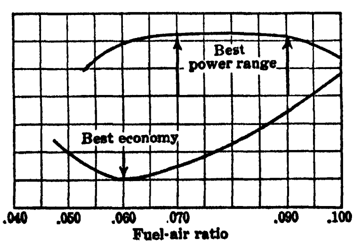


FIG. 3. Effect of fuel-air ratio on horsepower and fuel consumption.

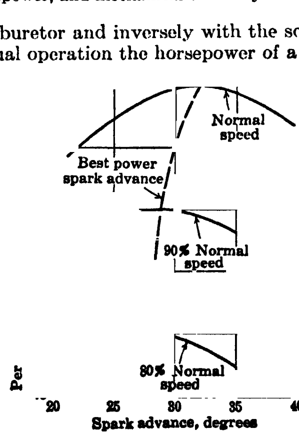


FIG. 4. Effect of spark advance on engine power.

The curves emphasize the importance of accurate ignition timing and illustrate the reasons for engine overheating with incorrect ignition timing. Inasmuch as the fuel quantity supplied to the engine is not affected by the spark advance, the power losses indicated by the curves also represent losses in thermal efficiency; the heat units not converted into useful work must be discharged to the cylinder walls and cooling air, or to the exhaust.

Compression ratio of a gasoline engine cylinder determines the amount of heat or work that can be extracted from the heated gases after combustion. Increasing the expansion (or compression) ratio raises the power of an internal-combustion engine and increases its thermal efficiency. A practical limit on compression ratio increase is established by the *detonation characteristics* of the fuel used (see also Section 15), and a practical limit of expansion ratio (at the low pressure end of the cycle) is imposed by the increasing increments of size, weight, and cost for decreasing increments of power. The improvement in power and efficiency of an internal-combustion engine with increase of compression ratio theoretically should follow the improvement in air cycle efficiency, as shown in Fig. 5. Actually, observed results in horsepower and specific fuel consumption, as shown also in Fig. 5, deviate only slightly from theoretical values, the deviation being accounted for largely by the conflicting influences of improved cylinder scavenging and increased friction losses with increased compression ratios.

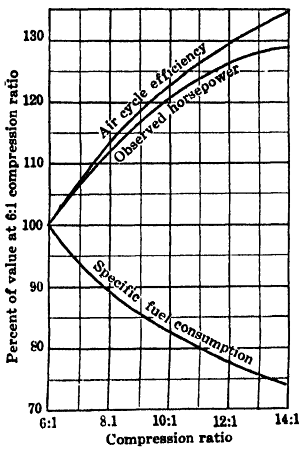


FIG. 5. Effect of compression ratio on power and specific fuel consumption.

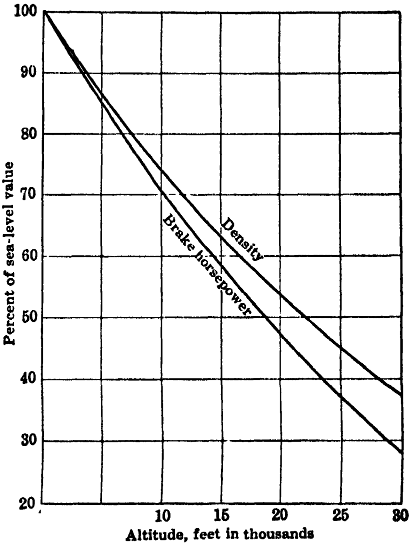


FIG. 6. Comparison of brake horsepower and atmospheric density variation with altitude.

Detonation and Preignition are frequently confused, but they are distinctly different phenomena. Preignition is premature ignition of the charge, ordinarily by incandescent particles of carbon or metal, such as spark plug points or exposed threads in the combustion chamber. The consequences of preignition are exactly the same as those of excessive spark advance—early pressure rise, overheating of the cylinder, low power. Preignition is self-aggravating because of the resulting overheating of the cylinder, and it rapidly increases in severity until further operation is impracticable. Preignition, unless it incites detonation, is not accompanied by excessively high cylinder pressures.

Detonation, the result of a distinct change in the nature of the combustion, is caused by characteristics of the chemical composition of the fuel. It is a cumulative spontaneous explosion of the fuel charge, very much more rapid than normal combustion and producing extremely high cylinder pressures and temperatures with consequent high stresses in the engine parts affected. Characteristic results of detonation are local overheating of cylinder combustion chamber and piston surfaces and severe erosion of piston crowns and sometimes of cylinder head inner surfaces. Detonation tendencies are aggravated by high compression ratio, high cylinder surface temperatures, and high mixture temperatures. However, the detonation tendencies of all ordinary fuels are now well known, and it is possible to supply a fuel with a suitable "knock rating" for any normal operating condition.

Altitude Performance. The density of the atmosphere decreases with altitude. The table on page 1-09 shows normal atmospheric conditions at various altitudes. Accordingly it is to be expected that the output of an aircraft engine will be reduced as the altitude increases. The actual performance of aircraft engines at altitude deviates somewhat from the density curve, as indicated in Fig. 6, which represents the average results of a large number of engine tests. Representative performance curves of a current production engine showing sea level and altitude power output at various speeds and manifold pressures are shown on Fig. 7, plotted on the standard SAE performance curve sheet.

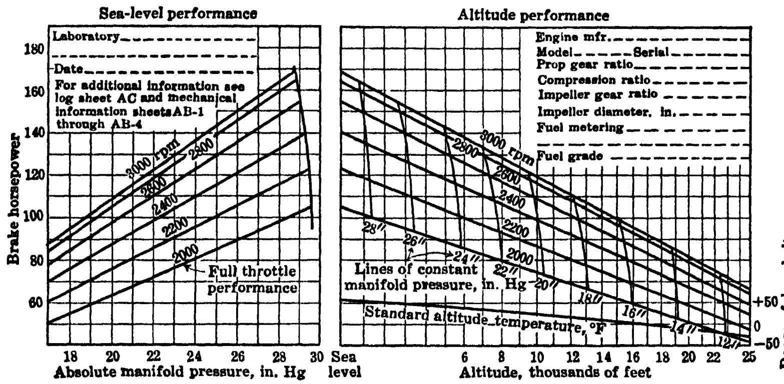


FIG. 7. Performance data of a typical aircraft engine shown on SAE aircraft engine test code performance curve sheet AD-7. (Reprinted by permission of the Society of Automotive Engineers)

17. ENGINE AUXILIARY SYSTEMS

IGNITION SYSTEMS. High-tension magnetos, with two independent electrical circuits and two spark plugs per cylinder, constitute the ignition system of most aircraft engines. The magneto may be a self-contained unit incorporating low-tension and high-tension circuits (the more familiar form), or it may consist of separate low-tension generator and high-tension distributor units. The latter construction is used principally when it will permit combining two units into one or will simplify and reduce the length of high-tension harnesses. The normal high-tension voltage required at the spark plug is in the neighborhood of 10,000 volts, and therefore the advantages of reducing the length and consequently the weight and leakage danger in the high tension conductors are obvious.

An early design, used on several automotive engines to avoid high-tension problems, carried low-tension impulses through the breaker and distribution system to individual high-tension coils at the spark plugs. Presumably that type of ignition was abandoned because of the excessive cost of multiple coils as soon as reliable high-tension cable was made available. Recently, however, a similar system has been under development (with important improvements) for use on aircraft engines. This type of ignition, called the *low-tension, high-frequency* type, produces impulses of sufficiently low voltage (1000 to 1500 volts) to avoid the principal difficulties encountered with high-tension harnesses and converts those impulses, by means of individual coils and condensers, to high-tension, high-frequency impulses at the spark plugs. The term "high frequency" refers to the cyclical frequency of the spark at the plug which, in the high-frequency system, is in the neighborhood of one million cycles per second, over 100 times the frequency of the conventional system. In addition to the reduction of high-tension problems in the spark plug leads, the high-frequency ignition seems to be very effective in avoiding the normal results of plug fouling. The explanation of this benefit is that the sparking voltage is impressed at the gaps in the high-frequency circuit for such a brief period that serious leakage through parallel paths over the surface of the insulator is impossible. Greatly reduced spark plug point erosion is also attributed to high-frequency ignition because of the short duration and low energy (somewhat less than 10% of that of the normal system) of the spark. Claims of greatly reduced size and weight in the low-tension, high-frequency system are not unanimously endorsed, however.

In an unshielded ignition system each high-tension wire forms a miniature broadcasting station and interferes seriously with any attempt to use radio communication. Therefore all ignition systems used in airplanes equipped with radio must be "shielded" by surrounding all elements of the high-tension circuit with grounded metallic screen or sheet to suppress electrical radiation. A resulting disadvantage, besides the weight and cost of the shielding, is the additional electrical capacity load that it introduces in the circuit, requiring an appreciable increase in initial voltage for the same terminal result.

Battery ignition, instead of magneto, for aircraft engines offers the advantages of low cost, mechanical simplicity, and improvement in low speed and starting voltages. However, adoption of this type of ignition has been discouraged by the disadvantages of greater complication in the radio shielding circuits, greater possibility of failure resulting from defects in other portions of the electrical system, lower high-speed voltages, and suspected increased risk of crash fires resulting from the presence of "live" circuits in the vicinity of the engine. Some interest in battery ignition systems has been exhibited, however, and many engines for personal and commercial aircraft have been produced using one magneto and one battery circuit, thus combining the operating advantages of the two systems and some, but not all, of the disadvantages of both.

Spark plugs for many years have been made with mica insulators (mica "cigarettes" wrapped around the center electrode and surrounded by stacked and compressed mica washers). Such plugs have given eminently satisfactory service, but obviously they were also very expensive. Plugs with insulators of porcelain, similar to those used in automobile engines, were found to be undependable in flight service. In recent years, however, the temperatures encountered in high-output engines and the attack of the lead compounds used in high-octane fuels have exceeded the capabilities of mica, and ceramic insulators have been developed that have entirely displaced mica in high-output engines on the grounds of performance, and seem likely to eliminate mica in low-output engines on the grounds of economy. Where radio is required, spark plugs also must be shielded. This is accomplished ordinarily by extending the body of the plug in the form of a tube over the outside of the insulator, with provision for the attachment of a metallic elbow that forms the end of the high-tension wire shielding.

CARBURETION AND FUEL INJECTION. Float and Pressure Carburetors. Two general types of carburetors used on aircraft engines are distinguished by their different methods of supplying fuel to the metering system. The earlier and more familiar type is the *float chamber carburetor*, in which the fuel level with reference to the fuel discharge nozzle is maintained in a float bowl by a float-operated needle valve. The fuel level is of necessity below the discharge nozzle opening, and flow of fuel from the discharge nozzle is induced by low pressure resulting from flow of intake air through a venturi that surrounds the discharge nozzle. The second type of carburetor derives its name, *pressure carburetor*, from the fact that fuel pressure is constantly maintained at the discharge nozzle when the engine is operating. The fuel pressure, produced by an engine-driven pump, is controlled by a diaphragm and admission valve which, in turn, respond to various automatic and manual forces to regulate the pressure, and consequently the fuel flow, to suit atmospheric and engine operating conditions. For two principal reasons the pressure carburetor is rapidly supplanting the float chamber type. (1) The float chamber carburetor obviously is limited in its operation with respect to attitude, i.e., angular tilt. (2) The depression that produces fuel flow in the float chamber carburetor also promotes fuel vaporization with consequent temperature reduction which, over a surprisingly wide range of atmospheric temperatures, will freeze the moisture from the air and clog the intake passages with ice. Both these defects are avoided to a large degree by the pressure carburetor. Claims have been made for reduced cost and weight in the pressure type of carburetor with reference to the float chamber type, but no concrete evidence of those advantages is yet available.

Direct Injection System. A third type of fuel metering that gives promise of replacing both carburetor types, with important operational advantages, is the *direct injection* of timed and measured quantities of fuel into each engine cylinder or into the intake manifold adjacent to each cylinder. In this system an engine-actuated pump plunger directs to the proper cylinder, by means of a distributor, the quantity of fuel required. In some of the larger pumps an individual plunger supplies each cylinder. Fuel injection requires a more complicated control system than is used with either carburetor but, in addition to the maneuverability and ice-free operation of the pressure carburetor, it offers additional advantages over both carburetor types in improved power and fuel economy, which, in extreme instances, may amount to 15 to 20%.

SUPERCHARGING AND COMPOUNDING. A nonsupercharged aircraft engine with normal compression ratio cannot be expected, under the most favorable circumstances, to produce more than approximately 140 psi brake mean effective pressure at sea level. As altitude is increased, the output diminishes rapidly, as indicated on Fig. 6. The supercharger, a compressor which increases the density, hence the weight of the intake charge, is used to improve one or both of those conditions. Originally the supercharger was fitted to compensate only for the reduction of power at altitude but, as the durability of aircraft engines improved, it was found possible to produce a marked improvement in sea-level performance without seriously affecting engine life or dependability. Hence nearly all current supercharger applications are for the purpose of improving ground-level performance and also for compensating for the normal loss of power at altitude.

Two basic types of superchargers have been used for this purpose, the **centrifugal compressor** and the **positive displacement** type. Largely for mechanical reasons, the positive displacement supercharger is being rapidly replaced by the centrifugal type. Two general types of centrifugal superchargers are in use, the **gear-driven** and the **exhaust turbo-driven** types. The first is driven by a train of gears from the crankshaft, and the second by a gas turbine actuated by the exhaust gases. The exhaust-driven supercharger has great flexibility and the compressor power required is furnished nearly free because the increased back pressure on the exhaust subtracts little from the net power of the engine. It does, however, involve exhaust piping problems and overheating difficulties in manifolds, gate valves, and turbine and diffuser blades. The gear-driven supercharger, to provide flexibility, range, and efficiency in all conditions of altitude and flight, is frequently built with two-speed drives, or with two-stage compressors, or both. The comparative performance characteristics of single-speed single-stage, two-speed single-stage, single-speed two-stage, and exhaust turbo-driven superchargers are shown in Fig. 8.

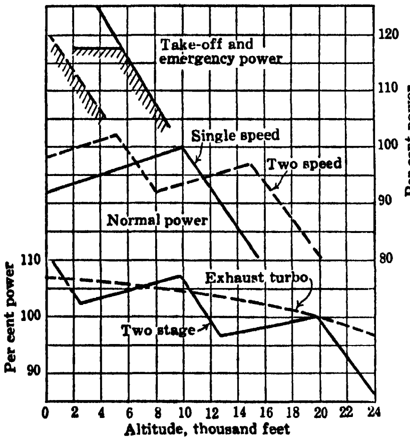


FIG. 8. Supercharger characteristics. (From Aircraft Power Plant Trends, by G. J. Mead, *SAE Journal*, Oct. 1937)

Compounding by use of the exhaust-driven turbine is an extension of the exhaust turbo supercharger development which promises a very considerable improvement in specific power and in thermal efficiency in the piston-type aircraft engine. In this process the exhaust gases of the engine are piped to one or more turbines geared to the crankshaft, so that power derived from further expansion of the exhaust gases is added to the power output of the engine. Full-scale compounded engines presently under experimental development have produced improvement, resulting from compounding, exceeding 35% in power output and 20% in fuel economy. Because of the marked improvement in fuel economy offered by this arrangement, it is expected that compounding will find greatest usefulness in long-range transport operation.

FUELS. The principal requirements of a fuel for aircraft use are exhibited in the following items abstracted from Army-Navy Aeronautical Specification AN-F-48a for reciprocating engine fuel (80 octane grade):

Freezing point—not higher than −76 F.
Sulfur—not greater than 0.05% by weight.
Specific gravity—not limited.
Reid vapor pressure—(at 100 F) not greater than 7.0 psi.
Accelerated gum content—not greater than 6.0 mg per 100 ml of fuel.
Heat of combustion—not less than 18,700 Btu per lb.
Distillation Range:
  At 167 F—fuel evaporated 10% min, 40% max.
  At 221 F—fuel evaporated 50% min.
  At 275 F—fuel evaporated 90% min.
  End point not over 338 F.
Maximum lead content—0.50 ml tetraethyl lead per U. S. gallon.
Knock rating—80 octane minimum.

Of the requirements listed above probably the most critical is the anti-knock rating because that is the quality that controls the permissible output of the engine by establishing the detonation limit. A common method of increasing the detonation resistance of a fuel is by the addition of tetraethyl lead compounds, but the lead attack on engine parts, the lead deposits on critical surfaces in the cylinder, and the diminishing response to lead addition with increasing quantities limit the general use of this ingredient to something less than 3 cc per gallon of fuel.

With the increasing demand for fuel economy more effective and less injurious procedures have been sought to permit increase in compression ratio and corresponding increase in power and economy. Present developments point to beneficial utilization of two similar processes, (1) supplementary injection of knock-suppressing fuels, when required, and (2) bifuel systems which will permit the use of anti-detonant fuels during take-off and in periods of high output operation. By either of these processes the use of special fuels, which might be expensive or injurious to engine parts in continuous service, can be limited to only those operating conditions that require special fuels. Both systems would be automatic in operation, their use mechanically applied by manifold pressure or other engine-operating condition. Special high-compression fuels (triptane, diisobutylene, etc.) which would permit use of extremely high compression ratios and provide corresponding increases in engine output and fuel economy have been proposed. Figure 5 illustrates the benefits of operation with the compression ratios permitted by such fuels (including also the use of supplementary injection and bifuel systems).

COOLING. A convenient approximate "rule of thumb" for the cooling air requirement of an air-cooled engine for aircraft is 10 cu ft of air per minute per horsepower. Inasmuch as the average cooling surface temperature of an air-cooled cylinder is about 100 F higher than the average temperature of the cooling surface of the liquid-cooled engine radiator, the air-to-surface temperature difference available for cooling will average some 50% greater in the case of the air-cooled engine, and consequently the volume of cooling air required by a liquid-cooled engine will be appreciably greater than that required by an air-cooled engine of equal power. The cooling air pressure required to circulate the air through the fins and other cooling surfaces of an air-cooled engine varies greatly with the installation, the dimensions of the fins, etc., but well-developed installations rarely require more than 8 in. of water pressure and requirements as low as 4 in. of water are not unusual. Inasmuch as the cooling pressure requirements for normal cruising operation are lower than those for full power output, it is customary to provide for cruising pressure at minimum airplane drag condition, and to furnish means for cooling pressure augmentation when required. In conventional nacelle installations, pressure augmentation is normally provided by the use of "cowl flaps," adjustable tabs at the trailing edge of the engine cowl, at the air exit from the cowl. These tabs, when tilted outward, create a low-pressure area at the exit and a correspondingly greater cooling air pressure difference through the engine. In installations where cooling air flow is not assured by the forward speed of the aircraft, such as in "pusher" installations and helicopters, cooling air flow is provided by engine-driven fans or by exhaust ejector jets directed into properly proportioned tubes, which eject a large volume of air at low pressure.

VIBRATION AND NOISE. Progress in the science of resilient mounts has practically eliminated the disturbing effect of dynamic unbalance of engines in aircraft, including the vibrations resulting from torque variation. The engine parts themselves, however, are not protected by such devices; in fact, the flexibility of the mounts frequently aggravates the stresses in engine members resulting from vibration. It is, therefore, of the utmost importance that engine vibration of all kinds be reduced to a minimum at the source. Two general types of vibration are involved: (1) forced vibration produced by a major disturbing force and not involving resonant frequency of any part (such as shaking forces produced by engine unbalance or irregular operation, oscillations resulting from torque variation at idling and low speed, etc.) and (2) resonant vibration produced by some minor disturbing force at a frequency which excites resonant vibration in some part of the engine or structure (such as resonant torsional vibration of the crankshaft produced by firing loads and engine mount vibration caused by propeller blades passing close to airplane structure). Forced vibrations ordinarily are readily recognizable and their solutions plain, though sometimes difficult and expensive. Resonant vibrations are usually rather more elusive and sometimes are discovered with difficulty because sources and manifestations are frequently inaccessible when the engine is in operation. A frequent victim of this type of vibration is the crankshaft, and it is now universal practice to analyze the crankshaft system for the most dangerous resonant frequencies likely to be encountered and to provide at the outset, as part of the crankshaft, a damper tuned to suppress those frequencies. In the examination of engine parts for suspected resonant vibrations a very valuable tool is the "strain gage," an electrical element whose resistance is altered by distortion of the element. Such elements can be calibrated to report directly the degree of distortion. When properly attached to the engine part under examination, and with electrical connections through slip rings if necessary, to visual or recording instruments, the elements will

Table 1. American Aircraft Engines (in Production)

Model	No. of Cylinders and Arrangement	Bore and Stroke	Displacement	Take-off, hp/rpm/alt.	Maximum Exhaust Take-off, hp/rpm/alt.	Cruising, hp	Cruising, rpm	Supercharger	Fuel Octane	Propeller Gear Ratio	Compression Ratio	Dry Weight	Dimensions: Length	Dimensions: Height or Diameter	Dimensions: Width
Allison															
V-1710-G6R and L	12 60° Vee	5 1/2 6	1710	1600/3200/—	1000/2700/27700	935	2400	2 - Speed	115/145	2.36	6 : 1	1595	103	29 9/32	38 5/8
Continental															
A-65	4 Hor.	3 7/8 3 5/8	171	65/2300/—	65/2300/SL	53	2150	None	73	None	6.3 : 1	170	30 13/32	20 5/16	31 1/2
C-75	4 Hor.	4 1/16 3 5/8	188	75/2275/—	75/2275/SL	58	2275	None	73	None	6.3 : 1	182	32	21 7/32	31 1/2
C-85	4 Hor.	4 1/16 3 5/8	188	85/2575/—	85/2575/SL	63	2400	None	73	None	6.3 : 1	182	32	21 7/32	31 1/2
C-90	4 Hor.	4 1/16 3 7/8	200.9	95/2625/—	90/2475/SL	77	2350	None	80	None	7.0 : 1	182	31.26	28 23/32	31 1/2
C-115	6 Hor.	4 1/16 3 5/8	282	115/2350/—	115/2350/SL	90	2200	None	73	None	6.3 : 1	257	39 3/4	23 1/4	31 1/2
C-125	6 Hor.	4 1/16 3 5/8	282	125/2550/—	125/2550/SL	98	2400	None	73	None	6.3 : 1	257	39 3/4	23 1/4	31 1/2
C-145	6 Hor.	4 1/16 3 7/8	301.3	145/2700/—	145/2700/SL	130	2600	None	80	None	7.0 : 1	254	36 3/8	27 13/32	31 1/2
E-165	6 Hor.	5 4	471	165/2050/—	165/2050/SL	115	1825	None	80	None	7.0 : 1	326	46.70	25.63	33.39
E-185	6 Hor.	5 4	471	185/2300/—	185/2300/SL	130	2050	None	80	None	7.0 : 1	326	46.70	25.63	33.39
W-670	7 Hor.	5 4	667.8	240/2400/—	240/2400/SL	175	2000	None	80	None	6.1 : 1	465	34.18	42.25	42.25
Franklin (Aircooled Motors, Inc.)															
4A4-75-A3	4 Hor.	4 1/2 3 1/2	225	75/2050/—	75/2050/SL	51	1870	None	73	None	6.5 : 1	214	30 11/16	20 29/32	30 13/16
4A4-85-A3	4 Hor.	4 1/2 3 1/2	225	85/2350/—	85/2350/SL	64	2140	None	73	None	6.5 : 1	214	30 11/16	20 29/32	30 13/16
4A4-95-A3	4 Hor.	4 1/2 3 1/2	225	95/2600/—	95/2600/SL	71	2360	None	73	None	6.5 : 1	214	30 11/16	20 29/32	30 13/16
4A4-75-B3	4 Hor.	4 1/2 3 1/2	225	75/1950/—	75/1950/SL	51	1775	None	80	None	7.0 : 1	214	30 11/16	20 29/32	30 13/16
4A4-85-B3	4 Hor.	4 1/2 3 1/2	225	85/2200/—	85/2200/SL	64	2000	None	80	None	7.0 : 1	214	30 11/16	20 29/32	30 13/16
4A4-100-B3	4 Hor.	4 1/2 3 1/2	225	100/2600/—	100/2600/SL	75	2370	None	80	None	7.0 : 1	214	30 11/16	20 29/32	30 13/16
6A4-125-A3	6 Hor.	4 1/2 3 1/2	335	125/2200/—	125/2200/SL	94	2000	None	73	None	6.5 : 1	291	37 3/8	22 19/32	30 13/16
6A4-135-A3	6 Hor.	4 1/2 3 1/2	335	135/2450/—	135/2450/SL	101	2230	None	73	None	6.5 : 1	291	37 3/8	22 19/32	30 13/16
6A4-145-A3	6 Hor.	4 1/2 3 1/2	335	145/2600/—	145/2600/SL	109	2370	None	73	None	6 5 : 1	291	37 3/8	22 19/32	30 13/16
6A4-130-B3	6 Hor.	4 1/2 3 1/2	335	130/2200/—	130/2200/SL	98	2000	None	80	None	7.0 : 1	291	37 3/8	22 19/32	30 13/16
6A4-140-B3	6 Hor.	4 1/2 3 1/2	335	140/2375/—	140/2375/SL	105	2160	None	80	None	7.0 : 1	291	37 3/8	22 19/32	30 13/16
6A4-150-B3	6 Hor.	4 1/2 3 1/2	335	150/2600/—	150/2600/SL	113	2370	None	80	None	7.0 : 1	291	37 3/8	22 19/32	30 13/16
6A8-175-B7	6 Hor.	5 4 1/4	500	175/2200/—	175/2200/SL	131	2000	None	80	None	7.0 : 1	382	45 17/64	30 1/16	33
6A8-200-B7	6 Hor.	5 4 1/4	500	200/2400/—	200/2400/SL	150	2180	None	80	None	7.0 : 1	382	45 17/64	30 1/16	33 27/32
6A8-225-B7	6 Hor.	5 4 1/4	500	225/2600/—	225/2600/SL	169	2370	None	80	None	7.0 : 1	382	45 17/64	30 1/16	33 27/32
6A8-215-B9F	6 Hor.	5 4 1/4	500	215/2500/—	215/2500/SL	161	2280	None	80	None	7.0 : 1	474	66 7/16	26 7/16	33 27/32
6V4-178-B3	6 Hor.	4 1/2 3 1/2	335	178/3000/—	178/3000/SL	....		None	80	None	7.0 : 1	294	36	26 13/16	30 13/16
6V4-165-B3F	6 Hor.	4 1/2 3 1/2	335	165/3000/—	165/3000/SL	....		None	82	None	7.0 : 1	339	40 5/8	29 13/16	30 13/16
6A4-165-B3	6 Hor.	4 1/2 3 1/2	335	165/2800/—	165/2800/SL	124	2550	None	80	None	7.0 : 1	307	37 3/8	25 5/16	30 13/16

Jacobs																	
O-240A	4	Hor.	4 3/8	4	241	100/2300/—	100/2300/SL	75	2090	None	80	None	6.5 : 1	200	39.63	18.13	33.6
O-360A	6	Hor.	4 3/8	4	361	165/2400/—	165/2400/SL	120	2160	None	80	None	6.5 : 1	300	47.44	18.13	33.6
R-755A	7	Rad.	5 1/4	5	757	300/2200/—	300/2200/SL	225	2000	None	80	None	6.0 : 1	505	39.5	44	44
R-755E	7	Rad.	5 1/4	5	757	350/2500/—	350/2500/SL	260	2250	None	91	.649	6.5 : 1	600	42.34	44	44
R-915A	7	Rad.	5 1/2	5 1/2	914	375/2300/—	375/2300/SL	275	2070	None	80	None	6.0 : 1	560	40.4	45.6	45.6
Lycoming																	
O-145-B2	4	Hor.	3 5/8	3 1/2	144.5	65/2550/—	65/2550/SL	49	2310	None	73	None	6.5 : 1	163	24.62	20.59	29.56
O-235-C	4	Hor.	4 3/8	3 7/8	233.3	100/2600/—	100/2600/SL	75	2360	None	73	None	6.5 : 1	207	29.56	22.53	32.00
O-235-C1	4	Hor.	4 3/8	3 7/8	233.3	115/2800/—	108/2600/SL	81	2360	None	80	None	6.75 : 1	207	29.56	22.53	32.00
O-290-A	4	Hor.	4 7/8	3 7/8	289.0	125/2600/—	125/2600/SL	94	2360	None	73	None	6.5 : 1	245	31.54	26.64	32.32
GO-290-A	4	Hor.	4 7/8	3 7/8	289.0	170/3400/—	160/3000/SL	120	2720	None	91/96	1.56 : 1	7.5 : 1	323	34.59	28.50	33.12
O-435-A	6	Hor.	4 7/8	3 7/8	434.0	190/2550/—	190/2550/SL	143	2310	None	73	None	6.5 : 1	364	38.10	29.61	32.24
GO-435-AC	6	Hor.	4 7/8	3 7/8	434.0	260/3400/—	240/3000/SL	180	2720	None	91/96	1.56 : 1	7.5 : 1	401	......	......	......
O-580	8	Hor.	4 7/8	3 7/8	580.0	350/3300/—	350/3300/SL	240	2730	None	91/96	None	7.5 : 1	573	......	19.97	33.18
SO-580	8	Hor.	4 7/8	3 7/8	580.0	370/3300/—	350/3300/SL	240	2730	7.91 : 1 ratio	91/96	None	7.5 : 1	570	52.09	19.97	33.18
R680E3A	9	Rad.	4 5/8	4 1/2	680.4	300/2300/—	285/2200/SL	222	2000	1 : 1 ratio	91/96	.64	7.0 : 1	515	37.50	43.50	......
R7755-3	36	Rad.	6 3/8	6 3/4	7755.0	5000/2600/—	4000/2300/—	3000	2100	6 : 1 ratio	100/130	.246	8.5 : 1	6050	87.1	61.0	......
Pratt and Whitney																	
Wasp Jr. R985B5	9	Rad.	5 3/16	5 3/16	985	450/2300/—	450/2300/SL	......		1 St 1 Sp	91/98	None	6 : 1	682	43	46.10	......
Wasp R1340S1H1	9	Rad.	5 3/4	5 3/4	1344	600/2250/6200	550/2200/6200	......		1 St 1 Sp	91/98	None	6 : 1	865	43	51.8	......
Wasp R1340S1H1G	9	Rad.	5 3/4	5 3/4	1344	600/2250/6200	550/2200/6200	......		1 St 1 Sp	91/98	.667	6 : 1	953	47.8	51.8	......
Wasp R1340S3H1	9	Rad.	5 3/4	5 3/4	1344	600/2250/3000	550/2200/5000	......		1 St 1 Sp	91/98	None	6 : 1	865	43	51.8	......
Wasp R1340S3H1G	9	Rad.	5 3/4	5 3/4	1344	600/2250/3000	550/2200/5000	......		1 St 1 Sp	91/98	.667	6 : 1	953	47.8	51.8	......
Wasp R1340S3H2	9	Rad.	5 3/4	5 3/4	1344	600/2250/3000	550/2200/5000	......		1 St 1 Sp	91/98	None	6 : 1	865	43	51.8	......
Twin Wasp R1830S1C3G	14	Rad.	5 1/2	5 1/2	1830	1200/2700/4900	1050/2550/7500	......		1 St 1 Sp	91/98	.5625	6.7 : 1	1467	61	48.19	......
Twin Wasp R2000D5	14	Rad.	5 3/4	5 1/2	2000	450/2700/2800	1200/2550/6400	......		1 St 1 Sp	100/130	.500	6.5 : 1	1570	61	49.10	......
Twin Wasp R2000-2SD13G	14	Rad.	5 3/4	5 1/2	2000	450/2700/1000	1200/2550/5000 1100/2550/14000	......		1 St 2 Sp	100/130	.500	6.5 : 1	1595	61	49.10	......
Twin Wasp R2180E1	14	Rad.	5 3/4	6	2181	1800/2800/1000	1400/2600/6000	......		1 St 1 Sp	100/130	.4375	6.7 : 1	1870	75.80	52.60	......
Twin Wasp R2180E12	14	Rad.	5 3/4	6	2181	1800/2800/1000 1300/2800/16000	1400/2600/6000 1250/2600/15500	......		1 St 2 Sp	100/130	.4375	6.7 : 1	1900	75.80	52.60	......
Double Wasp R2800CA5	18	Rad.	5 3/4	6	2804	2400/2800/—	1900/2600/4500	......		1 St 1 Sp	115/145	.450	6.7 : 1	2327	78.40	52.80	......
Double Wasp R2800CA15	18	Rad.	5 3/4	6	2804	2400/2800/— 2100/2800/3000	1800/2600/6000 1600/2600/16000	......		1 St 2 Sp	100/130	.450	6.7 : 1	2360	78.40	52.80	......
Double Wasp R2800CA17	18	Rad.	5 3/4	6	2804	2400/2800/— 2000/2600/10000	1900/2600/4500 1700/2600/14500	......		1 St 2 Sp	115/145	.450	6.7 : 1	2360	78.40	52.80	......
Double Wasp R2800CA18	18	Rad.	5 3/4	6	2804	2400/2800/— 1900/2600/10000	1900/2600/6000 1675/2600/13500	......		1 St 2 Sp	100/130	.450	6.7 : 1	2360	78.40	52.80	......
Double Wasp R2800CA19	18	Rad.	5 3/4	6	2804	2400/2800 2100/2600/7000	1900/2600/4500 1800/2600/11500	......		1 St 2 Sp	115/145	.450	6.7 : 1	2360	78.40	52.80	......
Wasp Major R4360TSB3G	28	Rad.	5 3/4	6	4363	3500/2700/500	2800/2550/3500	......		1 St 1 Sp	115/145	.375	6.7 : 1	3470	96.75	54.00	......
Wasp Major R4360VSB11G	28	Rad.	5 3/4	6	4363	3000/2700/1500	2500/2550/5000 2200/2550/14500	......		1 St Var. Sp	100/130	.425	7.0 : 1	3490	96.75	54.00	......

(Table continued on p. 13-54)

Table 1. American Aircraft Engines (in Production)—*Continued*

Model	No. of Cylinders and Arrangement	Bore and Stroke	Displacement	Take-off, hp/rpm/alt.	Maximum Exhaust Take-off, hp/rpm/alt.	Cruising, hp	rpm	Supercharger	Fuel Octane	Propeller Gear Ratio	Compression Ratio	Dry Weight	Dimensions: Length	Height or Diameter	Width
					RANGER										
6-440-C5	6 Line	4 1/8 5 1/2	441	200/2450/—	200/2450/SL	150	2230	None	87	None	7.5 : 1	376	53.16	33.50	21.87
SGV-770C-2A	12 60° Vee	4 5 1/8	773	550/3300/—	500/3150/9000	350	2560	1 St 1 Sp	100	.666	6.5 : 1	757	66.45	34.13	32.44
SGV-770D-4	12 60° Vee	4 5 1/8	773	575/3400/—	515/3400/13500 500/3400/22500	325	2600	1 St 2 Sp	100	.458	6.5 : 1	896	77.24	36.75	33.75
					WARNER										
Super Scarab 165	7 Rad.	4 5/8 4 1/4	500	175/2250/—	165/2100/SL	124	1910	None	73	None	6.4 : 1	341	30 1/2	37 1/4	....
Super Scarab 185	7 Rad.	4 7/8 4 1/4	550	200/2475/—	185/2175/SL	139	1978	None	73	None	6.2 : 1	344	30 1/2	37 1/4	....
					WRIGHT										
Cyclone 957C7BA1	7 Rad.	6 1/8 6 5/16	1300	800/2600/—	100/2400/5000	490	2200	1 St 1 Sp	91/98	.5625	6.2 : 1	1015	48.12	50.45	....
Cyclone 736C9HD	9 Rad.	6 1/8 6 1/8	1823	1425/2700/—	1275/2500/3500 1125/2500/10600	892	2300	1 St 2 Sp	100/130	.5625	6.8 : 1	1376	47.69	54.95	....
Cyclone 737C9HD	9 Rad.	6 1/8 6 7/8	1823	1425/2700/—	1275/2500/3500 975/2500/18300	890	2300	1 St 2 Sp	100/130	.5625	6.8 : 1	1376	47.69	56.91	....
Cyclone 740C9HD	9 Rad.	6 1/8 6 7/8	1823	1425/2700/—	1275/2500/3000	890	2200	1 St 1 Sp	100/130	.5625	6.8 : 1	1368	47.69	56.77	....
Cyclone 961C9HE1	9 Rad.	6 1/8 6 7/8	1823	1475/2800/—	1275/2500/3500 1125/2500/10600	890	2300	1 St 2 Sp	100/130	.4375	6.8 : 1	1398	48.50	54.95	....
Cyclone 959C9HE1	9 Rad.	6 1/8 6 7/8	1823	1525/2800/—	1275/2500/3500 1125/2500/10600	890	2300	1 St 2 Sp	122/145	.4375	6.8 : 1	1398	48.50	54.95	....
Cyclone 955C9HE1	9 Rad.	6 1/8 6 7/8	1823	1525/2800/—	1275/2500/3500 1125/2500/10600	890	2300	1 St 2 Sp	100/130	.4375	6.8 : 1	1413	48.50	54.95	....
Cyclone C18BA	18 Rad.	6 1/8 6 5/16	3347	2200/2800/—	2000/2400/4800 1800/2400/15000	1400	2300	1 St 2 Sp	100/130	.4375	6.85 : 1	2780	76.26	53.75	....
Cyclone 749C18BD	18 Rad.	6 1/8 6 5/16	3347	2500/2800/—	2100/2400/5500 1800/2400/16000	1470	2300	1 St 2 Sp	100/130	.4375	6.5 : 1	2884	78.47	55.62	....
Cyclone 836C18CA1	18 Rad.	6 1/8 6 5/16	3347	2700/2900/—	2300/2600/—	1600	2300	1 St 2 Sp	115/145	.4375	6.5 : 1	2848	81.93	55.62	....

report not only the amount of stress experienced by the part but also the location, the direction, and the frequency of application.

Aircraft engine noise is a frequent source of complaint. Recent exhaustive studies by the NACA and others indicate clearly that the principal source of noise, especially in direct-drive engines, is the propeller. Reduction of tip speed of the propeller by gearing or by increasing the number of blades, or both, has been demonstrated to be effective in reducing propeller noise to a very inoffensive level. Second in disturbing effect is the engine exhaust, and rapid progress is being made in the development of suitable mufflers, particularly for small engines for personal airplanes. Effective reduction of those two noise factors emphasizes the presence of mechanical noises, and it is to be expected that airplane engine designers will be forced to adopt the measures used by automobile engine designers in this respect (such as close clearance pistons, nonclattering gears, automatic valve adjustments, and the use of sound-absorbing material where resonant effects are found.

STRESS DETERMINATIONS. The science of stress determination in mechanical parts has made important progress in recent years, and it is now possible by the use of strain gages to determine the operating stresses at the critical points of all principal parts without waiting for the results of exhaustive endurance tests. Such determinations indicate not only where failures may be expected but also where stresses are low enough to permit weight reduction without hazard. Determinations may be made by reproducing statically the stresses expected during engine operation but, in the case of complex loading, it frequently is necessary to make determinations in parts in actual operation by bringing out leads through slip rings to the instruments. For checking stress concentrations in individual parts, a useful process is to coat the part with brittle lacquer and then to reproduce as accurately as possible the operating loads. Where strain occurs cracks will appear in the lacquer, the direction and spacing of the cracks indicating the location of stress concentration and its relative magnitude. This process does not require the use of the actual part in question but may be applied to a reproduction of the part in any homogeneous and easily worked material, such as aluminum or magnesium.

AUTOMOBILE ENGINES

(See Automotive Engineering, Section 14.)

GAS ENGINE COMPRESSORS

By J. N. MacKendrick

18. APPLICATION AND CONSTRUCTION

APPLICATION. Gas-engine-driven compressors are widely used in the petroleum and petroleum chemistry industry. Principal applications are for natural gasoline recovery, repressuring of oil fields, gas lifting of oil, recycling of gas for distillate recovery, for gasoline and other hydrocarbon gas recovery, and for refrigeration and general compressor service; in process industries for ammonia synthesis, butadiene, and other materials for synthetic rubber, helium recovery, etc.

In addition, these compressors are used for compressing natural gas for transmission through overland pipe lines, for collection systems, and for compressing both natural and manufactured gas in distribution systems.

TYPES. Gas-engine-driven compressors are built in sizes of 75 to 2500 hp for stationary installation. Skid-mounted units for semi-portable operation are built in sizes of 75 to 400 hp. On these units the engine-compressor unit, the radiator, the compressor intercoolers, traps, and auxiliary equipment are integral so that they can be installed or moved as a unit. Nearly all engine-driven compressor units have horizontal compressor cylinders so that liquid or condensate can pass out of the cylinders more readily.

Until 1930 gas-engine compressors were of the horizontal straight-line type with one- or two-power cylinders. Today most of them are built with horizontal compressor cylinders, and either vertical or V-type power cylinders. Units are available in two- to ten-power cylinders. Figure 1 illustrates the construction of the common angle-type compressor with V-type two-cycle power cylinders, and double-acting horizontal compressor cylinder.

Power cylinders are built in both two- and four-cycle types. The four-cycle engines draw in a mixture of gas and air through a proportioning valve to which gas is supplied

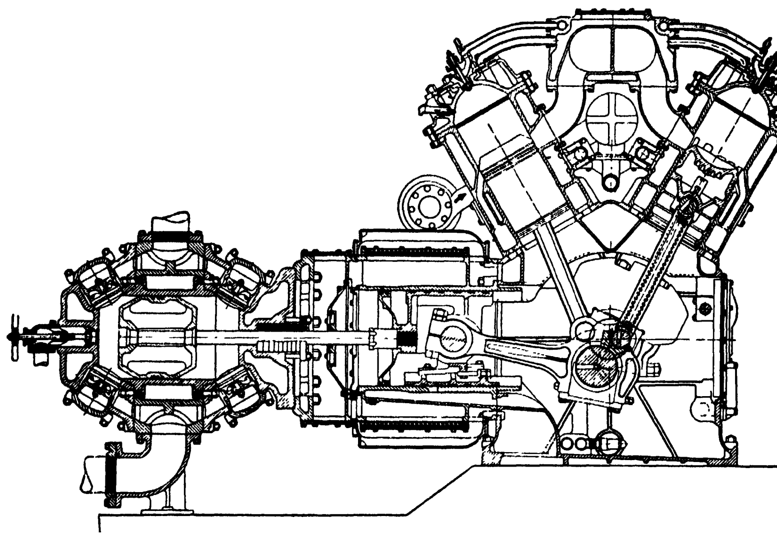


FIG. 1. Typical gas engine compressor. (Courtesy of The Cooper-Bessemer Corp.)

at 2 to 8 oz per sq in. pressure. Two-cycle units are air-scavenged and fuel gas is injected (at 15 to 40 psi) during the compression stroke through a valve in the cylinder head.

IGNITION SYSTEMS. The power cylinders are fired by a high-tension magneto or, alternatively, by individual coils for each power cylinder, supplied by either a low-tension magneto or a battery. Individual coils are often preferred to reduce the length of high-tension ignition wires. A cold-operating spark plug is used, as gas engines deposit very little carbon, compared to gasoline engines.

GOVERNING AND CONTROL. Compressor units are fitted with a mechanical governor or, where closer regulation is required, a hydraulic governor of the type used on large diesel engines. They are also built with pressure control, operated from either suction or discharge pressure of the compressor cylinders. This control holds the pressure constant by varying speed through a range of 60 to 100% of rated speed. If larger variation in capacity is required than is available through speed control, automatic clearance pockets or valve lifters, similar to those used on motor-driven compressors, may be used.

COMPRESSION PRESSURE. Gas engines are normally built with a compression pressure of 90 to 120 psi to enable them to use a wide range of fuels. Units today are being installed with compression pressures of 240 to 260 psia for operation on high-knock rating gas. Some spark-ignition gas engines have been built with compression pressures over 400 psi for use with selected fuel gas.

RATINGS. Gas engine compressors are rated for continuous full-load operation with a minimum outage time. Bmep ratings are 62 to 75 psi, piston speed 700 to 1000 ft per min. Piston speeds are kept low to minimize valve loss in the compressor cylinders.

FUEL CONSUMPTION. The standard fuel guarantee for gas-engine compressors is 10 cu ft of 1000 Btu gas per bhp-hr. For high-compression engines, the guarantee is 8.5 cu ft of 1000 Btu gas per bhp-hr. Because of the wide variation in composition of fuels used, guarantees are based on the *lower heating value* of the fuel gas.

FUELS most commonly used are natural gas, refinery residue gas, manufactured gas, propane, and butane. Natural gas normally has 900 to 1100 Btu per cu ft, consists primarily of ethane and methane. If it does not contain H_2S or heavier hydrocarbons, it is a good fuel for gas engines. Sulfur content should be less than 50 grains per 100 cu ft, since it causes the engine to detonate and forms sludge in cylinders and crankcase. Hydrocarbons heavier than butane, or having lower knock rating than butane, are undesirable.

Propane and butane, or a combination of the two, are satisfactory fuels for gas engines, and are frequently used as an emergency fuel when the natural gas supply fails.

Refinery residues, a general mixture of hydrocarbons, are satisfactory fuels if the sulfur content is low and they do not contain appreciable amounts of heavy, low-knock rating hydrocarbons.

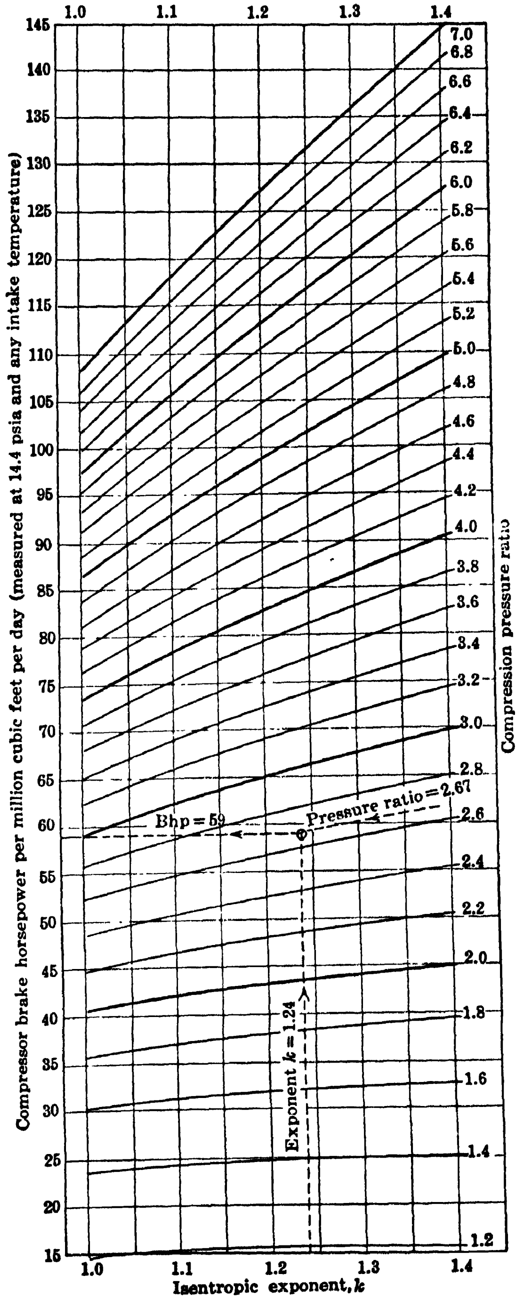


FIG. 2. Compressor horsepower chart. (Courtesy of Clark Bros. Co.)

Manufactured gas, producer gas, blast-furnace gas, and coke oven gas also are satisfactory fuels for gas engines. Although many engine-driven-compressors are used in manufacturing and distribution systems, few are used on commercial installations, owing to high fuel cost. Low Btu content gas usually requires lowering the engine rating; some two-cycle engines with gas injection can retain their normal rating by increasing the fuel-injection pressure.

COOLING SYSTEMS. Nearly all gas engine-driven compressors are water cooled; approximately 30% of the fuel energy goes into cylinder and head jacket water. In all but the smallest engines, the exhaust manifold also is cooled so that a total of around 3500 Btu per hp-hr is transmitted to the cooling water.

The success of an installation depends to a great extent on having a large flow of clean water going through the power cylinder jackets. Temperature rise through the engine should be not greater than 15 F, preferably 12 F. Piping should be designed so there is no possibility of air or steam pockets interrupting the water flow. Closed water systems using radiators or heat exchangers normally are used. Open water systems seldom are satisfactory because concentration of minerals by evaporation in the cooling tower causes deposits in the power cylinder jackets, causing overheating in turn. It is advisable to circulate at least 30 to 35 gallons of water per bhp-hr.

COMPRESSOR PIPING. Since inlet and discharge connections on compressor cylinders are designed for gas velocity appropriate to the cylinder's displacement and surge characteristics, pipe smaller than these connections should not be used. Sharp turns should be avoided in compressor piping, and long radius turns used whenever possible. The volume of inlet and discharge headers should be large enough to keep surges low. Any additional cost will be returned by improved economy of operation and lower maintenance.

Particular attention should be given to anchoring of the discharge piping and headers to prevent vibration, since they are subject to greater gas pulsation than the inlet.

Inlet and interstage piping must be thoroughly cleaned before installation, since compressor valves are readily damaged by impact and abrasion of foreign particles. It is advisable to insert a screen between inlet flanges of compressor cylinders during initial operation, to prevent passage of welding slugs, dirt, etc. If compression of dirty gas is anticipated, suitable scrubbers should be installed.

Provision should be made to drain condensate collected in low points of the compressor piping. Inlet piping should be arranged to prevent liquid from being carried into the compressor cylinder. Even though normal suction temperature may be above the dew point of the gas, the temperature of the pipe during starting may be low enough to cause considerable condensation.

Horsepower Chart. The horsepower required to compress 1,000,000 cu ft of gas per day to various pressures may be conveniently found from Fig. 2.

The compression ratio, R, is absolute discharge pressure divided by absolute suction pressure. Intersection of R on the chart and the specific heat ratio, c_p/c_v (= isentropic exponent), k value, indicate the brake horsepower required.

EXAMPLE. Find the power required to compress 1,000,000 cu ft per day of natural gas (N value = 1.24) from 10 psig suction to 150 psig discharge. $R = \frac{150 + 14.7}{10 + 14.7} = 6.65$. Compression ratios over 6 are not ordinarily used; hence two stages will be assumed. The compression ratio per stage is $\sqrt{6.65} = 2.58$. If 4 psi pressure drop is allowed in the intercooler, the ratio per stage is 2.67. At this pressure ratio and a k value of 1.24 we read 59.0 horsepower (per stage). Multiplying by the number of stages, 2, gives 118, the total brake horsepower required to compress 1,000,000 cu ft per day. Commonly used k (c_p/c_v) values are methane, 1.309; ethane, 1.22; propane, 1.161; butane, 1.11; carbon dioxide, 1.306; and air, 1.395. (See also Thermodynamics of Gases at High Velocity, Section 3.)

SECTION 14

LAND TRANSPORTATION

By

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STEAM ENGINE LOCOMOTIVES

By Alexander Ross

STEAM TURBINE LOCOMOTIVES

By R. P. Johnson

DIESEL LOCOMOTIVES

By T. F. Perkinson

ELECTRIC LOCOMOTIVES

By T. F. Perkinson

AUTOMOTIVE ENGINEERING

By Ralph A. Richardson

STEAM ENGINE LOCOMOTIVES

By Alexander Ross

1. TRAIN RESISTANCE

Under steady forward speed the total tractive effort developed by the cylinders of a locomotive must be equal to the total resistance of the locomotive, tender, and train. Gross train resistance is the pounds of tractive effort required per ton weight of locomotive, tender, and train to keep them in steady motion. It is made up of (1) net resistance on straight level track at uniform speed in still air; (2) grade resistance; (3) curve resistance; and (4) acceleration resistance. Most locomotive builders use the simple and practical train-resistance formulas compiled by W. J. Davis, Jr., of the General Electric Company. They are the ones used in this section.

RESISTANCE ON STRAIGHT LEVEL TRACK. These are the factors that resist movement of a train on level tangent track.

Journal resistance, which decreases as the weight of a car or a locomotive increases and as the temperature of the bearings and of the lubricating oil increases. It is highest at starting and is generally assumed for plain bearings to be about 20 lb per ton, diminishing to its minimum at 5 to 10 mph, and remaining constant at higher speeds. Low temperatures increase journal resistance at starting to about 30 to 35 lb per ton at freezing. Roller bearings affect journal friction resistance, particularly static friction at starting. Friction with roller bearings is reduced approximately 50% at starting, 10% at 5 to 35 mph, and 0% above 35 mph over the values with plain bearings. Values may be found from the following equations.

Plain bearings at 5 mph:

$$R_J = 1.3 + \frac{29}{W} \text{ in pounds per ton}$$

Roller bearings at 5 to 35 mph:

$$R_J = .90\left(1.3 + \frac{29}{W}\right) \text{ in pounds}$$

Plain and roller bearings above 35 mph

$$R_J = 1.3 + \frac{29}{W} \text{ in pounds per ton}$$

where W = average weight per axle in tons.

Air resistance has these components: (1) head pressure, (2) rear suction, and (3) friction resistance on the exterior surface. All are proportional to the square of the train speed. At high speeds air resistance becomes important, as it consumes a large proportion of the drawbar pull.

$$\text{Air resistance} = \frac{CAV^2}{Wn} \text{ (in pounds per ton)}$$

where A = effective cross-sectional area of the car or locomotive in square feet (120 for locomotives; for cars, see Tables 1 and 2); W = average weight per axle in tons; n = number of axles per car or locomotive; C = a constant, depending on equipment (for locomotives, C = 0.0024, for tenders and freight cars, C = 0.0005, for passenger cars, C = 0.00034); and V = speed in miles per hour.

For streamlined equipment, the coefficient C may be multiplied by these correction factors: Leading unit partially streamlined (locomotive), 0.65. Leading unit fully streamlined (locomotive), 0.50. Trailing unit fully streamlined (tender and all cars, including special rear end), 0.70.

Flange resistance has these components: (1) rolling of the wheel on the rail, (2) track resistance due to compression of the track, (3) concussions, (4) miscellaneous losses due to oscillations and vibrations that are absorbed, from which no return can be obtained,

Table 1. Resistance of Freight Cars and Locomotive Tenders on Straight Level Track

(From *Baldwin Locomotive Hand Book*)

(A = 87 sq ft)

Weight of Car, tons	Resistance in Pounds per Ton at Speeds in Miles per Hour										
		10	20		40	50	60	70	80	90	100
	TWO AXLES PER TRUCK—DOUBLE-TRUCK CARS										
20	7.3	7.7			12.3	14.7	17.6	20.9	24.6	29.0	33.4
30	5.4	5.8	6.7		9.3	11.0	13.2	15.3	17.9	21.2	24.2
40	4.4	4.8	5.5		7.8	9.2	10.9	12.6	14.7	17.3	19.6
50	3.8	4.2	4.9		6.8	8.1	9.5	10.9	12.7	15.0	16.8
60	3.4	3.7	4.4		6.2	7.3	8.5	9.9	11.4	13.4	15.0
70	3.2	3.4	4.1		5.7	6.7	7.9	9.1	10.5	12.3	13.7
80	3.0	3.3	3.9		5.4	6.4	7.4	8.5	9.8	11.4	12.7
90	2.8	3.1	3.7		5.1	6.0	7.0	8.1	9.2	10.8	11.9
100	2.7	3.0	3.5		4.9	5.8	6.7	7.7	8.8	10.3	11.3
120	2.5	2.7	3.3		4.6	5.4	6.3	7.2	8.2	9.5	10.4
140	2.4	2.6	3.2		4.5	5.2	5.9	6.8	7.7	8.9	9.7
	THREE AXLES PER TRUCK—DOUBLE-TRUCK CARS										
140	2.8	3.0	3.6	4.2	4.8	5.6	6.4	7.2	8.1	9.1	10.1
160	2.6	2.9	3.4	4.0	4.6	5.3	6.1	6.9	7.7	8.6	9.6
180	2.5	2.7	3.3	3.8	4.5	5.1	5.8	6.6	7.4	8.3	9.2
200	2.4	2.6	3.2	3.7	4.3	5.0	5.7	6.4	7.2	8.0	

Table 2. Resistance of Passenger Cars in Pounds per Ton on Straight Level Track

(From *Baldwin Locomotive Hand Book*)

(A = 120 sq ft)

Weight of Car, tons	Resistance in Pounds per Ton at Speeds in Miles per Hour										
	5	10	20	30	40	50	60	70	80	90	100
	TWO AXLES PER TRUCK—DOUBLE-TRUCK CARS										
20	7.3	7.6	8.5	9.8	11.6	13.7	16.2	19.2	22.6	26.3	30.5
30	5.4	5.6	6.3	7.3	8.5	10.1	11.9	13.9	16.3	18.9	21.8
40	4.4	4.5	5.2	6.0	7.0	8.3	9.7	11.3	13.1	15.2	17.4
50	3.8	4.0	4.6	5.2	6.1	7.1	8.3	9.7	11.2	12.9	14.7
60	3.4	3.6	4.1	4.7	5.5	6.4	7.5	8.7	10.0	11.4	13.0
	THREE AXLES PER TRUCK—DOUBLE-TRUCK CARS										
50	5.0	5.2	5.7	6.4	7.3	8.4	9.6	10.9	12.4	14.1	15.9
60	4.4	4.6	5.1	5.7	6.5	7.4	8.5	9.6	11.0	12.4	14.0
70	4.0	4.1	4.6	5.2	5.9	6.7	7.7	8.7	9.9	11.2	12.6
80	3.6	3.8	4.3	4.8	5.5	6.3	7.1	8.1	9.1	10.3	11.6
90	3.4	3.6	4.0	4.5	5.2	5.9	6.7	7.6	8.6	9.7	10.7

and (5) flange friction due to the pressure of the wheel flange against the rail. Flange resistance is calculated by using test constants and is expressed in pounds per ton:

$$R_F = 0.03V \quad \text{(for locomotives in train)}$$
$$R_F = 0.03V \quad \text{(for passenger cars)}$$
$$R_F = 0.045V \quad \text{(for freight cars and tenders)}$$

where V = speed in miles per hour.

Mechanical resistance, in pounds, equals $20T$, where T is the weight on drivers in tons.

Total resistance on straight level track is the sum of the above components, where R = total resistance in pounds per ton. Values of R are given in Table 2, p. 14-48.

GRADE RESISTANCE. The effort required to lift a ton of train up a grade of one foot per mile is 2000/5280 = 0.3788 lb. To find the total resistance due to grade in pounds per ton, multiply the rise in feet per mile by 0.3788. When the grade is represented in percentage, resistance in pounds per ton is 2000/100, or 20 lb for each one percent of grade.

CURVE RESISTANCE. It is impossible to give an accurate rule for calculating curve resistance. Most American railroads and locomotive builders base their calculations on tests made by the Pennsylvania Railroad, which show an average of 0.8 lb per ton per degree of level track. When a curve occurs on a grade most main-line tracks cancel its effect by reduction of the grade on the curve so that the combined resistance of grade and curve equals that of the straight grade. The grade is then said to be "compensated." The grade, when fully compensated, is reduced by 0.04% for each degree of curve. Since grade resistance is 20 lb per ton for each 1% of grade, and curve resistance is 0.8 lb per ton for each degree of curve, each degree of curve resistance is equivalent to 0.8/20% or 0.04% of grade. Usually the compensation in practice is 0.035% in grade for each degree of curve.

ACCELERATION RESISTANCE. The three resistances discussed, net resistance on straight level track, grade resistance, and curve resistance, are based on the train moving at uniform speed. The sum of these three, subtracted from the tractive effort available at the speed, is the tractive effort available for acceleration. The total force required to produce acceleration is made up of two parts. The first is the force needed to produce linear acceleration of the train as a whole, and the second is the force needed to produce rotational acceleration of the wheels and axles.

Acceleration resistance can be calculated from the formula:

$$R = \frac{70}{S}(V_2^2 - V_1^2)$$

or

$$R = \frac{95.6}{T}(V_2 - V_1)$$

where R = acceleration resistance, pounds per ton; S = distance, feet; T = time, seconds; V_2 = final velocity, miles per hour; and V_1 = original velocity, miles per hour.

2. CLASSIFICATION OF STEAM LOCOMOTIVES

Table 3 gives a generally used classification of steam locomotives, with the names in common use. (See also Electric Locomotives, Table 1, p. 14-47.) The type symbol was suggested by F. M. Whyte.

The articulated locomotive has a single boiler carried on a two-engine chassis. Each engine has its pair of cylinders, frames, driving wheels, trucks, and other equipment. The boiler is rigidly fastened to the rear engine frame, and the front end is carried on a boiler bearing support on the front engine. The two engine frames are connected by a hinged pin arrangement. With this type of locomotive, a high tractive effort is obtained with comparatively low axle loads and ability to negotiate sharp curves. An articulated 4–8–8–4 type built for the Union Pacific Railroad by the American Locomotive Company has a tractive effort of 135,375 lb. The weight of the engine alone in working order is 772,000 lb. All articulated engines built in this country in recent years are *simple engines*. The Mallet articulated locomotive, developed by Anatole Mallet, works as a *compound engine*, the high-pressure cylinders drive the rear engine, and the low-pressure cylinders drive the front engine. The 4–4–4–4 Duplex type differs from the articulated type, in that both engines operate in one rigid frame.

3. LOCOMOTIVE CHARACTERISTICS

HORSEPOWER. The steam locomotive is a self-contained power unit. As for all reciprocating piston power units, ihp = $PLAN/33{,}000$ (see p. 8-102), where P = mean effective cylinder pressure, pounds per square inch; L = length of stroke, feet; A = area of piston, square inches; and N = number of strokes (4 × number of revolutions per minute). P varies with engine speed and design. It would appear from the formula that all locomotives having the same boiler pressure and cylinders should produce the same ihp output at the same rpm, regardless of driving-wheel diameter. This is generally not possible because of restrictions in design for different locomotives, such as counterbalance, valve setting, riding stability, and boiler capacity.

Ihp output for any reciprocating power unit rises with the speed to a maximum and then falls, probably at a similar rate, to the point at which the speed is so great that only sufficient energy is admitted to the cylinders to keep the engine itself in motion; operating conditions prevent testing to this speed. Primarily, locomotive designers are interested

Table 3. Symbolic Notation Designating Wheel Arrangement for Various Types of Steam Locomotive

Type Symbol	Wheel Arrangement	Name
0-4-0	⊿ □ ◯◯	4-wheel switcher
0-6-0	⊿ □ ◯◯◯	6-wheel switcher
0-8-0	⊿ □ ◯◯◯◯	8-wheel switcher
0-10-0	⊿ □ ◯◯◯◯◯	10-wheel switcher
4-4-0	⊿ o □ o ◯◯	American
4-4-2	⊿ o □ o ◯◯ o	Atlantic
2-6-0	⊿ o □ ◯◯◯	Mogul
2-6-2	⊿ o □ ◯◯◯ o	Prairie
4-6-0	⊿ o □ o ◯◯◯	10-wheel
4-6-2	⊿ o □ o ◯◯◯ o	Pacific
4-6-4	⊿ o □ o ◯◯◯ o o	Hudson
2-8-0	⊿ o □ ◯◯◯◯	Consolidation
2-8-2	⊿ o □ ◯◯◯◯ o	Mikado
2-8-4	⊿ o □ ◯◯◯◯ o o	----------
4-8-0	⊿ o □ o ◯◯◯◯	----------
4-8-2	⊿ o □ o ◯◯◯◯ o	Mountain
4-8-4	⊿ o □ o ◯◯◯◯ o o	----------
2-10-0	⊿ o □ ◯◯◯◯◯	Decapod
2-10-2	⊿ o □ ◯◯◯◯◯ o	Santa Fe
2-10-4	⊿ o □ ◯◯◯◯◯ o o	Texas
4-12-2	⊿ o □ o ◯◯◯◯◯◯ o	----------
4-4-4-4	⊿ o □ o ◯◯ □ ◯◯ o o	Duplex
0-6-6-0	⊿ □ ◯◯◯ □ ◯◯◯	Articulated or Mallet
2-6-6-2	⊿ o □ ◯◯◯ □ ◯◯◯ o	Articulated or Mallet
4-6-6-4	⊿ o □ o ◯◯◯ □ ◯◯◯ o o	Articulated or Mallet
0-8-8-0	⊿ □ ◯◯◯◯ □ ◯◯◯◯	Articulated or Mallet
2-8-8-0	⊿ o □ ◯◯◯◯ □ ◯◯◯◯	Articulated or Mallet
2-8-8-2	⊿ o □ ◯◯◯◯ □ ◯◯◯◯ o	Articulated or Mallet
2-8-8-4	⊿ o □ ◯◯◯◯ □ ◯◯◯◯ o o	Articulated or Mallet
4-8-8-2	⊿ o □ o ◯◯◯◯ □ ◯◯◯◯ o	Articulated or Mallet
4-8-8-4	⊿ o □ o ◯◯◯◯ □ ◯◯◯◯ o o	Articulated or Mallet

only in the maximum ihp output as a measure of locomotive capacity. The characteristic curve is shown in Fig. 1.

The maximum ihp output for any locomotive is influenced by the cylinder size (a function of the weight on drivers), valve and valve gear characteristics, design restrictions, and boiler capacity (without which the curve peak cannot be reached).

The path of P (the mep of the indicator card) is theoretically a straight line from 100 to 0% boiler pressure, the slope varying with locomotive design characteristics because the path of a point influenced by two variables, both of which vary at the same rate, is a straight line; here the two variables are the quantity of steam admitted at each stroke and the piston speed. Both are a direct function of rpm. Locomotive design characteristics vary so widely that individual design tests are required to determine accurately the slope of the mep line and the point of maximum ihp output, after which the data may be safely used for other designs having similar characteristics and boiler capacity.

The ihp produced by any steam locomotive is a direct function of the total heat utilization at that time, which depends on the amount of fuel burned (heat liberation), the boiler efficiency (heat absorption), and the cylinder efficiency (heat utilization). Prime requirements in burning fuel are: (1) Sufficient grate area. It may be assumed for rating purposes in design that about 150 lb of coal can be burned per square foot of grate area. (2) Sufficient firebox volume to permit a design rating of about 320,000 Btu per cu ft (of net firebox volume) per hr.

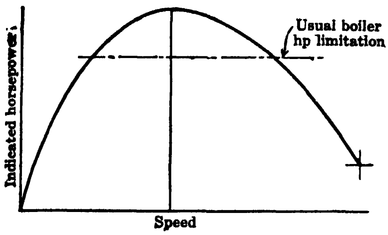


FIG. 1. Locomotive ihp-speed characteristic.

The boiler must have a sufficient number of tubes and flues to provide sufficient gas flow area for the products of combustion, sufficient steam flow area through the superheater elements, and sufficient heat absorption area. Properly proportioned grate area and firebox volume are of far greater importance than total heating surface if the required amount of coal is to be burned for maximum evaporation.

BOILER CAPACITY. Since their introduction in 1914, the *Cole ratios* for Steam Locomotives have been generally used for estimating evaporation of steam locomotive boilers. They proved satisfactory for locomotives in use at that time, which were arranged for low superheat, no firebox combustion chamber, and hand firing. The Cole ratios were based on the Coatesville Evaporation Tests made in 1912 on an oil-fired, saturated steam boiler without firebox combustion chamber. At maximum evaporation the heat release rate was 320,000 Btu per cu ft of net firebox volume per hour; it was at this rate that the evaporation values were established for firebox and tubes. The Cole ratios assume a heating surface to grate area ratio of 60 and a maximum coal rate of 120 lb per hr per sq ft of grate area, or 2 lb per hr of coal per square ft of heating surface, regardless of firebox volume. If the firebox volume of the test boiler had been increased by the addition of an internal combustion chamber, there would have been little difference in the total heat release, but the heat-release rate would have dropped below the 320,000 Btu rate at which the evaporation values were established. Improved combustion conditions would have increased evaporation materially, however, even with the lower heat-release rate. If enough more fuel had been burned to maintain the 320,000 Btu rate, there would have been more total heat released, with consequent greater evaporation and little change in smokebox temperature. The total heat released, and not the heat released per square foot of heating surface, is a measure of total evaporation, since both firebox and tube heating surface easily absorb much more heat per unit of area than indicated by the Cole ratios.

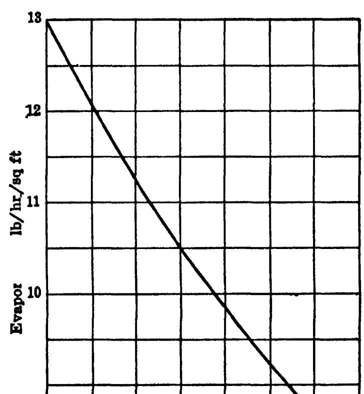


FIG. 2. The effect of fire-tube length on locomotive boiler evaporation.

Calculation of Evaporation. Briefly, these are the steps in calculating the expected locomotive boiler evaporation.

1. Establish estimated evaporation using the Cole ratios. For the firebox heating surface (*HS*), a value of 55 lb of steam per hour per square foot should be used. For tubes and flues, the value should be taken from Fig. 2.
2. Establish the coal rate (*CR*) in pounds per hour required to maintain the heat release rate of 320,000 Btu per cu ft of firebox volume (*FBV*).

$$CR = \frac{FBV \times 320{,}000}{13{,}000} = 24.6\ FBV$$

3. Adjust the estimated Cole ratio evaporation (1) to suit the coal required (2) to obtain the expected evaporation (*EE*).

$$EE = \text{Cole ratio evaporation}\left(\frac{7.5\ FBV}{HS} + 0.39\right)$$

The grate should be of sufficient size to burn not over 150 lb per sq ft per hr for mechanical stoker-fired locomotives and 120 lb per sq ft for hand-fired locomotives. At very high coal combustion rates of 200 to 250 lb of coal per square ft of grate area, the stack losses become very high. Table 4 shows the expected results for both a noncombustion-chamber boiler and an identical boiler arranged with a combustion chamber.

Table 4. Effect of Combustion Chamber

	Without Combustion Chamber	With Combustion Chamber
Expected evaporation, lb per hr	32,800	41,200
Coal burned, lb per hr	5,470	6,560
Heat release per cubic foot of firebox volume, Btu per cu ft per hr	320,000	320,000

Figure 3 shows the steam rate at various degrees of superheat. The superheat is determined by the tube and flue ratio as shown in Table 5. The expected evaporation divided by the amount of steam required per ihp determines the boiler capacity.

Table 5. Superheater Design

Tube Combination	Ratio of Small to Large Tubes	Degrees of Superheat, °F
5 3/8 and 2 in. 5 1/2 and 2 1/4 in.	7 or over 6 or over	150
5 3/8 and 2 in. 5 1/2 and 2 1/4 in.	5 to 7 4 to 6	200
5 3/8 and 2 in. 5 1/2 and 2 1/4 in.	Under 5	250
3 1/2 and 2 1/4 in.	....	250

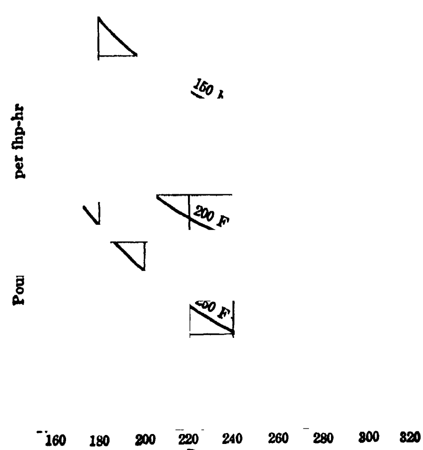


Fig. 3. Steam rate variation with pressure for various values of initial superheat.

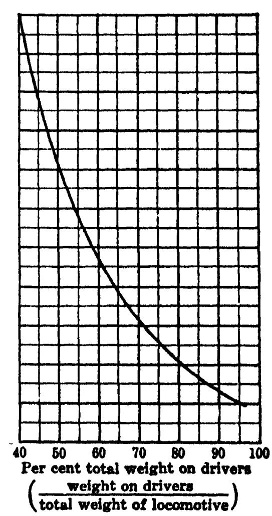


Fig. 4. Maximum ihp related to weight on drivers.

A method of determining the approximate maximum ihp in relation to the weight on drivers/total weight ratio is shown in Fig. 4. Test data from about fifty individual locomotives in general use at the present time were used in preparing the curve.

TRACTIVE EFFORT is defined as the average tangential force at the rail during one revolution of the driving wheels. Tractive effort and other data for typical American locomotives are given in Table 6. The formula for two-cylinder simple locomotives is

$$\text{Tractive effort} = \frac{d^2LP}{D}$$

where d = diameter of cylinder, inches; L = stroke, inches; P = mean effective pressure, pounds per square inch; and D = driving wheel diameter, inches. Alternatively,

$$\text{Tractive effort} = \frac{\text{ihp} \times 375}{\text{mph}}$$

Both equations are fundamental and retain the same variables as in the original ihp formula.

The cylinder TE-speed line can be drawn from only two datum points: (1) TE at 100% boiler pressure at standstill. (2) TE at some speed for which test data are available for some other existing design having similar characteristics. For locomotives of modern design, the tractive effort at 336 rpm can be determined from the formula by using 40% of the boiler pressure as P. The tractive effort line then appears as in Fig. 5.

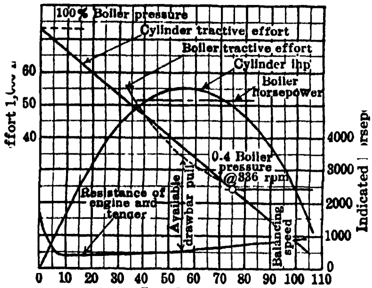


Fig. 5. Typical characteristic curves for a modern steam locomotive (4-8-4, rated tractive effort = 62,500 lb).

The generally accepted maximum tractive effort "rating factor" for all locomotives with conventional "long cut-offs" has been 80 to 90% of the boiler pressure for many years, as originally test-determined for slide-valve, saturated-steam, and hand-fired locomotives. With improved steam characteristics, this factor often reaches 90 to 95%.

Factor of adhesion is the weight on drivers divided by the tractive effort. A factor of 4.0 was satisfactory when the tractive effort rating factor did not exceed 85%, but with increased factors of 90 to 95% the factor of adhesion must be 4.25 if slippage is to be avoided.

Cut-offs. Whenever the maximum starting cut-offs are reduced below the usual 80 to 85% figures, the tractive effort rating factor of 85% also decreases and the tractive effort line changes shape. English, and some Continental practice, uses a rating factor of 75%. With "limited cut-off" and oversize cylinders, the cut-off may drop to 50 to 60%, requiring auxiliary starting ports operative only at speeds below 5 mph. These designs have proved sensitive and require careful handling to such an extent that few modern designs embody this feature.

While steam locomotives may be started with a maximum initial cut-off of 80 to 85% for conventional designs, with the usual types of outside crank-operated valve gears, acceleration requirements soon make shorter cut-offs necessary, down to 60 to 70% for running conditions. A still further reduction in cut-off with increasing speed is made until a rating factor of 20 to 25% is reached; beyond this unstable running conditions are encountered. Smooth running cannot be maintained at shorter cut-offs with conventional valve gear and a single valve controlling all four valve events. Multiple valves for different valve events materially improve this condition. Judgment of a high order is required to establish the proper cut-off for "drifting" to permit smooth riding; too long a cut-off results in dangerous inertia forces, and too short a cut-off produces too high a compression. Both conditions may become disastrous if uncontrolled.

For any multicylinder type, simple expansion locomotive,

$$\text{Tractive effort} \quad \frac{N}{2} \times \frac{d^2 \times L \times P}{D}$$

where N = number of cylinders.

For a two-cylinder compound locomotive

$$\text{Tractive effort} \quad 0.6\ \text{BP} \times d^2 \times L$$

where BP = boiler pressure, pounds per square inch; d = diameter of high pressure cylinder, inches; L = stroke of piston, inches; and D = diameter of driving wheels, inches.

The formula for two-cylinder compound locomotives is also applicable to *Mallet* type locomotives. Since the Mallet has four cylinders, the result must be doubled. The formula for two-cylinder and Mallet compounds assume a cylinder diameter ratio of 2.35 to 2.40.

Drawbar pull is the difference between the tractive effort and the total rolling and wind resistance of the engine and tender at any operating speed; in other words, it is the useful force exerted by the entire locomotive in hauling trains.

(*Continued on p. 14-11*)

Table 6. Dimensions of Typical American Standard Gage Locomotives

Railroad	P.R.R.	C.&N.W.	P.R.R.	Wab.	C.&O.	U.P.	C.M.ST.P. &P.	N.Y.C.	N.Y.C.	D.&H.	U.P.
Builder	P.R.R.	Baldwin	P.R.R.	Baldwin	Lima	Amer.	Amer.	Amer.	Amer.	Amer.	Amer.
Type	4-6-2	4-8-4	4-8-2	4-8-4	2-10-4	4-12-2	4-4-2	4-6-4	4-8-4	4-6-6-4	4-8-8-4
Service	Pass.	P. & F.	P. & F.	Frt.	Frt.	Frt.	Pass.	Pass.	Pass.	Frt.	Frt.
Year placed in service	1914-17	1929	1930	1931	1930	1930	1937	1937	1946	1946	1944
Fuel	Bit.	Bit.	Bit.	Bit.	Bit.	Bit.	Oil	Bit.	Bit.	Bit.	Bit.
Tractive force, lb	44,460	76,500	64,550	70,750	106,584	96,650	30,700	55,540	61,500	94,400	135,375
Weight in working order, lb	308,890	498,000	390,000	454,090	566,000	515,000	290,000	360,000	471,000	604,500	772,000
Weight on drivers, working order, lb	201,830	288,000	271,000	274 100	373,000	372,000	144,300	196,000	275,000	409,500	545,000
Weight on truck, leading, lb	53,600	87,000	59,000	78,590	61,000	81,000	72 700	66,000	91,400	77,500	99,800
Weight on truck, trailing, lb	53,420	123,000	60,000	101,400	132 000	62,000	73,000	98,000	104 600	117,500	127,200
Weight of engine and tender, w.o., lb	480,290	818,000	774,020	755,800	981,000	823,800	556,300	674,300	899,000	995,200	1,208,000
Wheel base, driving, ft and in.	13-10	20-6	18-10	18-3	24-4	30-8	8-6	14-0	20-6	35-1	47-3
Wheel base, total, ft and in.	36-2	48-7	41-9 1/2	45-0	49-3	52-4	37-7	40-4	48-5	59-11	72-5 1/2
Wheel base, engine and tender, ft and in.	72-9	91-1	96-6 3/8	86-10	99-5 3/4	91-6 1/2	78-10 1/2	83-7 1/2	97-2 1/2	103-6	117-7
Cylinders, diam and stroke, in.	27 x 28	27 x 32	27 x 30	27 x 32	29 x 34	Outside 27 x 32, Inside 27 x 31	19 x 28	22 1/2 x 29	25 1/2 x 32	20 1/2 x 32	23 3/4 x 32
Valves, kind	Piston	Piston	Piston	Piston	Piston	Piston Piston	Piston	Piston	Piston	Piston	Piston
Valve gear, kind	Walsch'rt	Walsch'rt	Walsch'rt	Walsch'rt	Baker	Walsch'rt	Walsch'rt	Baker	Baker	Walsch'rt	Walsch'rt
Valve diam (piston), in.	12	14	12	12	14	14 14	10	14	14	12	12
Steam lap, in.	1 5/16	1 1/2	1 7/16	1 3/8	1 15/16	1 1/4 1 1/16	1 1/8	1 5/8	1 9/16	1 1/4	1 3/8
Exhaust clearance, in.	3/8	1/8	7/16	1/8	0	1/8 C.L. 1/8 Lap.	1/4	3/16	3/16	0	1/8
Lead in full gear, in.	9/32	1/4	9/32	1/4	3/16	1/4 3/16	1/4	1/4	5/16	3/16	1/4
Valve travel, in.	7	7 3/4	7	8	9	7 5 7/8	6 1/2	8 1/2	8 1/2	7 1/2	7
Driving wheel diam, in.	80	76	72	70	69	67	84	79	79	69	68
Truck wheel diam, leading, in.	36	36	33	33	33	30	36	36	36	36	36
Truck wheel diam, trailing, in.	50	F.44, R.50	50	42 1/2	F.36, R.43	45	51	36-51	41	36-45	42
Driving axle journals, main, in.	11 x 15	13 1/2 x 14	12 x 16	13 x 14	13 1/2 x 14	12 x 13	11 13/16 roll	13 roll	13 roll	13 roll	13 3/4 roll
Driving axle journals, others, in.	11 x 15	12 x 14 1/2	11 x 16	11 1/2 x 14	12 x 14	10 x 3	11 13/16 roll	12 roll	12 1/8 roll	11 1/2 x 13	12 5/8 roll
Truck axle journals, leading, in.	6 1/2 x 12	7 1/2 x 14	6 1/2 x 12	6 x 11	7 x 14	7 1/2 x 13	7 1/2 roll	7 roll	8 1/4 roll	7 7/8 roll	8 1/4 roll
Truck axle journals, trailing, in.	6 1/2 x 12	9 x 14	6 1/2 x 12	9 x 14	7 & 9 x 14	9 x 14	7 roll	6 1/4-7 roll	6 1/2 roll	7-9 x 14	8 roll

(*Table continued on p. 14-10*)

Table 6. Dimensions of Typical American Standard Gage Locomotives—*Continued*

Railroad	P.R.R.	C.&N.W.	P.R.R.	Wab.	C.&O.	U.P.	C.M.ST.P. &P.	N.Y.C.	N.Y.C.	D.&H.	U.P.
Builder	P.R.R.	Baldwin	P.R.R.	Baldwin	Lima	Amer.	Amer.	Amer.	Amer.	Amer.	Amer.
Boiler, kind	Belpaire	Conical	Belpaire	Conical	Conical	Conical	Str. top	Conical	Conical	Str. top	Str. top
Working pressure, psig	205	250	250	250	260	220	300	275	275	285	300
Boiler diam, outside first ring, in.	82 1/2	90 3/16	84 1/2	86 1/2	99 3/4	90	78 5/16	82 1/4	91 7/8	97 1/16	101 1/16
Firebox, width and length, in.	80 x 126	96 x 150	80 x 126	96 x 144	108 x 162	108 x 185	132 1/16 x 75 3/16	130 13/16 x 90 1/4	151 1/16 x 96 1/4	143 23/32 x 108 3/16	225 x 96 3/16
Combustion chamber, length, in.	. . . . .	60	98	60	66	80 1/2	None	43	81 1/4	94	112
Tubes, number and outside diam, in.	236–2 1/4	51–2	120–2 1/4	49–2 1/4	59–2 1/4	40–3 1/2	160–2 1/4	59–2 1/4	55–2 1/4	222–2 1/4	212–2 1/4
Flues, number and outside diam, in.	40–5 1/2	214–3 1/2	170–3 1/2	214–3 1/2	275–3 1/2	222–3 1/2	43–5 1/2	183–3 1/2	177–4	60–5 1/2	73–5 1/2
Tubes and flues, length, ft and in.	19–0	21–0	19–0	21–0	21–0	22–0	19–0	19–0	19–11 1/4	22–0	22–0
Heating surface, firebox, sq ft	315	558	395	495	645	591	294	360	499	633	720
Heating surface, tubes and flues, sq ft	3731	4656	4303	4694	5990	5262	2951	3827	4320	4755	5035
Heating surface, total evaporative	4046	5214	4698	5189	6635	5853	3245	4187	4819	5388	5755
Heating surface, superheating	1147	2357	2052	2360	3030	2560	1029	1745	2073	1681	2043
Heating surface, combined superheating and evaporative	5193	7571	6750	7549	9665	8413	4274	5932	6892	7069	7798
Grate surface, sq ft	70	100	70	96	122	108	69	82	101	108	150
Booster	None	Trailer	None	None	Trailer	None	None	Trailer	None	None	None
Tender, frame	. . . . .	. .	Cast steel		. . . .	. . .	Cast steel	Cast steel	Cast steel	Cast steel	Cast steel
Tender, weight loaded, lb	171,400	320,000	384,020	301,710	415,000	308 800	266,300	314,300	418,000	390,700	436,000
Tender, wheel diam, in.	36	36	33	36	36	33	36	36	41	36	42
Tender, journals, in.	5 1/2 x 10	Roll. bear.	6 1/2 x 12	6 x 11	7 x 14	6 1/2 x 12	6 1/2 x 12 roll	6 roll	6 1/2 roll	7 x 14	6 3/4 roll
Tender, water capacity, gal	7850	18,000	22,090	15,000	23,500	18,000	13,000	14,000	18,000	22,500	25,000
Tender, fuel capacity, tons or gal	13 1/2 T.	20 T.	36 1/2 T.	18 T.	30 T.	20 T.	4000 gal	30	46	26	28
See notation below: W_d/TF =	4.5	4.4	4.2	3.9	4.1	3.8	4.70	4.53	4.47	4.34	4.03
TF/HS_E =	11.0	12.5	13.7	13.7	13.8	16.5	9.46	13.26	12.76	17.52	23.52
HS_E/G =	57 7	52.1	67.0	54 0	54 4	54.3	47.0	51.1	47.7	49.9	38.4
$TF \times D/HS_E$ =	879	950	989	954	952	1106	795	1048	1008	1208	1599
W_t/HS_E =	76.3	95 5	83.0	87.5	85.3	88.0	89.4	86.0	97.7	112.2	134.1
HS_S/HS_E =	0.283	0.452	0.437	0.455	0.456	0.437	0.317	0.417	0.430	0.312	0.355

NOTATION. W_d = weight on drivers, lb; W_t = total weight of locomotive, lb; TF = tractive force, lb; HS_E = evaporative heating surface, sq ft; HS_S = superheater heating surface, sq ft; G = grate area, sq ft; D = diameter of drivers, in.

It should be pointed out that the highest drawbar pull is exerted in starting and train acceleration, and not in maintaining train higher speeds except on heavy grades at slow speeds. Figure 5 shows typical characteristic curves for a modern steam locomotive design—in this case a 4-8-4 type with a rated (at 85%) tractive effort of 62,500 lb.

4. LOCOMOTIVE DETAILS

A comprehensive locomotive chart of a 4-8-4 type, showing essential details, is published by the Railway Educational Bureau of Omaha, Nebraska. It is reproduced in reduced size in Fig. 6 by special permission of the Bureau.

MAIN AND SIDE RODS. Nearly all main rod bodies are of I-section design, but side rod bodies are of either rectangular section or I section. Bearings may be of a fixed bronze type held in place with keys or keeper bolts, or of a double or floating bushing which consists of a bronze bearing free to rotate in a fixed steel or iron bushing. The latter type is usually used at least on the main crankpin and sometimes on other than main crankpins. Lubrication of the bearing must be provided either internally through the crankpin or a lubrication fitting attached to the rod. Where bosses are incorporated on the rod eyes for this purpose, they should be located at least 30 degrees from the vertical center line. Holes should have well-rounded edges to prevent the starting of cracks.

The maximum stresses shown in Figs. 7*A* and 7*B* are for carbon steel with a tensile strength of 80,000 psi and a yield point of 40,000 psi. Stresses may be increased for stronger steels, but the increase should not exceed the proportionate increase in ultimate tensile strength. Stresses due to centrifugal force are based on *diameter speed*, in mph at 336 rpm. Diameter speed is the speed in miles per hour which equals the diameter of the driving wheel in inches. Stresses at other than diameter speed are proportionate to the square of the revolutions per minute.

In Figs. 7*A* and 7*B*, S = stress, pounds per square inch; A = net area of section, square inches; P = piston thrust—area of cylinder times boiler pressure; B = load per square inch of area; L = length of rod between pin centers, inches; R = radius of gyration, inches; E = modulus of elasticity; h = depth of section, inches; C = crank radius, feet; and Z = section modulus.

CRANKPINS. The fiber stress in main crankpins due to bending should not exceed 16,000 psi when calculated by the following formula:

$$S = \frac{\text{Piston thrust} \times L}{\text{Section modulus}}$$

where L = distance from outside face of crankpin hub to center of main journal.

The diameter of the main journal should not exceed the length, and its *rubbing speed factor* should not exceed 1,100,000 at diameter speed as calculated by

$$\text{Rubbing speed factor} = \text{bearing pressure} \times 336 \times \pi \times \text{diameter of pin in feet}$$

The thrust for calculating the bearing pressures on the main side and other than main crankpins should be proportioned to the piston thrust in accordance with this table:

Type of Engine	Main Side Journal	Other Than Main Pin
4—coupled	60%	60%
6—coupled	66 2/3%	50%
8—coupled	75%	40%
10—coupled	80%	35%

In addition, it is recommended that the diameter of intermediate crankpins be increased 10% over front and back pins. Crankpins should be proportioned so as to make use of straight line rods if possible. The bearing pressure should not exceed 1600 lb.

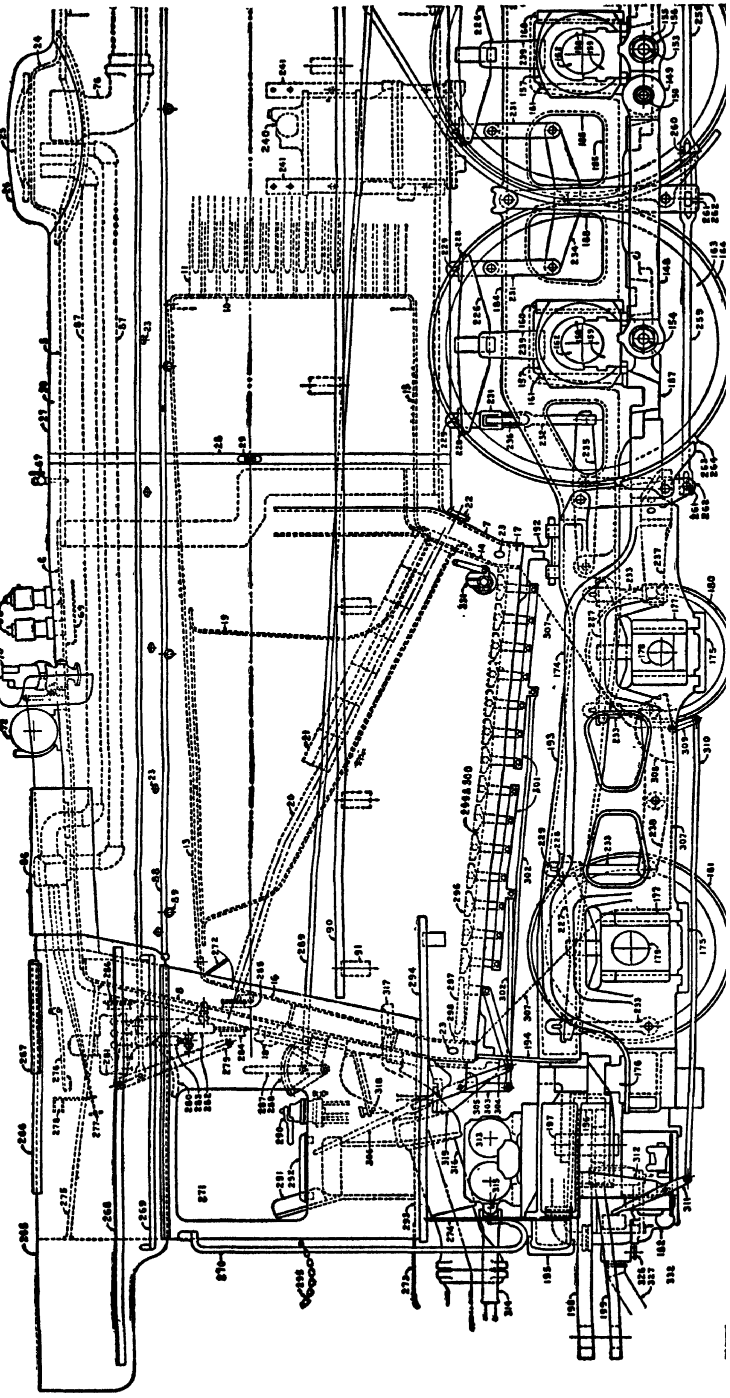
AXLES. The diameter of the journal of a main axle is determined by the formula:

$$D - {}^3/_8 \text{ in. allowance for wear} = \sqrt[3]{\frac{P(B + C)}{0.1964S}}$$

where D = nominal diameter of axle journal, inches; P = area of cylinder $\times$ boiler pressure; A = crank radius; B = one-half the difference between cylinder centers and frame centers; $C = \sqrt{A^2 + B^2}$; and S = stress (23,000 psi maximum). Maximum bearing pressures in pounds per square inch are shown below:

	Passenger	Freight	Switcher
Driving axles	175	200	200
Engine truck axles	225	250	...
Trailing truck axles	160	180	...

(Continued on p. 14-16)



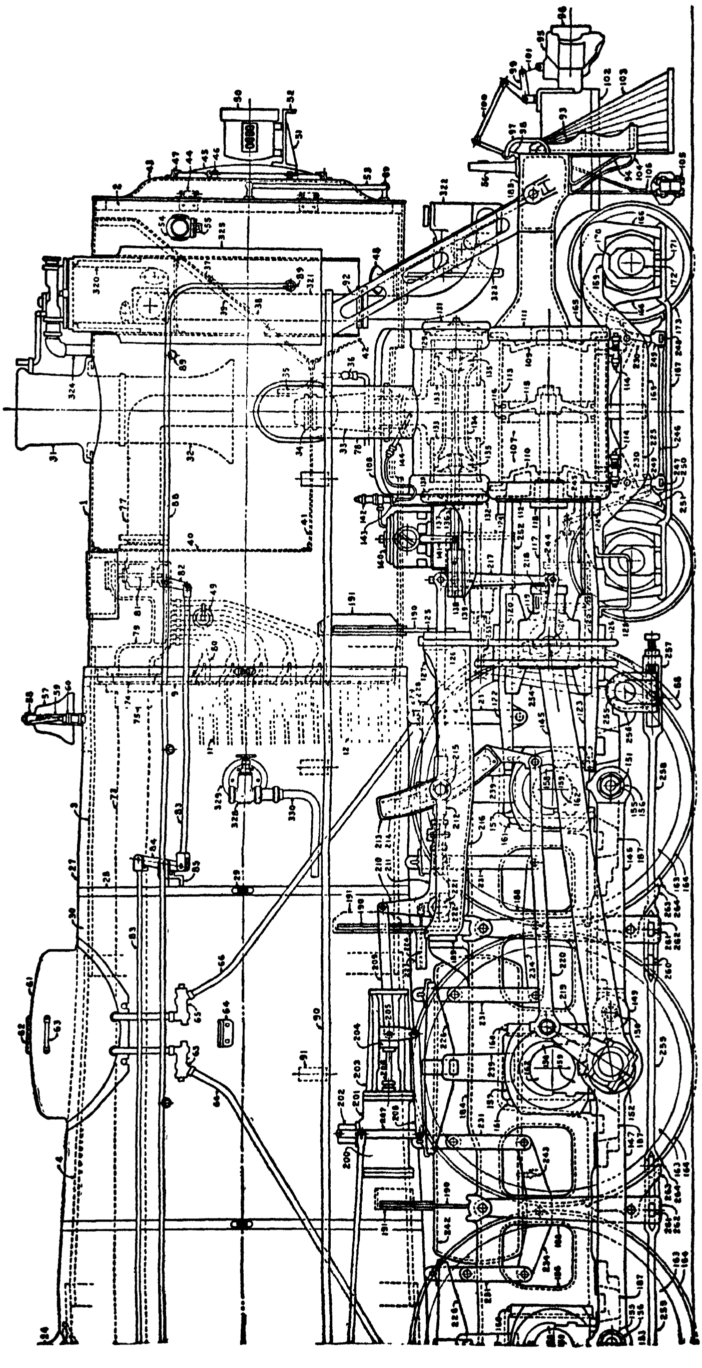


FIG. 6. Locomotive Chart, 4-8-4 Type. (The Railway Educational Bureau, Omaha, Nebraska.) (Copyright, 1948, by D. C. Buell)

(*Fig. 6 continued on next page*)

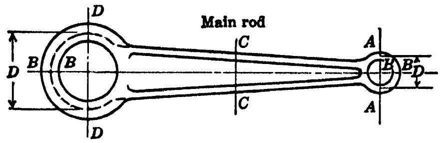
1. Smoke box
2. Smoke box ring
3. Boiler—first course
4. Boiler—second course
5. Boiler—third course
6. Boiler—roof sheet
7. Boiler—throat sheet
8. Boiler—back head
9. Boiler—front tube sheet
10. Boiler—back tube sheet
11. Boiler flues
12. Boiler tubes
13. Fire box crown sheet
14. Fire box throat sheet
15. Combustion chamber
16. Fire box door sheet
17. Fire box ring (or mud ring)
18. Fire door
19. Thermic syphon
20. Arch tubes
21. Arch brick
22. Arch tube washout cap (or plug)
23. Boiler washout plugs
24. Dome
25. Dome cap
26. Dome Casing
27. Boiler jacket
28. Boiler jacket bands
29. Boiler jacket band clamp
30. Boiler lagging
31. Smoke stack
32. Smoke stack extension
33. Exhaust pipe
34. Exhaust nozzle
35. Blower pipe
36. Blower pipe connection
37. Netting
38. Netting door
39. Netting angle
40. Diaphragm plates
41. Diaphragm table plate
42. Diaphragm apron
43. Smoke box front
44. Smoke box front hinge
45. Smoke box door
46. Smoke box door hinge
47. Smoke box door clamp
48. Smoke box brace
49. Inspection hole and cap
50. Head light
51. Head light bracket
52. Number plate
53. Hand rail on smoke box front
54. Classification lamp
55. Classification lamp socket
56. Flag fixture on bumper
57. Bell
58. Bell yoke
59. Bell stand
60. Bell ringer
61. Sand box
62. Sand box cover
63. Sand box hand hold
64. Sand box step
65. Sander
66. Sand pipe
67. Steam gage cock (for tests)
68. Safety valve
69. Safety valve baffle plate
70. Whistle
71. Whistle shield
72. Head light generator
73. Dry pipe
74. Dry pipe ring
75. Dry pipe sleeve
76. Dry pipe elbow
77. Steam pipes
78. Steam pipe casing
79. Superheater header
80. Superheater units
81. Multiple-valve throttle
82. Throttle valve crank
83. Throttle reach rod
84. Compensating lever
85. Compensating lever fulcrum
86. Turret
87. Turret dry pipe
88. Handrail
89. Handrail post
90. Running board
91. Running board bracket
92. Running board step
93. Bumper step
94. Bumper step bracket
95. Coupler
96. Coupler knuckle
97. Uncoupling lever
98. Uncoupling lever bracket
99. Uncoupling bell crank
100. Uncoupling connecting rod
101. Uncoupling chain
102. Coupler pocket
103. Pilot
104. Pilot brace
105. Train control receiver
106. Train control receiver bracket
•107. Cylinder
108. Cylinder saddle
109. Cylinder head—front
110. Cylinder head—back
111. Cylinder head casing—front
112. Cylinder head casing—back
113. Cylinder bushing
114. Cylinder cocks
115. Piston
116. Piston rings
117. Piston rod
118. Piston rod packing gland
119. Crosshead
120. Crosshead shoe
121. Crosshead pin—with nut and washer
122. Guide—top
123. Guide—bottom
124. Guide block
125. Guide yoke cross tie
126. Guide yoke
127. Guide yoke knee
128. Guide step
129. Steam chest head—front
130. Steam chest head—back
131. Steam chest head casing—front
132. Steam chest head casing—back
133. Piston valve bushing
134. Piston valve body
135. Piston valve spider
136. Valve stem
137. Valve stem packing gland
138. Valve stem crosshead
139. Valve stem crosshead guide
140. Force feed lubricator
141. Force feed lubricator lever arm
142. Terminal check
143. Terminal check support
144. Oil plug
145. Main rod
146. Side rod—front section
147. Side rod—intermediate section
148. Side rod—back section
149. Side rod—knuckle joint clevis
150. Knuckle pin—with nut and washer Main and side rods are fitted with brass bearings or bushings.
151. Crank pin—front
152. Crank pin—main
153. Crank pin—intermediate
154. Crank pin—back
155. Crank pin nut
156. Crank pin washer
157. Driving box (front, main, intermediate, and back)
158. Driving box brass
159. Driving box cellar
160. Driving box shoe
161. Driving box wedge
162. Driving axle or journal
163. Driving wheel center
164. Driving wheel tire
165. Engine truck frame
166. Engine truck pedestal
167. Engine truck pedestal tie bar
168. Engine truck equalizer
169. Engine truck box

170. Engine truck box bearing
171. Engine truck box cellar
172. Engine truck axle and journal
173. Engine truck wheel
174. Trailing truck frame
175. Trailing truck pedestal binder
176. Trailing truck rocker bearing
177. Trailing truck box
178. Trailing truck axle and journal—front
179. Trailing truck axle and journal—back
180. Trailing truck wheel—front
181. Trailing truck wheel—back
182. Locomotive booster
183. Front bumper
184. Frame—top rail
185. Frame—bottom rail
186. Frame pedestal
187. Frame pedestal binder
188. Frame filling
189. Frame crosstie
190. Waist sheet
191. Waist sheet angle
192. Expansion pad
193. Cradle
194. Expansion sheet
195. Radial buffer
196. Draw bar pocket
197. Draw bar pin
198. Draw bar
199. Draw bar safety bar
200. Power reverse gear
201. Cylinder
202. Cylinder valve
203. Cylinder head and guides
204. Crosshead
205. Crosshead arm
206. Piston rod
207. Floating lever
208. Floating lever rod
209. Reach rod
210. Reverse shaft arm—vertical
211. Reverse shaft arm—horizontal
212. Radius rod
213. Link
214. Link cheek
215. Link trunnion
216. Link support
217. Combination lever
218. Union link
219. Eccentric crank
220. Eccentric rod
221. Reverse shaft
222. Reverse shaft bearing
223. Counterbalance spring case
224. Counterbalance spring rod
225. Engine truck spring
226. Driving spring
227. Trailing truck spring
228. Spring clip
229. Spring hanger gib
230. Engine truck spring hanger
231. Driving spring hanger
232. Equalizer hanger
233. Trailing truck spring hanger
234. Spring equalizer
235. Rear equalizer
236. Transverse equalizer
237. Trailing truck equalizer
238. Trailing truck spring equalizer
239. Driving box saddle
240. Air compressor
241. Air compressor bracket
242. Air reservoir
243. Air reservoir drain cock
244. Engine truck brake cylinder
245. Cylinder lever (Engine truck)
246. Push rod (Engine truck)
247. Dead lever (Engine truck)
248. Live lever (Engine truck)
249. Brake hanger (Engine truck)
250. Brake head (Engine truck)
251. Brake shoe (Engine truck)
252. Driver brake cylinder
253. Push rod
254. Cylinder lever
255. Cylinder lever shaft
256. Cylinder lever shaft fulcrum
257. Slack adjuster
258. Slack adjuster pull rod
259. Pull rod
260. Brake equalizer
261. Brake beam
262. Brake hanger lever
263. Brake head
264. Brake shoe
265. Cab roof
266. Cab ventilator
267. Cab ventilator door
268. Cab gutter
269. Cab roof hand hold
270. Cab hand hold
271. Cab window
272. Clear vision window
273. Cab apron
274. Cab bracket
275. Cab back sheet brace
276. Whistle lever
277. Turret valve handles
278. Turret valve handle bracket
279. Throttle lever
280. Throttle lever quadrant
281. Lubricator—hydrostatic
282. Water column
283. Water glass
284. Steam gage
285. Air gage
286. Miscellaneous gages
287. Reverse lever
288. Reverse lever quadrant
289. Reverse lever reach rod
290. Engineer's brake valve
291. Cab seat
292. Cab arm rest
293. Cab deck
294. Cab running board
295 Gangway safety chain
296. Grate bars
297. Drop grate
298. Grate end frame
†299. Grate side frame
†300. Grate center frame
301. Grate connection
302. Grate connecting rod
303. Grate shaking lever
304. Drop grate lever
305. Grate lever fulcrum
306. Grate shaker bar
307. Ash pan
308. Ash pan door
309. Ash pan door crank
310. Ash pan door rod
311. Ash pan door lever
312. Ash pan door lever bracket
313. Stoker engine
314. Intermediate drive shaft
315. Universal joint
316. Intermediate conveyor
317. Distributor plate
318. Discharge box
319. Discharge box support
320. Feed water heater
321. Hot water suction pipe
322. Hot water pump
323. Hot water pump bracket
324. Cold water delivery pipe
325. Cover plate
326. Cold water pump
327. Cold water suction
328. Boiler check valve
329. Boiler check flange
330. Injector delivery pipe
331. Blowoff cock
†332. Non-lifting injector

*The locomotive bed is a unit steel casting including cylinders, frames, frame crossties, frame braces, bumper, foot plate cradle, air reservoirs, and other parts.

†Indicates name and location only—the part itself is not shown.

FIG. 6 (*continued*)



Section	Stress	Formulas	Allowable Stress, psi *
AA	Direct stress	$S = \frac{P}{A}$	7,000
AA	Bending stress	$S = \frac{P \times D}{20Z}$	20,000
BB	Bending stress	$S = \frac{P \times D}{12Z}$	20,000
CC	Direct stress	$S = \frac{P}{A}$	10,000
CC	Transverse bending	$S = \dfrac{B}{1 - \left(\frac{B}{40E} \times \frac{L^2}{R^2}\right)}$	11,000
CC	Bending, vertical column	$S = \dfrac{B \div 2}{1 - \left(\frac{B}{20E} \times \frac{L^2}{R^2}\right)}$	5,500
CC	Bending, centrifugal force	$S = 0.68 \left(\frac{L}{R}\right)^2 \times \frac{h}{2} \times C$	10,000
CC	Combined vertical bending	S = sum of above two formulas	15,000
DD	Direct stress	$S = \frac{P}{A}$	6,000
DD	Bending stress	$S = \frac{P \times D}{20Z}$	20,000

FIG. 7*A*. Allowable stress in main rods.

* Based on 80,000 psi ultimate; 40,000 psi yield point.

AXLE AND CRANKPIN MOUNTING PRESSURES. For mounting axles and crankpins into cast-iron centers, the desired pressure should be based on 10 tons per inch of diameter. For cast-steel centers, 15 tons per inch of diameter should be used, with an allowable variation of 10% over and under for both types. In mounting, a mixture of 12 $^1/_2$ lb of white lead to 1 gal of boiled linseed oil should be used as a lubricant.

CROSSHEADS AND GUIDES. The alligator type is recommended with removable upper and lower shoes. Since the center line of the cylinders is usually above the center line of the driving axles, the upward thrust, T, of the crosshead on the guide is determined by the equation

$$T = \frac{P\left(\frac{S}{2} + C\right)}{L}$$

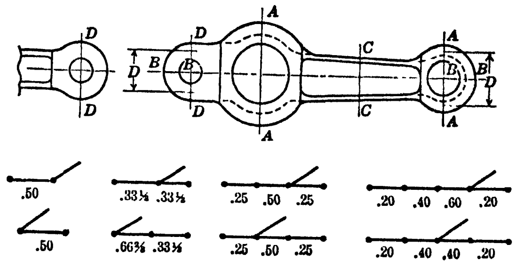
where P = piston thrust; S = stroke, inches; C = vertical distance, center line driving axles to center line cylinders, inches; and L = length of main rod in inches.

The size of the guide may then be determined by considering the guide as a simple beam with a concentrated load T. The fiber stress should not exceed 8000 psi. The bearing pressure of the crosshead shoe on the guide should not exceed 100 psi. The bearing pressure of the main rod on the crosshead pin should not exceed 4800 psi.

VALVE GEARS. The Walschaert gear, Fig. 8, invented by Egide Walschaert in 1844, is the gear most used in this country. It has constant lead and takes its motion from both the crosshead and the crankpin.

The Baker gear, shown in Fig. 9, has a constant lead and a variable preadmission.

FRAMES. Bar frames of cast or rolled steel, complete with cast-steel crossties, cylinders, foot plate, bumper bracket, and miscellaneous brackets, have been superseded in many cases by a one-piece steel casting similar to that manufactured by the General



Section	Stress	Formulas	Allowable Stress, psi
AA	Direct stress	$S = \frac{F}{A}$	4,000
AA	Bending stress	$S = \frac{F \times D}{20Z}$	16,000
BB	Bending stress	$S = \frac{F \times D}{12Z}$	16,000
CC	Direct stress	$S = \frac{F}{A}$	6,000
CC	Transverse bending	$S = \frac{B}{1 - \left(\frac{B}{40E} \times \frac{L^2}{R^2}\right)}$	7,000
CC	Bending, centrifugal force	$S = 0.68 \left(\frac{L}{R}\right)^2 \times h \times C$	10,000
CC	Combined direct stress and bending, centrifugal force		14,000
DD	Direct stress	$S = \frac{F}{A}$	4,000
	Knuckle pin bearing pressure		4,000

The load, F, to be used is a fraction of the piston thrust. The figures above represent main and side rod combinations for all types of locomotive. For example, the intermediate rod of an eight-coupled engine must be designed for a load of 0.50 × piston thrust. The thrust to be used in calculating the stress in the eyes is the same as the thrust carried by the rod.

FIG. 7B. Allowable stress in side rods.

* Based on 80,000 psi ultimate; 40,000 psi yield point.

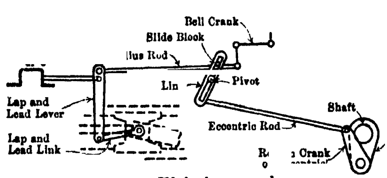


FIG. 8. Walschaert valve gear.

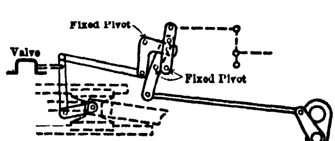


FIG. 9. Baker valve gear.

Steel Castings Corporation. An approximate equation for the minimum sections of bar frames in either wrought iron or cast steel is $S = P/C$, where S = sectional area of frame, square inches; P = piston thrust, pounds; and C = constant. (See Table 7.)

Table 7. Values of C in $S = P/C$

Section	Sections from Cylinders to Main Pedestals, Including Top Rail over Main Pedestal	Back of Main Pedestal
Top of pedestals	2700	3200
Top rail between pedestals	3200	3800
Lower rail between pedestals	4500	5300
Integral single rail at back of cylinder keying lug	1800	....

CONNECTION BETWEEN ENGINE AND TENDER. Drawbars between engine and tender should be straight throughout their full length. The size of the body is based on one square inch of area per 3000 lb of tractive effort, including tractive effort of booster if used, or per 18,000 lb of total weight of engine, whichever is greater.

A drawbar and safety chain are satisfactory for locomotives with a tractive effort under 24,600 lb, or a total engine weight of 147,000 lb. If these values are exceeded, unit drawbars with a safety bar located directly under the drawbar should be used. The area of the drawbar pin should be not less than 75% of the area of the drawbar body, assuming a vertical clearance of not over one inch between the drawbar eye and pocket.

BOILERS. In determination of boiler shell thicknesses, seam welts, riveting and bracing, American railroads follow the ASME Boiler Code, Section III.

Completely welded locomotive boilers that have given very satisfactory service have been built. Their advantage is in freedom from leaks, smooth contour which facilitates lagging and washouts, and saving in weight up to 6000 lb on modern road locomotives. Locomotive boilers are nearly always of the fire tube type; larger boilers are equipped with combustion chambers that increase the firebox volume.

All coal-fired locomotive boiler fireboxes are equipped with a firebrick arch placed on a line from the front of the firebox at the throat up to the back of the firebox near the crown. Approximately two-thirds of the distance, beginning at the throat, is covered with brick over the entire width of the firebox, causing all gases to pass around it. To facilitate circulation of water in the boiler, arch tubes may be fitted into the throat at the front of the firebox and into the backhead at the back of the firebox. The firebrick is usually supported on these tubes. If the boiler is equipped with the Nicholson Thermic Syphon or Security Circulators, the firebrick is supported by them.

SPRING RIGGING. The purpose of the spring rigging system is to provide a cushion to shocks and to distribute the weight of the locomotive over the wheels in predetermined proportions. Since the portion of the locomotive under the springs, commonly called dead weight, varies for different drivers, it is possible with a system of levers to distribute the load over the springs to the various wheels so that the driver rail loads are equal and the loads on the trucks are proportioned as desired. Since the length of the lever arms or equalizers depends on the weight and longitudinal center of gravity of the engine, it is customary to leave some of the equalizers undrilled until the engine has been weighed. The distribution of engine weight is then made in accordance with the actual drilling of the equalizers. Equalizing systems vary in accordance with the wheel arrangement, but in all cases an attempt is made to carry the load on three points for greatest stability.

NICHOLSON THERMIC SYPHONS are generally triangular-shaped devices used in fireboxes of steam locomotives and locomotive-type stationary boilers. One to five are used in each firebox, the number depending on the size and shape of the firebox and the combustion chamber. The syphon is made of firebox-quality steel and is arranged with the lower end in tubular form attached to a diaphragm in the throat or the floor of the combustion chamber and the upper part attached by means of a flanged connection to the crown sheet where it is supported by crown bolts through the flange. Syphons increase firebox heating surface and induce a circulation of water in the boiler. (See Fig. 10.)

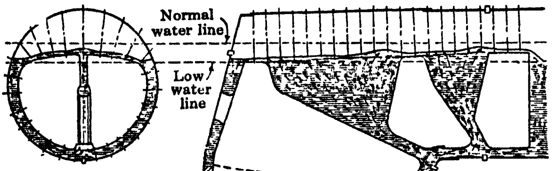


Fig. 10. Crown sheet protection by water flowing out of firebox and combustion chamber syphons. (Courtesy of Locomotive Firebox Co.)

THE SECURITY CIRCULATOR is a steam-generating and water-circulating element arranged in series in the locomotive firebox, connecting side water legs and the crown sheet, for use in any type of steam locomotive boiler. Located in the path of the hot gases, Security Circulators add to the effective heating area, increasing evaporation rates. Through thermic induction, there is a continuous flow of water from the side water legs through the Circulators, over the center of the crown sheet. Security Circulators, combined with Security arch brick arrangement, provide a series of baffles formed by the Circulator cross arms lying between the arched surfaces of the firebrick, which make an ideal cinder baffle and slag trap, reducing cinder cutting and slagging of the flue sheet, and other component parts of the locomotive boiler.

STOKERS are recommended for all coal-burning locomotives where firing rates exceed 4000–5000 lb of coal per hour. The advantages of mechanical firing are constant coal feed in proportion to air supply, and uniform firebox temperatures not attainable with intermittent hand firing. (See also Section 7.) Present-day stokers are of two general types. The first type is mounted on the backhead of the locomotive and fires through the conventional firedoor opening enlarged to accommodate the stoker. With this type a series of screws convey coal from the tender to a "table," where it is picked up by steam jets and distributed over the fuel bed. The second type of stoker consists of a vertical expanding conduit passing through the grates and mounted inside the firebox near the firedoor. Coal is fed by a series of conveying screws to this riser from which the coal spills over on to a plate or table, where it is picked up by steam jets and distributed to the various parts of the fuel bed. In both types of stoker run-of-mine coal may be handled because the tender portion of the stoker has provisions for crushing the coal as it is fed forward by the conveying screw. In all locomotive stokers the firing is done on the spreader principle in that the fuel is constantly sprinkled over the fire, the fuel bed being thin. Consequently, the firing rate may quickly be adjusted to load demand.

SUPERHEATERS are installed in locomotives to add heat to the steam after it has left the boiler; this added heat is called *superheat*. (See also Section 4.) The purpose is to save fuel by reducing steam consumption and to increase the power of the locomotive. The extra heat enables the steam to withstand the chilling effects of the steam chest and cylinder walls without serious losses from condensation, as in saturated-steam locomotives. The locomotive superheater comprises a steam heater having two chambers, one for distributing saturated steam to superheating pipes and another for receiving superheated steam from the elements and conducting it to the engine cylinders. A recent development of the superheater header has a multiple-valve throttle integral with the casting, which controls the flow of steam from the superheated steam compartment to the steam pipes and cylinders. For the effect of superheat see Figs. 11 and 12.

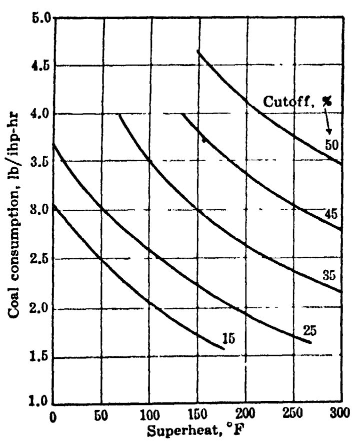


FIG. 11. Coal consumption as a function of superheat for various percentages of cutoff.

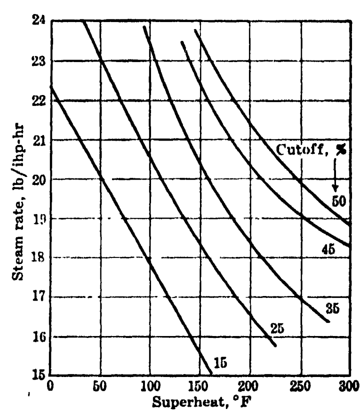


FIG. 12. Steam rate as a function of superheat for various percentages of cutoff.

ROLLER BEARING CRANKPINS. The first complete application of roller bearings to all rods and crankpins of a steam locomotive was made in 1934, when roller bearings were applied to the driving rods of a Pacific type locomotive on the Pennsylvania Railroad. Mechanical friction is reduced, making more power available for hauling the train. Crankpin fatigue resistance has been increased by improved design and use of high-strength materials.

There are no oil holes, knuckle pins, or other stress raisers in the rods themselves. Roller bearings on the crankpins eliminate the consideration of bearing pressure associated with friction bearings, and the width of the rod ends is consequently reduced, materially decreasing the bending stresses in the rods from eccentrically applied loads. The bearings, which are completely self-contained, are press fitted on the crankpins. The rods are fitted with $^3/_{16}$-in. thick, hard-rolled bronze bushings, and are loosely mounted on the crankpin bearings. A diameter clearance of approximately 0.018 in. is provided on the bore at one end, and the bore at the other end of the rod is slightly elongated. This allows the rods to follow the motion of the axles and crankpins without binding and compensates for inequalities in tram, quarter, and stroke between the respective crankpins.

The crankpin bearings are completely self-contained, and driving rods and wheels may be removed from the locomotive for maintenance purposes without disturbing the bearing assemblies. The bearings are lubricated with oil through fittings placed in the end of the pin and easily accessible for lubrication at engine terminals. The crankpin bores hold sufficient oil for 500-mile runs. The flow of oil from the bore of the crankpin to the bearings is controlled through felt rings in the oil-feed device. The rod bushings are lubricated with oil from the bearing chamber through oil holes in the bearing outer race.

POPPET VALVE GEAR. The Franklin system of steam distribution for locomotives uses double-seated poppet valves disposed horizontally in steam chests located at each end of each cylinder and actuated by cam motion. Two systems of actuation are used, one employing oscillating cams, known as type A, and the other employing revolving cams, known as type B.

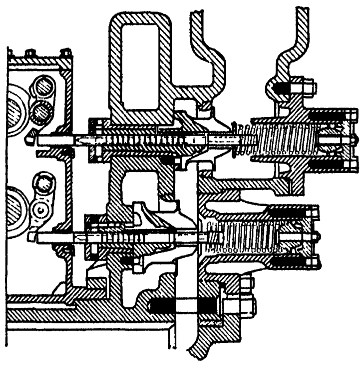


FIG. 13. Poppet valves and related parts in cylinder. (Courtesy of Franklin Railway Supply Co.)

Type A transfers the motion of the reciprocating locomotive crossheads through a *valve gear box* to a *cam box*, and thence to the valves. The type B uses the locomotive driving wheel as the source of motion. Both types produce independent intake and exhaust valve events and quick opening of valves with large passage area. Both types are equipped with drifting devices, whereby either intake or exhaust valves are held fully open during drifting periods.

The number of intake and exhaust valves used on each cylinder varies with the application—two or four intake valves and four or six exhaust valves can be provided. (See Fig. 13.)

LOCOMOTIVE FEEDWATER HEATERS. Exhaust steam is supplied to feedwater heaters at a pressure of 15 psi or higher, depending on the locomotive design and power output. Feedwater temperatures considerably higher than 212 F are usually obtained. Condensate from the exhaust heating steam is recovered, and a saving in both water and heat is effected. Table 8 gives data on the performance of a typical heater and a method of determining the heat and water saving or alternately the increased steaming capacity to be realized under the given conditions. It should be noted that, in the case of the locomotive, the heater and the feed pump are in competition with the injector they replace. Consequently, the heat in the steam required to drive the feed pump must in effect be charged against the heat recovered by the heater to determine the net heat recovered.

EXHAUST STEAM INJECTOR (see also The Injector, Art. 10, Section 7). The exhaust steam injector as used on a steam locomotive operates on exactly the same principle as a nonlifting live steam injector, except that it utilizes exhaust steam from main cylinders, supplemented by a small amount of live steam or supplementary steam which increases the pumping power of the injector. The live steam increases stability of operation and increases the water-regulation range. When the feedwater mixes with exhaust and live steam, the steam is condensed, giving a high velocity to the flow of steam to the point at which condensation takes place. The steam entrains the water, imparting a high velocity to its flow. By the use of properly designed nozzles, this velocity is converted into pressure for forcing water into the boiler; thus the injector is both a feedwater heating and a pumping device. The injector is usually located below the lowest water level of the tender tank so that water is fed to it by gravity. At times it is convenient to locate the injector above the lowest water level; a small pump is then used to feed the water to the injector. Live steam, at reduced pressure, is used to start the exhaust steam injector. If the locomotive is in operation and exhaust steam is available, the injector automatically switches to exhaust steam operation. When the locomotive throttle is closed, the injector automatically switches to live steam. When on live steam some of the steam is reduced in pressure to substitute for exhaust steam. When operating with exhaust steam, the injector functions as an open feedwater heater in that it returns to the boiler the heat contained in that portion of the exhaust used in the injector.

LOCOMOTIVE TENDERS. Small locomotive tenders are constructed by mounting the tank on a cast-steel or built-up frame. Modern high-capacity tenders utilize the cast-steel tender frame for increasing the water capacity by approximately 2500 gal. Tenders are carried on four- or six-wheel trucks, but recently a system of ten wheels on a rigid wheel base and a four-wheel truck has been used. The rigid wheel base system is connected by

Table 8. Heat and Water Saving or Increased Steaming Capacity with Feedwater Heaters

(Courtesy of Worthington Pump and Machinery Corp.)

Assumed temperature of water from tender, °F, average	55
Assumed pressure in boiler, psig	250
Assumed temperature superheated steam, °F	700
a. HEAT RECOVERED	
Assumed exhaust pressure from locomotive, 10 psig, condensing at 240 F and heating the water to	230 F
Heat added to feedwater, Btu per lb	175
Heat required to generate 1 lb of steam from water at 55°, Btu per lb	1339
Heat added to feedwater in heater compared to total heat required (reduction in heat required), %	13.07
Total heat required by locomotive with feed pump and 250 psig pressure in boiler compared to heat required for injector operation, %	102
Reduction in heat required by locomotive with heater (product last two figures), %	13.33
Heat required, %	88.67
Reduction in heat required compared to injector, %	11.33
b. SAVING OF WATER	
Heat required to heat 1 lb of water from 55 to 230 F, Btu	175
Heat given up by 1 lb of steam condensing from 10 psig to water at 230 F, Btu	962
Ratio water heated to steam condensed	5.50
Water saved ($1/6.5$), %	15.4
Water required (feed pump steam 2.25% of water pumped) 102.25 (100 − 15.4), %	86.6
Saving in water, %	13.4
c. INCREASED STEAM CAPACITY	
Assumed exhaust pressure, 15 psig, condensing at 250 F and heating the water to	240 F
Heat required to generate 1 lb of steam from water	
(*a*) at 55 F, Btu •	1339
(*b*) at 240 F, Btu	1154
Total heat (2% required to operate feed pump) (1339/1154) (100/102), %	113.75
Increased heat and increased steam capacity, %	13.75

an equalizing system so that the load is carried uniformly over the system. Capacities of tenders vary according to the railroad requirements. The New York Central 4–8–4 S-1 class has a capacity of 46 tons of coal and 18,000 gal of water, with a fully loaded weight of 418,000 lb for the tender and 471,000 lb for the engine. A Union Pacific 4–6–6–4 type has a capacity of 28 tons of coal and 25,000 gal of water, with a fully loaded weight of 436,500 lb for the tender and 627,000 lb for the engine.

TRUCK SWINGS. In order to permit a locomotive to negotiate a curve, provision must be made for the lateral displacement of a part of the locomotive from the center line of the track. When trucks are used, as on the modern passenger locomotive, great care must be given to their design so as to provide the necessary amount of lateral and yet retain enough lateral resistance to guide the locomotive around the curve. An approximate relationship between the truck swing, rigid wheel base, and radius of the curve is given by the equation $S = WT/2R$, where S = one-half total swing of the truck; W = distance from the center pin of the truck to rear of the rigid wheel base; T = distance from the center pin of the truck to front of the rigid wheel base; and R = radius of curve. All dimensions must be in the same units.

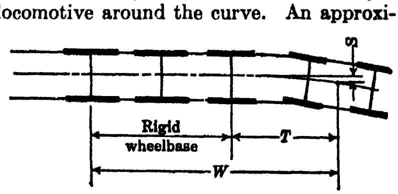


FIG. 14. Truck swing diagram.

In addition, further lateral displacement is possible owing to the lateral on the drivers of the rigid wheel base. This displacement is on a straight line basis and may easily be calculated. Gage widening and flange clearance should also be considered if an accurate study is to be made. (See Fig. 14.)

5. COUNTERBALANCE OF A RECIPROCATING STEAM LOCOMOTIVE *

(Method Recommended by *AAR Manual*, July 1, 1945)

Counterbalancing in a reciprocating steam locomotive includes provision of counterweights for revolving parts and compensation of the inertia forces set up by reciprocating parts. Counterbalancing has received greater attention as speeds have increased with the modern high-powered locomotive. Except in very small wheels, all the revolving weight can be balanced. Additional weight is added to compensate a portion of the reciprocating weight. When this additional weight is in a horizontal plane with the crankpin, it is effective in counterbalancing the reciprocating weights. However, when it is in a vertical plane with the crankpin, it creates an alternating decreasing and increasing force on the rail. The increase in force is commonly known as dynamic augment. A compromise must be made between the riding qualities of the locomotive and the effect of the unbalanced force on the track.

Inertia forces of the reciprocating parts produce two disturbances in the locomotive, one a fore and aft shaking and the other an angular motion about an axis perpendicular to the track, commonly called nosing. The effect of these forces on the locomotive depends on its mass, length of wheel base, weight, and acceleration of the reciprocating parts and the lateral resistances of the trucks and drivers if cushioning devices have been applied. The reciprocating parts include the piston, piston rod, crosshead complete with wrist pin, reciprocating portion of the main rod as determined by the center of percussion method and the union link. The unbalanced reciprocating weight per side should not exceed 4 lb per 1000 lb of locomotive weight, without tender. Also the equivalent weight producing dynamic augment should not exceed 150 lb in each main wheel or 200 lb in any of the other coupled wheels. The following method is recommended for determining the amount and distribution of the allowable reciprocating compensation per side of locomotive, in pounds:

Let W = weight of locomotive in working order in pounds; N = number of pairs of driving wheels; $R_1 = 200(N - 1) + 150$ = maximum allowable reciprocating compensation; $R_2 = (W \times 4)/1000$, maximum allowable reciprocating unbalance; and $WR = R_1 + R_2$ = maximum allowable weight of reciprocating parts per side of locomotive. If the weight of the reciprocating parts is less than the allowable maximum, the compensating weight per wheel should be reduced so that the dynamic augment and the reciprocating unbalance will be reduced in the same proportion.

Let WR_A = actual weight of reciprocating parts. Then the compensating weights should be distributed as follows:

$$\text{In all drivers other than main, } 200\left(\frac{WR_A}{WR}\right)$$

$$\text{In main drivers, } 150\left(\frac{WR_A}{WR}\right)$$

All references to weights in wheels are equivalent weights, defined as the weight of that mass which, revolving at crankpin radius about the center line of the axle, will develop the same centrifugal force as the mass under consideration revolving at its own center of gravity. For example, if a weight of 100 lb has its center of gravity at a distance 24 in. from the center of the axle and the locomotive has a 30-in. stroke, the equivalent weight at crankpin distance is $100 \times (24/15)$ or 160 lb. Counterbalance calculations are greatly simplified by reducing all weights to equivalent weights at crank radius.

Dynamic augment or increase in rail load can be calculated by the equation

$$\text{Dynamic augment in pounds} = \frac{1.603 \times S \times W \times V^2}{D^2}$$

where S = stroke, inches; W = equivalent weight producing dynamic augment; V = speed, miles per hour; D = diameter of driving wheel, inches.

The center of percussion method is recommended to determine the effect of the revolving and reciprocating portion of the main rod. The weight of the revolving portion of the main rod is given by the equation

$$\frac{K^2}{L^2} \times W$$

where K = radius of gyration of rod about the center of the crosshead pin, feet; L = length of main rod center to center, feet; and W = total weight of main rod, pounds. If

* Contributed by C. F. Schwenker.

the main rod is physically available, an accurate determination of the revolving weight may be made. (1) Obtain the scale weight at each end of the main rod from which the distance from the center of the crosshead pin to the center of gravity, m in feet, can be determined. (2) Swing the rod as a pendulum about the center of the crosshead pin and obtain the time, t, in seconds, for one full swing, i.e., over and back. (3) The time should be taken for fifty full swings in order to obtain an accurate value. Then $K^2 = 0.815mt^2$. (4) The reciprocating portion of the main rod is the difference between the total weight and the revolving weight.

If actual weights of the main rod cannot be obtained, a close approximation to the revolving weight may be had by using seven-eighths of the estimated scale weight of the back end.

BALANCING OF MAIN DRIVING WHEELS. The revolving parts on the main drivers to be balanced consist of the crankpin hub, side rods, main rod, and the large end weight of the eccentric crank along with their respective portion of the crankpin. From Fig. 15 it can be seen that these weights may be combined into a weight WR acting at a distance a from the mean plane in which the counterweight in the wheel revolves. When the relation between these planes is considered, it becomes obvious that no single equivalent weight W_1 in the plane of the counterweight can develop a centrifugal force which will balance the centrifugal force WR in its plane. A force developed by another equivalent weight W_2 in another plane will be required to complete this balance. The practical place for such a weight is in the opposite wheel with its plane of rotation at a distance b from the plane of the first balance. Then WR will be balanced if W_1 equals WR plus W_2 and $a \times WR = b \times W_2$. That is, if the equivalent weights on opposite sides of the axis of rotation of the axle are equal, and if the equivalent moments about any point in the system are equal, WR will be perfectly balanced by W_1 and W_2. Therefore, to balance the revolving parts at the right crank, *two* counterweights are required, one opposite the crankpin in the right wheel and one directly across the engine from the right crank in the left wheel. Similarly, to balance the revolving parts at the left pin, one counterweight is required in the left wheel opposite the crankpin, and another in the right wheel directly across the engine from the left crankpin. Consideration of the difference in planes of the revolving parts and the counterweights is known as *dynamic* or *cross counterbalancing* of the revolving parts.

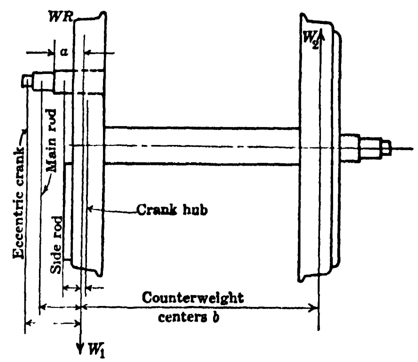


Fig. 15. Wheel balancing diagram.

These provisions take care of balancing of all revolving parts except the small end of the eccentric crank and the revolving portion of the eccentric rod. Since the small end of the eccentric crank is not revolving in line with the crankpin, the resultant weights in the adjacent and opposite wheels must be divided into two equivalent weights, one acting in line with the crankpin and the other at 90 degrees to the crankpin.

In Fig. 16, W_1 = counterweight required to balance effect of revolving parts on main crankpin in adjacent wheel; W_2 = counterweight required to balance effect of revolving parts on main crankpin in opposite wheel; W_3 = counterweight required to compensate portion of reciprocating weight; W_4 = counterweight required to balance the effect of the small end of the eccentric crank and revolving portion of eccentric rod in adjacent wheel; and W_5 = counterweight required to balance the effect of the small end of the eccentric crank and revolving portion of eccentric rod in opposite wheel.

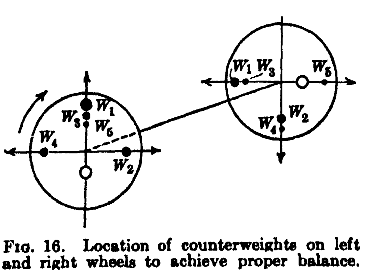


Fig. 16. Location of counterweights on left and right wheels to achieve proper balance.

For right-hand lead and trailing eccentric crank, Fig. 16 shows the counterweights necessary in the right and left wheels to correctly balance all the revolving weights and the desired portion of the reciprocating parts. It is also obvious that to balance a pair of wheels correctly different counterweights must be used in the right and left wheels. It is possible to design a main driving wheel with one counterweight by placing the counterweight at such an angle in the wheel that it satisfies the two weight requirements. Since it is not practicable to make two different castings, a wheel can be designed to satisfy the maximum value opposite the pin and the maximum value at right angles to the pin. The balances are then filled to meet the requirements of the respective wheel. This is known as using an *auxiliary balance*.

Coupled wheels other than the main may be balanced by the straight or static method, provided the equivalent weight producing dynamic augment does not exceed 200 lb when due consideration is given to the out-of-plane effect and the out-of-plane effect does not exceed 50% of the desired reciprocating compensation.

Lawford H. Fry (Locomotive Counterbalancing, *Trans. Am. Soc. Mech. Engrs.*, RR-56-2, June 1934) describes a detailed method of cross balancing, modifying, and extending the American Railway Association method given above. See also F. L. Baxter, The Balancing of Three-cylinder Locomotives, *The Engineer*, July 26, 1935, and R. Eksergian, *Trans. Am. Soc. Mech. Engrs.*, RR-51-5, May–Aug. 1929.

STEAM TURBINE LOCOMOTIVES

By R. P. Johnson

6. GEARED STEAM TURBINE LOCOMOTIVE

HISTORY. Various attempts to adapt the steam turbine, so successful in stationary and marine practice, to railroad motive power have been made both here and abroad. To derive the best results, the steam turbine should operate condensing, but this requires space and power not readily available within the clearance limits of railroads. Where high-horsepower locomotives are desired, as in the United States, it has also been felt necessary heretofore to use electrical transmission. As electric transmission is expensive, the use of a steam turbine directly geared to the driving wheels was studied by the Westinghouse Electric Corporation. In conjunction with Pennsylvania Railroad and The Baldwin Locomotive Works, a design was completed in 1941. The completed locomotive was delivered to the Pennsylvania Railroad in 1944.

The designers considered that a locomotive of this type offered several inherent advantages over the conventional reciprocating-type locomotive. The turbine eliminates pistons and all other reciprocating parts and, therefore, all parts are balanced and uniform torque is delivered to the wheels. Driving wheels can be made materially smaller, leaving more weight and space for the boiler and lowering the center of gravity. Maintenance should be materially reduced as a result of full rotative motion.

New and significant engineering advances made possible the geared locomotive design:

1. The power unit is a completely self-contained assembly.
2. Three-point support avoids transmitting frame distortion to the gear case.
3. A single lever controls both speed and direction of the locomotive.
4. Hardened and ground double helical gearing is used.
5. Tooth loading of the high-speed pinion is practically double that ordinarily used.

DESIGN. The propulsion equipment was designed specifically to operate with the conventional railroad fire-tube boiler and at pressures and temperatures commonly used with such boilers. The locomotive was originally designed as a 4–8–4 type, but the wartime shortage of alloy materials forced the use of a heavier boiler and other parts which required a 6–8–6 wheel arrangement to carry them. The rating of 6900 shaft horsepower was chosen as being the greatest practicable for a rigid-frame, four driving-axle unit.

The locomotive, known as Pennsylvania Railroad Class S-2, has a boiler capable of supplying 95,000 lb of steam per hour at 310 psig boiler pressure, or 285 psig and 750 F total temperature to the turbine nozzles. With this steam flow, the turbine develops 6550 hp at the rail at 70 mph and less at higher and lower speeds, as shown in Figs. 1 and 2, in which horsepower and tractive force at the rail for a conventional reciprocating engine, a diesel-electric, and a turbine locomotive are plotted against speed.

The propulsion unit comprises a forward turbine, a double-reduction gear for each of the two middle driving axles, flexible cup-drive elements between the final drive gears and two middle axles, a reverse turbine and gear unit clutched to the single high-speed pinion, and oil system auxiliaries. A pneumatic steam-admission control has overspeed and low-oil pressure protection. Both turbines are supported from the gear case, which in turn is supported from the main locomotive frame, making the power unit a complete assembly. The gear case is supported from the locomotive frame at three points so that distortions of the locomotive frame are not transmitted to the gear case.

The forward turbine, of the impulse type, consists of a Curtis stage followed by five full-admission Rateau stages. At 100 mph the turbine speed is approximately 9000 rpm. It is connected to the high-speed pinion at the reverse-turbine side of the unit, a quill shaft extending through the pinion. Steam enters the turbine through four 3-in. pipes, each pipe connected to a nozzle group covering approximately 25% of the blade periphery. There are four cam-operated valves for control of steam to the forward turbine, each controlling one of the four inlet pipes. The cams are arranged to open the valves in sequence.

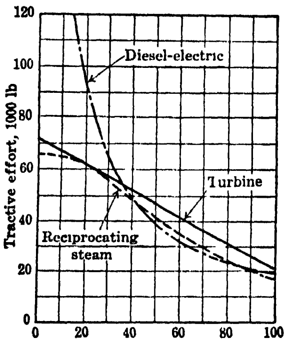


FIG. 1. Tractive force at rail for steam turbine, reciprocating steam, and diesel-electric locomotives.

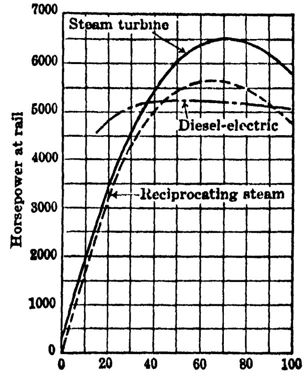


FIG. 2. Horsepower at rail for steam turbine, reciprocating steam, and diesel-electric locomotives.

The reverse turbine is a single Curtis stage, overhung on an extension of the reverse gear pinion shaft. Steam is admitted through a single inlet pipe by a cam-controlled valve. The maximum locomotive speed in reverse is 22-mph, at which speed the turbine develops 1500 hp at approximately 8300 rpm.

The maximum starting tractive force in reverse is 65,000 lb or 25% adhesion. This is obtained by addition of a reverse gear which multiplies the torque of the reverse turbine by four at the high-speed pinion. Power in reverse is transmitted to the main gear high-speed pinion through an hydraulically actuated positive-engagement clutch. The forward turbine is solidly connected to the high-speed pinion, but the reverse turbine is engaged only when reversing. A *zero-speed* interlock in the pneumatic control circuit prevents engagement or disengagement of the clutch when the locomotive is moving.

The main gear is a double-reduction unit of the nested type with two double-helical, high-speed gears, two low-speed spur pinions, two low-speed spur gears, and two cup drive elements all housed in a gear case. The high-speed pinion and the second reduction gearing are hardened and ground.

The cup drives, in addition to permitting up and down motion of the driving axles, are torsionally flexible, thus protecting the gearing and turbines from shock loads. To force an equal distribution of power flow to each of the geared axles, side rods are provided between the No. 2 and No. 3 driving axles.

SERVICE. The locomotive was delivered to the railroad in Oct. 1944. Adjustments for best steaming were made, and the locomotive was then placed in regular service hauling high-speed passenger trains. After 45,000 miles of service, the turbines and gearing were removed and carefully examined. The turbine blading and gear teeth were found to be in perfect condition. The locomotive was then returned to regular passenger service.

THE OVERALL EFFICIENCY of this type of power does not exceed that of an up-to-date reciprocating type of steam locomotive since the turbine is designed for best performance at one speed and a locomotive does not maintain this most efficient speed over a large proportion of its run. For this reason it is best suited to high-speed passenger service with few stops. However, if the turbine and gearing stand up for long periods without attention, the maintenance of this type of locomotive will be much less than a reciprocating locomotive.

Table 1. General Dimensions and Weights of Turbine-geared Locomotive

Type	6–8–6
Gage	4 ft 8 1/2 in.
Working pressure	310 psig
Fuel	Bituminous coal
Driving wheels, diameter	68 in.
Tractive force	70,500 lb forward
	65,000 lb reverse
Wheel base:	
Driving	19 ft 6 in.
Rigid	13 ft 6 in.
Total engine	53 ft 0 in.
Total engine and tender	107 ft 10 1/2 in.
Weight in working order:	
On drivers	260,000 lb
On front truck	143,000 lb
On back truck	177,000 lb
Total engine	580,000 lb
Total tender, two-thirds loaded	347,000 lb
Tender:	
No. of wheels	16
Diameter of wheels	36 in.
Water capacity	18,000 U. S. gal
Coal capacity	37 tons
Heating surface:	
Firebox	340 sq ft
Combustion chamber	190 sq ft
3 1/2-in. flues	3860 sq ft
2 1/4-in. flues	518 sq ft
Circulators	94 sq ft
Total	5002 sq ft
Superheater	2050 sq ft
Grate area	120 sq ft
Firebox volume	871 cu ft

7. STEAM TURBINE–ELECTRIC LOCOMOTIVE

ELECTRIC DRIVE. In planning new, high-speed passenger service between Washington and Cincinnati the Chesapeake and Ohio Railway desired a powerful coal-burning steam locomotive of a novel design. It was planned to use a noncondensing steam turbine, conventional firetube locomotive boiler, and electric transmission. The electric transmission was chosen because the railroad traverses a mountainous territory with long heavy grades. This type of transmission is basically more suited to these conditions than a geared drive. The turbine-electric locomotive is intrinsically heavier for a given output, but in mountainous territory this is not a drawback as the additional weight is available for adhesion. Three of these locomotives, designed and built by the Baldwin Locomotive Works and Westinghouse Electric Corporation for Chesapeake and Ohio Railway, were delivered in 1948.

DESIGN. While the main components of these locomotives are not in themselves novel, their arrangement in this locomotive is distinctly so. A coal bunker, carrying 29 1/2 tons of coal, is at the front of the locomotive, followed by the operator's cab, then the boiler (reversed from its usual position), the steam turbine, and generators. All these are carried on one chassis. This is followed by a water tender holding 25,000 gal, and

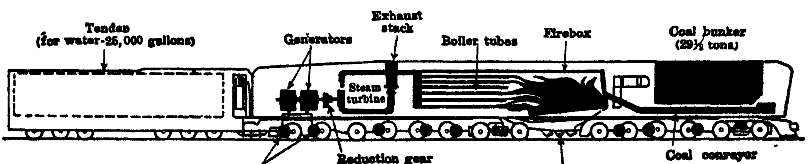


Fig. 3. Diagram of Chesapeake and Ohio turbine-electric locomotive.

designed to harmonize with the train rather than the locomotive, thus seemingly reducing the length of the motive power unit. The locomotives are fully streamlined. Overall length is 153 ft 0 in. (See Fig. 3.)

The running gear of the locomotive consists of a 4–8–0–4–8–4 wheel arrangement, comprising two separate units, each with three-point suspension for the spring-borne loads. The front unit consists of a four-wheel leading truck and a rigid eight-wheeled power truck with three axles motored. The rear unit has a four-wheeled guiding truck, a rigid eight-wheeled power unit, and a four-wheeled Delta trailing truck. Three axles of the power unit are motored as well as both axles of the trailing truck. The front and intermediate trucks have 36-in. diameter wheels. All others are 40 in. in diameter. All have roller bearings.

Trucks. The leading truck of each unit provides 20% initial and constant resistance by a combination of rollers on inclined seats and lateral resistance springs. Since the deflection of the main truck springs is about equal to the incline lift of the rollers, the truck can swing laterally, giving the necessary resistance without lifting the main cab structure. These trucks are not equalized with the driving wheel springs, thus serving as the front point of suspension for each unit. The motorized Delta trailing truck provides 10% initial and constant resistance and is equalized with the spring rigging of the rear main truck.

Each front and rear main truck has one pair of idler wheels, located to improve the frame construction and to permit the application of air ducts to the traction motors. The motors are carried through the frame with flexible connections. No provision is necessary for transferring weight from one unit to the other as coal is used, since the weight on drivers is high enough to develop full tractive power without the weight of the coal.

The cab underframe or body is divided into three sections. The center section, extending from the coal bunker to the end of the boiler, is a nickel steel casting and includes the two main center pins. It is designed to carry the buffing and traction loads when in operation. The boiler is anchored rigidly to the middle section of the cab underframe and has sliding supports under the barrel and at the front and rear of the firebox. Opposite each center pin on the frame are side bearings, lifting lugs, and safety clamps. The front and rear sections of the frame are high tensile steel weldments, attached to the cast-steel middle section with offset frame joints, with clamps used at top and bottom and vertical keys to prevent lateral movement. The end frame structures are of the cantilever type, joined to the middle frame section with I beams and pins to provide a frame that can be lifted vertically from the lifting lugs opposite the center pins. The front frame section is built integral with the coal bunker, and the rear section serves as a base for the turbine and generators.

The frame is supported by the running gears on the two center pins and nine sliding supports, arranged to distribute properly the weight of the cab unit on the various wheels in the running gears. The sliding supports allow universal movement by using a guided plunger resting on multiple-coil springs and having a spherical bearing as a contact head. These supports and center pins are provided with mechanical lubrication and are adjustable for varying capacity by liners under the coiled springs. This system permits the locomotive to take curves and rough track with very little vertical movement.

The power-generating unit consists of one 6000-hp, 6000-rpm steam turbine. At full load the turbine uses steam at the rate of 85,000 lb per hr, 290 psig inlet pressure, 750 F, exhausting to atmosphere through a conventional nozzle and stack as a draft source. The turbine is connected through a 6 : 1 reduction gear to two double-armature d-c generators.

The turbine is of the impulse type with five stages. Steam is supplied through a seven-valve steam chest, cast integral with the turbine cylinder cover. Each valve is connected by a cored passage to a nozzle group which admits steam to a portion of the control stage. The individual valves are opened in sequence by a lift bar regulated by a governor-operated hydraulic piston. The 8-in. throttle valve located on the side of the steam chest closes automatically if the turbine overspeeds. Of the two hundred gallons of lubricating oil carried in the gear case, part operates the governor, and the remainder is supplied through an orifice to journal bearings and gears.

GENERATORS AND MOTORS. The double-armature generators each have two eight-pole field assemblies in a common frame, with the two armatures mounted on a common shaft. This shaft has a single roller bearing at the outboard end. The inboard end is flanged to couple directly to its gear shaft, with its weight carried by the gear bearing. The generators are force ventilated by a separate 25,000 cu ft per min turbine-driven blower. Two sets of fields are provided, one for self-excitation and the other for separate excitation. Load variations are obtained by controlling the separately excited field strength.

Motors. Each of the four generator armatures supplies power to two 620-hp, 568-volt, 720-rpm traction motors connected in parallel. These are directly geared to eight of the locomotive axles with a gear ratio of 24 to 55, wheels of 40-in. diameter being used. This produces a continuous locomotive electrical rating of over 48,000 lb of tractive force, with a maximum tractive force of 98,000 lb for starting.

Each motor is built with a rolled steel frame, has six main and six commutating field poles, and roller armature bearings. Force ventilated, they are the largest traction motors built for locomotive use. The six-pole construction instead of the conventional four-pole design allows the motor to be built at a weight of 7380 lb complete with gear and gear case. The motor blower for the front unit is located in the front of the locomotive with the air compressors. The blowers for the rear unit are toward the rear in front of the steam turbine. The blowers are superheated steam-turbine-driven propeller-type units.

CONTROL. **The electrical control** differs from that used on diesel-electric locomotives in that acceleration is obtained partly by varying the strength of the separately excited fields of the main generators, and partly by controlling the speed of the turbine. To obtain a satisfactory steam rate, the speed of the turbine is not reduced below 60% of the full speed in the idling position of the controller. The control equipment for the main generators and the rear unit motors is in a compartment at the extreme rear of the locomotive; that for the front unit motors is located under the sloping bottom of the coal bunker.

The master controller located in the cab has two handles controlling speed and direction, respectively. The first notch admits steam to raise the turbine to *idling* speed, about 3850 rpm. Successive movement of the controller increases the power to the traction motors step by step to the point at which maximum separate excitation has been applied to the generators, and the turbine speed is increased to 75% of full speed. The self-excited field is also connected to the first notch but has little effect until the generator voltage increases. Further movement of the controller increases the turbine speed to the full amount.

The operator has before him, in addition to the regular gage equipment on locomotives, gages and meters showing traction motor current, turbine speed, lubricating oil temperature and pressure, and locomotive speed. Wheel slipping is indicated by a buzzer, and signal lights indicate tripping of overload relays, faults found by the ground detector, and malfunctioning of blowers.

Any generator armature and its associated motors can be disconnected should trouble occur in any of them, leaving intact three-quarters of the locomotive capacity.

Table 2. General Dimensions and Weights of Turbine-electric Locomotive

Type	4-8-0-4-8-4
Gage	4 ft 8 1/2 in.
Working pressure	310 psig
Fuel	Bituminous coal
Fuel capacity	29 1/2 tons
Driving wheel diameter	40 in.
Tractive force, maximum	98,000 lb
Tractive force, continuous	48,000 lb
Wheel base, rigid	17 ft 6 in.
Wheel base, total engine	90 ft 7 in.
Wheel base, engine and tender	140 ft 3 3/4 in.
Heating surface:	
Firebox	314 sq ft
Combustion chamber	100 sq ft
Flues, 4 in.	3121 sq ft
Tubes, 2 1/4 in.	736 sq ft
Syphons	126 sq ft
Total evaporative	4397 sq ft
Superheater	1770 sq ft
Grate area	112 sq ft
Firebox volume	699 cu ft
Tender:	
Number of wheels	12
Diameter of wheels	36 in.
Water capacity	25,000 gal
Weight in working order:	
On driving wheels	533,950 lb
On front truck	91,350 lb
On intermediate truck	97,400 lb
On idler wheels	134,500 lb
Total locomotive	857,200 lb
Tender (two-thirds full)	305,800 lb

DIESEL LOCOMOTIVES

By T. F. Perkinson

While diesel engines have been in commercial use in locomotives in the United States since 1925, general adoption of this type of prime mover was delayed by lack of suitable low-cost and reliable engines until the middle 1930's. The diesel locomotive has practically superseded the steam locomotive, so far as new orders for railroad switchers are concerned, and is making significant strides in displacing the steam locomotive in both passenger- and freight-train service. Virtually all new industrial locomotives are of the diesel type. Principal factors contributing to the acceptance of the diesel locomotive are (1) high reliability, (2) high availability, (3) low maintenance costs, and (4) general economy of operation relative to the conventional forms of steam motive power. The majority of diesel locomotives in railroad and industrial service in the United States employs electric transmission between the engine and the driving wheels. Mechanical transmissions, with or without hydraulic couplings or torque converters, are employed to a large extent in European switching locomotives, but electric transmission is employed on all European road locomotives above 500 hp (approximately) in rating, except for a few experimental machines.

8. TYPES OF DIESEL LOCOMOTIVES

ELECTRIC TRANSMISSION SYSTEMS. Since the diesel engine is, for practical purposes, inherently a constant-torque, constant-speed, and, as a consequence, constant-horsepower engine at its rating point, with little if any overload capacity, its use in railroad motive power must be attended by a transmission system that will convert the torque of the engine into tractive effort at the vehicle wheels at zero speed and permit a reduction in tractive effort with the increase in vehicle speed without exceeding the horsepower rating of the engine.

The constant horsepower characteristic of the diesel engine at full load and speed expressed in kilowatts can be plotted to coordinates of volts and amperes in the form of a rectangular hyperbola as shown in *a* of Fig. 1, in which a 500-hp engine has been used for illustrative purposes.

The volt-ampere characteristic of a d-c generator with fixed excitation at constant speed is shown in *b* of Fig. 1.

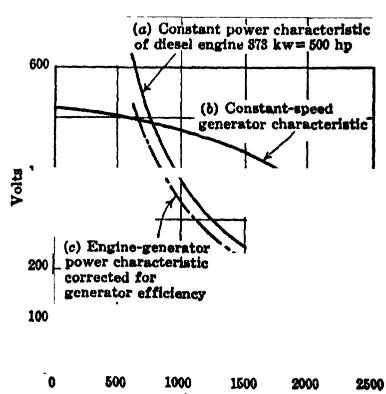


Fig. 1. Voltage current diagram illustrating engine, generator, and engine-generator characteristics.

In the engine-driven generator, load values up to 750 amp can be carried by the engine-generator set without overloading the engine, full load on the engine being secured where the two curves intersect. To avoid overloading the engine, excitation must be reduced as the load amperes increase so that an engine-generator characteristic corresponding to *c* of Fig. 1, corrected for generator efficiency, will be secured. Generator excitation regulation systems, responsive to slight changes in engine speed, are employed in traction service to effect the desired engine-generator characteristic.

The series-wound, d-c motor is ideally suited to work with the engine-generator characteristic described above, and to drive the vehicle wheels. Heavy ampere overloads can be imposed on this type of motor to furnish the high tractive efforts required for accelerations, while with decreasing tractive effort and increasing speed the load demand on the engine-generator set is substantially constant.

Traction motors are used in series, series-parallel, and parallel combinations to minimize the size, weight, and cost of the generator equipment.

The main traction generator is equipped with a series-wound field which is used in conjunction with the generator armature for engine-cranking purposes. The series field is cut out of circuit after the engine has been started. Starting energy is furnished from a storage battery. In addition to the main generator, a separate exciter (on all but small units, which are self-excited) and one or more auxiliary generators, all driven by direct connection, belting, or gearing from the engine, are provided.

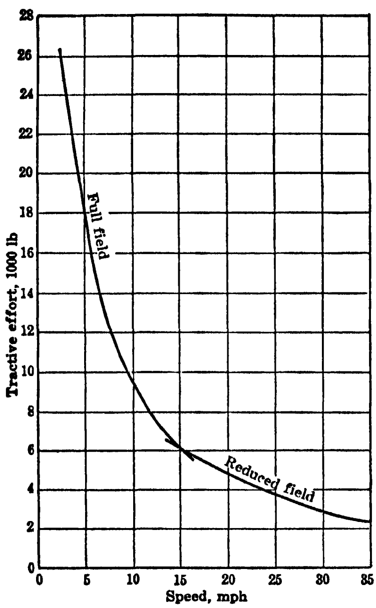


FIG. 2. Speed-tractive effort curve for 44-ton 380-hp diesel-electric railroad locomotive.

Electrodynamic braking is provided on many road locomotives, wherein the traction motors under controlled separate excitation from the traction generators are employed as generators dissipating their energy in special resistors. Added safety during braking operations and reduced wheel and brake-shoe maintenance are secured by the use of this method of braking. (See Fig. 4.)

Because of its many constructional and operating advantages, the electric-transmission system is widely used in all sizes and capacities of diesel locomotives.

SPEED-TRACTIVE EFFORT CHARACTERISTICS. **The speed-tractive effort characteristic** of the diesel-electric locomotive is essentially the same as the engine constant-power characteristic, with modifications interposed by the electric drive system, plotted in terms of tractive effort (pounds) and vehicle speed (miles per hour). Figure 2 shows the speed-tractive effort characteristic of a 380-hp locomotive with but one traction-motor connection (permanently parallel), whereas Fig. 3 illustrates a characteristic for a 2000-hp unit employing connections for traction motors of two motors in series—two groups in parallel, and four motors in parallel, with full-field and reduced-field strengths on the traction motors.

Calculation Method. E. E. Kimball (Ref. 1) describes a method developed in a committee of the American Railway Engineering Association (Ref. 2) for calculating the approximate speed-tractive effort characteristics of normally designed diesel-electric

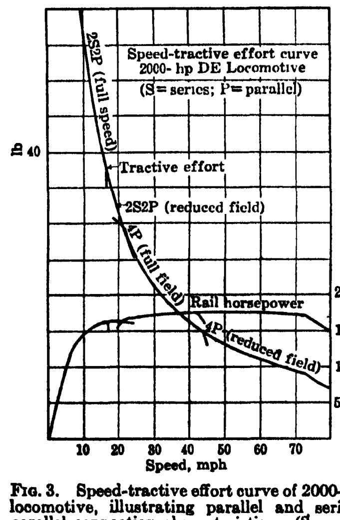


FIG. 3. Speed-tractive effort curve of 2000-hp locomotive, illustrating parallel and series-parallel connection characteristics. (See also Fig. 4.)

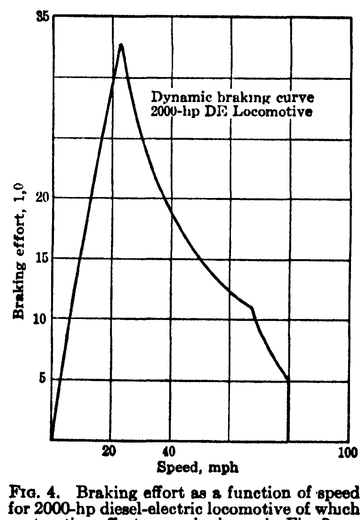


FIG. 4. Braking effort as a function of speed for 2000-hp diesel-electric locomotive of which tractive effort curve is shown in Fig. 3.

locomotives. The method condensed in tabular form is shown in the following, in which the characteristic data for a 2000-hp (nominal) locomotive are calculated. Some departures from actual design characteristics may be expected in using this approximate method because of individual electrical characteristics of the generator and traction-motor equipments.

Item		
1	Weight on drivers (tons)	102
2	Weight of locomotive (tons)	153
3	Rated horsepower of diesel engine	2100
4	Horsepower required for auxiliaries	
5	Nominal horsepower * output of locomotive (us 82%)	1722
6	Horsepower per ton on drivers, item 5/item 1	16.88
7	V_1 = 0.395 × item 6	6.67

Speed		Tractive Effort		Driver Horsepower
V	Mph	Per Ton on Drivers	Total	
(a)	(b)	(c)	(d)	(e)
0	0	600	61,200	0
$1.00V_1$	6.67	600	61,200	1080
$1.25V_1$	8.33	542	55,300	1227
$1.50V_1$	10.00	491	50,100	1336
$1.75V_1$	11.66	448	45,700	1421
$2.00V_1$	13.34	410	41,800	1490
$2.50V_1$	16.67	348	35,500	1577
$3.00V_1$	20.00	300	30,600	1631
$3.50V_1$	23.33	263	26,800	1667
$4.00V_1$	26.68	233	23,800	1695
$4.50V_1$	30.00	208	21,200	1698
$5.00V_1$	33.33	188	19,175	1700
$6.00V_1$	40.00	157	16,000	1705
$7.00V_1$	46.67	135	13,760	1710
$8.00V_1$	53.36	118	12,040	1715
$10.00V_1$	66.7	(x) 95	9,690	1720
$12.00V_1$	80.0	79	8,060	1720
$14.00V_1$		(y) 68		
$16.00V_1$		59		
$20.00V_1$		(z) 47		

* Nominal horsepower output of locomotive depends on allowances for auxiliaries and capacity of locomotive.

Assume: 80% (78–82) of item 3 for road locomotives and 75% (73–77) of item 3 for switch locomotive and rail cars.

(x)(y)(z). Indicate upper speed range where electrical characteristics may govern output of oil engine.

(x) Freight and switching locomotives, the limit is $10V_1$ to $12V_1$.

(y) Passenger locomotives, the limit is $14V_1$ to $16V_1$.

(z) High-speed railcars, the limit is $20V_1$ and above.

Column b = column a × item 7. Column d = column c × item 1. Column e = column b × column d/375.

(See Fig. 3 for curve of results.)

DYNAMIC-BRAKING CHARACTERISTICS. Figure 4 shows the dynamic-braking characteristic of the locomotive, the speed tractive-effort characteristic of which is shown in Fig. 3. Because traction-motor electrical losses are effective in providing braking effort, the braking effort procurable at a given speed on the full braking-effort portion of the curve is somewhat greater than the tractive effort procurable from the same motors at the same speed and amperes.

HORSEPOWER RATINGS. Diesel-electric locomotive horsepower ratings are generally given in terms of the horsepower input from the engine to the main generators for traction purposes. This is, however, not a rigid rule since many of the smaller locomotives are specified in terms of the brake horsepower output from the engine, which includes power used for auxiliary purposes (5 to 10% of the total) as well as that for traction. (See Ref. 3.)

WHEEL ARRANGEMENTS. The use of electric transmission and the absence of mechanical connections between the diesel engine and driving wheels permit employment of simple wheel and truck arrangements. The majority of diesel-electric locomotives in service in the United States (with the exception of the smaller-sized two-axle and a few rigid-frame four-axle units) employ the swivel truck with two or three axles in a rigid truck frame. Another exception to this generality is the 3000-hp unit of Fig. 10, which embodies a $2 - D + D - 2$ wheel arrangement (see p. 14-37, for nomenclature on electric locomotive and wheel arrangement classification). Various combinations and classifications are shown in the accompanying figures and Tables 1 and 2.

RAILROAD DIESEL-ELECTRIC LOCOMOTIVES. There has been a large degree of standardization attained among American locomotive builders in the matter of railroad switching and road-locomotive horsepower ratings and weights.

Several manufacturers make a 44-ton light switcher (Fig. 5), the weight being determined by the fact that labor agreements permit the use of one-man operation of diesel locomotives under 45 tons in weight.

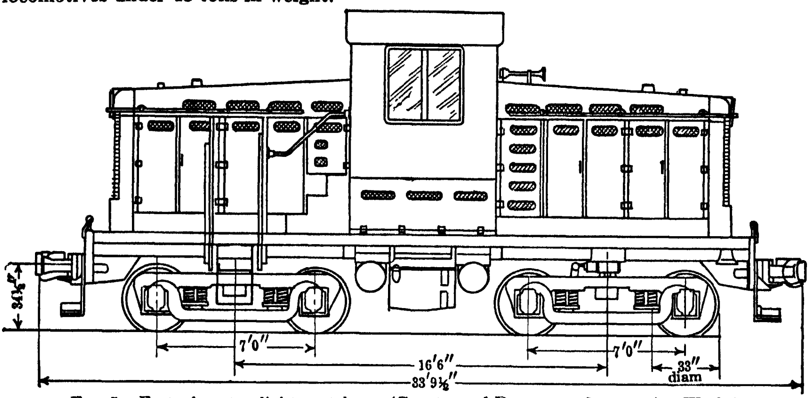


FIG. 5. Forty-four-ton light switcher. (Courtesy of Davenport Locomotive Works)

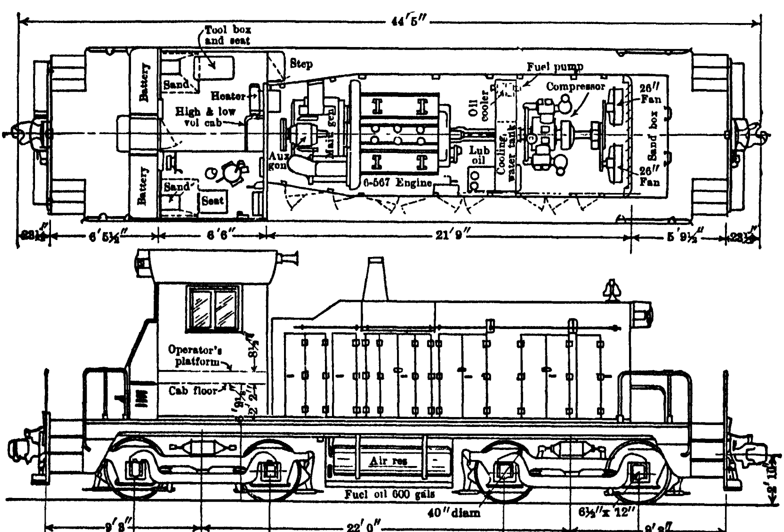


FIG. 6. Six-hundred-hp switching locomotive, model SW-1. (Courtesy of Electro-Motive Division, General Motors Corp.)

Standard 100-ton 600-hp (Fig. 6) and 115- to 120-ton 1000-hp switching units with a $B - B$ wheel arrangement, utilizing single engines, are made by several manufacturers.

Road units are made in 1500-hp and 2000-hp sizes and may be used as single-cab or multicab combinations of two-, three-, and, in rare cases, four-cab locomotives (Figs. 7, 8, and 9).

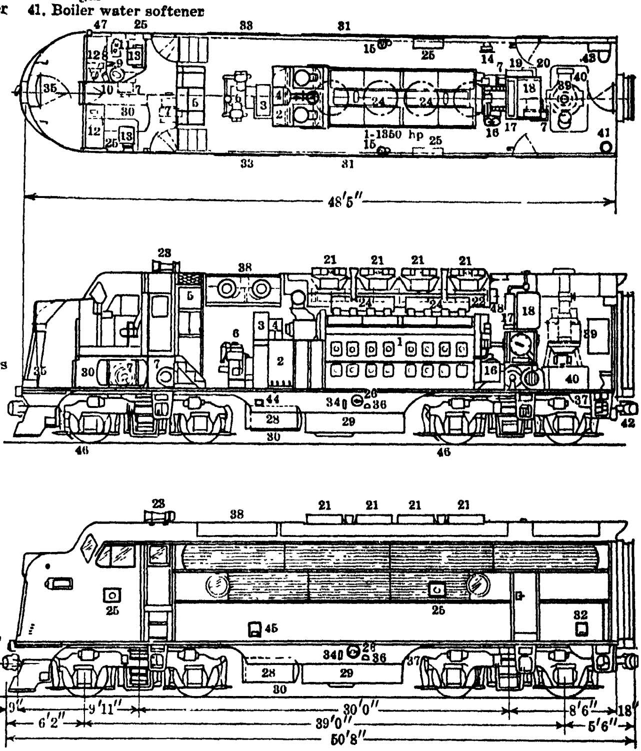
A single-cab, two-engined 3000-hp unit, which is used as a single-cab locomotive or in a combination of two cabs to produce a 6000-hp locomotive, is shown in Fig. 10.

(*Continued on p. 14-38*)

1. Engine E.M.D. model 16-567 B
2. Main generator and alternator
3. Generator blower
4. Aux. generator
5. Control cabinet
6. Air compressor
7. Traction motor blower
8. Instrument board
9. Control stand
10. Speedometer recorder
11. Air brake valve
12. Cab heater
13. Seat
14. Hand brake
15. Fuel tank vent with flame arrestor
16. Lub. oil filler
17. Lub. oil cooler
18. Engine water tank
19. Engine control and instrument panel
20. Load regulator
21. 34″ fan and motor
22. Radiator
23. Horn
24. Exhaust manifold
25. Sand box
26. Fuel filler
27. Head light (fixed beam)
28. Batteries
29. Fuel tank, 1200 gal
30. Main air reservoir
31. Air intake and shutters
32. Boiler water filler (500 gal tank only) both sides
33. Air intake for grids and engine room
34. Fuel tank gage
35. Door, (plain)
36. Emergency fuel cut-off
37. Engine water filler (both sides)
38. Dynamic brake grids and blowers
39. Boiler
40. Boiler water tank, 200 gal
41. Boiler water softener
42. Coupler between units
43. Toilet
44. Battery charging receptacle (left side only)
45. Boiler water tank filler (hatch tank) both sides
46. Sanding nozzles
47. Blue flag bracket
48. A-c contactor cabinet

Supplies

Fuel	1200 gal
Lub. oil	200 gal
Cooling water	230 gal
Sand	16 cu ft
Boiler water	200 or 800 gal



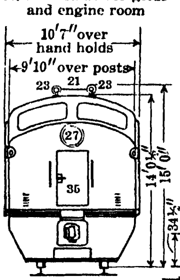


FIG. 7. 1500-hp locomotive "lead" unit, model F3. (Courtesy of Electro-Motive)

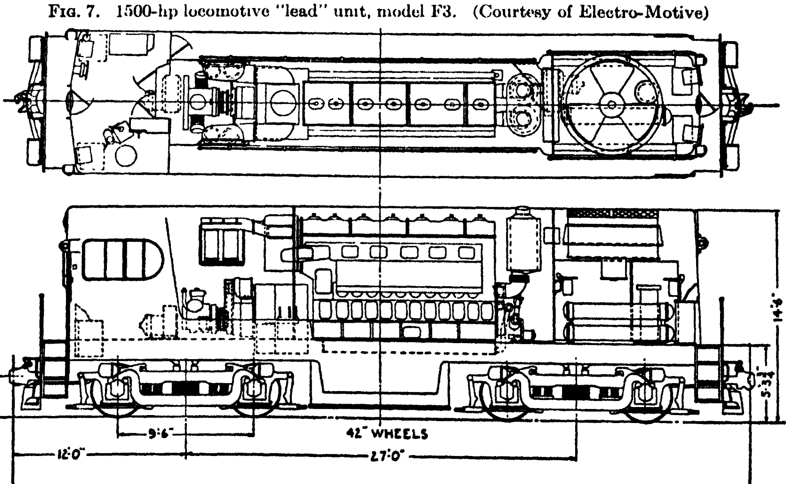


FIG. 8. Plan and elevation of locomotive, Model 99LEA4. (Courtesy of Fairbanks-Morse)

Table 1. Data on Representative American-built Diesel-electric Locomotive Units for Railroad Service

Figure number in text	(5)	. . . .	(6)	. . . . . .	. . . . . .	. . . . . .	. . . . . .	(7)	(8)	. . . . . .	(9)	(10)
Builder, Mechanical parts	Dav.	GE Co.	EMD	F-M	ALCO	ALCO	BLW	EMD	F-M	BLW	ALCO	BLW
Engine	Cat.	C-B	EMD	F-M	ALCO	ALCO	BLW	EMD	F-M	BLW	ALCO	BLW
Electrical equipment	West.	GE Co.	EMD	West.	GE Co.	GE Co.	West.	EMD	West.	West.	GE Co.	West.
Nominal hp rating	330	600	600	1000	1000	1500	1500	1500	2000	2000	2000	3000
Service	Br. and Sw.	Br. and Sw.	Sw.	Sw.	R. Sw.	R. Sw.	R. Sw.	Frt.	R. Sw.	T	P	F & P
Wheel arrangement	$B-B$	$B-B$	$B-B$	$B-B$	$B-B$	$B-B$	$A1A-A1A$	$B-B$	$B-B$	$C-C$	$A1A-A1A$	$2-D+D-2$
Nominal weight, tons	44	70	100	120	115	(115)	140	115	125	180	153	298
Engine Data:												
No. of engines	2	1	1	1	1	1	1	1	1	2	1	2
Total hp (approx.)	380	660	660	1050	1050	1600	1600	1600	2100	2100	2100	3150
No. cylinders (per engine)	8	6	8	6 (*OP*)	6	12	8	16	10(OP)	6	16	8
Bore and stroke, in.	$5\,3/4 \times 8$	$9 \times 10\,1/2$	8×10	$8\,1/8 \times 10$	$12\,1/2 \times 13$	$9 \times 10\,1/2$	$12\,3/4 \times 15\,1/2$	$8\,1/2 \times 10$	$8\,1/8 \times 10$	$12\,3/4 \times 15\,1/2$	$9 \times 10\,1/2$	$12\,3/4 \times 15\,1/2$
Speed, rpm	1000	1000	750	800	740	1000	625	800	850	625	1000	625
Cycles	4	4	2	2	4	4	4	2	2	4	4	4
Aspiration	N	SC	N	N	SC	SC	SC	N	N	SC	SC	SC
Weight, total, lb	88,000	139,000	201,000	240,000	230,000	230,000	280,000	230,000	250,000	360,000	306,000	595,000
Weight on drivers, lb	88,000	139,000	201,000	240,000	230,000	230,000	187,000	230,000	250,000	360,000	204,000	410,000
Length overall, ft-in.	33–10	37–0	43–7 1/2	48–10	54–11 3/4	55–0	58–0	50–8	51–0	71–0	66–2	91–6
Rigid-wheel base, ft-in.	7–0	6–10	8–0	8–0	9–4	9–4	11–6	9–0	9–6	13–0	15–6	16–9
Total-wheel base, ft-in.	23–6	26–3	30–0	33–6	40–4	39–4	43–9	39–0	36–6	51–3	49–8	73–2
Driving-wheel diameter, in.	33	36	40	40	40	40	42	40	42	42	40	42

Dav. = Davenport Locomotive Works. Cat. = Caterpillar Tractor Co. West. = Westinghouse Electric Corp. GE Co. = General Electric Co. C-B = Cooper-Bessemer Corp. EMD = Electro-Motive Division of General Motors Corp. F-M = Fairbanks-Morse & Co. ALCO = American Locomotive Co. BLW = Baldwin Locomotive Works. Br. = Branch line. Sw. = Switching. R. Sw. = Road switching. Frt. = Freight. P = Passenger. T = Transfer. N = Normal. SC = Supercharged. OP = Opposed piston.

Table 2. Data on Representative American-built Diesel-electric Locomotives for Industrial Service

Figure number	. . .	(11)	. . . . .	. . . . .	. . . . .	. . . . .	(12)	. . . . .	. . . . .	. . . . .
Builder, Mechanical parts	Dav.	GE Co.	Whit.	Atlas	Whit.	Vulc.	Port.	GE Co.	Port.	GE Co.
Engine	Cat.	Cumm.	Herc.	Cumm.	Herc.	Cumm.	C-B	Cumm.	Buda	C-B
Electrical equipment	West.	GE Co.	West	West.	West.	West.	West.	GE Co.	West.	GE Co.
Nominal hp rating	77	130	280	260	280	350	600	420	590	1500
Service	Const.	Ind Sw.	Min.	Ind. Sw.	Ind. Sw.	Ind. Sw.	Ind. Sw.	Ind. Sw.	Ind. Sw.	Min.
Wheel arrangement	*B*	*B*	*B* − *B*	*B* − *B*	*B*	*B* − *B*	*C* − *C*	*B* − *B*	*B* − *B*	*B* − *B*
Nominal weight, tons	10	25	35	45	50	65	75	80	100	130
Engine data:	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
No. of engines	1	1	2	2	2	2	1	2	2	2
Total hp (approx.)	86	150	320	300	320	400	660	470	650	1650
No. cylinders (per engine)	4	6	6	6	6	6	6	6	6	6
Bore and stroke, in.	$5\frac{3}{4} \times 8$	$4\frac{7}{8} \times 6$	5×6	$4\frac{7}{8} \times 6$	5×6	$4\frac{7}{8} \times 6$	$9 \times 10\frac{1}{2}$	$5\frac{1}{8} \times 6$	$6\frac{3}{4} \times 8\frac{3}{4}$	$10\frac{1}{2} \times 13\frac{1}{2}$
Speed, rpm	1000	1800	1600	1800	1600	1800	1000	2100	1200	750
Cycles	4	4	4	4	4	4	4	4	4	4
Aspiration	N	N	N	N	N	SC	SC	SC	SC	SC
Gage, in.	$56\frac{1}{2}$	M	36	$56\frac{1}{2}$	$56\frac{1}{2}$	$56\frac{1}{2}$	36	$56\frac{1}{2}$	$56\frac{1}{2}$	$56\frac{1}{2}$
Weight, total, lb	20,000	50,000	70,000	90,600	100,000	130,000	150,000	161,000	200,000	260,000
Weight on drivers, lb	20,000	50,000	70,000	90,600	100,000	130,000	150,000	161,000	200,000	260,000
Length overall, ft-in.	15–0	18–2	26–10	26–0	20–0	31–9	43–6	36–10	40–0	48–6
Rigid-wheel base, ft-in.	5–0	$6–0\frac{1}{2}$	7–0	5–3	8–0	5–0	10–0	6–10	7–6	8–0
Total-wheel base, ft-in.	5–0	$6–0\frac{1}{2}$	21–0	19–0	8–0	20–0	29–9	26–0	29–0	38–2
Driving-wheel diameter, in.	28	33	33	33	33	33	33	33	40	40

Atlas = Atlas Car & Mfg. Co. Dav. = Davenport Locomotive Works. Cat. = Caterpillar Tractor Co. West. = Westinghouse Electric Corp. GE Co. = General Electric Co. Cumm. = Cummins Engine Co. Whit. = Baldwin-Whitcomb Locomotive Works. Herc. = Hercules Motors Corp. Vulc. = Vulcan Iron Works. Port. = H. K. Porter Co., Inc. C-B = Cooper-Bessemer Corp. Buda = The Buda Co. Const. = Construction. Ind. Sw. = Industrial switching. Min. = Mining.

1. Engine
2. Main generator
3. Exciter
4. Auxiliary generators
5. Traction motors
6. Contractor compartment
7. Turbosupercharger
8. Turbosupercharger filters and silencers
9. Dynamic brake grids and blowers (modification)
10. Control stand
11. Brake valves
12. Traction motor blowers
13. Radiators
14. Radiator shutters
15. Radiator fan
16. Radiator fan clutch
17. Lubricating oil cooler
18. Lubricating oil filters
19. Engine water tank
20. Air compressor
21. Main air reservoirs
22. Steam generator
23. Batteries
24. Sanitary fixture
25. Fuel tank
26. Fuel tank filling connection
27. Fuel tank gage
28. Emergency fuel cut-off
29. Water tanks
30. Water tank filling connections
31. Sand boxes
32. Sand box filling holes
33. Generator air duct
34. Hand brake
35. Cab heaters
36. Seat
37. Horn
38. Bell
39. Number box
40. Tool box (modification)
41. Engine control panel
42. Water filling connection for sanitary fixture
43. Engine cooling water filling connections

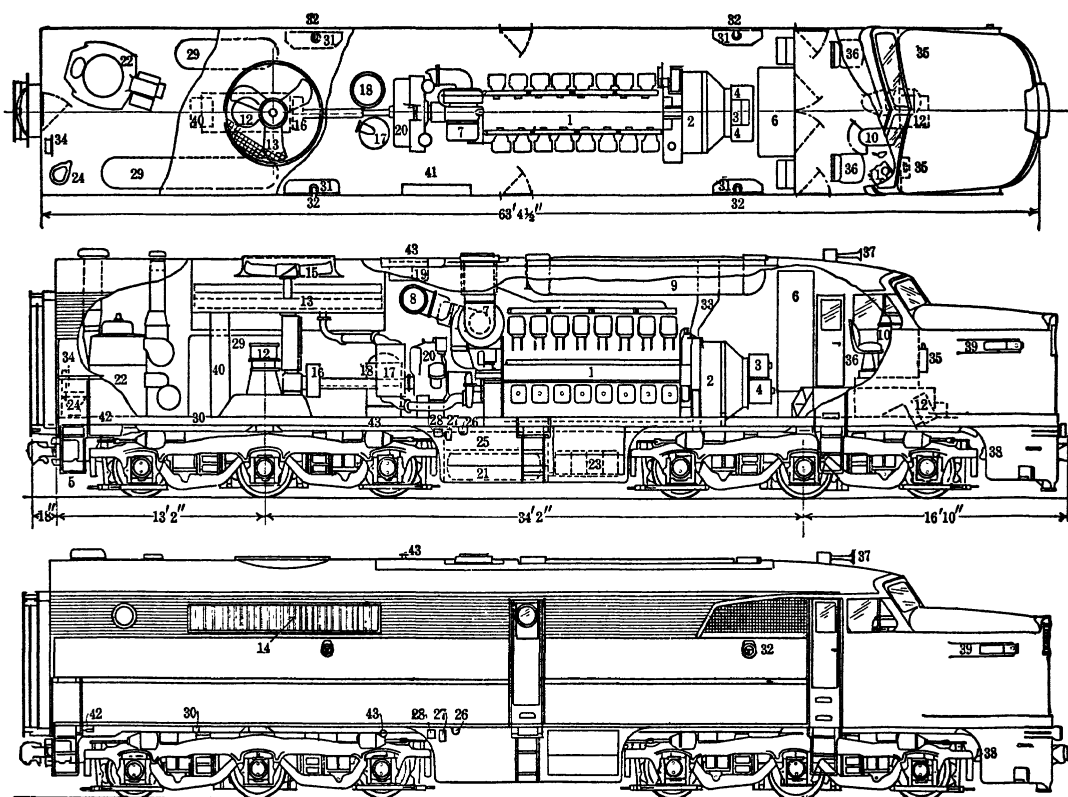


FIG. 9. Supercharged 16-cylinder road locomotive. (Courtesy of American Locomotive Co.)

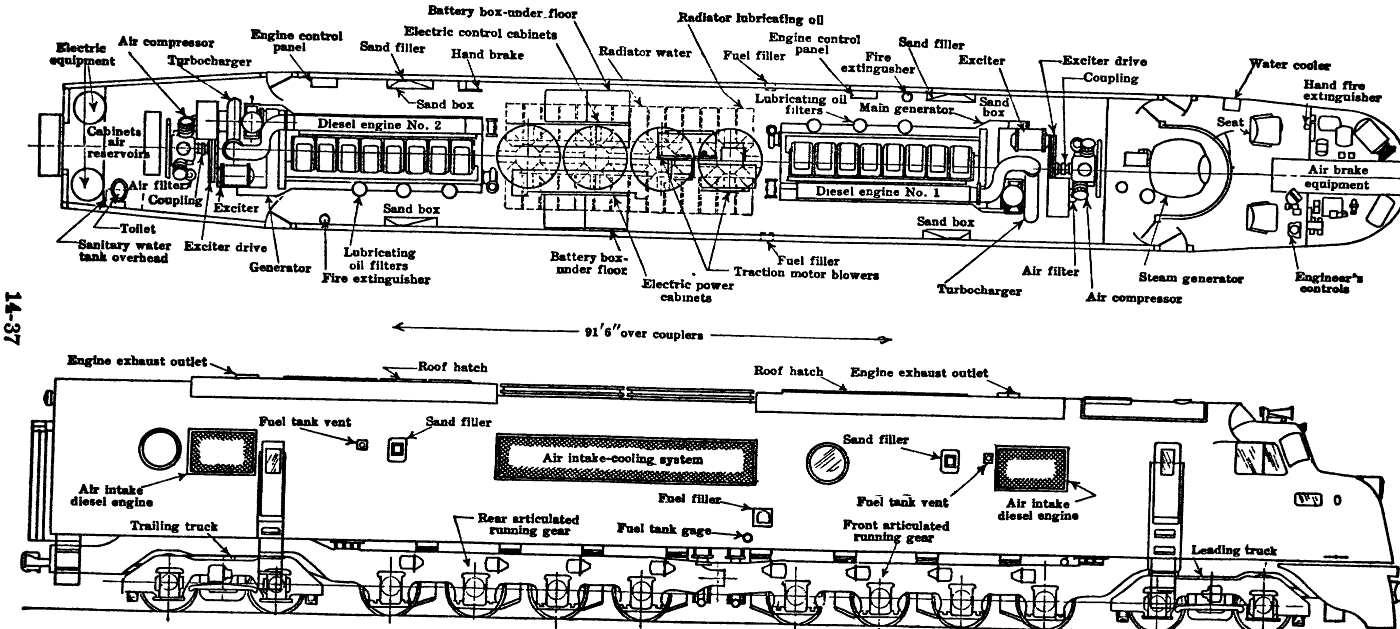


FIG. 10. Two-engine, single-cab 3000-hp unit, showing arrangement of equipment. (Courtesy of Baldwin Locomotive Works)

INDUSTRIAL DIESEL-ELECTRIC LOCOMOTIVES. The 100-ton 600-hp unit shown in Fig. 6 and a 115-ton 1000-hp unit are used in heavy industrial plant operations as well as in railroad yard switching. Sizes ranging from 10 tons 86 hp up to 130 tons 1000 to 1500 hp are in use in industrial services. The smaller sizes employ automotive-type high-speed engines; the larger sizes utilize the same types of engines used in railroad service. Typical industrial units are shown in Figs. 11 and 12.

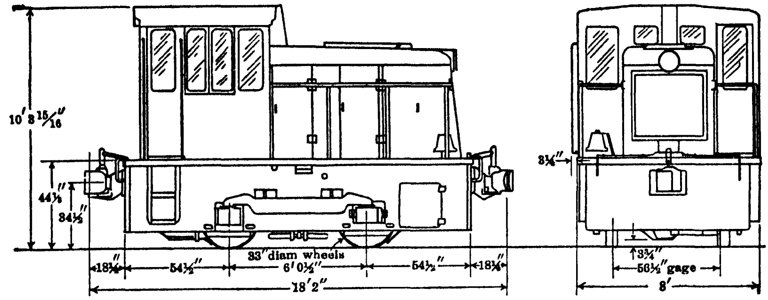


FIG. 11. Twenty-five-ton industrial switcher. (Courtesy of General Electric Co.)

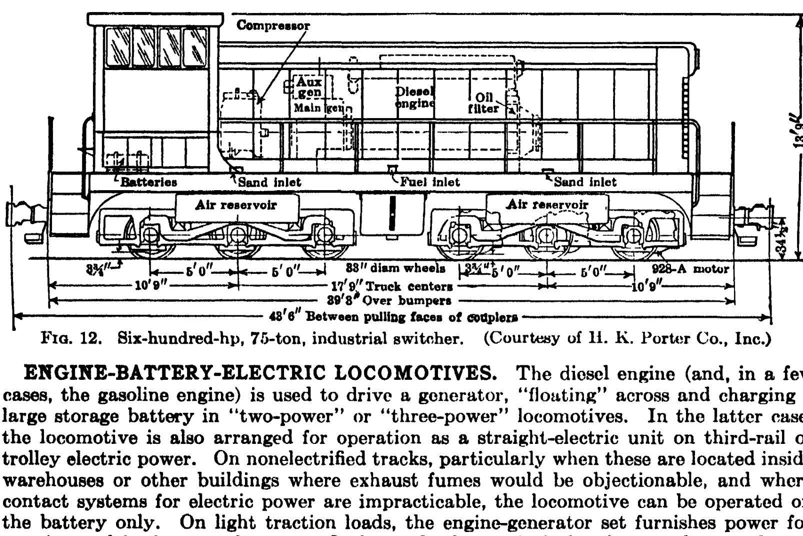


FIG. 12. Six-hundred-hp, 75-ton, industrial switcher. (Courtesy of H. K. Porter Co., Inc.)

ENGINE-BATTERY-ELECTRIC LOCOMOTIVES. The diesel engine (and, in a few cases, the gasoline engine) is used to drive a generator, "floating" across and charging a large storage battery in "two-power" or "three-power" locomotives. In the latter case, the locomotive is also arranged for operation as a straight-electric unit on third-rail or trolley electric power. On nonelectrified tracks, particularly when these are located inside warehouses or other buildings where exhaust fumes would be objectionable, and where contact systems for electric power are impracticable, the locomotive can be operated on the battery only. On light traction loads, the engine-generator set furnishes power for traction and for battery charging. On heavy loads, particularly when accelerating heavy trains, the battery assists the engine-generator set and carries the peak loads. Battery charging takes place when the locomotive is at rest. (See Ref. 4.)

In some installations the battery is omitted and the unit operates as a two-power locomotive, on the diesel-engine-generator set while working on nonelectrified trackage, and as a straight-electric locomotive when on tracks served by an electric contact system.

9. DIESEL-ELECTRIC LOCOMOTIVE AUXILIARIES

Brake air compressors on the earlier types of diesel-electric locomotives were of the electric-motor-driven type, with the motor powered from the main or an auxiliary generator. In the interests of lighter weights, lower costs, and better efficiency, compressors on modern locomotives are of the direct-driven type, driven from the main engine by direct or belted drives, and controlled by pressure-operated unloading valves. Capacities range from 20 to 300 cu ft per min per compressor at full engine speed, the smaller size being employed on 10- to 20-ton industrial sizes, and the latter being furnished on 2000-hp railroad locomotives.

Auxiliary generators for lighting, control, and battery-charging energy supply, for main-generator excitation, and for traction-motor blower motors are supplied on the larger units while the smaller sizes using automotive-type engines may be equipped with an automotive type of battery-charging generator. Auxiliary generators may be gear driven or belt driven, generally from extensions on the main generator shafts. Auxiliary generator voltages are nominally 32 or 64 to correspond to generally accepted railroad practice.

Storage batteries of the lead-acid or alkaline types are used for lighting and control energy and for initial engine-cranking service.

Traction-motor blowers for forced ventilation of the traction motors are furnished on all railroad locomotives and on the larger sizes of industrial units. Blowers may be belt driven from the main engine or electric motor driven from the auxiliary generator power system. In some of the larger road locomotives, driving motors are of the induction type supplied by an alternator direct driven by the main engine.

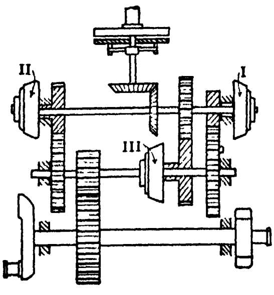
Engine water-cooling systems on the smaller railroad and industrial types are quite similar to those found in automotive practice wherein a radiator mounted ahead of the engine is cooled by a belt-driven fan from the main engine. In the larger locomotives special high-efficiency fans, direct driven from the main engine or through thermostatically controlled clutches from the engine, may be employed. Individually induction motor-driven roof-mounted fans, under thermostatic control, are employed in some instances.

Water-heating Boilers. Units used in passenger-train service are equipped with oil-fired boilers burning diesel fuel, with capacities ranging from 1600 to 3000 lb of steam per hour for car-heating purposes. Standby steam generators of 300-lb capacity are employed for engine cooling-water heating where the locomotive unit is subjected to low ambient temperatures when not in operation. The smaller railroad and industrial types may be equipped with kerosene-burning water heaters, or with immersion-type electric heaters supplied from plug-in wayside power, for cold-weather protection.

10. OTHER TRANSMISSIONS

MECHANICAL TRANSMISSIONS. Many attempts have been made to eliminate the electric-drive system by the substitution of various types of mechanical transmissions. To date relatively few diesel locomotives have been produced without electric drive except in introductory or experimental service.

In 1925 German locomotive builders produced, for service in Russia, two 1200-hp diesel locomotives employing *electromagnetic clutches* and a constant-mesh gear train between the engine shaft and a jack shaft which drove five rod-coupled driving axles through quartered cranks on the jackshaft, and connecting rods. The clutch-and-gear-train system, shown in Fig. 13, provided four speed stages, with a resultant stepped speed-tractive effort characteristic as shown in Fig. 14. Reversing of the locomotive was accomplished by reversing the diesel engine. A 45-hp, 1000-rpm auxiliary engine is employed for drive of an auxiliary electric generator, the main-engine cooling fan, and an air compressor.



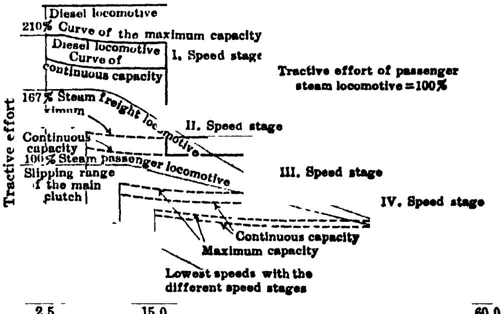


FIG. 13 (*left*). Clutch-and-gear-train system of 1200-hp German diesel locomotive.

FIG. 14 (*right*). Speed-tractive effort characteristic of mechanical drive diesel locomotive.

Figure 15 shows a 400-hp, 70-ton Plymouth diesel locomotive, employing a hydraulic coupling and a two-speed gear box with hydraulically operated clutches, driving a jack shaft to which the driving wheels are coupled by means of side rods. This is the largest mechanical-drive diesel locomotive built in the United States.

Smaller locomotives with engine ratings of 165 hp and under are built for light railroad service (principally in countries outside the United States) and industrial and construction switching. One British-built 25-ton 165-hp unit uses an automotive-type transmission system with a friction clutch and change-gear box.

A limited application of diesel locomotives has been made for mining motive power. Underground working with diesel locomotives requires special provisions for flame-proofing the exhaust and mitigating the effects of exhaust gases. The flame-proofing is accomplished by fitting baffling arrangements on the engine air inlet and by-passing the exhaust gases through water filters. European locomotives have been built weighing up to 20 tons; American practice to date involves units up to 10 tons in weight. Engine horsepower used in these locomotives ranges from 20 to 200, and the drive systems are generally of the mechanical type.

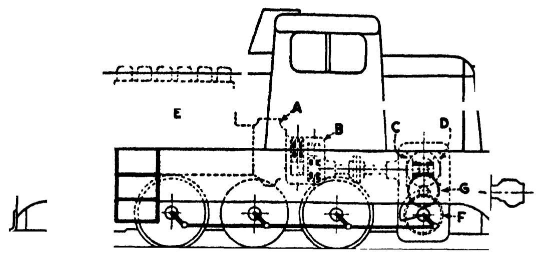


FIG. 15. Hydraulic-torque drive (E = engine; A = converter; B = two-speed gear; C = reverse clutch; G = gears; and F = jack shaft).

DIRECT DRIVE. An experimental direct-drive diesel locomotive was built in Germany in 1933. Three double-acting diesel cylinders are located in positions corresponding to the cylinders in a conventional three-cylinder steam locomotive, and drive directly to the driving axles by means of coupling and drive rods. Initial starting is accomplished by operating the engines with compressed air supplied by a diesel engine-driven compressor mounted within the locomotive superstructure. The locomotive operates as a compressed-air diesel unit after ignition of the fuel oil, up to a vehicle speed corresponding to 45 to 50 mph. The locomotive is rated at 1500 ihp.

11. SELF-PROPELLED CARS AND TRAINS

There are in the United States approximately 750 *gasoline-engine-propelled cars* with electric drive operating mainly in branch-line railroad service. These cars were built mainly in the decade of 1922 to 1932, but since the advent of reliable low-cost, high-speed diesel engines no new installations of gasoline engines in railcar service have been made. The gas-electric cars have an installed engine capacity ranging, in general, from 135 to 800 hp. Single engine-generator units are employed in the smaller sizes and up to approximately 400 hp, whereas the larger sizes include two engine-generator sets. Two d-c series-wound traction motors mounted in a two-axle swivel truck are used with single-power plants whereas four motors are employed with double power plants. Car bodies are built as straight passenger coaches, or in various combinations of passenger-baggage-mail compartments, depending on the service assignments. In many designs the cars are arranged as baggage-mail cars hauling one or more light passenger trailers. Motor-car weights range from the smaller sizes of 10 to 15 tons up to the larger sizes of 75 to 80 tons. Individual diesel-electric and diesel-mechanical cars differ from the gas-engine types essentially only in the prime mover.

Several trains with motive power built as an integral part of the train were placed in American and European services in the middle 1930's. Typical of this class is the Burlington Zephyr, described in detail in Ref. 5. In the United States this type of train has been superseded largely, in new construction, by individual stream-lined diesel-electric locomotives hauling stream-lined cars of modern nonarticulated lightweight design. Lack of flexibility in interchange of cars between trains and between connecting railroads, coupled with the difficulty of cutting out an unserviceable unit (power car or passenger car) has, in general, made obsolete the articulated form of self-propelled train, except where cars of a novel or experimental type are involved.

Builders and operators have favored the *diesel-powered railcar*, or "automotrice," to a greater extent in Europe than in the United States. European construction has utilized extensively truck-mounted engines (some as large as 600 hp) with hydraulic torque-converter drives. Smaller engines, truck mounted, have been used with mechanical drive. Table 3 lists a variety of railcars used by the Belgian State Railways. Drives, other than electric, for diesel-powered cars have not been employed in the United States to any great extent, except for a few relatively small low-powered individual railcars and car-trailer combinations.

Table 3. Diesel Railcars and Railcar Trains, Belgian National Railways

Year Placed in Service	No. Railcars or Trains	Type	Arrangement of Vehicle	Railcar Constructors: Class No.	Railcar Constructors: Mechanical Portion	Railcar Constructors: Engine	Railcar Constructors: Transmission	No. Engines and Power of Each	Capacity: Seats	Capacity: Standing	Light Weight, tons	Loaded Weight, tons	Max Speed, mph
1930	3	Diesel-mechanical	Bogie car	600	E.V.A.	Maybach	Maybach	1–150 *	92	80	35.7	49.2	40
1932	1	Diesel-electrical	Bogie car	650	D.E.V.A.	Burmeister & Wain	A.S.E.A.	1–200	80	50	39.1	49.2	50
1933	14	Diesel-mechanical	Bogie car	601	La Brugeoise	Maybach	Maybach	1–175	104	50	29.6	41.9	53
1934	1	Diesel-mechanical	Bogie car	602	La Brugeoise	Maybach	Maybach	1–210	100	50	27.5	39.5	53
1934	1	Diesel-electrical	Bogie car	651	La Brugeoise	Maybach	A.C.E.C.	1–210	100	50	28.4	40.4	56
1934	1	Diesel-electrical	Twin-car articulated	652	La Brugeoise	Maybach	Siemens-A.C.E.C.	1–410	185	30	61.4	79.3	94
1934	1	Diesel-mechanical	2-axle car	603	La Dyle	M.A.N.	T.A.G.	1–140	57	18	14.5	20.7	50
1934	5	Diesel-mechanical	2-axle car	604	Ganz	Ganz	Ganz	1–120	60	20	13.8	20.1	44
1935	1	Diesel-mechanical	Bogie car	605	Braine-le-Comte	Carels-Ganz	Ganz	1–220	100	50	26.4	38.0	51
1936	1	Diesel-mechanical	Bogie car	606	Baume-Marpent	Carels-Ganz	S.L.M.	1–320	70	60	33.3	44.3	53
1936	1	Diesel-mechanical	Bogie car	607	Haine-St. Pierre	Mercedes	S.L.M.	1–330 †	70	60	31.8	42.5	55
1936	3	Diesel-electrical	Triple-car articulated	653	La Brugeoise	Maybach	A.C.E.C.	2–410	229	40	115.0	138.0	94
1936	3	Diesel-electrical	Triple-car articulated	654	Baume-Marpent	Carels-Ganz	S.E.M.	2–365	229	40	121.0	145.0	85
1936	1	Diesel-electrical	Triple-car articulated	655	Baume-Marpent	Mercedes	A.C.E.C.	2–450 ‡	229	40	118.0	142.2	94
1936	1	Diesel-electrical	Triple-car articulated	656	Baume-Marpent	Frichs	A.C.E.C.	2–400 §	229	40	120 0	144.0	88
1939	6	Diesel-hydraulic	Triple-car non-articulated		La Brugeoise	Maybach	Voith	2–600 ‖	219	25	122.4	144.5	100
1939	12	Diesel-mechanical	Twin-car articulated§		Baume-Marpent	Carels-Ganz	S.L.M.	2–370	136	60	87.3	99.0	81
1939	6	Diesel-mechanical	Bogie car		Baume-Marpent	Carels-Ganz	S.L.M.	1–370	64	60	40.9	48.1	81
1939	6	Diesel-mechanical	Bogie car		Brossel	Brossel	Brossel	1–140	80	40	21.8	30.9	38
1939	6	Diesel-mechanical	2-axle car		Brossel	Brossel	Brossel	1–140	50	30	13.6	18.2	38
1940	50	Diesel-mechanical	2-axle car		S.N.C.F.B.	Brossel	Brossel	1–140	50	30	13.6	18.2	38

* Now 175 bhp. † Now replaced by ex-450-bhp engine derated. ‡ Now replaced by two 410-bhp Maybach engines. § Now a twin-car train with one Frichs 400-bhp engine. ‖ Pressure-charged.

Table 4. Diesel Engines in Use for Large Railroad-type Locomotives

Manufacturer	American Locomotive Company				Electro-Motive Division (GMC)			Fairbanks-Morse & Company			Baldwin Locomotive Works			Cooper-Bessemer Corporation		
Nominal horsepower (for traction)	2,000	1,500	1,000	600	1,500	1,000	600	2,000*	1,500*	1,000*	1,500	1,000	600	1,500	1,200	600
Full load, rpm	1,000	1,000	740	740	800	800	800	850	850	800	625	625	625	1,000	1,000	1,000
No. cylinders	16	12	6	6	16	12	6	10	8	6	8	8	6	16	12	6
Cylinder bore and stroke, in.	9 × 10 1/2	9 × 10 1/2	12 1/2 × 13	12 1/2 × 13	8 1/2 × 10	8 1/2 × 10	8 1/2 × 10	8 1/8 × 10	8 1/8 × 10	8 1/8 × 10	12 3/4 × 15 1/2	12 3/4 × 15 1/2	12 3/4 × 15 1/2	9 × 10 1/2	9 × 10 1/2	9 × 10 1/2
Cylinder arrangement	V	V	In line	In line	V	V	V	In line	In line	In line	In line	In line	In line	V	V	In line
Aspiration	SC	SC	SC	N	N	N	N	N	N	N	SC	N	N	SC	SC	SC
Type of supercharger drive	Turbo	Turbo	Turbo	.	.	.	.		.		Turbo	.	.	Turbo	Turbo	Turbo
Weight, dry, lb	37,000	29,650	35,000	33,000	31,000	25,823	16,565	36,700	28,500	23,250	39,300	37,700	30,200	39,000	30,000	22,000
Strokes per cycle	4	4	4	4	2	2	2	2	2	2	4	4	4	4	4	4

* These engines are of the opposed-piston design, using two pistons per cylinder.

Table 5. Representative Diesel Engines in Use for Railcars, Small Railroad-type, and Industrial Locomotives

Manufacturer	Cummins Engine Company						Caterpillar				Hercules			Buda			Sterling			H-O-R
Nominal horsepower	150	200	200	275	400	550	86	190	400	500	176	193	201	150	228	350	230	325	650	400
Full load, rpm	1,800	1,800	2,100	2,100	2,100	2,100	1,000	1,000	1,200	1,200	1,800	1,600	1,600	1,800	1,050	1,200	1,800	1,200	1,200	900
No. cylinders	6	6	6	6	12	12	4	8	12	12	6	6	6	6	6	6	6	6	8	6
Cylinder bore and stroke, in.	4 7/8 × 6	4 7/8 × 6	5 1/8 × 6	5 1/8 × 6	5 1/8 × 6	5 1/8 × 6	5 3/4 × 8	5 3/4 × 8	5 3/4 × 8	5 3/4 × 8	5 × 6	5 1/2 × 6	5 5/8 × 6	4 3/4 × 6 1/2	6 1/4 × 8 3/4	6 3/4 × 8 3/4	6 1/4 × 6 1/2	8 × 9	8 × 9	8 3/4 × 12
Cylinder arrangement	In line	In line	In line	In line	V	V	In line	V	V	V	In line	In line	In line	In line	In line	In line	In line	In line	In line	In line
Aspiration	N	SC	N	SC	N	SC	N	N	N	SC	N	N	N	N	N	SC	N	N	SC	N
Type of supercharger drive	.....	Mech.	.....	Mech.	....	Mech.	. . .	.	.	Mech.	.	.		.... .		Mech.	....	......	Turbo	......
Weight, dry, lb	2,200	2,500	2,400	2,850	5,500	6,000	4,400	6,800	8,800	9,200	2,300	2,600	2,600	2,270	6,025	7,400	2,350	8,000	9,600	9,875
Strokes per cycle	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Diesel engines have been used to some extent in the United States to replace the gasoline engines originally furnished for railcar service, and in some new railcar installations. In general, the engines are of the same types as employed in the smaller diesel-electric locomotives up to approximately 400 hp in rating.

12. LOCOMOTIVE ENGINES

Diesel engines used for locomotive service range in size and type from the smaller-sized automotive, high-speed, mass-produced engines of the order of 100 hp in rating to the larger medium-speed designs manufactured especially for traction service. Full-load speeds of 2100 rpm and corresponding specific weights of 11 lb per hp for supercharged engines up to 550 bhp rating are employed for some industrial switchers, whereas the majority of engines in the larger railroad types have engine speeds in the 800 to 1000 rpm range, with a tendency to go to the higher speed with new designs. Both 2-cycle and 4-cycle designs are employed for the railroad types, but the industrial types in the United States, exclusive of railroad types used in industrial service, invariably have 4-cycle engines. The largest engines in commercial production in the United States currently are of 2400-hp (nominal) rating. Single engines with double banking of cylinders having ratings of 2200 hp (1 hour) and 1900 hp (continuous) are installed in a few European locomotives. Solid injection is universally used in American locomotive diesels and in the majority of engines built by foreign manufacturers. Air injection has been used in early experimental units, but the inconvenience and operating hazards attendant on the procurement and maintenance of a high-pressure air supply in locomotive service have caused almost complete abandonment of this type of injection. Salient dimensions of representative American locomotive engines are given in Tables 4 and 5.

13. COSTS

DIESEL-ELECTRIC SWITCHING LOCOMOTIVES. Although the first cost of the diesel-electric is usually greater than the equivalent (weight and horsepower) steam locomotive, the operating economies realized with the former in switching service are generally of such magnitude as to more than offset the higher initial cost. Standard diesel-electric switching locomotives for railroad service carry prices of approximately $95 per hp for the 1000-hp size and $125 per hp for 600-hp units. Industrial switching units, carrying relatively high-speed engines, range in price from $107 for the smallest single-engine sizes (approximately 150 hp) to $120 per hp for the larger two-engine types.

Typical operating statistics for 1000-hp diesel-electrics in railroad yard-switching service and comparable steam figures are shown in the following tabulation.

	Diesel-electric	Comparable Steam
Locomotive hp	1,000	
Locomotive weight, tons	115	
Total locomotive hours	113,200	
Number of locomotives	19	
Availability, %	85.5	
Maintenance, cents per hr		
Engine and appurtenances	40.0	
Electrical	17.0	
Mechanical portion	21.0	
Total, cents per hr	78.0	170.0
Wages, enginemen, cents per hr	279.0	250.4
Fuel	44.7	100.0
Water	Neg.	1.1
Lubrication	5.9	0.5
Supplies	2.8	47.2
Cleaning and servicing	8.8	12.0
Total operation and maintenance, cents per hr	419.2	581.2
Gallons fuel oil used per hour	5.6	
Gallons lubricating oil used per hour	0.1	
Saving per hour, diesel-electric over steam		$1.62

Table 6. Comparison of Steam and Diesel-electric Operation of a Typical Major Class I Railroad (Annual Basis *)

(Adapted by permission from *AIEE Technical Paper* 49–42, by G. T. Bevan)

STEAM-OPERATING EXPENSE

Freight Service		*Passenger Service*		*Switching Service*	
Annual gross ton miles	6,911,249 (000)	Annual car miles	11,865,063	Annual switching hours	69,516
Annual train miles	3,824,355	Annual train miles	1,574,944	Annual locomotive miles	417,096
Annual locomotive miles	4,307,123	Annual locomotive miles	1,599,500		

ICC Account	Freight: Per Train Mile	Freight: Per 1000 GTM †	Freight: Per Annum	Passenger: Per Train Mile	Passenger: Per Car Mile	Passenger: Per Annum	ICC Account	Switching: Per Hour	Switching: Per Annum
308 Locomotive repairs	$ .4145	$ .2293	$1,585,021	$ .3199	$.0425	$ 503,842	308 Locomotive repairs	$ 1.2500	$ 86,895
392 Enginemen	.2680	.1483	1,025,095	.2050	.0273	322,864	380 Enginemen	3.1000	215,499
394 Fuel	.6882	.3808	2,631,798	.4605	.0611	725,245	382 Fuel	2.9132	202,233
397 Water	.0311	.0172	118,850	.0185	.0025	29,200	385 Water	0.1329	9,242
398 Lubricants	.0160	.0089	61,200	.0088	.0012	13,848	386 Lubricants	0.0140	974
399 Other supplies	.0081	.0045	30,900	.0053	.0007	8,262	387 Other supplies	0.0241	1,675
400 Enginehouse expense	.1539	.0852	588,500	.0861	.0114	135,540	388 Enginehouse expense	1.9693	136,900
401 Trainmen	.3409	.1887	1,304,105	.2150	.0286	338,613	378 Trainmen	4.8600	337,848
Total	$1.9207	$1.0629	$7,345,469	$1.3191	$.1753	$2,077,414	Total	$14.2635	$991.266

DIESEL-OPERATING EXPENSE

Freight Service		*Passenger Service*		*Switching Service*	
Annual gross ton miles	6,911,249 (000)	Annual car miles	11,865,063	Annual switching hours	69,516
Annual train miles	3,272,646	Annual train miles	1,387,318	Annual locomotive miles	417,096
Annual locomotive (unit) miles	5,581,978	Annual locomotive (unit) miles	1,724.619		

ICC Account	Freight: Per Train Mile	Freight: Per 1000 GTM †	Freight: Per Annum	Passenger: Per Train Mile	Passenger: Per Car Mile	Passenger: Per Annum	ICC Account	Switching: Per Hour	Switching: Per Annum
311 Locomotive repairs	$ .2040	$.0968	$ 669,837	$.1740	$.0240	$ 241,447	311 Locomotive repairs	$ .8450	$ 58,740
392 Enginemen	.3010	.1424	985,066	.2080	.0244	288,562	380 Enginemen	3.1000	215,500
394 Fuel	.3328	.1576	1,089,213	.2022	.0236	280,490	382 Fuel	.7960	55,335
397 Water	....	....	....	....	....	....	385 Water	....	....
398 Lubricants	.0401	.0190	131,249	.0252	.0029	34,947	386 Lubricants	.0796	5,533
399 Other supplies	.0081	.0038	26,508	.0053	.0006	7,353	387 Other supplies	.0210	1,460
400 Enginehouse expense	.0409	.0194	133,967	.0298	.0035	41,391	388 Enginehouse expense	.1440	10,010
401 Trainmen	.3409	.1615	1,115,645	.2150	.0252	298,274	378 Trainmen	4.8600	337,848
Total	$1.2678	$.6005	$4,151,485	$.8595	$.1022	$1,192.464	Total	$9.8456	$684.426

Annual Savings

Freight Service	$3,193,984
Passenger Service	884,950
Switching Service	306,840
Total	$4,385,774

* Prorated from July 13–19, 1946 Traffic. †GTM = gross ton-miles.

Where the diesel-electric locomotive replaces a steam-switcher which requires a two-man crew, and where operation of the diesel-electric with one engineman is permissible, the savings over steam operation are proportionately greater. In general, the overall operating costs, not including fixed charges, in industrial switching where one-man and two-men crews are used for diesel-electric and steam, respectively, are found to be 30 to 40% less for the diesel-electric.

Savings realizable are dependent to a large extent on the utilization at which the more expensive (first cost) motive power can be worked. The absence of boilers, fireboxes, and other components requiring time-consuming attention and the ability of the diesel-electric to work for extended periods without refueling and rewatering permit high degrees of utilization and availability. In many cases where an appreciable number of switching units are needed, as in large industrial or railroad yard switching operations, the overall work can be done with diesel-electrics with approximately 60 to 75% of the number of switching units normally assigned under steam operation.

DIESEL-ELECTRIC LOCOMOTIVES IN ROAD SERVICE. The high *availability and reliability* of the diesel-electric locomotive, with attendant lessened susceptibility to failure, reduced time for servicing en route, and lower operating costs than the steam locomotive have revolutionized railroad motive-power practices to such an extent that practically 90 to 95% of new railroad motive power being built for United States service is of the diesel-electric type. Availability and reliability characteristics permit the operation of fleets of diesel-electrics in passenger-train service, wherein relatively long runs are involved between terminals, with locomotives averaging as high as 250,000 to 300,000 miles per year. In freight-train service, monthly mileages of 10,000 to 15,000 per locomotive are not uncommon. These large mileages permit the displacement of steam locomotives by the diesel-electric in the ratio of 1.5 or 2 to 1, in favor of the diesel, thus offsetting to a large extent the higher first cost of the diesel-electric over the comparable steam unit.

Diesel-electric road locomotives in the larger (1500 and 2000 hp) sizes carry first costs ranging from $95 to $110 per horsepower, the lower figure holding for freight power and the higher for passenger locomotives with such "extras" as dynamic braking, train-heating boilers, cab-signal equipment, and other specialties. Comparable steam locomotive costs are approximately one-half these values.

P. W. Kiefer (Ref. 6) gives results of a comprehensive study involving steam, diesel-electric, and electric motive power as applied to the New York Central System. While lower (than steam) operating costs is the dominating factor in many cases, the advantages in most cases as compared with steam locomotives are to be found in greater effectiveness in train operation—both passenger and freight—rather than in appreciably lower operating costs. As of January 1948, diesel-electrics were handling approximately 10% of the road-freight tonnage, 25% of the passenger-train business, and 30% of the switching work of railroads in the United States.

Economics of Diesel-electric Locomotives in Railroad Service. Comparative operating costs of steam and diesel-electric motive power in railroad service for a typical application are shown in Table 6. The large savings are, in many cases, more than sufficient to balance the fixed charges involved in the purchase of higher first-cost power, with a resultant appreciable net return on the difference in motive-power investment. The operating accounts most affected are maintenance, fuel, and engine house expense. The greater availability of the diesel-electric for service, in many cases, reduces the number of motive-power units required to perform a given service, and permits greater mileages between inspections and shoppings, thus contributing greatly to reduction in operating expenses.

ELECTRIC LOCOMOTIVES

By T. F. Perkinson

14. GENERAL CLASSIFICATION

Electric locomotives may be grouped in several ways.

Alternating-current locomotives for operation from a contact system supplying either single-phase or polyphase a-c energy. These may be equipped with (1) series-wound commutator motors; (2) induction motors; or (3) d-c series-wound motors, fed by one or more (*a*) ac-dc motor-generator sets or (*b*) rectifiers.

Direct-current locomotives, which operate from a system supplying d-c energy (an external contact system or self-contained storage batteries) and use the energy without change of form on series-wound traction motors.

Ac-dc locomotives, which utilize series-wound commutator motors, suitable for either a-c or d-c operation, and are used where normal a-c motive power is operated on low-voltage third-rail contact systems in terminal work.

SYSTEM VOLTAGES, FREQUENCIES, AND PHASES FOR LOCOMOTIVE POWER SUPPLY. A-c Systems. In the United States the 11,000-volt, single-phase, 25-cycle system is the only system in current use. A variety of system voltages and frequencies is utilized in European electric railroading, the most important systems being these.

Voltage	Phases	Frequency	Where Used
20,000	1	50	Germany
16,000	1	15	Sweden, Norway
16,000	1	16 2/3	Sweden
16,000	1	25	Sweden
15,000	1	15	Germany
15,000	1	16 2/3	Austria, Germany, Italy, Switzerland
15,000	1	50	Hungary
11,000	1	16 2/3	Switzerland
10,000	3	45	Italy
10,000	1	16 2/3	Norway
10,000	1	25	Sweden
6,600	1	25	England
6,500	1	25	Austria
6,000	1	25	Germany, Norway
5,500	3	25	Spain
3,700	3	16 2/3	Italy
3,600	3	16 2/3	Italy
750	3	40	Switzerland

There are no a-c systems operated in countries other than those in Europe, the United States, and Canada (which has one 3300-volt, single-phase, 25-cycle operation). The three-phase system in Italy is being supplanted by 3000-volt, direct current.

D-c systems, classified according to contact system voltage, are these.

Voltage	Where Used
4000	Italy
3300	Russia
3000	United States, Russia, Brazil, Chile, South Africa, Spain, Germany, Belgium, Netherlands, Mexico, Poland, Morocco, Algeria, Italy
2400	United States, Canada, Chile
1650	Spain
1600	Venezuela
1500	United States, Australia, Brazil, Chile, Czechoslovakia, France, India, Japan, Java, Netherlands, New Zealand, Spain, South Africa, Russia, England
1200	United States, Manchuria, Cuba, Japan, Spain, England
1000	Germany
840	Switzerland
800	Italy, Germany
750	Switzerland, Italy, United States
660	Chile
650	United States, Cuba, France, Italy, Japan, England
630	England
600	United States, France, Mexico, England
575	Bolivia
500–250	Industrial in various countries

In the United States, the a-c system is confined to main-line railroads, while the d-c systems are used in industrial operations as well as on main-line railroads.

ELECTRIC LOCOMOTIVE CLASSIFICATION. General Electric Company–American Locomotive Company identify electric locomotives as follows.

	Example
(*a*) Wheel arrangement (see next paragraph)	$2 - C + C - 2$
(*b*) Weight on drivers (in thousands of pounds)	300
(*c*) Total weight (in thousands of pounds)	420
(*d*) Number of traction motors	6
(*e*) Manufacturer's traction motor model number	GEA-627
(*f*) Contact system voltage (nominal)	11,000

which is written: $2 - C + C - 2$–300/420-6GEA-627-11000.

Where the locomotive weight is given in metric units, the weight values are written in metric tons, followed by the letters MT.

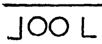
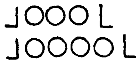
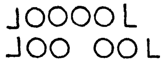
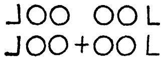
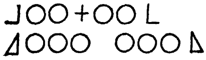
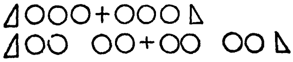
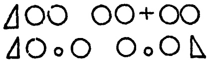
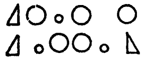
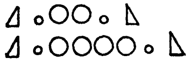
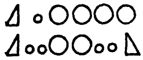
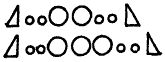
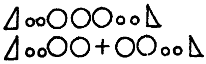
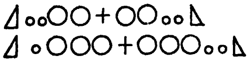
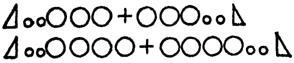
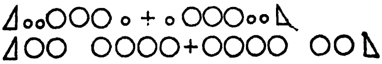
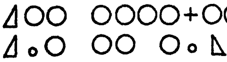
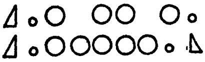
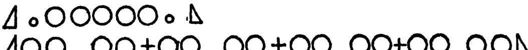
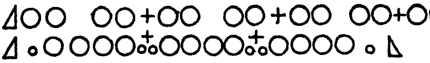
Example: $2 - C + C - 2$–123/165 MT-6GE-729-3000

(See *Gen. Elec. Review*, Oct. 1933, article by W. D. Bearce.)

WHEEL-ARRANGEMENT CLASSIFICATION. The American Railway Association (now Association of American Railroads) in 1932 recommended wheel classification as follows for electric locomotives. The system is also used for other types of locomotives employing electric drive.

	Example
(*a*) Arabic numerals designate nondriving axles	1, 2, 3, etc.
(*b*) Letters designate driving axles	*A*, *B*, *C*, *D*, etc.
(*c*) Plus sign indicates articulation between trucks	+
(*d*) Minus sign indicates absence of articulation between trucks	–
(*e*) Axles in rigid frame are shown together	1*A*1, *A*1*A*
(*f*) Parenthesis sign preceded by Arabic numeral indicates number of identical motive-power units in a locomotive	2(*B* – *B*)

Table 1. Symbolic Designation of Axle Arrangement for Electric Locomotives

Wheel arrangement	Whyte system	Continental system
	0-4-0	*B*
	0-6-0	*C*
	0-8-0	*D*
	0-4-4-0	*B-B*
	0-4+4-0	*B+B*
	0-6-6-0	*C-C'*
	0-6+6-0	*C+C*
	0-4-4+4-4-0	*B-B+B-B*
	0-6-6-0	*A1A-A1A*
	2-4-2	1-*B*-1
	2-8-2	1-*D*-1
	4-4-4	2-*B*-2
	4-6-4	2-*C*-2
	4-4+4-4	2-*B*+*B*-2
	2-6+6-2	1-*C*+*C*-1
	4-6+6-4	2-*C*+*C*-2
	4-8+8-4	2-*D*+*D*-2
	4-6-2+2-6-4	2-*C*-1+1-*C*-2
	4-8-8-4	*B-D+D-B*
	2-2-4-2-2	1*A*-*B*-*A*1
	2-10-2	1-*E*-1
	0-4-4+4-4+4-4+4-4-0	2 (*B*-*B*+*B*-*B*)
	2-8-2+2-8-2+2+8-2	3 (1-*D*+1)

The system is termed the *Continental System*, originating in Europe. Typical examples of wheel arrangements with classification in the Whyte System (for steam locomotives, and sometimes used for electric locomotives) are given in Table 1.

European practice uses a zero subscript to denote the absence of wheel coupling; e.g., $B_0 - B_0$ denotes noncoupled driving wheels in each of two trucks, whereas the absence of subscript zeros indicates that the wheels are coupled. United States practice does not utilize this refinement.

STANDARD METHOD OF RATING ELECTRIC LOCOMOTIVES (Ref. 1). Ratings are taken as at rims of drivers, with locomotive at constant speed on tangent, level track, and are (1) maximum start; (2) one-hour; (3) continuous. At each rating: (1) speed (V), mph; (2) tractive effort (T), pounds; and horsepower (hp) are given. Hp $= VT/375$.

Locomotive ratings are based on motor shaft ratings for maximum start (except as noted below) for 1 hr and continuously, with motors operating at rated voltage. The 1-hr motor ratings are with motors starting cold. Motor torque is reduced 3% when determining tractive effort, to compensate for mechanical losses, unless test data are available. Motor shaft ratings are determined by stand tests, and under agreed conditions of temperature rise, ventilation, etc., except with respect to 1-hr rating.

If test data are not available, tractive effort and speed are determined by $T = 0.97\,(24tG/dP)$; $V = SPd/336.1G$; where t = motor shaft torque, pound-feet; G = number of gear teeth; P = number of pinion teeth; d = driver diameter, inches; S = rpm of motor.

The maximum start rating of locomotive is at the maximum torque exerted by motors with any combination of connections, and at maximum speed attainable with this combination at maximum torque. If tractive effort so derived exceeds 25% of total weight on drivers, tractive effort is taken as 25% of total weight on drivers, and the speed is the maximum attainable at such tractive effort, with any combination of motor connections. The 1-hr and continuous ratings of locomotive are determined from the 1-hr and continuous ratings of motors. See Ref. 2 for details concerning railway motor ratings. In some instances, locomotive ratings are given in terms of speed and tractive effort for arbitrary short-time periods such as 5, 10, 15, 30 min with specified temperature rises on the traction motors.

TRACTIVE RESISTANCE OF ELECTRIC LOCOMOTIVES AND CARS (Ref. 3). Table 2 summarizes formulas derived from an analytical study of various tests and investigations of train resistance. The first two terms of the equations, derived from dynamometer and coasting tests on standard freight and passenger cars and electric locomotives,

Table 2. Train Resistance Formulas for Electric Locomotive and Motor Car Service

Notation. R = tractive resistance, lb per ton (2000 lb) on tangent, level track; A = area, sq ft, of cross section of locomotive or car body and trucks; V = speed, mph; n = No. of axles per car; w = average weight per axle, tons; wn = average weight of locomotive or car, tons.

Values of A. *Locomotives.* 50-ton, 105; 70-ton, 110; 100-ton and over, 120. *Freight cars.* 85–90. *Passenger cars.* 120. *Multiple-unit cars.* 100–110. *Motor cars.* 2-truck, 80–100; 1-truck, 70–75.

Where Used	Usual Formula Recommended for convenience in calculation. Approved for axle weights over 5 tons.	General Formula Applicable to all axle weights. To be used when axle weights are less than 5 tons.
Locomotives	$R = 1.3+\frac{29}{}+0.03V+\frac{0.0024AV^2}{}$	$\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.03V+\frac{0.0024AV^2}{wn}$
Freight cars	$R = 1.3+\frac{29}{}+0.045V+\frac{0.0005AV^2}{}$	$\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.045V+\frac{0.0005AV^2}{wn}$
Passenger cars (vestibuled)	$R = 1.3+\frac{29}{}+0.03V+\frac{0.00034AV^2}{}$	$R = \frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.03V+\frac{0.00034AV^2}{wn}$
Multiple-unit trains:		
Leading car (vestibuled)	$R = 1.3+\frac{29}{w}+0.045V+\frac{0.0024AV^2}{wn}$	$R = \frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.045V+\frac{0.0024AV^2}{wn}$
Trailing cars	$R = 1.3+\frac{29}{}+0.045V+\frac{0.00034AV^2}{}$	$R = \frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.045V+\frac{0.00034AV^2}{wn}$
Motor cars	$R = 1.3+\frac{29}{}+0.09V+\frac{0.0024AV^2}{}$	$\frac{9.4}{\sqrt{w}}+\frac{12.5}{w}+0.09V+\frac{0.0024AV^2}{wn}$

represent journal friction almost entirely, and are based on oil lubrication at average temperatures. Journal friction may increase 20 to 40% at temperatures below 32 F. The third term comprises resistances due to flange friction, concussion, swaying, and other frictions proportional to speed. The factor of this term decreases with increase of truck wheel base, and increases with poor roadbed conditions and inferior riding qualities of motor cars. The last term gives air resistance, pounds per ton, for average weight of car or locomotive for standard types of equipment. No allowance is made for head or strong side winds. Locomotive resistance represents tractive effort delivered to driving axles, excluding friction losses in gears, motor bearings, and other parts of driving equipment. These usually are covered in motive power efficiency. The formulas are based on tests in mild weather conditions. Values obtained from them may be used in calculations of electric distributing systems, substations, energy consumption, and power demand. In determining electric motor characteristics and gear reductions to meet particular speed requirements, it may be desirable to add a small percentage to the required speed, as insurance against unusual conditions.

TOTTEN MODIFICATIONS. Because of the extended use of streamlined shapes for passenger motive power and trains since the Davis formulas were developed, modifications to the last term in the Davis formulas are recommended. A. I. Totten (Ref. 4) recommends these substitutes for the V^2 (air resistance) term in the Davis formulas.

For Locomotives:

$$\frac{(0.023\sqrt[3]{L_L} + K)V^2}{W_L}$$

For Streamlined Lightweight Passenger Cars:

$$\frac{\left[0.0694\left(\frac{L_c}{100}\right)^{0.88} + K\right]V^2}{W_c}$$

For Standard Vestibule Cars Pulled by Streamlined Locomotive:

$$\frac{\left[0.0124\left(\frac{L_c}{100}\right)^{0.7} + K\right]V^2}{W_c}$$

For Streamlined Articulated Train with Power-unit Incorporated as Part of Train:

$$\frac{\left[0.07\left(\frac{L_t}{100}\right)^{0.8} + K\right]V^2}{W_t}$$

in which L_L = length of locomotive, feet; L_c = length of cars in train, feet; L_t = length of train, feet; W_L = weight of locomotive, tons; W_c = weight of cars, tons; W_t = weight of train, tons; V = speed, mph; and K = streamlining factor (see below).

The streamlining factors, K, ascertained by tests reported by DeBell and Lipetz in Ref. 5 are:

K_1 = For power or leading-car nose well streamlined = 0. For nose bluntly streamlined = 0.000036 × cross-sectional area of nose at full section, including trucks, in square feet.

K_2 = For tail shape of rear car well streamlined = 0. For tail bluntly streamlined = 0.000061 × cross-sectional area of tail at full section, including trucks, in square feet.

K_3 = For power-car trucks, both faired = 0. For unfaired trucks = 0.00026.

K_4 = For faired trailing-car trucks = 0. For unfaired trucks = 0.00013 × number of trailing-car trucks.

K_5 = For smooth diaphragms = 0. For cowled diaphragms = 0.000037 × P_e × number of diaphragms.

K_6 = For no bulge of power car = 0. For bulge of good streamline shape = 0.00032 × cross-sectional area of bulge in square feet. For bulge of relatively poor streamline shape = 0.00051 × cross-sectional area of bulge in square feet.

K_7 = For closed wheel shrouds on streamlined locomotives (all wheels completely enclosed) = 0. For open shrouds (2 ft × 2 ft 6 in. inspection openings over the driving-wheel journals) = 0.0005 × total number of openings. For short shrouds (driving wheels and tender trucks completely exposed) = 0.0182.

K_8 = For helmet nose on streamlined locomotive = 0. For straight nose = 0.0021. For round nose = 0.0026.

K_9 = For round-top boiler shape on streamlined locomotive = 0. For cowled top (domes and fittings enclosed in longitudinal cowl above boiler shroud) = 0.0035.

Train resistance in passenger cars with axle-driven electric generators and air-conditioning compressors is increased, depending on horsepower required for the machine drive and the speed of the car. It may be found by the formula

$$R = \frac{\text{hp} \times 375}{V \times W}$$

in which hp = horsepower input to generator, compressor, etc., V = speed, mph; W = weight of car, tons; and R = resistance, pounds per ton.

TYPICAL ELECTRIC LOCOMOTIVES FOR RAILROAD SERVICE. Characteristics and other data are given in Table 3, p. 14-54. Figure 1 illustrates the a-c locomotive of

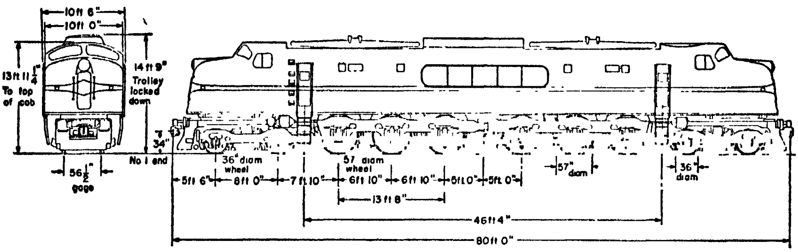


FIG. 1. Electric locomotive of 4780 hp (see Table 3, column *a*).

column (*a*) in Table 3. Characteristic performance curves, typical of an a-c locomotive with series-wound commutator motors, are shown in Fig. 2. Figure 3 depicts a 3000-volt d-c locomotive, column (*g*) in Table 3, with the same wheel arrangement as the a-c locomotive of Fig. 1. Characteristic curves for the 3000-volt d-c locomotive are shown in Fig. 4

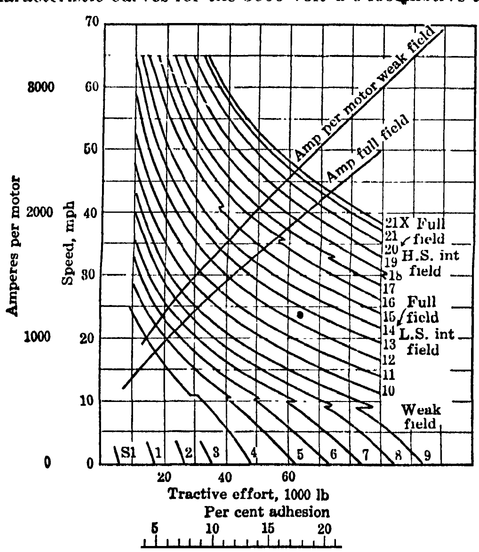


FIG. 2. Performance curves, a-c locomotive, series-wound commutator motors.

(motoring) and Fig. 5 (braking). The 5000-hp a-c motor-generator locomotive, column (*h*) in Table 3, is shown in semi-section in Fig. 6. Characteristic curves, motoring and braking, for this type of a-c locomotive are shown in Figs. 7 and 8, respectively.

(*Continued on p. 14-56*)

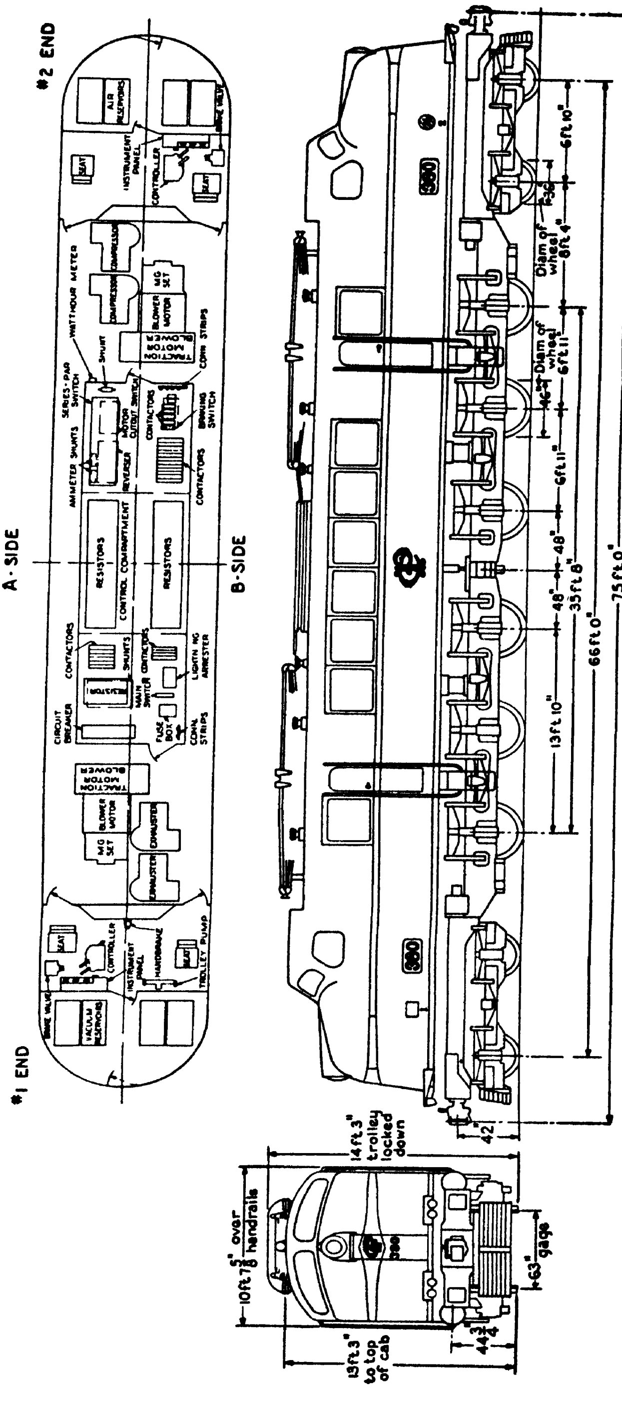


Fig. 3. Electric locomotive of 4050 hp (see Table 3, column *g*).

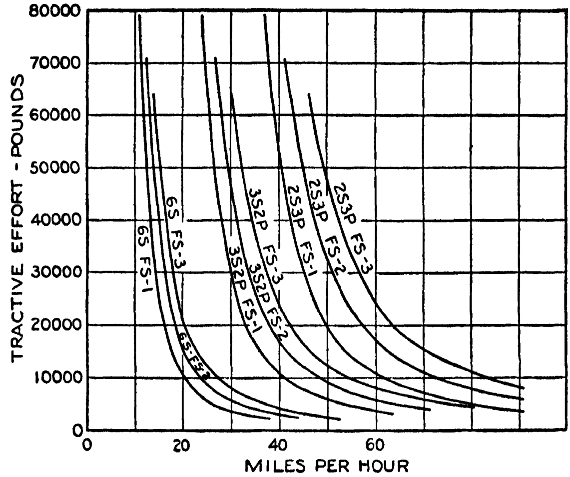


FIG. 4. Locomotive speed-tractive effort curves for Paulista electric locomotive (classification 2 − C + C − 2 − 270/364-6GE729A, 3000 volts).

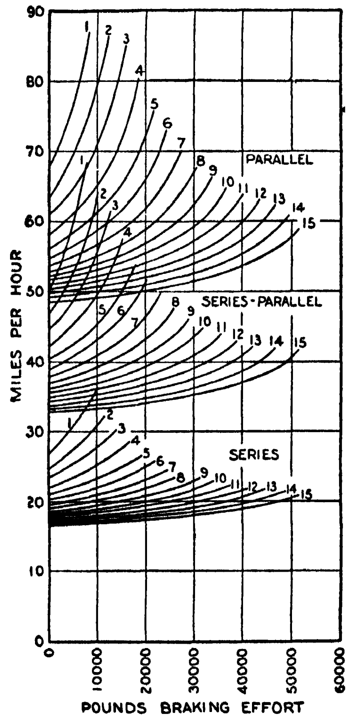


FIG. 5. Locomotive speed-braking effort curves of Paulista electric locomotive with regenerative braking (classification 2 − C + C − 2-270/364-6GE729A, 3000 volts).

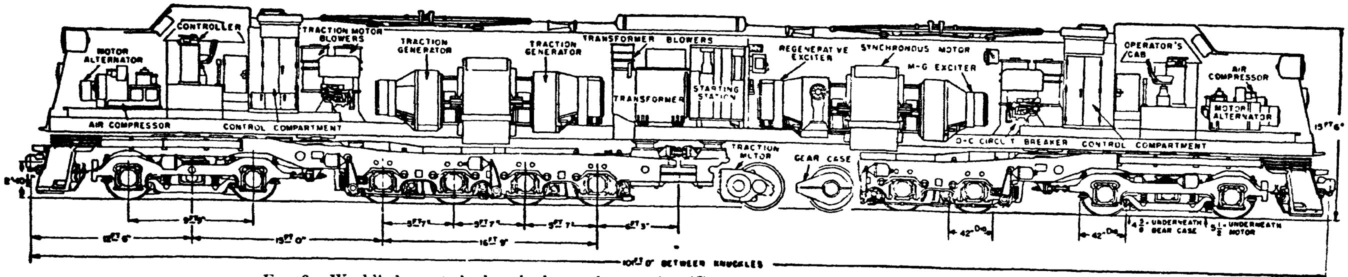


FIG. 6. World's largest single-cab electric locomotive (Great Northern Railway; see Table 3, column h).

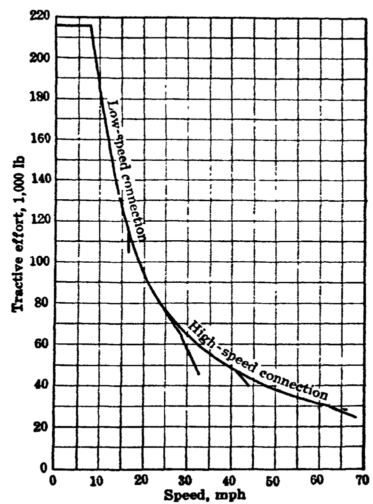


FIG. 7. Speed-tractive effort curve showing low-speed and high-speed connection.

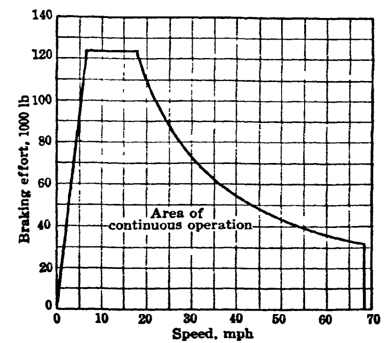


FIG. 8. Speed-braking effort curve.

Table 3. Data on Railroad Electric Locomotives

Column	(a)	(b)		(c)	(d)	(e)	(f)
Fig. No.	1	…		…	…	…	…
Railroad	New Haven	New Haven		N.&W.	P.R.R.	P.R.R.	P.R.R.
Builder, elec.	GE & West.	GE		West.	GE & West.	West.	GE & West.
Builder, mech.	GE & Baldwin	GE		Am. Loco.	P.R.R.	P.R.R.	GE & West.
Wheel arrangement	2–C+C–2	2–C+C–2		2(1–D–1)	2–C–2	2–D–2	2–C+C–2
Service	Frt.	Pass.		Frt.	Frt.	Pass.	Pass.
Series No. or class	0150–0159	0360–0365		LC-2	P-5-A	R-1	GG-1
Placed in service	1942–1943	1938		1925	1935	1936	1935–1946
A-c or d-c	A-c	A-c & D-c		A-c	A-c	A-c	A-c
Contact conductor:							
Voltage, kv	11	11	0.65	11	11	11	11
Type	Cat.	Cat.	3rd rail	Cat.	Cat.	Cat.	Cat.
Current collector	Panto.	Panto.	3rd rail shoe	Panto.	Panto.	Panto.	Panto.
Driving wheels:							
Number/diam, in.	12/56	12/56		16/62	6/72	8/62	12/57
Truck wheels:							
Number/diam, in.	8/36	8/36		8/33	8/36	8/36	8/36
Weight, thousands of pounds:							
Total	492	433.2		828.	394.	402.	477
On drivers	360	272.4		596.	229.	230.	303
Per driving axle	60	45.4		74.5	76 3	57.5	50.5
Mechanical portion	307	269.4		521	238.	266.2	314.2
Equipment	185	163.8		307	156.	135.7	162.8
Length overall	80′-0″	77′-0″		97′-2″	62′-8″	64′-8″	79′-6″
Width overall	10′-0″	10′-0″		10′-11 1/2″	10′-8 1/4″	10′-8 3/16″	10′-4 1/4″
Height, pantograph down	15′-0″	14′-8″		16′-0″	15′-0″	15′-0″	15′-0″
Rigid-wheel base	13′-0″	13′-8″		16′-6″	20′-0″	23′-0″	13′-8″
Total-wheel base	69′-0″	66′-0″		83′-0″	49′-10″	54′-0″	69′-0″
Traction motor:							
Armature, number	12	12		4	6	8	12
Armature mounted	Twin	Twin		Single	Twin	Twin	Twin
Type	SP	SP & DC		IND.	SP	SP	SP
Method of drive	Gear & quill	Gear & quill		Gear & side rods	Gear & quill	Gear & quill	Gear & quill
Gear ratio	4.94	3.39		4.76	3.88	2.74	3.21
Tractive force, thousands of pounds:							
At 25% adhesion	90	68.1		149	57.2	57.5	75.7
		AC	DC				
One-hour rating	…	…	…	108 0 63.0	…	…	…
Continuous rating	46.0	24.1	26.9	90.0 52.5	28.7	18.7	17.3
Horsepower:							
One-hour rating	…	…	…	4060 4750	…	…	…
Continuous rating	4780	3600	2840	3400 4000	3750	5000	4620
Speed, mph:							
One-hour rating	…	…	…	14.1 28.3	…	…	…
Continuous rating	39	56	39 6	14.1 28.3	49	100	100
Maximum	65	93		38	70	100	100
Equipped for:							
Regeneration	No	No		Yes	No	No	No
Multiple-unit	No	No		No	Yes	Yes	Yes

Table 3. Data on Railroad Electric Locomotives—*Continued*

Column	(g)	(h)	(j)	(k)	(l)	(m)
Fig. No.	3	6	. . . .	. . . .	. . . . . .	. . . . .
Railroad	Paulista	Gt. Northern	Virginian		Swiss Federal	Sorocabana
Builder, elec.	GE	GE	GE	GE	Brown Boveri	GE
Builder, mech.	GE	GE	GE	GE	SLW	GE
Wheel arrangement	$2-C+C-2$	$B-D+D-B$	$2(B-B+B-B)$	$2-D+D-2$	$B-B$	$1-C+C-1$
Service	Pass.	Pass. & Frt.	Frt.	Frt.	Pass.	Pass. & Frt.
Series No. or class	. . . . . .	5018–5019	125–128	. .	251–252	. . . .
Placed in service	1939–1947	1947	1947	1948	1944–1945	1944–1948
A-c or d-c	D-c	A-c	A-c	D-c	A-c	D-c
Contact conductor:						
Voltage, kv	3	11	11	3.3	15	3
Type	Cat.	Cat.	Cat.	Cat.	Cat.	Cat.
Current collector	Panto.	Panto.	Panto.	Panto.	Panto.	Panto.
Driving wheels:						
Number/diam, in.	12/46	24/42	32/42	16/47 1/4	8/49 1/8	12/44
Truck wheels:						
Number/diam, in.	8/36	. . . . . .	. . . .	8/37 3/8	. . . .	4/33
Weight, thousands of pounds:						
Total	364	720	1008	405	176	286
On drivers	270	720	1008	545	176	238
Per driving axle	45	60	63	50.7	44	39.7
Mechanical portion	. . . . . .	390	510	. . . . . .	97.5	. . . .
Equipment	. . .	330	498	. . . . .	78.5	. . . .
Length overall	75′-0″	101′-0″	150′-8″	90′-0″	51′-2 1/4″	61′-0″
Width overall	10′-7 1/2″	10′-4 3/8″	10′-4 3/8″	10′-0″	. . . .	9′-7 7/8″
Height, pantograph down	14′-3″	16′-0″	16′-3″	15′-0″	. . . .	13′-7 7/16″
Rigid-wheel base	13′-10″	16′-9″	9′-0″	21′-6″	10′-8″	13′-0″
Total-wheel base	66′-0″	85′-9″	133′-10″	77′-2″	37′-8 7/8″	50′-0″
Traction motor:						
Armature, number	6	12	16	8	4	6
Armature mounted	Single	Single	Single	Single	Single	Single
Type	DC	DC	DC	DC	SP	DC
Method of drive	Gear	Gear	Gear	Gear	Gear & quill	Gear
Gear ratio	2.71	4.11	4.11	3.81	2.22	4.41
Tractive force, thousands of pounds:						
At 25% adhesion	67.5	180	252	101.4	44	59.5
One-hour rating	34.5		. . .	79.5	30.4	29.5
Continuous rating	30	119	162	74	23.8	24.5
Horsepower:						
One-hour rating	4470	.	. . . . . .	5875	3850	2195
Continuous rating	4050	5000	6800	5560	3440	1910
Speed, mph:						
One-hour rating	48.8	. . . .	. . . .	27.7	47.5	27.9
Continuous rating	50.3	15.75	15.75	28.2	51.9	29.3
Maximum	90	65	50	68.8	81.3	56
Equipped for:						
Regeneration	Yes	Yes	Yes	Yes	Rheostatic	Yes
Multiple-unit	Yes	Yes	No	Yes	No	No

Abbreviations: SP = single-phase series-wound; DC = direct-current series-wound; IND. = three-phase induction; Cat. = Catenary; Panto. = Pantograph.

(*a*) These locomotives are designed for operation on alternating current only, and, while intended primarily for freight service, are arranged for addition of a train-heating boiler for passenger-train service.

(*b*) These locomotives are arranged for operation on both an a-c 11-kv overhead contact system and a d-c 650-volt third rail. They are used in passenger service running into Grand Central Terminal, New York City, on the New York Central System 650-volt third-rail terminal electrification.

(*c*) An a-c, split-phase, constant-speed (two running speeds) locomotive. Data given cover two identical cabs operated together as a single locomotive.

(*d*) Originally used in passenger service but later with changed gearing assigned to freight service.

(*e*) A "sample" locomotive noteworthy for the high horsepower rating per ton of locomotive weight on drivers.

(*Notes continued on next page*)

(*f*) Typical of a large number of freight and passenger locomotives operated by the Pennsylvania R.R. between New York, Washington, and Harrisburg, Pa.

(*g*) This locomotive is typical of a number operating in Brazil on 63-in. gage track. No train-heating equipment is provided.

(*h*) Designed for heavy-drag freight service in mountainous territory, these are the longest single-cab electric locomotives in existence. They are motor-generator locomotives with two 3000-hp single-phase a-c to d-c motor-generator sets furnishing power to d-c traction motors.

(*j*) These motor-generator locomotives carry a 4000-hp a-c/d-c motor-generator set in each of the two semipermanently coupled cabs comprising the locomotive. These are the first locomotives with a million pounds on drivers, and are operated in heavy-drag coal-train service.

(*k*) The largest and most powerful single-cab d-c locomotives built to date are for operation on 5-ft-gage track.

(*l*) Notable for the high horsepower per ton of locomotive weight (39.1 continuous). It is equipped with a high-voltage tapping transformer control and rheostatic braking.

(*m*) Illustrative of a large number operating in Brazil on meter-gage track.

15. CONSTRUCTION DETAILS

GENERAL MECHANICAL CONSTRUCTION—ELECTRIC LOCOMOTIVES. Welded construction is employed extensively in modern electric locomotives. With few exceptions, in which aluminum cab sheets have been used, the cab and cab platform are fabricated from rolled steel sheets, steel bars, and structural steel shapes by electric arc or gas welding. In some cases (the locomotive of Fig. 3 is an example) all-welded truck frames are used; in other instances the main truck and guiding truck frames are of cast steel. The latter form predominates in American railroading; fabricated or riveted-plate construction is the common European practice. Truck arrangements are used in almost endless variety (see Table 1 showing wheel classification nomenclature), with a tendency to carry all locomotive weight on driving axles except for high-speed designs which employ (in the United States) a two-axle nonmotored guiding truck at each end of the locomotive. High-speed diesel-electric locomotive operation has indicated the practicability of operating without special guiding trucks, and the tendency on new designs of electric locomotives is towards the utilization of all weight on driving wheels.

Cab shapes for electric road locomotives trend toward the general style made popular by the urge to streamlining employing a projecting portion or "nose" ahead of the operating positions, strongly constructed with "collision posts" for the protection of engine crews in the event of collisions. Where the older type of plain box cab is still employed in new construction, efforts are generally made to smooth out the lines for appearance improvement.

Wheels up to 48 in. in diameter for domestic locomotives are generally of the solid rolled-steel type, whereas locomotives for foreign service built in the United States are in many cases furnished with cast-steel spoked centers and steel tires. The type of drive between the motor and wheel, as well as the diameter, sometimes determines the wheel construction.

Journal bearings may be of the waste-packed oil-lubricated brass-bearing type, or of the antifriction roller grease-lubricated type; there is a tendency to use the latter type on new construction.

Traction motors are of the single-, double- (twin), or triple-armature type, the latter being used on a few European locomotives in which three armatures in a common motor frame drive two driving axles. Practically all modern d-c locomotives built in the United States employ the single-armature, series-wound, commutating-pole, externally ventilated, box-frame motor, axle-mounted and nose-suspended. Motors of this type rating 700 hp continuously are in current use. High-powered a-c locomotives in the United States having the series-wound commutator motor in general utilize twin-armature motors mounted on the running gear frames and driving through quill-gearing and flexible drives. The motor-generator a-c locomotives invariably have the conventional d-c series-wound, axle-hung, nose-suspended motor, with or without flexible gears. Locomotives having side-rod drive utilize frame-mounted a-c series-wound, d-c series-wound, or polyphase induction motors with wound rotors and slip rings.

Although there are many locomotives operating in Europe with rod drive, a marked tendency holds toward new construction eliminating such drives in favor of individual axle drive with gear reduction between each axle and its driving motor. Rod drive has disappeared from United States railroads, except for the induction-motored locomotives on the Virginian and Norfolk and Western. No rod drives have been built in the United States since 1925. Gearless locomotives with motor armatures concentric with the driving axles are no longer built because of the high unsprung weight carried on axles and because of the relatively low horsepower/weight ratio attainable with the slow-speed motors required for such drives.

For the simple single-reduction gear drive with nose-suspended axle-mounted traction motor, the gear may be of the solid or the flexible type. In the latter form a series of springs (leaf or coil) is provided around the circumference of the inner part of the gear and fitting into the outer part carrying the gear teeth. Quill-gear drives are employed wherewith a hollow axle (quill) concentric with the driving axle supports the gear which is connected to the driving-wheel-axle assembly through a system of springs, flexible shafts and disks, rubber pads, or sector-link combinations, which permit limited vertical movement of the wheel-axle assembly without a corresponding movement of the quill-mounted gear.

ELECTRICAL AND OTHER EQUIPMENT. Where contact-system voltages greater than 600 to 800 volts are employed, the pantograph type of current collector is employed, although the wheel-trolley or the bow collector is sometimes used on some smaller road and switching locomotives. Third-rail collector shoes are employed in some instances where voltages are of the order of 800 or lower. Oil-insulated circuit breakers or quick-operating air breakers may be used, but not always, between the pantograph and main transformer on a-c locomotives. Where breakers are not used, lowering of the pantograph on electrical faults in the locomotive is delayed by suitable protective relaying until the substation breakers have cleared the contact system. Direct-current locomotives generally employ a high-speed air-break circuit breaker between the pantograph and equipment. Transformers on new a-c locomotives built in the United States are invariably Askarel-insulated and cooled, the Askarel (a noninflammable oil-like coolant) being pump-circulated and cooled in air-blown heat exchangers. European practice uses the oil-insulated-and-cooled transformer, with primary-winding tap control, in some cases, or with low-voltage winding tap and preventive-coil control. To date, American practice employs only the latter form of control. Older American-built a-c locomotives use the air-blast and, in some cases, oil-insulated transformers.

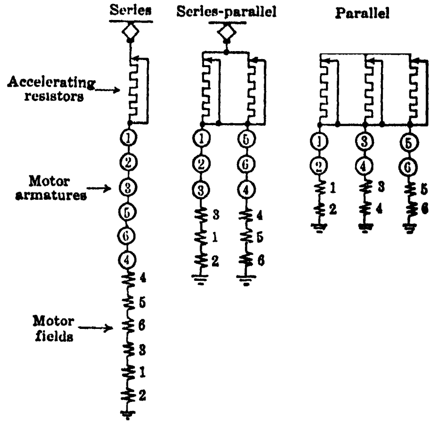


Fig. 9. Direct-current locomotive motor combinations.

Main power circuits and the larger auxiliary circuits are controlled by electropneumatic (electrically controlled, air-operated) contactors, as individual switches or group switches (in some cases cam-actuated with the camshaft pneumatically operated), while the smaller auxiliary circuits and control circuits are handled by electromagnetic contactors, under control of master controllers located at the operating positions. In d-c locomotives, accelerating control is secured by insertion and cutting out in steps resistance in series with the traction motors. A multiplicity of running points is secured by various combinations of motors in series/parallel connections, and by the shunting of traction-motor fields in one or more steps until a minimum of 30 to 40% field strength is secured (see Figs. 4 and 9).

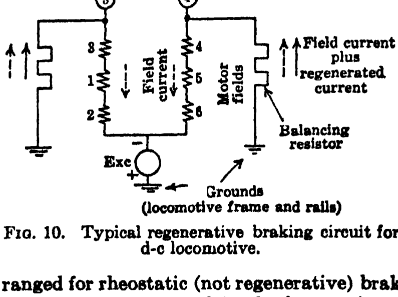


Fig. 10. Typical regenerative braking circuit for d-c locomotive.

Regenerative braking is generally provided for d-c locomotives designed to operate in mountainous territory involving gradients of 2% or greater. Some European a-c locomotives equipped with series-wound commutator motors operating in mountainous territory have in recent years been arranged for rheostatic (not regenerative) braking with the generated energy being dissipated in resistances mounted in the locomotives. In such instances the motor fields may be self-excited or excited from the main transformer. Regenerative braking is secured inherently in induction-motored locomotives by merely permitting the motors to run above synchronous speed. The motor-generator locomotive is readily adaptable to regenerative braking by separate excitation of the traction motor fields (see Fig. 10), and driving the main sets in reverse with the synchronous motors acting as synchronous alternators pumping through the transformers into the supply system. Figures 10, 11, and 12 are fundamental and simplified connection diagrams for d-c locomotives, for a-c series-wound motors with preventive-coil, low-voltage tap control, and for motor-generator locomotives, respectively.

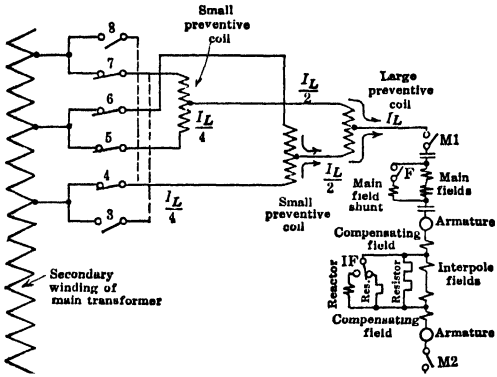


FIG. 11. Connection diagram for a-c series-wound motors with preventive-coil, low-voltage tap control, showing both traction motor and preventive coil circuits.

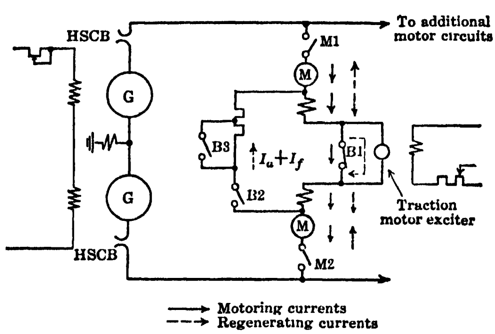


FIG. 12. Fundamental d-c circuits for motor-generator locomotive.

Locomotive mechanical brakes are, in the majority of cases, of the compressed-air-actuated type, regardless of the type of train braking (air or vacuum) employed. Combination systems of locomotive air brakes and vacuum train brakes are employed in many instances outside the United States. American practice employs only the compressed-air system for both locomotive and train braking. Air compressors and vacuum exhausters are of the individually motor-driven type under the control of pressure-operated contactors or relays controlling contactors in the motor circuits.

Motor-driven blowers are employed for furnishing ventilating air under relatively low pressure for the cooling of traction motors, motor-generator sets, and, in some cases, accelerating resistors. In some cases the blower motors are also used for driving d-c generators which furnish control power, battery charging, lighting energy, and, as in the locomotive of Fig. 3, traction-motor field excitation for regenerative braking.

16. INDUSTRIAL ELECTRIC LOCOMOTIVES

Table 4 gives characteristic and dimensional data on typical electric locomotives used in industrial service. They range in size from the small 1 1/2-ton "trammer" locomotive used in underground mining to the larger sizes approximating 125 tons in weight used in

Table 4. Industrial Electric Locomotives

Figure No.	13			14									
Builder, electrical	GE	West.	GE	GE	GE	GE	GE	GE	GE	GE	GE	GE	GE
Builder, mechanical	GE	BLW	GE	GE	GE	GE	GE	GE	GE	GE	GE	GE	GE
Service	Overhaul *	Open-pit Min.	Open-pit Min.	Mine Haulage	Coke Quenching	Dock Pusher	Mine Haulage	Mine Haulage	Mine Gathering	Tunnel Constr.	Mine Trammer	Ore Haulage	Quarry Haulage
Track gage	56 1/2"	56 1/2"	42"	44"	56 1/2"	36"	42"	42"	42"	36"	18–24"	56 1/2"	39 3/8"
Wheel arrangement	*B+B*	*B+B*	*B−B*	*B−B*	*B*	*B*	*C*	*B*	*B*	*B*	*B*	*B+B*	*B−B*
Contact conductor:													
Voltage	3000	1700	600	250	250	250	250	250	250	...	...	600	500
Type	Cat.	Cat.	Pin-suspended	Pin-suspended	3rd Rail	3rd Rail	Pin-suspended	Pin-suspended	Pin-suspended	...	...	Pin-suspended	Pin-suspended
Current collector	Panto.	Panto.	Panto.	Pole and slider	Shoe	Shoe	Pole and slider	Pole and slider	Pole and wheel	...	...	Panto.	Panto.
Driving wheels:													
Number	8	8	8	8	4	4	6	4	4	4	4	8	8
Diameter, in.	50	40	42	31	33	38	34	31	25	28	14	46	33
Weight, thousands of pounds:													
Total	250	242	132	50	40	60	40	30	16	20	3	200	80
On drivers	250	242	132	50	40	60	40	30	16	20	3	200	80
Per driving axle	62.5	60.5	33	12.5	20	30	13.3	15	8	10	1.5	50	20
Length overall	44'-0"	47'-4"	38'-6"	28'-0"	19'-4 1/2"	31'-2"	22'-6"	18'-9"	16'-2"	13'-4"	4'-10"	36'-11"	27'-9 1/2"
Width overall	10'-6"	10'-3"	8'-10"	6'-3"	11'-4 1/2"	4'-9 1/4"	5'-10"	6'-6"	5'-5"	5'-10"	2'-9"	9'-6"	8'-6"
Height overall	18'-6"	16'-10"	13'-0"	4'-2 5/8"	14'-11 1/2"	13'-0"	3'-7"	3'-1"	2'-10"	5'-0"	3'-9"	15'-0"	13'-10 1/2"
Rigid-wheel base	9'-0"	9'-6"	8'-8"	5'-9"	8'-0"	11'-0"	8'-4"	6'-8"	4'-2"	5'-6"	1'-10"	8'-0"	6'-2"
Total-wheel base	20'-0"	29'-6"	26'-2"	14'-11"	8'-0"	11'-0"	8'-4"	6'-8"	4'-2"	5'-6"	1'-10"	25'-2"	19'-3 1/2"
Traction motors:													
Armatures	4	4	4	4	2	2	3	2	2	2	1	4	4
Type	D-c	D-c	D-c	D-c	D-c	D-c	D-c	D-c	D-c	D-c	D-c	D-c	D-c
Method of drive	Gear	Gear	Gear	Gear	Gear	Gear	Gear	Gear	Gear	Gear	Gear	Gear	Gear
Gear ratio	5.46	3.69	3.4	4.5	4.8	5.0	4.8	4.5	4.42	6.35	12.1	4.47	4.8
Tractive force, thousands of pounds:													
At 25% adhesion	62.5	60.5	33	12.5	10	15	10	7.5	4	...	...	50	20
One-hour rating	...	...	23.2	16.8	6.6	10	11.5	8.4	4.1	3	0.4	32.3	16
Continuous	45.2	33	17.9	12.6	1.8	...	7.4	6.3	...	...	...	14.7	10.6
Speed:													
Maximum	40	40	35	40	...	...	...	...	...	...	...	30	35
One-hour rating	...	...	17.4	10.5	8.4	4.5	8.5	10.5	5.3	5.0	3.5	10.5	8.2
Continuous	25.8	15.2	18.7	11.5	13.5	...	9.9	11.5	...	...	...	13.2	9.3
Horsepower:													
One-hour rating	...	...	1075	480	150	125	260	236	60	32	5	908	350
Continuous	3100	1360	894	380	60	...	195	190	...	...	...	517	263

Abbreviations: Cat. = catenary; Panto. = pantograph.
* Equipped for multiple unit operation.

open-pit mining and in line-haul service between mine and mill—a service practically identical with railroad service. Figure 13 illustrates the 3000-volt 125-ton unit carried in Table 4. A "high-speed" mine-haulage locomotive of unusual size for this type of work is shown in Fig. 14, dimensions and apparatus layout.

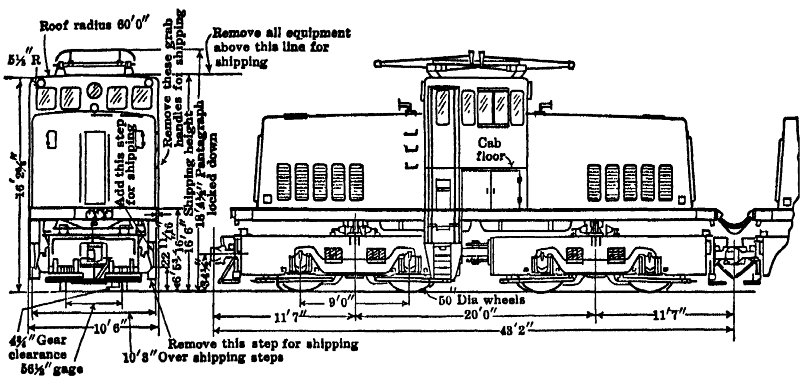


FIG. 13. 125-ton industrial locomotive. (Courtesy of General Electric Co., see Table 4)

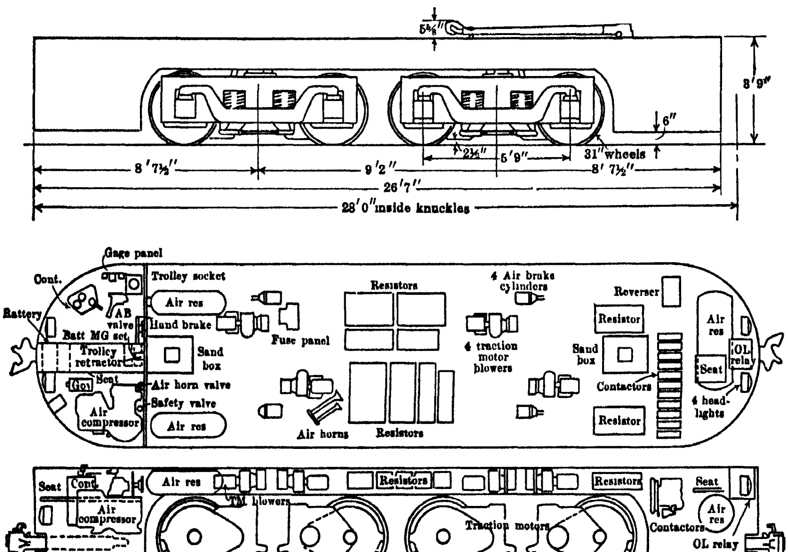


FIG. 14. Twenty-five-ton high-speed mine-haulage locomotive. (Courtesy of General Electric Co.; see Table 4)

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AUTOMOTIVE ENGINEERING *

By Ralph A. Richardson

17. GENERAL INFORMATION

Most automotive vehicles use 4-cycle spark ignition engines, although some trucks and buses use either 4-cycle or 2-cycle high-speed oil or diesel engines. The number of cylinders varies from 4 to 8 per engine, and some buses have 2 engines.

Passenger car engine displacements vary from about 200 to about 360 cu in. Bore and stroke range from 2 $^{15}/_{16}$ by 3 $^{3}/_{4}$ to 3 $^{1}/_{2}$ by 4 $^{5}/_{8}$, developing from 30 to 165 horsepower at a peak of 3600 to 4000 rpm. The wheel base on light cars of 2600 lb is about 112 in., and on the heaviest of 5000 lb about 146 in.

Truck engines are larger than passenger-car engines and are designed for high low-speed torque. The gross weight of trucks varies from 4400 to 95,000 lb.

Buses use one or two truck-type engines. They have a wheel base 148 to 270 in. and weigh 5300 to 20,000 lb.

The SAE recommends maximum body width of 96 in., a maximum vehicle width of fenders of 102 in., a maximum height of 13 ft 6 in. for motor vehicles. The recommended maximum length of a single unit is 35 ft, of a tractor semitrailer 45 ft, and of other combinations, such as a full trailer, 65 ft.

PERFORMANCE FACTORS. Several useful performance factors may be derived if an automobile engine is assumed to be a gas pump that at a given time has approximately the same output per cubic inch of displacement and the same specific fuel consumption as all other engines. Then from known specifications, factors representing car performance, economy, and wear may be determined by which to compare one car with another.

Engine rpm per car mile per hour (mph) is useful for comparing the ratio between engine speed and vehicle speed. It is also useful in deriving the other performance factors. The equation is: Engine rpm per mph $= E_r = (5280 \times R)/(C \times 60)$, where R = rear axle gear ratio; C = rear wheel rolling circumference, feet, $= \pi \times$ tire diameter (approx.). For conventional 3-speed transmissions, E_r varies from 44.6 to 59.2, averaging 50.6. For automatic and semi-automatic transmissions, E_r varies between 38.4 and 59.2, averaging 47.9.

Cubic feet per car mile (Q_c) is an economy factor showing how much mixture the engine pumps per car mile traveled. It assumes all engines to have the same carburetion, breathing capacity, and distribution characteristics, a safe assumption when comparing most automobile engines over a period of a few years. This factor represents gasoline used per mile of car travel. The equation is: $Q_c = (5280R/C) \times (D/2)$. D = engine displacement, cubic feet; R and C are as before. Q_c depends upon the size of the engine and varies widely in the several price fields. For conventional 3-speed transmissions, Q_c varies in the low-price field from 134.2 to 193.1; in the lower medium-price field from 185.4 to 253; and in the upper-price field from 266 to 277.4. For automatic and semi-automatic transmissions, Q_c varies in the low-price field from 123 to 193; in the lower medium-price field from 157.7 to 253; and in the upper-price field from 228 to 277.4. Low values indicate probable low fuel consumption. Figures show savings with automatic transmissions.

Cubic feet per ton mile (Q_t) is a performance factor showing the amount of mixture pumped per ton of vehicle moved per mile, and represents the power-weight ratio. It is assumed that all engines develop the same brake mep at the same road speeds. This is sufficiently accurate for purposes of comparison. The equation is: $Q_t = (5280RD/2C) \times (2000/W)$, where W = car weight, pounds, loaded with passengers, gas, oil, water, and equipment; R, D, and C are as before. For conventional 3-speed transmissions, Q_t ranges from 94.8 to 120.8, averaging 105.9. For automatic and semi-automatic transmissions it ranges from 73.2 to 120.8, averaging 101.1. A high value indicates high performance in high gear with conventional transmissions.

Feet of piston travel per car mile (T_p) is a factor indicative of engine wear. The equation is: $T_p = (5280R/C) \times 2S$, where S = stroke, feet; R and C are as before. For conventional 3-speed transmissions, T_p ranges from 1673 to 2566, averaging 2131. For automatic and semi-automatic transmissions, T_p ranges from 1540 to 2566, averaging 2010. Low values indicate low wear.

COEFFICIENT OF FRICTION BETWEEN TIRES AND ROAD SURFACE is the force required to cause tires to slide, divided by normal pressure between tires and road.

* For additional data on internal combustion engines, see Section 13.

It has two values: F, when sliding is impending; f, when sliding is under way. (See Table 1.) If P_s = force required to start sliding, P_u = force required for uniform sliding, w = normal pressure between tire and road, then $F = P_s/w$ and $f = P_u/w$. With hard-packed snow on pavement, F may be from 0.17 to 0.20; f from 0.12 to 0.15. Ice and sleet on pavement reduce F to as low as 0.08, and f to 0.07.

Table 1. Average Coefficients of Friction between Tires and Road Surfaces, Sliding in the Line of Travel

(*Bulletin* 88, Iowa State College)

Surface	Wet Road Surface		Dry Road Surface	
	F	f	F	f
Portland cement concrete, 2 years old	.89	.81	.96	.85
Portland cement concrete, 5 years old, greasy	.96	.89	.64	.54
Asphaltic concrete	.87	.79	.86	.82
Bitulithic	.69	.61	.73	.72
Wood block	.82	.75	.81	.60
Brick monolithic	.91	.82	.60	.54
Brick, sand filled	.87	.79	.62	.53
Brick, asphalt filled	.85	.75	.81	.75
Gravel	.75	.65	.79	.68
Earth	.68	.65	....	....

THE RESISTANCE OF MOTOR VEHICLES on level road is made up of rolling resistance and air resistance. Grade resistance is an additional factor on hills. Knowing total resistance, the horsepower required to propel the vehicle at any speed may be calculated.

Rolling resistance depends on road surface characteristics, type and condition of tires, and friction in bearings. It ranges from 10 to 30 lb per 1000 lb of vehicle on smooth roads. On rough gravel or dirt roads it may be 100 lb or more. An average value for smooth concrete or macadam is 12.

Let K equal rolling resistance coefficient in pounds per pound of car weight, W equal pounds car weight and R_r equal rolling resistance; then $R_r = KW$. For most purposes in design K is taken as 0.012.

Air resistance depends on the aerodynamic characteristics of the body. It varies as the square of the speed and is of greatest importance at speeds above 50 mph. Air resistance is directly proportional to the projected frontal area, the shape of the body, and the square of the wind velocity. Let R_a = air resistance; K_1 = coefficient of air resistance; A = projected frontal area in square feet; and V = velocity of the air past the body in mph. Then $R_a = K_1AV^2$. K_1 varies with the shape of the body, being greatest for shapes with sharp corners and large flat areas normal to the wind, and least for streamlined shapes. K_1 varies from 0.0010 to 0.0020. The value of K_1 for 4-door sedans is approximately 0.00125.

Frontal areas of passenger cars vary from 26 to 32 sq ft. This factor is determined largely by the passenger-seating capacity. Wide bodies tend to increase the value; low bodies decrease it.

Power required to drive the car at any speed is determined from the air and rolling resistance. Let hp = horsepower for overcoming total resistance. Then hp = $(KW + K_1AV^2)V/375$; K, K_1, W, and V, as before.

18. ENGINE DETAILS

THE COOLING SYSTEM dissipates waste heat. Several types are possible. Water cooling, with either thermosyphon or pump circulation, has been most used. Direct air cooling has been successful. An evaporative cooling system, in which the water surrounding cylinders is always at boiling temperature, also is possible. In this system the radiator is a condenser for the steam.

In design of the water-cooled system the most severe load is used, viz., that imposed by air temperature of 110 F, and car speed of 15 to 25 mph (1000 engine rpm) with wide-open throttle. The quantity of heat to be dissipated varies with engine design, being greatest for L-head, and least for valve-in-head engines. Specific heat of cooling water varies between 45 and 75 Btu per bhp per min at 1000 rpm. Design of the system depends on variables associated with the fan, water pump, and radiator section. Figure 1 shows heat dissipation from a typical passenger car radiator section. A temperature rise of 80 F is the maximum possible with existing sections, and is the difference between air and upper radiator tank water temperatures. For best operation, upper tank temperatures should be approximately 165 F, although under most severe load 210 F may be tolerated.

A propeller fan delivers the proper amount of air through the radiator. Fan horsepower varies as (speed)^3, while delivery, cubic feet per minute, varies directly with speed. (See Section 1.)

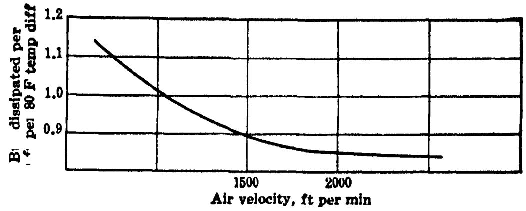


FIG. 1. Heat dissipation from a typical passenger car radiator section.

Thermostats to obtain quick warming of cylinder walls, especially in winter, prevent crankcase dilution. They begin to open at 135 F, and are fully open at 140 F. Radiator shutters, thermostatically operated, may be used to obtain rapid warming.

ANTI-FREEZE in the radiator is necessary in all Northern climates. Ethanol (denatured ethyl alcohol), methanol (synthetic methyl alcohol), isopropyl alcohol, and ethylene glycol are acceptable anti-freezes. They usually contain inhibitors to increase chemical stability and prevent corrosion. Figure 2 shows the freezing point of three common anti-freeze solutions. Oils, sugar, glucose, honey, or salt solutions are unsatisfactory for automobile radiators.

When water freezes its volume increases 9%, which might cause radiator or cylinder jacket breakage. Anti-freeze solutions tend to form slush slightly below their freezing points. When not enough anti-freeze is used, there is a greater hazard from overheating and heat cracking than from cracking by freezing.

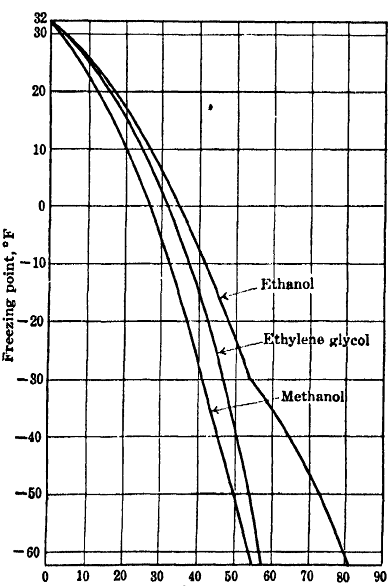


FIG. 2. Freezing points of common anti-freeze solutions.

Alcohol solutions cannot be used in systems having a thermostat opening at 165 F or higher. The boiling point of anti-freeze solutions drops about 2 F for every 1000 ft increase in altitude. Pressure caps holding radiator pressure at 5 psig increase the boiling point by 13 to 15 F.

CYLINDER ARRANGEMENT AND NUMBER are usually determined by the economics of manufacture, and by vibration characteristics. Figure 3 compares the forces in engines of most of the usual cylinder combinations. The analysis was made on engines of the same displacement, and the forces are plotted to scale.

Cylinders of passenger cars usually are cast iron, cast en block. In truck and bus engines, steel cylinder liners often are used. The minimum thickness of section recommended for cast iron is $^3/_{16}$ in., but up to $^1/_4$ in. may be used for large-bore cylinders for passenger cars and trucks. Average thickness for passenger cars is $^7/_{32}$ in. Minimum thickness of steel sleeves with radial stiffening ribs is $^1/_8$ in.; inserted steel sleeves usually are $^3/_{16}$ in. thick. Cylinder blocks should have a Brinell hardness of 187 to 202 in the cylinder bore and at the valve seats. If these parts are so located in the casting as to cool slowly, they will be hard at the edges, corners, and thin sections, and of hard machinability if made of unalloyed iron giving above hardness values. Additions of Ni and Cr counteract this tendency. An addition of 1 $^1/_2$% Ni also lowers the coefficient of expansion. Tensile strength should be 35,000 psi when cast in a $^1/_2$-in. section. Table 2 gives composition of cast irons much used for cylinders.

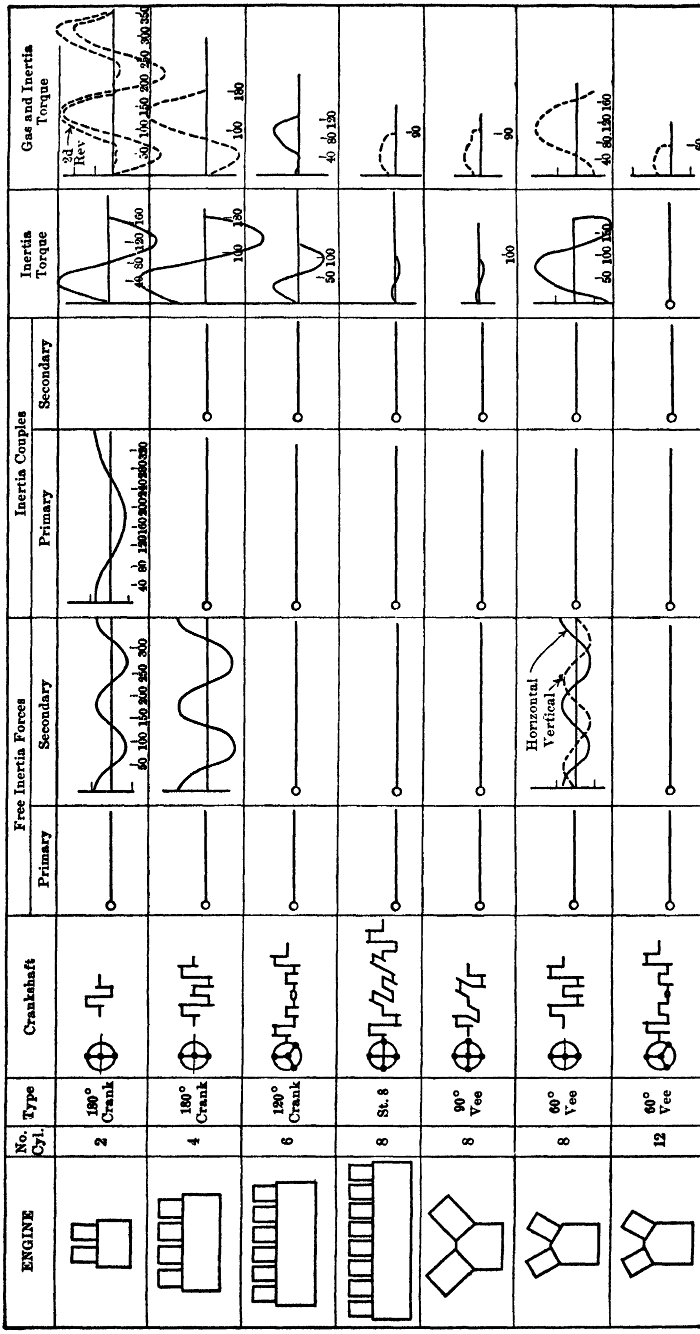


FIG. 3. Forces acting in engines.

Table 2. Compositions of Cast Irons for Automotive Engine Cylinders

(A. L. Boegehold, Regional Meeting, ASTM, Detroit, March 1930)

Cylinder Iron No.	C, total	Mn	P, max	S, max	Si	Cr	Ni
1	3.15–3.30	0.55–0.75	0.20	0.12	2.00–2.40	0.06–0.16	0.03–0.08
2	3.15–3.30	0.55–0.75	0.20	0.12	2.00–2.40	0.05–0.07	0.25–0.35
3	3.15–3.50	0.50–0.80	0.20	0.12	1.75–2.25		1.00–1.50

CRANKSHAFTS for passenger car and truck engines are usually steel forgings, although castings have been used successfully. Alloy steels are not necessary since the bearing requirements result in large enough sections to make plain carbon steel adequate in strength. In high-compression engines great stiffness is required to carry the high loads without excessive deflection which results in rough engine operation. A bearing between each crank throw or between each two or three throws have all been in successful use. For compression ratios above 8 to 1, a main bearing between each throw is desirable. Vee-type engines result in short stiff crankshafts.

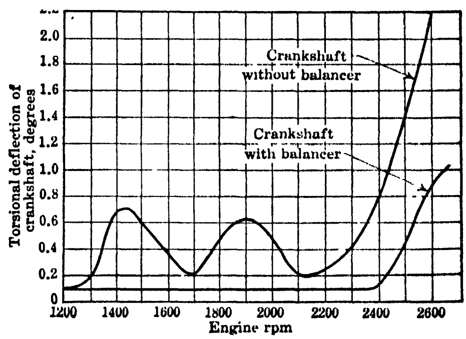


FIG. 4. Torsional vibration of crankshaft.

Crankshaft Vibration. Crankshafts are counterweighted to reduce centrifugal forces and bearing loads. In practice, 50% of the centrifugal forces should be balanced by the counterweights to obtain satisfactory smoothness. Static and dynamic balance are required. The limits vary from 1/4 to 1 oz-in. In long crankshafts (6 or more cylinders), torsional vibration at resonant speeds produces vibration which must be eliminated. Four-cylinder or V-8 engines usually have a short stiff crankshaft with a natural frequency above normal driving speeds. Torsional vibration in an automobile engine crankshaft is produced by a resonance of the torque impulses of the engine with the natural frequency of torsional vibration of the crankshaft; the impulses are produced by the gas pressure on the piston and by inertia forces of reciprocating parts. Gas pressures recur every two engine revolutions, and inertia forces every revolution. The forces are periodic and may be represented by harmonic sine functions, $f \times$ rpm, where f = frequency = 1/2, 1, 1 1/2, 2, 2 1/2, 3, 3 1/2, 4, 4 1/2, 5, 5 1/2, 6, etc. The crankshaft responds to excitation by the gas or inertia forces with a vibration whose frequency equals that of the exciting forces. The natural frequency of torsional vibration of a crankshaft is determined by the shaft stiffness and inertia of the shaft, connecting rods, and pistons. It is proportional to (stiffness ÷ inertia). Since long crankshafts cannot be built stiff enough to make the natural frequency of torsional vibration occur above normal driving range, a balancer is necessary. A torsion balancer or Lanchester damper are two types generally used. Figure 4 shows the amplitude of vibration of a crankshaft with and without a torsion balancer.

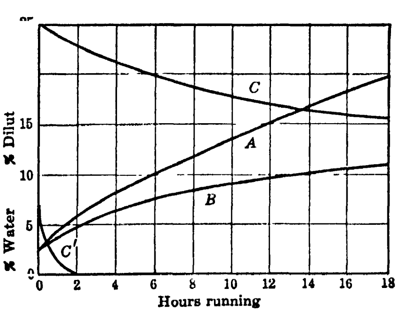


FIG. 5. Dilution of oil versus hours of running. *A*. Fresh oil, standard operation. *B*. Fresh oil, one-third of carburetor air drawn through crankcase. *C*. Polluted oil, one-third of carburetor air drawn through crankcase. *C'*. Water in oil, one-third of carburetor air drawn through crankcase. Fuel used; 70% gasoline, 30% kerosene.

CRANKCASE AND CRANKCASE VENTILATION. The upper half of the crankcase may be either cast iron or aluminum alloy, cast with the cylinders or separately. It must be sufficiently rigid to prevent deflection due to engine operation, misaligned crankshaft bearings, camshaft bearings, and camshaft drive mechanism. The lower half of the crankcase is an oil reservoir, holding 5 to 10 qt, depending on size of the engine. American practice provides ventilation through the crankcase to remove water and light ends of the fuel which blow past the pistons. This water is washed from the walls by the oil and deposited in the crankcase. During choking, especially in cold weather, large quantities of fuel also may be blown past the pistons into the oil. Water in the crankcase tends to corrode bearing surfaces of the crankshaft, piston pins, cylinder walls, and valve gear. Corrosion is most severe with gasolines high in sulfur. A blast of air through the crankcase holds oil dilution and water content to a minimum. (See Fig. 5.) Figure 6 shows tests of fuel dilution and water dilution on cylinder walls of an automobile engine which had stood out all night before the test. Initial jacket water temperature was 5 F.

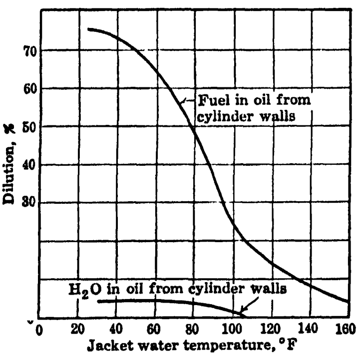


FIG. 6. Effect of jacket water temperature on dilution of oil from cylinder walls.

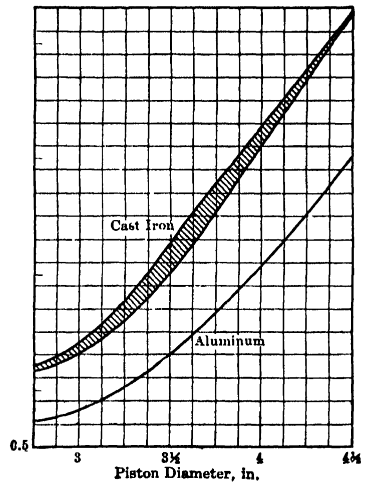


FIG. 7. Relative weight of cast-iron and aluminum pistons.

PISTONS AND RINGS. **Piston materials** in commonest use are aluminum alloys or cast iron. Steel may be used. Cast iron gives long life with low wear and can be fitted to closer limits in the cylinder. Aluminum alloys have the advantage of light weight and higher heat conductivity. Figure 7 compares the weights of typical aluminum and cast-iron pistons. The weight of the piston assembly is held to within limits of $^1/_{16}$ to $^3/_{16}$ oz to insure good engine balance. Clearance for cast-iron pistons can be taken as 0.00075 to 0.001 in. per in. of cylinder bore and as much as double this for aluminum, depending on the design. No exact rule is possible since clearance depends on piston design and duty.

In modern high-speed automotive engines, lubrication, scuffing, and piston wear are problems. Lead alloy, tin plating, or phosphate coatings are used on cast-iron or steel pistons. An anodic treatment or tin plating is used on aluminum pistons.

Rings. The piston is fitted with 3 or 4 piston rings. The upper rings are usually plain, and the lower one or two are oil-control rings. Table 3 shows the SAE recommended practice of ring widths for various cylinder diameters. Piston ring performance is dependent on proper piston design. When maximum temperatures behind the top ring groove are held below 350 to 400 F, no trouble is encountered with rings. It is general practice to drill a series of holes in the ring groove as drain holes behind the oil-control rings. These holes number 8 to 16, depending on the piston size, and are $^1/_{32}$ in. less in diameter than the ring width.

Table 3. Ring Widths for Cylinder Diameters

Cylinder Diameter, in.	Ring Width, in.	
	Compression Ring	Oil Ring
1 to 2 $^{15}/_{16}$	$^3/_{32}$	$^1/_8$ to $^3/_{16}$
3 to 3 $^{15}/_{16}$	$^3/_{32}$ to $^1/_8$	$^5/_{32}$ to $^1/_4$
4 to 5 $^{15}/_{16}$	$^3/_{32}$ to $^5/_{32}$	$^3/_{16}$ to $^1/_4$

VALVES AND VALVE MECHANISM. Figure 8 shows the most common arrangement of valves. The I head, commonly known as valve-in-head or overhead valve, gives a a simple combustion chamber with a minimum heat loss to cooling water due to the small water jacketing. The L head gives simplified valve action. The T head requires two camshafts, but permits large valves and low lifts. It requires the greatest amount of water jacketing. The F head has the intake valve over the exhaust valve, and is a combination of overhead and L valves.

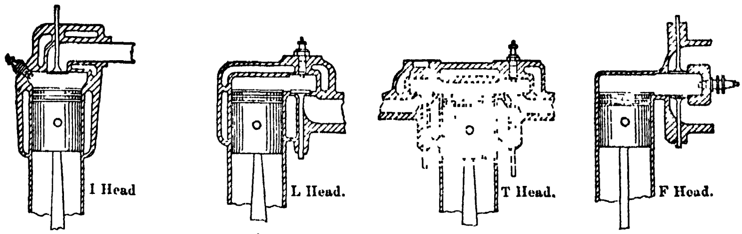


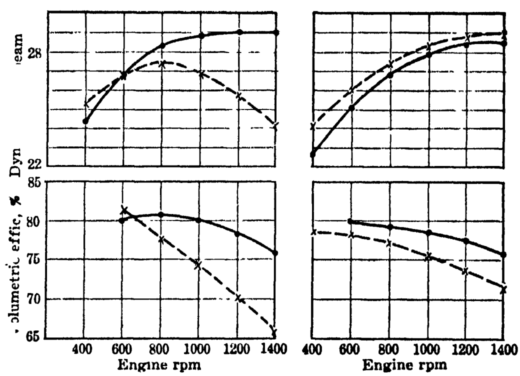
FIG. 8. Valve arrangements.

Valve material must withstand high temperature without corrosion. For intake valves, which operate cooler than exhaust valves, SAE 3140 or chrome-nickel steel is used. Exhaust valves are made of Silchrome, chrome nickel, or tungsten steel.

Valve mechanism design depends on many variables. The valve timing selected can give power either at high or at low engine speeds. To obtain best performance at high speed entails a sacrifice at low speed and vice versa. A compromise must be made to obtain the results desired. Figure 9 shows the effect of varying intake timing on a single-cylinder test engine, 3 $^1/_8$ in. bore, 4 $^1/_2$ in. stroke, with overhead valves.

Cam shape has much to do with the operation of the valves. Moving parts should be of light weight to reduce inertia forces. Good valve follow should be sought. This requires (1) a maximum of acceleration in as short a time as rigidity of the system will permit; (2) a minimum of deceleration continued over as long a time as is consistent with (1); and (3) a minimum of reversals of forces in the acceleration curve. A cam contour so designed should give maximum volumetric efficiency and optimum valve follow.

Valve Springs. Valves are closed by helical coiled springs. One of the greatest difficulties with such a spring is its tendency to vibrate or "surge" at definite speeds, depending on the natural vibration frequency of the spring. This frequency can be calculated by Ricardo's formula for helical springs, $n = 531\sqrt{R/W}$, where n = frequency, vibrations per minute, R = scale of the spring = load, pounds, necessary to deflect it 1 in., W = weight of active part, pounds. If (cam rpm × an integer) = n, a resonant vibration results. At low speeds, the amplitude is small, but at some operating speeds it becomes large enough to cause the coils to close and the springs to produce a sound.



Inlet	Opens	Closes	Total opening	Opens	Closes	Total opening
×——×	14°BTC	28°ABC	222°	14°BTC	68°ABC	262°
•——•	14°BTC	68°ABC	262°	5°BTC	77°ABC	262°
Exhaust	77°BBC	32°ATC	289°	77°BBC	32°ATC	289°

FIG. 9. The effect of varying intake valve timing on single-cylinder test engine.

Formulas used to calculate round wire springs are given in the *Design and Production* volume of Kent's *Mechanical Engineers' Handbook.*

Air Flow Through Valves. Valve port size, timing, and lift must be proportioned correctly to obtain high volumetric efficiency. Valve lift usually is about one-fourth port diameter; in slow-speed engines a material gain in air flow can be obtained by lifts of one-third port diameter. The air flow through a valve port with 30-degree seats is greater than through one with 45-degree seats. Gas velocity through the port is given by the approximate formula, $V = D^2sN/8qdh$, where D = cylinder diameter, inches; V = gas velocity, feet per second; s = piston stroke, inches; d = port diameter, inches; h = mean valve lift (inches from valve lift diagram); q = period of opening, degree of crankshaft rotation. This formula assumes the valve to be open for the duration shown on the diagram, and the valve annulus to be a cylindrical surface. If the valve is tapered, a correction factor should be applied as follows:

For 30-degree seats,

$$A_1 = A_2\left(0.866 + 0.375\frac{h}{d}\right)$$

For 45-degree seats,

$$A_1 = A_2\left(0.707 + 0.353\frac{h}{d}\right)$$

Then

$$V_2 \quad V_1\frac{A_2}{A_1}$$

where V_2 = velocity corrected for seat angle; V_1 = velocity, assuming a flat seat; A_1 = area of conical annulus, square inches; and A_2 = area of cylindrical annulus, square inches.

ENGINE LUBRICATION SYSTEM must supply oil to the major parts of the engine, such as the crankshaft main bearings, connecting rod bearings, piston pins, cylinder walls, camshaft, valve mechanism, and timing gear or chain. This is done either by splash or positive pressure or a combination of the two. The pressure system is most used for high-speed high-output engines. An oil pump located in the oil pan circulates oil under a pressure to the main connecting rod, camshaft and valve mechanism bearings. A pressure of 30 to 40 psi at 30-mph car speed is usual. A by-pass relief valve is used to avoid excessive pressure in cold weather and at high speed. The cylinder walls are lubricated by the spray thrown off by the connecting rod and piston pin bearings

An oil filter and **an oil cooler** may be used. The oil passes through the filter, which removes dirt, metal particles, and other contaminants that might cause rapid wear or bearing failure. The oil cooler keeps the temperature low in summer and provides for rapid warm-up and higher stabilized temperature in winter.

Motor oils for automobile engines are classified by SAE standards of viscosity (Table 4). An Automotive Manufacturers' viscosity classification is used for winter lubricating oils (Table 5). These tables classify oils by viscosity only. Oils are also classified as *Regular*,

Table 4. SAE Viscosity Classification

SAE Viscosity Number	Viscosity Range, SSU			
	At 130 F		At 210 F	
	Min	Max	Min	Max
10	90	120	......	......
20	120	185	......	......
30	185	255	......	......
40	255	......	......	80
50	......	......	80	105
60	......	......	105	125
70	......	......	125	150

Table 5. Automotive Manufacturers' Viscosity Classification

Viscosity Number	Viscosity Range at 0 F, SSU	
	Min	Max
10W	5,000	10,000
20W	10,000	40,000

Premium, or *Heavy Duty* type, by the Lubrication Committee, Division of Marketing of the American Petroleum Institute. Regular oils are suitable for normal use in most types of passenger-car engines when operated under ordinary driving conditions.

If the service is more severe, where full throttle operation is a larger percentage of the total, such as in mountain climbing, fast acceleration, or sustained high speed, temperature of practically all operating parts will be higher. Excessive heat is harmful to lubricating oils, resulting in oxidation products, which may contaminate the oil and form harmful deposits on engine parts. Oils having improved stability and oxidation resistance, designated at Premium type, may be required under these more severe conditions.

Oils possessing *detergent qualities*, in addition to improved oxidation resistance, may be required in heavy-duty operations, such as sustained high speed under heavy loads in some truck and bus operations. Heavy Duty type oils which have proved oxidation stability, bearing corrosion-resistance properties, and detergent-dispersant characteristics, are used in this service.

19. ENGINE DESIGN

KINEMATICS OF THE ENGINE (*Air Service Information Circ.* 421, by H. Camines and C. W. Isler). To determine the principal forces acting in the engine, the motion of the pistons and the connecting rods relative to the crankshaft is found; the angular velocity is taken as constant because of the large inertia of the flywheel and uniformity of torque in multicylinder engines. In Fig. 10*a*, which is a diagram of the usual arrangement of pistons, connecting rod, and crankshaft, let L = connecting-rod length, center-to-center, inches = BD; R = crank radius, inches = OD; θ = crank angle from top center position, degrees; ϕ = angle of connecting rod with center line of cylinder, degrees; s = piston travel, inches = AB.

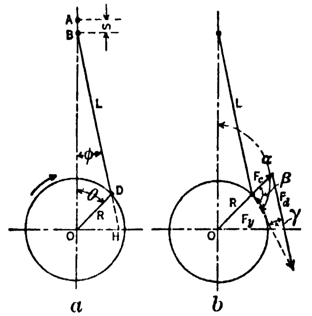


FIG. 10. Piston, crankshaft, and connecting rod geometric relations.

Piston- and connecting-rod position for various crank angles can be found by a diagram drawn to scale or by mathematical analysis. With the arrangement of Fig. 10*a*,

$$s = (R - R\cos\theta) + (L - L\cos\phi) \tag{1}$$

and

$$\phi = \sin^{-1}\left(\frac{R\sin\theta}{L}\right) \tag{2}$$

Combining eqs. 1 and 2,

$$\text{Piston travel} = s = R\left[1 - \cos\theta + (L/R) - \sqrt{(L/R)^2 - \sin^2\theta}\right] \tag{3}$$

and

$$\text{Percentage of piston travel} = \frac{s}{2R} = {}^1/_2\left[1 - \cos\theta + (L/R) - \sqrt{(L/R)^2 - \sin^2\theta}\right] \tag{4}$$

Equation 4 has been solved at every 10 degrees of crank angle for the usual ratios of (L/R) found in automobile engines. (See Table 6.)

Velocity. Let v_c = crankpin velocity, feet per second; v_p = piston velocity, feet per second; N = rpm; R = crank radius, inches; $(d\theta/dt)$ = angular velocity of connecting-rod; f_v = crank angle factor for piston velocity. Then, referring to Fig. 10*b*, and considering linear velocity of the crankpin as $v_c = R(d\theta/dt) = (2\pi N/60) \times (R/12)$,

$$\frac{d\theta}{dt} = \frac{R}{L}\frac{\cos\theta}{\cos\phi}\frac{d\theta}{dt} = \frac{\cos\theta}{L\cos\phi}, \quad \frac{\cos\theta}{\sqrt{L^2 - R^2\sin^2\theta}} \tag{5}$$

$$v_p = \frac{ds}{dt} = v_c(\sin\theta + \cos\theta\tan\phi) = v_c f_v \tag{6}$$

$$\sin\theta + \cos\theta\tan\phi = \sin\theta\left(1 + \frac{\cos\theta}{\sqrt{(L/R)^2 - \sin^2\theta}}\right) \tag{7}$$

The crank angle factor for piston velocity, f_v, may be found graphically. In Fig. 10*a* it is *OH/OD*. Table 7 gives values of f_v for ratios of L/R generally used in automobile engines.

Since ϕ, the connecting-rod angle, is small, the resulting error from substitution of sin ϕ for tan ϕ in eqs. 6 and 7 will be small. Making this substitution, the approximate equations are

$$v_p = v_c[\sin\theta + {}^1/_2(R/L)\sin 2\theta] \tag{8}$$

$$f_v = \sin\theta + {}^1/_2(R/L)\sin 2\theta \tag{9}$$

Acceleration. The exact expression for piston acceleration for the arrangement in Fig. 10a is complex and not commonly used. The approximate equation in general use is

$$a = (v_c^2/R)[\cos\theta + (R/L)\cos 2\theta] = (v_c^2/R) \times f_a \tag{10}$$

where a = linear acceleration of piston; f_a = crank angle factor for piston acceleration.

$$f_a = \cos\theta + (R/L)(\cos^2\theta/\cos^3\theta) - \sin\theta\tan\phi \quad \text{(exact)} \tag{11}$$

$$f_a = \cos\theta + (R/L)\cos 2\theta \quad \text{(approx.)} \tag{12}$$

Table 8 gives values of f_a for ratios of L/R generally used in automobile engines. Table 9 is a comparison of exact and approximate values.

Table 6. Percentage of Piston Travel, $s/2R$ (see Eq. 4)

Crank Angle, degrees		Values of L/R							
		3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4
		Percentage of Piston Travel							
0	360	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	350	1.0	1.0	1.0	1.0	0.9	0.9	0.9	0.9
20	340	4.0	3.9	3.8	3.8	3.7	3.7	3.7	3.6
30	330	8.8	8.7	8.6	8.5	8.4	8.3	8.2	8.1
40	320	15.2	15.0	14.8	14.6	14.4	14.3	14.2	14.1
50	310	22.8	22.5	22.3	22.1	21.8	21.6	21.4	21.2
60	300	31.4	31.0	30.6	30.3	30.0	29.7	29.5	29.3
70	290	40.5	40.0	39.6	39.2	38.8	38.5	38.2	37.9
80	280	49.6	49.1	48.7	48.2	47.8	47.5	47.2	46.9
90	270	58.6	58.0	57.5	57.1	56.7	56.4	56.1	55.8
100	260	67.0	66.5	66.0	65.6	65.2	64.8	64.5	64.2
110	250	74.7	74.2	73.8	73.4	73.0	72.7	72.4	72.1
120	240	81.4	81.0	80.6	80.3	80.0	79.7	79.5	79.3
130	230	87.1	86.8	86.5	86.3	86.0	85.8	85.6	85.3
140	220	91.8	91.6	91.4	91.2	91.0	90.9	90.8	90.7
150	210	95.4	95.3	95.2	95.1	95.0	94.9	94.8	94.7
160	200	98.0	97.9	97.8	97.8	97.7	97.7	97.7	97.7
170	190	99.5	99.5	99.4	99.4	99.3	99.3	99.3	99.3
180	180	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Table 7. Crank Angle Factor for Piston Velocity, f_v (see Eq. 6)

Crank Angle, degrees		Values of L/R							
		3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4
0	360	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	350	0.231	0.227	0.224	0.221	0.218	0.216	0.214	0.212
20	340	0.450	0.443	0.437	0.432	0.427	0.423	0.419	0.415
30	330	0.646	0.637	0.629	0.622	0.615	0.609	0.604	0.599
40	320	0.811	0.799	0.790	0.782	0.775	0.768	0.762	0.756
50	310	0.936	0.924	0.915	0.906	0.898	0.891	0.885	0.880
60	300	1.017	1.007	0.998	0.990	0.983	0.977	0.971	0.966
70	290	1.053	1.045	1.038	1.032	1.027	1.022	1.018	1.015
80	280	1.045	1.041	1.037	1.034	1.031	1.029	1.027	1.025
90	270	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	260	0.925	0.929	0.932	0.935	0.938	0.941	0.943	0.945
110	250	0.827	0.835	0.841	0.847	0.852	0.857	0.861	0.865
120	240	0.715	0.725	0.734	0.742	0.749	0.755	0.761	0.766
130	230	0.596	0.608	0.617	0.626	0.634	0.641	0.647	0.652
140	220	0.475	0.486	0.495	0.503	0.511	0.518	0.524	0.530
150	210	0.354	0.363	0.371	0.378	0.385	0.391	0.396	0.401
160	200	0.234	0.241	0.247	0.252	0.257	0.261	0.265	0.269
170	190	0.117	0.120	0.123	0.126	0.129	0.131	0.133	0.135
180	180	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 8. Crank Angle Factors for Piston Acceleration, f_a (see Eq. 10)

Crank Angle, degrees		Values of L/R 3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4
0	360	1.333	1.313	1.294	1.278	1.263	1.250	1.238	1.227
10	350	1.298	1.279	1.261	1.246	1.232	1.220	1.209	1.198
20	340	1.195	1.179	1.165	1.153	1.142	1.131	1.122	1.114
30	330	1.033	1.022	1.013	1.005	0.998	0.991	0.985	0.980
40	320	0.824	0.820	0.817	0.814	0.812	0.809	0.807	0.806
50	310	0.585	0.589	0.592	0.595	0.597	0.600	0.602	0.604
60	300	0.333	0.344	0.353	0.361	0.368	0.375	0.381	0.386
70	290	0.087	0.103	0.117	0.129	0.140	0.151	0.160	0.168
80	280	−0.139	−0.120	−0.103	−0.087	−0.073	−0.061	−0.050	−0.040
90	270	−0.333	−0.313	−0.294	−0.278	−0.263	−0.250	−0.238	−0.227
100	260	−0.486	−0.467	−0.450	−0.435	−0 421	−0.409	−0.397	−0.387
110	250	−0.597	−0.581	−0.567	−0.555	−0.544	−0.534	−0.524	−0.516
120	240	−0.667	−0.656	−0.647	−0.639	−0.632	−0.625	−0.619	−0.614
130	230	−0.701	−0.697	−0.694	−0.691	−0.688	−0.686	−0.684	−0.682
140	220	−0.708	−0.712	−0.715	−0.718	−0.720	−0.723	−0.725	−0.727
150	210	−0.699	−0.710	−0.719	−0.727	−0 734	−0.741	−0.747	−0.753
160	200	−0.684	−0.700	−0.714	−0.727	−0.738	−0.749	−0.757	−0.765
170	190	−0.672	−0.691	−0.708	−0.724	−0.738	−0.750	−0.761	−0.771
180	180	−0.666	−0.688	−0.706	−0.722	−0.737	−0.750	−0.762	−0.773

Table 9. Comparison of Values of Factors for Piston Velocity and Acceleration by Approximate and Exact Formulas

Crank Angle, degrees	Factors for Piston Velocity for $L/R = 4$		Factors for Piston Acceleration for $L/R = 4$	
	Exact Equation	Approximate Equation	Exact Equation	Approximate Equation
0	0.0	0.0	1.2500	1.2500
20	0.4226	0.4224	1.1336	1.1312
40	0.7676	0.7659	0.8139	0.8094
60	0.9768	0.9742	0.3572	0.3750
80	1.0289	1.0276	−0.0682	−0.0613
100	0.9407	0.9420	−0.4153	−0.4085
120	0 7552	0.7578	−0.6248	−0.6250
140	0.5180	0.5197	−0.7182	−0.7226
160	0.2614	0.2616	−0.7458	−0.7482
180	0.0	0.0	−0.7500	−0.7500

INERTIA AND CENTRIFUGAL FORCES are caused by motion of the pistons and connecting rods. In a fixed-cylinder engine, piston motion is linear; the resulting forces are purely inertia forces. The motion of the connecting rods is more complex. For an exact solution of the forces produced by the rod, it is necessary to consider the components of its motion. The motion of the rod may be analyzed as a translation of its center of gravity, with the linear velocity and acceleration of the piston combined with an angular velocity and acceleration about the piston pin. This motion of the rod sets up in each of its elements three separate forces: *a*, an inertia force due to linear acceleration in the direction of the cylinder axis; *b*, a centrifugal force due to angular velocity about the piston pin; *c*, an inertia force due to angular acceleration about the piston pin. These forces are closely approximated by assuming the mass of the connecting rod to be divided between the piston pin and the crankpin in inverse proportion to the distance of the respective pins from center of gravity of the rod. The crankpin portion produces a centrifugal force, the piston-pin portion produces an inertia force. To obtain the weight at either end, support the rod on knife edges directly over the center line of the bearings, the axis of the rod being horizontal. The knife edge under the end to be weighed rests on scales. Results can be verified by comparing the sum of the weights of the two ends with the total weight of the rod. The centrifugal force acting at the crankpin in the direction of the crank throw is

$$F_c = 0.0000284 W_c R N^2 \qquad (13)$$

where F_c = centrifugal force, pounds; W_c = weight of lower end of connecting rod, pounds;

R = crank radius, inches; N = rpm. The inertia force acting in the direction of the cylinder is

$$F_i = -0.0000284 W_i R N^2 f_a \tag{14}$$

where F_i = inertia force, pounds; W_i = reciprocating weight = weight of piston assembled plus upper end of connecting rod, pounds; f_a = crank angle factor for piston acceleration.

Resultant Forces on Piston. The gas pressure acting on the piston is obtained at various crank angles from the indicator diagram. If an experimental indicator card is not available, a theoretical card can be calculated. The total gas force is this pressure multiplied by piston area. Equation 14 determines the inertia force at various crank angles. Then if F_a = resultant force along cylinder axis, F_g = force on piston due to gas pressure, F_i = inertia force,

$$F_a = F_g + F_i \tag{15}$$

Piston side thrust due to the force acting along the cylinder axis is

$$F_s = F_a \times \tan \phi \tag{16}$$

where F_s = piston side thrust, pounds; F_a = resultant force along cylinder axis, pounds; ϕ = connecting-rod angle, degrees. For the arrangement of Fig. 10*a*, F_s may be expressed in terms of crank angle θ by

$$F_s = F_a\left[\sin \theta / \sqrt{(L/R)^2 - \sin^2 \theta}\right] \tag{17}$$

Piston side thrust is found throughout a complete cycle and is plotted versus crank angle and versus piston travel. The area of the latter curve is proportional to the piston friction loss if the coefficient of friction remains constant. From the average height of this curve, the average side thrust during the power stroke and during the complete cycle is determined.

Piston side pressure, pounds per square inch, equals (total piston side thrust) ÷ (projected bearing area). Since the piston diameter usually is relieved above the lower piston ring, only that portion below the lower ring is considered in determining effective bearing area.

Torque. The torque due to forces acting in the cylinder at any instant during the cycle is

$$T = F_a \times R \times f_v \tag{18}$$

where T = torque, pound-feet; F_a = resultant force along cylinder axis, pounds; R = crank radius, feet; f_v = crank angle factor for piston velocity. The resultant torque in a multicylinder engine is the algebraic sum of the instantaneous torques of the individual cylinders. To find this resultant, the angular relation of the cycles in the various cylinders must be considered. This analysis gives the indicated torque, as frictional forces are not considered.

The mean torque can be obtained from a curve of torque versus crank angle by using a planimeter. The ratio of the maximum to the mean torque of the engine is determined and used in later calculations.

Resultant force on crankpin is found by combining graphically the resultant force along the connecting-rod axis with the centrifugal force due to the weight of the lower end of the connecting rod. Use eq. 13. Let F_d = resultant force along the connecting-rod axis; F_a = resultant force along cylinder axis; ϕ = connecting-rod angle. Then

$$F_d = F_a / \cos \phi \tag{19}$$

In eq. 19 a plus (+) force denotes a force producing compression in the rod, a negative (−) force one producing tension. With the arrangement shown of Fig. 10*a*, F_d can be expressed in terms of the crank angle θ by

$$F_d = F_a / \sqrt{1 - (R/L) \sin^2 \theta]} \tag{20}$$

Figure 10*b* shows a graphic determination of the resultant force on the crankpin. F_c is laid off to scale along the center line of the crank throw. F_d represents a positive force acting along the connecting rod and is drawn parallel to it. The resultant force, F_r, represents, to scale, the magnitude of the force acting on the crankpin at the angle shown. The direction of F_r with respect to engine axis, crank throw, and connecting rod is given by angles α, β, and γ, respectively. F_r is obtained at various crank angles throughout the cycle, and plotted on a polar diagram with respect to the engine axis and crank throw, to determine maximum and mean forces. These forces give connecting-rod bearing loads.

Resultant Force on Crankshaft Bearings. The load distribution on the crankshaft bearings depends on the rigidity of the crankshaft and the crankcase, the alignment of the bearings, and the clearances between the journal and the bearings. These factors cannot be predetermined, and an exact analysis of the load distribution between the various crankshaft bearings is not possible where more than two main bearings are used.

An empirical method for computing the forces acting on the main bearings of a crankshaft where there is a main bearing at each side of the crankpin is as follows. The forces acting on the crankshaft bearings are obtained by assuming the force at the crankpin, together with the centrifugal force resulting from the weight of the crankpin and crank cheeks, to be equally divided between the two crankshaft bearings at each side of the crankpin. End bearings are loaded on only one side, by half the force of the crankpin combined with half the centrifugal force due to the crankpin and crank cheeks of the crank throw. The same loading is applied on both sides of the center and the intermediate bearings.

If more than one crankpin bearing or crank throw is located between each pair of main crankshaft bearings, the load distribution on the main bearings is found by treating each section of the crankshaft between two main bearings as a uniform beam rigidly supported at the center line of the main bearing bolts. The reactions on the main bearings A and B due to a force F_r on a crankpin located between them is

$$R_a = F_r \left[\frac{b^2(3a + b)}{(a + b)^3} \right] \tag{21}$$

$$R_b = F_r \left[\frac{a^2(a + 3b)}{(a + b)^3} \right] \tag{22}$$

where R_a and R_b = bearing reactions at bearings A and B, respectively; F_r = resultant force on crankpin; a and b = distance between center line of crankpin and center line of main bearings, respectively. The resultant force on a crankshaft bearing is the vector sum of the reactions due to the crankpin loads. In determining the resultant force, consideration must be given to the direction relative to the engine axis of each separate reaction. Both the relative positions of the crankpins and their cyclic relation also must be considered.

BEARING ANALYSIS. Maximum and mean bearing pressures are determined by dividing the maximum and mean resultant forces acting on the bearing by the projected area of the bearing. In determining projected bearing area, only the straight portion of the bearing length is considered as effective. The resultant pressures are expressed in pounds per square inch.

The rubbing factor PV on the bearing, pounds per square inch × feet per second, is the product of the rubbing velocity of the journal and the mean bearing pressure. The rubbing velocity in feet per second is

$$V_r = \pi(D/12) \times (\text{rpm}/60) \tag{23}$$

where V_r = rubbing velocity of bearing, feet per second; D = bearing diameter, inches. Table 10 gives bearing data for two typical automobile engines.

Table 10. Bearing Data of Typical Automobile Engines

Eight-cylinder Passenger-car Engine (Full pressure lubrication)		Six-cylinder Heavy-duty Truck Engine
Cylinder dimensions, in.	$3\,1/16 \times 4\,5/8$	$4\,7/8 \times 5\,1/2$
Maximum engine speed	4250 rpm	2100 rpm
Connecting rod, length, in.	9 3/4	12 1/8
Connecting rod, bearing diam, in.	2 3/16	2.5
Connecting rod, bearing width, in.	1 5/16	2.25
Connecting rod, bearing babbitt, width, in.	1 1/16	2.375
Connecting rod, bearing av load, psi	1,600	608
Connecting rod, bearing max, load, psi	2,500	993
Connecting rod, bearing av PV	63,000	16,600
Connecting rod, bearing max PV	98,700	27,100
At rpm	4,140	2,500

Main Bearings	Front	Front Center	Center	Rear Center	Rear	Front	Front Center	Center	Rear Center	Rear
Length, in.	1.605	1.183	1.681	1.183	2.386	2.281	1.562	2.812	1.562	3.250
Diameter, in.	2.311	2.374	2.436	2.499	2.561	2.750	2.750	2.750	2.750	2.750
Average load	450	590	730	560	280	277	466	477	466	202
Maximum load	900	1,180	1,460	1,120	560	462	826	653	826	336
Average PV	18,300	24,800	31,400	24,800	12,750	8,310	13,980	14,310	13,980	6,060
Maximum, PV	36,600	49,600	62,800	49,600	25,500	13,900	24,780	19,590	24,780	10,800
At rpm	4,140	4,140	4,140	4,140	4,140	2,500	2,500	2,500	2,500	2,500

Tin-base babbitt, lead-base alloys, cadmium alloys, copper-lead alloys, and a few aluminum alloys are suitable for automotive main and connecting-rod bearing materials. Tin-base babbitts have excellent characteristics. Lead-base bearings, while lower in cost, support less load in thick linings. When thicknesses less than 0.010 in. are used, lead-base bearings are similar in load-carrying capacity to tin-base bearings.

Copper-lead alloys are used where higher load-carrying capacity than babbitts are required. Cadmium alloys also give high load capacity but are attacked by corrosive oils.

20. AUTOMOBILE FUELS AND COMBUSTION

HEAT CONTENT. Gasoline, either cracked or straight run, is the ideal fuel for automobile engines. Its heat content is high. (See Section 2 and Table 11.) Columns 4 and 5 show the total, gross or high heating value, i.e., Btu liberated by a unit amount of gasoline burned with oxygen in a constant volume enclosure, the products of combustion CO_2, SO_2, and H_2O being cooled to the initial temperature and H_2O condensed to a liquid.

Table 11. Heating Value and Properties of Gasoline

(*Natl. Bur. Standards Miscellaneous Publication* 97)

Gravity		Density, lb per gal	Heat of Combustion at Constant Volume, Q_v		Heat of Combustion at Constant Pressure, Q_p	
Deg. API at 60 F	Specific at 60/60 F		Btu per lb	Btu per gal	Btu per lb	Btu per gal
55	.7587	6.326	20,140	127,400	18,810	119,000
60	.7389	6.160	20,260	124,800	18,900	116,400
65	.7201	6.004	20,360	122,200	18,980	113,900
69	.7057	5.884	20,440	120,200	19,040	112,000

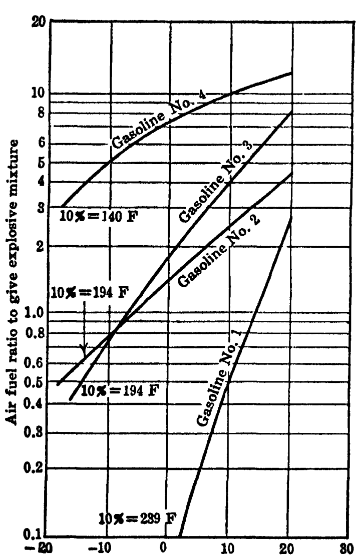


FIG. 11. Variation with temperature of air-fuel ratio for explosive mixtures.

Columns 6 and 7 show the net or low heating value, i.e., Btu liberated at constant pressure. This value is the most significant in calculating engine efficiencies, as, in most practical applications, combustion occurs at constant pressure and the H_2O formed is not condensed. The average specific gravity at 60 F of commercial gasoline is 0.74.

VAPOR LOCK. The 10% point of the ASTM distillation curve indicates the approximate temperature of the gasoline at which vapor lock troubles in the fuel line may occur. Summer gasolines range from 125 to 150 F and winter gasolines from 105 to 140 F. A more accurate indication of the tendency to vapor lock is the Reid vapor pressure.

STARTING CHARACTERISTICS. For easy starting, especially in cold weather, volatility is the most important fuel property. In winter, all the gasoline will not evaporate in the manifold; an excess must be supplied to enable the portion which does evaporate to form an explosive mixture. The excess is supplied by the choke, which changes the mixture ratio from the normal of about 13 : 1 to as low as 0.3 : 1. Figure 11 shows the air-fuel ratios required to produce an explosive mixture. The 10% points also are shown on the curve.

GUM CONTENT. Gum content in gasoline clogs valves and carburetor. Cracked gasolines have a greater tendency to form gum than straight-run gasolines. Permissible gum in gasoline should be under 5 mg per 100 cc.

KNOCK RATING AND OCTANE NUMBER. The anti-knock value of motor fuels for internal-combustion engines is one of its most important properties. This factor limits the compression ratio which can be designed into an engine and, therefore, its maximum specific output, efficiency, and fuel consumption. Fuels of the highest possible anti-knock quality are desired.

Knock Rating. The essentials of a uniform method of measuring the anti-knock qualities of fuels are (1) common reference fuels, (2) standardized engine and accessories, and (3) uniform test procedure. The anti-knock value is determined by the properties of the fuel under test and by the composition and characteristics of the standard fuel with which it is compared; and it is affected by the engine and conditions and method of engine operation.

Reference Fuels. The common reference scale, expressed in terms of octane number, has been adopted as a standard by the ASTM. Two pure hydrocarbons, *iso-octane* and *normal heptane*, are used. Normal heptane, a hydrocarbon of low anti-knock value, is zero on the octane scale. Iso-octane, a hydrocarbon of high anti-knock value, is 100 on the octane scale. *Octane number of a gasoline is the percentage by volume of iso-octane in a mixture of iso-octane and heptane that matches the gasoline in anti-knock quality, as determined in the standard engine under standard procedures.* Thus, if a gasoline is matched by a mixture of 80 parts iso-octane and 20 parts heptane, its octane number is 80.

Engine. The engine used for knock testing is a standard Cooperative Fuel Research Committee overhead valve variable compression engine, usually called simply *C.F.R. engine*. It has a bore of 3 1/4 in., a stroke of 4 1/2 in., and a displacement of 37.4 cu in. Figure 12 shows a cross section of the standard engine. Power is absorbed by an electric dynamometer. Fuels are compared by means of a bouncing-pin indicator.

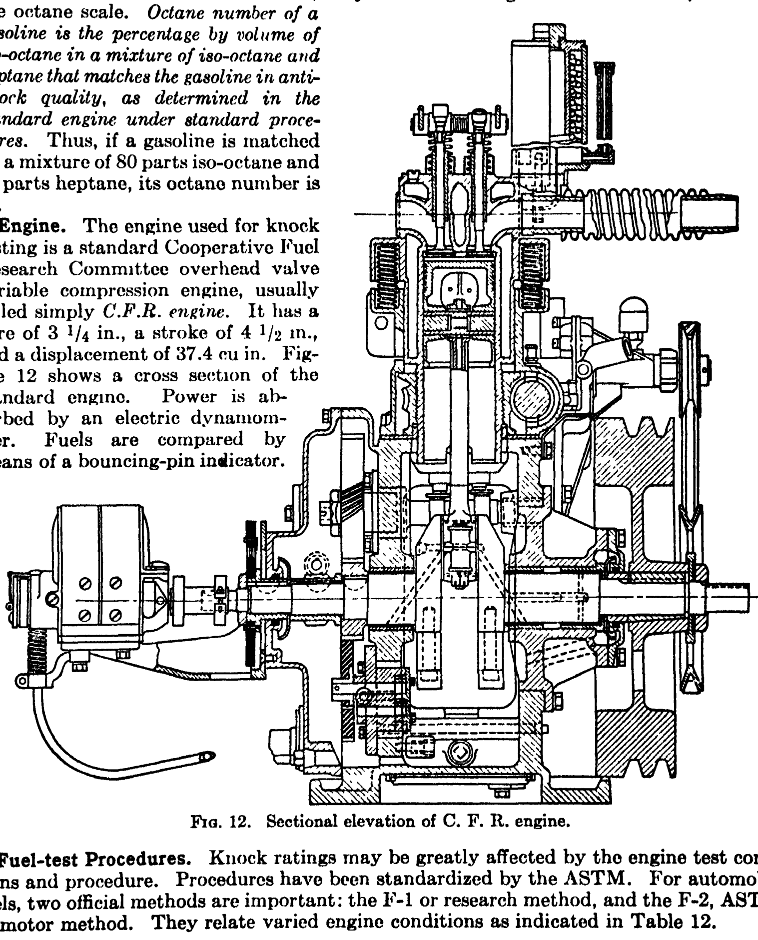


FIG. 12. Sectional elevation of C. F. R. engine.

Fuel-test Procedures. Knock ratings may be greatly affected by the engine test conditions and procedure. Procedures have been standardized by the ASTM. For automobile fuels, two official methods are important: the F-1 or research method, and the F-2, ASTM, or motor method. They relate varied engine conditions as indicated in Table 12.

Table 12. Test Conditions

	F-1 Research Method	F-2 Motor Method
Engine	C.F.R.	C.F.R.
Rpm	600	900
Water temperature, °F	212	212
Intake air temperature, °F	125	75–125
Mixture temperature, °F	125	300
Spark advance, degrees	13	Varies
Compression ratio	Varies	Varies

The F-1 or research method is quite mild, usually gives higher values of octane number; the F-2 or motor method is more severe. The two methods yield results which vary by as much as 35 octane numbers, with the research ratings the higher. In the 70 to 80 octane number range, it is seldom more than about 8 octane numbers. Sometimes this difference between motor and research octane number is used as a measure of the fuel sensitivity. A large difference indicates a sensitive fuel.

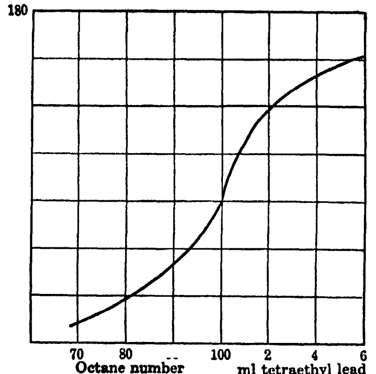


FIG. 13. Performance number as a function of Octane number and tetraethyl lead content per gallon of Iso-octane.

Since octane number depends on the degree of freedom from knock, and because the octane number scale stops at 100, a new concept called *performance number* has been introduced. The performance number of a fuel is a number obtained from a standard table from an equivalent octane number, as shown in Fig. 13. Performance number represents the average percentage of the power with pure iso-octane that can be obtained at borderline knock using the fuel in an average supercharged engine.

Many engine design factors affect the octane requirements of engines. Spark timing, piston temperatures, valve temperatures, mixture temperatures, jacket water temperature, valve timing, mixture ratio, combustion chamber shape, and combustion chamber deposits are all factors which must be considered. In addition, when an engine is installed in an automobile chassis, transmission characteristics affect octane number requirements. Engine designers must consider all these factors to utilize efficiently the octane numbers of available fuels.

21. COMPRESSION RATIO AND ENGINE EFFICIENCY

Compression ratio increase is the key to higher efficiencies and higher specific outputs. Figure 14 illustrates the gain in theoretical, indicated, and brake thermal efficiencies that may be expected. The theoretical curve shows the gain in indicated efficiency as compression ratio is increased, according to the curve represented by the equation $E = 1 - (1/r)^{n-1}$, where r is the compression ratio and n is the ratio of specific heats of the working fluid. In this case, $n = 1.21$ was taken, a value lying within the range of specific heats of the gases in the cylinder. The values for indicated and brake thermal efficiencies were obtained in a series of single-cylinder engine experiments with an overhead valve engine with a bore of 3 $^3/_8$ in., a stroke of 3 $^3/_8$ in., and a displacement of 30 cu in. The compression ratio was varied from 6.2 to 1 to 15 to 1.

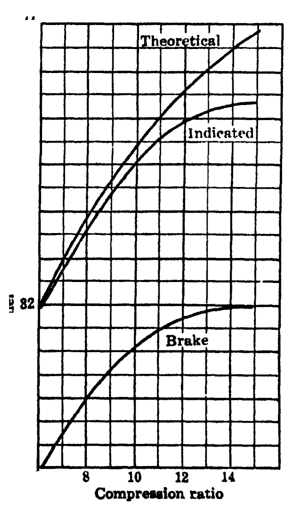


FIG. 14. Effect of compression ratio on thermal efficiency, illustrating the comparison between theoretical and true indicated values.

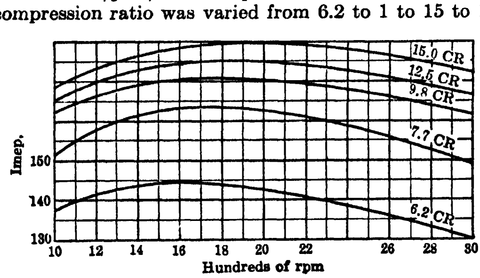


FIG. 15. Indicated mean effective pressure variation with speed for various compression ratios as determined from single-cylinder, high-compression test engine.

Mean Effective Pressure. Figure 15 shows the *indicated mean effective pressure* (imep) for speeds of 1000 to 3000 rpm and for compression ratios of 6.2 to 15. These data show the increase in full throttle power output as compression ratio was increased, although the rate of increase became less at the higher compression ratios.

Fuel Consumption. Figure 16 shows the decrease in full throttle *indicated specific fuel consumption* as the compression ratio was increased from 6.2 to 15. Smaller gains are again shown at the higher compression ratios. Similar large decreases in fuel consumption with increase in compression ratio are shown at part throttle. It is of interest that imep and specific fuel consumption for 15 to 1 compression ratio are comparative with diesel engine values.

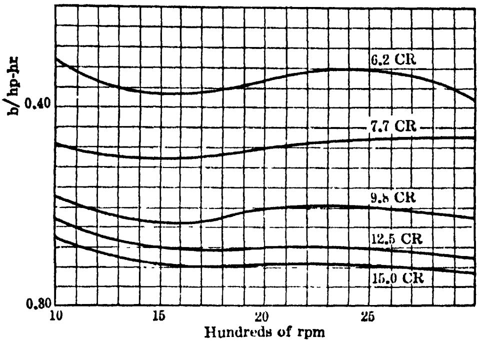


FIG. 16. Effect of speed on indicated specific fuel consumption for various compression ratios as determined from single-cylinder, high-compression engine.

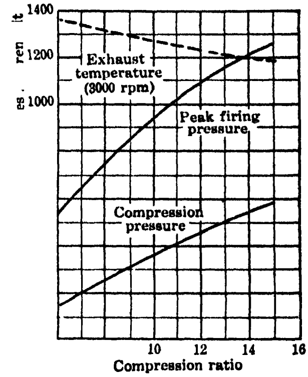


FIG. 17. Effect of compression ratio on compression and firing pressures as determined from single-cylinder, high-compression test engine.

Firing Pressures. Figure 17 illustrates the increase in *peak firing pressure* and *compression pressure*, and the decrease in *exhaust temperature* as the compression ratio is increased. As the compression ratio increases, the peak firing pressure increases from 580 psi at 6.2 to 1 compression ratio to 1230 psi at 15 to 1. These pressures greatly influence the mechanical design of the engine structure, smoothness, and bearing life. High compression ratios require adequate rigidity to carry the high loads.

Heat Balance. Table 13 is a heat balance comparison of two 6-cylinder engines, one with a compression ratio of 12.5 and the other with a ratio of 6.4, at 1000 rpm and 3000 rpm full throttle. The 12.5 engine had overhead valves and the 6.2 engine was L head. Figure 18 shows the heat rejection to the cooling water on both engines from 1000 to 3000 rpm. Throughout the speed range less heat is lost in the cooling water and exhaust gases, and more is utilized in producing power. This lower heat rejection would require about 30% less radiation ability to provide adequate cooling.

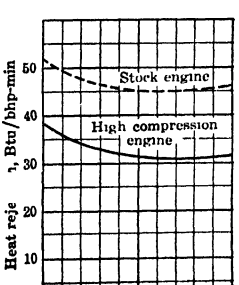


FIG. 18. Test data illustrating reduction in heat rejection by use of high-compression ratios.

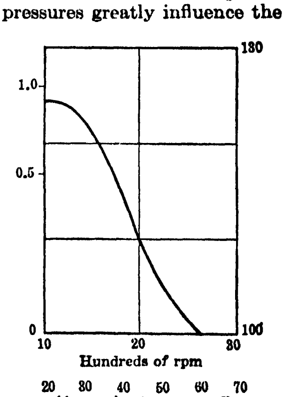


FIG. 19. Anti-knock requirement for high-compression (12.5 : 1) engines.

Table 13. Heat Balance Comparison

	12.5 Compression Ratio		6.4 Compression Ratio	
	1000 rpm	3000 rpm	1000 rpm	3000 rpm
Part of fuel heat in fhp, %	3.5	8.2	3.5	7.9
Part of fuel heat in bhp, %	33.2	32.8	24.9	23.3
Part of fuel heat in water, %	30.1	24.2	29.1	25.4
Balance of fuel heat, %	32.2	34.8	42.5	43.4

Anti-knock Requirements. Figure 19 shows the *anti-knock requirement* curve for a 6-cylinder 12.5 compression ratio engine, at full throttle operation of 800 to 4000 rpm. The maximum anti-knock requirement was equal to iso-octane plus 0.9 ml of tetraethyl lead per gallon.

22. FUEL SYSTEM

CARBURETORS. Carburetors act as metering devices and should: (1) Properly proportion the air-fuel ratio at different loads. (2) Correct for temperature changes. (3) Provide suitable accelerating mixtures. (4) Have low air resistance, to maintain high volumetric efficiency. Commercial carburetors embodying these principles may be classified according to metering principle as plain tube and air valve. The former uses a venturi tube to obtain a nearly constant mixture ratio throughout the major portion of its range. The latter uses the depression of an air valve for the same purpose. Other devices to obtain the desired characteristics over the entire speed and load ranges are metering pins, idle jets, compensating jets, double venturis, pressure bleeds, and numerous others. Many of them are common to both plain tube and air valve carburetors.

MIXTURE RATIOS at full throttle should be approximately 13 : 1 (lb of air per lb of fuel), and about 15 : 1 at part-throttle level-road operation to give greater economy. For all other loads, the ratio should be between these two. Part-throttle economizers and full-throttle enriching devices are added to carburetors to obtain this double range feature. For accelerating, a rich mixture is necessary and is provided by pressure on the float chamber or a fuel pump. For starting, especially in cold climates in winter, the carburetor must supply a very rich mixture. Only about 6% of the fuel can be evaporated to form a combustible mixture at 0 F. A choke to provide mixture ratios of 0.3 : 1 to 0.8 : 1 at 0 F is necessary for starting at low cranking speeds.

MANIFOLDING. An ideal manifold should supply an equal charge of identical fuel-air ratio established by the carburetor to all cylinders at all speeds, loads, and rates of acceleration. It is desirable that the manifold have equal demand intervals on a branch, established either in the firing order or in the manifold design.

Distribution of mixture to the cylinders is complicated by less than half the liquid being vaporized by the carburetor. To obtain better vaporization, heat is supplied in the manifold. Heat reduces volumetric efficiency and should be used as little as possible. Hot spots surrounding the throttle are effective, especially at part throttle. To vaporize liquid in suspension at full throttle, it must strike a hot spot at 90 degrees to the path of travel.

Liquid distribution concerns not only the manifold but also the carburetor, intake ports, and valves. Turbulent or spiraling air flow tends to upset liquid distribution. Violent spiraling tends, by centrifugal force, to throw all liquid drops to the walls of the manifolds. If spiraling is mild, distribution may be bettered by air straighteners. Manifolds of simple design are, in general, the best. In multicylinder engines, especially above 6 cylinders, multiple carburetors are necessary. Pronounced bumps or cavities at the tee, against which the mixture impinges, tend to increase turbulence and are undesirable. A straight section of at least (2 × manifold diameter) between elbow and siamese port is desirable to permit the mixture to resume normal flow.

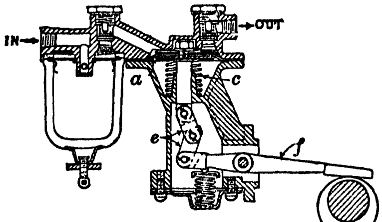


FIG. 20. Fuel pump.

FUEL FEED. A constant supply of gasoline must flow to the carburetor from the tank during operation of the engine. Figure 20 shows a typical positive engine-driven pump used for this purpose. Diaphragm *a* draws fuel from the tank and feeds it to the carburetor under pressure of the spring *c*. Diaphragm *a* is pulled down positively by lever *f* to draw fuel, but link *e* allows it to remain down if the carburetor float valve is closed.

23. ELECTRICAL SYSTEM

Electrical systems in American passenger cars operate at 6 volts; in European cars at 12 volts. The electrical circuit diagram (Fig. 21) is typical practice on a medium-priced 6-cylinder engine. The ignition coil converts the low voltage from the storage battery to a high voltage which will jump the spark plug gap. The spark plug gap should be 0.015 to 0.018 in. for high-compression engines and 0.018 to 0.022 in. for low-compression engines.

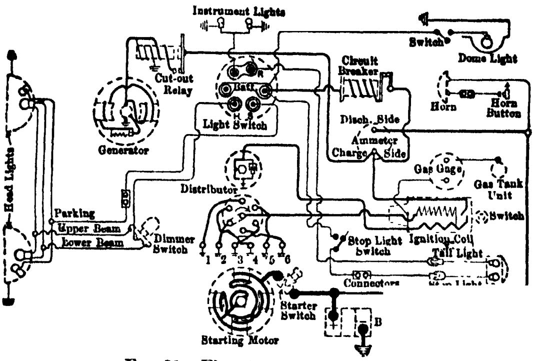


FIG. 21. Electrical circuit diagram.

Engagement of the starter pinion with the flywheel gear is made by a manual shift lever. Two other systems are used, a semi-automatic and an automatic engaging mechanism. The motor is series-wound to give high starting torque. The starter will crank a warm engine at about 165 rpm and a cold engine (0 F) at about 35 rpm.

24. CHASSIS

FRAMES AND SPRINGS. The body, fenders, radiator, engine, transmission, and wheels are attached to the motor vehicle frame. Strength and rigidity are the chief requirements of the frame. Usually, the two side members are channel section, with tubular or channel cross members riveted to them. To obtain greater torsional rigidity, X cross members sometimes are used. Figure 22 shows a so-called double drop frame which

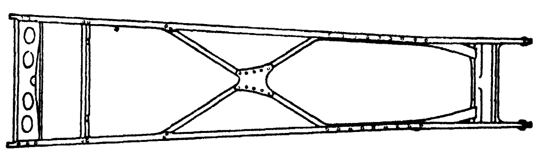


FIG. 22. Double drop frame with X cross bracing.

uses X cross bracing. The dimensions of frame section range as follows: depth, 5 to 9 in.; width, 1 $^3/_4$ to 3 in.; thickness, $^3/_{32}$ to $^3/_{16}$ in. The smaller values refer to light cars and the larger to the heaviest.

Methods of Drive. The three main methods of drive are Hotchkiss (Fig. 23), torque tube (Fig. 24), and independent spring suspension (Fig. 25). In the Hotchkiss, driving

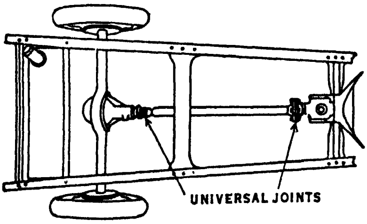


FIG. 23. Hotchkiss drive.

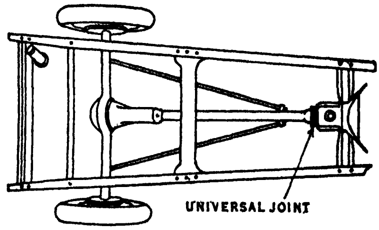


FIG. 24. Torque tube drive.

and braking torque are taken through the front and rear springs. In the torque tube, the drive is taken through a hollow tube enclosing the propeller shaft and firmly connected

at either end to the rear axle and transmission. In independent rear spring suspension, the rear axle housing is mounted on the frame. Universal joints, between the wheels and drive gears, are necessary to allow for the motion between the frame and wheels. The vertical motion of each rear wheel is therefore independent of the other. Front wheels may be similarly mounted so that their motion is independent.

Trucks, particularly military vehicles, often use four-wheel (4 by 4) or six-wheel drive (6 by 6), Figs. 26 and 27. These arrangements are used where more than the usual traction is required. This requires an extra set of gears called a transfer case behind the transmission. Sometimes the transfer case gives two speeds, which doubles the ratios available from the transmission.

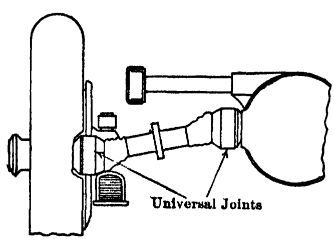


FIG. 25. Independent rear spring suspension drive.

Deflection of semi-elliptic springs may be found from the beam formula: $D = WL^3/48EI$, where D = deflection, inches; W = weight, pounds; L = length, inches; E = modulus of elasticity; I = moment of inertia. The initial deflection so found determines the period of vibration. An approximate formula for the period of semi-elliptic springs is $V = \sqrt{35{,}300/D}$, where D = initial deflection, inches; V = period, vibrations per minute. For rear springs, $V < 90$ gives good riding characteristics. In semi-elliptic front springs V should be more than double the rear to prevent resonance between the front and rear springs causing a pitching motion. In individual front wheel suspension, the front and rear springs can be about the same, which eliminates pitching. To control the springs, shock absorbers are used, some of which may be adjusted or are self-adjusting for different types of roads.

Factors other than springs contributing to good riding quality are (1) low unsprung weight; (2) low center of gravity; (3) long wheelbase; (4) soft springs; (5) good shock absorption; (6) rebound control; (7) large section tire; (8) proper weight distribution; (9) adjustable seats; and (10) proper seat cushions and springs.

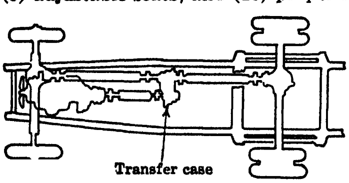


FIG. 26. Four-wheel (4 by 4) drive.

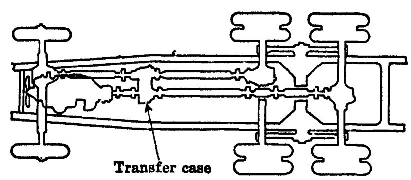


FIG. 27. Six-wheel (6 by 6) drive.

BRAKES must be able to stop the car quickly from any speed at which it may be driven, with good control and without undue change in the rate of deceleration. They must be able to withstand continuous operation to keep the speed low on long grades and to hold a parked car stationary. Legal requirements call for two brakes, a hand-lever-operated parking and emergency brake and a foot-pedal service brake. Brakes on all four wheels always are used.

Brake Types. The brakes used on American passenger cars are included in four classifications characterized by the type of shoe used, the arrangement of shoes, and location of the operating mechanism.

Considering only the shoes, there are three principal types:

(1) Rigidly mounted one-piece shoes which require an eccentric anchor pin or other means of adjustment to correct for manufacturing inaccuracies and to insure proper running clearances.

(2) One-piece shoes with flexible mounting which permits sufficient float to give the amount of movement required to eliminate adjustment. This arrangement does not always insure proper running clearance and, in most cases, reduces the braking ability of the shoe.

(3) Articulated shoes with friction means for retaining the clearance which is automatically determined by contact between shoe and drum. This type of shoe has two additional advantages: (1) unbalanced heel and toe pressures are impossible and (2) there is some correction for variation of the friction coefficient from the normal coefficient for which the angle of the link was selected.

The earliest form of internal-shoe brake was the one-piece shoe mounted on a fixed anchor pin. This type of brake was used for many years in Europe with a journaled cam and is still being used where power operation is required. It is not self-actuating when operated by a journaled cam, but, when operated by a floating cam or hydraulic cylinder, it becomes self-actuating and is used as such on many passenger cars.

In the Duo-Servo brake all shoes are forward-acting for either direction of rotation with one cylinder for each brake. The torque produced per unit of applied effort is greater than any known brake with a single operating means.

The two-cylinder brake has two single-ended cylinders. This arrangement is attractive for two reasons: (1) if the same shoes are used, the brake will produce approximately 50% more torque than the conventional one-cylinder brake, and (2) all forces for one shoe are equal and diametrically opposite to the corresponding forces on the other side.

The two-cylinder brake does not produce greater torque than the Duo-Servo brake. Therefore, the double-acting two-cylinder brake has no advantage as an alternative for use on all four wheels except that balanced forces and equal division of the torque reaction on the supporting structure tend to be more stable.

BRAKE SELF-ACTUATION is the essential factor that has made it possible to obtain sufficient braking power for passenger cars in the restricted space available. It has been the primary problem in brake design, and developments leading to the present standards of performance have been centered around development of design features and materials related to the use of self-actuation.

SKIDDING COEFFICIENT OF FRICTION. Figure 28 shows the range of straight skidding coefficients of friction for dry and wet road conditions with equivalent rates of deceleration. The boundary lines describe the upper and lower limits.

The curves show that the skidding coefficient decreases with speed to the extent that, at highway driving speeds, a coefficient of friction of 0.6 is all that can reasonably be expected and that the maximum rate of deceleration available without sliding wheels is about 20 ft per sec per sec.

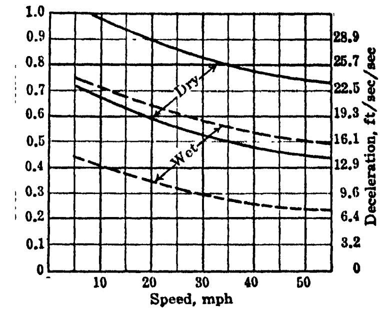


Fig. 28. Range of skidding coefficient of tire treads on concrete and bituminous pavements.

Also, it is of interest to note that in all but unusually poor conditions of road surfaces the normal rates of deceleration up to 10 ft per sec per sec can be obtained without danger of skidding if braking is properly distributed between front and rear wheels.

Division of applied effort between the front and the rear wheels depends on wheelbase, height of center of gravity, distribution of car weight on front and rear wheels, and coefficient of friction between tires and road. The weight transfer due to deceleration should also be considered, for which Fig. 29 may be used.

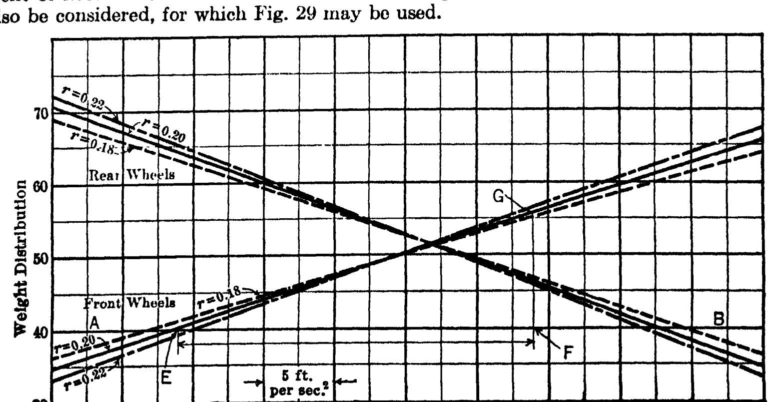


Fig. 29. Weight transfer due to deceleration.

Example. Assume static weight distribution to be: front wheels, 40%; rear wheels, 60%; r = (height of center of gravity) ÷ wheelbase = 0.20; deceleration = 25 ft per sec².

Solution. From the intersection E of the 40% line and line r = 0.20 for front wheels, lay off on the 40% line a distance EF = 25 ft per sec², according to scale shown. At F erect a perpendicular to EF,

intersecting r = 0.20 (front wheels) at G. Project G horizontally and read the new percentage of weight on front wheels as 55.5%. Percentage of weight on rear wheels = 100 − 55.5 = 44.5%. If the coefficient of friction between tires and road is such that 25 ft per sec² is the maximum deceleration possible, the distribution of brake effort should be 55.5% front and 44.5% rear in order to obtain the maximum deceleration.

PEDAL PRESSURE is transmitted to brake shoes by either a mechanical or a hydraulic system. Both should have high efficiency to obtain low pedal pressure. Retardation should be proportional to pedal pressure, with an initial pressure of not over 25 lb and preferably 10 to 15 lb. In the larger and heavier passenger cars, trucks, and busses, a servo or auxiliary mechanism is sometimes desirable to obtain low pedal pressures. Compressed air, manifold vacuum, or a mechanical device, driven from the engine or drive shaft, are used. The servo brake should give proportionality of effort equivalent to a mechanical system, and should positively and accurately control the auxiliary power.

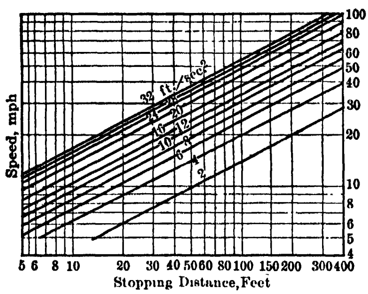


Fig. 30. Stopping distances at various deceleration rates.

Figure 30 shows the stopping distances at various deceleration rates.

To achieve desired low pedal pressures without resorting to the use of power devices, four means are available to the passenger-car brake designer. They are: (1) Mechanical advantage of brake diameter to wheel diameter. (2) Mechanical advantage of the brake operating system. (3) Efficiency of the operating system. (4) Amount of self-actuation that can safely be employed.

A large leverage ratio between the pedal and shoes is a sound and desirable means that should be employed to the maximum possible degree because it decreases pressure over the whole range without any loss of stability.

The high efficiency (about 98%) of the hydraulic fluid column for transmission of the effort at the pedal to the shoes after they are seated has contributed in large measure to the lack of necessity for power devices on passenger cars.

Mechanical losses which reduce overall system efficiency are (1) friction in the pedal bearing, (2) resistance to flow of fluid in the lines, (3) friction between the shoes and backing plate, and (4) unduly heavy and high rate springs at the pedal and between the shoes.

CLUTCHES. Conical, internal expanding, band, single, and multiple-disk clutches have been used on automobiles, the last two almost universally. For small and medium-sized cars, the single dry-disk clutch gives a simple design with few parts. For heavier cars, multiple-disk clutches sometimes are used.

The torque-carrying capacity of a clutch is approximately

$$T = \frac{fpA(r_2 + r_1)}{2}$$

where T = torque, in.-lb; f = coefficient of friction; p = unit pressure between plates, lb per sq in.; A = area of contact, sq in.; r_1 = outside diam of disk, in.; r_2 = inside diam of disk, in.

TRANSMISSIONS. Since automobile engines are essentially constant-speed machines it is necessary to provide a variable ratio between the engine and rear wheels. Many devices have been used including spur or helical gear trains, planetary gears, electric drive, pneumatic mechanisms, and hydraulic transmissions. These have been operated manually, semi-automatically, or completely automatically. Spur or helical gear trains with three or four forward speeds and reverse have been the basic drives in passenger cars and trucks for many years. Hydraulic mechanisms, either with or without gear sets, are becoming increasingly popular,

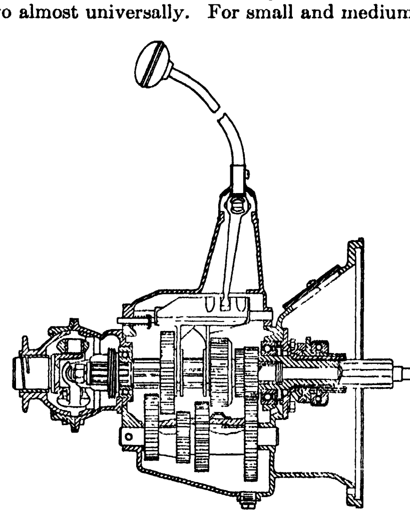


Fig. 31. Typical three-speed gear transmission.

however. For cruising operation an overdrive is sometimes used to lower the engine-wheel speed ratio and thus increase economy.

Conventional. Figure 31 is a typical three-speed and reverse spur gear transmission. Typical efficiencies range from 90% in low or first gear to over 99% in direct drive or high gear. To make gear shifting easier for the driver without clashing, synchro-mesh transmissions are used. Either two mating gears or dog clutches are brought up to an equal speed by a small cone clutch before they are engaged. A small differential in speed of the two mating members just before engagement insures that gear or clutch teeth will not meet end to end. In the constant mesh type, helical gears may be used to give quiet operation.

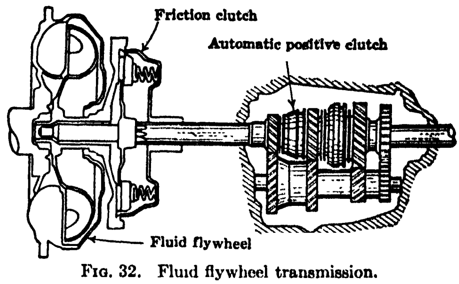


FIG. 32. Fluid flywheel transmission.

Automatic. Several types of automatic or semi-automatic transmissions use a hydraulic coupling more commonly known as a *fluid flywheel*. It may displace the friction clutch or be mounted between the clutch and engine (Fig. 32). The fluid flywheel is a torque *transmitter* not a torque *multiplier*. (See also Section 5.) Special oils are necessary for its operation. Since there is always some slip between the elements of the fluid flywheel, heat is generated which must be dissipated. A fluid flywheel used with planetary gear sets and with hydraulic controls which make the changes in ratio from one gear set to another is shown in Fig. 33. This type of transmission eliminates the clutch pedal and all forward control is in the foot throttle. The shift is made from one speed to another under load, the fluid flywheel acting as a cushion to the shock. Reverse is provided by a separate planetary gear set.

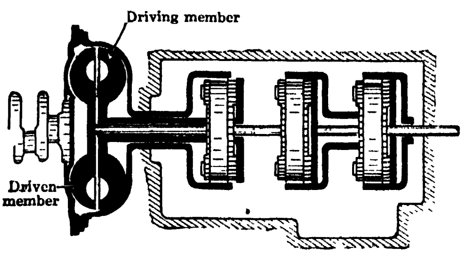


FIG. 33. Fluid flywheel with planetary gear sets and hydraulic controls.

Torque Converter. A hydraulic torque converter is also used in automatic transmissions. Unlike the fluid flywheel which it resembles in appearance, it can *multiply torque*. Figure 34 shows a diagram of a simple torque converter with the three main units indicated. The driving member is called the *pump*, the driven member the *turbine*, and the stationary member the *stator*. There may be several pumps and stator or reaction members. Gear sets may be used to provide more than one forward driving range and also reverse. Hydraulic controls are used to actuate gear mechanisms. This transmission is fully automatic, giving infinitely variable shift within its range. It eliminates the clutch pedal and full forward control is in the foot throttle.

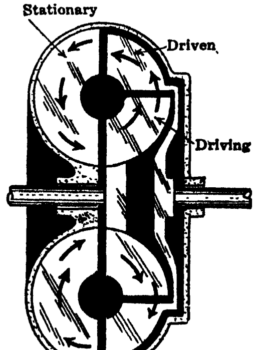


FIG. 34. Torque converter.

THE PROPELLER SHAFT or drive shaft transmits engine torque from the transmission to the rear wheels. It usually is of plain carbon steel. In the torque tube drive, it is enclosed in the torque tube, and in the Hotchkiss drive it is open. Propeller shafts are made both solid and hollow. The exposed shaft in the Hotchkiss drive is usually a hollow drawn tube.

For calculating stresses in the shaft, the following formula is used: $M_t = S_r I_p / r$, where M_t = torque, inch-pounds; S_r = shear stress, pounds per square inch; I_p = polar moment of inertia of shaft section; r = radius of shaft, inches.

One of the greatest difficulties in propeller shafts is the tendency to whip and cause noise at high engine speeds. Whip is due to shaft unbalance, lack of rigidity, and natural frequency of vibration. Hence high rigidity in the shaft and good initial balance are necessary, together with a resonant speed above that of the engine operating speeds. The natural frequency of a shaft with hinged ends. i.e., where universal joints are used, is $F = (\pi/2L^2)\sqrt{EIg/A\gamma}$, where F = natural frequency, cycles per second; L = length of shaft, inches; E = modulus of elasticity = 30×10^6 for steel; I = moment of inertia of the shaft section = $\pi d^4/64$ for solid round section; g = acceleration of gravity = 32.2; A = area of shaft section, square inches; γ = density of material, pounds per cubic inch.

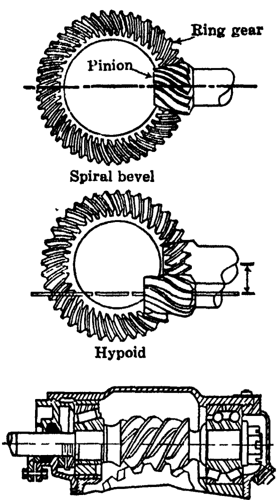


FIG. 35. Final drive gear arrangements.

THE FINAL DRIVE is through a gear reduction which varies with the type and use of the vehicle. Spiral bevel, hypoid, and worm gears may be used (Fig. 35). Hypoid under-drive worm gears allow the propeller shaft to be mounted lower, which in turn allows the body to be lowered. Worm gears are used on trucks and busses, which also use double reduction units with internal gears or combined spiral and spur gears. Both hypoid and worm gears have higher sliding friction than spiral bevel gears, run hotter, and require better lubrication.

In rear axle gear design, fatigue strength rather than static strength is considered. Studies in laboratory tests and in service have demonstrated that failures are due to fatigue. Destruction of the gear tooth surfaces by scoring is due to welding of small areas of the mating teeth under the influence of high pressure and high temperature. Pitting results from fatigue of the tooth surface due to repeated high compressive stresses.

Fatigue strength of spiral bevel and hypoid gears is affected by (1) bending stress; (2) stiffness of carrier, bearings, and gear; (3) material and heat treatment; (4) adjustment (position of total tooth contact area under load); (5) machine accuracy and finish. The resistance to scoring and pitting is affected by these factors, and also by compressive stress, surface hardeners, sliding velocity of teeth in contact, and properties of the lubricant.

Rear axle lubricants are specified by SAE viscosity numbers 80, 90, 140, and 250. SAE 80 is for extremely low temperatures and SAE 250 for extremely high temperatures. In gearing with high tooth loading or rubbing, special compounded oils are necessary. Small percentages of materials, such as sulfur or chlorine compounds, give the oil larger load-carrying capacities than are possible with mineral oil alone. Such oils, frequently called extreme pressure lubricants, are commonly specified for truck, bus, and passenger car service.

STEERING GEAR AND FRONT SUSPENSION. Automobiles are steered by the divided axle or Ackerman system. The front wheels, suspended from the ends of the front axle, pivot around the king pins. The front axle must sustain the weight of the car and the braking torque. It usually is of I section in the center, to carry the load, tapering to a circular

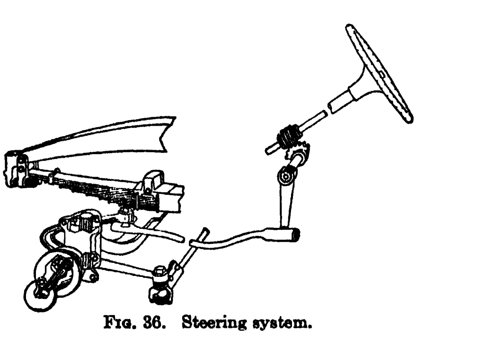


FIG. 36. Steering system.

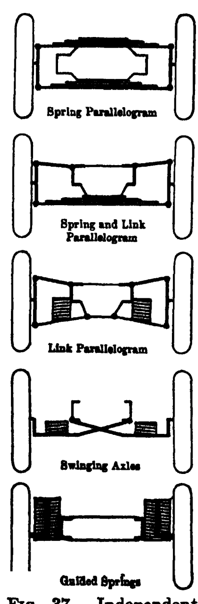
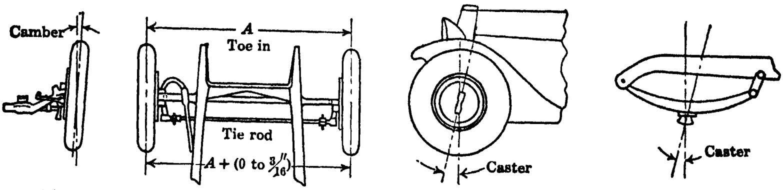


FIG. 37. Independent front wheel suspensions.

section outside the spring hangers to take the braking torque. Figure 36 shows a typical steering system.

Independent front-wheel suspension improves riding quality and helps eliminate front wheel shimmy, wheel fight, and tramp. Figure 37 shows diagrams of various types. In the types used, a lower rate spring can be used in front than with the semi-elliptic type, resulting in better riding qualities. The spring parallelogram gives parallel wheel motion, but a change in tread. The spring and link type relieves springs of brake torque,



(a) (b) (c)

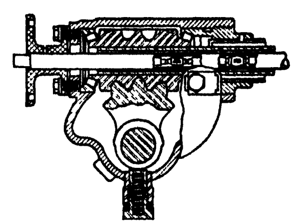
FIG. 38. Diagrams illustrating: (*a*) Camber. (*b*) Toe-in. (*c*) Caster.

but changes camber and track in proportion to the length of link. The link parallelogram type relieves springs of torque and allows proportioning length of links to obtain desired characteristics in camber and track changes. Swinging axles give large variations in camber and track, but small lean of wheels on curves. Guided springs produce no changes in camber or track when vertical guides are used, but give hard steering.

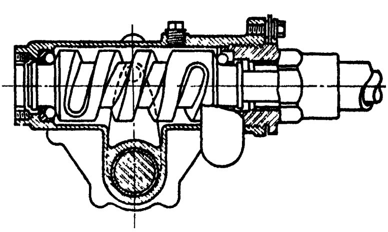
The geometry of the front wheels is chosen to give the best steering conditions on the steering system used. Wheels must be cambered, i.e., the tops slightly farther apart than the bottoms (Fig. 38*a*). The front of the wheels "toe-in," i.e., are closer together than the rear (Fig. 38*b*). The front axle and king pin are tilted backward (caster) so that the bottom is farther ahead than the top (Fig. 38*c*). Toe-in varies from 0 to $^3/_{16}$ in. measured at the outer diameter of the wheels. The tie rod is adjustable to vary this factor. Camber is used to bring the contact points of tires and road directly under the center of the king pins to give easy steering. It is designed into the car by making the angle between king pin and the front axle end other than 90 degrees. In practice, camber is measured as the angle between the king pin axis and the vertical axis of the wheel. Camber varies between 0.2 and 1.5 degrees.

Tilting the front axle gives a caster effect by bringing the point of tire contact with the road at a point back of a line drawn through the center of the king pin. Caster usually varies from 1 to 4 degrees, although a negative caster has been used.

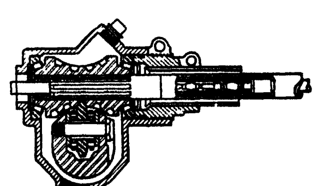
Steering Mechanism. The wheels are steered by a reduction mechanism. The most common types are (1) screw and nut; (2) worm and sector (Fig. 39*a*); (3) cam and lever (Fig. 39*b*); (4) worm and roller (Fig. 39*c*).



(*a*) Worm and sector



(*b*) Cam and lever.



(*c*) Worm and roller.

FIG. 39. Steering mechanisms.

The ratio of the reduction unit ranges from 12 : 1 to 19 : 1; it is highest on large cars. From full left to full right, the arc of the steering wheel varies from 1000 to 1500 degrees and the arc of the front wheels from 55 to 70 degrees. The efficiency of the gear unit has much to do with the ease of steering. Factors affecting efficiency include type of unit, temperature, lubrication, and use of anti-friction bearings. Average efficiencies for the various types of gear units are screw and nut, 20 to 40%; worm and sector, 40 to 50%; cam and lever, 40 to 65%; worm and roller, 60 to 70%. Under exceptionally good conditions, the efficiency of a worm and roller mechanism with anti-friction bearings may be as high as 85%.

UNIVERSAL JOINTS. Where torque tube drive is used, only one mechanical universal joint is required. Where a Hotchkiss drive is used, a universal joint on both ends of the propeller shaft, either mechanical or fabric, is necessary. Splines are used on the drive shaft end and a square section, key or spline, on the transmission or rear axle end. Figure 40 shows several types of universal joints.

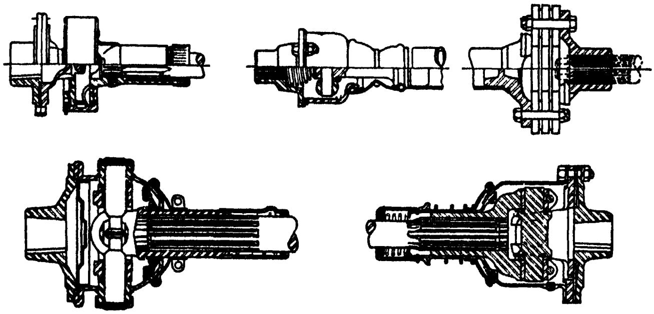


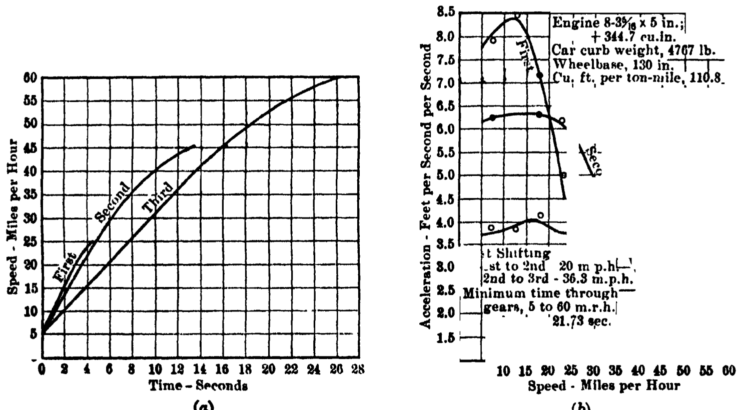
FIG. 40. Types of universal joint.

25. ROAD TESTS

PERFORMANCE TESTS. The final answer to all automotive engineering problems is the performance in a completed vehicle on the road. Final performance tests include (1) hill climb, (2) acceleration, (3) maximum speed, and (4) idle and minimum speed for flexibility. Economy tests include (1) constant-speed level-road economy, (2) open-highway economy, and (3) traffic economy. Brake performance, roadability, vapor lock, durability, octane requirement, cold start tests, and chassis dynamometer tests are other typical tests run on the complete automobile.

Instruments. Thermocouples or thermometers are used to take temperatures of oil, carburetor intake air, water, and mixture. A fifth wheel speedometer, stop watch, and chronograph also are used. A recording accelerometer may be used for acceleration tests. Intake manifold vacuum and exhaust pressure are measured and recorded. Temperatures are stabilized by driving 8 to 10 miles at speeds of 30 to 40 mph. Performance tests are not run if wind exceeds 10 mph.

Acceleration. The car is driven on a straight level road as nearly at a right angle to the wind as possible. Speed is reduced to about 2 mph below speed at which acceleration



(a) (b)

FIG. 41. Acceleration data.

is to be started. If car is braked to reduce speed, it should roll for an instant before accelerating. Record time in seconds with chronograph at 5-mph intervals to limit speed.

Runs in each direction are averaged. Repeat tests in all forward gears, starting at 5 mph and continuing to a speed at which acceleration is lower than in next higher gear ratio at same speed. Time to accelerate from 0 to 30 mph and 0 to 60 mph through the gears is measured by performing the shifts. Figure 41 illustrates typical acceleration data obtained by this test procedure.

Hill climb tests are run starting at 10 and 20 mph at the foot of a hill. Grades 7 to 30% and 1500 to 2000 ft long make good test hills. Tests are run in high gear only on standard transmissions. Driving range is used on automatic transmissions. The maximum speed and speed at the top are recorded. If the car goes over the hill, time required is recorded; if it stalls, the distance traveled is noted. Runs are made until results check consistently.

Maximum Speed. This test requires a smooth level roadway safe from other traffic. Tests are run in both directions to average out wind effects. Time is taken for at least 1/2 mile.

Idle and Minimum Speed for Flexibility. Find and record the minimum speed, in both directions, at which the car will idle in high gear, i.e., minimum speed with accelerator closed as far as possible. Find and record minimum speed in each direction from which car will accelerate smoothly without bucking in high gear or driving range. This speed is usually lower than the minimum idle speed, and the car must be braked to obtain it.

ECONOMY TESTS. In addition to the instruments used in performance tests, two calibrated burettes are required. Disconnect carburetor from gasoline pump and connect burettes mounted on dash to carburetor with flexible hose. Gasoline is pumped from car fuel tank by an auxiliary pump to fill burettes. A three-way valve connects either burette to carburetor or fuel tank.

Before test runs are started, the car is driven to warm up oil and water. The last one or two miles are run at speed of the first test. A smooth level roadway with a measured mile is required. As car enters measured mile, the burette valve is turned and the stop watch is started simultaneously. From data obtained, a curve of miles per gallon versus miles per hour is obtained (Fig. 42). Tests are started at 20 mph, and data are obtained at 10-mph intervals to maximum speed.

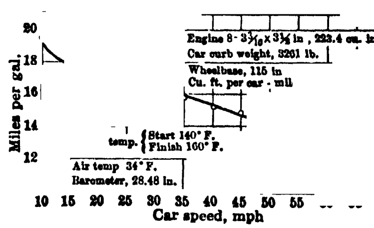


FIG. 42. Speed-fuel consumption curve.

BRAKE TESTS. Brake-performance tests are made to determine deceleration at various pedal pressures and pedal travels. Special equipment to measure and apply pedal pressures is required. This is usually a pneumatic cylinder and piston arrangement. Deceleration is measured by an accurate decelerometer. For stopping distance tests a

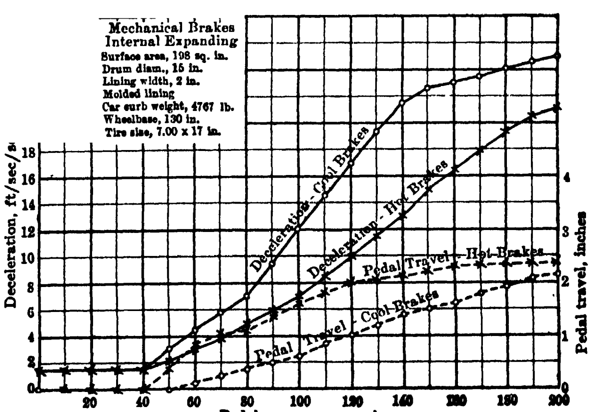


FIG. 43. Braking curve at 50 mph.

method of measuring distance on road from start of brake application to stop is required. A gun aimed at the pavement firing a wad of paint and tripped by the brake foot pedal may be used.

For deceleration versus pedal pressure data, stops are made from speeds of 25 mph and 50 mph. Pedal pressure is applied at a rate of 40 lb per sec. Deceleration and pedal travel are plotted against pedal pressure (Fig. 43).

A fade-out test consists of 4 stops from a speed of 70 mph, as rapidly as possible, with deceleration of 13 ft per sec per sec. Curves of deceleration versus time are plotted (Fig. 44.)

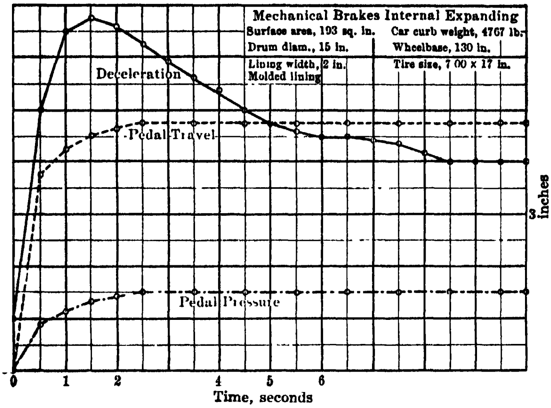


Fig. 44. Constant-pressure brake curve at 75 mph.

26. ENGINE TESTS

Hundreds of tests on engines and engine parts are used in automotive laboratories. Procedures used vary in different laboratories, often leading to confusion, since results are not directly comparable unless standard procedures are used. Most automotive engineering and development are done experimentally, using the results of dynamometer tests for evaluating results. General Motors Corporation has adopted a standard automotive engine test code, used by its divisions and suppliers, as well as most other units of the industry, to obtain results comparable to within 2%. A partial abstract of this code follows. Curves of the results of a few typical tests on a standard automobile engine are given to illustrate the use of the test procedures.

GENERAL ENGINE INFORMATION. Information on engine specifications, equipment, and settings is obtained at the time the test is made and permanently attached to the test data in the form of a log sheet listing items on which information is required. These items are listed here: bore and stroke; displacement; compression ratio (average volumetric compression ratio of all cylinders); where obtained (describe condition); valves, intake, exhaust (nominal head and port diameters, seat angle, maximum lift, cold lash, hot lash, and valve timing); valves, arrangement and description; camshaft data; cylinder head; manifolds, intake, and exhaust; carburetor data; pistons (piston clearance, etc.); exhaust system; ignition data; cooling system data; fuel; oil; car weight and frontal area; notes (append any notes on performance, operation, etc., also any notes required in further description of the foregoing items); valve lift diagrams (accompanying the data for any standard test).

PERFORMANCE CURVES. Instructions in Plotting Test Results. Data points obtained from fundamental observations are shown on the plotted curves. As the units of measurement are shown in the scale headings of the curve sheets, the curves are usually labeled only by the name. These labels are prefixed by the words "observed" or "corrected" ("obs." or "corr.") to distinguish between observed and corrected data. The label is placed to the right of the plotted curve and underlined, with an arrowhead pointing (right or left) to the units scale.

REQUIRED ENGINE EQUIPMENT. Transmission and clutch are omitted in dynamometer testing.

Intake system includes, when standard for the engine: (1) air cleaner; (2) carburetor; (3) choke valve (in open position); (4) riser; (5) manifold heat-control system; and (6) intake manifold.

Exhaust System. Tests are made with and without muffler. For tests with muffler, the standard exhaust system includes: (1) exhaust manifold; (2) automatic heat valve;

(3) exhaust pipe; (4) muffler; (5) tailpipe. For tests without muffler, the manifold is connected directly to the dynamometer exhaust system.

Cooling System. The standard water pump is used, operated at normal speed; water thermostats of the blocking type are not used; the car radiator is omitted except as required with indirect cooling by "hard" water; the standard cooling fan is used, driven at normal speed with normal belt tension.

Ignition System. The standard generator is used, operating at normal speed with the leads grounded to eliminate charging, or a generator charging a fully charged battery through the standard voltage regulator, set to control within the specified voltage limits.

Fuel and Lubricating System. The standard fuel pump, carburetor, air cleaner, and oil pump are used.

ENGINE ADJUSTMENTS AND SETTINGS. Throttle. Most tests are made at full throttle. For tests at part throttle, arbitrary road power data are used.

Carburetor. Both fixed-adjustment and externally adjustable carburetors are used. With either type, the best compromise settings are determined and maintained constant throughout a series of tests. The following adjustments are mentioned: (1) leanest for best torque, any condition; (2) richest for best torque, any condition; (3) fixed setting carburetors requiring only normal idle adjustment; (4) variable fuel flow, any condition, to obtain the progressive effect of this variable over the range from best torque to best economy; (5) the fuel flow varied at each speed to cover the range of usable air-fuel ratio.

Intake manifold heat control is maintained in the "summer" position unless it is automatic.

Ignition Settings. For all power tests, the spark-plug gaps and breaker clearances are adjusted to the standard position. Spark adjustment will depend on the type of test. The following type adjustments are enumerated: (1) full automatic; (2) manual, any condition, to give minimum advance for best torque; (3) manual to give borderline detonation in the most offensive cylinder; (4) manual to obtain the extremes of advance and retard, either side of best power advance, for a 1% loss in torque.

Valve lash is adjusted as specified by the manufacturer.

PRELIMINARY ENGINE-CHECKING PROCEDURE. Prior to any standard test (1) the engine is thoroughly conditioned, (2) all engine equipment and settings are checked and recorded, and (3) a performance check test is made.

Engine Condition. A new engine is run in under progressive loads and speeds (1) until there is no decrease in brake torque during a 15-min full throttle test at the predetermined limiting speed, (2) until there is no important increase in power over the speed range as checked periodically during the run-in.

Performance Check Test. As a final step, a performance check test is made under standard test conditions. (1) The carburetor is adjusted for good idling. (2) The engine is stabilized at full throttle, 1000 rpm, and the carburetor adjusted to the leanest position for best torque. (3) A 10-min test is made at full throttle, speed of peak power, recording full data.

FUELS AND LUBRICANTS. Fuels. Special fuels are used for engines requiring them; normally any good grade of fuel may be used. A fuel of given knock rating is not required, but the knock rating of the fuel must be stated.

Lubricants. Any good grade of engine oil is used. The only requirement is that the viscosity be that recommended by the manufacturer for summer conditions.

PRECISION. Directly measured data should be obtained with an instrumental precision of at least 1%. Instrumental precision is not to be confused with personal and probable errors.

ENGINE STABILITY. Performance data are obtained under stabilized operating conditions. Sudden heat changes should be avoided. In any test, the series of runs should progress continuously. For all power tests versus speed, a single series of stabilized runs at ascending speeds are sufficient. For any test run: (1) no data are taken until torque, speed, and temperatures have been maintained without noticeable change for at least one minute, and there should be no important change in these items during the test run; (2) no adjustments of engine settings are made during the run and as an overall check; (3) two separate torque readings are recorded—one before and another after recording all other data. The oil temperature increase during the run should not exceed 5 F.

TORQUE, SPEED, AND POWER. Instrumentation. *Torque.* The electric cradle dynamometer is used. The motoring capacity should permit making friction and compression tests over the speed range of the power test.

Speed. An accurately checked indicating tachometer is used for compression and friction tests. An accurately calibrated revolution counter operated from the dynamometer or engine shaft and synchronized with a time-measuring device is used for the power test. The counter and chronometer are operated from a single control.

Time. The conventional stop watch is satisfactory. Timer units operated from stabilized a-c circuits give a finer degree of precision.

Interval of Speed Measurement. A measuring interval of not less than 40 sec—preferably not less than 1 min—is used when measuring speed.

Limiting speed is defined either by maximum car speed or by limitations of the testing equipment.

COMPUTATIONS. Notation: B_c = carburetor air pressure, in. Hg abs; e = water vapor pressure, in. Hg abs; T_c = observed, and T_s = standard, carburetor air temperature, °F, abs; ME = mechanical efficiency.

Corrected Power. Full throttle power data, including power at borderline detonation, are corrected to standard conditions for carburetor air. Part throttle data are not corrected. Standard pressure is 29.92 in. Hg; standard for humidity is 0.5 in. Hg; standard temperatures are 60 and 100 F. The standard for stock tests is 100 F.

The combined correction factor for indicated power = $[29.42/(B_c - e)]\sqrt{T_c/T_s}$.

The combined correction factor for full throttle brake power =

$$\frac{1}{\mathrm{ME}}\{[29.42/(B_c - e)]\sqrt{(T_c/T_s)} + \mathrm{ME} - 1\}$$

Table 14 gives factors for application to *indicated power*, for the two standard datum temperatures of 60 and 100 F. Table 15 gives similar factors for full throttle *brake power*

Table 14. Indicated Power Correction Factors for Pressure, Temperature, and Humidity

(These factors are to be applied to indicated power or torque.)

60 F Datum Temperature

Carb. Temp., °F	Dry Carburetor Pressure, in. Hg									
	27.8	28.0	28.2	28.4	28.6	28.8	29.0	29.2	29.4	29.5
50	1.048	1.041	1.033	1.026	1.019	1.012	1.005	0.998	0.991	0.988
60	1.058	1.051	1.043	1.036	1.029	1.022	1.014	1.008	1.001	0.997
70	1.068	1.061	1.053	1.046	1.039	1.031	1.024	1.017	1.010	1.007
80	1.078	1.071	1.063	1.056	1.048	1.041	1.034	1.027	1.020	1.016
90	1.088	1.081	1.073	1.065	1.058	1.051	1.043	1.036	1.029	1.026
100	1.098	1.090	1.083	1.075	1.068	1.060	1.053	1.046	1.038	1.035
110	1.108	1.100	1.092	1.085	1.077	1.070	1.062	1.055	1.048	1.044
120	1.118	1.110	1.102	1.094	1.086	1.079	1.071	1.064	1.057	1.053

100 F Datum Temperature

Carb. Temp., °F	27.8	28.0	28.2	28.4	28.6	28.8	29.0	29.2	29.4	29.5
50	1.010	1.003	0.996	0.989	0.981	0.975	0.968	0.961	0.955	0.952
60	1.020	1.012	1.005	0.998	0.991	0.984	0.978	0.971	0.964	0.961
70	1.030	1.022	1.015	1.008	1.001	0.994	0.987	0.980	0.974	0.970
80	1.039	1.032	1.024	1.017	1.010	1.003	0.996	0.989	0.983	0.979
90	1.049	1.041	1.034	1.027	1.019	1.012	1.005	0.998	0.992	0.988
100	1.058	1.051	1.043	1.036	1.029	1.022	1.014	1.008	1.001	0.997
110	1.068	1.060	1.053	1.045	1.038	1.031	1.023	1.016	1.010	1.006
120	1.077	1.069	1.062	1.054	1.047	1.040	1.032	1.025	1.018	1.015
130	1.086	1.078	1.071	1.063	1.056	1.049	1.041	1.034	1.027	1.024
140	1.095	1.088	1.080	1.072	1.065	1.057	1.050	1.043	1.036	1.032
150	1.105	1.097	1.089	1.081	1.074	1.066	1.059	1.052	1.044	1.041

based on an *assumed average mechanical efficiency of 85%*. Table 16 gives correction factors for full throttle brake power for a range of mechanical efficiencies from 50 to 95%. The curve (Fig. 45) shows average mechanical efficiencies for automobile engines.

Fig. 45. Average mechanical efficiency, stock engines.

Table 15. Full Throttle Brake Power Correction Factors for Pressure, Temperature, and Humidity

(These factors are to be applied directly to full throttle brake power or torque. They should not be used for part throttle power.)

60 F Datum Temperature

Carb. Temp., °F	Dry Carburetor Pressure, in. Hg									
	27.8	28.0	28.2	28.4	28.6	28.8	29.0	29.2	29.4	29.5
50	1.056	1.048	1.039	1.031	1.022	1.014	1.006	0.998	0.989	0.986
60	1.068	1.060	1.051	1.042	1.034	1.026	1.016	1.009	1.001	0.996
70	1.080	1.072	1.062	1.054	1.046	1.036	1.028	1.020	1.012	1.008
80	1.092	1.084	1.074	1.066	1.056	1.048	1.040	1.032	1.024	1.019
90	1.104	1.095	1.086	1.076	1.068	1.060	1.051	1.042	1.034	1.031
100	1.115	1.106	1.098	1.088	1.080	1.071	1.062	1.054	1.045	1.041
110	1.127	1.118	1.108	1.100	1.091	1.082	1.073	1.065	1.056	1.052
120	1.139	1.129	1.120	1.111	1.102	1.093	1.084	1.075	1.067	1.062

100 F Datum Temperature

Carb. Temp., °F	27.8	28.0	28.2	28.4	28.6	28.8	29.0	29.2	29.4	29.5
50	1.012	1.004	0.995	0.987	0.978	0.971	0.963	0.954	0.947	0.944
60	1.024	1.014	1.006	0.998	0.989	0.981	0.974	0.966	0.958	0.954
70	1.035	1.026	1.018	1.009	1.001	0.993	0.985	0.976	0.969	0.965
80	1.046	1.038	1.028	1.020	1.012	1.004	0.995	0.987	0.980	0.975
90	1.058	1.048	1.040	1.032	1.022	1.014	1.006	0.998	0.991	0.986
100	1.068	1.060	1.051	1.042	1.034	1.026	1.016	1.009	1.001	0.996
110	1.080	1.071	1.062	1.053	1.045	1.036	1.027	1.019	1.012	1.007
120	1.091	1.081	1.073	1.064	1.055	1.047	1.038	1.029	1.021	1.018
130	1.101	1.092	1.084	1.074	1.066	1.058	1.048	1.040	1.032	1.028
140	1.112	1.104	1.094	1.085	1.076	1.067	1.059	1.051	1.042	1.038
150	1.124	1.114	1.105	1.095	1.087	1.078	1.069	1.061	1.052	1.048

The tables are used as follows:

1. Correct the observed barometer reading by subtracting the temperature correction, thus obtaining total pressure, B_c, at the carburetor entrance. If air-measuring apparatus causes a depression at the carburetor, this depression is subtracted from B_c to obtain total carburetor pressure, used in obtaining correction factors.
2. Determine the existing atmospheric water vapor pressure, e, and subtract from the corrected barometer reading to obtain dry air pressure $= B_c - e$. In general, the vapor pressure at the carburetor, even with a depression, is assumed equal to that in the room.
3. To obtain the correction factor for a mechanical efficiency of 0.85 use Table 14 or Table 15.

3-*A*. To obtain the *brake power correction factor* for any other mechanical efficiency, obtain the correction factor applicable to indicated power from Table 14. From Table 16, obtain the full throttle brake power correction factor applying to the test mechanical efficiency.

4. Multiply observed power by correction factor to obtain power corrected to standard datum.

Table 16. Correction Factors for Brake Power at Various Mechanical Efficiencies

Ihp, CF	Mechanical Efficiency									
	.95	.90	.85	.80	.75	.70	.65	.60	.55	.50
0.940	0.937	0.933	0.929	0.925	0.920	0.914	0.908	0.900	0.891	0.880
0.960	0.958	0.956	0.953	0.950	0.947	0.943	0.938	0.933	0.927	0.920
0.980	0.979	0.978	0.976	0.975	0.973	0.971	0.969	0.967	0.964	0.960
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.020	1.021	1.022	1.024	1.025	1.027	1.029	1.031	1.033	1.036	1.040
1.040	1.042	1.044	1.047	1.050	1.053	1.057	1.062	1.067	1.073	1.080
1.060	1.063	1.067	1.071	1.075	1.080	1.086	1.092	1.100	1.109	1.120
1.080	1.084	1.089	1.094	1.100	1.107	1.114	1.123	1.133	1.145	1.160
1.100	1.105	1.111	1.118	1.125	1.133	1.143	1.154	1.167	1.182	1.200
1.120	1.126	1.133	1.141	1.150	1.160	1.171	1.185	1.200	1.218	1.240

SPARK TIMING. The measured timing for a representative cylinder is sufficient, although timing for all cylinders is often desirable. The spark control is rigged to permit maintenance of any desired manual setting of the distributor head. For automatic control, the distributor head is clamped at required initial timing. A neon-bulb indicator is used, driven either from the engine side of the dynamometer coupling or an extension shaft at the front end of the engine. This indicator consists of a small neon bulb mounted behind a slit in an insulated rotor which rotates behind a stationary, metal, insulated, full-circle protractor ring, to which is connected, through an air condenser, a dead-end lead wire from the spark plug. Several neon bulbs are sometimes used, properly spaced for a given engine, with a lead wire from the common high-tension ignition wire.

DETONATION. Three arbitrary grades of intensity are used: (1) none (N); (2) incipient or borderline (BL), which is that condition where detonation is barely perceptible in the most offensive cylinder; (3) objectionable (OBJ) which is that condition where the intensity is greater than incipiency in any cylinder.

FUEL CONSUMPTION. Instrumentation. Either the weight or volumetric method is used. Where feasible, normal fuel pump pressure should be used.

Interval of Fuel Measurement. The minimum measuring interval is the same as for speed measurement. Speed and fuel are measured during the same interval.

AIR CONSUMPTION. Instrumentation. The standard intake system is used. Satisfactory instruments are the gasometer, smooth approach orifices or thin-plate, sharp-edged orifices of the Durley type. The gasometer is a primary standard for calibrating other meters. To reduce pulsations, it is customary to use a large-volume surge tank between engine and air meter. The effect of the air meter on engine performance should be checked.

TEMPERATURES AND PRESSURES. (See Section 18.)

27. SPECIFIC ENGINE TESTS

Purpose and typical engine test data are given below for several of the important standard tests.

TEST 1. FULL THROTTLE AS INSTALLED. This test gives representative full throttle performance over the speed range of the engine as installed in the car. It should not be used for purposes other than this. (See Fig. 46.)

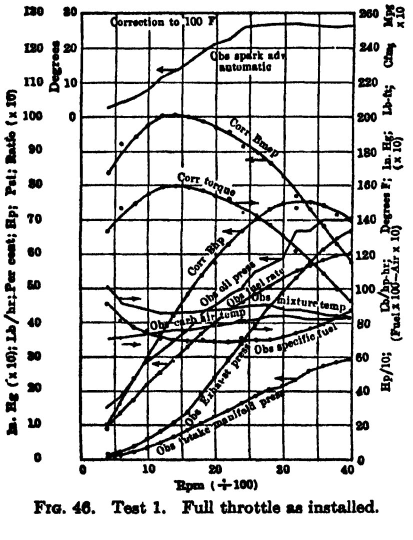


FIG. 46. Test 1. Full throttle as installed.

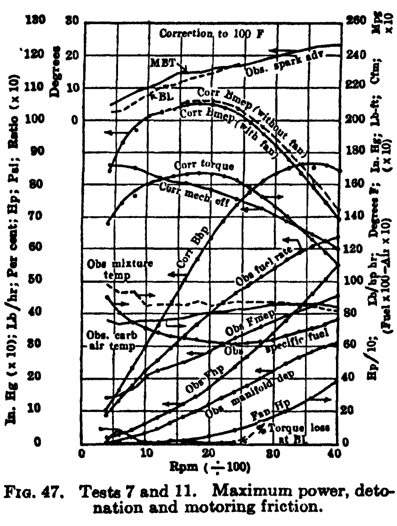


FIG. 47. Tests 7 and 11. Maximum power, detonation and motoring friction.

TEST 2. MUFFLER. This test is used to ascertain the effect of the muffler and tailpipe on full throttle performance. In order to segregate the muffler effect it is necessary: (1) to have the ram characteristics of the standard engine exhaust pipe common to both tests; (2) to use MBT (minimum best torque) spark to obtain the influence of the muffler on the best torque setting.

TEST 3. MAXIMUM POWER WITH BEST SPARK. This test is used to obtain maximum power performance with a given carburetor, with spark timing optimum, and the muffling system removed. This test should not be made to establish full throttle fuel consumption characteristics of a given design; for this purpose, Tests 4 or 8 are recommended. This test may be used to compare the fuel consumption characteristics of two given carburetors on the same engine.

TEST 4. MAXIMUM POWER WITH BEST SPARK AND ECONOMY. This test is used to obtain maximum power performance and best fuel economy at full throttle of a given engine design. It is used in preference to Test 3 to determine the effect of variables or design changes on full throttle fuel consumption. Test 8 is used when a more detailed analysis is desired.

TEST 5. BLOW-BY. This test is used to determine the rate of piston blow-by in cubic feet per minute. Since piston blow-by is closely related to piston ring characteristics, no rigid-load and speed schedule is set up for this test. The chief requirement is to determine carefully the critical speeds and loads at which to test.

TEST 6. SPARK ADVANCE. This test is used to obtain detailed information on the sensitivity of a given engine to spark advance, inclusive of its anti-detonation characteristics. The torque-spark advance relation is established at each speed over the spark range from retard for 1% torque loss or borderline detonation—whichever criterion governs—to over advance for 1% torque loss. Usually, there is no interest in borderline detonation occurring at over advance beyond that for 1% torque loss.

TEST 7. MAXIMUM POWER AND DETONATION. This test, used as a check on fundamental design, gives both the maximum power performance of Test 3 and engine detonation characteristics. (See Fig. 47.)

TEST 8. OPTIMUM POWER AND ECONOMY. This test is made to provide information on the optimum power and economy, which requires that tests, each speed, be made with variable fuel flow and that MBT spark advance be used for each fuel rate.

TEST 9. HEAT DISTRIBUTION. The heat distribution test involves making: (1) a water heat rejection test and (2) a motoring friction test made under the same conditions

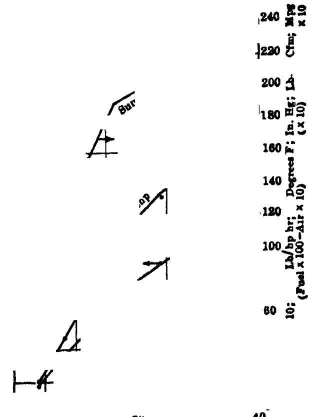


FIG. 48. Test 9. Engine performance.

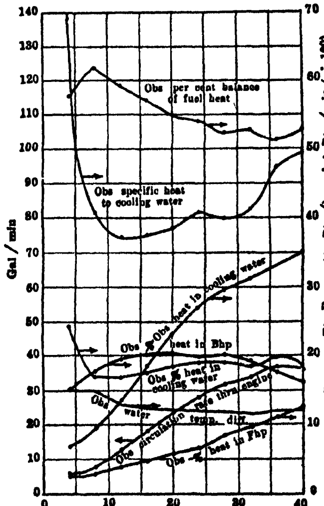


FIG. 49. Test 9. Heat distribution.

of water-out temperature, sump-oil temperature, and speed increments as the heat rejection tests. It is usually of no interest to separate exhaust, radiation and oil heat; hence it is required only that the fuel heat be divided into: (1) brake power; (2) friction power; (3) heat to cooling water; (4) the balance. The rate of water circulation through the engine is determined from the water heat rejection data. When a heat rejection test is made to ascertain radiator requirements in the car, the standard radiator is used to simulate water flow restrictions. (See Figs. 48 and 49.)

TEST 10. VOLUMETRIC EFFICIENCY. This test is used to determine the full throttle volumetric efficiency. It is made as a separate test in which power and economy are of secondary importance but in which all engine data are recorded with the air consumption data. (See Fig. 50.)

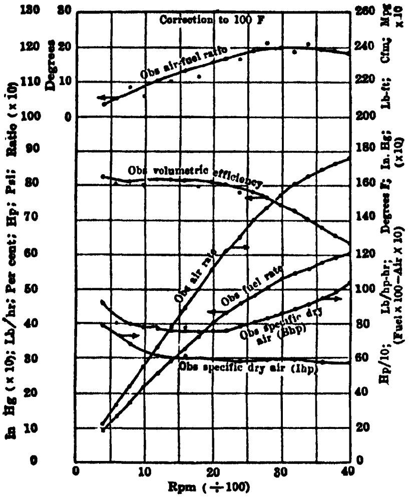


Fig. 50. Test 10. Volumetric efficiency.

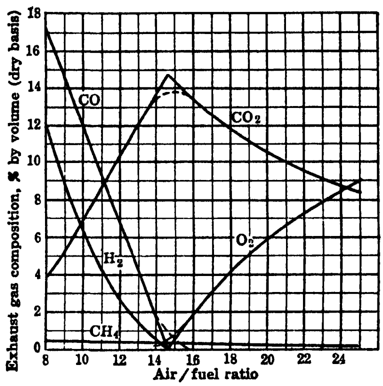


Fig. 51. Exhaust gas composition versus air/fuel ratio. (Computed relationships based on fuel compositions of C_8H_{17}, water-gas reaction equilibrium constant of 3.8, and formation of 0.15 mole of CH_4 per mole of C_8H_{17} burned.)

TEST 11. MOTORING FRICTION. The purpose of this test is to obtain an estimate of engine friction loss. Comparative results obtained by several methods justify the use of the full throttle motoring method.

TEST 12. MOTORING COMPRESSION. This test is used to show the relative air charge distribution among cylinders and to indicate ram effects.

TEST 13. FUEL DISTRIBUTION. Quantitative fuel distribution is indicated by comparison of relative combustion temperatures in individual cylinders as given by the plot of spark plug temperatures or thermocouple temperatures taken inside the cylinders versus hourly fuel rate for the engine. As temperature peaks are coincident with a 13 : 1 air-fuel ratio, the relative change in engine fuel consumption from the values corresponding to the individual peak locations to the value corresponding to the standard carburetor setting is used to evaluate cylinder air-fuel ratios. Two series of tests are required: (1) full throttle; (2) road power.

TEST 13-A. FUEL DISTRIBUTION. An optional method of determining fuel distribution is by the measurement of air-fuel ratios by gas sampling directly inside the individual cylinders. This is accomplished with solenoid operated sampling valves to obtain exhaust gas samples which are analyzed by either a direct reading air-fuel ratio meter or an Orsat apparatus. A solenoid operated sampling valve synchronized to engine crank movement is used to obtain the exhaust gas sample. Typical exhaust gas composition as a function of air-fuel ratio is shown in Fig. 51.

TEST 14. OIL CONSUMPTION. This test is used to obtain the absolute oil consumption rate but more particularly the rate of increase in consumption with speed. This latter factor has been found to give a more accurate index of the consumption characteristics than the absolute rate. Any comparisons should be made at like temperatures.

TEST 15. ROAD POWER AS INSTALLED. This test is used to obtain fuel economy as installed in the car under road load part throttle conditions. Interest in fuel consumption at high road speeds makes it desirable to test at engine speeds approaching maximum car speed.

TEST 16. ROAD POWER, BEST ECONOMY SPARK. This test is used to determine the best economy performance at part loads of a given carburetor on a given engine; therefore, it is used for comparing the part load performance of different carburetors on the same engine. This test should not be used as a measure of engine design; for this purpose, Test 17 is used which is planned to give optimum characteristics. Road loads are used for this test (see Test 15).

TEST 17. ROAD POWER, OPTIMUM ECONOMY. This test is used to give information on optimum part load economy, to serve as a basis for evaluating a specific engine design, for comparison with other designs, or for judging the economy performance obtained on the same engine under other conditions. Road loads are used for this test (see Test 15).

TEST 18. ECONOMY AT FRACTIONAL LOADS, AS INSTALLED. This test is used to obtain fuel economy at various part loads as installed in the car. Tests will be

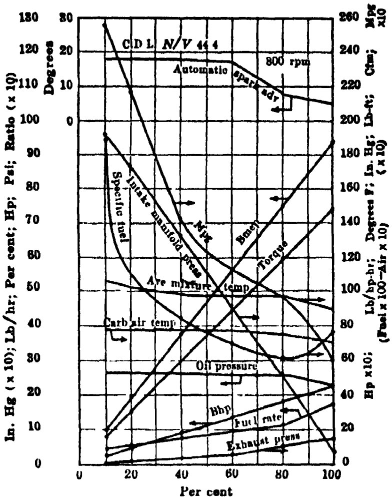


FIG. 52. Test 18. Economy at fractional loads, 800 rpm.

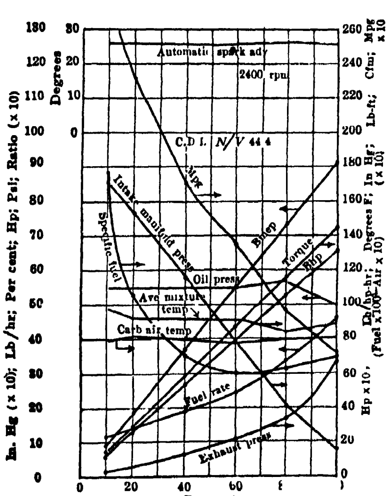


FIG. 53. Test 18. Economy at fractional loads, 2400 rpm.

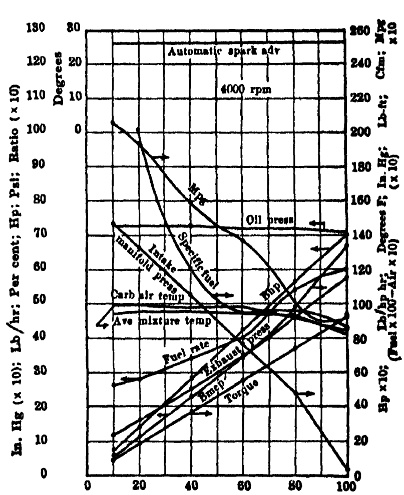


FIG. 54. Test 18. Economy at fractional loads, 4000 rpm.

run normally at $^1/_{20}$, $^1/_{10}$, $^1/_5$, $^2/_5$, $^3/_5$, and $^4/_5$ load or at other loads as desired for the problem under study. Full load from Test 1, "Full Throttle as Installed," will be used to determine fractional loads. The fractional loads are determined from the corrected full load. For exploratory work, set carburetor and spark for optimum conditions, either or both may be adjusted. (See Figs. 52 to 54.)

SECTION 15

AIR AND MARINE TRANSPORTATION

BY

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AIRCRAFT

BY FREDERICK K. TEICHMANN

HELICOPTERS

BY FREDERICK K. TEICHMANN

LIGHTER-THAN-AIR CRAFT

BY FREDERICK K. TEICHMANN

SUPERSONICS

BY NORMAN C. WITBECK

JET PROPULSION

BY C. W. SMITH

MARINE ENGINEERING

BY D. C. MACMILLAN

AIRCRAFT

By Frederick K. Teichmann

DEFINITIONS AND CLASSIFICATIONS. Aircraft fall into two general categories.

Lighter-than-air aircraft displace a mass of air greater in weight than their total weight, and, therefore, they can float in the atmosphere buoyed up by aerostatic forces.

Heavier-than-air aircraft do not displace enough air to achieve buoyancy, but derive their lift by the relative motion of air passing over appropriately designed surfaces. The sustaining forces are aerodynamic.

FORMS OF AIRCRAFT. Lighter-than-air aircraft assume two general forms, balloons and airships.

Balloons consist of containers for a suitable gas whose density is less than that of the atmosphere (heated air, hydrogen, helium, etc.), are usually without means of directional control, and always without means for mechanical propulsion.

Airships differ from balloons in that they have means for mechanical propulsion and apparatus for directional control. They consist of single or multiple containers for the supporting gases and the necessary structures to house the personnel, and to support the power plants and controlling surfaces. Structurally, they fall into three classes. (1) *Nonrigid*, in which the gas container maintains its shape solely by reason of the internal gas pressure. (2) *Semirigid*, in which the shape of the gas container is partially maintained by means of a structural keel, or backbone. (3) *Rigid*, in which appropriate structural members completely surround and are independent of the gas containers.

Heavier-than-air aircraft, in their more familiar forms, fall into these general classes: (1) *Airplanes*, fixed wing aircraft, equipped with some propulsive means, and supported by the dynamic action of the air. (2) *Gliders*, airplanes without power plants, which utilize the forces of gravity and the energy of air currents, usually thermal, to produce the required propulsive action. (3) *Pilotless aircraft, or guided missiles*, a special type of aircraft, carrying no personnel, designed to operate at high speeds, and usually gyroscopically and radio controlled. (4) *Helicopters*, aircraft designed to ascend vertically, and to hover, as well as to fly horizontally, because of the thrust of power-driven rotating airfoils or propeller-like blades. (5) *Autogyros*, resembling a helicopter but with blades rotating under the effect of aerodynamic forces only. There are other variants of the autogyro; for example, the gyroplane. Also proposed have been the (6) *Cyclogyro*, in which a series of blades rotate about a horizontal axis perpendicular to the direction of normal flight. (7) *Ornithopter*, which attempts to simulate the flapping action of birds' wings. (8) *Convertiplane, or Heliplane*, in which a fixed wing is released to permit its rotation, and vice versa. Such aircraft would rise as helicopters and fly forward in a manner similar to conventional aircraft.

1. CLASSIFICATION OF AIRPLANES

ELEMENTS OF AIRPLANES. Airplanes consist of five principal elements which may be combined in a variety of ways. They are (1) Main supporting or lifting surfaces (wings). (2) Auxiliary surfaces for stabilization and control (horizontal stabilizer, fin elevators, rudders, and ailerons). (3) Housing for the personnel, power plants, and cargo (body, fuselage, nacelles, or hull). (4) Undercarriage or landing gear (wheels, tail skids, floats, or hull). (5) Power plant (engines, propellers, fuel, oil, cooling system and controls). Airplanes may be classified under four major considerations, all of which are interrelated: (1) Structural arrangement. (2) The nature of their terrestrial base of operations. (3) Materials used. (4) Power-plant arrangement.

These principal elements may be broken down into subdivisions which afford a method of classifying airplanes. There are several of these subdivision classifications, as shown below.

Number of Wings. Airplanes may be either *monoplanes* (common), *biplanes* (rare), or *triplanes* (considered obsolete), depending on whether they have one, two, or three main supporting surfaces. Monoplanes may be of the *parasol*, the *high*, the *mid-*, or the *low-wing* type, depending on whether the wing is carried on struts above the fuselage or is attached to the upper, the middle, or the lower part of the fuselage (body). Also these wings may be of the internally braced or *full cantilever* variety, in which the structure is wholly enclosed within the wing contour; or they may be of the *semicantilever* variety, in which the wing is strut braced, or even wire braced, although the latter is now rare.

Biplanes may be either *orthogonal*, or *staggered*, depending on whether the top wing is placed directly over the bottom wing, or whether it is located forward or aft of the bottom wing.

Either monoplane or biplane wings may be tapered in section, or in plan, or in both; or they may be swept forward or back; that is, the spanwise axis of the wing may make an angle with respect to the plane of symmetry other than 90 degrees. A large amount of sweepback seems especially beneficial for airplanes flying at or above sonic speeds. To increase the lift or sustaining forces of these wings, lift-increase devices, referred to later, are added. Almost without exception the ailerons, or lateral control surfaces, are mounted at or near the tips of the main wings, although in the pure flying wing type, the aileron and elevator action may be combined.

Tail Surfaces. While the vertical (fin and rudder) and horizontal (stabilizer and elevator) surfaces are combined into a single tail unit, or *empennage*, they may be supported either at some distance aft of the wing (the common type) or at some distance ahead of the wing, referred to as "tail first" or *canard* type. The flying wing is often referred to as the tail-less airplane because the control surfaces are located laterally or at the wing tips instead of longitudinally aft (or ahead) of the wing.

Landing Gear. With respect to the landing gear, airplanes may be grouped as landplanes, seaplanes, or amphibians. *Landplanes* are fitted with an undercarriage, usually of three wheels, either disposed with one nose wheel ahead of two main wheels (as for the so-called *tricycle* gear) or with a tail wheel disposed behind the two main wheels, all connected to the body or fuselage through suitable shock-absorbing devices. *Seaplanes* take two practical forms: the *float seaplane*, essentially a land type with the undercarriage replaced by one or two suitably designed floats or pontoons; and the *flying boat*, in which the main body of the airplane is a watertight hull, with a bottom suitably shaped to give good take-off and landing characteristics. An *amphibian* is a composite type designed to operate from either water or land.

Power-plant Arrangement. Aircraft may be powered with the conventional internal-combustion engine, operating two-, three-, or four-bladed propellers, or counter-rotating propellers; or they may be powered with gas turbines operating propellers; or they may be powered with the jet-type engines.

These engines may be housed in nacelles along the wing, or entirely within the wing, or in the fuselage.

Propellers may be arranged either as *tractors*, or as *pushers*, depending on whether the propeller is ahead or behind the engine.

For take-off, the power of the airplane may be temporarily increased through the use of *rockets* or *catapults*. The former, known as jet-assisted take-off (Jato) is becoming of increasing importance in the take-off of heavily loaded aircraft.

Structural Design. Airplanes may be of composite design with the wing designed according to one method, the fuselage to another. However, either the complete airplane structure or the component parts may be classified as of *fabric covered truss* or *semimonocoque* (or reinforced monocoque) construction. Standard structure for the wings of small, light airplanes has been wooden spars, wooden or metal ribs, metal tie rods or drag wires, with the entire structure covered with fabric. A truss-type fuselage, employing essentially a Warren or Pratt truss, has been common for small, light airplanes. In heavier airplanes the covering, or skin, is usually metal, reinforced by longitudinal and transverse stiffeners with occasional transverse frames or bulkheads, to form the semimonocoque type of structure.

In still later designs, especially high-speed airplanes, sandwich construction, comprising two outer metal sheets attached to a soft (usually organic) core, is employed along with the plate-type structure where the skin or covering becomes so thick that stiffeners need not be used, or at least used sparingly.

TRANSPORT AND PERSONAL AIRCRAFT DATA are given in Tables 1 and 2 for some of the better-known aircraft.

Materials Used. The principal structural materials used in aircraft are wood, chiefly spruce (use of wood is gradually disappearing); fabrics, chiefly cotton; various soundproofing materials, made of felt, paper, fiber glass, etc.; and metals, chiefly steel, aluminum alloy, and magnesium alloy. For fittings, engine mounts, landing gear, and accessories, steel is commonly employed; for the main structure of the wing, fuselage, and tail surfaces, aluminum alloys are used, and occasionally magnesium alloys. See Table 3 for ratio of weights of common structural materials.

Table 1. Typical American Transport Airplane Data
(Based on data in *Aviation Week*, 1948)

Manufacturer	Designation	No. Passengers	Engines; Make; Model; hp	Propellers	Range, miles	Speed, mph	Cruising Speed, mph	Climb, ft/min	Gross Weight, lb	Empty Weight, lb	Wing Span	Length	Wing Area, sq ft	Wing Loading, lb per sq ft	Power Loading, lb per hp
Beech Aircraft Corp.	Twin-Quad	22–23	4 Lyc; 375	2 HS	1450	230	180	1000	19,500	NA	70′	53′	.....	.....	13.0
	D-18-C	4–9	2 Cont; 525	Hy	1370	240	224	1450	9,000	5,900	47′ 7″	33′ 11 1/2″	349	25.8	8.57
	D-18-S	4–9	2 P&W; 450	HS	1300	230	211	1250	8,500	5,615	47′ 7″	33′ 11 1/2″	349	24.3	9.45
Boeing Aircraft Co.	377 Stratocruiser	57–85	4 P&W R-4360; 3500	C	4200	375	340	1100	135,000	78,920	141′ 3″	110′ 4″	1751	77.0	9.65
Consolidated Vultee Aircraft Corp.	240 Convair-Liner	44	2 P&W R-2800-CA18; 2400	C rev	800	336	300	NA	39,500	8,509	91′ 9″	74′ 8″	.....	.....	8.23
Curtiss-Wright Corp., Airplane Div.	CW-32	(cargo)	4 P&W; 2100	C	1500	326	300	1100	98,450	45,415	130′ 2″	88′ 11 1/2″	.....	.....	11.7
Douglas Aircraft Co., Inc.	DC-3	23	2 P&W R-1830-92; 1050	HY	1510	234	202	1230	25,200	17,100	95′	64′ 5 1/2″	.....	.....	12.0
	DC-4	47	4 P&W 2 SD 13 G; 1200	HS	4250	246	231	1090	73,000	40,200	117′ 6″	93′ 5″	1457	50.0	15.2
	DC-6	55	4 P&W CA-15; 1800	HS or C	4480	352	301	1100	93,200	50,600	117′ 6″	100′ 7″	1457	64.0	12.95
	DC-6A	(cargo)	4 P&W R-2800 CA 15; 1800	HS or C rev	4000	365	310	1200	96,000	47,500	117′ 6″	105′ 7″	.....	.....	13.35
	DC-9	28	2 P&W 2180; 1650	HS	2125	257	242	NA	30,000	19,600	101′	75′	.....	.....	9.10
Grumman Aircraft Engineering Corp.	G-44-A * Widgeon	5	2 Ran 6-440C-5; 200	S or CR	640–810	165	130	1000	4,525	1,285	40′	31′	245	18.5	11.31
	G-73 * Mallard	12	2 P&W R-1340-S3H1; 550	Hy	721–1330	215	180	1290	12,750	9,350	66′ 8″	48′ 4″	...	.....	11.60
Lockheed Aircraft Corp.	749 Constellation	44–62	4 Wr 749C18BDI; 2500	C rev	5450	346	309	1280	102,000	58,971	123′	95′ 3″	1650	61.8	10.20
Glenn L. Martin Co.	2-0-2	40–44	2 P&W R-2800; 2400	HS	1435	310	260	1520	39,900	24,649	93′ 3 3/8″	71′ 4″	860	46.4	8.32
Northrop Aircraft Inc.	N-23 Pioneer	(cargo)	3 Wr; 700	HS	1700	175	150	1400	27,500	NA	87′	66′ 6″	.....	.....	13.10

Engines: Cont—Continental. Lyc—Lycoming. P&W—Pratt & Whitney. Ran—Ranger. Wr—Wright.
Propellers: HS—Hamilton Standard. C—Curtiss Electric. CR—Curtiss-Reed. S—Sensenich. Hy—Ham. Standard Hydromatic. rev—reversible pitch.
NA—Not available. * Flying Boat.

Table 2. Data on Leading American Personal Aircraft

(Data from *Aviation Week*, 1948)

Manufacturer	Designation	Engine	Horsepower	Speed, mph	Cruise Speed, mph	Range, miles	Gross Weight, lb	Empty Weight, lb	Wing Span	Length
Aeronca Aircraft Corp.	Champion	Cont	65	100	90	250	1220	750	35′ 2″	21′ 6″
	Super Chief	Cont	85	100	95	385	1350	820	36′ 1″	20′ 5″
	Sedan	Cont	145	120	105	445	2050	1150	37′ 6″	25′ 3″
All-American Aircraft, Inc.	Ensign	Cont	85	125	110	400	1550	1000	33′	22′
Beech Aircraft Corp.	Bonanza	Cont	185	184	172	750	2550	1558	32′ 10″	25′ 2″
Bellanca Aircraft Corp.	Cruisair Sr.	Frank	150	170	150	600	2150	1230	34′	21′ 3″
Call Aircraft Co.	Callair A-3	Cont	125	120	109	456	1550	975	35′ 9″	23′ 5″
Cessna Aircraft Co.	120	Cont	85	120	100	450	1450	785	32′ 10″	21′ 6″
	140	Cont	90	125	105	450	1450	860	32′ 10″	21′ 6″
	170	Cont	145		125		2200	1200	36′	24′ 11 1/2″
	190	Cont	240	170	160	750	3350	2015	36′ 2″	27′ 2″
	195	Jacobs	300	180	165	750	3350	2030	36′ 2″	27′ 4″
Consolidated Vultee-Stinson Div.	Voyager	Frank	165	NA	130	554	2400	1294	34′	25′ 2″
Engineering & Research Corp.	Ercoupe	Cont	85	120	110	350	1400	815	30′	20′ 9″
Funk Aircraft Co.	Customaire	Cont	85	115	100	350	1350	890	35′	20′
Luscombe Airplane Corp.	Silvaire 8A	Cont	65	115	105	300	1260	750	35′	20′
	Silvaire 8E	Cont	85	125	112	250	1400	765	35′	20′
	Silvaire 8F	Cont	90	128	115	475	1400	850	35′	20′
	Silvaire Sedan	Cont	165	145	130	500	2280	1280	38′	23′ 6″
Monocoupe Aircraft & Engine Corp.	Monocoupe	Lyc	108	145	130	520	1610	1000	32′	22′ 11″
	Monocoach	2 Lyc	160	180	155	750	3365	2100	36′	24′ 6″
Piper Aircraft Corp.	PA-11	Cont	65	100	87	300	1220	730	35′ 2 1/2″	22′ 4″
	PA-14	Lyc	108	123	110	600	1850	1000	35′ 5 1/2″	23′ 2 1/2″
	PA-15	Lyc	65	102	90	300	1100	620	29′ 3″	18′ 8″
Ryan Aeronautical Co.	Navion	Cont	185	157	150	500	2750	1680	33′ 4 1/2″	27′ 3″
Taylorcraft, Inc.	Model 47	Cont	65	105	95	380	1200	760	36′	22′
Texas Engineering & Mfg. Co., Inc.	Temco Swift	Cont	125	150	140	512	1710	1150	29′ 4″	20′ 10″

Engines: Cont—Continental; Frank—Franklin; Lyc—Lycoming.

Table 3. Ratio of Weights

	Aluminum Alloy	Steel	Spruce
Aluminum alloy	1.0000	2.7977	0.15430
Steel	0.3574	1.0000	0.05515
Spruce	6.4805	18.1312	1.00000
Weight of bar 1 sq in. by 1 ft, lb	1.2151	3.3996	0.18750

2. AERODYNAMICS

THE ATMOSPHERE. The layer of air which surrounds the earth is a nonhomogeneous fluid whose density varies inversely with the distance above sea level. The density at 20,000 ft is approximately one-half and at 40,000 ft about one-quarter the sea-level density. As with any gas, a very definite relationship exists among temperature, pressure, and density. The absolute values may vary considerably in any given locality, owing to local meteorological conditions. To have a basis for comparison of airplane performance or calibration of instruments, a purely arbitrary *standard atmosphere* has been assumed, roughly corresponding to average conditions at latitude 40° North, and defined by known altitude-temperature-pressure relations. (See The Standard Atmosphere, *NACA Report* 218 and 538.) In selecting bases for the standard atmosphere, international standards have been followed. The basic data are standard sea-level pressure, p_0, = 29.921 in. Hg (2116.4 lb per sq ft); standard temperature, t_0, = 59.0 F; standard specific weight of air, $g\rho$, = 0.07651 lb per cu ft; standard temperature gradient, a, = 0.003566 F per ft of altitude (1 F per 280 ft).

Based on the above assumption, properties of the standard atmosphere at intervals up to 100,000 ft are given in Table 4. Up to an altitude h of about 35,000 ft the density ratio is given correctly within 2 1/2% by the formula

$$\frac{\rho}{\rho_0} = 1 - 0.03\left[\left(\frac{}{1000}\right) - \left(\frac{}{10{,}000}\right)^2\right] \tag{1}$$

Table 4. Standard Atmosphere—English Units

Altitude	Temperature		Pressure			Density	
h, Thousands of feet	t, °F	t, °C	p, in. Hg abs	p, psia	p/p_0	$g\rho$, lb/cu ft	ρ/ρ_0
0	59.00	15.00	29.92	14.70	1.0000	.07651	1.0000
2	51.81	11.04	27.82	13.67	0.9298	.07213	0.9428
4	44.73	7.07	25.84	12.69	0.8636	.06794	0.8881
6	37.60	3.11	23.98	11.79	0.8013	.06395	0.8358
8	30.47	− 0.85	22.22	10.91	0.7427	.06013	0.7859
10	23.34	− 4.81	20.58	10.11	0.6876	.05649	0.7384
12	16.21	− 8.77	19.03	9.35	0.6359	.05303	0.6931
14	9.07	−12.74	17.57	8.46	0.5873	.04973	0.6499
16	1.94	−16.68	16.21	7.96	0.6418	.04658	0.6088
18	− 5.19	−20.65	14.94	7.19	0.4992	.04359	0.5698
20	−12.32	−24.62	13.74	6.76	0.4594	.04075	0.5327
25	−30.15	−34.53	11.10	5.45	0.3709	.03427	0.4480
30	−47.98	−44.44	8.88	4.36	0.2968	.02861	0.3740
40	−67.00	−55.00	5.54	2.72	0.1852	.01872	0.2447
50	−67.00	−55.00	3.436	1.69	0.1149	.01161	0.1517
60	−67.00	−55.00	2.132	1.05	0.0714	.0072	0.0929
70	−67.00	−55.00	1.322	0.65	0.0442	.0044	0.0575
80	−67.00	−55.00	0.820	0.40	0.0272	.0028	0.0366
90	−67.00	−55.00	0.568	0.28	0.0190	.0017	0.0222
100	−67.00	−55.00	0.326	0.16	0.0109	.0011	0.0144

FLUID RESISTANCE. Because atmospheric air is not a perfect fluid, any solid body passing through it at any speed is opposed by a certain fluid resistance, which depends on the density of the fluid, the dimensions and the form of the body, its velocity, and a number of other factors. That is,

$$R = \frac{\rho}{2} SV^2 f\left(AR, \alpha, \frac{\rho VL}{\mu}, \frac{V}{V_c}, \frac{v}{V}, \frac{l}{L}, \frac{V^2}{gL}\right) \tag{2}$$

In English units, the terms would be expressed as follows:

R is the total resistance or resultant force, pounds.

ρ is the mass density of the air, slugs per cubic foot.

S is the projected area of the body, usually square feet.

V is the velocity of motion, feet per second.

The function in parentheses indicates various parameters on which the resistance may depend, for example:

AR is the aspect ratio, commonly the ratio of two dimensions perpendicular to each other such as the ratio of the span to the chord.

α is the angle of attack or the attitude of the body with respect to the direction of relative motion of the fluid.

$(\rho VL)/\mu$ is known as Reynolds' number, which governs scale effects and is particularly useful in comparing data obtained at different speeds.

μ is the coefficient of viscosity.

V/V_c is known as the Mach number, and is the ratio of the fluid velocity to the velocity of sound in that fluid at the existing temperature. This factor is particularly important as the velocity of sound is approached or exceeded.

The turbulence factor v/V is the ratio of the velocity of the disturbance or turbulence to the velocity of the free stream.

The roughness factor l/L is the ratio of the linear dimension of an obstruction or surface roughness to the linear dimension of the object.

The Froude number $V^2/(gL)$ is concerned with wave motion and is of little importance except in curvilinear flight and in hydrodynamic problems. See also Section 5.

All the factors in eq. 2 can be more conveniently expressed by means of suitable curves or graphs which show variation of the force R (or some convenient coefficient) with angle of attack, α, or with Reynolds' number, or with Mach number. Equation 2 is usually expressed in this form:

$$R = C_R\left(\frac{\rho}{2}\right)SV^2 \tag{3}$$

where the terms have the same significance as before, and C_R is a resistance coefficient.

AIRFOILS are winglike surfaces, with cross sections designed to develop a useful dynamic reaction when relative motion is set up between them and the surrounding air. Figure 1 is a typical airfoil cross section, and indicates the location and direction of the resultant force. For a given angle of attack α, the resultant R intersects the chord c of the airfoil at the center of pressure O at a distance x from the leading edge. For convenience, it is customary to resolve the resultant force into its components L and D, respectively normal and parallel to the relative wind. Since the components of any force vary in accordance with the same laws that govern the force itself, fundamental equations may be written for the lift and drag:

$$L = C_L Sq \tag{4}$$

$$D = C_D Sq \tag{5}$$

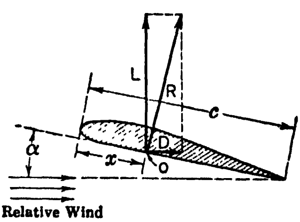


FIG. 1. Resolution of aerodynamic forces acting on a typical airfoil.

where C_L and C_D are known as NACA absolute lift and drag coefficients and are functions of the angle of attack, α. These coefficients are dimensionless; L and D are lift and drag, pounds; S is the projected area, square feet; $q = (\rho/2)V^2$ and is known as the dynamic pressure; V is the velocity of motion, *feet per second;* ρ must be mass units and is equal to $(0.07651/32.2) = 0.002378$ slug per cu ft at sea level.

For convenience, the engineering system may be employed when considering the drag or resistance of an object:

$$D = D_c SV^2 \tag{6}$$

where D is the drag, pounds; D_c is a coefficient (used in this form to distinguish it from the dimensionless coefficient C_D); S is in square feet; and V is in *miles per hour.*

The relation between the engineering and NACA absolute coefficients, is

Engineering	NACA Absolute
$L_c = \dfrac{L}{SV^2}$	$C_L = \dfrac{2L}{\rho SV^2}$
$D_c = \dfrac{D}{SV^2}$	$C_D = \dfrac{2D}{\rho SV^2}$

ρ must be in mass units. In foot-second units $\rho/2 = 0.001189$ under standard sea level conditions. To transpose coefficients from NACA absolute units to engineering units, multiply by 0.00255; to change engineering coefficients L_c and D_c to the dimensionless form, C_L and C_D, multiply by 392.

Figure 2 illustrates the usual method of plotting the section characteristics of an airfoil. The effect of various shapes on the lift and drag characteristics is shown qualitatively in Figs. 3 to 6, inclusive. See also p. 15-11 for distinction between c_l and C_L.

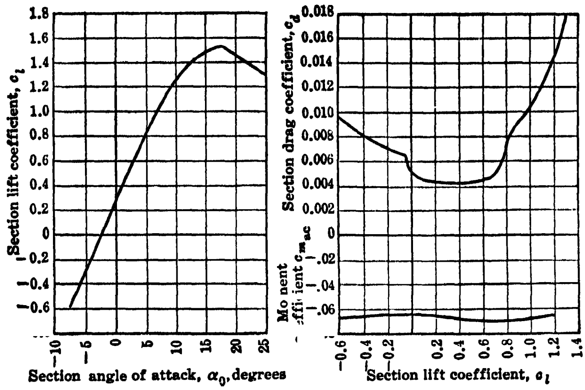


FIG. 2. Typical airfoil characteristics. (NACA 65_3-418 tested at Reynolds' number = 9×10^6)

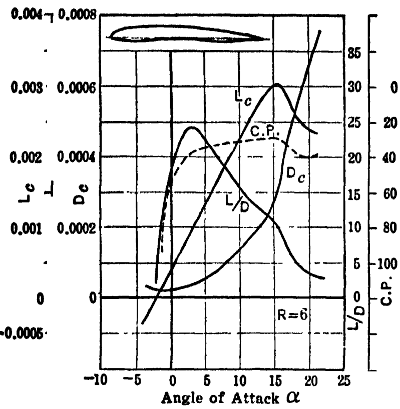


FIG. 3. Characteristics of a relatively thin airfoil with concave lower surface.

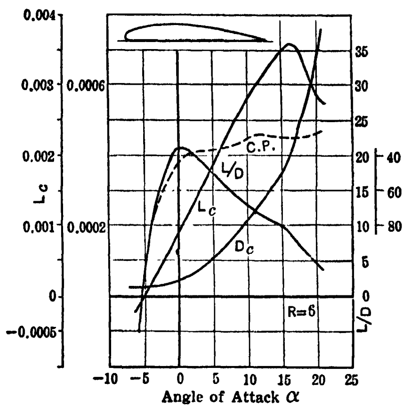


FIG. 4. Characteristics of medium thick airfoil with flat lower surface.

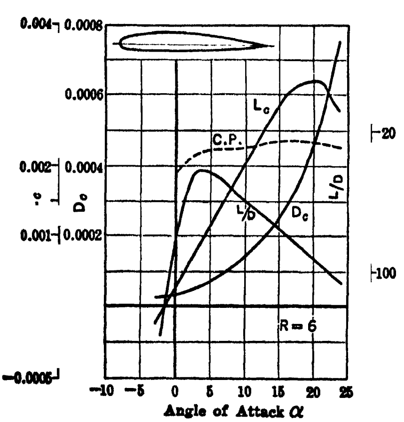


FIG. 5. Characteristics of medium thick airfoil with convex lower surface.

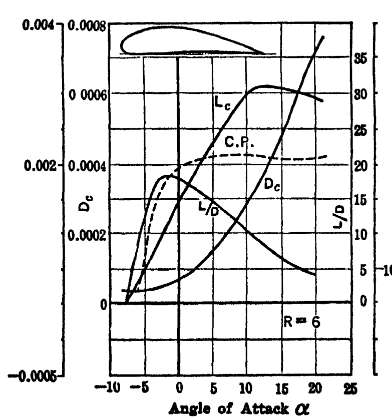


FIG 6. Characteristics of relatively thick airfoil with slightly concave lower surface.

In dealing with the lift and drag coefficients of wings, the area S involved is the area, in square feet, of the wing in plan; fundamentally, the product of the span (the distance from wing tip to wing tip) and the chord (distance from the leading edge to the trailing edge of the wing airfoil). In applying these equations to the calculation of parasite resistance of solid bodies (see Art. 3), any convenient area, or even dimension, may be used, although the area usually considered is the area of the body projected on a plane normal to the direction of relative motion.

CENTER OF PRESSURE TRAVEL AND MOMENT COEFFICIENT. The location of the vector of resultant force on an airfoil can be indicated either by specifying the distance from the leading edge to the intersection of the vector with the chord (called the *center of pressure*) or by giving the moment of the resultant force about some convenient axis. The moment is expressed by the equation

$$M = c_m q C S \tag{7}$$

where M is in foot-pounds; c_m is a dimensionless moment coefficient; q and S have the dimensions indicated previously; and C is the mean chord of the wing in feet. The first has the clearer physical significance and is valuable for purposes of illustration; the second is more convenient for calculation. Heretofore, the usual moment axis was a point on the chord 25% back from the leading edge because the moment coefficient about that point for any given airfoil is substantially independent of the angle of attack. Now the point along the chord is exactly chosen so as to obtain a constant value for the moment coefficient, independent of the angle of attack through the range of angles of normal interest.

The center of pressure coefficient, C_p, for an airfoil is the ratio of the leading-edge-to-center-of-pressure distance to the chord length or

$$\frac{x}{C} \tag{8}$$

The location of the center of pressure on an airfoil is a function of its section, planform, and angle of attack. For most common airfoils, as the angle of attack is increased from zero degrees to the angle of the maximum lift coefficient, the center of pressure tends to move forward from a point about 50% of the chord to a point near 30% of the chord behind the leading edge. Decreasing the angle of attack to very small values causes a very rapid rearward movement of the center of pressure for most airfoils. In fact, for angles within two or three degrees of that where the lift coefficient is zero, the center of pressure actually may pass behind the trailing edge. Thus airfoils are markedly unstable in pitch, and the addition of some form of auxiliary surface, usually a tail, is necessary to maintain equilibrium in flight. Airfoils whose bottom surfaces are substantially convex, and those in which the trailing edge is slightly reflexed upward, ordinarily exhibit a lower rate of center-of-pressure travel than nonsymmetrical positively cambered (curved) sections.

A typical distribution for an average airfoil of medium thickness toward the center of a span of rectangular planform is shown in Fig. 7. The curves represent the condition at

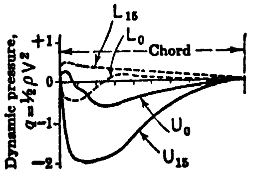


Fig. 7. Pressure distribution along chord of typical airfoil at low and high angles of attack.

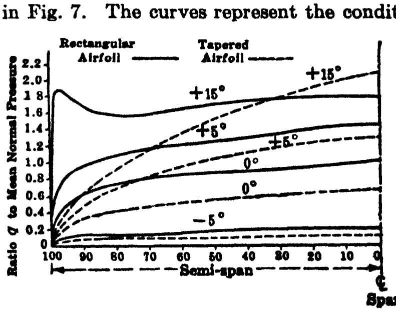


Fig. 8. Pressure distribution along the span of typical airfoils, rectangular and tapered.

approximately 0 degrees and at +15 degrees angle of attack. Typical pressure distributions along the span of rectangular and tapered airfoils are shown in Fig. 8. Note that for angles within the ordinary flying range the span loading is approximately elliptical. At high angles of attack, unusually high pressures tend to develop around the tips of a rectangular wing, which is undesirable from a structural standpoint. The average effects of tapering a wing in plan are indicated by the dotted lines in Fig. 8.

The shape of the wing tip influences the pressure distribution of the wing. However, by placing airfoils at different angles of incidence, so-called *aerodynamic twist*, at successive stations spanwise (i.e., each section has a different angle of incidence from the section at the root), it is possible to change the spanwise distribution. A specific pressure distribution may be desired for lateral control purposes, for structural reasons, or for improving certain performance requirements. For near-sonic or supersonic speeds (see NACA T.N. No. 1032 and No. 1033) it can be shown that, for an airfoil of infinite aspect ratio moving at an angle of sideslip, the pressure distribution over the wing is determined solely by the component of motion whose direction is normal to the leading edge. From this fact, it is further deduced that the pressure drag of an airfoil may be reduced if the planforms utilize sweepback angles greater than the Mach angle (the angle that plane shock waves make with the airfoil at sonic or supersonic speeds). See Art. 11, Fig. 1, p. 15-28.

If β represents the angle of sweepback, the velocity causing the pressure distribution over the airfoil for an airplane flying at velocity V is $V \cos \beta$. If $V \cos \beta$ is less than the sound velocity, V_c, the flow and distribution are similar to those occurring at subsonic speeds.

AIRFOIL DEVELOPMENT. A vast number of airfoil sections have been tested in aerodynamic laboratories. Much attention has been given lately to *laminar flow* airfoils and to airfoils designed for *compressible flow*. It can be shown that if the boundary layer, a comparatively thin layer of air adjacent to the airfoil, can be maintained as laminar (non-turbulent) flow over the airfoil, the drag coefficients of the airfoil are lower than for airfoils not so designed. While laminar flow can be maintained theoretically, practical difficulties are likely to arise because of surface irregularities, cut-outs, gaps in front of ailerons and flaps, and propeller slipstream. The commonly used laminar flow airfoils have been the NACA 24 series (NACA 2409, 2412, etc.), the NACA 44 series (NACA 4409, 4415, 4418), and the NACA 230 series (NACA 23009, 23015, etc.). For an excellent compendium of airfoil data, see *NACA Report* 824, entitled Summary of Airfoil Data.

For compressible flow of air encountered at speeds of aircraft exceeding 500 mph or more, a series of airfoils was developed by the NACA which attempts to prevent local airflow velocities over the airfoil from exceeding the speed of sound. For example, the airfoils designated, NACA 16-009, 16-109, 16-209, etc., belong to this series. The mean camber lines of these airfoils were designed to obtain uniform chordwise distribution, and then the thickness ordinates plotted perpendicular to the camber line. Whereas the commonly used airfoils (NACA 24009, for example) had their maximum ordinate at about 30% of the chord, the NACA 16-009 has its maximum thickness at about 50% of the chord.

SELECTION OF AIRFOILS. In selecting an airfoil for a particular purpose, the following properties are sought by designers. (1) High maximum lift coefficient, C_L. (2) Low drag coefficient, C_D, for the values of the lift coefficient that cover the normal flying range of the aircraft, especially at lift coefficients between 0.1 and 0.5. (3) Minimum center of pressure travel for lift coefficients from 0.1 to the maximum value. (4) Sufficient depth of the airfoil from about 10% of the chord to about 70% of the chord to facilitate structural design.

INDUCED DRAG AND ASPECT RATIO CORRECTION. For ordinary engineering purposes, lift and drag are referred to the direction of the relative wind at some point well outside the zone of influence of the airfoil. (See Fig. 1.) Actually the lines of flow immediately surrounding the airfoil are deflected downward. Thus a more exact axis of reference is found by bisecting the angle at which the air flows on to the section and the angle at which it leaves, as in Fig. 9. The components of the resultant force, taken with respect to the mean relative wind, are called the *profile lift*, L_o, and the *profile drag*, D_o. The ordinary lift and drag components, designated by L_a and D_a, may be found by taking components of the profile lift and drag parallel to the ordinary relative wind direction. Since profile drag is much smaller than profile lift, its component parallel to the lift axis may be neglected, and the profile lift may be assumed equal to the lift as ordinarily measured. The component of the profile lift along the drag axis, however, may have considerable magnitude, and is called the *induced drag*. The total drag thus is made up of two elements, profile drag and induced drag. Profile drag depends only on the shape of the section and its angle of attack. The induced drag, however, is purely a function of the *aspect ratio*, i.e., ratio of span to chord, and the lift coefficient. It is independent of the individual section considered. It can be shown

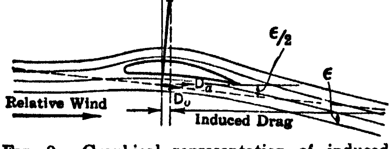


FIG. 9. Graphical representation of induced drag.

that the coefficient of induced drag C_{Di} may be expressed by

$$C_{Di} = \frac{C_L^2}{\pi AR} \tag{9}$$

where AR = aspect ratio. Thus the total drag coefficient for a given lift coefficient steadily decreases with increasing aspect ratio.

ASPECT RATIO, as usually defined, is the ratio of the span divided by the chord, but a more general relation is

$$AR = \frac{b^2}{S} \tag{10}$$

where AR is aspect ratio, b the extreme span, feet; S the area, square feet. It is customary to present airfoil lift and drag coefficients for infinite aspect ratio. Such coefficients are known as *section characteristics*, and are represented by c_l and c_d instead of C_L and C_D as for the wing of finite aspect ratio. For high aerodynamic efficiency, high values of AR are to be preferred. Present-day aircraft have aspect ratios of 6 to 14 while gliders have aspect ratios as high as 20.

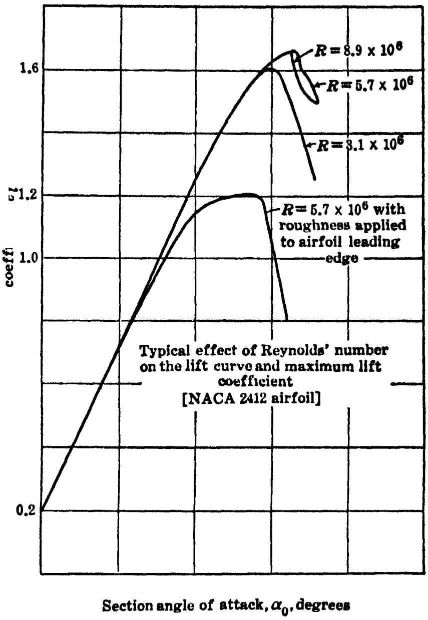


FIG. 10. Effect of Reynolds' number on the lift coefficient of an airfoil (NACA 2412).

SCALE EFFECTS OR REYNOLDS' NUMBER. Aerodynamic forces are affected by Reynolds' number, R.N. = $\rho VL/\mu$. In most problems the variation of the coefficients with scale is a secondary factor, but it becomes an important one when predicting full-scale aerodynamic characteristics from wind tunnel tests on models. As a general rule, the maximum lift for comparatively thin airfoil sections tends to increase slightly with increase in Reynolds' number; for thick sections the reverse is true. (See Fig. 10.) Induced drag does not change with Reynolds' number, but the effect on profile drag may be marked. (See Fig. 11.)

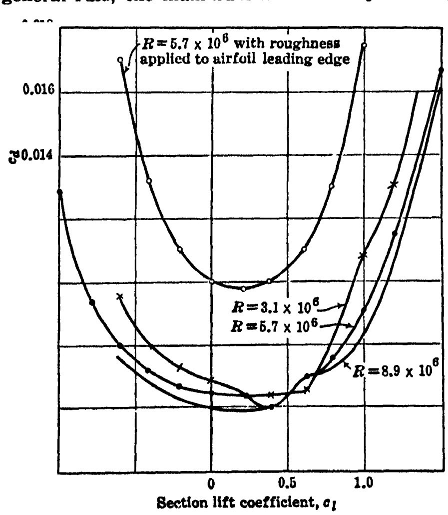


FIG. 11. Effect of Reynolds' number on the drag coefficient of an airfoil.

CORRECTION FOR MACH NUMBER. The section characteristics, lift, drag, and moment coefficients, for the lower speeds, where $M^2 = (V \cos \beta)^2/V_c^2$ is less than 1, can

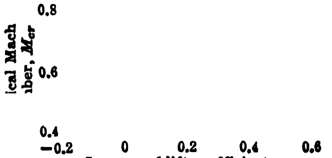


FIG. 12. Typical variation of airfoil critical Mach number with section lift coefficient. High-speed lift coefficient is obtained by multiplying by the factor

$$\frac{1}{\sqrt{1 - M_{cr}^2}}$$

be corrected to apply at other speeds V of the aircraft. The following expressions permit calculating the required corrections:

$$c_l = \frac{c_{l0}}{} \tag{11}$$

to give the new lift coefficient and

$$\frac{\quad}{\sqrt{1 - M^2}} : \frac{dC_L}{d\alpha} \tag{12}$$

to give the new slope of the lift curve. The subscript, 0, refers to subsonic tests. The slope is that of the c_l curve plotted versus α, the angle of attack. See Fig. 12.

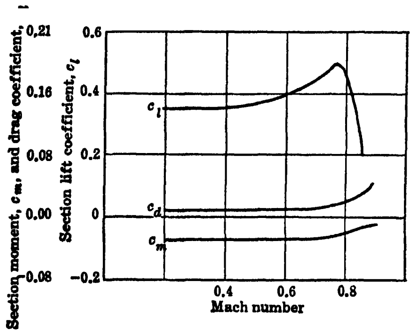


FIG. 13. Typical variation of airfoil section coefficients with Mach number.

The drag coefficient has to be found for an *equivalent* airfoil having a change in thickness and camber. The new equivalent "thickness" is

$$\frac{t_0}{\sqrt{1 - M^2}} \tag{13}$$

where t is the maximum thickness of the airfoil in question. For a cambered airfoil, the camber and the pitching moment coefficient, $c_{m_{a.c.}}$, change by the same ratio $1/\sqrt{1 - M^2}$. The sweepback angle, β, is incorporated in the design to obtain values of aerodynamic section characteristics for extreme speeds comparable to those of relatively thin airfoils at subsonic speeds. See Fig. 13 for variation of c_l, c_d, and c_m with Mach number.

HIGH LIFT DEVICES. To increase the speed range of airplanes, three general methods may be employed: (1) Flaps, either at leading or trailing edge. (2) Slots, either fixed or variable, located along the chord. (3) Boundary layer control either by applying suction, or expelling air rearwardly; or any combination of these methods. The advent of the jet engine, with its large quantities of compressed air, has made the third method a distinct possibility. Some experimental work has been done in this direction.

The general effect of moving trailing-edge flaps downward is to increase the concavity of the lower surface of the section and thus to increase the lift. Using trailing-edge flaps whose chord is approximately 20 to 25% of the chord of the basic airfoil, monoplane airfoils show an increase in maximum lift of about 40% with a flap setting of 45 to 60 degrees. In practice, however, the maximum flap travel usually is limited to about 30 degrees, under which conditions from 50 to 70% of the maximum possible increase in lift may be realized. The general effect of a trailing edge flap on the lift and drag is shown in Fig. 14. Flaps may be either simple, the whole trailing-edge section of the wing being pulled down as a unit, or split, the lower part of the trailing edge being depressed while the upper part is left rigid and the form of the upper surface of the wing remains undisturbed. The split type is more efficient, but the difference is not great. A nose flap has been proposed for airplanes designed for high speeds approaching $M = 1$. It is referred to, in common parlance, as the "droop snoot."

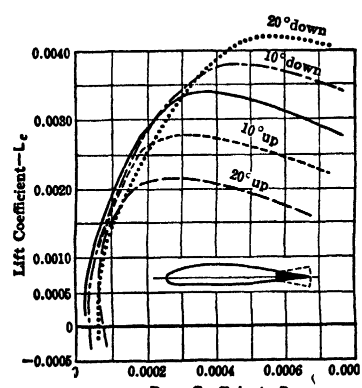
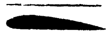
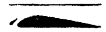
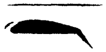
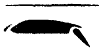
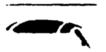
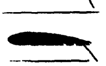
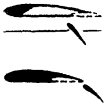
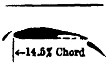


FIG. 14. Effect of trailing edge flap on lift and drag. (NACA M-6 airfoil with flap chord 20% of total wing chord.)

Fixed or variable leading-edge slots commonly are used only in that portion of the wing ahead of the ailerons. A variation of this is a slot immediately in front of the aileron for improving lateral control. The effect of the slot is to prevent "burbling" and thus maintain smooth airflow, with consequent increase in lift, at very large angles of attack. The fixed slot is preferred. Some of the combinations that have been tried, with their representative aerodynamic characteristics are shown in Fig. 15.

Another promising method is control of the boundary layer to reduce drag and delay burble, and thereby increase the maximum lift coefficient obtainable by sucking in the air along the airfoil surface. Similar advantages may be obtained by expelling air more or less tangentially (and rearwardly) to the airfoil surface. Neither approach has yet achieved a practical or economical means of controlling the boundary layer.

		Angle of lift increasing surface to basic airfoil	Flap chord in percent of basic airfoil chord	Max. lift coefficient C_L max.	Speed range ratio C_L max. / C_D min.	L/D at max. lift (2)	Angle of attack of basic airfoil at max. lift	Percent improvement in lift: Over plain airfoil (1)	Percent improvement in lift: Over simple flap	Percent improvement in speed-range ratio: Over plain airfoil (1)	Percent improvement in speed-range ratio: Over simple flap	Reference, N.A.C.A. report
Plain basic airfoil (1)				1.291	85.0	7.6	15°					T.R. 427
Simple flap		45°	30%	1.950	128.2	4.0	12°	51%		51%		T.R. 427
Slotted flap with cover plate		45°	30%	1.980	120.5	4.0	12°	53%	1.5%	42%	None	T.R. 427
Double slot and flap		45°	30%	2.442	117.5	4.0	16°	19%	25%	38%	None	T.R. 427
Fixed slot, cut in basic airfoil				1.772	73.8	5.3	24°	37%	None	None	None	T.R. 427
N.A.C.A. fixed auxiliary airfoil, ahead of basic airfoil (3)		0°	14.5%	(3) 1.705	104.5	3.5 Approx	24°	32%	None	23%	None	T.R. 428
N.A.C.A. optimum fixed slot (3)				(3) 1.648	76.4		24°	27%	None	None	None	T.R. 400
Handley-Page type automatic slot (3)				(3) 1.632	114.2		[illegible]	26%	None	34.5%	None	T.R. 400
Front slot and simple flap		45°	30%	2.182	91.0	3.8	19°	69%	12%	7%	None	T.R. 427
Front slot and slotted flap		45°	30%	2.261	93.2	3.8	19°	75%	16%	10%	None	T.R. 427
Triple slot and flap		45°	30%	2.600	87.3	3.8	20°	101%	33%	3%	None	T.R. 427
Split flap, rotated down, no backward movement		50°	30%	2.16	138.5	4.3	14°	70%	10.7%	63%	8%	T.N. 422
Split flap, trailing edge moved vertically downward (Zap)		60°	30%	2.35	150.8	3.7 Approx	13°	85%	20.5%	77%	17.5%	T.N. 428
Split flap, hinge point moved back to 90% of chord		54°	40%	(3) 2.222	142.2	3.8	13°	75%	14%	67%	11%	T.N. 422
Hall wing, front slot closed		48°	34%	2.08	138.8	3.6	13°	64%	6.7%	63%	8.1%	T.N. 417
Fowler wing projected (area increased approx. 31% over basic airfoil) (3)		40°	40%	(3) 2.422	155.3	4.25	15°	90%	24.3%	83%	21.2%	T.N. 419
Fowler wing with N.A.C.A. 22 slot and round nose of basic airfoil	 ←14.5% Chord	Slot −40° Flap +40°	Slot 14.5% Flap 40%	(3) 2.49	(3)(4) 137 (5) 199	3.76	21° to 25°	96%	28.1%	(3)(4) 61% (5) 134%	7% (5) 55%	T.N. 459
N.A.C.A. 22 slot on plain wing with rounded nose	 ←11.7% Chord	Slot −45°	Slot 14.5%	(3) 1.78	(4) 97.7 (5) 114.2	4.8	30°	40%	None	(3)(4) 15% (5) 35%	None	T.N. 459

NOTES.—1. In comparing properties of modified sections with the plain basic section, the coefficients used in each case were obtained under similar test conditions. Drag coefficients were taken with slot closed (if movable) and with flap neutral.

2. A low value of L/D at maximum lift indicates a steep glide angle and consequently a short landing. An L/D of 8 corresponds to a gliding angle of approximately 7 degrees, and a value of 3.5 means about 16 degrees. (T.R. 428.)

3. Based on total wing area; lift increasing device extended and projected on original chord line. Actually this area is necessary structural area and forms the basis for the comparison with the simple flap.

4. With slot and flap retracted the airfoil is not perfect, having a drag coefficient of 0.0182 compared with 0.0156 for the plain airfoil.

5. Based on contracted area.

FIG. 15. Characteristics of high lift devices applied to the Clark Y wing. (The Reynolds' number for all tests is 609,000, which corresponds to about one-third that for an ordinary small airplane at landing speed.)

3. PARASITE RESISTANCE

PARASITE DRAG is the air resistance of all parts of an aircraft except the wings. Drag arises from (1) skin friction and (2) turbulence. The latter, which arises from the breakdown in the smooth flow of air around the body and the subsequent formation of eddies, is usually larger.

SKIN FRICTION may be determined by either of two formulas. The choice between the two depends on the conditions existing in the layer of air immediately adjacent to the frictional surface. The effect of viscosity of fluid on the flow of air is confined to a *boundary layer*, within which the velocity gradient is exceedingly steep. The thickness of the boundary layer seldom exceeds $1/2$ in. in practice, except on the hulls of airships. At a distance of 2 ft from the leading edge of a flat plate at 200 mph it is about $3/16$ in. thick.

At low Reynolds' numbers the flow in the boundary layer is smooth and laminar, and the friction is determined by the formulas

$$D_f = C_{D_f} \frac{\rho}{2} S_e V^2 \tag{14}$$

$$C_{D_f} = \frac{1.333}{\sqrt{N}} \tag{15}$$

where N = Reynolds' number and S_e = total surface exposed to the air. In the case of a wing, or flat plate, S_e would be twice the area as ordinarily defined. At very high Reynolds' numbers the boundary-layer flow becomes turbulent. The friction then is much higher and varies according to a different law, following the formula

$$C_{D_f} = \frac{0.0745}{\sqrt[5]{N}} \tag{16}$$

At intermediate values of N the flow is laminar on the forward part of the surface, turbulent on the after part, and the friction has an intermediate value. The transition from laminar to turbulent flow is governed in part by the degree of roughness of the frictional surface, and in part by the general conditions existing in the stream. On the average it occurs at about N_x = 500,000. $N_x = \rho V x/\mu$, where x is distance from leading edge of the surface to the point under examination, and the other symbols have the same meaning as in the general definition of Reynolds' number. Figure 16 is a typical curve of friction coefficient against Reynolds' number showing the two distinct zones and the transition between them.

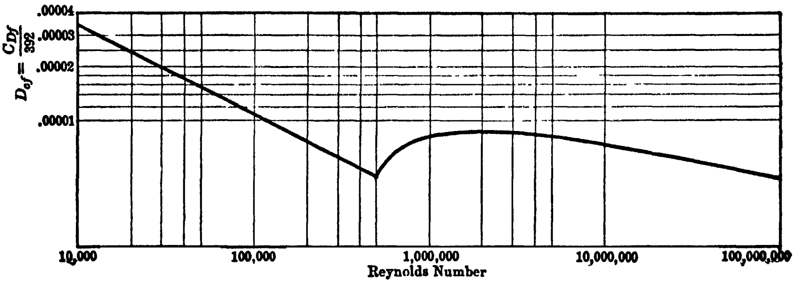


Fig. 16. Variation of skin friction coefficient.

SPHERES AND HEMISPHERES. The resistance of a sphere varies through a wide range with Reynolds' number. At Reynolds' number 500,000, corresponding to a sphere 1 ft in diameter and an air velocity of 45 miles per hour under standard atmospheric conditions at sea level, C_D = 0.078; at 1,000,000, C_D = .118; at 4,000,000, C_D = 0.145.

Eiffel found that if he attached a solid cone, whose base diameter was that of the sphere, and whose vertex angle was 20 degrees, to the downwind face of a hemisphere with convex side toward the wind, the total drag was approximately 50% of that of a complete sphere of the same diameter.

STREAMLINED BODIES. The resistance of typical streamlined bodies of revolution is shown in Fig. 17. The fineness ratio, or the maximum length divided by maximum diameter, of a streamlined body influences the total drag. In general, well-shaped streamlined bodies exhibit the best resistance values at fineness ratios of 2 to 3. The very best streamlined forms have a *disk ratio*, or ratio of their own resistance to that of a normal flat plate of area equal to the projected cross-section area of the body, as low as 1/40.

Good streamlined shapes are very sensitive to the effect of minor projections or slight surface discontinuities which disturb the smooth air flow. For example, the wrapping of a single turn of a thread 0.014 in. in diameter around a section 1 in. from the nose of a smooth streamlined body 8 in. in diameter and 24 in. long increases the total drag approximately 67%. Seven sets of similar thread rings placed at 1-in. intervals from the nose practically double the drag of the bare body.

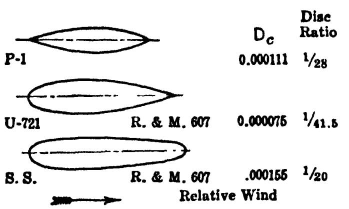
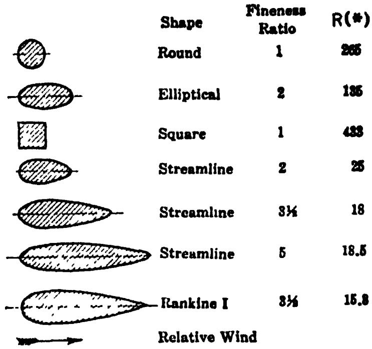


FIG. 17. Typical streamline bodies and their resistance coefficients ($392D_c = C_D$). Their resistances are compared with that of flat disks of equal diameters in the column noted "Disk Ratio."



(*)R = Resistance in pounds per 100 ft. of length per inch of width at 100 m.p.h.

FIG. 18. Relative drag of struts.

STRUTS. The relative drag of struts of several cross sections is shown in Fig. 18. Minimum drag for struts is associated with fineness ratios of 3 to 3 1/2. The drag coefficients for the best struts are about twice as high as for the best streamlined bodies, owing to the substitution of two-dimensional for three-dimensional flow.

Minimum strut resistance is obtained with the axis of the strut cross-section parallel to the relative wind. Yawing the average strut 7 degrees increases the drag by 50%; 8 degrees, approximately 100%. Round members may be used as struts without fairing if their axes lie at less than 45 degrees to the relative wind, as the section around which the air flows has then become elliptical.

FLAT PLATES NORMAL TO WIND. For a rectangular plate, of aspect ratio = 1, the drag coefficient C_D is 1.28. With increase of aspect ratio, the drag coefficient must be increased by a factor, as follows:

Aspect ratio	2	4	6	10	14	18
Coefficient correction factor	1.05	1.08	1.10	1.15	1.24	1.30

WIRES AND CABLES. The resistance coefficient for a single strand of round wire, cable, or streamlined wire is given in Table 5. The streamlined wire mentioned in the

Table 5. Drag for Wire

(Pounds per foot at 100 miles per hour in standard air)

Round		Cable		Streamlined	
Size	Drag	Size, in.	Drag	Size	Drag
No. 14 (B. & S.)	.142	1/16	0.180	6–40	.0551
10 "	.245	1/8	0.380	10–32	.0592
8 "	.318	3/16	0.575	1/4–28	.0734
6 "	.410	1/4	0.770	5/16–24	.0898
4 "	.525	3/8	1.165	3/8–24	.1121
....	....	1/2	1.555	7/16–20	.1388
....	. . .	.		1/2–20	.1530

table actually is rolled to a roughly lenticular section, with a major axis about 2 1/2 times the minor one. When round or streamlined wires are used in pairs, the resistance of the combination usually is below that of the total for the two wires singly, the saving by interference depending on the spacing between wires. The relative drag of wires in combination is shown in Fig. 19.

INTERFERENCE DRAG. The total drag of airplane components in close proximity to one another, for example, fuselage and wings, or fuselage and landing gear, always is considerably greater than the sum of the resistances of the individual units, because of mutual interference. In general, no members should intersect at sharp angles, but generous fairings or fillets should be provided. The importance of careful filleting between wings and fuselages has long been established. In one high-wing monoplane, studied at full scale, a simple 12-in. radius fillet at the intersection of the under surface of the wing and cabin effected a reduction in the total resistance of the combination of almost 2%. Research has indicated that the longitudinal rate of change of the radius of the fillet is important, and that the radius at the trailing edge of the wing should be much larger than at the leading edge.

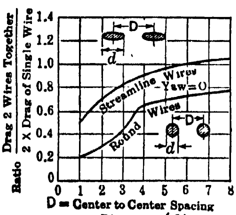


FIG. 19. Drag of adjacent wires and cables.

The location of an outboard engine nacelle with respect to a wing has an important bearing on interference effect and propulsive efficiency. NACA studies indicate that all nacelles should be completely cowled. In general, nacelles placed above a wing show much greater interference effect than those placed below. Nacelles placed closer than approximately one nacelle diameter below the lower surface of a wing should be completely faired into the wing, but in no case should the engine cowling hood be faired into the wing. The best location for a completely cowled nacelle for minimum drag and for greatest propulsive efficiency is with the propeller hub in line with, and about 25% ahead of, the leading edge of the wing. The location and fairing of nacelles have a great effect on top speeds, but little influence on climbing powers.

PARASITE RESISTANCE of airplanes of various types is given in Fig. 20 as a function of gross weight.

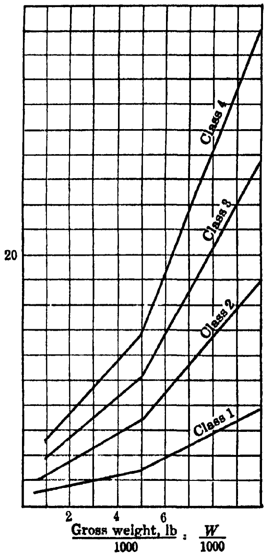


FIG. 20. Parasite resistance of airplanes (less wings) in terms of the equivalent flat-plate area. ($D_P = 1.28S_e q$)

Class 1. Cantilever monoplane with retractable chassis, streamlined fuselage, well-cowled engine, no external bracing.

Class 2. (*a*) Cantilever or wire-braced monoplane with cantilever or wire-braced chassis, wheel pants, streamlined fuselage, engine cowl or ring. (*b*) Biplane or externally braced monoplane with retractable chassis, streamlined fuselage, engine cowl or ring.

Class 3. Biplane or externally braced monoplane with streamlined fuselage, engine cowl or ring.

Class 4. Airplanes having excessive parasite drag.

4. PERFORMANCE ANALYSIS

All performance calculations are based on the relation between power required for flight and power available under the given condition. The power required, P, is a direct function of the total drag, i.e., wing drag + parasite drag.

$$\frac{DV}{375} \qquad (17)$$

where P = horsepower, D = total drag, pounds, and V = speed, miles per hour.

SLIP-STREAM EFFECT. In calculating total parasite drag at a given speed of flight, allowance must be made for slip-stream effect. The slip stream of the propeller is taken roughly as a hollow cylinder, concentric with the thrust axis, whose inside diameter is 0.2 × propeller diameter, and whose outside diameter is 0.8 to 0.9 × propeller diameter. The parasitic drag of all objects which lie within the slip stream must be calculated on the basis of slip-stream velocity, which always is greater than the speed of normal horizontal flight. Its value is

$$V_s = V\sqrt{1 + \left(\frac{490T}{D^2V^2}\right)} \tag{18}$$

where V_s = slip-stream velocity, miles per hour; V = speed of flight, miles per hour; T = propeller thrust, pounds; D = propeller diameter, feet. The propeller thrust for use in eq. 18 is

$$T = \frac{375P\eta}{V} \tag{19}$$

where η = fractional propeller efficiency, see Fig. 21, p. 15-20; other notation as before. The power available under any set of conditions equals the horsepower developed by the engine under those conditions, multiplied by propeller efficiency. Typical variation of propeller speed, N, with plane speed is shown in Fig. 22 for fixed pitch propellers.

EFFECT OF ALTITUDE ON PERFORMANCE. Increase in altitude is accompanied by a reduction in density of the air, with consequent reduction of power output for an unsupercharged engine. See Standard Atmosphere, Table 4, p. 15-06. The relation between power output, P, at constant rpm, and density ρ, based on average data of a number of tests, is

$$P = P_0\left(\frac{\rho}{\rho_0}\right)^{1.2} \tag{20}$$

where the subscript 0 indicates sea-level conditions. With controllable-pitch propellers the power available for propulsion follows the same formula. When the propeller is of the fixed-pitch type there is a further loss due to decrease of rpm at higher altitudes, and the effective propulsive power is more nearly proportional to $\rho^{1.3}$.

For all practical purposes, the total drag at any given angle of attack may be considered as independent of air density, and the power required for straight horizontal flight, P_r, is given by

$$P_r = P_{r0}\left(\frac{\rho_0}{\rho}\right)^{1/2} \tag{21}$$

since the speed of flight at a given angle of attack varies inversely as the square root of the air density. The subscript 0 indicates sea-level conditions.

Where engines are supercharged, sea-level output may be maintained to considerable altitudes. The power output at any given altitude then must be obtained from performance curves of the engine in question. It is common practice to supply engines supercharged to maintain as much power up to high altitude as it is safe to draw from them continuously at any altitude. Maximum speed will increase with altitude until the critical altitude is reached. For additional data on Supercharging, see Section 10.

MINIMUM OR STALLING SPEED for horizontal flight is

$$V_{\min} = 0.68\sqrt{\frac{2W}{\rho C_{L_{\max}}S}} \tag{22}$$

where $V_{\min}$ is in miles per hour; W is gross weight, pounds; S is effective wing area, square feet; ρ is mass density of air and equal to 0.002378 slug per cubic foot at sea level; $C_{L_{\max}}$ is the maximum lift coefficient of the wing. Its value may vary from 0.90 to 1.9 with the lower value for thin airfoils of 6 to 9% *thickness ratio* (where thickness ratio is ratio of maximum thickness to chord length) and the higher for airfoils of 15 to 21% thickness ratio.

The above formula can be further modified to read

$${}_{\min} = K\sqrt{\frac{W}{S}} \tag{23}$$

in which W/S is the *wing loading*, pounds per square foot. For personal airplanes without flaps or slots, and with average W/S between 5 and 20, K = 17; for transport airplanes, usually equipped with partial span flaps, and with W/S between 30 and 70, K = 12.

MAXIMUM HORIZONTAL SPEED. An empirical formula for maximum speed in miles per hour, based on published performance data of a large number of airplanes, is

$$= K\left(\frac{P}{S}\right)^{1/3} \tag{24}$$

where P = total nominal or rated horsepower; S = wing area, square feet; and K = a constant based upon performance of present-day airplanes. For personal airplanes, K = 140; for small flying boats or amphibians, K = 135; for transport airplanes, K = 210.

CRUISING SPEED is the normal flying speed of the aircraft for best all-round efficiency. For personal aircraft, cruising speed is usually about 90% of maximum speed; for transport airplanes, cruising speed varies from 85 to 95% of maximum.

RATE OF CLIMB at any altitude depends on the excess of engine power (over that required to overcome the aerodynamic resistance of the airplane along the flight path) that is available for doing work against gravity, or, in other words, raising the airplane from one altitude to another at a given rate of speed. At sea level this rate of climb for personal airplanes (nearly all of which have fixed pitch propellers), whose power loadings vary from about 14 to about 20, is given by the following empirical formula based upon existing data:

$$R_c = 18{,}000\frac{P}{W} - 45\frac{W}{S} \tag{25}$$

where R_c is rate of climb in feet per minute. For transport airplanes (which use controllable pitch propellers), having wing loadings, W/S, between 30 and 70, and power loadings, W/P, between 8 and 12, the empirical formula for rate of climb is

$$R_c = 18{,}000\frac{P}{W} - 11\frac{W}{S} \tag{26}$$

Ceiling is defined both as to *absolute ceiling* where the rate of climb is zero, and *service ceiling* where the rate of climb is 100 ft per min. The absolute ceiling, H, with a supercharged engine, can be expressed in terms of power available, P_{a0}, and power required, P_{r0}, at sea level, by

$$H = 40{,}000\log_{10}\left(\frac{P_{a0}}{P_{r0}}\right) \tag{27}$$

An empirical formula, based on the performance of a large number of airplanes with unsupercharged engines, for the *service ceiling* is

$$H = 40{,}000\log_{10}\frac{111}{(W/P)\sqrt{W/S}} \tag{28}$$

where H = service ceiling, feet; W = gross weight, pounds; P = horsepower; S = wing area, square feet.

The average value of $(W/P)\sqrt{W/S}$ or personal aircraft is 52. For airplanes equipped with supercharged engines and controllable pitch propellers, the empirical formula for service ceiling is

$$H = H_c + 40{,}000\log_{10}\frac{225}{(W/P)\sqrt{W/S}} \tag{29}$$

where H_c is the critical altitude of the supercharged engine. The value of $(W/P)\sqrt{W/S}$ for this group varies from 40 to 90.

RANGE of an airplane in miles can be calculated approximately by means of Breguet's formula:

$$R = 863.5\frac{\eta}{c}\left(\frac{L}{D}\right)\log_{10}\frac{W}{W-Q} \tag{30}$$

where R = range, miles; ρ = propeller efficiency; c = specific fuel consumption, pounds per horsepower per hour; W = gross weight, pounds; Q = total fuel weight, pounds; L/D = maximum lift to drag ratio of the airplane. An empirical formula, based upon existing data, for personal airplanes is

$$R = 11{,}500\log_{10}\frac{W}{W-Q} \tag{31}$$

For large air transport airplanes with supercharged engines, and controllable pitch propellers, the range may be approximated by

$$R = 60\left(\frac{W}{S}\right)\left(\frac{W}{P}\right)\log\frac{W}{W-Q} - 12{,}500\frac{W}{W-Q} \tag{32}$$

where W/S is wing loading, pounds per square foot and W/P is power loading, pounds per horsepower.

5. POWER PLANTS

(See also Jet Propulsion, p. 15-37, and Aircraft Engines, Section 13.)

Aircraft may be propelled by (1) internal-combustion, or reciprocating, engines operating propellers; (2) turbojet engines operating propellers; and (3) engines operating on the jet propulsion principle. The jet engines merit brief descriptions, although some of these engines are still undergoing extensive research to make them economically useful.

THE PULSE JET. The pulse jet makes use of intermittent explosions in a duct in which the entrance is equipped with a series of nonreturn admission valves and fuel-injection nozzles. As the duct travels through the air, air pressure on the nose opens the valves and the air is rammed into the duct to mix with the fuel for combustion. The pressure rise due to combustion closes the valves, while the relative instantaneous ejection of the gases through the tail pipe reduces the pressure sufficiently to re-open the valves and the cycle is repeated. These explosions occur at a frequency of about 3000 per min.

Present forms of the pulse jet rely on atmospheric air for oxygen so that their absolute ceiling is limited. There is also some doubt whether such a device will operate at or above sonic speed.

THE RAM JET OR ATHODYD (*A*ero-*Th*ermo-*Dy*namic *D*uct). In the athodyd or ram jet, compression is obtained by ram action of the air passing through a duct into which fuel is sprayed. Because of the high forward speed (sonic speeds or higher) of the aircraft, the air is rammed into a diverging intake where the air velocity is decreased and pressure increased. Once in the duct, it combines with injected liquid fuel and is ignited. Combustion gases and air are then driven out, by virtue of their own pressure, through a converging exhaust nozzle exhausting at atmospheric pressure, generating higher velocity than at the inlet, and producing net thrust, by the principle of change of momentum.

THE TURBOJET. The turbojet does not depend primarily upon the ram action of the high speed of the airplane, but utilizes a compressor that introduces the compressed air into the combustion chamber from which the expanding combustion gases and air pass through a multistage turbine, usually of the axial-flow type, and from the turbine blades into the exhaust cone from which it is exhausted into the atmosphere. A starting engine usually has to be supplied to start the compressor, but once the turbojet is in operation, the turbine, which takes out most of the energy of the expanding gases, provides the power to drive the compressor. In the turbojet the sole function of the turbine is to operate the compressor. (See p. 15-66 for table of turbojet data.)

THE PROPJET. A propeller may be added to the turbojet on the same shaft as the compressor and turbine. The propeller improves the take-off and low speed operation of the aircraft, as well as the fuel economy. (See p. 15-68 for table of propjet data.)

JET AND RECIPROCATING ENGINE COMBINATIONS. Various other designs have been either proposed or developed. The normal reciprocating engine, equipped with the conventional propeller, can be provided with a jet-type exhaust. Such an addition extends the permissible speed range of the conventional reciprocating engine. An exhaust-driven supercharger (compressor) and turbine can also be added to the reciprocating engine to augment propeller thrust with jet thrust.

ROCKETS carry their own fuel and oxygen supply, and so theoretically have limitless altitude possibilities. Rockets have been used to launch heavily loaded aircraft, or have assisted in the take-off of either heavily loaded airplanes or airplanes operating from airports located at relatively high altitudes (Jato).

RELATIVE SPEED RANGES. The reciprocating or conventional type internal-combustion engine is limited in its speed range primarily by its propeller whose tip speeds, approaching the speed of sound, cause a reduction in overall propulsive efficiency. The speed range of the conventional propeller-engine combination is still being pushed upward, and it may be that propeller developments may make even higher speeds possible than are foreseen at present. However, the upper limit of the speed attained is around 450 to 500 miles an hour. For data on modern aircraft engines see Table 1, p. 13-52.

The propeller-engine combination with a jet-type exhaust may reach a speed of about 480 miles an hour and higher.

The propeller and turbocharged internal-combustion engine combination raises the top speed up to about 500 miles an hour and higher.

Variable discharge turbine (VDT). See Section 10, Art 2.

The propjet boosts the speed range to about 650 miles an hour, the turbojet to 800 miles an hour, the ram jet to 1100 miles an hour.

A controllable airplane equipped with rocket engines (perhaps more properly a missile) could attain about 2000 miles an hour whereas the pure rocket can attain speeds of 7000 to 8000 miles per hour.

PROPULSIVE EFFICIENCY OF JET ENGINES. (See also p. 15-39.) It can be shown that the propulsive efficiency of jet engines (whether pulse jet, ram jet, or turbojet) can be determined from the relationship

$$\eta = \frac{2}{\dfrac{v_j}{v_a} + 1} \tag{33}$$

where η = propulsive efficiency; v_j = exit jet velocity, relative to the aircraft; and v_a = forward velocity of the aircraft.

The propulsive efficiency of the jet engine, therefore, increases as the forward velocity of the aircraft approaches the high jet exhaust velocity. This is one reason that this form of power plant is looked upon with such great favor for aircraft approaching sonic or supersonic speeds.

PROPELLERS. Propeller characteristics are expressed in terms of the function V/nD,

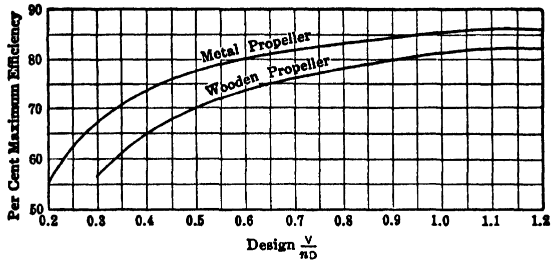


FIG. 21. Variation of efficiency with $\frac{V}{nD}$ for a fixed pitch propeller.

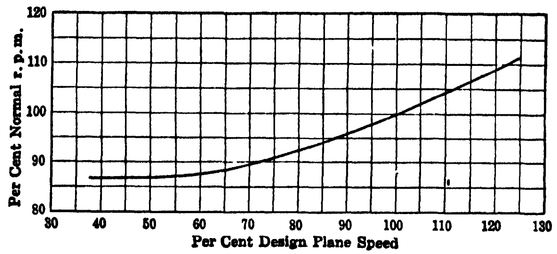


FIG. 22. Variation of full throttle rpm with speed of airplane when a fixed pitch propeller is used.

where V is the speed of flight, feet per second; n is the propeller rps; and D is the diameter, feet. (See Figs. 21 and 22.) The propeller characteristics or coefficients are

$$C_T = \frac{T}{\rho n^2 D^4} \tag{34}$$

$$C_Q = \frac{Q}{\rho n^2 D^5} \tag{35}$$

$$C_P = \frac{P}{\rho n^3 D^5} \tag{36}$$

where C_T, C_Q, and C_P are the thrust, torque, and power coefficients, respectively; T, Q, and P are thrust, torque, and power; the other terms have been previously defined. Propeller data are presented in terms of the *speed-power coefficient:*

$$C_S = \frac{0.638V}{P^{1/5}N^{2/5}}\left(\frac{\rho_0}{\rho}\right)^{1/5} \tag{37}$$

where V is aircraft speed, miles per hour; P is brake horsepower of engine; N is rpm of propeller. This formula is useful since a suitable propeller can be selected for a given aircraft if the engine horsepower, propeller rpm, and the desired maximum speed of the aircraft, are known. For the calculated value of C_S, the corresponding blade angle, propeller efficiency, and V/nD ratio (from which the propeller diameter can be determined since V and n are given) can be found from suitable graphs as given in NACA TR No. 350, and others (Fig. 23 is a typical chart). An empirical formula for the determination of the propeller diameter is

$$D = 1.75\sqrt{\left(\frac{P}{100}\right)\left(\frac{N}{1000}\right)^2\left(\frac{V}{100}\right)} \tag{38}$$

where D = propeller diameter, feet; P = horsepower of engine; N = rpm of propeller; V = maximum speed, miles per hour.

In order to keep the propeller tip speed at reasonable levels below the speed of sound, the number of blades is increased. The usual number of blades is 2, 3, or 4. *Counter-rotating* propellers not only supply more blades but also reduce the effect of propeller torque on the airplane, especially on highly powered craft whose take-off speed is insufficient to provide enough aerodynamic balancing moment through aileron operation.

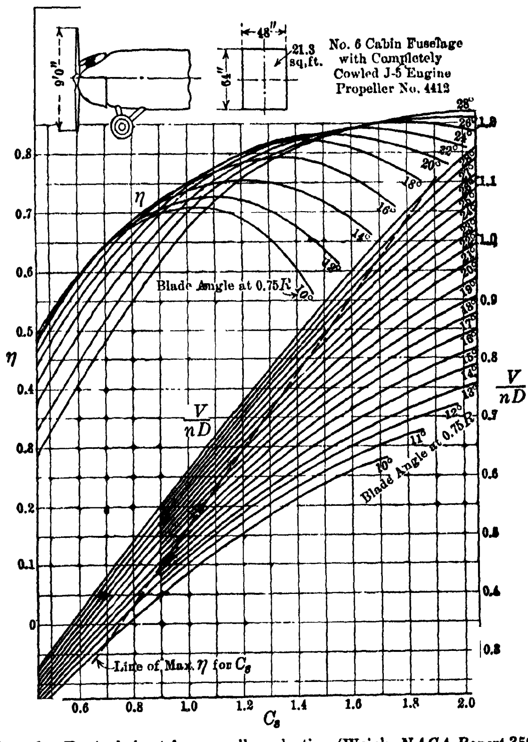


FIG. 23. Typical chart for propeller selection (Weick, *NACA Report* 350).

Reversible-pitch, controllable propellers may be used for *braking* purposes, reducing the landing run of airplanes. In Table 6 are comparative data on the effectiveness in this respect of reversing propellers on two airplanes of about 90,000 lb gross weight, equipped with conventional brakes, and four reversible propellers.

Table 6. Use of Reversible-pitch Propellers for Braking

(Data from *Aviation Week*, Dec. 1947)

Trial Number	Number of Propellers Reversed	Use of Brakes, %	Landing Run ft	Landing Run %
1	None	100	1908	100
	2	0	2482	130
	2	100	1357	71
	4	0	1748	91.5
	4	100	998	52.3
2	None	100	2230	100
	4	50	1078	48.4
	4	65.5	1000	44.9
	4	100	995	44.6

Table 7. Typical Propeller Weights and Ratings

Propeller Diameter	Number of Blades	Blade Material	Weight, lb	Maximum Continuous Rating hp	rpm
5′ 3″	2	Laminated wood	7	37	2550
5′ 11″	2	Laminated wood	10	40	2000
5′ 9″	2	Laminated wood	8	45	2575
5′ 9″	2	Laminated wood	8	50	2500
6′ 4″	2	Laminated wood	11	50	1900
6′ 0″	2	Laminated wood	9	75	2300
6′ 0″	2	Laminated wood	12	125	2600
6′ 4″	2	Laminated wood	15	125	2600
6′ 10″	2	Laminated wood	17	200	2450
7′ 2″	2	Fabric plastic	62	215	2600
7′ 6″	2	Aluminum alloy	175	550	2300
8′ 0″	2	Aluminum alloy	182	650	2300
9′ 0″	2	Aluminum alloy	196	650	2200
10′ 0″	3	Steel	330	1200	1667
12′ 0″	3	Steel	365	1500	1350

6. BALANCE, STABILITY, AND CONTROL

Stability, balance, control, and maneuverability are closely interrelated. The degree of stability and maneuverability required for an aircraft depends upon the service for which it is intended. Acrobatic airplanes, or fighters, for example, require less stability and more maneuverability. Since motion is obtained about three axes, (1) longitudinal, (2) lateral, and (3) normal, stability and control have to be considered about these axes. The moments produced about these axes when the airplane is disturbed from a normal position of equilibrium, or balance, is a measure of the stability of the craft. These moments may be calculated from

$$L = qC_l bS \tag{39}$$

$$M = qC_m CS \tag{40}$$

$$N = qC_n bS \tag{41}$$

where L, M, N are rolling moments about the longitudinal axis, pitching moments about the lateral axis, and yawing moments about the normal axis, all expressed in pounds feet; q is the dynamic pressure for the speed of the aircraft; C_l (not to be confused with the lift coefficient), C_m, and C_n are dimensionless coefficients; b is the wing span, feet; C is the wing chord, feet; and S is the wing area, square feet.

LONGITUDINAL STABILITY AND CONTROL. The airplane is designed to *trim* or balance longitudinally (i.e., in equilibrium) for any attitude in flight through the proper operation or deflection of the elevator, and to a limited degree by stabilizer adjustment (where used). In encountering a gust, or because of an intentional displacement, the airplane should return to its original attitude and line of flight of its own accord. This return is the physical phenomenon of longitudinal stability; the mathematical criterion requires a negative slope to the curve of pitching moments, $M = qC_mCS$ about the center of gravity plotted versus the angle of attack, α. It has been established that a suitable criterion is the evaluation of the ratio

$$\frac{(dC_m/d\alpha)}{W/S} = K \tag{42}$$

where $K = -0.0004$ average for highly maneuverable airplanes such as fighters; $K = -0.0006$ average for personal aircraft; and $K = -0.0008$ average for transport-type airplanes.

In order to achieve this stability, conventional airlanes are designed to have (1) the center of gravity of the airplane at 25% of the mean chord length; (2) the horizontal tail surfaces located two to three chord lengths behind the center of gravity of the airplane; (3) the horizontal tail surface area 12 to 16% of the wing area, if no flaps are used on the wing; 20% if flaps are used; (4) large-aspect ratios, 6 to 12 for the wing, 4 to 6 for the tail surfaces. In order to obtain an adequate degree of stability for different conditions of loading, the center of gravity location for different conditions of flight must not have a movement of more than 12% of the mean chord. Airplanes using a large degree of sweep-back, for subsonic, transonic, and supersonic designs, should have not more than 6% movement of the center of gravity. Low-wing monoplanes require relatively more tail surface area than high-wing monoplanes. Wings with deflected flaps tend to blanket horizontal tail surfaces so that here tail surfaces must again be larger than for an airplane not equipped with flaps.

Elevators comprise about 50% of the horizontal tail surface area and are required to move through $\pm$ 25 degrees to obtain control and trim at any flight attitude from about zero lift coefficient (maximum speed or low angle of attack condition) to maximum lift coefficient (minimum speed or high angle of attack condition).

LATERAL STABILITY AND CONTROL. Lateral stability is obtained primarily by incorporating *dihedral* in the wing, i.e., the wing tip placed well above the root of the wing. An angle of 3 to 6 degrees is usually sufficient if no sweepback of the wing is employed. To avoid the increased drag due to shock waves on high-speed aircraft, a high degree of sweepback (about 45 degrees, more or less) is employed, thereby obtaining excessive lateral stability. To counteract the stability produced by sweepback, it becomes necessary, therefore, to employ negative dihedral, because 5 degrees of sweepback is equivalent to 1 degree of dihedral, in producing stability.

Control is normally obtained by the differential action of the ailerons (one up and one down), a flapped surface located at the wing tip trailing edge. Lateral control could be obtained by any means which would increase or decrease the lift on one wing with relation to the other. Ailerons, which comprise about 8% of the wing area, are considered a part of the wing area.

DIRECTIONAL STABILITY AND CONTROL. Rolling of an aircraft about the longitudinal axis usually introduces yawing about the vertical axis. The deflection of the ailerons produces an adverse (out of the desired direction of turn) yawing moment. Engine torque produces rolling of the aircraft and is usually counteracted by partially deflected ailerons which cause a yawing moment. These yawing moments have to be counteracted by a suitable deflection of the rudder. In addition, the rudder (movable surface) in conjunction with the fin (fixed surface) has to produce enough yawing moment to turn the airplane. The function of the vertical tail surfaces is to maintain "weathercock" stability. Their area averages 8 to 12% of the wing area, with the rudder comprising about 50% of the total vertical area and required to deflect $\pm$25 degrees. Lateral and directional stability are intimately interconnected; too much of the former produces a so-called Dutch roll effect; too much of the latter produces a tight spiral spin (dive).

7. AIRWORTHINESS

Aircraft are designed to meet airworthiness requirements of an agency representing (1) the United States Air Forces or (2) the United States Navy, or (3) the Federal Government through the Civil Aeronautics Administration. These requirements may cover specifications with respect to any or all of the following: (1) drawings and general data; (2) structural analyses and reports; (3) instrumentation and equipment installation; (4) performance, stability, and control; and (5) inspection and certification. The most up-to-date information on requirements may be obtained by properly qualified inquirers from the pertinent military agency. Civil airworthiness requirements are published in a manual obtainable from the United States Government Printing Office.

STRUCTURAL ANALYSIS. The structural design of the airplane follows more or less orthodox procedures after the action of the loads on the various components of the structure have been determined. In general these loads are divided into three classifications: (1) flight loads, (2) ground or landing loads, and (3) handling or special loads. In flight the highest load expected during the normal operation of the airplane is the *limit load*. It may be encountered either in maneuvering the aircraft or in entering a region of turbulence or gusts. For civil airplanes the *limit load factor* (defined as the ratio of the limit load encountered to the gross weight of the airplane) for the gust condition may be determined from the formula

$$n = 1 + \Delta n = 1 + \frac{KUVm}{575W/S} \tag{43}$$

where n is the limit load factor; Δn is the load factor increment; U is the vertical gust (perpendicular to the line of flight) velocity, feet per second; V is the airplane speed, miles per hour; W is the gross weight of the airplane, pounds; S is the wing area, square feet; W/S is the wing loading, pounds per square foot; m is the slope of the lift curve per radian, and

$$K = \frac{1}{2}\left(\frac{W}{S}\right)^{1/4} \quad \text{for} \quad \frac{W}{S} < 16 \text{ lb per sq ft}$$

or

$$K = 1.33 - \frac{2.67}{(W/S)^{3/4}} \quad \text{for} \quad \frac{W}{S} > 16 \text{ lb per sq ft}$$

For maneuvering conditions, the limit load factor may be found from the formula

$$n = 1 + \Delta n = \left[0.77 + \frac{32{,}000}{W + 9200}\right]\left[\frac{3.25}{(W/P)^{0.435}}\right] \tag{44}$$

where P is the total brake horsepower of the engines, and the other terms have the same significance as before. These flight conditions are further specified as to the velocities at which such load factors are considered critical. With the load factors known, it is possible to correlate the load factors with the aerodynamic characteristics of the wing since

$$C_{N}qS = nW \tag{45}$$

from which the *normal force coefficient*, C_N, can be computed. The value of C_N can be obtained from the resolution of lift and drag forces of the wing to forces normal and parallel to a given geometric axis, usually the chord of the wing, so that

$$C_N = C_L \cos\alpha + C_D \sin\alpha \tag{46}$$

and

$$C_c = -C_L \sin\alpha + C_D \cos\alpha \tag{47}$$

where C_N and C_c are normal and chord force coefficients, and C_L, C_D, and α have the same significance as before.

For landing conditions, for civil airplanes, the primary limit load factors for level and three-point landing are determined from the lower of the values obtained from

$$n = 2.8 + \frac{9000}{W + 4000} \tag{48a}$$

$$n = 3.0 + 0.133\left(\frac{W}{S}\right) \tag{48b}$$

In neither case should the load factor exceed 4.33. There are further modifications for other types of landing gear or methods of landing.

When the structure is subjected to these limit loads, there should be no serious yielding of the structure. Stresses imposed by a limit load on any part of the structure should not exceed the *yield stress* of the material. The maximum load which any part of the structure can stand without failure is known as the *ultimate load*, and the stresses imposed by this ultimate load should not exceed the ultimate stress of the material. The ratio of the ultimate load to the limit load is called the *factor of safety;* it is usually taken as 1.5 and designated j_u. When the yield stress is used in design, the ratio of the yield load to limit load is also called a factor of safety, and is taken as 1, and designated j_y. The difference between the *design load* or stress (the product of the limit load or stress and the factor of safety of any part of the structure) and the allowable load or stress of the material used for that part of the structure is known as the *margin of safety.* It is usually presented as a percentage of the design load or stress. Because aircraft structures are required to be as light as possible, it is structurally economical to design for a margin of safety of zero, unless other factors, such as fatigue, wear, corrosion, maintenance, or design proportions, govern.

FLUTTER AND VIBRATION. Because the aircraft structure is a complex body, with comparatively high flexibility, subjected to a variety of forces, it becomes necessary as higher and higher aircraft speeds are reached, to investigate the structure for flutter and vibration. *Flutter* is a violent, self-induced vibration of a body resulting from a coupling of aerodynamic, elastic, and inertia forces acting upon the body. (Refer to NACA *Technical Reports* 685 and *Technical Memorandum* 782.) The natural frequencies in bending and torsion for different combinations of wing and aileron, fuselage and tail surfaces, have to be determined. Suitable ratios for design criteria have been established. Design details which may be employed to prevent flutter, especially when applied to control surfaces and control systems, call for structural stiffness, reduction of play in hinges, rigid interconnections between ailerons, and elevators, and complete static and dynamic balance of control surfaces, as well as some aerodynamic balancing and high damping qualities of the structure. For fixed surfaces, the center of gravity should be as close to the leading edge of the surfaces as possible.

HELICOPTERS

8. CLASSIFICATION OF HELICOPTERS

Helicopters are classified primarily according to their rotor configurations. (1) Single rotor with a torque compensating tail rotor. (2) Dual rotors, coaxially located. (3) Dual rotors, arranged in tandem. (4) Dual rotors, laterally displaced on pylons. (5) Dual rotors, intermeshing, either in tandem or side by side. Table 8 lists data on some typical American helicopters.

Table 8. Typical American Helicopter Specifications

(Data obtained from *Aviation Week*, Feb. 1948)

Manufacturer	Model Designation	Engine	Horsepower	Maximum Speed, mph	Cruising Speed, mph	Range, miles	Ceiling, ft	Gross Weight, lb	No. Rotor Blades	Diam., Rotor, ft	Blade Area, sq ft	Rotor rpm, Cruise	Antitorque Rotor	Power Loading, lb/hp	Disk Loading, lb/sq ft
Bell Aircraft Corp.	47D	Air	178	92	85	210	11,500	2,086	2	35.16	35.24	333	Yes	11.7	2.15
	48	P&W	600	105	90	300	13,000	6,000	2	47.5	81.6	256	Yes	10.0	3.38
Bendix Helicopter, Inc.	K	Cont	100	95	75	95	NA	1,007	2–2 *	25.0	21.0	412	No	10.0	2.35
	J	P&W	450	112	85	279	15,000	5,400	2–2 *	48	99.0	192	No	12.0	2.99
Doman Frasier Helicopters, Inc.	LZ2-A	Air	245	120	95	235	17,000	2,950	4	40	60.8	230	Yes	12.0	2.34
Helicopter Engineering Research Corp.	JOV-3	Lyc	100	100	73	138	12,000	1,200	2–3 †	18.5	54	470	No	12.0	2.23
Kaman Aircraft Corp.	K-190-A	Lyc	190	100	80	300	NA	2,500	2–2 ‡	38	44.4	221	No	13.2	2.21
	K-125-A	Lyc	125	100	80	NA	NA	NA	2–2 ‡	38	NA	NA	No	....	....
Kellett Aircraft Corp.	XR-10	Cont	500	NA	NA	NA	NA	11,000	6	65	17.76	140	No	22.0	3.31
Landgraf Helicopter Co.	H-2	Pob	85	100	100	150	NA	850	6	16	32.4	485	No	10.0	4.22
McDonnell Aircraft Corp.	XHJD-1	P&W	450	100	70	350	NA	11,000	6	46	200	190	No	24.5	6.63
	38	R-J	NA	50	50	NA	NA	610	2	18	7.6	640	No	....	2.39
Piasecki Helicopter Corp.	PV-3	P&W	600	100	90	300	12,000	6,900	3–3 †	41	NA	NA	No	11.5	2.61
Seibel Helicopter	S-3	Air	65	90	75	80	12,000	800	2	25	15.5	360	Yes	12.3	1.63
Sikorsky Aircraft Div.	R4	War	180	75	65	145	10,000	2,537	3	38	65.4	225	Yes	14.1	2.05
	R5	P&W	450	100	80	245	13,000	4,867	3	48	115.05	186	Yes	10.8	2.67
	R6	Air	235	100	75	375	11,600	2,704	3	38	65.4	252	Yes	11.5	2.19
	S-51	P&W	450	103	85	260	14,000	4,985	3	48	115.15	186	Yes	11.1	2.74
	S-52	Air	165	97	87	265	13,500	1,900	3	32	41.22	290	Yes	11.5	2.36
United Helicopters, Inc.	360	Air	178	105	85	212	12,000	2,100	2	34.5	NA	NA	Yes	11.8	2.24

Engines: Air—Air-cooled Motors. Cont—Continental. Lyc—Lycoming. P&W—Pratt & Whitney. R-J—Ram jets. War—Warner. Pob—Pobjoy.
Rotor Systems: * Coaxial. † Tandem. ‡ Contra-rotating, intermeshing.
NA—Not available.

The individual rotor blade is generally so hinged at the root that it may be permitted (1) to flap, or rotate in the vertical plane; (2) to feather, or rotate about the blade's spanwise axis; (3) to lag, or move for and aft in the horizontal plane. These movements are limited usually by limit stops. The area swept out by the rotor blades in one rotation is known as the disk area. The ratio of the gross weight of the craft to the disk area is known as the disk loading. The ratio of the total blade planform area to the disk area is known as the solidity.

9. PERFORMANCE OF HELICOPTERS

A few empirical formulas for single-rotor helicopter performance are the following:

$$V_{\max} = 170 \sqrt[3]{\frac{P}{S}} \tag{49}$$

$$R = 200 \left(\frac{W}{P}\right)\left(\frac{W}{S}\right) \log_{10} \frac{W}{W - Q} \tag{50}$$

$$H = 40{,}000 \log_{10} \frac{37}{\dfrac{W}{P}\sqrt{\dfrac{W}{S}}} \tag{51}$$

where the terms have the same significance as before, except that the area S refers to the disk area swept out by the rotor. (See eqs. 24, 27, and 30.) The value of $(W/P)\sqrt{W/S}$ for modern single-rotor helicopters is between 16 and 20; power loadings, W/P, vary from 10 to 14; and disk loadings, W/S from 1.5 to 3.5. Disk loadings of 1.8 to 2.4 usually yield very maneuverable designs as well as fairly high cruising speeds.

The average lift-to-drag ratio, L/D, for contemporary helicopters varies from 5 to 10, compared with values of 12 to 22 for present-day airplanes. The two types of craft, therefore, are not likely to compare favorably on the basis of performance, but the helicopter's forte is ability to hover, ascend, and descend steeply, and to make spot landings.

JET-PROPELLED HELICOPTERS. Considerable activity has been going on in this country in the field of jet-propelled rotors for helicopters. The advantages of this system are that a *compressed air transmission* is used, eliminating the heavy and expensive gearing of other types, and that the power is generated at the blade, eliminating large torques at the hub.

In this arrangement air is compressed by various means in a power plant contained within the fuselage. The compressed air is led through ducting and a hollow hub to hollow blades. Jet-engine type combustion chambers are mounted at the extremities of the helicopter blades. Combustion air enters these chambers, where it is mixed with jet-engine fuel, and burned. The product gases are ejected at high velocity from the rear of the blade tips, producing the thrust necessary to drive the blades.

One large helicopter has been designed in which two modified J-35 jet engines are used as an air source. Typical pressures at the blade tips are 2.5 to 3.0 atmospheres absolute. Helicopters of this type can be built in tremendous power ratings and lifting capacities. Large helicopters are of considerable value in transporting cargo or matériel across difficult or impassable terrain.

Although the fuel economy of this type of helicopter is reputed to be rather poor, the feasibility of construction of large helicopters by this method should overcome this disadvantage. No large helicopter of this type has yet flown, but intense activity in the field should make it a reality at the conclusion of the development program.

LIGHTER-THAN-AIR CRAFT

10. CHARACTERISTICS AND PERFORMANCE

Balloons and airships derive their lift from simple displacement of the air. A certain degree of dynamic lift may be developed from the forward motion of a power-driven streamlined airship envelope, but, in general, equilibrium conditions are reached at an altitude where the displaced volume of air equals the weight of the aerostat. The absolute altitude of equilibrium varies with atmospheric conditions. (See p. 15-02 for definitions and classification of airship types.)

AIRSHIP CHARACTERISTICS. The external shape of an airship envelope is dictated by structural economics and considerations of parasite drag and of control. Practically, pure streamlined shapes of optimum dimensions cannot be used, as it is necessary to modify the structure to attach control cars, power-plant housings, control surfaces, etc. For most practical purposes, the volume of the average airship envelope may be estimated from $V = \frac{1}{2}LD^2$, where V = volume, L = length, and D = maximum diameter. Under standard atmospheric conditions at sea level, buoyancy may be taken as 64 lb per 1000 cu ft for hydrogen and 58 lb per 1000 cu ft for helium. In a rigid ship, with a multiplicity of gas cells, the gas volume is usually about 5% less than the volume enclosed by the outer cover.

For rigid types the fineness ratio (length/maximum diameter) is tending downward. Early ships exhibited ratios of 8 1/2 or 9. In the Akron-Macon class this was reduced to 6. On nonrigid ships, structural factors favor a lower fineness ratio; it ranges between 3 and 5. Comparative statistics of three of the best-known rigid airships are given in Table 9.

Table 9. Characteristics of Typical Rigid Airships

	Los Angeles	Graf Zeppelin	Akron and Macon
Overall length, ft	658	776	785
Maximum diameter, ft	91	100	133
Gas volume, cu ft	2,470,000	3,700,000	6,500,000
Gross lift, lb	153,000	258,000	403,000
Total horsepower	2,000	2,750	4,480
Maximum speed, mph	73	80	84

AIRSHIP PERFORMANCE. The maximum speed of an airship depends simply upon its coefficient of aerodynamic resistance, complete with tail surfaces and cars. The coefficient commonly is stated in terms of the resistance per unit of volume of the airship envelope, rather than in terms of frontal area, as with airplane parts. Total resistance varies as the two-thirds power of the volume. Thus, $R = Cv^{2/3}V^2$, v being the volume of the envelope, cubic feet, and V the velocity, miles per hour. Using the customary miles per hour and square feet units, the value of C for the best rigid airship forms falls as low as 0.000035 for the bare envelope, or 0.00006 for a complete ship. Knowing the propeller efficiency, the power required for a given speed can be calculated and a maximum-speed formula obtained in the form $V_{max} = K\sqrt[3]{P/v^{2/3}} = Kv^{1/6}\sqrt[3]{P/v}$, where P = engine power and v = the volume. K attains a maximum value of approximately 165, and can be taken as 165 for large rigid airships of good design, and as 125 for typical nonrigid ships. Unlike airplanes, airships show an inherent increase in performance with increasing size. If power and volume and lift are doubled, the maximum speed, other things being equal, will be increased by 8%.

The ceiling attainable with an airship depends almost solely on the relation between the minimum fixed, or nondisposable, weight of the ship and the total lift that would exist at sea level if all the gas cells were filled to the limit. In attaining high altitudes the airship acts as a free balloon, and its altitude performance follows free-balloon laws. The ship will rise until the gas in the partly filled cells has expanded to fill them completely, and then continue to rise until the density of the surrounding air has decreased to a point where the ascensional force, or difference between the weight of air displaced by the ship and the weight of the gas doing the displacing, is barely equal to the solid weight of the ship at that moment. The altitude that can be attained is given approximately, including an allowance of 1000 ft for the effect of dynamic lift in carrying the ship above the level of static equilibrium, by $H = 29{,}000[1.75 - \sqrt{4(W_2/W_1) - 1}]$, where W_1 = maximum lift of airship at sea level with gas cells full; W_2 = total weight that has to be carried to the highest altitude.

The range of an airship in still air increases steadily with declining speed, the lowest speed always being the most economical under that condition. Other things being equal, it increases, like maximum speed, with increasing size. An approximate range formula, for ships using liquid fuel exclusively and inflated with helium, is

$$X = 21{,}000\left(\frac{W_f}{W_1}\right) \times \frac{\sqrt[3]{v \times 10^{-6}}}{(V/50)^2}$$

where X = range, miles; W_1 = maximum lift, pounds at sea level; W_f = weight of fuel carried, pounds; and V = cruising speed, miles per hour. For a hydrogen-filled ship the constant is increased to 24,000, but W_f/W_1 also is increased, as the extra lift of the lighter gas can all be put into extra fuel. The final result of replacing helium with hydrogen is likely to be an increase of range at a given speed of approximately 40%.

SUPERSONICS

By Norman C. Witbeck

NOMENCLATURE

a = velocity of sound, ft/sec
c = chord, ft
C_D = drag coefficient
C_L = lift coefficient
k = adiabatic exponent; ratio of specific heats, c_p/c_v
M = Mach number
p = pressure, lb/sq ft, static unless otherwise noted
R = gas constant = 53.3 ft/°R for air
S = entropy, Btu/(lb)(°R)
T = absolute temperature, °R, static unless otherwise noted
t = thickness, ft
V = velocity, ft/sec
α = deflection angle, expansion angle, or angle of attack
θ = angle between air flow and oblique or conical shock wave
μ = Mach angle
ρ = density, slugs/cu ft

Subscripts

n = normal
s = surface
t = total
1 = location before a shock or expansion
2 = location after a shock or expansion

11. SHOCK DATA

GENERAL. Sonic velocity, the rate of propagation of small pressure waves in a gaseous medium, is a function of the compressibility of the gas, and may be expressed

$$a = \sqrt{\frac{dp}{d\rho}} = \sqrt{kgRT} \tag{1}$$

Mach number, M, is defined as the ratio of a flow velocity to the velocity of sound in the same medium at the existing temperature.

$$M = \frac{V}{\sqrt{kgRT}} \tag{2}$$

Flow of compressible fluids is divided by sonic velocity into two regimes: *subsonic* flow when $M < 1.0$, and *supersonic* flow when $M > 1.0$. Analytically, the transition from subsonic to supersonic flow is characterized by a change in the partial differential equation describing the flow from the elliptic type to the hyperbolic type. Physically, supersonic flow is characterized by the fact that pressure waves cannot be propagated upstream as in subsonic flow; hence flow ahead of an immersed body cannot be disturbed by the presence of the body as it is in subsonic flow. This characteristic is illustrated in Fig. 1.

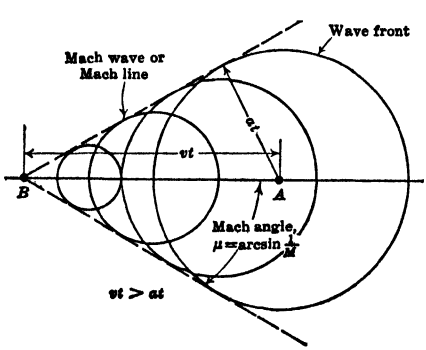


FIG. 1. Source of pressure disturbance moving at supersonic velocity.

Mach Waves. Consider a small source of disturbance moving at supersonic speed from point A to point B. At any time t, the wave front initiated at point A will have extended to a radius at, while the source of disturbance will have traversed the distance vt, greater than at. The envelope of the wave fronts will be a straight line in two dimensions, or a cone in three dimensions. This envelope is called a *Mach wave* or *Mach line*, and the angle it makes with the line of motion is called the *Mach angle*, μ. It is apparent from the figure that Mach angle may be expressed as

$$\mu = \arcsin \frac{1}{M} \tag{3}$$

In subsonic flow, the initial wave front is circular, or spherical, and always includes all subsequent wave fronts as well as the source of disturbance itself.

Shock waves exist when the source of disturbance in a supersonic stream causes a finite deflection, or compression, of the flow. Flow through a *shock* wave is characterized by an abrupt *increase* in pressure and entropy, and a *decrease* in velocity. *Expansion waves* or *expansion fans*, the expansion counterpart of shock waves, are characterized by a *decrease* in pressure and *increase* in velocity. In general, expansion waves may be considered isentropic.

NORMAL SHOCK WAVES or *plane shock waves* are oriented perpendicular to the flow direction. Flow behind a normal shock wave is always subsonic. The pressure rise,

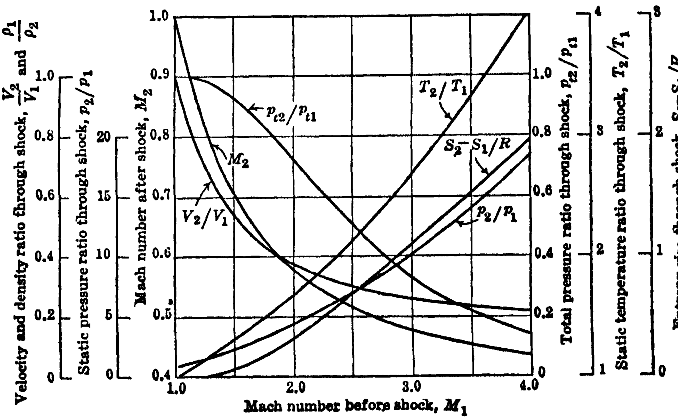


FIG. 2. Flow relations through a normal shock wave ($k = 1.40$).

entropy increase, and velocity decrease are greater through a normal shock wave than through any other type. The following relations, plotted in Fig. 2 and evaluated in Table 1, have been established for flow through a normal shock:

$$M_2^2 = \frac{2 + (k-1)M_1^2}{2kM_1^2 - k + 1} \tag{4}$$

$$\frac{p_2}{p_1} = \frac{2kM_1^2 - k + 1}{k+1} \tag{5}$$

$$\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{(k-1)M_1^2 + 2}{(k+1)M_1^2} \tag{6}$$

$$\frac{T_2}{T_1} = \frac{[2 + (k-1)M_1^2][2kM_1^2 - k + 1]}{(k+1)^2M_1^2} \tag{7}$$

$$\frac{p_{t2}}{p_{t1}} = \left[\frac{\frac{k+1}{2}M_1^2}{\frac{k-1}{2}M_1^2 + 1}\right]^{k/(k-1)} \left[\frac{\frac{k+1}{2}}{kM_1^2 - \frac{k-1}{2}}\right]^{1/(k-1)} \tag{8}$$

$$\frac{S_2 - S_1}{R} = \ln\left(\frac{p_{t1}}{p_{t2}}\right) \tag{9}$$

Table 1. Flow Relations through a Normal Shock Wave * †

(Adapted from *Handbook of Supersonic Aerodynamics*, Applied Physics Laboratory, The Johns Hopkins University, 1949. This material was compiled under Contract NOrd 7386 with the Bureau of Ordance, Department of the Navy.)

M_1	p_2/p_1	p_{t2}/p_{t1}	M_2	V_2/V_1, ρ_1/ρ_2	T_2/T_1	$\frac{S_2 - S_1}{R}$
1.0	1.00000	1.00000	1.00000	1.00000	1.00000	0.00000
1.1	1.24500	0.99893	0.91177	0.85537	1.06494	0.00107
1.2	1.51333	0.99280	0.84217	0.74537	1.12799	0.00723
1.3	1.80500	0.97937	0.78596	0.65976	1.19087	0.02084
1.4	2.12000	0.95819	0.73971	0.59184	1.25469	0.04270
1.5	2.45833	0.92979	0.70109	0.53704	1.32022	0.07280
1.6	2.82000	0.89520	0.66844	0.49219	1.38797	0.11071
1.7	3.20500	0.85572	0.64054	0.45502	1.45833	0.15581
1.8	3.61333	0.81268	0.61650	0.42387	1.53158	0.20741
1.9	4.04500	0.76736	0.59562	0.39751	1.60792	0.26480
2.0	4.50000	0.72087	0.57735	0.37500	1.68750	0.32729
2.1	4.97833	0.67420	0.56128	0.35563	1.77045	0.39422
2.2	5.48000	0.62814	0.54706	0.33884	1.85686	0.46500
2.3	6.00500	0.58329	0.53441	0.32420	1.94680	0.53906
2.4	6.55333	0.54014	0.52312	0.31134	2.04033	0.61592
2.5	7.12500	0.49901	0.51299	0.30000	2.13750	0.69512
2.6	7.72000	0.46012	0.50387	0.28994	2.23834	0.77626
2.7	8.33833	0.42359	0.49563	0.28098	2.34289	0.85899
2.8	8.98000	0.38946	0.48817	0.27296	2.45117	0.94298
2.9	9.64500	0.35773	0.48138	0.26576	2.56321	1.02797
3.0	10.33333	0.32834	0.47519	0.25926	2.67901	1.11369
3.1	11.04500	0.30121	0.46953	0.25338	2.79860	1.19994
3.2	11.78000	0.27623	0.46435	0.24805	2.92199	1.28653
3.3	12.53833	0.25328	0.45959	0.24319	3.04919	1.37328
3.4	13.32000	0.23223	0.45520	0.23875	3.18021	1.46005
3.5	14.12500	0.21295	0.45115	0.23469	3.31505	1.54671
3.6	14.95333	0.19531	0.44741	0.23097	3.45373	1.63316
3.7	15.80500	0.17919	0.44395	0.22754	3.59624	1.71929
3.8	16.68000	0.16447	0.44073	0.22438	3.74260	1.80503
3.9	17.57833	0.15103	0.43774	0.22146	3.89281	1.89030
4.0	18.50000	0.13876	0.43496	0.21875	4.04688	1.97504

* $k = 1.4$. † For more extensive tables, see Refs. 1 and 2.

TWO-DIMENSIONAL SHOCK PATTERNS. The simplest two-dimensional shock pattern is that associated with the angular deflection of a supersonic stream. A concave corner requires a single shock wave to deflect the flow to a direction parallel to the boundary, with a resultant pressure rise and velocity decrease. This shock wave is called an *angle shock* or *oblique shock,* because its orientation is not perpendicular to the approaching flow direction (see Fig. 3). The shock angle, θ, and the flow relations through the shock are a function of the deflection angle, α, and the approach Mach number, M_1. The following relations assume that the component of the approach velocity perpendicular to the shock front experiences a normal shock reaction while the tangential velocity component is unchanged. These relations also apply to flow around a wedge or past the lower side of an airfoil.

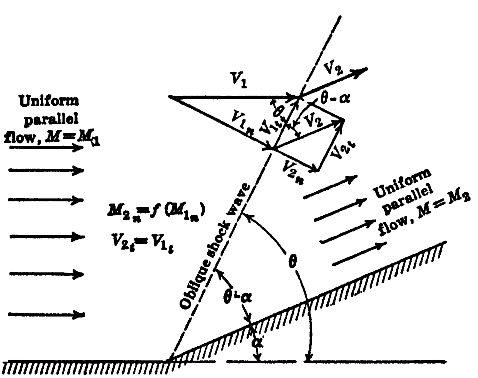


FIG. 3. Supersonic flow deflected by a concave corner.

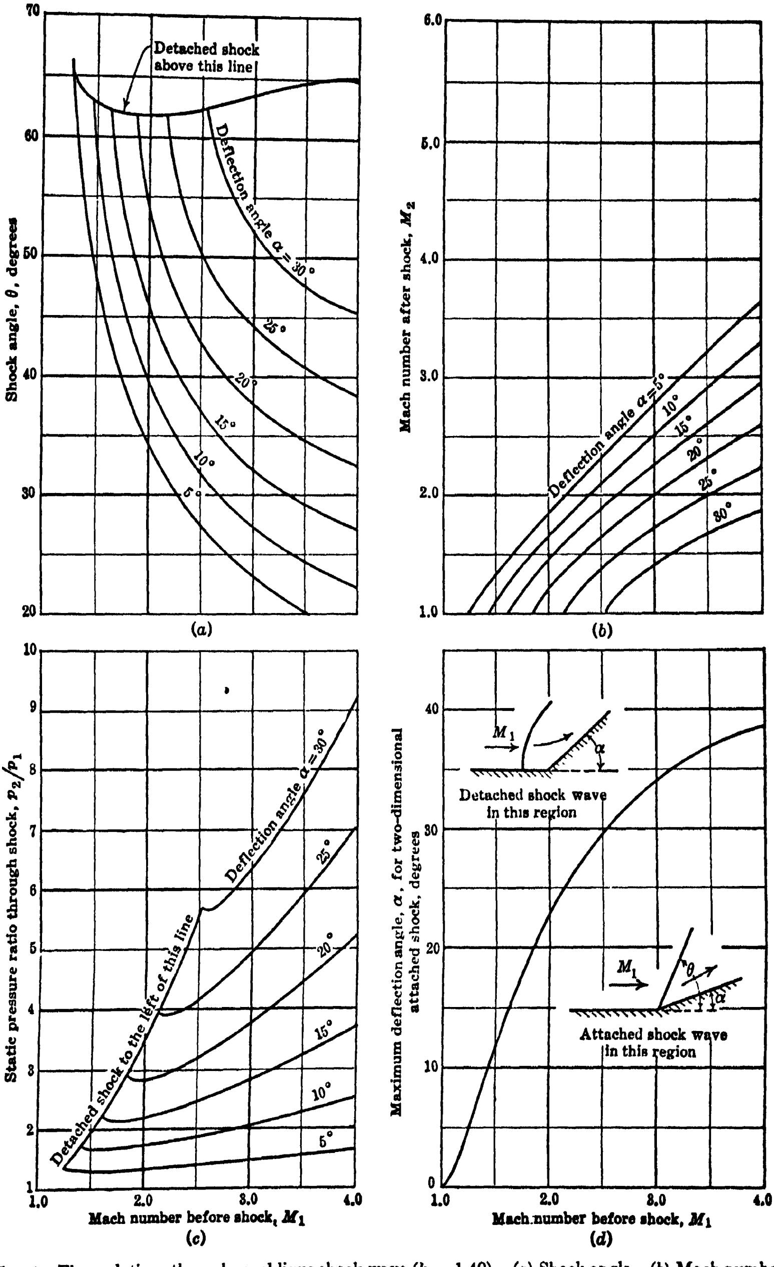


FIG. 4. Flow relations through an oblique shock wave ($k = 1.40$). (a) Shock angle. (b) Mach number after shock. (c) Static pressure ratio. (d) Maximum deflection angle for attached shock.

$$\tan(\theta - \alpha) = \left[\frac{(k-1)(M_1 \sin\theta)^2 + 2}{(k+1)(M_1 \sin\theta)^2}\right]\tan\theta \tag{10}$$

$$M_2^2 = \frac{1}{\dfrac{\left[1 + \left(\dfrac{k-1}{2}\right)M_1^2\right]\cos^2(\theta-\alpha)}{M_1^2\cos^2\theta} - \left(\dfrac{k-1}{2}\right)} \tag{11}$$

$$\frac{p_2}{p_1} = \frac{2k}{k+1}(M_1 \sin\theta)^2 - \frac{k-1}{k+1} \tag{12}$$

The Mach number behind an oblique shock is generally supersonic, although there is a small range of α and M_1 in which the flow is decelerated to subsonic. If the angle of deflection is increased above a critical value, the oblique shock will detach from the corner and move upstream, where it will stand perpendicular to the approaching flow direction and will, in the region near the boundary, exhibit the characteristics of a normal shock. This type of shock is called a *bow shock, bow wave,* or *detached shock.* The outer portion of such a shock curves away from the approaching flow, and the pressure rise, flow deflection, etc., through it vary with the distance from the boundary. Figure 4 graphically presents eqs. 10 to 12 and defines the critical value of M_1 and α for a two-dimensional attached oblique shock wave.

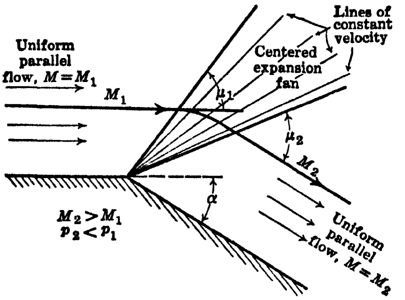


FIG. 5. Supersonic flow around a convex corner.

Supersonic flow around a convex corner is illustrated in Fig. 5. The flow in the disturbed region is called an *expansion fan.* It is characterized by the fact that along radial lines from the corner the velocity is constant. The expansion fan may be considered to be made up of an infinite number of infinitesimally small disturbance lines. In general

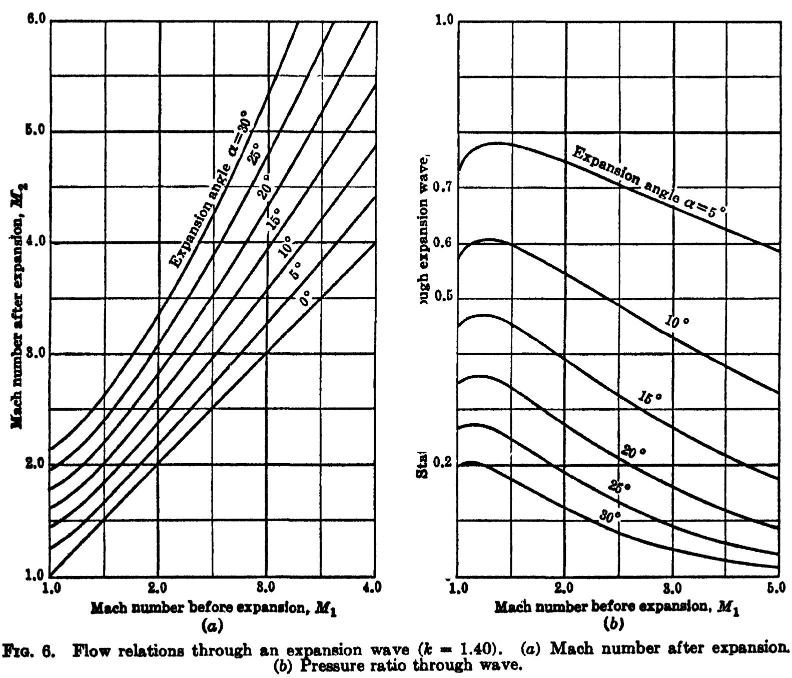


FIG. 6. Flow relations through an expansion wave ($k = 1.40$). (a) Mach number after expansion. (b) Pressure ratio through wave.

the flow around such a corner may be considered isentropic. If the approach Mach number to a convex corner is unity, the theoretically maximum allowable expansion angle, α_{max}, is given by

$$\alpha_{max} = \frac{\pi}{2}\left(\sqrt{\frac{k+1}{k-1}} - 1\right) \tag{13}$$

or about 130 degrees for air. This expansion angle corresponds to an expansion of the flow to a Mach number of infinity. Actually, long before this maximum angle is reached the flow will detach, forming a turbulent wake of considerably higher static pressure than would exist if the flow had continued to expand around the corner. Relations between velocity, pressure, and expansion angle are:

$$\alpha_i = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{M_2^2-1}\sqrt{\frac{k-1}{k+1}}\right) - \cos^{-1}\frac{1}{M_2} \tag{14}$$

$$\frac{p_2}{p_1} = \left[\frac{2+(k-1)M_1^2}{2+(k-1)M_2^2}\right]^{k/(k-1)} \tag{15}$$

where α_i is the angle required to increase the velocity from $M = 1.0$ to $M = M_2$. These relations are plotted in Fig. 6 and evaluated in table form in Ref. 3.

12. SHOCK APPLICATIONS

SUPERSONIC AIRFOIL SHAPES. In contrast to conventional subsonic airfoils, the drag of a supersonic lifting surface is primarily a function of the strength of the shock waves it generates rather than the frictional resistance of its surfaces. The characteristics of most supersonic airfoils can be readily calculated by use of eqs. 10 to 15 and Figs. 4 to 6. The leading edge of a supersonic airfoil is generally quite sharp to avoid the high drag that would be associated with a detached shock.

Flat Plate. The simplest supersonic airfoil is the flat plate (see Fig. 7). The lift-producing shock system consists of an oblique shock at the leading edge producing flow along the lower surface comparable to that produced by a concave corner of deflection α; and an expansion fan at the leading edge producing flow along the upper surface comparable to that produced by a convex corner of expansion angle α. The lift per unit of projected area is the pressure difference between upper and lower surfaces. The shock system at the trailing edge is necessary to bring the flow back toward its initial direction. Actually, some downwash exists immediately behind the flat plate, since shock losses on the upper and lower surfaces are unequal. The downwash becomes zero, however, if one considers the entire flow field surrounding the airfoil, including the intersections of the shock and expansion waves.

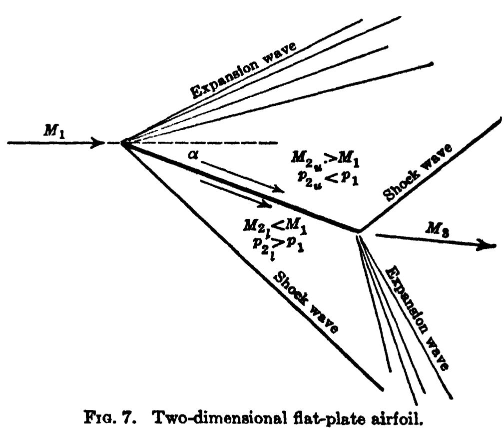


FIG. 7. Two-dimensional flat-plate airfoil.

Double-wedge and Circular-arc Types. The lift and drag of most other supersonic airfoils, such as the *double-wedge* and the *circular-arc* types, may be calculated by assuming the profile to be broken up into small straight-line elements and computing flow changes between each element and the next. It is important, however, that the deflection caused by the angle of attack and any camber near the leading edge be less than the critical value for shock detachment. If the shock is detached, the flow in the region of the leading edge becomes subsonic with a nonuniform velocity distribution.

A reasonable lift and drag approximation for a supersonic airfoil results from the *linearized theory* (see Ref. 4). According to this theory, lift is independent of thickness and camber, and may be expressed simply as

$$C_L = \frac{4\alpha}{\sqrt{M^2-1}} \quad (\alpha \text{ in radians}) \tag{16}$$

This approximation is very accurate for thin airfoils at low Mach numbers, but should be used with caution when $M > 4.0$ or when $t/c > 10\%$. The drag from linearized theory is

$$C_D = \frac{4\alpha^2}{\sqrt{M^2 - 1}} + \frac{16}{3}\frac{(t/c)^2}{\sqrt{M^2 - 1}} \quad (\alpha \text{ in radians}) \tag{17}$$

for the symmetrical circular-arc airfoil, and

$$C_D = \frac{4\alpha^2}{M^2 - 1} + 4\frac{(t/c)^2}{M^2 - 1} \quad (\alpha \text{ in radians}) \tag{18}$$

for the symmetrical double-wedge airfoil.

OTHER TWO-DIMENSIONAL SHOCK PATTERNS. Often two or more oblique shock waves intersect each other or are incident upon a normal shock or an expansion fan.

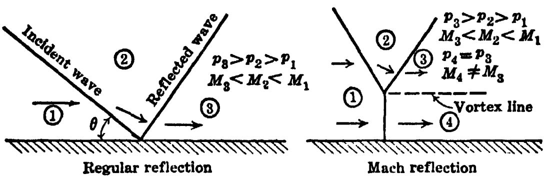


FIG. 8. Shock wave reflections.

The resulting shock pattern, while more complex than those previously treated, can usually be broken down into several components of readily analyzed types of interaction. An important point to remember is that whenever three shock waves exist at one point, a fourth surface of discontinuity must also be present. This discontinuity, called a *vortex sheet* or *slip stream*, separates zones of different velocity but equal static pressure, and is a streamline of the flow. The most frequent occurrence of the vortex sheet is in connection with the so-called *Mach reflection* of a shock wave from a solid boundary or other surface discontinuity. Figure 8 illustrates the difference between a Mach reflection and a *regular*

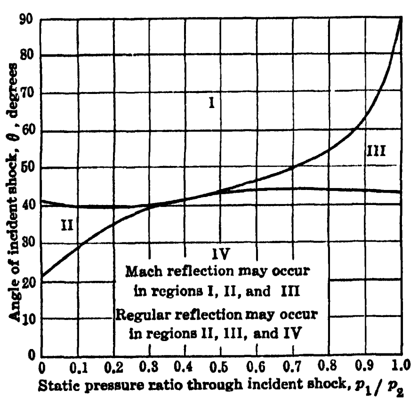
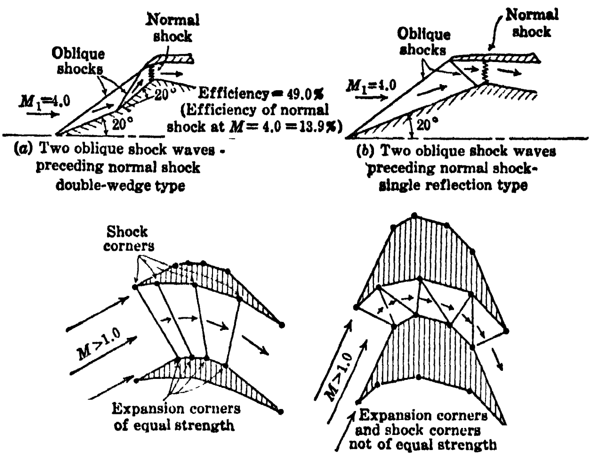


FIG. 9. Regions of regular and Mach reflections.

reflection. Whether a reflection is of the Mach or regular type depends primarily on the strength of the incident shock. Figure 9, adapted from Ref. 5, indicates the region in which Mach reflections are to be anticipated.

TWO-DIMENSIONAL DIFFUSERS. Combinations of oblique shock waves and a normal shock are often utilized to achieve more efficient supersonic diffusion than can be realized with a normal shock alone. Figure 10 illustrates two types of supersonic diffuser and compares their efficiency with that of a normal shock at the upstream Mach number. An efficient supersonic turbine blade passage may also be designed by utilizing oblique shock waves as illustrated in this figure.



(a) Two oblique shock waves - preceding normal shock double-wedge type

(b) Two oblique shock waves preceding normal shock-single reflection type

(c) Turbine blade for small turning angle

(d) Turbine blade for large turning angle

FIG. 10. Two-dimensional supersonic passages.

BOUNDARY LAYER-SHOCK WAVE INTERACTION. When a strong oblique or normal shock wave interacts with a *laminar* boundary layer, transition to a *turbulent* layer generally occurs. If the layer is initially turbulent the interaction will produce a pronounced thickening. Often a *forked shock wave* will result, and the boundary layer will separate from the boundary. (See Fig. 11.)

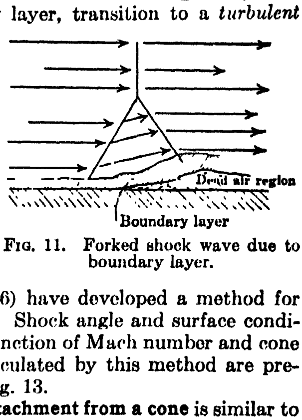


FIG. 11. Forked shock wave due to boundary layer.

THREE-DIMENSIONAL SHOCK PATTERNS. The simplest three-dimensional shock pattern is that caused by a pure cone immersed in a supersonic stream. The shock wave produced is also conical, and velocity and pressure changes through it are constant over its entire surface. Flow behind the shock wave is not parallel, but exhibits conical symmetry. Pressure, density, velocity, etc., are constant along straight lines through the apex (see Fig. 12). Taylor and Maccoll (see Ref. 6) have developed a method for determining the entire flow field by numerical iteration. Shock angle and surface conditions as a function of Mach number and cone angle as calculated by this method are presented in Fig. 13.

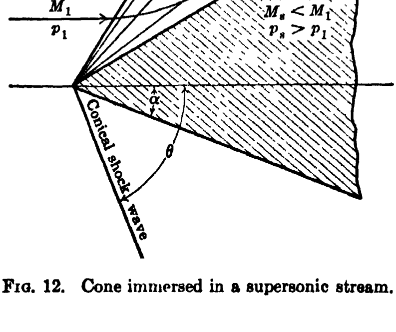


FIG. 12. Cone immersed in a supersonic stream.

Shock detachment from a cone is similar to that described for a concave corner or wedge, but occurs at a different value of α, as is shown in Fig. 13. Since the flow field behind a conical shock is not uniform, the shock produced by a second concave angle on a cone is not conical, but curves away from the approaching flow. Losses through this second shock vary with radius (see Fig. 14).

The flow over most nonconical bodies can be determined by one of the many modifications of the semigraphical *method of characteristics* (see Isenberg, Ref. 7). If the body is slender and moving at a Mach number below 4.0, the linearized theory yields excellent results. The linearized theory is also helpful in predicting the lift of slender bodies at small angles of attack.

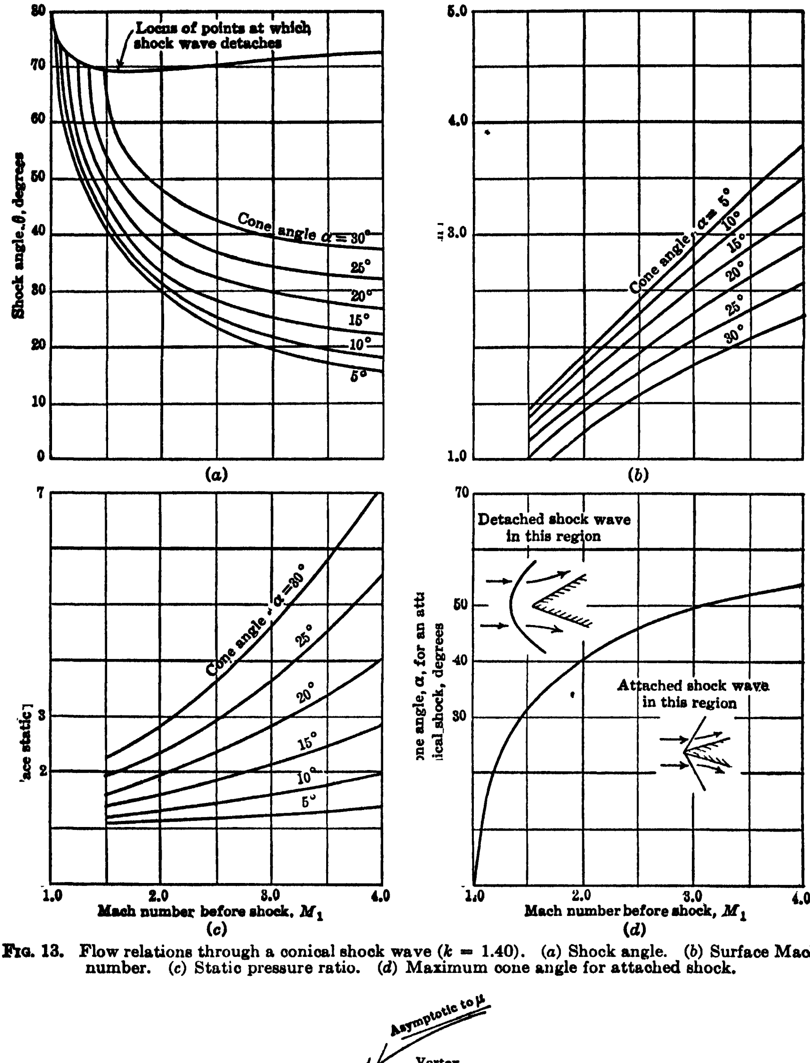


FIG. 13. Flow relations through a conical shock wave ($k = 1.40$). (a) Shock angle. (b) Surface Mach number. (c) Static pressure ratio. (d) Maximum cone angle for attached shock.

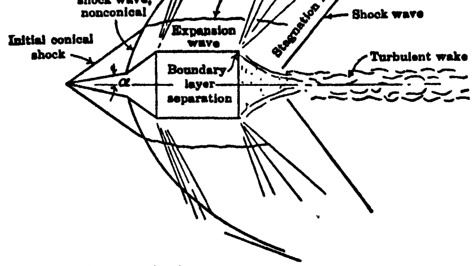


FIG. 14. Flow around a body with double-cone head shape immersed in a supersonic stream.

AIRCRAFT ENGINES

(See Section 13)

JET PROPULSION

By C. W. Smith

NOMENCLATURE

A = projected area (perpendicular to flight direction)
c_p = specific heat at constant pressure (any working medium)
c_{pc} = value of c_p for air during compression process
c_{pt} = value of c_p for gas during expansion process
c_{pm} = mean value of c_p over range of temperatures occurring in combustion chamber
c_v = specific heat at constant volume
D = diameter, or other characteristic dimension
F = thrust, force
ΔF = thrust correction
g = acceleration of gravity
k = isentropic exponent ($= c_p/c_v$)
N = speed of rotation
n = exponent of compression or expansion
P = power
P_c = actual power required for compression
P'_c = isentropic power required for compression
P_j = propulsive jet power
P_s = shaft power
P_t = actual power available from expansion
P'_t = isentropic power available from expansion
p = absolute pressure
p'/p = pressure ratio taken between any two specified points of compression or expansion process
q = volume flow
R = gas constant
r = pressure ratio (in eq. 14a, Table 1, and definition of X)
r = radius (in eqs. 25, 26, and 27)
T = absolute temperature
T'/T = temperature ratio taken between any two specified points of compression or expansion process
T_e = temperature of gas in tailpipe (exhaust gas temperature)
U = internal energy per unit mass.
V = velocity
V_a = absolute velocity of working medium
V_p = velocity of airplane or rocket in space
V_r = velocity of working medium relative to airplane or rocket
V_u = tangential component of gas velocity

v = specific volume
W_c = mechanical (shaft) work of compression
W_t = mechanical (shaft) work of expansion
w = mass flow (any working medium)
w_c = mass flow of air
w_f = mass flow of fuel
w_t = mass flow of gas (air flow plus fuel flow)
X = isentropic factor ($= r^{(k-1)/k} - 1$)
X_c = value of X for compression process
X_t = value of X for expansion process
δ = pressure factor ($\delta = p_t/p_0$)
η = propulsive efficiency
η_B = efficiency of Brayton cycle
η_B' = efficiency of modified Brayton cycle
η_c = efficiency of compression process
η_r = rocket efficiency
η_t = efficiency of expansion process
θ = temperature factor ($\theta = T_t/T_0$)
ρ = density
ρ'/ρ = density ratio taken between any two specified points of compression or expansion process

Subscripts and Superscripts

In eqs. 6 to 22 the numerical subscripts indicate points of the Brayton cycle (Fig. 6). Elsewhere:

0 = standard or reference value
1 = at inlet
2 = at exit
u = tangential component
t = test value

The primes (′) and double primes (″) in eq. 3 indicate different paths over or through the airplane.

13. GENERAL PRINCIPLES

ELEMENTS OF AIRCRAFT PROPULSION. All methods of aircraft and rocket propulsion are basically similar in that a forward thrust is obtained by the reaction of a backward-flowing column of air or other gas. In this respect an aircraft jet propulsion power plant does not differ fundamentally from a propeller power plant, since the stream of gases from the propulsion nozzle merely replaces the slip stream from the conventional propeller. In both cases air is "taken on board" at a relative velocity equal to the airplane velocity and discharged to the rear at a greater velocity. A retarding thrust results when air is taken on board, a propulsive thrust when it is discharged, and the difference is the useful thrust available to overcome fixed drag.

In the jet plant the working medium at the propulsion nozzle includes the fuel burned. Since the fuel is carried on the airplane, there is no retarding thrust from taking it on board, and theoretically the fuel and the air should be treated separately. Ordinarily, however, the mass of fuel is so small that it can be neglected.

Taken on board is a term applied technically to any air appreciably affected by the proximity of the airplane. This includes cooling air, cabin ventilating air, and the boundary layer air on the various airplane surfaces, all of which is taken on board at airplane velocity, and is usually discharged at a lower velocity. Although there is always some propulsive thrust if the air is discharged in a direction opposite the direction of flight, the net effect is a retarding thrust if the relative discharge velocity is less than the airplane velocity.

It is instructive to consider aircraft propulsion from both the absolute and the relative viewpoints. Consider first the usual case in which the working medium is taken from the atmosphere and discharged to the atmosphere (i.e., excluding rocket power plants). If an airplane is flying at constant velocity at constant altitude, the net thrust acting on it in the direction of flight is zero. The momentum of the enveloping atmospheric air (a vector quantity) will be zero after the airplane has passed, just as it was before. However, the boundary layer air and any other air that has been discharged with a relative velocity less than airplane velocity will follow the airplane, and the air that has been discharged with a greater velocity will flow in the opposite direction. The former represents a retarding thrust, the latter an equal propulsive thrust. In each case the airplane has exerted a certain force to accelerate the air from zero velocity up to its absolute velocity in space, and the air has exerted an equal and opposite force on the airplane.

The propulsive thrust is given by

$$F = \frac{w}{g} V_a \tag{1a}$$

or

$$F = \frac{w}{g} (V_r - V_p) \tag{1b}$$

where the symbols indicate only magnitudes, and not directions.

Equation 1*a* expresses the physical fact that the force acting on the airplane in the forward direction is equal in magnitude to the force accelerating the backward-flowing column of air from zero to the *absolute* velocity V_a. Equation 1*b* expresses the physical fact that when velocities *relative* to the airplane are considered, this force is the resultant of a propulsive force (proportional to jet *relative* velocity) and a retarding force (proportional to plane speed).

The fact that the air may be heated to a high temperature during its passage over or through the airplane *is of no consequence* insofar as thrust is concerned. It is merely a means of increasing the discharge velocity, and the resultant dynamical effects are the same as though the velocity were increased by some other means.

In general, the relative velocity with which the working medium is taken on board can be equal only to the airplane velocity in space, or, if the medium is carried on board, as in the case of a rocket, equal to zero. Sometimes it is proposed to take power-plant or auxiliary air from the boundary layer on some airplane surface. Although there is a retarding thrust when such air is taken on board, this is part of the normal aerodynamic drag, and use of the air for other purposes introduces no additional resistance. However, the method has not yet been perfected.

In order to increase the velocity of the medium, some kind of power plant is necessary, and the efficiency with which the kinetic energy is *produced* will depend upon the *power-plant cycle efficiency*. The efficiency with which it is *utilized* for propulsion will depend upon the *propulsive efficiency*, commonly defined as

$$\eta = \frac{(w/g) V_a V_p}{(w/g) V_a V_p + (w/g)(V_a^2/2)} = \frac{2}{2 + (V_a/V_p)} = \frac{2}{1 + (V_r/V_p)} \tag{2}$$

This equation considers the useful power output to be the useful thrust, as defined by eq. 1*a*, multiplied by the airplane velocity. The energy loss is the kinetic energy possessed by the medium after discharge, which is eventually dissipated. If $V_r = V_p$ (considering magnitudes only, not directions), the propulsive efficiency is 1.00, or 100%, but the thrust (eq. 1*b*) is zero. This relationship indicates that, for a given thrust, maximum propulsive efficiency is obtained when the mass flow is as large as possible and the discharge velocity is as small as possible. Thus, while V_r must be greater than V_p if positive thrust is to be obtained, the propulsive efficiency will increase as the ratio V_r/V_p becomes more nearly equal to unity.

FIELDS OF USEFULNESS OF PROPELLER AND JET PROPULSION POWER PLANTS. Equation 2 is equally applicable to a propeller or a jet propulsion power plant. It is evident that for high propulsive efficiency, the discharge velocity should be low when the airplane velocity is low. Although it is perfectly possible to have a low discharge velocity in a jet propulsion plant, the corresponding thrust would be so small as to be practically useless, since the mass flow must also be small because of the practical limitations on power plant size. On the other hand, the mass flow through a propeller can be very large because of its large diameter, even though the velocity is low. Thus both good efficiency and high thrust are possible with a propeller at low airplane speeds, whereas both cannot be simultaneously obtained with a jet plant.

As the airplane speed increases, the high discharge velocity inherent in a jet plant becomes less of a disadvantage. At very high speeds, conditions reverse to such an extent that it is possible to obtain both high thrust and high efficiency with a jet plant, whereas the thrust produced by the conventional propeller, even with widely variable pitch, falls off rapidly. The performance of the jet plant exceeds that of usual propeller plants at an airplane speed of about 500 to 600 miles per hour, and the margin of superiority will thereafter increase very rapidly.

Even at lower airplane speeds, a more general analysis may show that the overall propulsive efficiency is better with a jet plant than with a propeller. Thus if the flow of cooling air can be reduced, and if the aerodynamic drag of the airplane can be reduced (this amounts to reducing the boundary layer and other external air flow), the decreased retarding thrust may partially or wholly compensate for the loss of propulsive efficiency in the jet power plant path as compared with the corresponding propeller path.

In such a case a more useful formula for propulsive efficiency would be one that takes account of all paths of flow, and considers the useful thrust to be equal to the total minimum irreducible retarding thrust for all paths with the best possible design of the airplane under consideration. All air taken on board and discharged at a common velocity is lumped into the same path. The useful power output is a sum of terms of the form

$$FV_p = \frac{w}{g} V_a V_p$$

and the power loss is a sum of terms of the form

$$\frac{w}{g} \frac{V_a^2}{2}$$

One such useful power term and one energy loss term exists for each path, but the former may be either positive or negative, depending partly on what is considered to be the minimum irreducible retarding thrust. All the energy loss terms are positive. The general form of eq. 2 is then

$$\eta = \frac{(w'/g)V_a'V_p + (w''/g)V_a''V_p + \cdots}{(w'/g)V_a'V_p + (w''/g)V_a''V_p + \cdots + (w'/g)(V_a'^2/2) + (w''/g)(V_a''^2/2) + \cdots} \tag{3}$$

TYPES OF AIRCRAFT JET PROPULSION SYSTEMS. Turbojet. This is the most common type. It consists of a gas turbine (usually of the axial-flow type), a compressor (either of the centrifugal or axial-flow type) driven by the turbine, a combustion chamber between compressor and turbine, a discharge or tailpipe with propulsion nozzle, and various accessories. Such units are shown in Figs. 1, 2, 3, and 4.

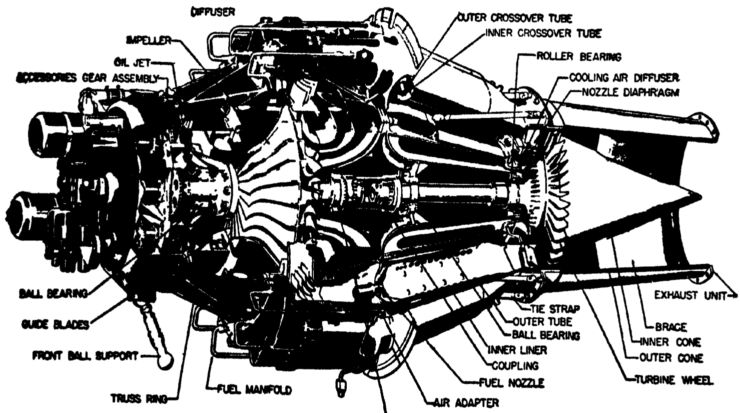


FIG. 1. Turbojet with centrifugal compressor. (Courtesy of Allison Division, General Motors Corp.)

Propjet. This is an aircraft gas turbine power plant that delivers most of its useful output to a propeller, but also discharges to the rear so as to produce some direct jet thrust. Present units of this type divide the total output in the ratio of roughly 80% propeller thrust to 20% pure jet thrust. A typical unit is shown in Figs. 5 and 6.

Ducted Fan. The ducted fan is a further compromise between the propeller and the pure jet propulsion power plant. The mass flow handled is less than that of a comparable propeller, but more than that of a jet power plant, whereas the discharge velocity is greater than that of the propeller, but less than that of the jet. Thus the mass flow and velocity can be adjusted to obtain a given thrust with a discharge velocity better suited to a given airplane speed.

Rocket. The rocket differs from the conventional jet propulsion power plant in that it carries its entire working medium with it. The force acting on the medium changes its absolute velocity from an initial value V_p in the forward direction to a final value $(V_r - V_p)$ in the backward direction. The magnitude of the increase is therefore equal to V_r, and the thrust is given by

$$F = \frac{w}{g} V_r \tag{4}$$

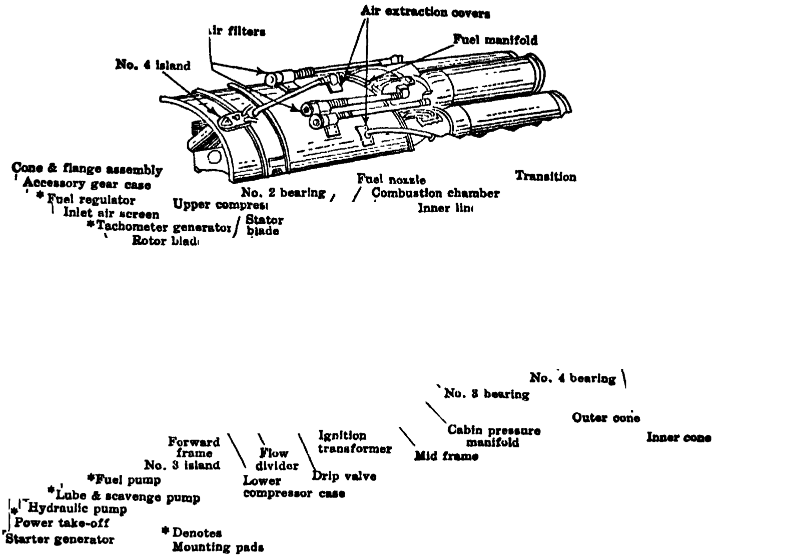


FIG. 2. Turbojet with axial-flow compressor. (Courtesy of Allison Division, General Motors Corp.)

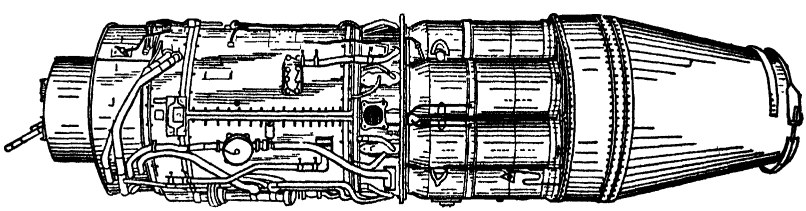


FIG. 3. Exterior view of turbojet with axial-flow compressor. (Courtesy of General Electric Co.)

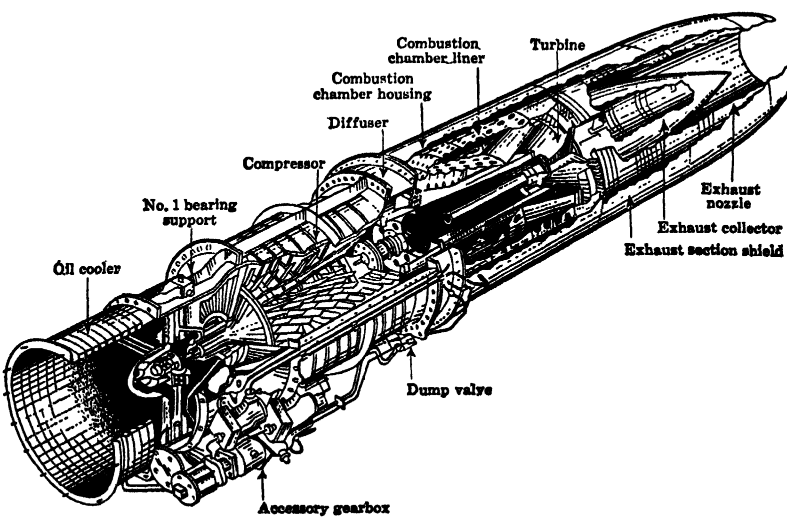


FIG. 4. Cutaway view of turbojet with axial-flow compressor. (Courtesy of Westinghouse Electric Corp.)

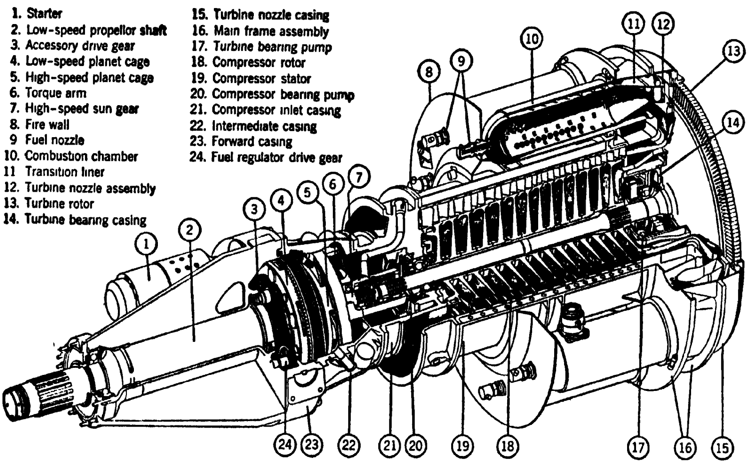


FIG. 5. Cutaway view of propjet with axial-flow compressor. (Courtesy of General Electric Co.)

There is some disagreement as to the best definition of efficiency for a rocket, but a definition of instantaneous propulsive efficiency consistent with the definition previously given for a jet propulsion power plant is

$$\eta_r = \frac{2V_rV_p}{V_r^2 + V_p^2} = \frac{2}{\frac{V_r}{V_p} + \frac{V_p}{V_r}} \tag{5}$$

The ram jet or athodyd is similar to the conventional jet propulsion power plant except that the increase of static pressure is obtained by diffusing the high-velocity head of the slip stream or ram air; no mechanical compressor is used. Hence no turbine is required; the only expansion occurs through the propulsion nozzle. Aside from the absence of rotating machinery, however, the ram jet is basically similar to the conventional jet propulsion power plant, and its thrust and propulsive efficiency are calculated in the same way.

Reciprocating Units. Although present-day aircraft jet propulsion units almost universally use the rotary compressor and turbine, it is not essential that rotary components be used. In theory, any conventional reciprocating engine could discharge to the rear and provide thrust directly rather than through a propeller. A higher efficiency of compression and expansion can usually be obtained in a reciprocating unit than in a rotary type, since no intermediate stage of velocity production is required, and the compression ratio may be higher. The permissible speed of a reciprocating engine is comparatively low, however, and the air-handling capacity is so low that if the engine flow constitutes the entire working medium, excessive size and weight would be required to obtain the same thrust as can be developed by a much smaller and lighter rotary unit, to say nothing of the greater mechanical complexity. However, one of the first jet propulsion units ever flown (the Italian Campini) used a reciprocating engine to drive a centrifugal compressor, and was essentially a ducted fan with a reciprocating driver.

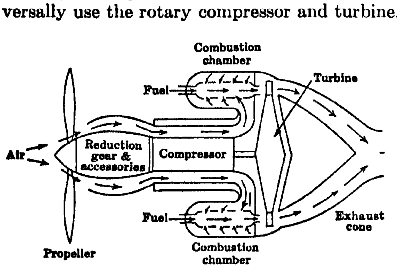


FIG. 6. Diagram of propjet.

ROTATING WINGS. A special application of jet propulsion is in driving, or assisting to drive, the rotating wings of such aircraft as helicopters. The jet propulsion nozzles are located at the tips of the rotor blades, where the peripheral velocity is high enough so that reasonably good propulsive efficiency is possible. The working medium may be supplied through ducts in the blades or may be taken directly from the atmosphere by a ram jet or pulse jet at the blade tip. See also p. 15-26.

ADVANTAGES AND LIMITATIONS OF VARIOUS TYPES. For airplane velocities less than about 250 miles per hour, there is little advantage in using anything except the reciprocating engine-propeller power plant for fixed-wing aircraft. If component efficiencies can be markedly improved, the gas turbine in some form will probably supplant the reciprocating engine even in low-power, low-speed aircraft. At present, however, the fuel consumption is prohibitive, and the period between overhauls not sufficiently great. It may be that a ducted fan driven by a small reciprocating engine will prove eventually to be a satisfactory power plant for such applications.

The propjet in general cannot compete with the reciprocating engine-propeller power plant at low airplane velocities, or with the turbojet at high airplane velocities, but is particularly suited to an intermediate range extending from about 250 up to 500 miles per hour. If the propeller can be suitably designed, it is possible that this range can be extended up to more than 600 miles per hour.

The turbojet is suitable for airplane velocities ranging from about 400 miles per hour as a minimum to some undetermined maximum. At velocities of 1000 miles per hour or more, the pressure ratio of the ram jet becomes sufficient for good thermal efficiency, and no doubt this power plant will eventually be used for aircraft as well as missiles. While it is simpler than the turbojet, and probably can be made extremely reliable, take-off requires either an auxiliary starting power plant or launching from another aircraft in motion, and landing presents particular problems because of the lack of thrust available as the velocity approaches zero. Moreover, the power plant is quite inflexible, since its possible output depends upon the aircraft speed. Thus it is possible that the turbojet will be used far up into the range of speeds that would otherwise appear better suited to the ram jet. On the other hand, the ram jet will probably be used as an auxiliary power plant to provide short bursts of power, regardless of low efficiency, even at rather low aircraft speeds.

All the power plants previously mentioned are dependent on the atmosphere for their working medium, and hence are limited to some maximum effective altitude. The rocket power plant carries its own medium, and thus is itself subject to no such limitation, although if it is used with a conventional type of airplane, sufficient air pressure is required for the necessary lift on the supporting surfaces. In general, the more suitable a power plant is for high airplane velocities, the more suitable it is for high altitude also. Thus the order of increasing effectiveness in both aircraft speed and altitude is: reciprocating engine-propeller, propjet, turbojet, ram jet, rocket. Although the ducted fan can theoretically be made to suit a wide range of velocities, it is probably best suited practically to the same range of velocities and altitudes as the propjet.

14. THERMODYNAMICS OF THE AIRCRAFT GAS TURBINE

IDEAL CYCLE. The ideal cycle is conventionally the Brayton (or Joule) cycle, which can be considered to be the resultant of a rotary compressor cycle (indicated by 0123 in Fig. 7) and a turbine cycle (indicated by 3450 in Fig. 7).

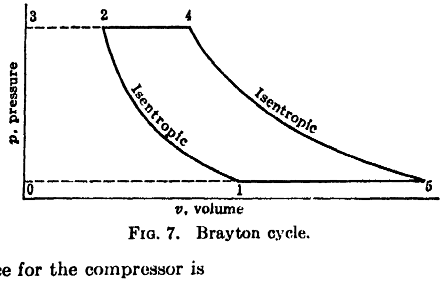


FIG. 7. Brayton cycle.

The energy balance for the compressor is

$$U_1 + p_1v_1 + \frac{V_1^2}{2g} + W_c = U_2 + p_2v_2 + \frac{V_2^2}{2g} \tag{6}$$

or

$$W_c + \frac{V_1^2 - V_2^2}{2g} = U_2 - U_1 + p_2v_2 - p_1v_1 \tag{7}$$

It follows from geometry that

$$W_c + \frac{V_1^2 - V_2^2}{2g} = \text{area } 0123 \tag{8}$$

since the area under the isentropic curve *1,2* is proportional to $U_2 - U_1$, and p_1v_1 and p_2v_2 represent obvious areas.

The energy balance for the turbine is

$$U_4 + p_4v_4 + \frac{V_4^2}{2g} = U_5 + p_5v_5 + \frac{V_5^2}{2g} + W_t \tag{9}$$

or

$$W_t + \frac{V_5^2 - V_4^2}{2g} = U_4 - U_5 + p_4v_4 - p_5v_5 \tag{10}$$

so that

$$W_t + \frac{V_5^2 - V_4^2}{2g} = \text{area } 3450 \tag{11}$$

The net output is given by the difference between eq. 11 and eq. 8.

$$W_t - W_c + \frac{V_5^2 - V_4^2 - V_1^2 + V_2^2}{2g} = \text{area } 1245 \tag{12}$$

The compressor exit velocity V_2 is theoretically equal in magnitude to the turbine inlet velocity V_4, and in any case these velocities are relatively small, so that eq. 12 becomes

$$W_t - W_c + \frac{V_5^2 - V_1^2}{2g} = \text{area } 1245 \tag{13}$$

That is, the area 1245 proportional to the net output of the Brayton cycle represents a certain amount of energy that may appear either in the form of mechanical energy $(W_t - W_c)$ or of kinetic energy $(V_5^2 - V_1^2)/2g$. In the case of the usual stationary power plant, $(V_5^2 - V_1^2)/2g = 0$, and the entire useful output appears as mechanical shaft power. For a pure jet propulsion power plant, $W_t = W_c$, and the entire useful output is represented by the kinetic energy $(V_5^2 - V_1^2)/2g$. In a propjet, W_t must be considerably greater than W_c, since most of the output is shaft power delivered to the propeller, and $(V_5^2 - V_1^2)/2g$ is relatively small.

The efficiency of the cycle represented by 1245 is easily determined by the customary method. It may be expressed as

$$\eta_B = 1 - \frac{1}{r^{(k-1)/k}} \tag{14a}$$

or

$$\eta_B = \frac{X}{X + 1} \tag{14b}$$

The second equation is a very convenient form, since tables of X are available for normal air (Table 1). Although based on a particular isentropic exponent $k = 1.3947$, the tabulated values are adequate for a comparison of cycle efficiencies. It is to be emphasized that, for a pure jet propulsion power plant, eqs. 14*a* and 14*b* give only the efficiency with which the increase of kinetic energy is produced. The efficiency with which the energy is utilized depends on the *propulsive* efficiency, and the overall thermal efficiency depends on both *cycle* and *propulsive* efficiencies. If the turbine drives a propeller, with no jet thrust, the cycle efficiency is the efficiency of production of the mechanical power delivered to the propeller, and the overall thermal efficiency depends on both cycle and propeller efficiencies.

Equations 14*a* and 14*b* also express the efficiency of the Otto cycle. The theoretical possibilities of the gas turbine cycle are therefore exactly the same as those of the conventional reciprocating engine operating on this cycle for the same pressure ratio. It should be noted that, for a given pressure ratio, the cycle efficiency is independent of any temperatures. This is true, however, *only* for the ideal cycle which includes two isentropics. When the compression and expansion processes are not isentropic the higher the turbine inlet temperature the more efficient the cycle, other factors remaining constant.

MODIFIED CYCLE EFFICIENCIES. The actual efficiencies with which kinetic energy and mechanical work are produced will of course be less than the ideal cycle efficiencies. Up to the present time, the actual efficiencies obtained with gas-turbine power plants have usually been lower than those obtainable with Otto cycle engines for the same pressure ratio, largely because of the lower efficiency of the processes of compression and expansion in the gas turbine power plants. A rotary compressor first imparts a velocity head to the air, and then converts this into pressure head. A rotary turbine first converts pressure head into velocity head, and then, if the turbine is designed primarily to develop shaft power, converts the velocity head into mechanical work. The necessity for an intermediate velocity conversion usually results in a lower efficiency than if mechanical work is done

(Continued on p. 15-48)

Table 1. Values of X for Normal Air and Perfect Diatomic Gases [$X = r^{0.283} - 1$]

(Reprinted by permission from Engineering Computations for Air and Gases, by Moss and Smith in *Trans. ASME*, Vol. 52, 1930, Paper APM-52-8)

r		0	1	2	3	4	5	6	7	8	9
1.00	0.00	000	028	057	085	113	141	169	198	226	254
1.01		282	310	338	366	394	422	450	478	506	534
1.02		562	590	618	646	673	701	729	757	785	812
1.03		840	868	895	923	951	978	006	034	061	089
1.04	0.01	116	144	171	199	226	253	281	308	336	363
1.05		390	418	445	472	500	527	554	581	608	636
1.06		663	690	717	744	771	798	825	852	879	906
1.07		933	960	987	014	041	068	095	122	148	175
1.08	0.02	202	229	255	282	309	336	362	389	416	442
1.09		469	495	522	549	575	602	628	655	681	708
1.10		734	760	787	813	840	866	892	919	945	971
1.11		997	024	050	076	102	129	155	181	207	233
1.12	0.03	259	285	311	337	363	389	415	441	467	493
1.13		519	545	571	597	623	649	675	700	726	752
1.14		778	804	829	855	881	906	932	958	983	009
1.15	0.04	035	060	086	111	137	162	188	213	239	264
1.16		290	315	341	366	391	417	442	467	493	518
1.17		543	569	594	619	644	670	695	720	745	770
1.18		796	821	846	871	896	921	946	971	996	021
1.19	0.05	046	071	096	121	146	171	196	221	245	270
1.20		295	320	345	370	394	419	444	469	493	518
1.21		543	567	592	617	641	666	691	715	740	764
1.22		789	813	838	862	887	911	936	960	985	009
1.23	0.06	034	058	082	107	131	155	180	204	228	253
1.24		277	301	325	350	374	398	422	446	470	495
1.25		519	543	567	591	615	639	663	687	711	735
1.26		759	783	807	831	855	879	903	927	951	974
1.27		998	022	046	070	094	117	141	165	189	212
1.28	0.07	236	260	283	307	331	354	378	402	425	449
1.29		472	496	520	543	567	590	614	637	661	684
1.30		708	731	754	778	801	825	848	871	895	918
1.31		941	965	988	011	035	058	081	104	128	151
1.32	0.08	174	197	220	243	267	290	313	336	359	382
1.33		405	428	451	474	497	520	543	566	589	612
1.34		635	658	681	704	727	750	773	795	818	841
1.35		864	887	910	932	955	978	001	023	046	069
1.36	0.09	092	114	137	160	182	205	228	250	273	295
1.37		318	341	363	386	408	431	453	476	498	521
1.38		543	566	588	611	633	655	678	700	723	745
1.39		767	790	812	834	857	879	901	923	946	968
1.40		990	012	035	057	079	101	123	145	168	190
1.41	0.10	212	234	256	278	300	322	344	366	389	411
1.42		433	455	477	499	521	542	564	586	608	630
1.43		652	674	696	718	740	761	783	805	827	849
1.44		871	892	914	936	958	979	001	023	045	066
1.45	0.11	088	110	131	153	175	196	218	239	261	283
1.46		304	326	347	369	390	412	433	455	476	498
1.47		520	541	562	584	605	627	648	669	691	712
1.48		734	755	776	798	819	840	862	883	904	925
1.49		947	968	989	010	032	053	074	095	116	138
1.50	0.12	159	180	201	222	243	264	286	307	328	349
1.51		370	391	412	433	454	475	496	517	538	559
1.52		580	601	622	643	664	685	706	726	747	768
1.53		789	810	831	852	872	893	914	935	956	977
1.54		997	018	039	060	080	101	122	142	163	184
1.55	0.13	205	225	246	266	287	308	328	349	370	390
1.56		411	431	452	472	493	513	534	554	575	595
1.57		616	636	657	677	698	718	739	759	780	800
1.58		820	841	861	881	902	922	942	963	983	003
1.59	0.14	024	044	064	085	105	125	145	165	186	206

Proportional Parts

	29		28		27		26
1	2.9	1	2.8	1	2.7	1	2.6
2	5.8	2	5.6	2	5.4	2	5.2
3	8.7	3	8.4	3	8.1	3	7.8
4	11.6	4	11.2	4	10.8	4	10.4
5	14.5	5	14.0	5	13.5	5	13.0
6	17.4	6	16.8	6	16.2	6	15.6
7	20.3	7	19.6	7	18.9	7	18.2
8	23.2	8	22.4	8	21.6	8	20.8
9	26.1	9	25.2	9	24.3	9	23.4

	25		24		23		22
1	2.5	1	2.4	1	2.3	1	2.2
2	5.0	2	4.8	2	4.6	2	4.4
3	7.5	3	7.2	3	6.9	3	6.6
4	10.0	4	9.6	4	9.2	4	8.8
5	12.5	5	12.0	5	11.5	5	11.0
6	15.0	6	14.4	6	13.8	6	13.2
7	17.5	7	16.8	7	16.1	7	15.4
8	20.0	8	19.2	8	18.4	8	17.6
9	22.5	9	21.6	9	20.7	9	19.8

	21		20		19		18
1	2.1	1	2.0	1	1.9	1	1.8
2	4.2	2	4.0	2	3.8	2	3.6
3	6.3	3	6.0	3	5.7	3	5.4
4	8.4	4	8.0	4	7.6	4	7.2
5	10.5	5	10.0	5	9.5	5	9.0
6	12.6	6	12.0	6	11.4	6	10.8
7	14.7	7	14.0	7	13.3	7	12.6
8	16.8	8	16.0	8	15.2	8	14.4
9	18.9	9	18.0	9	17.1	9	16.2

	17		16		15		14
1	1.7	1	1.6	1	1.5	1	1.4
2	3.4	2	3.2	2	3.0	2	2.8
3	5.1	3	4.8	3	4.5	3	4.2
4	6.8	4	6.4	4	6.0	4	5.6
5	8.5	5	8.0	5	7.5	5	7.0
6	10.2	6	9.6	6	9.0	6	8.4
7	11.9	7	11.2	7	10.5	7	9.8
8	13.6	8	12.8	8	12.0	8	11.2
9	15.3	9	14.4	9	13.5	9	12.6

	13		12
1	1.3	1	1.2
2	2.6	2	2.4
3	3.9	3	3.6
4	5.2	4	4.8
5	6.5	5	6.0
6	7.8	6	7.2
7	9.1	7	8.4
8	10.4	8	9.6
9	11.7	9	10.8

(*Table continued on p. 15-46*)

Table 1. Values of X for Normal Air and Perfect Diatomic Gases [$X = r^{0.283} - 1$]—*Continued*

(Reprinted by permission from Engineering Computations for Air and Gases, by Moss and Smith in *Trans. ASME*, Vol. 52, 1930, Paper APM-52-8)

r		0	1	2	3	4	5	6	7	8	9
1.60	0.14	226	246	267	287	307	327	347	367	387	408
1.61		428	448	468	488	508	528	548	568	588	608
1.62		628	648	668	688	708	728	748	768	788	808
1.63		828	848	868	888	908	928	948	968	988	007
1.64	0.15	027	047	067	087	107	126	146	166	186	206
1.65		225	245	265	284	304	324	344	363	383	403
1.66		423	442	462	481	501	521	540	560	580	599
1.67		619	638	658	678	697	717	736	756	775	795
1.68		814	834	853	873	892	912	931	951	970	990
1.69	0.16	009	028	048	067	087	106	125	145	164	184
1.70		203	222	242	261	280	299	319	338	357	377
1.71		396	415	434	454	473	492	511	531	550	569
1.72		588	607	626	646	665	684	703	722	741	760
1.73		780	799	818	837	856	875	894	913	932	951
1.74		970	989	008	027	046	065	084	103	122	141
1.75	0.17	160	179	198	217	236	255	274	292	311	330
1.76		349	368	387	406	425	443	462	481	500	519
1.77		538	556	575	594	613	631	650	669	688	706
1.78		725	744	762	781	800	818	837	856	874	893
1.79		912	930	949	968	986	005	023	042	061	079
1.80	0.18	098	116	135	153	172	191	209	228	246	265
1.81		283	302	320	339	357	376	394	412	431	449
1.82		468	486	505	523	541	560	578	596	615	633
1.83		652	670	688	707	725	743	762	780	798	816
1.84		835	853	871	890	908	926	944	962	981	999
1.85	0.19	017	035	054	072	090	108	126	144	163	181
1.86		199	217	235	253	271	289	308	326	344	362
1.87		380	398	416	434	452	470	488	506	524	542
1.88		560	578	596	614	632	650	668	686	704	722
1.89		740	758	776	794	811	829	847	865	883	901
1.90		919	937	954	972	990	008	026	044	061	079
1.91	0.20	097	115	133	150	168	186	204	221	239	257
1.92		275	292	310	328	345	363	381	399	416	434
1.93		452	469	487	504	522	540	557	575	593	610
1.94		628	645	663	681	698	716	733	751	768	786
1.95		804	821	839	856	874	891	909	926	944	961
1.96		979	996	013	031	048	066	083	101	118	135
1.97	0.21	153	170	188	205	222	240	257	275	292	309
1.98		327	344	361	379	396	413	431	448	465	482
1.99		500	517	534	552	569	586	603	620	638	655
2.00		672	689	707	724	741	758	775	792	810	827
2.01		844	861	878	895	913	930	947	964	981	998
2.02	0.22	015	032	049	066	084	101	118	135	152	169
2.03		186	203	220	237	254	271	288	305	322	339
2.04		356	373	390	407	424	441	458	474	491	508
2.05		525	542	559	576	593	610	627	644	660	677
2.06		694	711	728	745	762	778	795	812	829	846
2.07		863	879	896	913	930	946	963	980	997	013
2.08	0.23	030	047	064	080	097	114	130	147	164	181
2.09		197	214	231	247	264	281	297	314	331	347
2.10		364	380	397	414	430	447	463	480	497	513
2.11		530	546	563	579	596	613	629	646	662	679
2.12		695	712	728	745	761	778	794	811	827	844
2.13		860	877	893	909	926	942	959	975	992	008
2.14	0.24	024	041	057	074	090	106	123	139	155	172
2.15		188	204	221	237	253	270	286	302	319	335
2.16		351	368	384	400	416	433	449	465	481	498
2.17		514	530	546	563	579	595	611	627	644	660
2.18		676	692	708	724	741	757	773	789	805	821
2.19		838	854	870	886	902	918	934	950	966	983

r		0	1	2	3	4	5	6	7	8	9
2.20	0.24	999	015	031	047	063	079	095	111	127	143
2.21	0.25	159	175	191	207	223	239	255	271	287	303
2.22		319	335	351	367	383	399	415	431	447	463
2.23		479	495	511	526	542	558	574	590	606	622
2.24		638	654	669	685	701	717	733	749	765	780
2.25		796	812	828	844	859	875	891	907	923	938
2.26		954	970	986	001	017	033	049	064	080	096
2.27	0.26	112	127	143	159	175	190	206	222	237	253
2.28		269	284	300	316	331	347	363	378	394	409
2.29		425	441	456	472	488	503	519	534	550	566
2.30		581	597	612	628	643	659	675	690	706	721
2.31		737	752	768	783	799	814	830	845	861	876
2.32		892	907	923	938	954	969	984	000	015	031
2.33	0.27	046	062	077	092	108	123	139	154	169	185
2.34		200	216	231	246	262	277	292	308	323	338
2.35		354	369	384	400	415	430	446	461	476	492
2.36		507	522	538	553	568	583	599	614	629	644
2.37		660	675	690	705	721	736	751	766	781	797
2.38		812	827	842	857	873	888	903	918	933	948
2.39		964	979	994	009	024	039	054	070	085	100
2.40	0.28	115	130	145	160	175	190	205	220	236	251
2.41		266	281	296	311	326	341	356	371	386	401
2.42		416	431	446	461	476	491	506	521	536	551
2.43		566	581	596	611	626	641	656	671	686	701
2.44		716	730	745	760	775	790	805	820	835	850
2.45		865	879	894	909	924	939	954	969	984	998
2.46	0.29	013	028	043	058	073	087	102	117	132	147
2.47		162	176	191	206	221	235	250	265	280	295
2.48		309	324	339	353	368	383	398	412	427	442
2.49		457	471	486	501	515	530	545	559	574	589
2.50		604	618	633	647	662	677	691	706	721	735
2.51		750	765	779	794	808	823	838	852	867	881
2.52		896	911	925	940	954	969	984	998	013	027
2.53	0.30	042	056	071	085	100	114	129	144	158	173
2.54		187	202	216	231	245	260	274	289	303	318
2.55		332	346	361	375	390	404	419	433	448	462
2.56		476	491	505	520	534	548	563	577	592	606
2.57		620	635	649	663	678	692	707	721	735	750
2.58		764	778	793	807	821	836	850	864	879	893
2.59		907	921	936	950	964	979	993	007	021	036
2.60	0.31	050	064	079	093	107	121	136	150	164	178
2.61		193	207	221	235	249	264	278	292	306	320
2.62		335	349	363	377	391	405	420	434	448	462
2.63		476	490	505	519	533	547	561	575	589	603
2.64		618	632	646	660	674	688	702	716	730	744
2.65		759	773	787	801	815	829	843	857	871	885
2.66		899	913	927	941	955	969	983	997	011	025
2.67	0.32	039	053	067	081	095	109	123	137	151	165
2.68		179	193	207	221	235	249	262	276	290	304
2.69		318	332	346	360	374	388	402	416	429	443
2.70		457	471	485	499	513	527	540	554	568	582
2.71		596	610	624	637	651	665	679	693	707	720
2.72		734	748	762	776	789	803	817	831	845	858
2.73		872	886	900	913	927	941	955	968	982	996
2.74	0.33	010	023	037	051	065	078	092	106	119	133
2.75		147	161	174	188	202	215	229	243	256	270
2.76		284	297	311	325	338	352	366	379	393	407
2.77		420	434	448	461	475	488	502	516	529	543
2.78		556	570	584	597	611	624	638	651	665	679
2.79		692	706	719	733	746	760	773	787	801	814

Table 1. Values of X for Normal Air and Perfect Diatomic Gases [$X = r^{0.283} - 1$]—*Continued*

(Reprinted by permission from Engineering Computations for Air and Gases, by Moss and Smith in *Trans. ASME.* Vol. 52, 1930, Paper APM-52-8)

r		0	1	2	3	4	5	6	7	8	9
2.80	0.33	828	841	855	868	882	895	909	922	936	949
2.81		963	976	990	003	017	030	044	057	070	084
2.82	0.34	097	111	124	138	151	165	178	191	205	218
2.83		232	245	259	272	285	299	312	326	339	352
2.84		366	379	393	406	419	433	446	459	473	486
2.85		500	513	526	540	553	566	580	593	606	620
2.86		633	646	660	673	686	700	713	726	739	753
2.87		766	779	793	806	819	832	846	859	872	886
2.88		899	912	925	939	952	965	978	991	005	018
2.89	0.35	031	044	058	071	084	097	110	124	137	150
2.90	0.35	163	176	190	203	216	229	242	255	269	282
2.91		295	308	321	334	347	361	374	387	400	413
2.92		426	439	452	466	479	492	505	518	531	544
2.93		557	570	584	597	610	623	636	649	662	675
2.94		688	701	714	727	740	753	767	780	793	806
2.95		819	832	845	858	871	884	897	910	923	936
2.96		949	962	975	988	001	014	027	040	053	066
2.97	0.36	079	092	105	118	131	144	157	169	182	195
2.98		208	221	234	247	260	273	286	299	312	324
2.99		337	350	363	376	389	402	415	428	440	453

	0	1	2	3	4	5	6	7	8	9
3.0	0.3647	0.3659	0 3672	0 3685	0.3698	0.3711	0.3723	0 3736	0.3749	0.3761
3.1	0.3774	0.3786	0.3799	0.3811	0 3824	0.3836	0.3849	0.3861	0.3874	0.3886
3.2	0.3898	0.3911	0.3923	0.3935	0.3947	0.3959	0.3971	0 3984	0.3996	0.4008
3.3	0.4020	0.4032	0 4044	0 4056	0.4068	0.4080	0.4091	0.4103	0.4115	0.4127
3.4	0.4139	0.4150	0 4162	0.4174	0.4186	0.4197	0.4209	0 4220	0.4232	0.4244
3.5	0.4255	0.4267	0 4278	0.4290	0.4301	0.4313	0.4324	0.4335	0.4347	0.4358
3.6	0.4369	0.4380	0.4392	0.4403	0.4414	0.4425	0.4437	0.4448	0.4459	0.4470
3.7	0.4481	0.4492	0.4503	0.4514	0.4525	0.4536	0.4547	0 4558	0.4569	0.4580
3.8	0.4591	0.4602	0.4612	0.4623	0.4634	0.4645	0.4656	0 4666	0.4677	0.4688
3.9	0.4698	0.4709	0 4720	0.4730	0.4741	0 4752	0.4762	0.4773	0.4783	0.4794
4.0	0.4804	0.4815	0.4825	0.4835	0.4846	0 4856	0.4867	0.4877	0.4887	0.4898
4.1	0.4908	0.4918	0.4928	0.4939	0.4949	0.4959	0.4970	0.4980	0.4990	0.5000
4.2	0.5010	0.5020	0.5030	0.5040	0.5050	0 5060	0.5070	0.5080	0.5090	0.5100
4.3	0.5110	0.5120	0.5130	0.5140	0.5150	0 5160	0.5170	0.5179	0.5189	0.5199
4.4	0.5209	0.5219	0.5228	0.5238	0.5248	0.5258	0.5267	0.5277	0.5287	0.5296
4.5	0.5306	0.5316	0.5325	0.5335	0.5344	0.5354	0.5363	0.5373	0.5382	0.5392
4.6	0.5401	0.5411	0.5420	0.5430	0.5439	0 5449	0.5458	0.5467	0.5477	0.5486
4.7	0.5495	0.5505	0.5514	0.5523	0.5533	0 5542	0.5551	0 5560	0.5570	0.5579
4.8	0.5588	0.5597	0 5606	0.5616	0.5625	0.5634	0.5643	0 5652	0.5661	0.5670
4.9	0.5679	0.5688	0.5697	0 5706	0.5715	0.5724	0.5733	0.5742	0.5751	0.5760
5.0	0.5769	0.5778	0 5787	0.5796	0.5805	0.5814	0.5822	0.5831	0.5840	0.5849
5.1	0.5858	0.5867	0.5875	0.5884	0.5893	0.5902	0.5910	0.5919	0.5928	0.5936
5.2	0.5945	0.5954	0.5962	0.5971	0.5980	0.5988	0 5997	0.6006	0.6014	0.6023
5.3	0.6031	0.6040	0 6048	0.6057	0.6065	0.6074	0 6082	0.6091	0.6099	0.6108
5.4	0.6116	0.6125	0 6133	0.6142	0.6150	0.6159	0.6167	0.6175	0.6184	0.6192
5.5	0.6200	0.6209	0 6217	0.6225	0.6234	0 6242	0.6250	0.6258	0.6267	0.6275
5.6	0.6283	0.6291	0.6300	0 6308	0.6316	0.6324	0.6332	0.6340	0.6349	0.6357
5.7	0.6365	0.6373	0 6381	0 6389	0.6397	0 6405	0.6413	0.6421	0.6430	0.6438
5.8	0.6446	0.6454	0 6462	0.6470	0.6478	0.6486	0.6494	0.6502	0.6509	0.6517
5.9	0.6525	9.6533	0.6541	0.6549	0.6557	0.6565	0 6573	0.6581	0.6588	0.6596
6.0	0.6604	0 6612	0.6620	0.6628	0.6635	0.6643	0.6651	0.6659	0.6666	0.6674
6.1	0.6682	0 6690	0.6697	0.6705	0.6713	0 6721	0.6729	0 6736	0.6744	0.6752
6.2	0.6759	0 6767	0.6774	0.6782	0.6789	0.6797	0.6805	0 6812	0.6820	0.6827
6.3	0.6835	0 6843	0.6850	0.6858	0.6865	0.6873	0.6880	0.6888	0.6895	0.6903
6.4	0.6910	0.6918	0.6925	0.6933	0.6940	0.6948	0.6955	0.6963	0.6970	0.6978
6.5	0.6985	0.6992	0.7000	0.7007	0.7014	0.7021	0 7028	0.7036	0.7043	0.7050
6.6	0.7058	0.7065	0.7073	0.7080	0.7087	0.7095	0 7102	0.7110	0.7117	0.7124
6.7	0.7131	0 7138	0.7145	0.7153	0.7160	0.7167	0.7174	0 7181	0.7189	0.7196
6.8	0.7203	0.7210	0.7217	0.7224	0.7232	0.7239	0.7246	0.7253	0.7260	0.7267
6.9	0.7274	0.7281	0.7288	0.7295	0.7302	0.7309	0.7316	0 7323	0.7330	0.7338
7.0	0.7345	0.7352	0.7359	0.7366	0.7373	0.7380	0.7386	0.7393	0.7400	0.7407
7.1	0.7414	0.7421	0.7428	0.7435	0.7442	0.7449	0.7456	0.7463	0.7470	0.7477
7.2	0.7483	0.7490	0.7497	0.7504	0.7511	0.7518	0.7524	0.7531	0.7538	0.7545
7.3	0.7552	0.7559	0.7565	0.7572	0.7579	0.7586	0.7592	0.7599	0.7606	0.7613
7.4	0.7620	0.7626	0.7633	0.7640	0.7646	0.7653	0.7660	0.7666	0.7673	0.7680
7.5	0.7687	0.7693	0.7700	0.7706	0.7713	0.7720	0.7726	0.7733	0.7740	0.7746
7.6	0.7753	0.7760	0.7766	0.7773	0.7779	0.7786	0.7792	0.7799	0.7806	0.7812
7.7	0.7819	0.7825	0.7832	0.7838	0.7845	0.7851	0.7858	0.7864	0.7871	0.7877
7.8	0.7884	0.7890	0.7897	0.7903	0.7910	0.7916	0.7923	0.7929	9.7936	0.7942
7.9	0.7949	0.7955	0.7961	0.7968	0.7974	0.7981	0.7987	0.7993	0.8000	0.8006

(*Table continued on p. 15-48*)

Table 1. Values of X for Normal Air and Perfect Diatomic Gases [$X = r^{0.283} - 1$]—*Continued*

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r	0	1	2	3	4	5	6	7	8	9
8.0	0.8013	0.8019	0.8025	0.8032	0.8038	0.8044	0.8051	0.8057	0.8063	0.8070
8.1	0.8076	0.8082	0.8089	0.8095	0.8101	0.8108	0.8114	0.8120	0.8126	0.8133
8.2	0.8139	0.8145	0.8151	0.8158	0.8164	0.8170	0.8176	0.8183	0.8189	0.8195
8.3	0.8201	0.8207	0.8214	0.8220	0.8226	0.8232	0.8238	0.8245	0.8251	0.8257
8.4	0.8263	0.8269	0.8275	0.8281	0.8228	0.8294	0.8300	0.8306	0.8312	0.8318
8.5	0.8324	0.8330	0.8336	0.8343	0.8349	0.8355	0.8361	0.8367	0.8373	0.8379
8.6	0.8385	0.8391	0.8397	0.8403	0.8409	0.8415	0.8421	0.8427	0.8433	0.8439
8.7	0.8445	0.8451	0.8457	0.8463	0.8469	0.8475	0.8481	0.8487	0.8493	0.8499
8.8	0.8505	0.8511	0.8517	0.8523	0.8529	0.8535	0.8541	0.8547	0.8552	0.8558
8.9	0.8564	0.8570	0.8576	0.8582	0.8588	0.8594	0.8600	0.8605	0.8611	0.8617
9.0	0.8623	0.8629	0.8635	0.8641	0.8646	0.8652	0.8658	0.8664	0.8670	0.8676
9.1	0.8681	0.8687	0.8693	0.8699	0.8705	0.8710	0.8716	0.8722	0.8728	0.8734
9.2	0.8739	0.8745	0.8751	0.8757	0.8762	0.8768	0.8774	0.8779	0.8785	0.8791
9.3	0.8797	0.8802	0.8808	0.8814	0.8819	0.8825	0.8831	0.8837	0.8842	0.8848
9.4	0.8854	0.8859	0.8865	0.8871	0.8876	0.8882	0.8888	0.8893	0.8899	0.8905
9.5	0.8910	0.8916	0.8921	0.8927	0.8933	0.8938	0.8944	0.8949	0.8955	0.8961
9.6	0.8966	0.8972	0.8977	0.8983	0.8989	0.8994	0.9000	0.9005	0.9011	0.9016
9.7	0.9022	0.9028	0.9033	0.9039	0.9044	0.9050	0.9055	0.9061	0.9066	0.9072
9.8	0.9077	0.9083	0.9088	0.9094	0.9099	0.9105	0.9110	0.9116	0.9121	0.9127
9.9	0.9132	0.9138	0.9143	0.9149	0.9154	0.9159	0.9165	0.9170	0.9176	0.9181
10.0	0.9187	0.9192	0.9198	0.9203	0.9208	0.9214	0.9219	0.9225	0.9230	0.9235
10.1	0.9241	0.9246	0.9252	0.9257	0.9262	0.9268	0.9273	0.9278	0.9284	0.9289
10.2	0.9295	0.9300	0.9305	0.9311	0.9316	0.9321	0.9327	0.9332	0.9337	0.9343
10.3	0.9348	0.9353	0.9358	0.9364	0.9369	0.9374	0.9380	0.9385	0.9390	0.9396
10.4	0.9401	0.9406	0.9411	0.9417	0.9422	0.9427	0.9432	0.9438	0.9443	0.9448
10.5	0.9453	0.9459	0.9464	0.9469	0.9474	0.9480	0.9485	0.9490	0.9495	0.9500
10.6	0.9506	0.9511	0.9516	0.9521	0.9526	0.9532	0.9537	0.9542	0.9547	0.9552
10.7	0.9558	0.9563	0.9568	0.9573	0.9578	0.9583	0.9589	0.9594	0.9599	0.9604
10.8	0.9609	0.9614	0.9619	0.9625	0.9630	0.9635	0.9640	0.9645	0.9650	0.9655
10.9	0.9660	0.9665	0.9671	0.9676	0.9681	0.9686	0.9691	0.9696	0.9701	0.9706
11.0	0.9711	0.9716	0.9721	0.9726	0.9732	0.9737	0.9742	0.9747	0.9752	0.9757
11.1	0.9762	0.9767	0.9772	0.9777	0.9782	0.9787	0.9792	0.9797	0.9802	0.9807
11.2	0.9812	0.9817	0.9822	0.9827	0.9832	0.9837	0.9842	0.9847	0.9852	0.9857
11.3	0.9862	0.9867	0.9872	0.9877	0.9882	0.9887	0.9892	0.9897	0.9902	0.9907
11.4	0.9912	0.9916	0.9921	0.9926	0.9931	0.9936	0.9941	0.9946	0.9951	0.9956
11.5	0.9961	0.9966	0.9971	0.9975	0.9980	0.9985	0.9990	0.9995	1.0000	1.0005
11.6	1.0010	1.0015	1.0019	1.0024	1.0029	1.0034	1.0039	1.0044	1.0049	1.0054
11.7	1.0058	1.0063	1.0068	1.0073	1.0078	1.0083	1.0087	1.0092	1.0097	1.0102
11.8	1.0107	1.0112	1.0116	1.0121	1.0126	1.0131	1.0136	1.0140	1.0145	1.0150
11.9	1.0155	1.0160	1.0164	1.0169	1.0174	1.0179	1.0184	1.0188	1.0193	1.0198
12.0	1.0203	1.0207	1.0212	1.0217	1.0222	1.0226	1.0231	1.0236	1.0241	1.0245

directly on or by the fluid, as in a reciprocating machine. In a turbojet power plant part of the velocity head developed in the turbine is used directly for propulsive purposes; this is one reason why the gas turbine is particularly suitable for jet propulsion.

If the compression and turbine expansion efficiencies are taken into account, the actual power required for compression is

$$P_c = \frac{w_c c_{pc} X_c T_1}{\eta_c} \tag{15}$$

and the actual turbine power output is

$$P_t = \frac{\eta_t w_t c_{pt} X_t T_4}{X_t + 1} \tag{16}$$

The net output is the difference, $P_t - P_c$. The input is the heat required to raise the temperature of the air from the compressor discharge temperature $T_1 + X_c T_1/\eta_c$ to the turbine inlet temperature T_4. The modified cycle efficiency is then

$$\eta_B' = \frac{\eta_t w_t c_{pt} X_t T_4/(X_t + 1) - w_c c_{pc} X_c T_1/\eta_c}{w_t c_{pm}[T_4 - (1 + X_c/\eta_c)T_1]} \tag{17}$$

where c_{pm} is the mean specific heat during the addition of heat.

If as a first approximation it is assumed that

$$\frac{w_c}{w_t} = \frac{c_{pc}}{c_{pt}} = \frac{c_{pm}}{c_{pt}} = 1 \tag{18}$$

the equation reduces to

$$\eta_B' = \frac{\eta_t X_t/(X_t + 1) - (X_c/\eta_c)(T_1/T_4)}{1 - (1 + X_c/\eta_c)(T_1/T_4)} \tag{19}$$

This expression is a function of the temperature ratio T_1/T_4 as well as of the compression and expansion efficiencies. As a first approximation, both X_c and X_t may be read from Table 1. If this is not considered sufficiently accurate, X_t must be calculated, with the best available values of k used for the temperature range in question. In the typical aircraft gas turbine power plant, the volume flow is varied only by changing speed, since there is no throttle or other means provided for changing the area of the flow path. Therefore the volume flow remains approximately constant at a given speed, and the compression and expansion efficiencies are functions of pressure ratio only. Figure 8 shows values of the modified cycle efficiency as calculated from eq. 19 for a typical variation of compression and expansion efficiencies in a single physical unit.

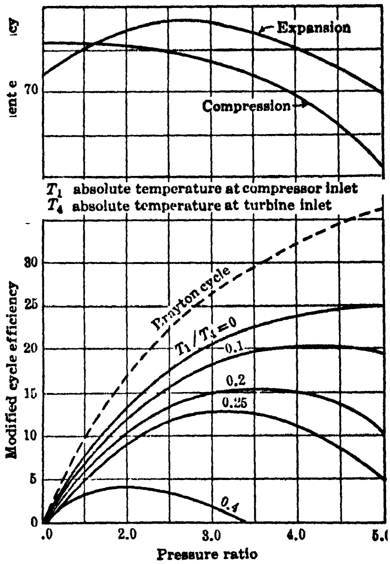

FIG. 8. Modified Brayton cycle efficiency as a function of pressure ratio for various temperature ratios. Component efficiencies are as shown by the upper curves.

Compression efficiency refers basically to the complete compression process from atmosphere up to the maximum pressure in the cycle, and expansion efficiency refers to the complete process of expansion from the maximum pressure down to atmosphere. Thus all losses in ducts, combustion chamber, propulsion nozzle, and the like should be reflected in the values of these efficiencies. However, such losses may be treated separately if desired. Even if the compression and expansion efficiency curves were applicable to a series of *different* machines, each designed for a different pressure ratio, the modified cycle efficiency curves would have the same general shape (though covering a broader range). Up to the present time it has been found difficult to maintain high component efficiencies at high pressure ratios, regardless of the number of compressor stages used.

SPECIAL CHARACTERISTICS OF MODIFIED CYCLE. Effect of Temperature. It is evident from Fig. 8 that a decrease of T_1/T_4 results in a considerable gain of efficiency when the ratio is high (0.4 or thereabouts), but the effect is much less at temperature ratios now customary. For example, the curve $T_1/T_4 = 0.25$ would be applicable to the temperatures $T_1 = 500$ R, $T_4 = 2000$ R, which are reasonable for present-day operation at low altitudes. Increase of T_4 by 500 F, to 2500 R would be necessary to give the efficiencies shown by the curve at $T_1/T_4 = 0.2$, and the resulting increase in peak efficiency would be relatively small. It is further evident that a given change in compressor inlet temperature is very much more effective in changing η_B' than the same change in turbine inlet temperature. The curves of Fig. 8 would be modified slightly if the variation of specific heat with temperature were taken into account, but the general conclusions would remain the same.

Comparison with Rankine Cycle. The gas turbine cycle is distinguished from the customary Rankine cycle used in steam turbine power plants chiefly by the amount of negative (compression) work required. The work of compression in the steam cycle is only that required to increase the pressure of water from condenser to boiler value. The negative work required to compress the liquid is extremely small compared with the positive work, and is frequently neglected in the conventional Rankine cycle, but the work required to compress air in the Brayton cycle is of the order of two-thirds of the positive output, even with a reasonably efficient unit. The net output of the steam power plant is directly proportional to the efficiency of expansion in the turbine, whereas the net output of the gas turbine plant is the difference of two large quantities, and is therefore very sensitive to changes in either of them. The efficiencies of both compression and expansion must be fairly high before the gas turbine power plant can even sustain itself. Furthermore, the effect of a change in either is considerably magnified in the modified cycle efficiency.

Effect of Machine Efficiency. Assuming the same pressure ratio for both compression and expansion, the ratio of *theoretical* power required for compression to *theoretical* power available from expansion is

$$P_c'/P_t' = (T_1/T_4)(X + 1) \tag{20}$$

If the pressure and temperature ratios are 4.0 and 3.0, respectively, $X = 0.48$ and $P_c'/P_t' \approx 0.5$.

By using this figure, the gain in modified (actual) cycle efficiency by reason of an increase in compression or expansion efficiency may be calculated for an illustrative case. Assume that $P_c' = 50$ hp, $P_t' = 100$ hp, $\eta_c = 0.80$, and $\eta_t = 0.80$. The table below shows the effect of individual increase of 0.01 in η_t and η_c.

Table 2. Illustration of the Effect of Changes in η_t and η_c *

η_t	η_c	$\eta_t P_t'$, hp	P_c'/η_c, hp	Net Output, hp	Percentage Increase in Power
.80	.80	80	62.5	17.5	...
.81	.80	81	62.5	18.5	5.7
.80	.81	80	61.7	18.3	4.6

* See also Section 10.

The percentage increase in net output is very nearly equal to the percentage increase in modified cycle efficiency. An increase of 0.01 (i.e., about 1.25%) in compression or expansion efficiency is therefore magnified about four times. The magnification would be less if the initial efficiencies were higher, and more if they were less. It can be shown that in nearly all cases of practical interest, an increase of 0.01 in expansion efficiency increases the modified cycle efficiency more than an equal increase in compression efficiency, so that, broadly speaking, increases of expansion efficiency pay somewhat greater dividends than equal increases of compression efficiency. The gain is by no means twice as great, however, even when P_t' is twice as great as P_c'.

Starting. A gas turbine power plant, like a reciprocating engine, must be started by external means. It becomes self-sustaining when the shaft output of the turbine becomes equal to the shaft input to the compressor, i.e., when

$$\frac{\eta_t X_t T_4}{X_t + 1} = \frac{X_c T_1}{\eta_c} \tag{21}$$

Assume that the pressure ratios of compression and expansion are equal. Then, for a pressure ratio of 4.0, $X = 0.480$ and

$$\frac{\eta_c \eta_t T_4}{T_1} \approx 1.50 \tag{22}$$

For a temperature ratio $T_4/T_1 = 3.0$, a minimum product of efficiencies $\eta_c\eta_t = 0.50$ is necessary; or, conversely, if an efficiency product of only 0.50 is available, the temperature ratio must be at least 3 : 1. If $T_1 = 500$ R, T_4 must be at least 1500 R. It will be found that most aircraft gas turbine power plants require a turbine inlet temperature at least this high before they become self-sustaining.

Effect of Temperature on Size or Output. Quite apart from its effect on efficiency, increase of turbine inlet temperature will increase the capacity of a unit of given size and weight by increasing the net work done per unit mass of working medium; that is, it will increase the difference between the compressor and turbine shaft powers. The efficiencies of compression and expansion also affect considerably the size of unit required for a given net output, as illustrated by the following numerical example.

Assume for a flow of 1 lb/sec			Then the flow required for a net output of 100 hp is
Actual Turbine Output, hp	Actual Compressor Input, hp	Net Output, hp	Lb/sec
6	5	1	100
6	4	2	50
6	3	3	33.3

15. COMPONENTS OF TURBOJETS

INTAKE DUCT. Installation requirements determine how short and direct a duct can be used. Usually a better flow path is possible when the compressor is of the axial-flow type. The velocity of jet-propelled airplanes in normal operation is nearly always greater than the velocity desired at the compressor inlet, so that there should be provision for diffusion in the duct. The flow may be unstable if the air is drawn from a thick boundary layer, and it is safer to take all air directly from the free stream. Because of the tendency to keep the frontal area a minimum at high airplane speeds, the intake duct should be carefully checked to make sure that there is no restricted section where sonic velocity may occur, since this would limit the output of the entire power plant.

COMPRESSOR. Usually the axial-flow compressor has a higher peak efficiency than the centrifugal type, but its operating range is less. The pressure ratio per stage of the usual axial-flow type is of the order of 1.1 to 1.2, so that from 10 to 15 stages are required to obtain the same pressure ratio as in a single centrifugal stage. By special designs, not yet perfected, the axial flow type ultimately may equal the centrifugal type in its pressure rise per stage. Although the increase of weight is not in the same proportion, normally the turbine would also be larger because of the lower speed, and the total weight of the power plant may be several hundred pounds greater than that of a plant using a centrifugal compressor.

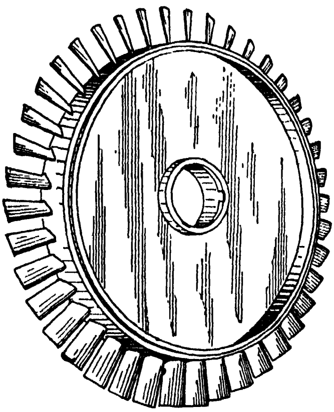

FIG. 9. Rotor disk and blades of an axial-flow compressor.

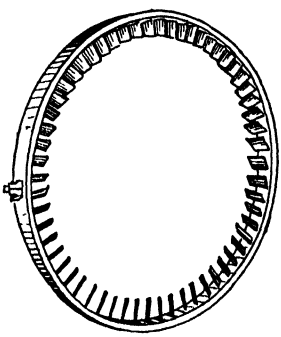

FIG. 10. Stator ring and blades of an axial-flow compressor.

A typical double-inlet centrifugal impeller is shown in Fig. 1, p. 15-40. Use of an inlet on each side increases the capacity for a given outer diameter, and also nearly eliminates impeller thrust, but has the disadvantage that it may be difficult to conduct air to the rear inlet without loss of ram pressure or increase of frontal area. The impeller is usually of duralumin, an aluminum alloy containing about 4 to 5% copper. If the temperature is likely to exceed 300 F, however, stainless steel is used.

The rotor and stator blades of an axial-flow compressor are shown in Figs. 9, 10, and 11. They may be cast, forged, or machined, but the tolerances must be closely held to insure adequate performance and life. The blades are not usually subject to exceptionally high centrifugal stresses, but may fail from vibration unless carefully designed. Figures 2, 4, and 5 (pp. 15-41 and 15-42) show the axial-flow compressor used as part of a power plant.

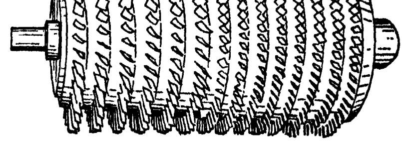

FIG. 11. Rotor of a typical axial-flow compressor.

An axial-flow rotor may be of *drum* or *disk* construction. The material may be either duralumin or a magnesium alloy, the latter being lighter, but more likely to distort. However, when the temperature is high, as it frequently is in the last few stages, steel is used.

Frequently the compressor blades are of steel in all stages, although duralumin may be used if the temperature is low and the fatigue strength is sufficiently high. The casing is usually made of an aluminum or magnesium alloy.

COMBUSTION CHAMBER. A typical *can-type* combustion chamber is shown in Fig. 12. It consists essentially of an outer shell and an inner shell, the latter often being called *flame tube* or *liner*. Air from the compressor flows through the annular space, thus cooling the inner shell, and then enters the inner chamber through a number of holes. The location and size of these holes greatly affect the chamber performance, and must be determined by careful experiment. Fuel is injected at one end of the chamber, called the combustion zone, where the amount of air admitted is approximately correct for complete combustion. Ordinarily a spark plug is used for ignition, but thereafter combustion is intended to be self-sustaining. The products of combustion mix with the rest of the air entering the inner chamber through the remaining holes, and the temperature of the mixture is reduced to a value safe for the turbine.

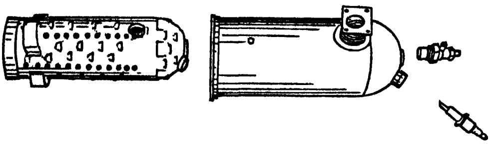

FIG. 12. Can-type combustion chamber, showing inner liner, outer casing, fuel nozzle, and spark plug.

It is particularly important to obtain a *uniform* temperature profile of the mixture leaving the chamber. Otherwise certain parts of the turbine will be subjected to excessively high temperatures, or the output will be limited by a high temperature at one point. Carbonization on the surface of the metal must be avoided, as it results in hot spots, distortion, and eventual failure. In a well-designed chamber, the air flow pattern insures sweeping away of carbon before it can form a large deposit.

The inner shell is exposed to high temperatures and high temperature gradients, and probably requires replacement more frequently than any other part of the present-day aircraft gas turbine power plant. It is usually made of Inconel. Possibly a suitable method of coating Inconel or some other high-temperature material with a ceramic will permit a much longer life.

TURBINE. The turbine usually has only a single stage. Somewhat higher efficiency might be obtained with more stages, but the gain would be small at the pressure ratios now customary, and the difficulties of keeping the turbine metal temperatures low enough for safety and long life would be greatly accentuated. Every effort is made to keep the turbine size a minimum, but the relative velocities in the rotor passages should not approach sonic values.

The rotor blades may have an integral shroud at the outer end, but frequently this is not used because of manufacturing difficulties and the extra centrifugal stress introduced. Thus there may be a loss of efficiency from leakage of the gas through the open space, as well as a lower natural frequency of vibration of the blades due to lack of support at the outer end. The leakage can be minimized by use of a stationary shroud (cooled if possible); the danger of failure from vibration can be greatly reduced by correctly shaping the blades and using suitable material.

The turbine disk is usually made of Timken 16-25-6 in the United States, although a composite disk of two different alloys may be preferred. Stayblade, Rex 78, and G18B have been used in England. Turbine buckets have been of Vitallium (cast), Hastelloy B, and S-816 in the United States, and of Rex 78 and Nimonic 80 in England. Austenitic steels are preferred to ferritic steels. The turbine casing may be of 347 stainless steel, Ka2SMo, 19-9 W-Mo, or other alloy. Alloy 25-20 in cast form has been used for diaphragm nozzle blades.

EXHAUST DUCT AND PROPULSION NOZZLE. It is not satisfactory to discharge the gases directly from the turbine to the atmosphere because (1) the gases probably have some rotary motion which should first be converted into motion in the line of flight; (2) ordinarily the turbine location is such that direct discharge is impracticable or dangerous; and (3) the velocity of the gas leaving the turbine must be limited for reasons of turbine efficiency, and it is necessary to discharge the gases from the airplane at a considerably higher velocity in order to obtain a reasonable thrust without an excessive mass flow.

An exhaust duct (usually called tailpipe) and nozzle are required to collect the gas and discharge it axially at a uniform velocity. An exhaust cone (or tailcone) covering the turbine disk is required both to protect the turbine disk from the hot gases and to avoid a sudden increase of area. The cone should be designed to provide a smooth transition from the turbine annulus to the duct, and frequently the flow passage area increases to permit some diffusion. The higher the static pressure from diffusion, the greater the efficiency of thrust augmentation when fuel is burned in the duct, and the less the friction losses.

However, in order to avoid too large and heavy a duct, as well as to minimize the possibility of distortion from large pressure differences, a fairly high velocity should be maintained. Such sheet metal parts are usually of 347 stainless steel or an alloy of similar properties.

A variable-area propulsion nozzle is always desirable from a performance standpoint, and becomes practically indispensable if extra fuel is burned in the exhaust duct for *thrust augmentation.* This feature, valuable for bursts of power over short periods, is known as *exhaust reheat* or *afterburning.* Such burning with a fixed nozzle area would result in excessive temperatures; the area must be increased to obtain a large increase of thrust with allowable temperatures. The area may be varied by the axial movement of a cone concentric with the nozzle opening, or by other means. The problem of designing such a nozzle is largely a mechanical one; free movement and reliability are required in spite of the effects of the high temperature gases. Preferably the movement should be automatically controlled most of the time by a device responsive either to gas temperature or to the rotary speed of the unit.

HEAT EXCHANGER. For maximum economy, especially in a propjet, the power plant should include a heat exchanger to recover heat from the exhaust gases by preheating the air entering the combustion chamber. Because of the size and weight of such an exchanger, it is seldom used. The Bristol Theseus 21 (British) power plant is a notable exception.

16. DESIGN AND PERFORMANCE CALCULATIONS

The aerodynamic and thermodynamic design of a conventional aircraft gas-turbine power plant, whether of the jet or propeller type, reduces in large part to the problem of designing the most efficient compressor and turbine for a specified capacity. Only certain aspects peculiar to aircraft gas turbines will be considered here.

CALCULATION OF THRUST. For design purposes it is usually assumed that the pressure over the external walls of a jet propulsion power plant is constant. In this case the net thrust acting on the unit is equal in magnitude to the force acting on the fluid passing through the unit, and is given by an equation of the same type as that used to find thrust (eq. 1*b*). Using the subscripts 1 and 2 to indicate inlet and exit, respectively, the net thrust is

$$F = \frac{w}{g}(V_2 - V_1) \tag{23}$$

where V_1 and V_2 are *relative* velocities in the direction of flight.

Frequently, the intake duct, especially when a double-inlet centrifugal impeller is used, has no projected area perpendicular to the direction of flight, so that $V_1 = 0$ whether the unit is in flight or stationary. However, if it is in flight, there is a retarding thrust exerted somewhere on the airplane equal to $(w/g)V_p$, and it is customary to calculate the net thrust from eq. 23 with $V_1 = V_p$ in all cases, regardless of how much of the retarding thrust actually acts on the unit itself. Thus the calculated net thrust in flight must be considered as a conventional value of significance only when the power plant and airplane are considered to be a single unit. There is an optimum entrance area of the intake duct for every combination of altitude, airplane speed, and rotor speed, but a variable area is impracticable. The area should be too large rather than too small, since free diffusion is more efficient than forced acceleration.

If the pressure ratio across the propulsion nozzle is greater than the critical ratio, so that a converging-diverging nozzle is theoretically required, but the nozzle actually used has only a converging section, the pressure at the exit opening will be higher than the pressure of the surrounding atmosphere. However, the fluid will expand in the atmosphere to a supersonic velocity, and there will be a corresponding reactive propulsive force, transmitted through the atmosphere and exerted on the power plant or adjacent aircraft structure. Thus V_2 should be taken as the velocity corresponding to complete expansion down to atmospheric pressure, regardless of whether the pressure ratio is subcritical or supercritical. If the pressure ratio is very much above critical, some allowance should be made for the greater loss in the free expansion process.

TURBINE. If a propeller is used, the turbine design will closely resemble that of a conventional steam turbine. Since most of the output is in the form of shaft power, a relatively large deflection of the gas must occur in the rotor buckets. In a pure jet propulsion unit, however, the deflection is only sufficient to provide the shaft power required to drive the compressor. The greater the deflection, the greater the friction and turning losses, in general, so that the nearly straight-through flow of the jet propulsion turbine is conducive to less internal loss.

Efficiency. In the ordinary stationary turbine, the kinetic energy at exit of the last stage is one of the most important losses. In aircraft gas turbines, exhaust energy can be utilized for propulsive purposes if the velocity is properly directed; it is, in fact, the entire net output of a pure jet plant, and may be an important fraction of the total output of a turbine-propeller plant. If the turbine is credited with this kinetic energy, its efficiency is considerably higher than that of a stationary turbine of the same capacity; frequently it is more than 90%. How efficiently this energy can be utilized depends on the airplane velocity.

The efficiency of a turbine in which the exit velocity head is not considered to be a loss may be defined in either of two ways:

$$\eta_t = \frac{P_s + P_j}{P_t'} \tag{24a}$$

or

$$\eta_t = \frac{P_s}{P_t' - P_j} \tag{24b}$$

in which the shaft and jet components of the power output are compared with the ideal turbine power, P_t'.

There is no general agreement as to the use of these formulas, but the first is probably used more frequently for a pure jet propulsion power plant, and the second chiefly for a turbine-propeller power plant.

Pressure Ratio. Pressure ratios vary from about 4:1 to 10:1 at maximum speed. A high pressure is desirable to obtain the maximum net output from a machine of given size, but it is difficult to maintain high efficiencies with very high pressure ratios even when the total expansion is divided into a number of stages. If only a single stage is used with a pressure ratio much above critical, converging-diverging nozzles are necessary for good efficiency, and such nozzles are very restricted in the range of pressure ratio over which they can operate efficiently. A nozzle designed for a pressure ratio of 7 is quite inefficient at ratios of 1.5 or 2, but since it is necessary to pass through these low ratios during starting, excessive external starting power may be required. High efficiency over a wider range may be obtained with a multistage turbine, but the temperature of the first stage moving blades will be higher because of the lower pressure drop, and the mechanical difficulties of cooling will be greater. Up to the present time the disadvantages of multistage turbines, including increased weight and length, have usually been considered to outweigh their advantages.

VORTEX DESIGN. (See also Section 1.) The *solid flow theory* widely used in the design of steam turbines assumes that a single diagram is applicable at all radii, so that the stator and rotor blade angles are constant. Sometimes this theory is modified to the extent that the actual linear velocity of the blade at each radius is used, resulting in different diagrams at different radii, and consequently in varying blade angle.

Many aircraft gas turbines (and recently several steam turbines), however, have been designed in accordance with *vortex flow theory*. This theory assumes that the gas is a continuous sheet of nonviscous fluid to which there has been imparted a constant velocity in an axial direction combined with a rotation about the axis. The fluid is confined within a cylindrical surface concentric with the axis, so that the component of its motion in any plane perpendicular to the axis must be circular, but it is otherwise free to follow its natural path. Under these conditions the only forces acting are pressure forces, and a radial pressure gradient is established of sufficient magnitude to provide the radial acceleration required to maintain the circular component of motion.

Condition of the Free Vortex. It can be shown that for such conditions to hold

$$rV_u = \text{constant} \tag{25}$$

where r is the radial distance from the shaft center line, and V_u is the tangential component of the fluid absolute velocity. This is the equation of a vortex. The greater the radius, the less the tangential component of fluid velocity, an inverse proportionality. The blades are then designed to suit this natural flow at all radii, the angles being such that the direction of the blade coincides with the natural direction of the fluid. This requires a variable blade angle and a twisted blade shape.

With such a flow, the amount of energy transferred from the gas to the rotor blades per unit mass of fluid is the same at all radii. So far as possible, the angular momentum is made zero at all radii at the exit of the last row of moving blades (axial exit velocity), since it is difficult to utilize efficiently the kinetic energy in a rotary component of motion. However, there is usually some unavoidable whirl as the gas leaves the turbine, and straighteners may be required.

Reaction in Vortex Design. A design based on vortex flow theory has variable reaction; that is, the energy release occurring in the rotor blades becomes a larger fraction of the total stage energy as the radius increases. Frequently the design is based on zero reaction (pure impulse) at the root of the blade. However, it is difficult to insure that the computed degree of reaction will actually be obtained.

If in a vortex-flow design the pressure and density along the blade length are assumed to vary in accordance with the law

$$\frac{p}{\rho^n} = \frac{p_0}{\rho_0^n}$$

where the subscript 0 indicates conditions at the inner radius, the pressure at any radius is given by the equation

$$\frac{p}{p_0} = \left[1 + \left(1 - \frac{1}{n}\right)\left(\frac{\rho_0 r_0^2 V_{0u}^2}{2gp_0}\right)\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right)\right]^{n/(n-1)} \tag{26}$$

and the density by

$$\frac{\rho}{\rho_0} = \left[1 + \left(1 - \frac{1}{n}\right)\left(\frac{\rho_0 r_0^2 V_{0u}^2}{2gp_0}\right)\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right)\right]^{1/(n-1)} \tag{27}$$

Some designs, especially in Germany, have been based on 50% reaction (symmetrical velocity diagrams) at all radii. Two other conditions must be specified to fix the velocity diagrams; these may be taken as equal transfer of energy per unit mass of fluid at all radii, and constant axial velocity component. Sometimes the *half-vortex theory* is used, the tangent of the blade angle at any radius being taken as the mean of the tangents of the angles given by the vortex flow and by the symmetrical diagram methods. A number of other theories have also been used or proposed.

Although the vortex theory appears superficially to conform to the physical facts more closely than most others, it does not take account of viscosity, leakage, boundary layer effects, other end effects, or the influence of wakes from preceding stages. It is difficult to get accurate correlation between the theory on which a design is based and test performance, but present indications are that the differences between designs based on the solid flow, vortex, and symmetrical velocity diagram methods are not great enough to be detectable without very accurate testing. Some authorities believe that any of these theories will give about the same result if the design is such as to avoid separation of flow from the blades.

COMPRESSORS used in aircraft gas turbine power plants are distinguished from compressors for industrial applications chiefly by the following features.

(1) The volume flow is almost directly proportional to the speed at all operating conditions, so that the normal operating point can be fairly close to peak efficiency without risk of instability. However, if the design unduly favors the normal operating speed, there may be instability at starting speeds that would require the use of "blow-off" valves to waste air. In such a case it is usually preferable to redesign the compressor.

(2) Although the total pressure ratio of the power plant increases with airplane speed because of the increase of ram pressure, the pressure ratio across the compressor itself remains approximately constant at a given rotary speed. The pressure ratio across the turbine usually varies as the ram varies, and the power plant design may require some compromise for this reason, especially for propjets.

(3) If the power plant is used only for jet propulsion, so that the amount of shaft power required is small (auxiliaries only), a centrifugal compressor may be efficient enough for the purpose, even though centrifugal compressors are seldom considered efficient enough for use in industrial-type gas turbines used as prime movers.

COMBUSTION CHAMBER. At the present time the design of a combustion chamber is almost entirely empirical. Largely for this reason, the use of a number of small separate chambers rather than a single annular chamber is favored. It is necessary in experimental development to make tests with full air and fuel flow through the chamber if normal pressure and velocity patterns are to be obtained; the required capacity of the test equipment is very much less if small individual chambers are used. However, an annular-type chamber ordinarily has less outer diameter, and is sometimes used.

A certain pressure drop is necessary in the combustion chamber to insure good combustion. This is usually about 3 to 6% of the compressor discharge absolute pressure.

TAILPIPE. Losses in both tailpipe and propulsion nozzle are magnified in the net output and specific fuel consumption, so that a reasonably large pipe is justified. The total pressure of the gas in the pipe must be that required to give the desired jet velocity, but this may be divided between static pressure and velocity head in various proportions.

From the standpoint of friction loss alone, the velocity head should be low and the static pressure high. However, the friction loss falls off approximately as the fifth power of the diameter of a round pipe, so that a point is soon reached at which further increase of diameter results in such rapidly diminishing returns that the greater size and weight cannot be justified on this ground alone. It may be justified if the thrust is to be augmented by burning fuel in the tailpipe, since the ease of accomplishment of this process increases with increasing static pressure.

There is no fixed relationship between the velocity of gas in the tailpipe and the absolute velocity of discharge from the turbine, since the latter may be converted to the former by the necessary diffusion or acceleration. However, in the absence of conflicting reasons, it is logical to design the turbine for an absolute discharge velocity approximately equal to the desired duct velocity. In most practical cases other considerations govern the design, primarily the following.

(1) The absolute discharge velocity should be low enough so that the gas velocity relative to the buckets is subsonic.

(2) The velocity should not be chosen much lower than that necessary to insure a subsonic relative value if doing so necessitates a larger and heavier turbine with highly stressed buckets.

(3) If the turbine diameter is already large for other reasons, and if the bucket stresses are relatively low, the discharge velocity may be low. In particular, the diameter of a turbine with an axial flow compressor may be greater than that of a comparable unit with a centrifugal compressor because of a lower rotary speed, and its buckets may be less highly stressed, so that a lower discharge velocity may be possible.

In some designs it may be important to keep the static pressure near the turbine low, to minimize distortion and insure the lowest possible temperature of gas in contact with the turbine. Values of the absolute discharge velocity range in practice from 600 to 1300 ft per sec.

The function of the tailcone is partly to protect the turbine disk from the hot gases, and partly to provide a smooth, continuous transition from the turbine discharge annulus to the tailpipe. Diffusion may or may not occur in this section.

PROPULSION NOZZLE. If a constant-area propulsion nozzle is used, it may be of a very simple type, consisting of hardly more than a conical taper at the end of the pipe. Sometimes the entire pipe tapers down to the nozzle diameter with a very short parallel section at the end to increase stiffness.

The area of the propulsion nozzle has a very marked effect on the performance of the power plant. The optimum area will depend not only on the particular power plant and on the operating conditions, but also on the characteristics of the installation. If the length of tailpipe used in a certain airplane is not the same as the length used in factory test, it may be that a different nozzle area is preferable for the airplane. In general, other elements of the power plant being fixed, the area should be as small as possible without exceeding allowable gas temperatures.

Decrease of nozzle area at constant rotor speed increases thrust, gas temperature, and, beyond a certain point, specific fuel consumption. The area should be small enough so that the high thrust corresponding to full allowable gas temperature is obtained at rated speed, even though the specific fuel consumption is somewhat high. This is essential to obtain maximum thrust for take-off and climb, as well as emergencies. During normal cruising a lower speed may be used, with lower gas temperatures and better fuel economy.

From the performance standpoint, it is still better to use a nozzle of variable area, which can be adjusted both to the particular installation and to the particular operating conditions. Variable nozzles have not been used much in the past because of the extra weight and mechanical complication involved, but they are essential when thrust is augmented by burning additional fuel in the tailpipe.

CALCULATION DETAILS. A design calculation of a gas-turbine power plant is more difficult than that of a steam power plant because the useful output is a relatively small difference of two large numbers, and errors in the estimated efficiencies of the components are magnified in the result. Moreover, a low estimate of efficiency is not necessarily conservative, but may lead to a design which is as unsuitable as if the efficiencies had been overestimated, since a departure from design conditions at one point affects conditions at all points. It is particularly difficult to predict accurately the overall performance in flight of a new design, since intake and exhaust duct losses vary with the installation.

Component efficiencies as defined below are useful in analyzing the performance. Other definitions may be used, and sometimes it is preferable to deal directly with the losses expressed in terms of pressure drop, velocity head, or dynamic pressure. Approximate values of the efficiencies are given in Table 3.

Table 3. Approximate Values of Component Efficiencies

Component	Efficiency, %
Intake duct (including diffuser, if used)	75–90
Compressor	
Centrifugal	72–80
Axial flow	80–88
Combustion chamber	
Aerodynamic efficiency	90–98
Combustion efficiency	94–98
Turbine (axial flow)	
Shaft	60–80
Shaft and jet	80–94
Tailpipe and propulsion nozzle	75–90

Intake duct (or ram) efficiency: fraction of theoretical ram energy available at compressor inlet.

Compressor efficiency: theoretical power required for isentropic compression divided by actual power required.

Combustion chamber aerodynamic efficiency: total pressure head of gas leaving combustion chamber divided by total pressure head of air entering.

Combustion chamber combustion efficiency: heat actually released by combustion divided by lower heating value of fuel.

Turbine shaft efficiency: turbine shaft power output divided by theoretical output corresponding to isentropic expansion from initial total pressure and temperature to static pressure at turbine exit.

Turbine shaft and jet efficiency: Two definitions are in use. (1) Total output (shaft power plus power represented by kinetic energy of discharge gases) divided by theoretical power corresponding to isentropic expansion from initial total pressure and temperature to static pressure at turbine exit. (2) Shaft power output divided by theoretical power corresponding to isentropic expansion from initial total pressure and temperature to total pressure at turbine exit. These definitions correspond to eqs. 24*a* and 24*b*, respectively.

Tailpipe and propulsion nozzle efficiency: fraction of total energy available at turbine exit (velocity head plus pressure head above atmosphere) which appears as kinetic energy in jet.

Methods of calculation are largely conventional once the efficiencies have been estimated. The compressor power input required is given by eq. 15. For a pure jet plant, the shaft output of the turbine must be just equal to the compressor input, and the blade angles are designed for only a relatively small gas deflection. If the turbine is to drive a propeller, the deflection must be correspondingly increased, and the turbine shaft efficiency, as defined above, will normally be higher because of the lower discharge velocity.

In designing turbine blades, velocity diagrams should be constructed at enough different radii, in accordance with the particular design theory used, to determine completely the shape of both rotor and stator blades. Even more important than the angles are the areas perpendicular to the direction of flow, which must be checked to insure their correctness for the corresponding gas velocities. Because of the finite thickness of the walls between passages, it will usually be necessary to increase somewhat the theoretical angles (measured from the tangential direction) in order to obtain the required areas.

Since experimentally determined coefficients and efficiencies must always be used, no great error is introduced, at least for design purposes, if it is assumed that the gas in the combustion chamber and turbine is air at a high temperature. Even constant specific heat might be assumed if consistency is observed, i.e., if the coefficients and efficiencies from previous tests are based upon such an assumption, and are then applied to new designs on the basis of the same assumption. Efficiencies so obtained are not directly comparable to steam turbine efficiencies.

The actual variation of the specific heat of air at high temperatures is still somewhat in question, but values determined from spectroscopic data are usually considered the most reliable. Keenan and Kaye's *Gas Tables* (Wiley) tabulates various thermodynamic quantities based on the variation of specific heat with temperature (see also p. 1-04) as given by Heck, *Mechanical Engineering*, Vol. 63, pp. 126–135 (1941).

The following published charts give values of various thermodynamic quantities for a mixture of gases representative of products of combustion.

(1) *Thermodynamic Charts for Combustion Processes*, Parts I and II, by Hottel, Williams, and Satterfield, Wiley, 1949.

(2) Charts of Thermodynamic Properties of Fluids Encountered in Calculations of Internal Combustion Engine Cycles, by Hottel and Williams, *Natl. Advisory Comm. Aeronaut. Tech. Note* 1026, May, 1946.

(3) Thermodynamic Charts for the Computation of Combustion and Mixture Temperatures at Constant Pressure, by Turner and Lord, *Natl. Advisory Comm. Aeronaut. Tech. Note* 1086, June, 1946.
(4) Enthalpy–Entropy Diagram, by Reindorf, *Technical Report* F-TR-1160-ND GS-USAF-Wright Field No. 60.
(5) Performance Charts for the Turbojet Engine, by Pinkel and Karp, *Fairchild Publication Fund Paper* 103, Institute of the Aeronautical Sciences.
(See also Section 2, Art. 28.)

17. TESTING OF AIRCRAFT GAS TURBINE POWER PLANTS

Tests of aircraft gas turbine power plants fall into three categories, between which, however, there is no sharp line of distinction: (1) factory research and development; (2) flight research and development; (3) factory production. To a large extent the testing technique is the same as for stationary gas turbines and similar apparatus. The following remarks are intended to point out some of the special features peculiar to aircraft power plants.

FACTORY RESEARCH AND DEVELOPMENT TESTS. Measurement of Output. Since the useful output of a pure jet propulsion power plant is represented by thrust rather than by shaft power, it is necessary in factory test to mount the unit in such a way that the thrust can be measured directly. This can be accomplished by suspending it by long, flexible supports, or by mounting it on ball or roller bearings which permit a slight movement. The thrust is indicated either by a calibrated gage measuring the pressure of oil acting on a piston opposing the motion, or by a bell crank and linkage indicating the thrust directly on a platform scales. Other methods have been proposed, but have not been widely used as yet.

The output of a gas-turbine power plant designed primarily to drive a propeller is most logically determined by a dynamometer. For approximate checks a calibrated propeller or the equivalent might be used. In such cases the additional thrust from the exhaust gases can usually be calculated with sufficient accuracy from test readings of pressure and temperature.

Test Setup. Usually a pure jet propulsion unit is mounted inside a test cell which is airtight except for one or more entrance nozzles or orifices to measure flow. Since all air used by the unit on test is drawn directly from the cell, the flow into the cell is equal to the flow through the unit. The pressure in the cell will be approximately atmospheric, so that this condition of operation corresponds nearly to zero airplane speed at sea level. Sometimes a more complicated arrangement is used whereby the air is supplied to the compressor inlet through a duct at a high velocity corresponding to some airplane speed.

The gases from the propulsion nozzle are usually discharged into a stack large enough at the base to correspond approximately to free discharge into atmosphere. If the location permits, no stack is necessary. The point where the tailpipe passes through the cell wall should be properly sealed, allowing sufficient flexibility for a slight motion. Since there is no air flow over the unit as in normal flight, it is frequently desirable to insert a partition between compressor and turbine to insure that the compressor does not draw in recirculated

A propjet unit may also be set up in such a cell, but since its air consumption is less than that of a comparable turbojet, it may be more convenient to supply the air through an intake duct. This makes it possible to test with different ram pressure ratios as well as to measure flow. If a propeller is used, the unit may be set up in an open cell or out-of-doors.

Corrections for Nonuniform Pressure Distribution on Outer Surface. It should be noted that the thrust is theoretically given by eq. 23 only if the pressure is everywhere constant over the outer surface of the unit under test, as well as at the inlet and exit openings. If the rated thrust is based upon this condition, and if for any reason (such as the use of partitions or seals) an abnormal pressure distribution is created, a correction to the measured thrust may be required. However, the variation of pressure which necessarily accompanies free diffusion or free expansion does not require any correction, since this merely compensates for the pressure and velocity changes at the inlet and exit openings.

The general formula for the net thrust is

$$F = \frac{w}{g}(V_2 - V_1) + p_2A_2 - p_1A_1 + \int_{Ext} p\, dA \tag{28}$$

where the last term is the integral of the pressure over the external surface. When the thrust at the reference conditions can be considered to be given by the test value of $(w/g)(V_2 - V_1)$ (in general, this would be when p_1 and p_2 do not differ very much from p_0), the correction to be applied is

$$\Delta F = p_1 A_1 - p_2 A_2 - \int_{Ext} p\, dA \tag{29}$$

The integrated pressure force $\int_{Ext} p\, dA$ is to be taken as positive if it is directed forward (i.e., in the direction of the resultant thrust), negative if it is directed backward. Usually this integral can be only approximately determined as the algebraic sum of two or more terms, each equal to the pressure measured in a certain region adjacent to the surface multiplied by the projected area on which it acts.

In other cases the theoretical correction is much more complicated, and perhaps will be impossible of accurate determination. Sometimes the measured or estimated excess or deficiency of external pressure, multiplied by the projected area affected, may be taken as the correction, to be added or subtracted in accordance with the particular distribution. No general rule can be given because of the variety of setups used.

FLIGHT RESEARCH AND DEVELOPMENT TESTS. Attempts are sometimes made to measure thrust directly by attaching the power plant to the airplane structure through a device which indicates thrust, as in factory test. Although it is possible to do this, the reading must be interpreted with care. Part of the resisting thrust associated with taking air on board may be exerted against other parts of the aircraft structure, and the pressure distribution on the external surfaces of the unit is very likely to be nonuniform. Thus the measured thrust will probably not be directly comparable with factory tests, and a conclusion as to the general efficiency of the installation cannot be drawn without considering the relative drag on the adjacent aircraft structure. Circumstances must decide whether an unfavorable pressure distribution should be charged to the installation, or to an excessive frontal area or poor streamlining of the unit itself, but no gain in measured thrust made by a modification of the installation will be of any avail if an equal or greater retarding thrust is thereby introduced elsewhere.

Sometimes an attempt is made to determine thrust in flight by measuring total pressure and temperature in the tailpipe. The static pressure at the nozzle exit is also measured, or is assumed equal to atmospheric pressure. These data, together with the nozzle area, are sufficient to determine the mass flow and discharge velocity, and hence the gross thrust. Usually a calibration factor is determined by taking the same measurements with the same nozzle during factory test. This method, of course, does not account for external pressure distribution. Its principal value seems to be in the correlation of mass flow in factory test with mass flow in flight at a given rotary speed, and thus in indicating possible restrictions in the intake system.

It is frequently convenient to make flight tests on a "flying test bed," or an airplane propelled by other power plants of sufficient capacity to provide the desired variation of altitude and airplane velocity. The unit under test can then be independently operated over its entire range.

Much flight testing is not for the purpose of determining performance, but to prove mechanical operation and endurance of parts under high altitude conditions. Flight tests of turbine-propeller and reciprocating engine-propeller power plants are of the same general nature; in either case the output may be measured in flight with a hub dynamometer or the equivalent.

FACTORY PRODUCTION TESTS are ordinarily made to verify guaranteed values of thrust, specific fuel consumption, and exhaust temperature, as well as to check mechanical operation. In order to obtain consistent results, the setup should be the same for each unit of a given type, including tailpipe and propulsion nozzle. It is desirable to make such tests with the same discharge pipe and nozzle used in service. Since the same type of unit may be used in several different types of airplanes, however, this is frequently not practicable. It should be borne in mind, therefore, that the values determined in factory test will perhaps differ considerably from those in actual installations.

Because of the extreme sensitivity of gas turbine power plants to small changes in dimensions of the flow path, and the large number of tolerances required in manufacture, there is likely to be a relatively wide variation of performance in a series of production units. One of the most critical items is the area of the turbine diaphragm nozzles, usually difficult to manufacture to close tolerances, and difficult to check. The inlet area of a centrifugal compressor diffuser is also rather critical. In many cases units that do not meet guaranteed values in one respect, but are better than required in another respect, can be made to meet all requirements by a change of either diaphragm nozzles or diffuser. These are normally separate parts, and hence can be easily changed. Sometimes it is possible to accomplish the same result by changing the propulsion nozzle.

The mutual effects of changes made anywhere in the system make it very difficult to predict with certainty the effect of any given change. However, if a unit has both low thrust and low temperature at the specified rotary speed, it is possible that the required performance can be obtained by using a propulsion nozzle of slightly less area. The effect of increasing turbine diaphragm nozzle area is frequently to increase thrust, temperature, and specific fuel consumption at all speeds. If the specific fuel consumption is high at low speeds, it may be possible to correct this without appreciably affecting full speed performance by decreasing the nozzle diaphragm area or, if a centrifugal compressor is used, by increasing the diffuser inlet area.

18. PRESENTATION AND CORRECTION OF PERFORMANCE DATA

CRITERIA OF DYNAMIC SIMILARITY. It is shown by the theory of dynamic similarity that, for nearly all practical purposes, the performance of a gas turbine power plant is a function of only two combinations of dimensionless variables. Each may be expressed in various ways, alternative forms being:

$$\textit{First criterion: } \frac{ND}{\sqrt{RT}}, \frac{p'}{p}, \frac{T'}{T}, \frac{\rho'}{\rho}$$

$$\textit{Second criterion: } \frac{q}{D^2\sqrt{RT}}, \frac{w_c\sqrt{RT}}{pD^2}, \frac{F}{pD^2}, \frac{w_f}{pD^2\sqrt{RT}}$$

(See Nomenclature, p. 15-37.)

D and R may be omitted when constant. Usually p, T, ρ and q are taken as compressor inlet (or atmospheric) values, but they may be taken elsewhere, if consistency is observed. It will be noted that each form of the second criterion is characterized by one physical quantity (q, w_c, F, or w_f) which occurs in that form only, and the form used in any particular case would ordinarily depend chiefly on which of these quantities happens to be of principal interest.

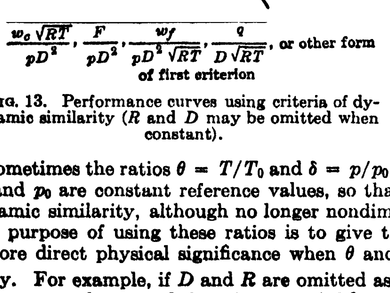

FIG. 13. Performance curves using criteria of dynamic similarity (R and D may be omitted when constant).

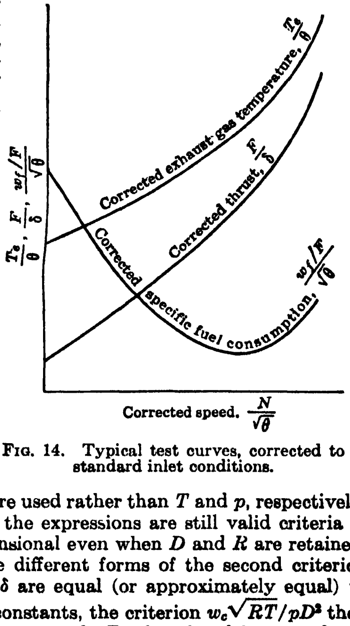

FIG. 14. Typical test curves, corrected to standard inlet conditions.

Sometimes the ratios $\theta = T/T_0$ and $\delta = p/p_0$ are used rather than T and p, respectively. T_0 and p_0 are constant reference values, so that the expressions are still valid criteria of dynamic similarity, although no longer nondimensional even when D and R are retained. The purpose of using these ratios is to give the different forms of the second criterion a more direct physical significance when θ and δ are equal (or approximately equal) to unity. For example, if D and R are omitted as constants, the criterion $w_c\sqrt{RT}/pD^2$ then becomes merely w_c, and the criterion F/pD^2 becomes merely F, when θ and δ are used and are equal to one.

Efficiency (as expressed by any one of the large number of measures of performance that may be defined) and the two criteria constitute a group of variables that may be plotted as abscissa, ordinate, and parameter in six different ways. Frequently both the first two forms of the first criterion are plotted. Although these forms are not independent, the calculation of one from the other is rather tedious, and the effect of plotting both curves is to make the calculation once and for all. The number of possible combinations of abscissa, ordinate, and efficiency is then 24. Of course, various forms of the second criterion may also be plotted on one chart. Representative plots based on the theory of dynamic similarity are shown in Figs. 13 and 14.

COMMERCIAL AND APPLICATION CURVES. Various modifications of the plots based on dynamic similarity will give more directly the physical quantities of interest, although some generality must be sacrificed. Such plots are obtained by assuming constant values of one or more of the factors in the criteria. Plots of this type are shown in Fig. 15.

CORRECTION OF TEST RESULTS. An aircraft gas turbine power plant is usually guaranteed to give a certain performance at standard sea level atmospheric or other specified conditions. It is therefore necessary to correct measured test values to these conditions. The required correction factors are obtained from the theory of dynamic similarity. Usually speed, thrust, specific fuel consumption, and temperature of the gas in the tailpipe are the quantities of chief interest, but certain other quantities are also included in Table 4.

Correction factors for products or quotients of the quantities in the left-hand column are obtained by multiplication or division of the respective correction factors.

The correction factor is the multiplier that converts the test value of the quantity in the first column to the corrected value at the specified reference conditions p_0, T_0, and R_0. Test values are indicated by p_t, T_t, and R_t. The factors are based on an ideal gas, and since only a single machine is in question, no size factor is included. Usually the gas constant R can also be considered as constant and omitted, and it is so taken in the column of special correction factors. However, R should theoretically be included if it is necessary to correct from test humidity to a standard humidity.

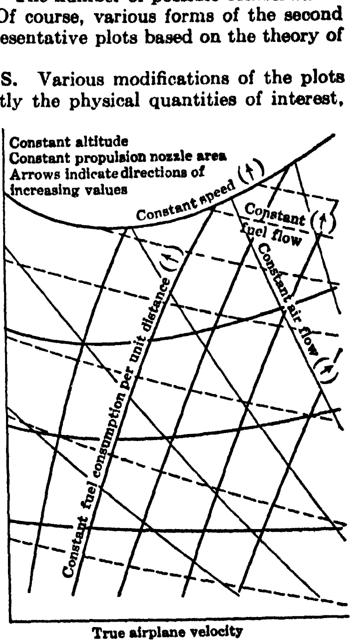

FIG. 15. Typical performance chart for jet-propulsion power plant.

Table 4. Correction Factors

Quantity Corrected	General Correction Factor	Special Correction Factor (R = Constant)
F	p_0/p_t	$1/\delta$
w_f	$(p_0/p_t)(\sqrt{R_0T_0}/\sqrt{R_tT_t})$	$1/(\delta\sqrt{\theta})$
T_e	T_0/T_t	$1/\theta$
N	$\sqrt{R_0T_0}/\sqrt{R_tT_t}$	$1/\sqrt{\theta}$
w_a	$(p_0/p_t)(\sqrt{R_tT_t}/\sqrt{R_0T_0})$	$\sqrt{\theta}/\delta$
q	$\sqrt{R_0T_0}/\sqrt{R_tT_t}$	$1/\sqrt{\theta}$

19. INSTALLATION AND OPERATION

LOCATION IN AIRPLANE. As in all aircraft installations, the propulsion unit must be located primarily with a view to good weight distribution. If a double-inlet centrifugal compressor is used, it should preferably be located in a plenum chamber, with sufficient space between the outer diameter of the casing and the aircraft structure to allow passage of air into the chamber feeding the rear inlet. However, a small difference of inlet pressure on the two sides will make no great difference. Usually even the front side of the impeller is not open directly to the slip stream, but must take its air from the side because of the barrier formed by a front bearing support and accessories mounted thereon.

MOUNTING. A unit is preferably supported at three points, the minimum number required for security. The use of more supports than are actually needed will increase the chances of damage from unforeseen strains. In most cases two of the supports will be near the center of gravity, and will take most of the load, while the third will steady the unit. Thermal expansions and contractions, as well as unavoidable deflections of the aircraft structure, make it necessary to provide considerable flexibility in the mounting.

The supports must be strong enough to transmit to the aircraft structure both the full thrust developed by the unit and the gyroscopic forces resulting from changes in the direction of flight. The power plant itself is usually designed to withstand gyroscopic forces occurring at full rotor speed with rates of precession of approximately 3 to 6 radians per second, or to withstand certain specified combinations of gyroscopic force and other kinds of loading.

FUELS. Both kerosene and gasoline have been used, though the former has been more common. Heavier fuels, such as diesel oils and other oils having large percentages of aromatics, could probably be used at sea level, but become too viscous at low temperatures. In large airplanes of the future it may be possible to make provision for heating the fuel.

Sometimes both a conventional reciprocating engine and a gas turbine are used on the same airplane, and then it is desirable to carry only one kind of fuel, which must necessarily be gasoline. Although the change from kerosene to gasoline appears superficially to be a minor one, a unit designed primarily for kerosene may be completely unsatisfactory for gasoline unless changes are made in the combustion chambers, pumps, fuel nozzles, piping, control devices, and other parts. Special care must be taken, therefore, that the power plant is designed for operation with the fuel used.

Recently a fuel known as AN-F-58 has been developed especially for gas turbine power plants. It resembles gasoline, but has a relatively low octane rating. Since crude oil yields a greater fraction of this fuel than of kerosene, it is expected that it will eventually supplant kerosene in the United States, at least for military purposes.

STARTING. Aircraft gas turbines are customarily furnished with an electric starter, battery, and generator. The starter must be of sufficient capacity so that, after ignition of the fuel, the combined output of the turbine and starter is sufficient to bring the unit up to a self-sustaining speed, which will depend on the efficiency of the compressor and turbine as well as turbine inlet temperature and pressure. Since the efficiency is low at low speeds, the temperature will be higher than normal. The more rapid the acceleration, the greater will be the power required, and the higher the required turbine inlet temperature.

A fairly high temperature is usually permitted during the short starting period, since the stresses are low. Otherwise a larger starter would be required, or it would be necessary to reduce the efficiency at normal speeds in order to favor low-speed operation. For a propjet, the propeller pitch should be reduced as nearly as possible to zero before starting. The maximum allowable starting temperature should be specified by the manufacturer for each type of unit, but it may be of the order of 1600 to 1800 F.

CONTROL OF TURBOJET. Basically the control of this unit is simple, consisting essentially of varying the fuel flow only. However, variation of altitude introduces some complications, since for a given power plant speed and volume flow the mass flow of air decreases with the decreased density, and the fuel flow should be correspondingly reduced. If this is done merely by means of the pilot's throttle, keeping the pressure at the fuel pump discharge constant by means of a relief valve, the position of the throttle lever corresponding to full speed moves toward the low-speed end of the quadrant as the altitude increases. This is undesirable, not only because it tends to confuse the pilot, but also because it narrows the range of adjustment between full speed and idling speed (thereby making the adjustment unduly sensitive), and also because the pilot may easily overspeed the unit by inadvertently moving the throttle beyond the maximum speed point.

To insure that a given position of the throttle lever shall always correspond to the same speed, regardless of altitude, either of two devices may be used.

(1) *Barometric Compensator.* The pressure held at the fuel pump outlet (throttle inlet) is automatically decreased as altitude increases by changing the setting of a spring-loaded valve which partially by-passes the fuel flow. The adjustment may be made responsive to the movement of an evacuated bellows which changes in length with change of atmospheric pressure. It is customary to include, in parallel with the barometric compensator, a governor to limit the unit to its maximum safe speed.

(2) *Wide-range Speed Governor.* The speed governor may be so modified that, instead of merely holding a safe maximum speed, it holds a certain definite speed corresponding to each position of the throttle lever. The speed held decreases regularly as the lever moves toward the closed position, and no barometric compensation is necessary. Beyond the normal full-throttle position, the governor restricts the speed to the safe maximum, at any altitude.

CONTROL OF PROPJET. A propjet power plant should operate at nearly constant speed at all loads for maximum economy and endurance of parts. Temperature, and not speed, should be reduced at light loads. With such operation it is not necessary to accelerate the rotor in increasing load, or to decelerate it in decreasing load, and the change from low power to full power can be made more rapidly.

It is possible that the conventional propeller governor, which maintains constant speed by changing propeller pitch, can be retained in a propjet installation, together with an emergency or limiting speed governor operating on the fuel supply. The usual propeller governor, however, acts so slowly that instability may result at some operating conditions when the pilot changes the fuel flow. Quick-acting propeller governors might be designed, but other methods of control are also possible.

In theory, changes in load might be obtained by manual change of propeller pitch, the speed being maintained constant by a governor acting on the fuel supply. Rather than relying on manual control, however, the change of pitch should preferably be made responsive to some quantity which is a measure of output, such as gas temperature, or the torque between the turbine shaft and propeller. With a sufficiently sensitive torque indicator, it should be possible to design a system in which the same position of the pilot's lever corresponds approximately to the same fraction of the total available output under all conditions of operation.

TEMPERATURE CONTROL. In both the turbojet and propjet the speed and output are ultimately limited by gas temperature, so that a control system responsive to temperature is in many respects the most logical. Even when some other basic method is used, it is desirable to provide a limiting or protective temperature control. However, it is difficult to design a temperature indicator sufficiently rugged for continuous service operation, and at the same time sufficiently sensitive for rapid response.

20. METHODS OF THRUST AUGMENTATION

The following methods have been proposed for augmenting the thrust of the conventional aircraft gas-turbine power plant.

JET OR ROTARY AUGMENTER. The simple jet augmenter is shown diagrammatically in Fig. 16. The total mass flow is increased by addition of atmospheric air to the power plant air by action of the viscous forces, and the common velocity at the end of the augmenter duct, which now becomes the exit point of the system, is less than the normal discharge velocity from the propulsion nozzle. Thus the effect of a greater mass flow and lower discharge velocity is achieved; and if the efficiency of the augmenter is high enough, the product of mass flow and velocity, and therefore the thrust, will be increased.

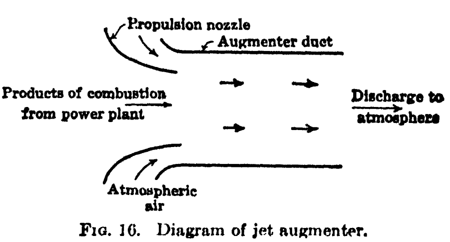

FIG. 16. Diagram of jet augmenter.

A rotary augmenter serves exactly the same purpose, but substitutes a mechanical pump for the viscous forces. Before discharge to atmosphere, the exhaust gases from the main turbine pass through an auxiliary turbine, where their velocity is decreased in furnishing the power required to drive the auxiliary pump, or fan, which would probably have only one stage. The efficiency can be made considerably higher than that of the jet type, but the augmenter will be larger, more costly, and probably heavier.

Both analysis and test show that a considerable gain in thrust (of the order of 30%) is possible with jet or rotary augmenters when the airplane is stationary or moving slowly, as at take-off. At normal operating speeds, this gain will be so reduced by the additional resistance from taking the extra air on board that there will be little or no net increase. Even in this case, it is possible that the discharge velocity will be better suited to the airplane velocity, so that a higher propulsive efficiency may be obtained; but the device then functions as a ducted fan rather than as a thrust augmenter.

The principal advantage of this method is that any increase of net thrust is obtained without burning additional fuel, so that the fuel economy is improved. However, unless a successful method can be devised for taking the additional air in flight from the boundary layer on the airplane surface, so that it can be regarded as obtained without increase of normal airplane drag, this method is evidently limited in practice to take-off and low airplane speeds, probably with provision for dropping the equipment or otherwise eliminating the excess drag at high airplane velocities.

INJECTION OF LIQUID AT COMPRESSOR INLET OR COMBUSTION CHAMBER. Injection of a liquid of high latent heat at the compressor inlet, with resultant evaporation, causes a considerable decrease of the air temperature. This decrease in temperature reduces the power required for compression and increases the total mass flow, so that an increase of thrust of the magnitude of 20 to 40% may be obtained.

Such a liquid may also be injected at the combustion chamber. Because the power required to pump a liquid from atmosphere up to combustion chamber pressure is small, the negative power of compression is much reduced. By this means also thrust may be increased 20 to 40%.

In both cases thrust will probably be increased at the expense of increased specific fuel consumption. With compressor injection, the decreased discharge temperature necessitates the burning of additional fuel to obtain the required temperature at the turbine inlet. With combustion chamber injection, additional fuel is required to furnish the heat of vaporization. It would be possible to improve the fuel economy if it were possible to recover the latent heat by subsequent condensation, but this is hardly practicable in an airplane.

Liquids suitable for injection are listed in Table 5.

Table 5. Liquids Suitable for Injection

(Adapted from Tables 285, 289, and 295, *Smithsonian Physical Tables*, 8th ed., 1934)

Liquid	Chemical Formula	Latent Heat of Vaporization, Btu/lb	Heating Value Lower, Btu/lb	Heating Value Upper, Btu/lb
Water	H_2O	970 (at 212 F)	......	.....
Methyl alcohol	CH_4O	481 (at 148 F)	8,415	9,605
Ethyl alcohol	C_2H_6O	369 (at 173 F)	11,600	12,840

The latent heat varies with temperature, and evaporation occurs over a range of temperatures. However, the above figures are sufficient for most practical purposes. Sometimes a mixture of water and alcohol is used.

Although alcohol is a fuel, conditions are not favorable for its complete combustion, and because of its low heating value, it is not an acceptable substitute for kerosene. Even the use of water will diminish the amount of the regular fuel which must be carried, but only by a relatively small amount. Thus injection of a liquid, whether water or alcohol, is limited to short periods of operation.

EXHAUST REHEAT. Since the fuel-air ratio in the conventional gas turbine power plant is always much less than the chemically correct ratio (usually 1 : 50 to 1 : 100, as compared with a chemically correct ratio of about 1 : 15), excess oxygen is available in the tailpipe to support further combustion. The advantage of burning the fuel here is that the permissible temperature is very much higher, since the gas does not have to pass through turbine buckets or other highly stressed parts. An adjustable propulsive nozzle should be provided so that the area can be increased when fuel is burned; otherwise the temperature and pressure may be too high.

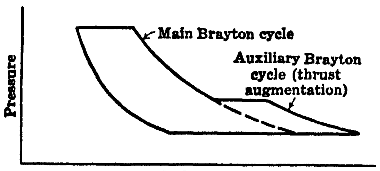

FIG. 17. Brayton cycle with exhaust reheat (afterburning or tail-cone burning) for thrust augmentation.

Increases in thrust of about 20 to 40% can be obtained by this method, but it is also limited to short periods of operation because of the low thermal efficiency. Thermodynamically, the process is equivalent to adding a small auxiliary Brayton cycle to the main Brayton cycle, as indicated in Fig. 17. The static pressure in the tailpipe is relatively low, and, since the efficiency of the auxiliary cycle is a function of the ratio of this pressure to atmospheric pressure, the corresponding theoretical specific fuel consumption will usually be at least two or three times greater than for the main cycle.

21. PERFORMANCE OF AIRPLANES WITH GAS TURBINE POWER PLANTS

TAKE-OFF. Because of the smaller thrust, an airplane with a turbojet power plant usually requires a longer take-off than a comparable airplane with a propeller. The velocity of the gas discharged from the propulsion nozzle is higher than that of the air discharged from a propeller, but the mass flow is so much less that the product of the two is considerably less. For this reason an assisted take-off may be necessary for a highly loaded jet airplane. A propjet, however, may have better characteristics than a reciprocating engine-propeller unit because of its extra jet thrust.

CLIMB. The initial rate of climb of an airplane with a turbojet will probably be less than that of a comparable airplane with a reciprocating engine-propeller power plant because of the lower thrust. However, the relative performance improves with increasing altitude, and the total time required to climb to a high altitude may be less.

ACCELERATION. Because of the inertia of the rotor, a gas turbine accelerates more slowly in practice than a reciprocating engine. However, a propjet would normally operate at full rated speed, changes of load being made by changing propeller pitch and fuel flow. Since the most economical operating speed of a turbojet is usually slightly lower than rated speed, it would ordinarily operate at a high rotary speed also, except when mere endurance is an objective. Hence the slower acceleration is not likely to be of much disadvantage except when speed has been reduced for landing and it is suddenly found that a successful landing cannot be made. With a quick-acting variable-area propulsion nozzle, however, it is possible to maintain a high rotor speed without excessive thrust for a normal landing.

GENERAL COMPARATIVE PERFORMANCE. At airplane speeds up to about 250 miles per hour the reciprocating engine-propeller power plant will probably remain supreme because of its high propulsive efficiency at these speeds and the relatively high efficiency of the power plant itself. For commercial airplanes flying at speeds of 250 up to about 500 or 600 miles per hour, either this power plant or the propjet may be used, depending on circumstances. So far as fuel consumption alone is concerned, this power plant may be superior, but the propjet has certain advantages that may be decisive. In general, the propjet weighs considerably less, and this may compensate wholly or partly for the greater specific fuel consumption, especially when the required range is small. The cost of fuel is normally less. The power plant is smaller in diameter, and better suited to streamlined installations and low drag. The amount of cooling air required is relatively small. Cabin supercharging is easily accomplished. The power plant is simpler in construction, and will undoubtedly be lower in cost for large production. The increased safety inherent in the use of kerosene rather than gasoline, as well as the increased passenger comfort from the reduction of vibration, may prove of considerable importance in commercial aviation.

For airplane speeds between 500 or 600 miles per hour and the speeds suitable for the ram jet and rocket, the propulsive efficiency of the turbojet becomes high enough so that, with its other advantages, this power plant is the best one for use in nearly all cases. In some applications it may be suitable for still lower airplane speeds—perhaps 400 miles per hour or less. For example, it may be used as an auxiliary power plant to furnish supplementary thrust for short periods of time, or it may be suitable as the main power plant of commercial airplanes for moderate operating ranges. However, excessive taxiing and delays in landing at airfields must be practically eliminated if the turbojet is to be used for commercial purposes, since otherwise the waste of fuel may become dangerous.

British-built commercial transport aircraft using the turbojet are in successful operation, although the largest field of application will probably be in military aircraft for a long time to come.

22. DATA FOR AIRCRAFT GAS TURBINE POWER PLANTS

Much of the data on aircraft gas turbines is not available because of military security restrictions. Furthermore, performance and ratings are changing rapidly, and any listed data may soon be obsolete. Data given in Table 6, therefore, cannot be considered as either complete or necessarily up to date, but are representative of the latest data released for publication at the time of writing.

Table 6. Aircraft Gas Turbines

(All data have been furnished by the respective manufacturers. The figures may not be directly comparable in all cases.)

Manufacturer	Model	Army or Navy Designation	Compressor Type/No. Stages	Turbine, No. Stages	Nominal Outside Diameter, in.	Nominal Overall Length, in.	Nominal Dry Weight, lb	Rotary Speed, rpm		S.L. Static Thrust (Dry), lb		Actual, Expected, or Guaranteed S.L. Spec. Fuel Consumption, lb/hr/lb thrust	
						AMERICAN TURBOJETS							
Allison Division General Motors Corp.	400-C5 (from G.E. I-40)	J33-A-23	Cent./1	1	50.5	107	1775	Mil. Nor.	11,750 11,250	Mil. Nor.	4600 3900	Mil. Nor.	1.13 1.12
Allison Division General Motors Corp.	GE TG 180-C1, -C2, -C3	J35-A-9, -11, -13	Ax./11	1	40.0	145	2455	Mil Nor.	7,700 7,400	Mil. Nor.	3750 3270	Mil. Nor.	1.115 1.080
Fredric Flader, Inc.	124	XJ-55-FF-1	. . .	1	15.75	79.0	300	Mil. Nor.	28,200 26,800	Mil. Nor.	770 700	Mil. Nor.	1.64 1.65
General Electric Co.	TG-190	J47	Ax./12	1	Other data not released, but size and weight approximately same as for J35.					Mil. Nor.	5000 . . .		
Pratt & Whitney Aircraft	JT-6B Turbo-Wasp	J42-P-4	Cent./1	1	49.5	103.3	1715	Mil. Nor.	12,300 11,600	Mil. Nor.	5000 4000	Mil. Nor. (guaranteed)	1.12 1.09
Westinghouse Electric Corp.	X9.5A	XJ32-WE-20	Ax./6	1	9.5	50	145	Mil. Nor.	34,000 28,000	Mil. Nor.	260 149	Mil. Nor.	1.6 1.7
Westinghouse Electric Corp.	X19-B	XJ30-WE-6	Ax./6	1	19	90	809	Mil. Nor.	18,000 17,000	Mil. Nor.	1365 1175	Mil. Nor.	. . . 1.28
Westinghouse Electric Corp.	19XB-2B	J30-WE-20	Ax./10	1	19	94	693	Mil. Nor.	17,000 14,700	Mil. Nor.	1560 1285	Mil. Nor.	. . . . . 1.17
Westinghouse Electric Corp.	24C4B	J34-WE-22	Ax./11	2	24	120	1200	Mil. Nor.	. . . .	Mil. Nor.	3000 2290	Mil. Nor.	. . . .

CANADIAN TURBOJETS													
A.V. Roe Canada Ltd.	Chinook	.. ..	Ax./9	1	32	125.1	1250	S.L. take-off S.L. climb S.L. cruise	10,100 9,800 9,500	Take-off Climb Cruise	2600 2100 1900	Cruise	1.00
A.V. Roe Canada Ltd.	Orenda	....	Ax./-	Other data not released									
ENGLISH TURBOJETS													
Armstrong Siddeley Motors Ltd.	Adder	. .	Ax./10	1	28	73	550	Max. Nor.	15,000 14,250	Max. take-off	1098		1.022
The De Havilland Aircraft Co. Ltd.	Goblin 2	. .	Cent./1	1	49.8	116	1550	Mil. Nor.	10,200 .	Mil. Nor.	3000 .	Mil. Nor.	1.25 ....
The De Havilland Aircraft Co. Ltd.	Goblin 3	. .	Cent./1	1	49.8	100.4	1570	Mil. Nor.	10,750 . . .	Mil. Nor.	3300 . . .	Mil. Nor.	..... 1.18
The De Havilland Aircraft Co. Ltd.	Ghost 2		Cent /1	1	53	116	2011	Mil. Nor.	10,000 8,500	Mil. Nor.	5000 2825	Mil. Nor.	..... 1.06
Metropolitan-Vickers Electrical Co. Ltd.	Beryl Mark I	. . . .	Ax./10	1	37 25	161.5	1785	Take-off and combat Climb (max) Cruise (max)	7,750 7,600 7,400	Take-off and combat Climb (max) Cruise (max)	3850 3670 3400	Take-off and combat Climb (max) Cruise (max)	1.10 1.08 1.07
Rolls-Royce Ltd.	Nene II (IC-09-1)	. .. .	Cent./1	1	49.5	97	1755	Mil. Nor.	12,300 12,000	Mil. Nor.	5000 3780	Mil. Nor.	.... 1.0
Rolls-Royce Ltd.	Derwent V (IC-09-1)		Cent./1	1	43	83 1	1280	Mil. Nor.	14,700 14,100	Mil. Nor.	3600 2560	Mil. Nor.	... . 1.0

(*Table continued on p. 15-68*)

Table 6. Aircraft Gas Turbines—*Continued*

(All data have been furnished by the respective manufacturers. The figures may not be directly comparable in all cases.)

Manufacturer	Model	Army or Navy Designation	Compressor Type/No. Stages	Turbine, No. Stages	Nominal Outside Diameter, in.	Nominal Overall Length, in.	Nominal Dry Weight, lb	Rotary Speed, rpm	S.L. Power to Propeller, hp	S.L. Static Thrust (Dry), lb	Actual, Expected, or Guaranteed Spec. Fuel Consumption
						AMERICAN PROPJETS					
Fredric Flader, Inc.	104	XT-33	Other data not released.								
General Electric Co.	TG-100B	T-31	Ax./14	1	35.1	84.7	2005	13,000	1700	420	0.87 lb/bhp-hr (S.L.)
Wright Aeronautical Corp.	Typhoon	T-35	Other data not released.								
						ENGLISH PROPJETS					
Armstrong Siddeley Motors Ltd.	Mamba 1	....	Ax./10	2	31	74	760	Max. 15,000 Nor. 14,000	1010 (Max. take-off)	307 (Max take-off)	0.734 lb/equivalent hp-hr (S.L.)
Armstrong Siddeley Motors Ltd.	Mamba 2	....	Ax./10	2	31	77	770	Max. 15,000 Nor. 14,000	1270 (Max take-off)	384 (Max take-off)	0.736 lb/equivalent hp-hr (S.L.)
Armstrong Siddeley Motors Ltd.	Double Mamba 1	....	Ax./10	2	....	99	2000	Max 15,000 Nor. 14,000	2540 (Max take-off)	770 (Max take-off)	0.737 lb/equivalent hp-hr (S.L.)
Armstrong Siddeley Motors Ltd.	Python 1	....	Ax./14	2	54.5	123	3150	Max 8,000 Nor. 7,600	3670 (Max take-off)	1150 (Max take-off)	0.717 lb/equivalent hp-hr (S.L.)
The Bristol Aeroplane Co., Ltd.	Theseus 501	....	Ax./8 Cent./1	2, plus 1 stage for driving propeller	54	81.85	2320	Max 8,200 (compressor)	1975	715	0.65 lb/bhp-hr (300 mph at 20,000 ft)
The Bristol Aeroplane Co., Ltd.	Proteus 2	....	Ax./- Cent./-	....	38.5	99.75	2900	Max 10,000 (compressor)	3200	800	....
D. Napier & Son Ltd.	Naiad	....	Ax./12	2	28	102	1095	18,250	1500	240	0.655 lb/equivalent bhp-hr (S.L.)
Rolls-Royce Ltd.	Dart RDa1 (IC-09-1)	....	Cent./2	2	38.5 (over cowling)	94.75	850	14,500	1000	325	....

Abbreviations: Cent.—centrifugal; Ax.—axial; Mil.—military; Nor.—normal; S.L.—sea level.

MARINE ENGINEERING

By D. C. MacMillan

Marine engineering is primarily application of mechanical and electrical engineering to propulsion and operation of ships. This chapter deals particularly with the marine features of subjects described in more detail elsewhere in their volume, and briefly reviews the characteristics of equipment peculiar to the marine industry. Marine engineering applications are comprehensively covered in the general references, p. 15-83.

23. GENERAL CHARACTERISTICS

CHARACTERISTICS OF SHIPS. Principal characteristics of the more important classes of American ships are given in Table 1.

Length overall is the extreme length of the ship.

Length between perpendiculars is the distance between two verticals erected normal to the *base line*, one at the forward edge of the stem at the designed *water line*, the other at the after edge of the stern or rudder post.

The beam (molded) is the breadth of the ship measured on the molded form, i.e., to the frame lines inside the shell plating on steel ships.

Depth is the vertical distance from the inner surface of the keel plating to the inner surface of the upper continuous deck at the side, amidships.

Draft is the vertical distance from the bottom of the keel to the water line.

Displacement is the weight (in long tons, 2240 lb) of water displaced by the immersed part of the ship, and is equal to the weight of the ship and everything on board.

Tonnage is an arbitrary measure of capacity, used for estimating navigational charges, and calculated assuming 100 cu ft represents 1 ton. *Gross tonnage* is the total enclosed capacity; *net tonnage* allows for deduction of spaces necessary to operate the ship, such as engine, fuel, crew, and similar spaces.

Deadweight is the difference between displacements of the ship when empty and loaded. *Cargo deadweight* is the total deadweight less fuel, stores, crew, and effects.

Sea speed is the sustained speed in nautical miles (6080 ft) per hour, expressed as knots (a unit of speed). It generally is defined as that speed obtained on trial using 80% of normal shaft horsepower.

Bale capacity is the cubic capacity of the cargo holds measured face to face of cargo battens and deck to underside of deck beams.

Power required to propel a ship depends on a great many variables, and if an accurate estimate is required it should be made by methods given in the references. Since ships generally are designed for minimum power at the design speed, some consistency of results might be expected; these are expressed by the *admiralty coefficient* (Table 2). The coefficient is affected by many items, including proportions of the ship, speed-length ratio, and propulsive coefficient. Like all overall coefficients, its values vary widely, but a judicious selection based on past performance permits a quick evaluation of power required. Table 2 lists some values. For modern single screw ships, this coefficient may range from 375 to 480, and for twin screw merchant ships from 360 to 390.

MARINE DESIGN CONSIDERATIONS. Probably the prime consideration in marine design is reliability. Other considerations become very insignificant where the consequences of failure may mean loss of service of the ship, or even loss of the ship. Weight of machinery receives careful consideration, especially for highly powered ships. Reduction of weight per horsepower can be accomplished by increasing rotative speeds, but this results in lower propeller efficiency and increased power, fuel consumption, and weight of machinery and fuel.

Space occupied by machinery is important, since the bulk of American cargo is carried on a measurement basis; it is essential to keep length of the machinery space at a minimum in cargo ships to increase the cargo capacity. Volume of the machinery space generally exceeds 13% of the gross tonnage, since when this is true a deduction of 32% is permitted in arriving at the net tonnage. Height of the machinery space influences stability; in passenger ships excessive height would occupy good passenger space.

Rolling and Pitching. The marine engineer is confronted with the dynamic effects of *rolling* and *pitching* of the ship, and must design for operation with *list* and *trim*. Machinery usually is designed to operate satisfactorily with a momentary roll of 30 degrees to either side, a permanent list of 15 degrees to either side, and a trim of 5 degrees fore and aft.

Table 1. Characteristics of U. S. Ships

Type	U.S.M.C. Designation or (Name)	Dimensions, ft and in.					Dis-place-ment, Tons	Gross Ton-nage	Net Ton-nage	Dead-weight Tons	Sea Speed, Knots	Total Normal Shp	No. of Pro-pellers	Type Ma-chin-ery	Cargo Capacities, 1000			Number	
		L.O.A.	L.B.P.	Beam	Depth	Draft									Bale, cu ft	Bar-rels	Refrig-erated, cu ft	Passen-gers	Crew
Cargo	C1-M-AV1	338-8	320	50-0	29-0	21-0	7,420	3,820	2,165	5,040	11.0	1,700	1	D.	228	....	9.8	0	39
Cargo	C1A	412	390	60-0	37-6	23-6	11,100	5,070	2,880	7,400	14.0	4,000	1	G.D.	446	....	0	8	41
Cargo	C1A	412	390	60-0	37-6	23-6	11,100	5,100	3,000	7,500	14.0	4,000	1	G.T.	449	....	0	8	43
Cargo	C1B	418	395	60-0	37-6	27-6	12,900	6,800	4,000	9,300	14.0	4,000	1	G.T.	448	....	0	8	43
Cargo	C1B	418	395	60-0	37-6	27-6	12,900	6,750	3,960	9,000	14.0	4,000	1	G.D.	441	....	0	8	41
Cargo	C2	459	435	63-0	40-6	25-9	13,900	6,100	3,600	9,700	15.5	6,000	1	G.T.	562	....	0	8	43
Cargo	C2	459	435	63-0	40-6	25-9	13,900	7,200	4,340	9,400	15.5	6,000	1	G.T.	471	...	29	8	43
Cargo	C2	459	435	63-0	40-6	25-9	13,900	6,020	3,560	9,270	15.5	6.000	1	G.D	558	...	0	0	41
Cargo	C2	459	435	63-0	40-6	25-9	13,900	6,200	3,650	8,660	15.5	6,000	1	D.	507	..	25	12	43
Cargo	C2	459	435	66-0	43-0	27-7	14,950	8,260	4,830	10,600	15.5	6,000	1	G.T.	583	....	0	0	43
Cargo	(Exporter)	473	450	68-0	42-3	27-0	14,500	6,740	4,000	9,500	16.5	8,000	1	G.T.	547	....	0	0	44
Cargo	C3	492	465	69-6	42-6	28-6	17,600	7,770	4,590	12,500	16.5	8,500	1	G.T.	700	....	0	12	43
Cargo	C3	492	465	69-6	42-6	28-6	17,600	7,770	4,590	12,100	16.5	8,500	1	G.T.	650	...	35	12	43
Cargo	C3	492	465	69-6	42-6	28-6	17,600	7,890	4,700	11,900	16.5	8,500	1	G.D.	656	....	33	12	41
Passenger and Cargo	C3	492	465	69-6	42-6	26-6	16,200	9,260	5,160	9,940	16.5	8,500	1	G.T.	478	...	43	96	111
Passenger and Cargo	C3	492	465	69-6	42-6	27-3	16,700	12,000	8,000	9,000	16.5	8,500	1	G.D.	446	...	45	197	136
Cargo	C4	500	496	71-6	43-6	32-10	21,950	10,680	7,500	14,900	16.5	9,000	1	G.T.	700	..	32	4	55
Passenger	(America)	723	660-6	93-3	64-0	32-6	35,400	26,450	14,300	14,300	22	34,000	2	G.T.	260	..	33.5	1200	640
Passenger and Cargo	P2	610	573	75-6	52-6	30-2	23,507	15,350	7,715	10,431	19	18,000	2	T.E.	174	5.4	35	550	352
Passenger and Cargo	(Panama)	493-6	471-6	64-0	46-9	26-0	14,200	9,980	5,865	6,550	16.5	9,000	2	G.T.	290		75	218	124
Cargo-Liberty	EC2	442	416	57-0	37-4	28-0	14,170	7,100	5,050	10,500	11	2,000	1	S.E.	500	....	0	0	54
Cargo-Victory	AP2	455	436-6	62-0	38-0	28-6	15,200	7,850	4,850	10,800	15.5	6.000	1	G.T.	453	...	0	0	54
Cargo-Victory	AP3	455	436-6	62-0	38-0	28-6	15,200	7,850	4,850	10,700	17	8,500	1	G.T.	453	....	0	0	54
Tanker	T2	523-6	503	68-0	39-3	30-0	21,800	10,200	6,000	16,500	14.5	6,000	1	T.E.	15	141	0	0	53
Tanker	T2	523-6	503	68-0	39-3	30-0	21,800	10,200	6,000	16,400	16.0	10,000	1	T.E.	15	141	0	0	53
Tanker	T3	502	487-6	68-0	37-0	26-8	21,200	10,000	6,000	16,400	15.0	7,000	1	G.T.	......	134	0	0	48
Tanker	(Cimarron)	553	525	75-0	39-0	31-6	24,800	11,300	6,600	18,300	18	13,500	2	G.T.	16	146	0	0	64
Ore Carrier	(Venore)	583	560	78-0	43-9	34-3	32,450	8,560	4,290	24,250	16.0	11,000	1	G.T.	420	....	0	0	49
Ore Carrier	....	620	605	60-0	35-0	24-0	20,880	9,500	7,000	15,600	10.5	2,500	1	S.E.	525	...	0	0	39

L.O.A. = length overall, L.B.P. = length between perpendiculars.
D. = direct drive diesel, G.D. = geared diesel, G.T. = geared turbine, T.E. = turboelectric, S.E. = steam engine.
U.S.M.C. = United States Maritime Commission.

Table 2. Speed and Power Characteristics Based on Model Tests

Ship Type	Length at Water Line	Displacement, Tons	Design Speed	Speed/Length Ratio ‡	Model Shp *	Admiralty Coefficient §	Normal Shp	Model Speed †	Speed/Length Ratio ‡	Admiralty Coefficient §
				SINGLE-SCREW SHIPS						
EC2	428	13,790	11	.532	1,600	503	2,000	11.75	.568	489
C1-M-AV1	320	7,400	11	.615	1,480	342	1,700	11.4	.637	330
C1B	395.5	12,825	14	.704	3,250	464	4,000	14.7	.740	437
C2	438.5	13,815	15.5	.740	4,750	451	6,000	16.45	.787	426
C3	469.5	16,725	16.5	.761	6,250	474	8,500	17.8	.822	437
C4	500.5	19,920	17	.760	7,650	475	9,000	17.9	.800	472
AP2	445	14,800	15.5	.735	5,000	449	6,000	16.2	.768	427
AP3	445	14,800	17	805	6,800	435	8,500	17.7	.839	393
Cargo	437	11,700	16	.765	4,750	433	6,000	17.15	.821	431
T2-A1	513	21,650	14 5	640	5,200	457	6,000	15 1	.667	448
T2-A2	513	21,650	16 0	.706	7,700	415	10,000	16.8	.742	370
Venore	572	32,450	16.0	.668	9,900	431	11,000	16.35	.683	414
				TWIN-SCREW SHIPS						
Panama	486.5	14,027	16.5	.747	6,350	411	9,000	18.05	.818	380
P2	590	22,380	19.0	.782	13,600	403	18,000	20.7	.853	391
Tanker	543	23,200	18 0	.771	12,100	392	13,500	18.5	.792	381
America	689	34,730	22 0	.839	28,500	402	34,000	23 0	.876	381

* Shp required to propel ship at design speed, as determined from model test.

† Speed attained when developing the normal shp actually installed, as determined from model test.

‡ Speed/length ratio = Speed, knots/$\sqrt{\text{water-line length, ft.}}$

§ Admiralty coefficient, $C = \dfrac{(\text{Displacement})^{2/3}\,(\text{speed})^3}{\text{Shp}}$.

Maneuvering. The marine prime mover must have satisfactory *maneuvering characteristics*, i.e., it must be able to operate at all speeds within its range, and to stop and reverse quickly; it must have sufficient torque throughout the reversing cycle to overcome the torque imposed on the propeller by the motion of the ship. Direct-connected steam and diesel engines are reversible, and usually are satisfactory in other respects, although the minimum speed of the diesel generally is about 30% of full speed. Geared turbines are provided with astern turbines for reversing. Electric drive also is used, and reversing accomplished electrically. Reverse reduction gears, with clutches, are used with non-reversing diesels. Controllable-pitch propellers also permit reversing the ship without reversing the prime mover.

TYPES OF PROPELLING MACHINERY. Both diesel and steam plants are used; these, with diverse transmission systems account for most of the types.

The speed of the propeller is fairly well determined by the speed and power of the ship. It is impossible to vary the normal relationship much without considerable sacrifice in efficiency and maneuverability. The prime mover must also be operated at its most suitable speed to avoid sacrifice in efficiency, weight, space, and cost. The prime mover generally requires higher rpm than the propeller, although the difference usually is negligible for reciprocating steam engines. Some diesels are direct-connected, but an appreciable saving results from using higher speed engines running at about two to eight times propeller rpm. The most suitable steam turbine speeds may be up to 80 times the propeller rpm. Types normally used at present are:

Steam	SHP per shaft
Boilers—water tube	
Main units—reciprocating engine	300 to 4000
geared turbine	4000 and up
turboelectric	5000 to 10,000
Diesel	
Main units—diesel direct	Up to 6000
diesel electric	Up to 6000
geared diesel	Up to 8000
geared diesel electric	Up to 6000

Steam plants are used for most new ships of 4000 shp and higher. The installation (per shaft) generally consists of two boilers and one main unit. Reciprocating engines are employed widely for steam installations in river, harbor, and lake craft of 4000 shp or less. Turboelectric drive has been used extensively for ocean-going tankers of 3000 to 10,000 shp.

Diesel engines are used for a large proportion of river, harbor, and lake craft, and for ocean-going ships of 4000 shp and less. Present trend is to use higher-speed reversing engines, or nonreversing engines with reverse-reduction gears or electric drive. Both d-c and a-c power is used. The propelling motor is direct-connected to the propeller shaft for a-c units, and for small d-c units. Where the installation requires more than one d-c generator, it has been found advantageous to use geared motors operating at generator speed, with one motor per generator. A flexible coupling is required for geared drives; types include mechanical, hydraulic, electromagnetic, and air flex. Extensive applications of reverse-reduction gears with air flex clutches have proved satisfactory. Electromagnetic couplings are preferred for reversing engine installations.

FUELS. Seagoing United States ships use oil fuel almost exclusively, although some modern coastwise colliers use coal. Most ships on the Great Lakes use coal.

A number of factors peculiar to the marine installation generally are of greater significance than cost of the fuel alone. The feature that generally determines the selection is the space required for fuel. Coal cannot be stowed in the double bottom of a ship as can oil, hence a coal-burning ship for a given performance must be larger and more expensive; conversely, a given ship cannot carry as much cargo. Where voyages are short (e.g., Great Lakes) bunker requirements are small, and the space requirement is not as serious; if the cost is favorable, coal is used. Other factors which influence the selection are (1) weight, (2) bunkering time and facilities, (3) cleanliness, and (4) crew required.

Coal used on shipboard is bituminous or semibituminous, burned by mechanical stokers; for small installations hand firing may be used.

The fuel oil used on shipboard generally falls into two types, Bunker C for steamships, and diesel oil for motorships. (See p. 2-45.) Oil sold as Bunker C may be a residual oil or a blend. Its specifications are essentially the same as Federal specifications for Grade 6 fuel oil. Several grades of oil fuel are used for diesel engines; the type required depends on the engine, high-speed engines requiring a lighter oil. See Table 3 for typical marine fuel oil characteristics.

Table 3. Typical Marine Fuel Oil Characteristics

Type	Bunker C Range	Bunker C Design	Heavy Diesel Oil	Light Diesel Oil
API gravity	7 to 25	10	22 to 30	30 to 36
Flash point, °F	160 to 260	150 min	150 min	150 min
Pour point, °F	− 10 to +65	40	20 max	15 max
Carbon residue, %	7 to 18	13	1.0 max	0.1
Ash, %	0 to 0.2	0.1	0.02 max	0.01 max
B.S. & W. by centrifuge, %	....	....	0.3 max	....
Carbon, %	86 to 90	....	....	....
Hydrogen, %	9 to 11.5	10.5	....	....
Sulfur, %	0 to 3	2	1.0 max	0.5 max
Water and sediment, %	0 to 2	2	1.0	0.1
Btu per lb	18,000 to 18,700	18,500	19,350	19,500
Viscosity, SSF @ 122 F	100 to 700	300 max	....	....
SSU @ 100 F	....	....	75 to 150	33 to 75
Cetane No.	....	....	35	40 to 60
Diesel index	....	....	....	43 to 52

REGULATORY BODIES. The marine power plant must be constructed in accordance with requirements of regulatory bodies, of which two are of major importance.

The American Bureau of Shipping is a private corporation organized by the maritime industry, managed by a Board elected from its membership, including shipowners, ship builders, and underwriters. The Bureau establishes standards to which "classed" ships must be built, and by which ships may be maintained in seaworthy condition. If a ship is to be "classed" the requirements of the rules, material specifications, and regulations must be met. Established standards include requirements applying to engines, boilers, propellers, shafting, piping systems, cargo refrigerating machinery, etc., i.e., those features of the marine power plant affecting seaworthiness of the ship or protection of cargo.

The U. S. Coast Guard, Marine Inspection Service, promulgates and enforces rules concerning safety of life at sea. Rules affecting the power plant apply to boilers, piping systems, fire protection systems, etc.

24. PROPELLERS AND TRANSMISSION SYSTEMS

PROPELLERS. Resistance of a ship to steady motion through the water is overcome by the propelling force. Propulsion energy is developed by the power plant and transmitted to the ship by a suitable propeller, nearly always a *screw* propeller.

The marine screw propeller has three, four, or five blades fastened to or integral with a hub. Propellers usually are cast in one piece, although built-up propellers are sometimes used on the Great Lakes because they are easily replaced if damaged or broken by ice. Solid propellers are more efficient because of the smaller, streamlined hub. Controllable pitch propellers have had very little marine application. Three-bladed propellers are used on multiscrew ships; four-bladed are most widely used for single-screw ships. Five-bladed propellers have been used when required to avoid vibration of a particular frequency.

The material used for blades and hub is manganese bronze or cast iron. Bronze is tough, corrosion-resistant to salt water, and takes a high polish, resulting in high efficiency. Cast iron is cheap, lacks corrosion resistance, and therefore is little used on salt water, but is extensively used on the Great Lakes.

Characteristics of a propeller are expressed by ratios:

p/d = ratio of pitch to diameter.
DAR = developed area ratio = ratio of developed area of all blades to the disk area.
PAR = projected area ratio = ratio of projected area of all blades to the disk area.
MWR = mean width ratio = ratio mean developed width of one blade to the diameter.
BTF = blade thickness fraction = ratio of the maximum blade thickness at shaft axis to the propeller diameter.

Pitch ratios range from 0.6 to 2.0 for tugboats to motorboats, respectively. The seagoing merchant ship range is from 0.8 to 1.2. Radial pitch distribution usually is uniform, but may be decreased towards the hub to compensate for the higher wake in the region of the hub.

Mean width ratio varies from 0.20 to 0.30 for four-bladed merchant ship propellers to 0.45 to 0.50 for three-bladed destroyer propellers.

The developed blade outline is generally elliptical, although wide or narrow-tipped blades have been used.

Blade Sections. Two types of blade section are used, airfoil and ogival. Ogival sections have a flat face and a back formed by a circular or parabolic arc with maximum thickness at midchord, and fairly sharp edges. Airfoil sections have maximum thickness at about one-third the chord length from the leading edge, a flat or curved face, and a well-rounded or blunt nose.

Blade thickness is determined by the required strength, and the BTF varies from 0.04 to 0.06. The bending moment acting at various radii decreases from a maximum near the hub to zero at the tip; usually the maximum blade section thickness is uniformly tapered from hub to tip. Blade strength may be calculated by curved beam theory with nonuniform load in special cases, but usually stresses at the root are calculated assuming that torque and thrust forces are concentrated at 0.60 and 0.66 radius, respectively, and then adding the effect of centrifugal force. The allowable stress for manganese bronze is about 6000 psi and for cast iron about 3000 psi.

One, two, three, or four propellers may be used, depending principally on the total power required. The highest propulsive efficiency is obtained with a single screw, making it preferable. Efficient transmission of power depends on propeller rpm and diameter. For good efficiency, higher powers require larger diameter and lower revolutions. Diameters normally do not exceed 22 ft; rpm is generally 85 or higher. Usually, twin screws are used if power exceeds 20,000 shp, and speed exceeds 20 knots. Other factors in selection of number of propellers are:

(1) Single prime mover has better efficiency than twin units of half power.
(2) Twin screws are an advantage when maneuvering in close quarters.
(3) A multiple screw ship is more reliable in the event of serious damage to propeller, shafting, or prime mover.
(4) For a given power, number of screws may influence the choice of type of machinery.

Generally, for maximum propulsive efficiency, the propeller diameter is as large as can be fitted on the ship. This is limited by draft of the ship, clearance between propeller tips and ship structure, immersion of blade tips, and manufacturing limitations (about 22 ft).

The rpm of the propeller is selected from propeller design charts, based on the power developed, diameter, speed of advance through the water, and best efficiency. Propeller design charts have been prepared from model tests. Propellers of various pitch ratios, number of blades, areas, thicknesses, blade section, etc., have been tested over a complete range of slip and the results plotted on charts like Fig. 1, which shows the results of tests by Admiral Taylor first published in 1924. These propellers had circular-back blade sec-

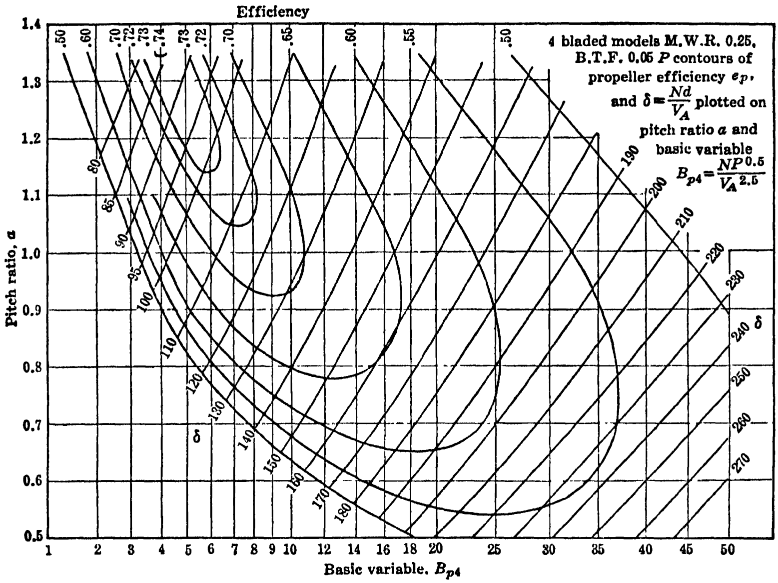

FIG. 1. Propeller chart.

N = rpm
d = diameter, ft
V_A = speed of advance of propeller through wake water, knots
P = shaft horsepower

$$B_{p4} = \frac{NP^{0.5}}{V_A^{2.5}}$$

tions, and they were not as efficient as some now used. The results of a modern series of four-bladed propellers having airfoil sections would show about 5% better efficiency.

For preliminary work, the best value of rpm or diameter may be determined from

$$50\sqrt[5]{\frac{\text{shp}}{\text{rpm}^3}}$$

where d = diameter in feet, rpm = revolutions per minute, and shp = shaft horsepower. Table 4 gives data for representative designs.

SHAFTING AND BEARINGS. In a ship power is transmitted from the prime mover to the propeller by shafting, sometimes half the ship in length. Merchant ships normally use solid carbon steel shafts with integral forged flanges. The allowable stress in line shafting is 8500 psi for material having an ultimate strength of 60,000 psi, yield point of 30,000 psi, and elongation of 30%. Shaft diameter usually is selected on the basis of allowable stress; deflection and torsional critical speed requirements are satisfied by adjusting shaft spans, rotating masses, etc., if necessary. The shaft length that carries the propeller (propeller or tail shaft) is larger in diameter than the line shaft because of the overhung propeller weight.

Bearings inside the ship normally are ring-oiled babbitt. A few installations of roller bearings have been made. Bearings outside the hull or outside the stern gland are lubricated by sea water. Outside the stern gland, the shaft is protected by a bronze sleeve, and the bearing generally is (1) lignum vitae blocks with grain on edge, (2) rubber bearing surfaces vulcanized to brass backing strips, and (3) phenolic composition blocks. Bearing pressures are about 30 to 40 psi of projected area for oil-lubricated bearings and 15 to 25 psi for water-lubricated bearings. For merchant ships, the span between bearings is about 10 to 15 diameters for propeller shafts and 20 diameters for the line shaft.

Table 4. Propeller Characteristics

Ship Type	Diameter, ft, d	Pitch, ft	Number Blades	Mean Width Ratio, MWR *	Projected Area Ratio, PAR	Blade Thickness Fraction, BTF	Shp	Rpm	K †
				SINGLE-SCREW SHIPS					
C1B	17.47	18.275	4	.225	.396	.044	4,000	90	49.4
C2	19.0	19.875	4	.225	.396	.0435	6,000	92	50.3
C3	21.67	21.67	4	.216	.386	.044	8,500	85	51.0
C4	21.67	21.67	4	.221	.348	.044	9,000	85	50.5
AP2	18.25	17.5	4	.25	.447	.0452	6,000	100	50.8
AP3	20.50	22.9	4	.237	.413	.047	8,500	85	48.2
C1-M-AV1	11.0	6.6	3	.28	.415	.0561	1,700	180	56.0
Liberty	18.5	16.0	4	.20	.	.0385	2,000	76	54.2
Cargo	19.0	20.0	4	.22	.38	.042	6,000	92	50.3
Cargo-Passenger	22.0	22.18	4	.22	.38	.047	12,500	92	50.5
				TWIN-SCREW SHIPS					
Panama	17.5	20.5	4	.21	.357	.040	4,500	90	48.5
P2	18.25	19.07	3	.30	.374	.0491	9,000	120	52.3
Tanker	17.5	22.19	4	.23	.372	.053	6,750	96	46.3
America	19.5	19.46	4	.24	.417	.0535	17,000	128	51.0
P2	18.0	22.67	4	.25	.389	.0463	8,500	100	46.9

* For definition, see text, p. 15-73.

† $d = K\sqrt[3]{\dfrac{\text{shp}}{\text{rpm}^3}}$.

Thrust developed by the propeller is transmitted through the shafting to the main thrust bearing. When the prime mover is a steam engine, diesel engine, or an electric motor, the thrust bearing usually is mounted in a separate casing just aft of the engine. With gear drive, it is usual to incorporate the main thrust bearing in the reduction gear casing. In either location, adequate foundations to distribute the load over the bottom structure is provided. The thrust load may be calculated from the propeller performance data, or approximated by the following:

$$T = 300\,\frac{\text{shp}}{V}$$

where T = maximum design thrust, pounds; shp = design shaft horsepower; and V = design speed, knots.

Thrust bearings usually are of the single-collar pivoted-shoe type with two to six shoes on each side of the collar. The allowable bearing pressure is about 300 psi for merchant ships.

REDUCTION GEARS. To reconcile the rotative speeds of a turbine prime mover and the propeller, mechanical reduction gearing generally is used. The power transmitted per unit is limited only by the size of the gear that can be hobbed or shaped. Units up to 50,000 shp have been built, and higher powers are contemplated.

Table 5 gives particulars of recent gear installations. The examples cover articulated, nested, and locked-train gears. Figure 2 illustrates a typical design.

In the articulated design, primary and secondary gears are separate, the first reduction wheels being in tandem with the second reduction pinions. The latter are hollow, to take a quill shaft which transmits the torque and provides torsional flexibility between high- and low-speed gears. Quill shafts usually are integral with or rigidly attached to their primary wheel shafts, but have flexible coupling connections to the secondary pinions. The turbines are connected to their respective high-speed pinions through flexible couplings.

Double-helical gearing is used throughout, except in unusual cases for which special provision must be made. Flexible couplings permit a certain axial movement of their mating elements, and the meshing pinions and wheels are "registered" axially with each other. A small thrust bearing must therefore be provided for each high-speed wheel shaft, whereas a main thrust bearing on the low-speed shaft locates the main gear and low-speed pinions, and transmits propeller thrust to the ship's structure.

In the nested type of gear, the two halves of the primary gears straddle the secondary gears, or vice versa. In both cases, the primary wheels are mounted on the same shafts

Table 5. Characteristics of Reduction Gears for Turbine Propulsion

(Data adapted by permission from Vols. 55 and 56, *Trans. Soc. Naval Architects* and *Marine Engrs.* From Marine Engineering, by Seward, and for C4 Ship by permission of The Falk Corporation.)

Ship or Type	Passenger and Cargo	C-1	C-2	C-4	*S.S. America* *	DD397 Class Destroyer	DD364 Class Destroyer
Type of Gear	Articulated	Nested	Articulated	Nested	Hp-Articulated	Nested	Locked-train
Single or double reduction	Double	Double	Double	Double	Combined S D	Double	Double
Shp per shaft	2500	4000	6000	9000	17000	25000	26000
Shp per pinion							
Hp—1st reduction	1250	2000	3000	4500	5667	12500	12000
Lp—1st reduction	1250	2000	3000	4500	....	12500	14000
Hp—2nd reduction	1250	2000	3000	4500	5667	12500	6000
Lp—2nd reduction	1250	2000	3000	4500	Ip 5667 Lp 5667	12500	7000
Rpm							
Hp—1st reduction pinion	7838	5995	6072	5004	3291	6041	5850
Lp—1st reduction pinion	6023	4979	4048	4289	....	4984	4926
2nd reduction pinions	1841	587	882	665.3	1506	1389	2283
Main gear	250	90	92	85	128	400	400
Gear ratio							
Hp—1st reduction	4.26	10.21	6.881	7.521	2.19	4.35	2.56
Lp—1st reduction	3.27	8.47	4.587	6.446	....	3.59	2.16
2nd reduction	7.37	6.53	9.592	7.82	11.77	3.47	5.71
Combined	31.4, 24.1	66.6, 55.3	66.0, 44.0	58.9, 50.4	25.7	15.1, 12.46	14.62, 12.31
Pitch circle diameter, inches							
Hp pinion—1st reduction	5.294	7.373	8.4	7.754	13.0	9.833	9.60
Lp pinion—1st reduction	6.515	8.878	12.6	9.046	....	11.919	11.40
1st reduction wheels	22.552, 21,311	75.25	57.8	58.318	28.4	42.761	24.60
2nd reduction pinions	8.459	15.975	14.88	20.250	14.786	24.000	15.064
Main gear	62.305	104.281	142.78	158.500	174	83.356	85.996
Face width, inches							
1st reduction	11.224	13	17	30.5	29.52	29.512	21.872
2nd reduction	18.020	28	33.25	33.75	50.39	29.703	19.503
Diametral pitch							
1st reduction	7.367	6.6456	5	6.1902	5 (hp only)	6.712	5.0
2nd reduction	6.147	5.6962	2.82	4	4.6667	4.667	3.85
Tooth pressure, lb per inch face							
Hp—1st reduction	338	438	436	478	565	898	615
Lp—1st reduction	357	438	436	478	....	898	717
2nd reduction	560	958	885	1247	637	1590	1126, 1312
K factor †							
Hp—1st reduction	78.9	68	59.5	69.8	63.4	112.3	89
Lp—1st reduction	71.6	67	42.1	61	....	96.3	92.1
2nd reduction	75.3	65	65.7	69.4	46.7	85.3	87.3, 102.3
Pitch line speed, fpm							
1st reduction	10860, 10260	11580	13380	10158	11220	15570	14710
2nd reduction	4080		3438	3527	5832	8718	9020
Helical angle							
1st reduction	about 30°	18° 18′ 36″	about 30°	40° 3′ 6″	45°	about 18°	45°
2nd reduction	about 30°	18° 18′ 36″	about 30°	23°	30°	30°	about 44.5°
Total weight of gear, lb ‡	16000	90000	127000	176000	239660	53367	48000
Weight of gear, lb/shp	6.40	22.5	21.16	19.55	14.1	2.14	1.85
Name of manufacturer		Westinghouse	General Elec.	Falk	DeLaval		

* *S.S. America*—Twin screw, 2 sets of triple series turbines, Hp double reduction, Ip and Lp single reduction gears.

† K factor $= \dfrac{\text{Tooth pressure per inch of face}}{\text{Pinion P.C.D.}} \times \dfrac{R+1}{R}$ where R = gear reduction ratio.

‡ Weight includes main thrust bearing except for *S.S. America*.

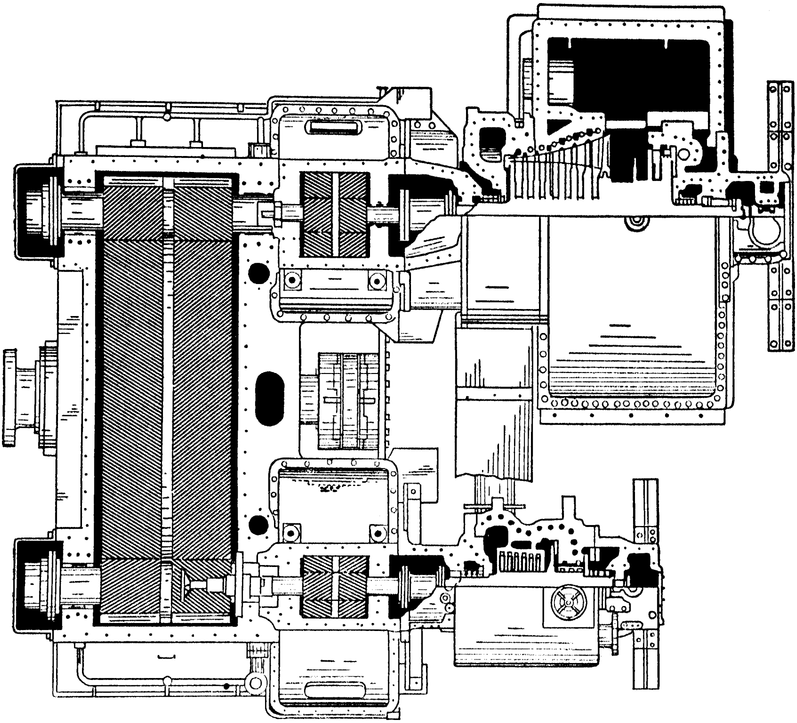

FIG. 2. Cross-compound geared (speed-reducing) steam turbine unit for ship propulsion. Unit is typical C3 design, see Table 1. (Courtesy of General Electric Co.)

as their respective secondary pinions. The arrangement in which the high-speed elements straddle the low-speed gear has the advantage of compactness and lower weight, but the span between the two helices of the high-speed pinions may necessitate a central bearing for these shafts. The main gear case structure, however, is of smaller proportions, and the housings for the primary gears are overhung from, but integral with, the main casing. Both designs of nested type gears have the disadvantage of lack of flexibility between primary and secondary gears.

Both the articulated and nested designs of double reduction gear have been referred to as the "free-train" type, i.e., "one in which the meshing contact of a pinion and a gear is not dependent upon nor restricted by the meshing contact of the same pinion and another gear." The opposite is true of the locked train type.

In the locked-train gear, also called the *divided-train* or *twin-drive,* each high-speed pinion transmits torque from a turbine through two primary wheels, one on either side, thence through quill shafts and low-speed pinions to the main gear wheel. An installation with high-pressure and low-pressure turbine thus would have two twin gear trains, each with one high-speed pinion, two primary wheels, and two secondary pinions, with a correspondingly larger number of bearings than a "free-train" gear.

In this type of gear it is essential, within limits, that the torque be divided equally between the halves of each train, both for ahead and astern operation. This has been accomplished by fitting a half coupling at each end of the quill shafts with fine teeth and choosing the number of teeth in the couplings, primary wheels, and secondary pinions so as to permit fine adjustment of the angular position of either of the primary wheels with its tandem secondary pinion. The locked-train gear by dividing the load on the main gear wheel over four pinions reduces the width of the gear faces and gives a compact, lightweight gear. It has been widely used in United States naval work. Some installations have been made for moderate powers with a single turbine driving through a locked-train, double-reduction gear.

In recent years single-reduction marine propulsion gears have been used for diesel engine and diesel-electric drive. Such arrangements permit the power to be divided among one, two, three, or four diesel engines or electric motors for each propeller shaft, permitting use of smaller and faster engines or motors.

With the accuracy attainable in modern gear cutting, pitch line speeds up to 15,000 ft per min are used satisfactorily, and K values of 75 for merchant work and higher values for naval work.

A planetary double-helical gear was used in United States *Eagle boats* during World War I, with a single sun pinion driven by a steam turbine transmitting power through three planet wheels mounted on a frame attached to the line shafting, gearing into a stationary internal gear.

Attention is again being given to planetary gearing and its possible development for marine propulsion, and also to the use of hardened gear teeth, permitting higher loads and smaller gears.

ELECTRIC DRIVE. This arrangement consists of one or more generators supplying current to electric motors directly connected to the propeller shaft. Installations from a few hundred up to 50,000 shp per shaft have been made.

Turboelectric installations generally have one three-phase two-pole high-speed a-c generator driving one low-speed multipole motor on the propeller shaft. Standard commercial frequencies do not have to be used, and design generator rpm typically may be from 2000 to 5400. With this drive there is a fixed ratio between generator and motor speed, and turbine speed must be varied to change propeller and ship speed. After experience was gained with induction motors in the first applications, it was possible to use the more desirable synchronous motor, with a pole face winding, so that it may be operated as a squirrel-cage induction motor during maneuvering. This has proved to be a satisfactory means of handling maneuvering of the propeller, hence this type of motor is now used on most a-c installations. Turboelectric a-c drive is applicable where (1) large amounts of power are needed at times for other than propulsion purposes (tankers, dredges, self-unloaders), (2) schedules permit running on part of the propelling units (passenger), and (3) full ahead power is required for astern (Coast Guard cutters, warships).

As compared with the geared-turbine drive, the turboelectric drive weighs more, takes up more space, costs slightly more, and its efficiency is about 6% less owing to the electrical losses. Electric drive permits greater flexibility in the machinery arrangement. The turbine rotates in one direction and idles when standing by, both of which tend toward a simple and rugged turbine design which minimizes thermal effects.

Alternating-current propulsion is also used with diesel engines. When several engines are used, this necessitates parallel operation of generator units on the propulsion motor bus over a speed range from about 30% to full speed on the engines, with varying load. This type of drive is relatively new with limited applications.

Direct-current propulsion has been used extensively, owing to its inherently greater control flexibility and superior torque characteristics. It has been used with turbine drive, but its present applications are confined to diesels, where several small, high-speed, lightweight, nonreversible engines are used to drive a single slow-speed propeller. Generators normally are shunt-wound, separately excited, and direct-connected to the engine. Motors usually are of the same type, may be direct-connected to the propeller, or one or more motors may be connected to the propeller through gearing. The overall transmission efficiency by either method is about 85%. The field of application for d-c diesel-electric propulsion is limited to the lower powers where flexibility of control is advantageous; this includes tugboats, ferryboats, fireboats, self-unloaders, small tankers, self-propelled dredges, ice-breakers, and tenders.

25. MARINE STEAM PLANTS

STEAM CONDITIONS used in marine steam plants are low in comparison to stationary plants, primarily owing to the smaller size of unit involved, on the average. At present, the following are justified economically: up to 4000 shp uniflow engines, 300 psig—600 F—4 in. Hg—240 F feedwater temperature; 4000 to 8500 shp geared turbine—450 psig—750 F—1.5 in. Hg—325 F feedwater temperature; 8500 to 20,000 shp geared turbine—600 psig—850 F—1.5 in. Hg—380 F feedwater temperature.

One 8000-shp experimental application of gas reheat, at 1200 psig, 750 F, has been made. Also one class of about ten ships with 11,000 shp geared turbines has been fitted with a double steam reheat plant at 1450 psig, 750 F/565 F/565 F.

ENGINES. At present, applications of steam engines to ship propulsion use the uniflow type. During World War II, the emergency fleet of Liberty ships was powered by conventional three-cylinder triple-expansion open-frame type marine engines. These are uneconomical for normal American operation.

The only problem associated with application of the uniflow steam engine to sea-going ships is removing the cylinder oil from condensate. Until World War II, applications were limited to those where large quantities of fresh water were available, either for 100% make-up, or for backwashing the sand-bed type filters used for removing emulsified oil from engine condensate. This now is accomplished by a pressure-leaf type of filter using activated diatomaceous earth as the filtering medium. The filter is cleaned manually, requires no fresh water. Approximately 1 to 1 1/2 lb of filter earth is required to remove 1 lb of cylinder lubricating oil.

While the rpm of the uniflow type engine is higher than the triple expansion type, it is not too high for connecting directly to the propeller, as is done in the normal installation.

The steam rate for uniflow engines for steam conditions stated above is about 10 to 11 lb per indicated horsepower. Mechanical efficiency is about 92 to 94%.

TURBINES. The essential difference between land and marine turbines is that the latter operate at variable speed. The transmission system necessary to reconcile the tur-

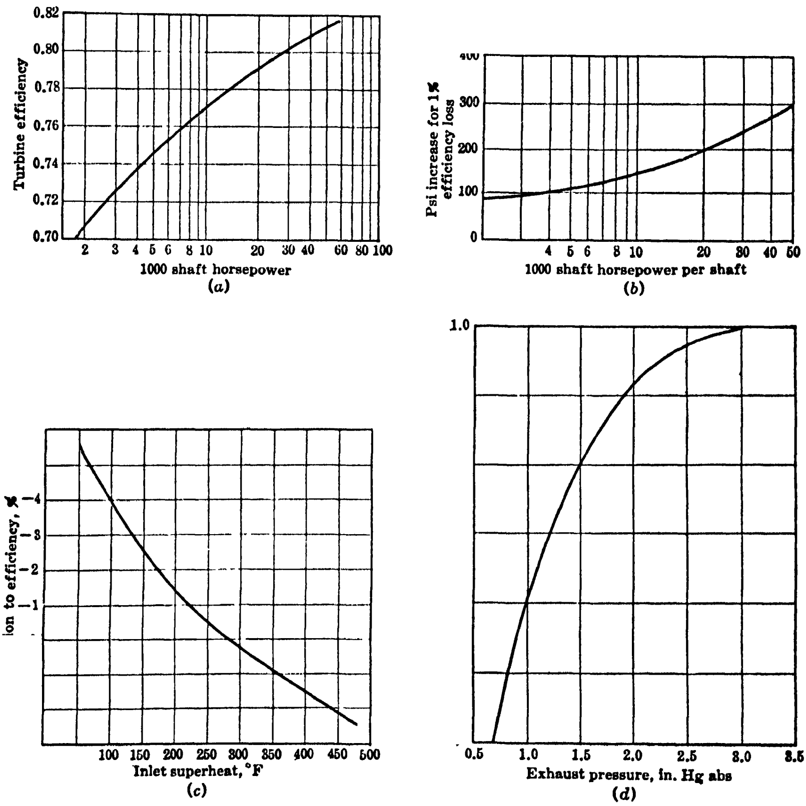

FIG. 3. Marine geared turbine efficiency. (*a*) Efficiency at 450 psia–740 F -1.5 in. Hg abs. (*b*) Pressure correction for *a*. (*c*) Superheat correction for *a*. (*d*) Exhaust pressure correction for *a*.

bine and propeller rpm gives the turbine designer a relatively free choice of turbine rotative speeds, although the tendency with electric drive is to use 3600 rpm. With electric drive, reversing is accomplished electrically; with geared drive it is necessary to use an astern turbine, usually in the same turbine casing as the ahead turbine.

American geared-turbine units consist of a high-pressure and a low-pressure turbine connected to a double reduction gear unit, in most installations. The astern turbine is incorporated in the low-pressure turbine casing where it rotates in a vacuum, when the unit is going ahead. For merchant ships, the astern turbine is designed for about 80% of full ahead torque at about 50% of full ahead rpm, with normal ahead steam flow. This results in a short simple design, usually satisfied by one two-row Curtis stage (see Section 8), followed by one Rateau stage or by another two-row Curtis stage. A *deflector* is installed between the last rows of the ahead and astern blades to prevent impingement of exhaust steam on the last stage of the ahead turbine. This arrangement of astern turbine results in low loss. Except for special cases, astern operation for periods exceeding one-half hour should be avoided, owing to the high temperatures generated in the ahead turbine when it is rotated backward.

The free choice of rpm permitted with gear drive has resulted in higher speeds, fewer stages, smaller diameters, and shorter, more rugged units. The separation of expansion into two separate casings permits another freedom of choice of rpm, with beneficial results.

Aside from the variable speed required for maneuvering, starting, and stopping, some marine applications, particularly naval ships, require that the turbines operate for long periods at about one-half rpm and one-eighth power. This requirement is not as important for merchant ships, which normally operate at full power. The merchant turbine therefore has its point of best economy at normal ship, and is provided with nozzle control (usually manual) down to about 60% load. Below this point control is by throttling. Naval turbines sometimes have the point of best economy at one-half power, and a steam rate at one-eighth load about equal to that at maximum load.

Typical efficiencies of marine geared-turbine units, measured at the gear output coupling, are given in Fig. 3.

BOILERS. In recent years there have been practically no installations of "Scotch" boilers (marine firetube). The choice for merchant ships generally is between the straight-tube sectional header and the bent-tube two-drum types. (See also Section 7.) For steam pressures of 400 psi and over, air heaters or economizers, or both, are used to keep efficiency high. (See Section 7.) Boilers frequently are specified for an efficiency of 87.5 to 88%. Furnace water walls are common, and a complete air casing usually is specified to give a cooler fire room and prevent furnace gas leakage. Combustion control and automatic feed control usually are fitted.

Two boilers per ship are installed in most lower-powered ships, and two boilers per shaft in the higher-powered ships. A larger number of boilers results in less loss of speed due to outage, and sometimes more than two are fitted for this reason, particularly in ships carrying perishable cargo. Very high-powered ships may require more than two boilers per shaft because of space restrictions limiting the size of individual units.

The commonest marine boilers have capacities of 15,000 to 50,000 lb per hr. Newer designs of highly powered merchant ships will increase this range to 100,000 lb per hr. Nearly all boilers are of the natural circulation type. Evaporation rates (including economizers) range from 4 to 8 lb per sq ft water heating surface. Heat releases range from 35,000 to 100,000 Btu per cu ft of furnace volume per hour at normal rating. Ratings of naval boilers are much higher, with heat releases of 500,000 Btu per cu ft per hr. Marine boilers usually are specified to have an overload rating of 25 to 50%, depending on the number of boilers installed. With one boiler of a two-boiler installation out of service, and the other operating at 150% rating, it is possible to develop approximately 75% power, about 90% speed. Merchant boilers weigh 2.5 to 4.0 lb per lb per hr evaporation, occupy 0.07 to 0.11 cu ft per lb per hr evaporation.

Forced draft is employed for oil-fired applications. Draft pressures at normal rating ranges up to 10 in. water pressure, naval boilers ranging up to 60 in. Usually one blower per boiler is used, with outlet damper or inlet vane control for combustion regulation. The present trend is to high stack exit velocities (4000 ft per min at normal rating) so gases and soot will not dirty the decks. Burners are of the mechanical pressure-atomizing type, capable of firing about 900 lb of oil per hour per burner. Some steam atomizing burner installations have been made. These have a wider range, desirable for combustion control and maneuvering.

CONDENSERS (see also Section 9). Marine condensers are of the surface type. In rivers, harbors, and shallow waters, circulating water may be dirty or sandy; therefore all installations have high and low sea suctions, controllable speed pumps, and strainers in the sea openings.

Design conditions normally are 26 in. vacuum for steam engines, 28½ in. vacuum for turbines, with sea temperatures of 70 to 75 F. For colder waters, such as the North Atlantic, 29.0 in. vacuum is used. In general, optimum design corresponds to about a 20-degree difference between vacuum temperature and sea temperature.

Condensers, mounted below the low-pressure turbine, may hang from the turbine or from beams supporting condenser and turbine. Construction usually is welded steel shells, naval brass or Muntz metal tube sheets, cast-iron heads, and aluminum brass or copper nickel tubes. The water sides are fitted with zinc plates to protect against electrolytic action.

Design water velocities generally do not exceed 6.5 to 7.0 ft per sec. Tubes for auxiliary condensers may be $^5/_8$ or $^3/_4$ in. in diameter, whereas main condensers usually are $^3/_4$ in. Heat transfer coefficients are in accordance with the Heat Exchange Institute Standards (see Section 9).

Condensers have integral-type air coolers; air is removed by twin two-stage steam ejectors mounted on condensate-cooled inter- and after-condensers.

FEED SYSTEMS for regenerative heating are used with two, three, or four extractions, corresponding to feed temperatures of about 240, 325, and 380 F. Air ejector inter- and after-condensers and the gland leak-off condenser are condensate-cooled. The first-stage heater uses extracted steam, normally is fitted with a drain cooler to cool drains to approximately 15 degrees above entering condensate temperature. The second-stage heater normally is of the deaerating type, having a reserve capacity of approximately 10 min supply of feedwater. The pressure in the heater is about 10 psig. The heater receives steam from an extraction opening on the main turbine and from the exhaust of steam turbine-driven pumps. Third-and fourth-stage heaters use extracted steam, and drains are cascaded to the deaerating heater.

Condensate pumps are motor-driven; boiler feed pumps usually are of the centrifugal turbine-driven type. Stand-by feed pumps generally are reciprocating steam-driven. Some installations of reciprocating feed pumps of the constant-stroke or variable-stroke type have been made, with motor drive. These units are efficient and economical of fuel in the smaller sizes, but operators prefer centrifugal pumps for 6000 shp and above.

AUXILIARIES. Auxiliary power is supplied by auxiliary generator sets driven by geared turbines or diesels for turbine-driven and diesel-driven ships, respectively. Each installation includes one stand-by generating set, and installations range from two 300-kw units in cargo ships to four 1200-kw units on high-powered passenger ships. Smaller installations use direct current at 240 volts, and usually three-wire sets are fitted to provide 120 volts for lighting, etc. Direct-current power is required for cargo winches since a-c equipment is not yet considered satisfactory for this purpose. On large installations the generator sets may include an a-c generator at 450 volts, three-phase, 60-cycle, and a d-c generator for supplying cargo winch requirements. Each generator set exhausts to its own condenser, or all sets may exhaust to an auxiliary condenser with a stand-by connection to the main condenser.

The lubricating-oil system serving the main turbines and gears is of the gravity type, where the lubricating-oil pumps take suction from the sump tank under the gear case and discharge through magnetic strainers and lubricating-oil coolers to tanks located at sufficient elevation to maintain pressure of 10 psig at turbine and gear oil inlet connections.

Pumps (see also Section 5) are of the vertical type to conserve floor space. Centrifugal, reciprocating, and rotary pumps are used when required. Oil pumps are of the rotary type with reciprocating steam-driven stand-bys. Other pumps are of the centrifugal type with some reciprocating steam-driven stand-bys, particularly in services requiring suction lift. Pump materials follow the recommendations of the Hydraulic Institute Standards, and sea-water pumps generally have all-bronze casings. (See Section 5.)

Each ship is fitted with air compressors for air-operated tools, operation of controls, and in some cases for operation of boiler soot blowers. Units up to 60 cu ft per min are air-cooled and larger units water-cooled.

26. MARINE DIESEL ENGINES

The principal difference between land and marine engines is that marine diesels must operate at variable speed, and means of reversing must be provided. Various means are available for reversing propeller thrust. Direct-connected engines or engines using reduction gears may be of the reversible type. Nonreversible engines may be used with electric drive in which the electric motor is reversed, or they may be used with reverse reduction gears having an electric, air, or hydraulically operated clutch for engaging or disengaging the gears.

Marine diesel engine auxiliaries are essentially the same as for stationary applications. However, an additional cooling system is necessary since the engines are fresh-water cooled. Heat is transferred by a cooler to sea water, which may also be circulated through the lubricating oil cooler. Compressed air for starting and maneuvering is stored in tanks at 250 to 390 psig. American Bureau of Shipping requires that the tanks have a capacity within starting limits to start reversible engines twelve consecutive times and to start nonreversing engines six consecutive times. Air compressors must have a capacity sufficient to charge the air tanks from atmospheric to full pressure in one hour.

LOW-SPEED ENGINES. Applications of the low-speed (90 to 180 rpm) direct-connected type are principally of two engine types, the Sun-Doxford opposed-piston and the two-cycle single-acting. They have been used in units up to 7500 shp and are large and heavy. Attempts to reduce size and weight by increasing rpm results in loss of propulsive efficiency. These engines have been adapted to burn centrifuged and heated boiler oil. The installation of direct-drive diesel engines of this type is not favored by United States owners.

MEDIUM- AND HIGH-SPEED ENGINES. Medium-speed engines (180 to 450 rpm) are used with reduction gearing with a flexible coupling between engine and gear. The engines are of reversible type and burn heavy diesel oil. Installations have been made of two engines per shaft for 4000 and 6000 shp and of two and four engines per shaft for 8500 shp. The clutch or coupling is preferably of the electric type, and maneuvering can be accomplished on multiple-engine drive without using air by operating one-half the engines ahead and one-half astern, and engaging the proper clutch. Not many American operators use this type of installation because of the personnel and maintenance problem. Where operators have organized their own maintenance staff, these installations have proved economical.

High-speed engines (500 to 800 rpm) are used with reverse reduction gearing or electric drive. The maximum size unit is approximately 2000 shp, and one, two, three, or four of these units may be coupled to one shaft. These engines are smaller and lighter in weight than those described above and require light diesel oil for satisfactory operation. They are used extensively for river, harbor, and lake craft in sizes up to 2000 to 3000 shp per shaft. There have been some applications to sea-going ships in sizes up to 6000 shp per shaft on twin-screw ships. However, under normal conditions they are not competitive with steam machinery at these high powers and not selected by American operators unless special conditions exist.

27. SHIP'S SERVICE MACHINERY

HULL MACHINERY ordinarily means machinery not serving the propulsion plant. It includes cargo winches, mooring winches and capstans, anchor windlass, steering gear, and towing machines.

Cargo Winches. Because of fineness of control and speed of reversing, steam-driven winches lend themselves well to cargo handling. Straight electric drive with direct or alternating current cannot readily equal the steam winch, but a d-c-motor-driven winch handles all normal requirements satisfactorily. The d-c-motor-driven winch is applied on all modern United States cargo and cargo-passenger ships.

Mooring Winches and Capstans. Winches for mooring usually have two gypsies and no drum. Mooring capstans have a single hoisting head, often with the motor located below the weather deck. These machines generally have a maximum service-line pull of 15,000 to 25,000 lb, depending on the size of lines, full load speeds of 25 to 50 ft per min, and light-line speeds of 50 to 100 ft per min.

The anchor windlass is usually required to hoist the anchor and chain at 30 to 36 ft per min from 180 to 360 ft depths. The anchor chain is heaved through a hawsepipe in which friction loss runs from 35 to 40%, in good designs. The chain is operated by a *wildcat*, a crude sprocket drive. Motors for direct-geared drive usually are series-wound with light shunt fields for better lowering characteristics and have a one-half hour full-load rating. Motors for hydraulic transmission may be d-c shunt wound or a-c squirrel cage.

Steering gears fall into two groups.

(1) Direct-driven gears with or without follow-up mechanism, for rudder torques up to about 175,000 ft-lb. They may be steam or electric driven, with little difference in weight or cost.

(2) Steering gears for higher torques are almost invariably built with full storage motion follow-ups and with electrohydraulic drive.

The steering-gear power plant is fitted in duplicate, each having sufficient capacity to turn the rudder at designed conditions. For hydraulic transmission, this consists of two electric motors, each driving a variable stroke pump of the parallel-piston "swash plate" type or the radial-piston eccentric type.

VENTILATION, HEATING, AND AIR CONDITIONING (see also Section 12). Modern shipboard practice combines ventilation and heating or cooling into a single system, using unheated or cooled air in warm weather and heated air in cold weather. Marine installations also involve the additional problems of watertight integrity, structural integrity, fire- and ratproofing, all of which are subject to the rules and regulations of various bodies, such as the American Bureau of Shipping, U. S. Coast Guard, U. S. Public Health Service.

Living quarters are mechanically ventilated and heated for personal comfort. Sufficient air is provided to limit temperature rise in summer to 10 F in living spaces and 15 F in working spaces (except machinery spaces). A complete air change is provided every 4 to 6 min in living spaces, and every 10 to 15 min in working spaces. The minimum air per person is usually 30 cu ft per min for public spaces and 40 to 50 cu ft per min for quarters. Heating may be by any one of three methods: direct radiation, hot blast, or unit heater, with steam at 35 psig as the usual heating medium.

Air conditioning is usually provided for passenger and crew quarters and public spaces on passenger ships. Refrigeration load is about one-half ton per person.

Cargo holds are mechanically ventilated, but may also be provided with a system that will ventilate, recirculate, and dehumidify. Dehumidification is obtained by introducing dry air, produced in a silica-gel type of dehumidifying unit, at a rate of 1000 cu ft per min dry air per 200,000 cu ft cargo space.

Main machinery spaces are fitted with a mechanical supply system that discharges air to the watch stations. Sufficient air is provided to limit temperature rise at watch stations to 15 to 20 F, and to an overall rise of 25 F. Heat liberated is approximately 1 1/2 to 2 1/2% of the heat in the fuel burned.

REFRIGERATION (see also Section 11). American ships are fitted with refrigeration for preservation of ship stores and with refrigerated boxes for carrying cargo. Normal installations use "Freon 12" as the refrigerant and usually are of the direct-expansion type, although brine systems are used in installations of large size, or for reliable and accurate control. All installations for cargo refrigeration must meet the requirements of the regulatory bodies. The usual size of cargo refrigeration installation is 15,000 to 100,000 cu ft, except for completely refrigerated ships engaged in special service.

DISTILLING PLANTS. Before World War II, cargo and passenger ships carried the fresh water required for the voyage in tanks. Small evaporating plants were used for boiler make-up, and emergency evaporators for evaporating salt water.

Large low-pressure distilling plants developed by the Navy received extensive application during the war on naval vessels and troopships, and were found to be satisfactory. Most post-war ships are fitted with such plants. These installations range in size from a single 6000-gallon per day single-effect plant for a cargo ship to two 60,000-gallon per day double-effect units for passenger ships. In normal commercial installations these plants are operated on a turbine bleeder at approximately 8 psia; in an emergency the capacity may be materially increased by operating from the next higher bleeder at approximately 5 psig. A single-effect plant operating on the 8 psia extraction point will distill approximately 43 tons of water per ton of fuel, and a large double-effect plant on the same bleeder will distill approximately 85 tons per ton of fuel. These plants are built of corrosion-resistant materials and meet the requirements of the U. S. Public Health Service for quality of drinking water.

The distilling plants, including pumps and controls, generally are constructed as an integral and automatic unit, requiring practically no attention. When operating from the low-pressure extraction point, scale forms slowly. A passenger-ship installation would consist of two or more units having a total clean-tube capacity when operating on the low-pressure extraction point of approximately 30% in excess of estimated requirements, usually 60 gallons per person per day plus make-up required by the machinery plant.

SECTION 16

ELECTRIC POWER

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ELECTRIC POWER

Power for an industrial or commercial plant is either purchased or generated by the user. From the source, power is distributed over the plant *distribution system*, which consists of *switchgear*, *transformers*, *substations*, *cable*, and *accessories*. Application of these is summarized in this section.

Alternating current is most widely used because of its availability and because it can be transmitted at high voltage and economically transformed to a lower voltage for utilization. Sixty cycles is now almost universal except for some large steel mills and the industrial area around Buffalo, N. Y., and in Ontario, Canada.

Direct current is utilized where the variable-speed characteristics of d-c motors is essential or desirable for a given application. Large amounts of d-c power are also used for electrolytic processes.

BASIC DATA *

Certain basic useful data and formulas are summarized here.

CIRCUIT CONSTANTS. Every circuit or transmission line has definite electrical characteristics or constants, depending on the material of conductors, their cross-sectional areas, outside diameters, and spacing. Electrically, each line consists of resistance and inductance in series, through which current must flow, and a capacitance between conductors into which, in a-c circuits, charging current flows.

Resistance of a unit length of line is a function of the material of the conductors and cross-sectional areas.

Inductance of a unit length of line is a function of the outside diameter of conductors and the distance between them. The inductance of one conductor of a single- or 3-phase circuit may be determined from

$$L = \left(1.41 \log_{10} \frac{s}{r} + 0.1524\right) \times 10^{-4} \tag{1}$$

where s/r is large as in overhead lines or

$$L = \left(1.41 \log_{10} \frac{s - r}{r} + 0.1524 + 0.304 \frac{d}{s}\right) \times 10^{-4} \tag{2}$$

where s/r is small as in multiconductor cables. L = inductance of one conductor, henrys per 1000 ft; s = spacing between center lines of conductors, inches; r = radius of conductor, inches; d = diameter of conductor, inches. If conductors of a 3-phase circuit are horizontally spaced, $s = \sqrt[3]{a \times b \times c}$, where a, b, c = respectively, distance between phases 1 and 2, 2 and 3, and 1 and 3.

Capacitance of a line is the function of the outside diameter of the conductors and their spacing. Where s/r is large,

$$C = \frac{7.354}{\log_{10} (s/r)} \times 10^{-9} \tag{3}$$

where C = capacitance, farads per 1000 ft, of one conductor to neutral. In industrial distribution systems, circuits are too short to cause appreciable error if capacitance is neglected.

* Revised by D. L. Beeman.

Kva power, voltage, and current relations in various types of circuits are given in Table 1.

Table 1. Formulas for Amperes, Horsepower, Kilowatts, and Kilovolt-amperes *

Desired Data	Alternating Current			Direct Current
	Single-phase	2-phase, 4-wire †	3-phase	
Kilowatts	$IEF/1000$	$2IEF/1000$	$1.73 \times IEF/1000$	$IE/1000$
Kilovolt-amperes	$IE/1000$	$2IE/1000$	$1.73IE/1000$	....
Horsepower	$IEeF/746$	$2IEeF/746$	$1.73 \times IEeF/746$	$IEe/746$
Amperes	$\text{Hp} \times 746/EeF$	$\text{Hp} \times 746/2EeF$	$\text{Hp} \times 746/1.73EeF$	$\text{Hp} \times 746/Ee$
Amperes	$\text{Kw} \times 1000/EF$	$\text{Kw} \times 1000/2EF$	$\text{Kw} \times 1000/1.73EF$	$\text{Kw} \times 1000/E$
Amperes	$\text{Kva} \times 1000/E$	$\text{Kva} \times 1000/2E$	$\text{Kva} \times 1000/1.73E$	....

* I = amperes; E = volts; e = efficiency; F = power factor; Hp = horsepower; Kw = kilowatts; Kva = kilovolt-amperes.

† In 3-wire, 2-phase circuits, the current in the common conductor is 1.41 times current in either of the other conductors.

VOLTAGE DROP, POWER LOSSES, AND POWER FACTOR IN POWER TRANSMISSION can be determined easily if line or cable constants and characteristics of power supplied to or received from the line are known. The most common methods of transmission are: Direct current, and single-phase, 2-phase, 4-wire, and 3-phase alternating current.

Notation. E = voltage between conductors; e = voltage line to neutral = $E/1.73$; f = frequency, cycles per second; I = current, amperes; kva = kilovolt-amperes; L = inductance of one conductor, henrys; P = power, kilowatts; R, r = resistance of one conductor, ohms; X = reactance of one conductor, ohms = $2\pi fL$; Z = impedance of one conductor, ohms = $\sqrt{r^2 + X^2}$; θ = displacement angle between voltage and current; $+\theta$ = lagging current; $-\theta$ = leading current; $\cos\theta$ = power factor. (See Art. 16.) Subscripts g, r, l, respectively, indicate: at generating end, at receiving end, and loss in line.

DIRECT CURRENT.

$$P_g = \frac{E_g I}{1000}; \quad P_r = \frac{E_r I}{1000}. \quad E_l = 2RI = E_g - E_r$$

$$P_l = \frac{E_l I}{1000} = \frac{2RI^2}{1000} = P_g - P_r$$

$$\text{Percentage regulation} = \frac{E_l}{E_r} \times 100$$

$$\text{Percentage power loss} = \frac{P_l}{P_g} \times 100$$

ALTERNATING CURRENT. Single Phase.

$$E_g = \sqrt{(E_r \cos\theta_r + 2rI)^2 + (\pm E_r \sin\theta_r + 2XI)^2}$$

$$\text{Percentage voltage regulation} = \frac{E_g - E_r}{E_r} \times 100$$

$$\cos\theta_g = \frac{E_r \cos\theta_r + 2rI}{E_g}$$

$$\text{Kva}_g = \frac{E_g I}{1000} = \frac{2e_g I}{1000}$$

$$P_g = \frac{E_g I \cos\theta_g}{1000} = \frac{2e_g I \cos\theta_g}{1000}$$

$$P_l = P_g - P_r = \frac{2rI^2}{1000}$$

$$\text{Percentage power loss} = \frac{P_l}{P_g} \times 100$$

Three Phase.

$$E_g = 1.73\sqrt{(e_r \cos\theta_r + rI)^2 + (\pm e_r \sin\theta_r + XI)^2}$$

$$\text{Percentage voltage regulation} = \frac{E_g - e_r}{E_r} \times 100$$

$$\cos\theta_g = \frac{e_r \cos\theta_r + rI}{e_g}$$

$$\text{Kva}_g = \frac{1.73E_gI}{1000} = \frac{3e_gI}{1000}$$

$$P_g = \frac{1.73E_gI\cos\theta_g}{1000} = \frac{3e_g\cos\theta_g}{1000}$$

$$P_l = P_g - P_r = \frac{3rI^2}{1000}$$

$$\text{Percentage power loss} = \frac{P_l}{P_g} \times 100$$

Example. A substation operates at 2200 volts, 60 cycles, 3-phase for a load of 250 kw at 80% power factor lagging. Power is received from a generating station, 5000 ft distant over a line of three No. 4 copper cable conductors spaced 24 in. apart in a horizontal plane.

From Table 5, R per conductor = 0.258 × 5 = 1.29 ohms. Equivalent equilateral spacing is $S = \sqrt[3]{a \times b \times c} = \sqrt[3]{24 \times 24 \times 48} = 30.2$ in. Reactance may be calculated from the inductance and frequency by means of eq. 1. From eq. 1, inductance per conductor =

$$L = [1.11 \log_{10}(30.2/0.117) + 0.1524] \times 5 \times 10^{-4} = 0.00178$$

$$\text{Reactance} = 2\pi fL = 2 \times 3.1416 \times 60 \times 0.00178 = 0.675 \text{ ohm per conductor}$$

$$\text{Impedance per conductor} = Z = \sqrt{r^2 + X^2} = \sqrt{1.29^2 + 0.675^2} = 1.454 \text{ ohms}$$

$$I = (250 \times 1000)/(1.73 \times 2200 \times 0.8) = 82 \text{ amperes}$$

$$\cos\theta_r = 0.8; \quad \theta_g = 36° 52'; \quad \sin\theta_r = 0.6$$

$$\text{Voltage at receiving end} = e_r = 2200/1.73 = 1270 \text{ volts}$$

Voltage at generating end = E_g

$$= 1.73\sqrt{(1270 \times 0.8 + 1.29 \times 82)^2 + (1270 \times 0.6 + 0.675 \times 82)^2} = 2400 \text{ volts}$$

$$\text{Percentage voltage regulation} = 100 \times (2400 - 2200)/2200 = 9.1\%$$

Power factor at generating end of line is

$$\cos\theta_g = (1270 \times 0.8 + 1.29 \times 82)/(2400/1.73) = 0.807$$

$$\text{Kva at generating end} = \text{Kva}_g = (1.73 \times 2400 \times 82)/1000 = 341$$

$$\text{Line loss} = P_l = (3 \times 1.29 \times 82^2)/1000 = 26 \text{ kw}$$

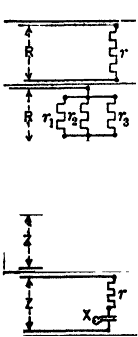
$$\text{Power supplied to line} = P_g = (1.73 \times 2400 \times 82 \times 0.807)/1000 = 276 \text{ kw}$$

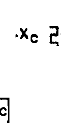
$$\text{Percentage power loss} = (26/276) \times 100 = 9.4\%$$

Two-phase, Four-wire. Calculations for transmission of power 2-phase, 4-wire, can be made by considering the 2-phase circuit as two independent single-phase circuits, each carrying half the power, and making use of the single-phase formulas.

Relation between resistance, inductive reactance, and capacitive reactance in a-c circuits is shown by the formulas and diagrams of Table 2, in which R, r = resistance; X_L, X_C = inductive and capacitive reactance, respectively; Z = impedance.

Table 2. Relation between Resistance and Reactance





$R = r$	$Z = \sqrt{r^2 + (X_L - X_C)^2}$
$R = 1/(1/r_1 + 1/r_2 + 1/r_3)$	$Z = 1/\sqrt{(1/r)^2 + (1/X_L)^2}$
$R = \{1/(1/r_1 + 1/r_2)\} + r_3$	$Z = 1/\sqrt{(1/r^2) + (1/X_C)^2}$
$Z = \sqrt{r^2 + X_L^2}$	$Z = 1/\sqrt{(1/r)^2 + (1/X_L - 1/X_C)^2}$
$Z = \sqrt{r^2 + X_C^2}$	

Table 3. Sizes, Weights and Current-carrying Capacity of Rectangular Copper Bars

Bar Size, in.	Cross Section, sq in.	Wt. per ft, lb	Current Density, amp per sq in. 750 — Current-carrying Capacity, amp	1000	Bar Size, in.	Cross Section, sq in.	Wt. per ft, lb	Current Density, amp per sq in. 750 — Current-carrying Capacity, amp	1000	Bar Size, in.	Cross Section, sq in.	Wt. per ft, lb	Current Density, amp per sq in. 750 — Current-carrying Capacity, amp	1000
2 x 1/8	.250	0.962	188	250	2 x 1/4	0.500	1.925	375	500	2 x 3/8	0.750	2.89	563	750
2 1/4 x 1/8	.281	1.080	211	281	2 1/4 x 1/4	0.562	2.165	422	562	2 1/4 x 3/8	0.842	3.24	632	842
2 1/2 x 1/8	.313	1.205	235	313	2 1/2 x 1/4	0.625	2.41	469	625	2 1/2 x 3/8	0.938	3.61	703	938
2 3/4 x 1/8	.344	1.324	258	344	2 3/4 x 1/4	0.688	2.65	516	688	2 3/4 x 3/8	1.030	3.97	772	1030
3 x 1/8	.375	1.444	281	375	3 x 1/4	0 750	2.89	563	750	3 x 3/8	1.125	4.33	844	1125
3 1/2 x 1/8	.437	1.680	328	437	3 1/2 x 1/4	0.875	3.37	656	875	3 1/2 x 3/8	1.314	5.06	985	1314
4 x 1/8	.500	1.925	375	500	4 x 1/4	1.000	3.85	750	1000	4 x 3/8	1.500	5.77	1125	1498

Table 4. Sizes and Weights of Copper Wire and Cable

Size		Rubber Insulation								Weatherproof Insulation							
		Double Braid				Triple Braid				Double Braid				Triple Braid			
		Solid		Stranded		Solid		Stranded		Solid		Stranded		Solid		Stranded	
A.W.G.	Cir. mils	Diam, in.	Lb per 1000 ft	Diam, in.	Lb per 1000 ft	Diam, in.	Lb per 1000 ft	Diam, in.	Lb per 1000 ft	Diam, in.	Lb per 1000 ft	Diam, in.	Lb per 1000 ft	Diam, in.	Lb per 1000 ft	Diam, in.	Lb per 1000 ft
...	1,000,000	...	...	1.46	3553	...	...	1.54	3637	...	...	1.37	3456	...	...	1.45	3674
...	900,000	...	...	1.40	3223	...	...	1.48	3304	...	...	1.31	3127	...	...	1.39	3332
...	800,000	...	...	1.33	2891	...	...	1.42	2968	...	...	1.24	2799	...	...	1.33	2992
...	700,000	...	...	1.27	2557	...	...	1.35	2631	...	...	1.18	2471	...	...	1.27	2650
...	600,000	...	...	1.19	2220	...	...	1.28	2290	...	...	1.11	2093	...	...	1.19	2235
...	500,000	...	...	1.09	1842	...	...	1.17	1906	...	...	1.03	1765	...	...	1.11	1894
...	400,000	...	...	1.00	1514	...	...	1.09	1573	...	...	0.94	1436	...	...	1.02	1553
...	300,000	...	...	0.90	1173	...	...	0.99	1226	...	...	0.85	1083	...		0.93	1174
0000	211,600	.70	793	0.77	833	.78	835	0.85	879	.61	723	0.71	745	.66	767	0.79	800
000	167,805	.65	646	0.71	675	.73	685	0.79	719	.56	587	0.65	604	.60	629	0.73	653
00	133,079	.61	528	0.66	556	.69	564	0.74	595	.52	467	0.60	482	.55	502	0.66	522
0	105,592	.57	439	0.61	457	.65	474	0.70	494	.47	377	0.56	388	.51	407	0.61	424
1	83,695	.53	363	0.57	377	.61	395	0.66	412	.41	294	0.47	303	.45	316	0.52	328
2	66,373	.45	276	0.50	293	.51	297	0.59	324	.37	239	0.42	246	.40	260	0.44	270
3	52,634	.42	228	0.45	238	.48	247	0.52	260	.35	185	0.38	190	.37	208	0.41	219
4	41,743	.39	190	0.42	198	.46	208	0.49	218	.32	151	0.35	155	.35	164	0.38	170
5	33,102	.36	154	0.40	166	.41	167	0.46	184	.30	122	0.32	126	.32	130	0.35	146
6	26,251	.34	130	0.36	136	.39	142	0.41	149	.28	100	0.31	103	.30	112	0.33	115
7	20,817	.30	105	0.32	108	...	...	...	...	...	...	...	...	...	...	...	...
8	16,510	.27	82.1	0.29	85.5	...	...	...	...	...	...	...	...	...	...	...	...
9	13,904	.26	68.4	0.27	70.0	...	...	...	...	...	...	...	...	...	...	...	...
10	10,382	.25	58.1	0.26	60.6	...	...	...	...	...	...	...	...	...	...	...	...
11	8,234	.24	50.0	0.25	52.3	...	...	...	...	...	...	...	...	...	...	...	...
12	6,230	.23	43.1	0.24	44.9	...	...	...	...	...	...	...	...	...	...	...	...
13	5,178	.22	38.5	0.23	39.6	...	...	...	...	...	...	...	...	...	...	...	...
14	4,107	.21	33.0	0.22	34.3		...	...	...	...	...	...	...	...	...	...	...

Table 5. Dimensions and Resistance of Copper Wire and Cable

(National Bureau of Standards)

American Wire Gage (A.W.G.)						Bare Concentric Cables of Standard Annealed Copper							
Gage No. A.W.G.	Diam, mils	Cross Section: Cir. mils	Cross Section: Sq in.	Ohms per 1000 ft at 25 C (77 F) *	Lb per 1000 ft	Size, 1000 cir. mils or A.W.G.	Ohms per 1000 ft at 25 C (77 F)†	Lb per 1000 ft †	Standard Strands: No. of Wires	Standard Strands: Diam of Wires, mils	Flexible Strands: No. of Wires	Flexible Strands: Diam of Wires, mils	Outside Diam, mils
0000	460	212,000	.166	0.0500	641	2000	0.00539	6180	127	125.5	169	108.8	1632
000	410	168,000	.132	0.0630	508	1750	0.00616	5410	127	117.4	169	101.8	1527
00	365	133,000	.105	0.0795	403	1500	0.00719	4630	91	128.4	127	108.7	1413
0	325	106,000	.0829	0.100	319	1250	0.00863	3860	91	117.2	127	99.2	1289
1	289	83,700	.0657	0.126	253	1000	0.0108	3090	61	128.0	91	104.8	1153
2	258	66,400	.0521	0 159	201	950	0.0114	2930	61	124.8	91	102.2	1124
3	229	52,600	.0413	0.201	159	900	0.0120	2780	61	121.5	91	99.4	1094
4	204	41,700	.0328	0.253	126	850	0.0127	2620	61	118.0	91	96.6	1063
5	182	33,100	.0260	0.320	100	800	0.0135	2470	61	114.5	91	93.8	1031
6	162	26,300	.0206	0.403	79.5	750	0.0144	2320	61	110.9	91	90.8	999
7	144	20,800	.0164	0.508	63.0	700	0.0154	2160	61	107.1	91	87.7	965
8	128	16,500	.0130	0.641	50.0	650	0.0166	2010	61	103.2	91	84.5	930
9	114	13,100	.0103	0.808	39.6	600	0.0180	1850	61	99.2	91	81.2	893
10	102	10,400	.00815	1.02	31.4	550	0.0196	1700	61	95.0	91	77.7	855
11	91	8,230	.00647	1.28	24.9	500	0.0216	1540	37	116.2	61	90.5	815
12	81	6,530	.00513	1.62	19 8	450	0.0240	1390	37	110.3	61	85.9	773
13	72	5,180	.00407	2.04	15.7	400	0.0270	1240	37	104.0	61	81.0	729
14	64	4,110	.00323	2.58	12 4	350	0.0308	1080	37	97.3	61	75.7	682
15	57	3,260	.00256	3.25	9.86	300	0.0360	926	37	90.0	61	70.1	631
16	51	2,580	.00203	4.09	7.82	250	0.0432	772	37	82.2	61	64.0	576
17	45	2,050	.00161	5.16	6.20								
18	40	1,620	.00128	6 51	4.92								
19	36	1,290	.00101	8.21	3.90								
20	32	1,020	.000802	10.4	3.09								
21	28.5	810	.000636	13.1	2.45								
22	25.3	642	.000505	16.5	1.94	0000	0.0510	653	19	105.5	37	75.6	533
23	22.6	509	.000400	20.8	1.54	000	0.0643	518	19	94.0	37	67.3	471
24	20.1	404	.000317	26.2	1.22	00	0.0811	411	19	83.7	37	60.0	420
25	17.9	320	.000252	33.0	0.970	0	0.102	326	19	74.5	37	53.4	374
26	15.9	254	.000200	41.6	0.769	1	0.129	258	19	66.4	37	47.6	333
27	14.2	202	.000158	52.5	0.610	2	0.163	205	7	97.4	19	59.1	296
28	12.6	160	.000126	66.2	0.484	3	0.205	163	7	86.7	19	52.6	263
29	11.3	127	.0000995	83.5	0.384	4	0.258	129	7	77.2	19	46.9	234
30	10.0	101.0	.0000789	105	0.304	5	0.326	102	7	68.8	19	41.7	209
31	8.9	79.7	.0000626	133	0.241	6	0.411	81	7	61.2	19	37.2	186
32	8.0	63.2	.0000496	167	0.191	7	0.518	64 3	7	54.5	19	33.1	166
33	7.1	50.1	.0000394	211	0.152	8	0.653	51	7	48.6	19	29.5	147
34	6.3	39.8	.0000312	266	0.120	10	1.039	32	7	38.5	19	23.4	117
35	5.6	31.5	.0000248	336	0.0954	12	1.652	20	7	30.5	19	18.5	92
36	5.0	25.0	.0000196	423	0.0757	14	2.626	12.7	7	24.2	19	14.7	73
37	4.5	19.8	.0000156	533	0.0600	16	4.176	8	7	19.2	19	11.7	58
38	4.0	15.7	.0000123	673	0.0476	......	....	......	...	.....	...	......	.....
39	3.5	12.5	.0000098	848	0.0377	.....	...	....	...	.....	...	......	.....
40	3.1	9.9	.0000078	1070	0.0299	......	.	.	....	.....	....	......	.....

* Values are for annealed copper of standard resistivity. Hard drawn copper may be taken as about 2.7% higher resistivity than annealed copper.

† The values are 2% greater than for a solid rod of cross section equal to the total cross section of the wires of the cable.

Resistivity of copper at 20 C = 0.6788 microhm-in.

Temperature coefficient of resistivity at 20 C = 0.00393 per deg C.

Table 6. Wire Gages

Gage No.	American Wire Gage B. & S. Diam, in.	American Wire Gage B. & S. Diam, mm	Steel Wire Gage (Washburn & Moen) Diam, in.	Steel Wire Gage (Washburn & Moen) Diam, mm	Birmingham Wire Gage (Stubs' Iron) Diam, in.	Birmingham Wire Gage (Stubs' Iron) Diam, mm	Old English Wire Gage (London) Diam, in.	Old English Wire Gage (London) Diam, mm	Stubs' Steel Wire Gage Diam, in.	Stubs' Steel Wire Gage Diam, mm	British Standard Wire Gage Diam, in.	British Standard Wire Gage Diam, mm	U. S. Std. Sheet Gage Thickness, in.
0 000 000	.....	.....	.4900	12.4	.....	.....	.....	.....	.....	.....	.500	12.7	.5000
000 000	.....	.....	.4615	11.7	.....	.....	.....	.....	.....	.....	.464	11.8	.4687
00 000	.....	.....	.4305	10.9	.....	.....	.....	.....	.....	.....	.432	11.0	.4375
0 000	.460	11.7	.3938	10.0	.454	11.5	.454	11.5	.....	.....	.400	10.2	.4062
000	.410	10.4	.3625	9.2	.425	10.8	.425	10.8	.....	.....	.372	9.4	.3750
00	.365	9.3	.3310	8.4	.380	9.7	.380	9.7	.....	.....	.348	8.8	.3437
0	.325	8.3	.3065	7.8	.340	8.6	.340	8.6	.....	.....	.324	8.2	.3125
1	.289	7.3	.2830	7.2	.300	7.6	.300	7.6	.227	5.77	.300	7.6	.2812
2	.258	6.5	.2625	6.7	.284	7.2	.284	7.2	.219	5.56	.276	7.0	.2656
3	.229	5.8	.2437	6.2	.259	6.6	.259	6.6	.212	5.38	.252	6.4	.2500
4	.204	5.2	.2253	5.7	.238	6.0	.238	6.0	.207	5.26	.232	5.9	.2344
5	.182	4.6	.2070	5.3	.220	5.6	.220	5.6	.204	5.18	.212	5.4	.2187
6	.162	4.1	.1920	4.9	.203	5.2	.203	5.2	.201	5.11	.192	4.9	.2035
7	.144	3.7	.1770	4.5	.180	4.6	.180	4.6	.199	5.05	.176	4.5	.1875
8	.128	3.3	.1620	4.1	.165	4.2	.165	4.2	.197	5.00	.160	4.1	.1719
9	.114	2.91	.1483	3.77	.148	3.76	.148	3.76	.194	4.93	.144	3.66	.1562
10	.102	2.59	.1350	3.43	.134	3.40	.134	3.40	.191	4.85	.128	3.25	.1406
11	.091	2.30	.1205	3.06	.120	3.05	.120	3.05	.188	4.78	.116	2.95	.1250
12	.081	2.05	.1055	2.68	.109	2.77	.109	2.77	.185	4.70	.104	2.64	.1094
13	.072	1.83	.0915	2.32	.095	2.41	.095	2.41	.182	4.62	.092	2.34	.0937
14	.064	1.63	.0800	2.03	.083	2.11	.083	2.11	.180	4.57	.080	2.03	.0781
15	.057	1.45	.0720	1.83	.072	1.83	.072	1.83	.178	4.52	.072	1.83	.0703
16	.051	1.29	.0625	1.59	.065	1.65	.065	1.65	.175	4.45	.064	1.63	.0625
17	.045	1.15	.0540	1.37	.058	1.47	.058	1.47	.172	4.37	.056	1.42	.0562
18	.040	1.02	.0475	1.21	.049	1.24	.049	1.24	.168	4.27	.048	1.22	.0500
19	.036	0.91	.0410	1.04	.042	1.07	.040	1.02	.164	4.17	.040	1.02	.0437
20	.032	0.81	.0348	0.88	.035	0.89	.035	0.89	.161	4.09	.036	0.91	.0375
21	.0285	0.72	.0317	0.81	.032	0.81	.0315	0.80	.157	3.99	.032	0.81	.0344
22	.0253	0.64	.0286	0.73	.028	0.71	.0295	0.75	.155	3.94	.028	0.71	.0312
23	.0226	0.57	.0258	0.66	.025	0.64	.0270	0.69	.153	3.89	.024	0.61	.0281
24	.0201	0.51	.0230	0.58	.022	0.56	.0250	0.64	.151	3.84	.022	0.56	.0250
25	.0179	0.45	.0204	0.52	.020	0.51	.0230	0.58	.148	3.76	.020	0.51	.0219
26	.0159	0.40	.0181	0.46	.018	0.46	.0205	0.52	.146	3.71	.018	0.46	.0187
27	.0142	0.36	.0173	0.439	.016	0.41	.01875	0.48	.143	3.63	.0164	0.42	.0172
28	.0126	0.32	.0162	0.411	.014	0.36	.01650	0.42	.139	3.53	.0148	0.38	.0156
29	.0113	0.29	.0150	0.381	.013	0.330	.01550	0.394	.134	3.40	.0136	0.345	.0141
30	.0100	0.25	.0140	0.356	.012	0.305	.01375	0.349	.127	3.23	.0124	0.315	.0125
31	.0089	0.227	.0132	0.335	.010	0.254	.01225	0.311	.120	3.05	.0116	0.295	.0109
32	.0080	0.202	.0128	0.325	.009	0.229	.01125	0.286	.115	2.92	.0108	0.274	.0101
33	.0071	0.180	.0118	0.300	.008	0.203	.01025	0.260	.112	2.84	.0100	0.254	.0094
34	.0063	0.160	.0104	0.264	.007	0.178	.00950	0.241	.110	2.79	.0092	0.234	.0086
35	.0056	0.143	.0095	0.241	.005	0.127	.00900	0.229	.108	2.74	.0084	.0213	.0078
36	.0050	0.127	.0090	0.229	.004	0.102	.00750	0.191	.106	2.69	.0076	0.193	.0070
37	.0045	0.113	.0085	0.216	.....	.....	.00650	0.165	.103	2.62	.0068	0.173	.0066
38	.0040	0.101	.0080	0.203	.....	.....	.00575	0.146	.101	2.57	.0060	0.152	.0062
39	.0035	0.090	.0075	0.191	.....	.....	.00500	0.127	.099	2.51	.0052	0.132	.....
40	.0031	0.080	.0070	0.178	.....	.....	.00450	0.114	.097	2.46	.0048	0.122	.....
41	.....	.....	.0066	0.168	.....	.....	.....	.....	.095	2.41	.0044	0.112	.....
42	.....	.....	.0062	0.157	.....	.....	.....	.....	.092	2.34	.0040	0.102	.....
43	.....	.....	.0060	0.152	.....	.....	.....	.....	.088	2.24	.0036	0.091	.....
44	.....	.....	.0058	0.147	.....	.....	.....	.....	.085	2.16	.0032	0.081	.....
45	.....	.....	.0055	0.140	.....	.....	.....	.....	.081	2.06	.0028	0.071	.....
46	.....	.....	.0052	0.132	.....	.....	.....	.....	.079	2.01	.0024	0.061	.....
47	.....	.....	.0050	0.127	.....	.....	.....	.....	.077	1.96	.0020	0.051	.....
48	.....	.....	.0048	0.122	.....	.....	.....	.....	.075	1.90	.0016	0.041	.....
49	.....	.....	.0046	0.117	.....	.....	.....	.....	.072	1.83	.0012	0.030	.....
50	.....	.....	.0044	0.112	.....	.....	.....	.....	.069	1.75	.0010	0.025	.....

POWER SOURCES

Industrial plant power supply can be divided into three broad groups: (1) plants that obtain all their power from a utility; (2) plants generating all power required for their own use with no connection to a utility system; and (3) plants generating part of their power requirements but maintaining a utility connection to supplement generation or for stand-by service. Some plants in this group sell small amounts of power to the utility during light-load conditions.

1. PURCHASED POWER IN INDUSTRY *

Purchased power refers specifically to that portion of electric energy purchased directly from a public utility for use in an industrial plant. *Power* is a term used to describe electricity in a general way; it is usually related to capacity or rate of doing work but sometimes used loosely to denote energy as well.

Purchased versus Generated Power. After utilities became large enough to supply the power demands of industry, the trend was toward more purchased power. Power is purchased rather than generated in many industrial plants because (1) purchased power is generally cheaper, (2) requires much less investment (usually investment in manufacturing equipment gives much higher rate of return), (3) industry does not have personnel trained in power plant operation and does not want maintenance and accounting problems, (4) a power plant requires too much space, and (5) a purchased power connection provides more reliable service and can be readily adjusted to meet fluctuations of demand caused by changes in production.

These reasons in favor of purchased power are by no means an indication that industrial generating plants will soon disappear. There are many places where industrial power plants are justified; they are covered fully under Article 2, p. 16-11.

MEASURING PURCHASED ENERGY. There are two broad classifications of the energy sold to industry. They are distinguished on the basis of the metering point.

Primary Power. When energy is sold by the utility at voltages above 600 volts, it is known as *primary power*.

Secondary Power. When the utility provides a voltage transformation to 600 volts or less, it is known as *secondary* power.

SUBSTATION INVESTMENT. When primary power is purchased the purchaser assumes responsibility for bringing power circuits into the plant at a voltage suitable for his equipment. The purchaser may have considerable investment in a substation for this purpose, depending on the voltages involved and the kilowatt capacity. The utility investment consists of metering equipment, transmission line to the customer's premises, and possibly the incoming-line circuit breakers.

In the sale of secondary power the public utility makes the investment in the substation to provide a suitable voltage for plant distribution. In this case the customer provides only the distribution system and connects it to the supply. Power rates are usually less for primary power than for secondary power.

The purchase of primary rather than secondary power is generally advantageous when there is a lower rate for primary power. With primary power the plant engineer has full freedom in selecting plant voltages and circuit layouts to fit his needs at lowest cost.

COST OF PURCHASED POWER. Public-utility rate schedules are closely controlled in most states by a Public Service Commission. The cost is affected by such factors as (1) geographical location, (2) whether primary or secondary power is purchased, (3) rate schedules available in the locality where the customer is located, (4) size of load to be served, and (5) character of load, i.e., load factor and power factor.

Rate Schedules. There is such a wide variety of rate schedules that generalizations only can be made. Often the customer is given a choice between two or more rate schedules. Selection is governed by size and type of load, possible future growth, or desire to purchase primary versus secondary power.

All rate schedules have a few common characteristics. Each schedule is a written contract between utility and customer, stating rates and the manner in which measurements are to be made. The contract usually covers a five-year period or more, with provision for termination within a specified time upon notice given by either party. Practically all rate schedules include three parts: (1) a charge proportional to kilowatt- or kilovolt-

* Contributed by R. M. Crenshaw.

ampere demand over a specified period of time, (2) a charge proportional to energy consumed, and (3) an adjustment feature proportional to fuel-cost variation from a base price.

Industrial plants often have short-time peak demands far above the average rate of energy consumption. The utility must provide adequate line and generating capacity to meet such demands. A charge proportional to such demand is justified because the energy used does not represent the true investment in capacity of the utility system. Many methods of measuring the maximum demand are used; the rate schedule describes the method used in the specific case.

Since the contract is in effect for several years, thus holding the demand and energy rates constant, the only protection that fuel-burning plants have against rising fuel cost is the fuel-cost adjustment clause.

EXAMPLE OF RATE SCHEDULE. The following primary power and light rate is typical for one class of rates.

Demand Charge

$1.70 per kva for the first 30 kva per month.
$1.50 per kva for all additional demand per month.

Energy Charge

2.0 cents per kilowatt-hour for the first (100 hr × kva demand)
1.75 cents per kilowatt-hour for the next (100 hr × kva demand)
1.50 cents per kilowatt-hour for the next (100 hr × kva demand)
1.0 cent per kilowatt-hour for all additional energy

Fuel Adjustment

An additional charge of 0.01 cent per kilowatt-hour for each full 10 cents increase above $3.00 per ton of coal shall be applied to the total kilowatt-hours used.

Minimum Monthly Charge

The minimum bill shall be $2.50 per kva of demand but not less than $75.00.

The complete rate contract contains details of the method of measuring demand and energy, computing fuel adjustment, prompt payment discount, etc.

EXAMPLE OF MONTHLY POWER BILL.

An industrial plant has a monthly maximum demand of 860 kva and energy consumption of 325,000 kw-hr. The cost of coal applying for this month is $4.25 per ton.

Demand Charge

1.70 × 30	$ 51.00
1.50 × (860 − 30)	1245.00
Total demand charge	$1296.00

Energy Charge

0.02 (100 × 860)	$1720.00
0.0175 (100 × 860)	1505.00
0.0150 (100 × 860)	1290.00

Each of these blocks accounts for 86,000 kw-hr. Then

325,000 − (3 × 86,000) = 67,000 kw-hr

remaining for the fourth block

0.010 × 67,000	670.00
Total energy charge	$5185.00

Fuel Adjustment

$4.25 − 3.00 = $1.25 over base price of fuel

This represents twelve times a full 10-cent increase in fuel cost; therefore,

12 × 0.01 = 0.12 cent is the additional charge per kilowatt-hour

0.0012 × 325,000	$ 390.00
Demand Charge	$1296.00
Energy Charge	5185.00
Fuel Adjustment	390.00
Total Monthly Bill	$6871.00

This represents an average cost of 2.11 cents per kilowatt-hour.

Although this example is representative of the structure of a rate schedule available for the purchase of primary power, the actual energy cost varies considerably in different sections of the country.

2. GENERATED POWER IN INDUSTRY *

Principal reasons for industrial power generation are: (1) By-products available that can be used for fuel or direct heat. (2) Industrial processes requiring steam that can conveniently be supplied from steam turbine exhaust. (3) Locations near abundant water power. (4) Locations remote from adequate public-utility service. (5) Public-utility supply not adequate to meet plant power demand or reliability. (6) Excessive purchased power cost because of rate contract restrictions on a particular type of load. (7) Odd frequency required by existing equipment.

WHEN TO GENERATE POWER. The decision to generate power in the industrial plant can be made only after considering all factors affecting the unit cost of energy, which may be divided into two major parts, *fixed charges* and *operating costs*.

Fixed charges are composed of the constant expenses of depreciation, insurance, taxes, and miscellaneous items. Depreciation, the largest factor in this group, is the method commonly used to write off or pay back the initial investment of a plant. It is expressed as an annual percentage of investment; the number of years used in depreciating the plant varies considerably. It may sometimes be less than normal plant life, because most industrial organizations depreciate their equipment in ten years or less, whereas the life of a generating plant is nearly always longer. If a 10% annual depreciation charge is made, the plant is written off in 10 years.

Taxes and insurance also are expressed as a percentage of investment but continue for the full life of the plant. Interest on investment is usually not included as part of fixed charges. If, however, circumstances are such that there is an annual saving of generated power over purchased power, this saving is considered the rate of return on investment.

Operating costs of a power plant include fuel, labor, maintenance, and supplies. Operating costs and fixed charges are combined to obtain the annual cost of operating the power plant. All the items of cost, except fuel, are practically constant for a given plant; therefore, the unit cost of energy goes down as load factor goes up. When plans are made for a new plant the importance of estimating load factor accurately is evident.

Since fuel is the largest item of cost (often over 50%) the trend in fuel costs must be considered carefully. Many current installations are providing boilers arranged to use either of two kinds of fuel, or both simultaneously, as coal and oil.

Plants having cheap fuel can usually show an attractive saving. The petroleum, coal mining, and steel industries have a source of cheap natural fuel. Steel mills with blast furnaces have large amounts of blast furnace gas that can be used in boilers to make steam. Some cement mills have hot kiln gases that can be passed through boilers to produce steam. Lumber and paper mills and certain chemical industries have by-products that can be used for fuel in boilers.

The paper, textile, steel, and some food industries require relatively large amounts of steam in process, i.e., cooking, drying, and heating. Here the application of a noncondensing or extraction steam turbine generator usually shows worthwhile return on investment. (See Section 8.)

Accounting procedure varies widely among individual plants in division of costs where waste fuel or process steam is involved. It is practically impossible to get a correct allocation of all charges; therefore, the case for a generating plant under these conditions can be influenced greatly by the allocation of fuel and labor costs.

TYPES OF POWER PLANT. Types of prime mover most used in industrial power plants are steam turbines (Section 8), water-wheel turbines (Section 5), mercury turbines (Section 8), gas turbines (Section 10), diesel engines (Section 13), and steam engines (Section 8).

Steam turbines are made in a wide range of ratings and steam conditions. The *extraction turbine*, with or without condenser, and the *noncondensing turbine* have found wide application where process steam is required. Extraction and noncondensing turbines are widely used because of the great increase in attainable thermal efficiency. This high efficiency is obtained because all heat in the steam leaving the turbine is charged to process, in contrast to the *straight condensing turbine* where about two-thirds of the heat is thrown away in the condenser, yet must be charged to the turbine. The fuel charged to an extraction or noncondensing turbine is the difference between total fuel and fuel required to supply process steam without using a turbine; hence these types of turbine offer attractive savings. Initial steam pressures in industrial plants vary between 150 and 850 psig. The trend is toward higher pressures, particularly where extraction or noncondensing turbines are used. Pressures of 400 and 600 psig are common for these applications.

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* Contributed by R. M. Crenshaw.

The water wheel turbine is not used extensively in industrial plants. It has, however, been used in some electrochemical plants and in a few paper and textile mills.

The mercury turbine (see Section 8) is expected to come into more common use because of its high efficiency. Fuel saving becomes more important as fuel costs increase. The by-product of the mercury cycle is process steam, making the cycle versatile and attractive.

The gas turbine (see Section 10) is still in the developmental stage, but indications are that it will have many industrial applications. It will be of particular interest where floor space is limited and where waste exhaust heat can be used.

The diesel engine (see Section 13) is used for power generation in smaller industrial plants that do not require process steam or where other conditions do not lend themselves to purchased power.

CAPACITY OF THE PLANT. The total plant generating capacity, number and size of units, influences the required investment. Standard definitions have been set up by ASA for *demand factor*, *load factor*, and *diversity factor* (see p. 16-99).

Demand factor is important in determining the required generating capacity. For a new plant it is customary to obtain the total connected load, then apply a demand factor gained from experience in a similar plant. The value is always less than the total connected load and is the capacity required to supply maximum demand.

Load factor is important in economical operation of a power plant. A high load factor (approaching unity) indicates steady 24-hour-per-day operation, with no peak demand periods high above average load. Generating units have best efficiency at or near full load, and fixed charges are constant regardless of load; hence lowest cost per unit of energy is obtained when load factor is high. It is desirable, therefore, to have several generating units, so that operating units can be changed to follow load variations, keeping generators well loaded.

Diversity factor is important in large plants having several distinct loads or distribution points. Each subdivision of the system has a demand factor; the advantage due to the probability that all peaks will not occur simultaneously is shown by diversity factor. Accurate determination of diversity factor is made only by actual measurements, all of which are made during a common time interval. By definition it is always greater than unity. Diversity factor is often used incorrectly in place of demand factor because the name suggests the difference between connected load and actual load.

When a new power plant is planned, some knowledge of the load is imperative in selection of units. It usually is possible to obtain the connected load; the demand factor, obtained from experience of another similar industry, ordinarily ranges from 30 to 80%. If there are two or more substations or separate load areas the diversity factor reduces the actual demand still further.

EXAMPLE. A certain plant has a connected load of 6000 kva. The demand factor is estimated at 60%; therefore, the maximum load at any time is $6000 \times 0.60 = 3600$ kva if all motors hit simultaneous peaks. But this plant has two separate operations so that the diversity factor is estimated at 120%; therefore the peak demands of both operations do not occur simultaneously. Then, $3600/1.20 = 3000$ kva, the actual load demand that is to be supplied by the generating station. This plant has full load operation in the daytime but limited night operation, resulting in a 24-hour average load of 2000 kva. The daily load factor is $2000/3000 = 66\,2/3\%$.

Since the plant load is quite steady for several hours per day, the full 3000-kva generating capacity is required. Two 1500-kva units or three 1000-kva units would be satisfactory to supply this load, the final decision depending on the best combination to allow one or two units to be shut down during light-load periods. But this generating station provides no *reserve* capacity; good practice dictates that reserve generating capacity equal to the largest single generator be maintained. In the example it would be satisfactory to install four 1000-kva units or three 1500-kva units. A general rule is that the rating of the largest unit must not be greater than one-fourth the total installed capacity.

MAINTENANCE. A power plant with a public-utility connection requires less reserve generating capacity than one that is self-contained. The ideal condition provides sufficient reserve to permit the largest unit to be shut down without affecting production. This condition is seldom attained in industry, however; either the cost of stand-by purchased power is too high or the required investment in generators is too great. When reserve capacity is not available, maintenance must be planned to coincide with reduced production schedules. A well-organized plan and efficient mechanics are essential because time is often limited. It is important to keep accurate records of repairs made, parts used, and notes indicating parts that will require attention at the next shutdown. An inventory record of repair parts is useful in assuring that parts are available when needed.

The generator, except exciters, requires little maintenance. The steam turbine is also reliable and will operate over long periods if protected from severe temperature changes, vibration, and given careful lubrication. Manufacturers recommend that a new turbine be given a complete internal inspection within six months to one year after installation. With normal operating conditions, subsequent inspections are recommended at two-year intervals or after 10,000 to 16,000 operating hours. A turbine actually requires less maintenance for continuous operation than for intermittent operation. A complete inspection requires a period of several days and should be supervised by an experienced turbine engineer. Maintenance repairs of condensers, coolers, exciters, and pumps must be made more often but require less time, often being done while the unit is in service. Boilers require more frequent but shorter inspections than turbine-generators, a period of two to five days being required unless major repairs are made. The care required by boilers increases rapidly as pressure and temperature are raised.

The diesel engine requires frequent maintenance. Usual recommendations are that a complete inspection be made every 3600 operating hours. Minor repairs and inspections are required at shorter intervals. (Consult Diesel Engines, Section 13, for details.) A survey of diesel-generator installations indicates an average maintenance cost of 1.5 to 2.0 mils per kilowatt-hour. (ASME Report on oil engine power costs.) Diesel units are particularly attractive where several small units are used to follow wide variations of load demand. This plan allows opportunity for periodic maintenance, necessary for satisfactory operation.

PLANT OPERATION. Satisfactory power plant operation requires teamwork from a skilled crew. In a steam plant rapid and reliable communication between boiler rooms, turbine room, and switchboard are essential. All operating personnel must be notified immediately of abnormal conditions. Modern practice links the parts of a station together with an audio frequency intercommunication set, using loud speakers to receive the signal above local noise. Telemetering or selsyn indicators are used to transmit load reading, steam conditions, etc., as required.

It is customary to take hourly readings of generator output, voltage, frequency, winding temperature, lubricating oil pressure and temperature, cooling water temperature, appropriate data concerning fuel input to the prime mover, etc. While readings are being taken, the operator inspects the machine for any abnormal conditions. As the readings are logged, comparison with previous readings readily shows changes in operating conditions which may indicate trouble.

Fuel is such an important item in all (except hydro) plants that it requires special attention. Adequate reserves for several weeks are sometimes required, depending on reliability of the supply. Labor can be saved and reliable service assured if adequate fuel-handling equipment is built into the plant. Often dual equipment is required for items such as conveyor, pulverizers and pumps.

The number of men required to operate a power plant varies considerably with type and size of plant and local labor regulations. If the plant operates 24 hours a day, it is customary to divide the time into three 8-hour shifts. The average industrial power plant has one man per generating unit for each shift, and a power plant superintendent on the daytime shift. Steam plants normally require two or more additional men to tend boilers. When the number of generating units exceeds four there may be less than one man per unit so that labor cost does not increase in direct proportion to the number of units. Most industrial installations have units rated 10,000 kw and smaller. In this range the operating personnel is a function of the number, rather than the size of units. Then labor cost per kilowatt hour decreases as plant capacity increases when load factor is essentially constant.

3. TURBINE-DRIVEN GENERATORS *

Standard turbine-driven generators usually are considered in connection with a steam turbine as a combined generating unit with respect to price, weight, dimensions, and efficiency.

REVOLVING FIELD TYPE A-C TURBINE GENERATORS. Standard generators are designed with enclosed revolving fields, 2-pole, 3-phase, 60-cycle, between 2000 and 7500 kw. They operate at 3600 rpm, direct-connected to the steam turbine, with direct-connected exciters of 125 or 250 volts. The frame of the generator is supported by feet at the sides. Within the frame, sheet steel laminations are built in sections and separated by spacers to form ventilating ducts. Laminations are slotted to receive armature coils. The generator is ventilated by fans at both ends of the rotor. Rotors are round, with field coils of copper ribbon wound edgewise, placed in slots in the periphery, and fastened by metal wedges. Collector rings are of steel.

* Contributed by E. W. Burstadt.

STANDARD RATINGS (in kilowatts at 0.8 power factor) and corresponding approximate weights are:

Rating, kw	2,000	2,500	3,000	3,500	4,000	5,000	6,000	7,500
Weight, lb	20,000	23,500	27,000	30,000	34,000	42,000	51,500	67,000

(See also Section 8.)

Usual practice includes dimensions of direct-connected turbine-driven generators in the complete dimensions of the combined unit, because base and foundation construction is laid out for the combined unit.

Temperature rise of generators for ambient temperatures of 40 C and altitude of 3300 ft or less is: armature, 60 C, measured by embedded detectors; field, 85 C, measured by resistance. Temperature rise of exciter is: core and windings, 40 C, measured by thermometer; commutator, 55 C, measured by thermometer.

REVOLVING-ARMATURE TYPE A-C TURBINE GENERATORS. Units of 10 to 60 kw are available for operation at 1800 rpm, 60-cycle, 3-phase. The armature is wound on the rotor, and power is taken off at collector rings on the shaft. Field poles are stationary. Voltages are limited to 120, 240, 480, and 600 volts.

Standard ratings at 0.8 power factor are: 10, 15, 20, 25, 30, 40, 50, and 60 kw. Direct-current units also are available in these ratings to operate at 3600 rpm, direct-connected, 125 and 250 volts.

GEARED TURBINE GENERATOR SETS from 75 to 1500 kw, inclusive, are available in standard units, consisting of high-speed turbines and speed-reducing gears direct-connected to either an a-c or d-c generator. These sets are used to obtain minimum steam consumption by operating the turbine at its most efficient speed. Alternating-current geared units operate at 1200 rpm for all ratings. Direct-current geared units operate at 1800 rpm for ratings of 75 to 100 kw and at 1200 rpm for ratings of 125 to 400 kw. For high ratings, generator speed varies with ratings and voltage.

Standard ratings of a-c (0.8 power factor) and d-c generators are 75, 100, 125, 150, 200, 250, 300, 400, 500, 700, 1000, 1500 kw.

Standard a-c voltage ratings are 240, 480, 600, 2400, 4160 volts; d-c, 125, 250, and 600 volts, for 2-wire; 125/250 volts for 3-wire.

Approximate prices of a-c and d-c turbine-driven generator units, including turbines, in standard industrial sizes are shown in Fig. 1.

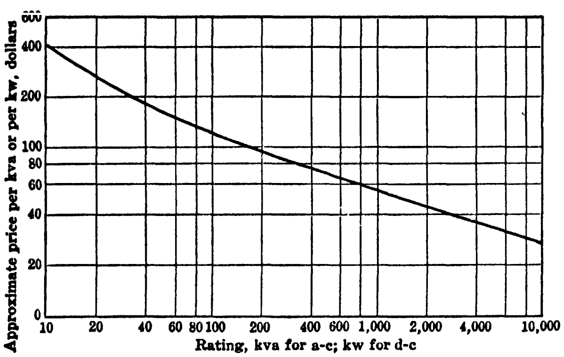


FIG. 1. Approximate price of standard turbine-driven a-c and d-c generator sets in industrial service, including condensing turbines.

Weights and dimensions of turbine generator sets corresponding to Fig. 1 are given in Tables 7 through 11.

Table 7. Dimensions and Weights of Condensing Turbine-generator Sets

(2000 to 7500 kw, 60 cycles, 3600 rpm)

Kilowatts	Overall Dimensions, in.			Approximate Weight, 1000 lb
	Long	Wide	High	
2000	273	105	86	68.0
2500	296	105	86	75.0
3000	311	124	86	88.0
3500	318	124	86	92.0
4000	335	132	97	107.0
5000	359	132	97	115.0
6000	353	132	74	134.0
7500	365	132	74	143.0

Table 8. Dimensions and Weights of Direct-connected Turbine-generator Sets
(10 to 60 kw; alternating current, 1800 rpm; direct current, 3600 rpm)

Kilo-watts	Alternating Current, 60 Cycles, 3 Phase, 0.8 Power Factor				Direct Current			
	Overall Dimensions, in.			Approximate Net Weight, lb	Overall Dimensions, in.			Approximate Net Weight, lb
	Long	Wide	High		Long	Wide	High	
10	82	25	26	1160	66	25	26	1100
15	87	25	26	1475	71	25	26	1180
20	87	25	26	1475	77	25	26	1400
25	88	40	33	2250	77	38	33	1750
30	...	...	...	...	79	38	33	1850
35	88	40	33	2250	...	...	...	...
40	...	...	...	...	96	46	42	2750
50	116	46	42	3250	96	46	42	2750
60	116	46	42	3800	96	46	42	3090

Table 9. Dimensions and Weights of A-c Condensing, Geared Turbine-generator Sets
(75 to 400 kw, 60 cycles, 3 phase, 1200 rpm)

Kilowatts	Overall Dimensions, in.			Approximate Net Weight, 1000 lb
	Long	Wide	High	
75	144	63	60	10.0
100	149	64	62	10.8
125	153	66	64	11.6
150	158	67	66	12.4
200	167	70	75	13.9
250	177	73	79	15.4
300	186	76	80	17.0
350	195	79	84	18.5
400	204	81	87	20.0

Table 10. Dimensions and Weights of D-c Condensing, Geared Turbine-generator Sets
(75 to 100 kw, 1800 rpm; 125 to 400 kw, 1200 rpm)

Kilo-watts	Overall Dimensions, in.					Approximate Net Weight, 1000 lb	
	Length			Width	Height	125 volt	250 and 125/250 volt
	125 volt	250 volt	125/250 volt				
75	144	144	150	63	60	10.0	10.0
100	157	149	155	64	62	11.3	11.1
125	..	153	159	66	64	12.6	12.3
125	161	...	...	66	64	12.6	12.3
150	...	158	164	67	66	13.9	13.5
150	166	...	...	67	66	13.9	13.5
200	175	167	173	70	75	16.5	15.8
250	185	177	183	73	79	19.2	18.0
300	194	186	192	77	84	21.8	20.4
400	212	204	210	81	87	27.0	25.0

Table 11. Dimensions and Weights of A-c Condensing, Geared Turbine-generator Sets
(500 to 1500 kw, 60 cycles, 3 phase, 1200 rpm)

Kilowatts	Overall Dimensions, in.			Approximate Net Weight, 1000 lb
	Long	Wide	High	
500	192	87	65	31.0
700	196	87	65	42.0
1000	244	94	65	47.0
1500	248	96	65	49.0

Exciters for a-c generators are direct-connected. The standard exciter is shunt-wound, rated at 125 volts, 40 C temperature rise, with 1.15 service factor. Generators of 60 kw rating and less usually have 64-volt exciters.

Standard exciter ratings are 1, $1^1/_2$, 2, 3, 5, $7^1/_2$, 10, 15, 20, and 25 kw. Higher ratings are usually specially designed for use with turbine-driven generators. With geared generators the exciter armatures are overhung from the generator shaft extension and the stator or field structure supported by a subbase built up from the main generator base. Exciters for direct-connected turbine generators have two pedestal bearings to support the armature and are mounted on a separate base.

Exciter prices are similar to those of small, standard high-speed d-c generators.

Motor-generator exciter sets often can be used to advantage where a separate source of power is available, especially to excite generators of very low speed. A direct-connected exciter here would be comparatively large.

4. ENGINE-DRIVEN GENERATORS *

STANDARD RATINGS. Standard kva ratings of synchronous (a-c) engine-driven generators are 1.25, 1.875, 2.5, 3.75, 6.25, 9.4, 12.5, 18.7, 25, 31.3, 37.5, 50, 62.5, 75, 93.8, 125, 156, 187, 219, 250, 312, 375, 438, 500, 625, 750, 875, 1000, 1125, 1250, 1563, 1875, 2188, 2500, 2812, 3125, 3750, 4375, 5000, 5625, 6250, 7500, 8750, and 10,000 kva.

STANDARD POWER-FACTOR RATING is 80% lagging, so the kilowatt ratings of the standard generators are 80% of the kva ratings listed above.

Effect of power factor on maximum continuous load which can be carried by an 80% power factor generator is shown in Fig. 2. Special generators which will deliver their full-rated kva at power factors as low as 30% lagging can also be obtained at increased cost.

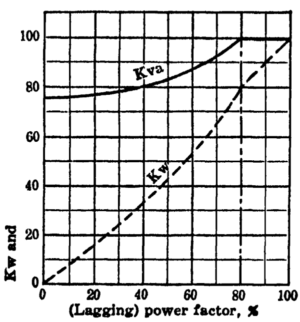


FIG. 2. Effect of power factor on maximum continuous kva and kw load that can be carried by typical 0.8 power-factor engine-driven generator.

STANDARD SPEED RATINGS OF 60-CYCLE GENERATORS are 80, 90, 100, 109, 120, 129, 138, 150, 164, 180, 200, 225, 240, 257, 277, 300, 327, 360, 400, 450, 514, 600, 720, 900, 1200, and 1800 rpm. The standard speeds for 50-cycle generators are five-sixths of the standard 60-cycle speed. The frequency (f), the rpm, and the number of poles (P) of a generator bear the relation

$$\text{Rpm} = \frac{120f}{P}$$

The number of poles (P) must be an even number.

STANDARD VOLTAGE RATINGS OF ENGINE-DRIVEN GENERATORS are 120, 240, 480, 600, 2400, 4160, 6900, and 13,800 volts. Most generators are designed for field excitation from a 125-volt d-c source, but small (75 kva and below) high-speed (1200 and 1800 rpm) generators are often designed with 64-volt fields, and very large generators (over 2500 kva) are frequently designed for 250-volt excitation.

CONSTRUCTION. Most engine-driven generators are of the *salient-pole* revolving field (stationary armature) type although revolving armature (stationary field) construction is sometimes employed on high-speed machines rated 25 kva and below.

Low-speed generators (450 rpm and below) are usually furnished without shaft or bearings, the generator rotor being mounted on an extension shaft furnished by the engine manufacturer. The higher-speed machines, on the other hand, are usually furnished with either a short shaft and single bearing (on the end opposite the engine) for rigid coupling

Table 12. Maximum Kilowatt Ratings of Generators for Two-bearing Belt Drive

	Maximum Kilowatt Rating	
Generator, rpm	Flat-belt Drive	V-belt Drive
1800	30	50
1200	50	75
900	75	125
720	125	200
600	125	200

* Contributed by M. N. Halberg.

to the engine shaft or a standard shaft and two bearings for flexible coupling to the engine shaft or for belt drive. Good practice limits the use of two-bearing generators with flat or multiple V belt to the ratings shown in Table 12.

EXCITERS. It is modern practice to provide each engine-driven generator with an individual exciter for supplying d-c excitation to its field winding. The exciter may be direct-connected to the generator shaft or it may be driven through a flat-belt, multiple V-belt, or chain drive. Standard kilowatt ratings of exciters are 1/8, 1/4, 1/2, 3/4, 1, 1 1/2, 2, 3, 5, 7 1/2, 10, 15, 20, 25, 30, 40, 50, 60, 75, and 100 kw. Standard speed ratings of exciters for belt drive are 500, 575, 700, 850, 1150, 1450, 1750, and 3600 rpm. Good practice limits the speed of belt-driven exciters to the values shown in Table 13. The standard voltage ratings of exciters are 125 and 250 volts, but 64-volt machines are also used.

Table 13. Maximum Speeds of Belted Exciters

Generator Speed, rpm	Maximum Speed (rpm) of Exciter	
	Flat-belt Drive	V-belt Drive
327 to 600, incl.	1750	1750
273 to 300	1450	1750
214 to 257	1150	1450
200	850	1450
164 to 187	850	1150
128 to 150	700	850
120 to 125	575	850
107 to 115	575	700

RHEOSTATS AND DISCHARGE RESISTOR. Engine-driven generators are normally furnished with a field discharge resistor and a generator-field rheostat whereas an exciter-field rheostat is furnished with the exciter. The generator-field rheostat is often omitted as it is not normally required if an individual exciter and voltage regulator are provided. If a regulator is not provided or is out of service for any reason so that manual control of the excitation is required, the generator-field rheostat may be essential, particularly when starting up a generator and attempting to synchronize it with other machines.

TEMPERATURE RISES of standard generators with Class A insulation are 50 C by thermometer (60 C by temperature detector) for the armature and 60 C by resistance for the field. These machines have no overload rating. Generators having a temperature rise of 40 C by thermometer (50 C by temperature detector) for the armature and 50 C by resistance for the field are also available at higher cost. These machines will carry 25% overload for 2 hr with an increase of 15 C in temperature rise.

VOLTAGE REGULATION (rise in voltage when rated kva load is removed from generator with excitation maintained at value giving rated voltage at rated kva load) of standard generators will not exceed the following:

Power Factor of Load	Voltage Regulation, %	
	Standard 50 C Generators	40 C Generators
0.8	40	34
0.9	35	30
1.0	25	20

Generators having lower values of regulation can be obtained at higher cost.

EFFICIENCIES. Approximate full-load efficiencies of low-speed generators are shown in Fig. 3. These do not include windage and friction, exciter or rheostat losses. Approximate full-load efficiencies of high-speed generators are shown in Fig. 4. These do not include rheostat or exciter losses, but *do* include windage and friction losses.

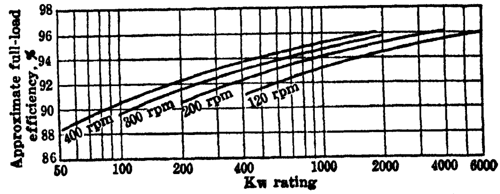


FIG. 3. Approximate full-load efficiency of 0.8 power factor, 3-phase 60-cycle 2400-volt (and below) synchronous generators. Values given do *not* include windage, friction, exciter or rheostat losses.

WEIGHTS AND DIMENSIONS. Typical weights and overall dimensions of low-speed engine-driven generators furnished without shaft or bearings

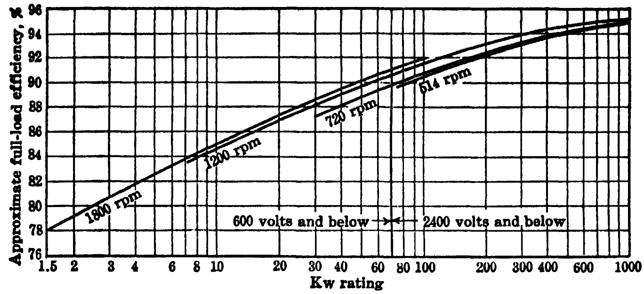


FIG. 4. Approximate full-load efficiency of 0.8 power factor, 3-phase 60-cycle high-speed synchronous generators. Values given do not include exciter or rheostat losses, but do include windage and friction losses.

are given in Table 14. Similar data for high-speed generators furnished with shaft and two bearings are listed in Table 15.

EFFECT OF MOTOR STARTING. The high current at low-power factor drawn by a squirrel-cage induction or synchronous motor when starting may result in excessive voltage drop on the generator. The effect is shown by the curves of Fig. 5 for typical 0.8 power factor engine-driven generators equipped with voltage regulators. The lower curves give the momentary drop in generator voltage which occurs when the motor is started, and the upper curves show the value to which the generator voltage recovers after the voltage regulator has acted to build up generator excitation to its maximum value. If two or more generators are operating in parallel when the motor is started, the motor starting kva should be expressed in percentage of the sum of the kva ratings of all the generators.

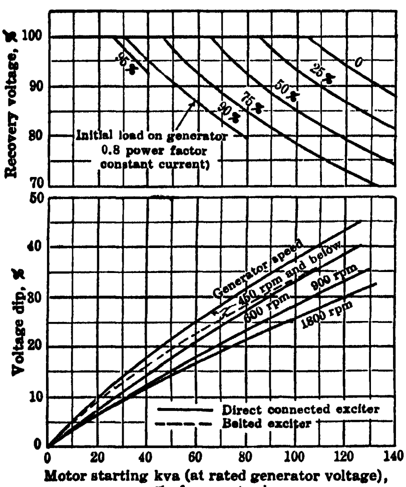


FIG. 5. Effect of motor starting on typical 0.8 power factor engine-driven generators equipped with voltage regulators.

The maximum voltage dip which can be tolerated is seldom over 25 or 30%. A greater voltage reduction may cause under-voltage devices on motor control to operate, tripping the motor off the line, or may cause stalling of heavily loaded motors. In some installations voltage variation must be held to much closer limits to prevent objectionable speed changes of motors in operation. If lamps are being supplied from the generator, voltage dips may have to be held to less than 5% to avoid objectionable flicker. Unless the generator voltage recovers to 100% after action of the voltage regulator, it is essential to compare the starting and accelerating torques of the motor being started with the torque requirements of its load to be certain that acceleration will be successful with less than normal voltage available from the power source.

To avoid difficulty with motor starting it is often necessary to select generators of a rating larger than required to supply the normal running load.

TORSIONAL VIBRATION OF ENGINE GENERATORS. A generator connected to an internal-combustion or steam engine forms a mechanical system consisting of two or more rotating masses (inertias) connected together by shafting and a coupling, which have torsional elasticity. The mechanical system, consequently, will have one or more natural frequencies of torsional vibration. If torque impulses of the engine have a frequency close to a natural frequency of the system, a severe oscillation will occur between the inertias, resulting in high stresses and possibly a failure of the shafting or coupling. This difficulty is avoided by selecting such shaft sizes, coupling characteristics, and flywheel inertias that the natural frequencies of the system are well removed from the frequency of the torque pulsations of the engine. Since most of the factors that affect

Table 14. Typical Weights and Overall Dimensions of Low-speed Generators (without Shaft or Bearings)

Kva Rating	Rpm	Weight, lb	Dimensions in Inches High	Wide	Long
31.3	450	1,750	43	45 1/2	31
	300	2,150	53	55 1/2	31
62.5	450	2,210	43	45 1/2	35
	300	2,920	61	62 1/2	31 3/4
125	450	2,920	61	62 1/2	31 3/4
	300	3,760	61	62 1/2	35 3/4
	200	5,220	74	75 1/2	35 3/4
187	450	3,760	61	62 1/2	35 3/4
	300	5,220	74	75 1/2	35 3/4
	200	6,870	88	89 1/2	36 1/4
250	450	4,250	61	63 1/2	40 1/2
	300	5,220	74	75 1/2	35 3/4
	200	6,870	88	89 1/2	36 1/4
500	450	6,700	74	76 1/2	48
	300	8,100	88	90 1/2	46
	200	11,100	98	99 1/2	42 1/2
	120	19,300	142	144 1/2	44 1/2
750	450	8,350	74	76 1/2	54 3/4
	300	11,650	88	90 1/2	54 3/4
	200	13,750	112	112 3/4	42 1/2
	120	19,300	142	144 1/2	44 1/2
1000	450	10,350	81	83 1/2	54 3/4
	300	14,200	98	100 1/2	56 3/4
	200	17,100	128	131	50
	120	23,800	142	144 1/2	51 1/4
2500	300	28,400	117	119 1/2	73 1/4
	200	29,500	144	147	62 3/4
	120	47,200	192	205	65
5000	120	89,840	240	262	75

Table 15. Typical Weights and Overall Dimensions of High-speed Generators (with Shaft and Two Bearings)

Kva Rating	Rpm	Weight, lb	Dimensions in Inches High	Wide	Long
9.4	1800	275	16	16	25
18.7	1800	300	16	16	27
37.5	1800	900	22	21	37
	1200	1,300	25	24	40
	720	1,600	29	28	49
62.5	1800	1,300	25	24	40
	1200	1,300	25	24	40
	720	2,150	29	28	47
125	1800	1,825	29	28	45
	1200	2,150	29	28	47
	720	2,900	34	34	51
	514	4,250	40	39	57
187	1200	2,900	34	34	51
	720	3,800	34	34	56
	514	5,140	40	39	62
250	1200	3,300	34	34	53
	720	5,140	40	39	62
	514	6,970	50	50	63
375	1200	5,140	40	39	62
	720	6,970	50	50	63
	514	7,900	50	50	66
625	1200	6,830	40	39	69
	720	9,050	50	50	69
	514	10,150	60	59	67
875	1200	9,050	50	50	69
	720	10,700	50	50	72
1250	1200	10,700	50	50	72
1875	1200	15,000	58	62	65

torsional vibration are contained in the engine, it is standard practice for the engine manufacturer to specify shaft sizes and assume responsibility for avoiding torsional vibration troubles. The generator manufacturer should furnish to the engine manufacturer generator drawings and data necessary for making the torsional vibration calculations.

CYCLIC SPEED VARIATION. Torque pulsations of an engine produce a cyclic variation in the speed and hence in the voltage of the generator driven by the engine. As shown in Fig. 6, a very small voltage variation in a certain range of frequencies will produce objectionable flickering of lamps supplied from the generator. This difficulty is avoided by providing sufficient inertia (wr^2) in the generator and engine flywheel. Because the amount necessary depends on the speed and other characteristics of the engine, it is determined by the engine manufacturer.

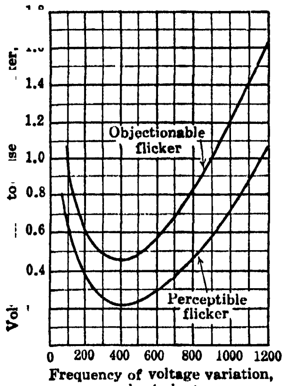


FIG. 6. Voltage variation required to cause flicker of incandescent lamps.

PARALLEL OPERATION. Synchronous generators operating in parallel are held in synchronism with each other by an electrical "coupling" which, acting through the common bus, tends to hold the rotors of the various machines together in their rotation. This coupling is not rigid but acts as though it contained a torsion spring. Thus the system is equivalent to a mechanical system consisting of a number of rotating masses (inertias) connected to a common shaft by torsionally flexible couplings. Such a system will have one or more natural frequencies of oscillation. If the torque impulses of an engine driving one of the generators has a frequency close to a natural frequency of the system, the system will oscillate and severe power pulsations may occur between the various generators. This may result in excessive heating of the generators, produce sufficient voltage variation to cause flickering of lamps on the power system, or even result in loss of synchronism between the various generators. Difficulty is avoided by providing each unit with proper inertia (wr^2) to limit the power pulsation to a safe value. The engine manufacturer normally assumes responsibility for successful parallel operation and furnishes a flywheel having the necessary wr^2 or specifies the wr^2 to be provided in the generator rotor. The generator data required for the calculations are furnished by the generator manufacturer.

The proper division of kilowatt load between generators operating in parallel is a function of the characteristics of the engine governors, and the proper division of reactive kva load is a function of the generator voltage regulators.

5. GENERATOR-VOLTAGE REGULATORS *

Alternating-current generators are usually provided with voltage regulators that control the excitation of the generator to maintain substantially constant voltage despite changes in load or other operating conditions. The regulator usually operates to vary the field excitation of the d-c exciter which serves to vary the exciter voltage, and hence the excitation of the a-c generator.

TYPES OF REGULATOR. There are two basic types of generator-voltage regulator in common use. The *direct-acting* type includes a rheostatic element connected in series with the exciter field winding; the effective resistance of this element is controlled *directly* by the action of the regulator. The *indirect-acting* type requires a motor-operated exciter field rheostat, and the regulator itself controls the operation of the rheostat motor, thus *indirectly* varying the resistance in series with the exciter field winding. The direct-acting regulators are used on the small and medium-sized generators (kva limits given in Table 16 are typical); the indirect-acting type is applied to the larger generating units.

Direct-acting Regulators. A popular form of direct-acting voltage regulator for use with self-excited exciters is shown in Fig. 7. The voltage sensitive element (torque element) is of the electromagnetic type and is balanced against a helical spring. It operates a rheostatic element consisting of two or more stacks of nonmetallic resistance material. Stabilization is provided electrically by means of a transformer whose primary is connected across the exciter armature and whose secondary is connected in series with the coils of

* Contributed by M. N. Halberg.

Table 16. Application Limits of Typical Direct-acting A-c Generator Voltage Regulators

Average of Generator and Exciter Speeds, rpm *	Maximum Rating of Generator, kva	Average of Generator and Exciter Speeds, rpm *	Maximum Rating of Generator, kva
80	625	257	3,125
100	750	277	3,500
112	875	300	3,750
120	1000	327	4,000
125	1125	360	4,375
128	1250	375	5,000
138	1375	400	6,250
150	1500	428	7,500
164	1625	450	9,375
180	1750	500	10,000
187	1875	550	12,500
200	2000	600	15,000
212	2250	650	18,750
225	2500	700	20,000
240	2750	750	25,000
250	3000	3600	25,000

* For intermediate speeds, maximum kva of generator corresponds to the next lower listed speed.

the voltage sensitive element. The connections are such that a rising exciter voltage forces a current through the voltage sensitive element in the same direction as the current produced by the generator voltage. This tends to oppose further increase in exciter voltage and hence stabilizes operation. The regulator is normally at rest, operating only when a change in excitation is required.

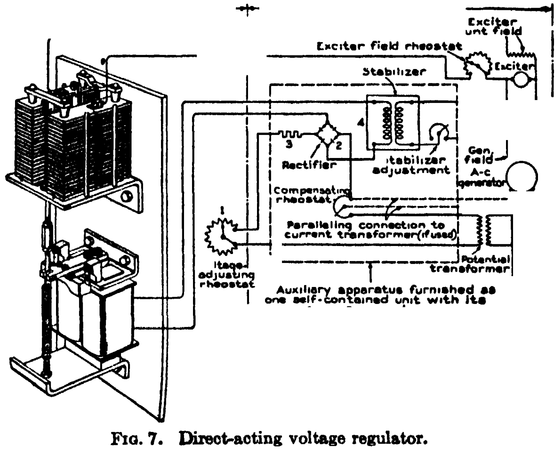


FIG. 7. Direct-acting voltage regulator.

Indirect-acting Regulators. A typical indirect-acting regulator is shown in Fig. 8. The voltage sensitive element is a polyphase torque motor which varies the position of two sets of contacts. A star wheel rotates continuously between one set of contacts, and a small change in generator voltage causes intermittent engagement of the star wheel with a contact to drive the motor-operated exciter field rheostat in the proper direction to restore the generator voltage. With a large change in generator voltage one of the second set of contacts engages its contact wheel, which energizes a high-speed relay to cut in or out all the regulating resistance. Damping is provided by action of a vane moving in a permanent magnet field. The motor-operated exciter field rheostat is arranged as a Wheatstone bridge with two fixed and two variable resistance legs. A constant voltage from a pilot exciter is impressed on two diagonally opposite terminals, and main exciter field winding is connected to the other two terminals. This arrangement gives a very wide range control of the main exciter voltage.

REGULATION OF GENERATORS IN PARALLEL. With two or more a-c generators operating in parallel it is modern practice to provide each generator with its own individual exciter and voltage regulator. The regulator may be either of the direct- or indirect-acting type. In either case means must be provided to insure that all generators take their share of the reactive kva load. A typical arrangement is that used on the direct-acting regulator described above. A current transformer and a compensating rheostat are provided, connected as shown in Fig. 7. The current transformer is connected in one lead, with the potential transformer across the other two leads. The phase relations are then such that the voltage drop across the compensating rheostat adds to the a-c potential on the regulator for lagging reactive kva output of the generator and subtracts for leading reactive kva output. This influences the regulator to reduce excitation for lagging current and to increase excitation for leading current. This action tends to divide the total reactive kva load among any number of machines in proportion to their ratings.

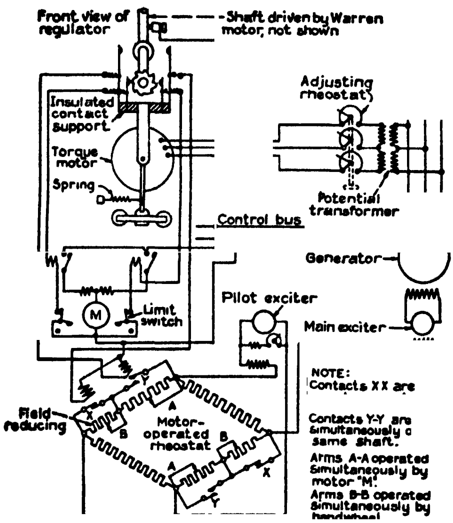


FIG. 8. Indirect-acting voltage regulator.

POWER DISTRIBUTION *

6. GENERAL PRINCIPLES

The load center system of power distribution used in the majority of new plants consists of: (1) high-voltage distribution (2.3 to 14.4 kv) in the plant to load center unit substations located at the center of load; (2) two or more small substations rather than one large substation when the load is larger than about 1000 to 1500 kva.

The load center system with the smaller substation is used because of its lower cost and operating advantages. To supply a plant from one 2500 kva substation rather than four

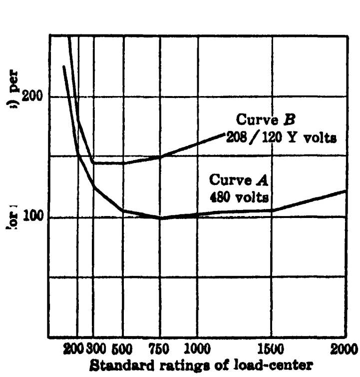


FIG. 9. Variation of relative cost of substations.

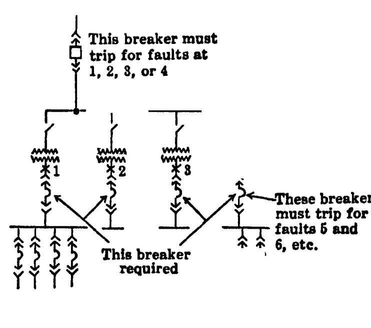


FIG. 10. Four unit substations connected to one primary feeder.

* Contributed by D. L. Beeman.

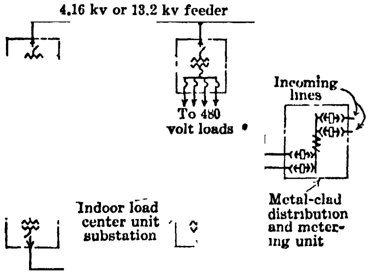
750-kva load center unit substations would cost about 17% more. The load center system has less voltage drop since the small substations are located close to the center of load and the secondary feeder runs are shorter; hence voltage drop is less than in large substation systems where some secondary feeders are of necessity very long and have large voltage drops. Close voltage regulation is essential for successful performance of the vast range of modern utilization equipment.

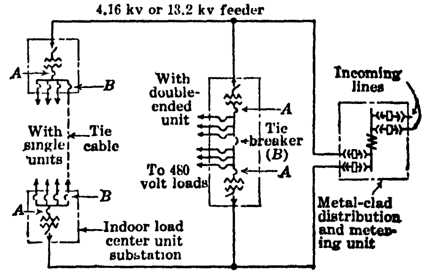
SIZE OF SUBSTATION. Many economic studies have shown that when the total load is larger than 1000 to 1500 kva at 480 volts or 500 to 750 kva at lower voltages, more than one substation should be employed. The most economical size is shown in Fig. 9.

Where a number of small substations are used, it is generally economically expedient to connect as many as four unit substations of the same size to one primary feeder without individual overcurrent protection (Fig. 10). When this is done, the 1947 Electrical Code requests that the main primary breaker be set at not more than six times the rating of the smallest transformer on the feeder. The National Electrical Code also requires that when there is not an individual primary breaker for each unit substation transformer, there shall be a main secondary breaker to provide adequate back-up protection for the feeder breakers and a certain degree of overload protection for the transformers.

7. CIRCUIT ARRANGEMENTS

A variety of circuit arrangements is used with the load center distribution system. The least costly and most common arrangement is the straight *radial* system shown in Fig. 11. It has no duplicate supply channels, and any given 480-volt bus can be supplied through only one primary feeder and one unit substation transformer. When such a system is





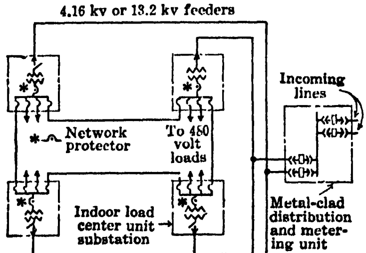


FIG. 11 (*upper left*). Radial system of circuit arrangement.
FIG. 12 (*upper right*). Secondary selective system of circuit arrangement.
FIG. 13 (*lower*). Secondary network system of circuit arrangement.

installed with good equipment, the service reliability is over 99%; hence is adequate for a great number of manufacturing plants that do not have continuous processes.

Emergency supply channels for either planned or unplanned outages of the supply circuit can be economically obtained by use of the secondary selective system (Fig. 12). This system is similar to the simpler radial arrangement except that there is a normally open tie between pairs of radial substations. The double-ended substation can be equipped with a normally open tie breaker. Should one supply channel be de-energized, all secondary buses can be re-energized by opening the proper transformer breaker *A* and closing the tie-feeder breakers, *B*. It is not customary to provide much excess transformer capacity to allow for emergency operation as the transformers are planned to be overloaded during these emergencies. This system is second in popularity to the straight radial system.

A few plants have used the secondary network system (Fig. 13). This system differs from the previous two in that all transformer secondaries are operated in parallel. It has the advantage that widely diverse loads can be handled with minimum transformer capacity. It has the disadvantage of extremely high cost and complexity of operation. The secondary network system would be particularly applicable to areas such as a ship pier, where nearly all load would be at one spot, but where the exact location of the load varies, depending on where the ships are docked.

8. PRIMARY DISTRIBUTION SYSTEMS

The supply for a load-center system can be either at utility voltage below 15 kv or through a master unit substation from some voltage higher than 15 kv. When voltage supplied by the utility is less than 15 kv the primary circuits are protected by metal-clad switchgear described in Art. 19. If utility voltage is above 15 kv, the transformation is made and the primary feeder circuits for the plant are controlled from a master unit substation described in Art. 18.

Radial primary feeders are used almost exclusively rather than so-called *looped feeders*. Radial feeders are far less expensive and usually adequate. Looped feeders, sectionalized with power circuit-breakers, are used in some very large steel mills.

Primary circuits should, for safety reasons, be enclosed in grounded metal or buried directly in the ground where operators or other personnel cannot accidentally contact live parts. Overhead aerial cable provides an inexpensive method of carrying power from one building to another. Where overhead cables cannot be tolerated because of obstruction, underground duct banks may be used.

Inside buildings it is preferable to have primary circuits enclosed in grounded metal and run overhead. Under-floor circuits are subject to oil accumulation and may obstruct the digging of foundations for machine tools.

Primary cables are run directly to the load-center unit substations. In general there is only a maintenance disconnect on the primary of the load-center substation, although power circuit-breakers may be employed where full short-circuit protection and switching means, for operating under any conditions, are required on the primary of each load-center unit substation transformer.

9. SECONDARY DISTRIBUTION SYSTEMS

Studies have shown that for lowest cost the size of secondary feeders from a load-center unit substation should be about 200 to 300 kva at 480 volts and about one-half that value at 208 or 240 volts (Fig. 14).

Secondary circuits may be cable in conduit, interlocked armor cable, or busway. Circuits in metal manufacturing plants are generally terminated at plug-in busway to permit easy installation of the electrical connections for machine tools.

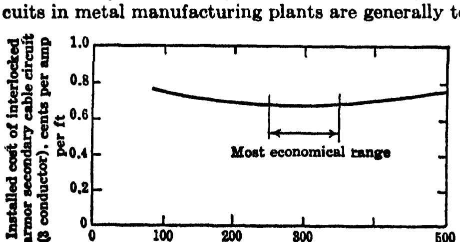


FIG. 14. Cost of secondary feeder circuits.

In the modern load-center system, power and lights are generally supplied from the same substations, a lower-cost method of distribution than separate substations. One method of supplying combined light and power is to install the substation with the secondary voltage of 480 volts, as in Fig. 15. One or more circuits for lighting feed small dry-type transformers, stepping down to 120/208 volts or 120/240 volts, single phase, for supplying panelboards that control feeders to the branch lighting circuits. This method has the advantage of 480 volt distribution of power, and of small lighting transformers which reduce the short-circuit current on the lighting panelboards.

In some cases where fluorescent lighting fixtures are used, the 480-volt secondaries of the unit substation transformers are wye-connected so that fluorescent lighting fixtures operate directly on line-to-neutral voltage, and no intermediate transformers are required.

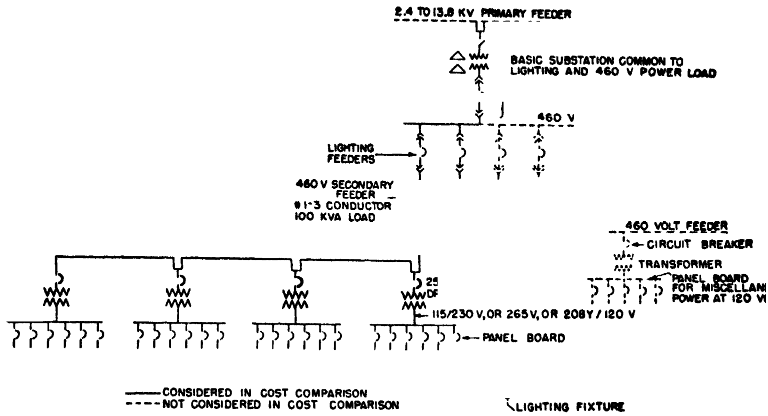


FIG. 15. Substation for combined light and power.

In combined light and power circuits, resistance welders should not be connected to the substation that supplies lights. Welders should be connected to one unit substation, and light in nonflicker-producing power to the other.

10. SELECTION OF VOLTAGE

One of the most important factors in designing industrial power distribution systems is to select the proper voltages, both primary and secondary. Where the utility voltage is below 15 kv, that voltage should be run directly to the load-center unit substations; there is no problem of selection of voltage, as it is determined by the voltage the utility has available. However, where the utility supply is above 15 kv, it is uneconomical to transmit the higher voltages through the plant to the unit substations; therefore, a transformation is made to some primary voltage below 15 kv. Here selection of voltage is extraordinarily important. Many economic studies have shown that either 4160 volts or 13,800 volts will suffice for practically all plants; 4160 volts for plants up to about 10,000 kva, and 13,800 volts for plants above 20,000 kva. Either voltage may be used in the intermediate size range.

For general plant distribution 2400 volts is generally not economical. It is economical for spot loads, however, such as pumping plants, provided the horsepower range of motors is in the order of 100 to 1000 hp or more.

Secondary utilization voltage for the average industrial plant should be 460 volts, chosen in preference to 575 volts because of the more ready availability of 440- and 460-volt utilization apparatus. It is chosen in preference to 230 volts because the 460-volt system is 25 to 50% lower in cost and generally gives lower losses and percentage voltage drop. The only suggested advantage of a 230-volt system is that there is less hazard to personnel. Either 230 or 460 volts is high enough to cause electrocution; hence neither can safely be worked when energized. With all circuits enclosed in grounded metal and maintenance work done only on de-energized parts, the safety record of 480-volt systems equals that of 240-volt systems, just as the safety record of 2300 volt and higher systems equals that of lower voltage systems.

Where there are a number of small hand tools, as in a radio assembly line, 208Y/120 volts may prove more economical than 460 volts. By using the 208Y/120-volt system instead of a 230-volt system, both lights and small hand tools can be supplied directly at 120 volts, and larger integral-horsepower motors at 3 phase, 208 volts.

VOLTAGE SPREAD has an effect on the performance of utilization equipment in a plant. By voltage spread is meant the spread between maximum light-load voltage and minimum full-load voltage existing anywhere in the plant.

Any deviation from rated voltage has an effect on the performance of utilization apparatus. For example, a 10% reduction below name plate voltage reduces the starting and running torque of standard induction motors by approximately 18%; and 5% overvoltage reduces the cathode life of electronic tubes by as much as 50%. A reduction of 10% in voltage applied to heating devices reduces their capacity by 20%, whereas excess voltage causes undue maintenance.

The modern industrial plant has so many electrical devices that are more sensitive to variation from rated voltage than motors and incandescent lamps that much attention should be given to voltage spread in the design of an industrial power distribution system.

It is almost impossible to design a power system without some voltage spread. Voltage spread is due to a number of causes. At no load, with the proper transformer voltage ratio and a transformer whose no-load secondary rating is 480 volts, 480 volts will exist throughout the plant. With practically no current flowing there is no voltage drop.

At full load there is a certain voltage drop (1) through the transformer, (2) in the secondary feeder, and (3) in the branch circuit. In a typical case, voltage drop may reduce the voltage at the end of a branch circuit from 480 to 450 volts, a spread of 30 volts.

Thus voltage spread is caused by drop in the plant system, even with constant primary system voltage. It is evident that care must be taken to design the plant power system for reasonable voltage drop.

There may be, in addition, a *primary voltage spread* which further increases the total spread. If the primary voltage spread is 10 volts, measured in terms of secondary voltage, the *total* voltage spread becomes 480 to 440 volts, or 40 volts.

If the combination of primary voltage spread and plant distribution system voltage drop is too great, induction voltage feeder regulators may be used to regulate particular feeders having critical load such as lighting load. Alternatively, load ratio control may be built into the master unit substation to regulate voltage for the entire plant.

The American Institute of Electrical Engineers' Industrial Power Systems Committee recommends certain maximum spreads for secondary utilization equipment. They are shown in Table 17 for circuits of 600 volts and less and in Table 18 for motors rated 2200 volts and higher. If voltages outside these limits occur in industrial plants steps should be taken to reduce the voltage spread; in some plants even closer spreads are desirable for economical operation of utilization equipment.

Table 17. Recommended Voltage Spread at Terminals of Utilization Devices in Industrial Distribution Systems of 600 Volts and Less

(Adapted from report of AIEE Industrial Power Systems Committee)

Nominal System Voltage	Commonly Used Utilization Device Voltage Ratings	Recommended Limits of Voltage at Terminals of Utilization Devices
120/208Y	115 or 120—1 phase 208 or 220—3 phase	197Y/114–217Y/125 *
230	† 220, 230	210–240
460	† 440, 460	420–480
575	† 550, 575	525–600

Designations for nominal system voltages are those commonly used in industrial plants.
* Polyphase power loads may not operate satisfactorily at this lower limit.
† Standard polyphase motor voltage ratings.

Table 18. Recommended Voltage Spread at the Terminals of Motors Served at Primary Voltage

(Adapted from report of AIEE Industrial Power Systems Committee)

Nominal System Voltage	Motor Name Plate Voltage Rating	Recommended Limits of Voltage at Terminals of High Voltage Motors: Min (−2%)	Max (+8% approx.)
2400	2200	2160	2380
2400	2300 *	2250	2480
4160	4000	3920	4320
4800	4600	4500	5000
6900	6600	6470	7130

* Present standard motor voltage rating.

SHORT-CIRCUIT CURRENT AND OVERCURRENT PROTECTION

11. CALCULATION OF SHORT-CIRCUIT CURRENT *

Determination of short-circuit currents in industrial power systems is one of the most important design considerations. Only after maximum short-circuit currents are known can protective devices be selected.

SHORT-CIRCUIT CURRENTS AND THEIR EFFECTS. Often only the load current is considered when selecting a circuit breaker or fuse. If adequate protection is to be provided for a plant electric system, the size of the electric power system must also be considered to determine how much short circuit it will deliver. This is done in order that circuit breakers or fuses may be selected with adequate IC (interrupting capacity). This IC should open safely the maximum short-circuit current which the power system can cause to flow through that breaker or fuse if a fault occurs in the feeder that it protects.

Magnitude of the load current is determined by the amount of work being done, and bears little relation to size of the system supplying the load. Magnitude of the short-circuit current is, however, directly related to the capacity of the power source. The larger the apparatus which supplies electric power to the system, the greater the short-circuit current.

EXAMPLE. A 440-volt, 3-phase, 10-hp motor draws about 13 amp of current at full load and will draw only this amount whether supplied by a 25-kva or a 2500-kva transformer bank. So, if only the load currents are considered when selecting motor branch circuit breakers, a 15- or 20-amp breaker would be specified. However, the size of the power system back of the breaker has a real bearing on the amount of the short-circuit current that can flow as a result of a fault on the load side of the circuit breaker. Hence, a much larger breaker would be required to handle the short-circuit current from a 25,000-kva bank than from a 25-kva bank of transformers. Consider Fig. 16. The data have been chosen for easy calculation rather than a representation of actual system voltages.

The impedance limiting the flow of load current is largely the 20 ohms apparent impedance of the motor. If a short circuit occurs at F, the only impedance to limit the flow of short-circuit current is the transformer impedance (0.1 ohm compared with 20 ohms for the motor); hence, the short-circuit current is 1000 amp or 200 times as great as the load current. Unless breaker A can open 1000 amp, the short-circuit current will continue to flow, doing great damage.

Suppose the plant grows and a larger transformer, rated at 1000 amp, is substituted for the 100-amp unit. A short-circuit at F (bottom in Fig. 16) will now be limited by only 0.01 ohm, the impedance of the larger transformer. Although the load current is still 5 amp the short-circuit current will now be 10,000 amp, and breaker A must be able to open that amount.

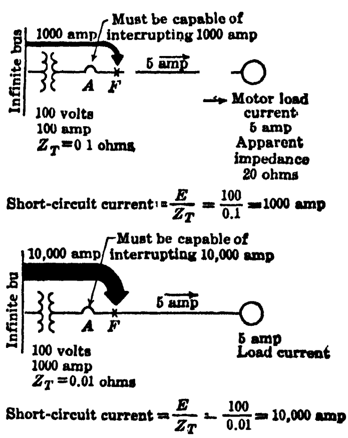


FIG. 16. Illustration of short-circuit current for two sizes of transformer.

Consequently, it is necessary to consider the size of the system supplying the plant as well as the load current, to be sure that breakers or fuses are selected that have adequate IC for stopping the flow of the short-circuit current.

SOURCES OF SHORT-CIRCUIT CURRENTS. When determining the magnitude of short-circuit currents, it is extremely important that all sources of short circuit be considered and that the reactance characteristics of these sources be known.

The three basic sources of short-circuit current are generators; synchronous motors and synchronous condensers; induction motors. All can feed short-circuit current into a fault.

Asymmetrical Short-circuit Currents. Most short-circuit currents are asymmetrical, as shown in Fig. 17. The accurate determination of such short-circuit currents is very complicated. These are assumed, for simplicity, to consist of two components, as shown in Fig. 18. One is the symmetrical a-c component, the other the d-c component. The symmetrical a-c component can be determined by dividing the line-to-neutral voltage by the proper reactance. The d-c component is taken care of by use of a multiplying factor.

* Contributed by D. L. Beeman.

Nearly all interrupting devices are so rated that either the momentary duty or the interrupting duty, or both, must be checked. The procedure is given below.

Momentary Rating. To determine the short-circuit current at the first half cycle for checking the circuit protective device on the basis of application on its momentary rating, it is necessary to consider all sources of short-circuit current, that is, generators, synchronous motors, induction motors, and utility connections. The *subtransient reactance* of generators, synchronous motors, and induction motors is employed in the reactance diagram. Since the d-c component is present at this time, it is necessary to account for it by use of a multiplying factor of either 1.4 or 1.6, as outlined in Table 19. The procedure is the same, regardless of the type of circuit protective device involved.

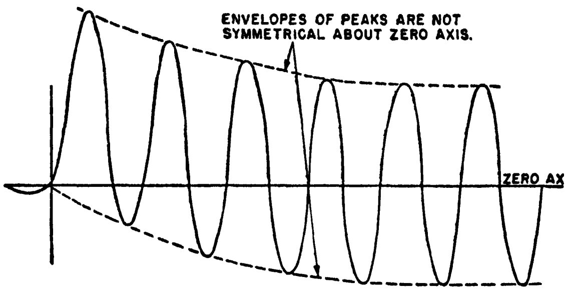


FIG. 17. Oscillographic trace of typical short circuit.

Multiplying Factors. Magnitude of the d-c component depends on the point on the voltage wave at which short circuit occurs. Only the maximum d-c component is considered, since the breaker must be applied to handle the maximum short-circuit current that can occur in a system. The maximum rms value of an offset current is 1.732 times the symmetrical value. In most industrial-plant power systems which operate above 600 volts, however, the ratio of the circuits is such that a multiplying factor of 1.6 is used to account for the d-c component of the first loop of current.

Interrupting Rating. To check the application of a circuit protective device on the basis of its interrupting rating, the short-circuit currents should be determined at the time that the devices open the circuit. The time required for the circuit-breaker contacts to part will vary over a considerable range, because of variation in relay time and in breaker operating speed. The fewer cycles required for the breaker contacts to part, the greater will be the current to interrupt. Therefore, the maximum interrupting duty is imposed on the breaker when the tripping relays operate instantaneously. In all short-circuit calculations, for the purpose of determining interrupting ratings, the relays are assumed to operate instantaneously. To account for variation in the breaker operating speed, power-circuit breakers have been grouped into classes, such as eight-cycle breakers, five-cycle breakers, three-cycle breakers. It is assumed that breakers of all manufacturers, in any one speed grouping, operate substantially the same with regard to contact parting time.

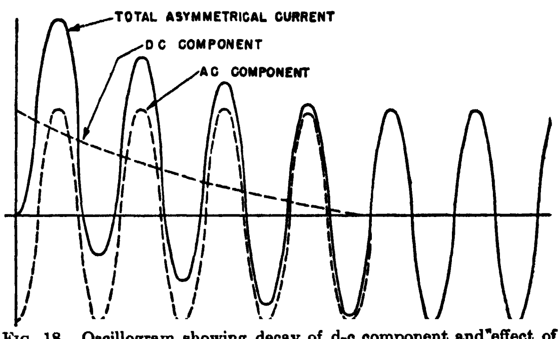


FIG. 18. Oscillogram showing decay of d-c component and effect of asymmetry of current.

Instead of specifying a time at which the short-circuit current is to be calculated, the simpler approach has been taken of specifying the generator and motor reactances, and multiplying factors to be used for determining the short-circuit current. These factors are listed in Table 19.

In industrial plants, eight-cycle breakers are generally used. In general, the induction motor contribution has disappeared, and that of the synchronous motors has changed from the subtransient to the transient condition before the contacts of the breakers part. Therefore, in calculating the interrupting duty on commonly used power circuit breakers, generator subtransient reactance and synchronous-motor transient reactance are used, and induction motors are neglected. The elapsed time is so long that nearly all the d-c component has disappeared. The remaining d-c component is more than offset by the

Table 19. Condensed Table of Multiplying Factors and Machine Reactances to Be Used for Calculating Short-circuit Currents for Circuit-breaker, Fuse, and Motor-starter Applications

Classification	Circuit Voltage	Location in System	Multiplying Factor	Machine Reactances to Use: Generators, Syn. Converters, Syn. Condenser, Frequency Changers	Machine Reactances to Use: Syn. Motors	Machine Reactances to Use: Induction Motors
Power Circuit-breakers						
				Interrupting Duty		
Eight cycle or slower (general case)	Above 600 volts	Any place where symmetrical short-circuit kva is less than 500,000 kva	1.0	Subtransient	Transient	Neglect
Five cycle	Above 600 volts		1.1	Subtransient	Transient	Neglect
				Momentary Duty		
General case	Above 600 volts	Near generating station	1.6	Subtransient	Subtransient	Subtransient
Less than 5 kv	601 volts to 5 kv	Remote from generating stations (X/R ratio less than 10)	1.4	Subtransient	Subtransient	Subtransient
High-voltage Fuses						
				3-Phase Kva Interrupting Duty		
All types, including all current-limiting fuses	Above 600 volts	Anywhere in system	1.0	Subtransient	Transient	Neglect
				Maximum Rms Ampere Interrupting Duty		
All types, including all current-limiting fuses	Above 600 volts	Anywhere in system	1.6	Subtransient	Subtransient	Subtransient
Noncurrent-limiting types only	601 to 15,000 volts	Remote from generating station (X/R ratio less than 4)	1.2	Subtransient	Subtransient	Subtransient
High-voltage Fused Motor Starters						
				3-Phase Kva Interrupting Duty		
All horsepower ratings	2400 and 4160Y	Anywhere in system	1.0	Subtransient	Transient	Neglect
				Maximum Rms Ampere Interrupting Duty		
All horsepower ratings	2400 and 4160Y	Anywhere in system	1.6	Subtransient	Subtransient	Subtransient
High-voltage Motor Starters						
				Interrupting Duty		
Circuit-breaker or contactor type	601 volts to 5 kv	Anywhere in system	1.0	Subtransient	Transient	Neglect
				Momentary Duty		
Circuit-breaker or contactor type	601 volts to 5 kv	Anywhere in system	1.6	Subtransient	Subtransient	Subtransient
Circuit-breaker or contactor type	601 volts to 5 kv	Remote from generating station (X/R ratio less than 10)	1.4	Subtransient	Subtransient	Subtransient
Apparatus 600 Volts and Below						
				Interrupting or Momentary Duty		
Air circuit-breakers or breaker-contactor combination motor starters	600 volts or less	Anywhere in system	1.25	Subtransient	Subtransient	Subtransient
Low-voltage fuses or fused combination motor starters	600 volts	Anywhere in system	1.0	Subtransient	Subtransient	Subtransient

reduction in a-c component due to increase in reactance of the generators. Hence a multiplying factor of 1 is used.

DIAGRAMS. One-line Diagram. The first step in making a short-circuit study is to prepare a one-line diagram showing all sources of short-circuit current, i.e., utility ties, generators, synchronous motors, induction motors, synchronous condensers, rotary converters, etc., and all significant circuit elements such as transformers, cables, and circuit breakers (Fig. 19).

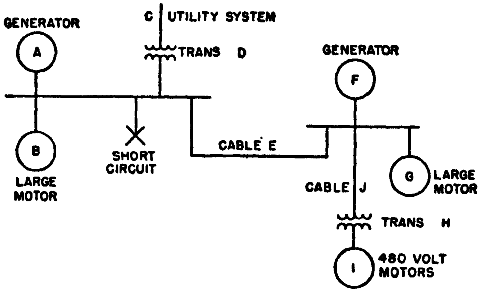


FIG. 19. Line diagram of typical power system.

Impedance or Reactance Diagram. The second step is to make an impedance or reactance diagram (Fig. 20), showing all significant reactances (and resistances if required) In the following discussion these are referred to as impedance diagrams. It is recognized, of course, that only reactances are used in many diagrams. The circuit elements and machines considered in the impedance diagram depend on many factors, i.e., circuit voltage, whether momentary or interrupting duty are to be checked, etc.

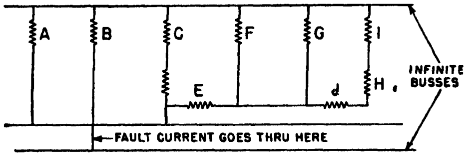


FIG. 20. Impedance diagram of system shown in Fig. 19.

The foregoing discussion and Table 19 explain when motors are to be considered and what motor reactances are to be used for checking the specific duty on a given breaker or fuses of a given voltage class. There are other problems, i.e., (1) selecting the type and location of the short circuit in the system, (2) determining the specific reactance of a given circuit element or machine, and (3) deciding whether or not circuit resistance should be considered.

THREE-PHASE SHORT CIRCUITS GENERALLY CONSIDERED. In most industrial systems, the maximum short-circuit current is obtained when a three-phase fault occurs. Short-circuit-current magnitudes are generally less for the line-to-neutral or line-to-line short circuits than for the three-phase fault. Thus the simple three-phase short-circuit current calculations will suffice for application of short-circuit protective devices in most industrial plants.

The proper reactances and multiplying factors are shown in Table 19.

12. OVERCURRENT PROTECTION *

GENERAL CONSIDERATIONS. The purpose of overcurrent protection is to prevent the attainment of excessive or dangerous temperatures in electrical conductors; to preserve continuity of service on all circuits that have been operating normally; and to limit the amount of energy liberated in the event of an electrical failure.

Serious fire and explosion hazards are incurred if electrical conductor temperatures are permitted to rise to the point where volatile gases are expelled from insulation. Reasonable values of maximum momentary feeder conductor temperatures (infrequently applied) are listed in Table 20. Continuous current ratings will be found in the National Electric Code and in cable application data handbooks.

*** Contributed by R. H. Kaufmann.**

Table 20. Cable Conductor Rated Maximum Continuous Operating Temperature and Peak Transient (Momentary) Temperature for Various Types and Operating Voltages

Cable Type	Voltage Class, kv	Max Continuous * Copper Temp., °C	Max Transient Copper Temp., °C
Vc type V or VL	1	85	150
1/c or 3/c	4.5	85	145
	7.5	84	135
	15	77	120
Impregnated	1	85	150
paper (solid)	4.5	85	145
1/c	7.5	85	140
	15	81	135
Impregnated	1	85	140
paper (solid)	4.5	85	135
3/c	7.5	85	130
	15	81	125
Type R †	1	60	140
	5	60	135
	8	60	130
	15	60	125
Type RH	1	75	150
	5	75	145
	8	75	140
Coronol	1	80	150
	5	80	145
	8	80	140
	15	80	130

* Actual operating temperature may be lower owing to conservative application or a favorable ambient temperature.

† Applies to new type R (1947 code specification).

The occurrence of severe overcurrent in one branch of an electrical system imposes a severe electrical load on the supply system. Unless promptly removed, other near-by load circuits may be disrupted owing either to prolonged undervoltage or to loss of power stability.

High values of current accompanying an electrical circuit fault result in the liberation of large quantities of heat at the fault location, and produces objectionable burning and splattering of molten metal. The duration of fault current flow should be reduced to the absolute minimum.

Protection, in all cases, is achieved by automatically interrupting the source of current supply to the particular circuit in distress. The protective equipment must not interfere with the normal flow of power current nor itself be damaged thereby. Normal system operation will entail short intervals of current flow exceeding the rated continuous value as exemplified by motor-starting demands.

PROTECTIVE INTERRUPTER. The power interrupter may consist of a recloseable power switching mechanism that can be tripped open by direct-acting trip coils or tripping relays (circuit-breaker), or it may consist of an expendable fusible member enclosed in a suitable arc chamber (fuse). The protective interrupter may be called upon to interrupt any value of current up to the maximum value associated with the available short-circuit capacity at its source terminals.

Required characteristics of the protective interrupter are defined by (1) The circuit operating voltage. (2) The electrical frequency. (3) The rated continuous current of the circuit. (4) The number of poles required. (5) The maximum fault current that can be supplied by the source system (available fault current). (6) The type of trip characteristic.

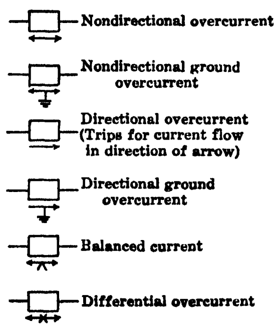
SELECTIVITY. The occurrence of excessive overcurrent in one member or circuit of an electrical system will be reflected in numerous other circuits between it and the source of power. The overcurrent protectors associated with these several circuits may detect current magnitudes that cannot be permitted to be sustained. Only the protector associated with the circuit member in which the overcurrent condition originated should be tripped open. The overcurrent condition in other circuits closer to the source will be simultaneously relieved, and these circuits can continue to serve their connected load circuits without interruption. *Selectivity* denotes the ability of the protective system to identify and interrupt current flow only in the individual system circuit which is responsible for the excessive overcurrent condition. Selectivity is achieved by appropriate selection of tripping characteristics in combination with circuit arrangement pattern. To acquire selectivity the protector associated with the circuit responsible for the distress must detect the condition, initiate, and complete the interruption of current before any of the other protectors operate.

TRIP CHARACTERISTICS. The varieties of trip characteristics considered to be of overcurrent character include:

- Overcurrent, nondirectional (line and ground).
  - Instantaneous.
  - Inverse time.
- Overcurrent, directional (line and ground).
  - Instantaneous.
  - Inverse time.
- Current Balance.
- Differential current.

The type of trip can be designated on one-line diagrams by the use of symbols as given in Fig. 21.

An overcurrent trip functions to initiate circuit interruption if the current magnitude exceeds a pre-assigned value. Instantaneous operation denotes no purposely delayed action, although a slight time is involved (typically about 0.01 sec at six times minimum operating value). An inverse time characteristic denotes a purposely delayed trip action in which the time delay becomes progressively less as the current magnitude becomes greater. The time characteristic may, in some devices, be adjustable; in others, it is fixed at the time of manufacture.



Note: For greater detail, there can be added below the arrow:
TD to denote time delay
INST to denote instantaneous

FIG. 21. Symbols for indicating type of overcurrent protection.

The directional overcurrent unit differs in that a sense of direction is imparted by an auxiliary source of potential or current. With the actuating current at the phase angle (relative to the reference quantity) to produce maximum trip action, the resulting characteristic will be almost identical with that obtained from the nondirectional unit. If the actuating current is displaced from this relative angle by more than 90 electrical degrees in either direction, no trip action results, regardless of the current magnitude. In d-c systems currents will be either directly in phase or in direct opposition.

The current balance trip compares two current values normally related by a fixed ratio. The trip unit operates on a deviation from the normal ratio. In the form of a separate trip relay, two sets of trip contacts are usually available that will identify which circuit current is greater than the correct ratio value.

The differential current trip in many respects closely resembles the current balance trip in that it compares two current values normally related by a fixed ratio and operates on a deviation from the correct ratio. *Rotating machine* and *transformer* differential relays are normally of the percentage differential type (10% for the former and 25% for the latter), in which the flow of normal current of correct ratio produces a small restraining force opposing operation. This allows greater sensitivity at light load without incurring the risk of false operation at high load owing to a slight mismatch in current transformer ratio or a change in the tap setting of the main transformers.

Fuses provide a fixed inverse-time trip characteristic. In general, the time delay at several times minimum operating current is relatively short. Current flow is not immediately stopped when the fuse link melts; hence, to obtain selectivity in a series chain of fuses of similar time characteristics requires a substantial difference in normal ampere rating; becoming progressively larger in the direction of the power source.

APPLICATION. In approaching application considerations it is desirable to divide the field into two classes: overloads (current less than ten times normal) and circuit faults (current of any value up to the maximum available).

Moderate Overload Protection (Class A). System overloads are the result of heavy current demands by healthy unfaulted load equipment. Induction motors demand five to eight times normal current when started at full voltage. Abnormal mechanical load imposed on running motors results in abnormal current demand, but not exceeding the full-voltage starting demand. Overcurrents of this sort are less than ten times normal, with some exceptions such as spot or seam-welder feeders. Protective equipment should be applied to permit these normal overloads to be carried so long as they do not exceed what can safely be tolerated without damage to equipment or circuits.

The ability of various types of electrical equipment to withstand safely occasional application of current in excess of normal continuous rating varies considerably. Typical characteristics are indicated in Fig. 22. For comparative purposes, the characteristics of a number of commonly used inverse-time overcurrent protectors are plotted on the same coordinates.

A line of thermal-type inverse-time overcurrent relays has been developed expressly for the purpose of providing moderate overcurrent protection (below ten times normal) to

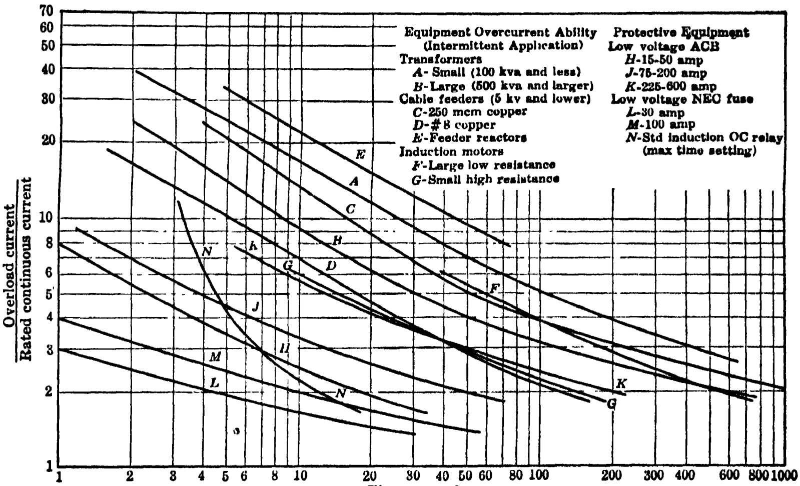


FIG. 22. Ability of electrical equipment to withstand greater than normal rated current.

synchronous motor stator and *induction motor windings*. *Large motors* (1000 hp and larger) generally employ embedded temperature detectors used in conjunction with a temperature relay.

Direct-current motors generally have a commutation limit of two or three times normal current which may dictate the use of instantaneous overcurrent protection set at the commutating limit.

Transformers, in the larger sizes, commonly make use of a direct oil temperature measurement in combination with a current actuated compensating mechanism to simulate internal hot-spot temperature. The combination relay frequently provides three-step control; the first operating to start supplemental cooling fans, the second to sound an alarm, and the third to de-energize the transformer.

In the absence of specialized thermal protective equipment for the moderate overcurrent region, it is desirable that all unsupervised circuits be equipped with automatic overcurrent protection with a minimum operating setting not in excess of 150% of the circuit continuous-current rating.

Circuit Fault Protection (Class B). Circuit faults represent a failure of insulation allowing unrestrained current flow in the fault arc. The magnitude of overcurrent may be small (less than normal) for a partial winding fault in a motor or transformer; or it may be high (one hundred times normal or more) for a feeder cable fault. In either case the circuit cannot be expected to regain normal conditions. Continued supply of current merely aggravates the severity of the fault condition and may adversely affect other circuits not faulted. It is, therefore, desirable to devise means of quickly identifying a fault condition and its location and immediately instituting interruption of current supply to the faulted circuit.

INVERSE-TIME OVERCURRENT. A system of inverse-time line overcurrent protectors, arranged to provide selective protection in a radial distribution system, is the most commonly employed method, especially in the smaller size branch circuit system. In

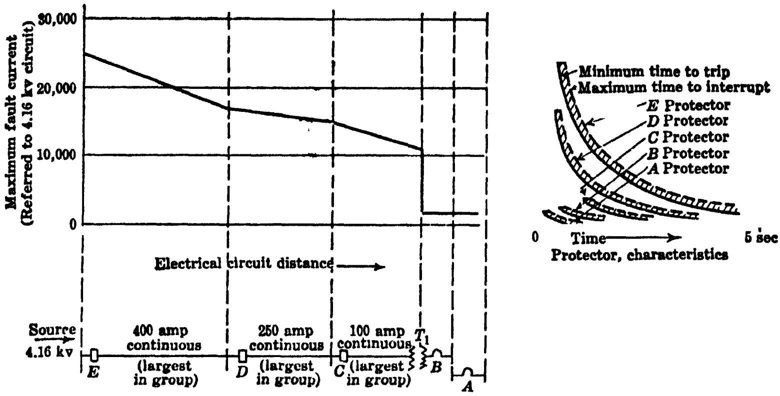


FIG. 23. Selective overcurrent system using only inverse-time and instantaneous overcurrent elements.

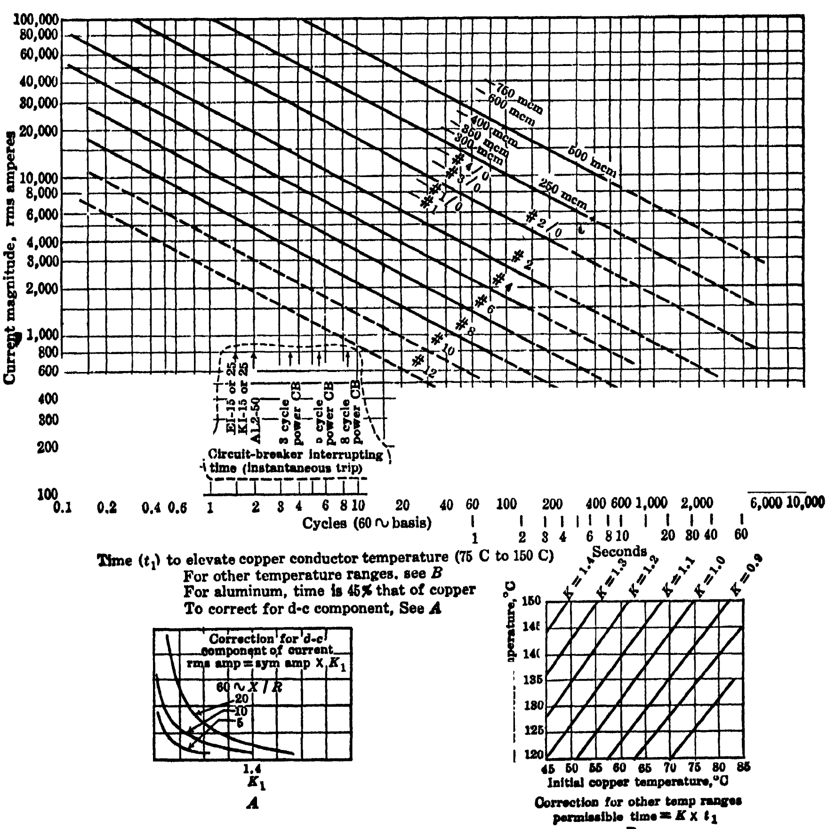


FIG. 24. Effect of current magnitude and conductor size on time to elevate copper conductor temperature from 75 to 150 C. For other temperatures, see correction, chart B.

Fig. 23, illustrating an inverse-time protective system, a number of feeder circuits radiates from each subdistribution bus. However, the selectivity problem is governed by the largest circuit in each group.

Both the necessary time delay for selectivity and possible fault-current magnitudes become progressively greater for protectors closer to the source. The severe damage that can accompany the prolonged flow of current to a circuit fault makes it desirable to seek methods of isolating faulted circuits more quickly.

The rate of temperature rise in feeder cable copper when carrying fault currents of large magnitude may make it necessary to use greatly oversize conductors unless the fault current conduction time can be reduced. Figure 24 designates the permissible time that a particular current magnitude can be maintained on various copper conductor sizes to elevate the copper temperature to 150 C, starting from a normal running temperature of 75 C. A 20,000-amp fault current could be safely permitted on a $^3/_0$ copper conductor for no more than 10 cycles ($^1/_6$ sec), which would require instantaneous trip action on an 8-cycle power circuit-breaker. Table 21 gives the minimum feeder conductor size that can safely be used in the presence of various available short-circuit currents.

Table 21. Estimation of Minimum Conductor Size

(Based on: Copper conductor. 75 C conductor temperature rise (75-150 C). A single interval of fault-current conduction. Constant current after 1/10 sec.)

Low-voltage Air Circuit-breaker Protection

IC (Amperes) (1.25 × Symmetrical)	Duration of Fault Current		
	1.5–2 Cycles (Instantaneous Trip)	1/4 Second	1/2 Second
5,000	No. 8 Awg	No. 4 Awg	No. 2 Awg
10,000	No. 4 Awg	No. 1 Awg	No. 1/0 Awg
15,000	No. 2 Awg	No. 2/0 Awg	No. 3/0 Awg
25,000	No. 1 Awg	No. 4/0 Awg	300 MCM
35,000	No. 1/0 Awg	300 MCM	400 MCM
50,000	No. 3/0 Awg	400 MCM	600 MCM
75,000	300 MCM	600 MCM	800 MCM
100,000	350 MCM	800 MCM	1000 MCM

High-voltage Power Circuit-breaker Protection

IC (Amperes) (1.0 × Symmetrical)	Interrupting Kva at:				Duration of Fault Current		
	2.4 Kv	4.16 Kv	6.9 Kv	13.8 Kv	8.5 Cycles (Instantaneous Trip)	1/2 Second	1 Second
3,000– 3,500		25 mva		75 mva	No. 6 Awg	No. 2 Awg	No. 2 Awg
3,500– 4,000					No. 4 Awg	No. 2 Awg	No. 1 Awg
4,000– 4,500			50 mva		No. 4 Awg	No. 2 Awg	No. 1 Awg
4,500– 5,000					No. 4 Awg	No. 2 Awg	No. 1/0 Awg
5,000– 6,000					No. 2 Awg	No. 1 Awg	No. 2/0 Awg
6,000– 7,000	25 mva	50 mva		150 mva	No. 2 Awg	No. 1 Awg	No. 2/0 Awg
7,000– 8,000					No. 2 Awg	No. 1/0 Awg	No. 3/0 Awg
8,000– 9,000			100 mva		No. 1 Awg	No. 2/0 Awg	No. 3/0 Awg
9,000–10,000					No. 1 Awg	No. 2/0 Awg	No. 4/0 Awg
10,000–12,500	50 mva	75 mva		250 mva	No. 1/0 Awg	No. 3/0 Awg	250 MCM
12,500–15,000		100 mva		350 mva	No. 2/0 Awg	No. 4/0 Awg	300 MCM
15,000–17,500					No. 2/0 Awg	250 MCM	350 MCM
17,500–20,000	75 mva				No. 3/0 Awg	300 MCM	400 MCM
20,000–25,000	100 mva	150 mva	250 mva	500 mva	No. 4/0 Awg	350 MCM	500 MCM
25,000–30,000					250 MCM	400 MCM	600 MCM
30,000–35,000		250 mva		750 mva	300 MCM	500 MCM	750 MCM
35,000–40,000	150 mva		500 mva		350 MCM	600 MCM	750 MCM

INSTANTANEOUS OVERCURRENT. A simple instantaneous overcurrent trip can be very useful in some cases for fault detection, but it must be used with caution lest it destroy selectivity. There are two conditions which allow it to be used effectively.

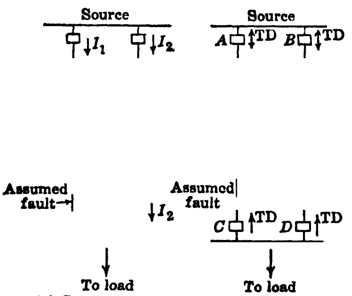
Individual Load Branch Circuits. In the absence of a local fault, the current flow in an individual load circuit will not exceed about ten times normal. Current flow in excess of that amount indicates a fault in that branch which can be promptly interrupted by applying an instantaneous overcurrent trip to that branch circuit protector set above the ten times normal level.

Ahead of a circuit section in which a large reduction in possible fault current magnitude occurs. This application can best be described by an example. Section C in Fig. 23 exemplifies the principle.

The occurrence of a fault beyond the transformer T_1 will not produce current flow through protector C in excess of 2000 amp. Thus the appearance of a current through protector C greater than 2000 amp indicates a fault between protector C and the transformer T_1 which can be cleared only by interruption at C. An instantaneous trip set somewhat above 2000 amp applied at protector C will speed up the high-current interruption performance of protector C and allow shorter time settings on protectors D and E.

DIRECTIONAL OVERCURRENT. There are circumstances in which current *magnitude* alone will not indicate the faulted branch but in which the fault location can be identified with the aid of *directional overcurrent.*

EXAMPLE. In the two parallel lines in a radial feeder system illustrated in Fig. 25, for a line fault near the remote end, the current magnitude at both ends of each line is practically identical, that is, $I_1 = I_2$. The direction of current flow at the remote terminal is opposite in the two lines. By application of fast-acting directional overcurrent at the remote terminal of each line directed to trip upon current flow toward the source, in combination with inverse-time overcurrent tripping at the source end of each line, selective isolation of the faulty line will be achieved. For the assumed fault location protector C will open first (protector D will not be operated, and the time characteristic of C is faster than that of B). Operation of protector C interrupts fault current flow through B. Continued fault current flow through protector A causes it to open after a slight additional time delay completing the correct isolation of the faulty line. (Note that the direction of normal load current is in the wrong direction to produce tripping at C or D, which permits the directional settings at C and D to be sensitive and fast without incurring the risk of false operation due to abnormal load current demands.)



(a) Current pattern (b) Relay application

FIG. 25. Parallel line relaying using directional overcurrent at the remote terminals.

BALANCED CURRENT. A current unbalance trip can be used effectively as a sensitive, fast fault detector if the system being protected can be arranged to consist of two branches that normally share the current flow in a fixed ratio. The normal ratio need not be 1 : 1, but frequently is. It can be applied to rotating machines or transformers if the windings can be brought out in two sections.

It can be applied to the source end protectors of the parallel line problem illustrated in Fig. 25. For through-current flow of any amount there is no tendency to trip. For all internal faults except those near the remote line terminal, immediate tripping of the proper source-end breaker occurs. For a fault near the far end of one line, the proper source-end breaker is opened immediately following operation of the directionally tripped breaker at the remote end. When used for parallel line protection it becomes necessary to revert automatically to inverse time overcurrent protection upon trip out of one line.

DIFFERENTIAL CURRENT. Differential overcurrent protection constitutes another variety of fault detection that permits very rapid interruption of fault current to a particular protected circuit upon occurrence of an internal fault within that section, but is practically insensitive to through currents, regardless of magnitude. A high order of sensitivity is commonly possible. Correct operation can be secured as a result of an internal fault whose magnitude is only a small fraction of rated continuous value.

Rotating Machine Differential Current. Each phase of the stator winding is provided with a differential relay that compares the current in one end of the winding with that in the other in the manner illustrated in Fig. 26. Any difference between the current in the two ends of the phase winding appears in the operating coil of the relay. The ability to detect fault current magnitudes of less than rated value is extremely valuable since an initial single coil failure may result in a fault current of less than rated value, and extensive coil and iron burning may occur before the fault current becomes large enough to actuate line overcurrent protective units.

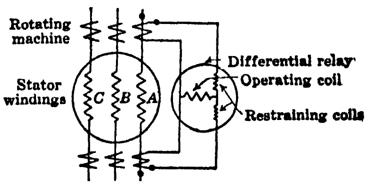
Transformer Differential Current. Except for magnetizing current, the primary current bears a definite ratio to the secondary current. Thus a differential current relay can be applied in the same manner as for machine windings. The transformer differential relay contains an internal autotransformer for producing a close match of primary and secondary currents that could not always be secured by current transformer ratio selection alone. As in the case of rotating machine protection, the ability to detect an internal fault of small current magnitude minimizes the magnitude of internal winding damage.

In some instances, when served from a high-capacity service bus, the transient magnetizing inrush current, when the transformer is switched on the line, may be sufficient to actuate the differential relay. This condition, if encountered, can be circumvented by briefly desensitizing the differential relay.

Line Differential Current. The same differential current principles are applicable to lines. Because of the long distance between the two ends of the circuit, the electrical quantities being compared are first reduced to miniature values and compared via an auxiliary circuit (either pilot wire or leased telephone circuit). If the terminals are more than about 10 miles apart, a carrier current comparison system may be more economical.

Bus Differential Current. Each phase of a power bus has connected to it a number of circuit taps, some of which may normally represent power sources and the remainder represent load feeders. In the absence of internal fault, the total current toward one phase bus must be zero. Should current either enter or leave this bus by means of an unintentional connection, this sum as measured on the intentional connections no longer equals zero.

By installing equal-ratio current transformers in each intentional tap connection, additively connecting the secondary circuits, and impressing the resultant total current on an overcurrent relay, as illustrated in Fig. 27, a bus differential-current fault-detector is obtained. An inverse-time overcurrent characteristic is employed to avoid false operation on transient current transformer errors.[9]



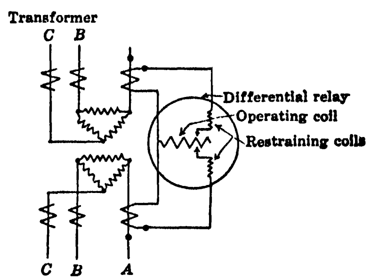


FIG. 26. Application of differential relays to rotating machinery and transformers showing detail of one phase.

During an external fault on one feeder circuit the full short-circuit current is flowing through the current transformer associated with that circuit. As the magnitude of this current becomes greater, there is a tendency for this heavily loaded current transformer to go off ratio, resulting in a false operating current in the differential relay. The through current at which false operation would result can be increased by (1) higher ratio current transformers, (2) low secondary lead burden, (3) no other burden on the differential current transformers (it is standard practice to reserve one set of current transformers for differential relays only), and (4) higher current setting on the overcurrent relay. At very high available fault current (above 50,000 amp) it may be desirable to adopt other methods of comparison to obtain acceptable sensitivity and speed of response combined with freedom from false operation.

FIG. 27. Bus differential-current fault-detector, showing detail of one phase.

Ground Overcurrent. The many benefits to be derived from resistance grounding the neutral of an industrial power distribution system are discussed in the next article, Neutral Grounding. Grounded-neutral operation provides the ability to improve fault protection greatly.

The effectiveness of machine differential protection is enhanced since ground current flow produces relay operation at the very inception of a fault which contacts the machine core iron.

A high order of machine fault protection approaching that provided by differential protection can be achieved by the use of a ground-current-responsive relay set to operate at a low current value, with short time delay. (In the absence of a stator winding fault, there will be zero ground current in the circuit of a machine whose neutral is ungrounded.)

More effective feeder circuit protection is made possible. It is an established fact that more than half the feeder circuit faults originate as a line-to-ground fault.

The extent of one ground current system will be limited; hence selectivity considerations of ground overcurrent protection need not extend beyond this boundary. In the example presented in Fig. 23 ground current flow in the local 4.16-kv system will not result for faults on the source side of the incoming power transformers or on the load side of unit substation transformers. Hence the appearance of ground current in a feeder serving unit substations is immediately indicative of a fault in the 4.16-kv feeder circuit, and a relay responsive to this current can be arranged to trip that feeder circuit-breaker without delay.

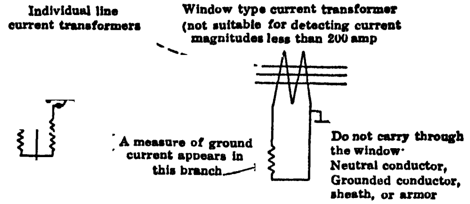


FIG. 28. Detection of ground current by window current transformers.

The available ground-fault current will usually be artificially restricted by the neutral grounding resistor to a value far below the available three-phase fault current. Hence, for all line-to-ground faults detection of location and isolation of the faulty section can be accomplished at the reduced value of current. Thus the violence of the fault arc is reduced, the disturbance in line-to-line voltages is lessened, and the duty imposed on the switching interrupter is reduced.

The detection of ground current can be accomplished by additive connection of line current transformers or by a window current transformer encircling all conductors, as illustrated in Fig. 28.

TYPICAL APPLICATION. A representative distribution system incorporating overcurrent protection as herein described is shown in Fig. 29.

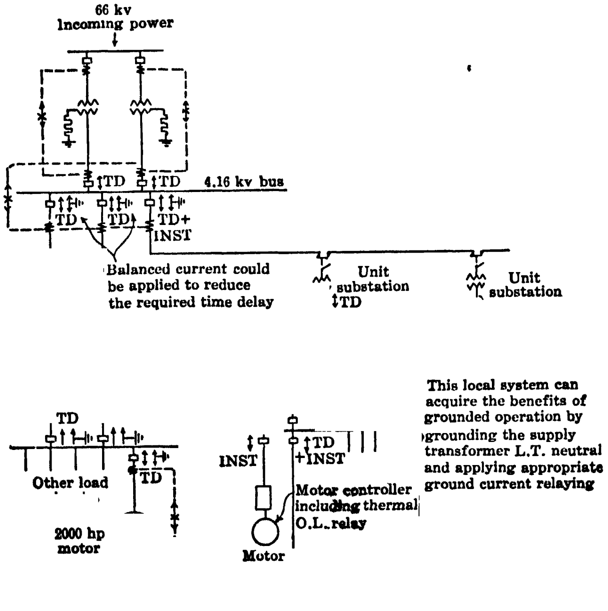


FIG. 29. Typical system with overcurrent protective relaying.

CODES. Various codes defining minimum permissible standards of overcurrent protection have been adopted. The National Electric Code is the fundamental national standard; however, some localities have adopted local codes and ordinances that supplement the national standard.

13. NEUTRAL GROUNDING *

It is generally preferable to ground the neutrals of all industrial power systems.

Grounding (1) enables fast location and isolation of ground faults; (2) minimizes overvoltage due to restriking; and (3) reduces steady-state voltage stresses to ground.

Systems should be grounded at the neutral of the supply transformers or generators. If the supply transformers and generators are delta-connected, grounding transformers may be employed as shown in Fig. 30. Where there are two or more sources of power, the neutrals of two or more of these sources should be grounded to insure having a neutral grounded in the event one of the sources is out of service.

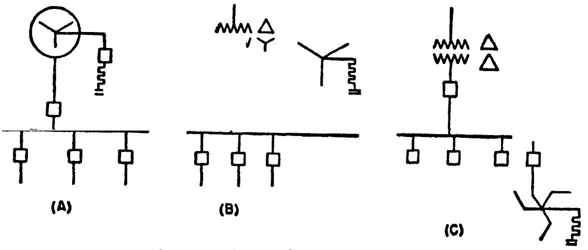


FIG. 30. Grounding arrangements.

There are many methods of grounding the neutral. (1) Solidly grounded—in which no intentional impedance is placed in the system neutral. (2) Reactor grounded—where a system is grounded through a reactor of a device such as a grounding transformer whose circuit impedance is mainly reactance. (3) Resistance grounding, which consists of a resistor in the neutral.

Solid grounding is used on low-voltage circuits, 600 volts and less, and on most circuits above 15 kv. In the range of 2.2 to 15 kv, any of the three types may be used. Small systems in this voltage range, where the supply capacity is of the order of 2000 kva or less, use solid grounding. On larger systems resistance grounding is employed to limit the ground fault currents and to lessen the damage when a line-to-ground fault occurs. Reactance grounding is used in this voltage range when it is desirable to use line-to-neutral lightning arresters to obtain the highest practical order of lightning protection for connected equipment. These factors are summarized in Fig. 31.

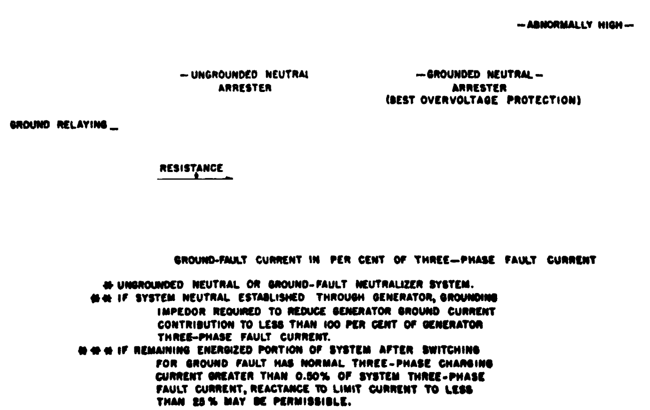


FIG. 31. Application guide for system grounding.

To use line-to-neutral lightning arresters, the ratio X_0/X_1 must be 0 to +3 and the ratio R_0/X_1 must be 1 or less. Generally, this means that the ground fault current must

* Contributed by D L. Beeman.

be 60% or more of the 3-phase fault current at the point of application of the line-to-neutral lightning arresters.

For further information on system grounding, see System Electrical Neutral Grounding, by Boice and Hunter, in the Nov. 1943 issue of *Electric Light and Power.*

14. LIGHTNING PROTECTION *

Lightning arresters are protective devices that limit surge voltage applied to the apparatus they protect and by-pass it to ground. Properly applied, the discharge voltage of the lightning arrester is below that of the transformer insulation or other insulation it is applied to protect. Laboratory tests and actual lightning discharges have demonstrated the ability of the modern lightning arrester to discharge surges commensurate with direct strokes of lightning.

Lightning arresters for a-c circuits are rated according to the maximum line-to-ground circuit voltage that they will withstand. There are four classifications available: the station-type (voltage ratings from 3 to 242 kv), the line-type (voltages from 20 to 73 kv), and the valve-type and the expulsion-type distribution arresters (voltages from 1 to 15 kv).

For most lightning arrester applications, overvoltages resulting from single line-to-ground faults may be used as the criterion in selecting the arrester with the proper maximum permissible line-to-ground voltage rating. When risk of arrester failure from excess system voltage is to be avoided, the system voltage applied to the arrester should in no case exceed the arrester's maximum permissible line-to-ground voltage rating. The standard ungrounded-neutral arrester is normally used when the system neutral is isolated, or when it is grounded through a ground-fault neutralizer, or through high values of resistance or reactance. The grounded-neutral arrester may be applied when the system neutral is effectively grounded, generally solidly grounded.

It is necessary that arresters be placed as close as possible to the equipment to be protected. For protecting transformers, line-type arresters are generally applied to transformer banks up to 1000 kva, and station-type arresters for banks over 1000 kva.

THYRITE ARRESTERS. Station-type arresters consist of stacks of arrester units, the number depending on the voltage of the circuit. This type is comprised of disks of Thyrite material and series gap elements, assembled as in Fig. 32. Thyrite is substantially an insulator at one voltage and a conductor at a higher voltage, its resistance being a function of voltage only. This characteristic permits Thyrite to act as valve material in an arrester to prevent system disturbances or cutages as the result of lightning discharges. The station-type arrester has the ability to withstand lightning-discharge currents of 100,000 amp, 5 × 10 microseconds wave shape.

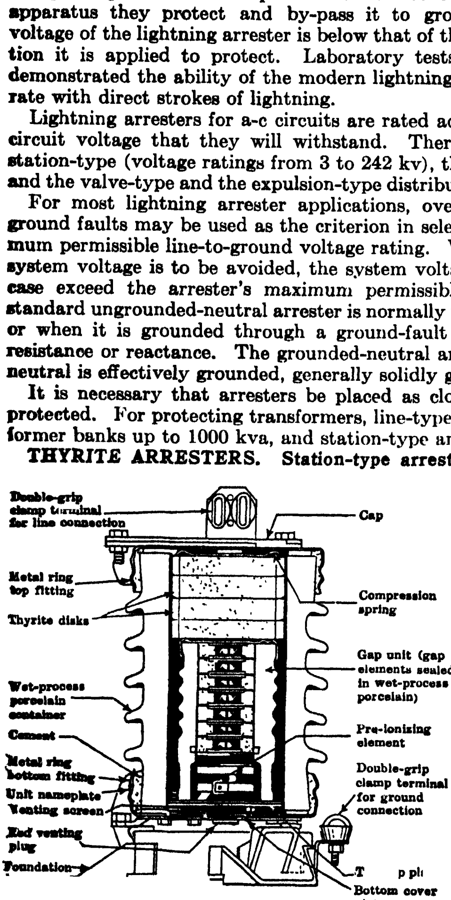


FIG. 32. Thyrite station-type lightning arrester unit.

Line-type arresters are a different arrangement of Thyrite disks and gap units. The line-type arrester is characterized by smaller diameter disks than the station-type unit, with a resulting decrease in the ability to withstand lightning-discharge currents. The value is 65,000 amp, 5 × 10 microseconds wave shape, for the line-type units.

DISTRIBUTION LIGHTNING ARRESTERS. Valve-type arresters, consisting of a valve-element and a gap unit, are used to protect distribution transformers, cable terminals, and other apparatus on distribution circuits 15 kv and below. Figure 33 illustrates typical connections for protecting distribution circuits and transformers.

Diagram *a* shows 3-phase and single-phase applications to 3-phase ungrounded-neutral systems or systems with the neutral grounded through resistance or reactance. The rating of the arrester is selected on the basis of line-to-line voltage. Diagram *b* shows

* Contributed by N. E. Dillow.

3-phase and single-phase applications to 3-phase, 3-wire, solidly grounded-neutral systems. Diagram *c* shows 3-phase and single-phase applications to 3-phase, 4-wire solidly grounded-neutral system with the neutral wire carried out. Diagram *d* shows single-phase applications to single-phase, multigrounded, common-neutral line, supplied from 3-phase, 4-wire, solidly grounded neutral source. The rating of the arresters, on the main circuit wires in *b*, *c*, and *d*, can be less than line-to-line voltage, usually of the next lower rating to that used on delta or ungrounded neutral systems. The maximum rating of the arrester on the neutral of *c*, designated *N*, is equal to, or higher than, the potential from neutral to ground due to any phase unbalancing. The ground at the neutral at the power source does not in any way reduce the lightning potentials occurring on the neutral wire out on the circuit. It is just as important to apply arresters to the neutral as to the phase wires, where the neutral is connected to the apparatus. Multigrounding precludes the necessity for such protection.

Expulsion-type arresters are used to protect distribution transformers primarily on rural distribution circuits. Constructed with an external gap and a rod gap in a fiber tube, they depend on the evolving of gas by the fiber and the resulting expulsion action to interrupt the arc. They are applied generally as in diagram *d* of Fig. 33.

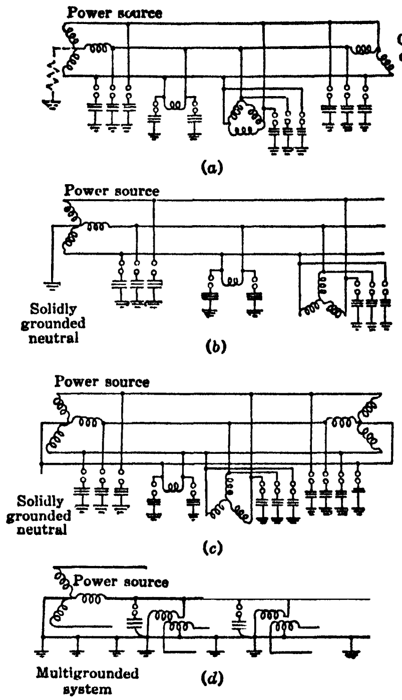


FIG. 33. Application of lightning arresters.

POWER-FACTOR IMPROVEMENT *

Reasons for Power-factor Improvement. The basic reasons for improving or maintaining good power factor in industrial plants are (1) to reduce the cost of purchased power when a power-factor clause is part of the rate structure; (2) to release electrical capacity; (3) to reduce electrical losses; and (4) to improve voltage levels.

Although most of the material and data included in this chapter refer to power-factor improvement by capacitors, they generally apply to other means of power-factor improvement, such as synchronous motors.

15. FUNDAMENTAL CONCEPTS

The current required by induction motors, transformers, fluorescent lights, induction heating furnaces, resistance welders, etc., may be considered to be made up of two separate components, *magnetizing current* and *power-producing current*. This concept of two kinds of alternating current is particularly helpful in understanding capacitor and power-factor applications. Some loads, such as incandescent lights, require only power-producing current.

Power-producing current (or working current) is current converted by the equipment into useful work, such as turning a lathe, making a weld, or pumping water. The unit of measurement of the power produced is the kilowatt (kw).

Magnetizing current (also known as *wattless*, *reactive*, or *nonworking* current) is current required to produce the flux necessary to the operation of induction devices. The unit of measurement of magnetizing volt-amperes is the kilovar (kvar; for kva, reactive).

* Contributed by W. C. Bloomquist.

Total current is current read on an ammeter in the circuit. It is generally made up of both magnetizing current and power-producing current. The unit of measurement of total volt-amperes or *apparent power* is kilovolt-amperes (kva).

Kilovar current and kilowatt current must be added *vectorially*, not *arithmetically*. If the kilowatt and kilovar are each 2 amp (Fig. 34), the total current may be found from the right-triangle relationship.

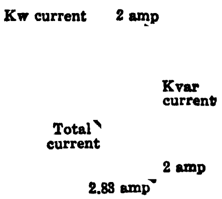


FIG. 34. Vector addition of magnetizing and power-producing currents.

$$(\text{Total current})^2 = (\text{kilovar current})^2 + (\text{kilowatt current})^2$$

$$\text{Total current} = \sqrt{8} = 2.43 \text{ amp}$$

If any two of the three currents or quantities are known, the third current can be obtained from the right-triangle relationship.

$$\text{Total kva} = \sqrt{(\text{kw})^2 + (\text{kvar})^2}$$

WHAT IS POWER FACTOR? Power factor is the ratio of power-producing current in a circuit to the total current in that circuit. Another definition of power factor, generally more useful, is the ratio of kw or working power to the total kva or apparent power. For the example illustrated in Fig. 34 the power factor is 2/2.83 = 0.707, or 70.7%. The angle included between the kva and the kilowatt components is called the *power-factor angle*, designated by θ. The cosine of this angle (cos θ) is the power factor.

$$\text{Power factor} = \frac{\text{kw}}{\text{kva}} = \cos\theta$$

The tangent of this angle is the ratio of kilovars to kilowatts. Hence, if kilowatts are known, the kilovars may be found by multiplying kilowatts by the tangent of the power-factor angle (tan θ).

$$\text{Kvar} = kw \times (\tan\theta)$$

Leading and Lagging Power Factor. The terms *leading power factor* and *lagging power factor* are confusing and meaningless unless the direction of both kilowatt and kilovar flow are known. Generally, in industrial plants, only the load power factor is considered. The following rule is helpful in differentiating between leading and lagging power factor. "The power factor is lagging if the load *requires* kilovars and leading if the load *furnishes* kilovars." Thus an induction motor has a lagging power factor; a synchronous motor has a leading power factor.

How to Improve Power Factor. When the kilovar current in a circuit is reduced, the total current is reduced. If the kilowatt current does not change, as is usually true, the power factor will improve as kilovar current is reduced. When the kilovar current becomes zero, all current is kilowatt current and the power factor is 1.0 or 100%. Thus the power factor may be improved by supplying the load kilovar requirements by a capacitor (or a synchronous motor) as in Fig. 35.

From Fig. 35, it might appear that the magnetizing requirements are the difference between the supply circuit kva and the motor kw or (100 − 80) = 20 kvar, and that a 20-kvar capacitor can be used to supply the magnetizing requirements. However, neither assumption is correct because of the triangle relationship. Simple subtraction of kilowatts from total kva never equals the kilovars.

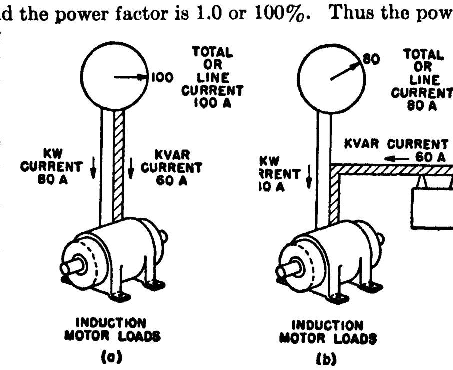


FIG. 35. Sources of kilovar current.

In actual practice, it is generally not necessary to improve the power factor to 100%; capacitors or other power-factor improvement means are used to supply part of the load kvar requirements and the power system the remainder.

There are several methods of improving power factor, but the two most common ones are by use of (1) capacitors and (2) synchronous motors. They have their respective advantages, which may be generalized as follows. Capacitors can be applied to any low power-factor load, are available in small kvar ratings for small loads, but can be combined into groups for large kvar ratings, are always in service (as contrasted with a synchronous motor which supplies kilovars only when it is running), and are generally the most economical means of overall power-factor improvement for most industrial plants. Synchronous motor applications are usually associated with certain types of motor drives, and for those applications they are usually the most economical means of obtaining power-factor improvement.

POWER FACTOR OF A GROUP OF LOADS. The power factor of an individual load generally is known or can be estimated. The power factor of a group of different loads is calculated, unless it can be measured.

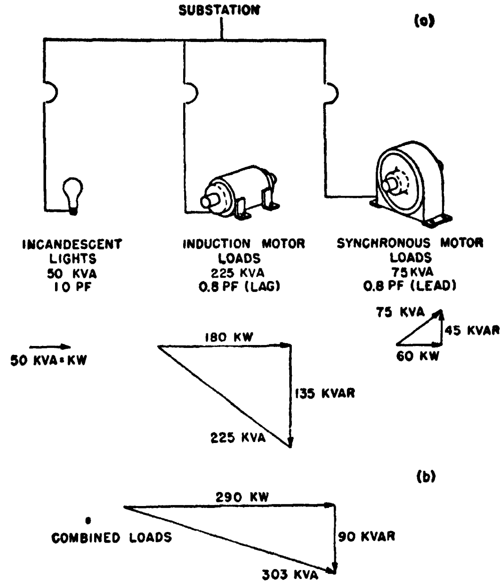


FIG. 36. Combining of substation loads.

EXAMPLE. Figure 36 shows a substation supplying different kinds of load. Their load values and combined components are:

Kilowatts:

Lights	=	50
Induction motor load	=	180
Synchronous motor	=	60
Total		290 kw

Kilovars:

Lights	=	0
Induction motor loads	=	135
Subtotal		135

Since synchronous motors have the ability to supply kilovars, the net kilovars that must be supplied by the substation is, therefore, the difference between the kilovars supplied by the synchronous motor and the kilovars required by the induction motor loads.

Induction motor loads require	135 kvar
Synchronous motor supply	45
Substation must supply	90 kvar

The total load and power factor are:

$$\text{Kva} = \sqrt{(290)^2 + (90)^2} = 303$$

$$\text{Power factor} = \frac{\text{kw}}{\text{kva}} = \frac{290}{303} = 0.956 \text{ or } 95.6\%, \text{ lagging.}$$

The various loads are added vectorially, as shown in Fig. 36. The directions of the kilovar components (down) for lagging power factor and (up) for leading power factor as shown are in accordance with accepted practice.

16. POWER-FACTOR CALCULATIONS

The right-triangle method is laborious for power-factor calculations. However, from the right-triangle relationship several simple and useful mathematical expressions may be written: $\cos\theta$ = pf = kw/kva, $\tan\theta$ = kvar/kw, $\sin\theta$ = kvar/kva.

Because the kilowatt component usually remains constant (kva and kvar components change with power factor), the most convenient expression is kvar = kw $\times$ $\tan\theta$.

For example, assume it is necessary to determine the kilovar rating to improve the load power factor.

$$\text{Kvar at original pf} = \text{kw} \times \tan\theta_1$$

$$\text{Kvar at improved pf} = \text{kw} \times \tan\theta_2$$

The capacitor rating required to improve the power factor is:

$$\text{kvar} = \text{kw} \times (\tan\theta_1 - \tan\theta_2)$$

All tables, charts, and curves which have a "kw multiplier" for determining the kilovar requirements for power-factor improvement are based on this equation.

POWER-FACTOR IMPROVEMENT TABLE. Kilowatt multipliers for determining the kilovar requirements necessary for improving the load power factor are given in Table 22.

EXAMPLE. Total kilowatt load of a plant is 100 kw at a power factor of 60%. The kvar of capacitors necessary to improve the power factor to 80% is found by multiplying the 100 kw by the factor found in the table, which is 0.583.

The kvar of capacitors required is 100 $\times$ (0.583) = 58 kvar. The nearest standard size is 60 kvar.

POWER-FACTOR IMPROVEMENT AND RATE CLAUSES. The primary reason for improving industrial plant power factor is to reduce power costs when a power-factor clause is part of the rate structure. The usual gross rate of return on the investment when capacitors are used for that purpose is 50 to 200%.

RELEASE OF ELECTRICAL CAPACITY BY POWER-FACTOR IMPROVEMENT. A system can carry maximum kilowatts when no kilovars are flowing, i.e., at unity power-factor. Alternating-current equipment such as generators, transformers, or cables can then deliver kilowatts equal to their kva rating. At any other power factor the kilowatt capacity is less than the kva capacity.

The amount of electrical capacity released by power-factor improvement is shown in Fig. 37. These curves also show the economical limits of releasing capacity with capacitors. (In industrial plants the S/C ratios range from 2/1 to 3/1 so that capacitors are almost always the most economical means of releasing capacity.)

ELECTRICAL LOSSES ARE REDUCED BY POWER-FACTOR IMPROVEMENT. In most industrial plant power-distribution systems, the kilowatt (I^2R) losses vary from 2.5 to 7.5% of the load kilowatt-hours, depending on hours of full load and no load plant operation, wire size, and length of main and branch circuits.

Losses are proportional to current squared, and since current is reduced in direct proportion to power-factor improvement, the losses are inversely proportional to the square of the power factor.

$$\text{Kw losses} \propto \left(\frac{\text{original pf}}{\text{improved pf}}\right)^2$$

$$\text{Loss reduction} = 1 - \left(\frac{\text{original pf}}{\text{improved pf}}\right)^2$$

Since capacitors, or other power-factor improvement means, cannot be practically and economically applied to every load, full benefit of loss reduction cannot be realized. For estimating purposes a net realization of 50 to 60% may be assumed.

VOLTAGE LEVEL RAISED BY POWER-FACTOR IMPROVEMENT. The expression for the approximate voltage drop in a circuit is

$$e = R \times (\text{kw current}) \pm X \times (\text{kvar current})$$

and from this it is seen that kilovar current operates only on the reactance. Since capacitors reduce the kilovar current they reduce the voltage drop (and therefore raise the voltage level) by an amount equal to the capacitor current times the reactance. Therefore, it is necessary only to know these two values to predict the voltage change due to capacitors.

Magnitude of Voltage Rise Due to Capacitors. The voltage rise due to capacitors in industrial plants with modern power distribution systems is not very great and is rarely more than 4 or 5%. The greatest gain in voltage improvement will be in plant-distribution

Table 22. Power-factor Improvement

(Figures below × kilowatt input = kvar of capacitance required to improve from one power factor to another.)

Original Power Factor, %	Desired Power Factor, %				
	100	95	90	85	80
50	1.732	1.403	1.248	1.112	.982
51	1.687	1.358	1.202	1.067	.936
52	1.643	1.314	1.158	1.023	.892
53	1.600	1.271	1.116	0.980	.850
54	1.559	1.230	1.074	0.939	.808
55	1.518	1.189	1.034	0.898	.768
56	1.479	1.150	0.995	0.859	.729
57	1.442	1.113	0.957	0.822	.691
58	1.405	1.076	0.920	0.785	.654
59	1.368	1.040	0.884	0.748	.618
60	1.333	1.004	0.849	0.713	.583
61	1.299	0.970	0.815	0.679	.549
62	1.266	0.937	0.781	0.646	.515
63	1.233	0.904	0.748	0.613	.482
64	1.201	0.872	0.716	0.581	.450
65	1.169	0.840	0.685	0.549	.419
66	1.138	0.810	0.654	0.518	.388
67	1.108	0.779	0.624	0.488	.358
68	1.078	0.750	0.594	0.458	.328
69	1.049	0.720	0.565	0.429	.298
70	1.020	0.691	0.536	0.400	.270
71	0.992	0.663	0.507	0.372	.241
72	0.964	0.635	0.480	0.344	.214
73	0.936	0.608	0.452	0.316	.186
74	0.909	0.580	0.425	0.289	.158
75	0.882	0.553	0.398	0.262	.132
76	0.855	0.527	0.371	0.235	.105
77	0.829	0.500	0.344	0.209	.078
78	0.802	0.474	0.318	0.182	.052
79	0.776	0.447	0.292	0.156	.026
80	0.750	0.421	0.266	0.130	
81	0.724	0.395	0.240	0.104	
82	0.698	0.369	0.214	0.078	
83	0.672	0.343	0.188	0.052	
84	0.646	0.317	0.162	0.026	
85	0.620	0.291	0.136		
86	0.593	0.265	0.109		
87	0.567	0.238	0.082		
88	0.540	0.211	0.056		
89	0.512	0.183	0.028		
90	0.484	0.155			
91	0.456	0.127			
92	0.426	0.097			
93	0.395	0.066			
94	0.363	0.034			
95	0.328				
96	0.292				
97	0.251				
98	0.203				
99	0.142				

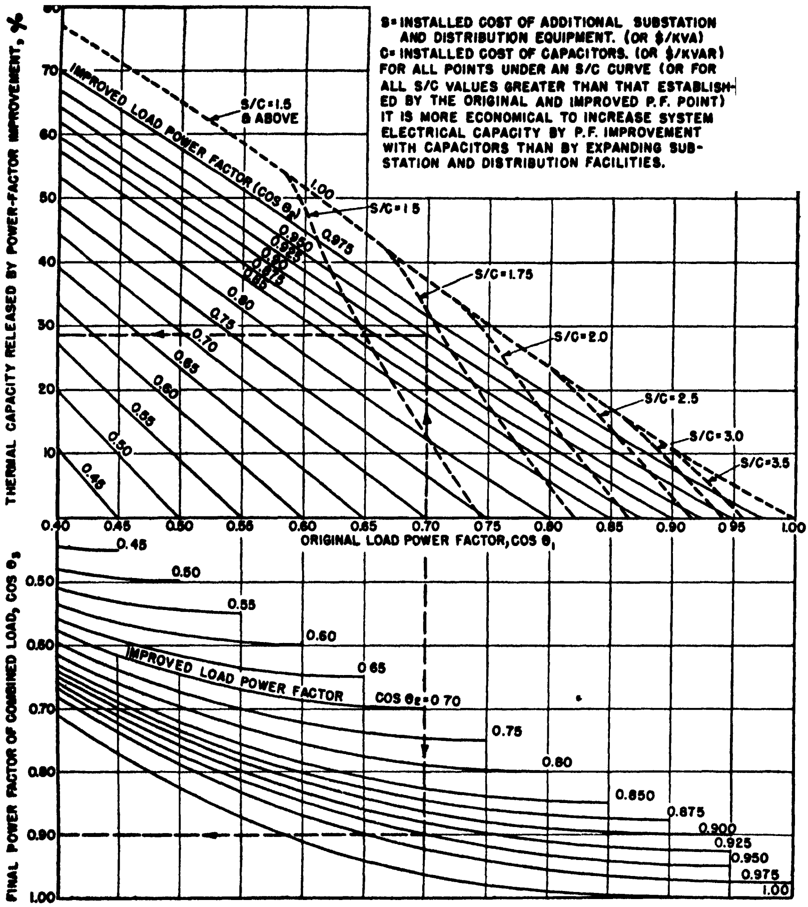


FIG. 37. Percentage thermal capacity released by improvement of load power factor and the relative economics of capacitors for increasing the electrical capacity.

Example. If the load power factor (cos θ_1) of a substation is improved from 70 to 95%, the system electrical capacity released is 28.5%; that is, the system can carry 28.5% more load (at 70% power factor) without exceeding the kva before the power factor was improved.

These curves also show that it is more economical to obtain this amount of system capacity by power-factor improvement with capacitors than by additional substation and distribution facilities for all S/C values greater than 1.7 (obtained by interpolation).

The final power factor (cos θ_3) of the original load plus the additional load is 90%.

circuits having high reactance and low system voltage, as in 230-volt systems having old-style feeder construction where the wires between phases are spaced far apart. When the load-center method of distribution is used for 460- and 575-volt systems, the voltage improvement will be small.

POWER-FACTOR IMPROVEMENT OF INDUCTION MOTORS. Power-factor improvement of induction-motor loads by shunt capacitors has been a common practice for many years. A recent development is the practice of connecting the capacitors directly at the motor terminals to permit switching the capacitors and motor as a unit.

The power factor of induction motors is quite high at full load, usually between 80 and 90%, depending on the motor speed and type of motor. At light loads, however, the power factor drops rapidly (see Fig. 38). Generally, induction motors do not operate at full load (often the drive is "overmotored"), resulting in a low operating power factor.

Even though the power factor of an induction motor varies materially from no load to full load, the motor kilovars are essentially constant as shown in Fig. 38. This characteristic makes the induction motor a particularly attractive capacitor application. With a properly selected capacitor the operating power factor is excellent over the entire range of the motor, generally 95 to 98% at full load and higher at partial loads for the capacitor ratings listed in Table 23.

Selection of Capacitor Rating for Induction Motors. Capacitors have been applied to induction motors and switched with the motor as a unit with good results except in a few cases. Experience has shown that when difficulties are encountered, it is because too large a capacitor bank is used. Table 23 lists the maximum recommended capacitor when the motor and capacitor are switched as a unit.

Table 23. Recommended Maximum Capacitor Rating When Capacitor and Motor Are Switched as a Unit *

NORMAL STARTING TORQUE MOTORS

Motor Horsepower Rating	Motor Speed in Rpm											
	3600		1800		1200		900		720		600	
	Kvar †	% AR ‡	Kvar	% AR	Kvar	% AR	Kvar	% AR	Kvar	% AR	Kvar	% AR
10	2.5	9	4	11	4	12	5	17	5	23	7.5	28
15	2.5	9	5	11	5	11	7.5	16	7.5	21	10	26
20	5	9	5	10	5	11	7.5	15	10	20	12.5	24
25	5	9	7.5	10	7.5	10	10	14	10	19	15	22
30	7.5	9	10	9	10	10	10	13	12.5	18	15	21
40	10	9	10	9	10	10	12.5	12	15	16	17.5	19
50	12.5	9	12.5	9	12.5	9	15	12	20	15	22.5	17
60	15	9	15	8	15	9	17.5	11	22.5	14	25	16
75	17.5	9	17.5	8	17.5	8	20	11	27.5	13	30	15
100	22.5	9	22.5	8	22.5	8	25	10	35	12	37.5	14
125	25	9	27.5	8	27.5	8	30	9	40	11	47.5	13
150	32.5	9	35	8	35	8	37.5	9	47.5	11	55	13
200	42.5	9	42.5	8	42.5	8	45	9	60	10	67.5	12

HIGH STARTING TORQUE MOTORS

Motor Horsepower Rating	Motor Speed in Rpm						Motor Horsepower Rating	Motor Speed in Rpm					
	1800		1200		900			1800		1200		900	
	Kvar †	% AR ‡	Kvar	% AR	Kvar	% AR		Kvar †	% AR ‡	Kvar	% AR	Kvar	% AR
5	2.5	16	3	22	4	30	30	10	12	10	13	15	17
7.5	3	14	4	19	5	27	40	12.5	11	15	12	20	17
10	4	13	5	17	7.5	25	50	15	11	17.5	12	22.5	16
15	5	12	5	15	10	22	60	17.5	11	20	11	25	16
20	5	12	7.5	14	10	19	75	20	11	25	11	30	15
25	7.5	12	10	13	12.5	18							

Notes. These data apply when the capacitor is connected at the motor terminals.

* Representative data for 3-phase, 60-cycle, general-purpose, or splashproof-type motors of 220, 440, or 2300 volt rating. For 50-cycle application the following representative data may be used:

(1) For standard 60-cycle motors operating at 50 cycles, multiply kvar from table by 1.3. Multiply per cent AR from table by 1.1.

(2) For standard 50-cycle motors operating at 50 cycles, multiply kvar from table by 1.1. Multiply per cent AR from table by 1.0.

The operating power factor, with capacitor ratings listed in these tables, ranges from 95 to 98% at full load and from 95 to 100% at partial loads.

† Kvar is rating of capacitors in kilovolt-amperes.

‡ % AR is percentage reduction in line current due to capacitors and is helpful for selecting the proper motor-overload relay.

COST COMPARISON OF INDUCTION MOTORS AND CAPACITORS VERSUS SYNCHRONOUS MOTORS. Initial cost is the most important guide in making the selection between the induction motor plus capacitors and the synchronous motor. The following tables show motor rating ranges where the cost of an induction motor plus capacitors (including a separate switching device for the capacitors) is less than an 0.8 pf synchronous motor and starter.

440- and 550-volt Equipment.

Motor Speed, rpm	Motor Rating, hp
1800	250 and less
1200	150 and less
900	150 and less
600	200 and less

2300-volt Motor and Control Equipment with 460- or 575-volt Capacitors. (Capacitors should be located on the 460- or 575-volt utilization systems rather than 2400- or 4160-volt systems for maximum overall benefit.)

Motor Speed, rpm	Motor Rating, hp
1800	250 and less
1200	175 and less
900	175 and less
600	200 and less

2300-volt Equipment. The synchronous motor and starter equipment costs less than the induction motor and capacitor combination over the entire speed range if a power circuit breaker is used to switch the capacitors.

For the above comparisons, the capacitor rating was selected on the basis that the induction-motor and capacitor combination will furnish the same amount of power-factor improvement at full load as an 0.8 power-factor synchronous motor of equal rating. At partial-load operation, the synchronous motor can supply slightly more kilovars to the system than the induction motor plus capacitor combination, but the difference is not great enough to be the basis of selection.

From the standpoint of total losses the two methods are a standoff. However, where efficiency is carefully evaluated individual comparisons should be made.

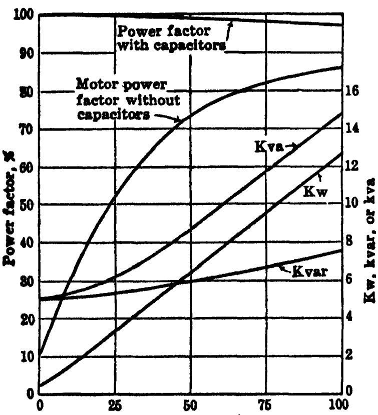


FIG. 38. Effect of capacitors on induction motor power factor.

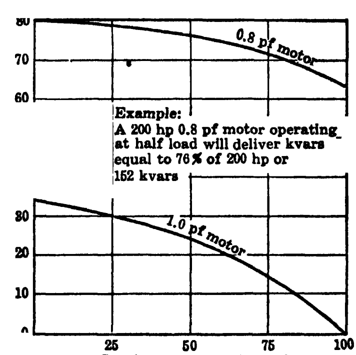


FIG. 39. Synchronous motor kilovars versus load at rated excitation.

POWER-FACTOR IMPROVEMENT FROM SYNCHRONOUS MOTORS. Synchronous motors may act as kilovar generators, and they generate kilovars in the same manner as a conventional generator does. Their ability to generate kilovars is a function of excitation and load. When underexcited they may not generate sufficient kilovars to supply their own needs and consequently must take additional kilovars from the system. When overexcited (normal operation), they can supply all their own kilovar requirements, and in addition can supply kilovars to the system. Thus they may be operated as kilovar generators. The curves of Fig. 39 show the kilovars that a synchronous motor is capable of delivering under various load conditions with normal excitation. As load increases, the number of kilovars the motor can supply decreases. At high overloads (not shown on these curves) a synchronous motor will take magnetizing current from the line.

MAXIMUM CAPACITOR RATING FOR GENERATORS. When generators are operated at *leading* power factor or *underexcited*, i.e., with less than normal excitation (as when too many kilovars are supplied by capacitors or synchronous motors), the generator will be unstable so that the prime mover pulls out of step and is unable to maintain voltage.

Figure 40 shows the approximate maximum kilovars for leading power-factor operation of generators for solid rotor (turbine-type) generators, and also for salient pole (engine-type) generators. Note that the kilovar and kilowatt values are in percentage of the *generator kva rating*. These data are approximate—there are too many variables to consider to be more specific. They serve as a useful guide, however, in establishing the maximum amount of unswitched capacitors.

LOCATION OF CAPACITORS. All the benefits that capacitors provide stem from the reduction of kilovar flow. This is true of power bill savings, release of system capacity, voltage improvement, and reduction of

Maximum reduction in kilovar flow is obtained when capacitors are located *at the load.* The concept of capacitors as kilovar generators is particularly helpful in understanding this point.

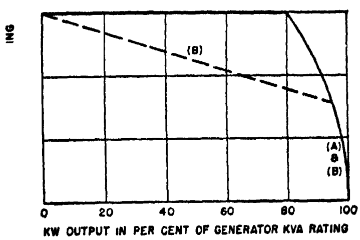


FIG. 40. Maximum kilovar output of generators. Data applicable to modern generators of normal design operated with voltage regulators. Curve *A* is for salient pole (engine-type) generators. Curve *B* is for round rotor (turbine-type) generators.

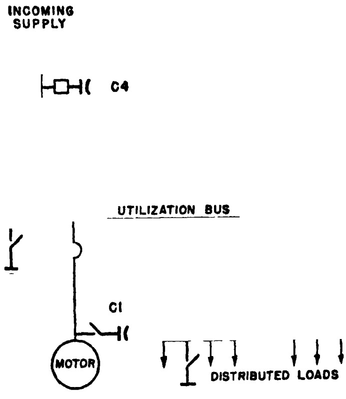


FIG. 41. Best location of capacitors. Order of preference: C_1, C_2, C_3, C_4.

Although maximum overall benefit is obtained when capacitors are located at the load, it is not always practical or economical to locate capacitors at each load. Whenever possible capacitors should be located near the load on 460- or 575-volt systems to obtain minimum cost and maximum benefits. These locations are shown in Fig. 41. The most desirable location is at the load as shown by C_1; next C_2, etc.

SUBSTATIONS *

An electric power substation is a grouping of electrical equipment through which electric energy is passed for the purpose of switching or of transforming voltage levels. Installations including generation or utilization equipment or minor distribution or transmission equipment are not usually classified as substations.

The general classification includes *primary substations* and *secondary substations.* A primary substation is defined as a substation having a low-side voltage between 601 and 15,000 volts. A secondary substation is one having a low-side voltage of 600 volts or less.

17. PRIMARY SUBSTATIONS

Primary substations provide the connecting link between transmission or subtransmission circuits and distribution circuits. They may be located outdoors, adjacent to the industrial plant, or only the high-voltage switchgear and transformers may be located outside the building and connected to indoor low-voltage switchgear by metal-enclosed bus passing through the building wall. In another arrangement, the entire primary substation is located outdoors whereas metering devices, relays, and control switches for the power circuit-breakers are located indoors on a central control panel. Factors that affect the arrangement selection are:

(1) Comparative value of space indoors and outdoors.
(2) Difference in the cost of weatherproof and indoor-type switchgear.

* Contributed by J. W. Yetter and L. D. Madsen.

(3) Frequency of operation of the power breakers and various other circuit elements.
(4) Prevailing weather conditions that may influence the desirability of operating outdoor installations.
(5) Presence of corrosive, dusty, or otherwise unsuitable atmospheres that may be encountered with some industrial processes.
(6) Local building codes and regulations.

A typical outdoor primary substation (Fig. 42) consists of *high-voltage switching equipment*, *transformers*, and *low-voltage metal-clad switchgear*. A description of these three components is given in the following paragraphs.

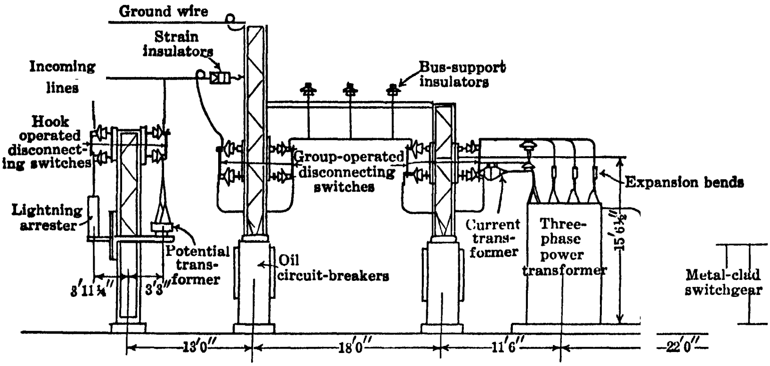


FIG. 42. Typical primary substation.

HIGH-VOLTAGE SWITCHING EQUIPMENT. For voltages above 15 kv, switchgear consisting of *outdoor circuit-breakers*, *disconnecting switches*, *lightning protective equipment*, *fuses*, and *open-type bus* are normally used. These elements are mounted on or within a steel structure. Incoming circuits are terminated on the structure by means of strain insulators. Outdoor power circuit-breakers for stationary service are available in voltage ratings of 7.5 kv and above. Interrupting ratings vary with voltage and range from 50,000 kva to 10,000,000 kva. These breakers are usually of the oil-filled tank type with a single mechanism for three-pole operation. They are provided with their own means of support so that they may be mounted on a concrete pier within the main substation structure.

Disconnecting switches mounted on porcelain insulators in the station structure may be of the *group-operated* or *hook-operated* type. Group-operated switches are somewhat more expensive but are usually preferred because they permit indirect operation either manually or by motor mechanisms. Hook-operated switches are commonly used with fuses or as unfused switches for single-phase devices such as lightning arresters, and potential and control transformers. It is usual practice to apply group-operated isolating switches with stationary-mounted power circuit-breakers ahead of power transformers. Although disconnecting switches have no published interrupting rating, it is recognized that they have certain limited interrupting ability. Group-operated switches equipped with arcing horns are frequently used at lower voltages to interrupt transformer exciting current, line charging current, and, in some cases, small load currents. For breaking load, heavy line-charging current, or exciting current of large transformers, particularly at the higher voltages, a power circuit-breaker is used.

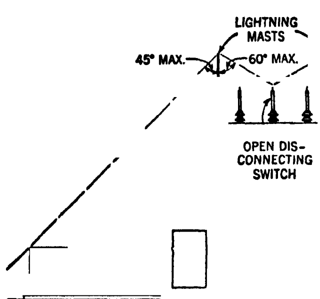


FIG. 43. Cones of protection provided by lightning masts or ground wires.

Where transformers are too small to justify individual high-voltage breakers, power fuses are frequently employed to remove a faulted circuit element from the system. These fuses should be chosen to provide adequate protection, should be rated to pass transformer magnetizing inrush currents, and should coordinate with low-voltage circuit-breaker relaying.

Surge Protection. Outdoor substations are provided with protection from both direct strokes and voltage surges arriving over transmission lines. Direct stroke protection is afforded by lightning masts extending above the substation structure as shown by **Fig. 43** or by overhead ground wires. Direct stroke protection should be designed so that all substation equipment is included within protective cones around the masts and/or ground wires. The protective cone should have angles not greater than those shown in **Fig. 43.**

The best protection against incoming surges is provided by lightning arresters. The type of arrester used depends primarily on the voltage of the circuit. The types available are: (1) station-type, 3 kv to 300 kv; (2) line-type, 20 kv to 73 kv; (3) distribution-type, 1 kv to 15 kv.

The station-type arrester affords a better protective level and has a higher discharge capacity than either of the other types. When the importance of equipment does not justify the expense of the station type arrester, the distribution or line types may be used.

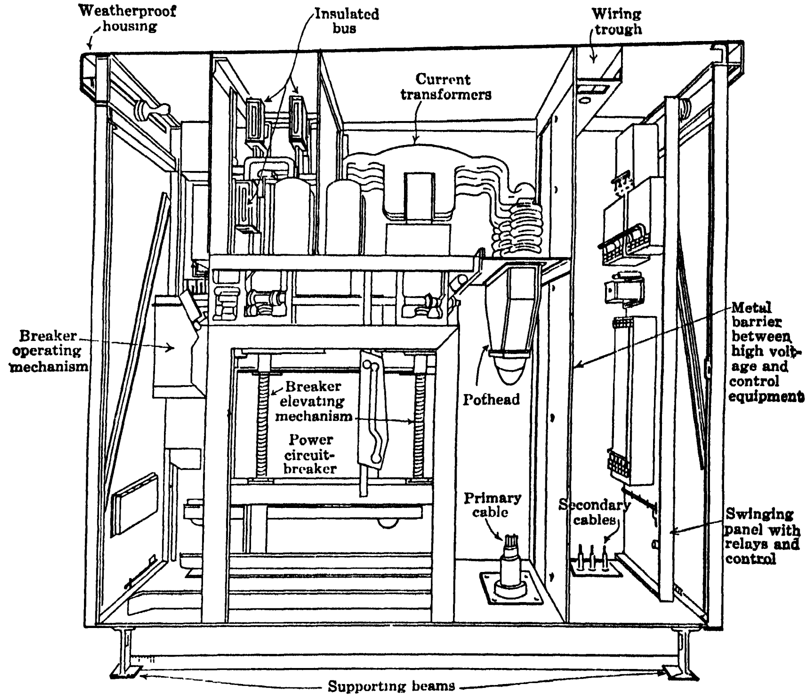


FIG. 44. Side view of typical metal-clad feeder breaker compartment.

The arrester should be located as close as possible electrically to the protected equipment. The arresters also should be located to protect normally open disconnecting switches or circuit-breakers.

POWER TRANSFORMERS integrally connected to metal-enclosed switchgear are of the three-phase type. Oil-filled transformers are used outdoors. Where power transformers are installed indoors, a noninflammable askarel-type insulating and cooling liquid is used. When relative location of transformers and switchgear permits, connections between components are made through a metal enclosure. This may be a direct connection, as shown in Fig. 42, or a length of metal-enclosed bus. In the latter case the transformers and switchgear may be separated by a building wall. In other arrangements, cable is used to connect transformer and switchgear. Incoming line connections to the transformer may be to cover-bushings or by means of cable to a junction box or switch box on the end of the tank. Outgoing connections are through bushings in the end of the tank to the switchgear, bus, or cable junction box. Standard substation transformers of the ratings normally used for industrial plants are listed in Table 24.

Where regulated primary feeders are required, transformers are available in listed ratings with self-contained automatic step-voltage regulators having a range of $\pm 10\%$.

Table 24. Ratings of Standard Power Transformers

Item	Nominal Incoming System Voltage	Preferred High-voltage Rating and Tap Voltages	Low-voltage Ratings	Self-cooled Kva Ratings	Fan-cooled Kva Ratings	Per Unit Impedance *
A	7,200	7,560 7,380 7,200 7,020 6,840	2,400 2,520 4,160 4,360 4,800 5,040 6,900 7,200	750 1,000 1,500 2,000 2,500 3,000 3,750 5,000	862 1,150 1,725 2,300 2,875 3,650 4,687 6,250	.055
B	7,620	8,000 7,810 7,620 7,430 7,240	As in A, plus 7,560	As in A	As in A	.055
C	8,320	8,736 8,528 8,320 8,112 7,904	As in B, plus 8,320	As in A	As in A	.055
D	12,000	12,600 12,300 12,000 11,700 11,400	As in C, plus 8,720 12,000	As in A	As in A	.055
E	13,200	13,860 13,530 13,200 12,870 12,540	As in D, plus 12,470 12,600 13,200	As in A	As in A	.055
F	14,400	14,400 14,100 13,800 13,500 13,200	As in E, plus 13,800 14,400	As in A	As in A	.055
G	23,000	24,100 23,500 22,900 22,300 21,700	As in F	As in A	As in A	.055
H	34,500	36,200 35,300 34,400 33,500 32,600	As in F	As in A, plus 6,000 7,500	As in A, plus 7,500 9,375	.060
J	46,000	46,200 45,000 43,800 42,600 41,400	As in F	As in H	As in H	.065
K	69,000	70,600 68,800 67,000 65,200 63,400	As in F	As in H	As in H	.070

* For transformers equipped with load-ratio control, impedance will be 0.005 greater.

Table 25. Circuit-breaker Ratings

INDOOR OIL-LESS POWER CIRCUIT-BREAKERS

Voltage Ratings			Short-time Current Ratings, amperes			Interrupting Ratings		
Rated Kv	Max Design Kv	Min Kv for Rated Interrupting Mva	Continuous 60 Cycles	Momentary	Four Second	3-Phase Rated Mva	Amperes at Rated Voltage	Max Amperes
4.16	4.76	2.3	600	20000	12500	50	7000	12500
4.16	4.76	2.3	1200	20000	12500	50	7000	12500
4.16	4.76	3.5	600	40000	25000	150	21000	25000
4.16	4.76	3.5	1200	40000	25000	150	21000	25000
4.16	4.76	3.5	2000	40000	25000	150	21000	25000
4.16	4.76	3.85	1200	60000	37500	250	35000	37500
4.16	4.76	3.85	2000	60000	37500	250	35000	37500
7.2	8.25	4.6	1200	51000	32000	250	20000	32000
7.2	8.25	4.6	2000	51000	32000	250	20000	32000
7.2	8.25	6.6	1200	70000	44000	500	40000	44000
7.2	8.25	6.6	2000	70000	44000	500	40000	44000
13.8	15.0	6.6	600	20000	13000	150	6300	13000
13.8	15.0	6.6	1200	20000	13000	150	6300	13000
13.8	15.0	6.6	1200	35000	22000	250	10600	22000
13.8	15.0	6.6	2000	35000	22000	250	10600	22000
13.8	15.0	11.5	1200	40000	25000	500	21000	25000
13.8	15.0	11.5	2000	40000	25000	500	21000	25000

INDOOR OIL POWER CIRCUIT-BREAKERS

Rated Kv	Max Design Kv	Min Kv for Rated Interrupting Mva	Continuous 60 Cycles	Momentary	Four Second	3-Phase Rated Mva	Amperes at Rated Voltage	Max Amperes
4.16	4.76	2.3	600	10000	6300	25	3500	6300
4.16	4.76	2.3	1200	20000	12500	50	7000	12500
7.2	8.25	2.3	600	20000	12500	50	4000	12500
7.2	8.25	2.3	600	40000	25000	100	8000	25000
7.2	8.25	2.3	1200	40000	25000	100	8000	25000
7.2	8.25	2.3	2000	40000	25000	100	8000	25000
13.8	15.0	4.0	600	35000	22000	150	6300	22000
13.8	15.0	4.0	1200	35000	22000	150	6300	22000
13.8	15.0	4.0	1200	60000	36000	250	10600	36000
13.8	15.0	6.6	1200	70000	44000	500	21000	44000
13.8	15.0	6.6	2000	70000	44000	500	21000	44000

The voltage maintained is a function of load current. This equipment is commonly called *load-ratio control equipment.*

LOW-VOLTAGE SWITCHING EQUIPMENT. For 15 kv and below, switching equipment may be of the open type described above or of the completely metal-enclosed factory-assembled type. Nearly all modern installations employ the latter type. An interior perspective view of a typical metal-clad feeder section is shown in Fig. 44. A typical installation includes several of these feeder compartments together with auxiliary compartments necessary to house control power and potential transformers, tripping, battery and charger, and associated accessory devices. Features of the metal-enclosed construction include separate compartments for the insulated bus, power circuit-breaker, relaying and control panel, current transformers, and the incoming cable. The power circuit-breakers are either of the vertical-lift type, as illustrated, or of the horizontal-drawout type, to make the breakers easily removable. Isolation from the circuit is accomplished without disconnecting switches. Power circuit-breakers are available in ratings shown in Table 25.

Metal-clad switchgear rated 5 or 15 kv is applicable for the high-voltage or low-voltage circuits of primary substations or as incoming line sections to secondary substations.

18. SECONDARY SUBSTATIONS

Secondary substations provide the connecting link between distribution circuits and utilization equipment. Standard secondary voltage ratings are 208Y/120, 240, 480, and 600 volts.

Secondary substations consist of incoming line sections, transformers, and low-voltage switchgear. These sections are usually joined to form an integral unit. A typical secondary substation including incoming liquid-filled switches, askarel transformers, and metal-enclosed drawout switchgear is illustrated in Fig. 45.

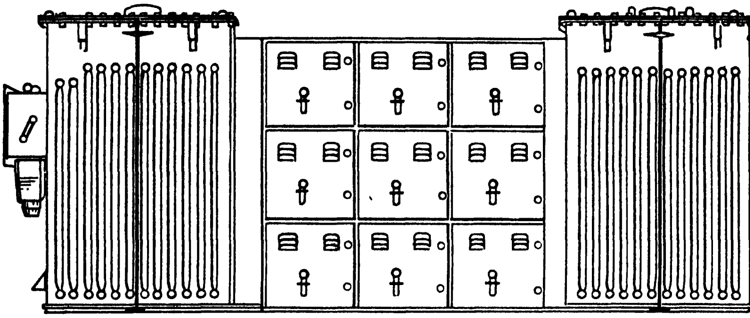


FIG. 45. Typical double-ended load center unit substation.

INCOMING LINE SECTIONS may be (1) metal-clad switchgear; (2) switch and fuse; (3) interrupter switch; (4) cable junction box; and (5) cover bushings.

Metal-clad switchgear, described under Primary Substations, Art. 17, can be directly connected on the high-voltage side of the transformer. This equipment has one or more transformer primary circuit-breakers and necessary auxiliary equipment. Power company metering equipment, if required, may be included.

Where necessary, the incoming section of each may consist of an interrupter switch and fuses. The fuses are supplied to remove automatically a faulted unit from the primary circuit. (See Art. 12.) The switch provides manual disconnection of a unit without interruption of service to the remaining units.

When suitable fault protection is afforded by a primary feeder breaker and an interruption of service to other substations on that circuit is not objectionable, the fuses are omitted and only the interrupter switch is furnished. This may be a liquid-filled or an air switch, depending on the type and kva rating of the transformer with which it is used. The interrupter switch is mechanically interlocked with the low-voltage main breaker to prevent its operation under load.

In simple radial distribution systems, junction boxes arranged to terminate one or more primary cables may be all that is required. At 15 kv and below, these junction boxes are usually air filled.

For outdoor installations, particularly at the higher voltages, the transformers may be equipped with cover bushings for the incoming lines.

TRANSFORMERS. Transformers for use in secondary substations may be of the liquid-filled or air type. Oil-filled transformers are used in those few installations made outdoors. Most secondary substations are installed indoors, and to avoid the expense of

Table 26. Ratings of Transformers for Secondary Substations

Item	Low Voltage Rating	Preferred High-voltage Ratings *	Kva Ratings	Per Unit Impedance
		2,400	100	.040
		4,160	150	.045
		4,800	200	.050
		6,900	300	.050
	208Y/120	12,000	500	.050
		13,200	750	.055
		13,800	1000	.055
	240	As in A	As in A, plus 1500	As in A, plus .055
	480 or 600	As in A	As in B, † plus 2000	As in B,† plus .055

* Standard transformers have four 2 1/2% rated kva taps in the high-voltage windings, two above and two below rated primary voltage.

† Except 100 kva not listed.

fireproof vaults askarel-filled or air-filled transformers are used. Three-phase transformers for use in secondary substations are available in ratings listed in Table 26. Increased capacity can be obtained by forced-air cooling. However, for the smaller ratings, it is usually more economical to purchase the next larger self-cooled rating than to add fans.

OUTGOING FEEDER SWITCHGEAR. Switchgear for 600 volts and lower is entirely enclosed in a grounded, dead-front metal structure, to permit installation of equipment in working areas without hazard to personnel. The assembly usually includes a transformer, main secondary breaker and feeder breakers, together with the necessary bus, current, and potential transformers, and meters and control devices. Where ungrounded shop circuits are used, an indicating ground detector is usually connected to the main bus and mounted on the front of the equipment.

The most frequently used arrangement employs breakers of the drawout (easily removable) construction. The breaker is mounted on a frame which rolls out or swings free of the structure. This drawout carriage is mechanically interlocked to prevent withdrawing the breaker from the connected position when it is closed. A test position is provided to permit operation of the breaker, inspection of the contacts, and general maintenance in the disconnected position without removing the breaker from the equipment. Double-pole and triple-pole drawout air circuit-breakers are available in ratings listed in Table 27.

Table 27. Ratings of Drawout Air Circuit-breakers

Current	Rating	Voltage Rating	
Interrupting Ratings, total rms amperes	Continuous Rating, amperes	A-C	D-C
15,000	15 to 225	600	250
25,000	35 to 600	600	250
50,000	100 to 1600	600	250 and 750
75,000	2000, 3000	600	250 and 750
100,000	4000	600	250 and 750

In smaller ratings, breakers are available for either manual or electrical operation. Those having a continuous rating of 2000 amp or more are usually electrically operated because of their large size and the difficulty of closing them manually.

Air circuit-breakers are provided with inherent direct-acting trip devices which trip as a function of current when faults occur. The overcurrent trip usually consists of a time-delay portion, adjustable down to breaker rating or below, and an instantaneous portion set to operate without intentional time delay for currents of short-circuit magnitude. The most common time delay tripping device consists of a sealed dashpot filled with oil and a piston with a definite size orifice. Current through the breaker is used to excite an electromagnet that moves the piston causing the oil to flow through the orifice. The time required for the oil to flow determines the tripping time since the piston trips the breaker latch at the end of its stroke. This produces an inverse time-tripping curve (see Article 12). The short-circuit trip is a similar electromagnet arranged to overcome the force on a restraining spring above a predetermined value of current. For selective operation between breakers in series, an escapement timer is used to delay tripping of the breakers near the source.

Cascade Operation. In general, the interrupting rating of air circuit-breakers should equal or exceed the fault current obtainable at the location where the breakers are installed, 0.5 cycle after inception of the fault. Where may feeder breakers are served through one or two sources of power, it may be economically justifiable to use feeder breakers of reduced interrupting rating, provided they can be safely applied without too seriously impairing the required selectivity of tripping.

Cascaded or backed-up breakers are applied above their interrupting rating as follows:

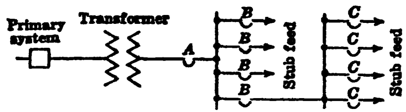
(1) All breakers connected directly to the incoming source must have interrupting ratings equal to the total fault current.

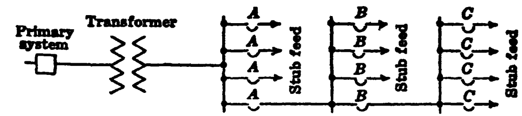
(2) Provided that all breakers between it and the power sources are set to trip instantaneously at not more than 80% of the rating of the second breaker, the breaker in the cascade may be applied up to 200% of its interrupting rating.

(3) Provided that the second breaker in cascade is set to trip instantaneously at not more than 80% of the rating of the third breaker, the third breaker in the cascade may be applied up to 300% of its interrupting rating. Further cascading cannot be done.

(Continued on p. 16-60)

Table 28. Application of Air Circuit Breakers—208 and 240 Volts





Transformer Rating 3-ph Kva and Impedance, %	Max Short-circuit Available from Primary System, mva	208 Volts: Normal Load Continuous Current, amp	208 Volts: Short-circuit Current, Thousands of Total Rms Amperes (Average 3-phase Current): Transformer Alone	50% Motor Load	Combined	208 Volts: Interrupting Rating of Air Circuit-breaker Recommended, Thousands of Amperes: A′ *	A *	B	C	240 Volts: Normal Load Continuous Current, amp	240 Volts: Short-circuit Current, Thousands of Total Rms Amperes: Transformer Alone	100% Motor Load	Combined	240 Volts: Interrupting Rating of Air Circuit-breaker Recommended, Thousands of Amperes: A′ *	A *	B	C
300 5%	15 25 50	834	14.9 16.7 18.6	2.1	17.0 18.8 20.7	50 50 50	25 25 25	15 15 15	15 15 15	722	12.9 14.6 16.1	3.6	16.5 18.2 19.7	50 50 50	25 25 25	15 15 15	15 15 15
	100 150 250		19.6 20.0 20.3		21.7 22.1 22.4	50 50 50	25 25 25	15 15 15	15 15 15		17.0 17.4 17.6		20.6 21.0 21.2	50 50 50	25 25 25	15 15 15	15 15 15
	500 Unlimited		20.5 20.8		22.6 22.9	50 50	25 25	15 15	15 15		17.9 18.1		21.5 21.7	50 50	25 25	15 15	15 15
450 5%	25 50 100	1250	22.9 26.5 28.6	3.1	26.0 29.6 31.7	50 50 50	50 50 50	15 15 25	15 15 15	1083	19.9 22.9 24.8	5.4	25.3 28.3 30.2	50 50 50	50 50 50	15 15 25	15 15 15
	150 250		29.4 30.1		32.5 33.2	50 50	50 50	• 25 25	15 15		25.4 26.0		30.8 31.4	50 50	50 50	25 25	15 15
	500 Unlimited		30.6 31.4		33.7 34.5	50 50	50 50	25 25	15 15		26.5 27.0		31.9 32.4	50 50	50 50	25 25	15 15
500 5%	25 50 100	1388	24.8 28.9 31.5	3.5	28.3 32.4 35.0	50 50 50	50 50 50	15 25 25	15 15 15	1200	21.5 25.1 27.3	6.0	27.5 31.1 33.3	50 50 50	50 50 50	15 25 25	15 15 15
	150 250		32.5 33.3		36.0 36.8	50 50	50 50	25 25	15 15		28.2 28.9		34.2 34.9	50 50	50 50	25 25	15 15
	500 Unlimited		34.0 34.6		37.5 38.1	50 50	50 50	25 25	15 15		29.5 30.1		35.5 36.1	50 50	50 50	25 25	15 15

600 5%	25	1668	28.2	4.2	32.4	75	50	25	15	1443	24.4	7.2	31.6	50	50	25	15
	50		33.5		37.7	75	50	25	15		29.0		36.2	50	50	25	15
	100		37.1		41.3	75	50	25	15		32.1		39.3	50	50	25	15
	150		38.5		42.7	75	50	25	15		33.3		40.5	50	50	25	15
	250		39.7		43.9	75	50	25	15		34.4		41.6	50	50	25	15
	500		40.6		44.8	75	50	25	15		35.1		42.3	50	50	25	15
	Unlimited		41.7		45.9	75	50	25	25		36.0		43.2	50	50	25	15
750 5 1/2%	25	2080	30.6	5.2	35.8	75	50	25	15	1800	26.6	9.0	35.6	75	50	25	15
	50		37.1		42.3	75	50	25	15		32.3		41.3	75	50	25	15
	100		41.6		46.8	75	50	25	25		36.1		45.1	75	50	25	25
	150		43.3		48.5	75	50	* 25	25		37.6		46.6	75	50	25	25
	250		44.8		50.0	75	50	25	25		39.0		48.0	75	50	25	25
	500		46.1		51.3	75	75	50	25		40.0		49.0	75	50	25	25
	Unlimited		47.3		52.5	75	75	50	25		41.1		50.1	75	75	50	25
1000 5 1/2%	25	2780	36.5	7.0	43.5	75	50	25	15	2400	31.7	12.0	43.7	75	50	25	15
	50		46.3		53.3	75	75	50	25		40.2		52.2	75	75	50	25
	100		53.4		60.4	75	75	50	25		46.3		58.3	75	75	50	25
	150		56.3		63.3	75	75	50	25		48.8		60.8	75	75	50	25
	250		58.9		65.9	75	75	50	25		51.0		63.0	75	75	50	25
	500		60.9		67.9	75	75	50	25		52.8		64.8	75	75	50	25
	Unlimited		63.2		70.2	75	75	50	25		54.7		66.7	75	75	50	25
1500 5 1/2%	25									3600	39.3	18.0	57.3	100	75	50	25
	50										53.2		71.2	100	75	50	25
	100										64.5		82.5	100	100	50	50
	150										69.5		87.5	100	100	50	50
	250										74.0		92.0	100	100	50	50
	500										77.9		95.9	100	100	50	50
	Unlimited										82.0		100.0	100	100	50	50

* Breaker *A'* selections are in all cases governed by the necessary continuous-current ratings of the transformer, and this is the reason that in some cases the interrupting ratings are higher than the ratings given for the corresponding *A* breaker. Interrupting ratings of *A* breakers are all based on the interrupting requirements only, as determined by the combined short-circuit total amperes.

Table 29. Application of Air Circuit Breakers—480 and 600 Volts

(See diagram of Table 28 for key to designations)

Transformer Rating 3-ph Kva and Impedance, %	Max Short-circuit Available from Primary System, mva	480 Volts: Normal Load Continuous Current, amp	480 Volts: Short-circuit Current, Thousands of Total Rms Amperes (Average 3-phase, current): Transformer Alone	100% Motor Load	Combined	480 Volts: Interrupting Rating of Air Circuit-breaker Recommended, Thousands of Amperes: *A′* *	*A* *	*B*	*C*	600 Volts: Normal Load Continuous Current, amp	600 Volts: Short-circuit Current, Thousands of Total Rms Amperes: Transformer Alone	100% Motor Load	Combined	600 Volts: Interrupting Rating of Air Circuit-breaker Recommended, Thousands of Amperes: *A′* *	*A* *	*B*	*C*
300 5%	25	361	7.3	1.8	9.1	25	15	15	15	289	5.8	1.4	7.3	25	15	15	15
	50		8.1		9.9	25	15	15	15		6.4		7.9	25	15	15	15
	100		8.5		10.3	25	15	15	15		6.8		8.3	25	15	15	15
	150		8.7		10.5	25	15	15	15		6.9		8.4	25	15	15	15
	250		8.8		10.6	25	15	15	15		7.0		8.5	25	15	15	15
	500		8.9		10.7	25	15	15	15		7.1		8.6	25	15	15	15
	Unlimited		9.0		10.8	25	15	15	15		7.2		8.7	25	15	15	15
450 5%	25	542	9.9	2.7	12.6	25	15	15	15	433	7.9	2.2	10.1	25	15	15	15
	50		11.5		14.2	25	15	15	15		9.0		11.2	25	15	15	15
	100		12.4		15.1	25	25	15	15		9.9		12.1	25	15	15	15
	150		12.7		15.4	25	25	15	15		10.2		12.4	25	15	15	15
	250		13.0		15.7	25	25	15	15		10.4		12.6	25	15	15	15
	500		13.3		16.0	25	25	15	15		10.6		12.8	25	15	15	15
	Unlimited		13.5		16.2	25	25	15	15		10.8		13.0	25	15	15	15
500 5%	25	600	10.8	3.0	13.8	25	15	15	15	481	8.6	2.4	11.0	25	15	15	15
	50		12.5		15.5	25	25	15	15		10.0		12.4	25	15	15	15
	100		13.7		16.7	25	25	15	15		11.0		13.4	25	15	15	15
	150		14.1		17.1	25	25	15	15		11.3		13.7	25	15	15	15
	250		14.5		17.5	25	25	15	15		11.6		14.0	25	15	15	15
	500		14.8		17.8	25	25	15	15		11.8		14.2	25	15	15	15
	Unlimited		15.1		18.1	25	25	15	15		12.0		14.4	25	15	15	15
600 5%	25	722	12.2	3.6	15.8	50	25	15	15	578	9.7	2.9	12.6	25	15	15	15
	50		14.6		18.2	50	25	15	15		11.6		14.5	25	15	15	15
	100		16.1		19.7	50	25	15	15		12.9		15.8	25	25	15	15

	150		16.7		20.3	50	25	15	15		13.3		16.2	25	25	15	15
	250		17.2		20.8	50	25	15	15		13.8		16.7	25	25	15	15
	500		17.6		21.2	50	25	15	15		14.1		17.0	25	25	15	15
	Unlimited		18.1		21.7	50	25	15	15		14.5		17.4	25	25	15	15
750 5 1/2%	25	900	13.3	4.5	17.8	50	25	15	15	722	10.6	3.6	14.2	50	15	15	15
	50		16.1		20.6	50	25	15	15		12.9		16.5	50	25	15	15
	100		18.0		22.5	50	25	15	15		14.5		18.1	50	25	15	15
	150		18.8		23.3	50	25	15	15		15.1		18.7	50	25	15	15
	250		19.5		24.0	50	25	15	15		15.6		19.2	50	25	15	15
	500		20.0		24.5	50	25	15	15		16 0		19.6	50	25	15	15
	Unlimited		20.5		25.0	50	25	15	15		16.4		20 0	50	25	15	15
1000 5 1/2%	25	1200	15.8	6.0	21.8	50	25	15	15	962	12.7	4.8	17.5	50	25	15	15
	50		20.1		26.1	50	50	15	15		16.1		20.9	50	25	15	15
	100		23.2		29.2	50	50	15	15		18.5		23.3	50	25	15	15
	150		24.4		30.4	50	50	25	15		19 5		24.3	50	25	15	15
	250		25.5		31.5	50	50	25	15		20.5		25.3	50	50	15	15
	500		26.4		32.4	50	50	25	15		21.1		25.9	50	50	15	15
	Unlimited		27.4		33.4	50	50	25	15		21 9		26.7	50	50	15	15
1500 5 1/2%	25	1800	19.6	9.0	28.6	75	50	15	15	1444	15 7	7.2	22.9	50	25	15	15
	50		26.6		35.6	75	50	25	15		21.3		28 5	50	50	15	15
	100		32.3		41.3	75	50	25	15		25 9		33.1	50	50	25	15
	150		34.8		43.8	75	50	25	15		27.9		35.1	50	50	25	15
	250		37.0		46.0	75	50	25	25		29.7		36.9	50	50	25	15
	500		38.9		47.9	75	50	25	25		31.2		38.4	50	50	25	15
	Unlimited		41.1		50.1	75	75	50	25		32.9		40.1	50	50	25	15
2000 5 1/2%	25	2400	22.1	12.0	34.1	75	50	25	15	1924	17.7	9.6	27.3	75	50	25	15
	50		31.7		43.7	75	50	25	15		25.3		34.9	75	50	25	15
	100		39.4		51.4	75	75	50	25		31.6		41 2	75	50	25	15
	150		44.1		56.1	75	75	50	25		35.3		44 9	75	50	25	15
	250		46.8		58.8	75	75	50	25		37.5		47.1	75	50	25	25
	500		50.7		62.7	75	75	50	25		40 5		50.1	75	75	50	25
	Unlimited		54.7		66.7	75	75	50	25		43.8		53 4	75	75	50	25

* Breaker A' selections are in all cases governed by the necessary continuous-current ratings of the transformer, and this is the reason that in some cases the interrupting ratings are higher than the ratings given for the corresponding A breaker. Interrupting ratings of A breakers are all based on the interrupting requirements only, as determined by the combined short-circuit total amperes.

Table 29. Application of Air Circuit Breakers—480 and 600 Volts

(See diagram of Table 28 for key to designations)

Transformer Rating 3-ph Kva and Impedance, %	Max Short-circuit Available from Primary System, mva	480 Volts								600 Volts							
		Normal Load Continuous Current, amp	Short-circuit Current, Thousands of Total Rms Amperes (Average 3-phase, current)			Interrupting Rating of Air Circuit-breaker Recommended, Thousands of Amperes				Normal Load Continuous Current, amp	Short-circuit Current, Thousands of Total Rms Amperes			Interrupting Rating of Air Circuit-breaker Recommended, Thousands of Amperes			
			Transformer Alone	100% Motor Load	Combined	*A′* *	*A* *	*B*	*C*		Transformer Alone	100% Motor Load	Combined	*A′* *	*A* *	*B*	*C*
300 5%	25	361	7.3	1.8	9.1	25	15	15	15	289	5.8	1.4	7.3	25	15	15	15
	50		8.1		9.9	25	15	15	15		6.4		7.9	25	15	15	15
	100		8.5		10.3	25	15	15	15		6.8		8.3	25	15	15	15
	150		8.7		10.5	25	15	15	15		6.9		8.4	25	15	15	15
	250		8.8		10.6	25	15	15	15		7.0		8.5	25	15	15	15
	500		8.9		10.7	25	15	15	15		7.1		8.6	25	15	15	15
	Unlimited		9.0		10.8	25	15	15	15		7.2		8.7	25	15	15	15
450 5%	25	542	9.9	2.7	12.6	25	15	15	15	433	7.9	2.2	10.1	25	15	15	15
	50		11.5		14.2	25	15	15	15		9.0		11.2	25	15	15	15
	100		12.4		15.1	25	25	15	15		9.9		12.1	25	15	15	15
	150		12.7		15.4	25	25	15	15		10.2		12.4	25	15	15	15
	250		13.0		15.7	25	25	15	15		10.4		12.6	25	15	15	15
	500		13.3		16.0	25	25	15	15		10.6		12.8	25	15	15	15
	Unlimited		13.5		16.2	25	25	15	15		10.8		13.0	25	15	15	15
500 5%	25	600	10.8	3.0	13.8	25	15	15	15	481	8.6	2.4	11.0	25	15	15	15
	50		12.5		15.5	25	25	15	15		10.0		12.4	25	15	15	15
	100		13.7		16.7	25	25	15	15		11.0		13.4	25	15	15	15
	150		14.1		17.1	25	25	15	15		11.3		13.7	25	15	15	15
	250		14.5		17.5	25	25	15	15		11.6		14.0	25	15	15	15
	500		14.8		17.8	25	25	15	15		11.8		14.2	25	15	15	15
	Unlimited		15.1		18.1	25	25	15	15		12.0		14.4	25	15	15	15
600 5%	25	722	12.2	3.6	15.8	50	25	15	15	578	9.7	2.9	12.6	25	15	15	15
	50		14.6		18.2	50	25	15	15		11.6		14.5	25	15	15	15
	100		16.1		19.7	50	25	15	15		12.9		15.8	25	25	15	15

	150 250		16.7 17.2		20.3 20.8	50 50	25 25	15 15	15 15		13.3 13.8		16.2 16.7	25 25	25 25	15 15	15 15
	500 Unlimited		17.6 18.1		21.2 21.7	50 50	25 25	15 15	15 15		14.1 14.5		17.0 17.4	25 25	25 25	15 15	15 15
750 5 1/2%	25 50 100	900	13.3 16.1 18.0	4.5	17.8 20.6 22.5	50 50 50	25 25 25	15 15 15	15 15 15	722	10.6 12.9 14.5	3.6	14.2 16.5 18.1	50 50 50	15 25 25	15 15 15	15 15 15
	150 250		18.8 19.5		23.3 24.0	50 50	25 25	15 15	15 15		15.1 15 6		18.7 19.2	50 50	25 25	15 15	15 15
	500 Unlimited		20.0 20.5		24.5 25.0	50 50	25 25	15 15	15 15		16.0 16.4		19.6 20.0	50 50	25 25	15 15	15 15
1000 5 1/2%	25 50 100	1200	15.8 20.1 23.2	6.0	21.8 26.1 29.2	50 50 50	25 50 50	15 15 15	15 15 15	962	12.7 16.1 18.5	4.8	17.5 20.9 23.3	50 50 50	25 25 25	15 15 15	15 15 15
	150 250		24.4 25.5		30.4 31.5	50 50	50 50	25 25	15 15		19.5 20 5		24.3 25.3	50 50	25 50	15 15	15 15
	500 Unlimited		26.4 27.4		32 4 33 4	50 50	50 50	25 25	15 15		21 1 21 9		25.9 26.7	50 50	50 50	15 15	15 15
1500 5 1/2%	25 50 100	1800	19.6 26.6 32.3	9.0	28.6 35.6 41.3	75 75 75	50 50 50	15 25 25	15 15 15	1444	15.7 21 3 25 9	7.2	22.9 28.5 33.1	50 50 50	25 50 50	15 15 25	15 15 15
	150 250		34.8 37.0		43.8 46.0	75 75	50 50	25 25	15 25		27.9 29.7		35.1 36.9	50 50	50 50	25 25	15 15
	500 Unlimited		38.9 41.1		47.9 50.1	75 75	50 75	25 50	25 25		31.2 32 9		38 4 40.1	50 50	50 50	25 25	15 15
2000 5 1/2%	25 50 100	2400	22.1 31.7 39.4	12.0	34.1 43.7 51.4	75 75 75	50 50 75	25 25 50	15 15 25	1924	17.7 25.3 31.6	9.6	27.3 34.9 41.2	75 75 75	50 50 50	25 25 25	15 15 15
	150 250		44.1 46.8		56.1 58.8	75 75	75 75	50 50	25 25		35.3 37.5		44 9 47.1	75 75	50 50	25 25	15 25
	500 Unlimited		50.7 54.7		62.7 66.7	75 75	75 75	50 50	25 25		40.5 43.8		50.1 53.4	75 75	75 75	50 50	25 25

* Breaker *A′* selections are in all cases governed by the necessary continuous-current ratings of the transformer, and this is the reason that in some cases the interupting ratings are higher than the ratings given for the corresponding *A* breaker. Interrupting ratings of *A* breakers are all based on the interrupting requirements nly, as determined by the combined short-circuit total amperes.

(4) No breaker, whether satisfying the conditions of 1, 2, and 3 or not, may be applied in any circuit where it may be called upon to open more than its interrupting rating without assistance of all other source breakers. In this respect, unusual conditions of load contribution to a fault must be carefully investigated. Unusually large (over 25%) synchronous motors, or a large percentage of synchronous loadings, will usually rule out cascade applications.

Where multiple-power sources are used, cascade rules usually dictate a very low instantaneous back-up setting. In direct contrast, multiple sources are usually employed to provide continuity of service under all conditions. A secondary selective system with tie circuits normally open often provides a better operating condition.

The principle of cascade operation depends on the back-up breakers operating with such speed that they assist the smaller breaker in interrupting the fault. For this reason, shunt trip coils operated by instantaneous relays are not satisfactory for back-up breakers. Furthermore, if a portion of the source current, but not all of it, is opened by back-up breakers, cascade cannot be used.

Application data for cascaded breakers are shown in Tables 28 and 29.

Transformer secondary breakers are usually selected with a continuous rating slightly above the transformer full-load current since transformers are required at times to carry loads in excess of their output rating.

Breakers serving one or more motors equipped with individual motor starters come under the general classification of motor branch feeder breakers. Their function is to disconnect the motor cables in case of fault. Motor overload and undervoltage protection are provided by the starter. The branch breakers may be equipped with standard inverse

Table 30. Air Circuit-breakers for Motor Circuits

Motor Hp	220 Volts, 3-phase, 60 Cycles				440 Volts, 3-phase, 60 Cycles				550 Volts, 3-phase, 60 Cycles			
	Induction and 0.8 P-F Synchronous Motors		1.0 P-F Synchronous Motors		Induction and 0.8 P-F Synchronous Motors		1.0 P-F Synchronous Motors		Induction and 0.8 P-F Synchronous Motors		1.0 P-F Synchronous Motors	
	Approx. Motor Full Load,* Amp	Recommended A.C.B. Ampere Rating	Approx. Motor Full Load,* Amp	Recommended A.C.B. Ampere Rating	Approx. Motor Full Load,* Amp	Recommended A.C.B. Ampere Rating	Approx. Motor Full Load,* Amp	Recommended A.C.B. Ampere Rating	Approx. Motor Full Load,* Amp	Recommended A.C.B. Ampere Rating	Approx. Motor Full Load,* Amp	Recommended A.C.B. Ampere Rating
5	13	20	11	15	7	15	6	15	5	15	4	15
7 1/2	20	25	17	20	10	15	8	15	8	15	7	15
10	26	35	21	25	13	20	11	15	11	15	9	15
15	39	50	33	50	20	25	17	20	16	20	13	20
20	52	70	45	70	26	35	21	25	21	25	18	25
25	66	90	56	70	33	50	28	35	26	35	21	35
30	78	100	67	90	39	50	33	50	30	35	27	35
40	105	150	87	125	53	70	45	70	42	50	36	50
50	130	175	108	150	66	90	56	70	52	70	45	70
60	157	200	130	175	78	100	67	90	63	90	54	70
75	195	250	167	225	98	125	84	125	78	100	67	90
100	260	350	217	300	130	175	108	150	105	150	87	125
125	325	400	278	350	164	225	139	175	130	175	108	150
150	390	500	327	400	195	250	167	225	157	200	130	175
175	459	600	390	500	229	300	195	250	184	250	152	200
200	520	....	435	600	260	350	217	300	210	300	173	225
225	590	....	501	600	295	400	251	350	236	300	195	250
250	650	....	556	....	325	400	278	350	260	350	217	300
300	780	....	654	....	390	500	327	500	314	400	260	350
350	918	....	780	....	459	600	390	500	367	500	304	400
400	1040	....	870	....	520	....	435	600	420	600	347	500
450	1180	....	1004	....	590	....	502	600	472	600	390	500
500	1300	....	1088	....	650	....	544	....	520	....	435	600
600	....	....	1180	....	780	....	650	....	628	....	520	600

* The "full-load current" values are approximate and should be used for estimating purposes only. Application should always be based only on actual currents taken by the specific machines in the circuit. The breaker required should have rating between 115% and 140% of the actual, full-load current, or the next higher standard rating.

time trips. If a small number of motors is fed from a branch breaker, or if one of the motors is relatively large, long time delay trips are required to "ride over" the starting inrush. Where several small motors of the same relative size are served, a branch breaker equipped with standard trips may give enough time to allow the motors to be started without danger of tripping. In this case it is important that the motor starters be arranged to open on loss of voltage; otherwise the return of voltage after a momentary loss would draw starting current of the entire group of motors through the branch breaker.

Where a single breaker is used as a motor starter and protector, motor overload and undervoltage protection are required as well as cable protection. Thermal relays operated from current transformers will usually provide more accurate overload protection than direct-acting trips, and should always be used where the motor serves large or important loads. If thermal relays are used, the breaker can be equipped with short-circuit trips set to trip instantaneously at some value above the maximum starting inrush. When reliable tripping source is available, an undervoltage relay is preferred to direct-acting undervoltage device for motor protection.

Air circuit-breaker ratings for motor circuits are listed in Table 30. The recommended air circuit-breaker ratings include the 115% service factor standard for most a-c motors.

In addition to drawout switchgear, the secondary substation may include a section of motor control units—full-voltage, reduced-voltage, wound rotor, single- and multispeed and reversing starters grouped and interlocked in any sequence necessary for a particular process. Each starter unit consists of a contactor and some type of short-circuit protection. Short-circuit protection for the large motors is provided by a breaker backing up the contactor. A switch and fuse is used to back up contactors for smaller motors.

SWITCHGEAR *

Definition. Switchgear is a general term covering switching, interrupting, control, metering, protective and regulating devices, also assemblies of those devices with associated interconnections, accessories, and supporting structures for use in connection with the generation, transmission, distribution, and conversion of electric power.

Selection of Switchgear.[*] In order to make a well-coordinated application of switchgear to any power system a one-line diagram of the system should be available. Such a diagram, together with information on maximum short-circuit currents at points of power entry, nature and ratings of loads, ratings and characteristics of transformers and generators, and the characteristics of power lines, gives a sound basis for the selection of suitable types and ratings of standard switchgear equipment.

SYSTEM CHARACTERISTICS. **Available short-circuit current or kva** evaluated at each point in the system where switchgear is to be used, determines the interrupting rating of the circuit-breakers or other circuit-interrupting devices used in the gear. Since standard gear is usually built with other parts designed to withstand the same or greater short-circuit currents than the breaker, gear having breakers of adequate interrupting ratings usually covers this phase of the selection.

Voltage and frequency of the system determine the general type of switchgear to be selected and the voltage rating within the type class.

Maximum Load Requirement of Each Circuit. From this information continuous ampere, kilowatt, kva, or horsepower ratings of the circuit units are determined.

Service Requirements. The degree of service continuity required for different circuits determines the type of circuit arrangement best suited to the application and, for low voltage circuits, the type of tripping system selected.

Location of equipment determines whether standard indoor, outdoor, drip-proof, or other type of switchgear enclosure is required. If there are unusual atmospheric conditions, such as the presence of explosive or corrosive gases, this should be called to the attention of the manufacturer.

The Nature of the Various Loads. From this information any unusual operating requirements can be determined, as for example the frequency with which circuit-breakers will be operated and under what conditions of load. Some types of load, such as electric-arc furnaces, require highly repetitive operation of the switchgear; the type of equipment required may differ from that used for less severe service. Manufacturers provide gear especially designed for such service.

TYPES OF SWITCHGEAR. Standardized types of switchgear equipment are available to meet a large percentage of the requirements of modern power systems, and leading

* Contributed by W. N. Gittings.

switchgear manufacturers are in a position to recommend the most advantageous modifications of such standard gear to meet other less frequently encountered situations. The salient features of these standard types are summarized here, with emphasis on functional characteristics rather than design details. Major consideration is given to types most widely used in industrial power distribution systems.

19. LOW-VOLTAGE SWITCHGEAR

Most industrial power is utilized at voltages ranging from 208 to 600 volts and from substations ranging in size from 100 to 1500 kva. For distributing power at these voltages, the standard type of switchgear is the drawout air circuit-breaker equipment. Figure 45, p. 16-54, shows a typical installation of this type of gear.

The circuit-interrupting devices of this type of gear are air circuit-breakers, produced in several sizes, depending on the continuous current-carrying capacity and the interrupting capacity required. Table 31 shows rating combinations standardized by NEMA for

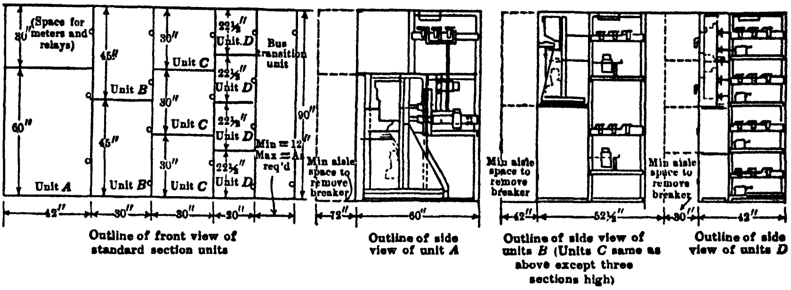
Table 31. 600-volt Air Circuit-breaker Combinations (NEMA Standard)

Continuous Rating, amp	Interrupting Ratings, amp				
	15,000	25,000	50,000	75,0	100,000
15	X				
20	X				
25	X				
35	X	X			
50	X	X			
70	X	X			
90	X	X			
100	X	X			
125	X	X			
150	X	X			
175	X	X			
200	X	X	X		
225	X	X	X		
250		X	X		
275		X	X		
300		X	X		
350		X	X		
400		X	X		
500		X	X		
600		X	X		
800			X		
1000			X		
1200			X		
1600			X		
2000				X	
3000				X	
4000					X

600-volt a-c air circuit-breakers of the type used in standard drawout metal-enclosed switchgear.

COMPLETE SWITCHGEAR ASSEMBLIES embodying the desired arrangements and number of circuits and circuit-breakers of suitable ratings are factory built by manufacturers from standardized subassemblies, so that the user provides only suitable foundations, and connects his cables to the proper terminals to complete the installation. These factory assemblies consist of tiers of standard compartments, each of which contains a circuit-breaker or an instrument and control panel. The circuit-breakers are built on removable carriages, and means are provided to draw the entire breaker out of its compartment for inspection, maintenance, replacement, or for isolation of the circuit. Interlocks are provided to prevent withdrawal or insertion unless the breaker is in the open position. Behind the tiers of breakers and instrument and control compartments is a structure carrying bus bars, connection bars, terminals, and necessary current and potential transformers. The entire structure is housed in a metal enclosure to insure maximum safety to operators and protection to equipment.

From complete listings of standard compartments provided in manufacturers' handbooks and bulletins, suitable combinations can readily be worked out to cover nearly all the usual circuit arrangements encountered in modern low-voltage power generation and distribution. Although space does not permit showing illustrations of all the units of one of these complete listings, Fig. 46 shows some of the more commonly used standard units. Figure 47 shows a typical assembly of standard units into a complete equipment.



Unit Type	Method of Operation	Maximum Number of Vertical Sections	Ratings of Circuit-breaker in Amperes		Type of Breaker
			Interrupting	Continuous	
A	Electric	1	75,000 and 100,000	2000 to 4000	AL-2-75 and AL-2-100
B	Electric	2	50,000	100 to 1600	AL-2-50
C	Manual	3			
D	Manual or Electric	4	25,000	35 to 600	AK-1-25
D	Manual or Electric	4	15,000	15 to 225	AK-1-15

FIG. 46. Standard metal-enclosed low-voltage a-c switchboards with drawout air circuit-breakers for 600-volt 3-phase 3-wire 60-cycle service. Feeder section units for incoming power line, power transformer, feeder, and motor-branch circuit application.

Standard units are available for the following types of circuit commonly encountered in systems in this voltage range (208 to 600 volts).

(1) Incoming lines or transformer secondaries up to 4000 amp.
(2) Generators up to 1650 kva at 240 volts, 3100 kva at 480 volts, or 4150 kva at 600 volts.
(3) Bus ties up to 4000 amp.
(4) Feeders up to 4000 amp.
(5) Synchronous or induction motors, full voltage or line-reactor automatic start up to 500 hp at 230 volts, 1000 hp at 440 volts, or 1250 hp at 550 volts.

Other standard units include the more common combinations of instruments and meters, and provisions for the control of electrically operated circuit-breakers. All circuit-breakers having interrupting ratings of 75,000 and 100,000 amp are electrically operated, and those of lower interrupting ratings are available for either electrical or manual operation. When electrical operation is used, control power transformers are available for control units. Standard auxiliary compartments are available to provide for bus transitions between different-sized units and for accommodation of totalizing metering equipments.

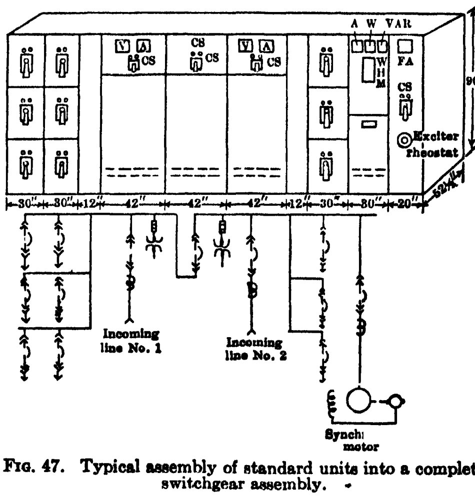


FIG. 47. Typical assembly of standard units into a complete switchgear assembly.

STANDARD UNIT. Each standard unit is provided with complete equipment necessary for the circuit. For example, a generator unit includes, in addition to the circuit-breakers, the usual complement of instruments: field switch, exciter rheostat, breaker, governor, and instrument transfer switches, voltage regulator, necessary protective relays, current and potential transformers, and complete main connections, buses, wiring, and terminal facilities. Although usually installed indoors, these equipments are available in weatherproof enclosures for outdoor installation.

One important consideration in the application of low-voltage switchgear is the degree of service continuity required. This determines, in addition to the circuit arrangement, which of two distinct systems of fault tripping in common use is selected. These systems are known as the *selective* and *cascade* systems.

20. MEDIUM-VOLTAGE SWITCHGEAR

Power is distributed to load centers at voltages ranging from 2300 to 13,800 volts by medium-voltage switchgear. The type of equipment now almost universally used in this class is known as *metal-clad switchgear*, which has been quite thoroughly standardized. Complete lines of standard circuit units are offered by all leading manufacturers. The circuit-interrupting device used in this type of gear is the power circuit-breaker. The most common form of power circuit-breaker in this voltage class is the magnetic air breaker, although its predecessor, the oil circuit-breaker, is still used to a limited extent, particularly where explosive atmosphere is prevalent as in oil refineries. Power circuit-breakers are produced in ratings shown in Table 32, and complete switchgear equipments are available

Table 32. Metal-clad Switchgear (Oilless)

Power Circuit-breaker Ratings *

Three-phase Interrupting Rating, mva	Rated kv †	Minimum kv for Rated Interrupting mva	Continuous 60-cycle Current Rating	Insulation Level Withstand Test ‡		Short Time § (Momentary) Ratings in thousands of amperes	Interrupting Ratings, in thousands of amperes	
				Low-Frequency rms, kv	Impulse Crest kv		At Rated Voltage	Maximum
50	4.16	2.3	600	19	60	20	7	12.5
50	4.16	2.3	1200	19	60	20	7	12.5
150	4.16	3.5	600	19	60	40	21	25
150	4.16	3.5	1200	19	60	40	21	25
150	4.16	3.5	2000	19	60	40	21	25
250	4.16	3.85	1200	19	60	60	35	37.5
250	4.16	3.85	2000	19	60	60	35	37.5
250	7.2	4.6	1200	36	95	51	20	32
250	7.2	4.6	2000	36	95	51	20	32
500	7.2	6.6	1200	36	95	70	40	44
500	7.2	6.6	2000	36	95	70	40	44
150	13.8	6.6	600	36	95	20	6.3	13
150	13.8	6.6	1200	36	95	20	6.3	13
250	13.8	6.6	1200	36	95	35	10.6	22
250	13.8	6.6	2000	36	95	35	10.6	22
500	13.8	11.5	1200	36	95	40	21	25
500	13.8	11.5	2000	36	95	40	21	25

* Data from ASA Standard Schedule of Preferred Ratings for Power Circuit Breakers C37.6–1949.

† Voltage ratings based on recommendations of EEI-NEMA Joint Committee on Preferred Voltage Ratings for A-C Systems and Equipment.

‡ 1.5 × 40 ms positive or negative. All impulse values are phase-to-phase, phase-to-ground, and across open contacts.

§ For definitions of short-time current ratings see ASA Standard for A-C Power Circuit Breakers C37.4–1945 or any subsequent edition approved by ASA.

in corresponding ratings. Figure 44, p. 16-51, shows a side view of a metal-clad switchgear unit with the more important components identified. Manufacturers provide complete listings of standard circuit units of this type of switchgear, with three different relative arrangements of the primary circuit equipment with respect to the secondary or control equipment. Selection of the arrangement is usually governed by the layout of the station.

Manufacturers' listings include standard units for the following types of circuits in commonly used ratings: (1) Generators. (2) Incoming lines or power transformer secondaries. (3) Synchronous or induction motors, full voltage, line-reactor, neutral-reactor, or autotransformer automatic start. (4) Feeders. (5) Bus sectionalizing. (6) Generator-neutral grounding. (7) Totalizing metering.

In addition, drawout potential-transformer compartments, bus-entrance compartments, and instrument brackets are available as standard components of complete equipments. For larger size breakers, and where quick removal of breaker units is required, electrically driven elevating mechanisms are available.

21. STATION-TYPE AND HIGH-VOLTAGE SWITCHGEAR

STATION TYPE SWITCHGEAR. Power usually is generated in large central generating stations at voltages ranging from 13.8 to 34.5 kv. The concentration of power is such as to require switchgear equipment having short-circuit interrupting ratings over the 500,000 kva maximum for which metal-clad switchgear with removable breakers is designed. For this type of service, station-type cubicle switchgear is generally used. The air-blast circuit-breaker is commonly used in this type of gear. This equipment is available in ratings ranging from 14.4 to 34.5 kv, 1200 to 5000 amp continuous, and 500,000 to 2,500,000 kva interrupting rating.

Station-type equipments embody stationary-type oil or air-blast breakers, safety-interlocked disconnecting switches, bushing current transformers, drawout potential transformers, and segregated phase construction. Control and secondary equipment are not built integral with the primary gear, but as a separately mounted duplex or bench-type switchboard.

The air-blast circuit-breaker interrupts the circuit by means of a blast of compressed air released across the contacts just as the mechanism opens them. The arc drawn between the contacts is rapidly cooled by the expanding air, and interruption occurs very quickly.

Station-type equipments have been standardized to a considerable extent. However, because of the magnitude and importance of installations of this type, manufacturers should be consulted for each application.

HIGH-VOLTAGE SWITCHGEAR. The transmission of power in large amounts over considerable distances involves voltages ranging from 34.5 to 287 kv.

Switchgear for such systems up to 69 kv is known in the industry as *package substations*, a more detailed discussion of which was given in Articles 17 and 18. The components of this type of equipment have been considerably standardized, and manufacturers supply coordinated complements to make up complete substations to fit many varieties of circuit arrangement.

The design and application of switchgear to circuits of voltages above 69 kv are usually the joint concern of manufacturers and large utility organizations. The more important components of high-voltage switchgear equipments are: (1) Outdoor power circuit breakers in ratings from 15 to 287 kv, and 50,000 to 10,000,000 kva interrupting rating. (2) Disconnecting switches of many ratings and arrangements. (3) Power fuses. (4) Lightning arresters. (5) Bus supports and insulators. (6) Connectors and fittings. (7) Supporting frame members.

CONTROL AND PROTECTIVE RELAYING SWITCHGEAR. In the low- and medium-voltage fields, standard factory-assembled switchgear usually embodies all necessary control and protective relaying functions. However, where station-type and high-voltage outdoor switchgear are used, it is usually impractical to include control and protective relaying functions as part of the main switchgear. For such applications separate switchboards are usually provided upon which control switches, instruments, meters, voltage regulators, and protective relays are mounted. Control and relaying switchboards are manufactured as completely factory-assembled equipments. Manufacturers have standardized structural components from which such switchboards are built. Upon these structures standard groups of control and protective devices for various types of circuit can be arranged to provide convenient and efficient centralized control of the circuits in large stations.

The most common type of switchboard in current use is the duplex board, like that shown in Fig. 48. It consists of front and rear panels separated by a passageway entered through doors at the ends. Control switches and instruments are mounted on the front panels, and relays and meters on the rear panels. All devices are dead-front to give maximum safety to operators. Complete wiring and terminal facilities are provided inside the structure.

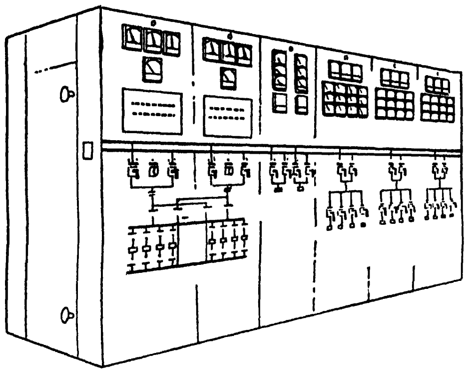


Fig. 48. Duplex switchboard.

The most important component of a control and protective relaying switchboard is the very important protective relay. It is the "eyes" of the power circuit-breakers in that it detects the presence and location of abnormal system conditions, such as short circuits or low voltage, and directs the tripping of the correct breakers to remove the faulty part of the system from service quickly and accurately. Standard combinations of relays are available to protect all components of the power system. (See the very complete list of literature on this subject contained in *Bibliography of Relay Literature*, AIEE, Nov. 1947.)

TRANSFORMERS *

22. TRANSFORMER CHARACTERISTICS

CONSTRUCTION. A transformer consists of two insulated coils of wire linked with a ring of iron. The coils are called *high-voltage* and *low-voltage windings*, or *primary* and *secondary windings*. The primary winding is connected to the *source of energy*, and the secondary is connected to the *load*. The high-voltage winding is designed for the higher voltage, and has the greater number of turns. The designations *high voltage* and *low voltage* are preferable to *primary* and *secondary*. The ring of iron is called the *core*.

Each coil consists of a number of loops of round or rectangular wire. Several strands may be used in parallel but electrically insulated from each other, from the core, and from the other coil.

The core consists of thin sheets of high-grade silicon steel. The thickness depends somewhat on the frequency at which the transformer is to operate. The thickness commonly used for 60 cycles is approximately 0.014 in.

The primary function of a transformer is to transform electrical energy from one alternating voltage to another. To transform large amounts of energy with maximum efficiency, many factors must be considered in determining the materials, design, and arrangement of the primary and secondary coils and the core.

RATING of a transformer or other induction apparatus consists of the output, together with other characteristics, such as voltage, current, frequency, and power factor assigned to it by the manufacturer. It is regarded as a test rating that defines an output which can be taken from the apparatus under prescribed conditions of test and within the limitations of established standards.

The established standard that applies to transformers is American Standards Association C-57. It represents standardized practices in the United States relating to transformers and other induction apparatus. Data in this standard were gathered principally from the established Standards of AIEE and NEMA. It contains information pertaining to definitions of terms, conditions on which the rating and behavior are based, conditions on which acceptance tests are made, and guides for testing and service operation of transformers and other induction apparatus.

TURN RATIO is the ratio of the number of turns in the high-voltage winding to the number in the low-voltage winding. At no load the voltage ratio is equal to the turn ratio, but under load the voltage ratio differs from the turn ratio because of the effect of regula-

* Contributed by E. V. DeBlieux.

tion. (Refer to Standards C-57 for methods for calculating regulation, voltage under load, and other characteristics.)

NO-LOAD LOSS AND EXCITING CURRENT. The no-load loss is the energy consumed in a transformer that is excited at rated voltage and frequency but which is not supplying load. The current that flows in the primary winding under this condition is the *exciting current.* The no-load loss varies from approximately 1% of rated output for small distribution transformers to approximately 0.3% for small power transformers, and to approximately 0.25% for very large power transformers. The no-load loss is somewhat less than these figures for low voltage and somewhat higher for high voltage, or three-phase transformers. Practically all no-load loss is in the core. It includes a small dielectric loss in the insulation and copper loss in the windings. For many years core steel used in transformers was hot-rolled into sheets and cut to the dimensions for assembly. Before assembly individual sheets are coated with enamel or other insulation to prevent circulation of eddy currents between the sheets. In recent years a new type of core steel, fabricated by cold rolling, has been developed. This steel has a high percentage of the grains oriented in the direction of rolling; this causes a large reduction in no-load loss and exciting current when the electrical flux is parallel to the direction of the grain. Core designs for power and distribution transformers that make use of this property have been developed.

LOAD LOSS AND IMPEDANCE VOLTAGE. *Load loss* is the energy loss incident to carrying load. It includes the losses produced in windings and other metallic parts as a result of the load currents flowing through windings. The voltage required to circulate rated current through one winding while another is short-circuited is the *impedance voltage* for the connection at which the test is made. It is usually expressed as a percentage of the rated winding voltage. It is the resultant of two components, one in phase with the load current, due to winding *resistance*, the other 90 degrees out of phase, due to winding *reactance*. Generally, resistance is small compared with reactance.

TOTAL LOSS is the sum of no-load and load loss. It is the loss that occurs during operation at rated voltage and load. Total loss varies from approximately 3% of the rated output for small distribution transformers, to approximately 1% for small power transformers, and to approximately 0.5% for very large power transformers. The total loss is somewhat less than these figures for low voltage and somewhat higher for high voltage or three-phase transformers.

Total loss is a measure of efficiency in transforming power. The figures given above indicate that the full load efficiency of transformers varies from 97% in small distribution transformers to 99% in small power transformers, and to 99.5% in very large power transformers.

POLARITY OF TRANSFORMERS refers to voltage vector relations of transformer leads brought outside the tank. Polarity is the relative direction of induced voltage from H_1 to H_2 as compared with that from X_1 to X_2, both being in the same order with respect to the tank. As indicated in Fig. 49, polarity can be additive or subtractive.

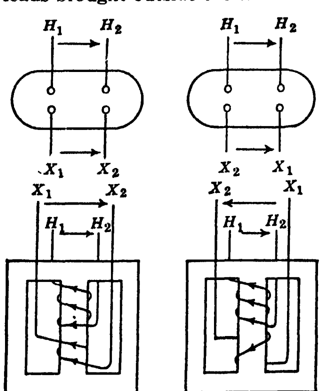


FIG. 49. Polarity of transformers.

ASA Standard C-57 requires that transformer lead designations indicate polarity. When leads are so marked, polarity of transformers is subtractive when H_1 and X_1 are adjacent, and additive when H_1 is diagonally opposite X_1.

Test for Polarity—Single-phase Transformers (see Fig. 50). Connect one high-voltage lead (B) to the opposite low-voltage lead (C). Apply voltage at AB. Measure voltage V between A and B, and V_1 between A and D. If V_1 is greater than V, polarity is additive; if less, it is subtractive.

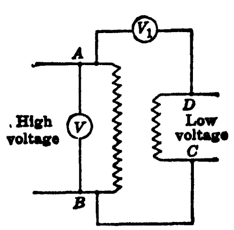


FIG. 50. Connections for polarity test.

Test for Polarity—Three-phase Transformers. Each phase is tested in the same manner as indicated for single-phase transformers. All phases should have the same polarity.

TYPES OF TRANSFORMER. Core and Shell Type. In the *core type* the windings surround and enclose most of the core. In the *shell type* the arrangement is reversed, and the core extends through and loops around the outside of the coils, thus largely surrounding them. Experience has shown that transformers of either type can be built with equal facility for most applications.

Dry Type and Liquid Insulated. In *dry-type transformers* air is the principal electrical insulation between the winding and the core and casing and is also the medium for removing the heat generated in the core and windings.

In *liquid-immersed transformers* these functions are performed by a liquid that may be either transformer oil or the noninflammable synthetic liquid Askarel.

Below is a comparison of the characteristics of transformer oil and Askarel:

	Askarel	Transformer Oil
Color	Straw yellow	Clear
Burning point	None	148 C
Viscosity at 37.8 C Saybolt	40 sec	57 sec.
Pour point	−32 C	−40 C
Specific gravity	1.55	.85

Air- or Askarel-insulated transformers are used in locations where the fire hazard of oil is objectionable. The advantages of Askarel transformers are that (1) they can be installed indoors without the expense of fireproof vaults; (2) in case of failure the products of decomposition of the insulating liquid are noninflammable and nonexplosive; and (3) they can be built for much higher voltage and capacities than air-insulated transformers.

Self- and Forced-cooled. Transformers are classified as *self-cooled* or *forced-cooled*, according to the means for dissipating the heat. In dry-type transformers the heat is transferred directly from the core and coils to the air. In liquid-immersed transformers the heat is first transferred from the core and coils to the insulating liquid and then to the radiating surface. A transformer is self-cooled if the heat is transferred directly from the core and coils or from the radiating surfaces by natural circulation of the air. A transformer is forced-cooled if the heat is transferred from the radiating surface by forced circulation of air, usually produced by fans. A transformer is water-cooled if the heat is transferred by forced circulation of water through a water cooler. To improve the transfer of heat from core and coils, the insulating liquid is sometimes forced-circulated between core and coils and the radiating surfaces by an oil pump.

Self-cooling is simplest and most reliable. If a transformer is required to carry overloads a combination of self and forced air cooling is preferable.

The comparative cost per kva for the different types of cooling given in Table 33 shows that high-capacity forced-cooled transformers are lower in first cost than self-cooled ones. Also, Table 33 shows that an oil-to-water cooling system is somewhat lower in first cost than an oil-to-air system except for self-cooled units below approximately 5000 kva. However, many other factors besides first cost must be considered when determining the most advantageous type of cooling for a specific service.

Table 33. Approximate Price of Standard Power Transformers, dollars per kva *

Primary Kv	15	34.5	115	15	34.5	115	15	34.5	115	15	34.5	115
Rating, kva	Self-cooled			Water-cooled			Forced Oil Forced Air-cooled			Forced Oil Water-cooled Cooler		
SINGLE-PHASE, 60-CYCLE, OIL-INSULATED												
1,000	3.3	4.0	8.4	3.9	4.4	9.1	5.3	5.7	10.5	4.5	5.0	9.9
2,000	2.8	3.1	5.5	2.9	3.2	5.5	3.4	3.6	6.1	3.0	3.3	5.6
5,000	2.4	2.5	3.2	2.5	2.7	3.4	2.0	2.1	3.3	1.8	1.9	3.0
10,000	2.1	2.2	2.7	1.9	2.0	2.6	1.5	1.6	2.3	1.4	1.5	2.1
20,000	2.0	2.1	2.4	1.6	1.8	2.2	1.3	1.4	1.8	1.2	1.3	1.7
50,000	...	1.8	2.1	...	1.6	1.8	...	1.2	1.4	...	1.1	1.3
THREE-PHASE, 60-CYCLE, OIL-INSULATED												
1,000	4.6	5.7	12.3	5.1	6.1	15.2	6.3	7.3	18.0	5.6	6.8	17.2
2,000	3.7	4.2	8.3	3.9	4.3	8.6	3.9	4.6	9.8	3.9	4.2	9.3
5,000	2.8	2.9	4.9	2.5	2.7	4.5	2.5	2.7	4.8	2.3	2.5	4.5
10,000	2.3	2.5	3.7	2.0	2.1	3.3	1.7	1.8	3.0	1.5	1.7	2.8
20,000	2.1	2.2	3.1	1.8	1.9	2.8	1.4	1.5	2.3	1.3	1.4	2.1
50,000	...	2.7	2.4	1.7	1.8	2.1	1.2	1.3	1.8	1.1	1.2	1.6

* Price additions are made for special features.

DISTRIBUTION AND POWER TRANSFORMERS. **Distribution transformers** are transformers that have a kva rating of 500 or less, used to distribute power to points of usage. Standard ratings of distribution transformers are given in Table 34. Approximate prices in dollars per kva for representative ratings of distribution transformers are given in Table 35. Price per kva increases with the voltage, and decreases as the transformer kva increases.

Table 34. Standard Distribution Transformer Ratings

Preferred Kva Ratings	Primary Voltage	Secondary Voltage	Kva Range
3			
5	480		
10	600	120/240	3–100
15	4160		
25	2400		
37.5	4800	120/240	
50	7200	240/120	3-167
75	12000	240/480	
100	13200	600	
167	14400		
250	13200 Gr. Y/7200		
333	13200 Gr. Y/7200	120/240	3–100
500	13200 Gr. Y/7200		

Voltages up to 110,000 volts are standard in distribution sizes. With 480 or 600 volt primary, two 5% tap voltages below the rated primary voltage are standard. Certain tap arrangements such as two 2 1/2% taps above and two 2 1/2% taps below or four taps below rated voltage are standard for var ous voltage and kva ratings. Preferred kva ratings of three-phase distribution transformers are 9, 15, 30, 45, 75, 112 1/2, 150, 225, 300, and 500 kva.

Table 35. Approximate Price of Distribution Transformers, Single-phase, 60-cycle, Self-cooled, Oil-insulated

Rating, kva	Primary Voltage			
	2400/4160Y	7200/12470Y	12,000	14,400
3	19.5	23.5	28.5	32.5
5	18.0	22.0	23.5	26.0
10	14.0	17.5	18.0	19.5
15	12.0	15.0	15.0	15.0
25	10.0	12.0	12.0	13.0
37.5	8.5	10.5	10.5	11.0
50	8.0	9.5	9.5	10.0
75	7.0	8.5	8.5	8.5
100	6.5	7.5	7.5	7.5
167	6.0	6.0	6.0	6.0
250	6.0	6.0	6.0	6.0
333	5.5	5.0	5.0	5.0
500	5.0	4.5	4.5	4.5

For 3 to 100 kva the standard secondary voltages are 120/240, 240/480, or 600 volts.
For 167 to 500 kva the standard secondary voltages are 240/480, or 600 volts.
All prices include standard taps.

Power transformers are transformers with a kva rating greater than 500. They are located at power-generating stations, at the terminals of power-transmission systems, at the voltage transformation points of the system, and at points of large power usage. Standard ratings of power transformers are given in Table 36. Approximate prices in dollars per kva of representative ratings of power transformers are given in Table 33. Price per kva increases with the voltage; decreases as the transformer kva increases. For the higher ratings the cost decreases with type of cooling in this order: self-cooled, water-cooled, forced-oil-cooled. For the lower kva ratings the order is reversed. Price additions are made for special electrical or mechanical features such as overload capacity, special accessories, extra voltage ratings, low noise level, and noninflammable insulating liquid.

Table 36. Standard Power Transformer Ratings

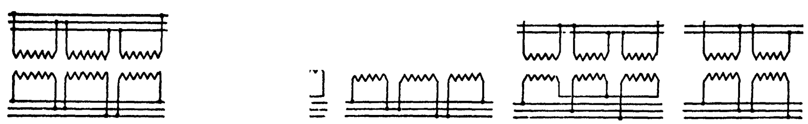
Voltage Ratings		Preferred Kva Ratings	
Nominal System Voltage	Preferred Voltage Rating	Single Phase	Three Phase
23,000	22,900	667	
34,500	34,400	833	750
46,000	43,800	1,000	1,000
69,000	67,000	1,250	1,200
115,000	110,000	1,667	1,500
138,000	132,000	2,000	2,000
161,000	154,000	2,500	2,500
230,000	220,000	3,333	3,000
		4,000	3,750
For voltages below 23,000 the preferred voltages are the distribution standard voltages		5,000 *	5,000 *
		6,667 *	6,000 *
		8,333 *	7,500 *
		10,000 *	10,000 *

* Higher standard ratings are multiples of these values.

Some manufacturers have standardized the design and manufacture of the lower voltage and kva ratings which they will supply on quicker delivery and at a lower price.

23. TRANSFORMER CONNECTIONS

Connections for three-phase voltage transformations most commonly used are illustrated in Fig. 51.



(a) Delta-delta (b) Star-star (c) Star-delta (d) Delta-star (e) Open-delta

FIG. 51. Transformer connections.

The advantages and disadvantages of these connections are described below.

DELTA-DELTA. Advantages. Most economical connection for large output at low voltage; a 3-phase bank of three single-phase transformers can operate in open-delta at 58% output if one unit fails; third harmonic voltages are eliminated; easy to phase in for parallel operations; heavily unbalanced 3-wire loads can be supplied without serious voltage unbalance.

Disadvantages. Copper cross section of both primary and secondary windings is a minimum whereas number of turns and insulation per phase is a maximum; neutral point of windings not available hence neutral of low-voltage winding cannot be grounded.

STAR-STAR. Used mainly for three-phase, core-type transformers for small power loads.

Advantages. Copper cross section is maximum, number of turns per phase is minimum; most economical for small output at high voltages; both neutrals available for grounding or for balanced 4-wire supply; easy to phase in for parallel operations; can be operated single-phase at 58% output.

Disadvantages. Neutral points are inherently unstable unless solidly grounded; unbalanced 4-wire load cannot be supplied unless primary and supply neutral points are tied together for shell-type or banks of single-phase transformers; a fault on one phase renders a 3-phase bank or unit inoperative for 3-phase output; third harmonic voltages exist but can be eliminated if transformer has a tertiary winding, connected delta, which supplies a circuit for third harmonic currents. This delta winding can also supply external loads, and if connected to a synchronous motor or condenser, improves power factor.

DELTA-STAR. Used for stepdown transformers to supply 4-wire distribution to motor and lighting loads, balanced or unbalanced; also used for stepping up voltage for power transmission.

Advantages. No third harmonic; secondary neutral available for grounding or for 3-phase, 4-wire supply; suitable for unbalanced 4-wire load, resulting unbalanced voltage being relatively small; balanced and unbalanced loads may be applied simultaneously. Transformers of widely different impedances can be used to form a 3-phase bank.

Disadvantages. No primary neutral available for grounding; a fault in one phase makes 3-phase unit or bank inoperative.

STAR-DELTA connection is used chiefly for stepping down voltages from high-voltage transmission lines.

Advantages. Third harmonic voltages eliminated by delta-connected secondary; primary can be grounded; most desirable for stepdown transformers for high-voltage transmission; secondary delta connection stabilizes the primary neutral. Transformers of widely different impedances can be used to form a 3-phase bank.

Disadvantages. No secondary neutral available for grounding or for 3-phase, 4-wire supply; a fault on one phase renders a 3-phase unit or bank inoperative.

OPEN-DELTA. Advantages. A 3-phase, shell-type transformer can operate in open-delta with one damaged phase; two units of a transformer bank consisting of three single-phase units can be operated in an open-delta, since a damaged single-phase unit can be removed entirely.

Disadvantages. With delta-connected, 3-phase, shell-type transformers, a damaged phase must be disconnected and short-circuited on itself to prevent voltage being induced by the good phases. To operate a 3-phase, core-type transformer in open-delta, the damaged phase must remain open-circuited and yet be capable of withstanding normal voltage induced in it from the other phase windings. When connected open-delta, current in each transformer is 30 degrees out of phase with voltage, and transformer operates at 86.6% power factor if load is noninductive. Capacity of a 3-phase transformer, or of a 3-phase bank, connected open-delta, with the damaged phase cut out, is 58% of the bank rating. Unbalance in voltages may cause burnout of three-phase motors.

CONNECTIONS FOR PHASE TRANSFORMATION. Two- or Three-phase to Single-phase. It is practically impossible to transform from polyphase to single-phase and obtain balanced conditions. The best method is simply to connect the transformer across one phase of a polyphase system. If resulting unbalance is serious, a polyphase motor, driving a single-phase generator, should be used.

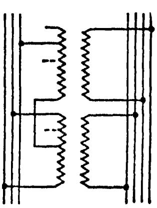


FIG. 52. Three-phase-two-phase Scott connection.

Three-phase to Two-phase. The most common connection is the Scott connection (Fig. 52), using two transformers, called *main* and *teaser*, T-connected on the 3-phase side. The teaser has 86.6% of the number of turns in the main winding. For supplying 2-phase, 4-wire system on the 2-phase side, both windings are identical and independent.

Usually main and teaser transformers are identical. The 3-phase winding of each has a 50% and 86.6% tap. When used as a teaser, one 3-phase line is connected to the 86.6% tap, and 13.4% of the winding is idle. The Scott connection thus requires 6.7% more copper than single-phase transformers delivering the same power.

PARALLEL OPERATION OF TRANSFORMERS. Two transformers having the same ratio, and proper impedance, can be connected in parallel if phase rotation and angu-

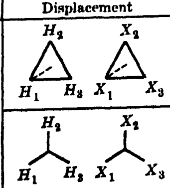
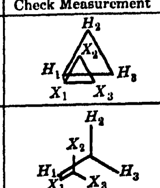
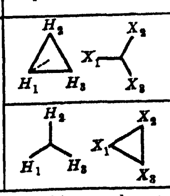
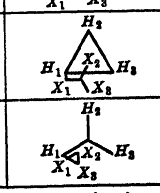
	Angular Displacement	Diagram for Check Measurement	Check Measurements
Angular displacement 0 degrees			**Connect** H_1 **to** X_1 **Measure** H_2-X_2, H_3-X_3, H_1-H_2, H_2-X_3 **Voltage relations** $(H_2-X_3)=(H_3-X_2)$ $(H_2-X_2)<(H_1-H_2)$ $(H_2-X_2)<(H_2-X_3)$
Angular displacement 30 degrees			**Connect** H_1 **to** X_1 **Measure** H_3-X_2, H_3-X_3, H_1-H_3, H_2-X_2, H_2-X_3 **Voltage relations** $(H_3-X_2)=(H_3-X_3)$ $(H_3-X_2)<(H_1-H_3)$ $(H_2-X_2)<(H_2-X_3)$ $(H_2-X_2)<(H_1-H_3)$

FIG. 53. Lead markings and vector diagrams for three-phase transformers.

lar displacement are the same. Delta-delta and star-star transformers have the same angular displacement, when polarity and phase rotation are the same. With delta-star or star-delta transformers, correct adjustment can be made by proper sequence of leads. If voltage diagrams are available, for the transformers to be paralleled it is necessary only that these diagrams coincide and that corresponding terminals be connected together. The basic requirement for parallel operation is that the leads to be connected are at the same potential. When transformer leads are marked in accordance with the ASA C-57 Standard markings, it is necessary only to connect similarly lettered leads together.

Three-phase transformers can be grouped according to their angular displacements. (See Fig. 53.) To operate in parallel, transformers must belong to the same group. One group cannot be changed to another by interchange of external leads. For instance, two delta-delta transformers, one of group 1 and the other of group 2, cannot be operated in parallel. Table 37 shows operative and inoperative connections.

Table 37. Transformer Operative and Inoperative Parallel Connections

Operative Parallel Connections				Inoperative Parallel Connections			
Low-voltage Side		High-voltage Side		Low-voltage Side		High-voltage Side	
Bank A	Bank B	Bank A	Bank B	Bank A	Bank B	Bank A	Bank B
Delta	Delta	Delta	Delta	Delta	Delta	Delta	Star
Star	Star	Star	Star	Delta	Delta	Star	Delta
Delta	Star	Delta	Star	Star	Star	Delta	Star
Star	Delta	Star	Delta	Star	Star	Star	Delta
Delta	Delta	Star	Star	....	....	....	....
Delta	Star	Star	Delta	....	....	....	......
Star	Star	Delta	Delta	....	....	....	....
Star	Delta	Delta	Star	....	... .	....	. ... .

Effect of Ratio on Parallel Operation. Circulating currents flow in the winding of parallel-connected transformers with unequal ratios of high and low voltage windings. The circulating current is equal to the difference of the two secondary voltages, e_1 and e_2 divided by the sum of the impedances Z_1 and Z_2, of the two transformers. That is

$$I_c = \frac{e_1 - e_2}{Z_1 + Z_2}$$

All values must be in like units, either ohms or percentages.

Example. Assume a voltage difference of 2% and an impedance of 4%, in each transformer. Percentage of circulating current I_c will then be

$$I_c = \frac{2 \times 100}{4 + 4} = 25\%$$

The circulating current is 25% of normal in both windings of the transformers. It adds to the load current in the transformer having the higher voltage and subtracts from the load current in the other.

AUTOTRANSFORMERS. When the high-voltage and low-voltage circuits of a transformer have parts of the winding in common, the transformer is called an *autotransformer*. With this arrangement the full winding is normally connected to the high-voltage circuit, and a tap lead is connected to the low-voltage circuits. The autotransformer differs from the conventional transformer in several important respects.

Only a part of the kva transferred from primary to secondary is transformed by the autotransformer. Generally, the percentage of the kva transformed is the same as the percentage voltage transformation, based on the high voltage; that is, if the autotransformer raises the voltage 10% it actually transforms only 10% of the kva transferred to the secondary, hence the autotransformer physical rating need be only 10% of a conventional transformer that transforms 100% of the kva. For these reasons an autotransformer has lower cost, greater efficiency, better regulation, smaller size, and less exciting current than a conventional transformer for the same application. The advantage is greater the smaller the difference between the high and low voltages. Difference in voltage of more than approximately two or three to one is undesirable in autotransformers for reasons of economy and voltage hazards.

A disadvantage of the autotransformer is that high- and low-voltage circuits are connected to a common transformer winding. The result is lower reactance and higher short-circuit currents and forces. Another disadvantage is that, owing to the metallic connection between the high- and low-voltage circuits, electric disturbances originating in one system affect the other. In particular grounds (especially of the high-voltage circuit), harmonic currents and line transients may produce dangerous overvoltages. For these reasons the high-voltage wye, low-voltage wye, with both neutrals grounded, and with a tertiary winding on the same core, is the preferred autotransformer connection.

INSTRUMENT TRANSFORMERS are especially designed to obtain high accuracy of ratio of primary and secondary current or voltage. Those designed for high accuracy of voltage ratio are *potential transformers.* Those designed for high accuracy of current ratio are *current transformers.* Instrument transformers are interposed between high-voltage power circuits and low-voltage instrument, meter, and relay circuits. They enable the meter and relay circuits to indicate accurately the conditions in the high-voltage power circuit.

Instrument transformers are classified in accordance with their ratio and phase-angle accuracy with specified meter and relay burden on the secondary. For details of these specifications, see ASA Standards C-57.

For approximately 15-kv application and below, instrument transformers usually have air or compound for the major electrical insulation. For higher voltages oil or Askarel liquid insulation is used. They are available for both indoor and outdoor service.

WIRE AND CABLE *

Insulated cables consist of three parts—*conductor, insulation,* and *protective finish.* The combinations that can be made from various materials are practically innumerable. This chapter deals with the more commonly used types of cable and their application. Special designs, however, can be manufactured to fit almost any special condition.

Although oil-impregnated, paper-insulated, lead-covered cable is the most economical type for high-voltage underground transmission and distribution, it will not be discussed in detail. It is used primarily by larger utility companies who have specially trained workmen required for its proper installation. The following discussion applies to industrial plants, railroads, and smaller utilities.

Rubber insulation, as a term, applies to compounds made from natural or synthetic rubber. No further distinction is made between these two types, because economic conditions at the time or future technological development could dictate which would be the more logical choice.

24. OVERHEAD DISTRIBUTION—OUTDOORS

OPEN-TYPE CONSTRUCTION using bare or weatherproof wire is usually the least expensive and is widely used. The wires are mounted on insulators supported on poles or attached to buildings.

The development of rubber-like jackets that provide extremely long life when exposed to outdoor conditions, and increased stress on continuity of service have contributed to a growing use of insulated aerial cable. Normally these cables are supported by a *messenger,* bound to it by the cable manufacturer with metal binding tape, as shown in Fig. 54, or supported in messenger rings. The insulation is usually a rubber compound, but may be varnished cambric. Rubber-insulated cables normally have a synthetic sheath, and at the higher voltages a metal shield may also be required to prevent corona discharge. Varnished-cambric insulated cables usually have a synthetic rubber or resin jacket for moisture protection, followed by a metal armor of material such as bronze or galvanized steel, depending on corrosion conditions.

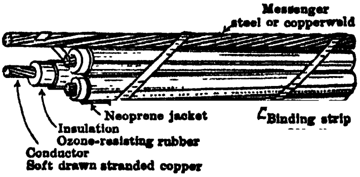


FIG. 54. Preassembled aerial cable supported by a messenger

AERIAL CABLE offers advantages over the open-wire construction, such as greater safety, better voltage regulation, better appearance, and greater reliability from faults caused by sleet and wind storms. It can be installed in congested locations, and on existing pole lines that would not carry an additional open-wire circuit.

* Contributed by R. B. McKinley.

25. UNDERGROUND AND GENERAL PLANT WIRING

Development of more versatile rubber insulation and synthetic jackets has greatly simplified the choice of cable construction for underground and general plant wiring when high reliability is desired. Rubber compounds that are both heat and moisture resistant can be supplied for low-voltage use. High-voltage rubber insulations have ozone resist-

Table 38. Types of Cable for Industrial Applications

Circuits	Voltage	Method of Installation	Type of Cable	Illustration
Incoming Lines or Yard Circuits	5001 to 15,000 volts	In conduit or duct	V.C. lead—1/C or 3/C; ozone-resisting rubber, shield, jacket 1/C or 3/C	Fig. 55, *A* and *B* Fig. 55, *C* and *D*
		Direct burial	V.C. lead-armor—3/C; ozone-resisting rubber, shield, jacket—3/C	Fig. 55*D*
	2001 to 5000 volts	In conduit or duct	V.C. lead—1/C or 3/C; ozone-resisting rubber jacket—1/C or 3/C	Fig. 55, *A* and *E* Fig. 55, *C*, *D*, and *F*
		Direct burial	V.C. lead-armor—3/C; ozone-resisting rubber, shield, jacket—3/C	Fig. 55*D*
	0 to 600 volts	In conduit or duct	V.C. lead—1/C or 3/C; moisture-resisting rubber, jacket—1/C or 3/C	Fig. 55, *A* and *E* Fig. 55, *F* and *G*
		Direct burial	Moisture-resisting rubber, jacket—1/C or 3/C	Fig. 55, *F* and *G*
Feeder Circuits Inside Buildings	5001 to 15,000 volts	In conduit or duct	V.C. lead—1/C or 3/C; ozone-resisting rubber, shield, jacket—1/C or 3/C	Fig. 55, *A* and *B* Fig. 55, *C* and *D*
	2001 to 5000 volts	In conduit or duct	V.C. lead—1/C or 3/C; ozone-resisting rubber, jacket—1/C	Fig. 55, *A* and *E* Fig. 55*F*
		Without conduit	V.C. interlocked armor—3/C	Fig. 55*J*
	0 to 600 volts	In conduit or duct	*Dry Locations*—V.C. braid; type T; rubber insulation, jacket—1/C *Wet Locations*—V.C. lead; type TW; moisture-resisting rubber, jacket—1/C	Fig. 55, *H*, *I*, *F* Fig. 55, *A*, *I*, *F*
		Without conduit	V.C. interlocked armor 3/C	Fig. 55*J*
Branch Circuits Inside Buildings	0 to 600 volts	(Same as 0 to 600 volt feeder circuits)		
Multiple Conductor Control	0 to 600 volts	In conduit or duct, direct burial, or aerial	Synthetic resin insulation and jacket; rubber insulation, jacket	Fig. 55*K*
Series Lighting Circuits	0 to 5000 volts	In conduit or duct, direct burial	Synthetic resin; ozone-resisting rubber, jacket	Fig. 55, *I* and *F*
Miscellaneous Circuits	0 to 600 volts		Machine tools, pickling plants, battery leads, etc.—type T	Fig. 55*I*
Portable Circuits	0 to 5000 volts		Synthetic jacketed portable cables—1/C, 3/C, multi-cond.	Fig. 55, *L*, *M*, and *N*

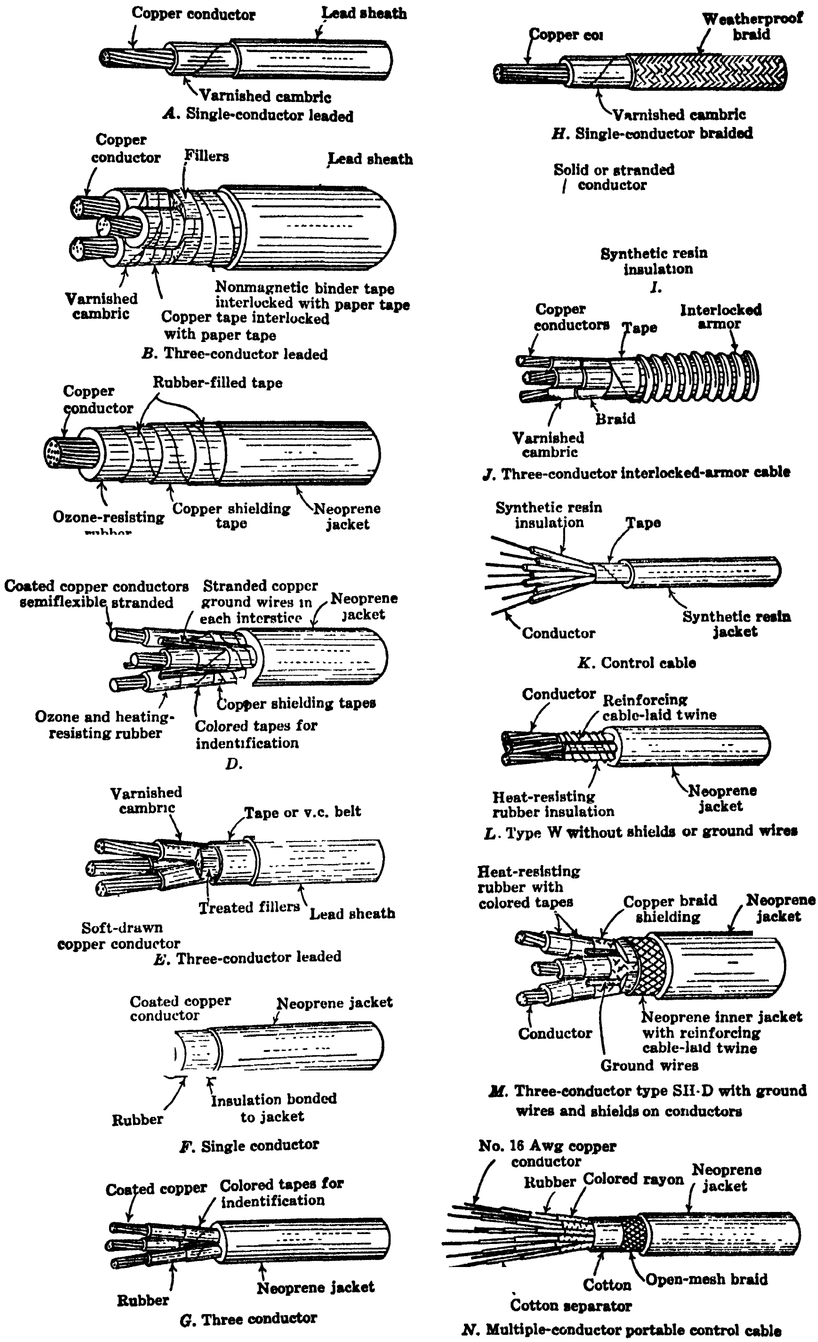


FIG. 55. Types of cable for industrial application.

ance in addition to heat- and moisture-resistant properties. Tough, synthetic jackets, in addition to providing mechanical protection against abrasion, provide oil resistance, retard flame travel, and offer lasting protection against moisture, ozone, and sunlight.

Synthetic resin insulations that do not require any additional finish are also available. They can be installed in wet or dry locations. These insulations do not oxidize or convey flame, are practically unaffected by sunlight or ozone, and are highly resistant to the usual oils, acids, and alkalies encountered in service.

It was formerly necessary to use lead-sheathed cable in wet locations, braid-finished cable in dry locations, and metallic-armor finish for burial use. Therefore, all runs had to be analyzed in detail, and two or three different types had to be purchased. Carrying stock for emergencies was a complicated matter. The synthetic-jacketed cables mentioned above can be installed for all these types of installations, greatly simplifying purchasing and stocking problems.

Varnished cambric insulated cable with interlocking armor finish has proved to be a very economical cable of great reliability. Its chief advantages, besides low installed costs, are high current-carrying capacity, less space occupied, time saved during original layout planning, and high re-use value.

Commonly used types of cable for the various applications encountered in general plant wiring are shown in Table 38. They are listed according to the application. Illustrations are shown in Fig. 55 to describe constructions shown in the table. Besides those shown in the table, the names of other suitable types may be obtained by reference to the latest edition of the National Electrical Code.

CONVERSION EQUIPMENT *

Electric power is frequently available in a form not suited for the application, and must be converted to the desired form. Four modes of electric power conversion are considered; a-c to d-c; d-c to a-c; d-c to d-c; a-c to a-c. Of the various forms of conversion, the most important is a-c to d-c.

In most locations the available power supply is a-c, which must be converted to supply electrolytic, traction, and general industrial d-c loads. These loads include both constant-voltage and variable-voltage loads.

D-c to a-c conversion is limited to special applications.

D-c to d-c conversion usually involves a voltage change. Typical applications are battery charging, d-c welding, and establishment of a neutral in three-wire systems.

A-c to a-c power conversion includes changes in voltage, frequency, or number of phases. Voltage changing by transformers is the commonest mode of a-c to a-c conversion. See Articles 22 and 23.

MEANS FOR CONVERTING POWER. The more important means currently in use for the various modes of power conversion are listed here.

A-c to d-c. Induction motor generators. Synchronous motor generators. Mercury arc rectifiers. Metallic rectifiers. Hot cathode rectifiers. Synchronous converters.

D-c to a-c. Motor generators. Inverted rotary converters. Mechanical inverters. Electronic inverters.

D-c to d-c. Motor generators.

A-c to a-c. *Voltage:* transformers. *Frequency:* constant-frequency ratio synchronous motor generator sets; variable-frequency ratio motor generator sets; induction generators; motor-driven inductor alternators; electronic converters. *Phase:* phase converting transformers; mechanical phase converters.

26. A-C TO D-C CONVERSION

INDUCTION MOTOR GENERATOR SETS. Induction motor-generator sets for general-purpose use are standardized for 25- and 60-cycle systems. They have a 2-bearing, squirrel cage induction motor with a short shaft extension, connected through a solid coupling to a single-bearing generator. Foundations to prevent deflection of bases are necessary. Sets larger than 125 kw have speed-limiting devices. D-c generators for three-wire service have two collector rings for obtaining the neutral.

Ratings. Induction motor generator sets can be built in ratings as high as several thousand kilowatts. Standard kilowatt ratings of 60- and 25-cycle sets are $^3/_4$, 1, 1 $^1/_2$, 2,

* Contributed by A. Schmidt, Jr.

3, 5, 7 1/2, 10, 15, 20, 25, 30, 40, 50, 60, 75, 100, 125, and 150 kw. At higher ratings, the drive is usually a synchronous motor because of its better power factor.

Momentary loads of 150% of rated generator current, in amperes, are permissible with successful commutation as defined by the AIEE. A service factor of 1.15 applies to all except 50-cycle sets. Temperature rise is 40 C, based on a 40 C ambient and continuous operation at rated load.

Efficiencies, dimensions, weights, and power factors of typical standard units are given in Table 39.

Table 39. Characteristics of General-purpose Induction Motor-generator Sets

Rating, kw	250 Volts D-c/220, 440, 550 Volts, 60 Cycles A-c					
	Full Load Efficiency	Full Load Power Factor	Net Weight, lb	Overall Dimensions, in.		
				Length	Width	Height
2	.665	.87	317	32 1/16	17 3/32	14 1/16
5	.70	.88	532	41 3/4	19 13/16	17 1/16
10	.74	.88	790	48 9/16	23 5/8	21
25	.783	.885	1460	58 1/16	27 13/16	25 3/4
50	.823	.88	3755	74 1/4	34	34 15/16
100	.847	.91	5880	85 3/4	38 1/2	40 1/4
150	.847	.885	8295	95 3/8	44 1/8	44 3/16

Parallel operation of motor generator sets usually is practicable. For satisfactory operation, adjustments in the field circuits may be necessary after installation.

SYNCHRONOUS MOTOR GENERATOR SETS. Synchronous motors are direct-connected to one or two d-c generators, depending on rating. Two unit sets have three bearings, and three unit sets have four bearings. The bases are not self-supporting and require foundations to prevent deflection. The range of sizes is:

No. Units in Set	D-c Voltage	Kw Range
2	125	50– 500
	250	50–2000
	600	200–4000
3	125	400–1000
	250	2000–4000
	600	2000–8000

Motor power factor is 0.8 leading; service factor is 1.15. Permissible load is 125% for 2 hr with a temperature rise of 55 C. Normal temperature rise is 40 C above 40 C ambient temperature. Momentary loads of 150% on units of 150 kw or less and 200% on larger units are permitted. Table 40 shows approximate efficiencies, weights, and dimensions for representative ratings.

Table 40. Characteristics of General-purpose 0.8 Power Factor Synchronous Motor-generator Sets

Rating, kw	250 Volts D-c/2300 Volts, 60 Cycles A-c				
	Full Load Efficiency	Net Weight, lb	Overall Dimensions, in.		
			Length	Width	Height
50	.81	3,855	77	36	38
100	.84	6,445	90	41	42
200	.868	9,050	110	44	48
300	.875	12,800	121	48	48
500	.885	23,400	140	66	59
1000	.891	38,600	161	84	68
1500	.895	71,800	180	113	82
2000	.896	103,000	210	136	103

27. RECTIFIERS

A rectifier is defined as an integral assembly of elementary devices, each consisting of an anode and its cathode, and having the characteristic of conducting current effectively in only one direction. The mechanism for unilateral conduction varies among different types of rectifiers.

MERCURY ARC RECTIFIERS use ionized mercury vapor in an evacuated tube to conduct the current between electrodes. At ratings up to 500 kw at 250 volts and 1000 kw at 600 volts, a typical rectifier consists of an assembly of permanently sealed, single-anode, water-cooled tubes. At higher ratings, single-anode tanks, coupled to a continuously operating vacuum pumping system, are used. Before 1940, most rectifiers were constructed as multiple-anode tanks. Manufacture since that time has been confined largely to the single-anode tank assembly because of its greater efficiency and ease of handling.

Pumped Rectifiers. Figure 56 shows a cross section of a typical pumped ignitron rectifier tank. A typical assembly includes six or twelve such tanks, vacuum pumps, circulating pump for cooling water, and water-to-water heat exchanger. When cooling water is not readily available, separately mounted water-to-air heat exchangers may be used. The excitation system is mounted separately. It includes an arrangement of capacitors and reactors for applying a power pulse of short duration and large magnitude to the ignitor, each cycle to start a cathode spot and means for adjusting the voltage of the rectifier by varying the time of ignition. A complete rectifier unit includes the rectifier with its auxiliaries and excitation equipment, power transformer, auxiliary transformer, and a-c and d-c switchgear. Figure 57 shows the connection diagram of a typical rectifier unit. A low-voltage high-current transformer is usually provided to bake out the rectifier and remove foreign gases before placing it in service.

Sealed Rectifiers. Figure 58 shows a cross section of a typical sealed ignitron rectifier tube. Six such tubes are assembled with the necessary transformer and a-c and d-c switchgear to form a unit d-c substation. Since tap water is frequently used for cooling, and no vacuum pumping system is needed, there is a considerable reduction in auxiliary equipment as compared with pumped rectifiers.

Ratings. D-c unit substations employing sealed ignitrons are supplied in ratings as follows:

D-c Volts	Kw
125	40– 200
250	75– 500
600	100–1000

These substations include transformer, auxiliaries, and complete switchgear.

Pumped rectifier units are supplied in the following ratings:

D-c Volts	Kw
250	500-2500
600	750–6000
1500	750–6000
3000	750-6000

Tables 41 and 42 show characteristics of typical pumped rectifiers and d-c unit substations using sealed rectifiers.

Table 41. Characteristics of General-purpose Pumped Ignitron Rectifiers *

Rating, kw	Rectifier Wt., lb	Rectifier Length	Rectifier Width	Rectifier Ht.	Transformer Wt., lb	Transformer Length	Transformer Width	Transformer Ht.	Efficiency Full Load	Efficiency 1/4 Load	Loss at No Load, kw
250 Volts D-c											
750	5900	96	59	78	7,200	132	90	122	.907	.902	7.7
1000	5900	96	59	78	9,000	132	95	122	.904	.906	8.7
1500	7500	101	61	78	10,500	138	130	128	.903	.909	10.5
600 Volts D-c											
1500	5900	96	59	78	10,500	138	130	128	.952	.946	10.7
2000	5900	96	59	78	13,500	162	158	128	.952	.948	12.5
3000	7500	101	61	78	17,000	172	163	136	.952	.950	16

* Dimensions are in inches.

(*Continued on p. 16-81*)

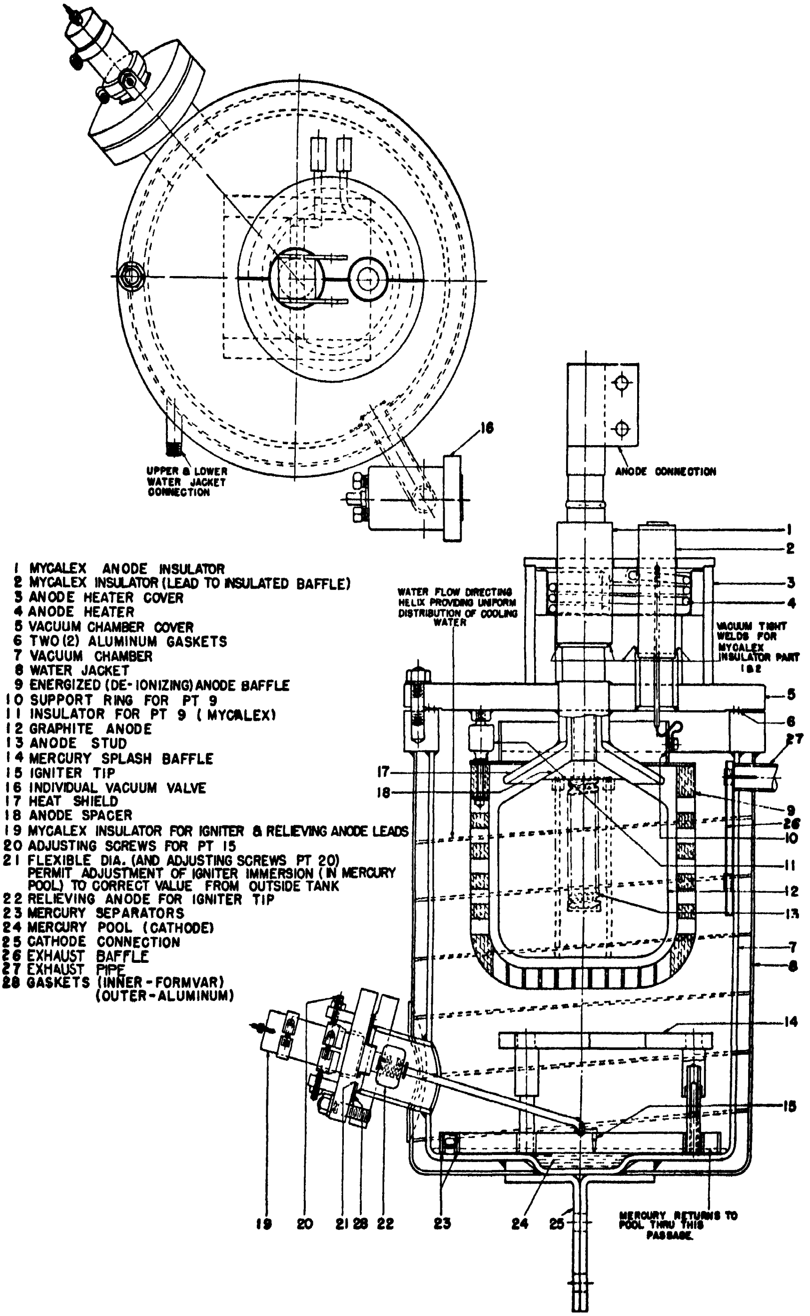


FIG. 56. Mercury arc rectifier, pumped ignitron type, 3000 kw 625 volts.

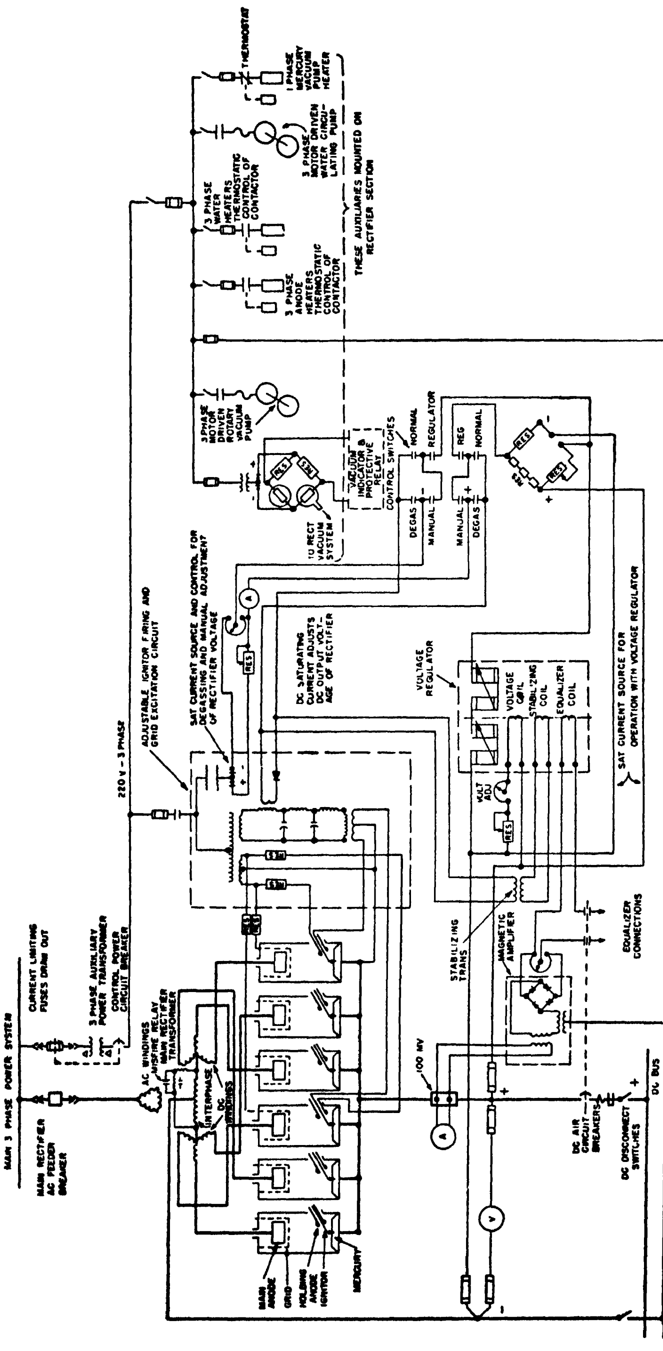


Fig. 57. Wiring diagram of pumped ignitron mercury arc rectifier.

Table 42. Characteristics of Typical 250-Volt D-c Unit Substations with Sealed Ignitron Rectifiers, Transformers, and A-c and D-c Switchgear

Rating, kw	Wt., lb	Dimensions, in.			Efficiency		Loss at No Load, kw
		Length	Width	Height	Full Load	1/4 Load	
100	12,000	204	75	90	.902	.862	2.5
300	18,500	222	75	90	.909	.902	4.1
500	30,500	276	75	112	.912	.907	5.0

Overload Rating. Rectifiers are generally given the same short-time and long-time overload ratings as corresponding motor generator sets. An important advantage of mercury arc rectifiers is that they are capable of withstanding very high momentary overloads without damage.

Losses. The principal losses in a rectifier are: (1) *Arc loss*, which is a d-c voltage drop in the mercury vapor, with a nearly constant value of 16 to 20 volts. (2) *Transformer losses*, which vary from 1 to $2\frac{1}{2}\%$ of the rectifier rating. (3) *Auxiliary losses*, which vary from $1\frac{1}{2}$ to 6 kw, depending on the type of rectifier.

Figures 59 and 60 show the total losses of various forms of conversion equipment over the load range for two typical equipment ratings. The low losses of rectifiers at light loads are noteworthy and account for their early acceptance in the traction field.

Since the arc drop is nearly constant, the efficiency of the rectifier increases with output voltage. Figure 61 shows the efficiency of rectifiers and motor generators as a function of d-c output voltage.

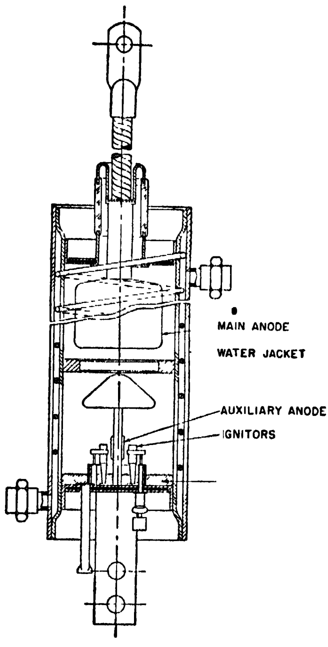


FIG. 58. Mercury arc rectifier, sealed ignitron type, 300 kw, 250 volts.

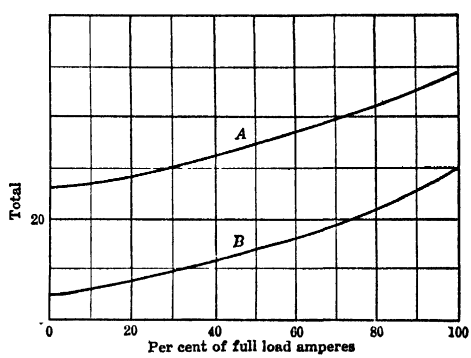


FIG. 59. Comparative losses in 300-kw 275-volt conversion equipment. Curve *A* is for synchronous motor generators. Curve *B* is for mercury-arc rectifiers

Figure 62 shows the relative efficiencies of rectifiers, rotary converters, and motor generators rated 1500 kw, 600 volts.

Regulation of an uncontrolled rectifier is similar to that of a shunt generator having a droop of about 6% from light load to full load. The excitation system can be arranged to shift the phase of anode firing to vary the output voltage. The control power for this circuit is regulated by a voltage regulator, and the output voltage can be controlled so as to be constant, or to rise or droop by a predetermined amount with increasing load. Current practice is to provide general-purpose rectifiers with voltage regulators to maintain constant output voltage.

Parallel Operation. When unregulated rectifiers are operated in parallel, they share the load in accordance with the droop in their respective regulation curves. If voltage regulators provided with equalizing windings are used, rectifiers may be operated in parallel with each other or with d-c generators, with correct load division over the load range.

Power Factor. The rectifier always has a lagging power factor. It takes reactive power in transferring current from one anode to another, and the use of phase control to reduce the d-c voltage increases the reactive power. The harmonic currents taken by the rectifier from the line reduce the power factor further. The value of power factor at full voltage depends on the transformer reactance. Representative full-load values are 0.93 for six-phase rectifiers and 0.96 for twelve-phase rectifiers.

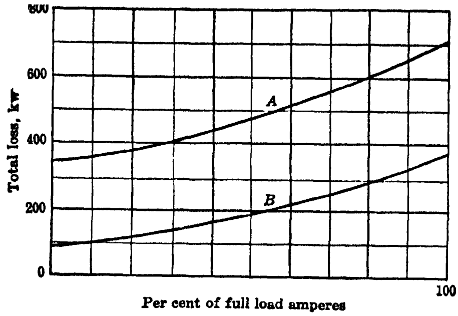


FIG. 60. Comparative losses in 7000-kw 600-volt conversion equipment. Curve *A* is for synchronous motor generators. Curve *B* is for mercury-arc rectifiers.

Protection. The mercury arc rectifier is subject to arc-back or loss of valve action. Arc-back may be the result of operation at excessive cooling water temperature, poor vacuum, or sustained overload. It is equivalent to a short circuit of the rectifier transformer. If the rectifier supplies a counter emf load, then arc-back is also equivalent to short circuit of the d-c bus. When the system potential fault current is excessive, high-speed reverse-current breakers are used to disconnect the rectifier in 0.016 sec or less. Medium-speed switchgear is used to clear faults in smaller systems.

Uses. With few exceptions, rectifiers may be employed as power converters wherever a-c to d-c motor generator sets or synchronous converters might be used and are generally preferred because of their better efficiency, quietness, lower maintenance cost, and ease of application. Rectifiers are not well suited to handle regenerative loads without considerable complication, if such loads have a large magnitude or high repetitive rate. Moderate or infrequent application of regeneration, as in the case of hoists or elevators, is handled with a regenerative braking resistor.

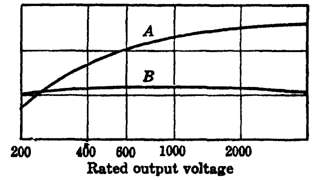


FIG. 61. Effect of output voltage on efficiency of conversion equipment. *A*. Rectifier. *B*. Synchronous motor generator.

The use of rectifiers is widespread in the electrochemical and transportation fields, and is gaining ground in the industrial field, particularly in the mining industry. In addition to industrial use as a constant voltage source, rectifiers are often arranged to control voltage from zero to full voltage, and are then used to supply individual motor drives where speed control over the entire speed range is required.

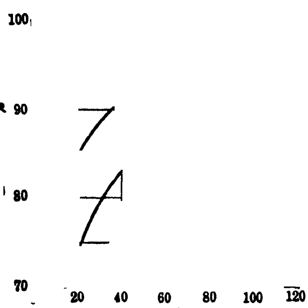


FIG. 62. Comparative efficiency of various types of conversion equipment rated 1500 kw, 600 volts. *A*. Mercury arc rectifier. *B*. Synchronous converter. *C*. Synchronous motor generator.

Life. The life of pumped rectifiers is indefinite. Under severe operating conditions, they may require cleaning at intervals of 5 to 10 years. The life expectancy of sealed ignitrons has not been definitely established, but appears to be at least 5 years.

HOT-CATHODE RECTIFIERS are thermionic units whose source of electrons is a filament or a cathode indirectly heated by a filament.

High-vacuum rectifier tubes have space-charge drops up to several hundred volts. They are used for applications where small currents are needed at high voltage, as in x-ray work and cable testing.

Vapor discharge tubes have a small amount of mercury or some inert gas, like argon, added to lower the space-charge drop. Such tubes may have a voltage drop of 5 to 15 volts.

Rating. Vapor discharge tubes are built with capacities of ½ to 20 amp. Assemblies of these are used in three classes of service: (1) *Low-voltage* applications, like battery charging, in ratings of 6 to 60 volts and 3 to 15 amp. (2) *General industrial applications* at 125 to 500 volts, such as supply for d-c motors, magnets, etc., at currents up to 40 amp. (3) *High-voltage* applications, principally broadcasting, at voltages up to 20 kv, and currents up to 30 amp.

Hot cathode rectifiers are often built with grids to regulate the output voltage. Such rectifiers may be used for operation of d-c motors up to 15 hp, with speed control over the entire speed range down to zero speed. Control may be made responsive to any one of a number of conditions of the output of the machine driven by the motor, depending on the application, such as tension, thickness, width, register (printing), speed, and weight. As a result, electronically controlled motors find wide use in a number of industrial applications, including paper, textile, rubber, machine tool, steel, printing, glass, and food.

Life expectancy of hot cathode rectifiers varies with the type of tube. It is usually a year or more.

METALLIC RECTIFIERS consist of plates of aluminum or copper, coated with selenium oxide or copper oxide, respectively. These combinations have unilateral conductivity.

Ratings. A unit rectifier, consisting of 4 plates, may be capable of delivering 0.1 to 4 amp or more at 4 to 16 volts d-c, depending on the size and type of the plate. A rectifier consists of a sufficient number of plates connected in series and parallel to deliver the desired output. The maximum efficiency of a metallic rectifier is approximately 0.80 so that it is competitive with other types of rectifier in smaller ratings. Rectifiers of this type are built with maximum ratings of a few kilowatts at voltages of 6 to 10,000 volts, except that at voltages under 50, ratings may be extended to about 100 kw.

Life of metallic rectifiers appears to be unlimited when they are operated within their rating. During the first few months of operation an aging process occurs, resulting in an increase in voltage drop and a consequent loss of 3 to 5% in efficiency.

The largest market for metallic rectifiers is in the battery charging field. An important application in the power field is for the supply of large currents at low voltages for electroplating and for the tinplating of steel, where currents of hundreds to thousands of amperes are required at 6 to 40 volts.

SYNCHRONOUS CONVERTERS comprise a synchronous motor and a d-c generator combined in one machine, with an armature winding common to both a-c and d-c circuits. Such a machine has an advantage over a motor generator set in efficiency, but is subject to the disadvantage that its output voltage varies with the a-c supply voltage and is not readily controlled except by use of transformer taps or an additional booster generator. Although the rotary converter has a higher efficiency than the rectifier at full load, its light load losses are considerably greater, so that the rotary converter is at a disadvantage in the transportation field, as compared to the rectifier. Because of the low light load losses, simplicity of voltage control, and ease of maintenance of the mercury arc rectifier, the use of rotary converters has declined considerably in favor of rectifiers in the last fifteen years.

28. D-C TO A-C CONVERSION

MOTOR GENERATOR SETS. Most applications requiring conversion from d-c to a-c are handled by motor generator sets. Data applying to synchronous motor d-c generator sets apply to these units, but when operating inverted the output of standard a-c to d-c sets is 80% of the rating of the d-c unit.

The synchronous rotary converter can be operated inverted to supply an a-c load. Machines are built for this service up to 20 kw. Their use is limited because of their poor voltage and frequency regulation, especially with lagging power factor.

MECHANICAL INVERTERS employing a vibrating reed are frequently used for conversion of small amounts of power from d-c to a-c, as for automobile radio receivers and railway coach fluorescent lights. Their capacity is limited by the power that can be handled by the vibrating contact that acts as a switch in connection with an oscillating circuit.

ELECTRONIC INVERTERS consist of hot-cathode grid-controlled rectifiers or mercury arc rectifiers excited in such a manner that they can convert d-c to a-c. When operated in this manner, they take reactive power from the a-c system so that they can be operated only where the a-c system has an excess of reactive power capacity available. This form of converter is occasionally used on an electric railway to return regenerative power to the a-c system, or as an element of an electronic frequency converter. It is also used as a part of electronic control of small d-c motors to provide regenerative braking.

29. A-C TO A-C CONVERSION

VOLTAGE. The commonest form of a-c to a-c conversion is voltage changing by transformers (see Art. 22).

FREQUENCY CONVERTERS. Interchange of power between two systems having constant, but different, frequencies is effected through a frequency changer consisting of two synchronous machines having a suitable ratio of number of poles. Power control is effected by rotation of the stator of one machine with respect to the other.

When the frequency ratio is not constant, one of the machines is a wound-rotor induction machine instead of a synchronous machine, and additional rotating equipment is required to control the transfer of power between the two systems.

When 3-phase power is required at a frequency up to a few times the supply frequency, an induction generator is frequently used. This consists of a wound-rotor induction motor, whose stator is excited at the supply frequency and whose rotor is driven by a second induction motor at a speed such that the desired frequency appears at the rotor slip rings. Such units are used for driving high-speed induction motors for wood working, grinders, etc.

Induction heating or melting requiring single-phase power at frequencies of 500 to 10,000 cycles is ordinarily supplied by inductor alternators driven by standard induction motors. For high-frequency heating applications requiring more than 10,000 cycles, electronic frequency changers are used. Such frequency changers use high-vacuum tubes and generating circuits similar to those of radio transmitters.

Frequency conversion at power frequencies may be carried out electronically by means of a rectifier and an inverter. This scheme requires no definite frequency ratio. Units have been constructed in capacities up to 20,000 kw.

PHASE CONVERSION. Polyphase phase conversion, as 3 to 2 phase, is accomplished with transformers with suitable winding ratios and taps (see Art. 23).

Phase conversion from polyphase to single phase, required to avoid unbalance due to excessive single-phase loads, as in a-c traction, is accomplished with a motor generator.

POWER-SUPPLY ECONOMICS *

30. ELECTRIC POWER DEVELOPMENT IN THE UNITED STATES

The electric system for the production, transmission, and distribution of electric energy was conceived by Thomas A. Edison and first applied by him to serve the Wall Street district in New York City. The electric power industry, as such, dates from Sept. 4, 1882, the date of initial service of the Pearl Street Station, the first central station built to generate electric power in bulk. The primary elements of an electric system as conceived by Thomas A. Edison have been retained to this day. Such differences as have since developed reflect technological progress in the art of design and application without change in fundamental concepts.

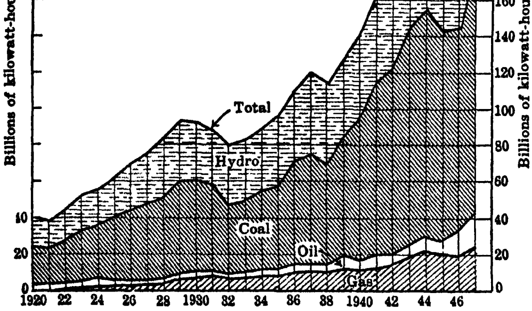


Fig. 1. Growth of electric utility output by sources of energy.

GROWTH of the industry, though slight in the years immediately following its inception, has accelerated since 1920. Electric energy production for public use from all sources, excluding industrial plants, increased from 39.4 billion kilowatt-hours in 1920 to 255.7 billion kilowatt-hours in 1947. This represents a 6.5 fold expansion during the 27-year period and an average increase of 24% of the 1920 power production each year. Alternatively, it represents a secular trend of 7.2% increase each year.

Plants producing electric energy may be classified as steam-electric, utilizing solid, liquid, and gaseous fuels, hydroelectric, utilizing the available energy from the flow of water, and internal-combustion plants. The steady increase in supply from these sources

* Contributed by M. J. Steinberg.

is illustrated in Fig. 1, and the relative outputs in Fig. 2. Figure 2 shows that the supply of electric energy from the several sources has been maintained at relatively constant proportions.

Hydroelectric generation has varied between 30.5 and 40% of the total generation, the average for the last five years being about 35%. Hydro sources of electric energy have been increased considerably by federally owned and operated hydro projects. This increase has been offset by additions of privately owned steam-electric plants, giving a relatively constant ratio. Growth in capacity of hydro and fuel-burning plants is shown in Fig. 3. A breakdown of plant capacity between public and private ownership is shown in Table 1. Nationwide, 19.8% of the total installed capacity is publicly owned. Public ownership extends to 40.3% of the hydro capacity; 9.3% of the steam capacity; and 67.7% of the internal-combustion capacity.

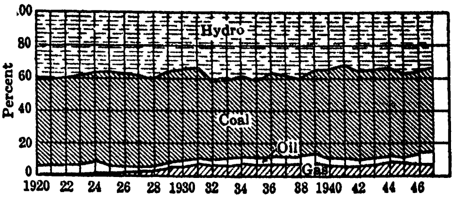


FIG. 2. Per cent of total power generation, by sources of energy.

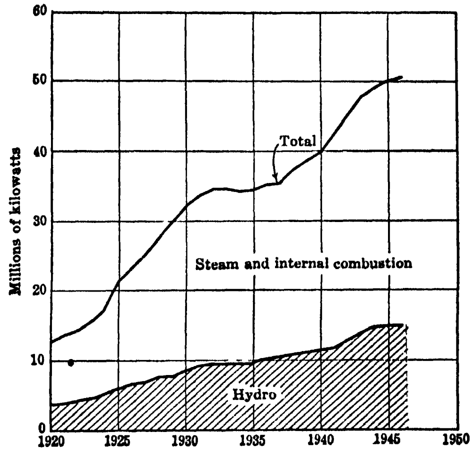


FIG. 3. Installed generating capacity of all-electric power plants contributing to the public supply

Table 1. Installed Capacity As of Dec. 31, 1946

(Adapted from Federal Power Commission *Report* S-49)

Ownership	Hydro		Steam		Internal Combustion		Total	
	Kilowatts	%	Kilowatts	%	Kilowatts	%	Kilowatts	%
Private	8,864,547	59.7	31,124,359	90.7	371,424	32.3	40,360,330	80.2
Public *	5,984,489	40.3	3,180,551	9.3	778,290	67.7	9,943,330	19.8
Total— kw	14,849,036	100.0	34,304,910	100.0	1,149,714	100.0	50,303,660	100.0
%	29.5		68.2		2.3		100.0	

* Includes Federal Hydroelectric Projects, Rural Cooperatives, and Municipal Systems.

Geographical distribution of electric energy production is shown in Table 2. Fuel consumption for electric energy production is shown by geographical divisions for the calendar year 1946 in Table 3. Consumption of coal in the Middle Atlantic and East North Central States accounts for 72% of the total coal used; 30% of the total oil was burned in the Pacific States, and 55% of the total gas was used in the West South Central States.

(*Continued on p. 16-88*)

Table 2. Generation by States and Type of Prime Mover of all Plants Contributing to the Public Supply

(Year 1946)

(Adapted from Federal Power Commission *Report* S-49)

State	Total		Hydro		Steam		Internal Combustion	
	1000 kwhr	%	1000 kwhr	%	1000 kwhr	%	1000 kwhr	%
Maine	1,370,979		1,220,497		143,500		6,982	
New Hampshire	1,014,111		793,286		217,825		3,000	
Vermont	758,930		746,565		12,229		136	
Massachusetts	4,888,513		632,030		4,227,672		28,811	
Rhode Island	1,060,315		8,565		1,051,606		144	
Connecticut	3,215,054		266,307		2,943,686		5,061	
New England	12,307,902	5.5	3,667,250	4.7	8,596,518	6.1	44,134	1.9
New York	22,892,398		7,549,360		15,293,181		49,857	
New Jersey	7,574,731		27,924		7,539,659		7,148	
Pennsylvania	16,882,095		1,804,385		15,059,304		18,406	
Middle Atlantic	47,349,224	21.2	9,381,669	12.0	37,892,144	26.6	75,411	3.2
Ohio	13,975,736		24,942		13,908,603		42,191	
Indiana	7,272,156		95,521		7,108,827		67,808	
Illinois	13,872,771		165,842		13,644,749		62,180	
Michigan	10,053,845		1,395,833		8,520,741		137,271	
Wisconsin	4,886,882		1,411,732		3,401,183		73,967	
East North Central	50,061,390	22.4	3,093,870	3.9	46,584,103	32.7	383,417	16.3
Minnesota	2,939,108		777,481		2,014,457		147,170	
Iowa	3,106,520		952,632		1,873,827		280,061	
Missouri	2,724,541		580,721		1,983,103		160,717	
North Dakota	351,530		0		326,748		24,782	
South Dakota	320,550		17,389		229,838		73,323	
Nebraska	1,393,105		503,912		796,350		92,843	
Kansas	2,344,444		23,985		2,153,977		166,482	
West North Central	13,179,798	5.9	2,856,120	3.6	9,378,300	6.6	945,378	40.2
Delaware	43,478		0		31,607		11,871	
Maryland	3,964,347		1,268,697		2,675,206		20,444	
Dist. of Columbia	1,942,431		5,571		1,936,860		0	
Virginia	3,385,359		646,624		2,710,460		28,275	
West Virginia	5,244,366		377,697		4,865,768		901	
North Carolina	5,678,117		2,862,696		2,798,863		16,558	
South Carolina	2,581,144		2,327,782		243,454		9,908	
Georgia	3,042,272		1,469,944		1,569,293		3,035	
Florida	2,661,581		51,063		2,528,536		81,982	
South Atlantic	28,543,095	12.8	9,010,074	11.5	19,360,047	13.6	172,974	7.4
Kentucky	2,379,427		1,516,868		858,151		4,408	
Tennessee	6,577,690		6,157,856		416,084		3,750	
Alabama	7,299,870		6,043,548		1,252,937		3,385	
Mississippi	225,626		0		210,349		15,277	
East South Central	16,482,613	7.4	13,718,272	17.5	2,737,521	1.9	26,820	1.1
Arkansas	836,095		396,371		401,267		38,457	
Louisiana	2,884,044		0		2,747,036		137,008	
Oklahoma	2,008,726		256,746		1,627,102		124,878	
Texas	7,340,608		796,836		6,363,205		180,567	
West South Central	13,069,473	5.9	1,449,953	1.9	11,138,610	7.8	480,910	20.4

Table 2. Generation by States and Type of Prime Mover of all Plants Contributing to the Public Supply—*Continued*

(Year 1946)

(Adapted from Federal Power Commission *Report* S-49)

State	Total		Hydro		Steam		Internal Combustion	
	1000 kwhr	%	1000 kwhr	%	1000 kwhr	%	1000 kwhr	%
Montana	2,460,586		2,431,942		26,919		1,725	
Idaho	1,333,719		1,331,362		0		2,357	
Wyoming	309,706		260,985		41,092		7,629	
Colorado	1,168,380		318,382		804,010		45,988	
New Mexico	600,927		101,458		456,218		43,251	
Arizona	2,852,606		2,469,286		304,364		78,956	
Utah	456,425		353,849		83,822		18,754	
Nevada	2,456,355		2,450,338		0		6,017	
Mountain	11,638,704	5.2	9,717,602	12.4	1,716,425	1.2	204,677	8.7
Washington	9,038,856		8,862,080		173,527		3,249	
Oregon	4,148,612		3,994,281		151,408		2,923	
California	17,310,002		12,646,149		4,651,384		12,469	
Pacific	30,497,470	13.7	25,502,510	32,5	4,976,319	3.5	18,641	0.8
United States	223,129,669	100.0	78,397,320	100.0	142,379,987	100.0	2,352,362	100.0

Table 3. Fuel Consumption for Electric Production

(Year 1946)

(Adapted from Federal Power Commission *Report* S-49)

Geographical Division	Coal *		Oil		Gas	
	Short Tons	%	42-gallon Barrels	%	MCF	%
New England	4,027,369	5.6	5,476,116	15.1	....	.....
Middle Atlantic	21,954,496	30.4	7,042,439	19.4	786,621	0.3
East North Central	30,086,232	41.7	1,418,575	3.9	5,318,006	1.7
West North Central	5,027,414	7.0	2,568,907	7.1	67,419,945	22.0
South Atlantic	8,657,427	12.0	7,020,375	19.3	15,934,016	5.2
East South Central	1,452,146	2.0	67,390	0.2	12,302,691	4.0
West South Central	232,531	0.3	1,029,295	2.8	169,731,061	55.3
Mountain	754,489	1.0	786,060	2.2	16,875,371	5.5
Pacific	4,626	..	10,906,714	30.0	18,577,854	6.0
Total—United States	72,196,730	100.0	36,315,871	100.0	306,941,565	100.0

* Includes bituminous coal, anthracite and lignite.

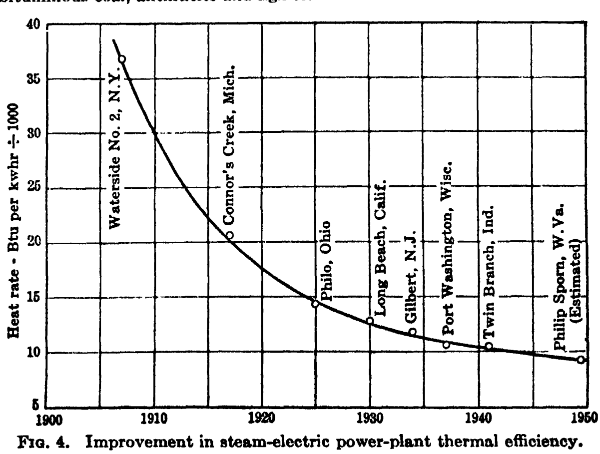


FIG. 4. Improvement in steam-electric power-plant thermal efficiency.

Improved thermal efficiency of steam-electric power plants is illustrated in Fig. 4 by the annual heat rates of selected plants, typical of the state of the art as of the year indicated. The steady downward trend of plant heat rate is reflected in Fig. 5, which shows the aver-

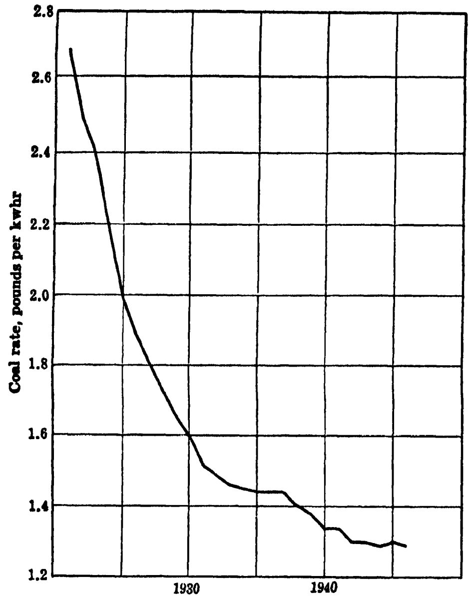


FIG. 5. Average coal rate for public supply of electric energy. (Based on total coal and coal equivalent of oil and gas.)

age rate of fuel consumption per kilowatt-hour based on total fuel consumption for public supply of electric energy, including the coal equivalent of oil and gas fuels. A similar trend with respect to the price paid for electric energy (Fig. 6) is in sharp contrast to the rising cost of living and to the rising cost of labor, materials, and supplies for the production of electric energy.

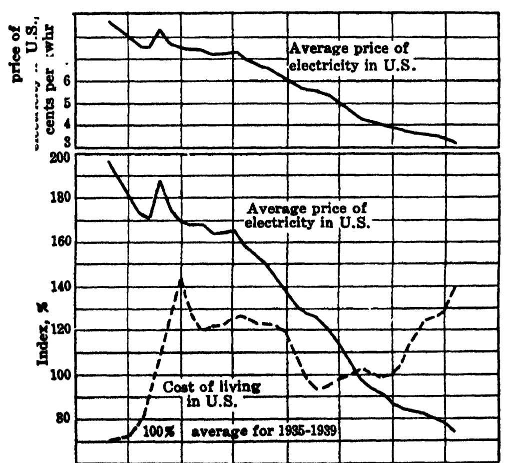


FIG. 6. Price of electricity in relation to the cost of living.

31. INVESTMENT

GENERATING-PLANT INVESTMENTS. Investment in electric utility plant covers facilities for production, transmission, distribution, and general purposes. Studies of unit investment costs are available in a report by the Federal Power Commission (Publication FPCS-18), derived from data for the year 1938 as submitted by 393 large companies comprising in excess of 95% of the nation's privately owned electric light and power industry. A summary of production plant investments is presented in Table 4 for the three major sources of supply. Relative distribution plant and overall plant investment are shown in Tables 5 and 6, respectively. In general, the unit investment per kilowatt of plant capacity is influenced by (1) plant site, (2) design, and (3) size of plant. For fuel-burning plants, the components of investment are land, structure, and equipment. In addition to these, hydro plants also require investments in reservoirs, dams, and waterways.

Table 4. Production Plant Investments

	Steam				Hydro				Internal Combustion			
	Min $/kw	Max $/kw	Average $/kw	Average %	Min $/kw	Max $/kw	Average $/kw	Average %	Min $/kw	Max $/kw	Average $/kw	Average %
Land	2	9	2	2	9	64	32	17.7	0.5	7.5	2.20	1.3
Reservoirs, dams, and waterways	....	....	.	..	100	162	99	54.7	.....	......	....	......
Structure	26	34	28	28	15	45	20	11.0	14	50	24	14.5
Equipment	68	126	70	70	21	74	30	16.6	110	209	139	84.2
Total	96	169	100	100	145	345	181	100	124.5	263.5	165.2	100

Table 5. Ratios of Distribution Plant Investment

Item	Percentage
Land and land rights	1.6
Poles, towers, conductors, etc.	30.5
Underground conductors, etc.	18.6
Transformers, services, and meters	27.1
Structures and equipment	17.9
Street lighting and signal systems	4.3
Total	100.0

Table 6. Ratios of Overall Plant Investment

Item	Percentage
Production	38.0
Distribution	40.6
Transmission	14.7
General	4.8
Intangible *	1.9
Total	100.0

* Capitalized value of rights, fees, franchise, etc., not subject to classification as physical plant.

Unit Investment Cost for Steam Stations. The study for steam-electric generating stations, based on reports for 150 generating stations (representing one-third of the total privately owned steam-electric plant capacity in the country), indicates that the unit investment per kilowatt of capacity decreases as the plant increases in size up to 50,000 kw. The decrease in unit investment is relatively slight and negligible as the size of plant increases above 50,000 kw. Land is a relatively minor item of the total investment. The unit investment in structure is but slightly affected by plant size. Unit investment in equipment, including boilers, prime movers and generators, accessory electric equipment, and miscellaneous power plant equipment, is largely dependent on economies in costs that are greatly influenced by size and design of the plant. Unit investment in structures is fairly constant for plants in excess of 2,000 kw capacity, and it may therefore be expected that, for a given design, the total cost of station structures will be in practically direct proportion to plant size.

Unit Investment Cost for Hydro Stations. The survey for hydroelectric generating stations indicates a higher investment per kilowatt than for steam plants, averaging $100 and $181 per kilowatt, respectively, for steam and hydro stations. The corresponding average plant capacities are 60,000 and 10,000 kw, respectively. For equal plant size, the difference in unit investment is reduced. Thus for plants of 100,000 kw capacity, the unit investments per kilowatt become $100 and $160, respectively, for steam and hydro installations. Unit investment for hydro stations decreases rapidly with size up to capacities of 50,000 kw, and only slightly for plant capacities in excess of this value. Investment in hydro equipment amounts to about one-half that shown for steam-generating stations. The larger investment in the steam stations is attributed to the investment in boiler plant equipment, which also accounts for the higher investment in structures. More than two-thirds of the total hydro plant investment is in reservoirs, dams, waterways, and land, the cost of which is the principal reason for the relatively higher investment in hydro stations than in steam stations of the same size.

Unit Investment Cost for Internal-combustion Stations. The survey of internal-combustion engine generating stations is based on reports for 95 stations ranging in size from 250 to over 5000 kw. Since this type of plant is limited to capacities considerably less than those for steam and hydro, comparisons of unit investment costs on an overall basis are not of much value. For a plant of 10,000 kw, however, comparative unit investments are $110, $200, and $130, respectively for steam, hydro, and internal-combustion engines. In general the unit investment per kilowatt decreases with plant size as for steam and hydro plants. Investment in equipment represents the largest item.

TRANSMISSION PLANT INVESTMENT. Unit Transmission Line Investment. Transmission plant includes high-voltage *transmission lines* and associated *substations*. Unit investments per structure mile for transmission lines (consisting of poles, towers, and conductors but excluding land, land rights, and clearing of land rights of way) increase *linearly* with transmission voltage, for a specified design. The ranges of investment for four common types are:

(1) Wood pole single-circuit lines, from approximately $1600 per mile at 11 kv, to $3800 per mile at 66 kv.
(2) Wood H-frame single-circuit lines, from $4100 per mile at 66 kv, to $6800 per mile at 132 kv.
(3) Steel tower single-circuit lines, from $5900 per mile at 33 kv, to $10,600 per mile at 132 kv.
(4) Steel tower double-circuit lines, from $11,800 per mile at 33 kv, to $18,000 per mile at 132 kv.

Transmission substation investment per kva of capacity, exclusive of land and land rights, tends to decrease but slightly with increase in capacity provided. The range of variation is from slightly in excess of $11 per kva for capacities under 10,000 kva to approximately $10 per kva for capacities of 100,000 kva and larger.

The cost of land, including land rights for both transmission lines and substations and the cost of clearing line rights-of-way, varies considerably, from a minimum value of 1.5% to a maximum value of 30% of the total transmission plant investment. The overall average for 89 companies as of Dec. 31, 1938, was 11.7%, and as of Dec. 31, 1943, 12.4%.

DISTRIBUTION PLANT INVESTMENT includes land and land rights, substations, overhead conductors and supporting structures, underground conduit and conductors, line transformers, services, and meters. In addition, it is the practice to include investment in street lighting and signal system equipment. Investments are affected to a considerable extent by customer density, which may be measured by the number of meters per wire mile or per circuit mile. A circuit mile is assumed to equal three wire miles. Investments per wire mile and per meter are shown in Fig. 7.

Street lighting and signal system investment includes equipment used wholly for public street and highway lighting, and traffic, fire alarm, police, and other signal systems. Ownership may vary from full ownership by the utility to varying degree of joint ownership with the community served. Investment is best related to distribution plant investment, which may vary from 3 to 5%, the average derived by the Federal Power Commission as of Dec. 31, 1938, being 3.6% of distribution plant investment.

GENERAL PLANT INVESTMENT includes land and structures for office buildings and general purposes, office furniture and equipment, transportation equipment, tools, communication, and other miscellaneous facilities incidental to but not otherwise assign-

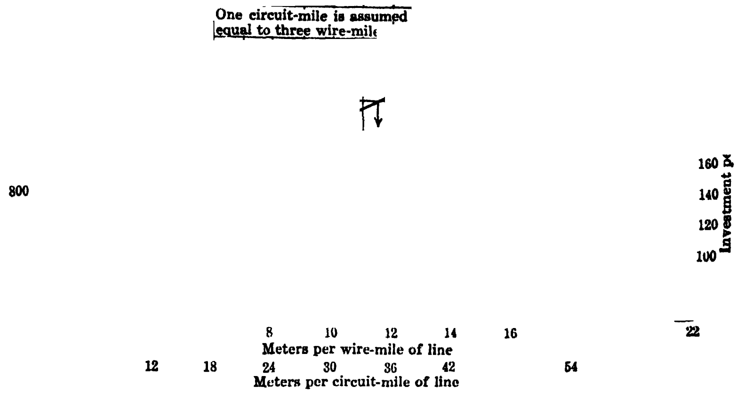


FIG. 7. Distribution-plant investment exclusive of street lighting and signal systems. (Data from the Federal Power Commission)

able to specific operations. General plant investment varies from 4 to 6% of total plant investment, depending on the type of production plant installed. A nationwide ratio of 4.8% was derived by the Federal Power Commission as of the end of the year 1943, as shown in Table 6.

32. ANNUAL FIXED CHARGES

Annual fixed charges or annual carrying charges on investment are the charges against revenue for depreciation of physical property, interest charges on capitalization, property, income, and miscellaneous taxes, and the cost of property insurance. The charges are said to be "fixed" because they are essentially a function of the amount of capital investment and practically independent of the quantity of electric energy produced.

The range of fixed charge rates applicable to book cost of investment are:

	Percentage
Depreciation	1 to 5
Taxes	2 to 5
Interest	4 to 8
Insurance	0.05 to 0.3
Total	to 18

DEPRECIATION is the reduction in value of property due to its decreased ability or capacity to perform present and future services. Depreciation has been defined as the consumption of investment in property; or the loss in service capacity of property because of use, wear and tear, physical deterioration, the current action of the elements, obsolescence, inadequacy, or the demands of public authority. Briefly, depreciation results from the usual forces and conditions which limit the service life of property and cause its retirement.

Actual depreciation is the true or actual decrease in value of property as determined from the physical inspection of the property by competent engineers qualified as valuation experts. Determination of actual depreciation is costly and impractical for properties of extensive magnitude and is therefore seldom resorted to.

Theoretical depreciation is the decrease in value of property as determined by the application of some theoretical method, generally by accountants for accounting purposes.

Depreciation expense (depreciation charge) is the amount of money that the accountant allows for depreciation as an expense against revenue in his financial statements.

Depreciable value is the depreciation of property from its value new. It is equal to the difference between its value when new and its salvage value at retirement.

Salvage value of property is the net sum of money realized over and above removal costs, upon retirement at the end of its service life.

Service life of property is the period from the date of initial service or installation to the date of retirement.

DEPRECIATING ACCOUNTING. The various methods in common use for determination of annual depreciation expense are briefly described below, for which the following notation is used:

P = initial investment, first cost, value new or book cost, dollars.
L = net salvage value of the property at the end of its service life, dollars.
n = estimated service life of the property, years.
i = annual earning rate of invested funds, expressed in percentage but used as a decimal in appropriate formulas.
D = annual depreciation expense, depreciation charge, or annual depreciation, dollars.
x = age of the property, years.
D_x = accrued depreciation to the end of x years, dollars.
D_n = depreciable value of the property, dollars.
B_x = book value of property as of the end of x years, dollars.

Then, regardless of the method used to determine the annual depreciation D,

$$D_n = P - L \tag{1}$$

$$B_x = P - D_x \tag{2}$$

Sinking-fund Depreciation. The annual depreciation D is that sum which, invested at the end of each year at interest rate i compounded annually, accumulates to the depreciable value of the property D_n at the end of its service life n.

$$D = (P - L)\left[\frac{i}{(1+i)^n - 1}\right] \tag{3}$$

$$D_x = (P - L)\left[\frac{(1+i)^x - 1}{(1+i)^n - 1}\right] = (P - L)\left[\frac{i}{(1+i)^n - 1}\right]\left[\frac{(1+i)^x - 1}{i}\right] \tag{4}$$

Straight-line Depreciation. Annual depreciation D is equal to the total depreciable value D_n divided by the estimated service life n.

$$D = \frac{P - L}{\ } \tag{5}$$

$$D_x = \frac{x}{n}(P - L) \tag{6}$$

This method assumes that the book value B_x decreases linearly with age.

Diminishing Value Depreciation. Also known as *Reducing Balance Depreciation* or *Fixed Percentage Depreciation.*

Annual depreciation D is a fixed percentage f of the book value of the property as of the beginning of the year. The percentage f remains constant, but the annual depreciation D decreases with the age of the property x.

$$f = 1 - \sqrt[n]{\frac{L}{D}} \tag{7}$$

$$D \quad f(B_{x-1}) \tag{8}$$

$$D_x \quad P\left[1 - \left(\frac{L}{P}\right)^{x/n}\right] \tag{9}$$

$$B_x \quad P\left(\frac{L}{P}\right)^{x/n} \tag{10}$$

In addition to the foregoing, other methods are employed. Some utilities set aside fixed percentages of the revenue for annual depreciation. Some do not employ any definite method of calculation. Instead, amounts are credited to reserve funds for retire-

Table 7. Relationship of Depreciation Reserves and Depreciation Expense to Total Electric Plant and Estimated Depreciable Electric Plant—by Depreciation Method

(Federal Power Commission, 1945)

Depreciation Method	Number of Utilities	Ratio of Depreciation Reserves: To Total Plant	To Depreciable Plant	Ratio of Annual Depreciation: To Total Plant	To Depreciable Plant
Straight-line method	132	24.6%	27.6%	2.63%	2.95%
Sinking-fund method	13	22.3	24.3	2.12	2.31
Revenue method	18	24.3	26.3	2.51	2.71
Other methods	90	18.2	20.6	2.25	2.55
Total	253	21.4	24.0	2.41	2.70

Table 8. Average Rates of Depreciation for Depreciable Electric Plant Accounts for Straight-line Depreciation

(Federal Power Commission, 1945)

Account Number	Class of Depreciable Property	Range of Depreciation Rates, %	Average of Depreciation Rates, %	Service Life of Average Depreciation Rate, years
	Steam Production Plant			
311	Structures and improvements	1.00- 2.50	1.94	51.55
312	Boiler plant equipment	2.00- 4.00	3.20	31.25
313	Engines and engine-driven generators	2.55- 3.00	2.87	34.84
314	Turbogenerator units	2.00- 4.00	2.95	33.90
315	Accessory electric equipment	2.00- 4.00	3.12	32.05
316	Miscellaneous power plant equipment	2.00- 8.14	3.79	26.39
	Average		2.98	33.56
	Hydraulic Production Plant			
321	Structures and improvements	1.00- 2.00	1.68	59.52
322	Reservoirs, dams, and waterways	0.86- 4.00	1.63	61.35
323	Water wheels, turbines, and generators	1.00- 4.00	2.84	35.21
324	Accessory electric equipment	1.50- 4.00	3.01	33.22
325	Miscellaneous power plant equipment	2.41- 4.00	3.05	32.79
326	Roads, railroads, and bridges	1.00- 2.00	1.47	68.03
	Average		2.28	43.86
	Internal Combustion Engine Production Plant			
331	Structures and improvements	2.00- 4.00	2.67	37.45
332	Fuel holders, producers, and accessories	2.00- 5.00	3.37	29.67
333	Internal-combustion engines	2.50- 5.00	4.05	24.69
334	Generators	2.50- 5.00	3.89	25.71
335	Accessory electric equipment	2.50- 5.00	3.81	26.25
336	Miscellaneous power plant equipment	2.50- 5.00	4.13	24.21
	Average		3.65	27.40
	Transmission Plant			
342	Structures and improvements	1.60- 4.00	2.55	39.22
343	Station equipment	2.50- 5.00	3.19	31.35
344	Towers and fixtures	1.75- 4.00	2.55	39.22
345	Poles and fixtures	2.50- 5.06	3.43	29.15
346	Overhead conductors and devices	1.42- 4.00	2.30	43.48
347	Underground conduits	1.67- 2.70	2.03	49.26
348	Underground conductors and devices	1.50- 3.03	2.25	44.44
349	Roads and trails	1.43- 5.00	2.49	40.16
	Average		2.60	38.46
	Distribution Plant			
351	Structures and improvements	0.90- 4.00	2.33	42.92
352	Station equipment	1.66- 5.00	3.34	29.94
353	Storage battery equipment	3.17	3.17	31.55
354	Poles, towers, and equipment	2.00- 5.37	3.76	26.60
355	Overhead conductors and devices	1.50- 4.00	2.75	36.36
356	Underground devices	1.00- 3.03	2.04	49.02
357	Underground conductors and devices	1.50- 3.03	2.23	44.84
358	Line transformers	2.00- 5.00	3.46	28.90
359	Services	1.00- 5.00	3.57	28.01
360	Meters	2.00- 5.00	3.50	28.57
361	Installations on customers' premises	2.00-10.00	4.05	24.69
362	Leased property on customers' premises	3.33- 5.00	4.44	22.52
363	Street lighting and signal systems	2.00- 6.00	4.06	24.63
	Average		3.28	30.49
	General Plant			
371	Structures and improvements	1.00- 3.00	2.21	45.25
372	Office furniture and equipment	3.00-15.00	5.93	16.86
373	Transportation equipment	. . . .	. . . .	. . . .
374	Stores equipment	2.00-10.00	5.85	17.09
375	Shop equipment	2.50-10.00	5.86	17.06
376	Laboratory equipment	2.47-15.00	6.26	15.97
377	Tools and work equipment	2.21-10.00	6.62	15.11
378	Communication equipment	1.80-10.00	5.45	18.35
379	Miscellaneous equipment	2.45-10.00	6.41	15.60
390	Other tangible property	1.85- 3.33	2.80	35.71
	Average		5.27	18.98

ments, in each year or from time to time after inspection of the property, which in the judgment of the management will be sufficient to provide for current retirements and to build up reserves against future retirements. A study by the Federal Power Commission for the year 1945 indicates that slightly more than 52% of the companies reporting used the straight-line method of computing annual depreciation charges. For 1937 only 15% of the companies used this method. The study reveals that there is a marked trend toward the general adoption of the straight-line method for providing for depreciation.

Table 7 shows the relationship of depreciation reserves and depreciation expense to total and depreciable plant investment as reported for the year 1945. The ratios to total plant are somewhat lower than the corresponding ratios to depreciable plant, since depreciable plant excludes land investments not generally considered to be depreciable.

A summary of the depreciation rates reported by 38 utilities employing the straight-line method for providing for depreciation for various plant accounts is presented in Table 8.

INTEREST is the cost of money use. As an item of the fixed charges on plant investment, interest includes payment of interest on long-term or bonded indebtedness, dividends on outstanding preferred and common stock, and taxes on revenue and income imposed by federal, state, and municipal authorities.

TAXES, one of the larger items chargeable against revenue, vary with location. A detailed tax statement from the annual report of the Consolidated Edison Company of New York, Inc., for the year 1947, presented in Table 9, indicates that tax payments amounted to 20.6% of total revenues.

Table 9. Detailed Tax Statement

(Consolidated Edison Co. of N. Y., Inc., 1947)

Local Taxes		
Real estate	$15,083,553	
Special franchise	13,072,766	
Gross receipts	710,738	
Public utilities excise	2,954,635	
Conduit companies excise	366,490	
Sales, compensating, and personal property	1,629,519	
Miscellaneous	2,125	
Total local taxes		$ 33,819,826
State Taxes		
Gross earnings	$ 1,723,776	
Excess dividends	169,323	
Franchise	81,758	
Unemployment insurance	1,435,652	
Public utilities gross income	6,203,391	
Miscellaneous	122,078	
Total state taxes		9,735,978
Federal Taxes		
Income	$17,780,000	
Capital stock	....	
Electrical energy	6,486,619	
Insurance contributions (old age benefits)	810,634	
Unemployment	243,190	
Miscellaneous	463,950	
Total federal taxes		25,784,393
Total Taxes		$ 69,340,197
Total Revenues		$336,377,000
Ratio—Taxes to Revenue		20.6%

33. COST OF PRODUCING POWER

PRODUCTION EXPENSE. Steam Plants. The production costs of steam plants vary with many factors which make comparison on a comparable basis difficult if not impossible. The thermal performance is a function primarily of station design as related to size of units, operating steam pressure and temperature, heat balance cycle, and type and method of fuel firing. For a fixed design, the thermal performance depends on such factors as load cycle, load factor, and plant factor. Production costs generally include the costs for fuel, labor, repairs, and supplies, of which fuel may represent 60 to 80% of the total production cost. Table 10 presents production cost data for selected large steam plants for the year 1946. It is readily seen by inspection of this table that a low production cost is due to either low heat rate (high thermal efficiency) or low fuel price or a combination of both.

Internal-combustion Engine Plants. The cost of producing electric energy with internal-combustion engines is influenced by the size and speed of the units, type of fuel injection, and utilization of the heat in the exhaust. Fuel oil, a major item, represents about 30 to 50% of the total cost. A survey by the Federal Power Commission for 1938 to determine the relation of production cost to plant size indicates the following:

Station capacity, kw	0–500	500–2000	Over 2000
Average capacity, kw	243	943	3287
Production expenses, mills/kwhr			
Fuel	7.1	4.9	3.9
Labor and materials	14.1	5.6	4.4
Total	21.2	10.5	8.3

Hydroelectric Plants. The items of production expense for hydroelectric generation are labor, maintenance, and supplies. The unit production cost is a function of the installed capacity as illustrated in Fig. 8.

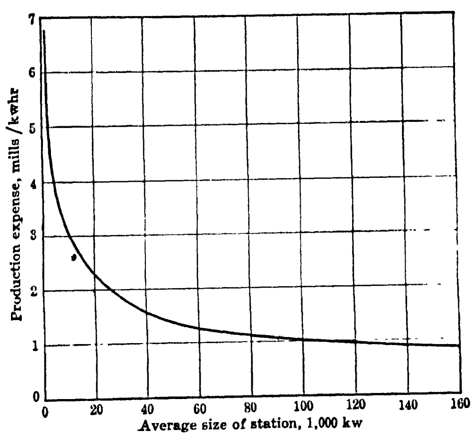


FIG. 8. Production expenses of hydroelectric stations, 1938. (Data from the Federal Power Commission)

TRANSMISSION AND DISTRIBUTION EXPENSE. **Transmission line operating expenses** include operation, supervision and engineering; load dispatching labor and expenses; operation of lines and maintenance of poles, towers, fixtures, and overhead conductors and devices. For a fixed design, the annual operating cost per structure mile of line varies with the operating line voltage. Referred to 1937 and 1938 prices, this expense may be determined as follows.

For single-circuit–H-frame construction:

$$C = 27 + 0.273\ (\text{kv})$$

For single-circuit–single-pole construction:

$$C = 24 + 0.767\ (\text{kv})$$

For single-circuit–steel-tower construction:

$$C = 50.5 + 0.752\ (\text{kv})$$

where C = operating expense per structure mile in dollars per year, and kv = line voltage, kilovolts.

Transmission substation operating expenses include operating supervision and engineering, load dispatching labor and expenses, operation of stations and maintenance of structures and station equipment. Referred to 1938 price levels, this expense averages 30 cents per kva of substation capacity.

(Continued on p. 16-98)

Table 10. Production Cost Data of Selected Large Steam-electric Power Plants

(1946)

Station Designation	A	B	C	D	E	F	G	H	I	J	K
Initial service date	1921	1925	1911	1925	1943	1930	1942	1920	1918	1920	1935
Main generator capacity, kw	630,000	285,000	179,000	160,000	100,000	180,000	60,000	290,000	210,000	310,800	160,000
Net station capacity, kw	625,000	273,000	172,600	156,600	94,000	180,000	60,000	312,000	203,500	....	....
Capacity of largest unit, kw	160,000	165,000	50,000	60,000	50,000	60,000	60,000	80,000	30,000	....	80,000
Net generation, 10^6 kwhr	2,682	1,519	953	712	638	1,151	583	1,588	912	1,124	920
Half-hour net maximum demand, kw	596,000	265,000	186,000	153,500	100,000	203,500	75,300	312,900	202,000	292,700	154,543
Operating load factor, %	68.9	....	....	69.6	86.0	79.4	93.2	81.8	77.2	65.1	79.7
Load factor, %	51.3	69.0	61.9	55.6	77.6	64.6	88.4	57.9	54.1	44.7	69.4
Plant factor, %	50.6	64 2	64.3	53.4	77.6	73.0	111.0	58.1	53.7	43.9	69.4
Coal price per short ton, $	6.931	6.768	6.864	7.200	7.569	3.261	2.413	2.824	5.720	4.931	5.740
Oil price per gallon, ¢	4.65	11.391 *	9.574 *	....	....	....	....	....	....	....	....
Fuel price per million Btu, ¢	26.60	26.49	28.53	26.93	28.23	12.95	10.78	11.37	21.67	21.95	24.27
Btu per lb coal	13,279	13,560	13,735	13,367	13,404	12,591	11,187	12,420	13,194	11,234	11,828
Btu per gal oil	148,732	....	....	....	....	....	....	....	....	....	....
Overall boiler room efficiency, %	85.8	....	....	....	....	80.9	86.1	84.1	84.0	82.7	86.9
Steam rate per kwhr, lb	11.14	10.20	10.69	....	....	10.69	10.06	10.59	10.36	10.11	7.93
Heat rate per kwhr, Btu	15,102	13,783	14,285	14,329	11,407	15,190	12,400	14,122	13,264	14,512	10,608
Unit production cost, mills/kwhr											
Labor	0.62	0.29	0.56	0.60	0.31	0.41	0.42	0.48	0.65	0.75	0.37
Fuel	4.21	3.65	4.08	3.94	3.23	2.02	1.37	1.61	2.94	3.19	2.57
Supplies	0.09	‡	‡	0.09	0.18	0.05	0.05	0.06	0.12	0.07	0.04
Maintenance	1.12	0.46	0.72	0.80	0.37	0.31	0.12	0.43	0.31	0.54	0.23
Miscellaneous	....	....	....	....	....	....	0.01	0.11	....	....	....
Total	6.04	4.40	5.36	5.43	4.09	2.79	1.97	2.69	4.02	4.55	3.21

Table 10. Production Cost Data of Selected Large Steam-electric Power Plants—*Continued*

(1946)

Station Designation	L	M	N	O	P	Q	R	S	T	U	V	W
Initial service date	1924	1945	1924	1929	1929	1928	1903	1925	...	1930	1942	1940
Main generator capacity, kw	415,000	100,000	300,000	385,000	350,000	315,000	323,000	339,600	256,000	160,000	80,000	160,000
Net station capacity, kw	405,000	100,000		...	342,000	342,000	308,000	339,600	.	160,000	90,000	180,000
Capacity of largest unit, kw	165,000	100,000	50,000	75,000	200,000	105,000	147,000	90,000	65,000	80,000	80,000	80,000
Net generation, 10^6 kwhr	1,768	690	945	1,915	2,161	2,309	1,452	1,464	1,173	791	455	885
Half-hour net maximum demand, kw	397,230	112,000	290,000	380,000	347,470	340,800	294,200	343,000	254,000	164,000	94,000	186,000
Operating load factor, %	...	...	57.7	74.8	81.3	87.6	81.2	...	73.0	...	...	...
Load factor, %	50.8	70.3	37.2	57.5	71.0	77.4	56.3	51.2	56.7	55.0	55.3	54.3
Plant factor, %	52.1	84.6	35.9	56.8	72.2	77.1	53.8	51.7	56.2	...	...	...
Coal price per short ton, $	2.439	2.385	5.820	5.510	3.990	3.000	4.170	6.811	4.691	5.551	5.551	5.799
Oil price per gallon, ¢	...	...	...	...	†	†	†	4.49	...	...	...	...
Fuel price per million Btu, ¢	12.20	10.47	24.23	21.78	19.60	15.13	20.34	27.54	17.80	21.93	21.34	22.64
Btu per lb coal	10,000	11,390	12,010	12,650	10,187	9,922	10,242	13,165	13,155	12,638	13,055	12,819
Btu per gal oil	...	...	...	...	.	...		148,238	...	...	...	...
Overall boiler room efficiency, %	80.0	88.0	89.9	85.2	83.6	85.0	80.7	...	85.1	...	...	...
Steam rate per kwhr, lb	9.63	9.52	10.77	9.64	9.51	8.88	10.09	...	9.83	...	...	...
Heat rate per kwhr, Btu	13,525	11,128	14,040	12,380	12,244	12,778	13,579	16,094	13,695	13,040	11,340	10,910
Unit production cost, mills/kwhr												
Labor	0.37	0.30	0.97	0.53	0.32	0.29	0.71	0.68	0.54	0.42	0.44	0.39
Fuel	1.67	1.21	3.47	2.69	2.40	1.93	2.76	4.43	2.44	2.86	2.42	2.47
Supplies	‡	‡	0.14	0.09	0.03	0.07	0.07	0.09	0.05	‡	‡	‡
Maintenance	0.47	0.22	0.66	0.34	0.53	0.31	0.53	0.56	0.39	0.35	0.32	0.31
Miscellaneous	...	...	0.02	0.01	...		...	...	...	...	...	...
Total	2.51	1.73	5.26	3.66	3.28	2.60	4.07	5.76	3.42	3.63	3.18	3.17

* Equivalent coal price, $/ton. † Included as equivalent coal. ‡ Included with labor cost.

Distribution expenses include operation supervision and engineering, load dispatching, distribution office expenses, and operation and maintenance of distribution substations and lines with appurtenant fixtures, conductors and devices, line transformers, services, meters and installations on customers' premises. Although this operating expense is influenced by the design of the distribution system, load density, and size of company, the variation in cost per customer with the number of customers served per pole mile of line is relatively small. The average expense per customer for 213 companies reporting to the Federal Power Commission for the year 1938 was $6.14. The average for 182 companies serving 100,000 customers or less amounted to $5.60, and for 31 companies serving more than 100,000 customers, $6.53.

Streeth lighting and signal system operating expenses include the cost of labor and materials incurred in the operation and maintenance of overhead and underground street lighting systems and traffic, fire, police, and other signal systems. This expense averages about 15% of the revenue derived from this class of service.

MISCELLANEOUS OPERATING EXPENSES. Accounting and collecting expenses include the cost of labor and materials used in connection with customers' contracts and orders, meter reading, billing, collecting and accounting, and charges for uncollectible accounts and rent expenses.

Sales promotion expenses include supervision expenses, salaries and commissions, demonstration, advertising, and other sales expenses and rents.

Administrative and general expenses include salaries and expenses of general officers, executives, and other general office employees, general office supplies and expenses, management and supervision fees and expenses, special services, legal services, regulatory commission expenses, insurance, injuries and damages, employees' welfare expenses and pensions, maintenance of general property, rents, franchise requirements, and miscellaneous general expenses.

Cost for the foregoing items range in value as follows:

	Range	Average
Accounting and collecting, per customer	$2.50–$ 4.86	$3.25
Sales promotion, per cent of operating revenue	1.1 – 3.7	2.5
Administrative and general, per cent of operating revenue	5.0 – 10.3	6.3

34. LOAD CURVES

***Chronological load curve** is a graphical representation of the variation in demand with respect to clock time.* Variation in demand may be due to one or more of the following factors: type of service, i.e., whether residential, railroad, industrial, commercial, etc.; time of day; day of the week; season of the year; and weather conditions. For nonfluctuating types of demand, it is customary to plot the instantaneous demand on the hour and half-hour and to assume linear variation between plotted points. Many systems currently use recording or graphic meters that continuously record on a chart the instantaneous demand. For fluctuating demands, it is customary to plot chronological load curves showing average values of demand over 10-, 15-, or 30-min intervals. The effect of season on the variation of the load is presented in Fig. 9.

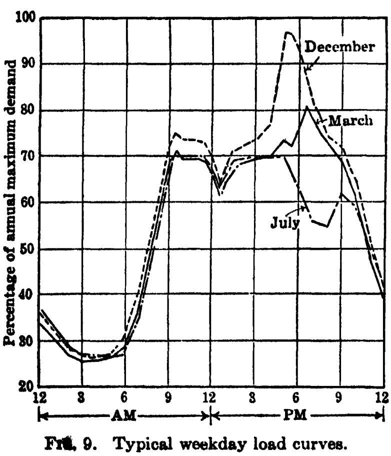


FIG. 9. Typical weekday load curves.

Load-duration curves represent the data of the chronological load curve with the ordinates rearranged in order of decreasing values. The horizontal distance of any point from the load axis indicates the duration of all loads equal to and greater than that indicated on the ordinate. The coordinates of any point on the curve are determined by the value of the load, and the length of the horizontal line at that load level as limited by the chronological load curve. The method is illustrated in Fig. 10 for points (1) and (2). The areas under the chronological load and load-duration curves are equal and represent the total energy for the period. For any increment of load, such as ΔL, the corresponding areas, and hence the corresponding incremental energies represented thereby, are equal. This is represented in Fig. 10 by the cross-hatched areas.

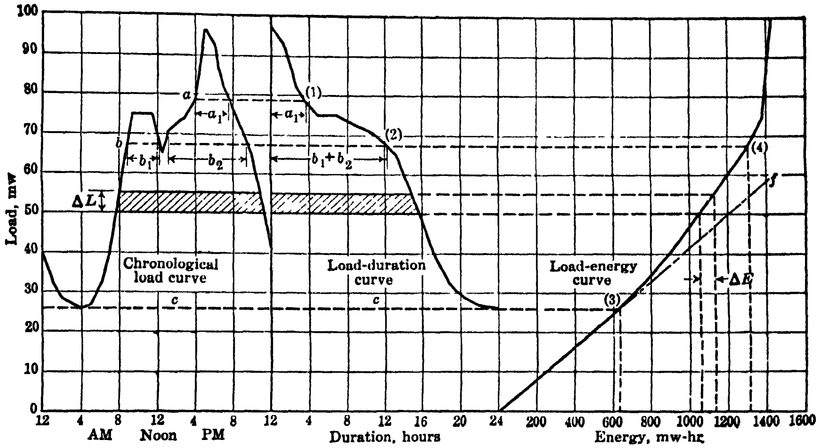


FIG. 10. Methods of showing load.

Load-energy curve indicates the total energy for a given load and all values less than that load. In Fig. 10, the abscissa corresponding to point (4) indicates the energy for all loads equal to and less than the load indicated by the position of line b; equivalent to the areas under the line b for both the chronological and load-duration curves. Similarly, ΔE is the energy value corresponding to ΔL or the cross-hatched areas. Point values for the curve are obtained from successive values of ΔE commencing with the lowest value of load.

35. TERMS AND DEFINITIONS

Load on a machine or apparatus is the power which it delivers.

Power is the time rate of transferring or transforming energy.

Energy of a system is measured by the amount of mechanical work which the system is capable of doing.

Demand of an installation or system is the load at the receiving terminals averaged over a specified interval of time.

Maximum demand is the greatest of all demands which have occurred during a given period of time.

Base load is the minimum load over a given period of time.

Connected load of a system is the sum of the continuous ratings of the load-consuming apparatus connected to the system.

Load diversity is the difference between the sum of the peaks of two or more individual loads and the peak of the combined loads.

Load coincidence of a system is the reciprocal of its load diversity.

Peak load is the maximum load consumed or produced by a unit or group of units in a stated period of time. It may be the maximum instantaneous load or the maximum average load over a designated interval of time.

Load factor is the ratio of the average load over a designated period of time to the peak load occurring in that period.

Plant factor is the ratio of the average load on the plant for the period of time considered, to the aggregate rating of all the generating equipment installed in the plant.

Capacity factor is the ratio of the average load on a machine or equipment for the period of time considered, to the rating of the machine or equipment.

Operation factor is the ratio of the duration of actual service of a machine or equipment to the total duration of the period of time considered.

Output factor is the ratio of the actual energy output, in the period of time considered, to the energy output which would have occurred if the machine or equipment had been operating at its full rating throughout its actual hours of service during the period.

Demand factor is the ratio of the maximum demand of a system, or part of a system, to the total connected load of the system, or part of system, under consideration.

Utilization factor is the ratio of the maximum demand of a system or part of a system, to the rated capacity of the system, or part of the system, under consideration.

Dump power is hydro power (in excess of load requirements) made available by surplus water.

Firm power is power intended always to be available, even under emergency conditions.

Prime power is the maximum potential power constantly available for transformation into electric power.

Cold reserve is reserve generating capacity that is available for service but not in operation.

Hot reserve is reserve generating capacity that is in operation but not in service.

Spinning reserve is reserve generating capacity that is connected to the bus and ready to take load.

Run-of-river station is a hydroelectric generating station that utilizes the stream flow without water storage.

Stream flow is the quantity rate of water passing a given point.

SECTION 17

ATOMIC POWER

BY

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ATOMIC POWER

FOREWORD. Public announcement of the production of atomic power in the form of electricity or mechanical power will be another milestone in the advance of our civilization. So far, the nuclear reactors which have been publicly described are for the conversion of fertile to fissionable material, for the production of radioactive isotopes, and for experimental purposes.

It has been announced that nuclear reactors to produce useful power are under development. The sections on power conversion systems and atomic power systems are written in anticipation of the successful completion of these developments.

Atomic Energy Act. The Atomic Energy Act of 1946 established the United States Atomic Energy Commission and set forth the law under which it operates. All information, critical materials, research, and development activities related to atomic power are controlled in the United States by the Atomic Energy Commission. The utilization of atomic power is also controlled by the Commission. Much of the information and the data necessary to designing atomic power plants has not yet been released to the public because of their military importance.

The information and data given here are from previously published sources. It is emphasized that the data on nuclear properties of materials are presented only for illustrative purposes. More accurate and complete data exist but have not yet been released.

Atomic Power Plant. An atomic power plant is an equipment for accomplishing the controlled release of atomic energy to produce useful mechanical or electrical power. The equipment includes a nuclear reactor (frequently called a pile) in which neutrons cause nuclear fissions. Heat energy from the fissions is extracted and used to drive suitable heat engines (usually turbines) to produce power.

1. NUCLEAR PROCESSES

THE FUNDAMENTAL PARTICLES AND RADIATION. In describing the workings of atomic power plants, it is useful to consider that all matter consists of various arrangements of a certain few fundamental particles. They are *electron*, *proton*, and *neutron*. Other particles are known or believed to exist, such as *positron*, *neutrino*, and *meson*, but they are not principally involved in the nuclear reactions resulting in useful power and will not be treated here. However, an element of energy, the *photon*, is involved in the relationships of these particles.

Electrons are the lightest known particles, having a mass when at rest, of 9.107×10^{-28} gram, and a negative electrical charge of 4.8025×10^{-10} electrostatic unit. They move with extremely high velocities, usually being constrained by electrostatic forces to move in orbits about other particles of opposite charge. Electrons ejected from nuclei are known as *beta rays*.

Protons are much heavier particles, having a positive electrical charge of the same magnitude as the negative charge of the electron. The mass of the proton is 1.67248×10^{-24} gram, being 1836 times greater than that of the electron.

Neutrons are particles slightly heavier than protons. Neutrons carry no electrical charge, and have a mass of 1.67472×10^{-24} gram. It is sometimes helpful to consider the neutron as a very closely joined proton and electron whose charges cancel. Neutrons in motion are not deflected by electrostatic forces and consequently penetrate matter much more readily than charged particles.

Photons are quanta of electromagnetic radiation having energy related to their frequency by the equation

$$E = h\nu \tag{1}$$

where E = energy of the photon, erg; h = Planck's constant, erg sec; ν = frequency of the photon, sec^{-1}.

In nuclear reactions, photons behave as particles traveling with the velocity of light. High-energy photons are known as *gamma rays*, and, like neutrons, are extremely penetrating.

Table 1. Properties of the Fundamental Particles

Particle	Charge, esu	Mass, Atomic Mass Units	Mass, Grams
Electron	-4.8025×10^{-10}	0.00054862	9.107×10^{-28}
Proton	$+4.8025 \times 10^{-10}$	1.007582	1.67248×10^{-24}
Neutron	0	1.00893	1.67472×10^{-24}
Photon	0	0.00107 *	1.762×10^{-27} *

* Equivalent mass of 1 Mev photon.

Table 2. Some Fundamental Units of Nuclear Physics

Ma
1 mass unit = 1.6604×10^{-24} gram
= 1/16 mass of $_8O^{16}$ (Oxygen, see Nuclear Structure, p. 17-04)
Charge:
1 electronic charge = 1.602×10^{-19} coulomb
= charge of one electron (−) or proton (+)
Energy:
1 electron volt = 1.602×10^{-12} erg
= kinetic energy of particle, having one electronic charge, when the particle has fallen freely through a potential drop of one volt
1 Mev = one million electron volts
Planck's constant:
$h = 6.624 \times 10^{-27}$ erg sec
Velocity of light:
$c = 2.9978 \times 10^{10}$ cm per sec

EQUIVALENCE OF MASS AND ENERGY. Mass and energy are equivalent and, under certain very special circumstances, convertible from one to the other. This fact was first confirmed by observation of radioactive substances. The direction of the conversion is such that the inertial mass of a moving body increases as its speed increases. The equivalence of mass and energy is stated by the very simple Einstein equation:

$$E = mc^2 \tag{2}$$

where E = energy, ergs; m = mass, grams; c = velocity of light, centimeters per second.

It may be seen from this that a small amount of mass is equivalent to a very great amount of energy. There are several processes which accomplish the conversion of mass to energy. The most important of them, nuclear fission, is the basis of atomic power.

Table 3. Conversion Table for Energy Units

Multiply Number of → by to Obtain ↓	Mev	mass units	ergs	g-cal	kw-hr	ft-lb	hp-hr
Mev	1	9.31×10^{2}	6.24×10^{5}	2.62×10^{13}	2.25×10^{19}	8.48×10^{12}	1.68×10^{19}
mass units	1.07×10^{-3}	1	6.71×10^{2}	2.81×10^{10}	2.41×10^{16}	9.08×10^{9}	1.80×10^{16}
ergs	1.60×10^{-6}	1.49×10^{-3}	1	4.19×10^{7}	3.60×10^{13}	1.36×10^{7}	2.68×10^{13}
g-cal	3.83×10^{-14}	3.56×10^{-11}	2.39×10^{-8}	1	8.60×10^{5}	3.24×10^{-1}	6.41×10^{5}
kw-hr	4.45×10^{-20}	4.15×10^{-17}	2.78×10^{-14}	1.16×10^{-6}	1	3.77×10^{-7}	7.46×10^{-1}
ft-lb	1.18×10^{-13}	1.10×10^{-10}	7.37×10^{-8}	3.09	2.65×10^{6}	1	1.98×10^{6}
hp-hr	5.96×10^{-20}	5.56×10^{-17}	3.72×10^{-14}	1.56×10^{-6}	1.34	5.05×10^{-7}	1

The increase in mass due to velocity, at relatively slow velocities (compared to the velocity of light), is according to the equation

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \qquad (3)$$

where m = mass of moving particle, grams; m_0 = rest mass of particle, grams; v = velocity of particle, centimeters per second; c = velocity of light, centimeters per second.

ATOMIC STRUCTURE. In matter the fundamental particles are arranged to form atoms. An atom consists of a number of closely grouped protons and neutrons surrounded by a number of electrons moving in orbits about the central nucleus of protons and neutrons.

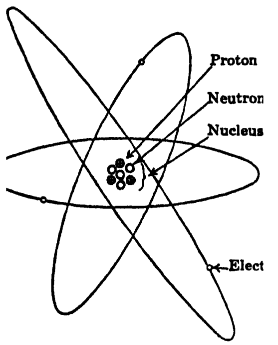


FIG. 1. Schematic representation of the Lithium atom ($_3Li^7$).

The nucleus typically has an approximate radius of 10^{-12} cm, while the electrons move in orbits averaging 10^{-8} cm in radius. There are equal numbers of electrons and protons in a stable atom, and it is electrically neutral. Some of the electrons may not be strongly held in the atom, and the near-by passing or direct collision of a charged particle or absorption of a photon may dislodge an electron, leaving the atom with a net positive charge. It is then said to be ionized.

The chemical behavior of an atom is determined by its number of electrons.

The simplest atom, hydrogen, consists of one proton and one electron. One of the most complicated atoms, Uranium^{238}, consists of 92 electrons, 92 protons, and 146 neutrons. Figure 1 is a schematic representation of the Lithium atom, which consists of 3 electrons, 3 protons, and 4 neutrons.

NUCLEAR STRUCTURE. The protons and neutrons of the nucleus are bound together by powerful forces to form a very dense and compact group. Under certain circumstances, however, these forces can be overcome and particles caused to leave the group.

The **atomic number,** Z, is the number of protons in the nucleus. Since this determines the number of electrons of the atom, it identifies the chemical properties of the atom. All atoms of a given atomic number have the same chemical properties, regardless of the structure of their nucleus.

The **mass number,** A, of an atom is the total number of protons and neutrons in its nucleus.

Isotopes are species of the same element having atoms of the same atomic number but different mass numbers.

The atomic number of an element is indicated by its name or symbol, whereas its mass number is indicated by a superscript following the symbol; for example, N^{14}. A more complete designation containing the atomic number as a prefixed subscript is sometimes used: $_7N^{14}$.

The actual mass of a nucleus is not the sum of the masses of the individual protons and neutrons which comprise it. Rather, the nucleus has somewhat less mass because the particles have given up some of their mass to create the *binding energy* of the nucleus.

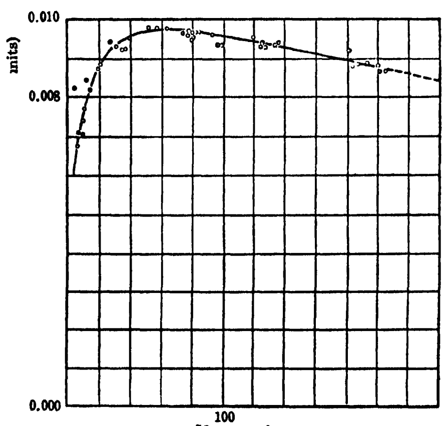


FIG. 2. Mass discrepancy per nuclear particle. (From Aston's data)

Figure 2 shows the discrepancy per nuclear particle between the measured mass of nuclei and the sum of the masses of the individual nuclear particles. The discrepancy is greatest for elements of intermediate mass number, and these elements have the most stable nuclei, since the binding energy per particle is greatest.

RADIOACTIVITY. Frequently substances occur with unstable nuclei which disintegrate spontaneously. The disintegration consists of the ejection of one or several particles from the nucleus, frequently accompanied by the emission of gamma-ray photons. Such substances are said to be *radioactive*. If the disintegration changes the number of protons in the nucleus (and consequently the number of orbital electrons), a new chemical element is formed. The new element may also be unstable and suffer further disintegration.

The particles emitted by the nuclei may be protons, neutrons, electrons (*beta rays*), or *alpha* particles. Alpha particles are helium nuclei, consisting of two protons and two neutrons. Some substances are naturally radioactive and may emit alpha (α), beta (β), or gamma (γ) rays. Many substances become artificially radioactive upon being bombarded by certain of the fundamental particles.

An example of a naturally radioactive series is

$$ {}_{92}\mathrm{UI}^{238} \xrightarrow[4.56 \times 10^9 y]{\alpha} {}_{90}\mathrm{UX}_1{}^{234} \xrightarrow[24.1d]{\beta} {}_{91}\mathrm{UX}_2{}^{234} \xrightarrow[1.14m]{\beta,\ \gamma} {}_{92}\mathrm{UII}^{234} $$

(The symbols y, d, h, m, and s are used to designate year, day, hour, minute, and second, respectively, in stating the half life of the disintegrating substance.)

An unstable nucleus has a definite and characteristic statistical probability of disintegrating within a given length of time. The number of disintegrations, ΔN, in an interval of time, Δt, is proportional to the total number of active nuclei N, the length of time, Δt, and the characteristic probability constant λ. Hence,

$$ \Delta N = -\lambda N \,\Delta t \tag{4} $$

The number N of nuclei remaining of an initial number N_0 after the lapse of time t is

$$ N = N_0 e^{-\lambda t} \tag{5} $$

It is usual to describe radioactive emitters by their ***half life***, $T_{1/2}$, the time necessary for half the initially present nuclei to disintegrate. In terms of the probability constant, this is

$$ T_{1/2} = \frac{0.693}{\lambda}\ \mathrm{sec} \tag{6} $$

In a radioactive series the decay of the original or parent substance and the growth and decay of the resulting new radioactive substance, the daughter, are described by these equations:

$$ N_1 = N_0 e^{-\lambda_1 t} \tag{7} $$

$$ N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t} - e^{-\lambda_2 t}] \tag{8} $$

Subscript 0 refers to the initial quantity of parent substance, subscript 1 to the parent substance, and subscript 2 to the daughter substance.

If the radioactive series is such that the daughter substance decays to form a stable substance, N_3, the process is described by the equation:

$$ N_3 = N_0 - N_1 - N_2 \tag{9} $$

or

$$ N_3 = N_0 \left[1 - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} \right] \tag{10} $$

Figure 3 represents graphically eqs. 7, 8, and 10 for a hypothetical radioactive series.

INTERACTION OF PARTICLES AND PHOTONS WITH MATTER. A particle or photon passing through matter will collide, sooner or later, with an atom. In an atomic power plant, an enormous number of collisions occur every second, causing a variety of interactions. The following are of particular interest.

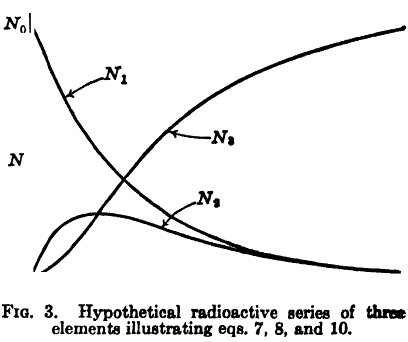


FIG. 3. Hypothetical radioactive series of three elements illustrating eqs. 7, 8, and 10.

Beta-ray Scattering and Absorption. A free electron moving at any considerable velocity will dislodge other electrons from their orbits as it passes through matter. In so doing, it gives up some of its energy, and after a number of collisions is slowed down and stopped. Typically, electrons ejected from radioactive nuclei will move a few feet in air or a few hundredths of an inch through aluminum before being stopped. The incident electron is deflected or scattered by its collisions, and follows a zigzag path, leaving behind it a trail of ionized particles.

Alpha Scattering and Absorption. Alpha particles emitted by radioactive substances are very quickly absorbed by matter. They create ions by dislodging electrons, and to extent are scattered by collisions with nuclei. Usually the coulomb forces acting between the positively charged nucleus and the positively charged alpha particle prevent their contact. The particle is deflected, and momentum is transferred to the nucleus in the manner of a billiard-ball collision. Only the most energetic can penetrate nuclei and be absorbed to form radioactive substances. Typically, alpha particles travel a few inches in air or a few mils in solids before being stopped.

Gamma-ray Reactions. Gamma-ray photons are absorbed in matter by collisions with atoms. The photon may give up all its energy to dislodging an electron from its orbit (the photoelectric effect), it may give up part of its energy to the electron and the remainder to the formation of a new, lower energy gamma-ray photon (the Compton effect), or it may collide with a nucleus, resulting in annihilation of the photon and formation of two new particles, an electron and a positron (pair production). The latter process is an interesting example of the conversion of energy into mass.

The absorption of gamma radiation by materials of different density varies almost directly with the density of material. There is a considerably more complex relationship between the absorption and the energy of the gamma radiation. Generally, high-energy gamma rays are more penetrating. If radiation of intensity I_0 strikes a layer of absorbing material, the intensity I at a distance x through the material is

$$I = I_0 e^{-\mu x} \tag{11}$$

where μ is the coefficient of absorption for the material for gamma radiation of that particular energy. The absorption coefficients for gamma radiation of energy 1.3 Mev in several materials are shown in Table 4.

Table 4. 1.3-Mev Gamma-ray Absorption Coefficients

Absorber	μ (cm^{-1})
Carbon	.115
Aluminum	.156
Iron	.455
Copper	.509
Tin	.345

Neutron Reactions. Neutrons in motion are not deflected by electronic or nuclear charge forces. When a neutron collides with a nucleus, one of several reactions may occur, and there is a definite probability of each, depending upon the material and the energy of the neutron.

The colliding neutron may undergo *elastic scattering*, in which it rebounds at a random angle, with conservation of energy and momentum of the system. It may undergo *inelastic scattering*, in which it rebounds after having transmitted a considerable amount of its energy to the nucleus, leaving the nucleus in an excited condition. Or, finally, the neutron may be *captured* by the nucleus, with one of the following results:

1. $(n - \alpha)$ This notation indicates capture of the neutron and instantaneous emission of an alpha particle from the nucleus.
2. $(n - p)$ Neutron capture—proton emission.
3. $(n - \gamma)$ Formation of a stable isotope.
4. $(n - \gamma)$ Formation of an unstable isotope, which subsequently decays by beta emission.

Nuclear Fission. If the nuclei of certain very heavy elements $-{}_{92}U^{233}$, ${}_{92}U^{235}$, or ${}_{94}Pu^{239}$ absorb neutrons, the nuclei will split approximately in half, releasing a large amount of energy. This process, known as *nuclear fission*, results in the formation of new nuclei (fission fragments) of roughly half the atomic weight of the original nuclei. The reaction also accomplishes the release of neutrons (averaging between 1 and 3) and subsequent emission of gamma-ray photons and electrons. Figure 4 represents a fundamental nuclear fission process.

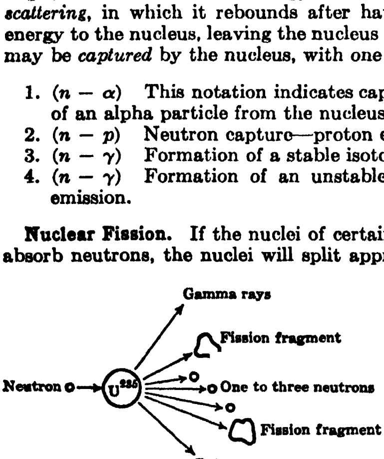


FIG. 4. Nuclear fission.

Occasionally one of the neutrons is not released at the instant of fission, but rather may be ejected from a fission fragment a short time later. These *delayed neutrons* play an important part in the controlled release of atomic energy.

When a great many fissions are taking place, the delayed neutrons appear in several groups according to the half lives of the radioactive disintegrations causing their appearance. Table 5 gives approximate half lives and percentages of the total number of neutrons (prompt + delayed) for delayed neutrons from fissions in U^{235}.

Table 5. Approximate Half Lives and Percentages of Delayed Neutrons

$T_{1/2}$, sec	%
55.6	.02
22.0	.14
4.51	.18
1.53	.20
0.42	.07
Total	.61

The fission fragments are elements of atomic mass ranging from 70 to 160. Many of them are radioactive. Their decay through a series of disintegrations to form stable nuclei may take months, or years.

A typical fission releases about 200 Mev of energy, which is distributed approximately as shown in Table 6.

Table 6. Approximate Distribution of Energy from Fission

Kinetic energy of fission fragments	160 Mev
Kinetic energy of neutrons	5
Prompt gamma radiation	5
Radioactive decay of fission fragments (mostly beta and gamma radiation)	20
Energy release due to ultimate absorption of neutrons from fission (neutron binding energy)	10
Total	200

NUCLEAR CROSS SECTIONS. The probability of a given type of nuclear reaction occurring is indicated by an equivalent cross-sectional area per nucleus, σ. For most substances this is a very small area, about 10^{-24} cm^2. *Cross sections* are frequently stated in *barns*, where 1 barn = 10^{-24} cm^2. The total cross section of a nucleus σ equals the sum of cross sections for the several nuclear reactions which can occur, such as σ_s (scattering), σ_c (capture), σ_f (fission).

$$\sigma = \sigma_s + \sigma_c + \sigma_f \tag{12}$$

In computing the progress of particles through matter, it is convenient to use the macroscopic cross section Σ, which is the total of the nuclear cross sections per unit volume, and has the dimension centimeter^{-1}.

$$\Sigma = \sigma N \tag{13}$$

where N is the number of nuclei per unit volume. The reciprocal of macroscopic cross section is the *mean free path* λ.

$$\frac{1}{\Sigma} = \lambda \tag{14}$$

This is the average distance the particle can travel through the substance before it suffers a collision of the type specified by the cross section.

The nuclear cross section bears little relationship to the actual projected area of the nucleus. Rather, it is thought of as the projected area of the region within which the short-range binding forces of the nucleus can influence the passing particle to collide. If a stream of particles, such as neutrons, penetrate material in which all collisions result in capture, the intensity I is related to the incident intensity I_0 (neutrons per cm^2 sec), and the distnce traveled, x cm, by

$$I = I_0 e^{-\Sigma_c x} = I_0 e^{-x/\lambda_c} \tag{15}$$

Cross sections vary with the velocity of the incident neutron. They are generally greater for low-velocity neutrons, and often are inversely proportional to neutron velocity.

$$\sigma \sim \frac{1}{v} \tag{16}$$

Frequently, however, the variation is erratic. There may be very high capture or fission cross sections at particular neutron velocities.

CHAIN REACTION. If a sufficient quantity of fissionable material is assembled, enormous amounts of energy can be released by means of a chain reaction. A neutron is absorbed, causing a fission producing several neutrons. They, in turn, are absorbed, causing fissions and producing still more neutrons. Thus the number of fissions may continue to increase tremendously. Figure 5 is a diagrammatic representation of such a chain.

There are always stray neutrons present in matter to initiate such chain reactions. They arise from cosmic-ray reactions or such radioactive effects as spontaneous fissions.

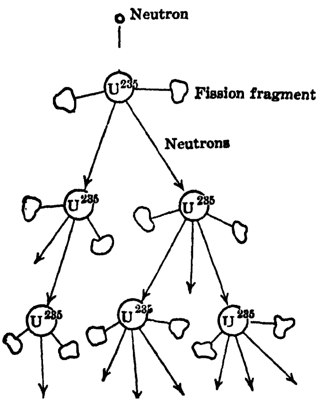


FIG. 5. Chain reaction.

Some of the neutrons may escape without being captured. Others may be captured without causing fission. Whether or not the chain reaction continues depends upon the competition among these processes: (1) escape, (2) nonfission capture, (3) fission. If the loss of neutrons due to the first two processes is less than the surplus from the third, the chain reaction will continue.

Table 7. Some Engineering Equivalents

1 watt	$\cong 3.12 \times 10^{10}$ U^{235} fissions/sec
1 gram U^{235} fissioned	$\cong$ 23,000 kwhr heat generated
Velocity of 1 Mev neutron	$\cong$ 8600 miles/sec
Velocity of thermal neutron	$\cong$ 1.36 miles/sec

NEUTRON DIFFUSION. In a chain-reacting system, neutrons diffuse through the material of the system much like a diffusing gas. Individual neutrons usually undergo many scattering collisions before finally being absorbed or escaping. Their direction of motion is random. Their energy varies from about 1 Mev to about $^1/_{40}$ electron volt. These least energetic neutrons are called *thermal neutrons* because they are in thermal equilibrium with the matter surrounding them.

In calculating diffusion of neutrons, the behavior of the average neutron is considered in each case, and the behavior of those deviating from the average is ignored. This method is accurate because there are always sufficient numbers of events to realize the statistical probabilities involved.

One-velocity Diffusion. The simplest diffusion theory assumes that all neutrons have the same velocity. This is a useful theory because in many nuclear reactors most of the neutrons are at thermal energies. In such reactors, it is reasonably accurate to ignore those neutrons having greater than thermal energies.

The continuity equation for neutrons being created, scattered isotropically, and absorbed is

$$\frac{\partial n}{\partial t} = q - \frac{n}{\tau} - \operatorname{div} \mathbf{j} \tag{17}$$

where n = instantaneous neutron density, q = rate of neutron creation, $\tau = 1/v\Sigma_a$ average lifetime of neutrons, t = time, v = neutron velocity, $D = v/3\Sigma_s$ = diffusion constant, $\mathbf{j} = -D \operatorname{grad} n$ = vector current of neutrons.

In a multiplying medium, creation = $k \times$ absorption, so for steady state $\left(\frac{\partial n}{\partial t} = 0\right)$:

$$\nabla^2 n + \frac{k-1}{L^2} n = 0 \tag{18}$$

$$\nabla^2 n = \frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + \frac{\partial^2 n}{\partial z^2} \quad \text{in rectangular coordinates}$$

$$L = \text{diffusion length}$$

$$L^2 = D = \frac{1}{3\Sigma_s\Sigma_a}$$

$$\Sigma_a = (\text{absorption}) = \Sigma_c + \Sigma_f$$

For a *point source* of neutrons in an infinite medium which scatters and captures but does not create ($\sigma_f = 0$), the distribution of neutrons is (in spherical coordinates):

$$n(r) \qquad \frac{Q\tau e^{-r/L}}{4\pi L r} \tag{19}$$

where Q is the strength of the point source.

For a *plane source* (in rectangular coordinates):

$$n(z) = \frac{Q\tau}{2L} e^{-|z|/L} \tag{20}$$

where Q is the strength of the plane source per unit area.

These expressions apply to homogeneous media which are not strongly absorbing, in regions remote from boundaries and sources.

Slowing Down of Neutrons. Neutrons emitted from fissions travel with great energy until they collide with a nucleus. If it is a scattering collision, the neutron loses some of its energy and travels at a lesser velocity until another collision occurs. This collision may take more energy away. Thus, if the neutron escapes absorption, it continues to lose energy through repeated scattering collisions, until it reaches thermal equilibrium with its surroundings. The energy loss per collision is greater when colliding with light nuclei. The *average logarithmic energy loss* per collision ξ is referred to, in computing the slowing down of neutrons:

$$\xi = \left[\ln \frac{E_{\text{before collision}}}{E_{\text{after collision}}}\right]_{\text{average}} \tag{21}$$

where E = neutron energy.

The relation between ξ and the mass of the nucleus M (in units of neutron mass) is

$$\xi = 1 - \frac{(M-1)^2}{2M} \ln \frac{M+1}{M-1} \tag{22}$$

(See Fig. 6.)

The *slowing down power* of a material, $\xi\Sigma_s$, is a measure of its effectiveness in reducing fast neutrons to thermal energies with a minimum number of collisions.

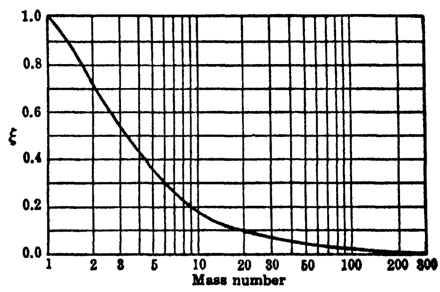


FIG. 6. Average logarithmic energy loss per collision.

Any one of these collisions may result in absorption of the neutron. The probability of absorption in a given material usually depends on the neutron energy. There are many so-called *resonance absorption bands*. These are ranges of incident neutron energy for which the absorption cross section is extremely large. For example, σ_a for U^{238} is very high at neutron energies of a few electron volts, and σ_f for U^{235} is high for thermal neutrons.

To improve the accuracy of the one velocity diffusion equation the diffusion length L may be modified to account for the diffusion of neutrons slowing down as well as their diffusion after reaching thermal energies:

$$L^2 = L_{st}^2 + L_{th}^2 \tag{23}$$

The square of the *slowing down length* L_{st}^2 is frequently called the *age*. It may be calculated if the variation of scattering cross section with neutron energy is known:

$$L_{st}^2 = \int_{E_{\text{thermal}}}^{E_{\text{fission}}} \frac{dE}{3(\Sigma_s)^2 \xi E} \tag{24}$$

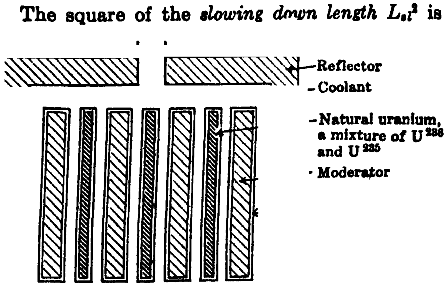


FIG. 7. Elements of a nuclear reactor.

2. NUCLEAR REACTORS

Nuclear reactors are assemblies of fissionable and other materials so arranged and in sufficient quantities as to be capable of supporting a chain reaction. They may be used to produce useful power and to create new fissionable material. The basic elements of a nuclear reactor are (1) fissionable material, (2) fertile material, (3) moderator, (4) coolant, (5) reflector, and (6) structural materials. (See Fig. 7 for a schematic representation of a nuclear reactor.)

FISSIONABLE AND FERTILE MATERIALS. The important *fissionable materials* are

Uranium ${}_{92}U^{235}$
Plutonium ${}_{94}Pu^{239}$
Uranium ${}_{92}U^{233}$

The important *fertile materials* are

Uranium ${}_{92}U^{238}$
Thorium ${}_{90}Th^{232}$

All of them are extremely heavy metals, quite active chemically, having relatively poor engineering properties. They may be used in nuclear reactors in metallic form, as oxides, carbides, or in alloys with other metals.

Natural uranium is a mixture of ${}_{92}U^{235}$ and ${}_{92}U^{238}$ in the proportion $\frac{U^{238}}{U^{235}} = \frac{139}{1}$.

U^{235} has a relatively large probability of absorbing neutrons to produce fissions. The fission cross section σ_f exists at all neutron energies but is greatest at thermal energies ($\sigma_f \cong 500$ barns).

U^{238} undergoes spontaneous fission at a very slow rate and also has a small probability of undergoing fission upon absorption of very high-energy neutrons. Neither of these fission reactions is of primary importance in maintaining chain reaction.

However, U^{238}, upon absorbing a low-energy neutron, undergoes a series of radioactive disintegrations leading to a new fissionable material. The following is a statement of the process:

$${}_{92}U^{238} + {}_0n^1 \rightarrow {}_{92}U^{239} \xrightarrow[23m]{\beta} {}_{93}Np^{239} \xrightarrow[2.3d]{\beta} {}_{94}Pu^{239}$$

Plutonium, ${}_{94}Pu^{239}$, is a fissionable material which disintegrates at a slow rate, by alpha emission. It is important that plutonium, being different chemically from U^{238}, may be separated by chemical means.

Thorium, ${}_{90}Th^{232}$, is another fertile material. Upon absorbing a neutron it decays to form the fissionable isotope of uranium, ${}_{92}U^{233}$. The following is a statement of the process:

$${}_{90}Th^{232} + {}_0n^1 \rightarrow {}_{90}Th^{233} \xrightarrow[23m]{\beta} {}_{91}Pa^{233} \xrightarrow[27d]{\beta} {}_{92}U^{233}$$

This uranium isotope may be chemically separated from the thorium.

Some nuclear reactors require a relatively pure concentration of fissionable material. In order to separate U^{235} from U^{238} for this purpose, several systems have been developed. The *gas diffusion separation* method utilizes the fact that the lighter isotope U^{235}, when part of a gas molecule, will penetrate a porous membrane more readily than the heavier isotope, U^{238}. The *electromagnetic separation* method employs the principle of the mass spectrometer to separate uranium isotopes. Both methods require extremely elaborate and expensive equipment. Other methods such as thermal diffusion and centrifuging have been developed to some small extent.

The chemical separations of Pu^{239} from U^{238} and of U^{233} from Th^{232} are more readily accomplished than the mechanical separation of U^{235} from U^{238}. Largely for this reason it is expected that the conversion of fertile material will be the more important method of producing concentrated fissionable material in the future.

MODERATOR. The probability of neutron collisions to produce fission is greater for thermal neutrons. For this reason a *moderator* is used in many nuclear reactors to slow down the neutrons to thermal energy. The moderator is a material of low atomic weight and very low-capture cross section. It is usually distributed through the reactor in such a way that high-energy neutrons emerging from fissions undergo numerous collisions within the moderator before encountering any fissionable or fertile material. The neutron loses more energy in colliding with a light nuclei than with a heavy one. Hence, in a moderator, fewer collisions are required to slow the neutron down to thermal energy. As a result, it has a much greater likelihood of escaping capture during the slowing process.

The most usual moderators are carbon, beryllium, or beryllium oxide, "heavy water" (D_2O) and ordinary water (H_2O). In a carbon-moderated reactor the neutrons undergo approximately 100 collisions in slowing down to thermal energies (for carbon $\xi = 0.15$).

COOLANT. In a nuclear reactor the energy which may be converted into useful power appears first in the form of kinetic energy of the fission fragments, neutrons, and beta and gamma radiation. Subsequent collisions of these particles with the material of the reactor very quickly convert the energy into thermal agitation of the atoms collided with. Thus the energy released by fissions must be extracted in the form of sensible heat.

Any of the methods of heat transfer may be used to accomplish this heat extraction, including radiation, conduction, free convection, and forced convection. Forced convection of a *coolant* fluid through channels in the nuclear reactor is most frequently employed. The coolant is heated upon passing through the reactor, and may leave at a relatively high temperature, permitting efficient utilization of the heat to drive a heat engine.

The coolant may have some effect on the chain reaction, since it may contribute to the scattering, slowing down, and capture of neutrons. The most desirable coolants have low-capture cross sections, good heat-transfer characteristics, and low tendency to corrode. Gases, such as air and helium, or liquids, such as water and mercury, may be used.

REFLECTOR. Some of the neutrons in a reactor may escape completely and hence make no contribution to continuing the chain reaction. To reduce this leakage loss, the reactor core may be surrounded with a reflector. This is a layer of material having a relatively large scattering cross section and small absorption cross section. Many neutrons leaving the core encounter scattering collisions in the reflector and are directed back into the core. These returned neutrons may cause fissions. Thus an assembly surrounded by a reflector requires less fissionable material to chain react.

Lead, carbon, and beryllium are typical reflector materials.

STRUCTURAL MATERIALS. Fissionable and fertile materials do not have sufficient strength and durability to form the mechanical structure of a reactor. Neither do many of the moderators and reflectors. For this reason it is necessary to employ some structural materials in the reactor. They may be used to support the fissionable material and other materials, to protect them from the corroding or eroding action of the coolant, and to provide means for manipulating them in controlling or refueling the reactor.

Structural materials must have adequate strength and durability and must have low absorption cross sections. Most organic materials and many other compounds are not suitable because of their inability to withstand the tremendous bombardment of neutrons and radiation in the reactor. Aluminum and iron have been used.

REACTIVITY. If a large number of fissions occur simultaneously in a typical nuclear reactor, the life history of the resulting group of neutrons is as follows.

A very few neutrons are absorbed quickly to cause *fast fissions*, slightly increasing the number of neutrons in the group. The neutrons are slowed down by predominately inelastic collisions while at high energies, and elastic collisions at lower energies. During the slowing down some of the neutrons suffer resonance capture in fertile material. After attaining thermal energy, the remaining neutrons continue to diffuse until they are absorbed to produce fissions or are captured by the impurities, structural materials, or other nonfissionable materials in the reactor. During the slowing down and thermal diffusion, some neutrons escape from the reactor. Those producing fissions cause a new group of neutrons, and the life cycle repeats.

The ratio of neutrons in the new generation to those in the old generation is called the *reactivity* or *multiplication constant, k*. For an infinite medium (no escape), the reactivity is

$$k = \epsilon p f \eta \tag{25}$$

where ϵ = fast fission multiplication factor, p = resonance escape probability, f = thermal utilization, η = number of neutrons released per neutron absorbed in U^{235}.

Fast fission multiplication, ϵ, depends on the manner in which the fissionable material is distributed in the reactor. In thermal reactors, the material is frequently arranged in lumps or rods surrounded by moderator. In them the fission neutrons very quickly encounter moderator and are slowed down before producing fast fissions. Therefore, ϵ is usually only very slightly greater than 1. A typical value is $\epsilon = 1.03$.

Resonance escape probability p, may be computed from data on the cross sections of materials in the reactor. It expresses numerically the likelihood of a fast neutron reaching thermal energy before being captured. It takes into account the variation of cross sections with energy of the neutrons, and the relative proportions of scattering nuclei (mostly moderator) and absorbing nuclei (mostly fertile material). For a homogeneous mixture of fertile material U^{238} and moderator, C,

$$p = e^{-\int_{E_{\text{thermal}}}^{E_{\text{fission}}} \frac{\Sigma_c(U^{238})}{\Sigma_c(U^{238}) + \Sigma_s(C)} \frac{dE}{\xi E}} \tag{26}$$

In this expression $\Sigma_c(U^{238})$ and $\Sigma_s(C)$ vary with neutron energy E in a manner which must be determined experimentally.

For typical ratios of moderator to fertile material (300 to 1000 atoms of carbon per atom of U^{238}) the resonance escape probability is of the order of 0.6 to 0.9.

Thermal utilization, f, is the probability that thermal neutrons will be absorbed to produce fissions. Competing with this are nonfission absorptions in fertile materials and other materials in the reactor. In a homogeneous mixture, the thermal utilization is simply the ratio of fission cross section to total absorption cross section:

$$f = \frac{\Sigma_f(U^{235})}{\Sigma_f(U^{235}) + \Sigma_a(U^{238}) + \Sigma_a(C + \text{other materials})} \tag{27}$$

Typically, the thermal utilization varies from 0.8 to 0.5.

Note that f increases when the relative proportion of moderator is decreased. The reverse is true of p. To obtain maximum reactivity a mixture must be selected to give the optimum balance between p and f.

Number of Neutrons per Neutron Absorbed in U^{235}. The number of neutrons released per neutron absorbed in U^{235}, η, is an inherent property of the fissionable material, and must be determined experimentally. On the average, η is between 1 and 3.

CRITICAL SIZE AND MASS. In an infinite medium, chain reaction will occur when $k = 1$. However, in a reactor of finite size, some neutrons escape from the surface. Hence, k must be somewhat greater than one to replace the neutrons lost by leakage. As the size of the reactor is increased, the ratio of surface to volume decreases, so that the relative effect of neutron leakage grows less. For every mixture having $k > 1$ there is a *critical size* at which the leakage is just compensated by the excess reactivity $(k - 1)$. This size may be computed from the diffusion equation:

$$\nabla^2 n + \frac{k-1}{L^2} n = 0 \tag{18}$$

When the boundary condition that $n = 0$ at the surface of the reactor is applied (a so-called bare reactor) the following expressions for critical size a and neutron distribution n obtain:

1. Cube having length of side a:

$$a_{\text{crit}} = \left[\frac{3\pi^2 L^2}{k-1}\right]^{1/2} \tag{28}$$

$$n = n_0 \sin\frac{\pi x}{a} \sin\frac{\pi y}{a} \sin\frac{\pi z}{a} \tag{29}$$

2. Sphere having radius a:

$$a_{\text{crit}} = \left[\frac{\pi^2 L^2}{k-1}\right]^{1/2} \tag{30}$$

$$\sin\frac{\pi r}{a} \tag{31}$$

3. Cylinder having radius a and length $2a$:

$$a_{\text{crit}} = \left[\frac{\left(\frac{1}{4}\pi^2 + 2.4\right)L^2}{k-1}\right]^{1/2} \tag{32}$$

$$n_0 J_(\left(2.4\frac{r}{a}\right) \cos\frac{\pi z}{a} \tag{33}$$

where n_0 = neutron density at center of reactor, J_0 = Bessel's function.

The boundary condition assumed in these solutions is slightly in error, and leads to values of a_{crit} which are a little too great. The curve of neutron distribution near a free surface (Fig. 8) is such that if it were extended it would reach zero at a point some distance outside the surface. This so-called *extrapolation distance* d is related to the properties of the material from which the neutrons are escaping:

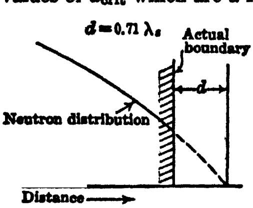


FIG. 8. Extrapolated end point of neutron distribution.

$$d = 0.71\lambda_s \tag{34}$$

where λ_s is the mean free path for scattering. This correction is readily applied to the values of critical dimension for bare reactors obtained from eqs. 28, 30, and 32.

In many practical reactor designs, k is only very slightly greater than one. It increases with concentration of fissionable material. It decreases with addition of fertile material, structural material, and other nonfission absorbers. Within limits k increases with the addition of moderator. However, as was pointed out, there is an optimum amount of moderator to give the most effective balance of p and f.

The reactivity, k, is decreased by diluting the reactor to provide space for the heat-removing coolant. Reductions of k make necessary increased reactor size and consequently increased mass of fissionable material.

If the reactor is surrounded by a reflector, fewer neutrons leak out, and the critical size is reduced. The saving depends on type and thickness of the reflector and size of the reactor.

REACTOR DYNAMICS. To control the release of energy in a nuclear reactor, it is necessary to control the rate at which fissions occur. This rate is proportional to the density of neutrons. Control is made possible by building the reactor to have somewhat greater excess reactivity than is necessary to make up for the leakage of neutrons. In steady-state operation this additional reactivity may be compensated by several methods, such as the introduction of absorbing material in the reactor or moving portions of the reflector to permit greater leakage.

When it is desired to increase the power output, some of the compensation is removed, permitting the number of neutrons to increase. When it is desired to decrease the power output, extra compensation is applied, and the number of neutrons decreases.

A typical neutron appears promptly at the instant of fission, undergoes slowing down collisions, and diffuses about until absorbed to produce another fission. It may experience as many as 200 collisions. The lifetime of this neutron is of the order of one or two thousandths of a second.

A few neutrons (about $^1/_2$ %) are delayed in appearing after the fission, so that their effective reproduction time may be very much longer than that of the prompt neutrons. These delayed neutrons have an important bearing on the rate of increase or decrease in neutron density.

When, in the operation of the reactor at steady state, a change in reactivity Δk is created by means of the controls, the neutron density will increase or decrease with time approximately according to the expression

$$n \cong n_0 e^{t/T} \tag{35}$$

where n = neutron density, n_0 = initial neutron density, T = reactor period related to Δk, and t = time after change in reactivity Δk.

The relationship between Δk and the period T is very approximately expressed as follows:

$$\Delta k \cong \frac{\tau}{T} + \frac{\beta \tau_d}{T + \tau_d} \tag{36}$$

where τ = average prompt neutron lifetime, T = reactor period, β = fraction of neutrons per generation which are delayed (approx. 0.005), τ_d = average time delay in appearance of delayed neutrons (approx. 10 sec).

From this equation it will be seen that when $\Delta k < \beta$ the reactor period T will be large, making it possible to actuate control mechanisms with enough speed to prevent a runaway condition. The physical explanation is that the significant effect of this small change in reactivity does not appear until the new delayed neutrons are released several seconds after the change was made.

On the other hand, if $\Delta k > \beta$ the reactor period will be very small, causing the neutron density to increase or decrease so rapidly as to make control extremely difficult. The reason for this is that when $\Delta k > \beta$, the reactor becomes chain reacting on the prompt neutrons alone. Since the generation time of prompt neutrons is very short, extremely rapid multiplication will ensue.

Because of thermal expansion of materials and changes of average neutron velocity with temperature, the reactivity will vary with temperature changes. The *temperature coefficient* C is the constant relating reactivity and temperature, $\Delta k = C\,\Delta$ temperature. For stable operation it must be zero or some negative value. A positive temperature coefficient may lead to an uncontrollable increase in reactivity. The value of temperature coefficient depends on the relative proportions of fuel, moderator, and other materials, and the manner in which heat is removed.

REACTOR TYPES. Reactors may be classified in several ways. One relates to the physical nature of the fuel (fissionable material), i.e., *solid fuel, liquid fuel, gaseous fuel.* Another relates to the manner in which the materials of the reactor are distributed, i.e., *homogeneous* or *heterogeneous.* Still another describes the energy of neutrons causing the fissions in the reactor, i.e., *thermal, fast,* or *intermediate* reactor.

Solid fuel reactors have fissionable material in the form of rods, plates, or lumps. It may be in concentrated form or it may be mixed or alloyed with fertile, moderator, or structural materials.

Liquid fuel reactors may have the fissionable material in the form of a salt such as uranyl sulfate dissolved in water (the moderator).

Gaseous fuel reactors are largely a matter of conjecture at present. The very dilute concentration of fissionable materials in such a reactor would make necessary a tremendous size to attain criticality.

Homogeneous reactors have an intimate and uniform mixture of fissionable, fertile, moderator, and structural materials. Certain liquid fuel reactors are of this type. The performance of such reactors is amenable to calculation, since the macroscopic cross sections are uniform throughout the reactor core.

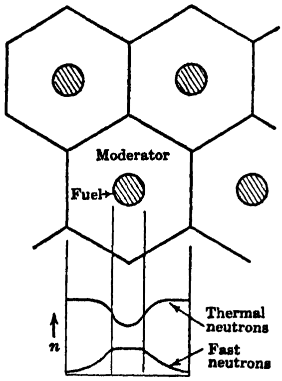


FIG. 9. Neutron distribution in a lattice cell.

In **heterogeneous reactors** the reactor materials are separated. In them the diffusion and absorption of neutrons are markedly affected by the varying scattering and absorption cross sections of the media encountered. For example, the fuel and fertile materials are frequently arranged in lumps embedded in a matrix of moderator.

The lumps and surrounding moderator in a series of *cells* form a *lattice* structure as shown in Fig. 9.

If fuel and fertile material are mixed in the lumps, as is the case in natural uranium, the lump emits fast neutrons from fissions and absorbs most of the thermal neutrons that penetrate it. Thus the density of fast neutrons is greatest at the lump and least in the moderator near the cell boundary. Conversely, the density of thermal neutrons is greatest near the cell boundary and least at the lump.

It is usual to calculate the reactivity of a single cell of the lattice structure, assuming no leakage of neutrons from the cell. If the reactivity of the cell can be determined, the critical size of the reactor may be calculated. This is done by assuming that the reactor is made up of a homogeneous medium having properties which would give the same k as was determined for the cell.

In **thermal reactors** almost all fissions are caused by neutrons at thermal energy. Most of the foregoing description pertains to reactors of this type.

In **fast reactors** most of the fissions are caused by fast neutrons. Such reactors contain no moderator. The fuel must be highly concentrated, and more fuel is required for criticality than in thermal reactors. This is because fission cross sections are very much smaller for fast neutrons than for thermal neutrons. Fast reactors lose many neutrons by leakage, and a reflector is necessary to reduce the amount of fuel required.

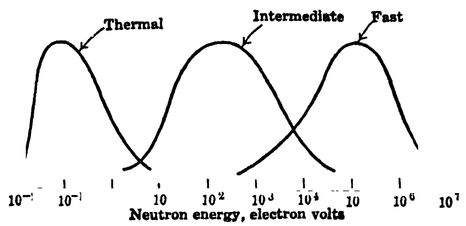


FIG. 10. Energy distribution of neutrons causing fissions in thermal, intermediate and fast reactors.

Intermediate reactors are reactors in which most of the fissions are caused by neutrons of energy somewhere between fast and thermal. Some moderator is used. The critical mass is less than that of a fast reactor and more than that of a thermal reactor having the same rating. Figure 10 compares the energy distribution in the three types of reactors.

REACTOR DESIGN. Reactors are designed according to the amount of power they are to produce and whether or not they are to *convert* fertile material. The desired power rating and maximum permissible temperatures determine the amount of heat transfer surface required to carry heat away from the fuel. This surface, in turn, fixes the volume necessary for the coolant. The resulting dilution of the structure, together with a consideration of the amount of moderator and structural material required, determine the macroscopic nuclear properties of the reactor. With these, the critical size and mass are determined.

Important structural features to be considered in reactor designs are these.

Fuel Structures. The fissionable material must be so arranged that it may be removed and replaced many times during the lifetime of the reactor. It must be protected from corrosion and erosion of the coolant stream. It must be contained so that radioactive fission fragments cannot leave the fuel and enter the coolant stream.

Frequently lumps or rods of fuel are jacketed with a metallic sheath. The coolant flows around the outside of the jacket to carry away the heat generated in the lump. To maintain effective heat removal, the jacket must be in intimate contact with the fuel.

Core structures must be designed to hold the fuel elements in position. Channels and manifolding must be provided for the coolant. The moderator must be supported. The structure must be capable of withstanding the terrific bombardment of neutrons which exists in a reactor. It must withstand thermal stresses, corrosion, and erosion. It must have adequate strength at the high temperature necessary for efficient utilization of heat evolved. Means must be provided for removing and replacing the fuel elements.

Fertile Material. If the fuel consists of natural or somewhat enriched uranium, the fertile material and fuel will be distributed together throughout the reactor core. In a reactor utilizing highly concentrated fuel, it may be advantageous to dispose the fertile material around the outside of the reactor core and reflector. In this way neutrons escaping through the reflector may be absorbed in the fertile material.

Control of the reactor may be accomplished by inserting and moving neutron absorbers. Boron-coated steel rods are frequently used. In a reactor where neutron leakage is an important loss, the control may be accomplished by moving sections of the reflector to permit more or less neutrons to escape. A third means of control is to move a portion of the fuel into or away from the reactor core.

In order to control a reactor it is necessary to know the instantaneous rate of power generation or the density of neutrons in the reactor. This measurement is frequently accomplished by detectors giving an output of electrical energy related to neutron density. One type of detector utilizes the reaction of a neutron penetrating a boron nucleus in a gas. The capture leads to emission of an energetic alpha particle which in turn creates ions in the gas. The charges are collected on electrodes so that the result of the neutron absorption is a pulse of electrical current. Such a device is called an *ion chamber*.

Suitable equipment must be provided to interpret measurements of neutron density and actuate control rods to maintain the desired power output.

Biological Shield. The core of a nuclear reactor emits a tremendous flux of neutrons, beta rays, and gamma rays. Exposure to such flux will quickly kill any living thing. For this reason the reactor must be surrounded by a *biological shield* which will absorb these radiations.

Biological shields may be constructed of many materials, but they must be selected so as to accomplish the absorption of all types of radiation emanating from the reactor. For example, lead, which effectively absorbs gamma radiation, scatters neutrons but does not absorb them strongly. Ordinary concrete is widely used as a biological shielding material. Typical concrete shields are several feet thick.

Radiation, particularly neutrons, has a great tendency to leak through holes or cracks in shielding; hence the shield must be designed to prevent this. Furthermore, it must be gastight because gas or air leaking out of the reactor may be dangerously radioactive.

It is not possible for persons to enter or reach into the reactor, because of the radioactivity. All operations, such as moving control rods or refueling, must be accomplished by remotely controlled mechanisms.

The reactor coolant is exposed to neutron flux in passing through the reactor. If it continuously recirculates, it may become dangerously radioactive, necessitating shielding the external machinery such as pipes, heat exchangers, or turbines through which it passes.

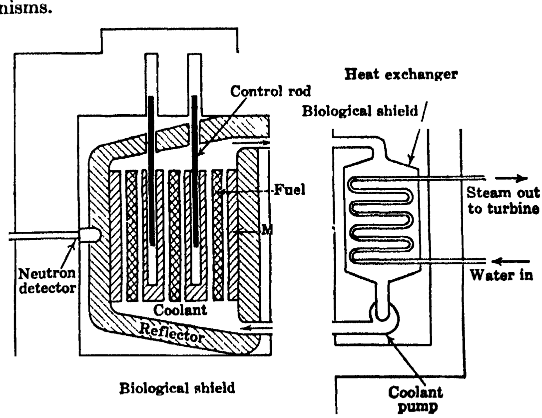


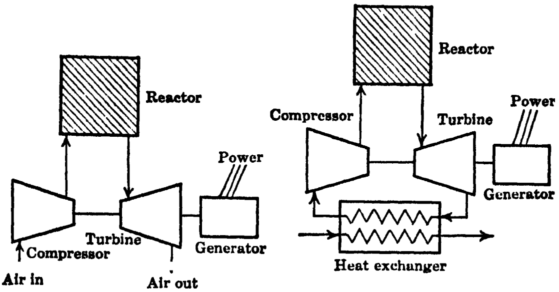
Fig. 11. Schematic representation of an atomic power plant.

3. POWER CONVERSION

The power-conversion system in an atomic power plant is used to convert heat from the nuclear reactor to electrical power or useful shaft work. This may be done by using the heated reactor coolant to drive a heat engine or by transferring heat from the reactor coolant to another fluid, used to drive the engine, as in Fig. 11.

An example of the first system is a gas-cooled reactor in which the coolant drives a gas turbine (Fig. 12*a*). Another possibility, involving considerable difficulties, is use of a liquid coolant, such as water or mercury, which is evaporated in the reactor, and used to drive a turbine (Fig. 12*b*).

An example of the second system is a gas or liquid metal cooled reactor in which the coolant passes through a heat exchanger to generate steam. The steam is used to drive a conventional steam turbine-generator set (Fig. 12*c*).



Open-cycle gas turbine Closed-cycle gas turbine

FIG. 12*a*. Power-conversion systems.

It does not appear likely that a direct conversion of atomic energy to useful power can be attained very soon because most of the energy released is in the form of kinetic energy of the fission fragments. These fragments are stopped quickly by collisions, converting the energy to heat. Even if it were possible to cause fissions in a gaseous media so that the fragments would travel farther, their direction would be perfectly random. All schemes to direct them so as to produce a net thrust or to generate electrical potential seem prohibitively inefficient.

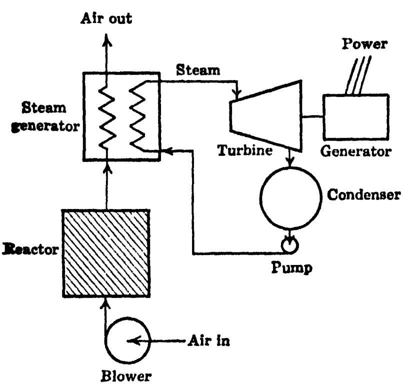


FIG. 12*b*. Open-cycle steam turbine power-conversion system.

4. ATOMIC POWER SYSTEMS

Accurate information as to the quantity of fissionable and fertile material in the world is not available for public release. Neither are data on the cost of preparing these materials for use. The cost of building and operating atomic power plants is also withheld. For this reason only very general estimates can be given about the economics and possible ultimate extent of atomic power.

Thorium and uranium are widely but thinly dispersed in the earth's crust. Unlike iron, copper, and many other metals, they do no occur in concentrated deposits. The processes for recovering and refining uranium and thorium are difficult and expensive. The amount of metals made available for atomic power plants in the future depends very greatly upon improving these processes. As they are improved, it will become economically feasible to mine and refine large amounts which are now too thinly distributed to recover.

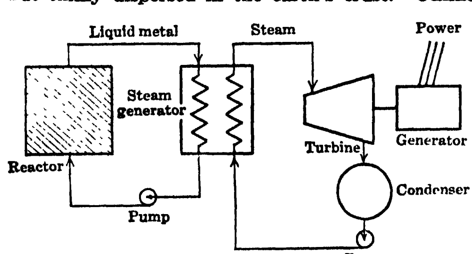


FIG. 12*c*. Closed-cycle steam turbine power-conversion system.

Generally, it can be said that the total energy available from complete utilization of presently mineable uranium and thorium in the world is at least of the same order of magnitude as the energy available in the world's reserves of coal and oil.

It is expected that the initial cost of atomic power plants will be greater than the cost of equivalent coal-burning power plants. The cost per kilowatt-hour produced is expected to be in the range of that produced by the more expensive grades of coal.

To utilize this tremendous store of energy, it will be necessary to operate reactors which not only create power but also produce more fissionable material than they consume. Such *primary reactors* might supply fuel to *secondary reactors*, which produce only power.

In addition to reactors and power-conversion equipment, *reprocessing plants* are required to separate unconsumed fissionable material from fission products. Fuel thus separated may be refabricated into fuel elements and returned to the nuclear reactor. Other reprocessing plants separate new fissionable material from partially converted fertile material and prepare both to be returned to reactors. (See Fig. 13.)

The material handled in these plants is intensely radioactive due to the large number of fission fragments contained. The work of dissolving the material and carrying out the chemical separations is done by remotely controlled apparatus behind biological shields. Elaborate and costly equipment is required. It is possible, however, for one reprocessing plant to serve several reactors, considerably reducing the reprocessing cost per reactor.

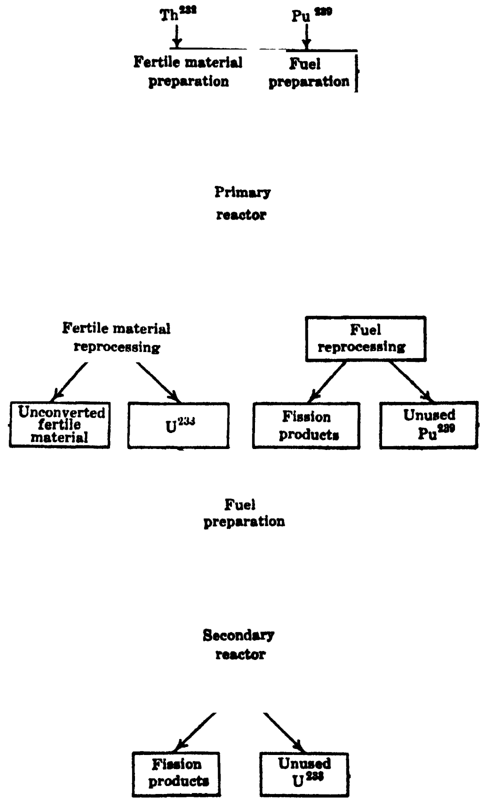


FIG. 13. Elements of an atomic power system using Plutonium and Thorium.

MOBILE POWER PLANTS. The size and weight of shielding required in atomic power plants at present limit their application. While size is not a serious handicap in land plants, it means that only the larger ships or aircraft can be considered feasible for atomic propulsion. Developments for these applications are being carried on. Even after the technical problems have been solved, it is likely that the high cost of atomic power plants will prohibit their use in smaller vehicles.

5. HEALTH PHYSICS

The measurement and control of radiation to protect the health of operating personnel are important in every phase of atomic power work. This highly developed science is called *health physics*.

Extremely small amounts of radiation are harmful, when repeated daily over long periods of time. Extremely small amounts of radioactive substances, such as uranium or plutonium, are deadly when taken into the body, as by breathing contaminated air or putting contaminated things in the mouth.

Very sensitive instruments have been developed to detect these hazards. Health physicists continually survey the air, earth, and water in the vicinity of atomic power plants and reprocessing plants, to determine whether or not they are becoming contaminated. The shielding of reactors and other equipment for handling radioactive materials is continually observed to discover and eliminate leakage of radiation. Persons working

Table 8. Some Nuclear Properties of the Elements *

(This table was compiled by Clark Goodman from data of Segre, Goldhaber, Briggs, Rainwater, Havens, and Bradbury and is reproduced from Volume I of *The Science and Engineering of Nuclear Power*, with the permission of the publisher, Addison-Wesley Press, Cambridge, Mass.)

Atomic Number, Z	Element	Density grams/cc	@ °C	Chem. Atomic Weight	Nuclei per cc $N \times 10^{24}$	Nuclear Cross Sections † in Barns σ	σ_c	σ_s	Cross Section † in cm^{-1} Σ_c	Σ_s
1	H	0.0709	−252.7	1.0078	.0424	20.	0.30	20.	0.013	.85
2	He	0.126	−268.9	4.003	.0190	1.5	0	1.5	0	.029
3	Li	0.53	20.	6.940	.0459	66.3	64.	2.	2.9	.092
4	Be	1.8	20.	9.02	.120	6.1	0.009	6.1	0.0011	.73
5	B	2.3	20.	10.82	.139	703.	700.	3.	97.	.42
6	C	1.62	20.	12.01	.0805	4.84	0.0045	4.8	0.00036	.39
7	N	0.808	−195.8	14.008	.0347	11.75	1.75	10.	0.061	.35
8	O	1.14	−183	16.000	.0429	4.1	0.0016	4.1	0.000068	.18
9	F	1.11	−187	19.00	.0351	4.1	0.01	4.1	0.00035	.14
10	Ne	0.9002		20.183	.0268	2.8		~2.5		.067
11	Na	0.93	97.5	22.997	.0243	4.	0.5	3.5	0.012	.085
12	Mg	1.57	650.	24.32	.0389	4.	0.4	3.6	0.016	.14
13	Al	2.4	658.	26.97	.0536	1.7	0.23	1.5	0.012	.080
14	Si	2.4	20.	28.06	.0515	1.95	0.25	1.7	0.013	.088
15	P	1.745	44.5	30.98	.0339	10.3	0.31	10.	0.01	.34
16	S	1.808	115.	32.06	.0339	2.	0.53	1.5	0.018	.051
17	Cl	1.557	−33.6	35.457	.0264	<43.	33.	~10.	0.87	~.26
18	A	1.402	−185.7	39.944	.0211	2.53	0.62	1.9	0.013	.040
19	K	0.83	62.	39.096	.0128	3.7	2.2	1.5	0.028	.019
20	Ca	1.55	20.	40.08	.0233	9.93	0.43	9.5	0.01	.22
21	Sc	2.5	20.	45.10	.0334		22.		0.74	
22	Ti	4.5	20.	47.90	.0566	11.2	5.2	6.	0.29	.34
23	V	5.68	20.	50.95	.0704	>13.	5.	>8.	0.35	>.56
24	Cr	7.14	20.	52.01	.0822	6.5	3.	4.	0.25	.33
25	Mn	7.2	20.	54.93	.0789	15.2	12.8	2.4	1.01	.19
26	Fe	6.9	1,530.	55.85	.0744	12.	2.5	11.	0.19	.82
27	Co	8.9	20.	58 94	.0909	38.	33.	5.	3.0	.45
28	Ni	8.90	20.	58.69	.0913	17.4	4.4	18.	0.40	1.6
29	Cu	8.3	1,083.	63.57	.0786	11.2	4.	8.	0.31	.63
30	Zn	6.7	463.	65.38	.0617	4.85	1.25	4.2	0.077	.26
31	Ga	5.91	20.	69.72	.0510	20.	2.3	18.	0.12	.92
32	Ge	5.36	20.	72.60	.0444	25.	~.6	22.	~0.03	.98
33	As	2.0	20.	74.91	.0161	12.6	5.6	7.	0.09	.11
34	Se	4.50	25.	78.96	.0343	28.	16.	10.	0.55	.34
35	Br	3.119	20.	79.916	.0235	9.5	7.	2.5	0.16	.06
36	Kr	(2)		83.7	.0144	27.	0.1	27.	0.001	.39
37	Rb	1.53	20.	85.48	.0104	12.	0.5	11.	0.005	.11
38	Sr	2.6	20.	87.63	.0179	11.	1.5	10.	0.027	.18
39	Y	5.51	20.	88.92	.0373		1.1		0.041	
40	Zr	6.4	20.	91.22	.0422	15.	>0.5	14.	>0.021	.59
41	Cb	8.57	20.	92.91	.0555	6.9	1.4	5.	0.078	.28
42	Mo	10.2	20.	95.95	.0638	7.9	3.9	6.5	0.25	.41
43	Tc			99.						
44	Ru	12.2	20.	101.7	.0502			6.		.30
45	Rh	12.5	20.	102.91	.0496	155.	150.	5.	7.4	.25
46	Pd	11.	1,550.	106.7	.0639	13.5	9.	4.5	.58	.29
47	Ag	10.5	20.	107.880	.0567	66.4	58.5	6.6	3.3	.37
48	Cd	8.6	20.	112.41	.0461	2,507.2	2,500.	5.3	115.	.24
49	In	7.3	20.	114.76	.0383	194.	190.	4.	7.3	.15
50	Sn	5.750	20.	118.70	.0292	4.89	0.69	5.0	0.020	.15

* In this table the gaseous elements are shown at such temperatures as to be liquid or solid so that their macroscopic cross sections may be compared with the other elements more readily. Some of the metals (Mg, Al, Fe, Cu, Zn) are shown at elevated temperatures such as might be encountered in nuclear reactors for power.

† For neutrons of approximately thermal energy.

Table 8. Some Nuclear Properties of the Elements *—*Continued*

Atomic Number, Z	Element	Density grams/cc	@ °C	Chem. Atomic Weight	Nuclei per cc $N \times 10^{24}$	Nuclear Cross Sections † in Barns			Cross Section † in cm^{-1}	
							σ_c	σ_s	Σ_c	Σ_s
51	Sb	6.684	25.	121.76	.0330	9.	4.7	4.2	0.155	.14
52	Te	(β)6.00	20.	127.61	.0283	10.	5.	5.	0.14	.14
53	I	4.93	20.	126.92	.0234	9.4	6.8	3.	0.16	.07
54	Xe	(2.7)		131.3	.0124	35.				
55	Cs	1.90	20.	132.91	.0086	50.	41.	9.	0.35	.08
56	Ba	3.5	20.	137.36	.0153	9.25	1.25	8.	0.019	.12
57	La	6.15	20.	138.92	.0267	25.	12.	13.	0.32	.35
58	Ce	6.9	20.	140.13	.0296	29.				
59	Pr	6.50	20.	140.92	.0278	>25.				
60	Nd	6.90	20.	144.27	.0288		90.		2.6	
61	Pm			147.						
62	Sm	7.7	20.	150.43	.0308		8,000.		246.	
63	Eu			152.0			2,500.			
64	Gd			156.9			~38,000.			
65	Tb			159.2		15.				
66	Dy			162.46			850.			
67	Ho			164.94			65.			
68	Er			167.2			260.			
69	Tm			169.4			130.			
70	Yb			173.04		65.				
71	Lu			174.99			195.			
72	Hf	11.4	20.	178.6	.0384	130.	120.	10.	4.6	.38
73	Ta	16.6	20.	180.88	.0552	24.5	20.	7.2	1.1	.40
74	W	19.3	20.	183.92	.0631	20.9	16.	5.7	1.01	.36
75	Re	20.	20.	186.31	.0691		130.		9.0	
76	Os	22.48	20.	190.2	.0707	30.	20.	10.	1.41	.71
77	Ir	22.4	20.	193.1	.0699		400.		27.9	
78	Pt	19.	1,755.	195.23	.0586	21.5	10.8	12.	.63	.70
79	Au	19.3	20.	197.2	.0589	101.	94.5	5.	5.57	.29
80	Hg	14.19	−38.9	200.61	.0426	440.	425.	15.	18.1	.64
81	Tl	11.85	20.	204.39	.0349	16.9	2.9	9.7	0.101	.34
82	Pb	11.34	20.	207.21	.0329	10.2	0.17	13.	0.006	.43
83	Bi	9.8	20.	209.00	.0282	8.9	0.016	9.2	0.0005	.26
84	Po			210.						
85	At			211.						
86	Rn	4.40	20.	222.	.0108					
87				223.						
88	Ra	5.0	20.	226.05	.0133					
89	Ac			227.						
90	Th	11.5	20.	232.12	.0290	16.	6.	~10.	0.17	~.29
91	Pa			231.						
92	U	18.7	20.	238.07	.0473	10.	~2.	8.2	0.095	.39
93	Np			237.						
94	Pu			239.						
95	Am			241.						
96	Cm			242.						

in these areas carry with them small radiation-detecting devices which indicate the occurrence of an inadvertent exposure.

One such detector is a small piece of sensitive photographic film in a light-tight container worn as a badge. These films are carried every day and periodically developed. Beta rays, gamma rays, and neutrons tend to expose the film. An estimate of the radiation received by the person can be made from the degree of exposure shown by the developed film.

The radioactivity of a source is measured in *curies.*

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ disintegrations/sec}$$

The damaging power of radiation is believed due to its creation of ions in living matter. The measure of quantity of gamma radiation is the roentgen, r. One roentgen of gamma radiation produces 1 esu of charge (on ions of either sign) per cu cm of air (NTP). It is widely accepted that the maximum permissible gamma-ray exposure per 8-hour day should be 0.1 r.

Other types of radiation, such as beta rays or neutrons, are measured in terms of the equivalent quantity of gamma radiation, rep (roentgen equivalent physical), required to produce the same ionization.

SECTION 18

INSTRUMENTATION

BY

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MEASUREMENT OF PROCESS VARIABLES

1. METHODS OF MEASURING TEMPERATURE

The thermodynamic temperature scale was defined by Kelvin as follows. The absolute values of two temperatures are to each other in the ratio of the heat supplied to the heat rejected in a reversible thermodynamic engine, working with a source and a refrigerator at the higher and lower temperatures, respectively. The interval between the freezing and boiling points of pure water is 100 C, giving the size of the unit degree. The scales in practical use are in general agreement with the thermodynamic scale or differ from it by small known quantities.

The fixed points of the International Temperature Scale are given in Table 1. Above 1945 F (1063 C) the scale is defined by a formula for the radiation of a black body. For all practical purposes one degree Kelvin is the same as one degree centigrade on the International Scale.

Table 1. Fixed Points of International Temperature Scale

Equilibrium at Standard Pressure of	Temperature °C	°F
Oxygen, liquid-gas	−182.97	−297.35
Ice and water	0.000	32.00
Steam and water	100.000	212.00
Sulfur, liquid-vapor	444.60	832.28
Silver, solid-liquid	960.8	1761.
Gold, solid-liquid	1063.	1945.

The temperature scales in industrial use are given in Table 2. The Kelvin scale is the centigrade absolute scale. The Rankine scale is the Fahrenheit absolute scale. Both scales assign 0 degrees as absolute zero.

Table 2. Common Temperature Scales

Name	Abbreviation	Water at Standard Conditions: Freezing Point	Boiling Point	Intervals
Fahrenheit	F	32.00	212.00	180
Centigrade	C	0.00	100.00	100
Kelvin	K	273.16	373.16	100
Rankine	R′	491.69	671.69	180
Réaumur	R	0.00	80.00	80

Conversion from one temperature scale to another is accomplished by these formulas: To convert Fahrenheit to centigrade:

$$T_F = (T_C + 40) \times \frac{9}{5} - 40 = 1.8(T_C + 40) - 40$$

To convert Fahrenheit to Rankine:

$$T_{R}' = T_F + 459.69$$

To convert centigrade to Fahrenheit:

$$T_C = (T_F + 40) \times \frac{5}{9} - 40 = 0.556(T_F + 40) - 40$$

To convert centigrade to Kelvin:

$$T_K = T_C + 273.16$$

where T_F = temperature in degrees Fahrenheit, T_C = temperature in degrees centigrade, T_R' = temperature in degrees Rankine, T_K = temperature in degrees Kelvin.

Conversion between Fahrenheit and centigrade scales is facilitated by Tables 3 and 4.

EXPANSION THERMOMETERS. Expansion thermometers operate on the principle

Table 3. Temperature Conversion—Centigrade to Fahrenheit

VALUES FOR INTERPOLATION IN TABLE

°C	1	2	3	4	5	6	7	8	9	10
°F	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4	16.2	18.0
Deg.→ C↓	0	10	20	30	40	50	60	70	80	90
					Degrees, Fahrenheit					
−200	−328	−346	−364	−382	−400	−418	−436	−454	.....	.....
−100	−148	−166	−184	−202	−220	−238	−256	−274	−292	−310
−0	+32	+14	−4	−22	−40	−58	−76	−94	−112	−130
+0	+32	+50	+68	+86	+104	+122	+140	+158	+176	+194
100	212	230	248	266	284	302	320	338	356	374
200	392	410	428	446	464	482	500	518	536	554
300	572	590	608	626	644	662	680	698	716	734
400	752	770	788	806	824	842	860	878	896	914
500	932	950	968	986	1004	1022	1040	1058	1076	1094
600	1112	1130	1148	1166	1184	1202	1220	1238	1256	1274
700	1292	1310	1328	1346	1364	1382	1400	1418	1436	1454
800	1472	1490	1508	1526	1544	1562	1580	1598	1616	1634
900	1652	1670	1688	1706	1724	1742	1760	1778	1796	1814
1000	1832	1850	1868	1886	1904	1922	1940	1958	1976	1994
1100	2012	2030	2048	2066	2084	2102	2120	2138	2156	2174
1200	2192	2210	2228	2246	2264	2282	2300	2318	2336	2354
1300	2372	2390	2408	2426	2444	2462	2480	2498	2516	2534
1400	2552	2570	2588	2606	2624	2642	2660	2678	2696	2714
1500	2732	2750	2768	2786	2804	2822	2840	2858	2876	2894
1600	2912	2930	2948	2966	2984	3002	3020	3038	3056	3074
1700	3092	3110	3128	3146	3164	3182	3200	3218	3236	3254
1800	3272	3290	3308	3326	3344	3362	3380	3398	3416	3434
1900	3452	3470	3488	3506	3524	3542	3560	3578	3596	3614
2000	3632	3650	3668	3686	3704	3722	3740	3758	3776	3794
2100	3812	3830	3848	3866	3884	3902	3920	3938	3956	3974
2200	3992	4010	4028	4046	4064	4082	4100	4118	4136	4154
2300	4172	4190	4208	4226	4244	4262	4280	4298	4316	4334
2400	4352	4370	4388	4406	4424	4442	4460	4478	4496	4514
2500	4532	4550	4568	4586	4604	4622	4640	4658	4676	4694
2600	4712	4730	4748	4766	4784	4802	4820	4838	4856	4874
2700	4892	4910	4928	4946	4964	4982	5000	5018	5036	5054
2800	5072	5090	5108	5126	5144	5162	5180	5198	5216	5234
2900	5252	5270	5288	5306	5324	5342	5360	5378	5396	5414
3000	5432	5450	5468	5486	5504	5522	5540	5558	5576	5594
3100	5612	5630	5648	5666	5684	5702	5720	5738	5756	5774
3200	5792	5810	5828	5846	5864	5882	5900	5918	5936	5954
3300	5972	5990	6008	6026	6044	6062	6080	6098	6116	6134
3400	6152	6170	6188	6206	6224	6242	6260	6278	6296	6314
3500	6332	6350	6368	6386	6404	6422	6440	6458	6476	6494
3600	6512	6530	6548	6566	6584	6602	6620	6638	6656	6674
3700	6692	6710	6728	6746	6764	6782	6800	6818	6836	6854
3800	6872	6890	6908	6926	6944	6962	6980	6998	7016	7034
3900	7052	7070	7088	7106	7124	7142	7160	7178	7196	7214

EXAMPLES: 1347 C = 1340 C + 7 C = 2444 F + 12.6 F = 2456.6 F
−194 C = −190 C + (−4 C) = −310 F + (−7.2 F) = −317.2 F
1852 F = 1850 F + 2 F = 1010 C + 1.11 C = 1011.11 C
−226 F = −220 F + (−6 F) = −134.44 C + (−3.33 C) = −137.77 C

Table 4. Temperature Conversion—Fahrenheit to Centigrade

VALUES FOR INTERPOLATION IN TABLE

°F	1	2	3	4	5	6	7	8	9	10
°C	0.555	1.111	1.666	2.222	2.777	3.333	3.888	4.444	5.000	5.555

Deg.→ F↓	0	10	20	30	40	50	60	70	80	90
	Degrees, Centigrade									
−400	−240.00	−245.56	−251.11	−256.67	−262.22	−267.78	−277.33	....	....	....
−300	−184.44	−190.00	−195.56	−201.11	−206.67	−212.22	−217.78	−223.23	−228.89	−234.44
−200	−128.89	−134.44	−140.00	−145.56	−151.11	−156.67	−162.22	−167.78	−173.33	−178.89
−100	−73.33	−78.89	−84.44	−90.00	−95.56	−101.11	−106.67	−112.22	−117.78	−123.33
−0	−17.78	−23.33	−28.89	−34.44	−40.00	−45.56	−51.11	−56.67	−62.22	−67.78
+0	−17.78	−12.22	−6.67	−1.11	+4.44	+10.00	+15.56	+21.11	+26.67	+32.22
100	37.78	43.33	48.89	54.44	60.00	65.56	71.11	76.67	82.22	87.78
200	93.33	98.89	104.44	110.00	115.56	121.11	126.67	132.22	137.78	143.33
300	148.89	154.44	160.00	165.56	171.11	176.67	182.22	187.78	193.33	198.89
400	204.44	210.00	215.56	221.11	226.67	232.22	237.78	243.33	248.89	254.44
500	260.00	265.56	271.11	276.67	282.22	287.78	293.33	298.89	304.44	310.00
600	315.56	321.11	326.67	332.22	337.78	343.33	348.89	354.44	360.00	365.56
700	371.11	376.67	382.22	387.78	393.33	398.89	404.44	410.00	415.56	421.11
800	426.67	432.22	437.78	443.33	448.89	454.44	460.00	465.56	471.11	476.67
900	482.22	487.78	493.33	498.89	504.44	510.00	515.56	521.11	526.67	532.22
1000	537.78	543.33	548.89	554.44	560.00	565.56	571.11	576.67	582.22	587.78
1100	593.33	598.89	604.44	610.00	615.56	621.11	626.67	632.22	637.78	643.33
1200	648.89	654.44	660.00	655.56	671.11	676.67	682.22	687.78	693.33	698.89
1300	704.44	710.00	715.56	721.11	726.67	732.22	737.78	743.33	748.89	754.44
1400	760.00	765.56	771.11	776.67	782.22	787.78	793.33	798.89	804.44	810.00
1500	815.56	821.11	826.67	832.22	837.78	843.33	848.89	854.44	860.00	865.56
1600	871.11	876.67	882.22	887.78	893.33	898.89	904.44	910.00	915.56	921.11
1700	926.67	932.22	937.78	943.33	948.89	954.44	960.00	965.56	971.11	976.67
1800	982.22	987.78	993.33	998.89	1004.4	1010.0	1015.6	1021.1	1026.7	1032.2
1900	1037.8	1043.3	1048.9	1054.4	1060.0	1065.6	1071.1	1076.7	1082.2	1087.8
2000	1093.3	1098.9	1104.4	1110.0	1115.6	1121.1	1126.7	1132.2	1137.8	1143.3
2100	1148.9	1154.4	1160.0	1165.6	1171.1	1176.7	1182.2	1187.8	1193.3	1198.9
2200	1204.4	1210.0	1215.6	1221.1	1226.7	1232.2	1237.8	1243.3	1248.9	1254.4
2300	1260.0	1265.6	1271.1	1276.7	1282.2	1287.8	1293.3	1298.9	1304.4	1310.0
2400	1315.6	1321.1	1326.7	1332.2	1337.8	1343.3	1348.9	1354.4	1360.0	1365.6
2500	1371.1	1376.7	1382.2	1387.8	1393.3	1398.9	1404.4	1410.0	1415.6	1421.1
2600	1426.7	1432.2	1437.8	1443.3	1448.9	1454.4	1460.0	1465.6	1471.1	1476.7
2700	1482.2	1487.8	1493.3	1498.9	1504.4	1510.0	1515.6	1521.1	1526.7	1532.2
2800	1537.8	1543.3	1548.9	1554.4	1560.0	1565.6	1571.1	1576.7	1582.2	1587.8
2900	1593.3	1598.9	1604.4	1610.0	1615.6	1621.1	1626.7	1632.2	1637.8	1643.3
3000	1648.9	1654.4	1660.0	1665.6	1671.1	1676.7	1682.2	1687.8	1693.3	1698.9
3100	1704.4	1710.0	1715.6	1721.1	1726.7	1732.2	1737.8	1743.3	1748.9	1754.4
3200	1760.0	1765.6	1771.1	1776.7	1782.2	1787.8	1793.3	1798.9	1804.4	1810.0
3300	1815.6	1821.1	1826.7	1832.2	1837.8	1843.3	1848.9	1854.4	1860.0	1865.6
3400	1871.1	1876.7	1882.2	1887.8	1893.3	1898.9	1904.4	1910.0	1915.6	1921.1
3500	1926.7	1932.2	1937.8	1943.3	1948.9	1954.4	1960.0	1965.6	1971.1	1976.7
3600	1982.2	1987.8	1993.3	1998.9	2004.4	2010.0	2015.6	2021.1	2026.7	2032.2
3700	2037.8	2043.3	2048.9	2054.4	2060.0	2065.6	2071.1	2076.7	2082.2	2087.8
3800	2093.3	2098.9	2104.4	2110.0	2115.6	2121.1	2126.7	2132.2	2137.8	2143.3
3900	2148.9	2154.4	2160.0	2165.6	2171.1	2176.7	2182.2	2187.8	2193.3	2198.9
4000	2204.4	2210.0	2215.6	2221.1	2226.7	2232.2	2237.8	2243.3	2248.9	2254.4
4100	2260.0	2265.6	2271.1	2276.7	2282.2	2287.8	2293.3	2298.9	2304.4	2310.0
4200	2315.6	2321.1	2326.7	2332.2	2337.8	2343.3	2348.9	2354.4	2360.0	2365.6
4300	2371.1	2376.7	2382.2	2387.8	2393.3	2398.9	2404.4	2410.0	2415.6	2421.1
4400	2426.7	2432.2	2437.8	2443.3	2448.9	2454.4	2460.0	2465.6	2471.1	2476.7
4500	2482.2	2487.8	2493.3	2498.9	2504.4	2510.0	2515.6	2521.1	2526.7	2532.2

Table 4. Temperature Conversion—Fahrenheit to Centigrade—*Continued*

VALUES FOR INTERPOLATION IN TABLE

°F	1	2	3	4	5	6	7	8	9	10
°C	0.555	1.111	1.666	2.222	2.777	3.333	3.888	4.444	5.000	5.555
Deg.→ F↓	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**
					Degrees, Centigrade					
4600	2537.8	2543.3	2548.9	2554.4	2560.0	2565.6	2571.1	2576.7	2582.2	2587.8
4700	2593.3	2598.9	2604.4	2610.0	2615.6	2621.1	2626.7	2632.2	2637.8	2643.3
4800	2648.9	2654.4	2660.0	2665.6	2671.1	2676.7	2682.2	2687.8	2693.3	2698.9
4900	2704.4	2710.0	2715.6	2721.1	2726.7	2732.2	2737.8	2743.3	2748.9	2754.4
5000	2760.0	2765.6	2771.1	2776.7	2782.2	2787.8	2793.3	2798.9	2804.4	2810.0
5100	2815.6	2821.1	2826.7	2832.2	2837.8	2843.3	2848.9	2854.4	2860.0	2865.6
5200	2871.1	2876.7	2882.2	2887.8	2893.3	2898.9	2904.4	2910.0	2915.6	2921.1
5300	2926.7	2932.2	2937.8	2943.3	2948.9	2954.4	2960.0	2965.6	2971.1	2976.7
5400	2982.2	2987.8	2993.3	2998.9	3004.4	3010.0	3015.6	3021.1	3026.7	3032.2
5500	3037.8	3043.3	3048.9	3054.4	3060.0	3065.6	3071.1	3076.7	3082.2	3087.8
5600	3093.3	3098.9	3104.4	3110.0	3115.6	3121.1	3126.7	3132.2	3137.8	3143.3
5700	3148.9	3154.4	3160.0	3165.6	3171.1	3176.7	3182.2	3187.8	3193.3	3198.9
5800	3204.4	3210.0	3215.6	3221.1	3226.7	3232.2	3237.8	3243.3	3248.9	3254.4
5900	3260.0	3265.6	3271.1	3276.7	3282.2	3287.8	3293.3	3298.9	3304.4	3310.0

that the expansion of solid, liquid, or gas is proportional to temperature. The types are liquid-in-glass thermometer, bimetallic expansion thermometer, and transmitting expansion thermometer. Ordinary industrial temperature ranges of these thermometers are given in Table 5.

Liquid-in-glass thermometer employs the thermal expansion of mercury or other liquid such as alcohol. The laboratory-type thermometer usually has a bare glass stem and is calibrated for full immersion of the mercury thread or for partial immersion. The industrial-type thermometer is contained in a metal casing, as shown in Fig. 1. They are available in a wide variety of styles and temperature ranges.

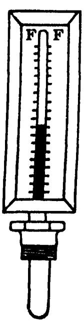


FIG. 1. Industrial mercury-in-glass thermometer.

Bimetallic thermometers employ the thermal expansion of a bimetal, i.e., two metals such as Invar, having a very low temperature coefficient of expansion, and brass, having a relatively high temperature coefficient of expansion, are welded together in a strip. Changes of temperature cause the bimetal strip to deflect. Bimetal thermometers ordinarily have a circular dial rather than a linear scale. They are available in a wide variety of styles and temperature ranges.

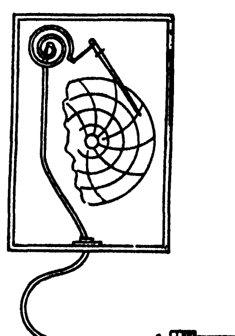


FIG. 2. Typical pressure thermometer.

Transmitting expansion thermometers employ the thermal expansion of a liquid, the pressure temperature relationship of a gas at substantially constant volume, or the vapor-pressure temperature relationship of a liquid. These three types are (1) liquid expansion, (2) gas expansion, and (3) vapor-actuated.

All types are constructed with a bulb connected by a length of metal capillary tubing to an instrument case as indicated in Fig. 2. A bourdon tube, spiral, or helix serves to

Table 5. Useful Temperature Range of Thermometers

Type	Lower Limit, °F	Upper Limit, °F
Liquid-in-glass		
Mercury	−40	950
Alcohol	−100	250
Bimetallic	−40	800
Transmitting-expansion		
Liquid (mercury)	−40	1000
Gas (nitrogen)	−200	800
Vapor *	−50	600

* Depends upon filling media. Full range not covered by any one filling medium.

convert the fluid pressure (expansion) into a motion for operating an indicating mechanism. The distance between bulb and instrument may be as great as 200 ft. The instrument may be:

1. Eccentric indicating, in which a pointer operates over a circular scale less than 180 geometrical degrees in extent.
2. Concentric indicating, in which a pointer operates over a circular scale more than 180 geometrical degrees in extent.
3. Recording, in which a pen operates on a circular recording chart.

The liquid-expansion thermometer nearly always employs mercury, the mercury completely filling the bulb, capillary, and bourdon tube. Often acetone, ethyl alcohol, ethyl ether, toluene, and water are used.

The gas-expansion thermometer nearly always uses nitrogen, but carbon dioxide and hydrogen are also used.

The vapor-actuated thermometer may use aniline, *n*-butane, ethyl alcohol, ethyl chloride, ethyl ether, *n*-hexane, methyl chloride, propane, sulfur dioxide, toluene, or water, depending upon the operating range desired.

IMPORTANT CHARACTERISTICS of the various transmitting expansion thermometers are noted in Table 6.

Table 6. Characteristics of Transmitting Expansion Thermometers

Characteristics	Mercury	Gas	Vapor
Scale shape (as used)	Linear	Linear	Nonlinear
Smallest span, °F approx.	100	160	80
Largest span, °F approx.	1000	800	350
Ambient effect	Yes	Yes	No
Cross-ambient effect	No	No	Yes
Head effect	No	No	Yes
Dip effect	Yes	No	No

A scale is linear if all temperature intervals are represented by equal incremental distances on the scale. The vapor-actuated thermometer does not have a linear scale since its shape depends upon the vapor-pressure versus temperature relationship of a liquid.

Span of a thermometer denotes the temperature interval between the lowest calibrated point and the highest calibrated point on the scale. The vapor-actuated thermometer can usually be made with the smallest span while the mercury thermometer can be made with the largest span.

Ambient effect in a transmitting thermometer is caused by variations in ambient temperature at or along the capillary or the bourdon tube. Ambient temperature changes do not affect the operation of the vapor-actuated thermometer because, by the nature of its operation, only the temperature which exists at the free surface of the liquid influences the vapor pressure in the system. This thermometer is constructed and filled in such a manner that the free surface of the liquid is always in the bulb. Ambient temperature changes affect the operation of the mercury- and gas-expansion thermometers. The mercury-expansion thermometer must usually be compensated either at the capillary or at the instrument case or both, depending upon the magnitude of ambient temperature changes, the span and operating temperature of the thermometer, and the length of capillary. Like the mercury-expansion thermometer, the gas-expansion thermometer sometimes requires compensation, but the ambient effect is not generally so large because the bulb volume and temperature span of the gas-expansion thermometer are usually large.

Cross-ambient effect exists only in the vapor-actuated thermometer and when the span of the thermometer is from below ambient to above ambient temperature. Then, when the measured temperature crosses the ambient temperature, a physical transfer of fluid between the "hotter" and the "colder" spaces in the thermometer system must take place, causing a serious delay in the operation of the thermometer.

Head effect exists only in the vapor-actuated thermometer and when the thermometer bulb is placed at a substantially higher or lower position (elevation) than that for which the instrument was calibrated. The head of liquid in the capillary adds to or subtracts from the vapor pressure in the thermometer system and thereby causes an error. This error is avoided by calibrating the thermometer with the bulb in the position in which it is to be used.

Dip effect exists mainly in the mercury thermometer and when the thermometer bulb is suddenly immersed in a fluid having a different temperature from the bulb. Because the metal casing of the bulb expands or contracts more quickly than the mercury, the thermometer may momentarily indicate a reversed change in temperature. This effect is not serious in the industrial use of the thermometer.

THERMOMETER WELLS. All expansion thermometers may be installed with a thermal well (as shown in Fig. 3) surrounding the thermometer bulb. Whenever possible, a thermometer bulb should be in direct contact with the substance whose temperature is being measured. When a thermometer well is necessary, all exposed parts of the well should be insulated, as should the wall of the pipe or other container for some distance on either side of the well, if such insulation will not affect the temperature of the fluid being measured. The well should be constructed so that it has the smallest possible metallic connection, consistent with strength, with the wall of the pipe or other container in order to reduce heat flow along the well. The well should have sufficient surface area to enable it to absorb heat from the substance being measured as rapidly as it is lost from the exposed end.

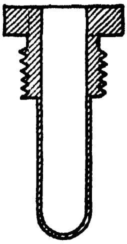


FIG. 3. Thermometer well.

Materials used in thermometer wells may be aluminum, brass, copper, glass, lead, nickel, plastic, rubber, stainless steel, or carbon steel, depending upon the fluid in which the well is immersed.

The response lag of a thermometer is nearly always increased to a considerable extent by the use of a thermal well. In many industrial applications the response lag may become an important factor.

ACCURACY OF EXPANSION THERMOMETERS. The accuracy of industrial liquid-in-glass and bimetallic thermometers is generally ±1.0% of span. The accuracy of industrial transmitting thermometers is generally ±0.5% of span. Thermometers of selected accuracy better than the figures given above can generally be obtained. The accuracy of any thermometer depends, among other factors, upon the conditions under which it is installed and used.

THERMOCOUPLE PYROMETERS. The thermocouple pyrometer consists essentially of a thermocouple of two different metals or alloys. The wires composing it are fused together at one end to form the measuring (hot) junction which is exposed to the temperature to be measured. The other ends are called the reference (cold) junction. The electromotive force, induced by the difference in temperature between the two ends, is proportional to the temperature difference and may be measured by an indicating instrument such as the millivoltmeter or potentiometer. These, together with leadwires connecting the thermocouple to the indicating instrument, complete the pyrometer. The thermocouple pyrometer is useful in measurement of temperatures between −300 F and 2800 F.

Thermocouples in most common industrial use are given in Table 7. The useful range given in Table 7 is for a bare thermocouple of the largest size wire. If a smaller size wire is used, the upper limit of temperature is appreciably reduced. If a thermal well is used, the upper temperature limit is sometimes increased. Platinum thermocouples are nearly always used with a gastight thermal well.

Temperature-emf data for thermocouples are given in Tables 8, 9, 10, 11, and 12. Nearly all thermocouples except a very few types from certain manufacturers are made to match these calibrations.

Table 7. Characteristics of Thermocouples

Thermocouple (Pos. Metal First)	Composition		Useful Temp. Range, °F	Melt. Temp., °F	Span, °F for 10 Mv	Average Accuracy, %
	Positive Metal	Negative Metal				
Chromel-alumel	90 Ni–9 Cr	97 Ni–3 Al	1000 to 2000	2570	440 at 1500	±3/4
Iron-constantan	100 Fe	55 Cu–44 Ni	0 to 1400	2210	325 at 700	±1
Copper-constantan	100 Cu	55 Cu–44 Ni	−300 to 600	1980	450 at 100	±3/4
Chromel-constantan	90 Ni–9 Cr	55 Cu–44 Ni	1000 to 2000	2210		±3/4
Pt 10% Rh-Platinum	90 Pt–10 Rh	100 Pt	0 to 2800	3220	1700 at 1400	±1/2
Pt 13% Rh-Platinum	87 Pt–13 Rh	100 Pt	0 to 2800	3200	1515 at 1400	±1/2

Special thermocouples have been used, mainly for the purpose of resisting corrosion and oxidation in particular applications. They are chromel versus KA2S, where KA2S is a metal similar to a stainless steel, iron versus alumel, and nickel versus nickel-molybdenum.

Platinum thermocouples are nearly always made from 24 B & S gage wire, 0.0201 in. in diameter. All other thermocouples are generally made of 8, 14, and 20 B & S gage wire.

Tube thermocouples are made of iron and constantan, with the iron in the form of a tube, the constantan wire running down the center of the tube. The purpose of this construction is to improve the speed of response by avoiding use of a thermal well. Tube thermocouples are generally $^1/_8$ in. outside diameter.

Accuracy of Thermocouples. The standard accuracy of thermocouples is given in Table 7. Special thermocouples selected for higher accuracy may be obtained at premium cost. These selected accuracies are: platinum couples—$\pm^1/_4$%; chromel-alumel couples—$\pm^3/_8$%; iron-constantan couples—$\pm^1/_2$%. In addition, thermocouples can be obtained with an error curve giving its deviation from standard calibration; some may be supplied with a National Bureau of Standards certification.

Table 8. Copper-constantan Thermocouple Emf *

(Cold Junction at 32 F)

°F	−200	−100	−0	+0	+100	+200	+300	+400	+500
				Millivolts					
0	−4.11	−2.56	−0.671	−0.671	1.52	3.97	6.64	9.52	12.57
10	−4.25	−2.73	−0.874	−0.465	1.75	4.22	6.92	9.82	12.88
20	−4.38	−2.90	−1.07	−0.255	1.99	4.48	7.21	10.12	13.20
30	−4.50	−3.06	−1.27	−0.043	2.23	4.75	7.49	10.42	13.52
40	−4.63	−3.22	−1.47	0.172	2.47	5.01	7.77	10.72	13.83
50	−4.75	−3.38	−1.66	0.389	2.71	5.28	8.06	11.03	14.15
60	−4.86	−3.53	−1.84	0.609	2.96	5.55	8.35	11.33	14.47
70	−4.97	−3.68	−2.03	0.832	3.21	5.82	8.64	11.64	14.79
80	−5.08	−3.83	−2.21	1.06	3.46	6.09	8.93	11.95	15.12
90	−5.18	−3.97	−2.39	1.29	3.71	6.37	9.23	12.26	15.44
100	−5.28	−4.11	−2.56	1.52	3.97	6.64	9.52	12.57	15.77
Mv per °F	0.0117	0.0155	0.0189	0.0219	0.0245	0.0268	0.0287	0.0305	0.0320

* Form of table by The Brown Instrument Company, Philadelphia, Pa. Data conform to Bureau of Standards specifications.

Table 9. Iron-constantan Thermocouple Data *

(Cold Junction at 32 F)

°F	0	100	200	300	400	500	600	700	800	900	1000
						Millivolts					
0	−0.922	1.96	4.92	7.95	11.02	14.09	17.16	20.23	23.31	26.41	29.54
10	−0.634	2.26	5.22	8.26	11.32	14.40	17.47	20.54	23.62	26.72	29.86
20	−0.346	2.55	5.53	8.56	11.63	14.70	17.77	20.85	23.93	27.04	30.18
30	−0.058	2.85	5.83	8.87	11.94	15.01	18.08	21.15	24.24	27.35	30.50
40	0.228	3.14	6.13	9.18	12.25	15.32	18.39	21.46	24.55	27.66	30.82
50	0.518	3.44	6.44	9.48	12.55	15.62	18.69	21.77	24.86	27.97	31.14
60	0.808	3.74	6.74	9.79	12.86	15.93	19.00	22.08	25.17	28.29	31.46
70	1.10	4.03	7.04	10.10	13.17	16.24	19.31	22.39	25.48	28.60	31.78
80	1.38	4.33	7.34	10.40	13.48	16.55	19.62	22.69	25.79	28.91	32.10
90	1.67	4.62	7.65	10.71	13.78	16.85	19.92	23.00	26.10	29.23	32.42
100	1.96	4.92	7.95	11.02	14.09	17.16	20.23	23.31	26.41	29.54	32.74
Mv per °F	0.0288	0.0296	0.0303	0.0307	0.0307	0.0307	0.0307	0.0308	0.0310	0.0313	0.0320

°F	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100
						Millivolts					
0	32.74	36.04	39.46	42.96	46.48	50.00	53.52	57.04	60.56	64.08	67.60
10	33.07	36.38	39.81	43.31	46.83	50.35	53.87	57.39	60.91	64.43	67.95
20	33.40	36.72	40.16	43.66	47.18	50.70	54.22	57.74	61.26	64.78	68.30
30	33.73	37.07	40.51	44.02	47.54	51.06	54.58	58.10	61.62	65.14	68.66
40	34.06	37.41	40.86	44.37	47.89	51.41	54.93	58.45	61.97	65.49	69.01
50	34.39	37.75	41.21	44.72	48.24	51.76	55.28	58.80	62.32	65.84	69.36
60	34.72	38.09	41.56	45.07	48.59	52.11	55.63	59.15	62.67	66.19	69.71
70	35.05	38.43	41.91	45.42	48.94	52.46	55.98	59.50	63.02	66.54	70.06
80	35.38	38.78	42.26	45.78	49.30	52.82	56.34	59.86	63.38	66.90	70.42
90	35.71	39.12	42.61	46.13	49.65	53.17	56.69	60.21	63.73	67.25	70.77
100	36.04	39.46	42.96	46.48	50.00	53.52	57.04	60.56	64.08	67.60	71.12
Mv per °F	0.0330	0.0342	0.0350	0.0352	0.0352	0.0352	0.0352	0.0352	0.0352	0.0352	0.0352

* Form of table by The Brown Instrument Company, Philadelphia, Pa. Data conform to specifications of most manufacturers. Bureau of Standards calibration is different, used in some government specifications.

Table 10. Chromel-alumel Thermocouple Emf *

(Cold Junction at 32 F)

°F	0	100	200	300	400	500	600	700	800	900	1000	1100	1200
	Millivolts												
0	−0.68	1.52	3.82	6.09	8.31	10.56	12.85	15.18	17.52	19.88	22.25	24.62	26.98
10	−0.47	1.74	4.05	6.31	8.53	10.79	13.08	15.41	17.75	20.12	22.49	24.85	27.21
20	−0.26	1.97	4.28	6.53	8.76	11.02	13.31	15.64	17.99	20.36	22.72	25.09	27.45
30	−0.04	2.20	4.51	6.75	8.98	11.25	13.55	15.88	18.22	20.59	22.96	25.33	27.68
40	0.18	2.43	4.74	6.98	9.20	11.47	13.78	16.11	18.46	20.83	23.20	25.57	27.92
50	0.40	2.66	4.97	7.20	9.43	11.70	14.01	16.35	18.70	21.07	23.43	25.80	28.15
60	0.62	2.89	5.19	7.42	9.66	11.93	14.24	16.58	18.93	21.30	23.67	26.04	28.39
70	0.84	3.12	5.42	7.64	9.88	12.16	14.48	16.82	19.17	21.54	23.91	26.27	28.62
80	1.06	3.36	5.64	7.87	10.11	12.39	14.71	17.05	19.41	21.78	24.14	28.51	28.86
90	1.29	3.59	5.87	8.09	10.33	12.62	14.94	17.29	19.64	22.01	24.38	26.74	29.09
100	1.52	3.82	6.09	8.31	10.56	12.85	15.18	17.52	19.88	22.25	24.62	26.98	29.33
Mv per °F	0.0220	0.0230	0.0227	0.0222	0.0225	0.0229	0.0233	0.0234	0.0236	0.0237	0.0237	0.0236	0.0235

°F	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400
	Millivolts											
0	29.33	31.65	33.94	36.20	38.43	40.62	42.77	44.89	46.97	49.01	51.00	52.95
10	29.56	31.88	34.17	36.42	38.65	40.83	42.98	45.10	47.18	49.21	51.20	53.14
20	29.79	32.11	34.40	36.65	38.87	41.05	43.20	45.31	47.38	49.41	51.39	53.33
30	30.02	32.34	34.62	36.87	39.09	41.27	43.41	45.52	47.59	49.61	51.59	53.52
40	30.26	32.57	34.85	37.10	39.31	41.48	43.62	45.73	47.79	49.81	51.78	53.71
50	30.49	32.80	35.08	37.32	39.53	41.70	43.83	45.93	47.99	50.01	51.98	53.90
60	30.72	33.03	35.30	37.54	39.75	41.91	44.04	46.14	48.20	50.21	52.17	54.09
70	30.96	33.26	35.53	37.76	39.96	42.13	44.26	46.35	48.40	50.41	52.37	54.28
80	31.19	33.49	35.75	37.99	40.18	42.34	44.47	46.56	48.61	50.61	52.56	54.47
90	31.42	33.71	35.98	38.21	40.40	42.56	44.68	46.76	48.81	50.80	52.75	54.66
100	31.65	33.94	36.20	38.43	40.62	42.77	44.89	46.97	49.01	51.00	52.95	54.85
Mv per °F	0.0232	0.0229	0.0226	0.0223	0.0219	0.0215	0.0212	0.0208	0.0204	0.0199	0.0195	0.0190

* Form of table by The Brown Instrument Company, Philadelphia, Pa. Data conform to Bureau of Standards specifications.

Thermocouple Leadwire. Thermocouples are calibrated for a certain reference-junction temperature. Any variation in this temperature will change the emf of the thermocouple and produce an error. Reference junction compensation is usually located at the millivoltmeter or potentiometer.

The hot junction of the thermocouple may be located several hundred feet from the instrument, and thermocouple leadwire is used to connect the thermocouple to the instrument. When cost is not prohibitive thermocouple leadwires are of the same materials as those composing the thermocouples.

Table 13 gives the thermocouple and leadwire combinations in common use. In Table 13 the first-named thermocouple wire is to be used with the first-named leadwire. Polarities of leadwires and thermocouples must be observed during installation.

Thermal Wells. In only a limited number of applications is it practical to expose the bare thermocouple to the fluid in which the temperature is being measured. Platinum thermocouples, especially, require careful protection against corrosion and contamination. Even with a thermal well, corrosion and oxidation may be so rapid that frequent replacement is necessary. The thermal well for a thermocouple is very similar in construction to the thermal well for a thermometer. Extra protection can be provided by using a secondary thermal well over the primary thermal well for the purpose of preventing sagging and making the assembly gastight.

Materials in common use for thermocouple protection are:

Metallic		Ceramic
Bronze	Iron, cast	Vycor
Monel	Iron, 28 chrome	Quartz
Steel	Nickel	Porcelain
Steel, calorized	Chromel	Firebrick
Iron, wrought	Chromax	Mullite
Iron, calorized	Nichrome	Sillimanite
Iron, 14 chrome	Inconel	Silica
Steel, stainless	Platinum	Silicon carbide

Table 11. Platinum-Pt 13% Rh Thermocouple Emf *

(Cold Junction at 32 F)

°F	0	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500
								Millivolts								
0	−0.0890	0.220	0.596	1.030	1.504	2.012	2.546	3.102	3.675	4.263	4.867	5.486	6.122	6.773	7.438	8.118
10	−0.0621	0.255	0.637	1.075	1.553	2.064	2.601	3.159	3.733	4.323	4.928	5.548	6.187	6.839	7.505	8.185
20	−0.0346	0.291	0.679	1.121	1.603	2.116	2.656	3.215	3.792	4.382	4.990	5.611	6.251	6.905	7.573	8.255
30	−0.0060	0.327	0.721	1.167	1.653	2.169	2.711	3.272	3.851	4.442	5.052	5.675	6.316	6.972	7.641	8.324
40	0.0244	0.364	0.764	1.214	1.703	2.223	2.767	3.329	3.910	4.502	5.114	5.738	6.381	7.038	7.708	8.394
50	0.0555	0.400	0.807	1.261	1.753	2.276	2.822	3.386	3.969	4.562	5.175	5.802	6.446	7.104	7.776	8.463
60	0.0871	0.439	0.851	1.309	1.804	2.330	2.878	3.444	4.028	4.623	5.237	5.866	6.511	7.170	7.845	8.533
70	0.119	0.477	0.895	1.357	1.856	2.383	2.934	3.501	4.087	4.684	5.299	5.930	6.577	7.237	7.913	8.602
80	0.152	0.516	0.939	1.405	1.907	2.438	2.990	3.559	4.145	4.745	5.361	5.994	6.642	7.303	7.981	8.672
90	0.186	0.556	0.984	1.454	1.959	2.492	3.046	3.617	4.204	4.806	5.423	6.058	6.707	7.371	8.049	8.741
100	0.220	0.596	1.030	1.504	2.012	2.546	3.102	3.675	4.263	4.867	5.486	6.122	6.773	7.438	8.118	8.811
Mv per °F	0.00323	0.00376	0.00434	0.00474	0.00508	0.00534	0.00556	0.00573	0.00588	0.00604	0.00619	0.00636	0.00651	0.00665	0.00680	0.00693

°F	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000
								Millivolts							
0	8.811	9.518	10.237	10.970	11.720	12.478	13.242	14.010	14.777	15.543	16.309	17.073	17.833	18.588	19.342
10	8.880	9.589	10.310	11.045	11.796	12.554	13.318	14.086	14.853	15.619	16.385	17.149	17.909	18.664	19.417
20	8.951	9.660	10.383	11.119	11.871	12.631	13.395	14.163	14.930	15.695	16.462	17.225	17.985	18.739	19.492
30	9.021	9.731	10.456	11.194	11.947	12.707	13.472	14.240	15.006	15.772	16.539	17.302	18.060	18.815	19.567
40	9.092	9.803	10.529	11.269	12.023	12.783	13.549	14.316	15.083	15.849	16.615	17.377	18.135	18.891	19.642
50	9.162	9.875	10.602	11.344	12.099	12.860	13.625	14.393	15.160	15.925	16.692	17.453	18.211	18.966	19.717
60	9.233	9.947	10.675	11.420	12.174	12.936	13.702	14.470	15.237	16.002	16.769	17.530	18.286	19.041	19.793
70	9.304	10.019	10.749	11.495	12.250	13.012	13.779	14.547	15.313	16.079	16.845	17.605	18.362	19.117	19.867
80	9.375	10.091	10.822	11.570	12.326	13.089	13.856	14.624	15.389	16.155	16.921	17.682	18.437	19.192	19.942
90	9.446	10.164	10.896	11.645	12.402	13.166	13.933	14.701	15.466	16.232	16.997	17.758	18.513	19.267	20.017
100	9.518	10.237	10.970	11.720	12.478	13.242	14.010	14.777	15.543	16.309	17.073	17.833	18.588	19.342	20.093
Mv per °F	0.00707	0.00719	0.00733	0.00750	0.00758	0.00764	0.00768	0.00767	0.00766	0.00766	0.00764	0.00760	0.00755	0.00754	0.00751

* Form of table by The Brown Instrument Company, Philadelphia, Pa. Data conform to Bureau of Standards specifications.

Table 12. Platinum-Pt 10% Rh Thermocouple Emf *

(Cold Junction at 32 F)

°F	0	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500
								Millivolts								
0	−0.0920	0.221	0.595	1.016	1.473	1.956	2.457	2.975	3.505	4.044	4.594	5.155	5.725	6.307	6.898	7.500
10	−0.0636	0.257	0.635	1.060	1.521	2.005	2.508	3.028	3.559	4.098	4.649	5.211	5.783	6.366	6.958	7.561
20	−0.0351	0.292	0.676	1.105	1.568	2.055	2.560	3.081	3.612	4.153	4.705	5.268	5.841	6.424	7.017	7.621
30	−0.0060	0.328	0.717	1.150	1.616	2.105	2.611	3.133	3.667	4.208	4.761	5.324	5.898	6.483	7.077	7.682
40	0.0243	0.365	0.758	1.196	1.664	2.155	2.663	3.186	3.720	4.263	4.817	5.381	5.956	6.542	7.137	7.744
50	0.0555	0.401	0.800	1.241	1.712	2.205	2.715	3.239	3.774	4.318	4.873	5.438	6.015	6.601	7.198	7.805
60	0.0875	0.439	0.843	1.287	1.760	2.255	2.767	3.293	3.828	4.373	4.929	5.495	6.073	6.660	7.258	7.866
70	0.120	0.477	0.886	1.333	1.808	2.305	2.819	3.346	3.882	4.428	4.985	5.553	6.132	6.720	7.318	7.928
80	0.153	0.516	0.929	1.380	1.857	2.356	2.871	3.399	3.936	4.483	5.042	5.610	6.190	6.779	7.379	7.989
90	0.187	0.555	0.972	1.426	1.906	2.406	2.923	3.452	3.990	4.539	5.098	5.667	6.249	6.838	7.439	8.051
100	0.221	0.595	1.016	1.473	1.956	2.457	2.975	3.505	4.044	4.594	5.155	5.725	6.307	6.898	7.500	8.112
Mv per °F	0.00325	0.00374	0.00421	0.00457	0.00483	0.00501	0.00518	0.00530	0.00539	0.00550	0.00561	0.00570	0.00582	0.00591	0.00602	0.00612

°F	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000	3100
								Millivolts								
0	8.112	8.734	9.365	10.007	10.657	11.316	11.977	12.642	13.305	13.968	14.629	15.288	15.943	16.596	17.247	17.892
10	8.174	8.796	9.429	10.071	10.723	11.382	12.043	12.708	13.372	14.034	14.695	15.353	16.009	16.661	17.311	17.957
20	8.236	8.859	9.493	10.136	10.789	11.448	12.110	12.775	13.438	14.100	14.761	15.418	16.074	16.726	17.376	18.021
30	8.298	8.922	9.557	10.201	10.855	11.514	12.177	12.841	13.505	14.166	14.826	15.484	16.139	16.791	17.440	......
40	8.360	8.895	9.621	10.266	10.920	11.580	12.243	12.907	13.571	14.233	14.892	15.550	16.205	16.856	17.505	......
50	8.422	9.048	9.685	10.331	10.986	11.646	12.310	12.974	13.637	14.299	14.958	15.615	16.270	16.922	17.570	......
60	8.484	9.111	9.749	10.396	11.052	11.712	12.376	13.040	13.703	14.365	15.024	15.680	16.335	16.987	17.634	......
70	8.546	9.175	9.813	10.461	11.118	11.778	12.442	13.107	13.770	14.431	15.090	15.746	16.401	17.052	17.699	......
80	8.609	9.238	9.877	10.526	11.184	11.844	12.509	13.173	13.836	14.497	15.156	15.812	16.466	17.117	17.763	......
90	8.671	9.302	9.942	10.591	11.250	11.911	12.575	13.239	13.902	14.563	15.222	15.878	16.531	17.182	17.828	......
100	8.734	9.365	10.007	10.657	11.316	11.977	12.642	13.305	13.968	14.629	15.288	15.943	16.596	17.247	17.892	......
Mv per °F	0.00622	0.00631	0.00642	0.00650	0.00659	0.00661	0.00665	0.00663	0.00663	0.00661	0.00659	0.00655	0.00653	0.00651	0.00645	....

* Form of table by The Brown Instrument Company, Philadelphia, Pa. Data conform to Bureau of Standards specifications.

The metals are listed in order of their temperature limit, bronze being the lowest, and platinum being the highest. The ceramics are also listed in order of their temperature limit.

A thermal well increases the response lag of the thermocouple, sometimes seriously. Careful attention should be given this matter during installation so that the most rapid heat transfer possible is effected. Radiation transfer of heat is important in nearly all thermocouple installations.

Table 13. Thermocouples and Leadwires

Thermocouple	Leadwire
Chromel-alumel	Chromel, alumel Copper, constantan Iron, copper-nickel alloy
Iron-constantan	Iron, constantan
Copper-constantan	Copper, constantan
Platinum-Pt 10% Rh Platinum-Pt 13% Rh	Cu-Ni alloy, copper

Millivoltmeter. The industrial millivoltmeter is a calibrated d-c galvanometer especially constructed for the measurement and indication of thermocouple emf. It is the simplest and least expensive of all instruments for use with thermocouples. The millivoltmeter is primarily an indicating instrument in which the pointer operates over a scale 5 or 6 in. long. Recording millivoltmeters are also in use but are becoming industrially less important.

Reference-junction temperature compensation is usually required in a millivoltmeter. Compensation is accomplished by a bimetallic strip which shifts the position of one of the hair springs controlling the millivoltmeter coil.

The accuracy of the millivoltmeter without a thermocouple is generally about $\pm 1\%$ of span.

Potentiometer. The automatically balancing potentiometer incorporates a null potentiometer circuit for automatic indication of thermocouple emf. The instrument is available in several styles:

1. Indicating-linear scale, in which a pointer operates over a linear scale 6 to 12 in. long.
2. Indicating-concentric scale, in which a pointer operates over a circular scale of about 300 geometrical degrees. In some types a circular dial is made to rotate past a fixed index.
3. Recording-strip chart, in which a pen operates on a strip chart approximately 10 in. wide.
4. Recording-circular chart, in which a pen operates on a circular recording chart.

Reference-junction compensation is accomplished automatically in the potentiometer by a nickel coil in the potentiometer circuit. Since the potentiometer circuit requires a battery to produce a standard or comparison emf, means must be provided for standardizing this source. Standardization is accomplished by comparing the battery emf to the emf from a standard cell. It may be done manually or, in most instruments, automatically. Automatic reference-junction compensation is generally desirable for industrial use of the instrument. The accuracy of the automatically balancing potentiometer is generally better than $\pm 0.25\%$ of span based on measurement of emf only. Potentiometers of selected accuracy can be obtained.

RESISTANCE THERMOMETERS. The resistance thermometer operates on the principle that the electrical resistance of a wire changes with temperature. Resistance of the thermometer bulb is measured by a Wheatstone bridge type instrument. The resistance thermometer is useful mainly in the measurement of temperatures between about -300 F and 1200 F.

The resistance-thermometer bulb consists essentially of a coil of fine wire wound on or in a frame of insulating material. Materials in common use for resistance-thermometer bulbs are given in Table 14.

Table 14. Characteristics of Resistance Bulbs

Metal	Resistance, ohms at 32 F	Useful Temperature Range, °F	Melting Temperature, °F
Platinum	10 to 35 *	−300 to 1200	3191
Copper	10	−40 to 250	2161
Nickel	100 to 300	−300 to 600	2646

* May sometimes be as high as 125 ohms.

Resistance-thermometer bulbs are generally used with a thermal well except when the temperature measurement is being made in dry air. Platinum bulbs may be fitted with a well of porcelain, brass, or stainless steel which is sometimes sealed. The thermal well for the resistance thermometer closely resembles those for thermocouples and expansion thermometer bulbs.

The accuracy of resistance-thermometer bulbs is better than the accuracy for either thermocouples or expansion thermometers. Standard industrial resistance-thermometer bulbs are generally accurate to ±0.25%, varying slightly for different temperature ranges. Resistance bulbs may be obtained of selected accuracy better than that given above.

Leadwire compensation for temperature variations is accomplished usually by the Siemens three-lead method for connecting the resistance bulb into the Wheatstone bridge circuit. Ordinary copper wires may be used to connect the resistance-thermometer bulb to the instrument. The distance between resistance-thermometer bulb and instrument may be almost any reasonable value.

The null-bridge resistance thermometer closely resembles physically the automatically balancing potentiometer but includes a Wheatstone bridge or similar bridge arranged for automatic balancing. However, a standard cell, standardization, and reference junction compensation are not required. The potential supply, either alternating or direct current, should be reasonably constant. The instrument may be obtained in either indicating or recording styles, as for the potentiometer. The accuracy of the null-bridge resistance thermometer is generally better than ±0.25% of span.

The deflectional resistance thermometer uses a circuit similar to the d-c Wheatstone bridge, except that there is no adjustable resistor for balancing the bridge. The instrument, a type of indicating millivoltmeter, is generally used for indicating such temperatures as are encountered in building installations and in air conditioning.

RADIATION METHODS. The intensity of the radiant energy emitted by a body is an indication of the temperature of that body. The intensity of radiation depends both upon the temperature of the body and upon the material composing it. If two similar hot bodies are at the same temperature, and the first is found to radiate energy at twice the rate of the second, the first is said to have twice the emissive power of the second. A material with the highest possible emissivity is known as a *black body*. Black-body conditions are closely approached by a uniformly heated hollow enclosure or by a small opening in a heated body. The numerical value 1.0 is usually applied to the emissivity of a black body, and all nonblack-body materials have an emissivity of less than 1.0 (see also Radiation, Section 3).

The radiation methods of measuring temperature are represented by the radiation pyrometer, the optical pyrometer, and the photoelectric pyrometer. The useful temperature ranges of these instruments are indicated in Table 15. Theoretically there is no upper temperature limit for radiation methods of measurement, but the values in Table 15 are those occurring in industrial use.

Table 15. Useful Temperature Ranges of Radiation Methods

Pyrometer Type	Lower Limit, °F	Upper Limit, °F
Radiation	200	3500
Optical	1000	5000
Photoelectric	1500	3000

The radiation pyrometer uses emission from a radiant body focused upon the hot junction of a small thermocouple in the pyrometer; the temperature to which this junction rises is approximately proportional to the rate at which the energy falls upon it. According to the Stefan-Boltzmann law, this temperature is proportional to the fourth power of the absolute temperature of the source of radiation. The electromotive force generated by this temperature rise is measured by a potentiometer, as in the thermocouple pyrometer, calibrated to form the relation between the emf and the temperature of the radiant body.

The effects which must be considered in the use of the radiation pyrometer are the distance effect, the emissivity effect, the absorbing media effect, and the reference-junction temperature effect.

Readings generally are independent of the distance of the instrument from the source of radiation, provided that the image more than covers the disk or black spot in the receiver. The radiation pyrometer tends to give a reading lower than the true reading as the sighting distance is increased or the size of the radiant body is decreased.

Radiation pyrometers are calibrated to read correctly when sighted on a black body. Most furnaces approximate black-body conditions. A material in a furnace at furnace temperature cannot be distinguished from the surroundings and may be considered a black body. If the coefficient of actual emissivity of the material is 0.45 its reflection coefficient will be 0.55, and the total emissivity of the body will be 1, or that of a black body. That is, 55% of the energy radiated from the body will be reflected from the surrounding walls which are at the same temperature. If, however, the material is removed from a furnace it will then emit only 45% of the radiant energy of a black body at the same temperature, and the radiation pyrometer will therefore read too low.

Materials in the open should have a correction applied to the observed temperature to convert it to the true temperature. Table 16 shows a corresponding true and apparent temperature observed with a radiation pyrometer sighted on materials in the open.

Table 16. True Temperature Corresponding to Apparent Temperature Measured by Radiation Pyrometers When Sighted upon Materials in the Open

(National Bureau of Standards, *Technologic Paper* 170)

Observed Temperature, °F	True Temperature, °F				
	Molten Iron	Molten Copper	Copper Oxide	Iron Oxide	Nickel Oxide
1110	....	2065	1330	1165	1310
1200	....	2210	1425	....	1390
1290	....	2355	1525	1355	1470
1380	....	....	1635	....	1555
1470	2190	....	1735	1545	1645
1560	2320	....	1830	... .	1725
1650	2445	....	1940	1735	1805
1740	2570	....	2040	....	1885
1830	2685	....	2140	1920	1965
1920	2820	....	....	....	2050
2010	2930	....	....	2110	2130
2100	3055	....	....	....	2210
2190	3180	....	....	2300	2290

Such media as lenses, glass windows, smoke, dirt, and gases between the source and the radiation receiver cause the instrument to read low. Hot gases, flame, and high-temperature carbon particles may cause the instrument to read high. If such conditions prevail it may be necessary to use a target tube with the radiation receiver. The target tube is a closed-end tube of metal or ceramic installed in the side of a furnace. The radiation receiver is sighted into the tube where black-body conditions are closely approached. Target tube materials are silicon carbide, sillimanite, inconel, nickel, chrome nickel, wrought iron, and steel.

The reference junction of the thermocouple is generally located in the radiation receiver. For this reason the housing of the radiation receiver should be maintained at substantially constant temperature. In some types automatic reference-junction compensation is supplied.

The accuracy of a radiation pyrometer calibrated under black-body conditions is usually better than ±0.5%. The accuracy under actual use depends considerably upon the details of the installation. The response of a radiation receiver is extremely fast and is one of the important advantages of this means of temperature measurement.

The industrial millivoltmeter or automatically balanced potentiometer instrument is used to measure the emf developed by the radiation receiver. The radiation receiver may be connected to the instrument with ordinary copper leadwire because both the hot and reference junctions of the thermocouple are located in the radiation receiving unit.

Optical pyrometers measure the energy radiated from incandescent bodies and utilize only that portion of the total energy in a narrow band near the visible red. In most types radiation from the target surface is focused by the lens onto a screen. The screen is viewed through a red filter glass so that only wavelengths of about 0.65 micron are seen. For comparison the radiation from a calibrated tungsten lamp is focused on a screen and also viewed through a red filter glass. The two screens are then compared by eye. There are two methods for varying the brightness of the screen images. First, the current through the standard lamp may be adjusted and the brightness of the standard screen matched against the brightness of the screen illuminated by radiation from the target surface. The current to the standard lamp may be measured and a scale calibrated in terms of temperature. Second, the intensity of the radiation from the target surface may be varied by using wedges of absorbing material. The wedge is moved to match the images on the two screens and a scale is arranged for indication of temperature.

Optical pyrometers are calibrated to read correctly when sighted on a black body. Bodies heated in a furnace approximate black-body conditions, and the temperature measurement is not seriously in error. Such error as exists will give a reading higher than the true reading if the furnace walls are brighter than the target surface, and lower than the true reading if the target surface is brighter than the walls. The pyrometer should be sighted into a deep wedge-shaped cavity or hole in the target surface.

The optical pyrometer sighted on glowing material in the open reads too low. Table 17 gives the corrections for various industrial materials when measured by optical pyrometers. The optical pyrometer will not give satisfactory readings when sighted through flames or smoke.

Table 17. True Temperature Corresponding to Observed Temperature Measured by Optical Pyrometers Using Red Light When Sighted on Materials in the Open

Observed Temperature, °F	True Temperature, °F						
	Molten Copper	Molten Iron	Solid Iron Oxide	Solid Nickel Oxide	Nichrome or Chromel	Molten Slag	Bright Platinum
1290	....	....	1290	1295	1295	....	1380
1470	....	....	1475	1475	1480	....	1580
1650	....	....	1655	1660	1660	....	1780
1740	1990	....	1745	1750	1755	....	1885
1830	2100	....	1840	1845	1850	....	1980
1920	2215	...	1930	1935	1945	....	2090
2010	2330	2160	2020	2030	2040	....	2195
2100	2445	2260	2115	2125	2140	....	2300
2190	2560	2365	2210	2220	2235	....	2405
2280	2680	2470	....	2310	....	....	2505
2370	2800	2570	....	2410	....	....	2615
2550	....	2775	...	....	....	2650	2830
2730	....	2985	....	....	....	2850	3045
2910	....	3195	....	....	....	3040	....
3090	....	3410	....	....	....	3235	....
3180	....	3515	....	....	....	3325	....

The accuracy of most optical pyrometers is very high; temperatures can be determined to within ±5.0 F. Optical pyrometers are manually operated instruments.

Photoelectric pyrometers are employed mainly where a simple, fast method of temperature measurement is desired. Some types use a photovoltaic cell and a millivoltmeter or potentiometer to measure the generated emf. Other types employ a photoemissive cell with an amplifying circuit, together with a comparison system based on a standard lamp as in an optical pyrometer. For further detail, consult one of the references at the end of this chapter (p. 18-22), particularly Rhodes or Weber.

2. METHODS OF MEASURING PRESSURE

The unit of pressure for nearly all engineering work is pounds per square inch.

There are three scales used for expression of pressure units: absolute pressure, gage pressure, and vacuum. Gage pressure is measured as the pressure above the atmospheric pressure existing at the time and place of measurement. Atmospheric or barometric pressure referred to standard conditions can be found in Table 18 in the vertical column under "atm." Shown also are pressure-conversion factors.

Absolute pressure is related to gage pressure and vacuum by

$$P_a = P_g + B = B - V$$

where P_a = absolute pressure, P_g = gage pressure, V = vacuum, B = barometric pressure, all expressed in the same units.

Differential pressure is a difference in pressure and is therefore independent of the particular scale used. Low absolute pressures (high vacuums) are generally expressed in microns, that is, in microns of mercury column where a micron is one-thousandth of a millimeter.

Measurement of pressure in terms of water or mercury column height must contain a specification of temperature, either implicitly or explicitly, since the height of a liquid column depends upon the density or specific gravity of the liquid which changes with temperature. The corrections are small as is shown in Table 19.

LIQUID MANOMETERS. The common types of liquid manometers are the U-tube manometer, the well manometer, the inclined tube manometer, and the ring manometer, all shown in Fig. 4. All manometers with one leg open to atmosphere measure gage pressure. For a U-tube manometer,

$$P_a \text{ (psia)} = \frac{hD}{1728} + B \text{ (psia)}$$

where h = the difference in height of columns, inches; D = density of manometer liquid, pounds per cubic foot at ambient temperature; B = barometric pressure. In a well-type manometer, one leg is enlarged so that the change of liquid level in the well or cistern is so small as to be negligible in all except precision measurements. In this case the same relation as above exists beween head and pressure.

Table 18. Conversion of Pressure Units *

Multiply Number of → by to Obtain ↓	psi	psf	Kg/sq cm	atm	in. H_2O, 4 C	ft H_2O, 4 C	in. Hg, 0 C	mm Hg, 0 C
psi		0.006944	14.22	14.70	0.03613	0.4336	0.4912	0.01934
psf	144.0		2048.	2117.	5.204	62.45	70.73	2.785
Kg/sq cm	0.07031	4.882 $\times 10^{-4}$		1.033	0.002540	0.03048	0.03453	0.001360
atm	0.06804	4.725 $\times 10^{-4}$	0.9678		0.002458	0.02950	0.03342	1.316 $\times 10^{-3}$
in. H_2O, 4 C	27.68	0.1922	393.7	406.8		12.	13.60	0.5354
ft H_2O, 4 C	2.307	0.01602	32.81	33.90	0.08333		1.133	0.04460
in. Hg, 0 C	2.036	0.01414	28.96	29.92	0.07355	0.8826		0.03937
mm Hg, 0 C	51.71	0.3591	735.6	760.0	1.868	22.42	25.30	

* Form of table after Eshbach's *Handbook of Engineering Fundamentals*, John Wiley and Sons.

See also in this book: (*a*) Conversion Table for Air Pressures, Table 7, p. 1-09. (*b*) Data in Table 6, p. 1-09, for pressure, temperature, and density variation with altitude of the NACA standard atmosphere. Additional data are given in Section 15 up to 100,000 feet altitude.

Temperature corrections for barometers and mercury columns, elevation corrections for barometers and pressure gages, correction of barometers to standard gravity, and other similar data are given in Section 19.

Conversion data for the metric equivalents of pressure in English units are given in Section 20.

Table 19. Pressure-column Temperature Correction

	Temperature			
Column of	0 C 32 F	4 C 39 F	15.5 C 60 F	20 C 68 F
H_2O	1.0001	1.0000	1.0010	1.0018
Hg	1.0000	1.0007	1.0028	1.0036

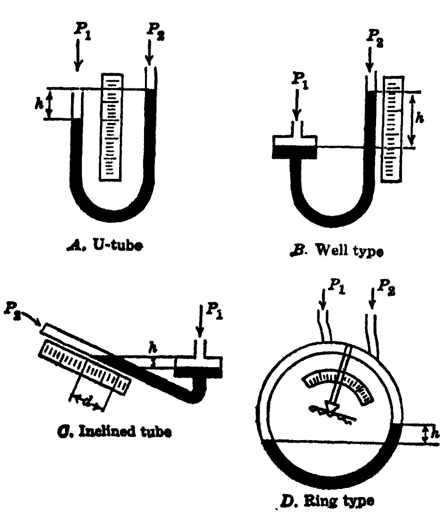


FIG. 4. Liquid manometers.

The inclined-tube manometer is used for very low pressures or vacuums. They are most commonly used as draft gages.

The ring-type manometer is constructed as shown in Fig. 4 so that the center of gravity of the system is below the pivot. When a pressure difference is applied the ring rotates until the moment caused by the additional liquid on one side equals the moment caused by the center of gravity on the other side.

Any of the manometers may be used for the measurement of gage pressure, vacuum, or differential pressure by suitable arrangement of the connections on the two sides of the manometer. Liquid manometers can be used up to about 300 psig pressure, depending upon the strength and sealing of the gage glasses.

The most common manometer liquids are water and mercury. Infrequently alcohol, kerosene, benzol, and oils of various composition are used.

INDICATING-RECORDING MANOMETERS. A mercury manometer may be made indicating or recording by employing a float in one leg of the manometer as shown in Fig. 5. The mechanical connection to the float is brought through the wall of the float chamber by means of a pressure-tight shaft. The change of level of mercury in the float chamber is generally about one-half inch. By changing the area of the leg not containing the float, ranges of differential pressure from 0 to 20 in. of water up to several hundred inches of water may be obtained. Recording manometers are made to withstand up to 2500 psig and sometimes even higher pressures. The instrument may be used for low gage pressures, vacuums, and low differential pressures. Eccentric-indicating styles and circular-chart recording styles are obtainable. Recording manometers of this kind are most commonly used for orifice flowmeters, and as such there are many varieties of mechanical and electrical forms.

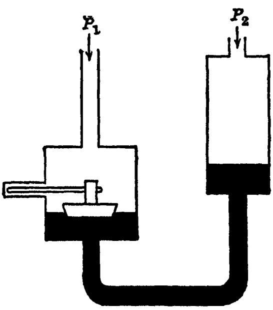


FIG. 5. Indicating manometer.

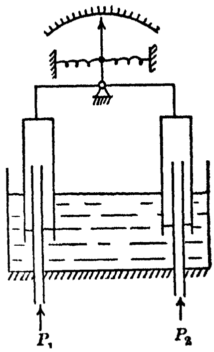


FIG. 6. Bell gage.

BELL GAGES. Bell-type gage for gage pressure, differential pressure, and vacuum measurements is shown in Fig. 6. The difference in pressures acting against the area of the bells causes the beam to deflect from its equilibrium position, and the instrument may be calibrated in pressure units. The function of the liquid is merely to seal the bells. Pressures and vacuums down to a fraction of an inch of water may be measured, but obviously the instrument is limited to low total pressures. Eccentric-indicating and circular-chart recording styles are available.

DIAPHRAGM GAGES. Gages for the measurement of draft pressures (vacuum or gage) employ a flexible diaphragm of oil-treated leather. The pressure acts against the effective area of a diaphragm and causes a spring to deflect an amount proportional to pressure. The range of such gages is usually in inches of water, and they are generally eccentric indicating. For higher pressures, flexible metallic diaphragms are sometimes used.

RECORDING ANEROID METER. For the measurement of low differential pressures, as in an orifice flowmeter, a bellows of large area may be used, one pressure being placed on the outside of the bellows and the other on the inside of the bellows. The differential pressure acting against the effective area of the bellows causes a spring to deflect an amount proportional to the differential pressure. The bellows is sealed in a cast-iron or steel housing, and the motion of the bellows is transmitted through the housing by a sealed torque tube. Indicating and circular-chart recording styles are available.

BELLOWS GAGES. For moderately low pressures of 1 to 50 psig, a bellows-type element may be used. The bellows may be of brass, phosphor bronze, beryllium copper, stainless steel, or monel. The range of the instrument is determined by the effective area of the bellows and the spring gradient. Eccentric-indicating, concentric-indicating, and circular-chart recording styles may be obtained.

BOURDON-TUBE GAGES. For pressures of 20 to 10,000 psig, a bourdon-tube element, a spiral element, or a helix element is used. A spiral is a bourdon tube of more than one turn wound in one plane at varying diameter. A helix is a bourdon tube of more than one turn wound at a common diameter. The bourdon-tube element may be of phosphor bronze, beryllium copper, or steel. Pressure gages with concentric or eccentric indicating dials may be obtained in a large variety of styles and in dial sizes of 2 in. up to 14 in. in diameter. They are in very common industrial use since they are inexpensive in moderate sizes. Circular-chart recording gages may also be obtained.

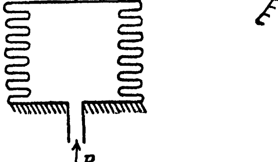


FIG. 7. Absolute-pressure gage.

ABSOLUTE-PRESSURE GAGES. An absolute-pressure gage is constructed by employing a bellows-type gage with a barometric compensator as shown in Fig. 7. The upper bellows is evacuated and sealed while the unknown pressure is applied to the lower bellows. Changes in barometric pressure are compensated by the action of the upper bellows. Obviously barometric compensation is important only in measurement of pressures up to about 300 psig. The instrument is usually circular-chart recording.

3. METHODS OF MEASURING HEAD

Head or liquid level is expressed in distance units, such as inches, feet, or meters, above a given reference plane. Sometimes, however, liquid level is expressed in volume units, such as cubic feet or gallons, or even in terms of weight units, such as pounds. In order to make these measurements explicit all necessary data, such as temperature, specific gravity, and static pressure, should be specified.

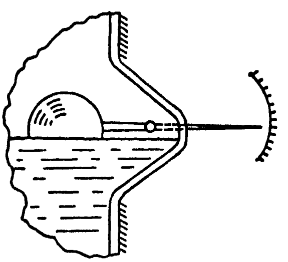


FIG. 8. Free-float liquid-level gage.

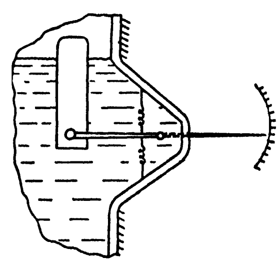


FIG. 9. Displacement-float liquid-level gage.

FREE-FLOAT LIQUID-LEVEL GAGE. The free-float type of gage is shown in Fig. 8. The float is free to follow the surface of the liquid, the motion of the float being brought out of the chamber through a pressure-tight shaft. The range of an instrument of this kind is limited to a few inches change in liquid level. The static pressure above the liquid may be as high as 2500 psig. Other types of free-float liquid-level gages are arranged with a chain or a tape so that changes of liquid level up to 20 or 30 ft may be measured. This latter type is more suitable for open vessels. Free-float liquid-level gages may be obtained in indicating and circular-chart recording styles.

DISPLACEMENT-FLOAT LIQUID-LEVEL GAGE. This type is shown in Fig. 9. The float is restrained by the springs attached to the arm, or by similar means, so that the buoyancy of the float causes the float arm to deflect as the liquid head at the float changes. This type is not restricted to small changes in level since the float may be several feet in length.

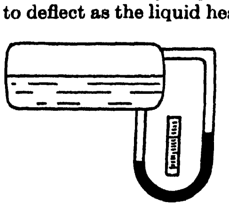


FIG. 10. A manometer used for measuring liquid level.

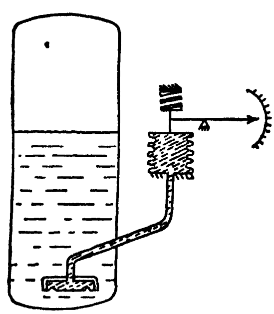


FIG. 11. Pressure-type liquid-level gage.

MANOMETER-TYPE LIQUID-LEVEL GAGE. In a closed vessel under pressure it is sometimes advantageous to use an indicating or recording manometer. This method is indicated in Fig. 10. The manometer may be constructed for as low as 20 in. of water up to several hundred inches of water change in head. In installing this kind of liquid-level gage it is important that careful consideration be given the effect of the various liquids in the legs of the manometer. Both mechanical and electrical types may be used.

STATIC-PRESSURE-TYPE LIQUID-LEVEL GAGE. Where the changes of liquid level are appreciable in extent it may be possible to use an ordinary pressure gage for the measurement of the liquid level as shown in Fig. 11. Here is shown a bellows pressure gage employing a diaphragm seal for the purpose of excluding the measured liquid from the pressure-gage system. Another method that is often employed is to use an air-purge system which maintains a slight positive pressure in the pressure-gage line, the excess air being allowed to bubble out of the gage line into the vessel.

4. METHODS OF MEASURING FLUID FLOW

There are four methods in common industrial use for the measurement of fluid flow in closed pipes. They are exemplified by quantity meters, velocity meters, head meters, and area meters. (For the measurement of liquid flow in open channels, see Hydraulics, Section 5, Art. 8.)

Units of fluid flow may be expressed in almost any volume or weight dimensions. However, total quantity of liquids is usually given in gallons, and total quantity of gases is usually given in cubic feet. Flow rate of liquids is usually given in gallons per minute (abbreviated gpm), and flow rate of gases is usually given in cubic feet per hour (abbreviated cfh). These quantities and flow rates are usually given at standard conditions such as 60 F and 30 in. Hg instead of operating conditions.

QUANTITY METERS. In quantity meters the fluid, gas, or liquid passes in discrete quantities, each quantity of identical volume. There are many types of quantity meters; among them are the reciprocating piston, the oscillating piston, the nutating disk, and the geared impeller types for liquids, and the geared impeller, bellows, and sealed drum types for gases.

Quantity meters make use of a counter which can be calibrated to give the total quantity in terms of either volume or weight passed at a given point.

Quantity meters for liquids will handle 0.05 up to 700 gpm. They may be operated under static pressures up to 1000 psig. Their accuracy is very good even at low flows. The piston types usually have an accuracy within about 0.2%. The accuracy of other types is usually about 0.5 to 1.0%. Quantity meters for gases will handle up to 50,000 cfh at a pressure of 500 psig. Their accuracy is usually one-half of 1%.

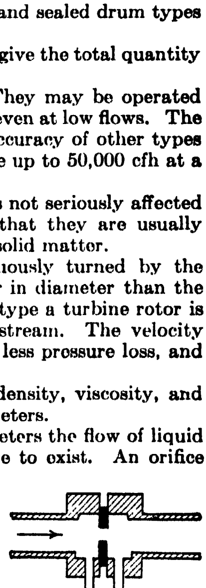
The advantages of a quantity meter are that it is accurate and is not seriously affected by temperature, pressure, and density. The disadvantages are that they are usually expensive in large sizes and are not suitable for liquids containing solid matter.

VELOCITY METERS. In velocity meters a rotor is continuously turned by the motion of the flow stream. In one type a propeller screw, smaller in diameter than the pipe, is rotated by the impact velocity of the stream. In another type a turbine rotor is made to turn by the force on the blades caused by the flowing stream. The velocity meter has a greater flow capacity than the quantity meter, causes less pressure loss, and may be used for the flow of abrasive materials.

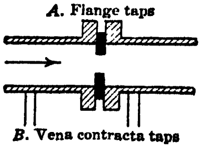
The accuracy of velocity meters is subject to the influence of density, viscosity, and temperature. They are not as accurate at low flows as quantity meters.

MEASUREMENT OF FLOW BY HEAD METERS. In head meters the flow of liquid across a resistance of constant area causes a difference in pressure to exist. An orifice plate, flow nozzle, or venturi tube is used for the resistance, and the differential pressure may be measured by any of the manometers or differential pressure gages discussed previously.

Orifice arrangements used in commercial practice are shown in Fig. 12. The thin-plate orifice is the simplest and the least expensive of any of the primary elements. It may be obtained in many different materials and in sizes for 2 to 14 in. diameter pipe. It causes the largest pressure loss of the three primary elements.



A. Flange taps



B. Vena contracta taps

FIG. 12. Orifice arrangements.

Orifice plates may be installed with vena contracta taps, flange taps, or pipe taps (see Section 1, Art. 7). Vena contracta taps are installed so that the downstream tap is located at the point of minimum static pressure. This arrangement results in the maximum pressure differential for a given flow but also requires that the taps be moved if the orifice size is changed. Flange taps are installed in a separate flange with the taps about one inch upstream and downstream from the orifice plate. This arrangement is the simplest and easiest to install. Pipe taps are installed sufficiently far from the orifice so that they measure the permanent pressure loss. This arrangement results in the smallest pressure differential for a given flow and is not as commonly used.

The theory of fluid flow is discussed in Sections 1 (Air), 3 (Heat, etc.), 4 (Steam, etc.) and 5 (Hydrodynamics, etc.) of this book. The following equations may be used for the flow of fluids (liquid or gas) through an orifice. The equations are based on the use of any kind of mercury manometer for the measurement of the differential pressure, and the manometer is assumed to be at 80 F ambient temperature. The equations will allow a computation correct to about 1%. In the equation for gases the expansion and compressibility effects are neglected, and the gas is assumed to be dry. The flow rate is given in terms of quantity at 60 F and, for a gas, at 30 in. Hg.

$$\left.\begin{aligned} W &= 1270\beta^2 K D^2 \sqrt{\frac{H}{v}} \\ N &= \frac{5.05W}{\mu\beta D} \end{aligned}\right\} \text{Steam}$$

$$\left.\begin{aligned} Q &= 2.54\beta^2 K D^2 \sqrt{H\rho} \\ N &= \frac{3160Q}{\mu\beta D} \end{aligned}\right\} \text{Water}$$

$$\left.\begin{aligned} Q &= 5.67\beta^2 K D^2 \sqrt{zH(13.53 - y)} \\ N &= \frac{3160xQ}{\mu\beta D} \end{aligned}\right\} \text{Liquids}$$

$$\left.\begin{aligned} V &= 7700\beta^2 K D^2 \sqrt{\frac{hP}{Tx}} \\ N &= \frac{13.64PxV}{T\mu\beta D} \end{aligned}\right\} \text{Gases}$$

$$\beta = \frac{d}{D} \text{ diameter ratio}$$

where β = diameter ratio; d = orifice internal diameter, inches; D = pipe internal diameter, inches; h = orifice differential pressure, inches of water; H = orifice differential pressure, inches of mercury; K = orifice flow coefficient (Tables 20 and 21); μ = fluid viscosity at flowing conditions, centipoise; N = Reynolds' number; P = static pressure at flowing conditions, psia; Q = quantity flow rate, gallons per minute; ρ = fluid density at flowing conditions, pounds per cubic foot; T = fluid temperature at flowing conditions, °R′; v = specific volume at flowing conditions, cubic feet per pound; V = volume flow rate, cubic feet per hour; W = weight flow rate, pounds per hour; x = fluid specific gravity at flowing conditions; y = fluid specific gravity at 80 F; z = reciprocal of fluid specific gravity at 60 F.

To determine pressure differential when orifice size and flow rate are known: (1) Compute β. (2) Compute N. (3) Find K in Tables 20 and 21 (interpolate if necessary). (4) Compute pressure differential (H or h).

To determine flow rate when orifice size and pressure differential are known: (1) Compute β. (2) Assume a value for flow rate (W, Q, or V). (3) Compute N. (4) Find K in Tables 20 and 21 (interpolate if necessary). (5) Repeat steps 2 to 4 if necessary.

To determine orifice size when flow rate and pressure differential are known: (1) Assume a value for orifice diameter (d). (2) Compute β. (3) Compute N. (4) Find K in Tables 20 and 21 (interpolate if necessary). (5) Compute flow rate. (6) Repeat steps 2 to 5 if necessary.

For further detail consult Gess, Spink, *Flow Measurement*, ASME, 1940, or *Fluid Meters, Theory and Application*, Part 1, ASME, 1937.

Table 20. Orifice Flow Coefficient (K)
Flange Taps

(Abstracted from *Fluid Meters, Theory and Application*, ASME, 1937.)

Pipe Size, in.	Reynolds' Number	β								
		.100	.200	.300	.400	.500	.600	.650	.700	.750
	1×10^4	.6134	.6104	.6150	.6278	.6521	.6945	.7251	.7630	......
2	1×10^5	.6046	.5987	.6018	.6104	.6275	.6558	.6765	.7025	......
	1×10^7	.6036	.5974	.6003	.6090	.6247	.6515	.6712	.6959	......
	1×10^4	.6120	.6119	.6183	.6314	.6580	.7056	.7400	.7828	......
3	1×10^5	.6001	.5968	.6019	.6109	.6271	.6555	.6761	.7022	......
	1×10^7	.5988	.5951	.6001	.6087	.6237	.6499	.6692	.6934	......
	1×10^4	.6118	.6151	.6212	.6343	.6631	.7158	.7540	.7985	.8645
4	1×10^5	.5971	.5970	.6023	.6110	.6271	.6556	.6764	.7001	.7391
	1×10^7	.5955	.5950	.6002	.6084	.6231	.6490	.6679	.6892	.7253
	2.5×10^4	.6018	.6061	.6107	.6209	.6430	.6836	.7132	.7500	.8017
6	2.5×10^5	.5936	.5965	.6013	.6095	.6246	.6515	.6711	.6958	.7294
	1×10^7	.5927	.5954	.6002	.6082	.6226	.6481	.6666	.6899	.7217
	2.5×10^4	.6035	.6088	.6127	.6229	.6466	.6912	.7239	.7646	.8214
8	2.5×10^5	.5932	.5969	.6016	.6095	.6246	.6517	.6715	.6966	.7299
	1×10^7	.5920	.5956	.6003	.6081	.6224	.6474	.6658	.6891	.7200

Table 21. Orifice Flow Coefficient (*K*)

Vena Contracta Taps

(Abstracted from *Fluid Meters, Theory and Application*, ASME, 1937)

Pipe Size, in.	Reynolds' Number	β									
		.100	.200	.300	.400	.500	.600	.650	.700	.750	.800
	1.5×10^4	.6148	.6067	.6082	.6185	.6361	......	......	......	......	......
2	1.5×10^5	......	.5971	.6001	.6093	.6257	.6541	.6753	.7034	.7417	.7960
	5×10^6	......	......	......	.6074	.6236	.6516	.6724	.7000	.7368	.7900
	1.5×10^4	.6081	.6037	.6081	.6185	......	......	......	......	......	......
3	1.5×10^5	.5972	.5955	.6000	.6092	.6257	.6542	.6756	.7039	.7424	.7975
	1×10^6	......	......	......	.6067	.6228	.6507	.6713	.6980	.7349	.7882
	1.5×10^4	.6063	.6034	.6081	.6185		......	......	......	......	......
4	1.5×10^5	.5959	.5954	.6000	.6092	.6257	.6542	.6756	.7041	.7429	.7983
	1×10^6	......	......	......	.6067	.6228	.6507	.6713	.6980	.7348	.7881
	1.5×10^4	.6024	.6028	.6080	.6185	......	......	......	......	......	......
6	1.5×10^5	......	.5951	.6000	.6093	.6258	.6542	.6757	.7044	.7441	.8001
	1×10^6	......	......	.5978	.6067	.6228	.6506	.6713	.6980	.7346	.7878
	1.5×10^4	.6023	.6029	.6083	.6189	......	......	......	......	......	......
8	1.5×10^5	......	.5952	.6001	.6093	.6258	.6542	.6759	.7050	.7447	.8010
	1.5×10^6	...	...				....	.6708	.6974	.7335	.7859

An orifice plate must be carefully installed in a straight run of pipe with no fittings or obstructions less than about 20 pipe diameters upstream and about 5 pipe diameters downstream. Straightening vanes may be required in some cases. Consult references for specific details.

A flow nozzle or venturi tube (Fig. 13) may also be used with a head meter, although they are not as common. The pressure loss with these elements is not as great as with the orifice plate. They are more expensive than an orifice plate, and fewer data are available on their coefficients. (See *Fluid Meters, Theory and Application*, p. 105, for flow nozzles and p. 99 for venturi tubes.)

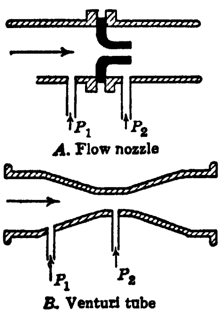


FIG. 13. Flow nozzle and venturi tube elements.

A pitot tube (Fig. 14) is also used with a head meter. It is not as accurate as an orifice or venturi, but its use becomes necessary when flow measurements are made in large pipes and ducts. It has the advantage that velocity traverses of the pipe or duct can be made. (See *Fluid Meters, Theory and Application*, p. 69.)

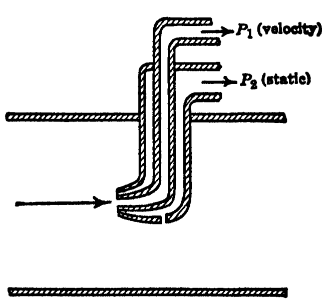


FIG. 14. Pitot-tube element.

The pressure connections between any of the primary elements and the manometer instrument must be considered in the calibration and use of a head meter. Often the manometer legs are filled with a liquid of different density from the manometer liquid, and liquid seals may be necessary to keep corrosive fluids out of the manometer. Such factors affect the calibration of the instrument. The manometer piping must be installed in accordance with standard practices.

In gas-flow measurement the flow rate measured by the head meter is dependent upon the static pressure of the gas. If this static pressure varies it must be measured independently so that flow readings may be compensated in accordance with the formula for flow of gases. Some head meters may be obtained with an additional static pressure-measuring element connected to them in such a way as to compensate for variations in gas pressure.

The scale calibration of a head meter has a square-root characteristic unless the manometer has a special means for extracting the square root to produce a semi-linear scale. The instrument may be either indicating or circular-chart recording. With one of the primary elements discussed above, flows of almost any reasonable values may be measured. The accuracy of a head meter is usually about ±0.5% of span above 30 or 40% of scale. At low readings on the scale the accuracy of a head meter cannot be depended upon although it will usually approach ±1%.

AREA METERS (ROTAMETER). In area meters a variation of area is presented to the flowing stream under constant differential pressure. The most common form is the rotameter shown in Fig. 15.

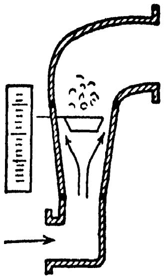


FIG. 15. Area meter or rotameter.

The float is supported in the flow stream by the pressure difference across the float, the greatest pressure being on the bottom of the float, and by the impact pressure on the bottom of the float. These forces upward must be balanced by the downward force of float weight. A rotameter is therefore installed in a vertical position. The float is usually carried in the center of a tapered glass tube, and the float may or may not be guided. The glass tube can be calibrated directly with a scale reading in terms of flow rate.

By suitable changes in float design the rotameter calibration may be compensated for changes in density and changes in viscosity of flowing fluid. The location of a rotameter close to fittings or obstructions in the line does not influence its accuracy as much as this location would influence the head meter.

The advantages of a rotameter are that they can handle many corrosive fluids, the scale calibration is nearly linear, and they are reasonably accurate at low flows. On the other hand, rotameters are not as rugged as head meters and quantity meters and they are expensive in large sizes.

For automatic indicating or recording of fluid flow the displacement of the float may be transmitted by pneumatic, electrical, or magnetic means to an indicating or circular-chart recording instrument.

The accuracy of rotameters is generally ±0.5% above 10% of maximum flow and with special calibration may be better than that, particularly at low flows.

INTEGRATION OF FLOW RATE. A head meter records the rate of flow in such units as gallons per minute. To obtain the total quantity it is necessary to integrate the flow rate over a period of time. This may be done manually, by a planimeter, or by an automatic integrator.

Most recording head meters use circular charts, which may be integrated manually by selecting small intervals of time, such as 15 minutes, and multiplying the average flow rate by the time period. For gas flow it is necessary to correct for variations in pressure by using the gas-flow formula.

A planimeter designed especially for use with circular charts of square-root characteristic is available from the manufacturers of recording meters.

Automatic integration may be obtained for nearly all recording head meters and area meters. Mechanical types of integrators employ the principle of the disk and wheel and are driven in a periodic manner from a small synchronous motor. For gas flow with varying static pressure, two integrators are used. On some head meters electrical integrators are used.

Integration of flow rate cannot generally be made more accurate than ±0.5% with either a planimeter or an automatic integrator. Over long periods of time, such as a week or month, the error of integration is cumulative, and accurate check of total quantity is difficult to establish.

AUTOMATIC CONTROL

An automatic controller, sometimes called a regulator, is a mechanism which measures the value of a variable quantity or condition and operates to correct or limit deviation of the measured value from a selected reference. Thus the purpose of an automatic controller, in addition to the controlling function, is to perform a measurement. Almost any of the instruments or meters discussed in the previous section may be used as an automatic controller by combining a controlling means with its measuring means.

An automatic controller, together with the process which it controls, constitutes a **closed loop of action and reaction** as shown in Fig. 1. The automatic controller measures, with a primary element, some variable quantity or condition associated with the balance or performance of the process. This quantity is called the controlled variable. The controller then moves or operates a final element in response to a function of the deviation. The final element determines the value of a manipulated variable, which in turn operates through the process to alter the value of the controlled variable. The effect of the manipulated variable on the controlled variable must be to oppose the change or trend already existing at the controlled variable.

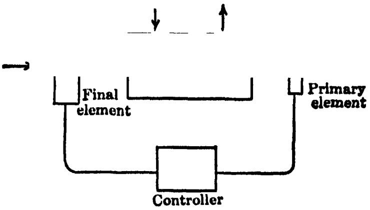


FIG. 1. Closed loop of automatic control.

Process disturbances, represented by the arrows in Fig. 1, cause the controlled variable to deviate from the desired value. It is the purpose of the automatic controller to eliminate as much as possible the effect of these disturbances.

The action and reaction in the closed loop are never instantaneous; these operations take time to occur. The main problem in automatic control is to select an automatic controller which will provide satisfactory performance in spite of these delays or lags.

5. ACTIONS OF AUTOMATIC CONTROLLERS

Every automatic controller has a specific method by which it accomplishes the purpose of automatic control. This is called the **action of control.**

TWO-POSITION ACTION is an action in which the final element is moved from one of two fixed positions to the other. This controller action is often called "on-off" or "open and shut." It is the simplest automatic-control action and is probably the most widely used. The simple thermostat which turns on a heater when the temperature is low and turns off a heater when the temperature is high is an example of this action.

TWO-POSITION DIFFERENTIAL ACTION is an action in which a final element is moved from one of two fixed positions to the other when the controlled variable reaches a predetermined value from one direction, and subsequently is moved to the other position only after the variable has passed in the opposite direction through a range of values to a second predetermined value. The differential gap is therefore similar to a hysteresis in the controller action, except that the differential is usually added intentionally to the controller action.

SINGLE-SPEED FLOATING ACTION is an action in which the final element is moved at a single rate. With these controllers the final element is in the act of gradually closing or gradually opening, and floats in a partly open position. A single-speed floating action does not recognize rate or magnitude of deviation but acts only upon the elapsed time of the deviation. In order to stabilize the action of the controller a neutral zone is often used so that no corrective action of the controller results when the controlled variable is within this zone.

PROPORTIONAL-SPEED FLOATING ACTION is an action in which there is a continuous linear relationship between the value of the controlled variable and rate of motion of the final element. The equation for this controller is

$$-\frac{dP}{dt} = fD$$

where P = position of final element, f = floating rate, D = deviation of measured variable, t = time.

Another form of the equation is:

$$-P = f\int D\,dt + K$$

where K is a constant of integration.

As these equations show, the controller recognizes not only *elapsed time* of deviation but also *magnitude* of deviation. A floating rate adjustment (f) is arranged so that the proportionality can be made between rate of change of final element position and deviation.

PROPORTIONAL ACTION is an action in which there is a continuous linear relationship between value of the controlled variable and position of the final element. The equation for this controller is

$$-P = \frac{1}{s}D + K$$

where s is the proportional band and K is a constant. Thus a proportional controller operates the final element in accordance with the magnitude only of the deviation. The proportional band adjustment (s) is similar to an amplification ratio which merely relates the motion of the final element to the magnitude of the controlled variable.

PROPORTIONAL PLUS RATE ACTION is an action in which a rate action is added to the proportional action. In rate action there is a continouus linear relationship between the rate of change of the controlled variable and the position of the final element. The equation for this controller is

$$-P = \frac{1}{s}D + \frac{q}{s}\frac{dD}{dt} + K$$

where q is the rate time. This action is often called a proportional plus derivative or a proportional plus rate-response action. The rate-time adjustment (q) adjusts the relationship between the rate of change of the controlled variable and the position of the final control element. In most controllers of this kind an adjustment of the proportional band (s) affects the proportional and the rate actions simultaneously.

PROPORTIONAL PLUS RESET ACTION is an action in which the proportional and proportional-speed floating actions are added. The equation for this action is

$$-P = \frac{r}{s}\int D\,dt + \frac{1}{s}D + K$$

where r is the reset rate. This is sometimes called a proportional plus integral action. The reset rate (r) determines the magnitude of the proportional-speed floating action. With most controllers of this kind the proportional band adjustment (s) affects both responses equally.

PROPORTIONAL PLUS RESET PLUS RATE ACTION is an action in which the proportional speed floating, proportional, and rate actions are added. The equation for this controller is

$$-P = \frac{r}{s}\int D\,dt + \frac{1}{s}D + \frac{q}{s}\frac{dD}{dt} + K$$

6. TYPES OF CONTROLLERS

The self-contained controller or regulator used in temperature control and pressure control is the most common type. These types are usually called the pressure regulator (Fig. 2) and the thermostatic valve (Fig. 3). The self-contained regulator is recognized by the fact that it requires no auxiliary supply of power since it operates entirely from the power developed in its measuring means. These devices, widely used in industry, are available in a great variety of sizes and operating ranges, all of them employing proportional control action.

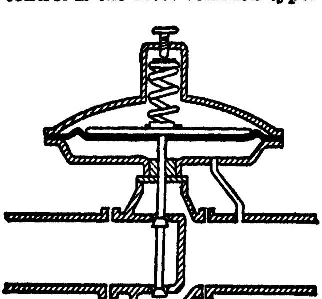


Fig. 2. Pressure regulator.

Pressure regulators are installed directly in the line through which a fluid, such usually as water, air, or steam, is flowing. They will operate to maintain pressures from a few inches of water up to thousands of psig at moderate rates of flow. Most simple pressure regulators do not operate well at very small flow rates far under their average capacity.

Thermostatic valves are installed directly in a steam or hot-water line leading to the tank or vessel in which the temperature is controlled. They are available in ranges for refrigeration service and for temperatures up to about 300 F. A thermostatic valve usually has a solid-filled liquid thermal system with a bulb and capillary.

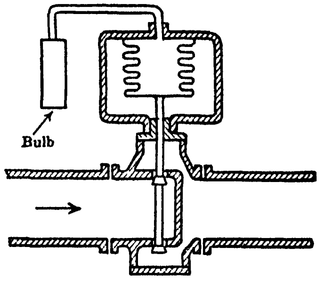


FIG. 3. Thermostatic valve.

PNEUMATIC CONTROLLERS are in wide use because of their simplicity, speed, and precision of operation. The pneumatic on-off controller is shown in Fig. 4. The bellows, as in a pressure gage, operates a light metal vane positioned close to the open end of a nozzle. The nozzle is supplied with air through a restriction. When the vane covers and uncovers the nozzle, the back pressure at the nozzle is changed, and this pressure may be used to operate a diaphragm control valve or other final control element.

The proportional controller (Fig. 5) operates on the same principles except that a feedback bellows is used to increase the stability of the pneumatic system. Other pneumatic controllers include reset action and rate action and their various combinations.

Pneumatic controllers require an air supply of about 15 to 25 psig. This air supply

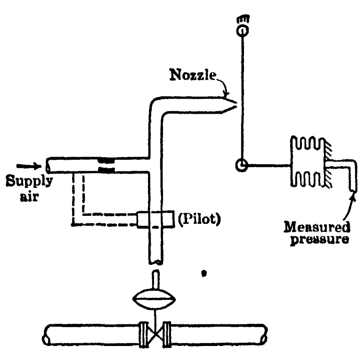


FIG. 4 Pneumatic two-position controller.

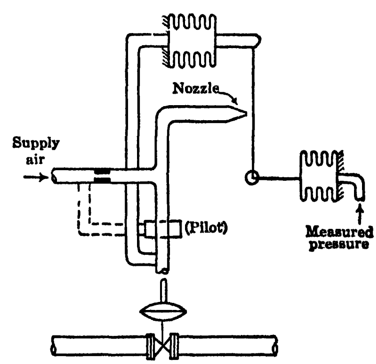


FIG. 5. Pneumatic proportional controller.

should be clean and dry and should be available at about 100 to 125 psig. An individual pressure regulator and air filter is ordinarily used in the air supply to each controller.

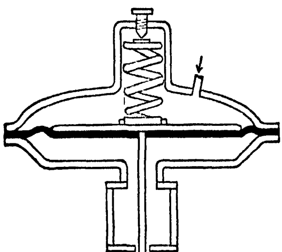


FIG. 6. Pneumatic diaphragm motor.

The output range of most pneumatic controllers is about 0 to 20 psig. Each pneumatic control requires about 0.5 cu ft of free air per min or less.

Sensitivity and accuracy of a pneumatic proportional controller are extremely high, and the equivalent of several thousand positions is available. For this reason they are more often used for proportional control (and various combination actions) than any other kind of controller.

Diaphragm motors of the type illustrated in Fig. 6 are most commonly used with pneumatic controllers. These motors may be used with plug valves, rotary valves, butterfly valves, dampers, and louvers. Diaphragm motors may provide up to about 2000 lb force at 2 or 3 in. stroke. The diaphragm motor very often is supplied with a positioning device in order to increase its power, speed, and sensitivity. Pneumatic positioning devices usually operate on about a 2 to 14 psig range from the pneumatic controller. The positioning device usually requires a separate filtered and regulated air supply from 20 to 100 psig; its air consumption is roughly about 1 to 2 cu ft of free air per min.

ELECTRIC CONTROLLERS of the two-position type are probably the most common type of controller in industrial service. There are essentially three methods of operation: the vane-type controller, the contact-type controller, and the proportional-type controller.

The vane-type controller is shown in Fig. 7. The bellows, as in a transmitting expansion thermometer or in a pressure gage, positions a light metal vane in or out of the space between two small coils. The change in mutual inductance between the coils controls an oscillator amplifier in such a manner that when the vane is between the coils a relay is closed and when the vane is out of the coils a relay is opened. (The operation may be reversed.) The relay can then be used to operate a solenoid valve or an electric-motor valve. Vane control may be used with almost any of the measuring instruments discussed in the first section, and they may be obtained with a large variety of relay actions.

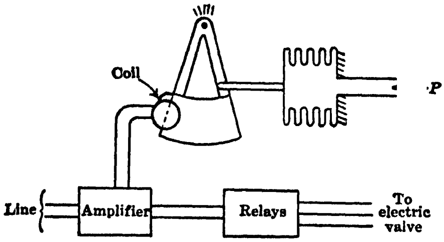


FIG. 7. Vane-type electric controller.

Contact-type controllers are made with open metal-to-metal contacts or mercury switch contacts operated directly by the measuring means, as in a home thermostat, or operated indirectly by a small auxiliary motor, as in a millivoltmeter controller. These controllers also may be obtained with a large variety of contact actions.

Solenoid valves are often used with vane-type and contact-type controllers, usually, however, in small sizes of single-seated plug valves. Electric-motor valves employ a motor of appropriate size for operating plug valves, butterfly valves, rotary valves, and dampers. Electric-motor valves are arranged so that they may be used as two-position or on-off controllers with a valve travel speed of 15, 30, and 60 sec. Some are constructed so that they may be used as single-speed floating controllers.

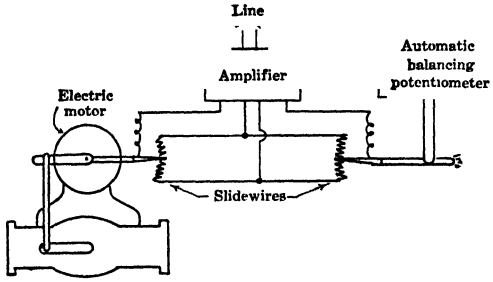


FIG. 8. Electric proportional controller.

Vane-type and contact-type controllers are often used for on-off control of small electric furnaces and electric heaters; sometimes power relays are used to handle heavier electric currents.

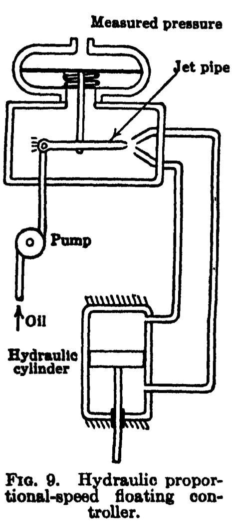


FIG. 9. Hydraulic proportional-speed floating controller.

The electric proportional controller is also widely used, generally with the potentiometer-type instrument. Their operation is indicated in Fig. 8. Two slidewires, one at the instrument and the other on the motor valve, are employed in a bridge-type circuit. The potentiometer measuring means positions the contactor along the slidewire, producing an unbalance in the bridge. This unbalance is amplified and applied, usually by relays, to an electric-motor valve. The electric-motor valve then moves to a new position corresponding to the position of the opposing slidewire and rebalances the bridge. Reset action and rate action are available in various combinations.

The sensitivity and accuracy of an electric proportional controller can be expressed as the equivalent of about 200 positions at the motor valve.

Electric controllers of the industrial type usually operate from a 110-volt a-c 50- to 60-cycle power supply; each controller may require up to 3 or 4 amp. Controllers operating on a 25-cycle a-c or on a d-c power supply are sometimes available.

Electric proportional controllers for electric furnaces are available in several types. One type adjusts a saturable-core reactor which varies the current flow to the heating electrodes of the furnace; another type interrupts the current flow to the furnace for various fractions of a cycle. They are available in single-speed floating control action and various combinations of proportional and proportional-speed floating actions.

HYDRAULIC CONTROLLERS may be operated by water pressure or from a separate oil-pressure supply system. The jet-pipe system is indicated in Fig. 9. Oil pressure is generated by the pump and the flow of oil passed through a movable nozzle; the oil jet impinges centrally on two receiving tubes. The measuring means, usually a pressure- or flow-measuring device, positions the jet pipe so that the oil is directed into one or the other of the two receiving tubes. This causes the piston in the operating cylinder to move in one direction or the other. The piston may be arranged to operate a valve or a damper. The action of a hydraulic controller is therefore proportional-speed floating, but other actions may also be obtained. An oil supply pressure of about 100 psig is used. The hydraulic controller is positive in action, and quite sensitive and accurate.

Because the hydraulic controller can create very large forces for the operation of large butterfly valves and dampers it is popular in power plants and combustion controls.

7. FINAL CONTROL ELEMENTS

The final control element is that portion of the controlling means which directly changes the value of the manipulated variable. The manipulated variable is that quantity or condition which is varied by the automatic controller so as to affect the value of the controlled variable. In controllers for temperature, pressure, flow, and liquid level, the manipulated variable is nearly always the rate of flow of a fluid. One exception is the temperature control of electric furnaces where the manipulated variable is the flow of electric current. The final control element for flowing fluids is usually a slip-stem valve, a rotary-stem valve, a butterfly valve, a gate valve, a louver, or a slide damper operated by an electric solenoid, an electric motor, a pneumatic diaphragm motor, or a pneumatic or hydraulic piston and cylinder.

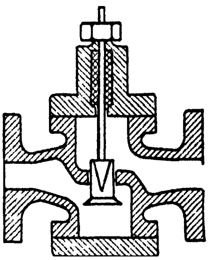


Fig. 10. Single-seated control valve.

SLIP-STEM VALVES are found in two types, the single-seated valve and the balanced double-seated valve. The single-seated valve is shown in Fig. 10. Its main advantage is that it can be made absolutely tight at shut-off. Double-seated valves are arranged so that the forces on the valve stem caused by the pressure differential are balanced. They cannot, in general, give absolutely tight shut-off. The slip-stem valve can be obtained with a variety of plugs or inner valves, among them the parabolic plug, the V-port plug, the bevel plug, and the rectangular-port plug. These variations are for the purpose of providing different flow characteristics and for handling different kinds of fluids.

Valve bodies and plugs may be obtained in a wide variety of materials, such as brass, bronze, carbon steel, and stainless steel. For example, a valve specified with bronze body and stainless steel trim means that the valve body is made of bronze and the valve plug and valve seats are made of stainless steel. Slip-stem valves may be obtained in sizes from $^1/_8$ in. to about 12 in. nominal diameter for pressure differentials up to about 1000 psi.

Slip-stem valves require a packing around the stem at the point where it passes out of the valve body. On valves handling corrosive fluids these packings require careful attention. Lubrication of the packing is usually supplied in order to reduce friction and leakage.

ROTARY-STEM VALVES are constructed much in the manner of the slip-stem valve except that the plug rotates in cylindrical seats in such a way that a rotary motion of the plug covers or uncovers an opening in the valve body. Otherwise their application is the same as for slip-stem valves. Rotary-stem valves are most commonly used with electric motor operators for the control of fuel oil to industrial furnaces.

BUTTERFLY VALVES consist of metal vanes rotating about their diameters inside a circular pipe, similar to the common stovepipe damper. The vane is operated by a shaft through a packing from a pneumatic diaphragm motor, an electric motor, or a hydraulic piston and cylinder. The butterfly valve, which is best for liquids or gases at low-pressure differentials, may be obtained in sizes (diameters) from 2 in. to about 60 in. A butterfly valve need not be opened more than 60 or 70 angular degrees from its closed position because little additional increase in flow is obtained beyond that point.

PRESSURE DIFFERENTIAL across the valve, louver, or damper is not an arbitrarily specified quantity. In operation a valve controlling the rate of flow of fluids is similar to an area flowmeter. Theoretically, the pressure differential remains constant while the area of the valve opening varies, thus changing the rate of flow. A small or zero-pressure differential at a valve cannot be obtained, nor is it desirable from the standpoint of valve operation.

The pressure differential at the valve is determined by the pressure at the supply point of the fluid minus the pressure at the terminal point of the fluid minus the friction pressure loss in the piping between the valve and the supply and terminal points.

It is very important that the pressure differential at the valve remain constant. In most cases these conditions are impossible to obtain. Under any conditions the following rules are helpful:

1. The supply pressure of the fluid should be constant.
2. The pressure of the fluid at the terminal point should be constant.
3. The friction pressure loss in the piping should be a minimum.

8. PROCESS CHARACTERISTICS

A process comprises the collective functions performed in and by the equipment in which a variable is to be controlled. From the standpoint of automatic control the process includes only the part of the industrial operation or processing unit that relates the manipulated variable to the controlled variable for one automatic controller. In the industrial sense of the word, a process may contain several operations, each under automatic control.

Processes under automatic control exhibit the characteristic that the action of the manipulated variable on the controlled variable is dependent upon time; that is, the reaction of the controlled variable lags the action of the manipulated variable. These so-called process lags are the result of four process characteristics: **capacitance, resistance, dead time,** and **self-regulation.**

CAPACITANCE is the change in quantity contained per unit of change in some reference variable. Capacitance describes the ability of a process to absorb or store up energy.

Thermal capacitance, derived from specific heat and mass, is expressed in Btu per degree. For example, the thermal capacitance of 10 lb of water is 10 Btu per °F.

Pressure capacitance of a gas, derived from the gas laws, is expressed in cubic feet per pound per square foot. For a given pressure vessel 10 cu ft of gas may be required to raise the pressure by one pound per square foot when operating at a particular pressure. The pressure capacitance of the vessel would be 10 cu ft per lb per sq ft.

Liquid capacitance, derived from the area of a vessel containing liquids, is expressed in cubic feet per foot. In a tank of 10 sq ft area, 10 cu ft of liquid would be required to raise the level by one foot. The liquid capacitance of the tank would be 10 sq ft.

RESISTANCE is opposition to flow. It describes the ability of a body to cause a drop in potential when there is a flow through the body.

Thermal resistance for a specific body is the reciprocal of thermal conductivity and is expressed in the units of degrees per Btu per minute. For a sheet of asbestos, for example, 25 F temperature drop might be required to cause a flow of heat of 1.0 Btu per min. The thermal resistance of the sheet would be 25° per Btu per min.

Resistance for the flow of fluids can be expressed in pounds per square foot per cubic foot per minute, that is, a given resistance to flow in a pipe might require 150 lb per sq ft pressure differential to cause a flow of 1.0 cu ft per min. The resistance would then be 150 lb per sq ft per cu ft per min. The units of resistance for fluid flow can also be expressed in terms of feet head per cubic foot per minute. The units of process capacitance and resistance are shown in Table 1.

Table 1. Units of Process Capacitance and Resistance

Characteristic	Dimensional Symbol	Temperature	Pressure	Liquid Level
Quantity	W	Btu	cu ft	cu ft
Potential	V	Degrees	psf	ft
Flow	W/T	Btu/min	cu ft/min	cu ft/min
Capacitance	W/V	Btu/deg	cu ft/lb/sq ft	sq ft
Resistance	VT/W	Degrees/Btu/min	lb/sq ft/cu ft/min	ft/cu ft/min

CAPACITANCE-RESISTANCE COMBINATIONS. Capacitance in industrial processes is not always lumped at a particular point but may be distributed. Likewise resistance may be either lumped or distributed. In thermal processes, capacitance and resistance are nearly always distributed. However, upon analysis of the process, it is usually found that either the capacitance or the resistance of a body is its main consideration. For example, insulation on the outside of a steam vessel is mainly thermal resistance.

Its thermal capacitance is so small relative to the rest of the process that it can be safely ignored. On the other hand, a volume of well-agitated water in a vessel is mainly thermal capacitance, and its resistance effects can be safely ignored. In the flow of gas through a pipe the resistance and capacitance effects may be about equally distributed.

In analyzing processes containing a flow of liquid or gas, resistance effects represented by friction and obstructions in the line and capacitance effects represented by tanks can usually be considered separately.

The reaction of a process to a change in manipulated variable and without automatic control (called the reaction curve) is shown in Fig. 11. A process involving a single capacitance and a single resistance always exhibits a rate of change that is a maximum at the time the change in manipulated variable is made. Processes with more than one capacitance, each separated by a resistance from the other, and processes with distributed capacitance and resistance always exhibit a rate of change that is zero at first and gradually increases to a maximum.

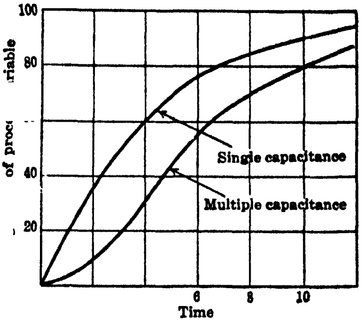


FIG. 11. Process reaction curves.

For the purpose of analysis it is usually assumed that process capacitance and resistance are linear. This is usually not the case, but serious error can be avoided by using values of capacitance and resistance that are related to the operating point of the process.

DEAD TIME, any definite delay period between two related actions, is measured in units of time. It occurs in processes where it is necessary to transfer energy by means of solids or fluids flowing over a given distance at a given speed. For example, if a control valve is located 50 ft from the process and the average velocity of flow is 250 ft per min, a dead time of 0.20 min will exist.

A dead time as small as 0.01 min may be significant in the operation of the automatic controller. On the other hand, a dead time as large as 5.0 min is sometimes encountered.

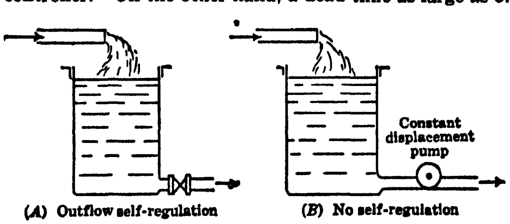


FIG. 12. Process self-regulation.

Self-regulation is a sustained reaction inherent in the process which assists or opposes the establishment of equilibrium, illustrated in Fig. 12 for a liquid-level process. The process on the left has no inflow self-regulation because changes in level in the tank do not affect the rate of flow of liquid to the tank. This process has outflow self-regulation because as the level of liquid in the tank increases the flow out of the tank increases. A process with self-regulation always exhibits the characteristic that the potential level in the process will reach such an equilibrium point that inflow is equal to outflow. The reaction curve of such a process is exponential in shape.

The process on the right in Fig. 12 has no inflow self-regulation and no outflow self-regulation. With the constant displacement pump at the outlet, the outflow is constant and is not affected by changes in liquid level. Processes without self-regulation do not, therefore, reach an equilibrium point if the inflow is not equal to the outflow. The reaction curve for such processes, if they are linear, is a straight line.

In most thermal processes there is no inflow self-regulation, since the temperature in the process usually does not affect the supply of heat to the process. Thermal processes generally exhibit outflow self-regulation since the amount of heat carried out of the process is proportional to the temperature.

Processes involving pressure generally exhibit inflow self-regulation because the rate of supply of fluid to the process depends upon the pressure in the process. The existence of inflow and outflow self-regulation in pressure-type processes depends upon the method by which fluid is supplied to and taken from the process, and whether separate flow control of the fluid to the process and from the process is used.

Liquid-level processes likewise may or may not have inflow and outflow self-regulation, depending upon the method by which liquid is supplied to and taken from the tank.

In control of rate of flow of fluids self-regulation exists to a considerable degree.

9. APPLICATION OF CONTROL

Through experiences in the application of automatic control to various processes it has been found that certain processes exhibit certain characteristics and that particular kinds of automatic controllers perform best on particular processes. To summarize these experiences:

Two-position control is most satisfactory when:

1. The process has only a single capacitance.
2. Dead time is negligible.
3. The process reacts slowly.
4. The automatic controller operates rapidly.

Single-speed floating control and proportional-speed floating control are most satisfactory when:

1. The process has only a single capacitance.
2. Dead time is negligible.
3. The process reacts quickly.
4. Process self-regulation is large.
5. The automatic controller operates rapidly.

Proportional control is most satisfactory when:

1. Large disturbances are not present.
2. The process reacts slowly.
3. Multiple capacity effects are small.
4. The automatic controller operates rapidly.

Reset action must be used when disturbances exist at the process since it serves to reduce the offset in proportional control. Offset is a sustained deviation due to the inherent characteristic of a proportional controller action.

Rate action must be used when either multiple-capacity effect or dead time in a process is large. It serves to speed up the reaction of the process, stabilize it more quickly, and reduce the magnitude of deviation of the controlled variable.

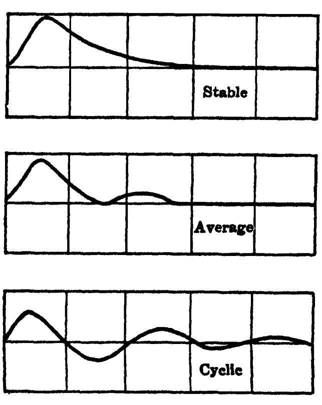


FIG. 13. Stability of control.

STABILITY OF CONTROL. Proportional controllers and proportional-speed floating controllers and any of their combinations contain means whereby their response may be adjusted to produce the kind of action by the controller that is desired. These adjustments are usually made after the controller is installed on the process. Various degrees of stability in the controller are shown in Fig. 13. Generally there are different criteria in judging the stability action of a controller, depending upon the kind of process under control. A stable operation of the controller is desirable when the process is connected into a series of continuous processes so that the cycling or oscillation of the variable in one process will not affect the operation of subsequent processes. In this case it is necessary to sacrifice somewhat on deviation because it will be somewhat larger. In order to reduce the magnitude of the deviation it is sometimes desirable to make the control "tighter." This will reduce somewhat the stability of the control. An average best setting of the controller will stabilize the variable in not more than 2 or 3 cycles, producing the greatest stability with the smallest deviation.

PRESSURE CONTROL. For pressure control or pressure-reducing service in pipe-line systems the self-contained pressure regulator operates satisfactorily. Controllers of this type have proportional action with a small fixed proportional band. For gas flow, pressure changes causing flow changes in the pipe line are usually not very fast, multiple capacity effects and dead time are very close to zero, and process self-regulation is high. For pressure control of vessels and tanks proportional control action with either a pneumatic or a hydraulic controller is usually adequate. If load changes of appreciable magnitude exist, reset action should be added to the controller. For pressure control where the volume of the system is small or the flow large, as in draft systems, proportional-speed floating control action with a hydraulic controller is satisfactory.

FLOW CONTROL. Quantity meters are not in common use for control of fluid flow. The orifice head meter and the area meter are most commonly used. With the orifice head meter it is best to arrange the control system so that the pressure most constant, either upstream or downstream, is one of the pressures at the orifice. For control of gas-flow rate, it is necessary that one of the differential pressures at the orifice should be absolutely constant. It may be necessary to use an auxiliary pressure controller to hold one of the pressures constant.

Proportional-speed floating control as in a hydraulic controller is best suited to flow control because the reaction of the flow rate to changes in valve position is fast, lags are small, and self-regulation is large. It should be noted that the main lags in flow control are those of the controller itself, particularly if a mercury manometer instrument is used. The same results may be achieved with the proportional plus reset action of pneumatic controllers, and it will generally be found that the proportional band of the controller will be large and the reset rate fast. Proportional control is not recommended for flow control.

LIQUID-LEVEL CONTROL. Two-position control of liquid level is often adequate if the area of the tank is large and a slight cycle in level is not objectionable. This can be accomplished with either an electric or a pneumatic controller. A great many float-type level controllers, self-contained, are used for liquid-level control where the float arm is connected mechanically to a balanced slip-stem valve. These controllers use proportional action and operate satisfactorily when the area of the tank is large. If the area of the tank is small or the through-flow is large a proportional-speed floating controller can be used. Pneumatic proportional control is satisfactory when the area of the tank is not small and when the through-flow is fairly constant. If the through-flow is not constant, a pneumatic proportional plus reset controller should be used.

TEMPERATURE CONTROL employs a wide variety of controllers, depending upon the results desired and the difficulty of the job. Most small heat-treating furnaces and high-temperature baths can be controlled satisfactorily with two-position control, using a thermocouple pyrometer or a radiation pyrometer with electric control. The main difficulty likely to be encountered is the measuring lag of the thermocouple element. The cycle of temperature which appears at the indicator or recorder will be less in amplitude compared to the actual changes of temperature as the measuring lag increases. Two-position control works best when the heating rate of the furnace is slow.

Single-speed floating control of furnaces and baths is adequate if the heating and cooling rates of the furnace or bath are fast.

When closer control of furnaces or bath temperature is desired, either electric proportional control or pneumatic proportional control can be used, provided that either the load changes are small or the heating and cooling rates of the furnace are small so that the proportional band of the controller will be narrow and the offset small. Reset action can be added to reduce the offset resulting from load changes. Rate action is usually not required for heat-treating furnaces and high-temperature baths.

For low-temperature baths, kettles, and heat exchangers there is a choice between the thermocouple pyrometer, the resistance thermometer, and the transmitting expansion thermometer. The selection depends upon cost and upon whether electric or pneumatic control is used. In such processes there will probably be an appreciable multiple-capacity effect, and in addition the measuring lag will probably not be small. In this case either electric proportional control or pneumatic proportional control can be used when the load changes are small. Reset response may be added to eliminate offset. Rate action may be required to overcome the multiple-capacity effect and the dead time.

For nearly all heat exchangers pneumatic proportional plus reset plus rate control is desirable because multiple-capacity effects are usually appreciable.

The application of automatic controllers is further summarized in Tables 2 and 3.

Table 2. Application of Controller Actions

Controller Action	Process Reaction	Multiple Capacity Effects and Dead Time	Process Disturbances	Process Self-regulation
Two-position	Slow	Very small	Small	*
Single-speed floating	Fast	Very small	Slow	Yes
Proportional-speed floating	Fast	Small	*	Yes
Proportional	Not fast	Not large	Small	*
Proportional + Rate	Not fast	*	Small	*
Proportional + Reset	*	Not large	Slow	*
Proportional + Reset + Rate	*	*	*	*

* Factor not critical.

Table 3. Selection of Automatic Controllers

Action and Type Controller	Flow Rate	Pressure: Pipe Line	Pressure: Small Volume or Large Flow	Pressure: Large Volume or Small Flow	Liquid Level: Small Area or Large Flow	Liquid Level: Large Area or Small Flow	High-temperature Furnaces and Baths: Fast Heating	High-temperature Furnaces and Baths: Slow Heating	Low Temperature: Large Ovens and Heat Exchangers	Low Temperature: Kettles, Vats and Small Heat Exchangers
Self-contained proportional action		*		*		*				*
On-off electrical				*		*		*		*
On-off pneumatic				*		*		*		*
		Use only if oscillation or cycling is not objectionable								
Single-speed floating electrical type							*			
Proportional—pneumatic	*	*	*	*	*	*	*	*	*	*
Proportional—electric				*		*	*	*		*
Proportional—hydraulic				*		*				
Add to above Reset action*	*	*	*		*		*		*	
Rate action†									*	
Proportional speed floating hydraulic type	*	*	*		*					

* Also add reset if process disturbances or load changes exist.
† Also add rate if sudden process disturbances or load changes exist.

GENERAL REFERENCES

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PROCESS INSTRUMENTATION

Manufacturing processes can be divided into two general classes: flow or *continuous* processes and nonflow or *batch* processes. Every degree between completely nonflow and completely flow-type processes may be found in the processing industries, such as oil, chemical, heavy chemicals, plastics, rubber, power, ceramic, glass, food, textile, ferrous metals, nonferrous metals, and paper manufacturing.

The flow or continuous process is characterized by a careful balance of materials control at every point in the process. Automatic flow control is therefore the "heart" of flow processes.

For example, in modern central generating stations, where electrical power is generated from coal, the manufacturing operation requires a balance between incoming coal, water, and air to produce steam that must be passed to the turbo-generators at the required rate to produce electrical power in amount corresponding to the rate of consumption. The main effort of production control is, in this case, to maintain a complete balance in and between units of the plant.

The batch or nonflow process is characterized by operations dependent upon time or careful control of volume or weight, and frequently both. Time-program control, either manual or automatic, and liquid-level and pressure control are usually the "heart" of batch processes.

For example, in a food-processing plant operations are usually conducted in batches. In processing each batch, as in cooking, the time and quantity are of main importance.

10. PLANT LAYOUT

The layout of the plant is determined largely by the economics of the product and the method of manufacturing it. Instrumentation is sometimes the deciding factor in the selection of a particular plant layout because the effectiveness of production control, which is the basic purpose of instrumentation, is dependent upon plant layout.

CENTRAL AND UNIT LAYOUT. Central layout of plant and instrumentation is shown in Fig. 1. Here all control operations are grouped at a single control center, often called the instrument room or control room. In this center are located all the instruments and automatic controllers that can be conveniently gathered there. (See also Combined Transmitter Controllers, p. 18-37.) Central layout is commonly used in continuous processes. The advantages of a central layout are that all instrument service and maintenance are centered at one point, and coordination of all processing units is made easier.

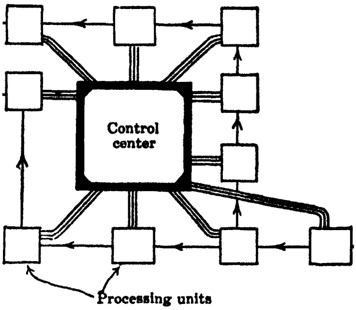

FIG. 1. Central instrumentation layout.

Unit layout of instrumentation is shown in Fig. 2. The control center is divided into a number of units, located near a given processing unit or units. This arrangement is most often used in batch or semibatch processes. It is also used in continuous processes of large-scale magnitude where a single control center would be too unwieldy for adequate production control. The advantage of unit layout is that the control center is close to the processing unit which it serves so that maintenance and trouble shooting can be accomplished more quickly. Also the connecting lines from the instruments and automatic controllers to the control center are shorter.

THE CONTROL CENTER may vary in size from a large building containing hundreds of instruments and automatic controllers down to a small enclosure or cubicle for a few instruments. Sometimes the control center will be simply a small group of instruments mounted on a panel and located in the open near a processing unit. In considering the layout of the control center, the following factors are important:

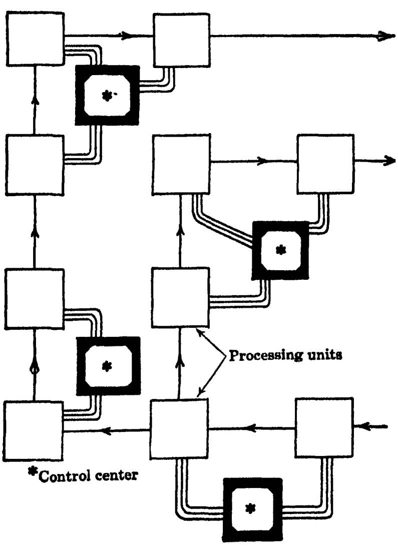

FIG. 2. Unit instrumentation layout.

1. The control center should preferably be completely enclosed.
2. Small control centers should have provisions for locking.
3. The control center must be clean and dry.
4. It must be temperature controlled and, in most cases, air conditioned.
5. It must be free from vibration.
6. It must be well lighted.
7. It must have an unfailing source of clean, dry air, of adequate pressure and capacity.
8. It must have an unfailing source of electrical power free of large surges in voltage and frequency.

These considerations apply to the control center whether or not it is housed in a separate enclosure.

It is desirable to make use of color dynamics, particularly in large control centers, in order to aid in the identification of various sections of the control network. Instrumentation maintenance must be accomplished quickly, and it is often necessary to trace particular thermocouple leads and manometer or pneumatic control lines. Instruments, automatic controllers, and panel boards are usually black except in certain industries, such as foods, where white is preferred for appearance and cleanliness.

Good coordination for the purpose of production control in a control center in a central layout and between control centers in a unit layout is not simple to achieve. Usually in large plants with either the central or unit layout a foreman is assigned to a particular group of instruments and automatic controllers controlling a particular processing unit. Coordination of the processing units is usually placed under a production-control manager. In small plants or units, particularly in batch processing, the control center is placed under the supervision of the process service operator.

Maintenance in large plants is usually directed from the control center. Service men may be stationed at the control center and may proceed to various points at the processing unit as the need arises. In even larger plants it has become necessary to station the service operators at a convenient point at the processing unit and direct these operators by telephone or "walkie-talkie" from the control center.

Maintenance in small plants is generally accomplished from a maintenance center subject to the call of the process operator.

INSTRUMENTATION DIAGRAM. A diagram can be made using the features of a process flow diagram but with all instrumentation and automatic control equipment outlined.

An instrumentation diagram should indicate (1) the variable being measured; (2) whether indication, recording, or other operations on the variable are required; (3) whether signaling of control functions are required; (4) any necessary auxiliary features; (5) type connecting lines; (6) location of point of measurement and point of control; (7) instrumentation at the control center and at the processing unit.

The purpose of an instrumentation diagram, however made, is to provide information quickly for use in process analysis, production control, and maintenance work. Therefore, it should not be loaded with useless information.

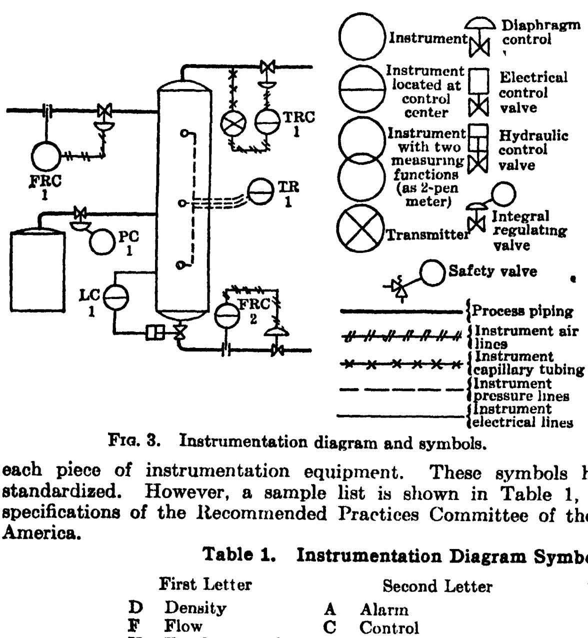

FIG. 3. Instrumentation diagram and symbols.

The main data of the instrumentation diagram are shown by means of symbols for each piece of instrumentation equipment. These symbols have not, as yet, been standardized. However, a sample list is shown in Table 1, based on the tentative specifications of the Recommended Practices Committee of the Instrument Society of America.

Table 1. Instrumentation Diagram Symbols

First Letter		Second Letter		Third Letter	
D	Density	A	Alarm	A	Alarm
F	Flow	C	Control	C	Control
H	Hand-actuated	E	Element (primary)	V	Valve
L	Level	G	Glass (no measurement)		
M	Moisture	I	Indicating		
P	Pressure	R	Recorder		
T	Temperature	S	Safety		
		W	Well		

Examples:

TRC-1 Temperature recording controller 1 on a given diagram.
FI-3 Flow rate indicating instrument, the third such instrument on a given diagram.
PCV-3 Valve for pressure controller 3; the instrument is "blind" (neither indicating nor recording).

An example of an instrumentation diagram is shown in Fig. 3, where the instrumentation symbols are shown. The diagram shows a schematic layout of the processing unit together with the necessary instrumentation.

The instrumentation diagram does not show exact location of equipment. For instruments which measure only, the general location of the measurement is given. For automatic controllers the location of the point of measurement and the location of the final element are specified in a general manner, and lines connecting the final element, instrument, and measuring element are shown. The location of the instrumentation at a control center or on the processing unit should be shown.

11. PLANT DESIGN

Many modern industrial operations must be designed with automatic control of the materials flow and process performance for the reason that automatic controllers are able to make operations more accurate, repetitive, and reproducible. Measuring instruments automatically provide information on which can be based the production control of the operation.

Instrumentation cannot be considered too early in the design of the process. Do not wait until the process design is complete and then request an instrumentation engineer to control it.

INSTRUMENTING THE PROCESS. Process instrumentation should be considered at about the design stage of the process, before the equipment is designed and calculated, but after the research stage has passed. In some processes which require operation under critical conditions it is even necessary to consider the instrumentation in the research stage.

The method of instrumenting a process generally proceeds as follows:

1. List all variables that are important to process performance, such as temperatures, pressures, flows, time operations, and volumes.
2. Divide this list into four groups:
   a. Variables neither measured nor controlled.
   b. Variables to be checked periodically. (Specify the frequency of measurement.)
   c. Variables to be continuously measured as an aid or check to performance.
   d. Variables to be controlled.
3. Decide, with the design engineer, if it has not already been done, what type of plant and unit layout would be most efficient.
4. Construct an instrumentation diagram.
5. Prepare brief tentative specifications of instrument and control equipment from the instrumentation diagram.
6. Determine, on the basis of the instrumentation diagram, the performance to be expected of the instruments, considering such factors as:
   a. The characteristics of the material being measured.
   b. The measuring lag of the instruments.
   c. The service maintenance of the instrument.
7. Determine, on the basis of the instrumentation diagram, the performance to be expected of the automatic controllers, considering such factors as:
   a. The measured variable.
   b. The manipulated variable.
   c. The service maintenance of the controller.
   d. The process load changes.
   e. The process lags.
8. Complete the tentative specifications for instrument and control equipment.

The design and instrumentation of the process should then progress through the final stages, including pilot plant or model stages, with the necessary corrections and revisions made as need arises.

A process should neither be overinstrumented or underinstrumented. Experience has indicated that the slight extra effort to achieve a sound and adequate instrumentation of the process builds the confidence of both management and operators in production control by means of instruments and automatic controllers.

PROCESS DYNAMICS. With a view to automatic control, the process design can be made in such a manner that the dynamics of the process aids automatic control and vice versa. General rules are difficult to state because of the infinite number of combinations of capacitance, resistance, self-regulation, and dead time that can be selected. The following guides may be used in process design if their effects are carefully considered in each particular design:

1. The process should have one large capacitance.
2. All capacitances except the large one should be reduced to a minimum.
3. All resistances should be reduced to a minimum.
4. Dead time should be reduced to a minimum.
5. Self-regulation should be used whenever possible.

If it is necessary to design a process that will allow a high or fast rate of change of the variable, all capacitances and resistances must be reduced to a minimum. This process will probably be more difficult to control and will be affected to a greater extent by load changes.

Such examples as the following are pertinent. Countercurrent heat exchangers are usually easier to control than co-current types. The "soaking" or tempering effect in co-current heat exchangers, where the supply and heated fluids are nearly at the same temperature, introduces a dead time not conducive to good automatic control. Liquid level in a liquid-storage tank is easier to control if the tank is installed so that the liquid surface area is a maximum. Massive construction of the fire box of a gas or oil-fired furnace makes automatic control difficult because of excessive heat storage in the supply. The automatic controller may turn down the flame to minimum without greatly affecting the heat-transfer rate.

An evaluation of the process design in terms of process resistance, capacitance, and dead time will automatically lead the way to improvements in process-control characteristics. The mere recognition of the existence of the dynamic characteristics of a process is a step toward better instrumentation.

PROCESS LOAD CHANGES are the reason for the use of automatic control. On a processing unit there is one or more variables under control. The remaining variables affecting the performance of the process are not under control, and it is expected that the automatic controller will compensate for changes in uncontrolled variables. These are load changes. A change in the through-flow or output of the process may also be a load change.

In considering the load changes in the process the effects of ambient conditions such as thermal radiation and sun radiation, barometric pressure, relative humidity, and wind conditions should not be overlooked. Three characteristics of load changes that are important are their location, their magnitude, and their rate.

The location of the load change in the control loop and its effect upon automatic control depend entirely upon the particular design of the process. Therefore, no general rule can be stated as to the relative effect of load changes at different points.

The magnitude of the load change should, of course, be reduced to a minimum since the deviation of the controlled variable is directly proportional to the magnitude of the load change. If the magnitude of the load change is so large as to cause a large deviation of the controlled variable, a separate automatic controller should be considered for reducing the effect on the main automatic controller.

The rate of the load change is probably the most important characteristic. Fast or sudden load changes are next to impossible to overcome by an automatic controller operating by itself. For sudden load changes it is better to review the process or plant design to determine if they can be eliminated. Slow load changes can be counteracted by most automatic controllers if the process is not difficult to control.

Cyclic load changes often are difficult to overcome. The design of most thermal processes does not allow the automatic controller to act quickly enough to reduce the deviation of the controlled variable. From this standpoint thermal processes have either too much distributed capacitance or too large capacitance. On the other hand, in many flow or pressure processes the automatic controller may be too slow acting to overcome cyclic load changes. It is possible to have a sinusoidal load change of a frequency that equals the frequency of the control loop and synchronous oscillation results.

The magnitude of the process load or through-put also affects the operation of the automatic controller as does any other load change. In addition, a change in through-put as in a heat exchanger usually changes the dynamic characteristics of the process because of a change in capacitance or a change in resistance (heat-transfer film coefficients). In some cases a large change in process through-put is sufficient to require different controller actions at high loads and at low loads.

12. TRANSMISSION AND CONTROL

Because an instrument or automatic controller must be located at a distance from the processing unit, it becomes necessary automatically to transmit both the measurement of a variable and the control effect. The need for transmission is dictated by the layout of the processing unit and the location of the control center. Transmission is often used in either continuous or batch processes.

TEMPERATURE-MEASUREMENT TRANSMISSION is accomplished by different methods, depending upon the type of instrument in use. The transmitting expansion thermometer is not generally used with greater than 200 ft of capillary because of filling limitations and cost. The thermocouple pyrometer, the resistance thermometer, and the radiation pyrometer have been generally used up to 500 ft distance between element and instrument and even up to 2000 ft in some cases. There is no practical limitation except the leadwire resistance which can be reduced by using heavier gage leadwire.

The addition of either pneumatic or electric transmission to the transmitting expansion thermometer allows the bulb to be placed up to 1000 ft distance from the instrument. With electric transmission there is little or no distance limitation. With pneumatic transmission, the measuring lag is increased at greater distances. In general, $^3/_8$-in. OD copper tubing for most pneumatic transmitters gives best results.

PRESSURE, FLOW, OR LEVEL-MEASUREMENT TRANSMISSION is usually accomplished by pneumatic means. Sometimes electrical transmission or hydraulic transmission is also used. Lines directly connected to pressure gages, flowmeters, and level meters should not be longer than about 25 to 50 ft (extremely variable, depending upon particular details). Long lines are costly, require extra maintenance and care, and are likely to cause a large measuring lag.

AUTOMATIC CONTROL AND TRANSMISSION. Controllers may be required to operate through long distances to the final element. Electric controllers such as the on-off and single-speed floating types are connected to solenoid valves or electric-motor valves. There is little or no limitation as to distance between controller and final element as long as the required voltage and current are available through the long leadwires. Electric proportional controllers using the Wheatstone bridge circuit are generally not used at distances greater than 1000 ft because of the resistance of the leads to the motor valve.

Pneumatic controllers will operate satisfactorily up to 500 ft distance between controller and final element, and even up to 1000 ft distance if the volume of the final element is small and if tubing of optimum size is used. A final element position device (positioner) should be used to increase the speed of operation. The optimum size tubing for connecting the controller to the final element is generally about $^3/_8$-in. OD copper tubing.

Hydraulic controllers will operate satisfactorily up to 500 ft and even 1000 ft distance between controller and final element.

COMBINED TRANSMITTER CONTROLLERS are coming into common use to avoid the lag imposed by transmission over long distances and also to unify the control center on large installations. The principle is shown in Fig. 4 for only one automatic controller. The automatic controller is constructed in such a manner that, although its control circuit operates in the normal manner, it is also able to transmit the measurement to a receiver at the control center. However, the control circuit (or loop) does not enter the control center. Thereby, the lags and delays encountered in transmission (particularly pneumatic transmission) are not included in the control loop. A set-point transmitter is used to effect the adjustment of the set point or control point of the automatic controller directly from the control center.

Where the control center is not too large, transmitter controllers may be of the indicating type and the receiver of the recording type. The control center is then constructed and employed in the usual manner.

For very large control centers, where unified production control is difficult, the transmitter controller may be made a recording type and the receiver may be an indicator of the smallest possible physical dimensions. Thus several hundred indicators can be placed on a small panel space under the supervision of only one or two operators.

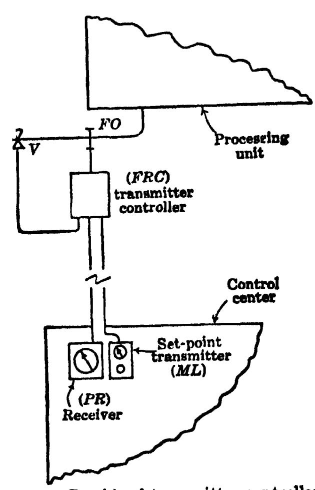

FIG. 4. Combined transmitter-controller.

REMOTE VALVE OPERATION for manual setting can be accomplished by either electric, pneumatic, or hydraulic means. Valves, dampers, louvers, and other final elements may be operated by a solenoid valve or an electric-motor valve on the processing unit with a two-position push-button station at the desired point. Throttling the valve or damper may be accomplished with a "floating" electric-motor valve operator and an "inching" push-button station. There is practically no limitation on distance.

Pneumatic-valve operation is accomplished with a diaphragm motor or other pneumatic device positioning a valve or damper, and a small precision-type pressure regulator or transmitter. Pneumatic means is generally used for throttling a valve at distances of not over 1000 ft.

Hydraulic systems for remote operation can also be used. Some are very similar to

the pneumatic type; others use a positive-displacement action. These are used for throttling action at any reasonable distance.

SIGNALING, INDICATING, RECORDING. Instruments and automatic controllers may or may not indicate the value of the variable which they measure. Often in industrial operations controllers are nonindicating (blind) simply because neither indication nor recording of the controlled variable is necessary. A good rule is never to use a recorder when an indicator will serve the purpose, and never to use an indicator when a nonindicating controller will suffice. In general, a recorder requires more maintenance than an indicator, and an indicator requires more maintenance than a nonindicating controller.

The purpose of a recorder is to provide a record or a history of past performance. A recording instrument without a chart in place is almost useless. Recording charts are nearly always kept in file as a record of process performance. Between circular-chart and strip-chart recorders there is little choice except personal preference. Circular charts are somewhat easier to file than strip charts, which are usually rolled. Circular-chart instruments show an 8-hour, 12-hour, or 24-hour performance at a single view on the instrument. Strip charts are easier to use when the variable must be recorded at a low position (lower 10 or 20%) on the scale or when a number of variables are to be recorded on a single chart.

Signaling is accomplished with an instrument arranged with contacts operating at several points on the instrument span. Electric lights, horns, or sirens may be used as necessary. For two-position signaling a red light signifies high or unsafe and a green light low or safe. Three-position and five-position signals may also be used. In general, it may be said that sufficient use is not made of this very simple and effective means of indication.

CONTROL COMBINATIONS. Often it is necessary to measure and control combinations of two variables or to control one variable in relation to another.

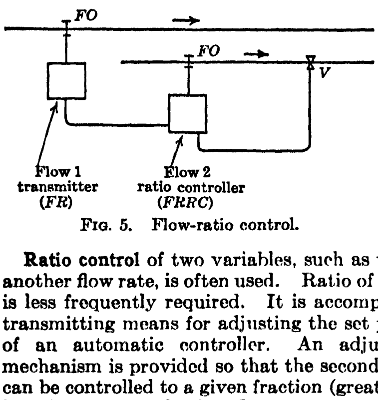

FIG. 5. Flow-ratio control.

Difference Measurement. The measurement of pressure differential (difference of two pressures) has been discussed. The measurement of temperature difference between two points can generally be accomplished by employing two thermocouples with the leads connected in opposition. A single instrument, usually a potentiometer, then operates from the difference in emf of the two thermocouples. Temperature difference can also be measured by using two resistance-thermometer bulbs in opposite arms of a resistance bridge.

Ratio control of two variables, such as the control of one flow rate to a desired ratio of another flow rate, is often used. Ratio of other variables such as temperature and pressure is less frequently required. It is accomplished (Fig. 5) by using one instrument with a transmitting means for adjusting the set point of an automatic controller. An adjusting mechanism is provided so that the second flow can be controlled to a given fraction (greater or less than one) of the first flow.

Program control is the control of a variable such as temperature to a desired value which varies with time. For example, in heat-treating furnaces and ovens it is often necessary to raise the temperature at a desired rate, hold it for a given time, and then cool at a certain rate as in annealing operations. This is accomplished by employing a cam or other means for adjusting the set point of an automatic controller to a fixed time schedule. Either thermometers or pyrometers may be used. In canning operations a pressure time cycle is sometimes required.

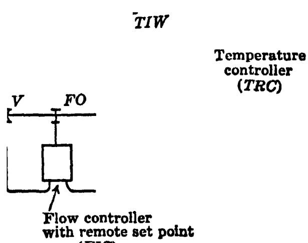

FIG. 6. Metered control.

Metered control is employed for the purpose of obtaining more exact control where fluctuations occur in the manipulated variable, other than the desired changes required for automatic control. The fuel-flow control arrangement (Fig. 6) is particularly susceptible to pressure changes at the valve. In this case, a flow controller operates a valve and controls the flow to the desired value. The set point of the flow controller is adjusted by a temperature controller for controlling the furnace temperature. If pressure fluctuations occur in the flow line, the flow controller practically eliminates the effect of these changes on the temperature of the furnace.

SECTION 19

POWER TEST CODES

ASME CODES

INSTRUMENTS AND APPARATUS

TEST CODE ABSTRACTS

ASME CODES

Tests to determine the performance of power-generating apparatus, or of equipment utilizing power, should be conducted under the Power Test Codes of the American Society of Mechanical Engineers. These codes prescribe the methods to be followed, the apparatus to be used, and the measurements and readings to be taken to determine completely the performance of power-generating or power-using apparatus.

Each type of apparatus is the subject of a separate code. In addition, supplementary information is published dealing with instruments and apparatus, whose use is common to all the codes. Complete copies of any code can be obtained from the American Society of Mechanical Engineers, 29 West 39th St., New York. Descriptions of codes in force are given herewith. A complete set of published Test Codes, and the Instruments and Apparatus Sections, may be purchased in two binders designed to hold them. The highest price of most individual codes, purchased separately, is about one dollar. A few are higher.

LIST OF ASME POWER TEST CODES.

Atmospheric Water-cooling Equipment (Pub. 1930). Applies only to equipment used for cooling of the comparatively large amounts of water required for power or industrial purposes.

Centrifugal, Mixed-flow and Axial-flow Compressors and Exhausters (Pub. 1950). In addition to a number of corrections and revisions, this edition includes new rules for the testing of superchargers and axial-flow compressors, in which the change in the gas specific weight exceeds 7%, and also for testing apparatus handling gases other than air. The code establishes rules for conducting and reporting tests which will determine under specified conditions one or more of the following quantities: (*a*) quantity of air or gas delivered, (*b*) pressure rise produced, (*c*) power required for compression, and (*d*) efficiency of the compressor.

Coal Pulverizers (Pub. 1944). Defines practice of testing pulverizers used for firing boiler furnaces, kilns, or industrial furnaces of various types.

Displacement Compressors, Vacuum Pumps, and Blowers (Pub. 1950). These rules show how to determine the performance of all positive displacement compressors, blowers, vacuum pumps, and all rotary type machines which operate on a positive displacement principle.

Dust-separating Apparatus (Pub. 1941). Designed for testing all types of dust-separating apparatus installed for operation in conjunction with solid-fuel fired furnaces in order to determine overall efficiency; efficiency according to size of particles; pressure loss; dust concentration at the inlet and outlet of separator; combustible content of dust entering, leaving, and caught by separator; quantity of gas passing through the separator.

Evaporating Apparatus (Pub. 1941). The testing of single- or multiple-effect evaporators according to these rules provides information on: (1) adaptability of apparatus, (2) best method of operation, (3) capacity or efficiency or both of new installation preparatory to acceptance.

Feedwater Heaters (Pub. 1927). Applies to open and closed feedwater heaters.

Fans (Pub. 1945). For conducting tests on blowers, fans, and exhausters of the centrifugal, axial, or mixed-flow types in which the fluid density change through the machine does not exceed 7%. Specifies the practice for conducting tests of fans to determine (1) pressure, (2) quantity of air or other gas, (3) power supplied to the fan shaft, and (4) efficiency, all under specified conditions of fan speed and air density. Provides instructions for arrangement of test equipment such as ducts, plenum chambers, flow straighteners, and instruments. See also Section 1, Art. 23.

Gas Producers (Pub. 1928). Intended primarily for testing producers whose gas is to be used for power purposes.

Gaseous Fuels (Pub. 1944). For determining the chemical and physical properties which serve as indicators of the value of those gaseous fuels which are extensively used in the generation of heat and power, or whose efficiency of utilization is to be ascertained.

Hydraulic Prime Movers (Pub. 1949). A set of standard rules for the testing of an individual reaction or impulse turbine unit of any type.

Internal-combustion Engines (Pub. 1949). This new code meets current requirements for a dependable set of rules for testing all forms of reciprocating internal-combustion engines, including gasoline engines, gas engines, and oil or dual fuel engines; and for evaluating the test results. Other recommendations cover the instruments, methods, and precautions to be employed.

Reciprocating Steam-driven Displacement Pumps (Pub. 1927). For determining the performance of the pump and engine, including reheaters, heaters, and jackets, if any, and jacket pumps, circulating pumps, condensate pumps, and vacuum pumps, which are concerned in their operation.

Reciprocating Steam Engines (Pub. 1935; reviewed and reaffirmed, 1949). Recommends standard testing methods for determining the performance of an engine, including steam jacket if any.

Stationary Steam-generating Units (Pub. 1946). For testing stationary steam-generating units defined as combinations of apparatus for producing, furnishing, or recovering heat, together with apparatus for transferring to a working fluid the heat thus made available. For purposes of this code such a unit may include boiler, water walls, water floor, water screens, superheater, reheater, economizer, air heater, furnace, and fuel-burning equipment. The code establishes rules for conducting tests to determine (*a*) capacity, (*b*) efficiency, (c) superheater characteristics, (*d*) any other operating characteristics. Instructions are given for two acceptable methods of testing for efficiency and capacity; (*a*) direct measurements of input and output, and (*b*) direct measurement of heat loss and either the input or output.

Steam Condensing Apparatus (Pub. 1938). Provides rules for determining (*a*) the absolute pressure the apparatus will maintain at the steam inlet nozzle when transferring heat rejected by the prime mover at a given rate in Btu per hour with a given flow and temperature of circulating water, and a given tube cleanliness, (*b*) thermal transmittance of surface condensers for given operating conditions, (c) the amount of undercooling of the condensate, (*d*) the percentage of dissolved oxygen in condensate.

Steam Locomotives (Pub. 1941). These two sets of rules outline (1) laboratory tests to determine the coal and steam consumption per unit of power when the locomotive is operated under fixed conditions, and (2) road tests to develop similar information under the conditions of road service. Consideration is given to measurements involved, instruments and apparatus required, preparations, operating conditions, duration of test, calculation of results, and method of calculating individual items.

Steam Turbines (Pub. 1949). This edition contains a number of revisions and changes, including important corrections in formulas for the determination of steam flow according to the enthalpy drop method. Rules provide for the testing of all types and applications of steam turbines, instruments to be employed and their applications, and methods of measurement. Instructions for determining the output of an electric generator driven by a steam turbine are also given.

Appendix to Test Code for Steam Turbine (Pub. 1949). The information in this 88-page pamphlet will facilitate the working up of test reports. Especially helpful are its numerical examples of many of the calculations involved in reporting tests conducted under the code rules, and filled-out hypothetical test forms.

POWER TEST CODES BEING REVISED are: Centrifugal and Rotary Pumps. Liquid Fuels. Refrigerating Systems. Solid Fuels. Speed Responsive Governors.

ABSTRACTED MATERIAL. Because it is impossible within reasonable space limits to quote each power test code completely, this section sets forth the general conditions of a few important codes. Selections have been made from these codes of only the more important material. In some instances portions of a paragraph have been deleted. It is necessary, therefore, that the reader consult the original code in any important case. The paragraph numbers of the original code are retained to facilitate reference, but since paragraphs have been deleted the numbers are not consecutive.

INSTRUMENTS AND APPARATUS bulletins are issued by ASME covering such broad fields as measurement of pressure, temperature, speed, flow, indicated horsepower, steam quality, time, linear measurements, surface areas, density, viscosity, humidity, smoke density, leakage, electrical quantities, and power. A few of the more important of the Instruments and Apparatus sections are abstracted herein, but for extensive information on even those abstracted, it is strongly recommended that the user consult the original Instruments and Apparatus sections.*

It is ASME practice to revise codes in accordance with developments in the field covered by the respective codes. The user should, therefore, be certain that the most recent code is used. The dates of approval of the material quoted herein precede the text.

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* Most of the following material is abstracted from official ASME publications. The original codes and sections of Instruments and Apparatus publications were prepared by the ASME Power Test Codes Committee.

INSTRUMENTS AND APPARATUS

1. TEMPERATURE MEASUREMENTS

(See Section 18, Instrumentation.)

2. SPEED MEASUREMENTS

(Approved, 1930)

Three methods of obtaining speed of rotating machinery are: (1) By a revolution counter and time piece. (2) By use of a tachometer. (3) By use of a stroboscope.

COUNTERS are instruments for determining number of revolutions or strokes of mechanisms. Continuous counters are used where a determination is sought over a considerable period of time. Hand counters are used for determinations over a short period. Continuous counters are of the following types.

A direct-geared counter comprises a series of gears arranged to rotate a set of dials or pointers that register number of revolutions. It is positive, accurate, and reliable.

A cyclometer-type counter is the next best type. The set-back type counts one for each revolution. A set-back device permits returning all figures to zero. It will add when turned forward but may not subtract when rotated backward. It is not recommended for determinations exceeding 500 counts per min. The locked-wheel type counts one for each revolution and has no set-back device. It is recommended for high count determinations, and may be operated at twice the maximum speed of the set-back counter.

Set-back rotary ratchet counters count 10 for each complete forward rotation of the driving shaft, which is rotated by an oscillating motion of a lever. An oscillation of 40 to 60 degrees will count one on the number wheels. Backward rotation will not disturb the number wheels. These counters may be operated at 150 counts per min, and for short periods at 200 or 300 counts per min. Without the set-back attachment, it may be run continuously at twice its normal speed if the driving shaft is fitted with ball-bearings.

Flat ratchet-type counters involve reciprocating motion and a ratchet drive. Unless the travel is controlled overtravel may occur, especially at high speed. This type, limited to slow count determinations, is unsuited to high-accuracy determinations.

Continuous counters usually are fastened rigidly to the machine. The primary drive is operated by gear, chain, or flexible shaft in determining revolutions, or a system of continuous or intermittent linkages in determining number of strokes.

Hand counters may be of either the dial or cyclometer type. If carefully handled and well made, they are fairly reliable over a range of 200 to 2000 rpm. The possibilities of errors in these instruments are slippage between the shaft and the point of the instrument where contact is made and inaccuracy in determining time between making and breaking contact. The counter should be allowed to operate at shaft speed for a few seconds, and, as the dial passes zero, the stop watch should be started. After running for about two minutes, the stop watch should be stopped as the dial again passes zero. This will give the time for an integral number of revolutions and eliminate errors due to slippage and sudden starting or stopping. The longer the period, the less the error.

THE TACHOSCOPE is an instrument in which revolution counter and stop watch are mounted in one frame and arranged to operate simultaneously. The range varies from 0 to 5000 rpm. Its accuracy depends on the stop watch incorporated in it.

The speed indicator is an instrument operating on the same principle as the tachoscope, and averages the speed over a short period of time, indicating directly the speed in rpm. It may be used to measure speeds up to 30,000 rpm. The errors usually do not exceed 0.3 to 1.0% for readings taken in the upper part of the scale.

TACHOMETERS. A tachometer indicates speed directly and continuously. Tachometers are made in several different types. They have a substantially constant rpm error over the upper 70 to 80% of the working range, which usually is 0 to 2500 or 0 to 1200 rpm. In most types the readings should be in the upper 75% of the range and preferably in the upper 50%. The driving gear ratio can be changed to meet any actual speed range.

Tachometers may be driven by any one of the following methods, the first three being preferred: (1) Gear or chain. (2) Flexible shaft or cable enclosed in flexible housing. (3) Speed pin threaded into the end of the revolving shaft under observation, with a metal sleeve, slotted in both ends, for a driving link between shaft and tachometer. These three are all positive drives. (4) Adapter on tachometer shaft pressed into contact with a so-called center on the rotating shaft of apparatus under test. (5) Belt and friction drive. This last method should be avoided as unreliable.

Centrifugal tachometers of the fly-ball and tilting-ring types operate through the centrifugal force produced by one balance ring or two or more revolving weights acting against springs. The compression of the springs moves the indicator or recorder pen. Good makes of this type of tachometer may have an error of 15 to 30 rpm between the range of 600 and 2500 rpm when new, and an error of 50 to 100 rpm, with a 25 to 50 or more rpm difference between up and down readings, after being in service for some time. The temperature effect may be approximately 1 rpm for each 10 F.

Liquid pump tachometers comprise a centrifugal pump, a reservoir for liquid, and an indicating tube into which the pump forces the liquid. Speed is determined by noting the position of the meniscus on the scale graduated in rpm. This instrument should be used only over the upper 40% of its range. The best instruments of this type, in good condition, can indicate speeds within 10 to 15 rpm at 6000 rpm.

Air tachometers are tachometers of the centrifugal type in which air is the working fluid as well as the transmitting medium. With it, indicating or recording mechanism may be located at a distance. An air tachometer should be used only in the upper 75% of its range. With a rotor making a maximum of 1500 rpm, it has an accuracy of 10 rpm when provided with a sensitive mechanism for indicating the air pressure produced and when operating under standard atmospheric conditions.

The autographic hydraulic pump tachometers use an autographic speed-recording mechanism which can portray speed changes incidental to changes of load, as in governor testing. It comprises three main elements, i.e., the tachometer pump, the indicating mechanism, and the auxiliary or return pump.

Force-drag tachometers depend on the drag produced by a rotating element which transmits a force from it to another movable part, to which the indicator needle is attached. The motion of the needle is limited by a spring. These tachometers are used largely on automobiles, but very little in power plant work. They are not recommended for use in connection with the Power Test Codes.

Chronometric tachometers comprise a repeating type of combination clock and revolution counter. They depend on direct revolution counting during an increment of time. They cannot be depended on closer than 6 to 7 rpm over the entire range of a scale of 2500 rpm maximum reading, and may have errors as great as 20 rpm when new, and an increased error of 5 or more rpm after a reasonable amount of continuous service, with possibility of complete failure.

Electric tachometers in commercial use are: (1) The electromagnetic type, consisting of a d-c generator with indicating volt meter. (2) An ordinary frequency meter calibrated to read speed instead of frequency. The electromagnetic type should be used with direct drive at not over 2500 rpm. Its accuracy is not closer than 20 rpm, and it may have a probable error of 0.5% for each 20 F temperature change.

The frequency meter tachometer may be of the vibrating reed type, the induction type, or the moving coil type. The reed type indicates only certain fixed speeds, corresponding to the individual reed, and these are rarely closer together than 0.5%. The range is from 800 to 12,500 rpm with an accuracy within 50 rpm for individual reeds when new. It may be accurate only within 100 rpm for individual reeds after use.

Vibration tachometers are similar in construction to vibrating reed frequency meter type tachometers, except that they depend only on the mechanical vibration produced by the rotating member. Their range and limitations are the same as those of the electrical instrument.

STROBOSCOPES utilize the persistence of vision when an object is viewed intermittently. The end of the rotating shaft under observation, or a disk attached to it, may be marked with several dots located equidistantly around the circumference of a circle whose center coincides with the center of the shaft. Vision is interrupted by using either a tuning fork arrangement or a rotating perforated disk driven by a separate machine. When sighting through a perforated disk, separately driven, the speed of this disk is varied until the marks or figures on the rotating shaft or disk appear to be stationary. Then the speed of the shaft under observation will be the same as that of the separately driven shaft, or some multiple of it, and equals (Indicated rpm × number of holes in disk) ÷ (number of images). The number of images is the number of times a single mark on the indicating shaft is seen through all the holes in the disk of the adjusted-to-speed stroboscope. There is practically no limit to the range over which the stroboscope can be used.

3. PRESSURE MEASUREMENTS *

(Approved, 1941)

BAROMETERS are used to measure pressure of the atmosphere. Readings, usually, are in inches of mercury. Barometric pressure plus gage pressure is the *absolute pressure.* Where measured pressure is less than atmospheric pressure, the pressure indicated by the reading of a U tube or vacuum gage is called the *vacuum.* The absolute pressure then is the difference between atmospheric pressure and the vacuum.

Mercurial barometer and mercury column readings always should be corrected to the value that would obtain if the mercury column were at 32 F. A calibration correction also should be applied. See below for methods of making these corrections.

The mercurial barometer is used generally and recommended. The diameter of bore of the glass tube of a barometer used under the test codes shall be not less than 0.25 in. The range usually is 25 to 32 in. Hg. Special barometers for mountainous regions begin at 15 or 20 in. A vernier attachment permits readings to 0.001 in., although for many engineering purposes readings to 0.01 in. are sufficiently accurate.

The aneroid barometer consists of an exhausted chamber whose ends are corrugated diaphragms. Atmospheric pressure on the diaphragms, balanced by a stiff spring, deflects them. The deflection is transmitted to a pointer which gives the reading, equivalent to that which would be given by a mercury barometer in inches or millimeters of mercury. Aneroid barometers are not as reliable as mercurial barometers, the temperature correction being uncertain. Their use is permissible only where accuracy in determination of barometric pressure is not of primary importance. The range of accuracy is 0.01 to 0.1 in.

The recording barometer or barograph is an aneroid barometer actuating a pen moving over a recording drum rotated by clockwork. It is subject to all the inaccuracies of the aneroid barometer, plus those due to the pen and its mechanism.

Installation of barometers. A barometer must be free from vibration, air currents, and violent temperature changes. It must be in good light, not directly exposed to the sun and not heated by nearby electric lamps. A mercurial barometer must hang free to insure a vertical position of the column. It must have a small vent to insure atmospheric pressure reaching the cistern.

BAROMETRIC CALIBRATION AND CORRECTION. Obtaining Barometric Pressure from Daily Weather Maps. Filed daily at all Weather Bureau stations, and in many post offices and other public places, are maps showing a set of "isobars" † or lines of equal barometric pressure, and "isotherms" or lines of equal temperature. These are for 7.20 A.M. Eastern Standard Time. Independent editions of the map are published at the principal Weather Bureau stations, such as New York, Boston, Chicago, and San Francisco, and copies for given days should be secured by application to the *nearest* such office.

If the station is at a different elevation from the barometer being calibrated, a correction should be made to compensate for this difference. Calibration also can be made by means of the weather maps. The location of the barometer is indicated by a dot on the map, and the barometric pressure is estimated by interpolation between the values shown on the two adjacent isobars. The following is an example of the calibration of a mercurial barometer at West Lynn, Mass.

Data	
Uncorrected barometric pressure	29.800 in. Hg
Check reading	29.801 in. Hg
Temperature	75 F
Height of cistern above floor	3 ft
Floor elevation above mean sea level	16.82 ft
Latitude	42.8 degrees N
Calibration	
Mean uncorrected reading	29.80 in. Hg
Temperature correction, Table 1	− 0.13 in. Hg
Elevation correction, Table 2	0.02 in. Hg
Gravity correction, Table 3	− 0.01 in. Hg
Corrected reading, reduced to sea level	29.68 in. Hg
Weather map reading, reported by Boston station of U. S. Weather Bureau reduced to sea level	29.67 in. Hg
Calibration correction	− 0.01 in. Hg

* See also Section 18.

† Since January 1, 1940, isobars on all published weather maps have been labeled in millibars in addition to inches of mercury. 1015.9 millibars equal 30.00 in. Hg; 1 millibar equals approximately one three hundredths of an inch (0.0295); one tenth of an inch of mercury equals 3.4 millibars, approximately.

Observed readings of barometer shall be corrected by the data in Tables 1 to 3, or by Tables in U. S. Weather Bureau Circular F, or by the Smithsonian Tables. The following is an example of a mercurial barometer reading, correction, and reduction at West Lynn, Mass.

Uncorrected barometric pressure (actual reading)	29 80 in. Hg
Barometer temperature	75 F
Temperature correction, Table 1	− 0.13 in. Hg
Gravity correction, Table 3	− 0.01 in. Hg
Calibration correction (*always* must be made)	− 0.01 in. Hg
Barometric pressure (at barometer elevation)	29.65 in. Hg
Level of barometer cistern *below* turbine center line	16 ft
Elevation correction, Table 2	− 0.02 in. Hg
Barometric pressure at elevation of turbine center line	29.63 in. Hg

Reduced reading, 29.63 × 0.4912 = 14.55 psi = absolute barometric pressure at elevation of turbine center line.

Temperature Corrections. Table 1 is used to correct, for temperature, the readings of a mercurial barometer, mercury gage, or U tube. The correction reduces the reading to the value that would obtain were the mercury at 32 F. The table also includes a slight correction for temperature expansion of a brass scale which is correct at 62 F.

Table 1. Temperature Corrections for Barometers and Mercury Columns

Temperature of Column, °F	Observed Reading of Column in Inches of Mercury								
	16	18	20	22	24	26	28	30	32
	Add								
−20	.07	.08	.09	.10	.11	.11	.12	.13	.14
−10	.06	.06	.07	.08	.08	.09	.10	.11	.11
0	.04	.05	.05	.06	.06	.07	.07	.08	.08
10	.03	.03	.03	.04	.0[illegible]	.04	.05	.05	.05
20	.01	.01	.02	.02	.02	.02	.02	.02	.02
30	.00	.00	.00	.00	.00	.00	.00	.00	.00
	Subtract								
35	.01	.01	.01	.01	.01	.01	.02	.02	.02
40	.02	.02	.02	.02	.02	.03	.03	.03	.03
45	.02	.03	.03	.03	.04	.04	.04	.04	.05
50	.03	.03	.04	.04	.05	.05	.05	.06	.06
55	.04	.04	.05	.05	.06	.06	.07	.07	.08
60	.05	.05	.06	.06	.07	.07	.08	.08	.09
65	.05	.06	.07	.07	.08	.09	.09	.10	.10
70	.06	.07	.07	.08	.09	.10	.10	.11	.12
75	.07	.07	.08	.09	.10	.11	.12	.13	.13
80	.07	.08	.09	.10	.11	.12	.13	.14	.15
85	.08	.09	.10	.11	.12	.13	.14	.15	.16
90	.09	.10	.11	.12	.13	.14	.15	.17	.18
95	.10	.11	.12	.13	.14	.16	.17	.18	.19
100	.10	.12	.13	.14	.15	.17	.18	.19	.20

Elevation Corrections. For exact work, gage readings must be corrected to account for weight of column of air between the level of the center line of apparatus under test,

Table 2. Elevation Corrections for Barometers and Pressure Gages

(Inches of mercury decrease of atmospheric pressure per 100 ft increase in elevation)

Mean Altitude, ft	Mean Atmospheric Temperature, °F						
	−20	0	20	40	60	80	100
0	.13	.12	.12	.11	.11	.10	.10
1000	.12	.12	.11	.11	.10	.10	.10
2000	.12	.11	.11	.10	.10	.10	.09
3000	.11	.11	.10	.10	.10	.09	.09
4000	.11	.10	.10	.10	.09	.08	.08
5000	.10	.10	.10	.09	.09	.08	.08
6000	.10	.10	.09	.09	.08	.08	.08
7000	.10	.09	.09	.09	.08	.08	.08

or any other reference plane, and the level where atmospheric pressure acts on the gage. (See Table 2.)

The correction is applied as follows. Let C = correction to be applied; c = correction value from Table 2; d = difference in elevation, feet. Then $C = (c \times d)/100$. If the

Table 3. Correction of Barometers and Mercury Columns to Standard Gravity

North Latitude, degrees	Elevation, ft											
	0	0	2000	2000	4000	4000	6000	6000	8000	8000	10,000	10,000
	Height of Column in Inches of Mercury											
	30	28	28	26	26	24	24	22	22	20	20	18
25	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.06	−0.05	−0.06	−0.05	−0.05	−0.05
30	−0.04	−0.04	−0.04	−0.04	−0.05	−0.04	−0.05	−0.04	−0.05	−0.04	−0.05	−0.04
35	−0.03	−0.03	−0.03	−0.03	−0.03	−0.03	−0.04	−0.03	−0.04	−0.03	−0.04	−0.03
40	−0.02	−0.01	−0.02	−0.02	−0.02	−0.02	−0.03	−0.02	−0.03	−0.03	−0.03	−0.03
45	−0.00	−0.00	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.02	−0.02	−0.02	−0.02
50	+0.01	+0.01	+0.01	+0.01	−0.00	−0.00	−0.00	−0.00	−0.01	−0.01	−0.01	−0.01

Table 4. Mercury-meniscus or Capillarity Corrections

(Differences in values given by different reliable authorities indicate uncertainties in these corrections.)

Internal Diameter of Tube, in.	Height of Meniscus, in.							
	.01	.02	.03	.04	.05	.06	.07	.08
	Correction in Inches to Be Added							
.40	.004	.007	.011	.014	.016	.019	.021	.023
.45	.002	.005	.007	.009	.012	.014	.016	.018
.50	.001	.003	.005	.006	.008	.009	.011	.012
.55	.001	.002	.003	.005	.006	.007	.008	.009
.60	.001	.002	.003	.004	.005	.005	.006	.007

Table 5. Multipliers for Water Columns for Precise Work *

(For work so precise that temperature of water columns needs to be taken into account, fresh distilled water shall be used, with precautions to avoid gradual dissolving of air. The effect of dissolved air on density does not seem to be known, but it is best to take no chances. The usual standard temperature of 68 F is a suitable standard for water columns.)

Temperature, °F	Density, lb per cu ft, ρ	Multipliers for Reducing Readings in Inches of Water Column to		
		Psi ($\rho/1728$)	In. Hg at 32 F (0.001178ρ)	In. H_2O at 68 F (ρ/ρ_{68})
32	62.420	.036122	.073530	1.0016
35	62.426	.036126	.073537	1.0017
40	62.429	.036127	.073540	1.0018
45	62.424	.036124	.073534	1.0017
50	62.412	.036117	.073520	1.0015
55	62.393	.036107	.073498	1.0012
60	62.368	.036093	.073470	1.0008
65	62.339	.036075	.073434	1.0003
68	62.318	.036063	.073410	1.0000
70	62.304	.036055	.073392	0.99976
75	62.263	.036032	.073345	0.99912
80	62.219	.036005	.073292	0.99840
85	62.169	.035977	.073234	0.99761
90	62.116	.035946	.073171	0.99674
95	62.058	.035912	.073103	0.99581
100	61.996	.035877	.073030	0.99483

* This table is based on the values of the density of water given in the International Critical Tables, Vol. III, p. 24. It is assumed that the inches of water column are standard inches, which requires a correction, not given here, of the actual readings, for variation of the measuring scale with temperature.

gage reading is below atmosphere and the gage is set below the reference plane, or if the gage reading is above atmospheric pressure and the gage is above the reference plane, C is *subtracted* from the gage reading. If the gage reading is below atmospheric pressure and the gage is above the reference plane, or if the gage reading is above atmospheric pressure and the gage is set below the reference plane, C is *added* to the gage reading.

Gravity corrections need not be made for most engineering work, but Weather Bureau barometric readings always include a gravity correction. Hence the corrections in Table 3 are necessary for accurate calibration by comparison with a Weather Bureau reading.

Meniscus corrections are not necessary with barometers, the correction being allowed for in setting of the scale. For mercury columns comprising a reservoir and a single leg, or for mercury U tubes with legs of different diameters, the corrections of Table 4 should be added to the reading taken at the top of the meniscus.

Head and Water Column Corrections. For precise work, the mean of temperatures at inlet and outlet of apparatus under test must be used. Values of density of water at

Table 6. Multipliers for Barometers and Mercury and Water U Tubes and Gages *

Instrument	Reading	Remarks	Multipliers for Reducing Readings to		
			Psi	In. Hg	Feet of Water
Barometer Mercury gage Mercury U tube	In. Hg corrected to 32 F	Correction made by Table 1	0.4912	1.00	$70.74/d$ exactly 1.134 for water †
Mercury gage Mercury U tube	In. Hg at t F	Use actual reading at actual temp. t	$0.4912 \times \left[1.0 - \frac{(t-32)}{10{,}000}\right]$	$1.0 - \frac{(t-32)}{10{,}000}$ ‡	$(70.74/d) \times \left[1.0 - \frac{(t-32)}{10{,}000}\right]$
Mercury gage Mercury U tube Barometer (in exceptional cases)	In. Hg †	Use uncorrected reading. Results are accurate at 56 F.	0.49		1.132 for water and mercury †
Mercury U-tube reading hydraulic pressure above atmosphere	In. Hg between levels of the two legs †	Water on top of lower column only, completely filling pressure pipe. Result is pressure at base of a column of water extending to zero level of mercury tube.	$0.49 - \frac{0.036}{2} = 0.472$	1.00 − 0.037 †	1.090 for water and mercury †
Differential mercury U-tube reading difference between two hydraulic pressures, both above atmosphere	In. Hg between levels in the two legs	Water on top of both columns completely filling both pressure pipes. Pressure is given between two points at same level.	0.49 − 0.036 = 0.454	1.00 − 0.074 †	1.048 for water and mercury †
Water U tube or water gage	In. water at t F.	Requires precision gage and precision computations. See Table 5.	$\frac{d}{1728}$	$0.001178d$	1/12 for same temperature of gage and column
Water U tube or water gage	In. water †	For usual engineering work	0.036	0.074 †	1/12
Bourdon pressure gage	Psi	Reading to be corrected per gage calibration	1.00	2.036 ‡ 2.04 †	$144/d$ exactly 2.31 for water †

* Readings are to be multiplied by the multipliers indicated. d is density of water, lb per cu ft, at actual temperature (see Table 5). † At usual room temperature. ‡ For in. Hg at 32 F.

various temperatures together with multipliers for reducing readings in inches of water to pounds per square inch or inches of mercury are given in Table 5. For ordinary engineering work variation of density with temperature need not be considered, and density is taken as corresponding to usual room temperatures. Multipliers for conversion of barometer and U-tube readings are given in Table 6.

(The following material is abstracted from Chapter 2, *Supplement on Instruments and Apparatus*, ASME. Approved Nov. 1945. Numbers are from original code.)

PRESSURE DEFINITIONS.

1. Static pressure is the force per unit area exerted on a wall by adjacent fluid (liquid or gas) which is at rest or which is flowing without disturbance along the wall of a conduit or *main*. It is measured by use of a *static hole* or *piezometer opening* in the wall, drilled perpendicularly to the surface adjacent to the fluid. *Static pressure* or merely *pressure*, as it is often called, must be distinguished from *total pressure* and *velocity pressure*.

7. Total pressure is greater than static pressure by an amount equivalent to the energy represented by the velocity of flow. It is exactly defined by the statement that frictionless, or isentropic expansion from fluid at rest with a pressure equal to the total pressure, into a region with pressure equal to the static pressure, would produce the existing velocity.

8. Total pressure is measured by use of an open ended tube pointing into the stream, known as an *impact tube*. The excess of total pressure as shown by an impact tube, over the static pressure, is the exact equivalent of the energy represented by the velocity, being the pressure rise due to perfect conversion of the velocity into pressure. This difference between total and static pressure is known as *velocity pressure* or *pressure equivalent of velocity head* or, sometimes, simply *velocity head.*

STATIC HOLES.

20. The size of static pressure holes (sometimes called piezometer openings) for engineering purposes usually should be the tap drill for $^1/_2$, $^3/_8$, or $^1/_4$ in. standard pipe thread, or $^1/_2$ to $^3/_4$ in. diameter. Smaller holes may be used if sufficient care is taken and are recommended for smooth-bore conduits having diameters of 3 in. and less.

23. A static pressure hole shall be drilled at right angles to the surface of the wall adjacent to the fluid. The hole shall be straight and of uniform size for a length from the wall adjacent to the fluid, equal to at least two hole diameters.

26. The lengths of straight main given in the following shall be provided between static holes (or holes for impact tubes) and irregularities such as valves, reducers, elbows, scrolls, or casings of pumps and compressors, etc.

A length of straight main at least equal to 5 diameters shall be provided between a symmetrical irregularity such as a reducer or a sudden change in main size, and a succeeding pressure hole.

A length of straight main at least equal to 10 diameters shall be provided between the further end of an unsymmetrical irregularity such as an ell, globe valve, etc., and a succeeding pressure hole.

A length of straight main at least equal to 2 diameters shall be provided between a pressure hole and a succeeding irregularity of any kind.

31. A *piezometer ring* which consists of a circular chamber or tube surrounding a conduit and connected to it by a number of static holes has often been used to obtain static pressure. A single pressure-measuring instrument is then attached so as to give the static pressure at one point in the piezometer ring.

32. A *static tube* may be used to measure static pressure in the midst of a fluid in a main with undisturbed flow. The static pressure is constant through the body of the fluid, and usually the easiest way to obtain it is by means of a static hole in the main wall, so that a static tube should be rarely used.

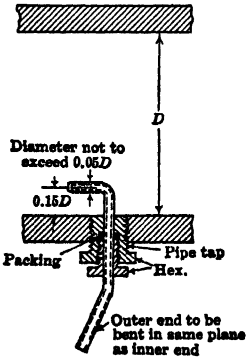


FIG. 1. Recommended location of impact tube.

IMPACT TUBES.

38. An impact tube is a tube with an open end pointing directly upstream, which, when connected to a pressure-measuring instrument, gives *total pressure*.

40. An impact tube is shown in Fig. 1. It shall be inserted in the main so that the open end is at a distance equal to 0.15 of the conduit width from the wall. This places it at a point 0.7 of the distance from the center of the conduit to the wall, where the velocity is approximately equal to the average value. Lengths of straight main shall precede and succeed the tube as specified in Par. 26. The outer diameter of the tube should not exceed about 5% of the conduit width to avoid restriction of flow.

46. The Pitot tube (sometimes called *Pitot-static tube*) is a combination impact tube and static tube which gives both total and static pressure through independent pipes assembled together.

(The following material is from Chapter 4, *Instruments and Apparatus*, ASME. Approved Aug. 1938.)

GENERAL.

1. Pressure and vacuum gages, as ordinarily used, are instruments for measuring the difference between atmospheric pressure and the pressure in a main or apparatus.

4. Bourdon gages transmit the pressure to a curved elastic tube closed at one end. The tendency of the tube to "straighten out" when pressure is applied causes motion of the closed end, which is transmitted to the pointer. In indicating gages, the Bourdon tube sometimes covers an arc of about 300 degrees. In some gages there are two Bourdon tubes whose differential motion operates the pointer. In some gages the Bourdon tube is in the form of a spiral or helix with a number of complete turns.

5. Bellows gages transmit pressure to an elastic chamber formed with two or more diaphragms or a corrugated cylinder. This construction is used principally for low-pressure gages.

6. Diaphragm gages use a chamber with a flexible diaphragm on one side, whose motion is transmitted by a mechanism or a mercury column. This type of gage was formerly much in use, but it is being displaced by Bourdon gages.

Another similar type, known as a chemical or protected gage, has the chamber above the diaphragm with a connected Bourdon tube, filled with mercury, glycerin, or other liquid.

47. Deadweight gages may be used directly for accurate pressure measurement to avoid the necessity for frequent calibration of an ordinary gage. One proprietary form is described in *Power*, April 7, 1931, p. 567.

Ordinary deadweight testers may be used, if there is provision for an upper stop to prevent excess pressure from bouncing the piston and weights out on the floor.

(The following material is from Chapter 5, *Instruments and Apparatus*, ASME. Approved March 1942.)

1. Liquid column gages, sometimes referred to as manometers, have a U with glass legs partly filled with liquid. At the top of one leg is a *gage pipe connection* for attachment of a *pipe for pressure measurement* leading to a point where pressure is to be measured. The liquid rises in the other leg and the measurement of the difference in level gives the pressure. A vacuum is measured in similar manner, the liquid level rising in the leg to which the connection is made.

3. Differential pressure, or the difference between two pressures applied to the two legs of the U, is measured by a *differential U tube* or *differential gage* which has a gage pipe connection at the top of each leg.

4. Absolute pressure of a vacuum occasionally is measured directly by applying it to one leg of the U of an *absolute gage* which has a completely evacuated space at the top of the other leg.

5. Vertical U tubes are the simplest and most common of the many types of liquid column gages. They are called U tubes, mercury columns, water columns, manometers, U gages, inverted U tubes, etc.

6. Single leg tubes are another common type, having one leg of the U shortened and increased in cross section so as to form a well, cistern, or reservoir at the lower level, which is connected at the bottom to the glass leg in which the liquid rises. They are called single leg tubes, mercury columns, water columns, mercury gages, water gages, manometers, well type manometers, mercury pots, etc.

7. Inclined gages or liquid draft gages are similar to single leg U tubes, except that the glass leg carrying the liquid is inclined so as to increase the length of the scale to give a magnified reading of small pressures. Other instruments for measurement of small pressures are two-liquid U tubes, hook gages, micromanometers, and bell gages.

24. Hook gages have many forms. They consist of two chambers, usually having exactly the same area, connected at the bottom, and filled with water so as to form a U tube. The maximum pressure which can be measured is usually 4 to 6 in. of water. One of the chambers is closed at the top, and the pressure to be measured is applied to it. The other chamber, open to atmosphere at the top, is provided with a hook, raised and lowered with a micrometer screw having a graduated head and scale, so as to give the level of the water to 0.001 in.

The hook gage is used primarily for calibrating inclined gages, bell gages, and the like. It may be used for direct measurement of very steady pressures.

25. Micromanometers are vertical, single leg tubes with change of liquid level given by adjustment of a micrometer screw reading to 0.001 or 0.0001 in. There are many forms.

26. Absolute pressure gages are mercury U tubes, one side being closed at the top and completely evacuated, and the other side connected to the vacuum to be measured. The difference in levels in the legs gives the absolute pressure and may be measured with suitable scales equipped with verniers and sliders.

Unless there is some precaution to preserve the evacuation, an absolute gage shall be used only a short time after evacuation or calibration.

TEST CODE ABSTRACTS

4. TEST CODE FOR STATIONARY STEAM-GENERATING UNITS

(July 1946)

SECTION 1. OBJECT AND SCOPE.

6. The purpose of this code is to establish rules for conducting tests to determine: (*a*) Capacity. (*b*) Efficiency. (*c*) Superheater characteristics. (*d*) Any other operating characteristic.

7. Instructions are given for two acceptable methods of testing for efficiency and capacity. Method (*a*), the direct measurement of input and output, is the preferred method of testing and shall be employed in all cases where it is practicable to do so. Method (*b*), the direct measurement of heat loss and either the input or output, shall be used only where the direct measurement of both input and output is not feasible or cannot be made within acceptable limits of accuracy, or where this method is mutually satisfactory to the parties concerned. The method followed in conducting the tests shall be clearly defined in the report.

The fundamental measurements needed for the input-output method are the quantity and heat value of the fuel, and the quantity, and heat absorption per pound, of steam generated. It also is necessary that sufficient heat loss data be taken to permit checking the input-output results. The heat-loss method requires especial attention to all factors of importance in the evaluation of losses in relation to input: sampling and analysis of fuel, flue gas, flue dust, and refuse; measurement of flue gas temperature; and determinations of humidity of combustion air and loss due to radiation and convection.

9. Method (*a*) of this code is applicable to all conditions of steam generation, its practical application being limited only by the measuring facilities to be made available.

Method (*b*) of this code applies to all types of steam-generating units operating with either solid, liquid, or gaseous fuels. In cases of tests to be conducted under Method (*b*) where steam generators are designed for normal operation on mixed fuels it will be necessary to make special agreement between the interested parties as to the method of test, or to perform separate tests using the different fuels one at a time. If separate tests are run using single fuels, this code will apply. If tests are made using mixed fuels, the guiding principles and general intent of this code should form the basis of the special agreement between the parties concerned.

SECTION 2. ENUMERATION AND DESCRIPTION OF TERMS.

12. Numerical Subscripts. The diagram of a steam-generating unit, shown in Fig. 1, is intended to serve as a key to numerical subscripts employed throughout this code to indicate the point or location referred to.

13. Letter Subscripts. The letter subscripts have the following meanings:

A = air	n = net
d = dust, or stack refuse	p = pit refuse
f = fuel	s = steam
g = gross	w = feedwater
G = gases of combustion	x = auxiliary

The chemical symbols are also used in some cases as subscripts.

With so many quantities involved and so many points of reference, it has been found impossible to restrict the code to the use of single subscripts. Where both letter and numerical subscripts are used, the numerical one is given second, for example, W_{s7}.

48. Test and Run. In order to eliminate the possibility of misunderstanding which might be caused by the use of the word "test" to mean either the entire investigation or one of its subdivisions, throughout this code the word "test" is applied only to the entire investigation, and the word "run" to a subdivision. A run consists of a complete set of observations made for a period of time with one or more of the independent variables maintained virtually constant.

SECTION 3. GUIDING PRINCIPLES.

53. Items on Which Agreement Shall Be Reached. The parties concerned in the test shall reach a definite agreement as to the specific objectives of the test.

Often the boiler proper and the different accessory parts of the steam-generating unit, such as fuel-burning equipment, air heater, and water-cooled walls of the furnace, are supplied by different manufacturers. This tends, in some cases, to complicate the responsibility for the performance guarantees. For instance, the manufacturer of the fuel-burning equipment might have given efficiency guarantees for the entire unit, based in part on assumed or guaranteed performance values for the heat-exchange apparatus. Even with only one manufacturer supplying the entire unit, it may be that not all the performance values given in the contract are guaranteed. It is not uncommon to see guarantees made at only one rate of output (usually the maximum continuous) and the other values labeled "expected performance." An agreement shall be made prior to the test which specifically allocates respective responsibilities for all performances and the operating conditions which are to be considered as part of the conditions of the test.

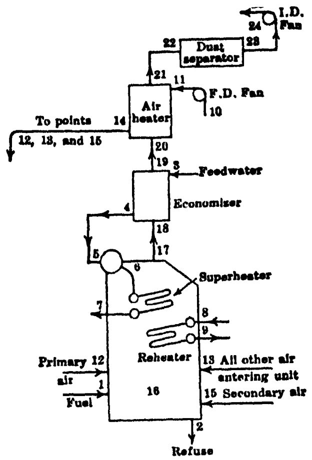


FIG. 1. Diagram of steam-generating unit showing locations in all circuits that correspond to numerical subscripts of the code.

Unless otherwise specified, efficiency shall be understood to mean gross efficiency. By agreement, however, the acceptability of a steam-generating unit may be based on the net efficiency, in which case the Appendix shall be followed.

In tests conducted to determine capacity characteristics the interested parties shall agree to a carefully prepared definition of the word "capacity" as it is intended to apply to the particular test.

Unless otherwise agreed, the thermodynamic properties of steam and water shall be taken from current editions of *Thermodynamic Properties of Steam*, by Keenan and Keyes, or *Vapor Charts*, by Ellenwood and Mackey.

Agreement shall be reached as to any instruments or methods of measurement to be employed which are not prescribed by this code. Any such deviations from the prescribed rules shall be fully explained in the test report.

57. The parties to the test shall agree upon a laboratory of recognized standing to make the necessary fuel analyses. Duplicate samples shall always be kept so that in case of mishap to the original samples duplicates will be available.

59. Preparation for Test. An acceptance test shall be undertaken only when the manufacturer or manufacturers certify that the unit is operating to their satisfaction and is, therefore, ready for test. Especially in the case of the fuel-burning equipment, adjustments and sometimes changes are necessary to obtain optimum performance. The preparations for an acceptance test preferably should be started as soon as the unit is in satisfactory condition for test, provided the load and other governing factors are suitable. Delays for longer periods may require maintenance in addition to cleaning in order to reproduce substantially new conditions.

60. In the case of an acceptance test all heat-transfer surfaces, both internal and external, shall be thoroughly cleaned before starting the test. During the test, only the normal cleaning shall be allowed.

The furnace setting, gas and air conduits, casing of the economizer and of the air heater shall be checked for leakage. Any leaks detected shall be stopped before the test is started.

62. A preliminary run shall be made for purposes of:

(*a*) Checking the operation of all instruments.
(*b*) Training the observers and other test personnel.

(c) Making minor adjustments, the needs for which were not evident during the preparation for test, and establishing proper combustion conditions for the particular fuel and rate of burning to be employed. If the manufacturer has passed upon the suitability of the equipment for test, as required in Par. 59, he shall be a party to such adjustments.

After a preliminary run has been made, it may be declared an acceptance run, upon mutual agreement, provided that all the requirements of a regular run have been met.

63. Starting and Stopping. Combustion conditions, rate of feeding fuel (also quantity of fuel on grate if hand or stoker-fired), rate of feeding water, water level in drum, excess air, and all controllable temperatures and pressures shall be, as nearly as possible, the same at the end of the run as at the beginning. These, and any other conditions in which variations might affect the results of the test, shall be caused to attain and hold the closest practical approximation to constancy for whatever period may be required to assure that the temperatures of the refractories of the setting and all other parts of the equipment have reached equilibrium before the run is started and shall be so maintained throughout the run. The time required to attain stabilization or equilibrium with respect to temperatures will vary widely with the design of the unit and character of materials in the setting, but, in general, at least three hours of operation at the output to be maintained during the run should be allowed for stabilization purposes. In some instances it may be necessary to terminate a run prematurely because of inability to maintain one or more of the above conditions at the desired value. The conditions which will call for terminating a run should be agreed upon before starting the test.

64. Duration of Runs. For determining the efficiency of a steam-generating unit when hand or stoker-fired, runs by Method (a) preferably should be of at least 24 hours duration. However, if conditions make it advisable or necessary, the length of a run may be reduced, but not to less than 10 hours, nor to a period less than that required to burn a total of 500 lb per sq ft of grate area. The longer the duration of runs the less will be the possibility of significant error from difference in water level or due to estimating the difference in amount of unburned fuel on the grate at the beginning and end of the run. In many cases it is difficult to estimate the change in thickness of a large fuel bed closer than 3 in. When the ratio of ash to unburned fuel is also indeterminate, the final estimate of effective change in thickness will frequently be in error by as much as 4 in. The possible error due to estimating the effective change in the amount of unburned fuel on the grate at the beginning and end of each run should be considered in determining the duration of each run. Runs by Method (b) shall be of at least four hours' duration.

For units fired with liquid or gaseous fuel, the runs preferably should be of not less than 10-hour duration; when conducted by Method (a), Par. 7, they shall be not shorter than 6 hours, and when conducted by Method (b) they shall be not shorter than 4 hours. These requirements apply also to pulverized fuel, provided the unit-mill system is used and, for tests conducted by Method (a), provided the fuel can be weighed as it is fed to the mills. For those stations having a centralized pulverizing plant, it is practically impossible to weigh the fuel fed to any one unit. In some cases a bunker can be segregated and the amount of fuel consumed during the run can be determined from the difference in amount of fuel in the bunker at the beginning and end of the run. Where Method (a) is used for such tests, the runs shall be of sufficient length to reduce the error due to estimating the amount of fuel fired to less than 1%.

For waste-heat boilers efficiency runs shall be for not less than 6 hours.

65. Number of Runs. For purposes of an acceptance test at least two runs shall be made approximately at each selected load. It is desirable, but not mandatory, that runs be made at not less than four loads so that a curve may be drawn to relate the test points, the loads being selected so as to embrace the load or loads for which guarantees are made.

66. Frequency and Consistency of Readings. Except for quantity measurements, the readings shall be taken at 15-min intervals. If, however, there are sudden and wide fluctuations, the readings shall be taken at 10-min intervals, or at such frequency as may be necessary to determine the true average. Where the amount of fuel or feedwater is determined from integrating instruments, a reading shall be taken every hour. If these quantities are weighed, the frequency of weighing is usually determined by the capacity of the scales, but the intervals shall be such that a total can be obtained for each hour of the test. The time of dump of each hopper of coal or of each tank of feedwater shall be recorded. The indicating element for venturi tubes, nozzles, or orifice plates shall be read at 5-min intervals or more frequently when the fluctuations are large.

67. Rejection of Runs. Should serious inconsistencies in the observed data be detected during a run, or during the computation of results, the run shall be rejected in whole or in part. A run that has been rejected shall be repeated if necessary to attain the objectives of the test.

SECTION 4. INSTRUMENTS AND METHODS OF MEASUREMENT.

68. The necessary instruments and mandatory rules for making measurements are prescribed herein. Specific references are made to ASME Power Test Codes, to Supplements on Instruments and Apparatus, and to other publications describing detailed methods and apparatus which shall be employed in testing stationary steam-generating units under the code. This code is confined to the essential requirements which shall be followed except with special agreement to the contrary. (See Section 3, Guiding Principles.)

The instruments generally required for an acceptance test of a steam-generating unit are included in the following list, which is presented merely for check purposes. Only those instruments necessary to attain the desired objective need be used, together with any others, not listed, which may be necessary.

(*a*) For Input Quantity Measurement: fuel-weighing scales, weigh hoppers, weigh larries, weighing tanks, volumetric tanks, flowmeters, etc.
(*b*) For Output Quantity Measurement: scales, weigh tanks, volumetric tanks or flowmeters, to measure quantities of steam generated and/or reheated.
(*c*) For Quality of Saturated Steam at Boiler Outlet and Reheater Inlet: suitable sampling and measuring apparatus.
(*d*) For Total Solids Determination in Steam: suitable sampling and conductivity apparatus.
(*e*) For Temperature of Steam at Superheater Outlet, Reheater Inlet, and Reheater Outlet: suitable temperature-measuring instruments.
(*f*) For Temperature of Feedwater Entering Economizer, Leaving Economizer, and Entering Boiler: suitable temperature-measuring instruments.
(*g*) For Steam Pressure at Boiler Drum, Boiler Outlet, Superheater Inlet, Superheater Outlet, Reheater Inlet, Reheater Outlet: Bourdon Gages, Deadweight Gages.
(*h*) For Feedwater Pressure: Bourdon Gages, Deadweight Gages.
(*i*) For Flue Gas Analysis at All Required Points: suitable apparatus for sampling and analysis.
(*j*) For Flue Gas and Air Temperatures: suitable thermometers, pyrometers, etc.
(*k*) For Flue Gas and Air Pressures: suitable manometers.
(*l*) For Flue Gas and Air Quantity Measurements: suitable flowmeters.
(*m*) For Refuse Measurement: suitable scales and containers.
(*n*) For Dust Measurements on Air, Fuel Gas, or Flue Gas: suitable sampling and separating apparatus.
(*o*) For Smoke-density Measurement: acceptable charts and other apparatus.
(*p*) For Auxiliary Energy Measurement: the necessary electrical instruments, orifices, nozzles, thermometers, pressure gages, etc.
(*q*) For Fuel Temperature Measurement: the necessary thermometers and wells.
(*r*) For Fuel Pressure Measurement: suitable gages and manometers.

SOLID FUEL.

69. Quantity Measurement. Fuel shall be weighed near the point where it is to be used. All loss of fuel between the point of weighing and the point of introduction to the equipment under test shall be eliminated or measured and accounted for to the satisfaction of all parties concerned. The weighing scales shall be checked prior to and after the test and caused to weigh to an accuracy within 2 in 1000 in the range of loads weighed. Checks and calibrations shall be made in accordance with Instruments and Apparatus, Part 5, Measurement of Quantity of Materials. Permissible types of scales are: weigh hoppers (stationary or traveling); platform scales in conjunction with suitable containers such as wheelbarrows or cars; and track scales. Complete emptying of hoppers, cars, or other containers between weighings shall be assured or proper corrections made. Amount of fuel in the bin or hopper to which the weighed fuel is discharged shall either be the same at the start and at the conclusion of the run (within limits satisfactory to the interested parties), or this bin or hopper shall be empty at the start of the run and the fuel remaining at the end of the run shall be dumped, weighed, and allowed for.

Arrangement and operation of fuel-weighing equipment shall preferably be such that checks on consumption during each hour of the run can be made as a matter of convenience and guide. Only the totals, however, are to be used in the final calculations.

70. Sampling. A representative sample of fuel shall be obtained in accordance with the Test Code for Solid Fuels. Loss of moisture due to strong drafts at the point of sampling (such as may occur at a pulverizer feeder for example) shall be guarded against. Samples of air-borne pulverized fuel shall be obtained as described in the Test Code for Coal Pulverizers, which requires sampling on traverses 90 degrees apart in the cross section of the fuel duct. The sampling time at each point should be proportional to the area represented by that point,* and the recovery should lie within limits of 90 to 110%, i.e., the rate of sampling should be so adjusted that the weight of the sample is to the total weight of the fuel as the cross-sectional area of the sampling tip is to the cross-sectional area of the fuel duct, within limits of ±10%.

71. Determination of Fuel Characteristics. Analysis and heat value determination shall be made in accordance with the Test Code for Solid Fuels, and Supplement on Instruments and Apparatus, Part 9 on Heat of Combustion. Fineness determination shall be made as specified in the latest revision of ASTM Standard Method of Sampling and Fineness Test of Powdered Coal, D-197-30, or by such superseding method as may be adopted by the ASME. Determination of such characteristics as grindability, etc., shall be in accordance with appropriate ASTM methods.

LIQUID FUEL.

72. Quantity Measurement. Liquid fuel quantity measurements shall, if possible, be made by weigh tanks or volumetric tanks. Meters may be used only upon agreement between the interested parties and shall be carefully calibrated under conditions simulating those existing during the test in regard to grade of oil, temperature, pressure, rate of flow, and meter position.

Leakage between point of measurement and point of firing shall be either eliminated or measured and accounted for. Branch connections to the piping shall be either blanked off or provided with valves and suitable tell-tale drains for detecting leakage. Leakage from valve stuffing boxes shall be eliminated. Any unavoidable leakage from pump stuffing boxes, or elsewhere, shall be collected and accounted for. In the event that the fuel piping leakage can be neither eliminated nor made small enough so that with due regard to possible influence on test results it may be ignored, the test shall be conducted in accordance with Method (*b*), Par. 7, for acceptance under this code.

Practice and precautions relative to the use of weigh tanks and volumetric tanks for liquid fuel measurement shall be those stated in Pars. 78 and 79.

73. Sampling. A representative sample of fuel shall be obtained in accordance with the Test Code for Liquid Fuels.

74. Determination of Fuel Characteristics. Analysis, high-heat value, density, viscosity, etc., shall be determined in accordance with the Test Code for Liquid Fuels, Instruments and Apparatus.

GASEOUS FUEL.

75. Quantity Measurement. Measurement of the relatively large volumes of gaseous fuel normally encountered in steam-generator testing requires the use of meters of the orifice, nozzle, or Pitot type. These methods of measurement should be undertaken only by those with knowledge and experience in the technique involved.

Pitot tube construction and operation shall be in accordance with Instruments and Apparatus.

The recommendations of Instruments and Apparatus shall be followed with reference not only to the design, construction, calibration, and use of flow nozzles and orifices, but also to their location, installation, and use in the pipe line and the installation of the connecting piping system between the primary element and the manometer. All computations of flow rate from the observed differentials, pressures, and temperatures shall be made in accordance with the provisions of Instruments and Apparatus.

If compressor pulsations or flow irregularities from control or regulating equipment are present, the differential pulsation shall be checked with a pulsometer, and, if in excess of 20 per cent of the average differential in amplitude, shall be reduced to this limit by introduction of capacity and resistance between the source of pulsation and the flow nozzle or orifice plate or by other means before measurement is considered acceptable.

Pressure of the gas at point of volume determination shall be measured by a suitable manometer as described in Instruments and Apparatus. Temperature shall be measured with thermometers in accordance with Instruments and Apparatus.

76. Sampling. The gas shall be properly sampled in accordance with the Test Code for Gaseous Fuels.

77. Determination of Fuel Characteristics. Gaseous fuel characteristics shall be determined in accordance with the Test Code for Gaseous Fuels; Instruments and Apparatus.

OUTPUT MEASUREMENT.

78. Weigh Tanks. The preferred method of measuring output is by weighing the feedwater. With suitable weigh tanks an accuracy of ±0.2% can be expected. If the quantity of water evaporated is measured by any method other than direct weighing, the measuring

* Sampling will be simplified if the sampling points are arranged to represent equal areas, thus making the sampling time the same for each point.

apparatus, its calibration and operation shall be agreed upon by the interested parties and fully described in the test report.

The water-weighing apparatus or other output measuring devices shall be arranged in an accessible place and made as convenient as possible.

Weigh-tank scales shall be calibrated over the entire range of loads at which they are to be used.

It shall be ascertained that inlet and outlet valves do not leak when closed.

The tank system shall be free from external force, so nothing can affect the weight reading except the tare and the water to be weighed. Tare shall be checked before each filling.

After each dumping it shall be made certain that dribbling has ceased before the outlet valve is closed.

Design, construction, calibration, and operation of weighing tanks shall be in accordance with Instruments and Apparatus.

79. Volumetric Tanks. Suitably designed volumetric tanks are capable of an accuracy within $\pm 0.5\%$.

Volumetric tanks shall be calibrated prior to the test with weighed increments of water at some fixed temperature. In the use of the tanks, suitable allowance shall be made for difference between the temperature of the water weighed and the temperature during calibration, cognizance being taken of thermal expansion of both the water and the tank metal.

The precautions given in Par. 78 shall be observed wherever they apply to volumetric tanks.

Design, construction, calibration, and operation of volumetric tanks shall be in accordance with Instruments and Apparatus.

80. Venturi Tube, Flow Nozzle, or Thin-plate Orifice for Feedwater. Feedwater quantity may be measured by venturi tube, flow nozzle, or thin-plate orifice upon agreement by the parties to the test.

The recommendations of Instruments and Apparatus shall be followed with reference not only to the design, construction, calibration, and use of flow nozzles and orifices, but also to their location, installation, and use in the pipe line and the installation of the connecting piping system between the primary element and the manometer.

Venturi tube, nozzle, or orifice design shall be such that the differential pressure as shown by the manometer is at least 6 in. of mercury.

If fluctuations in flow due to a reciprocating pump or other source of pulsation are present, these fluctuations shall be reduced to not more than 10 per cent of the average flow by the introduction of a cushion chamber, surge chamber, or other means of absorbing the surges between the source of pulsation and the primary device before measurement is considered acceptable.

Differential pressure from the primary device shall be measured by two complete manometer systems which shall check within $^1/_2$ per cent of the differential pressure, except that by agreement among all interested parties a single manometer and set of taps may be used. At least one manometer of a two-manometer system, and the calibrating manometer, if required, shall be in accordance with Instruments and Apparatus. If a single manometer is used, it shall be in strict accordance with Instruments and Apparatus and shall be calibrated before and after the test as specified therein. A commercial flowmeter may be used in place of one manometer of a two-manometer system, or by agreement for the single manometer of a one-manometer setup.

81. Flow Measurements of Steam. The output quantity may be determined by means of nozzles or thin-plate orifices upon agreement by the parties to the test.

The recommendations of Instruments and Apparatus shall be followed with reference not only to the design, construction, calibration, and use of flow nozzles and orifices, but also to their location, installation, and use in the pipe line and the installation of the connecting piping system between the primary element and the manometer.

Differential pressure from the primary device shall be measured by two complete manometer systems which shall check within $^1/_2$ per cent of the differential pressure, except that by agreement among all interested parties a single manometer and set of taps may be used. At least one manometer of a two-manometer system, and the calibrating manometer if required, shall be in accordance with Instruments and Apparatus. If a single manometer is used, it shall be in strict accordance with Instruments and Apparatus and shall be calibrated before and after the test as specified therein. A commercial flowmeter may be used in place of one manometer of a two-manometer system, or by agreement for the single manometer of a one-manometer setup.

If pulsations due to reciprocating engines or flow irregularities from control or regulating equipment are present, the differential pulsation shall be checked with a pulsometer, and, if in excess of 20 per cent of the average differential in amplitude, shall be reduced to this limit by introduction of capacity and resistance between the source of pulsation and the flow nozzle or orifice plate or by other means before measurement is considered acceptable.

FLUE GLAS SAMPLING AND ANALYSIS.

86. Sampling Locations. Analysis of the exit gases is the usual practice, but frequently analyses are required at other points. There may be considerable variation in gas analysis over the cross section of the gas passage due to stratification and air infiltration. The best practical method of obtaining a true sample is to divide the cross section of the gas passage into equal areas and to take velocity measurements and gas samples from the centers of these component areas. A weighted average can then be calculated, taking into consideration the gas temperature (Par. 90) as well as the velocity.

GAS AND AIR TEMPERATURE MEASUREMENTS.

90. Flue-gas Temperature. Flue-gas temperature is normally measured at the boiler outlet, economizer outlet, and air heater outlet but may in certain instances be measured at other points such as at the inlet to a waste-heat boiler.

Because of stratification, the accurate determination of flue-gas temperature generally requires that temperatures at several points over the cross section of the gas passage be measured and averaged. Number and location of the points depend upon the size and shape of the passage and upon the variation in temperature and velocity over its cross section. A preliminary survey of the passage shall be made as a guide in the location of temperature measurement points.

COMPUTATIONS. The section of the code covering computations is too lengthy for inclusion herein. The reader should consult the complete code.

5. TEST CODE FOR STEAM TURBINES

(Approved Sept. 14, 1948)

SECTION 1. OBJECT AND SCOPE.

1. The purpose of this code is to establish rules for conducting tests of a steam turbine to determine the following: (*a*) Capacity. (*b*) Steam or heat consumption. (*c*) Engine efficiency. (*d*) Emergency governor operation.

2. This code provides for the conduct of tests and for the computation and tabulation of the results, for turbines of the following types:

(*a*) Complete expansion condensing turbines in which all the steam enters at one pressure and all the steam leaves at a pressure less than that of the atmosphere.

(*b*) Condensing turbines similar to (*a*) except that the steam is reheated after partial expansion.

(*c*) Condensing turbines similar to (*a*) but operating on a regenerative cycle, i.e., steam being extracted from one or more stages solely for heating the unit's own feedwater. This class may include turbines that supply extraction steam for heating make-up feedwater, also evaporators and deaerators serving as extraction feedwater heaters.

(*d*) Condensing turbines similar to (*a*) but provided with both the special features described in (*b*) and (*c*).

(*e*) Noncondensing and back-pressure turbines in which all steam enters at one pressure and all steam leaves at a pressure equal to or greater than that of the atmosphere.

(*f*) Condensing extraction turbines in which some of the entering steam is extracted at one or more points after partial expansion, for purposes other than heating their own feedwater, the remainder leaving the exhaust flange at a pressure less than that of the atmosphere.

(*g*) Noncondensing extraction turbines in which some of the entering steam is extracted at one or more points after partial expansion, for purposes other than heating their own feedwater, the remainder leaving the exhaust flange at a pressure equal to or greater than that of the atmosphere.

(*h*) Condensing mixed-pressure turbines, in which steam is admitted at two or more pressures to two or more points, at the same time or at different times; all steam leaving the exhaust flange at a pressure less than that of the atmosphere.

(*i*) Noncondensing mixed-pressure turbines, in which steam is admitted at two or more pressures to two or more points, at the same time or at different times; all steam leaving the exhaust flange at a pressure equal to or greater than that of the atmosphere.

(*j*) Combination extraction and mixed-pressure turbines (*f*) and (*h*) or (*g*) and (*i*).

SECTION 2. DESCRIPTION AND DEFINITIONS OF TERMS.

See code for complete definitions. Selected symbols given in text as required.

SECTION 3. GUIDING PRINCIPLES.

81. Items on Which Agreement Should Be Reached. The parties to the test must reach a definite agreement as to its specific object and must agree as to the method of operation, that is, the intent of the contract or specification in this respect, upon which the guarantees have been based. They shall ascertain all the specified or contract operating conditions and warranties for all the values that are pertinent to the object of the test. Any omissions or ambiguities as to any of them in the contract or specification are to be eliminated or their values or intent agreed upon before the test is commenced.

83. Agreement must be reached as to the method of calibration of instruments and by whom.

87. The parties to the test may designate a person to direct the test and serve as arbitrator in event of disputes as to the accuracy of observations, conditions, or methods of operations.

88. Accredited representatives of the purchaser and the builder may at all times be present to verify that the tests are conducted in accordance with this code and the agreements made prior to the tests.

89. An acceptance test should be undertaken within two months of the time the turbine is first put into commercial service, provided no serious operating difficulty has been experienced. In any event, except with written agreement to the contrary, the acceptance test shall take place within the maintenance period specified in the contract.

90. Tolerances. This code does not include consideration of any over-all tolerances or margins on steam or heat consumption, or on engine efficiency guarantees written in a contract. The test results shall be reported as calculated from the test observations, instrument calibrations only having been applied.

91. Preparation for Test. Previous to an acceptance test, the turbine unit shall be placed at the disposal of the manufacturer for examination in order that he may ascertain that it is in a suitable condition for the conduct of an acceptance test. There must be assurance that nozzles and blading are free from scale or foreign matter.

92. Preliminary tests may be run for the purpose of: (*a*) Determining whether the turbine is in a suitable condition for the conduct of an acceptance test. (*b*) Checking all instruments. (*c*) Training personnel.

93. Unless otherwise agreed, for both preliminary and acceptance tests, the turbine shall be in commercial operating condition.

After a preliminary test is made, if mutually agreed, this test may be considered an acceptance test, provided it has complied with all requirements for an acceptance test.

102. Operating Conditions. Every effort shall be made to run the tests under the specified conditions such as output, pressures, temperatures, etc., in order to avoid as far as possible the application of corrections to the test results, some of which may be of doubtful accuracy.

109. Duration of Test. The duration of the test shall be such as to make it possible to obtain several check readings. Each test need continue only for a time sufficient to insure accurate and consistent results as required for certain of the measurements prescribed in Section 4, Par. 192.

114. Corrections shall be applied to the test results for any deviations of the test conditions from those specified. Correction factors for deviation of the operating conditions from those specified may be in the form of curves or numerical values. The method of applying corrections shall be carried out as required in Section 5.

115. The numerical values of corrections are preferably stated in the contract; if not, they shall be agreed upon by the parties to the test prior to the test. If mutually agreed upon by the parties to the test, auxiliary tests may be run for the purpose of determining the value of certain of the correction factors. Any such special tests shall be completely described in the test report, as to the methods employed and the results obtained.

SECTION 4. INSTRUMENTS AND METHODS OF MEASUREMENT.

Measurement of Mechanical Output

121. Absorption or transmission dynamometers are permissible, provided precautions are taken in their construction and use to insure accuracy. Included are electric or eddy-current generators, the input of which is measured by the reaction of the stator.

The dynamometer is preferably arranged so that the reaction due to friction of any bearings that are essentially a part of the dynamometer will be automatically included in the dynamometer readings. Otherwise the parties to the test shall agree upon an allowance for these losses which shall be stated in the test report.

Electrical Output Measurement

131. The net output of a turbine-driven generator is defined by the following formula:

$$\begin{bmatrix}\text{Net}\\ \text{output}\\ \text{(kw)}\end{bmatrix} = \begin{bmatrix}\text{Electrical}\\ \text{power output}\\ \text{of generator}\\ \text{(kw)}\end{bmatrix} - \begin{bmatrix}\text{That portion of}\\ \text{the excitation power}\\ \text{that is separately}\\ \text{supplied (kw)}\end{bmatrix} - \begin{bmatrix}\text{Power for ventilation}\\ \text{and other auxiliary}\\ \text{service if separately}\\ \text{supplied (kw)}\end{bmatrix}$$

The excitation power (kw) is the product of the current (amperes) supplied to the generator field and the sum of the voltage drops across the generator field and main field rheostats (volts), divided by 1000. Only that portion of the excitation power which is separately supplied from a source external to the turbine generator unit is to be charged against the prime mover.

If any generator auxiliary power service, such as that for driving ventilating fans, cooling water pumps, or lubricating oil pumps are electric motor driven, the power input to the motors shall be deducted from the main turbine generator output.

In the case of a steam rate test, if any generator auxiliary is engine or turbine driven, the auxiliary drive steam flow shall be added to the test steam flow of the main turbine, with no deduction from the main generator output.

In the case of a heat rate test, if any generator auxiliary is engine or turbine driven, the heat flow chargeable to the main turbine shall have added to it the heat flow chargeable to the auxiliary drive, with no deduction from the main generator output. This addition is the product of the steam flow to the auxiliary drive and the difference between the enthalpy of the steam supplied to the auxiliary drive and the enthalpy of saturated liquid corresponding to the measured exhaust pressure of the auxiliary drive.

132. Method of Power Measurement. For 3-phase a-c generators with grounded neutral, the power output of the main unit shall be measured by the 3-wattmeter method. For 3-phase a-c generators with isolated neutral, the power output of the main unit shall be measured by either the 2-wattmeter method or the 3-wattmeter method, either method being suitable. The power output of single-phase a-c generators shall be measured by the 1-wattmeter method. The power output of d-c generators shall be measured by the d-c voltmeter-ammeter method.

134. Instruments. Portable precision indicating wattmeters or watthour meters checked "in-place" shall be used with appropriate current and potential instrument transformers for measuring the electrical power output. Portable indicating ammeters and voltmeters shall be included in the measuring circuits to establish that the generator load conforms to rated conditions during the test, and to measure the current, voltage, and power factor for use in calculating the ratio and phase-angle corrections of the instrument transformers.

Steam Quantity Measurement

140. Turbines, Type (*a*) or (*b*). If the test is a steam or heat rate test, or an engine efficiency determination of a complete expansion or reheating turbine, type (*a*) or (*b*), the measurement may be made by:

(*a*) Actual weighing of the condensate discharged from a surface condenser by means of tanks and suitable scales, with the understanding that the limits of possible error may be ±0.25%.

(*b*) By measurement of the condensate discharged from a surface condenser by means of calibrated volumetric measuring tanks, with the understanding that the limits of possible error may be ±0.50%.

(*c*) By a steam flow measurement of the initial steam by means of nozzles or thin-plate orifices, provided the steam remains superheated not less than 25 F during its passage through the nozzle or orifice and upon agreement by the parties to the test and with the understanding that the limits of possible error may be ±1.50%.

(*d*) By a flow measurement of the condensate discharged from a surface condenser by means of either a nozzle, a thin-plate orifice, or venturi tube, upon agreement by the parties to the test and with the understanding that the limits of possible error may be ±1.25% and provided that during the passage of the water through the nozzle, orifice, or tube, either (1) the pressures remain not less than 50 psi above the vapor-pressure corresponding to boiling at the measured temperature, or (2) that the temperatures remain not less than 25 F below the boiling temperature corresponding to the lowest measured pressure.

(*e*) By means of a gravity tank arranged to receive the condensate pump delivery and having one or more calibrated nozzles in the base thereof combined with water-level measuring means, said nozzles discharging freely to the atmosphere. The quantity is determined from the measurement of head in the tank, together with the measurement of elapsed time, with the understanding that the limits of possible error may be ±0.50%.

141. Turbines, Type (c) or (d). If the test is a heat rate test, or an engine efficiency determination of a regenerative turbine, type (*c*) or (*d*), with extraction feedwater heaters included in the turbine guarantees, the measurement may be made by the means prescribed in item (*c*) of Par. 140, or

(*f*) By measurement of the feedwater discharged from the final heater or by measurement of condensate or feedwater at any point of the condensate or feedwater system where its flow is equal to that discharged from the final feedwater heater by means of either a nozzle, orifice, or venturi tube, as prescribed by item (*d*) of Par. 140.

(*g*) By measurement of the feedwater discharged from the final heater by any of the means prescribed by items (*a*), (*b*), or (*c*) of Par. 140, provided the water temperature does not exceed 175 F, or, for the purpose of the test, its temperature is reduced by means of a heat exchanger to a value not exceeding 175 F.

(*h*) By removing the condensate or feedwater from any point intermediate the heaters or intermediate a heater and the condensate pump where the temperature does not exceed 175 F, determining its quantity by means of a gravity tank (item [*e*] of Par. 140), and returning the water to a point between the same heaters with the precaution that the quantity measuring means are such that the flow through the succeeding hotter heaters is continuous and can, at all times, be maintained equal to the flow from the condenser. This requirement renders the use of weighing or volumetric tanks somewhat difficult. They may be employed, however, provided agreement is reached as to limits of possible error, and provided the weighing or volumetric tanks discharge into a secondary tank, whence the water is pumped to the next succeeding hotter feedwater heater by means of a pump, the delivery of which can be controlled. The mean level in the secondary tank before and after each discharge should be maintained approximately constant. The limits of possible error are dependent upon maintaining constant mean level in the secondary tank.

148. Extraction Feedwater Heaters. Condensate drains from heaters may be cascaded or delivered by other means to a point in the condensate system upstream of the point of measurement, provided this is consonant with the specification, contract, or intended method of operation. Disturbing the intended method of operation in this respect affects the flow to certain of the heaters so that the turbine operates under conditions other than those specified or intended. It is, therefore, sometimes necessary in the case of turbines types (*c*) and (*d*) to determine independently the condensate flow from certain extraction heaters.

149. Feed Heaters Steam Flow. The independent determination of steam flow to, or condensate flow from, a feedwater heater may be made by:

(*a*) A flow measurement in the line between the heater condensate pump and the feed line by the means and precautions prescribed by item (*d*) of Par. 140, or

(*b*) By a measurement of the rate of feedwater flow before it is joined by the heater condensate and measurements of the pressure and temperature of the heater condensate and the feedwater flow both before and after they join, provided the rise in temperature of the feedwater by the heater condensate is not less than 5 F and that the actual temperatures or the temperature difference can be measured within 0.2 F.

(*c*) By measurements of the pressure and temperature of the steam supplied to the heater (applicable only if such steam is superheated), the temperature of the heater condensate, the feedwater flow through the heater, and the pressures and temperatures of the water entering and leaving the heater.

Nozzles or Thin-plate Orifices for Measuring Steam or Water Quantity

160. Measurement of the flow of steam or water in a pipe by means of nozzles or thin-plate orifices should be undertaken only by those with knowledge and experience in the technique of such measurement and with the understanding that for a steam measurement the limits of possible error may be ±1.5% and for a water measurement, ±1.25%.

161. The technique of the method shall be carried out in detail as given in Instruments and Apparatus. This includes the selection of proper dimensional standards for nozzles and orifices, the use of the prescribed standard pressure tapping means for measuring the differential pressure, and the application of the prescribed formulas and coefficients for calculating steam or water flow. If these and the following precautions are observed, flow measurements may be made without calibration of the nozzle or orifice.

162. The ratio of nozzle throat or orifice diameter to pipe diameter shall not exceed 0.65.

Venturi Tubes for Measuring a Water Flow

176. For the measurement of water flow by a venturi tube, according to the means prescribed by item (*d*) of Par. 140 or item (*f*) of Par. 141, or for any flow the value of which influences the results of the test, the following precautions shall be followed:

177. Prior to the test, the tube shall be calibrated either by weighing the flow or by means of volumetric measuring tanks. The calibration shall be carried out over the range of flows and temperatures that will obtain during the tests.

178. Reynolds' number as calculated for the pipe flow shall be not less than that at which the flow coefficient is known to remain sensibly constant.

179. During calibration and test there shall be not less than 10 diameters of straight pipe upstream of the tube and not less than 3 diameters of straight pipe downstream of the tube. In both cases the diameters of the pipes shall be the same as that of the inlet and outlet of the tube.

Enthalpy Drop Method

188. In determining a steam flow according to the enthalpy drop method, the pressure and temperature of the initial steam and of the steam at the exhaust flange shall be measured with all the precautions required herein for pressure and temperature measurements. The temperatures may be determined by thermocouples, resistance thermometers, or mercury-in-glass thermometers, as may be agreed upon by the parties to the test. The precaution shall be taken that the means of measuring exhaust temperature is as little as possible affected by radiation or conduction to or from other portions of the turbine. It is for this reason that the enthalpy drop method may be employed only for turbines in which the flow at rated output is not less than 50,000 lb per hr. The exhaust pressure and temperature shall be observed at a number of points at the exhaust outlet according to the provisions for exhaust pressure measurements for condensing turbines.

190. The electrical, windage, and friction losses of the generator shall be computed from actual independent tests. The mechanical losses of the turbine may be calculated. Among these are:

(*a*) Friction of bearings.
(*b*) Power to revolve gland parts.
(*c*) Windage of all rotating parts that are external to the turbine casing.
(*d*) Power to operate lubricating oil pumps, speed-regulating mechanisms, and any other apparatus that is included in the steam or heat consumption guarantees.
(*e*) Heat loss by radiation, conduction, and convection.

The parties to the test shall assign and agree upon all these losses and shall state their values in the test report.

192. Duration of Tests. (*a*) For weighed or measured quantity of condensate or feedwater including a flow measurement of the same by any of the methods permissible herein and including the calibration of nozzle blocks removed from the turbine, the duration of the test shall be not less than three hours. This period may be curtailed upon agreement by the parties to the test, provided successive measurements at equal time intervals referred to constant output do not differ one from another by more than 1.0%.

(*b*) For the flow measurement of initial steam by means of a flow nozzle or an orifice in a thin plate or by means of first stage nozzle blocks that have been removed from the turbine and calibrated prior to the test, or for a steam flow determination by the enthalpy drop method, the duration of the test shall be for not less than twenty consecutive sets of simultaneous readings, occupying not less than 20 min.

(*c*) For a test to determine capacity, the duration of the test shall be not less than 15 min and shall comprise not less than five consecutive sets of simultaneous readings of output and the operating conditions as to pressures, temperatures, etc., that are required herein for consumption tests. Operating conditions shall remain sufficiently constant so that readings of the same values do not differ one from another by more than 2.0%. (See Par. 289.)

202. Vigilance shall be employed to discover sources of leakage which shall be either isolated, or measurement shall be made of them and proper allowances made therefor in the computation of results.

Among other sources of leakage may be the following:

(*a*) Extraneous or unused piping connections.
(*b*) Condensate pump leakage.
(*c*) *Ejector steam* unless the turbine guarantees include the operation of the ejectors.

Because this quantity is relatively small, it may be calculated from the measured areas of the nozzles and the steam conditions.

(*d*) *Make-up Feedwater.* For turbines type (*c*) or (*d*), if the turbine performance guarantees do not include specific provision for an amount of make-up feedwater and it is impracticable to operate the plant without admitting make-up feedwater, it is desirable that make-up be supplied to the boiler separately from the turbine condensate system. If this is impracticable, corrections shall be made to the test results. (See Par. 297.)

(*e*) *Surge tank*, if any, in connection with the condensate or feedwater system, should, if practicable, be disconnected and regarded as an unused connection.

(*f*) *Exhaust steam from auxiliaries*, if they discharge into the turbine or condenser system and the guarantees are not based on the inclusion of this steam, shall be diverted elsewhere for the period of the test. If this is impracticable, this steam consumption shall be determined. Because the amounts are relatively small, it is sufficient to measure it by means of a calibrated steam meter during the test of the main unit, provided the steam flow is steady as in the case of a turbine drive. With unsteady flow, as with reciprocating engine drives, a separate steam consumption test of the auxiliaries shall be made with precautions that the operating conditions shall be the same as obtain during the test of the main unit.

(*g*) *Water seals* when they cannot be supplied from condensate.

(*h*) *Water seal overflow* or leakage which is not returned to the condensate before the condensate is weighed or measured, if condensate is employed for sealing.

(*i*) *Steam escaping from glands, including valve stems, etc.*, if not agreed negligible by the parties to the test. This quantity may be condensed by any means and allowance made therefor in the computation of results.

(*j*) *Steam removed with air* by the air-removal means, if it is agreed by the parties to the parties to the test to be not negligible, as in the unusual case of disproportionately large air-removal means being employed. In such a case, the method of measurement of the quantity shall be agreed upon by the parties to the test.

(*k*) *Drips and drains* shall be (1) treated as unused connections, (2) returned to the condensate, or (3) cooled and weighed and allowance made therefor in the computation of results.

(*l*) *Water sealed atmospheric relief valves.*

(*m*) *Water sealed condenser connections.*

(*n*) *Water cooled stoker parts, etc.*, employing condensate.

(*o*) *Overflow outlets* from water seal tanks.

SECTION 5. COMPUTATION OF RESULTS.

254. Total Steam. Allowance must be made in the steam quantity measurement for:

(*a*) Condenser leakage.

(*b*) Water, other than condensate, condenser leakage, or make-up feedwater, entering the condenser.

(*c*) Condensate which does not pass to the quantity measuring means.

(*d*) Steam entering the condenser that has not been supplied to the turbine. (See Par. 202.)

255. Correction of the Test Result to Specified Conditions. Corrections should, if possible, be avoided by carrying out the test under the specified conditions. If the object of the test is to ascertain whether the turbine fulfills the performance guarantees under the specified conditions, the corrections to be applied should be agreed upon beforehand. (See Pars. 114 and 115.) It is far more satisfactory, however, to conduct a test under the specified conditions than to apply corrections which may be of doubtful accuracy.

After all efforts have been made, it may still be impossible to have every condition as desired, and the test results must be corrected so as to give, as nearly as possible, the values which would have obtained with specified conditions.

Correction values are preferably stated in the contract, if not they shall be agreed upon by the parties to the test before tests are commenced. If mutually agreed upon, separate tests may be carried out for the purpose of determining the value of certain of the corrections according to the requirements of Pars. 267 and 274.

When any of the corrections take the form of curves (not straight lines) for the variable, a smooth correction curve, without any abrupt changes of slope, should be drawn through the agreed correction values for the various points. The percentage correction between the specified and test value for the variable shall be taken as a single-valued slope between the two points on the curve.

260. A test of a steam turbine shall not be completed or if completed shall not be regarded as official, unless specifically agreed upon by the parties to the test, if the total arithmetic sum of all of the percentage corrections, regardless of the direction of the individual correction exceeds the following percentages:

Condensing and noncondensing turbines, types (*a*), (*b*), and (*e*)	10%
Regenerative turbines, with or without reheating, types (*c*) and (*d*)	15%

267. Correction Tests. Tests to determine correction values, if conducted (see Par. 115) will consist of runs with each of a series of values of the condition in question, with all other conditions except output as near as possible to the specified values.

Alternatively, correction tests, except those for initial pressure, may be run by maintaining steam flow substantially constant, i.e., by operating with constant turbine inlet pressure in which case the output will become variable.

274. The correction curves shall finally be plotted so as to show the test values of steam rate as ordinates for various values of a variable condition as abscissa. A straight line often represents the points but sometimes a curve must be drawn. The slope of the straight line or of a chord of the curve (the chord being drawn between test value and specified value of the variable) divided by the specified steam rate gives the percentage correction. Often a curve is also drawn with the loads which existed for each correction test as ordinates and the variable condition as abscissas.

Capacity Tests

289. The net output for each observation is to be corrected for the deviation of pressures, temperatures, etc., from the conditions of the test to those specified, as provided for in the case of steam or heat consumption tests. The test report shall state the average of the net outputs of the test, including the average of all the test operating conditions. The report shall also state the average of the corrected net outputs, including all of the operating conditions to which the test has been corrected. (See Par. 192[c].)

290. Average Performance. Guarantees of performance and the results of tests covering a series of outputs may be expressed as a single value based on weighted steam, heat consumptions, or engine efficiencies, for example:

1	2	3	4
Percentage of Rated Output (Example)	Steam or Heat Consumption, lb or Btu per kwhr or Engine Efficiency	Factor (Example)	Product of Columns (2) and (3)
100	....	3	....
80	....	4	....
60	....	3	....
40	....	2	....
		12	Sum

$$\text{Weighted average} = \frac{\text{Sum of column 4}}{\text{Sum of column 3}}$$

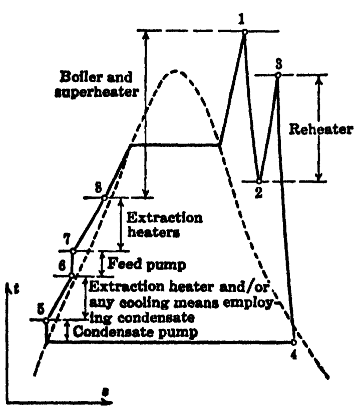


FIG. 2. Temperature-entropy diagram showing locations in the cycle corresponding to numerical subscripts.

The above percentages of rated output and their respective factors should be embodied in the contract, and established to suit the load conditions under which it is expected the turbine will operate. If not embodied in the contract, they shall be agreed upon by the parties to the contract, before any tests are commended.

The temperature-entropy diagram (Fig. 2) is intended to serve as a key to the numerical subscripts employed in the code, for reheating and regenerative condensing turbines, types (*b*), (*c*), and (*d*).

It is assumed that the pressure rise in the condensate pump is small, so that the accompanying rises in temperature and enthalpy of the condensate are negligible. In the diagram, the temperature rise has been grossly enlarged. All references to point 5 in this code assume that the temperature and the enthalpy of the condensate are sensibly the same on the inlet and on the discharge side of the condensate pump.

Steam Rates

291. For a complete expansion turbine, type (*a*), the steam rate under stated operating conditions, as to pressures, temperatures, output, etc., is:

$$\text{Steam rate, lb steam per kwhr}\quad \frac{W_1}{P_g \text{ or } P_c}$$

where

W_1 = the flow of steam entering the turbine, including leakages, if any, lb/hr

P_g = the net electrical output of turbine generator credited to the turbine under test, kw

P_c = the output measured at the turbine coupling, kw

Sometimes guarantees are expressed as a steam rate without extraction for regenerative turbines designed for the extraction of steam for heating feed, type (*c*), in which case this rate will be calculated as for type (*a*), but during the test the extraction openings from the turbine must be closed. The closing-off of extraction steam to the heaters introduces a different operating condition from that contemplated in the purchase of the turbine. It is, therefore, desirable that guarantees for this type of turbine (*c*) be expressed in terms of heat rate or as an engine efficiency. (See Par. 294.)

Heat Rates

294. For a complete expansion turbine, type (*a*), the performance may also be expressed as a heat rate, under stated operating conditions as to pressures, temperatures, output, etc.

$$\text{Heat rate, Btu per kwhr} = \frac{(W_1 - \Sigma W_g)(h_1 - h_{fx}) + \Sigma W_g(h_1 - h_{fg2})}{P_g \text{ or } P_c}$$

in which

* ΣW_g = the flow of steam measured in a leak-off pipe from glands, and from valve stems, etc., that receive steam from a point upstream of a reheater, which is led to waste or employed for any purpose extraneous to the turbine system under test and it or its heat is not delivered to any portion of the turbine steam or condensate path, and flow, if any, escaping to atmosphere.

h_1 = enthalpy of steam supplied.

h_{fx} = enthalpy of saturated liquid at exhaust pressure.

* h_{fg2} = enthalpy of saturated liquid at the pressure of the extraneous leak-off steam from glands.

(Other symbols as in Par. 291.)

296. For a reheating turbine without feed heating, type (*b*), the heat rate under stated operating conditions as to pressures, temperatures, output, etc., is:

Heat rate, Btu per kwhr =

$$\frac{\left\{\begin{matrix}(W_1 - \Sigma W_g)h_1 - (W_1 - \Sigma W_g - \Sigma W_{gr})h_{fx} + (W_3 - \Sigma W_{gr})h_3 - W_2h_2 \\ + \Sigma W_g(h_1 - h_{fg2}) + \Sigma W_{gr}(h_3 - h_{fg2})\end{matrix}\right\}}{P_g \text{ or } P_c}$$

in which

* ΣW_{gr} = flow of extraneous leak-off steam from glands, etc., that receive steam from downstream of a reheater, lb/hr.

W_2 = flow of steam to the reheater, lb/hr.

h_2 = enthalpy of steam to reheater, Btu/lb.

W_3 = flow of steam from reheater, lb/hr.

h_3 = enthalpy of steam returned from reheater, Btu/lb.

(Other symbols as in Pars. 291 and 294.)

297. (*a*) **For a regenerative turbine,** i.e., one provided for extraction for heating its own feedwater but without reheating, type (*c*), under stated operating conditions as to

* The subscript (*g*) designates "glands"; it is not to be confused with its standard significance (saturated vapor) which does not occur in this code.

pressures, temperatures, output, etc., the heat rate is, basically:

$$\text{Heat rate} = \frac{\text{Heat supplied} - \text{heat returned}}{\text{Output}}$$

which, expressed for a general case, is

$$\frac{W_1(h_1 - h_8)}{P_g \text{ or } P_c} \tag{1}$$

in which

$W_1 = W_8$.

W_8 = feedwater flow discharged from final heater, lb/hr.

h_8 = enthalpy of feedwater at discharge from final heater, Btu/lb.

(Other symbols as in Pars. 291, 294, and 296.)

(*b*) **Feed Pump.** Allowance shall be made in the heat rate formulas for the work and internal losses of a feed pump or pumps, if any, between the condenser and final feedwater heater. To accomplish this the value

$$+W_6(h_7 - h_6)$$

is added to the numerators of the heat rate formulas, where

W_6 = flow through feed pump, lb/hr.

h_6 = liquid enthalpy entering feed pump, Btu/lb.

h_7 = liquid enthalpy leaving feed pump, Btu/lb.

If two or more feed pumps are arranged to operate in series, between the condenser and final heater, an allowance for each of them shall be made.

Allowance for a condensate pump is not generally justified, because the enthalpy increase is negligible, unless the discharge pressure is more than 200 psi.

(*c*) **Make-up Feed and Allowance for Extraneous Leak-off Steam from Glands.** A regenerative turbine within a technical meaning of the words could not heat make-up feedwater. If water from an extraneous source (make-up feed) is heated by extracted steam, the turbine becomes, in part, an extraction turbine. For the turbine to be strictly regenerative, W_8 must be equal to W_1. Certain leakages, however, such as ΣW_g, included in the measurement of W_1, do not reach the condenser or condensate system, and so are not included in the measurement of W_8.

For the turbine to be regarded as strictly regenerative, make-up feedwater equivalent to the weight of ΣW_g must be considered as being admitted to the system. Condenser leakage, if any, is to be considered as make-up feed. If steam jet air pumps are included in the turbine performance guarantees, their steam flow shall be considered as a part of the turbine steam flow. If not included in the guarantees, the condensate therefrom, if admitted to the condensate system, shall be considered as make-up feed, taken at the enthalpy of the condenser condensate.

For the turbine to be strictly regenerative, the following equation must be satisfied in the respect to make-up feed:

$$W_m + W_l - \Sigma W_g = 0$$

in which

W_m = make-up feedwater from an extraneous source, lb per hr.

W_l = condenser leakage, including, if appropriate, condensate from steam jet air pumps, lb per hr.

(*d*) **Heat Rate.** If agreed by the parties to the test and if compatible with the contract or specification, the turbine may be regarded as strictly regenerative according to the foregoing item (*c*). If performance guarantees are based upon heating by extraction some quantity of make-up feed, not exceeding 5% of the initial flow, and/or it is impracticable to operate the plant without admitting make-up feed to the system, the guarantees and test result may be corrected to zero make-up feed, provided the make-up feed during the test does not exceed 5% of the initial steam flow.

(*e*) Any difference between W_8 and W_1 is to be considered in this case as make-up feed, either positive or negative, to be corrected to zero if the turbine is to be regarded as strictly regenerative.

(*f*) The code provides (see Par. 202[*d*]) that make-up feedwater shall, if practicable, be diverted elsewhere for the purpose of the test, in the case of the turbine being regarded as strictly regenerative.

(*g*) **Utilization Heat Rate.** Guarantees of performance of turbines, type (*c*) or (*d*), are frequently based upon considerable quantities of make-up feed being admitted to the condenser or condensate system and heated by extracted steam. If performance is to be determined under specified conditions as to make-up feed, the turbine becomes in part an extraction turbine. The term "heat rate" or "thermal efficiency" is not then applicable as a measure of performance.

If agreed by the parties to the test, and if compatible with the contract or specification, the performance may be expressed as a "utilization heat rate." This is the only permissible method of expressing performance when the specified quantity of make-up feed or that admitted during the test exceeds 5% of the initial steam flow.

(*h*) The complete formula for either the heat rate or the utilization heat rate, including allowances for extraneous leak-off steam from glands, etc., for make-up feedwater and for feed pump is:

$$\frac{(W_1 - \Sigma W_g)(h_1 - h_8) + W_6(h_7 - h_6) - W_e(h_8 - h_e) + \Sigma W_g(h_1 - h_{fg2})}{P_g \text{ or } P_c} \qquad (2)$$

where

W_e = make-up feed flow, lb/hr.

h_e = enthalpy of make-up feed, Btu/lb.

(Other symbols as above.)

(*i*) **For the heat rate** of a turbine regarded as strictly regenerative, corrections to the test result shall be made to render $W_8 = W_1$. In this case, $W_e = W_m + W_l$ and is rendered numerically equal to ΣW_g and

$$h_e = \frac{W_m h_m + W_l h_l}{W_m + W_l}$$

(See items [*c*] and [*f*].)

The thermal efficiency of the strictly regenerative turbine may be taken as:

$$\frac{3412.7}{\text{Heat rate calculated according to foregoing items } (h) \text{ and } (i)}$$

(*j*) **For the utilization heat rate** of a turbine regarded as not strictly regenerative, that is, the turbine heats by means of extraction some quantity of make-up feedwater, and W_8 is greater than W_1, the following values will apply in the foregoing formula (2) of item (*h*):

$$W_8 = W_1 + W_e - \Sigma W_g$$

$$W_e = W_m + W_l$$

$$h_e = \frac{W_m h_m + W_l h_l}{W_m + W_l}$$

The utilization heat efficiency may be taken as:

$$\frac{3412.7}{\text{Utilization heat rate calculated according to foregoing items } (h) \text{ and } (j)}$$

SECTION 6. REPORT OF TESTS.

For standard form see the complete code.

6. TEST CODE FOR STEAM-CONDENSING APPARATUS

1. The term "steam-condensing apparatus" is intended to include all apparatus whose primary function is to reduce the exhaust pressure of a prime mover to a pressure below atmospheric and, in the case of surface condensers, to condense the steam so that the condensate is available for re-use as boiler feed.

SECTION 1. OBJECT AND SCOPE.

4. The object of the test on steam-condensing apparatus shall be to verify the manufacturer's guarantees. The purpose of this code is to provide mandatory rules for determining the performance of a condenser with regard to one or more of the following:

The absolute pressure the apparatus will maintain at the steam inlet nozzle when transferring heat rejected by the prime mover at a given rate in Btu per hour with a given flow and temperature of circulating water, and a given tube cleanliness (see Sec. 5).

The thermal transmittance of surface condensers for given operating conditions noted in the preceding paragraph.

The amount of under-cooling of the condensate.

The percentage of dissolved oxygen in the condensate.

SECTION 3. GUIDING PRINCIPLES.

67. The isolating of the steam and condensate circuits shall be accomplished by one of the following means: (*a*) Blank flanges. (*b*) Double valves with vent valves between. (*c*) Single gate valves with bonnet vent valves between the double disks.

68. Air Leakage. All packing glands on valve stems under vacuum on any part of the unit including those on the regenerative feedwater heaters, all piping joints on the prime mover, and all joints in the turbine or condenser casing shall be thoroughly inspected for possible air leaks. An agreement shall be reached and recorded in writing before test on the maximum amount of "free air" leakage, as measured during test, that will be tolerated by the parties to the test. In no case should tests be undertaken until the leakage has been reduced to less than 5 cu ft per min.

69. Cooling Water Leakage. Since the load on a surface condenser is a function of the steam condensed per hour which is determined by measuring the condensate, any cooling water leakage into the steam space of a surface condenser shall be determined prior to the start of the test. No test shall be undertaken with leakage in excess of the following percentages of the steam flow at the rated load capacity of the prime mover as deduced from performance guarantees.

Turbine rating:

Less than	500 kw	1.00%
	500 to 1000 kw	0.75%
	1000 to 5000 kw	0.50%
Above	5000 kw	500 lb per hr

80. Frequency of Readings. With a one-hour run, the cooling water temperatures should be taken every 2 1/2 min. Other readings should be taken every 5 minutes. If the duration of a test run is more than one hour the frequency of the water temperature readings may be reduced accordingly.

SECTION 4. INSTRUMENTS AND METHODS OF MEASUREMENT.

85. Condenser Pressure Measurement. Mercury manometers, or by agreement absolute pressure gages, shall be used for measuring the condenser inlet pressure. The glass of these gages shall be high grade lead-free tubing, not smaller than 1/4 in. internal diameter, and preferably 1/2 in. at the point of measurement. The mercury used shall be clean and pure. The scales on the gages shall be divided into 0.02 in. divisions, but the smallest division may be 0.1 in. if the gages are equipped with verniers so that they can be readily read to 0.01 in. Either type of gage shall be carefully checked, and any error proved to be less than 0.01 in. Hg. Absolute pressure gages, if used, shall be checked at the beginning and end of the test by comparison with a mercury manometer and barometer.

87. The condenser inlet pressure shall be measured at or on either side of, but adjacent to, the joint connecting the turbine and condenser. If an approximately straight conduit connects the turbine exhaust to the condenser inlet the measurements shall be made near the condenser end of the conduit.

92. Barometer. The Fortin or Kew type of mercury-in-glass barometer shall be used exclusively for measuring the atmospheric pressure. The aneroid type of barometer is prohibited because of its unreliability, except where mercury-in-glass barometers are impractical, such as on shipboard during rough weather.

95. Cooling Water Temperatures. Mercury-in-glass thermometers with graduations etched on the stem shall be used for measuring the cooling water temperatures at inlet and outlet to the condenser water boxes. These thermometers shall be graduated in tenths or two-tenths of a degree and shall be accurate to within 0.1 F. The thermometers shall be inserted into oil-filled thermometer wells which project into the water conduits a distance above equal to 1/5 the conduit diameter. By agreement bare thermometers, suitably protected by wire cages, may be used. Also, by agreement properly calibrated resistance thermometers may be used.

97. Condensate and Air Temperatures. Mercury-in-glass thermometers, graduated in half degrees, shall be used for measuring the condensate temperature and the air outlet temperature. The spacing of the graduations on these thermometers shall be sufficiently wide to permit estimation to tenths of a degree and shall be accurate to within 0.5 F. Oil-filled thermometer wells shall be provided for the condensate thermometers. For measuring air outlet temperatures, insert the thermometers through rubber stoppers directly into the air stream.

98. Mercury-in-glass thermometers shall also be used for measuring the condensate (and raw water, if used) temperature entering and leaving air ejector condenser. These thermometers shall be graduated in half degrees, and shall be accurate to within 0.5 F.

104. Condensate Flow-rate Measurement. The condensate from the condenser shall be measured by one of the following means: (*a*) Tanks mounted on weighing scales. (*b*) Volumetric tanks. (*c*) Venturi tubes, nozzles, or orifice plates upon agreement of the parties to the test.

In all cases correction for condenser leakage shall be made.

119. Dissolved oxygen in the condensate shall be measured at the discharge of the condensate pump, first making sure no leaks exist at glands or casing joints of the pump. The Winkler method, as described in *Bulletin* 151 of the United States Public Health Service, shall be followed in collecting, fixing, and titrating the sample. At least three samples shall be taken during each run. Only a chemist or someone trained specially for this work should attempt to determine the dissolved oxygen in the condensate.

SECTION 5. CLEANLINESS-FACTOR DETERMINATION.

122. Definition of Cleanliness Factor. Cleanliness factor, a term used to express the degree of tube fouling, is defined as the ratio of the thermal transmittance of the tubes whose cleanliness factors are being determined to that of new tubes, operating under identical conditions of circulating water temperature and velocity, and the same external steam temperature and flow. The overall cleanliness factor of a condenser is the average cleanliness factor of all the tubes, but because of test limitations it is necessary to determine this value from measurements on a limited number of tubes in representative positions.

125. Procedure. For the cleanliness-factor determination, the tubes selected for test shall be isolated and their thermal transmittances determined. The data required for each tube are: the outside area of tube, the enveloping vapor temperature, the inlet and outlet temperatures of the cooling water, and the rate of flow of the water. These readings shall be taken simultaneously with the overall test in order to eliminate any question of possible changes in the cleanliness factor between the time of making the cleanliness factor determination and the overall test. This procedure will give a cleanliness factor for each run and will eliminate any errors in interpolating where the cleanliness factor decreases during the test period. Where different water velocities are used the cleanliness will be different even though no fouling occurs during the test period.

127. Selection of Sample Tubes. Groups of four tubes each shall be selected by the parties to the test at such points in the condenser as will give an average of the service conditions for all tubes in the condenser. The location of each of the four tubes in each group shall be such that they will be subjected to, as nearly as possible, the same service conditions on the steam side as possible and they shall not be separated from each other by more than one intervening tube. The number of groups selected shall be not less than one per 2000 tubes and in no case less than four groups. The selected sample tubes shall be considered representative of the entire condenser as to cleanliness when the average heat transmittance of the sample tubes is within ±7% of the value for the entire condenser, both computed on the same basis, i.e., using the steam temperature at the top of the condenser.

SECTION 7. STEAM JET AIR PUMPS.

148. Under actual operating conditions the vapor in the mixture removed from condensers is saturated and may even carry entrained moisture. The performance of steam-jet air pumps is affected materially by the moisture content of the air handled. Therefore, shop tests shall be made using air with its vapor saturated.

149. The performance of steam-jet air pumps depends upon:

(*a*) The quantity and temperature of the mixture removed from the main condenser.
(*b*) The steam pressure and temperature at the ejector nozzles.
(*c*) The quantity and temperature of the cooling water passing through the ejector's intermediate and after condensers.

151. Test Method. In so far as is possible the pump shall be operated under the conditions specified in the contract. All adjustments shall be made prior to the test and maintained throughout the test period. The duration of each run is usually governed by the time required to secure the necessary readings. At least three sets of consecutive readings, which give approximately consistent data, shall be taken. Since all instruments used are of the indicating type there is no need for long test runs.

152. The quantity of air handled by the pump can be governed by the size of nozzle used in the air intake pipe. It is customary to treat the quantity of air handled as the independent variable and to determine the absolute pressure maintained for a range of flows.

155. A bare-bulb thermometer inserted into the air intake pipe through a rubber stopper shall be used for measuring the temperature of the air and vapor mixture as received by the pump.

156. The steam consumption of the pump shall be measured by using the ejector nozzles as the steam metering device.

157. The quantity of cooling water circulating through the intermediate and after condensers shall be measured with venturi tube, nozzle, or orifice plate.

COMPUTATION AND REPORT OF RESULTS.

Consult the complete code for data on this subject.

SECTION 20

MATHEMATICAL TABLES

NUMBERS

1. LOGARITHMS

Logarithms (abbreviation *log*).—The log of a number is the exponent of the power to which it is necessary to raise a fixed number to produce the given number. The fixed number is called the *base*. Thus if the base is 10, the log of 1000 is 3, for $10^3 = 1000$. There are two systems of logs in general use, the *common*, in which the base is 10, and the Napierian, or *hyperbolic*, in which the base is 2.718281828. . . . The Napierian base is commonly denoted by e, as in the equation $e^y = x$, in which y is the Napierian log of x. The abbreviation $\log_e$ is commonly used to denote the Napierian log.

In any system of logs, the log of 1 is 0; the log of the base, taken in that system, is 1. In any system the base of which is greater than 1, the logs of all numbers greater than 1 are positive and the logs of all numbers less than 1 are negative.

The modulus of any system is equal to the reciprocal of the Napierian log of the base of that system. The modulus of the Napierian system is 1, that of the common system is 0.4342945. The log of a number in any system equals the modulus of that system $\times$ Napierian log of the number. The *hyperbolic* or *Napierian* log of any number equals the common log $\times$ 2.3025851.

Every log consists of two parts, an integral part called the *characteristic*, or index, and the decimal part, or *mantissa*. The mantissa only is given in the usual tables of common logs, with the decimal point omitted. The characteristic is one less than the number of figures to the left of the decimal point in the number whose log is to be found. The characteristic of numbers from 1 to 9.99+ is 0, from 10 to 99.99+ is 1, from 100 to 999+ is 2, from 0.1 to 0.99+ is -1, from 0.01 to 0.099+ is -2, etc. Thus,

log of 2000 is 3.30103;	log of 0.2	is	-1.30103,	or	$9.30103 - 10$
" " 200 " 2.30103;	" " 0.02	"	-2.30103,	"	$8.30103 - 10$
" " 20 " 1.30103;	" " 0.002	"	-3.30103,	"	$7.30103 - 10$
" " 2 " 0.30103;	" " 0.0002	"	-4.30103,	"	$6.30103 - 10$

The minus sign is frequently written above the characteristic thus: log 0.002 = $\bar{3}.30103$. The characteristic only is negative, the decimal part, or mantissa, being always positive.

When a log consists of a negative index and a positive mantissa, it is usual to write the negative sign over the index, or to add 10 to the index, and indicate the subtraction of 10 from the resulting logarithm. Thus log 0.2 = $\bar{1}.30103$, may be written $9.30103 - 10$. The difference between a logarithm and 10 is its *arithmetical complement* or **cologarithm.**

In tables of logarithmic sines, etc., the -10 is generally omitted, as being understood.

RULES FOR USE OF THE TABLE OF COMMON LOGARITHMS.—To Find the Log of a Decimal Fraction or of a Whole Number and a Decimal.—First find the log of the quantity as if there were no decimal point, then prefix the index according to rule; the index is one less than the number of figures to the left of the decimal point.

EXAMPLE. log of 3.14159.	log of 3.141	=	0.497068.	Diff. = 138.
From proportional parts	5	=	690	
" " "	09	=	1242	
	log 3.14159		0.4971494	

If the number is a decimal less than unity, the index is negative and is one more than the number of zeros to the right of the decimal point. Log of 0.0682 = $\bar{2}.833784 = 8.833784 - 10$.

To Find the Number Corresponding to a Given Log.—Find in the table the log nearest to the decimal part of the given log and take the first four digits of the required number from the column N and the top of the column containing the log which is the next less than the given log. To find the 5th and 6th digits subtract the log in the table from the given log, and multiply the difference by 100, and divide by the figure in the Diff. column opposite the log; annex the quotient to the four digits already found, and place the decimal point according to the rule; the number of figures to the left of the decimal point is one greater than the index. The number corresponding to a log is called the anti-logarithm.

Find the anti-log of.............................. 0 497150
Next lowest log in table corresponds to 3141........ 0 497068 Diff. = 82.

Tabular diff. = 138; 82 ÷ 138 = 0 59+

The index being 0, the number is therefore 3 14159+.

Multiplication by Means of Logarithms.—Add together the logs of the two numbers to be multiplied. The sum is the log of the product.

a. Where both factors are greater than unity.

EXAMPLE. 31 × 1274 = 39,494.

Solution. log 31 + log 1274 = 1.491362 + 3.105169 = 4.596531 = log 39,494.

b. Where one or more factors are less than unity, the logs with a negative characteristic can be handled most conveniently by adding and subtracting 10.

EXAMPLE. .000028961 × .084507 = .0000024474

Solution. log .000028961 + log .084507 = $\bar{5}$ 461814 + $\bar{2}$.926893.
= (5 461814 − 10) + (8.926893 − 10) = 14 388707 − 20 = $\bar{6}$ 388707
= log .0000024474.

Division by Means of Logarithms.—Subtract the log of the divisor from the log of the dividend. The remainder is the log of the quotient.

a. When the divisor is smaller than the dividend.

EXAMPLE. 2987 ÷ 63 = 47.41284.

Solution. log 2987 − log 63 = 3 475235 − 1.799341 = 1.675894 = log 47 41284.

b. When the divisor is larger than the dividend, add and subtract as many tens as may be necessary to the log of the dividend and proceed as before.

EXAMPLE. .000672 ÷ 263 = .00000255513.

Solution. log .000672 − log 263 = $\bar{4}$ 827369 − 2 419956
= 6.827369 − 2 419956 − 10 = $\bar{6}$.407413 = log .00000255513.

c. The log of a fraction is obtained by subtracting the log of the denominator from the log of the numerator. Thus,

$$\log \frac{a}{b} = \log a - \log b.$$

To Raise a Number to Any Given Power.—Multiply the log of the number by the exponent of the number, and find the number whose log is the product.

a Where the exponent consists of one figure.

EXAMPLE. 16 2^3 = 4251.528

Solution. 3 × log 16 2 = 3 × 1 209515 = 3 628545 = log 4251 528.

b. Where the exponent consists of two or more figures, it is best to multiply the characteristic and mantissa separately.

EXAMPLE. $005624^{.37}$ = .147067.

Solution. .37 × log .005624 = .37 × $\bar{3}$ 750045 = 37 × (− 3) + 37 × 750045
= − 1 11 + .277517 = $\bar{1}$.167517 = log .147067.

c. Where the number is a fraction, first find the log of the fraction and then multiply it by the exponent.

EXAMPLE. $\left(\frac{276}{.032}\right)^{.72}$ = 681.9396.

Solution. .72 (log 276 − log .032) = .72{ 2 440909 − (8.505150 − 10) }
= .72 × 3.935759 = 2.833746 = log 681.9396.

To Extract Any Root of a Number.—Divide the log of the number by the index of the root, and find the number whose log is the quotient.

To extract the root of a decimal: *a* When the root index is positive and evenly divisible into the negative characteristic of the log of the number, the division may be performed with the negative characteristic written in its usual place.

EXAMPLE. $\sqrt[4]{.0006954}$ = .16239.

Solution. log .0006954 ÷ 4 = $\bar{4}$ 842235 ÷ 4 = $\bar{1}$.210559 = log .16239.

b. When the root index is positive and not even divisible into the negative characteristic, add to the log of the number, and indicate the subtraction from it, the smallest integral multiple of the root which will eliminate the negative characteristic. Divide the result by the root index and ascertain the number whose log corresponds to the quotient.

EXAMPLE. $\sqrt[3.1]{.00002785}$ = .03393.

Solution. log .00002785 = $\bar{5}$.444825; $\bar{5}$.444825 ÷ 3.1 = {(2 × 3.1) + $\bar{5}$.444825 − 2 × 3.1} ÷ 3.1
= {1.644825 − 6.2} ÷ 3.1 = $\bar{2}$.530589 = log .03393.

c. When the root index is negative, determine the excess of the negative characteristic over the positive mantissa. Divide the result by the root index and ascertain the number whose log corresponds to the quotient.

EXAMPLE. $\sqrt[-4]{.000003976}$ = 22.394.

Solution. log .000003976 = $\bar{6}$.599446 = .599446 − 6 = − 5.400554;
− 5.400554 ÷ (− 4) = 1.350138 = log 22.394.

Solution of Exponential Equations.—In an exponential equation, the unknown quantity is the exponent; thus $a^x = b$. This may be transformed to $\log a^x = \log b$, or $x \log a = \log b$, whence $x = \log b \div \log a$.

a. When the base is greater than unity, put the equation in the form $x = \log b \div \log a$. Then $\log x = \log (\log b) - \log (\log a)$.

EXAMPLE. $32.6^x = 14.632$.

Solution. $\log x = \log (\log 14.632) - \log (\log 32.6) = \log 1.165303 - \log 1.513218$
$= .066439 - .179901 = \bar{1}.886538$; $x = .77008$.

b. When both the known quantities are decimal, put the equation in the form $x = \log b \div \log a$. Subtract the positive mantissa from the negative characteristic in both divisor and dividend, obtaining negative remainders. Change the signs of divisor and dividend and proceed as in Case *a*.

EXAMPLE. $.0729^x = .2693$.

Solution. $x = \log .2693 \div \log .0729 = \bar{1}.430236 \div \bar{2}\ 862728$
$= (-.569764) \div (-1.137272) = .569764 \div 1.137272$;
$\log x = \log .569764 - \log 1.137272 = \bar{1}.755695 - .055864 = \bar{1}.699831$
$x = .50099$.

c. When only one of the known quantities is a decimal, put the equation in the form $x = \dfrac{\log b}{\log a}$. Subtract the positive mantissa from the negative characteristic of the numerator or denominator as the case may be, and rewrite the fraction with the remainder so obtained as the new numerator or denominator. Make both numerator and denominator positive, but write a minus sign in front of the fraction, to signify that the result will be a negative quantity. Solve the fraction by logarithms and write a minus sign in front of the result.

EXAMPLE. $.726^x = 802.7$.

Solution. $x = \dfrac{\log 802.7}{\log .726} = \dfrac{2.904553}{\bar{1}.860937} = \dfrac{2.904553}{-.139063} = -\dfrac{2.904553}{.139063}$.

$\log x = -(\log 2.904553 - \log .139063) = -(.463079 - \bar{1}.143211) = -(1.319868)$.
$x = -20.8867$.

d. When the exponent is negative and one of the known quantities is less than unity, put the equation in the form $(-x) = \dfrac{\log b}{\log a}$. Subtract the positive mantissa from the negative characteristic, as in Case *c*, and multiply both sides of the resulting equation by (-1). Find the value of x as in Case *c*.

EXAMPLE. $10.78^{-x} = .09431$.

Solution. $x = \dfrac{\log .09431}{\log 10.78} = \dfrac{\bar{2}.974558}{1.032619}$; $x = \dfrac{1.025442}{1.032619}$.

$\log x = .010911 - .013940 = \bar{1}.996971$. $x = .99305$.

Table 1. Logarithms of Numbers from 1 to 100

N	Log	N	Log	N	Log	N	Log	N	Log
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.375061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1 986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

See pp. 20–05 to 20–22 for a complete table of six-place logarithms.

Table 2. Common Logarithms of Numbers

N	0	1	2	3	4	5	6	7	8	9	Diff.
100	**000000**	**000434**	**000868**	**001301**	**001734**	**002166**	**002598**	**003029**	**003461**	**003891**	432
1	004321	004751	005181	005609	006038	006466	006894	007321	007748	008174	428
2	008600	009026	009451	009876	010300	010724	011147	011570	011993	012415	424
3	012837	013259	013680	014100	014521	014940	015360	015779	016197	016616	420
4	017033	017451	017868	018284	018700	019116	019532	019947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	024486	024896	412
6	025306	025715	026125	026533	026942	027350	027757	028164	028571	028978	408
7	029384	029789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	034227	034628	035029	035430	035830	036230	036629	037028	400
9	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998	397
110	**041393**	**041787**	**042182**	**042576**	**042969**	**043362**	**043755**	**044148**	**044540**	**044932**	393
1	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830	390
2	049218	049606	049993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524	383
4	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	064083	376
6	064458	064832	065206	065580	065953	066326	066699	067071	067443	067815	373
7	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514	370
8	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182	366
9	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819	363
120	**079181**	**079543**	**079904**	**080266**	**080626**	**080987**	**081347**	**081707**	**082067**	**082426**	360
1	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004	357
2	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552	355
3	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
4	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562	349
5	096910	097257	097604	097951	098298	098644	098990	099335	099681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	103462	343
7	103804	104146	104487	104828	105169	105510	105851	106191	106531	106871	341
8	107210	107549	107888	108227	108565	108903	109241	109579	109916	110253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	113609	335

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173 6	217.0	260.4	303.8	347.2	390.6
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
416	41.6	83.2	124.8	166.4	208 0	249.6	291.2	332.8	374.4
414	41.4	82.8	124 2	165 6	207.0	248.4	289.8	331.2	372.6
412	41.2	82.4	123 6	164.8	206.0	247.2	288.4	329.6	370.8
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
408	40.8	81.6	122.4	163 2	204.0	244.8	285.6	326.4	367.2
406	40.6	81.2	121.8	162 4	203 0	243.6	284.2	324.8	365.4
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
402	40.2	80.4	120.6	160.8	201.0	241.2	281.4	321.6	361.8
400	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	358.2
396	39.6	79.2	118.8	158.4	198 0	237.6	277.2	316.8	356.4
394	39.4	78.8	118.2	157.6	197.0	236.4	275.8	315.2	354.6
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
360	36.0	72.0	108.0	144.0	180.0	216.0	252 0	288.0	324.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
130	**113943**	**114277**	**114611**	**114944**	**115278**	**115611**	**115943**	**116276**	**116608**	**116940**	333
1	117271	117603	117934	118265	118595	118926	119256	119586	119915	120245	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	123525	328
3	123852	124178	124504	124830	125156	125481	125806	126131	126456	126781	325
4	127105	127429	127753	128076	128399	128722	129045	129368	129690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219	321
6	133539	133858	134177	134496	134814	135133	135451	135769	136086	136403	318
7	136721	137037	137354	137671	137987	138303	138618	138934	139249	139564	316
8	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
9	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818	311
140	**146128**	**146438**	**146748**	**147058**	**147367**	**147676**	**147985**	**148294**	**148603**	**148911**	309
1	149219	149527	149835	150142	150449	150756	151063	151370	151676	151982	307
2	152288	152594	152900	153205	153510	153815	154120	154424	154728	155032	305
3	155336	155640	155943	156246	156549	156852	157154	157457	157759	158061	303
4	158362	158664	158965	159266	159567	159868	160168	160469	160769	161068	301
5	161368	161667	161967	162266	162564	162863	163161	163460	163758	164055	299
6	164353	164650	164947	165244	165541	165838	166134	166430	166726	167022	297
7	167317	167613	167908	168203	168497	168792	169086	169380	169674	169968	295
8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	173186	173478	173769	174060	174351	174641	174932	175222	175512	175802	291
150	**176091**	**176381**	**176670**	**176959**	**177248**	**177536**	**177825**	**178113**	**178401**	**178689**	289
1	178977	179264	179552	179839	180126	180413	180699	180986	181272	181558	287
2	181844	182129	182415	182700	182985	183270	183555	183839	184123	184407	285
3	184691	184975	185259	185542	185825	186108	186391	186674	186956	187239	283
4	187521	187803	188084	188366	188647	188928	189209	189490	189771	190051	281
5	190332	190612	190892	191171	191451	191730	192010	192289	192567	192846	279
6	193125	193403	193681	193959	194237	194514	194792	195069	195346	195623	278
7	195900	196176	196453	196729	197005	197281	197556	197832	198107	198382	276
8	198657	198932	199206	199481	199755	200029	200303	200577	200850	201124	274
9	201397	201670	201943	202216	202488	202761	203033	203305	203577	203848	272

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	254.4	286.2
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
292	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257.4
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
160	**204120**	**204391**	**204663**	**204934**	**205204**	**205475**	**205746**	**206016**	**206286**	**206556**	271
1	206826	207096	207365	207634	207904	208173	208441	208710	208979	209247	269
2	209515	209783	210051	210319	210586	210853	211121	211388	211654	211921	267
3	212188	212454	212720	212986	213252	213518	213783	214049	214314	214579	266
4	214844	215109	215373	215638	215902	216166	216430	216694	216957	217221	264
5	217484	217747	218010	218273	218536	218798	219060	219323	219585	219846	262
6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
7	222716	222976	223236	223496	223755	224015	224274	224533	224792	225051	259
8	225309	225568	225826	226084	226342	226600	226858	227115	227372	227630	258
9	227887	228144	228400	228657	228913	229170	229426	229682	229938	230193	256
170	**230449**	**230704**	**230960**	**231215**	**231470**	**231724**	**231979**	**232234**	**232488**	**232742**	255
1	232996	233250	233504	233757	234011	234264	234517	234770	235023	235276	253
2	235528	235781	236033	236285	236537	236789	237041	237292	237544	237795	252
3	238046	238297	238548	238799	239049	239299	239550	239800	240050	240300	250
4	240549	240799	241048	241297	241546	241795	242044	242293	242541	242790	249
5	243038	243286	243534	243782	244030	244277	244525	244772	245019	245266	248
6	245513	245759	246006	246252	246499	246745	246991	247237	247482	247728	246
7	247973	248219	248464	248709	248954	249198	249443	249687	249932	250176	245
8	250420	250664	250908	251151	251395	251638	251881	252125	252368	252610	243
9	252853	253096	253338	253580	253822	254064	254306	254548	254790	255031	242
180	**255273**	**255514**	**255755**	**255996**	**256237**	**256477**	**256718**	**256958**	**257198**	**257439**	241
1	257679	257918	258158	258398	258637	258877	259116	259355	259594	259833	239
2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
3	262451	262688	262925	263162	263399	263636	263873	264109	264346	264582	237
4	264818	265054	265290	265525	265761	265996	266232	266467	266702	266937	235
5	267172	267406	267641	267875	268110	268344	268578	268812	269046	269279	234
6	269513	269746	269980	270213	270446	270679	270912	271144	271377	271609	233
7	271842	272074	272306	272538	272770	273001	273233	273464	273696	273927	232
8	274158	274389	274620	274850	275081	275311	275542	275772	276002	276232	230
9	276462	276692	276921	277151	277380	277609	277838	278067	278296	278525	229
190	**278754**	**278982**	**279211**	**279439**	**279667**	**279895**	**280123**	**280351**	**280578**	**280806**	228
1	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075	227
2	283301	283527	283753	283979	284205	284431	284656	284882	285107	285332	226
3	285557	285782	286007	286232	286456	286681	286905	287130	287354	287578	225
4	287802	288026	288249	288473	288696	288920	289143	289366	289589	289812	223
5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
6	292256	292478	292699	292920	293141	293363	293584	293804	294025	294246	221
7	294466	294687	294907	295127	295347	295567	295787	296007	296226	296446	220
8	296665	296884	297104	297323	297542	297761	297979	298198	298416	298635	219
9	298853	299071	299289	299507	299725	299943	300161	300378	300595	300813	218

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
270	27.0	54.0	81.0	108.0	135.0	162.0	189.0	216.0	243.0
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
262	26 2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	230.4
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
250	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
230	23.0	46.0	69 0	92.0	115.0	138.0	161.0	184 0	207.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
200	**301030**	**301247**	**301464**	**301681**	**301898**	**302114**	**302331**	**302547**	**302764**	**302980**	217
1	303196	303412	303628	303844	304059	304275	304491	304706	304921	305136	216
2	305351	305566	305781	305996	306211	306425	306639	306854	307068	307282	215
3	307496	307710	307924	308137	308351	308564	308778	308991	309204	309417	213
4	309630	309843	310056	310268	310481	310693	310906	311118	311330	311542	212
5	311754	311966	312177	312389	312600	312812	313023	313234	313445	313656	211
6	313867	314078	314289	314499	314710	314920	315130	315340	315551	315760	210
7	315970	316180	316390	316599	316809	317018	317227	317436	317646	317854	209
8	318063	318272	318481	318689	318898	319106	319314	319522	319730	319938	208
9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	**322219**	**322426**	**322633**	**322839**	**323046**	**323252**	**323458**	**323665**	**323871**	**324077**	206
1	324282	324488	324694	324899	325105	325310	325516	325721	325926	326131	205
2	326336	326541	326745	326950	327155	327359	327563	327767	327972	328176	204
3	328380	328583	328787	328991	329194	329398	329601	329805	330008	330211	203
4	330414	330617	330819	331022	331225	331427	331630	331832	332034	332236	202
5	332438	332640	332842	333044	333246	333447	333649	333850	334051	334253	
6	334454	334655	334856	335057	335257	335458	335658	335859	336059	336260	201
7	336460	336660	336860	337060	337260	337459	337659	337858	338058	338257	200
8	338456	338656	338855	339054	339253	339451	339650	339849	340047	340246	199
9	340444	340642	340841	341039	341237	341435	341632	341830	342028	342225	198
220	**342423**	**342620**	**342817**	**343014**	**343212**	**343409**	**343606**	**343802**	**343999**	**344196**	197
1	344392	344589	344785	344981	345178	345374	345570	345766	345962	346157	196
2	346353	346549	346744	346939	347135	347330	347525	347720	347915	348110	195
3	348305	348500	348694	348889	349083	349278	349472	349666	349860	350054	194
4	350248	350442	350636	350829	351023	351216	351410	351603	351796	351989	193
5	352183	352375	352568	352761	352954	353147	353339	353532	353724	353916	
6	354108	354301	354493	354685	354876	355068	355260	355452	355643	355834	192
7	356026	356217	356408	356599	356790	356981	357172	357363	357554	357744	191
8	357935	358125	358316	358506	358696	358886	359076	359266	359456	359646	190
9	359835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	**361728**	**361917**	**362105**	**362294**	**362482**	**362671**	**362859**	**363048**	**363236**	**363424**	188
1	363612	363800	363988	364176	364363	364551	364739	364926	365113	365301	
2	365488	365675	365862	366049	366236	366423	366610	36[illegible]796	366983	367169	187
3	367356	367542	367729	367915	368101	368287	368[illegible]3	368659	368845	369030	186
4	369216	369401	369587	369772	369958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	371991	372175	372360	372544	372728	184
6	372912	373096	373280	373464	373647	373831	374015	374198	374382	374565	
7	374748	374932	375115	375298	375481	375664	375846	376029	376212	376394	183
8	376577	376759	376942	377124	377306	377488	377670	377852	378034	378216	182
9	378398	378580	378761	378943	379124	379306	379487	379668	379849	380030	181

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182 4	205.2
226	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	203.4
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.4
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
180	18.0	36.0	54.0	72.0	90.0	108.0	126 0	144 0	162.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
240	**380211**	**380392**	**380573**	**380754**	**380934**	**381115**	**381296**	**381476**	**381656**	**381837**	181
1	382017	382197	382377	382557	382737	382917	383097	383277	383456	383636	180
2	383815	383995	384174	384353	384533	384712	384891	385070	385249	385428	179
3	385606	385785	385964	386142	386321	386499	386677	386856	387034	387212	178
4	387390	387568	387746	387924	388101	388279	388456	388634	388811	388989	
5	389166	389343	389520	389698	389875	390051	390228	390405	390582	390759	177
6	390935	391112	391288	391464	391641	391817	391993	392169	392345	392521	176
7	392697	392873	393048	393224	393400	393575	393751	393926	394101	394277	
8	394452	394627	394802	394977	395152	395326	395501	395676	395850	396025	175
9	396199	396374	396548	396722	396896	397071	397245	397419	397592	397766	174
250	**397940**	**398114**	**398287**	**398461**	**398634**	**398808**	**398981**	**399154**	**399328**	**399501**	173
1	399674	399847	400020	400192	400365	400538	400711	400883	401056	401228	
2	401401	401573	401745	401917	402089	402261	402433	402605	402777	402949	172
3	403121	403292	403464	403635	403807	403978	404149	404320	404492	404663	171
4	404834	405005	405176	405346	405517	405688	405858	406029	406199	406370	
5	406540	406710	406881	407051	407221	407391	407561	407731	407901	408070	170
6	408240	408410	408579	408749	408918	409087	409257	409426	409595	409764	169
7	409933	410102	410271	410440	410609	410777	410946	411114	411283	411451	
8	411620	411788	411956	412124	412293	412461	412629	412796	412964	413132	168
9	413300	413467	413635	413803	413970	414137	414305	414472	414639	414806	167
260	**414973**	**415140**	**415307**	**415474**	**415641**	**415808**	**415974**	**416141**	**416308**	**416474**	
1	416641	416807	416973	417139	417306	417472	417638	417804	417970	418135	166
2	418301	418467	418633	418798	418964	419129	419295	419460	419625	419791	165
3	419956	420121	420286	420451	420616	420781	420945	421110	421275	421439	
4	421604	421768	421933	422097	422261	422426	422590	422754	422918	423082	164
5	423246	423410	423574	423737	423901	424065	424228	424392	424555	424718	
6	424882	425045	425208	425371	425534	425697	425860	426023	426186	426349	163
7	426511	426674	426836	426999	427161	427324	427486	427648	427811	427973	162
8	428135	428297	428459	428621	428783	428944	429106	429268	429429	429591	
9	429752	429914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	**431364**	**431525**	**431685**	**431846**	**432007**	**432167**	**432328**	**432488**	**432649**	**432809**	
1	432969	433130	433290	433450	433610	433770	433930	434090	434249	434409	160
2	434569	434729	434888	435048	435207	435367	435526	435685	435844	436004	159
3	436163	436322	436481	436640	436799	436957	437116	437275	437433	437592	
4	437751	437909	438067	438226	438384	438542	438701	438859	439017	439175	158
5	439333	439491	439648	439806	439964	440122	440279	440437	440594	440752	
6	440909	441066	441224	441381	441538	441695	441852	442009	442166	442323	157
7	442480	442637	442793	442950	443106	443263	443419	443576	443732	443889	
8	444045	444201	444357	444513	444669	444825	444981	445137	445293	445449	156
9	445604	445760	445915	446071	446226	446382	446537	446692	446848	447003	155
280	**447158**	**447313**	**447468**	**447623**	**447778**	**447933**	**448088**	**448242**	**448397**	**448552**	
1	448706	448861	449015	449170	449324	449478	449633	449787	449941	450095	154
2	450249	450403	450557	450711	450865	451018	451172	451326	451479	451633	
3	451786	451940	452093	452247	452400	452553	452706	452859	453012	453165	153
4	453318	453471	453624	453777	453930	454082	454235	454387	454540	454692	
5	454845	454997	455150	455302	455454	455606	455758	455910	456062	456214	152
6	456366	456518	456670	456821	456973	457125	457276	457428	457579	457731	
7	457882	458033	458184	458336	458487	458638	458789	458940	459091	459242	151
8	459392	459543	459694	459845	459995	460146	460296	460447	460597	460748	
9	460898	461048	461198	461348	461499	461649	461799	461948	462098	462248	150

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
162	16.2	32.4	48.6	64.8	81.0	97.2	113.4	129.6	145.8
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
290	**462398**	**462548**	**462697**	**462847**	**462997**	**463146**	**463296**	**463445**	**463594**	**463744**	
1	463893	464042	464191	464340	464490	464639	464788	464936	465085	465234	149
2	465383	465532	465680	465829	465977	466126	466274	466423	466571	466719	
3	466868	467016	467164	467312	467460	467608	467756	467904	468052	468200	148
4	468347	468495	468643	468790	468938	469085	469233	469380	469527	469675	
5	469822	469969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	471585	471732	471878	472025	472171	472318	472464	472610	146
7	472756	472903	473049	473195	473341	473487	473633	473779	473925	474071	
8	474216	474362	474508	474653	474799	474944	475090	475235	475381	475526	
9	475671	475816	475962	476107	476252	476397	476542	476687	476832	476976	145
300	**477121**	**477266**	**477411**	**477555**	**477700**	**477844**	**477989**	**478133**	**478278**	**478422**	
1	478566	478711	478855	478999	479143	479287	479431	479575	479719	479863	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	
3	481443	481586	481729	481872	482016	482159	482302	482445	482588	482731	143
4	482874	483016	483159	483302	483445	483587	483730	483872	484015	484157	
5	484300	484442	484585	484727	484869	485011	485153	485295	485437	485579	142
6	485721	485863	486005	486147	486289	486430	486572	486714	486855	486997	
7	487138	487280	487421	487563	487704	487845	487986	488127	488269	488410	141
8	488551	488692	488833	488974	489114	489255	489396	489537	489677	489818	
9	489958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	**491362**	**491502**	**491642**	**491782**	**491922**	**492062**	**492201**	**492341**	**492481**	**492621**	
1	492760	492900	493040	493179	493319	493458	493597	493737	493876	494015	139
2	494155	494294	494433	494572	494711	494850	494989	495128	495267	495406	
3	495544	495683	495822	495960	496099	496238	496376	496515	496653	496791	
4	496930	497068	497206	497344	497483	497621	497759	497897	498035	498173	138
5	498311	498448	498586	498724	498862	498999	499137	499275	499412	499550	
6	499687	499824	499962	500099	500236	500374	500511	500648	500785	500922	137
7	501059	501196	501333	501470	501607	501744	501880	502017	502154	502291	
8	502427	502564	502700	502837	502973	503109	503246	503382	503518	503655	136
9	503791	503927	504063	504199	504335	504471	504607	504743	504878	505014	
320	**505150**	**505286**	**505421**	**505557**	**505693**	**505828**	**505964**	**506099**	**506234**	**506370**	
1	506505	506640	506776	506911	507046	507181	507316	507451	507586	507721	135
2	507856	507991	508126	508260	508395	508530	508664	508799	508934	509068	
3	509203	509337	509471	509606	509740	509874	510009	510143	510277	510411	134
4	510545	510679	510813	510947	511081	511215	511349	511482	511616	511750	
5	511883	512017	512151	512284	512418	512551	512684	512818	512951	513084	133
6	513218	513351	513484	513617	513750	513883	514016	514149	514282	514415	
7	514548	514681	514813	514946	515079	515211	515344	515476	515609	515741	
8	515874	516006	516139	516271	516403	516535	516668	516800	516932	517064	132
9	517196	517328	517460	517592	517724	517855	517987	518119	518251	518382	
330	**518514**	**518646**	**518777**	**518909**	**519040**	**519171**	**519303**	**519434**	**519566**	**519697**	131
1	519828	519959	520090	520221	520353	520484	520615	520745	520876	521007	
2	521138	521269	521400	521530	521661	521792	521922	522053	522183	522314	
3	522444	522575	522705	522835	522966	523096	523226	523356	523486	523616	130
4	523746	523876	524006	524136	524266	524396	524526	524656	524785	524915	
5	525045	525174	525304	525434	525563	525693	525822	525951	526081	526210	129
6	526339	526469	526598	526727	526856	526985	527114	527243	527372	527501	
7	527630	527759	527888	528016	528145	528274	528402	528531	528660	528788	
8	528917	529045	529174	529302	529430	529559	529687	529815	529943	530072	128
9	530200	530328	530456	530584	530712	530840	530968	531096	531223	531351	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138.6
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136.8
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133.2
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131.4
144	14.4	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129.6
142	14.2	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127.8
140	14.0	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126.0
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
130	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
340	**531479**	**531607**	**531734**	**531862**	**531990**	**532117**	**532245**	**532372**	**532500**	**532627**	
1	532754	532882	533009	533136	533264	533391	533518	533645	533772	533899	127
2	534026	534153	534280	534407	534534	534661	534787	534914	535041	535167	
3	535294	535421	535547	535674	535800	535927	536053	536180	536306	536432	126
4	536558	536685	536811	536937	537063	537189	537315	537441	537567	537693	
5	537819	537945	538071	538197	538322	538448	538574	538699	538825	538951	
6	539076	539202	539327	539452	539578	539703	539829	539954	540079	540204	125
7	540329	540455	540580	540705	540830	540955	541080	541205	541330	541454	
8	541579	541704	541829	541953	542078	542203	542327	542452	542576	542701	
9	542825	542950	543074	543199	543323	543447	543571	543696	543820	543944	124
350	**544068**	**544192**	**544316**	**544440**	**544564**	**544688**	**544812**	**544936**	**545060**	**545183**	
1	545307	545431	545555	545678	545802	545925	546049	546172	546296	546419	
2	546543	546666	546789	546913	547036	547159	547282	547405	547529	547652	123
3	547775	547898	548021	548144	548267	548389	548512	548635	548758	548881	
4	549003	549126	549249	549371	549494	549616	549739	549861	549984	550106	
5	550228	550351	550473	550595	550717	550840	550962	551084	551206	551328	122
6	551450	551572	551694	551816	551938	552060	552181	552303	552425	552547	
7	552668	552790	552911	553033	553155	553276	553398	553519	553640	553762	121
8	553883	554004	554126	554247	554368	554489	554610	554731	554852	554973	
9	555094	555215	555336	555457	555578	555699	555820	555940	556061	556182	
360	**556303**	**556423**	**556544**	**556664**	**556785**	**556905**	**557026**	**557146**	**557267**	**557387**	120
1	557507	557627	557748	557868	557988	558108	558228	558349	558469	558589	
2	558709	558829	558948	559068	559188	559308	559428	559548	559667	559787	
3	559907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
4	561101	561221	561340	561459	561578	561698	561817	561936	562055	562174	
5	562293	562412	562531	562650	562769	562887	563006	563125	563244	563362	
6	563481	563600	563718	563837	563955	564074	564192	564311	564429	564548	
7	564666	564784	564903	565021	565139	565257	565376	565494	565612	565730	118
8	565848	565966	566084	566202	566320	566437	566555	566673	566791	566909	
9	567026	567144	567262	567379	567497	567614	567732	567849	567967	568084	
370	**568202**	**568319**	**568436**	**568554**	**568671**	**568788**	**568905**	**569023**	**569140**	**569257**	117
1	569374	569491	569608	569725	569842	569959	570076	570193	570309	570426	
2	570543	570660	570776	570893	571010	571126	571243	571359	571476	571592	
3	571709	571825	571942	572058	572174	572291	572407	572523	572639	572755	116
4	572872	572988	573104	573220	573336	573452	573568	573684	573800	573915	
5	574031	574147	574263	574379	574494	574610	574726	574841	574957	575072	
6	575188	575303	575419	575534	575650	575765	575880	575996	576111	576226	115
7	576341	576457	576572	576687	576802	576917	577032	577147	577262	577377	
8	577492	577607	577722	577836	577951	578066	578181	578295	578410	578525	
9	578639	578754	578868	578983	579097	579212	579326	579441	579555	579669	114
380	**579784**	**579898**	**580012**	**580126**	**580241**	**580355**	**580469**	**580583**	**580697**	**580811**	
1	580925	581039	581153	581267	581381	581495	581608	581722	581836	581950	
2	582063	582177	582291	582404	582518	582631	582745	582858	582972	583085	
3	583199	583312	583426	583539	583652	583765	583879	583992	584105	584218	
4	584331	584444	584557	584670	584783	584896	585009	585122	585235	585348	113
5	585461	585574	585686	585799	585912	586024	586137	586250	586362	586475	
6	586587	586700	586812	586925	587037	587149	587262	587374	587486	587599	
7	587711	587823	587935	588047	588160	588272	588384	588496	588608	588720	112
8	588832	588944	589056	589167	589279	589391	589503	589615	589726	589838	
9	589950	590061	590173	590284	590396	590507	590619	590730	590842	590953	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
122	12.2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	109.8
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0
118	11.8	23.6	35.4	47.2	59.0	70.8	82.6	94.4	106.2
116	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.4
114	11.4	22.8	34.2	45.6	57.0	68.4	79.8	91.2	102.6

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
390	**591065**	**591176**	**591287**	**591399**	**591510**	**591621**	**591732**	**591843**	**591955**	**592066**	
1	592177	592288	592399	592510	592621	592732	592843	592954	593064	593175	111
2	593286	593397	593508	593618	593729	593840	593950	594061	594171	594282	
3	594393	594503	594614	594724	594834	594945	595055	595165	595276	595386	
4	595496	595606	595717	595827	595937	596047	596157	596267	596377	596487	
5	596597	596707	596817	596927	597037	597146	597256	597366	597476	597586	110
6	597695	597805	597914	598024	598134	598243	598353	598462	598572	598681	
7	598791	598900	599009	599119	599228	599337	599446	599556	599665	599774	
8	599883	599992	600101	600210	600319	600428	600537	600646	600755	600864	109
9	600973	601082	601191	601299	601408	601517	601625	601734	601843	601951	
400	**602060**	**602169**	**602277**	**602386**	**602494**	**602603**	**602711**	**602819**	**602928**	**603036**	
1	603144	603253	603361	603469	603577	603686	603794	603902	604010	604118	108
2	604226	604334	604442	604550	604658	604766	604874	604982	605089	605197	
3	605305	605413	605521	605628	605736	605844	605951	606059	606166	606274	
4	606381	606489	606596	606704	606811	606919	607026	607133	607241	607348	
5	607455	607562	607669	607777	607884	607991	608098	608205	608312	608419	107
6	608526	608633	608740	608847	608954	609061	609167	609274	609381	609488	
7	609594	609701	609808	609914	610021	610128	610234	610341	610447	610554	
8	610660	610767	610873	610979	611086	611192	611298	611405	611511	611617	
9	611723	611829	611936	612042	612148	612254	612360	612466	612572	612678	106
410	**612784**	**612890**	**612996**	**613102**	**613207**	**613313**	**613419**	**613525**	**613630**	**613736**	
1	613842	613947	614053	614159	614264	614370	614475	614581	614686	614792	
2	614897	615003	615108	615213	615319	615424	615529	615634	615740	615845	
3	615950	616055	616160	616265	616370	616476	616581	616686	616790	616895	105
4	617000	617105	617210	617315	617420	617525	617629	617734	617839	617943	
5	618048	618153	618257	618362	618466	618571	618676	618780	618884	618989	
6	619093	619198	619302	619406	619511	619615	619719	619824	619928	620032	
7	620136	620240	620344	620448	620552	620656	620760	620864	620968	621072	104
8	621176	621280	621384	621488	621592	621695	621799	621903	622007	622110	
9	622214	622318	622421	622525	622628	622732	622835	622939	623042	623146	
420	**623249**	**623353**	**623456**	**623559**	**623663**	**623766**	**623869**	**623973**	**624076**	**624179**	
1	624282	624385	624488	624591	624695	624798	624901	625004	625107	625210	103
2	625312	625415	625518	625621	625724	625827	625929	626032	626135	626238	
3	626340	626443	626546	626648	626751	626853	626956	627058	627161	627263	
4	627366	627468	627571	627673	627775	627878	627980	628082	628185	628287	
5	628389	628491	628593	628695	628797	628900	629002	629104	629206	629308	102
6	629410	629512	629613	629715	629817	629919	630021	630123	630224	630326	
7	630428	630530	630631	630733	630835	630936	631038	631139	631241	631342	
8	631444	631545	631647	631748	631849	631951	632052	632153	632255	632356	101
9	632457	632559	632660	632761	632862	632963	633064	633165	633266	633367	
430	**633468**	**633569**	**633670**	**633771**	**633872**	**633973**	**634074**	**634175**	**634276**	**634376**	
1	634477	634578	634679	634779	634880	634981	635081	635182	635283	635383	
2	635484	635584	635685	635785	635886	635986	636087	636187	636287	636388	
3	636488	636588	636688	636789	636889	636989	637089	637189	637290	637390	
4	637490	637590	637690	637790	637890	637990	638090	638190	638290	638389	100
5	638489	638589	638689	638789	638888	638988	639088	639188	639287	639387	
6	639486	639586	639686	639785	639885	639984	640084	640183	640283	640382	
7	640481	640581	640680	640779	640879	640978	641077	641177	641276	641375	
8	641474	641573	641672	641771	641871	641970	642069	642168	642267	642366	
9	642465	642563	642662	642761	642860	642959	643058	643156	643255	643354	99

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
110	11.0	22.0	33.0	44.0	55.0	66.0	77.0	88.0	99.0
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.4
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6
102	10.2	20.4	30.6	40.8	51.0	61.2	71.4	81.6	91.8
100	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
440	**643453**	**643551**	**643650**	**643749**	**643847**	**643946**	**644044**	**644143**	**644242**	**644340**	
1	644439	644537	644636	644734	644832	644931	645029	645127	645226	645324	
2	645422	645521	645619	645717	645815	645913	646011	646110	646208	646306	
3	646404	646502	646600	646698	646796	646894	646992	647089	647187	647285	98
4	647383	647481	647579	647676	647774	647872	647969	648067	648165	648262	
5	648360	648458	648555	648653	648750	648848	648945	649043	649140	649237	
6	649335	649432	649530	649627	649724	649821	649919	650016	650113	650210	
7	650308	650405	650502	650599	650696	650793	650890	650987	651084	651181	
8	651278	651375	651472	651569	651666	651762	651859	651956	652053	652150	97
9	652246	652343	652440	652536	652633	652730	652826	652923	653019	653116	
450	**653213**	**653309**	**653405**	**653502**	**653598**	**653695**	**653791**	**653888**	**653984**	**654080**	
1	654177	654273	654369	654465	654562	654658	654754	654850	654946	655042	
2	655138	655235	655331	655427	655523	655619	655715	655810	655906	656002	96
3	656098	656194	656290	656386	656482	656577	656673	656769	656864	656960	
4	657056	657152	657247	657343	657438	657534	657629	657725	657820	657916	
5	658011	658107	658202	658298	658393	658488	658584	658679	658774	658870	
6	658965	659060	659155	659250	659346	659441	659536	659631	659726	659821	
7	659916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
8	660865	660960	661055	661150	661245	661339	661434	661529	661623	661718	
9	661813	661907	662002	662096	662191	662286	662380	662475	662569	662663	
460	**662758**	**662852**	**662947**	**663041**	**663135**	**663230**	**663324**	**663418**	**663512**	**663607**	
1	663701	663795	663889	663983	664078	664172	664266	664360	664454	664548	
2	664642	664736	664830	664924	665018	665112	665206	665299	665393	665487	94
3	665581	665675	665769	665862	665956	666050	666143	666237	666331	666424	
4	666518	666612	666705	666799	666892	666986	667079	667173	667266	667360	
5	667453	667546	667640	667733	667826	667920	668013	668106	668199	668293	
6	668386	668479	668572	668665	668759	668852	668945	669038	669131	669224	
7	669317	669410	669503	669596	669689	669782	669875	669967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	670988	671080	
9	671173	671265	671358	671451	671543	671636	671728	671821	671913	672005	
470	**672098**	**672190**	**672283**	**672375**	**672467**	**672560**	**672652**	**672744**	**672836**	**672929**	
1	673021	673113	673205	673297	673390	673482	673574	673666	673758	673850	
2	673942	674034	674126	674218	674310	674402	674494	674586	674677	674769	92
3	674861	674953	675045	675137	675228	675320	675412	675503	675595	675687	
4	675778	675870	675962	676053	676145	676236	676328	676419	676511	676602	
5	676694	676785	676876	676968	677059	677151	677242	677333	677424	677516	
6	677607	677698	677789	677881	677972	678063	678154	678245	678336	678427	
7	678518	678609	678700	678791	678882	678973	679064	679155	679246	679337	91
8	679428	679519	679610	679700	679791	679882	679973	680063	680154	680245	
9	680336	680426	680517	680607	680698	680789	680879	680970	681060	681151	
480	**681241**	**681332**	**681422**	**681513**	**681603**	**681693**	**681784**	**681874**	**681964**	**682055**	
1	682145	682235	682326	682416	682506	682596	682686	682777	682867	682957	
2	683047	683137	683227	683317	683407	683497	683587	683677	683767	683857	90
3	683947	684037	684127	684217	684307	684396	684486	684576	684666	684756	
4	684845	684935	685025	685114	685204	685294	685383	685473	685563	685652	
5	685742	685831	685921	686010	686100	686189	686279	686368	686458	686547	
6	686636	686726	686815	686904	686994	687083	687172	687261	687351	687440	
7	687529	687618	687707	687796	687886	687975	688064	688153	688242	688331	
8	688420	688509	688598	688687	688776	688865	688953	689042	689131	689220	89
9	689309	689398	689486	689575	689664	689753	689841	689930	690019	690107	
490	**690196**	**690285**	**690373**	**690462**	**690550**	**690639**	**690728**	**690816**	**690905**	**690993**	
1	691081	691170	691258	691347	691435	691524	691612	691700	691789	691877	
2	691965	692053	692142	692230	692318	692406	692494	692583	692671	692759	
3	692847	692935	693023	693111	693199	693287	693375	693463	693551	693639	88
4	693727	693815	693903	693991	694078	694166	694254	694342	694430	694517	
5	694605	694693	694781	694868	694956	695044	695131	695219	695307	695394	
6	695482	695569	695657	695744	695832	695919	696007	696094	696182	696269	
7	696356	696444	696531	696618	696706	696793	696880	696968	697055	697142	
8	697229	697317	697404	697491	697578	697665	697752	697839	697926	698014	87
9	698100	698188	698275	698362	698449	698535	698622	698709	698796	698883	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
500	**698970**	**699057**	**699144**	**699231**	**699317**	**699404**	**699491**	**699578**	**699664**	**699751**	
1	699838	699924	700011	700098	700184	700271	700358	700444	700531	700617	
2	700704	700790	700877	700963	701050	701136	701222	701309	701395	701482	
3	701568	701654	701741	701827	701913	701999	702086	702172	702258	702344	
4	702431	702517	702603	702689	702775	702861	702947	703033	703119	703205	
5	703291	703377	703463	703549	703635	703721	703807	703893	703979	704065	86
6	704151	704236	704322	704408	704494	704579	704665	704751	704837	704922	
7	705008	705094	705179	705265	705350	705436	705522	705607	705693	705778	
8	705864	705949	706035	706120	706206	706291	706376	706462	706547	706632	
9	706718	706803	706888	706974	707059	707144	707229	707315	707400	707485	
510	**707570**	**707655**	**707740**	**707826**	**707911**	**707996**	**708081**	**708166**	**708251**	**708336**	
1	708421	708506	708591	708676	708761	708846	708931	709015	709100	709185	85
2	709270	709355	709440	709524	709609	709694	709779	709863	709948	710033	
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	710879	
4	710963	711048	711132	711217	711301	711385	711470	711554	711639	711723	
5	711807	711892	711976	712060	712144	712229	712313	712397	712481	712566	
6	712650	712734	712818	712902	712986	713070	713154	713238	713323	713407	
7	713491	713575	713659	713742	713826	713910	713994	714078	714162	714246	84
8	714330	714414	714497	714581	714665	714749	714833	714916	715000	715084	
9	715167	715251	715335	715418	715502	715586	715669	715753	715836	715920	
520	**716003**	**716087**	**716170**	**716254**	**716337**	**716421**	**716504**	**716588**	**716671**	**716754**	
1	716838	716921	717004	717088	717171	717254	717338	717421	717504	717587	
2	717671	717754	717837	717920	718003	718086	718169	718253	718336	718419	83
3	718502	718585	718668	718751	718834	718917	719000	719083	719165	719248	
4	719331	719414	719497	719580	719663	719745	719828	719911	719994	720077	
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	720903	
6	720986	721068	721151	721233	721316	721398	721481	721563	721646	721728	
7	721811	721893	721975	722058	722140	722222	722305	722387	722469	722552	
8	722634	722716	722798	722881	722963	723045	723127	723209	723291	723374	
9	723456	723538	723620	723702	723784	723866	723948	724030	724112	724194	82
530	**724276**	**724358**	**724440**	**724522**	**724604**	**724685**	**724767**	**724849**	**724931**	**725013**	
1	725095	725176	725258	725340	725422	725503	725585	725667	725748	725830	
2	725912	725993	726075	726156	726238	726320	726401	726483	726564	726646	
3	726727	726809	726890	726972	727053	727134	727216	727297	727379	727460	
4	727541	727623	727704	727785	727866	727948	728029	728110	728191	728273	
5	728354	728435	728516	728597	728678	728759	728841	728922	729003	729084	
6	729165	729246	729327	729408	729489	729570	729651	729732	729813	729893	81
7	729974	730055	730136	730217	730298	730378	730459	730540	730621	730702	
8	730782	730863	730944	731024	731105	731186	731266	731347	731428	731508	
9	731589	731669	731750	731830	731911	731991	732072	732152	732233	732313	
540	**732394**	**732474**	**732555**	**732635**	**732715**	**732796**	**732876**	**732956**	**733037**	**733117**	
1	733197	733278	733358	733438	733518	733598	733679	733759	733839	733919	
2	733999	734079	734160	734240	734320	734400	734480	734560	734640	734720	80
3	734800	734880	734960	735040	735120	735200	735279	735359	735439	735519	
4	735599	735679	735759	735838	735918	735998	736078	736157	736237	736317	
5	736397	736476	736556	736635	736715	736795	736874	736954	737034	737113	
6	737193	737272	737352	737431	737511	737590	737670	737749	737829	737908	
7	737987	738067	738146	738225	738305	738384	738463	738543	738622	738701	
8	738781	738860	738939	739018	739097	739177	739256	739335	739414	739493	
9	739572	739651	739731	739810	739889	739968	740047	740126	740205	740284	79
550	**740363**	**740442**	**740521**	**740600**	**740678**	**740757**	**740836**	**740915**	**740994**	**741073**	
1	741152	741230	741309	741388	741467	741546	741624	741703	741782	741860	
2	741939	742018	742096	742175	742254	742332	742411	742489	742568	742647	
3	742725	742804	742882	742961	743039	743118	743196	743275	743353	743431	
4	743510	743588	743667	743745	743823	743902	743980	744058	744136	744215	
5	744293	744371	744449	744528	744606	744684	744762	744840	744919	744997	
6	745075	745153	745231	745309	745387	745465	745543	745621	745699	745777	78
7	745855	745933	746011	746089	746167	746245	746323	746401	746479	746556	
8	746634	746712	746790	746868	746945	747023	747101	747179	747256	747334	
9	747412	747489	747567	747645	747722	747800	747878	747955	748033	748110	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.6
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
560	**748188**	**748266**	**748343**	**748421**	**748498**	**748576**	**748653**	**748731**	**748808**	**748885**	
1	748963	749040	749118	749195	749272	749350	749427	749504	749582	749659	
2	749736	749814	749891	749968	750045	750123	750200	750277	750354	750431	
3	750508	750586	750663	750740	750817	750894	750971	751048	751125	751202	
4	751279	751356	751433	751510	751587	751664	751741	751818	751895	751972	77
5	752048	752125	752202	752279	752356	752433	752509	752586	752663	752740	
6	752816	752893	752970	753047	753123	753200	753277	753353	753430	753506	
7	753583	753660	753736	753813	753889	753966	754042	754119	754195	754272	
8	754348	754425	754501	754578	754654	754730	754807	754883	754960	755036	
9	755112	755189	755265	755341	755417	755494	755570	755646	755722	755799	
570	**755875**	**755951**	**756027**	**756103**	**756180**	**756256**	**756332**	**756408**	**756484**	**756560**	
1	756636	756712	756788	756864	756940	757016	757092	757168	757244	757320	76
2	757396	757472	757548	757624	757700	757775	757851	757927	758003	758079	
3	758155	758230	758306	758382	758458	758533	758609	758685	758761	758836	
4	758912	758988	759063	759139	759214	759290	759366	759441	759517	759592	
5	759668	759743	759819	759894	759970	760045	760121	760196	760272	760347	
6	760422	760498	760573	760649	760724	760799	760875	760950	761025	761101	
7	761176	761251	761326	761402	761477	761552	761627	761702	761778	761853	
8	761928	762003	762078	762153	762228	762303	762378	762453	762529	762604	75
9	762679	762754	762829	762904	762978	763053	763128	763203	763278	763353	
580	**763428**	**763503**	**763578**	**763653**	**763727**	**763802**	**763877**	**763952**	**764027**	**764101**	
1	764176	764251	764326	764400	764475	764550	764624	764699	764774	764848	
2	764923	764998	765072	765147	765221	765296	765370	765445	765520	765594	
3	765669	765743	765818	765892	765966	766041	766115	766190	766264	766338	
4	766413	766487	766562	766636	766710	766785	766859	766933	767007	767082	
5	767156	767230	767304	767379	767453	767527	767601	767675	767749	767823	
6	767898	767972	768046	768120	768194	768268	768342	768416	768490	768564	74
7	768638	768712	768786	768860	768934	769008	769082	769156	769230	769303	
8	769377	769451	769525	769599	769673	769746	769820	769894	769968	770042	
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	770778	
590	**770852**	**770926**	**770999**	**771073**	**771146**	**771220**	**771293**	**771367**	**771440**	**771514**	
1	771587	771661	771734	771808	771881	771955	772028	772102	772175	772248	
2	772322	772395	772468	772542	772615	772688	772762	772835	772908	772981	
3	773055	773128	773201	773274	773348	773421	773494	773567	773640	773713	
4	773786	773860	773933	774006	774079	774152	774225	774298	774371	774444	73
5	774517	774590	774663	774736	774809	774882	774955	775028	775100	775173	
6	775246	775319	775392	775465	775538	775610	775683	775756	775829	775902	
7	775974	776047	776120	776193	776265	776338	776411	776483	776556	776629	
8	776701	776774	776846	776919	776992	777064	777137	777209	777282	777354	
9	777427	777499	777572	777644	777717	777789	777862	777934	778006	778079	
600	**778151**	**778224**	**778296**	**778368**	**778441**	**778513**	**778585**	**778658**	**778730**	**778802**	
1	778874	778947	779019	779091	779163	779236	779308	779380	779452	779524	
2	779596	779669	779741	779813	779885	779957	780029	780101	780173	780245	
3	780317	780389	780461	780533	780605	780677	780749	780821	780893	780965	72
4	781037	781109	781181	781253	781324	781396	781468	781540	781612	781684	
5	781755	781827	781899	781971	782042	782114	782186	782258	782329	782401	
6	782473	782544	782616	782688	782759	782831	782902	782974	783046	783117	
7	783189	783260	783332	783403	783475	783546	783618	783689	783761	783832	
8	783904	783975	784046	784118	784189	784261	784332	784403	784475	784546	
9	784617	784689	784760	784831	784902	784974	785045	785116	785187	785259	
610	**785330**	**785401**	**785472**	**785543**	**785615**	**785686**	**785757**	**785828**	**785899**	**785970**	
1	786041	786112	786183	786254	786325	786396	786467	786538	786609	786680	71
2	786751	786822	786893	786964	787035	787106	787177	787248	787319	787390	
3	787460	787531	787602	787673	787744	787815	787885	787956	788027	788098	
4	788168	788239	788310	788381	788451	788522	788593	788663	788734	788804	
5	788875	788946	789016	789087	789157	789228	789299	789369	789440	789510	
6	789581	789651	789722	789792	789863	789933	790004	790074	790144	790215	
7	790285	790356	790426	790496	790567	790637	790707	790778	790848	790918	
8	790988	791059	791129	791199	791269	791340	791410	791480	791550	791620	
9	791691	791761	791831	791901	791971	792041	792111	792181	792252	792322	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
620	**792392**	**792462**	**792532**	**792602**	**792672**	**792742**	**792812**	**792882**	**792952**	**793022**	70
1	793092	793162	793231	793301	793371	793441	793511	793581	793651	793721	
2	793790	793860	793930	794000	794070	794139	794209	794279	794349	794418	
3	794488	794558	794627	794697	794767	794836	794906	794976	795045	795115	
4	795185	795254	795324	795393	795463	795532	795602	795672	795741	795811	
5	795880	795949	796019	796088	796158	796227	796297	796366	796436	796505	
6	796574	796644	796713	796782	796852	796921	796990	797060	797129	797198	
7	797268	797337	797406	797475	797545	797614	797683	797752	797821	797890	
8	797960	798029	798098	798167	798236	798305	798374	798443	798513	798582	
9	798651	798720	798789	798858	798927	798996	799065	799134	799203	799272	69
630	**799341**	**799409**	**799478**	**799547**	**799616**	**799685**	**799754**	**799823**	**799892**	**799961**	
1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	
2	800717	800786	800854	800923	800992	801061	801129	801198	801266	801335	
3	801404	801472	801541	801609	801678	801747	801815	801884	801952	802021	
4	802089	802158	802226	802295	802363	802432	802500	802568	802637	802705	
5	802774	802842	802910	802979	803047	803116	803184	803252	803321	803389	
6	803457	803525	803594	803662	803730	803798	803867	803935	804003	804071	
7	804139	804208	804276	804344	804412	804480	804548	804616	804685	804753	
8	804821	804889	804957	805025	805093	805161	805229	805297	805365	805433	68
9	805501	805569	805637	805705	805773	805841	805908	805976	806044	806112	
640	**806180**	**806248**	**806316**	**806384**	**806451**	**806519**	**806587**	**806655**	**806723**	**806790**	
1	806858	806926	806994	807061	807129	807197	807264	807332	807400	807467	
2	807535	807603	807670	807738	807806	807873	807941	808008	808076	808143	
3	808211	808279	808346	808414	808481	808549	808616	808684	808751	808818	
4	808886	808953	809021	809088	809156	809223	809290	809358	809425	809492	
5	809560	809627	809694	809762	809829	809896	809964	810031	810098	810165	
6	810233	810300	810367	810434	810501	810569	810636	810703	810770	810837	
7	810904	810971	811039	811106	811173	811240	811307	811374	811441	811508	67
8	811575	811642	811709	811776	811843	811910	811977	812044	812111	812178	
9	812245	812312	812379	812445	812512	812579	812646	812713	812780	812847	
650	**812913**	**812980**	**813047**	**813114**	**813181**	**813247**	**813314**	**813381**	**813448**	**813514**	
1	813581	813648	813714	813781	813848	813914	813981	814048	814114	814181	
2	814248	814314	814381	814447	814514	814581	814647	814714	814780	814847	
3	814913	814980	815046	815113	815179	815246	815312	815378	815445	815511	
4	815578	815644	815711	815777	815843	815910	815976	816042	816109	816175	
5	816241	816308	816374	816440	816506	816573	816639	816705	816771	816838	
6	816904	816970	817036	817102	817169	817235	817301	817367	817433	817499	
7	817565	817631	817698	817764	817830	817896	817962	818028	818094	818160	
8	818226	818292	818358	818424	818490	818556	818622	818688	818754	818820	66
9	818885	818951	819017	819083	819149	819215	819281	819346	819412	819478	
660	**819544**	**819610**	**819676**	**819741**	**819807**	**819873**	**819939**	**820004**	**820070**	**820136**	
1	820201	820267	820333	820399	820464	820530	820595	820661	820727	820792	
2	820858	820924	820989	821055	821120	821186	821251	821317	821382	821448	
3	821514	821579	821645	821710	821775	821841	821906	821972	822037	822103	
4	822168	822233	822299	822364	822430	822495	822560	822626	822691	822756	
5	822822	822887	822952	823018	823083	823148	823213	823279	823344	823409	
6	823474	823539	823605	823670	823735	823800	823865	823930	823996	824061	
7	824126	824191	824256	824321	824386	824451	824516	824581	824646	824711	
8	824776	824841	824906	824971	825036	825101	825166	825231	825296	825361	65
9	825426	825491	825556	825621	825686	825751	825815	825880	825945	826010	
670	**826075**	**826140**	**826204**	**826269**	**826334**	**826399**	**826464**	**826528**	**826593**	**826658**	
1	826723	826787	826852	826917	826981	827046	827111	827175	827240	827305	
2	827369	827434	827499	827563	827628	827692	827757	827821	827886	827951	
3	828015	828080	828144	828209	828273	828338	828402	828467	828531	828595	
4	828660	828724	828789	828853	828918	828982	829046	829111	829175	829239	
5	829304	829368	829432	829497	829561	829625	829690	829754	829818	829882	
6	829947	830011	830075	830139	830204	830268	830332	830396	830460	830525	
7	830589	830653	830717	830781	830845	830909	830973	831037	831102	831166	
8	831230	831294	831358	831422	831486	831550	831614	831678	831742	831806	64
9	831870	831934	831998	832062	832126	832189	832253	832317	832381	832445	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
66	6.6	13.2	19.8	26.4	33.0	39.6	46.2	52.8	59.4
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
680	**832509**	**832573**	**832637**	**832700**	**832764**	**832828**	**832892**	**832956**	**833020**	**833083**	
1	833147	833211	833275	833338	833402	833466	833530	833593	833657	833721	
2	833784	833848	833912	833975	834039	834103	834166	834230	834294	834357	
3	834421	834484	834548	834611	834675	834739	834802	834866	834929	834993	
4	835056	835120	835183	835247	835310	835373	835437	835500	835564	835627	
5	835691	835754	835817	835881	835944	836007	836071	836134	836197	836261	
6	836324	836387	836451	836514	836577	836641	836704	836767	836830	836894	
7	836957	837020	837083	837146	837210	837273	837336	837399	837462	837525	
8	837588	837652	837715	837778	837841	837904	837967	838030	838093	838156	63
9	838219	838282	838345	838408	838471	838534	838597	838660	838723	838786	
690	**838849**	**838912**	**838975**	**839038**	**839101**	**839164**	**839227**	**839289**	**839352**	**839415**	
1	839478	839541	839604	839667	839729	839792	839855	839918	839981	840043	
2	840106	840169	840232	840294	840357	840420	840482	840545	840608	840671	
3	840733	840796	840859	840921	840984	841046	841109	841172	841234	841297	
4	841359	841422	841485	841547	841610	841672	841735	841797	841860	841922	
5	841985	842047	842110	842172	842235	842297	842360	842422	842484	842547	
6	842609	842672	842734	842796	842859	842921	842983	843046	843108	843170	
7	843233	843295	843357	843420	843482	843544	843606	843669	843731	843793	
8	843855	843918	843980	844042	844104	844166	844229	844291	844353	844415	
9	844477	844539	844601	844664	844726	844788	844850	844912	844974	845036	
700	**845098**	**845160**	**845222**	**845284**	**845346**	**845408**	**845470**	**845532**	**845594**	**845656**	62
1	845718	845780	845842	845904	845966	846028	846090	846151	846213	846275	
2	846337	846399	846461	846523	846585	846646	846708	846770	846832	846894	
3	846955	847017	847079	847141	847202	847264	847326	847388	847449	847511	
4	847573	847634	847696	847758	847819	847881	847943	848004	848066	848128	
5	848189	848251	848312	848374	848435	848497	848559	848620	848682	848743	
6	848805	848866	848928	848989	849051	849112	849174	849235	849297	849358	
7	849419	849481	849542	849604	849665	849726	849788	849849	849911	849972	
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	
9	850646	850707	850769	850830	850891	850952	851014	851075	851136	851197	
710	**851258**	**851320**	**851381**	**851442**	**851503**	**851564**	**851625**	**851686**	**851747**	**851809**	
1	851870	851931	851992	852053	852114	852175	852236	852297	852358	852419	61
2	852480	852541	852602	852663	852724	852785	852846	852907	852968	853029	
3	853090	853150	853211	853272	853333	853394	853455	853516	853577	853637	
4	853698	853759	853820	853881	853941	854002	854063	854124	854185	854245	
5	854306	854367	854428	854488	854549	854610	854670	854731	854792	854852	
6	854913	854974	855034	855095	855156	855216	855277	855337	855398	855459	
7	855519	855580	855640	855701	855761	855822	855882	855943	856003	856064	
8	856124	856185	856245	856306	856366	856427	856487	856548	856608	856668	
9	856729	856789	856850	856910	856970	857031	857091	857152	857212	857272	
720	**857332**	**857393**	**857453**	**857513**	**857574**	**857634**	**857694**	**857755**	**857815**	**857875**	
1	857935	857995	858056	858116	858176	858236	858297	858357	858417	858477	
2	858537	858597	858657	858718	858778	858838	858898	858958	859018	859078	
3	859138	859198	859258	859318	859379	859439	859499	859559	859619	859679	60
4	859739	859799	859859	859918	859978	860038	860098	860158	860218	860278	
5	860338	860398	860458	860518	860578	860637	860697	860757	860817	860877	
6	860937	860996	861056	861116	861176	861236	861295	861355	861415	861475	
7	861534	861594	861654	861714	861773	861833	861893	861952	862012	862072	
8	862131	862191	862251	862310	862370	862430	862489	862549	862608	862668	
9	862728	862787	862847	862906	862966	863025	863085	863144	863204	863263	
730	**863323**	**863382**	**863442**	**863501**	**863561**	**863620**	**863680**	**863739**	**863799**	**863858**	
1	863917	863977	864036	864096	864155	864214	864274	864333	864392	864452	
2	864511	864570	864630	864689	864748	864808	864867	864926	864985	865045	
3	865104	865163	865222	865282	865341	865400	865459	865519	865578	865637	
4	865696	865755	865814	865874	865933	865992	866051	866110	866169	866228	
5	866287	866346	866405	866465	866524	866583	866642	866701	866760	866819	
6	866878	866937	866996	867055	867114	867173	867232	867291	867350	867409	59
7	867467	867526	867585	867644	867703	867762	867821	867880	867939	867998	
8	868056	868115	868174	868233	868292	868350	868409	868468	868527	868586	
9	868644	868703	868762	868821	868879	868938	868997	869056	869114	869173	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
740	**869232**	**869290**	**869349**	**869408**	**869466**	**869525**	**869584**	**869642**	**869701**	**869760**	
1	869818	869877	869935	869994	870053	870111	870170	870228	870287	870345	
2	870404	870462	870521	870579	870638	870696	870755	870813	870872	870930	
3	870989	871047	871106	871164	871223	871281	871339	871398	871456	871515	
4	871573	871631	871690	871748	871806	871865	871923	871981	872040	872098	
5	872156	872215	872273	872331	872389	872448	872506	872564	872622	872681	
6	872739	872797	872855	872913	872972	873030	873088	873146	873204	873262	
7	873321	873379	873437	873495	873553	873611	873669	873727	873785	873844	
8	873902	873960	874018	874076	874134	874192	874250	874308	874366	874424	58
9	874482	874540	874598	874656	874714	874772	874830	874888	874945	875003	
750	**875061**	**875119**	**875177**	**875235**	**875293**	**875351**	**875409**	**875466**	**875524**	**875582**	
1	875640	875698	875756	875813	875871	875929	875987	876045	876102	876160	
2	876218	876276	876333	876391	876449	876507	876564	876622	876680	876737	
3	876795	876853	876910	876968	877026	877083	877141	877199	877256	877314	
4	877371	877429	877487	877544	877602	877659	877717	877774	877832	877889	
5	877947	878004	878062	878119	878177	878234	878292	878349	878407	878464	
6	878522	878579	878637	878694	878752	878809	878866	878924	878981	879039	
7	879096	879153	879211	879268	879325	879383	879440	879497	879555	879612	
8	879669	879726	879784	879841	879898	879956	880013	880070	880127	880185	
9	880242	880299	880356	880413	880471	880528	880585	880642	880699	880756	
760	**880814**	**880871**	**880928**	**880985**	**881042**	**881099**	**881156**	**881213**	**881271**	**881328**	
1	881385	881442	881499	881556	881613	881670	881727	881784	881841	881898	
2	881955	882012	882069	882126	882183	882240	882297	882354	882411	882468	57
3	882525	882581	882638	882695	882752	882809	882866	882923	882980	883037	
4	883093	883150	883207	883264	883321	883377	883434	883491	883548	883605	
5	883661	883718	883775	883832	883888	883945	884002	884059	884115	884172	
6	884229	884285	884342	884399	884455	884512	884569	884625	884682	884739	
7	884795	884852	884909	884965	885022	885078	885135	885192	885248	885305	
8	885361	885418	885474	885531	885587	885644	885700	885757	885813	885870	
9	885926	885983	886039	886096	886152	886209	886265	886321	886378	886434	
770	**886491**	**886547**	**886604**	**886660**	**886716**	**886773**	**886829**	**886885**	**886942**	**886998**	
1	887054	887111	887167	887223	887280	887336	887392	887449	887505	887561	
2	887617	887674	887730	887786	887842	887898	887955	888011	888067	888123	
3	888179	888236	888292	888348	888404	888460	888516	888573	888629	888685	
4	888741	888797	888853	888909	888965	889021	889077	889134	889190	889246	
5	889302	889358	889414	889470	889526	889582	889638	889694	889750	889806	56
6	889862	889918	889974	890030	890086	890141	890197	890253	890309	890365	
7	890421	890477	890533	890589	890645	890700	890756	890812	890868	890924	
8	890980	891035	891091	891147	891203	891259	891314	891370	891426	891482	
9	891537	891593	891649	891705	891760	891816	891872	891928	891983	892039	
780	**892095**	**892150**	**892206**	**892262**	**892317**	**892373**	**892429**	**892484**	**892540**	**892595**	
1	892651	892707	892762	892818	892873	892929	892985	893040	893096	893151	
2	893207	893262	893318	893373	893429	893484	893540	893595	893651	893706	
3	893762	893817	893873	893928	893984	894039	894094	894150	894205	894261	
4	894316	894371	894427	894482	894538	894593	894648	894704	894759	894814	
5	894870	894925	894980	895036	895091	895146	895201	895257	895312	895367	
6	895423	895478	895533	895588	895644	895699	895754	895809	895864	895920	
7	895975	896030	896085	896140	896195	896251	896306	896361	896416	896471	
8	896526	896581	896636	896692	896747	896802	896857	896912	896967	897022	
9	897077	897132	897187	897242	897297	897352	897407	897462	897517	897572	
790	**897627**	**897682**	**897737**	**897792**	**897847**	**897902**	**897957**	**898012**	**898067**	**898122**	55
1	898176	898231	898286	898341	898396	898451	898506	898561	898615	898670	
2	898725	898780	898835	898890	898944	898999	899054	899109	899164	899218	
3	899273	899328	899383	899437	899492	899547	899602	899656	899711	899766	
4	899821	899875	899930	899985	900039	900094	900149	900203	900258	900312	
5	900367	900422	900476	900531	900586	900640	900695	900749	900804	900859	
6	900913	900968	901022	901077	901131	901186	901240	901295	901349	901404	
7	901458	901513	901567	901622	901676	901731	901785	901840	901894	901948	
8	902003	902057	902112	902166	902221	902275	902329	902384	902438	902492	
9	902547	902601	902655	902710	902764	902818	902873	902927	902981	903036	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
800	**903090**	**903144**	**903199**	**903253**	**903307**	**903361**	**903416**	**903470**	**903524**	**903578**	
1	903633	903687	903741	903795	903849	903904	903958	904012	904066	904120	
2	904174	904229	904283	904337	904391	904445	904499	904553	904607	904661	
3	904716	904770	904824	904878	904932	904986	905040	905094	905148	905202	
4	905256	905310	905364	905418	905472	905526	905580	905634	905688	905742	54
5	905796	905850	905904	905958	906012	906066	906119	906173	906227	906281	
6	906335	906389	906443	906497	906551	906604	906658	906712	906766	906820	
7	906874	906927	906981	907035	907089	907143	907196	907250	907304	907358	
8	907411	907465	907519	907573	907626	907680	907734	907787	907841	907895	
9	907949	908002	908056	908110	908163	908217	908270	908324	908378	908431	
810	**908485**	**908539**	**908592**	**908646**	**908699**	**908753**	**908807**	**908860**	**908914**	**908967**	
1	909021	909074	909128	909181	909235	909289	909342	909396	909449	909503	
2	909556	909610	909663	909716	909770	909823	909877	909930	909984	910037	
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	910571	
4	910624	910678	910731	910784	910838	910891	910944	910998	911051	911104	
5	911158	911211	911264	911317	911371	911424	911477	911530	911584	911637	
6	911690	911743	911797	911850	911903	911956	912009	912063	912116	912169	
7	912222	912275	912328	912381	912435	912488	912541	912594	912647	912700	
8	912753	912806	912859	912913	912966	913019	913072	913125	913178	913231	
9	913284	913337	913390	913443	913496	913549	913602	913655	913708	913761	53
820	**913814**	**913867**	**913920**	**913973**	**914026**	**914079**	**914132**	**914184**	**914237**	**914290**	
1	914343	914396	914449	914502	914555	914608	914660	914713	914766	914819	
2	914872	914925	914977	915030	915083	915136	915189	915241	915294	915347	
3	915400	915453	915505	915558	915611	915664	915716	915769	915822	915875	
4	915927	915980	916033	916085	916138	916191	916243	916296	916349	916401	
5	916454	916507	916559	916612	916664	916717	916770	916822	916875	916927	
6	916980	917033	917085	917138	917190	917243	917295	917348	917400	917453	
7	917506	917558	917611	917663	917716	917768	917820	917873	917925	917978	
8	918030	918083	918135	918188	918240	918293	918345	918397	918450	918502	
9	918555	918607	918659	918712	918764	918816	918869	918921	918973	919026	
830	**919078**	**919130**	**919183**	**919235**	**919287**	**919340**	**919392**	**919444**	**919496**	**919549**	
1	919601	919653	919706	919758	919810	919862	919914	919967	920019	920071	
2	920123	920176	920228	920280	920332	920384	920436	920489	920541	920593	
3	920645	920697	920749	920801	920853	920906	920958	921010	921062	921114	52
4	921166	921218	921270	921322	921374	921426	921478	921530	921582	921634	
5	921686	921738	921790	921842	921894	921946	921998	922050	922102	922154	
6	922206	922258	922310	922362	922414	922466	922518	922570	922622	922674	
7	922725	922777	922829	922881	922933	922985	923037	923089	923140	923192	
8	923244	923296	923348	923399	923451	923503	923555	923607	923658	923710	
9	923762	923814	923865	923917	923969	924021	924072	924124	924176	924228	
840	**924279**	**924331**	**924383**	**924434**	**924486**	**924538**	**924589**	**924641**	**924693**	**924744**	
1	924796	924848	924899	924951	925003	925054	925106	925157	925209	925261	
2	925312	925364	925415	925467	925518	925570	925621	925673	925725	925776	
3	925828	925879	925931	925982	926034	926085	926137	926188	926240	926291	
4	926342	926394	926445	926497	926548	926600	926651	926702	926754	926805	
5	926857	926908	926959	927011	927062	927114	927165	927216	927268	927319	
6	927370	927422	927473	927524	927576	927627	927678	927730	927781	927832	
7	927883	927935	927986	928037	928088	928140	928191	928242	928293	928345	
8	928396	928447	928498	928549	928601	928652	928703	928754	928805	928857	
9	928908	928959	929010	929061	929112	929163	929215	929266	929317	929368	
850	**929419**	**929470**	**929521**	**929572**	**929623**	**929674**	**929725**	**929776**	**929827**	**929879**	
1	929930	929981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	930542	930592	930643	930694	930745	930796	930847	930898	
3	930949	931000	931051	931102	931153	931204	931254	931305	931356	931407	
4	931458	931509	931560	931610	931661	931712	931763	931814	931865	931915	
5	931966	932017	932068	932118	932169	932220	932271	932322	932372	932423	
6	932474	932524	932575	932626	932677	932727	932778	932829	932879	932930	
7	932981	933031	933082	933133	933183	933234	933285	933335	933386	933437	
8	933487	933538	933589	933639	933690	933740	933791	933841	933892	933943	
9	933993	934044	934094	934145	934195	934246	934296	934347	934397	934448	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
860	**934498**	**934549**	**934599**	**934650**	**934700**	**934751**	**934801**	**934852**	**934902**	**934953**	
1	935003	935054	935104	935154	935205	935255	935306	935356	935406	935457	
2	935507	935558	935608	935658	935709	935759	935809	935860	935910	935960	
3	936011	936061	936111	936162	936212	936262	936313	936363	936413	936463	
4	936514	936564	936614	936665	936715	936765	936815	936865	936916	936966	
5	937016	937066	937116	937167	937217	937267	937317	937367	937418	937468	
6	937518	937568	937618	937668	937718	937769	937819	937869	937919	937969	
7	938019	938069	938119	938169	938219	938269	938320	938370	938420	938470	50
8	938520	938570	938620	938670	938720	938770	938820	938870	938920	938970	
9	939020	939070	939120	939170	939220	939270	939320	939369	939419	939469	
870	**939519**	**939569**	**939619**	**939669**	**939719**	**939769**	**939819**	**939869**	**939918**	**939968**	
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	
2	940516	940566	940616	940666	940716	940765	940815	940865	940915	940964	
3	941014	941064	941114	941163	941213	941263	941313	941362	941412	941462	
4	941511	941561	941611	941660	941710	941760	941809	941859	941909	941958	
5	942008	942058	942107	942157	942207	942256	942306	942355	942405	942455	
6	942504	942554	942603	942653	942702	942752	942801	942851	942901	942950	
7	943000	943049	943099	943148	943198	943247	943297	943346	943396	943445	
8	943495	943544	943593	943643	943692	943742	943791	943841	943890	943939	
9	943989	944038	944088	944137	944186	944236	944285	944335	944384	944433	
880	**944483**	**944532**	**944581**	**944631**	**944680**	**944729**	**944779**	**944828**	**944877**	**944927**	
1	944976	945025	945074	945124	945173	945222	945272	945321	945370	945419	
2	945469	945518	945567	945616	945665	945715	945764	945813	945862	945912	
3	945961	946010	946059	946108	946157	946207	946256	946305	946354	946403	
4	946452	946501	946551	946600	946649	946698	946747	946796	946845	946894	
5	946943	946992	947041	947090	947140	947189	947238	947287	947336	947385	
6	947434	947483	947532	947581	947630	947679	947728	947777	947826	947875	49
7	947924	947973	948022	948070	948119	948168	948217	948266	948315	948364	
8	948413	948462	948511	948560	948608	948657	948706	948755	948804	948853	
9	948902	948951	948999	949048	949097	949146	949195	949244	949292	949341	
890	**949390**	**949439**	**949488**	**949536**	**949585**	**949634**	**949683**	**949731**	**949780**	**949829**	
1	949878	949926	949975	950024	950073	950121	950170	950219	950267	950316	
2	950365	950414	950462	950511	950560	950608	950657	950706	950754	950803	
3	950851	950900	950949	950997	951046	951095	951143	951192	951240	951289	
4	951338	951386	951435	951483	951532	951580	951629	951677	951726	951775	
5	951823	951872	951920	951969	952017	952066	952114	952163	952211	952260	
6	952308	952356	952405	952453	952502	952550	952599	952647	952696	952744	
7	952792	952841	952889	952938	952986	953034	953083	953131	953180	953228	
8	953276	953325	953373	953421	953470	953518	953566	953615	953663	953711	
9	953760	953808	953856	953905	953953	954001	954049	954098	954146	954194	
900	**954243**	**954291**	**954339**	**954387**	**954435**	**954484**	**954532**	**954580**	**954628**	**954677**	
1	954725	954773	954821	954869	954918	954966	955014	955062	955110	955158	
2	955207	955255	955303	955351	955399	955447	955495	955543	955592	955640	
3	955688	955736	955784	955832	955880	955928	955976	956024	956072	956120	
4	956168	956216	956265	956313	956361	956409	956457	956505	956553	956601	
5	956649	956697	956745	956793	956840	956888	956936	956984	957032	957080	48
6	957128	957176	957224	957272	957320	957368	957416	957464	957512	957559	
7	957607	957655	957703	957751	957799	957847	957894	957942	957990	958038	
8	958086	958134	958181	958229	958277	958325	958373	958421	958468	958516	
9	958564	958612	958659	958707	958755	958803	958850	958898	958946	958994	
910	**959041**	**959089**	**959137**	**959185**	**959232**	**959280**	**959328**	**959375**	**959423**	**959471**	
1	959518	959566	959614	959661	959709	959757	959804	959852	959900	959947	
2	959995	960042	960090	960138	960185	960233	960280	960328	960376	960423	
3	960471	960518	960566	960613	960661	960709	960756	960804	960851	960899	
4	960946	960994	961041	961089	961136	961184	961231	961279	961326	961374	
5	961421	961469	961516	961563	961611	961658	961706	961753	961801	961848	
6	961895	961943	961990	962038	962085	962132	962180	962227	962275	962322	
7	962369	962417	962464	962511	962559	962606	962653	962701	962748	962795	
8	962843	962890	962937	962985	963032	963079	963126	963174	963221	963268	
9	963316	963363	963410	963457	963504	963552	963599	963646	963693	963741	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
920	**963788**	**963835**	**963882**	**963929**	**963977**	**964024**	**964071**	**964118**	**964165**	**964212**	
1	964260	964307	964354	964401	964448	964495	964542	964590	964637	964684	
2	964731	964778	964825	964872	964919	964966	965013	965061	965108	965155	
3	965202	965249	965296	965343	965390	965437	965484	965531	965578	965625	
4	965672	965719	965766	965813	965860	965907	965954	966001	966048	966095	
5	966142	966189	966236	966283	966329	966376	966423	966470	966517	966564	**47**
6	966611	966658	966705	966752	966799	966845	966892	966939	966986	967033	
7	967080	967127	967173	967220	967267	967314	967361	967408	967454	967501	
8	967548	967595	967642	967688	967735	967782	967829	967875	967922	967969	
9	968016	968062	968109	968156	968203	968249	968296	968343	968390	968436	
930	**968483**	**968530**	**968576**	**968623**	**968670**	**968716**	**968763**	**968810**	**968856**	**968903**	
1	968950	968996	969043	969090	969136	969183	969229	969276	969323	969369	
2	969416	969463	969509	969556	969602	969649	969695	969742	969789	969835	
3	969882	969928	969975	970021	970068	970114	970161	970207	970254	970300	
4	970347	970393	970440	970486	970533	970579	970626	970672	970719	970765	
5	970812	970858	970904	970951	970997	971044	971090	971137	971183	971229	
6	971276	971322	971369	971415	971461	971508	971554	971601	971647	971693	
7	971740	971786	971832	971879	971925	971971	972018	972064	972110	972157	
8	972203	972249	972295	972342	972388	972434	972481	972527	972573	972619	
9	972666	972712	972758	972804	972851	972897	972943	972989	973035	973082	
940	**973128**	**973174**	**973220**	**973266**	**973313**	**973359**	**973405**	**973451**	**973497**	**973543**	
1	973590	973636	973682	973728	973774	973820	973866	973913	973959	974005	
2	974051	974097	974143	974189	974235	974281	974327	974374	974420	974466	
3	974512	974558	974604	974650	974696	974742	974788	974834	974880	974926	
4	974972	975018	975064	975110	975156	975202	975248	975294	975340	975386	**46**
5	975432	975478	975524	975570	975616	975662	975707	975753	975799	975845	
6	975891	975937	975983	976029	976075	976121	976167	976212	976258	976304	
7	976350	976396	976442	976488	976533	976579	976625	976671	976717	976763	
8	976808	976854	976900	976946	976992	977037	977083	977129	977175	977220	
9	977266	977312	977358	977403	977449	977495	977541	977586	977632	977678	
950	**977724**	**977769**	**977815**	**977861**	**977906**	**977952**	**977998**	**978043**	**978089**	**978135**	
1	978181	978226	978272	978317	978363	978409	978454	978500	978546	978591	
2	978637	978683	978728	978774	978819	978865	978911	978956	979002	979047	
3	979093	979138	979184	979230	979275	979321	979366	979412	979457	979503	
4	979548	979594	979639	979685	979730	979776	979821	979867	979912	979958	
5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	
6	980458	980503	980549	980594	980640	980685	980730	980776	980821	980867	
7	980912	980957	981003	981048	981093	981139	981184	981229	981275	981320	
8	981366	981411	981456	981501	981547	981592	981637	981683	981728	981773	
9	981819	981864	981909	981954	982000	982045	982090	982135	982181	982226	
960	**982271**	**982316**	**982362**	**982407**	**982452**	**982497**	**982543**	**982588**	**982633**	**982678**	
1	982723	982769	982814	982859	982904	982949	982994	983040	983085	983130	
2	983175	983220	983265	983310	983356	983401	983446	983491	983536	983581	
3	983626	983671	983716	983762	983807	983852	983897	983942	983987	984032	
4	984077	984122	984167	984212	984257	984302	984347	984392	984437	984482	
5	984527	984572	984617	984662	984707	984752	984797	984842	984887	984932	**45**
6	984977	985022	985067	985112	985157	985202	985247	985292	985337	985382	
7	985426	985471	985516	985561	985606	985651	985696	985741	985786	985830	
8	985875	985920	985965	986010	986055	986100	986144	986189	986234	986279	
9	986324	986369	986413	986458	986503	986548	986593	986637	986682	986727	
970	**986772**	**986817**	**986861**	**986906**	**986951**	**986996**	**987040**	**987085**	**987130**	**987175**	
1	987219	987264	987309	987353	987398	987443	987488	987532	987577	987622	
2	987666	987711	987756	987800	987845	987890	987934	987979	988024	988068	
3	988113	988157	988202	988247	988291	988336	988381	988425	988470	988514	
4	988559	988604	988648	988693	988737	988782	988826	988871	988916	988960	
5	989005	989049	989094	989138	989183	989227	989272	989316	989361	989405	
6	989450	989494	989539	989583	989628	989672	989717	989761	989806	989850	
7	989895	989939	989983	990028	990072	990117	990161	990206	990250	990294	
8	990339	990383	990428	990472	990516	990561	990605	990650	990694	990738	
9	990783	990827	990871	990916	990960	991004	991049	991093	991137	991182	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
42	4.2	8.4	12.6	16.8	21.0	25.2	29.4	33.6	37.8

Table 2. Common Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9	Diff.
980	**991226**	**991270**	**991315**	**991359**	**991403**	**991448**	**991492**	**991536**	**991580**	**991625**	
1	991669	991713	991758	991802	991846	991890	991935	991979	992023	992067	
2	992111	992156	992200	992244	992288	992333	992377	992421	992465	992509	
3	992554	992598	992642	992686	992730	992774	992819	992863	992907	992951	
4	992995	993039	993083	993127	993172	993216	993260	993304	993348	993392	
5	993436	993480	993524	993568	993613	993657	993701	993745	993789	993833	
6	993877	993921	993965	994009	994053	994097	994141	994185	994229	994273	
7	994317	994361	994405	994449	994493	994537	994581	994625	994669	994713	**44**
8	994757	994801	994845	994889	994933	994977	995021	995065	995108	995152	
9	995196	995240	995284	995328	995372	995416	995460	995504	995547	995591	
990	**995635**	**995679**	**995723**	**995767**	**995811**	**995854**	**995898**	**995942**	**995986**	**996030**	
1	996074	996117	996161	996205	996249	996293	996337	996380	996424	996468	
2	996512	996555	996599	996643	996687	996731	996774	996818	996862	996906	
3	996949	996993	997037	997080	997124	997168	997212	997255	997299	997343	
4	997386	997430	997474	997517	997561	997605	997648	997692	997736	997779	
5	997823	997867	997910	997954	997998	998041	998085	998129	998172	998216	
6	998259	998303	998347	998390	998434	998477	998521	998564	998608	998652	
7	998695	998739	998782	998826	998869	998913	998956	999000	999043	999087	
8	999131	999174	999218	999261	999305	999348	999392	999435	999479	999522	
9	999565	999609	999652	999696	999739	999783	999826	999870	999913	999957	
1000	**000000**	**000043**	**000087**	**000130**	**000174**	**000217**	**000260**	**000304**	**000347**	**000391**	43

Table 3. Natural (Napierian or Hyperbolic) Logarithms of Numbers

Table gives natural logarithms of numbers from 1.0 to 9.99 directly. To find logarithms of numbers outside that range add or subtract natural logarithm of the powers of 10 as follows:

$\log_e 10 = 2.302585$	$\log_e 10^4 = 9.210340$	$\log_e 10^7 = 16.118096$
$\log_e 10^2 = 4.605170$	$\log_e 10^5 = 11.512925$	$\log_e 10^8 = 18.420681$
$\log_e 10^3 = 6.907755$	$\log_e 10^6 = 13.815511$	$\log_e 10^9 = 20.703266$

EXAMPLE.—$\log_e 679. = \log_e 6.79 + \log_e 10^2 = 1.9155 + 4.6052 = 5.5207$
$\log_e .0679 = \log_e 6.79 - \log_e 10^2 = 1.9155 - 4.6052 = -2.6897$

The common logarithm is the natural logarithm multiplied by the modulus of $\log_{10}$: $\log_{10} N = 0.434294 \log_e N$.

N	0	1	2	3	4	5	6	7	8	9
1.0	**0.0000**	**0.0100**	**0.0198**	**0.0296**	**0.0392**	**0.0488**	**0.0583**	**0.0677**	**0.0770**	**0.0862**
1.1	0.0953	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1740
1.2	0.1823	0.1906	0.1989	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4383	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366
1.9	0.6419	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881
2.0	**0.6931**	**0.6981**	**0.7031**	**0.7080**	**0.7129**	**0.7178**	**0.7227**	**0.7275**	**0.7324**	**0.7372**
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7885	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	**1.0986**	**1.1019**	**1.1053**	**1.1086**	**1.1119**	**1.1151**	**1.1184**	**1.1217**	**1.1249**	**1.1282**
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838

Table 3. Natural (Napierian or Hyperbolic) Logarithms of Numbers—*Continued*

N	0	1	2	3	4	5	6	7	8	9
4.0	**1.3863**	**1.3888**	**1.3913**	**1.3938**	**1.3962**	**1.3987**	**1.4012**	**1.4036**	**1.4061**	**1.4085**
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	**1.6094**	**1.6114**	**1.6134**	**1.6154**	**1.6174**	**1.6194**	**1.6214**	**1.6233**	**1.6253**	**1.6273**
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029
5.5	1.7047	1.7066	1.7084	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901
6.0	**1.7918**	**1.7934**	**1.7951**	**1.7967**	**1.7984**	**1.8001**	**1.8017**	**1.8034**	**1.8050**	**1.8066**
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445
7.0	**1.9459**	**1.9473**	**1.9488**	**1.9502**	**1.9516**	**1.9530**	**1.9544**	**1.9559**	**1.9573**	**1.9587**
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656
7.9	2.0669	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782
8.0	**2.0794**	**2.0807**	**2.0819**	**2.0832**	**2.0844**	**2.0857**	**2.0869**	**2.0882**	**2.0894**	**2.0906**
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961
9.0	**2.1972**	**2.1983**	**2.1994**	**2.2006**	**2.2017**	**2.2028**	**2.2039**	**2.2050**	**2.2061**	**2.2072**
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016

Table 4. Values and Logarithms of Exponentials and Hyperbolic Functions

The following tables give values of e^x, e^{-x}, sinh x, cosh x and tanh x for values of x from 0.00 to 6.00 in intervals of 0.01.

To facilitate computations involving multiplication, the common logarithms of e^x, sinh x, cosh x, and tanh x are also given.

For values of x greater than 6: e^x may be computed from the relationship $e^x = \log^{-1} (x \log_{10} e) = \log^{-1} 0.43429x$; e^{-x} approaches zero; sinh x and cosh x are approximately equal and become 0.5 e^x; and tanh x and coth x have values approximately equal to unity.

Where more accurate values of the exponentials and functions are required they may be computed from the following relationships.

$$e = 2.71828\ 18285 \qquad \frac{1}{e} = 0.36787\ 94412$$

$$M = \log_{10} e = 0.43429\ 44819 \qquad \frac{1}{M} = \log_e 10 \quad 2.30258\ 50930$$

$$e^x = \log^{-1} Mx \qquad = \log^{-1} - Mx$$

$$\text{Sinh } x = \frac{e^x - e^{-x}}{} \qquad \cosh x \quad e^x + e^- \qquad \tanh x = \frac{e^x - e^-}{e^x + e^-}$$

$$\text{csch } x = \frac{}{\sinh x} \qquad \text{sech } x = \frac{}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

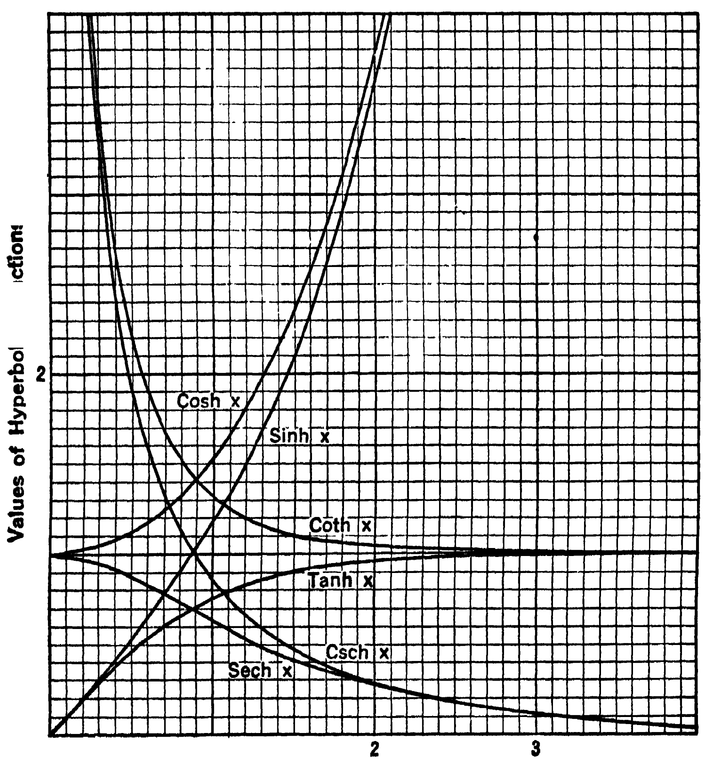


Chart of the Hyperbolic Functions.

Table 4. Values and Logarithms of Exponentials and Hyperbolic Functions—*Continued*

x	Values					Common Logarithms			
	e^x	e^{-x}	Sinh x	Cosh x	Tanh x	e^x	Sinh x	Cosh x	Tanh x
0.00	1.0000	1.0000	0.0000	1.0000	.00000	0.00000	$-\infty$	0.00000	$-\infty$
0.10	1.1052	.90484	0.1002	1.0050	.09967	0.04343	$\bar{1}.00072$	0.00217	$\bar{2}.99856$
0.20	1.2214	.81873	0.2013	1.0201	.19738	0.08686	$\bar{1}.30392$	0.00863	$\bar{1}.29529$
0.30	1.3499	.74082	0.3045	1.0453	.29131	0.13029	1.48362	0.10926	$\bar{1}.46436$
0.40	1.4918	.67032	0.4108	1.0811	.37995	0.17372	$\bar{1}.61358$	0.03385	$\bar{1}.57973$
0.50	1.6487	.60653	0.5211	1.1276	.46212	0.21715	1.71692	0.05217	1.66475
0.60	1.8221	.54881	0.6367	1.1855	.53705	0.26058	$\bar{1}.80390$	0.07389	$\bar{1}.73001$
0.70	2.0138	.49659	0.7586	1.2552	.60437	0.30401	$\bar{1}.88000$	0.09870	$\bar{1}.78130$
0.80	2.2255	.44933	0.8881	1.3374	.66404	0.34744	$\bar{1}.94846$	0.12627	1.82219
0.90	2.4596	.40657	1.0265	1.4331	.71630	0.39087	0.01137	0.15627	$\bar{1}.85509$
1.00	2.7183	.36788	1.1752	1.5431	.76159	0.43429	0.07011	0.18839	$\bar{1}.88172$
1.10	3.0042	.33287	1.3356	1.6685	.80050	0.47772	0.12569	0.22233	$\bar{1}.90336$
1.20	3.3201	.30119	1.5095	1.8107	.83365	0.52115	0.17882	0.25784	$\bar{1}.92099$
1.30	3.6693	.27253	1.6984	1.9709	.86172	0.56458	0.23004	0.29467	$\bar{1}.93537$
1.40	4.0552	.24660	1.9043	2.1509	.88535	0.60801	0.27974	0.33262	$\bar{1}.94712$
1.50	4.4817	.22313	2.1293	2.3524	.90515	0.65144	0.32823	0.37151	$\bar{1}.95672$
1.60	4.9530	.20190	2.3756	2.5775	.92167	0.69487	0.37577	0.41119	$\bar{1}.96457$
1.70	5.4739	.18268	2.6456	2.8283	.93541	0.73830	0.42253	0.45153	$\bar{1}.97100$
1.80	6.0496	.16530	2.9422	3.1075	.94681	0.78173	0.46867	0.49241	$\bar{1}.97626$
1.90	6.6859	.14957	3.2682	3.4177	.95624	0.82516	0.51430	0.53374	$\bar{1}.98057$
2.00	7.3891	.13534	3.6269	3.7622	.96403	0.86859	0.55953	0.57544	$\bar{1}.98409$
2.10	8.1662	.12246	4.0219	4.1443	.97045	0.91202	0.60443	0.61745	$\bar{1}.98697$
2.20	9.0250	.11080	4.4571	4.5679	.97574	0.95545	0.64905	0.65972	$\bar{1}.98934$
2.30	9.9742	.10026	4.9370	5.0372	.98010	0.99888	0.69346	0.70219	$\bar{1}.99127$
2.40	11.023	.09072	5.4662	5.5569	.98367	1.04231	0.73769	0.74484	$\bar{1}.99285$
2.50	12.182	.08208	6.0502	6.1323	.98661	1.08574	0.78177	0.78762	$\bar{1}.99415$
2.60	13.464	.07427	6.6947	6.7690	.98903	1.12917	0.82573	0.83052	$\bar{1}.99521$
2.70	14.880	.06721	7.4063	7.4735	.99101	1.17260	0.86960	0.87352	$\bar{1}.99608$
2.80	16.445	.06081	8.1919	8.2527	.99263	1.21602	0.91339	0.91660	$\bar{1}.99679$
2.90	18.174	.05502	9.0596	9.1146	.99396	1.25945	0.95711	0.95974	$\bar{1}.99737$
3.00	20.086	.04979	10.018	10.068	.99505	1.30288	1.00078	1.00293	$\bar{1}.99785$
3.10	22.198	.04505	11.077	11.122	.99595	1.34631	1.04440	1.04616	$\bar{1}.99824$
3.20	24.533	.04076	12.246	12.287	.99668	1.38974	1.08799	1.08943	$\bar{1}.99856$
3.30	27.113	.03688	13.538	13.575	.99728	1.43317	1.13155	1.13273	$\bar{1}.99882$
3.40	29.964	.03337	14.965	14.999	.99777	1.47660	1.17509	1.17605	$\bar{1}.99903$
3.50	33.115	.03020	16.543	16.573	.99818	1.52003	1.21860	1.21940	$\bar{1}.99921$
3.60	36.598	.02732	18.285	18.313	.99851	1.56346	1.26211	1.26275	$\bar{1}.99935$
3.70	40.447	.02472	20.211	20.236	.99878	1.60689	1.30559	1.30612	$\bar{1}.99947$
3.80	44.701	.02237	22.339	22.362	.99900	1.65032	1.34907	1.34951	$\bar{1}.99957$
3.90	49.402	.02024	24.691	24.711	.99918	1.69375	1.39254	1.39290	$\bar{1}.99964$
4.00	54.598	.01832	27.290	27.308	.99933	1.73718	1.43600	1.43629	$\bar{1}.99971$
4.10	60.340	.01657	30.162	30.178	.99945	1.78061	1.47946	1.47970	$\bar{1}.99976$
4.20	66.686	.01500	33.336	33.351	.99955	1.82404	1.52291	1.52310	$\bar{1}.99980$
4.30	73.700	.01357	36.843	36.857	.99963	1.86747	1.56636	1.56652	$\bar{1}.99984$
4.40	81.451	.01228	40.719	40.732	.99970	1.91090	1.60980	1.60993	$\bar{1}.99987$
4.50	90.017	.01111	45.003	45.014	.99975	1.95433	1.65324	1.65335	$\bar{1}.99989$
4.60	99.484	.01005	49.737	49.747	.99980	1.99775	1.69668	1.69677	$\bar{1}.99991$
4.70	109.95	.00910	54.969	54.978	.99983	2.04118	1.74012	1.74019	$\bar{1}.99993$
4.80	121.51	.00823	60.751	60.759	.99986	2.08461	1.78355	1.78361	$\bar{1}.99994$
4.90	134.29	.00745	67.141	67.149	.99989	2.12804	1.82699	1.82704	$\bar{1}.99995$
5.00	148.41	.00674	74.203	74.210	.99991	2.17147	1.87042	1.87046	$\bar{1}.99996$
5.10	164.02	.00610	82.008	82.014	.99993	2.21490	1.91386	1.91389	$\bar{1}.99997$
5.20	181.27	.00552	90.633	90.639	.99994	2.25833	1.95729	1.95731	$\bar{1}.99997$
5.30	200.34	.00499	100.17	100.17	.99995	2.30176	2.00072	2.00074	$\bar{1}.99998$
5.40	221.41	.00452	110.70	110.71	.99996	2.34519	2.04415	2.04417	$\bar{1}.99998$
5.50	244.69	.00409	122.34	122.35	.99997	2.38862	2.08758	2.08760	$\bar{1}.99999$
5.60	270.43	.00370	135.21	135.22	.99997	2.43205	2.13101	2.13103	$\bar{1}.99999$
5.70	298.87	.00335	149.43	149.44	.99998	2.47548	2.17444	2.17445	$\bar{1}.99999$
5.80	330.30	.00303	165.15	165.15	.99998	2.51891	2.21787	2.21788	$\bar{1}.99999$
5.90	365.04	.00274	182.52	182.52	.99998	2.56234	2.26130	2.26131	$\bar{1}.99999$
6.00	403.43	.00248	201.71	201.72	.99999	2.60577	2.30473	2.30474	$\bar{1}.99999$

2. PROPERTIES OF NUMBERS

Table 5. Certain Constants Containing e and π

$e = 2.7182818285$ $\qquad M = \log_{10} e = 0.4342944819$

$\pi = 3.1415926536$ $\qquad M^{-1} = \log_e 10 = 2.3025850930$

Powers of e			Multiples of π			Fractions of π		
e^n	Value	Logarithm	$n\pi$	Value	Logarithm	π/n	Value	Logarithm
e	2.718282	0.434294	π	3.141593	0.497150	$\pi/2$	1.570780	0.196120
e^{-1}	0.367879	$\bar{1}.565706$	2π	6.283185	0.798180	$\pi/3$	1.047198	0.020029
e^2	7.389057	0.868589	3π	9.424778	0.974271	$\pi/4$	0.785398	$\bar{1}.895090$
e^{-2}	0.135335	$\bar{1}.131411$	4π	12.566371	1.099210	$\pi/180$	0.017453*	$\bar{2}.241877$
$e^{1/2}$	1.648721	0.217147	5π	15.707963	1.196120			

Reciprocals of π			Powers of π			Roots of π		
n/π	Value	Logarithm	$\pi^{\pm n}$	Value	Logarithm	$\pi^{\pm 1/n}$	Value	Logarithm
$1/\pi$	0.318310	$\bar{1}.502850$	π^2	9.869604	0 994300	$\sqrt{\pi}$	1.772454	0.248575
$2/\pi$	0.636620	$\bar{1}.803880$	$1/\pi^2$	0.101321	$\bar{1}.005700$	$1/\sqrt{\pi}$	0.564190	$\bar{1}.751425$
$3/\pi$	0.954930	$\bar{1}.979971$	π^3	31.006277	1.491450	$\sqrt[3]{\pi}$	1.464592	0.165717
$180/\pi$	57.295780†	1.758123	$1/\pi^3$	0.032252	$\bar{2}.508550$	$1/\sqrt[3]{\pi}$	0.681784	$\bar{1}.834283$

* Number of radians per degree. † Number of degrees per radian.

Table 6. Factorials

n	$n! = 1 \cdot 2 \cdot 3 \ldots n$	$1/n!$	n	$n! = 1 \cdot 2 \cdot 3 \ldots n$	$1/n!$
1	1	1.	11	$399,168 \times 10^2$	0.250521×10^{-7}
2	2	0 5	12	$479,002 \times 10^3$	$.208768 \times 10^{-8}$
3	6	.166667	13	$622,702 \times 10^4$	$.160590 \times 10^{-9}$
4	24	$.416667 \times 10^{-1}$	14	$871,783 \times 10^5$	$.114707 \times 10^{-10}$
5	120	$.833333 \times 10^{-2}$	15	$130,767 \times 10^7$	$.764716 \times 10^{-12}$
6	720	$.138889 \times 10^{-2}$	16	$209,228 \times 10^8$	$.477948 \times 10^{-13}$
7	5,040	$.198413 \times 10^{-3}$	17	$355,687 \times 10^9$	$.281146 \times 10^{-14}$
8	40,320	$.248016 \times 10^{-4}$	18	$640,237 \times 10^{10}$	$.156192 \times 10^{-15}$
9	362,880	$.275573 \times 10^{-5}$	19	$121,645 \times 10^{12}$	$.822064 \times 10^{-17}$
10	3,628,800	$.275573 \times 10^{-6}$	20	$243,290 \times 10^{13}$	$.411032 \times 10^{-19}$

Table 7. Properties of Numbers

Decimal Equivalents, Squares, Cubes, Three-halves Powers, Square Roots, Cube Roots, Fifth Roots, Reciprocals, Circumference and Area of Circles

Number, N		N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N=Diam.)	
Fraction	Decimal								Circum	Area
1/64	.015625	0.000244	.381×10^-5	.1250	.2500	00195	.4353	64.0	0 04909	.00019
1/32	.03125	.000977	.305×10^-4	.1768	.3150	00552	.5000	32.0	.09818	.00077
3/64	.046875	.002197	.103×10^-3	.2165	.3606	.01015	.5422	21.3333	.14726	.00173
1/16	.0625	.003906	244×10^-3	.2500	.3969	01563	.5744	16.0	.19635	00307
5/64	078125	.006104	477×10^-3	.2795	.4275	02184	6006	12.80	.24544	00479
3/32	.09375	.008789	.824×10^-3	.3062	.4543	.02871	.6229	10.6667	.29452	.00690
	.10	.010	00100	.3162	.4642	.03162	.6310	10.0	.31416	.00785
7/64	.109375	.01196	001308	.3307	.4782	03617	.6424	9.1429	.34361	00939
1/8	.125	01563	001953	.3536	.5000	04419	6598	8 0	.39270	01227
9/64	.140625	01978	002782	.3750	5200	05273	6755	7.1111	.44179	01554
5/32	.15625	02441	003814	.3953	5386	06176	.6899	6 40	.49087	01917
11/64	.171875	.02954	005077	4146	5560	07126	.7031	5.8182	.53996	.02320
3/16	.1875	.03516	006592	.4330	5724	08119	.7155	5.3333	.58905	.02761
	.20	.040	.0080	.4472	5848	.08944	.7248	5.0	.62832	.03142
13/64	.203125	.04126	008381	4507	5878	.09155	7270	4.9231	.63814	.03241
7/32	.21875	.04785	01047	4677	.6025	.10231	7379	4 5714	.68722	.03758
15/64	.234375	.05493	.01287	.4841	.6166	11347	7481	4 2667	.73631	.04314
1/4	250	.0625	01563	.5000	.6300	12500	.7579	4 0	.78540	.04909
17/64	.265625	07056	01874	5154	6428	13690	.7671	3 7647	.83448	.05542
9/32	.28125	.07910	02225	5303	.6552	.14916	7759	3.5556	.88357	06213
19/64	.296875	.08813	.02616	.5449	.6671	.16176	.7844	3.3684	.93266	.06922
	.30	.090	.0270	.5477	.6694	.16432	.7860	3.3333	.94248	.07069
5/16	3125	.09766	03052	5590	.6786	.17469	.7925	3.2000	.98175	07670
21/64	328125	.10767	03533	5728	.6897	.18796	8002	3 0476	1 0308	08456
11/32	34375	.11816	04062	5863	7005	20154	8077	2.9091	1.0799	09281
23/64	359375	.12915	04641	5995	7110	.21544	.8149	2 7826	1.1290	.10143
3/8	375	.14063	05273	.6124	.7211	.22964	.8219	2.6667	1.1781	.11045
25/64	.390625	.15259	.05961	.6250	.7310	.24414	.8286	2.5600	1.2272	.11984
	.40	.16	.0640	.6325	.7368	.25298	.8326	2.50	1.2566	.12566
13/32	40625	.16504	06705	.6374	.7406	.25894	8351	2.4615	1.2763	.12962
27/64	421875	.17798	07508	.6495	7500	27402	.8415	2.3704	1.3254	13979
7/16	4375	19141	08374	.6614	7592	.28938	8476	2 2857	1.3744	15033
29/64	453125	.20532	09304	.6732	.7681	30502	8536	2.2069	1 4235	.16126
15/32	46875	.21973	10300	.6847	7768	.32093	8594	2.1333	1 4726	.17257
31/64	.484375	.23462	.11364	.6960	.7854	.33711	.8650	2.0645	1.5217	.18427
1/2	50	.2500	12500	.7071	.7937	.35355	.8706	2.0	1.5708	.19635
33/64	515625	.26587	13709	.7181	.8019	.37025	8759	1.9394	1.6199	20881
17/32	53125	.28223	14993	7289	8099	.38721	8812	1.8824	1.6690	.22166
35/64	.546875	.29907	.16355	7395	.8178	40442	.8863	1.8286	1.7181	23489
9/16	5625	.31641	17798	7500	.8255	42188	.8913	1.7778	1 7671	24850
37/64	578125	.33423	19323	7604	8331	43957	.8962	1.7297	1.8162	26250
19/32	59375	.35254	20932	7706	.8405	.45751	.9010	1.6842	1.8653	.27688
	60	.3600	.21600	7746	.8434	.46476	.9029	1.6667	1.8850	.28274
39/64	609375	.37134	22628	7806	8478	47569	.9057	1.6410	1.9144	.29165
5/8	625	.39063	24414	7906	8550	49410	9103	1.6000	1.9635	.30680
41/64	640625	.41040	26291	8004	8621	.51275	.9148	1.5610	2.0126	.32233
21/32	65625	.43066	28262	8101	8690	53162	9192	1.5238	2.0617	.33824
43/64	671875	.45142	30330	8197	8759	55072	9235	1.4884	2.1108	.35454
11/16	.6875	.47266	.32495	8297	.8826	57006	.9278	1.4545	2.1598	.37122
	.70	.4900	.34300	8367	.8879	.58566	.9312	1.4286	2.1991	.38485
45/64	703125	.49438	.34761	8385	8892	.58959	.9320	1.4222	2.2089	38829
23/32	71875	.51660	37131	8478	.8958	60935	.9361	1.3913	2.2580	.40574
47/64	734375	.53931	39605	8570	9022	.62933	.9401	1.3617	2 3071	.42357
3/4	750	.56250	42188	.8660	.9086	.64952	.9441	1.3333	2.3562	.44179
49/64	765625	.58618	44879	.8750	.9148	.66992	.9480	1.3061	2.4053	.46038
25/32	78125	.61035	47684	.8839	.9210	.69053	.9518	1.2800	2.4544	.47937
51/64	.796875	.63501	.50602	.8927	.9271	.71135	.9556	1.2549	2.5035	.49874
	.80	.6400	.51200	.8944	.9283	.71554	.9564	1.2500	2.5133	.50265
13/16	.8125	.66016	.53638	.9014	.9331	73238	9593	1.2308	2.5525	.51849
53/64	.828125	.68579	56792	.9100	.9391	.75361	.9630	1.2075	2.6016	.53862
27/32	.84375	.71191	60067	9186	.9449	.77503	9666	1.1852	2 6507	.55914
55/64	.859375	.73853	63467	9270	.9507	79666	9702	1.1636	2.6998	.58004
7/8	.875	.76563	66992	9354	.9565	.81849	9737	1.1429	2.7489	.60132
57/64	.890625	.79321	.70645	.9437	.9621	.84051	.9771	1.1228	2.7980	.62299
	.90	.81000	.72900	.9487	.9655	.85435	.9792	1.1111	2.8274	.63617
29/32	90625	.82129	.74429	.9520	.9677	.86272	.9805	1.1034	2.8471	.64504
59/64	.921875	.84985	.78346	.9601	.9733	.88513	.9839	1.0847	2.8962	.66747
15/16	.9375	.87891	.82398	.9683	.9787	.90773	.9872	1.0667	2.9452	.69029
61/64	953125	.90845	.86587	.9763	.9841	.93053	.9905	1.0492	2.9943	.71349
31/32	96875	.93848	.90915	.9843	.9895	.95349	.9937	1.0323	3.0434	.73708
63/64	.984375	.96899	.95385	.9922	.9948	.97666	.9969	1.0159	3.0925	.76104

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
1.	1.0000	**1.0000**	**1.0000**	1.0000	**1.0000**	**1.0000**	**1.0000000**	**3.1416**	**0.7854**
1.125	1.2656	1.4238	1.0606	1.0400	1.1932	1.0238	.8888888	3.5343	.9940
1.25	1.5625	1.9531	1.1180	1.0772	1.3975	1.0456	.80000000	3.9270	1.2272
1.375	1.8906	2.5996	1.1726	1.1120	1.6123	1.0658	.72727272	4.3197	1.4849
1.5	2.25	3.3750	1.2247	1.1447	1.8371	1.0845	.66666666	4.7124	1.7671
1.625	2.6406	4.2910	1.2748	1.1757	2.0715	1.1020	.61538462	5.1051	2.0739
1.75	3.0625	5.3594	1.3229	1.2051	2.3150	1.1186	.57142857	5.4978	2.4053
1.875	3.5156	6.5918	1.3693	1.2331	2.5675	1.1340	.53333333	5.8905	2.7612
2.	**4.0000**	**8.0000**	**1.4142**	**1.2599**	**2.8284**	**1.1487**	**.50000000**	**6.2832**	**3.1416**
2.125	4.5156	9.5957	1.4577	1.2856	3.0977	1.1627	.47058823	6.6759	3.5466
2.25	5.0625	11.3906	1.5000	1.3104	3.3750	1.1761	.44444444	7.0686	3.9761
2.375	5.6406	13.3965	1.5411	1.3342	3.6601	1.1889	.42105263	7.4613	4.4301
2.5	6.2500	15.6250	1.5811	1.3572	3.9529	1.2011	.40000000	7.8540	4.9087
2.625	6.8906	18.0879	1.6202	1.3795	4.2530	1.2129	.38095231	8.2467	5.4119
2.75	7.5625	20.7969	1.6583	1.4011	4.5604	1.2242	.36363636	8.6394	5.9396
2.875	8.2656	23.7637	1.6956	1.4219	4.8748	1.2352	.34782609	9.0321	6.4918
3.	**9.0000**	**27.0000**	**1.7321**	**1.4422**	**5.1962**	**1.2457**	**.33333333**	**9.4248**	**7.0686**
3.125	9.7656	30.5176	1.7678	1.4620	5.5243	1.2559	.32000000	9.8175	7.6699
3.25	10.5625	34.3281	1.8028	1.4813	5.8590	1.2658	.30769231	10.2102	8.2958
3.375	11.3906	38.4434	1.8371	1.5000	6.2003	1.2754	.29629629	10.6029	8.9462
3.5	12.2500	42.8750	1.8708	1.5183	6.5479	1.2847	.28571429	10.9956	9.6211
3.625	13.1406	47.6348	1.9039	1.5362	6.9018	1.2938	.27586207	11.3883	10.3206
3.75	14.0625	52.7344	1.9365	1.5536	7.2619	1.3026	.26666666	11.7810	11.0447
3.875	15.0156	58.1856	1.9685	1.5707	7.6279	1.3112	.25806452	12.1737	11.7932
4.	**16.0000**	**64.0000**	**2.0000**	**1.5874**	**8.0000**	**1.3195**	**.25000000**	**12.5664**	**12.5664**
4.125	17.0156	70.1895	2.0310	1.6038	8.3779	1.3277	.24242424	12.9591	13.3640
4.25	18.0625	76.7656	2.0616	1.6198	8.7616	1.3356	.23529412	13.3518	14.1863
4.375	19.1406	83.7402	2.0916	1.6355	9.1510	1.3434	.22857143	13.7445	15.0330
4.5	20.2500	91.1250	2.1213	1.6510	9.5460	1.3510	.22222222	14.1372	15.9043
4.625	21.3906	98.9317	2.1506	1.6661	9.9465	1.3584	.21621622	14.5299	16.8001
4.75	22.5625	107.1719	2.1795	1.6810	10.3524	1.3656	.21052632	14.9226	17.7205
4.875	23.7656	115.8574	2.2079	1.6956	10.7637	1.3728	.20512821	15.3153	18.6655
5.	**25.0000**	**125.0000**	**2.2361**	**1.7100**	**11.1803**	**1.3797**	**.20000000**	**15.7080**	**19.6350**
5.125	26.2656	134.6113	2.2638	1.7241	11.6022	1.3866	.19512195	16.1006	20.6289
5.25	27.5625	144.7031	2.2913	1.7380	12.0293	1.3933	.19047619	16.4933	21.6475
5.375	28.8906	155.2871	2.3184	1.7517	12.4614	1.3998	.18604651	16.8860	22.6906
5.5	30.2500	166.3750	2.3452	1.7652	12.8987	1.4063	.18181818	17.2787	23.7583
5.625	31.6406	177.9785	2.3727	1.7784	13.3409	1.4126	.17777777	17.6714	24.8505
5.75	33.0625	190.1094	2.3979	1.7915	13.7880	1.4188	.17391304	18.0641	25.9672
5.875	34.5156	202.7793	2.4238	1.8044	14.2400	1.4250	.17021277	18.4568	27.1085
6.	**36.0000**	**216.0000**	**2.4495**	**1.8171**	**14.6969**	**1.4310**	**.16666666**	**18.8496**	**28.2743**
6.125	37.5156	229.7832	2.4749	1.8297	15.1586	1.4369	.16326531	19.2422	29.4647
6.25	39.0625	244.1406	2.5000	1.8420	15.6250	1.4427	.16000000	19.6349	30.6796
6.375	40.6406	259.0840	2.5249	1.8542	16.0961	1.4484	.15686275	20.0276	31.9190
6.5	42.2500	274.6250	2.5495	1.8663	16.5718	1.4542	.15384615	20.4203	33.1831
6.625	43.8906	290.7754	2.5739	1.8781	17.0522	1.4596	.15094339	20.8130	34.4716
6.75	45.5625	307.5469	2.5981	1.8899	17.5370	1.4651	.14814815	21.2057	35.7847
6.875	47.2656	324.9512	2.6220	1.9015	18.0264	1.4705	.14545454	21.5984	37.1223
7.	**49.0000**	**343.0000**	**2.6458**	**1.9129**	**18.5203**	**1.4758**	**.14285714**	**21.9911**	**38.4845**
7.125	50.7656	361.7051	2.6693	1.9243	19.0186	1.4810	.14035088	22.3838	39.8712
7.25	52.5625	381.0781	2.6926	1.9354	19.5212	1.4862	.13793103	22.7765	41.2825
7.375	54.3906	401.1309	2.7157	1.9465	20.0283	1.4913	.13559322	23.1692	42.7183
7.5	56.2500	421.8750	2.7386	1.9574	20.5396	1.4963	.13333333	23.5619	44.1786
7.625	58.1406	443.3223	2.7613	1.9683	21.0552	1.5012	.13114754	23.9546	45.6635
7.75	60.0625	465.4844	2.7839	1.9789	21.5751	1.5061	.12903226	24.3473	47.1730
7.875	62.0156	488.3731	2.8063	1.9895	22.0992	1.5110	.12698413	24.7400	48.7069
8.	**64.0000**	**512.0000**	**2.8284**	**2.0000**	**22.6274**	**1.5157**	**.12500000**	**25.1327**	**50.2655**
8.125	66.0156	536.3770	2.8504	2.0104	23.1598	1.5204	.12307692	25.5254	51.8485
8.25	68.0625	561.5156	2.8723	2.0206	23.6963	1.5251	.12121212	25.9181	53.4562
8.375	70.1406	587.4278	2.8940	2.0308	24.2369	1.5297	.11940298	26.3108	55.0883
8.5	72.2500	614.1250	2.9155	2.0408	24.7816	1.5342	.11764706	26.7035	56.7450
8.625	74.3906	641.6192	2.9368	2.0508	25.3301	1.5387	.11594203	27.0962	58.4262
8.75	76.5625	669.9219	2.9580	2.0606	25.8828	1.5431	.11428571	27.4889	60.1320
8.875	78.7656	699.0450	2.9791	2.0704	26.4394	1.5475	.11267605	27.8816	61.8623
9.	**81.0000**	**729.0000**	**3.0000**	**2.0801**	**27.0000**	**1.5518**	**.11111111**	**28.2743**	**63.6172**
9.125	83.2656	759.7989	3.0207	2.0897	27.5645	1.5561	.10958904	28.6670	65.3966
9.25	85.5625	791.4531	3.0414	2.0992	28.1328	1.5604	.10810811	29.0597	67.2006
9.375	87.8906	823.9746	3.0619	2.1086	28.7050	1.5646	.10666666	29.4524	69.0291
9.5	90.2500	857.3750	3.0822	2.1179	29.2810	1.5687	.10526316	29.8451	70.8822
9.625	92.6406	891.6660	3.1024	2.1272	29.8608	1.5728	.10389610	30.2378	72.7597
9.75	95.0625	926.8594	3.1225	2.1363	30.4444	1.5769	.10256410	30.6305	74.6619
9.875	97.5156	962.9668	3.1425	2.1454	31.0317	1.5809	.10126582	31.0232	76.5886

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
10	**100**	**1000**	**3.1623**	**2.1544**	**31.623**	**1.5849**	**.10000000**	**31.4159**	**78.5398**
11	121	1331	3.3166	2.2240	36.483	1.6154	.09090909	34.5575	95.0332
12	144	1728	3.4641	2.2894	41.569	1.6438	.08333333	37.6991	113.0973
13	169	2197	3.6056	2.3513	46.873	1.6703	.07692308	40.8407	132.7323
14	196	2744	3.7417	2.4101	52.384	1.6953	.07142857	43.9823	153.9380
15	225	3375	3.8730	2.4662	58.095	1.7188	.06666667	47.1239	176.7146
16	256	4096	4.0000	2.5198	64.000	1.7411	.06250000	50.2654	201.0619
17	289	4913	4.1231	2.5713	70.093	1.7623	.05882353	53.4070	226.9801
18	324	5832	4.2426	2.6207	76.367	1.7826	.05555556	56.5486	254.4690
19	361	6859	4.3589	2.6684	82.819	1.8020	.05263158	59.6902	283.5287
20	**400**	**8000**	**4.4721**	**2.7144**	**89.442**	**1.8206**	**.05000000**	**62.8318**	**314.1593**
21	441	9261	4.5826	2.7589	96.235	1.8384	.04761905	65.9734	346.3606
22	484	10648	4.6904	2.8020	103.19	1.8556	.04545455	69.1150	380.1327
23	529	12167	4.7958	2.8439	110.30	1.8722	.04347826	72.2566	415.4756
24	576	13824	4.8990	2.8845	117.58	1.8882	.04166667	75.3982	452.3893
25	625	15625	5.0000	2.9240	125.00	1.9037	.04000000	78.5398	490.8739
26	676	17576	5.0990	2.9625	132.57	1.9186	.03846154	81.6813	530.9292
27	729	19683	5.1962	3.0000	140.30	1.9332	.03703704	84.8229	572.5553
28	784	21952	5.2915	3.0366	148.16	1.9473	.03571429	87.9645	615.7522
29	841	24389	5.3852	3.0723	156.17	1.9610	.03448276	91.1061	660.5198
30	**900**	**27000**	**5.4772**	**3.1072**	**164.32**	**1.9744**	**.03333333**	**94.2477**	**706.8583**
31	961	29791	5.5678	3.1414	172.60	1.9873	.03225806	97.3893	754.7676
32	1024	32768	5.6569	3.1748	181.02	2.0000	.03125000	100.5309	804.2477
33	1089	35937	5.7446	3.2075	189.57	2.0123	.03030303	103.6725	855.2986
34	1156	39304	5.8310	3.2396	198.25	2.0244	.02941176	106.8141	907.9203
35	1225	42875	5.9161	3.2711	207.06	2.0362	.02857143	109.9557	962.1127
36	1296	46656	6.0000	3.3019	216.00	2.0477	.02777778	113.0972	1017.8760
37	1369	50653	6.0828	3.3322	225.06	2.0589	.02702703	116.2388	1075.2101
38	1444	54872	6.1644	3.3620	234.25	2.0699	.02631579	119.3804	1134.1149
39	1521	59319	6.2450	3.3912	243.56	2.0807	.02564103	122.5220	1194.5906
40	**1600**	**64000**	**6.3246**	**3.4200**	**252.98**	**2.0913**	**.02500000**	**125.6636**	**1256.6371**
41	1681	68921	6.4031	3.4482	262.53	2.1016	.02439024	128.8052	1320.2543
42	1764	74088	6.4807	3.4760	272.19	2.1118	.02380952	131.9468	1385.4424
43	1849	79507	6.5574	3.5034	281.97	2.1218	.02325581	135.0884	1452.2012
44	1936	85184	6.6332	3.5303	291.86	2.1315	.02272727	138.2300	1520.5308
45	2025	91125	6.7082	3.5569	301.87	2.1411	.02222222	141.3716	1590.4313
46	2116	97336	6.7823	3.5830	311.99	2.1506	.02173913	144.5131	1661.9025
47	2209	103823	6.8557	3.6088	322.22	2.1598	.02127660	147.6547	1734.9445
48	2304	110592	6.9282	3.6342	332.55	2.1689	.02083333	150.7963	1809.5574
49	2401	117649	7.0000	3.6593	343.00	2.1779	.02040816	153.9379	1885.7410
50	**2500**	**125000**	**7.0711**	**3.6840**	**353.55**	**2.1867**	**.02000000**	**157.0795**	**1963.500**
51	2601	132651	7.1414	3.7084	364.21	2.1954	.01960784	160.2211	2042.820
52	2704	140608	7.2111	3.7325	374.98	2.2039	.01923077	163.3627	2123.716
53	2809	148877	7.2801	3.7563	385.85	2.2124	.01886792	166.5043	2206.183
54	2916	157464	7.3485	3.7798	396.82	2.2206	.01851852	169.6459	2290.221
55	3025	166375	7.4162	3.8030	407.89	2.2288	.01818182	172.7875	2375.829
56	3136	175616	7.4833	3.8259	419.07	2.2369	.01785714	175.9290	2463.008
57	3249	185193	7.5498	3.8485	430.35	2.2448	.01754386	179.0706	2551.758
58	3364	195112	7.6158	3.8709	441.72	2.2526	.01724138	182.2122	2642.079
59	3481	205379	7.6811	3.8930	453.19	2.2603	.01694915	185.3538	2733.970
60	**3600**	**216000**	**7.7460**	**3.9149**	**464.76**	**2.2679**	**.01666667**	**188.4954**	**2827.433**
61	3721	226981	7.8102	3.9365	476.43	2.2755	.01639344	191.6370	2922.466
62	3844	238328	7.8740	3.9579	488.19	2.2829	.01612903	194.7786	3019.070
63	3969	250047	7.9373	3.9791	500.05	2.2902	.01587302	197.9202	3117.245
64	4096	262144	8.0000	4.0000	512.00	2.2974	.01562500	201.0618	3216.990
65	4225	274625	8.0623	4.0207	524.05	2.3045	.01538462	204.2034	3318.307
66	4356	287496	8.1240	4.0412	536.19	2.3116	.01515152	207.3449	3421.194
67	4489	300763	8.1854	4.0615	548.42	2.3186	.01492537	210.4865	3525.652
68	4624	314432	8.2462	4.0817	560.74	2.3254	.01470588	213.6281	3631.680
69	4761	328509	8.3066	4.1016	573.16	2.3322	.01449275	216.7697	3739.280
70	**4900**	**343000**	**8.3666**	**4.1213**	**585.66**	**2.3389**	**.01428571**	**219.9118**	**3848.450**
71	5041	357911	8.4261	4.1408	598.26	2.3456	.01408451	223.0529	3959.191
72	5184	373248	8.4853	4.1602	610.94	2.3522	.01388889	226.1945	4071.503
73	5329	389017	8.5440	4.1793	623.71	2.3587	.01369863	229.3361	4185.386
74	5476	405224	8.6023	4.1983	636.57	2.3651	.01351351	232.4777	4300.839
75	5625	421875	8.6603	4.2172	649.52	2.3714	.01333333	235.6193	4417.864
76	5776	438976	8.7178	4.2358	662.55	2.3777	.01315789	238.7608	4536.459
77	5929	456533	8.7750	4.2543	675.68	2.3840	.01298701	241.9024	4656.625
78	6084	474552	8.8318	4.2727	688.88	2.3901	.01282051	245.0440	4778.361
79	6241	493039	8.8882	4.2908	702.17	2.3962	.01265823	248.1856	4901.669

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Area
80	**6400**	**512000**	**8.9443**	**4.3089**	**715.54**	**2.4022**	**.01250000**	**251.327**	**5026.547**
81	6561	531441	9.0000	4.3267	729.00	2.4082	.01234568	254.469	5152.998
82	6724	551368	9.0554	4.3445	742.54	2.4141	.01219512	257.610	5281.016
83	6889	571787	9.1104	4.3621	756.17	2.4200	.01204819	260.752	5410.607
84	7056	592704	9.1652	4.3795	769.88	2.4258	.01190476	263.894	5541.770
85	7225	614125	9.2195	4.3968	783.66	2.4315	.01176471	267.035	5674.501
86	7396	636056	9.2736	4.4140	797.53	2.4372	.01162791	270.177	5808.805
87	7569	658503	9.3274	4.4310	811.49	2.4429	.01149425	273.318	5944.679
88	7744	681472	9.3808	4.4480	825.52	2.4485	.01136364	276.460	6082.124
89	7921	704969	9.4340	4.4647	839.63	2.4540	.01123596	279.602	6221.138
90	**8100**	**729000**	**9.4868**	**4.4814**	**853.82**	**2.4595**	**.01111111**	**282.743**	**6361.725**
91	8281	753571	9.5394	4.4979	868.09	2.4650	.01098901	285.885	6503.882
92	8464	778688	9.5917	4.5144	882.44	2.4705	.01086957	289.026	6647.610
93	8649	804357	9.6437	4.5307	896.86	2.4758	.01075269	292.168	6792.909
94	8836	830584	9.6954	4.5468	911.36	2.4810	.01063830	295.309	6939.778
95	9025	857375	9.7468	4.5629	925.95	2.4863	.01052632	298.451	7088.219
96	9216	884736	9.7980	4.5789	940.61	2.4915	.01041667	301.593	7238.230
97	9409	912673	9.8489	4.5947	955.34	2.4966	.01030928	304.734	7389.812
98	9604	941192	9.8995	4.6104	970.15	2.5018	.01020408	307.876	7542.962
99	9801	970299	9.9499	4.6261	985.04	2.5069	.01010101	311.017	7697.688
100	**10000**	**1000000**	**10.0000**	**4.6416**	**1000.0**	**2.5119**	**.01000000**	**314.159**	**7853.982**
101	10201	1030301	10.0499	4.6570	1015.0	2.5169	.00990099	317.301	8011.85
102	10404	1061208	10.0995	4.6723	1030.1	2.5219	.00980392	320.442	8171.28
103	10609	1092727	10.1489	4.6875	1045.3	2.5268	.00970874	323.584	8332.29
104	10816	1124864	10.1980	4.7027	1060.6	2.5317	.00961538	326.725	8494.87
105	11025	1157625	10.2470	4.7177	1075.9	2.5365	.00952381	329.867	8659.01
106	11236	1191016	10.2956	4.7326	1091.3	2.5413	.00943396	333.009	8824.73
107	11449	1225043	10.3441	4.7475	1106.8	2.5461	.00934579	336.150	8992.02
108	11664	1259712	10.3923	4.7622	1122.4	2.5509	.00925926	339.292	9160.88
109	11881	1295029	10.4403	4.7769	1138.0	2.5556	.00917431	342.433	9331.32
110	**12100**	**1331000**	**10.4881**	**4.7914**	**1153.7**	**2.5602**	**.00909091**	**345.575**	**9503.32**
111	12321	1367631	10.5357	4.8059	1169.5	2.5649	.00900901	348.716	9676.89
112	12544	1404928	10.5830	4.8203	1185.3	2.5695	.00892857	351.858	9852.03
113	12769	1442897	10.6301	4.8346	1201.2	2.5740	.00884956	355.000	10028.75
114	12996	1481544	10.6771	4.8488	1217.2	2.5786	.00877193	358.141	10207.03
115	13225	1520875	10.7238	4.8629	1233.2	2.5831	.00869565	361.283	10386.89
116	13456	1560896	10.7703	4.8770	1249.4	2.5876	.00862069	364.424	10568.32
117	13689	1601613	10.8167	4.8910	1265.5	2.5920	.00854701	367.566	10751.31
118	13924	1643032	10.8628	4.9049	1281.8	2.5964	.00847458	370.708	10935.88
119	14161	1685159	10.9087	4.9187	1298.1	2.6008	.00840336	373.849	11122.02
120	**14400**	**1728000**	**10.9545**	**4.9324**	**1314.5**	**2.6052**	**.00833333**	**376.991**	**11309.73**
121	14641	1771561	11.0000	4.9461	1331.0	2.6095	.00826446	380.132	11499.01
122	14884	1815848	11.0454	4.9597	1347.5	2.6138	.00819672	383.274	11689.86
123	15129	1860867	11.0905	4.9732	1364.1	2.6181	.00813008	386.416	11882.29
124	15376	1906624	11.1355	4.9866	1380.8	2.6223	.00806452	389.557	12076.28
125	15625	1953125	11.1803	5.0000	1397.5	2.6265	.00800000	392.699	12271.84
126	15876	2000376	11.2250	5.0133	1414.4	2.6307	.00793651	395.840	12468.98
127	16129	2048383	11.2694	5.0265	1431.2	2.6349	.00787402	398.982	12667.68
128	16384	2097152	11.3137	5.0397	1448.2	2.6390	.00781250	402.124	12867.96
129	16641	2146689	11.3578	5.0528	1465.2	2.6431	.00775194	405.265	13069.81
130	**16900**	**2197000**	**11.4018**	**5.0658**	**1482.2**	**2.6472**	**.00769231**	**408.407**	**13273.23**
131	17161	2248091	11.4455	5.0788	1499.4	2.6513	.00763359	411.548	13478.22
132	17424	2299968	11.4891	5.0916	1516.6	2.6553	.00757576	414.690	13684.77
133	17689	2352637	11.5326	5.1045	1533.8	2.6593	.00751880	417.831	13892.91
134	17956	2406104	11.5758	5.1172	1551.2	2.6633	.00746269	420.973	14102.61
135	18225	2460375	11.6190	5.1299	1568.6	2.6673	.00740741	424.115	14313.88
136	18496	2515456	11.6619	5.1426	1586.0	2.6712	.00735294	427.256	14526.72
137	18769	2571353	11.7047	5.1551	1603.6	2.6751	.00729927	430.398	14741.14
138	19044	2628072	11.7473	5.1676	1621.1	2.6790	.00724638	433.539	14957.12
139	19321	2685619	11.7898	5.1801	1638.8	2.6829	.00719424	436.681	15174.67
140	**19600**	**2744000**	**11.8322**	**5.1925**	**1656.5**	**2.6867**	**.00714286**	**439.823**	**15393.80**
141	19881	2803221	11.8743	5.2048	1674.3	2.6906	.00709220	442.964	15614.50
142	20164	2863288	11.9164	5.2171	1692.1	2.6944	.00704225	446.106	15836.77
143	20449	2924207	11.9583	5.2293	1710.0	2.6981	.00699301	449.247	16060.60
144	20736	2985984	12.0000	5.2415	1728.0	2.7019	.00694444	452.389	16286.01
145	21025	3048625	12.0416	5.2536	1746.0	2.7057	.00689655	455.531	16512.99
146	21316	3112136	12.0830	5.2656	1764.1	2.7094	.00684932	458.672	16741.54
147	21609	3176523	12.1244	5.2776	1782.2	2.7131	.00680272	461.814	16971.67
148	21904	3241792	12.1655	5.2896	1800.5	2.7168	.00675676	464.955	17203.36
149	22201	3307949	12.2066	5.3015	1818.8	2.7204	.00671141	468.097	17436.62

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Area
150	**22500**	**3375000**	**12.2474**	**5.3133**	**1837.1**	**2.7241**	**.00666667**	**471.239**	**17671.46**
151	22801	3442951	12.2882	5.3251	1855.5	2.7277	.00662252	474.380	17907.86
152	23104	3511808	12.3288	5.3368	1874.0	2.7314	.00657895	477.522	18145.84
153	23409	3581577	12.3693	5.3485	1892.5	2.7349	.00653595	480.663	18385.38
154	23716	3652264	12.4097	5.3601	1911.1	2.7385	.00649351	483.805	18626.50
155	24025	3723875	12.4499	5.3717	1929.7	2.7420	.00645161	486.946	18869.19
156	24336	3796416	12.4900	5.3832	1948.4	2.7455	.00641026	490.088	19113.45
157	24649	3869893	12.5300	5.3947	1967.2	2.7490	.00636943	493.230	19359.28
158	24964	3944312	12.5698	5.4061	1986.0	2.7525	.00632911	496.371	19606.68
159	25281	4019679	12.6095	5.4175	2004.9	2.7560	.00628931	499.513	19855.65
160	**25600**	**4096000**	**12.6491**	**5.4288**	**2023.9**	**2.7595**	**.00625000**	**502.654**	**20106.19**
161	25921	4173281	12.6886	5.4401	2042.9	2.7629	.00621118	505.796	20358.30
162	26244	4251528	12.7279	5.4514	2061.9	2.7663	.00617284	508.938	20611.99
163	26569	4330747	12.7671	5.4626	2081.0	2.7697	.00613497	512.079	20867.24
164	26896	4410944	12.8062	5.4737	2100.2	2.7731	.00609756	515.221	21124.06
165	27225	4492125	12.8452	5.4848	2119.5	2.7765	.00606061	518.362	21382.46
166	27556	4574296	12.8841	5.4959	2138.8	2.7799	.00602410	521.504	21642.43
167	27889	4657463	12.9228	5.5069	2158.1	2.7832	.00598802	524.646	21903.96
168	28224	4741632	12.9615	5.5178	2177.5	2.7865	.00595238	527.787	22167.07
169	28561	4826809	13.0000	5.5288	2197.0	2.7898	.00591716	530.929	22431.75
170	**28900**	**4913000**	**13.0384**	**5.5397**	**2216.5**	**2.7931**	**.00588235**	**534.070**	**22698.00**
171	29241	5000211	13.0767	5.5505	2236.1	2.7964	.00584795	537.212	22965.82
172	29584	5088448	13.1149	5.5613	2255.8	2.7997	.00581395	540.353	23235.21
173	29929	5177717	13.1529	5.5721	2275.5	2.8029	.00578035	543.495	23506.18
174	30276	5268024	13.1909	5.5828	2295.2	2.8061	.00574713	546.637	23778.71
175	30625	5359375	13.2288	5.5934	2315.0	2.8094	.00571429	549.778	24052.81
176	30976	5451776	13.2665	5.6041	2334.9	2.8126	.00568182	552.920	24328.49
177	31329	5545233	13.3041	5.6147	2354.8	2.8158	.00564972	556.061	24605.73
178	31684	5639752	13.3417	5.6252	2374.8	2.8189	.00561798	559.203	24884.55
179	32041	5735339	13.3791	5.6357	2394.9	2.8221	.00558659	562.345	25164.94
180	**32400**	**5832000**	**13.4164**	**5.6462**	**2415.0**	**2.8252**	**.00555556**	**565.486**	**25446.90**
181	32761	5929741	13.4536	5.6567	2435.1	2.8284	.00552486	568.628	25730.42
182	33124	6028568	13.4907	5.6671	2455.3	2.8315	.00549451	571.769	26015.52
183	33489	6128487	13.5277	5.6774	2475.6	2.8346	.00546448	574.911	26302.19
184	33856	6229504	13.5647	5.6877	2495.9	2.8377	.00543478	578.053	26590.43
185	34225	6331625	13.6015	5.6980	2516.3	2.8408	.00540541	581.194	26880.25
186	34596	6434856	13.6382	5.7083	2536.7	2.8438	.00537634	584.336	27171.63
187	34969	6539203	13.6748	5.7185	2557.2	2.8469	.00534759	587.477	27464.58
188	35344	6644672	13.7113	5.7287	2577.7	2.8499	.00531915	590.619	27759.11
189	35721	6751269	13.7477	5.7388	2598.3	2.8529	.00529101	593.761	28055.20
190	**36100**	**6859000**	**13.7840**	**5.7489**	**2619.0**	**2.8560**	**.00526316**	**596.902**	**28352.87**
191	36481	6967871	13.8203	5.7590	2639.7	2.8590	.00523560	600.044	28652.10
192	36864	7077888	13.8564	5.7690	2660.4	2.8619	.00520833	603.185	28952.91
193	37249	7189057	13.8924	5.7790	2681.2	2.8649	.00518135	606.327	29255.29
194	37636	7301384	13.9284	5.7890	2702.1	2.8679	.00515464	609.468	29559.24
195	38025	7414875	13.9642	5.7989	2723.0	2.8708	.00512821	612.610	29864.76
196	38416	7529536	14.0000	5.8088	2744.0	2.8738	.00510204	615.752	30171.85
197	38809	7645373	14.0357	5.8186	2765.0	2.8767	.00507614	618.893	30480.51
198	39204	7762392	14.0712	5.8285	2786.1	2.8796	.00505051	622.035	30790.74
199	39601	7880599	14.1067	5.8383	2807.2	2.8825	.00502513	625.176	31102.55
200	**40000**	**8000000**	**14.1421**	**5.8480**	**2828.4**	**2.8854**	**.00500000**	**628.318**	**31415.93**
201	40401	8120601	14.1774	5.8578	2849.7	2.8883	.00497512	631.460	31730.87
202	40804	8242408	14.2127	5.8675	2871.0	2.8911	.00495050	634.601	32047.39
203	41209	8365427	14.2478	5.8771	2892.3	2.8940	.00492611	637.743	32365.47
204	41616	8489664	14.2829	5.8868	2913.7	2.8968	.00490196	640.884	32685.13
205	42025	8615125	14.3178	5.8964	2935.2	2.8997	.00487805	644.026	33006.36
206	42436	8741816	14.3527	5.9059	2956.7	2.9025	.00485437	647.168	33329.16
207	42849	8869743	14.3875	5.9155	2978.2	2.9053	.00483092	650.309	33653.53
208	43264	8998912	14.4222	5.9250	2999.8	2.9081	.00480769	653.451	33979.47
209	43681	9129329	14.4568	5.9345	3021.5	2.9109	.00478469	656.592	34306.98
210	**44100**	**9261000**	**14.4914**	**5.9439**	**3043.2**	**2.9137**	**.00476190**	**659.734**	**34636.06**
211	44521	9393931	14.5258	5.9533	3065.0	2.9165	.00473934	662.875	34966.71
212	44944	9528128	14.5602	5.9627	3086.8	2.9192	.00471698	666.017	35298.94
213	45369	9663597	14.5945	5.9721	3108.7	2.9220	.00469484	669.159	35632.73
214	45796	9800344	14.6287	5.9814	3130.6	2.9247	.00467290	672.300	35968.09
215	46225	9938375	14.6629	5.9907	3152.5	2.9274	.00465116	675.442	36305.03
216	46656	10077696	14.6969	6.0000	3174.5	2.9302	.00462963	678.583	36643.54
217	47089	10218313	14.7309	6.0092	3196.6	2.9329	.00460829	681.725	36983.61
218	47524	10360232	14.7648	6.0185	3218.7	2.9356	.00458716	684.867	37325.26
219	47961	10503459	14.7986	6.0277	3240.9	2.9383	.00456621	688.008	37668.48

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
220	**48400**	**10648000**	**14.8324**	**6.0368**	**3263.1**	**2.9409**	**.00454545**	**691.150**	**38013.27**
221	48841	10793861	14.8661	6.0459	3285.4	2.9436	.00452489	694.291	38359.63
222	49284	10941048	14.8997	6.0550	3307.7	2.9463	.00450450	697.433	38707.56
223	49729	11089567	14.9332	6.0641	3330.1	2.9489	.00448430	700.575	39057.07
224	50176	11239424	14.9666	6.0732	3352.5	2.9516	.00446429	703.716	39408.14
225	50625	11390625	15.0000	6.0822	3375.0	2.9542	.00444444	706.858	39760.78
226	51076	11543176	15.0333	6.0912	3397.5	2.9568	.00442478	709.999	40115.00
227	51529	11697083	15.0665	6.1002	3420.1	2.9594	00440529	713.141	40470.78
228	51984	11852352	15.0997	6.1091	3442.7	2.9620	.00438596	716.283	40828.14
229	52441	12008989	15.1327	6.1180	3465.4	2.9646	.00436681	719.424	41187.07
230	**52900**	**12167000**	**15.1658**	**6.1269**	**3488.1**	**2.9672**	**.00434783**	**722.566**	**41547.56**
231	53361	12326391	15.1987	6.1358	3510.9	2.9698	.00432900	725.707	41909.63
232	53824	12487168	15.2315	6.1446	3533.7	2.9723	.00431034	728.849	42273.27
233	54289	12649337	15.2643	6.1534	3556.6	2.9749	.00429185	731.990	42638.48
234	54756	12812904	15.2971	6.1622	3579.5	2.9774	.00427350	735.132	43005.26
235	55225	12977875	15.3297	6.1710	3602.5	2.9800	.00425532	738.274	43373.61
236	55696	13144256	15.3623	6.1797	3625.5	2.9825	.00423729	741.415	43743.54
237	56169	13312053	15.3948	6.1885	3648.6	2.9850	.00421941	744.557	44115.03
238	56644	13481272	15.4272	6.1972	3671.7	2 9875	.00420168	747.698	44488.09
239	57121	13651919	15.4596	6.2058	3694.8	2.9900	.00418410	750.840	44862.73
240	**57600**	**13824000**	**15.4919**	**6.2145**	**3718.0**	**2.9925**	**.00416667**	**753.982**	**45238.93**
241	58081	13997521	15.5242	6.2231	3741.3	2.9950	.00414938	757.123	45616.71
242	58564	14172488	15.5563	6.2317	3764 6	2.9975	00413223	760.265	45996.06
243	59049	14348907	15.5885	6.2403	3788.0	3.0000	.00411523	763.406	46376.98
244	59536	14526784	15.6205	6.2488	3811.4	3.0025	.00409836	766.548	46759.47
245	60025	14706125	15.6525	6.2573	3834.9	3.0049	.00408163	769.690	47143.52
246	60516	14886936	15.6844	6.2658	3858.4	3.0074	.00406504	772.831	47529.16
247	61009	15069223	15.7162	6.2743	3881.9	3.0098	.00404858	775.973	47916.36
248	61504	15252992	15.7480	6.2828	3905.5	3.0122	.00403226	779.114	48305.13
249	62001	15438249	15.7797	6.2912	3929.2	3.0147	.00401606	782.256	48695.47
250	**62500**	**15625000**	**15.8114**	**6.2996**	**3952.9**	**3.0171**	**.00400000**	**785.398**	**49087.39**
251	63001	15813251	15.8430	6.3080	3976.6	3.0195	00398406	788.539	49480.87
252	63504	16003008	15.8745	6.3164	4000.4	3.0219	.00396825	791.681	49875.92
253	64009	16194277	15.9060	6.3247	4024.2	3.0243	.00395257	794.822	50272.55
254	64516	16387064	15.9374	6.3330	4048.1	3.0267	.00393701	797.964	50670.75
255	65025	16581375	15 9687	6.3413	4072.0	3.0291	00392157	801.105	51070.52
256	65536	16777216	16.0000	6.3496	4096.0	3.0314	.00390625	804.247	51471.85
257	66049	16974593	16.0312	6.3579	4120.0	3.0338	.00389105	807.389	51874.76
258	66564	17173512	16.0624	6.3661	4144.1	3.0362	00387597	810.530	52279.24
259	67081	17373979	16.0935	6.3743	4168.2	3.0385	.00386100	813.672	52685.29
260	**67600**	**17576000**	**16.1245**	**6.3825**	**4192.4**	**3.0418**	**.00384615**	**816.813**	**53092.92**
261	68121	17779581	16.1555	6.3907	4216.6	3.0432	.00383142	819.955	53502.11
262	68644	17984728	16.1864	6.3988	4240.8	3.0455	.00381679	823.097	53912.87
263	69169	18191447	16.2173	6.4070	4265.1	3.0478	.00380228	826.238	54325.21
264	69696	18399744	16.2481	6.4151	4289.5	3.0501	.00378788	829.380	54739.11
265	70225	18609625	16.2788	6.4232	4313.9	3.0524	.00377358	832.521	55154.59
266	70756	18821096	16.3095	6.4312	4338.3	3.0547	.00375940	835.663	55571.63
267	71289	19034163	16.3401	6.4393	4362.8	3.0570	.00374532	838.805	55990.25
268	71824	19248832	16.3707	6.4473	4387.3	3.0593	00373134	841.946	56410.44
269	72361	19465109	16.4012	6.4553	4411.9	3.0616	.00371747	845.088	56832.20
270	**72900**	**19683000**	**16.4317**	**6.4633**	**4436.5**	**3.0639**	**.00370370**	**848.229**	**57255.53**
271	73441	19902511	16.4621	6.4713	4461.2	3.0662	.00369004	851.371	57680.43
272	73984	20123648	16.4924	6.4792	4485.9	3 0684	.00367647	854.512	58106.90
273	74529	20346417	16.5227	6.4872	4510.7	3.0707	.00366300	857.654	58534.94
274	75076	20570824	16.5529	6.4951	4535.5	3.0729	.00364964	860.796	58964.55
275	75625	20796875	16.5831	6.5030	4560.4	3.0752	.00363636	863.937	59395.74
276	76176	21024576	16.6132	6.5108	4585.3	3.0774	.00362319	867.079	59828.49
277	76729	21253933	16.6433	6.5187	4610.2	3.0796	.00361011	870.220	60262.82
278	77284	21484952	16.6733	6.5265	4635.2	3.0818	.00359712	873.362	60698.71
279	77841	21717639	16.7033	6.5343	4660.2	3.0840	.00358423	876.504	61136.18
280	**78400**	**21952000**	**16.7332**	**6.5421**	**4685.3**	**3.0863**	**.00357143**	**879.645**	**61575.22**
281	78961	22188041	16.7631	6.5499	4710.4	3.0885	.00355872	882.787	62015.82
282	79524	22425768	16.7929	6.5577	4735.6	3.0907	.00354610	885.928	62458.00
283	80089	22665187	16.8226	6.5654	4760.8	3.0928	.00353357	889.070	62901.75
284	80656	22906304	16.8523	6.5731	4786.0	3.0950	.00352113	892.212	63347.07
285	81225	23149125	16.8819	6.5808	4811.3	3.0972	.00350877	895.353	63793.97
286	81796	23393656	16.9115	6.5885	4836.7	3.0994	.00349650	898.495	64242.43
287	82369	23639903	16.9411	6.5962	4862.1	3.1015	.00348432	901.636	64692.46
288	82944	23887872	16.9706	6.6039	4887.5	3.1037	.00347222	904.778	65144.07
289	83521	24137569	17.0000	6.6115	4913.0	3.1058	.00346021	907.920	65597.24

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
290	**84100**	**24389000**	**17.0294**	**6.6191**	**4938.5**	**3.1080**	**.00344828**	**911.061**	**66051.99**
291	84681	24642171	17.0587	6.6267	4964.1	3.1101	.00343643	914.203	66508.30
292	85264	24897088	17.0880	6 6343	4989.7	3.1123	.00342466	917.344	66966.19
293	85849	25153757	17.1172	6.6419	5015.4	3.1144	.00341297	920.486	67425.65
294	86436	25412184	17.1464	6.6494	5041.1	3.1165	.00340136	923.627	67886.68
295	87025	25672375	17.1756	6.6569	5066.8	3.1186	.00338983	926.769	68349.28
296	87616	25934336	17.2047	6.6644	5092.6	3.1207	.00337838	929.911	68813.45
297	88209	26198073	17.2337	6.6719	5118.4	3.1228	.00336700	933.052	69279.19
298	88804	26463592	17 2627	6.6794	5144.3	3.1249	.00335570	936.194	69746.50
299	89401	26730899	17.2916	6.6869	5170.2	3.1270	.00334448	939.335	70215.38
300	**90000**	**27000000**	**17.3205**	**6.6943**	**5196.2**	**3.1291**	**.00333333**	**942.477**	**70685.83**
301	90601	27270901	17.3494	6.7018	5222.2	3.1312	.00332226	945.619	71157.86
302	91204	27543608	17.3781	6.7092	5248 2	3.1333	.00331126	948.760	71631.45
303	91809	27818127	17.4069	6.7166	5274.3	3.1354	.00330033	951.902	72106.62
304	92416	28094464	17.4356	6.7240	5300.4	3.1374	.00328947	955.043	72583.36
305	93025	28372625	17.4642	6.7313	5326 6	3.1395	.00327869	958.18[illegible]	73061.66
306	93636	28652616	17.4929	6.7387	5352.8	3.1416	.00326797	961.327	73541.54
307	94249	28934443	17 5214	6.7460	5379.1	3.1436	.00325733	964.468	74022.99
308	94864	29218112	17.5499	6.7533	5405 4	3.1456	.00324675	967.610	74506.01
309	95481	29503629	17.5784	6.7606	5431.7	3.1477	.00323625	970.751	74990.60
310	**96100**	**29791000**	**17.6068**	**6.7679**	**5458.1**	**3.1497**	**.00322581**	**973.893**	**75476.76**
311	96721	30080231	17.6352	6 7752	5484.5	3.1518	.00321543	977.034	75964.50
312	97344	30371328	17.6635	6 7824	5511.0	3.1538	.00320513	980 176	76453.80
313	97969	30664297	17.6918	6.7897	5537.5	3.1558	.00319489	983.318	76944.67
314	98596	30959144	17.7200	6.7969	5564.1	3.1578	.00318471	986.459	77437.12
315	99225	31255875	17.7482	6.8041	5590 7	3.1598	.00317460	989.601	77931.13
316	99856	31554496	17.7764	6.8113	5617.3	3.1618	.00316456	992.742	78426.72
317	100489	31855013	17.8045	6.8185	5644.0	3.1638	.00315457	995.884	78923.88
318	101124	32157432	17.8326	6 8256	5670.7	3.1658	.00314465	999 026	79422.60
319	101761	32461759	17.8606	6.8328	5697.5	3.1678	.00313480	1002.167	79922.90
320	**102400**	**32768000**	**17.8885**	**6.8399**	**5724.3**	**3.1698**	**.00312500**	**1005.309**	**80424.77**
321	103041	33076161	17.9165	6.8470	5751.2	3.1718	.00311526	1008.450	80928.21
322	103684	33386248	17.9444	6 8541	5778.1	3.1737	00310559	1011.592	81433.22
323	104329	33698267	17.9722	6.8612	5805.0	3.1757	.00309598	1014.734	81939.80
324	104976	34012224	18.0000	6.8683	5832.0	3.1777	.00308642	1017.875	82447.96
325	105625	34328125	18.0278	6.8753	5859.0	3.1796	.00307692	1021.017	82957.68
326	106276	34645976	18.0555	6.8824	5886.1	3.1816	.00306748	1024.158	83468.97
327	106929	34965783	18.0831	6 8894	5913.2	3.1835	.00305810	1027.300	83981.84
328	107584	35287552	18.1108	6 8964	5940 3	3.1855	.00304878	1030.442	84496.28
329	108241	35611289	18.1384	6.9034	5967.5	3.1874	.00303951	1033.583	85012.28
330	**108900**	**35937000**	**18.1659**	**6.9104**	**5994.7**	**3.1894**	**.00303030**	**1036.725**	**85529.86**
331	109561	36264691	18.1934	6.9174	6022 0	3.1913	.00302115	1039.866	86049.01
332	110224	36594368	18.2209	6.9244	6049.3	3.1932	.00301205	1043.008	86569.73
333	110889	36926037	18.2483	6.9313	6076.7	3.1951	.00300300	1046.149	87092.02
334	111556	37259704	18.2757	6.9382	6104.1	3.1970	.00299401	1049.291	87615.88
335	112225	37595375	18.3030	6 9451	6131.5	3.1989	.00298507	1052.433	88141.31
336	112896	37933056	18.3303	6.9521	6159.0	3.2009	.00297619	1055.574	88668.31
337	113569	38272753	18.3576	6.9589	6186.5	3.2028	.00296736	1058.716	89196.88
338	114244	38614472	18.3848	6.9658	6214.1	3.2047	.00295858	1061.857	89727.03
339	114921	38958219	18.4120	6.9727	6241.7	3.2066	.00294985	1064.999	90258.74
340	**115600**	**39304000**	**18.4391**	**6.9795**	**6269.3**	**3.2085**	**.00294118**	**1068.141**	**90792.03**
341	116281	39651821	18.4662	6.9864	6297.0	3.2103	.00293255	1071.282	91326.88
342	116964	40001688	18.4932	6.9932	6324.7	3.2122	.00292398	1074.424	91863.31
343	117649	40353607	18.5203	7.0000	6352.4	3.2141	.00291545	1077.565	92401.31
344	118336	40707584	18.5472	7.0068	6380.2	3.2160	.00290698	1080.707	92940.88
345	119025	41063625	18.5742	7.0136	6408.1	3.2178	.00289855	1083.849	93482.02
346	119716	41421736	18.6011	7.0203	6436.0	3.2197	.00289017	1086.990	94024.73
347	120409	41781923	18.6279	7.0271	6463.9	3.2216	.00288184	1090.132	94569.01
348	121104	42144192	18.6548	7.0338	6491.9	3.2234	.00287356	1093.273	95114.86
349	121801	42508549	18.6815	7.0406	6519.9	3.2253	.00286533	1096.415	95662.28
350	**122500**	**42875000**	**18.7083**	**7.0473**	**6547.9**	**3.2271**	**.00285714**	**1099.557**	**96211.28**
351	123201	43243551	18.7350	7.0540	6576.0	3.2289	.00284900	1102.698	96761.84
352	123904	43614208	18.7617	7.0607	6604.1	3.2308	.00284091	1105.840	97313.97
353	124609	43986977	18.7883	7.0674	6632.3	3.2326	.00283286	1108.981	97867.68
354	125316	44361864	18.8149	7.0740	6660.5	3.2345	.00282486	1112.123	98422.96
355	126025	44738875	18.8414	7.0807	6688.7	3.2363	.00281690	1115.264	98979.80
356	126736	45118016	18.8680	7.0873	6717.0	3.2381	.00280899	1118.406	99538.22
357	127449	45499293	18.8944	7.0940	6745.3	3.2399	.00280112	1121.548	100098.21
358	128164	45882712	18.9209	7.1006	6773.7	3.2417	.00279330	1124.689	100659.77
359	128881	46268279	18.9473	7.1072	6802.1	3.2435	.00278552	1127.831	101222.90

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	(Circle N = Diam.) Circum.	Area
360	**129600**	**46656000**	**18.9737**	**7.1138**	**6830.5**	**3.2453**	**.00277778**	**1130.973**	**101787.60**
361	130321	47045881	19.0000	7.1204	6859.0	3.2471	.00277008	1134.114	102353.87
362	131044	47437928	19.0263	7.1269	6887.5	3.2489	.00276243	1137.256	102921.72
363	131769	47832147	19.0526	7.1335	6916.1	3.2507	.00275482	1140.397	103491.13
364	132496	48228544	19.0788	7.1400	6944.7	3.2525	.00274725	1143.539	104062.12
365	133225	48627125	19.1050	7.1466	6973.3	3.2543	.00273973	1146.680	104634.67
366	133956	49027896	19.1311	7.1531	7002.0	3.2561	.00273224	1149.822	105208.80
367	134689	49430863	19.1572	7.1596	7030.7	3.2579	00272480	1152.964	105784.49
368	135424	49836032	19.1833	7.1661	7059.5	3.2597	00271739	1156.105	106361.76
369	136161	50243409	19.2094	7.1726	7088.3	3.2614	.00271003	1159.247	106940.60
370	**136900**	**50653000**	**19.2354**	**7.1791**	**7117.1**	**3.2632**	**.00270270**	**1162.388**	**107521.01**
371	137641	51064811	19.2614	7.1855	7146.0	3.2650	.00269542	1165.530	108102.99
372	138384	51478848	19.2873	7.1920	7174.9	3.2668	00268817	1168.671	108686.54
373	139129	51895117	19.3132	7.1984	7203.9	3.2685	.00268097	1171.813	109271.66
374	139876	52313624	19.3391	7.2048	7232.8	3.2702	.00267380	1174.955	109858.35
375	140625	52734375	19.3649	7.2112	7261.8	3.2719	.00266667	1178.096	110446.62
376	141376	53157376	19.3907	7.2177	7290.9	3.2737	.00265957	1181.238	111036.45
377	142129	53582633	19.4165	7.2240	7320.0	3.2754	.00265252	1184.379	111627.86
378	142884	54010152	19.4422	7.2304	7349.2	3.2772	.00264550	1187.521	112220.83
379	143641	54439939	19.4679	7.2368	7378.4	3.2789	.00263852	1190.663	112815.38
380	**144400**	**54872000**	**19.4936**	**7.2432**	**7407.6**	**3.2807**	**00263158**	**1193.804**	**113411.49**
381	145161	55306341	19.5192	7.2495	7436.8	3.2824	.00262467	1196.946	114009.18
382	145924	55742968	19.5448	7.2558	7466.1	3.2841	00261780	1200.087	114608.44
383	146689	56181887	19.5704	7.2622	7495.4	3.2858	.00261097	1203.229	115209.27
384	147456	56623104	19 5959	7.2685	7524.8	3.2875	00260417	1206.371	115811.67
385	148225	57066625	19.6214	7 2748	7554.2	3.2892	00259740	1209.512	116415.64
386	148996	57512456	19.6469	7.2811	7583.7	3.2909	.00259067	1212.654	117021.18
387	149769	57960603	19.6723	7.2874	7613.2	3.2926	00258398	1215.795	117628.30
388	150544	58411072	19.6977	7.2936	7642.7	3 2943	.00257732	1218 937	118236.98
389	151321	58863869	19.7231	7.2999	7672.3	3.2960	.00257069	1222.079	118847.24
390	**152100**	**59319000**	**19.7484**	**7.3061**	**7701.9**	**3.2977**	**.00256410**	**1225.220**	**119459.06**
391	152881	59776471	19.7737	7.3124	7731.5	3.2994	00255754	1228.362	120072.46
392	153664	60236288	19 7990	7.3186	7761.2	3.3011	.00255102	1231.503	120687.42
393	154449	60698457	19.8242	7.3248	7790.9	3.3028	.00254453	1234.645	121303.96
394	155236	61162984	19.8494	7.3310	7820.7	3.3045	.00253807	1237.786	121922.07
395	156025	61629875	19 8746	7.3372	7850.5	3.3061	00253165	1240 928	122541.75
396	156816	62099136	19.8997	7.3434	7880.3	3.3078	.00252525	1244.070	123163.00
397	157609	62570773	19 9249	7 3496	7910.2	3.3095	00251889	1247 211	123785.82
398	158404	63044792	19.9499	7.3558	7940 1	3.3111	.00251256	1250 353	124410.21
399	159201	63521199	19.9750	7.3619	7970.0	3.3128	.00250627	1253.494	125036.17
400	**160000**	**64000000**	**20.0000**	**7.3681**	**8000.0**	**3.3145**	**.00250000**	**1256.636**	**125663.71**
401	160801	64481201	20.0250	7.3742	8030.0	3.3161	.00249377	1259.778	126292.81
402	161604	64964808	20.0499	7.3803	8061.1	3.3178	00248756	1262 919	126923.48
403	162409	65450827	20.0749	7.3864	8090.2	3.3194	.00248139	1266.061	127555.73
404	163216	65939264	20.0998	7.3925	8120.3	3.3211	.00247525	1269.202	128189.55
405	164025	66430125	20.1246	7.3986	8150.5	3.3227	.00246914	1272.344	128824.93
406	164836	66923416	20.1494	7.4047	8180.7	3.3243	.00246305	1275.486	129461.89
407	165649	67419143	20.1742	7.4108	8210.9	3.3260	00245700	1278.627	130100.42
408	166464	67917312	20.1990	7.4169	8241.2	3.3276	00245098	1281.769	130740.52
409	167281	68417929	20.2237	7.4229	8271.5	3.3292	.00244499	1284.910	131382.19
410	**168100**	**68921000**	**20.2485**	**7.4290**	**8301.9**	**3.3308**	**.00243902**	**1288.052**	**132025.43**
411	168921	69426531	20.2731	7.4350	8332.3	3.3325	00243309	1291.193	132670.24
412	169744	69934528	20.2978	7.4410	8362.7	3 3341	.00242718	1294.335	133316.63
413	170569	70444997	20.3224	7.4470	8393.2	3.3357	.00242131	1297.477	133964.58
414	171396	70957944	20.3470	7.4530	8423.7	3.3373	00241546	1300.618	134614.10
415	172225	71473375	20.3715	7.4590	8454.2	3.3390	.00240964	1303.760	135265.20
416	173056	71991296	20.3961	7.4650	8484.8	3.3406	.00240385	1306.901	135917.86
417	173889	72511713	20.4206	7.4710	8515.4	3.3422	.00239808	1310.043	136572.10
418	174724	73034632	20.4450	7.4770	8546.0	3 3438	.00239234	1313.185	137227.91
419	175561	73560059	20.4695	7.4829	8576.7	3.3454	.00238663	1316.326	137885.29
420	**176400**	**74088000**	**20.4939**	**7.4889**	**8607.4**	**3.3470**	**.00238095**	**1319.468**	**138544.24**
421	177241	74618461	20.5183	7.4948	8638.2	3.3485	.00237530	1322.609	139204.76
422	178084	75151448	20.5426	7.5007	8669 0	3.3501	.00236967	1325.751	139866.85
423	178929	75686967	20.5670	7.5067	8699.8	3.3517	.00236407	1328.893	140530.51
424	179776	76225024	20.5913	7.5126	8730.7	3.3533	.00235849	1332.034	141195.74
425	180625	76765625	20.6155	7.5185	8761.6	3.3549	.00235294	1335.176	141862.54
426	181476	77308776	20.6398	7.5244	8792.5	3.3564	.00234742	1338.317	142530.92
427	182329	77854483	20.6640	7.5302	8823.5	3.3580	.00234192	1341.459	143200.86
428	183184	78402752	20.6882	7.5361	8854.5	3.3596	.00233645	1344.601	143872.38
429	184041	78953589	20.7123	7.5420	8885.6	3.3612	.00233100	1347.742	144545.46

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Area
430	**184900**	**79507000**	**20.7364**	**7.5478**	**8916.7**	**3.3627**	**.00232558**	**1350.884**	**145220.12**
431	185761	80062991	20.7605	7.5537	8947.8	3.3643	.00232019	1354.025	145896.35
432	186624	80621568	20.7846	7.5595	8979.0	3.3659	.00231481	1357.167	146574.15
433	187489	81182737	20.8087	7.5654	9010.1	3.3674	.00230947	1360.308	147253.52
434	188356	81746504	20.8327	7.5712	9041.4	3.3690	.00230415	1363.450	147934.46
435	189225	82312875	20.8567	7.5770	9072.7	3.3705	.00229885	1366.592	148616.97
436	190096	82881856	20.8806	7.5828	9104.0	3.3720	.00229358	1369.733	149301.05
437	190969	83453453	20.9045	7.5886	9135.3	3.3736	.00228833	1372.875	149986.70
438	191844	84027672	20.9284	7.5944	9166.7	3.3752	.00228311	1376.016	150673.92
439	192721	84604519	20.9523	7.6001	9198.1	3.3767	.00227790	1379.158	151362.72
440	**193600**	**85184000**	**20.9762**	**7.6059**	**9229.5**	**3.3783**	**.00227273**	**1382.300**	**152053.08**
441	194481	85766121	21.0000	7.6117	9261.0	3.3798	.00226757	1385.441	152745.02
442	195364	86350888	21.0238	7.6174	9292.5	3.3813	.00226244	1388.583	153438.53
443	196249	86938307	21.0476	7.6232	9324.1	3.3828	.00225734	1391.724	154133.60
444	197136	87528384	21.0713	7.6289	9355.7	3.3844	.00225225	1394.866	154830.25
445	198025	88121125	21.0950	7.6346	9387.3	3.3859	.00224719	1398.008	155528.47
446	198916	88716536	21.1187	7.6403	9419.0	3.3874	.00224215	1401.149	156228.26
447	199809	89314623	21.1424	7.6460	9450.7	3.3889	.00223714	1404.291	156929.62
448	200704	89915392	21.1660	7.6517	9482.4	3.3904	.00223214	1407.432	157632.55
449	201601	90518849	21.1896	7.6574	9514.2	3.3919	.00222717	1410.574	158337.05
450	**202500**	**91125000**	**21.2132**	**7.6631**	**9546.0**	**3.3935**	**.00222222**	**1413.716**	**159043.13**
451	203401	91733851	21.2368	7.6688	9577.8	3.3950	.00221729	1416.857	159750.77
452	204304	92345408	21.2603	7.6744	9609.6	3.3965	.00221239	1419.999	160459.99
453	205209	92959677	21.2838	7.6801	9641.5	3.3980	.00220751	1423.140	161170.77
454	206116	93576664	21.3073	7.6857	9673.5	3.3995	.00220264	1426.282	161883.13
455	207025	94196375	21.3307	7.6914	9705.5	3.4010	.00219780	1429.423	162597.05
456	207936	94818816	21.3542	7.6970	9737.5	3.4025	.00219298	1432.565	163312.55
457	208849	95443993	21.3776	7.7026	9769.5	3.4039	.00218818	1435.707	164029.62
458	209764	96071912	21.4009	7.7082	9801.6	3.4054	.00218341	1438.848	164748.26
459	210681	96702579	21.4243	7.7138	9833.8	3.4069	.00217865	1441.990	165468.47
460	**211600**	**97336000**	**21.4476**	**7.7194**	**9865.9**	**3.4084**	**.00217391**	**1445.131**	**166190.25**
461	212521	97972181	21.4709	7.7250	9898.1	3.4199	.00216920	1448.273	166913.60
462	213444	98611128	21.4942	7.7306	9930.3	3.4113	.00216450	1451.415	167638.52
463	214369	99252847	21.5174	7.7362	9962.6	3.4128	.00215983	1454.556	168365.02
464	215296	99897344	21.5407	7.7418	9994.8	3.4143	.00215517	1457.698	169093.08
465	216225	100544625	21.5639	7.7473	10027.	3.4158	.00215054	1460.839	169822.72
466	217156	101194696	21.5870	7.7529	10060.	3.4173	.00214592	1463.981	170553.92
467	218089	101847563	21.6102	7.7584	10092.	3.4187	.00214133	1467.123	171286.70
468	219024	102503232	21.6333	7.7639	10124.	3.4202	.00213675	1470.264	172021.05
469	219961	103161709	21.6564	7.7695	10157.	3.4217	.00213220	1473.406	172756.96
470	**220900**	**103823000**	**21.6795**	**7.7750**	**10189.**	**3.4231**	**.00212766**	**1476.547**	**173494.45**
471	221841	104487111	21.7025	7.7805	10222.	3.4246	.00212314	1479.689	174233.51
472	222784	105154048	21.7256	7.7860	10255.	3.4260	.00211864	1482.830	174974.14
473	223729	105823817	21.7486	7.7915	10287.	3.4275	.00211416	1485.972	175716.34
474	224676	106496424	21.7715	7.7970	10320.	3.4289	.00210970	1489.114	176460.12
475	225625	107171875	21.7945	7.8025	10352.	3.4304	.00210526	1492.255	177205.46
476	226576	107850176	21.8174	7.8079	10385.	3.4318	.00210084	1495.397	177952.37
477	227529	108531333	21.8403	7.8134	10418.	3.4332	.00209644	1498.538	178700.86
478	228484	109215352	21.8632	7.8188	10450.	3.4347	.00209205	1501.680	179450.91
479	229441	109902239	21.8861	7.8243	10483.	3.4361	.00208768	1504.822	180202.54
480	**230400**	**110592000**	**21.9089**	**7.8297**	**10516.**	**3.4375**	**.00208333**	**1507.963**	**180955.74**
481	231361	111284641	21.9317	7.8352	10549.	3.4390	.00207900	1511.105	181710.50
482	232324	111980168	21.9545	7.8406	10582.	3.4404	.00207469	1514.246	182466.84
483	233289	112678587	21.9773	7.8460	10615.	3.4418	.00207039	1517.388	183224.75
484	234256	113379904	22.0000	7.8514	10648.	3.4433	.00206612	1520.530	183984.23
485	235225	114084125	22.0227	7.8568	10681.	3.4447	.00206186	1523.671	184745.28
486	236196	114791256	22.0454	7.8622	10714.	3.4461	.00205761	1526.813	185507.90
487	237169	115501303	22.0681	7.8676	10747.	3.4475	.00205339	1529.954	186272.10
488	238144	116214272	22.0907	7.8730	10780.	3.4489	.00204918	1533.096	187037.86
489	239121	116930169	22.1133	7.8784	10813.	3.4504	.00204499	1536.238	187805.19
490	**240100**	**117649000**	**22.1359**	**7.8837**	**10847.**	**3.4518**	**.00204082**	**1539.379**	**188574.10**
491	241081	118370771	22.1585	7.8891	10880.	3.4532	.00203666	1542.521	189344.57
492	242064	119095488	22.1811	7.8944	10913.	3.4546	.00203252	1545.662	190116.62
493	243049	119823157	22.2036	7.8998	10946.	3.4560	.00202840	1548.804	190890.24
494	244036	120553784	22.2261	7.9051	10980.	3.4574	.00202429	1551.945	191665.43
495	245025	121287375	22.2486	7.9105	11013.	3.4588	.00202020	1555.087	192442.18
496	246016	122023936	22.2711	7.9158	11046.	3.4602	.00201613	1558.229	193220.51
497	247009	122763473	22.2935	7.9211	11080.	3.4616	.00201207	1561.370	194000.41
498	248004	123505992	22.3159	7.9264	11113.	3.4630	.00200803	1564.512	194781.89
499	249001	124251499	22.3383	7.9317	11147.	3.4643	.00200401	1567.653	195564.93

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
500	**250000**	**125000000**	**22.3607**	**7.9370**	**11180**	**3.4657**	**.00200000**	**1570.795**	**196349.54**
501	251001	125751501	22.3830	7.9423	11214	3.4671	.00199601	1573.937	197135.72
502	252004	126506008	22.4054	7.9476	11247	3.4685	.00199203	1577.078	197923.48
503	253009	127263527	22.4277	7.9528	11281	3.4699	.00198807	1580.220	198712.80
504	254016	128024064	22.4499	7.9581	11315	3.4713	.00198413	1583.361	199503.70
505	255025	128787625	22.4722	7.9634	11348	3.4726	.00198020	1586.503	200296.17
506	256036	129554216	22.4944	7.9686	11382	3.4740	.00197628	1589.645	201090.20
507	257049	130323843	22.5167	7.9739	11416	3.4754	.00197239	1592.786	201885.81
508	258064	131096512	22.5389	7.9791	11450	3.4768	.00196850	1595.928	202682.99
509	259081	131872229	22.5610	7.9843	11484	3.4781	.00196464	1599.069	203481.74
510	**260100**	**132651000**	**22.5832**	**7.9896**	**11517**	**3.4795**	**.00196078**	**1602.211**	**204282.06**
511	261121	133432831	22.6053	7.9948	11551	3.4808	.00195695	1605.352	205083.95
512	262144	134217728	22.6274	8.0000	11585	3.4822	.00195313	1608.494	205887.42
513	263169	135005697	22.6495	8.0052	11619	3.4836	.00194932	1611.636	206692.45
514	264196	135796744	22.6716	8.0104	11653	3.4849	.00194553	1614 777	207499.05
515	265225	136590875	22.6936	8.0156	11687	3.4863	.00194175	1617.919	208307.23
516	266256	137388096	22.7156	8.0208	11721	3.4876	.00193798	1621.060	209116.97
517	267289	138188413	22.7376	8.0260	11755	3.4890	.00193424	1624.202	209928.29
518	268324	138991832	22.7596	8.0311	11789	3.4904	.00193050	1627.344	210741.18
519	269361	139798359	22.7816	8.0363	11824	3.4917	.00192678	1630.485	211555.63
520	**270400**	**140608000**	**22.8035**	**8.0415**	**11858**	**3.4930**	**.00192308**	**1633.627**	**212371.66**
521	271441	141420761	22.8254	8.0466	11892	3.4944	.00191939	1636.768	213189.26
522	272484	142236648	22.8473	8.0517	11926	3.4957	.00191571	1639.910	214008.43
523	273529	143055667	22.8692	8.0569	11960	3.4970	.00191205	1643.052	214829.17
524	274576	143877824	22.8910	8.0620	11995	3.4984	.00190840	1646.193	215651.49
525	275625	144703125	22.9129	8.0671	12029	3.4997	.00190476	1649.335	216475.37
526	276676	145531576	22.9347	8.0723	12064	3.5010	.00190114	1652.476	217300.82
527	277729	146363183	22.9565	8.0774	12098	3 5024	.00189753	1655.618	218127.85
528	278784	147197952	22.9783	8.0825	12133	3.5037	.00189394	1658 760	218956.44
529	279841	148035889	23.0000	8.0876	12167	3.5050	.00189036	1661.901	219786.61
530	**280900**	**148877000**	**23.0217**	**8.0927**	**12202**	**3.5064**	**.00188679**	**1665.043**	**220618.34**
531	281961	149721291	23.0434	8.0978	12236	3.5077	.00188324	1668.184	221451.65
532	283024	150568768	23.0651	8.1028	12271	3.5090	00187970	1671.326	222286.53
533	284089	151419437	23.0868	8.1079	12305	3.5103	•00187617	1674.467	223122.98
534	285156	152273304	23.1084	8.1130	12340	3.5116	.00187266	1677.609	223961.00
535	286225	153130375	23.1301	8.1180	12375	3.5130	.00186916	1680.751	224800.59
536	287296	153990656	23.1517	8.1231	12410	3.5143	.00186567	1683.892	225641.75
537	288369	154854153	23.1733	8.1281	12444	3.5156	.00186220	1687.034	226484.48
538	289444	155720872	23.1948	8.1332	12479	3.5169	.00185874	1690.175	227328.79
539	290521	156590819	23.2164	8.1382	12514	3.5182	.00185529	1693.317	228174.66
540	**291600**	**157464000**	**23.2379**	**8.1433**	**12549**	**3.5195**	**.00185185**	**1696.459**	**229022.10**
541	292681	158340421	23.2594	8.1483	12583	3.5208	.00184843	1699.600	229871.12
542	293764	159220088	23.2809	8.1533	12618	3.5221	.00184502	1702.742	230721.71
543	294849	160103007	23.3024	8.1583	12653	3.5234	.00184162	1705.883	231573.86
544	295936	160989184	23.3238	8.1633	12688	3.5247	.00183824	1709.025	232427.59
545	297025	161878625	23.3452	8.1683	12723	3.5260	.00183486	1712.167	233282.89
546	298116	162771336	23.3666	8.1733	12758	3.5273	.00183150	1715.308	234139.76
547	299209	163667323	23.3880	8.1783	12793	3.5286	.00182815	1718.450	234998.20
548	300304	164566592	23.4094	8.1833	12828	3.5299	.00182482	1721.591	235858.21
549	301401	165469149	23.4307	8.1882	12863	3.5311	.00182149	1724.733	236719.79
550	**302500**	**166375000**	**23.4521**	**8.1932**	**12899**	**3.5324**	**.00181818**	**1727.876**	**237582.94**
551	303601	167284151	23.4734	8.1982	12934	3.5337	.00181488	1731.016	238447.67
552	*304704*	*168196608*	*23.4947*	*8.2031*	*12969*	*3.5350*	*.00181159*	*1734.158*	*239313.96*
553	*305809*	*169112377*	*23.5160*	*8.2081*	*13004*	*3.5363*	*.00180832*	*1737.299*	*240181.83*
554	306916	170031464	23.5372	8.2130	13040	3.5376	.00180505	1740.441	241051.26
555	308025	170953875	23.5584	8.2180	13075	3.5388	.00180180	1743.582	241922.27
556	309136	171879616	23.5797	8.2229	13110	3.5401	.00179856	1746.724	242794.85
557	310249	172808693	23.6008	8.2278	13146	3.5414	.00179533	1749.866	243668.99
558	311364	173741112	23.6220	8.2327	13181	3.5426	.00179211	1753.007	244544.71
559	312481	174676879	23.6432	8.2377	13217	3.5439	.00178891	1756.149	245422.00
560	**313600**	**175616000**	**23.6643**	**8.2426**	**13252**	**3.5451**	**.00178571**	**1759.290**	**246300.86**
561	314721	176558481	23.6854	8.2475	13288	3.5464	.00178253	1762.432	247181.30
562	315844	177504328	23.7065	8.2524	13323	3.5477	.00177936	1765.574	248063.30
563	316969	178453547	23.7276	8.2573	13359	3.5490	.00177620	1768.715	248946.87
564	318096	179406144	23.7487	8.2621	13394	3.5502	.00177305	1771.857	249832.01
565	319225	180362125	23.7697	8.2670	13430	3.5515	.00176991	1774.998	250718.73
566	320356	181321496	23.7908	8.2719	13466	3.5527	.00176678	1778.140	251607.01
567	321489	182284263	23.8118	8.2768	13501	3.5540	.00176367	1781.282	252496.87
568	322624	183250432	23.8328	8.2816	13537	3.5553	.00176056	1784.423	253388.30
569	323761	184220009	23.8537	8.2865	13573	3.5565	.00175747	1787.565	254281.29

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Area
570	**324900**	**185193000**	**23.8747**	**8.2913**	**13609**	**3.5577**	**.00175439**	**1790.706**	**255175.86**
571	326041	186169411	23.8956	8.2962	13644	3.5590	.00175131	1793.848	256072.00
572	327184	187149248	23.9165	8.3010	13680	3.5602	.00174825	1796.989	256969.71
573	328329	188132517	23.9374	8.3059	13716	3.5615	.00174520	1800.131	257868.99
574	329476	189119224	23.9583	8.3107	13752	3.5627	.00174216	1803.273	258769.85
575	330625	190109375	23.9792	8.3155	13788	3.5640	.00173913	1806.414	259672.27
576	331776	191102976	24.0000	8.3203	13824	3.5652	.00173611	1809.556	260576.26
577	332929	192100033	24.0208	8.3251	13860	3.5664	.00173310	1812.697	261481.83
578	334084	193100552	24.0416	8.3300	13896	3.5677	.00173010	1815.839	262388.96
579	335241	194104539	24.0624	8.3348	13932	3.5689	.00172712	1818.981	263297.67
580	**336400**	**195112000**	**24.0832**	**8.3396**	**13968**	**3.5702**	**.00172414**	**1822.122**	**264207.94**
581	337561	196122941	24.1039	8.3443	14004	3.5714	.00172117	1825.264	265119.79
582	338724	197137368	24.1247	8.3491	14040	3.5726	.00171821	1828.405	266033.21
583	339889	198155287	24.1454	8.3539	14077	3.5738	.00171527	1831.547	266948.20
584	341056	199176704	24.1661	8.3587	14113	3.5751	.00171233	1834.689	267864.76
585	342225	200201625	24.1868	8.3634	14149	3.5763	.00170940	1837.830	268782.89
586	343396	201230056	24.2074	8.3682	14186	3.5775	.00170648	1840.972	269702.59
587	344569	202262003	24.2281	8.3730	14222	3.5787	.00170358	1844.113	270623.86
588	345744	203297472	24.2487	8.3777	14258	3.5799	.00170068	1847.255	271546.70
589	346921	204336469	24.2693	8.3825	14295	3.5812	.00169779	1850.397	272471.12
590	**348100**	**205379000**	**24.2899**	**8.3872**	**14331**	**3.5824**	**.00169492**	**1853.538**	**273397.10**
591	349281	206425071	24.3105	8.3919	14368	3.5886	.00169205	1856.680	274324.66
592	350464	207474688	24.3311	8.3967	14404	3.5848	.00168919	1859.821	275253.78
593	351649	208527857	24.3516	8.4014	14440	3.5860	.00168634	1862.963	276184.48
594	352836	209584584	24.3721	8.4061	14477	3.5872	.00168350	1866.104	277116.75
595	354025	210644875	24.3926	8.4108	14514	3.5884	.00168067	1869.246	278050.58
596	355216	211708736	24.4131	8.4155	14550	3.5896	.00167785	1872.388	278985.99
597	356409	212776173	24.4336	8.4202	14587	3.5908	.00167504	1875.529	279922.97
598	357604	213847192	24.4540	8.4249	14624	3.5920	.00167224	1878.671	280861.52
599	358801	214921799	24.4745	8.4296	14660	3.5932	.00166945	1881.812	281801.65
600	**360000**	**216000000**	**24.4949**	**8.4343**	**14697**	**3.5944**	**.00166667**	**1884.954**	**282743.34**
601	361201	217081801	24.5153	8.4390	14734	3.5956	.00166389	1888.096	283686.60
602	362404	218167208	24.5357	8.4437	14770	3.5958	.00166113	1891.237	284631.44
603	363609	219256227	24.5561	8.4484	14807	3.5980	.00165837	1894.379	285577.84
604	364816	220348864	24.5764	8.4530	14844	3.5992	.00165563	1897.520	286525.82
605	366025	221445125	24.5967	8.4577	14881	3.6004	.00165289	1900.662	287475.36
606	367236	222545016	24.6171	8.4623	14918	3.6016	.00165017	1903.804	288426.48
607	368449	223648543	24.6374	8.4670	14955	3.6028	.00164745	1906.945	289379.17
608	369664	224755712	24.6577	8.4716	14992	3.6040	.00164474	1910.087	290333.43
609	370881	225866529	24.6779	8.4763	15029	3.6052	.00164204	1913.228	291289.26
610	**372100**	**226981000**	**24.6982**	**8.4809**	**15066**	**3.6063**	**.00163934**	**1916.370**	**292246.66**
611	373321	228099131	24.7184	8.4856	15103	3.6075	.00163666	1919.511	293205.63
612	374544	229220928	24.7386	8.4902	15140	3.6087	.00163399	1922.653	294166.17
613	375769	230346397	24.7588	8.4948	15177	3.6099	.00163132	1925.795	295128.28
614	376996	231475544	24.7790	8.4994	15214	3.6111	.00162866	1928.936	296091.97
615	378225	232608375	24.7992	8.5040	15252	3.6122	.00162602	1932.078	297057.22
616	379456	233744896	24.8193	8.5086	15289	3.6134	.00162338	1935.219	298024.05
617	380689	234885113	24.8395	8.5132	15326	3.6146	.00162075	1938.361	298992.44
618	381924	236029032	24.8596	8.5178	15363	3.6158	.00161812	1941.503	299962.41
619	383161	237176659	24.8797	8.5224	15400	3.6169	.00161551	1944.644	300933.95
620	**384400**	**238328000**	**24.8998**	**8.5270**	**15437**	**3.6181**	**.00161290**	**1947.786**	**301907.05**
621	385641	239483061	24.9199	8.5316	15475	3.6192	.00161031	1950.927	302881.73
622	386884	240641848	24.9399	8.5362	15513	3.6204	.00160772	1954.069	303857.98
623	388129	241804367	24.9600	8.5408	15550	3.6216	.00160514	1957.211	304835.80
624	389376	242970624	24.9800	8.5453	15588	3.6227	.00160256	1960.352	305815.20
625	390625	244140625	25.0000	8.5499	15625	3.6239	.00160000	1963.494	306796.16
626	391876	245314376	25.0200	8.5544	15663	3.6250	.00159744	1966.635	307778.69
627	393129	246491883	25.0400	8.5590	15700	3.6262	.00159490	1969.777	308762.79
628	394384	247673152	25.0599	8.5635	15738	3.6274	.00159236	1972.919	309748.47
629	395641	248858189	25.0799	8.5681	15775	3.6285	.00158983	1976.060	310735.71
630	**396900**	**250047000**	**25.0998**	**8.5726**	**15813**	**3.6297**	**.00158730**	**1979.203**	**311724.53**
631	398161	251239591	25.1197	8.5772	15850	3.6309	.00158479	1982.343	312714.92
632	399424	252435968	25.1396	8.5817	15888	3.6320	.00158228	1985.485	313706.88
633	400689	253636137	25.1595	8.5862	15926	3.6331	.00157978	1988.626	314700.40
634	401956	254840104	25.1794	8.5907	15964	3.6343	.00157729	1991.768	315695.50
635	403225	256047875	25.1992	8.5952	16002	3.6354	.00157480	1994.910	316692.17
636	404496	257259456	25.2190	8.5997	16040	3.6366	.00157233	1998.051	317690.42
637	405769	258474853	25.2389	8.6043	16077	3.6377	.00156986	2001.193	318690.23
638	407044	259694072	25.2587	8.6088	16115	3.6389	.00156740	2004.334	319691.61
639	408321	260917119	25.2784	8.6132	16153	3.6400	.00156495	2007.476	320694.56

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
640	**409600**	**262144000**	**25.2982**	**8.6177**	**16191**	**3.6411**	**.00156250**	**2010.618**	**321699.09**
641	410881	263374721	25.3180	8.6222	16229	3.6423	.00156006	2013.759	322705.18
642	412164	264609288	25.3377	8.6267	16267	3.6435	.00155763	2016.901	323712.85
643	413449	265847707	25.3574	8.6312	16305	3.6446	.00155521	2020.042	324722.09
644	414736	267089984	25.3772	8.6357	16343	3.6457	.00155280	2023.184	325732.89
645	416025	268336125	25.3969	8.6401	16381	3.6468	.00155039	2026.326	326745.27
646	417316	269586136	25.4165	8.6446	16419	3.6479	.00154799	2029.467	327759.22
647	418609	270840023	25.4362	8.6490	16457	3.6499	.00154560	2032.609	328774.74
648	419904	272097792	25.4558	8.6535	16495	3.6502	.00154321	2035.750	329791.83
649	421201	273359449	25.4755	8.6579	16534	3.6513	.00154083	2038.892	330810.49
650	**422500**	**274625000**	**25.4951**	**8.6624**	**16572**	**3.6524**	**.00153846**	**2042.034**	**331830.72**
651	423801	275894451	25.5147	8.6668	16610	3.6536	.00153610	2045.175	332852.53
652	425104	277167808	25.5343	8.6713	16648	3.6547	.00153374	2048.317	333875.90
653	426409	278445077	25.5539	8.6757	16687	3.6558	.00153139	2051.458	334900.85
654	427716	279726264	25.5734	8.6801	16725	3.6569	.00152905	2054.600	335927.36
655	429025	281011375	25.5930	8.6845	16764	3.6580	.00152672	2057.741	336955.45
656	430336	282300416	25.6125	8.6890	16802	3.6592	.00152439	2060.883	337985.10
657	431649	283593393	25.6320	8.6934	16840	3.6603	.00152207	2064.025	339016.33
658	432964	284890312	25.6515	8.6978	16879	3.6614	.00151976	2067.166	340049.13
659	434281	286191179	25.6710	8.7022	16917	3.6625	.00151745	2070.308	341083.50
660	**435600**	**287496000**	**25.6905**	**8.7066**	**16956**	**3.6636**	**.00151515**	**2073.449**	**342119.44**
661	436921	288804781	25.7099	8.7110	16994	3.6647	.00151286	2076.591	343156.95
662	438244	290117528	25.7294	8.7154	17033	3.6658	.00151057	2079.733	344196.03
663	439569	291434247	25.7488	8.7198	17071	3.6669	.00150830	2082.874	345236.69
664	440896	292754944	25.7682	8.7241	17110	3.6680	.00150602	2086.016	346278.91
665	442225	294079625	25.7876	8.7285	17149	3.6691	.00150376	2089.157	347322.70
666	443556	295408296	25.8070	8.7329	17187	3.6702	.00150150	2092.299	348368.07
667	444889	296740963	25.8263	8.7373	17226	3.6713	.00149925	2095.441	349415.00
668	446224	298077632	25.8457	8.7416	17265	3.6724	.00149701	2098.582	350463.51
669	447561	299418309	25.8650	8.7460	17304	3.6735	.00149477	2101.724	351513.59
670	**448900**	**300763000**	**25.8844**	**8.7503**	**17343**	**3.6746**	**.00149254**	**2104.865**	**352565.24**
671	450241	302111711	25.9037	8.7547	17381	3.6757	.00149031	2108.007	353618.45
672	451584	303464448	25.9230	8.7590	17420	3.6768	.00148810	2111.148	354673.24
673	452929	304821217	25.9422	8.7634	17459	3.6779	.00148588	2114.290	355729.60
674	454276	306182024	25.9615	8.7677	17498	3.6790	.00148368	2117.432	356787.54
675	455625	307546875	25.9808	8.7721	17537	3.6801	.00148148	2120.573	357847.04
676	456976	308915776	26.0000	8.7764	17576	3.6812	.00147929	2123.715	358908.11
677	458329	310288733	26.0192	8.7807	17615	3.6823	.00147710	2126.856	359970.75
678	459684	311665752	26.0384	8.7850	17654	3.6834	.00147493	2129.998	361034.97
679	461041	313046839	26.0576	8.7893	17693	3.6845	.00147275	2133.140	362100.75
680	**462400**	**314432000**	**26.0768**	**8.7937**	**17732**	**3.6856**	**.00147059**	**2136.281**	**363168.11**
681	463761	315821241	26.0960	8.7980	17771	3.6866	.00146843	2139.423	364237.04
682	465124	317214568	26.1151	8.8023	17810	3.6877	.00146628	2142.564	365307.54
683	466489	318611987	26.1343	8.8066	17850	3.6888	.00146413	2145.706	366379.60
684	467856	320013504	26.1534	8.8109	17889	3.6899	.00146199	2148.848	367453.24
685	469225	321419125	26.1725	8.8152	17928	3.6909	.00145985	2151.989	368528.45
686	470596	322828856	26.1916	8.8194	17967	3.6920	.00145773	2155.131	369605.23
687	471969	324242703	26.2107	8.8237	18007	3.6931	.00145560	2158.272	370683.59
688	473344	325660672	26.2298	8.8280	18046	3.6942	.00145349	2161.414	371763.51
689	474721	327082769	26.2488	8.8323	18085	3.6953	.00145138	2164.556	372845.00
690	**476100**	**328509000**	**26.2679**	**8.8366**	**18125**	**3.6963**	**.00144928**	**2167.697**	**373928.07**
691	477481	329939371	26.2869	8.8408	18164	3.6974	.00144718	2170.839	375012.70
692	478864	331373888	26.3059	8.8451	18204	3.6985	.00144509	2173.980	376098.91
693	480249	332812557	26.3249	8.8493	18243	3.6995	.00144300	2177.122	377186.68
694	481636	334255384	26.3439	8.8536	18283	3.7006	.00144092	2180.263	378276.03
695	483025	335702375	26.3629	8.8578	18322	3.7016	.00143885	2183.405	379366.95
696	484416	337153536	26.3818	8.8621	18362	3.7027	.00143678	2186.547	380459.44
697	485809	338608873	26.4008	8.8663	18401	3.7038	.00143472	2189.688	381553.50
698	487204	340068392	26.4197	8.8706	18441	3.7049	.00143266	2192.830	382649.13
699	488601	341532099	26.4386	8.8748	18480	3.7059	.00143062	2195.971	383746.33
700	**490000**	**343000000**	**26.4575**	**8.8790**	**18520**	**3.7070**	**.00142857**	**2199.113**	**384845.10**
701	491401	344472101	26.4764	8.8833	18560	3.7080	.00142653	2202.255	385945.44
702	492804	345948408	26.4953	8.8875	18600	3.7091	.00142450	2205.396	387047.36
703	494209	347428927	26.5141	8.8917	18640	3.7101	.00142248	2208.538	388150.84
704	495616	348913664	26.5330	8.8959	18679	3.7112	.00142045	2211.679	389255.90
705	497025	350402625	26.5518	8.9001	18719	3.7123	.00141844	2214.821	390362.52
706	498436	351895816	26.5707	8.9043	18759	3.7133	.00141643	2217.963	391470.72
707	499849	353393243	26.5895	8.9085	18799	3.7144	.00141443	2221.104	392580.49
708	501264	354894912	26.6083	8.9127	18839	3.7154	.00141243	2224.246	393691.82
709	502681	356400829	26.6271	8.9169	18879	3.7165	.00141044	2227.387	394804.73

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
710	**504100**	**357911000**	**26.6458**	**8.9211**	**18919**	**3.7175**	**.00140845**	**2230.529**	**395919.21**
711	505521	359425431	26.6646	8.9253	18959	3.7185	.00140647	2233.670	397035.26
712	506944	360944128	26.6833	8.9295	18999	3.7196	.00140449	2236.812	398152.89
713	508369	362467097	26.7021	8.9337	19039	3.7206	.00140252	2239.954	399272.08
714	509796	363994344	26.7208	8.9378	19079	3.7217	.00140056	2243.095	400392.84
715	511225	365525875	26.7395	8.9420	19119	3.7227	.00139860	2246.237	401515.18
716	512656	367061696	26.7582	8.9462	19159	3.7238	.00139665	2249.378	402639.08
717	514089	368601813	26.7769	8 9503	19199	3.7248	.00139470	2252.520	403764.56
718	515524	370146232	26.7955	8.9545	19239	3.7258	.00139276	2255.662	404891.60
719	516961	371694959	26.8142	8.9587	19280	3.7269	.00139082	2258.803	406020.22
720	**518400**	**373248000**	**26.8328**	**8.9628**	**19320**	**3.7279**	**.00138889**	**2261.945**	**407150.41**
721	519841	374805361	26.8514	8.9670	19360	3.7290	.00138696	2265.086	408282.17
722	521284	376367048	26.8701	8 9711	19400	3.7300	00138504	2268.228	409415.50
723	522729	377933067	26.8887	8.9752	19440	3.7310	.00138313	2271.370	410550.40
724	524176	379503424	26.9072	8 9794	19481	3.7321	.00138122	2274.511	411686.87
725	525625	381078125	26 9258	8.9835	19521	3.7331	.00137931	2277.653	412824.91
726	527076	382657176	26.9444	8.9876	19562	3.7341	.00137741	2280.794	413964.52
727	528529	384240583	26 9629	8 9918	19602	3.7351	.00137552	2283 936	415105.71
728	529984	385828352	26.9815	8.9959	19643	3.7362	.00137363	2287 078	416248.46
729	531441	387420489	27.0000	9.0000	19683	3.7372	.00137174	2290.219	417392.79
730	**532900**	**389017000**	**27.0185**	**9.0041**	**19724**	**3.7382**	**00136986**	**2293.361**	**418538.68**
731	534361	390617891	27.0370	9.0082	19764	3.7392	00136799	2296 502	419686.15
732	535824	392223168	27.0555	9.0123	19805	3 7403	.00136612	2299.644	420835.19
733	537289	393832837	27.0740	9.0164	19845	3.7413	.00136426	2302.785	421985.79
734	538756	395446904	27.0924	9.0205	19886	3.7423	.00136240	2305 927	423137.97
735	540225	397065375	27.1109	9.0246	19927	3.7433	00136054	2309.069	424291.72
736	541696	398688256	27.1293	9.0287	19967	3.7443	00135870	2312.210	425447.04
737	543169	400315553	27.1477	9.0328	20008	3.7454	00135685	2315 352	426603.94
738	544644	401947272	27.1662	9.0369	20049	3.7464	00135501	2318.493	427762.40
739	546121	403583419	27.1846	9.0410	20090	3.7474	.00135318	2321.635	428922.43
740	**547600**	**405224000**	**27.2029**	**9.0450**	**20130**	**3.7484**	**.00135135**	**2324.777**	**430084.02**
741	549081	406869021	27.2213	9.0491	20171	3.7494	.00134953	2327.918	431247.21
742	550564	408518488	27.2397	9 0532	20212	3.7504	.00134771	2331.060	432411.95
743	552049	410172407	27.2580	9.0572	20253	3.7514	.00134590	2334.201	433578.27
744	553536	411830784	27.2764	9.0613	20294	3.7524	.00134409	2337.343	434746.16
745	555025	413493625	27.2947	9.0654	20335	3.7534	00134228	2340.485	435915.62
746	556516	415160936	27.3130	9.0694	20376	3.7545	.00134048	2343.626	437086.64
747	558009	416832723	27.3313	9.0735	20417	3.7555	.00133869	2346.768	438259.24
748	559504	418508992	27.3496	9 0775	20458	3.7565	.00133690	2349.909	439433.41
749	561001	420189749	27.3679	9.0816	20499	3.7575	.00133511	2353.051	440609.16
750	**562500**	**421875000**	**27.3861**	**9.0856**	**20540**	**3.7585**	**.00133333**	**2356.193**	**441786.47**
751	564001	423564751	27.4044	9.0896	20581	3.7595	.00133156	2359.334	442965.35
752	565504	425259008	27.4226	9 0937	20622	3 7605	.00132979	2362.476	444145.80
753	567009	426957777	27.4408	9.0977	20663	3.7615	.00132802	2365.617	445327.83
754	568516	428661064	27.4591	9.1017	20704	3.7625	00132626	2368.759	446511.42
755	570025	430368875	27.4773	9.1057	20745	3.7635	00132450	2371.900	447696.59
756	571536	432081216	27.4955	9.1098	20787	3.7645	.00132275	2375.042	448883.32
757	573049	433798093	27.5136	9 1138	20828	3.7655	.00132100	2378.184	450071.63
758	574564	435519512	27.5318	9.1178	20869	3.7665	.00131926	2381.325	451261.51
759	576081	437245479	27.5500	9.1218	20910	3.7675	.00131752	2384.467	452452.96
760	**577600**	**438976000**	**27.5681**	**9.1258**	**20952**	**3.7685**	**.00131579**	**2387.608**	**453645.98**
761	579121	440711081	27.5862	9.1298	20993	3.7694	00131406	2390.750	454840.57
762	580644	442450728	27.6043	9.1338	21035	3.7704	.00131234	2393.892	456036.73
763	582169	444194947	27.6225	9.1378	21076	3.7714	.00131062	2397.033	457234.46
764	583696	445943744	27.6405	9.1418	21117	3.7724	00130890	2400.175	458433.77
765	585225	447697125	27.6586	9.1458	21159	3.7734	.00130719	2403.316	459634.64
766	586756	449455096	27.6767	9.1498	21200	3.7744	.00130548	2406.458	460837.08
767	588289	451217663	27.6948	9.1537	21242	3.7754	.00130378	2409.600	462041.10
768	589824	452984832	27.7128	9.1577	21283	3.7764	.00130208	2412.741	463246.69
769	591361	454756609	27.7308	9.1617	21325	3.7774	.00130039	2415.883	464453.84
770	**592900**	**456533000**	**27.7489**	**9.1657**	**21367**	**3.7784**	**.00129870**	**2419.024**	**465662.57**
771	594441	458314011	27.7669	9.1696	21408	3.7793	.00129702	2422.166	466872.87
772	595984	460099648	27.7849	9.1736	21450	3.7803	.00129534	2425.307	468084.74
773	597529	461889917	27.8029	9.1775	21492	3.7813	.00129366	2428.449	469298.18
774	599076	463684824	27.8209	9.1815	21533	3.7822	.00129199	2431.591	470513.19
775	600625	465484375	27.8388	9.1855	21575	3.7832	.00129032	2434.732	471729.77
776	602176	467288576	27.8568	9.1894	21617	3.7842	.00128866	2437.874	472947.92
777	603729	469097433	27.8747	9.1933	21658	3.7852	.00128700	2441.015	474167.65
778	605284	470910952	27.8927	9.1973	21700	3.7861	.00128535	2444.157	475388.94
779	606841	472729139	27.9106	9.2012	21742	3.7871	.00128370	2447.299	476611.81

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.)	
								Circum.	Area
780	**608400**	**474552000**	**27.9285**	**9.2052**	**21784**	**3.7881**	**.00128205**	**2450.440**	**477836.24**
781	609961	476379541	27.9464	9.2091	21826	3.7890	.00128041	2453.582	479062.25
782	611524	478211768	27.9643	9.2130	21868	3.7900	.00127877	2456.723	480289.83
783	613089	480048687	27.9821	9.2170	21910	3.7910	.00127714	2459.865	481518.97
784	614656	481890304	28.0000	9.2209	21952	3.7920	.00127551	2463.007	482749.69
785	616225	483736625	28.0179	9.2248	21994	3.7929	.00127389	2466.148	483981.98
786	617796	485587656	28.0357	9.2287	22036	3.7939	.00127226	2469.290	485215.84
787	619369	487443403	28.0535	9.2326	22078	3.7949	.00127065	2472.431	486451.28
788	620944	489303872	28.0713	9.2365	22120	3.7959	.00126904	2475.573	487688.28
789	622521	491169069	28.0891	9.2404	22162	3.7969	.00126743	2478.715	488926.85
790	**624100**	**493039000**	**28.1069**	**9.2443**	**22205**	**3.7978**	**.00126582**	**2481.856**	**490166.99**
791	625681	494913671	28.1247	9.2482	22247	3.7987	.00126422	2484.998	491408.71
792	627264	496793088	28.1425	9.2521	22289	3.7997	.00126263	2488.139	492651.99
793	628849	498677257	28.1603	9.2560	22331	3.8006	.00126103	2491.281	493896.85
794	630436	500566184	28.1780	9.2599	22373	3.8016	.00125945	2494.422	495143.28
795	632025	502459875	28.1957	9.2638	22416	3.8025	00125786	2497.564	496391.27
796	633616	504358336	28.2135	9.2677	22458	3.8035	.00125628	2500.706	497640.84
797	635209	506261573	28.2312	9.2716	22500	3.8044	.00125471	2503.847	498891.98
798	636804	508169592	28.2489	9.2754	22543	3.8054	.00125313	2506.989	500144.69
799	638401	510082399	28.2666	9.2793	22585	3.8064	.00125156	2510.130	501398.97
800	**640000**	**512000000**	**28.2843**	**9.2832**	**22627**	**3.8073**	**.00125000**	**2513.272**	**502654.82**
801	641601	513922401	28.3019	9.2870	22670	3.8083	.00124844	2516.414	503912.25
802	643204	515849608	28.3196	9.2909	22712	3.8092	.00124688	2519.555	505171.24
803	644809	517781627	28.3373	9.2948	22755	3.8102	.00124533	2522.697	506431.80
804	646416	519718464	28.3549	9.2986	22797	3.8111	.00124378	2525.838	507693.94
805	648025	521660125	28.3725	9.3025	22840	3.8121	.00124224	2528.980	508957.64
806	649636	523606616	28.3901	9.3063	22883	3.8130	.00124069	2532.122	510222.92
807	651249	525557943	28.4077	9.3102	22925	3.8139	.00123916	2535.263	511489.77
808	652864	527514112	28.4253	9.3140	22968	3.8149	.00123762	2538.405	512758.19
809	654481	529475129	28.4429	9.3179	23010	3.8158	.00123609	2541.546	514028.18
810	**656100**	**531441000**	**28.4605**	**9.3217**	**23053**	**3.8168**	**.00123457**	**2544.688**	**515299.74**
811	657721	533411731	28.4781	9.3255	23096	3.8177	.00123305	2547.829	516572.87
812	659344	535387328	28.4956	9.3294	23138	3.8186	.00123153	2550.971	517847.57
813	660969	537367797	28.5132	9.3332	23181	3.8196	.00123001	2554.113	519123.84
814	662596	539353144	28.5307	9.3370	23224	3.8205	.00122850	2557.254	520401.68
815	664225	541343375	28.5482	9.3408	23267	3.8215	.00122699	2560.396	521681.10
816	665856	543338496	28.5657	9.3447	23310	3.8224	.00122549	2563.537	522962.08
817	667489	545338513	28.5832	9.3485	23352	3.8234	.00122399	2566.679	524244.63
818	669124	547343432	28.6007	9.3523	23395	3.8243	.00122249	2569.821	525528.76
819	670761	549353259	28.6182	9.3561	23438	3.8252	.00122100	2572.962	526814.46
820	**672400**	**551368000**	**28.6356**	**9.3599**	**23481**	**3.8262**	**.00121951**	**2576.104**	**528101.73**
821	674041	553387661	28.6531	9.3637	23524	3.8271	.00121803	2579.245	529390.56
822	675684	555412248	28.6705	9.3675	23567	3.8280	.00121655	2582.387	530680.97
823	677329	557441767	28.6880	9.3713	23610	3.8290	.00121507	2585.529	531972.95
824	678976	559476224	28.7054	9.3751	23653	3.8299	.00121359	2588.670	533266.50
825	680625	561515625	28.7228	9.3789	23696	3.8308	.00121212	2591 812	534561.62
826	682276	563559976	28.7402	9.3827	23740	3.8317	.00121065	2594.953	535858.32
827	683929	565609283	28.7576	9.3865	23783	3.8327	.00120919	2598.095	537156.58
828	685584	567663552	28.7750	9.3902	23826	3.8336	.00120773	2601.237	538456.41
829	687241	569722789	28.7924	9.3940	23869	3.8345	.00120627	2604.378	539757.82
830	**688900**	**571787000**	**28.8097**	**9.3978**	**23912**	**3.8355**	**.00120482**	**2607.520**	**541060.79**
831	690561	573856191	28.8271	9.4016	23955	3.8364	.00120337	2610.661	542365.34
832	692224	575930368	28.8444	9.4053	23999	3.8373	.00120192	2613.803	543671.46
833	693889	578009537	28.8617	9.4091	24042	3.8382	.00120048	2616.944	544979.15
834	695556	580093704	28.8791	9.4129	24085	3.8391	.00119904	2620.086	546288.40
835	697225	582182875	28.8964	9.4166	24128	3.8401	.00119760	2623.228	547599.23
836	698896	584277056	28.9137	9.4204	24172	3.8410	.00119617	2626.369	548911.63
837	700569	586376253	28.9310	9.4241	24215	3.8419	.00119474	2629.511	550225.61
838	702244	588480472	28.9482	9.4279	24259	3.8428	.00119332	2632.652	551541.15
839	703921	590589719	28.9655	9.4316	24302	3.8437	.00119190	2635.794	552858.26
840	**705600**	**592704000**	**28.9828**	**9.4354**	**24346**	**3.8446**	**.00119048**	**2638.936**	**554176.94**
841	707281	594823321	29.0000	9.4391	24389	3.8456	.00118906	2642.077	555497.20
842	708964	596947688	29.0172	9.4429	24432	3.8465	.00118765	2645.219	556819.02
843	710649	599077107	29.0345	9.4466	24476	3.8474	.00118624	2648.360	558142.42
844	712336	601211584	29.0517	9.4503	24520	3.8483	.00118483	2651.502	559467.39
845	714025	603351125	29.0689	9.4541	24563	3.8492	.00118343	2654.644	560793.92
846	715716	605495736	29.0861	9.4578	24607	3.8501	.00118203	2657.785	562122.03
847	717409	607645423	29.1033	9.4615	24650	3.8510	.00118064	2660.927	563451.71
848	719104	609800192	29.1204	9.4652	24694	3.8519	.00117925	2664.068	564782.96
849	720801	611960049	29.1376	9.4690	24738	3.8528	.00117786	2667.210	566115.78

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
850	**722500**	**614125000**	**29.1548**	**9.4727**	**24782**	**3.8538**	**.00117647**	**2670.352**	**567450.17**
851	724201	616295051	29.1719	9.4764	24825	3.8547	.00117509	2673.493	568786.14
852	725904	618470208	29.1890	9.4801	24869	3.8556	.00117371	2676.635	570123.67
853	727609	620650477	29.2062	9.4838	24913	3.8565	.00117233	2679.776	571462.77
854	729316	622835864	29.2233	9.4875	24957	3.8574	.00117096	2682.918	572803.45
855	731025	625026375	29.2404	9.4912	25000	3.8582	.00116959	2686.059	574145.69
856	732736	627222016	29.2575	9.4949	25044	3.8592	.00116822	2689.201	575489.51
857	734449	629422793	29.2746	9.4986	25088	3.8601	.00116686	2692.343	576834.90
858	736164	631628712	29.2916	9.5023	25132	3.8610	.00116550	2695.484	578181.85
859	737881	633839779	29.3087	9.5060	25176	3.8619	.00116414	2698.626	579530.38
860	**739600**	**636056000**	**29.3258**	**9.5097**	**25220**	**3.8628**	**.00116279**	**2701.767**	**580880.48**
861	741321	638277381	29.3428	9.5134	25264	3.8637	.00116144	2704.909	582232.15
862	743044	640503928	29.3598	9.5171	25308	3.8646	.00116009	2708.051	583585.39
863	744769	642735647	29.3769	9.5207	25352	3.8655	.00115875	2711.192	584940.20
864	746496	644972544	29.3939	9.5244	25396	3.8664	.00115741	2714.334	586296.59
865	748225	647214625	29.4109	9.5281	25440	3.8673	.00115607	2717.475	587654.54
866	749956	649461896	29.4279	9.5317	25485	3.8682	.00115473	2720.617	589014.07
867	751689	651714363	29.4449	9.5354	25529	3.8691	.00115340	2723.759	590375.16
868	753424	653972032	29.4618	9.5391	25573	3.8700	.00115207	2726.900	591737.83
869	755161	656234909	29.4788	9.5427	25617	3.8708	.00115075	2730.042	593102.06
870	**756900**	**658503000**	**29.4958**	**9.5464**	**25661**	**3.8717**	**.00114943**	**2733.185**	**594467.87**
871	758641	660776311	29.5127	9.5501	25706	3.8726	.00114811	2736.325	595835.25
872	760384	663054848	29.5296	9.5537	25750	3.8735	.00114679	2739.466	597204.20
873	762129	665338617	29.5466	9.5574	25794	3.8744	.00114548	2742.608	598574.72
874	763876	667627624	29.5635	9.5610	25839	3.8753	.00114416	2745.750	599946.81
875	765625	669921875	29.5804	9.5647	25883	3.8762	.00114286	2748.891	601320.47
876	767376	672221376	29.5973	9.5683	25927	3.8771	.00114155	2752.033	602695.70
877	769129	674526133	29.6142	9.5719	25972	3.8780	.00114025	2755.174	604072.50
878	770884	676836152	29.6311	9.5756	26016	3.8789	.00113895	2758.316	605450.88
879	772641	679151439	29.6479	9.5792	26061	3.8797	.00113766	2761.458	606830.82
880	**774400**	**681472000**	**29.6648**	**9.5828**	**26105**	**3.8806**	**.00113636**	**2764.599**	**608212.34**
881	776161	683797841	29.6816	9.5865	26150	3.8815	.00113507	2767.741	609595.42
882	777924	686128968	29.6985	9.5901	26194	3.8823	.00113379	2770.882	610980.08
883	779689	688465387	29.7153	9.5937	26239	3.8832	.00113250	2774.024	612366.31
884	781456	690807104	29.7321	9.5973	26283	3.8841	.00113122	2777.166	613754.11
885	783225	693154125	29.7489	9.6010	26328	3.8850	.00112994	2780.307	615143.48
886	784996	695506456	29.7658	9.6046	26373	3.8859	.00112867	2783.449	616534.42
887	786769	697864103	29.7825	9.6082	26417	3.8868	.00112740	2786.590	617926.93
888	788544	700227072	29.7993	9.6118	26462	3.8877	.00112613	2789.732	619321.01
889	790321	702595369	29.8161	9.6154	26507	3.8885	.00112486	2792.874	620716.66
890	**792100**	**704969000**	**29.8329**	**9.6190**	**26551**	**3.8894**	**.00112360**	**2796.015**	**622113.89**
891	793881	707347971	29.8496	9.6226	26596	3.8902	.00112233	2799.157	623512.68
892	795664	709732288	29.8664	9.6262	26641	3.8911	.00112108	2802.298	624913.04
893	797449	712121957	29.8831	9.6298	26686	3.8920	.00111982	2805.440	626314.98
894	799236	714516984	29.8998	9.6334	26730	3.8929	.00111857	2808.581	627718.49
895	801025	716917375	29.9166	9.6370	26775	3.8937	.00111732	2811.723	629123.56
896	802816	719323136	29.9333	9.6406	26820	3.8946	.00111607	2814.865	630530.21
897	804609	721734273	29.9500	9.6442	26865	3.8955	.00111483	2818.006	631938.43
898	806404	724150792	29.9666	9.6477	26910	3.8963	.00111359	2821.148	633348.22
899	808201	726572699	29.9833	9.6513	26955	3.8972	.00111235	2824.289	634759.58
900	**810000**	**729000000**	**30.0000**	**9.6549**	**27000**	**3.8981**	**.00111111**	**2827.431**	**636172.51**
901	811801	731432701	30.0167	9.6585	27045	3.8989	.00110988	2830.573	637587.01
902	813604	733870808	30.0333	9.6620	27090	3.8998	.00110865	2833.714	639003.09
903	815409	736314327	30.0500	9.6656	27135	3.9007	.00110742	2836.856	640420.73
904	817216	738763264	30.0666	9.6692	27180	3.9015	.00110619	2839.997	641839.95
905	819025	741217625	30.0832	9.6727	27225	3.9024	.00110497	2843.139	643260.73
906	820836	743677416	30.0998	9.6763	27270	3.9032	.00110375	2846.281	644683.09
907	822649	746142643	30.1164	9.6799	27316	3.9041	.00110254	2849.422	646107.01
908	824464	748613312	30.1330	9.6834	27361	3.9050	.00110132	2852.564	647532.51
909	826281	751089429	30.1496	9.6870	27406	3.9059	.00110011	2855.705	648959.58
910	**828100**	**753571000**	**30.1662**	**9.6905**	**27451**	**3.9067**	**.00109890**	**2858.847**	**650388.22**
911	829921	756058031	30.1828	9.6941	27497	3.9076	.00109769	2861.988	651818.43
912	831744	758550528	30.1993	9.6976	27542	3.9084	.00109649	2865.130	653250.21
913	833569	761048497	30.2159	9.7012	27587	3.9093	.00109529	2868.272	654683.56
914	835396	763551944	30.2324	9.7047	27632	3.9101	.00109409	2871.413	656118.48
915	837225	766060875	30.2490	9.7082	27678	3.9110	.00109290	2874.555	657554.98
916	839056	768575296	30.2655	9.7118	27723	3.9118	.00109170	2877.696	658993.04
917	840889	771095213	30.2820	9.7153	27769	3.9127	.00109051	2880.838	660432.68
918	842724	773620632	30.2985	9.7188	27814	3.9135	.00108932	2883.980	661873.88
919	844561	776151559	30.3150	9.7224	27859	3.9144	.00108814	2887.121	663316.66

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Circle (N = Diam.) Area
920	846400	778688000	30.3315	9.7259	27905	3.9153	.00108696	2890.263	664761.01
921	848241	781229961	30.3480	9.7294	27950	3.9161	.00108578	2893.404	666206.92
922	850084	783777448	30.3645	9.7329	27996	3.9169	.00108460	2896.546	667654.41
923	851929	786330467	30.3809	9.7364	28042	3.9178	.00108342	2899.688	669103.47
924	853776	788889024	30.3974	9.7400	28087	3.9186	.00108225	2902.829	670554.10
925	855625	791453125	30.4138	9.7435	28133	3.9194	.00108108	2905.971	672006.30
926	857476	794022776	30.4302	9.7470	28179	3.9203	.00107991	2909.112	673460.08
927	859329	796597983	30.4467	9.7505	28224	3.9212	.00107875	2912.254	674915.42
928	861184	799178752	30.4631	9.7540	28270	3.9220	.00107759	2915.396	676372.33
929	863041	801765089	30.4795	9.7575	28315	3.9229	.00107643	2918.537	677830.82
930	864900	804357000	30.4959	9.7610	28361	3.9237	.00107527	2921.679	679290.87
931	866761	806954491	30.5123	9.7645	28407	3.9246	.00107411	2924.820	680752.50
932	868624	809557568	30.5287	9.7680	28453	3.9254	.00107296	2927.962	682215.69
933	870489	812166237	30.5450	9.7715	28499	3.9262	.00107181	2931.103	683680.46
934	872356	814780504	30.5614	9.7750	28544	3.9271	.00107066	2934.245	685146.80
935	874225	817400375	30.5778	9.7785	28590	3.9279	.00106952	2937.387	686614.71
936	876096	820025856	30.5941	9.7819	28636	3.9288	.00106838	2940.528	688084.19
937	877969	822656953	30.6105	9.7854	28682	3.9296	.00106724	2943.670	689555.24
938	879844	825293672	30.6268	9.7889	28728	3.9304	.00106610	2946.811	691027.86
939	881721	827936019	30.6431	9.7924	28774	3.9313	.00106496	2949.953	692502.05
940	883600	830584000	30.6594	9.7959	28820	3.9321	.00106383	2953.095	693977.82
941	885481	833237621	30.6757	9.7993	28866	3.9329	.00106270	2956.236	695455.15
942	887364	835896888	30.6920	9.8028	28912	3.9338	.00106157	2959.378	696934.06
943	889249	838561807	30.7083	9.8063	28958	3.9346	.00106045	2962.519	698414.53
944	891136	841232384	30.7246	9.8097	29004	3.9354	.00105932	2965.661	699896.58
945	893025	843908625	30.7409	9.8132	29050	3.9363	.00105820	2968.803	701380.19
946	894916	846590536	30.7571	9.8167	29096	3.9371	.00105708	2971.944	702865.38
947	896809	849278123	30.7734	9.8201	29142	3.9379	.00105597	2975.086	704352.14
948	898704	851971392	30.7896	9.8236	29189	3.9388	.00105485	2978.227	705840.47
949	900601	854670349	30.8058	9.8270	29235	3.9396	.00105374	2981.369	707330.37
950	902500	857375000	30.8221	9.8305	29281	3.9404	.00105263	2984.511	708821.84
951	904401	860085351	30.8383	9.8339	29327	3.9413	.00105152	2987.652	710314.88
952	906304	862801408	30.8545	9.8374	29374	3.9421	.00105042	2990.794	711809.50
953	908209	865523177	30.8707	9.8408	29420	3.9429	.00104932	2993.935	713305.68
954	910116	868250664	30.8869	9.8443	29466	3.9438	.00104822	2997.077	714803.43
955	912025	870983875	30.9031	9.8477	29513	3.9446	.00104712	3000.218	716302.76
956	913936	873722816	30.9192	9.8511	29559	3.9454	.00104603	3003.360	717803.66
957	915849	876467493	30.9354	9.8546	29605	3.9462	.00104493	3006.502	719306.12
958	917764	879217912	30.9516	9.8580	29652	3.9471	.00104384	3009.643	720810.16
959	919681	881974079	30.9677	9.8614	29698	3.9479	.00104275	3012.785	722315.77
960	921600	884736000	30.9839	9.8648	29745	3.9487	.00104167	3015.926	723822.95
961	923521	887503681	31.0000	9.8683	29791	3.9495	.00104058	3019.068	725331.70
962	925444	890277128	31.0161	9.8717	29838	3.9503	.00103950	3022.210	726842.02
963	927369	893056347	31.0322	9.8751	29884	3.9512	.00103842	3025.351	728353.91
964	929296	895841344	31.0483	9.8785	29931	3.9520	.00103734	3028.493	729867.37
965	931225	898632125	31.0644	9.8819	29977	3.9528	.00103627	3031.634	731382.40
966	933156	901428696	31.0805	9.8854	30024	3.9536	.00103520	3034.776	732899.01
967	935089	904231063	31.0966	9.8888	30070	3.9544	.00103413	3037.918	734417.18
968	937024	907039232	31.1127	9.8922	30117	3.9553	.00103306	3041.059	735936.93
969	938961	909853209	31.1288	9.8956	30164	3.9561	.00103199	3044.201	737458.24
970	940900	912673000	31.1448	9.8990	30210	3.9569	.00103093	3047.342	738981.13
971	942841	915498611	31.1609	9.9024	30257	3.9577	.00102987	3050.484	740505.59
972	944784	918330048	31.1769	9.9058	30304	3.9585	.00102881	3053.625	742031.62
973	946729	921167317	31.1929	9.9092	30351	3.9593	.00102775	3056.767	743559.22
974	948676	924010424	31.2090	9.9126	30398	3.9602	.00102669	3059.909	745088.39
975	950625	926859375	31.2250	9.9160	30444	3.9610	.00102564	3063.050	746619.13
976	952576	929714176	31.2410	9.9194	30491	3.9618	.00102459	3066.192	748151.44
977	954529	932574833	31.2570	9.9227	30538	3.9626	.00102354	3069.333	749685.32
978	956484	935441352	31.2730	9.9261	30585	3.9634	.00102249	3072.475	751220.78
979	958441	938313739	31.2890	9.9295	30632	3.9642	.00102145	3075.617	752757.80
980	960400	941192000	31.3050	9.9329	30679	3.9650	.00102041	3078.758	754296.40
981	962361	944076141	31.3209	9.9363	30726	3.9658	.00101937	3081.900	755836.59
982	964324	946966168	31.3369	9.9396	30773	3.9666	.00101833	3085.041	757378.30
983	966289	949862087	31.3528	9.9430	30820	3.9674	.00101729	3088.183	758921.61
984	968256	952763904	31.3688	9.9464	30867	3.9682	.00101626	3091.325	760466.48
985	970225	955671625	31.3847	9.9497	30914	3.9691	.00101523	3094.466	762012.93
986	972196	958585256	31.4006	9.9531	30961	3.9699	.00101420	3097.608	763560.95
987	974169	961504803	31.4166	9.9565	31008	3.9707	.00101317	3100.749	765110.54
988	976144	964430272	31.4325	9.9598	31055	3.9715	.00101215	3103.891	766661.70
989	978121	967361669	31.4484	9.9632	31102	3.9723	.00101112	3107.033	768214.44

Table 7. Properties of Numbers—*Continued*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
								Circum.	Area
990	**980100**	**970299000**	**31.4643**	**9.9666**	**31150**	**3.9731**	**.00101010**	**3110.174**	**769768.74**
991	982081	973242271	31.4802	9.9699	31197	3.9739	.00100908	3113.316	771324.61
992	984064	976191488	31.4960	9.9733	31244	3.9747	.00100806	3116.457	772882.06
993	986049	979146657	31.5119	9.9766	31291	3.9755	.00100705	3119.599	774441.07
994	988036	982107784	31.5278	9.9800	31339	3.9763	.00100604	3122.740	776001.66
995	990025	985074875	31.5436	9.9833	31386	3.9771	.00100503	3125.882	777563.82
996	992016	988047936	31.5595	9.9866	31433	3.9779	.00100402	3129.024	779127.54
997	994009	991026973	31.5753	9.9900	31480	3.9787	.00100301	3132.165	780692.84
998	996004	994011992	31.5911	9.9933	31528	3.9795	.00100200	3135.307	782259.71
999	998001	997002999	31.6070	9.9967	31575	3.9803	.00100100	3138.448	783828.15
1000	**1000000**	**1000000000**	**31.6228**	**10.0000**	**31623**	**3.9811**	**.00100000**	**3141.593**	**785398.16**

3. MEASURES, WEIGHTS, AND UNITS

Flow and capacity equivalents, Tables 8 and 9, are based on the following values.

1 cubic foot = 7.480519 U. S. gallons
1 U. S. gallon = 231 cu in = 0.13368056 cu ft
1 lb of water at 62° F = 0.01603489 cu ft

The British Imperial gallon = 277.418 cu in (U.S.) = 277.420 cu in (British) = 1.20094 U. S. gallons.

The flow equivalents of Table 8 are based on water at 62° F. These flow equivalents for a given weight of water can be converted to flow equivalents for any other fluid by multiplying by the ratio of the density of that fluid to the density of water. For example the flow per minute of 20 lb of oil of specific gravity 0.90 (water = 1.0) is

$$(1/0.90) \times 20 \times 0.1200 = 2.67 \text{ gal. per min.}$$

Table 8. Flow Equivalents

	Cu. ft. per sec.	Cu. ft. per min.	U. S. Gallons per min.	U. S. Gals. per hr.	U. S. Gallons per 24 hr.	Lb. H_2O per min.	Lb. H_2O per hr.
1 cu. ft. per sec. =	1	60	448 83	26,929 9	646,316.8	3741.00	224,460
1 cu. ft. per min. =	1/60	1	7 4805	448.83	10,771.2	62 35	3741.00
1 gal. per min. =	0 002228	0 1337	1	60	1440	8 3350	500 1
1 gal. per hr. =	3713×10^{-5}	0 002280	1/60	1	24	0.1389	8 3350
1 gal. per 24 hr. =	1547×10^{-6}	9283×10^{-5}	6944×10^{-4}	1/24	1	0 005788	0.3473
1 lb H_2O per min. =	2673×10^{-3}	0 01604	0 1200	7 1986	172 7658	1	60
1 lb. H_2O per hr. =	4455×10^{-5}	2673×10^{-3}	0 0020	0 1200	2 8794	1/60	1

Table 9. Gallon and Cubic Feet Equivalents

Gallons or cu. ft.	Gallons equivalent to cu. ft.	Cu. ft. equivalent to gallons	Gallons or cu. ft.	Gallons equivalent to cu. ft.	Cu ft. equivalent to gallons	Gallons or cu. ft.	Gallons equivalent to cu. ft.	Cu. ft. equivalent to gallons
0.1	0.75	0.0134	100	748.0	13.368	100,000	748,051.9	13,368 1
.2	1.50	.0267	200	1,496.1	26.736	200,000	1,496,103.8	26,736 1
.3	2.24	.0401	300	2,244.2	40.104	300,000	2,244,155.7	40,104.2
.4	2.99	.0535	400	2,992.2	53.472	400,000	2,992,207.6	53,472.2
.5	3.74	.0668	500	3,740.3	66.840	500,000	3,740,259.5	66,840.3
.6	4.49	.0802	600	4,488.3	80.208	600,000	4,488,311.4	80,208.3
.7	5.24	.0936	700	5,236.4	93.576	700,000	5,236,363.3	93,576.4
.8	5.98	.1069	800	5,984.4	106 944	800,000	5,984,415.2	106,944.4
.9	6.73	.1203	900	6,732.5	120.312	900,000	6,732,467.1	120,312.5
1.0	7.48	.134	1,000	7,480 5	133.681	1,000,000	7,480,519	133,680.6
2	14.96	.267	2,000	14,961.0	267.361	2,000,000	14,961,038	267,361.1
3	22.44	.401	3,000	22,441.6	401.042	3,000,000	22,441,557	401,041.7
4	29.92	.535	4,000	29,922.1	534.722	4,000,000	29,922,076	534,722.2
5	37.40	.668	5,000	37,402.6	668.403	5,000,000	37,402,595	668,402.8
6	44.88	.802	6,000	44,883.1	802.083	6,000,000	44,883,114	802,083.3
7	52.36	.936	7,000	52,363.6	935.764	7,000,000	52,363,633	935,763.9
8	59.84	1.069	8,000	59,844.2	1,069.444	8,000,000	59,844,152	1,069,444
9	67.32	1.203	9,000	67,324.7	1,203.125	9,000,000	67,324,671	1,203,125
10	74.80	1.337	10,000	74,805.2	1,336.806	10,000,000	74,805,190	1,336,806
20	149.6	2.674	20,000	149,610 4	2,673.61	20,000,000	149,610,380	2,673,611
30	224.4	4.010	30,000	224,415.6	4,010.42	30,000,000	224,415,570	4,010,417
40	299.2	5.347	40,000	299,220 8	5,347.22	40,000,000	299,220,760	5,347,222
50	374.0	6.684	50,000	374,025.9	6,684.03	50,000,000	374,025,950	6,684,028
60	448.8	8.021	60,000	448,831.1	8,020.83	60,000,000	448,831,140	8,020,833
70	523.6	9.358	70,000	523,636.3	9,357.64	70,000,000	523,636,330	9,357,639
80	598.4	10.694	80,000	598,441.5	10,694.44	80,000,000	598,441,520	10,694,444
90	673.2	12.031	90,000	673,246 7	12,031 25	90,000,000	673,246,710	12,031,250

MEASURES AND WEIGHTS COMMONLY USED IN THE UNITED STATES AND GREAT BRITAIN

Abbreviations

bbl.	= barrel	dwt.	= pennyweight	hr.	= hour	quin.	= quintal
bu.	= bushel	fath.	= fathom	Imp.	= Imperial (bu., gal.)	qr.	= quarter
chn.	= chain	ft.	= foot	in.	= inch	qt.	= quart
circm.	= circumference	fl. drm.	= fluid drachm	lea	= league	scr.	= scruple
cir. in.	= circular inch	fl. oz.	= fluid ounce	m. pace	= military pace	sec.	= second
cir. mil.	= circular mil	furl.	= furlong	min.	= minute	sq.	= square (in., ft., etc.)
cwt	= hundred weight	gal.	= gallon (U.S.)	naut. mi.	= nautical mile	stat. mi.	= statute mile
deg.	= degree	grn.	= grain	pnch	= puncheon	trce.	= tierce
drm.	= drachm (dram)	hghd.	= hogshead	quad.	= quadrant	yd.	= yard

Table 10. Measures of Length

Long Measure

12 in. = 1 ft.
3 ft. = 1 yd.
5 1/2 yd. = 16 1/2 ft. = 1 rod, pole or perch
40 poles = 220 yd. = 1 furl.
8 furl. = 1760 yd. = 5280 ft. = 1 mile
4 in. = 1 hand
9 in. = 1 span
2 1/2 ft. = 1 m. pace.

Land, Surveyors, or Gunter's Measure

7.92 in. = 1 link
100 links = 66 ft. = 4 rods = 1 chn.
10 chn. = 220 yd = 1 furl.
8 furl. = 80 chn. = 1 mile.

Nautical Measure

6080.26 ft. = 1.15156 stat. mi. = 1 naut. mi.
3 naut. mi. = 1 lea.
60 naut. mi. = 1 deg. (at equator)
6 ft. = 1 fath.
120 fath. = 1 cable
1 naut. mi. per hr. = 1 knot.

Table 11. Measures of Area

Square Measure

144 sq. in. = 183 35 cir. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd
30 1/4 sq. yd. = 272 1/4 sq. ft. = 1 sq. rod
160 sq. rods = 10 chn. } = 1 acre
4840 sq. yd. = 43560 sq. ft. } = 1 acre
640 acres = 27,878,400 sq. ft. = 1 sq. mile
1 acre = $(208.71\text{ ft})^2$
1 cir. in. = {area of circle 1 in. diam. = 0 7854 sq in
1 cir. mil = {area of circle 0 001 in. diam. = 0.0_57854 sq. in.
1,000,000 cir. mil = 1 cir. in.

Board Measure

144 cu. in. = 12 × 12 × 1 in. = 1 board ft.

The above definition of a board foot applies to rough green lumber. American practice uses a yard standard of 29/32 in. thick and an industrial standard of 7/8 in. thick for rough dry lumber; for dressed lumber the corresponding thicknesses are 25/32 in. and 13/16 in. For example, a rough dry piece 12 × 12 × 29/32 in. contains 1 board ft. Board ft. in round timber = $(1/4\,c \times d \times l)/144$. c = mean circm., d = diam., l = length, all in ft.

Table 12. Measures of Weight

(The grain is the same in all systems.)

Avoirdupois Weight

16 drm. = 437.5 grn. = 1 oz.
16 oz. = 7000 grn. = 1 lb.
14 lb. = 1 stone
28 lb. = 2 stone = 1 qr.
100 lb. = 1 quin.
4 qr. = 112 lb. = 1 cwt.
20 cwt. = 2240 lb. = 1 long ton
2000 lb. = 1 short ton.

Troy Weight (for gold and silver)

24 grn. = 1 dwt.
20 dwt. = 480 grn. = 1 oz.
12 oz = 5760 grn. = 1 lb.
1 Assay ton = 29,167 milligrams = {Troy oz. per 2000 lb. ton avoirdupois

Carat Weight (for precious stones)

1 carat = 3.086 grn. = 0.200 gram

Apothecaries' Weight

20 grn. = 1 scr. ℈
3 scr. = 60 grn. = 1 drm. ʒ
8 drm. = 480 grn. = 1 oz. ℥
12 oz. = 5760 grn. = 1 lb.

Table 13. Measures of Volume

Cubic Measure

1728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.
1 cord of wood = {4 × 4 × 8 ft. = 128 cu. ft.
1 perch masonry = {16 1/2 × 1 1/2 × 1 ft. = 23 3/4 cu. ft.

Liquid Measure

4 gills = 1 pint
2 pints = 1 qt.
4 qt. = 231 cu. in. = 8 3356 lb. H_2O = 1 U S gal. = 0 8327 Imp. gal.
1 Imp. gal. = 277.420 cu. in. = 10 lb. H_2O
1.20094 U. S. gal. = 1 Imp. gal.
7.4805 U. S. gal. = 1 cu. ft.

Old Liquid Measure

31 1/2 gal. = 1 bbl.
42 gal. = 1 trce.
2 bbl. = 63 gal. = 1 hghd.
84 gal. = 2 trce = 1 pchn.
2 hghd. = 126 gal. = 1 pipe or butt
2 pipes = 3 pchn. = 1 tun.

Apothecaries' Fluid Measure

60 minims = 1 fl. drm.
8 drm. = 1 fl. oz.
1 fl. oz. = 1/128 U. S. gal. = 1.8047 cu. in. = 456.3 grn. H_2O at 39° F.
16 fl. oz. = 1 pint
1 fl. oz. (British) = 1.732 cu. in. = 437.5 grn. H_2O at 62° F.

Dry Measure

2 pints = 1 qt.
8 qts. = 1 peck
4 pecks = 2150.42 cu. in. = 1 bu. (struck)
1 1/4 bu. (struck) = 1 bu. (heaped)
105 qts. (struck) = 7056 cu. in. = 1 bbl.
8 Imp. gal. = 2218.192 cu. in. = 1 Imp. bu.
8 Imp. bu. = 1 qr.

Shipping Measure

100 cu. ft. = 1 Register ton *
40 cu. ft. = 31.143 U. S. bu. = 1 shipping ton (U. S.) †
42 cu. ft. = 32.719 Imp. bu. = 1 shipping ton (British) †

* For measurement of entire internal capacity † For measurement of cargo.

Miner's Inch

A variable unit, defined by statute in different states as the quantity of water that will flow through an orifice of 1 sq. in. area, under a head ranging from 4 1/2 to 6 in. U. S. Reclamation Service defines the miner's inch as 1 cu. in. per sec. The law governing the locality in which the measurement is to be made should be consulted.

Table 14. Circular Measure

60 sec.	= 1 min.
60 min.	= 1 deg.
90 deg.	= 1 quad.
360 deg.	= circm.
57.2957795 deg.	= 1 radian
	= 57° 17′ 44.806″
1 radian	= angle of arc equal to radius.

Table 15. Time

60 sec.	= 1 min.	7 days	= 1 week
60 min.	= 1 hr.	52 weeks	= 1 year
24 hr.	= 1 day		

1 year (exact) = 365 days, 5 hr., 48 min., 45.6 sec.

A year divisible by 4 is a leap year and contains 366 days, except centesimal years, of which only those divisible by 400 are leap years.

365.24734 mean solar days
= 366.008515 sidereal days
24 hr. mean solar time
= 23 hr. 56 min., 49.9 sec. sidereal time
1 mean solar day
= 1 mean sidereal day + 3 min., 10.1 sec.

THE METRIC SYSTEM

The fundamental standard of length in the metric system is the International Standard Meter, derived from the Metre des Archives, Paris. Its length is defined as the distance between two lines at 0° C. on a platinum-iridium bar deposited at the International Bureau of Weights and Measures near Paris. 1 meter = 39.37 in.

The fundamental standard of mass is the International Standard Kilogram, which is a mass of platinum-iridium, deposited at the International Bureau, whose weight *in vacuo* is the same as that of the Kilogram des Archives.

The fundamental unit of volume is the liter, which is defined as the volume of a kilogram of water at the temperature of maximum density, 4° C., under a pressure of 76 cm. of mercury. The liter originally was defined as a cubic decimeter, but is slightly greater. The generally accepted relation between the cubic decimeter and the liter is 1 liter = 1.000027 cu. decimeters

The unit of force is the **dyne**, which is the force which acting for 1 second on a mass of one gram will produce a velocity of 1 centimeter per second. 1 dyne = (1/980 665) gram. 1,000,000 dynes = 1.020 kilograms force = 2.248 lb. force. 1 lb. force = 0.4536 kilogram force = 448,800 dynes.

The unit of heat is the **gram-calorie,** which is the quantity of heat required to raise 1 gram of water 1° C. See Section 3.

Table 16. Units of Metric Measures *

Length

Unit	Symbol	Value in Meters
Micron	μ	0.000 001
Millimeter	mm.	0.001
Centimeter	cm.	0.01
Decimeter	dm.	0.1
Meter (unit)	m.	1.0
Dekameter	dkm.	10.0
Hectometer	hm.	100.0
Kilometer	km.	1,000.0
Myriameter	Mm.	10,000.0
Megameter		1,000,000.0

Area

Unit	Symbol	Value in Sq. Meters
Sq. millimeter	sq. mm.	0.000 001
Sq. centimeter	sq. cm.	0.000 1
Sq. decimeter	sq. dm.	0.01
Sq. meter (centiare)	sq. m.	1.0
Sq. dekameter (are)	sq. dkm.	100.0
Hectare	ha.	10,000.0
Sq. kilometer	sq. km.	1,000,000.0

Cubic Measure

Unit	Symbol	Value in Cubic Meters
Cubic kilometer	cu. km.	1,000,000,000
Cubic hectometer	cu. hm.	1,000,000
Cubic dekameter	cu. dkm.	1,000
Cubic meter	cu. m.	1
Cubic decimeter	cu. dm.	0.001
Cubic centimeter	cu. cm.	0.000 001
Cubic millimeter	cu. mm.	0.000 000 001
Cubic micron	μ^3	$0.0_{17}1$

Volume

Unit	Symbol	Value in Liters
Milliliter	ml.	0.001
Centiliter	cl.	0.01
Deciliter	dl.	0.1
Liter (unit)	l.	1.0
Dekaliter	dkl.	10.0
Hectoliter	hl.	100.0
Kiloliter	kl.	1,000.0

Weight

Unit	Symbol	Value in Grams	Unit	Symbol	Value in Grams
Microgram	γ	0.000 001	Gram (unit)	g.	1.0
Milligram	mg.	0.001	Dekagram	dkg.	10.0
Centigram	cg.	0.01	Hectogram	hg.	100.0
Decigram	dg.	0.1	Kilogram	kg.	1,000.0
Gram (unit)	g.	1.0	Myriagram	Mg.	10,000.0
			Quintal	q.	100,000.0
			Ton	t.	1,000,000.0

* A subscript after a figure indicates the number of times it is repeated. Thus $0.0_{3}8$ = 0.0008

Table 17. Metric Equivalents of Length *

English to Metric			Log.	Metric to English			Log.
1 inch =	25.4	mm.	1.404834	1 mm. =	0.03937	in.	$\bar{2}$.595165
=	2.54	cm.	0.404834	=	0.003281	ft.	$\bar{3}$.515984
=	0.0254	m.	$\bar{2}$.404834	=	0.001094	yd.	$\bar{3}$.038863
1 foot =	304.800	mm.	2.484015	1 cm. =	0.3937	in.	$\bar{1}$.595165
=	30.480	cm.	1 484015	=	0.03281	ft.	$\bar{2}$.515984
=	0.3048	m.	$\bar{1}$.484015	=	0.01094	yd.	$\bar{2}$.038863
1 yard =	91.4402	cm.	1.961137	1 meter =	39 37	in.	1.595162
=	0.9144	m.	$\bar{1}$.961137	=	3.2808	ft.	0.515984
=	0.0_3914	km.	$\bar{4}$.961137	=	1.0936	yd.	0.038863
1 rod =	502.9211	cm.	2.701500	=	0.1988	rd.	$\bar{1}$.298500
=	5.0292	m.	0.701500	=	0.04971	chain	$\bar{2}$.696440
=	0.005029	km.	$\bar{3}$.701500	=	0.0_36214	mi.	$\bar{4}$.793350
1 chain =	2011.68	cm.	3.303559	1 kilometer =	3280.833	ft.	3.515984
=	20.1168	m.	1.303559	=	1093 611	yd.	3.038863
=	0.02012	km.	$\bar{2}$.303559	=	198 838	rods	2.298500
1 mile =	1609.344	m.	3 206650	=	49 7095	chains	1.696440
=	1 6093	km	0 206650	=	0 6214	mi	$\bar{1}$ 793350

Table 18. Metric Equivalents of Area *

English to Metric			Log.	Metric to English			Log.
1 cir. mil =	0 0_35067	sq. mm.	4 704751	1 sq. mm. =	1,973 55	cir. mils	3.295249
1 sq. in. =	645 163	sq. mm.	2.809669	=	0 001550	sq. in.	$\bar{3}$.190331
=	6.4516	sq. cm.	0.809669	=	0 0_410764	sq. ft.	$\bar{5}$.031968
=	0.0_36452	sq. m.	4.809669	=	0 0_51196	sq. yd.	$\bar{6}$.077726
1 sq. ft. =	92,903.41	sq mm.	4.968032	1 sq. cm. =	0 1550	sq. in.	$\bar{1}$.190331
=	929 0341	sq. cm.	2.968032	=	0 001076	sq. ft.	$\bar{3}$.031968
=	0 0929	sq. m.	$\bar{2}$.968032	=	0 0_31196	sq. yd.	$\bar{4}$.077726
=	0 0_5929	hectare	$\bar{6}$.968032	1 sq. m. =	1,549 9969	sq in.	3.190331
=	0 0_7929	sq. km.	8 968032	=	10 7639	sq. ft.	1.031968
1 sq. yd. =	836,130 74	sq. mm.	5.922274	=	1 1960	sq. yd.	0.077726
=	8,361 307	sq. cm.	3.922274	=	0 002471	sq. chain	$\bar{3}$.392882
=	0 83613	sq. m.	1 922274	=	0 0_32471	acre	$\bar{4}$.392882
=	0 0_4836	hectare	$\bar{5}$.922274	=	0 0_63861	sq. mi.	$\bar{7}$.586700
=	0 0_6836	sq. km.	$\bar{7}$.922274	1 hectare =	107,638 7	sq. ft.	5.031968
1 sq. chain =	404 686	sq. m.	2 607118	=	11,959 85	sq. yd.	4.077726
=	0 04047	hectare	$\bar{2}$ 607118	=	24 710	sq. chain	1.392882
=	0 0_34047	sq. km.	4 607118	=	2 4710	acres	0.392882
1 acre =	4,046 86	sq. m.	3.607118	=	0.003861	sq. mi.	3.586700
=	0 4047	hectare	1 607118	1 sq. km. =	10,763,867 38	sq. ft.	7.031968
=	0 004047	sq. km	3.607118	=	1,195,985.26	sq. yd.	6.077726
1 sq. mile =	2,589,998	sq. m	6 413300	=	2,471 050	sq. chains	3.392882
=	259 0	hectare	2 413300	=	247 1045	acres	2.392882
=	2 590	sq km.	0.413300	=	0 3861	sq. mi.	$\bar{1}$.586700

Table 19. Metric Equivalents of Weight or Mass *

English to Metric			Log.	Metric to English			Log.
1 grain =	64 797	mg.	1.811568	1 milligram =	0 01543	grain	$\bar{2}$.188433
=	0 0648	gm.	$\bar{2}$.811568	=	0 0_43215	oz. Troy	$\bar{5}$.507191
1 oz. Troy or				=	0 0_43527	oz. avoir.	$\bar{5}$.547454
apothecary =	31,103 5	mg.	4.492809	1 gram =	15 4324	grains	1.188433
=	31 103	gm.	1.492809	=	0 03215	oz. Troy	$\bar{2}$.507191
=	0 03110	kg.	$\bar{2}$.492809	=	0 03527	oz. avoir.	$\bar{2}$.547454
1 oz. avoirdupois =	28,349 5	mg.	4.452546	=	0 0_22679	lb. Troy	$\bar{3}$.428010
=	28 3495	gm.	1.452546	=	0 0_22205	lb. avoir.	$\bar{3}$.343334
=	0.02835	kg.	$\bar{2}$.452546	1 kilogram =	32 1508	oz. Troy	1.507191
1 pound Troy or				=	35 2740	oz. avoir.	1.547454
apothecary =	373.2417	gm.	2.571990	=	2 6792	lb. Troy	0.428010
=	0 3732	kg.	$\bar{1}$.571990	=	2 2046	lb. avoir.	0.343334
1 pound avoirdupois =	453.5925	gm.	2.656666	=	0 0_21102	short ton	$\bar{3}$.043304
=	0 4536	kg.	$\bar{1}$.656666	=	0 0_39842	long ton	$\bar{4}$.993086
=	0 0_34536	metric ton	$\bar{4}$.656666	1 metric ton =	2204 62	lb. avoir.	3.343334
1 short ton =	907 2	kg.	2.957696	=	1.1023	short tons	0.043304
=	0 9072	metric ton	$\bar{1}$.957696	=	0 9842	long ton	$\bar{1}$.993086
1 long ton =	1,016 06	kg.	3.006914				
=	1 0161	metric ton	0.006914				

* A subscript after a figure indicates the number of times it is repeated. Thus 0.0_38 = 0.0008

Table 20. Metric Equivalents of Capacity or Volume *

English to Metric	Log.	Metric to English	Log.
1 cu. in. = 16,387 17 cu. mm.	4.214504	1 cu. mm. = 0.0_46102 cu. in.	$\bar{5}$.785496
= 16 3872 cu. cm.	1.214504	= $0\ 0_32705$ fluid dr.	$\bar{4}$.432185
= 0.016387 l.	$\bar{2}$.214504	= $0\ 0_43381$ fluid oz.	$\bar{5}$.529094
= $0\ 0_316387$ hl.	4.214504	1 cu. cm. = 0 06102 cu. in.	$\bar{2}$.785496
= 0.0_416387 cu. m.	$\bar{5}$.214504	= $0\ 0_43531$ cu. ft.	$\bar{5}$.547951
1 cu. ft. = 28,317 08 cu. cm.	4.452049	= $0\ 0_51308$ cu. yd.	$\bar{6}$.116589
= 28 3169 l.	1.452049	= 0.0_42838 bushel	$\bar{5}$.452972
= 0 2832 hl.	$\bar{1}$.452049	= 0.2705 fluid dr.	$\bar{1}$.432185
= 0 02832 cu.m.	$\bar{2}$.452049	= 0.03381 fluid oz.	$\bar{2}$.529094
1 cu. yd. = 764,559 5 cu. cm.	5.883411	= 0 001057 quart	$\bar{3}$.023944
= 764 5595 l.	2.883411	= $0\ 0_32642$ gallon	$\bar{4}$.421884
= 7 6456 hl.	0.883411	1 liter = 61 02398 cu. in.	1.785496
= 0 7646 cu. m.	$\bar{1}$.883411	= 0.035313 cu. ft.	$\bar{2}$.547951
1 bushel = 35,239 3 cu. cm.	4.547027	= 0 0013079 cu. yd.	$\bar{3}$.116589
= 35 2393 l.	1.547027	= 0 028377 bushel	$\bar{2}$.452972
= 0 3524 hl.	1.547027	= 1 0567 quart	0.023944
= 0 03524 cu. m.	$\bar{2}$.547027	= 0 2642 gallon	$\bar{1}$.421884
1 fluid drachm = 3,696 7 cu. mm.	3 567815	1 hectoliter = 6,102.398 cu. in.	3.785496
= 3 6967 cu. cm.	0.567815	= 3 5313 cu. ft.	0.547951
1 fluid oz. = 29,573 7 cu. mm.	4.470906	= 0 13079 cu. yd.	$\bar{1}$.116589
= 29 5737 cu. cm.	1.470906	= 2 8377 bushels	0.452972
1 quart = 946 359 cu. cm.	2.976056	1 cu. meter = 61,023 38 cu. in.	4.785496
= 0 9464 l.	$\bar{1}$ 976056	= 35 3133 cu. ft.	1.547951
= $0\ 0_3946$ cu. m.	4.976056	= 1 3079 cu. yd.	0.116589
1 U. S. gallon = 3,785 43 cu. cm.	3.578116	= 28 3773 bushels	1.452972
= 3 7854 l.	0.578116	= 1,056 682 quarts	3.023944
= 0 003785 cu. m.	3.578116	= 264 170 gallons	2.421884

Table 21. Metric Equivalents of Density *

English to Metric	Log.	Metric to English	Log.
1 lb. per cu. in. = 27.680 gm. per cu. cm.	1.442162	1 gm. per cu. cm. = 0.03613 lb. per cu. in.	2.557838
= 27,679.7 kg. per cu. m.	4.442162	= 62.430 lb. per cu. ft.	1.795381
1 lb. per cu. ft. = 0.01602 gm. per cu. cm.	$\bar{2}$.204619	= 8.3454 lb. per U. S. gal.	0 921450
= 16 0183 kg. per cu. m.	1.204619	1 kg. per cu. m. = 0.0_43613 lb. per cu. in.	$\bar{5}$.557838
= 0 01602 metric ton per cu. m.	$\bar{2}$.204619	= 0.062430 lb. per cu. ft.	$\bar{2}$.795381
1 lb. per cu. yd. = 0 5933 kg. per cu. m.	$\bar{1}$.773254	= 1.6856 lb. per cu. yd.	0.226746
= 0.0_35933 metric tons per cu. m.	$\bar{4}$ 773254	= 0.0_28345 lb. per U. S. gal.	$\bar{3}$.921450
1 lb. per U. S. gal. = 0.1198 gm. per cu. cm.	$\bar{1}$.078550	1 metric ton per cu. m = 62.4286 lb. per cu. ft.	1.795381
= 119.826 kg. per cu. m.	2.078550	= 1685.487 lb. per cu. yd.	3 226746
1 short ton per cu. yd. = 1 1865 metric tons per cu. m.	0.074285	= 0.8428 short ton per cu. yd.	$\bar{1}$.925715
1 long ton per cu. yd. = 1.3289 metric tons per cu. m.	0.123502	= 0.7525 long ton per cu. yd.	$\bar{1}$.876498

Table 22. Metric Equivalents of Velocity *

English to Metric	Log.	Metric to English	Log.
1 in. per sec., = 2.54001 cm. per sec.	0.404834	1 cm. per sec. = 0.3937 in. per sec.	$\bar{1}$ 595165
= 0.0254 m. per sec.	$\bar{2}$.404834	= 0.03281 ft. per sec.	$\bar{2}$.515984
= 1.5240 m. per min.	0.182985	= 1.9685 ft. per min.	0.294136
1 ft. per sec. = 30.4801 cm. per sec.	1.484015	= 0 02237 mi. per hr.	$\bar{2}$.349653
= 0.3048 m. per sec.	$\bar{1}$.484015	= 0.01943 knot	$\bar{2}$.288367
= 18.2880 m. per min.	1.262166	1 m. per sec. = 39.37 in. per sec.	1.595165
= 1.0973 km. per hr.	0.040318	= 3.2808 ft. per sec.	0.515984
1 ft. per min. = 0.5080 cm. per sec.	$\bar{1}$.705864	= 196.8500 ft. per min.	2.294136
= 0.00508 m. per sec.	$\bar{3}$.705864	= 2.2369 mi. per hr.	0.349653
= 0.3048 m. per min.	$\bar{1}$.484015	= 1.9426 knots	0.288367
= 0.018288 km. per hr.	$\bar{2}$.262166	1 m. per min. = 0.6562 in. per sec.	$\bar{1}$.817014
1 mile per hr. = 44.704 cm. per sec.	1.650348	= 0.05468 ft. per sec.	$\bar{2}$.737833
= 0.4470 m. per sec.	$\bar{1}$.650348	= 3.2808 ft. per min.	0.515984
= 26.8222 m. per min.	1.428495	= 0.03728 mi. per hr.	$\bar{2}$.571503
= 1.6093 km. per hr.	0.206650	= 0.03238 knot	$\bar{2}$.510218
1 knot = 51.4791 cm. per sec.	1.711631	1 km. per hr. = 0.9113 ft. per sec.	$\bar{1}$.959681
= 0.5148 m. per sec.	$\bar{1}$.711631	= 54.6806 ft. per min.	1.737833
= 30.8875 m. per min.	1.489782	= 0.62138 mi. per hr.	$\bar{1}$.793357
= 1.8532 km. per hr.	0.267933	= 0.5396 knot	$\bar{1}$.732072

* A subscript after a figure indicates the number of times it is repeated. Thus $0.0_38 = 0.0008$

Table 23. Metric Equivalents of Pressure *

English to Metric	Log.	Metric to English	Log.
1 lb. per sq. in. = 0.7031 gm. per sq. mm.	$\bar{1}.846996$	1 gm. per sq. mm. = 1.4223 lb. per sq. in.	0.153004
= 70.3066 gm. per sq. cm.	2.846996	= 204.8170 lb. per sq. ft.	2.311366
= 0.07031 kg. per sq. cm.	$\bar{2}.846996$	1 gm. per sq. cm. = 0.01422 lb. per sq. in.	$\bar{2}.153004$
= 0.7031 metric ton per sq. m.	$\bar{1}.846996$	= 2.0482 lb. per sq. ft.	0.311366
= 0.07031 metric atmosphere †	$\bar{2}.846996$	1 kg. per sq. cm. = 14.2234 lb. per sq. in.	1.153004
1 lb. per sq. ft. = 0.004882 gm. per sq. mm.	$\bar{3}.688598$	= 2048.1696 lb. per sq. ft.	3.311366
= 0.4882 gm. per sq. cm.	$\bar{1}.688598$	1 metric ton per sq. m. = 1.4223 lb. per sq. in.	0.153004
= 0.0_34882 kg. per sq. cm.	$\bar{4}.688598$	= 204.8170 lb. per sq. ft.	2.311366
= 0.004882 metric ton per sq. m.	$\bar{3}.688598$	= 0.1024 ton per sq. ft. §	$\bar{1}.010342$
= 0.0_34882 metric atmosphere †	$\bar{4}.688598$	1 metric atmosphere † = 14.2234 lb. per sq. in.	1.153004
1 ton per sq. ft. † = 9.7648 metric tons per sq. m.	0.989663	= 2048.1696 lb. per sq. ft.	3.311366
= 0.9765 metric atmosphere †	$\bar{1}.989663$	= 1.0241 tons per sq. ft.	0.010342
1 atmosphere ‡ = 1.0335 metric atmospheres †	0.014310	= 0.9676 atmosphere ‡	$\bar{1}.985696$

† 1 metric atmosphere = 1 kg. per sq. cm. ‡ 1 atmosphere = 14.7 lb. per sq. in. § 1 ton = 2000 lb.

Table 24. Metric Equivalents of Force *

English to Metric		Log.	Metric to English		Log.
1 lb.	= 444,800 dynes	5.648165	1 dyne	= 0.0_52248 lb.	$\bar{6}.351796$
	= 453.6 grams	2.656673		= 0.0_47233 poundal	$\bar{5}.859318$
	= 0.04448 joule per cm.	$\bar{2}.648165$		= 0.0_21020 gram	$\bar{3}.008600$
	= 0.4536 kg.	$\bar{1}.656673$		= 0.0_51020 kg.	$\bar{6}.008600$
	= 32.17 poundals	1.507451		= 0 0_61 joule per cm.	$\bar{7}.000000$
1 poundal	= 13,826 dynes	4.140696	1 gram	= 0.0_22205 lb.	$\bar{3}.343409$
	= 14.10 grams	1.149219		= 0.07093 poundal	$\bar{2}.850830$
	= 0 0_213826 joule per cm.	$\bar{3}.140696$		= 980.7 dynes	2.991536
	= 0 01410 kg.	$\bar{2}.149219$		= 0.001 kg.	$\bar{3}.000000$
	= 0.03108 lb.	$\bar{2}.492481$		= 0.0_49807 joule per cm.	$\bar{5}.991536$
			1 joule per cm.	= 22.48 lb.	1.351796
				= 723.3 poundals	2.859318
				= 10,000,000 dynes	7.000000
				= 10,200 grams	4.008600
				= 10.20 kg.	1.008600
			1 kg.	= 2.205 lb.	0.343409
				= 70.93 poundals	1.850830
				= 980,700 dynes	5.991536
				= 1000 grams	3.000000
				= 0.09807 joule per cm.	2.991536

Table 25. Metric Equivalents of Power, Work, and Energy *

English to Metric	Log	Metric to English	Log
1 ft-lb = 0.138255 kg-m	$\bar{1}.140680$	1 kg-m = 7.2330 ft-lb	0.859318
= 0.0_637650 kwhr	$\bar{7}.575765$	= $0.0_5365304$ hp-hr	$\bar{6}.562655$
= 0.32379 IT cal	$\bar{1}.510263$	= 0.0_292938 Btu	$\bar{3}.968193$
1 hp = 76.042 kg-m per sec	1.881053	1 kwhr = 2.656000 ft-lb	6.424228
= 0.76042 poncelet	$\bar{1}.881053$	= 1.3414 hp-hr	0.127558
= 1.01389 cheval vapeur	0.005994	= 3412.75 Btu	3.533103
1 hp-hr = 273,750 kg-m	5.437355	1 cheval vapeur = 0.9863 hp	$\bar{1}.994009$
= 0.74548 kwhr	$\bar{1}.872435$	1 poncelet = 1.3151 hp	0.118959
= 641,100 IT cal	5.806926	1 IT cal † = 3.0884 ft-lb	0.489738
1 Poncelet = 100 kg-m per sec	2.000000	= $0.0_5155980$ hp-hr	$\bar{6}.193070$
1 cheval-vapeur = 75 kg-m per sec	1.875061	= 0.0_239683 Btu	$\bar{3}.598605$
1 Btu = 251.996 IT cal	2.401400	= 1.00037 mean calories	0.000160
= 107.599 kg-m	2.031808		
= 0.0_329302	$\bar{4}.466898$		

* A subscript after a figure indicates the number of times it is repeated. Thus 0.0_38 = 0.0008.

† The IT cal is defined as 1/860 of the international watthour, and is nearly equal to the mean calorie; 1 IT cal = 1.00037 mean calories, or 1 Btu = 251.996 IT cal. (See *Mechanical Engineering*, Nov. 1935, p. 710.)

GEOMETRY

4. CIRCULAR ARCS, CHORDS, SEGMENTS, AND SECTORS

Notation.—A = central angle, degrees; C = chord of arc; c = chord of 1/2 arc; D = diameter; $G = \dfrac{\text{area of segment}}{R^2}$; H = rise of arc or height of segment; L = length of arc; R = radius; S = area of sector.

L (exact) $= 2\pi RA/360 = 0.017453RA$

L (approx.) $= (8c - C)/3 = \{(2c \times 10H)/(60D - 27A)\} + 2c$

$= \{(\sqrt{C^2 + 4H^2} \times 10H^2)/(15C^2 + 33H^2)\} + 2c$

$C = 2\sqrt{c^2 - H^2} = \sqrt{D^2 - (D - 2H)^2} = (8c - 3L) = 2\sqrt{R^2 - (R - H)^2} = 2\sqrt{(D - H) \times H}$

$C/2 = \sqrt{H \times (D - H)}$; $c = 1/2\sqrt{C^2 - 4H^2} = \sqrt{D \times H} = (3L + C)/8$

$D = c^2/H = (1/4\,C^2 + H^2)/H$

$H = c^2/D = 1/2\,(D - \sqrt{D^2 - C^2})$ when $H < R$; $= 1/2\,(D + \sqrt{D^2 - C^2}) = \sqrt{c^2 - 1/4\,C^2}$ when $H > R$

$G = S - 1/2\,C\sqrt{R^2 - 1/4\,C^2}$ when $G <$ semi-circle; $= S + 1/2\,C\sqrt{R^2 - 1/4\,C^2}$ when $S >$ semi-circle

$= S - (C/2)\,(R - H) = 1/2\,\{(L - C)\,(D/2) + CH\}$

$S = 1/2\,LR = \pi AR^2/360 = (\pi R^2/360) \times 2[\sin^{-1}\{(C/2)/R\}]$; $R = (C^2 + 4H^2)/8H$

Table 26. Circular Arcs, Chords, and Segments. Radius = 1

Central Angle in Degrees	Arc Length L	Arc Rise H	Chord, Length C	$\frac{H}{C}$	Segment Area $\frac{}{R^2} = G$*	Central Angle in Degrees	Arc Length L	Arc Rise H	Chord, Length C	$\frac{H}{C}$	Segment Area $\frac{}{R^2} = G$*
1	0.0175	0.0000	0.0175	0.0022	0.00000	31	0.5411	0.0364	0.5345	0.0680	0.01301
2	.0349	.0002	.0349	.0044	.00000	32	.5585	.0387	.5513	.0703	.01429
3	.0524	.0003	.0524	.0066	.00001	33	.5760	.0412	.5680	.0725	.01566
4	.0698	.0006	.0698	.0087	.00003	34	.5934	.0437	.5847	.0747	.01711
5	.0873	.0010	.0872	.0109	.00006	35	.6109	.0463	.6014	.0770	.01864
6	.1047	.0014	.1047	.0131	.00010	36	.6283	.0489	.6180	.0792	.02027
7	.1222	.0019	.1221	.0153	.00015	37	.6458	.0517	.6346	.0814	.02198
8	.1396	.0024	.1395	.0175	.00023	38	.6632	.0545	.6511	.0837	.02378
9	.1571	.0031	.1569	.0196	.00032	39	.6807	.0574	.6676	.0859	.02568
10	**.1745**	**.0038**	**.1743**	**.0218**	**.00044**	**40**	**.6981**	**.0603**	**.6840**	**.0882**	**.02767**
11	.1920	.0046	.1917	.0240	.00059	41	.7156	.0633	.7004	.0904	.02976
12	.2094	.0055	.2091	.0262	.00076	42	.7330	.0664	.7167	.0927	.03195
13	.2269	.0064	.2264	.0284	.00097	43	.7505	.0696	.7330	.0949	.03425
14	.2443	.0075	.2437	.0306	.00121	44	.7679	.0728	.7492	.0972	.03664
15	.2618	.0086	.2611	.0328	.00149	45	.7854	.0761	.7654	.0995	.03915
16	.2793	.0097	.2783	.0350	.00181	46	.8029	.0795	.7815	.1017	.04176
17	.2967	.0110	.2956	.0372	.00217	47	.8203	.0829	.7975	.1040	.04448
18	.3142	.0123	.3129	.0394	.00257	48	.8378	.0865	.8135	.1063	.04731
19	.3316	.0137	.3301	.0415	.00302	49	.8552	.0900	.8294	.1086	.05025
20	**.3491**	**.0152**	**.3473**	**.0437**	**.00352**	**50**	**.8727**	**.0937**	**.8452**	**.1108**	**.05331**
21	.3665	.0167	.3645	.0459	.00408	51	.8901	.0974	.8610	.1131	.05649
22	.3840	.0184	.3816	.0481	.00468	52	.9076	.1012	.8767	.1154	.05978
23	.4014	.0201	.3987	.0503	.00535	53	.9250	.1051	.8924	.1177	.06319
24	.4189	.0219	.4158	.0526	.00607	54	.9425	.1090	.9080	.1200	.06673
25	.4363	.0237	.4329	.0548	.00686	55	.9599	.1130	.9235	.1223	.07039
26	.4538	.0256	.4499	.0570	.00771	56	.9774	.1171	.9389	.1247	.07417
27	.4712	.0276	.4669	.0592	.00862	57	.9848	.1212	.9543	.1270	.07808
28	.4887	.0297	.4838	.0614	.00961	58	1.0123	.1254	.9696	.1293	.08212
29	.5061	.0319	.5008	.0636	.01067	59	1.0297	.1296	.9848	.1316	.08629
30	**.5236**	**.0341**	**.5176**	**.0658**	**.01180**	**60**	**1.0472**	**.1340**	**1.0000**	**.1340**	**.09059**

*Area of segment of any radius = factor G × square of radius

Table 26. Circular Arcs, Chords, and Segments—*Continued*

Central Angle in Degrees	Arc Length L	Arc Rise H	Chord, Length C	$\frac{H}{C}$	Segment Area $\frac{}{R^2} = G$*	Central Angle in Degrees	Arc Length L	Arc Rise H	Chord, Length C	$\frac{H}{C}$	Segment Area $\frac{}{R^2} = G$*
61	1.0647	0.1384	1.015	0.1363	0.09502	121	2.1118	0.5076	1.741	0.2916	0.62734
62	1.0821	.1428	1.030	.1387	.09958	122	2.1293	.5152	1.749	.2945	.64063
63	1.0996	.1474	1.045	.1410	.10428	123	2.1468	.5228	1.758	.2975	.65404
64	1.1170	.1520	1.060	.1434	.10911	124	2.1642	.5305	1.766	.3004	.66759
65	1.1345	.1566	1.075	.1457	.11408	125	2.1817	.5383	1.774	.3034	.68125
66	1.1519	.1613	1.089	.1481	.11919	126	2.1991	.5460	1.782	.3064	.69505
67	1.1694	.1661	1.104	.1505	.12443	127	2.2166	.5538	1.790	.3094	.70897
68	1.1868	.1710	1.118	.1529	.12982	128	2.2340	.5616	1.798	.3124	.72301
69	1.2043	.1759	1.133	.1553	.13535	129	2.2515	.5695	1.805	.3155	.73716
70	**1.2217**	**.1808**	**1.147**	**.1576**	**.14102**	**130**	**2.2689**	**.5774**	**1.813**	**.3185**	**.75144**
71	1.2392	.1859	1.161	.1601	.14683	131	2.2864	.5853	1.820	.3216	.76584
72	1.2566	.1910	1.176	.1625	.15279	132	2.3038	.5933	1.827	.3247	.78034
73	1.2741	.1961	1.190	.1649	.15889	133	2.3213	.6013	1.834	.3278	.79497
74	1.2915	.2014	1.204	.1673	.16514	134	2.3387	.6093	1.841	.3309	.80970
75	1.3090	.2066	1.218	.1697	.17154	135	2.3562	.6173	1.848	.3341	.82454
76	1.3265	.2120	1.231	.1722	.17808	136	2.3736	.6254	1.854	.3373	.83949
77	1.3439	.2174	1.245	.1746	.18477	137	2.3911	.6335	1.861	.3404	.85455
78	1.3614	.2229	1.259	.1771	.19160	138	2.4086	.6416	1.867	.3436	.86971
79	1.3788	.2284	1.272	.1795	.19859	139	2.4260	.6498	1.873	.3469	.88497
80	**1.3963**	**.2340**	**1.286**	**.1820**	**.20573**	**140**	**2.4435**	**.6580**	**1.879**	**.3501**	**.90034**
81	1.4137	.2396	1.299	.1845	.21301	141	2.4609	.6662	1.885	.3534	.91580
82	1.4312	.2453	1.312	.1869	.22045	142	2.4784	.6744	1.891	.3566	.93135
83	1.4486	.2510	1.325	.1894	.22804	143	2.4958	.6827	1.897	.3599	.94700
84	1.4661	.2569	1.338	.1919	.23578	144	2.5133	.6910	1.902	.3633	.96274
85	1.4835	.2627	1.351	.1944	.24367	145	2.5307	.6993	1.907	.3666	.97858
86	1.5010	.2686	1.364	.1970	.25171	146	2.5482	.7076	1.913	.3700	.99449
87	1.5184	.2746	1.377	.1995	.25990	147	2.5656	.7160	1.918	.3734	1.0105
88	1.5359	.2807	1.389	.2020	.26825	148	2.5831	.7244	1.923	.3768	1.0266
89	1.5533	.2867	1.402	.2046	.27675	149	2.6005	.7328	1.927	.3802	1.0428
90	**1.5708**	**.2929**	**1.414**	**.2071**	**.28540**	**150**	**2.6180**	**.7412**	**1.932**	**.3837**	**1.0590**
91	1.5882	.2991	1.427	.2097	.29420	151	2.6354	.7496	1.936	.3871	1.0753
92	1.6057	.3053	1.439	.2122	.30316	152	2.6529	.7581	1.941	.3906	1.0917
93	1.6232	.3116	1.451	.2148	.31226	153	2.6704	.7666	1.945	.3942	1.1082
94	1.6406	.3180	1.463	.2174	.32152	154	2.6878	.7750	1.949	.3977	1.1247
95	1.6581	.3244	1.475	.2200	.33093	155	2.7053	.7836	1.953	.4013	1.1413
96	1.6755	.3309	1.486	.2226	.34050	156	2.7227	.7921	1.956	.4049	1.1580
97	1.6930	.3374	1.498	.2252	.35021	157	2.7402	.8006	1.960	.4085	1.1747
98	1.7104	.3439	1.509	.2279	.36008	158	2.7576	.8092	1.963	.4122	1.1915
99	1.7279	.3506	1.521	.2305	.37009	159	2.7751	.8178	1.967	.4158	1.2084
100	**1.7453**	**.3572**	**1.532**	**.2332**	**.38026**	**160**	**2.7925**	**.8264**	**1.970**	**.4195**	**1.2253**
101	1.7628	.3639	1.543	.2358	.39058	161	2.8100	.8350	1.973	.4233	1.2422
102	1.7802	.3707	1.554	.2385	.40104	162	2.8274	.8436	1.975	.4270	1.2592
103	1.7977	.3775	1.565	.2412	.41166	163	2.8449	.8522	1.978	.4308	1.2763
104	1.8151	.3843	1.576	.2439	.42242	164	2.8623	.8608	1.981	.4346	1.2934
105	1.8326	.3912	1.587	.2466	.43333	165	2.8798	.8695	1.983	.4385	1.3105
106	1.8500	.3982	1.597	.2493	.44439	166	2.8972	.8781	1.985	.4424	1.3277
107	1.8675	.4052	1.608	.2520	.45560	167	2.9147	.8868	1.987	.4463	1.3449
108	1.8850	.4122	1.618	.2548	.46695	168	2.9322	.8955	1.989	.4502	1.3621
109	1.9024	.4193	1.628	.2575	.47844	169	2.9496	.9042	1.991	.4542	1.3794
110	**1.9199**	**.4264**	**1.638**	**.2603**	**.49008**	**170**	**2.9671**	**.9128**	**1.992**	**.4582**	**1.3967**
111	1.9373	.4336	1.648	.2631	.50187	171	2.9845	.9215	1.994	.4622	1.4140
112	1.9548	.4408	1.658	.2659	.51379	172	3.0020	.9302	1.995	.4663	1.4314
113	1.9722	.4481	1.668	.2687	.52586	173	3.0194	.9390	1.996	.4704	1.4488
114	1.9897	.4554	1.677	.2715	.53806	174	3.0369	.9477	1.997	.4745	1.4662
115	2.0071	.4627	1.687	.2743	.55041	175	3.0543	.9564	1.998	.4786	1.4836
116	2.0246	.4701	1.696	.2772	.56289	176	3.0718	.9651	1.9988	.4828	1.5010
117	2.0420	.4775	1.705	.2800	.57551	177	3.0892	.9738	1.9993	.4871	1.5184
118	2.0595	.4850	1.714	.2829	.58827	178	3.1067	.9825	1.9997	.4913	1.5359
119	2.0769	.4925	1.723	.2858	.60116	179	3.1241	.9913	1.9999	.4957	1.5533
120	**2.0944**	**.5000**	**1.732**	**.2887**	**.61419**	**180**	**3.1416**	**1.0000**	**2.000**	**.5000**	**1.5708**

* Area of segment of any radius = factor $G \times$ square of radius

Table 27. Areas of Segments of a Circle

Area = $F \times D^2$ where segment < semicircle; = $\frac{1}{4}\pi D^2$ − (area of segment of rise h). H = rise or height of segment; D = diam. of circle; F = factor corresponding to H/D. $h = (D - H)$

H/D	F	H/D	F	H/D	F	H/D	F	H/D	F	H/D	F
0.000	0.00000	0.07	0.02417	0.14	0.06683	0.21	0.11990	0.28	0.18002	0.35	0.24498
.001	.00004	.071	.02468	.141	.06753	.211	.12071	.281	.18092	.351	.24593
.002	.00012	.072	.02520	.142	.06822	.212	.12153	.282	.18182	.352	.24689
.003	.00022	.073	.02571	.143	.06892	.213	.12235	.283	.18272	.353	.24784
.004	.00034	.074	.02624	.144	.06963	.214	.12317	.284	.18362	.354	.24880
.005	.00047	.075	.02676	.145	.07033	.215	.12399	.285	.18452	.355	.24976
.006	.00062	.076	.02729	.146	.07103	.216	.12481	.286	.18542	.256	.25071
.007	.00078	.077	.02782	.147	.07174	.217	.12563	.287	.18633	.357	.25167
.008	.00095	.078	.02836	.148	.07245	.218	.12646	.288	.18723	.358	.25263
.009	.00113	.079	.02889	.149	.07316	.219	.12729	.289	.18814	.359	.25359
.01	.00133	.08	.02943	.15	.07387	.22	.12811	.29	.18905	.36	.25455
.011	.00153	.081	.02998	.151	.07459	.221	.12894	.291	.18996	.361	.25551
.012	.00175	.082	.03053	.152	.07531	.222	.12977	.292	.19086	.362	.25647
.013	.00197	.083	.03108	.153	.07603	.223	.13060	.293	.19177	.363	.25743
.014	.00220	.084	.03163	.154	.07675	.224	.13144	.294	.19268	.364	.25839
.015	.00244	.085	.03219	.155	.07747	.225	.13227	.295	.19360	.365	.25936
.016	.00268	.086	.03275	.156	.07819	.226	.13311	.296	.19451	.366	.26032
.017	.00294	.087	.03331	.157	.07892	.227	.13395	.297	.19542	.367	.26128
.018	.00320	.088	.03387	.158	.07965	.228	.13478	.298	.19634	.368	.26225
.019	.00347	.089	.03444	.159	.08038	.229	.13562	.299	.19725	.369	.26321
.02	.00375	.09	.03501	.16	.08111	.23	.13646	.3	.19817	.37	.26418
.021	.00403	.091	.03559	.161	.08185	.231	.13731	.301	.19908	.371	.26514
.022	.00432	.092	.03616	.162	.08258	.232	.13815	.302	.20000	.372	.26611
.023	.00462	.093	.03674	.163	.08332	.233	.13900	.303	.20092	.373	.26708
.024	.00492	.094	.03732	.164	.08406	.234	.13984	.304	.20184	.374	.26805
.025	.00523	.095	.03791	.165	.08480	.235	.14069	.305	.20276	.375	.26901
.026	.00555	.096	.03850	.166	.08554	.236	.14154	.306	.20368	.376	.26998
.027	.00587	.097	.03909	.167	.08629	.237	.14239	.307	.20460	.377	.27095
.028	.00619	.098	.03968	.168	.08704	.238	.14324	.308	.20553	.378	.27192
.029	.00653	.099	.04028	.169	.08779	.239	.14409	.309	.20645	.379	.27289
.03	.00687	.1	.04087	.17	.08854	.24	.14494	.31	.20738	.38	.27386
.031	.00721	.101	.04148	.171	.08929	.241	.14580	.311	.20830	.381	.27483
.032	.00756	.102	.04208	.172	.09004	.242	.14666	.312	.20923	.382	.27580
.033	.00791	.103	.04269	.173	.09080	.243	.14751	.313	.21015	.383	.27678
.034	.00827	.104	.04330	.174	.09155	.244	.14837	.314	.21108	.384	.27775
.035	.00864	.105	.04391	.175	.09231	.245	.14923	.315	.21201	.385	.27872
.036	.00901	.106	.04452	.176	.09307	.246	.15009	.316	.21294	.386	.27969
.037	.00938	.107	.04514	.177	.09384	.247	.15095	.317	.21387	.387	.28067
.038	.00976	.108	.04576	.178	.09460	.248	.15182	.318	.21480	.388	.28164
.039	.01015	.109	.04638	.179	.09537	.249	.15268	.319	.21573	.389	.28262
.04	.01054	.11	.04701	.18	.09613	.25	.15355	.32	.21667	.39	.28359
.041	.01093	.111	.04763	.181	.09690	.251	.15441	.321	.21760	.391	.28457
.042	.01133	.112	.04826	.182	.09767	.252	.15528	.322	.21853	.392	.28554
.043	.01173	.113	.04889	.183	.09845	.253	.15615	.323	.21947	.393	.28652
.044	.01214	.114	.04953	.184	.09922	.254	.15702	.324	.22040	.394	.28750
.045	.01255	.115	.05016	.185	.10000	.255	.15789	.325	.22134	.395	.28848
.046	.01297	.116	.05080	.186	.10077	.256	.15876	.326	.22228	.396	.28945
.047	.01339	.117	.05145	.187	.10155	.257	.15964	.327	.22322	.397	.29043
.048	.01382	.118	.05209	.188	.10233	.258	.16051	.328	.22415	.398	.29141
.049	.01425	.119	.05274	.189	.10312	.259	.16139	.329	.22509	.399	.29239
.05	.01468	.12	.05338	.19	.10390	.26	.16226	.33	.22603	.4	.29337
.051	.01512	.121	.05404	.191	.10469	.261	.16314	.331	.22697	.401	.29435
.052	.01556	.122	.05469	.192	.10547	.262	.16402	.332	.22792	.402	.29533
.053	.01601	.123	.05535	.193	.10626	.263	.16490	.333	.22886	.403	.29631
.054	.01646	.124	.05600	.194	.10705	.264	.16578	.334	.22980	.404	.29729
.055	.01691	.125	.05666	.195	.10784	.265	.16666	.335	.23074	.405	.29827
.056	.01737	.126	.05733	.196	.10864	.266	.16755	.336	.23169	.406	.29926
.057	.01783	.127	.05799	.197	.10943	.267	.16843	.337	.23263	.407	.30024
.058	.01830	.128	.05866	.198	.11023	.268	.16932	.338	.23358	.408	.30122
.059	.01877	.129	.05933	.199	.11102	.269	.17020	.339	.23453	.409	.30220
.06	.01924	.13	.06000	.2	.11182	.27	.17109	.34	.23547	.41	.30319
.061	.01972	.131	.06067	.201	.11262	.271	.17198	.341	.23642	.411	.30417
.062	.02020	.132	.06135	.202	.11343	.272	.17287	.342	.23737	.412	.30516
.063	.02068	.133	.06203	.203	.11423	.273	.17376	.343	.23832	.413	.30614
.064	.02117	.134	.06271	.204	.11504	.274	.17465	.344	.23927	.414	.30712
.065	.02166	.135	.06339	.205	.11584	.275	.17554	.345	.24022	.415	.30811
.066	.02215	.136	.06407	.206	.11665	.276	.17644	.346	.24117	.416	.30910
.067	.02265	.137	.06476	.207	.11746	.277	.17733	.347	.24212	.417	.31008
.068	.02315	.138	.06545	.208	.11827	.278	.17823	.348	.24307	.418	.31107
.069	.02366	.139	.06614	.209	11908	.279	.17912	.349	.24403	.419	.31205

Table 27. Area of Segments—*Continued*

H/D	F	H/D	F	H/D	F	H/D	F	H/D	F	H/D	F
0.42	0.31304	0.435	0.32788	0.45	0.34278	0.465	0.35773	0.48	0.37270	0.495	0.38770
.421	.31403	.436	.32887	.451	.34378	.466	.35873	.481	.37370	.496	.38870
.422	.31502	.437	.32987	.452	.34477	.467	.35972	.482	.37470	.497	.38970
.423	.31600	.438	.33086	.453	.34577	.468	.36072	.483	.37570	.498	.39070
.424	.31699	.439	.33185	.454	.34676	.469	.36172	.484	.37670	.499	.39170
.425	.31798	.44	.33284	.455	.34776	.47	.36272	.485	.37770	.500	.39270
.426	.31897	.441	.33384	.456	.34876	.471	.36372	.486	.37870		
.427	.31996	.442	.33483	.457	.34975	.472	.36471	.487	.37970		
.428	.32095	.443	.33582	.458	.35075	.473	.36571	.488	.38070		
.429	.32194	.444	.33682	.459	.35175	.474	.36671	.489	.38170		
.43	.32293	.445	.33781	.46	.35274	.475	.36771	.49	.38270		
.431	.32392	.446	.33880	.461	.35374	.476	.36871	.491	.38370		
.432	.32491	.447	.33980	.462	.35474	.477	.36971	.492	.38470		
.433	.32590	.448	.34079	.463	.35573	.478	.37071	.493	.38570		
.434	.32689	.449	.34179	.464	.35673	.479	37171	.494	.38670		

Table 28. Values of Degrees and Minutes in Radians

Deg	Radians	Deg	Radians	Deg	Radians	Deg	Radians	Deg	Radians	Deg	Radians	Min	Radians	Min	Radians
1	0 01745	31	0 54105	61	1 06465	91	1 58825	121	2 11185	151	2 63545	1	0 00029	31	0.00902
2	.03491	32	.55851	62	1 08210	92	1 60570	122	2 12930	152	2 65290	2	.00058	32	.00931
3	.05236	33	.57596	63	1 09956	93	1.62316	123	2 14676	153	2 67035	3	.00087	33	.00960
4	.06981	34	.59341	64	1.11701	94	1.64061	124	2.16421	154	2 68781	4	.00116	34	.00989
5	.08727	35	.61087	65	1.13446	95	1.65806	125	2.18166	155	2 70526	5	.00145	35	.01018
6	.10472	36	.62832	66	1.15192	96	1 67552	126	2.19912	156	2.72271	6	.00175	36	.01047
7	.12217	37	.64577	67	1.16937	97	1 69297	127	2.21657	157	2.74017	7	.00204	37	.01076
8	.13963	38	.66323	68	1.18682	98	1.71042	128	2 23402	158	2 75762	8	.00233	38	.01105
9	.15708	39	68068	69	1.20428	99	1 72788	129	2 25148	159	2 77507	9	.00262	39	.01134
10	.17453	40	69813	70	1.22173	100	1.74533	130	2 26893	160	2 79253	10	00291	40	.01164
11	.19199	41	71559	71	1 23918	101	1.76278	131	2 28638	161	2 80998	11	00320	41	.01193
12	.20944	42	.73304	72	1 25664	102	1.78024	132	2 30384	162	2 82743	12	.00349	42	.01222
13	.22689	43	.75049	73	1 27409	103	1.79769	133	2 32129	163	2 84489	13	.00378	43	.01251
14	.24435	44	.76795	74	1 29154	104	1.81514	134	2 33874	164	2 86234	14	00407	44	.01280
15	.26180	45	.78540	75	1 30900	105	1 83260	135	2 35620	165	2 87979	15	00436	45	.01309
16	.27925	46	.80285	76	1.32645	106	1 85005	136	2 37365	166	2 89725	16	.00465	46	.01338
17	29671	47	82031	77	1 34390	107	1 86750	137	2 39110	167	2 91470	17	.00495	47	.01367
18	.31416	48	83776	78	1 36136	108	1 88496	138	2 40856	168	2 93215	18	.00524	48	.01396
19	.33161	49	85521	79	1.37881	109	1 90241	139	2 42601	169	2 94961	19	.00553	49	.01425
20	34907	50	87267	80	1 39626	110	1 91986	140	2 44346	170	2 96706	20	00582	50	.01454
21	.36652	51	89012	81	1 41372	111	1 93732	141	2 46092	171	2 98451	21	00611	51	.01484
22	38397	52	90757	82	1 43117	112	1 95477	142	2 47837	172	3 00197	22	00640	52	.01513
23	.40143	53	92502	83	1 44862	113	1 97222	143	2 49582	173	3 01942	23	00669	53	.01542
24	.41888	54	94248	84	1 46608	114	1 98968	144	2 51328	174	3 03687	24	.00698	54	.01571
25	.43633	55	95993	85	1 48353	115	2 00713	145	2 53073	175	3 05433	25	00727	55	01600
26	.45379	56	.97738	86	1 50098	116	2 02458	146	2 54818	176	3 07178	26	.00756	56	.01629
27	.47124	57	99484	87	1 51844	117	2 04204	147	2 56564	177	3.08923	27	.00785	57	.01658
28	.48869	58	1 01299	88	1 53589	118	2 05949	148	2.58309	178	3.10669	28	.00814	58	.01687
29	50615	59	1 02974	89	1 55334	119	2 07694	149	2.60054	179	3 12414	29	00844	59	.01716
30	52360	60	1 04720	90	1 57030	120	2 09440	150	2 61800	180	3 14159	30	00873	60	01745

Table 29. Values of Radians in Degrees

Rad.	00	01	02	03	04	05	06	07	08	09
	Deg	Deg.	Deg.	Deg.	Deg	Deg.	Deg.	Deg.	Deg	Deg.
0.0	0 0000	0 5730	1 1459	1 7189	2 2918	2 8648	3 4377	4 0107	4 5837	5.1566
.1	5.7296	6 3025	6 8755	7 4485	8 0214	8 5944	9 1673	9 7403	10.3132	10.8862
.2	11.4591	12 0321	12 6051	13 1780	13 7510	14 3239	14 8969	15 4699	16.0428	16.6158
.3	17 1887	17.7617	18 3346	18 9076	19 4806	20 0535	20 6265	21 1994	21 7724	22.3454
.4	22.9183	23 4913	24 0642	24 6372	25 2101	25 7831	26 3561	26 9290	27 5020	28.0749
.5	28 6479	29 2208	29 7938	30 3668	30 9397	31 1527	32 0856	32 6586	33.2316	33.8045
.6	34.3775	34 9504	35 5234	36 0963	36 6693	37 2423	37 8152	38 3882	38 9611	39.5341
.7	40.1070	40 6800	41 2530	41 8259	42 3989	42 9718	43 5448	44.1178	44.6907	45.2637
.8	45.8366	46.4096	46 9825	47 5555	48 1285	48 7014	49 2744	49 8473	50.4203	50.9932
.9	51 5662	52 1392	52 7121	53 2851	53 8580	54 4310	55 0039	55 5769	56 1499	56.7228

1 Radian = 57.29578 deg | 2 Radians = 114.59156 deg. | 3 Radians = 171.88734 deg.

Table 30. Decimals of a Degree in Minutes and Seconds

Decimal	00		01		02		03		04		.05		.06		.07		.08		.09	
	Min.	Sec.	Min.	Sec.	Min.	Sec	Min.	Sec	Min.	Sec	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec	Min.	Sec.
0.0	0	0	0	36	1	12	1	48	2	24	3	0	3	36	4	12	4	48	5	24
.1	6	0	6	36	7	12	7	48	8	24	9	0	9	36	10	12	10	48	11	24
.2	12	0	12	36	13	12	13	48	14	24	15	0	15	36	16	12	16	48	17	24
.3	18	0	18	36	19	12	19	48	20	24	21	0	21	36	22	12	22	48	23	24
.4	24	0	24	36	25	12	25	48	26	24	27	0	27	36	28	12	28	48	29	24
.5	30	0	30	36	31	12	31	48	32	24	33	0	33	36	34	12	24	48	35	24
.6	36	0	36	36	37	12	37	48	38	24	39	0	39	36	40	12	40	48	41	24
.7	42	0	42	36	43	12	43	48	44	24	45	0	45	36	46	12	46	48	47	24
.8	48	0	48	36	49	12	49	48	50	24	51	0	51	36	52	12	52	48	53	24
.9	54	0	54	36	55	12	55	48	56	24	57	0	57	36	58	12	58	48	59	24

Table 31. Circumferences and Areas of Circles—Diameters in Feet and Inches

Circumferences in feet; areas in square feet

Diam., feet	Diameter, inches 0	1	2	3	4	5	6	7	8	9	10	11
	Circumference											
0	0 0000	0 2618	0 5236	0 7854	1 0472	1 3090	1.5708	1 8326	2 0944	2 3562	2 6180	2 8798
1	3 1416	3.4034	3.6653	3 9270	4 1877	4 4507	4 7124	4 9741	5 2361	5 4978	5 7595	6 0215
2	6 2832	6 5450	6 8069	7 0686	7 3293	7 5923	7 8540	8 1157	8 3777	8 6394	8 9011	9 1631
3	9 4248	9 6866	9 9485	10 210	10 471	10.734	10 996	11 257	11.519	11 781	12 043	12 305
4	12 566	12.828	13 090	13 352	13 613	13 875	14 137	14 399	14.661	14 923	15 184	15 446
5	15 708	15.970	16 232	16 493	16 754	17 017	17.279	17 540	17 802	18 064	18.326	18 588
6	18 850	19.111	19 373	19 635	19 896	20 159	20 420	20 682	20 944	21 206	21 467	21 729
7	21.991	22.253	22 515	22 777	23 037	23 300	23 562	23 824	24 086	24 347	24 609	24 871
8	25 133	25.395	25 656	25 918	26 179	26 442	26 704	26 965	27 227	27 489	27 751	28 013
9	28 274	28 536	28 798	29 060	29 320	29 583	29 845	30 107	30 369	30 631	30 892	31 154
10	31.416	31 678	31 940	32 201	32 462	32 725	32 987	33 248	33 510	33 772	34 034	34.296
11	34.557	34 819	35 081	35 343	35 604	35 867	36 128	36 390	36 652	36 914	37 175	37.437
12	37 699	37.961	38 223	38 484	38 745	39 008	39 270	39 532	39 794	40.055	40 317	40 579
13	40.841	41 102	41 364	41.626	41 887	42 150	42 411	42 673	42 935	43 197	43 459	43 721
14	43.982	44 244	44.506	44 768	45 028	45 291	45 553	45 815	46 077	46 338	46 600	46 862
15	47.124	47 386	47.648	47 909	48 170	48 433	48 695	48 956	49 218	49 480	49 742	50 004
16	50.265	50 527	50 789	51 051	51.312	51 575	51 836	52 098	52.360	52 622	52 883	53 145
17	53.407	53.669	53 931	54.192	54 453	54 716	54 978	55 240	55 502	55 763	56 025	56 287
18	56.549	56 810	57 072	57 334	57 595	57 858	58 119	58 381	58 643	58 905	59 167	59 429
19	59 690	59 952	60.214	60 476	60 736	61 000	61 261	61 523	61.785	62 046	62 308	62.570
20	62.832	63.094	63.356	63 617	63 878	64 141	64 403	64 664	64 926	65 188	65 450	65 712
21	65 973	66 235	66.497	66 759	67 020	67.282	67 544	67 806	68 068	68 330	68 591	68 853
22	69 115	69 377	69 639	69 900	70.161	70 424	70 686	70 947	71 209	71.471	71 733	71 995
23	72 257	72 518	72 780	73 042	73 303	73 566	73 827	74 089	74 351	74 613	74 874	75 136
24	75 398	75 660	75 922	76 184	76 444	76 707	76 969	77 231	77 493	77 754	78 016	78 278
25	78 540	78 802	79 063	79 325	79 586	79 849	80 111	80 372	80 634	80 896	81 158	81 420
26	81 681	81 943	82 205	82 467	82 727	82 990	83 252	83 514	83 776	84 038	84 299	84 561
27	84 823	85 085	85 347	85.608	85 869	86 132	86 394	86 655	86 917	87 179	87 441	87 703
28	87 965	88 226	88.488	88 749	89 011	89 274	89 535	89 797	90 059	90 321	90 582	90 844
29	91.106	91 367	91 630	91 892	92 152	92.415	92 677	92 939	93 201	93 462	93 724	93 986
30	94 248	94 509	94 771	95 033	95 294	95 557	95 818	96 080	96 342	96.604	96 866	97 128

•

Area

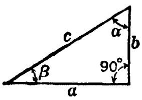
Diam., feet	0	1	2	3	4	5	6	7	8	9	10	11
0	0 0000	0 0055	0 0218	0 0491	0 0873	0 1364	0 1963	0 2673	0 3491	0 4418	0 5454	0 6600
1	0 7854	0 9218	1 069	1 227	1 396	1 576	1 767	1 969	2 182	2 405	2 640	2 885
2	3.142	3.409	3 687	3 976	4 276	4 587	4 909	5 241	5 585	5 940	6 305	6 681
3	7.069	7 467	7 876	8.296	8 727	9 168	9 621	10 08	10 56	11 04	11 54	12 05
4	12.57	13.10	13 64	14 19	14 75	15 32	15 90	16 50	17 10	17 72	18 35	18 99
5	19 63	20 29	20 97	21 65	22 34	23 04	23 76	24 48	25 22	25 97	26 73	27 49
6	28.27	29 07	29 87	30.68	31 50	32 34	33 18	34 04	34 91	35 78	36 67	37 57
7	38.48	39.41	40.34	41.28	42 24	43 20	44 18	45 17	46 16	47 17	48 19	49 22
8	50 27	51.32	52 38	53.46	54 54	55 64	56 75	57 86	58 99	60 13	61 28	62 44
9	63 62	64 80	66 00	67 20	68 42	69 64	70 88	72 13	73 39	74 66	75.94	77 24
10	78 54	79 85	81 18	82 52	83 86	85 22	86 59	87.97	89 36	90 76	92.18	93.60
11	95 03	96.48	97 93	99 40	100 9	102.4	103.9	105 4	106 9	108 4	110. 0	111 5
12	113.1	114 7	116.3	117 9	119 5	121.1	122 7	124.4	126 0	127 7	129 4	131.0
13	132 7	134 4	136 2	137 9	139 6	141 4	143 1	144 9	146.7	148 5	150 3	152.1
14	153.9	155 8	157.6	159 5	161.4	163.2	165 1	167 0	168 9	170.9	172 8	174.8
15	176 7	178 7	180.7	182.7	184 7	186 7	188.7	190.7	192.8	194.8	196 9	199.0
16	201.1	203.2	205.3	207.4	209 5	211.7	213 8	216 0	218.2	220 4	222 5	224.8
17	227.0	229.2	231 4	233.7	236 0	238.2	240 5	242 8	245.1	247.5	249 8	252 1
18	254.5	256.8	259.2	261 6	264.0	266.4	268.8	271.2	273.7	276.1	278 6	281 0
19	283.5	286.0	288 5	291.0	293.6	296.1	298.7	301 2	303.8	306.4	308 9	311.5
20	314.2	316.8	319.4	322.1	324.7	327.4	330.1	332.7	335.4	338.2	340.9	343 6
21	346.4	349.1	351 9	354.7	357.4	360.2	363.1	365 9	368.7	371.5	374.4	377.2
22	380.1	383 0	386 0	388 8	391.7	394 7	397.6	400 5	403.5	406 5	409.5	412.5
23	415.5	418 5	421.5	424.6	427.6	430.7	433.7	436.8	439 9	443 0	446.1	449.2
24	452 4	455.5	458 7	461 9	465 0	468 2	471 4	474 6	477.9	481.1	484 3	487.6
25	490 9	494.1	497.4	500 7	504 0	507.4	510.7	514 0	517.4	520 8	524.1	527.5
26	530 9	534.3	537 7	541 2	544.6	548 1	551.6	555 0	558.5	562.0	565.5	569 0
27	572 6	576.1	579 6	583.2	586 8	590 3	594 0	597 5	601.2	604.8	608 4	612.1
28	615.8	619 4	623.1	626.8	630 5	634.2	637.9	641 7	645.4	649.2	652.9	656.7
29	660 5	664.3	668.1	672.0	675.8	679.6	683.5	687 3	691.2	695.1	699.0	702 9
30	706 9	710 8	714.7	718.7	722 6	726.6	730 6	734 6	738.6	742 6	746 7	750 7

5. MENSURATION

Table 32a. Plane Rectilinear Figures

Notation: Lines, $a, b, c, \ldots$; angles, $\alpha, \beta, \gamma, \ldots$; altitude (perpendicular height), h; side, s; diagonals, $d, d_1, \ldots$; perimeter, p; radius of inscribed circle, r; radius of circumscribed circle, R; area, A.

1. Right Triangle



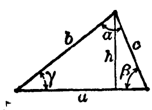
(One angle 90°)

$p = a + b + c$; $c^2 = a^2 + b^2$;

$$A = \frac{ab}{2} = \frac{a^2}{2}\tan\beta = \frac{c^2}{4}\sin 2\beta = \frac{c^2}{4}\sin 2\alpha.$$

For additional formulas, see *General Triangle* below, and also trigonometry.

2. General Triangle (and Equilateral Triangle)



For General Triangle:

$p = a + b + c$. Let $s = \frac{1}{2}(a + b + c)$.

$$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s};\quad R = \frac{a}{2\sin\alpha} = \frac{abc}{4rs};$$

$$A = \frac{ah}{2} = \frac{ab}{2}\sin\gamma = \frac{b^2\sin\gamma\sin\alpha}{2\sin\beta} = rs = \frac{abc}{4R}.$$

Length of median to side $c = \frac{1}{2}\sqrt{2(a^2+b^2) - c^2}$.

Length of bisector of angle $\gamma = \dfrac{\sqrt{ab[(a+b)^2 - c^2]}}{a+b}$.

For Equilateral Triangle ($a = b = c = s$ and $\alpha = \beta = \gamma = 60°$):

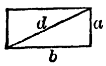
(Equal sides and equal angles)

$$p = 3s;\quad r = \frac{s}{2\sqrt{3}};\quad R = \frac{s}{\sqrt{3}} = 2r;$$

$$h = \frac{s\sqrt{3}}{2};\quad s = \frac{2h}{\sqrt{3}};\quad A = \frac{s^2\sqrt{3}}{4}.$$

For additional formulas, see trigonometry.

3. Rectangle (and Square)



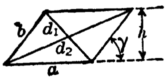
For Rectangle:

$p = 2(a + b)$; $d = \sqrt{a^2 + b^2}$; $A = ab$.

For Square ($a = b = s$):

$$p = 4s;\quad d = s\sqrt{2};\quad s = \frac{d}{\sqrt{2}};\quad A = s^2 = \frac{d^2}{2}.$$

4. General Parallelogram (Rhomboid) (and Rhombus)



For General Parallelogram (Rhomboid):

(Opposite sides parallel)

$p = 2(a + b)$; $d_1 = \sqrt{a^2 + b^2 - 2ab\cos\gamma}$;

$d_2 = \sqrt{a^2 + b^2 + 2ab\cos\gamma}$; $d_1^2 + d_2^2 = 2(a^2 + b^2)$;

$A = ah = ab\sin\gamma$.

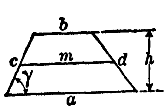
For Rhombus ($a = b = s$):

(Opposite sides parallel and all sides equal)

$$p = 4s;\quad d_1 = 2s\sin\frac{\gamma}{2};\quad d_2 = 2s\cos\frac{\gamma}{2};\quad d_1^2 + d_2^2 = 4s^2;$$

$$d_1 d_2 = 2s^2\sin\gamma;\quad A = sh = s^2\sin\gamma = \frac{d_1 d_2}{2}.$$

5. General Trapezoid (and Isosceles Trapezoid)



Let mid-line bisecting non-parallel sides $= m$. Then $m = \dfrac{a+b}{2}$.

For General Trapezoid:

(Only one pair of opposite sides parallel)

$$p = a + b + c + d;\quad A = \frac{(a+b)h}{2} = mh.$$

For Isosceles Trapezoid ($d = c$):

(Non-parallel sides equal)

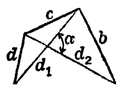
$$A = \frac{(a+b)h}{2} = mh = \frac{(a+b)c\sin\gamma}{2}$$

$$= (a - c\cos\gamma)c\sin\gamma = (b + c\cos\gamma)c\sin\gamma.$$

(*Table continued on p. 20-56*)

Table 32a. Plane Rectilinear Figures—*Continued*

6. General Quadrilateral (Trapezium)

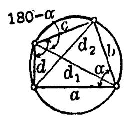


(No sides parallel)

$p = a + b + c + d$

$A = \frac{1}{2} d_1 d_2 \sin\alpha$ = sum of areas of the two triangles formed by either diagonal and the four sides.

7. Quadrilateral Inscribed in Circle



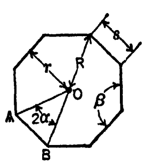
(Sum of opposite angles = 180°)

$ac + bd = d_1 d_2.$

Let $s = \frac{1}{2}(a + b + c + d) = \frac{p}{2}$ and α = angle between sides a and b.

$A = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \frac{1}{2}(ab + cd)\sin\alpha.$

8. Regular Polygon (and General Polygon)



For Regular Polygon:

(Equal sides and equal angles)

Let n = number of sides.

Central angle $= 2\alpha = \frac{2\pi}{n}$ radians;

Vertex angle $= \beta = \frac{(n-2)}{n}\pi$ radians.

$p = ns;\ s = 2r\tan\alpha = 2R\sin\alpha;$

$r = \frac{s}{2}\cot\alpha;\ R = \frac{s}{2}\csc\alpha;$

$A = \frac{nsr}{2} = nr^2\tan\alpha = \frac{nR^2}{2}\sin 2\alpha = \frac{ns^2}{4}\cot\alpha$ = sum of areas of the n equal triangles such as OAB.

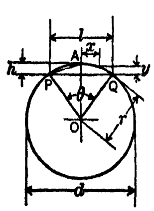
For General Polygon:

A = sum of areas of constituent triangles into which it can be divided

Table 32b. Plane Curvilinear Figures

Notation. Lines, $a, b, \ldots$; radius, r; diameter, d; perimeter, p; circumference, c; central angle in radians, θ; arc, s; chord of arc (s), l; chord of half arc ($s/2$), l'; rise, h; area, A.

9. Circle (and Circular Arc)



For Circle:

$d = 2r;\ c = 2\pi r = \pi d;\ A = \pi r^2 = \frac{\pi d^2}{4} = \frac{c^2}{4\pi}.$

For Circular Arc:

Let arc $PAQ = s$; and chord $PA = \left(\text{chord of } \frac{s}{2}\right) = l'$. Then,

$s = r\theta = \frac{d\theta}{2};\ s = \frac{8l' - l}{3}.$ (The latter equation is Huyghen's approximate formula. For θ small, error is very small; for $\theta = 120°$, error equals about 1 part in 400; for $\theta = 180°$, error is less than 1.25%.)

$l = 2r\sin\frac{\theta}{2};\ l = 2\sqrt{2hr - h^2}$ (approximate formula)

$r = \frac{4h^2 + l^2}{8h}$ (approximate formula)

$2\sin\frac{\upsilon}{2}$

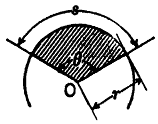
$h = r \mp \sqrt{r^2 - \frac{l^2}{4}}$ (− if $\theta \leq 180°$; + if $\theta \geq 180°$) $= r\left(1 - \cos\frac{\theta}{2}\right)$

$= r \operatorname{versin}\frac{\theta}{2} = 2r\sin^2\frac{\theta}{4} = \frac{l}{2}\tan\frac{\theta}{4} = r + y - \sqrt{r^2 - x^2}.$

Side ordinate $y = h - r + \sqrt{r^2 - x^2}.$

Table 32b. Plane Curvilinear Figures—*Continued*

10. Circular Sector (and Semicircle)



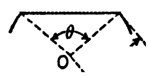
For Circular Sector:

$$A = \frac{\theta r^2}{2} = \frac{sr}{2}.$$

For Semicircle:

$$A = \frac{\pi r^2}{2}$$

11. Circular Segment

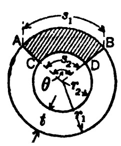


$$A = \frac{r^2}{2}(\theta - \sin\theta)$$

$$= \tfrac{1}{2}[sr \mp l(r-h)] \ (-\text{ if } h \leqq r;\ +\text{ if } h \geqq r).$$

$A = \frac{2lh}{3}$ or $\frac{h}{15}(8l' + 6l)$. (Approximate formulas. For h small compared with r, error is very small; for $h = \frac{r}{4}$, first formula errs about 3.5% and second less than 1.0%.)

12. Annulus



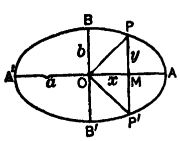
(Surface between two concentric circles)

$$A = \pi(r_1^2 - r_2^2) = \pi(r_1 + r_2)(r_1 - r_2);$$

$$A \text{ of sector } ABCD = \frac{\theta}{2}(r_1^2 - r_2^2) = \frac{\theta}{2}(r_1 + r_2)(r_1 - r_2)$$

$$= \frac{t}{2}(s_1 + s_2).$$

13. Ellipse



$$p = \pi(a+b)\left(1 + \frac{R^2}{4} + \frac{R^4}{64} + \frac{R^6}{256} + \ldots\right) \text{ where } R = \frac{a-b}{a+b}.$$

$$p = \pi(a+b)\frac{64 - 3R^4}{64 - 16R^2} \text{ (approximate formula).}$$

$A = \pi ab$; A of quadrant $AOB = \frac{\pi ab}{4}$.

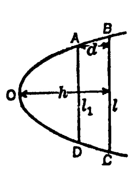
A of sector $AOP = \frac{ab}{2}\cos^{-1}\frac{x}{a}$; A of sector $POB = \frac{ab}{2}\sin^{-1}\frac{x}{a}$;

A of section $BPP'B' = xy + ab\sin^{-1}\frac{x}{a}$;

A of segment $PAP'P = -xy + ab\cos^{-1}\frac{x}{a}$.

For additional formulas, see analytic geometry.

14. Parabola



$$\text{Arc } BOC = s = \tfrac{1}{2}\sqrt{l^2 + 16h^2} + \frac{l^2}{8h}\log_e\frac{4h + \sqrt{l^2 + 16h^2}}{l}.$$

Let $R = \frac{h}{l}$. Then,

$$s = l\left(1 + \frac{8R^2}{3} - \frac{32R^4}{5} + \ldots\right) \text{ (approximate formula).}$$

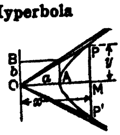
$$d = \frac{h}{l^2}(l^2 - l_1^2);\ l_1 = l\sqrt{\frac{h-d}{h}};\ h = \frac{dl^2}{l^2 - l_1^2};$$

A of segment $BOC = \frac{2hl}{3}$;

A of section $ABCD = \frac{2}{3}d\left(\frac{l^3 - l_1^3}{l^2 - l_1^2}\right)$.

For additional formulas, see analytic geometry.

15. Hyperbola



A of figure $OPAP'O = ab\log_e\left(\frac{x}{a} + \frac{y}{b}\right) = ab\cosh^{-1}\frac{x}{a}$;

A of segment $PAP' = xy - ab\log_e\left(\frac{x}{a} + \frac{y}{b}\right) = xy - ab\cosh^{-1}\frac{x}{a}$.

For additional formulas, see analytic geometry.

(*Table continued on p. 20-58*)

Table 32b. Plane Curvilinear Figures—*Continued*

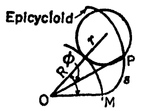
16. Cycloid

Arc $OP = s = 4\,r\left(1 - \cos\frac{\phi}{2}\right)$; Arc $OMN = 8\,r$;

A under curve $OMN = 3\,\pi\,r^2$.

For additional formulas, see analytic geometry.

17. Epicycloid

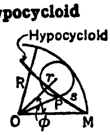


Arc $MP = s = \frac{4\,r}{R}(R + r)\left(1 - \cos\frac{R\phi}{2\,r}\right)$;

Area $MOP = A = \frac{r}{2\,R}(R + r)(R + 2\,r)\left(\frac{R\phi}{r} - \sin\frac{R\phi}{r}\right)$.

For additional formulas, see analytic geometry.

18. Hypocycloid

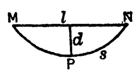


Arc $MP = s = \frac{4\,r}{R}(R - r)\left(1 - \cos\frac{R\phi}{2\,r}\right)$;

Area $MOP = A = \frac{r}{2\,R}(R - r)(R - 2\,r)\left(\frac{R\phi}{r} - \sin\frac{R\phi}{r}\right)$.

For additional formulas, see analytic geometry.

19. Catenary

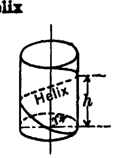


If d is small compared with l:

Arc $MPN = s = l\left[1 + \frac{2}{3}\left(\frac{2d}{l}\right)^2\right]$ (approximately).

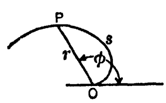
For additional formulas, see analytic geometry.

20. Helix



Let length of helix $= s$; radius of coil ($=$ radius of cylinder in figure) $= r$; distance advanced in one revolution $=$ pitch $= h$; and number of revolutions $= n$. Then,

21. Spiral of Archimedes



Let $a = \frac{r}{\phi}$. Then,

Arc $OP = s = \frac{a}{2}\left[\phi\sqrt{1 + \phi^2} + \log_e\left(\phi + \sqrt{1 + \phi^2}\right)\right]$.

For additional formulas, see analytic geometry.

22. Irregular Figure

Divide the figure into an *even* number, n, of strips by means of $(n + 1)$ ordinates, y_i, spaced equal distances, w. The area can then be determined approximately by any of the following formulas, which are presented in the order of usual increasing approach to accuracy. In any of the first three cases, the greater the number of strips used, the more nearly accurate will be the result.

(Approximate Formulas)

Trapezoidal Rule...

$$A = w\left[\frac{y_0 + y_n}{2} + y_1 + y_2 + \ldots + y_{n-1}\right];$$

Durand's Rule.....

$$A = w\,[0.4(y_0 + y_n) + 1.1(y_1 + y_{n-1}) + y_2 + y_3 + \ldots + y_{n-2}];$$

Simpson's Rule.....
(*n must* be even)

$$A = \frac{w}{3}\,[(y_0 + y_n) + 4\,(y_1 + y_3 + \ldots + y_{n-1}) + 2\,(y_2 + y_4 + \ldots + y_{n-2})];$$

Weddle's Rule.....
(for 6 strips only)

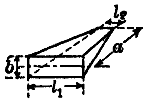
$$A = \frac{3w}{10}\,[5(y_1 + y_5) + 6\,y_3 + y_0 + y_2 + y_4 + y_6].$$

Areas of irregular surfaces can often be determined more quickly by such methods as plotting on squared paper and counting the squares; graphical coordinate representation (see analytic geometry); or use of a planimeter.

Table 32c. Solids Having Plane Surfaces

Notation. Lines, a, b, c, . . . ; altitude (perpendicular height), h; slant height, s; perimeter of base, p_b or p_B; perimeter of a right section, p_r; area of base, A_b or A_B; area of a right section, A_r; total area of lateral surfaces, A_l; total area of all surfaces, A_t; volume, V.

23. Wedge (and Right Triangular Prism)



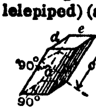
For Wedge:

(Narrow-side rectangular); $V = \frac{ab}{6}(2l_1 + l_2)$.

For Right Triangular Prism (or wedge having parallel triangular bases perpendicular to sides) $l_2 = l_1 = l$:

$V = \frac{abl}{2}$.

24. Rectangular Prism (or Rectangular Parallelepiped) (and Cube)



For Rectangular Prism or Rectangular Parallelepiped:

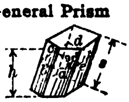
$A_l = 2c(a + b)$; $A_t = 2(de + ac + bc)$;

$V = A_r c = abc$.

For Cube (letting $b = c = a$):

$A_t = 6a^2$; $V = a^3$; Diagonal $= a\sqrt{3}$.

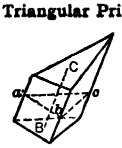
25. General Prism



$A_l = hp_b = sp_r = s(a + b + \ldots + n)$;

$V = hA_b = sA_r$.

26. General Truncated Prism (and Truncated Triangular Prism)



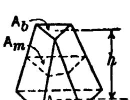
For General Truncated Prism:

$V = A_r \cdot$(length of line BC joining centers of gravity of bases).

For Truncated Triangular Prism:

$V = \frac{A_r}{3}(a + b + c)$.

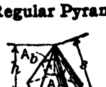
27. Prismatoid



Let area of mid-section $= A_m$.

$V = \frac{h}{6}(A_B + A_b + 4A_m)$.

28. Right Regular Pyramid (and Frustum of Right Regular Pyramid)



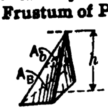
For Right Regular Pyramid:

$A_l = \frac{sp_B}{2}$; $V = \frac{hA_B}{3}$.

For Frustum of Right Regular Pyramid:

$A_l = \frac{s}{2}(p_B + p_b)$; $V = \frac{h}{3}\left(A_B + A_b + \sqrt{A_B A_b}\right)$.

29. General Pyramid (and Frustum of Pyramid)



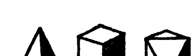
For General Pyramid:

$V = \frac{hA_B}{3}$.

For Frustum of General Pyramid:

$V = \frac{h}{3}\left(A_B + A_b + \sqrt{A_B A_b}\right)$.

30. Regular Polyhedrons



Tetrahedron Cube Octahedron

Dodecahedron Icosahedron

Let edge $= a$, and radius of inscribed sphere $= r$. Then,

$r = \frac{3V}{A_t}$, and:

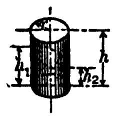
Number of Faces	*Form of Faces*	*Total Area A_t*	*Volume V*
4	Equilateral triangle	$1.7321\ a^2$	$0.1179\ a^3$
6	Square	$6.0000\ a^2$	$1.0000\ a^3$
8	Equilateral triangle	$3.4641\ a^2$	$0.4714\ a^3$
12	Regular pentagon	$20.6457\ a^2$	$7.6631\ a^3$
20	Equilateral triangle	$8.6603\ a^2$	$2.1817\ a^3$

(Factors shown only to four decimal places.)

Table 32d. Solids Having Curved Surfaces

Notation. Lines, a, b, c, . . . ; altitude (perpendicular height), h, h_1, . . . ; slant height, s; radius, r; perimeter of base, p_b; perimeter of a right section, p_r; angle in radians, ϕ; arc, s; chord of segment, l; rise, h; area of base, A_b or A_B; area of a right section, A_r; total area of convex surface, A_l; total area of all surfaces, A_t; volume, V.

31. Right Circular Cylinder (and Truncated Right Circular Cylinder)



For Right Circular Cylinder:

$A_l = 2\pi rh$; $A_t = 2\pi r\,(r + h)$;

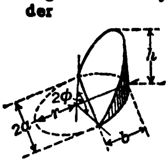
$V = \pi r^2 h$.

For Truncated Right Circular Cylinder:

$$A_l = \pi r\,(h_1 + h_2);\quad A_t = \pi r\left[h_1 + h_2 + r + \sqrt{r^2 + \left(\frac{h_1 - h_2}{2}\right)^2}\right];$$

$$V = \frac{\pi r^2}{2}(h_1 + h_2).$$

32. Ungula (Wedge) of Right Circular Cylinder



$$A_l = \frac{2\,rh}{b}\,[a + (b - r)\phi];$$

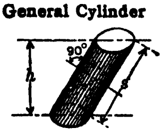
$$V = \frac{h}{3b}\,[a\,(3r^2 - a^2) + 3\,r^2\,(b - r)\phi]$$

$$= \frac{hr^3}{b}\left[\sin\phi - \frac{\sin^3\phi}{3} - \phi\cos\phi\right].$$

For Semicircular Base (letting $a = b = r$):

$A_l = 2\,rh$; $V = \frac{2\,r^2h}{3}$.

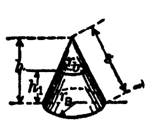
33. General Cylinder



$A_l = p_b h = p_r s$;

$V = A_b h = A_r s$.

34. Right Circular Cone (and Frustum of Right Circular Cone)



For Right Circular Cone:

$A_l = \pi r_B s = \pi r_B\sqrt{r_B^2 + h^2}$; $A_t = \pi r_B\,(r_B + s)$;

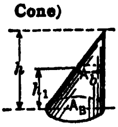
$V = \frac{\pi r_B^2 h}{3}$.

For Frustum of Right Circular Cone:

$s = \sqrt{h_1^2 + (r_B - r_b)^2}$; $A_l = \pi s\,(r_B + r_b)$;

$V = \frac{\pi h_1}{3}(r_B^2 + r_b^2 + r_B r_b)$.

35. General Cone (and Frustum of General Cone)



For General Cone:

$V = \frac{A_B h}{3}$.

For Frustum of General Cone:

$$V = \frac{h_1}{3}\left(A_B + A_b + \sqrt{A_B A_b}\right).$$

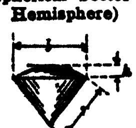
36. Sphere

Let diameter $= d$.

$A_t = 4\pi r^2 = \pi d^2$;

$V = \frac{4\pi r^3}{3} = \frac{\pi d^3}{6}$.

37. Spherical Sector (and Hemisphere)



For Spherical Sector:

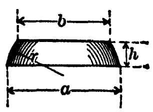
$A_t = \frac{\pi r}{2}(4h + l)$; $V = \frac{2\pi r^2 h}{3}$.

For Hemisphere (letting $h = \frac{l}{2} = r$):

$A_t = 3\pi r^2$; $V = \frac{2\pi r^3}{3}$.

Table 32d. Solids Having Curved Surfaces—*Continued*

38. Spherical Zone (and Spherical Segment)



For Spherical Zone Bounded by Two Planes:

$A_l = 2\pi rh;\ A_t = \frac{\pi}{4}(8\,rh + a^2 + b^2).$

For Spherical Zone Bounded by One Plane ($b = 0$):

$A_l = 2\pi rh = \frac{\pi}{4}(4\,h^2 + a^2);$

$A_t = \frac{\pi}{4}(8\,rh + a^2) = \frac{\pi}{2}(2\,h^2 + a^2).$

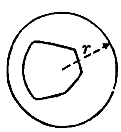
For Spherical Segment with Two Bases:

$V = \frac{\pi h}{24}(3\,a^2 + 3\,b^2 + 4\,h^2).$

For Spherical Segment with One Base ($b = 0$):

$V = \frac{\pi h}{24}(3\,a^2 + 4\,h^2) = \pi h^2\left(r - \frac{h}{3}\right).$

39. Spherical Polygon (and Spherical Triangle)



For Spherical Polygon:

Let sum of angles in radians $= \theta$ and number of sides $= n$.

$A = [\theta - (n - 2)\pi]r^2$

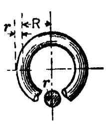
(The quantity $[\theta - (n - 2)\pi]$ is called "spherical excess.")

For Spherical Triangle ($n = 3$):

$A = (\theta - \pi)\,r^2$

For additional formulas, see trigonometry.

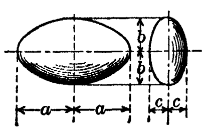
40. Torus



$A_l = 4\pi^2 Rr;$

$V = 2\pi^2 Rr^2.$

41. Ellipsoid (and Spheroids)



For Ellipsoid:

$V = \frac{4}{3}\pi abc.$

For Prolate Spheroid:

Let $c = b$ and $\frac{\sqrt{a^2 - b^2}}{a} = e.$

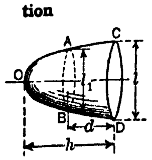
$A_t = 2\pi b^2 + 2\pi ab\frac{\sin^{-1} e}{e};\ V = \frac{4}{3}\pi ab^2.$

For Oblate Spheroid:

Let $c = a$ and $\frac{\sqrt{a^2 - b^2}}{a} = e.$

$A_t = 2\pi a^2 + \frac{\pi b^2}{e}\ln\left(\frac{1 + e}{1 - e}\right);\ V = \frac{4}{3}\pi a^2 b.$

42. Paraboloid of Revolution



A_l of segment $DOC = \frac{2\pi l}{3h^2}\left[\left(\frac{l^2}{16} + h^2\right)^{3/2} - \left(\frac{l}{4}\right)^3\right].$

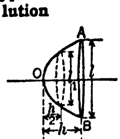
For Paraboloidal Segment with Two Bases:

V of $ABCD = \frac{\pi d}{8}(l^2 + l_1^2).$

For Paraboloidal Segment with One Base ($l_1 = 0$ and $d = h$):

V of $DOC = \frac{\pi h l^2}{8}.$

43. Hyperboloid of Revolution

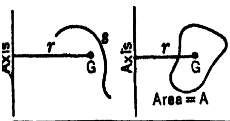


V of segment $AOB = \frac{\pi h}{24}(l^2 + 4l_1^2).$

(*Table continued on p. 20-62*)

Table 32d. Solids Having Curved Surfaces—*Continued*

44. Surface and Solid of Revolution



Let perpendicular distance from axis to center of gravity (G) of curve (or surface) $= r$. Curve (or surface) must not cross axis. Then,

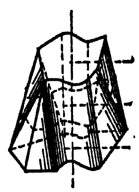
Area of Surface generated by curve revolving about axis:

$A_l = 2\pi rs.$

Volume of Solid generated by surface revolving about axis:

$V = 2\pi rA.$

45. Irregular Solid



One of the following methods can often be employed to determine the volume of an irregular solid with a reasonable approach to accuracy:

(a) Divide the solid into prisms, cylinders, etc., and sum their individual volumes.

(b) Divide one surface into triangles, after replacing curved lines by straight ones and curved surfaces by plane ones. Then multiply the area of each triangle by the mean depth of the section beneath it (which generally approximates the average of the depths at its corners). Sum the volumes thus obtained.

(c) If two surfaces are parallel, replace any curved lateral surfaces by plane surfaces best suited to the contour and then employ the prismatoidal formula.

6. ANALYTIC GEOMETRY

See Eshbach's *Handbook of Engineering Fundamentals*, pages 2-61 and 2-72 (Point and Line; Transformation of Coordinates; Conic Sections; Point, Line, and Plane; Quadric Surfaces).

TRIGONOMETRY

7. CIRCULAR FUNCTIONS OF PLANE ANGLES

Trigonometric Functions. The angle α in Fig. 1, is measured in degrees or radians, as defined in Table 14. The ratio of any two of the quantities x, y, or r determines the extent of the opening between the lines OP and OX. Since these ratios are functions of the angle they may be used to measure or construct it. The definitions and terms used to designate the functions are as follows:

$$\text{Sine } \alpha = \frac{y}{r} = \sin \alpha$$

$$\text{Cosine } \alpha = \frac{x}{r} = \cos \alpha$$

$$\text{Tangent } \alpha = \frac{y}{x} = \tan \alpha$$

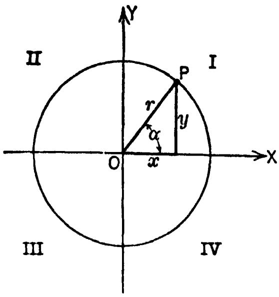


FIG. 1

Cotangent $\alpha = \frac{x}{y} = \cot\alpha$

Secant $\alpha = \frac{r}{x} = \sec\alpha$

Cosecant $\alpha = \frac{r}{y} = \csc\alpha$

Versine $\alpha = \frac{r-x}{r} = \text{vers}\,\alpha = 1 - \cos\alpha$

Coversine $\alpha = \frac{r-y}{r} = \text{covers}\,\alpha = 1 - \sin\alpha$

Haversine $\alpha = \frac{r-x}{2r} = \text{hav}\,\alpha = \frac{1}{2}\,\text{vers}\,\alpha$

Positive and Negative Values. An angle α (Fig. 1), if measured in a *counter-clockwise* direction, is said to be *positive*; if measured *clockwise, negative.* Following the convention that x is positive if measured along OX to the right of the OY axis and negative if measured to the left, and similarly, y is positive if measured along OY above the OX axis and negative if measured below, the signs of the trigonometric functions are different for angles in the quadrants I, II, III, and IV. The signs of the six most common functions are tabulated in Table 33.

Table 33. Signs of Trigonometric Functions

Quadrant	sin	cos	tan	cot	sec	csc
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−

Values of Trigonometric Functions are periodic, the period of the sin, cos, sec, csc being 2π radians, and that of the tan and cot, π radians. For example, in Fig. 2, (n an integer)

$$\sin(\alpha + 2\pi n) = \sin\alpha$$
$$\tan(\alpha + \pi n) = \tan\alpha$$

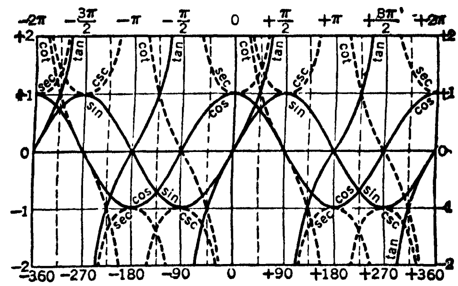


FIG. 2

Functions of angles in any quadrant (Fig. 1) in terms of angles in the first quadrant, and values of the functions for certain angles are given in Tables 34 and 35, respectively. Values of the functions for angles in increments of one-tenth of a degree may be obtained from the table of Trigonometric Functions beginning on p. 20-67.

Table 34. Functions of Angles in Any Quadrant in Terms of Angles in the First Quadrant

	$-\alpha$	$90° \pm \alpha$	$180° \pm \alpha$	$270° \pm \alpha$	$360° \pm \alpha$
sin	$-\sin\alpha$	$+\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$
cos	$+\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$	$+\cos\alpha$
tan	$-\tan\alpha$	$\mp\cot\alpha$	$\pm\tan\alpha$	$\mp\cot\alpha$	$\pm\tan\alpha$
cot	$-\cot\alpha$	$\mp\tan\alpha$	$\pm\cot\alpha$	$\mp\tan\alpha$	$\pm\cot\alpha$
sec	$+\sec\alpha$	$\mp\csc\alpha$	$-\sec\alpha$	$\pm\csc\alpha$	$+\sec\alpha$
csc	$-\csc\alpha$	$+\sec\alpha$	$\mp\csc\alpha$	$-\sec\alpha$	$\pm\csc\alpha$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha \quad \text{(Law of Cosines)}$$

$$\frac{a-b}{a+b} = \frac{\tan {}^1/_2 (\alpha - \beta)}{\tan {}^1/_2 (\alpha + \beta)} \quad \text{(Law of Tangents)}$$

$$\alpha + \beta + \gamma = 180°$$

$$a = b \cos \gamma + c \cos \beta;\ b = c \cos \alpha + a \cos \gamma;\ c = a \cos \beta + b \cos \alpha$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\sin \alpha = \frac{2}{bc} A;\ \sin \beta = \frac{2}{ca} A;\ \sin \gamma = \frac{2}{ab} A$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}};\ \sin \frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}};\ \sin \frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\cos \frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}};\ \cos \frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}};\ \cos \frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\tan \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}};\ \tan \frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}};\ \tan \frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

Solution of Plane Oblique Triangles.

Given a, b, c. (If logarithms are to be used, use 1.)

1. $r = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$; $A = \sqrt{s(s-a)(s-b)(s-c)} = rs$;

   $\tan \dfrac{\alpha}{2} = \dfrac{r}{s-a}$; $\tan \dfrac{\beta}{2} = \dfrac{r}{s-b}$; $\tan \dfrac{\gamma}{2} = \dfrac{r}{s-c}$.

2. $\cos \alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$; $\cos \beta = \dfrac{a^2 + c^2 - b^2}{2ac}$;

   $\cos \gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$, or $\gamma = 180° - (\alpha + \beta)$.

Given a, b, α.

$\sin \beta = \dfrac{b \sin \alpha}{a}$ (if $a > b$, $\beta < \dfrac{\pi}{2}$ and has only one value; if $b > a$, β has two values, β_1 and $\beta_2 = 180° - \beta_1$); $\gamma = 180° - (\alpha + \beta)$; $c = \dfrac{a \sin \gamma}{\sin \alpha}$; $A = {}^1/_2\, ab \sin \gamma$.

Given a, α, β.

$b = \dfrac{a \sin \beta}{\sin \alpha}$; $\gamma = 180° - (\alpha + \beta)$; $c = \dfrac{a \sin \gamma}{\sin \alpha}$; $A = {}^1/_2\, ab \sin \gamma$.

Given a, b, γ. (If logarithms are to be used, use 1.)

1. $\tan {}^1/_2 (\alpha - \beta) = \dfrac{a-b}{a+b} \cot {}^1/_2 \gamma$; ${}^1/_2 (\alpha + \beta) = 90° - {}^1/_2 \gamma$; $c = \dfrac{a \sin \gamma}{\sin \alpha}$

   $A = {}^1/_2\, ab \sin \gamma$.

2. $c = \sqrt{a^2 + b^2 - 2\, ab \cos \gamma}$; $\sin \alpha = \dfrac{a \sin \gamma}{c}$; $\beta = 180° - (\alpha + \gamma)$.

3. $\tan \alpha = \dfrac{a \sin \gamma}{b - a \cos \gamma}$; $\beta = 180° - (\alpha + \gamma)$; $c = \dfrac{a \sin \gamma}{\sin \alpha}$.

Solution of Plane Right Triangles. Let $\gamma = 90°$ and c be the hypotenuse. Given any two sides or one side and an acute angle α.

$$a = \sqrt{c^2 - b^2} = \sqrt{(c+b)(c-b)} = b \tan \alpha = c \sin \alpha.$$

$$b = \sqrt{c^2 - a^2} = \sqrt{(c+a)(c-a)} = \frac{a}{\tan \alpha} = c \cos \alpha.$$

$$c = \sqrt{a^2 + b^2} = \frac{a}{\sin \alpha} = \frac{b}{\cos \alpha}.$$

$$\alpha = \sin^{-1} \frac{a}{c} = \cos^{-1} \frac{b}{c} = \tan^{-1} \frac{a}{b};\ \beta = 90° - \alpha.$$

$$A = \frac{ab}{2} = \frac{a^2}{2 \tan \alpha} = \frac{b^2 \tan \alpha}{2} = \frac{c^2 \sin 2\alpha}{4}.$$

9. TRIGONOMETRIC FUNCTIONS

The following tables give the values of sin x, cos x, and tan x for values of x from 0 to 90° in intervals of 0.1 degree. By making use of the periodic character of these functions, the values can be determined from these tables for all values of x to an accuracy of 0.1 degree. (See Trigonometric Formulas.)

If the angle is given in radians multiply the number of radians by $\frac{180}{\pi}$ (57.295) to obtain the number of degrees.

Table 37. Trigonometric Functions **0.0°–15.9°**

Angle in Degrees	Name of Function	Value of Function for Each Tenth of a Degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210
7	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	9.1299	0.1317	0.1334	0.1352	0.1370	0.1388
8	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
9	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1663	0.1685	0.1702	0.1719
	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
10	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
11	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
12	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
13	sin	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402
	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
14	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
15	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849

Table 37. Trigonometric Functions—*Continued* **16.0°–35.9°**

Angle in Degrees	Name of Function	Value of Function for Each Tenth of a Degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0 2840	0 2857	0.2874	0.2890	0.2907
16	cos	0.9613	0.9608	0 9603	0 9598	0.9593	0 9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
	sin	0.2924	0 2940	0 2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
17	cos	0.9563	0 9558	0.9553	0.9548	0 9542	0.9537	0.9532	0 9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
	sin	0.3090	0 3107	0.3123	0.3140	0.3156	0 3173	0.3190	0.3206	0.3223	0.3239
18	cos	0 9511	0.9505	0 9500	0.9494	0 9489	0.9483	0.9478	0.9472	0.9466	0 9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
	sin	0 3256	0.3272	0 3289	0.3305	0 3322	0.3338	0 3355	0.3371	0.3387	0.3404
19	cos	0 9455	0 9449	0 9444	0 9438	0.9432	0.9426	0 9421	0 9415	0.9409	0 9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
	sin	0.3420	0.3437	0.3453	0.3469	0 3486	0.3502	0.3518	0.3535	0.3551	0 3567
20	cos	0.9397	0 9391	0.9385	0 9379	0 9373	0.9367	0.9361	0.9354	0 9348	0 9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819
	sin	0 3584	0.3600	0.3616	0.3633	0.3649	0 3665	0 3681	0.3697	0.3714	0 3730
21	cos	0 9336	0.9330	0.9323	0.9317	0 9311	0 9304	0 9298	0.9291	0 9285	0 9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
	sin	0 3746	0.3762	0.3778	0.3795	0.3811	0 3827	0 3843	0.3859	0 3875	0.3891
22	cos	0 9272	0.9265	0.9259	0.9252	0 9245	0.9239	0 9232	0.9225	0.9219	0 9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0 3987	0.4003	0.4019	0.4035	0.4051
23	cos	0.9205	0.9198	0 9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0 9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
24	cos	0.9135	0.9128	0.9121	0.9114	0 9107	0 9100	0 9092	0.9085	0 9078	0 9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
	sin	0.4226	0.4242	0 4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
25	cos	0.9063	0.9056	0 9048	0 9041	0.9033	0 9026	0 9018	0 9011	0.9003	0 8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0 4493	0.4509	0.4524
26	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0 8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
27	cos	0.8910	0.8902	0.8894	0.8886	0 8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
28	cos	0.8829	0.8821	0.8813	0.8805	0 8796	0.8788	0.8780	0.8771	0.8763	0 8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
29	cos	0.8746	0 8738	0.8729	0.8721	0.8712	0.8704	0.8695	0 8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
30	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
31	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
32	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
33	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
34	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976
	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
35	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239

Table 37. Trigonometric Functions—*Continued* 36.0°-55.9°

Angle in Degrees	Name of Function	Value of Function for Each Tenth of a Degree									
		0 0	0 1	0 2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0 5878	0 5892	0 5906	0 5920	0 5934	0 5948	0 5962	0 5976	0 5990	0 6004
36	cos	0.8090	0.8080	0 8070	0 8059	0 8049	0 8039	0 8028	0.8018	0 8007	0 7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
	sin	0 6018	0.6032	0.6046	0.6060	0 6074	0.6088	0 6101	0 6115	0 6129	0 6143
37	cos	0 7986	0.7976	0 7965	0 7955	0 7944	0.7934	0 7923	0 7912	0 7902	0 7891
	tan	0 7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
	sin	0 6157	0.6170	0 6184	0 6198	0.6211	0 6225	0 6239	0 6252	0 6266	0 6280
38	cos	0 7880	0 7869	0 7859	0 7848	0.7837	0 7826	0 7815	0 7804	0.7793	0 7782
	tan	0.7813	0.7841	0 7869	0 7898	0.7926	0.7954	0 7983	0.8012	0.8040	0.8069
	sin	0 6293	0 6307	0 6320	0 6334	0.6347	0 6361	0 6374	0 6388	0 6401	0 6414
39	cos	0 7771	0 7760	0 7749	0 7738	0 7727	0 7716	0 7705	0 7694	0 7683	0 7672
	tan	0 8098	0 8127	0 8156	0 8185	0 8214	0.8243	0 8273	0.8302	0 8332	0.8361
	sin	0 6428	0 6441	0 6455	0 6468	0 6481	0.6494	0.6508	0 6521	0 6534	0 6547
40	cos	0 7660	0 7649	0 7638	0 7627	0 7615	0 7604	0 7593	0 7581	0 7570	0 7559
	tan	0.8391	0.8421	0 8451	0.8481	0.8511	0.8541	0 8571	0 8601	0.8632	0.8662
	sin	0 6561	0 6574	0 6587	0 6600	0 6613	0 6626	0 6639	0 6653	0 6665	0 6678
41	cos	0 7547	0 7536	0 7524	0 7513	0 7501	0 7490	0 7478	0 7466	0 7455	0 7443
	tan	0 8693	0 8724	0.8754	0 8785	0 8816	0.8847	0 8878	0 8910	0.8941	0 8972
	sin	0 6691	0 6704	0 6717	0.6730	0 6743	0 6756	0 6769	0 6782	0 6794	0 6807
42	cos	0 7431	0 7420	0 7408	0 7396	0 7385	0 7373	0 7361	0 7349	0 7337	0 7325
	tan	0 9004	0.9036	0 9067	0.9099	0 9131	0.9163	0 9195	0 9228	0.9260	0.9293
	sin	0 6820	0 6833	0 6845	0 6858	0 6871	0 6884	0 6896	0 6909	0 6921	0 6934
43	cos	0 7314	0 7302	0 7290	0 7278	0 7266	0 7254	0 7242	0 7230	0 7218	0 7206
	tan	0.9325	0 9358	0 9391	0.9424	0 9457	0.9490	0.9523	0 9556	0.9590	0 9623
	sin	0 6947	0 6959	0 6972	0 6984	0 6997	0 7009	0 7022	0 7034	0 7046	0 7059
44	cos	0 7193	0 7181	0 7169	0 7157	0 7145	0 7133	0 7120	0 7108	0 7096	0 7083
	tan	0.9657	0 9691	0 9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0 9965
	sin	0 7071	0 7083	0 7096	0 7108	0 7120	0 7133	0.7145	0 7157	0 7169	0 7181
45	cos	0 7071	0.7059	0 7046	0 7034	0 7022	0 7009	0 6997	0 6984	0 6972	0 6959
	tan	1 0000	1 0035	1 0070	1.0105	1 0141	1 0176	1 0212	1.0247	1.0283	1.0319
	sin	0 7193	0 7206	0 7218	0 7230	0 7242	0.7254	0 7266	0 7278	0 7290	0 7302
46	cos	0 6947	0 6934	0 6921	0 6909	0 6896	0 6884	0 6871	0 6858	0 6845	0 6833
	tan	1 0355	1.0392	1.0428	1.0464	1 0501	1.0538	1.0575	1.0612	1.0649	1.0686
	sin	0 7314	0 7325	0 7337	0 7349	0 7361	0 7373	0.7385	0 7396	0.7408	0.7420
47	cos	0 6820	0 6807	0 6794	0 6782	0 6769	0.6756	0 6743	0 6730	0.6717	0 6704
	tan	1 0724	1 0761	1 0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
	sin	0 7431	0 7443	0 7455	0.7466	0 7478	0 7490	0 7501	0.7513	0.7524	0 7536
48	cos	0 6691	0 6678	0 6665	0 6652	0 6639	0 6626	0 6613	0 6600	0 6587	0 6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
	sin	0 7547	0 7559	0 7570	0 7581	0 7593	0.7604	0 7615	0 7627	0 7638	0 7649
49	cos	0 6561	0 6547	0 6534	0 6521	0 6508	0.6494	0 6481	0 6468	0 6455	0.6441
	tan	1 1504	1.1544	1.1585	1.1626	1.1667	1.1708	1 1750	1.1792	1.1833	1.1875
	sin	0 7660	0.7672	0 7683	0 7694	0 7705	0 7716	0 7727	0 7738	0.7749	0 7760
50	cos	0 6428	0 6414	0 6401	0 6388	0 6374	0 6361	0 6347	0 6334	0 6320	0 6307
	tan	1 1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
	sin	0.7771	0 7782	0 7793	0 7804	0 7815	0 7826	0 7837	0 7848	0 7859	0 7869
51	cos	0 6293	0 6280	0.6266	0.6252	0 6239	0 6225	0 6211	0 6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
	sin	0 7880	0.7891	0 7902	0 7912	0 7923	0.7934	0 7944	0.7955	0.7965	0.7976
52	cos	0 6157	0 6143	0 6129	0 6115	0 6101	0.6088	0 6074	0.6060	0 6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
	sin	0 7986	0 7997	0 8007	0 8018	0 8028	0.8039	0.8049	0 8059	0.8070	0.8080
53	cos	0.6018	0 6004	0 5990	0.5976	0 5962	0.5948	0 5934	0 5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
	sin	0.8090	0 8100	0 8111	0.8121	0.8131	0.8141	0 8151	0.8161	0.8171	0 8181
54	cos	0.5878	0 5864	0 5850	0 5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
	sin	0 8192	0.8202	0 8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
55	cos	0 5736	0 5721	0.5707	0 5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1 4335	1 4388	1.4442	1.4496	1.4550	1 4605	1.4659	1.4715	1.4770

Table 37. Trigonometric Functions—*Continued* **56.0°–75.9°**

Angle in Degrees	Name of Function	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.8290	0.8300	0 8310	0.8320	0.8329	0 8339	0.8348	0.8358	0.8368	0 8377
56	cos	0.5592	0.5577	0.5563	0.5548	0 5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
	sin	0 8387	0.8396	0.8406	0 8415	0.8425	0.8434	0 8443	0.8453	0 8462	0 8471
57	cos	0 5446	0 5432	0.5417	0 5402	0 5388	0.5373	0.5358	0.5344	0.5329	0 5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
	sin	0.8480	0 8490	0 8499	0 8508	0 8517	0.8526	0 8536	0 8545	0.8554	0 8563
58	cos	0.5299	0 5284	0 5270	0 5255	0 5240	0 5225	0 5210	0 5195	0 5180	0 5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
	sin	0 8572	0.8581	0.8590	0 8599	0.8607	0.8616	0 8625	0 8634	0 8643	0 8652
59	cos	0 5150	0 5135	0 5120	0 5105	0.5090	0 5075	0 5060	0.5045	0 5030	0 5015
	tan	1.6643	1.6709	1.6775	1 6842	1.6909	1.6977	1 7045	1.7113	1.7182	1.7251
	sin	0 8660	0.8669	0.8678	0 8686	0.8695	0.8704	0 8712	0.8721	0.8729	0 8738
60	cos	0 5000	0 4985	0 4970	0 4955	0.4939	0.4924	0 4909	0 4894	0 4879	0.4863
	tan	1.7321	1.7391	1.7461	1 7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
	sin	0.8746	0 8755	0 8763	0 8771	0 8780	0 8788	0 8796	0.8805	0.8813	0.8821
61	cos	0 4848	0 4833	0 4818	0 4802	0 4787	0.4772	0 4756	0 4741	0 4726	0 4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
	sin	0 8829	0 8838	0 8846	0.8854	0 8862	0 8870	0 8878	0 8886	0 8894	0 8902
62	cos	0 4695	0.4679	0 4664	0 4648	0 4633	0 4617	0.4602	0 4586	0 4571	0 4555
	tan	1.8807	1.8887	1.8967	1 9047	1.9128	1.9210	1.9292	1.9375	1.9458	1 9542
	sin	0 8910	0 8918	0 8926	0 8934	0 8942	0 8949	0 8957	0.8965	0.8973	0 8980
63	cos	0 4540	0.4524	0 4509	0 4493	0 4478	0 4462	0 4446	0 4431	0 4415	0 4399
	tan	1 9626	1.9711	1 9797	1.9883	1 9970	2.0057	2 0145	2 0233	2.0323	2.0413
	sin	0 8988	0 8996	0 9003	0 9011	0 9018	0 9026	0 9033	0 9041	0 9048	0 9056
64	cos	0 4384	0 4368	0 4352	0 4337	0 4321	0 4305	0 4289	0 4274	0 4258	0 4242
	tan	2 0503	2.0594	2 0686	2.0778	2 0872	2 0965	2.1060	2.1155	2.1251	2.1348
	sin	0 9063	0 9070	0.9078	0 9085	0 9092	0 9100	0.9107	0 9114	0 9121	0 9128
65	cos	0 4226	0.4210	0 4195	0 4179	0 4163	0 4147	0 4131	0 4115	0 4099	0 4083
	tan	2.1445	2.1543	2.1642	2 1742	2.1842	2.1943	2.2045	2 2148	2 2251	2 2355
	sin	0.9135	0.9143	0 9150	0 9157	0 9164	0 9171	0 9178	0 9184	0 9191	0 9198
66	cos	0.4067	0.4051	0 4035	0 4019	0 4003	0 3987	0 3971	0 3955	0 3939	0 3923
	tan	2 2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
	sin	0 9205	0.9212	0 9219	0 9225	0.9232	0 9239	0 9245	0.9252	0 9259	0 9265
67	cos	0 3907	0.3891	0.3875	0 3859	0.3843	0 3827	0.3811	0.3795	0 3778	0 3762
	tan	2 3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
	sin	0.9272	0.9278	0 9285	0 9291	0.9298	0 9304	0 9311	0 9317	0.9323	0 9330
68	cos	0 3746	0 3730	0 3714	0 3697	0.3681	0.3665	0 3649	0 3633	0 3616	0 3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
	sin	0 9336	0 9342	0.9348	0.9354	0.9361	0 9367	0 9373	0 9379	0 9385	0.9391
69	cos	0.3584	0.3567	0 3551	0.3535	0.3518	0.3502	0 3486	0.3469	0 3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
	sin	0.9397	0.9403	0 9409	0.9415	0 9421	0 9426	0 9432	0 9438	0 9444	0 9449
70	cos	0.3420	0.3404	0 3387	0.3371	0.3355	0 3338	0 3322	0.3305	0 3289	0 3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
	sin	0.9455	0.9461	0 9466	0.9472	0.9478	0 9483	0 9489	0.9494	0.9500	0 9505
71	cos	0.3256	0.3239	0 3223	0 3206	0 3190	0 3173	0 3156	0.3140	0 3123	0 3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
	sin	0.9511	0.9516	0 9521	0.9527	0 9532	0.9537	0.9542	0.9548	0.9553	0.9558
72	cos	0 3090	0.3074	0.3057	0.3040	0 3024	0.3007	0 2990	0.2974	0.2957	0 2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
73	cos	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
74	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
75	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0 2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3 9520	3.9812

Table 37. Trigonometric Functions—*Continued* 76.0°–89.9°

Angle in Degrees	Name of Function	Value of Function for Each Tenth of a Degree									
		0 0	0.1	0.2	0 3	0 4	0 5	0 6	0 7	0 8	0 9
	sin	0 9703	0 9707	0 9711	0 9715	0 9720	0 9724	0 9728	0 9732	0 9736	0.9740
76	cos	0 2419	0 2402	0 2385	0 2368	0 2351	0 2334	0 2317	0 2300	0.2284	0 2267
	tan	4.0108	4.0408	4 0713	4.1022	4.1335	4.1653	4.1976	4 2303	4 2635	4.2972
	sin	0.9744	0 9748	0 9751	0 9755	0 9759	0 9763	0 9767	0 9770	0 9774	0 9778
77	cos	0 2250	0 2232	0 2215	0 2198	0 2181	0.2164	0 2147	0 2130	0 2113	0 2096
	tan	4.3315	4 3662	4 4015	4.4374	4 4737	4.5107	4.5483	4 5864	4.6252	4.6646
	sin	0 9781	0 9785	0 9789	0 9792	0 9796	0 9799	0.9803	0.9806	0 9810	0.9813
78	cos	0 2079	0 2062	0 2045	0 2028	0 2011	0 1994	0 1977	0 1959	0 1942	0.1925
	tan	4 7046	4.7453	4 7867	4.8288	4.8716	4.9152	4 9594	5.0045	5.0504	5.0970
	sin	0 9816	0 9820	0 9823	0 9826	0 9829	0 9833	0 9836	0.9839	0 9842	0.9845
79	cos	0 1908	0 1891	0 1874	0 1857	0 1840	0 1822	0 1805	0 1788	0 1771	0.1754
	tan	5.1446	5.1929	5 2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
	sin	0 9848	0 9851	0 9854	0 9857	0 9860	0 9863	0 9866	0 9869	0.9871	0.9874
80	cos	0 1736	0 1719	0 1702	0 1685	0 1668	0 1650	0 1633	0 1616	0 1599	0.1582
	tan	5 6713	5.7297	5.7894	5 8502	5.9124	5.9758	6 0405	6.1066	6.1742	6.2432
	sin	0 9877	0.9880	0 9882	0 9885	0 9888	0 9890	0 9893	0 9895	0 9898	0 9900
81	cos	0.1564	0 1547	0 1530	0 1513	0 1495	0 1478	0 1461	0 1444	0.1426	0 1409
	tan	6.3138	6.3859	6 4596	6.5350	6 6122	6.6912	6.7720	6.8548	6.9395	7.0264
	sin	0 9903	0 9905	0 9907	0.9910	0 9912	0 9914	0 9917	0 9919	0.9921	0 9923
82	cos	0 1392	0 1374	0 1357	0 1340	0 1323	0 1305	0 1288	0 1271	0.1253	0 1236
	tan	7 1154	7.2066	7.3002	7 3962	7.4947	7 5958	7 6996	7 8062	7.9158	8.0285
	sin	0.9925	0 9928	0 9930	0 9932	0 9934	0 9936	0 9938	0 9940	0 9942	0.9943
83	cos	0 1219	0 1201	0 1184	0 1167	0 1149	0 1132	0 1115	0 1097	0 1080	0.1063
	tan	8.1443	8 2636	8 3863	8 5126	8 6427	8.7769	8 9152	9.0579	9.2052	9.3572
	sin	0 9945	0 9947	0 9949	0 9951	0 9952	0 9954	0 9956	0 9957	0 9959	0.9960
84	cos	0 1045	0 1028	0 1011	0 0993	0 0976	0 0958	0 0941	0 0924	0 0906	0.0889
	tan	9.5144	9 6768	9 8448	10.02	10 20	10.39	10.58	10 78	10.99	11.20
	sin	0 9962	0 9963	0 9965	0 9966	0 9968	0 9969	0 9971	0 9972	0.9973	0.9974
85	cos	0 0872	0 0854	0 0837	0 0819	0.0802	0 0785	0.0767	0 0750	0 0732	0 0715
	tan	11.43	11.66	11 91	12.16	12 43	12 71	13 00	13.30	13.62	13.95
	sin	0 9976	0 9977	0 9978	0 9979	0 9980	0 9981	0 9982	0 9983	0.9984	0.9985
86	cos	0 0698	0 0680	0 0663	0 0645	0 0628	0 0610	0 0593	0 0576	0 0558	0.0541
	tan	14.30	14 67	15.06	15 89	15 46	16 35	16 83	17 34	17 89	18.46
	sin	0 9986	0 9987	0 9988	0 9989	0 9990	0 9990	0 9991	0 9992	0.9993	0.9993
87	cos	0 0523	0 0506	0 0488	0 0471	0 0454	0 0436	0 0419	0 0401	0 0384	0 0366
	tan	19.08	19 74	20 45	21.20	22 02	22 90	23.86	24 90	26.03	27.27
	sin	0 9994	0 9995	0 9995	0 9996	0 9996	0 9997	0 9997	0 9997	0.9998	0.9998
88	cos	0 0349	0 0332	0 0314	0 0297	0 0279	0 0262	0 0244	0 0227	0 0209	0.0192
	tan	28.64	30 14	31 82	33 69	35 80	38 19	40.92	44 07	47.74	52 08
	sin	0 9998	0 9999	0 9999	0 9999	0 9999	1.000	1 000	1.000	1 000	1 000
89	cos	0 0175	0 0157	0 0140	0 0122	0 0105	0 0087	0 0070	0 0052	0.0035	0 0017
	tan	57 29	63 66	71 62	81 85	95 49	114 6	143 2	191 0	286 5	573 0

CALCULUS

10. DIFFERENTIAL CALCULUS

Definition of a Function. A variable y is said to be a function of a variable x if the value of y is determined when the value of x is given. In this definition, x is called the independent variable and y the dependent variable. The symbols $F(x), f(x), \phi(x)$, etc., are used to represent various functions of x, while the symbol $f(a)$ represents the value of $f(x)$ when $x = a$.

Limit, Derivative, Differential, Continuity. The *constant* a is said to be the *limit of a variable* x, if, as the variable changes its value, the numerical difference between the variable and constant becomes and remains less than any small positive constant which may be assigned. The symbol $x \to a$ or $\lim x = a$ is used for this definition. An example of a variable becoming equal to its limit is a swinging pendulum finally coming to rest. An illustration of a variable never reaching its limit is a polygon of n sides inscribed in a circle. No matter how large n is taken, the circumference of area of the polygon never equals that of the circle.

x becomes *infinitely large*, $x \to \infty$, means that the value of x becomes *larger* than any assigned *positive* number. $x \to -\infty$ means that the value of x becomes *smaller* than any assigned *negative* number.

Usually a change in x causes a change in y. A change in x is called an *increment* of x and is denoted by Δx. Similarly a change in y is denoted by Δy. If

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

has a *definite* value, it is called the *derivative* of y with respect to x and is denoted by $\frac{dy}{dx}$ or $f'(x)$.

The geometric interpretation of $f'(x)$ is

$$f'(x) = \frac{dy}{dx} = \tan \theta \tag{1}$$

or $f'(x)$ is equal to the slope of the tangent to the curve $y = f(x)$ at the point of contact $P(x, y)$ (Fig. 1).

$$\lim_{PR \to 0} \frac{RQ}{PR} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{dy}{dx} = f'(x) = \tan \theta \tag{2}$$

The differentials of x and y, respectively, are

$$dx = \Delta x$$

$$dy = f'(x)dx$$

Continuity of a Function in an Interval. A function is called *continuous* at $x = b$ if it has a definite value at b and approaches that value as a limit whenever x approaches b as a limit. The notion of continuity at a point suggests that the graph of the function is a smooth curve in the neighborhood of the point. The analytic conditions that $f(x)$ be continuous at b are that $f(b)$ have a definite value and that

$$|f(x) - f(b)| < \epsilon \text{ for } |x - b| < \delta(\epsilon) \tag{3}$$

where ϵ is any positive number which can be chosen as small as desired, while $\delta(\epsilon)$ depends on ϵ. The bars outside of $|f(x) - f(b)|$ show that the *absolute value* or value without the algebraic sign is to be taken; thus $|2 - 5| = |5 - 2| = 3$. A function which is continuous at each point of an interval is said to be continuous in that interval. An example of a continuous function is $f(x) = x^2$. The function $\phi(x) = \dfrac{1}{x - a}$ is continuous for all values of x other than $x = a$, at which point it becomes

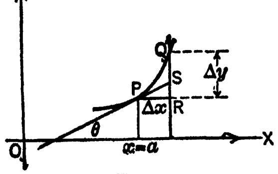


Fig. 1

infinite. Every differentiable function is continuous, although the reverse is not always true.

If, in the above definition of continuity, the number δ can be chosen the same for all points in the interval, the function is said to be *uniformly continuous* in that interval.

Derivatives of Higher Order. The *derivative* of the *first derivative* of y with respect to x is called the *second derivative of y with respect to x* and is denoted by

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \quad \text{or} \quad f''(x) \tag{4}$$

$$\frac{d^3y}{dx^3} \quad \text{or} \quad f'''(x) \tag{5}$$

is the third derivative of y with respect to x. If $y = f(x)$, the second differential of $f(x)$ is

$$d^2f = f''(x)dx^2 \tag{6}$$

Indeterminate Forms

If a function $f(x)$ for $x = a$ (where a can also be ∞) has no determined value but appears in one of the meaningless forms

$$\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty, \infty - \infty, 0^0, \infty^0, 0^\infty, 1^\infty$$

then it may happen that the $\lim\limits_{x\to a} f(x)$ has a definite value. For the determination of this limiting value, if it exists, the following rules can be used:

$\frac{0}{0}$. If $f(x) = \frac{\phi(x)}{\psi(x)}$, $\phi(a) = 0$, and $\psi(a) = 0$, then

$$\lim_{x\to a} f(x) = \lim_{x\to a}\frac{\phi'(x)}{\psi'(x)} \qquad \text{(L'Hospital's rule)} \tag{7}$$

If, however, $\phi'(a) = 0$ and $\psi'(a) = 0$, the rule is applied again, with the result

$$\lim_{x\to a}\frac{\phi(x)}{\psi(x)} = \lim_{\xi\to a}\frac{\phi'(\xi)}{\psi'(\xi)} = \frac{\phi''(a)}{\psi''(a)} \tag{8}$$

unless $\phi''(a) = 0$ and $\psi''(a) = 0$. In this case, the rule is applied again.

Example. Find the value of $\frac{\sin x}{x}$ for $x = 0$.

$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\cos x}{1} = 1.$$

$\frac{\infty}{\infty}$. If $f(x) = \frac{\phi(x)}{\psi(x)}$, $\phi(a) = \infty$, and $\psi(a) = \infty$, then

$$\lim_{x\to a}\frac{\phi(x)}{\psi(x)} = \lim_{x\to a}\frac{\phi'(x)}{\psi'(x)} \tag{9}$$

as before.

$0\cdot\infty$. If $f(x) = \phi(x)\cdot\psi(x)$, $\phi(a) = 0$, and $\psi(a) = \infty$, then place $\frac{1}{\psi(x)} = \omega(x)$ and obtain the previous case $\frac{0}{0}$.

$\infty - \infty$. If $f(x) = \phi(x) - \psi(x)$, $\phi(a) = \infty$, and $\psi(a) = \infty$, then place $\phi(x) = \frac{1}{u(x)}$, $\psi(x) = \frac{1}{v(x)}$ and obtain

$$f(x) = \frac{v(x) - u(x)}{u(x)v(x)} \tag{10}$$

which takes the form $\frac{0}{0}$.

0^0, ∞^0, 0^∞, 1^∞. An expression of the type $[\psi(x)]^{\phi(x)}$ may, for $x = a$, give rise to the forms 0^0, ∞^0, 0^∞, 1^∞.

Such an expression may be reduced to a type $\frac{0}{0}$ or $\frac{\infty}{\infty}$ by the use of logarithms. Thus,

$$\left.\begin{aligned} u &= [\psi(x)]^{\phi(x)} \\ \log_e u &= \phi(x)\cdot\log_e \psi(x) \end{aligned}\right\} \qquad (11)$$

If $\lim\limits_{x\to a} \phi(x)\cdot\log_e \psi(x)$ can be found by the previous methods, the limit approached by u can be found.

Example. $u = (1 - x)^{1/x}$ for $x = 0$

$$\log_e u = \frac{\log_e (1-x)}{x}$$

$$\lim_{x\to 0} \frac{\log_e (1-x)}{x} = -1$$

Therefore $\lim\limits_{x\to 0} \log_e u = -1$ and $\lim\limits_{x\to 0} u = e^{-1}$

Table 38. Differentiation Formulas

Let $u, v, w, \ldots$ be functions of x; a and n be constants; and e be the base of the natural or Napierian logarithms. Then $e = 2.7183^-$.

$$\frac{d}{dx} a = 0$$

$$\frac{d}{dx}(u + v + w + \ldots) = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} + \ldots$$

$$\frac{d}{dx} au = a\frac{du}{dx}$$

$$\frac{d}{dx} uv = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw\ldots) = \left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx} + \ldots\right)(uvw\ldots)$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx} u^n = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}\log_e u = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\log_{10} u = \frac{1}{u}\frac{du}{dx}\log_{10} e = (0.4343)\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx} e^u = e^u\frac{du}{dx}$$

$$\frac{d}{dx} u^v = vu^{v-1}\frac{du}{dx} + u^v\frac{dv}{dx}\log_e u$$

$$\frac{d}{dx} f(u) = \frac{df(u)}{du}\cdot\frac{du}{dx}$$

$$\frac{d^2 f(u)}{dx^2} = \frac{df(u)}{du}\cdot\frac{d^2 u}{dx^2} + \frac{d^2 f(u)}{du^2}\left(\frac{du}{dx}\right)^2$$

$$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u\tan u\frac{du}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u\cot u\frac{du}{dx}$$

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}\quad\left(-\frac{\pi}{2} \leqq \sin^{-1} u \leqq \frac{\pi}{2}\right)$$

$$\frac{d}{dx}\cos^{-1} u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}\quad(0 \leqq \cos^{-1} u \leqq \pi)$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\cot^{-1} u = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}\ *$$

$$\frac{d}{dx}\csc^{-1} u = -\frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}\ *$$

$$\frac{d}{dx}\sinh u = \cosh u\frac{du}{dx}$$

$$\frac{d}{dx}\cosh u = \sinh u\frac{du}{dx}$$

$$\frac{d}{dx}\tanh u = \operatorname{sech}^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\coth u = -\operatorname{csch}^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u\tanh u\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u\coth u\frac{du}{dx}$$

$$\frac{d}{dx}\sinh^{-1} u = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$$

$$\frac{d}{dx}\cosh^{-1} u = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}\tanh^{-1} u = \frac{1}{1-u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\coth^{-1} u = \frac{1}{1-u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sech}^{-1} u = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{csch}^{-1} u = -\frac{1}{u\sqrt{u^2+1}}\frac{du}{dx}$$

* For angles in the first and third quadrants. Use the opposite sign in the second and fourth quadrants.

SERIES EXPANSION OF FUNCTIONS. Taylor's Formula. If $f(x)$ and all its derivatives are continuous in the neighborhood of the point $x = a$, then $f(x)$ can be developed into a power series arranged according to ascending powers of $x - a$. The series is:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n-1)}(a)}{(n-1)!}(x-a)^{n-1} + R_n \tag{12}$$

where $n! = n(n-1)(n-2)\dots 2\cdot 1$, and is called factorial n, and

$$R_n = \frac{f^n(\xi)}{n!}(x-a)^n \tag{13}$$

is the *remainder* after n terms of the series and

$$\xi = a + \theta(x-a),\ 0 < \theta < 1 \tag{14}$$

Another form:

$$f(x+h) = f(x) + \frac{h}{1!}f'(x) + \frac{h^2}{2!}f''(x) + \dots + \frac{h^{n-1}}{(n-1)!}f^{(n-1)}(x) + R_n \tag{15}$$

where $$R_n = \frac{h^n}{n!}f^n(\xi),\ \xi = x + \theta h,\ 0 < \theta < 1 \tag{16}$$

Maclaurin's Form of Taylor's Formula (for $a = 0$).

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + R_n \tag{17}$$

where $$R_n = \frac{f^n(\xi)}{n!}x^n,\ \xi = \theta x,\ 0 < \theta < 1 \tag{18}$$

Care must be exercised in using these formulas that the ***series converges***, that is, $\lim_{n\to\infty} R_n = 0$. If Taylor's series converges rapidly, the sum of the first few terms gives a good approximation to $f(x)$ for values of x near $x = a$. If Maclaurin's series converges rapidly, the sum of the first few terms gives a good approximation to $f(x)$ for values of x near $x = 0$. That not all functions can be expanded into Maclaurin's series is shown by the examples; $f(x) = \frac{1}{x}$; $\frac{1}{x^2}$; $\sqrt{x}$, $\frac{1}{\sqrt{x}}$; $\log_e x$; $\cot x$; etc.

Example. Expand e^{nx} into a series of ascending powers of x.

$$\begin{aligned} f(x) &= e^{nx}, & f(0) &= 1\\ f'(x) &= ne^{nx}, & f'(0) &= n\\ f''(x) &= n^2e^{nx}, & f''(0) &= n^2\\ f'''(x) &= n^3e^{nx}, & f'''(0) &= n^3\\ f^{IV}(x) &= n^4e^{nx}, & f^{IV}(0) &= n^4 \end{aligned}$$

$$e^{nx} = 1 + \frac{n}{2!}x + \frac{n^2}{2!}x^2 + \frac{n^3}{3!}x^3 + \dots$$

If x and n are less than unity, this series converges rapidly.

Table 39. Functions Expanded into Series
($\log = \log_e$)

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}x^3 + \dots \qquad (x^2 < a^2)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \qquad (-\infty < x < \infty)$$

$$a^x = 1 + x\log a + \frac{(x\log a)^2}{2!} + \frac{(x\log a)^3}{3!} + \dots \qquad (-\infty < x < \infty)$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \dots \qquad (-\infty < x < \infty)$$

$$e^{\sin x} = 1 + x + \frac{x^2}{2!} - \frac{3x^4}{4!} - \frac{8x^5}{5!} - \frac{3x^6}{6!} + \frac{56x^7}{7!} + \dots \qquad (-\infty < x < \infty)$$

$$e^{\cos x} = e\left(1 - \frac{x^2}{2!} + \frac{4x^4}{4!} - \frac{31x^6}{6!} + \dots\right) \qquad (-\infty < x < \infty)$$

$$e^{\tan x} = 1 + x + \frac{x^2}{2!} + \frac{3x^3}{3!} + \frac{9x^4}{4!} + \frac{37x^5}{5!} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\log x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \dots \qquad \left(x > \frac{1}{2}\right)$$

$$\log x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \dots\right] \qquad (x > 0)$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \qquad (-1 < x < 1)$$

$$\log\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots\right] \qquad (-1 < x < 1)$$

Table 39. Functions Expanded into Series—*Continued*

$$\log\left(\frac{x+1}{x-1}\right) = 2\left[\frac{1}{x} + \frac{1}{3}\left(\frac{1}{x}\right)^3 + \frac{1}{5}\left(\frac{1}{x}\right)^5 + \dots\right] \qquad (-1 < x < 1)$$

$$\log\sin x = \log x - \frac{x^2}{6} - \frac{x^4}{180} - \frac{x^6}{2835} - \dots \qquad (-\pi < x < \pi)$$

$$\log\cos x = -\frac{x^2}{2} - \frac{x^4}{12} - \frac{x^6}{45} - \frac{17x^8}{2520} - \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\log\tan x = \log x + \frac{x^2}{3} + \frac{7x^4}{90} + \frac{62x^6}{2835} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \qquad (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \qquad (-\infty < x < \infty)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \frac{x^7}{4725} - \dots \qquad (-\pi < x < \pi)$$

$$\sec x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\csc x = \frac{1}{x} + \frac{x}{3!} + \frac{7x^3}{3\cdot 5!} + \frac{31x^5}{3\cdot 7!} + \dots \qquad (-\pi < x < \pi)$$

$$\sin^{-1} x = x + \frac{x^3}{2\cdot 3} + \frac{3x^5}{2\cdot 4\cdot 5} + \frac{3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} + \dots \qquad (-1 < x < 1)$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\tan^{-1} x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \dots \qquad (-1 > x > 1)$$

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

$$\sec^{-1} x = \frac{\pi}{2} - \frac{1}{x} - \frac{1}{6x^3} - \frac{3}{2\cdot 4\cdot 5x^5} - \frac{3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} - \dots \qquad (-1 > x > 1)$$

$$\csc^{-1} x = \frac{\pi}{2} - \sec^{-1} x$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots \qquad (-\infty < x < \infty)$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \dots \qquad (-\infty < x < \infty)$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} - \frac{x^7}{4725} + \dots \qquad (-\pi < x < \pi)$$

$$\operatorname{sech} x = 1 - \frac{x^2}{2!} + \frac{5x^4}{4!} - \frac{61x^6}{6!} + \frac{1385x^8}{8!} - \dots \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\operatorname{csch} x = \frac{1}{x} - \frac{x}{6} + \frac{7x^3}{360} - \frac{31x^5}{15120} + \dots \qquad (-\pi < x < \pi)$$

$$\sinh^{-1} x = x - \frac{x^3}{2\cdot 3} + \frac{3x^5}{2\cdot 4\cdot 5} - \frac{3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} + \dots \qquad (-1 < x < 1)$$

$$\sinh^{-1} x = \log 2x + \frac{1}{2\cdot 2x^2} - \frac{3}{2\cdot 4\cdot 4x^4} + \frac{3\cdot 5}{2\cdot 4\cdot 6\cdot 6x^6} + \dots \qquad (x > 1)$$

$$\cosh^{-1} x = \pm\left(\log 2x - \frac{1}{2\cdot 2x^2} - \frac{1\cdot 3}{2\cdot 4\cdot 4x^4} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6x^6} - \dots\right) \qquad (x > 1)$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \qquad (-1 < x < 1)$$

$$\coth^{-1} x = \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} + \dots \qquad (-1 > x > 1)$$

$$\operatorname{sech}^{-1} x = \pm\left(\log\frac{2}{x} - \frac{1}{2\cdot 2}x^2 - \frac{1\cdot 3}{2\cdot 4\cdot 4}x^4 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 6}x^6 - \dots\right) \qquad (0 < x < 1)$$

$$\operatorname{csch}^{-1} x = \frac{1}{x} - \frac{1}{2\cdot 3x^3} + \frac{3}{2\cdot 4\cdot 5x^5} - \frac{3\cdot 5}{2\cdot 4\cdot 6\cdot 7x^7} + \dots \qquad (-1 > x > 1)$$

11. INTEGRAL CALCULUS

Integration is the inverse operation of differentiation. It is indicated by the symbol $\int$ followed by the differential function to be integrated. For example,

$$d(x^3) = 3x^2\,dx \tag{1}$$

and

$$3\int x^2\,dx = x^3 + c \tag{2}$$

or in general

$$\int f'(x)\,dx = f(x) + c \tag{3}$$

where $f(x) + c$ is *indefinite* owing to the necessity of adding the arbitrary constant c whenever the operation of integration is performed.

$$\int_a^b f'(x)\,dx = f(b) - f(a) \tag{4}$$

is called the *definite integral*, where $f'(x)$ is continuous in the interval a to b or has at most a finite number of finite discontinuities in the interval.

Fundamental Forms. Since integration is an anti-differentiation operation, facility in integrating depends upon the ability to recognize the forms of the derivatives of elementary functions and also the knowledge of how to transform a given function into an elementary form. Table 40 is a list of fundamental forms to which many integrals may be reduced by simple transformation.

Integration by Parts is a method frequently employed if it is advantageous to consider the integral of a function as the integral of the product of a function by the differential of another function; then

$$\int u\,dv = uv - \int v\,du \tag{5}$$

since

$$d(uv) = u\,dv + v\,du \tag{6}$$

where u and v are both functions of a variable x.

Integration of Rational Fractions. If the degree of the numerator is not less than that of the denominator in the equation

$$R(x) = \frac{\phi(x)}{f(x)}$$

where $\phi(x)$ and $f(x)$ are rational polynomials, $R(x)$ can be put in the form of a polynomial and a remainder by performing the division indicated. The remainder may then be represented by partial fractions, and both the polynomial and remainder integrated directly by application of formulas 5, 6, 17, and 18 in Table 40.

Integration of Irrational Functions may frequently be accomplished by reducing them to rational integrals by changing the variable. The method is called integration by rationalization or integration by substitution.

For example, integrals containing the following forms may be rationalised by the substitutions indicated:

Form	*Substitution*
$f[(ax+b)^{p/q}]\,dx$	let $ax + b = y^q$
$f[(ax+b)^{p/q}(ax+b)^{r/s}]\,dx$	let $ax + b = y^n$, where n is the L.C.M. of q, s
$f[x, \sqrt{x^2+ax+b}]\,dx$	let $\sqrt{x^2+ax+b} = y - x$
$f[x, \sqrt{-x^2+ax+b}]\,dx$	let $\sqrt{-x^2+ax+b} = \sqrt{(\alpha - x)(\beta + x)}$ $= (\alpha - x)y$ or $= (\beta + x)y$
$f[\sin x, \cos x]\,dx$	let $\tan \frac{x}{2} = y$
$f[x, \sqrt{a^2 - x^2}]\,dx$	let $x = a \sin y$
$f[x, \sqrt{x^2 - a^2}]\,dx$	let $x = a \sec y$ or $x = a \cosh y$
$f[x, \sqrt{x^2 + a^2}]\,dx$	let $x = a \tan y$ or $x = a \sinh y$

Table 40. Table of Integrals

Elementary Indefinite Integrals

1. $\int a\,dx = ax + c$
2. $\int (u + v + w + \dots)dx = \int u\,dx + \int v\,dx + \int w\,dx + \dots$
3. $\int u\,dv = uv - \int v\,du$, integration by parts
4. $\int f(x)\,dx = \int f[\phi(y)]\phi'(y)dy,\ x = \phi(y)$, change of variable
5. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c,\ (n \neq -1)$
6. $\int \dfrac{dx}{x} = \log_e x + c = \log_e c_1 x,\ [\log_e x = \log_e(-x) + (2k+1)\pi i]$
7. $\int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$
8. $\int a^x\,dx = \dfrac{a^x}{\log_e a} + c$
9. $\int a^x \log_e a\,dx = a^x + c$
10. $\int \sin ax\,dx = -\dfrac{1}{a}\cos ax + c$
11. $\int \cos ax\,dx = \dfrac{1}{a}\sin ax + c$
12. $\int \tan ax\,dx = -\dfrac{1}{a}\log_e \cos ax + c = \dfrac{1}{a}\log_e \sec ax + c$
13. $\int \cot ax\,dx = \dfrac{1}{a}\log_e \sin ax + c = -\dfrac{1}{a}\log_e \csc ax + c$
14. $\int \sec ax\,dx = \dfrac{1}{a}\log_e(\sec ax + \tan ax) + c = \dfrac{1}{a}\log_e \tan\left(\dfrac{ax}{2} + \dfrac{\pi}{4}\right) + c$
15. $\int \csc ax\,dx = \dfrac{1}{a}\log_e(\csc ax - \cot ax) + c = \dfrac{1}{a}\log_e \tan\dfrac{ax}{2} + c$
16. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\dfrac{x}{a} + c = -\cos^{-1}\dfrac{x}{a} + c \quad (x^2 < a^2)$
17. $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c = -\dfrac{1}{a}\cot^{-1}\dfrac{x}{a} + c$
18. $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\log\dfrac{x-a}{x+a} = -\dfrac{1}{a}\tanh^{-1}\dfrac{x}{a} = -\dfrac{1}{a}\coth^{-1}\dfrac{x}{a}$
19. $\int \sinh ax\,dx = \dfrac{1}{a}\cosh ax + c$
20. $\int \cosh ax\,dx = \dfrac{1}{a}\sinh ax + c$
21. $\int \tanh ax\,dx = \dfrac{1}{a}\log_e(\cosh ax) + c$
22. $\int \coth ax\,dx = \dfrac{1}{a}\log_e(\sinh ax) + c$
23. $\int \operatorname{sech} ax\,dx = \dfrac{1}{a}\sin^{-1}(\tanh ax) + c$
24. $\int \operatorname{csch} ax\,dx = \dfrac{1}{a}\log_e\left(\tanh\dfrac{ax}{2}\right) + c$
25. $\int \sin^2 ax\,dx = \dfrac{1}{2}x - \dfrac{1}{2a}\sin ax\cos ax + c = \dfrac{1}{2}x - \dfrac{1}{4a}\sin 2ax + c$
26. $\int \cos^2 ax\,dx = \dfrac{1}{2}x + \dfrac{1}{2a}\sin ax\cos ax + c = \dfrac{1}{2}x + \dfrac{1}{4a}\sin 2ax + c$
27. $\int \tan^2 ax\,dx = \dfrac{1}{a}\tan ax - x + c$
28. $\int \cot^2 ax\,dx = -\dfrac{1}{a}\cot ax - x + c$

Elementary Indefinite Integrals—*Continued*

29. $\int \sec^2 ax\, dx = \frac{1}{a} \tan ax + c$

30. $\int \csc^2 ax\, dx = -\frac{1}{a} \cot ax + c$

31. $\int \sin^{-1} ax\, dx = x \sin^{-1} ax + \frac{1}{a}\sqrt{1 - a^2 x^2} + c$

32. $\int \cos^{-1} ax\, dx = x \cos^{-1} ax - \frac{1}{a}\sqrt{1 - a^2 x^2} + c$

33. $\int \tan^{-1} ax\, dx = x \tan^{-1} ax - \frac{1}{2a} \log_e (1 + a^2 x^2) + c$

34. $\int \cot^{-1} ax\, dx = x \cot^{-1} ax + \frac{1}{2a} \log_e (1 + a^2 x^2) + c$

35. $\int \sec^{-1} ax\, dx = x \sec^{-1} ax - \frac{1}{a} \log_e (ax + \sqrt{a^2 x^2 - 1}) + c$

36. $\int \csc^{-1} ax\, dx = x \csc^{-1} ax + \frac{1}{a} \log_e (ax + \sqrt{a^2 x^2 - 1}) + c$

Integrals Involving $(ax + b)$

37. $\int (ax + b)^n\, dx = \frac{1}{a(n + 1)} (ax + b)^{n+1} \ (n \neq -1)$

38. $\int \frac{dx}{ax + b} = \frac{1}{a} \log_e (ax + b)$

39. $\int x(ax + b)^n\, dx = \frac{1}{a^2(n + 2)} (ax + b)^{n+2} - \frac{b}{a^2(n + 1)} (ax + b)^{n+1} (n \neq -1, -2)$

40. $\int \frac{x\, dx}{ax + b} = \frac{x}{a} - \frac{b}{a^2} \log_e (ax + b)$

41. $\int \frac{x\, dx}{(ax + b)^2} = \frac{b}{a^2(ax + b)} + \frac{1}{a^2} \log_e (ax + b)$

42. $\int \frac{x^2\, dx}{ax + b} = \frac{1}{a^3}\left[\frac{1}{2}(ax + b)^2 - 2b(ax + b) + b^2 \log_e (ax + b)\right]$

43. $\int \frac{x^2\, dx}{(ax + b)^2} = \frac{1}{a^3}\left[(ax + b) - 2b \log_e (ax + b) - \frac{b^2}{ax + b}\right]$

44. $\int \frac{x^2\, dx}{(ax + b)^3} = \frac{1}{a^3}\left[\log_e (ax + b) + \frac{2b}{ax + b} - \frac{b^2}{2(ax + b)^2}\right]$

45.

46. $\int \frac{dx}{x^2 (ax + b)} = -\frac{1}{bx} + \frac{a}{b^2} \log_e \frac{ax + b}{x}$

47. $\int \frac{dx}{x(ax + b)^2} = \frac{1}{b(ax + b)} - \frac{1}{b^2} \log_e \frac{ax + b}{x}$

48. $\int \frac{dx}{x^2 (ax + b)^2} = -\frac{b + 2ax}{b^2 x(ax + b)} + \frac{2a}{b^3} \log_e \frac{ax + b}{x}$

49. $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \log_e \sqrt{ax + b} - \sqrt{b}$ (*b* positive)

50. $\int \frac{}{x\sqrt{ax + b}} = \frac{}{\sqrt{-b}} \quad \sqrt{-b}$ (*b* negative)

51. $\int \frac{\sqrt{ax + b}}{x}\, dx = 2\sqrt{ax + b} + \sqrt{b} \log_e \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}}$ (*b* positive)

52. $\int \frac{\sqrt{ax + b}}{x}\, dx = 2\sqrt{ax + b} - 2\sqrt{-b} \tan^{-1} \sqrt{\frac{ax + b}{-b}}$ (*b* negative)

(Table continued on p. 20-80)

Integrals Involving $(ax+b)$—*Continued*

53. $$\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b\sqrt{b}}\log_e \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}} \quad (b \text{ positive})$$

54. $$\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{b\sqrt{-b}}\tan^{-1}\sqrt{\frac{ax+b}{-b}} \quad (b \text{ negative})$$

55. $$\int \frac{ax+b}{fx+g}\,dx = \frac{ax}{f} + \frac{bf-ag}{f^2}\log_e(fx+g)$$

56. $$\int \frac{dx}{(ax+b)(fx+g)} = \frac{1}{bf-ag}\log_e\left(\frac{fx+g}{ax+b}\right) \quad (ag \neq bf)$$

57. $$\int \frac{x\,dx}{(ax+b)(fx+g)} = \frac{1}{bf-ag}\left[\frac{b}{a}\log_e(ax+b) - \frac{g}{f}\log_e(fx+g)\right] \quad (ag \neq bf)$$

58. $$\int \frac{dx}{(ax+b)^2(fx+g)} = \frac{1}{bf-ag}\left(\frac{1}{ax+b} + \frac{f}{bf-ag}\log_e\frac{fx+g}{ax+b}\right) \quad (ag \neq bf)$$

Integrals Involving (ax^n+b)

59. $$\int (ax^2+b)^n x\,dx = \frac{1}{2a}\frac{(ax^2+b)^{n+1}}{n+1} \quad (n \neq -1)$$

60. $$\int \frac{dx}{ax^2+b} = \frac{1}{\sqrt{ab}}\tan^{-1}\left(x\sqrt{\frac{a}{b}}\right) \quad (a \text{ and } b \text{ positive})$$

61. $$\int \frac{dx}{ax^2+b} = \frac{1}{2\sqrt{-ab}}\log_e\frac{x\sqrt{a}-\sqrt{-b}}{x\sqrt{a}+\sqrt{-b}} \quad (a \text{ positive, } b \text{ negative})$$
$$= \frac{1}{2\sqrt{-ab}}\log_e\frac{\sqrt{b}+x\sqrt{-a}}{\sqrt{b}-x\sqrt{-a}} \quad (a \text{ negative, } b \text{ positive})$$

62. $$\int \frac{dx}{x(ax^2+b)} = \frac{1}{2b}\log_e\frac{x^2}{ax^2+b}$$

63. $$\int \frac{dx}{(ax^2+b)^n} = \frac{1}{2(n-1)b}\frac{x}{(ax^2+b)^{n-1}} + \frac{2n-3}{2(n-1)b}\int \frac{dx}{(ax^2+b)^{n-1}} \quad (n \text{ integer} > 1)$$

64. $$\int \frac{x^2\,dx}{ax^2+b} = \frac{x}{a} - \frac{b}{a}\int \frac{dx}{ax^2+b}$$

65. $$\int \frac{x^2\,dx}{(ax^2+b)^n} = -\frac{1}{2(n-1)a}\frac{x}{(ax^2+b)^{n-1}} + \frac{1}{2(n-1)a}\int \frac{dx}{(ax^2+b)^{n-1}} \quad (n \text{ integer} > 1)$$

66. $$\int \frac{dx}{x^2(ax^2+b)^n} = \frac{1}{b}\int \frac{dx}{x^2(ax^2+b)^{n-1}} - \frac{a}{b}\int \frac{dx}{(ax^2+b)^n} \quad (n = \text{positive integer})$$

67. $$\int \sqrt{ax^2+b}\,dx = \frac{x}{2}\sqrt{ax^2+b} + \frac{b}{2\sqrt{a}}\log_e(x\sqrt{a}+\sqrt{ax^2+b}) \quad (a \text{ positive})$$

68. $$\int \sqrt{ax^2+b}\,dx = \frac{x}{2}\sqrt{ax^2+b} + \frac{b}{2\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right) \quad (a \text{ negative})$$

69. $$\int \frac{dx}{\sqrt{ax^2+b}} = \frac{1}{\sqrt{a}}\log_e(x\sqrt{a}+\sqrt{ax^2+b}) \quad (a \text{ positive})$$

70. $$\int \frac{dx}{\sqrt{ax^2+b}} = \frac{1}{\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right) \quad (a \text{ negative})$$

71. $$\int \frac{x\,dx}{\sqrt{ax^2+b}} = \frac{1}{a}\sqrt{ax^2+b}$$

72. $$\int \frac{\sqrt{ax^2+b}}{x}\,dx = \sqrt{ax^2+b} + \sqrt{b}\log_e\frac{\sqrt{ax^2+b}-\sqrt{b}}{x} \quad (b \text{ positive})$$

73. $$\int \frac{\sqrt{ax^2+b}}{x}\,dx = \sqrt{ax^2+b} - \sqrt{-b}\tan^{-1}\frac{\sqrt{ax^2+b}}{\sqrt{-b}} \quad (b \text{ negative})$$

74. $$\int x\sqrt{ax^2+b}\,dx = \frac{1}{3a}(ax^2+b)^{3/2}$$

Integrals Involving $(ax^n + b)$—*Continued*

75. $\int x^2 \sqrt{ax^2+b}\, dx = \frac{x}{4a}(ax^2+b)^{3/2} - \frac{bx}{8a}\sqrt{ax^2+b} - \frac{b^2}{8a\sqrt{a}} \log_e (x\sqrt{a} + \sqrt{ax^2+b})$ (a positive)

76. $\int x^2 \sqrt{ax^2+b}\, dx = \frac{x}{4a}(ax^2+b)^{3/2} - \frac{bx}{8a}\sqrt{ax^2+b} - \frac{b^2}{8a\sqrt{-a}} \sin^{-1}\left(x\sqrt{\frac{-a}{b}}\right)$ (a negative)

77. $\int \frac{dx}{x\sqrt{ax^2+b}} = \frac{1}{\sqrt{b}} \log_e \frac{\sqrt{ax^2+b} - \sqrt{b}}{x}$ (b positive)

78. $\int \frac{dx}{x\sqrt{ax^2+b}} = \frac{1}{\sqrt{-b}} \sec^{-1}\left(x\sqrt{-\frac{a}{b}}\right)$ (b negative)

79. $\int \frac{x^2\, dx}{\sqrt{ax^2+b}} = \frac{x}{2a}\sqrt{ax^2+b} - \frac{b}{2a\sqrt{a}} \log_e (x\sqrt{a} + \sqrt{ax^2+b})$ (a positive)

80. $\int \frac{x^2\, dx}{\sqrt{ax^2+b}} = \frac{x}{2a}\sqrt{ax^2+b} - \frac{b}{2a\sqrt{-a}} \sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right)$ (a negative)

81. $\int \frac{\sqrt{ax^2+b}}{x^2}\, dx = -\frac{\sqrt{ax^2+b}}{x} + \sqrt{a} \log_e (x\sqrt{a} + \sqrt{ax^2+b})$ (a positive)

82. $\int \frac{\sqrt{ax^2+b}}{x^2}\, dx = -\frac{\sqrt{ax^2+b}}{x} - \sqrt{-a} \sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right)$ (a negative)

83. $\int \frac{dx}{x(ax^n+b)} = \frac{1}{bn} \log_e \frac{x^n}{ax^n+b}$

84. $\int \frac{dx}{x\sqrt{ax^n+b}} = \frac{1}{n\sqrt{b}} \log_e \frac{\sqrt{ax^n+b} - \sqrt{b}}{\sqrt{ax^n+b} + \sqrt{b}}$ (b positive)

85. $\int \frac{dx}{x\sqrt{ax^n+b}} = \frac{2}{n\sqrt{-b}} \sec^{-1}\sqrt{-\frac{ax^n}{b}}$ (b negative)

Integrals Involving $(ax^2 + bx + d)$

86. $\int \frac{dx}{ax^2+bx+d} = \frac{1}{\sqrt{b^2-4ad}} \log_e \frac{2ax+b-\sqrt{b^2-4ad}}{2ax+b+\sqrt{b^2-4ad}}$ ($b^2 > 4ad$)

87. $\int \frac{dx}{ax^2+bx+d} = \frac{2}{\sqrt{4ad-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ad-b^2}}$ ($b^2 < 4ad$)

88. $\int \frac{dx}{ax^2+bx+d} = -\frac{2}{2ax+b}$ ($b^2 = 4ad$)

89. $\int \frac{dx}{\sqrt{ax^2+bx+d}} = \frac{1}{\sqrt{a}} \log_e \left(2ax + b + 2\sqrt{a(ax^2+bx+d)}\right)$ (a positive)

90. $\int \frac{dx}{\sqrt{ax^2+bx+d}} = \frac{1}{\sqrt{-a}} \sin^{-1} \frac{-2ax-b}{\sqrt{b^2-4ad}}$ (a negative)

91. $\int \frac{x\, dx}{ax^2+bx+d} = \frac{1}{2a} \log_e (ax^2+bx+d) - \frac{b}{2a}\int \frac{dx}{ax^2+bx+d}$

92. $\int \frac{x\, dx}{\sqrt{ax^2+bx+d}} = \frac{\sqrt{ax^2+bx+d}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{ax^2+bx+d}}$

93. $\int \frac{dx}{x\sqrt{ax^2+bx+d}} = -\frac{1}{\sqrt{d}} \log_e \left(\frac{\sqrt{ax^2+bx+d}+\sqrt{d}}{x} + \frac{b}{2\sqrt{d}}\right)$ (d positive)

94. $\int \frac{dx}{x\sqrt{ax^2+bx+d}} = \frac{1}{\sqrt{-d}} \sin^{-1} \frac{bx+2d}{x\sqrt{b^2-4ad}}$ (d negative)

95. $\int \frac{dx}{x\sqrt{ax^2+bx}} = -\frac{2}{bx}\sqrt{ax^2+bx}$

(Table continued on p. 20-82)

Integrals Involving $(ax^2 + bx + d)$—*Continued*

96. $\int \sqrt{ax^2 + bx + d}\, dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + d} + \frac{4ad - b^2}{8a} \int \frac{dx}{\sqrt{ax^2 + bx + d}}$

97. $\int x \sqrt{ax^2 + bx + d}\, dx = \frac{(ax^2 + bx + d)^{3/2}}{3a} - \frac{b}{2a} \int \sqrt{ax^2 + bx + d}\, dx$

Integrals Involving $\sin^n ax$

98. $\int \sin^3 ax\, dx = -\frac{1}{a} \cos ax + \frac{1}{3a} \cos^3 ax$

99. $\int \sin^4 ax\, dx = \frac{3}{8} x - \frac{1}{4a} \sin 2ax + \frac{1}{32a} \sin 4ax$

100. $\int \sin^n ax\, dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax\, dx$ (n = positive integer)

101. $\int x \sin ax\, dx = \quad \sin ax \qquad x \cos ax$

102. $\int x^2 \sin ax\, dx = \frac{2x}{a^2} \sin ax - \left(\frac{x^2}{a} - \frac{2}{a^3}\right) \cos ax$

103. $\int x^3 \sin ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right) \sin ax - \left(\frac{x^3}{a} - \frac{6x}{a^3}\right) \cos ax$

104. $\int x^n \sin ax\, dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax\, dx \quad (n > 0)$

105. $\int \frac{\sin ax}{x^n}\, dx = -\frac{1}{n-1} \frac{\sin ax}{x^{n-1}} + \frac{a}{n-1} \int \frac{\cos ax}{x^{n-1}}\, dx$

106. $\int \frac{dx}{\sin^n ax} = -\frac{1}{a(n-1)} \frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} ax}$ (n integer > 1)

107. $\int \frac{x\, dx}{\sin^2 ax} = -\frac{x}{a} \cot ax + \frac{1}{a^2} \log_e \sin ax$

108. $\int \frac{dx}{1 + \sin ax} = -\frac{1}{a} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

109. $\int \frac{dx}{1 - \sin ax} = \frac{1}{a} \cot\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

110. $\int \frac{x\, dx}{1 + \sin ax} = -\frac{x}{a} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2} \log_e \cos\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

111. $\int \frac{x\, dx}{1 - \sin ax} = \frac{x}{a} \cot\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2} \log_e \sin\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

112. $\int \frac{dx}{b + d \sin ax} = \frac{-2}{a\sqrt{b^2 - d^2}} \tan^{-1}\left[\sqrt{\frac{b-d}{b+d}} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right] \quad (b^2 > d^2)$

113. $\int \frac{dx}{b + d \sin ax} = \frac{-1}{a\sqrt{d^2 - b^2}} \log_e \frac{d + b \sin ax + \sqrt{d^2 - b^2} \cos ax}{b + d \sin ax} \quad (d^2 > b^2)$

114. $\int \sin ax \sin bx\, dx = \frac{\sin (a-b)x}{2(a-b)} - \frac{\sin (a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$

Integrals Involving $\cos^n ax$

115. $\int \cos^3 ax\, dx = \frac{1}{a} \sin ax - \frac{1}{3a} \sin^3 ax$

116. $\int \cos^4 ax\, dx = \frac{3}{8} x + \frac{1}{4a} \sin 2ax + \frac{1}{32a} \sin 4ax$

117. $\int \cos^n ax\, dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax\, dx$ (n = positive integer)

118. $\int x \cos ax\, dx = \frac{\cos ax}{a^2} + x \sin ax$

Integrals Involving $\cos^n ax$—Continued

119. $\int x^2 \cos ax\, dx = \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right) \sin ax$

120. $\int x^3 \cos ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right) \cos ax + \left(\frac{x^3}{a} - \frac{6x}{a^3}\right) \sin ax$

121. $\int x^n \cos ax\, dx = \frac{x^n \sin ax}{a} - \frac{n}{a}\int x^{n-1} \sin ax\, dx \quad (n > 0)$

122. $\int \frac{\cos ax}{x^n} dx = -\frac{1}{n-1} \frac{\cos ax}{x^{n-1}} - \frac{a}{n-1}\int \frac{\sin ax}{x^{n-1}} dx$

123. $\int \frac{dx}{\cos^n ax} = \frac{1}{a(n-1)} \frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} ax} \quad (n \text{ integer} > 1)$

124. $\int \frac{x\, dx}{\cos^2 ax} = \frac{x}{a} \tan ax + \frac{1}{a^2} \log_e \cos ax$

125. $\int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2}$

126. $\int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot \frac{ax}{2}$

127. $\int \frac{x\, dx}{1 + \cos ax} = \frac{x}{a} \tan \frac{ax}{2} + \frac{2}{a^2} \log_e \cos \frac{ax}{2}$

128. $\int \frac{x\, dx}{1 - \cos ax} = -\frac{x}{a} \cot \frac{ax}{2} + \frac{2}{a^2} \log_e \sin \frac{ax}{2}$

129. $\int \frac{dx}{b + d \cos ax} = \frac{2}{a\sqrt{b^2 - d^2}} \tan^{-1}\left(\sqrt{\frac{b-d}{b+d}} \tan \frac{ax}{2}\right) \quad (b^2 > d^2)$

130. $\int \frac{dx}{b + d \cos ax} = \frac{1}{a\sqrt{d^2 - b^2}} \log_e \frac{d + b \cos ax + \sqrt{d^2 - b^2} \sin ax}{b + d \cos ax} \quad (d^2 > b^2)$

131. $\int \cos ax \cos bx\, dx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$

Integrals Involving $\sin^n ax$, $\cos^n ax$

132. $\int \sin ax \cos bx\, dx = -\frac{1}{2}\left[\frac{\cos (a-b)x}{a-b} + \frac{\cos (a+b)x}{a+b}\right] \quad (a^2 \neq b^2)$

133. $\int \sin^2 ax \cos^2 ax\, dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$

134. $\int \sin^n ax \cos ax\, dx = \frac{1}{a(n+1)} \sin^{n+1} ax \quad (n \neq -1)$

135. $\int \sin ax \cos^n ax\, dx = -\frac{1}{a(n+1)} \cos^{n+1} ax \quad (n \neq -1)$

136. $\int \sin^n ax \cos^m ax\, dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(n+m)} + \frac{n-1}{n+m}\int \sin^{n-2} ax \cos^m ax\, dx \quad (m, n \text{ pos})$

137. $\int \frac{\sin^{1}}{\cos^m ax} dx = \qquad n - \frac{m+2}{-1}\int \frac{\sin^n ax}{\cos^{m-2} ax} dx \quad (m, n \text{ pos}, m \neq 1)$

138. $\int \frac{\cos^m ax}{\sin^n ax} dx = \frac{-\cos^{m+1} ax}{a(n-1)\sin^{n-1} ax} + \frac{n-m-2}{(n-1)}\int \frac{\cos^m ax}{\sin^{n-2} ax} dx \quad (m, n \text{ pos}, n \neq 1)$

139. $\int \frac{dx}{\sin ax \cos ax} = \frac{1}{a} \log_e \tan ax$

140. $\int \frac{dx}{b \sin ax + d \cos ax} - \frac{1}{a\sqrt{b^2 + d^2}} \log_e \tan {}^1\!/\!_2\left(ax + \tan^{-1} \frac{d}{b}\right)$

141. $\int \frac{\sin ax}{b + d \cos ax} dx = -\frac{1}{ad} \log_e (b + d \cos ax)$

142. $\int \frac{\cos ax}{b + d \sin ax} dx = \frac{1}{ad} \log_e (b + d \sin ax)$

(Table continued on p. 20-84)

Integrals Involving $\tan^n ax$, $\cot^n ax$, $\sec^n ax$, $\csc^n ax$

143. $\int \tan^n ax\, dx = \frac{1}{a(n-1)} \tan^{n-1} ax - \int \tan^{n-2} ax\, dx \quad (n \text{ integer} > 1)$

144. $\int \cot^n ax\, dx = -\frac{1}{a(n-1)} \cot^{n-1} ax - \int \cot^{n-2} ax\, dx \quad (n \text{ integer} > 1)$

145. $\int \sec^n ax\, dx = \frac{1}{a(n-1)} \frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1} \int \sec^{n-2} ax\, dx \quad (n \text{ integer} > 1)$

146. $\int \csc^n ax\, dx = -\frac{1}{a(n-1)} \frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1} \int \csc^{n-2} ax\, dx \quad (n \text{ integer} > 1)$

147. $\int \frac{dx}{b + d \tan ax} = \frac{1}{b^2 + d^2}\left[bx + \frac{d}{a} \log_e (b \cos ax + d \sin ax) \right]$

148. $\int \frac{dx}{\sqrt{b + d \tan^2 ax}} = \frac{1}{a\sqrt{b-d}} \sin^{-1} \left[\sqrt{\frac{b-d}{b}} \sin ax \right] \quad (b \text{ pos}, b^2 > d^2)$

149. $\int \tan ax \sec ax\, dx = \frac{1}{a} \sec ax$

150. $\int \tan^n ax \sec^2 ax\, dx = \frac{1}{a(n+1)} \tan^{n+1} ax \quad (n \neq -1)$

151. $\int \frac{\sec^2 ax\, dx}{\tan ax} = \frac{1}{a} \log_e \tan ax$

152. $\int \cot ax \csc ax\, dx = -\frac{1}{a} \csc ax$

153. $\int \cot^n ax \csc^2 ax\, dx = -\frac{1}{a(n+1)} \cot^{n+1} ax \quad (n \neq -1)$

154. $\int \frac{\csc^2 ax}{\cot ax}\, dx = -\frac{1}{a} \log_e \cot ax$

Integrals Involving b^{ax}, e^{ax}, $\sin bx$, $\cos bx$

155. $\int x b^{ax}\, dx = \frac{xb^{ax}}{a \log_e b} \quad \frac{\quad}{a^2(\log_e b)^2}$

156. $\int x e^{ax}\, dx = \frac{e^{ax}}{a^2} (ax - 1)$

157. $\int x^n b^{ax}\, dx = \frac{x^n b^{ax}}{a \log_e b} - \frac{n}{a \log_e b} \int x^{n-1} b^{ax}\, dx \quad (n \text{ positive})$

158. $\int x^n e^{ax}\, dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax}\, dx \quad (n \text{ positive})$

159. $\int \frac{dx}{b + de^{ax}} = \frac{1}{ab}\left[ax - \log_e (b + de^{ax}) \right]$

160. $\int \frac{e^{ax}\, dx}{b + de^{ax}} = \frac{1}{ad} \log_e (b + de^{ax})$

161. $\int \frac{dx}{be^{ax} + de^{-ax}} = \frac{1}{a\sqrt{bd}} \tan^{-1}\left(e^{ax} \sqrt{\frac{b}{d}} \right) \quad (b \text{ and } d \text{ positive})$

162. $\int \frac{e^{ax}}{x}\, dx = \log_e x + ax + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^3}{3 \cdot 3!} + \cdots$

163 $\int \frac{e^{ax}}{x^n}\, dx = \frac{1}{n-1}\left(-\frac{e^{ax}}{x^{n-1}} + a \int \frac{e^{ax}}{x^{n-1}}\, dx \right) \quad (n \text{ integer} > 1)$

164. $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$

165. $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$

166. $\int xe^{ax} \sin bx\, dx = \frac{xe^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$

$$- \frac{e^{ax}}{(a^2 + b^2)^2} [(a^2 - b^2) \sin bx - 2ab \cos bx]$$

167. $\int xe^{ax} \cos bx\, dx = \frac{xe^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$

$$- \frac{e^{ax}}{(a^2 + b^2)^2} \left[(a^2 - b^2) \cos bx + 2ab \sin bx \right]$$

Some Definite Integrals

1. $\int_0^a \sqrt{a^2 - x^2}\, dx = \frac{\pi a^2}{4}$

2. $\int_0^a \sqrt{2\, ax - x^2}\, dx = \frac{\pi a^2}{4}$

3. $\int_0^\infty \frac{dx}{a + bx^2} = \frac{\pi}{2\sqrt{ab}}$ (*a* and *b* positive)

4. $\int_0^{\sqrt{a/b}} \frac{dx}{a + bx^2} dx = \int_{\sqrt{a/b}}^\infty \frac{dx}{a + bx^2} = \frac{\pi}{4\sqrt{ab}}$ (*a* and *b* positive)

5. $\int_0^{\sqrt{a/b}} \frac{dx}{\sqrt{a - bx^2}} = \frac{\pi}{2\sqrt{b}}$ (*a* and *b* positive)

6. $\int_0^\infty \frac{\sin bx}{x}\, dx = \frac{\pi}{2}$ $(b > 0)$
$= 0$ $(b = 0)$
$= -\frac{\pi}{2}$ $(b < 0)$

7. $\int_0^\infty \frac{\tan x}{x}\, dx = \frac{\pi}{2}$

8. $\int_0^{\pi/2} \sin^{2n+1} x\, dx = \int_0^{\pi/2} \cos^{2n+1} x\, dx = \frac{2\cdot 4\cdot 6\cdot \ldots \cdot 2n}{3\cdot 5\cdot 7\cdot \ldots \cdot (2n+1)}$ $(n > 0)$

9. $\int_0^{\pi/2} \sin^{2n} x\, dx = \int_0^{\pi/2} \cos^{2n} x\, dx = \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot 6\cdot \ldots \cdot 2n}\cdot\frac{\pi}{2}$ $(n > 0)$

10. $\int_0^\pi \sin ax \sin bx\, dx = \int_0^\pi \cos ax \cos bx\, dx = 0$ $(a \neq b)$

11. $\int_0^\pi \sin^2 ax\, dx = \int_0^\pi \cos^2 ax\, dx = \frac{\pi}{2}$

12. $\int_0^{\pi/2} \log_e \cos x\, dx = \int_0^{\pi/2} \log_e \sin x\, dx = -\frac{\pi}{2}\log_e 2$

13. $\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

14. $\int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}}$ $(a > 0,\ n = 1, 2, 3, \ldots)$

15. $\int_0^1 \frac{\log_e x}{1 - x}\, dx = -\frac{\pi^2}{6}$

16. $\int_0^1 \frac{\log_e x}{1 + x}\, dx = -\frac{\pi^2}{12}$

17. $\int_0^1 \frac{\log_e x}{1 - x^2}\, dx = \frac{\pi^2}{8}$

ELLIPTIC INTEGRALS

Reference: *Smithsonian Mathematical Formulae and Tables of Elliptic Functions* by E. P. Adams and R. L. Hippisley, Washington, D.C., 1922.

SYMBOLS AND ABBREVIATIONS

Greek Letters

Α	α	Alpha	Η	η	Eta	Ν	ν	Nu	Τ	τ	Tau
Β	β	Beta	Θ	ϑ θ	Theta	Ξ	ξ	Xi	Υ	υ	Upsilon
Γ	γ	Gamma	Ι	ι	Iota	Ο	ο	Omicron	Φ	φ	Phi
Δ	δ	Delta	Κ	κ	Kappa	Π	π	Pi	Χ	χ	Chi
Ε	ε	Epsilon	Λ	λ	Lambda	Ρ	ρ	Rho	Ψ	ψ	Psi
Ζ	ζ	Zeta	Μ	μ	Mu	Σ	σ ς	Sigma	Ω	ω	Omega

Mathematical Signs and Abbreviations

$+$ plus (addition).
$+$ positive.
$-$ minus (subtraction).
$-$ negative.
$\pm$ plus or minus.
$\mp$ minus or plus.
$=$ equals.
$\geqq$ equals or greater than.
$\leqq$ equals or is less than.
$\approx$ approximately equals.
$\times$ multiplied by.
ab or $a.b = a \times b$.
$\div$ divided by.
$/$ divided by.
$\frac{a}{b} = a/b = a \div b$. $15/16 = \frac{15}{16}$.
$0.2 = \frac{2}{10}$; $0.002 = \frac{2}{1000}$.
$\sqrt{\ }$ square root.
$\sqrt[3]{\ }$ cube root.
$\sqrt[4]{\ }$ 4th root.
: is to, : : so is, : to (proportion).
2 : 4 : : 3 : 6, 2 is to 4 as 3 is to 6.
: ratio; divided by.
2 : 4, ratio of 2 to 4 = 2/4.
$>$ greater than.
$<$ less than.
° degrees, arc or thermometer.
′ minutes or feet.
″ seconds or inches.
′ ″ ‴ accents to distinguish letters, as a', a'', a'''.
a_1, a_2, a_3, a_b, a_c, read a sub 1, a sub b, etc.
( ) [ ] { } —— parentheses, brackets, braces, vinculum; denoting that the numbers enclosed are to be taken together; as, $(a + b)c = \overline{4 + 3} \times 5 = 35$.
a^2, a^3, a squared, a cubed.

a^n, a raised to the nth power.
$a^{2/3} = \sqrt[3]{a^2}$, $a^{3/2} = \sqrt{a^3}$.
$a^{-1} = \frac{1}{a}$, $a^{-2} = \frac{1}{a^2}$.
$10^9 = 10$ to the 9th power $= 1{,}000{,}000{,}000$.
$\sin a =$ the sine of a.
$\sin^{-1} a =$ the arc whose sine is a.
$\sin a^{-1} = \frac{1}{\sin a}$.
log = logarithm.
$\log_e$ or hyp log = hyperbolic logarithm.
% per cent.
∠ angle.
△ triangle.
sin, sine.
cos, cosine.
tan, tangent.
sec, secant
versin, versed sine.
cot, cotangent.
cosec, cosecant.
covers, co-versed sine.

In Algebra, the first letters of the alphabet, a, b, c, d, etc, are generally used to denote known quantities, and the last letters, w, x, y, z, etc., unknown quantities.

Abbreviations and Symbols commonly used.

d, differential (in calculus).
$\int$, integral (in calculus).
$\int_b^a$, integral between limits a and b.
Δ, delta, difference.
Σ, sigma, sign of summation.
π, pi, ratio of circumference of circle to diameter $= 3.14159$.
g, acceleration due to gravity $= 32.16$ ft. per second per second.

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