MACHINERY'S HANDBOOK

Thage Dahlgren

1926.

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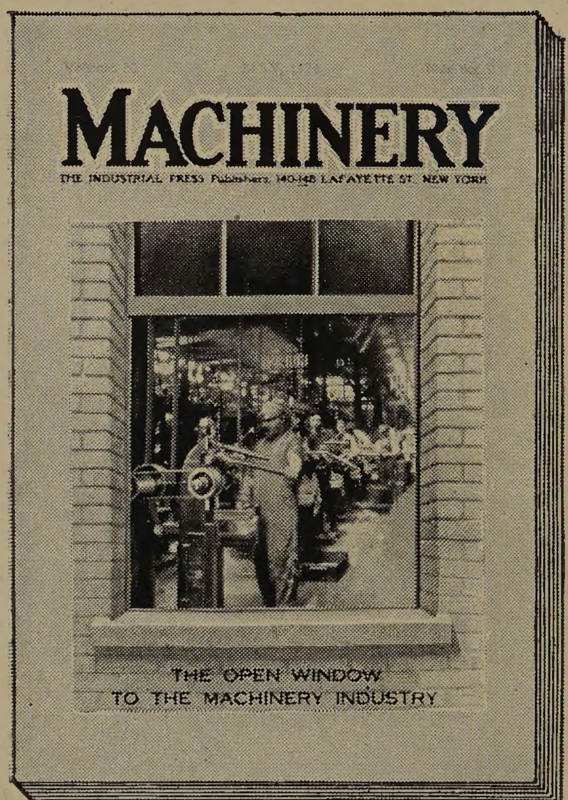
MACHINERY
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THE OPEN WINDOW
TO THE MACHINERY INDUSTRY

MACHINERY'S HANDBOOK

FOR MACHINE SHOP AND DRAFTING-ROOM

A REFERENCE BOOK ON MACHINE DESIGN AND SHOP PRACTICE FOR THE MECHANICAL ENGINEER, DRAFTSMAN, TOOLMAKER AND MACHINIST

SIXTH EDITION

REVISED AND ENLARGED

ONE HUNDRED EIGHTIETH THOUSAND

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PREFACE TO ENLARGED EDITION

This thoroughly revised and enlarged edition of Machinery's Handbook contains many standards, tables, rules, formulas and practical data, systematically collated since the publication of earlier editions. The work is enriched by this new material, most of which is published because of repeated requests from engineers, shop executives, and skilled workmen, who have indicated its need in a standard work of reference that is relied upon to contain all the essential working data and fundamental principles of the machine-building industries.

To indicate concisely the extent of this revision, the more important changes may be summarized: The total number of pages has been increased from 1400 to 1592; there are 230 pages of entirely new matter; 97 new tables; 77 new illustrations; 35 revised tables; 55 pages of revised text, exclusive of new material; and several hundred smaller but important changes.

The publishers here express their appreciation and thanks for the co-operation of many friends in the machinery industries, who have assisted in this work of revision by offering practical suggestions or constructive criticisms of former editions. Since Machinery's Handbook was first published in 1914, all letters containing suggestions or material for its improvement have been filed systematically, and have been most valuable in the work of editing this new edition.

For more than ten years, Machinery's Handbook has been a standard reference work for the machinery industries, and the publishers have spared neither labor nor expense to keep it abreast of mechanical progress.

This preface would be incomplete without the names of the men who compiled and edited this book. For the capable and long-continued labors bestowed on this work by Erik Oberg, editor-in-chief, and by Franklin D. Jones, his associate, the publishers here express their sincere thanks and appreciation.

The Industrial Press

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GENERAL CONTENTS

MACHINERY'S HANDBOOK
FOR MACHINE SHOP AND DRAFTING ROOM

MATHEMATICAL TABLES

Square and Cube Roots of Decimal Numbers

Decimal	Square Root	Cube Root	Decimal	Square Root	Cube Root	Decimal	Square Root	Cube Root
0.01	0.1000	0.2154	0.34	0.5831	0.6980	0.67	0.8185	0.8750
0.02	0.1414	0.2714	0.35	0.5916	0.7047	0.68	0.8246	0.8794
0.03	0.1732	0.3107	0.36	0.6000	0.7114	0.69	0.8307	0.8837
0.04	0.2000	0.3420	0.37	0.6083	0.7179	0.70	0.8367	0.8879
0.05	0.2236	0.3684	0.38	0.6164	0.7243	0.71	0.8426	0.8921
0.06	0.2449	0.3915	0.39	0.6245	0.7306	0.72	0.8485	0.8963
0.07	0.2646	0.4121	0.40	0.6325	0.7368	0.73	0.8544	0.9004
0.08	0.2828	0.4309	0.41	0.6403	0.7429	0.74	0.8602	0.9045
0.09	0.3000	0.4481	0.42	0.6481	0.7489	0.75	0.8660	0.9086
0.10	0.3162	0.4642	0.43	0.6557	0.7548	0.76	0.8718	0.9126
0.11	0.3317	0.4791	0.44	0.6633	0.7606	0.77	0.8775	0.9166
0.12	0.3464	0.4932	0.45	0.6708	0.7663	0.78	0.8832	0.9205
0.13	0.3606	0.5066	0.46	0.6782	0.7719	0.79	0.8888	0.9244
0.14	0.3742	0.5192	0.47	0.6856	0.7775	0.80	0.8944	0.9283
0.15	0.3873	0.5313	0.48	0.6928	0.7830	0.81	0.9000	0.9322
0.16	0.4000	0.5429	0.49	0.7000	0.7884	0.82	0.9055	0.9360
0.17	0.4123	0.5540	0.50	0.7071	0.7937	0.83	0.9110	0.9398
0.18	0.4243	0.5646	0.51	0.7141	0.7990	0.84	0.9165	0.9435
0.19	0.4359	0.5749	0.52	0.7211	0.8041	0.85	0.9220	0.9473
0.20	0.4472	0.5848	0.53	0.7280	0.8093	0.86	0.9274	0.9510
0.21	0.4583	0.5944	0.54	0.7348	0.8143	0.87	0.9327	0.9546
0.22	0.4690	0.6037	0.55	0.7416	0.8193	0.88	0.9381	0.9583
0.23	0.4796	0.6127	0.56	0.7483	0.8243	0.89	0.9434	0.9619
0.24	0.4899	0.6214	0.57	0.7550	0.8291	0.90	0.9487	0.9655
0.25	0.5000	0.6300	0.58	0.7616	0.8340	0.91	0.9539	0.9691
0.26	0.5099	0.6383	0.59	0.7681	0.8387	0.92	0.9592	0.9726
0.27	0.5196	0.6463	0.60	0.7746	0.8434	0.93	0.9644	0.9761
0.28	0.5292	0.6542	0.61	0.7810	0.8481	0.94	0.9695	0.9796
0.29	0.5385	0.6619	0.62	0.7874	0.8527	0.95	0.9747	0.9830
0.30	0.5477	0.6694	0.63	0.7937	0.8573	0.96	0.9798	0.9865
0.31	0.5568	0.6768	0.64	0.8000	0.8618	0.97	0.9849	0.9899
0.32	0.5657	0.6840	0.65	0.8062	0.8662	0.98	0.9899	0.9933
0.33	0.5745	0.6910	0.66	0.8124	0.8707	0.99	0.9950	0.9967

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1	1	1	1.00000	1.00000	1.0000000	1
2	4	8	1.41421	1.25992	0.5000000	2
3	9	27	1.73205	1.44225	0.3333333	3
4	16	64	2.00000	1.58740	0.2500000	4
5	25	125	2.23607	1.70998	0.2000000	5
6	36	216	2.44949	1.81712	0.1666667	6
7	49	343	2.64575	1.91293	0.1428571	7
8	64	512	2.82843	2.00000	0.1250000	8
9	81	729	3.00000	2.08008	0.1111111	9
10	100	1,000	3.16228	2.15443	0.1000000	10
11	121	1,331	3.31662	2.22398	0.0909091	11
12	144	1,728	3.46410	2.28943	0.0833333	12
13	169	2,197	3.60555	2.35133	0.0769231	13
14	196	2,744	3.74166	2.41014	0.0714286	14
15	225	3,375	3.87298	2.46621	0.0666667	15
16	256	4,096	4.00000	2.51984	0.0625000	16
17	289	4,913	4.12311	2.57128	0.0588235	17
18	324	5,832	4.24264	2.62074	0.0555556	18
19	361	6,859	4.35890	2.66840	0.0526316	19
20	400	8,000	4.47214	2.71442	0.0500000	20
21	441	9,261	4.58258	2.75892	0.0476190	21
22	484	10,648	4.69042	2.80204	0.0454545	22
23	529	12,167	4.79583	2.84387	0.0434783	23
24	576	13,824	4.89898	2.88450	0.0416667	24
25	625	15,625	5.00000	2.92402	0.0400000	25
26	676	17,576	5.09902	2.96250	0.0384615	26
27	729	19,683	5.19615	3.00000	0.0370370	27
28	784	21,952	5.29150	3.03659	0.0357143	28
29	841	24,389	5.38516	3.07232	0.0344828	29
30	900	27,000	5.47723	3.10723	0.0333333	30
31	961	29,791	5.56776	3.14138	0.0322581	31
32	1,024	32,768	5.65685	3.17480	0.0312500	32
33	1,089	35,937	5.74456	3.20753	0.0303030	33
34	1,156	39,304	5.83095	3.23961	0.0294118	34
35	1,225	42,875	5.91608	3.27107	0.0285714	35
36	1,296	46,656	6.00000	3.30193	0.0277778	36
37	1,369	50,653	6.08276	3.33222	0.0270270	37
38	1,444	54,872	6.16441	3.36198	0.0263158	38
39	1,521	59,319	6.24500	3.39121	0.0256410	39
40	1,600	64,000	6.32456	3.41995	0.0250000	40
41	1,681	68,921	6.40312	3.44822	0.0243902	41
42	1,764	74,088	6.48074	3.47603	0.0238095	42
43	1,849	79,507	6.55744	3.50340	0.0232558	43
44	1,936	85,184	6.63325	3.53035	0.0227273	44
45	2,025	91,125	6.70820	3.55689	0.0222222	45
46	2,116	97,336	6.78233	3.58305	0.0217391	46
47	2,209	103,823	6.85565	3.60883	0.0212766	47
48	2,304	110,592	6.92820	3.63424	0.0208333	48
49	2,401	117,649	7.00000	3.65931	0.0204082	49
50	2,500	125,000	7.07107	3.68403	0.0200000	50

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
51	2,601	132,651	7.14143	3.70843	0.0196078	51
52	2,704	140,608	7.21110	3.73251	0.0192308	52
53	2,809	148,877	7.28011	3.75629	0.0188679	53
54	2,916	157,464	7.34847	3.77976	0.0185185	54
55	3,025	166,375	7.41620	3.80295	0.0181818	55
56	3,136	175,616	7.48331	3.82586	0.0178571	56
57	3,249	185,193	7.54983	3.84850	0.0175439	57
58	3,364	195,112	7.61577	3.87088	0.0172414	58
59	3,481	205,379	7.68115	3.89300	0.0169492	59
60	3,600	216,000	7.74597	3.91487	0.0166667	60
61	3,721	226,981	7.81025	3.93650	0.0163934	61
62	3,844	238,328	7.87401	3.95789	0.0161290	62
63	3,969	250,047	7.93725	3.97906	0.0158730	63
64	4,096	262,144	8.00000	4.00000	0.0156250	64
65	4,225	274,625	8.06226	4.02073	0.0153846	65
66	4,356	287,496	8.12404	4.04124	0.0151515	66
67	4,489	300,763	8.18535	4.06155	0.0149254	67
68	4,624	314,432	8.24621	4.08166	0.0147059	68
69	4,761	328,509	8.30662	4.10157	0.0144928	69
70	4,900	343,000	8.36660	4.12129	0.0142857	70
71	5,041	357,911	8.42615	4.14082	0.0140845	71
72	5,184	373,248	8.48528	4.16017	0.0138889	72
73	5,329	389,017	8.54400	4.17934	0.0136986	73
74	5,476	405,224	8.60233	4.19834	0.0135135	74
75	5,625	421,875	8.66025	4.21716	0.0133333	75
76	5,776	438,976	8.71780	4.23582	0.0131579	76
77	5,929	456,533	8.77496	4.25432	0.0129870	77
78	6,084	474,552	8.83176	4.27266	0.0128205	78
79	6,241	493,039	8.88819	4.29084	0.0126582	79
80	6,400	512,000	8.94427	4.30887	0.0125000	80
81	6,561	531,441	9.00000	4.32675	0.0123457	81
82	6,724	551,368	9.05539	4.34448	0.0121951	82
83	6,889	571,787	9.11043	4.36207	0.0120482	83
84	7,056	592,704	9.16515	4.37952	0.0119048	84
85	7,225	614,125	9.21954	4.39683	0.0117647	85
86	7,396	636,056	9.27362	4.41400	0.0116279	86
87	7,569	658,503	9.32738	4.43105	0.0114943	87
88	7,744	681,472	9.38083	4.44797	0.0113636	88
89	7,921	704,969	9.43398	4.46475	0.0112360	89
90	8,100	729,000	9.48683	4.48140	0.0111111	90
91	8,281	753,571	9.53939	4.49794	0.0109890	91
92	8,464	778,688	9.59166	4.51436	0.0108696	92
93	8,649	804,357	9.64365	4.53065	0.0107527	93
94	8,836	830,584	9.69536	4.54684	0.0106383	94
95	9,025	857,375	9.74679	4.56290	0.0105263	95
96	9,216	884,736	9.79796	4.57886	0.0104167	96
97	9,409	912,673	9.84886	4.59470	0.0103093	97
98	9,604	941,192	9.89949	4.61044	0.0102041	98
99	9,801	970,299	9.94987	4.62607	0.0101010	99
100	10,000	1,000,000	10.00000	4.64159	0.0100000	100

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
101	10,201	1,030,301	10.0499	4.65701	0.0099010	101
102	10,404	1,061,208	10.0995	4.67233	0.0098039	102
103	10,609	1,092,727	10.1489	4.68755	0.0097087	103
104	10,816	1,124,864	10.1980	4.70267	0.0096154	104
105	11,025	1,157,625	10.2470	4.71769	0.0095238	105
106	11,236	1,191,016	10.2956	4.73262	0.0094340	106
107	11,449	1,225,043	10.3441	4.74746	0.0093458	107
108	11,664	1,259,712	10.3923	4.76220	0.0092593	108
109	11,881	1,295,029	10.4403	4.77686	0.0091743	109
110	12,100	1,331,000	10.4881	4.79142	0.0090909	110
111	12,321	1,367,631	10.5357	4.80590	0.0090090	111
112	12,544	1,404,928	10.5830	4.82028	0.0089286	112
113	12,769	1,442,897	10.6301	4.83459	0.0088496	113
114	12,996	1,481,544	10.6771	4.84881	0.0087719	114
115	13,225	1,520,875	10.7238	4.86294	0.0086957	115
116	13,456	1,560,896	10.7703	4.87700	0.0086207	116
117	13,689	1,601,613	10.8167	4.89097	0.0085470	117
118	13,924	1,643,032	10.8628	4.90487	0.0084746	118
119	14,161	1,685,159	10.9087	4.91868	0.0084034	119
120	14,400	1,728,000	10.9545	4.93242	0.0083333	120
121	14,641	1,771,561	11.0000	4.94609	0.0082645	121
122	14,884	1,815,848	11.0454	4.95968	0.0081967	122
123	15,129	1,860,867	11.0905	4.97319	0.0081301	123
124	15,376	1,906,624	11.1355	4.98663	0.0080645	124
125	15,625	1,953,125	11.1803	5.00000	0.0080000	125
126	15,876	2,000,376	11.2250	5.01330	0.0079365	126
127	16,129	2,048,383	11.2694	5.02653	0.0078740	127
128	16,384	2,097,152	11.3137	5.03968	0.0078125	128
129	16,641	2,146,689	11.3578	5.05277	0.0077519	129
130	16,900	2,197,000	11.4018	5.06580	0.0076923	130
131	17,161	2,248,091	11.4455	5.07875	0.0076336	131
132	17,424	2,299,968	11.4891	5.09164	0.0075758	132
133	17,689	2,352,637	11.5326	5.10447	0.0075188	133
134	17,956	2,406,104	11.5758	5.11723	0.0074627	134
135	18,225	2,460,375	11.6190	5.12993	0.0074074	135
136	18,496	2,515,456	11.6619	5.14256	0.0073529	136
137	18,769	2,571,353	11.7047	5.15514	0.0072993	137
138	19,044	2,628,072	11.7473	5.16765	0.0072464	138
139	19,321	2,685,619	11.7898	5.18010	0.0071942	139
140	19,600	2,744,000	11.8322	5.19249	0.0071429	140
141	19,881	2,803,221	11.8743	5.20483	0.0070922	141
142	20,164	2,863,288	11.9164	5.21710	0.0070423	142
143	20,449	2,924,207	11.9583	5.22932	0.0069930	143
144	20,736	2,985,984	12.0000	5.24148	0.0069444	144
145	21,025	3,048,625	12.0416	5.25359	0.0068966	145
146	21,316	3,112,136	12.0830	5.26564	0.0068493	146
147	21,609	3,176,523	12.1244	5.27763	0.0068027	147
148	21,904	3,241,792	12.1655	5.28957	0.0067568	148
149	22,201	3,307,949	12.2066	5.30146	0.0067114	149
150	22,500	3,375,000	12.2474	5.31329	0.0066667	150

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
151	22,801	3,442,951	12.2882	5.32507	0.0066225	151
152	23,104	3,511,808	12.3288	5.33680	0.0065789	152
153	23,409	3,581,577	12.3693	5.34848	0.0065359	153
154	23,716	3,652,264	12.4097	5.36011	0.0064935	154
155	24,025	3,723,875	12.4499	5.37169	0.0064516	155
156	24,336	3,796,416	12.4900	5.38321	0.0064103	156
157	24,649	3,869,893	12.5300	5.39469	0.0063694	157
158	24,964	3,944,312	12.5698	5.40612	0.0063291	158
159	25,281	4,019,679	12.6095	5.41750	0.0062893	159
160	25,600	4,096,000	12.6491	5.42884	0.0062500	160
161	25,921	4,173,281	12.6886	5.44012	0.0062112	161
162	26,244	4,251,528	12.7279	5.45136	0.0061728	162
163	26,569	4,330,747	12.7671	5.46256	0.0061350	163
164	26,896	4,410,944	12.8062	5.47370	0.0060976	164
165	27,225	4,492,125	12.8452	5.48481	0.0060606	165
166	27,556	4,574,296	12.8841	5.49586	0.0060241	166
167	27,889	4,657,463	12.9228	5.50688	0.0059880	167
168	28,224	4,741,632	12.9615	5.51785	0.0059524	168
169	28,561	4,826,809	13.0000	5.52877	0.0059172	169
170	28,900	4,913,000	13.0384	5.53966	0.0058823	170
171	29,241	5,000,211	13.0767	5.55050	0.0058480	171
172	29,584	5,088,448	13.1149	5.56130	0.0058140	172
173	29,929	5,177,717	13.1529	5.57205	0.0057803	173
174	30,276	5,268,024	13.1909	5.58277	0.0057471	174
175	30,625	5,359,375	13.2288	5.59344	0.0057143	175
176	30,976	5,451,776	13.2665	5.60408	0.0056818	176
177	31,329	5,545,233	13.3041	5.61467	0.0056497	177
178	31,684	5,639,752	13.3417	5.62523	0.0056180	178
179	32,041	5,735,339	13.3791	5.63574	0.0055866	179
180	32,400	5,832,000	13.4164	5.64622	0.0055556	180
181	32,761	5,929,741	13.4536	5.65665	0.0055249	181
182	33,124	6,028,568	13.4907	5.66705	0.0054945	182
183	33,489	6,128,487	13.5277	5.67741	0.0054645	183
184	33,856	6,229,504	13.5647	5.68773	0.0054348	184
185	34,225	6,331,625	13.6015	5.69802	0.0054054	185
186	34,596	6,434,856	13.6382	5.70827	0.0053763	186
187	34,969	6,539,203	13.6748	5.71848	0.0053476	187
188	35,344	6,644,672	13.7113	5.72865	0.0053191	188
189	35,721	6,751,269	13.7477	5.73879	0.0052910	189
190	36,100	6,859,000	13.7840	5.74890	0.0052632	190
191	36,481	6,967,871	13.8203	5.75897	0.0052356	191
192	36,864	7,077,888	13.8564	5.76900	0.0052083	192
193	37,249	7,189,057	13.8924	5.77900	0.0051813	193
194	37,636	7,301,384	13.9284	5.78896	0.0051546	194
195	38,025	7,414,875	13.9642	5.79889	0.0051282	195
196	38,416	7,529,536	14.0000	5.80879	0.0051020	196
197	38,809	7,645,373	14.0357	5.81865	0.0050761	197
198	39,204	7,762,392	14.0712	5.82849	0.0050505	198
199	39,601	7,880,599	14.1067	5.83827	0.0050251	199
200	40,000	8,000,000	14.1421	5.84804	0.0050000	200

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
201	40,401	8,120,601	14.1774	5.85777	0.0049751	201
202	40,804	8,242,408	14.2127	5.86747	0.0049505	202
203	41,209	8,365,427	14.2478	5.87713	0.0049261	203
204	41,616	8,489,664	14.2829	5.88677	0.0049020	204
205	42,025	8,615,125	14.3178	5.89637	0.0048780	205
206	42,436	8,741,816	14.3527	5.90594	0.0048544	206
207	42,849	8,869,743	14.3875	5.91548	0.0048309	207
208	43,264	8,998,912	14.4222	5.92499	0.0048077	208
209	43,681	9,129,329	14.4568	5.93447	0.0047847	209
210	44,100	9,261,000	14.4914	5.94392	0.0047619	210
211	44,521	9,393,931	14.5258	5.95334	0.0047393	211
212	44,944	9,528,128	14.5602	5.96273	0.0047170	212
213	45,369	9,663,597	14.5945	5.97209	0.0046948	213
214	45,796	9,800,344	14.6287	5.98142	0.0046729	214
215	46,225	9,938,375	14.6629	5.99073	0.0046512	215
216	46,656	10,077,696	14.6969	6.00000	0.0046296	216
217	47,089	10,218,313	14.7309	6.00925	0.0046083	217
218	47,524	10,360,232	14.7648	6.01846	0.0045872	218
219	47,961	10,503,459	14.7986	6.02765	0.0045662	219
220	48,400	10,648,000	14.8324	6.03681	0.0045455	220
221	48,841	10,793,861	14.8661	6.04594	0.0045249	221
222	49,284	10,941,048	14.8997	6.05505	0.0045045	222
223	49,729	11,089,567	14.9332	6.06413	0.0044843	223
224	50,176	11,239,424	14.9666	6.07318	0.0044643	224
225	50,625	11,390,625	15.0000	6.08220	0.0044444	225
226	51,076	11,543,176	15.0333	6.09120	0.0044248	226
227	51,529	11,697,083	15.0665	6.10017	0.0044053	227
228	51,984	11,852,352	15.0997	6.10911	0.0043860	228
229	52,441	12,008,989	15.1327	6.11803	0.0043668	229
230	52,900	12,167,000	15.1658	6.12693	0.0043478	230
231	53,361	12,326,391	15.1987	6.13579	0.0043290	231
232	53,824	12,487,168	15.2315	6.14463	0.0043103	232
233	54,289	12,649,337	15.2643	6.15345	0.0042918	233
234	54,756	12,812,904	15.2971	6.16224	0.0042735	234
235	55,225	12,977,875	15.3297	6.17101	0.0042553	235
236	55,696	13,144,256	15.3623	6.17975	0.0042373	236
237	56,169	13,312,053	15.3948	6.18846	0.0042194	237
238	56,644	13,481,272	15.4272	6.19715	0.0042017	238
239	57,121	13,651,919	15.4596	6.20582	0.0041841	239
240	57,600	13,824,000	15.4919	6.21447	0.0041667	240
241	58,081	13,997,521	15.5242	6.22308	0.0041494	241
242	58,564	14,172,488	15.5563	6.23168	0.0041322	242
243	59,049	14,348,907	15.5885	6.24025	0.0041152	243
244	59,536	14,526,784	15.6205	6.24880	0.0040984	244
245	60,025	14,706,125	15.6525	6.25732	0.0040816	245
246	60,516	14,886,936	15.6844	6.26583	0.0040650	246
247	61,009	15,069,223	15.7162	6.27431	0.0040486	247
248	61,504	15,252,992	15.7480	6.28276	0.0040323	248
249	62,001	15,438,249	15.7797	6.29119	0.0040161	249
250	62,500	15,625,000	15.8114	6.29961	0.0040000	250

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
251	63,001	15,813,251	15.8430	6.30799	0.0039841	251
252	63,504	16,003,008	15.8745	6.31636	0.0039683	252
253	64,009	16,194,277	15.9060	6.32470	0.0039526	253
254	64,516	16,387,064	15.9374	6.33303	0.0039370	254
255	65,025	16,581,375	15.9687	6.34133	0.0039216	255
256	65,536	16,777,216	16.0000	6.34960	0.0039063	256
257	66,049	16,974,593	16.0312	6.35786	0.0038911	257
258	66,564	17,173,512	16.0624	6.36610	0.0038760	258
259	67,081	17,373,979	16.0935	6.37431	0.0038610	259
260	67,600	17,576,000	16.1245	6.38250	0.0038462	260
261	68,121	17,779,581	16.1555	6.39068	0.0038314	261
262	68,644	17,984,728	16.1864	6.39883	0.0038168	262
263	69,169	18,191,447	16.2173	6.40696	0.0038023	263
264	69,696	18,399,744	16.2481	6.41507	0.0037879	264
265	70,225	18,609,625	16.2788	6.42316	0.0037736	265
266	70,756	18,821,096	16.3095	6.43123	0.0037594	266
267	71,289	19,034,163	16.3401	6.43928	0.0037453	267
268	71,824	19,248,832	16.3707	6.44731	0.0037313	268
269	72,361	19,465,109	16.4012	6.45531	0.0037175	269
270	72,900	19,683,000	16.4317	6.46330	0.0037037	270
271	73,441	19,902,511	16.4621	6.47127	0.0036900	271
272	73,984	20,123,648	16.4924	6.47922	0.0036765	272
273	74,529	20,346,417	16.5227	6.48715	0.0036630	273
274	75,076	20,570,824	16.5529	6.49507	0.0036496	274
275	75,625	20,796,875	16.5831	6.50296	0.0036364	275
276	76,176	21,024,576	16.6132	6.51083	0.0036232	276
277	76,729	21,253,933	16.6433	6.51868	0.0036101	277
278	77,284	21,484,952	16.6733	6.52652	0.0035971	278
279	77,841	21,717,639	16.7033	6.53434	0.0035842	279
280	78,400	21,952,000	16.7332	6.54213	0.0035714	280
281	78,961	22,188,041	16.7631	6.54991	0.0035587	281
282	79,524	22,425,768	16.7929	6.55767	0.0035461	282
283	80,089	22,665,187	16.8226	6.56541	0.0035336	283
284	80,656	22,906,304	16.8523	6.57314	0.0035211	284
285	81,225	23,149,125	16.8819	6.58084	0.0035088	285
286	81,796	23,393,656	16.9115	6.58853	0.0034965	286
287	82,369	23,639,903	16.9411	6.59620	0.0034843	287
288	82,944	23,887,872	16.9706	6.60385	0.0034722	288
289	83,521	24,137,569	17.0000	6.61149	0.0034602	289
290	84,100	24,389,000	17.0294	6.61911	0.0034483	290
291	84,681	24,642,171	17.0587	6.62671	0.0034364	291
292	85,264	24,897,088	17.0880	6.63429	0.0034247	292
293	85,849	25,153,757	17.1172	6.64185	0.0034130	293
294	86,436	25,412,184	17.1464	6.64940	0.0034014	294
295	87,025	25,672,375	17.1756	6.65693	0.0033898	295
296	87,616	25,934,336	17.2047	6.66444	0.0033784	296
297	88,209	26,198,073	17.2337	6.67194	0.0033670	297
298	88,804	26,463,592	17.2627	6.67942	0.0033557	298
299	89,401	26,730,899	17.2916	6.68688	0.0033445	299
300	90,000	27,000,000	17.3205	6.69433	0.0033333	300

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
301	90,601	27,270,901	17.3494	6.70176	0.0033223	301
302	91,204	27,543,608	17.3781	6.70917	0.0033113	302
303	91,809	27,818,127	17.4069	6.71657	0.0033003	303
304	92,416	28,094,464	17.4356	6.72395	0.0032895	304
305	93,025	28,372,625	17.4642	6.73132	0.0032787	305
306	93,636	28,652,616	17.4929	6.73866	0.0032680	306
307	94,249	28,934,443	17.5214	6.74600	0.0032573	307
308	94,864	29,218,112	17.5499	6.75331	0.0032468	308
309	95,481	29,503,629	17.5784	6.76061	0.0032362	309
310	96,100	29,791,000	17.6068	6.76790	0.0032258	310
311	96,721	30,080,231	17.6352	6.77517	0.0032154	311
312	97,344	30,371,328	17.6635	6.78242	0.0032051	312
313	97,969	30,664,297	17.6918	6.78966	0.0031949	313
314	98,596	30,959,144	17.7200	6.79688	0.0031847	314
315	99,225	31,255,875	17.7482	6.80409	0.0031746	315
316	99,856	31,554,496	17.7764	6.81128	0.0031646	316
317	100,489	31,855,013	17.8045	6.81846	0.0031546	317
318	101,124	32,157,432	17.8326	6.82562	0.0031447	318
319	101,761	32,461,759	17.8606	6.83277	0.0031348	319
320	102,400	32,768,000	17.8885	6.83990	0.0031250	320
321	103,041	33,076,161	17.9165	6.84702	0.0031153	321
322	103,684	33,386,248	17.9444	6.85412	0.0031056	322
323	104,329	33,698,267	17.9722	6.86121	0.0030960	323
324	104,976	34,012,224	18.0000	6.86829	0.0030864	324
325	105,625	34,328,125	18.0278	6.87534	0.0030769	325
326	106,276	34,645,976	18.0555	6.88239	0.0030675	326
327	106,929	34,965,783	18.0831	6.88942	0.0030581	327
328	107,584	35,287,552	18.1108	6.89643	0.0030488	328
329	108,241	35,611,289	18.1384	6.90344	0.0030395	329
330	108,900	35,937,000	18.1659	6.91042	0.0030303	330
331	109,561	36,264,691	18.1934	6.91740	0.0030211	331
332	110,224	36,594,368	18.2209	6.92436	0.0030120	332
333	110,889	36,926,037	18.2483	6.93131	0.0030030	333
334	111,556	37,259,704	18.2757	6.93823	0.0029940	334
335	112,225	37,595,375	18.3030	6.94515	0.0029851	335
336	112,896	37,933,056	18.3303	6.95205	0.0029762	336
337	113,569	38,272,753	18.3576	6.95894	0.0029674	337
338	114,244	38,614,472	18.3848	6.96582	0.0029586	338
339	114,921	38,958,219	18.4120	6.97268	0.0029499	339
340	115,600	39,304,000	18.4391	6.97953	0.0029412	340
341	116,281	39,651,821	18.4662	6.98637	0.0029326	341
342	116,964	40,001,688	18.4932	6.99319	0.0029240	342
343	117,649	40,353,607	18.5203	7.00000	0.0029155	343
344	118,336	40,707,584	18.5472	7.00680	0.0029070	344
345	119,025	41,063,625	18.5742	7.01358	0.0028986	345
346	119,716	41,421,736	18.6011	7.02035	0.0028902	346
347	120,409	41,781,923	18.6279	7.02711	0.0028818	347
348	121,104	42,144,192	18.6548	7.03385	0.0028736	348
349	121,801	42,508,549	18.6815	7.04059	0.0028653	349
350	122,500	42,875,000	18.7083	7.04730	0.0028571	350

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
351	123,201	43,243,551	18.7350	7.05400	0.0028490	351
352	123,904	43,614,208	18.7617	7.06070	0.0028409	352
353	124,609	43,986,977	18.7883	7.06738	0.0028329	353
354	125,316	44,361,864	18.8149	7.07404	0.0028249	354
355	126,025	44,738,875	18.8414	7.08070	0.0028169	355
356	126,736	45,118,016	18.8680	7.08734	0.0028090	356
357	127,449	45,499,293	18.8944	7.09397	0.0028011	357
358	128,164	45,882,712	18.9209	7.10059	0.0027933	358
359	128,881	46,268,279	18.9473	7.10719	0.0027855	359
360	129,600	46,656,000	18.9737	7.11379	0.0027778	360
361	130,321	47,045,881	19.0000	7.12037	0.0027701	361
362	131,044	47,437,928	19.0263	7.12694	0.0027624	362
363	131,769	47,832,147	19.0526	7.13349	0.0027548	363
364	132,496	48,228,544	19.0788	7.14004	0.0027473	364
365	133,225	48,627,125	19.1050	7.14657	0.0027397	365
366	133,956	49,027,896	19.1311	7.15309	0.0027322	366
367	134,689	49,430,863	19.1572	7.15960	0.0027248	367
368	135,424	49,836,032	19.1833	7.16610	0.0027174	368
369	136,161	50,243,409	19.2094	7.17258	0.0027100	369
370	136,900	50,653,000	19.2354	7.17905	0.0027027	370
371	137,641	51,064,811	19.2614	7.18552	0.0026954	371
372	138,384	51,478,848	19.2873	7.19197	0.0026882	372
373	139,129	51,895,117	19.3132	7.19841	0.0026810	373
374	139,876	52,313,624	19.3391	7.20483	0.0026738	374
375	140,625	52,734,375	19.3649	7.21125	0.0026667	375
376	141,376	53,157,376	19.3907	7.21765	0.0026596	376
377	142,129	53,582,633	19.4165	7.22405	0.0026525	377
378	142,884	54,010,152	19.4422	7.23043	0.0026455	378
379	143,641	54,439,939	19.4679	7.23680	0.0026385	379
380	144,400	54,872,000	19.4936	7.24316	0.0026316	380
381	145,161	55,306,341	19.5192	7.24950	0.0026247	381
382	145,924	55,742,968	19.5448	7.25584	0.0026178	382
383	146,689	56,181,887	19.5704	7.26217	0.0026110	383
384	147,456	56,623,104	19.5959	7.26848	0.0026042	384
385	148,225	57,066,625	19.6214	7.27479	0.0025974	385
386	148,996	57,512,456	19.6469	7.28108	0.0025907	386
387	149,769	57,960,603	19.6723	7.28736	0.0025840	387
388	150,544	58,411,072	19.6977	7.29363	0.0025773	388
389	151,321	58,863,869	19.7231	7.29989	0.0025707	389
390	152,100	59,319,000	19.7484	7.30614	0.0025641	390
391	152,881	59,776,471	19.7737	7.31238	0.0025575	391
392	153,664	60,236,288	19.7990	7.31861	0.0025510	392
393	154,449	60,698,457	19.8242	7.32483	0.0025445	393
394	155,236	61,162,984	19.8494	7.33104	0.0025381	394
395	156,025	61,629,875	19.8746	7.33723	0.0025316	395
396	156,816	62,099,136	19.8997	7.34342	0.0025253	396
397	157,609	62,570,773	19.9249	7.34960	0.0025189	397
398	158,404	63,044,792	19.9499	7.35576	0.0025126	398
399	159,201	63,521,199	19.9750	7.36192	0.0025063	399
400	160,000	64,000,000	20.0000	7.36806	0.0025000	400

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
401	160,801	64,481,201	20.0250	7.37420	0.0024938	401
402	161,604	64,964,808	20.0499	7.38032	0.0024876	402
403	162,409	65,450,827	20.0749	7.38644	0.0024814	403
404	163,216	65,939,264	20.0998	7.39254	0.0024752	404
405	164,025	66,430,125	20.1246	7.39864	0.0024691	405
406	164,836	66,923,416	20.1494	7.40472	0.0024631	406
407	165,649	67,419,143	20.1742	7.41080	0.0024570	407
408	166,464	67,917,312	20.1990	7.41686	0.0024510	408
409	167,281	68,417,929	20.2237	7.42291	0.0024450	409
410	168,100	68,921,000	20.2485	7.42896	0.0024390	410
411	168,921	69,426,531	20.2731	7.43499	0.0024331	411
412	169,744	69,934,528	20.2978	7.44102	0.0024272	412
413	170,569	70,444,997	20.3224	7.44703	0.0024213	413
414	171,396	70,957,944	20.3470	7.45304	0.0024155	414
415	172,225	71,473,375	20.3715	7.45904	0.0024096	415
416	173,056	71,991,296	20.3961	7.46502	0.0024038	416
417	173,889	72,511,713	20.4206	7.47100	0.0023981	417
418	174,724	73,034,632	20.4450	7.47697	0.0023923	418
419	175,561	73,560,059	20.4695	7.48292	0.0023866	419
420	176,400	74,088,000	20.4939	7.48887	0.0023810	420
421	177,241	74,618,461	20.5183	7.49481	0.0023753	421
422	178,084	75,151,448	20.5426	7.50074	0.0023697	422
423	178,929	75,686,967	20.5670	7.50666	0.0023641	423
424	179,776	76,225,024	20.5913	7.51257	0.0023585	424
425	180,625	76,765,625	20.6155	7.51847	0.0023529	425
426	181,476	77,308,776	20.6398	7.52437	0.0023474	426
427	182,329	77,854,483	20.6640	7.53025	0.0023419	427
428	183,184	78,402,752	20.6882	7.53612	0.0023364	428
429	184,041	78,953,589	20.7123	7.54199	0.0023310	429
430	184,900	79,507,000	20.7364	7.54784	0.0023256	430
431	185,761	80,062,991	20.7605	7.55369	0.0023202	431
432	186,624	80,621,568	20.7846	7.55953	0.0023148	432
433	187,489	81,182,737	20.8087	7.56535	0.0023095	433
434	188,356	81,746,504	20.8327	7.57117	0.0023041	434
435	189,225	82,312,875	20.8567	7.57698	0.0022989	435
436	190,096	82,881,856	20.8806	7.58279	0.0022936	436
437	190,969	83,453,453	20.9045	7.58858	0.0022883	437
438	191,844	84,027,672	20.9284	7.59436	0.0022831	438
439	192,721	84,604,519	20.9523	7.60014	0.0022779	439
440	193,600	85,184,000	20.9762	7.60590	0.0022727	440
441	194,481	85,766,121	21.0000	7.61166	0.0022676	441
442	195,364	86,350,888	21.0238	7.61741	0.0022624	442
443	196,249	86,938,307	21.0476	7.62315	0.0022573	443
444	197,136	87,528,384	21.0713	7.62888	0.0022523	444
445	198,025	88,121,125	21.0950	7.63461	0.0022472	445
446	198,916	88,716,536	21.1187	7.64032	0.0022422	446
447	199,809	89,314,623	21.1424	7.64603	0.0022371	447
448	200,704	89,915,392	21.1660	7.65172	0.0022321	448
449	201,601	90,518,849	21.1896	7.65741	0.0022272	449
450	202,500	91,125,000	21.2132	7.66309	0.0022222	450

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
451	203,401	91,733,851	21.2368	7.66877	0.0022173	451
452	204,304	92,345,408	21.2603	7.67443	0.0022124	452
453	205,209	92,959,677	21.2838	7.68009	0.0022075	453
454	206,116	93,576,664	21.3073	7.68573	0.0022026	454
455	207,025	94,196,375	21.3307	7.69137	0.0021978	455
456	207,936	94,818,816	21.3542	7.69700	0.0021930	456
457	208,849	95,443,993	21.3776	7.70262	0.0021882	457
458	209,764	96,071,912	21.4009	7.70824	0.0021834	458
459	210,681	96,702,579	21.4243	7.71384	0.0021786	459
460	211,600	97,336,000	21.4476	7.71944	0.0021739	460
461	212,521	97,972,181	21.4709	7.72503	0.0021692	461
462	213,444	98,611,128	21.4942	7.73061	0.0021645	462
463	214,369	99,252,847	21.5174	7.73619	0.0021598	463
464	215,296	99,897,344	21.5407	7.74175	0.0021552	464
465	216,225	100,544,625	21.5639	7.74731	0.0021505	465
466	217,156	101,194,696	21.5870	7.75286	0.0021459	466
467	218,089	101,847,563	21.6102	7.75840	0.0021413	467
468	219,024	102,503,232	21.6333	7.76394	0.0021368	468
469	219,961	103,161,709	21.6564	7.76946	0.0021322	469
470	220,900	103,823,000	21.6795	7.77498	0.0021277	470
471	221,841	104,487,111	21.7025	7.78049	0.0021231	471
472	222,784	105,154,048	21.7256	7.78599	0.0021186	472
473	223,729	105,823,817	21.7486	7.79149	0.0021142	473
474	224,676	106,496,424	21.7715	7.79697	0.0021097	474
475	225,625	107,171,875	21.7945	7.80245	0.0021053	475
476	226,576	107,850,176	21.8174	7.80793	0.0021008	476
477	227,529	108,531,333	21.8403	7.81339	0.0020964	477
478	228,484	109,215,352	21.8632	7.81885	0.0020921	478
479	229,441	109,902,239	21.8861	7.82429	0.0020877	479
480	230,400	110,592,000	21.9089	7.82974	0.0020833	480
481	231,361	111,284,641	21.9317	7.83517	0.0020790	481
482	232,324	111,980,168	21.9545	7.84059	0.0020747	482
483	233,289	112,678,587	21.9773	7.84601	0.0020704	483
484	234,256	113,379,904	22.0000	7.85142	0.0020661	484
485	235,225	114,084,125	22.0227	7.85683	0.0020619	485
486	236,196	114,791,256	22.0454	7.86222	0.0020576	486
487	237,169	115,501,303	22.0681	7.86761	0.0020534	487
488	238,144	116,214,272	22.0907	7.87299	0.0020492	488
489	239,121	116,930,169	22.1133	7.87837	0.0020450	489
490	240,100	117,649,000	22.1359	7.88374	0.0020408	490
491	241,081	118,370,771	22.1585	7.88909	0.0020367	491
492	242,064	119,095,488	22.1811	7.89445	0.0020325	492
493	243,049	119,823,157	22.2036	7.89979	0.0020284	493
494	244,036	120,553,784	22.2261	7.90513	0.0020243	494
495	245,025	121,287,375	22.2486	7.91046	0.0020202	495
496	246,016	122,023,936	22.2711	7.91578	0.0020161	496
497	247,009	122,763,473	22.2935	7.92110	0.0020121	497
498	248,004	123,505,992	22.3159	7.92641	0.0020080	498
499	249,001	124,251,499	22.3383	7.93171	0.0020040	499
500	250,000	125,000,000	22.3607	7.93701	0.0020000	500

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
501	251,001	125,751,501	22.3830	7.94229	0.0019960	501
502	252,004	126,506,008	22.4054	7.94757	0.0019920	502
503	253,009	127,263,527	22.4277	7.95285	0.0019881	503
504	254,016	128,024,064	22.4499	7.95811	0.0019841	504
505	255,025	128,787,625	22.4722	7.96337	0.0019802	505
506	256,036	129,554,216	22.4944	7.96863	0.0019763	506
507	257,049	130,323,843	22.5167	7.97387	0.0019724	507
508	258,064	131,096,512	22.5389	7.97911	0.0019685	508
509	259,081	131,872,229	22.5610	7.98434	0.0019646	509
510	260,100	132,651,000	22.5832	7.98957	0.0019608	510
511	261,121	133,432,831	22.6053	7.99479	0.0019569	511
512	262,144	134,217,728	22.6274	8.00000	0.0019531	512
513	263,169	135,005,697	22.6495	8.00520	0.0019493	513
514	264,196	135,796,744	22.6716	8.01040	0.0019455	514
515	265,225	136,590,875	22.6936	8.01559	0.0019417	515
516	266,256	137,388,096	22.7156	8.02078	0.0019380	516
517	267,289	138,188,413	22.7376	8.02596	0.0019342	517
518	268,324	138,991,832	22.7596	8.03113	0.0019305	518
519	269,361	139,798,359	22.7816	8.03629	0.0019268	519
520	270,400	140,608,000	22.8035	8.04145	0.0019231	520
521	271,441	141,420,761	22.8254	8.04660	0.0019194	521
522	272,484	142,236,648	22.8473	8.05175	0.0019157	522
523	273,529	143,055,667	22.8692	8.05689	0.0019120	523
524	274,576	143,877,824	22.8910	8.06202	0.0019084	524
525	275,625	144,703,125	22.9129	8.06714	0.0019048	525
526	276,676	145,531,576	22.9347	8.07226	0.0019011	526
527	277,729	146,363,183	22.9565	8.07737	0.0018975	527
528	278,784	147,197,952	22.9783	8.08248	0.0018939	528
529	279,841	148,035,889	23.0000	8.08758	0.0018904	529
530	280,900	148,877,000	23.0217	8.09267	0.0018868	530
531	281,961	149,721,291	23.0434	8.09776	0.0018832	531
532	283,024	150,568,768	23.0651	8.10284	0.0018797	532
533	284,089	151,419,437	23.0868	8.10791	0.0018762	533
534	285,156	152,273,304	23.1084	8.11298	0.0018727	534
535	286,225	153,130,375	23.1301	8.11804	0.0018692	535
536	287,296	153,990,656	23.1517	8.12310	0.0018657	536
537	288,369	154,854,153	23.1733	8.12814	0.0018622	537
538	289,444	155,720,872	23.1948	8.13319	0.0018587	538
539	290,521	156,590,819	23.2164	8.13822	0.0018553	539
540	291,600	157,464,000	23.2379	8.14325	0.0018519	540
541	292,681	158,340,421	23.2594	8.14828	0.0018484	541
542	293,764	159,220,088	23.2809	8.15329	0.0018450	542
543	294,849	160,103,007	23.3024	8.15831	0.0018416	543
544	295,936	160,989,184	23.3238	8.16331	0.0018382	544
545	297,025	161,878,625	23.3452	8.16831	0.0018349	545
546	298,116	162,771,336	23.3666	8.17330	0.0018315	546
547	299,209	163,667,323	23.3880	8.17829	0.0018282	547
548	300,304	164,566,592	23.4094	8.18327	0.0018248	548
549	301,401	165,469,149	23.4307	8.18824	0.0018215	549
550	302,500	166,375,000	23.4521	8.19321	0.0018182	550

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
551	303,601	167,284,151	23.4734	8.19818	0.0018149	551
552	304,704	168,196,608	23.4947	8.20313	0.0018116	552
553	305,809	169,112,377	23.5160	8.20808	0.0018083	553
554	306,916	170,031,464	23.5372	8.21303	0.0018051	554
555	308,025	170,953,875	23.5584	8.21797	0.0018018	555
556	309,136	171,879,616	23.5797	8.22290	0.0017986	556
557	310,249	172,808,693	23.6008	8.22783	0.0017953	557
558	311,364	173,741,112	23.6220	8.23275	0.0017921	558
559	312,481	174,676,879	23.6432	8.23766	0.0017889	559
560	313,600	175,616,000	23.6643	8.24257	0.0017857	560
561	314,721	176,558,481	23.6854	8.24747	0.0017825	561
562	315,844	177,504,328	23.7065	8.25237	0.0017794	562
563	316,969	178,453,547	23.7276	8.25726	0.0017762	563
564	318,096	179,406,144	23.7487	8.26215	0.0017731	564
565	319,225	180,362,125	23.7697	8.26703	0.0017699	565
566	320,356	181,321,496	23.7908	8.27190	0.0017668	566
567	321,489	182,284,263	23.8118	8.27677	0.0017637	567
568	322,624	183,250,432	23.8328	8.28163	0.0017606	568
569	323,761	184,220,009	23.8537	8.28649	0.0017575	569
570	324,900	185,193,000	23.8747	8.29134	0.0017544	570
571	326,041	186,169,411	23.8956	8.29619	0.0017513	571
572	327,184	187,149,248	23.9165	8.30103	0.0017483	572
573	328,329	188,132,517	23.9374	8.30587	0.0017452	573
574	329,476	189,119,224	23.9583	8.31069	0.0017422	574
575	330,625	190,109,375	23.9792	8.31552	0.0017391	575
576	331,776	191,102,976	24.0000	8.32034	0.0017361	576
577	332,929	192,100,033	24.0208	8.32515	0.0017331	577
578	334,084	193,100,552	24.0416	8.32995	0.0017301	578
579	335,241	194,104,539	24.0624	8.33476	0.0017271	579
580	336,400	195,112,000	24.0832	8.33955	0.0017241	580
581	337,561	196,122,941	24.1039	8.34434	0.0017212	581
582	338,724	197,137,368	24.1247	8.34913	0.0017182	582
583	339,889	198,155,287	24.1454	8.35390	0.0017153	583
584	341,056	199,176,704	24.1661	8.35868	0.0017123	584
585	342,225	200,201,625	24.1868	8.36345	0.0017094	585
586	343,396	201,230,056	24.2074	8.36821	0.0017065	586
587	344,569	202,262,003	24.2281	8.37297	0.0017036	587
588	345,744	203,297,472	24.2487	8.37772	0.0017007	588
589	346,921	204,336,469	24.2693	8.38247	0.0016978	589
590	348,100	205,379,000	24.2899	8.38721	0.0016949	590
591	349,281	206,425,071	24.3105	8.39194	0.0016920	591
592	350,464	207,474,688	24.3311	8.39667	0.0016892	592
593	351,649	208,527,857	24.3516	8.40140	0.0016863	593
594	352,836	209,584,584	24.3721	8.40612	0.0016835	594
595	354,025	210,644,875	24.3926	8.41083	0.0016807	595
596	355,216	211,708,736	24.4131	8.41554	0.0016779	596
597	356,409	212,776,173	24.4336	8.42025	0.0016750	597
598	357,604	213,847,192	24.4540	8.42494	0.0016722	598
599	358,801	214,921,799	24.4745	8.42964	0.0016694	599
600	360,000	216,000,000	24.4949	8.43433	0.0016667	600

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
601	361,201	217,081,801	24.5153	8.43901	0.0016639	601
602	362,404	218,167,208	24.5357	8.44369	0.0016611	602
603	363,609	219,256,227	24.5561	8.44836	0.0016584	603
604	364,816	220,348,864	24.5764	8.45303	0.0016556	604
605	366,025	221,445,125	24.5967	8.45769	0.0016529	605
606	367,236	222,545,016	24.6171	8.46235	0.0016502	606
607	368,449	223,648,543	24.6374	8.46700	0.0016474	607
608	369,664	224,755,712	24.6577	8.47165	0.0016447	608
609	370,881	225,866,529	24.6779	8.47629	0.0016420	609
610	372,100	226,981,000	24.6982	8.48093	0.0016393	610
611	373,321	228,099,131	24.7184	8.48556	0.0016367	611
612	374,544	229,220,928	24.7386	8.49018	0.0016340	612
613	375,769	230,346,397	24.7588	8.49481	0.0016313	613
614	376,996	231,475,544	24.7790	8.49942	0.0016287	614
615	378,225	232,608,375	24.7992	8.50404	0.0016260	615
616	379,456	233,744,896	24.8193	8.50864	0.0016234	616
617	380,689	234,885,113	24.8395	8.51324	0.0016207	617
618	381,924	236,029,032	24.8596	8.51784	0.0016181	618
619	383,161	237,176,659	24.8797	8.52243	0.0016155	619
620	384,400	238,328,000	24.8998	8.52702	0.0016129	620
621	385,641	239,483,061	24.9199	8.53160	0.0016103	621
622	386,884	240,641,848	24.9399	8.53618	0.0016077	622
623	388,129	241,804,367	24.9600	8.54075	0.0016051	623
624	389,376	242,970,624	24.9800	8.54532	0.0016026	624
625	390,625	244,140,625	25.0000	8.54988	0.0016000	625
626	391,876	245,314,376	25.0200	8.55444	0.0015974	626
627	393,129	246,491,883	25.0400	8.55899	0.0015949	627
628	394,384	247,673,152	25.0599	8.56354	0.0015924	628
629	395,641	248,858,189	25.0799	8.56808	0.0015898	629
630	396,900	250,047,000	25.0998	8.57262	0.0015873	630
631	398,161	251,239,591	25.1197	8.57715	0.0015848	631
632	399,424	252,435,968	25.1396	8.58168	0.0015823	632
633	400,689	253,636,137	25.1595	8.58620	0.0015798	633
634	401,956	254,840,104	25.1794	8.59072	0.0015773	634
635	403,225	256,047,875	25.1992	8.59524	0.0015748	635
636	404,496	257,259,456	25.2190	8.59975	0.0015723	636
637	405,769	258,474,853	25.2389	8.60425	0.0015699	637
638	407,044	259,694,072	25.2587	8.60875	0.0015674	638
639	408,321	260,917,119	25.2784	8.61325	0.0015649	639
640	409,600	262,144,000	25.2982	8.61774	0.0015625	640
641	410,881	263,374,721	25.3180	8.62222	0.0015601	641
642	412,164	264,609,288	25.3377	8.62671	0.0015576	642
643	413,449	265,847,707	25.3574	8.63118	0.0015552	643
644	414,736	267,089,984	25.3772	8.63566	0.0015528	644
645	416,025	268,336,125	25.3969	8.64012	0.0015504	645
646	417,316	269,586,136	25.4165	8.64459	0.0015480	646
647	418,609	270,840,023	25.4362	8.64904	0.0015456	647
648	419,904	272,097,792	25.4558	8.65350	0.0015432	648
649	421,201	273,359,449	25.4755	8.65795	0.0015408	649
650	422,500	274,625,000	25.4951	8.66239	0.0015385	650

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
651	423,801	275,894,451	25.5147	8.66683	0.0015361	651
652	425,104	277,167,808	25.5343	8.67127	0.0015337	652
653	426,409	278,445,077	25.5539	8.67570	0.0015314	653
654	427,716	279,726,264	25.5734	8.68012	0.0015291	654
655	429,025	281,011,375	25.5930	8.68455	0.0015267	655
656	430,336	282,300,416	25.6125	8.68896	0.0015244	656
657	431,649	283,593,393	25.6320	8.69338	0.0015221	657
658	432,964	284,890,312	25.6515	8.69778	0.0015198	658
659	434,281	286,191,179	25.6710	8.70219	0.0015175	659
660	435,600	287,496,000	25.6905	8.70659	0.0015152	660
661	436,921	288,804,781	25.7099	8.71098	0.0015129	661
662	438,244	290,117,528	25.7294	8.71537	0.0015106	662
663	439,569	291,434,247	25.7488	8.71976	0.0015083	663
664	440,896	292,754,944	25.7682	8.72414	0.0015060	664
665	442,225	294,079,625	25.7876	8.72852	0.0015038	665
666	443,556	295,408,296	25.8070	8.73289	0.0015015	666
667	444,889	296,740,963	25.8263	8.73726	0.0014993	667
668	446,224	298,077,632	25.8457	8.74162	0.0014970	668
669	447,561	299,418,309	25.8650	8.74598	0.0014948	669
670	448,900	300,763,000	25.8844	8.75034	0.0014925	670
671	450,241	302,111,711	25.9037	8.75469	0.0014903	671
672	451,584	303,464,448	25.9230	8.75904	0.0014881	672
673	452,929	304,821,217	25.9422	8.76338	0.0014859	673
674	454,276	306,182,024	25.9615	8.76772	0.0014837	674
675	455,625	307,546,875	25.9808	8.77205	0.0014815	675
676	456,976	308,915,776	26.0000	8.77638	0.0014793	676
677	458,329	310,288,733	26.0192	8.78071	0.0014771	677
678	459,684	311,665,752	26.0384	8.78503	0.0014749	678
679	461,041	313,046,839	26.0576	8.78935	0.0014728	679
680	462,400	314,432,000	26.0768	8.79366	0.0014706	680
681	463,761	315,821,241	26.0960	8.79797	0.0014684	681
682	465,124	317,214,568	26.1151	8.80227	0.0014663	682
683	466,489	318,611,987	26.1343	8.80657	0.0014641	683
684	467,856	320,013,504	26.1534	8.81087	0.0014620	684
685	469,225	321,419,125	26.1725	8.81516	0.0014599	685
686	470,596	322,828,856	26.1916	8.81945	0.0014577	686
687	471,969	324,242,703	26.2107	8.82373	0.0014556	687
688	473,344	325,660,672	26.2298	8.82801	0.0014535	688
689	474,721	327,082,769	26.2488	8.83229	0.0014514	689
690	476,100	328,509,000	26.2679	8.83656	0.0014493	690
691	477,481	329,939,371	26.2869	8.84082	0.0014472	691
692	478,864	331,373,888	26.3059	8.84509	0.0014451	692
693	480,249	332,812,557	26.3249	8.84934	0.0014430	693
694	481,636	334,255,384	26.3439	8.85360	0.0014409	694
695	483,025	335,702,375	26.3629	8.85785	0.0014388	695
696	484,416	337,153,536	26.3818	8.86210	0.0014368	696
697	485,809	338,608,873	26.4008	8.86634	0.0014347	697
698	487,204	340,068,392	26.4197	8.87058	0.0014327	698
699	488,601	341,532,099	26.4386	8.87481	0.0014306	699
700	490,000	343,000,000	26.4575	8.87904	0.0014286	700

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
701	491,401	344,472,101	26.4764	8.88327	0.0014265	701
702	492,804	345,948,408	26.4953	8.88749	0.0014245	702
703	494,209	347,428,927	26.5141	8.89171	0.0014225	703
704	495,616	348,913,664	26.5330	8.89592	0.0014205	704
705	497,025	350,402,625	26.5518	8.90013	0.0014184	705
706	498,436	351,895,816	26.5707	8.90434	0.0014164	706
707	499,849	353,393,243	26.5895	8.90854	0.0014144	707
708	501,264	354,894,912	26.6083	8.91274	0.0014124	708
709	502,681	356,400,829	26.6271	8.91693	0.0014104	709
710	504,100	357,911,000	26.6458	8.92112	0.0014085	710
711	505,521	359,425,431	26.6646	8.92531	0.0014065	711
712	506,944	360,944,128	26.6833	8.92949	0.0014045	712
713	508,369	362,467,097	26.7021	8.93367	0.0014025	713
714	509,796	363,994,344	26.7208	8.93784	0.0014006	714
715	511,225	365,525,875	26.7395	8.94201	0.0013986	715
716	512,656	367,061,696	26.7582	8.94618	0.0013966	716
717	514,089	368,601,813	26.7769	8.95034	0.0013947	717
718	515,524	370,146,232	26.7955	8.95450	0.0013928	718
719	516,961	371,694,959	26.8142	8.95866	0.0013908	719
720	518,400	373,248,000	26.8328	8.96281	0.0013889	720
721	519,841	374,805,361	26.8514	8.96696	0.0013870	721
722	521,284	376,367,048	26.8701	8.97110	0.0013850	722
723	522,729	377,933,067	26.8887	8.97524	0.0013831	723
724	524,176	379,503,424	26.9072	8.97938	0.0013812	724
725	525,625	381,078,125	26.9258	8.98351	0.0013793	725
726	527,076	382,657,176	26.9444	8.98764	0.0013774	726
727	528,529	384,240,583	26.9629	8.99176	0.0013755	727
728	529,984	385,828,352	26.9815	8.99589	0.0013736	728
729	531,441	387,420,489	27.0000	9.00000	0.0013717	729
730	532,900	389,017,000	27.0185	9.00411	0.0013699	730
731	534,361	390,617,891	27.0370	9.00822	0.0013680	731
732	535,824	392,223,168	27.0555	9.01233	0.0013661	732
733	537,289	393,832,837	27.0740	9.01643	0.0013643	733
734	538,756	395,446,904	27.0924	9.02053	0.0013624	734
735	540,225	397,065,375	27.1109	9.02462	0.0013605	735
736	541,696	398,688,256	27.1293	9.02871	0.0013587	736
737	543,169	400,315,553	27.1477	9.03280	0.0013569	737
738	544,644	401,947,272	27.1662	9.03689	0.0013550	738
739	546,121	403,583,419	27.1846	9.04097	0.0013532	739
740	547,600	405,224,000	27.2029	9.04504	0.0013514	740
741	549,081	406,869,021	27.2213	9.04911	0.0013495	741
742	550,564	408,518,488	27.2397	9.05318	0.0013477	742
743	552,049	410,172,407	27.2580	9.05725	0.0013459	743
744	553,536	411,830,784	27.2764	9.06131	0.0013441	744
745	555,025	413,493,625	27.2947	9.06537	0.0013423	745
746	556,516	415,160,936	27.3130	9.06942	0.0013405	746
747	558,009	416,832,723	27.3313	9.07347	0.0013387	747
748	559,504	418,508,992	27.3496	9.07752	0.0013369	748
749	561,001	420,189,749	27.3679	9.08156	0.0013351	749
750	562,500	421,875,000	27.3861	9.08560	0.0013333	750

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
751	564,001	423,564,751	27.4044	9.08964	0.0013316	751
752	565,504	425,259,008	27.4226	9.09367	0.0013298	752
753	567,009	426,957,777	27.4408	9.09770	0.0013280	753
754	568,516	428,661,064	27.4591	9.10173	0.0013263	754
755	570,025	430,368,875	27.4773	9.10575	0.0013245	755
756	571,536	432,081,216	27.4955	9.10977	0.0013228	756
757	573,049	433,798,093	27.5136	9.11378	0.0013210	757
758	574,564	435,519,512	27.5318	9.11779	0.0013193	758
759	576,081	437,245,479	27.5500	9.12180	0.0013175	759
760	577,600	438,976,000	27.5681	9.12581	0.0013158	760
761	579,121	440,711,081	27.5862	9.12981	0.0013141	761
762	580,644	442,450,728	27.6043	9.13380	0.0013123	762
763	582,169	444,194,947	27.6225	9.13780	0.0013106	763
764	583,696	445,943,744	27.6405	9.14179	0.0013089	764
765	585,225	447,697,125	27.6586	9.14577	0.0013072	765
766	586,756	449,455,096	27.6767	9.14976	0.0013055	766
767	588,289	451,217,663	27.6948	9.15374	0.0013038	767
768	589,824	452,984,832	27.7128	9.15771	0.0013021	768
769	591,361	454,756,609	27.7308	9.16169	0.0013004	769
770	592,900	456,533,000	27.7489	9.16566	0.0012987	770
771	594,441	458,314,011	27.7669	9.16962	0.0012970	771
772	595,984	460,099,648	27.7849	9.17359	0.0012953	772
773	597,529	461,889,917	27.8029	9.17754	0.0012937	773
774	599,076	463,684,824	27.8209	9.18150	0.0012920	774
775	600,625	465,484,375	27.8388	9.18545	0.0012903	775
776	602,176	467,288,576	27.8568	9.18940	0.0012887	776
777	603,729	469,097,433	27.8747	9.19335	0.0012870	777
778	605,284	470,910,952	27.8927	9.19729	0.0012853	778
779	606,841	472,729,139	27.9106	9.20123	0.0012837	779
780	608,400	474,552,000	27.9285	9.20516	0.0012821	780
781	609,961	476,379,541	27.9464	9.20910	0.0012804	781
782	611,524	478,211,768	27.9643	9.21303	0.0012788	782
783	613,089	480,048,687	27.9821	9.21695	0.0012771	783
784	614,656	481,890,304	28.0000	9.22087	0.0012755	784
785	616,225	483,736,625	28.0179	9.22479	0.0012739	785
786	617,796	485,587,656	28.0357	9.22871	0.0012723	786
787	619,369	487,443,403	28.0535	9.23262	0.0012706	787
788	620,944	489,303,872	28.0713	9.23653	0.0012690	788
789	622,521	491,169,069	28.0891	9.24043	0.0012674	789
790	624,100	493,039,000	28.1069	9.24434	0.0012658	790
791	625,681	494,913,671	28.1247	9.24823	0.0012642	791
792	627,264	496,793,088	28.1425	9.25213	0.0012626	792
793	628,849	498,677,257	28.1603	9.25602	0.0012610	793
794	630,436	500,566,184	28.1780	9.25991	0.0012594	794
795	632,025	502,459,875	28.1957	9.26380	0.0012579	795
796	633,616	504,358,336	28.2135	9.26768	0.0012563	796
797	635,209	506,261,573	28.2312	9.27156	0.0012547	797
798	636,804	508,169,592	28.2489	9.27544	0.0012531	798
799	638,401	510,082,399	28.2666	9.27931	0.0012516	799
800	640,000	512,000,000	28.2843	9.28318	0.0012500	800

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
801	641,601	513,922,401	28.3019	9.28704	0.0012484	801
802	643,204	515,849,608	28.3196	9.29091	0.0012469	802
803	644,809	517,781,627	28.3373	9.29477	0.0012453	803
804	646,416	519,718,464	28.3549	9.29862	0.0012438	804
805	648,025	521,660,125	28.3725	9.30248	0.0012422	805
806	649,636	523,606,616	28.3901	9.30633	0.0012407	806
807	651,249	525,557,943	28.4077	9.31018	0.0012392	807
808	652,864	527,514,112	28.4253	9.31402	0.0012376	808
809	654,481	529,475,129	28.4429	9.31786	0.0012361	809
810	656,100	531,441,000	28.4605	9.32170	0.0012346	810
811	657,721	533,411,731	28.4781	9.32553	0.0012330	811
812	659,344	535,387,328	28.4956	9.32936	0.0012315	812
813	660,969	537,367,797	28.5132	9.33319	0.0012300	813
814	662,596	539,353,144	28.5307	9.33702	0.0012285	814
815	664,225	541,343,375	28.5482	9.34084	0.0012270	815
816	665,856	543,338,496	28.5657	9.34466	0.0012255	816
817	667,489	545,338,513	28.5832	9.34847	0.0012240	817
818	669,124	547,343,432	28.6007	9.35229	0.0012225	818
819	670,761	549,353,259	28.6182	9.35610	0.0012210	819
820	672,400	551,368,000	28.6356	9.35990	0.0012195	820
821	674,041	553,387,661	28.6531	9.36370	0.0012180	821
822	675,684	555,412,248	28.6705	9.36751	0.0012165	822
823	677,329	557,441,767	28.6880	9.37130	0.0012151	823
824	678,976	559,476,224	28.7054	9.37510	0.0012136	824
825	680,625	561,515,625	28.7228	9.37889	0.0012121	825
826	682,276	563,559,976	28.7402	9.38268	0.0012107	826
827	683,929	565,609,283	28.7576	9.38646	0.0012092	827
828	685,584	567,663,552	28.7750	9.39024	0.0012077	828
829	687,241	569,722,789	28.7924	9.39402	0.0012063	829
830	688,900	571,787,000	28.8097	9.39780	0.0012048	830
831	690,561	573,856,191	28.8271	9.40157	0.0012034	831
832	692,224	575,930,368	28.8444	9.40534	0.0012019	832
833	693,889	578,009,537	28.8617	9.40911	0.0012005	833
834	695,556	580,093,704	28.8791	9.41287	0.0011990	834
835	697,225	582,182,875	28.8964	9.41663	0.0011976	835
836	698,896	584,277,056	28.9137	9.42039	0.0011962	836
837	700,569	586,376,253	28.9310	9.42414	0.0011947	837
838	702,244	588,480,472	28.9482	9.42789	0.0011933	838
839	703,921	590,589,719	28.9655	9.43164	0.0011919	839
840	705,600	592,704,000	28.9828	9.43538	0.0011905	840
841	707,281	594,823,321	29.0000	9.43913	0.0011891	841
842	708,964	596,947,688	29.0172	9.44287	0.0011876	842
843	710,649	599,077,107	29.0345	9.44661	0.0011862	843
844	712,336	601,211,584	29.0517	9.45034	0.0011848	844
845	714,025	603,351,125	29.0689	9.45407	0.0011834	845
846	715,716	605,495,736	29.0861	9.45780	0.0011820	846
847	717,409	607,645,423	29.1033	9.46152	0.0011806	847
848	719,104	609,800,192	29.1204	9.46525	0.0011792	848
849	720,801	611,960,049	29.1376	9.46897	0.0011779	849
850	722,500	614,125,000	29.1548	9.47268	0.0011765	850

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
851	724,201	616,295,051	29.1719	9.47640	0.0011751	851
852	725,904	618,470,208	29.1890	9.48011	0.0011737	852
853	727,609	620,650,477	29.2062	9.48381	0.0011723	853
854	729,316	622,835,864	29.2233	9.48752	0.0011710	854
855	731,025	625,026,375	29.2404	9.49122	0.0011696	855
856	732,736	627,222,016	29.2575	9.49492	0.0011682	856
857	734,449	629,422,793	29.2746	9.49861	0.0011669	857
858	736,164	631,628,712	29.2916	9.50231	0.0011655	858
859	737,881	633,839,779	29.3087	9.50600	0.0011641	859
860	739,600	636,056,000	29.3258	9.50969	0.0011628	860
861	741,321	638,277,381	29.3428	9.51337	0.0011614	861
862	743,044	640,503,928	29.3598	9.51705	0.0011601	862
863	744,769	642,735,647	29.3769	9.52073	0.0011587	863
864	746,496	644,972,544	29.3939	9.52441	0.0011574	864
865	748,225	647,214,625	29.4109	9.52808	0.0011561	865
866	749,956	649,461,896	29.4279	9.53175	0.0011547	866
867	751,689	651,714,363	29.4449	9.53542	0.0011534	867
868	753,424	653,972,032	29.4618	9.53908	0.0011521	868
869	755,161	656,234,909	29.4788	9.54274	0.0011507	869
870	756,900	658,503,000	29.4958	9.54640	0.0011494	870
871	758,641	660,776,311	29.5127	9.55006	0.0011481	871
872	760,384	663,054,848	29.5296	9.55371	0.0011468	872
873	762,129	665,338,617	29.5466	9.55736	0.0011455	873
874	763,876	667,627,624	29.5635	9.56101	0.0011442	874
875	765,625	669,921,875	29.5804	9.56466	0.0011429	875
876	767,376	672,221,376	29.5973	9.56830	0.0011416	876
877	769,129	674,526,133	29.6142	9.57194	0.0011403	877
878	770,884	676,836,152	29.6311	9.57557	0.0011390	878
879	772,641	679,151,439	29.6479	9.57921	0.0011377	879
880	774,400	681,472,000	29.6648	9.58284	0.0011364	880
881	776,161	683,797,841	29.6816	9.58647	0.0011351	881
882	777,924	686,128,968	29.6985	9.59009	0.0011338	882
883	779,689	688,465,387	29.7153	9.59372	0.0011325	883
884	781,456	690,807,104	29.7321	9.59734	0.0011312	884
885	783,225	693,154,125	29.7489	9.60095	0.0011299	885
886	784,996	695,506,456	29.7658	9.60457	0.0011287	886
887	786,769	697,864,103	29.7825	9.60818	0.0011274	887
888	788,544	700,227,072	29.7993	9.61179	0.0011261	888
889	790,321	702,595,369	29.8161	9.61540	0.0011249	889
890	792,100	704,969,000	29.8329	9.61900	0.0011236	890
891	793,881	707,347,971	29.8496	9.62260	0.0011223	891
892	795,664	709,732,288	29.8664	9.62620	0.0011211	892
893	797,449	712,121,957	29.8831	9.62980	0.0011198	893
894	799,236	714,516,984	29.8998	9.63339	0.0011186	894
895	801,025	716,917,375	29.9166	9.63698	0.0011173	895
896	802,816	719,323,136	29.9333	9.64057	0.0011161	896
897	804,609	721,734,273	29.9500	9.64415	0.0011148	897
898	806,404	724,150,792	29.9666	9.64774	0.0011136	898
899	808,201	726,572,699	29.9833	9.65132	0.0011123	899
900	810,000	729,000,000	30.0000	9.65489	0.0011111	900

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
901	811,801	731,432,701	30.0167	9.65847	0.0011099	901
902	813,604	733,870,808	30.0333	9.66204	0.0011086	902
903	815,409	736,314,327	30.0500	9.66561	0.0011074	903
904	817,216	738,763,264	30.0666	9.66918	0.0011062	904
905	819,025	741,217,625	30.0832	9.67274	0.0011050	905
906	820,836	743,677,416	30.0998	9.67630	0.0011038	906
907	822,649	746,142,643	30.1164	9.67986	0.0011025	907
908	824,464	748,613,312	30.1330	9.68342	0.0011013	908
909	826,281	751,089,429	30.1496	9.68697	0.0011001	909
910	828,100	753,571,000	30.1662	9.69052	0.0010989	910
911	829,921	756,058,031	30.1828	9.69407	0.0010977	911
912	831,744	758,550,528	30.1993	9.69762	0.0010965	912
913	833,569	761,048,497	30.2159	9.70116	0.0010953	913
914	835,396	763,551,944	30.2324	9.70470	0.0010941	914
915	837,225	766,060,875	30.2490	9.70824	0.0010929	915
916	839,056	768,575,296	30.2655	9.71177	0.0010917	916
917	840,889	771,095,213	30.2820	9.71531	0.0010905	917
918	842,724	773,620,632	30.2985	9.71884	0.0010893	918
919	844,561	776,151,559	30.3150	9.72236	0.0010881	919
920	846,400	778,688,000	30.3315	9.72589	0.0010870	920
921	848,241	781,229,961	30.3480	9.72941	0.0010858	921
922	850,084	783,777,448	30.3645	9.73293	0.0010846	922
923	851,929	786,330,467	30.3809	9.73645	0.0010834	923
924	853,776	788,889,024	30.3974	9.73996	0.0010823	924
925	855,625	791,453,125	30.4138	9.74348	0.0010811	925
926	857,476	794,022,776	30.4302	9.74699	0.0010799	926
927	859,329	796,597,983	30.4467	9.75049	0.0010787	927
928	861,184	799,178,752	30.4631	9.75400	0.0010776	928
929	863,041	801,765,089	30.4795	9.75750	0.0010764	929
930	864,900	804,357,000	30.4959	9.76100	0.0010753	930
931	866,761	806,954,491	30.5123	9.76450	0.0010741	931
932	868,624	809,557,568	30.5287	9.76799	0.0010730	932
933	870,489	812,166,237	30.5450	9.77148	0.0010718	933
934	872,356	814,780,504	30.5614	9.77497	0.0010707	934
935	874,225	817,400,375	30.5778	9.77846	0.0010695	935
936	876,096	820,025,856	30.5941	9.78195	0.0010684	936
937	877,969	822,656,953	30.6105	9.78543	0.0010672	937
938	879,844	825,293,672	30.6268	9.78891	0.0010661	938
939	881,721	827,936,019	30.6431	9.79239	0.0010650	939
940	883,600	830,584,000	30.6594	9.79586	0.0010638	940
941	885,481	833,237,621	30.6757	9.79933	0.0010627	941
942	887,364	835,896,888	30.6920	9.80280	0.0010616	942
943	889,249	838,561,807	30.7083	9.80627	0.0010604	943
944	891,136	841,232,384	30.7246	9.80974	0.0010593	944
945	893,025	843,908,625	30.7409	9.81320	0.0010582	945
946	894,916	846,590,536	30.7571	9.81666	0.0010571	946
947	896,809	849,278,123	30.7734	9.82012	0.0010560	947
948	898,704	851,971,392	30.7896	9.82357	0.0010549	948
949	900,601	854,670,349	30.8058	9.82703	0.0010537	949
950	902,500	857,375,000	30.8221	9.83048	0.0010526	950

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
951	904,401	860,085,351	30.8383	9.83392	0.0010515	951
952	906,304	862,801,408	30.8545	9.83737	0.0010504	952
953	908,209	865,523,177	30.8707	9.84081	0.0010493	953
954	910,116	868,250,664	30.8869	9.84425	0.0010482	954
955	912,025	870,983,875	30.9031	9.84769	0.0010471	955
956	913,936	873,722,816	30.9192	9.85113	0.0010460	956
957	915,849	876,467,493	30.9354	9.85456	0.0010449	957
958	917,764	879,217,912	30.9516	9.85799	0.0010438	958
959	919,681	881,974,079	30.9677	9.86142	0.0010428	959
960	921,600	884,736,000	30.9839	9.86485	0.0010417	960
961	923,521	887,503,681	31.0000	9.86827	0.0010406	961
962	925,444	890,277,128	31.0161	9.87169	0.0010395	962
963	927,369	893,056,347	31.0322	9.87511	0.0010384	963
964	929,296	895,841,344	31.0483	9.87853	0.0010373	964
965	931,225	898,632,125	31.0644	9.88195	0.0010363	965
966	933,156	901,428,696	31.0805	9.88536	0.0010352	966
967	935,089	904,231,063	31.0966	9.88877	0.0010341	967
968	937,024	907,039,232	31.1127	9.89217	0.0010331	968
969	938,961	909,853,209	31.1288	9.89558	0.0010320	969
970	940,900	912,673,000	31.1448	9.89898	0.0010309	970
971	942,841	915,498,611	31.1609	9.90238	0.0010299	971
972	944,784	918,330,048	31.1769	9.90578	0.0010288	972
973	946,729	921,167,317	31.1929	9.90918	0.0010277	973
974	948,676	924,010,424	31.2090	9.91257	0.0010267	974
975	950,625	926,859,375	31.2250	9.91596	0.0010256	975
976	952,576	929,714,176	31.2410	9.91935	0.0010246	976
977	954,529	932,574,833	31.2570	9.92274	0.0010235	977
978	956,484	935,441,352	31.2730	9.92612	0.0010225	978
979	958,441	938,313,739	31.2890	9.92950	0.0010215	979
980	960,400	941,192,000	31.3050	9.93288	0.0010204	980
981	962,361	944,076,141	31.3209	9.93626	0.0010194	981
982	964,324	946,966,168	31.3369	9.93964	0.0010183	982
983	966,289	949,862,087	31.3528	9.94301	0.0010173	983
984	968,256	952,763,904	31.3688	9.94638	0.0010163	984
985	970,225	955,671,625	31.3847	9.94975	0.0010152	985
986	972,196	958,585,256	31.4006	9.95311	0.0010142	986
987	974,169	961,504,803	31.4166	9.95648	0.0010132	987
988	976,144	964,430,272	31.4325	9.95984	0.0010121	988
989	978,121	967,361,669	31.4484	9.96320	0.0010111	989
990	980,100	970,299,000	31.4643	9.96655	0.0010101	990
991	982,081	973,242,271	31.4802	9.96991	0.0010091	991
992	984,064	976,191,488	31.4960	9.97326	0.0010081	992
993	986,049	979,146,657	31.5119	9.97661	0.0010070	993
994	988,036	982,107,784	31.5278	9.97996	0.0010060	994
995	990,025	985,074,875	31.5436	9.98331	0.0010050	995
996	992,016	988,047,936	31.5595	9.98665	0.0010040	996
997	994,009	991,026,973	31.5753	9.98999	0.0010030	997
998	996,004	994,011,992	31.5911	9.99333	0.0010020	998
999	998,001	997,002,999	31.6070	9.99667	0.0010010	999
1000	1,000,000	1,000,000,000	31.6228	10.00000	0.0010000	1000

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1001	1,002,001	1,003,003,001	31.6386	10.0033	0.0009990	1001
1002	1,004,004	1,006,012,008	31.6544	10.0067	0.0009980	1002
1003	1,006,009	1,009,027,027	31.6702	10.0100	0.0009970	1003
1004	1,008,016	1,012,048,064	31.6860	10.0133	0.0009960	1004
1005	1,010,025	1,015,075,125	31.7017	10.0166	0.0009950	1005
1006	1,012,036	1,018,108,216	31.7175	10.0200	0.0009940	1006
1007	1,014,049	1,021,147,343	31.7333	10.0233	0.0009930	1007
1008	1,016,064	1,024,192,512	31.7490	10.0266	0.0009921	1008
1009	1,018,081	1,027,243,729	31.7648	10.0299	0.0009911	1009
1010	1,020,100	1,030,301,000	31.7805	10.0332	0.0009901	1010
1011	1,022,121	1,033,364,331	31.7962	10.0365	0.0009891	1011
1012	1,024,144	1,036,433,728	31.8119	10.0398	0.0009881	1012
1013	1,026,169	1,039,509,197	31.8277	10.0431	0.0009872	1013
1014	1,028,196	1,042,590,744	31.8434	10.0465	0.0009862	1014
1015	1,030,225	1,045,678,375	31.8591	10.0498	0.0009852	1015
1016	1,032,256	1,048,772,096	31.8748	10.0531	0.0009843	1016
1017	1,034,289	1,051,871,913	31.8904	10.0563	0.0009833	1017
1018	1,036,324	1,054,977,832	31.9061	10.0596	0.0009823	1018
1019	1,038,361	1,058,089,859	31.9218	10.0629	0.0009814	1019
1020	1,040,400	1,061,208,000	31.9374	10.0662	0.0009804	1020
1021	1,042,441	1,064,332,261	31.9531	10.0695	0.0009794	1021
1022	1,044,484	1,067,462,648	31.9687	10.0728	0.0009785	1022
1023	1,046,529	1,070,599,167	31.9844	10.0761	0.0009775	1023
1024	1,048,576	1,073,741,824	32.0000	10.0794	0.0009766	1024
1025	1,050,625	1,076,890,625	32.0156	10.0826	0.0009756	1025
1026	1,052,676	1,080,045,576	32.0312	10.0859	0.0009747	1026
1027	1,054,729	1,083,206,683	32.0468	10.0892	0.0009737	1027
1028	1,056,784	1,086,373,952	32.0624	10.0925	0.0009728	1028
1029	1,058,841	1,089,547,389	32.0780	10.0957	0.0009718	1029
1030	1,060,900	1,092,727,000	32.0936	10.0990	0.0009709	1030
1031	1,062,961	1,095,912,791	32.1092	10.1023	0.0009699	1031
1032	1,065,024	1,099,104,768	32.1248	10.1055	0.0009690	1032
1033	1,067,089	1,102,302,937	32.1403	10.1088	0.0009681	1033
1034	1,069,156	1,105,507,304	32.1559	10.1121	0.0009671	1034
1035	1,071,225	1,108,717,875	32.1714	10.1153	0.0009662	1035
1036	1,073,296	1,111,934,656	32.1870	10.1186	0.0009653	1036
1037	1,075,369	1,115,157,653	32.2025	10.1218	0.0009643	1037
1038	1,077,444	1,118,386,872	32.2180	10.1251	0.0009634	1038
1039	1,079,521	1,121,622,319	32.2335	10.1283	0.0009625	1039
1040	1,081,600	1,124,864,000	32.2490	10.1316	0.0009615	1040
1041	1,083,681	1,128,111,921	32.2645	10.1348	0.0009606	1041
1042	1,085,764	1,131,366,088	32.2800	10.1381	0.0009597	1042
1043	1,087,849	1,134,626,507	32.2955	10.1413	0.0009588	1043
1044	1,089,936	1,137,893,184	32.3110	10.1446	0.0009579	1044
1045	1,092,025	1,141,166,125	32.3265	10.1478	0.0009569	1045
1046	1,094,116	1,144,445,336	32.3419	10.1510	0.0009560	1046
1047	1,096,209	1,147,730,823	32.3574	10.1543	0.0009551	1047
1048	1,098,304	1,151,022,592	32.3728	10.1575	0.0009542	1048
1049	1,100,401	1,154,320,649	32.3883	10.1607	0.0009533	1049
1050	1,102,500	1,157,625,000	32.4037	10.1640	0.0009524	1050

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1051	1,104,601	1,160,935,651	32.4191	10.1672	0.0009515	1051
1052	1,106,704	1,164,252,608	32.4345	10.1704	0.0009506	1052
1053	1,108,809	1,167,575,877	32.4500	10.1736	0.0009497	1053
1054	1,110,916	1,170,905,464	32.4654	10.1769	0.0009488	1054
1055	1,113,025	1,174,241,375	32.4808	10.1801	0.0009479	1055
1056	1,115,136	1,177,583,616	32.4962	10.1833	0.0009470	1056
1057	1,117,249	1,180,932,193	32.5115	10.1865	0.0009461	1057
1058	1,119,364	1,184,287,112	32.5269	10.1897	0.0009452	1058
1059	1,121,481	1,187,648,379	32.5423	10.1929	0.0009443	1059
1060	1,123,600	1,191,016,000	32.5576	10.1961	0.0009434	1060
1061	1,125,721	1,194,389,981	32.5730	10.1993	0.0009425	1061
1062	1,127,844	1,197,770,328	32.5883	10.2025	0.0009416	1062
1063	1,129,969	1,201,157,047	32.6037	10.2057	0.0009407	1063
1064	1,132,096	1,204,550,144	32.6190	10.2089	0.0009398	1064
1065	1,134,225	1,207,949,625	32.6343	10.2121	0.0009390	1065
1066	1,136,356	1,211,355,496	32.6497	10.2153	0.0009381	1066
1067	1,138,489	1,214,767,763	32.6650	10.2185	0.0009372	1067
1068	1,140,624	1,218,186,432	32.6803	10.2217	0.0009363	1068
1069	1,142,761	1,221,611,509	32.6956	10.2249	0.0009355	1069
1070	1,144,900	1,225,043,000	32.7109	10.2281	0.0009346	1070
1071	1,147,041	1,228,480,911	32.7261	10.2313	0.0009337	1071
1072	1,149,184	1,231,925,248	32.7414	10.2345	0.0009328	1072
1073	1,151,329	1,235,376,017	32.7567	10.2376	0.0009320	1073
1074	1,153,476	1,238,833,224	32.7719	10.2408	0.0009311	1074
1075	1,155,625	1,242,296,875	32.7872	10.2440	0.0009302	1075
1076	1,157,776	1,245,766,976	32.8024	10.2472	0.0009294	1076
1077	1,159,929	1,249,243,533	32.8177	10.2503	0.0009285	1077
1078	1,162,084	1,252,726,552	32.8329	10.2535	0.0009276	1078
1079	1,164,241	1,256,216,039	32.8481	10.2567	0.0009268	1079
1080	1,166,400	1,259,712,000	32.8634	10.2599	0.0009259	1080
1081	1,168,561	1,263,214,441	32.8786	10.2630	0.0009251	1081
1082	1,170,724	1,266,723,368	32.8938	10.2662	0.0009242	1082
1083	1,172,889	1,270,238,787	32.9090	10.2693	0.0009234	1083
1084	1,175,056	1,273,760,704	32.9242	10.2725	0.0009225	1084
1085	1,177,225	1,277,289,125	32.9393	10.2757	0.0009217	1085
1086	1,179,396	1,280,824,056	32.9545	10.2788	0.0009208	1086
1087	1,181,569	1,284,365,503	32.9697	10.2820	0.0009200	1087
1088	1,183,744	1,287,913,472	32.9848	10.2851	0.0009191	1088
1089	1,185,921	1,291,467,969	33.0000	10.2883	0.0009183	1089
1090	1,188,100	1,295,029,000	33.0151	10.2914	0.0009174	1090
1091	1,190,281	1,298,596,571	33.0303	10.2946	0.0009166	1091
1092	1,192,464	1,302,170,688	33.0454	10.2977	0.0009158	1092
1093	1,194,649	1,305,751,357	33.0606	10.3009	0.0009149	1093
1094	1,196,836	1,309,338,584	33.0757	10.3040	0.0009141	1094
1095	1,199,025	1,312,932,375	33.0908	10.3071	0.0009132	1095
1096	1,201,216	1,316,532,736	33.1059	10.3103	0.0009124	1096
1097	1,203,409	1,320,139,673	33.1210	10.3134	0.0009116	1097
1098	1,205,604	1,323,753,192	33.1361	10.3165	0.0009107	1098
1099	1,207,801	1,327,373,299	33.1512	10.3197	0.0009099	1099
1100	1,210,000	1,331,000,000	33.1662	10.3228	0.0009091	1100

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1101	1,212,201	1,334,633,301	33.1813	10.3259	0.0009083	1101
1102	1,214,404	1,338,273,208	33.1964	10.3291	0.0009074	1102
1103	1,216,609	1,341,919,727	33.2114	10.3322	0.0009066	1103
1104	1,218,816	1,345,572,864	33.2265	10.3353	0.0009058	1104
1105	1,221,025	1,349,232,625	33.2415	10.3384	0.0009050	1105
1106	1,223,236	1,352,899,016	33.2566	10.3415	0.0009042	1106
1107	1,225,449	1,356,572,043	33.2716	10.3447	0.0009033	1107
1108	1,227,664	1,360,251,712	33.2866	10.3478	0.0009025	1108
1109	1,229,881	1,363,938,029	33.3017	10.3509	0.0009017	1109
1110	1,232,100	1,367,631,000	33.3167	10.3540	0.0009009	1110
1111	1,234,321	1,371,330,631	33.3317	10.3571	0.0009001	1111
1112	1,236,544	1,375,036,928	33.3467	10.3602	0.0008993	1112
1113	1,238,769	1,378,749,897	33.3617	10.3633	0.0008985	1113
1114	1,240,996	1,382,469,544	33.3766	10.3664	0.0008977	1114
1115	1,243,225	1,386,195,875	33.3916	10.3695	0.0008969	1115
1116	1,245,456	1,389,928,896	33.4066	10.3726	0.0008961	1116
1117	1,247,689	1,393,668,613	33.4215	10.3757	0.0008953	1117
1118	1,249,924	1,397,415,032	33.4365	10.3788	0.0008945	1118
1119	1,252,161	1,401,168,159	33.4515	10.3819	0.0008937	1119
1120	1,254,400	1,404,928,000	33.4664	10.3850	0.0008929	1120
1121	1,256,641	1,408,694,561	33.4813	10.3881	0.0008921	1121
1122	1,258,884	1,412,467,848	33.4963	10.3912	0.0008913	1122
1123	1,261,129	1,416,247,867	33.5112	10.3943	0.0008905	1123
1124	1,263,376	1,420,034,624	33.5261	10.3973	0.0008897	1124
1125	1,265,625	1,423,828,125	33.5410	10.4004	0.0008889	1125
1126	1,267,876	1,427,628,376	33.5559	10.4035	0.0008881	1126
1127	1,270,129	1,431,435,383	33.5708	10.4066	0.0008873	1127
1128	1,272,384	1,435,249,152	33.5857	10.4097	0.0008865	1128
1129	1,274,641	1,439,069,689	33.6006	10.4127	0.0008857	1129
1130	1,276,900	1,442,897,000	33.6155	10.4158	0.0008850	1130
1131	1,279,161	1,446,731,091	33.6303	10.4189	0.0008842	1131
1132	1,281,424	1,450,571,968	33.6452	10.4219	0.0008834	1132
1133	1,283,689	1,454,419,637	33.6601	10.4250	0.0008826	1133
1134	1,285,956	1,458,274,104	33.6749	10.4281	0.0008818	1134
1135	1,288,225	1,462,135,375	33.6898	10.4311	0.0008811	1135
1136	1,290,496	1,466,003,456	33.7046	10.4342	0.0008803	1136
1137	1,292,769	1,469,878,353	33.7194	10.4373	0.0008795	1137
1138	1,295,044	1,473,760,072	33.7342	10.4403	0.0008787	1138
1139	1,297,321	1,477,648,619	33.7491	10.4434	0.0008780	1139
1140	1,299,600	1,481,544,000	33.7639	10.4464	0.0008772	1140
1141	1,301,881	1,485,446,221	33.7787	10.4495	0.0008764	1141
1142	1,304,164	1,489,355,288	33.7935	10.4525	0.0008757	1142
1143	1,306,449	1,493,271,207	33.8083	10.4556	0.0008749	1143
1144	1,308,736	1,497,193,984	33.8231	10.4586	0.0008741	1144
1145	1,311,025	1,501,123,625	33.8378	10.4617	0.0008734	1145
1146	1,313,316	1,505,060,136	33.8526	10.4647	0.0008726	1146
1147	1,315,609	1,509,003,523	33.8674	10.4678	0.0008718	1147
1148	1,317,904	1,512,953,792	33.8821	10.4708	0.0008711	1148
1149	1,320,201	1,516,910,949	33.8969	10.4739	0.0008703	1149
1150	1,322,500	1,520,875,000	33.9116	10.4769	0.0008696	1150

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1151	1,324,801	1,524,845,951	33.9264	10.4799	0.0008688	1151
1152	1,327,104	1,528,823,808	33.9411	10.4830	0.0008681	1152
1153	1,329,409	1,532,808,577	33.9559	10.4860	0.0008673	1153
1154	1,331,716	1,536,800,264	33.9706	10.4890	0.0008666	1154
1155	1,334,025	1,540,798,875	33.9853	10.4921	0.0008658	1155
1156	1,336,336	1,544,804,416	34.0000	10.4951	0.0008651	1156
1157	1,338,649	1,548,816,893	34.0147	10.4981	0.0008643	1157
1158	1,340,964	1,552,836,312	34.0294	10.5011	0.0008636	1158
1159	1,343,281	1,556,862,679	34.0441	10.5042	0.0008628	1159
1160	1,345,600	1,560,896,000	34.0588	10.5072	0.0008621	1160
1161	1,347,921	1,564,936,281	34.0735	10.5102	0.0008613	1161
1162	1,350,244	1,568,983,528	34.0881	10.5132	0.0008606	1162
1163	1,352,569	1,573,037,747	34.1028	10.5162	0.0008598	1163
1164	1,354,896	1,577,098,944	34.1174	10.5192	0.0008591	1164
1165	1,357,225	1,581,167,125	34.1321	10.5223	0.0008584	1165
1166	1,359,556	1,585,242,296	34.1467	10.5253	0.0008576	1166
1167	1,361,889	1,589,324,463	34.1614	10.5283	0.0008569	1167
1168	1,364,224	1,593,413,632	34.1760	10.5313	0.0008562	1168
1169	1,366,561	1,597,509,809	34.1906	10.5343	0.0008554	1169
1170	1,368,900	1,601,613,000	34.2053	10.5373	0.0008547	1170
1171	1,371,241	1,605,723,211	34.2199	10.5403	0.0008540	1171
1172	1,373,584	1,609,840,448	34.2345	10.5433	0.0008532	1172
1173	1,375,929	1,613,964,717	34.2491	10.5463	0.0008525	1173
1174	1,378,276	1,618,096,024	34.2637	10.5493	0.0008518	1174
1175	1,380,625	1,622,234,375	34.2783	10.5523	0.0008511	1175
1176	1,382,976	1,626,379,776	34.2929	10.5553	0.0008503	1176
1177	1,385,329	1,630,532,233	34.3074	10.5583	0.0008496	1177
1178	1,387,684	1,634,691,752	34.3220	10.5612	0.0008489	1178
1179	1,390,041	1,638,858,339	34.3366	10.5642	0.0008482	1179
1180	1,392,400	1,643,032,000	34.3511	10.5672	0.0008475	1180
1181	1,394,761	1,647,212,741	34.3657	10.5702	0.0008467	1181
1182	1,397,124	1,651,400,568	34.3802	10.5732	0.0008460	1182
1183	1,399,489	1,655,595,487	34.3948	10.5762	0.0008453	1183
1184	1,401,856	1,659,797,504	34.4093	10.5791	0.0008446	1184
1185	1,404,225	1,664,006,625	34.4238	10.5821	0.0008439	1185
1186	1,406,596	1,668,222,856	34.4384	10.5851	0.0008432	1186
1187	1,408,969	1,672,446,203	34.4529	10.5881	0.0008425	1187
1188	1,411,344	1,676,676,672	34.4674	10.5910	0.0008418	1188
1189	1,413,721	1,680,914,269	34.4819	10.5940	0.0008410	1189
1190	1,416,100	1,685,159,000	34.4964	10.5970	0.0008403	1190
1191	1,418,481	1,689,410,871	34.5109	10.6000	0.0008396	1191
1192	1,420,864	1,693,669,888	34.5254	10.6029	0.0008389	1192
1193	1,423,249	1,697,936,057	34.5398	10.6059	0.0008382	1193
1194	1,425,636	1,702,209,384	34.5543	10.6088	0.0008375	1194
1195	1,428,025	1,706,489,875	34.5688	10.6118	0.0008368	1195
1196	1,430,416	1,710,777,536	34.5832	10.6148	0.0008361	1196
1197	1,432,809	1,715,072,373	34.5977	10.6177	0.0008354	1197
1198	1,435,204	1,719,374,392	34.6121	10.6207	0.0008347	1198
1199	1,437,601	1,723,683,599	34.6266	10.6236	0.0008340	1199
1200	1,440,000	1,728,000,000	34.6410	10.6266	0.0008333	1200

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1201	1,442,401	1,732,323,601	34.6554	10.6295	0.0008326	1201
1202	1,444,804	1,736,654,408	34.6699	10.6325	0.0008319	1202
1203	1,447,209	1,740,992,427	34.6843	10.6354	0.0008313	1203
1204	1,449,616	1,745,337,664	34.6987	10.6384	0.0008306	1204
1205	1,452,025	1,749,690,125	34.7131	10.6413	0.0008299	1205
1206	1,454,436	1,754,049,816	34.7275	10.6443	0.0008292	1206
1207	1,456,849	1,758,416,743	34.7419	10.6472	0.0008285	1207
1208	1,459,264	1,762,790,912	34.7563	10.6501	0.0008278	1208
1209	1,461,681	1,767,172,329	34.7707	10.6531	0.0008271	1209
1210	1,464,100	1,771,561,000	34.7851	10.6560	0.0008264	1210
1211	1,466,521	1,775,956,931	34.7994	10.6590	0.0008258	1211
1212	1,468,944	1,780,360,128	34.8138	10.6619	0.0008251	1212
1213	1,471,369	1,784,770,597	34.8281	10.6648	0.0008244	1213
1214	1,473,796	1,789,188,344	34.8425	10.6678	0.0008237	1214
1215	1,476,225	1,793,613,375	34.8569	10.6707	0.0008230	1215
1216	1,478,656	1,798,045,696	34.8712	10.6736	0.0008224	1216
1217	1,481,089	1,802,485,313	34.8855	10.6765	0.0008217	1217
1218	1,483,524	1,806,932,232	34.8999	10.6795	0.0008210	1218
1219	1,485,961	1,811,386,459	34.9142	10.6824	0.0008203	1219
1220	1,488,400	1,815,848,000	34.9285	10.6853	0.0008197	1220
1221	1,490,841	1,820,316,861	34.9428	10.6882	0.0008190	1221
1222	1,493,284	1,824,793,048	34.9571	10.6911	0.0008183	1222
1223	1,495,729	1,829,276,567	34.9714	10.6940	0.0008177	1223
1224	1,498,176	1,833,767,424	34.9857	10.6970	0.0008170	1224
1225	1,500,625	1,838,265,625	35.0000	10.6999	0.0008163	1225
1226	1,503,076	1,842,771,176	35.0143	10.7028	0.0008157	1226
1227	1,505,529	1,847,284,083	35.0286	10.7057	0.0008150	1227
1228	1,507,984	1,851,804,352	35.0428	10.7086	0.0008143	1228
1229	1,510,441	1,856,331,989	35.0571	10.7115	0.0008137	1229
1230	1,512,900	1,860,867,000	35.0714	10.7144	0.0008130	1230
1231	1,515,361	1,865,409,391	35.0856	10.7173	0.0008123	1231
1232	1,517,824	1,869,959,168	35.0999	10.7202	0.0008117	1232
1233	1,520,289	1,874,516,337	35.1141	10.7231	0.0008110	1233
1234	1,522,756	1,879,080,904	35.1283	10.7260	0.0008104	1234
1235	1,525,225	1,883,652,875	35.1426	10.7289	0.0008097	1235
1236	1,527,696	1,888,232,256	35.1568	10.7318	0.0008091	1236
1237	1,530,169	1,892,819,053	35.1710	10.7347	0.0008084	1237
1238	1,532,644	1,897,413,272	35.1852	10.7376	0.0008078	1238
1239	1,535,121	1,902,014,919	35.1994	10.7405	0.0008071	1239
1240	1,537,600	1,906,624,000	35.2136	10.7434	0.0008065	1240
1241	1,540,081	1,911,240,521	35.2278	10.7463	0.0008058	1241
1242	1,542,564	1,915,864,488	35.2420	10.7491	0.0008052	1242
1243	1,545,049	1,920,495,907	35.2562	10.7520	0.0008045	1243
1244	1,547,536	1,925,134,784	35.2704	10.7549	0.0008039	1244
1245	1,550,025	1,929,781,125	35.2846	10.7578	0.0008032	1245
1246	1,552,516	1,934,434,936	35.2987	10.7607	0.0008026	1246
1247	1,555,009	1,939,096,223	35.3129	10.7635	0.0008019	1247
1248	1,557,504	1,943,764,992	35.3270	10.7664	0.0008013	1248
1249	1,560,001	1,948,441,249	35.3412	10.7693	0.0008006	1249
1250	1,562,500	1,953,125,000	35.3553	10.7722	0.0008000	1250

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1251	1,565,001	1,957,816,251	35.3695	10.7750	0.0007994	1251
1252	1,567,504	1,962,515,008	35.3836	10.7779	0.0007987	1252
1253	1,570,009	1,967,221,277	35.3977	10.7808	0.0007981	1253
1254	1,572,516	1,971,935,064	35.4119	10.7837	0.0007974	1254
1255	1,575,025	1,976,656,375	35.4260	10.7865	0.0007968	1255
1256	1,577,536	1,981,385,216	35.4401	10.7894	0.0007962	1256
1257	1,580,049	1,986,121,593	35.4542	10.7922	0.0007955	1257
1258	1,582,564	1,990,865,512	35.4683	10.7951	0.0007949	1258
1259	1,585,081	1,995,616,979	35.4824	10.7980	0.0007943	1259
1260	1,587,600	2,000,376,000	35.4965	10.8008	0.0007937	1260
1261	1,590,121	2,005,142,581	35.5106	10.8037	0.0007930	1261
1262	1,592,644	2,009,916,728	35.5246	10.8065	0.0007924	1262
1263	1,595,169	2,014,698,447	35.5387	10.8094	0.0007918	1263
1264	1,597,696	2,019,487,744	35.5528	10.8122	0.0007911	1264
1265	1,600,225	2,024,284,625	35.5668	10.8151	0.0007905	1265
1266	1,602,756	2,029,089,096	35.5809	10.8179	0.0007899	1266
1267	1,605,289	2,033,901,163	35.5949	10.8208	0.0007893	1267
1268	1,607,824	2,038,720,832	35.6090	10.8236	0.0007886	1268
1269	1,610,361	2,043,548,109	35.6230	10.8265	0.0007880	1269
1270	1,612,900	2,048,383,000	35.6371	10.8293	0.0007874	1270
1271	1,615,441	2,053,225,511	35.6511	10.8322	0.0007868	1271
1272	1,617,984	2,058,075,648	35.6651	10.8350	0.0007862	1272
1273	1,620,529	2,062,933,417	35.6791	10.8378	0.0007855	1273
1274	1,623,076	2,067,798,824	35.6931	10.8407	0.0007849	1274
1275	1,625,625	2,072,671,875	35.7071	10.8435	0.0007843	1275
1276	1,628,176	2,077,552,576	35.7211	10.8463	0.0007837	1276
1277	1,630,729	2,082,440,933	35.7351	10.8492	0.0007831	1277
1278	1,633,284	2,087,336,952	35.7491	10.8520	0.0007825	1278
1279	1,635,841	2,092,240,639	35.7631	10.8548	0.0007819	1279
1280	1,638,400	2,097,152,000	35.7771	10.8577	0.0007813	1280
1281	1,640,961	2,102,071,041	35.7911	10.8605	0.0007806	1281
1282	1,643,524	2,106,997,768	35.8050	10.8633	0.0007800	1282
1283	1,646,089	2,111,932,187	35.8190	10.8661	0.0007794	1283
1284	1,648,656	2,116,874,304	35.8329	10.8690	0.0007788	1284
1285	1,651,225	2,121,824,125	35.8469	10.8718	0.0007782	1285
1286	1,653,796	2,126,781,656	35.8608	10.8746	0.0007776	1286
1287	1,656,369	2,131,746,903	35.8748	10.8774	0.0007770	1287
1288	1,658,944	2,136,719,872	35.8887	10.8802	0.0007764	1288
1289	1,661,521	2,141,700,569	35.9026	10.8831	0.0007758	1289
1290	1,664,100	2,146,689,000	35.9166	10.8859	0.0007752	1290
1291	1,666,681	2,151,685,171	35.9305	10.8887	0.0007746	1291
1292	1,669,264	2,156,689,088	35.9444	10.8915	0.0007740	1292
1293	1,671,849	2,161,700,757	35.9583	10.8943	0.0007734	1293
1294	1,674,436	2,166,720,184	35.9722	10.8971	0.0007728	1294
1295	1,677,025	2,171,747,375	35.9861	10.8999	0.0007722	1295
1296	1,679,616	2,176,782,336	36.0000	10.9027	0.0007716	1296
1297	1,682,209	2,181,825,073	36.0139	10.9055	0.0007710	1297
1298	1,684,804	2,186,875,592	36.0278	10.9083	0.0007704	1298
1299	1,687,401	2,191,933,899	36.0416	10.9111	0.0007698	1299
1300	1,690,000	2,197,000,000	36.0555	10.9139	0.0007692	1300

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1301	1,692,601	2,202,073,901	36.0694	10.9167	0.0007686	1301
1302	1,695,204	2,207,155,608	36.0832	10.9195	0.0007680	1302
1303	1,697,809	2,212,245,127	36.0971	10.9223	0.0007675	1303
1304	1,700,416	2,217,342,464	36.1109	10.9251	0.0007669	1304
1305	1,703,025	2,222,447,625	36.1248	10.9279	0.0007663	1305
1306	1,705,636	2,227,560,616	36.1386	10.9307	0.0007657	1306
1307	1,708,249	2,232,681,443	36.1525	10.9335	0.0007651	1307
1308	1,710,864	2,237,810,112	36.1663	10.9363	0.0007645	1308
1309	1,713,481	2,242,946,629	36.1801	10.9391	0.0007639	1309
1310	1,716,100	2,248,091,000	36.1939	10.9418	0.0007634	1310
1311	1,718,721	2,253,243,231	36.2077	10.9446	0.0007628	1311
1312	1,721,344	2,258,403,328	36.2215	10.9474	0.0007622	1312
1313	1,723,969	2,263,571,297	36.2353	10.9502	0.0007616	1313
1314	1,726,596	2,268,747,144	36.2491	10.9530	0.0007610	1314
1315	1,729,225	2,273,930,875	36.2629	10.9557	0.0007605	1315
1316	1,731,856	2,279,122,496	36.2767	10.9585	0.0007599	1316
1317	1,734,489	2,284,322,013	36.2905	10.9613	0.0007593	1317
1318	1,737,124	2,289,529,432	36.3043	10.9641	0.0007587	1318
1319	1,739,761	2,294,744,759	36.3180	10.9668	0.0007582	1319
1320	1,742,400	2,299,968,000	36.3318	10.9696	0.0007576	1320
1321	1,745,041	2,305,199,161	36.3456	10.9724	0.0007570	1321
1322	1,747,684	2,310,438,248	36.3593	10.9752	0.0007564	1322
1323	1,750,329	2,315,685,267	36.3731	10.9779	0.0007559	1323
1324	1,752,976	2,320,940,224	36.3868	10.9807	0.0007553	1324
1325	1,755,625	2,326,203,125	36.4005	10.9834	0.0007547	1325
1326	1,758,276	2,331,473,976	36.4143	10.9862	0.0007541	1326
1327	1,760,929	2,336,752,783	36.4280	10.9890	0.0007536	1327
1328	1,763,584	2,342,039,552	36.4417	10.9917	0.0007530	1328
1329	1,766,241	2,347,334,289	36.4555	10.9945	0.0007524	1329
1330	1,768,900	2,352,637,000	36.4692	10.9972	0.0007519	1330
1331	1,771,561	2,357,947,691	36.4829	11.0000	0.0007513	1331
1332	1,774,224	2,363,266,368	36.4966	11.0028	0.0007508	1332
1333	1,776,889	2,368,593,037	36.5103	11.0055	0.0007502	1333
1334	1,779,556	2,373,927,704	36.5240	11.0083	0.0007496	1334
1335	1,782,225	2,379,270,375	36.5377	11.0110	0.0007491	1335
1336	1,784,896	2,384,621,056	36.5513	11.0138	0.0007485	1336
1337	1,787,569	2,389,979,753	36.5650	11.0165	0.0007479	1337
1338	1,790,244	2,395,346,472	36.5787	11.0193	0.0007474	1338
1339	1,792,921	2,400,721,219	36.5923	11.0220	0.0007468	1339
1340	1,795,600	2,406,104,000	36.6060	11.0247	0.0007463	1340
1341	1,798,281	2,411,494,821	36.6197	11.0275	0.0007457	1341
1342	1,800,964	2,416,893,688	36.6333	11.0302	0.0007452	1342
1343	1,803,649	2,422,300,607	36.6470	11.0330	0.0007446	1343
1344	1,806,336	2,427,715,584	36.6606	11.0357	0.0007440	1344
1345	1,809,025	2,433,138,625	36.6742	11.0384	0.0007435	1345
1346	1,811,716	2,438,569,736	36.6879	11.0412	0.0007429	1346
1347	1,814,409	2,444,008,923	36.7015	11.0439	0.0007424	1347
1348	1,817,104	2,449,456,192	36.7151	11.0466	0.0007418	1348
1349	1,819,801	2,454,911,549	36.7287	11.0494	0.0007413	1349
1350	1,822,500	2,460,375,000	36.7423	11.0521	0.0007407	1350

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1351	1,825,201	2,465,846,551	36.7560	11.0548	0.0007402	1351
1352	1,827,904	2,471,326,208	36.7696	11.0575	0.0007396	1352
1353	1,830,609	2,476,813,977	36.7831	11.0603	0.0007391	1353
1354	1,833,316	2,482,309,864	36.7967	11.0630	0.0007386	1354
1355	1,836,025	2,487,813,875	36.8103	11.0657	0.0007380	1355
1356	1,838,736	2,493,326,016	36.8239	11.0684	0.0007375	1356
1357	1,841,449	2,498,846,293	36.8375	11.0712	0.0007369	1357
1358	1,844,164	2,504,374,712	36.8511	11.0739	0.0007364	1358
1359	1,846,881	2,509,911,279	36.8646	11.0766	0.0007358	1359
1360	1,849,600	2,515,456,000	36.8782	11.0793	0.0007353	1360
1361	1,852,321	2,521,008,881	36.8917	11.0820	0.0007348	1361
1362	1,855,044	2,526,569,928	36.9053	11.0847	0.0007342	1362
1363	1,857,769	2,532,139,147	36.9188	11.0875	0.0007337	1363
1364	1,860,496	2,537,716,544	36.9324	11.0902	0.0007331	1364
1365	1,863,225	2,543,302,125	36.9459	11.0929	0.0007326	1365
1366	1,865,956	2,548,895,896	36.9594	11.0956	0.0007321	1366
1367	1,868,689	2,554,497,863	36.9730	11.0983	0.0007315	1367
1368	1,871,424	2,560,108,032	36.9865	11.1010	0.0007310	1368
1369	1,874,161	2,565,726,409	37.0000	11.1037	0.0007305	1369
1370	1,876,900	2,571,353,000	37.0135	11.1064	0.0007299	1370
1371	1,879,641	2,576,987,811	37.0270	11.1091	0.0007294	1371
1372	1,882,384	2,582,630,848	37.0405	11.1118	0.0007289	1372
1373	1,885,129	2,588,282,117	37.0540	11.1145	0.0007283	1373
1374	1,887,876	2,593,941,624	37.0675	11.1172	0.0007278	1374
1375	1,890,625	2,599,609,375	37.0810	11.1199	0.0007273	1375
1376	1,893,376	2,605,285,376	37.0945	11.1226	0.0007267	1376
1377	1,896,129	2,610,969,633	37.1080	11.1253	0.0007262	1377
1378	1,898,884	2,616,662,152	37.1214	11.1280	0.0007257	1378
1379	1,901,641	2,622,362,939	37.1349	11.1307	0.0007252	1379
1380	1,904,400	2,628,072,000	37.1484	11.1334	0.0007246	1380
1381	1,907,161	2,633,789,341	37.1618	11.1361	0.0007241	1381
1382	1,909,924	2,639,514,968	37.1753	11.1387	0.0007236	1382
1383	1,912,689	2,645,248,887	37.1887	11.1414	0.0007231	1383
1384	1,915,456	2,650,991,104	37.2022	11.1441	0.0007225	1384
1385	1,918,225	2,656,741,625	37.2156	11.1468	0.0007220	1385
1386	1,920,996	2,662,500,456	37.2290	11.1495	0.0007215	1386
1387	1,923,769	2,668,267,603	37.2424	11.1522	0.0007210	1387
1388	1,926,544	2,674,043,072	37.2559	11.1548	0.0007205	1388
1389	1,929,321	2,679,826,869	37.2693	11.1575	0.0007199	1389
1390	1,932,100	2,685,619,000	37.2827	11.1602	0.0007194	1390
1391	1,934,881	2,691,419,471	37.2961	11.1629	0.0007189	1391
1392	1,937,664	2,697,228,288	37.3095	11.1655	0.0007184	1392
1393	1,940,449	2,703,045,457	37.3229	11.1682	0.0007179	1393
1394	1,943,236	2,708,870,984	37.3363	11.1709	0.0007174	1394
1395	1,946,025	2,714,704,875	37.3497	11.1736	0.0007168	1395
1396	1,948,816	2,720,547,136	37.3631	11.1762	0.0007163	1396
1397	1,951,609	2,726,397,773	37.3765	11.1789	0.0007158	1397
1398	1,954,404	2,732,256,792	37.3898	11.1816	0.0007153	1398
1399	1,957,201	2,738,124,199	37.4032	11.1842	0.0007148	1399
1400	1,960,000	2,744,000,000	37.4166	11.1869	0.0007143	1400

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1401	1,962,801	2,749,884,201	37.4299	11.1896	0.0007138	1401
1402	1,965,604	2,755,776,808	37.4433	11.1922	0.0007133	1402
1403	1,968,409	2,761,677,827	37.4566	11.1949	0.0007128	1403
1404	1,971,216	2,767,587,264	37.4700	11.1975	0.0007123	1404
1405	1,974,025	2,773,505,125	37.4833	11.2002	0.0007117	1405
1406	1,976,836	2,779,431,416	37.4967	11.2028	0.0007112	1406
1407	1,979,649	2,785,366,143	37.5100	11.2055	0.0007107	1407
1408	1,982,464	2,791,309,312	37.5233	11.2082	0.0007102	1408
1409	1,985,281	2,797,260,929	37.5366	11.2108	0.0007097	1409
1410	1,988,100	2,803,221,000	37.5500	11.2135	0.0007092	1410
1411	1,990,921	2,809,189,531	37.5633	11.2161	0.0007087	1411
1412	1,993,744	2,815,166,528	37.5766	11.2188	0.0007082	1412
1413	1,996,569	2,821,151,997	37.5899	11.2214	0.0007077	1413
1414	1,999,396	2,827,145,944	37.6032	11.2241	0.0007072	1414
1415	2,002,225	2,833,148,375	37.6165	11.2267	0.0007067	1415
1416	2,005,056	2,839,159,296	37.6298	11.2293	0.0007062	1416
1417	2,007,889	2,845,178,713	37.6431	11.2320	0.0007057	1417
1418	2,010,724	2,851,206,632	37.6563	11.2346	0.0007052	1418
1419	2,013,561	2,857,243,059	37.6696	11.2373	0.0007047	1419
1420	2,016,400	2,863,288,000	37.6829	11.2399	0.0007042	1420
1421	2,019,241	2,869,341,461	37.6962	11.2425	0.0007037	1421
1422	2,022,084	2,875,403,448	37.7094	11.2452	0.0007032	1422
1423	2,024,929	2,881,473,967	37.7227	11.2478	0.0007027	1423
1424	2,027,776	2,887,553,024	37.7359	11.2504	0.0007022	1424
1425	2,030,625	2,893,640,625	37.7492	11.2531	0.0007018	1425
1426	2,033,476	2,899,736,776	37.7624	11.2557	0.0007013	1426
1427	2,036,329	2,905,841,483	37.7757	11.2583	0.0007008	1427
1428	2,039,184	2,911,954,752	37.7889	11.2610	0.0007003	1428
1429	2,042,041	2,918,076,589	37.8021	11.2636	0.0006998	1429
1430	2,044,900	2,924,207,000	37.8153	11.2662	0.0006993	1430
1431	2,047,761	2,930,345,991	37.8286	11.2689	0.0006988	1431
1432	2,050,624	2,936,493,568	37.8418	11.2715	0.0006983	1432
1433	2,053,489	2,942,649,737	37.8550	11.2741	0.0006978	1433
1434	2,056,356	2,948,814,504	37.8682	11.2767	0.0006974	1434
1435	2,059,225	2,954,987,875	37.8814	11.2793	0.0006969	1435
1436	2,062,096	2,961,169,856	37.8946	11.2820	0.0006964	1436
1437	2,064,969	2,967,360,453	37.9078	11.2846	0.0006959	1437
1438	2,067,844	2,973,559,672	37.9210	11.2872	0.0006954	1438
1439	2,070,721	2,979,767,519	37.9342	11.2898	0.0006949	1439
1440	2,073,600	2,985,984,000	37.9473	11.2924	0.0006944	1440
1441	2,076,481	2,992,209,121	37.9605	11.2950	0.0006940	1441
1442	2,079,364	2,998,442,888	37.9737	11.2977	0.0006935	1442
1443	2,082,249	3,004,685,307	37.9868	11.3003	0.0006930	1443
1444	2,085,136	3,010,936,384	38.0000	11.3029	0.0006925	1444
1445	2,088,025	3,017,196,125	38.0132	11.3055	0.0006920	1445
1446	2,090,916	3,023,464,536	38.0263	11.3081	0.0006916	1446
1447	2,093,809	3,029,741,623	38.0395	11.3107	0.0006911	1447
1448	2,096,704	3,036,027,392	38.0526	11.3133	0.0006906	1448
1449	2,099,601	3,042,321,849	38.0657	11.3159	0.0006901	1449
1450	2,102,500	3,048,625,000	38.0789	11.3185	0.0006897	1450

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1451	2,105,401	3,054,936,851	38.0920	11.3211	0.0006892	1451
1452	2,108,304	3,061,257,408	38.1051	11.3237	0.0006887	1452
1453	2,111,209	3,067,586,677	38.1182	11.3263	0.0006882	1453
1454	2,114,116	3,073,924,664	38.1314	11.3289	0.0006878	1454
1455	2,117,025	3,080,271,375	38.1445	11.3315	0.0006873	1455
1456	2,119,936	3,086,626,816	38.1576	11.3341	0.0006868	1456
1457	2,122,849	3,092,990,993	38.1707	11.3367	0.0006863	1457
1458	2,125,764	3,099,363,912	38.1838	11.3393	0.0006859	1458
1459	2,128,681	3,105,745,579	38.1969	11.3419	0.0006854	1459
1460	2,131,600	3,112,136,000	38.2099	11.3445	0.0006849	1460
1461	2,134,521	3,118,535,181	38.2230	11.3471	0.0006845	1461
1462	2,137,444	3,124,943,128	38.2361	11.3496	0.0006840	1462
1463	2,140,369	3,131,359,847	38.2492	11.3522	0.0006835	1463
1464	2,143,296	3,137,785,344	38.2623	11.3548	0.0006831	1464
1465	2,146,225	3,144,219,625	38.2753	11.3574	0.0006826	1465
1466	2,149,156	3,150,662,696	38.2884	11.3600	0.0006821	1466
1467	2,152,089	3,157,114,563	38.3014	11.3626	0.0006817	1467
1468	2,155,024	3,163,575,232	38.3145	11.3652	0.0006812	1468
1469	2,157,961	3,170,044,709	38.3275	11.3677	0.0006807	1469
1470	2,160,900	3,176,523,000	38.3406	11.3703	0.0006803	1470
1471	2,163,841	3,183,010,111	38.3536	11.3729	0.0006798	1471
1472	2,166,784	3,189,506,048	38.3667	11.3755	0.0006793	1472
1473	2,169,729	3,196,010,817	38.3797	11.3780	0.0006789	1473
1474	2,172,676	3,202,524,424	38.3927	11.3806	0.0006784	1474
1475	2,175,625	3,209,046,875	38.4057	11.3832	0.0006780	1475
1476	2,178,576	3,215,578,176	38.4187	11.3858	0.0006775	1476
1477	2,181,529	3,222,118,333	38.4318	11.3883	0.0006770	1477
1478	2,184,484	3,228,667,352	38.4448	11.3909	0.0006766	1478
1479	2,187,441	3,235,225,239	38.4578	11.3935	0.0006761	1479
1480	2,190,400	3,241,792,000	38.4708	11.3960	0.0006757	1480
1481	2,193,361	3,248,367,641	38.4838	11.3986	0.0006752	1481
1482	2,196,324	3,254,952,168	38.4968	11.4012	0.0006748	1482
1483	2,199,289	3,261,545,587	38.5097	11.4037	0.0006743	1483
1484	2,202,256	3,268,147,904	38.5227	11.4063	0.0006739	1484
1485	2,205,225	3,274,759,125	38.5357	11.4089	0.0006734	1485
1486	2,208,196	3,281,379,256	38.5487	11.4114	0.0006729	1486
1487	2,211,169	3,288,008,303	38.5616	11.4140	0.0006725	1487
1488	2,214,144	3,294,646,272	38.5746	11.4165	0.0006720	1488
1489	2,217,121	3,301,293,169	38.5876	11.4191	0.0006716	1489
1490	2,220,100	3,307,949,000	38.6005	11.4216	0.0006711	1490
1491	2,223,081	3,314,613,771	38.6135	11.4242	0.0006707	1491
1492	2,226,064	3,321,287,488	38.6264	11.4268	0.0006702	1492
1493	2,229,049	3,327,970,157	38.6394	11.4293	0.0006698	1493
1494	2,232,036	3,334,661,784	38.6523	11.4319	0.0006693	1494
1495	2,235,025	3,341,362,375	38.6652	11.4344	0.0006689	1495
1496	2,238,016	3,348,071,936	38.6782	11.4370	0.0006684	1496
1497	2,241,009	3,354,790,473	38.6911	11.4395	0.0006680	1497
1498	2,244,004	3,361,517,992	38.7040	11.4421	0.0006676	1498
1499	2,247,001	3,368,254,499	38.7169	11.4446	0.0006671	1499
1500	2,250,000	3,375,000,000	38.7298	11.4471	0.0006667	1500

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1501	2,253,001	3,381,754,501	38.7427	11.4497	0.0006662	1501
1502	2,256,004	3,388,518,008	38.7556	11.4522	0.0006658	1502
1503	2,259,009	3,395,290,527	38.7685	11.4548	0.0006653	1503
1504	2,262,016	3,402,072,064	38.7814	11.4573	0.0006649	1504
1505	2,265,025	3,408,862,625	38.7943	11.4598	0.0006645	1505
1506	2,268,036	3,415,662,216	38.8072	11.4624	0.0006640	1506
1507	2,271,049	3,422,470,843	38.8201	11.4649	0.0006636	1507
1508	2,274,064	3,429,288,512	38.8330	11.4675	0.0006631	1508
1509	2,277,081	3,436,115,229	38.8458	11.4700	0.0006627	1509
1510	2,280,100	3,442,951,000	38.8587	11.4725	0.0006623	1510
1511	2,283,121	3,449,795,831	38.8716	11.4751	0.0006618	1511
1512	2,286,144	3,456,649,728	38.8844	11.4776	0.0006614	1512
1513	2,289,169	3,463,512,697	38.8973	11.4801	0.0006609	1513
1514	2,292,196	3,470,384,744	38.9102	11.4826	0.0006605	1514
1515	2,295,225	3,477,265,875	38.9230	11.4852	0.0006601	1515
1516	2,298,256	3,484,156,096	38.9358	11.4877	0.0006596	1516
1517	2,301,289	3,491,055,413	38.9487	11.4902	0.0006592	1517
1518	2,304,324	3,497,963,832	38.9615	11.4927	0.0006588	1518
1519	2,307,361	3,504,881,359	38.9744	11.4953	0.0006583	1519
1520	2,310,400	3,511,808,000	38.9872	11.4978	0.0006579	1520
1521	2,313,441	3,518,743,761	39.0000	11.5003	0.0006575	1521
1522	2,316,484	3,525,688,648	39.0128	11.5028	0.0006570	1522
1523	2,319,529	3,532,642,667	39.0256	11.5054	0.0006566	1523
1524	2,322,576	3,539,605,824	39.0384	11.5079	0.0006562	1524
1525	2,325,625	3,546,578,125	39.0512	11.5104	0.0006557	1525
1526	2,328,676	3,553,559,576	39.0640	11.5129	0.0006553	1526
1527	2,331,729	3,560,550,183	39.0768	11.5154	0.0006549	1527
1528	2,334,784	3,567,549,952	39.0896	11.5179	0.0006545	1528
1529	2,337,841	3,574,558,889	39.1024	11.5204	0.0006540	1529
1530	2,340,900	3,581,577,000	39.1152	11.5230	0.0006536	1530
1531	2,343,961	3,588,604,291	39.1280	11.5255	0.0006532	1531
1532	2,347,024	3,595,640,768	39.1408	11.5280	0.0006527	1532
1533	2,350,089	3,602,686,437	39.1535	11.5305	0.0006523	1533
1534	2,353,156	3,609,741,304	39.1663	11.5330	0.0006519	1534
1535	2,356,225	3,616,805,375	39.1791	11.5355	0.0006515	1535
1536	2,359,296	3,623,878,656	39.1918	11.5380	0.0006510	1536
1537	2,362,369	3,630,961,153	39.2046	11.5405	0.0006506	1537
1538	2,365,444	3,638,052,872	39.2173	11.5430	0.0006502	1538
1539	2,368,521	3,645,153,819	39.2301	11.5455	0.0006498	1539
1540	2,371,600	3,652,264,000	39.2428	11.5480	0.0006494	1540
1541	2,374,681	3,659,383,421	39.2556	11.5505	0.0006489	1541
1542	2,377,764	3,666,512,088	39.2683	11.5530	0.0006485	1542
1543	2,380,849	3,673,650,007	39.2810	11.5555	0.0006481	1543
1544	2,383,936	3,680,797,184	39.2938	11.5580	0.0006477	1544
1545	2,387,025	3,687,953,625	39.3065	11.5605	0.0006472	1545
1546	2,390,116	3,695,119,336	39.3192	11.5630	0.0006468	1546
1547	2,393,209	3,702,294,323	39.3319	11.5655	0.0006464	1547
1548	2,396,304	3,709,478,592	39.3446	11.5680	0.0006460	1548
1549	2,399,401	3,716,672,149	39.3573	11.5705	0.0006456	1549
1550	2,402,500	3,723,875,000	39.3700	11.5729	0.0006452	1550

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1551	2,405,601	3,731,087,151	39.3827	11.5754	0.0006447	1551
1552	2,408,704	3,738,308,608	39.3954	11.5779	0.0006443	1552
1553	2,411,809	3,745,539,377	39.4081	11.5804	0.0006439	1553
1554	2,414,916	3,752,779,464	39.4208	11.5829	0.0006435	1554
1555	2,418,025	3,760,028,875	39.4335	11.5854	0.0006431	1555
1556	2,421,136	3,767,287,616	39.4462	11.5879	0.0006427	1556
1557	2,424,249	3,774,555,693	39.4588	11.5903	0.0006423	1557
1558	2,427,364	3,781,833,112	39.4715	11.5928	0.0006418	1558
1559	2,430,481	3,789,119,879	39.4842	11.5953	0.0006414	1559
1560	2,433,600	3,796,416,000	39.4968	11.5978	0.0006410	1560
1561	2,436,721	3,803,721,481	39.5095	11.6003	0.0006406	1561
1562	2,439,844	3,811,036,328	39.5221	11.6027	0.0006402	1562
1563	2,442,969	3,818,360,547	39.5348	11.6052	0.0006398	1563
1564	2,446,096	3,825,694,144	39.5474	11.6077	0.0006394	1564
1565	2,449,225	3,833,037,125	39.5601	11.6102	0.0006390	1565
1566	2,452,356	3,840,389,496	39.5727	11.6126	0.0006386	1566
1567	2,455,489	3,847,751,263	39.5854	11.6151	0.0006382	1567
1568	2,458,624	3,855,123,432	39.5980	11.6176	0.0006378	1568
1569	2,461,761	3,862,503,009	39.6106	11.6200	0.0006373	1569
1570	2,464,900	3,869,893,000	39.6232	11.6225	0.0006369	1570
1571	2,468,041	3,877,292,411	39.6358	11.6250	0.0006365	1571
1572	2,471,184	3,884,701,248	39.6485	11.6274	0.0006361	1572
1573	2,474,329	3,892,119,517	39.6611	11.6299	0.0006357	1573
1574	2,477,476	3,899,547,224	39.6737	11.6324	0.0006353	1574
1575	2,480,625	3,906,984,375	39.6863	11.6348	0.0006349	1575
1576	2,483,776	3,914,430,976	39.6989	11.6373	0.0006345	1576
1577	2,486,929	3,921,887,033	39.7115	11.6398	0.0006341	1577
1578	2,490,084	3,929,352,552	39.7240	11.6422	0.0006337	1578
1579	2,493,241	3,936,827,539	39.7366	11.6447	0.0006333	1579
1580	2,496,400	3,944,312,000	39.7492	11.6471	0.0006329	1580
1581	2,499,561	3,951,805,941	39.7618	11.6496	0.0006325	1581
1582	2,502,724	3,959,309,368	39.7744	11.6520	0.0006321	1582
1583	2,505,889	3,966,822,287	39.7869	11.6545	0.0006317	1583
1584	2,509,056	3,974,344,704	39.7995	11.6570	0.0006313	1584
1585	2,512,225	3,981,876,625	39.8121	11.6594	0.0006309	1585
1586	2,515,396	3,989,418,056	39.8246	11.6619	0.0006305	1586
1587	2,518,569	3,996,969,003	39.8372	11.6643	0.0006301	1587
1588	2,521,744	4,004,529,472	39.8497	11.6668	0.0006297	1588
1589	2,524,921	4,012,099,469	39.8623	11.6692	0.0006293	1589
1590	2,528,100	4,019,679,000	39.8748	11.6717	0.0006289	1590
1591	2,531,281	4,027,268,071	39.8873	11.6741	0.0006285	1591
1592	2,534,464	4,034,866,688	39.8999	11.6765	0.0006281	1592
1593	2,537,649	4,042,474,857	39.9124	11.6790	0.0006277	1593
1594	2,540,836	4,050,092,584	39.9249	11.6814	0.0006274	1594
1595	2,544,025	4,057,719,875	39.9375	11.6839	0.0006270	1595
1596	2,547,216	4,065,356,736	39.9500	11.6863	0.0006266	1596
1597	2,550,409	4,073,003,173	39.9625	11.6887	0.0006262	1597
1598	2,553,604	4,080,659,192	39.9750	11.6912	0.0006258	1598
1599	2,556,801	4,088,324,799	39.9875	11.6936	0.0006254	1599
1600	2,560,000	4,096,000,000	40.0000	11.6961	0.0006250	1600

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1601	2,563,201	4,103,684,801	40.0125	11.6985	0.0006246	1601
1602	2,566,404	4,111,379,208	40.0250	11.7009	0.0006242	1602
1603	2,569,609	4,119,083,227	40.0375	11.7034	0.0006238	1603
1604	2,572,816	4,126,796,864	40.0500	11.7058	0.0006234	1604
1605	2,576,025	4,134,520,125	40.0625	11.7082	0.0006231	1605
1606	2,579,236	4,142,253,016	40.0749	11.7107	0.0006227	1606
1607	2,582,449	4,149,995,543	40.0874	11.7131	0.0006223	1607
1608	2,585,664	4,157,747,712	40.0999	11.7155	0.0006219	1608
1609	2,588,881	4,165,509,529	40.1123	11.7180	0.0006215	1609
1610	2,592,100	4,173,281,000	40.1248	11.7204	0.0006211	1610
1611	2,595,321	4,181,062,131	40.1373	11.7228	0.0006207	1611
1612	2,598,544	4,188,852,928	40.1497	11.7252	0.0006203	1612
1613	2,601,769	4,196,653,397	40.1622	11.7277	0.0006200	1613
1614	2,604,996	4,204,463,544	40.1746	11.7301	0.0006196	1614
1615	2,608,225	4,212,283,375	40.1871	11.7325	0.0006192	1615
1616	2,611,456	4,220,112,896	40.1995	11.7349	0.0006188	1616
1617	2,614,689	4,227,952,113	40.2119	11.7373	0.0006184	1617
1618	2,617,924	4,235,801,032	40.2244	11.7398	0.0006180	1618
1619	2,621,161	4,243,659,659	40.2368	11.7422	0.0006177	1619
1620	2,624,400	4,251,528,000	40.2492	11.7446	0.0006173	1620
1621	2,627,641	4,259,406,061	40.2616	11.7470	0.0006169	1621
1622	2,630,884	4,267,293,848	40.2741	11.7494	0.0006165	1622
1623	2,634,129	4,275,191,367	40.2865	11.7518	0.0006161	1623
1624	2,637,376	4,283,098,624	40.2989	11.7543	0.0006158	1624
1625	2,640,625	4,291,015,625	40.3113	11.7567	0.0006154	1625
1626	2,643,876	4,298,942,376	40.3237	11.7591	0.0006150	1626
1627	2,647,129	4,306,878,883	40.3361	11.7615	0.0006146	1627
1628	2,650,384	4,314,825,152	40.3485	11.7639	0.0006143	1628
1629	2,653,641	4,322,781,189	40.3609	11.7663	0.0006139	1629
1630	2,656,900	4,330,747,000	40.3733	11.7687	0.0006135	1630
1631	2,660,161	4,338,722,591	40.3856	11.7711	0.0006131	1631
1632	2,663,424	4,346,707,968	40.3980	11.7735	0.0006127	1632
1633	2,666,689	4,354,703,137	40.4104	11.7759	0.0006124	1633
1634	2,669,956	4,362,708,104	40.4228	11.7783	0.0006120	1634
1635	2,673,225	4,370,722,875	40.4351	11.7807	0.0006116	1635
1636	2,676,496	4,378,747,456	40.4475	11.7831	0.0006112	1636
1637	2,679,769	4,386,781,853	40.4599	11.7855	0.0006109	1637
1638	2,683,044	4,394,826,072	40.4722	11.7879	0.0006105	1638
1639	2,686,321	4,402,880,119	40.4846	11.7903	0.0006101	1639
1640	2,689,600	4,410,944,000	40.4969	11.7927	0.0006098	1640
1641	2,692,881	4,419,017,721	40.5093	11.7951	0.0006094	1641
1642	2,696,164	4,427,101,288	40.5216	11.7975	0.0006090	1642
1643	2,699,449	4,435,194,707	40.5339	11.7999	0.0006086	1643
1644	2,702,736	4,443,297,984	40.5463	11.8023	0.0006083	1644
1645	2,706,025	4,451,411,125	40.5586	11.8047	0.0006079	1645
1646	2,709,316	4,459,534,136	40.5709	11.8071	0.0006075	1646
1647	2,712,609	4,467,667,023	40.5832	11.8095	0.0006072	1647
1648	2,715,904	4,475,809,792	40.5956	11.8119	0.0006068	1648
1649	2,719,201	4,483,962,449	40.6079	11.8143	0.0006064	1649
1650	2,722,500	4,492,125,000	40.6202	11.8167	0.0006061	1650

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1651	2,725,801	4,500,297,451	40.6325	11.8190	0.0006057	1651
1652	2,729,104	4,508,479,808	40.6448	11.8214	0.0006053	1652
1653	2,732,409	4,516,672,077	40.6571	11.8238	0.0006050	1653
1654	2,735,716	4,524,874,264	40.6694	11.8262	0.0006046	1654
1655	2,739,025	4,533,086,375	40.6817	11.8286	0.0006042	1655
1656	2,742,336	4,541,308,416	40.6940	11.8310	0.0006039	1656
1657	2,745,649	4,549,540,393	40.7063	11.8333	0.0006035	1657
1658	2,748,964	4,557,782,312	40.7185	11.8357	0.0006031	1658
1659	2,752,281	4,566,034,179	40.7308	11.8381	0.0006028	1659
1660	2,755,600	4,574,296,000	40.7431	11.8405	0.0006024	1660
1661	2,758,921	4,582,567,781	40.7554	11.8429	0.0006020	1661
1662	2,762,244	4,590,849,528	40.7676	11.8452	0.0006017	1662
1663	2,765,569	4,599,141,247	40.7799	11.8476	0.0006013	1663
1664	2,768,896	4,607,442,944	40.7922	11.8500	0.0006010	1664
1665	2,772,225	4,615,754,625	40.8044	11.8524	0.0006006	1665
1666	2,775,556	4,624,076,296	40.8167	11.8547	0.0006002	1666
1667	2,778,889	4,632,407,963	40.8289	11.8571	0.0005999	1667
1668	2,782,224	4,640,749,632	40.8412	11.8595	0.0005995	1668
1669	2,785,561	4,649,101,309	40.8534	11.8618	0.0005992	1669
1670	2,788,900	4,657,463,000	40.8656	11.8642	0.0005988	1670
1671	2,792,241	4,665,834,711	40.8779	11.8666	0.0005984	1671
1672	2,795,584	4,674,216,448	40.8901	11.8689	0.0005981	1672
1673	2,798,929	4,682,608,217	40.9023	11.8713	0.0005977	1673
1674	2,802,276	4,691,010,024	40.9145	11.8737	0.0005974	1674
1675	2,805,625	4,699,421,875	40.9268	11.8760	0.0005970	1675
1676	2,808,976	4,707,843,776	40.9390	11.8784	0.0005967	1676
1677	2,812,329	4,716,275,733	40.9512	11.8808	0.0005963	1677
1678	2,815,684	4,724,717,752	40.9634	11.8831	0.0005959	1678
1679	2,819,041	4,733,169,839	40.9756	11.8855	0.0005956	1679
1680	2,822,400	4,741,632,000	40.9878	11.8878	0.0005952	1680
1681	2,825,761	4,750,104,241	41.0000	11.8902	0.0005949	1681
1682	2,829,124	4,758,586,568	41.0122	11.8926	0.0005945	1682
1683	2,832,489	4,767,078,987	41.0244	11.8949	0.0005942	1683
1684	2,835,856	4,775,581,504	41.0366	11.8973	0.0005938	1684
1685	2,839,225	4,784,094,125	41.0487	11.8996	0.0005935	1685
1686	2,842,596	4,792,616,856	41.0609	11.9020	0.0005931	1686
1687	2,845,969	4,801,149,703	41.0731	11.9043	0.0005928	1687
1688	2,849,344	4,809,692,672	41.0853	11.9067	0.0005924	1688
1689	2,852,721	4,818,245,769	41.0974	11.9090	0.0005921	1689
1690	2,856,100	4,826,809,000	41.1096	11.9114	0.0005917	1690
1691	2,859,481	4,835,382,371	41.1218	11.9137	0.0005914	1691
1692	2,862,864	4,843,965,888	41.1339	11.9161	0.0005910	1692
1693	2,866,249	4,852,559,557	41.1461	11.9184	0.0005907	1693
1694	2,869,636	4,861,163,384	41.1582	11.9208	0.0005903	1694
1695	2,873,025	4,869,777,375	41.1704	11.9231	0.0005900	1695
1696	2,876,416	4,878,401,536	41.1825	11.9255	0.0005896	1696
1697	2,879,809	4,887,035,873	41.1947	11.9278	0.0005893	1697
1698	2,883,204	4,895,680,392	41.2068	11.9301	0.0005889	1698
1699	2,886,601	4,904,335,099	41.2189	11.9325	0.0005886	1699
1700	2,890,000	4,913,000,000	41.2311	11.9348	0.0005882	1700

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1701	2,893,401	4,921,675,101	41.2432	11.9372	0.0005879	1701
1702	2,896,804	4,930,360,408	41.2553	11.9395	0.0005875	1702
1703	2,900,209	4,939,055,927	41.2674	11.9418	0.0005872	1703
1704	2,903,616	4,947,761,664	41.2795	11.9442	0.0005869	1704
1705	2,907,025	4,956,477,625	41.2916	11.9465	0.0005865	1705
1706	2,910,436	4,965,203,816	41.3038	11.9489	0.0005862	1706
1707	2,913,849	4,973,940,243	41.3159	11.9512	0.0005858	1707
1708	2,917,264	4,982,686,912	41.3280	11.9535	0.0005855	1708
1709	2,920,681	4,991,443,829	41.3401	11.9559	0.0005851	1709
1710	2,924,100	5,000,211,000	41.3521	11.9582	0.0005848	1710
1711	2,927,521	5,008,988,431	41.3642	11.9605	0.0005845	1711
1712	2,930,944	5,017,776,128	41.3763	11.9628	0.0005841	1712
1713	2,934,369	5,026,574,097	41.3884	11.9652	0.0005838	1713
1714	2,937,796	5,035,382,344	41.4005	11.9675	0.0005834	1714
1715	2,941,225	5,044,200,875	41.4126	11.9698	0.0005831	1715
1716	2,944,656	5,053,029,696	41.4246	11.9722	0.0005828	1716
1717	2,948,089	5,061,868,813	41.4367	11.9745	0.0005824	1717
1718	2,951,524	5,070,718,232	41.4488	11.9768	0.0005821	1718
1719	2,954,961	5,079,577,959	41.4608	11.9791	0.0005817	1719
1720	2,958,400	5,088,448,000	41.4729	11.9815	0.0005814	1720
1721	2,961,841	5,097,328,361	41.4849	11.9838	0.0005811	1721
1722	2,965,284	5,106,219,048	41.4970	11.9861	0.0005807	1722
1723	2,968,729	5,115,120,067	41.5090	11.9884	0.0005804	1723
1724	2,972,176	5,124,031,424	41.5211	11.9907	0.0005800	1724
1725	2,975,625	5,132,953,125	41.5331	11.9931	0.0005797	1725
1726	2,979,076	5,141,885,176	41.5452	11.9954	0.0005794	1726
1727	2,982,529	5,150,827,583	41.5572	11.9977	0.0005790	1727
1728	2,985,984	5,159,780,352	41.5692	12.0000	0.0005787	1728
1729	2,989,441	5,168,743,489	41.5812	12.0023	0.0005784	1729
1730	2,992,900	5,177,717,000	41.5933	12.0046	0.0005780	1730
1731	2,996,361	5,186,700,891	41.6053	12.0069	0.0005777	1731
1732	2,999,824	5,195,695,168	41.6173	12.0093	0.0005774	1732
1733	3,003,289	5,204,699,837	41.6293	12.0116	0.0005770	1733
1734	3,006,756	5,213,714,904	41.6413	12.0139	0.0005767	1734
1735	3,010,225	5,222,740,375	41.6533	12.0162	0.0005764	1735
1736	3,013,696	5,231,776,256	41.6653	12.0185	0.0005760	1736
1737	3,017,169	5,240,822,553	41.6773	12.0208	0.0005757	1737
1738	3,020,644	5,249,879,272	41.6893	12.0231	0.0005754	1738
1739	3,024,121	5,258,946,419	41.7013	12.0254	0.0005750	1739
1740	3,027,600	5,268,024,000	41.7133	12.0277	0.0005747	1740
1741	3,031,081	5,277,112,021	41.7253	12.0300	0.0005744	1741
1742	3,034,564	5,286,210,488	41.7373	12.0323	0.0005741	1742
1743	3,038,049	5,295,319,407	41.7493	12.0346	0.0005737	1743
1744	3,041,536	5,304,438,784	41.7612	12.0369	0.0005734	1744
1745	3,045,025	5,313,568,625	41.7732	12.0392	0.0005731	1745
1746	3,048,516	5,322,708,936	41.7852	12.0415	0.0005727	1746
1747	3,052,009	5,331,859,723	41.7971	12.0438	0.0005724	1747
1748	3,055,504	5,341,020,992	41.8091	12.0461	0.0005721	1748
1749	3,059,001	5,350,192,749	41.8210	12.0484	0.0005718	1749
1750	3,062,500	5,359,375,000	41.8330	12.0507	0.0005714	1750

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1751	3,066,001	5,368,567,751	41.8450	12.0530	0.0005711	1751
1752	3,069,504	5,377,771,008	41.8569	12.0553	0.0005708	1752
1753	3,073,009	5,386,984,777	41.8688	12.0576	0.0005705	1753
1754	3,076,516	5,396,209,064	41.8808	12.0599	0.0005701	1754
1755	3,080,025	5,405,443,875	41.8927	12.0622	0.0005698	1755
1756	3,083,536	5,414,689,216	41.9047	12.0645	0.0005695	1756
1757	3,087,049	5,423,945,093	41.9166	12.0668	0.0005692	1757
1758	3,090,564	5,433,211,512	41.9285	12.0690	0.0005688	1758
1759	3,094,081	5,442,488,479	41.9404	12.0713	0.0005685	1759
1760	3,097,600	5,451,776,000	41.9524	12.0736	0.0005682	1760
1761	3,101,121	5,461,074,081	41.9643	12.0759	0.0005679	1761
1762	3,104,644	5,470,382,728	41.9762	12.0782	0.0005675	1762
1763	3,108,169	5,479,701,947	41.9881	12.0805	0.0005672	1763
1764	3,111,696	5,489,031,744	42.0000	12.0828	0.0005669	1764
1765	3,115,225	5,498,372,125	42.0119	12.0850	0.0005666	1765
1766	3,118,756	5,507,723,096	42.0238	12.0873	0.0005663	1766
1767	3,122,289	5,517,084,663	42.0357	12.0896	0.0005659	1767
1768	3,125,824	5,526,456,832	42.0476	12.0919	0.0005656	1768
1769	3,129,361	5,535,839,609	42.0595	12.0942	0.0005653	1769
1770	3,132,900	5,545,233,000	42.0714	12.0964	0.0005650	1770
1771	3,136,441	5,554,637,011	42.0833	12.0987	0.0005647	1771
1772	3,139,984	5,564,051,648	42.0951	12.1010	0.0005643	1772
1773	3,143,529	5,573,476,917	42.1070	12.1033	0.0005640	1773
1774	3,147,076	5,582,912,824	42.1189	12.1056	0.0005637	1774
1775	3,150,625	5,592,359,375	42.1307	12.1078	0.0005634	1775
1776	3,154,176	5,601,816,576	42.1426	12.1101	0.0005631	1776
1777	3,157,729	5,611,284,433	42.1545	12.1124	0.0005627	1777
1778	3,161,284	5,620,762,952	42.1663	12.1146	0.0005624	1778
1779	3,164,841	5,630,252,139	42.1782	12.1169	0.0005621	1779
1780	3,168,400	5,639,752,000	42.1900	12.1192	0.0005618	1780
1781	3,171,961	5,649,262,541	42.2019	12.1215	0.0005615	1781
1782	3,175,524	5,658,783,768	42.2137	12.1237	0.0005612	1782
1783	3,179,089	5,668,315,687	42.2256	12.1260	0.0005609	1783
1784	3,182,656	5,677,858,304	42.2374	12.1283	0.0005605	1784
1785	3,186,225	5,687,411,625	42.2493	12.1305	0.0005602	1785
1786	3,189,796	5,696,975,656	42.2611	12.1328	0.0005599	1786
1787	3,193,369	5,706,550,403	42.2729	12.1350	0.0005596	1787
1788	3,196,944	5,716,135,872	42.2847	12.1373	0.0005593	1788
1789	3,200,521	5,725,732,069	42.2966	12.1396	0.0005590	1789
1790	3,204,100	5,735,339,000	42.3084	12.1418	0.0005587	1790
1791	3,207,681	5,744,956,671	42.3202	12.1441	0.0005583	1791
1792	3,211,264	5,754,585,088	42.3320	12.1464	0.0005580	1792
1793	3,214,849	5,764,224,257	42.3438	12.1486	0.0005577	1793
1794	3,218,436	5,773,874,184	42.3556	12.1509	0.0005574	1794
1795	3,222,025	5,783,534,875	42.3674	12.1531	0.0005571	1795
1796	3,225,616	5,793,206,336	42.3792	12.1554	0.0005568	1796
1797	3,229,209	5,802,888,573	42.3910	12.1576	0.0005565	1797
1798	3,232,804	5,812,581,592	42.4028	12.1599	0.0005562	1798
1799	3,236,401	5,822,285,399	42.4146	12.1622	0.0005559	1799
1800	3,240,000	5,832,000,000	42.4264	12.1644	0.0005556	1800

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1801	3,243,601	5,841,725,401	42.4382	12.1667	0.0005552	1801
1802	3,247,204	5,851,461,608	42.4500	12.1689	0.0005549	1802
1803	3,250,809	5,861,208,627	42.4617	12.1712	0.0005546	1803
1804	3,254,416	5,870,966,464	42.4735	12.1734	0.0005543	1804
1805	3,258,025	5,880,735,125	42.4853	12.1757	0.0005540	1805
1806	3,261,636	5,890,514,616	42.4971	12.1779	0.0005537	1806
1807	3,265,249	5,900,304,943	42.5088	12.1802	0.0005534	1807
1808	3,268,864	5,910,106,112	42.5206	12.1824	0.0005531	1808
1809	3,272,481	5,919,918,129	42.5323	12.1846	0.0005528	1809
1810	3,276,100	5,929,741,000	42.5441	12.1869	0.0005525	1810
1811	3,279,721	5,939,574,731	42.5558	12.1891	0.0005522	1811
1812	3,283,344	5,949,419,328	42.5676	12.1914	0.0005519	1812
1813	3,286,969	5,959,274,797	42.5793	12.1936	0.0005516	1813
1814	3,290,596	5,969,141,144	42.5911	12.1959	0.0005513	1814
1815	3,294,225	5,979,018,375	42.6028	12.1981	0.0005510	1815
1816	3,297,856	5,988,906,496	42.6146	12.2003	0.0005507	1816
1817	3,301,489	5,998,805,513	42.6263	12.2026	0.0005504	1817
1818	3,305,124	6,008,715,432	42.6380	12.2048	0.0005501	1818
1819	3,308,761	6,018,636,259	42.6497	12.2071	0.0005498	1819
1820	3,312,400	6,028,568,000	42.6615	12.2093	0.0005495	1820
1821	3,316,041	6,038,510,661	42.6732	12.2115	0.0005491	1821
1822	3,319,684	6,048,464,248	42.6849	12.2138	0.0005488	1822
1823	3,323,329	6,058,428,767	42.6966	12.2160	0.0005485	1823
1824	3,326,976	6,068,404,224	42.7083	12.2182	0.0005482	1824
1825	3,330,625	6,078,390,625	42.7200	12.2205	0.0005479	1825
1826	3,334,276	6,088,387,976	42.7317	12.2227	0.0005476	1826
1827	3,337,929	6,098,396,283	42.7434	12.2249	0.0005473	1827
1828	3,341,584	6,108,415,552	42.7551	12.2272	0.0005470	1828
1829	3,345,241	6,118,445,789	42.7668	12.2294	0.0005467	1829
1830	3,348,900	6,128,487,000	42.7785	12.2316	0.0005464	1830
1831	3,352,561	6,138,539,191	42.7902	12.2338	0.0005461	1831
1832	3,356,224	6,148,602,368	42.8019	12.2361	0.0005459	1832
1833	3,359,889	6,158,676,537	42.8135	12.2383	0.0005456	1833
1834	3,363,556	6,168,761,704	42.8252	12.2405	0.0005453	1834
1835	3,367,225	6,178,857,875	42.8369	12.2427	0.0005450	1835
1836	3,370,896	6,188,965,056	42.8486	12.2450	0.0005447	1836
1837	3,374,569	6,199,083,253	42.8602	12.2472	0.0005444	1837
1838	3,378,244	6,209,212,472	42.8719	12.2494	0.0005441	1838
1839	3,381,921	6,219,352,719	42.8836	12.2516	0.0005438	1839
1840	3,385,600	6,229,504,000	42.8952	12.2539	0.0005435	1840
1841	3,389,281	6,239,666,321	42.9069	12.2561	0.0005432	1841
1842	3,392,964	6,249,839,688	42.9185	12.2583	0.0005429	1842
1843	3,396,649	6,260,024,107	42.9302	12.2605	0.0005426	1843
1844	3,400,336	6,270,219,584	42.9418	12.2627	0.0005423	1844
1845	3,404,025	6,280,426,125	42.9535	12.2649	0.0005420	1845
1846	3,407,716	6,290,643,736	42.9651	12.2672	0.0005417	1846
1847	3,411,409	6,300,872,423	42.9767	12.2694	0.0005414	1847
1848	3,415,104	6,311,112,192	42.9884	12.2716	0.0005411	1848
1849	3,418,801	6,321,363,049	43.0000	12.2738	0.0005408	1849
1850	3,422,500	6,331,625,000	43.0116	12.2760	0.0005405	1850

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1851	3,426,201	6,341,898,051	43.0232	12.2782	0.0005402	1851
1852	3,429,904	6,352,182,208	43.0349	12.2804	0.0005400	1852
1853	3,433,609	6,362,477,477	43.0465	12.2826	0.0005397	1853
1854	3,437,316	6,372,783,864	43.0581	12.2849	0.0005394	1854
1855	3,441,025	6,383,101,375	43.0697	12.2871	0.0005391	1855
1856	3,444,736	6,393,430,016	43.0813	12.2893	0.0005388	1856
1857	3,448,449	6,403,769,793	43.0929	12.2915	0.0005385	1857
1858	3,452,164	6,414,120,712	43.1045	12.2937	0.0005382	1858
1859	3,455,881	6,424,482,779	43.1161	12.2959	0.0005379	1859
1860	3,459,600	6,434,856,000	43.1277	12.2981	0.0005376	1860
1861	3,463,321	6,445,240,381	43.1393	12.3003	0.0005373	1861
1862	3,467,044	6,455,635,928	43.1509	12.3025	0.0005371	1862
1863	3,470,769	6,466,042,647	43.1625	12.3047	0.0005368	1863
1864	3,474,496	6,476,460,544	43.1741	12.3069	0.0005365	1864
1865	3,478,225	6,486,889,625	43.1856	12.3091	0.0005362	1865
1866	3,481,956	6,497,329,896	43.1972	12.3113	0.0005359	1866
1867	3,485,689	6,507,781,363	43.2088	12.3135	0.0005356	1867
1868	3,489,424	6,518,244,032	43.2204	12.3157	0.0005353	1868
1869	3,493,161	6,528,717,909	43.2319	12.3179	0.0005350	1869
1870	3,496,900	6,539,203,000	43.2435	12.3201	0.0005348	1870
1871	3,500,641	6,549,699,311	43.2551	12.3223	0.0005345	1871
1872	3,504,384	6,560,206,848	43.2666	12.3245	0.0005342	1872
1873	3,508,129	6,570,725,617	43.2782	12.3267	0.0005339	1873
1874	3,511,876	6,581,255,624	43.2897	12.3289	0.0005336	1874
1875	3,515,625	6,591,796,875	43.3013	12.3311	0.0005333	1875
1876	3,519,376	6,602,349,376	43.3128	12.3333	0.0005330	1876
1877	3,523,129	6,612,913,133	43.3244	12.3354	0.0005328	1877
1878	3,526,884	6,623,488,152	43.3359	12.3376	0.0005325	1878
1879	3,530,641	6,634,074,439	43.3474	12.3398	0.0005322	1879
1880	3,534,400	6,644,672,000	43.3590	12.3420	0.0005319	1880
1881	3,538,161	6,655,280,841	43.3705	12.3442	0.0005316	1881
1882	3,541,924	6,665,900,968	43.3820	12.3464	0.0005313	1882
1883	3,545,689	6,676,532,387	43.3935	12.3486	0.0005311	1883
1884	3,549,456	6,687,175,104	43.4051	12.3508	0.0005308	1884
1885	3,553,225	6,697,829,125	43.4166	12.3529	0.0005305	1885
1886	3,556,996	6,708,494,456	43.4281	12.3551	0.0005302	1886
1887	3,560,769	6,719,171,103	43.4396	12.3573	0.0005299	1887
1888	3,564,544	6,729,859,072	43.4511	12.3595	0.0005297	1888
1889	3,568,321	6,740,558,369	43.4626	12.3617	0.0005294	1889
1890	3,572,100	6,751,269,000	43.4741	12.3639	0.0005291	1890
1891	3,575,881	6,761,990,971	43.4856	12.3660	0.0005288	1891
1892	3,579,664	6,772,724,288	43.4971	12.3682	0.0005285	1892
1893	3,583,449	6,783,468,957	43.5086	12.3704	0.0005283	1893
1894	3,587,236	6,794,224,984	43.5201	12.3726	0.0005280	1894
1895	3,591,025	6,804,992,375	43.5316	12.3747	0.0005277	1895
1896	3,594,816	6,815,771,136	43.5431	12.3769	0.0005274	1896
1897	3,598,609	6,826,561,273	43.5546	12.3791	0.0005271	1897
1898	3,602,404	6,837,362,792	43.5660	12.3813	0.0005269	1898
1899	3,606,201	6,848,175,699	43.5775	12.3835	0.0005266	1899
1900	3,610,000	6,859,000,000	43.5890	12.3856	0.0005263	1900

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1901	3,613,801	6,869,835,701	43.6005	12.3878	0.0005260	1901
1902	3,617,604	6,880,682,808	43.6119	12.3900	0.0005258	1902
1903	3,621,409	6,891,541,327	43.6234	12.3921	0.0005255	1903
1904	3,625,216	6,902,411,264	43.6348	12.3943	0.0005252	1904
1905	3,629,025	6,913,292,625	43.6463	12.3965	0.0005249	1905
1906	3,632,836	6,924,185,416	43.6578	12.3986	0.0005247	1906
1907	3,636,649	6,935,089,643	43.6692	12.4008	0.0005244	1907
1908	3,640,464	6,946,005,312	43.6807	12.4030	0.0005241	1908
1909	3,644,281	6,956,932,429	43.6921	12.4051	0.0005238	1909
1910	3,648,100	6,967,871,000	43.7035	12.4073	0.0005236	1910
1911	3,651,921	6,978,821,031	43.7150	12.4095	0.0005233	1911
1912	3,655,744	6,989,782,528	43.7264	12.4116	0.0005230	1912
1913	3,659,569	7,000,755,497	43.7379	12.4138	0.0005227	1913
1914	3,663,396	7,011,739,944	43.7493	12.4160	0.0005225	1914
1915	3,667,225	7,022,735,875	43.7607	12.4181	0.0005222	1915
1916	3,671,056	7,033,743,296	43.7721	12.4203	0.0005219	1916
1917	3,674,889	7,044,762,213	43.7836	12.4225	0.0005216	1917
1918	3,678,724	7,055,792,632	43.7950	12.4246	0.0005214	1918
1919	3,682,561	7,066,834,559	43.8064	12.4268	0.0005211	1919
1920	3,686,400	7,077,888,000	43.8178	12.4289	0.0005208	1920
1921	3,690,241	7,088,952,961	43.8292	12.4311	0.0005206	1921
1922	3,694,084	7,100,029,448	43.8406	12.4332	0.0005203	1922
1923	3,697,929	7,111,117,467	43.8520	12.4354	0.0005200	1923
1924	3,701,776	7,122,217,024	43.8634	12.4376	0.0005198	1924
1925	3,705,625	7,133,328,125	43.8748	12.4397	0.0005195	1925
1926	3,709,476	7,144,450,776	43.8862	12.4419	0.0005192	1926
1927	3,713,329	7,155,584,983	43.8976	12.4440	0.0005189	1927
1928	3,717,184	7,166,730,752	43.9090	12.4462	0.0005187	1928
1929	3,721,041	7,177,888,089	43.9204	12.4483	0.0005184	1929
1930	3,724,900	7,189,057,000	43.9318	12.4505	0.0005181	1930
1931	3,728,761	7,200,237,491	43.9431	12.4526	0.0005179	1931
1932	3,732,624	7,211,429,568	43.9545	12.4548	0.0005176	1932
1933	3,736,489	7,222,633,237	43.9659	12.4569	0.0005173	1933
1934	3,740,356	7,233,848,504	43.9773	12.4591	0.0005171	1934
1935	3,744,225	7,245,075,375	43.9886	12.4612	0.0005168	1935
1936	3,748,096	7,256,313,856	44.0000	12.4634	0.0005165	1936
1937	3,751,969	7,267,563,953	44.0114	12.4655	0.0005163	1937
1938	3,755,844	7,278,825,672	44.0227	12.4676	0.0005160	1938
1939	3,759,721	7,290,099,019	44.0341	12.4698	0.0005157	1939
1940	3,763,600	7,301,384,000	44.0454	12.4719	0.0005155	1940
1941	3,767,481	7,312,680,621	44.0568	12.4741	0.0005152	1941
1942	3,771,364	7,323,988,888	44.0681	12.4762	0.0005149	1942
1943	3,775,249	7,335,308,807	44.0795	12.4784	0.0005147	1943
1944	3,779,136	7,346,640,384	44.0908	12.4805	0.0005144	1944
1945	3,783,025	7,357,983,625	44.1022	12.4826	0.0005141	1945
1946	3,786,916	7,369,338,536	44.1135	12.4848	0.0005139	1946
1947	3,790,809	7,380,705,123	44.1248	12.4869	0.0005136	1947
1948	3,794,704	7,392,083,392	44.1362	12.4891	0.0005133	1948
1949	3,798,601	7,403,473,349	44.1475	12.4912	0.0005131	1949
1950	3,802,500	7,414,875,000	44.1588	12.4933	0.0005128	1950

Powers, Roots and Reciprocals

No.	Square	Cube	Sq. Root	Cube Root	Reciprocal	No.
1951	3,806,401	7,426,288,351	44.1701	12.4955	0.0005126	1951
1952	3,810,304	7,437,713,408	44.1814	12.4976	0.0005123	1952
1953	3,814,209	7,449,150,177	44.1928	12.4997	0.0005120	1953
1954	3,818,116	7,460,598,664	44.2041	12.5019	0.0005118	1954
1955	3,822,025	7,472,058,875	44.2154	12.5040	0.0005115	1955
1956	3,825,936	7,483,530,816	44.2267	12.5061	0.0005112	1956
1957	3,829,849	7,495,014,493	44.2380	12.5083	0.0005110	1957
1958	3,833,764	7,506,509,912	44.2493	12.5104	0.0005107	1958
1959	3,837,681	7,518,017,079	44.2606	12.5125	0.0005105	1959
1960	3,841,600	7,529,536,000	44.2719	12.5146	0.0005102	1960
1961	3,845,521	7,541,066,681	44.2832	12.5168	0.0005099	1961
1962	3,849,444	7,552,609,128	44.2945	12.5189	0.0005097	1962
1963	3,853,369	7,564,163,347	44.3058	12.5210	0.0005094	1963
1964	3,857,296	7,575,729,344	44.3170	12.5232	0.0005092	1964
1965	3,861,225	7,587,307,125	44.3283	12.5253	0.0005089	1965
1966	3,865,156	7,598,896,696	44.3396	12.5274	0.0005086	1966
1967	3,869,089	7,610,498,063	44.3509	12.5295	0.0005084	1967
1968	3,873,024	7,622,111,232	44.3621	12.5317	0.0005081	1968
1969	3,876,961	7,633,736,209	44.3734	12.5338	0.0005079	1969
1970	3,880,900	7,645,373,000	44.3847	12.5359	0.0005076	1970
1971	3,884,841	7,657,021,611	44.3959	12.5380	0.0005074	1971
1972	3,888,784	7,668,682,048	44.4072	12.5401	0.0005071	1972
1973	3,892,729	7,680,354,317	44.4185	12.5423	0.0005068	1973
1974	3,896,676	7,692,038,424	44.4297	12.5444	0.0005066	1974
1975	3,900,625	7,703,734,375	44.4410	12.5465	0.0005063	1975
1976	3,904,576	7,715,442,176	44.4522	12.5486	0.0005061	1976
1977	3,908,529	7,727,161,833	44.4635	12.5507	0.0005058	1977
1978	3,912,484	7,738,893,352	44.4747	12.5528	0.0005056	1978
1979	3,916,441	7,750,636,739	44.4860	12.5550	0.0005053	1979
1980	3,920,400	7,762,392,000	44.4972	12.5571	0.0005051	1980
1981	3,924,361	7,774,159,141	44.5084	12.5592	0.0005048	1981
1982	3,928,324	7,785,938,168	44.5197	12.5613	0.0005045	1982
1983	3,932,289	7,797,729,087	44.5309	12.5634	0.0005043	1983
1984	3,936,256	7,809,531,904	44.5421	12.5655	0.0005040	1984
1985	3,940,225	7,821,346,625	44.5533	12.5676	0.0005038	1985
1986	3,944,196	7,833,173,256	44.5646	12.5697	0.0005035	1986
1987	3,948,169	7,845,011,803	44.5758	12.5719	0.0005033	1987
1988	3,952,144	7,856,862,272	44.5870	12.5740	0.0005030	1988
1989	3,956,121	7,868,724,669	44.5982	12.5761	0.0005028	1989
1990	3,960,100	7,880,599,000	44.6094	12.5782	0.0005025	1990
1991	3,964,081	7,892,485,271	44.6206	12.5803	0.0005023	1991
1992	3,968,064	7,904,383,488	44.6318	12.5824	0.0005020	1992
1993	3,972,049	7,916,293,657	44.6430	12.5845	0.0005018	1993
1994	3,976,036	7,928,215,784	44.6542	12.5866	0.0005015	1994
1995	3,980,025	7,940,149,875	44.6654	12.5887	0.0005013	1995
1996	3,984,016	7,952,095,936	44.6766	12.5908	0.0005010	1996
1997	3,988,009	7,964,053,973	44.6878	12.5929	0.0005008	1997
1998	3,992,004	7,976,023,992	44.6990	12.5950	0.0005005	1998
1999	3,996,001	7,988,005,999	44.7102	12.5971	0.0005003	1999
2000	4,000,000	8,000,000,000	44.7214	12.5992	0.0005000	2000

Squares of Mixed Numbers from 1/64 to 12, by 64ths

I. Squares of Mixed Numbers from 1/64 to 6

	0	1	2	3	4	5
1/64	0.00024	1.03149	4.06274	9.09399	16.12524	25.15649
1/32	0.00098	1.06348	4.12598	9.18848	16.25098	25.31348
3/64	0.00220	1.09595	4.18970	9.28345	16.37720	25.47095
1/16	0.00391	1.12891	4.25391	9.37891	16.50391	25.62891
5/64	0.00610	1.16235	4.31860	9.47485	16.63110	25.78735
3/32	0.00879	1.19629	4.38379	9.57129	16.75879	25.94629
7/64	0.01196	1.23071	4.44946	9.66821	16.88696	26.10571
1/8	0.01562	1.26562	4.51562	9.76562	17.01562	26.26562
9/64	0.01978	1.30103	4.58228	9.86353	17.14478	26.42603
5/32	0.02441	1.33691	4.64941	9.96191	17.27441	26.58691
11/64	0.02954	1.37329	4.71704	10.06079	17.40454	26.74829
3/16	0.03516	1.41016	4.78516	10.16016	17.53516	26.91016
13/64	0.04126	1.44751	4.85376	10.26001	17.66626	27.07251
7/32	0.04785	1.48535	4.92285	10.36035	17.79785	27.23535
15/64	0.05493	1.52368	4.99243	10.46118	17.92993	27.39868
1/4	0.06250	1.56250	5.06250	10.56250	18.06250	27.56250
17/64	0.07056	1.60181	5.13306	10.66431	18.19556	27.72681
9/32	0.07910	1.64160	5.20410	10.76660	18.32910	27.89160
19/64	0.08813	1.68188	5.27563	10.86938	18.46313	28.05688
5/16	0.09766	1.72266	5.34766	10.97266	18.59766	28.22266
21/64	0.10767	1.76392	5.42017	11.07642	18.73267	28.38892
11/32	0.11816	1.80566	5.49316	11.18066	18.86816	28.55566
23/64	0.12915	1.84790	5.56663	11.28540	19.00415	28.72290
3/8	0.14062	1.89062	5.64062	11.39062	19.14062	28.89062
25/64	0.15259	1.93384	5.71509	11.49634	19.27759	29.05884
13/32	0.16504	1.97754	5.79004	11.60254	19.41504	29.22754
27/64	0.17798	2.02173	5.86548	11.70923	19.55298	29.39673
7/16	0.19141	2.06641	5.94141	11.81641	19.69141	29.56641
29/64	0.20532	2.11157	6.01782	11.92407	19.83032	29.73657
15/32	0.21973	2.15723	6.09473	12.03223	19.96973	29.90723
31/64	0.23462	2.20337	6.17212	12.14087	20.10962	30.07837
1/2	0.25000	2.25000	6.25000	12.25000	20.25000	30.25000
33/64	0.26587	2.29712	6.32837	12.35962	20.39087	30.42212
17/32	0.28223	2.34473	6.40723	12.46973	20.53223	30.59473
35/64	0.29907	2.39282	6.48657	12.58032	20.67407	30.76782
9/16	0.31641	2.44141	6.56641	12.69141	20.81641	30.94141
37/64	0.33423	2.49048	6.64673	12.80298	20.95923	31.11548
19/32	0.35254	2.54004	6.72754	12.91504	21.10254	31.29004
39/64	0.37134	2.59009	6.80884	13.02759	21.24634	31.46509
5/8	0.39062	2.64062	6.89062	13.14062	21.39062	31.64062
41/64	0.41040	2.69165	6.97290	13.25415	21.53540	31.81665
21/32	0.43066	2.74316	7.05566	13.36816	21.68066	31.99316

The tables of squares of mixed numbers from 1/64 to 12 are arranged in as compact a manner as possible, and a few words may be necessary to explain their use. Assume, for example, that the square of 8 5/64 is required; 8 is located at the

Squares of Mixed Numbers from 1/64 to 6 (Continued)

	0	1	2	3	4	5
43/64	0.45142	2.79517	7.13892	13.48267	21.82642	32.17017
11/16	0.47266	2.84766	7.22266	13.59766	21.97266	32.34766
45/64	0.49438	2.90063	7.30688	13.71313	22.11938	32.52563
23/32	0.51660	2.95410	7.39160	13.82910	22.26660	32.70410
47/64	0.53931	3.00806	7.47681	13.94556	22.41431	32.88306
3/4	0.56250	3.06250	7.56250	14.06250	22.56250	33.06250
49/64	0.58618	3.11743	7.64868	14.17993	22.71118	33.24243
25/32	0.61035	3.17285	7.73535	14.29785	22.86035	33.42285
51/64	0.63501	3.22876	7.82251	14.41626	23.01001	33.60376
13/16	0.66016	3.28516	7.91016	14.53516	23.16016	33.78516
53/64	0.68579	3.34204	7.99829	14.65454	23.31079	33.96704
27/32	0.71191	3.39941	8.08691	14.77441	23.46191	34.14941
55/64	0.73853	3.45728	8.17603	14.89478	23.61363	34.33228
7/8	0.76562	3.51562	8.26562	15.01562	23.76562	34.51562
57/64	0.79321	3.57446	8.35571	15.13696	23.91821	34.69946
29/32	0.82129	3.63379	8.44629	15.25879	24.07129	34.88379
59/64	0.84985	3.69360	8.53735	15.38110	24.22485	35.06860
15/16	0.87891	3.75391	8.62891	15.50391	24.37891	35.25391
61/64	0.90845	3.81470	8.72095	15.62720	24.53345	35.43970
31/32	0.93848	3.87598	8.81348	15.75098	24.68848	35.62598
63/64	0.96899	3.93774	8.90649	15.87524	24.84399	35.81274

II. Squares of Mixed Numbers from 6 1/64 to 12

	6	7	8	9	10	11
1/64	36.18774	49.21899	64.25024	81.28149	100.31274	121.34399
1/32	36.37598	49.43848	64.50098	81.56348	100.62598	121.68848
3/64	36.56470	49.65845	64.75220	81.84595	100.93970	122.03345
1/16	36.75391	49.87891	65.00391	82.12891	101.25391	122.37891
5/64	36.94360	50.09985	65.25610	82.41235	101.56860	122.72485
3/32	37.13379	50.32129	65.50879	82.69629	101.88379	123.07129
7/64	37.32446	50.54321	65.76196	82.98071	102.19946	123.41821
1/8	37.51562	50.76562	66.01562	83.26562	102.51562	123.76562
9/64	37.70728	50.98853	66.26978	83.55103	102.83228	124.11353
5/32	37.89941	51.21191	66.52441	83.83691	103.14941	124.46191
11/64	38.09204	51.43579	66.77954	84.12329	103.46704	124.81079
3/16	38.28516	51.66016	67.03516	84.41016	103.78516	125.16016
13/64	38.47876	51.88501	67.29126	84.69751	104.10376	125.51001
7/32	38.67285	52.11035	67.54785	84.98535	104.42285	125.86035
15/64	38.86743	52.33618	67.80493	85.27368	104.74243	126.21110
1/4	39.06250	52.56250	68.06250	85.56250	105.06250	126.56250

top of its column, and 5/64 in the left-hand column. The square is then found to equal 65.25610. In the same way, the square of 3 3/16 is found to equal 10.16016.

Squares of Mixed Numbers from 6 1/64 to 12 (Continued)

	6	7	8	9	10	11
17/64	39.25806	52.78931	68.32056	85.85181	105.38306	126.91431
9/32	39.45410	53.01660	68.57910	86.14160	105.70410	127.26660
19/64	39.65063	53.24438	68.83813	86.43188	106.02563	127.61938
5/16	39.84766	53.47266	69.09766	86.72266	106.34766	127.97266
21/64	40.04517	53.70142	69.35767	87.01392	106.67017	128.32642
11/32	40.24316	53.93066	69.61816	87.30566	106.99316	128.68066
23/64	40.44165	54.16040	69.87915	87.59790	107.31665	129.03540
3/8	40.64062	54.39062	70.14062	87.89062	107.64062	129.39062
25/64	40.84009	54.62134	70.40259	88.18384	107.96509	129.74634
13/32	41.04004	54.85254	70.66504	88.47754	108.29004	130.10254
27/64	41.24048	55.08423	70.92798	88.77173	108.61548	130.45923
7/16	41.44141	55.31641	71.19141	89.06641	108.94141	130.81641
29/64	41.64282	55.54907	71.45532	89.36157	109.26782	131.17407
15/32	41.84473	55.78223	71.71973	89.65723	109.59473	131.53223
31/64	42.04712	56.01587	71.98462	89.95337	109.92212	131.89087
1/2	42.25000	56.25000	72.25000	90.25000	110.25000	132.25000
33/64	42.45337	56.48462	72.51587	90.54712	110.57837	132.60962
17/32	42.65723	56.71973	72.78223	90.84473	110.90723	132.96973
35/64	42.86157	56.95532	73.04907	91.14282	111.23657	133.33032
9/16	43.06641	57.19141	73.31641	91.44141	111.56641	133.69141
37/64	43.27173	57.42798	73.58423	91.74048	111.89673	134.05298
19/32	43.47754	57.66504	73.85254	92.04004	112.22754	134.41504
39/64	43.68384	57.90259	74.12134	92.34009	112.55884	134.77759
5/8	43.89062	58.14062	74.39062	92.64062	112.89062	135.14062
41/64	44.09790	58.37915	74.66040	92.94165	113.22290	135.50415
21/32	44.30566	58.61816	74.93066	93.24316	113.55566	135.86816
43/64	44.51392	58.85767	75.20142	93.54517	113.88892	136.23267
11/16	44.72266	59.09766	75.47266	93.84766	114.22266	136.59766
45/64	44.93188	59.33813	75.74438	94.15063	114.55688	136.96313
23/32	45.14160	59.57910	76.01660	94.45410	114.89160	137.32910
47/64	45.35181	59.82056	76.28931	94.75806	115.22681	137.69556
3/4	45.56250	60.06250	76.56250	95.06250	115.56250	138.06250
49/64	45.77368	60.30493	76.83618	95.36743	115.89868	138.42993
25/32	45.98535	60.54785	77.11035	95.67285	116.23535	138.79785
51/64	46.19751	60.79126	77.38501	95.97876	116.57251	139.16626
13/16	46.41016	61.03516	77.66016	96.28516	116.91016	139.53516
53/64	46.62329	61.27954	77.93579	96.59204	117.24829	139.90454
27/32	46.83691	61.52441	78.21191	96.89941	117.58691	140.27441
55/64	47.05103	61.76978	78.48853	97.20728	117.92603	140.64478
7/8	47.26562	62.01562	78.76562	97.51562	118.26562	141.01562
57/64	47.48071	62.26196	79.04321	97.82446	118.60571	141.38696
29/32	47.69629	62.50879	79.32129	98.13379	118.94629	141.75879
59/64	47.91235	62.75610	79.59985	98.44360	119.28735	142.13110
15/16	48.12891	63.00391	79.87891	98.75391	119.62891	142.50391
61/64	48.34595	63.25220	80.15845	99.06470	119.97095	142.87720
31/32	48.56348	63.50098	80.43848	99.37598	120.31348	143.25098
63/64	48.78149	63.75024	80.71899	99.68774	120.65649	143.62524

Squares and Cubes of Numbers from 1/32 to 100

Advancing by 32nds to 2; from 2 to 10 by 16ths; from 10 to 100 by 8ths

No.	Square	Cube	No.	Square	Cube	No.	Square	Cube
1/32	0.000976	0.000031	1 17/32	2.344727	3.590363	4	16.0000	64.0000
1/16	0.003906	0.000244	9/16	2.441406	3.814697	1/16	16.5039	67.0471
3/32	0.008789	0.000824	19/32	2.540039	4.048187	1/8	17.0156	70.1895
1/8	0.015625	0.001953	5/8	2.640625	4.291016	3/16	17.5352	73.4285
5/32	0.024414	0.003815	21/32	2.743164	4.543365	1/4	18.0625	76.7656
3/16	0.035156	0.006592	11/16	2.847656	4.805419	5/16	18.5977	80.2024
7/32	0.047852	0.010468	23/32	2.954102	5.077362	3/8	19.1406	83.7402
1/4	0.062500	0.015625	3/4	3.062500	5.359375	7/16	19.6914	87.3806
9/32	0.079102	0.022247	25/32	3.172852	5.651642	1/2	20.2500	91.1250
5/16	0.097656	0.030518	13/16	3.285156	5.954346	9/16	20.8164	94.9749
11/32	0.118164	0.040619	27/32	3.399414	6.267660	5/8	21.3906	98.9316
3/8	0.140625	0.052734	7/8	3.515625	6.591797	11/16	21.9727	102.9968
13/32	0.165039	0.067047	29/32	3.633789	6.926910	3/4	22.5625	107.1719
7/16	0.191406	0.083740	15/16	3.753906	7.273193	13/16	23.1602	111.4583
15/32	0.219727	0.102997	31/32	3.875977	7.630828	7/8	23.7656	115.8574
1/2	0.250000	0.125000	2	4.00000	8.00000	15/16	24.3789	120.3708
17/32	0.282227	0.149933	1/32	4.12598	8.38089	5	25.0000	125.0000
9/16	0.316406	0.177979	1/16	4.25391	8.77368	1/16	25.6289	129.7463
19/32	0.352539	0.209320	1/8	4.51563	9.59570	1/8	26.2656	134.6113
5/8	0.390625	0.244141	3/16	4.78516	10.46754	3/16	26.9102	139.5964
21/32	0.430664	0.282623	1/4	5.06250	11.39063	1/4	27.5625	144.7031
11/16	0.472656	0.324951	5/16	5.34766	12.36646	5/16	28.2227	149.9329
23/32	0.516602	0.371307	3/8	5.64063	13.39648	3/8	28.8906	155.2871
3/4	0.562500	0.421875	7/16	5.94141	14.48218	7/16	29.5664	160.7673
25/32	0.610352	0.476837	1/2	6.25000	15.62500	1/2	30.2500	166.3750
13/16	0.660156	0.536377	9/16	6.56641	16.82642	9/16	30.9414	172.1116
27/32	0.711914	0.600678	5/8	6.89063	18.08789	5/8	31.6406	177.9785
7/8	0.765625	0.669922	11/16	7.22266	19.41089	11/16	32.3477	183.9773
29/32	0.821289	0.744293	3/4	7.56250	20.79688	3/4	33.0625	190.1094
15/16	0.878906	0.823975	13/16	7.91016	22.24731	13/16	33.7852	196.3762
31/32	0.938477	0.909149	7/8	8.26563	23.76367	7/8	34.5156	202.7793
1	1.000000	1.000000	15/16	8.62891	25.34741	15/16	35.2539	209.3201
1/32	1.063477	1.096800	3	9.00000	27.00000	6	36.0000	216.0000
1/16	1.128906	1.199463	1/16	9.37891	28.72290	1/16	36.7539	222.8206
3/32	1.196289	1.308441	1/8	9.76563	30.51758	1/8	37.5156	229.7832
1/8	1.265625	1.423828	3/16	10.16016	32.38550	3/16	38.2852	236.8894
5/32	1.336914	1.545807	1/4	10.56250	34.32813	1/4	39.0625	244.1406
3/16	1.410156	1.674561	5/16	10.97266	36.34692	5/16	39.8477	251.5383
7/32	1.485352	1.810272	3/8	11.39063	38.44336	3/8	40.6406	259.0840
1/4	1.562500	1.953125	7/16	11.81641	40.61889	7/16	41.4414	266.7791
9/32	1.641602	2.103302	1/2	12.25000	42.87500	1/2	42.2500	274.6250
5/16	1.722656	2.260986	9/16	12.69141	45.21313	9/16	43.0664	282.6233
11/32	1.805664	2.426361	5/8	13.14063	47.63477	5/8	43.8906	290.7754
3/8	1.890625	2.599609	11/16	13.59766	50.14135	11/16	44.7227	299.0828
13/32	1.977539	2.780914	3/4	14.06250	52.73438	3/4	45.5625	307.5469
7/16	2.066406	2.970459	13/16	14.53516	55.41528	13/16	46.4102	316.1692
15/32	2.157227	3.168927	7/8	15.01563	58.18555	7/8	47.2656	324.9512
1/2	2.250000	3.375000	15/16	15.50391	61.04663	15/16	48.1289	333.8943

Squares and Cubes of Numbers from 1/32 to 100 (Continued)

No.	Square	Cube	No.	Square	Cube	No.	Square	Cube
7	49.0000	343.0000	10	100.0000	1000.0000	16	256.0000	4096.000
1/16	49.8789	352.2698	1/8	102.5156	1037.9707	1/8	260.0156	4192.752
1/8	50.7656	361.7051	1/4	105.0625	1076.8906	1/4	264.0625	4291.015
3/16	51.6602	371.3074	3/8	107.6406	1116.7715	3/8	268.1406	4390.802
1/4	52.5625	381.0781	1/2	110.2500	1157.6250	1/2	272.2500	4492.125
5/16	53.4727	391.0188	5/8	112.8906	1199.4629	5/8	276.3906	4594.994
3/8	54.3906	401.1309	3/4	115.5625	1242.2969	3/4	280.5625	4699.421
7/16	55.3164	411.4158	7/8	118.2656	1286.1387	7/8	284.7656	4805.419
1/2	56.2500	421.8750	11	121.0000	1331.0000	17	289.0000	4913.000
9/16	57.1914	432.5100	1/8	123.7656	1376.8926	1/8	293.2656	5022.173
5/8	58.1406	443.3223	1/4	126.5625	1423.8281	1/4	297.5625	5132.953
11/16	59.0977	454.3132	3/8	129.3906	1471.8184	3/8	301.8906	5245.349
3/4	60.0625	465.4844	1/2	132.2500	1520.8750	1/2	306.2500	5359.375
13/16	61.0352	476.8372	5/8	135.1406	1571.0098	5/8	310.6406	5475.041
7/8	62.0156	488.3730	3/4	138.0625	1622.2344	3/4	315.0625	5592.359
15/16	63.0039	500.0935	7/8	141.0156	1674.5605	7/8	319.5156	5711.341
8	64.0000	512.0000	12	144.0000	1728.0000	18	324.0000	5832.000
1/16	65.0039	524.0940	1/8	147.0156	1782.5645	1/8	328.5156	5954.345
1/8	66.0156	536.3770	1/4	150.0625	1838.2656	1/4	333.0625	6078.390
3/16	67.0352	548.8503	3/8	153.1406	1895.1152	3/8	337.6406	6204.146
1/4	68.0625	561.5156	1/2	156.2500	1953.1250	1/2	342.2500	6331.625
5/16	69.0977	574.3743	5/8	159.3906	2012.3066	5/8	346.8906	6460.837
3/8	70.1406	587.4277	3/4	162.5625	2072.6719	3/4	351.5625	6591.796
7/16	71.1914	600.6775	7/8	165.7656	2134.2324	7/8	356.2656	6724.513
1/2	72.2500	614.1250	13	169.0000	2197.0000	19	361.0000	6859.000
9/16	73.3164	627.7717	1/8	172.2656	2260.9863	1/8	365.7656	6995.267
5/8	74.3906	641.6191	1/4	175.5625	2326.2031	1/4	370.5625	7133.328
11/16	75.4727	655.6687	3/8	178.8906	2392.6621	3/8	375.3906	7273.193
3/4	76.5625	669.9219	1/2	182.2500	2460.3750	1/2	380.2500	7414.875
13/16	77.6602	684.3801	5/8	185.6406	2529.3535	5/8	385.1406	7558.384
7/8	78.7656	699.0449	3/4	189.0625	2599.6094	3/4	390.0625	7703.734
15/16	79.8789	713.9177	7/8	192.5156	2671.1543	7/8	395.0156	7850.935
9	81.0000	729.0000	14	196.0000	2744.0000	20	400.0000	8000.000
1/16	82.1289	744.2932	1/8	199.5156	2818.1582	1/8	405.0156	8150.939
1/8	83.2656	759.7988	1/4	203.0625	2893.6406	1/4	410.0625	8303.765
3/16	84.4102	775.5183	3/8	206.6406	2970.4590	3/8	415.1406	8458.490
1/4	85.5625	791.4531	1/2	210.2500	3048.6250	1/2	420.2500	8615.125
5/16	86.7227	807.6047	5/8	213.8906	3128.1504	5/8	425.3906	8773.681
3/8	87.8906	823.9746	3/4	217.5625	3209.0469	3/4	430.5625	8934.171
7/16	89.0664	840.5642	7/8	221.2656	3291.3262	7/8	435.7656	9096.607
1/2	90.2500	857.3750	15	225.0000	3375.0000	21	441.0000	9261.000
9/16	91.4414	874.4084	1/8	228.7656	3460.0801	1/8	446.2656	9427.361
5/8	92.6406	891.6660	1/4	232.5625	3546.5781	1/4	451.5625	9595.703
11/16	93.8477	909.1492	3/8	236.3906	3634.5059	3/8	456.8906	9766.037
3/4	95.0625	926.8594	1/2	240.2500	3723.8750	1/2	462.2500	9,938.375
13/16	96.2852	944.7981	5/8	244.1406	3814.6973	5/8	467.6406	10,112.728
7/8	97.5156	962.9668	3/4	248.0625	3906.9844	3/4	473.0625	10,289.109
15/16	98.7539	981.3669	7/8	252.0156	4000.7480	7/8	478.5156	10,467.529

Squares and Cubes of Numbers from 1/32 to 100 (Continued)

No.	Square	Cube	No.	Square	Cube	No.	Square	Cube
22	484.0000	10,648.000	28	784.000	21,952.000	34	1156.000	39,304.000
⅛	489.5156	10,830.533	⅛	791.015	22,247.314	⅛	1164.515	39,739.095
¼	495.0625	11,015.140	¼	798.062	22,545.265	¼	1173.062	40,177.390
⅜	500.6406	11,201.834	⅜	805.140	22,845.865	⅜	1181.640	40,618.896
½	506.2500	11,390.625	½	812.250	23,149.125	½	1190.250	41,063.625
⅝	511.8906	11,581.525	⅝	819.390	23,455.056	⅝	1198.890	41,511.587
¾	517.5625	11,774.546	¾	826.562	23,763.671	¾	1207.562	41,962.796
⅞	523.2656	11,969.701	⅞	833.765	24,074.982	⅞	1216.265	42,417.263
23	529.0000	12,167.000	29	841.000	24,389.000	35	1225.000	42,875.000
⅛	534.7656	12,366.455	⅛	848.265	24,705.736	⅛	1233.765	43,336.017
¼	540.5625	12,568.078	¼	855.562	25,025.203	¼	1242.562	43,800.328
⅜	546.3906	12,771.880	⅜	862.890	25,347.412	⅜	1251.390	44,267.943
½	552.2500	12,977.875	½	870.250	25,672.375	½	1260.250	44,738.875
⅝	558.1406	13,186.072	⅝	877.640	26,000.103	⅝	1269.140	45,213.134
¾	564.0625	13,396.484	¾	885.062	26,330.609	¾	1278.062	45,690.734
⅞	570.0156	13,609.123	⅞	892.515	26,663.904	⅞	1287.015	46,171.685
24	576.0000	13,824.000	30	900.000	27,000.000	36	1296.000	46,656.000
⅛	582.0156	14,041.127	⅛	907.515	27,338.908	⅛	1305.015	47,143.689
¼	588.0625	14,260.515	¼	915.062	27,680.640	¼	1314.062	47,634.765
⅜	594.1406	14,482.177	⅜	922.640	28,025.209	⅜	1323.140	48,129.240
½	600.2500	14,706.125	½	930.250	28,372.625	½	1332.250	48,627.125
⅝	606.3906	14,932.369	⅝	937.890	28,722.900	⅝	1341.390	49,128.431
¾	612.5625	15,160.921	¾	945.562	29,076.046	¾	1350.562	49,633.171
⅞	618.7656	15,391.794	⅞	953.265	29,432.076	⅞	1359.765	50,141.357
25	625.0000	15,625.000	31	961.000	29,791.000	37	1369.000	50,653.000
⅛	631.2656	15,860.548	⅛	968.765	30,152.830	⅛	1378.265	51,168.111
¼	637.5625	16,098.453	¼	976 562	30,517.578	¼	1387.562	51,686.703
⅜	643.8906	16,338.724	⅜	984.390	30,885.255	⅜	1396.890	52,208.787
½	650.2500	16,581.375	½	992.250	31,255.875	½	1406.250	52,734.375
⅝	656.6406	16,826.416	⅝	1000.140	31,629.447	⅝	1415.640	53,263.478
¾	663.0625	17,073.859	¾	1008.062	32,005.984	¾	1425.062	53,796.109
⅞	669.5156	17,323.716	⅞	1016.015	32,385.498	⅞	1434.515	54,332.279
26	676.0000	17,576.000	32	1024.000	32,768.000	38	1444.000	54,872.000
⅛	682.5156	17,830.720	⅛	1032.015	33,153.502	⅛	1453.515	55,415.283
¼	689.0625	18,087.890	¼	1040.062	33,542.015	¼	1463.062	55,962.140
⅜	695.6406	18,347.521	⅜	1048.140	33,933.552	⅜	1472.640	56,512.584
½	702.2500	18,609.625	½	1056.250	34,328.125	½	1482.250	57,066.625
⅝	708.8906	18,874.212	⅝	1064.390	34,725.744	⅝	1491.890	57,624.275
¾	715.5625	19,141.296	¾	1072.562	35,126.421	¾	1501.562	58,185.546
⅞	722.2656	19,410.888	⅞	1080.765	35,530.169	⅞	1511.265	58,750.451
27	729.0000	19,683.000	33	1089.000	35,937.000	39	1521.000	59,319.000
⅛	735.7656	19,957.642	⅛	1097.265	36,346.923	⅛	1530.765	59,891.205
¼	742.5625	20,234.828	¼	1105.562	36,759.953	¼	1540.562	60,467.078
⅜	749.3906	20,514.568	⅜	1113.890	37,176.099	⅜	1550.390	61,046.630
½	756.2500	20,796.875	½	1122.250	37,595.375	½	1560.250	61,629.875
⅝	763.1406	21,081.759	⅝	1130.640	38,017.791	⅝	1570.140	62,216.822
¾	770.0625	21,369.234	¾	1139.062	38,443.359	¾	1580.062	62,807.484
⅞	777.0156	21,659.310	⅞	1147.515	38,872.091	⅞	1590.015	63,401.873

Squares and Cubes of Numbers from 1/32 to 100 (Continued)

No.	Square	Cube	No.	Square	Cube	No.	Square	Cube
40	1600.000	64,000.000	46	2116.000	97,336.00	52	2704.000	140,608.00
⅛	1610.015	64,601.877	⅛	2127.515	98,131.65	⅛	2717.015	141,624.43
¼	1620.062	65,207.516	¼	2139.062	98,931.64	¼	2730.062	142,645.76
⅜	1630.140	65,816.928	⅜	2150.640	99,735.95	⅜	2743.140	143,671.99
½	1640.250	66,430.125	½	2162.250	100,544.62	½	2756.250	144,703.12
⅝	1650.390	67,047.119	⅝	2173.890	101,357.65	⅝	2769.390	145,739.18
¾	1660.562	67,667.922	¾	2185.562	102,175.04	¾	2782.562	146,780.17
⅞	1670.765	68,292.545	⅞	2197.265	102,996.82	⅞	2795.765	147,826.10
41	1681.000	68,921.000	47	2209.000	103,823.00	53	2809.000	148,877.00
⅛	1691.265	69,553.299	⅛	2220.765	104,653.58	⅛	2822.265	149,932.86
¼	1701.562	70,189.453	¼	2232.562	105,488.57	¼	2835.562	150,993.70
⅜	1711.890	70,829.475	⅜	2244.390	106,328.00	⅜	2848.890	152,059.53
½	1722.250	71,473.375	½	2256.250	107,171.87	½	2862.250	153,130.37
⅝	1732.640	72,121.166	⅝	2268.140	108,020.19	⅝	2875.640	154,206.22
¾	1743.062	72,772.859	¾	2280.062	108,872.98	¾	2889.062	155,287.10
⅞	1753.515	73,428.467	⅞	2292.015	109,730.24	⅞	2902.515	156,373.02
42	1764.000	74,088.000	48	2304.000	110,592.00	54	2916.000	157,464.00
⅛	1774.515	74,751.471	⅛	2316.015	111,458.25	⅛	2929.515	158,560.03
¼	1785.062	75,418.891	¼	2328.062	112,329.01	¼	2943.062	159,661.14
⅜	1795.640	76,090.271	⅜	2340.140	113,204.30	⅜	2956.640	160,767.33
½	1806.250	76,765.625	½	2352.250	114,084.12	½	2970.250	161,878.62
⅝	1816.890	77,444.963	⅝	2364.390	114,968.49	⅝	2983.890	162,995.02
¾	1827.562	78,128.297	¾	2376.562	115,857.42	¾	2997.562	164,116.54
⅞	1838.265	78,815.639	⅞	2388.765	116,750.92	⅞	3011.265	165,243.20
43	1849.000	79,507.000	49	2401.000	117,649.00	55	3025.000	166,375.00
⅛	1859.765	80,202.393	⅛	2413.265	118,551.67	⅛	3038.765	167,511.95
¼	1870.562	80,901.828	¼	2425.562	119,458.95	¼	3052.562	168,654.07
⅜	1881.390	81,605.318	⅜	2437.890	120,370.85	⅜	3066.390	169,801.38
½	1892.250	82,312.875	½	2450.250	121,287.37	½	3080.250	170,953.87
⅝	1903.140	83,024.510	⅝	2462.640	122,208.54	⅝	3094.140	172,111.57
¾	1914.062	83,740.234	¾	2475.062	123,134.35	¾	3108.062	173,274.48
⅞	1925.015	84,460.061	⅞	2487.515	124,064.84	⅞	3122.015	174,442.62
44	1936.000	85,184.000	50	2500.000	125,000.00	56	3136.000	175,616.00
⅛	1947.015	85,912.064	⅛	2512.515	125,939.84	⅛	3150.015	176,794.62
¼	1958.062	86,644.266	¼	2525.062	126,884.39	¼	3164.062	177,978.51
⅜	1969.140	87,380.615	⅜	2537.640	127,833.64	⅜	3178.140	179,167.67
½	1980.250	88,121.125	½	2550.250	128,787.62	½	3192.250	180,362.12
⅝	1991.390	88,865.807	⅝	2562.890	129,746.33	⅝	3206.390	181,561.86
¾	2002.562	89,614.672	¾	2575.562	130,709.79	¾	3220.562	182,766.92
⅞	2013.765	90,367.732	⅞	2588.265	131,678.01	⅞	3234.765	183,977.29
45	2025.000	91,125.000	51	2601.000	132,651.00	57	3249.000	185,193.00
⅛	2036.265	91,886.486	⅛	2613.765	133,628.76	⅛	3263.265	186,414.04
¼	2047.562	92,652.203	¼	2626.562	134,611.32	¼	3277.562	187,640.45
⅜	2058.890	93,422.162	⅜	2639.390	135,598.69	⅜	3291.890	188,872.22
½	2070.250	94,196.375	½	2652.250	136,590.87	½	3306.250	190,109.37
⅝	2081.640	94,974.854	⅝	2665.140	137,587.88	⅝	3320.640	191,351.91
¾	2093.062	95,757.609	¾	2678.062	138,589.73	¾	3335.062	192,599.85
⅞	2104.515	96,544.654	⅞	2691.015	139,596.43	⅞	3349.515	193,853.21

Squares and Cubes of Numbers from 1/32 to 100 (Continued)

No.	Square	Cube	No.	Square	Cube	No.	Square	Cube
58	3364.000	195,112.00	64	4096.000	262,144.00	70	4900.000	343,000.00
⅛	3378.515	196,376.22	⅛	4112.015	263,683.00	⅛	4917.515	344,840.78
¼	3393.062	197,645.89	¼	4128.062	265,228.01	¼	4935.062	346,688.14
⅜	3407.640	198,921.02	⅜	4144.140	266,779.05	⅜	4952.640	348,542.08
½	3422.250	200,201.62	½	4160.250	268,336.12	½	4970.250	350,402.62
⅝	3436.890	201,487.71	⅝	4176.390	269,899.24	⅝	4987.890	352,269.77
¾	3451.562	202,779.29	¾	4192.562	271,468.42	¾	5005.562	354,143.54
⅞	3466.265	204,076.38	⅞	4208.765	273,043.67	⅞	5023.265	356,023.95
59	3481.000	205,379.00	65	4225.000	274,625.00	71	5041.000	357,911.00
⅛	3495.765	206,687.14	⅛	4241.265	276,212.42	⅛	5058.765	359,804.70
¼	3510.562	208,000.82	¼	4257.562	277,805.95	¼	5076.562	361,705.07
⅜	3525.390	209,320.06	⅜	4273.890	279,405.60	⅜	5094.390	363,612.13
½	3540.250	210,644.87	½	4290.250	281,011.37	½	5112.250	365,525.87
⅝	3555.140	211,975.25	⅝	4306.640	282,623.29	⅝	5130.140	367,446.32
¾	3570.062	213,311.23	¾	4323.062	284,241.35	¾	5148.062	369,373.48
⅞	3585.015	214,652.81	⅞	4339.515	285,865.59	⅞	5166.015	371,307.37
60	3600.000	216,000.00	66	4356.000	287,496.00	72	5184.000	373,248.00
⅛	3615.015	217,352.81	⅛	4372.515	289,132.59	⅛	5202.015	375,195.37
¼	3630.062	218,711.26	¼	4389.062	290,775.39	¼	5220.062	377,149.51
⅜	3645.140	220,075.36	⅜	4405.640	292,424.39	⅜	5238.140	379,110.42
½	3660.250	221,445.12	½	4422.250	294,079.62	½	5256.250	381,078.12
⅝	3675.390	222,820.55	⅝	4438.890	295,741.08	⅝	5274.390	383,052.61
¾	3690.562	224,201.67	¾	4455.562	297,408.79	¾	5292.562	385,033.92
⅞	3705.765	225,588.48	⅞	4472.265	299,082.76	⅞	5310.765	387,022.04
61	3721.000	226,981.00	67	4489.000	300,763.00	73	5329.000	389,017.00
⅛	3736.265	228,379.23	⅛	4505.765	302,449.51	⅛	5347.265	391,018.79
¼	3751.562	229,783.20	¼	4522.562	304,142.32	¼	5365.562	393,027.45
⅜	3766.890	231,192.91	⅜	4539.390	305,841.44	⅜	5383.890	395,042.97
½	3782.250	232,608.37	½	4556.250	307,546.87	½	5402.250	397,065.37
⅝	3797.640	234,029.60	⅝	4573.140	309,258.63	⅝	5420.635	399,094.29
¾	3813.062	235,456.60	¾	4590.062	310,976.73	¾	5439.062	401,130.85
⅞	3828.515	236,889.40	⅞	4607.015	312,701.18	⅞	5457.515	403,173.96
62	3844.000	238,328.00	68	4624.000	314,432.00	74	5476.000	405,204.00
⅛	3859.515	239,772.40	⅛	4641.015	316,169.18	⅛	5494.515	407,280.97
¼	3875.062	241,222.64	¼	4658.062	317,912.76	¼	5513.062	409,344.89
⅜	3890.640	242,678.70	⅜	4675.140	319,662.74	⅜	5531.640	411,415.77
½	3906.250	244,140.62	½	4692.250	321,419.12	½	5550.250	413,493.62
⅝	3921.890	245,608.40	⅝	4709.390	323,181.93	⅝	5568.890	415,578.46
¾	3937.562	247,082.04	¾	4726.562	324,951.17	¾	5587.562	417,670.29
⅞	3953.265	248,561.57	⅞	4743.765	326,726.85	⅞	5606.265	419,769.13
63	3969.000	250,047.00	69	4761.000	328,509.00	75	5625.000	421,875.00
⅛	3984.765	251,538.33	⅛	4778.265	330,297.61	⅛	5643.765	423,987.89
¼	4000.562	253,035.57	¼	4795.562	332,092.70	¼	5662.562	426,107.82
⅜	4016.390	254,538.75	⅜	4812.890	333,894.28	⅜	5681.390	428,234.81
½	4032.250	256,047.87	½	4830.250	335,702.37	½	5700.250	430,368.87
⅝	4048.140	257,562.94	⅝	4847.640	337,516.97	⅝	5719.140	432,510.01
¾	4064.062	259,083.98	¾	4865.062	339,338.10	¾	5738.062	434,658.23
⅞	4080.015	260,610.99	⅞	4882.515	341,165.77	⅞	5757.015	436,813.56

Squares and Cubes of Numbers from 1/32 to 100 (Continued)

No.	Square	Cube	No.	Square	Cube	No.	Square	Cube
76	5776.000	438,976.00	82	6724.000	551,368.00	88	7744.000	681,472.00
1/8	5795.015	441,145.56	1/8	6744.515	553,893.34	1/8	7766.015	684,380.12
1/4	5814.062	443,322.26	1/4	6765.062	556,426.39	1/4	7788.062	687,296.51
3/8	5833.140	445,506.11	3/8	6785.640	558,967.14	3/8	7810.140	690,221.17
1/2	5852.250	447,697.12	1/2	6806.250	561,515.62	1/2	7832.250	693,154.12
5/8	5871.390	449,895.30	5/8	6826.890	564,071.83	5/8	7854.390	696,095.36
3/4	5890.562	452,100.67	3/4	6847.562	566,635.79	3/4	7876.562	699,044.92
7/8	5909.765	454,313.23	7/8	6868.265	569,207.51	7/8	7898.765	702,002.79
77	5929.000	456,533.00	83	6889.000	571,787.00	89	7921.000	704,969.00
1/8	5948.265	458,759.98	1/8	6909.765	574,374.26	1/8	7943.265	707,943.54
1/4	5967.562	460,994.20	1/4	6930.562	576,969.32	1/4	7965.562	710,926.45
3/8	5986.890	463,235.66	3/8	6951.390	579,572.19	3/8	7987.890	713,917.72
1/2	6006.250	465,484.37	1/2	6972.250	582,182.87	1/2	8010.250	716,917.37
5/8	6025.640	467,740.35	5/8	6993.140	584,801.38	5/8	8032.640	719,925.41
3/4	6045.062	470,003.60	3/4	7014.062	587,427.73	3/4	8055.062	722,941.85
7/8	6064.515	472,274.15	7/8	7035.015	590,061.93	7/8	8077.515	725,966.71
78	6084.000	474,552.00	84	7056.000	592,704.00	90	8100.000	729.000.00
1/8	6103.515	476,837.15	1/8	7077.015	595,353.93	1/8	8122.515	732,041.72
1/4	6123.062	479,129.64	1/4	7098.062	598,011.76	1/4	8145.062	735,091.89
3/8	6142.640	481,429.45	3/8	7119.140	600,677.49	3/8	8167.640	738,150.52
1/2	6162.250	483,736.62	1/2	7140.250	603,351.12	1/2	8190.250	741,217.62
5/8	6181.890	486,051.15	5/8	7161.390	606,032.68	5/8	8212.890	744,293.21
3/4	6201.562	488,373.04	3/4	7182.562	608,722.17	3/4	8235.562	747,377.29
7/8	6221.265	490,702.32	7/8	7203.765	611,419.60	7/8	8258.265	750,469.88
79	6241.000	493,039.00	85	7225.000	614,125.00	91	8281.000	753,571.00
1/8	6260.765	495,383.08	1/8	7246.265	616,838.36	1/8	8303.765	756,680.64
1/4	6280.562	497,734.57	1/4	7267.562	619,559.70	1/4	8326.562	759,798.82
3/8	6300.390	500,093.50	3/8	7288.890	622,289.03	3/8	8349.390	762,925.56
1/2	6320.250	502,459.87	1/2	7310.250	625,026.37	1/2	8372.250	766,060.87
5/8	6340.140	504,833.69	5/8	7331.640	627,771.72	5/8	8395.140	769,204.76
3/4	6360.062	507,214.98	3/4	7353.062	630,525.10	3/4	8418.062	772,357.23
7/8	6380.015	509,603.74	7/8	7374.515	633,286.52	7/8	8441.015	775,518.31
80	6400.000	512,000.00	86	7396.000	636,056.00	92	8464.000	778,688.00
1/8	6420.015	514,403.75	1/8	7417.515	638,833.53	1/8	8487.015	781,866.31
1/4	6440.062	516,815.01	1/4	7439.062	641,619.14	1/4	8510.062	785,053.26
3/8	6460.140	519,233.80	3/8	7460.640	644,412.83	3/8	8533.140	788,248.86
1/2	6480.250	521,660.12	1/2	7482.250	647,214.62	1/2	8556.250	791,453.12
5/8	6500.390	524,093.99	5/8	7503.890	650,024.52	5/8	8579.390	794,666.05
3/4	6520.562	526,535.42	3/4	7525.562	652,842.54	3/4	8602.562	797,887.67
7/8	6540.765	528,984.42	7/8	7547.265	655,668.70	7/8	8625.765	801,117.98
81	6561.000	531,441.00	87	7569.000	658,503.00	93	8649.000	804,357.00
1/8	6581.265	533,905.17	1/8	7590.765	661,345.45	1/8	8672.265	807,604.73
1/4	6601.562	536,376.95	1/4	7612.562	664,196.07	1/4	8695.562	810,861.20
3/8	6621.890	538,856.35	3/8	7634.390	667,054.88	3/8	8718.890	814,126.41
1/2	6642.250	541,343.37	1/2	7656.250	669,921.87	1/2	8742.250	817,400.37
5/8	6662.640	543,838.04	5/8	7678.140	672,797.07	5/8	8765.640	820,683.10
3/4	6683.062	546,340.35	3/4	7700.062	675,680.48	3/4	8789.062	823,974.61
7/8	6703.515	548,850.34	7/8	7722.015	678,572.12	7/8	8812.515	827,274.90

Squares and Cubes of Numbers from 1/32 to 100 (Continued)

No.	Square	Cube	No.	Square	Cube	No.	Square	Cube
94	8836.000	830,584.00	96	9216.000	884,736.00	98	9604.00	941,192.0
1/8	8859.515	833,901.90	1/8	9240.015	888,196.50	1/8	9628.51	944,798.0
1/4	8883.062	837,228.64	1/4	9264.062	891,666.01	1/4	9653.06	948,413.3
3/8	8906.640	840,564.20	3/8	9288.140	895,144.55	3/8	9677.64	952,037.8
1/2	8930.250	843,908.62	1/2	9312.250	898,632.12	1/2	9702.25	955,671.6
5/8	8953.890	847,261.90	5/8	9336.390	902,128.74	5/8	9726.89	959,314.5
3/4	8977.562	850,624.04	3/4	9360.562	905,634.42	3/4	9751.56	962,966.7
7/8	9001.265	853,995.07	7/8	9384.765	909,149.17	7/8	9776.26	966,628.2
95	9025.000	857,375.00	97	9409.000	912,673.00	99	9801.00	970,299.0
1/8	9048.765	860,763.83	1/8	9433.265	916,205.92	1/8	9825.76	973,979.0
1/4	9072.562	864,161.57	1/4	9457.562	919,747.95	1/4	9850.56	977,668.3
3/8	9096.390	867,568.25	3/8	9481.890	923,299.10	3/8	9875.39	981,366.9
1/2	9120.250	870,983.87	1/2	9506.250	926,859.37	1/2	9900.25	985,074.8
5/8	9144.140	874,408.44	5/8	9530.640	930,428.79	5/8	9925.14	988,792.1
3/4	9168.062	877,841.98	3/4	9555.062	934,007.35	3/4	9950.06	992,518.7
7/8	9192.015	881,284.49	7/8	9579.515	937,595.09	7/8	9975.01	996,254.6
						100	10,000.00	1,000,000.0

Table of Fractions of $\pi = 3.14159265$

a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$
1	3.14159	21	0.14960	41	0.07662	61	0.05150	81	0.03879
2	1.57080	22	0.14280	42	0.07480	62	0.05067	82	0.03831
3	1.04720	23	0.13659	43	0.07306	63	0.04987	83	0.03785
4	0.78540	24	0.13090	44	0.07140	64	0.04909	84	0.03740
5	0.62832	25	0.12566	45	0.06981	65	0.04833	85	0.03696
6	0.52360	26	0.12083	46	0.06830	66	0.04760	86	0.03653
7	0.44880	27	0.11636	47	0.06684	67	0.04689	87	0.03611
8	0.39270	28	0.11220	48	0.06545	68	0.04620	88	0.03570
9	0.34907	29	0.10833	49	0.06411	69	0.04553	89	0.03530
10	0.31416	30	0.10472	50	0.06283	70	0.04488	90	0.03491
11	0.28560	31	0.10134	51	0.06160	71	0.04425	91	0.03452
12	0.26180	32	0.09817	52	0.06042	72	0.04363	92	0.03415
13	0.24166	33	0.09520	53	0.05928	73	0.04304	93	0.03378
14	0.22440	34	0.09240	54	0.05818	74	0.04245	94	0.03342
15	0.20944	35	0.08976	55	0.05712	75	0.04189	95	0.03307
16	0.19635	36	0.08727	56	0.05610	76	0.04134	96	0.03272
17	0.18480	37	0.08491	57	0.05512	77	0.04080	97	0.03239
18	0.17453	38	0.08267	58	0.05417	78	0.04028	98	0.03206
19	0.16535	39	0.08055	59	0.05325	79	0.03977	99	0.03173
20	0.15708	40	0.07854	60	0.05236	80	0.03927	100	0.03142

Pi (π). — The ratio of the circumference of a circle to its diameter, which is represented by the Greek letter pi (π), is an incommensurable quantity. The value 3.1416 is accurate enough for ordinary purposes and the value $^{22}/_{7}$ is convenient for rough calculations. The fractions of π given in the above table will be found convenient in certain calculations and also the values in the table of constants on page 76.

Table of Decimal Equivalents, Squares, Cubes, Square Roots, Cube Roots and Logarithms of Fractions from 1/64 to 1, by 64ths

Fraction	Decimal Equivalent	Log.	Square	Log.	Cube	Log.	Sq. Root	Log.	Cube Root	Log.
1/64	0.015625	$\bar{2}.19382$	0.0002441	$\bar{4}.38764$	0.000003815	$\bar{6}.58146$	0.1250	$\bar{1}.09691$	0.2500	$\bar{1}.39794$
1/32	0.03125	$\bar{2}.49485$	0.0009765	$\bar{4}.98970$	0.00003052	$\bar{5}.48455$	0.1768	$\bar{1}.24743$	0.3150	$\bar{1}.49828$
3/64	0.046875	$\bar{2}.67094$	0.002197	$\bar{3}.34188$	0.0001030	$\bar{4}.01282$	0.2165	$\bar{1}.33547$	0.3606	$\bar{1}.55698$
1/16	0.0625	$\bar{2}.79588$	0.003906	$\bar{3}.59176$	0.0002442	$\bar{4}.38764$	0.2500	$\bar{1}.39794$	0.3968	$\bar{1}.59863$
5/64	0.078125	$\bar{2}.89279$	0.006104	$\bar{3}.78558$	0.0004768	$\bar{4}.67837$	0.2795	$\bar{1}.44639$	0.4275	$\bar{1}.63093$
3/32	0.09375	$\bar{2}.97197$	0.008789	$\bar{3}.94394$	0.0008240	$\bar{4}.91591$	0.3062	$\bar{1}.48598$	0.4543	$\bar{1}.65732$
7/64	0.109375	$\bar{1}.03892$	0.01196	$\bar{2}.07784$	0.001308	$\bar{3}.11676$	0.3307	$\bar{1}.51946$	0.4782	$\bar{1}.67964$
1/8	0.125	$\bar{1}.09691$	0.01563	$\bar{2}.19382$	0.001953	$\bar{3}.29073$	0.3535	$\bar{1}.54845$	0.5000	$\bar{1}.69897$
9/64	0.140625	$\bar{1}.14807$	0.01978	$\bar{2}.29614$	0.002781	$\bar{3}.44421$	0.3750	$\bar{1}.57403$	0.5200	$\bar{1}.71602$
5/32	0.15625	$\bar{1}.19382$	0.02441	$\bar{2}.38764$	0.003815	$\bar{3}.58146$	0.3953	$\bar{1}.59691$	0.5386	$\bar{1}.73127$
11/64	0.171875	$\bar{1}.23522$	0.02954	$\bar{2}.47044$	0.005078	$\bar{3}.70566$	0.4161	$\bar{1}.61761$	0.5560	$\bar{1}.74507$
3/16	0.1875	$\bar{1}.27300$	0.03516	$\bar{2}.54600$	0.006592	$\bar{3}.81900$	0.4330	$\bar{1}.63650$	0.5724	$\bar{1}.75767$
13/64	0.203125	$\bar{1}.30776$	0.04126	$\bar{2}.61553$	0.008381	$\bar{3}.92329$	0.4507	$\bar{1}.65388$	0.5878	$\bar{1}.76925$
7/32	0.21875	$\bar{1}.33995$	0.04786	$\bar{2}.67990$	0.01047	$\bar{2}.01985$	0.4677	$\bar{1}.66998$	0.6025	$\bar{1}.77998$
15/64	0.234375	$\bar{1}.36992$	0.05493	$\bar{2}.73984$	0.01287	$\bar{2}.10976$	0.4841	$\bar{1}.68496$	0.6166	$\bar{1}.78997$
1/4	0.250	$\bar{1}.39794$	0.06250	$\bar{2}.79588$	0.01562	$\bar{2}.19382$	0.5000	$\bar{1}.69897$	0.6300	$\bar{1}.79931$
17/64	0.265625	$\bar{1}.42427$	0.07056	$\bar{2}.84854$	0.01874	$\bar{2}.27281$	0.5154	$\bar{1}.71213$	0.6428	$\bar{1}.80809$
9/32	0.28125	$\bar{1}.44910$	0.07910	$\bar{2}.89820$	0.02225	$\bar{2}.34730$	0.5303	$\bar{1}.72455$	0.6552	$\bar{1}.81636$
19/64	0.296875	$\bar{1}.47258$	0.08813	$\bar{2}.94516$	0.02616	$\bar{2}.41774$	0.5449	$\bar{1}.73629$	0.6671	$\bar{1}.82419$
5/16	0.3125	$\bar{1}.49485$	0.09766	$\bar{2}.98970$	0.03052	$\bar{2}.48455$	0.5590	$\bar{1}.74742$	0.6786	$\bar{1}.83161$

Table of Decimal Equivalents, Squares, Cubes, Etc., of Fractions

Fraction	Decimal Equivalent	Log.	Square	Log.	Cube	Log.	Sq. Root	Log.	Cube Root	Log.
21/64	0.328125	$\bar{1}$.51604	0.1077	$\bar{1}$.03208	0.03533	$\bar{2}$.54812	0.5728	$\bar{1}$.75802	0.6897	$\bar{1}$.83868
11/32	0.34375	$\bar{1}$.53625	0.1182	$\bar{1}$.07250	0.04062	$\bar{2}$.60875	0.5863	$\bar{1}$.76812	0.7005	$\bar{1}$.84541
23/64	0.359375	$\bar{1}$.55555	0.1291	$\bar{1}$.11110	0.04641	$\bar{2}$.66665	0.5995	$\bar{1}$.77777	0.7110	$\bar{1}$.85185
3/8	0.375	$\bar{1}$.57403	0.1406	$\bar{1}$.14806	0.05273	$\bar{2}$.72209	0.6124	$\bar{1}$.78701	0.7211	$\bar{1}$.85801
25/64	0.390625	$\bar{1}$.59176	0.1526	$\bar{1}$.18352	0.05960	$\bar{2}$.77528	0.6250	$\bar{1}$.79588	0.7310	$\bar{1}$.86392
13/32	0.40625	$\bar{1}$.60879	0.1650	$\bar{1}$.21758	0.06705	$\bar{2}$.82637	0.6374	$\bar{1}$.80439	0.7406	$\bar{1}$.86959
27/64	0.421875	$\bar{1}$.62519	0.1780	$\bar{1}$.25037	0.07508	$\bar{2}$.87555	0.6495	$\bar{1}$.81259	0.7500	$\bar{1}$.87506
7/16	0.4375	$\bar{1}$.64098	0.1914	$\bar{1}$.28196	0.08374	$\bar{2}$.92294	0.6614	$\bar{1}$.82049	0.7592	$\bar{1}$.88032
29/64	0.453125	$\bar{1}$.65622	0.2053	$\bar{1}$.31244	0.09304	$\bar{2}$.96866	0.6732	$\bar{1}$.82811	0.7681	$\bar{1}$.88540
15/32	0.46875	$\bar{1}$.67094	0.2197	$\bar{1}$.34188	0.1030	$\bar{1}$.01282	0.6847	$\bar{1}$.83547	0.7768	$\bar{1}$.89031
31/64	0.484375	$\bar{1}$.68518	0.2346	$\bar{1}$.37036	0.1136	$\bar{1}$.05554	0.6960	$\bar{1}$.84259	0.7853	$\bar{1}$.89506
1/2	0.500	$\bar{1}$.69897	0.2500	$\bar{1}$.39794	0.1250	$\bar{1}$.09691	0.7071	$\bar{1}$.84948	0.7937	$\bar{1}$.89966
33/64	0.515625	$\bar{1}$.71233	0.2659	$\bar{1}$.42466	0.1371	$\bar{1}$.13699	0.7181	$\bar{1}$.85616	0.8019	$\bar{1}$.90411
17/32	0.53125	$\bar{1}$.72530	0.2822	$\bar{1}$.45060	0.1499	$\bar{1}$.17590	0.7289	$\bar{1}$.86265	0.8099	$\bar{1}$.90843
35/64	0.546875	$\bar{1}$.73789	0.2991	$\bar{1}$.47578	0.1636	$\bar{1}$.21367	0.7395	$\bar{1}$.86894	0.8178	$\bar{1}$.91263
9/16	0.5625	$\bar{1}$.75012	0.3164	$\bar{1}$.50024	0.1780	$\bar{1}$.25036	0.7500	$\bar{1}$.87506	0.8255	$\bar{1}$.91670
37/64	0.578125	$\bar{1}$.76202	0.3342	$\bar{1}$.52404	0.1932	$\bar{1}$.28606	0.7603	$\bar{1}$.88101	0.8331	$\bar{1}$.92067
19/32	0.59375	$\bar{1}$.77361	0.3525	$\bar{1}$.54722	0.2093	$\bar{1}$.32083	0.7706	$\bar{1}$.88680	0.8405	$\bar{1}$.92453
39/64	0.609375	$\bar{1}$.78488	0.3713	$\bar{1}$.56976	0.2263	$\bar{1}$.35464	0.7806	$\bar{1}$.89244	0.8478	$\bar{1}$.92829
5/8	0.625	$\bar{1}$.79588	0.3906	$\bar{1}$.59176	0.2441	$\bar{1}$.38764	0.7906	$\bar{1}$.89794	0.8550	$\bar{1}$.93196
41/64	0.640625	$\bar{1}$.80661	0.4104	$\bar{1}$.61322	0.2629	$\bar{1}$.41983	0.8004	$\bar{1}$.90330	0.8621	$\bar{1}$.93553
21/32	0.65625	$\bar{1}$.81707	0.4307	$\bar{1}$.63414	0.2826	$\bar{1}$.45121	0.8101	$\bar{1}$.90853	0.8690	$\bar{1}$.93902

Table of Decimal Equivalents, Squares, Cubes, Etc., of Fractions

Fraction	Decimal Equivalent	Log.	Square	Log.	Cube	Log.	Sq. Root	Log.	Cube Root	Log.
43/64	0.671875	$\bar{1}$.82729	0.4514	$\bar{1}$.65458	0.3033	$\bar{1}$.48187	0.8197	$\bar{1}$.91364	0.8758	$\bar{1}$.94243
11/16	0.6875	$\bar{1}$.83727	0.4727	$\bar{1}$.67454	0.3250	$\bar{1}$.51181	0.8292	$\bar{1}$.91864	0.8826	$\bar{1}$.94575
45/64	0.703125	$\bar{1}$.84704	0.4944	$\bar{1}$.69408	0.3476	$\bar{1}$.54112	0.8385	$\bar{1}$.92352	0.8892	$\bar{1}$.94901
23/32	0.71875	$\bar{1}$.85658	0.5166	$\bar{1}$.71316	0.3713	$\bar{1}$.56974	0.8478	$\bar{1}$.92829	0.8958	$\bar{1}$.95219
47/64	0.734375	$\bar{1}$.86592	0.5393	$\bar{1}$.73184	0.3961	$\bar{1}$.59776	0.8569	$\bar{1}$.93296	0.9022	$\bar{1}$.95530
3/4	0.750	$\bar{1}$.87506	0.5625	$\bar{1}$.75012	0.4219	$\bar{1}$.62518	0.8660	$\bar{1}$.93753	0.9086	$\bar{1}$.95835
49/64	0.765625	$\bar{1}$.88402	0.5862	$\bar{1}$.76804	0.4488	$\bar{1}$.65206	0.8750	$\bar{1}$.94201	0.9148	$\bar{1}$.96134
25/32	0.78125	$\bar{1}$.89279	0.6104	$\bar{1}$.78558	0.4768	$\bar{1}$.67837	0.8839	$\bar{1}$.94640	0.9210	$\bar{1}$.96426
51/64	0.796875	$\bar{1}$.90139	0.6350	$\bar{1}$.80278	0.5060	$\bar{1}$.70417	0.8927	$\bar{1}$.95069	0.9271	$\bar{1}$.96713
13/16	0.8125	$\bar{1}$.90982	0.6602	$\bar{1}$.81964	0.5364	$\bar{1}$.72946	0.9014	$\bar{1}$.95491	0.9331	$\bar{1}$.96994
53/64	0.828125	$\bar{1}$.91810	0.6858	$\bar{1}$.83620	0.5679	$\bar{1}$.75430	0.9100	$\bar{1}$.95905	0.9391	$\bar{1}$.97270
27/32	0.84375	$\bar{1}$.92622	0.7119	$\bar{1}$.85244	0.6007	$\bar{1}$.77866	0.9186	$\bar{1}$.96311	0.9449	$\bar{1}$.97540
55/64	0.859375	$\bar{1}$.93419	0.7385	$\bar{1}$.86838	0.6347	$\bar{1}$.80257	0.9270	$\bar{1}$.96709	0.9507	$\bar{1}$.97806
7/8	0.875	$\bar{1}$.94201	0.7656	$\bar{1}$.88402	0.6699	$\bar{1}$.82603	0.9354	$\bar{1}$.97101	0.9565	$\bar{1}$.98067
57/64	0.890625	$\bar{1}$.94969	0.7932	$\bar{1}$.89938	0.7064	$\bar{1}$.84907	0.9437	$\bar{1}$.97484	0.9621	$\bar{1}$.98323
29/32	0.90625	$\bar{1}$.95725	0.8213	$\bar{1}$.91450	0.7443	$\bar{1}$.87175	0.9520	$\bar{1}$.97862	0.9677	$\bar{1}$.98575
59/64	0.921875	$\bar{1}$.96467	0.8499	$\bar{1}$.92934	0.7835	$\bar{1}$.89401	0.9601	$\bar{1}$.98233	0.9732	$\bar{1}$.98822
15/16	0.9375	$\bar{1}$.97197	0.8789	$\bar{1}$.94394	0.8240	$\bar{1}$.91591	0.9682	$\bar{1}$.98598	0.9787	$\bar{1}$.99065
61/64	0.953125	$\bar{1}$.97915	0.9084	$\bar{1}$.95830	0.8659	$\bar{1}$.93745	0.9763	$\bar{1}$.98957	0.9841	$\bar{1}$.99305
31/32	0.96875	$\bar{1}$.98621	0.9385	$\bar{1}$.97242	0.9091	$\bar{1}$.95863	0.9843	$\bar{1}$.99310	0.9895	$\bar{1}$.99540
63/64	0.984375	$\bar{1}$.99316	0.9690	$\bar{1}$.98632	0.9539	$\bar{1}$.97948	0.9922	$\bar{1}$.99658	0.9948	$\bar{1}$.99772
1	1	0	1	0	1	0	1	0	1	0

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
1/64	0.0491	0.0002	2	6.2832	3.1416	5	15.7080	19.635
1/32	0.0982	0.0008	1/16	6.4795	3.3410	1/16	15.9043	20.129
1/16	0.1964	0.0031	1/8	6.6759	3.5466	1/8	16.1007	20.629
3/32	0.2945	0.0069	3/16	6.8722	3.7583	3/16	16.2970	21.135
1/8	0.3927	0.0123	1/4	7.0686	3.9761	1/4	16.4934	21.648
5/32	0.4909	0.0192	5/16	7.2649	4.2000	5/16	16.6897	22.166
3/16	0.5890	0.0276	3/8	7.4613	4.4301	3/8	16.8861	22.691
7/32	0.6872	0.0376	7/16	7.6576	4.6664	7/16	17.0824	23.221
1/4	0.7854	0.0491	1/2	7.8540	4.9087	1/2	17.2788	23.758
9/32	0.8836	0.0621	9/16	8.0503	5.1572	9/16	17.4751	24.301
5/16	0.9817	0.0767	5/8	8.2467	5.4119	5/8	17.6715	24.850
11/32	1.0799	0.0928	11/16	8.4430	5.6727	11/16	17.8678	25.406
3/8	1.1781	0.1105	3/4	8.6394	5.9396	3/4	18.0642	25.967
13/32	1.2763	0.1296	13/16	8.8357	6.2126	13/16	18.2605	26.535
7/16	1.3745	0.1503	7/8	9.0321	6.4918	7/8	18.4569	27.109
15/32	1.4726	0.1726	15/16	9.2284	6.7771	15/16	18.6532	27.688
1/2	1.5708	0.1964	3	9.4248	7.0686	6	18.8496	28.274
17/32	1.6690	0.2217	1/16	9.6211	7.3662	1/8	19.2423	29.465
9/16	1.7672	0.2485	1/8	9.8175	7.6699	1/4	19.6350	30.680
19/32	1.8653	0.2769	3/16	10.0138	7.9798	3/8	20.0277	31.919
5/8	1.9635	0.3068	1/4	10.2102	8.2958	1/2	20.4204	33.183
21/32	2.0617	0.3382	5/16	10.4065	8.6179	5/8	20.8131	34.472
11/16	2.1598	0.3712	3/8	10.6029	8.9462	3/4	21.2058	35.785
23/32	2.2580	0.4057	7/16	10.7992	9.2806	7/8	21.5984	37.122
3/4	2.3562	0.4418	1/2	10.9956	9.6211	7	21.9911	38.485
25/32	2.4544	0.4794	9/16	11.1919	9.9678	1/8	22.3838	39.871
13/16	2.5525	0.5185	5/8	11.3883	10.321	1/4	22.7765	41.282
27/32	2.6507	0.5591	11/16	11.5846	10.680	3/8	23.1692	42.718
7/8	2.7489	0.6013	3/4	11.7810	11.045	1/2	23.5619	44.179
29/32	2.8471	0.6450	13/16	11.9773	11.416	5/8	23.9546	45.664
15/16	2.9452	0.6903	7/8	12.1737	11.793	3/4	24.3473	47.173
31/32	3.0434	0.7371	15/16	12.3700	12.177	7/8	24.7400	48.707
1	3.1416	0.7854	4	12.5664	12.566	8	25.1327	50.265
1/16	3.3379	0.8866	1/16	12.7627	12.962	1/8	25.5254	51.849
1/8	3.5343	0.9940	1/8	12.9591	13.364	1/4	25.9181	53.456
3/16	3.7306	1.1075	3/16	13.1554	13.772	3/8	26.3108	55.088
1/4	3.9270	1.2272	1/4	13.3518	14.186	1/2	26.7035	56.745
5/16	4.1233	1.3530	5/16	13.5481	14.607	5/8	27.0962	58.426
3/8	4.3197	1.4849	3/8	13.7445	15.033	3/4	27.4889	60.132
7/16	4.5160	1.6230	7/16	13.9408	15.466	7/8	27.8816	61.862
1/2	4.7124	1.7671	1/2	14.1372	15.904	9	28.2743	63.617
9/16	4.9087	1.9175	9/16	14.3335	16.349	1/8	28.6670	65.397
5/8	5.1051	2.0739	5/8	14.5299	16.800	1/4	29.0597	67.201
11/16	5.3014	2.2365	11/16	14.7262	17.257	3/8	29.4524	69.029
3/4	5.4978	2.4053	3/4	14.9226	17.721	1/2	29.8451	70.882
13/16	5.6941	2.5802	13/16	15.1189	18.190	5/8	30.2378	72.760
7/8	5.8905	2.7612	7/8	15.3153	18.665	3/4	30.6305	74.662
15/16	6.0868	2.9483	15/16	15.5116	19.147	7/8	31.0232	76.589

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
10	31.4159	78.540	16	50.2655	201.06	22	69.1150	380.13
⅛	31.8086	80.516	⅛	50.6582	204.22	⅛	69.5077	384.46
¼	32.2013	82.516	¼	51.0509	207.39	¼	69.9004	388.82
⅜	32.5940	84.541	⅜	51.4436	210.60	⅜	70.2931	393.20
½	32.9867	86.590	½	51.8363	213.82	½	70.6858	397.61
⅝	33.3794	88.664	⅝	52.2290	217.08	⅝	71.0785	402.04
¾	33.7721	90.763	¾	52.6217	220.35	¾	71.4712	406.49
⅞	34.1648	92.886	⅞	53.0144	223.65	⅞	71.8639	410.97
11	34.5575	95.033	17	53.4071	226.98	23	72.2566	415.48
⅛	34.9502	97.205	⅛	53.7998	230.33	⅛	72.6493	420.00
¼	35.3429	99.402	¼	54.1925	233.71	¼	73.0420	424.56
⅜	35.7356	101.62	⅜	54.5852	237.10	⅜	73.4347	429.13
½	36.1283	103.87	½	54.9779	240.53	½	73.8274	433.74
⅝	36.5210	106.14	⅝	55.3706	243.98	⅝	74.2201	438.36
¾	36.9137	108.43	¾	55.7633	247.45	¾	74.6128	443.01
⅞	37.3064	110.75	⅞	56.1560	250.95	⅞	75.0055	447.69
12	37.6991	113.10	18	56.5487	254.47	24	75.3982	452.39
⅛	38.0918	115.47	⅛	56.9414	258.02	⅛	75.7909	457.11
¼	38.4845	117.86	¼	57.3341	261.59	¼	76.1836	461.86
⅜	38.8772	120.28	⅜	57.7268	265.18	⅜	76.5763	466.64
½	39.2699	122.72	½	58.1195	268.80	½	76.9690	471.44
⅝	39.6626	125.19	⅝	58.5122	272.45	⅝	77.3617	476.26
¾	40.0553	127.68	¾	58.9049	276.12	¾	77.7544	481.11
⅞	40.4480	130.19	⅞	59.2976	279.81	⅞	78.1471	485.98
13	40.8407	132.73	19	59.6903	283.53	25	78.5398	490.87
⅛	41.2334	135.30	⅛	60.0830	287.27	⅛	78.9325	495.79
¼	41.6261	137.89	¼	60.4757	291.04	¼	79.3252	500.74
⅜	42.0188	140.50	⅜	60.8684	294.83	⅜	79.7179	505.71
½	42.4115	143.14	½	61.2611	298.65	½	80.1106	510.71
⅝	42.8042	145.80	⅝	61.6538	302.49	⅝	80.5033	515.72
¾	43.1969	148.49	¾	62.0465	306.35	¾	80.8960	520.77
⅞	43.5896	151.20	⅞	62.4392	310.24	⅞	81.2887	525.84
14	43.9823	153.94	20	62.8319	314.16	26	81.6814	530.93
⅛	44.3750	156.70	⅛	63.2246	318.10	⅛	82.0741	536.05
¼	44.7677	159.48	¼	63.6173	322.06	¼	82.4668	541.19
⅜	45.1604	162.30	⅜	64.0100	326.05	⅜	82.8595	546.35
½	45.5531	165.13	½	64.4026	330.06	½	83.2522	551.55
⅝	45.9458	167.99	⅝	64.7953	334.10	⅝	83.6449	556.76
¾	46.3385	170.87	¾	65.1880	338.16	¾	84.0376	562.00
⅞	46.7312	173.78	⅞	65.5807	342.25	⅞	84.4303	567.27
15	47.1239	176.71	21	65.9734	346.36	27	84.8230	572.56
⅛	47.5166	179.67	⅛	66.3661	350.50	⅛	85.2157	577.87
¼	47.9093	182.65	¼	66.7588	354.66	¼	85.6084	583.21
⅜	48.3020	185.66	⅜	67.1515	358.84	⅜	86.0011	588.57
½	48.6947	188.69	½	67.5442	363.05	½	86.3938	593.96
⅝	49.0874	191.75	⅝	67.9369	367.28	⅝	86.7865	599.37
¾	49.4801	194.83	¾	68.3296	371.54	¾	87.1792	604.81
⅞	49.8728	197.93	⅞	68.7223	375.83	⅞	87.5719	610.27

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
28	87.9646	615.75	34	106.814	907.92	40	125.664	1256.6
⅛	88.3573	621.26	⅛	107.207	914.61	⅛	126.056	1264.5
¼	88.7500	626.80	¼	107.600	921.32	¼	126.449	1272.4
⅜	89.1427	632.36	⅜	107.992	928.06	⅜	126.842	1280.3
½	89.5354	637.94	½	108.385	934.82	½	127.235	1288.2
⅝	89.9281	643.55	⅝	108.778	941.61	⅝	127.627	1296.2
¾	90.3208	649.18	¾	109.170	948.42	¾	128.020	1304.2
⅞	90.7135	654.84	⅞	109.563	955.25	⅞	128.413	1312.2
29	91.1062	660.52	35	109.956	962.11	41	128.805	1320.3
⅛	91.4989	666.23	⅛	110.348	969.00	⅛	129.198	1328.3
¼	91.8916	671.96	¼	110.741	975.91	¼	129.591	1336.4
⅜	92.2843	677.71	⅜	111.134	982.84	⅜	129.983	1344.5
½	92.6770	683.49	½	111.527	989.80	½	130.376	1352.7
⅝	93.0697	689.30	⅝	111.919	996.78	⅝	130.769	1360.8
¾	93.4624	695.13	¾	112.312	1003.8	¾	131.161	1369.0
⅞	93.8551	700.98	⅞	112.705	1010.8	⅞	131.554	1377.2
30	94.2478	706.86	36	113.097	1017.9	42	131.947	1385.4
⅛	94.6405	712.76	⅛	113.490	1025.0	⅛	132.340	1393.7
¼	95.0332	718.69	¼	113.883	1032.1	¼	132.732	1402.0
⅜	95.4259	724.64	⅜	114.275	1039.2	⅜	133.125	1410.3
½	95.8186	730.62	½	114.668	1046.3	½	133.518	1418.6
⅝	96.2113	736.62	⅝	115.061	1053.5	⅝	133.910	1427.0
¾	96.6040	742.64	¾	115.454	1060.7	¾	134.303	1435.4
⅞	96.9967	748.69	⅞	115.846	1068.0	⅞	134.696	1443.8
31	97.3894	754.77	37	116.239	1075.2	43	135.088	1452.2
⅛	97.7821	760.87	⅛	116.632	1082.5	⅛	135.481	1460.7
¼	98.1748	766.99	¼	117.024	1089.8	¼	135.874	1469.1
⅜	98.5675	773.14	⅜	117.417	1097.1	⅜	136.267	1477.6
½	98.9602	779.31	½	117.810	1104.5	½	136.659	1486.2
⅝	99.3529	785.51	⅝	118.202	1111.8	⅝	137.052	1494.7
¾	99.7456	791.73	¾	118.596	1119.2	¾	137.445	1503.3
⅞	100.138	797.98	⅞	118.988	1126.7	⅞	137.837	1511.9
32	100.531	804.25	38	119.381	1134.1	44	138.230	1520.5
⅛	100.924	810.54	⅛	119.773	1141.6	⅛	138.623	1529.2
¼	101.316	816.86	¼	120.166	1149.1	¼	139.015	1537.9
⅜	101.709	823.21	⅜	120.559	1156.6	⅜	139.408	1546.6
½	102.102	829.58	½	120.951	1164.2	½	139.801	1555.3
⅝	102.494	835.97	⅝	121.344	1171.7	⅝	140.194	1564.0
¾	102.887	842.39	¾	121.737	1179.3	¾	140.586	1572.8
⅞	103.280	848.83	⅞	122.129	1186.9	⅞	140.979	1581.6
33	103.673	855.30	39	122.522	1194.6	45	141.372	1590.4
⅛	104.065	861.79	⅛	122.915	1202.3	⅛	141.764	1599.3
¼	104.458	868.31	¼	123.308	1210.0	¼	142.157	1608.2
⅜	104.851	874.85	⅜	123.700	1217.7	⅜	142.550	1617.0
½	105.243	881.41	½	124.093	1225.4	½	142.942	1626.0
⅝	105.636	888.00	⅝	124.486	1233.2	⅝	143.335	1634.9
¾	106.029	894.62	¾	124.878	1241.0	¾	143.728	1643.9
⅞	106.421	901.26	⅞	125.271	1248.8	⅞	144.121	1652.9

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
46	144.513	1661.9	52	163.363	2123.7	58	182.212	2642.1
⅛	144.906	1670.9	⅛	163.756	2133.9	⅛	182.605	2653.5
¼	145.299	1680.0	¼	164.148	2144.2	¼	182.998	2664.9
⅜	145.691	1689.1	⅜	164.541	2154.5	⅜	183.390	2676.4
½	146.084	1698.2	½	164.934	2164.8	½	183.783	2687.8
⅝	146.477	1707.4	⅝	165.326	2175.1	⅝	184.176	2699.3
¾	146.869	1716.5	¾	165.719	2185.4	¾	184.569	2710.9
⅞	147.262	1725.7	⅞	166.112	2195.8	⅞	184.961	2722.4
47	147.655	1734.9	53	166.504	2206.2	59	185.354	2734.0
⅛	148.048	1744.2	⅛	166.897	2216.6	⅛	185.747	2745.6
¼	148.440	1753.5	¼	167.290	2227.0	¼	186.139	2757.2
⅜	148.833	1762.7	⅜	167.683	2237.5	⅜	186.532	2768.8
½	149.226	1772.1	½	168.075	2248.0	½	186.925	2780.5
⅝	149.618	1781.4	⅝	168.468	2258.5	⅝	187.317	2792.2
¾	150.011	1790.8	¾	168.861	2269.1	¾	187.710	2803.9
⅞	150.404	1800.1	⅞	169.253	2279.6	⅞	188.103	2815.7
48	150.796	1809.6	54	169.646	2290.2	60	188.496	2827.4
⅛	151.189	1819.0	⅛	170.039	2300.8	⅛	188.888	2839.2
¼	151.582	1828.5	¼	170.431	2311.5	¼	189.281	2851.0
⅜	151.975	1837.9	⅜	170.824	2322.1	⅜	189.674	2862.9
½	152.367	1847.5	½	171.217	2332.8	½	190.066	2874.8
⅝	152.760	1857.0	⅝	171.609	2343.5	⅝	190.459	2886.6
¾	153.153	1866.5	¾	172.002	2354.3	¾	190.852	2898.6
⅞	153.545	1876.1	⅞	172.395	2365.0	⅞	191.244	2910.5
49	153.938	1885.7	55	172.788	2375.8	61	191.637	2922.5
⅛	154.331	1895.4	⅛	173.180	2386.6	⅛	192.030	2934.5
¼	154.723	1905.0	¼	173.573	2397.5	¼	192.423	2946.5
⅜	155.116	1914.7	⅜	173.966	2408.3	⅜	192.815	2958.5
½	155.509	1924.4	½	174.358	2419.2	½	193.208	2970.6
⅝	155.902	1934.2	⅝	174.751	2430.1	⅝	193.601	2982.7
¾	156.294	1943.9	¾	175.144	2441.1	¾	193.993	2994.8
⅞	156.687	1953.7	⅞	175.536	2452.0	⅞	194.386	3006.9
50	157.080	1963.5	56	175.929	2463.0	62	194.779	3019.1
⅛	157.472	1973.3	⅛	176.322	2474.0	⅛	195.171	3031.3
¼	157.865	1983.2	¼	176.715	2485.0	¼	195.564	3043.5
⅜	158.258	1993.1	⅜	177.107	2496.1	⅜	195.957	3055.7
½	158.650	2003.0	½	177.500	2507.2	½	196.350	3068.0
⅝	159.043	2012.9	⅝	177.893	2518.3	⅝	196.742	3080.3
¾	159.436	2022.8	¾	178.285	2529.4	¾	197.135	3092.6
⅞	159.829	2032.8	⅞	178.678	2540.6	⅞	197.528	3104.9
51	160.221	2042.8	57	179.071	2551.8	63	197.920	3117.2
⅛	160.614	2052.8	⅛	179.463	2563.0	⅛	198.313	3129.6
¼	161.007	2062.9	¼	179.856	2574.2	¼	198.706	3142.0
⅜	161.399	2073.0	⅜	180.249	2585.4	⅜	199.098	3154.5
½	161.792	2083.1	½	180.642	2596.7	½	199.491	3166.9
⅝	162.185	2093.2	⅝	181.034	2608.0	⅝	199.884	3179.4
¾	162.577	2103.3	¾	181.427	2619.4	¾	200.277	3191.9
⅞	162.970	2113.5	⅞	181.820	2630.7	⅞	200.669	3204.4

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
64	201.062	3217.0	70	219.911	3848.5	76	238.761	4536.5
⅛	201.455	3229.6	⅛	220.304	3862.2	⅛	239.154	4551.4
¼	201.847	3242.2	¼	220.697	3876.0	¼	239.546	4566.4
⅜	202.240	3254.8	⅜	221.090	3889.8	⅜	239.939	4581.3
½	202.633	3267.5	½	221.482	3903.6	½	240.332	4596.3
⅝	203.025	3280.1	⅝	221.875	3917.5	⅝	240.725	4611.4
¾	203.418	3292.8	¾	222.268	3931.4	¾	241.117	4626.4
⅞	203.811	3305.6	⅞	222.660	3945.3	⅞	241.510	4641.5
65	204.204	3318.3	71	223.053	3959.2	77	241.903	4656.6
⅛	204.596	3331.1	⅛	223.446	3973.1	⅛	242.295	4671.8
¼	204.989	3343.9	¼	223.838	3987.1	¼	242.688	4686.9
⅜	205.382	3356.7	⅜	224.231	4001.1	⅜	243.081	4702.1
½	205.774	3369.6	½	224.624	4015.2	½	243.473	4717.3
⅝	206.167	3382.4	⅝	225.017	4029.2	⅝	243.866	4732.5
¾	206.560	3395.3	¾	225.409	4043.3	¾	244.259	4747.8
⅞	206.952	3408.2	⅞	225.802	4057.4	⅞	244.652	4763.1
66	207.345	3421.2	72	226.195	4071.5	78	245.044	4778.4
⅛	207.738	3434.2	⅛	226.587	4085.7	⅛	245.437	4793.7
¼	208.131	3447.2	¼	226.980	4099.8	¼	245.830	4809.0
⅜	208.523	3460.2	⅜	227.373	4114.0	⅜	246.222	4824.4
½	208.916	3473.2	½	227.765	4128.2	½	246.615	4839.8
⅝	209.309	3486.3	⅝	228.158	4142.5	⅝	247.008	4855.2
¾	209.701	3499.4	¾	228.551	4156.8	¾	247.400	4870.7
⅞	210.094	3512.5	⅞	228.944	4171.1	⅞	247.793	4886.2
67	210.487	3525.7	73	229.336	4185.4	79	248.186	4901.7
⅛	210.879	3538.8	⅛	229.729	4199.7	⅛	248.579	4917.2
¼	211.272	3552.0	¼	230.122	4214.1	¼	248.971	4932.7
⅜	211.665	3565.2	⅜	230.514	4228.5	⅜	249.364	4948.3
½	212.058	3578.5	½	230.907	4242.9	½	249.757	4963.9
⅝	212.450	3591.7	⅝	231.300	4257.4	⅝	250.149	4979.5
¾	212.843	3605.0	¾	231.692	4271.8	¾	250.542	4995.2
⅞	213.236	3618.3	⅞	232.085	4286.3	⅞	250.935	5010.9
68	213.628	3631.7	74	232.478	4300.8	80	251.327	5026.5
⅛	214.021	3645.0	⅛	232.871	4315.4	⅛	251.720	5042.3
¼	214.414	3658.4	¼	233.263	4329.9	¼	252.113	5058.0
⅜	214.806	3671.8	⅜	233.656	4344.5	⅜	252.506	5073.8
½	215.199	3685.3	½	234.049	4359.2	½	252.898	5089.6
⅝	215.592	3698.7	⅝	234.441	4373.8	⅝	253.291	5105.4
¾	215.984	3712.2	¾	234.834	4388.5	¾	253.684	5121.2
⅞	216.377	3725.7	⅞	235.227	4403.1	⅞	254.076	5137.1
69	216.770	3739.3	75	235.619	4417.9	81	254.469	5153.0
⅛	217.163	3752.8	⅛	236.012	4432.6	⅛	254.862	5168.9
¼	217.555	3766.4	¼	236.405	4447.4	¼	255.254	5184.9
⅜	217.948	3780.0	⅜	236.798	4462.2	⅜	255.647	5200.8
½	218.341	3793.7	½	237.190	4477.0	½	256.040	5216.8
⅝	218.733	3807.3	⅝	237.583	4491.8	⅝	256.433	5232.8
¾	219.126	3821.0	¾	237.976	4506.7	¾	256.825	5248.9
⅞	219.519	3834.7	⅞	238.368	4521.5	⅞	257.218	5264.9

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
82	257.611	5281.0	88	276.460	6082.1	94	295.310	6939.8
1/8	258.003	5297.1	1/8	276.853	6099.4	1/8	295.702	6958.2
1/4	258.396	5313.3	1/4	277.246	6116.7	1/4	296.095	6976.7
3/8	258.789	5329.4	3/8	277.638	6134.1	3/8	296.488	6995.3
1/2	259.181	5345.6	1/2	278.031	6151.4	1/2	296.881	7013.8
5/8	259.574	5361.8	5/8	278.424	6168.8	5/8	297.273	7032.4
3/4	259.967	5378.1	3/4	278.816	6186.2	3/4	297.666	7051.0
7/8	260.359	5394.3	7/8	279.209	6203.7	7/8	298.059	7069.6
83	260.752	5410.6	89	279.602	6221.1	95	298.451	7088.2
1/8	261.145	5426.9	1/8	279.994	6238.6	1/8	298.844	7106.9
1/4	261.538	5443.3	1/4	280.387	6256.1	1/4	299.237	7125.6
3/8	261.930	5459.6	3/8	280.780	6273.7	3/8	299.629	7144.3
1/2	262.323	5476.0	1/2	281.173	6291.2	1/2	300.022	7163.0
5/8	262.716	5492.4	5/8	281.565	6308.8	5/8	300.415	7181.8
3/4	263.108	5508.8	3/4	281.958	6326.4	3/4	300.807	7200.6
7/8	263.501	5525.3	7/8	282.351	6344.1	7/8	301.200	7219.4
84	263.894	5541.8	90	282.743	6361.7	96	301.593	7238.2
1/8	264.286	5558.3	1/8	283.136	6379.4	1/8	301.986	7257.1
1/4	264.679	5574.8	1/4	283.529	6397.1	1/4	302.378	7276.0
3/8	265.072	5591.4	3/8	283.921	6414.9	3/8	302.771	7294.9
1/2	265.465	5607.9	1/2	284.314	6432.6	1/2	303.164	7313.8
5/8	265.857	5624.5	5/8	284.707	6450.4	5/8	303.556	7332.8
3/4	266.250	5641.2	3/4	285.100	6468.2	3/4	303.949	7351.8
7/8	266.643	5657.8	7/8	285.492	6486.0	7/8	304.342	7370.8
85	267.035	5674.5	91	285.885	6503.9	97	304.734	7389.8
1/8	267.428	5691.2	1/8	286.278	6521.8	1/8	305.127	7408.9
1/4	267.821	5707.9	1/4	286.670	6539.7	1/4	305.520	7428.0
3/8	268.213	5724.7	3/8	287.063	6557.6	3/8	305.913	7447.1
1/2	268.606	5741.5	1/2	287.456	6575.5	1/2	306.305	7466.2
5/8	268.999	5758.3	5/8	287.848	6593.5	5/8	306.698	7485.3
3/4	269.392	5775.1	3/4	288.241	6611.5	3/4	307.091	7504.5
7/8	269.784	5791.9	7/8	288.634	6629.6	7/8	307.483	7523.7
86	270.177	5808.8	92	289.027	6647.6	98	307.876	7543.0
1/8	270.570	5825.7	1/8	289.419	6665.7	1/8	308.269	7562.2
1/4	270.962	5842.6	1/4	289.812	6683.8	1/4	308.661	7581.5
3/8	271.355	5859.6	3/8	290.205	6701.9	3/8	309.054	7600.8
1/2	271.748	5876.5	1/2	290.597	6720.1	1/2	309.447	7620.1
5/8	272.140	5893.5	5/8	290.990	6738.2	5/8	309.840	7639.5
3/4	272.533	5910.6	3/4	291.383	6756.4	3/4	310.232	7658.9
7/8	272.926	5927.6	7/8	291.775	6774.7	7/8	310.625	7678.3
87	273.319	5944.7	93	292.168	6792.9	99	311.018	7697.7
1/8	273.711	5961.8	1/8	292.561	6811.2	1/8	311.410	7717.1
1/4	274.104	5978.9	1/4	292.954	6829.5	1/4	311.803	7736.6
3/8	274.497	5996.0	3/8	293.346	6847.8	3/8	312.196	7756.1
1/2	274.889	6013.2	1/2	293.739	6866.1	1/2	312.588	7775.6
5/8	275.282	6030.4	5/8	294.132	6884.5	5/8	312.981	7795.2
3/4	275.675	6047.6	3/4	294.524	6902.9	3/4	313.374	7814.8
7/8	276.067	6064.9	7/8	294.917	6921.3	7/8	313.767	7834.4

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
100	314.16	7,854.0	150	471.24	17,671.5	200	628.32	31,415.9
101	317.30	8,011.8	151	474.38	17,907.9	201	631.46	31,730.9
102	320.44	8,171.3	152	477.52	18,145.8	202	634.60	32,047.4
103	323.58	8,332.3	153	480.66	18,385.4	203	637.74	32,365.5
104	326.73	8,494.9	154	483.81	18,626.5	204	640.88	32,685.1
105	329.87	8,659.0	155	486.95	18,869.2	205	644.03	33,006.4
106	333.01	8,824.7	156	490.09	19,113.4	206	647.17	33,329.2
107	336.15	8,992.0	157	493.23	19,359.3	207	650.31	33,653.5
108	339.29	9,160.9	158	496.37	19,606.7	208	653.45	33,979.5
109	342.43	9,331.3	159	499.51	19,855.7	209	656.59	34,307.0
110	345.58	9,503.3	160	502.65	20,106.2	210	659.73	34,636.1
111	348.72	9,676.9	161	505.80	20,358.3	211	662.88	34,966.7
112	351.86	9,852.0	162	508.94	20,612.0	212	666.02	35,298.9
113	355.00	10,028.7	163	512.08	20,867.2	213	669.16	35,632.7
114	358.14	10,207.0	164	515.22	21,124.1	214	672.30	35,968.1
115	361.28	10,386.9	165	518.36	21,382.5	215	675.44	36,305.0
116	364.42	10,568.3	166	521.50	21,642.4	216	678.58	36,643.5
117	367.57	10,751.3	167	524.65	21,904.0	217	681.73	36,983.6
118	370.71	10,935.9	168	527.79	22,167.1	218	684.87	37,325.3
119	373.85	11,122.0	169	530.93	22,431.8	219	688.01	37,668.5
120	376.99	11,309.7	170	534.07	22,698.0	220	691.15	38,013.3
121	380.13	11,499.0	171	537.21	22,965.8	221	694.29	38,359.6
122	383.27	11,689.9	172	540.35	23,235.2	222	697.43	38,707.6
123	386.42	11,882.3	173	543.50	23,506.2	223	700.58	39,057.1
124	389.56	12,076.3	174	546.64	23,778.7	224	703.72	39,408.1
125	392.70	12,271.8	175	549.78	24,052.8	225	706.86	39,760.8
126	395.84	12,469.0	176	552.92	24,328.5	226	710.00	40,115.0
127	398.98	12,667.7	177	556.06	24,605.7	227	713.14	40,470.8
128	402.12	12,868.0	178	559.20	24,884.6	228	716.28	40,828.1
129	405.27	13,069.8	179	562.35	25,164.9	229	719.42	41,187.1
130	408.41	13,273.2	180	565.49	25,446.9	230	722.57	41,547.6
131	411.55	13,478.2	181	568.63	25,730.4	231	725.71	41,909.6
132	414.69	13,684.8	182	571.77	26,015.5	232	728.85	42,273.3
133	417.83	13,892.9	183	574.91	26,302.2	233	731.99	42,638.5
134	420.97	14,102.6	184	578.05	26,590.4	234	735.13	43,005.3
135	424.12	14,313.9	185	581.19	26,880.3	235	738.27	43,373.6
136	427.26	14,526.7	186	584.34	27,171.6	236	741.42	43,743.5
137	430.40	14,741.1	187	587.48	27,464.6	237	744.56	44,115.0
138	433.54	14,957.1	188	590.62	27,759.1	238	747.70	44,488.1
139	436.68	15,174.7	189	593.76	28,055.2	239	750.84	44,862.7
140	439.82	15,393.8	190	596.90	28,352.9	240	753.98	45,238.9
141	442.96	15,614.5	191	600.04	28,652.1	241	757.12	45,616.7
142	446.11	15,836.8	192	603.19	28,952.9	242	760.27	45,996.1
143	449.25	16,060.6	193	606.33	29,255.3	243	763.41	46,377.0
144	452.39	16,286.0	194	609.47	29,559.2	244	766.55	46,759.5
145	455.53	16,513.0	195	612.61	29,864.8	245	769.69	47,143.5
146	458.67	16,741.5	196	615.75	30,171.9	246	772.83	47,529.2
147	461.81	16,971.7	197	618.89	30,480.5	247	775.97	47,916.4
148	464.96	17,203.4	198	622.04	30,790.7	248	779.11	48,305.1
149	468.10	17,436.6	199	625.18	31,102.6	249	782.26	48,695.5

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
250	785.40	49,087.4	300	942.48	70,685.8	350	1099.56	96,211.3
251	788.54	49,480.9	301	945.62	71,157.9	351	1102.70	96,761.8
252	791.68	49,875.9	302	948.76	71,631.5	352	1105.84	97,314.0
253	794.82	50,272.6	303	951.90	72,106.6	353	1108.98	97,867.7
254	797.96	50,670.7	304	955.04	72,583.4	354	1112.12	98,423.0
255	801.11	51,070.5	305	958.19	73,061.7	355	1115.27	98,979.8
256	804.25	51,471.9	306	961.33	73,541.5	356	1118.41	99,538.2
257	807.39	51,874.8	307	964.47	74,023.0	357	1121.55	100,098
258	810.53	52,279.2	308	967.61	74,506.0	358	1124.69	100,660
259	813.67	52,685.3	309	970.75	74,990.6	359	1127.83	101,223
260	816.81	53,092.9	310	973.89	75,476.8	360	1130.97	101,788
261	819.96	53,502.1	311	977.04	75,964.5	361	1134.11	102,354
262	823.10	53,912.9	312	980.18	76,453.8	362	1137.26	102,922
263	826.24	54,325.2	313	983.32	76,944.7	363	1140.40	103,491
264	829.38	54,739.1	314	986.46	77,437.1	364	1143.54	104,062
265	832.52	55,154.6	315	989.60	77,931.1	365	1146.68	104,635
266	835.66	55,571.6	316	992.74	78,426.7	366	1149.82	105,209
267	838.81	55,990.2	317	995.88	78,923.9	367	1152.96	105,784
268	841.95	56,410.4	318	999.03	79,422.6	368	1156.11	106,362
269	845.09	56,832.2	319	1002.17	79,922.9	369	1159.25	106,941
270	848.23	57,255.5	320	1005.31	80,424.8	370	1162.39	107,521
271	851.37	57,680.4	321	1008.45	80,928.2	371	1165.53	108,103
272	854.51	58,106.9	322	1011.59	81,433.2	372	1168.67	108,687
273	857.65	58,534.9	323	1014.73	81,939.8	373	1171.81	109,272
274	860.80	58,964.6	324	1017.88	82,448.0	374	1174.96	109,858
275	863.94	59,395.7	325	1021.02	82,957.7	375	1178.10	110,447
276	867.08	59,828.5	326	1024.16	83,469.0	376	1181.24	111,036
277	870.22	60,262.8	327	1027.30	83,981.8	377	1184.38	111,628
278	873.36	60,698.7	328	1030.44	84,496.3	378	1187.52	112,221
279	876.50	61,136.2	329	1033.58	85,012.3	379	1190.66	112,815
280	879.65	61,575.2	330	1036.73	85,529.9	380	1193.81	113,411
281	882.79	62,015.8	331	1039.87	86,049.0	381	1196.95	114,009
282	885.93	62,458.0	332	1043.01	86,569.7	382	1200.09	114,608
283	889.07	62,901.8	333	1046.15	87,092.0	383	1203.23	115,209
284	892.21	63,347.1	334	1049.29	87,615.9	384	1206.37	115,812
285	895.35	63,794.0	335	1052.43	88,141.3	385	1209.51	116,416
286	898.50	64,242.4	336	1055.58	88,668.3	386	1212.65	117,021
287	901.64	64,692.5	337	1058.72	89,196.9	387	1215.80	117,628
288	904.78	65,144.1	338	1061.86	89,727.0	388	1218.94	118,237
289	907.92	65,597.2	339	1065.00	90,258.7	389	1222.08	118,847
290	911.06	66,052.0	340	1068.14	90,792.0	390	1225.22	119,459
291	914.20	66,508.3	341	1071.28	91,326.9	391	1228.36	120,072
292	917.35	66,966.2	342	1074.42	91,863.3	392	1231.50	120,687
293	920.49	67,425.6	343	1077.57	92,401.3	393	1234.65	121,304
294	923.63	67,886.7	344	1080.71	92,940.9	394	1237.79	121,922
295	926.77	68,349.3	345	1083.85	93,482.0	395	1240.93	122,542
296	929.91	68,813.4	346	1086.99	94,024.7	396	1244.07	123,163
297	933.05	69,279.2	347	1090.13	94,569.0	397	1247.21	123,786
298	936.19	69,746.5	348	1093.27	95,114.9	398	1250.35	124,410
299	939.34	70,215.4	349	1096.42	95,662.3	399	1253.50	125,036

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
400	1256.64	125,664	450	1413.72	159,043	500	1570.80	196,350
401	1259.78	126,293	451	1416.86	159,751	501	1573.94	197,136
402	1262.92	126,923	452	1420.00	160,460	502	1577.08	197,923
403	1266.06	127,556	453	1423.14	161,171	503	1580.22	198,713
404	1269.20	128,190	454	1426.28	161,883	504	1583.36	199,504
405	1272.35	128,825	455	1429.42	162,597	505	1586.50	200,296
406	1275.49	129,462	456	1432.57	163,313	506	1589.65	201,090
407	1278.63	130,100	457	1435.71	164,030	507	1592.79	201,886
408	1281.77	130,741	458	1438.85	164,748	508	1595.93	202,683
409	1284.91	131,382	459	1441.99	165,468	509	1599.07	203,482
410	1288.05	132,025	460	1445.13	166,190	510	1602.21	204,282
411	1291.19	132,670	461	1448.27	166,914	511	1605.35	205,084
412	1294.34	133,317	462	1451.42	167,639	512	1608.50	205,887
413	1297.48	133,965	463	1454.56	168,365	513	1611.64	206,692
414	1300.62	134,614	464	1457.70	169,093	514	1614.78	207,499
415	1303.76	135,265	465	1460.84	169,823	515	1617.92	208,307
416	1306.90	135,918	466	1463.98	170,554	516	1621.06	209,117
417	1310.04	136,572	467	1467.12	171,287	517	1624.20	209,928
418	1313.19	137,228	468	1470.27	172,021	518	1627.35	210,741
419	1316.33	137,885	469	1473.41	172,757	519	1630.49	211,556
420	1319.47	138,544	470	1476.55	173,494	520	1633.63	212,372
421	1322.61	139,205	471	1479.69	174,234	521	1636.77	213,189
422	1325.75	139,867	472	1482.83	174,974	522	1639.91	214,008
423	1328.89	140,531	473	1485.97	175,716	523	1643.05	214,829
424	1332.04	141,196	474	1489.11	176,460	524	1646.20	215,651
425	1335.18	141,863	475	1492.26	177,205	525	1649.34	216,475
426	1338.32	142,531	476	1495.40	177,952	526	1652.48	217,301
427	1341.46	143,201	477	1498.54	178,701	527	1655.62	218,128
428	1344.60	143,872	478	1501.68	179,451	528	1658.76	218,956
429	1347.74	144,545	479	1504.82	180,203	529	1661.90	219,787
430	1350.88	145,220	480	1507.96	180,956	530	1665.04	220,618
431	1354.03	145,896	481	1511.11	181,711	531	1668.19	221,452
432	1357.17	146,574	482	1514.25	182,467	532	1671.33	222,287
433	1360.31	147,254	483	1517.39	183,225	533	1674.47	223,123
434	1363.45	147,934	484	1520.53	183,984	534	1677.61	223,961
435	1366.59	148,617	485	1523.67	184,745	535	1680.75	224,801
436	1369.73	149,301	486	1526.81	185,508	536	1683.89	225,642
437	1372.88	149,987	487	1529.96	186,272	537	1687.04	226,484
438	1376.02	150,674	488	1533.10	187,038	538	1690.18	227,329
439	1379.16	151,363	489	1536.24	187,805	539	1693.32	228,175
440	1382.30	152,053	490	1539.38	188,574	540	1696.46	229,022
441	1385.44	152,745	491	1542.52	189,345	541	1699.60	229,871
442	1388.58	153,439	492	1545.66	190,117	542	1702.74	230,722
443	1391.73	154,134	493	1548.81	190,890	543	1705.88	231,574
444	1394.87	154,830	494	1551.95	191,665	544	1709.03	232,428
445	1398.01	155,528	495	1555.09	192,442	545	1712.17	233,283
446	1401.15	156,228	496	1558.23	193,221	546	1715.31	234,140
447	1404.29	156,930	497	1561.37	194,000	547	1718.45	234,998
448	1407.43	157,633	498	1564.51	194,782	548	1721.59	235,858
449	1410.58	158,337	499	1567.65	195,565	549	1724.73	236,720

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
550	1727.88	237,583	600	1884.96	282,743	650	2042.04	331,831
551	1731.02	238,448	601	1888.10	283,687	651	2045.18	332,853
552	1734.16	239,314	602	1891.24	284,631	652	2048.32	333,876
553	1737.30	240,182	603	1894.38	285,578	653	2051.46	334,901
554	1740.44	241,051	604	1897.52	286,526	654	2054.60	335,927
555	1743.58	241,922	605	1900.66	287,475	655	2057.74	336,955
556	1746.73	242,795	606	1903.81	288,426	656	2060.88	337,985
557	1749.87	243,669	607	1906.95	289,379	657	2064.03	339,016
558	1753.01	244,545	608	1910.09	290,333	658	2067.17	340,049
559	1756.15	245,422	609	1913.23	291,289	659	2070.31	341,083
560	1759.29	246,301	610	1916.37	292,247	660	2073.45	342,119
561	1762.43	247,181	611	1919.51	293,206	661	2076.59	343,157
562	1765.58	248,063	612	1922.65	294,166	662	2079.73	344,196
563	1768.72	248,947	613	1925.80	295,128	663	2082.88	345,237
564	1771.86	249,832	614	1928.94	296,092	664	2086.02	346,279
565	1775.00	250,719	615	1932.08	297,057	665	2089.16	347,323
566	1778.14	251,607	616	1935.22	298,024	666	2092.30	348,368
567	1781.28	252,497	617	1938.36	298,992	667	2095.44	349,415
568	1784.42	253,388	618	1941.50	299,962	668	2098.58	350,464
569	1787.57	254,281	619	1944.65	300,934	669	2101.73	351,514
570	1790.71	255,176	620	1947.79	301,907	670	2104.87	352,565
571	1793.85	256,072	621	1950.93	302,882	671	2108.01	353,618
572	1796.99	256,970	622	1954.07	303,858	672	2111.15	354,673
573	1800.13	257,869	623	1957.21	304,836	673	2114.29	355,730
574	1803.27	258,770	624	1960.35	305,815	674	2117.43	356,788
575	1806.42	259,672	625	1963.50	306,796	675	2120.58	357,847
576	1809.56	260,576	626	1966.64	307,779	676	2123.72	358,908
577	1812.70	261,482	627	1969.78	308,763	677	2126.86	359,971
578	1815.84	262,389	628	1972.92	309,748	678	2130.00	361,035
579	1818.98	263,298	629	1976.06	310,736	679	2133.14	362,101
580	1822.12	264,208	630	1979.20	311,725	680	2136.28	363,168
581	1825.27	265,120	631	1982.35	312,715	681	2139.42	364,237
582	1828.41	266,033	632	1985.49	313,707	682	2142.57	365,308
583	1831.55	266,948	633	1988.63	314,700	683	2145.71	366,380
584	1834.69	267,865	634	1991.77	315,696	684	2148.85	367,453
585	1837.83	268,783	635	1994.91	316,692	685	2151.99	368,528
586	1840.97	269,703	636	1998.05	317,690	686	2155.13	369,605
587	1844.11	270,624	637	2001.19	318,690	687	2158.27	370,684
588	1847.26	271,547	638	2004.34	319,692	688	2161.42	371,764
589	1850.40	272,471	639	2007.48	320,695	689	2164.56	372,845
590	1853.54	273,397	640	2010.62	321,699	690	2167.70	373,928
591	1856.68	274,325	641	2013.76	322,705	691	2170.84	375,013
592	1859.82	275,254	642	2016.90	323,713	692	2173.98	376,099
593	1862.96	276,184	643	2020.04	324,722	693	2177.12	377,187
594	1866.11	277,117	644	2023.19	325,733	694	2180.27	378,276
595	1869.25	278,051	645	2026.33	326,745	695	2183.41	379,367
596	1872.39	278,986	646	2029.47	327,759	696	2186.55	380,459
597	1875.53	279,923	647	2032.61	328,775	697	2189.69	381,554
598	1878.67	280,862	648	2035.75	329,792	698	2192.83	382,649
599	1881.81	281,802	649	2038.89	330,810	699	2195.97	383,746

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
700	2199.11	384,845	750	2356.19	441,786	800	2513.27	502,655
701	2202.26	385,945	751	2359.34	442,965	801	2516.42	503,912
702	2205.40	387,047	752	2362.48	444,146	802	2519.56	505,171
703	2208.54	388,151	753	2365.62	445,328	803	2522.70	506,432
704	2211.68	389,256	754	2368.76	446,511	804	2525.84	507,694
705	2214.82	390,363	755	2371.90	447,697	805	2528.98	508,958
706	2217.96	391,471	756	2375.04	448,883	806	2532.12	510,223
707	2221.11	392,580	757	2378.19	450,072	807	2535.27	511,490
708	2224.25	393,692	758	2381.33	451,262	808	2538.41	512,758
709	2227.39	394,805	759	2384.47	452,453	809	2541.55	514,028
710	2230.53	395,919	760	2387.61	453,646	810	2544.69	515,300
711	2233.67	397,035	761	2390.75	454,841	811	2547.83	516,573
712	2236.81	398,153	762	2393.89	456,037	812	2550.97	517,848
713	2239.96	399,272	763	2397.04	457,234	813	2554.11	519,124
714	2243.10	400,393	764	2400.18	458,434	814	2557.26	520,402
715	2246.24	401,515	765	2403.32	459,635	815	2560.40	521,681
716	2249.38	402,639	766	2406.46	460,837	816	2563.54	522,962
717	2252.52	403,765	767	2409.60	462,041	817	2566.68	524,245
718	2255.66	404,892	768	2412.74	463,247	818	2569.82	525,529
719	2258.81	406,020	769	2415.88	464,454	819	2572.96	526,814
720	2261.95	407,150	770	2419.03	465,663	820	2576.11	528,102
721	2265.09	408,282	771	2422.17	466,873	821	2579.25	529,391
722	2268.23	409,416	772	2425.31	468,085	822	2582.39	530,681
723	2271.37	410,550	773	2428.45	469,298	823	2585.53	531,973
724	2274.51	411,687	774	2431.59	470,513	824	2588.67	533,267
725	2277.65	412,825	775	2434.73	471,730	825	2591.81	534,562
726	2280.80	413,965	776	2437.88	472,948	826	2594.96	535,858
727	2283.94	415,106	777	2441.02	474,168	827	2598.10	537,157
728	2287.08	416,248	778	2444.16	475,389	828	2601.24	538,456
729	2290.22	417,393	779	2447.30	476,612	829	2604.38	539,758
730	2293.36	418,539	780	2450.44	477,836	830	2607.52	541,061
731	2296.50	419,686	781	2453.58	479,062	831	2610.66	542,365
732	2299.65	420,835	782	2456.73	480,290	832	2613.81	543,671
733	2302.79	421,986	783	2459.87	481,519	833	2616.95	544,979
734	2305.93	423,138	784	2463.01	482,750	834	2620.09	546,288
735	2309.07	424,292	785	2466.15	483,982	835	2623.23	547,599
736	2312.21	425,447	786	2469.29	485,216	836	2626.37	548,912
737	2315.35	426,604	787	2472.43	486,451	837	2629.51	550,226
738	2318.50	427,762	788	2475.58	487,688	838	2632.65	551,541
739	2321.64	428,922	789	2478.72	488,927	839	2635.80	552,858
740	2324.78	430,084	790	2481.86	490,167	840	2638.94	554,177
741	2327.92	431,247	791	2485.00	491,409	841	2642.08	555,497
742	2331.06	432,412	792	2488.14	492,652	842	2645.22	556,819
743	2334.20	433,578	793	2491.28	493,897	843	2648.36	558,142
744	2337.34	434,746	794	2494.42	495,143	844	2651.50	559,467
745	2340.49	435,916	795	2497.57	496,391	845	2654.65	560,794
746	2343.63	437,087	796	2500.71	497,641	846	2657.79	562,122
747	2346.77	438,259	797	2503.85	498,892	847	2660.93	563,452
748	2349.91	439,433	798	2506.99	500,145	848	2664.07	564,783
749	2353.05	440,609	799	2510.13	501,399	849	2667.21	566,116

Circumferences and Areas of Circles

Diameter	Circumference	Area	Diameter	Circumference	Area	Diameter	Circumference	Area
850	2670.35	567,450	900	2827.43	636,173	950	2984.51	708,822
851	2673.50	568,786	901	2830.58	637,587	951	2987.65	710,315
852	2676.64	570,124	902	2833.72	639,003	952	2990.80	711,809
853	2679.78	571,463	903	2836.86	640,421	953	2993.94	713,306
854	2682.92	572,803	904	2840.00	641,840	954	2997.08	714,803
855	2686.06	574,146	905	2843.14	643,261	955	3000.22	716,303
856	2689.20	575,490	906	2846.28	644,683	956	3003.36	717,804
857	2692.34	576,835	907	2849.42	646,107	957	3006.50	719,306
858	2695.49	578,182	908	2852.57	647,533	958	3009.65	720,810
859	2698.63	579,530	909	2855.71	648,960	959	3012.79	722,316
860	2701.77	580,880	910	2858.85	650,388	960	3015.93	723,823
861	2704.91	582,232	911	2861.99	651,818	961	3019.07	725,332
862	2708.05	583,585	912	2865.13	653,250	962	3022.21	726,842
863	2711.19	584,940	913	2868.27	654,684	963	3025.35	728,354
864	2714.34	586,297	914	2871.42	656,118	964	3028.50	729,867
865	2717.48	587,655	915	2874.56	657,555	965	3031.64	731,382
866	2720.62	589,014	916	2877.70	658,993	966	3034.78	732,899
867	2723.76	590,375	917	2880.84	660,433	967	3037.92	734,417
868	2726.90	591,738	918	2883.98	661,874	968	3041.06	735,937
869	2730.04	593,102	919	2887.12	663,317	969	3044.20	737,458
870	2733.19	594,468	920	2890.27	664,761	970	3047.34	738,981
871	2736.33	595,835	921	2893.41	666,207	971	3050.49	740,506
872	2739.47	597,204	922	2896.55	667,654	972	3053.63	742,032
873	2742.61	598,575	923	2899.69	669,103	973	3056.77	743,559
874	2745.75	599,947	924	2902.83	670,554	974	3059.91	745,088
875	2748.89	601,320	925	2905.97	672,006	975	3063.05	746,619
876	2752.04	602,696	926	2909.11	673,460	976	3066.19	748,151
877	2755.18	604,073	927	2912.26	674,915	977	3069.34	749,685
878	2758.32	605,451	928	2915.40	676,372	978	3072.48	751,221
879	2761.46	606,831	929	2918.54	677,831	979	3075.62	752,758
880	2764.60	608,212	930	2921.68	679,291	980	3078.76	754,296
881	2767.74	609,595	931	2924.82	680,752	981	3081.90	755,837
882	2770.88	610,980	932	2927.96	682,216	982	3085.04	757,378
883	2774.03	612,366	933	2931.11	683,680	983	3088.19	758,922
884	2777.17	613,754	934	2934.25	685,147	984	3091.33	760,466
885	2780.31	615,143	935	2937.39	686,615	985	3094.47	762,013
886	2783.45	616,534	936	2940.53	688,084	986	3097.61	763,561
887	2786.59	617,927	937	2943.67	689,555	987	3100.75	765,111
888	2789.73	619,321	938	2946.81	691,028	988	3103.89	766,662
889	2792.88	620,717	939	2949.96	692,502	989	3107.04	768,214
890	2796.02	622,114	940	2953.10	693,978	990	3110.18	769,769
891	2799.16	623,513	941	2956.24	695,455	991	3113.32	771,325
892	2802.30	624,913	942	2959.38	696,934	992	3116.46	772,882
893	2805.44	626,315	943	2962.52	698,415	993	3119.60	774,441
894	2808.58	627,718	944	2965.66	699,897	994	3122.74	776,002
895	2811.73	629,124	945	2968.81	701,380	995	3125.88	777,564
896	2814.87	630,530	946	2971.95	702,865	996	3129.03	779,128
897	2818.01	631,938	947	2975.09	704,352	997	3132.17	780,693
898	2821.15	633,348	948	2978.23	705,840	998	3135.31	782,260
899	2824.29	634,760	949	2981.37	707,330	999	3138.45	783,828

Diameters, Circumferences and Areas of Circles in Feet and Inches

Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. In.	Area Sq. Ft.	Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. In.	Area Sq. Ft.
1	6	4	8½	254.469	1.7671	2	0	6	3⅜	452.390	3.1416
	6⅛	4	9	258.016	1.7918		0¼	6	4⅛	461.864	3.2074
	6¼	4	9⅜	261.587	1.8166		0½	6	5	471.436	3.2739
	6⅜	4	9¾	265.182	1.8415		0¾	6	5¾	481.106	3.3410
	6½	4	10⅛	268.803	1.8667	2	1	6	6½	490.875	3.4089
	6⅝	4	10½	272.447	1.8920		1¼	6	7⅜	500.741	3.4774
	6¾	4	10⅞	276.117	1.9175		1½	6	8⅛	510.706	3.5466
	6⅞	4	11¼	279.811	1.9431		1¾	6	8⅞	520.769	3.6165
1	7	4	11¾	283.529	1.9690	2	2	6	9⅝	530.930	3.6176
	7⅛	5	0⅛	287.272	1.9949		2¼	6	10½	541.189	3.7583
	7¼	5	0½	291.039	2.0211		2⅓	6	11¼	551.547	3.8302
	7⅜	5	0⅞	294.831	2.0474		2¾	7	0	562.002	3.9028
	7½	5	1¼	298.648	2.0739	2	3	7	0⅞	572.556	3.9761
	7⅝	5	1⅝	302.489	2.1006		3¼	7	1⅝	583.208	4.0501
	7¾	5	2	306.355	2.1275		3½	7	2⅜	593.958	4.1247
	7⅞	5	2½	310.245	2.1545		3¾	7	3⅛	604.807	4.2000
1	8	5	2⅞	314.160	2.1817	2	4	7	4	615.753	4.2761
	8⅛	5	3¼	318.099	2.2090		4¼	7	4¾	626.798	4.3528
	8¼	5	3⅝	322.063	2.2365		4½	7	5½	637.941	4.4301
	8⅜	5	4	326.051	2.2642		4¾	7	6⅜	649.182	4.5082
	8½	5	4⅜	330.064	2.2921	2	5	7	7⅛	660.521	4.5870
	8⅝	5	4¾	334.101	2.3201		5¼	7	7⅞	671.958	4.6665
	8¾	5	5¼	338.163	2.3480		5½	7	8⅝	683.494	4.7465
	8⅞	5	5⅝	342.250	2.3768		5¾	7	9½	695.128	4.8273
1	9	5	6	346.361	2.4053	2	6	7	10¼	706.860	4.9088
	9⅛	5	6⅜	350.497	2.4340		6¼	7	11	718.690	4.9909
	9¼	5	6¾	354.657	2.4629		6½	7	11⅞	730.618	5.0737
	9⅜	5	7⅛	358.841	2.4920		6¾	8	0⅝	742.644	5.1573
	9½	5	7½	363.051	2.5212	2	7	8	1⅜	754.769	5.2415
	9⅝	5	7⅞	367.284	2.5506		7¼	8	2⅛	766.992	5.3263
	9¾	5	8⅜	371.543	2.5802		7½	8	3	779.313	5.4112
	9⅞	5	8¾	375.826	2.6099		7¾	8	3¾	791.732	5.4995
1	10	5	9⅛	380.133	2.6640	2	8	8	4½	804.249	5.5851
	10⅛	5	9½	384.465	2.6699		8¼	8	5⅜	816.865	5.6727
	10¼	5	9⅞	388.822	2.7002		8½	8	6⅛	829.578	5.7610
	10⅜	5	10¼	393.203	2.7306		8¾	8	6⅞	842.390	5.8499
	10½	5	10⅝	397.608	2.7612	2	9	8	7⅝	855.300	5.9396
	10⅝	5	11⅛	402.038	2.7919		9¼	8	8½	868.308	6.0299
	10¾	5	11½	406.493	2.8229		9½	8	9¼	881.415	6.1209
	10⅞	5	11⅞	410.972	2.8540		9¾	8	10	894.619	6.2126
1	11	6	0¼	415.476	2.8853	2	10	8	10⅞	907.922	6.3050
	11⅛	6	0⅝	420.004	2.9167		10¼	8	11⅝	921.323	6.3981
	11¼	6	1	424.557	2.9483		10½	9	0⅜	934.822	6.4918
	11⅜	6	1⅜	429.135	2.9801		10¾	9	1⅛	948.419	6.5862
	11½	6	1⅞	433.737	3.0121	2	11	9	2	962.115	6.6814
	11⅝	6	2¼	438.363	3.0442		11¼	9	2¾	975.908	6.7771
	11¾	6	2⅝	443.014	3.0765		11½	9	3½	989.800	6.8736
	11⅞	6	3	447.690	3.1090		11¾	9	4¼	1003.790	6.9707

* Circumference to nearest ⅛ inch.

Diameters, Circumferences and Areas of Circles in Feet and Inches

Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. In.	Area Sq. Ft.	Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. In.	Area Sq. Ft.
3	0	9	5⅛	1017.87	7.068	4	0	12	6¾	1809.56	12.566
	0¼	9	5⅞	1032.06	7.167		0¼	12	7⅝	1828.46	12.697
	0½	9	6⅝	1046.35	7.266		0½	12	8⅜	1847.45	12.829
	0¾	9	7½	1060.73	7.366		0¾	12	9⅛	1866.55	12.962
3	1	9	8¼	1075.21	7.466	4	1	12	10	1885.74	13.095
	1¼	9	9	1089.79	7.568		1¼	12	10¾	1905.03	13.229
	1½	9	9¾	1104.46	7.669		1½	12	11½	1924.42	13.364
	1¾	9	10½	1119.24	7.772		1¾	13	0¼	1943.91	13.499
3	2	9	11⅜	1134.12	7.875	4	2	13	1⅛	1963.50	13.635
	2¼	10	0⅛	1149.09	7.979		2¼	13	1⅞	1983.18	13.772
	2½	10	1	1164.16	8.084		2½	13	2⅝	2002.96	13.909
	2¾	10	1¾	1179.32	8.189		2¾	13	3⅜	2022.84	14.047
3	3	10	2½	1194.59	8.295	4	3	13	4¼	2042.82	14.186
	3¼	10	3¼	1209.95	8.402		3¼	13	5	2062.90	14.325
	3½	10	4⅛	1225.42	8.509		3½	13	5¾	2083.07	14.465
	3¾	10	4⅞	1240.98	8.617		3¾	13	6⅝	2103.35	14.606
3	4	10	5⅝	1256.64	8.726	4	4	13	7⅜	2123.72	14.748
	4¼	10	6½	1272.39	8.836		4¼	13	8⅛	2144.19	14.890
	4½	10	7¼	1288.25	8.946		4½	13	8⅞	2164.75	15.033
	4¾	10	8	1304.20	9.056		4¾	13	9¾	2185.42	15.176
3	5	10	8¾	1320.25	9.168	4	5	13	10½	2206.18	15.320
	5¼	10	9⅝	1336.40	9.280		5¼	13	11¼	2227.05	15.465
	5½	10	10⅜	1352.65	9.393		5½	14	0⅛	2248.01	15.611
	5¾	10	11⅛	1369.00	9.506		5¾	14	0⅞	2269.06	15.757
3	6	11	0	1385.44	9.635	4	6	14	1⅝	2290.22	15.904
	6¼	11	0¾	1401.98	9.736		6¼	14	2⅜	2311.48	16.051
	6½	11	1½	1418.62	9.851		6½	14	3¼	2332.83	16.200
	6¾	11	2¼	1435.36	9.967		6¾	14	4	2354.28	16.349
3	7	11	3⅛	1452.20	10.084	4	7	14	4¾	2375.83	16.498
	7¼	11	3⅞	1469.14	10.202		7¼	14	5⅝	2397.48	16.649
	7½	11	4⅝	1486.17	10.320		7½	14	6⅜	2419.22	16.800
	7¾	11	5½	1503.30	10.439		7¾	14	7⅛	2441.07	16.951
3	8	11	6¼	1520.53	10.559	4	8	14	7⅞	2463.01	17.104
	8¼	11	7	1537.86	10.679		8¼	14	8¾	2485.05	17.257
	8½	11	7¾	1555.28	10.800		8½	14	9½	2507.19	17.411
	8¾	11	8⅝	1572.81	10.922		8¾	14	10¼	2529.42	17.565
3	9	11	9⅜	1590.43	11.044	4	9	14	11⅛	2551.76	17.720
	9¼	11	10⅛	1608.15	11.137		9¼	14	11⅞	2574.19	17.876
	9½	11	11	1625.97	11.291		9½	15	0⅝	2596.72	18.032
	9¾	11	11¾	1643.89	11.415		9¾	15	1⅜	2619.35	18.189
3	10	12	0½	1661.90	11.541	4	10	15	2¼	2642.08	18.347
	10¼	12	1¼	1680.02	11.666		10¼	15	3	2664.91	18.506
	10½	12	2⅛	1698.23	11.793		10½	15	3¾	2687.83	18.665
	10¾	12	2⅞	1716.54	11.920		10¾	15	4⅝	2710.85	18.825
3	11	12	3⅝	1734.94	12.048	4	11	15	5⅜	2733.97	18.985
	11¼	12	4½	1753.45	12.176		11¼	15	6⅛	2757.19	19.147
	11½	12	5¼	1772.05	12.305		11½	15	6⅞	2780.51	19.309
	11¾	12	6	1790.76	12.435		11¾	15	7¾	2803.92	19.471

* Circumference to nearest ⅛ inch.

Diameters, Circumferences and Areas of Circles in Feet and Inches

Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. In.	Area Sq. Ft.	Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. In.	Area Sq. Ft.
5	0	15	8½	2827.44	19.635	6	0	18	10¼	4071.51	28.274
	0¼	15	9¼	2851.05	19.798		0¼	18	11	4099.83	28.471
	0½	15	10⅛	2874.76	19.963		0½	18	11¾	4128.25	28.668
	0¾	15	10⅞	2898.56	20.128		0¾	19	0½	4156.77	28.866
5	1	15	11⅝	2922.47	20.294	6	1	19	1⅜	4185.39	29.065
	1¼	16	0⅜	2946.47	20.461		1¼	19	2⅛	4214.11	29.264
	1½	16	1¼	2970.57	20.629		1½	19	2⅞	4242.92	29.464
	1¾	16	2	2994.77	20.797		1¾	19	3¾	4271.83	29.665
5	2	16	2¾	3019.07	20.965	6	2	19	4½	4300.85	29.867
	2¼	16	3⅝	3043.47	21.135		2¼	19	5¼	4329.95	30.069
	2½	16	4⅜	3067.96	21.305		2½	19	6	4359.16	30.271
	2¾	16	5⅛	3092.56	21.476		2¾	19	6⅞	4388.47	30.475
5	3	16	5⅞	3117.25	21.647	6	3	19	7⅝	4417.87	30.679
	3¼	16	6¾	3142.04	21.819		3¼	19	8⅜	4447.37	30.884
	3½	16	7½	3166.92	21.992		3½	19	9¼	4476.97	31.090
	3¾	16	8¼	3191.91	22.166		3¾	19	10	4506.67	31.296
5	4	16	9	3216.99	22.340	6	4	19	10¾	4536.47	31.503
	4¼	16	9⅞	3242.17	22.512		4¼	19	11½	4566.36	31.710
	4½	16	10⅝	3267.46	22.690		4½	20	0⅜	4596.35	31.919
	4¾	16	11⅜	3292.83	22.866		4¾	20	1⅛	4626.44	32.128
5	5	17	0¼	3318.31	23.043	6	5	20	1⅞	4656.63	32.337
	5¼	17	1	3343.88	23.221		5¼	20	2¾	4686.92	32.548
	5½	17	1¾	3369.56	23.399		5½	20	3½	4717.30	32.759
	5¾	17	2½	3395.33	23.578		5¾	20	4¼	4747.79	32.970
5	6	17	3⅜	3421.20	23.758	6	6	20	5	4778.37	33.183
	6¼	17	4⅛	3447.16	23.938		6¼	20	5⅞	4809.05	33.396
	6½	17	4⅞	3473.23	24.119		6½	20	6⅝	4839.83	33.609
	6¾	17	5¾	3499.39	24.301		6¾	20	7⅜	4870.70	33.824
5	7	17	6½	3525.66	24.483	6	7	20	8⅛	4901.68	34.039
	7¼	17	7¼	3552.01	24.666		7¼	20	9	4932.75	34.255
	7½	17	8	3578.47	24.850		7½	20	9¾	4963.92	34.471
	7¾	17	8⅞	3605.03	25.034		7¾	20	10½	4995.19	34.688
5	8	17	9⅝	3631.68	25.220	6	8	20	11⅜	5026.56	34.906
	8¼	17	10⅜	3658.44	25.405		8¼	21	0⅛	5058.02	35.125
	8½	17	11¼	3685.29	25.592		8½	21	0⅞	5089.58	35.344
	8¾	18	0	3712.24	25.779		8¾	21	1⅝	5121.24	35.564
5	9	18	0¾	3739.28	25.967	6	9	21	2½	5153.00	35.784
	9¼	18	1½	3766.43	26.155		9¼	21	3¼	5184.86	36.006
	9½	18	2⅜	3793.67	26.344		9½	21	4	5216.82	36.227
	9¾	18	3⅛	3821.02	26.534		9¾	21	4⅞	5248.87	36.450
5	10	18	3⅞	3848.46	26.725	6	10	21	5⅝	5281.02	36.674
	10¼	18	4¾	3875.99	26.916		10¼	21	6⅜	5313.27	36.897
	10½	18	5½	3903.63	27.108		10½	21	7⅛	5345.62	37.122
	10¾	18	6¼	3931.36	27.301		10¾	21	8	5378.07	37.347
5	11	18	7⅛	3959.20	27.494	6	11	21	8¾	5410.62	37.573
	11¼	18	7⅞	3987.13	27.688		11¼	21	9½	5443.26	37.800
	11½	18	8⅝	4015.16	27.883		11½	21	10⅜	5476.00	38.027
	11¾	18	9⅜	4043.28	28.078		11¾	21	11⅛	5508.84	38.256

* Circumference to nearest ⅛ inch.

Diameters, Circumferences and Areas of Circles in Feet and Inches

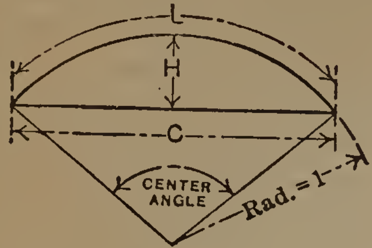
Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. Ft.	Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. Ft.	Diam. Ft.	Diam. In.	Circum.* Ft.	Circum.* In.	Area Sq. Ft.
7	0	21	11⅞	38.48	11	0	34	6¾	95.03	15	0	47	1½	176.71
	1	22	3	39.40		1	34	9⅞	96.47		1	47	4⅝	178.68
	2	22	6⅛	40.33		2	35	1	97.93		2	47	7¾	180.66
	3	22	9⅜	41.28		3	35	4⅛	99.40		3	47	10⅞	182 65
	4	23	0½	42.23		4	35	7¼	100.87		4	48	2	184.65
	5	23	3⅝	43.20		5	35	10⅝	102.36		5	48	5¼	186.66
	6	23	6¾	44.17		6	36	1½	103.86		6	48	8⅜	188.69
	7	23	9⅞	45.16		7	36	4⅝	105.37		7	48	11½	190.72
	8	24	1	46.16		8	36	7⅞	106.90		8	49	2⅝	192.77
	9	24	4⅛	47.17		9	36	11	108.43		9	49	5¾	194.82
	10	24	7¼	48.19		10	37	2⅛	109.97		10	49	8⅞	196.89
	11	24	10½	49.22		11	37	5¼	111.53		11	50	0	198.97
8	0	25	1⅝	50.26	12	0	37	8⅜	113.09	16	0	50	3¼	201.06
	1	25	4¾	51.31		1	37	11½	114.67		1	50	6⅜	203.16
	2	25	7⅞	52.38		2	38	2⅝	116.26		2	50	9½	205.27
	3	25	11	53.45		3	38	5¾	117.85		3	51	0⅝	207.39
	4	26	2⅛	54.54		4	38	9	119.46		4	51	3¾	209.52
	5	26	5¼	55.63		5	39	0⅛	121.08		5	51	6⅞	211.67
	6	26	8½	56.74		6	39	3¼	122.71		6	51	10	213.82
	7	26	11⅝	57.86		7	39	6⅜	124.36		7	52	1⅛	215.99
	8	27	2¾	58.99		8	39	9½	126.01		8	52	4⅜	218.16
	9	27	5⅞	60.13		9	40	0⅝	127.67		9	52	7½	220.35
	10	27	9	61.28		10	40	3¾	129.35		10	52	10⅝	222.55
	11	28	0⅛	62.44		11	40	7	131.03		11	53	1¾	224.76
9	0	28	3¼	63.61	13	0	40	10⅛	132.73	17	0	53	4⅞	226.98
	1	28	6⅜	64.80		1	41	1¼	134.43		1	53	8	229.21
	2	28	9⅝	65.99		2	41	4⅜	136.15		2	53	11⅛	231.45
	3	29	0¾	67.20		3	41	7½	137.88		3	54	2¼	233.70
	4	29	3⅞	68.41		4	41	10⅝	139.62		4	54	5½	235.96
	5	29	7	69.64		5	42	1¾	141.37		5	54	8⅝	238.24
	6	29	10⅛	70.88		6	42	5	143.13		6	54	11¾	240.52
	7	30	1¼	72.13		7	42	8⅛	144.91		7	55	2⅞	242.82
	8	30	4⅜	73.39		8	42	11¼	146.69		8	55	6	245.13
	9	30	7⅝	74.66		9	43	2⅜	148.48		9	55	9⅛	247.45
	10	30	10¾	75.94		10	43	5½	150.29		10	56	0¼	249.77
	11	31	1⅞	77.23		11	43	8⅝	152.11		11	56	3½	252.11
10	0	31	5	78.54	14	0	43	11¾	153.93	18	0	56	6⅝	254.46
	1	31	8⅛	79.85		1	44	2⅞	155.77		1	56	9¾	256.83
	2	31	11¼	81.17		2	44	6⅛	157.62		2	57	0⅞	259.20
	3	32	2⅜	82.51		3	44	9¼	159.48		3	57	4	261.58
	4	32	5½	83.86		4	45	0⅜	161.35		4	57	7⅛	263.98
	5	32	8¾	85.22		5	45	3½	163.23		5	57	10¼	266.38
	6	32	11⅞	86.59		6	45	6⅝	165.13		6	58	1⅜	268.80
	7	33	3	87.97		7	45	9¾	167.03		7	58	4⅝	271.22
	8	33	6⅛	89.36		8	46	0⅞	168.94		8	58	7¾	273.66
	9	33	9¼	90.76		9	46	4	170.87		9	58	10⅞	276.11
	10	34	0⅜	92.17		10	46	7¼	172.80		10	59	2	278.57
	11	34	3½	93.59		11	46	10⅜	174.75		11	59	5⅛	281.04

* Circumference to nearest ⅛ inch.

Circumferences and Corresponding Diameters of Circles

Circum.	Diameter	Circum.	Diameter	Circum.	Diameter	Circum.	Diameter	Circum.	Diameter
1	0.3183	51	16.2338	101	32.149	151	48.065	201	63.980
2	0.6366	52	16.5521	102	32.468	152	48.383	202	64.299
3	0.9549	53	16.8704	103	32.786	153	48.701	203	64.617
4	1.2732	54	17.1887	104	33.104	154	49.020	204	64.935
5	1.5915	55	17.5070	105	33.422	155	49.338	205	65.253
6	1.9099	56	17.8254	106	33.741	156	49.656	206	65.572
7	2.2282	57	18.1437	107	34.059	157	49.975	207	65.890
8	2.5465	58	18.4620	108	34.377	158	50.293	208	66.208
9	2.8648	59	18.7803	109	34.696	159	50.611	209	66.527
10	3.1831	60	19.0986	110	35.014	160	50.930	210	66.845
11	3.5014	61	19.4169	111	35.332	161	51.248	211	67.163
12	3.8197	62	19.7352	112	35.651	162	51.566	212	67.482
13	4.1380	63	20.0535	113	35.969	163	51.884	213	67.800
14	4.4563	64	20.3718	114	36.287	164	52.203	214	68.118
15	4.7746	65	20.6901	115	36.606	165	52.521	215	68.437
16	5.0930	66	21.0085	116	36.924	166	52.839	216	68.755
17	5.4113	67	21.3268	117	37.242	167	53.158	217	69.073
18	5.7296	68	21.6451	118	37.561	168	53.476	218	69.392
19	6.0479	69	21.9634	119	37.879	169	53.794	219	69.710
20	6.3662	70	22.2817	120	38.197	170	54.113	220	70.028
21	6.6845	71	22.6000	121	38.515	171	54.431	221	70.346
22	7.0028	72	22.9183	122	38.834	172	54.749	222	70.665
23	7.3211	73	23.2366	123	39.152	173	55.068	223	70.983
24	7.6394	74	23.5549	124	39.470	174	55.386	224	71.301
25	7.9577	75	23.8732	125	39.789	175	55.704	225	71.620
26	8.2761	76	24.1916	126	40.107	176	56.022	226	71.938
27	8.5944	77	24.5099	127	40.425	177	56.341	227	72.256
28	8.9127	78	24.8282	128	40.744	178	56.659	228	72.575
29	9.2310	79	25.1465	129	41.062	179	56.977	229	72.893
30	9.5493	80	25.4648	130	41.380	180	57.296	230	73.211
31	9.8676	81	25.7831	131	41.699	181	57.614	231	73.530
32	10.1859	82	26.1014	132	42.017	182	57.932	232	73.848
33	10.5042	83	26.4197	133	42.335	183	58.251	233	74.166
34	10.8225	84	26.7380	134	42.653	184	58.569	234	74.484
35	11.1408	85	27.0563	135	42.972	185	58.887	235	74.803
36	11.4592	86	27.3747	136	43.290	186	59.206	236	75.121
37	11.7775	87	27.6930	137	43.608	187	59.524	237	75.439
38	12.0958	88	28.0113	138	43.927	188	59.842	238	75.758
39	12.4141	89	28.3296	139	44.245	189	60.161	239	76.076
40	12.7324	90	28.6479	140	44.563	190	60.479	240	76.394
41	13.0507	91	28.9662	141	44.882	191	60.797	241	76.713
42	13.3690	92	29.2845	142	45.200	192	61.115	242	77.031
43	13.6873	93	29.6028	143	45.518	193	61.434	243	77.349
44	14.0056	94	29.9211	144	45.837	194	61.752	244	77.668
45	14.3239	95	30.2394	145	46.155	195	62.070	245	77.986
46	14.6423	96	30.5577	146	46.473	196	62.389	246	78.304
47	14.9606	97	30.8761	147	46.792	197	62.707	247	78.622
48	15.2789	98	31.1944	148	47.110	198	63.025	248	78.941
49	15.5972	99	31.5127	149	47.428	199	63.344	249	79.259
50	15.9155	100	31.8310	150	47.746	200	63.662	250	79.577

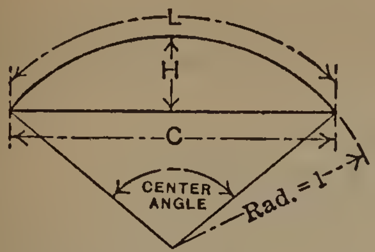
Segments of Circles



Length of Arc, Height of Segment, Length of Chord and Area of Segment for Angles from 1 to 180 degrees, and Radius = 1. — For other radii, multiply the values of *L*, *H* and *C* in the table by the given radius, and the values for areas, by the square of the radius.

Center Angle, Degrees	*L*	*H*	*C*	Area of Segment	Center Angle, Degrees	*L*	*H*	*C*	Area of Segment
1	0.017	0.0000	0.017	0.00000	46	0.803	0.0795	0.781	0.04176
2	0.035	0.0002	0.035	0.00000	47	0.820	0.0829	0.797	0.04448
3	0.052	0.0003	0.052	0.00001	48	0.838	0.0865	0.813	0.04731
4	0.070	0.0006	0.070	0.00003	49	0.855	0.0900	0.829	0.05025
5	0.087	0.0010	0.087	0.00006	50	0.873	0.0937	0.845	0.05331
6	0.105	0.0014	0.105	0.00010	51	0.890	0.0974	0.861	0.05649
7	0.122	0.0019	0.122	0.00015	52	0.908	0.1012	0.877	0.05978
8	0.140	0.0024	0.139	0.00023	53	0.925	0.1051	0.892	0.06319
9	0.157	0.0031	0.157	0.00032	54	0.942	0.1090	0.908	0.06673
10	0.174	0.0038	0.174	0.00044	55	0.960	0.1130	0.923	0.07039
11	0.192	0.0046	0.192	0.00059	56	0.977	0.1171	0.939	0.07417
12	0.209	0.0055	0.209	0.00076	57	0.995	0.1212	0.954	0.07808
13	0.227	0.0064	0.226	0.00097	58	1.012	0.1254	0.970	0.08212
14	0.244	0.0075	0.244	0.00121	59	1.030	0.1296	0.985	0.08629
15	0.262	0.0086	0.261	0.00149	60	1.047	0.1340	1.000	0.09059
16	0.279	0.0097	0.278	0.00181	61	1.065	0.1384	1.015	0.09502
17	0.297	0.0110	0.296	0.00217	62	1.082	0.1428	1.030	0.09958
18	0.314	0.0123	0.313	0.00257	63	1.100	0.1474	1.045	0.10428
19	0.332	0.0137	0.330	0.00302	64	1.117	0.1520	1.060	0.10911
20	0.349	0.0152	0.347	0.00352	65	1.134	0.1566	1.075	0.11408
21	0.366	0.0167	0.364	0.00408	66	1.152	0.1613	1.089	0.11919
22	0.384	0.0184	0.382	0.00468	67	1.169	0.1661	1.104	0.12443
23	0.401	0.0201	0.399	0.00535	68	1.187	0.1710	1.118	0.12982
24	0.419	0.0219	0.416	0.00607	69	1.204	0.1759	1.133	0.13535
25	0.436	0.0237	0.433	0.00686	70	1.222	0.1808	1.147	0.14102
26	0.454	0.0256	0.450	0.00771	71	1.239	0.1859	1.161	0.14683
27	0.471	0.0276	0.467	0.00862	72	1.257	0.1910	1.176	0.15279
28	0.489	0.0297	0.484	0.00961	73	1.274	0.1961	1.190	0.15889
29	0.506	0.0319	0.501	0.01067	74	1.291	0.2014	1.204	0.16514
30	0.524	0.0341	0.518	0.01180	75	1.309	0.2066	1.217	0.17154
31	0.541	0.0364	0.534	0.01301	76	1.326	0.2120	1.231	0.17808
32	0.558	0.0387	0.551	0.01429	77	1.344	0.2174	1.245	0.18477
33	0.576	0.0412	0.568	0.01566	78	1.361	0.2229	1.259	0.19160
34	0.593	0.0437	0.585	0.01711	79	1.379	0.2284	1.272	0.19859
35	0.611	0.0463	0.601	0.01864	80	1.396	0.2340	1.286	0.20573
36	0.628	0.0489	0.618	0.02027	81	1.414	0.2396	1.299	0.21301
37	0.646	0.0517	0.635	0.02198	82	1.431	0.2453	1.312	0.22045
38	0.663	0.0545	0.651	0.02378	83	1.449	0.2510	1.325	0.22804
39	0.681	0.0574	0.668	0.02568	84	1.466	0.2569	1.338	0.23578
40	0.698	0.0603	0.684	0.02767	85	1.483	0.2627	1.351	0.24367
41	0.716	0.0633	0.700	0.02976	86	1.501	0.2686	1.364	0.25171
42	0.733	0.0664	0.717	0.03195	87	1.518	0.2746	1.377	0.25990
43	0.750	0.0696	0.733	0.03425	88	1.536	0.2807	1.389	0.26825
44	0.768	0.0728	0.749	0.03664	89	1.553	0.2867	1.402	0.27677
45	0.785	0.0761	0.765	0.03915	90	1.571	0.2929	1.414	0.28540

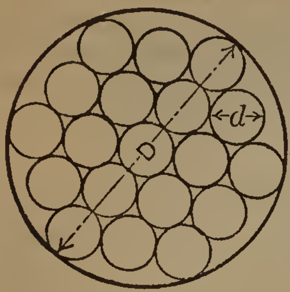
Segments of Circles



Length of Arc, Height of Segment, Length of Chord and Area of Segment for Angles from 1 to 180 degrees, and Radius = 1. — For other radii, multiply the values of L, H and C in the table by the given radius, and the values for areas, by the square of the radius.

Center Angle, Degrees	L	H	C	Area of Segment	Center Angle, Degrees	L	H	C	Area of Segment
91	1.588	0.2991	1.426	0.2942	136	2.374	0.6254	1.854	0.8395
92	1.606	0.3053	1.439	0.3032	137	2.391	0.6335	1.861	0.8545
93	1.623	0.3116	1.451	0.3123	138	2.409	0.6416	1.867	0.8697
94	1.641	0.3180	1.463	0.3215	139	2.426	0.6498	1.873	0.8850
95	1.658	0.3244	1.475	0.3309	140	2.443	0.6580	1.879	0.9003
96	1.675	0.3309	1.486	0.3405	141	2.461	0.6662	1.885	0.9158
97	1.693	0.3374	1.498	0.3502	142	2.478	0.6744	1.891	0.9313
98	1.710	0.3439	1.509	0.3601	143	2.496	0.6827	1.897	0.9470
99	1.728	0.3506	1.521	0.3701	144	2.513	0.6910	1.902	0.9627
100	1.745	0.3572	1.532	0.3803	145	2.531	0.6993	1.907	0.9786
101	1.763	0.3639	1.543	0.3906	146	2.548	0.7076	1.913	0.9945
102	1.780	0.3707	1.554	0.4010	147	2.566	0.7160	1.918	1.0105
103	1.798	0.3775	1.565	0.4117	148	2.583	0.7244	1.922	1.0266
104	1.815	0.3843	1.576	0.4224	149	2.600	0.7328	1.927	1.0427
105	1.833	0.3912	1.587	0.4333	150	2.618	0.7412	1.932	1.0590
106	1.850	0.3982	1.597	0.4444	151	2.635	0.7496	1.936	1.0753
107	1.867	0.4052	1.608	0.4556	152	2.653	0.7581	1.941	1.0917
108	1.885	0.4122	1.618	0.4669	153	2.670	0.7666	1.945	1.1082
109	1.902	0.4193	1.628	0.4784	154	2.688	0.7750	1.949	1.1247
110	1.920	0.4264	1.638	0.4901	155	2.705	0.7836	1.953	1.1413
111	1.937	0.4336	1.648	0.5019	156	2.723	0.7921	1.956	1.1580
112	1.955	0.4408	1.658	0.5138	157	2.740	0.8006	1.960	1.1747
113	1.972	0.4481	1.668	0.5259	158	2.758	0.8092	1.963	1.1915
114	1.990	0.4554	1.677	0.5381	159	2.775	0.8178	1.966	1.2083
115	2.007	0.4627	1.687	0.5504	160	2.792	0.8264	1.970	1.2252
116	2.025	0.4701	1.696	0.5629	161	2.810	0.8350	1.973	1.2422
117	2.042	0.4775	1.705	0.5755	162	2.827	0.8436	1.975	1.2592
118	2.059	0.4850	1.714	0.5883	163	2.845	0.8522	1.978	1.2763
119	2.077	0.4925	1.723	0.6012	164	2.862	0.8608	1.980	1.2933
120	2.094	0.5000	1.732	0.6142	165	2.880	0.8695	1.983	1.3105
121	2.112	0.5076	1.741	0.6273	166	2.897	0.8781	1.985	1.3277
122	2.129	0.5152	1.749	0.6406	167	2.915	0.8868	1.987	1.3449
123	2.147	0.5228	1.758	0.6540	168	2.932	0.8955	1.989	1.3621
124	2.164	0.5305	1.766	0.6676	169	2.950	0.9042	1.991	1.3794
125	2.182	0.5383	1.774	0.6812	170	2.967	0.9128	1.992	1.3967
126	2.199	0.5460	1.782	0.6950	171	2.984	0.9215	1.994	1.4140
127	2.217	0.5538	1.790	0.7090	172	3.002	0.9302	1.995	1.4314
128	2.234	0.5616	1.798	0.7230	173	3.019	0.9390	1.996	1.4488
129	2.251	0.5695	1.805	0.7372	174	3.037	0.9477	1.997	1.4662
130	2.269	0.5774	1.813	0.7514	175	3.054	0.9564	1.998	1.4836
131	2.286	0.5853	1.820	0.7658	176	3.072	0.9651	1.999	1.5010
132	2.304	0.5933	1.827	0.7803	177	3.089	0.9738	1.999	1.5185
133	2.321	0.6013	1.834	0.7950	178	3.107	0.9825	2.000	1.5359
134	2.339	0.6093	1.841	0.8097	179	3.124	0.9913	2.000	1.5533
135	2.356	0.6173	1.848	0.8245	180	3.142	1.0000	2.000	1.5708

Table of Circles that Can be Inscribed within an Enclosing Circle



From the table below the number of circles of a given diameter that can be inscribed in a larger circle of known diameter, can be found.

N = number of inscribed circles.

$$R = \text{ratio} = \frac{\text{diameter of large circle}}{\text{diameter of small circle}} = \frac{D}{d}.$$

Example: How many wires, ½ inch in diameter, can be placed inside a pipe, 5 inches in diameter? — $R = 5 \div ½ = 10$, and finding the value nearest to this number in the columns of R in the table, $N = 76$ is determined by interpolation.

Approximate Formulas:

$$N = 0.907\left(\frac{D}{d} - 0.94\right)^2 + 3.7 \qquad R = 0.94 + \sqrt{\frac{N - 3.7}{0.907}}$$

N	R	N	R	N	R	N	R	N	R
2	2.00	34	6.76	130	12.80	290	18.75	600	26.65
3	2.15	35	6.86	135	13.06	295	18.90	610	26.86
4	2.41	36	7.00	140	13.26	300	19.05	620	27.07
5	2.70	37	7.00	145	13.49	310	19.35	630	27.28
6	3.00	38	7.08	150	13.72	320	19.65	640	27.49
7	3.00	39	7.18	155	13.95	330	19.94	650	27.70
8	3.31	40	7.31	160	14.17	340	20.23	660	27.91
9	3.61	41	7.39	165	14.39	350	20.52	670	28.12
10	3.80	42	7.43	170	14.60	360	20.81	680	28.33
11	3.92	43	7.61	175	14.81	370	21.09	690	28.54
12	4.05	44	7.70	180	15.01	380	21.36	700	28.75
13	4.23	45	7.72	185	15.20	390	21.63	720	29.14
14	4.41	46	7.81	190	15.39	400	21.90	740	29.52
15	4.55	47	7.92	195	15.57	410	22.17	760	29.90
16	4.70	48	8.00	200	15.75	420	22.44	780	30.28
17	4.86	49	8.03	205	15.93	430	22.70	800	30.65
18	5.00	50	8.13	210	16.11	440	22.96	820	31.02
19	5.00	55	8.21	215	16.29	450	23.21	840	31.39
20	5.18	60	8.94	220	16.46	460	23.47	860	31.75
21	5.31	65	9.25	225	16.63	470	23.72	880	32.11
22	5.49	70	9.61	230	16.80	480	23.97	900	32.46
23	5.61	75	9.93	235	16.97	490	24.21	920	32.80
24	5.72	80	10.20	240	17.14	500	24.45	940	33.14
25	5.81	85	10.46	245	17.30	510	24.68	960	33.48
26	5.92	90	10.73	250	17.46	520	24.91	980	33.82
27	6.00	95	11.15	255	17.63	530	25.13	1000	34.15
28	6.13	100	11.34	260	17.79	540	25.35	1100	35.75
29	6.23	105	11.60	265	17.95	550	25.57	1200	37.30
30	6.40	110	11.85	270	18.11	560	25.79	1300	38.80
31	6.44	115	12.10	275	18.27	570	26.01	1400	40.20
32	6.55	120	12.34	280	18.43	580	26.23	1500	41.60
33	6.70	125	12.57	285	18.59	590	26.44	1600	42.95

Rapid Proof of Multiplications and Divisions

To prove that the product of a multiplication is correct, add together the individual figures in each of the two factors (proceeding to add until a single digit is obtained, as indicated below), and multiply the sum of the figures of one of the factors by the sum of the other factor. This product, reduced by similar addition of digits to one figure, should then equal the sum of the digits in the product.

Example: $3617 \times 2034 = 7{,}356{,}978$.

Adding digits in one factor: $3+6+1+7=17$; $1+7=8$.

Adding digits in other factor: $2+0+3+4=9$.

Multiplying the sums: $8 \times 9 = 72$. Adding digits: $7+2=9$. Adding digits of product: $7+3+5+6+9+7+8=45$; $4+5=9$. The fact that the sum of the digits of the product equals the sum of the digits in the product of the sum of the digits in the factors, is an indication that the product is correct. If the final sums are not equal, the product is incorrect.

A division can be proved in a similar manner, by considering the divisor and quotient as factors, and the dividend as the product.

Example: $131{,}872 \div 317 = 416$.

$3+1+7=11$; $1+1=2$. $4+1+6=11$; $1+1=2$.

Then $2 \times 2 = 4$, and $1+3+1+8+7+2=22$; $2+2=4$.

If there is a remainder, the sum of its digits (reduced to one digit as before) should be added to the product of the final digits of the divisor and quotient. The final digit of this sum should equal the final digit of the dividend.

Useful Constants Multiplied and Divided by 1 to 10

Constant	Multiplied by:							
	2	3	4	5	6	7	8	9
0.7854	1.5708	2.3562	3.1416	3.9270	4.7124	5.4978	6.2832	7.0686
3.1416	6.2832	9.4248	12.566	15.708	18.850	21.991	25.133	28.274
14.7	29.4	44.1	58.8	73.5	88.2	102.9	117.6	132.3
32.16	64.32	96.48	128.64	160.80	192.96	225.12	257.28	289.44
64.32	128.64	192.96	257.28	321.60	385.92	450.24	514.56	578.88
144	288	432	576	720	864	1,008	1,152	1,296
778	1,556	2,334	3,112	3,890	4,668	5,446	6,224	7,002
1,728	3,456	5,184	6,912	8,640	10,368	12,096	13,824	15,552
33,000	66,000	99,000	132,000	165,000	198,000	231,000	264,000	297,000

Constant	Divided by:							
	2	3	4	5	6	7	8	9
0.7854	0.3927	0.2618	0.1964	0.1571	0.1309	0.1122	0.0982	0.0873
3.1416	1.5708	1.0472	0.7854	0.6283	0.5236	0.4488	0.3927	0.3490
14.7	7.350	4.900	3.625	2.940	2.450	2.100	1.838	1.633
32.16	16.080	10.720	8.040	6.432	5.360	4.594	4.020	3.573
64.32	32.160	21.440	16.080	12.864	10.720	9.189	8.040	7.147
144	72	48	36	28.800	24	20.571	18	16
778	389	259.33	194.50	155.60	129.67	111.14	97.25	86.44
1,728	864	576	432	345.60	288	246.86	216	192
33,000	16,500	11,000	8250	6600	5500	4714.3	4125	3666.7

Table of Commonly Used Constants

Constant	Numerical Value	Logarithm	Constant	Numerical Value	Logarithm
π	3.141593	0.49715	Weight in pounds of:		
2π	6.283185	0.79818	Water column, 1″×1″×1 ft.	0.4335	$\bar{1}$.63699
$\pi \div 4$	0.785398	$\bar{1}$.89509	1 U.S. gallon of water, 39.1°F.	8.34	0.92117
π^2	9.869604	0.99430	1 cu. ft. of water, 39.1° F...	62.4245	1.79535
π^3	31.006277	1.49145	1 cu. in. of water, 39.1° F...	0.0361	$\bar{2}$.55751
$1 \div \pi$	0.318310	$\bar{1}$.50285	1 cu. ft. of air, 32° F., atmospheric pressure	0.08073	$\bar{2}$.90703
$1 \div \pi^2$	0.101321	$\bar{1}$.00570	Volume in cu. ft. of:		
$1 \div \pi^3$	0.032252	$\bar{2}$.50855	1 pound of water, 39.1° F...	0.01602	$\bar{2}$.20465
$\sqrt{\pi}$	1.772454	0.24858	1 pound of air, 32° F., atmospheric pressure	12.387	1.09297
$\sqrt[3]{\pi}$	1.464592	0.16572	Volume in gallons of 1 pound of water, 39.1° F.........	0.1199	$\bar{1}$.07883
g	32.16	1.50732	Volume in cu. in. of 1 pound of water, 39.1° F.	27.70	1.44249
g^2	1034.266	3.01463	One cubic ft. in gallons	7.4805	0.87393
$2g$	64.32	1.80835	Atmospheric pressure in pounds per sq. in........	14.696	1.16720
$1 \div 2g$	0.01555	$\bar{2}$.19165			
$\sqrt{2g}$	8.01998	0.90417			
$1 \div \sqrt{g}$	0.17634	$\bar{1}$.24635			
$\pi \div \sqrt{g}$	0.55399	$\bar{1}$.74350			
e	2.71828	0.43429			

Lengths of Chords for Spacing off the Circumference of Circles

On the following pages are given tables of the lengths of chords for spacing off the circumference of circles. The object of these tables is to make possible the division of the periphery into a number of equal parts without trials with the dividers. The first table is calculated for circles having a diameter equal to 1. For circles of other diameters, the length of chord given in the table should be multiplied by the diameter of the circle. This first table may be used by toolmakers when setting "buttons" in circular formation. Assume that it is required to divide the periphery of a circle of 20 inches diameter into thirty-two equal parts. From the table the length of the chord is found to be 0.098017 inch, if the diameter of the circle were 1 inch. With a diameter of 20 inches the length of the chord for one division would be $20 \times 0.098017 = 1.9603$ inch.

The two following pages give an additional table for the spacing off of circles, the table, in this case, being worked out for diameters from 1/16 inch to 14 inches. As an example, assume that it is required to divide a circle having a diameter of 6½ inches into seven equal parts. Find first, in the column headed "6" and in line with 7 divisions, the length of the chord for a 6-inch circle, which is 2.604 inches. Then find the length of the chord for a ½-inch diameter circle, 7 divisions, which is 0.217. The sum of these two values, $2.604 + 0.217 = 2.821$ inches, is the length of the chord required for spacing off the circumference of a 6½-inch circle into seven equal divisions.

As another example, assume that it is required to divide a circle having a diameter of 9 23/32 inches into 15 equal divisions. First find the length of the chord for a 9-inch circle, which is 1.871 inch. The length of the chord for a 23/32-inch circle can easily be estimated from the table by taking the value that is exactly between those given for 11/16 and ¾ inch. The value for 11/16 inch is 0.143, and for ¾ inch, 0.156. Hence for 23/32, the value would be 0.150. Then, $1.871 + 0.150 = 2.021$ inches.

Lengths of Chords for Spacing Off the Circumference of Circles with a Diameter Equal to 1

For Circles of Other Diameters Multiply Length Given in Table by Diameter of Circle

No. of Spaces	Length of Chord	No. of Spaces	Length of Chord	No. of Spaces	Length of Chord	No. of Spaces	Length of Chord
3	0.866025	51	0.061560	99	0.031727	147	0.021369
4	0.707106	52	0.060378	100	0.031410	148	0.021225
5	0.587785	53	0.059240	101	0.031099	149	0.021082
6	0.500000	54	0.058144	102	0.030795	150	0.020942
7	0.433883	55	0.057088	103	0.030496	151	0.020803
8	0.382683	56	0.056070	104	0.030202	152	0.020666
9	0.342020	57	0.055087	105	0.029915	153	0.020531
10	0.309017	58	0.054138	106	0.029633	154	0.020398
11	0.281732	59	0.053222	107	0.029356	155	0.020266
12	0.258819	60	0.052336	108	0.029084	156	0.020137
13	0 239315	61	0.051478	109	0.028817	157	0.020008
14	0.222520	62	0.050649	110	0.028556	158	0.019882
15	0.207911	63	0.049845	111	0.028296	159	0.019757
16	0.195090	64	0.049067	112	0.028046	160	0.019633
17	0.183749	65	0.048313	113	0.027798	161	0.019511
18	0.173648	66	0.047581	114	0.027554	162	0.019391
19	0.164594	67	0.046872	115	0.027314	163	0.019272
20	0.156434	68	0.046183	116	0.027079	164	0.019154
21	0.149042	69	0.045514	117	0.026847	165	0.019038
22	0.142314	70	0.044864	118	0.026620	166	0.018924
23	0.136166	71	0.044233	119	0.026396	167	0.018810
24	0.130526	72	0.043619	120	0.026176	168	0.018698
25	0.125333	73	0.043022	121	0.025960	169	0.018588
26	0.120536	74	0.042441	122	0.025747	170	0.018478
27	0.116092	75	0.041875	123	0.025538	171	0.018370
28	0.111964	76	0.041324	124	0.025332	172	0.018264
29	0.108118	77	0.040788	125	0.025130	173	0.018158
30	0.104528	78	0.040265	126	0.024930	174	0.018054
31	0.101168	79	0.039756	127	0.024734	175	0.017950
32	0.098017	80	0.039259	128	0.024541	176	0.017848
33	0.095056	81	0.038775	129	0.024350	177	0.017748
34	0.092268	82	0.038302	130	0.024163	178	0.017648
35	0.089639	83	0.037841	131	0.023979	179	0.017549
36	0.087155	84	0 037391	132	0.023797	180	0.017452
37	0.084805	85	0.036951	133	0.023618	181	0.017355
38	0.082579	86	0.036522	134	0.023442	182	0.017260
39	0.080466	87	0.036102	135	0.023268	183	0.017166
40	0.078459	88	0.035692	136	0.023097	184	0.017073
41	0.076549	89	0.035291	137	0.022929	185	0.016980
42	0.074730	90	0.034899	138	0.022763	186	0.016889
43	0.072995	91	0.034516	139	0.022599	187	0.016799
44	0.071339	92	0.034141	140	0.022438	188	0.016709
45	0.069756	93	0.033774	141	0.022278	189	0.016621
46	0.068242	94	0.033414	142	0.022122	190	0.016533
47	0.066792	95	0.033063	143	0.021967	191	0.016447
48	0.065403	96	0.032719	144	0.021814	192	0.016361
49	0.064070	97	0.032381	145	0.021664	193	0.016276
50	0.062790	98	0.032051	146	0.021516	194	0.016193

Table for Spacing Off the Circumference of Circles

(See page 76 for explanatory matter.)

No. of Divisions	Degrees in Arc	Diameter of Circle to be Spaced Off														
		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16
		Length of Chord														
3	120	0.054	0.108	0.162	0.216	0.270	0.324	0.378	0.432	0.486	0.541	0.595	0.649	0.703	0.757	0.811
4	90	0.044	0.088	0.132	0.176	0.221	0.265	0.309	0.353	0.397	0.442	0.486	0.530	0.574	0.618	0.662
5	72	0.037	0.073	0.110	0.146	0.183	0.220	0.257	0.293	0.330	0.367	0.403	0.440	0.477	0.514	0.551
6	60	0.031	0.063	0.094	0.125	0.156	0.188	0.219	0.250	0.281	0.312	0.343	0.375	0.406	0.438	0.469
7	51 4/7	0.027	0.054	0.081	0.108	0.135	0.162	0.189	0.217	0.244	0.271	0.298	0.325	0.352	0.379	0.406
8	45	0.024	0.048	0.072	0.096	0.120	0.143	0.167	0.191	0.215	0.239	0.263	0.287	0.311	0.335	0.359
9	40	0.021	0.043	0.064	0.086	0.107	0.128	0.149	0.171	0.192	0.214	0.235	0.257	0.278	0.299	0.320
10	36	0.019	0.039	0.058	0.077	0.097	0.116	0.135	0.155	0.174	0.193	0.212	0.232	0.251	0.270	0.289
11	32 8/11	0.018	0.035	0.053	0.070	0.088	0.105	0.123	0.141	0.158	0.176	0.193	0.211	0.228	0.246	0.264
12	30	0.016	0.032	0.048	0.065	0.081	0.097	0.114	0.130	0.146	0.162	0.178	0.194	0.211	0.227	0.243
13	27 9/13	0.015	0.030	0.045	0.059	0.074	0.089	0.104	0.119	0.134	0.149	0.164	0.179	0.194	0.209	0.224
14	25 5/7	0.014	0.028	0.042	0.056	0.069	0.083	0.097	0.111	0.125	0.139	0.153	0.167	0.180	0.194	0.208
15	24	0.013	0.026	0.039	0.052	0.065	0.078	0.091	0.104	0.117	0.130	0.143	0.156	0.169	0.182	0.195
16	22 1/2	0.012	0.024	0.037	0.049	0.061	0.073	0.085	0.098	0.109	0.122	0.134	0.146	0.158	0.170	0.183
17	21 3/17	0.011	0.023	0.034	0.046	0.057	0.069	0.080	0.092	0.103	0.115	0.126	0.138	0.149	0.160	0.172
18	20	0.011	0.022	0.032	0.043	0.054	0.065	0.076	0.087	0.097	0.108	0.119	0.130	0.141	0.152	0.163
19	18 18/19	0.010	0.021	0.031	0.041	0.051	0.062	0.072	0.082	0.092	0.103	0.113	0.123	0.133	0.144	0.154
20	18	0.010	0.020	0.029	0.039	0.049	0.059	0.068	0.078	0.088	0.098	0.107	0.117	0.127	0.136	0.146
21	17 1/7	0.009	0.019	0.028	0.037	0.047	0.056	0.065	0.075	0.084	0.093	0.102	0.112	0.121	0.130	0.139
22	16 4/11	0.009	0.018	0.027	0.036	0.045	0.053	0.062	0.071	0.080	0.089	0.098	0.107	0.115	0.124	0.133
23	15 15/23	0.009	0.017	0.026	0.034	0.043	0.051	0.059	0.068	0.077	0.085	0.094	0.102	0.111	0.119	0.128
24	15	0.008	0.016	0.024	0.033	0.041	0.049	0.057	0.065	0.073	0.082	0.090	0.098	0.106	0.114	0.122
25	14 2/5	0.008	0.016	0.023	0.031	0.039	0.047	0.055	0.063	0.070	0.078	0.086	0.094	0.102	0.109	0.117
26	13 11/13	0.008	0.015	0.023	0.030	0.038	0.045	0.053	0.060	0.068	0.075	0.083	0.090	0.098	0.105	0.113
28	12 6/7	0.007	0.014	0.021	0.028	0.035	0.042	0.049	0.056	0.063	0.070	0.077	0.084	0.091	0.098	0.105
30	12	0.007	0.013	0.019	0.026	0.033	0.039	0.046	0.052	0.059	0.065	0.072	0.078	0.085	0.091	0.098
32	11 1/4	0.006	0.012	0.018	0.024	0.031	0.037	0.043	0.049	0.055	0.061	0.067	0.074	0.080	0.086	0.092

Table for Spacing Off the Circumference of Circles

No. of Divisions	Degrees in Arc	Diameter of Circle to be Spaced Off													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
		Length of Chord													
3	120	0.866	1.732	2.598	3.464	4.330	5.196	6.062	6.928	7.794	8.660	9.526	10.392	11.258	12.124
4	90	0.707	1.414	2.121	2.828	3.536	4.243	4.950	5.657	6.364	7.071	7.778	8.485	9.192	9.900
5	72	0.588	1.176	1.763	2.351	2.939	3.527	4.115	4.702	5.290	5.878	6.465	7.053	7.641	8.229
6	60	0.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	5.500	6.000	6.500	7.000
7	51 3/7	0.434	0.868	1.302	1.736	2.170	2.604	3.037	3.471	3.905	4.339	4.773	5.207	5.641	6.075
8	45	0.383	0.765	1.148	1.531	1.913	2.296	2.679	3.061	3.444	3.827	4.210	4.592	4.975	5.358
9	40	0.342	0.684	1.026	1.368	1.710	2.052	2.394	2.736	3.078	3.420	3.762	4.104	4.446	4.788
10	36	0.309	0.618	0.927	1.236	1.545	1.854	2.163	2.472	2.781	3.090	3.399	3.708	4.017	4.326
11	32 8/11	0.282	0.564	0.845	1.127	1.409	1.691	1.973	2.254	2.536	2.818	3.100	3.381	3.663	3.945
12	30	0.259	0.518	0.776	1.035	1.294	1.553	1.812	2.070	2.329	2.588	2.847	3.106	3.365	3.624
13	27 9/13	0.239	0.479	0.718	0.958	1.197	1.436	1.676	1.915	2.154	2.393	2.633	2.873	3.112	3.352
14	25 5/7	0.222	0.445	0.667	0.890	1.112	1.334	1.557	1.779	2.001	2.224	2.446	2.669	2.891	3.114
15	24	0.208	0.416	0.624	0.832	1.040	1.247	1.455	1.663	1.871	2.079	2.287	2.495	2.703	2.911
16	22 1/2	0.195	0.390	0.585	0.780	0.975	1.171	1.366	1.561	1.756	1.951	2.146	2.341	2.536	2.731
17	21 3/17	0.184	0.367	0.551	0.735	0.918	1.102	1.286	1.469	1.653	1.837	2.020	2.204	2.388	2.571
18	20	0.174	0.347	0.521	0.695	0.868	1.041	1.215	1.389	1.563	1.736	1.910	2.084	2.257	2.431
19	18 18/19	0.165	0.329	0.493	0.658	0.822	0.987	1.151	1.316	1.480	1.645	1.809	1.974	2.138	2.303
20	18	0.156	0.313	0.469	0.626	0.782	0.938	1.095	1.251	1.408	1.564	1.721	1.877	2.033	2.190
21	17 1/7	0.149	0.298	0.447	0.596	0.745	0.894	1.043	1.192	1.341	1.490	1.639	1.788	1.936	2.085
22	16 4/11	0.142	0.286	0.428	0.570	0.712	0.855	0.997	1.139	1.281	1.423	1.566	1.708	1.850	1.993
23	15 15/23	0.136	0.273	0.409	0.545	0.681	0.818	0.954	1.091	1.227	1.362	1.499	1.635	1.772	1.908
24	15	0.131	0.261	0.392	0.522	0.653	0.783	0.914	1.044	1.175	1.305	1.436	1.566	1.697	1.827
25	14 2/5	0.125	0.251	0.376	0.501	0.627	0.752	0.877	1.003	1.128	1.253	1.379	1.504	1.629	1.755
26	13 11/13	0.120	0.241	0.361	0.482	0.602	0.723	0.843	0.964	1.084	1.205	1.325	1.445	1.566	1.686
28	12 6/7	0.112	0.224	0.336	0.448	0.560	0.672	0.784	0.896	1.008	1.120	1.232	1.344	1.456	1.568
30	12	0.105	0.209	0.314	0.418	0.523	0.627	0.732	0.836	0.941	1.045	1.150	1.254	1.359	1.463
32	11 1/4	0.098	0.196	0.294	0.392	0.490	0.588	0.686	0.784	0.882	0.980	1.078	1.176	1.274	1.372

Formulas and Table for Regular Polygons

N = number of sides.
S = length of side.
R = radius of circumscribed circle.
r = radius of inscribed circle.
A = area of polygon.
$\alpha = 180° \div N$ = one-half center angle of one side.

Formulas:

$$A = \frac{N \times \cot\alpha \times S^2}{4} = N \times \sin\alpha \times \cos\alpha \times R^2 = N \times \tan\alpha \times r^2$$

$$R = \frac{S}{2\sin\alpha} = \frac{r}{\cos\alpha} \qquad S = 2R \times \sin\alpha = 2r \times \tan\alpha$$

$$r = \frac{S \times \cot\alpha}{2} = R \times \cos\alpha$$

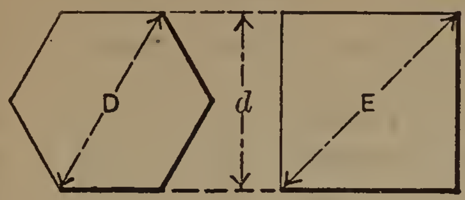
Examples of Use of Table.

A regular hexagon is inscribed in a circle of 6 inches diameter. Find the area and the radius of the inscribed circle. — Here $R = 3$. From the table, area $(A) = 2.5981\ R^2 = 2.5981 \times 9 = 23.3829$ square inches. Radius of inscribed circle, $r = 0.866\ R = 0.866 \times 3 = 2.598$ inches.

Thirty-two bolts are to be equally spaced on the periphery of a bolt-circle, 16 inches in diameter. Find the chordal distance between the bolts. — Chordal distance equals the side (S) of a polygon with 32 sides. $R = 8$. Hence, $S = 0.196\ R = 0.196 \times 8 = 1.568$ inch.

No. of Sides	$A=$	$A=$	$A=$	$R=$	$R=$	$S=$	$S=$	$r=$	$r=$	No. of Sides
3	0.4330 S^2	1.2990 R^2	5.1962 r^2	0.5774 S	2.0000 r	1.7321 R	3.4641 r	0.5000 R	0.2887 S	3
4	1.0000 S^2	2.0000 R^2	4.0000 r^2	0.7071 S	1.4142 r	1.4142 R	2.0000 r	0.7071 R	0.5000 S	4
5	1.7205 S^2	2.3776 R^2	3.6327 r^2	0.8507 S	1.2361 r	1.1756 R	1.4531 r	0.8090 R	0.6882 S	5
6	2.5981 S^2	2.5981 R^2	3.4641 r^2	1.0000 S	1.1547 r	1.0000 R	1.1547 r	0.8660 R	0.8660 S	6
7	3.6339 S^2	2.7364 R^2	3.3710 r^2	1.1524 S	1.1099 r	0.8678 R	0.9631 r	0.9010 R	1.0383 S	7
8	4.8284 S^2	2.8284 R^2	3.3137 r^2	1.3066 S	1.0824 r	0.7654 R	0.8284 r	0.9239 R	1.2071 S	8
9	6.1818 S^2	2.8925 R^2	3.2757 r^2	1.4619 S	1.0642 r	0.6840 R	0.7279 r	0.9397 R	1.3737 S	9
10	7.6942 S^2	2.9389 R^2	3.2492 r^2	1.6180 S	1.0515 r	0.6180 R	0.6498 r	0.9511 R	1.5388 S	10
12	11.196 S^2	3.0000 R^2	3.2154 r^2	1.9319 S	1.0353 r	0.5176 R	0.5359 r	0.9659 R	1.8660 S	12
16	20.109 S^2	3.0615 R^2	3.1826 r^2	2.5629 S	1.0196 r	0.3902 R	0.3978 r	0.9808 R	2.5137 S	16
20	31.569 S^2	3.0902 R^2	3.1677 r^2	3.1962 S	1.0125 r	0.3129 R	0.3168 r	0.9877 R	3.1569 S	20
24	45.575 S^2	3.1058 R^2	3.1597 r^2	3.8306 S	1.0086 r	0.2611 R	0.2633 r	0.9914 R	3.7979 S	24
32	81.225 S^2	3.1214 R^2	3.1517 r^2	5.1011 S	1.0048 r	0.1960 R	0.1970 r	0.9952 R	5.0766 S	32
48	183.08 S^2	3.1326 R^2	3.1461 r^2	7.6449 S	1.0021 r	0.1308 R	0.1311 r	0.9979 R	7.6285 S	48
64	325.69 S^2	3.1365 R^2	3.1441 r^2	10.190 S	1.0012 r	0.0981 R	0.0983 r	0.9988 R	10.178 S	64

Distance Across Corners of Squares and Hexagons



$D = 1.1547\,d$

$E = 1.4142\,d$

d	D	E	d	D	E	d	D	E
1/4	0.2886	0.3535	1 1/4	1.4434	1.7677	2 5/16	2.6702	3.2703
9/32	0.3247	0.3977	1 9/32	1.4794	1.8119	2 3/8	2.7424	3.3587
5/16	0.3608	0.4419	1 5/16	1.5155	1.8561	2 7/16	2.8145	3.4471
11/32	0.3968	0.4861	1 11/32	1.5516	1.9003	2 1/2	2.8867	3.5355
3/8	0.4329	0.5303	1 3/8	1.5877	1.9445	2 9/16	2.9589	3.6239
13/32	0.4690	0.5745	1 13/32	1.6238	1.9887	2 5/8	3.0311	3.7123
7/16	0.5051	0.6187	1 7/16	1.6598	2.0329	2 11/16	3.1032	3.8007
15/32	0.5412	0.6629	1 15/32	1.6959	2.0771	2 3/4	3.1754	3.8891
1/2	0.5773	0.7071	1 1/2	1.7320	2.1213	2 13/16	3.2476	3.9794
17/32	0.6133	0.7513	1 17/32	1.7681	2.1655	2 7/8	3.3197	4.0658
9/16	0.6494	0.7955	1 9/16	1.8042	2.2097	2 15/16	3.3919	4.1542
19/32	0.6855	0.8397	1 19/32	1.8403	2.2539	3	3.4641	4.2426
5/8	0.7216	0.8839	1 5/8	1.8764	2.2981	3 1/16	3.5362	4.3310
21/32	0.7576	0.9281	1 21/32	1.9124	2.3423	3 1/8	3.6084	4.4194
11/16	0.7937	0.9723	1 11/16	1.9485	2.3865	3 3/16	3.6806	4.5078
23/32	0.8298	1.0164	1 23/32	1.9846	2.4306	3 1/4	3.7527	4.5962
3/4	0.8659	1.0606	1 3/4	2.0207	2.4708	3 5/16	3.8249	4.6846
25/32	0.9020	1.1048	1 25/32	2.0568	2.5190	3 3/8	3.8971	4.7729
13/16	0.9380	1.1490	1 13/16	2.0929	2.5632	3 7/16	3.9692	4.8613
27/32	0.9741	1.1932	1 27/32	2.1289	2.6074	3 1/2	4.0414	4.9497
7/8	1.0102	1.2374	1 7/8	2.1650	2.6516	3 9/16	4.1136	5.0381
29/32	1.0463	1.2816	1 29/32	2.2011	2.6958	3 5/8	4.1857	5.1265
15/16	1.0824	1.3258	1 15/16	2.2372	2.7400	3 11/16	4.2579	5.2149
31/32	1.1184	1.3700	1 31/32	2.2733	2.7842	3 3/4	4.3301	5.3033
1	1.1547	1.4142	2	2.3094	2.8284	3 13/16	4.4023	5.3917
1 1/32	1.1907	1.4584	2 1/32	2.3453	2.8726	3 7/8	4.4744	5.4801
1 1/16	1.2268	1.5026	2 1/16	2.3815	2.9168	3 15/16	4.5466	5.5684
1 3/32	1.2629	1.5468	2 3/32	2.4176	2.9610	4	4.6188	5.6568
1 1/8	1.2990	1.5910	2 1/8	2.4537	3.0052	4 1/8	4.7631	5.8336
1 5/32	1.3351	1.6352	2 5/32	2.4898	3.0494	4 1/4	4.9074	6.0104
1 3/16	1.3712	1.6793	2 3/16	2.5259	3.0936	4 3/8	5.0518	6.1872
1 7/32	1.4073	1.7235	2 1/4	2.5981	3.1820	4 1/2	5.1961	6.3639

Surface and Volume of Spheres

d = diameter. Surface = $3.1416\, d^2$. Volume = $0.5236\, d^3$.

Diam.	Surface	Volume	Diam.	Surface	Volume	Diam.	Surface	Volume
1/64	0.00077	0.000002	2	12.566	4.1888	6 1/2	132.73	143.79
1/32	0.00307	0.00002	2 1/16	13.364	4.5939	6 5/8	137.89	152.25
1/16	0.01227	0.00013	2 1/8	14.186	5.0243	6 3/4	143.14	161.03
3/32	0.02761	0.00043	2 3/16	15.033	5.4809	6 7/8	148.49	170.14
1/8	0.04909	0.00102	2 1/4	15.904	5.9641	7	153.94	179.59
5/32	0.07670	0.00200	2 5/16	16.800	6.4751	7 1/8	159.49	189.39
3/16	0.11045	0.00345	2 3/8	17.721	7.0144	7 1/4	165.13	199.53
7/32	0.15033	0.00548	2 7/16	18.666	7.5829	7 3/8	170.87	210.03
1/4	0.19635	0.00818	2 1/2	19.635	8.1813	7 1/2	176.71	220.89
9/32	0.24851	0.01165	2 9/16	20.629	8.8103	7 5/8	182.66	232.13
5/16	0.30680	0.01598	2 5/8	21.648	9.4708	7 3/4	188.69	243.73
11/32	0.37123	0.02127	2 11/16	22.691	10.164	7 7/8	194.83	255.72
3/8	0.44179	0.02761	2 3/4	23.758	10.889	8	201.06	268.08
13/32	0.51848	0.03511	2 13/16	24.850	11.649	8 1/8	207.39	280.85
7/16	0.60132	0.04385	2 7/8	25.967	12.443	8 1/4	213.82	294.01
15/32	0.69028	0.05393	2 15/16	27.109	13.272	8 3/8	220.36	307.58
1/2	0.78540	0.06545	3	28.274	14.137	8 1/2	226.98	321.56
17/32	0.88664	0.07850	3 1/16	29.465	15.039	8 5/8	233.71	335.95
9/16	0.99403	0.09319	3 1/8	30.680	15.979	8 3/4	240.53	350.77
19/32	1.1075	0.10960	3 3/16	31.919	16.957	8 7/8	247.45	366.02
5/8	1.2272	0.12783	3 1/4	33.183	17.974	9	254.47	381.70
21/32	1.3530	0.14798	3 5/16	34.472	19.031	9 1/8	261.59	397.83
11/16	1.4849	0.17014	3 3/8	35.784	20.129	9 1/4	268.81	414.41
23/32	1.6230	0.19442	3 7/16	37.122	21.268	9 3/8	276.12	431.44
3/4	1.7671	0.22089	3 1/2	38.484	22.449	9 1/2	283.53	448.92
25/32	1.9175	0.24967	3 5/8	41.283	24.942	9 5/8	291.04	466.87
13/16	2.0739	0.28084	3 3/4	44.179	27.611	9 3/4	298.65	485.31
27/32	2.2365	0.31451	3 7/8	47.173	30.466	9 7/8	306.36	504.21
7/8	2.4053	0.35077	4	50.265	33.510	10	314.16	523.60
29/32	2.5802	0.38971	4 1/8	53.456	36.751	10 1/4	330.06	563.86
15/16	2.7611	0.43143	4 1/4	56.745	40.195	10 1/2	346.36	606.13
31/32	2.9483	0.47603	4 3/8	60.133	43.847	10 3/4	363.05	650.46
1	3.1416	0.52360	4 1/2	63.617	47.713	11	380.13	696.91
1 1/16	3.5466	0.62804	4 5/8	67.201	51.801	11 1/4	397.61	745.51
1 1/8	3.9761	0.74551	4 3/4	70.883	56.116	11 1/2	415.48	796.33
1 3/16	4.4301	0.87681	4 7/8	74.663	60.663	11 3/4	433.73	849.40
1 1/4	4.9088	1.0227	5	78.540	65.450	12	452.39	904.78
1 5/16	5.4119	1.1839	5 1/8	82.516	70.482	12 1/4	471.44	962.52
1 3/8	5.9396	1.3611	5 1/4	86.591	75.767	12 1/2	490.87	1022.7
1 7/16	6.4919	1.5553	5 3/8	90.763	81.308	12 3/4	510.71	1085.3
1 1/2	7.0686	1.7671	5 1/2	95.033	87.113	13	530.93	1150.3
1 9/16	7.6699	1.9974	5 5/8	99.401	93.189	13 1/4	551.55	1218.0
1 5/8	8.2957	2.2468	5 3/4	103.87	99.541	13 1/2	572.55	1288.3
1 11/16	8.9461	2.5161	5 7/8	108.44	106.18	13 3/4	593.95	1361.2
1 3/4	9.6211	2.8062	6	113.10	113.10	14	615.75	1436.8
1 13/16	10.321	3.1177	6 1/8	117.87	120.31	14 1/4	637.95	1515.1
1 7/8	11.044	3.4514	6 1/4	122.72	127.83	14 1/2	660.52	1596.3
1 15/16	11.793	3.8083	6 3/8	127.68	135.66	14 3/4	683.49	1680.3

Surface and Volume of Spheres

Diam.	Surface	Volume	Diam.	Surface	Volume	Diam.	Surface	Volume
15	706.85	1,767.2	27½	2375.8	10,889	51	8,171.2	69,456
15¼	730.63	1,857.0	27¾	2419.2	11,189	51½	8,332.3	71,519
15½	754.77	1,949.8	28	2463.0	11,494	52	8,494.8	73,622
15¾	779.32	2,045.7	28¼	2507.2	11,805	52½	8,658.9	75,767
16	804.25	2,144.7	28½	2551.8	12,121	53	8,824.8	77,952
16¼	829.57	2,246.8	28¾	2596.7	12,443	53½	8,992.0	80,178
16½	855.29	2,352.1	29	2642.1	12,770	54	9,160.8	82,448
16¾	881.42	2,460.6	29½	2734.0	13,442	54½	9,331.2	84,760
17	907.93	2,572.4	30	2827.4	14,137	55	9,503.2	87,114
17¼	934.83	2,687.6	30½	2922.5	14,856	55½	9,676.8	89,511
17½	962.12	2,806.2	31	3019.1	15,599	56	9,852.0	91,953
17¾	989.80	2,928.2	31½	3117.3	16,366	56½	10,029	94,438
18	1017.9	3,053.6	32	3217.0	17,157	57	10,207	96,967
18¼	1046.4	3,182.6	32½	3318.3	17,974	57½	10,387	99,541
18½	1075.2	3,315.3	33	3421.2	18,817	58	10,568	102,161
18¾	1104.5	3,451.5	33½	3525.7	19,685	58½	10,751	104,826
19	1134.1	3,591.4	34	3631.7	20,580	59	10,936	107,536
19¼	1164.2	3,735.0	34½	3739.3	21,501	59½	11,122	110,294
19½	1194.6	3,882.5	35	3848.5	22,449	60	11,310	113,098
19¾	1225.4	4,033.7	35½	3959.2	23,425	60½	11,499	115,949
20	1256.7	4,188.8	36	4071.5	24,429	61	11,690	118,847
20¼	1288.3	4,347.8	36½	4185.5	25,461	61½	11,882	121,794
20½	1320.3	4,510.9	37	4300.9	26,522	62	12,076	124,789
20¾	1352.7	4,677.9	37½	4417.9	27,612	62½	12,272	127,832
21	1385.5	4,849.1	38	4536.5	28,731	63	12,469	130,925
21¼	1418.6	5,024.3	38½	4656.7	29,880	63½	12,668	134,067
21½	1452.2	5,203.7	39	4778.4	31,059	64	12,868	137,259
21¾	1486.2	5,387.4	39½	4901.7	32,270	64½	13,070	140,501
22	1520.5	5,575.3	40	5026.5	33,510	65	13,273	143,794
22¼	1555.3	5,767.6	40½	5153.1	34,783	65½	13,478	147,138
22½	1590.4	5,964.1	41	5281.1	36,087	66	13,685	150,533
22¾	1626.0	6,165.2	41½	5410.7	37,423	66½	13,893	153,980
23	1661.9	6,370.6	42	5541.9	38,792	67	14,103	157,480
23¼	1698.2	6,580.6	42½	5674.5	40,194	67½	14,314	161,032
23½	1735.0	6,795.2	43	5808.8	41,630	68	14,527	164,637
23¾	1772.1	7,014.3	43½	5944.7	43,099	68½	14,741	168,295
24	1809.6	7,238.2	44	6082.1	44,602	69	14,957	172,007
24¼	1847.5	7,466.7	44½	6221.2	46,141	69½	15,175	175,774
24½	1885.8	7,700.1	45	6361.7	47,713	70	15,394	179,595
24¾	1924.4	7,938.3	45½	6503.9	49,321	70½	15,615	183,471
25	1963.5	8,181.3	46	6647.6	50,965	71	15,837	187,402
25¼	2002.9	8,429.2	46½	6792.9	52,645	71½	16,061	191,389
25½	2042.8	8,682.0	47	6939.9	54,362	72	16,286	195,433
25¾	2083.0	8,939.9	47½	7088.3	56,115	72½	16,513	199,532
26	2123.7	9,202.8	48	7238.3	57,906	73	16,742	203,689
26¼	2164.7	9,470.8	48½	7389.9	59,734	73½	16,972	207,903
26½	2206.2	9,744.0	49	7543.1	61,601	74	17,204	212,175
26¾	2248.0	10,022	49½	7697.7	63,506	74½	17,437	216,505
27	2290.2	10,306	50	7854.0	65,450	75	17,672	220,894
27¼	2332.8	10,595	50½	8011.8	67,433	75½	17,908	225,341

Surface and Volume of Spheres

Diam.	Surface	Volume	Diam.	Surface	Volume	Diam.	Surface	Volume
76	18,146	229,848	101	32,047	539,464	151	71,631	1,802,725
76½	18,386	234,414	102	32,685	555,647	152	72,583	1,838,778
77	18,626	239,041	103	33,329	572,150	153	73,542	1,875,309
77½	18,869	243,728	104	33,979	588,977	154	74,506	1,912,321
78	19,114	248,475	105	34,636	606,131	155	75,477	1,949,816
78½	19,360	253,284	106	35,299	623,614	156	76,454	1,987,799
79	19,607	258,155	107	35,968	641,431	157	77,437	2,026,271
79½	19,856	263,088	108	36,644	659,584	158	78,427	2,065,237
80	20,106	268,083	109	37,325	678,076	159	79,423	2,104,699
80½	20,358	273,141	110	38,013	696,910	160	80,425	2,144,660
81	20,612	278,263	111	38,708	716,090	161	81,433	2,185,125
81½	20,867	283,447	112	39,408	735,619	162	82,448	2,226,094
82	21,124	288,696	113	40,115	755,499	163	83,469	2,267,574
82½	21,382	294,010	114	40,828	775,735	164	84,496	2,309,565
83	21,642	299,388	115	41,548	796,328	165	85,530	2,352,071
83½	21,904	304,831	116	42,273	817,283	166	86,569	2,395,096
84	22,167	310,340	117	43,005	838,603	167	87,616	2,438,642
84½	22,432	315,915	118	43,744	860,289	168	88,668	2,482,713
85	22,698	321,556	119	44,488	882,347	169	89,729	2,527,311
85½	22,966	327,264	120	45,239	904,779	170	90,792	2,572,441
86	23,235	333,039	121	45,996	927,587	171	91,863	2,618,104
86½	23,506	338,882	122	46,759	950,776	172	92,941	2,664,305
87	23,779	344,792	123	47,529	974,348	173	94,025	2,711,046
87½	24,053	350,771	124	48,305	998,306	174	95,115	2,758,331
88	24,328	356,819	125	49,087	1,022,654	175	96,211	2,806,162
88½	24,606	362,935	126	49,876	1,047,394	176	97,314	2,854,543
89	24,885	369,122	127	50,671	1,072,531	177	98,423	2,903,477
89½	25,165	375,378	128	51,472	1,098,066	178	99,538	2,952,967
90	25,447	381,704	129	52,279	1,124,004	179	100,660	3,003,006
90½	25,730	388,102	130	53,093	1,150,347	180	101,788	3,053,628
91	26,016	394,570	131	53,913	1,177,098	181	102,922	3,104,805
91½	26,302	401,109	132	54,739	1,204,260	182	104,062	3,156,551
92	26,590	407,721	133	55,572	1,231,838	183	105,209	3,208,869
92½	26,880	414,405	134	56,410	1,259,833	184	106,362	3,261,761
93	27,172	421,161	135	57,256	1,288,249	185	107,521	3,315,231
93½	27,464	427,991	136	58,107	1,317,090	186	108,687	3,369,282
94	27,759	434,894	137	58,965	1,346,357	187	109,858	3,423,919
94½	28,055	441,871	138	59,829	1,376,055	188	111,090	3,479,142
95	28,353	448,920	139	60,699	1,406,187	189	112,221	3,534,956
95½	28,652	456,047	140	61,575	1,436,755	190	113,411	3,591,364
96	28,953	463,248	141	62,458	1,467,763	191	114,609	3,648,369
96½	29,255	470,524	142	63,347	1,499,214	192	115,812	3,705,973
97	29,559	477,874	143	64,242	1,531,112	193	117,021	3,764,181
97½	29,865	485,302	144	65,144	1,563,457	194	118,237	3,822,996
98	30,172	492,808	145	66,052	1,596,256	195	119,459	3,882,419
98½	30,481	500,388	146	66,966	1,629,511	196	120,687	3,942,456
99	30,791	508,047	147	67,887	1,663,224	197	121,922	4,003,108
99½	31,103	515,785	148	68,813	1,697,398	198	123,163	4,064,379
100	31,416	523,598	149	69,747	1,732,038	199	124,420	4,126,272
100½	31,731	531,492	150	70,686	1,767,146	200	125,664	4,188,790

Table for Finding Volume of Spherical Segments

Multiply factor C in table by the cube of the length of the chord of the segment; the product equals the volume.

Center Angle of Segment, Deg.	C	Center Angle of Segment, Deg.	C	Center Angle of Segment, Deg.	C	Center Angle of Segment, Deg.	C	Center Angle of Segment, Deg.	C
3	0.0026	39	0.0341	75	0.0692	111	0.1128	147	0.1739
6	0.0051	42	0.0368	78	0.0724	114	0.1171	150	0.1802
9	0.0077	45	0.0396	81	0.0757	117	0.1215	153	0.1869
12	0.0103	48	0.0424	84	0.0791	120	0.1260	156	0.1937
15	0.0129	51	0.0452	87	0.0825	123	0.1306	159	0.2010
18	0.0155	54	0.0480	90	0.0860	126	0.1354	162	0.2085
21	0.0181	57	0.0509	93	0.0895	129	0.1403	165	0.2163
24	0.0207	60	0.0539	96	0.0932	132	0.1454	168	0.2246
27	0.0233	63	0.0568	99	0.0969	135	0.1507	171	0.2332
30	0.0260	66	0.0599	102	0.1008	138	0.1562	174	0.2423
33	0.0287	69	0.0629	105	0.1047	141	0.1619	177	0.2518
36	0.0314	72	0.0660	108	0.1087	144	0.1678	180	0.2618

Example: Find the volume of a spherical segment having a center angle of 30 degrees, if the length of the chord is 10 inches.

$$10^3 \times 0.026 = 1000 \times 0.026 = 26 \text{ cubic inches}$$

Prime Numbers — Factors

The *factors* of a given number are those numbers which when multiplied together give a product equal to that number; thus, 2 and 3 are factors of 6; and 5 and 7 are factors of 35.

A *prime number* is one which has no factors except itself and 1. Thus, 3, 5, 7, 11, etc., are prime numbers. A factor which is a prime number is called a *prime factor*.

The accompanying "Prime Number and Factor Table" gives the smallest prime factor of all odd numbers from 1 to 9600, and can be used for finding all the factors for numbers up to this limit. For example, find the factors of 931. In the column headed "900," and in the line indicated by "31" in the left-hand column, the smallest prime factor is found to be 7. As this leaves another factor 133 (since $931 \div 7 = 133$), find the smallest prime factor of this number. In the column headed "100" and in the line "33," this is found to be 7, leaving a factor 19. This latter is a prime number; hence, the factors of 931 are $7 \times 7 \times 19$. Where no factor is given for a number in the factor table, it indicates that the number is a prime number. Tables of prime numbers and factors are especially useful in calculating the gearing for unusual gear ratios and for spiral gear generating machines, etc.

For factoring, the following general rules will be found useful:

2 is a factor of any number the right-hand figure of which is an even number or 0. Thus, $28 = 2 \times 14$, and $210 = 2 \times 105$.

3 is a factor of any number the sum of the figures of which is evenly divisible by 3. Thus, 3 is a factor of 1869, because $1 + 8 + 6 + 9 = 24$, and $24 \div 3 = 8$.

4 is a factor of any number the two right-hand figures of which, considered as one number, are evenly divisible by 4. Thus, 1844 has a factor 4, because $44 \div 4 = 11$.

5 is a factor of any number the right-hand figure of which is 0 or 5. Thus, $85 = 5 \times 17$; $70 = 5 \times 14$.

Prime Number and Factor Table

From to	0 100	100 200	200 300	300 400	400 500	500 600	600 700	700 800	800 900	900 1000	1000 1100	1100 1200
1	P	P	3	7	P	3	P	P	3	17	7	3
3	P	P	7	3	13	P	3	19	11	3	17	P
5	P	3	5	5	3	5	5	3	5	5	3	5
7	P	P	3	P	11	3	P	7	3	P	19	3
9	3	P	11	3	P	P	3	P	P	3	P	P
11	P	3	P	P	3	7	13	3	P	P	3	11
13	P	P	3	P	7	3	P	23	3	11	P	3
15	3	5	5	3	5	5	3	5	5	3	5	5
17	P	3	7	P	3	11	P	3	19	7	3	P
19	P	7	3	11	P	3	P	P	3	P	P	3
21	3	11	13	3	P	P	3	7	P	3	P	19
23	P	3	P	17	3	P	7	3	P	13	3	P
25	5	5	3	5	5	3	5	5	3	5	5	3
27	3	P	P	3	7	17	3	P	P	3	13	7
29	P	3	P	7	3	23	17	3	P	P	3	P
31	P	P	3	P	P	3	P	17	3	7	P	3
33	3	7	P	3	P	13	3	P	7	3	P	11
35	5	3	5	5	3	5	5	3	5	5	3	5
37	P	P	3	P	19	3	7	11	3	P	17	3
39	3	P	P	3	P	7	3	P	P	3	P	17
41	P	3	P	11	3	P	P	3	29	P	3	7
43	P	11	3	7	P	3	P	P	3	23	7	3
45	3	5	5	3	5	5	3	5	5	3	5	5
47	P	3	13	P	3	P	P	3	7	P	3	31
49	7	P	3	P	P	3	11	7	3	13	P	3
51	3	P	P	3	11	19	3	P	23	3	P	P
53	P	3	11	P	3	7	P	3	P	P	3	P
55	5	5	3	5	5	3	5	5	3	5	5	3
57	3	P	P	3	P	P	3	P	P	3	7	13
59	P	3	7	P	3	13	P	3	P	7	3	19
61	P	7	3	19	P	3	P	P	3	31	P	3
63	3	P	P	3	P	P	3	7	P	3	P	P
65	5	3	5	5	3	5	5	3	5	5	3	5
67	P	P	3	P	P	3	23	13	3	P	11	3
69	3	13	P	3	7	P	3	P	11	3	P	7
71	P	3	P	7	3	P	11	3	13	P	3	P
73	P	P	3	P	11	3	P	P	3	7	29	3
75	3	5	5	3	5	5	3	5	5	3	5	5
77	7	3	P	13	3	P	P	3	P	P	3	11
79	P	P	3	P	P	3	7	19	3	11	13	3
81	3	P	P	3	13	7	3	11	P	3	23	P
83	P	3	P	P	3	11	P	3	P	P	3	7
85	5	5	3	5	5	3	5	5	3	5	5	3
87	3	11	7	3	P	P	3	P	P	3	P	P
89	P	3	17	P	3	19	13	3	7	23	3	29
91	7	P	3	17	P	3	P	7	3	P	P	3
93	3	P	P	3	17	P	3	13	19	3	P	P
95	5	3	5	5	3	5	5	3	5	5	3	5
97	P	P	3	P	7	3	17	P	3	P	P	3
99	3	P	13	3	P	P	3	17	29	3	7	11

Prime Number and Factor Table

From to	1200 1300	1300 1400	1400 1500	1500 1600	1600 1700	1700 1800	1800 1900	1900 2000	2000 2100	2100 2200	2200 2300	2300 2400
1	P	P	3	19	P	3	P	P	3	11	31	3
3	3	P	23	3	7	13	3	11	P	3	P	7
5	5	3	5	5	3	5	5	3	5	5	3	5
7	17	P	3	11	P	3	13	P	3	7	P	3
9	3	7	P	3	P	P	3	23	7	3	47	P
11	7	3	17	P	3	29	P	3	P	P	3	P
13	P	13	3	17	P	3	7	P	3	P	P	3
15	3	5	5	3	5	5	3	5	5	3	5	5
17	P	3	13	37	3	17	23	3	P	29	3	7
19	23	P	3	7	P	3	17	19	3	13	7	3
21	3	P	7	3	P	P	3	17	43	3	P	11
23	P	3	P	P	3	P	P	3	7	11	3	23
25	5	5	3	5	5	3	5	5	3	5	5	3
27	3	P	P	3	P	11	3	41	P	3	17	13
29	P	3	P	11	3	7	31	3	P	P	3	17
31	P	11	3	P	7	3	P	P	3	P	23	3
33	3	31	P	3	23	P	3	P	19	3	7	P
35	5	3	5	5	3	5	5	3	5	5	3	5
37	P	7	3	29	P	3	11	13	3	P	P	3
39	3	13	P	3	11	37	3	7	P	3	P	P
41	17	3	11	23	3	P	7	3	13	P	3	P
43	11	17	3	P	31	3	19	29	3	P	P	3
45	3	5	5	3	5	5	3	5	5	3	5	5
47	29	3	P	7	3	P	P	3	23	19	3	P
49	P	19	3	P	17	3	43	P	3	7	13	3
51	3	7	P	3	13	17	3	P	7	3	P	P
53	7	3	P	P	3	P	17	3	P	P	3	13
55	5	5	3	5	5	3	5	5	3	5	5	3
57	3	23	31	3	P	7	3	19	11	3	37	P
59	P	3	P	P	3	P	11	3	29	17	3	7
61	13	P	3	7	11	3	P	37	3	P	7	3
63	3	29	7	3	P	41	3	13	P	3	31	17
65	5	3	5	5	3	5	5	3	5	5	3	5
67	7	P	3	P	P	3	P	7	3	11	P	3
69	3	37	13	3	P	29	3	11	P	3	P	23
71	31	3	P	P	3	7	P	3	19	13	3	P
73	19	P	3	11	7	3	P	P	3	41	P	3
75	3	5	5	3	5	5	3	5	5	3	5	5
77	P	3	7	19	3	P	P	3	31	7	3	P
79	P	7	3	P	23	3	P	P	3	P	43	3
81	3	P	P	3	41	13	3	7	P	3	P	P
83	P	3	P	P	3	P	7	3	P	37	3	P
85	5	5	3	5	5	3	5	5	3	5	5	3
87	3	19	P	3	7	P	3	P	P	3	P	7
89	P	3	P	7	3	P	P	3	P	11	3	P
91	P	13	3	37	19	3	31	11	3	7	29	3
93	3	7	P	3	P	11	3	P	7	3	P	P
95	5	3	5	5	3	5	5	3	5	5	3	5
97	P	11	3	P	P	3	7	P	3	13	P	3
99	3	P	P	3	P	7	3	P	P	3	11	P

Prime Number and Factor Table

From to	2400 2500	2500 2600	2600 2700	2700 2800	2800 2900	2900 3000	3000 3100	3100 3200	3200 3300	3300 3400	3400 3500	3500 3600
1	7	41	3	37	P	3	P	7	3	P	19	3
3	3	P	19	3	P	P	3	29	P	3	41	31
5	5	3	5	5	3	5	5	3	5	5	3	5
7	29	23	3	P	7	3	31	13	3	P	P	3
9	3	13	P	3	53	P	3	P	P	3	7	11
11	P	3	7	P	3	41	P	3	13	7	3	P
13	19	7	3	P	29	3	23	11	3	P	P	3
15	3	5	5	3	5	5	3	5	5	3	5	5
17	P	3	P	11	3	P	7	3	P	31	3	P
19	41	11	3	P	P	3	P	P	3	P	13	3
21	3	P	P	3	7	23	3	P	P	3	11	7
23	P	3	43	7	3	37	P	3	11	P	3	13
25	5	5	3	5	5	3	5	5	3	5	5	3
27	3	7	37	3	11	P	3	53	7	3	23	P
29	7	3	11	P	3	29	13	3	P	P	3	P
31	11	P	3	P	19	3	7	31	3	P	47	3
33	3	17	P	3	P	7	3	13	53	3	P	P
35	5	3	5	5	3	5	5	3	5	5	3	5
37	P	43	3	7	P	3	P	P	3	47	7	3
39	3	P	7	3	17	P	3	43	41	3	19	P
41	P	3	19	P	3	17	P	3	7	13	3	P
43	7	P	3	13	P	3	17	7	3	P	11	3
45	3	5	5	3	5	5	3	5	5	3	5	5
47	P	3	P	41	3	7	11	3	17	P	3	P
49	31	P	3	P	7	3	P	47	3	17	P	3
51	3	P	11	3	P	13	3	23	P	3	7	53
53	11	3	7	P	3	P	43	3	P	7	3	11
55	5	5	3	5	5	3	5	5	3	5	5	3
57	3	P	P	3	P	P	3	7	P	3	P	P
59	P	3	P	31	3	11	7	3	P	P	3	P
61	23	13	3	11	P	3	P	29	3	P	P	3
63	3	11	P	3	7	P	3	P	13	3	P	7
65	5	3	5	5	3	5	5	3	5	5	3	5
67	P	17	3	P	47	3	P	P	3	7	P	3
69	3	7	17	3	19	P	3	P	7	3	P	43
71	7	3	P	17	3	P	37	3	P	P	3	P
73	P	31	3	47	13	3	7	19	3	P	23	3
75	3	5	5	3	5	5	3	5	5	3	5	5
77	P	3	P	P	3	13	17	3	29	11	3	7
79	37	P	3	7	P	3	P	11	3	31	7	3
81	3	29	7	3	43	11	3	P	17	3	59	P
83	13	3	P	11	3	19	P	3	7	17	3	P
85	5	5	3	5	5	3	5	5	3	5	5	3
87	3	13	P	3	P	29	3	P	19	3	11	17
89	19	3	P	P	3	7	P	3	11	P	3	37
91	47	P	3	P	7	3	11	P	3	P	P	3
93	3	P	P	3	11	41	3	31	37	3	7	P
95	5	3	5	5	3	5	5	3	5	5	3	5
97	11	7	3	P	P	3	19	23	3	43	13	3
99	3	23	P	3	13	P	3	7	P	3	P	59

Prime Number and Factor Table

From to	3600 3700	3700 3800	3800 3900	3900 4000	4000 4100	4100 4200	4200 4300	4300 4400	4400 4500	4500 4600	4600 4700	4700 4800
1	13	P	3	47	P	3	P	11	3	7	43	3
3	3	7	P	3	P	11	3	13	7	3	P	P
5	5	3	5	5	3	5	5	3	5	5	3	5
7	P	11	3	P	P	3	7	59	3	P	17	3
9	3	P	13	3	19	7	3	31	P	3	11	17
11	23	3	37	P	3	P	P	3	11	13	3	7
13	P	47	3	7	P	3	11	19	3	P	7	3
15	3	5	5	3	5	5	3	5	5	3	5	5
17	P	3	11	P	3	23	P	3	7	P	3	53
19	7	P	3	P	P	3	P	7	3	P	31	3
21	3	61	P	3	P	13	3	29	P	3	P	P
23	P	3	P	P	3	7	41	3	P	P	3	P
25	5	5	3	5	5	3	5	5	3	5	5	3
27	3	P	43	3	P	P	3	P	19	3	7	29
29	19	3	7	P	3	P	P	3	43	7	3	P
31	P	7	3	P	29	3	P	61	3	23	11	3
33	3	P	P	3	37	P	3	7	11	3	41	P
35	5	3	5	5	3	5	5	3	5	5	3	5
37	P	37	3	31	11	3	19	P	3	13	P	3
39	3	P	11	3	7	P	3	P	23	3	P	7
41	11	3	23	7	3	41	P	3	P	19	3	11
43	P	19	3	P	13	3	P	43	3	7	P	3
45	3	5	5	3	5	5	3	5	5	3	5	5
47	7	3	P	P	3	11	31	3	P	P	3	47
49	41	23	3	11	P	3	7	P	3	P	P	3
51	3	11	P	3	P	7	3	19	P	3	P	P
53	13	3	P	59	3	P	P	3	61	29	3	7
55	5	5	3	5	5	3	5	5	3	5	5	3
57	3	13	7	3	P	P	3	P	P	3	P	67
59	P	3	17	37	3	P	P	3	7	47	3	P
61	7	P	3	17	31	3	P	7	3	P	59	3
63	3	53	P	3	17	23	3	P	P	3	P	11
65	5	3	5	5	3	5	5	3	5	5	3	5
67	19	P	3	P	7	3	17	11	3	P	13	3
69	3	P	53	3	13	11	3	17	41	3	7	19
71	P	3	7	11	3	43	P	3	17	7	3	13
73	P	7	3	29	P	3	P	P	3	17	P	3
75	3	5	5	3	5	5	3	5	5	3	5	5
77	P	3	P	41	3	P	7	3	11	23	3	17
79	13	P	3	23	P	3	11	29	3	19	P	3
81	3	19	P	3	7	37	3	13	P	3	31	7
83	29	3	11	7	3	47	P	3	P	P	3	P
85	5	5	3	5	5	3	5	5	3	5	5	3
87	3	7	13	3	61	53	3	41	7	3	43	P
89	7	3	P	P	3	59	P	3	67	13	3	P
91	P	17	3	13	P	3	7	P	3	P	P	3
93	3	P	17	3	P	7	3	23	P	3	13	P
95	5	3	5	5	3	5	5	3	5	5	3	5
97	P	P	3	7	17	3	P	P	3	P	7	3
99	3	29	7	3	P	13	3	53	11	3	37	P

Prime Number and Factor Table

From to	4800 4900	4900 5000	5000 5100	5100 5200	5200 5300	5300 5400	5400 5500	5500 5600	5600 5700	5700 5800	5800 5900	5900 6000
1	P	13	3	P	7	3	11	P	3	P	P	3
3	3	P	P	3	11	P	3	P	13	3	7	P
5	5	3	5	5	3	5	5	3	5	5	3	5
7	11	7	3	P	41	3	P	P	3	13	P	3
9	3	P	P	3	P	P	3	7	71	3	37	19
11	17	3	P	19	3	47	7	3	31	P	3	23
13	P	17	3	P	13	3	P	37	3	29	P	3
15	3	5	5	3	5	5	3	5	5	3	5	5
17	P	3	29	7	3	13	P	3	41	P	3	61
19	61	P	3	P	17	3	P	P	3	7	11	3
21	3	7	P	3	23	17	3	P	7	3	P	31
23	7	3	P	47	3	P	11	3	P	59	3	P
25	5	5	3	5	5	3	5	5	3	5	5	3
27	3	13	11	3	P	7	3	P	17	3	P	P
29	11	3	47	23	3	73	61	3	13	17	3	7
31	P	P	3	7	P	3	P	P	3	11	7	3
33	3	P	7	3	P	P	3	11	43	3	19	17
35	5	3	5	5	3	5	5	3	5	5	3	5
37	7	P	3	11	P	3	P	7	3	P	13	3
39	3	11	P	3	13	19	3	29	P	3	P	P
41	47	3	71	53	3	7	P	3	P	P	3	13
43	29	P	3	37	7	3	P	23	3	P	P	3
45	3	5	5	3	5	5	3	5	5	3	5	5
47	37	3	7	P	3	P	13	3	P	7	3	19
49	13	7	3	19	29	3	P	31	3	P	P	3
51	3	P	P	3	59	P	3	7	P	3	P	11
53	23	3	31	P	3	53	7	3	P	11	3	P
55	5	5	3	5	5	3	5	5	3	5	5	3
57	3	P	13	3	7	11	3	P	P	3	P	7
59	43	3	P	7	3	23	53	3	P	13	3	59
61	P	11	3	13	P	3	43	67	3	7	P	3
63	3	7	61	3	19	31	3	P	7	3	11	67
65	5	3	5	5	3	5	5	3	5	5	3	5
67	31	P	3	P	23	3	7	19	3	73	P	3
69	3	P	37	3	11	7	3	P	P	3	P	47
71	P	3	11	P	3	41	P	3	53	29	3	7
73	11	P	3	7	P	3	13	P	3	23	7	3
75	3	5	5	3	5	5	3	5	5	3	5	5
77	P	3	P	31	3	19	P	3	7	53	3	43
79	7	13	3	P	P	3	P	7	3	P	P	3
81	3	17	P	3	P	P	3	P	13	3	P	P
83	19	3	13	71	3	7	P	3	P	P	3	31
85	5	5	3	5	5	3	5	5	3	5	5	3
87	3	P	P	3	17	P	3	37	11	3	7	P
89	P	3	7	P	3	17	11	3	P	7	3	53
91	67	7	3	29	11	3	17	P	3	P	43	3
93	3	P	11	3	67	P	3	7	P	3	71	13
95	5	3	5	5	3	5	5	3	5	5	3	5
97	59	19	3	P	P	3	23	29	3	11	P	3
99	3	P	P	3	7	P	3	11	41	3	17	7

Prime Number and Factor Table

From to	6000 6100	6100 6200	6200 6300	6300 6400	6400 6500	6500 6600	6600 6700	6700 6800	6800 6900	6900 7000	7000 7100	7100 7200
1	17	P	3	P	37	3	7	P	3	67	P	3
3	3	17	P	3	19	7	3	P	P	3	47	P
5	5	3	5	5	3	5	5	3	5	5	3	5
7	P	31	3	7	43	3	P	19	3	P	7	3
9	3	41	7	3	13	23	3	P	11	3	43	P
11	P	3	P	P	3	17	11	3	7	P	3	13
13	7	P	3	59	11	3	17	7	3	31	P	3
15	3	5	5	3	5	5	3	5	5	3	5	5
17	11	3	P	P	3	7	13	3	17	P	3	11
19	13	29	3	71	7	3	P	P	3	11	P	3
21	3	P	P	3	P	P	3	11	19	3	7	P
23	19	3	7	P	3	11	37	3	P	7	3	17
25	5	5	3	5	5	3	5	5	3	5	5	3
27	3	11	13	3	P	61	3	7	P	3	P	P
29	P	3	P	P	3	P	7	3	P	13	3	P
31	37	P	3	13	59	3	19	53	3	29	79	3
33	3	P	23	3	7	47	3	P	P	3	13	7
35	5	3	5	5	3	5	5	3	5	5	3	5
37	P	17	3	P	41	3	P	P	3	7	31	3
39	3	7	17	3	47	13	3	23	7	3	P	11
41	7	3	79	17	3	31	29	3	P	11	3	37
43	P	P	3	P	17	3	7	11	3	53	P	3
45	3	5	5	3	5	5	3	5	5	3	5	5
47	P	3	P	11	3	P	17	3	41	P	3	7
49	23	11	3	7	P	3	61	17	3	P	7	3
51	3	P	7	3	P	P	3	43	13	3	11	P
53	P	3	13	P	3	P	P	3	7	17	3	23
55	5	5	3	5	5	3	5	5	3	5	5	3
57	3	47	P	3	11	79	3	29	P	3	P	17
59	73	3	11	P	3	7	P	3	19	P	3	P
61	11	61	3	P	7	3	P	P	3	P	23	3
63	3	P	P	3	23	P	3	P	P	3	7	13
65	5	3	5	5	3	5	5	3	5	5	3	5
67	P	7	3	P	29	3	59	67	3	P	37	3
69	3	31	P	3	P	P	3	7	P	3	P	67
71	13	3	P	23	3	P	7	3	P	P	3	71
73	P	P	3	P	P	3	P	13	3	19	11	3
75	3	5	5	3	5	5	3	5	5	3	5	5
77	59	3	P	7	3	P	11	3	13	P	3	P
79	P	37	3	P	11	3	P	P	3	7	P	3
81	3	7	11	3	P	P	3	P	7	3	73	43
83	7	3	61	13	3	29	41	3	P	P	3	11
85	5	5	3	5	5	3	5	5	3	5	5	3
87	3	23	P	3	13	7	3	11	71	3	19	P
89	P	3	19	P	3	11	P	3	83	29	3	7
91	P	41	3	7	P	3	P	P	3	P	7	3
93	3	11	7	3	43	19	3	P	61	3	41	P
95	5	3	5	5	3	5	5	3	5	5	3	5
97	7	P	3	P	73	3	37	7	3	P	47	3
99	3	P	P	3	67	P	3	13	P	3	31	23

Prime Number and Factor Table

From to	7200 7300	7300 7400	7400 7500	7500 7600	7600 7700	7700 7800	7800 7900	7900 8000	8000 8100	8100 8200	8200 8300	8300 8400
1	19	7	3	13	11	3	29	P	3	P	59	3
3	3	67	11	3	P	P	3	7	53	3	13	19
5	5	3	5	5	3	5	5	3	5	5	3	5
7	P	P	3	P	P	3	37	P	3	11	29	3
9	3	P	31	3	7	13	3	11	P	3	P	7
11	P	3	P	7	3	11	73	3	P	P	3	P
13	P	71	3	11	23	3	13	41	3	7	43	3
15	3	5	5	3	5	5	3	5	5	3	5	5
17	7	3	P	P	3	P	P	3	P	P	3	P
19	P	13	3	73	19	3	7	P	3	23	P	3
21	3	P	41	3	P	7	3	89	13	3	P	53
23	31	3	13	P	3	P	P	3	71	P	3	7
25	5	5	3	5	5	3	5	5	3	5	5	3
27	3	17	7	3	29	P	3	P	23	3	19	11
29	P	3	17	P	3	59	P	3	7	11	3	P
31	7	P	3	17	13	3	41	7	3	47	P	3
33	3	P	P	3	17	11	3	P	29	3	P	13
35	5	3	5	5	3	5	5	3	5	5	3	5
37	P	11	3	P	7	3	17	P	3	79	P	3
39	3	41	43	3	P	71	3	17	P	3	7	31
41	13	3	7	P	3	P	P	3	11	7	3	19
43	P	7	3	19	P	3	11	13	3	17	P	3
45	3	5	5	3	5	5	3	5	5	3	5	5
47	P	3	11	P	3	61	7	3	13	P	3	17
49	11	P	3	P	P	3	47	P	3	29	73	3
51	3	P	P	3	7	23	3	P	83	3	37	7
53	P	3	29	7	3	P	P	3	P	31	3	P
55	5	5	3	5	5	3	5	5	3	5	5	3
57	3	7	P	3	13	P	3	73	7	3	23	61
59	7	3	P	P	3	P	29	3	P	41	3	13
61	53	17	3	P	47	3	7	19	3	P	11	3
63	3	37	17	3	79	7	3	P	11	3	P	P
65	5	3	5	5	3	5	5	3	5	5	3	5
67	13	53	3	7	11	3	P	31	3	P	7	3
69	3	P	7	3	P	17	3	13	P	3	P	P
71	11	3	31	67	3	19	17	3	7	P	3	11
73	7	73	3	P	P	3	P	7	3	11	P	3
75	3	5	5	3	5	5	3	5	5	3	5	5
77	19	3	P	P	3	7	P	3	41	13	3	P
79	29	47	3	11	7	3	P	79	3	P	17	3
81	3	11	P	3	P	31	3	23	P	3	7	17
83	P	3	7	P	3	43	P	3	59	7	3	83
85	5	5	3	5	5	3	5	5	3	5	5	3
87	3	83	P	3	P	13	3	7	P	3	P	P
89	37	3	P	P	3	P	7	3	P	19	3	P
91	23	19	3	P	P	3	13	61	3	P	P	3
93	3	P	59	3	7	P	3	P	P	3	P	7
95	5	3	5	5	3	5	5	3	5	5	3	5
97	P	13	3	71	43	3	53	11	3	7	P	3
99	3	7	P	3	P	11	3	19	7	3	43	37

Prime Number and Factor Table

From to	8400 8500	8500 8600	8600 8700	8700 8800	8800 8900	8900 9000	9000 9100	9100 9200	9200 9300	9300 9400	9400 9500	9500 9600
1	31	P	3	7	13	3	P	19	3	71	7	3
3	3	11	7	3	P	29	3	P	P	3	P	13
5	5	3	5	5	3	5	5	3	5	5	3	5
7	7	47	3	P	P	3	P	7	3	41	23	3
9	3	67	P	3	23	59	3	P	P	3	97	37
11	13	3	79	31	3	7	P	3	61	P	3	P
13	47	P	3	P	7	3	P	13	3	67	P	3
15	3	5	5	3	5	5	3	5	5	3	5	5
17	19	3	7	23	3	37	71	3	13	7	3	31
19	P	7	3	P	P	3	29	11	3	P	P	3
21	3	P	37	3	P	11	3	7	P	3	P	P
23	P	3	P	11	3	P	7	3	23	P	3	89
25	5	5	3	5	5	3	5	5	3	5	5	3
27	3	P	P	3	7	79	3	P	P	3	11	7
29	P	3	P	7	3	P	P	3	11	19	3	13
31	P	19	3	P	P	3	11	23	3	7	P	3
33	3	7	89	3	11	P	3	P	7	3	P	P
35	5	3	5	5	3	5	5	3	5	5	3	5
37	11	P	3	P	P	3	7	P	3	P	P	3
39	3	P	53	3	P	7	3	13	P	3	P	P
41	23	3	P	P	3	P	P	3	P	P	3	7
43	P	P	3	7	37	3	P	41	3	P	7	3
45	3	5	5	3	5	5	3	5	5	3	5	5
47	P	3	P	P	3	23	83	3	7	13	3	P
49	7	83	3	13	P	3	P	7	3	P	11	3
51	3	17	41	3	53	P	3	P	11	3	13	P
53	79	3	17	P	3	7	11	3	19	47	3	41
55	5	5	3	5	5	3	5	5	3	5	5	3
57	3	43	11	3	17	13	3	P	P	3	7	19
59	11	3	7	19	3	17	P	3	47	7	3	11
61	P	7	3	P	P	3	13	P	3	11	P	3
63	3	P	P	3	P	P	3	7	59	3	P	73
65	5	3	5	5	3	5	5	3	5	5	3	5
67	P	13	3	11	P	3	P	89	3	17	P	3
69	3	11	P	3	7	P	3	53	13	3	17	7
71	43	3	13	7	3	P	47	3	73	P	3	17
73	37	P	3	31	19	3	43	P	3	7	P	3
75	3	5	5	3	5	5	3	5	5	3	5	5
77	7	3	P	67	3	47	29	3	P	P	3	61
79	61	23	3	P	13	3	7	67	3	83	P	3
81	3	P	P	3	83	7	3	P	P	3	19	11
83	17	3	19	P	3	13	31	3	P	11	3	7
85	5	5	3	5	5	3	5	5	3	5	5	3
87	3	31	7	3	P	11	3	P	37	3	53	P
89	13	3	P	11	3	89	61	3	7	41	3	43
91	7	11	3	59	17	3	P	7	3	P	P	3
93	3	13	P	3	P	17	3	29	P	3	11	53
95	5	3	5	5	3	5	5	3	5	5	3	5
97	29	P	3	19	7	3	11	17	3	P	P	3
99	3	P	P	3	11	P	3	P	17	3	7	29

Transposition of Formulas

A formula is a rule for a calculation expressed by using letters and signs instead of writing out the rule in words; by this means it is possible to condense, in a very small space, the essentials of long and cumbersome rules. The letters used in formulas simply stand in place of the figures which are to be substituted when solving a specific problem.

An important method for facilitating the use of formulas is known as *transposition.* As an example, the formula for the horsepower transmitted by belting may be written:

$$\text{H.P.} = \frac{SVW}{33{,}000}$$

in which

H.P. = horsepower transmitted;
S = working stress of belt per inch of width, in pounds;
V = velocity of belt in feet per minute;
W = width of belt in inches.

If the working stress S, the velocity V, and the width W, are known, the horsepower can be found directly from this formula by inserting the given values. Assume $S = 33$; $V = 600$; and $W = 5$. Then:

$$\text{H.P.} = \frac{33 \times 600 \times 5}{33{,}000} = 3.$$

Assume, however, that the horsepower, the stress S, and the velocity V are known, and that the width of belt, W, is to be found. The formula must then be transposed so that the symbol W will be on one side of the equals sign and all the known quantities on the other. The transposed formula is as follows:

$$\frac{\text{H.P.} \times 33{,}000}{SV} = W.$$

The quantities (S and V) that were in the numerator on the right side of the equals sign are transposed to the denominator on the left side, and "33,000" which was in the denominator on the right side of the equals sign is transposed to the numerator on the other side. This is in conformity with the general rule for transposition. Symbols which are not part of a fraction, like "H.P." in the formula first given, are to be considered as being numerators (having the denominator 1).

According to the rule given, any formula of the form $A = \frac{B}{C}$ can be transposed as below:

$$A \times C = B, \quad \text{and} \quad C = \frac{B}{A}$$

Suppose a formula to be of the form:

$$A = \frac{B \times C}{D}$$

Then:

$$D = \frac{B \times C}{A}; \quad \frac{A \times D}{C} = B; \quad \frac{A \times D}{B} = C.$$

The method given is only directly applicable when all the quantities in the numerator or denominator are standing independently or are *factors of a product.* If connected by + or − signs, the transposition can be made by the method shown only when the *whole sum* or *difference* is transposed.

Example:

$$A = \frac{B + C}{D}; \quad \text{then} \quad D = \frac{B + C}{A} \quad \text{and} \quad A \times D = B + C.$$

A quantity preceded by a + or − sign can be transposed to the opposite side of the equals sign by changing its sign; if the sign is +, change it to − on the other side; if it is −, change it to +.

Example:

$$B+C=A-D;\quad \text{then}\quad B+C+D=A;$$
$$B=A-D-C;$$
$$C=A-D-B;$$
$$D=A-B-C.$$

When several numbers or quantities in a formula are connected with signs indicating that additions, subtractions, multiplications or divisions are to be made, the multiplications should be carried out before any of the other operations. Division also precedes addition and subtraction if written in line with these. The other operations are carried out in the order written.

Examples:

$$10+26\times 7-2=10+182-2=190.$$
$$18\div 6+15\times 3=3+45=48.$$
$$12+14\div 2-4=12+7-4=15.$$

When it is required that certain additions and subtractions should precede multiplications and divisions, use is made of parentheses ( ) and brackets [ ]. These indicate that the calculation inside the parentheses or brackets should be carried out complete by itself before the remaining calculations are commenced. If one bracket is placed inside of another, the one inside is first calculated.

Examples:

$$(6-2)\times 5+8=4\times 5+8=20+8=28.$$
$$6\times(4+7)\div 22=6\times 11\div 22=66\div 22=3.$$
$$2+[10\times 6(8+2)-4]\times 2=2+[10\times 6\times 10-4]\times 2$$
$$=2+[600-4]\times 2=2+596\times 2=2+1192=1194.$$

The parentheses are considered as a sign of multiplication; for example, $6\,(8+2) = 6\times(8+2)$.

The line or bar between the numerator and denominator in a fractional expression is to be considered as a division sign. For example,

$$\frac{12+16+22}{10}=(12+16+22)\div 10=50\div 10=5.$$

In formulas the multiplication sign (×) is often left out between symbols or letters, the values of which are to be multiplied. Thus

$$AB=A\times B,\quad \text{and}\quad \frac{ABC}{D}=(A\times B\times C)\div D$$

Ratio and Proportion

The *ratio* between two quantities is the quotient obtained by dividing the first quantity by the second. For example, the ratio between 3 and 12 is $\frac{1}{4}$, and the ratio between 12 and 3 is 4. Ratio is generally indicated by the sign (:); thus 12 : 3 indicates the ratio of 12 to 3.

A *reciprocal* or *inverse* ratio is the reciprocal of the original ratio. Thus, the inverse ratio of 5 : 7 is 7 : 5.

In a *compound* ratio each term is the product of the corresponding terms in two or more simple ratios. Thus, when

$$8:2=4,\qquad 9:3=3,\qquad 10:5=2,$$

then the compound ratio is:

$$8\times 9\times 10:2\times 3\times 5=4\times 3\times 2,$$
$$720:30=24.$$

Proportion is the equality of ratios. Thus,

$$6 : 3 = 10 : 5, \quad \text{or} \quad 6 : 3 :: 10 : 5.$$

The first and last terms in a proportion are called the *extremes;* the second and third, the *means.* The product of the extremes is equal to the product of the means. Thus,

$$25 : 2 = 100 : 8 \quad \text{and} \quad 25 \times 8 = 2 \times 100.$$

If three terms in a proportion are known, the remaining term may be found by the following rules:

The first term is equal to the product of the second and third terms, divided by the fourth.

The second term is equal to the product of the first and fourth terms, divided by the third.

The third term is equal to the product of the first and fourth terms, divided by the second.

The fourth term is equal to the product of the second and third terms, divided by the first.

Examples: — Let x be the term to be found, then,

$$x : 12 = 3.5 : 21 \qquad x = \frac{12 \times 3.5}{21} = \frac{42}{21} = 2$$

$$\tfrac{1}{4} : x = 14 : 42 \qquad x = \frac{\frac{1}{4} \times 42}{14} = \frac{1}{4} \times 3 = \frac{3}{4}$$

$$5 : 9 = x : 63 \qquad x = \frac{5 \times 63}{9} = \frac{315}{9} = 35$$

$$\tfrac{1}{4} : \tfrac{7}{8} = 4 : x \qquad x = \frac{\frac{7}{8} \times 4}{\frac{1}{4}} = \frac{3\frac{1}{2}}{\frac{1}{4}} = 14.$$

If the second and third terms are the same, either is said to be the *mean proportional* between the other two. Thus, $8 : 4 = 4 : 2$, and 4 is the mean proportional between 8 and 2. The mean proportional between two numbers may be found by multiplying the numbers together, and extracting the square root of the product. Thus, the mean proportional between 3 and 12 is found as below:

$$3 \times 12 = 36, \quad \text{and} \quad \sqrt{36} = 6,$$

which is the mean proportional.

Practical Examples Involving Simple Proportion. — If it takes 18 days to assemble 4 lathes, how long would it require to assemble 14 lathes?

Let the number of days to be found be x. Then write out the proportion as below:

$$4 : 18 = 14 : x$$
$$(\text{lathes} : \text{days} = \text{lathes} : \text{days})$$

Find now the fourth term by the rule given:

$$x = \frac{18 \times 14}{4} = 63 \text{ days.}$$

Thirty-four linear feet of bar stock are required for the blanks for 100 clamping bolts. How many feet of stock would be required for 912 bolts?

Let x = total length of stock required for 912 bolts.

$$34 : 100 = x : 912$$
$$(\text{feet} : \text{bolts} = \text{feet} : \text{bolts})$$

Then, the third term $x = \dfrac{34 \times 912}{100} = 310$ feet, approximately.

Example of Inverse Proportion. — A factory employing 270 men completes a given number of typewriters weekly, the number of working hours being 60 per week. How many men would be required for the same production if the working hours were reduced to 54 per week?

The time per week is in an inverse proportion to the number of men employed; the *shorter* the time, the *more* men. The inverse proportion is written:

$$270 : x = 54 : 60$$

(men, 10-hour basis : men, 9-hour basis = time, 9-hour basis : time, 10-hour basis)

The second term $x = \dfrac{270 \times 60}{54} = 300$ men.

Example of Compound Proportion. — If a man capable of turning 65 studs in a day of 10 hours is paid 32.5 cents per hour, how much ought a man be paid who turns 72 studs in a 9-hour day, if compensated in the same proportion?

When solving problems involving compound proportion, the following method of analysis tends to simplify the solution. Make up a table with four columns headed, "First Cause," "First Effect," "Second Cause," "Second Effect," and place under each the respective factors given in the problem. In the example above, the table would be arranged as below:

First Cause	First Effect	Second Cause	Second Effect
1 man 10 hours 32.5 cents	65 studs	1 man 9 hours x cents	72 studs

Consider as *causes* the number of men working, the length of time they work, and their capacity for work; the pay received or the amount of product turned out in a unit of time indicates the capacity for work. The effect is the total product given either in numbers, or by the dimensions of the work carried out. The unknown quantity is called x.

When the table is completed, take all the quantities in the first and fourth columns and place them as the numerator of a fraction with multiplication signs between them, and all the quantities in the second and third columns and place them as the denominator of a fraction with multiplication signs between them. Put this fraction equal to 1. Then cancel and reduce the fraction to its simplest form as below.

$$\frac{1 \times 10 \times 32.5 \times 72}{65 \times 1 \times 9 \times x} = 1; \quad \frac{40}{x} = 1, \text{ or } x = 40 \text{ cents.}$$

Percentage

If out of 100 pieces made, 12 do not pass inspection, it is said that 12 per cent (12 on the hundred) are rejected. If a quantity of steel is bought for $100 and sold for $140, the profit is 40 per cent.

The per cent of gain or loss is found by dividing the amount of gain or loss by the *original* number of which the percentage is wanted, and multiplying the quotient by 100.

Examples: — Out of a total output of 280 castings a day, 30 castings are, on an average, rejected. What is the percentage of bad castings?

$$\frac{30}{280} \times 100 = 10.7 \text{ per cent.}$$

If by a new process 100 pieces can be made in the same time as 60 could formerly be made, what is the gain in output of the new process over the old, expressed in per cent?

Original number, 60; gain $100 - 60 = 40$. Hence,

$$\frac{40}{60} \times 100 = 66.7 \text{ per cent.}$$

Care should be taken always to use the original number, or the number of which the percentage is wanted, as the divisor in all percentage calculations. In the example just given, it is the percentage of gain over the old output 60 that is wanted, and not the percentage with relation to the new output 100. Mistakes are often made by overlooking this important point.

Interest

Interest is the money paid for the use of money lent for a certain time. *Simple* interest is the interest paid on the principal (money lent) only. When simple interest that is due is not paid, and its amount is added to the interest-bearing principal, the interest calculated on this new principal is called *compound* interest. The compounding of the interest into the principal may take place yearly or oftener, according to circumstances.

Simple Interest. — The following formulas are applicable to the calculations involving simple interest. Let:

P = principal or amount of money lent;
p = per cent of interest;
r = interest rate = the interest, expressed decimally, on $1.00 for one year = the per cent of interest divided by 100; thus, if the interest is 6 per cent, the rate $r = 6/100 = 0.06$;
n = the number of years for which interest is calculated;
I = the amount of interest for n years at the given rate;
P_n = principal with interest for n years added, or the total amount after n years.

Then:

Interest for n years, $I = Prn$.
Total amount after n years, $P_n = P + Prn = P(1 + rn)$.
Interest rate $r = I \div Pn$.
Number of years $n = I \div Pr$.
Principal, or amount lent $= I \div rn$.

Example: — Assume that $250 has been loaned for three years at 6 per cent simple interest. Then: $P = 250$; $p = 6$; $r = p \div 100 = 0.06$; $n = 3$.

$$I = Prn = 250 \times 0.06 \times 3 = \$45.$$
$$P_n = P + I = 250 + 45 = \$295.$$

The accurate interest for one day is 1/365 of the interest for one year. Banks, however, customarily take the year as composed of 12 months of 30 days, making a total of 360 days to a year.

Compound Interest. — The following formulas are applicable when compound interest is to be computed, using the same notation as for simple interest, and assuming that the interest is compounded annually.

The total amount after n years, $P_n = P(1 + r)^n$.

The principal $P = \dfrac{P_n}{(1 + r)^n}$ The rate $r = \sqrt[n]{\dfrac{P_n}{P}} - 1$

The number of years during which the money is lent

$$n = \frac{\log P_n - \log P}{\log (1 + r)}$$

Logarithms are especially useful in calculating compound interest. To find the total amount P_n of principal and interest after n years, the formula just given can be transcribed as below:

$$\log P_n = \log P + n \log (1 + r).$$

If the interest is payable q times a year, it will be computed q times during each year, or nq times during n years. The rate for each compounding will be $r \div q$, if r is the annual rate. Hence, at the end of n years the amount due will be:

$$P_n = P \left(1 + \frac{r}{q}\right)^{nq}$$

Thus, if the term be five years, the interest be payable quarterly, and the annual rate be 6 per cent, then, $n = 5$; $q = 4$; $r = 0.06$; $r \div q = 0.06 \div 4 = 0.015$; and $nq = 5 \times 4 = 20$.

Example: — In what time will \$500 become \$1000 at 6 per cent interest compounded yearly?

$$P_n = 1000; \quad P = 500; \quad r = 0.06.$$

Substituting these values in the formula:

$$1000 = 500\,(1 + 0.06)^n, \quad \text{or} \quad 2 = 1.06^n, \quad \text{and} \quad n \times \log 1.06 = \log 2.$$

Hence
$$n = \frac{0.30103}{0.02531} = 11.9 \text{ years.}$$

This is the number of years in which any principal will double itself at 6 per cent compound interest.

Present Value and Discount. — The present value V of a given amount due in a given time, is the sum which placed at interest for the given time, will produce the given amount. Hence,

$$\text{At simple interest, } V = \frac{P_n}{1 + nr}$$

$$\text{At compound interest, } V = \frac{P_n}{(1 + r)^n}$$

in which P_n is the amount due in n years time, and r is the rate of simple interest, or the per cent divided by 100.

The *true discount* D is the difference between the amount due at the end of n years and the present value, or,

$$\text{At simple interest, } D = P_n - V = \frac{P_n nr}{1 + nr}$$

$$\text{At compound interest, } D = P_n - V = P_n \left[1 - \frac{1}{(1 + r)^n}\right]$$

These formulas are for interest compounded annually. If the interest is payable and compounded semi-annually, or quarterly, modify the formulas as indicated in the formulas for compound interest.

Example: — Required the present value and discount of $500 due in six months at 6 per cent simple interest. Here, $P_n = 500$; $n = 6/12$ years $= 1/2$; $r = 0.06$; then,

$$V = \frac{500}{1 + 0.5 \times 0.06} = \$485.44.$$

$$D = 500 - 485.44 = \$14.56.$$

Example: — Required the sum which placed at 5 per cent compound interest, will in three years produce $5000. Here, $P_n = 5000$; $r = 0.05$; $n = 3$. Then,

$$V = \frac{5000}{(1 + 0.05)^3} = 4319.19.$$

Bank discount is calculated at simple interest on the total amount of a promissory note for the term of the note and on the basis of a year of 360 days.

Annuities. — An annuity is a fixed sum paid at regular intervals. In the formulas given below, yearly payments are assumed. It is customary to calculate annuities on the basis of compound interest.

If an annuity A is to be paid out for n consecutive years, the interest rate being r, then the present value P of the annuity is:

$$P = A\,\frac{(1 + r)^n - 1}{(1 + r)^n r}$$

Example: — If an annuity of $200 is to be paid for 10 years, what is the present amount of money that need be deposited if the interest is 5 per cent? Here,

$$A = 200; \quad r = 5 \div 100 = 0.05; \quad n = 10.$$

$$P = 200\,\frac{1.05^{10} - 1}{1.05^{10} \times 0.05} = 1544.36.$$

The annuity that a principal P, drawing interest at the rate r, will give for a period of n years, is:

$$A = \frac{Pr\,(1 + r)^n}{(1 + r)^n - 1}$$

Example: — A sum of $10,000 is placed at 4 per cent interest. What is the amount of the annuity which can be paid for 20 years out of this sum? Here,

$$P = 10{,}000; \quad r = 0.04; \quad n = 20.$$

$$A = \frac{10{,}000 \times 0.04 \times 1.04^{20}}{1.04^{20} - 1} = 735.82.$$

If at the beginning of each year a sum A is set aside at an interest rate r, then the total value of the sum set aside, with interest, will be at the end of n years:

$$P_n = A\,\frac{(1 + r)\,[(1 + r)^n - 1]}{r}$$

If at the end of each year a sum A is set aside at an interest rate r, then the total value of the principal, with interest, at the end of n years will be:

$$P_n = A\,\frac{(1 + r)^n - 1}{r}$$

If a principal P is increased or decreased by a sum A at the end of each year, then the value of the principal after n years will be:

$$P_n = P\,(1 + r)^n \pm A\,\frac{(1 + r)^n - 1}{r}$$

If the sum A by which the principal P is decreased each year is greater than the total yearly interest on the principal, then the principal, with the accumulated interest, will be entirely used up in n years:

$$n = \frac{\log A - \log (A - Pr)}{\log (1 + r)}$$

Sinking Funds.— Amortization is "the extinction of a debt, usually by means of a sinking fund." The sinking fund is created by a fixed investment S placed annually at compound interest for a term of years, and is hence an annuity of sufficient size to produce at the end of the term of years the amount necessary for the repayment of the principal of the debt, or to provide a definite sum for other purposes. Let:

S = the annual investment;

r = rate of interest (the per cent divided by 100);

P = the amount of the sinking fund;

n = the number of years for its creation.

Then:

$$P = S\frac{(1+r)^n - 1}{r}, \quad \text{and} \quad S = \frac{Pr}{(1+r)^n - 1}$$

which formulas correspond to those given above, where a sum A was laid aside at the end of each year.

Example: — If $2000 is invested annually for 10 years, at 4 per cent compound interest, as a sinking fund, what would be the total amount of the fund at the expiration of the term? Here, $S = 2000$; $n = 10$; $r = 0.04$.

$$P = 2000\frac{1.04^{10} - 1}{0.04} = 24{,}012.25$$

Alligation

When an alloy is composed of several metals varying in price, the price per pound of the alloy can be found as in the following example: An alloy is composed of 50 pounds of copper at 14 cents a pound, 10 pounds of tin at 29 cents a pound, 20 pounds of zinc at 5 cents a pound, and 5 pounds of lead at 4 cents a pound. What is the cost of the alloy per pound, no account being taken of the cost of mixing it?

Multiply the number of pounds of each of the ingredients by its price per pound, add these products together, and divide the sum by the total weight of all the ingredients. The quotient is the price per pound of the alloy.

$50 \times 14 + 10 \times 29 + 20 \times 5 + 5 \times 4 = 700 + 290 + 100 + 20 = 1110.$

Total weight of metal in alloy = $50 + 10 + 20 + 5 = 85$.

Price per pound of alloy = $1110/85 = 13$ cents, approximately.

In general, let a, b, c and d be the weights of each of the ingredients, and w, x, y and z be their respective values per unit weight. Then the average price P per unit weight of the alloy is found by the formula:

$$P = \frac{aw + bx + cy + dz}{a + b + c + d}$$

Example: — Find the average price per pound of an alloy containing 40 pounds of tin at 30 cents per pound, 48 pounds of lead at 4 cents per pound, 10 pounds of antimony at 8 cents per pound, and 2 pounds of copper at 15 cents per pound.

$$P = \frac{40 \times 30 + 48 \times 4 + 10 \times 8 + 2 \times 15}{40 + 48 + 10 + 2} = \frac{1502}{100} = 15.02 \text{ cents.}$$

Formulas for Arithmetical Progression

To Find	Given	Use Equation
a	$d \quad l \quad n$	$a = l - (n-1)d$
	$d \quad n \quad S$	$a = \frac{S}{n} - \frac{n-1}{2} \times d$
	$d \quad l \quad S$	$a = \frac{d}{2} \pm \frac{1}{2}\sqrt{(2l+d)^2 - 8dS}$
	$l \quad n \quad S$	$a = \frac{2S}{n} - l$
d	$a \quad l \quad n$	$d = \frac{l-a}{n-1}$
	$a \quad n \quad S$	$d = \frac{2S - 2an}{n(n-1)}$
	$a \quad l \quad S$	$d = \frac{l^2 - a^2}{2S - l - a}$
	$l \quad n \quad S$	$d = \frac{2nl - 2S}{n(n-1)}$
l	$a \quad d \quad n$	$l = a + (n-1)d$
	$a \quad d \quad S$	$l = -\frac{d}{2} \pm \frac{1}{2}\sqrt{8dS + (2a-d)^2}$
	$a \quad n \quad S$	$l = \frac{2S}{n} - a$
	$d \quad n \quad S$	$l = \frac{S}{n} + \frac{n-1}{2} \times d$
n	$a \quad d \quad l$	$n = 1 + \frac{l-a}{d}$
	$a \quad d \quad S$	$n = \frac{d-2a}{2d} \pm \frac{1}{2d}\sqrt{8dS + (2a-d)^2}$
	$a \quad l \quad S$	$n = \frac{2S}{a+l}$
	$d \quad l \quad S$	$n = \frac{2l+d}{2d} \pm \frac{1}{2d}\sqrt{(2l+d)^2 - 8dS}$
S	$a \quad d \quad n$	$S = \frac{n}{2}[2a + (n-1)d]$
	$a \quad d \quad l$	$S = \frac{a+l}{2} + \frac{l^2 - a^2}{2d} = \frac{a+l}{2d}(l + d - a)$
	$a \quad l \quad n$	$S = \frac{n}{2}(a+l)$
	$d \quad l \quad n$	$S = \frac{n}{2}[2l - (n-1)d]$

Arithmetical Progression

An arithmetical progression is a series of numbers in which each consecutive term differs from the preceding one by a fixed amount called the *common difference, d*. Thus, 1, 3, 5, 7, etc., is an arithmetical progression where the difference d is 2. The difference in this case is *added* to the preceding term, and the progression is called increasing. In the series 13, 10, 7, 4, etc., the difference is (-3), and the progression is called decreasing. In any arithmetical progression (or part of progression) let

a = the first term considered;
l = the last term considered;
n = the number of terms;
d = the common difference;
S = the sum of n terms.

Then the general formulas are:

$$l = a + (n - 1)\,d \quad \text{and} \quad S = \frac{a+l}{2} \times n$$

In these formulas d is positive in an increasing and negative in a decreasing progression. When any three of the five quantities above are given, the other two can be found by the formulas in the accompanying table of arithmetical progression.

Example: — In an arithmetical progression, the first term equals 5, and the last term 40. The difference is 7. Find the sum of the progression.

$$S = \frac{a+l}{2d}(l + d - a) = \frac{5+40}{2 \times 7}(40 + 7 - 5) = 135.$$

Geometrical Progression

A geometrical progression or a geometrical series is a series in which each term is derived by multiplying the preceding term by a constant multiplier called the *ratio*. When the ratio is greater than 1, the progression is increasing; when smaller than 1, it is decreasing. Thus, 2, 6, 18, 54, etc., is an increasing geometrical progression with a ratio of 3, while 24, 12, 6, etc., is a decreasing progression with a ratio of ½.

In any geometrical progression (or part of progression) let

a = the first term;
l = the last (or nth) term;
n = the number of terms;
r = the ratio of the progression;
S = the sum of n terms.

Then the general formulas are:

$$l = ar^{n-1} \quad \text{and} \quad S = \frac{rl - a}{r - 1}$$

When any three of the five quantities above are given, the other two can be found by the formulas tabulated in the accompanying table. Geometrical progressions are used for finding the successive speeds in machine tool drives, in interest calculations, etc.

Example: — The lowest speed of a lathe is 20 R.P.M. The highest speed is 225 R.P.M. There are 18 speeds. Find the ratio between successive speeds.

$$\text{Ratio, } r = \sqrt[n-1]{\frac{l}{a}} = \sqrt[17]{\frac{225}{20}} = \sqrt[17]{11.25} = 1.153.$$

Formulas for Geometrical Progression

To Find	Given	Use Equation
a	$l \quad n \quad r$	$a = \dfrac{l}{r^{n-1}}$
	$n \quad r \quad S$	$a = \dfrac{(r-1)S}{r^n - 1}$
	$l \quad r \quad S$	$a = lr - (r-1)S$
	$l \quad n \quad S$	$a(S-a)^{n-1} = l(S-l)^{n-1}$
l	$a \quad n \quad r$	$l = ar^{n-1}$
	$a \quad r \quad S$	$l = \dfrac{1}{r}[a + (r-1)S]$
	$a \quad n \quad S$	$l(S-l)^{n-1} = a(S-a)^{n-1}$
	$n \quad r \quad S$	$l = \dfrac{S(r-1)r^{n-1}}{r^n - 1}$
n	$a \quad l \quad r$	$n = \dfrac{\log l - \log a}{\log r} + 1$
	$a \quad r \quad S$	$n = \dfrac{\log[a + (r-1)S] - \log a}{\log r}$
	$a \quad l \quad S$	$n = \dfrac{\log l - \log a}{\log(S-a) - \log(S-l)} + 1$
	$l \quad r \quad S$	$n = \dfrac{\log l - \log[lr - (r-1)S]}{\log r} + 1$
r	$a \quad l \quad n$	$r = \sqrt[n-1]{\dfrac{l}{a}}$
	$a \quad n \quad S$	$r^n = \dfrac{Sr}{a} + \dfrac{a-S}{a}$
	$a \quad l \quad S$	$r = \dfrac{S-a}{S-l}$
	$l \quad n \quad S$	$r^n = \dfrac{Sr^{n-1}}{S-l} - \dfrac{l}{S-l}$
S	$a \quad n \quad r$	$S = \dfrac{a(r^n - 1)}{r - 1}$
	$a \quad l \quad r$	$S = \dfrac{lr - a}{r - 1}$
	$a \quad l \quad n$	$S = \dfrac{\sqrt[n-1]{l^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{l} - \sqrt[n-1]{a}}$
	$l \quad n \quad r$	$S = \dfrac{l(r^n - 1)}{(r-1)r^{n-1}}$

Mathematical Signs and Commonly Used Abbreviations

$+$	Plus (sign of addition)	
$+$	Positive	
$-$	Minus (sign of subtraction)	
$-$	Negative	
$\pm$ ($\mp$)	Plus or minus (minus or plus)	
$\times$	Multiplied by (multiplication sign)	
$\cdot$	Multiplied by (multiplication sign)	
$\div$	Divided by (division sign)	
$:$	Divided by (division sign)	
$:$	Is to (in proportion)	
$=$	Equals	
$::$	Equals (in proportion)	
$\cong$	Approximately equals	
$>$	Greater than	
$<$	Less than	
$\geqq$	Greater than or equal to	
$\leqq$	Less than or equal to	
$\therefore$	Therefore	
$\sqrt{\ }$	Square root	
$\sqrt[3]{\ }$	Cube root	
$\sqrt[4]{\ }$	4th root	
$\sqrt[n]{\ }$	*n*th root	
a^2	*a* squared (2d power of *a*)	
a^3	*a* cubed (3d power of *a*)	
a^4	4th power of *a*	
a^n	*n*th power of *a*	
$\frac{1}{n}$	Reciprocal value of *n*	
log	Logarithm	
hyp. log nat. log $\log_e$ ln	Hyperbolic, natural or Napierian logarithm	
lim.	Limit value (of an expression)	
∞	Infinity	
α	Alpha	commonly used to denote angles
β	Beta	
γ	Gamma	
θ	Theta	
ϕ	Phi	
Δ	Delta (difference)	
δ	Delta (differential)	
μ	Mu (coefficient of friction)	
π	Pi (3.1416)	
Σ	Sigma (sign of summation)	
ω	Omega (angles measured in radians)	
d	Differential (in calculus)	
e	Base of hyp. logarithms (2.71828)	

g	Acceleration due to gravity (32.16 ft. per sec.)
i	Imaginary quantity ($\sqrt{-1}$)
sin	Sine
cos	Cosine
tan (tg) (tang)	Tangent
cot (ctg)	Cotangent
sec	Secant
cosec	Cosecant
versin	Versed sine
covers	Coversed sine
$\sin^{-1} a$ arc sin a	Arc the sine of which is a
$(\sin a)^{-1}$	Reciprocal of sin a ($1 \div \sin a$)
$\sinh x$	Hyperbolic sine of x
$\cosh x$	Hyperbolic cosine of x
$\int$	Integral (in calculus)
$\int_b^a$	Integral between the limits a and b
!	$5! = 1 \times 2 \times 3 \times 4 \times 5$
$\angle$	Angle
$\llcorner$	Right angle
$\perp$	Perpendicular to
$^\circ$	Degree (circular arc or thermometer)
$'$	Minutes or feet
$''$	Seconds or inches
a'	*a* prime
a''	*a* double prime
a_1	*a* sub one
a_2	*a* sub two
a_n	*a* sub *n*
( )	Parentheses
[ ]	Brackets
{ }	Braces
B.H.P.	Brake horsepower
B.T.U.	British thermal units
H.P.	Horsepower
I.H.P.	Indicated horsepower
K.W.	Kilowatt
M.E.P.	Mean effective pressure
R.P.M.	Revolutions per minute
C.G.S.	Centimeter-gram-second system

Greek Letters

The Greek letters are frequently used in mathematical expressions and formulas. The Greek alphabet is given below.

Α	α	Alpha	Η	η	Eta	Ν	ν	Nu	Τ	τ	Tau
Β	β	Beta	Θ	ϑ θ	Theta	Ξ	ξ	Xi	Υ	υ	Upsilon
Γ	γ	Gamma	Ι	ι	Iota	Ο	o	Omicron	Φ	ϕ	Phi
Δ	δ	Delta	Κ	κ	Kappa	Π	π	Pi	Χ	χ	Chi
Ε	ϵ	Epsilon	Λ	λ	Lambda	Ρ	ρ	Rho	Ψ	ψ	Psi
Ζ	ζ	Zeta	Μ	μ	Mu	Σ	σ ς	Sigma	Ω	ω	Omega

Positive and Negative Numbers

The degrees on a thermometer scale extending upward from the zero point may be called *positive* and may be preceded by a plus sign; thus $+5$ degrees means 5 degrees above zero. The degrees below zero may be called *negative* and may be preceded by a minus sign; thus -5 degrees means 5 degrees below zero. In the same way, the ordinary numbers 1, 2, 3, etc., which are larger than 0, are called positive numbers; but numbers can be conceived of as extending in the other direction from 0, numbers that, in fact, are less than 0, and these are called negative. As these numbers must be expressed by the same figures as the positive numbers they are designated by a minus sign placed before them, thus: (-3). A negative number should always be enclosed within parentheses whenever it is written in line with other numbers; for example: $17+(-13)-3\times(-0.76)$.

Negative numbers are most commonly met with in the use of logarithms and natural trigonometric functions. The following rules govern calculations with negative numbers.

A negative number can be added to a positive number by subtracting its numerical value from the positive number.

Example: $4+(-3)=4-3=1.$

A negative number can be subtracted from a positive number by adding its numerical value to the positive number.

Example: $4-(-3)=4+3=7.$

A negative number can be added to a negative number by adding the numerical values and making the sum negative.

Example: $(-4)+(-3)=-7.$

A negative number can be subtracted from a negative number by subtracting the numerical values and making the difference negative.

Example: $(-4)-(-3)=-1.$

If in a subtraction the number to be subtracted is larger than the number from which it is to be subtracted, the calculation can be carried out by subtracting the smaller number from the larger, and indicating that the remainder is negative.

Example: $3-5=-(5-3)=-2.$

When a positive number is to be multiplied or divided by a negative number, multiply or divide the numerical values as usual; the product or quotient, respectively, is negative. The same rule is true if a negative number is multiplied or divided by a positive number.

Examples: $4\times(-3)=-12;\quad(-4)\times 3=-12;$

$15\div(-3)=-5;\quad(-15)\div 3=-5.$

When two negative numbers are to be multiplied by each other, the product is positive. When a negative number is divided by a negative number, the quotient is positive.

Examples: $(-4) \times (-3) = 12$; $(-4) \div (-3) = 1.333$.

The two last rules are often expressed for memorizing as follows: "Equal signs make plus, unequal signs make minus."

Powers and Roots

The *square* of a number is the product of that number multiplied by itself. Thus, the square of 9 is $9 \times 9 = 81$. The square of a number is indicated by the *exponent* (^{2}), thus: $9^2 = 9 \times 9 = 81$.

The *cube* or *third power* of a number is obtained if the number itself is repeated as a factor three times. Thus, the cube of 4 is $4 \times 4 \times 4 = 64$, and is written 4^3.

If a number is repeated as a factor four or five times, respectively, the product is the fourth or fifth power. Thus $3^4 = 3 \times 3 \times 3 \times 3 = 81$, and $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. A number can be raised to any power by repeating it as a factor the required number of times.

The *square root* of a given number is that number which, when multiplied by itself, will give a product equal to the given number. The square root of 16 (written $\sqrt{16}$) equals 4, because $4 \times 4 = 16$.

The *cube root* of a given number is that number which, when repeated as a factor three times, will give a product equal to the given number. Thus, the cube root of 64 (written $\sqrt[3]{64}$) equals 4, because $4 \times 4 \times 4 = 64$.

The fourth, fifth, etc., roots of a given number are those numbers which when repeated as factors four, five, etc., times, will give as a product the given number. Thus $\sqrt[4]{16} = 2$, because $2 \times 2 \times 2 \times 2 = 16$.

The multiplications required for raising numbers to powers and the extracting of roots are greatly facilitated by the use of logarithms. The extracting of the square root and cube root by the regular arithmetical methods is a slow and cumbersome operation, and any roots can be more rapidly found by using logarithms.

The tables of squares and cubes, and square roots and cube roots, found at the beginning of this book, give these values directly for all whole numbers up to 2000. For ordinary practical calculations the squares, cubes, etc., for fractional values between whole numbers can usually be estimated. These tables also give the *reciprocals* of numbers from 1 to 2000. The reciprocal of a number is the quotient obtained by dividing 1 by the number. Thus the reciprocal of 4 is $1 \div 4 = 0.25$. The reciprocal values given in the tables can be used to save labor in division, as the quotient can be obtained by multiplying the dividend by the reciprocal of the divisor. Thus, the reciprocal of 244 is 0.0040984. To divide 13 by 244, or to reduce $13/244$ to a decimal, multiply as follows: $13 \times 0.0040984 = 0.0532792$.

Extracting the Square Root. — Assume that the square root of 119,716 is to be found. Beginning at the unit figure or decimal point, point off the number into periods of two figures each. Should there be an odd number of figures in the given number, the last period to the left will have only one figure.

Find the greatest whole number, the square of which does not exceed the value of the figures in the left-hand period (11), and write this number as the first figure in the root. In this example, this number is 3, the square of which is 9. Subtract this square from the left-hand period, and move down the next period of two figures and annex it to the remainder.

$$\begin{array}{rr|l} & 11'97'16 & 346 \\ 3 \times 3 = & 9 & \\ \hline 3 \times 20 = 60 & 297 & \\ (60 + 4) \times 4 = & 256 & \\ \hline 34 \times 20 = 680 & 4116 & \\ (680 + 6) \times 6 = & 4116 & \\ \hline \end{array}$$

Multiply the figure of the root obtained by the constant 20; $(3 \times 20 = 60)$; find how many times this product is contained in the number 297. This gives a trial figure for the second figure of the root; 60 is contained 4 whole times in 297, and 4 is, therefore, placed as the next figure of the root.

Subtract from 297 the product of 60 plus the figure of the root just obtained (4), multiplied by the same figure (4); $(60 + 4) \times 4 = 256$. If this product were larger than 297 it would indicate that the trial figure is too large, and a figure one unit smaller should be used. Then move down the next period of two figures and annex it to the remainder.

Multiply the figures of the root thus far obtained by 20; $(34 \times 20 = 680)$; find how many times this product is contained in 4116. This gives a trial figure for the third figure of the root. Then proceed as before.

If the last subtraction leaves no remainder, and if there are no more periods of figures to move down from the given number, the obtained root 346 is the exact square root of 119,716. If there is a remainder when the last period of figures has been moved down, place a decimal point after the figures already obtained in the root, annex two ciphers (00) to the remainder, and proceed as before until a sufficient number of decimals have been obtained.

Extracting the Cube Root. — Assume that the cube root of 80,621,568 is to be found. Beginning at the unit figure or decimal point, point off the number into periods of three figures each.

$$\begin{array}{rr|l} & 80'621'568 & 432 \\ 4 \times 4 \times 4 = & 64 & \\ \hline 4^2 \times 300 = 4{,}800 & 16621 & \\ 4^2 \times 300 \times 3 + 4 \times 30 \times 3^2 + 3^3 = & 15507 & \\ \hline 43^2 \times 300 = 554{,}700 & 1114568 & \\ 43^2 \times 300 \times 2 + 43 \times 30 \times 2^2 + 2^3 = & 1114568 & \\ \hline \end{array}$$

Find the greatest whole number, the cube of which does not exceed the value of the figures in the left-hand period (80), and write this number as the first figure in the root; 4 is the greatest whole number the cube of which does not exceed 80; subtract the cube of 4 from the left-hand period and move down the next period of three figures, and annex it to the remainder.

Multiply the square of the figure in the root by the constant 300; $(4^2 \times 300 = 4800)$, and find how many times this product is contained in the number 16,621. This gives a trial figure for the second figure of the root.

Now subtract from 16,621 the *sum* of the following products:

1. The square of the figure or figures already obtained in the root, except the last one, multiplied by 300 and this product multiplied by the figure just obtained in the root: $4^2 \times 300 \times 3 = 16 \times 300 \times 3 = 14{,}400$.

2. The figure or figures already obtained in the root, except the last one, multiplied by 30, and this product multiplied by the square of the last figure obtained: $4 \times 30 \times 3^2 = 4 \times 30 \times 9 = 1{,}080$.

3. The cube of the last figure obtained: $3^3 = 3 \times 3 \times 3 = 27$.

If the sum of these various products is larger than 16,621, it indicates that the trial figure is too large, and a figure one unit smaller should be used. After having subtracted as directed, move down the next period of three figures, and annex it to the remainder, and proceed as before.

Principal Algebraic Expressions and Formulas

$$a \times a = aa = a^2$$
$$a \times a \times a = aaa = a^3$$
$$a \times b = ab$$
$$a^2b^2 = (ab)^2$$
$$a^2a^3 = a^{2+3} = a^5$$
$$a^4 \div a^3 = a^{4-3} = a$$
$$a^\circ = 1$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$(a+b)^2 = a^2 + 2\,ab + b^2$$
$$(a-b)^2 = a^2 - 2\,ab + b^2$$
$$\frac{a^3}{b^3} = \left(\frac{a}{b}\right)^3$$
$$\frac{1}{a^3} = \left(\frac{1}{a}\right)^3 = a^{-3}$$
$$(a^2)^3 = a^{2\times 3} = (a^3)^2 = a^6$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$(a+b)^3 = a^3 + 3\,a^2b + 3\,ab^2 + b^3$$
$$(a-b)^3 = a^3 - 3\,a^2b + 3\,ab^2 - b^3$$

$$\sqrt{a} \times \sqrt{a} = a$$
$$\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$$
$$(\sqrt[3]{a})^3 = a$$
$$\sqrt[3]{a^2} = (\sqrt[3]{a})^2 = a^{\frac{2}{3}}$$
$$\sqrt[4]{\sqrt[3]{a}} = \sqrt[4\times 3]{a} = \sqrt[3]{\sqrt[4]{a}}$$
$$\sqrt{a} + \sqrt{b} = \sqrt{a + b + 2\sqrt{ab}}$$
$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$
$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$
$$\sqrt[3]{\frac{1}{a}} = \frac{1}{\sqrt[3]{a}} = a^{-\frac{1}{3}}$$

When $a \times b = x$, then $\log a + \log b = \log x$

$a \div b = x$, then $\log a - \log b = \log x$

$a^3 = x$, then $3 \log a = \log x$

$\sqrt[3]{a} = x$, then $\dfrac{\log a}{3} = \log x$

Equations

An equation is a statement of equality between two expressions, as $5\,x = 105$. The unknown quantity in an equation is generally designated by the letter x. If there is more than one unknown quantity, the others are designated by letters also selected at the end of the alphabet, as y, z, u, t, etc.

An equation of the first degree is one which contains the unknown quantity only in the first power, as $3\,x = 9$. A quadratic equation is one which contains the unknown quantity in the second, but no higher, power, as $x^2 + 3\,x = 10$.

Solving Equations of the First Degree with One Unknown. — Transpose all the terms containing the unknown x to one side of the equals sign, and all the other terms to the other side. Combine and simplify the expressions as far as possible, and divide both sides by the coefficient of the unknown x. (See the rules given for transposition of formulas.)

Example:

$$22\,x - 11 = 15\,x + 10$$
$$22\,x - 15\,x = 10 + 11$$
$$7\,x = 21$$
$$x = 3$$

Solution of Equations of the First Degree with Two Unknowns. — The form of the simplified equations is:

$$ax + by = c$$
$$a_1x + b_1y = c_1$$

Then,

$$x = \frac{cb_1 - c_1b}{ab_1 - a_1b} \qquad y = \frac{ac_1 - a_1c}{ab_1 - a_1b}$$

Example:

$$3x + 4y = 17$$
$$5x - 2y = 11$$
$$x = \frac{17 \times (-2) - 11 \times 4}{3 \times (-2) - 5 \times 4} = \frac{-34 - 44}{-6 - 20} = \frac{-78}{-26} = 3.$$

The value of y can now be most easily found by inserting the value of x in one of the equations:

$$5 \times 3 - 2y = 11; \quad 2y = 15 - 11 = 4; \quad y = 2.$$

Solution of Quadratic Equations with One Unknown. — If the form of the simplified equation is:

$$x^2 + px = q, \quad \text{then} \quad x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} + q}$$

If the form of the equation is:

$$ax^2 + bx = c, \quad \text{then} \quad x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$$

Example 1:

$$x^2 + 12x = 108$$
$$x = -\frac{12}{2} \pm \sqrt{\frac{144}{4} + 108} = -6 \pm \sqrt{36 + 108} = -6 \pm \sqrt{144} = -6 \pm 12$$
$$x = 6, \quad \text{or} \quad x = -18.$$

Example 2:

$$3x^2 - 15x = 0.$$
$$x = \frac{+15 \pm \sqrt{225 + 0}}{2 \times 3} = \frac{15 \pm 15}{6}.$$
$$x = \frac{30}{6} = 5, \quad \text{or} \quad x = \frac{0}{6} = 0.$$

Cubic Equations. — If the given equation has the form:

$$x^3 + ax + b = 0$$

then,

$$x = \left(-\frac{b}{2} + \sqrt{\frac{a^3}{27} + \frac{b^2}{4}}\right)^{\frac{1}{3}} + \left(-\frac{b}{2} - \sqrt{\frac{a^3}{27} + \frac{b^2}{4}}\right)^{\frac{1}{3}}$$

The equation $x^3 + px^2 + qx + r = 0$, may be reduced to the form $x_1^3 + ax_1 + b = 0$, by substituting $x_1 - \frac{p}{3}$ for x in the given equation.

THE SLIDE-RULE

By means of the slide-rule, various calculations may be made mechanically with greater ease and rapidity than by ordinary arithmetical methods, and usually with sufficient accuracy for all practical requirements. Slide-rules are used principally for performing multiplication and division, but they may also be used for finding powers, roots, logarithms, and trigonometrical functions, and for various other purposes. The slide-rule in its most common form consists of three main parts. There is a main body or rule, a slide, and a runner or "cursor." Scales *A* and *D* (see Fig. 1) are on the rule, and scales *B* and *C* on the slide. Fig. 1 shows the right-hand half of the rule and the left-hand half of the slide, this sectional view being shown in order to secure a larger reproduction. The runner, which is in the form of a light metal frame that is free to slide endwise along the rule, is also shown in Fig. 1. Scales *A* and *B* are alike, as are scales *C* and *D*. All four scales are of the same length, but the graduations on scales *A* and *B* are different from those on scales *C* and *D*. The graduation 1 (seen at the extreme left in Fig. 1) is in the center of the rule, and scale *A* has two parts on opposite sides of this middle point 1 that are graduated exactly alike. The left-hand half from 1 to 1 is called the *left-hand A scale*, and the right-hand half from 1 to 1, the *right-hand A scale*. As scale *B* is exactly like scale *A*, its sections are designated as the *left-hand B scale* and the *right-hand B scale*. Each end of each scale is marked by the figure 1 which is known as the *index*. Thus, the figure 1 at the left-hand end of a scale is called the *left-hand index*, and figure 1 at the right-hand end is called the *right-hand index*.

As will be seen, the divisions on a slide-rule are not uniform. For example, that part of the left-hand *D* scale between divisions 1 and 2 (see enlarged detailed view, Fig. 2) is divided into ten main parts, no two of which are equal. Nevertheless, each of these main spaces represents 0.1. Each main space is subdivided into ten unequal parts, each of which represents one-tenth of 0.1, or 0.01. On the scales *A* and *B*, each of the ten main spaces between 1 and 2 is divided into only five parts; therefore, each subdivision represents 0.02. It is not possible to divide the spaces between graduations 4 and 5, 6 and 7, 8 and 9, etc., into as many parts as between 1 and 2, or 2 and 3, because the graduation lines would be too close together; therefore, the subdivisions near the right-hand end of any of the scales represent greater values than those near the left-hand end. This fact must be kept in mind or errors will result in the use of the slide-rule.

Reading the Slide-rule Graduations. — The method of reading the graduations is indicated in Fig. 2. A small figure 1 to the right of the left-hand index 1, represents the value 1.1, the small figure 2 marks the line that represents 1.2, and so on, to the small figure 9, which denotes 1.9. The second line to the right of the left-hand index denotes 1.02, as it is the second of the ten marks between 1 and 1.1. Several other lines at different points of the scale have the values indicated. It will be noticed that in each case the reading is accurate to at least three figures, and in some cases to four. The divisions on all four of the scales on the slide-rule are such that values can be read correctly to three figures; but beyond this it is not possible to proceed accurately, although the fourth figure may be obtained by estimating with the eye the fractional part of the subdivision. For instance, the fourth figure of the reading 1.535 is possible because the point is halfway between 1.53 and 1.54.

The values of these readings are based on the assumption that the left-hand index represents 1. The value of the left-hand index of any scale may be taken as any power of 0.1 or of 10; that is, it may be taken to represent 0.1, 0.01, 0.001, etc., or 10, 100, 1000, etc. If the left-hand index of the *D* scale represents 0.01, the readings in Fig. 2 must be multiplied by 0.01, so that they become 0.0102,

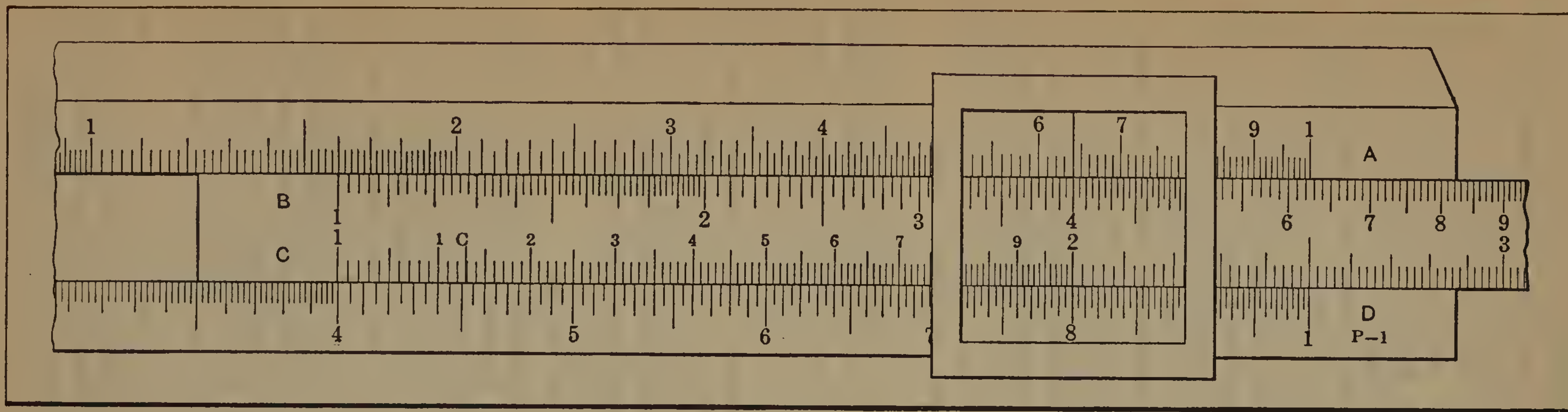


Fig. 1

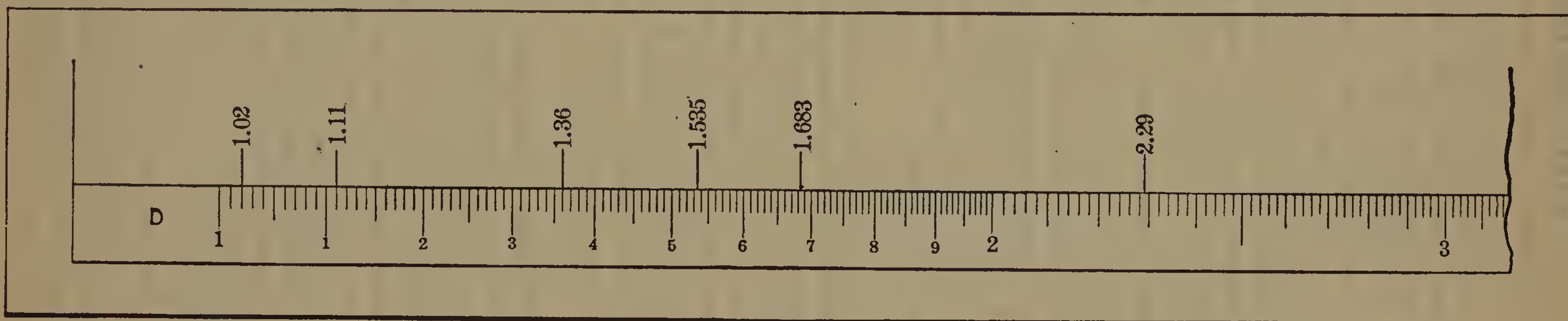


Fig. 2

0.0111, 0.0136, 0.01535, 0.01683, and 0.0229. If the left-hand index represents 1000, the readings must be multiplied by 1000. When a value has been assigned to the index, the same ratio must be maintained throughout the remainder of the scale, a line on any scale always representing the same sequence of figures, but not always the same value. The scales *C* and *D* have a greater number of subdivisions between two consecutive numbers than have the scales *A* and *B* between the same numbers; therefore, it is customary to use the *C* and *D* scales for multiplications and divisions, as they can be read with greater ease and accuracy.

Multiplication of Whole Numbers. — In multiplying two numbers with the slide-rule, one index of the *C* scale is set opposite one of the numbers on the *D* scale, and the product is read on the *D* scale directly in line with the other number on the *C* scale.

Example: — Find the product of 4×2. Move the slide to the right, as shown in Fig. 1, until the left-hand index of the *C* scale is exactly in line with 4 on the *D* scale; then, directly opposite 2 on the *C* scale, read the product, 8, on the *D* scale. The hair-line on the glass plate of the runner should be used, as shown, to determine what reading on the *D* scale is opposite 2 on the *C* scale.

Example: — Find the product of 7×2. If the left-hand index of the *C* scale is set to 7 on the *D* scale, the 2 on the *C* scale will be far to the right of the end of the *D* scale, and no result can be read. In such a case as this, move the slide to the left until the right-hand index of the *C* scale is opposite 2 on the *D* scale; then, opposite 7 on the *C* scale will be found the small figure 4 on the *D* scale, which is the point corresponding to 1.4; but 1.4 is not the product of 7 and 2, as $7 \times 2 = 14$. However, neglecting the decimal point, the figures of the result are correct. It becomes evident, therefore, that while the slide-rule will correctly show the first two, three, or four figures of the result, it will not always indicate the correct number of figures preceding the decimal point. The following simple rule may be used to find the number of figures in the product of two whole numbers:

Rule: — If the result is obtained with the slide projecting to the left, the number of figures in the product is equal to the sum of the numbers of figures in the two numbers; but, if the slide projects to the right, the number of figures in the product is one less than the sum of the numbers of figures in the two numbers.

For instance, apply this rule to the example illustrated by Fig. 1. In this case, each number contains one figure, and the slide projects to the right; therefore, the number of figures in the result must be $(1 + 1) - 1 = 1$.

Example: — Let the product of 223×7285 be required. Set the right-hand index of the *C* scale to 223 on the *D* scale, as shown in Fig. 3. It is not possible to determine the exact point on the *C* scale corresponding to 7285, as that scale, between 7 and 8, can be read accurately to only two figures, while the third must be approximated. So the runner is set with its hair-line at the position corresponding to 729 on the *C* scale, as nearly as can be judged. Below, on the *D* scale, the hair-line points to 1625, as nearly as can be read; hence, 1625 forms the first four figures of the result. As the slide projects to the left, the product must contain $3 + 4 = 7$ figures, according to the rule. Therefore the first four figures 1625 must be increased to seven by the addition of ciphers at the right, giving 1,625,000. The actual product, worked out in the usual way, is 1,624,555, which shows that the slide-rule gives the correct result to the third figure.

Multiplication of Decimals. — If the numbers to be multiplied are decimals or mixed numbers, disregard the decimal points for the time being and treat each as a whole number. Find the product of the whole numbers as already explained, and then point off as many decimal places as there are in both original numbers.

Example: — Let the product of 335.75×0.00264 be required. Neglecting the decimal points, these numbers become 33,575 and 264. As the scales cannot be read accurately to more than three places, the first of the numbers may be written 33,600. Set the left-hand index of the *C* scale to 336 on the *D* scale, and opposite 264 on the *C* scale, as indicated by the hair-line, read 886 on the *D* scale. As the slide projects to the right, the number of figures in the product is $5 + 3 - 1 = 7$; that is, four ciphers must be added to the three figures read on the *D* scale, giving 8,860,000 as the product of 33,600 and 264 on the rule. But the original numbers, 335.75 and 0.00264, have two decimal places and five decimal places, respectively, or seven decimal places in all; hence, seven places must be pointed off in the product of the whole numbers, giving 0.8860000, or simply 0.886, as the required result.

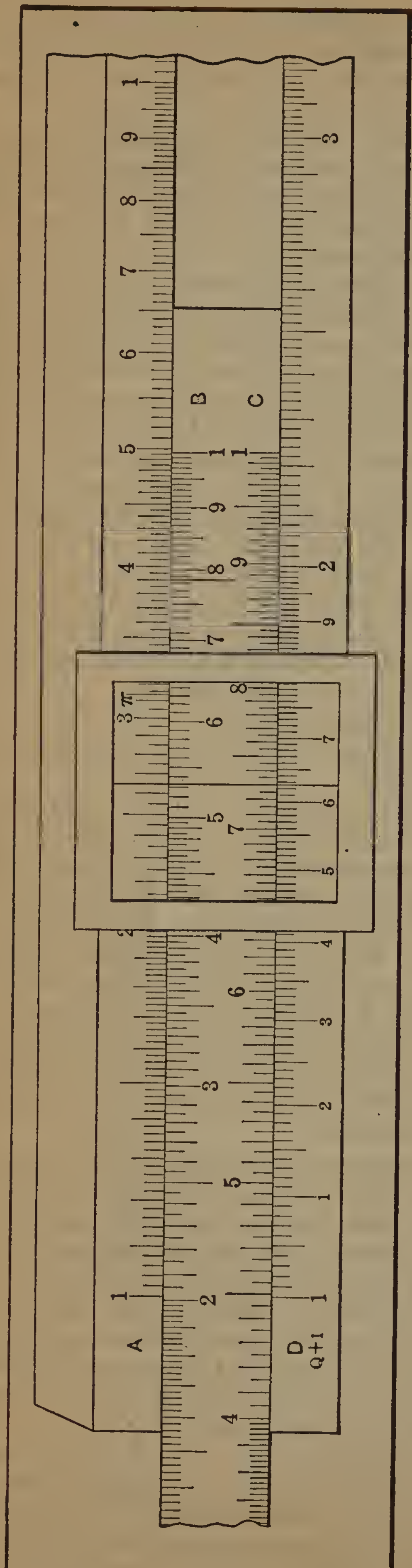


Fig. 3

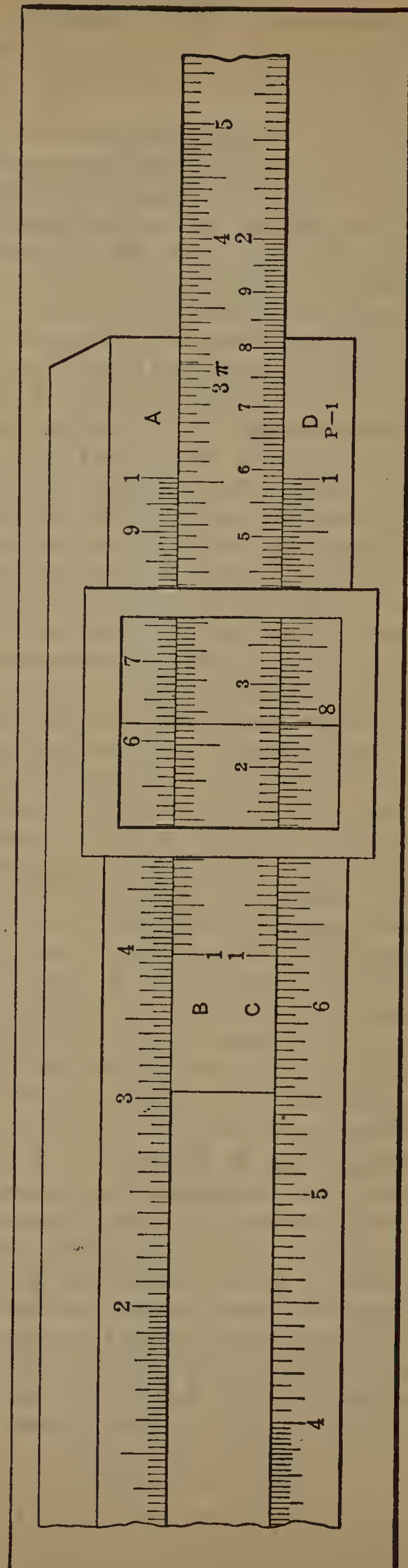


Fig. 4

Continued Multiplication. — Continued multiplication can be performed very quickly by the use of the slide-rule. For example, suppose that the result of $6.3 \times 1.25 \times 965.5 \times 0.47$ is required. Neglecting decimal points for the time being, and writing all the numbers as whole numbers, the problem becomes $63 \times 125 \times 9655 \times 47$. Set the left-hand index of the *C* scale to 63 on the *D* scale and move the runner until the hair-line is over 125 on the *C* scale, as shown in Fig. 4. Now, without disturbing the runner, move the slide to the left until the right-hand index of the *C* scale is under the hair-line, and then, leaving the slide stationary, move the runner until the hair-line is over 9655 on the *C* scale. Finally, without moving the runner set the slide with the right-hand index of the *C* scale under the hair line. Then, without moving the slide, set the hair-line over 47 on the *C* scale. The result, read on the *D* scale under the hair-line, is 357. In obtaining this result, however, the slide projected once to the right and twice to the left; so taking the sum of the numbers of figures in all the factors, treated as whole numbers, and subtracting 1 for the time that the slide projected to the right, the number of figures in the product is found to be $(2 + 3 + 4 + 2) - 1 = 10$; that is, the product of the whole numbers is 3,570,000,000. But, the original factors contain $1 + 2 + 1 + 2 = 6$ decima places, which must be pointed off in the final result; hence, the required product is 3570.000000, or simply 3570.

In this example there are three settings of the slide, each representing one process of multiplication. The first gives the product of 63 and 125; the second gives the product of (63×125) and 9655; and the third gives the product of $(63 \times 125 \times 9655)$ and 47. The hair-line on the runner serves to designate the product obtained at each setting.

Division of Whole Numbers. — Division is simply a reversal of multiplication. The number to be divided, or the dividend, is found on the *D* scale, and the hair-line of the runner is set over it. The divisor is found on the *C* scale and is set directly under the hair-line, also. The quotient is then read from the *D* scale at a point opposite the index of the *C* scale. In performing the division of whole numbers by the aid of the slide-rule, two cases may arise: The dividend may exceed the divisor or the divisor may exceed the dividend. If the dividend is the greater, the quotient will be a whole number or a mixed number. To find the number of figures in the whole-number part of the quotient, the following simple rule may be used:

Rule: — If the slide projects to the left, the number of figures in the whole-number part of the quotient is equal to the difference between the number of figures in the dividend and the number of figures in the divisor; but, if the slide projects to the right, the number of figures in the whole-number part of the quotient is 1 plus this difference.

Example: — To illustrate the application of this rule, suppose that the quotient of $1325 \div 592$ is required. Set the runner so that the hair-line is directly over 1325 on the *D* scale, as in Fig. 5. Next, without moving the runner, set the slide so that 592 on the *C* scale is directly under the hair-line. Then, opposite the right-hand index of the *C* scale will be found 224 on the *D* scale. As there are four figures in the dividend and three in the divisor, and the slide projects to the left, the number of figures in the whole-number part of the quotient, according to the rule, is $4 - 3 = 1$; therefore, the result is 2.24.

When Divisor is Greater than Dividend. — If the divisor is greater than the dividend, a different rule must be used, because the result will then be wholly a decimal. The slide and the runner are set in the same way as previously described, but the decimal point is located according to the following rule:

Rule: — If the slide projects to the left, the number of ciphers between the decimal point and the first figure of the quotient is equal to the difference between

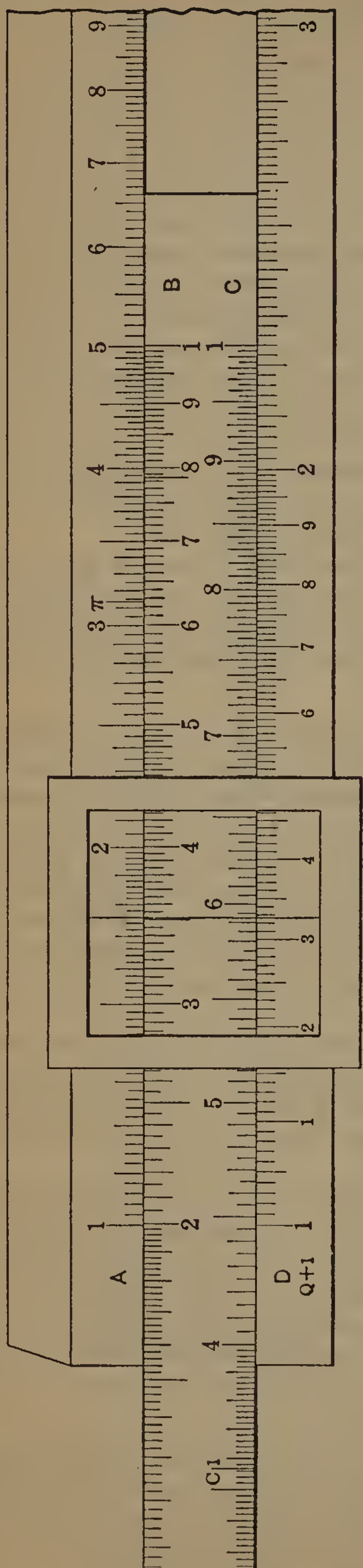


Fig. 5

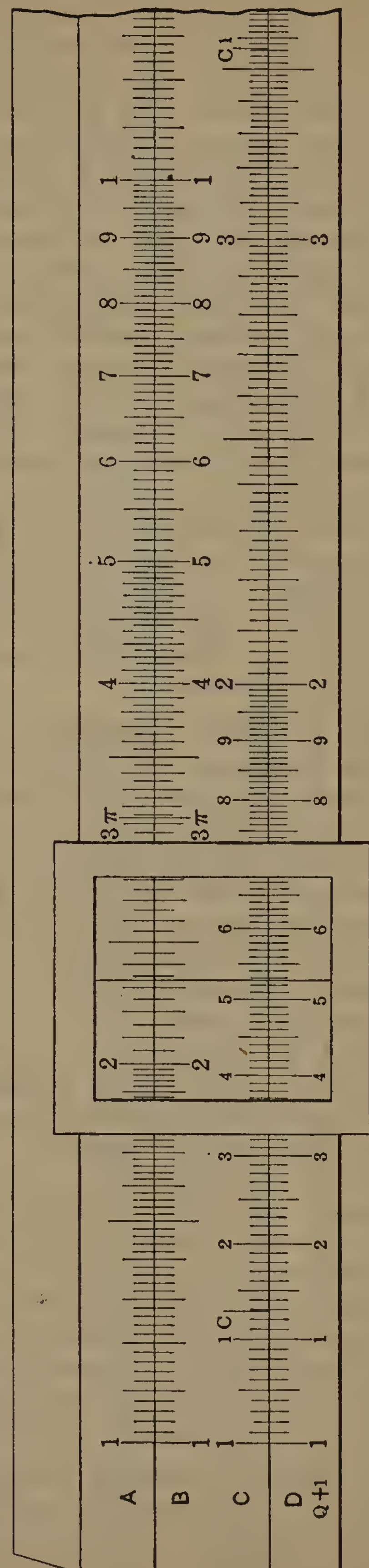


Fig. 6

the number of figures in the divisor and the number of figures in the dividend; but, if the slide projects to the right, the number of ciphers is 1 less than this difference.

Example: — Let the quotient of 176 ÷ 625 be required. Setting the runner and the slide as already explained, the result is found to be 282, with the slide projecting to the left. Then, the number of ciphers between the decimal point and the first figure must be $3 - 3 = 0$, since the divisor and the dividend each contain three figures; that is, the first figure immediately follows the decimal point, and the result is 0.282.

Division of Decimals. — The division of decimals and mixed numbers by means of the slide-rule does not differ in principle from the division of whole numbers. The only difficulty that arises is that of locating the decimal point in the quotient. As the adding of ciphers to the right of a decimal number does not affect its value — that is, 0.2 = 0.20 = 0.200, and so on — the following rule may be formulated:

Rule: — If the problem requires the division of decimals, add ciphers to either the dividend or the divisor until both have the same number of decimal places. Then ignore the decimal points in the new numbers thus obtained, converting them into whole numbers, perform the division as already explained, and locate the decimal point by the proper rule as given in connection with the division of whole numbers.

Example: — As an illustration of the method to be followed, let the quotient of 0.236 ÷ 926 be required. The divisor, 926, may be written 926.000 without changing its value, and the problem then becomes 0.236 ÷ 926.000; or, dropping the decimal points, and treating both as whole numbers, the problem is 236 ÷ 926,000. The runner and slide are set as explained for whole numbers. The slide projects to the left and the quotient on the D scale is 255. There are six figures in the divisor and three in the dividend; hence, the number of ciphers intervening between the decimal point and the first figure is $6 - 3 = 3$. The result, therefore, must be 0.000255.

Squares of Numbers. — The squares and square roots of numbers can readily be found by using the slide-rule. The A and D scales only are used in finding the square of a number. Set the runner so that the hair-line stands over the number on the D scale and the square of that number will be found on the A scale under the hair-line.

Example: — Find the square of 152.7. Set the runner, as shown in Fig. 6, so that the hair-line is over 1527 on the D scale, and under the hair-line on the A scale will be found 233. The number of whole-number places and decimal places in the square can easily be found. If the result is read from the left-hand A scale, the square contains one less than twice as many figures as there are in the original number; whereas, if it is read from the right-hand A scale, the square contains twice as many figures as the original number; and if there is a decimal, there will be twice as many places as in the original number. In the above problem, the original number contains four figures and the result is found on the left-hand A scale; hence, the number of figures in the square is $(2 \times 4) - 1 = 7$. There must also be $2 \times 1 = 2$ decimal places in the final result. Therefore, adding ciphers to 233 to make seven figures and then pointing off two decimal places, the required square is found to be 23,300.00, or 23,300.

Square Root. — The square root of a number is found by reversing the process of finding the square; that is, the number is located on the A scale and the root is found directly below, on the D scale, the hair-line being used to make the readings accurately. If the number is a whole number or a mixed number and contains an *odd* number of figures in the whole-number part, or if it is wholly a decimal and has an *odd* number of ciphers directly following the decimal point, the left-hand A

scale must be used; in all other cases, the right-hand A scale is used. Let N represent the number of figures in the whole-number part, N' the number of ciphers directly following the decimal point, in the root, and n the same numbers in the original number. Then, if the left-hand A scale is used, $N = \frac{n+1}{2}$, and $N' = \frac{n-1}{2}$. If the right-hand A scale is used, $N = N' = \frac{n}{2}$.

The finding of a square root is illustrated in Fig. 6.

Example: — Let the square root of 2.33 be required. As there is an odd number of figures in the whole-number part, the left-hand A scale is used, and below, on the D scale, 1527 is read. With the left-hand A scale in use, $N = \frac{n+1}{2} = \frac{1+1}{2} = 1$; hence, the square root is 1.527.

It is very probable that the number of figures preceding the decimal point, or the number of ciphers following the decimal point, in a square root can be determined more easily and quickly by inspection than by the foregoing rules, because by pointing off the given number into periods of two figures each, as taught in arithmetic, the desired information as to the location of the decimal point in the root can be seen at a glance. It is abso utely necessary, however, to use some form of rule, such as that given, to determine whether the left-hand or the right-hand A scale is to be used.

Cubes of Numbers. — Finding the cube of a number requires the use of all four scales. The index of the C scale is set to the given number on the D scale and, opposite the same number on the *left-hand B* scale, the cube is read from the A scale. Three cases may arise, as follows:

Case I. — The slide may project to the left and the cube be found on the left-hand A scale.

Case II. — The slide may project to the right and the cube be found on the right-hand A scale.

Case III. — The slide may project to the right and the cube be found on the left-hand A scale.

If the number contains a decimal, ignore the decima point for the time being and treat it as a whole number. Let N represent the number of figures in the cube and n the number of figures in the given number. Then:

$$N = 3n, \text{ for Case I;}$$
$$N = 3n - 1, \text{ for Case II;}$$
$$N = 3n - 2, \text{ for Case III.}$$

After the total number of figures has been found by one of these rules, point off three times as many places as there are decimal places in the given number, and the result will be the required cube of the given number.

Example: — To illustrate, suppose that 36.8^3 is to be found. Treat it as a whole number, 368, and set the left-hand index of the C scale to 368 on the D scale, as in Fig. 7. Then, opposite 368 on the left-hand B scale will be found 5 on the A scale. As this setting falls under Case II, $N = (3 \times 3) - 1 = 8$; hence, the reading is 50,000,000. There is one decimal place in the given number, so that three places must be pointed off in the result, giving 50,000.000, or simply 50,000, as the required cube.

The cube can be found quite as readily by multiplying the number together three times; that is, $36.8^3 = 36.8 \times 36.8 \times 36.8$. This continued multiplication can be performed as previously described, and the position of the decimal point be determined by the rules given in that connection. This method will doubtless be found easier to remember than the one outlined above, involving three cases with as many different rules.

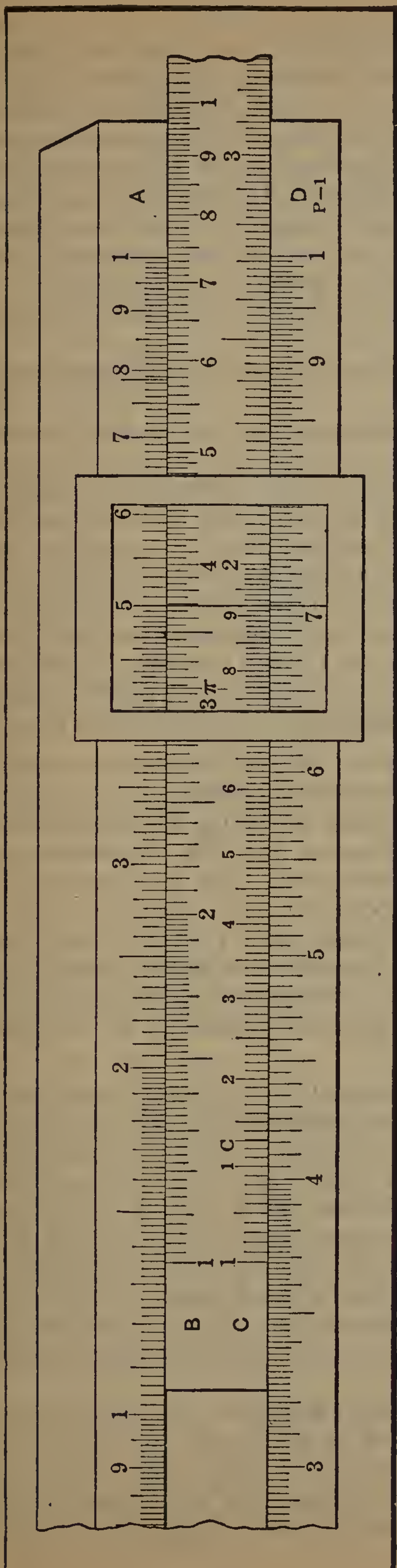


Fig. 7

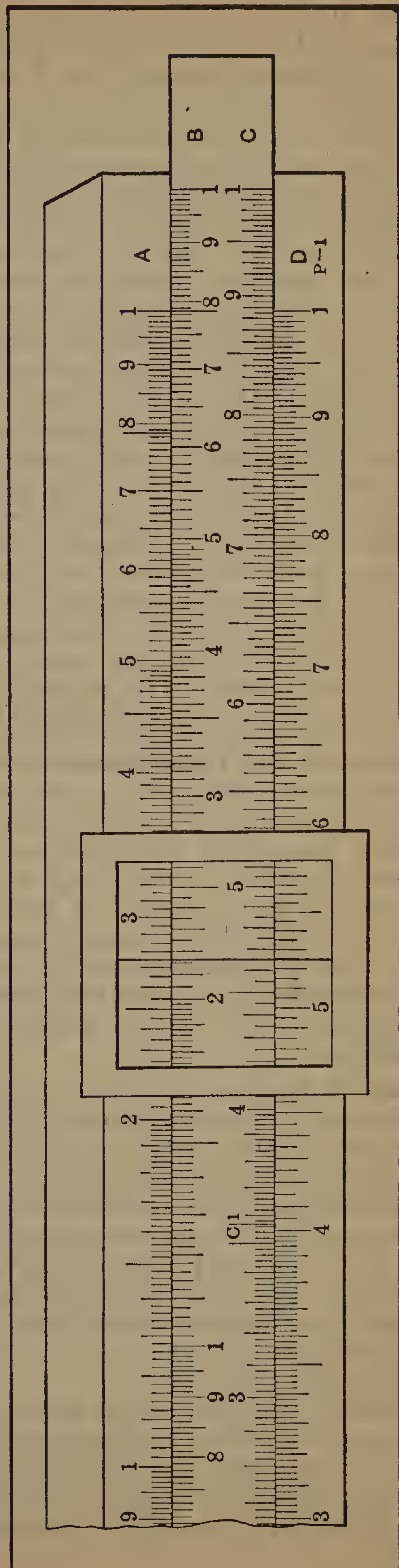


Fig. 8

Cube Root. — The operation of finding the cube root is the reverse of finding the cube, and the method of determining the number of figures in the root, preceding or following the decimal point, is the same as that used in arithmetic; that is, the number is pointed off into periods of three figures each, beginning at the decimal point, and the root then contains one figure for each period or part of a period. If the number contains a decimal, add ciphers to the right of it, if necessary, so that the number of figures in the decimal part will be exactly divisible by 3, without a remainder. The scales and indexes to be used in finding the cube root depend upon the number of figures in the first period at the left, not counting ciphers when they *immediately* follow the decimal point. The rules to be followed are:

Case I. — If there are three figures in the first period, use the left-hand A scale and the right-hand index of the C scale.

Case II. — If there are two figures in the first period, use the right-hand A scale and the left-hand index of the C scale.

Case III. — If there is one figure in the first period, use the left-hand A scale and the left-hand index of the C scale.

Example: — Find the cube root of 0.05. Add ciphers to make three figures, giving 0.050, thus forming one period of three figures. This indicates that the first figure of the root immediately follows the decimal point. As there are two figures, 50, in the first period, "050," the right-hand A scale and the left-hand index of the C scale are used. Set the runner so that the hair-line is across 5 on the right-hand A scale, as in Fig. 7. Then move the slide until the reading on the B scale under the hair-line is exactly equal to that on the D scale opposite the left-hand index of the C scale. This reading on the D scale, 368, is the result required. The first figure of the root must follow the decimal point; therefore, the root is 0.368.

Diameters and Circumferences of Circles. — The ratio of the circumference of a circle to the diameter is $3.1416 = \pi$, and this value is marked by an extra line on the A and B scales, on many rules. It is used in finding the circumference from the diameter by multiplication, or the diameter from the circumference by division. In either instance, the A and B scales are used. The left-hand index of the B scale is set to π on the A scale, and opposite the given diameter on the B scale is read the circumference on the A scale; or, the value π on the B scale is set to the given circumference on the A scale, and opposite the index of the B scale the required diameter is read from the A scale. This multiplication or division is performed in exactly the same way as in using the C and D scales for the same kind of operation.

Areas of Circles. — The area of a circle is equal to 0.7854 times the square of the diameter; consequently, the value 0.7854 is marked by an extra line on the right-hand A and B scales on many rules. To find the area of a circle, having given the diameter, set the line 0.7854 of the B scale to either the left-hand or the right-hand index of the A scale and read the area from the B scale, above the given diameter on the D scale, the hair-line on the runner being used to transfer from D to B. If the area is given, set the slide as before, and read the required diameter from the D scale directly below the given area on the B scale. If the number of figures preceding the decimal point, or the number of ciphers immediately following the decimal point, in the area, is odd, use the left-hand B scale; otherwise, use the right-hand B scale.

Example: — Find the area of a circle $5\frac{1}{4}$ inches in diameter. Set the line marking 0.7854 on the B scale to the right-hand index of the A scale, as in Fig. 8, and above 5.25 on the D scale will be found 21.6 on the B scale; hence, the area is 21.6 square inches. The same setting, taken in reverse order, can be used to illustrate the method of finding the diameter of a circle the area of which is 21.6 square inches.

LOGARITHMS

The object of logarithms is to facilitate and shorten calculations involving multiplication, division, the extraction of roots and the obtaining of powers of numbers. A logarithm consists of two parts, a whole number and a decimal. The whole number, which may be either a positive or negative number, or zero, is called the *characteristic;* the decimal is called the *mantissa.* As a rule, the decimal or mantissa only is given in tables of logarithms. The characteristic is prefixed to the mantissa according to the following rules:

For 1 and for all numbers greater than 1, the characteristic is one less than the number of places to the left of the decimal point in the given number. For example, the characteristic of the logarithm of 237 is 2, and of 2536.5 is 3.

For numbers smaller than 1, that is for numbers wholly decimal, the characteristic is negative and its numerical value is one more than the number of ciphers between the decimal point and the first decimal which is not a cipher. For example, the characteristic of the logarithm of 0.036 is (− 2), and the characteristic of the logarithm of 0.0006 is (− 4). Instead of writing the minus sign (−) in front of the figure, as (− 2), it is frequently written over the figure, thus: ($\bar{2}$). This method

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
400	60	206	217	228	239	249	260	271	282	293	304	
401		314	325	336	347	358	369	379	390	401	412	
402		423	433	444	455	466	477	487	498	509	520	
403		531	541	552	563	574	584	595	606	617	627	
404		638	649	660	670	681	692	703	713	724	735	

Fig. 1

is used because the minus sign refers only to the characteristic and not to the mantissa, which is always positive.

The logarithmic tables in the following give, in the body of the tables, the mantissa of the logarithms of numbers from 1 to 10,000. When finding the mantissa, the decimal point in a number is disregarded. The mantissa of the logarithms of 2716, 271.6, 27.16, 2.716, or 0.02716, for example, is the same. The tables give directly the mantissa of logarithms of numbers with four figures or less; the logarithms for numbers with more than four figures can be approximated.

To find the logarithm of a number from the tables, locate the first three figures of the number in the left-hand column, and then find the fourth figure at the top of the columns of the page. Then follow the column down from this last figure until opposite the three first figures in the left-hand column. The figure thus found in the body of the table is the mantissa of the logarithm. If the number of which the logarithm is required does not contain four figures, annex ciphers to the right so as to obtain four figures. If the mantissa of the logarithm of 6 is required, for example, find the mantissa for 6000.

Example: — Find the logarithm of 4032. Locate 403 in the left-hand column of the logarithmic tables, then follow downward the column headed "2" at the top of the page, and find the required mantissa opposite 403. The mantissa is .60552 the "group" figures 60 being found in the column under "L" and prefixed to the figures 552 found directly in the column under "2." The characteristic of the logarithm being 3, log 4032 = 3.60552. (See Fig. 1.)

All the mantissas, or the numbers in the tables, are decimals, and the decimal point has, therefore, been omitted in the tables, since no confusion could arise from this; but it should always be put before the figures of the mantissa as soon as taken from the table.

In the tables it will be found that, in some cases, the figures are preceded by the sign (*). The sign (*) indicates that the two figures to be prefixed are those given in the next line below that in which the last three figures are read. For example the logarithm of 5018 is 3.70053, the two figures to be prefixed being 70 and not 69 as would ordinarily be the case. (See Fig. 2.)

Finding a Number the Logarithm of which is Given.—When a logarithm is given and it is required to find the corresponding number, find the first two figures of the mantissa in the column headed "L" in the tables; then find in the group of mantissas, all having the same first two figures, the remaining three figures. These may appear in any of the columns headed "0" to "9". The number heading the column in which the last three figures of the mantissa are found is the last figure in the number sought, and the number in the left-hand column, headed "N," in line with the last three figures of the mantissa, gives the three first figures in the number sought. When the actual figures in the number sought have been deter-

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
500	69	897	906	914	923	932	940	949	958	966	975	
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70	070	079	088	096	105	114	122	131	140	148	
503		157	165	174	183	191	200	209	217	226	234	
504		243	252	260	269	278	286	295	303	312	321	

Fig. 2

mined, locate the decimal point according to the rules given for the characteristic of logarithms. If the characteristic is greater than 3, add ciphers. For example, if the figures corresponding to a given mantissa are 3765 and the characteristic is 5, then the number sought has 6 figures to the left of the decimal point, and is 376,500. If the characteristic had been $\bar{3}$, then the number sought would, in this case, have been 0.003765. If the mantissa is not exactly obtainable in the tables, find the mantissa in the table which is the nearest to the one given and determine the number corresponding to this. In most cases, this gives results accurate enough. By interpolation, as will be explained later, more accurate results can be obtained.

If the three last figures of the mantissa, as found in the table, are preceded by a (*), it indicates that these three figures belong to the group preceded by the two figures in the "L" column in the line next below.

Example: — Find the number the logarithm of which is 2.70053.

First find the two figures of the mantissa (70) in the column headed "L" in the tables. Then find the remaining three figures (053) in the mantissas which all have 70 for their first two figures. The (*) in front of the figure *053 in the line next above that in which 70 is found indicates that these figures belong to the group preceded by 70. Therefore, the number corresponding to the logarithm 2.70053 is 501.8. (See Fig. 2.)

Multiplication by Logarithms.—If two or more numbers are to be multiplied together, find the logarithms of the numbers to be multiplied, and add these logarithms. The sum is the logarithm of the product, and the number corresponding to this logarithm, as found from the logarithmic tables, is the required product.

Example: — Find the product of $2831 \times 2.692 \times 29.69 \times 19.4$.

This calculation is carried out by means of logarithms as follows:

$$\begin{array}{ll} \log 2831. & = 3.45194 \\ \log 2.692 & = 0.43008 \\ \log 29.69 & = 1.47261 \\ \log 19.4 & = 1.28780 \\ \hline & 6.64243 \end{array}$$

The product, as found from the tables, with ciphers added, then is 4,390,000.

Division by Logarithms. — When dividing one number by another, subtract the logarithm of the divisor from the logarithm of the dividend; the remainder is the logarithm of the quotient. For example, to find the quotient of $7658 \div 935.3$, find log 7658 and subtract from it log 935.3; the remainder is the logarithm of the quotient.

A modified method, especially useful when the dividend and divisor are composed of several factors, is to use the negative value of the logarithms of all the numbers or factors of the divisor. These negative values can then be *added* to the logarithms of the factors composing the dividend. As the mantissa must always remain positive, however, in order to permit direct addition, the negative value of the logarithm cannot be obtained by simply placing a minus sign before it. Instead it is obtained in the following manner:

If the characteristic is positive, add 1 to its numerical value and place a minus sign over it. To obtain the mantissa, subtract the given mantissa from 1.00000.

Example: — The logarithm of $950 = 2.97772$. Find $(-\log 950)$. According to the rule, the characteristic will be $\bar{3}$. The mantissa will be $1.00000 - .97772 = .02228$. Hence, $(-\log 950) = \bar{3}.02228$.

If the characteristic of the logarithm is negative, subtract 1 from its numerical value and make it positive. The mantissa is obtained by the same rule as given above. For example, the logarithm of $0.003 = \bar{3}.47712$; hence, $(-\log 0.003) = 2.52288$. The following example shows the usefulness of this method in an actual calculation:

$$\frac{0.0272 \times 27.1 \times 12.6}{2.371 \times 0.007}$$

$$\begin{array}{ll} \log 0.0272 & = \bar{2}.43457 \\ \log 27.1 & = 1.43297 \\ \log 12.6 & = 1.10037 \\ -\log 2.371 & = \bar{1}.62507 \\ -\log 0.007 & = 2.15490 \\ \hline & 2.74788 \end{array}$$

The result, then, is 559.6.

Obtaining the Powers of Numbers. — A number may be raised to any power by simply multiplying the logarithm of the number by the exponent of the number. The product gives the logarithm of the value of the power.

Example 1. — Find the value of 6.51^3.

$$\log 6.51 = 0.81358$$

$$3 \times 0.81358 = 2.44074$$

The logarithm 2.44074 is then the logarithm of 6.51^3. Hence 6.51^3 equals the number corresponding to this logarithm, as found from the tables, or $6.51^3 = 275.9$.

Example 2. — Find the value of $12^{1.29}$.

$$\log 12 = 1.07918$$

$$1.29 \times 1.07918 = 1.39214$$

Hence, $12^{1.29} = 24.67$.

The multiplication 1.29×1.07918 is carried out in the usual arithmetical way. The example above is one of a type which cannot be solved by any means except by the use of logarithms.

One difficulty is met with when raising a number less than 1 to a given power. The logarithm is then composed of a negative term, the characteristic, and a positive term, the mantissa. For example: Find the value 0.31^5. The logarithm of $0.31 = \bar{1}.49136$. In this case, multiply, separately, the characteristic and the mantissa by the exponent, as shown below. Then add the products.

$$\log 0.31 = \bar{1}.49136$$

Multiplying characteristic and mantissa separately by 5 we have:

$$5 \times \bar{1} = \bar{5}$$

$$5 \times 0.49136 = 2.45680$$

$$\log 0.31^5 = \bar{3}.45680$$

Hence, $0.31^5 = 0.002863$.

Extracting Roots by Logarithms. — Roots of numbers, as for example $\sqrt[5]{37}$, can easily be extracted by means of logarithms. The small (5) in the radical ($\sqrt{\ }$) of the root-sign is called the index of the root. Any root of a number may be found by dividing its logarithm by the index of the root; the quotient is the logarithm of the root.

Example 1. — Find $\sqrt[3]{276}$.

$$\log 276 = 2.44091$$

$$2.44091 \div 3 = 0.81364$$

Hence $\log \sqrt[3]{276} = 0.81364$, and $\sqrt[3]{276} = 6.511$.

Example 2. — Find $\sqrt[3]{0.67}$.

$$\log 0.67 = \bar{1}.82607$$

In this case it is not possible to divide directly, because there is a negative characteristic and a positive mantissa. Proceed as follows: Add numerically as many *negative* units or parts of units to the characteristic as is necessary to make it evenly contain the index of the root. Then add the *same* number of *positive* units or parts of units to the mantissa. Divide each separately by the index. The quotients give the characteristic and mantissa, respectively, of the logarithm of the root. Proceeding with the example above according to this rule, we have:

$$\bar{1} + \bar{2} = \bar{3}; \quad \bar{3} \div 3 = \bar{1}.$$

$$.82607 + 2 = 2.82607; \quad 2.82607 \div 3 = .94202.$$

Hence, $\log \sqrt[3]{0.67} = \bar{1}.94202$, and $\sqrt[3]{0.67} = 0.875$.

Example 3. — Find $\sqrt[1.7]{0.2}$.

$$\log 0.2 = \bar{1}.30103.$$

If we add (-0.7) to the characteristic of the logarithm found, it will be evenly divisible by the index of the root.

Hence:

$$\bar{1} + (-0.7) = -1.7; \quad -1.7 \div 1.7 = \bar{1}.$$

$$.30103 + 0.7 = 1.00103; \quad 1.00103 \div 1.7 = .58884.$$

Hence, $\log \sqrt[1.7]{0.2} = \bar{1}.58884$, and $\sqrt[1.7]{0.2} = 0.388$.

When exponents or indices are given in common fractions, it is usually best to change them to decimal fractions before proceeding further with the problem.

Interpolation. — If the number for which the logarithm is required consists of five figures, it is possible by means of the small tables in the right-hand column of the logarithmic tables, headed "P.P." (proportional parts), to obtain the logarithm more accurately than by taking the nearest value for four figures. The logarithm of 1524.2, for example, is found as follows:

First find the difference between the nearest larger and the nearest smaller logarithms in the table. Log 1524 = 3.18298 and log 1525 = 3.18327. (See Fig. 3.) The difference is 0.00029. Then in the small table headed "29" in the right-hand

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.		
150	17	609	638	667	696	725	754	782	811	840	869			
151		898	926	955	984	*013	*041	*070	*099	*127	*156		29	28
152	18	184	213	241	270	298	327	355	384	412	441	1	2,9	2,8
153		469	498	526	554	583	611	639	667	696	724	2	5,8	5,6
154		752	780	808	837	865	893	921	949	977	*005	3	8,7	8,4

Fig. 3

column, find the figure opposite 2 (2 being the last or fifth figure in the given number). This figure is 5.8. Add this to the mantissa of the smaller of the two logarithms already found, disregarding the decimal point in the mantissa, and considering it, for the while being, as a whole number. Then, 18298 + 5.8 = 18303.8, or approximately, 18304. This is the mantissa of the logarithm of 1524.2 and the complete logarithm is 3.18304.

To find a number more accurately than to four figures, when the mantissa cannot be found exactly in the tables, find the mantissa which is nearest to, but less than, the given mantissa. Subtract this mantissa from the nearest larger mantissa in the tables and find in the right-hand column the small table headed by this difference. Then subtract the nearest smaller mantissa from the given logarithm and find the exact or approximate difference in the "proportional part" table. The corresponding figure in the left-hand column of the "proportional part" table is the fifth figure in the number sought, the other four figures being those corresponding to the logarithm next smaller than the given logarithm. In accordance with this rule, the number corresponding to the logarithm 4.46262 is found to be 29,015.

Hyperbolic Logarithms. — In many calculations, notably those involving calculations of the mean effective pressure of steam in engine cylinders, use is made of logarithms termed *hyperbolic*, *Napierian*, or *natural*; the preferable name, and that most commonly used in the United States, is hyperbolic logarithms. The hyperbolic logarithms are usually designated "hyp. log." Sometimes the hyperbolic logarithm is also designated "$\log_e$" and "nat. log." Tables are given in the following for hyperbolic logarithms from 1 to 100.

To convert the common logarithms to hyperbolic logarithms, the former should be multiplied by 2.30258. To convert hyperbolic logarithms to common logarithms, multiply by 0.43429. Hyperbolic logarithms find extensive use in higher mathematics.

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9
100	00	000	043	087	130	173	217	260	303	346	389
101		432	475	518	561	604	647	689	732	775	817
102		860	903	945	988	*030	*072	*115	*157	*199	*242
103	01	284	326	368	410	452	494	536	578	620	662
104		703	745	787	828	870	912	953	995	*036	*078
105	02	119	160	202	243	284	325	366	407	449	490
106		531	572	612	653	694	735	776	816	857	898
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03	342	383	423	463	503	543	583	623	663	703
109		743	782	822	862	902	941	981	*021	*060	*100
110	04	139	179	218	258	297	336	376	415	454	493
111		532	571	610	650	689	727	766	805	844	883
112		922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05	308	346	385	423	461	500	538	576	614	652
114		690	729	767	805	843	881	918	956	994	*032
115	06	070	108	145	183	221	258	296	333	371	408
116		446	483	521	558	595	633	670	707	744	781
117		819	856	893	930	967	*004	*041	*078	*115	*151
118	07	188	225	262	298	335	372	408	445	482	518
119		555	591	628	664	700	737	773	809	846	882
120		918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08	279	314	350	386	422	458	493	529	565	600
122		636	672	707	743	778	814	849	884	920	955
123		991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09	342	377	412	447	482	517	552	587	621	656
125		691	726	760	795	830	864	899	934	968	*003
126	10	037	072	106	140	175	209	243	278	312	346
127		380	415	449	483	517	551	585	619	653	687
128		721	755	789	823	857	890	924	958	992	*025
129	11	059	093	126	160	193	227	261	294	327	361
130		394	428	461	494	528	561	594	628	661	694
131		727	760	793	826	860	893	926	959	992	*024
132	12	057	090	123	156	189	222	254	287	320	352
133		385	418	450	483	516	548	581	613	646	678
134		710	743	775	808	840	872	905	937	969	*001
135	13	033	066	098	130	162	194	226	258	290	322
136		354	386	418	450	481	513	545	577	609	640
137		672	704	735	767	799	830	862	893	925	956
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14	301	333	364	395	426	457	489	520	551	582
140		613	644	675	706	737	768	799	829	860	891
141		922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15	229	259	290	320	351	381	412	442	473	503
143		534	564	594	625	655	685	715	746	776	806
144		836	866	897	927	957	987	*017	*047	*077	*107
145	16	137	167	197	227	256	286	316	346	376	406
146		435	465	495	524	554	584	613	643	673	702
147		732	761	791	820	850	879	909	938	967	997
148	17	026	056	085	114	143	173	202	231	260	289
149		319	348	377	406	435	464	493	522	551	580
150		609	638	667	696	725	754	782	811	840	869

P. P.			
	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8
	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1
	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4
	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7
	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Tables of Logarithms

N.	L. 0		1	2	3	4	5	6	7	8	9
150	17	609	638	667	696	725	754	782	811	840	869
151		898	926	955	984	*013	*041	*070	*099	*127	*156
152	18	184	213	241	270	298	327	355	384	412	441
153		469	498	526	554	583	611	639	667	696	724
154		752	780	808	837	865	893	921	949	977	*005
155	19	033	061	089	117	145	173	201	229	257	285
156		312	340	368	396	424	451	479	507	535	562
157		590	618	645	673	700	728	756	783	811	838
158		866	893	921	948	976	*003	*030	*058	*085	*112
159	20	140	167	194	222	249	276	303	330	358	385
160		412	439	466	493	520	548	575	602	629	656
161		683	710	737	763	790	817	844	871	898	925
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21	219	245	272	299	325	352	378	405	431	458
164		484	511	537	564	590	617	643	669	696	722
165		748	775	801	827	854	880	906	932	958	985
166	22	011	037	063	089	115	141	167	194	220	246
167		272	298	324	350	376	401	427	453	479	505
168		531	557	583	608	634	660	686	712	737	763
169		789	814	840	866	891	917	943	968	994	*019
170	23	045	070	096	121	147	172	198	223	249	274
171		300	325	350	376	401	426	452	477	502	528
172		553	578	603	629	654	679	704	729	754	779
173		805	830	855	880	905	930	955	980	*005	*030
174	24	055	080	105	130	155	180	204	229	254	279
175		304	329	353	378	403	428	452	477	502	527
176		551	576	601	625	650	674	699	724	748	773
177		797	822	846	871	895	920	944	969	993	*018
178	25	042	066	091	115	139	164	188	212	237	261
179		285	310	334	358	382	406	431	455	479	503
180		527	551	575	600	624	648	672	696	720	744
181		768	792	816	840	864	888	912	935	959	983
182	26	007	031	055	079	102	126	150	174	198	221
183		245	269	293	316	340	364	387	411	435	458
184		482	505	529	553	576	600	623	647	670	694
185		717	741	764	788	811	834	858	881	905	928
186		951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27	184	207	231	254	277	300	323	346	370	393
188		416	439	462	485	508	531	554	577	600	623
189		646	669	692	715	738	761	784	807	830	852
190		875	898	921	944	967	989	*012	*035	*058	*081
191	28	103	126	149	171	194	217	240	262	285	307
192		330	353	375	398	421	443	466	488	511	533
193		556	578	601	623	646	668	691	713	735	758
194		780	803	825	847	870	892	914	937	959	981
195	29	003	026	048	070	092	115	137	159	181	203
196		226	248	270	292	314	336	358	380	403	425
197		447	469	491	513	535	557	579	601	623	645
198		667	688	710	732	754	776	798	820	842	863
199		885	907	929	951	973	994	*016	*038	*060	*081
200	30	103	125	146	168	190	211	233	255	276	298

P. P.

	29	28	27	26	25	24	23	22	21
1	2.9	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.1
2	5.8	5.6	5.4	5.2	5.0	4.8	4.6	4.4	4.2
3	8.7	8.4	8.1	7.8	7.5	7.2	6.9	6.6	6.3
4	11.6	11.2	10.8	10.4	10.0	9.6	9.2	8.8	8.4
5	14.5	14.0	13.5	13.0	12.5	12.0	11.5	11.0	10.5
6	17.4	16.8	16.2	15.6	15.0	14.4	13.8	13.2	12.6
7	20.3	19.6	18.9	18.2	17.5	16.8	16.1	15.4	14.7
8	23.2	22.4	21.6	20.8	20.0	19.2	18.4	17.6	16.8
9	26.1	25.2	24.3	23.4	22.5	21.6	20.7	19.8	18.9

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9
200	30	103	125	146	168	190	211	233	255	276	298
201		320	341	363	384	406	428	449	471	492	514
202		535	557	578	600	621	643	664	685	707	728
203		750	771	792	814	835	856	878	899	920	942
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31	175	197	218	239	260	281	302	323	345	366
206		387	408	429	450	471	492	513	534	555	576
207		597	618	639	660	681	702	723	744	765	785
208		806	827	848	869	890	911	931	952	973	994
209	32	015	035	056	077	098	118	139	160	181	201
210		222	243	263	284	305	325	346	366	387	408
211		428	449	469	490	510	531	552	572	593	613
212		634	654	675	695	715	736	756	777	797	818
213		838	858	879	899	919	940	960	980	*001	*021
214	33	041	062	082	102	122	143	163	183	203	224
215		244	264	284	304	325	345	365	385	405	425
216		445	465	486	506	526	546	566	586	606	626
217		646	666	686	706	726	746	766	786	806	826
218		846	866	885	905	925	945	965	985	*005	*025
219	34	044	064	084	104	124	143	163	183	203	223
220		242	262	282	301	321	341	361	380	400	420
221		439	459	479	498	518	537	557	577	596	616
222		635	655	674	694	713	733	753	772	792	811
223		830	850	869	889	908	928	947	967	986	*005
224	35	025	044	064	083	102	122	141	160	180	199
225		218	238	257	276	295	315	334	353	372	392
226		411	430	449	468	488	507	526	545	564	583
227		603	622	641	660	679	698	717	736	755	774
228		793	813	832	851	870	889	908	927	946	965
229		984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36	173	192	211	229	248	267	286	305	324	342
231		361	380	399	418	436	455	474	493	511	530
232		549	568	586	605	624	642	661	680	698	717
233		736	754	773	791	810	829	847	866	884	903
234		922	940	959	977	996	*014	*033	*051	*070	*088
235	37	107	125	144	162	181	199	218	236	254	273
236		291	310	328	346	365	383	401	420	438	457
237		475	493	511	530	548	566	585	603	621	639
238		658	676	694	712	731	749	767	785	803	822
239		840	858	876	894	912	931	949	967	985	*003
240	38	021	039	057	075	093	112	130	148	166	184
241		202	220	238	256	274	292	310	328	346	364
242		382	399	417	435	453	471	489	507	525	543
243		561	578	596	614	632	650	668	686	703	721
244		739	757	775	792	810	828	846	863	881	899
245		917	934	952	970	987	*005	*023	*041	*058	*076
246	39	094	111	129	146	164	182	199	217	235	252
247		270	287	305	322	340	358	375	393	410	428
248		445	463	480	498	515	533	550	568	585	602
249		620	637	655	672	690	707	724	742	759	777
250		794	811	829	846	863	881	898	915	933	950

P. P.

	22	21	20	19	18	17
1	2.2	2.1	2.0	1.9	1.8	1.7
2	4.4	4.2	4.0	3.8	3.6	3.4
3	6.6	6.3	6.0	5.7	5.4	5.1
4	8.8	8.4	8.0	7.6	7.2	6.8
5	11.0	10.5	10.0	9.5	9.0	8.5
6	13.2	12.6	12.0	11.4	10.8	10.2
7	15.4	14.7	14.0	13.3	12.6	11.9
8	17.6	16.8	16.0	15.2	14.4	13.6
9	19.8	18.9	18.0	17.1	16.2	15.3

Tables of Logarithms

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.	
250	39 794	811	829	846	863	881	898	915	933	950		18
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	1	1.8
252	40 140	157	175	192	209	226	243	261	278	295	2	3.6
253	312	329	346	364	381	398	415	432	449	466	3	5.4
254	483	500	518	535	552	569	586	603	620	637	4	7.2
255	654	671	688	705	722	739	756	773	790	807	5	9.0
256	824	841	858	875	892	909	926	943	960	976	6	10.8
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	7	12.6
258	41 162	179	196	212	229	246	263	280	296	313	8	14.4
259	330	347	363	380	397	414	430	447	464	481	9	16.2
260	497	514	531	547	564	581	597	614	631	647		17
261	664	681	697	714	731	747	764	780	797	814	1	1.7
262	830	847	863	880	896	913	929	946	963	979	2	3.4
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	3	5.1
264	42 160	177	193	210	226	243	259	275	292	308	4	6.8
265	325	341	357	374	390	406	423	439	455	472	5	8.5
266	488	504	521	537	553	570	586	602	619	635	6	10.2
267	651	667	684	700	716	732	749	765	781	797	7	11.9
268	813	830	846	862	878	894	911	927	943	959	8	13.6
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	9	15.3
270	43 136	152	169	185	201	217	233	249	265	281		16
271	297	313	329	345	361	377	393	409	425	441	1	1.6
272	457	473	489	505	521	537	553	569	584	600	2	3.2
273	616	632	648	664	680	696	712	727	743	759	3	4.8
274	775	791	807	823	838	854	870	886	902	917	4	6.4
275	933	949	965	981	996	*012	*028	*044	*059	*075	5	8.0
276	44 091	107	122	138	154	170	185	201	217	232	6	9.6
277	248	264	279	295	311	326	342	358	373	389	7	11.2
278	404	420	436	451	467	483	498	514	529	545	8	12.8
279	560	576	592	607	623	638	654	669	685	700	9	14.4
280	716	731	747	762	778	793	809	824	840	855		15
281	871	886	902	917	932	948	963	979	994	*010	1	1.5
282	45 025	040	056	071	086	102	117	133	148	163	2	3.0
283	179	194	209	225	240	255	271	286	301	317	3	4.5
284	332	347	362	378	393	408	423	439	454	469	4	6.0
285	484	500	515	530	545	561	576	591	606	621	5	7.5
286	637	652	667	682	697	712	728	743	758	773	6	9.0
287	788	803	818	834	849	864	879	894	909	924	7	10.5
288	939	954	969	984	*000	*015	*030	*045	*060	*075	8	12.0
289	46 090	105	120	135	150	165	180	195	210	225	9	13.5
290	240	255	270	285	300	315	330	345	359	374		
291	389	404	419	434	449	464	479	494	509	523		14
292	538	553	568	583	598	613	627	642	657	672	1	1.4
293	687	702	716	731	746	761	776	790	805	820	2	2.8
294	835	850	864	879	894	909	923	938	953	967	3	4.2
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	4	5.6
296	47 129	144	159	173	188	202	217	232	246	261	5	7.0
297	276	290	305	319	334	349	363	378	392	407	6	8.4
298	422	436	451	465	480	494	509	524	538	553	7	9.8
299	567	582	596	611	625	640	654	669	683	698	8	11.2
300	712	727	741	756	770	784	799	813	828	842	9	12.6

Tables of Logarithms

N.	L. 0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	015	029	044	058	073	087	101	116	130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	430	444	458	473	487	501	515	530	544	558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	831	845	859	872	886	900	914	927	941	955
316	969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50 106	120	133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	*001	*014	*028	*041
324	51 055	068	081	095	108	121	135	148	162	175
325	188	202	215	228	242	255	268	282	295	308
326	322	335	348	362	375	388	402	415	428	441
327	455	468	481	495	508	521	534	548	561	574
328	587	601	614	627	640	654	667	680	693	706
329	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52 114	127	140	153	166	179	192	205	218	231
333	244	257	270	284	297	310	323	336	349	362
334	375	388	401	414	427	440	453	466	479	492
335	504	517	530	543	556	569	582	595	608	621
336	634	647	660	673	686	699	711	724	737	750
337	763	776	789	802	815	827	840	853	866	879
338	892	905	917	930	943	956	969	982	994	*007
339	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
341	275	288	301	314	326	339	352	364	377	390
342	403	415	428	441	453	466	479	491	504	517
343	529	542	555	567	580	593	605	618	631	643
344	656	668	681	694	706	719	732	744	757	769
345	782	794	807	820	832	845	857	870	882	895
346	908	920	933	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	158	170	183	195	208	220	233	245	258	270
349	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518

P. P.

	15	14	13	12
1	1.5	1.4	1.3	1.2
2	3.0	2.8	2.6	2.4
3	4.5	4.2	3.9	3.6
4	6.0	5.6	5.2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.0	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9
350	54	407	419	432	444	456	469	481	494	506	518
351		531	543	555	568	580	593	605	617	630	642
352		654	667	679	691	704	716	728	741	753	765
353		777	790	802	814	827	839	851	864	876	888
354		900	913	925	937	949	962	974	986	998	*011
355	55	023	035	047	060	072	084	096	108	121	133
356		145	157	169	182	194	206	218	230	242	255
357		267	279	291	303	315	328	340	352	364	376
358		388	400	413	425	437	449	461	473	485	497
359		509	522	534	546	558	570	582	594	606	618
360		630	642	654	666	678	691	703	715	727	739
361		751	763	775	787	799	811	823	835	847	859
362		871	883	895	907	919	931	943	955	967	979
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56	110	122	134	146	158	170	182	194	205	217
365		229	241	253	265	277	289	301	312	324	336
366		348	360	372	384	396	407	419	431	443	455
367		467	478	490	502	514	526	538	549	561	573
368		585	597	608	620	632	644	656	667	679	691
369		703	714	726	738	750	761	773	785	797	808
370		820	832	844	855	867	879	891	902	914	926
371		937	949	961	972	984	996	*008	*019	*031	*043
372	57	054	066	078	089	101	113	124	136	148	159
373		171	183	194	206	217	229	241	252	264	276
374		287	299	310	322	334	345	357	368	380	392
375		403	415	426	438	449	461	473	484	496	507
376		519	530	542	553	565	576	588	600	611	623
377		634	646	657	669	680	692	703	715	726	738
378		749	761	772	784	795	807	818	830	841	852
379		864	875	887	898	910	921	933	944	955	967
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58	092	104	115	127	138	149	161	172	184	195
382		206	218	229	240	252	263	274	286	297	309
383		320	331	343	354	365	377	388	399	410	422
384		433	444	456	467	478	490	501	512	524	535
385		546	557	569	580	591	602	614	625	636	647
386		659	670	681	692	704	715	726	737	749	760
387		771	782	794	805	816	827	838	850	861	872
388		883	894	906	917	928	939	950	961	973	984
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59	106	118	129	140	151	162	173	184	195	207
391		218	229	240	251	262	273	284	295	306	318
392		329	340	351	362	373	384	395	406	417	428
393		439	450	461	472	483	494	506	517	528	539
394		550	561	572	583	594	605	616	627	638	649
395		660	671	682	693	704	715	726	737	748	759
396		770	780	791	802	813	824	835	846	857	868
397		879	890	901	912	923	934	945	956	966	977
398		988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60	097	108	119	130	141	152	163	173	184	195
400		206	217	228	239	249	260	271	282	293	304

P. P.

	13
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5
6	7.8
7	9.1
8	10.4
9	11.7

	12
1	1.2
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9
400	60	206	217	228	239	249	260	271	282	293	304
401		314	325	336	347	358	369	379	390	401	412
402		423	433	444	455	466	477	487	498	509	520
403		531	541	552	563	574	584	595	606	617	627
404		638	649	660	670	681	692	703	713	724	735
405		746	756	767	778	788	799	810	821	831	842
406		853	863	874	885	895	906	917	927	938	949
407		959	970	981	991	*002	*013	*023	*034	*045	*055
408	61	066	077	087	098	109	119	130	140	151	162
409		172	183	194	204	215	225	236	247	257	268
410		278	289	300	310	321	331	342	352	363	374
411		384	395	405	416	426	437	448	458	469	479
412		490	500	511	521	532	542	553	563	574	584
413		595	606	616	627	637	648	658	669	679	690
414		700	711	721	731	742	752	763	773	784	794
415		805	815	826	836	847	857	868	878	888	899
416		909	920	930	941	951	962	972	982	993	*003
417	62	014	024	034	045	055	066	076	086	097	107
418		118	128	138	149	159	170	180	190	201	211
419		221	232	242	252	263	273	284	294	304	315
420		325	335	346	356	366	377	387	397	408	418
421		428	439	449	459	469	480	490	500	511	521
422		531	542	552	562	572	583	593	603	613	624
423		634	644	655	665	675	685	696	706	716	726
424		737	747	757	767	778	788	798	808	818	829
425		839	849	859	870	880	890	900	910	921	931
426		941	951	961	972	982	992	*002	*012	*022	*033
427	63	043	053	063	073	083	094	104	114	124	134
428		144	155	165	175	185	195	205	215	225	236
429		246	256	266	276	286	296	306	317	327	337
430		347	357	367	377	387	397	407	417	428	438
431		448	458	468	478	488	498	508	518	528	538
432		548	558	568	579	589	599	609	619	629	639
433		649	659	669	679	689	699	709	719	729	739
434		749	759	769	779	789	799	809	819	829	839
435		849	859	869	879	889	899	909	919	929	939
436		949	959	969	979	988	998	*008	*018	*028	*038
437	64	048	058	068	078	088	098	108	118	128	137
438		147	157	167	177	187	197	207	217	227	237
439		246	256	266	276	286	296	306	316	326	335
440		345	355	365	375	385	395	404	414	424	434
441		444	454	464	473	483	493	503	513	523	532
442		542	552	562	572	582	591	601	611	621	631
443		640	650	660	670	680	689	699	709	719	729
444		738	748	758	768	777	787	797	807	816	826
445		836	846	856	865	875	885	895	904	914	924
446		933	943	953	963	972	982	992	*002	*011	*021
447	65	031	040	050	060	070	079	089	099	108	118
448		128	137	147	157	167	176	186	196	205	215
449		225	234	244	254	263	273	283	292	302	312
450		321	331	341	350	360	369	379	389	398	408

P. P.

	11	10	9
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
450	65	321	331	341	350	360	369	379	389	398	408	
451		418	427	437	447	456	466	475	485	495	504	
452		514	523	533	543	552	562	571	581	591	600	
453		610	619	629	639	648	658	667	677	686	696	
454		706	715	725	734	744	753	763	772	782	792	
455		801	811	820	830	839	849	858	868	877	887	
456		896	906	916	925	935	944	954	963	973	982	10
457		992	*001	*011	*020	*030	*039	*049	*058	*068	*077	1 | 1.0
458	66	087	096	106	115	124	134	143	153	162	172	2 | 2.0
459		181	191	200	210	219	229	238	247	257	266	3 | 3.0
460		276	285	295	304	314	323	332	342	351	361	4 | 4.0
461		370	380	389	398	408	417	427	436	445	455	5 | 5.0
462		464	474	483	492	502	511	521	530	539	549	6 | 6.0
463		558	567	577	586	596	605	614	624	633	642	7 | 7.0
464		652	661	671	680	689	699	708	717	727	736	8 | 8.0
465		745	755	764	773	783	792	801	811	820	829	9 | 9.0
466		839	848	857	867	876	885	894	904	913	922	
467		932	941	950	960	969	978	987	997	*006	*015	
468	67	025	034	043	052	062	071	080	089	099	108	
469		117	127	136	145	154	164	173	182	191	201	
470		210	219	228	237	247	256	265	274	284	293	9
471		302	311	321	330	339	348	357	367	376	385	1 | 0.9
472		394	403	413	422	431	440	449	459	468	477	2 | 1.8
473		486	495	504	514	523	532	541	550	560	569	3 | 2.7
474		578	587	596	605	614	624	633	642	651	660	4 | 3.6
475		669	679	688	697	706	715	724	733	742	752	5 | 4.5
476		761	770	779	788	797	806	815	825	834	843	6 | 5.4
477		852	861	870	879	888	897	906	916	925	934	7 | 6.3
478		943	952	961	970	979	988	997	*006	*015	*024	8 | 7.2
479	68	034	043	052	061	070	079	088	097	106	115	9 | 8.1
480		124	133	142	151	160	169	178	187	196	205	
481		215	224	233	242	251	260	269	278	287	296	
482		305	314	323	332	341	350	359	368	377	386	
483		395	404	413	422	431	440	449	458	467	476	
484		485	494	502	511	520	529	538	547	556	565	
485		574	583	592	601	610	619	628	637	646	655	8
486		664	673	681	690	699	708	717	726	735	744	1 | 0.8
487		753	762	771	780	789	797	806	815	824	833	2 | 1.6
488		842	851	860	869	878	886	895	904	913	922	3 | 2.4
489		931	940	949	958	966	975	984	993	*002	*011	4 | 3.2
490	69	020	028	037	046	055	064	073	082	090	099	5 | 4.0
491		108	117	126	135	144	152	161	170	179	188	6 | 4.8
492		197	205	214	223	232	241	249	258	267	276	7 | 5.6
493		285	294	302	311	320	329	338	346	355	364	8 | 6.4
494		373	381	390	399	408	417	425	434	443	452	9 | 7.2
495		461	469	478	487	496	504	513	522	531	539	
496		548	557	566	574	583	592	601	609	618	627	
497		636	644	653	662	671	679	688	697	705	714	
498		723	732	740	749	758	767	775	784	793	801	
499		810	819	827	836	845	854	862	871	880	888	
500		897	906	914	923	932	940	949	958	966	975	

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9
500	69	897	906	914	923	932	940	949	958	966	975
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70	070	079	088	096	105	114	122	131	140	148
503		157	165	174	183	191	200	209	217	226	234
504		243	252	260	269	278	286	295	303	312	321
505		329	338	346	355	364	372	381	389	398	406
506		415	424	432	441	449	458	467	475	484	492
507		501	509	518	526	535	544	552	561	569	578
508		586	595	603	612	621	629	638	646	655	663
509		672	680	689	697	706	714	723	731	740	749
510		757	766	774	783	791	800	808	817	825	834
511		842	851	859	868	876	885	893	902	910	919
512		927	935	944	952	961	969	978	986	995	*003
513	71	012	020	029	037	046	054	063	071	079	088
514		096	105	113	122	130	139	147	155	164	172
515		181	189	198	206	214	223	231	240	248	257
516		265	273	282	290	299	307	315	324	332	341
517		349	357	366	374	383	391	399	408	416	425
518		433	441	450	458	466	475	483	492	500	508
519		517	525	533	542	550	559	567	575	584	592
520		600	609	617	625	634	642	650	659	667	675
521		684	692	700	709	717	725	734	742	750	759
522		767	775	784	792	800	809	817	825	834	842
523		850	858	867	875	883	892	900	908	917	925
524		933	941	950	958	966	975	983	991	999	*008
525	72	016	024	032	041	049	057	066	074	082	090
526		099	107	115	123	132	140	148	156	165	173
527		181	189	198	206	214	222	230	239	247	255
528		263	272	280	288	296	304	313	321	329	337
529		346	354	362	370	378	387	395	403	411	419
530		428	436	444	452	460	469	477	485	493	501
531		509	518	526	534	542	550	558	567	575	583
532		591	599	607	616	624	632	640	648	656	665
533		673	681	689	697	705	713	722	730	738	746
534		754	762	770	779	787	795	803	811	819	827
535		835	843	852	860	868	876	884	892	900	908
536		916	925	933	941	949	957	965	973	981	989
537		997	*006	*014	*022	*030	*038	*046	*054	*062	*070
538	73	078	086	094	102	111	119	127	135	143	151
539		159	167	175	183	191	199	207	215	223	231
540		239	247	255	263	272	280	288	296	304	312
541		320	328	336	344	352	360	368	376	384	392
542		400	408	416	424	432	440	448	456	464	472
543		480	488	496	504	512	520	528	536	544	552
544		560	568	576	584	592	600	608	616	624	632
545		640	648	656	664	672	679	687	695	703	711
546		719	727	735	743	751	759	767	775	783	791
547		799	807	815	823	830	838	846	854	862	870
548		878	886	894	902	910	918	926	933	941	949
549		957	965	973	981	989	997	*005	*013	*020	*028
550	74	036	044	052	060	068	076	084	092	099	107

P. P.

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9
550	74	036	044	052	060	068	076	084	092	099	107
551		115	123	131	139	147	155	162	170	178	186
552		194	202	210	218	225	233	241	249	257	265
553		273	280	288	296	304	312	320	327	335	343
554		351	359	367	374	382	390	398	406	414	421
555		429	437	445	453	461	468	476	484	492	500
556		507	515	523	531	539	547	554	562	570	578
557		586	593	601	609	617	624	632	640	648	656
558		663	671	679	687	695	702	710	718	726	733
559		741	749	757	764	772	780	788	796	803	811
560		819	827	834	842	850	858	865	873	881	889
561		896	904	912	920	927	935	943	950	958	966
562		974	981	989	997	*005	*012	*020	*028	*035	*043
563	75	051	059	066	074	082	089	097	105	113	120
564		128	136	143	151	159	166	174	182	189	197
565		205	213	220	228	236	243	251	259	266	274
566		282	289	297	305	312	320	328	335	343	351
567		358	366	374	381	389	397	404	412	420	427
568		435	442	450	458	465	473	481	488	496	504
569		511	519	526	534	542	549	557	565	572	580
570		587	595	603	610	618	626	633	641	648	656
571		664	671	679	686	694	702	709	717	724	732
572		740	747	755	762	770	778	785	793	800	808
573		815	823	831	838	846	853	861	868	876	884
574		891	899	906	914	921	929	937	944	952	959
575		967	974	982	989	997	*005	*012	*020	*027	*035
576	76	042	050	057	065	072	080	087	095	103	110
577		118	125	133	140	148	155	163	170	178	185
578		193	200	208	215	223	230	238	245	253	260
579		268	275	283	290	298	305	313	320	328	335
580		343	350	358	365	373	380	388	395	403	410
581		418	425	433	440	448	455	462	470	477	485
582		492	500	507	515	522	530	537	545	552	559
583		567	574	582	589	597	604	612	619	626	634
584		641	649	656	664	671	678	686	693	701	708
585		716	723	730	738	745	753	760	768	775	782
586		790	797	805	812	819	827	834	842	849	856
587		864	871	879	886	893	901	908	916	923	930
588		938	945	953	960	967	975	982	989	997	*004
589	77	012	019	026	034	041	048	056	063	070	078
590		085	093	100	107	115	122	129	137	144	151
591		159	166	173	181	188	195	203	210	217	225
592		232	240	247	254	262	269	276	283	291	298
593		305	313	320	327	335	342	349	357	364	371
594		379	386	393	401	408	415	422	430	437	444
595		452	459	466	474	481	488	495	503	510	517
596		525	532	539	546	554	561	568	576	583	590
597		597	605	612	619	627	634	641	648	656	663
598		670	677	685	692	699	706	714	721	728	735
599		743	750	757	764	772	779	786	793	801	808
600		815	822	830	837	844	851	859	866	873	880

P. P.

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9
600	77	815	822	830	837	844	851	859	866	873	880
601		887	895	902	909	916	924	931	938	945	952
602		960	967	974	981	988	996	*003	*010	*017	*025
603	78	032	039	046	053	061	068	075	082	089	097
604		104	111	118	125	132	140	147	154	161	168
605		176	183	190	197	204	211	219	226	233	240
606		247	254	262	269	276	283	290	297	305	312
607		319	326	333	340	347	355	362	369	376	383
608		390	398	405	412	419	426	433	440	447	455
609		462	469	476	483	490	497	504	512	519	526
610		533	540	547	554	561	569	576	583	590	597
611		604	611	618	625	633	640	647	654	661	668
612		675	682	689	696	704	711	718	725	732	739
613		746	753	760	767	774	781	789	796	803	810
614		817	824	831	838	845	852	859	866	873	880
615		888	895	902	909	916	923	930	937	944	951
616		958	965	972	979	986	993	*000	*007	*014	*021
617	79	029	036	043	050	057	064	071	078	085	092
618		099	106	113	120	127	134	141	148	155	162
619		169	176	183	190	197	204	211	218	225	232
620		239	246	253	260	267	274	281	288	295	302
621		309	316	323	330	337	344	351	358	365	372
622		379	386	393	400	407	414	421	428	435	442
623		449	456	463	470	477	484	491	498	505	511
624		518	525	532	539	546	553	560	567	574	581
625		588	595	602	609	616	623	630	637	644	650
626		657	664	671	678	685	692	699	706	713	720
627		727	734	741	748	754	761	768	775	782	789
628		796	803	810	817	824	831	837	844	851	858
629		865	872	879	886	893	900	906	913	920	927
630		934	941	948	955	962	969	975	982	989	996
631	80	003	010	017	024	030	037	044	051	058	065
632		072	079	085	092	099	106	113	120	127	134
633		140	147	154	161	168	175	182	188	195	202
634		209	216	223	229	236	243	250	257	264	271
635		277	284	291	298	305	312	318	325	332	339
636		346	353	359	366	373	380	387	393	400	407
637		414	421	428	434	441	448	455	462	468	475
638		482	489	496	502	509	516	523	530	536	543
639		550	557	564	570	577	584	591	598	604	611
640		618	625	632	638	645	652	659	665	672	679
641		686	693	699	706	713	720	726	733	740	747
642		754	760	767	774	781	787	794	801	808	814
643		821	828	835	841	848	855	862	868	875	882
644		889	895	902	909	916	922	929	936	943	949
645		956	963	969	976	983	990	996	*003	*010	*017
646	81	023	030	037	043	050	057	064	070	077	084
647		090	097	104	111	117	124	131	137	144	151
648		158	164	171	178	184	191	198	204	211	218
649		224	231	238	245	251	258	265	271	278	285
650		291	298	305	311	318	325	331	338	345	351

P. P.

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
650	81	291	298	305	311	318	325	331	338	345	351	
651		358	365	371	378	385	391	398	405	411	418	
652		425	431	438	445	451	458	465	471	478	485	
653		491	498	505	511	518	525	531	538	544	551	
654		558	564	571	578	584	591	598	604	611	617	
655		624	631	637	644	651	657	664	671	677	684	
656		690	697	704	710	717	723	730	737	743	750	
657		757	763	770	776	783	790	796	803	809	816	
658		823	829	836	842	849	856	862	869	875	882	
659		889	895	902	908	915	921	928	935	941	948	
660		954	961	968	974	981	987	994	*000	*007	*014	
661	82	020	027	033	040	046	053	060	066	073	079	7
662		086	092	099	105	112	119	125	132	138	145	1 0.7
663		151	158	164	171	178	184	191	197	204	210	2 1.4
664		217	223	230	236	243	249	256	263	269	276	3 2.1
665		282	289	295	302	308	315	321	328	334	341	4 2.8
666		347	354	360	367	373	380	387	393	400	406	5 3.5
667		413	419	426	432	439	445	452	458	465	471	6 4.2
668		478	484	491	497	504	510	517	523	530	536	7 4.9
669		543	549	556	562	569	575	582	588	595	601	8 5.6
670		607	614	620	627	633	640	646	653	659	666	9 6.3
671		672	679	685	692	698	705	711	718	724	730	
672		737	743	750	756	763	769	776	782	789	795	
673		802	808	814	821	827	834	840	847	853	860	
674		866	872	879	885	892	898	905	911	918	924	
675		930	937	943	950	956	963	969	975	982	988	
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83	059	065	072	078	085	091	097	104	110	117	
678		123	129	136	142	149	155	161	168	174	181	
679		187	193	200	206	213	219	225	232	238	245	
680		251	257	264	270	276	283	289	296	302	308	
681		315	321	327	334	340	347	353	359	366	372	6
682		378	385	391	398	404	410	417	423	429	436	1 0.6
683		442	448	455	461	467	474	480	487	493	499	2 1.2
684		506	512	518	525	531	537	544	550	556	563	3 1.8
685		569	575	582	588	594	601	607	613	620	626	4 2.4
686		632	639	645	651	658	664	670	677	683	689	5 3.0
687		696	702	708	715	721	727	734	740	746	753	6 3.6
688		759	765	771	778	784	790	797	803	809	816	7 4.2
689		822	828	835	841	847	853	860	866	872	879	8 4.8
690		885	891	897	904	910	916	923	929	935	942	9 5.4
691		948	954	960	967	973	979	985	992	998	*004	
692	84	011	017	023	029	036	042	048	055	061	067	
693		073	080	086	092	098	105	111	117	123	130	
694		136	142	148	155	161	167	173	180	186	192	
695		198	205	211	217	223	230	236	242	248	255	
696		261	267	273	280	286	292	298	305	311	317	
697		323	330	336	342	348	354	361	367	373	379	
698		386	392	398	404	410	417	423	429	435	442	
699		448	454	460	466	473	479	485	491	497	504	
700		510	516	522	528	535	541	547	553	559	566	

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
700	84	510	516	522	528	535	541	547	553	559	566	
701		572	578	584	590	597	603	609	615	621	628	
702		634	640	646	652	658	665	671	677	683	689	
703		696	702	708	714	720	726	733	739	745	751	
704		757	763	770	776	782	788	794	800	807	813	
705		819	825	831	837	844	850	856	862	868	874	
706		880	887	893	899	905	911	917	924	930	936	7
707		942	948	954	960	967	973	979	985	991	997	1 | 0.7
708	85	003	009	016	022	028	034	040	046	052	058	2 | 1.4
709		065	071	077	083	089	095	101	107	114	120	3 | 2.1
710		126	132	138	144	150	156	163	169	175	181	4 | 2.8
711		187	193	199	205	211	217	224	230	236	242	5 | 3.5
712		248	254	260	266	272	278	285	291	297	303	6 | 4.2
713		309	315	321	327	333	339	345	352	358	364	7 | 4.9
714		370	376	382	388	394	400	406	412	418	425	8 | 5.6
715		431	437	443	449	455	461	467	473	479	485	9 | 6.3
716		491	497	503	509	516	522	528	534	540	546	
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	6
721		794	800	806	812	818	824	830	836	842	848	1 | 0.6
722		854	860	866	872	878	884	890	896	902	908	2 | 1.2
723		914	920	926	932	938	944	950	956	962	968	3 | 1.8
724		974	980	986	992	998	*004	*010	*016	*022	*028	4 | 2.4
725	86	034	040	046	052	058	064	070	076	082	088	5 | 3.0
726		094	100	106	112	118	124	130	136	141	147	6 | 3.6
727		153	159	165	171	177	183	189	195	201	207	7 | 4.2
728		213	219	225	231	237	243	249	255	261	267	8 | 4.8
729		273	279	285	291	297	303	308	314	320	326	9 | 5.4
730		332	338	344	350	356	362	368	374	380	386	
731		392	398	404	410	415	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	5
736		688	694	700	705	711	717	723	729	735	741	1 | 0.5
737		747	753	759	764	770	776	782	788	794	800	2 | 1.0
738		806	812	817	823	829	835	841	847	853	859	3 | 1.5
739		864	870	876	882	888	894	900	906	911	917	4 | 2.0
740		923	929	935	941	947	953	958	964	970	976	5 | 2.5
741		982	988	994	999	*005	*011	*017	*023	*029	*035	6 | 3.0
742	87	040	046	052	058	064	070	075	081	087	093	7 | 3.5
743		099	105	111	116	122	128	134	140	146	151	8 | 4.0
744		157	163	169	175	181	186	192	198	204	210	9 | 4.5
745		216	221	227	233	239	245	251	256	262	268	
746		274	280	286	291	297	303	309	315	320	326	
747		332	338	344	349	355	361	367	373	379	384	
748		390	396	402	408	413	419	425	431	437	442	
749		448	454	460	466	471	477	483	489	495	500	
750		506	512	518	523	529	535	541	547	552	558	

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
750	87	506	512	518	523	529	535	541	547	552	558	
751		564	570	576	581	587	593	599	604	610	616	
752		622	628	633	639	645	651	656	662	668	674	
753		679	685	691	697	703	708	714	720	726	731	
754		737	743	749	754	760	766	772	777	783	789	
755		795	800	806	812	818	823	829	835	841	846	
756		852	858	864	869	875	881	887	892	898	904	
757		910	915	921	927	933	938	944	950	955	961	
758		967	973	978	984	990	996	*001	*007	*013	*018	
759	88	024	030	036	041	047	053	058	064	070	076	
760		081	087	093	098	104	110	116	121	127	133	6
761		138	144	150	156	161	167	173	178	184	190	1 0.6
762		195	201	207	213	218	224	230	235	241	247	2 1.2
763		252	258	264	270	275	281	287	292	298	304	3 1.8
764		309	315	321	326	332	338	343	349	355	360	4 2.4
765		366	372	377	383	389	395	400	406	412	417	5 3.0
766		423	429	434	440	446	451	457	463	468	474	6 3.6
767		480	485	491	497	502	508	513	519	525	530	7 4.2
768		536	542	547	553	559	564	570	576	581	587	8 4.8
769		593	598	604	610	615	621	627	632	638	643	9 5.4
770		649	655	660	666	672	677	683	689	694	700	
771		705	711	717	722	728	734	739	745	750	756	
772		762	767	773	779	784	790	795	801	807	812	
773		818	824	829	835	840	846	852	857	863	868	
774		874	880	885	891	897	902	908	913	919	925	
775		930	936	941	947	953	958	964	969	975	981	
776		986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89	042	048	053	059	064	070	076	081	087	092	
778		098	104	109	115	120	126	131	137	143	148	
779		154	159	165	170	176	182	187	193	198	204	
780		209	215	221	226	232	237	243	248	254	260	5
781		265	271	276	282	287	293	298	304	310	315	1 0.5
782		321	326	332	337	343	348	354	360	365	371	2 1.0
783		376	382	387	393	398	404	409	415	421	426	3 1.5
784		432	437	443	448	454	459	465	470	476	481	4 2.0
785		487	492	498	504	509	515	520	526	531	537	5 2.5
786		542	548	553	559	564	570	575	581	586	592	6 3.0
787		597	603	609	614	620	625	631	636	642	647	7 3.5
788		653	658	664	669	675	680	686	691	697	702	8 4.0
789		708	713	719	724	730	735	741	746	752	757	9 4.5
790		763	768	774	779	785	790	796	801	807	812	
791		818	823	829	834	840	845	851	856	862	867	
792		873	878	883	889	894	900	905	911	916	922	
793		927	933	938	944	949	955	960	966	971	977	
794		982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90	037	042	048	053	059	064	069	075	080	086	
796		091	097	102	108	113	119	124	129	135	140	
797		146	151	157	162	168	173	179	184	189	195	
798		200	206	211	217	222	227	233	238	244	249	
799		255	260	266	271	276	282	287	293	298	304	
800		309	314	320	325	331	336	342	347	352	358	

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
800	90	309	314	320	325	331	336	342	347	352	358	
801		363	369	374	380	385	390	396	401	407	412	
802		417	423	428	434	439	445	450	455	461	466	
803		472	477	482	488	493	499	504	509	515	520	
804		526	531	536	542	547	553	558	563	569	574	
805		580	585	590	596	601	607	612	617	623	628	
806		634	639	644	650	655	660	666	671	677	682	
807		687	693	698	703	709	714	720	725	730	736	
808		741	747	752	757	763	768	773	779	784	789	
809		795	800	806	811	816	822	827	832	838	843	
810		849	854	859	865	870	875	881	886	891	897	6
811		902	907	913	918	924	929	934	940	945	950	1 0.6
812		956	961	966	972	977	982	988	993	998	*004	2 1.2
813	91	009	014	020	025	030	036	041	046	052	057	3 1.8
814		062	068	073	078	084	089	094	100	105	110	4 2.4
815		116	121	126	132	137	142	148	153	158	164	5 3.0
816		169	174	180	185	190	196	201	206	212	217	6 3.6
817		222	228	233	238	243	249	254	259	265	270	7 4.2
818		275	281	286	291	297	302	307	312	318	323	8 4.8
819		328	334	339	344	350	355	360	365	371	376	9 5.4
820		381	387	392	397	403	408	413	418	424	429	
821		434	440	445	450	455	461	466	471	477	482	
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	5
831		960	965	971	976	981	986	991	997	*002	*007	1 0.5
832	92	012	018	023	028	033	038	044	049	054	059	2 1.0
833		065	070	075	080	085	091	096	101	106	111	3 1.5
834		117	122	127	132	137	143	148	153	158	163	4 2.0
835		169	174	179	184	189	195	200	205	210	215	5 2.5
836		221	226	231	236	241	247	252	257	262	267	6 3.0
837		273	278	283	288	293	298	304	309	314	319	7 3.5
838		324	330	335	340	345	350	355	361	366	371	8 4.0
839		376	381	387	392	397	402	407	412	418	423	9 4.5
840		428	433	438	443	449	454	459	464	469	474	
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	716	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
850	92	942	947	952	957	962	967	973	978	983	988	
851		993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	
856		247	252	258	263	268	273	278	283	288	293	6
857		298	303	308	313	318	323	328	334	339	344	1 | 0.6
858		349	354	359	364	369	374	379	384	389	394	2 | 1.2
859		399	404	409	414	420	425	430	435	440	445	3 | 1.8
860		450	455	460	465	470	475	480	485	490	495	4 | 2.4
861		500	505	510	515	520	526	531	536	541	546	5 | 3.0
862		551	556	561	566	571	576	581	586	591	596	6 | 3.6
863		601	606	611	616	621	626	631	636	641	646	7 | 4.2
864		651	656	661	666	671	676	682	687	692	697	8 | 4.8
865		702	707	712	717	722	727	732	737	742	747	9 | 5.4
866		752	757	762	767	772	777	782	787	792	797	
867		802	807	812	817	822	827	832	837	842	847	
868		852	857	862	867	872	877	882	887	892	897	
869		902	907	912	917	922	927	932	937	942	947	
870		952	957	962	967	972	977	982	987	992	997	
871	94	002	007	012	017	022	027	032	037	042	047	5
872		052	057	062	067	072	077	082	086	091	096	1 | 0.5
873		101	106	111	116	121	126	131	136	141	146	2 | 1.0
874		151	156	161	166	171	176	181	186	191	196	3 | 1.5
875		201	206	211	216	221	226	231	236	240	245	4 | 2.0
876		250	255	260	265	270	275	280	285	290	295	5 | 2.5
877		300	305	310	315	320	325	330	335	340	345	6 | 3.0
878		349	354	359	364	369	374	379	384	389	394	7 | 3.5
879		399	404	409	414	419	424	429	433	438	443	8 | 4.0
880		448	453	458	463	468	473	478	483	488	493	9 | 4.5
881		498	503	507	512	517	522	527	532	537	542	
882		547	552	557	562	567	571	576	581	586	591	
883		596	601	606	611	616	621	626	630	635	640	
884		645	650	655	660	665	670	675	680	685	689	
885		694	699	704	709	714	719	724	729	734	738	
886		743	748	753	758	763	768	773	778	783	787	4
887		792	797	802	807	812	817	822	827	832	836	1 | 0.4
888		841	846	851	856	861	866	871	876	880	885	2 | 0.8
889		890	895	900	905	910	915	919	924	929	934	3 | 1.2
890		939	944	949	954	959	963	968	973	978	983	4 | 1.6
891		988	993	998	*002	*007	*012	*017	*022	*027	*032	5 | 2.0
892	95	036	041	046	051	056	061	066	071	075	080	6 | 2.4
893		085	090	095	100	105	109	114	119	124	129	7 | 2.8
894		134	139	143	148	153	158	163	168	173	177	8 | 3.2
895		182	187	192	197	202	207	211	216	221	226	9 | 3.6
896		231	236	240	245	250	255	260	265	270	274	
897		279	284	289	294	299	303	308	313	318	323	
898		328	332	337	342	347	352	357	361	366	371	
899		376	381	386	390	395	400	405	410	415	419	
900		424	429	434	439	444	448	453	458	463	468	

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9
900	95	424	429	434	439	444	448	453	458	463	468
901		472	477	482	487	492	497	501	506	511	516
902		521	525	530	535	540	545	550	554	559	564
903		569	574	578	583	588	593	598	602	607	612
904		617	622	626	631	636	641	646	650	655	660
905		665	670	674	679	684	689	694	698	703	708
906		713	718	722	727	732	737	742	746	751	756
907		761	766	770	775	780	785	789	794	799	804
908		809	813	818	823	828	832	837	842	847	852
909		856	861	866	871	875	880	885	890	895	899
910		904	909	914	918	923	928	933	938	942	947
911		952	957	961	966	971	976	980	985	990	995
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042
913	96	047	052	057	061	066	071	076	080	085	090
914		095	099	104	109	114	118	123	128	133	137
915		142	147	152	156	161	166	171	175	180	185
916		190	194	199	204	209	213	218	223	227	232
917		237	242	246	251	256	261	265	270	275	280
918		284	289	294	298	303	308	313	317	322	327
919		332	336	341	346	350	355	360	365	369	374
920		379	384	388	393	398	402	407	412	417	421
921		426	431	435	440	445	450	454	459	464	468
922		473	478	483	487	492	497	501	506	511	515
923		520	525	530	534	539	544	548	553	558	562
924		567	572	577	581	586	591	595	600	605	609
925		614	619	624	628	633	638	642	647	652	656
926		661	666	670	675	680	685	689	694	699	703
927		708	713	717	722	727	731	736	741	745	750
928		755	759	764	769	774	778	783	788	792	797
929		802	806	811	816	820	825	830	834	839	844
930		848	853	858	862	867	872	876	881	886	890
931		895	900	904	909	914	918	923	928	932	937
932		942	946	951	956	960	965	970	974	979	984
933		988	993	997	*002	*007	*011	*016	*021	*025	*030
934	97	035	039	044	049	053	058	063	067	072	077
935		081	086	090	095	100	104	109	114	118	123
936		128	132	137	142	146	151	155	160	165	169
937		174	179	183	188	192	197	202	206	211	216
938		220	225	230	234	239	243	248	253	257	262
939		267	271	276	280	285	290	294	299	304	308
940		313	317	322	327	331	336	340	345	350	354
941		359	364	368	373	377	382	387	391	396	400
942		405	410	414	419	424	428	433	437	442	447
943		451	456	460	465	470	474	479	483	488	493
944		497	502	506	511	516	520	525	529	534	539
945		543	548	552	557	562	566	571	575	580	585
946		589	594	598	603	607	612	617	621	626	630
947		635	640	644	649	653	658	663	667	672	676
948		681	685	690	695	699	704	708	713	717	722
949		727	731	736	740	745	749	754	759	763	768
950		772	777	782	786	791	795	800	804	809	813

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Tables of Logarithms

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
950	97	772	777	782	786	791	795	800	804	809	813	
951		818	823	827	832	836	841	845	850	855	859	
952		864	868	873	877	882	886	891	896	900	905	
953		909	914	918	923	928	932	937	941	946	950	
954		955	959	964	968	973	978	982	987	991	996	
955	98	000	005	009	014	019	023	028	032	037	041	
956		046	050	055	059	064	068	073	078	082	087	
957		091	096	100	105	109	114	118	123	127	132	
958		137	141	146	150	155	159	164	168	173	177	
959		182	186	191	195	200	204	209	214	218	223	
960		227	232	236	241	245	250	254	259	263	268	
961		272	277	281	286	290	295	299	304	308	313	5
962		318	322	327	331	336	340	345	349	354	358	1 | 0.5
963		363	367	372	376	381	385	390	394	399	403	2 | 1.0
964		408	412	417	421	426	430	435	439	444	448	3 | 1.5
965		453	457	462	466	471	475	480	484	489	493	4 | 2.0
966		498	502	507	511	516	520	525	529	534	538	5 | 2.5
967		543	547	552	556	561	565	570	574	579	583	6 | 3.0
968		588	592	597	601	605	610	614	619	623	628	7 | 3.5
969		632	637	641	646	650	655	659	664	668	673	8 | 4.0
970		677	682	686	691	695	700	704	709	713	717	9 | 4.5
971		722	726	731	735	740	744	749	753	758	762	
972		767	771	776	780	784	789	793	798	802	807	
973		811	816	820	825	829	834	838	843	847	851	
974		856	860	865	869	874	878	883	887	892	896	
975		900	905	909	914	918	923	927	932	936	941	
976		945	949	954	958	963	967	972	976	981	985	
977		989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99	034	038	043	047	052	056	061	065	069	074	
979		078	083	087	092	096	100	105	109	114	118	
980		123	127	131	136	140	145	149	154	158	162	
981		167	171	176	180	185	189	193	198	202	207	4
982		211	216	220	224	229	233	238	242	247	251	1 | 0.4
983		255	260	264	269	273	277	282	286	291	295	2 | 0.8
984		300	304	308	313	317	322	326	330	335	339	3 | 1.2
985		344	348	352	357	361	366	370	374	379	383	4 | 1.6
986		388	392	396	401	405	410	414	419	423	427	5 | 2.0
987		432	436	441	445	449	454	458	463	467	471	6 | 2.4
988		476	480	484	489	493	498	502	506	511	515	7 | 2.8
989		520	524	528	533	537	542	546	550	555	559	8 | 3.2
990		564	568	572	577	581	585	590	594	599	603	9 | 3.6
991		607	612	616	621	625	629	634	638	642	647	
992		651	656	660	664	669	673	677	682	686	691	
993		695	699	704	708	712	717	721	726	730	734	
994		739	743	747	752	756	760	765	769	774	778	
995		782	787	791	795	800	804	808	813	817	822	
996		826	830	835	839	843	848	852	856	861	865	
997		870	874	878	883	887	891	896	900	904	909	
998		913	917	922	926	930	935	939	944	948	952	
999		957	961	965	970	974	978	983	987	991	996	
1000	00	000	004	009	013	017	022	026	030	035	039	

Hyperbolic Logarithms

No.	H. Log.	No.	H. Log.	No.	H. Log.	No.	H. Log.	No.	H. Log.
1.01	0.0099	1.51	0.4121	2.01	0.6981	2.51	0.9203	3.01	1.1019
1.02	0.0198	1.52	0.4187	2.02	0.7031	2.52	0.9243	3.02	1.1053
1.03	0.0296	1.53	0.4253	2.03	0.7080	2.53	0.9282	3.03	1.1086
1.04	0.0392	1.54	0.4318	2.04	0.7129	2.54	0.9322	3.04	1.1119
1.05	0.0488	1.55	0.4383	2.05	0.7178	2.55	0.9361	3.05	1.1151
1.06	0.0583	1.56	0.4447	2.06	0.7227	2.56	0.9400	3.06	1.1184
1.07	0.0677	1.57	0.4511	2.07	0.7275	2.57	0.9439	3.07	1.1216
1.08	0.0770	1.58	0.4574	2.08	0.7324	2.58	0.9478	3.08	1.1249
1.09	0.0862	1.59	0.4637	2.09	0.7372	2.59	0.9517	3.09	1.1282
1.10	0.0953	1.60	0.4700	2.10	0.7419	2.60	0.9555	3.10	1.1314
1.11	0.1044	1.61	0.4762	2.11	0.7467	2.61	0.9594	3.11	1.1346
1.12	0.1133	1.62	0.4824	2.12	0.7514	2.62	0.9632	3.12	1.1378
1.13	0.1222	1.63	0.4886	2.13	0.7561	2.63	0.9670	3.13	1.1410
1.14	0.1310	1.64	0.4947	2.14	0.7608	2.64	0.9708	3.14	1.1442
1.15	0.1398	1.65	0.5008	2.15	0.7655	2.65	0.9746	3.15	1.1474
1.16	0.1484	1.66	0.5068	2.16	0.7701	2.66	0.9783	3.16	1.1506
1.17	0.1570	1.67	0.5128	2.17	0.7747	2.67	0.9821	3.17	1.1537
1.18	0.1655	1.68	0.5188	2.18	0.7793	2.68	0.9858	3.18	1.1569
1.19	0.1740	1.69	0.5247	2.19	0.7839	2.69	0.9895	3.19	1.1600
1.20	0.1823	1.70	0.5306	2.20	0.7885	2.70	0.9933	3.20	1.1632
1.21	0.1906	1.71	0.5365	2.21	0.7930	2.71	0.9969	3.21	1.1663
1.22	0.1988	1.72	0.5423	2.22	0.7975	2.72	1.0006	3.22	1.1694
1.23	0.2070	1.73	0.5481	2.23	0.8020	2.73	1.0043	3.23	1.1725
1.24	0.2151	1.74	0.5539	2.24	0.8065	2.74	1.0080	3.24	1.1756
1.25	0.2231	1.75	0.5596	2.25	0.8109	2.75	1.0116	3.25	1.1787
1.26	0.2311	1.76	0.5653	2.26	0.8154	2.76	1.0152	3.26	1.1817
1.27	0.2390	1.77	0.5710	2.27	0.8198	2.77	1.0188	3.27	1.1848
1.28	0.2469	1.78	0.5766	2.28	0.8242	2.78	1.0225	3.28	1.1878
1.29	0.2546	1.79	0.5822	2.29	0.8286	2.79	1.0260	3.29	1.1909
1.30	0.2624	1.80	0.5878	2.30	0.8329	2.80	1.0296	3.30	1.1939
1.31	0.2700	1.81	0.5933	2.31	0.8372	2.81	1.0332	3.31	1.1969
1.32	0.2776	1.82	0.5988	2.32	0.8416	2.82	1.0367	3.32	1.1999
1.33	0.2852	1.83	0.6043	2.33	0.8458	2.83	1.0403	3.33	1.2030
1.34	0.2927	1.84	0.6098	2.34	0.8502	2.84	1.0438	3.34	1.2060
1.35	0.3001	1.85	0.6152	2.35	0.8544	2.85	1.0473	3.35	1.2090
1.36	0.3075	1.86	0.6206	2.36	0.8587	2.86	1.0508	3.36	1.2119
1.37	0.3148	1.87	0.6259	2.37	0.8629	2.87	1.0543	3.37	1.2149
1.38	0.3221	1.88	0.6313	2.38	0.8671	2.88	1.0578	3.38	1.2179
1.39	0.3293	1.89	0.6366	2.39	0.8713	2.89	1.0613	3.39	1.2208
1.40	0.3365	1.90	0.6419	2.40	0.8755	2.90	1.0647	3.40	1.2238
1.41	0.3436	1.91	0.6471	2.41	0.8796	2.91	1.0682	3.41	1.2267
1.42	0.3507	1.92	0.6523	2.42	0.8838	2.92	1.0716	3.42	1.2296
1.43	0.3577	1.93	0.6575	2.43	0.8879	2.93	1.0750	3.43	1.2326
1.44	0.3646	1.94	0.6627	2.44	0.8920	2.94	1.0784	3.44	1.2355
1.45	0.3716	1.95	0.6678	2.45	0.8961	2.95	1.0818	3.45	1.2384
1.46	0.3784	1.96	0.6729	2.46	0.9002	2.96	1.0852	3.46	1.2413
1.47	0.3853	1.97	0.6780	2.47	0.9042	2.97	1.0886	3.47	1.2442
1.48	0.3920	1.98	0.6831	2.48	0.9083	2.98	1.0919	3.48	1.2470
1.49	0.3988	1.99	0.6881	2.49	0.9123	2.99	1.0953	3.49	1.2499
1.50	0.4055	2.00	0.6931	2.50	0.9163	3.00	1.0986	3.50	1.2528

Hyperbolic Logarithms

No.	H. Log.	No.	H. Log.	No.	H. Log.	No.	H. Log.	No.	H. Log.
3.51	1.2556	4.01	1.3888	4.51	1.5063	5.01	1.6114	5.51	1.7066
3.52	1.2585	4.02	1.3913	4.52	1.5085	5.02	1.6134	5.52	1.7084
3.53	1.2613	4.03	1.3938	4.53	1.5107	5.03	1.6154	5.53	1.7102
3.54	1.2641	4.04	1.3962	4.54	1.5129	5.04	1.6174	5.54	1.7120
3.55	1.2669	4.05	1.3987	4.55	1.5151	5.05	1.6194	5.55	1.7138
3.56	1.2698	4.06	1.4012	4.56	1.5173	5.06	1.6214	5.56	1.7156
3.57	1.2726	4.07	1.4036	4.57	1.5195	5.07	1.6233	5.57	1.7174
3.58	1.2754	4.08	1.4061	4.58	1.5217	5.08	1.6253	5.58	1.7192
3.59	1.2782	4.09	1.4085	4.59	1.5239	5.09	1.6273	5.59	1.7210
3.60	1.2809	4.10	1.4110	4.60	1.5261	5.10	1.6292	5.60	1.7228
3.61	1.2837	4.11	1.4134	4.61	1.5282	5.11	1.6312	5.61	1.7246
3.62	1.2865	4.12	1.4159	4.62	1.5304	5.12	1.6332	5.62	1.7263
3.63	1.2892	4.13	1.4183	4.63	1.5326	5.13	1.6351	5.63	1.7281
3.64	1.2920	4.14	1.4207	4.64	1.5347	5.14	1.6371	5.64	1.7299
3.65	1.2947	4.15	1.4231	4.65	1.5369	5.15	1.6390	5.65	1.7317
3.66	1.2975	4.16	1.4255	4.66	1.5390	5.16	1.6409	5.66	1.7334
3.67	1.3002	4.17	1.4279	4.67	1.5412	5.17	1.6429	5.67	1.7352
3.68	1.3029	4.18	1.4303	4.68	1.5433	5.18	1.6448	5.68	1.7370
3.69	1.3056	4.19	1.4327	4.69	1.5454	5.19	1.6467	5.69	1.7387
3.70	1.3083	4.20	1.4351	4.70	1.5476	5.20	1.6487	5.70	1.7405
3.71	1.3110	4.21	1.4375	4.71	1.5497	5.21	1.6506	5.71	1.7422
3.72	1.3137	4.22	1.4398	4.72	1.5518	5.22	1.6525	5.72	1.7440
3.73	1.3164	4.23	1.4422	4.73	1.5539	5.23	1.6544	5.73	1.7457
3.74	1.3191	4.24	1.4446	4.74	1.5560	5.24	1.6563	5.74	1.7475
3.75	1.3218	4.25	1.4469	4.75	1.5581	5.25	1.6582	5.75	1.7492
3.76	1.3244	4.26	1.4493	4.76	1.5602	5.26	1.6601	5.76	1.7509
3.77	1.3271	4.27	1.4516	4.77	1.5623	5.27	1.6620	5.77	1.7527
3.78	1.3297	4.28	1.4540	4.78	1.5644	5.28	1.6639	5.78	1.7544
3.79	1.3324	4.29	1.4563	4.79	1.5665	5.29	1.6658	5.79	1.7561
3.80	1.3350	4.30	1.4586	4.80	1.5686	5.30	1.6677	5.80	1.7579
3.81	1.3376	4.31	1.4609	4.81	1.5707	5.31	1.6696	5.81	1.7596
3.82	1.3403	4.32	1.4633	4.82	1.5728	5.32	1.6715	5.82	1.7613
3.83	1.3429	4.33	1.4656	4.83	1.5748	5.33	1.6734	5.83	1.7630
3.84	1.3455	4.34	1.4679	4.84	1.5769	5.34	1.6752	5.84	1.7647
3.85	1.3481	4.35	1.4702	4.85	1.5790	5.35	1.6771	5.85	1.7664
3.86	1.3507	4.36	1.4725	4.86	1.5810	5.36	1.6790	5.86	1.7681
3.87	1.3533	4.37	1.4748	4.87	1.5831	5.37	1.6808	5.87	1.7699
3.88	1.3558	4.38	1.4770	4.88	1.5851	5.38	1.6827	5.88	1.7716
3.89	1.3584	4.39	1.4793	4.89	1.5872	5.39	1.6845	5.89	1.7733
3.90	1.3610	4.40	1.4816	4.90	1.5892	5.40	1.6864	5.90	1.7750
3.91	1.3635	4.41	1.4839	4.91	1.5913	5.41	1.6882	5.91	1.7766
3.92	1.3661	4.42	1.4861	4.92	1.5933	5.42	1.6901	5.92	1.7783
3.93	1.3686	4.43	1.4884	4.93	1.5953	5.43	1.6919	5.93	1.7800
3.94	1.3712	4.44	1.4907	4.94	1.5974	5.44	1.6938	5.94	1.7817
3.95	1.3737	4.45	1.4929	4.95	1.5994	5.45	1.6956	5.95	1.7834
3.96	1.3762	4.46	1.4951	4.96	1.6014	5.46	1.6974	5.96	1.7851
3.97	1.3788	4.47	1.4974	4.97	1.6034	5.47	1.6993	5.97	1.7867
3.98	1.3813	4.48	1.4996	4.98	1.6054	5.48	1.7011	5.98	1.7884
3.99	1.3838	4.49	1.5019	4.99	1.6074	5.49	1.7029	5.99	1.7901
4.00	1.3863	4.50	1.5041	5.00	1.6094	5.50	1.7047	6.00	1.7918

Hyperbolic Logarithms

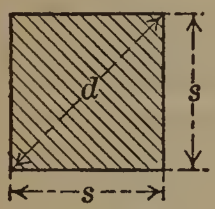
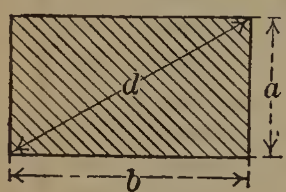
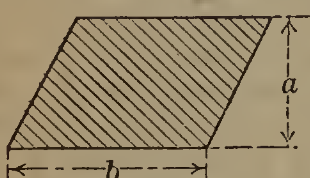
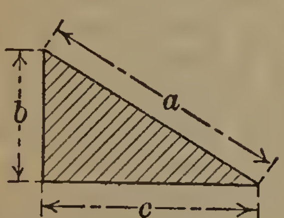
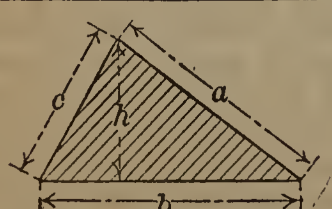
No.	H. Log.	No.	H. Log.	No.	H. Log.	No.	H. Log.	No.	H. Log.
6.01	1.7934	6.51	1.8733	7.01	1.9473	7.51	2.0162	8.01	2.0807
6.02	1.7951	6.52	1.8749	7.02	1.9488	7.52	2.0176	8.02	2.0819
6.03	1.7967	6.53	1.8764	7.03	1.9502	7.53	2.0189	8.03	2.0832
6.04	1.7984	6.54	1.8779	7.04	1.9516	7.54	2.0202	8.04	2.0844
6.05	1.8001	6.55	1.8795	7.05	1.9530	7.55	2.0215	8.05	2.0857
6.06	1.8017	6.56	1.8810	7.06	1.9544	7.56	2.0229	8.06	2.0869
6.07	1.8034	6.57	1.8825	7.07	1.9559	7.57	2.0242	8.07	2.0882
6.08	1.8050	6.58	1.8840	7.08	1.9573	7.58	2.0255	8.08	2.0894
6.09	1.8066	6.59	1.8856	7.09	1.9587	7.59	2.0268	8.09	2.0906
6.10	1.8083	6.60	1.8871	7.10	1.9601	7.60	2.0281	8.10	2.0919
6.11	1.8099	6.61	1.8886	7.11	1.9615	7.61	2.0295	8.11	2.0931
6.12	1.8116	6.62	1.8901	7.12	1.9629	7.62	2.0308	8.12	2.0943
6.13	1.8132	6.63	1.8916	7.13	1.9643	7.63	2.0321	8.13	2.0956
6.14	1.8148	6.64	1.8931	7.14	1.9657	7.64	2.0334	8.14	2.0968
6.15	1.8165	6.65	1.8946	7.15	1.9671	7.65	2.0347	8.15	2.0980
6.16	1.8181	6.66	1.8961	7.16	1.9685	7.66	2.0360	8.16	2.0992
6.17	1.8197	6.67	1.8976	7.17	1.9699	7.67	2.0373	8.17	2.1005
6.18	1.8213	6.68	1.8991	7.18	1.9713	7.68	2.0386	8.18	2.1017
6.19	1.8229	6.69	1.9006	7.19	1.9727	7.69	2.0399	8.19	2.1029
6.20	1.8245	6.70	1.9021	7.20	1.9741	7.70	2.0412	8.20	2.1041
6.21	1.8262	6.71	1.9036	7.21	1.9755	7.71	2.0425	8.21	2.1054
6.22	1.8278	6.72	1.9051	7.22	1.9769	7.72	2.0438	8.22	2.1066
6.23	1.8294	6.73	1.9066	7.23	1.9782	7.73	2.0451	8.23	2.1078
6.24	1.8310	6.74	1.9081	7.24	1.9796	7.74	2.0464	8.24	2.1090
6.25	1.8326	6.75	1.9095	7.25	1.9810	7.75	2.0477	8.25	2.1102
6.26	1.8342	6.76	1.9110	7.26	1.9824	7.76	2.0490	8.26	2.1114
6.27	1.8358	6.77	1.9125	7.27	1.9838	7.77	2.0503	8.27	2.1126
6.28	1.8374	6.78	1.9140	7.28	1.9851	7.78	2.0516	8.28	2.1138
6.29	1.8390	6.79	1.9155	7.29	1.9865	7.79	2.0528	8.29	2.1150
6.30	1.8405	6.80	1.9169	7.30	1.9879	7.80	2.0541	8.30	2.1163
6.31	1.8421	6.81	1.9184	7.31	1.9892	7.81	2.0554	8.31	2.1175
6.32	1.8437	6.82	1.9199	7.32	1.9906	7.82	2.0567	8.32	2.1187
6.33	1.8453	6.83	1.9213	7.33	1.9920	7.83	2.0580	8.33	2.1199
6.34	1.8469	6.84	1.9228	7.34	1.9933	7.84	2.0592	8.34	2.1211
6.35	1.8485	6.85	1.9242	7.35	1.9947	7.85	2.0605	8.35	2.1223
6.36	1.8500	6.86	1.9257	7.36	1.9961	7.86	2.0618	8.36	2.1235
6.37	1.8516	6.87	1.9272	7.37	1.9974	7.87	2.0631	8.37	2.1247
6.38	1.8532	6.88	1.9286	7.38	1.9988	7.88	2.0643	8.38	2.1258
6.39	1.8547	6.89	1.9301	7.39	2.0001	7.89	2.0656	8.39	2.1270
6.40	1.8563	6.90	1.9315	7.40	2.0015	7.90	2.0669	8.40	2.1282
6.41	1.8579	6.91	1.9330	7.41	2.0028	7.91	2.0681	8.41	2.1294
6.42	1.8594	6.92	1.9344	7.42	2.0041	7.92	2.0694	8.42	2.1306
6.43	1.8610	6.93	1.9359	7.43	2.0055	7.93	2.0707	8.43	2.1318
6.44	1.8625	6.94	1.9373	7.44	2.0069	7.94	2.0719	8.44	2.1330
6.45	1.8641	6.95	1.9387	7.45	2.0082	7.95	2.0732	8.45	2.1342
6.46	1.8656	6.96	1.9402	7.46	2.0096	7.96	2.0744	8.46	2.1353
6.47	1.8672	6.97	1.9416	7.47	2.0109	7.97	2.0757	8.47	2.1365
6.48	1.8687	6.98	1.9430	7.48	2.0122	7.98	2.0769	8.48	2.1377
6.49	1.8703	6.99	1.9445	7.49	2.0136	7.99	2.0782	8.49	2.1389
6.50	1.8718	7.00	1.9459	7.50	2.0149	8.00	2.0794	8.50	2.1401

Hyperbolic Logarithms

No.	H. Log.	No.	H. Log.	No.	H. Log.	No.	H. Log.	No.	H. Log.
8.51	2.1412	9.01	2.1983	9.51	2.2523	10.25	2.3273	41	3.7136
8.52	2.1424	9.02	2.1994	9.52	2.2534	10.50	2.3514	42	3.7377
8.53	2.1436	9.03	2.2006	9.53	2.2544	10.75	2.3749	43	3.7612
8.54	2.1448	9.04	2.2017	9.54	2.2555	11.00	2.3979	44	3.7842
8.55	2.1459	9.05	2.2028	9.55	2.2565	11.25	2.4204	45	3.8067
8.56	2.1471	9.06	2.2039	9.56	2.2576	11.50	2.4423	46	3.8286
8.57	2.1483	9.07	2.2050	9.57	2.2586	11.75	2.4638	47	3.8501
8.58	2.1494	9.08	2.2061	9.58	2.2597	12.00	2.4849	48	3.8712
8.59	2.1506	9.09	2.2072	9.59	2.2607	12.25	2.5055	49	3.8918
8.60	2.1518	9.10	2.2083	9.60	2.2618	12.50	2.5257	50	3.9120
8.61	2.1529	9.11	2.2094	9.61	2.2628	12.75	2.5455	51	3.9318
8.62	2.1541	9.12	2.2105	9.62	2.2638	13.00	2.5649	52	3.9512
8.63	2.1552	9.13	2.2116	9.63	2.2649	13.25	2.5840	53	3.9703
8.64	2.1564	9.14	2.2127	9.64	2.2659	13.50	2.6027	54	3.9890
8.65	2.1576	9.15	2.2138	9.65	2.2670	13.75	2.6210	55	4.0073
8.66	2.1587	9.16	2.2148	9.66	2.2680	14.00	2.6391	56	4.0254
8.67	2.1599	9.17	2.2159	9.67	2.2690	14.25	2.6568	57	4.0431
8.68	2.1610	9.18	2.2170	9.68	2.2701	14.50	2.6741	58	4.0604
8.69	2.1622	9.19	2.2181	9.69	2.2711	14.75	2.6912	59	4.0775
8.70	2.1633	9.20	2.2192	9.70	2.2721	15.00	2.7081	60	4.0943
8.71	2.1645	9.21	2.2203	9.71	2.2732	15.50	2.7408	61	4.1109
8.72	2.1656	9.22	2.2214	9.72	2.2742	16.00	2.7726	62	4.1271
8.73	2.1668	9.23	2.2225	9.73	2.2752	16.50	2.8034	63	4.1431
8.74	2.1679	9.24	2.2235	9.74	2.2762	17.00	2.8332	64	4.1589
8.75	2.1691	9.25	2.2246	9.75	2.2773	17.50	2.8622	65	4.1744
8.76	2.1702	9.26	2.2257	9.76	2.2783	18.00	2.8904	66	4.1897
8.77	2.1713	9.27	2.2268	9.77	2.2793	18.50	2.9178	67	4.2047
8.78	2.1725	9.28	2.2279	9.78	2.2803	19.00	2.9444	68	4.2195
8.79	2.1736	9.29	2.2289	9.79	2.2814	19.50	2.9704	69	4.2341
8.80	2.1748	9.30	2.2300	9.80	2.2824	20.00	2.9957	70	4.2485
8.81	2.1759	9.31	2.2311	9.81	2.2834	21	3.0445	71	4.2627
8.82	2.1770	9.32	2.2322	9.82	2.2844	22	3.0910	72	4.2767
8.83	2.1782	9.33	2.2332	9.83	2.2854	23	3.1355	73	4.2905
8.84	2.1793	9.34	2.2343	9.84	2.2865	24	3.1781	74	4.3041
8.85	2.1804	9.35	2.2354	9.85	2.2875	25	3.2189	75	4.3175
8.86	2.1815	9.36	2.2364	9.86	2.2885	26	3.2581	76	4.3307
8.87	2.1827	9.37	2.2375	9.87	2.2895	27	3.2958	77	4.3438
8.88	2.1838	9.38	2.2386	9.88	2.2905	28	3.3322	78	4.3567
8.89	2.1849	9.39	2.2396	9.89	2.2915	29	3.3673	79	4.3694
8.90	2.1861	9.40	2.2407	9.90	2.2925	30	3.4012	80	4.3820
8.91	2.1872	9.41	2.2418	9.91	2.2935	31	3.4340	82	4.4067
8.92	2.1883	9.42	2.2428	9.92	2.2946	32	3.4657	84	4.4308
8.93	2.1894	9.43	2.2439	9.93	2.2956	33	3.4965	86	4.4543
8.94	2.1905	9.44	2.2450	9.94	2.2966	34	3.5264	88	4.4773
8.95	2.1917	9.45	2.2460	9.95	2.2976	35	3.5553	90	4.4998
8.96	2.1928	9.46	2.2471	9.96	2.2986	36	3.5835	92	4.5218
8.97	2.1939	9.47	2.2481	9.97	2.2996	37	3.6109	94	4.5433
8.98	2.1950	9.48	2.2492	9.98	2.3006	38	3.6376	96	4.5643
8.99	2.1961	9.49	2.2502	9.99	2.3016	39	3.6636	98	4.5850
9.00	2.1972	9.50	2.2513	10.00	2.3026	40	3.6889	100	4.6052

Mensuration

In the following tables are given the areas of plane figures, together with other formulas relating to their dimensions and properties; the surfaces of solids; and the volumes of solids. The notation used in the formulas is, as far as possible, given in the illustration accompanying them; where this has not been possible, it is given at the beginning of each set of formulas.

 **Square**	A = area. $A = s^2$ $A = \frac{1}{2} d^2$ $s = 0.7071\, d = \sqrt{A}$ $d = 1.414\, s = 1.414 \sqrt{A}$
 **Rectangle**	A = area. $A = ab$ $A = a\sqrt{d^2 - a^2} = b\sqrt{d^2 - b^2}$ $d = \sqrt{a^2 + b^2}$ $a = \sqrt{d^2 - b^2} = A \div b$ $b = \sqrt{d^2 - a^2} = A \div a$
 **Parallelogram**	A = area. $A = ab$ $a = A \div b$ $b = A \div a$ Note that dimension a is measured at right angles to line b.
 **Right-angled Triangle**	A = area. $A = \frac{bc}{2}$ $a = \sqrt{b^2 + c^2}$ $b = \sqrt{a^2 - c^2}$ $c = \sqrt{a^2 - b^2}$
 **Acute-angled Triangle**	A = area. $A = \frac{bh}{2} = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2\,b}\right)^2}$ If $S = \frac{1}{2}(a + b + c)$, then $A = \sqrt{S(S-a)(S-b)(S-c)}$

Examples of the Use of the Formulas

Below are given a number of examples showing the use of the formulas on the opposite page. Each section of the page corresponds to the opposite section on the previous page, and the illustration on that page should be referred to. The notation used in the illustrations is also used in the examples given.

Square. — Assume that the side s of a square is 15 inches. Find the area and the length of the diagonal.

$$\text{Area} = A = s^2 = 15^2 = 225 \text{ square inches.}$$

$$\text{Diagonal} = d = 1.414\, s = 1.414 \times 15 = 21.21 \text{ inches.}$$

The area of a square is 625 square inches. Find the length of the side s and the diagonal d.

$$s = \sqrt{A} = \sqrt{625} = 25 \text{ inches.}$$

$$d = 1.414 \sqrt{A} = 1.414 \times 25 = 35.35 \text{ inches.}$$

Rectangle. — The side a of a rectangle is 12 inches, and the area 70.5 square inches. Find the length of the side b, and the diagonal d.

$$b = A \div a = 70.5 \div 12 = 5.875 \text{ inches.}$$

$$d = \sqrt{a^2 + b^2} = \sqrt{12^2 + 5.875^2} = \sqrt{178.516} = 13.361 \text{ inches.}$$

The sides of a rectangle are 30.5 and 11 inches long. Find the area.

$$\text{Area} = a \times b = 30.5 \times 11 = 335.5 \text{ square inches.}$$

Parallelogram. — The base b of a parallelogram is 16 feet. The height a is 5.5 feet. Find the area.

$$\text{Area} = A = a \times b = 5.5 \times 16 = 88 \text{ square feet.}$$

The area of a parallelogram is 12 square inches. The height is 1.5 inch. Find the length of the base b.

$$b = A \div a = 12 \div 1.5 = 8 \text{ inches.}$$

Right-angled Triangle. — The sides b and c in a right-angled triangle are 6 and 8 inches. Find side a and the area.

$$a = \sqrt{b^2 + c^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ inches.}$$

$$A = \frac{b \times c}{2} = \frac{6 \times 8}{2} = \frac{48}{2} = 24 \text{ square inches.}$$

If $a = 10$ and $b = 6$, had been known, but not c, the latter would have been found as follows:

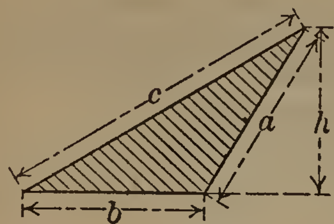
$$c = \sqrt{a^2 - b^2} = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8 \text{ inches.}$$

Acute-angled Triangle. — If $a = 10$, $b = 9$, and $c = 8$ inches, what is the area of the triangle?

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2\,b}\right)^2} = \frac{9}{2}\sqrt{10^2 - \left(\frac{10^2 + 9^2 - 8^2}{2 \times 9}\right)^2} = 4.5\sqrt{100 - \left(\frac{117}{18}\right)^2}$$

$$= 4.5\sqrt{100 - 42.25} = 4.5\sqrt{57.75} = 4.5 \times 7.60 = 34.20 \text{ square inches.}$$

Mensuration



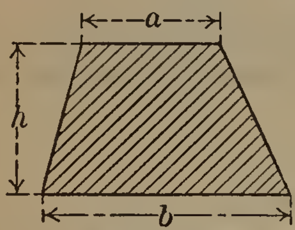
Obtuse-angled Triangle

A = area.

$$A = \frac{bh}{2} = \frac{b}{2}\sqrt{a^2 - \left(\frac{c^2 - a^2 - b^2}{2b}\right)^2}$$

If $S = \frac{1}{2}(a + b + c)$, then

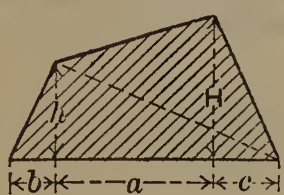
$$A = \sqrt{S(S-a)(S-b)(S-c)}$$



Trapezoid

A = area.

$$A = \frac{(a+b)\,h}{2}$$



Trapezium

A = area.

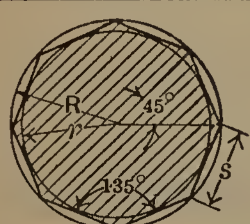
$$A = \frac{(H+h)\,a + bh + cH}{2}$$

A trapezium can also be divided into two triangles as indicated by the dotted line. The area of each of these triangles is computed, and the results added to find the area of the trapezium.

Regular Hexagon

A = area;
R = radius of circumscribed circle;
r = radius of inscribed circle.

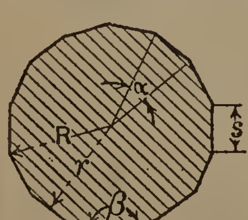
$A = 2.598\,s^2 = 2.598\,R^2 = 3.464\,r^2$
$R = s = 1.155\,r$
$r = 0.866\,s = 0.866\,R$
$s = R = 1.155\,r$



Regular Octagon

A = area;
R = radius of circumscribed circle;
r = radius of inscribed circle.

$A = 4.828\,s^2 = 2.828\,R^2 = 3.314\,r^2$
$R = 1.307\,s = 1.082\,r$
$r = 1.207\,s = 0.924\,R$
$s = 0.765\,R = 0.828\,r$



Regular Polygon

A = area; n = number of sides.
$\alpha = 360° \div n$ $\beta = 180° - \alpha$

$$A = \frac{nsr}{2} = \frac{ns}{2}\sqrt{R^2 - \frac{s^2}{4}}$$

$$R = \sqrt{r^2 + \frac{s^2}{4}};\quad r = \sqrt{R^2 - \frac{s^2}{4}};\quad s = 2\sqrt{R^2 - r^2}$$

Examples of the Use of the Formulas

Obtuse-angled Triangle. — The side $a = 5$, side $b = 4$, and side $c = 8$ inches. Find the area.

$$S = \tfrac{1}{2}(a + b + c) = \tfrac{1}{2}(5 + 4 + 8) = \tfrac{1}{2} \times 17 = 8.5$$

$$A = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{8.5\,(8.5-5)\,(8.5-4)\,(8.5-8)}$$

$$= \sqrt{8.5 \times 3.5 \times 4.5 \times 0.5} = \sqrt{66.937} = 8.18 \text{ square inches.}$$

Trapezoid. — Side $a = 23$ feet, side $b = 32$ feet, and height $h = 12$ feet. Find the area.

$$A = \frac{(a+b)\,h}{2} = \frac{(23+32)\,12}{2} = \frac{55 \times 12}{2} = \frac{660}{2} = 330 \text{ square feet.}$$

Trapezium. — Let $a = 10$, $b = 2$, $c = 3$, $h = 8$, and $H = 12$ inches. Find the area.

$$A = \frac{(H+h)\,a + bh + cH}{2} = \frac{(12+8)\,10 + 2 \times 8 + 3 \times 12}{2}$$

$$= \frac{20 \times 10 + 16 + 36}{2} = \frac{252}{2} = 126 \text{ square inches.}$$

Regular Hexagon. — The side s of a regular hexagon is 4 inches. Find the area and the radius r of the inscribed circle.

$$A = 2.598\,s^2 = 2.598 \times 4^2 = 2.598 \times 16 = 41.568 \text{ square inches.}$$

$$r = 0.866\,s = 0.866 \times 4 = 3.464 \text{ inches.}$$

What is the length of the side of a hexagon that is described about a circle of 5 inches radius? — Here $r = 5$. Hence,

$$s = 1.155\,r = 1.155 \times 5 = 5.775 \text{ inches.}$$

Regular Octagon. — Find the area and the length of the side of an octagon that is inscribed in a circle of 12 inches diameter.

Diameter of circumscribed circle = 12 inches; hence, $R = 6$ inches.

$$A = 2.828\,R^2 = 2.828 \times 6^2 = 2.828 \times 36 = 101.81 \text{ square inches.}$$

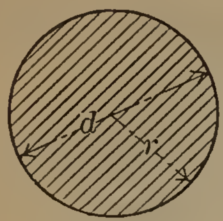
$$s = 0.765\,R = 0.765 \times 6 = 4.590 \text{ inches.}$$

Regular Polygon. — Find the area of a polygon having 12 sides, inscribed in a circle of 8 inches radius. The length of the side s is 4.141 inches.

$$A = \frac{ns}{2}\sqrt{R^2 - \frac{s^2}{4}} = \frac{12 \times 4.141}{2}\sqrt{8^2 - \frac{4.141^2}{4}} = 24.846\sqrt{59.713}$$

$$= 24.846 \times 7.727 = 191.98 \text{ square inches.}$$

Mensuration



Circle

A = area; C = circumference.

$$A = \pi r^2 = 3.1416\, r^2 = 0.7854\, d^2$$

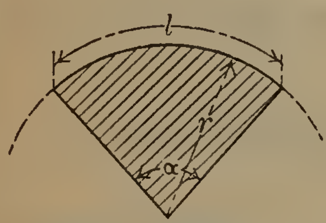
$$C = 2\pi r = 6.2832\, r = 3.1416\, d$$

$$r = C \div 6.2832 = \sqrt{A \div 3.1416} = 0.564\sqrt{A}$$

$$d = C \div 3.1416 = \sqrt{A \div 0.7854} = 1.128\sqrt{A}$$

Length of arc for center-angle of $1° = 0.008727\, d$

Length of arc for center-angle of $n° = 0.008727\, nd$



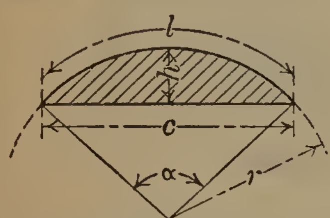
Circular Sector

A = area; l = length of arc; α = angle, in degrees.

$$l = \frac{r \times \alpha \times 3.1416}{180} = 0.01745\, r\alpha = \frac{2A}{r}$$

$$A = \tfrac{1}{2} rl = 0.008727\, \alpha r^2$$

$$\alpha = \frac{57.296\, l}{r} \qquad r = \frac{2A}{l} = \frac{57.296\, l}{\alpha}$$



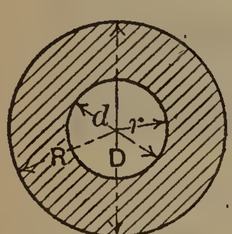
Circular Segment

A = area; l = length of arc; α = angle, in degrees.

$$c = 2\sqrt{h(2r - h)} \qquad A = \tfrac{1}{2}[rl - c(r - h)]$$

$$r = \frac{c^2 + 4h^2}{8h} \qquad l = 0.01745\, r\alpha$$

$$h = r - \tfrac{1}{2}\sqrt{4r^2 - c^2} \qquad \alpha = \frac{57.296\, l}{r}$$



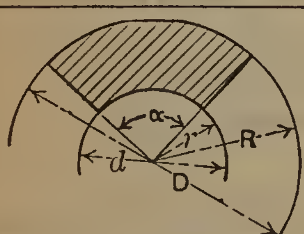
Circular Ring

A = area.

$$A = \pi(R^2 - r^2) = 3.1416\,(R^2 - r^2)$$

$$= 3.1416\,(R + r)(R - r)$$

$$= 0.7854\,(D^2 - d^2) = 0.7854\,(D + d)(D - d)$$

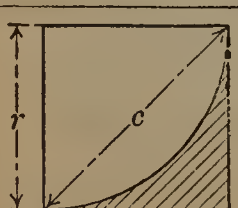


Circular Ring Sector

A = area; α = angle, in degrees.

$$A = \frac{\alpha\pi}{360}(R^2 - r^2) = 0.00873\, \alpha\,(R^2 - r^2)$$

$$= \frac{\alpha\pi}{4 \times 360}(D^2 - d^2) = 0.00218\, \alpha\,(D^2 - d^2)$$



Spandrel or Fillet

A = area.

$$A = r^2 - \frac{\pi r^2}{4} = 0.215\, r^2$$

$$= 0.1075\, c^2$$

Examples of the Use of the Formulas

Circle. — Find the area A and circumference C of a circle with a diameter of 2¾ inches.

$A = 0.7854\,d^2 = 0.7854 \times 2.75^2 = 0.7854 \times 2.75 \times 2.75 = 5.9396$ square inches.

$C = 3.1416\,d = 3.1416 \times 2.75 = 8.6394$ inches.

The area of a circle is 16.8 square inches. Find its diameter.

$$d = 1.128\sqrt{A} = 1.128\sqrt{16.8} = 1.128 \times 4.099 = 4.624 \text{ inches.}$$

Circular Sector. — The radius of a circle is 1½ inch, and angle α of a sector of the circle is 60 degrees. Find the area of the sector and the length of arc l.

$A = 0.008727\,\alpha r^2 = 0.008727 \times 60 \times 1.5^2 = 0.5236 \times 1.5 \times 1.5 = 1.178$ sq. inch.

$l = 0.01745\,r\alpha = 0.01745 \times 1.5 \times 60 = 1.5705$ inch.

Circular Segment. — The radius r of a circular segment is 60 inches and the height h is 8 inches. Find the length of the chord c.

$c = 2\sqrt{h(2r-h)} = 2\sqrt{8 \times (2 \times 60 - 8)} = 2\sqrt{896} = 2 \times 29.93 = 59.86$ inches.

If $c = 16$, and $h = 6$ inches, what is the radius of the circle of which the segment is a part?

$$r = \frac{c^2 + 4h^2}{8h} = \frac{16^2 + 4 \times 6^2}{8 \times 6} = \frac{256 + 144}{48} = \frac{400}{48} = 8\tfrac{1}{3} \text{ inches.}$$

Circular Ring. — Let the outside diameter $D = 12$ inches and the inside diameter $d = 8$ inches. Find area of ring.

$A = 0.7854\,(D^2 - d^2) = 0.7854\,(12^2 - 8^2) = 0.7854\,(144 - 64) = 0.7854 \times 80$
$= 62.83$ square inches.

By the alternative formula:

$A = 0.7854\,(D + d)\,(D - d) = 0.7854\,(12 + 8)\,(12 - 8) = 0.7854 \times 20 \times 4$
$= 62.83$ square inches.

Circular Ring Sector. — Find the area, if the outside radius $R = 5$ inches, the inside radius $r = 2$ inches, and $\alpha = 72$ degrees.

$$A = 0.00873\,\alpha\,(R^2 - r^2) = 0.00873 \times 72\,(5^2 - 2^2)$$
$$= 0.6286\,(25 - 4) = 0.6286 \times 21 = 13.2 \text{ square inches.}$$

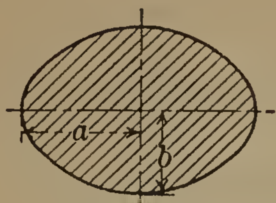
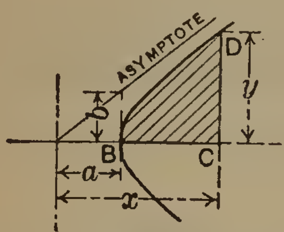
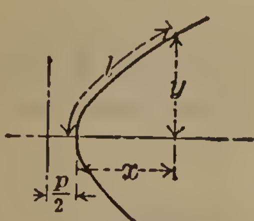
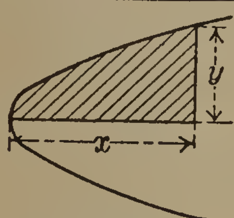
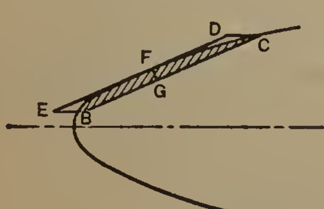
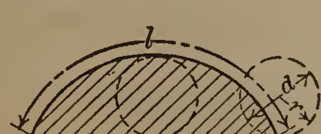
Spandrel or Fillet. — Find the area of a spandrel, the radius of which is 0.7 inch.

$$A = 0.215\,r^2 = 0.215 \times 0.7^2 = 0.215 \times 0.7 \times 0.7 = 0.105 \text{ square inch.}$$

If chord c were given as 2.2 inches, what would be the area?

$$A = 0.1075\,c^2 = 0.1075 \times 2.2^2 = 0.1075 \times 4.84 = 0.520 \text{ square inch.}$$

Mensuration

<table>
<tr><td>
Ellipse</td><td>A = area; P = perimeter or circumference.
$A = \pi ab = 3.1416\,ab.$
An approximate formula for the perimeter is:
$P = 3.1416\sqrt{2\,(a^2+b^2)}$
A closer approximation is:
$P = 3.1416\sqrt{2\,(a^2+b^2) - \frac{(a-b)^2}{2.2}}$</td></tr>
<tr><td>
Hyperbola</td><td>A = area BCD.
$A = \frac{xy}{2} - \frac{ab}{2}\text{ hyp. log}\left(\frac{x}{a}+\frac{y}{b}\right)$</td></tr>
<tr><td>
Parabola</td><td>l = length of arc.
$l = \frac{p}{2}\left[\sqrt{\frac{2x}{p}\left(1+\frac{2x}{p}\right)} + \text{hyp. log}\left(\sqrt{\frac{2x}{p}} + \sqrt{1+\frac{2x}{p}}\right)\right]$
When x is small in proportion to y, the following is a close approximation:
$l = y\left[1 + \frac{2}{3}\left(\frac{x}{y}\right)^2 - \frac{2}{5}\left(\frac{x}{y}\right)^4\right]$, or $l = \sqrt{y^2 + \frac{4}{3}x^2}$</td></tr>
<tr><td>
Parabola</td><td>A = area.
$A = \frac{2}{3}xy$
(The area is equal to two-thirds of the rectangle which has x for its base and y for its height.)</td></tr>
<tr><td>
Segment of Parabola</td><td>A = area.
Area $BFC = A = \frac{2}{3}$ area of parallelogram $BCDE$.
If FG is the height of the segment, measured at right angles to BC, then:
Area of segment $BFC = \frac{2}{3}\,BC \times FG$</td></tr>
<tr><td>
Cycloid</td><td>A = area; l = length of cycloid.
$A = 3\pi r^2 = 9.4248\,r^2 = 2.3562\,d^2$
$= 3 \times$ area of generating circle
$l = 8r = 4d$</td></tr>
</table>

Examples of the Use of the Formulas

Ellipse. — The larger or major axis is 8 inches. The smaller or minor axis is 6 inches. Find the area and the approximate circumference. Here, then, $a = 4$, and $b = 3$.

$$A = 3.1416\,ab = 3.1416 \times 4 \times 3 = 37.699 \text{ square inches.}$$

$$P = 3.1416\sqrt{2\,(a^2 + b^2)} = 3.1416 \times \sqrt{2\,(4^2 + 3^2)} = 3.1416 \times \sqrt{2 \times 25}$$
$$= 3.1416\sqrt{50} = 3.1416 \times 7.071 = 22.214 \text{ inches.}$$

Hyperbola. — The half-axes a and b are 3 and 2 inches, respectively. Find area shown shaded in illustration for $x = 8$ and $y = 5$.

Inserting the known values in the formula:

$$A = \frac{8 \times 5}{2} - \frac{3 \times 2}{2} \times \text{hyp. log}\left(\frac{8}{3} + \frac{5}{2}\right) = 20 - 3 \times \text{hyp. log } 5.167$$
$$= 20 - 3 \times 1.6423 = 20 - 4.927 = 15.073 \text{ square inches.}$$

Parabola. — If $x = 2$ and $y = 24$ feet, what is the approximate length l of the parabolic curve?

$$l = y\left[1 + \frac{2}{3}\left(\frac{x}{y}\right)^2 - \frac{2}{5}\left(\frac{x}{y}\right)^4\right] = 24\left[1 + \frac{2}{3}\left(\frac{2}{24}\right)^2 - \frac{2}{5}\left(\frac{2}{24}\right)^4\right]$$
$$= 24\left[1 + \frac{2}{3} \times \frac{1}{144} - \frac{2}{5} \times \frac{1}{20{,}736}\right] = 24 \times 1.0046 = 24.11 \text{ feet.}$$

Parabola. — Let the dimension x in the illustration be 15 inches, and y, 9 inches. Find the area of the shaded portion ot the parabola.

$$A = \tfrac{2}{3} \times xy = \tfrac{2}{3} \times 15 \times 9 = 10 \times 9 = 90 \text{ square inches.}$$

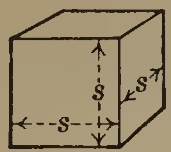
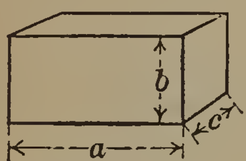
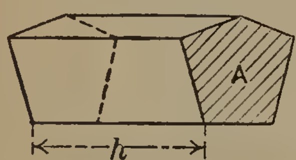
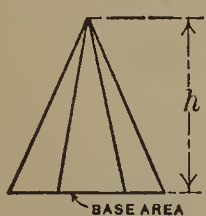
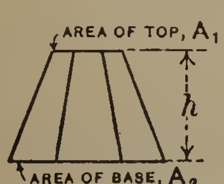
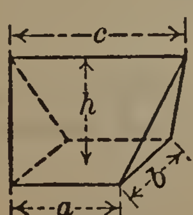
Segment of Parabola. — The length of the chord $BC = 19.5$ inches. The distance between lines BC and DE, measured at right angles to BC, is 2.25 inches. This is the height of the segment. Find the area.

$$\text{Area} = A = \tfrac{2}{3}\,BC \times FG = \tfrac{2}{3} \times 19.5 \times 2.25 = 29.25 \text{ square inches.}$$

Cycloid. — The diameter of the generating circle of a cycloid is 6 inches. Find the length l of the cycloidal curve, and the area enclosed between the curve and the base line.

$$l = 4\,d = 4 \times 6 = 24 \text{ inches.}$$
$$A = 2.3562\,d^2 = 2.3562 \times 6^2 = 2.3562 \times 36 = 84.82 \text{ square inches.}$$

Volumes of Solids

 **Cube**	V = volume. $V = s^3$ $s = \sqrt[3]{V}$
 **Square Prism**	V = volume. $V = abc$ $a = \dfrac{V}{bc}$ $\quad b = \dfrac{V}{ac}$ $\quad c = \dfrac{V}{ab}$
 **Prism**	V = volume; A = area of end surface. $V = h \times A$ The area A of the end surface is found by the formulas for areas of plane figures on the preceding pages. Height h must be measured perpendicular to end surface.
 **Pyramid**	V = volume. $V = \frac{1}{3} h \times$ area of base. If the base is a regular polygon with n sides, and s = length of side, r = radius of inscribed circle, and R = radius of circumscribed circle, then: $V = \dfrac{nsrh}{6} = \dfrac{nsh}{6}\sqrt{R^2 - \dfrac{s^2}{4}}$
 **Frustum of Pyramid**	V = volume. $V = \dfrac{h}{3}(A_1 + A_2 + \sqrt{A_1 \times A_2})$
 **Wedge**	V = volume. $V = \dfrac{(2a + c)\,bh}{6}$

Examples of the Use of the Formulas

Cube. — The side of a cube equals 9.5 inches. Find its volume.

$$\text{Volume} = V = s^3 = 9.5^3 = 9.5 \times 9.5 \times 9.5 = 857.375 \text{ cubic inches.}$$

The volume of a cube is 231 cubic inches. What is the length of the side?

$$s = \sqrt[3]{V} = \sqrt[3]{231} = 6.136 \text{ inches.}$$

Square Prism. — In a square prism, $a = 6$, $b = 5$, $c = 4$. Find the volume.

$$V = a \times b \times c = 6 \times 5 \times 4 = 120 \text{ cubic inches.}$$

How high should a box be made to contain 25 cubic feet, if it is 4 feet long and $2\frac{1}{2}$ feet wide? Here, $a = 4$, $c = 2.5$, and $V = 25$. Then,

$$b = \text{depth} = \frac{V}{ac} = \frac{25}{4 \times 2.5} = \frac{25}{10} = 2.5 \text{ feet.}$$

Prism. — A prism having for its base a regular hexagon with a side s of 3 inches, is 10 inches high. Find the volume.

$$\text{Area of hexagon} = A = 2.598\,s^2 = 2.598 \times 9 = 23.382 \text{ square inches.}$$

$$\text{Volume of prism} = h \times A = 10 \times 23.382 = 233.82 \text{ cubic inches.}$$

Pyramid. —A pyramid, having a height of 9 feet, has a base formed by a rectangle, the sides of which are 2 and 3 feet, respectively. Find the volume.

$$\text{Area of base} = 2 \times 3 = 6 \text{ square feet;} \quad h = 9 \text{ feet.}$$

$$\text{Volume} = V = \tfrac{1}{3}\,h \times \text{area of base} = \tfrac{1}{3} \times 9 \times 6 = 18 \text{ cubic feet.}$$

Frustum of Pyramid. — The pyramid in the previous example is cut off $4\frac{1}{2}$ feet from the base, the upper part being removed. The sides of the rectangle forming the top surface of the frustum are, then, 1 and $1\frac{1}{2}$ foot long, respectively. Find the volume of the frustum.

$$\text{Area of top} = A_1 = 1 \times 1\tfrac{1}{2} = 1\tfrac{1}{2} \text{ sq. ft.} \quad \text{Area of base} = A_2 = 2 \times 3 = 6 \text{ sq. ft.}$$

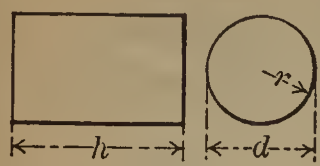
$$V = \frac{4.5}{3}\left(1.5 + 6 + \sqrt{1.5 \times 6}\right) = 1.5\left(7.5 + \sqrt{9}\right) = 1.5 \times 10.5 = 15.75 \text{ cubic feet.}$$

Wedge. — Let $a = 4$ inches, $b = 3$ inches, and $c = 5$ inches. The height $h = 4.5$ inches. Find the volume.

$$V = \frac{(2a + c)\,bh}{6} = \frac{(2 \times 4 + 5) \times 3 \times 4.5}{6} = \frac{(8 + 5) \times 13.5}{6} = \frac{13 \times 13.5}{6}$$

$$= \frac{175.5}{6} = 29.25 \text{ cubic inches.}$$

Volumes of Solids



Cylinder

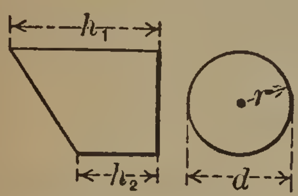
V = volume; S = area of cylindrical surface.

$V = 3.1416\, r^2h = 0.7854\, d^2h$

$S = 6.2832\, rh = 3.1416\, dh$

Total area A of cylindrical surface and end surfaces:

$A = 6.2832\, r\,(r + h) = 3.1416\, d\,(\frac{1}{2}\, d + h)$

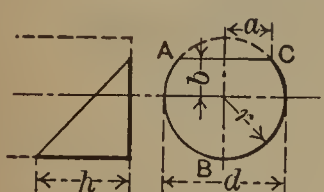


Portion of Cylinder

V = volume; S = area of cylindrical surface.

$V = 1.5708\, r^2\,(h_1 + h_2) = 0.3927\, d^2\,(h_1 + h_2)$

$S = 3.1416\, r\,(h_1 + h_2) = 1.5708\, d\,(h_1 + h_2)$



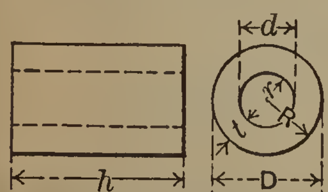
Portion of Cylinder

V = volume; S = area of cylindrical surface.

$$V = \left(\frac{2}{3}\, a^3 \pm b \times \text{area } ABC\right) \frac{h}{r \pm b}$$

$$S = (ad \pm b \times \text{length of arc } ABC)\, \frac{h}{r \pm b}$$

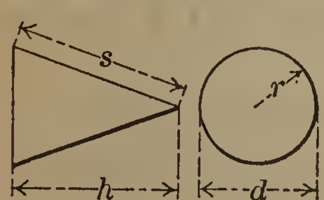
Use $+$ when base area is larger, and $-$ when base area is less than one-half the base circle.



Hollow Cylinder

V = volume.

$$\begin{aligned} V &= 3.1416\, h\,(R^2 - r^2) = 0.7854\, h\,(D^2 - d^2) \\ &= 3.1416\, ht\,(2\,R - t) = 3.1416\, ht\,(D - t) \\ &= 3.1416\, ht\,(2\,r + t) = 3.1416\, ht\,(d + t) \\ &= 3.1416\, ht\,(R + r) = 1.5708\, ht\,(D + d) \end{aligned}$$



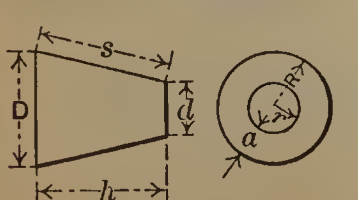
Cone

V = volume; A = area of conical surface.

$$V = \frac{3.1416\, r^2h}{3} = 1.0472\, r^2h = 0.2618\, d^2h$$

$$A = 3.1416\, r\sqrt{r^2 + h^2} = 3.1416\, rs = 1.5708\, ds$$

$$s = \sqrt{r^2 + h^2} = \sqrt{\frac{d^2}{4} + h^2}$$



Frustum of Cone

V = volume; A = area of conical surface.

$V = 1.0472\, h\,(R^2 + Rr + r^2) = 0.2618\, h\,(D^2 + Dd + d^2)$

$A = 3.1416\, s\,(R + r) = 1.5708\, s\,(D + d)$

$a = R - r \qquad s = \sqrt{a^2 + h^2} = \sqrt{(R - r)^2 + h^2}$

Examples of the Use of the Formulas

Cylinder. — The diameter of a cylinder is $2\frac{1}{2}$ inches. The length or height is 20 inches. Find the volume, and the area of the cylindrical surface S.

$$V = 0.7854\, d^2h = 0.7854 \times 2\tfrac{1}{2}^2 \times 20 = 0.7854 \times 6.25 \times 20 = 98.17 \text{ cubic inches.}$$
$$S = 3.1416\, dh = 3.1416 \times 2\tfrac{1}{2} \times 20 = 157.08 \text{ square inches.}$$

Portion of Cylinder. — A cylinder 5 inches in diameter, is cut off at an angle, as shown in the illustration. Dimension $h_1 = 6$, and $h_2 = 4$ inches. Find the volume and the area S of the cylindrical surface.

$$V = 0.3927\, d^2\,(h_1 + h_2) = 0.3927 \times 5^2 \times (6 + 4) = 0.3927 \times 25 \times 10$$
$$= 98.175 \text{ cubic inches.}$$
$$S = 1.5708\, d\,(h_1 + h_2) = 1.5708 \times 5 \times 10 = 78.54 \text{ square inches.}$$

Portion of Cylinder. — Find the volume of a cylinder so cut off that line AC passes through the center of the base circle — that is, the base area is a half-circle. The diameter of the cylinder = 5 inches, and height h = 2 inches.

In this case $a = 2.5$; $b = 0$; area $ABC = \frac{1}{2} \times 0.7854 \times 5^2 = 9.82$; $r = 2.5$.

$$V = \left(\frac{2}{3} \times 2.5^3 + 0 \times 9.82\right) \frac{2}{2.5 + 0} = \frac{2}{3} \times 15.625 \times 0.8 = 8.33 \text{ cubic inches.}$$

Hollow Cylinder. — A cylindrical shell, 28 inches high, is 36 inches in outside diameter, and 4 inches thick. Find its volume.

$$V = 3.1416\, ht\,(D - t) = 3.1416 \times 28 \times 4\,(36 - 4) = 3.1416 \times 28 \times 4 \times 32$$
$$= 11{,}259.5 \text{ cubic inches.}$$

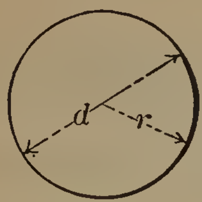
Cone. — Find the volume and area of conical surface of a cone, the base of which is a circle of 6 inches diameter, and the height of which is 4 inches.

$$V = 0.2618\, d^2h = 0.2618 \times 6^2 \times 4 = 0.2618 \times 36 \times 4 = 37.7 \text{ cubic inches.}$$
$$A = 3.1416\, r\sqrt{r^2 + h^2} = 3.1416 \times 3 \times \sqrt{3^2 + 4^2} = 9.4248 \times \sqrt{25}$$
$$= 47.124 \text{ square inches.}$$

Frustum of Cone. — Find the volume of a frustum of a cone of the following dimensions: D = 8 inches; d = 4 inches; h = 5 inches.

$$V = 0.2618 \times 5\,(8^2 + 8 \times 4 + 4^2) = 0.2618 \times 5\,(64 + 32 + 16)$$
$$= 0.2618 \times 5 \times 112 = 146.61 \text{ cubic inches.}$$

Volumes of Solids



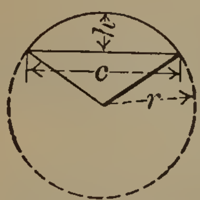
Sphere

V = volume; A = area of surface.

$$V = \frac{4\pi r^3}{3} = \frac{\pi d^3}{6} = 4.1888\, r^3 = 0.5236\, d^3$$

$$A = 4\pi r^2 = \pi d^2 = 12.5664\, r^2 = 3.1416\, d^2$$

$$r = \sqrt[3]{\frac{3\,V}{4\pi}} = 0.6204\sqrt[3]{V}$$



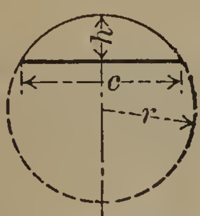
Spherical Sector

V = volume; A = total area of conical and spherical surface.

$$V = \frac{2\pi r^2 h}{3} = 2.0944\, r^2 h$$

$$A = 3.1416\, r\left(2\,h + \tfrac{1}{2}\,c\right)$$

$$c = 2\sqrt{h\,(2\,r - h)}$$



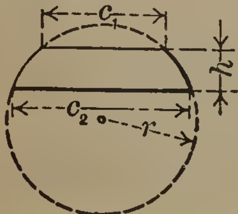
Spherical Segment

V = volume; A = area of spherical surface.

$$V = 3.1416\, h^2\left(r - \frac{h}{3}\right) = 3.1416\, h\left(\frac{c^2}{8} + \frac{h^2}{6}\right)$$

$$A = 2\pi r h = 6.2832\, rh = 3.1416\left(\frac{c^2}{4} + h^2\right)$$

$$c = 2\sqrt{h\,(2\,r - h)};\quad r = \frac{c^2 + 4\,h^2}{8\,h}$$



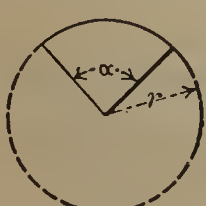
Spherical Zone

V = volume; A = area of spherical surface.

$$V = 0.5236\, h\left(\frac{3\,c_1^2}{4} + \frac{3\,c_2^2}{4} + h^2\right)$$

$$A = 2\pi r h = 6.2832\, rh$$

$$r = \sqrt{\frac{c_2^2}{4} + \left(\frac{c_2^2 - c_1^2 - 4\,h^2}{8\,h}\right)^2}$$



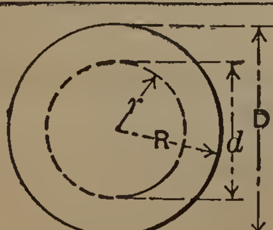
Spherical Wedge

V = volume; A = area of spherical surface;

α = center angle in degrees.

$$V = \frac{\alpha}{360} \times \frac{4\pi r^3}{3} = 0.0116\,\alpha r^3$$

$$A = \frac{\alpha}{360} \times 4\pi r^2 = 0.0349\,\alpha r^2$$



Hollow Sphere

V = volume.

$$V = \frac{4\pi}{3}\left(R^3 - r^3\right) = 4.1888\left(R^3 - r^3\right)$$

$$= \frac{\pi}{6}\left(D^3 - d^3\right) = 0.5236\left(D^3 - d^3\right)$$

Examples of the Use of the Formulas

Sphere. — Find the volume and surface of a sphere 6.5 inches in diameter.

$V = 0.5236\, d^3 = 0.5236 \times 6.5^3 = 0.5236 \times 6.5 \times 6.5 \times 6.5 = 143.79$ cubic inches.

$A = 3.1416\, d^2 = 3.1416 \times 6.5^2 = 3.1416 \times 6.5 \times 6.5 = 132.73$ square inches.

The volume of a sphere is 64 cubic inches. Find its radius.

$r = 0.6204 \sqrt[3]{64} = 0.6204 \times 4 = 2.4816$ inches.

Spherical Sector. — Find the volume of a sector of a sphere 6 inches in diameter, the height h of the sector being 1.5 inch. Also find length of chord c. — Here $r = 3$, and $h = 1.5$.

$V = 2.0944\, r^2h = 2.0944 \times 3^2 \times 1.5 = 2.0944 \times 9 \times 1.5 = 28.27$ cubic inches.

$c = 2\sqrt{h\,(2\,r - h)} = 2\sqrt{1.5\,(2 \times 3 - 1.5)} = 2\sqrt{6.75} = 2 \times 2.598$

$= 5.196$ square inches.

Spherical Segment. — A segment of a sphere has the following dimensions: $h = 2$ inches; $c = 5$ inches. Find the volume V and the radius of the sphere of which the segment is a part.

$$V = 3.1416 \times 2 \times \left(\frac{5^2}{8} + \frac{2^2}{6}\right) = 6.2832 \times \left(\frac{25}{8} + \frac{4}{6}\right) = 6.2832 \times 3.792$$

$= 23.825$ cubic inches.

$$r = \frac{5^2 + 4 \times 2^2}{8 \times 2} = \frac{25 + 16}{16} = \frac{41}{16} = 2\tfrac{9}{16} \text{ inches.}$$

Spherical Zone. — In a spherical zone, let $c_1 = 3$; $c_2 = 4$; and $h = 1.5$ inch. Find the volume.

$$V = 0.5236 \times 1.5 \times \left(\frac{3 \times 3^2}{4} + \frac{3 \times 4^2}{4} + 1.5^2\right) = 0.5236 \times 1.5 \times \left(\frac{27}{4} + \frac{48}{4} + 2.25\right)$$

$= 0.5236 \times 1.5 \times 21 = 16.493$ cubic inches.

Spherical Wedge. — Find the area of the spherical surface and the volume of a wedge of a sphere. The diameter of the sphere is 4 inches, and the center angle α is 45 degrees.

$V = 0.0116 \times 45 \times 2^3 = 0.0116 \times 45 \times 8 = 4.176$ cubic inches.

$A = 0.0349 \times 45 \times 2^2 = 0.0349 \times 45 \times 4 = 6.282$ square inches.

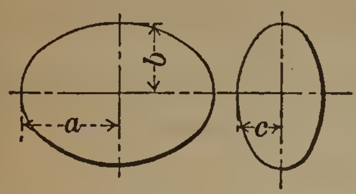
Hollow Sphere. — Find the volume of a hollow sphere, 8 inches in outside diameter, with a thickness of material of 1.5 inch.

Here $R = 4$; $r = 4 - 1.5 = 2.5$.

$V = 4.1888\,(4^3 - 2.5^3) = 4.1888\,(64 - 15.625) = 4.1888 \times 48.375$

$= 202.63$ cubic inches.

Volumes of Solids



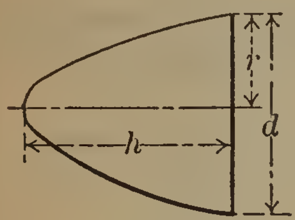
Ellipsoid

V = volume; A = area of surface.

$$V = \frac{4\pi}{3} abc = 4.1888\, abc$$

In an ellipsoid of revolution, or spheroid, where $b = c$:

$$V = 4.1888\, ab^2, \quad \text{and} \quad A = \frac{4\pi}{\sqrt{2}} b \sqrt{a^2 + b^2}$$

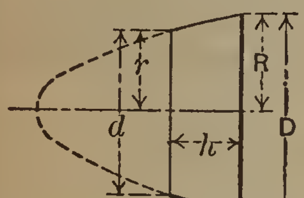


Paraboloid

V = volume; $V = \frac{1}{2}\pi r^2 h = 0.3927\, d^2 h$

A = area; $A = \frac{2\pi}{3p}\left[\sqrt{\left(\frac{d^2}{4} + p^2\right)^3} - p^3\right]$ in which

$$p = \frac{d^2}{8h}$$

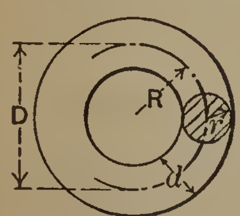


Paraboloidal Segment

V = volume.

$$V = \frac{\pi}{2} h\,(R^2 + r^2) = 1.5708\, h\,(R^2 + r^2)$$

$$= \frac{\pi}{8} h\,(D^2 + d^2) = 0.3927\, h\,(D^2 + d^2)$$



Torus

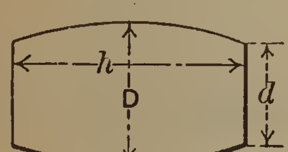
V = volume; A = area of surface.

$$V = 2\pi^2 Rr^2 = 19.739\, Rr^2$$

$$= \frac{\pi^2}{4} Dd^2 = 2.4674\, Dd^2$$

$$A = 4\pi^2 Rr = 39.478\, Rr$$

$$= \pi^2 Dd = 9.8696\, Dd$$



Barrel

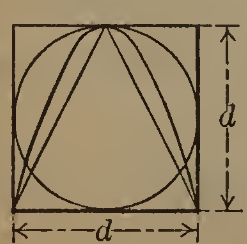
V = approximate volume.

If the sides are bent to the arc of a circle:

$$V = \tfrac{1}{12}\pi h\,(2D^2 + d^2) = 0.262\, h\,(2D^2 + d^2)$$

If the sides are bent to the arc of a parabola:

$$V = 0.209\, h\,(2D^2 + Dd + \tfrac{3}{4} d^2)$$



If d = base diameter and height of a cone, a paraboloid and a cylinder, and the diameter of a sphere, then the volumes of these bodies are to each other as below:

Cone: paraboloid: sphere: cylinder $= \frac{1}{3} : \frac{1}{2} : \frac{2}{3} : 1$

Examples of the Use of the Formulas

Ellipsoid or Spheroid. — Find the volume and area of surface of a spheroid in which $a = 5$, and $b = c = 1.5$ inch.

$$V = 4.1888 \times 5 \times 1.5^2 = 4.1888 \times 5 \times 2.25 = 47.124 \text{ cubic inches.}$$

$$A = \frac{4 \times 3.1416}{\sqrt{2}} \times 1.5 \times \sqrt{5^2 + 1.5^2} = \frac{4 \times 3.1416}{1.414} \times 1.5 \times 5.22 = 69.57 \text{ sq. inches.}$$

Paraboloid. — Find the volume of a paraboloid in which $h = 12$ and $d = 5$ inches.

$$V = 0.3927\, d^2h = 0.3927 \times 5^2 \times 12 = 0.3927 \times 25 \times 12$$
$$= 0.3927 \times 300 = 117.81 \text{ cubic inches.}$$

Segment of Paraboloid. — Find the volume of a segment of a paraboloid in which $D = 5$ inches, $d = 3$ inches, and $h = 6$ inches.

$$V = 0.3927\, h\,(D^2 + d^2) = 0.3927 \times 6 \times (5^2 + 3^2) = 0.3927 \times 6 \times (25 + 9)$$
$$= 0.3927 \times 6 \times 34 = 80.11 \text{ cubic inches.}$$

Torus. — Find the volume and area of surface of a torus in which $d = 1.5$ and $D = 5$ inches.

$$V = 2.4674 \times 5 \times 1.5^2 = 2.4674 \times 5 \times 2.25 = 27.76 \text{ cubic inches.}$$

$$A = 9.8696 \times 5 \times 1.5 = 74.022 \text{ square inches}$$

Barrel. — Find the approximate contents of a barrel, the inside dimensions of which are $D = 24$ inches; $d = 20$ inches; $h = 48$ inches.

$$V = 0.262\, h\,(2\,D^2 + d^2) = 0.262 \times 48 \times (2 \times 24^2 + 20^2) = 0.262 \times 48$$
$$\times (1152 + 400) = 0.262 \times 48 \times 1552 = 19{,}518 \text{ cubic inches.}$$

Assume, as an example, that the diameter of the base of a cone, paraboloid and cylinder is 2 inches, that the height is 2 inches, and that the diameter of a sphere is 2 inches. Then the volumes, written in formula-form, are as below:

Cone	Paraboloid	Sphere	Cylinder

$$\frac{3.1416 \times 2^2 \times 2}{12} : \frac{3.1416 \times 2^2 \times 2}{8} : \frac{3.1416 \times 2^3}{6} : \frac{3.1416 \times 2^2 \times 2}{4} = \frac{1}{3} : \frac{1}{2} : \frac{2}{3} : 1$$

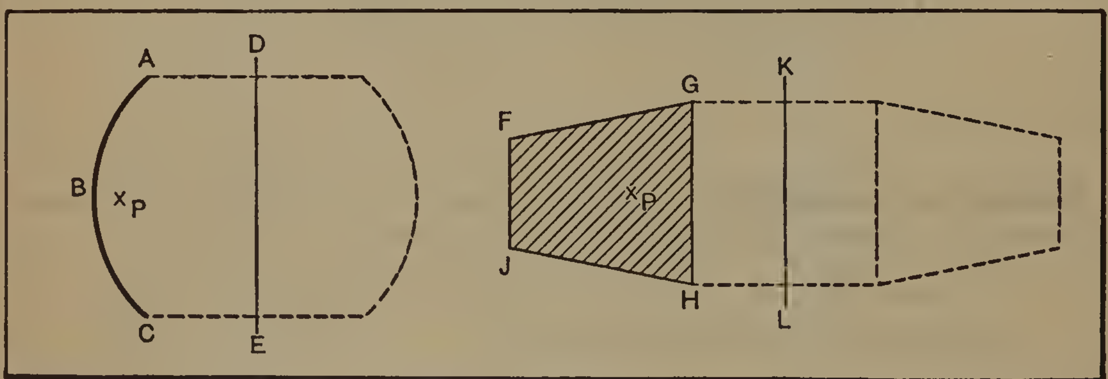
The Prismoidal Formula. — The prismoidal formula is a general formula by which the volume of any prism, pyramid or frustum of a pyramid may be found.

A_1 = area at one end of the body;
A_2 = area at the other end;
A_m = area of middle section between the two end surfaces;
h = height of body.

Then, volume V of the body is

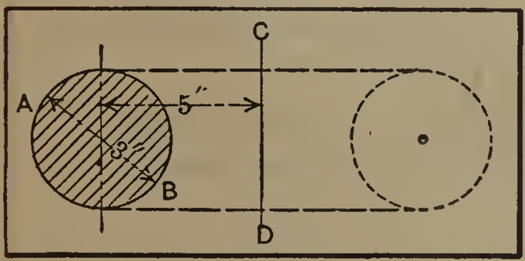
$$V = \frac{h}{6}(A_1 + 4A_m + A_2)$$

Pappus or Guldinus Rules. — By means of these rules the area of any surface of revolution and the volume of any solid of revolution may be found. The area of the surface swept out by the revolution of a line ABC (see illustration) about the axis DE equals the length of the line multiplied by the length of the path



of its center of gravity, P. If the line is of such a shape that it is difficult to determine its center of gravity, then the line may be divided into a number of short sections, each of which may be considered as a straight line, and the areas swept out by these different sections, as computed by the rule given, may be added to find the total area. The line must lie wholly on one side of the axis of revolution and must be in the same plane.

The volume of a solid body formed by the revolution of a surface $FGHJ$ about axis KL equals the area of the surface multiplied by the length of the path of its center of gravity. The surface must lie wholly on one side of the axis of revolution and in the same plane.



Example: — By means of these rules the area and volume of a cylindrical ring or torus may be found. The torus is formed by a circle AB being rotated about axis CD. The center of gravity of the circle is at its center. Hence, with the dimensions given in the illustration, the length of the path of the center of gravity of the circle is $3.1416 \times 10 = 31.416$ inches. This multiplied by the length of the circumference of the circle, which is $3.1416 \times 3 = 9.4248$ inches, equals:

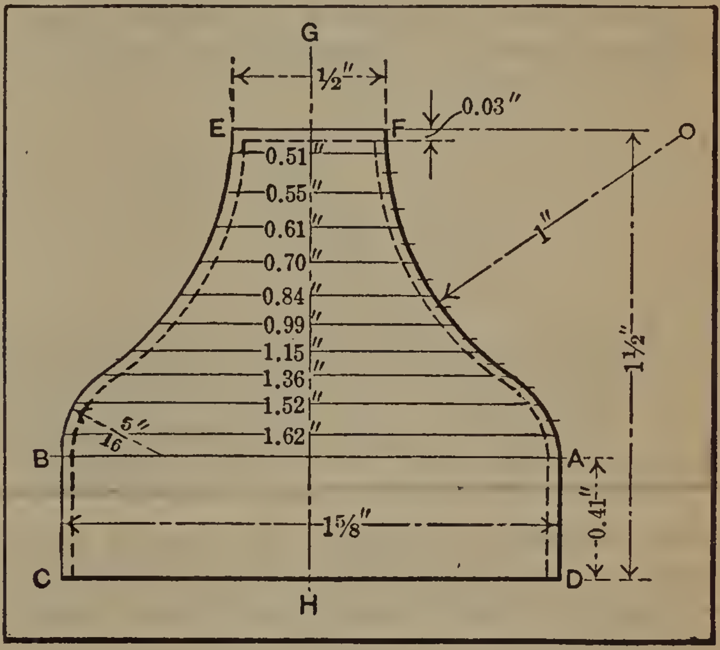
$$31.416 \times 9.4248 = 296.089 \text{ square inches}$$

which is the area of the torus.

The volume equals the area of the circle, which is $0.7854 \times 9 = 7.0686$ square inches, multiplied by the path of the center of gravity, which is 31.416, as before; hence,

$$\text{volume} = 7.0686 \times 31.416 = 222.067 \text{ cubic inches.}$$

Example of Approximate Method for Finding the Area of a Surface of Revolution. — The accompanying illustration is shown in order to give an example of the approximate method based on Guldinus' rule, that can be used for finding the area of a symmetrical body. In the illustration, the dimensions in common fractions are the known dimensions; those in decimals are found by actual measurements on a figure drawn to scale. The method for finding the area is as follows: First separate such areas as are cylindrical, conical or spherical, as these can be found by exact formulas. In the illustration *ABCD* is a cylinder, the area of the surface of which can be easily found. The top area *EF* is simply a circular area, and can thus be computed separately. The remainder of the surface generated by rotating line *AF* about the axis *GH* is found by the approximate method explained in the previous section. From point *A*, set off equal distances on line *AF*. In the present case each division indicated is ⅛ inch long. From the central or middle point of each of these parts draw a line at right angles to the axis of rotation *GH*, measure the length of these lines or diameters (the length of each is given in decimals), add all these lengths together and multiply the sum by the length of one division set off on line *AF* (in this case, ⅛ inch), and multiply this product by π. This gives the approximate area of the surface of revolution.



In setting off divisions ⅛ inch long along line *AF*, the last division does not reach exactly to point *F*, but only to a point 0.03 inch below it. The part 0.03 inch high at the top of the cup, can be considered as a cylinder of ½ inch diameter and 0.03 inch height, the area of the cylindrical surface of which is easily computed. By adding the various surfaces together the total surface of the cup is found as below:

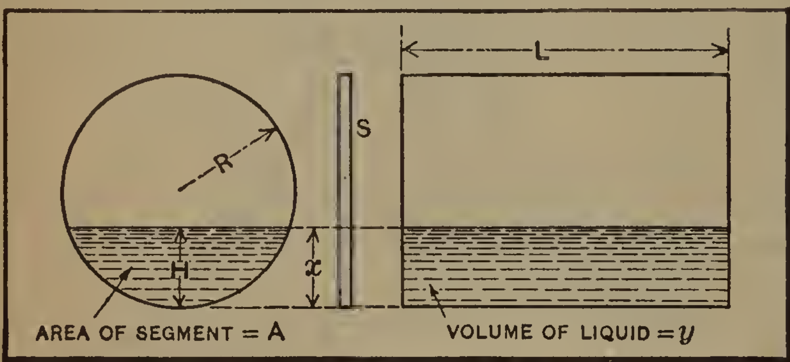
Cylinder, 1⅝ inch diameter, 0.41 inch high........	2.093 square inches
Circle, ½ inch diameter..........................	0.196 square inches
Cylinder, ½ inch diameter, 0.03 inch high.........	0.047 square inches
Irregular surface................................	3.868 square inches
Total..	6.204 square inches

Plane Surfaces of Irregular Outline. — The areas of plane surfaces of irregular outline can best be found by dividing the surfaces into a number of geometrical figures, the areas of which can be found by the general rules and formulas. The most convenient method is, as a rule, to divide the area into a number of narrow strips which may be regarded as rectangles, the base of the rectangle being the width of the strip, and a line through the center of the strip its mean height. Surfaces of irregular outline, having straight boundary lines, are most conveniently divided into a number of triangles, the areas of which can be readily found and added together.

Areas Enclosed by Cycloidal Curves. — The area between a cycloid and the straight line upon which the generating circle rolls, equals three times the area of the generating circle (see diagram, page 154). The areas between epicycloidal and hypocycloidal curves and the "fixed circle" upon which the generating circle is rolled, may be determined by the following formulas, in which a = radius of the fixed circle upon which the generating circle rolls; b = radius of the generating circle; A = the area for the epicycloidal curve; and A_1 = the area for the hypocycloidal curve.

$$A = \frac{3.1416\, b^2\,(3\,a + 2\,b)}{a}; \qquad A_1 = \frac{3.1416\, b^2\,(3\,a - 2\,b)}{a}$$

Finding the Contents of Cylindrical Tanks at Different Levels. — The following method for determining the contents of a cylindrical tank at any given level may be employed for locating the graduations on gage sticks such as are used for measuring approximately the contents, by inserting the gage vertically in the center of the tank with the lower end against the bottom and noting the relation between the level of the liquid and the graduation marks which indicate the number of gallons. The table "Segments of Circles" (see pages 72 and 73) is used to simplify the calculations. Column H of this table gives the values for the heights of segments with different center angles and for unit radius; or, in other words, the ratio: $\frac{\text{height of segment}}{\text{radius}}$. Another column gives the area of the segment in a circle of unit radius. The area of a similar segment of larger radius is this area multiplied by the given radius squared. Let



L = length of tank, in inches (see accompanying illustration);

R = radius in inches;

H = height of segment for unit radius = $\frac{\text{height}}{R}$;

y = capacity, in gallons, for which graduation is to be found on scale S;

x = distance from graduation mark to nearest end of scale;

$U = \frac{L}{231}$ gallons in one-inch square section, the length of which equals the length of the tank;

a = corresponding area in table giving areas for segments of unit radius;

A = area of segment of given radius.

As $H = \frac{\text{height}}{R} = \frac{x}{R}$, $x = HR$; also, by the rule at the head of the table, the area of a segment of given radius, or A, equals aR^2. The number of gallons represented by a graduation on the scale, or y, equals $AU = aR^2U$, and $a = \frac{y}{R^2U}$. Therefore, R^2U is a constant for any given tank. The method of procedure will be illustrated by a practical example.

Example: — A tank is 20 feet long and 6 feet in diameter, and the graduation mark representing 1000 gallons is to be located on a scale. Find the distance from the graduation mark to the nearest end of the scale.

As the radius of the tank is 36 inches and the length 240 inches, the total capacity, in this case, equals:

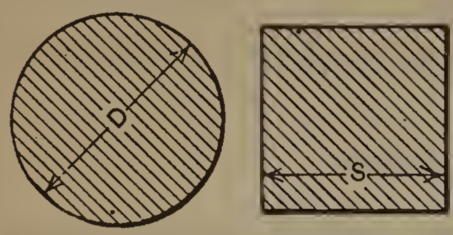
$$\frac{3.1416 \times 36^2 \times 240}{231} = 4230 \text{ gallons.}$$

As the graduation mark is to represent 1000 gallons, which is less than one-half the total capacity, this graduation will be located with reference to the lower end of the scale. The constant R^2U, for this tank, is $36^2 \times \frac{240}{231} = 1346$. As $a = \frac{y}{R^2U}$, the corresponding area of a segment of unit radius is $1000 \div 1346 = 0.743$. The table shows that this area is greater than the corresponding area of a segment of 129 degrees, and less than the area of a 130-degree segment. By interpolation, the value of H is found to be about 0.573. As $x = HR$, the graduation mark on the scale for 1000 gallons is $0.573 \times 36 = 20.62$ inches from the lower end.

In the preceding example, if a graduation mark were required to represent, say, 3000 gallons, this mark would be located from the top of the scale, because 3000 is more than one-half the total capacity of the tank. In this case, the value of y equals the difference between the total capacity and the number of gallons which the graduation is to represent. Thus, in this case, $y = 4230 - 3000 = 1230$, and $a = 1230 \div 1346 = 0.914$. By interpolation, the corresponding value of H is found to be 0.665. Therefore, $x = 0.665 \times 36 = 23.94$ inches for the 3000-gallon graduation mark.

Determining Areas with a Planimeter. — The area of a plane figure may be determined readily by means of a planimeter even though the figure is very irregular in outline. This type of instrument is extensively used for obtaining the areas of steam engine indicator cards. (After finding the area, in square inches, this area is divided by the length of the card, in inches, to obtain the average height; the quotient is then multiplied by the scale of the indicator spring and the result equals the mean effective pressure.) When using the planimeter the "tracing point" is moved along the outline of the area or surface to be measured. The "Amsler polar planimeter," which is a common form, has two arms; the end of one arm has a needle point by means of which it is attached to the paper on which the outline is drawn. The arm that is fixed to the paper has a standard length, while the tracing arm can be adjusted so that the results may be read in different units. The instrument is so connected with a graduated wheel, the periphery of which rests upon the paper, that the area can be read off on the wheel and its vernier. The planimeter may also have a disk for counting the number of revolutions made by the wheel. The tracing point must be moved carefully around the outline until the starting point is again reached. The area is determined by the difference of the reading on the disk and the graduated wheel at the beginning of the measuring movement and at the end. If the planimeter is too small to measure a given area, the area can be divided and the sections measured separately. The results obtained by the polar planimeter are correct within an error of about one per cent. The rolling planimeter, which is a more expensive type, gives results that are correct within about 0.1 per cent. The whole instrument rolls forward and backward in a straight line while the tracing point follows the outline of the surface to be measured. This type is adapted for measuring figures of indefinite length and limited breadth. Some planimeters give results in various denominations of value, such as square inches, square feet, and square decimeters, and also the average height of an indicator diagram at one measurement or by a direct reading, after tracing the outline.

Diameters of Circles and Sides of Squares of Equal Area



The table below will be found useful for determining the diameter of a circle of an area equal to that of a square, the side of which is known, or for determining the side of a square which has an area equal to that of a circle, the area or diameter of which is known. For example, if the diameter of a circle is 17½ inches, it is found from the table that the side of a square of the same area is 15.51 inches.

Diam. of Circle, *D*	Side of Square, *S*	Area of Circle or Square	Diam. of Circle, *D*	Side of Square, *S*	Area of Circle or Square	Diam. of Circle, *D*	Side of Square *S*	Area of Circle or Square
½	0.44	0.196	20½	18.17	330.06	40½	35.89	1288.25
1	0.89	0.785	21	18.61	346.36	41	36.34	1320.25
1½	1.33	1.767	21½	19.05	363.05	41½	36.78	1352.65
2	1.77	3.142	22	19.50	380.13	42	37.22	1385.44
2½	2.22	4.909	22½	19.94	397.61	42½	37.66	1418.63
3	2.66	7.069	23	20.38	415.48	43	38.11	1452.20
3½	3.10	9.621	23½	20.83	433.74	43½	38.55	1486.17
4	3.54	12.566	24	21.27	452.39	44	38.99	1520.53
4½	3.99	15.904	24½	21.71	471.44	44½	39.44	1555.28
5	4.43	19.635	25	22.16	490.87	45	39.88	1590.43
5½	4.87	23.758	25½	22.60	510.71	45½	40.32	1625.97
6	5.32	28.274	26	23.04	530.93	46	40.77	1661.90
6½	5.76	33.183	26½	23.49	551.55	46½	41.21	1698.23
7	6.20	38.485	27	23.93	572.56	47	41.65	1734.94
7½	6.65	44.179	27½	24.37	593.96	47½	42.10	1772.05
8	7.09	50.265	28	24.81	615.75	48	42.54	1809.56
8½	7.53	56.745	28½	25.26	637.94	48½	42.98	1847.45
9	7.98	63.617	29	25.70	660.52	49	43.43	1885.74
9½	8.42	70.882	29½	26.14	683.49	49½	43.87	1924.42
10	8.86	78.540	30	26.59	706.86	50	44.31	1963.50
10½	9.31	86.590	30½	27.03	730.62	50½	44.75	2002.96
11	9.75	95.033	31	27.47	754.77	51	45.20	2042.82
11½	10.19	103.87	31½	27.92	779.31	51½	45.64	2083.07
12	10.64	113.10	32	28.36	804.25	52	46.08	2123.72
12½	11.08	122.72	32½	28.80	829.58	52½	46.53	2164.75
13	11.52	132.73	33	29.25	855.30	53	46.97	2206.18
13½	11.96	143.14	33½	29.69	881.41	53½	47.41	2248.01
14	12.41	153.94	34	30.13	907.92	54	47.86	2290.22
14½	12.85	165.13	34½	30.57	934.82	54½	48.30	2332.83
15	13.29	176.71	35	31.02	962.11	55	48.74	2375.83
15½	13.74	188.69	35½	31.46	989.80	55½	49.19	2419.22
16	14.18	201.06	36	31.90	1017.88	56	49.63	2463.01
16½	14.62	213.82	36½	32.35	1046.35	56½	50.07	2507.19
17	15.07	226.98	37	32.79	1075.21	57	50.51	2551.76
17½	15.51	240.53	37½	33.23	1104.47	57½	50.96	2596.72
18	15.95	254.47	38	33.68	1134.11	58	51.40	2642.08
18½	16.40	268.80	38½	34.12	1164.16	58½	51.84	2687.83
19	16.84	283.53	39	34.56	1194.59	59	52.29	2733.97
19½	17.28	298.65	39½	35.01	1225.42	59½	52.73	2780.51
20	17.72	314.16	40	35.45	1256.64	60	53.17	2827.43

SOLUTION OF TRIANGLES

Any figure bounded by three straight lines is called a triangle. Any one of the three lines may be called the base, and the line drawn from the angle opposite the base at right angles to it is called the height or altitude of the triangle.

If all the three sides of a triangle are of equal length, the triangle is called *equilateral*. Each one of the three angles in an equilateral triangle equals 60 degrees. If two sides are of equal length, the triangle is an *isosceles* triangle. If one angle is a right or 90-degree angle, the triangle is a *right* or *right-angled* triangle. The side opposite the right angle is called the *hypotenuse*.

If all the angles are less than 90 degrees, the triangle is called an *acute* or *acute-angled* triangle. If one of the angles is larger than 90 degrees, the triangle is called an *obtuse-angled* triangle. Both acute and obtuse-angled triangles are known under the common name of *oblique-angled* triangles. The sum of the three angles in every triangle is 180 degrees.

The sides and angles of any triangle which are not known can be found when: 1. All the three sides; 2. Two sides and one angle; or, 3. One side and two angles, are given. In other words, if a triangle is considered as consisting of six parts, three angles and three sides, the unknown parts can be determined when any three parts are given, provided at least one of the given parts is a side.

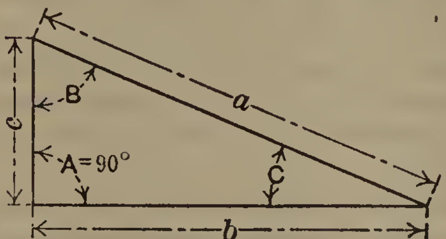
Functions of Angles. — The functions of angles used in solving triangles are sine, cosine, tangent, cotangent, secant, and cosecant. These expressions are usually abbreviated as follows:

sin = sine,	cot = cotangent,
cos = cosine,	sec = secant,
tan = tangent,	cosec = cosecant.

If in a right-angled triangle (see the illustration in the table below), the lengths of the three sides are represented by a, b and c, and the angles opposite each of these sides by A, B and C, then the side a opposite the right angle is the hypotenuse;

Trigonometrical Functions of Angles

The *sine* of an angle equals the opposite side divided by the hypotenuse. Hence, $\sin B = b \div a$, and $\sin C = c \div a$.



The *cosine* of an angle equals the adjacent side divided by the hypotenuse. Hence, $\cos B = c \div a$, and $\cos C = b \div a$.

The *tangent* of an angle equals the opposite side divided by the adjacent side. Hence, $\tan B = b \div c$, and $\tan C = c \div b$.

The *cotangent* of an angle equals the adjacent side divided by the opposite side. Hence, $\cot B = c \div b$, and $\cot C = b \div c$.

The *secant* of an angle equals the hypotenuse divided by the adjacent side. Hence, $\sec B = a \div c$, and $\sec C = a \div b$.

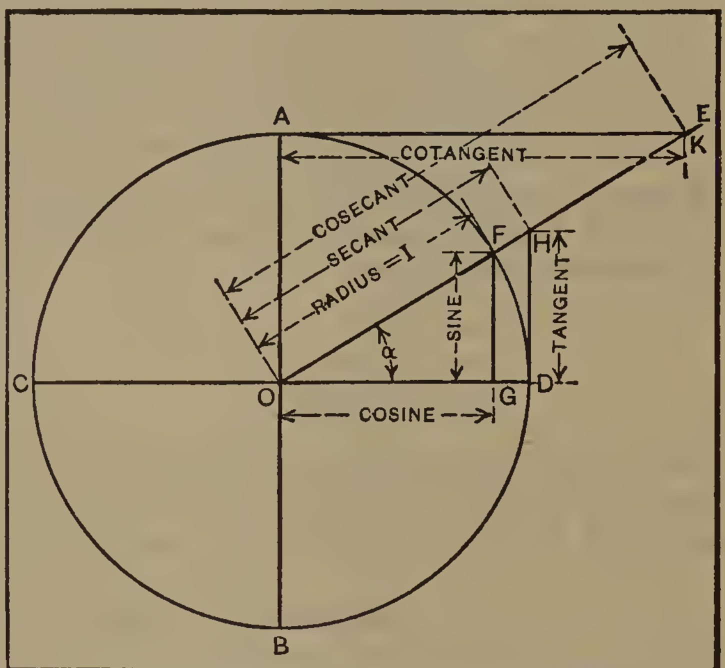
The *cosecant* of an angle equals the hypotenuse divided by the opposite side. Hence, $\operatorname{cosec} B = a \div b$, and $\operatorname{cosec} C = a \div c$.

It should be noted that the functions of the angles can be found in this manner only when the triangle is right-angled.

side b is called the *side adjacent* to angle C and is also the *side opposite* to angle B; side c is the side adjacent to angle B and the side opposite to angle C. The meanings of the various functions of angles can be explained by the aid of a right-angled triangle.

The following relation exists between the angular functions of the two acute angles in a right-angled triangle: The sine of angle B equals the cosine of angle C; the tangent of angle B equals the cotangent of angle C, and *vice versa*. The sum of the two acute angles in a right-angled triangle always equals 90 degrees; hence, when one angle is known, the other can easily be found. When any two angles together make 90 degrees, one is called the *complement* of the other, and in that case the sine of the one equals the cosine of the other, and the tangent of the one equals the cotangent of the other.

Graphic Illustrations of the Functions of Angles. — Draw a circle with a radius 1, as shown in the accompanying illustration. Draw two lines AB and CD at right angles to each other through the center of the circle; then draw a line OE from the center O forming an angle α with line OD. The sine for this angle is represented by line FG, drawn from point F where OE intersects the periphery of the circle, at right angles to CD. The cosine of angle α is represented by line OG. The tangent of the angle is represented by line DH drawn from point D at right angles to CD. The cotangent is represented by line AK drawn from point A at right angles to AB. The secant is represented by OH, and the cosecant by OK.



In graphically illustrating the functions of angles, it is assumed that all distances measured in the horizontal direction to the right of line AB are positive. Those measured horizontally to the left of AB are negative. All distances measured vertically, are positive above line CD and negative below it. It can then be readily seen that the sine is positive for all angles less than 180 degrees. For angles larger than 180 degrees, the sine would be measured below CD, and is negative. The cosine is positive up to 90 degrees, but for angles larger than 90 but less than 270 degrees, the cosine is measured to the left of line AB and is negative.

On the next page a table "Changes in Value and Sign of Trigonometrical Functions" is given. This table is arranged to show directly whether the function of any given angle is positive or negative. It also gives the limits between which the numerical values of the function vary. For example, it will be seen from the table that the cosine of an angle between 90 and 180 degrees is negative, and that its value will be somewhere between 0 and − 1. In the same way, the cotangent of an angle between 180 and 270 degrees is positive and has a value between infinity and 0; in other words, the cotangent for 180 degrees is infinitely large and then the cotangent gradually decreases for increasing angles, so that the cotangent for 270 degrees equals 0.

Tables of Trigonometric Functions. — The numerical values for the natural or trigonometric functions for all degrees and minutes are given in the tables in the following. When the sine, cosine, tangent, etc., of an angle between 0 and 45 degrees is to be found, the given number of degrees is found at the top of the table and the given number of minutes in the extreme left-hand column. Then read off the figures opposite the given number of minutes, in the columns headed sine, cosine, tangent, etc.

When the sine, cosine, etc., of an angle between 45 and 90 degrees is to be found, the number of degrees is found at the bottom of the table and the number of minutes in the extreme right-hand column. Then read off the required function opposite the number of minutes in the column marked with the required function at the *bottom*. The functions for angles greater than 90 degrees are also found from these

Changes in Value and Sign of Trigonometrical Functions

Function	Between 0° and 90°	Between 90° and 180°	Between 180° and 270°	Between 270° and 360°
Sine	Positive From 0 to 1	Positive From 1 to 0	Negative From 0 to − 1	Negative From − 1 to 0
Cosine	Positive From 1 to 0	Negative From 0 to − 1	Negative From − 1 to 0	Positive From 0 to 1
Tangent	Positive From 0 to ∞	Negative From ∞ to 0	Positive From 0 to ∞	Negative From ∞ to 0
Cotangent	Positive From ∞ to 0	Negative From 0 to ∞	Positive From ∞ to 0	Negative From 0 to ∞
Secant	Positive From 1 to ∞	Negative From ∞ to − 1	Negative From − 1 to ∞	Positive From ∞ to 1
Cosecant	Positive From ∞ to 1	Positive From 1 to ∞	Negative From ∞ to − 1	Negative From − 1 to ∞

tables, as indicated by the numbers in the upper right-hand and lower left-hand corners of the tables. For angles between 90 and 135 degrees, the minutes are read in the extreme left-hand column but the functions are read at the bottom of the table. For angles between 135 and 180 degrees, the minutes are read in the extreme right-hand column and the functions at the head of the table.

The sine is positive for all angles up to 180 degrees. The cosine, tangent and cotangent for angles between 90 and 180 degrees, while they have the same numerical values as for angles from 0 to 90 degrees, are negative. These should be preceded by a minus sign; thus tan 123 degrees 20 minutes = − 1.5204.

Formulas for the solution of right-angled and oblique-angled triangles, arranged in tabular form, are given on the following pages.

Measuring Angles in Radians. — While in practical work angles are always measured in degrees and minutes, the system for measuring angles in what is termed *circular measure* is often employed in theoretical investigations and in formulas relating to revolving bodies. In this system the unit of measurement is the *radian*, that is, the angle at the center of a circle which embraces an arc equal in length to the length of the radius. The value of the radian in degrees equals $180 \div \pi = 57.2958$ degrees. In this measurement, then, π denotes an angle of 180 degrees, and $\pi \div 2$, an angle of 90 degrees.

It is especially convenient to measure angles in radians when dealing with angular velocity. If ω = angular velocity per second of the revolving body, in radians; v = velocity of a point on the periphery of the body, in feet per second; and r = the radius, in feet, then:

$$\omega = \frac{v}{r}$$

For example, assume that the velocity of a point on the periphery is 20 feet per second and the radius, 2 feet. Then the angular velocity is found as below:

$$\omega = \frac{20}{2} = 10 \text{ radians.}$$

The simple manner in which the relation between the angular velocity, the linear velocity, and the radius or diameter of the revolving body can be expressed, is the reason for using the radian as a unit of angular measurement.

The Law of Sines. — In a triangle, any side is to any other side as the sine of the angle opposite the first side is to the sine of the angle opposite the other side; or, if a and b be the sides, and A and B the angles opposite them:

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$

The Law of Cosines. — In a triangle, the square of any side is equal to the sum of the squares of the other two sides minus twice their product times the cosine of the included angle; or if a, b and c be the sides and the angle opposite side a be denoted A, then:

$$a^2 = b^2 + c^2 - 2\,bc \cos A$$

These two laws, together with the proposition that the sum of the three angles equals 180 degrees, are the basis of all formulas relating to the solution of triangles.

Use of Tables of Squares in Solving Right-angled Triangles. — The tables of squares at the beginning of the book may be used to advantage in solving right-angled triangles. Assume that the sides including the right angle are known, and that they are $1\frac{5}{8}$ and $1\frac{7}{8}$ inch, respectively. Find the side opposite the right angle.

$$\text{Side to be found} = \sqrt{1.625^2 + 1.875^2}$$

From tables of squares:

$$\begin{aligned} 1.625^2 &= 2.640625 \\ 1.875^2 &= \underline{3.515625} \\ & 6.156250 \end{aligned}$$

By looking up the figures 615 in the number column in the tables, and finding the square root, we get the figures 24.799. The square root of 616 is 24.819. Hence, by estimating, the square root of $615.625 = 24.812$. As we want the square root of 6.15625, move the decimal point in the root one step to the left; then $\sqrt{6.15625} = 2.4812$. This is the length of the side opposite the right angle.

Important Trigonometric Formulas

$$\sin^2 A + \cos^2 A = 1 \qquad \tan A = \frac{\sin A}{\cos A} = \frac{1}{\cot A}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{1}{\tan A} \qquad \sec A = \frac{1}{\cos A} \qquad \operatorname{cosec} A = \frac{1}{\sin A}$$

$$\sin A = \sqrt{1 - \cos^2 A} = \frac{\tan A}{\sqrt{1 + \tan^2 A}} = \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$\cos A = \sqrt{1 - \sin^2 A} = \frac{1}{\sqrt{1 + \tan^2 A}} = \frac{\cot A}{\sqrt{1 + \cot^2 A}}$$

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cot (A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A} \qquad \cot (A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\tan A + \tan B = \frac{\sin (A + B)}{\cos A \cos B} \qquad \tan A - \tan B = \frac{\sin (A - B)}{\cos A \cos B}$$

$$\cot A + \cot B = \frac{\sin (B + A)}{\sin A \sin B} \qquad \cot A - \cot B = \frac{\sin (B - A)}{\sin A \sin B}$$

$$\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin (A + B) \sin (A - B)$$

$$\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos (A + B) \cos (A - B).$$

$$\sin A \sin B = \tfrac{1}{2} \cos (A - B) - \tfrac{1}{2} \cos (A + B).$$

$$\cos A \cos B = \tfrac{1}{2} \cos (A - B) + \tfrac{1}{2} \cos (A + B)$$

$$\sin A \cos B = \tfrac{1}{2} \sin (A + B) + \tfrac{1}{2} \sin (A - B)$$

$$\tan A \tan B = \frac{\tan A + \tan B}{\cot A + \cot B} \qquad \cot A \cot B = \frac{\cot A + \cot B}{\tan A + \tan B}$$

$$\sin A = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A \qquad \sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$$

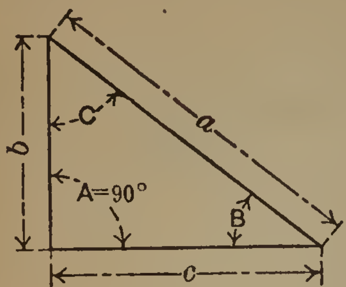
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2}{\cot A - \tan A}$$

$$\cot 2A = \frac{\cot^2 A - 1}{2 \cot A} = \frac{\cot A - \tan A}{2}$$

$$\sin A = \frac{2 \tan \tfrac{1}{2} A}{1 + \tan^2 \tfrac{1}{2} A} \qquad \cos A = \frac{1 - \tan^2 \tfrac{1}{2} A}{1 + \tan^2 \tfrac{1}{2} A}$$

$$2 \sin^2 A = 1 - \cos 2A \qquad 2 \cos^2 A = 1 + \cos 2A$$

Solution of Right-angled Triangles

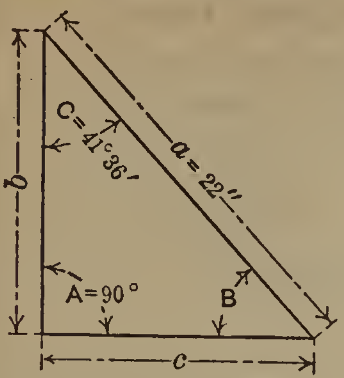


As shown in the illustration, the sides of the right-angled triangle are designated a, b and c. The angles opposite each of these sides are designated A, B and C, respectively.

Angle A, opposite the hypotenuse a is the right angle, and is therefore always one of the known quantities.

Sides and Angles Known	Formulas for Sides and Angles to be Found		
Sides a and b..........	$c = \sqrt{a^2 - b^2}$	$\sin B = \frac{b}{a}$	$C = 90° - B$
Sides a and c..........	$b = \sqrt{a^2 - c^2}$	$\sin C = \frac{c}{a}$	$B = 90° - C$
Sides b and c..........	$a = \sqrt{b^2 + c^2}$	$\tan B = \frac{b}{c}$	$C = 90° - B$
Side a; angle B........	$b = a \times \sin B$	$c = a \times \cos B$	$C = 90° - B$
Side a; angle C........	$b = a \times \cos C$	$c = a \times \sin C$	$B = 90° - C$
Side b; angle B........	$a = \frac{b}{\sin B}$	$c = b \times \cot B$	$C = 90° - B$
Side b; angle C........	$a = \frac{b}{\cos C}$	$c = b \times \tan C$	$B = 90° - C$
Side c; angle B........	$a = \frac{c}{\cos B}$	$b = c \times \tan B$	$C = 90° - B$
Side c; angle C........	$a = \frac{c}{\sin C}$	$b = c \times \cot C$	$B = 90° - C$

Examples of the Solution of Right-angled Triangles



Sides and angles known:

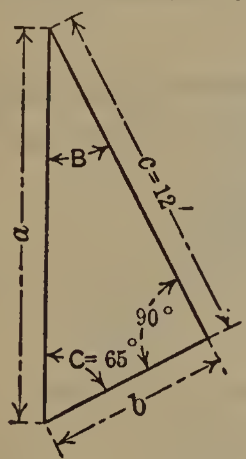
$$a = 22 \text{ inches}; \quad C = 41^\circ\ 36'.$$

Then, by the formulas given on the preceding page:

$$b = a \times \cos C = 22 \times \cos 41^\circ\ 36' = 22 \times 0.74780$$
$$= 16.4516 \text{ inches.}$$

$$c = a \times \sin C = 22 \times \sin 41^\circ\ 36' - 22 \times 0.66393$$
$$= 14.6065 \text{ inches.}$$

$$B = 90^\circ - 41^\circ\ 36' = 48^\circ\ 24'.$$



Sides and angles known:

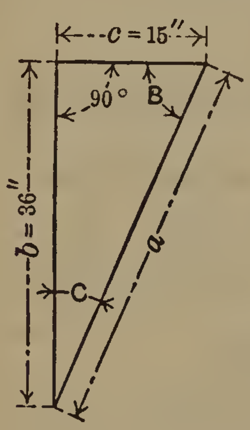
$$c = 12 \text{ feet}; \quad C = 65^\circ.$$

Then, by the formulas given on the preceding page:

$$a = \frac{c}{\sin C} = \frac{12}{\sin 65^\circ} = \frac{12}{0.90631} = 13.2405 \text{ feet.}$$

$$b = c \times \cot C = 12 \times \cot 65^\circ = 12 \times 0.46631$$
$$= 5.5957 \text{ feet.}$$

$$B = 90^\circ - 65^\circ = 25^\circ.$$



Sides known:

$$b = 36 \text{ inches}; \quad c = 15 \text{ inches.}$$

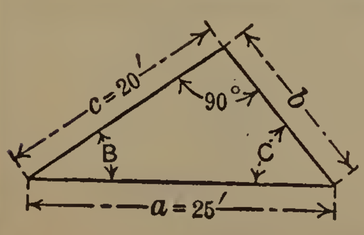
Then, by the formulas given on the preceding page:

$$a = \sqrt{b^2 + c^2} = \sqrt{36^2 + 15^2} = \sqrt{1296 + 225}$$
$$= \sqrt{1521} = 39 \text{ inches.}$$

$$\tan B = \frac{b}{c} = \frac{36}{15} = 2.4$$

Hence, $B = 67^\circ\ 23'$.

$$C = 90^\circ - 67^\circ\ 23' = 22^\circ\ 37'.$$



Sides known:

$$a = 25 \text{ feet}; \quad c = 20 \text{ feet.}$$

From the formulas on the preceding page:

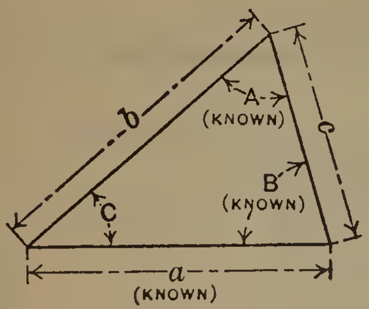
$$b = \sqrt{a^2 - c^2} = \sqrt{25^2 - 20^2} = \sqrt{625 - 400}$$
$$= \sqrt{225} = 15 \text{ feet.}$$

$$\sin C = \frac{c}{a} = \frac{20}{25} = 0.8$$

Hence, $C = 53^\circ\ 8'$.

$$B = 90^\circ - 53^\circ\ 8' = 36^\circ\ 52'.$$

Solution of Oblique-angled Triangles



One side and two angles known.

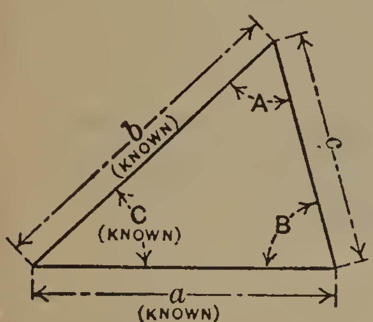
Call the known side a, the angle opposite it A, and the other known angle B. Then:

$$C = 180^\circ - (A + B)$$

$$b = \frac{a \times \sin B}{\sin A} \qquad c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

If angles B and C are given, but not A, then $A = 180^\circ - (B + C)$, the other formulas being the same.



Two sides and the angle between them known.

Call the known sides a and b, and the known angle between them C. Then:

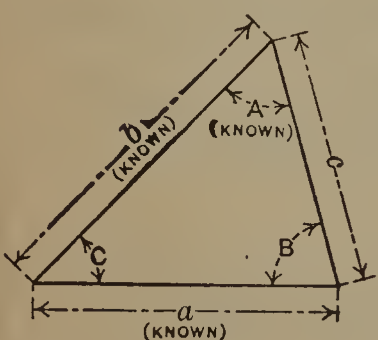
$$\tan A = \frac{a \times \sin C}{b - a \times \cos C}$$

$$B = 180^\circ - (A + C) \qquad c = \frac{a \times \sin C}{\sin A}$$

Side c may also be found directly as below:

$$c = \sqrt{a^2 + b^2 - 2\,ab \times \cos C}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$



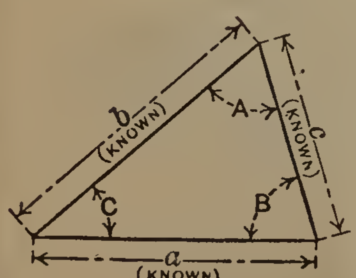
Two sides and the angle opposite one of the sides known.

Call the known angle A, the side opposite it a, and the other known side b. Then:

$$\sin B = \frac{b \times \sin A}{a} \qquad C = 180^\circ - (A + B)$$

$$c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$



All three sides known.

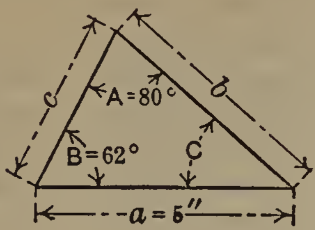
Call the sides a, b and c, and the angles opposite them, A, B and C. Then:

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}$$

$$\sin B = \frac{b \times \sin A}{a} \qquad C = 180^\circ - (A + B)$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

Examples of the Solution of Oblique-angled Triangles



Sides and angles known:

$$a = 5 \text{ inches};\; A = 80°;\; B = 62°.$$

Then, by the formulas on the opposite page:

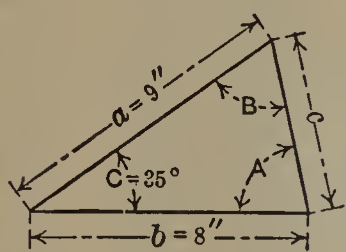
$$C = 180° - (80° + 62°) = 180° - 142° = 38°.$$

$$b = \frac{a \times \sin B}{\sin A} = \frac{5 \times \sin 62°}{\sin 80°} = \frac{5 \times 0.88295}{0.98481} = 4.483$$

inches.

$$c = \frac{a \times \sin C}{\sin A} = \frac{5 \times \sin 38°}{\sin 80°} = \frac{5 \times 0.61566}{0.98481} = 3.126$$

inches.



Sides and angles known:

$$a = 9 \text{ inches};\; b = 8 \text{ inches};\; C = 35°.$$

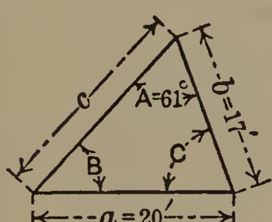
$$\tan A = \frac{a \times \sin C}{b - a \times \cos C} = \frac{9 \times \sin 35°}{8 - 9 \times \cos 35°}$$

$$= \frac{9 \times 0.57358}{8 - 9 \times 0.81915} = \frac{5.16222}{0.62765} = 8.22468.$$

Hence, $A = 83°\ 4'.$

$$B = 180° - (A + C) = 180° - 118°\ 4' = 61°\ 56'.$$

$$c = \frac{a \times \sin C}{\sin A} = \frac{9 \times 0.57358}{0.99269} = 5.2 \text{ inches}.$$



Sides and angles known:

$$a = 20 \text{ feet};\; b = 17 \text{ feet};\; A = 61°.$$

$$\sin B = \frac{b \times \sin A}{a} = \frac{17 \times \sin 61°}{20}$$

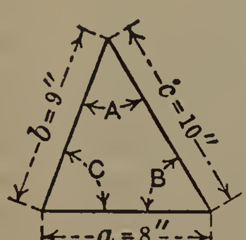
$$= \frac{17 \times 0.87462}{20} = 0.74343.$$

Hence, $B = 48°\ 1'.$

$$C = 180° - (A + B) = 180° - 109°\ 1' = 70°\ 59'.$$

$$c = \frac{a \times \sin C}{\sin A} = \frac{20 \times \sin 70°\ 59'}{\sin 61°} = \frac{20 \times 0.94542}{0.87462}$$

$$= 21.62 \text{ feet}.$$



Sides known:

$$a = 8 \text{ inches};\; b = 9 \text{ inches};\; c = 10 \text{ inches}.$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc} = \frac{9^2 + 10^2 - 8^2}{2 \times 9 \times 10}$$

$$= \frac{81 + 100 - 64}{180} = \frac{117}{180} = 0.65000.$$

Hence, $A = 49°\ 27'.$

$$\sin B = \frac{b \times \sin A}{a} = \frac{9 \times 0.75984}{8} = 0.85482.$$

Hence, $B = 58°\ 44'.$

$$C = 180° - (A + B) = 180° - 108°\ 11' = 71°\ 49'.$$

Table for Converting Minutes into Decimals of a Degree

Min.	Dec. of Degree	Min.	Dec. of Degree	Min.	Dec. of Degree	Min.	Dec. of Degree	Min.	Dec. of Degree
¼	0.00416	12¼	0.20416	24¼	0.40416	36¼	0.60416	48¼	0.80416
½	0.00833	12½	0.20833	24½	0.40833	36½	0.60833	48½	0.80833
¾	0.01250	12¾	0.21250	24¾	0.41250	36¾	0.61250	48¾	0.81250
1	0.01666	13	0.21666	25	0.41666	37	0.61666	49	0.81666
1¼	0.02083	13¼	0.22083	25¼	0.42083	37¼	0.62083	49¼	0.82083
1½	0.02500	13½	0.22500	25½	0.42500	37½	0.62500	49½	0.82500
1¾	0.02916	13¾	0.22916	25¾	0.42916	37¾	0.62916	49¾	0.82916
2	0.03333	14	0.23333	26	0.43333	38	0.63333	50	0.83333
2¼	0.03750	14¼	0.23750	26¼	0.43750	38¼	0.63750	50¼	0.83750
2½	0.04166	14½	0.24166	26½	0.44166	38½	0.64166	50½	0.84166
2¾	0.04583	14¾	0.24583	26¾	0.44583	38¾	0.64583	50¾	0.84583
3	0.05000	15	0.25000	27	0.45000	39	0.65000	51	0.85000
3¼	0.05416	15¼	0.25416	27¼	0.45416	39¼	0.65416	51¼	0.85416
3½	0.05833	15½	0.25833	27½	0.45833	39½	0.65833	51½	0.85833
3¾	0.06250	15¾	0.26250	27¾	0.46250	39¾	0.66250	51¾	0.86250
4	0.06666	16	0.26666	28	0.46666	40	0.66666	52	0.86666
4¼	0.07083	16¼	0.27083	28¼	0.47083	40¼	0.67083	52¼	0.87083
4½	0.07500	16½	0.27500	28½	0.47500	40½	0.67500	52½	0.87500
4¾	0.07916	16¾	0.27916	28¾	0.47916	40¾	0.67916	52¾	0.87916
5	0.08333	17	0.28333	29	0.48333	41	0.68333	53	0.88333
5¼	0.08750	17¼	0.28750	29¼	0.48750	41¼	0.68750	53¼	0.88750
5½	0.09166	17½	0.29166	29½	0.49166	41½	0.69166	53½	0.89166
5¾	0.09583	17¾	0.29583	29¾	0.49583	41¾	0.69583	53¾	0.89583
6	0.10000	18	0.30000	30	0.50000	42	0.70000	54	0.90000
6¼	0.10416	18¼	0.30416	30¼	0.50416	42¼	0.70416	54¼	0.90416
6½	0.10833	18½	0.30833	30½	0.50833	42½	0.70833	54½	0.90833
6¾	0.11250	18¾	0.31250	30¾	0.51250	42¾	0.71250	54¾	0.91250
7	0.11666	19	0.31666	31	0.51666	43	0.71666	55	0.91666
7¼	0.12083	19¼	0.32083	31¼	0.52083	43¼	0.72083	55¼	0.92083
7½	0.12500	19½	0.32500	31½	0.52500	43½	0.72500	55½	0.92500
7¾	0.12916	19¾	0.32916	31¾	0.52916	43¾	0.72916	55¾	0.92916
8	0.13333	20	0.33333	32	0.53333	44	0.73333	56	0.93333
8¼	0.13750	20¼	0.33750	32¼	0.53750	44¼	0.73750	56¼	0.93750
8½	0.14166	20½	0.34166	32½	0.54166	44½	0.74166	56½	0.94166
8¾	0.14583	20¾	0.34583	32¾	0.54583	44¾	0.74583	56¾	0.94583
9	0.15000	21	0.35000	33	0.55000	45	0.75000	57	0.95000
9¼	0.15416	21¼	0.35416	33¼	0.55416	45¼	0.75416	57¼	0.95416
9½	0.15833	21½	0.35833	33½	0.55833	45½	0.75833	57½	0.95833
9¾	0.16250	21¾	0.36250	33¾	0.56250	45¾	0.76250	57¾	0.96250
10	0.16666	22	0.36666	34	0.56666	46	0.76666	58	0.96666
10¼	0.17083	22¼	0.37083	34¼	0.57083	46¼	0.77083	58¼	0.97083
10½	0.17500	22½	0.37500	34½	0.57500	46½	0.77500	58½	0.97500
10¾	0.17916	22¾	0.37916	34¾	0.57916	46¾	0.77916	58¾	0.97916
11	0.18333	23	0.38333	35	0.58333	47	0.78333	59	0.98333
11¼	0.18750	23¼	0.38750	35¼	0.58750	47¼	0.78750	59¼	0.98750
11½	0.19166	23½	0.39166	35½	0.59166	47½	0.79166	59½	0.99166
11¾	0.19583	23¾	0.39583	35¾	0.59583	47¾	0.79583	59¾	0.99583
12	0.20000	24	0.40000	36	0.60000	48	0.80000	60	1.00000

0° **Natural Trigonometric Functions** **179°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.00000	1.0000	0.00000	Infinite	1.0000	Infinite	0.00000	1.00000	60
1	.00029	.0000	.00029	3437.7	.0000	3437.7	.00000	0.99971	59
2	.00058	.0000	.00058	1718.9	.0000	1718.9	.00000	.99942	58
3	.00087	.0000	.00087	1145.9	.0000	1145.9	.00000	.99913	57
4	.00116	.0000	.00116	859.44	.0000	859.44	.00000	.99884	56
5	0.00145	1.0000	0.00145	687.55	1.0000	687.55	0.00000	0.99854	55
6	.00174	.0000	.00174	572.96	.0000	572.96	.00000	.99825	54
7	.00204	.0000	.00204	491.11	.0000	491.11	.00000	.99796	53
8	.00233	.0000	.00233	429.72	.0000	429.72	.00000	.99767	52
9	.00262	.0000	.00262	381.97	.0000	381.97	.00000	.99738	51
10	0.00291	0.99999	0.00291	343.77	1.0000	343.77	0.00000	0.99709	50
11	.00320	.99999	.00320	312.52	.0000	312.52	.00000	.99680	49
12	.00349	.99999	.00349	286.48	.0000	286.48	.00001	.99651	48
13	.00378	.99999	.00378	264.44	.0000	264.44	.00001	.99622	47
14	.00407	.99999	.00407	245.55	.0000	245.55	.00001	.99593	46
15	0.00436	0.99999	0.00436	229.18	1.0000	229.18	0.00001	0.99564	45
16	.00465	.99999	.00465	214.86	.0000	214.86	.00001	.99534	44
17	.00494	.99999	.00494	202.22	.0000	202.22	.00001	.99505	43
18	.00524	.99999	.00524	190.98	.0000	190.99	.00001	.99476	42
19	.00553	.99998	.00553	180.93	.0000	180.93	.00001	.99447	41
20	0.00582	0.99998	0.00582	171.88	1.0000	171.89	0.00002	0.99418	40
21	.00611	.99998	.00611	163.70	.0000	163.70	.00002	.99389	39
22	.00640	.99998	.00640	156.26	.0000	156.26	.00002	.99360	38
23	.00669	.99998	.00669	149.46	.0000	149.47	.00002	.99331	37
24	.00698	.99997	.00698	143.24	.0000	143.24	.00002	.99302	36
25	0.00727	0.99997	0.00727	137.51	1.0000	137.51	0.00003	0.99273	35
26	.00756	.99997	.00756	132.22	.0000	132.22	.00003	.99244	34
27	.00785	.99997	.00785	127.32	.0000	127.32	.00003	.99215	33
28	.00814	.99997	.00814	122.77	.0000	122.78	.00003	.99185	32
29	.00843	.99996	.00844	118.54	.0000	118.54	.00003	.99156	31
30	0.00873	0.99996	0.00873	114.59	1.0000	114.59	0.00004	0.99127	30
31	.00902	.99996	.00902	110.89	.0000	110.90	.00004	.99098	29
32	.00931	.99996	.00931	107.43	.0000	107.43	.00004	.99069	28
33	.00960	.99995	.00960	104.17	.0000	104.17	.00005	.99040	27
34	.00989	.99995	.00989	101.11	.0000	101.11	.00005	.99011	26
35	0.01018	0.99995	0.01018	98.218	1.0000	98.223	0.00005	0.98982	25
36	.01047	.99994	.01047	95.489	.0000	95.495	.00005	.98953	24
37	.01076	.99994	.01076	92.908	.0000	92.914	.00006	.98924	23
38	.01105	.99994	.01105	90.463	.0001	90.469	.00006	.98895	22
39	.01134	.99993	.01134	88.143	.0001	88.149	.00006	.98865	21
40	0.01163	0.99993	0.01164	85.940	1.0001	85.946	0.00007	0.98836	20
41	.01193	.99993	.01193	83.843	.0001	83.849	.00007	.98807	19
42	.01222	.99992	.01222	81.847	.0001	81.853	.00007	.98778	18
43	.01251	.99992	.01251	79.943	.0001	79.950	.00008	.98749	17
44	.01280	.99992	.01280	78.126	.0001	78.133	.00008	.98720	16
45	0.01309	0.99991	0.01309	76.390	1.0001	76.396	0.00008	0.98691	15
46	.01338	.99991	.01338	74.729	.0001	74.736	.00009	.98662	14
47	.01367	.99991	.01367	73.139	.0001	73.146	.00009	.98633	13
48	.01396	.99990	.01396	71.615	.0001	71.622	.00010	.98604	12
49	.01425	.99990	.01425	70.153	.0001	70.160	.00010	.98575	11
50	0.01454	0.99989	0 01454	68.750	1.0001	68.757	0.00010	0.98546	10
51	.01483	.99989	.01484	67.402	.0001	67.409	.00011	.98516	9
52	.01512	.99988	.01513	66.105	.0001	66.113	.00011	.98487	8
53	.01542	.99988	.01542	64.858	.0001	64.866	.00012	.98458	7
54	.01571	.99988	.01571	63.657	.0001	63.664	.00012	.98429	6
55	0.01600	0.99987	0.01600	62.499	1.0001	62.507	0.00013	0.98400	5
56	.01629	.99987	.01629	61.383	.0001	61.391	.00013	.98371	4
57	.01658	.99987	.01658	60.306	.0001	60.314	.00014	.98342	3
58	.01687	.99986	.01687	59.266	.0001	59.274	.00014	.98313	2
59	.01716	.99985	.01716	58.261	.0001	58.270	.00015	.98284	1
60	0.01745	0.99985	0.01745	57.290	1.0001	57.299	0.00015	0.98255	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

90° **89°**

1° **Natural Trigonometric Functions** 178°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.01745	0.99985	0.01745	57.290	1.0001	57.299	0.00015	0.98255	60
1	.01774	.99984	.01775	56.350	.0001	56.359	.00016	.98226	59
2	.01803	.99984	.01804	55.441	.0001	55.450	.00016	.98196	58
3	.01832	.99983	.01833	54.561	.0002	54.570	.00017	.98167	57
4	.01861	.99983	.01862	53.708	.0002	53.718	.00017	.98138	56
5	0.01891	0.99982	0.01891	52.882	1.0002	52.891	0.00018	0.98109	55
6	.01920	.99981	.01920	52.081	.0002	52.090	.00018	.98080	54
7	.01949	.99981	.01949	51.303	.0002	51.313	.00019	.98051	53
8	.01978	.99980	.01978	50.548	.0002	50.558	.00019	.98022	52
9	.02007	.99980	.02007	49.816	.0002	49.826	.00020	.97993	51
10	0.02036	0.99979	0.02036	49.104	1.0002	49.114	0.00021	0.97964	50
11	.02065	.99979	.02066	48.412	.0002	48.422	.00021	.97935	49
12	.02094	.99978	.02095	47.739	.0002	47.750	.00022	.97906	48
13	.02123	.99977	.02124	47.085	.0002	47.096	.00022	.97877	47
14	.02152	.99977	.02153	46.449	.0002	46.460	.00023	.97847	46
15	0.02181	0.99976	0.02182	45.829	1.0002	45.840	0.00024	0.97818	45
16	.02210	.99975	.02211	45.226	.0002	45.237	.00024	.97789	44
17	.02240	.99975	.02240	44.638	.0002	44.650	.00025	.97760	43
18	.02269	.99974	.02269	44.066	.0002	44.077	.00026	.97731	42
19	.02298	.99974	.02298	43.508	.0003	43.520	.00026	.97702	41
20	0.02326	0.99973	0.02327	42.964	1.0003	42.976	0.00027	0.97673	40
21	.02356	.99972	.02357	42.433	.0003	42.445	.00028	.97644	39
22	.02385	.99971	.02386	41.916	.0003	41.928	.00028	.97615	38
23	.02414	.99971	.02415	41.410	.0003	41.423	.00029	.97586	37
24	.02443	.99970	.02444	40.917	.0003	40.930	.00030	.97557	36
25	0.02472	0.99969	0.02473	40.436	1.0003	40.448	0.00030	0.97528	35
26	.02501	.99969	.02502	39.965	.0003	39.978	.00031	.97499	34
27	.02530	.99968	.02531	39.506	.0003	39.518	.00032	.97469	33
28	.02559	.99967	.02560	39.057	.0003	39.069	.00033	.97440	32
29	.02589	.99966	.02589	38.618	.0003	38.631	.00033	.97411	31
30	0.02618	0.99966	0.02618	38.188	1.0003	38.201	0.00034	0.97382	30
31	.02647	.99965	.02648	37.769	.0003	37.782	.00035	.97353	29
32	.02676	.99964	.02677	37.358	.0003	37.371	.00036	.97324	28
33	.02705	.99963	.02706	36.956	.0004	36.969	.00036	.97295	27
34	.02734	.99963	.02735	36.563	.0004	36.576	.00037	.97266	26
35	0.02763	0.99962	0.02764	36.177	1.0004	36.191	0.00038	0.97237	25
36	.02792	.99961	.02793	35.800	.0004	35.814	.00039	.97208	24
37	.02821	.99960	.02822	35.431	.0004	35.445	.00040	.97179	23
38	.02850	.99959	.02851	35.069	.0004	35.084	.00041	.97150	22
39	.02879	.99958	.02880	34.715	.0004	34.729	.00041	.97121	21
40	0.02908	0.99958	0.02910	34.368	1.0004	34.382	0.00042	0.97091	20
41	.02937	.99957	.02939	34.027	.0004	34.042	.00043	.97062	19
42	.02967	.99956	.02968	33.693	.0004	33.708	.00044	.97033	18
43	.02996	.99955	.02997	33.366	.0004	33.381	.00045	.97004	17
44	.03025	.99954	.03026	33.045	.0004	33.060	.00046	.96975	16
45	0.03054	0.99953	0.03055	32.730	1.0005	32.745	0.00046	0.96946	15
46	.03083	.99952	.03084	32.421	.0005	32.437	.00047	.96917	14
47	.03112	.99951	.03113	32.118	.0005	32.134	.00048	.96888	13
48	.03141	.99951	.03143	31.820	.0005	31.836	.00049	.96859	12
49	.03170	.99950	.03172	31.528	.0005	31.544	.00050	.96830	11
50	0.03199	0.99949	0.03201	31.241	1.0005	31.257	0.00051	0.96801	10
51	.03228	.99948	.03230	30.960	.0005	30.976	.00052	.96772	9
52	.03257	.99947	.03259	30.683	.0005	30.699	.00053	.96743	8
53	.03286	.99946	.03288	30.411	.0005	30.428	.00054	.96713	7
54	.03315	.99945	.03317	30.145	.0005	30.161	.00055	.96684	6
55	0.03344	0.99944	0.03346	29.882	1.0005	29.899	0.00056	0.96655	5
56	.03374	.99943	.03375	29.624	.0006	29.641	.00057	.96626	4
57	.03403	.99942	.03405	29.371	.0006	29.388	.00058	.96597	3
58	.03432	.99941	.03434	29.122	.0006	29.139	.00059	.96568	2
59	.03461	.99940	.03463	28.877	.0006	28.894	.00060	.96539	1
60	0.03490	0.99939	0.03492	28.636	1.0006	28.654	0.00061	0.96510	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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2° **Natural Trigonometric Functions** **177°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.03490	0.99939	0.03492	28.636	1.0006	28.654	0.00061	0.96510	60
1	.03519	.99938	.03521	28.399	.0006	28.417	.00062	.96481	59
2	.03548	.99937	.03550	28.166	.0006	28.184	.00063	.96452	58
3	.03577	.99936	.03579	27.937	.0006	27.955	.00064	.96423	57
4	.03606	.99935	.03608	27.712	.0006	27.730	.00065	.96394	56
5	0.03635	0.99934	0.03638	27.490	1.0007	27.508	0.00066	0.96365	55
6	.03664	.99933	.03667	27.271	.0007	27.290	.00067	.96336	54
7	.03693	.99932	.03696	27.056	.0007	27.075	.00068	.96306	53
8	.03722	.99931	.03725	26.845	.0007	26.864	.00069	.96277	52
9	.03751	.99930	.03754	26.637	.0007	26.655	.00070	.96248	51
10	0.03781	0.99928	0.03783	26.432	1.0007	26.450	0.00071	0.96219	50
11	.03810	.99927	.03812	26.230	.0007	26.249	.00073	.96190	49
12	.03839	.99926	.03842	26.031	.0007	26.050	.00074	.96161	48
13	.03868	.99925	.03871	25.835	.0007	25.854	.00075	.96132	47
14	.03897	.99924	.03900	25.642	.0008	25.661	.00076	.96103	46
15	0.03926	0.99923	0.03929	25.452	1.0008	25.471	0.00077	0.96074	45
16	.03955	.99922	.03958	25.264	.0008	25.284	.00078	.96045	44
17	.03984	.99921	.03987	25.080	.0008	25.100	.00079	.96016	43
18	.04013	.99919	.04016	24.898	.0008	24.918	.00080	.95987	42
19	.04042	.99918	.04045	24.718	.0008	24.739	.00082	.95958	41
20	0.04071	0.99917	0.04075	24.542	1.0008	24.562	0.00083	0.95929	40
21	.04100	.99916	.04104	24.367	.0008	24.388	.00084	.95900	39
22	.04129	.99915	.04133	24.196	.0008	24.216	.00085	.95870	38
23	.04158	.99913	.04162	24.026	.0009	24.047	.00086	.95841	37
24	.04187	.99912	.04191	23.859	.0009	23.880	.00088	.95812	36
25	0.04217	0.99911	0.04220	23.694	1.0009	23.716	0.00089	0.95783	35
26	.04246	.99910	.04249	23.532	.0009	23.553	.00090	.95754	34
27	.04275	.99908	.04279	23.372	.0009	23.393	.00091	.95725	33
28	.04304	.99907	.04308	23.214	.0009	23.235	.00093	.95696	32
29	.04333	.99906	.04337	23.058	.0009	23.079	.00094	.95667	31
30	0.04362	0.99905	0.04366	22.904	1.0009	22.925	0.00095	0.95638	30
31	.04391	.99903	.04395	22.752	.0010	22.774	.00096	.95609	29
32	.04420	.99902	.04424	22.602	.0010	22.624	.00098	.95580	28
33	.04449	.99901	.04453	22.454	.0010	22.476	.00099	.95551	27
34	.04478	.99900	.04483	22.308	.0010	22.330	.00100	.95522	26
35	0.04507	0.99898	0.04512	22.164	1.0010	22.186	0.00102	0.95493	25
36	.04536	.99897	.04541	22.022	.0010	22.044	.00103	.95464	24
37	.04565	.99896	.04570	21.881	.0010	21.904	.00104	.95435	23
38	.04594	.99894	.04599	21.742	.0010	21.765	.00106	.95405	22
39	.04623	.99893	.04628	21.606	.0011	21.629	.00107	.95376	21
40	0.04652	0.99892	0.04657	21.470	1.0011	21.494	0.00108	0.95347	20
41	.04681	.99890	.04687	21.337	.0011	21.360	.00110	.95318	19
42	.04711	.99889	.04716	21.205	.0011	21.228	.00111	.95289	18
43	.04740	.99888	.04745	21.075	.0011	21.098	.00112	.95260	17
44	.04769	.99886	.04774	20.946	.0011	20.970	.00114	.95231	16
45	0.04798	0.99885	0.04803	20.819	1.0011	20.843	0.00115	0.95202	15
46	.04827	.99883	.04832	20.693	.0012	20.717	.00116	.95173	14
47	.04856	.99882	.04862	20.569	.0012	20.593	.00118	.95144	13
48	.04885	.99881	.04891	20.446	.0012	20.471	.00119	.95115	12
49	.04914	.99879	.04920	20.325	.0012	20.350	.00121	.95086	11
50	0.04943	0.99878	0.04949	20.205	1.0012	20.230	0.00122	0.95057	10
51	.04972	.99876	.04978	20.087	.0012	20.112	.00124	.95028	9
52	.05001	.99875	.05007	19.970	.0012	19.995	.00125	.94999	8
53	.05030	.99873	.05037	19.854	.0013	19.880	.00127	.94970	7
54	.05059	.99872	.05066	19.740	.0013	19.766	.00128	.94941	6
55	0.05088	0.99870	0.05095	19.627	1.0013	19.653	0.00129	0.94912	5
56	.05117	.99869	.05124	19.515	.0013	19.541	.00131	.94883	4
57	.05146	.99867	.05153	19.405	.0013	19.431	.00132	.94853	3
58	.05175	.99866	.05182	19.296	.0013	19.322	.00134	.94824	2
59	.05204	.99864	.05212	19.188	.0013	19.214	.00135	.94795	1
60	0.05234	0.99863	0.05241	19.081	1.0014	19.107	0.00137	0.94766	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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3° **Natural Trigonometric Functions** 176°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.05234	0.99863	0.05241	19.081	1.0014	19.107	0.00137	0.94766	60
1	.05263	.99861	.05270	18.975	.0014	19.002	.00138	.94737	59
2	.05292	.99860	.05299	18.871	.0014	18.897	.00140	.94708	58
3	.05321	.99858	.05328	18.768	.0014	18.794	.00142	.94679	57
4	.05350	.99857	.05357	18.665	.0014	18.692	.00143	.94650	56
5	0.05379	0.99855	0.05387	18.564	1.0014	18.591	0.00145	0.94621	55
6	.05408	.99854	.05416	18.464	.0015	18.491	.00146	.94592	54
7	.05437	.99852	.05445	18.365	.0015	18.393	.00148	.94563	53
8	.05466	.99850	.05474	18.268	.0015	18.295	.00149	.94534	52
9	.05495	.99849	.05503	18.171	.0015	18.198	.00151	.94505	51
10	0.05524	0.99847	0.05532	18.075	1.0015	18.103	0.00153	0.94476	50
11	.05553	.99846	.05562	17.980	.0015	18.008	.00154	.94447	49
12	.05582	.99844	.05591	17.886	.0016	17.914	.00156	.94418	48
13	.05611	.99842	.05620	17.793	.0016	17.821	.00157	.94389	47
14	.05640	.99841	.05649	17.701	.0016	17.730	.00159	.94360	46
15	0.05669	0.99839	0.05678	17.610	1.0016	17.639	0.00161	0.94331	45
16	.05698	.99837	.05707	17.520	.0016	17.549	.00162	.94302	44
17	.05727	.99836	.05737	17.431	.0016	17.460	.00164	.94273	43
18	.05756	.99834	.05766	17.343	.0017	17.372	.00166	.94244	42
19	.05785	.99832	.05795	17.256	.0017	17.285	.00167	.94214	41
20	0.05814	0.99831	0.05824	17.169	1.0017	17.198	0.00169	0.94185	40
21	.05843	.99829	.05853	17.084	.0017	17.113	.00171	.94156	39
22	.05872	.99827	.05883	16.999	.0017	17.028	.00172	.94127	38
23	.05902	.99826	.05912	16.915	.0017	16.944	.00174	.94098	37
24	.05931	.99824	.05941	16.832	.0018	16.861	.00176	.94069	36
25	0.05960	0.99822	0.05970	16.750	1.0018	16.779	0.00178	0.94040	35
26	.05989	.99820	.05999	16.668	.0018	16.698	.00179	.94011	34
27	.06018	.99819	.06029	16.587	.0018	16.617	.00181	.93982	33
28	.06047	.99817	.06058	16.507	.0018	16.538	.00183	.93953	32
29	.06076	.99815	.06087	16.428	.0018	16.459	.00185	.93924	31
30	0.06105	0.99813	0.06116	16.350	1.0019	16.380	0.00186	0.93895	30
31	.06134	.99812	.06145	16.272	.0019	16.303	.00188	.93866	29
32	.06163	.99810	.06175	16.195	.0019	16.226	.00190	.93837	28
33	.06192	.99808	.06204	16.119	.0019	16.150	.00192	.93808	27
34	.06221	.99806	.06233	16.043	.0019	16.075	.00194	.93777	26
35	0.06250	0.99804	0.06262	15.969	1.0019	16.000	0.00195	0.93750	25
36	.06279	.99803	.06291	15.894	.0020	15.926	.00197	.93721	24
37	.06308	.99801	.06321	15.821	.0020	15.853	.00199	.93692	23
38	.06337	.99799	.06350	15.748	.0020	15.780	.00201	.93663	22
39	.06366	.99797	.06379	15.676	.0020	15.708	.00203	.93634	21
40	0.06395	0.99795	0.06408	15.605	1.0020	15.637	0.00205	0.93605	20
41	.06424	.99793	.06437	15.534	.0021	15.566	.00206	.93576	19
42	.06453	.99791	.06467	15.464	.0021	15.496	.00208	.93547	18
43	.06482	.99790	.06496	15.394	.0021	15.427	.00210	.93518	17
44	.06511	.99788	.06525	15.325	.0021	15.358	.00212	.93489	16
45	0.06540	0.99786	0.06554	15.257	1.0021	15.290	0.00214	0.93460	15
46	.06569	.99784	.06583	15.189	.0022	15.222	.00216	.93431	14
47	.06598	.99782	.06613	15.122	.0022	15.155	.00218	.93402	13
48	.06627	.99780	.06642	15.056	.0022	15.089	.00220	.93373	12
49	.06656	.99778	.06671	14.990	.0022	15.023	.00222	.93343	11
50	0.06685	0.99776	0.06700	14.924	1.0022	14.958	0.00224	0.93314	10
51	.06714	.99774	.06730	14.860	.0023	14.893	.00226	.93285	9
52	.06743	.99772	.06759	14.795	.0023	14.829	.00228	.93256	8
53	.06772	.99770	.06788	14.732	.0023	14.765	.00230	.93227	7
54	.06801	.99768	.06817	14.668	.0023	14.702	.00231	.93198	6
55	0.06830	0.99766	0.06846	14.606	1.0023	14.640	0.00233	0.93169	5
56	.06859	.99764	.06876	14.544	.0024	14.578	.00235	.93140	4
57	.06888	.99762	.06905	14.482	.0024	14.517	.00237	.93111	3
58	.06918	.99760	.06934	14.421	.0024	14.456	.00239	.93082	2
59	.06947	.99758	.06963	14.361	.0024	14.395	.00241	.93053	1
60	0.06976	0.99756	0.06993	14.301	1.0024	14.335	0.00243	0.93024	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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4° **Natural Trigonometric Functions** **175°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.06976	0.99756	0.06993	14.301	1.0024	14.335	0.00243	0.93024	60
1	.07005	.99754	.07022	14.241	.0025	14.276	.00246	.92995	59
2	.07034	.99752	.07051	14.182	.0025	14.217	.00248	.92966	58
3	.07063	.99750	.07080	14.123	.0025	14.159	.00250	.92937	57
4	.07092	.99748	.07110	14.065	.0025	14.101	.00252	.92908	56
5	0.07121	0.99746	0.07139	14.008	1.0025	14.043	0.00254	0.92879	55
6	.07150	.99744	.07168	13.951	.0026	13.986	.00256	.92850	54
7	.07179	.99742	.07197	13.894	.0026	13.930	.00258	.92821	53
8	.07208	.99740	.07226	13.838	.0026	13.874	.00260	.92792	52
9	.07237	.99738	.07256	13.782	.0026	13.818	.00262	.92763	51
10	0.07266	0.99736	0.07285	13.727	1.0026	13.763	0.00264	0.92734	50
11	.07295	.99733	.07314	13.672	.0027	13.708	.00266	.92705	49
12	.07324	.99731	.07343	13.617	.0027	13.654	.00268	.92676	48
13	.07353	.99729	.07373	13.563	.0027	13.600	.00271	.92647	47
14	.07382	.99727	.07402	13.510	.0027	13.547	.00273	.92618	46
15	0.07411	0.99725	0.07431	13.457	1.0027	13.494	0.00275	0.92589	45
16	.07440	.99723	.07460	13.404	.0028	13.441	.00277	.92560	44
17	.07469	.99721	.07490	13.351	.0028	13.389	.00279	.92531	43
18	.07498	.99718	.07519	13.299	.0028	13.337	.00281	.92502	42
19	.07527	.99716	.07548	13.248	.0028	13.286	.00284	.92473	41
20	0.07556	0.99714	0.07577	13.197	1.0029	13.235	0.00286	0.92444	40
21	.07585	.99712	.07607	13.146	.0029	13.184	.00288	.92415	39
22	.07614	.99710	.07636	13.096	.0029	13.134	.00290	.92386	38
23	.07643	.99707	.07665	13.046	.0029	13.084	.00292	.92357	37
24	.07672	.99705	.07694	12.996	.0029	13.034	.00295	.92328	36
25	0.07701	0.99703	0.07724	12.947	1.0030	12.985	0.00297	0.92299	35
26	.07730	.99701	.07753	12.898	.0030	12.937	.00299	.92270	34
27	.07759	.99698	.07782	12.849	.0030	12.888	.00301	.92241	33
28	.07788	.99696	.07812	12.801	.0030	12.840	.00304	.92212	32
29	.07817	.99694	.07841	12.754	.0031	12.793	.00306	.92183	31
30	0.07846	0.99692	0.07870	12.706	1.0031	12.745	0.00308	0.92154	30
31	.07875	.99689	.07899	12.659	.0031	12.698	.00310	.92125	29
32	.07904	.99687	.07929	12.612	.0031	12.652	.00313	.92096	28
33	.07933	.99685	.07958	12.566	.0032	12.606	.00315	.92067	27
34	.07962	.99682	.07987	12.520	.0032	12.560	.00317	.92038	26
35	0.07991	0.99680	0.08016	12.474	1.0032	12.514	0.00320	0.92009	25
36	.08020	.99678	.08046	12.429	.0032	12.469	.00322	.91980	24
37	.08049	.99675	.08075	12.384	.0032	12.424	.00324	.91951	23
38	.08078	.99673	.08104	12.339	.0033	12.379	.00327	.91922	22
39	.08107	.99671	.08134	12.295	.0033	12.335	.00329	.91893	21
40	0.08136	0.99668	0.08163	12.250	1.0033	12.291	0.00331	0.91864	20
41	.08165	.99666	.08192	12.207	.0033	12.248	.00334	.91835	19
42	.08194	.99664	.08221	12.163	.0034	12.204	.00336	.91806	18
43	.08223	.99661	.08251	12.120	.0034	12.161	.00339	.91777	17
44	.08252	.99659	.08280	12.077	.0034	12.118	.00341	.91748	16
45	0.08281	0.99656	0.08309	12.035	1.0034	12.076	0.00343	0.91719	15
46	.08310	.99654	.08339	11.992	.0035	12.034	.00346	.91690	14
47	.08339	.99652	.08368	11.950	.0035	11.992	.00348	.91661	13
48	.08368	.99649	.08397	11.909	.0035	11.950	.00351	.91632	12
49	.08397	.99647	.08426	11.867	.0035	11.909	.00353	.91603	11
50	0.08426	0.99644	0.08456	11.826	1.0036	11.868	0.00356	0.91574	10
51	.08455	.99642	.08485	11.785	.0036	11.828	.00358	.91545	9
52	.08484	.99639	.08514	11.745	.0036	11.787	.00360	.91516	8
53	.08513	.99637	.08544	11.704	.0036	11.747	.00363	.91487	7
54	.08542	.99634	.08573	11.664	.0037	11.707	.00365	.91458	6
55	0.08571	0.99632	0.08602	11.625	1.0037	11.668	0.00368	0.91429	5
56	.08600	.99629	.08632	11.585	.0037	11.628	.00370	.91400	4
57	.08629	.99627	.08661	11.546	.0037	11.589	.00373	.91371	3
58	.08658	.99624	.08690	11.507	.0038	11.550	.00375	.91342	2
59	.08687	.99622	.08719	11.468	.0038	11.512	.00378	.91313	1
60	0.08715	0.99619	0.08749	11.430	1.0038	11.474	0.00380	0.91284	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

94° **85°**

5° **Natural Trigonometric Functions** 174°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.08715	0.99619	0.08749	11.430	1.0038	11.474	0.00380	0.91284	60
1	.08744	.99617	.08778	11.392	.0038	11.436	.00383	.91255	59
2	.08773	.99614	.08807	11.354	.0039	11.398	.00386	.91226	58
3	.08802	.99612	.08837	11.316	.0039	11.360	.00388	.91197	57
4	.08831	.99609	.08866	11.279	.0039	11.323	.00391	.91168	56
5	0.08860	0.99607	0.08895	11.242	1.0039	11.286	0.00393	0.91139	55
6	.08889	.99604	.08925	11.205	.0040	11.249	.00396	.91110	54
7	.08918	.99601	.08954	11.168	.0040	11.213	.00398	.91082	53
8	.08947	.99599	.08983	11.132	.0040	11.176	.00401	.91053	52
9	.08976	.99596	.09013	11.095	.0040	11.140	.00404	.91024	51
10	0.09005	0.99594	0.09042	11.059	1.0041	11.104	0.00406	0.90995	50
11	.09034	.99591	.09071	11.024	.0041	11.069	.00409	.90966	49
12	.09063	.99588	.09101	10.988	.0041	11.033	.00411	.90937	48
13	.09092	.99586	.09130	10.953	.0041	10.998	.00414	.90908	47
14	.09121	.99583	.09159	10.918	.0042	10.963	.00417	.90879	46
15	0.09150	0.99580	0.09189	10.883	1.0042	10.929	0.00419	0.90850	45
16	.09179	.99578	.09218	10.848	.0042	10.894	.00422	.90821	44
17	.09208	.99575	.09247	10.814	.0043	10.860	.00425	.90792	43
18	.09237	.99572	.09277	10.780	.0043	10.826	.00427	.90763	42
19	.09266	.99570	.09306	10.746	.0043	10.792	.00430	.90734	41
20	0.09295	0.99567	0.09335	10.712	1.0043	10.758	0.00433	0.90705	40
21	.09324	.99564	.09365	10.678	.0044	10.725	.00436	.90676	39
22	.09353	.99562	.09394	10.645	.0044	10.692	.00438	.90647	38
23	.09382	.99559	.09423	10.612	.0044	10.659	.00441	.90618	37
24	.09411	.99556	.09453	10.579	.0044	10.626	.00444	.90589	36
25	0.09440	0.99553	0.09482	10.546	1.0045	10.593	0.00446	0.90560	35
26	.09469	.99551	.09511	10.514	.0045	10.561	.00449	.90531	34
27	.09498	.99548	.09541	10.481	.0045	10.529	.00452	.90502	33
28	.09527	.99545	.09570	10.449	.0046	10.497	.00455	.90473	32
29	.09556	.99542	.09599	10.417	.0046	10.465	.00458	.90444	31
30	0.09584	0.99540	0.09629	10.385	1.0046	10.433	0.00460	0.90415	30
31	.09613	.99537	.09658	10.354	.0046	10.402	.00463	.90386	29
32	.09642	.99534	.09688	10.322	.0047	10.371	.00466	.90357	28
33	.09671	.99531	.09717	10.291	.0047	10.340	.00469	.90328	27
34	.09700	.99528	.09746	10.260	.0047	10.309	.00472	.90300	26
35	0.09729	0.99525	0.09776	10.229	1.0048	10.278	0.00474	0.90271	25
36	.09758	.99523	.09805	10.199	.0048	10.248	.00477	.90242	24
37	.09787	.99520	.09834	10.168	.0048	10.217	.00480	.90213	23
38	.09816	.99517	.09864	10.138	.0048	10.187	.00483	.90184	22
39	.09845	.99514	.09893	10.108	.0049	10.157	.00486	.90155	21
40	0.09874	0.99511	0.09922	10.078	1.0049	10.127	0.00489	0.90126	20
41	.09903	.99508	.09952	10.048	.0049	10.098	.00491	.90097	19
42	.09932	.99505	.09981	10.019	.0050	10.068	.00494	.90068	18
43	.09961	.99503	.10011	9.9893	.0050	10.039	.00497	.90039	17
44	.09990	.99500	.10040	9.9601	.0050	10.010	.00500	.90010	16
45	0.10019	0.99497	0.10069	9.9310	1.0050	9.9812	0.00503	0.89981	15
46	.10048	.99494	.10099	9.9021	.0051	9.9525	.00506	.89952	14
47	.10077	.99491	.10128	9.8734	.0051	9.9239	.00509	.89923	13
48	.10106	.99488	.10158	9.8448	.0051	9.8955	.00512	.89894	12
49	.10134	.99485	.10187	9.8164	.0052	9.8672	.00515	.89865	11
50	0.10163	0.99482	0.10216	9.7882	1.0052	9.8391	0.00518	0.89836	10
51	.10192	.99479	.10246	9.7601	.0052	9.8112	.00521	.89807	9
52	.10221	.99476	.10275	9.7322	.0053	9.7834	.00524	.89779	8
53	.10250	.99473	.10305	9.7044	.0053	9.7558	.00527	.89750	7
54	.10279	.99470	.10334	9.6768	.0053	9.7283	.00530	.89721	6
55	0.10308	0.99467	0.10363	9.6493	1.0053	9.7010	0.00533	0.89692	5
56	.10337	.99464	.10393	9.6220	.0054	9.6739	.00536	.89663	4
57	.10366	.99461	.10422	9.5949	.0054	9.6469	.00539	.89634	3
58	.10395	.99458	.10452	9.5679	.0054	9.6200	.00542	.89605	2
59	.10424	.99455	.10481	9.5411	.0055	9.5933	.00545	.89576	1
60	0.10453	0.99452	0.10510	9.5144	1.0055	9.5668	0.00548	0.89547	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

95° 84°

6° **Natural Trigonometric Functions** 173°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.10453	0.99452	0.10510	9.5144	1.0055	9.5668	0.00548	0.89547	60
1	.10482	.99449	.10540	.4878	.0055	.5404	.00551	.89518	59
2	.10511	.99446	.10569	.4614	.0056	.5141	.00554	.89489	58
3	.10540	.99443	.10599	.4351	.0056	.4880	.00557	.89460	57
4	.10568	.99440	.10628	.4090	.0056	.4620	.00560	.89431	56
5	0.10597	0.99437	0.10657	9.3831	1.0057	9.4362	0.00563	0.89402	55
6	.10626	.99434	.10687	.3572	.0057	.4105	.00566	.89373	54
7	.10655	.99431	.10716	.3315	.0057	.3850	.00569	.89345	53
8	.10684	.99428	.10746	.3060	.0057	.3596	.00572	.89316	52
9	.10713	.99424	.10775	.2806	.0058	.3343	.00575	.89287	51
10	0.10742	0.99421	0.10805	9.2553	1.0058	9.3092	0.00579	0.89258	50
11	.10771	.99418	.10834	.2302	.0058	.2842	.00582	.89229	49
12	.10800	.99415	.10863	.2051	.0059	.2593	.00585	.89200	48
13	.10829	.99412	.10893	.1803	.0059	.2346	.00588	.89171	47
14	.10858	.99409	.10922	.1555	.0059	.2100	.00591	.89142	46
15	0.10887	0.99406	0.10952	9.1309	1.0060	9.1855	0.00594	0.89113	45
16	.10916	.99402	.10981	.1064	.0060	.1612	.00597	.89084	44
17	.10944	.99399	.11011	.0821	.0060	.1370	.00601	.89055	43
18	.10973	.99396	.11040	.0579	.0061	.1129	.00604	.89026	42
19	.11002	.99393	.11069	.0338	.0061	.0890	.00607	.88998	41
20	0.11031	0.99390	0.11099	9.0098	1.0061	9.0651	0.00610	0.88969	40
21	.11060	.99386	.11128	8.9860	.0062	.0414	.00613	.88940	39
22	.11089	.99383	.11158	.9623	.0062	.0179	.00617	.88911	38
23	.11118	.99380	.11187	.9387	.0062	8.9944	.00620	.88882	37
24	.11147	.99377	.11217	.9152	.0063	.9711	.00623	.88853	36
25	0.11176	0.99373	0.11246	8.8918	1.0063	8.9479	0.00626	0.88824	35
26	.11205	.99370	.11276	.8686	.0063	.9248	.00630	.88795	34
27	.11234	.99367	.11305	.8455	.0064	.9018	.00633	.88766	33
28	.11262	.99364	.11335	.8225	.0064	.8790	.00636	.88737	32
29	.11291	.99360	.11364	.7996	.0064	.8563	.00639	.88708	31
30	0.11320	0.99357	0.11393	8.7769	1.0065	8.8337	0.00643	0.88680	30
31	.11349	.99354	.11423	.7542	.0065	.8112	.00646	.88651	29
32	.11378	.99350	.11452	.7317	.0065	.7888	.00649	.88622	28
33	.11407	.99347	.11482	.7093	.0066	.7665	.00653	.88593	27
34	.11436	.99344	.11511	.6870	.0066	.7444	.00656	.88564	26
35	0.11465	0.99341	0.11541	8.6648	1.0066	8.7223	0.00659	0.88535	25
36	.11494	.99337	.11570	.6427	.0067	.7004	.00663	.88506	24
37	.11523	.99334	.11600	.6208	.0067	.6786	.00666	.88477	23
38	.11551	.99330	.11629	.5989	.0067	.6569	.00669	.88448	22
39	.11580	.99327	.11659	.5772	.0068	.6353	.00673	.88420	21
40	0.11609	0.99324	0.11688	8.5555	1.0068	8.6138	0.00676	0.88391	20
41	.11638	.99320	.11718	.5340	.0068	.5924	.00679	.88362	19
42	.11667	.99317	.11747	.5126	.0069	.5711	.00683	.88333	18
43	.11696	.99314	.11777	.4913	.0069	.5499	.00686	.88304	17
44	.11725	.99310	.11806	.4701	.0069	.5289	.00690	.88272	16
45	0.11754	0.99307	0.11836	8.4489	1.0070	8.5079	0.00693	0.88246	15
46	.11783	.99303	.11865	.4279	.0070	.4871	.00696	.88217	14
47	.11811	.99300	.11895	.4070	.0070	.4663	.00700	.88188	13
48	.11840	.99296	.11924	.3862	.0071	.4457	.00703	.88160	12
49	.11869	.99293	.11954	.3655	.0071	.4251	.00707	.88131	11
50	0.11898	0.99290	0.11983	8.3449	1.0071	8.4046	0.00710	0.88102	10
51	.11927	.99286	.12013	.3244	.0072	.3843	.00714	.88073	9
52	.11956	.99283	.12042	.3040	.0072	.3640	.00717	.88044	8
53	.11985	.99279	.12072	.2837	.0073	.3439	.00721	.88015	7
54	.12014	.99276	.12101	.2635	.0073	.3238	.00724	.87986	6
55	0.12042	0.99272	0.12131	8.2434	1.0073	8.3039	0.00728	0.87957	5
56	.12071	.99269	.12160	.2234	.0074	.2840	.00731	.87928	4
57	.12100	.99265	.12190	.2035	.0074	.2642	.00735	.87900	3
58	.12129	.99262	.12219	.1837	.0074	.2446	.00738	.87871	2
59	.12158	.99258	.12249	.1640	.0075	.2250	.00742	.87842	1
60	0.12187	0.99255	0.12278	8.1443	1.0075	8.2055	0.00745	0.87813	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

96° 83°

7° **Natural Trigonometric Functions** 172°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.12187	0.99255	0.12278	8.1443	1.0075	8.2055	0.00745	0.87813	60
1	.12216	.99251	.12308	.1248	.0075	.1861	.00749	.87787	59
2	.12245	.99247	.12337	.1053	.0076	.1668	.00752	.87755	58
3	.12273	.99244	.12367	.0860	.0076	.1476	.00756	.87726	57
4	.12302	.99240	.12396	.0667	.0076	.1285	.00760	.87697	56
5	0.12331	0.99237	0.12426	8.0476	1.0077	8.1094	0.00763	0.87669	55
6	.12360	.99233	.12456	.0285	.0077	.0905	.00767	.87640	54
7	.12389	.99229	.12485	.0095	.0078	.0717	.00770	.87611	53
8	.12418	.99226	.12515	7.9906	.0078	.0529	.00774	.87582	52
9	.12447	.99222	.12544	.9717	.0078	.0342	.00778	.87553	51
10	0.12476	0.99219	0.12574	7.9530	1.0079	8.0156	0.00781	0.87524	50
11	.12504	.99215	.12603	.9344	.0079	7.9971	.00785	.87495	49
12	.12533	.99211	.12633	.9158	.0079	.9787	.00788	.87467	48
13	.12562	.99208	.12662	.8973	.0080	.9604	.00792	.87438	47
14	.12591	.99204	.12692	.8789	.0080	.9421	.00796	.87409	46
15	0.12620	0.99200	0.12722	7.8606	1.0080	7.9240	0.00799	0.87380	45
16	.12649	.99197	.12751	.8424	.0081	.9059	.00803	.87351	44
17	.12678	.99193	.12781	.8243	.0081	.8879	.00807	.87322	43
18	.12706	.99189	.12810	.8062	.0082	.8700	.00810	.87293	42
19	.12735	.99186	.12840	.7882	.0082	.8522	.00814	.87265	41
20	0.12764	0.99182	0.12869	7.7703	1.0082	7.8344	0.00818	0.87236	40
21	.12793	.99178	.12899	.7525	.0083	.8168	.00822	.87207	39
22	.12822	.99174	.12928	.7348	.0083	.7992	.00825	.87178	38
23	.12851	.99171	.12958	.7171	.0084	.7817	.00829	.87149	37
24	.12879	.99167	.12988	.6996	.0084	.7642	.00833	.87120	36
25	0.12908	0.99163	0.13017	7.6821	1.0084	7.7469	0.00837	0.87091	35
26	.12937	.99160	.13047	.6646	.0085	.7296	.00840	.87063	34
27	.12966	.99156	.13076	.6473	.0085	.7124	.00844	.87034	33
28	.12995	.99152	.13106	.6300	.0085	.6953	.00848	.87005	32
29	.13024	.99148	.13136	.6129	.0086	.6783	.00852	.86976	31
30	0.13053	0.99144	0.13165	7.5957	1.0086	7.6613	0.00855	0.86947	30
31	.13081	.99141	.13195	.5787	.0087	.6444	.00859	.86918	29
32	.13110	.99137	.13224	.5617	.0087	.6276	.00863	.86890	28
33	.13139	.99133	.13254	.5449	.0087	.6108	.00867	.86861	27
34	.13168	.99129	.13284	.5280	.0088	.5942	.00871	.86832	26
35	0.13197	0.99125	0.13313	7.5113	1.0088	7.5776	0.00875	0.86803	25
36	.13226	.99121	.13343	.4946	.0089	.5611	.00878	.86774	24
37	.13254	.99118	.13372	.4780	.0089	.5446	.00882	.86745	23
38	.13283	.99114	.13402	.4615	.0089	.5282	.00886	.86717	22
39	.13312	.99110	.13432	.4451	.0090	.5119	.00890	.86688	21
40	0.13341	0.99106	0.13461	7.4287	1.0090	7.4957	0.00894	0.86659	20
41	.13370	.99102	.13491	.4124	.0090	.4795	.00898	.86630	19
42	.13399	.99098	.13520	.3961	.0091	.4634	.00902	.86601	18
43	.13427	.99094	.13550	.3800	.0091	.4474	.00905	.86572	17
44	.13456	.99090	.13580	.3639	.0092	.4315	.00909	.86544	16
45	0.13485	0.99086	0.13609	7.3479	1.0092	7.4156	0.00913	0.86515	15
46	.13514	.99083	.13639	.3319	.0092	.3998	.00917	.86486	14
47	.13543	.99079	.13669	.3160	.0093	.3840	.00921	.86457	13
48	.13571	.99075	.13698	.3002	.0093	.3683	.00925	.86428	12
49	.13600	.99071	.13728	.2844	.0094	.3527	.00929	.86400	11
50	0.13629	0.99067	0.13757	7.2687	1.0094	7.3372	0.00933	0.86371	10
51	.13658	.99063	.13787	.2531	.0094	.3217	.00937	.86342	9
52	.13687	.99059	.13817	.2375	.0095	.3063	.00941	.86313	8
53	.13716	.99055	.13846	.2220	.0095	.2909	.00945	.86284	7
54	.13744	.99051	.13876	.2066	.0096	.2757	.00949	.86255	6
55	0.13773	0.99047	0.13906	7.1912	1.0096	7.2604	0.00953	0.86227	5
56	.13802	.99043	.13935	.1759	.0097	.2453	.00957	.86198	4
57	.13831	.99039	.13965	.1607	.0097	.2302	.00961	.86169	3
58	.13860	.99035	.13995	.1455	.0097	.2152	.00965	.86140	2
59	.13888	.99031	.14024	.1304	.0098	.2002	.00969	.86111	1
60	0.13917	0.99027	0.14054	7.1154	1.0098	7.1853	0.00973	0.86083	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

97° 82°

8° **Natural Trigonometric Functions** 171°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.13917	0.99027	0.14054	7.1154	1.0098	7.1853	0.00973	0.86083	60
1	.13946	.99023	.14084	.1004	.0099	.1704	.00977	.86054	59
2	.13975	.99019	.14113	.0854	.0099	.1557	.00981	.86025	58
3	.14004	.99015	.14143	.0706	.0099	.1409	.00985	.85996	57
4	.14032	.99010	.14173	.0558	.0100	.1263	.00989	.85967	56
5	0.14061	0.99006	0.14202	7.0410	1.0100	7.1117	0.00993	0.85939	55
6	.14090	.99002	.14232	.0264	.0101	.0972	.00998	.85910	54
7	.14119	.98998	.14262	.0117	.0101	.0827	.01002	.85881	53
8	.14148	.98994	.14291	6.9972	.0102	.0683	.01006	.85852	52
9	.14176	.98990	.14321	.9827	.0102	.0539	.01010	.85823	51
10	0.14205	0.98986	0.14351	6.9682	1.0102	7.0396	0.01014	0.85795	50
11	.14234	.98982	.14380	.9538	.0103	.0254	.01018	.85766	49
12	.14263	.98978	.14410	.9395	.0103	.0112	.01022	.85737	48
13	.14292	.98973	.14440	.9252	.0104	6.9971	.01026	.85708	47
14	.14320	.98969	.14470	.9110	.0104	.9830	.01031	.85679	46
15	0.14349	0.98965	0.14499	6.8969	1.0104	6.9690	0.01035	0.85651	45
16	.14378	.98961	.14529	.8828	.0105	.9550	.01039	.85622	44
17	.14407	.98957	.14559	.8687	.0105	.9411	.01043	.85593	43
18	.14436	.98952	.14588	.8547	.0106	.9273	.01047	.85564	42
19	.14464	.98948	.14618	.8408	.0106	.9135	.01052	.85536	41
20	0.14493	0.98944	0.14648	6.8269	1.0107	6.8998	0.01056	0.85507	40
21	.14522	.98940	.14677	.8131	.0107	.8861	.01060	.85478	39
22	.14551	.98936	.14707	.7993	.0107	.8725	.01064	.85449	38
23	.14579	.98931	.14737	.7856	.0108	.8589	.01068	.85420	37
24	.14608	.98927	.14767	.7720	.0108	.8454	.01073	.85392	36
25	0.14637	0.98923	0.14796	6.7584	1.0109	6.8320	0.01077	0.85363	35
26	.14666	.98919	.14826	.7448	.0109	.8185	.01081	.85334	34
27	.14695	.98914	.14856	.7313	.0110	.8052	.01085	.85305	33
28	.14723	.98910	.14886	.7179	.0110	.7919	.01090	.85277	32
29	.14752	.98906	.14915	.7045	.0111	.7787	.01094	.85248	31
30	0.14781	0.98901	0.14945	6.6911	1.0111	6.7655	0.01098	0.85219	30
31	.14810	.98897	.14975	.6779	.0111	.7523	.01103	.85190	29
32	.14838	.98893	.15004	.6646	.0112	.7392	.01107	.85161	28
33	.14867	.98889	.15034	.6514	.0112	.7262	.01111	.85133	27
34	.14896	.98884	.15064	.6383	.0113	.7132	.01116	.85104	26
35	0.14925	0.98880	0.15094	6.6252	1.0113	6.7003	0.01120	0.85075	25
36	.14953	.98876	.15123	.6122	.0114	.6874	.01124	.85046	24
37	.14982	.98871	.15153	.5992	.0114	.6745	.01129	.85018	23
38	.15011	.98867	.15183	.5863	.0115	.6617	.01133	.84989	22
39	.15040	.98862	.15213	.5734	.0115	.6490	.01137	.84960	21
40	0.15068	0.98858	0.15243	6.5605	1.0115	6.6363	0.01142	0.84931	20
41	.15097	.98854	.15272	.5478	.0116	.6237	.01146	.84903	19
42	.15126	.98849	.15302	.5350	.0116	.6111	.01151	.84874	18
43	.15155	.98845	.15332	.5223	.0117	.5985	.01155	.84845	17
44	.15183	.98840	.15362	.5097	.0117	.5860	.01159	.84816	16
45	0.15212	0.98836	0.15391	6.4971	1.0118	6.5736	0.01164	0.84788	15
46	.15241	.98832	.15421	.4845	.0118	.5612	.01168	.84759	14
47	.15270	.98827	.15451	.4720	.0119	.5488	.01173	.84730	13
48	.15298	.98823	.15481	.4596	.0119	.5365	.01177	.84701	12
49	.15328	.98818	.15511	.4472	.0119	.5243	.01182	.84672	11
50	0.15356	0.98814	0.15540	6.4348	1.0120	6.5121	0.01186	0.84644	10
51	.15385	.98809	.15570	.4225	.0120	.4999	.01190	.84615	9
52	.15413	.98805	.15600	.4103	.0121	.4878	.01195	.84586	8
53	.15442	.98800	.15630	.3980	.0121	.4757	.01199	.84558	7
54	.15471	.98796	.15659	.3859	.0122	.4637	.01204	.84529	6
55	0.15500	0.98791	0.15689	6.3737	1.0122	6.4517	0.01208	0.84500	5
56	.15528	.98787	.15719	.3616	.0123	.4398	.01213	.84471	4
57	.15557	.98782	.15749	.3496	.0123	.4279	.01217	.84443	3
58	.15586	.98778	.15779	.3376	.0124	.4160	.01222	.84414	2
59	.15615	.98773	.15809	.3257	.0124	.4042	.01227	.84385	1
60	0.15643	0.98769	0.15838	6.3137	1.0125	6.3924	0.01231	0.84356	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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9° **Natural Trigonometric Functions** 170°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.15643	0.98769	0.15838	6.3137	1.0125	6.3924	0.01231	0.84356	60
1	.15672	.98764	.15868	.3019	.0125	.3807	.01236	.84328	59
2	.15701	.98760	.15898	.2901	.0125	.3690	.01240	.84299	58
3	.15730	.98755	.15928	.2783	.0126	.3574	.01245	.84270	57
4	.15758	.98750	.15958	.2665	.0126	.3458	.01249	.84242	56
5	0.15787	0.98746	0.15987	6.2548	1.0127	6.3343	0.01254	0.84213	55
6	.15816	.98741	.16017	.2432	.0127	.3228	.01259	.84184	54
7	.15844	.98737	.16047	.2316	.0128	.3113	.01263	.84155	53
8	.15873	.98732	.16077	.2200	.0128	.2999	.01268	.84127	52
9	.15902	.98727	.16107	.2085	.0129	.2885	.01272	.84098	51
10	0.15931	0.98723	0.16137	6.1970	1.0129	6.2772	0.01277	0.84069	50
11	.15959	.98718	.16167	.1856	.0130	.2659	.01282	.84041	49
12	.15988	.98714	.16196	.1742	.0130	.2546	.01286	.84012	48
13	.16017	.98709	.16226	.1628	.0131	.2434	.01291	.83983	47
14	.16045	.98704	.16256	.1515	.0131	.2322	.01296	.83954	46
15	0.16074	0.98700	0.16286	6.1402	1.0132	6.2211	0.01300	0.83926	45
16	.16103	.98695	.16316	.1290	.0132	.2100	.01305	.83897	44
17	.16132	.98690	.16346	.1178	.0133	.1990	.01310	.83868	43
18	.16160	.98685	.16376	.1066	.0133	.1880	.01314	.83840	42
19	.16189	.98681	.16405	.0955	.0134	.1770	.01319	.83811	41
20	0.16218	0.98676	0.16435	6.0844	1.0134	6.1661	0.01324	0.83782	40
21	.16246	.98671	.16465	.0734	.0135	.1552	.01328	.83753	39
22	.16275	.98667	.16495	.0624	.0135	.1443	.01333	.83725	38
23	.16304	.98662	.16525	.0514	.0136	.1335	.01338	.83696	37
24	.16333	.98657	.16555	.0405	.0136	.1227	.01343	.83667	36
25	0.16361	0.98652	0.16585	6.0296	1.0136	6.1120	0.01347	0.83639	35
26	.16390	.98648	.16615	.0188	.0137	.1013	.01352	.83610	34
27	.16419	.98643	.16644	.0080	.0137	.0906	.01357	.83581	33
28	.16447	.98638	.16674	5.9972	.0138	.0800	.01362	.83553	32
29	.16476	.98633	.16704	.9865	.0138	.0694	.01367	.83524	31
30	0.16505	0.98628	0.16734	5.9758	1.0139	6.0588	0.01371	0.83495	30
31	.16533	.98624	.16764	.9651	.0139	.0483	.01376	.83466	29
32	.16562	.98619	.16794	.9545	.0140	.0379	.01381	.83438	28
33	.16591	.98614	.16824	.9439	.0140	.0274	.01386	.83409	27
34	.16619	.98609	.16854	.9333	.0141	.0170	.01391	.83380	26
35	0.16648	0.98604	0.16884	5.9228	1.0141	6.0066	0.01395	0.83352	25
36	.16677	.98600	.16914	.9123	.0142	5.9963	.01400	.83323	24
37	.16705	.98595	.16944	.9019	.0142	.9860	.01405	.83294	23
38	.16734	.98590	.16973	.8915	.0143	.9758	.01410	.83266	22
39	.16763	.98585	.17003	.8811	.0143	.9655	.01415	.83237	21
40	0.16791	0.98580	0.17033	5.8708	1.0144	5.9554	0.01420	0.83208	20
41	.16820	.98575	.17063	.8605	.0144	.9452	.01425	.83180	19
42	.16849	.98570	.17093	.8502	.0145	.9351	.01430	.83151	18
43	.16878	.98565	.17123	.8400	.0145	.9250	.01434	.83122	17
44	.16906	.98560	.17153	.8298	.0146	.9150	.01439	.83094	16
45	0.16935	0.98556	0.17183	5.8196	1.0146	5.9049	0.01444	0.83065	15
46	.16964	.98551	.17213	.8095	.0147	.8950	.01449	.83036	14
47	.16992	.98546	.17243	.7994	.0147	.8850	.01454	.83008	13
48	.17021	.98541	.17273	.7894	.0148	.8751	.01459	.82979	12
49	.17050	.98536	.17303	.7794	.0148	.8652	.01464	.82950	11
50	0.17078	0.98531	0.17333	5.7694	1.0149	5.8554	0.01469	0.82922	10
51	.17107	.98526	.17363	.7594	.0150	.8456	.01474	.82893	9
52	.17136	.98521	.17393	.7495	.0150	.8358	.01479	.82864	8
53	.17164	.98516	.17423	.7396	.0151	.8261	.01484	.82836	7
54	.17193	.98511	.17453	.7297	.0151	.8163	.01489	.82807	6
55	0.17221	0.98506	0.17483	5.7199	1.0152	5.8067	0.01494	0.82778	5
56	.17250	.98501	.17513	.7101	.0152	.7970	.01499	.82750	4
57	.17279	.98496	.17543	.7004	.0153	.7874	.01504	.82721	3
58	.17307	.98491	.17573	.6906	.0153	.7778	.01509	.82692	2
59	.17336	.98486	.17603	.6809	.0154	.7683	.01514	.82664	1
60	0.17365	0.98481	0.17633	5.6713	1.0154	5.7588	0.01519	0.82635	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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10° **Natural Trigonometric Functions** 169°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.17365	0.98481	0.17633	5.6713	1.0154	5.7588	0.01519	0.82635	60
1	.17393	.98476	.17663	.6616	.0155	.7493	.01524	.82606	59
2	.17422	.98471	.17693	.6520	.0155	.7398	.01529	.82578	58
3	.17451	.98465	.17723	.6425	.0156	.7304	.01534	.82549	57
4	.17479	.98460	.17753	.6329	.0156	.7210	.01539	.82521	56
5	0.17508	0.98455	0.17783	5.6234	1.0157	5.7117	0.01544	0.82492	55
6	.17537	.98450	.17813	.6140	.0157	.7023	.01550	.82463	54
7	.17565	.98445	.17843	.6045	.0158	.6930	.01555	.82435	53
8	.17594	.98440	.17873	.5951	.0158	.6838	.01560	.82406	52
9	.17622	.98435	.17903	.5857	.0159	.6745	.01565	.82377	51
10	0.17651	0.98430	0.17933	5.5764	1.0159	5.6653	0.01570	0.82349	50
11	.17680	.98425	.17963	.5670	.0160	.6561	.01575	.82320	49
12	.17708	.98419	.17993	.5578	.0160	.6470	.01580	.82291	48
13	.17737	.98414	.18023	.5485	.0161	.6379	.01585	.82263	47
14	.17766	.98409	.18053	.5393	.0162	.6288	.01591	.82234	46
15	0.17794	0.98404	0.18083	5.5301	1.0162	5.6197	0.01596	0.82206	45
16	.17823	.98399	.18113	.5209	.0163	.6107	.01601	.82177	44
17	.17852	.98394	.18143	.5117	.0163	.6017	.01606	.82148	43
18	.17880	.98388	.18173	.5026	.0164	.5928	.01611	.82120	42
19	.17909	.98383	.18203	.4936	.0164	.5838	.01617	.82091	41
20	0.17937	0.98378	0.18233	5.4845	1.0165	5.5749	0.01622	0.82062	40
21	.17966	.98373	.18263	.4755	.0165	.5660	.01627	.82034	39
22	.17995	.98368	.18293	.4665	.0166	.5572	.01632	.82005	38
23	.18023	.98362	.18323	.4575	.0166	.5484	.01638	.81977	37
24	.18052	.98357	.18353	.4486	.0167	.5396	.01643	.81948	36
25	0.18080	0.98352	0.18383	5.4396	1.0167	5.5308	0.01648	0.81919	35
26	.18109	.98347	.18413	.4308	.0168	.5221	.01653	.81891	34
27	.18138	.98341	.18444	.4219	.0169	.5134	.01659	.81862	33
28	.18166	.98336	.18474	.4131	.0169	.5047	.01664	.81834	32
29	.18195	.98331	.18504	.4043	.0170	.4960	.01669	.81805	31
30	0.18223	0.98325	0.18534	5.3955	1.0170	5.4874	0.01674	0.81776	30
31	.18252	.98320	.18564	.3868	.0171	.4788	.01680	.81748	29
32	.18281	.98315	.18594	.3780	.0171	.4702	.01685	.81719	28
33	.18309	.98309	.18624	.3694	.0172	.4617	.01690	.81691	27
34	.18338	.98304	.18654	.3607	.0172	.4532	.01696	.81662	26
35	0.18366	0.98299	0.18684	5.3521	1.0173	5.4447	0.01701	0.81633	25
36	.18395	.98293	.18714	.3434	.0174	.4362	.01706	.81605	24
37	.18424	.98288	.18745	.3349	.0174	.4278	.01712	.81576	23
38	.18452	.98283	.18775	.3263	.0175	.4194	.01717	.81548	22
39	.18481	.98277	.18805	.3178	.0175	.4110	.01722	.81519	21
40	0.18509	0.98272	0.18835	5.3093	1.0176	5.4026	0.01728	0.81490	20
41	.18538	.98267	.18865	.3008	.0176	.3943	.01733	.81462	19
42	.18567	.98261	.18895	.2923	.0177	.3860	.01739	.81433	18
43	.18595	.98256	.18925	.2839	.0177	.3777	.01744	.81405	17
44	.18624	.98250	.18955	.2755	.0178	.3695	.01749	.81376	16
45	0.18652	0.98245	0.18985	5.2671	1.0179	5.3612	0.01755	0.81348	15
46	.18681	.98240	.19016	.2588	.0179	.3530	.01760	.81319	14
47	.18709	.98234	.19046	.2505	.0180	.3449	.01766	.81290	13
48	.18738	.98229	.19076	.2422	.0180	.3367	.01771	.81262	12
49	.18767	.98223	.19106	.2339	.0181	.3286	.01777	.81233	11
50	0.18795	0.98218	0.19136	5.2257	1.0181	5.3205	0.01782	0.81205	10
51	.18824	.98212	.19166	.2174	.0182	.3124	.01788	.81176	9
52	.18852	.98207	.19197	.2092	.0182	.3044	.01793	.81147	8
53	.18881	.98201	.19227	.2011	.0183	.2963	.01799	.81119	7
54	.18909	.98196	.19257	.1929	.0184	.2883	.01804	.81090	6
55	0.18938	0.98190	0.19287	5.1848	1.0184	5.2803	0.01810	0.81062	5
56	.18967	.98185	.19317	.1767	.0185	.2724	.01815	.81033	4
57	.18995	.98179	.19347	.1686	.0185	.2645	.01821	.81005	3
58	.19024	.98174	.19378	.1606	.0186	.2566	.01826	.80976	2
59	.19052	.98168	.19408	.1525	.0186	.2487	.01832	.80948	1
60	0.19081	0.98163	0.19438	5.1445	1.0187	5.2408	0.01837	0.80919	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

100° 79°

11° **Natural Trigonometric Functions** **168°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.19081	0.98163	0.19438	5.1445	1.0187	5.2408	0.01837	0.80919	60
1	.19109	.98157	.19468	.1366	.0188	.2330	.01843	.80890	59
2	.19138	.98152	.19498	.1286	.0188	.2252	.01848	.80862	58
3	.19166	.98146	.19529	.1207	.0189	.2174	.01854	.80833	57
4	.19195	.98140	.19559	.1128	.0189	.2097	.01859	.80805	56
5	0.19224	0.98135	0.19589	5.1049	1.0190	5.2019	0.01865	0.80776	55
6	.19252	.98129	.19619	.0970	.0191	.1942	.01871	.80748	54
7	.19281	.98124	.19649	.0892	.0191	.1865	.01876	.80719	53
8	.19309	.98118	.19680	.0814	.0192	.1788	.01882	.80691	52
9	.19338	.98112	.19710	.0736	.0192	.1712	.01887	.80662	51
10	0.19366	0.98107	0.19740	5.0658	1.0193	5.1636	0.01893	0.80634	50
11	.19395	.98101	.19770	.0581	.0193	.1560	.01899	.80605	49
12	.19423	.98095	.19800	.0504	.0194	.1484	.01904	.80576	48
13	.19452	.98090	.19831	.0427	.0195	.1409	.01910	.80548	47
14	.19480	.98084	.19861	.0350	.0195	.1333	.01916	.80519	46
15	0.19509	0.98078	0.19891	5.0273	1.0196	5.1258	0.01921	0.80491	45
16	.19537	.98073	.19921	.0197	.0196	.1183	.01927	.80462	44
17	.19566	.98067	.19952	.0121	.0197	.1109	.01933	.80434	43
18	.19595	.98061	.19982	.0045	.0198	.1034	.01938	.80405	42
19	.19623	.98056	.20012	4.9969	.0198	.0960	.01944	.80377	41
20	0.19652	0.98050	0.20042	4.9894	1.0199	5.0886	0.01950	0.80348	40
21	.19680	.98044	.20073	.9819	.0199	.0812	.01956	.80320	39
22	.19709	.98039	.20103	.9744	.0200	.0739	.01961	.80291	38
23	.19737	.98033	.20133	.9669	.0201	.0666	.01967	.80263	37
24	.19766	.98027	.20163	.9594	.0201	.0593	.01973	.80234	36
25	0.19794	0.98021	0.20194	4.9520	1.0202	5.0520	0.01979	0.80206	35
26	.19823	.98016	.20224	.9446	.0202	.0447	.01984	.80177	34
27	.19851	.98010	.20254	.9372	.0203	.0375	.01990	.80149	33
28	.19880	.98004	.20285	.9298	.0204	.0302	.01996	.80120	32
29	.19908	.97998	.20315	.9225	.0204	.0230	.02002	.80092	31
30	0.19937	0.97992	0.20345	4.9151	1.0205	5.0158	0.02007	0.80063	30
31	.19965	.97987	.20375	.9078	.0205	.0087	.02013	.80035	29
32	.19994	.97981	.20406	.9006	.0206	.0015	.02019	.80006	28
33	.20022	.97975	.20436	.8933	.0207	4.9944	.02025	.79978	27
34	.20051	.97969	.20466	.8860	.0207	.9873	.02031	.79949	26
35	0.20079	0.97963	0.20497	4.8788	1.0208	4.9802	0.02037	0.79921	25
36	.20108	.97957	.20527	.8716	.0208	.9732	.02042	.79892	24
37	.20136	.97952	.20557	.8644	.0209	.9661	.02048	.79863	23
38	.20165	.97946	.20588	.8573	.0210	.9591	.02054	.79835	22
39	.20193	.97940	.20618	.8501	.0210	.9521	.02060	.79807	21
40	0.20222	0.97934	0.20648	4.8430	1.0211	4.9452	0.02066	0.79778	20
41	.20250	.97928	.20679	.8359	.0211	.9382	.02072	.79750	19
42	.20279	.97922	.20709	.8288	.0212	.9313	.02078	.79721	18
43	.20307	.97916	.20739	.8217	.0213	.9243	.02084	.79693	17
44	.20336	.97910	.20770	.8147	.0213	.9175	.02089	.79664	16
45	0.20364	0.97904	0.20800	4.8077	1.0214	4.9106	0.02095	0.79636	15
46	.20393	.97899	.20830	.8007	.0215	.9037	.02101	.79607	14
47	.20421	.97893	.20861	.7937	.0215	.8969	.02107	.79579	13
48	.20450	.97887	.20891	.7867	.0216	.8901	.02113	.79550	12
49	.20478	.97881	.20921	.7798	.0216	.8833	.02119	.79522	11
50	0.20506	0.97875	0.20952	4.7728	1.0217	4.8765	0.02125	0.79493	10
51	.20535	.97869	.20982	.7659	.0218	.8697	.02131	.79465	9
52	.20563	.97863	.21012	.7591	.0218	.8630	.02137	.79436	8
53	.20592	.97857	.21043	.7522	.0219	.8563	.02143	.79408	7
54	.20620	.97851	.21073	.7453	.0220	.8496	.02149	.79379	6
55	0.20649	0.97845	0.21104	4.7385	1.0220	4.8429	0.02155	0.79351	5
56	.20677	.97839	.21134	.7317	.0221	.8362	.02161	.79323	4
57	.20706	.97833	.21164	.7249	.0221	.8296	.02167	.79294	3
58	.20734	.97827	.21195	.7181	.0222	.8229	.02173	.79266	2
59	.20763	.97821	.21225	.7114	.0223	.8163	.02179	.79237	1
60	0.20791	0.97815	0.21256	4.7046	1.0223	4.8097	0.02185	0.79209	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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12° **Natural Trigonometric Functions** **167°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.20791	0.97815	0.21256	4.7046	1.0223	4.8097	0.02185	0.79209	60
1	.20820	.97809	.21286	.6979	.0224	.8032	.02191	.79180	59
2	.20848	.97803	.21316	.6912	.0225	.7966	.02197	.79152	58
3	.20876	.97797	.21347	.6845	.0225	.7901	.02203	.79123	57
4	.20905	.97790	.21377	.6778	.0226	.7835	.02209	.79105	56
5	0.20933	0.97784	0.21408	4.6712	1.0226	4.7770	0.02215	0.79066	55
6	.20962	.97778	.21438	.6646	.0227	.7706	.02222	.79038	54
7	.20990	.97772	.21468	.6580	.0228	.7641	.02228	.79010	53
8	.21019	.97766	.21499	.6514	.0228	.7576	.02234	.78981	52
9	.21047	.97760	.21529	.6448	.0229	.7512	.02240	.78953	51
10	0.21076	0.97754	0.21560	4.6382	1.0230	4.7448	0.02246	0.78924	50
11	.21104	.97748	.21590	.6317	.0230	.7384	.02252	.78896	49
12	.21132	.97741	.21621	.6252	.0231	.7320	.02258	.78867	48
13	.21161	.97735	.21651	.6187	.0232	.7257	.02264	.78839	47
14	.21189	.97729	.21682	.6122	.0232	.7193	.02271	.78811	46
15	0.21218	0.97723	0.21712	4.6057	1.0233	4.7130	0.02277	0.78782	45
16	.21246	.97717	.21742	.5993	.0234	.7067	.02283	.78754	44
17	.21275	.97711	.21773	.5928	.0234	.7004	.02289	.78725	43
18	.21303	.97704	.21803	.5864	.0235	.6942	.02295	.78697	42
19	.21331	.97698	.21834	.5800	.0235	.6879	.02302	.78668	41
20	0.21360	0.97692	0.21864	4.5736	1.0236	4.6817	0.02308	0.78640	40
21	.21388	.97686	.21895	.5673	.0237	.6754	.02314	.78612	39
22	.21417	.97680	.21925	.5609	.0237	.6692	.02320	.78583	38
23	.21445	.97673	.21956	.5546	.0238	.6631	.02326	.78555	37
24	.21473	.97667	.21986	.5483	.0239	.6569	.02333	.78526	36
25	0.21502	0.97661	0.22017	4.5420	1.0239	4.6507	0.02339	0.78508	35
26	.21530	.97655	.22047	.5357	.0240	.6446	.02345	.78470	34
27	.21559	.97648	.22078	.5294	.0241	.6385	.02351	.78441	33
28	.21587	.97642	.22108	.5232	.0241	.6324	.02358	.78413	32
29	.21615	.97636	.22139	.5169	.0242	.6263	.02364	.78384	31
30	0.21644	0.97630	0.22169	4.5107	1.0243	4.6201	0.02370	0.78356	30
31	.21672	.97623	.22200	.5045	.0243	.6142	.02377	.78328	29
32	.21701	.97617	.22230	.4983	.0244	.6081	.02383	.78299	28
33	.21729	.97611	.22261	.4921	.0245	.6021	.02389	.78271	27
34	.21757	.97604	.22291	.4860	.0245	.5961	.02396	.78242	26
35	0.21786	0.97598	0.22322	4.4799	1.0246	4.5901	0.02402	0.78214	25
36	.21814	.97592	.22353	.4737	.0247	.5841	.02408	.78186	24
37	.21843	.97585	.22383	.4676	.0247	.5782	.02415	.78154	23
38	.21871	.97579	.22414	.4615	.0248	.5722	.02421	.78129	22
39	.21899	.97573	.22444	.4555	.0249	.5663	.02427	.78100	21
40	0.21928	0.97566	0.22475	4.4494	1.0249	4.5604	0.02434	0.78072	20
41	.21956	.97560	.22505	.4434	.0250	.5545	.02440	.78043	19
42	.21985	.97553	.22536	.4373	.0251	.5486	.02446	.78015	18
43	.22013	.97547	.22566	.4313	.0251	.5428	.02453	.77987	17
44	.22041	.97541	.22597	.4253	.0252	.5369	.02459	.77959	16
45	0.22070	0.97534	0.22628	4.4194	1.0253	4.5311	0.02466	0.77930	15
46	.22098	.97528	.22658	.4134	.0253	.5253	.02472	.77902	14
47	.22126	.97521	.22689	.4074	.0254	.5195	.02479	.77873	13
48	.22155	.97515	.22719	.4015	.0255	.5137	.02485	.77845	12
49	.22183	.97508	.22750	.3956	.0255	.5079	.02491	.77817	11
50	0.22211	0.97502	0.22781	4.3897	1.0256	4.5021	0.02498	0.77788	10
51	.22240	.97495	.22811	.3838	.0257	.4964	.02504	.77760	9
52	.22268	.97489	.22842	.3779	.0257	.4907	.02511	.77732	8
53	.22297	.97483	.22872	.3721	.0258	.4850	.02517	.77703	7
54	.22325	.97476	.22903	.3662	.0259	.4793	.02524	.77675	6
55	0.22353	0.97470	0.22934	4.3604	1.0260	4.4736	0.02530	0.77647	5
56	.22382	.97463	.22964	.3546	.0260	.4679	.02537	.77618	4
57	.22410	.97457	.22995	.3488	.0261	.4623	.02543	.77590	3
58	.22438	.97450	.23025	.3430	.0262	.4566	.02550	.77561	2
59	.22467	.97443	.23056	.3372	.0262	.4510	.02556	.77533	1
60	0.22495	0.97437	0.23087	4.3315	1.0263	4.4454	0.02563	0.77505	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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13° **Natural Trigonometric Functions** 166°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.22495	0.97437	0.23087	4.3315	1.0263	4.4454	0.02563	0.77505	60
1	.22523	.97430	.23117	.3257	.0264	.4398	.02569	.77476	59
2	.22552	.97424	.23148	.3200	.0264	.4342	.02576	.77448	58
3	.22580	.97417	.23179	.3143	.0265	.4287	.02583	.77420	57
4	.22608	.97411	.23209	.3086	.0266	.4231	.02589	.77391	56
5	0.22637	0.97404	0.23240	4.3029	1.0266	4.4176	0.02596	0.77363	55
6	.22665	.97398	.23270	.2972	.0267	.4121	.02602	.77335	54
7	.22693	.97391	.23301	.2916	.0268	.4065	.02609	.77306	53
8	.22722	.97384	.23332	.2859	.0268	.4011	.02616	.77278	52
9	.22750	.97378	.23363	.2803	.0269	.3956	.02622	.77250	51
10	0.22778	0.97371	0.23393	4.2747	1.0270	4.3901	0.02629	0.77221	50
11	.22807	.97364	.23424	.2691	.0271	.3847	.02635	.77193	49
12	.22835	.97358	.23455	.2635	.0271	.3792	.02642	.77165	48
13	.22863	.97351	.23485	.2579	.0272	.3738	.02649	.77136	47
14	.22892	.97344	.23516	.2524	.0273	.3684	.02655	.77108	46
15	0.22920	0.97338	0.23547	4.2468	1.0273	4.3630	0.02662	0.77080	45
16	.22948	.97331	.23577	.2413	.0274	.3576	.02669	.77052	44
17	.22977	.97324	.23608	.2358	.0275	.3522	.02675	.77023	43
18	.23005	.97318	.23639	.2303	.0276	.3469	.02682	.76995	42
19	.23033	.97311	.23670	.2248	.0276	.3415	.02689	.76967	41
20	0.23061	0.97304	0.23700	4.2193	1.0277	4.3362	0.02695	0.76938	40
21	.23090	.97298	.23731	.2139	.0278	.3309	.02702	.76910	39
22	.23118	.97291	.23762	.2084	.0278	.3256	.02709	.76882	38
23	.23146	.97284	.23793	.2030	.0279	.3203	.02716	.76853	37
24	.23175	.97277	.23823	.1976	.0280	.3150	.02722	.76825	36
25	0.23203	0.97271	0.23854	4.1921	1.0280	4.3098	0.02729	0.76797	35
26	.23231	.97264	.23885	.1867	.0281	.3045	.02736	.76769	34
27	.23260	.97257	.23916	.1814	.0282	.2993	.02743	.76740	33
28	.23288	.97250	.23946	.1760	.0283	.2941	.02749	.76712	32
29	.23316	.97244	.23977	.1706	.0283	.2888	.02756	.76684	31
30	0.23344	0.97237	0.24008	4.1653	1.0284	4.2836	0.02763	0.76655	30
31	.23373	.97230	.24039	.1600	.0285	.2785	.02770	.76627	29
32	.23401	.97223	.24069	.1546	.0285	.2733	.02777	.76599	28
33	.23429	.97216	.24100	.1493	.0286	.2681	.02783	.76571	27
34	.23458	.97210	.24131	.1440	.0287	.2630	.02790	.76542	26
35	0.23486	0.97203	0.24162	4.1388	1.0288	4.2579	0.02797	0.76514	25
36	.23514	.97196	.24192	.1335	.0288	.2527	.02804	.76486	24
37	.23542	.97189	.24223	.1282	.0289	.2476	.02811	.76457	23
38	.23571	.97182	.24254	.1230	.0290	.2425	.02818	.76429	22
39	.23599	.97175	.24285	.1178	.0291	.2375	.02824	.76401	21
40	0.23627	0.97169	0.24316	4.1126	1.0291	4.2324	0.02831	0.76373	20
41	.23655	.97162	.24346	.1073	.0292	.2273	.02838	.76344	19
42	.23684	.97155	.24377	.1022	.0293	.2223	.02845	.76316	18
43	.23712	.97148	.24408	.0970	.0293	.2173	.02852	.76288	17
44	.23740	.97141	.24439	.0918	.0294	.2122	.02859	.76260	16
45	0.23768	0.97134	0.24470	4.0867	1.0295	4.2072	0.02866	0.76231	15
46	.23797	.97127	.24501	.0815	.0296	.2022	.02873	.76203	14
47	.23825	.97120	.24531	.0764	.0296	.1972	.02880	.76175	13
48	.23853	.97113	.24562	.0713	.0297	.1923	.02886	.76147	12
49	.23881	.97106	.24593	.0662	.0298	.1873	.02893	.76118	11
50	0.23910	0.97099	0.24624	4.0611	1.0299	4.1824	0.02900	0.76090	10
51	.23938	.97092	.24655	.0560	.0299	.1774	.02907	.76062	9
52	.23966	.97086	.24686	.0509	.0300	.1725	.02914	.76034	8
53	.23994	.97079	.24717	.0458	.0301	.1676	.02921	.76005	7
54	.24023	.97072	.24747	.0408	.0302	.1627	.02928	.75977	6
55	0.24051	0.97065	0.24778	4.0358	1.0302	4.1578	0.02935	0.75949	5
56	.24079	.97058	.24809	.0307	.0303	.1529	.02942	.75921	4
57	.24107	.97051	.24840	.0257	.0304	.1481	.02949	.75892	3
58	.24136	.97044	.24871	.0207	.0305	.1432	.02956	.75864	2
59	.24164	.97037	.24902	.0157	.0305	.1384	.02963	.75836	1
60	0.24192	0.97029	0.24933	4.0108	1.0306	4.1336	0.02970	0.75808	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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14° **Natural Trigonometric Functions** 165°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.24192	0.97029	0.24933	4.0108	1.0306	4.1336	0.02970	0.75808	60
1	.24220	.97022	.24964	.0058	.0307	.1287	.02977	.75779	59
2	.24249	.97015	.24995	.0009	.0308	.1239	.02984	.75751	58
3	.24277	.97008	.25025	3.9959	.0308	.1191	.02991	.75723	57
4	.24305	.97001	.25056	.9910	.0309	.1144	.02999	.75695	56
5	0.24333	0.96994	0.25087	3.9861	1.0310	4.1096	0.03006	0.75667	55
6	.24361	.96987	.25118	.9812	.0311	.1048	.03013	.75638	54
7	.24390	.96980	.25149	.9763	.0311	.1001	.03020	.75610	53
8	.24418	.96973	.25180	.9714	.0312	.0953	.03027	.75582	52
9	.24446	.96966	.25211	.9665	.0313	.0906	.03034	.75554	51
10	0.24474	0.96959	0.25242	3.9616	1.0314	4.0859	0.03041	0.75526	50
11	.24502	.96952	.25273	.9568	.0314	.0812	.03048	.75497	49
12	.24531	.96944	.25304	.9520	.0315	.0765	.03055	.75469	48
13	.24559	.96937	.25335	.9471	.0316	.0718	.03063	.75441	47
14	.24587	.96930	.25366	.9423	.0317	.0672	.03070	.75413	46
15	0.24615	0.96923	0.25397	3.9375	1.0317	4.0625	0.03077	0.75385	45
16	.24643	.96916	.25428	.9327	.0318	.0579	.03084	.75356	44
17	.24672	.96909	.25459	.9279	.0319	.0532	.03091	.75328	43
18	.24700	.96901	.25490	.9231	.0320	.0486	.03098	.75300	42
19	.24728	.96894	.25521	.9184	.0320	.0440	.03106	.75272	41
20	0.24756	0.96887	0.25552	3.9136	1.0321	4.0394	0.03113	0.75244	40
21	.24784	.96880	.25583	.9089	.0322	.0348	.03120	.75215	39
22	.24813	.96873	.25614	.9042	.0323	.0302	.03127	.75187	38
23	.24841	.96865	.25645	.8994	.0323	.0256	.03134	.75159	37
24	.24869	.96858	.25676	.8947	.0324	.0211	.03142	.75131	36
25	0.24897	0.96851	0.25707	3.8900	1.0325	4.0165	0.03149	0.75103	35
26	.24925	.96844	.25738	.8853	.0326	.0120	.03156	.75075	34
27	.24953	.96836	.25769	.8807	.0327	.0074	.03163	.75046	33
28	.24982	.96829	.25800	.8760	.0327	.0029	.03171	.75018	32
29	.25010	.96822	.25831	.8713	.0328	3.9984	.03178	.74990	31
30	0.25038	0.96815	0.25862	3.8667	1.0329	3.9939	0.03185	0.74962	30
31	.25066	.96807	.25893	.8621	.0330	.9894	.03192	.74934	29
32	.25094	.96800	.25924	.8574	.0330	.9850	.03200	.74906	28
33	.25122	.96793	.25955	.8528	.0331	.9805	.03207	.74877	27
34	.25151	.96785	.25986	.8482	.0332	.9760	.03214	.74849	26
35	0.25179	0.96778	0.26017	3.8436	1.0333	3.9716	0.03222	0.74821	25
36	.25207	.96771	.26048	.8390	.0334	.9672	.03229	.74793	24
37	.25235	.96763	.26079	.8345	.0334	.9627	.03236	.74765	23
38	.25263	.96756	.26110	.8299	.0335	.9583	.03244	.74737	22
39	.25291	.96749	.26141	.8254	.0336	.9539	.03251	.74709	21
40	0.25319	0.96741	0.26172	3.8208	1.0337	3.9495	0.03258	0.74680	20
41	.25348	.96734	.26203	.8163	.0338	.9451	.03266	.74652	19
42	.25376	.96727	.26234	.8118	.0338	.9408	.03273	.74624	18
43	.25404	.96719	.26266	.8073	.0339	.9364	.03281	.74596	17
44	.25432	.96712	.26297	.8027	.0340	.9320	.03288	.74568	16
45	0.25460	0.96704	0.26328	3.7983	1.0341	3.9277	0.03295	0.74540	15
46	.25488	.96697	.26359	.7938	.0341	.9234	.03303	.74512	14
47	.25516	.96690	.26390	.7893	.0342	.9190	.03310	.74483	13
48	.25544	.96682	.26421	.7848	.0343	.9147	.03318	.74455	12
49	.25573	.96675	.26452	.7804	.0344	.9104	.03325	.74427	11
50	0.25601	0.96667	0.26483	3.7759	1.0345	3.9061	0.03332	0.74399	10
51	.25629	.96660	.26514	.7715	.0345	.9018	.03340	.74371	9
52	.25657	.96652	.26546	.7671	.0346	.8976	.03347	.74344	8
53	.25685	.96645	.26577	.7627	.0347	.8933	.03355	.74315	7
54	.25713	.96638	.26608	.7583	.0348	.8890	.03362	.74287	6
55	0.25741	0.96630	0.26639	3.7539	1.0349	3.8848	0.03370	0.74259	5
56	.25769	.96623	.26670	.7495	.0349	.8805	.03377	.74230	4
57	.25798	.96615	.26701	.7451	.0350	.8763	.03385	.74202	3
58	.25826	.96608	.26732	.7407	.0351	.8721	.03392	.74174	2
59	.25854	.96600	.26764	.7364	.0352	.8679	.03400	.74146	1
60	0.25882	0.96592	0.26795	3.7320	1.0353	3.8637	0.03407	0.74118	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

104° 75°

15° **Natural Trigonometric Functions** **164°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.25882	0.96592	0.26795	3.7320	1.0353	3.8637	0.03407	0.74118	60
1	.25910	.96585	.26826	.7277	.0353	.8595	.03415	.74090	59
2	.25938	.96577	.26857	.7234	.0354	.8553	.03422	.74062	58
3	.25966	.96570	.26888	.7191	.0355	.8512	.03430	.74034	57
4	.25994	.96562	.26920	.7147	.0356	.8470	.03438	.74006	56
5	0.26022	0.96555	0.26951	3.7104	1.0357	3.8428	0.03445	0.73978	55
6	.26050	.96547	.26982	.7062	.0358	.8387	.03453	.73949	54
7	.26078	.96540	.27013	.7019	.0358	.8346	.03460	.73921	53
8	.26107	.96532	.27044	.6976	.0359	.8304	.03468	.73893	52
9	.26135	.96524	.27076	.6933	.0360	.8263	.03475	.73865	51
10	0.26163	0.96517	0.27107	3.6891	1.0361	3.8222	0.03483	0.73837	50
11	.26191	.96509	.27138	.6848	.0362	.8181	.03491	.73809	49
12	.26219	.96502	.27169	.6806	.0362	.8140	.03498	.73781	48
13	.26247	.96494	.27201	.6764	.0363	.8100	.03506	.73753	47
14	.26275	.96486	.27232	.6722	.0364	.8059	.03514	.73725	46
15	0.26303	0.96479	0.27263	3.6679	1.0365	3.8018	0.03521	0.73697	45
16	.26331	.96471	.27294	.6637	.0366	.7978	.03529	.73669	44
17	.26359	.96463	.27326	.6596	.0367	.7937	.03536	.73641	43
18	.26387	.96456	.27357	.6554	.0367	.7897	.03544	.73613	42
19	.26415	.96448	.27388	.6512	.0368	.7857	.03552	.73585	41
20	0.26443	0.96440	0.27419	3.6470	1.0369	3.7816	0.03560	0.73556	40
21	.26471	.96433	.27451	.6429	.0370	.7776	.03567	.73528	39
22	.26499	.96425	.27482	.6387	.0371	.7736	.03575	.73500	38
23	.26527	.96417	.27513	.6346	.0371	.7697	.03583	.73472	37
24	.26556	.96409	.27544	.6305	.0372	.7657	.03590	.73444	36
25	0.26584	0.96402	0.27576	3.6263	1.0373	3.7617	0.03598	0.73416	35
26	.26612	.96394	.27607	.6222	.0374	.7577	.03606	.73388	34
27	.26640	.96386	.27638	.6181	.0375	.7538	.03614	.73360	33
28	.26668	.96378	.27670	.6140	.0376	.7498	.03621	.73332	32
29	.26696	.96371	.27701	.6100	.0376	.7459	.03629	.73304	31
30	0.26724	0.96363	0.27732	3.6059	1.0377	3.7420	0.03637	0.73276	30
31	.26752	.96355	.27764	.6018	.0378	.7380	.03645	.73248	29
32	.26780	.96347	.27795	.5977	.0379	.7341	.03652	.73220	28
33	.26808	.96340	.27826	.5937	.0380	.7302	.03660	.73192	27
34	.26836	.96332	.27858	.5896	.0381	.7263	.03668	.73164	26
35	0.26864	0.96324	0.27889	3.5856	1.0382	3.7224	0.03676	0.73136	25
36	.26892	.96316	.27920	.5816	.0382	.7186	.03684	.73108	24
37	.26920	.96308	.27952	.5776	.0383	.7147	.03691	.73080	23
38	.26948	.96301	.27983	.5736	.0384	.7108	.03699	.73052	22
39	.26976	.96293	.28014	.5696	.0385	.7070	.03707	.73024	21
40	0.27004	0.96285	0.28046	3.5656	1.0386	3.7031	0.03715	0.72996	20
41	.27032	.96277	.28077	.5616	.0387	.6993	.03723	.72968	19
42	.27060	.96269	.28109	.5576	.0387	.6955	.03731	.72940	18
43	.27088	.96261	.28140	.5536	.0388	.6917	.03739	.72912	17
44	.27116	.96253	.28171	.5497	.0389	.6878	.03746	.72884	16
45	0.27144	0.96245	0.28203	3.5457	1.0390	3.6840	0.03754	0.72856	15
46	.27172	.96238	.28234	.5418	.0391	.6802	.03762	.72828	14
47	.27200	.96230	.28266	.5378	.0392	.6765	.03770	.72800	13
48	.27228	.96222	.28297	.5339	.0393	.6727	.03778	.72772	12
49	.27256	.96214	.28328	.5300	.0393	.6689	.03786	.72744	11
50	0.27284	0.96206	0.28360	3.5261	1.0394	3.6651	0.03794	0.72716	10
51	.27312	.96198	.28391	.5222	.0395	.6614	.03802	.72688	9
52	.27340	.96190	.28423	.5183	.0396	.6576	.03810	.72660	8
53	.27368	.96182	.28454	.5144	.0397	.6539	.03818	.72632	7
54	.27396	.96174	.28486	.5105	.0398	.6502	.03826	.72604	6
55	0.27424	0.96166	0.28517	3.5066	1.0399	3.6464	0.03834	0.72576	5
56	.27452	.96158	.28549	.5028	.0399	.6427	.03842	.72548	4
57	.27480	.96150	.28580	.4989	.0400	.6390	.03850	.72520	3
58	.27508	.96142	.28611	.4951	.0401	.6353	.03858	.72492	2
59	.27536	.96134	.28643	.4912	.0402	.6316	.03866	.72464	1
60	0.27564	0.96126	0.28674	3.4874	1.0403	3.6279	0.03874	0.72436	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

105° **74°**

16° **Natural Trigonometric Functions** **163°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.27564	0.96126	0.28674	3.4874	1.0403	3.6279	0.03874	0.72436	60
1	.27592	.96118	.28706	.4836	.0404	.6243	.03882	.72408	59
2	.27620	.96110	.28737	.4798	.0405	.6206	.03890	.72380	58
3	.27648	.96102	.28769	.4760	.0406	.6169	.03898	.72352	57
4	.27675	.96094	.28800	.4722	.0406	.6133	.03906	.72324	56
5	0.27703	0.96086	0.28832	3.4684	1.0407	3.6096	0.03914	0.72296	55
6	.27731	.96078	.28863	.4646	.0408	.6060	.03922	.72268	54
7	.27759	.96070	.28895	.4608	.0409	.6024	.03930	.72240	53
8	.27787	.96062	.28926	.4570	.0410	.5987	.03938	.72213	52
9	.27815	.96054	.28958	.4533	.0411	.5951	.03946	.72185	51
10	0.27843	0.96045	0.28990	3.4495	1.0412	3.5915	0.03954	0.72157	50
11	.27871	.96037	.29021	.4458	.0413	.5879	.03962	.72129	49
12	.27899	.96029	.29053	.4420	.0413	.5843	.03971	.72101	48
13	.27927	.96021	.29084	.4383	.0414	.5807	.03979	.72073	47
14	.27955	.96013	.29116	.4346	.0415	.5772	.03987	.72045	46
15	0.27983	0.96005	0.29147	3.4308	1.0416	3.5736	0.03995	0.72017	45
16	.28011	.95997	.29179	.4271	.0417	.5700	.04003	.71989	44
17	.28039	.95989	.29210	.4234	.0418	.5665	.04011	.71961	43
18	.28067	.95980	.29242	.4197	.0419	.5629	.04019	.71933	42
19	.28094	.95972	.29274	.4160	.0420	.5594	.04028	.71905	41
20	0.28122	0.95964	0.29305	3.4124	1.0420	3.5559	0.04036	0.71877	40
21	.28150	.95956	.29337	.4087	.0421	.5523	.04044	.71849	39
22	.28178	.95948	.29368	.4050	.0422	.5488	.04052	.71822	38
23	.28206	.95940	.29400	.4014	.0423	.5453	.04060	.71794	37
24	.28234	.95931	.29432	.3977	.0424	.5418	.04069	.71766	36
25	0.28262	0.95923	0.29463	3.3941	1.0425	3.5383	0.04077	0.71738	35
26	.28290	.95915	.29495	.3904	.0426	.5348	.04085	.71710	34
27	.28318	.95907	.29526	.3868	.0427	.5313	.04093	.71682	33
28	.28346	.95898	.29558	.3832	.0428	.5279	.04101	.71654	32
29	.28374	.95890	.29590	.3795	.0428	.5244	.04110	.71626	31
30	0.28401	0.95882	0.29621	3.3759	1.0429	3.5209	0.04118	0.71608	30
31	.28429	.95874	.29653	.3723	.0430	.5175	.04126	.71570	29
32	.28457	.95865	.29685	.3687	.0431	.5140	.04134	.71543	28
33	.28485	.95857	.29716	.3651	.0432	.5106	.04143	.71515	27
34	.28513	.95849	.29748	.3616	.0433	.5072	.04151	.71487	26
35	0.28541	0.95840	0.29780	3.3580	1.0434	3.5037	0.04159	0.71459	25
36	.28569	.95832	.29811	.3544	.0435	.5003	.04168	.71431	24
37	.28597	.95824	.29843	.3509	.0436	.4969	.04176	.71403	23
38	.28624	.95816	.29875	.3473	.0437	.4935	.04184	.71375	22
39	.28652	.95807	.29906	.3438	.0438	.4901	.04193	.71347	21
40	0.28680	0.95799	0.29938	3.3402	1.0438	3.4867	0.04201	0.71320	20
41	.28708	.95791	.29970	.3367	.0439	.4833	.04209	.71292	19
42	.28736	.95782	.30001	.3332	.0440	.4799	.04218	.71264	18
43	.28764	.95774	.30033	.3296	.0441	.4766	.04226	.71236	17
44	.28792	.95765	.30065	.3261	.0442	.4732	.04234	.71208	16
45	0.28820	0.95757	0.30096	3.3226	1.0443	3.4698	0.04243	0.71180	15
46	.28847	.95749	.30128	.3191	.0444	.4665	.04251	.71152	14
47	.28875	.95740	.30160	.3156	.0445	.4632	.04260	.71125	13
48	.28903	.95732	.30192	.3121	.0446	.4598	.04268	.71097	12
49	.28931	.95723	.30223	.3087	.0447	.4565	.04276	.71069	11
50	0.28959	0.95715	0.30255	3.3052	1.0448	3.4532	0.04285	0.71041	10
51	.28987	.95707	.30287	.3017	.0448	.4498	.04293	.71013	9
52	.29014	.95698	.30319	.2983	.0449	.4465	.04302	.70985	8
53	.29042	.95690	.30350	.2948	.0450	.4432	.04310	.70958	7
54	.29070	.95681	.30382	.2914	.0451	.4399	.04319	.70930	6
55	0.29098	0.95673	0.30414	3.2879	1.0452	3.4366	0.04327	0.70902	5
56	.29126	.95664	.30446	.2845	.0453	.4334	.04335	.70874	4
57	.29154	.95656	.30478	.2811	.0454	.4301	.04344	.70846	3
58	.29181	.95647	.30509	.2777	.0455	.4268	.04352	.70818	2
59	.29209	.95639	.30541	.2742	.0456	.4236	.04361	.70791	1
60	0.29237	0.95630	0.30573	3.2708	1.0457	3.4203	0.04369	0.70763	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

106° **73°**

17° Natural Trigonometric Functions **162°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.29237	0.95630	0.30573	3.2708	1.0457	3.4203	0.04369	0.70763	60
1	.29265	.95622	.30605	.2674	.0458	.4170	.04378	.70735	59
2	.29293	.95613	.30637	.2640	.0459	.4138	.04386	.70707	58
3	.29321	.95605	.30668	.2607	.0460	.4106	.04395	.70679	57
4	.29348	.95596	.30700	.2573	.0461	.4073	.04404	.70651	56
5	0.29376	0.95588	0.30732	3.2539	1.0461	3.4041	0.04412	0.70624	55
6	.29404	.95579	.30764	.2505	.0462	.4009	.04421	.70596	54
7	.29432	.95571	.30796	.2472	.0463	.3977	.04426	.70568	53
8	.29460	.95562	.30828	.2438	.0464	.3945	.04438	.70540	52
9	.29487	.95554	.30859	.2405	.0465	.3913	.04446	.70512	51
10	0.29515	0.95545	0.30891	3.2371	1.0466	3.3881	0.04455	0.70485	50
11	.29543	.95536	.30923	.2338	.0467	.3849	.04463	.70457	49
12	.29571	.95528	.30955	.2305	.0468	.3817	.04472	.70429	48
13	.29598	.95519	.30987	.2271	.0469	.3785	.04481	.70401	47
14	.29626	.95511	.31019	.2238	.0470	.3754	.04489	.70374	46
15	0.29654	0.95502	0.31051	3.2205	1.0471	3.3722	0.04498	0.70346	45
16	.29682	.95493	.31083	.2172	.0472	.3690	.04507	.70318	44
17	.29710	.95485	.31115	.2139	.0473	.3659	.04515	.70290	43
18	.29737	.95476	.31146	.2106	.0474	.3627	.04524	.70262	42
19	.29765	.95467	.31178	.2073	.0475	.3596	.04532	.70235	41
20	0.29793	0.95459	0.31210	3.2041	1.0476	3.3565	0.04541	0.70207	40
21	.29821	.95450	.31242	.2008	.0477	.3534	.04550	.70179	39
22	.29848	.95441	.31274	.1975	.0478	.3502	.04558	.70151	38
23	.29876	.95433	.31306	.1942	.0478	.3471	.04567	.70124	37
24	.29904	.95424	.31338	.1910	.0479	.3440	.04576	.70096	36
25	0.29932	0.95415	0.31370	3.1877	1.0480	3.3409	0.04585	0.70068	35
26	.29959	.95407	.31402	.1845	.0481	.3378	.04593	.70040	34
27	.29987	.95398	.31434	1813	.0482	.3347	.04602	.70013	33
28	.30015	.95389	.31466	.1780	.0483	.3316	.04611	.69982	32
29	.30043	.95380	.31498	.1748	.0484	.3286	.04619	.69957	31
30	0.30070	0.95372	0.31530	3.1716	1.0485	3.3255	0.04628	0.69929	30
31	.30098	.95363	.31562	.1684	.0486	.3224	.04637	.69902	29
32	.30126	.95354	.31594	.1652	.0487	.3194	.04646	.69874	28
33	.30154	.95345	.31626	.1620	.0488	.3163	.04654	.69846	27
34	.30181	.95337	.31658	.1588	.0489	.3133	.04663	.69818	26
35	0.30209	0.95328	0.31690	3.1556	1.0490	3.3102	0.04672	0.69791	25
36	.30237	.95319	.31722	.1524	.0491	.3072	.04681	.69763	24
37	.30265	.95310	.31754	.1492	.0492	.3042	.04690	.69735	23
38	.30292	.95301	.31786	.1460	.0493	.3011	.04698	.69707	22
39	.30320	.95293	.31818	.1429	.0494	.2981	.04707	.69680	21
40	0.30348	0.95284	0.31850	3.1397	1.0495	3.2951	0.04716	0.69652	20
41	.30375	.95275	.31882	.1366	.0496	.2921	.04725	.69624	19
42	.30403	.95266	.31914	.1334	.0497	.2891	.04734	.69597	18
43	.30431	.95257	.31946	.1303	.0498	.2861	.04743	.69569	17
44	.30459	.95248	.31978	.1271	.0499	.2831	.04751	.69541	16
45	0.30486	0.95239	0.32010	3.1240	1.0500	3.2801	0.04760	0.69513	15
46	.30514	.95231	.32042	.1209	.0501	.2772	.04769	.69486	14
47	.30542	.95222	.32074	.1177	.0502	.2742	.04778	.69458	13
48	.30569	.95213	.32106	.1146	.0503	.2712	.04787	.69430	12
49	.30597	.95204	.32138	.1115	.0504	.2683	.04796	.69403	11
50	0.30625	0.95195	0.32171	3.1084	1.0505	3.2653	0.04805	0.69375	10
51	.30653	.95186	.32203	.1053	.0506	.2624	.04814	.69347	9
52	.30680	.95177	.32235	.1022	.0507	.2594	.04823	.69320	8
53	.30708	.95168	.32267	.0991	.0508	.2565	.04832	.69292	7
54	.30736	.95159	.32299	.0960	.0509	.2535	.04840	.69264	6
55	0.30763	0.95150	0.32331	3.0930	1.0510	3.2506	0.04849	0.69237	5
56	.30791	.95141	.32363	.0899	.0511	.2477	.04858	.69209	4
57	.30819	.95132	.32395	.0868	.0512	.2448	.04867	.69181	3
58	.30846	.95124	.32428	.0838	.0513	.2419	.04876	.69154	2
59	.30874	.95115	.32460	.0807	.0514	.2390	.04885	.69126	1
60	0.30902	0.95106	0.32492	3.0777	1.0515	3.2361	0.04894	0.69098	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

107° **72°**

18° **Natural Trigonometric Functions** **161°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.30902	0.95106	0.32492	3.0777	1.0515	3.2361	0.04894	0.69098	60
1	.30929	.95097	.32524	.0746	.0516	.2332	.04903	.69071	59
2	.30957	.95088	.32556	.0716	.0517	.2303	.04912	.69043	58
3	.30985	.95079	.32588	.0686	.0518	.2274	.04921	.69015	57
4	.31012	.95070	.32621	.0655	.0519	.2245	.04930	.68988	56
5	0.31040	0.95061	0.32653	3.0625	1.0520	3.2216	0.04939	0.68960	55
6	.31068	.95051	.32685	.0595	.0521	.2188	.04948	.68932	54
7	.31095	.95042	.32717	.0565	.0522	.2159	.04957	.68905	53
8	.31123	.95033	.32749	.0535	.0523	.2131	.04966	.68877	52
9	.31150	.95024	.32782	.0505	.0524	.2102	.04975	.68849	51
10	0.31178	0.95015	0.32814	3.0475	1.0525	3.2074	0.04985	0.68822	50
11	.31206	.95006	.32846	.0445	.0526	.2045	.04994	.68794	49
12	.31233	.94997	.32878	.0415	.0527	.2017	.05003	.68766	48
13	.31261	.94988	.32910	.0385	.0528	.1989	.05012	.68739	47
14	.31289	.94979	.32943	.0356	.0529	.1960	.05021	.68711	46
15	0.31316	0.94970	0.32975	3.0326	1.0530	3.1932	0.05030	0.68684	45
16	.31344	.94961	.33007	.0296	.0531	.1904	.05039	.68656	44
17	.31372	.94952	.33039	.0267	.0532	.1876	.05048	.68628	43
18	.31399	.94942	.33072	.0237	.0533	.1848	.05057	.68601	42
19	.31427	.94933	.33104	.0208	.0534	.1820	.05066	.68573	41
20	0.31454	0.94924	0.33136	3.0178	1.0535	3.1792	0.05076	0.68545	40
21	.31482	.94915	.33169	.0149	.0536	.1764	.05085	.68518	39
22	.31510	.94906	.33201	.0120	.0537	.1736	.05094	.68490	38
23	.31537	.94897	.33233	.0090	.0538	.1708	.05103	.68463	37
24	.31565	.94888	.33265	.0061	.0539	.1681	.05112	.68435	36
25	0.31592	0.94878	0.33298	3.0032	1.0540	3.1653	0.05121	0.68407	35
26	.31620	.94869	.33330	.0003	.0541	.1625	.05131	.68380	34
27	.31648	.94860	.33362	2.9974	.0542	.1598	.05140	.68352	33
28	.31675	.94851	.33395	.9945	.0543	.1570	.05149	.68325	32
29	.31703	.94841	.33427	.9916	.0544	.1543	.05158	.68297	31
30	0.31730	0.94832	0.33459	2.9887	1.0545	3.1515	0.05168	0.68269	30
31	.31758	.94823	.33492	.9858	.0546	.1488	.05177	.68242	29
32	.31786	.94814	.33524	.9829	.0547	.1461	.05186	.68214	28
33	.31813	.94805	.33557	.9800	.0548	.1433	.05195	.68187	27
34	.31841	.94795	.33589	.9772	.0549	.1406	.05205	.68159	26
35	0.31868	0.94786	0.33621	2.9743	1.0550	3.1379	0.05214	0.68132	25
36	.31896	.94777	.33654	.9714	.0551	.1352	.05223	.68104	24
37	.31923	.94767	.33686	.9686	.0552	.1325	.05232	.68076	23
38	.31951	.94758	.33718	.9657	.0553	.1298	.05242	.68049	22
39	.31978	.94749	.33751	.9629	.0554	.1271	.05251	.68021	21
40	0.32006	0.94740	0.33783	2.9600	1.0555	3.1244	0.05260	0.67994	20
41	.32034	.94730	.33816	.9572	.0556	.1217	.05270	.67966	19
42	.32061	.94721	.33848	.9544	.0557	.1190	.05279	.67939	18
43	.32089	.94712	.33880	.9515	.0558	.1163	.05288	.67911	17
44	.32116	.94702	.33913	.9487	.0559	.1137	.05297	.67884	16
45	0.32144	0.94693	0.33945	2.9459	1.0560	3.1110	0.05307	0.67856	15
46	.32171	.94684	.33978	.9431	.0561	.1083	.05316	.67828	14
47	.32199	.94674	.34010	.9403	.0562	.1057	.05326	.67801	13
48	.32226	.94665	.34043	.9375	.0563	.1030	.05335	.67773	12
49	.32254	.94655	.34075	.9347	.0565	.1004	.05344	.67746	11
50	0.32282	0.94646	0.34108	2.9319	1.0566	3.0977	0.05354	0.67718	10
51	.32309	.94637	.34140	.9291	.0567	.0951	.05363	.67691	9
52	.32337	.94627	.34173	.9263	.0568	.0925	.05373	.67663	8
53	.32364	.94618	.34205	.9235	.0569	.0898	.05382	.67636	7
54	.32392	.94608	.34238	.9208	.0570	.0872	.05391	.67608	6
55	0.32419	0.94599	0.34270	2.9180	1.0571	3.0846	0.05401	0.67581	5
56	.32447	.94590	.34303	.9152	.0572	.0820	.05410	.67553	4
57	.32474	.94580	.34335	.9125	.0573	.0793	.05420	.67526	3
58	.32502	.94571	.34368	.9097	.0574	.0767	.05429	.67498	2
59	.32529	.94561	.34400	.9069	.0575	.0741	.05439	.67471	1
60	0.32557	0.94552	0.34433	2.9042	1.0576	3.0715	0.05448	0.67443	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

108° **71°**

19° **Natural Trigonometric Functions** 160°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.32557	0.94552	0.34433	2.9042	1.0576	3.0715	0.05448	0.67443	60
1	.32584	.94542	.34465	.9015	.0577	.0690	.05458	.67416	59
2	.32612	.94533	.34498	.8987	.0578	.0664	.05467	.67388	58
3	.32639	.94523	.34530	.8960	.0579	.0638	.05476	.67361	57
4	.32667	.94514	.34563	.8933	.0580	.0612	.05486	.67333	56
5	0.32694	0.94504	0.34595	2.8905	1.0581	3.0586	0.05495	0.67306	55
6	.32722	.94495	.34628	.8878	.0582	.0561	.05505	.67278	54
7	.32749	.94485	.34661	.8851	.0584	.0535	.05515	.67251	53
8	.32777	.94476	.34693	.8824	.0585	.0509	.05524	.67223	52
9	.32804	.94466	.34726	.8797	.0586	.0484	.05534	.67196	51
10	0.32832	0.94457	0.34758	2.8770	1.0587	3.0458	0.05543	0.67168	50
11	.32859	.94447	.34791	.8743	.0588	.0433	.05553	.67141	49
12	.32887	.94438	.34824	.8716	.0589	.0407	.05562	.67113	48
13	.32914	.94428	.34856	.8689	.0590	.0382	.05572	.67086	47
14	.32942	.94418	.34889	.8662	.0591	.0357	.05581	.67058	46
15	0.32969	0.94409	0.34921	2.8636	1.0592	3.0331	0.05591	0.67031	45
16	.32996	.94399	.34954	.8609	.0593	.0306	.05601	.67003	44
17	.33024	.94390	.34987	.8582	.0594	.0281	.05610	.66976	43
18	.33051	.94380	.35019	.8555	.0595	.0256	.05620	.66948	42
19	.33079	.94370	.35052	.8529	.0596	.0231	.05629	.66921	41
20	0.33106	0.94361	0.35085	2.8502	1.0598	3.0206	0.05639	0.66894	40
21	.33134	.94351	.35117	.8476	.0599	.0181	.05649	.66866	39
22	.33161	.94341	.35150	.8449	.0600	.0156	.05658	.66839	38
23	.33189	.94332	.35183	.8423	.0601	.0131	.05668	.66811	37
24	.33216	.94322	.35215	.8396	.0602	.0106	.05678	.66784	36
25	0.33243	0.94313	0.35248	2.8370	1.0603	3.0081	0.05687	0.66756	35
26	.33271	.94303	.35281	.8344	.0604	.0056	.05697	.66729	34
27	.33298	.94293	.35314	.8318	.0605	.0031	.05707	.66701	33
28	.33326	.94283	.35346	.8291	.0606	.0007	.05716	.66674	32
29	.33353	.94274	.35379	.8265	.0607	2.9982	.05726	.66647	31
30	0.33381	0.94264	0.35412	2.8239	1.0608	2.9957	0.05736	0.66619	30
31	.33408	.94254	.35445	.8213	.0609	.9933	.05745	.66592	29
32	.33435	.94245	.35477	.8187	.0611	.9908	.05755	.66564	28
33	.33463	.94235	.35510	.8161	.0612	.9884	.05765	.66537	27
34	.33490	.94225	.35543	.8135	.0613	.9859	.05775	.66510	26
35	0.33518	0.94215	0.35576	2.8109	1.0614	2.9835	0.05784	0.66482	25
36	.33545	.94206	.35608	.8083	.0615	.9810	.05794	.66455	24
37	.33572	.94196	.35641	.8057	.0616	.9786	.05804	.66427	23
38	.33600	.94186	.35674	.8032	.0617	.9762	.05814	.66400	22
39	.33627	.94176	.35707	.8006	.0618	.9738	.05823	.66373	21
40	0.33655	0.94167	0.35739	2.7980	1.0619	2.9713	0.05833	0.66345	20
41	.33682	.94157	.35772	.7954	.0620	.9689	.05843	.66318	19
42	.33709	.94147	.35805	.7929	.0622	.9665	.05853	.66290	18
43	.33737	.94137	.35838	.7903	.0623	.9641	.05863	.66263	17
44	.33764	.94127	.35871	.7878	.0624	.9617	.05872	.66236	16
45	0.33792	0.94118	0.35904	2.7852	1.0625	2.9593	0.05882	0.66208	15
46	.33819	.94108	.35936	.7827	.0626	.9569	.05892	.66181	14
47	.33846	.94098	.35969	.7801	.0627	.9545	.05902	.66153	13
48	.33874	.94088	.36002	.7776	.0628	.9521	.05912	.66126	12
49	.33901	.94078	.36035	.7751	.0629	.9497	.05922	.66099	11
50	0.33928	0.94068	0.36068	2.7725	1.0630	2.9474	0.05932	0.66071	10
51	.33956	.94058	.36101	.7700	.0632	.9450	.05941	.66044	9
52	.33983	.94049	.36134	.7675	.0633	.9426	.05951	.66017	8
53	.34011	.94039	.36167	.7650	.0634	.9402	.05961	.65989	7
54	.34038	.94029	.36199	.7625	.0635	.9379	.05971	.65962	6
55	0.34065	0.94019	0.36232	2.7600	1.0636	2.9355	0.05981	0.65935	5
56	.34093	.94009	.36265	.7575	.0637	.9332	.05991	.65907	4
57	.34120	.93999	.36298	.7550	.0638	.9308	.06001	.65880	3
58	.34147	.93989	.36331	.7525	.0639	.9285	.06011	.65853	2
59	.34175	.93979	.36364	.7500	.0641	.9261	.06021	.65825	1
60	0.34202	0.93969	0.36397	2.7475	1.0642	2.9238	0.06031	0.65798	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

109° 70°

20° **Natural Trigonometric Functions** 159°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.34202	0.93969	0.36397	2.7475	1.0642	2.9238	0.06031	0.65798	60
1	.34229	.93959	.36430	.7450	.0643	.9215	.06041	.65771	59
2	.34257	.93949	.36463	.7425	.0644	.9191	.06051	.65743	58
3	.34284	.93939	.36496	.7400	.0645	.9168	.06061	.65716	57
4	.34311	.93929	.36529	.7376	.0646	.9145	.06071	.65689	56
5	0.34339	0.93919	0.36562	2.7351	1.0647	2.9122	0.06080	0.65661	55
6	.34366	.93909	.36595	.7326	.0648	.9098	.06090	.65634	54
7	.34393	.93899	.36628	.7302	.0650	.9075	.06100	.65607	53
8	.34421	.93889	.36661	.7277	.0651	.9052	.06110	.65579	52
9	.34448	.93879	.36694	.7252	.0652	.9029	.06121	.65552	51
10	0.34475	0.93869	0.36727	2.7228	1.0653	2.9006	0.06131	0.65525	50
11	.34502	.93859	.36760	.7204	.0654	.8983	.06141	.65497	49
12	.34530	.93849	.36793	.7179	.0655	.8960	.06151	.65470	48
13	.34557	.93839	.36826	.7155	.0656	.8937	.06161	.65443	47
14	.34584	.93829	.36859	.7130	.0658	.8915	.06171	.65415	46
15	0.34612	0.93819	0.36892	2.7106	1.0659	2.8892	0.06181	0.65388	45
16	.34639	.93809	.36925	.7082	.0660	.8869	.06191	.65361	44
17	.34666	.93799	.36958	.7058	.0661	.8846	.06201	.65334	43
18	.34693	.93789	.36991	.7033	.0662	.8824	.06211	.65306	42
19	.34721	.93779	.37024	.7009	.0663	.8801	.06221	.65279	41
20	0.34748	0.93769	0.37057	2.6985	1.0664	2.8778	0.06231	0.65252	40
21	.34775	.93758	.37090	.6961	.0666	.8756	.06241	.65225	39
22	.34803	.93748	.37123	.6937	.0667	.8733	.06251	.65197	38
23	.34830	.93738	.37156	.6913	.0668	.8711	.06262	.65170	37
24	.34857	.93728	.37190	.6889	.0669	.8688	.06272	.65143	36
25	0.34884	0.93718	0.37223	2.6865	1.0670	2.8666	0.06282	0.65115	35
26	.34912	.93708	.37256	.6841	.0671	.8644	.06292	.65088	34
27	.34939	.93698	.37289	.6817	.0673	.8621	.06302	.65061	33
28	.34966	.93687	.37322	.6794	.0674	.8599	.06312	.65034	32
29	.34993	.93677	.37355	.6770	.0675	.8577	.06323	.65006	31
30	0.35021	0.93667	0.37388	2.6746	1.0676	2.8554	0.06333	0.64979	30
31	.35048	.93657	.37422	.6722	.0677	.8532	.06343	.64952	29
32	.35075	.93647	.37455	.6699	.0678	.8510	.06353	.64925	28
33	.35102	.93637	.37488	.6675	.0679	.8488	.06363	.64897	27
34	.35130	.93626	.37521	.6652	.0681	.8466	.06373	.64870	26
35	0.35157	0.93616	0.37554	2.6628	1.0682	2.8444	0.06384	0.64843	25
36	.35184	.93606	.37587	.6604	.0683	.8422	.06394	.64816	24
37	.35211	.93596	.37621	.6581	.0684	.8400	.06404	.64789	23
38	.35239	.93585	.37654	.6558	.0685	.8378	.06414	.64761	22
39	.35266	.93575	.37687	.6534	.0686	.8356	.06425	.64734	21
40	0.35293	0.93565	0.37720	2.6511	1.0688	2.8334	0.06435	0.64707	20
41	.35320	.93555	.37754	.6487	.0689	.8312	.06445	.64680	19
42	.35347	.93544	.37787	.6464	.0690	.8290	.06456	.64652	18
43	.35375	.93534	.37820	.6441	.0691	.8269	.06466	.64625	17
44	.35402	.93524	.37853	.6418	.0692	.8247	.06476	.64598	16
45	0.35429	0.93513	0.37887	2.6394	1.0694	2.8225	0.06486	0.64571	15
46	.35456	.93503	.37920	.6371	.0695	.8204	.06497	.64544	14
47	.35483	.93493	.37953	.6348	.0696	.8182	.06507	.64516	13
48	.35511	.93482	.37986	.6325	.0697	.8160	.06517	.64489	12
49	.35538	.93472	.38020	.6302	.0698	.8139	.06528	.64462	11
50	0.35565	0.93462	0.38053	2.6279	1.0699	2.8117	0.06538	0.64435	10
51	.35592	.93451	.38086	.6256	.0701	.8096	.06548	.64408	9
52	.35619	.93441	.38120	.6233	.0702	.8074	.06559	.64380	8
53	.35647	.93431	.38153	.6210	.0703	.8053	.06569	.64353	7
54	.35674	.93420	.38186	.6187	.0704	.8032	.06579	.64326	6
55	0.35701	0.93410	0.38220	2.6164	1.0705	2.8010	0.06590	0.64299	5
56	.35728	.93400	.38253	.6142	.0707	.7989	.06600	.64272	4
57	.35755	.93389	.38286	.6119	.0708	.7968	.06611	.64245	3
58	.35782	.93379	.38320	.6096	.0709	.7947	.06621	.64217	2
59	.35810	.93368	.38353	.6073	.0710	.7925	.06631	.64190	1
60	0.35837	0.93358	0.38386	2.6051	1.0711	2.7904	0.06642	0.64163	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

110° 69°

21° **Natural Trigonometric Functions** 158°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.35837	0.93358	0.38386	2.6051	1.0711	2.7904	0.06642	0.64163	60
1	.35864	.93348	.38420	.6028	.0713	.7883	.06652	.64136	59
2	.35891	.93337	.38453	.6006	.0714	.7862	.06663	.64109	58
3	.35918	.93327	.38486	.5983	.0715	.7841	.06673	.64082	57
4	.35945	.93316	.38520	.5960	.0716	.7820	.06684	.64055	56
5	0.35972	0.93306	0.38553	2.5938	1.0717	2.7799	0.06694	0.64027	55
6	.36000	.93295	.38587	.5916	.0719	.7778	.06705	.64000	54
7	.36027	.93285	.38620	.5893	.0720	.7757	.06715	.63973	53
8	.36054	.93274	.38654	.5871	.0721	.7736	.06726	.63946	52
9	.36081	.93264	.38687	.5848	.0722	.7715	.06736	.63919	51
10	0.36108	0.93253	0.38720	2.5826	1.0723	2.7694	0.06747	0.63892	50
11	.36135	.93243	.38754	.5804	.0725	.7674	.06757	.63865	49
12	.36162	.93232	.38787	.5781	.0726	.7653	.06768	.63837	48
13	.36189	.93222	.38821	.5759	.0727	.7632	.06778	.63810	47
14	.36217	.93211	.38854	.5737	.0728	.7611	.06789	.63783	46
15	0.36244	0.93201	0.38888	2.5715	1.0729	2.7591	0.06799	0.63756	45
16	.36271	.93190	.38921	.5693	.0731	.7570	.06810	.63729	44
17	.36298	.93180	.38955	.5671	.0732	.7550	.06820	.63702	43
18	.36325	.93169	.38988	.5649	.0733	.7529	.06831	.63675	42
19	.36352	.93158	.39022	.5627	.0734	.7509	.06841	.63648	41
20	0.36379	0.93148	0.39055	2.5605	1.0736	2.7488	0.06852	0.63621	40
21	.36406	.93137	.39089	.5583	.0737	.7468	.06863	.63593	39
22	.36433	.93127	.39122	.5561	.0738	.7447	.06873	.63566	38
23	.36460	.93116	.39156	.5539	.0739	.7427	.06884	.63539	37
24	.36488	.93105	.39189	.5517	.0740	.7406	.06894	.63512	36
25	0.36515	0.93095	0.39223	2.5495	1.0742	2.7386	0.06905	0.63485	35
26	.36542	.93084	.39257	.5473	.0743	.7366	.06916	.63458	34
27	.36569	.93074	.39290	.5451	.0744	.7346	.06926	.63431	33
28	.36596	.93063	.39324	.5430	.0745	.7325	.06937	.63404	32
29	.36623	.93052	.39357	.5408	.0747	.7305	.06947	.63377	31
30	0.36650	0.93042	0.39391	2.5386	1.0748	2.7285	0.06958	0.63350	30
31	.36677	.93031	.39425	.5365	.0749	.7265	.06969	.63323	29
32	.36704	.93020	.39458	.5343	.0750	.7245	.06979	.63296	28
33	.36731	.93010	.39492	.5322	.0751	.7225	.06990	.63269	27
34	.36758	.92999	.39525	.5300	.0753	.7205	.07001	.63242	26
35	0.36785	0.92988	0.39559	2.5278	1.0754	2.7185	0.07012	0.63214	25
36	.36812	.92978	.39593	.5257	.0755	.7165	.07022	.63187	24
37	.36839	.92967	.39626	.5236	.0756	.7145	.07033	.63160	23
38	.36866	.92956	.39660	.5214	.0758	.7125	.07044	.63133	22
39	.36893	.92945	.39694	.5193	.0759	.7105	.07054	.63106	21
40	0.36921	0.92935	0.39727	2.5171	1.0760	2.7085	0.07065	0.63079	20
41	.36948	.92924	.39761	.5150	.0761	.7065	.07076	.63052	19
42	.36975	.92913	.39795	.5129	.0763	.7045	.07087	.63025	18
43	.37002	.92902	.39828	.5108	.0764	.7026	.07097	.62998	17
44	.37029	.92892	.39862	.5086	.0765	.7006	.07108	.62971	16
45	0.37056	0.92881	0.39896	2.5065	1.0766	2.6986	0.07119	0.62944	15
46	.37083	.92870	.39930	.5044	.0768	.6967	.07130	.62917	14
47	.37110	.92859	.39963	.5023	.0769	.6947	.07141	.62890	13
48	.37137	.92848	.39997	.5002	.0770	.6927	.07151	.62863	12
49	.37164	.92838	.40031	.4981	.0771	.6908	.07162	.62836	11
50	0.37191	0.92827	0.40065	2.4960	1.0773	2.6888	0.07173	0.62809	10
51	.37218	.92816	.40098	.4939	.0774	.6869	.07184	.62782	9
52	.37245	.92805	.40132	.4918	.0775	.6849	.07195	.62755	8
53	.37272	.92794	.40166	.4897	.0776	.6830	.07205	.62728	7
54	.37299	.92784	.40200	.4876	.0778	.6810	.07216	.62701	6
55	0.37326	0.92773	0.40233	2.4855	1.0779	2.6791	0.07227	0.62674	5
56	.37353	.92762	.40267	.4834	.0780	.6772	.07238	.62647	4
57	.37380	.92751	.40301	.4813	.0781	.6752	.07249	.62620	3
58	.37407	.92740	.40335	.4792	.0783	.6733	.07260	.62593	2
59	.37434	.92729	.40369	.4772	.0784	.6714	.07271	.62566	1
60	0.37461	0.92718	0.40403	2.4751	1.0785	2.6695	0.07282	0.62539	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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22° **Natural Trigonometric Functions** **157°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.37461	0.92718	0.40403	2.4751	1.0785	2.6695	0.07282	0.62539	60
1	.37488	.92707	.40436	.4730	.0787	.6675	.07292	.62512	59
2	.37514	.92696	.40470	.4709	.0788	.6656	.07303	.62485	58
3	.37541	.92686	.40504	.4689	.0789	.6637	.07314	.62458	57
4	.37568	.92675	.40538	.4668	.0790	.6618	.07325	.62431	56
5	0.37595	0.92664	0.40572	2.4647	1.0792	2.6599	0.07336	0.62404	55
6	.37622	.92653	.40606	.4627	.0793	.6580	.07347	.62377	54
7	.37649	.92642	.40640	.4606	.0794	.6561	.07358	.62351	53
8	.37676	.92631	.40673	.4586	.0795	.6542	.07369	.62324	52
9	.37703	.92620	.40707	.4565	.0797	.6523	.07380	.62297	51
10	0.37730	0.92609	0.40741	2.4545	1.0798	2.6504	0.07391	0.62270	50
11	.37757	.92598	.40775	.4525	.0799	.6485	.07402	.62243	49
12	.37784	.92587	.40809	.4504	.0801	.6466	.07413	.62216	48
13	.37811	.92576	.40843	.4484	.0802	.6447	.07424	.62189	47
14	.37838	.92565	.40877	.4463	.0803	.6428	.07435	.62162	46
15	0.37865	0.92554	0.40911	2.4443	1.0804	2.6410	0.07446	0.62135	45
16	.37892	.92543	.40945	.4423	.0806	.6391	.07457	.62108	44
17	.37919	.92532	.40979	.4403	.0807	.6372	.07468	.62081	43
18	.37946	.92521	.41013	.4382	.0808	.6353	.07479	.62054	42
19	.37972	.92510	.41047	.4362	.0810	.6335	.07490	.62027	41
20	0.37999	0.92499	0.41081	2.4342	1.0811	2.6316	0.07501	0.62000	40
21	.38026	.92488	.41115	.4322	.0812	.6297	.07512	.61974	39
22	.38053	.92477	.41149	.4302	.0813	.6279	.07523	.61947	38
23	.38080	.92466	.41183	.4282	.0815	.6260	.07534	.61920	37
24	.38107	.92455	.41217	.4262	.0816	.6242	.07545	.61893	36
25	0.38134	0.92443	0.41251	2.4242	1.0817	2.6223	0.07556	0.61866	35
26	.38161	.92432	.41285	.4222	.0819	.6205	.07567	.61839	34
27	.38188	.92421	.41319	.4202	.0820	.6186	.07579	.61812	33
28	.38214	.92410	.41353	.4182	.0821	.6168	.07590	.61785	32
29	.38241	.92399	.41387	.4162	.0823	.6150	.07601	.61758	31
30	0.38268	0.92388	0.41421	2.4142	1.0824	2.6131	0.07612	0.61732	30
31	.38295	.92377	.41455	.4122	.0825	.6113	.07623	.61705	29
32	.38322	.92366	.41489	.4102	.0826	.6095	.07634	.61678	28
33	.38349	.92354	.41524	.4083	.0828	.6076	.07645	.61651	27
34	.38376	.92343	.41558	.4063	.0829	.6058	.07657	.61624	26
35	0.38403	0.92332	0.41592	2.4043	1.0830	2.6040	0.07668	0.61597	25
36	.38429	.92321	.41626	.4023	.0832	.6022	.07679	.61570	24
37	.38456	.92310	.41660	.4004	.0833	.6003	.07690	.61544	23
38	.38483	.92299	.41694	.3984	.0834	.5985	.07701	.61517	22
39	.38510	.92287	.41728	.3964	.0836	.5967	.07712	.61490	21
40	0.38537	0.92276	0.41762	2.3945	1.0837	2.5949	0.07724	0.61463	20
41	.38564	.92265	.41797	.3925	.0838	.5931	.07735	.61436	19
42	.38591	.92254	.41831	.3906	.0840	.5913	.07746	.61409	18
43	.38617	.92242	.41865	.3886	.0841	.5895	.07757	.61382	17
44	.38644	.92231	.41899	.3867	.0842	.5877	.07769	.61356	16
45	0.38671	0.92220	0.41933	2.3847	1.0844	2.5859	0.07780	0.61329	15
46	.38698	.92209	.41968	.3828	.0845	.5841	.07791	.61302	14
47	.38725	.92197	.42002	.3808	.0846	.5823	.07802	.61275	13
48	.38751	.92186	.42036	.3789	.0847	.5805	.07814	.61248	12
49	.38778	.92175	.42070	.3770	.0849	.5787	.07825	.61222	11
50	0.38805	0.92164	0.42105	2.3750	1.0850	2.5770	0.07836	0.61195	10
51	.38832	.92152	.42139	.3731	.0851	.5752	.07847	.61168	9
52	.38859	.92141	.42173	.3712	.0853	.5734	.07859	.61141	8
53	.38886	.92130	.42207	.3692	.0854	.5716	.07870	.61114	7
54	.38912	.92118	.42242	.3673	.0855	.5699	.07881	.61088	6
55	0.38939	0.92107	0.42276	2.3654	1.0857	2.5681	0.07893	0.61061	5
56	.38966	.92096	.42310	.3635	.0858	.5663	.07904	.61034	4
57	.38993	.92084	.42344	.3616	.0859	.5646	.07915	.61007	3
58	.39019	.92073	.42379	.3597	.0861	.5628	.07927	.60980	2
59	.39046	.92062	.42413	.3577	.0862	.5610	.07938	.60954	1
60	0.39073	0.92050	0.42447	2.3558	1.0864	2.5593	0.07949	0.60927	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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23° **Natural Trigonometric Functions** **156°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.39073	0.92050	0.42447	2.3558	1.0864	2.5593	0.07949	0.60927	60
1	.39100	.92039	.42482	.3539	.0865	.5575	.07961	.60900	59
2	.39126	.92028	.42516	.3520	.0866	.5558	.07972	.60873	58
3	.39153	.92016	.42550	.3501	.0868	.5540	.07984	.60846	57
4	.39180	.92005	.42585	.3482	.0869	.5523	.07995	.60820	56
5	0.39207	0.91993	0.42619	2.3463	1.0870	2.5506	0.08006	0.60793	55
6	.39234	.91982	.42654	.3445	.0872	.5488	.08018	.60766	54
7	.39260	.91971	.42688	.3426	.0873	.5471	.08029	.60739	53
8	.39287	.91959	.42722	.3407	.0874	.5453	.08041	.60713	52
9	.39314	.91948	.42757	.3388	.0876	.5436	.08052	.60686	51
10	0.39341	0.91936	0.42791	2.3369	1.0877	2.5419	0.08063	0.60659	50
11	.39367	.91925	.42826	.3350	.0878	.5402	.08075	.60632	49
12	.39394	.91913	.42860	.3332	.0880	.5384	.08086	.60606	48
13	.39421	.91902	.42894	.3313	.0881	.5367	.08098	.60579	47
14	.39448	.91891	.42929	.3294	.0882	.5350	.08109	.60552	46
15	0.39474	0.91879	0.42963	2.3276	1.0884	2.5333	0.08121	0.60526	45
16	.39501	.91868	.42998	.3257	.0885	.5316	.08132	.60499	44
17	.39528	.91856	.43032	.3238	.0886	.5299	.08144	.60472	43
18	.39554	.91845	.43067	.3220	.0888	.5281	.08155	.60445	42
19	.39581	.91833	.43101	.3201	.0889	.5264	.08167	.60419	41
20	0.39608	0.91822	0.43136	2.3183	1.0891	2.5247	0.08178	0.60392	40
21	.39635	.91810	.43170	.3164	.0892	.5230	.08190	.60365	39
22	.39661	.91798	.43205	.3145	.0893	.5213	.08201	.60339	38
23	.39688	.91787	.43239	.3127	.0895	.5196	.08213	.60312	37
24	.39715	.91775	.43274	.3109	.0896	.5179	.08224	.60285	36
25	0.39741	0.91764	0.43308	2.3090	1.0897	2.5163	0.08236	0.60258	35
26	.39768	.91752	.43343	.3072	.0899	.5146	.08248	.60232	34
27	.39795	.91741	.43377	.3053	.0900	.5129	.08259	.60205	33
28	.39821	.91729	.43412	.3035	.0902	.5112	.08271	.60178	32
29	.39848	.91718	.43447	.3017	.0903	.5095	.08282	.60152	31
30	0.39875	0.91706	0.43481	2.2998	1.0904	2.5078	0.08294	0.60125	30
31	.39901	.91694	.43516	.2980	.0906	.5062	.08306	.60098	29
32	.39928	.91683	.43550	.2962	.0907	.5045	.08317	.60072	28
33	.39955	.91671	.43585	.2944	.0908	.5028	.08329	.60045	27
34	.39981	.91659	.43620	.2925	.0910	.5011	.08340	.60018	26
35	0.40008	0.91648	0.43654	2.2907	1.0911	2.4995	0.08352	0.59992	25
36	.40035	.91636	.43689	.2889	.0913	.4978	.08364	.59965	24
37	.40061	.91625	.43723	.2871	.0914	.4961	.08375	.59938	23
38	.40088	.91613	.43758	.2853	.0915	.4945	.08387	.59912	22
39	.40115	.91601	.43793	.2835	.0917	.4928	.08399	.59885	21
40	0.40141	0.91590	0.43827	2.2817	1.0918	2.4912	0.08410	0.59858	20
41	.40168	.91578	.43862	.2799	.0920	.4895	.08422	.59832	19
42	.40195	.91566	.43897	.2781	.0921	.4879	.08434	.59805	18
43	.40221	.91554	.43932	.2763	.0922	.4862	.08445	.59778	17
44	.40248	.91543	.43966	.2745	.0924	.4846	.08457	.59752	16
45	0.40275	0.91531	0.44001	2.2727	1.0925	2.4829	0.08469	0.59725	15
46	.40301	.91519	.44036	.2709	.0927	.4813	.08480	.59699	14
47	.40328	.91508	.44070	.2691	.0928	.4797	.08492	.59672	13
48	.40354	.91496	.44105	.2673	.0929	.4780	.08504	.59645	12
49	.40381	.91484	.44140	.2655	.0931	.4764	.08516	.59619	11
50	0.40408	0.91472	0.44175	2.2637	1.0932	2.4748	0.08527	0.59592	10
51	.40434	.91461	.44209	.2619	.0934	.4731	.08539	.59566	9
52	.40461	.91449	.44244	.2602	.0935	.4715	.08551	.59539	8
53	.40487	.91437	.44279	.2584	.0936	.4699	.08563	.59512	7
54	.40514	.91425	.44314	.2566	.0938	.4683	.08575	.59486	6
55	0.40541	0.91414	0.44349	2.2548	1.0939	2.4666	0.08586	0.59459	5
56	.40567	.91402	.44383	.2531	.0941	.4650	.08598	.59433	4
57	.40594	.91390	.44418	.2513	.0942	.4634	.08610	.59406	3
58	.40620	.91378	.44453	.2495	.0943	.4618	.08622	.59379	2
59	.40647	.91366	.44488	.2478	.0945	.4602	.08634	.59353	1
60	0.40674	0.91354	0.44523	2.2460	1.0946	2.4586	0.08645	0.59326	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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24° **Natural Trigonometric Functions** **155°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.40674	0.91354	0.44523	2.2460	1.0946	2.4586	0.08645	0.59326	60
1	.40700	.91343	.44558	.2443	.0948	.4570	.08657	.59300	59
2	.40727	.91331	.44593	.2425	.0949	.4554	.08669	.59273	58
3	.40753	.91319	.44627	.2408	.0951	.4538	.08681	.59247	57
4	.40780	.91307	.44662	.2390	.0952	.4522	.08693	.59220	56
5	0.40806	0.91295	0.44697	2.2373	1.0953	2.4506	0.08705	0.59193	55
6	.40833	.91283	.44732	.2355	.0955	.4490	.08716	.59167	54
7	.40860	.91271	.44767	.2338	.0956	.4474	.08728	.59140	53
8	.40886	.91260	.44802	.2320	.0958	.4458	.08740	.59114	52
9	.40913	.91248	.44837	.2303	.0959	.4442	.08752	.59087	51
10	0.40939	0.91236	0.44872	2.2286	1.0961	2.4426	0.08764	0.59061	50
11	.40966	.91224	.44907	.2268	.0962	.4411	.08776	.59034	49
12	.40992	.91212	.44942	.2251	.0963	.4395	.08788	.59008	48
13	.41019	.91200	.44977	.2234	.0965	.4379	.08800	.58981	47
14	.41045	.91188	.45012	.2216	.0966	.4363	.08812	.58955	46
15	0.41072	0.91176	0.45047	2.2199	1.0968	2.4347	0.08824	0.58928	45
16	.41098	.91164	.45082	.2182	.0969	.4332	.08836	.58901	44
17	.41125	.91152	.45117	.2165	.0971	.4316	.08848	.58875	43
18	.41151	.91140	.45152	.2147	.0972	.4300	.08860	.58848	42
19	.41178	.91128	.45187	.2130	.0973	.4285	.08872	.58822	41
20	0.41204	0.91116	0.45222	2.2113	1.0975	2.4269	0.08884	0.58795	40
21	.41231	.91104	.45257	.2096	.0976	.4254	.08896	.58769	39
22	.41257	.91092	.45292	.2079	.0978	.4238	.08908	.58742	38
23	.41284	.91080	.45327	.2062	.0979	.4222	.08920	.58716	37
24	.41310	.91068	.45362	.2045	.0981	.4207	.08932	.58689	36
25	0.41337	0.91056	0.45397	2.2028	1.0982	2.4191	0.08944	0.58663	35
26	.41363	.91044	.45432	.2011	.0984	.4176	.08956	.58636	34
27	.41390	.91032	.45467	.1994	.0985	.4160	.08968	.58610	33
28	.41416	.91020	.45502	.1977	.0986	.4145	.08980	.58584	32
29	.41443	.91008	.45537	.1960	.0988	.4130	.08992	.58557	31
30	0.41469	0.90996	0.45573	2.1943	1.0989	2.4114	0.09004	0.58531	30
31	.41496	.90984	.45608	.1926	.0991	.4099	.09016	.58504	29
32	.41522	.90972	.45643	.1909	.0992	.4083	.09028	.58478	28
33	.41549	.90960	.45678	.1892	.0994	.4068	.09040	.58451	27
34	.41575	.90948	.45713	.1875	.0995	.4053	.09052	.58425	26
35	0.41602	0.90936	0.45748	2.1859	1.0997	2.4037	0.09064	0.58398	25
36	.41628	.90924	.45783	.1842	.0998	.4022	.09076	.58372	24
37	.41654	.90911	.45819	.1825	.1000	.4007	.09088	.58345	23
38	.41681	.90899	.45854	.1808	.1001	.3992	.09101	.58319	22
39	.41707	.90887	.45889	.1792	.1003	.3976	.09113	.58292	21
40	0.41734	0.90875	0.45924	2.1775	1.1004	2.3961	0.09125	0.58266	20
41	.41760	.90863	.45960	.1758	.1005	.3946	.09137	.58240	19
42	.41787	.90851	.45995	.1741	.1007	.3931	.09149	.58213	18
43	.41813	.90839	.46030	.1725	.1008	.3916	.09161	.58187	17
44	.41839	.90826	.46065	.1708	.1010	.3901	.09173	.58160	16
45	0.41866	0.90814	0.46101	2.1692	1.1011	2.3886	0.09186	0.58134	15
46	.41892	.90802	.46136	.1675	.1013	.3871	.09198	.58108	14
47	.41919	.90790	.46171	.1658	.1014	.3856	.09210	.58081	13
48	.41945	.90778	.46206	.1642	.1016	.3841	.09222	.58055	12
49	.41972	.90765	.46242	.1625	.1017	.3826	.09234	.58028	11
50	0.41998	0.90753	0.46277	2.1609	1.1019	2.3811	0.09247	0.58002	10
51	.42024	.90741	.46312	.1592	.1020	.3796	.09259	.57975	9
52	.42051	.90729	.46348	.1576	.1022	.3781	.09271	.57949	8
53	.42077	.90717	.46383	.1559	.1023	.3766	.09283	.57923	7
54	.42103	.90704	.46418	.1543	.1025	.3751	.09296	.57896	6
55	0.42130	0.90692	0.46454	2.1527	1.1026	2.3736	0.09308	0.57870	5
56	.42156	.90680	.46489	.1510	.1028	.3721	.09320	.57844	4
57	.42183	.90668	.46524	.1494	.1029	.3706	.09332	.57817	3
58	.42209	.90655	.46560	.1478	.1031	.3691	.09345	.57791	2
59	.42235	.90643	.46595	.1461	.1032	.3677	.09357	.57764	1
60	0.42262	0.90631	0.46631	2.1445	1.1034	2.3662	0.09369	0.57738	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

114° **65°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.42262	0.90631	0.46631	2.1445	1.1034	2.3662	0.09369	0.57738	60
1	.42288	.90618	.46666	.1429	.1035	.3647	.09381	.57712	59
2	.42314	.90606	.46702	.1412	.1037	.3632	.09394	.57685	58
3	.42341	.90594	.46737	.1396	.1038	.3618	.09406	.57659	57
4	.42367	.90581	.46772	.1380	.1040	.3603	.09418	.57633	56
5	0.42394	0.90569	0.46808	2.1364	1.1041	2.3588	0.09431	0.57606	55
6	.42420	.90557	.46843	.1348	.1043	.3574	.09443	.57580	54
7	.42446	.90544	.46879	.1331	.1044	.3559	.09455	.57554	53
8	.42473	.90532	.46914	.1315	.1046	.3544	.09468	.57527	52
9	.42499	.90520	.46950	.1299	.1047	.3530	.09480	.57501	51
10	0.42525	0.90507	0.46985	2.1283	1.1049	2.3515	0.09492	0.57475	50
11	.42552	.90495	.47021	.1267	.1050	.3501	.09505	.57448	49
12	.42578	.90483	.47056	.1251	.1052	.3486	.09517	.57422	48
13	.42604	.90470	.47092	.1235	.1053	.3472	.09530	.57396	47
14	.42630	.90458	.47127	.1219	.1055	.3457	.09542	.57369	46
15	0.42657	0.90445	0.47163	2.1203	1.1056	2.3443	0.09554	0.57343	45
16	.42683	.90433	.47199	.1187	.1058	.3428	.09567	.57317	44
17	.42709	.90421	.47234	.1171	.1059	.3414	.09579	.57290	43
18	.42736	.90408	.47270	.1155	.1061	.3399	.09592	.57264	42
19	.42762	.90396	.47305	.1139	.1062	.3385	.09604	.57238	41
20	0.42788	0.90383	0.47341	2.1123	1.1064	2.3371	0.09617	0.57212	40
21	.42815	.90371	.47376	.1107	.1065	.3356	.09629	.57185	39
22	.42841	.90358	.47412	.1092	.1067	.3342	.09641	.57159	38
23	.42867	.90346	.47448	.1076	.1068	.3328	.09654	.57133	37
24	.42893	.90333	.47483	.1060	.1070	.3313	.09666	.57106	36
25	0.42920	0.90321	0.47519	2.1044	1.1072	2.3299	0.09679	0.57080	35
26	.42946	.90308	.47555	.1028	.1073	.3285	.09691	.57054	34
27	.42972	.90296	.47590	.1013	.1075	.3271	.09704	.57028	33
28	.42998	.90283	.47626	.0997	.1076	.3256	.09716	.57001	32
29	.43025	.90271	.47662	.0981	.1078	.3242	.09729	.56975	31
30	0.43051	0.90258	0.47697	2.0965	1.1079	2.3228	0.09741	0.56949	30
31	.43077	.90246	.47733	.0950	.1081	.3214	.09754	.56923	29
32	.43104	.90233	.47769	.0934	.1082	.3200	.09766	.56896	28
33	.43130	.90221	.47805	.0918	.1084	.3186	.09779	.56870	27
34	.43156	.90208	.47840	.0903	.1085	.3172	.09792	.56844	26
35	0.43182	0.90196	0.47876	2.0887	1.1087	2.3158	0.09804	0.56818	25
36	.43208	.90183	.47912	.0872	.1088	.3143	.09817	.56791	24
37	.43235	.90171	.47948	.0856	.1090	.3129	.09829	.56765	23
38	.43261	.90158	.47983	.0840	.1092	.3115	.09842	.56739	22
39	.43287	.90145	.48019	.0825	.1093	.3101	.09854	.56713	21
40	0.43313	0.90133	0.48055	2.0809	1.1095	2.3087	0.09867	0.56686	20
41	.43340	.90120	.48091	.0794	.1096	.3073	.09880	.56660	19
42	.43366	.90108	.48127	.0778	.1098	.3059	.09892	.56634	18
43	.43392	.90095	.48162	.0763	.1099	.3046	.09905	.56608	17
44	.43418	.90082	.48198	.0747	.1101	.3032	.09917	.56582	16
45	0.43444	0.90070	0.48234	2.0732	1.1102	2.3018	0.09930	0.56555	15
46	.43471	.90057	.48270	.0717	.1104	.3004	.09943	.56529	14
47	.43497	.90044	.48306	.0701	.1106	.2990	.09955	.56503	13
48	.43523	.90032	.48342	.0686	.1107	.2976	.09968	.56477	12
49	.43549	.90019	.48378	.0671	.1109	.2962	.09981	.56451	11
50	0.43575	0.90006	0.48414	2.0655	1.1110	2.2949	0.09993	0.56424	10
51	.43602	.89994	.48449	.0640	.1112	.2935	.10006	.56398	9
52	.43628	.89981	.48485	.0625	.1113	.2921	.10019	.56372	8
53	.43654	.89968	.48521	.0609	.1115	.2907	.10031	.56346	7
54	.43680	.89956	.48557	.0594	.1116	.2894	.10044	.56320	6
55	0.43706	0.89943	0.48593	2.0579	1.1118	2.2880	0.10057	0.56294	5
56	.43732	.89930	.48629	.0564	.1120	.2866	.10070	.56267	4
57	.43759	.89918	.48665	.0548	.1121	.2853	.10082	.56241	3
58	.43785	.89905	.48701	.0533	.1123	.2839	.10095	.56215	2
59	.43811	.89892	.48737	.0518	.1124	.2825	.10108	.56189	1
60	0.43837	0.89879	0.48773	2.0503	1.1126	2.2812	0.10121	0.56163	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

26° **Natural Trigonometric Functions** 153°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.43837	0.89879	0.48773	2.0503	1.1126	2.2812	0.10121	0.56163	60
1	.43863	.89867	.48809	.0488	.1127	.2798	.10133	.56137	59
2	.43889	.89854	.48845	.0473	.1129	.2784	.10146	.56111	58
3	.43915	.89841	.48881	.0458	.1131	.2771	.10159	.56084	57
4	.43942	.89828	.48917	.0443	.1132	.2757	.10172	.56058	56
5	0.43968	0.89815	0.48953	2.0427	1.1134	2.2744	0.10184	0.56032	55
6	.43994	.89803	.48989	.0412	.1135	.2730	.10197	.56006	54
7	.44020	.89790	.49025	.0397	.1137	.2717	.10210	.55980	53
8	.44046	.89777	.49062	.0382	.1139	.2703	.10223	.55954	52
9	.44072	.89764	.49098	.0367	.1140	.2690	.10236	.55928	51
10	0.44098	0.89751	0.49134	2.0352	1.1142	2.2676	0.10248	0.55902	50
11	.44124	.89739	.49170	.0338	.1143	.2663	.10261	.55875	49
12	.44150	.89726	.49206	.0323	.1145	.2650	.10274	.55849	48
13	.44177	.89713	.49242	.0308	.1147	.2636	.10287	.55823	47
14	.44203	.89700	.49278	.0293	.1148	.2623	.10300	.55797	46
15	0.44229	0.89687	0.49314	2.0278	1.1150	2.2610	0.10313	0.55771	45
16	.44255	.89674	.49351	.0263	.1151	.2596	.10326	.55745	44
17	.44281	.89661	.49387	.0248	.1153	.2583	.10338	.55719	43
18	.44307	.89649	.49423	.0233	.1155	.2570	.10351	.55693	42
19	.44333	.89636	.49459	.0219	.1156	.2556	.10364	.55667	41
20	0.44359	0.89623	0.49495	2.0204	1.1158	2.2543	0.10377	0.55641	40
21	.44385	.89610	.49532	.0189	.1159	.2530	.10390	.55615	39
22	.44411	.89597	.49568	.0174	.1161	.2517	.10403	.55589	38
23	.44437	.89584	.49604	.0159	.1163	.2503	.10416	.55562	37
24	.44463	.89571	.49640	.0145	.1164	.2490	.10429	.55536	36
25	0.44489	0.89558	0.49677	2.0130	1.1166	2.2477	0.10442	0.55510	35
26	.44516	.89545	.49713	.0115	.1167	.2464	.10455	.55484	34
27	.44542	.89532	.49749	.0101	.1169	.2451	.10468	.55458	33
28	.44568	.89519	.49785	.0086	.1171	.2438	.10481	.55432	32
29	.44594	.89506	.49822	.0071	.1172	.2425	.10493	.55406	31
30	0.44620	0.89493	0.49858	2.0057	1.1174	2.2411	0.10506	0.55380	30
31	.44646	.89480	.49894	.0042	.1176	.2398	.10519	.55354	29
32	.44672	.89467	.49931	.0028	.1177	.2385	.10532	.55328	28
33	.44698	.89454	.49967	.0013	.1179	.2372	.10545	.55302	27
34	.44724	.89441	.50003	1.9998	.1180	.2359	.10558	.55276	26
35	0.44750	0.89428	0.50040	1.9984	1.1182	2.2346	0.10571	0.55250	25
36	.44776	.89415	.50076	.9969	.1184	.2333	.10584	.55224	24
37	.44802	.89402	.50113	.9955	.1185	.2320	.10598	.55198	23
38	.44828	.89389	.50149	.9940	.1187	.2307	.10611	.55172	22
39	.44854	.89376	.50185	.9926	.1189	.2294	.10624	.55146	21
40	0.44880	0.89363	0.50222	1.9912	1.1190	2.2282	0.10637	0.55120	20
41	.44906	.89350	.50258	.9897	.1192	.2269	.10650	.55094	19
42	.44932	.89337	.50295	.9883	.1193	.2256	.10663	.55068	18
43	.44958	.89324	.50331	.9868	.1195	.2243	.10676	.55042	17
44	.44984	.89311	.50368	.9854	.1197	.2230	.10689	.55016	16
45	0.45010	0.89298	0.50404	1.9840	1.1198	2.2217	0.10702	0.54990	15
46	.45036	.89285	.50441	.9825	.1200	.2204	.10715	.54964	14
47	.45062	.89272	.50477	.9811	.1202	.2192	.10728	.54938	13
48	.45088	.89258	.50514	.9797	.1203	.2179	.10741	.54912	12
49	.45114	.89245	.50550	.9782	.1205	.2166	.10754	.54886	11
50	0.45140	0.89232	0.50587	1.9768	1.1207	2.2153	0.10768	0.54860	10
51	.45166	.89219	.50623	.9754	.1208	.2141	.10781	.54834	9
52	.45191	.89206	.50660	.9739	.1210	.2128	.10794	.54808	8
53	.45217	.89193	.50696	.9725	.1212	.2115	.10807	.54782	7
54	.45243	.89180	.50733	.9711	.1213	.2103	.10820	.54756	6
55	0.45269	0.89166	0.50769	1.9697	1.1215	2.2090	0.10833	0.54730	5
56	.45295	.89153	.50806	.9683	.1217	.2077	.10846	.54705	4
57	.45321	.89140	.50843	.9668	.1218	.2065	.10860	.54679	3
58	.45347	.89127	.50879	.9654	.1220	.2052	.10873	.54653	2
59	.45373	.89114	.50916	.9640	.1222	.2039	.10886	.54627	1
60	0.45399	0.89101	0.50952	1.9626	1.1223	2.2027	0.10899	0.54601	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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27° **Natural Trigonometric Functions** **152°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.45399	0.89101	0.50952	1.9626	1.1223	2.2027	0.10899	0.54601	60
1	.45425	.89087	.50989	.9612	.1225	.2014	.10912	.54575	59
2	.45451	.89074	.51026	.9598	.1226	.2002	.10926	.54549	58
3	.45477	.89061	.51062	.9584	.1228	.1989	.10939	.54523	57
4	.45503	.89048	.51099	.9570	.1230	.1977	.10952	.54497	56
5	0.45528	0.89034	0.51136	1.9556	1.1231	2.1964	0.10965	0.54471	55
6	.45554	.89021	.51172	.9542	.1233	.1952	.10979	.54445	54
7	.45580	.89008	.51209	.9528	.1235	.1939	.10992	.54420	53
8	.45606	.88995	.51246	.9514	.1237	.1927	.11005	.54394	52
9	.45632	.88981	.51283	.9500	.1238	.1914	.11018	.54368	51
10	0.45658	0.88968	0.51319	1.9486	1.1240	2.1902	0.11032	0.54342	50
11	.45684	.88955	.51356	.9472	.1242	.1889	.11045	.54316	49
12	.45710	.88942	.51393	.9458	.1243	.1877	.11058	.54290	48
13	.45736	.88928	.51430	.9444	.1245	.1865	.11072	.54264	47
14	.45761	.88915	.51466	.9430	.1247	.1852	.11085	.54238	46
15	0.45787	0.88902	0.51503	1.9416	1.1248	2.1840	0.11098	0.54213	45
16	.45813	.88888	.51540	.9402	.1250	.1828	.11112	.54187	44
17	.45839	.88875	.51577	.9388	.1252	.1815	.11125	.54161	43
18	.45865	.88862	.51614	.9375	.1253	.1803	.11138	.54135	42
19	.45891	.88848	.51651	.9361	.1255	.1791	.11152	.54109	41
20	0.45917	0.88835	0.51687	1.9347	1.1257	2.1778	0.11165	0.54083	40
21	.45942	.88822	.51724	.9333	.1258	.1766	.11178	.54057	39
22	.45968	.88808	.51761	.9319	.1260	.1754	.11192	.54032	38
23	.45994	.88795	.51798	.9306	.1262	.1742	.11205	.54006	37
24	.46020	.88781	.51835	.9292	.1264	.1730	.11218	.53980	36
25	0.46046	0.88768	0.51872	1.9278	1.1265	2.1717	0.11232	0.53954	35
26	.46072	.88755	.51909	.9264	.1267	.1705	.11245	.53928	34
27	.46097	.88741	.51946	.9251	.1269	.1693	.11259	.53902	33
28	.46123	.88728	.51983	.9237	.1270	.1681	.11272	.53877	32
29	.46149	.88714	.52020	.9223	.1272	.1669	.11285	.53851	31
30	0.46175	0.88701	0.52057	1.9210	1.1274	2.1657	0.11299	0.53825	30
31	.46201	.88688	.52094	.9196	.1275	.1645	.11312	.53799	29
32	.46226	.88674	.52131	.9182	.1277	.1633	.11326	.53773	28
33	.46252	.88661	.52168	.9169	.1279	.1620	.11339	.53748	27
34	.46278	.88647	.52205	.9155	.1281	.1608	.11353	.53722	26
35	0.46304	0.88634	0.52242	1.9142	1.1282	2.1596	0.11366	0.53696	25
36	.46330	.88620	.52279	.9128	.1284	.1584	.11380	.53670	24
37	.46355	.88607	.52316	.9115	.1286	.1572	.11393	.53645	23
38	.46381	.88593	.52353	.9101	.1287	.1560	.11407	.53619	22
39	.46407	.88580	.52390	.9088	.1289	.1548	.11420	.53593	21
40	0.46433	0.88566	0.52427	1.9074	1.1291	2.1536	0.11434	0.53567	20
41	.46458	.88553	.52464	.9061	.1293	.1525	.11447	.53541	19
42	.46484	.88539	.52501	.9047	.1294	.1513	.11461	.53516	18
43	.46510	.88526	.52538	.9034	.1296	.1501	.11474	.53490	17
44	.46536	.88512	.52575	.9020	.1298	.1489	.11488	.53464	16
45	0.46561	0.88499	0.52612	1.9007	1.1299	2.1477	0.11501	0.53438	15
46	.46587	.88485	.52650	.8993	.1301	.1465	.11515	.53413	14
47	.46613	.88472	.52687	.8980	.1303	.1453	.11528	.53387	13
48	.46639	.88458	.52724	.8967	.1305	.1441	.11542	.53361	12
49	.46664	.88444	.52761	.8953	.1306	.1430	.11555	.53336	11
50	0.46690	0.88431	0.52798	1.8940	1.1308	2.1418	0.11569	0.53310	10
51	.46716	.88417	.52836	.8927	.1310	.1406	.11583	.53284	9
52	.46741	.88404	.52873	.8913	.1312	.1394	.11596	.53258	8
53	.46767	.88390	.52910	.8900	.1313	.1382	.11610	.53233	7
54	.46793	.88376	.52947	.8887	.1315	.1371	.11623	.53207	6
55	0.46819	0.88363	0.52984	1.8873	1.1317	2.1359	0.11637	0.53181	5
56	.46844	.88349	.53022	.8860	.1319	.1347	.11651	.53156	4
57	.46870	.88336	.53059	.8847	.1320	.1335	.11664	.53130	3
58	.46896	.88322	.53096	.8834	.1322	.1324	.11678	.53104	2
59	.46921	.88308	.53134	.8820	.1324	.1312	.11691	.53078	1
60	0.46947	0.88295	0.53171	1.8807	1.1326	2.1300	0.11705	0.53053	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

117° **62°**

28° **Natural Trigonometric Functions** 151°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.46947	0.88295	0.53171	1.8807	1.1326	2.1300	0.11705	0.53053	60
1	.46973	.88281	.53208	.8794	.1327	.1289	.11719	.53027	59
2	.46998	.88267	.53245	.8781	.1329	.1277	.11732	.53001	58
3	.47024	.88254	.53283	.8768	.1331	.1266	.11746	.52976	57
4	.47050	.88240	.53320	.8754	.1333	.1254	.11760	.52950	56
5	0.47075	0.88226	0.53358	1.8741	1.1334	2.1242	0.11774	0.52924	55
6	.47101	.88213	.53395	.8728	.1336	.1231	.11787	.52899	54
7	.47127	.88199	.53432	.8715	.1338	.1219	.11801	.52873	53
8	.47152	.88185	.53470	.8702	.1340	.1208	.11815	.52847	52
9	.47178	.88171	.53507	.8689	.1341	.1196	.11828	.52822	51
10	0.47204	0.88158	0.53545	1.8676	1.1343	2.1185	0.11842	0.52796	50
11	.47229	.88144	.53582	.8663	.1345	.1173	.11856	.52770	49
12	.47255	.88130	.53619	.8650	.1347	.1162	.11870	.52745	48
13	.47281	.88117	.53657	.8637	.1349	.1150	.11883	.52719	47
14	.47306	.88103	.53694	.8624	.1350	.1139	.11897	.52694	46
15	0.47332	0.88089	0.53732	1.8611	1.1352	2.1127	0.11911	0.52668	45
16	.47357	.88075	.53769	.8598	.1354	.1116	.11925	.52642	44
17	.47383	.88061	.53807	.8585	.1356	.1104	.11938	.52617	43
18	.47409	.88048	.53844	.8572	.1357	.1093	.11952	.52591	42
19	.47434	.88034	.53882	.8559	.1359	.1082	.11966	.52565	41
20	0.47460	0.88020	0.53919	1.8546	1.1361	2.1070	0.11980	0.52540	40
21	.47486	.88006	.53957	.8533	.1363	.1059	.11994	.52514	39
22	.47511	.87992	.53995	.8520	.1365	.1048	.12007	.52489	38
23	.47537	.87979	.54032	.8507	.1366	.1036	.12021	.52463	37
24	.47562	.87965	.54070	.8495	.1368	.1025	.12035	.52437	36
25	0.47588	0.87951	0.54107	1.8482	1.1370	2.1014	0.12049	0.52412	35
26	.47613	.87937	.54145	.8469	.1372	.1002	.12063	.52386	34
27	.47639	.87923	.54183	.8456	.1373	.0991	.12077	.52361	33
28	.47665	.87909	.54220	.8443	.1375	.0980	.12090	.52335	32
29	.47690	.87895	.54258	.8430	.1377	.0969	.12104	.52310	31
30	0.47716	0.87882	0.54295	1.8418	1.1379	2.0957	0.12118	0.52284	30
31	.47741	.87868	.54333	.8405	.1381	.0946	.12132	.52258	29
32	.47767	.87854	.54371	.8392	.1382	.0935	.12146	.52233	28
33	.47792	.87840	.54409	.8379	.1384	.0924	.12160	.52207	27
34	.47818	.87826	.54446	.8367	.1386	.0912	.12174	.52182	26
35	0.47844	0.87812	0.54484	1.8354	1.1388	2.0901	0.12188	0.52156	25
36	.47869	.87798	.54522	.8341	.1390	.0890	.12202	.52131	24
37	.47895	.87784	.54559	.8329	.1391	.0879	.12216	.52105	23
38	.47920	.87770	.54597	.8316	.1393	.0868	.12229	.52080	22
39	.47946	.87756	.54635	.8303	.1395	.0857	.12243	.52054	21
40	0.47971	0.87742	0.54673	1.8291	1.1397	2.0846	0.12257	0.52029	20
41	.47997	.87728	.54711	.8278	.1399	.0835	.12271	.52003	19
42	.48022	.87715	.54748	.8265	.1401	.0824	.12285	.51978	18
43	.48048	.87701	.54786	.8253	.1402	.0812	.12299	.51952	17
44	.48073	.87687	.54824	.8240	.1404	.0801	.12313	.51927	16
45	0.48099	0.87673	0.54862	1.8227	1.1406	2.0790	0.12327	0.51901	15
46	.48124	.87659	.54900	.8215	.1408	.0779	.12341	.51876	14
47	.48150	.87645	.54937	.8202	.1410	.0768	.12355	.51850	13
48	.48175	.87631	.54975	.8190	.1411	.0757	.12369	.51825	12
49	.48201	.87617	.55013	.8177	.1413	.0746	.12383	.51799	11
50	0.48226	0.87603	0.55051	1.8165	1.1415	2.0735	0.12397	0.51774	10
51	.48252	.87588	.55089	.8152	.1417	.0725	.12411	.51748	9
52	.48277	.87574	.55127	.8140	.1419	.0714	.12425	.51723	8
53	.48303	.87560	.55165	.8127	.1421	.0703	.12439	.51697	7
54	.48328	.87546	.55203	.8115	.1422	.0692	.12453	.51672	6
55	0.48354	0.87532	0.55241	1.8102	1.1424	2.0681	0.12468	0.51646	5
56	.48379	.87518	.55279	.8090	.1426	.0670	.12482	.51621	4
57	.48405	.87504	.55317	.8078	.1428	.0659	.12496	.51595	3
58	.48430	.87490	.55355	.8065	.1430	.0648	.12510	.51570	2
59	.48455	.87476	.55393	.8053	.1432	.0637	.12524	.51544	1
60	0.48481	0.87462	0.55431	1.8040	1.1433	2.0627	0.12538	0.51519	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

118° 61°

29° **Natural Trigonometric Functions** **150°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.48481	0.87462	0.55431	1.8040	1.1433	2.0627	0.12538	0.51519	60
1	.48506	.87448	.55469	.8028	.1435	.0616	.12552	.51493	59
2	.48532	.87434	.55507	.8016	.1437	.0605	.12566	.51468	58
3	.48557	.87420	.55545	.8003	.1439	.0594	.12580	.51443	57
4	.48583	.87405	.55583	.7991	.1441	.0583	.12594	.51417	56
5	0.48608	0.87391	0.55621	1.7979	1.1443	2.0573	0.12609	0.51392	55
6	.48633	.87377	.55659	.7966	.1445	.0562	.12623	.51366	54
7	.48659	.87363	.55697	.7954	.1446	.0551	.12637	.51341	53
8	.48684	.87349	.55735	.7942	.1448	.0540	.12651	.51316	52
9	.48710	.87335	.55774	.7930	.1450	.0530	.12665	.51290	51
10	0.48735	0.87320	0.55812	1.7917	1.1452	2.0519	0.12679	0.51265	50
11	.48760	.87306	.55850	.7905	.1454	.0508	.12694	.51239	49
12	.48786	.87292	.55888	.7893	.1456	.0498	.12708	.51214	48
13	.48811	.87278	.55926	.7881	.1458	.0487	.12722	.51189	47
14	.48837	.87264	.55964	.7868	.1459	.0476	.12736	.51163	46
15	0.48862	0.87250	0.56003	1.7856	1.1461	2.0466	0.12750	0.51138	45
16	.48887	.87235	.56041	.7844	.1463	.0455	.12765	.51112	44
17	.48913	.87221	.56079	.7832	.1465	.0444	.12779	.51087	43
18	.48938	.87207	.56117	.7820	.1467	.0434	.12793	.51062	42
19	.48964	.87193	.56156	.7808	.1469	.0423	.12807	.51036	41
20	0.48989	0.87178	0.56194	1.7795	1.1471	2.0413	0.12821	0.51011	40
21	.49014	.87164	.56232	.7783	.1473	.0402	.12836	.50986	39
22	.49040	.87150	.56270	.7771	.1474	.0392	.12850	.50960	38
23	.49065	.87136	.56309	.7759	.1476	.0381	.12864	.50935	37
24	.49090	.87121	.56347	.7747	.1478	.0370	.12879	.50910	36
25	0.49116	0.87107	0.56385	1.7735	1.1480	2.0360	0.12893	0.50884	35
26	.49141	.87093	.56424	.7723	.1482	.0349	.12907	.50859	34
27	.49166	.87078	.56462	.7711	.1484	.0339	.12921	.50834	33
28	.49192	.87064	.56500	.7699	.1486	.0329	.12936	.50808	32
29	.49217	.87050	.56539	.7687	.1488	.0318	.12950	.50783	31
30	0.49242	0.87035	0.56577	1.7675	1.1489	2.0308	0.12964	0.50758	30
31	.49268	.87021	.56616	.7663	.1491	.0297	.12979	.50732	29
32	.49293	.87007	.56654	.7651	.1493	.0287	.12993	.50707	28
33	.49318	.86992	.56692	.7639	.1495	.0276	.13007	.50682	27
34	.49343	.86978	.56731	.7627	.1497	.0266	.13022	.50656	26
35	0.49369	0.86964	0.56769	1.7615	1.1499	2.0256	0.13036	0.50631	25
36	.49394	.86949	.56808	.7603	.1501	.0245	.13050	.50606	24
37	.49419	.86935	.56846	.7591	.1503	.0235	.13065	.50580	23
38	.49445	.86921	.56885	.7579	.1505	.0224	.13079	.50555	22
39	.49470	.86906	.56923	.7567	.1507	.0214	.13094	.50530	21
40	0.49495	0.86892	0.56962	1.7555	1.1508	2.0204	0.13108	0.50505	20
41	.49521	.86877	.57000	.7544	.1510	.0194	.13122	.50479	19
42	.49546	.86863	.57039	.7532	.1512	.0183	.13137	.50454	18
43	.49571	.86849	.57077	.7520	.1514	.0173	.13151	.50429	17
44	.49596	.86834	.57116	.7508	.1516	.0163	.13166	.50404	16
45	0.49622	0.86820	0.57155	1.7496	1.1518	2.0152	0.13180	0.50378	15
46	.49647	.86805	.57193	.7484	.1520	.0142	.13194	.50353	14
47	.49672	.86791	.57232	.7473	.1522	.0132	.13209	.50328	13
48	.49697	.86776	.57270	.7461	.1524	.0122	.13223	.50303	12
49	.49723	.86762	.57309	.7449	.1526	.0111	.13238	.50277	11
50	0.49748	0.86748	0.57348	1.7437	1.1528	2.0101	0.13252	0.50252	10
51	.49773	.86733	.57386	.7426	.1530	.0091	.13267	.50227	9
52	.49798	.86719	.57425	.7414	.1531	.0081	.13281	.50202	8
53	.49823	.86704	.57464	.7402	.1533	.0071	.13296	.50176	7
54	.49849	.86690	.57502	.7390	.1535	.0061	.13310	.50151	6
55	0.49874	0.86675	0.57541	1.7379	1.1537	2.0050	0.13325	0.50126	5
56	.49899	.86661	.57580	.7367	.1539	.0040	.13339	.50101	4
57	.49924	.86646	.57619	.7355	.1541	.0030	.13354	.50076	3
58	.49950	.86632	.57657	.7344	.1543	.0020	.13368	.50050	2
59	.49975	.86617	.57696	.7332	.1545	.0010	.13383	.50025	1
60	0.50000	0.86603	0.57735	1.7320	1.1547	2.0000	0.13397	0.50000	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

119° **60°**

30° **Natural Trigonometric Functions** 149°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.50000	0.86603	0.57735	1.7320	1.1547	2.0000	0.13397	0.50000	60
1	.50025	.86588	.57774	.7309	.1549	1.9990	.13412	.49975	59
2	.50050	.86573	.57813	.7297	.1551	.9980	.13426	.49950	58
3	.50075	.86559	.57851	.7286	.1553	.9970	.13441	.49924	57
4	.50101	.86544	.57890	.7274	.1555	.9960	.13456	.49899	56
5	0.50126	0.86530	0.57929	1.7262	1.1557	1.9950	0.13470	0.49874	55
6	.50151	.86515	.57968	.7251	.1559	.9940	.13485	.49849	54
7	.50176	.86500	.58007	.7239	.1561	.9930	.13499	.49824	53
8	.50201	.86486	.58046	.7228	.1562	.9920	.13514	.49799	52
9	.50226	.86471	.58085	.7216	.1564	.9910	.13529	.49773	51
10	0.50252	0.86457	0.58123	1.7205	1.1566	1.9900	0.13543	0.49748	50
11	.50277	.86442	.58162	.7193	.1568	.9890	.13558	.49723	49
12	.50302	.86427	.58201	.7182	.1570	.9880	.13572	.49698	48
13	.50327	.86413	.58240	.7170	.1572	.9870	.13587	.49673	47
14	.50352	.86398	.58279	.7159	.1574	.9860	.13602	.49648	46
15	0.50377	0.86383	0.58318	1.7147	1.1576	1.9850	0.13616	0.49623	45
16	.50402	.86369	.58357	.7136	.1578	.9840	.13631	.49597	44
17	.50428	.86354	.58396	.7124	.1580	.9830	.13646	.49572	43
18	.50453	.86339	.58435	.7113	.1582	.9820	.13660	.49547	42
19	.50478	.86325	.58474	.7101	.1584	.9811	.13675	.49522	41
20	0.50503	0.86310	0.58513	1.7090	1.1586	1.9801	0.13690	0.49497	40
21	.50528	.86295	.58552	.7079	.1588	.9791	.13704	.49472	39
22	.50553	.86281	.58591	.7067	.1590	.9781	.13719	.49447	38
23	.50578	.86266	.58630	.7056	.1592	.9771	.13734	.49422	37
24	.50603	.86251	.58670	.7044	.1594	.9761	.13749	.49397	36
25	0.50628	0.86237	0.58709	1.7033	1.1596	1.9752	0.13763	0.49371	35
26	.50653	.86222	.58748	.7022	.1598	.9742	.13778	.49346	34
27	.50679	.86207	.58787	.7010	.1600	.9732	.13793	.49321	33
28	.50704	.86192	.58826	.6999	.1602	.9722	.13807	.49296	32
29	.50729	.86178	.58865	.6988	.1604	.9713	.13822	.49271	31
30	0.50754	0.86163	0.58904	1.6977	1.1606	1.9703	0.13837	0.49246	30
31	.50779	.86148	.58944	.6965	.1608	.9693	.13852	.49221	29
32	.50804	.86133	.58983	.6954	.1610	.9683	.13867	.49196	28
33	.50829	.86118	.59022	.6943	.1612	.9674	.13881	.49171	27
34	.50854	.86104	.59061	.6931	.1614	.9664	.13896	.49146	26
35	0.50879	0.86089	0.59100	1.6920	1.1616	1.9654	0.13911	0.49121	25
36	.50904	.86074	.59140	.6909	.1618	.9645	.13926	.49096	24
37	.50929	.86059	.59179	.6898	.1620	.9635	.13941	.49071	23
38	.50954	.86044	.59218	.6887	.1622	.9625	.13955	.49046	22
39	.50979	.86030	.59258	.6875	.1624	.9616	.13970	.49021	21
40	0.51004	0.86015	0.59297	1.6864	1.1626	1.9606	0.13985	0.48996	20
41	.51029	.86000	.59336	.6853	.1628	.9596	.14000	.48971	19
42	.51054	.85985	.59376	.6842	.1630	.9587	.14015	.48946	18
43	.51079	.85970	.59415	.6831	.1632	.9577	.14030	.48921	17
44	.51104	.85955	.59454	.6820	.1634	.9568	.14044	.48896	16
45	0.51129	0.85941	0.59494	1.6808	1.1636	1.9558	0.14059	0.48871	15
46	.51154	.85926	.59533	.6797	.1638	.9549	.14074	.48846	14
47	.51179	.85911	.59572	.6786	.1640	.9539	.14089	.48821	13
48	.51204	.85896	.59612	.6775	.1642	.9530	.14104	.48796	12
49	.51229	.85881	.59651	.6764	.1644	.9520	.14119	.48771	11
50	0.51254	0.85866	0.59691	1.6753	1.1646	1.9510	0.14134	0.48746	10
51	.51279	.85851	.59730	.6742	.1648	.9501	.14149	.48721	9
52	.51304	.85836	.59770	.6731	.1650	.9491	.14164	.48696	8
53	.51329	.85821	.59809	.6720	.1652	.9482	.14178	.48671	7
54	.51354	.85806	.59849	.6709	.1654	.9473	.14193	.48646	6
55	0.51379	0.85791	0.59888	1.6698	1.1656	1.9463	0.14208	0.48621	5
56	.51404	.85777	.59928	.6687	.1658	.9454	.14223	.48596	4
57	.51429	.85762	.59967	.6676	.1660	.9444	.14238	.48571	3
58	.51454	.85747	.60007	.6665	.1662	.9435	.14253	.48546	2
59	.51479	.85732	.60046	.6654	.1664	.9425	.14268	.48521	1
60	0.51504	0.85717	0.60086	1.6643	1.1666	1.9416	0.14283	0.48496	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

120° 59°

31° **Natural Trigonometric Functions** 148°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.51504	0.85717	0.60086	1.6643	1.1666	1.9416	0.14283	0.48496	60
1	.51529	.85702	.60126	.6632	.1668	.9407	.14298	.48471	59
2	.51554	.85687	.60165	.6621	.1670	.9397	.14313	.48446	58
3	.51578	.85672	.60205	.6610	.1672	.9388	.14328	.48421	57
4	.51603	.85657	.60244	.6599	.1674	.9378	.14343	.48396	56
5	0.51628	0.85642	0.60284	1.6588	1.1676	1.9369	0.14358	0.48371	55
6	.51653	.85627	.60324	.6577	.1678	.9360	.14373	.48347	54
7	.51678	.85612	.60363	.6566	.1681	.9350	.14388	.48322	53
8	.51703	.85597	.60403	.6555	.1683	.9341	.14403	.48297	52
9	.51728	.85582	.60443	.6544	.1685	.9332	.14418	.48272	51
10	0.51753	0.85566	0.60483	1.6534	1.1687	1.9322	0.14433	0.48247	50
11	.51778	.85551	.60522	.6523	.1689	.9313	.14448	.48222	49
12	.51803	.85536	.60562	.6512	.1691	.9304	.14463	.48197	48
13	.51827	.85521	.60602	.6501	.1693	.9295	.14479	.48172	47
14	.51852	.85506	.60642	.6490	.1695	.9285	.14494	.48147	46
15	0.51877	0.85491	0.60681	1.6479	1.1697	1.9276	0.14509	0.48123	45
16	.51902	.85476	.60721	.6469	.1699	.9267	.14524	.48098	44
17	.51927	.85461	.60761	.6458	.1701	.9258	.14539	.48073	43
18	.51952	.85446	.60801	.6447	.1703	.9248	.14554	.48048	42
19	.51977	.85431	.60841	.6436	.1705	.9239	.14569	.48023	41
20	0.52002	0.85416	0.60881	1.6425	1.1707	1.9230	0.14584	0.47998	40
21	.52026	.85400	.60920	.6415	.1709	.9221	.14599	.47973	39
22	.52051	.85385	.60960	.6404	.1712	.9212	.14615	.47949	38
23	.52076	.85370	.61000	.6393	.1714	.9203	.14630	.47924	37
24	.52101	.85355	.61040	.6383	.1716	.9193	.14645	.47899	36
25	0.52126	0.85340	0.61080	1.6372	1.1718	1.9184	0.14660	0.47874	35
26	.52151	.85325	.61120	.6361	.1720	.9175	.14675	.47849	34
27	.52175	.85309	.61160	.6350	.1722	.9166	.14690	.47824	33
28	.52200	.85294	.61200	.6340	.1724	.9157	.14706	.47800	32
29	.52225	.85279	.61240	.6329	.1726	.9148	.14721	.47775	31
30	0.52250	0.85264	0.61280	1.6318	1.1728	1.9139	0.14736	0.47750	30
31	.52275	.85249	.61320	.6308	.1730	.9130	.14751	.47725	29
32	.52299	.85234	.61360	.6297	.1732	.9121	.14766	.47700	28
33	.52324	.85218	.61400	.6286	.1734	.9112	.14782	.47676	27
34	.52349	.85203	.61440	.6276	.1737	.9102	.14797	.47651	26
35	0.52374	0.85188	0.61480	1.6265	1.1739	1.9093	0.14812	0.47626	25
36	.52398	.85173	.61520	.6255	.1741	.9084	.14827	.47601	24
37	.52423	.85157	.61560	.6244	.1743	.9075	.14842	.47577	23
38	.52448	.85142	.61601	.6233	.1745	.9066	.14858	.47552	22
39	.52473	.85127	.61641	.6223	.1747	.9057	.14873	.47527	21
40	0.52498	0.85112	0.61681	1.6212	1.1749	1.9048	0.14888	0.47502	20
41	.52522	.85096	.61721	.6202	.1751	.9039	.14904	.47477	19
42	.52547	.85081	.61761	.6191	.1753	.9030	.14919	.47453	18
43	.52572	.85066	.61801	.6181	.1756	.9021	.14934	.47428	17
44	.52597	.85050	.61842	.6170	.1758	.9013	.14949	.47403	16
45	0.52621	0.85035	0.61882	1.6160	1.1760	1.9004	0.14965	0.47379	15
46	.52646	.85020	.61922	.6149	.1762	.8995	.14980	.47354	14
47	.52671	.85004	.61962	.6139	.1764	.8986	.14995	.47329	13
48	.52695	.84989	.62003	.6128	.1766	.8977	.15011	.47304	12
49	.52720	.84974	.62043	.6118	.1768	.8968	.15026	.47280	11
50	0.52745	0.84959	0.62083	1.6107	1.1770	1.8959	0.15041	0.47255	10
51	.52770	.84943	.62123	.6097	.1772	.8950	.15057	.47230	9
52	.52794	.84928	.62164	.6086	.1775	.8941	.15072	.47205	8
53	.52819	.84912	.62204	.6076	.1777	.8932	.15087	.47181	7
54	.52844	.84897	.62244	.6066	.1779	.8924	.15103	.47156	6
55	0.52868	0.84882	0.62285	1.6055	1.1781	1.8915	0.15118	0.47131	5
56	.52893	.84866	.62325	.6045	.1783	.8906	.15133	.47107	4
57	.52918	.84851	.62366	.6034	.1785	.8897	.15149	.47082	3
58	.52942	.84836	.62406	.6024	.1787	.8888	.15164	.47057	2
59	.52967	.84820	.62446	.6014	.1790	.8879	.15180	.47033	1
60	0.52992	0.84805	0.62487	1.6003	1.1792	1.8871	0.15195	0.47008	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

121° 58°

32° **Natural Trigonometric Functions** **147°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos	M
0	0.52992	0.84805	0.62487	1.6003	1.1792	1.8871	0.15195	0.47008	60
1	.53016	.84789	.62527	.5993	.1794	.8862	.15211	.46983	59
2	.53041	.84774	.62568	.5983	.1796	.8853	.15226	.46959	58
3	.53066	.84758	.62608	.5972	.1798	.8844	.15241	.46934	57
4	.53090	.84743	.62649	.5962	.1800	.8836	.15257	.46909	56
5	0.53115	0.84728	0.62689	1.5952	1.1802	1.8827	0.15272	0.46885	55
6	.53140	.84712	.62730	.5941	.1805	.8818	.15288	.46860	54
7	.53164	.84697	62770	.5931	.1807	.8809	.15303	.46835	53
8	.53189	.84681	.62811	.5921	.1809	.8801	.15319	.46811	52
9	.53214	.84666	.62851	.5910	.1811	.8792	.15334	.46786	51
10	0.53238	0.84650	0.62892	1.5900	1.1813	1.8783	0.15350	0.46762	50
11	.53263	.84635	.62933	.5890	.1815	.8775	.15365	.46737	49
12	.53288	.84619	.62973	.5880	.1818	.8766	.15381	.46712	48
13	.53312	.84604	.63014	.5869	.1820	.8757	.15396	.46688	47
14	.53337	.84588	.63055	.5859	.1822	.8749	.15412	.46663	46
15	0.53361	0.84573	0.63095	1.5849	1.1824	1.8740	0.15427	0.46638	45
16	.53386	.84557	.63136	.5839	.1826	.8731	.15443	.46614	44
17	.53411	.84542	.63177	.5829	.1828	.8723	.15458	.46589	43
18	.53435	.84526	.63217	.5818	.1831	.8714	.15474	.46565	42
19	.53460	.84511	.63258	.5808	.1833	.8706	.15489	.46540	41
20	0.53484	0.84495	0.63299	1.5798	1.1835	1.8697	0.15505	0.46516	40
21	.53509	.84479	.63339	.5788	.1837	.8688	.15520	.46491	39
22	.53533	.84464	.63380	.5778	.1839	.8680	.15536	.46466	38
23	.53558	.84448	.63421	.5768	.1841	.8671	.15552	.46442	37
24	.53583	.84433	.63462	.5757	.1844	.8663	.15567	.46417	36
25	0.53607	0.84417	0.63503	1.5747	1.1846	1.8654	0.15583	0.46393	35
26	.53632	.84402	.63543	.5737	.1848	.8646	.15598	.46368	34
27	.53656	.84386	.63584	.5727	.1850	.8637	.15614	.46344	33
28	.53681	.84370	.63625	.5717	.1852	.8629	.15630	.46319	32
29	.53705	.84355	.63666	.5707	.1855	.8620	.15645	.46294	31
30	0.53730	0.84339	0.63707	1.5697	1.1857	1.8611	0.15661	0.46270	30
31	.53754	.84323	.63748	.5687	.1859	.8603	.15676	.46245	29
32	.53779	.84308	.63789	.5677	.1861	.8595	.15692	.46221	28
33	.53803	.84292	.63830	.5667	.1863	.8586	.15708	.46196	27
34	.53828	.84276	.63871	.5657	.1866	.8578	.15723	.46172	26
35	0.53852	0.84261	0.63912	1.5646	1.1868	1.8569	0.15739	0.46147	25
36	.53877	.84245	.63953	.5636	.1870	.8561	.15755	.46123	24
37	.53901	.84229	.63994	.5626	.1872	.8552	.15770	.46098	23
38	.53926	.84214	.64035	.5616	.1874	.8544	.15786	.46074	22
39	.53950	.84198	.64076	.5606	.1877	.8535	.15802	.46049	21
40	0.53975	0.84182	0.64117	1.5596	1.1879	1.8527	0.15817	0.46025	20
41	.53999	.84167	.64158	.5586	.1881	.8519	.15833	.46000	19
42	.54024	.84151	.64199	.5577	.1883	.8510	.15849	.45976	18
43	.54048	.84135	.64240	.5567	.1886	.8502	.15865	.45951	17
44	.54073	.84120	.64281	.5557	.1888	.8493	.15880	.45927	16
45	0.54097	0.84104	0.64322	1.5547	1.1890	1.8485	0.15896	0.45902	15
46	.54122	.84088	.64363	.5537	.1892	.8477	.15912	.45878	14
47	.54146	.84072	.64404	.5527	.1894	.8468	.15927	.45854	13
48	.54171	.84057	.64446	.5517	.1897	.8460	.15943	.45829	12
49	.54195	.84041	.64487	.5507	.1899	.8452	.15959	.45805	11
50	0.54220	0.84025	0.64528	1.5497	1.1901	1.8443	0.15975	0.45780	10
51	.54244	.84009	.64569	.5487	.1903	.8435	.15991	.45756	9
52	.54268	.83993	.64610	.5477	.1906	.8427	.16006	.45731	8
53	.54293	.83978	.64652	.5467	.1908	.8418	.16022	.45707	7
54	.54317	.83962	.64693	.5458	.1910	.8410	.16038	.45682	6
55	0.54342	0.83946	0.64734	1.5448	1.1912	1.8402	0.16054	0.45658	5
56	.54366	.83930	.64775	.5438	.1915	.8394	.16070	.45634	4
57	.54391	.83914	.64817	.5428	.1917	.8385	.16085	.45609	3
58	.54415	.83899	.64858	.5418	.1919	.8377	.16101	.45585	2
59	.54439	.83883	.64899	.5408	.1921	.8369	.16117	.45560	1
60	0.54464	0.83867	0.64941	1.5399	1.1922	1.8361	0.16133	0.45536	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

122° **57°**

33° **Natural Trigonometric Functions** 146°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.54464	0.83867	0.64941	1.5399	1.1924	1.8361	0.16133	0.45536	60
1	.54488	.83851	.64982	.5389	.1926	.8352	.16149	.45512	59
2	.54513	.83835	.65023	.5379	.1928	.8344	.16165	.45487	58
3	.54537	.83819	.65065	.5369	.1930	.8336	.16180	.45463	57
4	.54561	.83804	.65106	.5359	.1933	.8328	.16196	.45438	56
5	0.54586	0.83788	0.65148	1.5350	1.1935	1.8320	0.16212	0.45414	55
6	.54610	.83772	.65189	.5340	.1937	.8311	.16228	.45390	54
7	.54634	.83756	.65231	.5330	.1939	.8303	16244	.45365	53
8	.54659	.83740	.65272	.5320	.1942	.8295	.16260	.45341	52
9	.54683	.83724	.65314	.5311	.1944	.8287	.16276	.45317	51
10	0.54708	0.83708	0.65355	1.5301	1.1946	1.8279	0.16292	0.45292	50
11	.54732	.83692	.65397	.5291	.1948	.8271	.16308	.45268	49
12	.54756	.83676	.65438	.5282	.1951	.8263	.16323	.45244	48
13	.54781	.83660	.65480	.5272	.1953	.8255	.16339	.45219	47
14	.54805	.83644	.65521	.5262	.1955	.8246	.16355	.45195	46
15	0.54829	0.83629	0.65563	1.5252	1.1958	1.8238	0.16371	0.45171	45
16	.54854	.83613	.65604	.5243	.1960	.8230	.16387	.45146	44
17	.54878	.83597	.65646	.5233	.1962	.8222	.16403	.45122	43
18	.54902	.83581	.65688	.5223	.1964	.8214	.16419	.45098	42
19	.54926	.83565	.65729	.5214	.1967	.8206	.16435	.45073	41
20	0.54951	0.83549	0.65771	1.5204	1.1969	1.8198	0.16451	0.45049	40
21	.54975	.83533	.65813	.5195	.1971	.8190	.16467	.45025	39
22	.54999	.83517	.65854	.5185	.1974	.8182	.16483	.45000	38
23	.55024	.83501	.65896	.5175	.1976	.8174	.16499	.44976	37
24	.55048	.83485	.65938	.5166	.1978	.8166	.16515	.44952	36
25	0.55072	0.83469	0.65980	1.5156	1.1980	1.8158	0.16531	0.44928	35
26	.55097	.83453	.66021	.5147	.1983	.8150	.16547	.44903	34
27	.55121	.83437	.66063	.5137	.1985	.8142	.16563	.44879	33
28	.55145	.83421	.66105	.5127	.1987	.8134	.16579	.44855	32
29	.55169	.83405	.66147	.5118	.1990	.8126	.16595	.44830	31
30	0.55194	0.83388	0.66188	1.5108	1.1992	1.8118	0.16611	0.44806	30
31	.55218	.83372	.66230	.5099	.1994	.8110	.16627	.44782	29
32	.55242	.83356	.66272	.5089	.1997	.8102	.16643	.44758	28
33	.55266	.83340	.66314	.5080	.1999	.8094	.16660	.44733	27
34	.55291	.83324	.66356	.5070	.2001	.8086	.16676	.44709	26
35	0.55315	0.83308	0.66398	1.5061	1.2004	1.8078	0.16692	0.44685	25
36	.55339	.83292	.66440	.5051	.2006	.8070	.16708	.44661	24
37	.55363	.83276	.66482	.5042	.2008	.8062	.16724	.44637	23
38	.55388	.83260	.66524	.5032	.2010	.8054	.16740	.44612	22
39	.55412	.83244	.66566	.5023	.2013	.8047	.16756	.44588	21
40	0.55436	0.83228	0.66608	1.5013	1.2015	1.8039	0.16772	0.44564	20
41	.55460	.83211	.66650	.5004	.2017	.8031	.16788	.44540	19
42	.55484	.83195	.66692	.4994	.2020	.8023	.16804	.44515	18
43	.55509	.83179	.66734	.4985	.2022	.8015	.16821	.44491	17
44	.55533	.83163	.66776	.4975	.2024	.8007	.16837	.44467	16
45	0.55557	0.83147	0.66818	1.4966	1.2027	1.7999	0.16853	0.44443	15
46	.55581	.83131	.66860	.4957	.2029	.7992	.16869	.44419	14
47	.55605	.83115	.66902	.4947	.2031	.7984	.16885	.44395	13
48	.55629	.83098	.66944	.4938	.2034	.7976	.16901	.44370	12
49	.55654	.83082	.66986	.4928	.2036	.7968	.16918	.44346	11
50	0.55678	0.83066	0.67028	1.4919	1.2039	1.7960	0.16934	0.44322	10
51	.55702	.83050	.67071	.4910	.2041	.7953	.16950	.44298	9
52	.55726	.83034	.67113	.4900	.2043	.7945	.16966	.44274	8
53	.55750	.83017	.67155	.4891	.2046	.7937	.16982	.44250	7
54	.55774	.83001	.67197	.4881	.2048	.7929	.16999	.44225	6
55	0.55799	0.82985	0.67239	1.4872	1.2050	1.7921	0.17015	0.44201	5
56	.55823	.82969	.67282	.4863	.2053	.7914	.17031	.44177	4
57	.55847	.82952	.67324	.4853	.2055	.7906	.17047	.44153	3
58	.55871	.82936	.67366	.4844	.2057	.7898	.17064	.44129	2
59	.55895	.82920	.67408	.4835	.2060	.7891	.17080	.44105	1
60	0.55919	0.82904	0.67451	1.4826	1.2062	1.7883	0.17096	0.44081	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

123° 56°

34° **Natural Trigonometric Functions** 145°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.55919	0.82904	0.67451	1.4826	1.2062	1.7883	0.17096	0.44081	60
1	.55943	.82887	.67493	.4816	.2064	.7875	.17112	.44057	59
2	.55967	.82871	.67535	.4807	.2067	.7867	.17129	.44032	58
3	.55992	.82855	.67578	.4798	.2069	.7860	.17145	.44008	57
4	.56016	.82839	.67620	.4788	.2072	.7852	.17161	.43984	56
5	0.56040	0.82822	0.67663	1.4779	1.2074	1.7844	0.17178	0.43960	55
6	.56064	.82806	.67705	.4770	.2076	.7837	.17194	.43936	54
7	.56088	.82790	.67747	.4761	.2079	.7829	.17210	.43912	53
8	.56112	.82773	.67790	.4751	.2081	.7821	.17227	.43888	52
9	.56136	.82757	.67832	.4742	.2083	.7814	.17243	.43864	51
10	0.56160	0.82741	0.67875	1.4733	1.2086	1.7806	0.17259	0.43840	50
11	.56184	.82724	.67917	.4724	.2088	.7798	.17276	.43816	49
12	.56208	.82708	.67960	.4714	.2091	.7791	.17292	.43792	48
13	.56232	.82692	.68002	.4705	.2093	.7783	.17308	.43768	47
14	.56256	.82675	.68045	.4696	.2095	.7776	.17325	.43743	46
15	0.56280	0.82659	0.68087	1.4687	1.2098	1.7768	0.17341	0.43719	45
16	.56304	.82643	.68130	.4678	.2100	.7760	.17357	.43695	44
17	.56328	.82626	.68173	.4669	.2103	.7753	.17374	.43671	43
18	.56353	.82610	.68215	.4659	.2105	.7745	.17390	.43647	42
19	.56377	.82593	.68258	.4650	.2107	.7738	.17406	.43623	41
20	0.56401	0.82577	0.68301	1.4641	1.2110	1.7730	0.17423	0.43599	40
21	.56425	.82561	.68343	.4632	.2112	.7723	.17439	.43575	39
22	.56449	.82544	.68386	.4623	.2115	.7715	.17456	.43551	38
23	.56473	.82528	.68429	.4614	.2117	.7708	.17472	.43527	37
24	.56497	.82511	.68471	.4605	.2119	.7700	.17489	.43503	36
25	0.56521	0.82495	0.68514	1.4595	1.2122	1.7693	0.17505	0.43479	35
26	.56545	.82478	.68557	.4586	.2124	.7685	.17521	.43455	34
27	.56569	.82462	.68600	.4577	.2127	.7678	.17538	.43431	33
28	.56593	.82445	.68642	.4568	.2129	.7670	.17554	.43407	32
29	.56617	.82429	.68685	.4559	.2132	.7663	.17571	.43383	31
30	0.56641	0.82413	0.68728	1.4550	1.2134	1.7655	0.17587	0.43359	30
31	.56664	.82396	.68771	.4541	.2136	.7648	.17604	.43335	29
32	.56688	.82380	.68814	.4532	.2139	.7640	.17620	.43311	28
33	.56712	.82363	.68857	.4523	.2141	.7633	.17637	.43287	27
34	.56736	.82347	.68899	.4514	.2144	.7625	.17653	.43263	26
35	0.56760	0.82330	0.68942	1.4505	1.2146	1.7618	0.17670	0.43239	25
36	.56784	.82314	.68985	.4496	.2149	.7610	.17686	.43216	24
37	.56808	.82297	.69028	.4487	.2151	.7603	.17703	.43192	23
38	.56832	.82280	.69071	.4478	.2153	.7596	.17719	.43168	22
39	.56856	.82264	.69114	.4469	.2156	.7588	.17736	.43144	21
40	0.56880	0.82247	0.69157	1.4460	1.2158	1.7581	0.17752	0.43120	20
41	.56904	.82231	.69200	.4451	.2161	.7573	.17769	.43096	19
42	.56928	.82214	.69243	.4442	.2163	.7566	.17786	.43072	18
43	.56952	.82198	.69286	.4433	.2166	.7559	.17802	.43048	17
44	.56976	.82181	.69329	.4424	.2168	.7551	.17819	.43024	16
45	0.57000	0.82165	0.69372	1.4415	1.2171	1.7544	0.17835	0.43000	15
46	.57023	.82148	.69415	.4406	.2173	.7537	.17852	.42976	14
47	.57047	.82131	.69459	.4397	.2175	.7529	.17868	.42952	13
48	.57071	.82115	.69502	.4388	.2178	.7522	.17885	.42929	12
49	.57095	.82098	.69545	.4379	.2180	.7514	.17902	.42905	11
50	0.57119	0.82082	0.69588	1.4370	1.2183	1.7507	0.17918	0.42881	10
51	.57143	.82065	.69631	.4361	.2185	.7500	.17935	.42857	9
52	.57167	.82048	.69674	.4352	.2188	.7493	.17951	.42833	8
53	.57191	.82032	.69718	.4343	.2190	.7485	.17968	.42809	7
54	.57214	.82015	.69761	.4335	.2193	.7478	.17985	.42785	6
55	0.57238	0.81998	0.69804	1.4326	1.2195	1.7471	0.18001	0.42761	5
56	.57262	.81982	.69847	.4317	.2198	.7463	.18018	.42738	4
57	.57286	.81965	.69891	.4308	.2200	.7456	.18035	.42714	3
58	.57310	.81948	.69934	.4299	.2203	.7449	.18051	.42690	2
59	.57334	.81932	.69977	.4290	.2205	.7442	.18068	.42666	1
60	0.57358	0.81915	0.70021	1.4281	1.2208	1.7434	0.18085	0.42642	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

124° 55°

35° **Natural Trigonometric Functions** 144°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.57358	0.81915	0.70021	1.4281	1.2208	1.7434	0.18085	0.42642	60
1	.57381	.81898	.70064	.4273	.2210	.7427	.18101	.42618	59
2	.57405	.81882	.70107	.4264	.2213	.7420	.18118	.42595	58
3	.57429	.81865	.70151	.4255	.2215	.7413	.18135	.42571	57
4	.57453	.81848	.70194	.4246	.2218	.7405	.18151	.42547	56
5	0.57477	0.81832	0.70238	1.4237	1.2220	1.7398	0.18168	0.42523	55
6	.57500	.81815	.70281	.4228	.2223	.7391	.18185	.42499	54
7	.57524	.81798	.70325	.4220	.2225	.7384	.18202	.42476	53
8	.57548	.81781	.70368	.4211	.2228	.7377	.18218	.42452	52
9	.57572	.81765	.70412	.4202	.2230	.7369	.18235	.42428	51
10	0.57596	0.81748	0.70455	1.4193	1.2233	1.7362	0.18252	0.42404	50
11	.57619	.81731	.70499	.4185	.2235	.7355	.18269	.42380	49
12	.57643	.81714	.70542	.4176	.2238	.7348	.18285	.42357	48
13	.57667	.81698	.70586	.4167	.2240	.7341	.18302	.42333	47
14	.57691	.81681	.70629	.4158	.2243	.7334	.18319	.42309	46
15	0.57714	0.81664	0.70673	1.4150	1.2245	1.7327	0.18336	0.42285	45
16	.57738	.81647	.70717	.4141	.2248	.7319	.18353	.42262	44
17	.57762	.81630	.70760	.4132	.2250	.7312	.18369	.42238	43
18	.57786	.81614	.70804	.4123	.2253	.7305	.18386	.42214	42
19	.57809	.81597	.70848	.4115	.2255	.7298	.18403	.42190	41
20	0.57833	0.81580	0.70891	1.4106	1.2258	1.7291	0.18420	0.42167	40
21	.57857	.81563	.70935	.4097	.2260	.7284	.18437	.42143	39
22	.57881	.81546	.70979	.4089	.2263	.7277	.18453	.42119	38
23	.57904	.81530	.71022	.4080	.2265	.7270	.18470	.42096	37
24	.57928	.81513	.71066	.4071	.2268	.7263	.18487	.42072	36
25	0.57952	0.81496	0.71110	1.4063	1.2270	1.7256	0.18504	0.42048	35
26	.57975	.81479	.71154	.4054	.2273	.7249	.18521	.42024	34
27	.57999	.81462	.71198	.4045	.2276	.7242	.18538	.42001	33
28	.58023	.81445	.71241	.4037	.2278	.7234	.18555	.41977	32
29	.58047	.81428	.71285	.4028	.2281	.7227	.18571	.41953	31
30	0.58070	0.81411	0.71329	1.4019	1.2283	1.7220	0.18588	0.41930	30
31	.58094	.81395	.71373	.4011	.2286	.7213	.18605	.41906	29
32	.58118	.81378	.71417	.4002	.2288	.7206	.18622	.41882	28
33	.58141	.81361	.71461	.3994	.2291	.7199	.18639	.41859	27
34	.58165	.81344	.71505	.3985	.2293	.7192	.18656	.41835	26
35	0.58189	0.81327	0.71549	1.3976	1.2296	1.7185	0.18673	0.41811	25
36	.58212	.81310	.71593	.3968	.2298	.7178	.18690	.41788	24
37	.58236	.81293	.71637	.3959	.2301	.7171	.18707	.41764	23
38	.58259	.81276	.71681	.3951	.2304	.7164	.18724	.41740	22
39	.58283	.81259	.71725	.3942	.2306	.7157	.18741	.41717	21
40	0.58307	0.81242	0.71769	1.3933	1.2309	1.7151	0.18758	0.41693	20
41	.58330	.81225	.71813	.3925	.2311	.7144	.18775	.41669	19
42	.58354	.81208	.71857	.3916	.2314	.7137	.18792	.41646	18
43	.58378	.81191	.71901	.3908	.2316	.7130	.18809	.41622	17
44	.58401	.81174	.71945	.3899	.2319	.7123	.18826	.41599	16
45	0.58425	0.81157	0.71990	1.3891	1.2322	1.7116	0.18843	0.41575	15
46	.58448	.81140	.72034	.3882	.2324	.7109	.18860	.41551	14
47	.58472	.81123	.72078	.3874	.2327	.7102	.18877	.41528	13
48	.58496	.81106	.72122	.3865	.2329	.7095	.18894	.41504	12
49	.58519	.81089	.72166	.3857	.2332	.7088	.18911	.41481	11
50	0.58543	0.81072	0.72211	1.3848	1.2335	1.7081	0.18928	0.41457	10
51	.58566	.81055	.72255	.3840	.2337	.7075	.18945	.41433	9
52	.58590	.81038	.72299	.3831	.2340	.7068	.18962	.41410	8
53	.58614	.81021	.72344	.3823	.2342	.7061	.18979	.41386	7
54	.58637	.81004	.72388	.3814	.2345	.7054	.18996	.41363	6
55	0.58661	0.80987	0.72432	1.3806	1.2348	1.7047	0.19013	0.41339	5
56	.58684	.80970	.72477	.3797	.2350	.7040	.19030	.41316	4
57	.58708	.80953	.72521	.3789	.2353	.7033	.19047	.41292	3
58	.58731	.80936	.72565	.3781	.2355	.7027	.19064	.41268	2
59	.58755	.80919	.72610	.3772	.2358	.7020	.19081	.41245	1
60	0.58778	0.80902	0.72654	1.3764	1.2361	1.7013	0.19098	0.41221	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

125° 54°

36° **Natural Trigonometric Functions** 143°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos	M
0	0.58778	0.80902	0.72654	1.3764	1.2361	1.7013	0.19098	0.41221	60
1	.58802	.80885	.72699	.3755	.2363	.7006	.19115	.41198	59
2	.58825	.80867	.72743	.3747	.2366	.6999	.19132	.41174	58
3	.58849	.80850	.72788	.3738	.2368	.6993	.19150	.41151	57
4	.58873	.80833	.72832	.3730	.2371	.6986	.19167	.41127	56
5	0.58896	0.80816	0.72877	1.3722	1.2374	1.6979	0.19184	0.41104	55
6	.58920	.80799	.72921	.3713	.2376	.6972	.19201	.41080	54
7	.58943	.80782	.72966	.3705	.2379	.6965	.19218	.41057	53
8	.58967	.80765	.73010	.3697	.2382	.6959	.19235	.41033	52
9	.58990	.80747	.73055	.3688	.2384	.6952	.19252	.41010	51
10	0.59014	0.80730	0.73100	1.3680	1.2387	1.6945	0.19270	0.40986	50
11	.59037	.80713	.73144	.3672	.2389	.6938	.19287	.40963	49
12	.59060	.80696	.73189	.3663	.2392	.6932	.19304	.40939	48
13	.59084	.80679	.73234	.3655	.2395	.6925	.19321	.40916	47
14	.59107	.80662	.73278	.3647	.2397	.6918	.19338	.40892	46
15	0.59131	0.80644	0.73323	1.3638	1.2400	1.6912	0.19355	0.40869	45
16	.59154	.80627	.73368	.3630	.2403	.6905	.19373	.40845	44
17	.59178	.80610	.73412	.3622	.2405	.6898	.19390	.40822	43
18	.59201	.80593	.73457	.3613	.2408	.6891	.19407	.40799	42
19	.59225	.80576	.73502	.3605	.2411	.6885	.19424	.40775	41
20	0.59248	0.80558	0.73547	1.3597	1.2413	1.6878	0.19442	0.40752	40
21	.59272	.80541	.73592	.3588	.2416	.6871	.19459	.40728	39
22	.59295	.80524	.73637	.3580	.2419	.6865	.19476	.40705	38
23	.59318	.80507	.73681	.3572	.2421	.6858	.19493	.40681	37
24	.59342	.80489	.73726	.3564	.2424	.6851	.19511	.40658	36
25	0.59365	0.80472	0.73771	1.3555	1.2427	1.6845	0.19528	0.40635	35
26	.59389	.80455	.73816	.3547	.2429	.6838	.19545	.40611	34
27	.59412	.80437	.73861	.3539	.2432	.6831	.19562	.40588	33
28	.59435	.80420	.73906	.3531	.2435	.6825	.19580	.40564	32
29	.59459	.80403	.73951	.3522	.2437	.6818	.19597	.40541	31
30	0.59482	0.80386	0.73996	1.3514	1.2440	1.6812	0.19614	0.40518	30
31	.59506	.80368	.74041	.3506	.2443	.6805	.19632	.40494	29
32	.59529	.80351	.74086	.3498	.2445	.6798	.19649	.40471	28
33	.59552	.80334	.74131	.3489	.2448	.6792	.19666	.40447	27
34	.59576	.80316	.74176	.3481	.2451	.6785	.19683	.40424	26
35	0.59599	0.80299	0.74221	1.3473	1.2453	1.6779	0.19701	0.40401	25
36	.59622	.80282	.74266	.3465	.2456	.6772	.19718	.40377	24
37	.59646	.80264	.74312	.3457	.2459	.6766	.19736	.40354	23
38	.59669	.80247	.74357	.3449	.2461	.6759	.19753	.40331	22
39	.59692	.80230	.74402	.3440	.2464	.6752	.19770	.40307	21
40	0.59716	0.80212	0.74447	1.3432	1.2467	1.6746	0.19788	0.40284	20
41	.59739	.80195	.74492	.3424	.2470	.6739	.19805	.40261	19
42	.59762	.80177	.74538	.3416	.2472	.6733	.19822	.40237	18
43	.59786	.80160	.74583	.3408	.2475	.6726	.19840	.40214	17
44	.59809	.80143	.74628	.3400	.2478	.6720	.19857	.40191	16
45	0.59832	0.80125	0.74673	1.3392	1.2480	1.6713	0.19875	0.40167	15
46	.59856	.80108	.74719	.3383	.2483	.6707	.19892	.40144	14
47	.59879	.80090	.74764	.3375	.2486	.6700	.19909	.40121	13
48	.59902	.80073	.74809	.3367	.2488	.6694	.19927	.40098	12
49	.59926	.80056	.74855	.3359	.2491	.6687	.19944	.40074	11
50	0.59949	0.80038	0.74900	1.3351	1.2494	1.6681	0.19962	0.40051	10
51	.59972	.80021	.74946	.3343	.2497	.6674	.19979	.40028	9
52	.59995	.80003	.74991	.3335	.2499	.6668	.19997	.40004	8
53	.60019	.79986	.75037	.3327	.2502	.6661	.20014	.39981	7
54	.60042	.79968	.75082	.3319	.2505	.6655	.20031	.39958	6
55	0.60065	0.79951	0.75128	1.3311	1.2508	1.6648	0.20049	0.39935	5
56	.60088	.79933	.75173	.3303	.2510	.6642	.20066	.39911	4
57	.60112	.79916	.75219	.3294	.2513	.6636	.20084	.39888	3
58	.60135	.79898	.75264	.3286	.2516	.6629	.20101	.39865	2
59	.60158	.79881	.75310	.3278	.2519	.6623	.20119	.39842	1
60	0.60181	0.79863	0.75355	1.3270	1.2521	1.6616	0.20136	0.39818	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

126° 53°

37° **Natural Trigonometric Functions** 142°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.60181	0.79863	0.75355	1.3270	1.2521	1.6616	0.20136	0.39818	60
1	.60205	.79846	.75401	.3262	.2524	.6610	.20154	.39795	59
2	.60228	.79828	.75447	.3254	.2527	.6603	.20171	.39772	58
3	.60251	.79811	.75492	.3246	.2530	.6597	.20189	.39749	57
4	.60274	.79793	.75538	.3238	.2532	.6591	.20206	.39726	56
5	0.60298	0.79776	0.75584	1.3230	1.2535	1.6584	0.20224	0.39702	55
6	.60320	.79758	.75629	.3222	.2538	.6578	.20242	.39679	54
7	.60344	.79741	.75675	.3214	.2541	.6572	.20259	.39656	53
8	.60367	.79723	.75721	.3206	.2543	.6565	.20277	.39633	52
9	.60390	.79706	.75767	.3198	.2546	.6559	.20294	.39610	51
10	0.60413	0.79688	0.75812	1.3190	1.2549	1.6552	0.20312	0.39586	50
11	.60437	.79670	.75858	.3182	.2552	.6546	.20329	.39563	49
12	.60460	.79653	.75904	.3174	.2554	.6540	.20347	.39540	48
13	.60483	.79635	.75950	.3166	.2557	.6533	.20365	.39517	47
14	.60506	.79618	.75996	.3159	.2560	.6527	.20382	.39494	46
15	0.60529	0.79600	0.76042	1.3151	1.2563	1.6521	0.20400	0.39471	45
16	.60552	.79582	.76088	.3143	.2565	.6514	.20417	.39447	44
17	.60576	.79565	.76134	.3135	.2568	.6508	.20435	.39424	43
18	.60599	.79547	.76179	.3127	.2571	.6502	.20453	.39401	42
19	.60622	.79530	.76225	.3119	.2574	.6496	.20470	.39378	41
20	0.60645	0.79512	0.76271	1.3111	1.2577	1.6489	0.20488	0.39355	40
21	.60668	.79494	.76317	.3103	.2579	.6483	.20505	.39332	39
22	.60691	.79477	.76364	.3095	.2582	.6477	.20523	.39309	38
23	.60714	.79459	.76410	.3087	.2585	.6470	.20541	.39285	37
24	.60737	.79441	.76456	.3079	.2588	.6464	.20558	.39262	36
25	0.60761	0.79424	0.76502	1.3071	1.2591	1.6458	0.20576	0.39239	35
26	.60784	.79406	.76548	.3064	.2593	.6452	.20594	.39216	34
27	.60807	.79388	.76594	.3056	.2596	.6445	.20611	.39193	33
28	.60830	.79371	.76640	.3048	.2599	.6439	.20629	.39170	32
29	.60853	.79353	.76686	.3040	.2602	.6433	.20647	.39147	31
30	0.60876	0.79335	0.76733	1.3032	1.2605	1.6427	0.20665	0.39124	30
31	.60899	.79318	.76779	.3024	.2607	.6420	.20682	.39101	29
32	.60922	.79300	.76825	.3016	.2610	.6414	.20700	.39078	28
33	.60945	.79282	.76871	.3009	.2613	.6408	.20718	.39055	27
34	.60968	.79264	.76918	.3001	.2616	.6402	.20735	.39031	26
35	0.60991	0 79247	0.76964	1.2993	1.2619	1.6396	0.20753	0.39008	25
36	.61014	.79229	.77010	.2985	.2622	.6389	.20771	.38985	24
37	.61037	.79211	.77057	.2977	.2624	.6383	.20789	.38962	23
38	.61061	.79193	.77103	.2970	.2627	.6377	.20806	.38939	22
39	.61084	.79176	.77149	.2962	.2630	.6371	.20824	.38916	21
40	0.61107	0.79158	0.77196	1.2954	1.2633	1.6365	0.20842	0.38893	20
41	.61130	.79140	.77242	.2946	.2636	.6359	.20860	.38870	19
42	.61153	.79122	.77289	.2938	.2639	.6352	.20878	.38847	18
43	.61176	.79104	.77335	.2931	.2641	.6346	.20895	.38824	17
44	.61199	.79087	.77382	.2923	.2644	.6340	.20913	.38801	16
45	0.61222	0.79069	0.77428	1.2915	1.2647	1.6334	0.20931	0.38778	15
46	.61245	.79051	.77475	.2907	.2650	.6328	.20949	.38755	14
47	.61268	.79033	.77521	.2900	.2653	.6322	.20967	.38732	13
48	.61290	.79015	.77568	.2892	.2656	.6316	.20984	.38709	12
49	.61314	.78998	.77614	.2884	.2659	.6309	.21002	.38686	11
50	0.61337	0.78980	0.77661	1.2876	1.2661	1.6303	0.21020	0.38663	10
51	.61360	.78962	.77708	.2869	.2664	.6297	.21038	.38640	9
52	.61383	.78944	.77754	.2861	.2667	.6291	.21056	.38617	8
53	.61405	.78926	.77801	.2853	.2670	.6285	.21074	.38594	7
54	.61428	.78908	.77848	.2845	.2673	.6279	.21091	.38571	6
55	0.61451	0.78890	0.77895	1.2838	1.2676	1.6273	0.21109	0.38548	5
56	.61474	.78873	.77941	.2830	.2679	.6267	.21127	.38525	4
57	.61497	.78855	.77988	.2822	.2681	.6261	.21145	.38503	3
58	.61520	.78837	.78035	.2815	.2684	.6255	.21163	.38480	2
59	.61543	.78819	.78082	.2807	.2687	.6249	.21181	.38457	1
60	0.61566	0.78801	0.78128	1.2799	1.2690	1.6243	0.21199	0.38434	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

127° 52°

38° **Natural Trigonometric Functions** 141°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.61566	0.78801	0.78128	1.2799	1.2690	1.6243	0.21199	0.38434	60
1	.61589	.78783	.78175	.2792	.2693	.6237	.21217	.38411	59
2	.61612	.78765	.78222	.2784	.2696	.6231	.21235	.38388	58
3	.61635	.78747	.78269	.2776	.2699	.6224	.21253	.38365	57
4	.61658	.78729	.78316	.2769	.2702	.6218	.21271	.38342	56
5	0.61681	0.78711	0.78363	1.2761	1.2705	1.6212	0.21288	0.38319	55
6	.61703	.78693	.78410	.2753	.2707	.6206	.21306	.38296	54
7	.61726	.78675	.78457	.2746	.2710	.6200	.21324	.38273	53
8	.61749	.78657	.78504	.2738	.2713	.6194	.21342	.38251	52
9	.61772	.78640	.78551	.2730	.2716	.6188	.21360	.38228	51
10	0.61795	0.78622	0.78598	1.2723	1.2719	1.6182	0.21378	0.38205	50
11	.61818	.78604	.78645	.2715	.2722	.6176	.21396	.38182	49
12	.61841	.78586	.78692	.2708	.2725	.6170	.21414	.38159	48
13	.61864	.78568	.78739	.2700	.2728	.6164	.21432	.38136	47
14	.61886	.78550	.78786	.2692	.2731	.6159	.21450	.38113	46
15	0.61909	0.78532	0.78834	1.2685	1.2734	1.6153	0.21468	0.38091	45
16	.61932	.78514	.78881	.2677	.2737	.6147	.21486	.38068	44
17	.61955	.78496	.78928	.2670	.2739	.6141	.21504	.38045	43
18	.61978	.78478	.78975	.2662	.2742	.6135	.21522	.38022	42
19	.62001	.78460	.79022	.2655	.2745	.6129	.21540	.37999	41
20	0.62023	0.78441	0.79070	1.2647	1.2748	1.6123	0.21558	0.37976	40
21	.62046	.78423	.79117	.2639	.2751	.6117	.21576	.37954	39
22	.62069	.78405	.79164	.2632	.2754	.6111	.21594	.37931	38
23	.62092	.78387	.79212	.2624	.2757	.6105	.21612	.37908	37
24	.62115	.78369	.79259	.2617	.2760	.6099	.21631	.37885	36
25	0.62137	0.78351	0.79306	1.2609	1.2763	1.6093	0.21649	0.37862	35
26	.62160	.78333	.79354	.2602	.2766	.6087	.21667	.37840	34
27	.62183	.78315	.79401	.2594	.2769	.6081	.21685	.37817	33
28	.62206	.78297	.79449	.2587	.2772	.6077	.21703	.37794	32
29	.62229	.78279	.79496	.2579	.2775	.6070	.21721	.37771	31
30	0.62251	0.78261	0.79543	1.2572	1.2778	1.6064	0.21739	0.37748	30
31	.62274	.78243	.79591	.2564	.2781	.6058	.21757	.37726	29
32	.62297	.78224	.79639	.2557	.2784	.6052	.21775	.37703	28
33	.62320	.78206	.79686	.2549	.2787	.6046	.21793	.37680	27
34	.62342	.78188	.79734	.2542	.2790	.6040	.21812	.37657	26
35	0.62365	0.78170	0.79781	1.2534	1.2793	1.6034	0.21830	0.37635	25
36	.62388	.78152	.79829	.2527	.2795	.6029	.21848	.37612	24
37	.62411	.78134	.79876	.2519	.2798	.6023	.21866	.37589	23
38	.62433	.78116	.79924	.2512	.2801	.6017	.21884	.37566	22
39	.62456	.78097	.79972	.2504	.2804	.6011	.21902	.37544	21
40	0.62479	0.78079	0.80020	1.2497	1.2807	1.6005	0.21921	0.37521	20
41	.62501	.78061	.80067	.2489	.2810	.6000	.21939	.37498	19
42	.62524	.78043	.80115	.2482	.2813	.5994	.21957	.37476	18
43	.62547	.78025	.80163	.2475	.2816	.5988	.21975	.37453	17
44	.62570	.78007	.80211	.2467	.2819	.5982	.21993	.37430	16
45	0.62592	0.77988	0.80258	1.2460	1.2822	1.5976	0.22011	0.37408	15
46	.62615	.77970	.80306	.2452	.2825	.5971	.22030	.37385	14
47	.62638	.77952	.80354	.2445	.2828	.5965	.22048	.37362	13
48	.62660	.77934	.80402	.2437	.2831	.5959	.22066	.37340	12
49	.62683	.77915	.80450	.2430	.2834	.5953	.22084	.37317	11
50	0.62706	0.77897	0.80498	1.2423	1.2837	1.5947	0.22103	0.37294	10
51	.62728	.77879	.80546	.2415	.2840	.5942	.22121	.37272	9
52	.62751	.77861	.80594	.2408	.2843	.5936	.22139	.37249	8
53	.62774	.77842	.80642	.2400	.2846	.5930	.22157	.37226	7
54	.62796	.77824	.80690	.2393	.2849	.5924	.22176	.37204	6
55	0.62819	0.77806	0.80738	1.2386	1.2852	1.5919	0.22194	0.37181	5
56	.62841	.77788	.80786	.2378	.2855	.5913	.22212	.37158	4
57	.62864	.77769	.80834	.2371	.2858	.5907	.22230	.37136	3
58	.62887	.77751	.80882	.2364	.2861	.5901	.22249	.37113	2
59	.62909	.77733	.80930	.2356	.2864	.5896	.22267	.37090	1
60	0.62932	0.77715	0.80978	1.2349	1.2867	1.5890	0.22285	0.37068	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

128° 51°

39° **Natural Trigonometric Functions** 140°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.62932	0.77715	0.80978	1.2349	1.2867	1.5890	0.22285	0.37068	60
1	.62955	.77696	.81026	.2342	.2871	.5884	.22304	.37045	59
2	.62977	.77678	.81075	.2334	.2874	.5879	.22322	.37023	58
3	.63000	.77660	.81123	.2327	.2877	.5873	.22340	.37000	57
4	.63022	.77641	.81171	.2320	.2880	.5867	.22359	.36977	56
5	0.63045	0.77623	0.81219	1.2312	1.2883	1.5862	0.22377	0.36955	55
6	.63067	.77605	.81268	.2305	.2886	.5856	.22395	.36932	54
7	.63090	.77586	.81316	.2297	.2889	.5850	.22414	.36910	53
8	.63113	.77568	.81364	.2290	.2892	.5845	.22432	.36887	52
9	.63135	.77549	.81413	.2283	.2895	.5839	.22450	.36865	51
10	0.63158	0.77531	0.81461	1.2276	1.2898	1.5833	0.22469	0.36842	50
11	.63180	.77513	.81509	.2268	.2901	.5828	.22487	.36820	49
12	.63203	.77494	.81558	.2261	.2904	.5822	.22505	.36797	48
13	.63225	.77476	.81606	.2254	.2907	.5816	.22524	.36774	47
14	.63248	.77458	.81655	.2247	.2910	.5811	.22542	.36752	46
15	0.63270	0.77439	0.81703	1.2239	1.2913	1.5805	0.22561	0.36729	45
16	.63293	.77421	.81752	.2232	.2916	.5799	.22579	.36707	44
17	.63315	.77402	.81800	.2225	.2919	.5794	.22597	.36684	43
18	.63338	.77384	.81849	.2218	.2922	.5788	.22616	.36662	42
19	.63360	.77365	.81898	.2210	.2926	.5783	.22634	.36639	41
20	0.63383	0.77347	0.81946	1.2203	1.2929	1.5777	0.22653	0.36617	40
21	.63405	.77329	.81995	.2196	.2932	.5771	.22671	.36594	39
22	.63428	.77310	.82043	.2189	.2935	.5766	.22690	.36572	38
23	.63450	.77292	.82092	.2181	.2938	.5760	.22708	.36549	37
24	.63473	.77273	.82141	.2174	.2941	.5755	.22727	.36527	36
25	0.63495	0.77255	0.82190	1.2167	1.2944	1.5749	0.22745	0.36504	35
26	.63518	.77236	.82238	.2160	.2947	.5743	.22763	.36482	34
27	.63540	.77218	.82287	.2152	.2950	.5738	.22782	.36459	33
28	.63563	.77199	.82336	.2145	.2953	.5732	.22800	.36437	32
29	.63585	.77181	.82385	.2138	.2956	.5727	.22819	.36415	31
30	0.63608	0.77162	0.82434	1.2131	1.2960	1.5721	0.22837	0.36392	30
31	.63630	.77144	.82482	.2124	.2963	.5716	.22856	.36370	29
32	.63653	.77125	.82531	.2117	.2966	.5710	.22874	.36347	28
33	.63675	.77107	.82580	.2109	.2969	.5705	.22893	.36325	27
34	.63697	.77088	.82629	.2102	.2972	.5699	.22912	.36302	26
35	0.63720	0.77070	0.82678	1.2095	1.2975	1.5694	0.22930	0.36280	25
36	.63742	.77051	.82727	.2088	.2978	.5688	.22949	.36258	24
37	.63765	.77033	.82776	.2081	.2981	.5683	.22967	.36235	23
38	.63787	.77014	.82825	.2074	.2985	.5677	.22986	.36213	22
39	.63810	.76996	.82874	.2066	.2988	.5672	.23004	.36190	21
40	0.63832	0.76977	0.82923	1.2059	1.2991	1.5666	0.23023	0.36168	20
41	.63854	.76958	.82972	.2052	.2994	.5661	.23041	.36146	19
42	.63877	.76940	.83022	.2045	.2997	.5655	.23060	.36123	18
43	.63899	.76921	.83071	.2038	.3000	.5650	.23079	.36101	17
44	.63921	.76903	.83120	.2031	.3003	.5644	.23097	.36078	16
45	0.63944	0.76884	0.83169	1.2024	1.3006	1.5639	0.23116	0.36056	15
46	.63966	.76865	.83218	.2016	.3010	.5633	.23134	.36034	14
47	.63989	.76847	.83267	.2009	.3013	.5628	.23153	.36011	13
48	.64011	.76828	.83317	.2002	.3016	.5622	.23172	.35989	12
49	.64033	.76810	.83366	.1995	.3019	.5617	.23190	.35967	11
50	0.64056	0.76791	0.83415	1.1988	1.3022	1.5611	0.23209	0.35944	10
51	.64078	.76772	.83465	.1981	.3025	.5606	.23227	.35922	9
52	.64100	.76754	.83514	.1974	.3029	.5600	.23246	.35900	8
53	.64123	.76735	.83563	.1967	.3032	.5595	.23265	.35877	7
54	.64145	.76716	.83613	.1960	.3035	.5590	.23283	.35855	6
55	0.64167	0.76698	0.83662	1.1953	1.3038	1.5584	0.23302	0.35833	5
56	.64189	.76679	.83712	.1946	.3041	.5579	.23321	.35810	4
57	.64212	.76660	.83761	.1939	.3044	.5573	.23339	.35788	3
58	.64234	.76642	.83811	.1932	.3048	.5568	.23358	.35766	2
59	.64256	.76623	.83860	.1924	.3051	.5563	.23377	.35743	1
60	0.64279	0.76604	0.83910	1.1917	1.3054	1.5557	0.23395	0.35721	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

129° 50°

40° **Natural Trigonometric Functions** **139°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.64279	0.76604	0.83910	1.1917	1.3054	1.5557	0.23395	0.35721	60
1	.64301	.76586	.83959	.1910	.3057	.5552	.23414	.35699	59
2	.64323	.76567	.84009	.1903	.3060	.5546	.23433	.35677	58
3	.64345	.76548	.84059	.1896	.3064	.5541	.23452	.35654	57
4	.64368	.76530	.84108	.1889	.3067	.5536	.23470	.35632	56
5	0.64390	0.76511	0.84158	1.1882	1.3070	1.5530	0.23489	0.35610	55
6	.64412	.76492	.84208	.1875	.3073	.5525	.23508	.35588	54
7	.64435	.76473	.84257	.1868	.3076	.5520	.23527	.35565	53
8	.64457	.76455	.84307	.1861	.3080	.5514	.23545	.35543	52
9	.64479	.76436	.84357	.1854	.3083	.5509	.23564	.35521	51
10	0.64501	0.76417	0.84407	1.1847	1.3086	1.5503	0.23583	0.35499	50
11	.64523	.76398	.84457	.1840	.3089	.5498	.23602	.35476	49
12	.64546	.76380	.84506	.1833	.3092	.5493	.23620	.35454	48
13	.64568	.76361	.84556	.1826	.3096	.5487	.23639	.35432	47
14	.64590	.76342	.84606	.1819	.3099	.5482	.23658	.35410	46
15	0.64612	0.76323	0.84656	1.1812	1.3102	1.5477	0.23677	0.35388	45
16	.64635	.76304	.84706	.1805	.3105	.5471	.23695	.35365	44
17	.64657	.76286	.84756	.1798	.3109	.5466	.23714	.35343	43
18	.64679	.76267	.84806	.1791	.3112	.5461	.23733	.35321	42
19	.64701	.76248	.84856	.1785	.3115	.5456	.23752	.35299	41
20	0.64723	0.76229	0.84906	1.1778	1.3118	1.5450	0.23771	0.35277	40
21	.64745	.76210	.84956	.1771	.3121	.5445	.23790	.35254	39
22	.64768	.76191	.85006	.1764	.3125	.5440	.23808	.35232	38
23	.64790	.76173	.85056	.1757	.3128	.5434	.23827	.35210	37
24	.64812	.76154	.85107	.1750	.3131	.5429	.23846	.35188	36
25	0.64834	0.76135	0.85157	1.1743	1.3134	1.5424	0.23865	0.35166	35
26	.64856	.76116	.85207	.1736	.3138	.5419	.23884	.35144	34
27	.64878	.76097	.85257	.1729	.3141	.5413	.23903	.35121	33
28	.64900	.76078	.85307	.1722	.3144	.5408	.23922	.35099	32
29	.64923	.76059	.85358	.1715	.3148	.5403	.23940	.35077	31
30	0.64945	0.76041	0.85408	1.1708	1.3151	1.5398	0.23959	0.35055	30
31	.64967	.76022	.85458	.1702	.3154	.5392	.23978	.35033	29
32	.64989	.76003	.85509	.1695	.3157	.5387	.23997	.35011	28
33	.65011	.75984	.85559	.1688	.3161	.5382	.24016	.34989	27
34	.65033	.75965	.85609	.1681	.3164	.5377	.24035	.34967	26
35	0.65055	0.75946	0.85660	1.1674	1.3167	1.5371	0.24054	0.34945	25
36	.65077	.75927	.85710	.1667	.3170	.5366	.24073	.34922	24
37	.65100	.75908	.85761	.1660	.3174	.5361	.24092	.34900	23
38	.65121	.75889	.85811	.1653	.3177	.5356	.24111	.34878	22
39	.65144	.75870	.85862	.1647	.3180	.5351	.24130	.34856	21
40	0.65166	0.75851	0.85912	1.1640	1.3184	1.5345	0.24149	0.34834	20
41	.65188	.75832	.85963	.1633	.3187	.5340	.24168	.34812	19
42	.65210	.75813	.86013	.1626	.3190	.5335	.24186	.34790	18
43	.65232	.75794	.86064	.1619	.3193	.5330	.24205	.34768	17
44	.65254	.75775	.86115	.1612	.3197	.5325	.24224	.34746	16
45	0.65276	0.75756	0.86165	1.1605	1.3200	1.5319	0.24243	0.34724	15
46	.65298	.75737	.86216	.1599	.3203	.5314	.24262	.34702	14
47	.65320	.75718	.86267	.1592	.3207	.5309	.24281	.34680	13
48	.65342	.75700	.86318	.1585	.3210	.5304	.24300	.34658	12
49	.65364	.75680	.86368	.1578	.3213	.5299	.24319	.34636	11
50	0.65386	0.75661	0.86419	1.1571	1.3217	1.5294	0.24338	0.34614	10
51	.65408	.75642	.86470	.1565	.3220	.5289	.24357	.34592	9
52	.65430	.75623	.86521	.1558	.3223	.5283	.24376	.34570	8
53	.65452	.75604	.86572	.1551	.3227	.5278	.24396	.34548	7
54	.65474	.75585	.86623	.1544	.3230	.5273	.24415	.34526	6
55	0.65496	0.75566	0.86674	1.1537	1.3233	1.5268	0.24434	0.34504	5
56	.65518	.75547	.86725	.1531	.3237	.5263	.24453	.34482	4
57	.65540	.75528	.86775	.1524	.3240	.5258	.24472	.34460	3
58	.65562	.75509	.86826	.1517	.3243	.5253	.24491	.34438	2
59	.65584	.75490	.86878	.1510	.3247	.5248	.24510	.34416	1
60	0.65606	0.75471	0.86929	1.1504	1.3250	1.5242	0.24529	0.34394	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

130° **49°**

41° **Natural Trigonometric Functions** **138°**

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.65606	0.75471	0.86929	1.1504	1.3250	1.5242	0.24529	0.34394	60
1	.65628	.75452	.86980	.1497	.3253	.5237	.24548	.34372	59
2	.65650	.75433	.87031	.1490	.3257	.5232	.24567	.34350	58
3	.65672	.75414	.87082	.1483	.3260	.5227	.24586	.34328	57
4	.65694	.75394	.87133	.1477	.3263	.5222	.24605	.34306	56
5	0.65716	0.75375	0.87184	1.1470	1.3267	1.5217	0.24624	0.34284	55
6	.65737	.75356	.87235	.1463	.3270	.5212	.24644	.34262	54
7	.65759	.75337	.87287	.1456	.3274	.5207	.24663	.34240	53
8	.65781	.75318	.87338	.1450	.3277	.5202	.24682	.34219	52
9	.65803	.75299	.87389	.1443	.3280	.5197	.24701	.34197	51
10	0.65825	0.75280	0.87441	1.1436	1.3284	1.5192	0.24720	0.34175	50
11	.65847	.75261	.87492	.1430	.3287	.5187	.24739	.34153	49
12	.65869	.75241	.87543	.1423	.3290	.5182	.24758	.34131	48
13	.65891	.75222	.87595	.1416	.3294	.5177	.24778	.34109	47
14	.65913	.75203	.87646	.1409	.3297	.5171	.24797	.34087	46
15	0.65934	0.75184	0.87698	1.1403	1.3301	1.5166	0.24816	0.34065	45
16	.65956	.75165	.87749	.1396	.3304	.5161	.24835	.34043	44
17	.65978	.75146	.87801	.1389	.3307	.5156	.24854	.34022	43
18	.66000	.75126	.87852	.1383	.3311	.5151	.24873	.34000	42
19	.66022	.75107	.87904	.1376	.3314	.5146	.24893	.33978	41
20	0.66044	0.75088	0.87955	1.1369	1.3318	1.5141	0.24912	0.33956	40
21	.66066	.75069	.88007	.1363	.3321	.5136	.24931	.33934	39
22	.66087	.75049	.88058	.1356	.3324	.5131	.24950	.33912	38
23	.66109	.75030	.88110	.1349	.3328	.5126	.24970	.33891	37
24	.66131	.75011	.88162	.1343	.3331	.5121	.24989	.33869	36
25	0.66153	0.74992	0.88213	1.1336	1.3335	1.5116	0.25008	0.33847	35
26	.66175	.74973	.88265	.1329	.3338	.5111	.25027	.33825	34
27	.66197	.74953	.88317	.1323	.3342	.5106	.25047	.33803	33
28	.66218	.74934	.88369	.1316	.3345	.5101	.25066	.33781	32
29	.66240	.74915	.88421	.1309	.3348	.5096	.25085	.33760	31
30	0.66262	0.74895	0.88472	1.1303	1.3352	1.5092	0.25104	0.33738	30
31	.66284	.74876	.88524	.1296	.3355	.5087	.25124	.33716	29
32	.66305	.74857	.88576	.1290	.3359	.5082	.25143	.33694	28
33	.66327	.74838	.88628	.1283	.3362	.5077	.25162	.33673	27
34	.66349	.74818	.88680	.1276	.3366	.5072	.25181	.33651	26
35	0.66371	0.74799	0.88732	1.1270	1.3369	1.5067	0.25201	0.33629	25
36	.66393	.74780	.88784	.1263	.3372	.5062	.25220	.33607	24
37	.66414	.74760	.88836	.1257	.3376	.5057	.25239	.33586	23
38	.66436	.74741	.88888	.1250	.3379	.5052	.25259	.33564	22
39	.66458	.74722	.88940	.1243	.3383	.5047	.25278	.33542	21
40	0.66479	0.74702	0.88992	1.1237	1.3386	1.5042	0.25297	0.33520	20
41	.66501	.74683	.89044	.1230	.3390	.5037	.25317	.33499	19
42	.66523	.74664	.89097	.1224	.3393	.5032	.25336	.33477	18
43	.66545	.74644	.89149	.1217	.3397	.5027	.25355	.33455	17
44	.66566	.74625	.89201	.1211	.3400	.5022	.25375	.33433	16
45	0.66588	0.74606	0.89253	1.1204	1.3404	1.5018	0.25394	0.33412	15
46	.66610	.74586	.89306	.1197	.3407	.5013	.25414	.33390	14
47	.66631	.74567	.89358	.1191	.3411	.5008	.25433	.33368	13
48	.66653	.74548	.89410	.1184	.3414	.5003	.25452	.33347	12
49	.66675	.74528	.89463	.1178	.3418	.4998	.25472	.33325	11
50	0.66697	0.74509	0.89515	1.1171	1.3421	1.4993	0.25491	0.33303	10
51	.66718	.74489	.89567	.1165	.3425	.4988	.25510	.33282	9
52	.66740	.74470	.89620	.1158	.3428	.4983	.25530	.33260	8
53	.66762	.74450	.89672	.1152	.3432	.4979	.25549	.33238	7
54	.66783	.74431	.89725	.1145	.3435	.4974	.25569	.33217	6
55	0.66805	0.74412	0.89777	1.1139	1.3439	1.4969	0.25588	0.33195	5
56	.66826	.74392	.89830	.1132	.3442	.4964	.25608	.33173	4
57	.66848	.74373	.89882	.1126	.3446	.4959	.25627	.33152	3
58	.66870	.74353	.89935	.1119	.3449	.4954	.25647	.33130	2
59	.66891	.74334	.89988	.1113	.3453	.4949	.25666	.33108	1
60	0.66913	0.74314	0.90040	1.1106	1.3456	1.4945	0.25685	0.33087	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

131° **48°**

42° **Natural Trigonometric Functions** 137°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.66913	0.74314	0.90040	1.1106	1.3456	1.4945	0.25685	0.33087	60
1	.66935	.74295	.90093	.1100	.3460	.4940	.25705	.33065	59
2	.66956	.74275	.90146	.1093	.3463	.4935	.25724	.33044	58
3	.66978	.74256	.90198	.1086	.3467	.4930	.25744	.33022	57
4	.66999	.74236	.90251	.1080	.3470	.4925	.25763	.33000	56
5	0.67021	0.74217	0.90304	1.1074	1.3474	1.4921	0.25783	0.32979	55
6	.67043	.74197	.90357	.1067	.3477	.4916	.25802	.32957	54
7	.67064	.74178	.90410	.1061	.3481	.4911	.25822	.32936	53
8	.67086	.74158	.90463	.1054	.3485	.4906	.25841	.32914	52
9	.67107	.74139	.90515	.1048	.3488	.4901	.25861	.32893	51
10	0.67129	0.74119	0.90568	1.1041	1.3492	1.4897	0.25880	0.32871	50
11	.67150	.74100	.90621	.1035	.3495	.4892	.25900	.32849	49
12	.67172	.74080	.90674	.1028	.3499	.4887	.25919	.32828	48
13	.67194	.74061	.90727	.1022	.3502	.4882	.25939	.32806	47
14	.67215	.74041	.90780	.1015	.3506	.4877	.25959	.32785	46
15	0.67237	0.74022	0.90834	1.1009	1.3509	1.4873	0.25978	0.32763	45
16	.67258	.74002	.90887	.1003	.3513	.4868	.25998	.32742	44
17	.67280	.73983	.90940	.0996	.3517	.4863	.26017	.32720	43
18	.67301	.73963	.90993	.0990	.3520	.4858	.26037	.32699	42
19	.67323	.73943	.91046	.0983	.3524	.4854	.26056	.32677	41
20	0.67344	0.73924	0.91099	1.0977	1.3527	1.4849	0.26076	0.32656	40
21	.67366	.73904	.91153	.0971	.3531	.4844	.26096	.32634	39
22	.67387	.73885	.91206	.0964	.3534	.4839	.26115	.32613	38
23	.67409	.73865	.91259	.0958	.3538	.4835	.26135	.32591	37
24	.67430	.73845	.91312	.0951	.3542	.4830	.26154	.32570	36
25	0.67452	0.73826	0.91366	1.0945	1.3545	1.4825	0.26174	0.32548	35
26	.67473	.73806	.91419	.0939	.3549	.4821	.26194	.32527	34
27	.67495	.73787	.91473	.0932	.3552	.4816	.26213	.32505	33
28	.67516	.73767	.91526	.0926	.3556	.4811	.26233	.32484	32
29	.67537	.73747	.91580	.0919	.3560	.4806	.26253	.32462	31
30	0.67559	0.73728	0.91633	1.0913	1.3563	1.4802	0.26272	0.32441	30
31	.67580	.73708	.91687	.0907	.3567	.4797	.26292	.32419	29
32	.67602	.73688	.91740	.0900	.3571	.4792	.26311	.32398	28
33	.67623	.73669	.91794	.0894	.3574	.4788	.26331	.32377	27
34	.67645	.73649	.91847	.0888	.3578	.4783	.26351	.32355	26
35	0.67666	0.73629	0.91901	1.0881	1.3581	1.4778	0.26371	0.32334	25
36	.67688	.73610	.91955	.0875	.3585	.4774	.26390	.32312	24
37	.67709	.73590	.92008	.0868	.3589	.4769	.26410	.32291	23
38	.67730	.73570	.92062	.0862	.3592	.4764	.26430	.32269	22
39	.67752	.73551	.92116	.0856	.3596	.4760	.26449	.32248	21
40	0.67773	0.73531	0.92170	1.0849	1.3600	1.4755	0.26469	0.32227	20
41	.67794	.73511	.92223	.0843	.3603	.4750	.26489	.32205	19
42	.67816	.73491	.92277	.0837	.3607	.4746	.26508	.32184	18
43	.67837	.73472	.92331	.0830	.3611	.4741	.26528	.32163	17
44	.67859	.73452	.92385	.0824	.3614	.4736	.26548	.32141	16
45	0.67880	0.73432	0.92439	1.0818	1.3618	1.4732	0.26568	0.32120	15
46	.67901	.73412	.92493	.0812	.3622	.4727	.26587	.32098	14
47	.67923	.73393	.92547	.0805	.3625	.4723	.26607	.32077	13
48	.67944	.73373	.92601	.0799	.3629	.4718	.26627	.32056	12
49	.67965	.73353	.92655	.0793	.3633	.4713	.26647	.32034	11
50	0.67987	0.73333	0.92709	1.0786	1.3636	1.4709	0.26666	0.32013	10
51	.68008	.73314	.92763	.0780	.3640	.4704	.26686	.31992	9
52	.68029	.73294	.92817	.0774	.3644	.4699	.26706	.31970	8
53	.68051	.73274	.92871	.0767	.3647	.4695	.26726	.31949	7
54	.68072	.73254	.92926	.0761	.3651	.4690	.26746	.31928	6
55	0.68093	0.73234	0.92980	1.0755	1.3655	1.4686	0.26765	0.31907	5
56	.68115	.73215	.93034	.0749	.3658	.4681	.26785	.31885	4
57	.68136	.73195	.93088	.0742	.3662	.4676	.26805	.31864	3
58	.68157	.73175	.93143	.0736	.3666	.4672	.26825	.31843	2
59	.68178	.73155	.93197	.0730	.3669	.4667	.26845	.31821	1
60	0.68200	0.73135	0.93251	1.0724	1.3673	1.4663	0.26865	0.31800	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

43° **Natural Trigonometric Functions** 136°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.68200	0.73135	0.93251	1.0724	1.3673	1.4663	0.26865	0.31800	60
1	.68221	.73115	.93306	.0717	.3677	.4658	.26884	.31779	59
2	.68242	.73096	.93360	.0711	.3681	.4654	.26904	.31758	58
3	.68264	.73076	.93415	.0705	.3684	.4649	.26924	.31736	57
4	.68285	.73056	.93469	.0699	.3688	.4644	.26944	.31715	56
5	0.68306	0.73036	0.93524	1.0692	1.3692	1.4640	0.26964	0.31694	55
6	.68327	.73016	.93578	.0686	.3695	.4635	.26984	.31673	54
7	.68349	.72996	.93633	.0680	.3699	.4631	.27004	.31651	53
8	.68370	.72976	.93687	.0674	.3703	.4626	.27023	.31630	52
9	.68391	.72956	.93742	.0667	.3707	.4622	.27043	.31609	51
10	0.68412	0.72937	0.93797	1.0661	1.3710	1.4617	0.27063	0.31588	50
11	.68433	.72917	.93851	.0655	.3714	.4613	.27083	.31566	49
12	.68455	.72897	.93906	.0649	.3718	.4608	.27103	.31545	48
13	.68476	.72877	.93961	.0643	.3722	.4604	.27123	.31524	47
14	.68497	.72857	.94016	.0636	.3725	.4599	.27143	.31503	46
15	0.68518	0.72837	0.94071	1.0630	1.3729	1.4595	0.27163	0.31482	45
16	.68539	.72817	.94125	.0624	.3733	.4590	.27183	.31460	44
17	.68561	.72797	.94180	.0618	.3737	.4586	.27203	.31439	43
18	.68582	.72777	.94235	.0612	.3740	.4581	.27223	.31418	42
19	.68603	.72757	.94290	.0605	.3744	.4577	.27243	.31397	41
20	0.68624	0.72737	0.94345	1.0599	1.3748	1.4572	0.27263	0.31376	40
21	.68645	.72717	.94400	.0593	.3752	.4568	.27283	.31355	39
22	.68666	.72697	.94455	.0587	.3756	.4563	.27302	.31333	38
23	.68688	.72677	.94510	.0581	.3759	.4559	.27322	.31312	37
24	.68709	.72657	.94565	.0575	.3763	.4554	.27342	.31291	36
25	0.68730	0.72637	0.94620	1.0568	1.3767	1.4550	0.27362	0.31270	35
26	.68751	.72617	.94675	.0562	.3771	.4545	.27382	.31249	34
27	.68772	.72597	.94731	.0556	.3774	.4541	.27402	.31228	33
28	.68793	.72577	.94786	.0550	.3778	.4536	.27422	.31207	32
29	.68814	.72557	.94841	.0544	.3782	.4532	.27442	.31186	31
30	0.68835	0.72537	0.94896	1.0538	1.3786	1.4527	0.27462	0.31164	30
31	.68856	.72517	.94952	.0532	.3790	.4523	.27482	.31143	29
32	.68878	.72497	.95007	.0525	.3794	.4518	.27503	.31122	28
33	.68899	.72477	.95062	.0519	.3797	.4514	.27523	.31101	27
34	.68920	.72457	.95118	.0513	.3801	.4510	.27543	.31080	26
35	0.68941	0.72437	0.95173	1.0507	1.3805	1.4505	0.27563	0.31059	25
36	.68962	.72417	.95229	.0501	.3809	.4501	.27583	.31038	24
37	.68983	.72397	.95284	.0495	.3813	.4496	.27603	.31017	23
38	.69004	.72377	.95340	.0489	.3816	.4492	.27623	.30996	22
39	.69025	.72357	.95395	.0483	.3820	.4487	.27643	.30975	21
40	0.69046	0.72337	0.95451	1.0476	1.3824	1.4483	0.27663	0.30954	20
41	.69067	.72317	.95506	.0470	.3828	.4479	.27683	.30933	19
42	.69088	.72297	.95562	.0464	.3832	.4474	.27703	.30912	18
43	.69109	.72277	.95618	.0458	.3836	.4470	.27723	.30891	17
44	.69130	.72256	.95673	.0452	.3839	.4465	.27743	.30870	16
45	0.69151	0.72236	0.95729	1.0446	1.3843	1.4461	0.27764	0.30849	15
46	.69172	.72216	.95785	.0440	.3847	.4457	.27784	.30828	14
47	.69193	.72196	.95841	.0434	.3851	.4452	.27804	.30807	13
48	.69214	.72176	.95896	.0428	.3855	.4448	.27824	.30786	12
49	.69235	.72156	.95952	.0422	.3859	.4443	.27844	.30765	11
50	0.69256	0.72136	0.96008	1.0416	1.3863	1.4439	0.27864	0.30744	10
51	.69277	.72115	.96064	.0410	.3867	.4435	.27884	.30723	9
52	.69298	.72095	.96120	.0404	.3870	.4430	.27904	.30702	8
53	.69319	.72075	.96176	.0397	.3874	.4426	.27925	.30681	7
54	.69340	.72055	.96232	.0391	.3878	.4422	.27945	.30660	6
55	0.69361	0.72035	0.96288	1.0385	1.3882	1.4417	0.27965	0.30639	5
56	.69382	.72015	.96344	.0379	.3886	.4413	.27985	.30618	4
57	.69403	.71994	.96400	.0373	.3890	.4408	.28005	.30597	3
58	.69424	.71974	.96456	.0367	.3894	.4404	.28026	.30576	2
59	.69445	.71954	.96513	.0361	.3898	.4400	.28046	.30555	1
60	0.69466	0.71934	0.96569	1.0355	1.3902	1.4395	0.28066	0.30534	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

133° 46°

44° **Natural Trigonometric Functions** 135°

M	Sine	Cosine	Tan.	Cotan.	Secant	Cosec.	Vrs. Sin.	Vrs. Cos.	M
0	0.69466	0.71934	0.96569	1.0355	1.3902	1.4395	0.28066	0.30534	60
1	.69487	.71914	.96625	.0349	.3905	.4391	.28086	.30513	59
2	.69508	.71893	.96681	.0343	.3909	.4387	.28106	.30492	58
3	.69528	.71873	.96738	.0337	.3913	.4382	.28127	.30471	57
4	.69549	.71853	.96794	.0331	.3917	.4378	.28147	.30450	56
5	0.69570	0.71833	0.96850	1.0325	1.3921	1.4374	0.28167	0.30430	55
6	.69591	.71813	.96907	.0319	.3925	.4370	.28187	.30409	54
7	.69612	.71792	.96963	.0313	.3929	.4365	.28208	.30388	53
8	.69633	.71772	.97020	.0307	.3933	.4361	.28228	.30367	52
9	.69654	.71752	.97076	.0301	.3937	.4357	.28248	.30346	51
10	0.69675	0.71732	0.97133	1.0295	1.3941	1.4352	0.28268	0.30325	50
11	.69696	.71711	.97189	.0289	.3945	.4348	.28289	.30304	49
12	.69716	.71691	.97246	.0283	.3949	.4344	.28309	.30283	48
13	.69737	.71671	.97302	.0277	.3953	.4339	.28329	.30263	47
14	.69758	.71650	.97359	.0271	.3957	.4335	.28349	.30242	46
15	0.69779	0.71630	0.97416	1.0265	1.3960	1.4331	0.28370	0.30221	45
16	.69800	.71610	.97472	.0259	.3964	.4327	.28390	.30200	44
17	.69821	.71589	.97529	.0253	.3968	.4322	.28410	.30179	43
18	.69841	.71569	.97586	.0247	.3972	.4318	.28431	.30158	42
19	.69862	.71549	.97643	.0241	.3976	.4314	.28451	.30138	41
20	0.69883	0.71529	0.97700	1.0235	1.3980	1.4310	0.28471	0.30117	40
21	.69904	.71508	.97756	.0229	.3984	.4305	.28492	.30096	39
22	.69925	.71488	.97813	.0223	.3988	.4301	.28512	.30075	38
23	.69945	.71468	.97870	.0218	.3992	.4297	.28532	.30054	37
24	.69966	.71447	.97927	.0212	.3996	.4292	.28553	.30034	36
25	0.69987	0.71427	0.97984	1.0206	1.4000	1.4288	0.28573	0.30013	35
26	.70008	.71406	.98041	.0200	.4004	.4284	.28593	.29992	34
27	.70029	.71386	.98098	.0194	.4008	.4280	.28614	.29971	33
28	.70049	.71366	.98155	.0188	.4012	.4276	.28634	.29950	32
29	.70070	.71345	.98212	.0182	.4016	.4271	.28654	.29930	31
30	0.70091	0.71325	0.98270	1.0176	1.4020	1.4267	0.28675	0.29909	30
31	.70112	.71305	.98327	.0170	.4024	.4263	.28695	.29888	29
32	.70132	.71284	.98384	.0164	.4028	.4259	.28716	.29867	28
33	.70153	.71264	.98441	.0158	.4032	.4254	.28736	.29847	27
34	.70174	.71243	.98499	.0152	.4036	.4250	.28756	.29826	26
35	0.70194	0.71223	0.98556	1.0146	1.4040	1.4246	0.28777	0.29805	25
36	.70215	.71203	.98613	.0141	.4044	.4242	.28797	.29785	24
37	.70236	.71182	.98671	.0135	.4048	.4238	.28818	.29764	23
38	.70257	.71162	.98728	.0129	.4052	.4233	.28838	.29743	22
39	.70277	.71141	.98786	.0123	.4056	.4229	.28859	.29722	21
40	0.70298	0.71121	0.98843	1.0117	1.4060	1.4225	0.28879	0.29702	20
41	.70319	.71100	.98901	.0111	.4065	.4221	.28899	.29681	19
42	.70339	.71080	.98958	.0105	.4069	.4217	.28920	.29660	18
43	.70360	.71059	.99016	.0099	.4073	.4212	.28940	.29640	17
44	.70381	.71039	.99073	.0093	.4077	.4208	.28961	.29619	16
45	0.70401	0.71018	0.99131	1.0088	1.4081	1.4204	0.28981	0.29598	15
46	.70422	.70998	.99189	.0082	.4085	.4200	.29002	.29578	14
47	.70443	.70977	.99246	.0076	.4089	.4196	.29022	.29557	13
48	.70463	.70957	.99304	.0070	.4093	.4192	.29043	.29536	12
49	.70484	.70936	.99362	.0064	.4097	.4188	.29063	.29516	11
50	0.70505	0.70916	0.99420	1.0058	1.4101	1.4183	0.29084	0.29495	10
51	.70525	.70895	.99478	.0052	.4105	.4179	.29104	.29475	9
52	.70546	.70875	.99536	.0047	.4109	.4175	.29125	.29454	8
53	.70566	.70854	.99593	.0041	.4113	.4171	.29145	.29433	7
54	.70587	.70834	.99651	.0035	.4117	.4167	.29166	.29413	6
55	0.70608	0.70813	0.99709	1.0029	1.4122	1.4163	0.29186	0.29392	5
56	.70628	.70793	.99767	.0023	.4126	.4159	.29207	.29372	4
57	.70649	.70772	.99826	.0017	.4130	.4154	.29228	.29351	3
58	.70669	.70752	.99884	.0012	.4134	.4150	.29248	.29330	2
59	.70690	.70731	.99942	.0006	.4138	.4146	.29269	.29310	1
60	0.70711	0.70711	1.00000	1.0000	1.4142	1.4142	0.29289	0.29289	0
M	Cosine	Sine	Cotan.	Tan.	Cosec.	Secant	Vrs. Cos.	Vrs. Sin.	M

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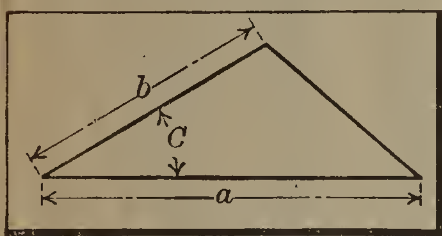
Use of Logarithms in Solving Triangles

The following tables "Logarithms of Trigonometrical Functions" may be used in the solution of triangles. The calculations are worked out in the same manner as with logarithms in general. In these tables, the characteristic is given in all cases, together with the mantissa. The complete logarithm of the functions, therefore, is found directly from the tables; however, as the values of the natural functions of sines and cosines, and of tangents for angles less than 45 degrees, are always less than 1, the characteristics would always be negative for these functions. In order to avoid these negative characteristics, the logarithm, as generally given, has had 10 added to its value; consequently, the actual value of the logarithm for "cosine 3 degrees," for example, is 9.99940 − 10.

When using these logarithms in calculations with other logarithms, the calculations can be carried out exactly as explained under "LOGARITHMS." When writing down the logarithm taken from the tables of "Logarithms of Trigonometrical Functions," $\bar{1}.99940$ is written for 9.99940, $\bar{2}.71940$ for 8.71940, $\bar{3}.30882$ for 7.30882, etc., changing the form to that which was made use of in the section on "LOGARITHMS." The examples which follow will show the method of procedure.



Example: — Find the area of a triangle where the lengths of two sides are 53 and 82 inches, and the angle between them is 30 degrees.

The area is found by the formula (see accompanying illustration):

$$\text{Area} = \frac{a \times b \times \sin C}{2} = \frac{53 \times 82 \times \sin 30^\circ}{2}$$

The logarithms of the numbers 53 and 82 are obtained from the regular tables in the usual manner. The logarithm for the sine of 30 degrees, as given in the table of "Logarithms of Trigonometrical Functions" is 9.69897 which, as previously explained, is written as $\bar{1}.69897$. The logarithm of the divisor is given a negative value, as explained in the section on "LOGARITHMS." (See "Division by Logarithms.") Proceed now to find the logarithm of the area.

$$\begin{aligned} \log 53 &= 1.72428 \\ \log 82 &= 1.91381 \\ \log \sin 30^\circ &= \bar{1}.69897 \\ -\log 2 &= \bar{1}.69897 \\ \hline &\quad 3.03603 \end{aligned}$$

The logarithm of the area thus is 3.03603, and from a logarithmic table it is found, by interpolation, that the area equals 1086.5 square inches.

Angles A and C and side a in a triangle are known. (See table "Solution of Oblique-angled Triangles.") $A = 37^\circ\ 42'$; $C = 68^\circ\ 12'$; $a = 12$ inches. Find side c.

The formula for finding side c is:

$$c = \frac{a \times \sin C}{\sin A} = \frac{12 \times \sin 68^\circ\ 12'}{\sin 37^\circ\ 42'}$$

When finding the logarithms, note that as log sine $37^\circ\ 42' = \bar{1}.78642$, the negative value of the logarithm equals 0.21358.

$$\begin{aligned} \log 12 &= 1.07918 \\ \log \sin 68^\circ\ 12' &= \bar{1}.96778 \\ -\log \sin 37^\circ\ 42' &= 0.21358 \\ \hline &\quad 1.26054 \end{aligned}$$

Thus $\log c = 1.26054$, and, hence, $c = 18.22$ inches.

0° **Logarithms of Trigonometrical Functions** 179°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	Inf. Neg.	10.00000	Inf. Neg.	Infinite	10.00000	Infinite	60
1	6.46373	.00000	6.46373	13.53627	.00000	13.53627	59
2	.76476	.00000	.76476	.23524	.00000	.23524	58
3	.94085	.00000	.94085	.05915	.00000	.05915	57
4	7.06579	.00000	7.06579	12.93421	.00000	12.93421	56
5	7.16270	10.00000	7.16270	12.83730	10.00000	12.83730	55
6	.24188	.00000	.24188	.75812	.00000	.75812	54
7	.30882	.00000	.30882	.69118	.00000	.69118	53
8	.36682	.00000	.36682	.63318	.00000	.63318	52
9	.41797	.00000	.41797	.58203	.00000	.58203	51
10	7.46373	10.00000	7.46373	12.53627	10.00000	12.53627	50
11	.50512	.00000	.50512	.49488	.00000	.49488	49
12	.54291	.00000	.54291	.45709	.00000	.45709	48
13	.57767	.00000	.57767	.42233	.00000	.42233	47
14	.60985	.00000	.60986	.39014	.00000	.39015	46
15	7.63982	10.00000	7.63982	12.36018	10.00000	12.36018	45
16	.66784	.00000	.66785	.33215	.00000	.33216	44
17	.69417	9.99999	.69418	.30582	.00001	.30583	43
18	.71900	.99999	.71900	.28100	.00001	.28100	42
19	.74248	.99999	.74248	.25752	.00001	.25752	41
20	7.76475	9.99999	7.76476	12.23524	10.00001	12.23525	40
21	.78594	.99999	.78595	.21405	.00001	.21406	39
22	.80615	.99999	.80615	.19385	.00001	.19385	38
23	.82545	.99999	.82546	.17454	.00001	.17455	37
24	.84393	.99999	.84394	.15606	.00001	.15607	36
25	7.86166	9.99999	7.86167	12.13833	10.00001	12.13834	35
26	.87870	.99999	.87871	.12129	.00001	.12130	34
27	.89509	.99999	.89510	.10490	.00001	.10491	33
28	.91088	.99999	.91089	.08911	.00001	.08912	32
29	.92612	.99998	.92613	.07387	.00002	.07388	31
30	7.94084	9.99998	7.94086	12.05914	10.00002	12.05916	30
31	.95508	.99998	.95510	.04490	.00002	.04492	29
32	.96887	.99998	.96889	.03111	.00002	.03113	28
33	.98223	.99998	.98225	.01775	.00002	.01777	27
34	.99520	.99998	.99522	.00478	.00002	.00480	26
35	8.00779	9.99998	8.00781	11.99219	10.00002	11.99221	25
36	.02002	.99998	.02004	.97996	.00002	.97998	24
37	.03192	.99997	.03194	.96806	.00003	.96808	23
38	.04350	.99997	.04353	.95647	.00003	.95650	22
39	.05478	.99997	.05481	.94519	.00003	.94522	21
40	8.06578	9.99997	8.06581	11.93419	10.00003	11.93422	20
41	.07650	.99997	.07653	.92347	.00003	.92350	19
42	.08696	.99997	.08700	.91300	.00003	.91304	18
43	.09718	.99997	.09722	.90278	.00003	.90282	17
44	.10717	.99996	.10720	.89280	.00004	.89283	16
45	8.11693	9.99996	8.11696	11.88304	10.00004	11.88307	15
46	.12647	.99996	.12651	.87349	.00004	.87353	14
47	.13581	.99996	.13585	.86415	.00004	.86419	13
48	.14495	.99996	.14500	.85500	.00004	.85505	12
49	.15391	.99996	.15395	.84605	.00004	.84609	11
50	8.16268	9.99995	8.16273	11.83727	10.00005	11.83732	10
51	.17128	.99995	.17133	.82867	.00005	.82872	9
52	.17971	.99995	.17976	.82024	.00005	.82029	8
53	.18798	.99995	.18804	.81196	.00005	.81202	7
54	.19610	.99995	.19616	.80384	.00005	.80390	6
55	8.20407	9.99994	8.20413	11.79587	10.00006	11.79593	5
56	.21189	.99994	.21195	.78805	.00006	.78811	4
57	.21958	.99994	.21964	.78036	.00006	.78042	3
58	.22713	.99994	.22720	.77280	.00006	.77287	2
59	.23456	.99994	.23462	.76538	.00006	.76544	1
60	8.24186	9.99993	8.24192	11.75808	10.00007	11.75814	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

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1° **Logarithms of Trigonometrical Functions** **178°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	8.24186	9.99993	8.24192	11.75808	10.00007	11.75814	60
1	.24903	.99993	.24910	.75090	.00007	.75097	59
2	.25609	.99993	.25616	.74384	.00007	.74391	58
3	.26304	.99993	.26312	.73688	.00007	.73696	57
4	.26988	.99992	.26996	.73004	.00008	.73012	56
5	8.27661	9.99992	8.27669	11.72331	10.00008	11.72339	55
6	.28324	.99992	.28332	.71668	.00008	.71676	54
7	.28977	.99992	.28986	.71014	.00008	.71023	53
8	.29621	.99992	.29629	.70371	.00008	.70379	52
9	.30255	.99991	.30263	.69737	.00009	.69745	51
10	8.30879	9.99991	8.30888	11.69112	10.00009	11.69121	50
11	.31495	.99991	.31505	.68495	.00009	.68505	49
12	.32103	.99990	.32112	.67888	.00010	.67897	48
13	.32702	.99990	.32711	.67289	.00010	.67298	47
14	.33292	.99990	.33302	.66698	.00010	.66708	46
15	8.33875	9.99990	8.33886	11.66114	10.00010	11.66125	45
16	.34450	.99989	.34461	.65539	.00011	.65550	44
17	.35018	.99989	.35029	.64971	.00011	.64982	43
18	.35578	.99989	.35590	.64410	.00011	.64422	42
19	.36131	.99989	.36143	.63857	.00011	.63869	41
20	8.36678	9.99988	8.36689	11.63311	10.00012	11.63322	40
21	.37217	.99988	.37229	.62771	.00012	.62783	39
22	.37750	.99988	.37762	.62238	.00012	.62250	38
23	.38276	.99987	.38289	.61711	.00013	.61724	37
24	.38796	.99987	.38809	.61191	.00013	.61204	36
25	8.39310	9.99987	8.39323	11.60677	10.00013	11.60690	35
26	.39818	.99986	.39832	.60168	.00014	.60182	34
27	.40320	.99986	.40334	.59666	.00014	.59680	33
28	.40816	.99986	.40830	.59170	.00014	.59184	32
29	.41307	.99985	.41321	.58679	.00015	.58693	31
30	8.41792	9.99985	8.41807	11.58193	10.00015	11.58208	30
31	.42272	.99985	.42287	.57713	.00015	.57728	29
32	.42746	.99984	.42762	.57238	.00016	.57254	28
33	.43216	.99984	.43232	.56768	.00016	.56784	27
34	.43680	.99984	.43696	.56304	.00016	.56320	26
35	8.44139	9.99983	8.44156	11.55844	10.00017	11.55861	25
36	.44594	.99983	.44611	.55389	.00017	.55406	24
37	.45044	.99983	.45061	.54939	.00017	.54956	23
38	.45489	.99982	.45507	.54493	.00018	.54511	22
39	.45930	.99982	.45948	.54052	.00018	.54070	21
40	8.46366	9.99982	8.46385	11.53615	10.00018	11.53634	20
41	.46799	.99981	.46817	.53183	.00019	.53201	19
42	.47226	.99981	.47245	.52755	.00019	.52774	18
43	.47650	.99981	.47669	.52331	.00019	.52350	17
44	.48069	.99980	.48089	.51911	.00020	.51931	16
45	8.48485	9.99980	8.48505	11.51495	10.00020	11.51515	15
46	.48896	.99979	.48917	.51083	.00021	.51104	14
47	.49304	.99979	.49325	.50675	.00021	.50696	13
48	.49708	.99979	.49729	.50271	.00021	.50292	12
49	.50108	.99978	.50130	.49870	.00022	.49892	11
50	8.50504	9.99978	8.50527	11.49473	10.00022	11.49496	10
51	.50897	.99977	.50920	.49080	.00023	.49103	9
52	.51287	.99977	.51310	.48690	.00023	.48713	8
53	.51673	.99977	.51696	.48304	.00023	.48327	7
54	.52055	.99976	.52079	.47921	.00024	.47945	6
55	8.52434	9.99976	8.52459	11.47541	10.00024	11.47566	5
56	.52810	.99975	.52835	.47165	.00025	.47190	4
57	.53183	.99975	.53208	.46792	.00025	.46817	3
58	.53552	.99974	.53578	.46422	.00026	.46448	2
59	.53919	.99974	.53945	.46055	.00026	.46081	1
60	8.54282	9.99974	8.54308	11.45692	10.00026	11.45718	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

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2° **Logarithms of Trigonometrical Functions** **177°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	8.54282	9.99974	8.54308	11.45692	10.00026	11.45718	60
1	.54642	.99973	.54669	.45331	.00027	.45358	59
2	.54999	.99973	.55027	.44973	.00027	.45001	58
3	.55354	.99972	.55382	.44618	.00028	.44646	57
4	.55705	.99972	.55734	.44266	.00028	.44295	56
5	8.56054	9.99971	8.56083	11.43917	10.00029	11.43946	55
6	.56400	.99971	.56429	.43571	.00029	.43600	54
7	.56743	.99970	.56773	.43227	.00030	.43257	53
8	.57084	.99970	.57114	.42886	.00030	.42916	52
9	.57421	.99969	.57452	.42548	.00031	.42579	51
10	8.57757	9.99969	8.57788	11.42212	10.00031	11.42243	50
11	.58089	.99968	.58121	.41879	.00032	.41911	49
12	.58419	.99968	.58451	.41549	.00032	.41581	48
13	.58747	.99967	.58779	.41221	.00033	.41253	47
14	.59072	.99967	.59105	.40895	.00033	.40928	46
15	8.59395	9.99967	8.59428	11.40572	10.00033	11.40605	45
16	.59715	.99966	.59749	.40251	.00034	.40285	44
17	.60033	.99966	.60068	.39932	.00034	.39967	43
18	.60349	.99965	.60384	.39616	.00035	.39651	42
19	.60662	.99964	.60698	.39302	.00036	.39338	41
20	8.60973	9.99964	8.61009	11.38991	10.00036	11.39027	40
21	.61282	.99963	.61319	.38681	.00037	.38718	39
22	.61589	.99963	.61626	.38374	.00037	.38411	38
23	.61894	.99962	.61931	.38069	.00038	.38106	37
24	.62196	.99962	.62234	.37766	.00038	.37804	36
25	8.62497	9.99961	8.62535	11.37465	10.00039	11.37503	35
26	.62795	.99961	.62834	.37166	.00039	.37205	34
27	.63091	.99960	.63131	.36869	.00040	.36909	33
28	.63385	.99960	.63426	.36574	.00040	.36615	32
29	.63678	.99959	.63718	.36282	.00041	.36322	31
30	8.63968	9.99959	8.64009	11.35991	10.00041	11.36032	30
31	.64256	.99958	.64298	.35702	.00042	.35744	29
32	.64543	.99958	.64585	.35415	.00042	.35457	28
33	.64827	.99957	.64870	.35130	.00043	.35173	27
34	.65110	.99956	.65154	.34846	.00044	.34890	26
35	8.65391	9.99956	8.65435	11.34565	10.00044	11.34609	25
36	.65670	.99955	.65715	.34285	.00045	.34330	24
37	.65947	.99955	.65993	.34007	.00045	.34053	23
38	.66223	.99954	.66269	.33731	.00046	.33777	22
39	.66497	.99954	.66543	.33457	.00046	.33503	21
40	8.66769	9.99953	8.66816	11.33184	10.00047	11.33231	20
41	.67039	.99952	.67087	.32913	.00048	.32961	19
42	.67308	.99952	.67356	.32644	.00048	.32692	18
43	.67575	.99951	.67624	.32376	.00049	.32425	17
44	.67841	.99951	.67890	.32110	.00049	.32159	16
45	8.68104	9.99950	8.68154	11.31846	10.00050	11.31896	15
46	.68367	.99949	.68417	.31583	.00051	.31633	14
47	.68627	.99949	.68678	.31322	.00051	.31373	13
48	.68886	.99948	.68938	.31062	.00052	.31114	12
49	.69144	.99948	.69196	.30804	.00052	.30856	11
50	8.69400	9.99947	8.69453	11.30547	10.00053	11.30600	10
51	.69654	.99946	.69708	.30292	.00054	.30346	9
52	.69907	.99946	.69962	.30038	.00054	.30093	8
53	.70159	.99945	.70214	.29786	.00055	.29841	7
54	.70409	.99944	.70465	.29535	.00056	.29591	6
55	8.70658	9.99944	8.70714	11.29286	10.00056	11.29342	5
56	.70905	.99943	.70962	.29038	.00057	.29095	4
57	.71151	.99942	.71208	.28792	.00058	.28849	3
58	.71395	.99942	.71453	.28547	.00058	.28605	2
59	.71638	.99941	.71697	.28303	.00059	.28362	1
60	8.71880	9.99940	8.71940	11.28060	10.00060	11.28120	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

92° **87°**

3° **Logarithms of Trigonometrical Functions** 176°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	8.71880	9.99940	8.71940	11.28060	10.00060	11.28120	60
1	.72120	.99940	.72181	.27819	.00060	.27880	59
2	.72359	.99939	.72420	.27580	.00061	.27641	58
3	.72597	.99938	.72659	.27341	.00062	.27403	57
4	.72834	.99938	.72896	.27104	.00062	.27166	56
5	8.73069	9.99937	8.73132	11.26868	10.00063	11.26931	55
6	.73303	.99936	.73366	.26634	.00064	.26697	54
7	.73535	.99936	.73600	.26400	.00064	.26465	53
8	.73767	.99935	.73832	.26168	.00065	.26233	52
9	.73997	.99934	.74063	.25937	.00066	.26003	51
10	8.74226	9.99934	8.74292	11.25708	10.00066	11.25774	50
11	.74454	.99933	.74521	.25479	.00067	.25546	49
12	.74680	.99932	.74748	.25252	.00068	.25320	48
13	.74906	.99932	.74974	.25026	.00068	.25094	47
14	.75130	.99931	.75199	.24801	.00069	.24870	46
15	8.75353	9.99930	8.75423	11.24577	10.00070	11.24647	45
16	.75575	.99929	.75645	.24355	.00071	.24425	44
17	.75795	.99929	.75867	.24133	.00071	.24205	43
18	.76015	.99928	.76087	.23913	.00072	.23985	42
19	.76234	.99927	.76306	.23694	.00073	.23766	41
20	8.76451	9.99926	8.76525	11.23475	10.00074	11.23549	40
21	.76667	.99926	.76742	.23258	.00074	.23333	39
22	.76883	.99925	.76958	.23042	.00075	.23117	38
23	.77097	.99924	.77173	.22827	.00076	.22903	37
24	.77310	.99923	.77387	.22613	.00077	.22690	36
25	8.77522	9.99923	8.77600	11.22400	10.00077	11.22478	35
26	.77733	.99922	.77811	.22189	.00078	.22267	34
27	.77943	.99921	.78022	.21978	.00079	.22057	33
28	.78152	.99920	.78232	.21768	.00080	.21848	32
29	.78360	.99920	.78441	.21559	.00080	.21640	31
30	8.78568	9.99919	8.78649	11.21351	10.00081	11.21432	30
31	.78774	.99918	.78855	.21145	.00082	.21226	29
32	.78979	.99917	.79061	.20939	.00083	.21021	28
33	.79183	.99917	.79266	.20734	.00083	.20817	27
34	.79386	.99916	.79470	.20530	.00084	.20614	26
35	8.79588	9.99915	8.79673	11.20327	10.00085	11.20412	25
36	.79789	.99914	.79875	.20125	.00086	.20211	24
37	.79990	.99913	.80076	.19924	.00087	.20010	23
38	.80189	.99913	.80277	.19723	.00087	.19811	22
39	.80388	.99912	.80476	.19524	.00088	.19612	21
40	8.80585	9.99911	8.80674	11.19326	10.00089	11.19415	20
41	.80782	.99910	.80872	.19128	.00090	.19218	19
42	.80978	.99909	.81068	.18932	.00091	.19022	18
43	.81173	.99909	.81264	.18736	.00091	.18827	17
44	.81367	.99908	.81459	.18541	.00092	.18633	16
45	8.81560	9.99907	8.81653	11.18347	10.00093	11.18440	15
46	.81752	.99906	.81846	.18154	.00094	.18248	14
47	.81944	.99905	.82038	.17962	.00095	.18056	13
48	.82134	.99904	.82230	.17770	.00096	.17866	12
49	.82324	.99904	.82420	.17580	.00096	.17676	11
50	8.82513	9.99903	8.82610	11.17390	10.00097	11.17487	10
51	.82701	.99902	.82799	.17201	.00098	.17299	9
52	.82888	.99901	.82987	.17013	.00099	.17112	8
53	.83075	.99900	.83175	.16825	.00100	.16925	7
54	.83261	.99899	.83361	.16639	.00101	.16739	6
55	8.83446	9.99898	8.83547	11.16453	10.00102	11.16554	5
56	.83630	.99898	.83732	.16268	.00102	.16370	4
57	.83813	.99897	.83916	.16084	.00103	.16187	3
58	.83996	.99896	.84100	.15900	.00104	.16004	2
59	.84177	.99895	.84282	.15718	.00105	.15823	1
60	8.84358	9.99894	8.84464	11.15536	10.00106	11.15642	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

93° 86°

4° **Logarithms of Trigonometrical Functions** **175°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	8.84358	9.99894	8.84464	11.15536	10.00106	11.15642	60
1	.84539	.99893	.84646	.15354	.00107	.15461	59
2	.84718	.99892	.84826	.15174	.00108	.15282	58
3	.84897	.99891	.85006	.14994	.00109	.15103	57
4	.85075	.99891	.85185	.14815	.00109	.14925	56
5	8.85252	9.99890	8.85363	11.14637	10.00110	11.14748	55
6	.85429	.99889	.85540	.14460	.00111	.14571	54
7	.85605	.99888	.85717	.14283	.00112	.14395	53
8	.85780	.99887	.85893	.14107	.00113	.14220	52
9	.85955	.99886	.86069	.13931	.00114	.14045	51
10	8.86128	9.99885	8.86243	11.13757	10.00115	11.13872	50
11	.86301	.99884	.86417	.13583	.00116	.13699	49
12	.86474	.99883	.86591	.13409	.00117	.13526	48
13	.86645	.99882	.86763	.13237	.00118	.13355	47
14	.86816	.99881	.86935	.13065	.00119	.13184	46
15	8.86987	9.99880	8.87106	11.12894	10.00120	11.13013	45
16	.87156	.99879	.87277	.12723	.00121	.12844	44
17	.87325	.99879	.87447	.12553	.00121	.12675	43
18	.87494	.99878	.87616	.12384	.00122	.12506	42
19	.87661	.99877	.87785	.12215	.00123	.12339	41
20	8.87829	9.99876	8.87953	11.12047	10.00124	11.12171	40
21	.87995	.99875	.88120	.11880	.00125	.12005	39
22	.88161	.99874	.88287	.11713	.00126	.11839	38
23	.88326	.99873	.88453	.11547	.00127	.11674	37
24	.88490	.99872	.88618	.11382	.00128	.11510	36
25	8.88654	9.99871	8.88783	11.11217	10.00129	11.11346	35
26	.88817	.99870	.88948	.11052	.00130	.11183	34
27	.88980	.99869	.89111	.10889	00131	.11020	33
28	.89142	.99868	.89274	.10726	.00132	.10858	32
29	.89304	99867	.89437	.10563	.00133	.10696	31
30	8.89464	9.99866	8.89598	11.10402	10.00134	11.10536	30
31	.89625	.99865	.89760	.10240	.00135	.10375	29
32	.89784	.99864	.89920	.10080	.00136	.10216	28
33	.89943	.99863	.90080	.09920	.00137	.10057	27
34	.90102	.99862	.90240	.09760	.00138	.09898	26
35	8.90260	9.99861	8.90399	11.09601	10.00139	11.09740	25
36	.90417	.99860	.90557	.09443	.00140	.09583	24
37	.90574	.99859	.90715	.09285	.00141	.09426	23
38	.90730	.99858	.90872	.09128	.00142	.09270	22
39	.90885	.99857	.91029	.08971	.00143	.09115	21
40	8.91040	9.99856	8.91185	11.08815	10.00144	11.08960	20
41	.91195	.99855	.91340	.08660	.00145	.08805	19
42	.91349	.99854	.91495	.08505	.00146	.08651	18
43	.91502	.99853	.91650	.08350	.00147	.08498	17
44	.91655	.99852	.91803	.08197	.00148	.08345	16
45	8.91807	9.99851	8.91957	11.08043	10.00149	11.08193	15
46	.91959	.99850	.92110	.07890	.00150	.08041	14
47	.92110	.99848	.92262	.07738	.00152	.07890	13
48	.92261	.99847	.92414	.07586	.00153	.07739	12
49	.92411	.99846	.92565	.07435	00154	.07589	11
50	8.92561	9.99845	8.92716	11.07284	10.00155	11.07439	10
51	.92710	.99844	.92866	.07134	.00156	.07290	9
52	.92859	.99843	.93016	.06984	.00157	.07141	8
53	.93007	.99842	.93165	.06835	.00158	.06993	7
54	.93154	.99841	.93313	.06687	.00159	.06846	6
55	8.93301	9.99840	8.93462	11.06538	10.00160	11.06699	5
56	.93448	.99839	.93609	.06391	.00161	.06552	4
57	.93594	.99838	.93756	.06244	.00162	.06406	3
58	.93740	.99837	.93903	.06097	.00163	.06260	2
59	.93885	.99836	.94049	.05951	.00164	.06115	1
60	8.94030	9.99834	8.94195	11.05805	10.00166	11.05970	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

94° **85°**

5° **Logarithms of Trigonometrical Functions** 174°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	8.94030	9.99834	8.94195	11.05805	10.00166	11.05970	60
1	.94174	.99833	.94340	.05660	.00167	.05826	59
2	.94317	.99832	.94485	.05515	.00168	.05683	58
3	.94461	.99831	.94630	.05370	.00169	.05539	57
4	.94603	.99830	.94773	.05227	.00170	.05397	56
5	8.94746	9.99829	8.94917	11.05083	10 00171	11.05254	55
6	.94887	.99828	.95060	.04940	.00172	.05113	54
7	.95029	.99827	.95202	.04798	.00173	.04971	53
8	.95170	.99825	.95344	.04656	.00175	.04830	52
9	.95310	.99824	.95486	.04514	.00176	.04690	51
10	8.95450	9.99823	8.95627	11.04373	10.00177	11.04550	50
11	.95589	.99822	.95767	.04233	.00178	.04411	49
12	.95728	.99821	.95908	.04092	.00179	.04272	48
13	.95867	.99820	.96047	.03953	.00180	.04133	47
14	.96005	.99819	.96187	.03813	.00181	.03995	46
15	8.96143	9.99817	8.96325	11.03675	10.00183	11.03857	45
16	.96280	.99816	.96464	.03536	.00184	.03720	44
17	.96417	.99815	.96602	.03398	.00185	.03583	43
18	.96553	.99814	.96739	.03261	.00186	.03447	42
19	.96689	.99813	.96877	.03123	.00187	.03311	41
20	8.96825	9.99812	8.97013	11.02987	10.00188	11.03175	40
21	.96960	.99810	.97150	.02850	.00190	.03040	39
22	.97095	.99809	.97285	.02715	.00191	.02905	38
23	.97229	.99808	.97421	.02579	.00192	.02771	37
24	.97363	.99807	.97556	.02444	.00193	.02637	36
25	8.97496	9.99806	8.97691	11.02309	10.00194	11.02504	35
26	.97629	.99804	.97825	.02175	.00196	.02371	34
27	.97762	.99803	.97959	.02041	.00197	.02238	33
28	.97894	.99802	.98092	.01908	.00198	.02106	32
29	.98026	.99801	.98225	.01775	.00199	.01974	31
30	8.98157	9.99800	8.98358	11.01642	10.00200	11.01843	30
31	.98288	.99798	.98490	.01510	.00202	.01712	29
32	.98419	.99797	.98622	.01378	.00203	.01581	28
33	.98549	.99796	.98753	.01247	.00204	.01451	27
34	.98679	.99795	.98884	.01116	.00205	.01321	26
35	8.98808	9.99793	8.99015	11.00985	10.00207	11.01192	25
36	.98937	.99792	.99145	.00855	.00208	.01063	24
37	.99066	.99791	.99275	.00725	.00209	.00934	23
38	.99194	.99790	.99405	.00595	.00210	.00806	22
39	.99322	.99788	.99534	.00466	.00212	.00678	21
40	8.99450	9.99787	8.99662	11.00338	10.00213	11.00550	20
41	.99577	.99786	.99791	.00209	.00214	.00423	19
42	.99704	.99785	.99919	.00081	.00215	.00296	18
43	.99830	.99783	9.00046	10.99954	.00217	.00170	17
44	.99956	.99782	.00174	.99826	.00218	.00044	16
45	9.00082	9.99781	9.00301	10.99699	10 00219	10.99918	15
46	.00207	.99780	.00427	.99573	.00220	.99793	14
47	.00332	.99778	.00553	.99447	.00222	.99668	13
48	.00456	.99777	.00679	.99321	.00223	.99544	12
49	.00581	.99776	.00805	.99195	.00224	.99419	11
50	9.00704	9.99775	9.00930	10.99070	10.00225	10.99296	10
51	.00828	.99773	.01055	.98945	.00227	.99172	9
52	.00951	.99772	.01179	.98821	.00228	.99049	8
53	.01074	.99771	.01303	.98697	.00229	.98926	7
54	.01196	.99769	.01427	.98573	.00231	.98804	6
55	9.01318	9.99768	9.01550	10.98450	10.00232	10.98682	5
56	.01440	.99767	.01673	.98327	.00233	.98560	4
57	.01561	.99765	.01796	.98204	.00235	.98439	3
58	.01682	.99764	.01918	.98082	.00236	.98318	2
59	.01803	.99763	.02040	.97960	.00237	.98197	1
60	9.01923	9.99761	9.02162	10.97838	10.00239	10.98077	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

85° 84°

6° **Logarithms of Trigonometrical Functions** **173°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.01923	9.99761	9.02162	10.97838	10.00239	10.98077	60
1	.02043	.99760	.02283	.97717	.00240	.97957	59
2	.02163	.99759	.02404	.97596	.00241	.97837	58
3	.02283	.99757	.02525	.97475	.00243	.97717	57
4	.02402	.99756	.02645	.97355	.00244	.97598	56
5	9.02520	9.99755	9.02766	10.97234	10.00245	10.97480	55
6	.02639	.99753	.02885	.97115	.00247	.97361	54
7	.02757	.99752	.03005	.96995	.00248	.97243	53
8	.02874	.99751	.03124	.96876	.00249	.97126	52
9	.02992	.99749	.03242	.96758	.00251	.97008	51
10	9.03109	9.99748	9.03361	10.96639	10.00252	10.96891	50
11	.03226	.99747	.03479	.96521	.00253	.96774	49
12	.03342	.99745	.03597	.96403	.00255	.96658	48
13	.03458	.99744	.03714	.96286	.00256	.96542	47
14	.03574	.99742	.03832	.96168	.00258	.96426	46
15	9.03690	9.99741	9.03948	10.96052	10.00259	10.96310	45
16	.03805	.99740	.04065	.95935	.00260	.96195	44
17	.03920	.99738	.04181	.95819	.00262	.96080	43
18	.04034	.99737	.04297	.95703	.00263	.95966	42
19	.04149	.99736	.04413	.95587	.00264	.95851	41
20	9.04262	9.99734	9.04528	10.95472	10.00266	10.95738	40
21	.04376	.99733	.04643	.95357	.00267	.95624	39
22	.04490	.99731	.04758	.95242	.00269	.95510	38
23	.04603	.99730	.04873	.95127	.00270	.95397	37
24	.04715	.99728	.04987	.95013	.00272	.95285	36
25	9.04828	9.99727	9.05101	10.94899	10.00273	10.95172	35
26	.04940	99726	.05214	.94786	.00274	.95060	34
27	.05052	.99724	.05328	.94672	.00276	.94948	33
28	.05164	.99723	.05441	.94559	.00277	.94836	32
29	.05275	.99721	.05553	.94447	.00279	.94725	31
30	9.05386	9.99720	9.05666	10.94334	10.00280	10.94614	30
31	.05497	.99718	.05778	.94222	.00282	.94503	29
32	.05607	.99717	.05890	.94110	.00283	.94393	28
33	.05717	.99716	.06002	.93998	.00284	.94283	27
34	.05827	.99714	.06113	.93887	.00286	.94173	26
35	9.05937	9.99713	9.06224	10.93776	10.00287	10.94063	25
36	.06046	.99711	.06335	.93665	.00289	.93954	24
37	.06155	.99710	.06445	.93555	.00290	.93845	23
38	.06264	.99708	.06556	.93444	.00292	.93736	22
39	.06372	.99707	.06666	.93334	.00293	.93628	21
40	9.06481	9.99705	9.06775	10.93225	10.00295	10.93519	20
41	.06589	.99704	.06885	.93115	.00296	.93411	19
42	.06696	.99702	.06994	.93006	.00298	.93304	18
43	.06804	.99701	.07103	.92897	.00299	.93196	17
44	.06911	.99699	.07211	.92789	.00301	.93089	16
45	9.07018	9.99698	9.07320	10.92680	10.00302	10.92982	15
46	.07124	.99696	.07428	.92572	.00304	.92876	14
47	.07231	.99695	.07536	.92464	.00305	.92769	13
48	.07337	.99693	.07643	.92357	.00307	.92663	12
49	.07442	.99692	.07751	.92249	.00308	.92558	11
50	9.07548	9.99690	9.07858	10.92142	10.00310	10.92452	10
51	.07653	.99689	.07964	.92036	.00311	.92347	9
52	.07758	.99687	.08071	.91929	.00313	.92242	8
53	.07863	.99686	.08177	.91823	.00314	.92137	7
54	.07968	.99684	.08283	.91717	.00316	.92032	6
55	9.08072	9.99683	9.08389	10.91611	10.00317	10.91928	5
56	.08176	.99681	.08495	.91505	.00319	.91824	4
57	.08280	.99680	.08600	.91400	.00320	.91720	3
58	.08383	.99678	.08705	.91295	.00322	.91617	2
59	.08486	.99677	.08810	.91190	.00323	.91514	1
60	9.08589	9.99675	9.08914	10.91086	10.00325	10.91411	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

96° **83°**

7° **Logarithms of Trigonometrical Functions** 172°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.08589	9.99675	9.08914	10.91086	10.00325	10.91411	60
1	.08692	.99674	.09019	.90981	.00326	.91308	59
2	.08795	.99672	.09123	.90877	.00328	.91205	58
3	.08897	.99670	.09227	.90773	.00330	.91103	57
4	.08999	.99669	.09330	.90670	.00331	.91001	56
5	9.09101	9.99667	9.09434	10.90566	10.00333	10.90899	55
6	.09202	.99666	.09537	.90463	.00334	.90798	54
7	.09304	.99664	.09640	.90360	.00336	.90696	53
8	.09405	.99663	.09742	.90258	.00337	.90595	52
9	.09506	.99661	.09845	.90155	.00339	.90494	51
10	9.09606	9.99659	9.09947	10.90053	10.00341	10.90394	50
11	.09707	.99658	.10049	.89951	.00342	.90293	49
12	.09807	.99656	.10150	.89850	.00344	.90193	48
13	.09907	.99655	.10252	.89748	.00345	.90093	47
14	.10006	.99653	.10353	.89647	.00347	.89994	46
15	9.10106	9.99651	9.10454	10.89546	10.00349	10.89894	45
16	.10205	.99650	.10555	.89445	.00350	.89795	44
17	.10304	.99648	.10656	.89344	.00352	.89696	43
18	.10402	.99647	.10756	.89244	.00353	.89598	42
19	.10501	.99645	.10856	.89144	.00355	.89499	41
20	9.10599	9.99643	9.10956	10.89044	10.00357	10.89401	40
21	.10697	.99642	.11056	.88944	.00358	.89303	39
22	.10795	.99640	.11155	.88845	.00360	.89205	38
23	.10893	.99638	.11254	.88746	.00362	.89107	37
24	.10990	.99637	.11353	.88647	.00363	.89010	36
25	9.11087	9.99635	9.11452	10.88548	10.00365	10.88913	35
26	.11184	.99633	.11551	.88449	.00367	.88816	34
27	.11281	.99632	.11649	.88351	.00368	.88719	33
28	.11377	.99630	.11747	.88253	.00370	.88623	32
29	.11474	.99629	.11845	.88155	.00371	.88526	31
30	9.11570	9.99627	9.11943	10.88057	10.00373	10.88430	30
31	.11666	.99625	.12040	.87960	.00375	.88334	29
32	.11761	.99624	.12138	.87862	.00376	.88239	28
33	.11857	.99622	.12235	.87765	.00378	.88143	27
34	.11952	.99620	.12332	.87668	.00380	.88048	26
35	9.12047	9.99618	9.12428	10.87572	10.00382	10.87953	25
36	.12142	.99617	.12525	.87475	.00383	.87858	24
37	.12236	.99615	.12621	.87379	.00385	.87764	23
38	.12331	.99613	.12717	.87283	.00387	.87669	22
39	.12425	.99612	.12813	.87187	.00388	.87575	21
40	9.12519	9.99610	9.12909	10.87091	10.00390	10.87481	20
41	.12612	.99608	.13004	.86996	.00392	.87388	19
42	.12706	.99607	.13099	.86901	.00393	.87294	18
43	.12799	.99605	.13194	.86806	.00395	.87201	17
44	.12892	.99603	.13289	.86711	.00397	.87108	16
45	9.12985	9.99601	9.13384	10.86616	10.00399	10.87015	15
46	.13078	.99600	.13478	.86522	.00400	.86922	14
47	.13171	.99598	.13573	.86427	.00402	.86829	13
48	.13263	.99596	.13667	.86333	.00404	.86737	12
49	.13355	.99595	.13761	.86239	.00405	.86645	11
50	9.13447	9.99593	9.13854	10.86146	10.00407	10.86553	10
51	.13539	.99591	.13948	.86052	.00409	.86461	9
52	.13630	.99589	.14041	.85959	.00411	.86370	8
53	.13722	.99588	.14134	.85866	.00412	.86278	7
54	.13813	.99586	.14227	.85773	.00414	.86187	6
55	9.13904	9.99584	9.14320	10.85680	10.00416	10.86096	5
56	.13994	.99582	.14412	.85588	.00418	.86006	4
57	.14085	.99581	.14504	.85496	.00419	.85915	3
58	.14175	.99579	.14597	.85403	.00421	.85825	2
59	.14266	.99577	.14688	.85312	.00423	.85734	1
60	9.14356	9.99575	9.14780	10.85220	10.00425	10.85644	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

97° 82°

8° **Logarithms of Trigonometrical Functions** 171°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.14356	9.99575	9.14780	10.85220	10.00425	10.85644	60
1	.14445	.99574	.14872	.85128	.00426	.85555	59
2	.14535	.99572	.14963	.85037	.00428	.85465	58
3	.14624	.99570	.15054	.84946	.00430	.85376	57
4	.14714	.99568	.15145	.84855	.00432	.85286	56
5	9.14803	9.99566	9.15236	10.84764	10.00434	10.85197	55
6	.14891	.99565	.15327	.84673	.00435	.85109	54
7	.14980	.99563	.15417	.84583	.00437	.85020	53
8	.15069	.99561	.15508	.84492	.00439	.84931	52
9	.15157	.99559	.15598	.84402	.00441	.84843	51
10	9.15245	9.99557	9.15688	10.84312	10.00443	10.84755	50
11	.15333	.99556	.15777	.84223	.00444	.84667	49
12	.15421	.99554	.15867	.84133	.00446	.84579	48
13	.15508	.99552	.15956	.84044	.00448	.84492	47
14	.15596	.99550	.16046	.83954	.00450	.84404	46
15	9.15683	9.99548	9.16135	10.83865	10.00452	10.84317	45
16	.15770	.99546	.16224	.83776	.00454	.84230	44
17	.15857	.99545	.16312	.83688	.00455	.84143	43
18	.15944	.99543	.16401	.83599	.00457	.84056	42
19	.16030	.99541	.16489	.83511	.00459	.83970	41
20	9.16116	9.99539	9.16577	10.83423	10.00461	10.83884	40
21	.16203	.99537	.16665	.83335	.00463	.83797	39
22	.16289	.99535	.16753	.83247	.00465	.83711	38
23	.16374	.99533	.16841	.83159	.00467	.83626	37
24	.16460	.99532	.16928	.83072	.00468	.83540	36
25	9.16545	9.99530	9.17016	10.82984	10.00470	10.83455	35
26	.16631	.99528	.17103	.82897	.00472	.83369	34
27	.16716	.99526	.17190	.82810	.00474	.83284	33
28	.16801	.99524	.17277	.82723	.00476	.83199	32
29	.16886	.99522	.17363	.82637	.00478	.83114	31
30	9.16970	9.99520	9.17450	10.82550	10.00480	10.83030	30
31	.17055	.99518	.17536	.82464	.00482	.82945	29
32	.17139	.99517	.17622	.82378	.00483	.82861	28
33	.17223	.99515	.17708	.82292	.00485	.82777	27
34	.17307	.99513	.17794	.82206	.00487	.82693	26
35	9.17391	9.99511	9.17880	10.82120	10.00489	10.82609	25
36	.17474	.99509	.17965	.82035	.00491	.82526	24
37	.17558	.99507	.18051	.81949	.00493	.82442	23
38	.17641	.99505	.18136	.81864	.00495	.82359	22
39	.17724	.99503	.18221	.81779	.00497	.82276	21
40	9.17807	9.99501	9.18306	10.81694	10.00499	10.82193	20
41	.17890	.99499	.18391	.81609	.00501	.82110	19
42	.17973	.99497	.18475	.81525	.00503	.82027	18
43	.18055	.99495	.18560	.81440	.00505	.81945	17
44	.18137	.99494	.18644	.81356	.00506	.81863	16
45	9.18220	9.99492	9.18728	10.81272	10.00508	10.81780	15
46	.18302	.99490	.18812	.81188	.00510	.81698	14
47	.18383	.99488	.18896	.81104	.00512	.81617	13
48	.18465	.99486	.18979	.81021	.00514	.81535	12
49	.18547	.99484	.19063	.80937	.00516	.81453	11
50	9.18628	9.99482	9.19146	10.80854	10.00518	10.81372	10
51	.18709	.99480	.19229	.80771	.00520	.81291	9
52	.18790	.99478	.19312	.80688	.00522	.81210	8
53	.18871	.99476	.19395	.80605	.00524	.81129	7
54	.18952	.99474	.19478	.80522	.00526	.81048	6
55	9.19033	9.99472	9.19561	10.80439	10.00528	10.80967	5
56	.19113	.99470	.19643	.80357	.00530	.80887	4
57	.19193	.99468	.19725	.80275	.00532	.80807	3
58	.19273	.99466	.19807	.80193	.00534	.80727	2
59	.19353	.99464	.19889	.80111	.00536	.80647	1
60	9.19433	9.99462	9.19971	10.80029	10.00538	10.80567	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

98° 81°

9° **Logarithms of Trigonometrical Functions** 170°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.19433	9.99462	9.19971	10.80029	10.00538	10.80567	60
1	.19513	.99460	.20053	.79947	.00540	.80487	59
2	.19592	.99458	.20134	.79866	.00542	.80408	58
3	.19672	.99456	.20216	.79784	.00544	.80328	57
4	.19751	.99454	.20297	.79703	.00546	.80249	56
5	9.19830	9.99452	9.20378	10.79622	10.00548	10.80170	55
6	.19909	.99450	.20459	.79541	.00550	.80091	54
7	.19988	.99448	.20540	.79460	.00552	.80012	53
8	.20067	.99446	.20621	.79379	.00554	.79933	52
9	.20145	.99444	.20701	.79299	.00556	.79855	51
10	9.20223	9.99442	9.20782	10.79218	10.00558	10.79777	50
11	.20302	.99440	.20862	.79138	.00560	.79698	49
12	.20380	.99438	.20942	.79058	.00562	.79620	48
13	.20458	.99436	.21022	.78978	.00564	.79542	47
14	.20535	.99434	.21102	.78898	.00566	.79465	46
15	9.20613	9.99432	9.21182	10.78818	10.00568	10.79387	45
16	.20691	.99429	.21261	.78739	.00571	.79309	44
17	.20768	.99427	.21341	.78659	.00573	.79232	43
18	.20845	.99425	.21420	.78580	.00575	.79155	42
19	.20922	.99423	.21499	.78501	.00577	.79078	41
20	9.20999	9.99421	9.21578	10.78422	10.00579	10.79001	40
21	.21076	.99419	.21657	.78343	.00581	.78924	39
22	.21153	.99417	.21736	.78264	.00583	.78847	38
23	.21229	.99415	.21814	.78186	.00585	.78771	37
24	.21306	.99413	.21893	.78107	.00587	.78694	36
25	9.21382	9.99411	9.21971	10.78029	10.00589	10.78618	35
26	.21458	.99409	.22049	.77951	.00591	.78542	34
27	.21534	.99407	.22127	.77873	.00593	.78466	33
28	.21610	.99404	.22205	.77795	.00596	.78390	32
29	.21685	.99402	.22283	.77717	.00598	.78315	31
30	9.21761	9.99400	9.22361	10.77639	10.00600	10.78239	30
31	.21836	.99398	.22438	.77562	.00602	.78164	29
32	.21912	.99396	.22516	.77484	.00604	.78088	28
33	.21987	.99394	.22593	.77407	.00606	.78013	27
34	.22062	.99392	.22670	.77330	.00608	.77938	26
35	9.22137	9.99390	9.22747	10.77253	10.00610	10.77863	25
36	.22211	.99388	.22824	.77176	.00612	.77789	24
37	.22286	.99385	.22901	.77099	.00615	.77714	23
38	.22361	.99383	.22977	.77023	.00617	.77639	22
39	.22435	.99381	.23054	.76946	.00619	.77565	21
40	9.22509	9.99379	9.23130	10.76870	10.00621	10.77491	20
41	.22583	.99377	.23206	.76794	.00623	.77417	19
42	.22657	.99375	23283	.76717	.00625	.77343	18
43	.22731	.99372	.23359	.76641	.00628	.77269	17
44	.22805	.99370	.23435	.76565	.00630	.77195	16
45	9.22878	9.99368	9.23510	10.76490	10.00632	10.77122	15
46	.22952	.99366	.23586	.76414	.00634	.77048	14
47	.23025	.99364	.23661	.76339	.00636	.76975	13
48	.23098	.99362	.23737	.76263	.00638	.76902	12
49	.23171	.99359	.23812	.76188	.00641	.76829	11
50	9.23244	9.99357	9.23887	10.76113	10.00643	10.76756	10
51	.23317	.99355	.23962	.76038	.00645	.76683	9
52	.23390	.99353	.24037	.75963	.00647	.76610	8
53	.23462	.99351	.24112	.75888	.00649	.76538	7
54	.23535	.99348	.24186	.75814	.00652	.76465	6
55	9.23607	9.99346	9.24261	10.75739	10.00654	10.76393	5
56	.23679	.99344	.24335	.75665	.00656	.76321	4
57	.23752	.99342	.24410	.75590	.00658	.76248	3
58	.23823	.99340	.24484	.75516	.00660	.76177	2
59	.23895	.99337	.24558	.75442	.00663	.76105	1
60	9.23967	9.99335	9.24632	10.75368	10.00665	10.76033	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

99° 80°

10° **Logarithms of Trigonometrical Functions** 169°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.23967	9.99335	9.24632	10.75368	10.00665	10.76033	60
1	.24039	.99333	.24706	.75294	.00667	.75961	59
2	.24110	.99331	.24779	.75221	.00669	.75890	58
3	.24181	.99328	.24853	.75147	.00672	.75819	57
4	.24253	.99326	.24926	.75074	.00674	.75747	56
5	9.24324	9.99324	9.25000	10.75000	10.00676	10.75676	55
6	.24395	.99322	.25073	.74927	.00678	.75605	54
7	.24466	.99319	.25146	.74854	.00681	.75534	53
8	.24536	.99317	.25219	.74781	.00683	.75464	52
9	.24607	.99315	.25292	.74708	.00685	.75393	51
10	9.24677	9.99313	9.25365	10.74635	10.00687	10.75323	50
11	.24748	.99310	.25437	.74563	.00690	.75252	49
12	.24818	.99308	.25510	.74490	.00692	.75182	48
13	.24888	.99306	.25582	.74418	.00694	.75112	47
14	.24958	.99304	.25655	.74345	.00696	.75042	46
15	9.25028	9.99301	9.25727	10.74273	10.00699	10.74972	45
16	.25098	.99299	.25799	.74201	.00701	.74902	44
17	.25168	.99297	.25871	.74129	.00703	.74832	43
18	.25237	.99294	.25943	.74057	.00706	.74763	42
19	.25307	.99292	.26015	.73985	.00708	.74693	41
20	9.25376	9.99290	9.26086	10.73914	10.00710	10.74624	40
21	.25445	.99288	.26158	.73842	.00712	.74555	39
22	.25514	.99285	.26229	.73771	.00715	.74486	38
23	.25583	.99283	.26301	.73699	.00717	.74417	37
24	.25652	.99281	.26372	.73628	.00719	.74348	36
25	9.25721	9 99278	9.26443	10.73557	10.00722	10.74279	35
26	.25790	.99276	.26514	.73486	.00724	.74210	34
27	.25858	.99274	.26585	.73415	.00726	.74142	33
28	.25927	.99271	.26655	.73345	.00729	.74073	32
29	.25995	.99269	.26726	.73274	.00731	.74005	31
30	9.26063	9.99267	9.26797	10.73203	10.00733	10.73937	30
31	.26131	.99264	.26867	.73133	.00736	.73869	29
32	.26199	.99262	.26937	.73063	.00738	.73801	28
33	.26267	.99260	.27008	.72992	.00740	.73733	27
34	.26335	.99257	.27078	.72922	.00743	.73665	26
35	9.26403	9.99255	9.27148	10.72852	10.00745	10.73597	25
36	.26470	.99252	.27218	.72782	.00748	.73530	24
37	.26538	.99250	.27288	.72712	.00750	.73462	23
38	.26605	.99248	.27357	.72643	.00752	.73395	22
39	.26672	.99245	.27427	.72573	.00755	.73328	21
40	9.26739	9.99243	9.27496	10.72504	10.00757	10.73261	20
41	.26806	.99241	.27566	.72434	.00759	.73194	19
42	.26873	.99238	.27635	.72365	.00762	.73127	18
43	.26940	.99236	.27704	.72296	.00764	.73060	17
44	.27007	.99233	.27773	.72227	.00767	.72993	16
45	9.27073	9.99231	9.27842	10.72158	10.00769	10.72927	15
46	.27140	.99229	.27911	.72089	.00771	.72860	14
47	.27206	.99226	.27980	.72020	.00774	.72794	13
48	.27273	.99224	.28049	.71951	.00776	.72727	12
49	.27339	.99221	.28117	.71883	.00779	.72661	11
50	9.27405	9.99219	9.28186	10.71814	10.00781	10.72595	10
51	.27471	.99217	.28254	.71746	.00783	.72529	9
52	.27537	.99214	.28323	.71677	.00786	.72463	8
53	.27602	99212	.28391	.71609	.00788	.72398	7
54	.27668	.99209	.28459	.71541	.00791	.72332	6
55	9.27734	9.99207	9.28527	10.71473	10.00793	10.72266	5
56	.27799	.99204	.28595	.71405	.00796	.72201	4
57	.27864	.99202	.28662	.71338	.00798	.72136	3
58	.27930	.99200	.28730	.71270	.00800	.72070	2
59	.27995	.99197	.28798	.71202	.00803	.72005	1
60	9.28060	9.99195	9.28865	10.71135	10.00805	10.71940	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

100° 79°

11° **Logarithms of Trigonometrical Functions** 168°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.28060	9.99195	9.28865	10.71135	10.00805	10.71940	60
1	.28125	.99192	.28933	.71067	.00808	.71875	59
2	.28190	.99190	.29000	.71000	.00810	.71810	58
3	.28254	.99187	.29067	.70933	.00813	.71746	57
4	.28319	.99185	.29134	.70866	.00815	.71681	56
5	9.28384	9.99182	9.29201	10.70799	10.00818	10.71616	55
6	.28448	.99180	.29268	.70732	.00820	.71552	54
7	.28512	.99177	.29335	.70665	.00823	.71488	53
8	.28577	.99175	.29402	.70598	.00825	.71423	52
9	.28641	.99172	.29468	.70532	.00828	.71359	51
10	9.28705	9.99170	9.29535	10.70465	10.00830	10.71295	50
11	.28769	.99167	.29601	.70399	.00833	.71231	49
12	.28833	.99165	.29668	.70332	.00835	.71167	48
13	.28896	.99162	.29734	.70266	.00838	.71104	47
14	.28960	.99160	.29800	.70200	.00840	.71040	46
15	9.29024	9.99157	9.29866	10.70134	10.00843	10.70976	45
16	.29087	.99155	.29932	.70068	.00845	.70913	44
17	.29150	.99152	.29998	.70002	.00848	.70850	43
18	.29214	.99150	.30064	.69936	.00850	.70786	42
19	.29277	.99147	.30130	.69870	.00853	.70723	41
20	9.29340	9.99145	9.30195	10.69805	10.00855	10.70660	40
21	.29403	.99142	.30261	.69739	.00858	.70597	39
22	.29466	.99140	.30326	.69674	.00860	.70534	38
23	.29529	.99137	.30391	.69609	.00863	.70471	37
24	.29591	.99135	.30457	.69543	.00865	.70409	36
25	9.29654	9.99132	9.30522	10.69478	10.00868	10.70346	35
26	.29716	.99130	.30587	.69413	.00870	.70284	34
27	.29779	.99127	.30652	.69348	.00873	.70221	33
28	.29841	.99124	.30717	.69283	.00876	.70159	32
29	.29903	.99122	.30782	.69218	.00878	.70097	31
30	9.29966	9.99119	9.30846	10.69154	10.00881	10.70034	30
31	.30028	.99117	.30911	.69089	.00883	.69972	29
32	.30090	.99114	.30975	.69025	.00886	.69910	28
33	.30151	.99112	.31040	.68960	.00888	.69849	27
34	.30213	.99109	.31104	.68896	.00891	.69787	26
35	9.30275	9.99106	9.31168	10.68832	10.00894	10.69725	25
36	.30336	.99104	.31233	.68767	.00896	.69664	24
37	.30398	.99101	.31297	.68703	.00899	.69602	23
38	.30459	.99099	.31361	.68639	.00901	.69541	22
39	.30521	.99096	.31425	.68575	.00904	.69479	21
40	9.30582	9.99093	9.31489	10.68511	10.00907	10.69418	20
41	.30643	.99091	.31552	.68448	.00909	.69357	19
42	.30704	.99088	.31616	.68384	.00912	69296	18
43	.30765	.99086	.31679	.68321	.00914	.69235	17
44	.30826	99083	.31743	.68257	.00917	.69174	16
45	9.30887	9.99080	9.31806	10.68194	10.00920	10.69113	15
46	.30947	.99078	.31870	.68130	.00922	.69053	14
47	.31008	.99075	.31933	.68067	.00925	.68992	13
48	.31068	.99072	.31996	.68004	.00928	.68932	12
49	.31129	.99070	.32059	.67941	.00930	.68871	11
50	9.31189	9.99067	9.32122	10.67878	10.00933	10.68811	10
51	.31250	.99064	.32185	.67815	.00936	.68750	9
52	.31310	.99062	.32248	.67752	.00938	.68690	8
53	.31370	.99059	.32311	.67689	.00941	.68630	7
54	.31430	.99056	.32373	.67627	.00944	.68570	6
55	9.31490	9.99054	9.32436	10.67564	10.00946	10.68510	5
56	.31549	.99051	.32498	.67502	.00949	.68451	4
57	.31609	.99048	.32561	.67439	.00952	.68391	3
58	.31669	.99046	.32623	.67377	.00954	.68331	2
59	.31728	.99043	.32685	.67315	.00957	.68272	1
60	9.31788	9.99040	9.32747	10.67253	10.00960	10.68212	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

101° 78°

12° **Logarithms of Trigonometrical Functions** 167°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.31788	9.99040	9.32747	10.67253	10.00960	10.68212	60
1	.31847	.99038	.32810	.67190	.00962	.68153	59
2	.31907	.99035	.32872	.67128	.00965	.68093	58
3	.31966	.99032	.32933	.67067	.00968	.68034	57
4	.32025	.99030	.32995	.67005	.00970	.67975	56
5	9.32084	9.99027	9.33057	10.66943	10.00973	10.67916	55
6	.32143	.99024	.33119	.66881	.00976	.67857	54
7	.32202	.99022	.33180	.66820	.00978	.67798	53
8	.32261	.99019	.33242	.66758	.00981	.67739	52
9	.32319	.99016	.33303	.66697	.00984	.67681	51
10	9.32378	9.99013	9.33365	10.66635	10.00987	10.67622	50
11	.32437	.99011	.33426	.66574	.00989	.67563	49
12	.32495	.99008	.33487	.66513	.00992	.67505	48
13	.32553	.99005	.33548	.66452	.00995	.67447	47
14	.32612	.99002	.33609	.66391	.00998	.67388	46
15	9.32670	9.99000	9.33670	10.66330	10.01000	10.67330	45
16	.32728	.98997	.33731	.66269	.01003	.67272	44
17	.32786	.98994	.33792	.66208	.01006	.67214	43
18	.32844	.98991	.33853	.66147	.01009	.67156	42
19	.32902	.98989	.33913	.66087	.01011	.67098	41
20	9.32960	9.98986	9.33974	10.66026	10.01014	10.67040	40
21	.33018	.98983	.34034	.65966	.01017	.66982	39
22	.33075	.98980	.34095	.65905	.01020	.66925	38
23	.33133	.98978	.34155	.65845	.01022	.66867	37
24	.33190	.98975	.34215	.65785	.01025	.66810	36
25	9.33248	9.98972	9.34276	10.65724	10.01028	10.66752	35
26	.33305	.98969	.34336	.65664	.01031	.66695	34
27	.33362	.98967	.34396	.65604	.01033	.66638	33
28	.33420	.98964	.34456	.65544	.01036	.66580	32
29	.33477	.98961	.34516	.65484	.01039	.66523	31
30	9.33534	9.98958	9.34576	10.65424	10.01042	10.66466	30
31	.33591	.98955	.34635	.65365	.01045	.66409	29
32	.33647	.98953	.34695	.65305	.01047	.66353	28
33	.33704	.98950	.34755	.65245	.01050	.66296	27
34	.33761	.98947	.34814	.65186	.01053	.66239	26
35	9.33818	9.98944	9.34874	10.65126	10.01056	10.66182	25
36	.33874	.98941	.34933	.65067	.01059	.66126	24
37	.33931	.98938	.34992	.65008	.01062	.66069	23
38	.33987	.98936	.35051	.64949	.01064	.66013	22
39	.34043	.98933	.35111	.64889	.01067	.65957	21
40	9.34100	9.98930	9.35170	10.64830	10.01070	10.65900	20
41	.34156	.98927	.35229	.64771	.01073	.65844	19
42	.34212	.98924	.35288	.64712	.01076	.65788	18
43	.34268	.98921	.35347	.64653	.01079	.65732	17
44	.34324	.98919	.35405	.64595	.01081	.65676	16
45	9.34380	9.98916	9.35464	10.64536	10.01084	10.65620	15
46	.34436	.98913	35523	.64477	.01087	.65564	14
47	.34491	.98910	.35581	.64419	.01090	.65509	13
48	.34547	.98907	.35640	.64360	.01093	.65453	12
49	.34602	.98904	.35698	.64302	.01096	.65398	11
50	9.34658	9.98901	9.35757	10.64243	10.01099	10.65342	10
51	.34713	.98898	.35815	.64185	.01102	.65287	9
52	.34769	.98896	.35873	.64127	.01104	.65231	8
53	.34824	.98893	.35931	.64069	.01107	.65176	7
54	.34879	.98890	.35989	.64011	.01110	.65121	6
55	9.34934	9.98887	9.36047	10.63953	10.01113	10.65066	5
56	.34989	.98884	.36105	.63895	.01116	.65011	4
57	.35044	.98881	.36163	.63837	.01119	.64956	3
58	.35099	.98878	.36221	.63779	.01122	.64901	2
59	.35154	.98875	.36279	.63721	.01125	.64846	1
60	9.35209	9.98872	9.36336	10.63664	10.01128	10.64791	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

102° 77°

13° **Logarithms of Trigonometrical Functions** 166°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.35209	9.98872	9.36336	10.63664	10.01128	10.64791	60
1	.35263	.98869	.36394	.63606	.01131	.64737	59
2	.35318	.98867	.36452	.63548	.01133	.64682	58
3	.35373	.98864	.36509	.63491	.01136	.64627	57
4	.35427	.98861	.36566	.63434	.01139	.64573	56
5	9.35481	9.98858	9.36624	10.63376	10.01142	10.64519	55
6	.35536	.98855	.36681	.63319	.01145	.64464	54
7	.35590	.98852	.36738	.63262	.01148	.64410	53
8	.35644	.98849	.36795	.63205	.01151	.64356	52
9	.35698	.98846	.36852	.63148	.01154	.64302	51
10	9.35752	9.98843	9.36909	10.63091	10.01157	10.64248	50
11	.35806	.98840	.36966	.63034	.01160	.64194	49
12	.35860	.98837	.37023	.62977	.01163	.64140	48
13	.35914	.98834	.37080	.62920	.01166	.64086	47
14	.35968	.98831	.37137	.62863	.01169	.64032	46
15	9.36022	9.98828	9.37193	10.62807	10.01172	10.63978	45
16	.36075	.98825	.37250	.62750	.01175	.63925	44
17	.36129	.98822	.37306	.62694	.01178	.63871	43
18	.36182	.98819	.37363	.62637	.01181	.63818	42
19	.36236	.98816	.37419	.62581	.01184	.63764	41
20	9.36289	9.98813	9.37476	10.62524	10.01187	10.63711	40
21	.36342	.98810	.37532	.62468	.01190	.63658	39
22	.36395	.98807	.37588	.62412	.01193	.63605	38
23	.36449	.98804	.37644	.62356	.01196	.63551	37
24	.36502	.98801	.37700	.62300	.01199	.63498	36
25	9.36555	9.98798	9.37756	10.62244	10.01202	10.63445	35
26	.36608	.98795	.37812	.62188	.01205	.63392	34
27	.36660	.98792	.37868	.62132	.01208	.63340	33
28	.36713	.98789	.37924	.62076	.01211	.63287	32
29	.36766	.98786	.37980	.62020	.01214	.63234	31
30	9.36819	9.98783	9.38035	10.61965	10.01217	10.63181	30
31	.36871	.98780	.38091	.61909	.01220	.63129	29
32	.36924	.98777	.38147	.61853	.01223	.63076	28
33	.36976	.98774	.38202	.61798	.01226	.63024	27
34	.37028	.98771	.38257	.61743	.01229	.62972	26
35	9.37081	9.98768	9.38313	10.61687	10.01232	10.62919	25
36	.37133	.98765	.38368	.61632	.01235	.62867	24
37	.37185	.98762	.38423	.61577	.01238	.62815	23
38	.37237	98759	.38479	.61521	.01241	.62763	22
39	.37289	.98756	.38534	.61466	.01244	.62711	21
40	9.37341	9.98753	9.38589	10.61411	10.01247	10.62659	20
41	.37393	.98750	.38644	.61356	.01250	.62607	19
42	.37445	.98746	.38699	.61301	.01254	.62555	18
43	.37497	.98743	.38754	.61246	.01257	.62503	17
44	.37549	.98740	.38808	.61192	.01260	.62451	16
45	9.37600	9.98737	9.38863	10.61137	10.01263	10.62400	15
46	.37652	.98734	.38918	.61082	.01266	.62348	14
47	.37703	.98731	.38972	.61028	.01269	.62297	13
48	.37755	.98728	.39027	.60973	.01272	.62245	12
49	.37806	.98725	.39082	.60918	.01275	.62194	11
50	9.37858	9.98722	9.39136	10.60864	10.01278	10.62142	10
51	.37909	98719	.39190	.60810	.01281	.62091	9
52	.37960	.98715	.39245	.60755	.01285	.62040	8
53	.38011	.98712	.39299	.60701	.01288	.61989	7
54	.38062	.98709	.39353	.60647	.01291	.61938	6
55	9.38113	9.98706	9.39407	10.60593	10.01294	10.61887	5
56	.38164	.98703	.39461	.60539	.01297	.61836	4
57	.38215	.98700	.39515	.60485	.01300	.61785	3
58	.38266	.98697	.39569	.60431	.01303	.61734	2
59	.38317	.98694	.39623	.60377	.01306	.61683	1
60	9.38368	9.98690	9.39677	10.60323	10.01310	10.61632	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

103° 76°

14° **Logarithms of Trigonometrical Functions** **165°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.38368	9.98690	9.39677	10.60323	10.01310	10.61632	60
1	.38418	.98687	.39731	.60269	.01313	.61582	59
2	.38469	.98684	.39785	.60215	.01316	.61531	58
3	.38519	.98681	.39838	.60162	.01319	.61481	57
4	.38570	.98678	.39892	.60108	.01322	.61430	56
5	9.38620	9.98675	9.39945	10.60055	10.01325	10.61380	55
6	.38670	.98671	.39999	.60001	.01329	.61330	54
7	.38721	.98668	.40052	.59948	.01332	.61279	53
8	.38771	.98665	.40106	.59894	.01335	.61229	52
9	.38821	.98662	.40159	.59841	.01338	.61179	51
10	9.38871	9.98659	9.40212	10.59788	10.01341	10.61129	50
11	.38921	.98656	.40266	.59734	.01344	.61079	49
12	.38971	.98652	.40319	.59681	.01348	.61029	48
13	.39021	.98649	.40372	.59628	.01351	.60979	47
14	.39071	.98646	.40425	.59575	.01354	.60929	46
15	9.39121	9.98643	9.40478	10.59522	10.01357	10.60879	45
16	.39170	.98640	.40531	.59469	.01360	.60830	44
17	.39220	.98636	.40584	.59416	.01364	.60780	43
18	.39270	.98633	.40636	.59364	.01367	.60730	42
19	.39319	.98630	.40689	.59311	.01370	.60681	41
20	9.39369	9.98627	9.40742	10.59258	10.01373	10.60631	40
21	.39418	.98623	.40795	.59205	.01377	.60582	39
22	.39467	.98620	.40847	.59153	.01380	.60533	38
23	.39517	.98617	.40900	.59100	.01383	.60483	37
24	.39566	.98614	.40952	.59048	.01386	.60434	36
25	9.39615	9.98610	9.41005	10.58995	10.01390	10.60385	35
26	.39664	.98607	.41057	.58943	.01393	.60336	34
27	.39713	.98604	.41109	.58891	.01396	.60287	33
28	.39762	.98601	.41161	.58839	.01399	.60238	32
29	.39811	.98597	.41214	.58786	.01403	.60189	31
30	9.39860	9.98594	9.41266	10.58734	10.01406	10.60140	30
31	.39909	.98591	.41318	.58682	.01409	.60091	29
32	.39958	.98588	.41370	.58630	.01412	.60042	28
33	.40006	.98584	.41422	.58578	.01416	.59994	27
34	.40055	.98581	.41474	.58526	.01419	.59945	26
35	9.40103	9.98578	9.41526	10.58474	10.01422	10.59897	25
36	.40152	.98574	.41578	.58422	.01426	.59848	24
37	.40200	.98571	.41629	.58371	.01429	.59800	23
38	.40249	.98568	.41681	.58319	.01432	.59751	22
39	.40297	.98565	.41733	.58267	.01435	.59703	21
40	9.40346	9.98561	9.41784	10.58216	10.01439	10.59654	20
41	.40394	.98558	.41836	.58164	.01442	.59606	19
42	.40442	.98555	.41887	.58113	.01445	.59558	18
43	.40490	.98551	.41939	.58061	.01449	.59510	17
44	.40538	.98548	.41990	.58010	.01452	.59462	16
45	9.40586	9.98545	9.42041	10.57959	10.01455	10.59414	15
46	.40634	.98541	.42093	.57907	.01459	.59366	14
47	.40682	.98538	.42144	.57856	.01462	.59318	13
48	.40730	.98535	.42195	.57805	.01465	.59270	12
49	.40778	.98531	.42246	.57754	.01469	.59222	11
50	9.40825	9.98528	9.42297	10.57703	10.01472	10.59175	10
51	.40873	.98525	.42348	.57652	.01475	.59127	9
52	.40921	.98521	.42399	.57601	.01479	.59079	8
53	.40968	.98518	.42450	.57550	.01482	.59032	7
54	.41016	.98515	.42501	.57499	.01485	.58984	6
55	9.41063	9.98511	9.42552	10.57448	10.01489	10.58937	5
56	.41111	.98508	.42603	.57397	.01492	.58889	4
57	.41158	.98505	.42653	.57347	.01495	.58842	3
58	.41205	.98501	.42704	.57296	.01499	.58795	2
59	.41252	.98498	.42755	.57245	.01502	.58748	1
60	9.41300	9.98494	9.42805	10.57195	10.01506	10.58700	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

104° **75°**

15° **Logarithms of Trigonometrical Functions** 164°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.41300	9.98494	9.42805	10.57195	10.01506	10.58700	60
1	.41347	.98491	.42856	.57144	.01509	.58653	59
2	.41394	.98488	.42906	.57094	.01512	.58606	58
3	.41441	.98484	.42957	.57043	.01516	.58559	57
4	.41488	.98481	.43007	.56993	.01519	.58512	56
5	9.41535	9.98477	9.43057	10.56943	10.01523	10.58465	55
6	.41582	.98474	.43108	.56892	.01526	.58418	54
7	.41628	.98471	.43158	.56842	.01529	.58372	53
8	.41675	.98467	.43208	.56792	.01533	.58325	52
9	.41722	.98464	.43258	.56742	.01536	.58278	51
10	9.41768	9.98460	9.43308	10.56692	10.01540	10.58232	50
11	.41815	.98457	.43358	.56642	.01543	.58185	49
12	.41861	.98453	.43408	.56592	.01547	.58139	48
13	.41908	.98450	.43458	.56542	.01550	.58092	47
14	.41954	.98447	.43508	.56492	.01553	.58046	46
15	9.42001	9.98443	9.43558	10.56442	10.01557	10.57999	45
16	.42047	.98440	.43607	.56393	.01560	.57953	44
17	.42093	.98436	.43657	.56343	.01564	.57907	43
18	.42140	.98433	.43707	.56293	.01567	.57860	42
19	.42186	.98429	.43756	.56244	.01571	.57814	41
20	9.42232	9.98426	9.43806	10.56194	10.01574	10.57768	40
21	.42278	.98422	.43855	.56145	.01578	.57722	39
22	.42324	.98419	.43905	.56095	.01581	.57676	38
23	.42370	.98415	.43954	.56046	.01585	.57630	37
24	.42416	.98412	.44004	.55996	.01588	.57584	36
25	9.42461	9.98409	9.44053	10.55947	10.01591	10.57539	35
26	.42507	.98405	.44102	.55898	.01595	.57493	34
27	.42553	.98402	.44151	.55849	.01598	.57447	33
28	.42599	.98398	.44201	.55799	.01602	.57401	32
29	.42644	.98395	.44250	.55750	.01605	.57356	31
30	9.42690	9.98391	9.44299	10.55701	10.01609	10.57310	30
31	.42735	.98388	.44348	.55652	.01612	.57265	29
32	.42781	.98384	.44397	.55603	.01616	.57219	28
33	.42826	.98381	.44446	.55554	.01619	.57174	27
34	.42872	.98377	.44495	.55505	.01623	.57128	26
35	9.42917	9.98373	9.44544	10.55456	10.01627	10.57083	25
36	.42962	.98370	.44592	.55408	.01630	.57038	24
37	.43008	.98366	.44641	.55359	.01634	.56992	23
38	.43053	.98363	.44690	.55310	.01637	.56947	22
39	.43098	.98359	.44738	.55262	.01641	.56902	21
40	9.43143	9.98356	9.44787	10.55213	10.01644	10.56857	20
41	.43188	.98352	.44836	.55164	.01648	.56812	19
42	.43233	.98349	.44884	.55116	.01651	.56767	18
43	.43278	.98345	.44933	.55067	.01655	.56722	17
44	.43323	.98342	.44981	.55019	.01658	.56677	16
45	9.43367	9.98338	9.45029	10.54971	10.01662	10.56633	15
46	.43412	.98334	.45078	.54922	.01666	.56588	14
47	.43457	.98331	.45126	.54874	.01669	.56543	13
48	.43502	.98327	.45174	.54826	.01673	.56498	12
49	.43546	.98324	.45222	.54778	.01676	.56454	11
50	9.43591	9.98320	9.45271	10.54729	10.01680	10.56409	10
51	.43635	.98317	.45319	.54681	.01683	.56365	9
52	.43680	.98313	.45367	.54633	.01687	.56320	8
53	.43724	.98309	.45415	.54585	.01691	.56276	7
54	.43769	.98306	.45463	.54537	.01694	.56231	6
55	9.43813	9.98302	9.45511	10.54489	10.01698	10.56187	5
56	.43857	.98299	.45559	.54441	.01701	.56143	4
57	.43901	.98295	.45606	.54394	.01705	.56099	3
58	.43946	.98291	.45654	.54346	.01709	.56054	2
59	.43990	.98288	.45702	.54298	.01712	.56010	1
60	9.44034	9.98284	9.45750	10.54250	10.01716	10.55966	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

105° 74°

16° **Logarithms of Trigonometrical Functions.** **163°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.44034	9.98284	9.45750	10.54250	10.01716	10.55966	60
1	.44078	.98281	.45797	.54203	.01719	.55922	59
2	.44122	.98277	.45845	.54155	.01723	.55878	58
3	.44166	.98273	.45892	.54108	.01727	.55834	57
4	.44210	.98270	.45940	.54060	.01730	.55790	56
5	9.44253	9.98266	9.45987	10.54013	10 01734	10.55747	55
6	.44297	.98262	.46035	.53965	.01738	.55703	54
7	.44341	.98259	.46082	.53918	.01741	.55659	53
8	.44385	.98255	.46130	.53870	.01745	.55615	52
9	.44428	.98251	.46177	.53823	.01749	.55572	51
10	9.44472	9.98248	9.46224	10.53776	10.01752	10.55528	50
11	.44516	.98244	.46271	.53729	.01756	.55484	49
12	.44559	.98240	.46319	.53681	.01760	.55441	48
13	.44602	.98237	.46366	.53634	.01763	.55398	47
14	.44646	.98233	.46413	.53587	.01767	.55354	46
15	9.44689	9.98229	9.46460	10.53540	10.01771	10.55311	45
16	.44733	.98226	.46507	.53493	.01774	.55267	44
17	.44776	.98222	.46554	.53446	.01778	.55224	43
18	.44819	.98218	.46601	.53399	.01782	.55181	42
19	.44862	.98215	.46648	.53352	.01785	.55138	41
20	9.44905	9.98211	9.46694	10.53306	10.01789	10.55095	40
21	.44948	.98207	.46741	.53259	.01793	.55052	39
22	.44992	.98204	.46788	.53212	.01796	.55008	38
23	.45035	.98200	.46835	.53165	.01800	.54965	37
24	.45077	.98196	.46881	.53119	.01804	.54923	36
25	9.45120	9.98192	9.46928	10.53072	10.01808	10.54880	35
26	.45163	.98189	.46975	.53025	.01811	.54837	34
27	.45206	.98185	.47021	.52979	.01815	.54794	33
28	.45249	.98181	.47068	.52932	.01819	.54751	32
29	.45292	.98177	.47114	.52886	.01823	.54708	31
30	9.45334	9.98174	9.47160	10.52840	10.01826	10.54666	30
31	.45377	.98170	.47207	.52793	.01830	.54623	29
32	.45419	.98166	.47253	.52747	.01834	.54581	28
33	.45462	.98162	.47299	.52701	.01838	.54538	27
34	.45504	.98159	.47346	.52654	.01841	.54496	26
35	9.45547	9.98155	9.47392	10.52608	10.01845	10.54453	25
36	.45589	.98151	.47438	.52562	.01849	.54411	24
37	.45632	.98147	.47484	.52516	.01853	.54368	23
38	.45674	.98144	.47530	.52470	.01856	.54326	22
39	.45716	.98140	.47576	.52424	.01860	.54284	21
40	9.45758	9.98136	9.47622	10.52378	10.01864	10.54242	20
41	.45801	.98132	.47668	.52332	.01868	.54199	19
42	.45843	.98129	.47714	.52286	.01871	.54157	18
43	.45885	.98125	.47760	.52240	.01875	.54115	17
44	.45927	.98121	.47806	.52194	.01879	.54073	16
45	9.45969	9.98117	9.47852	10.52148	10.01883	10.54031	15
46	.46011	.98113	.47897	.52103	.01887	.53989	14
47	.46053	.98110	.47943	.52057	.01890	.53947	13
48	.46095	.98106	.47989	.52011	.01894	.53905	12
49	.46136	.98102	.48035	.51965	.01898	.53864	11
50	9.46178	9.98098	9.48080	10.51920	10.01902	10.53822	10
51	.46220	.98094	.48126	.51874	.01906	.53780	9
52	.46262	.98090	.48171	.51829	.01910	.53738	8
53	.46303	.98087	.48217	.51783	.01913	.53697	7
54	.46345	.98083	.48262	.51738	.01917	.53655	6
55	9.46386	9.98079	9.48307	10.51693	10.01921	10.53614	5
56	.46428	.98075	.48353	.51647	.01925	.53572	4
57	.46469	.98071	.48398	.51602	.01929	.53531	3
58	.46511	.98067	.48443	.51557	.01933	.53489	2
59	.46552	.98063	.48489	.51511	.01937	.53448	1
60	9.46594	9.98060	9.48534	10.51466	10.01940	10.53406	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

106° **73°**

17° **Logarithms of Trigonometrical Functions** 162°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.46594	9.98060	9.48534	10.51466	10.01940	10.53406	60
1	.46635	.98056	.48579	.51421	.01944	.53365	59
2	.46676	.98052	.48624	.51376	.01948	.53324	58
3	.46717	.98048	.48669	.51331	.01952	.53283	57
4	.46758	.98044	.48714	.51286	.01956	.53242	56
5	9.46800	9.98040	9.48759	10.51241	10.01960	10.53200	55
6	.46841	.98036	.48804	.51196	.01964	.53159	54
7	.46882	.98032	.48849	.51151	.01968	.53118	53
8	.46923	.98029	.48894	.51106	.01971	.53077	52
9	.46964	.98025	.48939	.51061	.01975	.53036	51
10	9.47005	9.98021	9.48984	10.51016	10.01979	10.52995	50
11	.47045	.98017	.49029	.50971	.01983	.52955	49
12	.47086	.98013	.49073	.50927	.01987	.52914	48
13	.47127	.98009	.49118	.50882	.01991	.52873	47
14	.47168	.98005	.49163	.50837	.01995	.52832	46
15	9.47209	9.98001	9.49207	10.50793	10.01999	10.52791	45
16	.47249	.97997	.49252	.50748	.02003	.52751	44
17	.47290	.97993	.49296	.50704	.02007	.52710	43
18	.47330	.97989	.49341	.50659	.02011	.52670	42
19	.47371	.97986	.49385	.50615	.02014	.52629	41
20	9.47411	9.97982	9.49430	10.50570	10.02018	10.52589	40
21	.47452	.97978	.49474	.50526	.02022	.52548	39
22	.47492	.97974	.49519	.50481	.02026	.52508	38
23	.47533	.97970	.49563	.50437	.02030	.52467	37
24	.47573	.97966	.49607	.50393	.02034	.52427	36
25	9.47613	9.97962	9.49652	10.50348	10.02038	10.52387	35
26	.47654	.97958	.49696	.50304	.02042	.52346	34
27	.47694	.97954	.49740	.50260	.02046	.52306	33
28	.47734	.97950	.49784	.50216	.02050	.52266	32
29	.47774	.97946	.49828	.50172	.02054	.52226	31
30	9.47814	9.97942	9.49872	10.50128	10.02058	10.52186	30
31	.47854	.97938	.49916	.50084	.02062	.52146	29
32	.47894	.97934	.49960	.50040	.02066	.52106	28
33	.47934	.97930	.50004	.49996	.02070	.52066	27
34	.47974	.97926	.50048	.49952	.02074	.52026	26
35	9.48014	9.97922	9.50092	10.49908	10.02078	10.51986	25
36	.48054	.97918	.50136	.49864	.02082	.51946	24
37	.48094	.97914	.50180	.49820	.02086	.51906	23
38	.48133	.97910	.50223	.49777	.02090	.51867	22
39	.48173	.97906	.50267	.49733	.02094	.51827	21
40	9.48213	9.97902	9.50311	10.49689	10.02098	10.51787	20
41	.48252	.97898	.50355	.49645	.02102	.51748	19
42	.48292	.97894	.50398	.49602	.02106	.51708	18
43	.48332	.97890	.50442	.49558	.02110	.51668	17
44	.48371	.97886	.50485	.49515	.02114	.51629	16
45	9.48411	9.97882	9.50529	10.49471	10.02118	10.51589	15
46	.48450	.97878	.50572	.49428	.02122	.51550	14
47	.48490	.97874	.50616	.49384	.02126	.51510	13
48	.48529	.97870	.50659	.49341	.02130	.51471	12
49	.48568	.97866	.50703	.49297	.02134	.51432	11
50	9.48607	9.97861	9.50746	10.49254	10.02139	10.51393	10
51	.48647	.97857	.50789	.49211	.02143	.51353	9
52	.48686	.97853	.50833	.49167	.02147	.51314	8
53	.48725	.97849	.50876	.49124	.02151	.51275	7
54	.48764	.97845	.50919	.49081	.02155	.51236	6
55	9.48803	9.97841	9.50962	10.49038	10.02159	10.51197	5
56	.48842	.97837	.51005	.48995	.02163	.51158	4
57	.48881	.97833	.51048	.48952	.02167	.51119	3
58	.48920	.97829	.51092	.48908	.02171	.51080	2
59	.48959	.97825	.51135	.48865	.02175	.51041	1
60	9.48998	9.97821	9.51178	10.48822	10.02179	10.51002	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

107° 72°

18° **Logarithms of Trigonometrical Functions** 161°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.48998	9.97821	9.51178	10.48822	10.02179	10.51002	60
1	.49037	.97817	.51221	.48779	.02183	.50963	59
2	.49076	.97812	.51264	.48736	.02188	.50924	58
3	.49115	.97808	.51306	.48694	.02192	.50885	57
4	.49153	.97804	.51349	.48651	.02196	.50847	56
5	9.49192	9.97800	9.51392	10.48608	10.02200	10.50808	55
6	.49231	.97796	.51435	.48565	.02204	.50769	54
7	.49269	.97792	.51478	.48522	.02208	.50731	53
8	.49308	.97788	.51520	.48480	.02212	.50692	52
9	.49347	.97784	.51563	.48437	.02216	.50653	51
10	9.49385	9.97779	9.51606	10.48394	10.02221	10.50615	50
11	.49424	.97775	.51648	.48352	.02225	.50576	49
12	.49462	.97771	.51691	.48309	.02229	.50538	48
13	.49500	.97767	.51734	.48266	.02233	.50500	47
14	.49539	.97763	.51776	.48224	.02237	.50461	46
15	9.49577	9.97759	9.51819	10.48181	10.02241	10.50423	45
16	.49615	.97754	.51861	.48139	.02246	.50385	44
17	.49654	.97750	.51903	.48097	.02250	.50346	43
18	.49692	.97746	.51946	.48054	.02254	.50308	42
19	.49730	.97742	.51988	.48012	.02258	.50270	41
20	9.49768	9.97738	9.52031	10.47969	10.02262	10.50232	40
21	.49806	.97734	.52073	.47927	.02266	.50194	39
22	.49844	.97729	.52115	.47885	.02271	.50156	38
23	.49882	.97725	.52157	.47843	.02275	.50118	37
24	.49920	.97721	.52200	.47800	.02279	.50080	36
25	9.49958	9.97717	9.52242	10.47758	10.02283	10.50042	35
26	.49996	.97713	.52284	.47716	.02287	.50004	34
27	.50034	.97708	.52326	.47674	.02292	.49966	33
28	.50072	.97704	.52368	.47632	.02296	.49928	32
29	.50110	.97700	.52410	.47590	.02300	.49890	31
30	9.50148	9.97696	9.52452	10.47548	10.02304	10.49852	30
31	.50185	.97691	.52494	.47506	.02309	.49815	29
32	.50223	.97687	.52536	.47464	.02313	.49777	28
33	.50261	.97683	.52578	.47422	.02317	.49739	27
34	.50298	.97679	.52620	.47380	.02321	.49702	26
35	9.50336	9.97674	9.52661	10.47339	10.02326	10.49664	25
36	.50374	.97670	.52703	.47297	.02330	.49626	24
37	.50411	.97666	.52745	.47255	.02334	.49589	23
38	.50449	.97662	.52787	.47213	.02338	.49551	22
39	.50486	.97657	.52829	.47171	.02343	.49514	21
40	9.50523	9.97653	9.52870	10.47130	10.02347	10.49477	20
41	.50561	.97649	.52912	.47088	.02351	.49439	19
42	.50598	.97645	.52953	.47047	.02355	.49402	18
43	.50635	.97640	.52995	.47005	.02360	.49365	17
44	.50673	.97636	.53037	.46963	.02364	.49327	16
45	9.50710	9.97632	9.53078	10.46922	10.02368	10.49290	15
46	.50747	.97628	.53120	.46880	.02372	.49253	14
47	.50784	.97623	.53161	.46839	.02377	.49216	13
48	.50821	.97619	.53202	.46798	.02381	.49179	12
49	.50858	.97615	.53244	.46756	.02385	.49142	11
50	9.50896	9.97610	9.53285	10.46715	10.02390	10.49104	10
51	.50933	.97606	.53327	.46673	.02394	.49067	9
52	.50970	.97602	.53368	.46632	.02398	.49030	8
53	.51007	.97597	.53409	.46591	.02403	.48993	7
54	.51043	.97593	.53450	.46550	.02407	.48957	6
55	9.51080	9.97589	9.53492	10.46508	10.02411	10.48920	5
56	.51117	.97584	.53533	.46467	.02416	.48883	4
57	.51154	.97580	.53574	.46426	.02420	.48846	3
58	.51191	.97576	.53615	.46385	.02424	.48809	2
59	.51227	.97571	.53656	.46344	.02429	.48773	1
60	9.51264	9.97567	9.53697	10.46303	10.02433	10.48736	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

108° 71°

19° **Logarithms of Trigonometrical Functions** **160°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.51264	9.97567	9.53697	10.46303	10.02433	10.48736	60
1	.51301	.97563	.53738	.46262	.02437	.48699	59
2	.51338	.97558	.53779	.46221	.02442	.48662	58
3	.51374	.97554	.53820	.46180	.02446	.48626	57
4	.51411	.97550	.53861	.46139	.02450	.48589	56
5	9.51447	9.97545	9.53902	10.46098	10.02455	10.48553	55
6	.51484	.97541	.53943	.46057	.02459	.48516	54
7	.51520	.97536	.53984	.46016	.02464	.48480	53
8	.51557	.97532	.54025	.45975	.02468	.48443	52
9	.51593	.97528	.54065	.45935	.02472	.48407	51
10	9.51629	9.97523	9.54106	10.45894	10.02477	10.48371	50
11	.51666	.97519	.54147	.45853	.02481	.48334	49
12	.51702	.97515	.54187	.45813	.02485	.48298	48
13	.51738	.97510	.54228	.45772	.02490	.48262	47
14	.51774	.97506	.54269	.45731	.02494	.48226	46
15	9.51811	9.97501	9.54309	10.45691	10.02499	10.48189	45
16	.51847	.97497	.54350	.45650	.02503	.48153	44
17	.51883	.97492	.54390	.45610	.02508	.48117	43
18	.51919	.97488	.54431	.45569	.02512	.48081	42
19	.51955	.97484	.54471	.45529	.02516	.48045	41
20	9.51991	9.97479	9.54512	10.45488	10.02521	10.48009	40
21	.52027	.97475	.54552	.45448	.02525	.47973	39
22	.52063	.97470	.54593	.45407	.02530	.47937	38
23	.52099	.97466	.54633	.45367	.02534	.47901	37
24	.52135	.97461	.54673	.45327	.02539	.47865	36
25	9.52171	9.97457	9.54714	10.45286	10.02543	10.47829	35
26	.52207	.97453	54754	.45246	.02547	.47793	34
27	.52242	.97448	.54794	.45206	.02552	.47758	33
28	.52278	.97444	.54835	.45165	.02556	.47722	32
29	.52314	.97439	.54875	.45125	.02561	.47686	31
30	9.52350	9.97435	9.54915	10.45085	10.02565	10.47650	30
31	.52385	.97430	.54955	.45045	.02570	.47615	29
32	.52421	.97426	.54995	.45005	.02574	.47579	28
33	.52456	.97421	.55035	.44965	.02579	.47544	27
34	.52492	.97417	.55075	.44925	.02583	.47508	26
35	9.52527	9.97412	9.55115	10.44885	10.02588	10.47473	25
36	.52563	.97408	.55155	.44845	.02592	.47437	24
37	.52598	.97403	.55195	.44805	.02597	.47402	23
38	.52634	.97399	.55235	.44765	.02601	.47366	22
39	.52669	.97394	.55275	.44725	.02606	.47331	21
40	9.52705	9.97390	9.55315	10.44685	10.02610	10.47295	20
41	.52740	.97385	.55355	.44645	.02615	.47260	19
42	.52775	.97381	.55395	.44605	.02619	.47225	18
43	.52811	.97376	.55434	.44566	.02624	.47189	17
44	.52846	.97372	.55474	.44526	.02628	.47154	16
45	9.52881	9.97367	9.55514	10.44486	10.02633	10.47119	15
46	.52916	.97363	.55554	.44446	.02637	.47084	14
47	.52951	.97358	.55593	.44407	.02642	.47049	13
48	.52986	.97353	.55633	.44367	.02647	.47014	12
49	.53021	.97349	.55673	.44327	.02651	.46979	11
50	9.53056	9.97344	9.55712	10.44288	10.02656	10.46944	10
51	.53092	.97340	.55752	.44248	.02660	.46908	9
52	.53126	.97335	.55791	.44209	.02665	.46874	8
53	.53161	.97331	.55831	.44169	.02669	.46839	7
54	.53196	.97326	.55870	.44130	.02674	.46804	6
55	9.53231	9.97322	9.55910	10.44090	10.02678	10.46769	5
56	.53266	.97317	.55949	.44051	.02683	.46734	4
57	.53301	.97312	.55989	.44011	.02688	.46699	3
58	.53336	.97308	.56028	.43972	.02692	.46664	2
59	.53370	.97303	.56067	.43933	.02697	.46630	1
60	9.53405	9.97299	9.56107	10.43893	10.02701	10.46595	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

109° **70°**

20° **Logarithms of Trigonometrical Functions** 159°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.53405	9.97299	9.56107	10.43893	10.02701	10.46595	60
1	.53440	.97294	.56146	.43854	.02706	.46560	59
2	.53475	.97289	.56185	.43815	.02711	.46525	58
3	.53509	.97285	.56224	.43776	.02715	.46491	57
4	.53544	.97280	.56264	.43736	.02720	.46456	56
5	9.53578	9.97276	9.56303	10.43697	10.02724	10.46422	55
6	.53613	.97271	.56342	.43658	.02729	.46387	54
7	.53647	.97266	.56381	.43619	.02734	.46353	53
8	.53682	.97262	.56420	.43580	.02738	.46318	52
9	.53716	.97257	.56459	.43541	.02743	.46284	51
10	9.53751	9.97252	9.56498	10.43502	10.02748	10.46249	50
11	.53785	.97248	.56537	.43463	.02752	.46215	49
12	.53819	.97243	.56576	.43424	.02757	.46181	48
13	.53854	.97238	.56615	.43385	.02762	.46146	47
14	.53888	.97234	.56654	.43346	.02766	.46112	46
15	9.53922	9.97229	9.56693	10.43307	10.02771	10.46078	45
16	.53957	.97224	.56732	.43268	.02776	.46043	44
17	.53991	.97220	.56771	.43229	.02780	.46009	43
18	.54025	.97215	.56810	.43190	.02785	.45975	42
19	.54059	.97210	.56849	.43151	.02790	.45941	41
20	9.54093	9.97206	9.56887	10.43113	10.02794	10.45907	40
21	.54127	.97201	.56926	.43074	.02799	.45873	39
22	.54161	.97196	.56965	.43035	.02804	.45839	38
23	.54195	.97192	.57004	.42996	.02808	.45805	37
24	.54229	.97187	.57042	.42958	.02813	.45771	36
25	9.54263	9.97182	9.57081	10.42919	10.02818	10.45737	35
26	.54297	.97178	.57120	.42880	.02822	.45703	34
27	.54331	.97173	.57158	.42842	.02827	.45669	33
28	.54365	.97168	.57197	.42803	.02832	.45635	32
29	.54399	.97163	.57235	.42765	.02837	.45601	31
30	9.54433	9.97159	9.57274	10.42726	10.02841	10.45567	30
31	.54466	.97154	.57312	.42688	.02846	.45534	29
32	.54500	.97149	.57351	.42649	.02851	.45500	28
33	.54534	.97145	.57389	.42611	.02855	.45466	27
34	.54567	.97140	.57428	.42572	.02860	.45433	26
35	9.54601	9.97135	9.57466	10.42534	10.02865	10.45399	25
36	.54635	.97130	.57504	42496	.02870	.45365	24
37	.54668	.97126	.57543	.42457	.02874	.45332	23
38	.54702	.97121	.57581	.42419	.02879	.45298	22
39	.54735	.97116	.57619	.42381	.02884	.45265	21
40	9.54769	9.97111	9.57658	10.42342	10.02889	10.45231	20
41	.54802	.97107	.57696	.42304	.02893	.45198	19
42	.54836	.97102	.57734	.42266	.02898	.45164	18
43	.54869	.97097	.57772	.42228	.02903	.45131	17
44	.54903	.97092	.57810	.42190	.02908	.45097	16
45	9.54936	9.97087	9.57849	10.42151	10.02913	10.45064	15
46	.54969	.97083	.57887	.42113	.02917	.45031	14
47	.55003	.97078	.57925	.42075	.02922	.44997	13
48	.55036	.97073	.57963	.42037	.02937	.44964	12
49	.55069	.97068	.58001	.41999	.02932	.44931	11
50	9.55102	9.97063	9.58039	10.41961	10.02937	10.44898	10
51	.55136	.97059	.58077	.41923	.02941	.44864	9
52	.55169	.97054	.58115	.41885	.02946	.44831	8
53	.55202	.97049	.58153	.41847	.02951	.44798	7
54	.55235	.97044	.58191	.41809	.02956	.44765	6
55	9.55268	9.97039	9.58229	10.41771	10.02961	10.44732	5
56	.55301	.97035	.58267	.41733	.02965	.44699	4
57	.55334	.97030	.58304	.41696	.02970	.44666	3
58	.55367	.97025	.58342	.41658	.02975	.44633	2
59	.55400	.97020	.58380	.41620	.02980	.44600	1
60	9.55433	9.97015	9.58418	10.41582	10.02985	10.44567	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

110° 69°

21° **Logarithms of Trigonometrical Functions** 158°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.55433	9.97015	9.58418	10.41582	10.02985	10.44567	60
1	.55466	.97010	.58455	.41545	.02990	.44534	59
2	.55499	.97005	.58493	.41507	.02995	.44501	58
3	.55532	.97001	.58531	.41469	.02999	.44468	57
4	.55564	.96996	.58569	.41431	.03004	.44436	56
5	9.55597	9.96991	9.58606	10.41394	10.03009	10.44403	55
6	.55630	.96986	.58644	.41356	.03014	.44370	54
7	.55663	.96981	.58681	.41319	.03019	.44337	53
8	.55695	.96976	.58719	.41281	.03024	.44305	52
9	.55728	.96971	.58757	.41243	.03029	.44272	51
10	9.55761	9.96966	9.58794	10.41206	10.03034	10.44239	50
11	.55793	.96962	.58832	.41168	.03038	.44207	49
12	.55826	.96957	.58869	.41131	.03043	.44174	48
13	.55858	.96952	.58907	.41093	.03048	.44142	47
14	.55891	.96947	.58944	.41056	.03053	.44109	46
15	9.55923	9.96942	9.58981	10.41019	10.03058	10.44077	45
16	.55956	.96937	.59019	.40981	.03063	.44044	44
17	.55988	.96932	.59056	.40944	.03068	.44012	43
18	.56021	.96927	.59094	.40906	.03073	.43979	42
19	.56053	.96922	.59131	.40869	.03078	.43947	41
20	9.56085	9.96917	9.59168	10.40832	10.03083	10.43915	40
21	.56118	.96912	.59205	.40795	.03088	.43882	39
22	.56150	.96907	.59243	.40757	.03093	.43850	38
23	.56182	.96903	.59280	.40720	.03097	.43818	37
24	.56215	.96898	.59317	.40683	.03102	.43785	36
25	9.56247	9.96893	9.59354	10.40646	10.03107	10.43753	35
26	.56279	.96888	.59391	.40609	.03112	.43721	34
27	.56311	.96883	.59429	.40571	.03117	.43689	33
28	.56343	.96878	.59466	.40534	.03122	.43657	32
29	.56375	.96873	.59503	.40497	.03127	.43625	31
30	9.56408	9.96868	9.59540	10.40460	10.03132	10.43592	30
31	.56440	.96863	.59577	.40423	.03137	.43560	29
32	.56472	.96858	.59614	.40386	.03142	.43528	28
33	.56504	.96853	.59651	.40349	.03147	.43496	27
34	.56536	.96848	.59688	.40312	.03152	.43464	26
35	9.56568	9.96843	9.59725	10.40275	10.03157	10.43432	25
36	.56599	.96838	.59762	.40238	.03162	.43401	24
37	.56631	.96833	.59799	.40201	.03167	.43369	23
38	.56663	.96828	.59835	.40165	.03172	.43337	22
39	.56695	.96823	.59872	.40128	.03177	.43305	21
40	9.56727	9.96818	9.59909	10.40091	10.03182	10.43273	20
41	.56759	.96813	.59946	.40054	.03187	.43241	19
42	.56790	.96808	.59983	.40017	.03192	.43210	18
43	.56822	.96803	.60019	.39981	.03197	.43178	17
44	.56854	.96798	.60056	.39944	.03202	.43146	16
45	9.56886	9.96793	9.60093	10.39907	10.03207	10.43114	15
46	.56917	.96788	.60130	.39870	.03212	.43083	14
47	.56949	.96783	.60166	.39834	.03217	.43051	13
48	.56980	.96778	.60203	.39797	.03222	.43020	12
49	.57012	.96772	.60240	.39760	.03228	.42988	11
50	9.57044	9.96767	9.60276	10.39724	10.03233	10.42956	10
51	.57075	.96762	.60313	.39687	.03238	.42925	9
52	.57107	.96757	.60349	.39651	.03243	.42893	8
53	.57138	.96752	.60386	.39614	.03248	.42862	7
54	.57169	.96747	.60422	.39578	.03253	.42831	6
55	9.57201	9.96742	9.60459	10.39541	10.03258	10.42799	5
56	.57232	.96737	.60495	.39505	.03263	.42768	4
57	.57264	.96732	.60532	.39468	.03268	.42736	3
58	.57295	.96727	.60568	.39432	.03273	.42705	2
59	.57326	.96722	.60605	.39395	.03278	.42674	1
60	9.57358	9.96717	9.60641	10.39359	10.03283	10.42642	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

111° 68°

22° **Logarithms of Trigonometrical Functions** **157°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.57358	9.96717	9.60641	10.39359	10.03283	10.42642	60
1	.57389	.96711	.60677	.39323	.03289	.42611	59
2	.57420	.96706	.60714	.39286	.03294	.42580	58
3	.57451	.96701	.60750	.39250	.03299	.42549	57
4	.57482	.96696	.60786	.39214	.03304	.42518	56
5	9.57514	9.96691	9.60823	10.39177	10.03309	10.42486	55
6	.57545	.96686	.60859	.39141	.03314	.42455	54
7	.57576	.96681	.60895	.39105	.03319	.42424	53
8	.57607	.96676	.60931	.39069	.03324	.42393	52
9	.57638	.96670	.60967	.39033	.03330	.42362	51
10	9.57669	9.96665	9.61004	10.38996	10.03335	10.42331	50
11	.57700	.96660	.61040	.38960	.03340	.42300	49
12	.57731	.96655	.61076	.38924	.03345	.42269	48
13	.57762	.96650	.61112	.38888	.03350	.42238	47
14	.57793	.96645	.61148	.38852	.03355	.42207	46
15	9.57824	9.96640	9.61184	10.38816	10.03360	10.42176	45
16	.57855	.96634	.61220	.38780	.03366	.42145	44
17	.57885	.96629	.61256	.38744	.03371	.42115	43
18	.57916	.96624	.61292	.38708	.03376	.42084	42
19	.57947	.96619	.61328	.38672	.03381	.42053	41
20	9.57978	9.96614	9.61364	10.38636	10.03386	10.42022	40
21	.58008	.96608	.61400	.38600	.03392	.41992	39
22	.58039	.96603	.61436	.38564	.03397	.41961	38
23	.58070	.96598	.61472	.38528	.03402	.41930	37
24	.58101	.96593	.61508	.38492	.03407	.41899	36
25	9.58131	9.96588	9.61544	10.38456	10.03412	10.41869	35
26	.58162	.96582	.61579	.38421	.03418	.41838	34
27	.58192	.96577	.61615	.38385	.03423	.41808	33
28	.58223	.96572	.61651	.38349	.03428	.41777	32
29	.58253	.96567	.61687	.38313	.03433	.41747	31
30	9.58284	9.96562	9.61722	10.38278	10.03438	10.41716	30
31	.58314	.96556	.61758	.38242	.03444	.41686	29
32	.58345	.96551	.61794	.38206	.03449	.41655	28
33	.58375	.96546	.61830	.38170	.03454	.41625	27
34	.58406	.96541	.61865	.38135	.03459	.41594	26
35	9.58436	9.96535	9.61901	10.38099	10.03465	10.41564	25
36	.58467	.96530	.61936	.38064	.03470	.41533	24
37	.58497	.96525	.61972	.38028	.03475	.41503	23
38	.58527	.96520	.62008	.37992	.03480	.41473	22
39	.58557	.96514	.62043	.37957	.03486	.41443	21
40	9.58588	9.96509	9.62079	10.37921	10.03491	10.41412	20
41	.58618	.96504	.62114	37886	.03496	41382	19
42	.58648	.96498	.62150	.37850	.03502	.41352	18
43	.58678	.96493	.62185	.37815	.03507	.41322	17
44	.58709	.96488	.62221	.37779	.03512	.41291	16
45	9.58739	9.96483	9.62256	10.37744	10.03517	10.41261	15
46	.58769	.96477	.62292	.37708	.03523	.41231	14
47	.58799	.96472	.62327	.37673	.03528	.41201	13
48	.58829	.96467	.62362	.37638	.03533	.41171	12
49	.58859	.96461	.62398	.37602	.03539	.41141	11
50	9.58889	9.96456	9.62433	10.37567	10.03544	10.41111	10
51	.58919	.96451	.62468	.37532	.03549	.41081	9
52	.58949	.96445	.62504	.37496	.03555	41051	8
53	.58979	.96440	.62539	.37461	.03560	.41021	7
54	.59009	.96435	.62574	.37426	.03565	.40991	6
55	9.59039	9.96429	9.62609	10.37391	10.03571	10.40961	5
56	.59069	.96424	.62645	.37355	.03576	.40931	4
57	.59098	.96419	.62680	.37320	.03581	.40902	3
58	.59128	.96413	.62715	.37285	.03587	.40872	2
59	.59158	.96408	.62750	.37250	.03592	.40842	1
60	9.59188	9.96403	9.62785	10.37215	10.03597	10.40812	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

112° **67°**

23° **Logarithms of Trigonometrical Functions** **156°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.59188	9.96403	9.62785	10.37215	10.03597	10.40812	60
1	.59218	.96397	.62820	.37180	.03603	.40782	59
2	.59247	.96392	.62855	.37145	.03608	.40753	58
3	.59277	.96387	.62890	.37110	.03613	.40723	57
4	.59307	.96381	.62926	.37074	.03619	.40693	56
5	9.59336	9.96376	9.62961	10.37039	10.03624	10.40664	55
6	.59366	.96370	.62996	.37004	.03630	.40634	54
7	.59396	.96365	.63031	.36969	.03635	.40604	53
8	.59425	.96360	.63066	.36934	.03640	.40575	52
9	.59455	.96354	.63101	.36899	.03646	.40545	51
10	9.59484	9.96349	9.63135	10.36865	10.03651	10.40516	50
11	.59514	.96343	.63170	.36830	.03657	.40486	49
12	.59543	.96338	.63205	.36795	.03662	.40457	48
13	.59573	.96333	.63240	.36760	.03667	.40427	47
14	.59602	.96327	.63275	.36725	.03673	.40398	46
15	9.59632	9.96322	9.63310	10.36690	10.03678	10.40368	45
16	.59661	.96316	.63345	.36655	.03684	.40339	44
17	.59690	.96311	.63379	.36621	.03689	.40310	43
18	.59720	.96305	.63414	.36586	.03695	.40280	42
19	.59749	.96300	.63449	.36551	.03700	.40251	41
20	9.59778	9.96294	9.63484	10.36516	10.03706	10.40222	40
21	.59808	.96289	.63519	.36481	.03711	.40192	39
22	.59837	.96284	.63553	.36447	.03716	.40163	38
23	.59866	.96278	.63588	.36412	.03722	.40134	37
24	.59895	.96273	.63623	.36377	.03727	.40105	36
25	9.59924	9.96267	9.63657	10.36343	10.03733	10.40076	35
26	.59954	.96262	.63692	.36308	.03738	.40046	34
27	.59983	.96256	.63726	.36274	.03744	.40017	33
28	.60012	.96251	.63761	.36239	.03749	.39988	32
29	.60041	.96245	.63796	.36204	.03755	.39959	31
30	9.60070	9.96240	9.63830	10.36170	10.03760	10.39930	30
31	.60099	.96234	.63865	.36135	.03766	.39901	29
32	.60128	.96229	.63899	.36101	.03771	.39872	28
33	.60157	.96223	.63934	.36066	.03777	.39843	27
34	.60186	.96218	.63968	.36032	.03782	.39814	26
35	9.60215	9.96212	9.64003	10.35997	10.03788	10.39785	25
36	.60244	.96207	.64037	.35963	.03793	.39756	24
37	.60273	.96201	.64072	.35928	.03799	.39727	23
38	.60302	.96196	.64106	.35894	.03804	.39698	22
39	.60331	.96190	.64140	.35860	.03810	.39669	21
40	9.60359	9.96185	9.64175	10.35825	10.03815	10.39641	20
41	.60388	.96179	.64209	.35791	.03821	.39612	19
42	.60417	.96174	.64243	.35757	.03826	.39583	18
43	.60446	.96168	.64278	.35722	.03832	.39554	17
44	.60474	.96162	.64312	.35688	.03838	.39526	16
45	9.60503	9.96157	9.64346	10.35654	10.03843	10.39497	15
46	.60532	.96151	.64381	.35619	.03849	.39468	14
47	.60561	.96146	.64415	.35585	.03854	.39439	13
48	.60589	.96140	.64449	.35551	.03860	.39411	12
49	.60618	.96135	.64483	.35517	.03865	.39382	11
50	9.60646	9.96129	9.64517	10.35483	10.03871	10.39354	10
51	.60675	.96123	.64552	.35448	.03877	.39325	9
52	.60704	.96118	.64586	.35414	.03882	.39296	8
53	.60732	.96112	.64620	.35380	.03888	.39268	7
54	.60761	.96107	.64654	.35346	.03893	.39239	6
55	9.60789	9.96101	9.64688	10.35312	10.03899	10.39211	5
56	.60818	.96095	.64722	.35278	.03905	.39182	4
57	.60846	.96090	.64756	.35244	.03910	.39154	3
58	.60875	.96084	.64790	.35210	.03916	.39125	2
59	.60903	.96079	.64824	.35176	.03921	.39097	1
60	9.60931	9.96073	9.64858	10.35142	10.03927	10.39069	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

113° **66°**

24° **Logarithms of Trigonometrical Functions** **155°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.60931	9.96073	9.64858	10.35142	10.03927	10.39069	60
1	.60960	.96067	.64892	.35108	.03933	.39040	59
2	.60988	.96062	.64926	.35074	.03938	.39012	58
3	.61016	.96056	.64960	.35040	.03944	.38984	57
4	.61045	.96050	.64994	.35006	.03950	.38955	56
5	9.61073	9.96045	9.65028	10.34972	10.03955	10.38927	55
6	.61101	.96039	.65062	.34938	.03961	.38899	54
7	.61129	.96034	.65096	.34904	.03966	.38871	53
8	.61158	.96028	.65130	.34870	.03972	.38842	52
9	.61186	.96022	.65164	.34836	.03978	.38814	51
10	9.61214	9.96017	9.65197	10.34803	10.03983	10.38786	50
11	.61242	.96011	.65231	.34769	.03989	.38758	49
12	.61270	.96005	.65265	.34735	.03995	.38730	48
13	.61298	.96000	.65299	.34701	.04000	.38702	47
14	.61326	.95994	.65333	.34667	.04006	.38674	46
15	9.61354	9.95988	9.65366	10.34634	10.04012	10.38646	45
16	.61382	.95982	.65400	.34600	.04018	.38618	44
17	.61411	.95977	.65434	.34566	.04023	.38589	43
18	.61438	.95971	.65467	.34533	.04029	.38562	42
19	.61466	.95965	.65501	.34499	.04035	.38534	41
20	9.61494	9.95960	9.65535	10.34465	10.04040	10.38506	40
21	.61522	.95954	.65568	.34432	.04046	.38478	39
22	.61550	.95948	.65602	.34398	.04052	.38450	38
23	.61578	.95942	.65636	.34364	.04058	.38422	37
24	.61606	.95937	.65669	.34331	.04063	.38394	36
25	9.61634	9.95931	9.65703	10.34297	10.04069	10.38366	35
26	.61662	.95925	.65736	.34264	.04075	.38338	34
27	.61689	.95920	.65770	.34230	.04080	.38311	33
28	.61717	.95914	.65803	.34197	.04086	.38283	32
29	.61745	.95908	.65837	.34163	.04092	.38255	31
30	9.61773	9.95902	9.65870	10.34130	10.04098	10.38227	30
31	.61800	.95897	.65904	.34096	.04103	.38200	29
32	.61828	.95891	.65937	.34063	.04109	.38172	28
33	.61856	.95885	.65971	.34029	.04115	.38144	27
34	.61883	.95879	.66004	.33996	.04121	.38117	26
35	9.61911	9.95873	9.66038	10.33962	10.04127	10.38089	25
36	.61939	.95868	.66071	.33929	.04132	.38061	24
37	.61966	.95862	.66104	.33896	.04138	.38034	23
38	.61994	.95856	.66138	.33862	.04144	.38006	22
39	.62021	.95850	.66171	.33829	.04150	.37979	21
40	9.62049	9.95844	9.66204	10.33796	10.04156	10.37951	20
41	.62076	.95839	.66238	.33762	.04161	.37924	19
42	.62104	.95833	.66271	.33729	.04167	.37896	18
43	.62131	.95827	.66304	.33696	.04173	.37869	17
44	.62159	.95821	.66337	.33663	.04179	.37841	16
45	9.62186	9.95815	9.66371	10.33629	10.04185	10.37814	15
46	.62214	.95810	.66404	.33596	.04190	.37786	14
47	.62241	.95804	.66437	.33563	.04196	.37759	13
48	.62268	.95798	.66470	.33530	.04202	.37732	12
49	.62296	.95792	.66503	.33497	.04208	.37704	11
50	9.62323	9.95786	9.66537	10.33463	10.04214	10.37677	10
51	.62350	.95780	.66570	.33430	.04220	.37650	9
52	.62377	.95775	.66603	.33397	.04225	.37623	8
53	.62405	.95769	.66636	.33364	.04231	.37595	7
54	.62432	.95763	.66669	.33331	.04237	.37568	6
55	9.62459	9.95757	9.66702	10.33298	10.04243	10.37541	5
56	.62486	.95751	.66735	.33265	.04249	.37514	4
57	.62513	.95745	.66768	.33232	.04255	.37487	3
58	.62541	.95739	.66801	.33199	.04261	.37459	2
59	.62568	.95733	.66834	.33166	.04267	.37432	1
60	9.62595	9.95728	9.66867	10.33133	10.04272	10.37405	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

114° **65°**

25° **Logarithms of Trigonometrical Functions** 154°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.62595	9.95728	9.66867	10.33133	10.04272	10.37405	60
1	.62622	.95722	.66900	.33100	.04278	.37378	59
2	.62649	.95716	.66933	.33067	.04284	.37351	58
3	.62676	.95710	.66966	.33034	.04290	.37324	57
4	.62703	.95704	.66999	.33001	.04296	.37297	56
5	9.62730	9.95698	9.67032	10.32968	10.04302	10.37270	55
6	.62757	.95692	.67065	.32935	.04308	.37243	54
7	.62784	.95686	.67098	.32902	.04314	.37216	53
8	.62811	.95680	.67131	.32869	.04320	.37189	52
9	.62838	.95674	.67163	.32837	.04326	.37162	51
10	9.62865	9.95668	9.67196	10.32804	10.04332	10.37135	50
11	.62892	.95663	.67229	.32771	.04337	.37108	49
12	.62918	.95657	.67262	.32738	.04343	.37082	48
13	.62945	.95651	.67295	.32705	.04349	.37055	47
14	.62972	.95645	.67327	.32673	.04355	.37028	46
15	9.62999	9.95639	9.67360	10.32640	10.04361	10.37001	45
16	.63026	.95633	.67393	.32607	.04367	.36974	44
17	.63052	.95627	.67426	.32574	.04373	.36948	43
18	.63079	.95621	.67458	.32542	.04379	.36921	42
19	.63106	.95615	.67491	.32509	.04385	.36894	41
20	9.63133	9.95609	9.67524	10.32476	10.04391	10.36867	40
21	.63159	.95603	.67556	.32444	.04397	.36841	39
22	.63186	.95597	.67589	.32411	.04403	.36814	38
23	.63213	.95591	.67622	.32378	.04409	.36787	37
24	.63239	.95585	.67654	.32346	.04415	.36761	36
25	9.63266	9.95579	9.67687	10.32313	10.04421	10.36734	35
26	.63292	.95573	.67719	.32281	.04427	.36708	34
27	.63319	.95567	.67752	.32248	.04433	.36681	33
28	.63345	.95561	.67785	.32215	.04439	.36655	32
29	.63372	.95555	.67817	.32183	.04445	.36628	31
30	9.63398	9.95549	9.67850	10.32150	10.04451	10.36602	30
31	.63425	.95543	.67882	.32118	.04457	.36575	29
32	.63451	.95537	.67915	.32085	.04463	.36549	28
33	.63478	.95531	.67947	.32053	.04469	.36522	27
34	.63504	.95525	.67980	.32020	.04475	.36496	26
35	9.63531	9.95519	9.68012	10.31988	10.04481	10.36469	25
36	.63557	.95513	.68044	.31956	.04487	.36443	24
37	.63583	.95507	.68077	.31923	.04493	.36417	23
38	.63610	.95500	.68109	.31891	.04500	.36390	22
39	.63636	.95494	.68142	.31858	.04506	.36364	21
40	9.63662	9.95488	9.68174	10.31826	10.04512	10.36338	20
41	.63689	.95482	.68206	.31794	.04518	.36311	19
42	.63715	.95476	.68239	.31761	.04524	.36285	18
43	.63741	.95470	.68271	.31729	.04530	.36259	17
44	.63767	.95464	.68303	.31697	.04536	.36233	16
45	9.63794	9.95458	9.68336	10.31664	10.04542	10.36206	15
46	.63820	.95452	.68368	.31632	.04548	.36180	14
47	.63846	.95446	.68400	.31600	.04554	.36154	13
48	.63872	.95440	.68432	.31568	.04560	.36128	12
49	.63898	.95434	.68465	.31535	.04566	.36102	11
50	9.63924	9.95427	9.68497	10.31503	10.04573	10.36076	10
51	.63950	.95421	.68529	.31471	.04579	.36050	9
52	.63976	.95415	.68561	.31439	.04585	.36024	8
53	.64002	.95409	.68593	.31407	.04591	.35998	7
54	.64028	.95403	.68626	.31374	.04597	.35972	6
55	9.64054	9.95397	9.68658	10.31342	10.04603	10.35946	5
56	.64080	.95391	.68690	.31310	.04609	.35920	4
57	.64106	.95384	.68722	.31278	.04616	.35894	3
58	.64132	.95378	.68754	.31246	.04622	.35868	2
59	.64158	.95372	.68786	.31214	.04628	.35842	1
60	9.64184	9.95366	9.68818	10.31182	10.04634	10.35816	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

115° 64°

26° **Logarithms of Trigonometrical Functions** 153°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.64184	9.95366	9.68818	10.31182	10.04634	10.35816	60
1	.64210	.95360	.68850	.31150	.04640	.35790	59
2	.64236	.95354	.68882	.31118	.04646	.35764	58
3	.64262	.95348	.68914	.31086	.04652	.35738	57
4	.64288	.95341	.68946	.31054	.04659	.35712	56
5	9.64313	9.95335	9.68978	10.31022	10.04665	10.35687	55
6	.64339	.95329	.69010	.30990	.04671	.35661	54
7	.64365	.95323	.69042	.30958	.04677	.35635	53
8	.64391	.95317	.69074	.30926	.04683	.35609	52
9	.64417	.95310	.69106	.30894	.04690	.35583	51
10	9.64442	9.95304	9.69138	10.30862	10.04696	10.35558	50
11	.64468	.95298	.69170	.30830	.04702	.35532	49
12	.64494	.95292	.69202	.30798	.04708	.35506	48
13	.64519	.95286	.69234	.30766	.04714	.35481	47
14	.64545	.95279	.69266	.30734	.04721	.35455	46
15	9.64571	9.95273	9.69298	10.30702	10.04727	10.35429	45
16	.64596	.95267	.69329	.30671	.04733	.35404	44
17	.64622	.95261	.69361	.30639	.04739	.35378	43
18	.64647	.95254	.69393	.30607	.04746	.35353	42
19	.64673	.95248	.69425	.30575	.04752	.35327	41
20	9.64698	9.95242	9.69457	10.30543	10.04758	10.35302	40
21	.64724	.95236	.69488	.30512	.04764	.35276	39
22	.64749	.95229	.69520	.30480	.04771	.35251	38
23	.64775	.95223	.69552	.30448	.04777	.35225	37
24	.64800	.95217	.69584	.30416	.04783	.35200	36
25	9.64826	9.95211	9.69615	10.30385	10.04789	10.35174	35
26	.64851	.95204	.69647	.30353	.04796	.35149	34
27	.64877	.95198	.69679	.30321	.04802	.35123	33
28	.64902	.95192	.69710	.30290	.04808	.35098	32
29	.64927	.95185	.69742	.30258	.04815	.35073	31
30	9.64953	9.95179	9.69774	10.30226	10.04821	10.35047	30
31	.64978	.95173	.69805	.30195	.04827	.35022	29
32	.65003	.95167	.69837	.30163	.04833	.34997	28
33	.65029	.95160	.69868	.30132	.04840	.34971	27
34	.65054	.95154	.69900	.30100	.04846	.34946	26
35	9.65079	9.95148	9.69932	10.30068	10.04852	10.34921	25
36	.65104	.95141	.69963	.30037	.04859	.34896	24
37	.65130	.95135	.69995	.30005	.04865	.34870	23
38	.65155	.95129	.70026	.29974	.04871	.34845	22
39	.65180	.95122	.70058	.29942	.04878	.34820	21
40	9.65205	9.95116	9.70089	10.29911	10.04884	10.34795	20
41	.65230	.95110	.70121	.29879	.04890	.34770	19
42	.65255	.95103	.70152	.29848	.04897	.34745	18
43	.65281	.95097	.70184	.29816	.04903	.34719	17
44	.65306	.95090	.70215	.29785	.04910	.34694	16
45	9.65331	9.95084	9.70247	10.29753	10.04916	10.34669	15
46	.65356	.95078	.70278	.29722	.04922	.34644	14
47	.65381	.95071	.70309	.29691	.04929	.34619	13
48	.65406	.95065	.70341	.29659	.04935	.34594	12
49	.65431	.95059	.70372	.29628	.04941	.34569	11
50	9.65456	9.95052	9.70404	10.29596	10.04948	10.34544	10
51	.65481	.95046	.70435	.29565	.04954	.34519	9
52	.65506	.95039	.70466	.29534	.04961	.34494	8
53	.65531	.95033	.70498	.29502	.04967	.34469	7
54	.65556	.95027	.70529	.29471	.04973	.34444	6
55	9.65580	9.95020	9.70560	10.29440	10.04980	10.34420	5
56	.65605	.95014	.70592	.29408	.04986	.34395	4
57	.65630	.95007	.70623	.29377	.04993	.34370	3
58	.65655	.95001	.70654	.29346	.04999	.34345	2
59	.65680	.94995	.70685	.29315	.05005	.34320	1
60	9.65705	9.94988	9.70717	10.29283	10.05012	10.34295	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

116° 63°

27° **Logarithms of Trigonometrical Functions** 152°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.65705	9.94988	9.70717	10.29283	10.05012	10.34295	60
1	.65729	.94982	.70748	.29252	.05018	.34271	59
2	.65754	.94975	.70779	.29221	.05025	.34246	58
3	.65779	.94969	.70810	.29190	.05031	.34221	57
4	.65804	.94962	.70841	.29159	.05038	.34196	56
5	9.65828	9.94956	9.70873	10.29127	10.05044	10.34172	55
6	.65853	.94949	.70904	.29096	.05051	.34147	54
7	.65878	.94943	.70935	.29065	.05057	.34122	53
8	.65902	.94936	.70966	.29034	.05064	.34098	52
9	.65927	.94930	.70997	.29003	.05070	.34073	51
10	9.65952	9.94923	9.71028	10.28972	10.05077	10.34048	50
11	.65976	.94917	.71059	.28941	.05083	.34024	49
12	.66001	.94911	.71090	.28910	.05089	.33999	48
13	.66025	.94904	.71121	.28879	.05096	.33975	47
14	.66050	.94898	.71153	.28847	.05102	.33950	46
15	9.66075	9.94891	9.71184	10.28816	10.05109	10.33925	45
16	.66099	.94885	.71215	.28785	.05115	.33901	44
17	.66124	.94878	.71246	.28754	.05122	.33876	43
18	.66148	.94871	.71277	.28723	.05129	.33852	42
19	.66173	.94865	.71308	.28692	.05135	.33827	41
20	9.66197	9.94858	9.71339	10.28661	10.05142	10.33803	40
21	.66221	.94852	.71370	.28630	.05148	.33779	39
22	.66246	.94845	.71401	.28599	.05155	.33754	38
23	.66270	.94839	.71431	.28569	.05161	.33730	37
24	.66295	.94832	.71462	.28538	.05168	.33705	36
25	9.66319	9.94826	9.71493	10.28507	10.05174	10.33681	35
26	.66343	.94819	.71524	.28476	.05181	.33657	34
27	.66368	.94813	.71555	.28445	.05187	.33632	33
28	.66392	.94806	.71586	.28414	.05194	.33608	32
29	.66416	.94799	.71617	.28383	.05201	.33584	31
30	9.66441	9.94793	9.71648	10.28352	10.05207	10.33559	30
31	.66465	.94786	.71679	.28321	.05214	.33535	29
32	.66489	.94780	.71709	.28291	.05220	.33511	28
33	.66513	.94773	.71740	.28260	.05227	.33487	27
34	.66537	.94767	.71771	.28229	.05233	.33463	26
35	9.66562	9.94760	9.71802	10.28198	10.05240	10.33438	25
36	.66586	.94753	.71833	.28167	.05247	.33414	24
37	.66610	.94747	.71863	.28137	.05253	.33390	23
38	.66634	.94740	.71894	.28106	.05260	.33366	22
39	.66658	.94734	.71925	.28075	.05266	33342	21
40	9.66682	9.94727	9.71955	10.28045	10.05273	10.33318	20
41	.66706	.94720	.71986	.28014	.05280	.33294	19
42	.66731	.94714	.72017	.27983	.05286	.33269	18
43	.66755	.94707	.72048	.27952	.05293	.33245	17
44	.66779	.94700	.72078	.27922	.05300	.33221	16
45	9.66803	9.94694	9.72109	10.27891	10.05306	10.33197	15
46	.66827	.94687	.72140	.27860	.05313	.33173	14
47	.66851	.94680	.72170	.27830	.05320	.33149	13
48	.66875	.94674	.72201	.27799	.05326	.33125	12
49	.66899	.94667	.72231	.27769	.05333	.33101	11
50	9.66922	9.94660	9.72262	10.27738	10.05340	10.33078	10
51	.66946	.94654	.72293	.27707	.05346	.33054	9
52	.66970	.94647	.72323	.27677	.05353	.33030	8
53	.66994	.94640	.72354	.27646	.05360	.33006	7
54	.67018	.94634	.72384	.27616	.05366	.32982	6
55	9.67042	9.94627	9.72415	10.27585	10.05373	10.32958	5
56	.67066	.94620	.72445	.27555	.05380	.32934	4
57	.67090	.94614	.72476	.27524	.05386	.32910	3
58	.67113	.94607	.72506	.27494	.05393	.32887	2
59	.67137	.94600	.72537	.27463	.05400	.32863	1
60	9.67161	9.94593	9.72567	10.27433	10.05407	10.32839	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

28° **Logarithms of Trigonometrical Functions** **151°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.67161	9.94593	9.72567	10.27433	10.05407	10.32839	60
1	.67185	.94587	.72598	.27402	.05413	.32815	59
2	.67208	.94580	.72628	.27372	.05420	.32792	58
3	.67232	.94573	.72659	.27341	.05427	.32768	57
4	.67256	.94567	.72689	.27311	.05433	.32744	56
5	9.67280	9.94560	9.72720	10.27280	10.05440	10.32720	55
6	.67303	.94553	.72750	.27250	.05447	.32697	54
7	.67327	.94546	.72780	.27220	.05454	.32673	53
8	.67350	.94540	.72811	.27189	.05460	.32650	52
9	.67374	.94533	.72841	.27159	.05467	.32626	51
10	9.67398	9.94526	9.72872	10.27128	10.05474	10.32602	50
11	.67421	.94519	.72902	.27098	.05481	.32579	49
12	.67445	.94513	.72932	.27068	.05487	.32555	48
13	.67468	.94506	.72963	.27037	.05494	.32532	47
14	.67492	.94499	.72993	.27007	.05501	.32508	46
15	9.67515	9.94492	9.73023	10.26977	10.05508	10.32485	45
16	.67539	.94485	.73054	.26946	.05515	.32461	44
17	.67562	.94479	.73084	.26916	.05521	.32438	43
18	.67586	.94472	.73114	.26886	.05528	.32414	42
19	.67609	.94465	.73144	.26856	.05535	.32391	41
20	9.67633	9.94458	9.73175	10.26825	10.05542	10.32367	40
21	.67656	.94451	.73205	.26795	.05549	.32344	39
22	.67680	.94445	.73235	.26765	.05555	.32320	38
23	.67703	.94438	.73265	.26735	.05562	.32297	37
24	.67726	.94431	.73295	.26705	.05569	.32274	36
25	9.67750	9.94424	9.73326	10.26674	10.05576	10.32250	35
26	.67773	.94417	.73356	.26644	.05583	.32227	34
27	.67796	.94410	.73386	.26614	.05590	.32204	33
28	.67820	.94404	.73416	.26584	.05596	.32180	32
29	.67843	.94397	.73446	.26554	.05603	.32157	31
30	9.67866	9.94390	9.73476	10.26524	10.05610	10.32134	30
31	.67890	.94383	.73507	.26493	.05617	.32110	29
32	.67913	.94376	.73537	.26463	.05624	.32087	28
33	.67936	.94369	.73567	.26433	.05631	.32064	27
34	.67959	.94362	.73597	.26403	.05638	.32041	26
35	9.67982	9.94355	9.73627	10.26373	10.05645	10.32018	25
36	.68006	.94349	.73657	.26343	.05651	.31994	24
37	.68029	.94342	.73687	.26313	.05658	.31971	23
38	.68052	.94335	.73717	.26283	.05665	.31948	22
39	.68075	.94328	.73747	.26253	.05672	.31925	21
40	9.68098	9.94321	9.73777	10.26223	10.05679	10.31902	20
41	.68121	.94314	.73807	.26193	.05686	.31879	19
42	.68144	.94307	.73837	.26163	.05693	.31856	18
43	.68167	.94300	.73867	.26133	.05700	.31833	17
44	.68190	.94293	.73897	.26103	.05707	.31810	16
45	9.68213	9.94286	9.73927	10.26073	10.05714	10.31787	15
46	.68237	.94279	.73957	.26043	.05721	.31763	14
47	.68260	.94273	.73987	.26013	.05727	.31740	13
48	.68283	.94266	.74017	.25983	.05734	.31717	12
49	.68305	.94259	.74047	.25953	.05741	.31695	11
50	9.68328	9.94252	9.74077	10.25923	10.05748	10.31672	10
51	.68351	.94245	.74107	.25893	.05755	.31649	9
52	.68374	.94238	.74137	.25863	.05762	.31626	8
53	.68397	.94231	.74166	.25834	.05769	.31603	7
54	.68420	.94224	.74196	.25804	.05776	.31580	6
55	9.68443	9.94217	9.74226	10.25774	10.05783	10.31557	5
56	.68466	.94210	.74256	.25744	.05790	.31534	4
57	.68489	.94203	.74286	.25714	.05797	.31511	3
58	.68512	.94196	.74316	.25684	.05804	.31488	2
59	.68534	.94189	.74345	.25655	.05811	.31466	1
60	9.68557	9.94182	9.74375	10.25625	10.05818	10.31443	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

118° **61°**

29° **Logarithms of Trigonometrical Functions** **150°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.68557	9.94182	9.74375	10.25625	10.05818	10.31443	60
1	.68580	.94175	.74405	.25595	.05825	.31420	59
2	.68603	.94168	.74435	.25565	.05832	.31397	58
3	.68625	.94161	.74465	.25535	.05839	.31375	57
4	.68648	.94154	.74494	.25506	.05846	.31352	56
5	9.68671	9.94147	9.74524	10.25476	10.05853	10.31329	55
6	.68694	.94140	.74554	.25446	.05860	.31306	54
7	.68716	.94133	.74583	.25417	.05867	.31284	53
8	.68739	.94126	.74613	.25387	.05874	.31261	52
9	.68762	.94119	.74643	.25357	.05881	.31238	51
10	9.68784	9.94112	9.74673	10.25327	10.05888	10.31216	50
11	.68807	.94105	.74702	.25298	.05895	.31193	49
12	.68829	.94098	.74732	.25268	.05902	.31171	48
13	.68852	.94090	.74762	.25238	.05910	.31148	47
14	.68875	.94083	.74791	.25209	.05917	.31125	46
15	9.68897	9.94076	9.74821	10.25179	10.05924	10.31103	45
16	.68920	.94069	.74851	.25149	.05931	.31080	44
17	.68942	.94062	.74880	.25120	.05938	.31058	43
18	.68965	.94055	.74910	.25090	.05945	.31035	42
19	.68987	.94048	.74939	.25061	.05952	.31013	41
20	9.69010	9.94041	9.74969	10.25031	10.05959	10.30990	40
21	.69032	.94034	.74998	.25002	.05966	.30968	39
22	.69055	.94027	.75028	.24972	.05973	.30945	38
23	.69077	.94020	.75058	.24942	.05980	.30923	37
24	.69100	.94012	.75087	.24913	.05988	.30900	36
25	9.69122	9.94005	9.75117	10.24883	10.05995	10.30878	35
26	.69144	.93998	.75146	.24854	.06002	.30856	34
27	.69167	.93991	.75176	.24824	.06009	.30833	33
28	.69189	.93984	.75205	.24795	.06016	.30811	32
29	.69212	.93977	.75235	.24765	.06023	.30788	31
30	9.69234	9.93970	9.75264	10.24736	10.06030	10.30766	30
31	.69256	.93963	.75294	.24706	.06037	.30744	29
32	.69279	.93955	.75323	.24677	.06045	.30721	28
33	.69301	.93948	.75353	.24647	.06052	.30699	27
34	.69323	.93941	.75382	.24618	.06059	.30677	26
35	9.69345	9.93934	9.75411	10.24589	10.06066	10.30655	25
36	.69368	.93927	.75441	.24559	.06073	.30632	24
37	.69390	.93920	.75470	.24530	.06080	.30610	23
38	.69412	.93912	.75500	.24500	.06088	.30588	22
39	.69434	.93905	.75529	.24471	.06095	.30566	21
40	9.69456	9.93898	9.75558	10.24442	10.06102	10.30544	20
41	.69479	.93891	.75588	.24412	.06109	.30521	19
42	.69501	.93884	.75617	.24383	.06116	.30499	18
43	.69523	.93876	.75647	.24353	.06124	.30477	17
44	.69545	.93869	.75676	.24324	.06131	.30455	16
45	9.69567	9.93862	9.75705	10.24295	10.06138	10.30433	15
46	.69589	.93855	.75735	.24265	.06145	.30411	14
47	.69611	.93847	.75764	.24236	.06153	.30389	13
48	.69633	.93840	.75793	.24207	.06160	.30367	12
49	.69655	.93833	.75822	.24178	.06167	.30345	11
50	9.69677	9.93826	9.75852	10.24148	10.06174	10.30323	10
51	.69699	.93819	.75881	.24119	.06181	.30301	9
52	.69721	.93811	.75910	.24090	.06189	.30279	8
53	.69743	.93804	.75939	.24061	.06196	.30257	7
54	.69765	.93797	.75969	.24031	.06203	.30235	6
55	9.69787	9.93789	9.75998	10.24002	10.06211	10.30213	5
56	.69809	.93782	.76027	.23973	.06218	.30191	4
57	.69831	.93775	.76056	.23944	.06225	.30169	3
58	.69853	.93768	.76086	.23914	.06232	.30147	2
59	.69875	.93760	.76115	.23885	.06240	.30125	1
60	9.69897	9.93753	9.76144	10.23856	10.06247	10.30103	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

119° **60°**

30° **Logarithms of Trigonometrical Functions** **149°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.69897	9.93753	9.76144	10.23856	10.06247	10.30103	60
1	.69919	.93746	.76173	.23827	.06254	.30081	59
2	.69941	.93738	.76202	.23798	.06262	.30059	58
3	.69963	.93731	.76231	.23769	.06269	.30037	57
4	.69984	.93724	.76261	.23739	.06276	.30016	56
5	9.70006	9.93717	9.76290	10.23710	10.06283	10.29994	55
6	.70028	.93709	.76319	.23681	.06291	.29972	54
7	.70050	.93702	.76348	.23652	.06298	.29950	53
8	.70072	.93695	.76377	.23623	.06305	.29928	52
9	.70093	.93687	.76406	.23594	.06313	.29907	51
10	9.70115	9.93680	9.76435	10.23565	10.06320	10.29885	50
11	.70137	.93673	.76464	.23536	.06327	.29863	49
12	.70159	.93665	.76493	.23507	.06335	.29841	48
13	.70180	.93658	.76522	.23478	.06342	.29820	47
14	.70202	.93650	.76551	.23449	.06350	.29798	46
15	9.70224	9.93643	9.76580	10.23420	10.06357	10.29776	45
16	.70245	.93636	.76609	.23391	.06364	.29755	44
17	.70267	.93628	.76639	.23361	.06372	.29733	43
18	.70288	.93621	.76668	.23332	.06379	.29712	42
19	.70310	.93614	.76697	.23303	.06386	.29690	41
20	9.70332	9.93606	9.76725	10.23275	10.06394	10.29668	40
21	.70353	.93599	.76754	.23246	.06401	.29647	39
22	.70375	.93591	.76783	.23217	.06409	.29625	38
23	.70396	.93584	.76812	.23188	.06416	.29604	37
24	.70418	.93577	.76841	.23159	.06423	.29582	36
25	9.70439	9.93569	9.76870	10.23130	10.06431	10.29561	35
26	.70461	.93562	.76899	.23101	.06438	.29539	34
27	.70482	.93554	.76928	.23072	.06446	.29518	33
28	.70504	.93547	.76957	.23043	.06453	.29496	32
29	.70525	.93539	.76986	.23014	.06461	.29475	31
30	9.70547	9.93532	9.77015	10.22985	10.06468	10.29453	30
31	.70568	.93525	.77044	.22956	.06475	.29432	29
32	.70590	.93517	.77073	.22927	.06483	.29410	28
33	.70611	.93510	.77101	.22899	.06490	.29389	27
34	.70633	.93502	.77130	.22870	.06498	.29367	26
35	9.70654	9.93495	9.77159	10.22841	10.06505	10.29346	25
36	.70675	.93487	.77188	.22812	.06513	.29325	24
37	.70697	.93480	.77217	.22783	.06520	.29303	23
38	.70718	.93472	.77246	.22754	.06528	.29282	22
39	.70739	.93465	.77274	.22726	.06535	.29261	21
40	9.70761	9.93457	9.77303	10.22697	10.06543	10.29239	20
41	.70782	.93450	.77332	.22668	.06550	.29218	19
42	.70803	.93442	.77361	.22639	.06558	.29197	18
43	.70824	.93435	.77390	.22610	.06565	.29176	17
44	.70846	.93427	.77418	.22582	.06573	.29154	16
45	9.70867	9.93420	9.77447	10.22553	10.06580	10.29133	15
46	.70888	.93412	.77476	.22524	.06588	.29112	14
47	.70909	.93405	.77505	.22495	.06595	.29091	13
48	.70931	.93397	.77533	.22467	.06603	.29069	12
49	.70952	.93390	.77562	.22438	.06610	.29048	11
50	9.70973	9.93382	9.77591	10.22409	10.06618	10.29027	10
51	.70994	.93375	.77619	.22381	.06625	.29006	9
52	.71015	.93367	.77648	.22352	.06633	.28985	8
53	.71036	.93360	.77677	.22323	.06640	.28964	7
54	.71058	.93352	.77706	.22294	.06648	.28942	6
55	9.71079	9.93344	9.77734	10.22266	10.06656	10.28921	5
56	.71100	.93337	.77763	.22237	.06663	.28900	4
57	.71121	.93329	.77791	.22209	.06671	.28879	3
58	.71142	.93322	.77820	.22180	.06678	.28858	2
59	.71163	.93314	.77849	.22151	.06686	.28837	1
60	9.71184	9.93307	9.77877	10.22123	10.06693	10.28816	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

120° **59°**

31° **Logarithms of Trigonometrical Functions** 148°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.71184	9.93307	9.77877	10.22123	10.06693	10.28816	60
1	.71205	.93299	.77906	.22094	.06701	.28795	59
2	.71226	.93291	.77935	.22065	.06709	.28774	58
3	.71247	.93284	.77963	.22037	.06716	.28753	57
4	.71268	.93276	.77992	.22008	.06724	.28732	56
5	9.71289	9.93269	9.78020	10.21980	10.06731	10.28711	55
6	.71310	.93261	.78049	.21951	.06739	.28690	54
7	.71331	.93253	.78077	.21923	.06747	.28669	53
8	.71352	.93246	.78106	.21894	.06754	.28648	52
9	.71373	.93238	.78135	.21865	.06762	.28627	51
10	9.71393	9.93230	9.78163	10.21837	10.06770	10.28607	50
11	.71414	.93223	.78192	.21808	.06777	.28586	49
12	.71435	.93215	.78220	.21780	.06785	.28565	48
13	.71456	.93207	.78249	.21751	.06793	.28544	47
14	.71477	.93200	.78277	.21723	.06800	.28523	46
15	9.71498	9.93192	9.78306	10.21694	10.06808	10.28502	45
16	.71519	.93184	.78334	.21666	.06816	.28481	44
17	.71539	.93177	.78363	.21637	.06823	.28461	43
18	.71560	.93169	.78391	.21609	.06831	.28440	42
19	.71581	.93161	.78419	.21581	.06839	.28419	41
20	9.71602	9.93154	9.78448	10.21552	10.06846	10.28398	40
21	.71622	.93146	.78476	.21524	.06854	.28378	39
22	.71643	.93138	.78505	.21495	.06862	.28357	38
23	.71664	.93131	.78533	.21467	.06869	.28336	37
24	.71685	.93123	.78562	.21438	.06877	.28315	36
25	9.71705	9.93115	9.78590	10.21410	10.06885	10.28295	35
26	.71726	.93108	.78618	.21382	.06892	.28274	34
27	.71747	.93100	.78647	.21353	.06900	.28253	33
28	.71767	.93092	.78675	.21325	.06908	.28233	32
29	.71788	.93084	.78704	.21296	.06916	.28212	31
30	9.71809	9.93077	9.78732	10.21268	10.06923	10.28191	30
31	.71829	.93069	.78760	.21240	.06931	.28171	29
32	.71850	.93061	.78789	.21211	.06939	.28150	28
33	.71870	.93053	.78817	.21183	.06947	.28130	27
34	.71891	.93046	.78845	.21155	.06954	.28109	26
35	9.71911	9.93038	9.78874	10.21126	10.06962	10.28089	25
36	.71932	.93030	.78902	.21098	.06970	.28068	24
37	.71952	.93022	.78930	.21070	.06978	.28048	23
38	.71973	.93014	.78959	.21041	.06986	.28027	22
39	.71994	.93007	.78987	.21013	.06993	.28006	21
40	9.72014	9.92999	9.79015	10.20985	10.07001	10.27986	20
41	.72034	.92991	.79043	.20957	.07009	.27966	19
42	.72055	.92983	.79072	.20928	.07017	.27945	18
43	.72075	.92976	.79100	.20900	.07024	.27925	17
44	.72096	.92968	.79128	.20872	.07032	.27904	16
45	9.72116	9.92960	9.79156	10.20844	10.07040	10.27884	15
46	.72137	.92952	.79185	.20815	.07048	.27863	14
47	.72157	.92944	.79213	.20787	.07056	.27843	13
48	.72177	.92936	.79241	.20759	.07064	.27823	12
49	.72198	.92929	.79269	.20731	.07071	.27802	11
50	9.72218	9.92921	9.79297	10.20703	10.07079	10.27782	10
51	.72238	.92913	.79326	.20674	.07087	.27762	9
52	.72259	.92905	.79354	.20646	.07095	.27741	8
53	.72279	.92897	.79382	.20618	.07103	.27721	7
54	.72299	.92889	.79410	.20590	.07111	.27701	6
55	9.72320	9.92881	9.79438	10.20562	10.07119	10.27680	5
56	.72340	.92874	.79466	.20534	.07126	.27660	4
57	.72360	.92866	.79495	.20505	.07134	.27640	3
58	.72381	.92858	.79523	.20477	.07142	.27619	2
59	.72401	.92850	.79551	.20449	.07150	.27599	1
60	9.72421	9.92842	9.79579	10.20421	10.07158	10.27579	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

121° 58°

32° **Logarithms of Trigonometrical Functions** **147°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.72421	9.92842	9.79579	10.20421	10.07158	10.27579	60
1	.72441	.92834	.79607	.20393	.07166	.27559	59
2	.72461	.92826	.79635	.20365	.07174	.27539	58
3	.72482	.92818	.79663	.20337	.07182	.27518	57
4	.72502	.92810	.79691	.20309	.07190	.27498	56
5	9.72522	9.92803	9.79719	10.20281	10.07197	10.27478	55
6	.72542	.92795	.79747	.20253	.07205	.27458	54
7	.72562	.92787	.79776	.20224	.07213	.27438	53
8	.72582	.92779	.79804	.20196	.07221	.27418	52
9	.72602	.92771	.79832	.20168	.07229	.27398	51
10	9.72622	9.92763	9.79860	10.20140	10.07237	10.27378	50
11	.72643	.92755	.79888	.20112	.07245	.27357	49
12	.72663	.92747	.79916	.20084	.07253	.27337	48
13	.72683	.92739	.79944	.20056	.07261	.27317	47
14	.72703	.92731	.79972	.20028	.07269	.27297	46
15	9.72723	9.92723	9.80000	10.20000	10.07277	10.27277	45
16	.72743	.92715	.80028	.19972	.07285	.27257	44
17	.72763	.92707	.80056	.19944	.07293	.27237	43
18	.72783	.92699	.80084	.19916	.07301	.27217	42
19	72803	.92691	.80112	.19888	.07309	.27197	41
20	9.72823	9.92683	9.80140	10.19860	10.07317	10.27177	40
21	.72843	.92675	.80168	.19832	.07325	.27157	39
22	.72863	.92667	.80195	.19805	.07333	.27137	38
23	.72883	.92659	.80223	.19777	.07341	.27117	37
24	.72902	.92651	.80251	.19749	.07349	.27098	36
25	9.72922	9.92643	9.80279	10.19721	10.07357	10.27078	35
26	.72942	.92635	.80307	.19693	.07365	.27058	34
27	.72962	.92627	.80335	.19665	.07373	.27038	33
28	.72982	.92619	.80363	.19637	.07381	.27018	32
29	.73002	.92611	.80391	.19609	.07389	.26998	31
30	9.73022	9.92603	9.80419	10.19581	10.07397	10.26978	30
31	.73041	.92595	.80447	.19553	.07405	26959	29
32	.73061	.92587	.80474	.19526	.07413	.26939	28
33	.73081	.92579	.80502	.19498	.07421	.26919	27
34	.73101	.92571	.80530	.19470	.07429	.26899	26
35	9.73121	9.92563	9.80558	10.19442	10.07437	10.26879	25
36	.73140	.92555	.80586	.19414	.07445	.26860	24
37	.73160	.92546	.80614	.19386	.07454	.26840	23
38	.73180	.92538	.80642	.19358	.07462	.26820	22
39	.73200	.92530	.80669	.19331	.07470	.26800	21
40	9.73219	9.92522	9.80697	10.19303	10.07478	10.26781	20
41	.73239	.92514	.80725	.19275	.07486	.26761	19
42	.73259	.92506	.80753	.19247	.07494	.26741	18
43	.73278	.92498	.80781	.19219	.07502	.26722	17
44	.73298	.92490	.80808	.19192	.07510	.26702	16
45	9.73318	9.92482	9.80836	10.19164	10.07518	10.26682	15
46	.73337	.92473	.80864	.19136	.07527	.26663	14
47	.73357	.92465	.80892	.19108	.07535	.26643	13
48	.73377	.92457	.80919	.19081	.07543	.26623	12
49	.73396	.92449	.80947	.19053	.07551	.26604	11
50	9.73416	9.92441	9.80975	10.19025	10.07559	10.26584	10
51	.73435	.92433	.81003	.18997	.07567	.26565	9
52	.73455	.92425	.81030	.18970	.07575	.26545	8
53	.73474	.92416	.81058	.18942	.07584	.26526	7
54	.73494	.92408	.81086	.18914	.07592	.26506	6
55	9.73513	9.92400	9.81113	10.18887	10.07600	10.26487	5
56	.73533	.92392	.81141	.18859	.07608	.26467	4
57	.73552	.92384	.81169	.18831	.07616	.26448	3
58	.73572	.92376	.81196	.18804	.07624	.26428	2
59	.73591	.92367	.81224	.18776	.07633	.26409	1
60	9.73611	9.92359	9.81252	10.18748	10.07641	10.26389	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

122° **57°**

33° **Logarithms of Trigonometrical Functions** **146°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.73611	9.92359	9.81252	10.18748	10.07641	10.26389	60
1	.73630	.92351	.81279	.18721	.07649	.26370	59
2	.73650	.92343	.81307	.18693	.07657	.26350	58
3	.73669	.92335	.81335	.18665	.07665	.26331	57
4	.73689	.92326	.81362	.18638	.07674	.26311	56
5	9.73708	9.92318	9.81390	10.18610	10.07682	10.26292	55
6	.73727	.92310	.81418	.18582	.07690	.26273	54
7	.73747	.92302	.81445	.18555	.07698	.26253	53
8	.73766	.92293	.81473	.18527	.07707	.26234	52
9	.73785	.92285	.81500	.18500	.07715	.26215	51
10	9.73805	9.92277	9.81528	10.18472	10.07723	10.26195	50
11	.73824	.92269	.81556	.18444	.07731	.26176	49
12	.73843	.92260	.81583	.18417	.07740	.26157	48
13	.73863	.92252	.81611	.18389	.07748	.26137	47
14	.73882	.92244	.81638	.18362	.07756	.26118	46
15	9.73901	9.92235	9.81666	10.18334	10.07765	10.26099	45
16	.73921	.92227	.81693	.18307	.07773	.26079	44
17	.73940	.92219	.81721	.18279	.07781	.26060	43
18	.73959	.92211	.81748	.18252	.07789	.26041	42
19	.73978	.92202	.81776	.18224	.07798	.26022	41
20	9.73997	9.92194	9.81803	10.18197	10.07806	10.26003	40
21	.74017	.92186	.81831	.18169	.07814	.25983	39
22	.74036	.92177	.81858	.18142	.07823	.25964	38
23	.74055	.92169	.81886	.18114	.07831	.25945	37
24	.74074	.92161	.81913	.18087	.07839	.25926	36
25	9.74093	9.92152	9.81941	10.18059	10.07848	10.25907	35
26	.74113	.92144	.81968	.18032	.07856	.25887	34
27	.74132	.92136	.81996	.18004	.07864	.25868	33
28	.74151	.92127	.82023	.17977	.07873	.25849	32
29	.74170	.92119	.82051	.17949	.07881	.25830	31
30	9.74189	9.92111	9.82078	10.17922	10.07889	10.25811	30
31	.74208	.92102	.82106	.17894	.07898	.25792	29
32	.74227	.92094	.82133	.17867	.07906	.25773	28
33	.74246	.92086	.82161	.17839	.07914	.25754	27
34	.74265	.92077	.82188	.17812	.07923	.25735	26
35	9.74284	9.92069	9.82215	10.17785	10.07931	10.25716	25
36	.74303	.92060	.82243	.17757	.07940	.25697	24
37	.74322	.92052	.82270	.17730	.07948	.25678	23
38	.74341	.92044	.82298	.17702	.07956	.25659	22
39	.74360	.92035	.82325	.17675	.07965	.25640	21
40	9.74379	9.92027	9.82352	10.17648	10.07973	10.25621	20
41	.74398	.92018	.82380	.17620	.07982	.25602	19
42	.74417	.92010	.82407	.17593	.07990	.25583	18
43	.74436	.92002	.82435	.17565	.07998	.25564	17
44	.74455	.91993	.82462	.17538	.08007	.25545	16
45	9.74474	9.91985	9.82489	10.17511	10.08015	10.25526	15
46	.74493	.91976	.82517	.17483	.08024	.25507	14
47	.74512	.91968	.82544	.17456	.08032	.25488	13
48	.74531	.91959	.82571	.17429	.08041	.25469	12
49	.74549	.91951	.82599	.17401	.08049	.25451	11
50	9.74568	9.91942	9.82626	10.17374	10.08058	10.25432	10
51	.74587	.91934	.82653	.17347	.08066	.25413	9
52	.74606	.91925	.82681	.17319	.08075	.25394	8
53	.74625	.91917	.82708	.17292	.08083	.25375	7
54	.74644	.91908	.82735	.17265	.08092	.25356	6
55	9.74662	9.91900	9.82762	10.17238	10.08100	10.25338	5
56	.74681	.91891	.82790	.17210	.08109	.25319	4
57	.74700	.91883	.82817	.17183	.08117	.25300	3
58	.74719	.91874	.82844	.17156	.08126	.25281	2
59	.74737	.91866	.82871	.17129	.08134	.25263	1
60	9.74756	9.91857	9.82899	10.17101	10.08143	10.25244	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

123° **56°**

34° **Logarithms of Trigonometrical Functions** 145°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.74756	9.91857	9.82899	10.17101	10.08143	10.25244	60
1	.74775	.91849	.82926	.17074	.08151	.25225	59
2	.74794	.91840	.82953	.17047	.08160	.25206	58
3	.74812	.91832	.82980	.17020	.08168	.25188	57
4	.74831	.91823	.83008	.16992	.08177	.25169	56
5	9.74850	9.91815	9.83035	10.16965	10.08185	10.25150	55
6	.74868	.91806	.83062	.16938	.08194	.25132	54
7	.74887	.91798	.83089	.16911	.08202	.25113	53
8	.74906	.91789	.83117	.16883	.08211	.25094	52
9	.74924	.91781	.83144	.16856	.08219	.25076	51
10	9.74943	9.91772	9.83171	10.16829	10.08228	10.25057	50
11	.74961	.91763	.83198	.16802	.08237	.25039	49
12	.74980	.91755	.83225	.16775	.08245	.25020	48
13	.74999	.91746	.83252	.16748	.08254	.25001	47
14	.75017	.91738	.83280	.16720	.08262	.24983	46
15	9.75036	9.91729	9.83307	10.16693	10.08271	10.24964	45
16	.75054	.91720	.83334	.16666	.08280	.24946	44
17	.75073	.91712	.83361	.16639	.08288	.24927	43
18	.75091	.91703	.83388	.16612	.08297	.24909	42
19	.75110	.91695	.83415	.16585	.08305	.24890	41
20	9.75128	9.91686	9.83442	10.16558	10.08314	10.24872	40
21	.75147	.91677	.83470	.16530	.08323	.24853	39
22	.75165	.91669	.83497	.16503	.08331	.24835	38
23	.75184	.91660	.83524	.16476	.08340	.24816	37
24	.75202	.91651	.83551	.16449	.08349	.24798	36
25	9.75221	9.91643	9.83578	10.16422	10.08357	10.24779	35
26	.75239	.91634	.83605	.16395	.08366	.24761	34
27	.75258	.91625	.83632	.16368	.08375	.24742	33
28	.75276	.91617	.83659	.16341	.08383	.24724	32
29	.75294	.91608	.83686	.16314	.08392	.24706	31
30	9.75313	9.91599	9.83713	10.16287	10.08401	10.24687	30
31	.75331	.91591	.83740	.16260	.08409	.24669	29
32	.75350	.91582	.83768	.16232	.08418	.24650	28
33	.75368	.91573	.83795	.16205	.08427	.24632	27
34	.75386	.91565	.83822	.16178	.08435	.24614	26
35	9.75405	9.91556	9.83849	10.16151	10.08444	10.24595	25
36	.75423	.91547	.83876	.16124	.08453	.24577	24
37	.75441	.91538	.83903	.16097	.08462	.24559	23
38	.75459	.91530	.83930	.16070	.08470	.24541	22
39	.75478	.91521	.83957	.16043	.08479	.24522	21
40	9.75496	9.91512	9.83984	10.16016	10.08488	10.24504	20
41	.75514	.91504	.84011	.15989	.08496	.24486	19
42	.75533	.91495	.84038	.15962	.08505	.24467	18
43	.75551	.91486	.84065	.15935	.08514	.24449	17
44	.75569	.91477	.84092	.15908	.08523	.24431	16
45	9.75587	9.91469	9.84119	10.15881	10.08531	10.24413	15
46	.75605	.91460	.84146	.15854	.08540	.24395	14
47	.75624	.91451	.84173	.15827	.08549	.24376	13
48	.75642	.91442	.84200	.15800	.08558	.24358	12
49	.75660	.91433	.84227	.15773	.08567	.24340	11
50	9.75678	9.91425	9.84254	10.15746	10.08575	10.24322	10
51	.75696	.91416	.84280	.15720	.08584	.24304	9
52	.75714	.91407	.84307	.15693	.08593	.24286	8
53	.75733	.91398	.84334	.15666	.08602	.24267	7
54	.75751	.91389	.84361	.15639	.08611	.24249	6
55	9.75769	9.91381	9.84388	10.15612	10.08619	10.24231	5
56	.75787	.91372	.84415	.15585	.08628	.24213	4
57	.75805	.91363	.84442	.15558	.08637	.24195	3
58	.75823	.91354	.84469	.15531	.08646	.24177	2
59	.75841	.91345	.84496	.15504	.08655	.24159	1
60	9.75859	9.91336	9.84523	10.15477	10.08664	10.24141	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

124° 55°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.75859	9.91336	9.84523	10.15477	10.08664	10.24141	60
1	.75877	.91328	.84550	.15450	.08672	.24123	59
2	.75895	.91319	.84576	.15424	.08681	.24105	58
3	.75913	.91310	.84603	.15397	.08690	.24087	57
4	.75931	.91301	.84630	.15370	.08699	.24069	56
5	9.75949	9.91292	9.84657	10.15343	10.08708	10.24051	55
6	.75967	.91283	.84684	.15316	.08717	.24033	54
7	.75985	.91274	.84711	.15289	.08726	.24015	53
8	.76003	.91266	.84738	.15262	.08734	.23997	52
9	.76021	.91257	.84764	.15236	.08743	.23979	51
10	9.76039	9.91248	9.84791	10.15209	10.08752	10.23961	50
11	.76057	.91239	.84818	.15182	.08761	.23943	49
12	.76075	.91230	.84845	.15155	.08770	.23925	48
13	.76093	.91221	.84872	.15128	.08779	.23907	47
14	.76111	.91212	.84899	.15101	.08788	.23889	46
15	9.76129	9.91203	9.84925	10.15075	10.08797	10.23871	45
16	.76146	.91194	.84952	.15048	.08806	.23854	44
17	.76164	.91185	.84979	.15021	.08815	.23836	43
18	.76182	.91176	.85006	.14994	.08824	.23818	42
19	.76200	.91167	.85033	.14967	.08833	.23800	41
20	9.76218	9.91158	9.85059	10.14941	10.08842	10.23782	40
21	.76236	.91149	.85086	.14914	.08851	.23764	39
22	.76253	.91141	.85113	.14887	.08859	.23747	38
23	.76271	.91132	.85140	.14860	.08868	.23729	37
24	.76289	.91123	.85166	.14834	.08877	.23711	36
25	9.76307	9.91114	9.85193	10.14807	10.08886	10.23693	35
26	.76324	.91105	.85220	.14780	.08895	.23676	34
27	.76342	.91096	.85247	.14753	.08904	.23658	33
28	.76360	.91087	.85273	.14727	.08913	.23640	32
29	.76378	.91078	.85300	.14700	.08922	.23622	31
30	9.76395	9.91069	9.85327	10.14673	10.08931	10.23605	30
31	.76413	.91060	.85354	.14646	.08940	.23587	29
32	.76431	.91051	.85380	.14620	.08949	.23569	28
33	.76448	.91042	.85407	.14593	.08958	.23552	27
34	.76466	.91033	.85434	.14566	.08967	.23534	26
35	9.76484	9.91023	9.85460	10.14540	10.08977	10.23516	25
36	.76501	.91014	.85487	.14513	.08986	.23499	24
37	.76519	.91005	.85514	.14486	.08995	.23481	23
38	.76537	.90996	.85540	.14460	.09004	.23463	22
39	.76554	.90987	.85567	.14433	.09013	.23446	21
40	9.76572	9.90978	9.85594	10.14406	10.09022	10.23428	20
41	.76590	.90969	.85620	.14380	.09031	.23410	19
42	.76607	.90960	.85647	.14353	.09040	.23393	18
43	.76625	.90951	.85674	.14326	.09049	.23375	17
44	.76642	.90942	.85700	.14300	.09058	.23358	16
45	9.76660	9.90933	9.85727	10.14273	10.09067	10.23340	15
46	.76677	.90924	.85754	.14246	.09076	.23323	14
47	.76695	.90915	.85780	.14220	.09085	.23305	13
48	.76712	.90906	.85807	.14193	.09094	.23288	12
49	.76730	.90896	.85834	.14166	.09104	.23270	11
50	9.76747	9.90887	9.85860	10.14140	10.09113	10.23253	10
51	.76765	.90878	.85887	.14113	.09122	.23235	9
52	.76782	.90869	.85913	.14087	.09131	.23218	8
53	.76800	.90860	.85940	.14060	.09140	.23200	7
54	.76817	.90851	.85967	.14033	.09149	.23183	6
55	9.76835	9.90842	9.85993	10.14007	10.09158	10.23165	5
56	.76852	.90832	.86020	.13980	.09168	.23148	4
57	.76870	.90823	.86046	.13954	.09177	.23130	3
58	.76887	.90814	.86073	.13927	.09186	.23113	2
59	.76904	.90805	.86100	.13900	.09195	.23096	1
60	9.76922	9.90796	9.86126	10.13874	10.09204	10.23078	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

36° **Logarithms of Trigonometrical Functions** 143°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.76922	9.90796	9.86126	10.13874	10.09204	10.23078	60
1	.76939	.90787	.86153	.13847	.09213	.23061	59
2	.76957	.90777	.86179	.13821	.09223	.23043	58
3	.76974	.90768	.86206	.13794	.09232	.23026	57
4	.76991	.90759	.86232	.13768	.09241	.23009	56
5	9.77009	9.90750	9.86259	10.13741	10.09250	10.22991	55
6	.77026	.90741	.86285	.13715	.09259	.22974	54
7	.77043	.90731	.86312	.13688	.09269	.22957	53
8	.77061	.90722	.86338	.13662	.09278	.22939	52
9	.77078	.90713	.86365	.13635	.09287	.22922	51
10	9.77095	9.90704	9.86392	10.13608	10.09296	10.22905	50
11	.77112	.90694	.86418	.13582	.09306	.22888	49
12	.77130	.90685	.86445	.13555	.09315	.22870	48
13	.77147	.90676	.86471	.13529	.09324	.22853	47
14	.77164	.90667	.86498	.13502	.09333	.22836	46
15	9.77181	9.90657	9.86524	10.13476	10.09343	10.22819	45
16	.77199	.90648	.86551	.13449	.09352	.22801	44
17	.77216	.90639	.86577	.13423	.09361	.22784	43
18	.77233	.90630	.86603	.13397	.09370	.22767	42
19	.77250	.90620	.86630	.13370	.09380	.22750	41
20	9.77268	9.90611	9.86656	10.13344	10.09389	10.22732	40
21	.77285	.90602	.86683	.13317	.09398	.22715	39
22	.77302	.90592	.86709	.13291	.09408	.22698	38
23	.77319	.90583	.86736	.13264	.09417	.22681	37
24	.77336	.90574	.86762	.13238	.09426	.22664	36
25	9.77353	9.90565	9.86789	10.13211	10.09435	10.22647	35
26	.77370	.90555	.86815	.13185	.09445	.22630	34
27	.77387	.90546	.86842	.13158	.09454	.22613	33
28	.77405	.90537	.86868	.13132	.09463	.22595	32
29	.77422	.90527	.86894	.13106	.09473	.22578	31
30	9.77439	9.90518	9.86921	10.13079	10.09482	10.22561	30
31	.77456	.90509	.86947	.13053	.09491	.22544	29
32	.77473	.90499	.86974	.13026	.09501	.22527	28
33	.77490	.90490	.87000	.13000	.09510	.22510	27
34	.77507	.90480	.87027	.12973	.09520	.22493	26
35	9.77524	9.90471	9.87053	10.12947	10.09529	10.22476	25
36	.77541	.90462	.87079	.12921	.09538	.22459	24
37	.77558	.90452	.87106	.12894	.09548	.22442	23
38	.77575	.90443	.87132	.12868	.09557	.22425	22
39	.77592	.90434	.87158	.12842	.09566	.22408	21
40	9.77609	9.90424	9.87185	10.12815	10.09576	10.22391	20
41	.77626	.90415	.87211	.12789	.09585	.22374	19
42	.77643	.90405	.87238	.12762	.09595	.22357	18
43	.77660	.90396	.87264	.12736	.09604	.22340	17
44	.77677	.90386	.87290	.12710	.09614	.22323	16
45	9.77694	9.90377	9.87317	10.12683	10.09623	10.22306	15
46	.77711	.90368	.87343	.12657	.09632	.22289	14
47	.77728	.90358	.87369	.12631	.09642	.22272	13
48	.77744	.90349	.87396	.12604	.09651	.22256	12
49	.77761	.90339	.87422	.12578	.09661	.22239	11
50	9.77778	9.90330	9.87448	10.12552	10.09670	10.22222	10
51	.77795	.90320	.87475	.12525	.09680	.22205	9
52	.77812	.90311	.87501	.12499	.09689	.22188	8
53	.77829	.90301	.87527	.12473	.09699	.22171	7
54	.77846	.90292	.87554	.12446	.09708	.22154	6
55	9.77862	9.90282	9.87580	10.12420	10.09718	10.22138	5
56	.77879	.90273	.87606	.12394	.09727	.22121	4
57	.77896	.90263	.87633	.12367	.09737	.22104	3
58	.77913	.90254	.87659	.12341	.09746	.22087	2
59	.77930	.90244	.87685	.12315	.09756	.22070	1
60	9.77946	9.90235	9.87711	10.12289	10.09765	10.22054	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

126° 53°

37° **Logarithms of Trigonometrical Functions** **142°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.77946	9.90235	9.87711	10.12289	10.09765	10.22054	60
1	.77963	.90225	.87738	.12262	.09775	.22037	59
2	.77980	.90216	.87764	.12236	.09784	.22020	58
3	.77997	.90206	.87790	.12210	.09794	.22003	57
4	.78013	.90197	.87817	.12183	.09803	.21987	56
5	9.78030	9.90187	9.87843	10.12157	10.09813	10.21970	55
6	.78047	.90178	.87869	.12131	.09822	.21953	54
7	.78063	.90168	.87895	.12105	.09832	.21937	53
8	.78080	.90159	.87922	.12078	.09841	.21920	52
9	.78097	.90149	.87948	.12052	.09851	.21903	51
10	9.78113	9.90139	9.87974	10.12026	10.09861	10.21887	50
11	.78130	.90130	.88000	.12000	.09870	.21870	49
12	.78147	.90120	.88027	.11973	.09880	.21853	48
13	.78163	.90111	.88053	.11947	.09889	.21837	47
14	.78180	.90101	.88079	.11921	.09899	.21820	46
15	9.78197	9.90091	9.88105	10.11895	10.09909	10.21803	45
16	.78213	.90082	.88131	.11869	.09918	.21787	44
17	.78230	.90072	.88158	.11842	.09928	.21770	43
18	.78246	.90063	.88184	.11816	.09937	.21754	42
19	.78263	.90053	.88210	.11790	.09947	.21737	41
20	9.78280	9.90043	9.88236	10.11764	10.09957	10.21720	40
21	.78296	.90034	.88262	.11738	.09966	.21704	39
22	.78313	.90024	.88289	.11711	.09976	.21687	38
23	.78329	.90014	.88315	.11685	.09986	.21671	37
24	.78346	.90005	.88341	.11659	.09995	.21654	36
25	9.78362	9.89995	9.88367	10.11633	10.10005	10.21638	35
26	.78379	.89985	.88393	.11607	.10015	.21621	34
27	.78395	.89976	.88420	.11580	.10024	.21605	33
28	.78412	.89966	.88446	.11554	.10034	.21588	32
29	.78428	.89956	.88472	.11528	.10044	.21572	31
30	9.78445	9.89947	9.88498	10.11502	10.10053	10.21555	30
31	.78461	.89937	.88524	.11476	.10063	.21539	29
32	.78478	.89927	.88550	.11450	.10073	.21522	28
33	.78494	.89918	.88577	.11423	.10082	.21506	27
34	.78510	.89908	.88603	.11397	.10092	.21490	26
35	9.78527	9.89898	9.88629	10.11371	10.10102	10.21473	25
36	.78543	.89888	.88655	.11345	.10112	.21457	24
37	.78560	.89879	.88681	.11319	.10121	.21440	23
38	.78576	.89869	.88707	.11293	.10131	.21424	22
39	.78592	.89859	.88733	.11267	.10141	.21408	21
40	9.78609	9.89849	9.88759	10.11241	10.10151	10.21391	20
41	.78625	.89840	.88786	.11214	.10160	.21375	19
42	.78642	.89830	.88812	.11188	.10170	.21358	18
43	.78658	.89820	.88838	.11162	.10180	.21342	17
44	.78674	.89810	.88864	.11136	.10190	.21326	16
45	9.78691	9.89801	9.88890	10.11110	10.10199	10.21309	15
46	.78707	.89791	.88916	.11084	.10209	.21293	14
47	.78723	.89781	.88942	.11058	.10219	.21277	13
48	.78739	.89771	.88968	.11032	.10229	.21261	12
49	.78756	.89761	.88994	.11006	.10239	.21244	11
50	9.78772	9.89752	9.89020	10.10980	10.10248	10.21228	10
51	.78788	.89742	.89046	.10954	.10258	.21212	9
52	.78805	.89732	.89073	.10927	.10268	.21195	8
53	.78821	.89722	.89099	.10901	.10278	.21179	7
54	.78837	.89712	.89125	.10875	.10288	.21163	6
55	9.78853	9.89702	9.89151	10.10849	10.10298	10.21147	5
56	.78869	.89693	.89177	.10823	.10307	.21131	4
57	.78886	.89683	.89203	.10797	.10317	.21114	3
58	.78902	.89673	.89229	.10771	.10327	.21098	2
59	.78918	.89663	.89255	.10745	.10337	.21082	1
60	9.78934	9.89653	9.89281	10.10719	10.10347	10.21066	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

127° **52°**

38° **Logarithms of Trigonometrical Functions** 141°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.78934	9.89653	9.89281	10.10719	10.10347	10.21066	60
1	.78950	.89643	.89307	.10693	.10357	.21050	59
2	.78967	.89633	.89333	.10667	.10367	.21033	58
3	.78983	.89624	.89359	.10641	.10376	.21017	57
4	.78999	.89614	.89385	.10615	.10386	.21001	56
5	9.79015	9.89604	9.89411	10.10589	10.10396	10.20985	55
6	.79031	.89594	.89437	.10563	.10406	.20969	54
7	.79047	.89584	.89463	.10537	.10416	.20953	53
8	.79063	.89574	.89489	.10511	.10426	.20937	52
9	.79079	.89564	.89515	.10485	.10436	.20921	51
10	9.79095	9.89554	9.89541	10.10459	10.10446	10.20905	50
11	.79111	.89544	.89567	.10433	.10456	.20889	49
12	.79128	.89534	.89593	.10407	.10466	.20872	48
13	.79144	.89524	.89619	.10381	.10476	.20856	47
14	.79160	.89514	.89645	.10355	.10486	.20840	46
15	9.79176	9.89504	9.89671	10.10329	10.10496	10.20824	45
16	.79192	.89495	.89697	.10303	.10505	.20808	44
17	.79208	.89485	.89723	.10277	.10515	.20792	43
18	.79224	.89475	.89749	.10251	.10525	.20776	42
19	.79240	.89465	.89775	.10225	.10535	.20760	41
20	9.79256	9.89455	9.89801	10.10199	10.10545	10.20744	40
21	.79272	.89445	.89827	.10173	.10555	.20728	39
22	.79288	.89435	.89853	.10147	.10565	.20712	38
23	.79304	.89425	.89879	.10121	.10575	.20696	37
24	.79319	.89415	.89905	.10095	.10585	.20681	36
25	9.79335	9.89405	9.89931	10.10069	10.10595	10.20665	35
26	.79351	.89395	.89957	.10043	.10605	.20649	34
27	.79367	.89385	.89983	.10017	.10615	.20633	33
28	.79383	.89375	.90009	.09991	.10625	.20617	32
29	.79399	.89364	.90035	.09965	.10636	.20601	31
30	9.79415	9.89354	9.90061	10.09939	10.10646	10.20585	30
31	.79431	.89344	.90086	.09914	.10656	.20569	29
32	.79447	.89334	.90112	.09888	.10666	.20553	28
33	.79463	.89324	.90138	.09862	.10676	.20537	27
34	.79478	.89314	.90164	.09836	.10686	.20522	26
35	9.79494	9.89304	9.90190	10.09810	10.10696	10.20506	25
36	.79510	.89294	.90216	.09784	.10706	.20490	24
37	.79526	.89284	.90242	.09758	.10716	.20474	23
38	.79542	.89274	.90268	.09732	.10726	.20458	22
39	.79558	.89264	.90294	.09706	.10736	.20442	21
40	9.79573	9.89254	9.90320	10.09680	10.10746	10.20427	20
41	.79589	.89244	.90346	.09654	.10756	.20411	19
42	.79605	.89233	.90371	.09629	.10767	.20395	18
43	.79621	.89223	.90397	.09603	.10777	.20379	17
44	.79636	.89213	.90423	.09577	.10787	.20364	16
45	9.79652	9.89203	9.90449	10.09551	10.10797	10.20348	15
46	.79668	.89193	.90475	.09525	.10807	.20332	14
47	.79684	.89183	.90501	.09499	.10817	.20316	13
48	.79699	.89173	.90527	.09473	.10827	.20301	12
49	.79715	.89162	.90553	.09447	.10838	.20285	11
50	9.79731	9.89152	9.90578	10.09422	10.10848	10.20269	10
51	.79746	.89142	.90604	.09396	.10858	.20254	9
52	.79762	.89132	.90630	.09370	.10868	.20238	8
53	.79778	.89122	.90656	.09344	.10878	.20222	7
54	.79793	.89112	.90682	.09318	.10888	.20207	6
55	9.79809	9.89101	9.90708	10.09292	10.10899	10.20191	5
56	.79825	.89091	.90734	.09266	.10909	.20175	4
57	.79840	.89081	.90759	.09241	.10919	.20160	3
58	.79856	.89071	.90785	.09215	.10929	.20144	2
59	.79872	.89060	.90811	.09189	.10940	.20128	1
60	9.79887	9.89050	9.90837	10.09163	10.10950	10.20113	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

128° 51°

39° **Logarithms of Trigonometrical Functions** 140°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.79887	9.89050	9.90837	10.09163	10.10950	10.20113	60
1	.79903	.89040	.90863	.09137	.10960	.20097	59
2	.79918	.89030	.90889	.09111	.10970	.20082	58
3	.79934	.89020	.90914	.09086	.10980	.20066	57
4	.79950	.89009	.90940	.09060	.10991	.20050	56
5	9.79965	9.88999	9.90966	10.09034	10.11001	10.20035	55
6	.79981	.88989	.90992	.09008	.11011	.20019	54
7	.79996	.88978	.91018	.08982	.11022	.20004	53
8	.80012	.88968	.91043	.08957	.11032	.19988	52
9	.80027	.88958	.91069	.08931	.11042	.19973	51
10	9.80043	9.88948	9.91095	10.08905	10.11052	10.19957	50
11	.80058	.88937	.91121	.08879	.11063	.19942	49
12	.80074	.88927	.91147	.08853	.11073	.19926	48
13	.80089	.88917	.91172	.08828	.11083	.19911	47
14	.80105	.88906	.91198	.08802	.11094	.19895	46
15	9.80120	9.88896	9.91224	10.08776	10.11104	10.19880	45
16	.80136	.88886	.91250	.08750	.11114	.19864	44
17	.80151	.88875	.91276	.08724	.11125	.19849	43
18	.80166	.88865	.91301	.08699	.11135	.19834	42
19	.80182	.88855	.91327	.08673	.11145	.19818	41
20	9.80197	9.88844	9.91353	10.08647	10.11156	10.19803	40
21	.80213	.88834	.91379	.08621	.11166	.19787	39
22	.80228	.88824	.91404	.08596	.11176	.19772	38
23	.80244	.88813	.91430	.08570	.11187	.19756	37
24	.80259	.88803	.91456	.08544	.11197	.19741	36
25	9.80274	9.88793	9.91482	10.08518	10.11207	10.19726	35
26	.80290	.88782	.91507	.08493	.11218	.19710	34
27	.80305	.88772	.91533	.08467	.11228	.19695	33
28	.80320	.88761	.91559	.08441	.11239	.19680	32
29	.80336	.88751	.91585	.08415	.11249	.19664	31
30	9.80351	9.88741	9.91610	10.08390	10.11259	10.19649	30
31	.80366	.88730	.91636	.08364	.11270	.19634	29
32	.80382	.88720	.91662	.08338	.11280	.19618	28
33	.80397	.88709	.91688	.08312	.11291	.19603	27
34	.80412	.88699	.91713	.08287	.11301	.19588	26
35	9.80428	9.88688	9.91739	10.08261	10.11312	10.19572	25
36	.80443	.88678	.91765	.08235	.11322	.19557	24
37	.80458	.88668	.91791	.08209	.11332	.19542	23
38	.80473	.88657	.91816	.08184	.11343	.19527	22
39	.80489	.88647	.91842	.08158	.11353	.19511	21
40	9.80504	9.88636	9.91868	10.08132	10.11364	10.19496	20
41	.80519	.88626	.91893	.08107	.11374	.19481	19
42	.80534	.88615	.91919	.08081	.11385	.19466	18
43	.80550	.88605	.91945	.08055	.11395	.19450	17
44	.80565	.88594	.91971	.08029	.11406	.19435	16
45	9.80580	9.88584	9.91996	10.08004	10.11416	10.19420	15
46	.80595	.88573	.92022	.07978	.11427	.19405	14
47	.80610	.88563	.92048	.07952	.11437	.19390	13
48	.80625	.88552	.92073	.07927	.11448	.19375	12
49	.80641	.88542	.92099	.07901	.11458	.19359	11
50	9.80656	9.88531	9.92125	10.07875	10.11469	10.19344	10
51	.80671	.88521	.92150	.07850	.11479	.19329	9
52	.80686	.88510	.92176	.07824	.11490	.19314	8
53	.80701	.88499	.92202	.07798	.11501	.19299	7
54	.80716	.88489	.92227	.07773	.11511	.19284	6
55	9.80731	9.88478	9.92253	10.07747	10.11522	10.19269	5
56	.80746	.88468	.92279	.07721	.11532	.19254	4
57	.80762	.88457	.92304	.07696	.11543	.19238	3
58	.80777	.88447	.92330	.07670	.11553	.19223	2
59	.80792	.88436	.92356	.07644	.11564	.19208	1
60	9.80807	9.88425	9.92381	10.07619	10.11575	10.19193	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

129° 50°

40° **Logarithms of Trigonometrical Functions** **139°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.80807	9.88425	9.92381	10.07619	10.11575	10.19193	60
1	.80822	.88415	.92407	.07593	.11585	.19178	59
2	.80837	.88404	.92433	.07567	.11596	.19163	58
3	.80852	.88394	.92458	.07542	.11606	.19148	57
4	.80867	.88383	.92484	.07516	.11617	.19133	56
5	9.80882	9.88372	9.92510	10.07490	10.11628	10.19118	55
6	.80897	.88362	.92535	.07465	.11638	.19103	54
7	.80912	.88351	.92561	.07439	.11649	.19088	53
8	.80927	.88340	.92587	.07413	.11660	.19073	52
9	.80942	.88330	.92612	.07388	.11670	.19058	51
10	9.80957	9.88319	9.92638	10.07362	10.11681	10.19043	50
11	.80972	.88308	.92663	.07337	.11692	.19028	49
12	.80987	.88298	.92689	.07311	.11702	.19013	48
13	.81002	.88287	.92715	.07285	.11713	.18998	47
14	.81017	.88276	.92740	.07260	.11724	.18983	46
15	9.81032	9.88266	9.92766	10.07234	10.11734	10.18968	45
16	.81047	.88255	.92792	.07208	.11745	.18953	44
17	.81061	.88244	.92817	.07183	.11756	.18939	43
18	.81076	.88234	.92843	.07157	.11766	.18924	42
19	.81091	.88223	.92868	.07132	.11777	.18909	41
20	9.81106	9.88212	9.92894	10.07106	10.11788	10.18894	40
21	.81121	.88201	.92920	.07080	.11799	.18879	39
22	.81136	.88191	.92945	.07055	.11809	.18864	38
23	.81151	.88180	.92971	.07029	.11820	.18849	37
24	.81166	.88169	.92996	.07004	.11831	.18834	36
25	9.81180	9.88158	9.93022	10.06978	10.11842	10.18820	35
26	.81195	.88148	.93048	.06952	.11852	.18805	34
27	.81210	.88137	.93073	.06927	.11863	.18790	33
28	.81225	.88126	.93099	.06901	.11874	.18775	32
29	.81240	.88115	.93124	.06876	.11885	.18760	31
30	9.81254	9.88105	9.93150	10.06850	10.11895	10.18746	30
31	.81269	.88094	.93175	.06825	.11906	.18731	29
32	.81284	.88083	.93201	.06799	.11917	.18716	28
33	.81299	.88072	.93227	.06773	.11928	.18701	27
34	.81314	.88061	.93252	.06748	.11939	.18686	26
35	9.81328	9.88051	9.93278	10.06722	10.11949	10.18672	25
36	.81343	.88040	.93303	.06697	.11960	.18657	24
37	.81358	.88029	.93329	.06671	.11971	.18642	23
38	.81372	.88018	.93354	.06646	.11982	.18628	22
39	.81387	.88007	.93380	.06620	.11993	.18613	21
40	9.81402	9.87996	9.93406	10.06594	10.12004	10.18598	20
41	.81417	.87985	.93431	.06569	.12015	.18583	19
42	.81431	.87975	.93457	.06543	.12025	.18569	18
43	.81446	.87964	.93482	.06518	.12036	.18554	17
44	.81461	.87953	.93508	.06492	.12047	.18539	16
45	9.81475	9.87942	9.93533	10.06467	10.12058	10.18525	15
46	.81490	.87931	.93559	.06441	.12069	.18510	14
47	.81505	.87920	.93584	.06416	.12080	.18495	13
48	.81519	.87909	.93610	.06390	.12091	.18481	12
49	.81534	.87898	.93636	.06364	.12102	.18466	11
50	9.81549	9.87887	9.93661	10.06339	10.12113	10.18451	10
51	.81563	.87877	.93687	.06313	.12123	.18437	9
52	.81578	.87866	.93712	.06288	.12134	.18422	8
53	.81592	.87855	.93738	.06262	.12145	.18408	7
54	.81607	.87844	.93763	.06237	.12156	.18393	6
55	9.81622	9.87833	9.93789	10.06211	10.12167	10.18378	5
56	.81636	.87822	.93814	.06186	.12178	.18364	4
57	.81651	.87811	.93840	.06160	.12189	.18349	3
58	.81665	.87800	.93865	.06135	.12200	.18335	2
59	.81680	.87789	.93891	.06109	.12211	.18320	1
60	9.81694	9.87778	9.93916	10.06084	10.12222	10.18306	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

130° **49°**

41° **Logarithms of Trigonometrical Functions** 138°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.81694	9.87778	9.93916	10.06084	10.12222	10.18306	60
1	.81709	.87767	.93942	.06058	.12233	.18291	59
2	.81723	.87756	.93967	.06033	.12244	.18277	58
3	.81738	.87745	.93993	.06007	.12255	.18262	57
4	.81752	.87734	.94018	.05982	.12266	.18248	56
5	9.81767	9.87723	9.94044	10.05956	10.12277	10.18233	55
6	.81781	.87712	.94069	.05931	.12288	.18219	54
7	.81796	.87701	.94095	.05905	.12299	.18204	53
8	.81810	.87690	.94120	.05880	.12310	.18190	52
9	.81825	.87679	.94146	.05854	.12321	.18175	51
10	9.81839	9.87668	9.94171	10.05829	10.12332	10.18161	50
11	.81854	.87657	.94197	.05803	.12343	.18146	49
12	.81868	.87646	.94222	.05778	.12354	.18132	48
13	.81882	.87635	.94248	.05752	.12365	.18118	47
14	.81897	.87624	.94273	.05727	.12376	.18103	46
15	9.81911	9.87613	9.94299	10.05701	10.12387	10.18089	45
16	.81926	.87601	.94324	.05676	.12399	.18074	44
17	.81940	.87590	.94350	.05650	.12410	.18060	43
18	.81955	.87579	.94375	.05625	.12421	.18045	42
19	.81969	.87568	.94401	.05599	.12432	.18031	41
20	9.81983	9.87557	9.94426	10.05574	10.12443	10.18017	40
21	.81998	.87546	.94452	.05548	.12454	.18002	39
22	.82012	.87535	.94477	.05523	.12465	.17988	38
23	.82026	.87524	.94503	.05497	.12476	.17974	37
24	.82041	.87513	.94528	.05472	.12487	.17959	36
25	9.82055	9.87501	9.94554	10.05446	10.12499	10.17945	35
26	.82069	.87490	.94579	.05421	.12510	.17931	34
27	.82084	.87479	.94604	.05396	.12521	.17916	33
28	.82098	.87468	.94630	.05370	.12532	.17902	32
29	.82112	.87457	.94655	.05345	.12543	.17888	31
30	9.82126	9.87446	9.94681	10.05319	10.12554	10.17874	30
31	.82141	.87434	.94706	.05294	.12566	.17859	29
32	.82155	.87423	.94732	.05268	.12577	.17845	28
33	.82169	.87412	.94757	.05243	.12588	.17831	27
34	.82184	.87401	.94783	.05217	.12599	.17816	26
35	9.82198	9.87390	9.94808	10.05192	10.12610	10.17802	25
36	.82212	.87378	.94834	.05166	.12622	.17788	24
37	.82226	.87367	.94859	.05141	.12633	.17774	23
38	.82240	.87356	.94884	.05116	.12644	.17760	22
39	.82255	.87345	.94910	.05090	.12655	.17745	21
40	9.82269	9.87334	9.94935	10.05065	10.12666	10.17731	20
41	.82283	.87322	.94961	.05039	.12678	.17717	19
42	.82297	.87311	.94986	.05014	.12689	.17703	18
43	.82311	.87300	.95012	.04988	.12700	.17689	17
44	.82326	.87288	.95037	.04963	.12712	.17674	16
45	9.82340	9.87277	9.95062	10.04938	10.12723	10.17660	15
46	.82354	.87266	.95088	.04912	.12734	.17646	14
47	.82368	.87255	.95113	.04887	.12745	.17632	13
48	.82382	.87243	.95139	.04861	.12757	.17618	12
49	.82396	.87232	.95164	.04836	.12768	.17604	11
50	9.82410	9.87221	9.95190	10.04810	10.12779	10.17590	10
51	.82424	.87209	.95215	.04785	.12791	.17576	9
52	.82439	.87198	.95240	.04760	.12802	.17561	8
53	.82453	.87187	.95266	.04734	.12813	.17547	7
54	.82467	.87175	.95291	.04709	.12825	.17533	6
55	9.82481	9.87164	9.95317	10.04683	10.12836	10.17519	5
56	.82495	.87153	.95342	.04658	.12847	.17505	4
57	.82509	.87141	.95368	.04632	.12859	.17491	3
58	.82523	.87130	.95393	.04607	.12870	.17477	2
59	.82537	.87119	.95418	.04582	.12881	.17463	1
60	9.82551	9.87107	9.95444	10.04556	10.12893	10.17449	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

131° 48°

42° Logarithms of Trigonometrical Functions **137°**

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.82551	9.87107	9.95444	10.04556	10.12893	10.17449	60
1	.82565	.87096	.95469	.04531	.12904	.17435	59
2	.82579	.87085	.95495	.04505	.12915	.17421	58
3	.82593	.87073	.95520	.04480	.12927	.17407	57
4	.82607	.87062	.95545	.04455	.12938	.17393	56
5	9.82621	9.87050	9.95571	10.04429	10.12950	10.17379	55
6	.82635	.87039	.95596	.04404	.12961	.17365	54
7	.82649	.87028	.95622	.04378	.12972	.17351	53
8	.82663	.87016	.95647	.04353	.12984	.17337	52
9	.82677	.87005	.95672	.04328	.12995	.17323	51
10	9.82691	9.86993	9.95698	10.04302	10.13007	10.17309	50
11	.82705	.86982	.95723	.04277	.13018	.17295	49
12	.82719	.86970	.95748	.04252	.13030	.17281	48
13	.82733	.86959	.95774	.04226	.13041	.17267	47
14	.82747	.86947	.95799	.04201	.13053	.17253	46
15	9.82761	9.86936	9.95825	10.04175	10.13064	10.17239	45
16	.82775	.86924	.95850	.04150	.13076	.17225	44
17	.82788	.86913	.95875	.04125	.13087	.17212	43
18	.82802	.86902	.95901	.04099	.13098	.17198	42
19	.82816	.86890	.95926	.04074	.13110	.17184	41
20	9.82830	9.86879	9.95952	10.04048	10.13121	10.17170	40
21	.82844	.86867	.95977	.04023	.13133	.17156	39
22	.82858	.86855	.96002	.03998	.13145	.17142	38
23	.82872	.86844	.96028	.03972	.13156	.17128	37
24	.82885	.86832	.96053	.03947	.13168	.17115	36
25	9.82899	9.86821	9.96078	10.03922	10.13179	10.17101	35
26	.82913	.86809	.96104	.03896	.13191	.17087	34
27	.82927	.86798	.96129	.03871	.13202	.17073	33
28	.82941	.86786	.96155	.03845	.13214	.17059	32
29	.82955	.86775	.96180	.03820	.13225	.17045	31
30	9.82968	9.86763	9.96205	10.03795	10.13237	10.17032	30
31	.82982	.86752	.96231	.03769	.13248	.17018	29
32	.82996	.86740	.96256	.03744	.13260	.17004	28
33	.83010	.86728	.96281	.03719	.13272	.16990	27
34	.83023	.86717	.96307	.03693	.13283	.16977	26
35	9.83037	9.86705	9.96332	10.03668	10.13295	10.16963	25
36	.83051	.86694	.96357	.03643	.13306	.16949	24
37	.83065	.86682	.96383	.03617	.13318	.16935	23
38	.83078	.86670	.96408	.03592	.13330	.16922	22
39	.83092	.86659	.96433	.03567	.13341	.16908	21
40	9.83106	9.86647	9.96459	10.03541	10.13353	10.16894	20
41	.83120	.86635	.96484	.03516	.13365	.16880	19
42	.83133	.86624	.96510	.03490	.13376	.16867	18
43	.83147	.86612	.96535	.03465	.13388	.16853	17
44	.83161	.86600	.96560	.03440	.13400	.16839	16
45	9.83174	9.86589	9.96586	10.03414	10.13411	10.16826	15
46	.83188	.86577	.96611	.03389	.13423	.16812	14
47	.83202	.86565	.96636	.03364	.13435	.16798	13
48	.83215	.86554	.96662	.03338	.13446	.16785	12
49	.83229	.86542	.96687	.03313	.13458	.16771	11
50	9.83242	9.86530	9.96712	10.03288	10.13470	10.16758	10
51	.83256	.86518	.96738	.03262	.13482	.16744	9
52	.83270	.86507	.96763	.03237	.13493	.16730	8
53	.83283	.86495	.96788	.03212	.13505	.16717	7
54	.83297	.86483	.96814	.03186	.13517	.16703	6
55	9.83310	9.86472	9.96839	10.03161	10.13528	10.16690	5
56	.83324	.86460	.96864	.03136	.13540	.16676	4
57	.83338	.86448	.96890	.03110	.13552	.16662	3
58	.83351	.86436	.96915	.03085	.13564	.16649	2
59	.83365	.86425	.96940	.03060	.13575	.16635	1
60	9.83378	9.86413	9.96966	10.03034	10.13587	10.16622	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

132° **47°**

43° **Logarithms of Trigonometrical Functions** 136°

M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.83378	9.86413	9.96966	10.03034	10.13587	10.16622	60
1	.83392	.86401	.96991	.03009	.13599	.16608	59
2	.83405	.86389	.97016	.02984	.13611	.16595	58
3	.83419	.86377	.97042	.02958	.13623	.16581	57
4	.83432	.86366	.97067	.02933	.13634	.16568	56
5	9.83446	9.86354	9.97092	10.02908	10.13646	10.16554	55
6	.83459	.86342	.97118	.02882	.13658	.16541	54
7	.83473	.86330	.97143	.02857	.13670	.16527	53
8	.83486	.86318	.97168	.02832	.13682	.16514	52
9	.83500	.86306	.97193	.02807	.13694	.16500	51
10	9.83513	9.86295	9.97219	10.02781	10.13705	10.16487	50
11	.83527	.86283	.97244	.02756	.13717	.16473	49
12	.83540	.86271	.97269	.02731	.13729	.16460	48
13	.83554	.86259	.97295	.02705	.13741	.16446	47
14	.83567	.86247	.97320	.02680	.13753	.16433	46
15	9.83581	9.86235	9.97345	10.02655	10.13765	10.16419	45
16	.83594	.86223	.97371	.02629	.13777	.16406	44
17	.83608	.86211	.97396	.02604	.13789	.16392	43
18	.83621	.86200	.97421	.02579	.13800	.16379	42
19	.83634	.86188	.97447	.02553	.13812	.16366	41
20	9.83648	9.86176	9.97472	10.02528	10.13824	10.16352	40
21	.83661	.86164	.97497	.02503	.13836	.16339	39
22	.83674	.86152	.97523	.02477	.13848	.16326	38
23	.83688	.86140	.97548	.02452	.13860	.16312	37
24	.83701	.86128	.97573	.02427	.13872	.16299	36
25	9.83715	9.86116	9.97598	10.02402	10.13884	10.16285	35
26	.83728	.86104	.97624	.02376	.13896	.16272	34
27	.83741	.86092	.97649	.02351	.13908	.16259	33
28	.83755	.86080	.97674	.02326	.13920	.16245	32
29	.83768	.86068	.97700	.02300	.13932	.16232	31
30	9.83781	9.86056	9.97725	10.02275	10.13944	10.16219	30
31	.83795	.86044	.97750	.02250	.13956	.16205	29
32	.83808	.86032	.97776	.02224	.13968	.16192	28
33	.83821	.86020	.97801	.02199	.13980	.16179	27
34	.83834	.86008	.97826	.02174	.13992	.16166	26
35	9.83848	9.85996	9.97851	10.02149	10.14004	10.16152	25
36	.83861	.85984	.97877	.02123	.14016	.16139	24
37	.83874	.85972	.97902	.02098	.14028	.16126	23
38	.83887	.85960	.97927	.02073	.14040	.16113	22
39	.83901	.85948	.97953	.02047	.14052	.16099	21
40	9.83914	9.85936	9.97978	10.02022	10.14064	10.16086	20
41	.83927	.85924	.98003	.01997	.14076	.16073	19
42	.83940	.85912	.98029	.01971	.14088	.16060	18
43	.83954	.85900	.98054	.01946	.14100	.16046	17
44	.83967	.85888	.98079	.01921	.14112	.16033	16
45	9.83980	9.85876	9.98104	10.01896	10.14124	10.16020	15
46	.83993	.85864	.98130	.01870	.14136	.16007	14
47	.84006	.85851	.98155	.01845	.14149	.15994	13
48	.84020	.85839	.98180	.01820	.14161	.15980	12
49	.84033	.85827	.98206	.01794	.14173	.15967	11
50	9.84046	9.85815	9.98231	10.01769	10.14185	10.15954	10
51	.84059	.85803	.98256	.01744	.14197	.15941	9
52	.84072	.85791	.98281	.01719	.14209	.15928	8
53	.84085	.85779	.98307	.01693	.14221	.15915	7
54	.84098	.85766	.98332	.01668	.14234	.15902	6
55	9.84112	9.85754	9.98357	10.01643	10.14246	10.15888	5
56	.84125	.85742	.98383	.01617	.14258	.15875	4
57	.84138	.85730	.98408	.01592	.14270	.15862	3
58	.84151	.85718	.98433	.01567	.14282	.15849	2
59	.84164	.85706	.98458	.01542	.14294	.15836	1
60	9.84177	9.85693	9.98484	10.01516	10.14307	10.15823	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

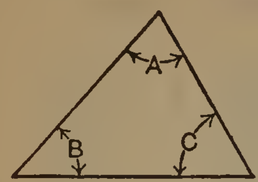
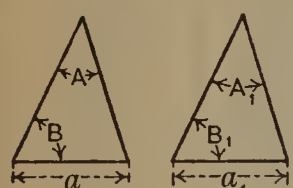
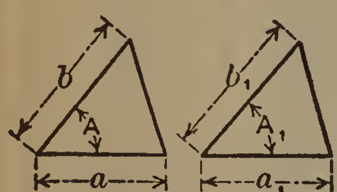
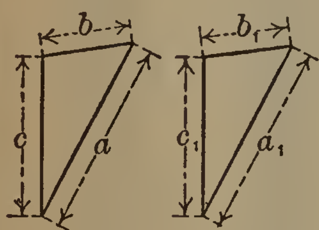
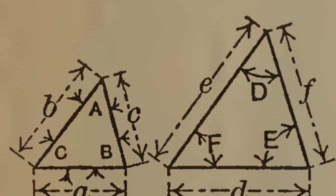
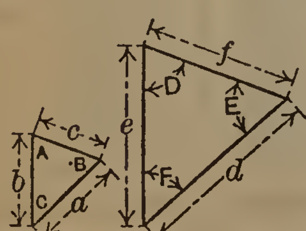
133° 46°

44° **Logarithms of Trigonometrical Functions** 135°

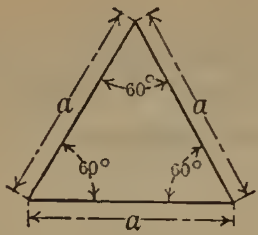
M	Sine	Cosine	Tangent	Cotangent	Secant	Cosecant	M
0	9.84177	9.85693	9.98484	10.01516	10.14307	10.15823	60
1	.84190	.85681	.98509	.01491	.14319	.15810	59
2	.84203	.85669	.98534	.01466	.14331	.15797	58
3	.84216	.85657	.98560	.01440	.14343	.15784	57
4	.84229	.85645	.98585	.01415	.14355	.15771	56
5	9.84242	9.85632	9.98610	10.01390	10.14368	10.15758	55
6	.84255	.85620	.98635	.01365	.14380	.15745	54
7	.84269	.85608	.98661	.01339	.14392	.15731	53
8	.84282	.85596	.98686	.01314	.14404	.15718	52
9	.84295	.85583	.98711	.01289	.14417	.15705	51
10	9.84308	9.85571	9.98737	10.01263	10.14429	10.15692	50
11	.84321	.85559	.98762	.01238	.14441	.15679	49
12	.84334	.85547	.98787	.01213	.14453	.15666	48
13	.84347	.85534	.98812	.01188	.14466	.15653	47
14	.84360	.85522	.98838	.01162	.14478	.15640	46
15	9.84373	9.85510	9.98863	10.01137	10.14490	10.15627	45
16	.84385	.85497	.98888	.01112	.14503	.15615	44
17	.84398	.85485	.98913	.01087	.14515	.15602	43
18	.84411	.85473	.98939	.01061	.14527	.15589	42
19	.84424	.85460	.98964	.01036	.14540	.15576	41
20	9.84437	9.85448	9.98989	10.01011	10.14552	10.15563	40
21	.84450	.85436	.99015	.00985	.14564	.15550	39
22	.84463	.85423	.99040	.00960	.14577	.15537	38
23	.84476	.85411	.99065	.00935	.14589	.15524	37
24	.84489	.85399	.99090	.00910	.14601	.15511	36
25	9.84502	9.85386	9.99116	10.00884	10.14614	10.15498	35
26	.84515	.85374	.99141	.00859	.14626	.15485	34
27	.84528	.85361	.99166	.00834	.14639	.15472	33
28	.84540	.85349	.99191	.00809	.14651	.15460	32
29	.84553	.85337	.99217	.00783	.14663	.15447	31
30	9.84566	9.85324	9.99242	10.00758	10.14676	10.15434	30
31	.84579	.85312	.99267	.00733	.14688	.15421	29
32	.84592	.85299	.99293	.00707	.14701	.15408	28
33	.84605	.85287	.99318	.00682	.14713	.15395	27
34	.84618	.85274	.99343	.00657	.14726	.15382	26
35	9.84630	9.85262	9.99368	10.00632	10.14738	10.15370	25
36	.84643	.85250	.99394	.00606	.14750	.15357	24
37	.84656	.85237	.99419	.00581	.14763	.15344	23
38	.84669	.85225	.99444	.00556	.14775	.15331	22
39	.84682	.85212	.99469	.00531	.14788	.15318	21
40	9.84694	9.85200	9.99495	10.00505	10.14800	10.15306	20
41	.84707	.85187	.99520	.00480	.14813	.15293	19
42	.84720	.85175	.99545	.00455	.14825	.15280	18
43	.84733	.85162	.99570	.00430	.14838	.15267	17
44	.84745	.85150	.99596	.00404	.14850	.15255	16
45	9.84758	9.85137	9.99621	10.00379	10.14863	10.15242	15
46	.84771	.85125	.99646	.00354	.14875	.15229	14
47	.84784	.85112	.99672	.00328	.14888	.15216	13
48	.84796	.85100	.99697	.00303	.14900	.15204	12
49	.84809	.85087	.99722	.00278	.14913	.15191	11
50	9.84822	9.85074	9.99747	10.00253	10.14926	10.15178	10
51	.84835	.85062	.99773	.00227	.14938	.15165	9
52	.84847	.85049	.99798	.00202	.14951	.15153	8
53	.84860	.85037	.99823	.00177	.14963	.15140	7
54	.84873	.85024	.99848	.00152	.14976	.15127	6
55	9.84885	9.85012	9.99874	10.00126	10.14988	10.15115	5
56	.84898	.84999	.99899	.00101	.15001	.15102	4
57	.84911	.84986	.99924	.00076	.15014	.15089	3
58	.84923	.84974	.99949	.00051	.15026	.15077	2
59	.84936	.84961	.99975	.00025	.15039	.15064	1
60	9.84949	9.84949	10.00000	10.00000	10.15051	10.15051	0
M	Cosine	Sine	Cotangent	Tangent	Cosecant	Secant	M

134° 45°

Geometrical Propositions

	The sum of the three angles in a triangle always equals 180 degrees. Hence, if two angles are known, the third angle can always be found. $A + B + C = 180°$ $\quad A = 180° - (B + C)$ $B = 180° - (A + C)$ $\quad C = 180° - (A + B)$
	If one side and two angles in one triangle are equal to one side and similarly located angles in another triangle, then the remaining two sides and angle are also equal. If $a = a_1$, $A = A_1$ and $B = B_1$, then the two other sides and the remaining angle are also equal.
	If two sides and the angle between them in one triangle are equal to two sides and a similarly located angle in another triangle, then the remaining side and angles are also equal. If $a = a_1$, $b = b_1$ and $A = A_1$, then the remaining side and angles are also equal.
	If the three sides in one triangle are equal to the three sides of another triangle, then the angles in the two triangles are also equal. If $a = a_1$, $b = b_1$ and $c = c_1$, then the angles between the respective sides are also equal.
	If the three sides of one triangle are proportional to corresponding sides in another triangle, then the triangles are called *similar*, and the angles in the one are equal to the angles in the other. If $a : b : c = d : e : f$, then $A = D$, $B = E$ and $C = F$.
	If the angles in one triangle are equal to the angles of another triangle, then the triangles are similar and their corresponding sides are proportional. If $A = D$, $B = E$ and $C = F$, then $a : b : c = d : e : f$.

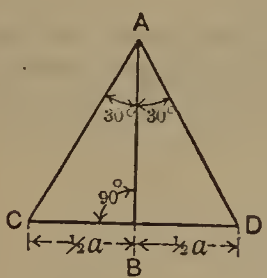
Geometrical Propositions



If the three sides in a triangle are equal — that is, if the triangle is *equilateral* — then the three angles are also equal.

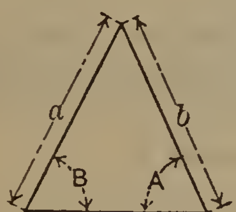
Each of the three equal angles in an equilateral triangle is 60 degrees.

If the three angles in a triangle are equal, then the three sides are also equal.



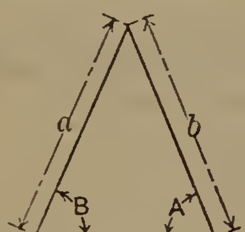
A line which in an equilateral triangle bisects or divides any of the angles into two equal parts, bisects also the side opposite the angle and is at right angles to it.

If line AB divides angle CAD into two equal parts, it also divides line CD into two equal parts and is at right angles to it.



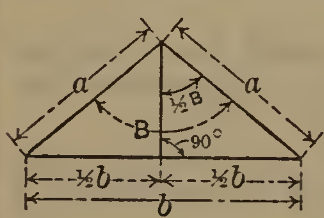
If two sides in a triangle are equal — that is, if the triangle is an *isosceles* triangle — then the angles opposite these sides are also equal.

If side a equals side b, then angle A equals angle B.

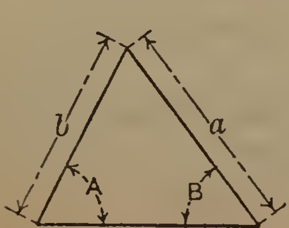


If two angles in a triangle are equal, then the sides opposite these angles are also equal.

If angles A and B are equal, then side a equals side b.



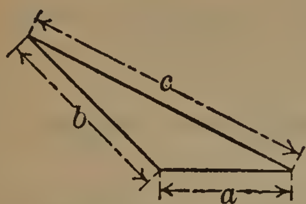
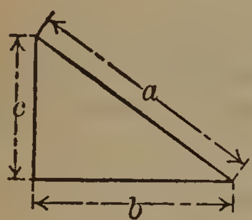
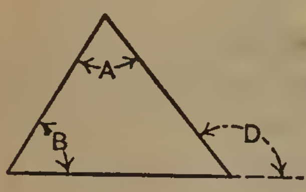
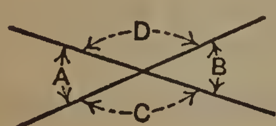
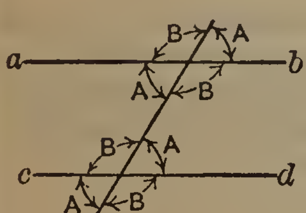
In an isosceles triangle, if a straight line is drawn from the point where the two equal sides meet, so that it bisects the third side or base of the triangle, then it also bisects the angle between the equal sides and is perpendicular to the base.



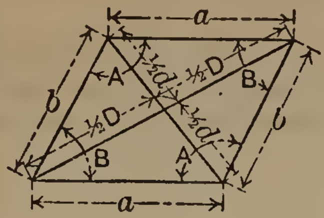
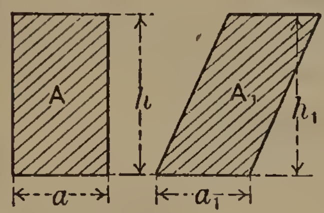
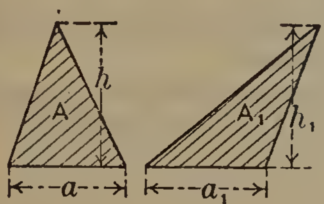
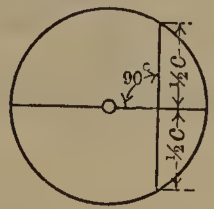
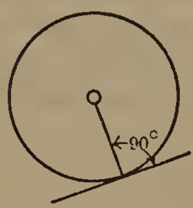
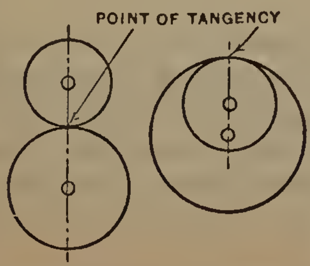
In every triangle, that angle is greater which is opposite a longer side. — In every triangle, that side is greater which is opposite a greater angle.

If a is longer than b, then angle A is greater than B. If angle A is greater than B, then side a is longer than b.

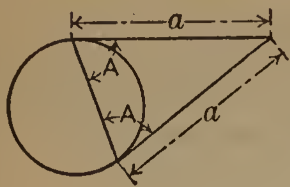
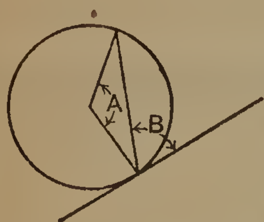
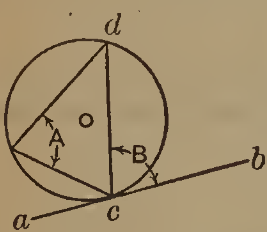
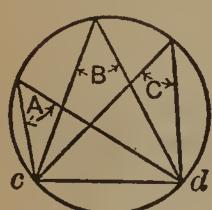
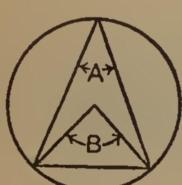
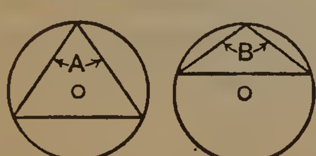
Geometrical Propositions

	In every triangle, the sum of the lengths of two sides is always greater than the length of the third. Side $a +$ side b is always greater than side c.
	In a right-angled triangle, the square of the hypotenuse or the side opposite the right angle is equal to the sum of the squares on the two sides which form the right angle. $a^2 = b^2 + c^2$.
	If one side of a triangle is produced, then the exterior angle is equal to the sum of the two interior opposite angles. Angle $D =$ angle $A +$ angle B.
	If two lines intersect, then the opposite angles formed by the intersecting lines are equal. Angle $A =$ angle B. Angle $C =$ angle D.
	If a line intersects two parallel lines, then the corresponding angles formed by the intersecting line and the parallel lines are equal. Lines ab and cd are parallel. Then all the angles designated A are equal, and all those designated B are equal.
	In any figure having four sides, the sum of the interior angles equals 360 degrees. $A + B + C + D = 360$ degrees.

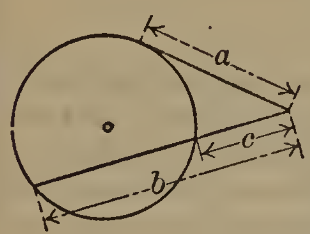
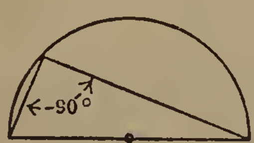
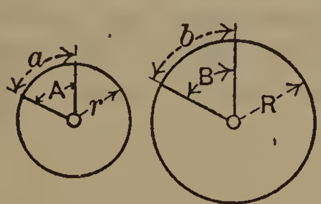
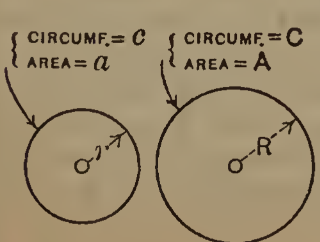
Geometrical Propositions

	The sides which are opposite each other in a parallelogram are equal; the angles which are opposite each other are equal; the diagonal divides it into two equal parts. If two diagonals are drawn, they bisect each other.
	The areas of two parallelograms which have equal base and equal height, are equal. If $a = a_1$ and $h = h_1$, then area A = area A_1.
	The areas of triangles having equal base and equal height are equal. If $a = a_1$ and $h = h_1$, then area A = area A_1.
	If a diameter of a circle is at right angles to a chord, then it bisects or divides the chord into two equal parts.
	If a line is tangent to a circle, then it is also at right angles to a line drawn from the center of the circle to the point of tangency — that is, to a radial line through the point of tangency.
	If two circles are tangent to each other, then the straight line which passes through the centers of the two circles must also pass through the point of tangency.

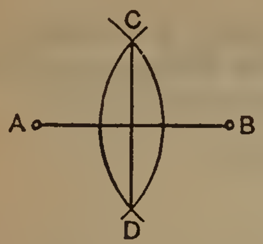
Geometrical Propositions

	If from a point without a circle tangents are drawn to a circle, the two tangents are equal and make equal angles with the chord joining the points of tangency.
	The angle between a tangent and a chord drawn from the point of tangency equals one-half the angle at the center subtended by the chord. Angle $B = \frac{1}{2}$ angle A.
	The angle between a tangent and a chord drawn from the point of tangency equals the angle at the periphery subtended by the chord. Angle B, between tangent ab and chord cd, equals angle A subtended at the periphery by chord cd.
	All angles having their vertex at the periphery of a circle and subtended by the same chord are equal. Angles A, B and C, all subtended by chord cd, are equal.
	If an angle at the circumference of a circle, between two chords, is subtended by the same arc as the angle at the center, between two radii, then the angle at the circumference is equal to one-half of the angle at the center. Angle $A = \frac{1}{2}$ angle B.
A=LESS THAN 90° B=MORE THAN 90° 	An angle subtended by a chord in a circular segment larger than one-half the circle is an acute angle — an angle less than 90 degrees. An angle subtended by a chord in a circular segment less than one-half the circle is an obtuse angle — an angle greater than 90 degrees.

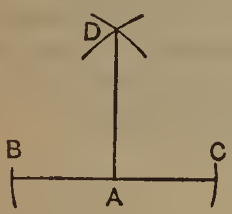
Geometrical Propositions

	If two chords intersect each other in a circle, then the rectangle of the segments of the one equals the rectangle of the segments of the other. $a \times b = c \times d.$
	If from a point outside of a circle two lines are drawn, one of which intersects the circle while the other is tangent to it, then the rectangle contained by the total length of the intersecting line, and that part of it which is between the outside point and the periphery, equals the square of the tangent. $a^2 = b \times c.$
	If a triangle is inscribed in a semi-circle, the angle opposite the diameter is a right (90-degree) angle. All angles at the periphery of a circle, subtended by the diameter, are right (90-degree) angles.
	The length of circular arcs of the same circle are proportional to the corresponding angles at the center. $A : B = a : b.$
	The length of circular arcs having the same center angle are proportional to the length of the radii. If $A = B$, then $a : b = r : R.$
	The circumferences of two circles are proportional to their radii. The areas of two circles are proportional to the squares of their radii. $c : C = r : R.$ $a : A = r^2 : R^2.$

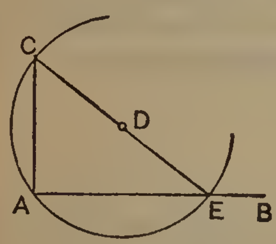
Geometrical Problems



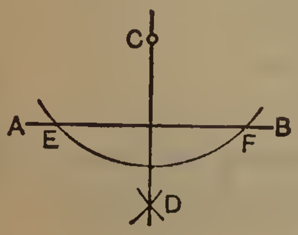
To divide a line AB into two equal parts: With the ends A and B as centers and a radius greater than one-half the line, draw circular arcs. Through the intersections C and D, draw line CD. This line divides AB into two equal parts and is also perpendicular to AB.



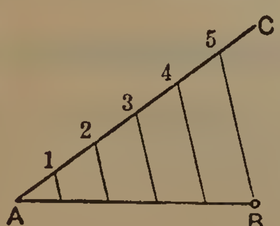
To draw a perpendicular to a straight line from a point A on that line: With A as a center and with any radius, draw circular arcs intersecting the given line at B and C. Then, with B and C as centers and a radius longer than AB, draw circular arcs intersecting at D. Line DA is perpendicular to BC at A.



To draw a perpendicular line from a point A at the end of a line AB: With any point D, outside of the line AB, as a center, and with AD as a radius, draw a circular arc intersecting AB at E. Draw a line through E and D intersecting the arc at C; then join AC. This line is the required perpendicular.

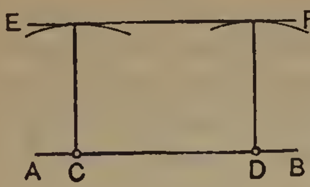


To draw a perpendicular to a line AB from a point C at a distance from it: With C as a center, draw a circular arc intersecting the given line at E and F. With E and F as centers, draw circular arcs with a radius longer than one-half the distance between E and F. These arcs intersect at D. Line CD is the required perpendicular.

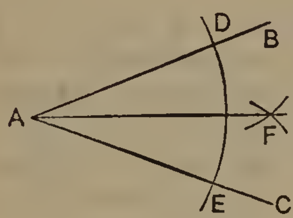


To divide a straight line AB into a number of equal parts: Let it be required to divide AB into five equal parts. Draw line AC at an angle with AB. Set off on AC five equal parts of any convenient length. Draw $B\,5$ and then draw lines parallel with $B\,5$ through the other division points on AC. The points where these lines intersect AB are the required division points.

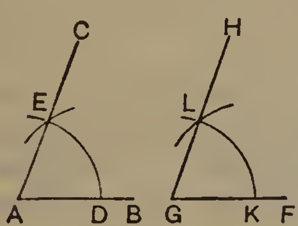
Geometrical Problems



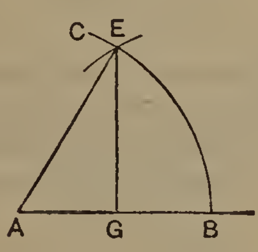
To draw a straight line parallel to a given line AB, at a given distance from it: With any points C and D on AB as centers, draw circular arcs with the given distance as radius. Line EF, drawn to touch the circular arcs, is the required parallel line.



To bisect or divide an angle BAC into two equal parts: With A as a center and any radius, draw arc DE. With D and E as centers and a radius greater than one-half DE, draw circular arcs intersecting at F. Line AF divides the angle into two equal parts.

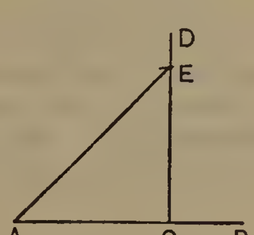


To draw an angle upon a line AB, equal to a given angle FGH: With point G as a center and with any radius, draw arc KL. With A as a center and with the same radius, draw arc DE. Make arc DE equal to KL and draw AC through E. Angle BAC then equals angle FGH.

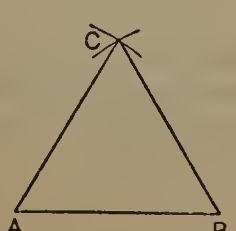


To lay out a 60-degree angle: With A as a center and any radius, draw an arc BC. With point B as a center and AB as a radius, draw an arc intersecting at E the arc just drawn. EAB is a 60-degree angle.

A 30-degree angle may be obtained either by dividing a 60-degree angle into two equal parts, or by drawing a line EG perpendicular to AB. Angle AEG is then 30 degrees.

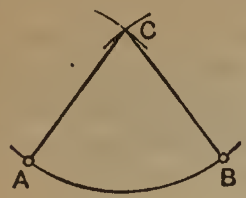


To draw a 45-degree angle: From point A on line AB set off a distance AC. Draw the perpendicular DC and set off a distance CE equal to AC. Draw AE. Angle EAC is a 45-degree angle.

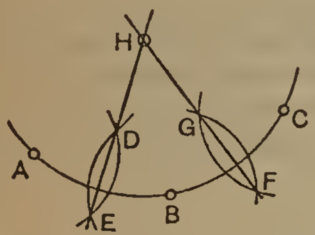


To draw an equilateral triangle, the length of the sides of which equals AB: With A and B as centers and AB as radius, draw circular arcs intersecting at C. Draw AC and BC. Then ABC is an equilateral triangle.

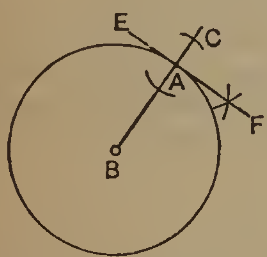
Geometrical Problems



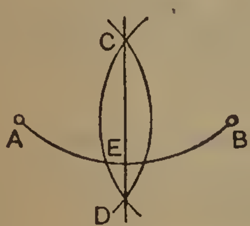
To draw a circular arc with a given radius through two given points A and B: With A and B as centers, and the given radius as radius, draw circular arcs intersecting at C. With C as a center, and the same radius, draw a circular arc through A and B.



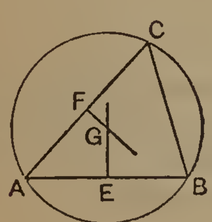
To find the center of a circle or of an arc of a circle: Select three points on the periphery of the circle, as A, B and C. With each of these points as a center and the same radius, describe arcs intersecting each other. Through the points of intersection draw lines DE and FG. Point H where these lines intersect is the center of the circle.



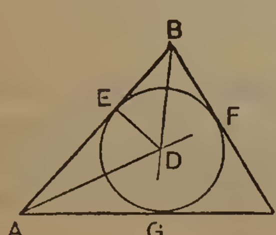
To draw a tangent to a circle from a given point on the circumference: Through the point of tangency A, draw a radial line BC. At point A, draw a line EF at right angles to BC. This line is the required tangent.



To divide a circular arc AB into two equal parts: With A and B as centers, and a radius larger than half the distance between A and B, draw circular arcs intersecting at C and D. Line CD divides arc AB into two equal parts at E.

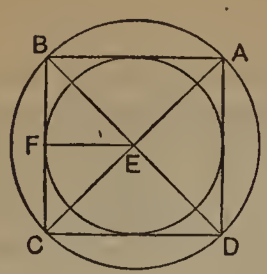


To describe a circle about a triangle: Divide the sides AB and AC into two equal parts, and from the division points E and F draw lines at right angles to the sides. These lines intersect at G. With G as a center and GA as a radius, draw circle ABC.

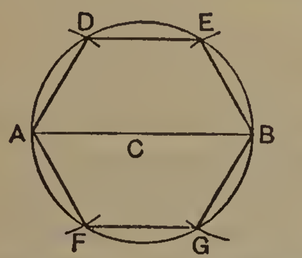


To inscribe a circle in a triangle: Bisect two of the angles, A and B, by lines intersecting at D. From D draw a line DE perpendicular to one of the sides, and with DE as a radius, draw circle EFG.

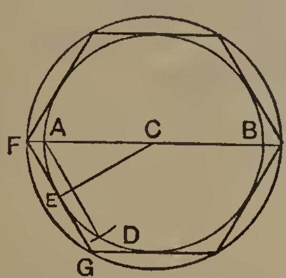
Geometrical Problems



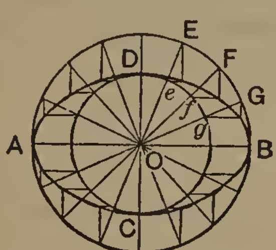
To describe a circle about a square and to inscribe a circle in a square: The center of both the circumscribed and inscribed circle is located at the point *E*, where the two diagonals of the square intersect. The radius of the circumscribed circle is *AE*, and of the inscribed circle, *EF*.



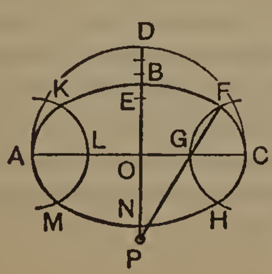
To inscribe a hexagon in a circle: Draw a diameter *AB*. With *A* and *B* as centers and with the radius of the circle as radius, describe circular arcs intersecting the given circle at *D*, *E*, *F* and *G*. Draw lines *AD*, *DE*, etc., forming the required hexagon.



To describe a hexagon about a circle: Draw a diameter *AB*, and with *A* as a center and the radius of the circle as radius, cut the circumference of the given circle at *D*. Join *AD* and bisect it with radius *CE*. Through *E*, draw *FG* parallel to *AD* and intersecting line *AB* at *F*. With *C* as a center and *CF* as radius, draw a circle. Within this circle inscribe the hexagon as in the preceding problem.

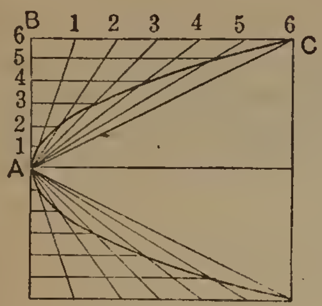


To describe an ellipse with the given axes *AB* and *CD*: Describe circles with *O* as a center and *AB* and *CD* as diameters. From a number of points, *E*, *F*, *G*, etc., on the outer circle draw radii intersecting the inner circle at *e*, *f*, *g*. From *E*, *F* and *G* draw lines perpendicular to *AB*, and from *e*, *f* and *g* draw lines parallel to *AB*. The intersections of these perpendicular and parallel lines are points on the curve of the ellipse.

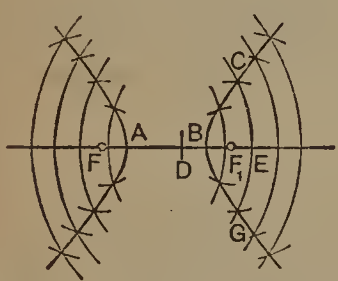


To construct an approximate ellipse by circular arcs: Let *AC* be the major axis and *BN* the minor. Draw half circle *ADC* with *O* as a center. Divide *BD* into three equal parts and set off *BE* equal to one of these parts. With *A* and *C* as centers and *OE* as radius, describe circular arcs *KLM* and *FGH*; with *G* and *L* as centers, and the same radius, describe arcs *FCH* and *KAM*. Through *F* and *G* draw line *FP*, and with *P* as a center draw the arc *FBK*. Arc *HNM* is drawn in the same manner.

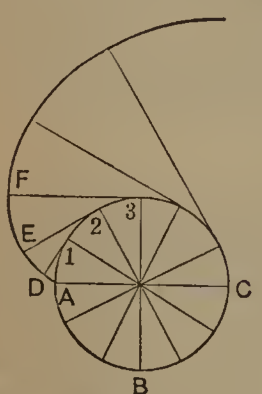
Geometrical Problems



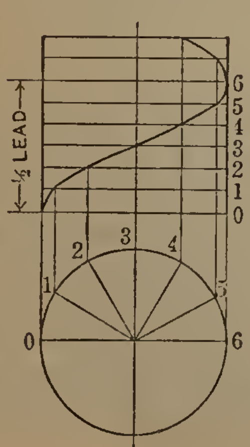
To construct a parabola: Divide line AB into a number of equal parts and divide BC into the same number of parts. From the division points on AB draw horizontal lines. From the division points on BC draw lines to point A. The points of intersection between lines drawn from points numbered alike are points on the parabola.



To construct a hyperbola: From focus F lay off a distance FD equal to the transverse axis, or the distance AB between the two branches of the curve. With F as a center and any distance FE greater than FB as a radius, describe a circular arc. Then with F_1 as a center and DE as a radius, describe arcs intersecting at C and G the arc just described. C and G are points on the hyperbola. Any number of points can be found in a similar manner.



To construct an involute: Divide the circumference of the base circle ABC into a number of equal parts. Through the division points 1, 2, 3, etc., draw tangents to the circle and make the lengths D 1, E 2, F 3, etc., of these tangents equal to the actual length of the arcs A 1, A 2, A 3, etc.



To construct a helix: Divide half the circumference of the cylinder on the surface of which the helix is to be described into a number of equal parts. Divide half the lead of the helix into the same number of equal parts. From the division points on the circle representing the cylinder draw vertical lines, and from the division points on the lead draw horizontal lines as shown. The intersections between lines numbered alike are points on the helix.

MECHANICS

Mechanics treats of the action of forces and their effect. A *force* is defined as any cause tending to produce or modify motion. The units by which a force is usually measured are pounds or tons. Besides force there are two other elementary quantities in mechanics from which numerous compound quantities are derived. These are *distance*, measured in linear units, as inches, feet, etc., and *time*, expressed in hours, minutes or seconds.

Work, in mechanics, is the product of force by distance, and is expressed by a combination of units of weight (force) and distance, as inch-pounds, foot-pounds, foot-tons, etc.

Power, in mechanics, is the product of force by distance divided by time, or the performance of a given amount of work in a given time, and is expressed as inch-pounds per minute, foot-pounds per minute or second, etc.

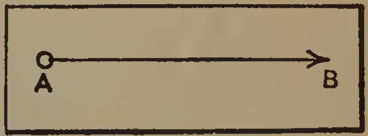
The term *power* is frequently used by writers on mechanics to designate a *force*. In connection with the so-called "mechanical powers" — the lever, wheel and axle, wedge, screw, etc. — it is usual to speak of the applied force as the power; this is, however, not strictly correct, as power should always, in mechanics, be used in accordance with the definition given above.

Horsepower (abbreviated H.P.) is the unit of power adopted for engineering work. One horsepower is equal to 33,000 foot-pounds per minute, or 550 foot-pounds per second. The metric horsepower, used in countries where the metric system is employed, is equal to 75 kilogram-meters per second, or 542.5 foot-pounds per second, or 32,550 foot-pounds per minute. The *kilowatt*, used in electrical work, equals 1.34 horsepower; or one horsepower equals 0.746 kilowatt.

Velocity is distance divided by time, and is expressed in feet per minute, miles per hour, etc.

Inertia is that property of a body which causes it to tend to continue in its present state of rest or motion, unless acted upon by some force.

Graphical Representation of Forces. — A force has three characteristics which, when known, determine it. They are direction, place of application, and magnitude. The *direction* of a force is the direction in which it tends to move the body upon which it acts. The *place of application* is generally assumed to be a point, as the center of gravity. The *magnitude* is measured in pounds, as already stated. Forces may conveniently be represented by straight lines and arrow heads. The arrow head indicates the direction of the force, and the length of the line its magnitude to any suitable scale. The point of application may be at any point on the line, but it is generally convenient to assume it to be at one end. In the accompanying

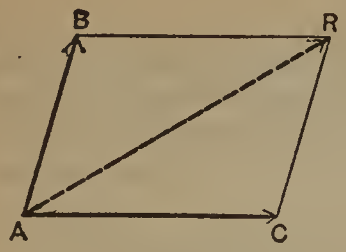


illustration, a force is supposed to act along line AB in a direction from left to right. The length of line AB shows the magnitude of the force. If point A is the point of application, the force is exerted as a pull, but if point B be assumed to be the point of application, it would indicate that the force is exerted as a push.

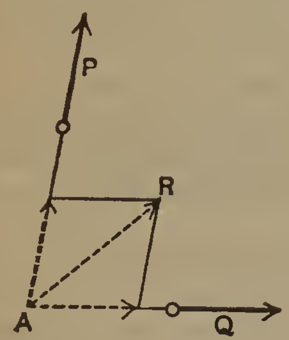
The single force which produces the same effect upon a body as two or more forces acting together, is called their *resultant*. The separate forces which can be so combined are called the *components*. The finding of the resultant of two or more forces is called the *composition* of forces, and the finding of two or more components of a given force, the *resolution* of forces.

On the two following pages are shown the methods used in the solution of those problems in the composition of forces that are most frequently met with. The resolution of forces is carried out in a similar manner to the composition, except that the work is carried out in a reverse order.

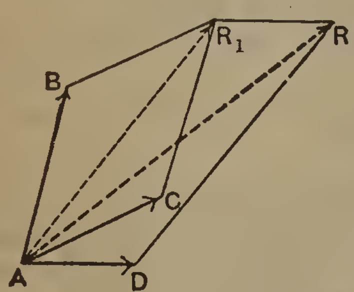
Composition and Resolution of Forces



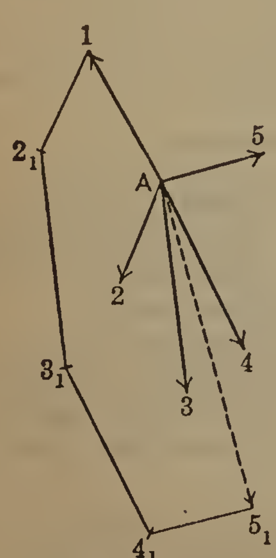
Parallelogram of Forces. — If two forces applied at a point are represented in magnitude and direction by the adjacent sides of a parallelogram (AB and AC in the accompanying illustration), their resultant will be represented in magnitude and direction by the diagonal AR drawn from the intersection of the two component forces.



If two forces P and Q do not have the same point of application, but the lines indicating their directions intersect, the forces may be imagined as applied at the point of intersection between the lines (as at A), and the resultant of the two forces may be found by constructing the parallelogram of forces. Line AR shows the direction and magnitude of the resultant, the point of application of which may be assumed to be at any point on line AR or its extension.

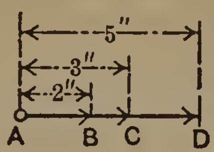


If the resultant of three or more forces having the same point of application is to be found, first find the resultant of any two of the forces (AB and AC) and then find the resultant of the resultant just found (AR_1) and the third force (AD). If there be more than three forces, continue in this manner until the resultant of all the forces has been found.

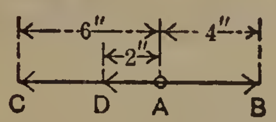


Polygon of Forces. — When several forces are applied at a point and act in a single plane, their resultant may be found more simply than by the method just described, as follows: From the extreme end of the line representing the first force, draw a line representing the second force, parallel to it and of the same length and in the direction of the second force. Then through the extreme end of this line draw a line parallel to, and of the same length and direction as the third force, and continue this until all the forces have been thus represented. Then draw a line from the point of application of the forces (as A) to the extreme point (as 5_1) of the line last drawn. This line ($A\ 5_1$) is the resultant of the forces.

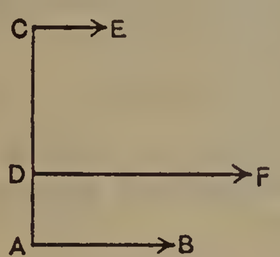
Composition and Resolution of Forces



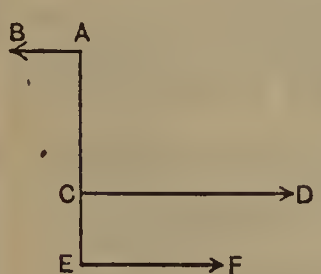
The resultant of two forces applied at the same point and acting in the same direction, is equal to the sum of the forces. For example, if the two forces AB and AC, one equal to two and the other equal to three pounds, are applied at point A, then their resultant AD equals the sum of these forces, or five pounds.



If two forces act in opposite directions, then their resultant is equal to their difference, and the direction of the resultant is the same as the direction of the greater of the two forces. For example: AB and AC are both applied at point A; then, if AB equals four and AC equals six pounds, the resultant AD equals two pounds and acts in the direction of AC.

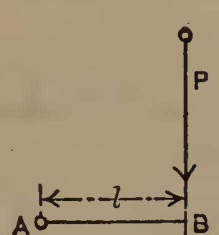


Parallel Forces. — If two forces are parallel and act in the same direction, then their resultant is parallel to both lines, is located between them, and is equal to the sum of the two components. The point of application of the resultant divides the line joining the points of application of the components inversely as the magnitude of the components. Thus, $AB:CE = CD:AD$.



The resultant of two parallel forces acting in opposite directions is parallel to both lines, is located outside of them on the side of the greater of the components, has the same direction as the greater component, and is equal in magnitude to the difference between the two components. The point of application on the line AC produced is found from the proportion:

$$AB : CD = CE : AE.$$

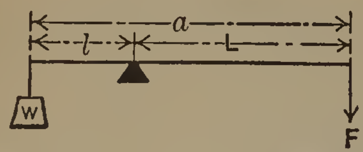
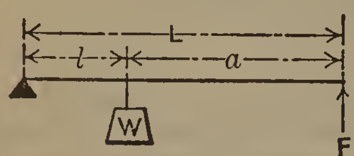
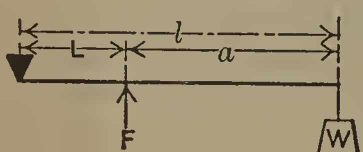
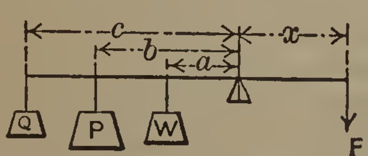


Moment of P about A Equals $P \times l$

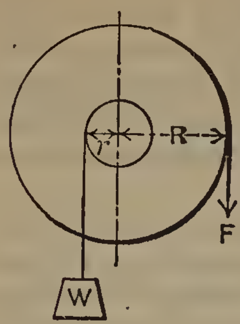
Moment of a Force. — The moment of a force with respect to a point is the product of the force multiplied by the perpendicular distance from the given point to the direction of the force. In the illustration, the moment of the force P with relation to point A is $P \times AB$. The perpendicular distance AB is called the lever-arm of the force. The moment is the measure of the tendency of the force to produce rotation about the given point, which is termed the center of moments. If the force is measured in pounds and the distance in inches, the moment is expressed in inch-pounds.

The moment of the resultant of any number of forces acting together in the same plane is equal to the algebraic sum of the moments of the separate forces.

Levers

Types of Levers	Examples
 $F:W=l:L \qquad F\times L=W\times l$ $F=\frac{W\times l}{L} \qquad W=\frac{F\times L}{l}$ $L=\frac{W\times a}{W+F}=\frac{W\times l}{F};\quad l=\frac{F\times a}{W+F}=\frac{F\times L}{W}$	A pull of 80 pounds is exerted at the end of the lever, at W; $l=12$ inches and $L=32$ inches. Find the value of force F required to balance the lever. $F=\frac{80\times 12}{32}=\frac{960}{32}=30$ pounds. If $F=20$; $W=180$; and $l=3$; how long must L be made to secure equilibrium? $L=\frac{180\times 3}{20}=27.$
 $F:W=l:L \qquad F\times L=W\times l$ $F=\frac{W\times l}{L} \qquad W=\frac{F\times L}{l}$ $L=\frac{W\times a}{W-F}=\frac{W\times l}{F};\quad l=\frac{F\times a}{W-F}=\frac{F\times L}{W}$	Total length L of a lever is 25 inches. A weight of 90 pounds is supported at W; l is 10 inches. Find the value of F. $F=\frac{90\times 10}{25}=36$ pounds. If $F=100$ pounds, $W=2200$ pounds, and $a=5$ feet, what should L equal to secure equilibrium? $L=\frac{2200\times 5}{2200-100}=5.24$ feet.
 $F:W=l:L \qquad F\times L=W\times l$ $F=\frac{W\times l}{L} \qquad W=\frac{F\times L}{l}$ $L=\frac{W\times a}{F-W}=\frac{W\times l}{F};\quad l=\frac{F\times a}{F-W}=\frac{F\times L}{W}$	$F=28$ pounds; $L=10$ inches; $a=24$ inches. What weight W can be supported? $l=a+L=24+10=34$ inches. $W=\frac{28\times 10}{34}=8.23$ pounds. Let $F=12$ tons; $W=4.5$ tons; $a=16$ feet. Find L and l. $L=\frac{4.5\times 16}{12-4.5}=9.6$ feet; $l=16+9.6=25.6$ feet.
 When three or more forces act on a lever: $F\times x=W\times a+P\times b+Q\times c$ $x=\frac{W\times a+P\times b+Q\times c}{F}$ $F=\frac{W\times a+P\times b+Q\times c}{x}$	Let $W=20$, $P=30$, and $Q=15$ pounds; $a=4$, $b=7$, and $c=10$ inches. If $x=6$ inches, find F. $F=\frac{20\times 4+30\times 7+15\times 10}{6}=73\frac{1}{3}$ lbs. Assuming $F=20$ in the example above, how long must lever arm x be made? $x=\frac{20\times 4+30\times 7+15\times 10}{20}=22$ ins.

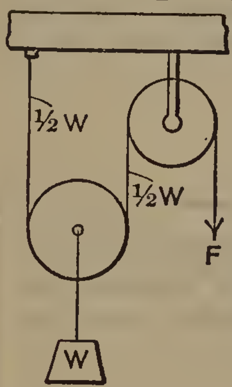
Wheels and Pulleys



$$F : W = r : R$$
$$F \times R = W \times r$$
$$F = \frac{W \times r}{R}$$
$$W = \frac{F \times R}{r}$$
$$R = \frac{W \times r}{F}$$
$$r = \frac{F \times R}{W}$$

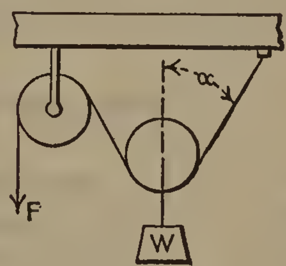
The radius of a drum on which is wound the lifting rope of a windlass is 2 inches. What force will be exerted at the periphery of a gear of 24 inches diameter, mounted on the same shaft as the drum and transmitting power to it, if one ton (2000 pounds) is to be lifted? Here $W = 2000$; $R = 12$; $r = 2$.

$$F = \frac{2000 \times 2}{12} = 333 \text{ pounds.}$$

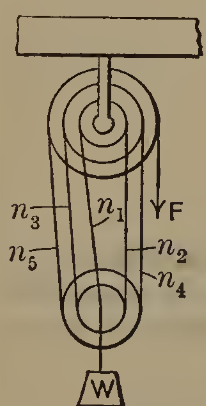


$$F = \tfrac{1}{2} W$$

The velocity with which weight W will be raised equals one-half the velocity of the force applied at F.



$$F : W = \sec \alpha : 2$$
$$F = \frac{W \times \sec \alpha}{2}$$
$$W = 2F \times \cos \alpha$$



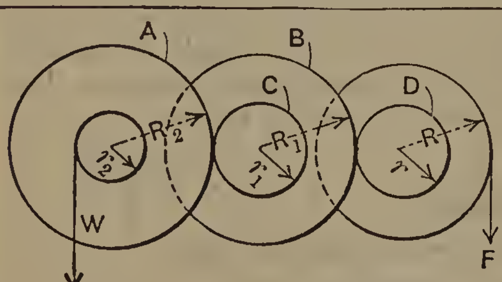
n = number of strands or parts of rope (n_1, n_2, etc.).

$$F = \frac{1}{n} \times W$$

The velocity with which W will be raised equals $\frac{1}{n}$ of the velocity of the force applied at F.

In the illustration is shown a combination of a double and triple block. The pulleys each turn freely on a pin as axis, and are drawn with different diameters, to show the parts of the rope more clearly. There are 5 parts of rope. Therefore, if 200 pounds is to be lifted, the force F required at the end of the rope is:

$$F = \tfrac{1}{5} \times 200 = 40 \text{ pounds.}$$



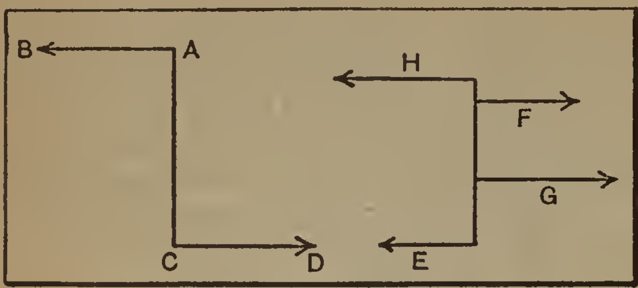
A, *B*, *C* and *D* are the pitch circles of gears.

$$F = \frac{W \times r \times r_1 \times r_2}{R \times R_1 \times R_2}$$
$$W = \frac{F \times R \times R_1 \times R_2}{r \times r_1 \times r_2}$$

Let the pitch diameters of gears A, B, C and D be 30, 28, 12 and 10 inches, respectively. Then $R_2 = 15$; $R_1 = 14$; $r_1 = 6$; and $r = 5$. Let $R = 12$, and $r_2 = 4$. Then the force F required to lift a weight W of 2000 pounds, friction being neglected, is:

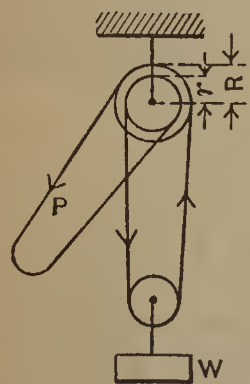
$$F = \frac{2000 \times 5 \times 6 \times 4}{12 \times 14 \times 15} = 95 \text{ pounds.}$$

Couples. — If the forces AB and CD are equal and act in opposite directions, then the resultant equals o, or, in other words, the two forces have no resultant and are called a couple. A couple tends to produce rotation. The measure of this tendency is called the moment of the couple and is the product of one of the forces multiplied by the distance between the two.



As a couple has no resultant, no single force can balance or counteract the tendency of the couple to produce rotation. To prevent the rotation of a body acted upon by a couple, two other forces are therefore required, forming a second couple. In the illustration, E and F form one couple and G and H are the balancing couple. The body on which they act is in equilibrium if the moments of the two couples are equal and tend to rotate the body in opposite directions.

Differential Pulley — Screw

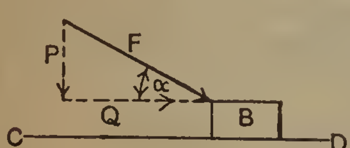


Differential Pulley. — In the differential pulley a chain must be used, engaging sprockets, so as to prevent the chain from slipping over the pulley faces.

$$P \times R = \tfrac{1}{2} W (R - r)$$

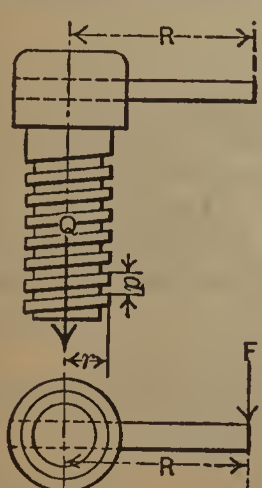
$$P = \frac{W (R - r)}{2 R}$$

$$W = \frac{2 PR}{R - r}$$



Force Moving Body on Horizontal Plane. — F tends to move B along line CD; Q is the component which actually moves B; P is the pressure, due to F, of the body on CD.

$$Q = F \times \cos\alpha; \qquad P = \sqrt{F^2 - Q^2}$$



Screw. — F = force at end of handle or wrench; R = lever-arm of F; r = pitch radius of screw; p = lead of thread. Then, neglecting friction:

$$F = Q \times \frac{p}{6.2832\,R} \qquad Q = F \times \frac{6.2832\,R}{p}$$

If μ is the coefficient of friction, then:

For motion in same direction as Q:

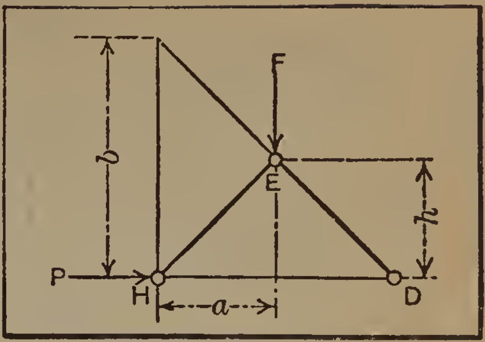
$$F = Q \times \frac{p - 6.2832\,\mu r}{6.2832\,r + \mu p} \times \frac{r}{R}$$

For motion in a direction opposite to Q:

$$F = Q \times \frac{p + 6.2832\,\mu r}{6.2832\,r - \mu p} \times \frac{r}{R}$$

Toggle-joint. — If arms ED and EH are of unequal length:

$$P = \frac{Fa}{b}$$



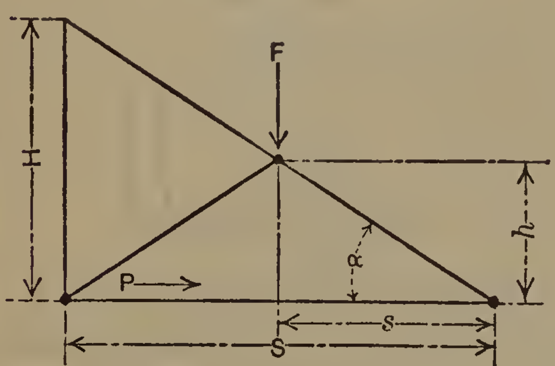
The relation between P and F changes constantly as F moves downwards.

If arms ED and EH are equal:

$$P = \frac{Fa}{2h}$$

A double toggle-joint does not increase the pressure exerted so long as the relative distances moved by F and P remain the same.

Toggle-joints with Equal Arms



F = power applied;
P = resistance;
α = given angle.

$$2P\sin\alpha = F\cos\alpha;$$

$$\frac{P}{F} = \frac{\cos\alpha}{2\sin\alpha} = \text{coefficient};$$

or, $P = F \times \text{coefficient}$.

Equivalent expressions:

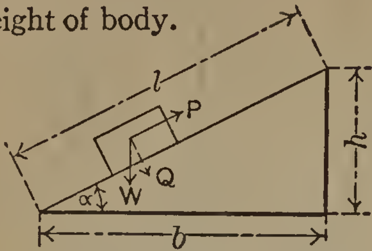
$$P = \frac{FS}{4h}; \qquad P = \frac{Fs}{H}, \text{ as per diagram.}$$

To use the table, measure angle α, and find the coefficient in the table corresponding to the angle found. The coefficient is the ratio of the resistance to the power, and multiplying the power by the coefficient gives the resistance, neglecting friction.

Angle	Coefficient	Angle	Coefficient	Angle	Coefficient	Angle	Coefficient
0° 2′	862	0° 50′	34.4	2° 45′	10.4	8° 0′	3.58
0 4	456	0 55	31.2	2 50	10.1	8 30	3.35
0 6	285	1 0	28.6	3 0	9.54	9 0	3.15
0 8	216	1 10	24.6	3 15	8.81	9 30	2.99
0 10	171	1 15	22.9	3 30	8.17	10 0	2.84
0 12	143	1 20	21.5	3 45	7.63	11 0	2.57
0 14	122	1 30	19.1	4 0	7.25	12 0	2.35
0 15	115	1 40	17.2	4 15	6.73	13 0	2.17
0 16	107	1 45	16.4	4 30	6.35	14 0	2.00
0 18	95.4	1 50	15.6	4 45	6.02	15 0	1.87
0 20	85.8	2 0	14.3	5 0	5.71	16 0	1.74
0 25	68.6	2 10	13.2	5 30	5.19	17 0	1.64
0 30	57.3	2 15	12.7	6 0	4.76	18 0	1.54
0 35	49.1	2 20	12.5	6 30	4.39	19 0	1.45
0 40	42.8	2 30	11.5	7 0	4.07	20 0	1.37
0 45	38.2	2 40	10.7	7 30	3.79	....	....

Inclined Plane — Wedge

W = weight of body.



Neglecting friction:

$$P = W \times \frac{h}{l} = W \times \sin\alpha$$

$$W = P \times \frac{l}{h} = \frac{P}{\sin\alpha} = P \times \operatorname{cosec}\alpha$$

$$Q = W \times \frac{b}{l} = W \times \cos\alpha$$

If friction is taken into account, then force P to pull body up is:

$$P = W\,(\mu\cos\alpha + \sin\alpha)$$

Force P_1 to pull body down is:

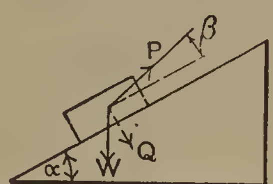
$$P_1 = W\,(\mu\cos\alpha - \sin\alpha)$$

Force P_2 to hold body stationary:

$$P_2 = W\,(\sin\alpha - \mu\cos\alpha)$$

in which μ is the coefficient of friction.

W = weight of body.



Neglecting friction:

$$P = W \times \frac{\sin\alpha}{\cos\beta}$$

$$W = P \times \frac{\cos\beta}{\sin\alpha}$$

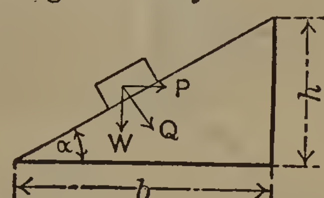
$$Q = W \times \frac{\cos(\alpha+\beta)}{\cos\beta}$$

With friction:

Coefficient of friction = $\mu = \tan\phi$.

$$P = W \times \frac{\sin(\alpha+\phi)}{\cos(\beta-\phi)}$$

W = weight of body.



Neglecting friction:

$$P = W \times \frac{h}{b} = W \times \tan\alpha$$

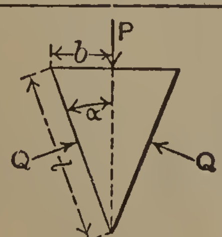
$$W = P \times \frac{b}{h} = P \times \cot\alpha$$

$$Q = \frac{W}{\cos\alpha} = W \times \sec\alpha$$

With friction:

Coefficient of friction = $\mu = \tan\phi$.

$$P = W\tan(\alpha+\phi)$$



Neglecting friction:

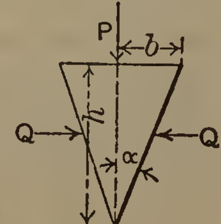
$$P = 2Q \times \frac{b}{l} = 2Q \times \sin\alpha$$

$$Q = P \times \frac{l}{2b} = \tfrac{1}{2}P \times \operatorname{cosec}\alpha$$

With friction:

Coefficient of friction = μ.

$$P = 2Q\,(\mu\cos\alpha + \sin\alpha)$$



Neglecting friction:

$$P = 2Q \times \frac{b}{h} = 2Q \times \tan\alpha$$

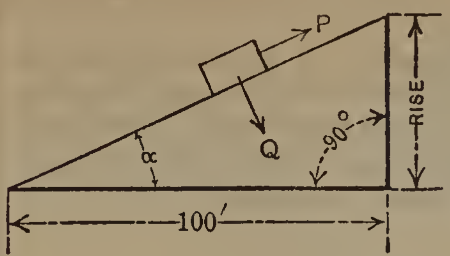
$$Q = P \times \frac{h}{2b} = \tfrac{1}{2}P \times \cot\alpha$$

With friction:

Coefficient of friction = $\mu = \tan\phi$.

$$P = 2Q\tan(\alpha+\phi)$$

Table of Forces on Inclined Planes



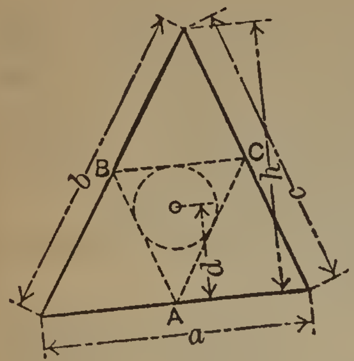
The table below makes it possible to find the force required for moving a body on an inclined plane. The friction on the plane is not taken into account. The column headed "Tension *P* in Cable per Ton of 2000 Pounds" gives the pull in pounds required for moving one ton along the inclined surface. The fourth column gives the perpendicular or normal pressure. If the coefficient of friction is known, the added pull required to overcome friction is thus easily determined:

$Q \times$ coefficient of friction = additional pull required.

Tensions and Pressures in Pounds

Per Cent of Grade. Rise, Ft. per 100 Ft.	Angle α	Tension P in Cable per Ton of 2000 Lbs.	Perpendicular Pressure Q on Plane per Ton of 2000 Lbs.	Per Cent of Grade. Rise, Ft. Per 100 Ft.	Angle α	Tension P in Cable per Ton of 2000 Lbs.	Perpendicular Pressure Q on Plane per Ton of 2000 Lbs.
1	0° 35′	20.2	1999.8	39	21° 19′	727.0	1863.0
2	1 9	40.0	1999.4	40	21 49	743.2	1856.6
3	1 44	60.4	1999.0	41	22 18	758.8	1850.4
4	2 18	80.2	1998.2	42	22 47	774.4	1843.8
5	2 52	100.0	1997.4	43	23 17	790.4	1837.0
6	3 27	120.2	1996.2	44	23 45	805.4	1830.6
7	4 1	140.0	1995.0	45	24 14	820.8	1823.6
8	4 35	159.8	1993.6	46	24 43	836.2	1816.6
9	5 9	179.4	1991.8	47	25 11	851.0	1809.8
10	5 43	199.2	1990.0	48	25 39	865.6	1802.8
11	6 17	218.8	1987.8	49	26 7	880.4	1795.6
12	6 51	238.4	1985.6	50	26 34	894.4	1788.8
13	7 25	258.0	1983.2	51	27 2	909.0	1781.4
14	7 59	277.6	1980.6	52	27 29	922.8	1774.2
15	8 32	296.6	1977.8	53	27 56	936.8	1766.8
16	9 6	316.2	1974.8	54	28 23	950.6	1759.4
17	9 39	335.2	1971.6	55	28 49	964.0	1752.2
18	10 13	354.6	1968.2	56	29 15	977.2	1744.8
19	10 46	373.6	1964.6	57	29 41	990.4	1737.4
20	11 19	392.4	1961.0	58	30 7	1003.4	1730.0
21	11 52	411.2	1957.2	59	30 33	1016.4	1722.2
22	12 25	430.0	1953.2	60	30 58	1029.0	1714.8
23	12 58	448.6	1949.0	61	31 23	1041.4	1707.4
24	13 30	466.8	1944.6	62	31 48	1053.8	1699.6
25	14 3	485.4	1940.0	63	32 13	1066.2	1692.0
26	14 35	503.4	1935.4	64	32 38	1078.4	1684.2
27	15 7	521.4	1930.6	65	33 2	1090.2	1676.6
28	15 39	539.4	1925.8	66	33 26	1101.8	1669.0
29	16 11	557.4	1920.6	67	33 50	1113.4	1661.2
30	16 42	574.6	1915.6	68	34 13	1124.6	1653.8
31	17 14	592.4	1910.2	69	34 37	1136.0	1645.8
32	17 45	609.6	1904.6	70	35 0	1147.0	1638.2
33	18 16	626.8	1899.2	71	35 23	1158.0	1630.4
34	18 47	643.8	1893.4	72	35 46	1168.8	1622.8
35	19 18	661.0	1887.6	73	36 8	1179.2	1615.2
36	19 48	677.4	1881.6	74	36 31	1190.0	1607.2
37	20 19	694.4	1875.4	75	36 53	1200.4	1599.6
38	20 49	710.6	1869.4	..	.. ..	......	......

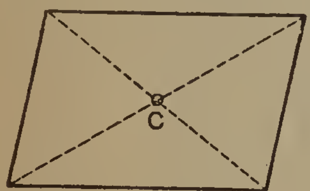
Center of Gravity



Perimeter of a Triangle. — If A, B and C are the middle points of the sides of the triangle, then the center of gravity is at the center of the circle that can be inscribed in triangle ABC. The distance d of the center of gravity from side a is:

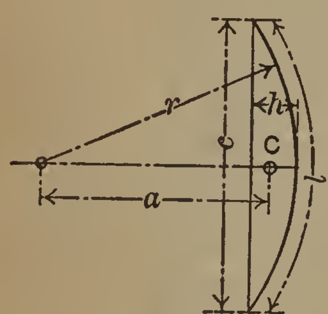
$$d = \frac{h\,(b + c)}{2\,(a + b + c)}$$

where h is the height perpendicular to a.



Perimeter of a Parallelogram. — The center of gravity is at the intersection of the diagonals.

Area of a Parallelogram. — The center of gravity is at the intersection of the diagonals.



Circular Arc. — The center of gravity is on the line that bisects the arc, at a distance

$$a = \frac{r \times c}{l} = \frac{c\,(c^2 + 4\,h^2)}{8\,lh}$$ from the center of the circle.

For an arc equal to one-half the periphery:

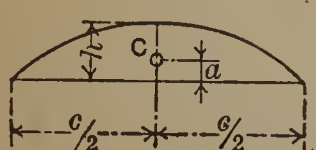
$$a = 2\,r \div \pi = 0.6366\,r$$

For an arc equal to one-quarter of the periphery:

$$a = 2\,r\sqrt{2} \div \pi = 0.9003\,r$$

For an arc equal to one-sixth of the periphery:

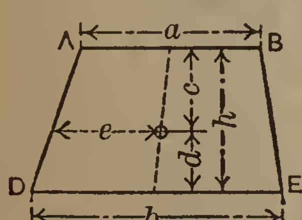
$$a = 3\,r \div \pi = 0.9549\,r$$

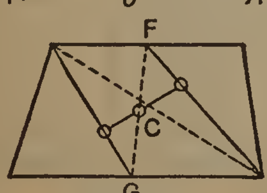


Circular Arc (approximate). —

$$a = \tfrac{2}{3}\,h$$

This formula is very nearly exact for all arcs less than one-quarter of the periphery. The error is only about one per cent for a quarter circle, and decreases for smaller arcs.





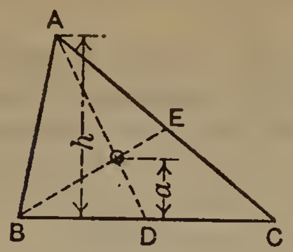
Area of Trapezoid. — The center of gravity is on the line joining the middle points of parallel lines AB and DE.

$$c = \frac{h\,(a + 2\,b)}{3\,(a + b)} \qquad d = \frac{h\,(2\,a + b)}{3\,(a + b)}$$

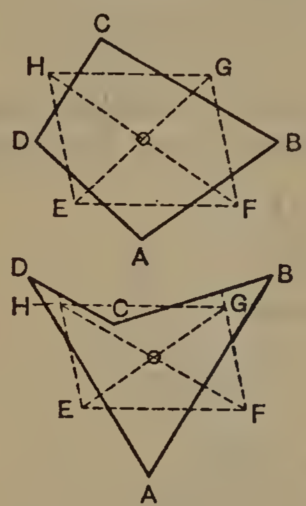
$$e = \frac{a^2 + ab + b^2}{3\,(a + b)}$$

The trapezoid can also be divided into two triangles. The center of gravity is at the intersection of the line joining the centers of gravity of the triangles, and the middle line FG.

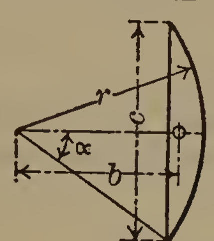
Center of Gravity



Area of Triangle. — The center of gravity is at the intersection of lines AD and BE, which bisect the sides BC and AC. The perpendicular distance from the center of gravity to any one of the sides is equal to one-third the height perpendicular to that side. Hence, $a = h \div 3$.



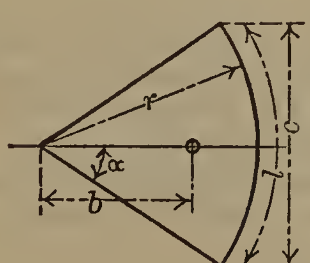
Any Four-sided Figure. — Two cases are possible, as shown in the illustration. To find the center of gravity of the four-sided figure $ABCD$, each of the sides is divided into three equal parts. A line is then drawn through each pair of division points next to the points of intersection A, B, C, and D of the sides of the figure. These lines form a parallelogram $EFGH$; the intersection of the diagonals EG and FH locates the required center of gravity.



Circle Segment. — The distance of the center of gravity from the center of the circle is:

$$b = \frac{c^3}{12\,A} = \frac{2}{3} \times \frac{r^3 \sin^3 \alpha}{A}$$

in which A = area of segment.



Circle Sector. — Distance b from center of gravity to center of circle is:

$$b = \frac{2\,rc}{3\,l} = \frac{r^2c}{3\,A} = 38.197\,\frac{r \sin \alpha}{\alpha}$$

in which A = area of sector, and α is expressed in degrees.

For the area of a half-circle:

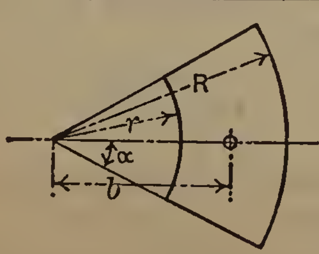
$$b = 4\,r \div 3\,\pi = 0.4244\,r$$

For the area of a quarter circle:

$$b = 4\,\sqrt{2} \times r \div 3\,\pi = 0.6002\,r$$

For the area of a sixth of a circle:

$$b = 2\,r \div \pi = 0.6366\,r$$

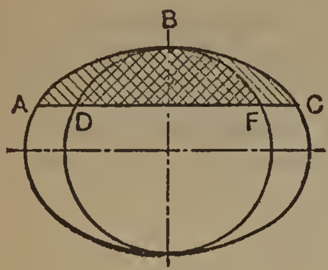


Part of Circle Ring. — Distance b from center of gravity to center of circle is:

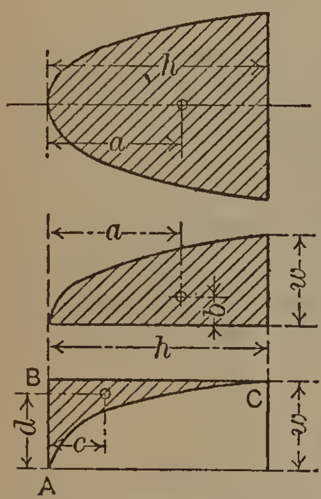
$$b = 38.197\,\frac{(R^3 - r^3) \sin \alpha}{(R^2 - r^2)\,\alpha}$$

Angle α is expressed in degrees.

Center of Gravity



Segment of an Ellipse. — The center of gravity of an elliptic segment ABC, symmetrical about one of the axes, coincides with the center of gravity of the segment DBF of a circle, the diameter of which is equal to that axis of the ellipse about which the elliptic segment is symmetrical.



Area of a Parabola. — For the complete parabolic area, the center of gravity is on the center line or axis, and

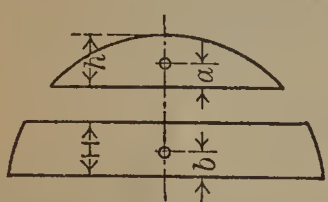
$$a = \frac{3h}{5}$$

For one-half of the parabola:

$$a = \frac{3h}{5} \quad \text{and} \quad b = \frac{3w}{8}$$

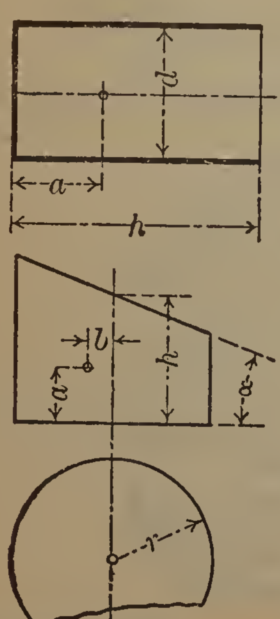
For the complement area ABC:

$$c = 0.3h \quad \text{and} \quad d = 0.75w$$



Spherical Surface of Segments and Zones of Spheres. — Distances a and b which determine the center of gravity, are:

$$a = \frac{h}{2} \qquad b = \frac{H}{2}$$



Cylinder. — The center of gravity of a solid cylinder (or prism) with parallel end surfaces, is located at the middle of the line that joins the centers of gravity of the end surfaces.

The center of gravity of a cylindrical surface or shell, with the base or end surface in one end, is found from:

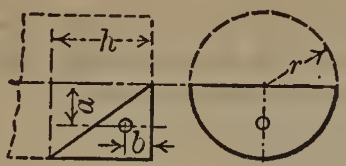
$$a = \frac{2h^2}{4h + d}$$

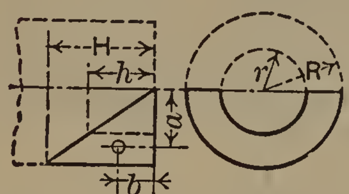
The center of gravity of a cylinder cut off by an inclined plane is located by:

$$a = \frac{h}{2} + \frac{r^2 \tan^2 \alpha}{8h} \qquad b = \frac{r^2 \tan \alpha}{4h}$$

where α is the angle between the obliquely cut off surface and the base surface.

Center of Gravity





Portion of Cylinder. — For a solid portion of a cylinder, as shown, the center of gravity is determined by:

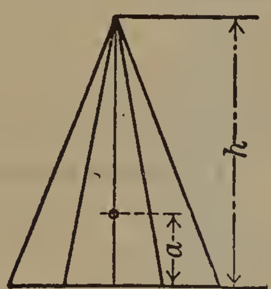
$$a = \tfrac{3}{16} \times 3.1416\, r \qquad b = \tfrac{3}{32} \times 3.1416\, h$$

For the cylindrical surface only:

$$a = \tfrac{1}{4} \times 3.1416\, r \qquad b = \tfrac{1}{8} \times 3.1416\, h$$

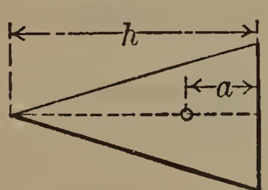
If the cylinder is hollow, the center of gravity of the solid shell is found by:

$$a = \tfrac{3}{16} \times 3.1416\, \frac{R^4 - r^4}{R^3 - r^3}; \quad b = \tfrac{3}{32} \times 3.1416\, \frac{H^4 - h^4}{H^3 - h^3}$$



Pyramid. — In a solid pyramid the center of gravity is located on the line joining the apex with the center of gravity of the base surface, at a distance from the base equal to one-quarter of the height; or $a = \frac{1}{4} h$.

The center of gravity of the triangular surfaces forming the pyramid is located on the line joining the apex with the center of gravity of the base surface, at a distance from the base equal to one-third of the height; or $a = \frac{1}{3} h$.

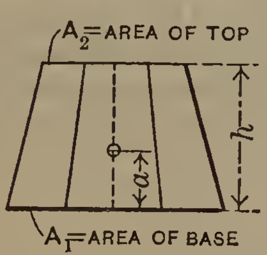


Cone. — The same rules apply as for the pyramid. For the solid cone:

$$a = \tfrac{1}{4} h$$

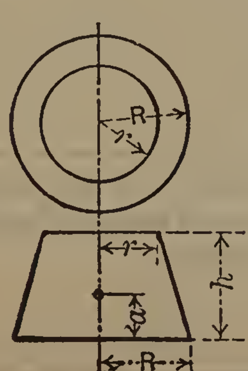
For the conical surface:

$$a = \tfrac{1}{3} h$$



Frustum of Pyramid. — The center of gravity is located on the line that joins the centers of gravity of the end surfaces. If A_1 = area of base surface, and A_2 area of top surface,

$$a = \frac{h\,(A_1 + 2\sqrt{A_1 \times A_2} + 3\,A_2)}{4\,(A_1 + \sqrt{A_1 \times A_2} + A_2)}.$$



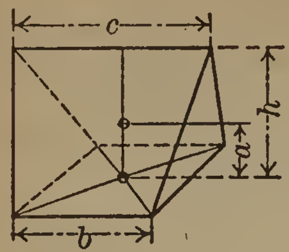
Frustum of Cone. — The same rules apply as for the frustum of a pyramid. For a solid frustum of a circular cone the formula below is also used:

$$a = \frac{h\,(R^2 + 2\,Rr + 3\,r^2)}{4\,(R^2 + Rr + r^2)}$$

The location of the center of gravity of the conical surface of a frustum of a cone is determined by:

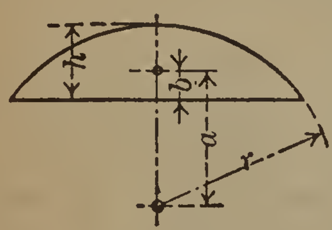
$$a = \frac{h\,(R + 2\,r)}{3\,(R + r)}$$

Center of Gravity



Wedge. — The center of gravity is on the line joining the center of gravity of the base with the middle point of the edge, and is located at:

$$a = \frac{h\,(b + c)}{2\,(2\,b + c)}$$

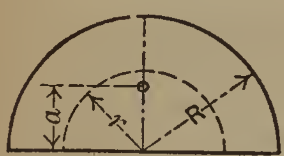


Spherical Segment. — The center of gravity of a solid segment is determined by:

$$a = \frac{3\,(2\,r - h)^2}{4\,(3\,r - h)}$$

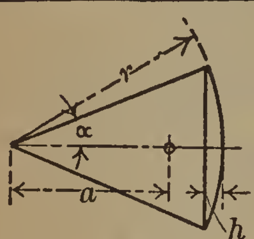
$$b = \frac{h\,(4\,r - h)}{4\,(3\,r - h)}$$

For a half-sphere, $a = b = \frac{3}{8}\,r$



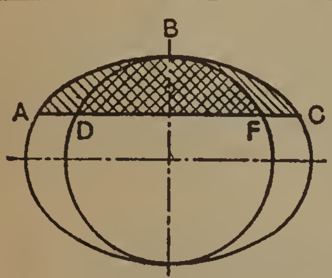
Half of a Hollow Sphere. — The center of gravity is located at:

$$a = \frac{3\,(R^4 - r^4)}{8\,(R^3 - r^3)}$$

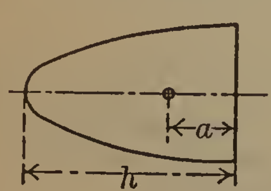


Spherical Sector. — The center of gravity of a solid sector is at:

$$a = \tfrac{3}{8}\,(1 + \cos\alpha)\,r = \tfrac{3}{8}\,(2\,r - h)$$

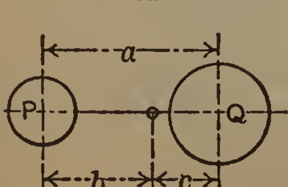


Segment of Ellipsoid or Spheroid. — The center of gravity of a solid segment ABC, symmetrical about the axis of rotation, coincides with the center of gravity of the segment DBF of a sphere, the diameter of which is equal to the axis of rotation of the spheroid.



Paraboloid. — The center of gravity of a solid paraboloid of rotation is at:

$$a = \tfrac{1}{3}\,h$$



Center of Gravity of Two Bodies. — If the weights of the bodies are P and Q, and the distance between their centers of gravity is a, then:

$$b = \frac{Qa}{P + Q} \qquad c = \frac{Pa}{P + Q}$$

Center of Gravity of Figures of any Outline. — If the figure is symmetrical about a center line, as in Fig. 1, the center of gravity will be located on that line. To find the exact location on that line, the simplest method is by taking moments with reference to any convenient axis at right angles to this center line. Divide the area into geometrical figures, the centers of gravity of which can be easily found. In this case, divide the figure into three rectangles *KLMN*, *EFGH* and *OPRS*. Call the areas of these rectangles *A*, *B* and *C*, respectively, and find the center of gravity of each. Then select any convenient axis, as *XX*, at right angles to the center line *YY*, and determine distances *a*, *b* and *c*. The distance *y* of the center of gravity of the complete figure from the axis *XX* is then found from the equation:

$$y = \frac{Aa + Bb + Cc}{A + B + C}$$

As an example, assume that the area *A* is 24 square inches, *B*, 14 square inches,

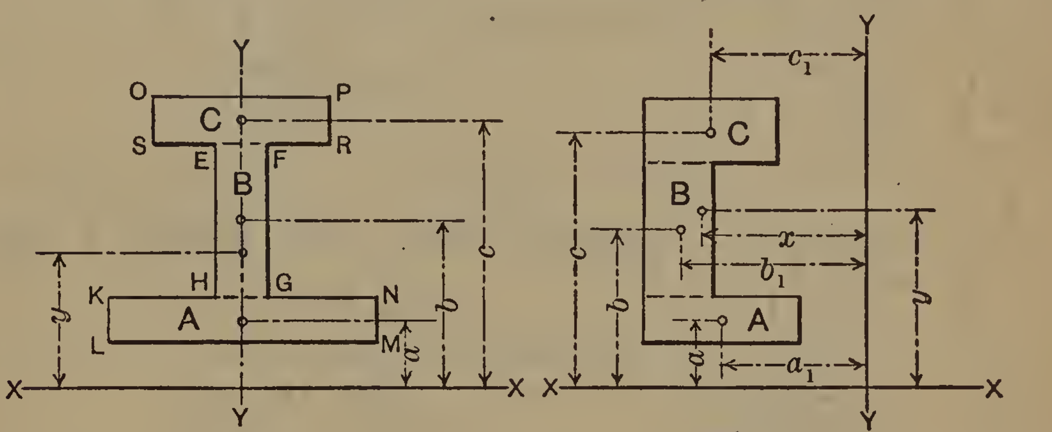


Fig. 1 Fig. 2

and *C*, 16 square inches, and that $a = 3$ inches, $b = 7.5$ inches, and $c = 12$ inches. Then:

$$y = \frac{24 \times 3 + 14 \times 7.5 + 16 \times 12}{24 + 14 + 16} = \frac{369}{54} = 6.83 \text{ inches.}$$

If the figure, the center of gravity of which is to be found, is not symmetrical about any axis, then moments must be taken with relation to two axes *XX* and *YY*, as shown in Fig. 2. The figure is divided into convenient geometrical figures, the centers of gravity of which can be easily found, the same as before. The center of gravity is determined by the equations:

$$x = \frac{Aa_1 + Bb_1 + Cc_1}{A + B + C} \qquad y = \frac{Aa + Bb + Cc}{A + B + C}$$

As an example, let $A = 14$ square inches, $B = 18$ square inches, and $C = 20$ square inches. Let $a = 3$ inches, $b = 7$ inches, and $c = 11.5$ inches. Let $a_1 = 6.5$ inches, $b_1 = 8.5$ inches, and $c_1 = 7$ inches. Then:

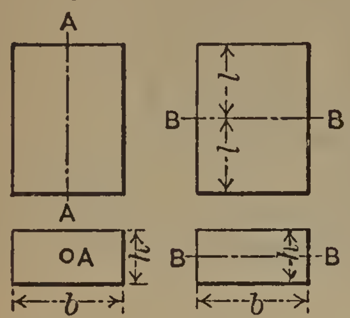
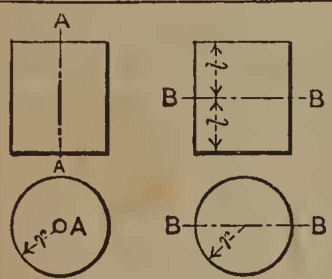
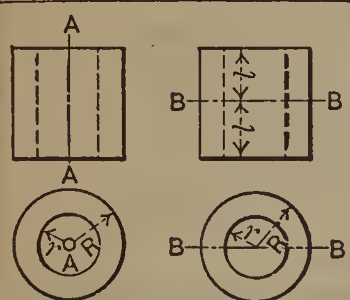
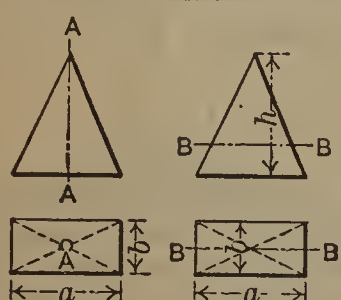
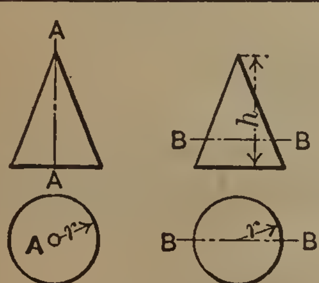
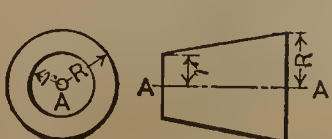
$$x = \frac{14 \times 6.5 + 18 \times 8.5 + 20 \times 7}{14 + 18 + 20} = \frac{384}{52} = 7.38 \text{ inches.}$$

$$y = \frac{14 \times 3 + 18 \times 7 + 20 \times 11.5}{14 + 18 + 20} = \frac{398}{52} = 7.65 \text{ inches.}$$

In other words, the center of gravity is located at a distance of 7.65 inches from the axis *XX* and 7.38 inches from the axis *YY*.

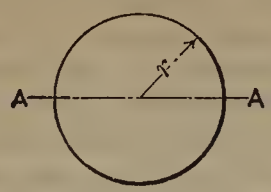
Moments of Inertia

(W = weight of body)

	*Prism.* — With reference to axis $A - A$: $I = \frac{W}{12}(h^2 + b^2)$ With reference to axis $B - B$: $I = W\left(\frac{l^2}{3} + \frac{h^2}{12}\right)$
	*Cylinder.* — With reference to axis $A - A$: $I = \frac{1}{2} Wr^2$ With reference to axis $B - B$: $I = W\left(\frac{l^2}{3} + \frac{r^2}{4}\right)$
	*Hollow Cylinder.* — With reference to axis $A - A$: $I = \frac{1}{2} W(R^2 + r^2)$ With reference to axis $B - B$: $I = W\left(\frac{l^2}{3} + \frac{R^2 + r^2}{4}\right)$
	*Pyramid,* rectangular base. — With reference to axis $A - A$: $I = \frac{W}{20}(a^2 + b^2)$ With reference to axis $B - B$ (through the center of gravity): $I = W\left(\frac{3}{80}h^2 + \frac{b^2}{20}\right)$
	*Cone.* — With reference to axis $A - A$: $I = \frac{3W}{10}r^2$ With reference to axis $B - B$ (through the center of gravity): $I = \frac{3W}{20}\left(r^2 + \frac{h^2}{4}\right)$
	*Frustum of Cone.* — With reference to axis $A - A$: $I = \frac{3W(R^5 - r^5)}{10(R^3 - r^3)}$

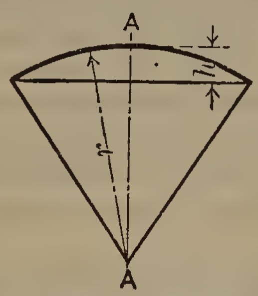
Moments of Inertia

(W = weight of body)



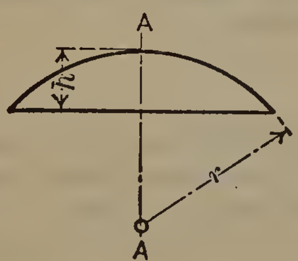
Sphere. — With reference to any axis through the center:

$$I = \tfrac{2}{5} Wr^2$$



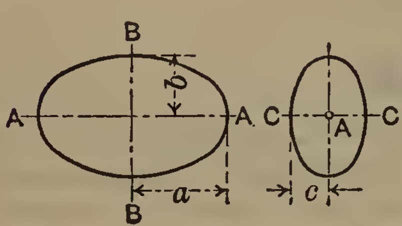
Spherical Sector. — With reference to axis $A-A$:

$$I = \frac{W}{5}(3\,rh - h^2)$$



Spherical Segment. — With reference to axis $A-A$:

$$I = W\left(r^2 - \frac{3\,rh}{4} + \frac{3\,h^2}{20}\right)\frac{2\,h}{3\,r-h}$$



Ellipsoid. — With reference to axis $A-A$:

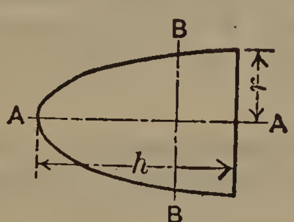
$$I = \frac{W}{5}(b^2 + c^2)$$

With reference to axis $B-B$:

$$I = \frac{W}{5}(a^2 + c^2)$$

With reference to axis $C-C$:

$$I = \frac{W}{5}(a^2 + b^2)$$

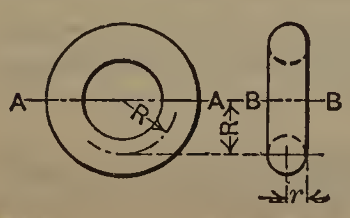


Paraboloid. — With reference to axis $A-A$:

$$I = \tfrac{1}{3} Wr^2$$

With reference to axis $B-B$ (through the center of gravity):

$$I = W\left(\frac{r^2}{6} + \frac{h^2}{18}\right)$$



Torus. — With reference to axis $A-A$:

$$I = W\left(\frac{R^2}{2} + \frac{5\,r^2}{8}\right)$$

With reference to axis $B-B$:

$$I = W\left(R^2 + \tfrac{3}{4} r^2\right)$$

Moment of Inertia. — The moment of inertia of a body, with respect to an axis, is the sum of the products obtained by multiplying the weights of each elementary particle by the square of its distance from the axis. Hence, the moment of inertia of the same body varies according to the position of the axis. It has its minimum value when the axis passes through the center of gravity. The moment of inertia is numerically equal to the weight of a body which, if it could be conceived of as concentrated at a distance of unity from the axis of rotation, would, if actuated by the same forces, rotate with the same angular velocity as that of the actual body. In other words, the moment of inertia bears the same relation to angular acceleration as weight does to linear acceleration. When the term moment of inertia is used in regard to areas, it is equal to the sum of the products obtained by multiplying each elementary area by the square of its distance from the axis. The moments of inertia of a number of solids are given on the two preceding pages. The moments of inertia of surfaces are especially useful in calculating the strength of beams, and the moments for a number of different cross-sections are given later in connection with the beam formulas.

If the moment of inertia I of a solid or surface, with respect to an axis through its center of gravity, is known, then the moment of inertia with respect to any parallel axis at a distance a from the axis through the center of gravity, is:

$$I_a = I + A \times a^2$$

in which A = the weight or area of the solid or figure of which the moment of inertia is to be found. For example, assume that the moment of inertia of a body weighing 3 pounds is 18 with reference to an axis through the center of gravity. Find the moment of inertia with reference to a parallel axis at a distance of 5 inches from the axis through the center of gravity.

$$I_a = 18 + 3 \times 5^2 = 18 + 75 = 93.$$

Motion, Force and Work

Motion is a progressive change of position of a body. Velocity is the rate of motion. When the velocity of a body is the same at every moment during which the motion takes place, the latter is called *uniform*. When the velocity is variable and constantly increasing, the rate at which it changes is called *acceleration;* that is, acceleration is the rate at which the velocity of a body changes in a unit of time, as the change in feet per second, in one second. When the motion is decreasing instead of increasing, it is called *retarded* motion, and the rate at which the motion is retarded is frequently called the *de-acceleration.* If the acceleration is uniform, the motion is called *uniformly accelerated* motion. An example of such motion is found in that of falling bodies.

Newton's Laws of Motion. — *First Law.* Every body continues in a state of rest or in uniform motion in a straight line, except if it is compelled by a force to change its state of rest or motion.

Second Law. If a body is acted upon by several forces, it is acted upon by each of these as if the others did not exist. This is true whether the body is at rest or in motion. In other words, if two or more forces act upon a body at the same time, each produces exactly the same effect as if it acted alone; the total effect or resultant motion of *all* the forces may be found by a diagram in the same way as the resultant of forces is found.

Third Law. To every action there is always an equal reaction, or, in other words, if a force acts to change the state of motion of a body, the body offers a resistance equal and directly opposite to the force.

Mass. — The *mass* of a body equals the weight divided by the acceleration due to gravity, or:

$$\text{Mass} = \frac{\text{weight}}{32.16}$$

General Formulas for Motion. — In the following formulas:

F = force in pounds;
S = space in feet;
T = time in seconds;
V = velocity in feet per second;
P = power in foot-pounds per second;
K = work in foot-pounds;
H.P. = horsepower.

The relations between these various quantities are given in the following formulas:

$$F = \frac{P}{V} = \frac{K}{S} = \frac{K}{VT} = \frac{550\,\text{H.P.}}{V}$$

$$S = VT = \frac{PT}{F} = \frac{K}{F} = \frac{550\,T\,\text{H.P.}}{F}$$

$$T = \frac{S}{V} = \frac{FS}{P} = \frac{K}{FV} = \frac{FS}{550\,\text{H.P.}}$$

$$V = \frac{S}{T} = \frac{P}{F} = \frac{K}{FT} = \frac{550\,\text{H.P.}}{F}$$

$$P = FV = \frac{FS}{T} = \frac{K}{T} = 550\,\text{H.P.}$$

$$K = FS = PT = FVT = 550\,T\,\text{H.P.}$$

$$\text{H.P.} = \frac{P}{550} = \frac{FV}{550} = \frac{FS}{550\,T} = \frac{K}{550\,T}$$

Example: — A casting weighing 300 pounds is to be lifted by means of an overhead crane. The casting is lifted 10 feet in 12 seconds. What is the horsepower developed? Here $F = 300$; $S = 10$; $T = 12$.

$$\text{H.P.} = \frac{F \times S}{550\,T} = \frac{300 \times 10}{550 \times 12} = 0.45.$$

Momentum. — The momentum of a moving body is equivalent to that constant force which would bring the body to rest in one second by resisting its movement.

$$\text{Momentum} = \text{mass} \times \text{velocity in feet per second}$$

$$= \frac{\text{weight}}{32.16} \times \text{velocity in feet per second.}$$

(Momentum should not be confused with the moment of a force, which is equal to the force multiplied by its lever arm.)

Falling Bodies

Under the influence of gravity alone, all bodies fall to the earth with the same velocity and with the same acceleration. The value of the acceleration is commonly denoted by the letter g. The acceleration increases with the latitude and decreases with the elevation above the level of the sea. Its value at the level of the sea in the latitude of New York is 32.16 feet per second. (In the metric system g equals 9.81 meters per second at 45 degrees latitude and sea level.)

Formulas for Accelerated Motion. — In the following formulas:

V = velocity in feet per second of a falling body at the end of time T;
T = time in seconds the body is falling;
S = space in feet which the falling body passes through in time T;
u = space in feet which the body falls in the Tth second;
g = acceleration due to gravity.

$$V = gT = \frac{2S}{T} = \sqrt{2gS} = 8.02\sqrt{S}$$

$$S = \frac{1}{2}gT^2 = \frac{1}{2}VT = \frac{V^2}{2g} = \frac{V^2}{64.32}$$

$$T = \frac{V}{g} = \frac{2S}{V} = \sqrt{\frac{2S}{g}} = \frac{\sqrt{S}}{4.01} = \frac{u}{g} + \frac{1}{2}$$

$$u = g(T - \tfrac{1}{2})$$

Examples: What velocity has a body attained after having fallen for a time of 5 seconds?

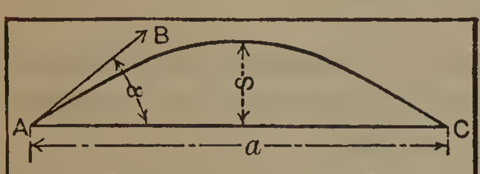
$$V = gT = 32.16 \times 5 = 160.8 \text{ feet per second.}$$

A metal ball falls from the top of a tower 300 feet high. What length of time will be required before it reaches the ground?

$$T = \sqrt{\frac{2S}{g}} = \sqrt{\frac{2 \times 300}{32.16}} = \sqrt{18.66} = 4.32 \text{ seconds.}$$

What is the velocity of the ball in the previous example when it reaches the ground?

$$V = \sqrt{2gS} = \sqrt{2 \times 32.16 \times 300} = \sqrt{19{,}296} = 139 \text{ feet, nearly.}$$



Bodies thrown at an Angle with the Vertical. — When a body is thrown in a direction other than vertical, it describes a parabolic curve. If V (represented by line AB) is the velocity at which the body is thrown from a given point, T the time it requires to pass from A to C, and the other quantities are as noted in the accompanying engraving, then:

$$a = \frac{2V^2 \sin\alpha \cos\alpha}{g}; \qquad T = \frac{2V \sin\alpha}{g}; \qquad S = \frac{V^2 \sin^2\alpha}{2g}.$$

Example: A bullet is fired in the air at an angle of 45 degrees. The initial velocity is 600 feet per second. How far will it pass before striking the ground?

$$a = \frac{2 \times 600^2 \times 0.707 \times 0.707}{g} = 11{,}200 \text{ feet, approx.}$$

The distance a projectile will travel is greatest when its initial direction is at an angle of 45 degrees with the horizontal.

Retarded Motion. — When a body is thrown vertically upwards, its motion will be retarded in the same ratio as it is accelerated when falling, the retardation or de-acceleration being 32.16 feet per second. In the following formulas: V = velocity at which the body begins to ascend; T = time in seconds passing before the body reaches the highest point, or the point of return; t = any time less than T; S = height in feet to which the body will ascend before reaching its greatest height; s = space body ascends in time t; v = velocity at the end of time t.

$$V = v + gt = \frac{s}{t} + \frac{gt}{2} = \sqrt{v^2 + 2\,gs}$$

$$v = V - gt = \frac{s}{t} - \frac{gt}{2} = \sqrt{V^2 - 2\,gs}$$

$$s = t\left(V - \frac{gt}{2}\right) = t\left(v + \frac{gt}{2}\right)$$

$$t = \frac{V - v}{g} = \frac{V}{g} - \sqrt{\frac{V^2}{g^2} - \frac{2\,s}{g}}$$

The formulas for T and S are the same as for accelerated motion.

Examples: With what velocity should a ball start to ascend in order to strike an object 30 feet above with a velocity of 20 feet per second?

$$V = \sqrt{v^2 + 2\,gs} = \sqrt{20^2 + 2 \times 32.16 \times 30} = 48.27 \text{ feet.}$$

An object is thrown vertically into the air with a velocity of 100 feet per second. When does it reach the highest point and start to return?

$$T = \frac{V}{g} = \frac{100}{32.16} = 3.11 \text{ seconds.}$$

Energy. — A body is said to possess energy when it is capable of doing work or of overcoming resistance. The energy may be either *kinetic* or *potential*. Thus, energy possessed by a body on account of its motion is kinetic or actual energy, and it is expressed in foot-pounds. Potential or latent energy is the capacity for doing work possessed by a body on account of its position or chemical composition. For example, a weight that has been lifted to some point possesses potential energy, and when the weight falls, this potential energy is changed to kinetic energy. Water stored in a reservoir and the heat or chemical energy in fuel and gunpowder are other examples of potential energy. If E = energy in foot-pounds; M = mass (weight ÷ 32.16); V = velocity in feet per second; S = total space passed through in feet; and W = weight in pounds; then:

$$E = \frac{1}{2} MV^2 = \frac{WV^2}{2\,g} = \frac{WV^2}{64.32}$$

For a falling body, acted upon by gravity alone, $E = SW$.

As an example of the use of the formulas involving velocities of falling bodies and energy, the following example is given: A projectile is fired from a 12-inch gun vertically into the air. It strikes the ground, coming down, exactly 1 minute and 40 seconds after leaving the muzzle. Disregarding air resistance, what height did the projectile reach? What was its velocity when leaving the muzzle? And what is the energy of the projectile when it strikes the ground, if its weight is assumed to be 600 pounds?

The time required for the projectile to reach its greatest height is one-half of the total time for the upward and downward journey. Thus, in 50 seconds, the projectile has reached the point where its velocity is zero, and where it begins to fall.

Hence, using the formulas given for falling bodies:

$$S = \frac{32.16 \times 50^2}{2} = \frac{32.16 \times 2500}{2} = 40,200 \text{ feet,}$$

or

$$\frac{40,200}{5,280} = 7.6 \text{ miles, approximately.}$$

The velocity of the projectile when leaving the muzzle is the same as the velocity acquired when again reaching the ground. This velocity is found by the formula:

$$V = gT = 32.16 \times 50 = 1608 \text{ feet per second.}$$

The energy of the projectile when it strikes the ground equals its weight multiplied by the distance through which it has fallen. If W = weight, and E = energy, we have:

$$E = W \times S = 600 \times 40,200 = 24,120,000 \text{ foot-pounds;}$$

or by another formula for the energy:

$$E = \frac{WV^2}{2g} = \frac{600 \times 1608^2}{2 \times 32.16} = \frac{600 \times 2,585,664}{2 \times 32.16} = 24,120,000 \text{ foot-pounds.}$$

Motion on Inclined Planes. — When a body descends an inclined plane, actuated by the force of gravity, its velocity, friction being neglected, is equal to that acquired by a body falling freely the height of the plane. The formulas for uniformly accelerated motion apply to motion on inclined planes.

V = velocity of body sliding down an inclined plane, in feet per second, at end of time T;
S = space traversed in feet;
T = time in seconds;
α = the angle of the plane with the horizontal;
g = acceleration due to gravity = 32.16.

$$V = gT \sin\alpha = \sqrt{2\,gS \sin\alpha}$$

$$S = \frac{gT^2 \sin\alpha}{2} = \frac{V^2}{2\,g \sin\alpha}$$

$$T = \frac{V}{g \sin\alpha} = \sqrt{\frac{2\,S}{g \sin\alpha}}$$

Force of a Blow. — The energy of a body raised to a given height and permitted to fall, as in the case of a drop hammer, is equal to the weight multiplied by the height through which it falls. Hence, the force of a blow cannot be expressed directly in pounds, but the energy with which a hammer will strike a piece of work can be expressed in foot-pounds. The average force of the blow, then, is equal to the number of foot-pounds, divided by the amount of the penetration. When the force of a blow is calculated, the weight of the falling body should always be added to the energy due to the fall. If W = weight of falling body in pounds; S = the height through which it has fallen in feet; and d = distance in feet the object struck is moved (or penetrated), then:

$$\text{average force of blow} = \frac{WS}{d} + W.$$

Example 1: — A pile driver weighing 200 pounds strikes the top of the pile after having fallen from a height of 20 feet. It forces the pile into the ground a distance of 6 inches. Before the ram is brought to rest, it will then have performed $200 \times 20 = 4000$ foot-pounds of work, and as this energy is expended in a distance of one-half foot, the average force of the blow equals $4000 \div \frac{1}{2} + 200 = 8200$ pounds.

Example 2: — If, upon reaching the ground, the projectile in the example given on page 277, buries itself to a depth of 8 feet, what is the average force of the blow with which it strikes the ground?

$$F = \frac{E}{d} + W = \frac{24,120,000}{8} + 600 = 3,015,600 \text{ pounds.}$$

Center and Radius of Oscillation. — If a body oscillates about a horizontal axis which does not pass through its center of gravity, there will be a point on the line drawn from the center of gravity, perpendicular to the axis, the motion of which will be the same as if the whole mass were concentrated at that point. This point is called the *center of oscillation*. The *radius of oscillation* is the distance between the center of oscillation and the point of suspension. In a straight line, or in a bar of small diameter, suspended at one end and oscillating about it, the center of oscillation is at two-thirds the length of the rod from the end by which it is suspended.

When the vibrations are perpendicular to the plane of the figure, and the figure is suspended by the vertex of an angle or its uppermost point, the radius of oscillation of an isosceles triangle is equal to ¾ of the height of the triangle; of a circle, ⅝ of the diameter; of a parabola, 5/7 of the height.

If the vibrations are in the plane of the figure, then the radius of oscillation of a circle equals ¾ of the diameter; of a rectangle, suspended at the vertex of one angle, ⅔ of the diagonal.

Center of Percussion. — If a body oscillates about an axis, then the point at which, if a blow is struck by the body, the percussive action is the same as if the whole mass of the body were concentrated at that point, is called the *center of percussion.* This point is located at the same point as the center of oscillation.

Center and Radius of Gyration. — The center of gyration with reference to an axis is the point at which the entire weight of a body may be considered as concentrated, the moment of inertia, meanwhile, remaining unchanged; or, in a revolving body, the center of gyration is the point at which the whole weight of the body may be considered as concentrated, the angular velocity remaining the same. The *radius of gyration* is the distance from this point to the axis of rotation. If W is the weight of a body; I, its moment of inertia; and k, the radius of gyration, then:

$$k = \sqrt{\frac{I}{W}} \quad \text{and} \quad I = Wk^2.$$

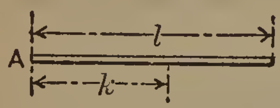
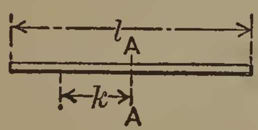
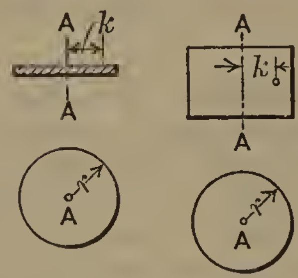
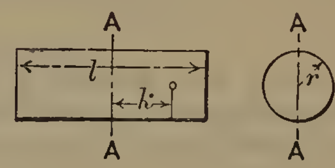
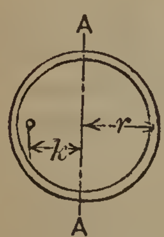
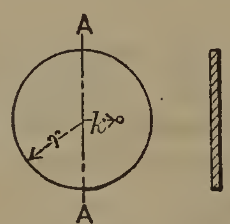
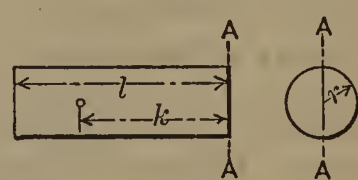
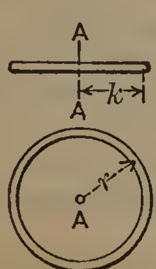
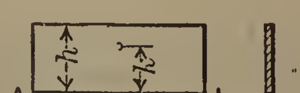
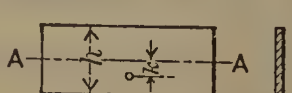
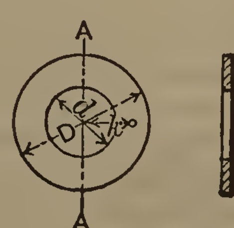
To find the radius of gyration of an area, as the cross-section of a beam, divide the moment of inertia of the area by the area and extract the square root.

The square of the radius of gyration of an oscillating body is equal to the product of the radius of oscillation multiplied by the distance of the center of gravity of the suspended body from the point of suspension.

When the axis, with reference to which the radius of gyration is taken, passes through the center of gravity, the radius of gyration is the least possible and is called the *principal* radius of gyration.

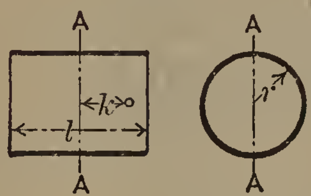
For a solid cylindrical body, like a disk or emery wheel, the radius of gyration is equal to the radius of the disk divided by $\sqrt{2}$ (= radius $\times$ 0.707). For a flywheel rim, it is sufficiently accurate to assume the radius of gyration to be the distance from the center to a point halfway between the outer and inner edges of the rim.

Radius of Gyration

Bar of Small Diameter. Axis at end.  $k = 0.5773\,l$ $k^2 = \frac{1}{3}\,l^2$ Axis at center.  $k = 0.2886\,l$ $k^2 = \frac{1}{12}\,l^2$	Thin Circular Disk. Axis through center. Cylinder. Axis through center.  $k = 0.7071\,r$ $k^2 = \frac{1}{2}\,r^2$	Cylinder. Axis, diameter at mid-length.  $k = 0.289\sqrt{l^2 + 3\,r^2}$ $k^2 = \frac{l^2}{12} + \frac{r^2}{4}$
Bar of Small Diameter, bent to Circular Shape. Axis, a diameter of the ring.  $k = 0.7071\,r$ $k^2 = \frac{1}{2}\,r^2$	Thin Circular Disk. Axis its diameter.  $k = \frac{1}{2}\,r$ $k^2 = \frac{1}{4}\,r^2$	Cylinder. Axis, diameter at end.  $k = 0.289\sqrt{4\,l^2 + 3\,r^2}$ $k^2 = \frac{l^2}{3} + \frac{r^2}{4}$
Bar of Small Diameter, bent to Circular Shape. Axis through center of ring.  $k = r$; $k^2 = r^2$	Parallelogram (Thin flat plate). Axis at base.  $k = 0.5773\,h$; $k^2 = \frac{1}{3}\,h^2$ Axis at mid-height.  $k = 0.2886\,h$; $k^2 = \frac{1}{12}\,h^2$	Thin, Flat, Circular Ring. Axis its diameter.  $k = \frac{1}{4}\sqrt{D^2 + d^2}$ $k^2 = \frac{D^2 + d^2}{16}$

Radius of Gyration

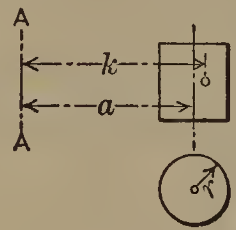
Thin Hollow Cylinder.
Axis, diameter at mid-length.



$$k = 0.289\sqrt{l^2 + 6r^2}$$

$$k^2 = \frac{l^2}{12} + \frac{r^2}{2}$$

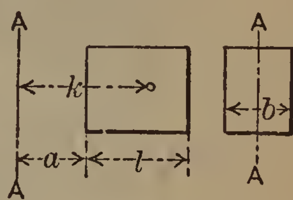
Cylinder.
Axis at a distance.



$$k = \sqrt{a^2 + \tfrac{1}{2}r^2}$$

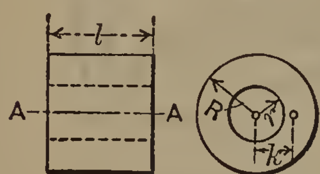
$$k^2 = a^2 + \tfrac{1}{2}r^2$$

Parallelepiped.
Axis at distance from end.



$$k = \sqrt{\frac{4l^2 + b^2}{12} + a^2 + al}$$

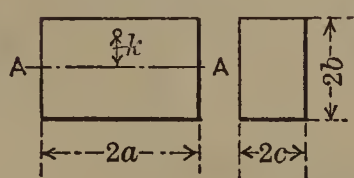
Hollow Cylinder.
Longitudinal Axis.



$$k = 0.7071\sqrt{R^2 + r^2}$$

$$k^2 = \tfrac{1}{2}(R^2 + r^2)$$

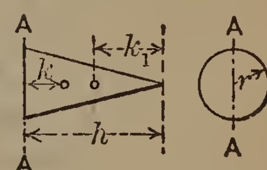
Rectangular Prism.
Axis through center.



$$k = 0.577\sqrt{b^2 + c^2}$$

$$k^2 = \tfrac{1}{3}(b^2 + c^2)$$

Cone.
Axis at base.

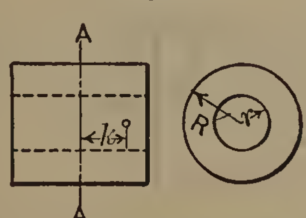


$$k = \sqrt{\frac{2h^2 + 3r^2}{20}}$$

Axis at apex.

$$k_1 = \sqrt{\frac{12h^2 + 3r^2}{20}}$$

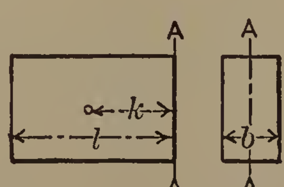
Hollow Cylinder.
Axis, diameter at mid-length.



$$k = 0.289\sqrt{l^2 + 3(R^2 + r^2)}$$

$$k^2 = \frac{l^2}{12} + \frac{R^2 + r^2}{4}$$

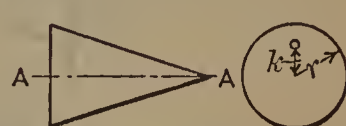
Parallelepiped.
Axis at one end, central.



$$k = 0.289\sqrt{4l^2 + b^2}$$

$$k^2 = \frac{4l^2 + b^2}{12}$$

Cone.
Axis through its center line.

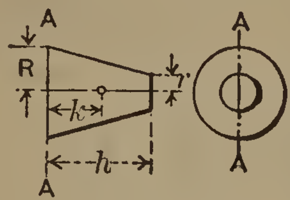


$$k = 0.5477\,r$$

$$k^2 = 0.3\,r^2$$

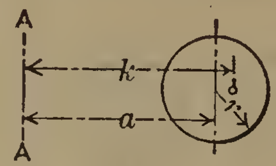
Radius of Gyration

Frustum of Cone.
Axis at large end.



$$k=\sqrt{\frac{h}{10}\left(\frac{R^2+3\,Rr+r^2}{R^2+Rr+r^2}\right)+\frac{3}{20}\left(\frac{R^5-r^5}{R^3-r^3}\right)}$$

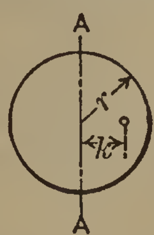
Sphere.
Axis at a distance.



$k=\sqrt{a^2+\frac{2}{5}r^2}$

$k^2=a^2+\frac{2}{5}r^2$

Sphere.
Axis its diameter.

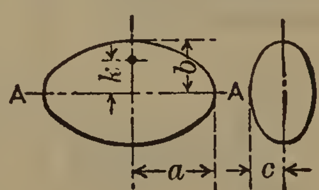


$k=0.6325\,r;\quad k^2=\frac{2}{5}r^2$

Thin Spherical Shell.

$k=0.8165\,r;\quad k^2=\frac{2}{3}r^2$

Ellipsoid.
Axis through center.



$k=0.447\sqrt{b^2+c^2}$

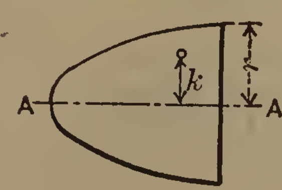
$k^2=\frac{1}{5}(b^2+c^2)$

Hollow Sphere.
Axis its diameter.

$$k=0.6325\sqrt{\frac{R^5-r^5}{R^3-r^3}}$$

$$k^2=\frac{2\,(R^5-r^5)}{5\,(R^3-r^3)}$$

Paraboloid.
Axis through center.



$k=0.5773\,r$

$k^2=\frac{1}{3}r^2$

Formulas for Rotary Motion

In the following formulas:

F = force in pounds, acting in the direction of the tangent;
P = power in foot-pounds, per second;
V = velocity in feet per second;
S = distance passed through in feet by point of application of force F;
T = time in seconds;
K = work in foot-pounds;
R = radius of revolution (radius to point of application of force F) in feet;
n = revolutions per minute;
N = total number of revolutions in time T;
H.P. = horsepower.

$$F = \frac{60\,P}{2\,\pi Rn} = \frac{9.55\,P}{Rn} = \frac{9.55\,K}{RnT} = \frac{5252\text{ H.P.}}{Rn}$$

$$P = \frac{2\,\pi RnF}{60} = \frac{RnF}{9.55} = \frac{FRN}{0.159\,T}$$

$$V = \frac{2\,\pi Rn}{60} = 0.1047\,Rn$$

$$S = \frac{2\,\pi RnT}{60} = \frac{RnT}{9.55} = 2\,\pi RN$$

$$T = \frac{60\,S}{2\,\pi Rn} = \frac{9.55\,S}{Rn} = \frac{9.55\,K}{FRn} = \frac{FRN}{87.5\text{ H.P.}}$$

$$K = \frac{2\,\pi FRnT}{60} = \frac{FRnT}{9.55} = 2\,\pi RNF$$

$$R = \frac{60\,V}{2\,\pi n} = \frac{9.55\,V}{n} = \frac{9.55\,P}{Fn} = \frac{5252\text{ H.P.}}{Fn} = \frac{9.55\,K}{FnT}$$

$$n = \frac{60\,V}{2\,\pi R} = \frac{9.55\,V}{R} = \frac{5252\text{ H.P.}}{FR}$$

$$N = \frac{87.5\,T\text{ H.P.}}{FR} = \frac{S}{2\,\pi R} = \frac{K}{2\,\pi RF} = \frac{9.55\,PT}{FR}$$

$$\text{H.P.} = \frac{FRn}{5252} = \frac{FRN}{87.5\,T}$$

Examples: — A pulley, 36 inches (3 feet) in diameter, makes 110 revolutions per minute. The pull of the belt on the pulley is 200 pounds. Find the horsepower transmitted. Here $F = 200$; $R = 1.5$; $n = 110$.

$$\text{H.P.} = \frac{FRn}{5252} = \frac{200 \times 1.5 \times 110}{5252} = 6.3.$$

What is the force exerted at the periphery of a gear 18 inches in diameter, revolving at 60 revolutions per minute and transmitting 4 horsepower? Here $R = 0.75$; $n = 60$.

$$F = \frac{5252 \times \text{H.P.}}{Rn} = \frac{5252 \times 4}{0.75 \times 60} = 467\text{ pounds.}$$

General Formulas for Accelerated or Retarded Motion

In the following formulas:

F = force in pounds causing acceleration, acting on a body free to move;
T = time in seconds during which the force acts on the body;
G = constant acceleration in feet per second, due to the force F;
V = final velocity at the end of time T, or uniform velocity of a moving body;
S = distance in feet passed through while the force acts on the body;
W = force required to move body at uniform velocity, in pounds;
P = average power exerted during time T, in foot-pounds per second;
K = work in foot-pounds (or energy) concentrated in the moving body;
g = acceleration due to gravity = 32.16.

$$F = \frac{GW}{g} = \frac{VW}{gT} = \frac{2\,WS}{gT^2} = \frac{WV^2}{2\,gS} = \frac{PT}{S} = \frac{2\,K}{GT^2} = \frac{K}{S} = \sqrt{\frac{2\,PW}{gT}}$$

$$T = \frac{V}{G} = \frac{VW}{gF} = \sqrt{\frac{2\,WS}{gF}} = \sqrt{\frac{2\,S}{G}} = \frac{K}{P} = \sqrt{\frac{2\,WK}{gF^2}}$$

$$G = \frac{gF}{W} = \frac{V}{T} = \frac{V^2}{2\,S} = \frac{2\,S}{T^2} = \frac{gPT}{WS} = \frac{gK}{WS} = \frac{2\,K}{FT^2}$$

$$V = GT = \sqrt{2\,GS} = \frac{2\,S}{T} = \frac{gFT}{W} = \sqrt{\frac{2\,gPT}{W}} = \sqrt{\frac{2\,gK}{W}} = \sqrt{\frac{2\,gSF}{W}}$$

$$S = \frac{VT}{2} = \frac{V^2}{2\,G} = \frac{PT}{F} = \frac{GT^2}{2} = \frac{K}{F} = \frac{gK}{GW} = \frac{gFT^2}{2\,W}$$

$$W = \frac{gF}{G} = \frac{gFT}{V} = \frac{2\,gK}{V^2} = \frac{2\,gFS}{V^2} = \frac{gFT^2}{2\,S} = \frac{gPT^3}{2\,S^2} = \frac{gT^2K}{2\,S^2}$$

$$P = \frac{FS}{T} = \frac{K}{T} = \frac{FV^2}{2\,GT} = \frac{WV^2}{2\,gT} = \frac{2\,WS^2}{gT^3} = \frac{gF^2T}{2\,W}$$

$$K = FS = \frac{FVT}{2} = \frac{FGT^2}{2} = PT = \frac{WV^2}{2\,g} = \frac{GWVT}{2\,g} = \frac{gF^2T^2}{2\,W}$$

Examples: — A body weighing 300 pounds is to be lifted. The velocity with which it is to be hoisted is 3 feet per second, and this velocity is to be reached in 5 seconds. Find the force required for hoisting during the time the motion is accelerated. Here $W = 300$; $V = 3$; $T = 5$. Then, force required for acceleration:

$$F = \frac{VW}{gT} = \frac{3 \times 300}{32.16 \times 5} = 5.6 \text{ pounds.}$$

To this is added the force required to hold the body suspended, or 300 pounds; $300 + 5.6 = 305.6$ pounds.

The force required to move a crane weighing 5000 pounds upon its runways at a uniform speed of 10 feet per second is 200 pounds. What is the force required to start the crane, if this speed is to be acquired in 6 seconds? Here $W = 5000$; $V = 10$; $T = 6$. Then:

$$F = \frac{VW}{gT} = \frac{5000 \times 10}{32.16 \times 6} = 260 \text{ pounds, approx.}$$

To this must be added the 200 pounds required to move the crane at a uniform speed of 10 feet per second, making the total force required during acceleration equal to $260 + 200 = 460$ pounds.

Formulas for Accelerated Rotary Motion

In the following formulas:

F = force in pounds, acting tangentially at end of radius R;
R = radius at end of which F is applied, in feet;
k = radius of gyration in feet;
T = time of acceleration in seconds;
n = revolutions per minute at end of time T;
N = total number of revolutions in time T;
W = weight of revolving body in pounds;
K = work in foot-pounds (or energy) concentrated in revolving body.

From the general formulas for accelerated or retarded motion on the preceding page:

$$F = \frac{VW}{gT}, \text{ from which, for rotary motion, } F = \frac{2\pi knW}{60\,gT}$$

and

$$FR = \frac{Wk \times 2\pi kn}{60\,gT} = \frac{W \times 2\pi k^2 n}{60\,gT}$$

Assuming $g = 32.16$, we have, by substitution:

$$F = \frac{0.00326\,Wnk^2}{TR} = \frac{0.391\,WNk^2}{T^2R} = \frac{60\,K}{3.1416\,RnT} = \frac{K}{6.2832\,RN}$$

$$T = \frac{0.00326\,Wnk^2}{FR} = \sqrt{\frac{0.391\,WNk^2}{FR}} = \frac{60\,K}{3.1416\,RnF} = \frac{0.25\,k\sqrt{WK}}{FR}$$

$$n = \frac{120\,N}{T} = \frac{307.1\,FTR}{Wk^2} = \frac{60\,K}{3.1416\,RTF} = \frac{76.6}{k}\sqrt{\frac{K}{W}}$$

$$N = \frac{Tn}{120} = \frac{2.559\,FT^2R}{Wk^2} = \frac{K}{6.2832\,RF} = \frac{0.64\,T}{k}\sqrt{\frac{K}{W}}$$

$$k = \sqrt{\frac{307.1\,FRT}{WN}} = \sqrt{\frac{2.559\,FRT^2}{WN}} = \frac{0.256\,KT}{N\sqrt{WNFR}} = \frac{337\,K}{n\sqrt{WnTFR}}$$

Example: — A circular disk 18 inches in diameter and weighing 12 pounds revolves about its axis. The revolving force is applied at the end of a lever of 8 inch radius. Find the force required to bring the disk to a speed of 300 revolutions per minute in 10 seconds, friction being neglected.

$$W = 12; \quad R = \tfrac{2}{3}; \quad n = 300; \quad T = 10; \quad k = 0.707 \times \tfrac{3}{4} = 0.53.$$

$$F = \frac{0.00326\,Wnk^2}{TR} = \frac{0.00326 \times 12 \times 300 \times 0.53^2}{10 \times \frac{2}{3}} = 0.49 \text{ pound.}$$

Example: — A flywheel, having an outside diameter of 7 feet and weighing 3000 pounds, is to be brought from a condition of rest to a speed of 200 R.P.M. A belt pull of 100 pounds is available at the periphery of the flywheel rim for this purpose; this belt pull is required entirely for acceleration. If the radius of gyration is assumed to be equal to the mean radius of the flywheel rim, which is 3 feet 3 inches, what time will be required (from rest) to obtain a speed of 200 R.P.M.?

$$F = 100; \quad R = 3.5; \quad k = 3.25; \quad n = 200; \quad W = 3000.$$

Then, time required for acceleration:

$$T = \frac{0.00326\,Wnk^2}{FR} = \frac{0.00326 \times 3000 \times 200 \times 3.25^2}{100 \times 3.5} = 59 \text{ seconds.}$$

Centrifugal Force

Centrifugal Force. — When a body revolves in a curved path, it exerts a force called the *centrifugal force* upon the arm or cord which restrains it from moving in a straight (tangential) line. In the following formulas:

F = centrifugal force in pounds;
W = weight of revolving body in pounds;
v = velocity of revolving body in feet per second;
R = radius of the circle in which the body revolves, (or, in general, radius of gyration), in feet;
n = number of revolutions per minute;
g = acceleration due to gravity = 32.16.

$$F = \frac{Wv^2}{gR} = \frac{Wv^2}{32.16\,R} = \frac{4\,WR\pi^2n^2}{60 \times 60\,g} = \frac{WRn^2}{2933} = 0.000341\,WRn^2$$

$$W = \frac{FRg}{v^2} = \frac{2933\,F}{Rn^2} \qquad v = \sqrt{\frac{FRg}{W}}$$

$$R = \frac{Wv^2}{Fg} = \frac{2933\,F}{Wn^2} \qquad n = \sqrt{\frac{2933\,F}{WR}}$$

(If n is the number of revolutions per second instead of per minute, then $F = 1.227\,WRn^2$.)

For a thin disk or cylinder (emery-wheel, grindstone, etc.), rotating about its center, the resultant of all the centrifugal or radial forces that tend to rupture the disk equals $0.0000767\,WRn^2$, if R is the outside radius of the disk.

Calculating Centrifugal Force. — In the ordinary formula for centrifugal force, $F = 0.000341\,WRn^2$; the mean radius R of the flywheel or pulley rim is given in feet. For small dimensions, it is more convenient to have the formula in the form:

$$F = 0.000028416\,Wrn^2$$

in which F = centrifugal force, in pounds; W = weight of rim, in pounds; r = mean radius of rim, in inches; n = number of revolutions per minute.

In this formula let $C = 0.000028416\,n^2$. This, then, is the centrifugal force of one pound, one inch from the axis. The formula can now be written in the form,

$$F = WrC$$

C is calculated for various values of the revolutions per minute n, and the calculated values of C are tabulated in the accompanying "Table for Calculating Centrifugal Force." To find the centrifugal force in any given case, simply find the value of C in the table and multiply it by the product of W and r, the four multiplications in the original formula given thus having been reduced to two.

Example. — A cast-iron pulley having a mean rim radius of 8 inches and a rim area of 4½ square inches, is to revolve 460 revolutions per minute. Determine the centrifugal force.

The value of C as given in the table for a speed of 460 R.P.M., is 6 approximately; as a cubic inch of cast iron weighs 0.26 pound, the rim weight = $16 \times 3.1416 \times 4\frac{1}{2} \times 0.26$ = 60 pounds nearly; therefore

$$F = 60 \times 8 \times 6 = 2880 \text{ pounds.}$$

Table for Calculating Centrifugal Force

n	C	n	C	n	C	n	C
50.0	0.07104	75	0.15984	125	0.44400	200	1.1367
50.5	0.07247	76	0.16413	126	0.45113	202	1.1595
51.0	0.07391	77	0.16848	127	0.45832	204	1.1826
51.5	0.07537	78	0.17288	128	0.46557	206	1.2059
52.0	0.07684	79	0.17734	129	0.47287	208	1.2294
52.5	0.07832	80	0.18186	130	0.48023	210	1.2531
53.0	0.07982	81	0.18644	131	0.48765	212	1.2771
53.5	0.08133	82	0.19107	132	0.49512	214	1.3013
54.0	0.08286	83	0.19576	133	0.50265	216	1.3258
54.5	0.08440	84	0.20050	134	0.51023	218	1.3505
55.0	0.08596	85	0.20530	135	0.51788	220	1.3753
55.5	0.08753	86	0.21016	136	0.52558	222	1.4005
56.0	0.08911	87	0.21508	137	0.53334	224	1.4258
56.5	0.09071	88	0.22005	138	0.54115	226	1.4514
57.0	0.09232	89	0.22508	139	0.54902	228	1.4772
57.5	0.09395	90	0.23017	140	0.55695	230	1.5032
58.0	0.09559	91	0.23531	141	0.56493	235	1.5693
58.5	0.09725	92	0.24051	142	0.57298	240	1.6358
59.0	0.09892	93	0.24577	143	0.58108	245	1.7057
59.5	0.10060	94	0.25108	144	0.58923	250	1.7760
60.0	0.10230	95	0.25645	145	0.59744	255	1.8478
60.5	0.10401	96	0.26188	146	0.60571	260	1.9209
61.0	0.10573	97	0.26737	147	0.61404	265	1.9955
61.5	0.10748	98	0.27291	148	0.62242	270	2.0715
62.0	0.10923	99	0.27851	149	0.63086	275	2.1490
62.5	0.11100	100	0.28416	150	0.63936	280	2.2278
63.0	0.11278	101	0.28987	152	0.65652	285	2.3081
63.5	0.11458	102	0.29564	154	0.67391	290	2.3898
64.0	0.11639	103	0.30147	156	0.69169	295	2.4729
64.5	0.11822	104	0.30735	158	0.70928	300	2.5574
65.0	0.12006	105	0.31328	160	0.72745	310	2.7308
65.5	0.12192	106	0.31928	162	0.74575	320	2.9098
66.0	0.12378	107	0.32533	164	0.76427	330	3.0945
66.5	0.12566	108	0.33144	166	0.78306	340	3.2849
67.0	0.12756	109	0.33761	168	0.80201	350	3.4809
67.5	0.12947	110	0.34383	170	0.82122	360	3.6823
68.0	0.13140	111	0.35011	172	0.84065	370	3.8901
68.5	0.13334	112	0.35645	174	0.86032	380	4.1032
69.0	0.13529	113	0.36284	176	0.88021	390	4.3220
69.5	0.13726	114	0.36929	178	0.90033	400	4.5466
70.0	0.13924	115	0.37580	180	0.92067	410	4.7767
70.5	0.14124	116	0.38236	182	0.94124	420	5.0126
71.0	0.14325	117	0.38899	184	0.96204	430	5.2541
71.5	0.14527	118	0.39566	186	0.98307	440	5.5013
72.0	0.14731	119	0.40240	188	1.00433	450	5.7542
72.5	0.14936	120	0.40921	190	1.02590	460	6.0128
73.0	0.15143	121	0.41604	192	1.04752	470	6.2770
73.5	0.15351	122	0.42294	194	1.06946	480	6.5470
74.0	0.15561	123	0.42991	196	1.09160	490	6.8227
74.5	0.15772	124	0.43692	198	1.11400	500	7.1040

FLYWHEELS

Flywheels may be classified either as *balance wheels* or as *flywheel pulleys*. The object of all flywheels is to equalize the energy exerted and the work done and thereby prevent excessive or sudden changes of speed. The permissible speed variation is an important factor in all flywheel designs. The allowable speed change varies considerably for different classes of machinery; for instance, it is about 1 or 2 per cent in modern steam engines, while in punching and shearing machinery a speed variation of 20 per cent may be allowed.

As the function of a balance wheel is to absorb and equalize energy in case the resistance to motion, or driving power, varies throughout the cycle, the rim section is generally quite heavy and is designed with reference to the energy that must be stored in it to prevent excessive speed variations and also with reference to the strength necessary to withstand safely the stresses resulting from the required speed. The rims of most balance wheels are either square or nearly square in section, but flywheel pulleys are commonly made wide to accommodate a belt and relatively thin in a radial direction, although this is not an invariable rule.

Flywheels, in general, may either be formed of a solid or one-piece section, or they may be of sectional construction. Flywheels in diameters up to about eight feet are usually cast solid, the hubs being divided in some cases to relieve cooling stresses. Flywheels ranging from, say, eight feet to fifteen feet in diameter, are commonly cast in half sections, and the larger sizes in several sections, the number of which may equal the number of arms in the wheel. The sectional flywheels may be divided into two general classes. One class includes cast wheels which are formed of sections principally because a solid casting would be too large to transport readily. The second class includes wheels of sectional construction which, by reason of the materials used and the special arrangement of the sections, enables much higher peripheral speeds to be obtained safely than would be possible with ordinary sectional wheels of the type not designed especially for high speeds. Various designs have been built to withstand the extreme stresses encountered in some classes of service. The rims in some cases are laminated, being partly or entirely formed of numerous segment-shaped steel plates. Another type of flywheel, which is superior to an ordinary sectional wheel, has a solid cast-iron rim connected to the hub by disk-shaped steel plates instead of cast spokes. Steel wheels may be divided into three distinct types, including (1) those having the center and rim built up entirely of steel plates, (2) those having a cast-iron center and steel rim, and (3) those having a cast-steel center and rim formed of steel plates. Wheels having wire-wound rims have been used to a limited extent when extremely high speeds have been necessary.

When the rim is formed of sections held together by joints it is very important to design these joints properly. The ordinary bolted and flanged rim joints located between the arms average about 20 per cent of the strength of a solid rim and about 25 per cent is the maximum strength obtainable for a joint of this kind. However, by placing the joints at the ends of the arms instead of between them, an efficiency of 50 per cent of the strength of the rim may be obtained. This is due to the fact that the joint is not subjected to the outward bending stresses between the arms but is directly supported by the arm, the end of which is secured to the rim just beneath the joint. When the rim sections of heavy balance wheels are held together by steel links shrunk into place, an efficiency of 60 per cent may be obtained; and by using a rim of box or I-section, a link type of joint connection may have an efficiency of 100 per cent.

Energy Due to Changes of Velocity. — When a flywheel absorbs energy from a variable driving force, as in the case of a steam engine, the velocity increases; and when this stored energy is given out, the velocity diminishes. When the driven member of a machine encounters a variable resistance in performing its work, as when the punch of a punching machine is passing through a steel plate, the flywheel gives up energy while the punch is at work, and, consequently, the speed of the flywheel is reduced. The total energy that a flywheel would give out if brought to a standstill is given by the formula:

$$E = \frac{Wv^2}{2g} = \frac{Wv^2}{64.32}$$

in which E = total energy of flywheel, in foot-pounds;
W = weight of flywheel rim, in pounds;
v = velocity at mean radius of flywheel rim, in feet per second;
g = acceleration due to gravity = 32.16.

If the velocity of a flywheel changes, the energy it will absorb or give up is proportional to the difference between the squares of its initial and final speeds, and is equal to the difference between the energy which it would give out if brought to a full stop and that which is still stored in it at the reduced velocity. Hence:

$$E_1 = \frac{Wv_1^2}{2g} - \frac{Wv_2^2}{2g} = \frac{W(v_1^2 - v_2^2)}{64.32}$$

in which E_1 = energy in foot-pounds which a flywheel will give out while the speed is reduced from v_1 to v_2;
W = weight of flywheel rim, in pounds;
v_1 = velocity at mean radius of flywheel rim before any energy has been given out, in feet per second;
v_2 = velocity of flywheel rim at end of period during which the energy has been given out, in feet per second.

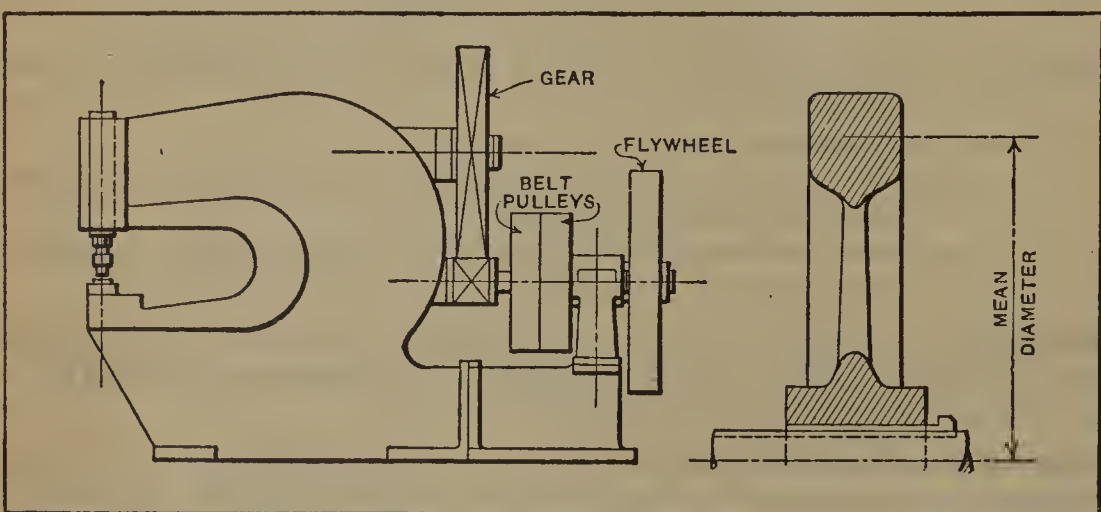
Ordinarily, the effect of the arms and hub does not enter into flywheel calculations, and only the weight of the rim is considered. In computing the velocity, the mean radius of the rim is commonly used.

General Procedure in Flywheel Design. — The general method of designing a flywheel is to determine first the value of E_1 or the energy the flywheel must either supply or absorb for a given change in velocity, which, in turn, varies for different classes of service. The mean diameter of the flywheel may be assumed, or it may be fixed within certain limits by the general design of the machine. Ordinarily the speed of the flywheel shaft is known, at least approximately; the values of v_1 and v_2 can then be determined, the latter depending upon the allowable percentage of speed variation. When these values are known, the weight of the rim and the cross-sectional area required to obtain this weight may be computed. The general procedure will be illustrated more in detail by considering the design of flywheels for punching and shearing machinery.

Flywheels for Presses, Punches, Shears, Etc. — In these classes of machinery, the work that the machine performs is of an intermittent nature and is done during a small part of the time required for the driving shaft of the machine to make a complete revolution. In order to distribute the work of the machine over the entire period of revolution of the driving shaft, a heavy-rimmed flywheel is placed on the shaft, giving the belt an opportunity to perform an almost uniform amount of work during the whole revolution. During the greater part of the revolution of the driving shaft, the belt power is used to accelerate the speed of the flywheel. During the part of the revolution when the work is done, the energy thus stored up in the flywheel is given out at the expense of its velocity. The problem is to determine the weight and cross-sectional area of the rim when the conditions affecting the design of the flywheel are known.

Example: — A flywheel is required for a punching machine capable of punching ¾-inch holes through structural steel plates ¾ inch thick. This machine (see accompanying diagram) is of the general type having a belt-driven shaft at the rear which carries a flywheel and a pinion that meshes with a large gear on the main shaft at the top of the machine. It is assumed that the relative speeds of the pinion and large gear are 7 to 1, respectively, and that the slide is to make 30 working strokes per minute. The preliminary lay-out shows that the flywheel should have a mean diameter (see enlarged detail) of about 30 inches. Find the weight of the flywheel and the size of the rim.

Energy Supplied by Flywheel. — The energy which the flywheel must give up for a given change in velocity, and the weight of rim necessary to supply that energy, must be determined. The maximum pressure for shearing a ¾-inch hole through ¾-inch structural steel equals approximately the circumference of the hole multiplied by the thickness of the stock multiplied by the tensile strength, which is nearly the same as the shearing resistance of the steel. Thus, in this case, $3.1416 \times 3/4 \times 3/4 \times 60{,}000 = 106{,}000$ pounds. The average pressure will be much less than the maximum. Some designers assume that the average pressure is about



one-half the maximum, although experiments show that the material is practically sheared off when the punch has entered the sheet a distance equal to about one-third the sheet thickness. On this latter basis, the average energy E_a in foot-pounds is 2200 in this case. Thus:

$$E_a = \frac{106{,}000 \times 1/3 \times 3/4}{12} = \frac{106{,}000}{4 \times 12} = 2200 \text{ foot-pounds.}$$

If the efficiency of the machine is taken at 85 per cent, the energy required will equal $2200 \div 0.85 = 2600$ foot-pounds nearly. Assume that the energy supplied by the belt while the punch is at work is determined by calculation to equal 175 foot-pounds. Then the flywheel must supply $2600 - 175 \doteq 2425$ foot-pounds $= E_1$.

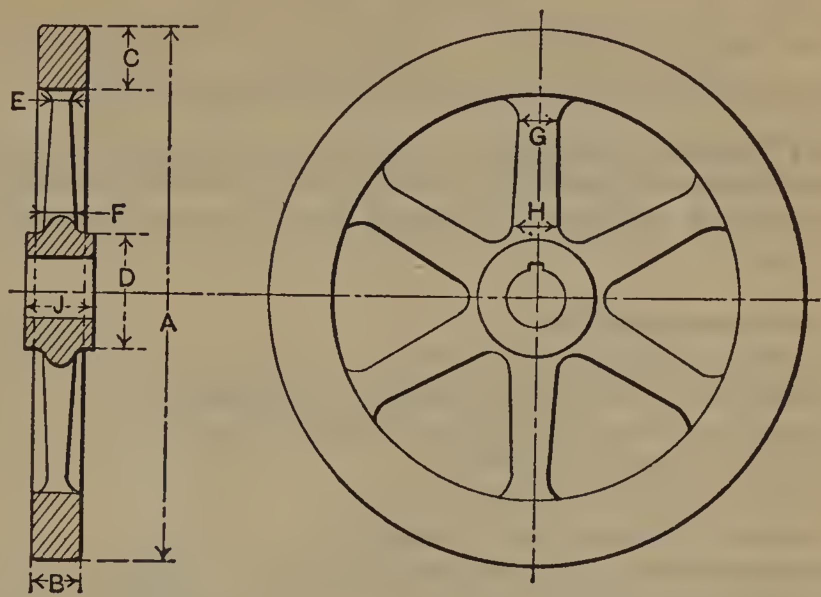
Rim Velocity at Mean Radius. — When the mean radius of the flywheel is known, the velocity of the rim at the mean radius, in feet per second, is:

$$v = \frac{2 \times 3.1416 \times R \times n}{60}$$

in which v = velocity at mean radius of flywheel, in feet per second;
R = mean radius of flywheel rim, in feet;
n = number of revolutions per minute.

Dimensions of Flywheels for Punches and Shears

(Maximum number of revolutions per minute given in table should never be exceeded for cast-iron flywheels.)



A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*J*	Max. R.P.M.
24	3	3½	6	1¼	1⅜	2¾	3¼	3½	955
30	3½	4	7	1⅜	1½	3	3¾	4	796
36	4	4½	8	1½	1¾	3¼	4¼	4½	637
42	4¼	4¾	9	1¾	2	3½	4½	5	557
48	4½	5	10	1¾	2	3¾	4¾	5½	478
54	4¾	5½	11	2	2¼	4	5	6	430
60	5	6	12	2¼	2½	4½	5½	6½	382
72	5½	7	13	2½	2¾	5	6½	7	318
84	6	8	14	3	3½	5½	7½	8	273
96	7	9	15	3½	4	6	9	9	239
108	8	10	16½	3¾	4½	6½	10½	10	212
120	9	11	18	4	5	7½	12	12	191

According to the preliminary lay-out the mean diameter in this case should be about 30 inches and the driving shaft is to make 210 R.P.M.; hence,

$$v = \frac{2 \times 3.1416 \times 1.25 \times 210}{60} = 27.5 \text{ feet per second.}$$

Weight of Flywheel Rim. — Assuming that the allowable variation in velocity when punching is about 15 per cent, and values of v_1 and v_2 are respectively 27.5 and 23.4 feet per second $(27.5 \times 0.85 = 23.4)$, the weight of a flywheel rim necessary to supply a given amount of energy in foot-pounds while the speed is reduced from v_1 to v_2 would be:

$$W = \frac{E_1 \times 64.32}{v_1^2 - v_2^2} = \frac{2425 \times 64.32}{27.5^2 - 23.4^2} = 750 \text{ pounds.}$$

Size of Rim for Given Weight. — Since 1 cubic inch of cast iron weighs 0.26 pound, a flywheel rim weighing 750 pounds contains $\frac{750}{0.26} = 2884$ cubic inches. The cross-sectional area of the rim in square inches equals the total number of cubic inches divided by the mean circumference, or $\frac{2884}{92.24} = 31$ square inches nearly, which is approximately the area of a rim 5⅛ inches wide and 6 inches deep.

Simplified Flywheel Calculations. — Calculations for designing the flywheels of punches and shears are simplified by the following formulas and the accompanying table of constants applying to different percentages of speed reduction. In these formulas let:

H.P. = horsepower required;
N = number of strokes per minute;
E = total energy required per stroke, in foot-pounds;
E_1 = energy given up by flywheel, in foot-pounds;
T = time in seconds per stroke;
T_1 = time in seconds of actual cut;
W = weight of flywheel rim, in pounds;
D = mean diameter of flywheel rim, in feet;
R = number of revolutions per minute of flywheel before speed reduction;
C and C_1 = values as given in table;
a = width of flywheel rim;
b = depth of flywheel rim;
y = ratio of depth to width of rim.

$$\text{H.P.} = \frac{EN}{33{,}000} = \frac{E}{T \times 550} \qquad E_1 = E\left(1 - \frac{T_1}{T}\right)$$

$$W = \frac{E_1}{CD^2R^2} \qquad a = \sqrt{\frac{1.22\,W}{12\,Dy}} \qquad b = ay$$

For cast-iron flywheels, with a maximum stress of 1000 pounds per square inch:

$$W = C_1E_1 \qquad R = 1940 \div D$$

Values of C and C_1 in the Previous Formulas

Per Cent Reduction	C	C_1	Per Cent Reduction	C	C_1
2½	0.00000213	0.1250	10	0.00000810	0.0328
5	0.00000426	0.0625	15	0.00001180	0.0225
7½	0.00000617	0.0432	20	0.00001535	0.0173

Example 1: — A hot slab shear is required to cut a slab 4 × 15 inches which, at a shearing stress of 6000 pounds per square inch, gives a pressure between the knives of 360,000 pounds. The total energy required for the cut will then be 360,000 × 4/12 = 120,000 foot-pounds. The shear is to make 20 strokes per minute; the actual cutting time is 0.75 second, and the balance of the stroke is 2.25 seconds.

The flywheel is to have a mean diameter of 6 feet 6 inches and is to run at a speed of 200 R.P.M.; the reduction in speed to be 10 per cent per stroke when cutting.

$$H.P. = \frac{120{,}000 \times 20}{33{,}000} = 72.7 \text{ horsepower;}$$

$$E_1 = 120{,}000 \times \left(1 - \frac{0.75}{3}\right) = 90{,}000 \text{ foot-pounds;}$$

$$W = \frac{90{,}000}{0.0000081 \times 6.5^2 \times 200^2} = 6570 \text{ pounds.}$$

Assuming a ratio of 1.22 between depth and width of rim,

$$a = \sqrt{\frac{6570}{12 \times 6.5}} = 9.18 \text{ inches;}$$

$$b = 1.22 \times 9.18 = 11.2 \text{ inches;}$$

or size of rim, say, $9 \times 11\frac{1}{2}$ inches.

Example 2: — Suppose that the flywheel in Example 1 is to be made with a stress of 1000 pounds, due to centrifugal force, per square inch of rim section.

$$C_1 \text{ for 10 per cent} = 0.0328;$$

$$W = 0.0328 \times 90{,}000 = 2950 \text{ pounds.}$$

$$R = \frac{1940}{D}. \quad \text{If } D = 6 \text{ feet, } R = \frac{1940}{6} = 323 \text{ R.P.M.}$$

Assuming a ratio of 1.22 between depth and width of rim, as before:

$$a = \sqrt{\frac{2950}{12 \times 6}} = 6.4 \text{ inches;}$$

$$b = 1.22 \times 6.4 = 7.8 \text{ inches;}$$

or size of rim, say, $6\frac{1}{4} \times 8$ inches.

Stresses in Flywheel Rims. — In general, high speed is desirable for flywheels in order to avoid using wheels which are unnecessarily large and heavy. The stress which tends to rupture a flywheel rim depends solely upon the rim velocity, and is independent of the rim radius, which can be proved as follows: The sum of the centrifugal (radial) forces of the whole rim of a flywheel is:

$$F = \frac{Wv^2}{gR} = \frac{4\,WR\pi^2n^2}{60 \times 60\,g} = 0.000341\,WRn^2$$

where F = centrifugal force, in pounds; W = weight of rim in pounds; R = mean radius of rim in feet, which is approximately equal to radius of gyration; n = revolutions per minute.

The resultant of half of this force tends to disrupt one-half of the rim from the other half. The rupture is resisted by the two sections of the rim at each end of the diameter. The resultant of half the radial forces is to the sum of half of the radial forces as the diameter of the flywheel is to half its circumference, or:

$$\frac{\text{resultant}}{\text{sum of half the radial forces}} = \frac{1}{\frac{1}{2}\pi};$$

Hence,

$$\text{resultant} = \frac{2}{\pi} \times \text{sum of half the radial forces} = \frac{2}{\pi} \times \frac{0.000341\,WRn^2}{2}$$

$$= 0.00010854\,WRn^2.$$

As this resultant force is resisted by the section at each end of the diameter, each section must resist a force

$$S = \frac{0.00010854\, WRn^2}{2} = 0.00005427\, WRn^2.$$

The weight of a rim of cast iron, one square inch in section, is $2\pi R \times 3.125 = 19.635\, R$ pounds, R being in feet. Consequently

$$S = 0.00005427 \times 19.635\, R \times Rn^2 = 0.0010656\, R^2n^2.$$

But as $v = \frac{2\pi Rn}{60}$, and $v^2 = \frac{4\pi^2R^2n^2}{3600}$, where v = velocity of rim in feet per second, it follows that

$$S = \frac{0.0010656\, v^2 \times 3600}{4\pi^2} = 0.0972\, v^2.$$

Thus the stress S in the flywheel rim is independent of the radius and depends only on the rim velocity.

Relation between Centrifugal Force and Disruptive Force in a Flywheel Rim. — When the total centrifugal force has been calculated for a flywheel rim, the result is analogous to that obtained when the total internal pressure acting on a certain length of a section of a steam boiler shell is figured. The total centrifugal force, therefore, is not equal to the disruptive force tending to tear the rim apart. For example, the total internal pressure on a 1-inch long section of a boiler shell, 50 inches in diameter (157 inches in circumference), with a pressure of 100 pounds per square inch, is 15,700 pounds. The actual bursting or tangential stress is $15{,}700 \div 3.1416 = 5000$ pounds. This bursting stress is resisted by two thicknesses of the shell, one on each side. In the case of the flywheel, the total centrifugal force developed in the rim must be divided by 3.1416 to get the measure of the disruptive force.

Thickness of Flywheel Rims. — A mathematical analysis of the stresses in flywheel rims is not conclusive owing to the uncertainty of shrinkage stresses in castings or the strength of the joint in the case of sectional wheels. When a flywheel of ordinary design is revolving at high speed, the tendency of the rim is to bend or bow outward between the arms, and the bending stresses may be serious, especially if the rim is wide and thin and the spokes are rather widely spaced. When the rims are thick, this tendency does not need to be considered, but in the case of a thin rim, running at a high rate of speed, the stress in the middle might become sufficiently great to cause the wheel to fail. The proper thickness of a cast-iron rim to resist this tendency is given for solid rims by the formula:

$$t = \frac{0.475\, d}{n^2\left(\frac{6000}{v^2} - \frac{1}{10}\right)}$$

For a jointed rim, the formula is:

$$t = \frac{0.95\, d}{n^2\left(\frac{6000}{v^2} - \frac{1}{10}\right)}$$

In these formulas, t = thickness of rim, in inches; d = diameter of flywheel, in inches; n = number of arms; v = peripheral speed, in feet per second.

Safe Speeds for Flywheels. — One hundred feet per second may be regarded as a safe rim speed for cast-iron wheels made in one piece, providing the design is such that there are no severe shrinkage strains in the casting. Ordinarily, there are strains, and, therefore, about 85 feet per second is as high a rim speed as should be considered good practice. If the wheel is made in halves or sections, the efficiency of the rim joint must be taken into consideration. A steel-casting flywheel made in one piece and free from shrinkage strains should run with safety at the rim speed of 200 feet per second. It should be noted that the stress in the rim increases with the square of the speed, so that even a small increase in speed causes a considerable increase in the stress.

Safe speeds for flywheels are given in the accompanying tables. The safe speeds for cast-iron flywheels with solid rims are based on a rim speed of 5280 feet per minute, or 88 feet per second. The second table of safe speeds for cast-iron flywheels with jointed rims is based on a maximum efficiency of 25 per cent for flanged joints, 50 per cent for pad joints, and 60 per cent for link joints. These speeds are recommended by the flywheel insurance department of the Fidelity & Casualty Co. of New York.

Safe Speeds for Cast-iron Flywheels with Solid Rims

Diam. of Flywheel, Feet	Max. Safe Speed, R.P.M.	Diam. of Flywheel, Feet	Max. Safe Speed, R.P.M.	Diam. of Flywheel, Feet	Max. Safe Speed, R.P.M.	Diam. of Flywheel, Feet	Max. Safe Speed, R.P.M.	Diam. of Flywheel, Feet	Max. Safe Speed, R.P.M.
1	1680	6½	258	12½	134	18½	91	24½	68
1¼	1344	7	240	13	129	19	89	25	67
1½	1120	7½	224	13½	124	19½	86	25½	66
2	840	8	210	14	120	20	84	26	65
2½	672	8½	198	14½	116	20½	82	26½	63
3	560	9	187	15	112	21	80	27	62
3½	480	9½	177	15½	108	21½	78	27½	61
4	420	10	168	16	105	22	76	28	60
4½	373	10½	160	16½	102	22½	74	28½	59
5	336	11	153	17	99	23	73	29	58
5½	305	11½	146	17½	96	23½	72	29½	57
6	280	12	140	18	93	24	70	30	56

As the stresses in the rim due to centrifugal force increase with the square of the speed, a relatively slight increase in speed may be dangerous and even cause the wheel to burst or "explode." The margin of safety on speed is the square root of the factor of safety for strength. If the speed should be tripled, the rim stress would be nine times as great as before, and in the case of a flywheel originally designed with a strength factor of safety of 27 for normal speed, this factor would be reduced to 3 if the speed were tripled. If a sectional wheel having flanged and bolted joints located between the arms had a factor of safety of 12 in the solid rim for normal speeds, the factor for the joints would be 3 or less at normal speeds, since the average strength of such joints is only about one-fifth to one-fourth the strength of the rim; therefore, in this case the flywheel would be liable to burst if the speed were increased only 73 per cent, and comparatively few flywheels have as large a margin of safety as this.

Since the material which has the greatest strength for a given weight will withstand the highest speed, wood is in this respect a better material for flywheels than cast-iron, and it has been used to some extent for large wheels. A well-constructed maple wheel may be run safely at a rim speed of 150 feet per second, provided the wheel is not made in halves or sections with rim joints which permit taking the wheel apart. The wooden rims are built up of segments which are sawed so as to obtain as much of the straight grain of the wood as possible.

Safe Speeds for Cast-iron Flywheels with Jointed Rims*

Diam. in Ft.	Revolutions per Minute			Diam. in Ft.	Revolutions per Minute		
	Flange Joint	Pad Joint	Link Joint		Flange Joint	Pad Joint	Link Joint
1	955	1350	1480	16	60	84	92
2	478	675	740	17	56	79	87
3	318	450	493	18	53	75	82
4	239	338	370	19	50	71	78
5	191	270	296	20	48	68	74
6	159	225	247	21	46	65	70
7	136	193	212	22	44	62	67
8	119	169	185	23	42	59	64
9	106	150	164	24	40	56	62
10	96	135	148	25	38	54	59
11	87	123	135	26	37	52	57
12	80	113	124	27	35	50	55
13	73	104	114	28	34	48	53
14	68	96	106	29	33	47	51
15	64	90	99	30	32	45	49

* If the revolutions given in the table be increased 20 per cent the margin of safety on speed will be reduced to *two and one-half;* if the revolutions be increased 50 per cent the margin of safety will be reduced to *two.*

Factors of Safety for Flywheels. — Cast-iron flywheels commonly have a factor of safety of 10. When flywheel rims are made of wood, the full tensile strength of the wood is not obtained, and partly for this reason a factor of safety of 20 is used for wood. In the case of a wooden rim, the real factor of safety is less than 20, because part of the tensile strength is lost in sawing, and on account of the joints between the segments.

Wheels do not often fail from torsional stress or from twisting action in pulling their load, because enough material can be put in the wheel to resist successfully any load required. There is, however, no possible way to overcome the centrifugal force due to speed. Increasing the thickness of the rim of the wheel does not strengthen it so far as centrifugal force is concerned, because the weight added also increases the centrifugal force, leaving the wheel no stronger than before. There is, therefore, a definite speed at which any wheel, however sound, will explode, regardless of the amount of material it contains.

Tests to Determine Flywheel Bursting Speeds. — Tests made by Prof. C. H. Benjamin, to determine the bursting speeds of flywheels, showed the following results:

Cast-iron Wheels with Solid Rims. — Cast-iron wheels having solid rims burst at a rim speed of 395 feet per second, corresponding to a centrifugal tension of about 15,600 pounds per square inch.

Wheels with Jointed Rims. — Four wheels were tested with joints and bolts inside the rim, after the familiar design ordinarily employed for band wheels, but with the joints located at points one-fourth of the distance from one arm to the next, these being the points of least bending moment, and, consequently, the points at which the deflection due to centrifugal force would be expected to have the least effect. The tests, however, did not bear out this conclusion. The wheels burst at a rim speed of 194 feet per second, corresponding to a centrifugal tension of about 3750 pounds per square inch. These wheels, therefore, were only about one-quarter as strong as the wheels with solid rims, and burst at practically the same speed as wheels in a previous series of tests in which the rim joints were midway between the arms. This is doubtless due to the fact that the heavy mass of the flanges and bolts locates the bending moment near them. In these wheels the combined tensile strength of the bolts in the flange joints was slightly less than one-third the strength of the rim, which is about the maximum ratio possible with this style of joint.

Bursting Speed for Link Joints. — Another type of wheel with deep rim, fastened together at the joints midway between the arms by links shrunk into recesses, after the manner of flywheels for massive engines, gave much superior results. This wheel burst at a speed of 256 feet per second, indicating a centrifugal tension of about 6600 pounds per square inch.

Wheel having Tie-rods. — Tests were made on a band wheel having joints inside the rim, midway between the arms, and in all respects like others of this design previously tested, except that tie-rods were used to connect the joints with the hub. It burst at a speed of 225 feet per second, showing an increase of strength of from 30 to 40 per cent over similar wheels without the tie-rods.

Wheel Rim of I-section. — Several wheels of special design, not in common use, were also tested, the one giving the greatest strength being an English wheel, with solid rim of I-section, made of high-grade cast iron and with the rim tied to the hub by steel wire spokes. These spokes were adjusted to have a uniform tension by "tuning," and the wheel gave way at a rim speed of 424 feet per second, which is slightly higher than the speed of rupture of the solid rim wheels with ordinary style of spokes.

Bursting Speeds of Cast-iron Pulleys. — The pulley tested was of cast iron, well proportioned, and of the type used on shafting for transmitting power. It was 48 inches in diameter, had six arms and weighed 194 pounds. The rim was whole and was 8½ inches wide and about ⅜ inch thick, finished on the outside. The arms were elliptical in section, 3⅛ inches by 1 1/16 inch at the hub, and 2 inches by ¾ inch at the rim. On the whole the wheel was well designed and showed no signs of shrinkage strains. It had, however, been balanced by riveting a cast-iron washer inside the rim at the lighter side. The pulley burst at a speed of 1100 revolutions per minute, the linear speed of the rim at rupture being 230 feet per second.

Wheel No. 2 was a cast-iron pulley of the same general style and dimensions as No. 1, but with a split hub and rim. The flanges were located midway between the arms and bolted at some little distance inside the rim; thus the joints were in the worst possible position to withstand the bending action due to centrifugal force, and their own weight only aggravated the difficulty. The flanges weighed with their bolts 7½ pounds. This wheel burst at less than 700 revolutions per minute, the tachometer not recording below this speed. It was estimated that the speed was only about 600 revolutions per minute. At 600 revolutions per minute the linear speed of the rim would be only 125 feet per second.

Tests on Flywheel of Special Construction. — The third test was on a flywheel 49 inches in diameter and weighing about 900 pounds. The rim was 6¾ inches wide and 1⅛ inches thick, and was built of ten segments, the material being steel casting. Each joint was secured by three "prisoners" of an I-section on the outside face, by link prisoners on each edge, and by a dovetailed bronze clamp on the inside, fitting over lugs on the rim. The arms were of phosphor-bronze, twenty in number, ten on each side, and were a cross in section. These arms came midway between the rim joints and were bolted to plane faces on the polygonal hub. The rim was further reinforced by a system of diagonal bracing, each section of the rim being supported at five points on each side, in such a way as to relieve it almost entirely from bending. The braces, like the arms, were of phosphor-bronze, and all bolts and connecting links of steel. This wheel was designed as a model of a proposed 30-foot flywheel. On account of the excessive air resistance the wheel was enclosed at the sides between sheet-metal disks. This wheel burst at 1775 revolutions per minute or at a linear speed of 372 feet per second. The hub and main spokes of the wheel remained nearly in place, but parts of the rim were found two hundred feet away. This sudden failure of the rim casing was unexpected, as it was thought the flange bolts were the parts to give way first. The tensile strength of the casing at the point of fracture was about four times the strength of the wheel rim at a solid section.

Bursting Speeds of Disks. — Let N = bursting speed of a disk in revolutions per minute; S = ultimate tensile strength of material in pounds per square inch; R = outside radius of disk; r = radius of bore or hole in disk; w = weight of material in pounds per cubic inch; $g = 32.16$. Then:

$$N = \frac{180}{3.1416}\sqrt{\frac{Sg}{(R^2 + Rr + r^2)\,w}} = 57.3\sqrt{\frac{32.16\,S}{(R^2 + Rr + r^2)\,w}}$$

Steam Engine Flywheels. — The variable amount of energy during each stroke and the allowable percentage of speed variation are of especial importance in designing steam engine flywheels. The earlier the point of cut-off, the greater the variation in energy and the larger the flywheel that will be required. The weight of the reciprocating parts and the length of the connecting-rod also affect the variation. The following formula is used for computing the weight of the flywheel rim:

Let W = weight of rim in pounds;
D = mean diameter of rim in feet;
N = number of revolutions per minute;
$\frac{1}{n}$ = allowable variation in speed (from 1/50 to 1/100);
E = excess and deficiency of energy in foot-pounds;
c = factor of energy excess, from the accompanying table;
H.P. = indicated horsepower.

Then, if the indicated horsepower is given:

$$W = \frac{387{,}587{,}500 \times cn \times \text{H.P.}}{D^2N^3} \tag{1}$$

If the work in foot-pounds is given, then:

$$W = \frac{11{,}745\, nE}{D^2N^2} \qquad (2)$$

In the second formula, E equals the average work in foot-pounds done by the engine in one revolution, multiplied by the decimal given in the accompanying

Factors for Engine Flywheel Calculations

Condensing Engines						
Fraction of stroke at which steam is cut off........	1/3	1/4	1/5	1/6	1/7	1/8
Factor of energy excess...	0.163	0.173	0.178	0.184	0.189	0.191

Non-condensing Engines				
Steam cut off at........................	1/2	1/3	1/4	1/5
Factor of energy excess...................	0.160	0.186	0.209	0.232

table, "Factors for Engine Flywheel Calculations," which covers both the condensing and non-condensing engines:

Example 1. — A non-condensing engine of 150 indicated horsepower is to make 200 revolutions per minute, with a speed variation of 2 per cent. The average cut-off is to be at one-quarter stroke, and the flywheel is to have a mean diameter of 6 feet. Required, the necessary weight of rim in pounds.

From the table $c = 0.209$, and from the data given H.P. = 150; $N = 200$; $\frac{1}{n} = 1/50$ or $n = 50$; $D = 6$.

Substituting these values in equation (1):

$$W = \frac{387{,}587{,}500 \times 0.209 \times 50 \times 150}{6^2 \times 200^3} = 2110 \text{ pounds, nearly.}$$

Example 2. — A condensing engine, 24×42 inches, cuts off at one-third stroke and has a mean effective pressure of 50 pounds per square inch. The flywheel is to be 18 feet in mean diameter and make 75 revolutions per minute with a variation of 1 per cent. Required, weight of rim.

The work done on the piston in one revolution is equal to the pressure on the piston multiplied by the distance traveled or twice the stroke in feet. The area of the piston in this case is 452.4 square inches, and twice the stroke is 7 feet. The work done on the piston in one revolution is, therefore, $452.4 \times 50 \times 7 = 158{,}340$ foot-pounds. From the table $c = 0.163$, and therefore:

$$E = 158{,}340 \times 0.163 = 25{,}810 \text{ foot-pounds.}$$

From the data given: $n = 100$; $D = 18$; $N = 75$. Substituting these values in equation (2):

$$W = \frac{11{,}745 \times 100 \times 25{,}810}{18^2 \times 75^2} = 16{,}650 \text{ pounds, nearly.}$$

Spokes or Arms of Flywheels. — Flywheel arms are usually of elliptical cross-section. The major axis of the ellipse is in the plane of rotation to give the arms greater resistance to bending stresses and reduce the air resistance which may be considerable at high velocity. The stresses in the arms may be severe, due to the inertia of a heavy rim when sudden load changes occur. The strength of the arms should equal three-fourths the strength of the shaft in torsion.

If W equals the width of the arm at the hub (length of major axis) and D equals the shaft diameter, then W equals 1.3 D for a wheel having 6 arms; and for an 8-arm wheel W equals 1.2 D. The thickness of the arm at the hub (length of minor axis) equals one-half the width. The arms usually taper toward the rim. The cross-sectional area at the rim should not be less than two-thirds the area at the hub.

Flywheels for Motor-driven Planers. — The primary function of a flywheel for a motor-driven planer is not so much for maintaining a constant speed as relieving the motor from excessive shocks at the points of reversal. Tests made at the Worcester Polytechnic Institute with a 36- by 36- by 10-foot planer driven by a 10-horsepower induction motor showed a current consumption of 1.85 kilowatt-hour when no flywheel was used and the length of the stroke was five feet. With a ten-foot stroke, the consumption was 1.63 kilowatt-hour without a flywheel. When a flywheel was used, the consumption was 1.3 and 1.24 kilowatt-hour, respectively, for the two lengths of stroke mentioned. Thus, with the flywheel, 29.5 per cent less power was required with the short stroke, and 24 per cent less power with the long stroke. The flywheel was also an advantage in increasing the average rate of production and preventing "slow-downs" and tardy reversals.

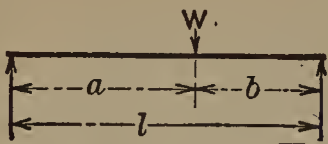
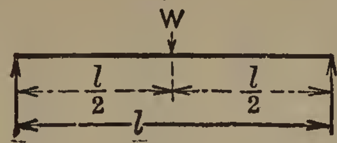
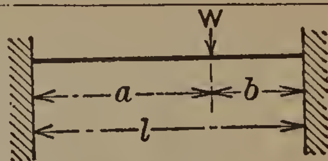
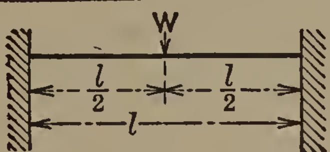
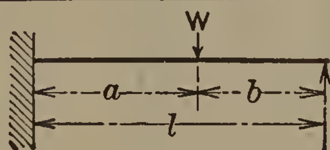
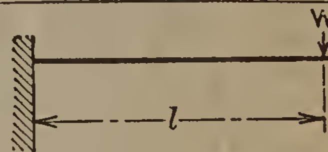
Critical Speed of Rotating Body. — If a body or disk mounted upon a shaft rotates about it, the center of gravity of the body or disk must be at the center of the shaft, if a perfect running balance is to be obtained. In most cases, however, the center of gravity of the disk will be slightly removed from the center of the shaft, owing to the difficulty of perfect balancing. Now, if the shaft and disk be rotated, the centrifugal force generated by the heavier side will be greater than that generated by the lighter side geometrically opposite to it, and the shaft will deflect toward the heavier side, causing the center of the disk to rotate in a small circle. These conditions hold true up to a comparatively high speed; but a point is eventually reached (at several thousand revolutions per minute) when momentarily there will be excessive vibration, and then the parts will run quietly again. The speed at which this occurs is called the *critical speed* of the wheel, and the phenomenon itself is called the *settling* of the wheel. The explanation of the settling is that at this speed the axis of rotation changes, and the wheel and shaft, instead of rotating about their geometrical center, begin to rotate about an axis through their center of gravity. The shaft itself is then deflected so that for every revolution its geometrical center traces a circle around the center of gravity of the rotating mass.

Critical speeds depend upon the magnitude or location of the load or loads carried by the shaft, the length of the shaft, its diameter and the kind of supporting bearings. The normal operating speed of a machine may or may not be higher than the critical speed. For instance, some steam turbines exceed the critical speed, although they do not run long enough at the critical speed for the vibrations to build up to an excessive amplitude. The practice of the General Electric Co. at Schenectady is to keep below the critical speeds. It is assumed that the maximum speed of a machine may be within 20 per cent of the critical speed without vibration troubles. Thus, in a design of steam turbine sets, critical speed is a factor that determines the size of the shafts, both for the generators and turbines. While a machine may run very close to the critical speed, the alignment and play of the bearings, the balance and construction generally, will require extra care, resulting in a more expensive machine; moreover, while such a machine may run smoothly for a considerable time, any looseness or play that may develop later, causing a slight unbalance, will immediately set up excessive vibrations.

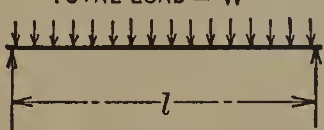
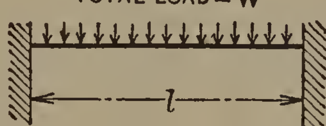
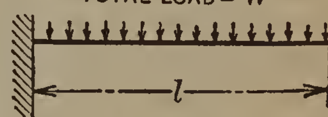
Formulas for Critical Speeds. — The critical speed formulas given in the accompanying table (from the paper on Critical Speed Calculation presented

Critical Speed Formulas

Formulas for Single Concentrated Load

 $N = 387{,}000 \dfrac{d^2}{ab}\sqrt{\dfrac{l}{W}}$ Bearings supported	 $N = 1{,}550{,}500 \dfrac{d^2}{l\sqrt{Wl}}$ Bearings supported	 $N = 387{,}000 \dfrac{d^2 l}{ab}\sqrt{\dfrac{l}{Wab}}$ Bearings fixed
 $N = 3{,}100{,}850 \dfrac{d^2}{l\sqrt{Wl}}$ Bearings fixed	 $N = 775{,}200 \dfrac{d^2 l}{ab}\sqrt{\dfrac{l}{Wa(3l+b)}}$ One fixed — One supported	 $N = 387{,}000 \dfrac{d^2}{l\sqrt{Wl}}$ One fixed — One free end

Formulas for Distributed Loads — First Critical Speed

 $N = 2{,}232{,}500 \dfrac{d^2}{l\sqrt{Wl}}$ $N_1 = 4{,}760{,}000 \dfrac{d}{l^2}$ Bearings supported	 $N = 4{,}979{,}250 \dfrac{d^2}{l\sqrt{Wl}}$ $N_1 = 10{,}616{,}740 \dfrac{d}{l^2}$ Bearings fixed	 $N = 795{,}200 \dfrac{d^2}{l\sqrt{Wl}}$ $N_1 = 1{,}695{,}500 \dfrac{d}{l^2}$ One fixed — One free end

N = critical speed, R.P.M.; N_1 = critical speed of shaft alone; d = diameter of shaft, in inches; W = load applied to shaft, in pounds; l = distance between centers of bearings, in inches; a and b = distances from bearings to load.

before the A.S.M.E. by S. H. Weaver) apply to (1) shafts with single concentrated loads and (2) shafts carrying uniformly distributed loads. These formulas also cover different conditions as regards bearings. If the bearings are self-aligning or very short, the shaft is considered supported at the ends; whereas, if the bearings are long and rigid, the shaft is considered fixed. These formulas, for both concentrated and distributed loads, apply to vertical shafts as well as horizontal shafts, the critical speeds having the same value in both cases. The data required for the solution of critical speed problems are the same as for shaft deflection. As the shaft is usually of variable diameter and its stiffness is increased by a long hub, an ideal shaft of uniform diameter and equal stiffness must be assumed.

In calculating critical speeds, the weight of the shaft is either neglected or, say, one-half to two-thirds of the weight is added to the concentrated load. The formulas apply to steel shafts having a modulus of elasticity $E = 29{,}000{,}000$. While a shaft carrying a number of loads or a distributed load may have an infinite number of critical speeds, ordinarily it is the first critical speed that is of importance in engineering work, which is the speed obtained by the formulas given in the table for distributed loads.

Angular Velocity in Radians for Given Number of Revolutions per Minute

R.P.M.	Angular Velocity in Radians									
	0	1	2	3	4	5	6	7	8	9
0	0.00	0.10	0.21	0.31	0.42	0.52	0.63	0.73	0.84	0.94
10	1.05	1.15	1.26	1.36	1.47	1.57	1.67	1.78	1.88	1.99
20	2.09	2.20	2.30	2.41	2.51	2.62	2.72	2.83	2.93	3.04
30	3.14	3.25	3.35	3.46	3.56	3.66	3.77	3.87	3.98	4.08
40	4.19	4.29	4.40	4.50	4.61	4.71	4.82	4.92	5.03	5.13
50	5.24	5.34	5.44	5.55	5.65	5.76	5.86	5.97	6.07	6.18
60	6.28	6.39	6.49	6.60	6.70	6.81	6.91	7.02	7.12	7.23
70	7.33	7.43	7.54	7.64	7.75	7.85	7.96	8.06	8.17	8.27
80	8.38	8.48	8.59	8.69	8.80	8.90	9.01	9.11	9.21	9.32
90	9.42	9.53	9.63	9.74	9.84	9.95	10.05	10.16	10.26	10.37
100	10.47	10.58	10.68	10.79	10.89	11.00	11.10	11.20	11.31	11.41
110	11.52	11.62	11.73	11.83	11.94	12.04	12.15	12.25	12.36	12.46
120	12.57	12.67	12.78	12.88	12.98	13.09	13.19	13.30	13.40	13.51
130	13.61	13.72	13.82	13.93	14.03	14.14	14.24	14.35	14.45	14.56
140	14.66	14.76	14.87	14.97	15.08	15.18	15.29	15.39	15.50	15.60
150	15.71	15.81	15.92	16.02	16.13	16.23	16.34	16.44	16.55	16.65
160	16.75	16.86	16.96	17.07	17.17	17.28	17.38	17.49	17.59	17.70
170	17.80	17.91	18.01	18.12	18.22	18.33	18.43	18.53	18.64	18.74
180	18.85	18.95	19.06	19.16	19.27	19.37	19.48	19.58	19.69	19.79
190	19.90	20.00	20.11	20.21	20.32	20.42	20.52	20.63	20.73	20.84
200	20.94	21.05	21.15	21.26	21.36	21.47	21.57	21.68	21.78	21.89
210	21.99	22.10	22.20	22.30	22.41	22.51	22.62	22.72	22.83	22.93
220	23.04	23.14	23.25	23.35	23.46	23.56	23.67	23.77	23.88	23.98
230	24.09	24.19	24.29	24.40	24.50	24.61	24.71	24.82	24.92	25.03
240	25.13	25.24	25.34	25.45	25.55	25.66	25.76	25.87	25.97	26.07
250	26.18	26.28	26.39	26.49	26.60	26.70	26.81	26.91	27.02	27.12
260	27.23	27.33	27.44	27.54	27.65	27.75	27.85	27.96	28.06	28.17
270	28.27	28.38	28.48	28.59	28.69	28.80	28.90	29.01	29.11	29.22
280	29.32	29.43	29.53	29.64	29.74	29.84	29.95	30.05	30.16	30.26
290	30.37	30.47	30.58	30.68	30.79	30.89	31.00	31.10	31.21	31.31
300	31.42	31.52	31.62	31.73	31.83	31.94	32.04	32.15	32.25	32.36
310	32.46	32.57	32.67	32.78	32.88	32.99	33.09	33.20	33.30	33.41
320	33.51	33.61	33.72	33.82	33.93	34.03	34.14	34.24	34.35	34.45
330	34.56	34.66	34.77	34.87	34.98	35.08	35.19	35.29	35.39	35.50
340	35.60	35.71	35.81	35.92	36.02	36.13	36.23	36.34	36.44	36.55
350	36.65	36.76	36.86	36.97	37.07	37.18	37.28	37.38	37.49	37.59
360	37.70	37.80	37.91	38.01	38.12	38.22	38.33	38.43	38.54	38.64
370	38.75	38.85	38.96	39.06	39.16	39.27	39.37	39.48	39.58	39.69
380	39.79	39.90	40.00	40.11	40.21	40.32	40.42	40.53	40.63	40.74
390	40.84	40.94	41.05	41.15	41.26	41.36	41.47	41.57	41.68	41.78
400	41.89	41.99	42.10	42.20	42.31	42.41	42.52	42.62	42.73	42.83
410	42.93	43.04	43.14	43.25	43.35	43.46	43.56	43.67	43.77	43.88
420	43.98	44.09	44.19	44.30	44.40	44.51	44.61	44.71	44.82	44.92
430	45.03	45.13	45.24	45.34	45.45	45.55	45.66	45.76	45.87	45.97
440	46.08	46.18	46.29	46.39	46.50	46.60	46.70	46.81	46.91	47.02
450	47.12	47.23	47.33	47.44	47.54	47.65	47.75	47.86	47.96	48.07
460	48.17	48.28	48.38	48.48	48.59	48.69	48.80	48.90	49.01	49.11
470	49.22	49.32	49.43	49.53	49.64	49.74	49.85	49.95	50.06	50.16
480	50.26	50.37	50.47	50.58	50.68	50.79	50.89	51.00	51.10	51.21
490	51.31	51.42	51.52	51.63	51.73	51.84	51.94	52.05	52.15	52.26

Degrees Expressed in Radians

Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.	Deg.	Rad.
1	0.0175	31	0.5411	61	1.0647	91	1.5882	121	2.1118	151	2.6354
2	0.0349	32	0.5585	62	1.0821	92	1.6057	122	2.1293	152	2.6529
3	0.0524	33	0.5760	63	1.0996	93	1.6232	123	2.1468	153	2.6704
4	0.0698	34	0.5934	64	1.1170	94	1.6406	124	2.1642	154	2.6878
5	0.0873	35	0.6109	65	1.1345	95	1.6581	125	2.1817	155	2.7053
6	0.1047	36	0.6283	66	1.1519	96	1.6755	126	2.1991	156	2.7227
7	0.1222	37	0.6458	67	1.1694	97	1.6930	127	2.2166	157	2.7402
8	0.1396	38	0.6632	68	1.1868	98	1.7104	128	2.2340	158	2.7576
9	0.1571	39	0.6807	69	1.2043	99	1.7279	129	2.2515	159	2.7751
10	0.1745	40	0.6981	70	1.2217	100	1.7453	130	2.2689	160	2.7925
11	0.1920	41	0.7156	71	1.2392	101	1.7628	131	2.2864	161	2.8100
12	0.2094	42	0.7330	72	1.2566	102	1.7802	132	2.3038	162	2.8274
13	0.2269	43	0.7505	73	1.2741	103	1.7977	133	2.3213	163	2.8449
14	0.2443	44	0.7679	74	1.2915	104	1.8151	134	2.3387	164	2.8623
15	0.2618	45	0.7854	75	1.3090	105	1.8326	135	2.3562	165	2.8798
16	0.2793	46	0.8029	76	1.3265	106	1.8500	136	2.3736	166	2.8972
17	0.2967	47	0.8203	77	1.3439	107	1.8675	137	2.3911	167	2.9147
18	0.3142	48	0.8378	78	1.3614	108	1.8850	138	2.4086	168	2.9322
19	0.3316	49	0.8552	79	1.3788	109	1.9024	139	2.4260	169	2.9496
20	0.3491	50	0.8727	80	1.3963	110	1.9199	140	2.4435	170	2.9671
21	0.3665	51	0.8901	81	1.4137	111	1.9373	141	2.4609	171	2.9845
22	0.3840	52	0.9076	82	1.4312	112	1.9548	142	2.4784	172	3.0020
23	0.4014	53	0.9250	83	1.4486	113	1.9722	143	2.4958	173	3.0194
24	0.4189	54	0.9425	84	1.4661	114	1.9897	144	2.5133	174	3.0369
25	0.4363	55	0.9599	85	1.4835	115	2.0071	145	2.5307	175	3.0543
26	0.4538	56	0.9774	86	1.5010	116	2.0246	146	2.5482	176	3.0718
27	0.4712	57	0.9948	87	1.5184	117	2.0420	147	2.5656	177	3.0892
28	0.4887	58	1.0123	88	1.5359	118	2.0595	148	2.5831	178	3.1067
29	0.5061	59	1.0297	89	1.5533	119	2.0769	149	2.6005	179	3.1241
30	0.5236	60	1.0472	90	1.5708	120	2.0944	150	2.6180	180	3.1416

Minutes Expressed in Radians

Min.	Rad.	Min.	Rad.	Min.	Rad.	Min.	Rad.	Min.	Rad.	Min.	Rad.
1	0.0003	11	0.0032	21	0.0061	31	0.0090	41	0.0119	51	0.0148
2	0.0006	12	0.0035	22	0.0064	32	0.0093	42	0.0122	52	0.0151
3	0.0009	13	0.0038	23	0.0067	33	0.0096	43	0.0125	53	0.0154
4	0.0012	14	0.0041	24	0.0070	34	0.0099	44	0.0128	54	0.0157
5	0.0015	15	0.0044	25	0.0073	35	0.0102	45	0.0131	55	0.0160
6	0.0017	16	0.0047	26	0.0076	36	0.0105	46	0.0134	56	0.0163
7	0.0020	17	0.0049	27	0.0079	37	0.0108	47	0.0137	57	0.0166
8	0.0023	18	0.0052	28	0.0081	38	0.0111	48	0.0140	58	0.0169
9	0.0026	19	0.0055	29	0.0084	39	0.0113	49	0.0143	59	0.0172
10	0.0029	20	0.0058	30	0.0087	40	0.0116	50	0.0145	60	0.0175

Radians Expressed in Degrees, Minutes and Seconds

Rad.	Angle	Rad.	Angle	Rad.	Angle	Rad.	Angle	Rad.	Angle
0.001	0° 3′ 26″	0.008	0° 27′ 30″	0.06	3° 26′ 16″	0.4	22° 55′ 6″	2.0	114° 35′ 30″
0.002	0 6 53	0.009	0 30 56	0.07	4 0 39	0.5	28 38 52	3.0	171 53 14
0.003	0 10 19	0.01	0 34 23	0.08	4 35 1	0.6	34 22 39	4.0	229 10 59
0.004	0 13 45	0.02	1 8 45	0.09	5 9 24	0.7	40 6 25	5.0	286 28 44
0.005	0 17 11	0.03	1 43 8	0.1	5 43 46	0.8	45 50 12	6.0	343 46 29
0.006	0 20 38	0.04	2 17 31	0.2	11 27 33	0.9	51 33 58	7.0	401 4 14
0.007	0 24 4	0.05	2 51 53	0.3	17 11 19	1.0	57 17 45	8.0	458 21 58

Angular Velocity. — The angular velocity of a rotating body is expressed in angular measure and equals the angle through which any radius of the body turns in one second. This angle is generally expressed in radians.

$$\text{One radian} = \frac{180}{\pi} = \frac{180}{3.1416} = 57.3 \text{ degrees.}$$

The angular velocity is generally denoted by the Greek letter ω. Let ω = angular velocity in radians; r = radius of revolving body in feet; n = number of revolutions per minute; v = velocity of a point on the periphery, in feet per second; then,

$$v = \frac{2\pi r n}{60}; \quad \omega = \frac{v}{r} = \frac{2\pi n}{60}; \quad v = \omega r.$$

Example: — A flywheel, 12 feet in diameter, revolves at 60 revolutions per minute. Find the angular velocity ω. Here, $r = 6$; $n = 60$.

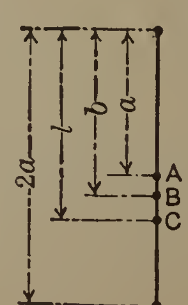
$$\omega = \frac{2 \times 3.1416 \times 60}{60} = 6.2832 \text{ radians.}$$

The table of angular velocity gives this velocity directly in radians for all numbers of revolutions per minute from 1 to 499. The method in which this table is used may be best explained by an example: Find the angular velocity in radians of a flywheel making 97 revolutions per minute. Locate 90 in the left-hand column and 7 at the top of the columns; at the intersection of the two lines, the angular velocity is read off as equal to 10.16 radians.

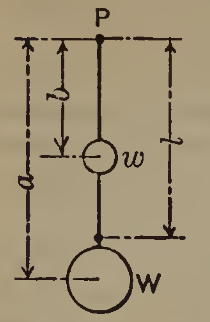
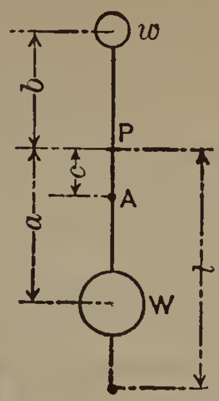
The Pendulum

A *simple pendulum* is a material point which is supposed to be suspended from a fixed point by a string without weight. A *compound pendulum* is a material body suspended from a fixed axis about which it oscillates by the force of gravity. The center of oscillation is the point at which, if all the matter in the compound pendulum were concentrated there, it would make a simple pendulum which would oscillate in the same periods of time. The angle included between the extreme positions of the line drawn from the point of suspension to the center of oscillation, is called the angle of oscillation. The time of vibration of a pendulum varies directly as the square root of the length and inversely as the square root of the acceleration due to gravity at the given latitude and elevation above the earth's surface. Hence, there is a definite length of a simple pendulum that vibrates seconds at any given place. At New York this length is 39.1017 inches or 3.2585 feet. In the formulas in the tables: l = the length of simple pendulum or distance between point of suspension and center of oscillation, in inches; t = time in seconds for n oscillations; n = number of single oscillations in time t.

Simple Pendulum

	$l = \frac{12\,gt^2}{\pi^2 n^2} = \frac{39.1\,t^2}{n^2}$ $t = \frac{n\sqrt{l}}{6.25}$ $n = \frac{6.25\,t}{\sqrt{l}}$		A, center of gravity. B, center of gyration. C, center of oscillation. $a : b = b : l$ $b = \sqrt{al} = 1.155\,a$ $l = 1\tfrac{1}{3}\,a$

Compound Pendulum

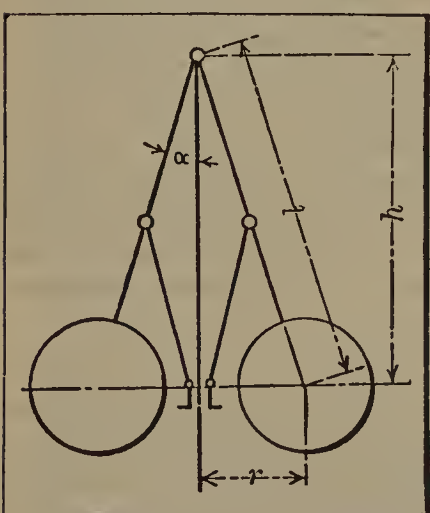
	W and w, weights in pounds. P, point of suspension. $l = \frac{a^2W + b^2w}{aW + bw}$		W and w, weights in pounds. P, point of suspension. A, center of gravity. $c = \frac{aW - bw}{W + w}$ $l = \frac{a^2W + b^2w}{c\,(W + w)}$

Conical Pendulum. — If a weight suspended by a cord revolves at a uniform speed along the circumference of a circle in a horizontal plane, this weight forms a conical pendulum and is held in equilibrium by three forces; the tension in the cord, the centrifugal force, and the force of gravity. Let r be the radius of the circular path in feet which the weight follows; h, the distance in feet of the plane in which the weight moves from the point of suspension; v, the velocity in feet per second of the center of gravity of the weight. Then, time t, in seconds for one revolution, is:

$$t = \frac{2\pi r}{v} = 6.283\sqrt{\frac{h}{g}} \qquad h = \frac{gt^2}{4\pi^2} = 0.8146\,t^2$$

The principle of the conical pendulum is employed in the design of fly-ball governors for steam engines.

Application to Governors. — In the following formulas:



n = number of revolutions per minute;
g = acceleration due to gravity = 32.16;
α = angle made by the arm with the vertical axis;
h = vertical distance from center of ball to point of suspension, in feet;
l = length of arm, in feet.

$$n = \frac{60}{2\pi}\sqrt{\frac{g}{h}} = \frac{54.16}{\sqrt{h}} = \frac{54.16}{\sqrt{l \cos \alpha}}$$

$$h = \frac{2933}{n^2}.$$

Example: — In a governor, of the type shown in the illustration, the balls fly out so that angle α = 45 degrees; how many revolutions per minute are the balls making, if l = 6 inches?

$$n = \frac{54.16}{\sqrt{\frac{1}{2} \times \cos 45^\circ}} = 91.1 \text{ rev. per minute.}$$

Engine Governors. — Governors may be of a purely centrifugal type such as the fly-ball or pendulum design previously referred to, or the principle of inertia may be introduced to secure better speed regulation. Thus, there are two general classes of governors known as centrifugal and inertia governors. The method of utilizing the motion of the governing element for regulating the speed varies; as applied to steam engines, there is the general type of governor which controls the speed by operating a throttling valve which increases or diminishes the amount of steam admitted to the steam-chest, and another general type which regulates the speed by changing the point of cut-off and consequently the amount of expansion in the cylinders.

In the design of governors, the sensitiveness, effort, and stability of the governor are important factors. The sensitiveness of a fly-ball governor is indicated by the amount that the governing sleeve is displaced for a given change in speed, the displacement being relatively large for a given speed change if the governor is sensitive.

The term "effort" as applied to a governor relates to the energy it is capable of exerting upon the governing mechanism. Thus, in the case of a fly-ball governor, the effort indicates the energy exerted on the sleeve while the governor speed is increasing or diminishing. If the energy stored in a revolving governor is small, its sensitiveness will be reduced, because a larger speed change is necessary to obtain the power for operating the governing mechanism than would be required with a governor which exerts greater energy for a given speed change.

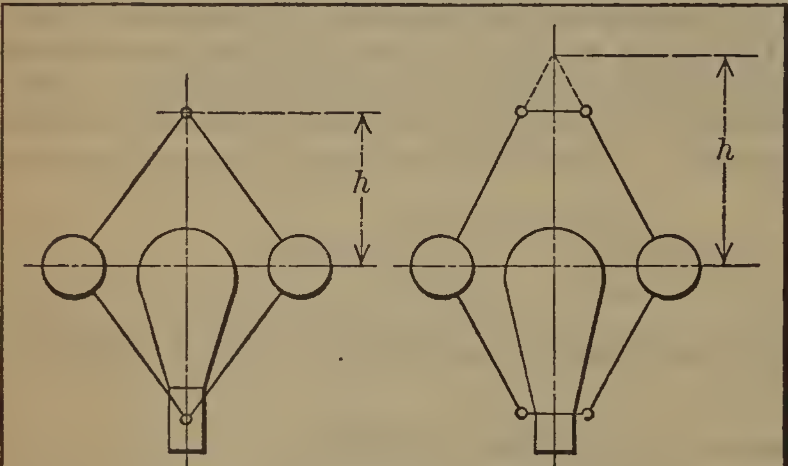


FIG. 1. FIG. 2.

When a governor occupies a definite position of equilibrium for any speed within the range of speeds controlled by the governor, it is said to be "stable." If the load on an engine having a fly-ball governor is diminished, the balls of a stable governor will move outward to a new position as the speed increases, although there will usually be a temporary oscillating movement on each side of this new position, the oscillations gradually diminishing. If the governor were instable (and therefore useless) the oscillations would increase until the limiting points were reached.

Loaded or Weighted Fly-ball Governors. — As the arms of a governor of the conical pendulum type swing outward toward the horizontal position as the result of increasing speed, the change in height h (see Fig. 1) is small for given changes in speed. For instance, if the speed is changed from 50 to 70 revolutions per minute, the difference between the values of h is nearly 7 inches, whereas if the speed changes from 200 to 300 revolutions per minute, the difference in height h for the two speeds is only about ½ inch. Hence, the simple pendulum governor is not suitable for the higher speeds, because then the movement which accompanies the speed changes is too small to secure proper regulation through the governing mechanism. Fly-ball governors are adapted for much higher speeds by loading them. The load may be in the form of a weight which surrounds the spindle, as illustrated by Fig. 1. This is known as a Porter governor.

In the following formula, w = the weight of one governor ball in pounds; e = the weight of the additional load; h = the height in feet indicated by the diagram, Fig. 1; n = speed of governor in revolutions per minute:

$$h = \frac{2933}{n^2} \times \frac{e+w}{w}$$

If the governor is constructed as indicated by the diagram Fig. 2, the height h is not measured from the points at which the arms or rods are suspended, but from the point where the axes of the rods intersect with the vertical center line. The outward movement of the balls may be resisted by a spring instead of a weight, as in the case of the Hartnell governor, which is known as a spring-loaded type.

Sensitiveness and Stability of Governors. — The sensitiveness of one governor may be compared with that of another by determining the *coefficient of speed variations*. If C = the coefficient of speed variations, M = maximum speed within limits of the governor action; M_1 = minimum speed within limits of governor action; m = mean speed within these limits; then,

$$C = \frac{M - M_1}{m}$$

The minimum value of coefficient C necessary to obtain stability in a pendulum type of governor is given by the following formula in which y = distance the fly-balls move horizontally in feet; F = mean centrifugal force of fly-balls in pounds; H = indicated horsepower of engine; W = the weight of engine flywheel in pounds; S = revolutions per minute of main shaft; R = the flywheel radius in feet.

$$C = 4000 \sqrt[3]{\frac{xy}{F}} \times \sqrt[3]{\frac{H^2}{S^4R^4W^2}}$$

The factor x in this formula represents that weight which would be equivalent to the weights of the various moving parts, if it were centered at a point corresponding to the center of gravity of the fly-balls. To determine the value of x, first determine the weights of the different moving parts of the governor, such as the balls, the central weight or load (in the case of a Porter governor), the sleeve, etc.; multiply the weight of each part by the square of the distance it moves from one position to the other; add the various products thus obtained, and divide the total sum by the square of the corresponding movement of the fly-balls at right angles to the governor spindle.

Shaft Governors. — Shaft governors are so named because the governing mechanism is carried by the main shaft and is commonly attached in some way to the flywheel. One type is so arranged that, in the case of a steam engine, the action of centrifugal force on a pivoted and weighted lever, to which a spring is attached, changes the position of the eccentric which operates the slide valve, thus increasing or decreasing the valve travel and changing the point of cut-off. Another type is so designed that the inertia of a pivoted "weight arm" accelerates the governing action by acting in conjunction with the effect of centrifugal force, thus increasing the sensitiveness of the governor. With the inertia governor, the effort or force needed to actuate the governing mechanism increases as the rate of velocity change increases; hence this type is adapted to engines liable to sudden load changes. When the load remains practically constant, the centrifugal type of shaft governor is often employed in preference to the inertia type. The design of these governors depends upon the arrangement of the governing mechanism and upon varying factors.

STRENGTH OF MATERIALS

Elastic Limit; Modulus of Elasticity. — When external forces act upon a material, they produce tension, compression, bending, shearing, or torsional stresses within the material. In most instances, a combination of two or more of these stresses is produced. All stresses to which a material is subjected cause a deformation in it. If the stress is not too great, however, the material will return to its original shape and dimensions when the external stress is removed. The property which enables a material to return to its original shape and dimensions is called its *elasticity*. If a material has been stressed to such an extent that, upon the removal of the load, it does not fully return to its original shape and dimensions, its *elastic limit* has been exceeded. Up to the elastic limit the deformation is directly proportional to the load. The elastic limit is defined as the point at which the deformation ceases to be proportional to the stress. This point, however, is difficult to determine with accuracy.

The *modulus of elasticity* of a material is the quotient obtained by dividing the stress per square inch by the elongation in one inch caused by this stress. The modulus of elasticity is generally denoted by E. If an elongation of 0.015 inch is produced in a steel bar, ten inches long, by a load of 45,000 pounds per each square inch of cross-section of the bar, then the modulus of elasticity will be:

$$E = \frac{45{,}000}{0.0015} = 30{,}000{,}000.$$

As the elongation is assumed to be proportional to the load up to the elastic limit, the modulus of elasticity of a material may be used for finding the elongation e produced by any load per square inch, S, or: $e = S \div E$.

For example, the modulus of elasticity of wrought iron is 27,000,000. Find the elongation per inch produced by a stress of 15,000 pounds per square inch.

$$e = 15{,}000 \div 27{,}000{,}000 = 0.00055 \text{ inch}.$$

Factor of Safety. — The factor of safety may be considered as the product of four primary factors which may be designated as factors a, b, c and d. Designating the factor of safety by F,

$$F = a \times b \times c \times d.$$

The first of these factors a is the ratio of the ultimate strength of the material to the elastic limit, meaning, in this case, by the elastic limit, that boundary line within which the material is perfectly elastic and takes no permanent set. For ordinary materials, the factor a is 2; for nickel steel and oil tempered forgings, it is reduced to 1½.

The second factor b depends on the character of the stress within the material. This factor is 1, for a dead load; 2, for a load varying between zero and maximum; and 3, for a load which produces alternately a tension and a compression equal in amount.

The third factor c depends upon the manner in which the load is applied to the piece under stress. For a load gradually applied, this factor is 1. For a load suddenly applied, the factor is 2. If the load is applied not only suddenly but with impact, this factor must be still further increased in value. (See "Stresses in Beams Produced by Shocks.")

The last factor d may be called the factor of ignorance. The other factors provide against known conditions and this provides against the unknown. It commonly varies in value between 1½ and 3 and occasionally should be given as high a value as 10. It provides against accidental overload, against unexpectedly severe service and unreliable or imperfect materials, etc. When all the conditions are thoroughly known and there is no danger of overload, this factor may be made equal to 1½ for wrought iron and mild steel, and 2, for cast iron.

As an example of the use of the formula given for the factor of safety, find the factor of safety that ought to be used for a forged steel steam-engine piston-rod. The elastic limit will probably be slightly more than one-half the ultimate strength; hence, $a = 2$. The rod will be alternately in tension and compression; hence, $b = 3$. The steam pressure will be applied suddenly or nearly so; hence, $c = 2$. The material is of a reliable kind; hence, $d = 1\frac{1}{2}$. Then:

$$F = 2 \times 3 \times 2 \times 1\tfrac{1}{2} = 18.$$

A table of factors of safety determined by the analytical method outlined, is given herewith.

Table of Factors of Safety

Class of Service	Factors				
	a	*b*	*c*	*d*	*F*
Boilers	2	1	1	2¼–3	4½– 6
Piston-and connecting-rods for double-acting engines	1½–2	3	2	1½	13½–18
Piston- and connecting-rod for single-acting engines	1½–2	2	2	1½	9–12
Shaft carrying bandwheel, flywheel, or armature	1½–2	3	1	1½	6¾– 9
Lathe spindles	2	2	2	1½	12
Mill shafting	2	3	2	2	24
Steel work in buildings	2	1	1	2	4
Steel work in bridges	2	1	1	2½	5
Steel work for small work	2	1	2	1½	6
Cast-iron wheel rims	2	1	1	10	20
Steel wheel rims	2	1	1	4	8
Materials	**Minimum Values**				
Cast-iron and other castings	2	1	1	2	4
Wrought iron or mild steel	2	1	1	1½	3
Oil tempered or nickel steel	1½	1	1	1½	2¼
Hardened steel	1½	1	1	2	3
Bronze and brass, rolled or forged	2	1	1	1½	3

Strength and Properties of Tobin Bronze. — This special bronze contains from 59 to 63 per cent of copper, from ½ to 1½ per cent of tin, the remainder being zinc. It has a specific gravity of 8.4, the weight per cubic inch being 0.304 pound. On account of its tensile strength and its resistance to the corrosive action of sea water, it is used to a great extent in marine engineering. The ultimate tensile strength varies from 60,000 to 65,000 pounds per square inch, and the compressive strength from 170,000 to 180,000 pounds per square inch. The melting point is at 1600 degrees F. The non-liability of Tobin bronze to give forth sparks makes it valuable for powder plates and powder-mill tools.

Average Ultimate Strength of Common Metals; Pounds per Square Inch *

Material	Tension	Compression	Shear	Modulus of Elasticity
Aluminum castings	15,000	12,000	12,000	11,000,000
Brass, cast	24,000	30,000	36,000	9,000,000
Bronze, gun-metal	32,000	20,000	....	10,000,000
Bronze, manganese	60,000	120,000	....	....
Bronze, phosphor	50,000	....	....	14,000,000
Copper, cast	24,000	40,000	30,000	10,000,000
Copper wire, annealed	36,000	....	....	15,000,000
Copper wire, unannealed	60,000	....	....	18,000,000
Iron, cast	15,000	80,000	18,000	12,000,000
Iron wire, annealed	60,000	....	....	15,000,000
Iron wire, unannealed	80,000	....	....	25,000,000
Iron, wrought	48,000	46,000	40,000	27,000,000
Lead, cast	2,000	....	....	1,000,000
Steel castings	70,000	70,000	60,000	30,000,000
Steel, structural	60,000	60,000	50,000	29,000,000
Steel wire, annealed	80,000	....	....	29,000,000
Steel wire, unannealed	120,000	....	....	30,000,000
Steel wire, crucible	180,000	....	....	30,000,000
Steel wire, susp. bridge	200,000	....	....	30,000,000
Steel wire, piano	300,000	....	....	....
Steel wire, plow	270,000	....	....	....
Tin, cast	3,500	6,000	....	4,000,000
Zinc, cast	5,000	20,000	....	13,000,000

* For strength of heat-treated alloy steels, see table, "Physical Properties of Heat-treated Alloy Steels."

Average Ultimate Strength of Common Materials other than Metals
(Pounds per square inch)

Material	Compression	Tension
Bricks, best hard	12,000	400
Bricks, light red	1,000	40
Brickwork, common	1,000	50
Brickwork, best	2,000	300
Cement, Portland, one month old	2,000	400
Cement, Portland, one year old	3,000	500
Concrete, Portland	1,000	200
Concrete, Portland, one year old	2,000	400
Granite	19,000	700
Limestone and sandstone	9,000	300
Trap rock	20,000	800
Slate	14,000	500
Vulcanized Fiber	39,000	13,000

Tensile and Compressive Strength of Different Kinds of Steel. — The ultimate strength of steel in tension and compression is practically the same, and may, for different kinds of steel, be assumed as follows:

Kind of Steel	Ultimate Strength, Lbs. per Sq. In.	Kind of Steel	Ultimate Strength, Lbs. per Sq. In.
Structural steel for rivets......	55,000	Machine steel.......	75,000
Structural steel for beams......	60,000	Gun steel...........	90,000
Boiler steel for rivets..........	50,000	Axle steel...........	100,000
Boiler steel for plates..........	60,000	Spring steel.........	125,000

Influence of Temperature on the Strength of Metals

Material	Degrees Fahrenheit							
	210	400	570	750	930	1100	1300	1475
	Strength of Metal in Per Cent of the Strength at 70 Degrees F.							
Wrought iron.......	104	112	116	96	76	42	25	15
Cast iron..........	....	100	99	92	76	42	....	....
Steel castings......	109	125	121	97	57	....	....	....
Structural steel.....	103	132	122	86	49	28	....	....
Copper............	95	85	73	59	42	....	....	....
Bronze............	101	94	57	26	18	....	....	....

Strength of Special Alloy Steel. — Chrome-vanadium steel is made by the Erie Forge Co. of the following composition: Silicon, 0.131 per cent; sulphur, 0.017 per cent; phosphorus, 0.015 per cent; manganese, 0.60 per cent; carbon, 0.32 per cent; chromium, 0.87 per cent; vanadium, 0.18 per cent.

In the annealed state a steel of this composition shows a tensile strength of 134,000 pounds per square inch, with an elastic limit of 97,000 pounds per square inch. The elongation in 2 inches is 21.1 per cent, and the reduction in area, 57.3 per cent. After having been heat treated, quenched in oil, and annealed, the steel possesses an ultimate strength of 210,000 pounds per square inch; an elastic limit of 191,500 pounds per square inch; an elongation in 2 inches of 12 per cent, and a reduction in area of 45 per cent. A forging of a connecting-rod made from this steel was twisted through an angle of 270 degrees, giving it the appearance of a cork-screw, but showed no fracture or flaw of any kind. (For strength of a number of different heat-treated alloy steels, see also the table, "Physical Properties of Heat-treated Alloy Steels.")

General Factors of Safety

Material	Steady Load	Load Varying from Zero to Maximum in one Direction	Load Varying from Zero to Maximum in both Directions	Suddenly Varying Loads and Shocks
Cast iron	6	10	15	20
Wrought iron	4	6	8	12
Steel	5	6	8	12
Wood	8	10	15	20
Brick	15	20	25	30
Stone	15	20	25	30

General Formulas for the Strength of Materials.— In the following formulas:

A = area of cross-section of material in square inches;
E = modulus of elasticity;
I = moment of inertia of section about an axis passing through the center of gravity;
I_p = polar moment of inertia of section;
M_b = maximum bending moment in inch-pounds;
M_t = moment of force tending to twist (torsional moment) in inch-pounds;
P = total stress in pounds;
y = distance from center of gravity to most remote fiber;
S = permissible working stress in pounds per square inch;
Z = section modulus for bending (moment of resistance);
Z_p = section modulus for torsion;
e = elongation or shortening in inches;
l = length in inches.

For tension and compression:

$$P = A \times S; \qquad e = \frac{Pl}{AE}.$$

For shear:

$$P = A \times S.$$

Assume permissible working stress for shear to equal about four-fifths the permissible stress in tension.

For bending:

$$M_b = \frac{SI}{y} = SZ.$$

For torsion:

$$M_t = \frac{SI_p}{y} = SZ_p.$$

The rule expressed by the formula above, that the section modulus for torsion Z_p equals the polar moment of inertia I_p, divided by the distance y from the center of gravity to the most remote fiber, holds true only for circular sections, but may also be applied with fair accuracy to sections nearly circular. For other cross-sections, the section modulus of torsion does not equal the polar moment of inertia divided by the distance from the center of gravity to the most remote fiber. For-

Strength of Bronzes

(U. S. Government Tests)

Copper, Per Cent	Tin, Per Cent	Tensile Strength, Lbs. per Sq. In.	Yield-point, Lbs. per Sq. In.	Compressive Strength, Lbs. per Sq. In.	Elongation, Per Cent	Compression, Per Cent
100	....	27,000	14,000	41,000	8.0	44
95	5	31,000	17,000	46,000	10.0	41
90	10	29,000	21,000	54,000	4.0	31
85	15	33,000	26,000	74,000	1.6	24
80	20	32,000	28,000	124,000	0.5	14
75	25	18,000	18,000	150,000	....	8
70	30	6,500	6,500	143,000	....	2
65	35	2,800	2,800	75,000	....	4

Strength of Brasses

(U. S. Government Tests)

Copper, Per Cent	Zinc, Per Cent	Tensile Strength, Lbs. per Sq. In.	Yield-point, Lbs. per Sq. In.	Compressive Strength, Lbs. per Sq. In.	Elongation, Per Cent
100	....	27,000	14,000	41,000	7
95	5	28,000	12,000	28,000	12
90	10	30,000	10,000	29,000	18
85	15	32,000	9,000	33,000	25
80	20	34,000	8,000	39,000	33
75	25	37,000	9,000	46,000	38
70	30	41,000	10,000	54,000	38
65	35	46,000	13,000	63,000	33
60	40	49,000	17,000	74,000	19
55	45	44,000	20,000	90,000	10
50	50	30,000	24,000	116,000	4
45	55	14,000	14,000	126,000	..

Strength of Copper-zinc-tin Alloys

(U. S. Government Tests)

Percentage of			Tensile Strength, Lbs. per Sq. In.	Percentage of			Tensile Strength, Lbs. per Sq. In.	Percentage of			Tensile Strength, Lbs. per Sq. In.
Copper	Zinc	Tin		Copper	Zinc	Tin		Copper	Zinc	Tin	
45	50	5	15,000	60	20	20	10,000	75	20	5	45,000
50	45	5	50,000	65	30	5	50,000	75	15	10	45,000
50	40	10	15,000	65	25	10	42,000	75	10	15	43,000
55	43	2	65,000	65	20	15	30,000	75	5	20	41,000
55	40	5	62,000	65	15	20	18,000	80	15	5	45,000
55	35	10	32,500	65	10	25	12,000	80	10	10	45,000
55	30	15	15,000	70	25	5	45,000	80	5	15	47,500
60	37	3	60,000	70	20	10	44,000	85	10	5	43,500
60	35	5	52,500	70	15	15	37,000	85	5	10	46,500
60	30	10	40,000	70	10	20	30,000	90	5	5	42,000

mulas giving the approximate section modulus for torsion for other than circular cross-sections, will be found under the head, "Polar Moment of Inertia and Polar Section Modulus." The permissible working stress for torsion may be assumed as four-fifths the permissible stress in tension.

Examples: — 1. A wrought-iron bar is to support (in tension) a load of 40,000 pounds. The load is gradually applied and then, after having reached its maximum value, gradually removed. Find the diameter of bar required.

From the accompanying tables: Ultimate strength of wrought iron = 48,000 pounds per square inch. Factor of safety to use in present case = 6. Hence, safe working stress = 48,000 ÷ 6 = 8000 pounds per square inch. Inserting the known values in formula $P = A \times S$:

$$40{,}000 = A \times 8000, \quad \text{or} \quad A = 5 \text{ square inches.}$$

The diameter of a bar, the cross-section of which is 5 square inches, is 2½ inches, approximately.

2. What would be the total elongation of the bar in the previous example under full load, if the bar were 5 feet long?

$$e = \frac{Pl}{AE} = \frac{40{,}000 \times 5 \times 12}{5 \times 27{,}000{,}000} = 0.018 \text{ inch.}$$

3. A square bar, firmly held at one end, is supporting a load of 3000 pounds at the outer free end. The length of the bar is 2½ feet. The bar is made of structural steel, and the load is steady. Find the size of bar required for safe loading.

M_b = load × lever arm in inches = 3000 × 30 = 90,000.
S = safe stress = 60,000 ÷ 5 = 12,000.
$I = s^4 \div 12$ for a square, if s = side of square.
$y = s \div 2$ in present case.

Hence:

$$M_b = \frac{SI}{y}, \quad \text{or} \quad 90{,}000 = \frac{12{,}000 \times s^4}{12 \times \frac{1}{2} s} = \frac{12{,}000\, s^3}{6}.$$

$$s^3 = 45, \quad \text{or} \quad s = 3.56 \text{ inches.}$$

4. A square bar is subjected to a steady torsional moment of 90,000 inch-pounds. The bar is made of structural steel. Find the size of bar required for safe loading.

M_t = 90,000.
$S = (60{,}000 \times \frac{4}{5}) \div 5 = 9600$.
$Z_p = \frac{2}{9} s^3$ for a square, if s = side of square.

Hence:

$$M_t = SZ_p, \quad \text{or} \quad 90{,}000 = 9600 \times \tfrac{2}{9} s^3;$$
$$s^3 = 42.2, \text{ approx.,} \quad \text{or} \quad s = 3.48 \text{ inches.}$$

Formulas for Combined Bending and Torsion. — The subject of combined bending and torsion is one that has proved more confusing to machine designers than any other phase of the strength of materials that is usually encountered by the mechanical engineer engaged in ordinary machine design. Prof. A. Lewis Jenkins, in a paper presented before the annual meeting of the American Society of Mechanical Engineers in December, 1917, thoroughly analyzes the subject and gives formulas which may be directly applied by machine designers when the bending moment, the twisting moment and the permissible working stress are known. Two formulas are given, one for the design of shafts of brittle materials and one for soft ductile materials.

Grashof Formula for Brittle Materials. — The following formula, known as the Grashof formula, may be used for the design of shafts and similar machine parts constructed of brittle materials, such as cast iron, hardened or annealed tool steel, hard bronze, and other materials having a small contraction of area when tested in tension:

$$SZ = \tfrac{3}{8} M_b + \tfrac{5}{8} \sqrt{M_b^2 + M_t^2} \qquad (1)$$

in which S = permissible or working stress in tension;
Z = rectangular (ordinary) section modulus;
M_b = bending moment to which member is subjected;
M_t = torsional moment to which member is subjected.

Formula for Soft Ductile Materials. — For the design of parts made of soft or ductile materials, such as mild (low-carbon or machine) steel, copper, soft brass and soft steel tubing, the following formulas should be used:

$$S_s Z_p = 1.3 \sqrt{M_b^2 + M_t^2} \qquad (2)$$

in which Z_p = polar section modulus,
S_s = permissible or working stress in shear,

the notation otherwise being the same as above.

The accompanying table gives values for the unit tensile and unit shearing stresses at the yield point of a number of different materials. By using a suitable factor of safety, the permissible working stresses may be obtained from this table.

Stresses at Yield Point of Different Materials

Material	Average Stress at Yield Point, Lbs. per Sq. In.	
	Tensile	Shearing
Machine steel (mild carbon steel)	47,000	30,000
Rivet steel	39,000	24,000
High-carbon steel	60,000	27,000
Nickel steel	70,000	37,000
Steel tubing	22,000	13,000

When the preceding formulas are applicable, the Guest formula, which follows, and the use of which has been advocated often, will be found unnecessary. It would perhaps be desirable if the Guest formula were not so generally recommended, because it applies only to ductile materials of circular cross-section and if used for hard or brittle materials will result in erroneous and unsuitable dimensions.

Guest Formula. — The Guest formula for parts subjected to combined bending and torsional stresses was published in 1900 by J. J. Guest as the result of experiments made by him. It is applicable to parts of circular cross-section and when such material as mild steel (machine steel) is used. The safe stress S in this formula may be assumed to be equal to the safe stress in bending. The formula is:

$$\text{Combined moment} = \sqrt{M_b^2 + M_t^2} = SZ \qquad (3)$$

This formula is used in computing the values found in the table "Diameters of Shafts for Combined Torsional and Bending Stresses."

Example: — Assume a square bar of structural steel to be subjected to combined bending and torsional moments. The bending moment is 90,000 inch-pounds and the torsional moment is 90,000 inch-pounds. Find the size of a square bar required to withstand the combined moment safely. Using formula (2) for mild steel parts subjected to combined bending and torsional stresses:

$$S_sZ_p = 1.3\sqrt{90{,}000^2 + 90{,}000^2} = 165{,}460 \text{ approximately.}$$

The safe unit-stress for shear is assumed as four-fifths or five-sixths of the permissible stress in tension; hence for structural steel the safe unit-stress equals $(60{,}000 \times \frac{4}{5} \div 5)$ or 9600 pounds per square inch; $Z_p = 0.22\ S^3$ in which S equals the side of the square; therefore

$$165{,}460 = S_sZ_p = 9600 \times 0.22\ S^3,$$

$$S^3 = 78.34 \text{ and } S = 4\tfrac{9}{32} \text{ inches.}$$

Stresses in Machine Parts

Shape of Machine Parts. — While the size of machine parts depends mainly upon the magnitude of the stresses, their shape depends to a large extent upon the manner or direction in which the load or strain is brought to bear upon them. If a part is subjected to tension only, that is, merely resists a force tending to pull it apart, then the shape is not very material, although a round rod, which is most compact and generally cheapest, is the best. Almost any other shape is satisfactory however, but it is well to avoid using thin and broad parts, as the strain might then be brought upon one edge instead of uniformly distributed over the whole area, and cause a stress in one part of the cross-section greater than that for which the material is adapted.

A machine part that is to resist compression only, should have a shape similar to that required for resisting tension, except when the proportion of its length to its diameter or thickness is such that it will be likely to buckle or bend. This will take place, in many cases, when the length exceeds five or six times the diameter and it then becomes desirable to use a hollow or cross-ribbed form of construction, with the metal as far from the axis of the piece as possible. A hollow cylindrical form is most effective, although a hollow, square or cross-ribbed form may be adopted for reasons of appearance or cheapness of production.

When a piece is designed to resist bending, it should have its greatest depth of material in the direction in which the force is applied, because the capacity of a piece to resist bending as indicated by the formula for the section modulus, increases as the square of its thickness or depth in the direction of the force, and only directly as its width. Thus, to increase the depth of a beam two or three times in the direction of the force, would increase its capacity to resist bending four or nine times; while to increase its width two or three times, would only increase its strength two or three times. The proportion of depth and width must, of course, not be carried to an extreme, as there then would be a tendency for the piece to buckle or yield sideways.

Stresses in Castings. — The stresses in castings due to shrinkage in cooling often increase, to a considerable extent, the stresses due to the load. If all parts of a casting could be made to cool equally fast, there would be little trouble from this source, but as different parts of the casting vary in thickness, the time required for cooling varies, and stressses are set up which are sometimes great enough to rupture the casting without any additional load being placed upon it. In the case of a pulley, the hub, on cooling, tends to draw the arms away from the rim. As these strains are primarily due to unequal cooling, it is evident that in order to reduce them to the lowest point, it is necessary to make the different parts of the casting as nearly uniform in thickness as possible. This, however, is not always feasible and in cases when it is not, a liberal allowance should be made for the internal stresses, by using a larger factor of safety when calculating for the external load.

Economical Sections for a Given Strength or Stiffness to Resist Bending. — When a material such as steel is used, which has practically the same properties in tension as compression, the most economical form of beam cross-section for vertical loading only, that is, a load in a single direction, is a beam of I-section, as shown in Fig. 1. For both vertical and horizontal loading, that is, for loading in two directions at right angles to each other, a beam of hollow rectangular section, as shown in Fig. 2, will require the least amount of material for a given strength and stiffness. If the load is the same in the horizontal as in the vertical direction then the cross-section will be a hollow square. For equal loading in any direction, a hollow circular section, Fig. 3, should be used.

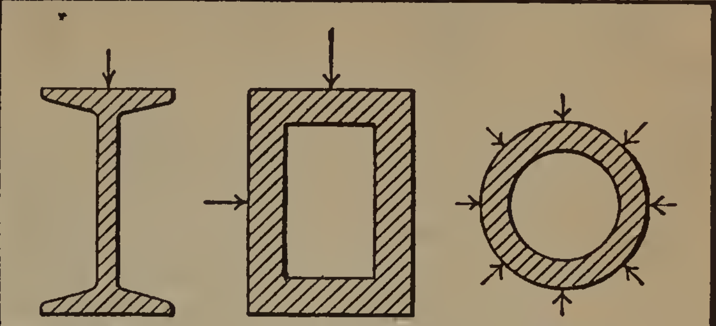


Fig. 1 Fig. 2 Fig. 3

Stresses Due to Changes of Temperature. — If a bar of metal is confined in a space so that it is prevented from expanding or contracting, stresses will be induced in it if it is subjected to temperature changes. These stresses are termed "temperature stresses," and their magnitude is measured by the amount of compression or elongation. If T = change in temperature in degrees F.; S = stress produced by the temperature change, per square inch; C = coefficient of linear expansion; and E = modulus of elasticity, then: $S = C \times T \times E$.

The values of the coefficient of linear expansion for one degree F. equals 0.0000074, for high-carbon steel; 0.0000065, for mild (machine) steel; 0.0000062, for cast iron; and 0.0000068, for wrought iron.

Bending Stresses Combined with Direct Tension or Compression. — In U-shaped machine parts, such as, for example, punch or shear frames, the metal in the back of the frame resists a uniformly distributed tensile stress, due to the pressure between the jaws, and also a stress due to the bending moment set up by the same pressure. The maximum tensile stress, therefore, is composed of the sum of this uniformly distributed stress and the stress due to the bending moment. If the pressure on a machine part is in such a direction that the stresses induced are partly compression and partly bending stresses, then the stress due to direct compression is added to the compressive stress due to bending, in order to find the total compressive stress to which the machine part is subjected.

Shear Stresses Combined with Tension or Compression Stresses. — The rather complicated calculations necessary in cases where shear stresses are combined with tension or compression stresses may be avoided by using the accompanying table which gives factors by means of which the maximum combined unit shear and the maximum combined unit tension or compression may be determined, when the forces causing shear and tension or compression are known. For example, assume that the unit shear S, as produced by the force causing shear alone, is 9000 pounds per square inch, and that the unit tension or compression T, produced by the force that causes tension or compression only, is 12,000 pounds per square inch. Then the ratio $S \div T = 0.75$, and from the table it is then found that the tension (or compression) factor $x = 1.401$ for this ratio of S to T. This means that in this case the maximum combined tension will be 1.401 times what it would have been if there had been no shear. The shear factor y is 1.20, indicating that the maximum combined shear stress is 1.20 times what it would have been if there had been no tension or compression stresses. If the separate unit stresses are known, therefore, the total combined stresses may be quickly determined by this table.

Shear Stresses Combined with Tension or Compression Stresses

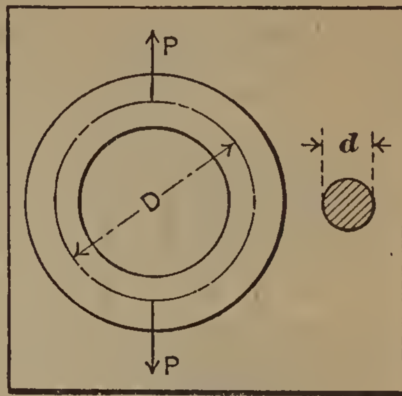
Ratio $\frac{S}{T}$	Tension Factor x	Shear Factor y	Ratio $\frac{S}{T}$	Tension Factor x	Shear Factor y
0.05	1.002	10.05	0.80	1.44	1.18
0.10	1.010	5.10	0.85	1.49	1.16
0.15	1.022	3.48	0.90	1.53	1.14
0.20	1.038	2.69	0.95	1.57	1.13
0.25	1.059	2.24	1.00	1.62	1.12
0.30	1.083	1.94	1.05	1.66	1.11
0.35	1.110	1.74	1.10	1.71	1.10
0.40	1.140	1.60	1.15	1.75	1.09
0.45	1.173	1.49	1.20	1.80	1.08
0.50	1.207	1.41	1.25	1.85	1.08
0.55	1.243	1.35	1.30	1.89	1.07
0.60	1.281	1.30	1.35	1.94	1.07
0.65	1.320	1.26	1.40	1.99	1.06
0.70	1.360	1.23	1.45	2.03	1.06
0.75	1.401	1.20	1.50	2.08	1.05
Ratio $\frac{T}{S}$	**Shear Factor y**	**Tension Factor x**	**Ratio $\frac{T}{S}$**	**Shear Factor y**	**Tension Factor x**
0.05	1.0003	20.50	0.80	1.077	1.85
0.10	1.0012	10.51	0.85	1.086	1.78
0.15	1.0028	7.18	0.90	1.096	1.72
0.20	1.0050	5.52	0.95	1.107	1.67
0.25	1.0078	4.53	1.00	1.118	1.62
0.30	1.0112	3.87	1.05	1.129	1.57
0.35	1.0152	3.40	1.10	1.141	1.54
0.40	1.0198	3.04	1.15	1.153	1.50
0.45	1.0250	2.77	1.20	1.166	1.47
0.50	1.0308	2.56	1.25	1.179	1.44
0.55	1.0371	2.38	1.30	1.193	1.42
0.60	1.0440	2.24	1.35	1.206	1.39
0.65	1.0515	2.11	1.40	1.221	1.37
0.70	1.0595	2.01	1.45	1.235	1.35
0.75	1.0680	1.92	1.50	1.250	1.33

Combined Torsion and Compression. — Propeller shafts of steamers and vertical shafts carrying considerable weight, are subjected to combined torsion and compression. Let P_1 = maximum resultant compressive stress; P_2 = maximum resultant shearing stress; C = the compressive stress due to the thrust; S = the shearing stress due to the twisting moment. Then,

$$P_1 = \tfrac{1}{2}\left(C + \sqrt{C^2 + 4S^2}\right); \qquad P_2 = \tfrac{1}{2}\sqrt{C^2 + 4S^2}$$

Stresses in a Loaded Ring. — The load that may be safely carried by a ring loaded as indicated by the accompanying engraving is found from the formula:

$$P = \frac{2\,S\pi I}{Dy}$$



in which P = load on ring, in pounds; D = mean diameter of ring, in inches; S = allowable working stress in pounds per square inch; I = moment of inertia of section; y = distance from the center of gravity of the section to the most remote fiber, in inches.

For a ring of circular section, where d equals the diameter of the bar from which the ring is made:

$$P = 0.617\,S\frac{d^3}{D} \qquad S = 1.621\,P\frac{D}{d^3}$$

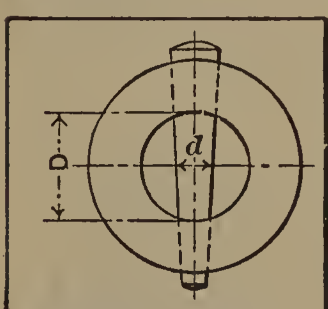
These formulas are especially applicable to loaded rings in which the diameter d is small in proportion to the mean diameter D, because it is only in such cases that the bending action becomes important. For loaded rings having an internal diameter small in proportion to the diameter of the stock, as in the case of eyebolts, the formulas would not apply, because then the metal is stressed largely in direct tension, and the formulas would give excessive dimensions.

Strength of Taper Pins. — The mean diameter of taper pin required to safely transmit a known turning moment, may be found from the formulas:

$$d = 1.13\sqrt{\frac{PR}{DS}} \quad . \quad . \quad . \quad (1), \quad \text{and} \quad d = 283\sqrt{\frac{\text{H.P.}}{NDS}} \quad . \quad . \quad . \quad (2)$$

in which formulas PR = turning moment in inch-pounds; S = safe unit stress = 6000 pounds per square inch; H.P. = horsepower transmitted; N = number of revolutions per minute; and d and D denote dimensions shown in the engraving.

Examples: — A lever secured to a 2-inch round shaft by a steel tapered pin (dimension d = 3/8 inch) has a pull of 50 pounds at a 30-inch radius from shaft center. Find S, the unit working stress on the pin. By transposing Formula (1):



$$S = \frac{1.27\,PR}{Dd^2} = \frac{1.27 \times 50 \times 30}{2 \times (\frac{3}{8})^2} = 6770$$

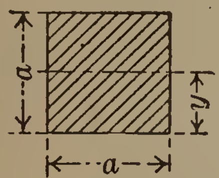
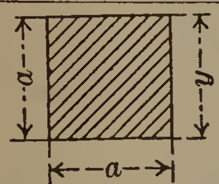
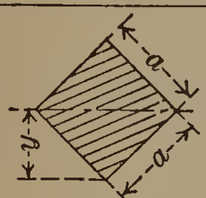
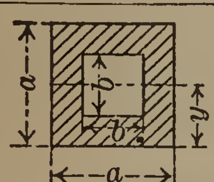
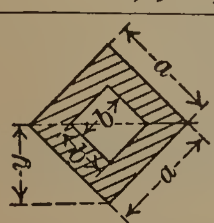
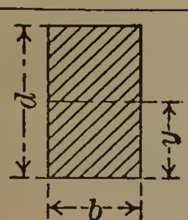
pounds per square inch (nearly), which is a safe unit working stress for machine steel in shear.

Let P = 50 pounds, R = 30 inches, D = 2 inches, and S = 6000 pounds unit working stress. Using Formula (1) to find d:

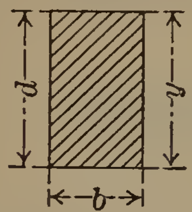
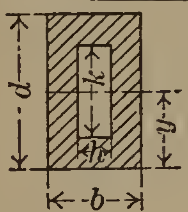
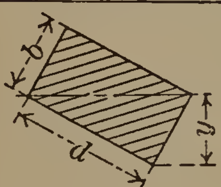
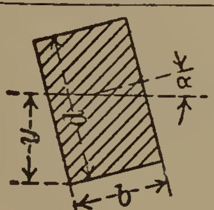
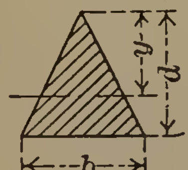
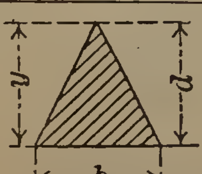
$$d = 1.13\sqrt{\frac{PR}{DS}} = 1.13\sqrt{\frac{50 \times 30}{2 \times 6000}} = 1.13\sqrt{\frac{1}{8}} = 0.4 \text{ inch.}$$

Tables of Moments of Inertia, Section Moduli, etc. — On the following pages are given tables of the moments of inertia and other properties of forty-two different cross-sections of such outlines as are most frequently met with in structural steel shapes or in cast-iron designs. The tables give the area of the section and the distance y from the neutral axis to the extreme fiber, in each case. In some cases, where the formulas for the section modulus and radius of gyration are very lengthy, the formula for the section modulus, for example, has been simply given as $\frac{I}{y}$. The radius of gyration is sometimes given as $\sqrt{\frac{I}{A}}$, when the complete formula would be too long to put into the space available.

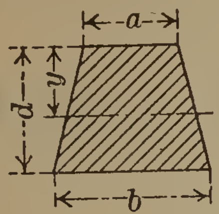
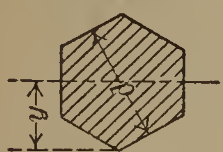
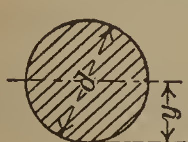
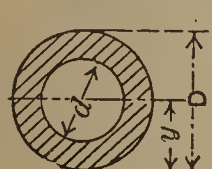
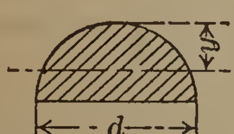
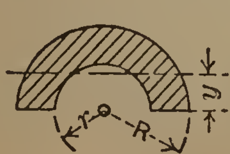
Moments of Inertia, Section Moduli, etc., of Sections

Section A = area y = distance from axis to extreme fiber	Moment of Inertia I	Section Modulus $Z = \frac{I}{y}$	Radius of Gyration $r = \sqrt{\frac{I}{A}}$
 $A = a^2$; $y = \frac{1}{2}a$	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}} = 0.289\,a$
 $A = a^2$; $y = a$	$\frac{a^4}{3}$	$\frac{a^3}{3}$	$\frac{a}{\sqrt{3}} = 0.577\,a$
 $A = a^2$ $y = \frac{a}{\sqrt{2}} = 0.707\,a$	$\frac{a^4}{12}$	$\frac{a^3}{6\sqrt{2}} = 0.118\,a^3$	$\frac{a}{\sqrt{12}} = 0.289\,a$
 $A = a^2 - b^2$; $y = \frac{1}{2}a$	$\frac{a^4 - b^4}{12}$	$\frac{a^4 - b^4}{6\,a}$	$\sqrt{\frac{a^2 + b^2}{12}}$ $= 0.289\sqrt{a^2 + b^2}$
 $A = a^2 - b^2$ $y = \frac{a}{\sqrt{2}} = 0.707\,a$	$\frac{a^4 - b^4}{12}$	$\frac{\sqrt{2}\,(a^4 - b^4)}{12\,a}$ $= 0.118\,\frac{a^4 - b^4}{a}$	$\sqrt{\frac{a^2 + b^2}{12}}$ $= 0.289\sqrt{a^2 + b^2}$
 $A = bd$; $y = \frac{1}{2}d$	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	$\frac{d}{\sqrt{12}} = 0.289\,d$

Moments of Inertia, Section Moduli, etc., of Sections

A = area y = distance from axis to extreme fiber	Moment of Inertia I	Section Modulus $Z = \frac{I}{y}$	Radius of Gyration $r = \sqrt{\frac{I}{A}}$
 $A = bd;\ y = d$	$\frac{bd^3}{3}$	$\frac{bd^2}{3}$	$\frac{d}{\sqrt{3}} = 0.577\,d$
 $A = bd - hk$ $y = \frac{1}{2}d$	$\frac{bd^3 - hk^3}{12}$	$\frac{bd^3 - hk^3}{6\,d}$	$\sqrt{\frac{bd^3 - hk^3}{12\,(bd - hk)}}$ $= 0.289\sqrt{\frac{bd^3 - hk^3}{bd - hk}}$
 $A = bd$ $y = \frac{bd}{\sqrt{b^2 + d^2}}$	$\frac{b^3d^3}{6\,(b^2 + d^2)}$	$\frac{b^2d^2}{6\sqrt{b^2 + d^2}}$	$\frac{bd}{\sqrt{6\,(b^2 + d^2)}}$ $= 0.408\frac{bd}{\sqrt{b^2 + d^2}}$
 $A = bd$ $y = \frac{1}{2}\,(d\cos\alpha + b\sin\alpha)$	$\frac{bd}{12}\,(d^2\cos^2\alpha + b^2\sin^2\alpha)$	$\frac{bd}{6}\left(\frac{d^2\cos^2\alpha + b^2\sin^2\alpha}{d\cos\alpha + b\sin\alpha}\right)$	$\sqrt{\frac{d^2\cos^2\alpha + b^2\sin^2\alpha}{12}}$ $= 0.289 \times \sqrt{d^2\cos^2\alpha + b^2\sin^2\alpha}$
 $A = \frac{1}{2}bd;\ y = \frac{2}{3}d$	$\frac{bd^3}{36}$	$\frac{bd^2}{24}$	$\frac{d}{\sqrt{18}} = 0.236\,d$
 $A = \frac{1}{2}bd;\ y = d$	$\frac{bd^3}{12}$	$\frac{bd^2}{12}$	$\frac{d}{\sqrt{6}} = 0.408\,d$

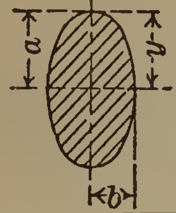
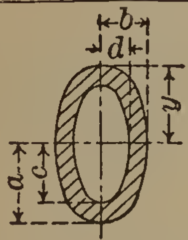
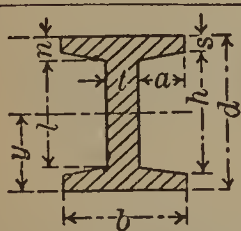
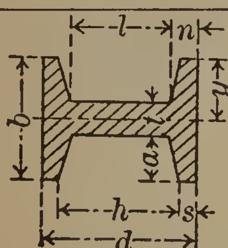
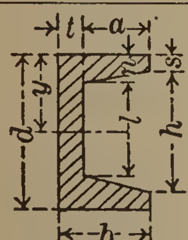
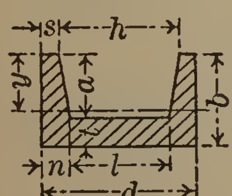
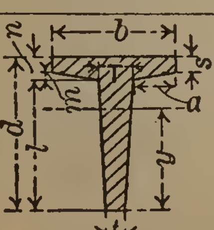
Moments of Inertia, Section Moduli, etc.

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y
	$\frac{d(a+b)}{2}$	$\frac{d(a+2b)}{3(a+b)}$
	$\frac{3d^2 \tan 30^\circ}{2} = 0.866\,d^2$	$\frac{d}{2}$
	$\frac{3d^2 \tan 30^\circ}{2} = 0.866\,d^2$	$\frac{d}{2 \cos 30^\circ} = 0.577\,d$
	$2d^2 \tan 22\frac{1}{2}^\circ = 0.828\,d^2$	$\frac{d}{2}$
	$\frac{\pi d^2}{4} = 0.7854\,d^2$	$\frac{d}{2}$
	$\frac{\pi(D^2 - d^2)}{4}$ $= 0.7854\,(D^2 - d^2)$	$\frac{D}{2}$
	$\frac{\pi d^2}{8} = 0.393\,d^2$	$\frac{(3\pi - 4)d}{6\pi}$ $= 0.288\,d$
	$\frac{\pi(R^2 - r^2)}{2}$ $= 1.5708\,(R^2 - r^2)$	$\frac{4(R^3 - r^3)}{3\pi(R^2 - r^2)}$ $= 0.424\,\frac{R^3 - r^3}{R^2 - r^2}$

Moments of Inertia, Section Moduli, etc.

Moment of Inertia, I	Section Modulus, $Z = \frac{I}{y}$	Radius of Gyration, $r = \sqrt{\frac{I}{A}}$
$\frac{d^3(a^2+4ab+b^2)}{36(a+b)}$	$\frac{d^2(a^2+4ab+b^2)}{12(a+2b)}$	$\sqrt{\frac{d^2(a^2+4ab+b^2)}{18(a+b)^2}}$
$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 30^\circ)}{4\cos^2 30^\circ}\right]$ $= 0.06\,d^4$	$\frac{A}{6}\left[\frac{d(1+2\cos^2 30^\circ)}{4\cos^2 30^\circ}\right]$ $= 0.12\,d^3$	$\sqrt{\frac{d^2(1+2\cos^2 30^\circ)}{48\cos^2 30^\circ}}$ $= 0.264\,d$
$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 30^\circ)}{4\cos^2 30^\circ}\right]$ $= 0.06\,d^4$	$\frac{A}{6.9}\left[\frac{d(1+2\cos^2 30^\circ)}{4\cos^2 30^\circ}\right]$ $= 0.104\,d^3$	$\sqrt{\frac{d^2(1+2\cos^2 30^\circ)}{48\cos^2 30^\circ}}$ $= 0.264\,d$
$\frac{A}{12}\left[\frac{d^2(1+2\cos^2 22\frac{1}{2}^\circ)}{4\cos^2 22\frac{1}{2}^\circ}\right]$ $= 0.055\,d^4$	$\frac{A}{6}\left[\frac{d(1+2\cos^2 22\frac{1}{2}^\circ)}{4\cos^2 22\frac{1}{2}^\circ}\right]$ $= 0.109\,d^3$	$\sqrt{\frac{d^2(1+2\cos^2 22\frac{1}{2}^\circ)}{48\cos^2 22\frac{1}{2}^\circ}}$ $= 0.257\,d$
$\frac{\pi d^4}{64} = 0.049\,d^4$	$\frac{\pi d^3}{32} = 0.098\,d^3$	$\frac{d}{4}$
$\frac{\pi(D^4-d^4)}{64}$ $= 0.049\,(D^4-d^4)$	$\frac{\pi(D^4-d^4)}{32\,D}$ $= 0.098\,\frac{D^4-d^4}{D}$	$\frac{\sqrt{D^2+d^2}}{4}$
$\frac{(9\pi^2-64)\,d^4}{1152\pi}$ $= 0.007\,d^4$	$\frac{(9\pi^2-64)\,d^3}{192\,(3\pi-4)}$ $= 0.024\,d^3$	$\frac{\sqrt{(9\pi^2-64)\,d^2}}{12\pi}$ $= 0.132\,d$
$0.1098\,(R^4-r^4)$ $-\frac{0.283\,R^2r^2\,(R-r)}{R+r}$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$

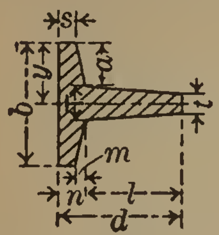
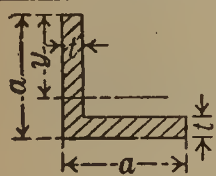
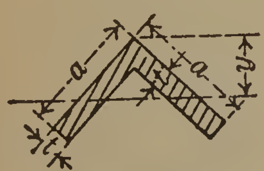
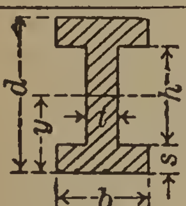
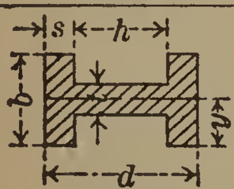
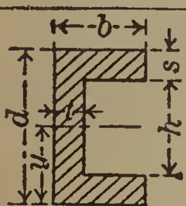
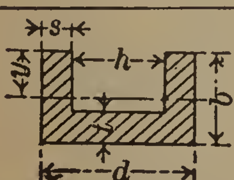
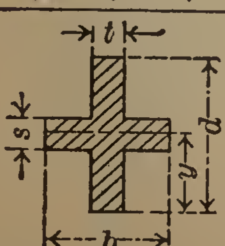
Moments of Inertia, Section Moduli, etc.

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y
	$\pi ab = 3.1416\,ab$	a
	$\pi(ab - cd)$ $= 3.1416\,(ab - cd)$	a
	$dt + 2a(s + n)$	$\frac{d}{2}$
	$dt + 2a(s + n)$	$\frac{b}{2}$
	$dt + a(s + n)$	$\frac{d}{2}$
	$dt + a(s + n)$	$b - [b^2 s + \frac{ht^2}{2} + \frac{g}{3}(b - t)^2 \times (b + 2t)] \div A$ in which g = slope of flange $= \frac{h - l}{2(b - t)}$
	$\frac{l(T + t)}{2} + Tn + a(s + n)$	$d - [3s^2(b + T) + 2am(m + 3s) + 3Td^2 - l(T - t)(3d - l)] \div 6A$

Moments of Inertia, Section Moduli, etc.

Moment of Inertia, I	Section Modulus, $Z = \frac{I}{y}$	Radius of Gyration, $r = \sqrt{\frac{I}{A}}$
$\frac{\pi a^3 b}{4} = 0.7854\, a^3 b$	$\frac{\pi a^2 b}{4} = 0.7854\, a^2 b$	$\frac{a}{2}$
$\frac{\pi}{4}(a^3 b - c^3 d)$ $= 0.7854\,(a^3 b - c^3 d)$	$\frac{\pi\,(a^3 b - c^3 d)}{4\,a}$ $= 0.7854\,\frac{a^3 b - c^3 d}{a}$	$\frac{1}{2}\sqrt{\frac{a^3 b - c^3 d}{ab - cd}}$
$\frac{1}{12}\left[bd^3 - \frac{1}{4\,g}(h^4 - l^4)\right]$ in which g = slope of flange $= \frac{h-l}{b-t} = \frac{1}{6}$ for standard I-beams.	$\frac{1}{6\,d}\left[bd^3 - \frac{1}{4\,g}(h^4 - l^4)\right]$	$\sqrt{\frac{\frac{1}{12}\left[bd^3 - \frac{1}{4\,g}(h^4 - l^4)\right]}{dt + 2\,a\,(s+n)}}$
$\frac{1}{12}\left[b^3(d-h) + lt^3 + \frac{g}{4}(b^4 - t^4)\right]$ in which g = slope of flange (see above).	$\frac{1}{6\,b}\left[b^3(d-h) + lt^3 + \frac{g}{4}(b^4 - t^4)\right]$	$\sqrt{\frac{I}{A}}$
$\frac{1}{12}\left[bd^3 - \frac{1}{8\,g}(h^4 - l^4)\right]$ in which g = slope of flange $= \frac{h-l}{2\,(b-t)} = \frac{1}{6}$ for standard channels.	$\frac{1}{6\,d}\left[bd^3 - \frac{1}{8\,g}(h^4 - l^4)\right]$	$\sqrt{\frac{\frac{1}{12}\left[bd^3 - \frac{1}{8\,g}(h^4 - l^4)\right]}{dt + a\,(s+n)}}$
$\frac{1}{3}\left[2\,sb^3 + lt^3 + \frac{g}{2}(b^4 - t^4)\right] - A\,(b-y)^2$ in which g = slope of flange (see above).	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
$\frac{1}{12}\,[l^3(T + 3\,t) + 4\,bn^3 - 2\,am^3] - A\,(d - y - n)^2$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$

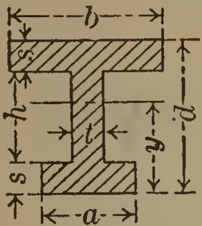
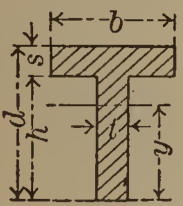
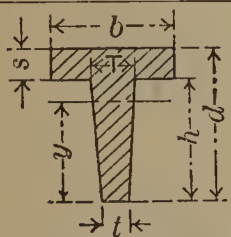
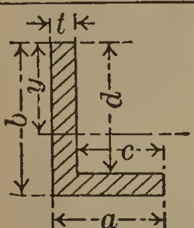
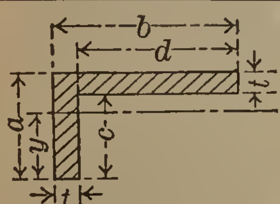
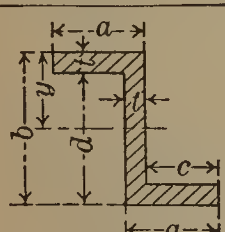
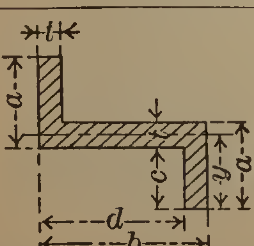
Moments of Inertia, Section Moduli, etc.

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y
	$\frac{l(T+t)}{2} + Tn$ $+ a(s+n)$	$\frac{b}{2}$
	$t(2a-t)$	$a - \frac{a^2+at-t^2}{2(2a-t)}$
	$t(2a-t)$	$\frac{a^2+at-t^2}{2(2a-t)\cos 45^\circ}$
	$bd-h(b-t)$	$\frac{d}{2}$
	$bd-h(b-t)$	$\frac{b}{2}$
	$bd-h(b-t)$	$\frac{d}{2}$
	$bd-h(b-t)$	$b - \frac{2b^2s+ht^2}{2bd-2h(b-t)}$
	$dt+s(b-t)$	$\frac{d}{2}$

Moments of Inertia, Section Moduli, etc.

Moment of inertia, I	Section Modulus, $Z = \frac{I}{y}$	Radius of Gyration, $r = \sqrt{\frac{I}{A}}$
$\frac{sb^3 + mT^3 + lt^3}{12} + \frac{am[2a^2 + (2a + 3T)^2]}{36} + \frac{l(T - t)[(T - t)^2 + 2(T + 2t)^2]}{144}$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
$\frac{1}{3}[ty^3 + a(a - y)^3 - (a - t)(a - y - t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
$\frac{1}{3}[2x^4 - 2(x - t)^4 + t[a - (2x - \frac{1}{2}t)]^3]$ in which $x = \frac{a^2 + at - t^2}{2(2a - t)}$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
$\frac{bd^3 - h^3(b - t)}{12}$	$\frac{bd^3 - h^3(b - t)}{6d}$	$\sqrt{\frac{bd^3 - h^3(b - t)}{12[bd - h(b - t)]}}$
$\frac{2sb^3 + ht^3}{12}$	$\frac{2sb^3 + ht^3}{6b}$	$\sqrt{\frac{2sb^3 + ht^3}{12[bd - h(b - t)]}}$
$\frac{bd^3 - h^3(b - t)}{12}$	$\frac{bd^3 - h^3(b - t)}{6d}$	$\sqrt{\frac{bd^3 - h^3(b - t)}{12[bd - h(b - t)]}}$
$\frac{2sb^3 + ht^3}{3} - A(b - y)^2$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
$\frac{td^3 + s^3(b - t)}{12}$	$\frac{td^3 + s^3(b - t)}{6d}$	$\sqrt{\frac{td^3 + s^3(b - t)}{12[td + s(b - t)]}}$

Moments of Inertia, Section Moduli, etc.

Section	Area of Section, A	Distance from Neutral Axis to Extreme Fiber, y
	$bs + ht + as$	$d - [td^2 + s^2 (b - t) + s (a - t)(2d - s)] \div 2A$
	$bs + ht$	$d - \dfrac{d^2t + s^2 (b - t)}{2(bs + ht)}$
	$bs + \dfrac{h(T + t)}{2}$	$d - [3bs^2 + 3ht(d + s) + h(T - t)(h + 3s)] \div 6A$
	$t(a + b - t)$	$b - \dfrac{t(2d + a) + d^2}{2(d + a)}$
	$t(a + b - t)$	$a - \dfrac{t(2c + b) + c^2}{2(c + b)}$
	$t[b + 2(a - t)]$	$\dfrac{b}{2}$
	$t[b + 2(a - t)]$	$\dfrac{2a - t}{2}$

Moments of Inertia, Section Moduli, etc.

Moment of Inertia, I	Section Modulus, $Z = \frac{I}{y}$	Radius of Gyration, $r = \sqrt{\frac{I}{A}}$
$\frac{1}{3}[b(d-y)^3 + ay^3 - (b-t)(d-y-s)^3 - (a-t)(y-s)^3]$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
$\frac{1}{3}[ty^3 + b(d-y)^3 - (b-t)(d-y-s)^3]$	$\frac{I}{y}$	$\sqrt{\frac{1}{3(bs+ht)}[ty^3 + b(d-y)^3 - (b-t)(d-y-s)^3]}$
$\frac{1}{12}[4bs^3 + h^3(3t+T)] - A(d-y-s)^2$	$\frac{I}{y}$	$\sqrt{\frac{I}{A}}$
$\frac{1}{3}[ty^3 + a(b-y)^3 - (a-t)(b-y-t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{1}{3t(a+b-t)}[ty^3+a(b-y)^3 -(a-t)(b-y-t)^3]}$
$\frac{1}{3}[ty^3 + b(a-y)^3 - (b-t)(a-y-t)^3]$	$\frac{I}{y}$	$\sqrt{\frac{1}{3t(a+b-t)}[ty^3+b(a-y)^3 -(b-t)(a-y-t)^3]}$
$\frac{ab^3 - c(b-2t)^3}{12}$	$\frac{ab^3 - c(b-2t)^3}{6b}$	$\sqrt{\frac{ab^3-c(b-2t)^3}{12t[b+2(a-t)]}}$
$\frac{b(a+c)^3 - 2c^3d - 6a^2cd}{12}$	$\frac{b(a+c)^3 - 2c^3d - 6a^2cd}{6(2a-t)}$	$\sqrt{\frac{b(a+c)^3-2c^3d-6a^2cd}{12t[b+2(a-t)]}}$

Section Modulus, Area, etc., of Sections for Punch and Shear Frames.— Machine frames cannot be standardized so as to permit tables of section modulus, area, etc., of the sections to be made up in the same way as for standard structural steel sections, but it is possible to arrange a table for punch and shear frames so as to simplify the work of selecting proper sections. A table of these quantities is, therefore, given. To illustrate the use of the table, take as an example the punch frame shown diagrammatically in Fig. 1. The distance from the center line of the punch to the back of the gap is 24 inches. Assume that the maximum pressure *P*, tending to force the jaws apart, is that due to punching a 1-inch circular hole in soft steel plate 1 inch thick, or say about 157,000 pounds. Consider the section at *TX*. The action of *P* is such as to produce a tensile stress on the section to the left of the neutral axis *N*, with a compressive stress to the right of *N*, both due to

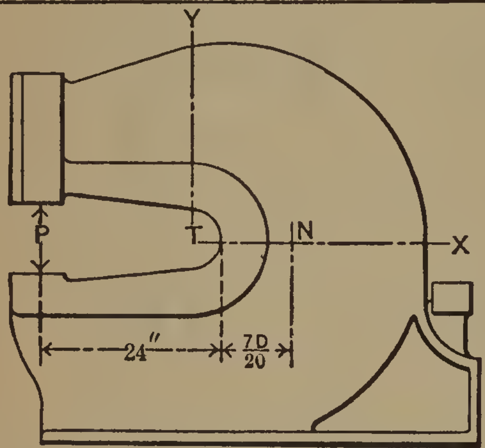


Fig. 1

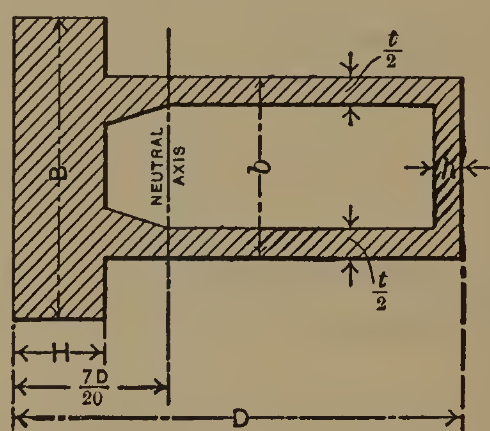


Fig. 2

flexure; and, besides, there is a tensile stress distributed uniformly over the section. It is usually sufficient to determine the maximum tensile stress to the left of *N*.

Maximum tensile stress = flexure tensile stress + uniformly distributed tensile stress.

$$\text{Flexure tensile stress} = \frac{\text{Moment of } P \text{ about } N}{\text{Tensile section modulus of section } TX}$$

$$\text{Uniformly distributed tensile stress} = \frac{P}{\text{Area of section } TX}$$

Assume that *D*, in Fig. 2, is about 30 inches. Then $\frac{7\,D}{20} = 10\frac{1}{2}$ inches. If 3000 pounds per square inch is the allowable fiber stress, and the stress due to flexure only is considered, the required section modulus for tension would be:

$$\frac{157{,}000 \times (24 + 10\frac{1}{2})}{3000} = 1800, \text{ about.}$$

As no allowance has been made for the additional tensile stress uniformly distributed over the section, a section must be selected the section modulus of which is somewhat greater than 1800, say about 2500.

The table of "Properties of Sections for Punch and Shear Frames" gives a great variety of shapes and proportions, the only dimensions common to all being the depth of section, *D*, and the distance of the neutral axis from the extreme tension fiber. This location of the neutral axis represents average practice and insures an economical distribution of metal. Generous fillets and rounded corners should, of course, be used on the actual section.

Suppose that a deep narrow section is desired, similar to that in Fig. 2, in which the dimensions are as follows: $D = 10$ inches; $B = 7$ inches; $b = 4$ inches; $H = 1.98$ inch; $\frac{1}{2}t = h = \frac{5}{8}$ inch; F (area) = 26.3 square inches; and Z_t (section modulus for tension) = 78.6. This section, taken from the table, is not, of course, large enough, but a similar one which is large enough may be easily found as follows:

If two sections A and B are similar in all respects, then

$$\frac{\text{Area of } A}{\text{Area of } B} = \frac{(\text{Any dimension of } A)^2}{(\text{Corresponding dimension of } B)^2};$$

and

$$\frac{\text{Section modulus of } A}{\text{Section modulus of } B} = \frac{(\text{Any dimension of } A)^3}{(\text{Corresponding dimension of } B)^3};$$

$$\frac{\text{Required section modulus}}{\text{Modulus of section from the table}} = \frac{(\text{Required } D)^3}{(D \text{ of section from the table})^3}$$

Hence, $\frac{2500}{78.6} = \frac{(\text{Required } D)^3}{(10)^3}$. This last equation solved for D gives $D = 31.7$ inches, nearly. As the large section will be exactly similar to the small one, the area of the large section is found from the equation:

$$\frac{\text{Area of required section}}{26.3} = \frac{(31.7)^2}{(10)^2}$$

and area = 264 square inches, about.

The neutral axis will be $\frac{7}{20} \times 31.7$, or 11.1 inches from the extreme tension fiber.

This trial section may now be tested for the maximum stress on it.

Flexure tensile stress $= \frac{157{,}000 \times (24 + 11.1)}{2500} = 2200$ pounds per square inch, about.

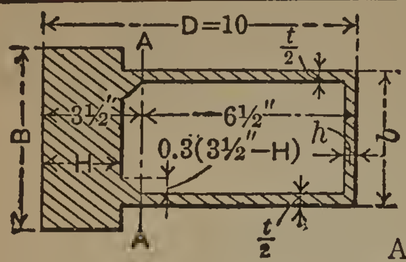
Uniformly distributed tensile stress $= \frac{157{,}000}{264} = 600$ pounds per square inch, about.

The total tensile stress is, therefore, 2800 pounds per square inch.

If this result had not been near enough to the 3000 pounds per square inch assumed, another section could have been selected and worked out in the same way to get a closer result. The depth D of the required section being 31.7 inches as compared with 10 inches of the similar section given in the table, each of the dimensions of the required section will be 3.17 times the corresponding one given in the table. Hence $D = 31.7$, say 32 inches; $B = 22.2$, say 22 inches; $b = 12.7$, say 13 inches; $H = 6.3$, say $6\frac{1}{2}$ inches, $\frac{1}{2}t = h = 1.98$, say 2 inches. The webs thicken gradually from the neutral axis to the tension flange so as to avoid too sudden a change in the section. For selecting the section at TY, Fig. 1, the procedure would be the same except that there would be no uniformly distributed stress to be added as in the case of section TX.

The method here given for determining the stress on the section TX is the one which is generally used in calculating the strength of shear frames and other designs of similar type. It is correct for straight beams, and until recently it was considered approximately correct for curved beams, but investigations by Bach and others indicate that the maximum stresses in curved beams are very much greater than found by the methods generally used. A quick and accurate way of applying the new theory has, however, not yet been devised, and for this reason the only possible way is to use the method outlined and provide an ample factor of safety. This is a safe course, because cast-iron punch frames, the sections of which have been determined by these methods, have stood up under their loads for years.

Properties of Sections for Punch and Shear Frames

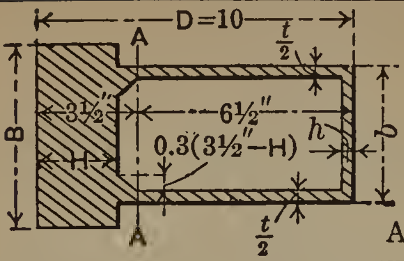


Z_c = Section Modulus for Compression;
Z_t = Section Modulus for Tension;
F = Area of Section;
I = Moment of Inertia about Gravity Axis $A-A$.

All dimensions in inches

B	b	$h = \frac{1}{2}t$	H	F	I	Z_c	Z_t
10	10	¼	0.57	15.36	228.51	35.20	65.40
		⅜	1.10	23.43	311.78	47.95	89.10
		½	1.80	31.82	397.83	61.20	113.70
	9	¼	0.51	14.66	200.77	30.89	57.36
		⅜	0.99	21.64	290.32	44.66	82.95
		½	1.61	29.56	371.95	57.22	106.27
		⅝	2.41	38.69	438.24	67.44	125.21
	8	¼	0.44	13.87	180.98	27.80	51.50
		⅜	0.88	20.41	272.07	41.90	77.70
		½	1.38	27.27	345.47	53.20	98.50
		⅝	2.04	35.20	410.37	63.10	117.00
		¾	3.50	49.63	462.98	71.20	132.00
	7	¼	0.38	13.15	172.76	26.50	49.30
		⅜	0.77	19.21	261.46	40.30	74.60
		½	1.24	25.69	320.52	49.30	91.50
		⅝	1.74	32.24	378.80	58.30	108.00
		¾	2.34	39.42	428.57	65.90	122.60
9	9	¼	0.59	14.66	204.06	31.40	58.30
		⅜	1.20	21.57	291.47	44.80	83.40
		½	2.00	30.68	363.50	55.80	103.60
	8	¼	0.50	13.83	185.91	28.60	53.65
		⅜	1.00	20.34	268.15	41.28	76.65
		½	1.70	28.07	338.66	52.15	96.81
		⅝	2.60	37.12	403.27	62.10	115.12
	7	¼	0.42	13.00	173.33	26.60	49.80
		⅜	0.89	19.25	250.41	38.40	71.60
		½	1.42	25.65	317.24	48.80	90.50
		⅝	2.11	33.03	375.00	57.70	107.10
		¾	3.06	42.13	420.36	64.60	120.10
	6	¼	0.36	12.40	161.70	24.90	46.20
		⅜	0.75	17.92	226.20	34.80	64.70
		½	1.23	23.89	290.50	44.70	83.00
		⅝	1.79	30.22	345.70	53.10	100.60
		¾	2.62	38.26	392.00	60.30	112.00
8	8	¼	0.62	14.09	187.46	28.90	53.55
		⅜	1.20	20.50	268.38	41.30	76.70
		½	2.10	28.79	336.75	51.80	96.20
	7	¼	0.55	13.69	172.25	26.50	49.20
		⅜	1.10	19.50	248.56	38.30	71.10
		½	1.90	27.07	315.01	48.50	90.00

Properties of Sections for Punch and Shear Frames



Z_c = Section Modulus for Compression;
Z_t = Section Modulus for Tension;
F = Area of Section;
I = Moment of Inertia about Gravity Axis $A - A$.

All dimensions in inches

B	b	$h = \frac{1}{2}t$	H	F	I	Z_c	Z_t
8	7	⅝	3.00	36.41	377.05	58.0	108.0
	6	¼	0.43	12.43	155.80	23.9	44.5
		⅜	0.91	18.07	221.14	34.0	63.1
		½	1.50	24.20	283.77	43.5	80.8
		⅝	2.36	31.79	339.75	52.3	96.9
	5	¼	0.33	11.61	139.76	21.42	39.9
		⅜	0.75	16.81	204.37	31.41	58.4
		½	1.25	22.27	261.65	40.25	75.2
		⅝	2.00	29.00	310.74	49.7	88.8
		¾	2.75	35.66	350.65	54.0	100.0
7	7	¼	0.70	13.52	169.40	26.04	48.4
		⅜	1.40	19.99	243.80	37.5	69.6
		½	2.44	27.28	310.80	47.8	88.8
	6	¼	0.55	12.55	147.20	22.6	42.1
		⅜	1.14	18.27	220.03	33.8	62.8
		½	2.00	25.17	282.60	43.4	80.8
	5	¼	0.41	12.28	136.92	20.0	39.1
		⅜	0.95	16.98	203.40	31.0	58.0
		½	1.65	23.03	258.64	40.0	73.8
		⅝	2.55	30.36	302.60	46.5	86.5
	4	¼	0.31	10.96	124.20	19.12	35.5
		⅜	0.76	15.71	183.00	28.15	52.3
		½	1.31	20.80	232.20	35.7	66.4
		⅝	1.98	26.30	275.00	42.3	78.6
6	6	¼	0.68	12.56	156.25	24.0	44.6
		⅜	1.56	18.79	222.10	32.6	60.5
		½	3.50	30.00	275.00	42.3	78.6
	5	¼	0.53	11.68	180.83	29.0	51.6
		⅜	1.27	17.25	214.79	33.0	61.4
		½	2.35	24.15	253.85	39.0	73.4
	4	¼	0.38	10.88	125.59	19.3	35.8
		⅜	1.00	15.84	181.36	27.9	51.8
		½	1.80	20.87	228.96	35.2	65.4
5	5	¼	0.73	11.76	139.73	21.5	40.0
		⅜	1.70	17.28	196.98	30.2	56.2
	4	¼	0.55	10.97	125.40	19.3	35.9
		⅜	1.45	16.13	181.52	28.0	52.0
	3	¼	0.84	10.16	110.14	17.25	32.0
		⅜	1.12	14.90	167.70	25.7	47.6
		½	2.10	19.99	198.57	30.5	56.8

Section Moduli of Rectangles

Depth of Rectangle, Inches	Width of Rectangle, Inches												
	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
2	0.17	0.21	0.25	0.29	0.33	0.37	0.42	0.46	0.50	0.54	0.58	0.63	0.67
2½	0.26	0.33	0.39	0.45	0.52	0.58	0.65	0.71	0.78	0.84	0.91	0.97	1.04
3	0.38	0.47	0.56	0.65	0.75	0.84	0.94	1.03	1.13	1.22	1.31	1.41	1.50
3½	0.51	0.64	0.76	0.89	1.02	1.15	1.28	1.40	1.53	1.66	1.79	1.92	2.04
4	0.66	0.83	1.00	1.16	1.33	1.50	1.67	1.83	2.00	2.17	2.34	2.50	2.67
4½	0.84	1.05	1.27	1.48	1.69	1.90	2.11	2.32	2.53	2.74	2.95	3.16	3.38
5	1.04	1.30	1.56	1.82	2.08	2.34	2.60	2.86	3.13	3.39	3.65	3.91	4.17
5½	1.26	1.57	1.89	2.20	2.52	2.83	3.15	3.46	3.78	4.10	4.41	4.73	5.05
6	1.50	1.87	2.25	2.62	3.00	3.37	3.75	4.12	4.50	4.88	5.25	5.63	6.00
6½	1.76	2.20	2.64	3.08	3.52	3.96	4.40	4.84	5.28	5.72	6.16	6.60	7.04
7	2.04	2.55	3.06	3.57	4.08	4.59	5.11	5.62	6.13	6.64	7.15	7.66	8.17
7½	2.34	2.93	3.52	4.10	4.69	5.27	5.86	6.45	7.03	7.62	8.20	8.79	9.38
8	2.66	3.33	4.00	4.66	5.33	6.00	6.67	7.33	8.00	8.67	9.34	10.00	10.67
8½	3.01	3.76	4.52	5.27	6.02	6.77	7.53	8.28	9.04	9.79	10.54	11.29	12.04
9	3.37	4.22	5.06	5.90	6.75	7.59	8.44	9.28	10.12	10.97	11.81	12.66	13.50
9½	3.76	4.70	5.64	6.58	7.52	8.46	9.40	10.34	11.28	12.22	13.16	14.10	15.04
10	4.17	5.21	6.25	7.29	8.33	9.38	10.42	11.46	12.50	13.54	14.58	15.62	16.67
10½	4.59	5.74	6.89	8.04	9.19	10.33	11.48	12.63	13.77	14.92	16.07	17.22	18.37
11	5.04	6.30	7.56	8.82	10.08	11.34	12.60	13.86	15.12	16.38	17.64	18.90	20.16
11½	5.51	6.88	8.26	9.64	11.02	12.39	13.77	15.15	16.53	17.91	19.28	20.66	22.04
12	6.00	7.50	9.00	10.50	12.00	13.50	15.00	16.50	18.00	19.50	21.00	22.50	24.00
12½	6.51	8.14	9.76	11.39	13.02	14.65	16.27	17.90	19.52	21.15	22.78	24.41	26.04
13	7.04	8.80	10.56	12.32	14.08	15.84	17.60	19.36	21.12	22.88	24.64	26.40	28.16
13½	7.59	9.49	11.39	13.29	15.19	17.09	18.99	20.89	22.79	24.69	26.59	28.49	30.39
14	8.17	10.21	12.25	14.29	16.33	18.37	20.41	22.46	24.50	26.54	28.58	30.62	32.66
14½	8.76	10.95	13.14	15.33	17.52	19.71	21.90	24.09	26.28	28.47	30.66	32.85	35.04

Section Moduli of Rectangles

Depth of Rectangle, Inches	Width of Rectangle, Inches: 1 1/16	1 1/8	1 3/16	1 1/4	1 5/16	1 3/8	1 7/16	1 1/2	1 9/16	1 5/8	1 11/16	1 3/4	1 13/16	1 7/8	1 15/16	2
2	0.71	0.75	0.79	0.83	0.87	0.92	0.96	1.00	1.04	1.08	1.13	1.17	1.21	1.25	1.29	1.33
2½	1.10	1.17	1.24	1.30	1.37	1.43	1.50	1.56	1.63	1.69	1.76	1.82	1.89	1.95	2.02	2.08
3	1.59	1.69	1.78	1.87	1.97	2.06	2.15	2.25	2.34	2.44	2.54	2.63	2.72	2.82	2.91	3.00
3½	2.17	2.30	2.42	2.55	2.67	2.80	2.93	3.06	3.19	3.32	3.44	3.57	3.69	3.82	3.95	4.08
4	2.83	3.00	3.17	3.33	3.49	3.66	3.83	4.00	4.16	4.33	4.50	4.66	4.83	5.00	5.16	5.33
4½	3.59	3.80	4.01	4.22	4.43	4.64	4.85	5.06	5.27	5.48	5.69	5.90	6.11	6.32	6.53	6.75
5	4.43	4.69	4.95	5.21	5.47	5.73	5.99	6.25	6.51	6.77	7.03	7.29	7.55	7.81	8.07	8.33
5½	5.36	5.67	5.99	6.30	6.62	6.93	7.25	7.56	7.88	8.19	8.51	8.83	9.14	9.45	9.76	10.08
6	6.37	6.75	7.13	7.50	7.88	8.25	8.63	9.00	9.38	9.75	10.12	10.50	10.88	11.26	11.63	12.00
6½	7.48	7.92	8.36	8.80	9.24	9.68	10.12	10.56	11.00	11.44	11.88	12.32	12.76	13.20	13.64	14.08
7	8.67	9.18	9.70	10.21	10.72	11.23	11.74	12.25	12.76	13.27	13.79	14.30	14.81	15.32	15.83	16.34
7½	9.96	10.55	11.14	11.72	12.30	12.88	13.47	14.06	14.64	15.23	15.82	16.41	16.99	17.58	18.17	18.75
8	11.33	12.00	12.67	13.34	14.00	14.67	15.34	16.00	16.67	17.34	18.00	18.67	19.34	20.00	20.67	21.34
8½	12.79	13.54	14.29	15.04	15.80	16.55	17.30	18.06	18.82	19.57	20.32	21.08	21.83	22.58	23.33	24.08
9	14.34	15.18	16.02	16.87	17.71	18.55	19.40	20.25	21.10	21.94	22.79	23.63	24.47	25.31	26.15	27.00
9½	15.98	16.92	17.86	18.80	19.74	20.68	21.62	22.56	23.50	24.44	25.38	26.32	27.26	28.20	29.14	30.08
10	17.71	18.75	19.79	20.83	21.87	22.91	23.95	25.00	26.04	27.08	28.12	29.16	30.20	31.24	32.28	33.33
10½	19.52	20.67	21.82	22.97	24.12	25.27	26.42	27.56	28.71	29.86	31.01	32.16	33.31	34.45	35.60	36.75
11	21.43	22.69	23.95	25.21	26.47	27.73	28.99	30.25	31.51	32.77	34.03	35.29	36.55	37.81	39.07	40.33
11½	23.42	24.80	26.18	27.55	28.93	30.31	31.69	33.06	34.44	35.82	37.20	38.57	39.95	41.33	42.70	44.08
12	25.50	27.00	28.50	30.00	31.50	33.00	34.50	36.00	37.50	39.00	40.50	42.00	43.50	45.00	46.50	48.00
12½	27.67	29.30	30.93	32.55	34.18	35.80	37.43	39.06	40.69	42.32	43.95	45.57	47.20	48.83	50.46	52.08
13	29.93	31.69	33.45	35.21	36.97	38.73	40.49	42.25	44.01	45.77	47.53	49.29	51.05	52.81	54.57	56.33
13½	32.27	34.17	36.07	37.97	39.87	41.77	43.67	45.56	47.46	49.36	51.26	53.16	55.06	56.96	58.86	60.75
14	34.71	36.75	38.79	40.83	42.87	44.91	46.95	49.00	51.04	53.08	55.12	57.17	59.21	61.25	63.29	65.33
14½	37.23	39.42	41.61	43.80	45.99	48.18	50.37	52.56	54.75	56.94	59.13	61.32	63.51	65.70	67.89	70.08

Section Moduli of Rectangles

Depth of Rectangle, Inches	Width of Rectangle, Inches															
	2 1/16	2 1/8	2 3/16	2 1/4	2 5/16	2 3/8	2 7/16	2 1/2	2 9/16	2 5/8	2 11/16	2 3/4	2 13/16	2 7/8	2 15/16	3
2	1.38	1.42	1.46	1.50	1.54	1.58	1.62	1.66	1.70	1.75	1.79	1.83	1.87	1.91	1.95	2.00
2½	2.15	2.21	2.28	2.34	2.41	2.47	2.53	2.60	2.67	2.73	2.80	2.86	2.93	2.99	3.06	3.13
3	3.09	3.19	3.28	3.37	3.46	3.56	3.65	3.75	3.84	3.93	4.03	4.12	4.22	4.32	4.41	4.50
3½	4.21	4.33	4.46	4.59	4.71	4.84	4.97	5.10	5.22	5.35	5.48	5.61	5.74	5.87	6.00	6.13
4	5.50	5.67	5.84	6.00	6.16	6.33	6.50	6.66	6.83	7.00	7.16	7.33	7.50	7.66	7.83	8.00
4½	6.96	7.17	7.38	7.59	7.80	8.01	8.22	8.44	8.65	8.86	9.07	9.28	9.49	9.70	9.91	10.13
5	8.59	8.86	9.12	9.38	9.64	9.90	10.16	10.42	10.68	10.94	11.20	11.46	11.72	11.98	12.24	12.50
5½	10.40	10.72	11.03	11.34	11.66	11.97	12.28	12.60	12.92	13.23	13.54	13.85	14.17	14.49	14.81	15.13
6	12.38	12.75	13.13	13.50	13.87	14.25	14.62	15.00	15.37	15.74	16.12	16.50	16.87	17.24	17.62	18.00
6½	14.52	14.96	15.40	15.84	16.28	16.72	17.16	17.60	18.04	18.48	18.92	19.36	19.80	20.24	20.68	21.13
7	16.85	17.36	17.87	18.38	18.89	19.40	19.91	20.42	20.93	21.44	21.95	22.46	22.97	23.48	23.99	24.50
7½	19.34	19.93	20.51	21.10	21.68	22.26	22.84	23.43	24.02	24.60	25.19	25.78	26.37	26.96	27.54	28.13
8	22.00	22.67	23.33	24.00	24.66	25.33	26.00	26.67	27.34	28.01	28.67	29.33	30.00	30.67	31.33	32.00
8½	24.83	25.58	26.33	27.09	27.85	28.60	29.35	30.10	30.85	31.60	32.35	33.11	33.86	34.62	35.38	36.13
9	27.84	28.68	29.53	30.38	31.22	32.06	32.91	33.75	34.60	35.45	36.29	37.13	37.98	38.82	39.66	40.50
9½	31.02	31.96	32.90	33.84	34.78	35.72	36.66	37.60	38.54	39.48	40.42	41.36	42.30	43.24	44.18	45.13
10	34.37	35.41	36.45	37.50	38.54	39.58	40.62	41.67	42.71	43.75	44.79	45.83	46.87	47.92	48.96	50.00
10½	37.90	39.05	40.20	41.34	42.49	43.64	44.79	45.94	47.09	48.24	49.38	50.53	51.68	52.83	53.98	55.13
11	41.59	42.85	44.11	45.37	46.63	47.90	49.15	50.41	51.67	52.93	54.20	55.46	56.72	57.98	59.24	60.50
11½	45.46	46.84	48.21	49.59	50.97	52.35	53.73	55.10	56.48	57.86	59.24	60.62	61.99	63.37	64.75	66.13
12	49.50	51.00	52.50	54.00	55.50	57.00	58.50	60.00	61.50	63.00	64.50	66.00	67.50	69.00	70.50	72.00
12½	53.71	55.34	56.97	58.59	60.22	61.85	63.48	65.10	66.73	68.36	69.99	71.61	73.24	74.87	76.50	78.13
13	58.09	59.85	61.61	63.38	65.14	66.90	68.66	70.42	72.18	73.94	75.70	77.46	79.22	80.98	82.74	84.50
13½	62.65	64.55	66.45	68.35	70.25	72.15	74.05	75.94	77.83	79.73	81.63	83.53	85.43	87.33	89.23	91.13
14	67.38	69.42	71.46	73.50	75.54	77.58	79.62	81.66	83.71	85.75	87.79	89.83	91.88	93.92	95.96	98.00
14½	72.27	74.46	76.65	78.84	81.03	83.22	85.41	87.60	89.79	91.98	94.17	96.36	98.55	100.74	102.93	105.13

Section Moduli of Rectangles

Depth of Rectangle, Inches	Width of Rectangle, Inches 3 1/16	3 1/8	3 3/16	3 1/4	3 5/16	3 3/8	3 7/16	3 1/2	3 9/16	3 5/8	3 11/16	3 3/4	3 13/16	3 7/8	3 15/16	4
2	2.04	2.08	2.12	2.17	2.21	2.25	2.29	2.33	2.37	2.41	2.46	2.50	2.54	2.58	2.63	2.67
2½	3.19	3.26	3.32	3.39	3.45	3.52	3.58	3.65	3.71	3.78	3.84	3.91	3.97	4.04	4.10	4.17
3	4.60	4.69	4.78	4.87	4.96	5.06	5.15	5.25	5.35	5.44	5.53	5.63	5.72	5.81	5.90	6.00
3½	6.26	6.38	6.51	6.63	6.76	6.89	7.02	7.15	7.27	7.40	7.53	7.66	7.78	7.91	8.04	8.17
4	8.17	8.33	8.50	8.67	8.83	9.00	9.17	9.33	9.50	9.67	9.83	10.00	10.16	10.33	10.50	10.67
4½	10.34	10.55	10.76	10.97	11.18	11.39	11.60	11.81	12.02	12.23	12.45	12.66	12.87	13.08	13.29	13.50
5	12.76	13.02	13.28	13.54	13.80	14.06	14.32	14.58	14.84	15.10	15.36	15.63	15.89	16.15	16.41	16.67
5½	15.44	15.75	16.07	16.39	16.70	17.01	17.32	17.63	17.95	18.27	18.59	18.91	19.22	19.54	19.85	20.17
6	18.38	18.75	19.13	19.50	19.88	20.25	20.63	21.00	21.37	21.75	22.13	22.50	22.87	23.25	23.62	24.00
6½	21.57	22.01	22.45	22.89	23.33	23.77	24.21	24.65	25.09	25.53	25.97	26.41	26.85	27.29	27.73	28.17
7	25.01	25.52	26.03	26.54	27.05	27.56	28.07	28.58	29.09	29.60	30.11	30.63	31.14	31.65	32.16	32.67
7½	28.71	29.30	29.88	30.47	31.05	31.64	32.22	32.81	33.39	33.98	34.57	35.15	35.74	36.32	36.91	37.50
8	32.67	33.33	34.00	34.67	35.33	36.00	36.67	37.33	38.00	38.67	39.33	40.00	40.67	41.33	42.00	42.67
8½	36.87	37.62	38.37	39.12	39.87	40.63	41.38	42.14	42.90	43.65	44.40	45.16	45.91	46.66	47.41	48.17
9	41.34	42.18	43.02	43.87	44.72	45.57	46.41	47.25	48.09	48.93	49.78	50.63	51.47	52.31	53.15	54.00
9½	46.06	47.00	47.94	48.88	49.82	50.76	51.70	52.65	53.59	54.53	55.47	56.41	57.35	58.30	59.25	60.20
10	51.04	52.08	53.12	54.16	55.21	56.25	57.29	58.33	59.37	60.41	61.46	62.50	63.54	64.58	65.62	66.67
10½	56.27	57.42	58.57	59.72	60.87	62.01	63.16	64.31	65.46	66.61	67.76	68.91	70.05	71.20	72.35	73.50
11	61.76	63.02	64.28	65.54	66.80	68.06	69.32	70.58	71.84	73.10	74.36	75.63	76.89	78.15	79.41	80.67
11½	67.50	68.88	70.26	71.64	73.02	74.40	75.78	77.15	78.52	79.90	81.28	82.66	84.03	85.41	86.79	88.17
12	73.50	75.00	76.50	78.00	79.50	81.00	82.50	84.00	85.50	87.00	88.50	90.00	91.50	93.00	94.50	96.00
12½	79.75	81.38	83.00	84.63	86.26	87.89	89.52	91.15	92.78	94.40	96.03	97.66	99.29	100.92	102.55	104.17
13	86.26	88.02	89.78	91.54	93.30	95.06	96.82	98.58	100.34	102.10	103.86	105.63	107.39	109.15	110.91	112.67
13½	93.02	94.92	96.82	98.72	100.62	102.52	104.42	106.31	108.21	110.11	112.01	113.91	115.81	117.71	119.61	121.50
14	100.04	102.08	104.12	106.16	108.20	110.25	112.29	114.33	116.37	118.41	120.45	122.50	124.54	126.58	128.62	130.67
14½	107.32	109.51	111.70	113.89	116.08	118.27	120.46	122.65	124.84	127.03	129.22	131.41	133.60	135.79	137.98	140.17

Section Moduli and Moments of Inertia for Round Shafts

Diam.	Section Modulus	Moment of Inertia	Diam.	Section Modulus	Moment of Inertia	Diam.	Section Modulus	Moment of Inertia
1.00	0.0981	0.0490	1.50	0.3313	0.2485	2.00	0.7854	0.7854
1.01	0.1011	0.0510	1.51	0.3380	0.2552	2.01	0.7972	0.8012
1.02	0.1041	0.0531	1.52	0.3447	0.2620	2.02	0.8092	0.8172
1.03	0.1072	0.0552	1.53	0.3516	0.2689	2.03	0.8212	0.8335
1.04	0.1104	0.0574	1.54	0.3585	0.2760	2.04	0.8334	0.8501
1.05	0.1136	0.0596	1.55	0.3655	0.2833	2.05	0.8457	0.8669
1.06	0.1169	0.0619	1.56	0.3727	0.2907	2.06	0.8523	0.8839
1.07	0.1202	0.0643	1.57	0.3799	0.2982	2.07	0.8707	0.9012
1.08	0.1236	0.0667	1.58	0.3872	0.3059	2.08	0.8834	0.9188
1.09	0.1271	0.0692	1.59	0.3946	0.3137	2.09	0.8962	0.9366
1.10	0.1307	0.0718	1.60	0.4021	0.3217	2.10	0.9092	0.9547
1.11	0.1342	0.0745	1.61	0.4097	0.3298	2.11	0.9222	0.9729
1.12	0.1379	0.0772	1.62	0.4173	0.3380	2.12	0.9354	0.9915
1.13	0.1416	0.0800	1.63	0.4251	0.3465	2.13	0.9509	1.0103
1.14	0.1454	0.0829	1.64	0.4330	0.3550	2.14	0.9621	1.0295
1.15	0.1493	0.0860	1.65	0.4410	0.3638	2.15	0.9757	1.0488
1.16	0.1532	0.0888	1.66	0.4490	0.3727	2.16	0.9894	1.0685
1.17	0.1572	0.0919	1.67	0.4572	0.3818	2.17	1.0031	1.0884
1.18	0.1613	0.0951	1.68	0.4655	0.3910	2.18	1.0171	1.1086
1.19	0.1654	0.0984	1.69	0.4738	0.4004	2.19	1.0311	1.1291
1.20	0.1696	0.1018	1.70	0.4823	0.4100	2.20	1.0450	1.1499
1.21	0.1739	0.1052	1.71	0.4908	0.4197	2.21	1.0596	1.1709
1.22	0.1782	0.1087	1.72	0.4995	0.4296	2.22	1.0741	1.1923
1.23	0.1826	0.1123	1.73	0.5083	0.4397	2.23	1.0887	1.2139
1.24	0.1871	0.1160	1.74	0.5171	0.4499	2.24	1.1034	1.2358
1.25	0.1917	0.1198	1.75	0.5261	0.4603	2.25	1.1183	1.2580
1.26	0.1963	0.1237	1.76	0.5352	0.4710	2.26	1.1332	1.2806
1.27	0.2011	0.1277	1.77	0.5444	0.4818	2.27	1.1483	1.3034
1.28	0.2058	0.1317	1.78	0.5536	0.4927	2.28	1.1636	1.3265
1.29	0.2107	0.1359	1.79	0.5630	0.5039	2.29	1.1790	1.3499
1.30	0.2157	0.1402	1.80	0.5726	0.5153	2.30	1.1940	1.3737
1.31	0.2207	0.1445	1.81	0.5821	0.5268	2.31	1.2101	1.3977
1.32	0.2258	0.1490	1.82	0.5918	0.5385	2.32	1.2259	1.4234
1.33	0.2309	0.1535	1.83	0.6016	0.5505	2.33	1.2418	1.4468
1.34	0.2362	0.1582	1.84	0.6115	0.5626	2.34	1.2579	1.4718
1.35	0.2415	0.1630	1.85	0.6216	0.5749	2.35	1.2741	1.4971
1.36	0.2469	0.1679	1.86	0.6317	0.5875	2.36	1.2904	1.5227
1.37	0.2524	0.1729	1.87	0.6419	0.6002	2.37	1.3072	1.5487
1.38	0.2580	0.1780	1.88	0.6524	0.6132	2.38	1.3235	1.5750
1.39	0.2636	0.1832	1.89	0.6628	0.6263	2.39	1.3403	1.6016
1.40	0.2694	0.1886	1.90	0.6734	0.6397	2.40	1.3570	1.6286
1.41	0.2752	0.1940	1.91	0.6840	0.6532	2.41	1.3742	1.6559
1.42	0.2811	0.1995	1.92	0.6948	0.6670	2.42	1.3914	1.6836
1.43	0.2870	0.2052	1.93	0.7057	0.6810	2.43	1.4087	1.7116
1.44	0.2931	0.2110	1.94	0.7168	0.6953	2.44	1.4262	1.7399
1.45	0.2993	0.2170	1.95	0.7279	0.7097	2.45	1.4438	1.7686
1.46	0.3055	0.2230	1.96	0.7392	0.7244	2.46	1.4615	1.7977
1.47	0.3118	0.2292	1.97	0.7505	0.7393	2.47	1.4794	1.8271
1.48	0.3182	0.2355	1.98	0.7620	0.7544	2.48	1.4975	1.8526
1.49	0.3247	0.2419	1.99	0.7736	0.7698	2.49	1.5156	1.8870

Section Moduli and Moments of Inertia for Round Shafts

Diam.	Section Modulus	Moment of Inertia	Diam.	Section Modulus	Moment of Inertia	Diam.	Section Modulus	Moment of Inertia
2.50	1.5340	1.9175	3.00	2.6510	3.9761	3.50	4.2090	7.3662
2.51	1.5489	1.9483	3.01	2.6773	4.0293	3.51	4.2455	7.4507
2.52	1.5711	1.9796	3.02	2.7041	4.0831	3.52	4.2818	7.5360
2.53	1.5899	2.0112	3.03	2.7310	4.1375	3.53	4.3184	7.6220
2.54	1.6088	2.0431	3.04	2.7581	4.1924	3.54	4.3552	7.7087
2.55	1.6279	2.0755	3.05	2.7855	4.2478	3.55	4.3922	7.7962
2.56	1.6471	2.1083	3.06	2.8130	4.3038	3.56	4.4294	7.8845
2.57	1.6665	2.1414	3.07	2.8406	4.3604	3.57	4.4669	7.9734
2.58	1.6860	2.1749	3.08	2.8685	4.4175	3.58	4.5045	8.0631
2.59	1.7057	2.2088	3.09	2.8965	4.4751	3.59	4.5423	8.1536
2.60	1.7260	2.2432	3.10	2.9250	4.5333	3.60	4.5800	8.2448
2.61	1.7455	2.2779	3.11	2.9531	4.5921	3.61	4.6187	8.3367
2.62	1.7656	2.3130	3.12	2.9817	4.6514	3.62	4.6572	8.4296
2.63	1.7859	2.3485	3.13	3.0104	4.7113	3.63	4.6959	8.5231
2.64	1.8064	2.3844	3.14	3.0394	4.7718	3.64	4.7347	8.6174
2.65	1.8270	2.4208	3.15	3.0685	4.8330	3.65	4.7739	8.7125
2.66	1.8478	2.4575	3.16	3.0978	4.8946	3.66	4.8133	8.8084
2.67	1.8686	2.4947	3.17	3.1274	4.9568	3.67	4.8529	8.9050
2.68	1.8897	2.5322	3.18	3.1570	5.0197	3.68	4.8926	9.0025
2.69	1.9110	2.5702	3.19	3.1869	5.0832	3.69	4.9325	9.1007
2.70	1.9320	2.6087	3.20	3.2170	5.1472	3.70	4.9730	9.1998
2.71	1.9539	2.6476	3.21	3.2472	5.2119	3.71	5.0133	9.2996
2.72	1.9756	2.6868	3.22	3.2777	5.2771	3.72	5.0540	9.4003
2.73	1.9975	2.7266	3.23	3.3083	5.3430	3.73	5.0948	9.5018
2.74	2.0195	2.7668	3.24	3.3391	5.4094	3.74	5.1359	9.6041
2.75	2.0417	2.8074	3.25	3.3701	5.4765	3.75	5.1771	9.7072
2.76	2.0641	2.8484	3.26	3.4014	5.5442	3.76	5.2187	9.8112
2.77	2.0866	2.8899	3.27	3.4328	5.6126	3.77	5.2605	9.9160
2.78	2.1093	2.9319	3.28	3.4644	5.6815	3.78	5.3024	10.0216
2.79	2.1321	2.9743	3.29	3.4961	5.7511	3.79	5.3407	10.1281
2.80	2.1550	3.0172	3.30	3.5280	5.8214	3.80	5.3870	10.2350
2.81	2.1783	3.0605	3.31	3.5603	5.8923	3.81	5.4297	10.3436
2.82	2.2016	3.1043	3.32	3.5926	5.9638	3.82	5.4726	10.4526
2.83	2.2251	3.1486	3.33	3.6252	6.0359	3.83	5.5156	10.5624
2.84	2.2488	3.1933	3.34	3.6580	6.1088	3.84	5.5590	10.6486
2.85	2.2727	3.2385	3.35	3.6909	6.1823	3.85	5.6025	10.7848
2.86	2.2966	3.2842	3.36	3.7241	6.2564	3.86	5.6462	10.8970
2.87	2.3208	3.3304	3.37	3.7575	6.3312	3.87	5.6903	11.0110
2.88	2.3452	3.3771	3.38	3.7909	6.4067	3.88	5.7345	11.1250
2.89	2.3697	3.4242	3.39	3.8246	6.4829	3.89	5.7789	11.2400
2.90	2.3940	3.4719	3.40	3.8590	6.5597	3.90	5.8240	11.3560
2.91	2.4192	3.5200	3.41	3.8928	6.6372	3.91	5.8685	11.4730
2.92	2.4442	3.5686	3.42	3.9272	6.7154	3.92	5.9137	11.5910
2.93	2.4695	3.6178	3.43	3.9617	6.7943	3.93	5.9590	11.7100
2.94	2.4949	3.6674	3.44	3.9965	6.8739	3.94	6.0046	11.8290
2.95	2.5204	3.7175	3.45	4.0314	6.9542	3.95	6.0505	11.9500
2.96	2.5461	3.7682	3.46	4.0666	7.0352	3.96	6.0966	12.0690
2.97	2.5720	3.8194	3.47	4.1019	7.1168	3.97	6.1429	12.1930
2.98	2.5981	3.8711	3.48	4.1375	7.1976	3.98	6.1894	12.3170
2.99	2.6243	3.9233	3.49	4.1732	7.2824	3.99	6.2361	12.4410

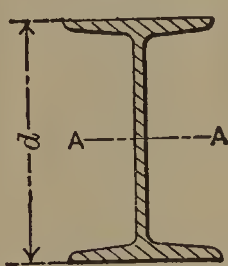
Section Moduli and Moments of Inertia for Round Shafts

Diam.	Section Modulus	Moment of Inertia	Diam.	Section Modulus	Moment of Inertia	Diam.	Section Modulus	Moment of Inertia
4.00	6.2830	12.566	4.50	8.946	20.129	5.00	12.272	30.680
4.01	6.3304	12.692	4.51	9.006	20.308	5.01	12.345	30.926
4.02	6.3779	12.820	4.52	9.066	20.489	5.02	12.420	31.173
4.03	6.4256	12.948	4.53	9.126	20.671	5.03	12.493	31.423
4.04	6.4736	13.077	4.54	9.186	20.854	5.04	12.568	31.673
4.05	6.5217	13.207	4.55	9.247	20.990	5.05	12.644	31.925
4.06	6.5701	13.337	4.56	9.308	21.224	5.06	12.718	32.179
4.07	6.6188	13.469	4.57	9.370	21.411	5.07	12.794	32.434
4.08	6.6677	13.602	4.58	9.431	21.599	5.08	12.870	32.691
4.09	6.7169	13.736	4.59	9.493	21.788	5.09	12.946	32.949
4.10	6.7660	13.871	4.60	9.556	21.979	5.10	13.023	33.209
4.11	6.8159	14.007	4.61	9.618	22.170	5.11	13.099	33.470
4.12	6.8657	14.143	4.62	9.681	22.363	5.12	13.177	33.733
4.13	6.9164	14.281	4.63	9.744	22.557	5.13	13.254	33.997
4.14	6.9663	14.420	4.64	9.807	22.753	5.14	13.332	34.263
4.15	7.0169	14.560	4.65	9.870	22.950	5.15	13.410	34.530
4.16	7.0677	14.701	4.66	9.934	23.148	5.16	13.488	34.799
4.17	7.1188	14.843	4.67	9.998	23.347	5.17	13.567	35.070
4.18	7.1702	14.985	4.68	10.063	23.548	5.18	13.645	35.342
4.19	7.2217	15.129	4.69	10.127	23.750	5.19	13.725	35.615
4.20	7.2740	15.274	4.70	10.193	23.953	5.20	13.804	35.891
4.21	7.3256	15.420	4.71	10.258	24.157	5.21	13.884	36.168
4.22	7.3779	15.568	4.72	10.323	24.363	5.22	13.964	36.446
4.23	7.4305	15.715	4.73	10.389	24.570	5.23	14.045	36.726
4.24	7.4833	15.865	4.74	10.455	24.779	5.24	14.125	37.008
4.25	7.5364	16.015	4.75	10.521	24.989	5.25	14.206	37.291
4.26	7.5898	16.166	4.76	10.588	25.200	5.26	14.287	37.576
4.27	7.6433	16.319	4.77	10.654	25.412	5.27	14.369	37.863
4.28	7.6972	16.472	4.78	10.722	25.626	5.28	14.451	38.151
4.29	7.7513	16.626	4.79	10.789	25.841	5.29	14.534	38.440
4.30	7.8060	16.782	4.80	10.857	26.058	5.30	14.616	38.732
4.31	7.8602	16.938	4.81	10.925	26.275	5.31	14.699	39.025
4.32	7.9149	17.096	4.82	10.994	26.495	5.32	14.782	39.320
4.33	7.9701	17.255	4.83	11.062	26.715	5.33	14.866	39.617
4.34	8.0294	17.415	4.84	11.131	26.937	5.34	14.949	39.915
4.35	8.0810	17.576	4.85	11.199	27.160	5.35	15.034	40.215
4.36	8.1369	17.738	4.86	11.269	27.385	5.36	15.118	40.516
4.37	8.1930	17.902	4.87	11.339	27.611	5.37	15.202	40.819
4.38	8.2494	18.066	4.88	11.409	27.839	5.38	15.288	41.124
4.39	8.3060	18.231	4.89	11.479	28.067	5.39	15.373	41.526
4.40	8.3630	18.398	4.90	11.550	28.298	5.40	15.459	41.739
4.41	8.4200	18.566	4.91	11.621	28.530	5.41	15.545	42.049
4.42	8.4775	18.735	4.92	11.692	28.763	5.42	15.631	42.361
4.43	8.5351	18.905	4.93	11.763	28.997	5.43	15.718	42.674
4.44	8.5930	19.077	4.94	11.835	29.233	5.44	15.805	42.990
4.45	8.6513	19.249	4.95	11.907	29.471	5.45	15.893	43.307
4.46	8.7097	19.423	4.96	11.979	29.710	5.46	15.980	43.626
4.47	8.7685	19.598	4.97	12.052	29.950	5.47	16.068	43.946
4.48	8.8274	19.773	4.98	12.124	30.192	5.48	16.157	44.268
4.49	8.8867	19.950	4.99	12.198	30.435	5.49	16.245	44.572

Section Moduli and Moments of Inertia for Round Shafts

Diam.	Section Modulus	Moment of Inertia	Diam.	Section Modulus	Moment of Inertia	Diam.	Section Modulus	Moment of Inertia
5.50	16.334	44.918	6.70	29.527	98.917	8.40	58.189	244.392
5.55	16.783	46.574	6.80	30.869	104.956	8.50	60.292	256.239
5.60	17.241	48.275	6.90	32.251	111.266	8.60	62.445	268.512
5.65	17.707	50.022	7.00	33.674	117.859	8.70	64.648	281.220
5.70	18.181	51.817	7.10	35.138	124.739	8.80	66.903	294.375
5.75	18.664	53.659	7.20	36.644	131.917	8.90	69.210	307.985
5.80	19.155	55.550	7.30	38.192	139.399	9.00	71.569	322.062
5.85	19.654	57.490	7.40	39.783	147.196	9.10	73.982	336.616
5.90	20.163	59.481	7.50	41.417	155.316	9.20	76.448	351.659
5.95	20.680	61.523	7.60	43.096	163.766	9.30	78.968	367.199
6.00	21.206	63.617	7.70	44.820	172.557	9.40	81.542	383.249
6.10	22.284	67.965	7.80	46.589	181.697	9.50	84.173	399.820
6.20	23.398	72.533	7.90	48.404	191.197	9.60	86.859	416.922
6.30	24.548	77.327	8.00	50.265	201.062	9.70	89.601	434.567
6.40	25.736	82.355	8.10	52.174	211.305	9.80	92.401	452.766
6.50	26.961	87.624	8.20	54.130	221.935	9.90	95.259	471.531
6.60	28.225	93.142	8.30	56.135	232.960	10.00	98.175	490.874

Section Moduli, Weights, etc., of Standard I-beams



Depth of Beam, d Inches	Weight per Foot, Pounds	Area of Section, Sq. Ins.	Section Modulus, Axis $A-A$
3	5.50	1.63	1.7
3	6.50	1.91	1.8
3	7.50	2 21	1.9
4	7.50	2.21	3.0
4	8.50	2.50	3.2
4	9.50	2.79	3 4
4	10.50	3.09	3.6
5	9.75	2.87	4.8
5	12.25	3.60	5.4
5	14.75	4.34	6.1

Depth of Beam, d Inches	Weight per Foot, Pounds	Area of Section, Sq. Ins.	Section Modulus, Axis $A-A$	Depth of Beam, d Inches	Weight per Foot, Pounds	Area of Section, Sq. Ins.	Section Modulus, Axis $A-A$
6	12.25	3.61	7.3	12	35	10.29	38.0
6	14.75	4.34	8.0	12	40	11.76	41.0
6	17.25	5.07	8.7	15	42	12.48	58.9
7	15.00	4.42	10.4	15	45	13.24	60.8
7	17.50	5.15	11.2	15	50	14.71	64.5
7	20.00	5.88	12.1	15	55	16.18	68.1
8	18.00	5.33	14.2	15	60	17.65	71.8
8	20.25	5.96	15.0	18	55	15.93	88.4
8	22.75	6.69	16.0	18	60	17.65	93.5
8	25.25	7.43	17.0	18	65	19.12	97.9
9	21.00	6.31	18.9	18	70	20.59	102.4
9	25.00	7.35	20.4	20	65	19.08	117.0
9	30.00	8.82	22.6	20	70	20.59	122.0
9	35.00	10.29	24.8	20	75	22.06	126.9
10	25.00	7.37	24.4	24	80	23.32	173.9
10	30.00	8.82	26.8	24	85	25.00	180.7
10	35.00	10.29	29.3	24	90	26.47	186.5
10	40.00	11.76	31.7	24	95	27.94	192.4
12	31.50	9.26	36.0	24	100	29.41	198.3

Properties of Standard Pipe Sections (National Tube Co.)

Nom. Size	Actual Outside Diam.	Internal Diam.	Thickness of Wall	Weight per Foot, Lbs.	Area of Metal, Square Ins.	Moment of Inertia	Section Modulus	Radius of Gyration	Square of Radius of Gyration
1/8	0.405	0.269	0.068	0.244	0.072	0.0011	0.005	0.121	0.015
1/4	0.540	0.364	0.088	0.424	0.125	0.0033	0.012	0.163	0.027
3/8	0.675	0.493	0.091	0.567	0.167	0.0073	0.022	0.209	0.044
1/2	0.840	0.622	0.109	0.850	0.250	0.0171	0.041	0.261	0.068
3/4	1.050	0.824	0.113	1.130	0.333	0.0370	0.071	0.334	0.111
1	1.315	1.049	0.133	1.678	0.494	0.0873	0.133	0.420	0.177
1 1/4	1.660	1.380	0.140	2.272	0.668	0.1947	0.235	0.540	0.291
1 1/2	1.900	1.610	0.145	2.717	0.800	0.3099	0.326	0.623	0.388
2	2.375	2.067	0.154	3.652	1.075	0.6657	0.561	0.787	0.620
2 1/2	2.875	2.469	0.203	5.793	1.704	1.530	1.064	0.947	0.898
3	3.500	3.068	0.216	7.575	2.228	3.017	1.724	1.164	1.354
3 1/2	4.000	3.548	0.226	9.109	2.680	4.788	2.394	1.337	1.787
4	4.500	4.026	0.237	10.790	3.174	7.233	3.214	1.510	2.279
4 1/2	5.000	4.506	0.247	12.538	3.688	10.44	4.177	1.683	2.832
5	5.563	5.047	0.258	14.617	4.300	15.16	5.451	1.878	3.526
6	6.625	6.065	0.280	18.974	5.581	28.14	8.496	2.245	5.042
7	7.625	7.023	0.301	23.544	6.926	46.52	12.20	2.592	6.716
8	8.625	8.071	0.277	24.696	7.265	63.35	14.69	2.953	8.721
8	8.625	7.981	0.322	28.554	8.399	72.49	16.81	2.938	8.630
9	9.625	8.941	0.342	33.907	9.974	107.6	22.35	3.284	10.79
10	10.750	10.192	0.279	31.201	9.178	125.9	23.42	3.703	13.71
10	10.750	10.136	0.307	34.240	10.07	137.4	25.57	3.694	13.64
10	10.750	10.020	0.365	40.483	11.91	160.7	29.90	3.674	13.50
11	11.750	11.000	0.375	45.557	13.40	217.0	36.93	4.024	16.19
12	12.750	12.090	0.330	43.773	12.88	248.5	38.97	4.393	19.30
12	12.750	12.000	0.375	49.562	14.58	279.3	43.82	4.377	19.16
13	14.000	13.250	0.375	54.568	16.05	372.8	53.25	4.819	23.22
14	15.000	14.250	0.375	58.573	17.23	461.0	61.46	5.172	26.75
15	16.000	15.250	0.375	62.579	18.41	562.1	70.26	5.526	30.54

Properties of Special Pipe Sections (National Tube Co.)

Actual Outside Diam., Ins.	Internal Diam.	Thickness of Wall	Weight per Foot, Lbs.	Area of Metal, Square Ins.	Moment of Inertia	Section Modulus	Radius of Gyration	Square of Radius of Gyration
17	16.520	0.240	42.96	12.64	444	52.21	5.926	35.12
18	17.510	0.245	46.46	13.67	539	59.85	6.278	39.41
18	17.182	0.409	76.84	22.60	875	97.2	6.221	38.70
19	18.482	0.259	51.84	15.25	670	70.5	6.627	43.91
20	19.456	0.272	57.31	16.86	820	82.0	6.976	48.66
20	19.182	0.409	85.58	25.17	1208	120.8	6.928	48.00
22	21.398	0.301	69.76	20.52	1208	109.8	7.672	58.87
24	23.340	0.330	83.42	24.54	1719	143.2	8.369	70.05
26	25.276	0.362	99.12	29.16	2396	184.3	9.065	82.18
28	27.208	0.396	116.75	34.34	3272	233.7	9.760	95.27
30	29.136	0.432	136.42	40.13	4386	292.4	10.450	109.30

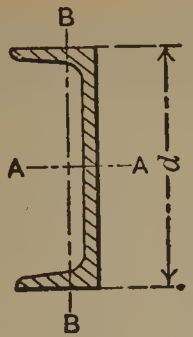
Properties of Extra Strong Pipe Sections (National Tube Co.)

Nom. Size	Actual Outside Diam.	Internal Diam.	Thickness of Wall	Weight per Foot, Lbs.	Area of Metal, Square Ins.	Moment of Inertia	Section Modulus	Radius of Gyration	Square of Radius of Gyration
⅛	0.405	0.215	0.095	0.31	0.093	0.0012	0.006	0.115	0.013
¼	0.540	0.302	0.119	0.53	0.157	0.0038	0.014	0.155	0.024
⅜	0.675	0.423	0.126	0.74	0.217	0.0086	0.026	0.199	0.040
½	0.840	0.546	0.147	1.09	0.320	0.0201	0.048	0.250	0.063
¾	1.050	0.742	0.154	1.47	0.433	0.0448	0.085	0.321	0.103
1	1.315	0.957	0.179	2.17	0.639	0.1056	0.161	0.407	0.165
1¼	1.660	1.278	0.191	3.00	0.881	0.2418	0.291	0.524	0.274
1½	1.900	1.500	0.200	3.63	1.068	0.3912	0.412	0.605	0.366
2	2.375	1.939	0.218	5.02	1.477	0.8679	0.731	0.766	0.587
2½	2.875	2.323	0.276	7.66	2.254	1.924	1.339	0.924	0.854
3	3.500	2.900	0.300	10.25	3.016	3.894	2.225	1.136	1.291
3½	4.000	3.364	0.318	12.50	3.678	6.280	3.140	1.307	1.707
4	4.500	3.826	0.337	14.98	4.407	9.610	4.271	1.477	2.181
4½	5.000	4.290	0.355	17.61	5.180	14.05	5.621	1.647	2.712
5	5.563	4.813	0.375	20.78	6.112	20.67	7.431	1.839	3.382
6	6.625	5.761	0.432	28.57	8.405	40.49	12.22	2.195	4.817
7	7.625	6.625	0.500	38.05	11.19	71.37	18.72	2.525	6.377
8	8.625	7.625	0.500	43.39	12.76	105.7	24.51	2.878	8.283
9	9.625	8.625	0.500	48.73	14.33	149.6	31.09	3.231	10.44
10	10.750	9.750	0.500	54.73	16.10	212.0	39.43	3.628	13.16
11	11.750	10.750	0.500	60.07	17.67	280.1	47.68	3.981	15.85
12	12.750	11.750	0.500	65.41	19.24	361.5	56.71	4.335	18.79
13	14.000	13.000	0.500	72.09	21.21	483.8	69.11	4.776	22.81
14	15.000	14.000	0.500	77.43	22.78	599.3	79.91	5.130	26.31
15	16.000	15.000	0.500	82.77	24.35	731.9	91.49	5.483	30.06

Properties of Double Extra Strong Pipe Sections (National Tube Co.)

Nom. Size	Actual Outside Diam.	Internal Diam.	Thickness of Wall	Weight per Foot, Lbs.	Area of Metal, Square Ins.	Moment of Inertia	Section Modulus	Radius of Gyration	Square of Radius of Gyration
½	0.840	0.252	0.294	1.71	0.504	0.0242	0.058	0.219	0.048
¾	1.050	0.434	0.308	2.44	0.718	0.0579	0.110	0.284	0.081
1	1.315	0.599	0.358	3.66	1.076	0.1405	0.214	0.361	0.130
1¼	1.660	0.896	0.382	5.21	1.534	0.3411	0.411	0.472	0.222
1½	1.900	1.100	0.400	6.41	1.885	0.5678	0.598	0.549	0.301
2	2.375	1.503	0.436	9.03	2.656	1.311	1.104	0.703	0.494
2½	2.875	1.771	0.552	13.69	4.028	2.871	1.997	0.844	0.713
3	3.500	2.300	0.600	18.58	5.466	5.993	3.424	1.047	1.096
3½	4.000	2.728	0.636	22.85	6.721	9.848	4.924	1.210	1.465
4	4.500	3.152	0.674	27.54	8.101	15.28	6.793	1.374	1.887
4½	5.000	3.580	0.710	32.53	9.569	22.62	9.047	1.537	2.364
5	5.563	4.063	0.750	38.55	11.34	33.63	12.09	1.722	2.966
6	6.625	4.897	0.864	53.16	15.64	66.33	20.02	2.060	4.242
7	7.625	5.875	0.875	63.08	18.56	107.5	28.18	2.406	5.791
8	8.625	6.875	0.875	72.42	21.30	162.0	37.56	2.757	7.604

Section Moduli, Weights, etc., of Standard Channels

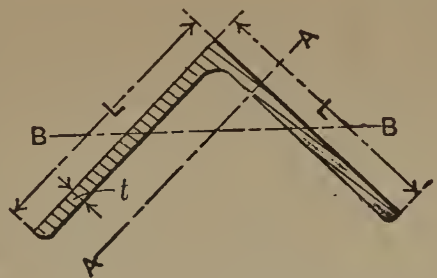


Depth of Channel, *d* Inches	Weight per Foot, Pounds	Area of Section, Sq. Ins.	Section Modulus, Axis *A–A*	Section Modulus, Axis *B–B*
3	4.00	1.19	1.1	0.21
3	5.00	1.47	1.2	0.24
3	6.00	1.76	1.4	0.27
4	5.25	1.55	1.9	0.29
4	6.25	1.84	2.1	0.32
4	7.25	2.13	2.3	0.35
5	6.50	1.95	3.0	0.38
5	9.00	2.65	3.5	0.45
5	11.50	3.38	4.2	0.54
6	8.00	2.38	4.3	0.50
6	10.50	3.09	5.0	0.57
6	13.00	3.82	5.8	0.65
6	15.50	4.56	6.5	0.74
7	9.75	2.85	6.0	0.63
7	12.25	3.60	6.9	0.71
7	14.75	4.34	7.8	0.79
7	17.25	5.07	8.6	0.87
7	19.75	5.81	9.5	0.96
8	11.25	3.35	8.1	0.79
8	13.75	4.04	9.0	0.87
8	16.25	4.78	10.0	0.95
8	18.75	5.51	11.0	1.02
8	21.25	6.25	11.9	1.11
9	13.25	3.89	10.5	0.97
9	15.00	4.41	11.3	1.03
9	20.00	5.88	13.5	1.19
9	25.00	7.35	15.7	1.36
10	15.00	4.46	13.4	1.17
10	20.00	5.88	15.7	1.34
10	25.00	7.35	18.2	1.50
10	30.00	8.82	20.6	1.67
10	35.00	10.29	23.1	1.87
12	20.50	6.03	21.4	1.75
12	25.00	7.35	24.0	1.91
12	30.00	8.82	26.9	2.09
12	35.00	10.29	29.9	2.27
12	40.00	11.76	32.8	2.46
15	33.00	9.90	41.7	3.16
15	35.00	10.29	42.7	3.22
15	40.00	11.76	46.3	3.43
15	45.00	13.24	50.0	3.63
15	50.00	14.71	53.7	3.85
15	55.00	16.18	57.4	4.07

Section Moduli and Weights per Foot of Square Beams

Side of Square	Weight per Foot, Pounds	Section Modulus	Side of Square	Weight per Foot, Pounds	Section Modulus	Side of Square	Weight per Foot, Pounds	Section Modulus
1	3.40	0.17	2⅝	23.43	3.00	6	122.4	36.0
1⅛	4.30	0.24	2¾	25.70	3.46	6½	143.6	45.7
1¼	5.31	0.33	2⅞	28.10	3.95	7	166.6	57.1
1⅜	6.43	0.43	3	30.60	4.50	7½	191.3	70.3
1½	7.65	0.56	3¼	35.92	5.72	8	217.6	85.3
1⅝	8.98	0.71	3½	41.65	7.14	8½	245.6	102.3
1¾	10.41	0.89	3¾	47.82	8.79	9	275.4	121.5
1⅞	11.95	1.10	4	54.40	10.70	9½	306.8	142.7
2	13.60	1.33	4¼	61.41	12.80	10	340.0	166.7
2⅛	15.35	1.60	4½	68.85	15.20	10½	374.9	192.7
2¼	17.22	1.90	4¾	76.71	17.80	11	411.4	221.7
2⅜	19.18	2.23	5	85.00	20.80	11½	449.6	253.5
2½	21.25	2.60	5½	102.80	27.70	12	489.6	288.0

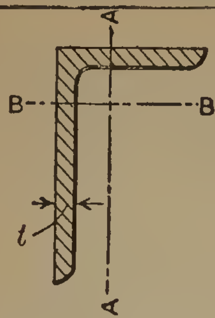
Section Moduli, Weights, etc., of Standard Angles with Equal Legs*



Length of Leg, L Inches	Thickness, t Inches	Weight per Foot, Pounds	Area of Section, Sq. Ins.	Section, Modulus, Axis A–A	Section Modulus, Axis B–B
¾	⅛	0.6	0.18	0.017	0.011
¾	3/16	0.9	0.25	0.024	0.014
1	⅛	0.8	0.24	0.031	0.021
1	3/16	1.2	0.34	0.044	0.028
1	¼	1.5	0.44	0.056	0.034
1¼	⅛	1.1	0.30	0.049	0.035
1¼	3/16	1.5	0.44	0.071	0.047
1¼	¼	2.0	0.57	0.091	0.057
1¼	5/16	2.4	0.69	0.109	0.066
1½	⅛	1.3	0.36	0.072	0.053
1½	3/16	1.8	0.53	0.104	0.072
1½	¼	2.4	0.69	0.134	0.088
1½	5/16	2.9	0.84	0.162	0.101
1½	⅜	3.4	0.99	0.188	0.114
1½	7/16	3.9	1.13	0.214	0.126
1¾	3/16	2.2	0.63	0.14	0.10
1¾	¼	2.8	0.82	0.19	0.13
1¾	5/16	3.4	1.00	0.23	0.15
1¾	⅜	4.0	1.18	0.26	0.16
1¾	7/16	4.6	1.34	0.30	0.18
1¾	½	5.1	1.50	0.33	0.20
2	3/16	2.5	0.72	0.19	0.14
2	¼	3.2	0.94	0.25	0.17
2	5/16	4.0	1.16	0.30	0.20
2	⅜	4.7	1.36	0.35	0.22
2	7/16	5.3	1.56	0.40	0.25
2	½	6.0	1.75	0.45	0.27
2½	3/16	3.1	0.91	0.30	0.22
2½	¼	4.1	1.19	0.39	0.28
2½	5/16	5.0	1.47	0.48	0.33
2½	⅜	5.9	1.74	0.57	0.38
2½	7/16	6.8	2.00	0.65	0.42
2½	½	7.7	2.25	0.72	0.46
2½	9/16	8.5	2.50	0.80	0.49
3	¼	4.9	1.44	0.58	0.42
3	5/16	6.1	1.78	0.71	0.50
3	⅜	7.2	2.11	0.83	0.57
3	7/16	8.3	2.44	0.95	0.64
3	½	9.4	2.75	1.07	0.70
3	9/16	10.4	3.06	1.19	0.76
3	⅝	11.5	3.36	1.30	0.81
3	11/16	12.5	3.66	1.40	0.86
3½	5/16	7.2	2.09	0.98	0.71
3½	⅜	8.5	2.49	1.15	0.81
3½	7/16	9.8	2.88	1.32	0.91
3½	½	11.1	3.25	1.49	1.00
3½	9/16	12.4	3.63	1.65	1.09
3½	⅝	13.6	3.99	1.81	1.17
3½	11/16	14.8	4.34	1.96	1.24
3½	¾	16.0	4.69	2.11	1.31
3½	13/16	17.1	5.03	2.25	1.38
3½	⅞	18.3	5.36	2.39	1.45
4	5/16	8.2	2.41	1.29	0.95
4	⅜	9.8	2.86	1.52	1.10
4	7/16	11.3	3.31	1.75	1.23
4	½	12.8	3.75	1.97	1.36
4	9/16	14.3	4.19	2.19	1.48
4	⅝	15.7	4.62	2.40	1.59
4	11/16	17.1	5.03	2.61	1.70
4	¾	18.5	5.44	2.81	1.80
4	13/16	19.9	5.84	3.01	1.89
4	⅞	21.2	6.24	3.20	1.99
6	⅜	14.9	4.36	3.53	2.67
6	7/16	17.2	5.06	4.07	3.04
6	½	19.6	5.75	4.61	3.37
6	9/16	21.9	6.44	5.14	3.70
6	⅝	24.2	7.11	5.66	4.01
6	11/16	26.5	7.78	6.17	4.31
6	¾	28.7	8.44	6.66	4.59
6	13/16	31.0	9.09	7.15	4.86
6	⅞	33.1	9.74	7.63	5.12
6	15/16	35.3	10.38	8.11	5.37
6	1	37.4	11.00	8.57	5.61
8	½	26.4	7.75	8.37	6.33
8	9/16	29.6	8.69	9.34	6.98
8	⅝	32.7	9.61	10.30	7.60
8	11/16	35.8	10.53	11.25	8.20
8	¾	38.9	11.44	12.18	8.77
8	13/16	42.0	12.34	13.11	9.33
8	⅞	45.0	13.24	14.02	9.86
8	15/16	48.1	14.13	14.91	10.38
8	1	51.0	15.00	15.80	10.88
8	1 1/16	54.0	15.88	16.67	11.36
8	1⅛	56.9	16.74	17.53	11.83

* Sizes opposite the braces are not included in the 1910 standard.

Section Moduli, Weights, etc., of Standard Angles with Unequal Legs*



Dimensions of Angle, Inches	Thickness t, Inches	Weight per Foot, Pounds	Area of Section, Sq. Ins.	Section Modulus, Axis A–A	Section Modulus, Axis B–B
2½×2	3/16	2.8	0.81	0.20	0.29
2½×2	¼	3.7	1.07	0.25	0.38
2½×2	5/16	4.5	1.31	0.31	0.47
2½×2	⅜	5.3	1.55	0.36	0.55
2½×2	7/16	6.1	1.78	0.41	0.62
2½×2	½	6.8	2.00	0.46	0.70
2½×2	9/16	7.6	2.22	0.51	0.77
3 ×2½	¼	4.5	1.32	0.40	0.56
3 ×2½	5/16	5.6	1.63	0.49	0.69
3 ×2½	⅜	6.6	1.93	0.58	0.81
3 ×2½	7/16	7.6	2.22	0.66	0.93
3 ×2½	½	8.5	2.50	0.74	1.04
3 ×2½	9/16	9.5	2.78	0.82	1.15
3 ×2½	⅝	10.4	3.05	0.90	1.26
3½×2½	¼	4.9	1.44	0.41	0.75
3½×2½	5/16	6.1	1.78	0.50	0.93
3½×2½	⅜	7.2	2.11	0.59	1.09
3½×2½	7/16	8.3	2.44	0.68	1.26
3½×2½	½	9.4	2.75	0.76	1.41
3½×2½	9/16	10.4	3.06	0.84	1.56
3½×2½	⅝	11.5	3.36	0.92	1.71
3½×2½	11/16	12.5	3.66	0.99	1.85
3½×2½	¾	13.4	3.94	1.07	1.99
3½×3	5/16	6.6	1.94	0.72	0.95
3½×3	⅜	7.9	2.30	0.85	1.13
3½×3	7/16	9.1	2.66	0.98	1.29
3½×3	½	10.2	3.00	1.10	1.45
3½×3	9/16	11.4	3.34	1.21	1.61
3½×3	⅝	12.5	3.68	1.33	1.76
3½×3	11/16	13.6	4.00	1.44	1.91
3½×3	¾	14.7	4.32	1.54	2.05
3½×3	13/16	15.8	4.63	1.65	2.20
3½×3	⅞	16.8	4.93	1.75	2.33
4 ×3	5/16	7.2	2.09	0.73	1.23
4 ×3	⅜	8.5	2.49	0.87	1.46
4 ×3	7/16	9.8	2.88	0.99	1.68
4 ×3	½	11.1	3.25	1.12	1.89
4 ×3	9/16	12.4	3.63	1.23	2.09
4 ×3	⅝	13.6	3.99	1.35	2.30

Dimensions of Angle, Inches	Thickness t, Inches	Weight per Foot, Pounds	Area of Section, Sq. Ins.	Section Modulus, Axis A–A	Section Modulus, Axis B–B
4×3	11/16	14.8	4.34	1.46	2.49
4×3	¾	16.0	4.69	1.57	2.68
4×3	13/16	17.1	5.03	1.68	2.87
4×3	⅞	18.3	5.36	1.79	3.05
5×3	5/16	8.2	2.41	0.75	1.89
5×3	⅜	9.8	2.86	0.89	2.24
5×3	7/16	11.3	3.31	1.02	2.58
5×3	½	12.8	3.75	1.15	2.91
5×3	9/16	14.3	4.19	1.27	3.23
5×3	⅝	15.7	4.61	1.39	3.55
5×3	11/16	17.1	5.03	1.51	3.86
5×3	¾	18.5	5.44	1.62	4.16
5×3	13/16	19.9	5.84	1.74	4.46
5×3	⅞	21.2	6.24	1.85	4.75
5×3½	5/16	8.7	2.56	1.02	1.94
5×3½	⅜	10.4	3.05	1.21	2.29
5×3½	7/16	12.0	3.53	1.39	2.64
5×3½	½	13.6	4.00	1.56	2.99
5×3½	9/16	15.2	4.47	1.73	3.32
5×3½	⅝	16.8	4.93	1.90	3.65
5×3½	11/16	18.3	5.38	2.06	3.97
5×3½	¾	19.8	5.82	2.22	4.28
5×3½	13/16	21.3	6.25	2.37	4.58
5×3½	⅞	22.7	6.68	2.52	4.88
5×3½	15/16	24.2	7.09	2.67	5.17
6×3½	⅜	11.7	3.43	1.23	3.24
6×3½	7/16	13.5	3.97	1.41	3.75
6×3½	½	15.3	4.50	1.59	4.24
6×3½	9/16	17.1	5.03	1.77	4.72
6×3½	⅝	18.9	5.55	1.94	5.19
6×3½	11/16	20.6	6.06	2.11	5.65
6×3½	¾	22.4	6.57	2.27	6.10
6×3½	13/16	24.0	7.06	2.43	6.55
6×3½	⅞	25.7	7.55	2.59	6.98
6×3½	15/16	27.3	8.03	2.74	7.41
6×3½	1	28.9	8.50	2.90	7.80
6×4	⅜	12.3	3.61	1.60	3.32
6×4	7/16	14.3	4.19	1.85	3.83
6×4	½	16.2	4.75	2.08	4.33
6×4	9/16	18.1	5.31	2.31	4.83
6×4	⅝	20.0	5.86	2.54	5.31
6×4	11/16	21.8	6.41	2.76	5.78
6×4	¾	23.6	6.94	2.97	6.25
6×4	13/16	25.4	7.47	3.18	6.70
6×4	⅞	27.2	7.99	3.39	7.15
6×4	15/16	28.9	8.50	3.59	7.59
6×4	1	30.6	9.00	3.79	8.02

* Sizes opposite the braces are not included in the 1910 standard.

Length of Angles Bent to Circular Shape. — When it is required to calculate the length of an angle-iron used either inside or outside of a tank or smokestack, the following method and table of constants may be used: Assume, for example, that a stand-pipe, 20 feet inside diameter, is provided with a 3 by 3 by 3/8 inch angle-iron on the inside at the top. The circumference of a circle 20 feet in diameter is 754 inches. From the table of constants, find the constant for a 3 by 3 by 3/8 inch angle-iron, which is 4.319. The length of the angle then is 754 − 4.319 = 749.681 inches. Should the angle be on the outside, add the constant instead of subtracting it; thus, 754 + 4.319 = 758.319 inches.

Table of Constants Used for Calculating Length of Angles Bent to Circular Shape

Size of Angle	Constant	Size of Angle	Constant
1/4 × 2 × 2	2.879	1/2 × 3 1/2 × 3 1/2	5.235
5/16 × 2 × 2	3.076	3/8 × 4 × 4	5.366
3/8 × 2 × 2	3.272	1/2 × 4 × 4	5.758
1/4 × 2 1/2 × 2 1/2	3.403	3/8 × 5 × 5	6.414
5/16 × 2 1/2 × 2 1/2	3.600	1/2 × 5 × 5	6.804
3/8 × 2 1/2 × 2 1/2	3.796	3/8 × 6 × 6	7.461
1/2 × 2 1/2 × 2 1/2	4.188	1/2 × 6 × 6	7.854
1/4 × 3 × 3	3.926	3/4 × 6 × 6	8.639
5/16 × 3 × 3	4.123	1/2 × 8 × 8	9.949
3/8 × 3 × 3	4.319	3/4 × 8 × 8	10.734
1/2 × 3 × 3	4.711	1 × 8 × 8	11.520
3/8 × 3 1/2 × 3 1/2	4.843	....	......

Moment of Inertia of Built-up Sections. — The usual method of calculating the moment of inertia of a built-up section involves the calculations of the moment of inertia for each element of the section about its own neutral axis, and the transferring of this moment of inertia to the previously found neutral axis of the whole built-up section. A much simpler method that can be used in the case of any section which can be divided into rectangular elements bounded by lines parallel

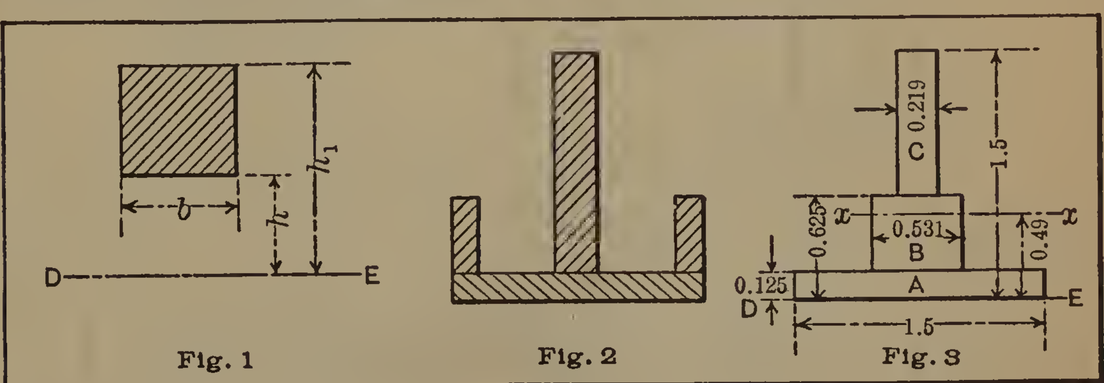


Fig. 1 Fig. 2 Fig. 3

and perpendicular to the neutral axis, is the so-called tabular method based upon the formula: $I = \frac{b\,(h_1^3 - h^3)}{3}$ in which I = the moment of inertia about axis DE, Fig. 1, and b, h and h_1 are dimensions as given in the same illustration.

The method may be illustrated by applying it to the section shown in Fig. 2, and for simplicity of calculation shown "massed" in Fig. 3. The calculation may then be tabulated as shown in the accompanying table. The distance from the axis *DE* to the neutral axis *xx* (which will be designated as *d*) is found by dividing the sum of the geometrical moments by the area. The moment of inertia about the neutral axis is then found in the usual way by subtracting the area multiplied by d^2 from the moment of inertia about the axis *DE*.

Tabulated Calculation of Moment of Inertia

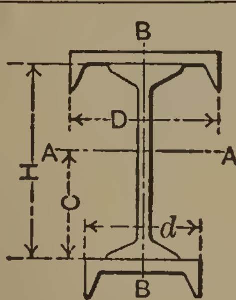
Section	Breadth b	Height h_1	Area $b\,(h_1 - h)$	h_1^2	Moment $\dfrac{b\,(h_1^2 - h^2)}{2}$	h_1^3	I about axis DE $\dfrac{b\,(h_1^3 - h^3)}{3}$
A	1.500	0.125	0.187	0.016	0.012	0.002	0.001
B	0.531	0.625	0.266	0.391	0.100	0.244	0.043
C	0.219	1.500	0.191	2.250	0.203	3.375	0.228
			$A = 0.644$		$M = 0.315$		$I_{DE} = 0.272$

Distance d from DE to $xx = \dfrac{M}{A} = \dfrac{0.315}{0.644} = 0.49.$

Moment of inertia of whole section with reference to its neutral axis: $I_n = I_{DE} - Ad^2 = 0.272 - 0.644 \times 0.49^2 = 0.117.$

Sections for Crane and Telpher Runways. — In the design of crane, telpher and similar runways, suitable provision should be made for the lateral strength. The three types of section most commonly used for the purposes mentioned consist of: 1. An I-beam for vertical strength with a channel riveted to the compression flange for lateral stiffness. 2. The same construction, with the addition of a smaller

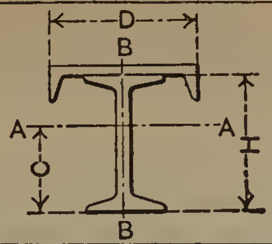
Sections for Crane and Telpher Runways



Properties of Sections Consisting of One I-beam and Two Channels

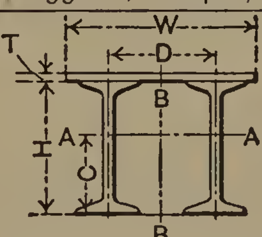
Upper Channel		I-beam		Lower Channel		Section Modulus				Moment of Inertia	Distance, C
D, Inches	Weight per Foot, Pounds	H, Inches	Weight per Foot, Pounds	d, Inches	Weight per Foot, Pounds	Upper Chord, Axis A–A	Lower Chord, Axis A–A	Upper Chord, Axis B–B	Lower Chord, Axis B–B		
10	15	10	25	8	11.25	62.87	43.62	14.09	8.97	321.89	5.12
12	20.5	10	25	10	15	76.93	50.92	21.98	14.09	394.66	5.15
12	20.5	12	31.5	10	15	99.54	68.40	22.19	14.35	603.23	6.22
12	20.5	15	42	10	15	139.08	100.03	22.62	14.86	1040.34	7.80
15	33	15	42	12	20.5	185.61	119.97	42.65	22.62	1336.49	8.20
12	20.5	18	55	10	15	185.03	138.27	23.16	15.52	1652.32	9.35
15	33	18	55	12	20.5	241.06	163.01	43.11	23.16	2075.17	9.79

Sections for Crane and Telpher Runways



Properties of Sections Consisting of One I-beam and One Channel

Channel: D, Inches	Channel: Weight per Foot, Lbs.	I-beam: H, Inches	I-beam: Weight per Foot, Lbs.	Section Modulus: Upper Chord, Axis A–A	Section Modulus: Lower Chord, Axis A–A	Section Modulus: Upper Chord, Axis B–B	Moment of Inertia	Distance C
10	15	10	25	52.06	27.15	14.09	182.72	6.73
10	15	12	31.5	70.22	39.97	14.35	311.78	7.80
12	20.5	12	31.5	81.71	40.66	22.19	333.39	8.20
12	20.5	12	40	90.41	50.31	22.55	396.91	7.89
10	15	15	42	103.55	64.87	14.86	607.83	9.37
12	20.5	15	42	118.80	66.06	22.62	648.68	9.82
15	33	15	42	151.94	68.18	42.65	724.77	10.63
12	25	15	50	135.28	75.02	25.33	742.67	9.90
12	20.5	15	60	140.17	90.13	23.56	838.22	9.30
15	33	15	60	173.59	93.20	43.43	933.90	10.02
12	20.5	18	55	161.57	99.11	23.16	1122.90	11.33
15	33	18	55	203.18	102.50	43.11	1253.60	12.23
15	33	15	80	197.81	120.17	44.48	1151.27	9.58
12	20.5	20	65	199.98	129.60	23.72	1594.03	12.30
15	33	20	65	247.50	133.85	43.56	1772.11	13.24
15	33	20	80	278.43	164.98	44.52	2113.31	12.81
15	40	20	80	305.44	168.05	49.12	2226.62	13.25
15	33	24	80	339.17	196.13	44.56	3032.18	15.46
15	55	24	100	455.00	239.53	60.64	3894.80	16.26

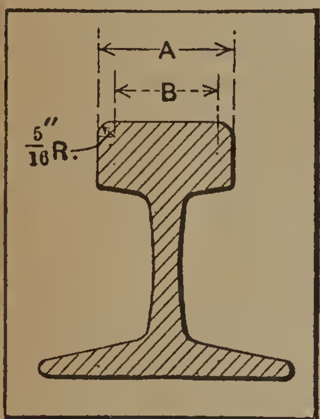


Properties of Sections Consisting of Two I-beams and One Connecting Plate

Plate: W, Inches	Plate: T, Inches	I-beam: H, Inches	I-beam: Weight per Foot, Lbs.	Section Modulus: Upper Chord, Axis A–A	Section Modulus: Lower Chord, Axis A–A	Section Modulus: Upper Chord, Axis B–B	Distance, D	Moment of Inertia	Distance, C
12	3/8	10	25	81.01	54.18	20.27	5.75	337.02	6.22
14	1/2	12	31.5	131.53	81.72	36.30	7.50	630.03	7.71
14	1/2	12	40	147.64	100.75	40.38	7.25	748.53	7.43
14	1/2	15	42	192.53	131.50	40.03	7.00	1211.00	9.21
15	1/2	15	60	239.39	179.40	55.30	7.50	1589.54	8.86
15	1/2	15	70	251.22	196.35	61.22	7.50	1708.27	8.70
15	1/2	15	80	285.65	230.98	65.55	7.25	1979.53	8.57
16	1/2	18	55	279.02	197.60	60.51	8.50	2140.06	10.83
16	1/2	20	65	347.57	257.80	65.48	8.25	3034.28	11.77
16	1/2	20	80	406.10	318.45	67.99	7.50	3658.97	11.49
18	1/2	24	80	504.42	379.58	90.22	9.50	5306.49	13.98
18	1/2	24	100	545.59	435.11	106.12	9.50	5930.54	13.63

channel to the tension flange to increase the vertical strength. 3. Two I-beams side by side, with a cover plate on the top flanges only. An illustration of each of these three types is shown in the tables of "Sections for Crane and Telpher Runways," where the section modulus, moment of inertia and other properties of the three types of built-up sections are given.

Size of Rail Necessary to Carry a Given Load. — The following formulas may be employed for determining the size of rail and wheel suitable for carrying a given load. Let, A = the width of the head of the rail in inches; B = width of the tread of the rail in inches; C = the wheel-load in pounds; D = the diameter of the wheel in inches.



Then the width of the tread of the rail in inches is found from the formula:

$$B = \frac{C}{1250\,D} \tag{1}$$

The width A of the head equals $B + \frac{5}{8}$ inch. The diameter D of the smallest track wheel that will safely carry the load is found from the formula:

$$D = \frac{C}{A \times K} \tag{2}$$

in which K = 600 to 800 for steel castings; K = 300 to 400 for cast iron.

As an example, assume that the wheel-load in a given case is 10,000 pounds; the diameter of the wheel is 20 inches; and the material steel casting. Determine the size of rail necessary to carry this load. From Formula (1):

$$B = \frac{10{,}000}{1250 \times 20} = 0.4 \text{ inch.}$$

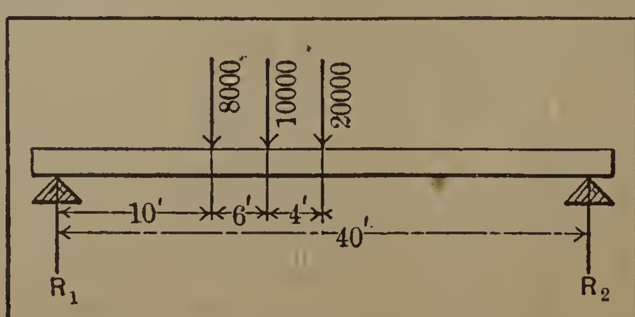
Hence the width of the rail required equals $0.4 + \frac{5}{8}$ inch = 1.025 inch. Determine also whether a wheel 20 inches in diameter is large enough to safely carry the load. From Formula (2):

$$D = \frac{10{,}000}{1.025 \times 600} = 16\tfrac{1}{4} \text{ inches.}$$

This is the smallest diameter of track wheel that will safely carry the load; hence a 20-inch wheel is ample.

BEAMS

Reaction at the Supports. — When a beam is loaded by vertical loads or forces, the sum of the reactions at the supports equals the sum of the loads. In a simple beam, when the loads are symmetrically placed with reference to the supports, or when the load is uniformly distributed, the reaction at each end will equal one-half of the sum of the loads. When the loads are not symmetrically placed, the reaction at each support may be ascertained from the fact that the algebraic sum of the moments must equal zero. In the accompanying illustration, if moments are taken about the support to the left, then:



$$R_2 \times 40 - 8000 \times 10 - 10{,}000 \times 16 - 20{,}000 \times 20 = 0;$$
$$R_2 = 16{,}000 \text{ pounds.}$$

Moments taken about the support at the right will, in the same way, give

$$R_1 = 22{,}000 \text{ pounds.}$$

The sum of the reactions equals 38,000 pounds, which is also the sum of the loads. If part of the load is uniformly distributed over the beam, this part is first equally divided between the two supports, or the uniform load may be considered as concentrated at its center of gravity.

Stresses and Deflections in Beams. — On the following pages is given an extensive table of formulas for stresses and deflections in beams, shafts, etc. It is assumed that all the dimensions are in inches, all loads in pounds, and all stresses in pounds per square inch. In the tables:

E = modulus of elasticity of the material;
I = moment of inertia of the cross-section of the beam;
Z = section modulus of the cross-section of the beam = $I \div$ distance from neutral axis to extreme fiber;
W = load on beam;
s = stress in extreme fiber, or maximum stress in the cross-section considered, due to load W. A positive value of s denotes tension in the upper fibers and compression in the lower ones (as in a cantilever). A negative value of s denotes the reverse (as in a beam supported at the ends). The greatest safe load is that value of W which causes a maximum stress equal to, but not exceeding, the greatest safe value of s;
y = deflection measured from the position occupied if the load causing the deflection were removed. A positive value of y denotes deflection below this position; a negative value, deflection upward;
u, v, w, x = variable distances along the beam from a given support to any point.

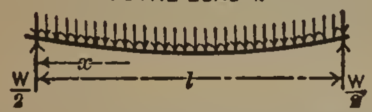
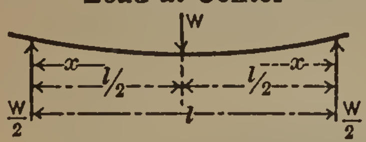
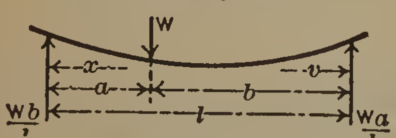
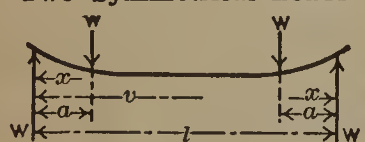
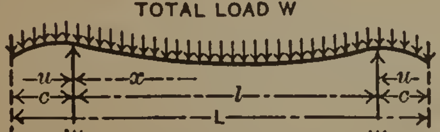
If there are several kinds of loads, as, for instance, a uniform load and a load at any point, or separate loads at different points, the total stress and the total deflection at any point is found by adding together the various stresses or deflections at the point considered due to each load acting by itself. If the stress or deflection due to any one of the loads is negative, it must be subtracted instead of added.

Remarks Relative to the Use of the Tables. — In the diagrammatical illustrations of the beams and their loading, the values indicated near, but below, the supports are the "reactions" or upward forces at the supports. For Cases 1 to 12, inclusive, the reactions, as well as the formulas for the stresses, are the same whether the beam is of constant or variable cross-section. For the other cases, the reactions and the stresses given are for constant cross-section beams only.

The bending moment at any point in inch-pounds is $s \times Z$ and can be found by omitting the divisor Z in the formula for the stress given in the tables. A positive value of the bending moment denotes tension in the upper fibers and compression in the lower ones. A negative value denotes the reverse. The value of W corresponding to a given stress is found by transposition of the formula. For example, in Case 1, the stress at the critical point is $s = -Wl \div 8Z$. From this we find $W = -8Zs \div l$. Of course, the negative sign of W may be ignored.

The deflections given in the tables apply only to cases where the cross-section of the beam is constant for its entire length.

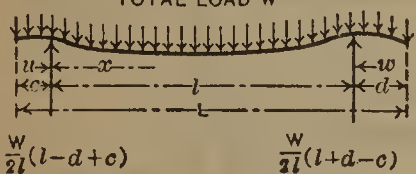
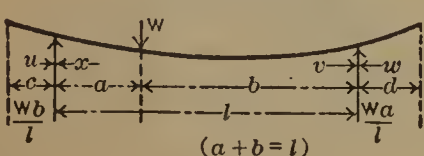
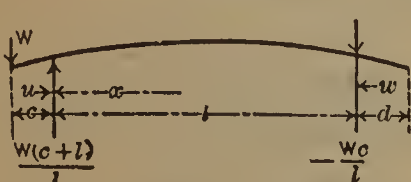
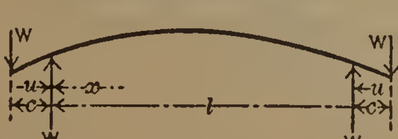
Stresses and Deflections in Beams

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points
Case 1. — Supported at Both Ends, Uniform Load TOTAL LOAD W 	$s = -\dfrac{W}{2Zl}x(l-x)$	Stress at center, $-\dfrac{Wl}{8Z}$. If cross-section is constant, this is the maximum stress.
Case 2. — Supported at Both Ends, Load at Center 	Between each support and load, $s = -\dfrac{Wx}{2Z}$	Stress at center, $-\dfrac{Wl}{4Z}$ If cross-section is constant, this is the maximum stress.
Case 3. — Supported at Both Ends, Load at any Point  $a+b=l$	For segment of length a, $s = -\dfrac{Wbx}{Zl}$ For segment of length b, $s = -\dfrac{Wav}{Zl}$	Stress at load, $-\dfrac{Wab}{Zl}$ If cross-section is constant, this is the maximum stress.
Case 4. — Supported at Both Ends, Two Symmetrical Loads 	Between each support and adjacent load, $s = -\dfrac{Wx}{Z}$ Between loads, $s = -\dfrac{Wa}{Z}$	Stress at each load, and at all points between, $-\dfrac{Wa}{Z}$
Case 5. — Both Ends Overhanging Supports Symmetrically, Uniform Load TOTAL LOAD W  $L=l+2c$	Between each support and adjacent end, $s = \dfrac{W}{2ZL}(c-u)^2$ Between supports, $s = \dfrac{W}{2ZL}[c^2 - x(l-x)]$	Stress at each support, $\dfrac{Wc^2}{2ZL}$ Stress at center, $\dfrac{W}{2ZL}(c^2 - \frac{1}{4}l^2)$ If cross-section is constant, the greater of these is the maximum stress. If l is greater than $2c$, the stress is zero at points $\sqrt{\frac{1}{4}l^2 - c^2}$ on both sides of the center. If cross-section is constant and if $l = 2.828c$, the stresses at supports and center are equal and opposite, and are $\pm\dfrac{WL}{46.62Z}$

Stresses and Deflections in Beams

Deflections	
General Formula for Deflection at any Point	Deflections at Critical Points
$y = \frac{Wx(l-x)}{24\,EIl}[l^2 + x(l-x)]$	Maximum deflection, at center, $\frac{5}{384}\frac{Wl^3}{EI}$
Between each support and load, $y = \frac{Wx}{48\,EI}(3\,l^2 - 4\,x^2)$	Maximum deflection, at load, $\frac{Wl^3}{48\,EI}$
For segment of length a, $y = \frac{Wbx}{6\,EIl}(l^2 - x^2 - b^2)$ For segment of length b, $y = \frac{Wav}{6\,EIl}(l^2 - v^2 - a^2)$	Deflection at load, $\frac{Wa^2b^2}{3\,EIl}$ Let a be the length of the shorter segment and b of the longer one. The maximum deflection is in the longer segment, at $v = b\sqrt{\frac{1}{3} + \frac{2\,a}{3\,b}} = v_1$, and is $\frac{Wav_1^3}{3\,EIl}$
Between each support and adjacent load. $y = \frac{Wx}{6\,EI}[3\,a(l-a) - x^2]$ Between loads, $y = \frac{Wa}{6\,EI}[3\,v(l-v) - a^2]$	Maximum deflection at center, $\frac{Wa}{24\,EI}(3\,l^2 - 4\,a^2)$ Deflection at loads $\frac{Wa^2}{6\,EI}(3\,l - 4\,a)$
Between each support and adjacent end, $y = \frac{Wu}{24\,EIL}[6\,c^2(l+u) - u^2(4\,c-u) - l^3]$ Between supports, $y = \frac{Wx(l-x)}{24\,EIL}[x(l-x) + l^2 - 6\,c^2]$	Deflection at ends, $\frac{Wc}{24\,EIL}[3\,c^2(c+2\,l) - l^3]$ Deflection at center, $\frac{Wl^2}{384\,EIL}(5\,l^2 - 24c^2)$ If l is between $2\,c$ and $2.449\,c$, there are maximum upward deflections at points $\sqrt{3\,(\frac{1}{4}\,l^2 - c^2)}$ on both sides of the center, which are, $-\frac{W}{96\,EIL}(6\,c^2 - l^2)^2$

Stresses and Deflections in Beams

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points
Case 6. — Both Ends Overhanging Supports Unsymmetrically, Uniform Load TOTAL LOAD W 	For overhanging end of length c, $s = \frac{W}{2ZL}(c-u)^2$ Between supports, $s = \frac{W}{2ZL}\left\{c^2\left(\frac{l-x}{l}\right) + d^2\frac{x}{l} - x(l-x)\right\}$ For overhanging end of length d, $s = \frac{W}{2ZL}(d-w)^2$	Stress at support next end of length c, $\frac{Wc^2}{2ZL}$ Critical stress between supports is at $x = \frac{l^2+c^2-d^2}{2l} = x_1$ and is $\frac{W}{2ZL}(c^2 - x_1^2)$ Stress at support next end of length d, $\frac{Wd^2}{2ZL}$ If cross-section is constant, the greatest of these three is the maximum stress. If $x_1 > c$, the stress is zero at points $\sqrt{x_1^2-c^2}$ on both sides of $x = x_1$.
Case 7. — Both Ends Overhanging Supports, Load at any Point Between 	Between supports: For segment of length a, $s = -\frac{Wbx}{Zl}$ For segment of length b, $s = -\frac{Wav}{Zl}$ Beyond supports $s = 0$.	Stress at load, $-\frac{Wab}{Zl}$ If cross-section is constant, this is the maximum stress.
Case 8. — Both Ends Overhanging Supports, Single Overhanging Load 	Between load and adjacent support, $s = \frac{W}{Z}(c-u)$ Between supports, $s = \frac{Wc}{Zl}(l-x)$ Between unloaded end and adjacent support, $s = 0$.	Stress at support adjacent to load, $\frac{Wc}{Z}$ If cross-section is constant, this is the maximum stress. Stress is zero at other support.
Case 9. — Both Ends Overhanging Supports, Symmetrical Overhanging Loads 	Between each load and adjacent support, $s = \frac{W}{Z}(c-u)$ Between supports, $s = \frac{Wc}{Z}$	Stress at supports and at all points between, $\frac{Wc}{Z}$ If cross-section is constant, this is the maximum stress.

Stresses and Deflections in Beams

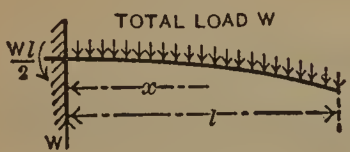
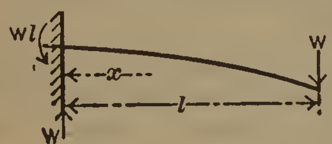
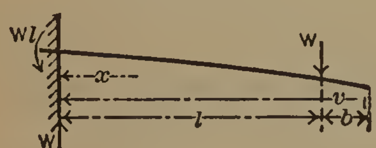
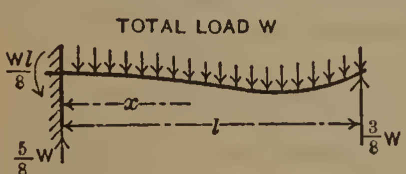
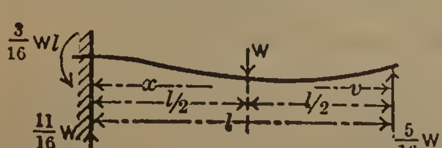
Deflections	
General Formula for Deflections at any Point	Deflections at Critical Points
For overhanging end of length c, $y=\frac{Wu}{24\,EIL}[2\,l\,(d^2+2\,c^2)+6\,c^2u-u^2\,(4\,c-u)-l^3]$ Between supports, $y=\frac{Wx\,(l-x)}{24\,EIL}\left\{x\,(l-x)+l^2-2\,(d^2+c^2)-\frac{2}{l}[d^2x+c^2\,(l-x)]\right\}$ For overhanging end of length d, $y=\frac{Ww}{24\,EIL}[2\,l\,(c^2+2\,d^2)+6\,d^2w-w^2\,(4\,d-w)-l^3]$	Deflection at end c, $\frac{Wc}{24\,EIL}[2\,l\,(d^2+2\,c^2)+3\,c^3-l^3]$ Deflection at end d, $\frac{Wd}{24\,EIL}[2\,l\,(c^2+2\,d^2)+3\,d^3-l^3]$ This case is so complicated that convenient general expressions for the critical deflections between supports cannot be obtained.
Between supports, same as Case 3. For overhanging end of length c, $y=-\frac{Wabu}{6\,EIl}(l+b)$ For overhanging end of length d, $y=-\frac{Wabw}{6\,EIl}(l+a)$	Between supports, same as Case 3. Deflection at end c, $-\frac{Wabc}{6\,EIl}(l+b)$ Deflection at end d, $-\frac{Wabd}{6\,EIl}(l+a)$
Between load and adjacent support, $y=\frac{Wu}{6\,EI}(3\,cu-u^2+2\,cl)$ Between supports, $y=-\frac{Wcx}{6\,EIl}(l-x)(2\,l-x)$ Between unloaded end and adjacent support, $y=\frac{Wclw}{6\,EI}$	Deflection at load, $\frac{Wc^2}{3\,EI}(c+l)$ Maximum upward deflection is at $x=0.42265\,l$, and is $-\frac{Wcl^2}{15.55\,EI}$ Deflection at unloaded end, $\frac{Wcld}{6\,EI}$
Between each load and adjacent support, $y=\frac{Wu}{6\,EI}[3\,c\,(l+u)-u^2]$ Between supports, $y=-\frac{Wcx}{2\,EI}(l-x)$	Deflections at loads, $\frac{Wc^2}{6\,EI}(2\,c+3\,l)$ Deflection at center, $-\frac{Wcl^2}{8\,EI}$

The above expressions involve the usual approximations of the theory of flexure, and hold only for small deflections. Exact expressions for deflections of any magnitude are as follows:

Between supports the curve is a circle of radius $r=\frac{EI}{Wc}$; $y=\sqrt{r^2-\frac{1}{4}l^2}-\sqrt{r^2-(\frac{1}{2}l-x)^2}$

Deflection at center, $\sqrt{r^2-\frac{1}{4}l^2}-r$

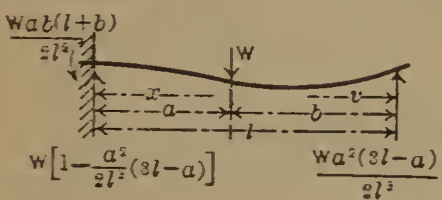
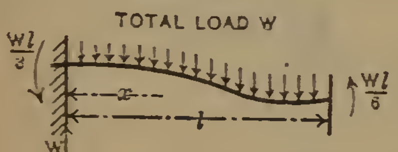
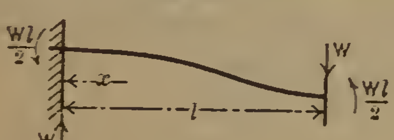
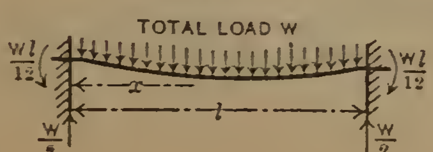
Stresses and Deflections in Beams

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points
Case 10. — Fixed at One End, Uniform Load 	$s = \frac{W}{2\,Zl}(l-x)^2$	Stress at support, $\frac{Wl}{2\,Z}$ If cross-section is constant, this is the maximum stress.
Case 11. — Fixed at One End, Load at Other 	$s = \frac{W}{Z}(l-x)$	Stress at support, $\frac{Wl}{Z}$ If cross-section is constant, this is the maximum stress.
Case 12. — Fixed at One End, Intermediate Load 	Between support and load, $s = \frac{W}{Z}(l-x)$ Beyond load, $s = 0$.	Stress at support, $\frac{Wl}{Z}$ If cross-section is constant, this is the maximum stress.
Case 13. — Fixed at One End, Supported at the Other, Uniform Load 	$s = \frac{W(l-x)}{2\,Zl}(\tfrac{1}{4}\,l - x)$	Maximum stress at point of fixture, $\frac{Wl}{8\,Z}$ Stress is zero at $x = \tfrac{1}{4}\,l$. Greatest negative stress is at $x = \tfrac{5}{8}\,l$ and is $-\frac{9}{128}\,\frac{Wl}{Z}$
Case 14. — Fixed at One End, Supported at the Other, Load at Center 	Between point of fixture and load, $s = \frac{W}{16\,Z}(3\,l - 11\,x)$ Between support and load, $s = -\frac{5}{16}\,\frac{Wv}{Z}$	Maximum stress at point of fixture, $\frac{3}{16}\,\frac{Wl}{Z}$ Stress is zero at $x = \frac{3}{11}\,l$ Greatest negative stress at center, $-\frac{5}{32}\,\frac{Wl}{Z}$

Stresses and Deflections in Beams

Deflections	
General Formula for Deflection at any Point	Deflections at Critical Points
$y = \frac{Wx^2}{24\,EIl}\,[2\,l^2 + (2\,l - x)^2]$	Maximum deflection, at end, $\frac{Wl^3}{8\,EI}$
$y = \frac{Wx^2}{6\,EI}\,(3\,l - x)$	Maximum deflection, at end, $\frac{Wl^3}{3\,EI}$
Between support and load, $y = \frac{Wx^2}{6\,EI}\,(3\,l - x)$ Beyond load, $y = \frac{Wl^2}{6\,EI}\,(3\,v - l)$	Deflection at load, $\frac{Wl^3}{3\,EI}$ Maximum deflection, at end, $\frac{Wl^2}{6\,EI}\,(2\,l + 3\,b)$
$y = \frac{Wx^2\,(l - x)}{48\,EIl}\,(3\,l - 2\,x)$	Maximum deflection is at $x = 0.5785\,l$, and is $\frac{Wl^3}{185\,EI}$ Deflection at center, $\frac{Wl^3}{192\,EI}$ Deflection at point of greatest negative stress, at $x = \frac{5}{8}\,l$ is $\frac{Wl^3}{187\,EI}$
Between point of fixture and load, $y = \frac{Wx^2}{96\,EI}\,(9\,l - 11\,x)$ Between support and load, $y = \frac{Wv}{96\,EI}\,(3\,l^2 - 5\,v^2)$	Maximum deflection is at $v = 0.4472\,l$, and is $\frac{Wl^3}{107.33\,EI}$ Deflection at load, $\frac{7}{768}\,\frac{Wl^3}{EI}$

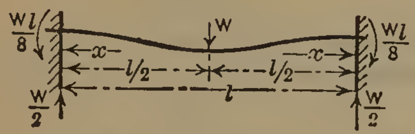
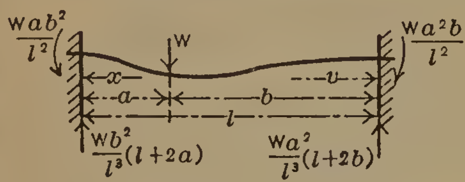
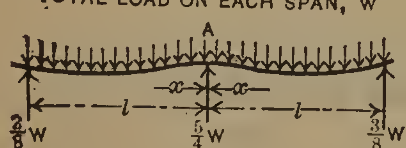
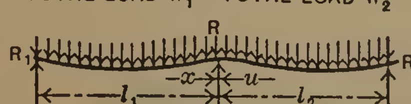
Stresses and Deflections in Beams

Type of Beam	Stresses	
	General Formula for Stress at any Point	Stresses at Critical Points
Case 15. — Fixed at One End, Supported at the Other, Load at any Point $m = (l + a)(l + b) + al$ $n = al(l + b)$ 	Between point of fixture and load, $s = \frac{Wb}{2Zl^3}(n - mx)$ Between support and load, $s = -\frac{Wa^2v}{2Zl^3}(3l-a)$	Greatest positive stress, at point of fixture, $\frac{Wab}{2Zl^2}(l+b)$ Greatest negative stress, at load, $-\frac{Wa^2b}{2Zl^3}(3l-a)$ If $a < 0.5858\,l$, the first is the maximum stress. If $a = 0.5858\,l$, the two are equal and are $\pm\frac{Wl}{5.83\,Z}$. If $a > 0.5858\,l$, the second is the maximum stress. Stress is zero at $x = \frac{n}{m}$
Case 16. — Fixed at One End, Free but Guided at the Other, Uniform Load 	$s = \frac{Wl}{Z}\left\{\frac{1}{3} - \frac{x}{l} + \frac{1}{2}\left(\frac{x}{l}\right)^2\right\}$	Maximum stress, at support, $\frac{Wl}{3Z}$ Stress is zero for $x = 0.4227\,l$ Greatest negative stress, at free end, $-\frac{Wl}{6Z}$
Case 17. — Fixed at One End, Free but Guided at the Other, with Load 	$s = \frac{W}{Z}(\tfrac{1}{2}\,l - x)$	Stress at support, $\frac{Wl}{2Z}$ Stress at free end $-\frac{Wl}{2Z}$ These are the maximum stresses and are equal and opposite. Stress is zero at center.
Case 18. — Fixed at Both Ends, Uniform Load 	$s = \frac{Wl}{2Z}\left\{\frac{1}{6} - \frac{x}{l} + \left(\frac{x}{l}\right)^2\right\}$	Maximum stress, at ends, $\frac{Wl}{12Z}$ Stress is zero at $x = 0.7887\,l$ and at $x = 0.2113\,l$ Greatest negative stress, at center, $-\frac{Wl}{24\,Z}$

Stresses and Deflections in Beams

Deflections	
General Formula for Deflections at any Point	Deflections at Critical Points
Between point of fixture and load, $y = \frac{Wx^2b}{12\,EIl^3}(3\,n - mx)$ Between support and load, $y = \frac{Wa^2v}{12\,EIl^3}[3\,l^2b - v^2(3\,l - a)]$	Deflection at load, $\frac{Wa^3b^2}{12\,EIl^3}(3\,l + b)$ If $a < 0.5858\,l$, maximum deflection is between load and support, at $v = l\sqrt{\frac{b}{2\,l + b}}$ and is $\frac{Wa^2b}{6\,EI}\sqrt{\frac{b}{2\,l + b}}$ If $a = 0.5858\,l$, maximum deflection is at load and is $\frac{Wl^3}{101.9\,EI}$ If $a > 0.5858\,l$, maximum deflection is between load and point of fixture, at $x = \frac{2\,n}{m}$, and is $\frac{Wbn^3}{3\,EIm^2l^3}$
$y = \frac{Wx^2}{24\,EIl}(2\,l - x)^2$	Maximum deflection, at free end, $\frac{Wl^3}{24\,EI}$
$y = \frac{Wx^2}{12\,EI}(3\,l - 2\,x)$	Maximum deflection, at free end, $\frac{Wl^3}{12\,EI}$
$y = \frac{Wx^2}{24\,EIl}(l - x)^2$	Maximum deflection, at center, $\frac{Wl^3}{384\,EI}$

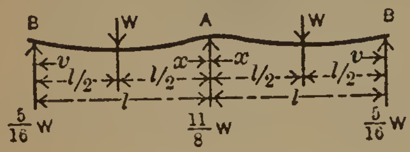
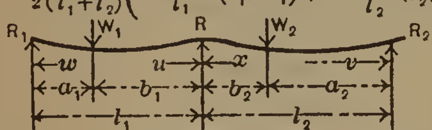
Stresses and Deflections in Beams

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points
Case 19. — Fixed at Both Ends, Load at Center 	Between each end and load, $s = \frac{W}{2Z}(\frac{1}{4}l - x)$	Stress at ends $\frac{Wl}{8Z}$; at load $-\frac{Wl}{8Z}$ These are the maximum stresses and are equal and opposite. Stress is zero at $x = \frac{1}{4}l$
Case 20. — Fixed at Both Ends, Load at any Point 	For segment of length a, $s = \frac{Wb^2}{Zl^3}[al - x(l+2a)]$ For segment of length b, $s = \frac{Wa^2}{Zl^3}[bl - v(l+2b)]$	Stress at end next segment of length a, $\frac{Wab^2}{Zl^2}$ Stress at end next segment of length b, $\frac{Wa^2b}{Zl^2}$ Maximum stress is at end next shorter segment. Stress is zero for $x = \frac{al}{l+2a}$ and $v = \frac{bl}{l+2b}$ Greatest negative stress, at load, $-\frac{2Wa^2b^2}{Zl^3}$
Case 21. — Continuous Beam, with Two Equal Spans, Uniform Load TOTAL LOAD ON EACH SPAN, W 	$s = \frac{W(l-x)}{2Zl}(\frac{1}{4}l - x)$	Maximum stress at point A, $\frac{Wl}{8Z}$ Stress is zero at $x = \frac{1}{4}l$. Greatest negative stress is at $x = \frac{5}{8}$ and is, $-\frac{9}{128}\frac{Wl}{Z}$
Case 22. — Continuous Beam, with Two Unequal Spans, Unequal, Uniform Loads TOTAL LOAD W_1 TOTAL LOAD W_2  $\frac{l_1W_1(3l_1+4l_2)-W_2l_2^2}{8l_1(l_1+l_2)}$ $\frac{l_2W_2(3l_2+4l_1)-W_1l_1^2}{8l_2(l_1+l_2)}$ $\left(\frac{W_1+W_2}{2}\right)+\frac{1}{8}\left(\frac{W_1l_1}{l_2}+\frac{W_2l_2}{l_1}\right)$	Between R_1 and R, $s = \frac{l_1-x}{Z}\left\{\frac{(l_1-x)W_1}{2l_1} - r_1\right\}$ Between R_2 and R, $s = \frac{l_2-u}{Z}\left\{\frac{(l_2-u)W_2}{2l_2} - r_2\right\}$	Stress at support R, $\frac{W_1l_1^2 + W_2l_2^2}{8Z(l_1+l_2)}$ Greatest stress in the first span is at $x = \frac{l_1}{W_1}(W_1 - r_1)$, and is, $-\frac{r_1^2l_1}{2ZW_1}$ Greatest stress in the second span is at $u = \frac{l_2}{W_2}(W_2 - r_2)$, and is, $-\frac{r_2^2l_2}{2ZW_2}$

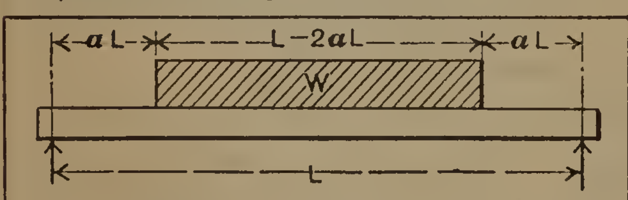
Stresses and Deflections in Beams

Deflections	
General Formula for Deflections at any Point	Deflections at Critical Points
$y = \frac{Wx^2}{48\,EI}(3\,l - 4\,x)$	Maximum deflection, at load, $\frac{Wl^3}{192\,EI}$
For segment of length a, $y = \frac{Wx^2b^2}{6\,EIl^3}[2\,a\,(l - x) + l\,(a - x)]$ For segment of length b, $v = \frac{Wv^2a^2}{6\,EIl^3}[2\,b\,(l - v) + l\,(b - v)]$	Deflection at load, $\frac{Wa^3b^3}{3\,EIl^3}$ Let b be the length of the longer segment and a of the shorter one. The maximum deflection is in the longer segment, at $v = \frac{2\,bl}{l + 2\,b}$, and is $\frac{2\,Wa^2b^3}{3\,EI\,(l + 2\,b)^2}$
$y = \frac{Wx^2\,(l - x)}{48\,EIl}(3\,l - 2\,x)$	Maximum deflection is at $x = 0.5785\,l$, and is $\frac{Wl^3}{185\,EI}$ Deflection at center of span, $\frac{Wl^3}{192\,EI}$ Deflection at point of greatest negative stress, at $x = \frac{5}{8}\,l$ is $\frac{Wl^3}{187\,EI}$
Between R_1 and R, $y = \frac{x\,(l_1 - x)}{24\,EI}\left\{(2\,l_1 - x)\,(4\,r_1 - W_1) - \frac{W_1\,(l_1 - x)^2}{l_1}\right\}$ Between R_2 and R, $y = \frac{u\,(l_2 - u)}{24\,EI}\left\{(2\,l_2 - u)\,(4\,r_2 - W_2) - \frac{W_2\,(l_2 - u)^2}{l_2}\right\}$	This case is so complicated that convenient general expressions for the critical deflections cannot be obtained.

Stresses and Deflections in Beams

Type of Beam	Stresses: General Formula for Stress at any Point	Stresses: Stresses at Critical Points
Case 23. — Continuous Beam, with Two Equal Spans, Equal Loads at Center of Each 	Between point A and load, $s = \frac{W}{16Z}(3l - 11x)$ Between point B and load, $s = -\frac{5}{16}\frac{Wv}{Z}$	Maximum stress at point A, $\frac{3}{16}\frac{Wl}{Z}$ Stress is zero at $x = \frac{3}{11}l$ Greatest negative stress at center of span, $-\frac{5}{32}\frac{Wl}{Z}$
Case 24. — Continuous Beam, with Two Unequal Spans, Unequal Loads at any Point of Each $m = \frac{1}{2(l_1+l_2)}\left(\frac{W_1a_1b_1}{l_1}(l_1+a_1) + \frac{W_2a_2b_2}{l_2}(l_2+a_2)\right)$  $\frac{W_1b_1 - m}{l_1} = r_1$ $\quad \frac{W_1a_1+m}{l_1} + \frac{W_2a_2+m}{l_2} = r$ $\quad \frac{W_2b_2-m}{l_2} = r_2$	Between R_1 and W_1, $s = -\frac{wr_1}{Z}$ Between R and W_1, $s = \frac{1}{l_1Z}[m(l_1-u) - W_1a_1u]$ Between R and W_2, $s = \frac{1}{l_2Z}[m(l_2-x) - W_2a_2x]$ Between R_2 and W_2, $s = -\frac{vr_2}{Z}$	Stress at load W_1, $-\frac{a_1r_1}{Z}$ Stress at support R, $\frac{m}{Z}$ Stress at load W_2, $-\frac{a_2r_2}{Z}$ The greatest of these is the maximum stress.

Deflection of Beam Uniformly Loaded for Part of Its Length. — In the following formulas, all lengths are in inches and all weights in pounds. W = total load; L = total length between supports; E = modulus of elasticity; I = moments of inertia of beam section; a = fraction of length of beam at each end, that is not loaded; f = deflection.



$$f = \frac{WL^3}{EI\,384\,(1-2a)}(5 - 24a^2 + 16a^4)$$

The expression for maximum bending moment is: $M_{max.} = \frac{1}{8}WL(1+2a)$. These formulas apply to simple beams resting on supports at the ends.

Beams of Uniform Strength Throughout Their Length. — In nearly all cases, the bending moment in a beam is not uniform throughout its length, but varies. Therefore, a beam of uniform cross-section which is made strong enough at its most strained section, will have an excess of material at every other section. Sometimes it may be desirable to have the cross-section uniform, while in other cases the metal can be more advantageously distributed if the beam is so designed that its cross-section varies from point to point, so that it is at every point just great enough to take care of the bending stresses at that point. A table is given showing beams

Stresses and Deflections in Beams

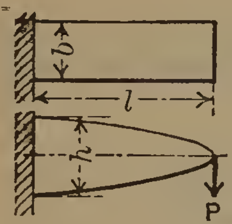
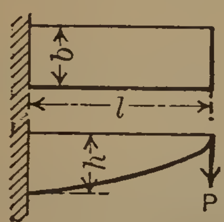
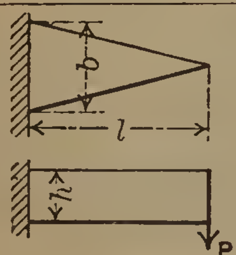
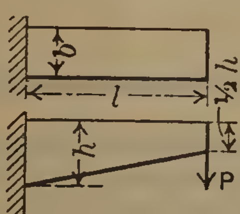
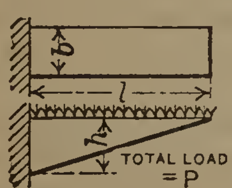
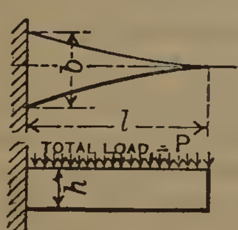
Deflections	
General Formula for Deflections at any Point	Deflections at Critical Points
Between point A and load, $y = \frac{Wx^2}{96\,EI}(9\,l - 11\,x)$ Between point B and load, $y = \frac{Wv}{96\,EI}(3\,l^2 - 5\,v^2)$	Maximum deflection is at $v = 0.4472\,l$, and is $\frac{Wl^3}{107.33\,EI}$ Deflection at load, $\frac{7}{768}\frac{Wl^3}{EI}$
Between R_1 and W_1, $y = \frac{w}{6\,EI}\left\{(l_1 - w)(l_1 + w)\,r_1 - \frac{W_1b_1^3}{l_1}\right\}$ Between R and W_1, $y = \frac{u}{6\,EIl_1}[W_1a_1b_1(l_1 + a_1) - W_1a_1u^2 - m(2\,l_1 - u)(l_1 - u)]$ Between R and W_2, $y = \frac{x}{6\,EIl_2}[W_2a_2b_2(l_2 + a_2) - W_2a_2x^2 - m(2\,l_2 - x)(l_2 - x)]$ Between R_2 and W_2, $y = \frac{v}{6\,EI}\left\{(l_2 - v)(l_2 + v)\,r_2 - \frac{W_2b_2^3}{l_2}\right\}$	Deflection at load W_1, $\frac{a_1b_1}{6\,EIl_1}[2\,a_1b_1W_1 - m(l_1 + a_1)]$ Deflection at load W_2, $\frac{a_2b_2}{6\,EIl_2}[2\,a_2b_2W_2 - m(l_2 + a_2)]$ This case is so complicated that convenient general expressions for the maximum deflections cannot be obtained.

in which the load is applied in different ways and which are supported by different methods, and the shape of the beam required for uniform strength is indicated. It should be noted that the shape given is the theoretical shape required to resist bending only. It is apparent that sufficient cross-section of beam must also be added either at the points of support (in the case of beams supported at both ends), or at the point of application of the load (in the case of beams loaded at one end), to take care of the vertical shear.

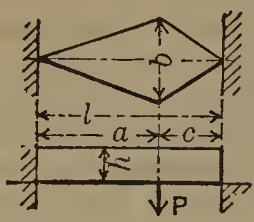
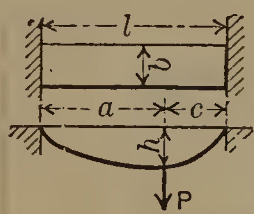
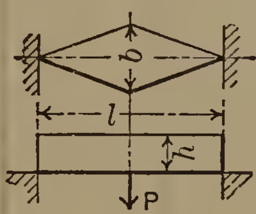
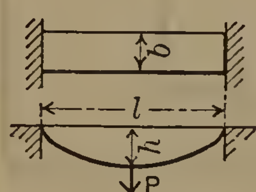
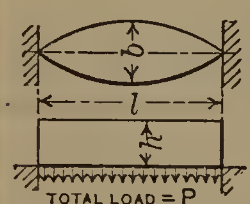
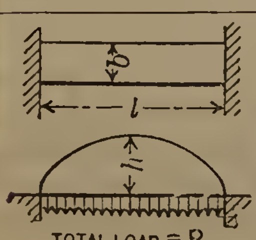
It should be noted that the theoretical shapes of the beams given in the tables on the two following pages are based on the stated assumptions of uniformity of width or depth of cross-section, and unless these are observed in the design, the theoretical outlines do not apply without modifications. For example, in a cantilever with the load at one end, the outline is a parabola only when the width of the beam is uniform. It is not correct to use a strictly parabolic shape when the thickness is not uniform, as, for instance, when the beam is made of an I- or T-section. In such cases, some modification may be necessary; but it is evident that whatever the shape adopted, the correct depth of the section can be obtained by an investigation of the bending moment and the shearing load at a number of points, and then a line can be drawn through the points thus ascertained, which will provide for a beam of practically uniform strength whether the cross-section be of uniform width or not.

Beams of Uniform Strength Throughout Their Length

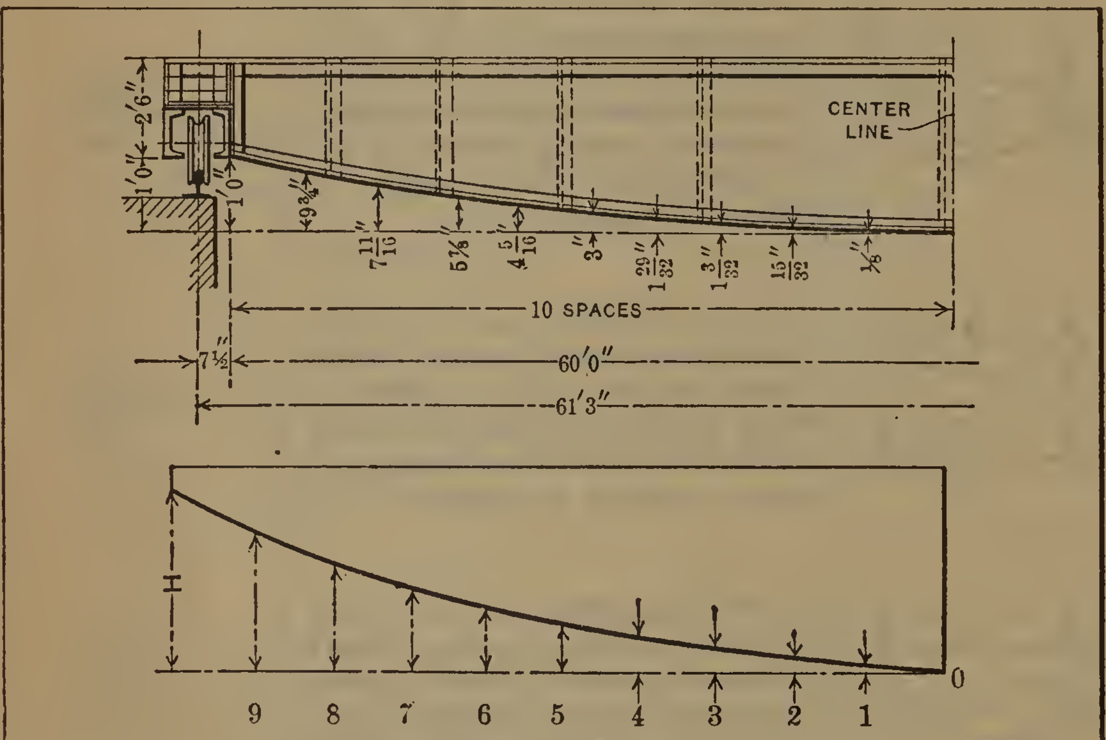
(All loads in pounds, all dimensions in inches.)

Type of Beam	Description	P = Carrying Capacity S = Safe Stress per Square Inch
	Load at one end. Width of beam uniform. Depth of beam decreasing towards loaded end. Outline of beam-shape, parabola with vertex at loaded end.	$P = \frac{Sbh^2}{6l}$
	Load at one end. Width of beam uniform. Depth of beam decreasing towards loaded end. Outline of beam, one-half of a parabola with vertex at loaded end. Beam may be reversed so that upper edge is parabolic.	$P = \frac{Sbh^2}{6l}$
	Load at one end. Depth of beam uniform. Width of beam decreasing towards loaded end. Outline of beam triangular, with apex at loaded end.	$P = \frac{Sbh^2}{6l}$
	Beam of *approximately* uniform strength. Load at one end. Width of beam uniform. Depth of beam decreasing towards loaded end, but not tapering to a sharp point.	$P = \frac{Sbh^2}{6l}$
	Uniformly distributed load. Width of beam uniform. Depth of beam decreasing towards outer end. Outline of beam, right-angled triangle.	$P = \frac{Sbh^2}{3l}$
	Uniformly distributed load. Depth of beam uniform. Width of beam gradually decreasing towards outer end. Outline of beam is formed by two parabolas which tangent each other at their vertices at the outer end of the beam.	$P = \frac{Sbh^2}{3l}$

Beams of Uniform Strength Throughout Their Length

Type of Beam	Description	P = Carrying Capacity S = Safe Stress per Square Inch
	Beam supported at both ends. Load concentrated at any point. Depth of beam uniform. Width of beam maximum at point of loading. Outline of beam, two triangles with apexes at points of support.	$P = \frac{Sbh^2l}{6\,ac}$
	Beam supported at both ends. Load concentrated at any point. Width of beam uniform. Depth of beam maximum at point of loading. Outline of beam is formed by two parabolas with their vertexes at points of support.	$P = \frac{Sbh^2l}{6\,ac}$
	Beam supported at both ends. Load concentrated in the middle. Depth of beam uniform. Width of beam maximum at point of loading. Outline of beam, two triangles with apexes at points of support.	$P = \frac{2\,Sbh^2}{3\,l}$
	Beam supported at both ends. Load concentrated at center. Width of beam uniform. Depth of beam maximum at point of loading. Outline of beam, two parabolas with vertices at points of support.	$P = \frac{2\,Sbh^2}{3\,l}$
	Beam supported at both ends. Load uniformly distributed. Depth of beam uniform. Width of beam maximum at center. Outline of beam, two parabolas with vertexes at middle of beam.	$P = \frac{4\,Sbh^2}{3\,l}$
	Beam supported at both ends. Load uniformly distributed. Width of beam uniform. Depth of beam maximum at center. Outline of beam one-half of an ellipse.	$P = \frac{4\,Sbh^2}{3\,l}$

Crane Girders with Curved Lower Chords. — An example of a design which makes use of the principles of beams of uniform strength is found in the ordinary fish-belly type of crane girder. When laying out crane girders, the accompanying tables will be found convenient. The engraving will explain the use of the tables. A crane girder having a span of 61 feet 3 inches has been assumed as an example. The curved part has a span of 60 feet; one-half of this distance, or 30 feet, is divided for the convenience of the templet makers into ten spaces of 3 feet each. The end ordinate, assumed here to be 1 foot, will be found at the extreme left of the tables under the heading H. The lengths of the remaining nine ordinates follow in order. For short spans, say about 30 feet, it is most convenient to divide the base of the curve into five spaces, as it is the usual practice to give ordinates about every 3 feet. In this case, we would use only every other ordinate in the tables, or, beginning with the left-hand column, the ordinates would be as found in the columns headed H, 8, 6, 4 and 2.



The tables are calculated from the formula:

$$X = H \times (M^2 \div N^2)$$

in which H = end ordinate; X = required ordinate; N = number of equal spaces into which the base line is divided; M = number of spaces from 0 to the required ordinate. When $N = 10$, as in the case for which the tables are calculated, $N^2 = 100$, and

$$X = H \times 0.01\, M^2.$$

Hence, ordinate No. 8 equals $H \times 0.01 \times 64 = 0.64\, H$. Ordinate No. 4 equals $H \times 0.01 \times 16 = 0.16\, H$.

Opinions vary considerably as to the allowable working stress in crane girders. Many cranes have girders which are designed for a stress of only 8000 pounds per square inch, while in others the stress will be over 14,000 pounds. However, a general factor of safety of 5 is the most usual and desirable in crane work, and if that factor of safety is adopted, the working stress should be anywhere from 11,000 to 12,000 pounds per square inch.

Ordinates of Parabolas for Crane Girder Design — I

	Ordinates								
H	9	8	7	6	5	4	3	2	1
Ft. Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
6	4 7/8	3 27/32	2 15/16	2 5/32	1 1/2	31/32	17/32	1/4	1/16
6 1/4	5 1/16	4	3 1/16	2 1/4	1 9/16	1	9/16	1/4	1/16
6 1/2	5 1/4	4 5/32	3 3/16	2 11/32	1 5/8	1 1/32	19/32	1/4	1/16
6 3/4	5 15/32	4 5/16	3 5/16	2 7/16	1 11/16	1 3/32	19/32	9/32	1/16
7	5 11/16	4 15/32	3 7/16	2 17/32	1 3/4	1 1/8	5/8	9/32	1/16
7 1/4	5 7/8	4 21/32	3 9/16	2 5/8	1 13/16	1 5/32	21/32	9/32	1/16
7 1/2	6 1/16	4 13/16	3 11/16	2 11/16	1 7/8	1 7/32	11/16	5/16	1/16
7 3/4	6 9/32	4 31/32	3 13/16	2 25/32	1 15/16	1 1/4	11/16	5/16	1/16
8	6 15/32	5 1/8	3 15/16	2 7/8	2	1 9/32	23/32	5/16	3/32
8 1/4	6 19/32	5 9/32	4 1/32	2 31/32	2 1/16	1 5/16	3/4	11/32	3/32
8 1/2	6 7/8	5 7/16	4 5/32	3 1/16	2 1/8	1 3/8	3/4	11/32	3/32
8 3/4	7 3/32	5 19/32	4 9/32	3 5/32	2 3/16	1 13/32	25/32	11/32	3/32
9	7 9/32	5 3/4	4 13/32	3 1/4	2 1/4	1 7/16	13/16	3/8	3/32
9 1/4	7 1/2	5 15/16	4 17/32	3 9/32	2 5/16	1 15/32	27/32	3/8	3/32
9 1/2	7 11/16	6 3/32	4 21/32	3 7/16	2 3/8	1 1/2	27/32	3/8	3/32
9 3/4	7 29/32	6 1/4	4 25/32	3 1/2	2 7/16	1 9/16	7/8	13/32	3/32
10	8 3/32	6 13/32	4 29/32	3 19/32	2 1/2	1 19/32	29/32	13/32	3/32
10 1/4	8 5/16	6 9/16	5	3 11/16	2 9/16	1 5/8	15/16	13/32	3/32
10 1/2	8 1/2	6 23/32	5 5/32	3 25/32	2 5/8	1 11/16	15/16	13/32	3/32
10 3/4	8 11/16	6 7/8	5 1/4	3 7/8	2 11/16	1 23/32	31/32	7/16	3/32
11	8 29/32	7 1/32	5 13/32	3 31/32	2 3/4	1 3/4	1	7/16	1/8
11 1/4	9 1/8	7 3/16	5 1/2	4 1/16	2 13/16	1 13/16	1	7/16	1/8
11 1/2	9 5/16	7 3/8	5 5/8	4 5/32	2 7/8	1 27/32	1 1/32	15/32	1/8
11 3/4	9 1/2	7 17/32	5 3/4	4 7/32	2 15/16	1 7/8	1 1/16	15/32	1/8
12	9 3/4	7 11/16	5 7/8	4 5/16	3	1 29/32	1 3/32	15/32	1/8
1 0 1/4	9 15/16	7 27/32	6	4 13/32	3 1/16	1 31/32	1 3/32	1/2	1/8
1 0 1/2	10 1/8	8	6 1/8	4 1/2	3 1/8	2	1 1/8	1/2	1/8
1 0 3/4	10 5/16	8 5/32	6 1/4	4 19/32	3 3/16	2 3/32	1 1/8	1/2	1/8
1 1	10 17/32	8 5/16	6 3/8	4 11/16	3 1/4	2 3/32	1 5/32	17/32	1/8
1 1 1/4	10 3/4	8 15/32	6 1/2	4 25/32	3 5/16	2 1/8	1 3/16	17/32	1/8
1 1 1/2	10 15/16	8 5/8	6 5/8	4 7/8	3 3/8	2 5/32	1 7/32	17/32	1/8
1 1 3/4	11 1/8	8 7/8	6 3/4	4 15/16	3 7/16	2 3/16	1 1/4	9/16	1/8
1 2	11 11/32	8 31/32	6 7/8	5 1/32	3 1/2	2 1/4	1 1/4	9/16	5/32
1 2 1/4	11 17/32	9 1/8	7	5 1/8	3 9/16	2 9/32	1 9/32	9/16	5/32
1 2 1/2	11 3/4	9 9/32	7 3/32	5 7/32	3 5/8	2 5/16	1 5/16	19/32	5/32
1 2 3/4	11 15/16	9 7/16	7 7/32	5 5/16	3 11/16	2 3/8	1 5/16	19/32	5/32
1 3	12 5/32	9 19/32	7 11/32	5 13/32	3 3/4	2 13/32	1 11/32	19/32	5/32
1 3 1/4	12 11/32	9 3/4	7 15/32	5 1/2	3 13/16	2 9/16	1 3/8	19/32	5/32
1 3 1/2	12 9/16	9 15/16	7 19/32	5 19/32	3 7/8	2 1/2	1 13/32	5/8	5/32
1 3 3/4	12 3/4	10 3/32	7 23/32	5 21/32	3 15/16	2 17/32	1 13/32	5/8	5/32
1 4	12 31/32	10 1/4	7 27/32	5 3/4	4	2 9/16	1 7/16	5/8	5/32
1 4 1/4	13 5/32	10 13/32	7 31/32	5 27/32	4 1/16	2 19/32	1 15/32	21/32	5/32
1 4 1/2	13 3/8	10 9/16	8 3/32	5 15/16	4 1/8	2 5/8	1 15/32	21/32	5/32
1 4 3/4	13 9/16	10 23/32	8 5/32	6 1/32	4 3/16	2 11/16	1 1/2	11/16	5/32
1 5	13 25/32	10 7/8	8 11/32	6 1/8	4 1/4	2 23/32	1 17/32	11/16	5/32
1 5 1/4	13 31/32	11 1/32	8 7/16	6 7/32	4 5/16	2 3/4	1 9/16	11/16	3/16
1 5 1/2	14 3/16	11 3/16	8 9/16	6 5/16	4 3/8	2 13/16	1 9/16	11/16	3/16
1 5 3/4	14 3/8	11 3/8	8 11/16	6 13/32	4 7/16	2 27/32	1 19/32	23/32	3/16
1 6	14 19/32	11 17/32	8 13/16	6 15/32	4 1/2	2 7/8	1 5/8	23/32	3/16

Ordinates of Parabolas for Crane Girder Design — 2

H		9		8		7		6	5	4	3	2	1
Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ft.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
1	6	1	2 19/32		11 17/32		8 13/16	6 15/32	4 1/2	2 7/8	1 5/8	23/32	3/16
1	6 1/4	1	2 25/32		11 11/16		8 15/16	6 9/16	4 9/16	2 15/16	1 21/32	23/32	3/16
1	6 1/2	1	3		11 27/32		9 1/16	6 21/32	4 5/8	2 31/32	1 21/32	3/4	3/16
1	6 3/4	1	3 3/16	1	0		9 3/16	6 3/4	4 11/16	3	1 11/16	3/4	3/16
1	7	1	3 13/32	1	0 5/32		9 5/16	6 27/32	4 3/4	3 1/32	1 23/32	3/4	3/16
1	7 1/4	1	3 19/32	1	0 5/16		9 7/16	6 15/16	4 13/16	3 3/32	1 23/32	3/4	3/16
1	7 1/2	1	3 25/32	1	0 15/32		9 9/16	7	4 7/8	3 1/8	1 3/4	25/32	3/16
1	7 3/4	1	4	1	0 21/32		9 11/16	7 1/8	4 15/16	3 5/32	1 25/32	25/32	3/16
1	8	1	4 3/16	1	0 13/16		9 13/16	7 3/16	5	3 3/16	1 13/16	13/16	3/16
1	8 1/4	1	4 13/32	1	0 31/32		9 15/16	7 9/32	5 1/16	3 1/4	1 13/16	13/16	3/16
1	8 1/2	1	4 19/32	1	1 1/8		10 1/32	7 3/8	5 1/8	3 9/32	1 27/32	13/16	7/32
1	8 3/4	1	4 13/16	1	1 9/32		10 5/32	7 15/32	5 3/16	3 5/16	1 7/8	27/32	7/32
1	9	1	5	1	1 7/16		10 9/32	7 9/16	5 1/4	3 3/8	1 7/8	27/32	7/32
1	9 1/4	1	5 7/32	1	1 19/32		10 13/32	7 21/32	5 5/16	3 13/32	1 29/32	27/32	7/32
1	9 1/2	1	5 13/32	1	1 3/4		10 17/32	7 3/4	5 3/8	3 7/16	1 15/16	7/8	7/32
1	9 3/4	1	5 5/8	1	1 15/16		10 21/32	7 27/32	5 7/16	3 15/32	1 31/32	7/8	7/32
1	10	1	5 13/16	1	2 3/32		10 25/32	7 15/16	5 1/2	3 17/32	1 31/32	7/8	7/32
1	10 1/4	1	6 1/32	1	2 1/4		10 29/32	8	5 9/16	3 9/16	2	29/32	7/32
1	10 1/2	1	6 7/32	1	2 7/16		11 1/32	8 3/32	5 5/8	3 19/32	2 1/32	29/32	7/32
1	10 3/4	1	6 7/16	1	2 9/16		11 5/32	8 3/16	5 11/16	3 5/8	2 1/16	29/32	7/32
1	11	1	6 5/8	1	2 23/32		11 9/32	8 9/32	5 3/4	3 11/16	2 1/16	15/16	7/32
1	11 1/4	1	6 15/16	1	2 7/8		11 13/32	8 3/8	5 13/16	3 23/32	2 3/32	15/16	7/32
1	11 1/2	1	7 1/32	1	3 1/32		11 17/32	8 15/32	5 7/8	3 3/4	2 1/8	15/16	1/4
1	11 3/4	1	7 1/4	1	3 3/16		11 5/8	8 9/16	5 15/16	3 13/16	2 1/8	31/32	1/4
2	0	1	7 7/16	1	3 3/8		11 3/4	8 21/32	6	3 27/32	2 5/32	31/32	1/4
2	0 1/4	1	7 21/32	1	3 17/32		11 7/8	8 23/32	6 1/16	3 7/8	2 3/16	31/32	1/4
2	0 1/2	1	7 27/32	1	3 11/16	1	0	8 13/16	6 1/8	3 15/16	2 7/32	31/32	1/4
2	0 3/4	1	8 1/32	1	3 27/32	1	0 1/8	8 29/32	6 3/16	3 31/32	2 7/32	1	1/4
2	1	1	8 1/4	1	4	1	0 1/4	9	6 1/4	4	2 1/4	1	1/4
2	1 1/4	1	8 15/32	1	4 5/32	1	0 3/8	9 3/32	6 5/16	4 1/32	2 9/32	1	1/4
2	1 1/2	1	8 21/32	1	4 5/16	1	0 1/2	9 3/16	6 3/8	4 3/32	2 9/32	1 1/32	1/4
2	1 3/4	1	8 27/32	1	4 15/32	1	0 5/8	9 9/32	6 7/16	4 1/8	2 5/16	1 1/32	1/4
2	2	1	9 1/16	1	4 21/32	1	0 3/4	9 3/8	6 1/2	4 5/32	2 11/32	1 1/32	1/4
2	2 1/4	1	9 1/4	1	4 13/16	1	0 7/8	9 15/32	6 9/16	4 7/32	2 3/8	1 1/16	1/4
2	2 1/2	1	9 15/32	1	4 31/32	1	1	9 17/32	6 5/8	4 1/4	2 3/8	1 1/16	1/4
2	2 3/4	1	9 21/32	1	5 1/8	1	1 3/32	9 5/8	6 11/16	4 9/32	2 13/32	1 1/16	1/4
2	3	1	9 7/8	1	5 9/32	1	1 1/4	9 23/32	6 3/4	4 5/16	2 7/16	1 3/32	9/32
2	3 1/4	1	10 1/16	1	5 7/16	1	1 11/32	9 13/16	6 13/16	4 3/8	2 15/32	1 3/32	9/32
2	3 1/2	1	10 9/32	1	5 19/32	1	1 15/32	9 29/32	6 7/8	4 13/32	2 15/32	1 3/32	9/32
2	3 3/4	1	10 15/32	1	5 3/4	1	1 19/32	10	6 15/16	4 7/16	2 1/2	1 1/8	9/32
2	4	1	10 11/16	1	5 15/16	1	1 23/32	10 3/32	7	4 15/32	2 17/32	1 1/8	9/32
2	4 1/4	1	10 7/8	1	6 3/32	1	1 27/32	10 3/16	7 1/16	4 17/32	2 17/32	1 1/8	9/32
2	4 1/2	1	11 3/32	1	6 1/4	1	1 31/32	10 1/4	7 1/8	4 9/16	2 9/16	1 5/32	9/32
2	4 3/4	1	11 9/32	1	6 13/32	1	2 3/32	10 11/32	7 3/16	4 19/32	2 19/32	1 5/32	9/32
2	5	1	11 1/2	1	6 9/16	1	2 7/32	10 7/16	7 1/4	4 21/32	2 5/8	1 5/32	9/32
2	5 1/4	1	11 11/16	1	6 23/32	1	2 11/32	10 17/32	7 5/16	4 11/16	2 5/8	1 3/16	9/32
2	5 1/2	1	11 29/32	1	6 7/8	1	2 15/32	10 5/8	7 3/8	4 23/32	2 21/32	1 3/16	5/16
2	5 3/4	2	0 3/32	1	7 1/32	1	2 19/32	10 23/32	7 7/16	4 3/4	2 11/16	1 3/16	5/16
2	6	2	0 5/16	1	7 7/32	1	2 23/32	10 13/16	7 1/2	4 13/16	2 23/32	1 7/32	5/16

Permissible Working Stresses for Structural Timbers

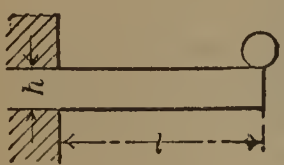
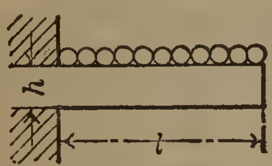
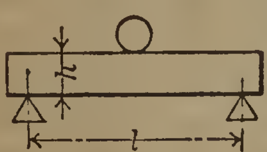
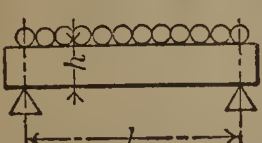
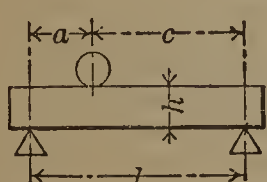
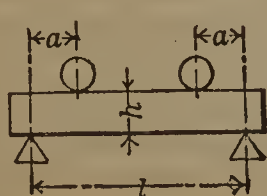
(U. S. Government Tests)

Kind of Timber	Bending, Pounds per Sq. In.			Compression, Pounds per Sq. In.			
	Allowable Stress in Extreme Fiber		Allowable Horizontal Shear Stress	Allowable Stress Parallel to Grain "Short Columns"		Allowable Stress Perpendicular to Grain	
	Outside Location	Dry Location	All Locations	Outside Location	Dry Location	Outside Location	Inside Location
Cedar, western red	800	900	80	700	700	150	200
Cedar, northern white	650	750	70	500	550	140	175
Chestnut	850	950	90	700	800	200	300
Cypress	1100	1300	100	1100	1100	250	350
Douglas fir (No. 1 str'l) *	1400	1600	100	1100	1200	250	350
Douglas fir (No. 2 str'l)	1100	1300	90	900	1000	225	300
Fir, balsam	750	900	70	600	700	125	150
Gum, red	900	1100	100	750	800	200	300
Hemlock, western	1100	1300	75	900	900	225	300
Hemlock, eastern	900	1000	70	700	700	225	300
Hickory	1500	1900	140	1200	1500	400	600
Maple, sugar or hard	1300	1500	150	1100	1200	375	500
Maple, silver or soft	900	1000	100	700	800	250	350
Oak, white or red	1200	1400	125	900	1000	375	500
Pine, s. yellow (dense) †	1400	1600	125	1100	1200	250	350
Pine, s. yellow (sound)	1100	1300	105	900	1000	225	300
Pine, eastern white	800	900	85	750	750	150	250
Pine, western white	800	900	85	750	750	150	250
Pine, Norway	1000	1100	85	800	800	175	300
Redwood	1000	1200	70	900	1000	150	250
Spruce, red or white	900	1100	85	750	800	150	250
Spruce, Englemann	650	750	70	550	600	140	175

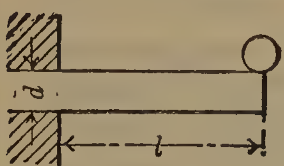
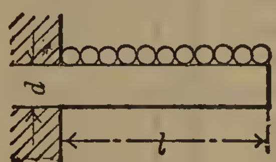
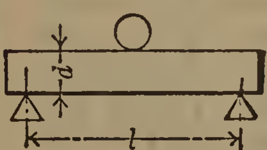
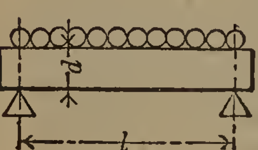
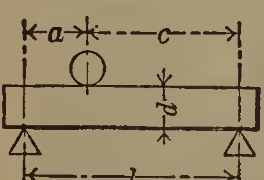
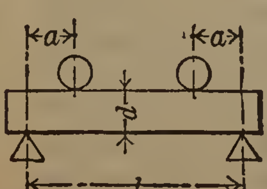
* The strength of large timbers depends chiefly upon the density or weight per cubic foot of the dry wood and upon the character, size, number and location of defects. "Dense" Douglas fir of the "No. 1 structural grade" shows on one end an average of at least six annual rings per inch and at least one-third "summer wood," measured over 3 inches on a line extending from the pith to the corner farthest from the pith when the least dimension of the timber is 5 inches or more. The point where the 3-inch line begins is found by the formula $A = \frac{1}{2} D - 2$, where A = distance in inches from pith to beginning of 3-inch line and D = minimum dimension of timber in inches. The "No. 2 structural grade" for Douglas fir includes timbers not passing the No. 1 grade, because (1) there is less density than required or (2) greater defects than are permitted.

† The term "southern yellow pine" includes the species known heretofore as long-leaf pine, short-leaf pine, loblolly pine, Cuban pine and pond pine. "Dense" southern yellow pine shows on either end an average of at least six annual rings per inch and at least one-third summer wood, or else the greater number of rings shows at least one-third summer wood all as measured over the third, fourth, and fifth inches of a radial line extending from the pith. Wide-ringed material, excluded by this rule, is acceptable, provided the amount of summer wood measured as previously specified is at least one-half. "Sound" southern yellow pine includes pieces without any ring or summer wood requirement.

Rectangular Solid Beams

Style of Loading and Support	Breadth of Beam in Inches b	Height of Beam in Inches h	Stress per Sq. In. in Extreme Fibers of Beam f	Length of Beam in Inches l	Load in Pounds W
	Beam fixed at one end, loaded at the other				
	$\frac{6\,lW}{fh^2} = b$	$\sqrt{\frac{6\,lW}{bf}} = h$	$\frac{6\,lW}{bh^2} = f$	$\frac{bfh^2}{6\,W} = l$	$\frac{bfh^2}{6\,l} = W$
	Beam fixed at one end, uniformly loaded				
	$\frac{3\,lW}{fh^2} = b$	$\sqrt{\frac{3\,lW}{bf}} = h$	$\frac{3\,lW}{bh^2} = f$	$\frac{bfh^2}{3\,W} = l$	$\frac{bfh^2}{3\,l} = W$
	Beam supported at both ends, single load in middle				
	$\frac{3\,lW}{2\,fh^2} = b$	$\sqrt{\frac{3\,lW}{2\,bf}} = h$	$\frac{3\,lW}{2\,bh^2} = f$	$\frac{2\,bfh^2}{3\,W} = l$	$\frac{2\,bfh^2}{3\,l} = W$
	Beam supported at both ends, uniformly loaded				
	$\frac{3\,lW}{4\,fh^2} = b$	$\sqrt{\frac{3\,lW}{4\,bf}} = h$	$\frac{3\,lW}{4\,bh^2} = f$	$\frac{4\,bfh^2}{3\,W} = l$	$\frac{4\,bfh^2}{3\,l} = W$
	Beam supported at both ends, single unsymmetrical load				
	$\frac{6\,Wac}{fh^2l} = b$	$\sqrt{\frac{6Wac}{bfl}} = h$	$\frac{6\,Wac}{bh^2l} = f$	$a + c = l$	$\frac{bh^2fl}{6\,ac} = W$
	Beam supported at both ends, two symmetrical loads				
	$\frac{3\,Wa}{fh^2} = b$	$\sqrt{\frac{3\,Wa}{bf}} = h$	$\frac{3\,Wa}{bh^2} = f$	l, any length $\frac{bh^2f}{3\,W} = a$	$\frac{bh^2f}{3\,a} = W$

Round Solid Beams

Style of Loading and Support	Diameter of Beam in Inches d	Stress per Sq. In. in Extreme Fibers of Beam f	Length of Beam in Inches l	Load in Pounds W
	Beam fixed at one end, loaded at the other			
	$\sqrt[3]{\frac{10.18\,lW}{f}}=d$	$\frac{10.18\,lW}{d^3}=f$	$\frac{d^3f}{10.18\,W}=l$	$\frac{d^3f}{10.18\,l}=W$
	Beam fixed at one end, uniformly loaded			
	$\sqrt[3]{\frac{5.092\,Wl}{f}}=d$	$\frac{5.092\,Wl}{d^3}=f$	$\frac{d^3f}{5.092\,W}=l$	$\frac{d^3f}{5.092\,l}=W$
	Beam supported at both ends, single load in middle			
	$\sqrt[3]{\frac{2.546\,Wl}{f}}=d$	$\frac{2.546\,Wl}{d^3}=f$	$\frac{d^3f}{2.546\,W}=l$	$\frac{d^3f}{2.546\,l}=W$
	Beam supported at both ends, uniformly loaded			
	$\sqrt[3]{\frac{1.273\,Wl}{f}}=d$	$\frac{1.273\,Wl}{d^3}=f$	$\frac{d^3f}{1.273\,W}=l$	$\frac{d^3f}{1.273\,l}=W$
	Beam supported at both ends, single unsymmetrical load			
	$\sqrt[3]{\frac{10.18\,Wac}{fl}}=d$	$\frac{10.18\,Wac}{d^3l}=f$	$a+c=l$	$\frac{d^3fl}{10.18\,ac}=W$
	Beam supported at both ends, two symmetrical loads			
	$\sqrt[3]{\frac{5.092\,Wa}{f}}=d$	$\frac{5.092\,Wa}{d^3}=f$	l, any length $\frac{d^3f}{5.092\,W}=a$	$\frac{d^3f}{5.092\,a}=W$

Relation of Depth to Span of a Girder. — When in the design of a girder or cantilever, the specifications do not prescribe a particular depth of girder, the accompanying table of relation between depth and span will be found useful. The table is directly applicable when the girder is symmetrical in section about its neutral axis, and for a modulus of elasticity of 29,000,000, which is very general for structural shapes. The deflections used for calculating the table are those in most general use, being 1 inch per 100 feet span, or $\frac{L}{1200}$, and 2 inches per 100 feet span, or

Relation Between Depth of Girder and Length of Span

(L=length of span in inches)

Conditions	Safe Stress, Pounds per Square Inch	Depth, Inches
Beam supported at ends; load concentrated at middle; deflection, $L \div 1200$ or 1 inch per 100 feet.	10,000	$L \div 14.5$
	12,500	$L \div 11.6$
	16,000	$L \div 9.05$
	20,000	$L \div 7.25$
Beam supported at ends; load concentrated at middle; deflection, $L \div 600$ or 2 inches per 100 feet.	10,000	$L \div 29.0$
	12,500	$L \div 23.2$
	16,000	$L \div 18.1$
	20,000	$L \div 14.5$
Beam supported at ends; load uniformly distributed; deflection, $L \div 1200$ or 1 inch per 100 feet.	10,000	$L \div 11.6$
	12,500	$L \div 9.3$
	16,000	$L \div 7.25$
	20,000	$L \div 5.8$
Beam supported at ends; load uniformly distributed; deflection, $L \div 600$ or 2 inches per 100 feet.	10,000	$L \div 23.2$
	12,500	$L \div 18.6$
	16,000	$L \div 14.5$
	20,000	$L \div 11.6$
Cantilever; load concentrated at end; deflection, $L \div 1200$ or 1 inch per 100 feet.	10,000	$L \div 3.625$
	12,500	$L \div 2.9$
	16,000	$L \div 2.27$
	20,000	$L \div 1.81$
Cantilever; load concentrated at end; deflection, $L \div 600$ or 2 inches per 100 feet.	10,000	$L \div 7.25$
	12,500	$L \div 5.8$
	16,000	$L \div 4.54$
	20,000	$L \div 3.62$
Cantilever; load uniformly distributed; deflection, $L \div 1200$ or 1 inch per 100 feet.	10,000	$L \div 4.83$
	12,500	$L \div 3.87$
	16,000	$L \div 3.03$
	20,000	$L \div 2.42$
Cantilever; load uniformly distributed; deflection, $L \div 600$ or 2 inches per 100 feet.	10,000	$L \div 9.66$
	12,500	$L \div 7.74$
	16,000	$L \div 6.06$
	20,000	$L \div 4.84$

$\frac{L}{600}$, L being in inches. The lower limit is used where stiffness is necessary, and the higher limit where stiffness is not of primary importance.

If the modulus of elasticity is other than that used in these calculations, the divisor of the expression in the last column of the table will be directly proportional to the value of the modulus. For each condition of loading, the divisor is inversely proportional to the assumed safe stresses.

Having arrived at the required depth for a given deflection, it is an easy matter to find a suitable section for the given stress; but in all cases where exact work is required, the actual deflection and stress should be obtained after the design has been completed.

Strength of Channels. — Experiments on standard channels carried out by Bach (published in 1909) show that the regular bending formula for beams freely supported at their ends and loaded in the center gives too high a value for the strength of structural channels. The experiments show that the amount by which the value obtained from the formula is greater than that obtained by experiments, is, for channels 4¾ inches high, 7 per cent; for channels 8¾ inches high, 18 per cent; and for channels 11¾ inches high, 26 per cent. These values are those found when the load is assumed to be applied in the center line of the web of the channel as shown in Fig. 1. If the load is placed along the line of the vertical neutral axis of the channel as shown in Fig. 2, the permissible load according to the beam formula is 10, 25.5 and 34 per cent greater than that shown by the experiments. These experiments, therefore, indicate that when the usual formulas are employed in calculations, for channels or other structural shapes, a liberal factor of safety should be allowed in order to compensate for the difference of the results given by the formula and those of actual experiments. It should be noted that the formula for bending is fully correct whenever the section of the member is such that the load is fully distributed over the whole sectional area, as in a rectangular section, Fig. 3; but in the case of channels as well as many other structural shapes, the load is not, as a rule, properly distributed over the whole section, but stresses certain portions of the section in a higher degree than others.

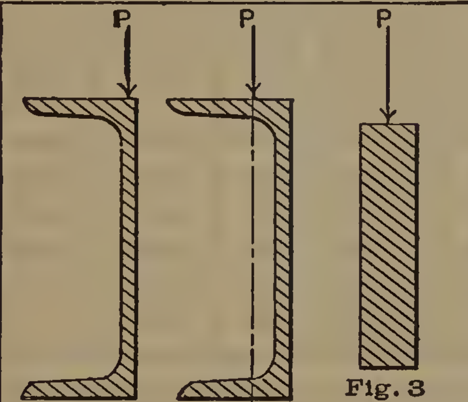


Fig. 1 Fig. 2 Fig. 3

Strength of Columns or Struts. — When a body that has considerable length in proportion to its width, breadth, or diameter is subjected to compression stresses, the ordinary formula for compression is not applicable, because bending stresses are set up on account of the length of the column or strut. The accompanying table gives formulas for calculating the ultimate load in pounds per square inch for columns. These formulas are based on the Rankine formula which is the one generally used by engineers in the design of columns unless the latter are unusually slender. When the length of a compressive member exceeds 8 or 10 times its smallest lateral dimension it is ordinarily known as a column, strut or post. Columns may, for the purpose of analysis, be divided into *short columns* and *long columns*. Short columns fail when the combined bending and compressive stresses reach the yield point of the material, but with long columns, failure occurs by buckling, before the load reaches the elastic limit. The *slenderness ratio* of a column is found by dividing the length by the least radius of gyration (l/r), the length of the column being in inches since the radius of gyration is commonly given in inches. The Rankine formula is generally applied for slenderness ratios between

20 and 100 and sometimes for ratios up to 120. In general, when the value of l/r exceeds 100, columns begin to fail by buckling, although the ratio varies according to the material and the support at the ends, as explained in the following: For ratios of l/r from 20 to about 100, there is little difference in the ultimate loads, especially for columns having fixed, flat, and hinged ends; and the same formulas may be applied when the ratios do not exceed the values given.

Euler's Formulas for Slender Columns. — Euler's formulas (see accompanying table) are commonly applied instead of Rankine's when the slenderness ratio of a column exceeds certain values which depend not only upon the material but upon whether the ends are flat, hinged, rounded, or fixed. According to one series of experiments, Euler's formulas should be used when the values of l/r exceed the following ratios for columns of different materials: Structural steel with flat ends, 195; with hinged ends, 155; round ends, 120. Cast iron with flat ends, 120; hinged ends, 100; round ends, 75. Oak with flat ends, 130. The difference in loading capacities for different values of l/r is shown by the accompanying table, "Ultimate Loads on Columns for Different Ratios of Slenderness." This table represents tests made at the Pencoyd Iron Works on wrought-iron columns.

Ultimate Loads on Columns for Different Ratios of Slenderness

Ratio l/r	Fixed Ends	Flat Ends	Hinged Ends	Round Ends	Ratio l/r	Fixed Ends	Flat Ends	Hinged Ends	Round Ends
20	46,000	46,000	46,000	44,000	260	11,000	9800	6500	3800
60	36,000	36,000	36,000	30,500	300	9,000	7200	5000	2800
100	30,000	29,800	28,000	20,000	340	7,000	5100	4000	2100
140	25,500	23,500	21,000	12,800	380	5,800	3500	3000	1700
180	20,000	16,800	12,800	7,500	420	4,800	2500	2300	1300
220	15,000	12,700	8,800	5,000	440	4,300	2200	2100	....

Eccentrically Loaded Columns. — In the application of the column formulas previously referred to, it is assumed that the action of the load coincides with the axis of the column. If the load is offset relative to the column axis, the column is said to be eccentrically loaded, and its strength is then calculated by using a modification of the Rankine formula, the quantity cz/r^2 being added to the denominator, as shown in the table on the next page. This modified formula is applicable to columns having a slenderness ratio varying from 20 or 30 to about 100.

Pipe Columns. — The allowable compressive stress for steel pipe columns may be determined from the formula:

$$S = 15{,}200 - 58\,L \div R$$

in which S = allowable compressive stress in pounds per square inch; L = length of column in inches; R = radius of gyration in inches. This formula is applicable to steel pipe columns with flat ends. No columns should be used having an unsupported length greater than 120 times its radius of gyration. The formula is based upon the requirements of the New York Building Code.

A similar formula, based upon the Chicago Building Ordinances, is:

$$S = 16{,}000 - 70\,L \div R$$

in which the letters denote the same quantities as in the previous formula.

Flat Stayed Surfaces. — In many cases, large flat areas are held against pressure by stays distributed at regular intervals over the surface. In boiler work, these stays are usually screwed into the plate and the projecting end riveted over to insure

Rankine's and Euler's Formulas for Columns

p = ultimate load in lbs. per square inch; P = total ultimate load in lbs.
S = ultimate compressive strength of material in pounds per square inch;
l = length of column or strut in inches;
r = least radius of gyration in inches; $r^2 = \frac{\text{moment of inertia}}{\text{area of section}}$;
I = least moment of inertia;
E = modulus of elasticity of material;
c = distance in inches from neutral axis of cross-section to side under compression;
z = distance in inches from axis of load to axis coinciding with center of gravity of cross-section.

Rankine's Formulas

Material	Both Ends of Column Fixed	One End Fixed and One End Rounded	Both Ends Rounded
Steel..........	$p = \dfrac{S}{1 + \dfrac{l^2}{25{,}000\ r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{12{,}500\ r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{6250\ r^2}}$
Cast Iron......	$p = \dfrac{S}{1 + \dfrac{l^2}{5000\ r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{2500\ r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{1250\ r^2}}$
Wrought Iron..	$p = \dfrac{S}{1 + \dfrac{l^2}{35{,}000\ r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{17{,}500\ r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{8750\ r^2}}$
Timber........	$p = \dfrac{S}{1 + \dfrac{l^2}{3000\ r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{1500\ r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{750\ r^2}}$

Formulas Modified for Eccentrically Loaded Columns

Material *	Both Ends of Column Fixed	One End Fixed and One End Rounded	Both Ends Rounded
Steel..........	$p = \dfrac{S}{1 + \dfrac{l^2}{25{,}000\ r^2} + \dfrac{cz}{r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{12{,}500\ r^2} + \dfrac{cz}{r^2}}$	$p = \dfrac{S}{1 + \dfrac{l^2}{6250\ r^2} + \dfrac{cz}{r^2}}$

* For other materials such as cast iron, etc., use the Rankine formulas given in the upper table and add to the denominator the quantity $\frac{cz}{r^2}$.

Euler's Formulas for Slender Columns

Both Ends of Column Fixed	One End Fixed and One End Rounded	Both Ends Rounded	One End Fixed and One End Free
$P = \dfrac{4\pi^2 IE}{l^2}$	$P = \dfrac{2\pi^2 IE}{l^2}$	$P = \dfrac{\pi^2 IE}{l^2}$	$P = \dfrac{\pi^2 IE}{4 l^2}$

To find the total safe load for a given section, multiply the value of p (as found by any of the above formulas), by the area of the section and divide by a suitable factor of safety. (See table "General Factors of Safety.")

steam tightness. The U. S. Board of Supervising Inspectors and the American Boiler Makers Association rules give the following formula for flat stayed surfaces:

$$P = \frac{C \times t^2}{S^2}$$

in which P = pressure in pounds per square inch;

C = a constant which equals 112, for plates 7/16 inch and under; 120, for plates over 7/16 inch thick; 140, for plates with stays having a nut and bolt on the inside and outside; and 160, for plates with stays having washers of at least one-half the thickness of the plate, and with a diameter at least one-half of the greatest pitch.

t = thickness of plate in 16ths of an inch (thickness = 7/16, $t = 7$);

S = greatest pitch of stays in inches.

Strength of Flat Plates. — Exact formulas for finding the bending moments of flat plates supported along their edges and subjected to stresses created by pressures normal to their surfaces have not been determined. The formulas given by

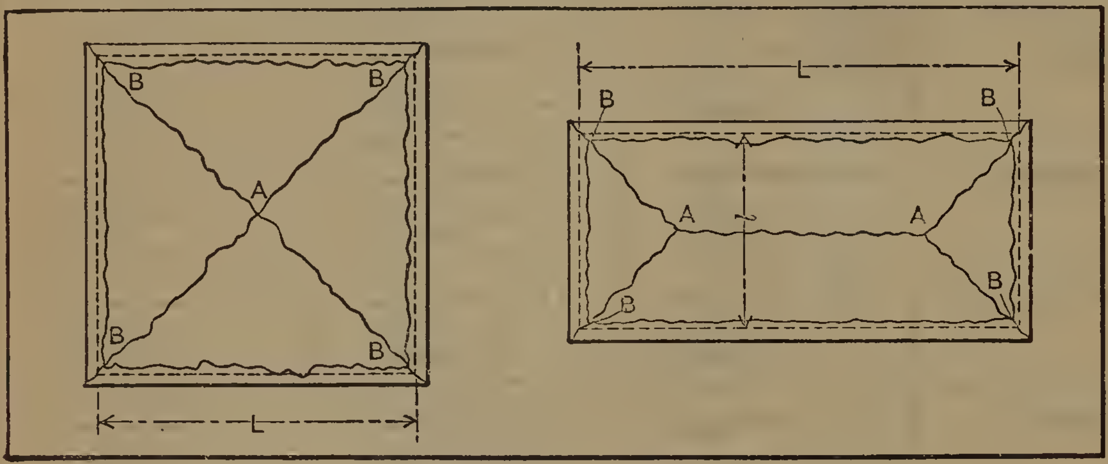


Fig. 1 Fig. 2

different authorities are founded on assumptions and should be considered as approximations only; they should be used with caution, as the results obtained are not likely to be very accurate.

A square cast-iron plate fixed or rigidly held at the edges and loaded with a uniformly distributed load, or a load concentrated at the center, would be likely to fail as shown in Fig. 1. It would first fracture along the diagonal lines from A to B and then fail at or near the fixed edges along lines BB. The plate might also shear off along the edges BB, depending upon the method of loading and the thickness of the plate. If the plate were merely supported along all the four edges, but not rigidly held, it would be likely to fail by breaking along the diagonal lines AB only.

In Fig. 2 is illustrated the probable manner of failure of a flat rectangular plate of cast iron, loaded with a uniformly distributed load. The plate, if secured along all the four edges, would probably fail by fracturing along the center line AA of the long axis of the plate and along the diagonal lines AB, and then fail at or near the edges of the support along the lines BB. If the plate were merely supported along all four edges, it would fail simply by fracturing along the center line AA and the diagonal lines AB. A plate firmly secured at the edges offers greater resistance to the stress created by the load than does a plate merely supported at the edges. While the formulas given in the following are approximate only, it is important that formulas be deduced and used for designs of this character, because they indicate, in a general way, the dimensions required, and the factor of safety assumed will always be taken large enough so that, practically, the approximate nature of the formulas does not detract from their value.

Square and Rectangular Flat Plates. — As the formulas of different authorities vary, formulas giving the higher or safer values have, as a rule, been selected. All of the flat-plate formulas given apply to cast iron, since they are usually required for this material by designers. In the following formulas: W = total load in pounds; P = load per square inch of surface, in pounds; L = span, or distance between supports, in inches; S = fiber stress in pounds per square inch; t = thickness of plate in inches. For rectangular plates, L and l are the long and short spans, or distances between the supported edges, respectively.

1. Square flat plate, supported at all four edges, with a load uniformly distributed over the unsupported surface of the plate. (Based on Grashof's formulas.)

$$W = 3.56\, St^2 \qquad S = 0.28\frac{W}{t^2} = 0.28\frac{PL^2}{t^2}$$

$$L = 1.89\, t\sqrt{\frac{S}{P}} \qquad t = 0.53\sqrt{\frac{W}{S}} = 0.53\, L\sqrt{\frac{P}{S}}$$

2. Square flat plate, firmly secured along all four edges, with a load uniformly distributed over the unsupported surface of the plate. (Based on Unwin's formulas.)

$$W = 4\, St^2 \qquad S = 0.25\frac{W}{t^2} = 0.25\frac{PL^2}{t^2}$$

$$L = 2\, t\sqrt{\frac{S}{P}} \qquad t = 0.5\sqrt{\frac{W}{S}} = 0.5\, L\sqrt{\frac{P}{S}}$$

3. Square flat plate, supported at all four edges, with a load W concentrated at the center. (Based on Grashof's formulas.)

$$W = 0.67\, St^2 \qquad S = 1.5\frac{W}{t^2} \qquad t = 1.23\sqrt{\frac{W}{S}}$$

4. Square flat plate, firmly secured along all four edges, with a load W concentrated at the center. (Based on Grashof's formulas.)

$$W = 0.76\, St^2 \qquad S = 1.31\frac{W}{t^2} \qquad t = 1.14\sqrt{\frac{W}{S}}$$

5. Flat rectangular plate, supported at all four edges, with a load uniformly distributed over the unsupported surface of the plate. (Based on Grashof's formulas.)

$$W = 1.77\frac{St^2(L^2+l^2)}{Ll} \qquad P = 1.77\frac{St^2(L^2+l^2)}{L^2l^2}$$

$$S = 0.56\frac{WLl}{t^2(L^2+l^2)} \qquad t = 0.75\sqrt{\frac{WLl}{S(L^2+l^2)}}$$

6. Flat rectangular plate, firmly secured at all four edges, with a load uniformly distributed over the unsupported area of the plate. (Based on Grashof's formulas.)

$$W = 2.67\frac{St^2(L^2+l^2)}{Ll} \qquad P = 2.67\frac{St^2(L^2+l^2)}{L^2l^2}$$

$$S = 0.375\frac{WLl}{t^2(L^2+l^2)} \qquad t = 0.62\sqrt{\frac{WLl}{S(L^2+l^2)}}$$

7. Flat rectangular plate, supported at all four edges, with a load W concentrated at the center. (Based on Grashof's formulas.)

$$W = 0.33\,\frac{St^2(L^2+l^2)}{Ll} \qquad S = \frac{3\,WLl}{t^2(L^2+l^2)} \qquad t = 1.73\sqrt{\frac{WLl}{S(L^2+l^2)}}$$

8. Flat rectangular plate, firmly secured at all four edges, with a load W concentrated at the center. (Based on Grashof's formulas.)

$$W = 0.38\,\frac{St^2(L^2+l^2)}{Ll} \qquad S = 2.62\,\frac{WLl}{t^2(L^2+l^2)} \qquad t = 1.6\sqrt{\frac{WLl}{S(L^2+l^2)}}$$

Formulas for Circular Flat Plates. — In the following formulas:

W = total load in pounds;
P = load in pounds per square inch;
R = radius of plate, to the supporting edge, in inches;
S = fiber stress in pounds per square inch;
t = thickness of plate in inches;
d = deflection at center of plate in inches;
E = modulus of elasticity.

1. Circular flat plate, supported all around the edge, with a load uniformly distributed over the unsupported area of the plate. (Based on Reuleaux's formulas.)

$$W = 3.14\,St^2 \qquad S = \frac{PR^2}{t^2} = 0.318\,\frac{W}{t^2}$$

$$R = t\sqrt{\frac{S}{P}} \qquad t = R\sqrt{\frac{P}{S}} = 0.56\sqrt{\frac{W}{S}}$$

$$P = \frac{St^2}{R^2} \qquad d = \frac{5\,PR^4}{6\,Et^3} = 0.265\,\frac{WR^2}{Et^3}$$

2. Circular flat plate, firmly secured all around the edge, with a load uniformly distributed over the unsupported area of the plate. (Based on Reuleaux's formulas.)

$$W = 4.7\,St^2 \qquad S = 0.67\,\frac{PR^2}{t^2} = 0.21\,\frac{W}{t^2}$$

$$R = 1.22\,t\sqrt{\frac{S}{P}} \qquad t = 0.81\,R\sqrt{\frac{P}{S}} = 0.46\sqrt{\frac{W}{S}}$$

$$P = 1.5\,\frac{St^2}{R^2} \qquad d = \frac{PR^4}{6\,Et^3} = 0.053\,\frac{WR^2}{Et^3}$$

3. Circular flat plate, supported all around the edge, with a load concentrated at the center of the plate upon a circular area with radius r. (Based on Bach's formulas.)

$$W = 0.7\,\frac{St^2}{1-\dfrac{2\,r}{3\,R}} \qquad S = 1.43\,\frac{W\left(1-\dfrac{2\,r}{3\,R}\right)}{t^2}$$

$$t = 1.2\sqrt{\frac{W\left(1-\dfrac{2\,r}{3\,R}\right)}{S}} \qquad d = 0.5\,\frac{WR^2}{Et^3}$$

4. Circular flat plate, firmly secured all around the edge, with a load concentrated at the center of the plate upon a circular area with radius r. (Based on Grashof's formulas.)

$$W = 2.36 \frac{St^2}{\text{hyp. log} \dfrac{R}{r}} \qquad S = 0.424 \frac{W}{t^2} \text{hyp. log} \frac{R}{r}$$

$$t = 0.65 \sqrt{\frac{W \text{ hyp. log} \dfrac{R}{r}}{S}} \qquad d = 0.48 \frac{WR^2}{Et^3}$$

Strength of Cylinders Subjected to Internal Pressure. — In low-pressure work, the general practice is to make the thickness of the metal equal to the internal diameter in inches times the pressure in pounds per square inch, and this product divided by twice the allowable working stress of the material. To this is added a variable quantity to allow for unsound castings and possible unknown stresses. Hence, if t = thickness in inches; d = inside diameter in inches; P = pressure in pounds per square inch; S = allowable tensile stress in pounds per square inch, then:

$$t = \frac{dP}{2S}$$

To the value of t thus obtained must then be added an amount to allow for variations in the material and possible excessive stresses when the cylinder is in operation.

Find the thickness required for a cast-iron cylinder, 15 inches in diameter (inside), to withstand an internal pressure of 200 pounds per square inch. Assume the allowable working stress for cast iron to be 4000 pounds per square inch. Then:

$$t = \frac{15 \times 200}{2 \times 4000} = \frac{3}{8} \text{ inch.}$$

The material being cast iron, a liberal allowance must be added to this thickness to take care of possible defects in the casting.

The formula given should be used only for low pressures. When the pressures rise, the Barlow formula is preferable. This formula is similar in form to the one already given, but it gives results quite different when applied to tubes and pipes having walls of considerable thickness in proportion to the diameter, because the Barlow formula is expressed in terms of the outside diameter, whereas the formula given above is expressed in terms of the inside diameter. The Barlow formula is:

$$t = \frac{DP}{2S}$$

in which t = thickness in inches; D = outside diameter in inches; P = pressure in pounds per square inch; S = allowable tensile stress in pounds per square inch.

This formula is based on assumptions which cannot be considered as theoretically correct, but the error is on the side of safety, and experiments have proved that of the various formulas proposed for the strength of tubes and pipes subjected to moderate pressures, the Barlow formula gives the most reliable results.

The average ultimate tensile strength of seamless steel tubes may be assumed at 55,000 pounds per square inch; that for butt-welded steel pipe at 40,000; that for lap-welded steel pipe at 50,000; and that for wrought-iron pipe (butt-welded or lap-welded) at 28,000 pounds per square inch.

If seamless steel tubes are assumed to have a strength of 100 per cent, butt-welded steel pipe has a comparative strength of 73 per cent, and lap-welded steel pipe of 92 per cent. From this it will be seen that the strength of a butt-weld is only about 80 per cent of that of a lap-weld. The relative strengths of wrought-iron and steel pipe are as follows: Butt-welded wrought-iron pipe has 70 per cent of the strength of similar butt-welded steel pipe, and lap-welded wrought-iron pipe has 57 per cent of the strength of similar lap-welded steel pipe.

Cylinders Subjected to High Internal Pressure. — For high pressures, Lamé's formula is used. This formula is in its usual form,

$$t = r\left(\sqrt{\frac{S+P}{S-P}} - 1\right)$$

sometimes inconvenient to use. The following forms of the same formula obtained by substitution are often useful:

$$S = P\,\frac{R^2+r^2}{R^2-r^2} \qquad R = r\sqrt{\frac{S+P}{S-P}}$$

$$P = S\,\frac{R^2-r^2}{R^2+r^2} \qquad r = R\sqrt{\frac{S-P}{S+P}}$$

In these formulas:

S = maximum allowable fiber stress per square inch;
R = outer radius of cylinder in inches;
r = inner radius of cylinder in inches;
P = pressure within the cylinder in pounds per square inch;
$t = R - r$ = thickness of cylinder in inches.

A table of ratios of outside radius to inside radius of thick cylinders is given for convenience in calculating the dimensions of cylinders under high internal pressure without the use of the formulas. This table is based on the Lamé formula. As an example of the use of the table, assume that a cylinder of 10 inches inside diameter is to withstand a pressure of 2500 pounds per square inch; the material is cast iron, the allowable stress in this case being 6000 pounds per square inch. To solve the problem, locate the allowable stress per square inch in the left-hand column of the table and the working pressure at the top of the columns. Then find the ratio between the outside and inside radii in the body of the table. In this case, the ratio is 1.558, and hence the outside diameter of the cylinder should be 10×1.558, or about 15⅝ inches.

Unless very high-grade material is used and sound castings assured, cast iron should not be used for pressures exceeding 2000 pounds per square inch. When pressures exceed 2500 pounds per square inch, the packings are likely to leak and the valves and pipe fittings give trouble. It is, therefore, advisable to keep the pressure below this point, if possible. It is well to leave more metal in the bottom of a hydraulic cylinder than is indicated by the results of calculations, because a hole of some size must be cored in the bottom to permit the entrance of a boring bar when finishing the cylinder, and when this hole is subsequently tapped and plugged it often gives trouble if the precaution mentioned is not taken.

For steady or gradually applied stresses, the maximum allowable fiber stress S in the formulas above may be assumed from 3500 to 4000 pounds per square inch for cast iron; from 6000 to 7000 pounds per square inch for brass; and as 12,000 pounds per square inch for steel castings. For intermittent stresses, such as in cylinders for steam and hydraulic work, 3000 pounds per square inch for cast iron; 5000 pounds per square inch for brass; and 10,000 pounds per square inch for steel castings, is ordinarily used. These values give ample factors of safety.

Ratio of Outside Radius to Inside Radius, Thick Cylinders

Allowable Stress in Metal per Sq. In. of Section	Working Pressure in Cylinder, Pounds per Square Inch												
	1000	1500	2000	2500	3000	3500	4000	4500	5000	5500	6000	6500	7000
2,000	1.732	....	....	....	....	....	....	....	....	....	....	....	....
2,500	1.527	2.000	....	....	....	....	....	....	....	....	....	....	....
3,000	1.414	1.732	2.236	....	....	....	....	....	....	....	....	....	....
3,500	1.341	1.581	1.915	2.449	....	....	....	....	....	....	....	....	....
4,000	1.291	1.483	1.732	2.081	2.645	....	....	....	....	....	....	....	....
4,500	1.253	1.414	1.612	1.871	2.236	2.828	....	....	....	....	....	....	....
5,000	1.224	1.362	1.527	1.732	2.000	2.380	3.000	....	....	....	....	....	....
5,500	1.201	1.322	1.464	1.633	1.844	2.121	2.516	3.162	....	....	....	....	....
6,000	1.183	1.291	1.414	1.558	1.732	1.949	2.236	2.645	3.316	....	....	....	....
6,500	....	1.264	1.374	1.500	1.647	1.825	2.049	2.345	2.768	3.464	....	....	....
7,000	....	1.243	1.341	1.453	1.581	1.732	1.914	2.144	2.449	2.886	3.605	....	....
7,500	....	1.224	1.314	1.414	1.527	1.658	1.813	2.000	2.236	2.549	3.000	3.741	....
8,000	....	1.209	1.291	1.381	1.483	1.599	1.732	1.889	2.081	2.323	2.645	3.109	3.872
8,500	....	1.194	1.271	1.354	1.446	1.548	1.666	1.802	1.963	2.160	2.408	2.738	3.214
9,000	....	1.183	1.253	1.330	1.414	1.507	1.612	1.732	1.871	2.035	2.236	2.440	2.828
9,500	....	....	1.235	1.306	1.386	1.472	1.566	1.673	1.795	1.936	2.104	2.309	2.569
10,000	....	....	1.224	1.291	1.362	1.441	1.527	1.623	1.732	1.856	2.000	2.171	2.380
10,500	....	....	1.212	1.274	1.341	1.414	1.493	1.581	1.678	1.789	1.915	2.061	2.236
11,000	....	....	1.201	1.260	1.322	1.390	1.464	1.544	1.633	1.732	1.844	1.972	2.121
11,500	....	....	1.193	1.247	1.306	1.369	1.437	1.511	1.593	1.683	1.784	1.897	2.027
12,000	....	....	1.183	1.235	1.291	1.359	1.414	1.483	1.558	1.640	1.732	1.834	1.949
12,500	....	....	....	1.224	1.277	1.333	1.393	1.457	1.527	1.603	1.687	1.779	1.878
13,000	....	....	....	1.215	1.264	1.318	1.374	1.434	1.500	1.570	1.647	1.732	1.825
13,500	....	....	....	1.206	1.253	1.303	1.357	1.414	1.475	1.541	1.612	1.690	1.775
14,000	....	....	....	1.197	1.243	1.291	1.341	1.395	1.453	1.514	1.581	1.653	1.732
14,500	....	....	....	1.189	1.233	1.279	1.327	1.378	1.432	1.490	1.553	1.620	1.693
15,000	....	....	....	1.183	1.224	1.268	1.314	1.362	1.414	1.469	1.527	1.590	1.658
16,000	....	....	....	1.170	1.209	1.249	1.291	1.335	1.381	1.431	1.483	1.538	1.599

Spherical Shells Subjected to Internal Pressure. — Let:

D = internal diameter of shell in inches;
P = internal pressure in pounds per square inch;
S = safe tensile stress per square inch;
t = the thickness of metal in the shell in inches. Then:

$$P\frac{\pi D^2}{4} = \pi DtS, \quad \text{and} \quad t = \frac{PD}{4S}$$

This formula also applies to hemi-spherical shells, such as the hemi-spherical head of a cylindrical container subjected to internal pressure, etc.

Example: — Find the thickness of metal required in the hemi-spherical end of a cylindrical vessel, 2 feet in diameter, subjected to an internal pressure of 500 pounds per square inch. The material is mild steel and a tensile stress of 10 000 pounds per square inch is allowable.

$$t = \frac{500 \times 2 \times 12}{4 \times 10{,}000} = 0.3 \text{ inch.}$$

If the radius of curvature of the dome head of a boiler or container subjected to internal pressure is made equal to the diameter of the boiler, the thickness of the cylindrical shell and of the spherical head should be made the same. For example, if a boiler is 3 feet in diameter, the radius of curvature of its head should be made 3 feet, if material of the same thickness is to be used and the stresses are to be equal in both the head and cylindrical portion.

Collapsing Pressures of Cylinders and Tubes Subjected to External Pressures. — The following formulas may be used for finding the collapsing pressures of modern lap-welded Bessemer steel tubes:

$$P = 86{,}670\frac{t}{D} - 1386 \quad . \quad . \quad . \quad (1) \qquad\qquad P = 50{,}210{,}000\left(\frac{t}{D}\right)^3 \quad . \quad . \quad . \quad (2)$$

in which P = collapsing pressure in pounds per square inch; D = outside diameter of tube or cylinder in inches; t = thickness of wall in inches.

Formula (1) is for values of P greater than 580 pounds per square inch, and Formula (2) is for values of P less than 580 pounds per square inch. These formulas are substantially correct for all lengths of pipe greater than six diameters between transverse joints that tend to hold the pipe to a circular form. The pressure P found is the actual collapsing pressure, and a suitable factor of safety must be used. Ordinarily, a factor of safety of 5 is sufficient. In cases where there are repeated fluctuations of the pressure, vibration, shocks and other stresses, a factor of safety of from 6 to 12 should be used.

The table "Tubes Subjected to External Pressure" is based upon the requirements of the Steam Boat Inspection Service of the Department of Commerce and Labor and gives the permissible working pressures and corresponding minimum thickness of wall for long, plain, lap-welded and seamless steel flues subjected to external pressure only. The thicknesses in the table have been calculated from the formula:

$$T = \frac{[(F \times P) + 1386]\,D}{86{,}670}$$

in which D = outside diameter of flue or tube in inches; T = thickness of wall in inches; P = working pressure in pounds per square inch; F = factor of safety. The formula is applicable to working pressures greater than 100 pounds per square inch, to outside diameters from 7 to 18 inches, and to temperatures less than 650° F.

The Formulas (1) and (2) given on the preceding page were determined by Prof. R. T. Stewart, Dean of the Mechanical Engineering Department of the University of Pittsburg, in a series of experiments carried out at the plant of the National Tube Co., McKeesport, Pa. These tests occupied a period of four years. A full report of the details of these experiments will be found in a paper presented by Prof. Stewart before the American Society of Mechanical Engineers in May, 1906. The principal conclusions to be drawn from the results of this research may be briefly stated as follows:

The length of tube, between transverse joints tending to hold it to a circular form, has no practical influence upon the collapsing pressure of a commercial lap-welded steel tube, so long as this length is not less than about six times the diameter of the tube.

The apparent fiber stress under which the different tubes failed varied from about 7000 pounds per square inch for the relatively thinnest to 35,000 pounds per square inch for the relatively thickest walls. Since the average yield point of the material tested was 37,000 pounds and the tensile strength 58,000 pounds per square inch, it is evident that the strength of a tube subjected to external fluid collapsing pressure is not dependent alone upon the elastic limit or ultimate strength of the material from which it is made.

Tubes Subjected to External Pressure

Outside Diameter of Tube, Inches	Working Pressure in Pounds per Square Inch						
	100	120	140	160	180	200	220
	Thickness of Tube in Inches. Safety Factor, 5						
7	0.152	0.160	0.168	0.177	0.185	0.193	0.201
8	0.174	0.183	0.193	0.202	0.211	0.220	0.229
9	0.196	0.206	0.217	0.227	0.237	0.248	0.258
10	0.218	0.229	0.241	0.252	0.264	0.275	0.287
11	0.239	0.252	0.265	0.277	0.290	0.303	0.316
12	0.261	0.275	0.289	0.303	0.317	0.330	0.344
13	0.283	0.298	0.313	0.328	0.343	0.358	0.373
14	0.301	0.320	0.337	0.353	0.369	0.385	0.402
15	0.323	0.343	0.361	0.378	0.396	0.413	0.430
16	0.344	0.366	0.385	0.404	0.422	0.440	0.459
17	0.366	0.389	0.409	0.429	0.448	0.468	0.488
18	0.387	0.412	0.433	0.454	0.475	0.496	0.516

Dimensions and Maximum Allowable Pressure of Tubes Subjected to External Pressure

Outside Diam., Inches	Thickness of Material, Inches	Maximum Pressure Allowed, Pounds	Outside Diam., Inches	Thickness of Material, Inches	Maximum Pressure Allowed, Pounds	Outside Diam., Inches	Thickness of Material, Inches	Maximum Pressure Allowed, Pounds
2	0.095	427	3	0.109	327	4	0.134	303
2¼	0.095	380	3¼	0.120	332	4½	0.134	238
2½	0.109	392	3½	0.120	308	5	0.148	235
2¾	0.109	356	3¾	0.120	282	6	0.165	199

RIVETING AND RIVETED JOINTS

Classes of Riveted Joints. — When the plates to be joined by riveting overlap each other and are held together by one or more rows of rivets, a *lap-joint* is formed. In a *butt-joint* the plates are in the same plane and are united by a cover plate or butt strap, which is riveted to each plate. A combination lap-joint consists of a cover plate inside or outside the lap, and three rows of rivets, the central row passing through the two plates and the cover, and having twice as many rivets as the other two rows. The term *single riveting* means one row of rivets in a lap-joint or one row on each side of a butt-joint; *double riveting* means two rows of rivets in a lap-joint or two rows on each side of the joint in butt riveting. Joints are also triple and quadruple riveted.

Pitch of Rivets. — The pitch is the distance from center to center of adjacent rivets. The pitch of rivets should be as large as possible without impairing the tightness of the joint when under pressure. For single-riveted lap-joints in the circular seams of boilers which have double-riveted longitudinal lap-joints:

$$\text{pitch} = d \times 2.25 = t \times 5, \text{ approximately,}$$

in which d = the actual diameter of rivet (in parallel hole); t = thickness of plate.

For double-riveted lap-joints:

$$\text{pitch} = 8\,t$$

The following formulas for determining the pitch are given by Unwin:

For single-riveted joints in single shear (mild steel):

$$\text{pitch} = 0.644\,\frac{d^2}{t} + d.$$

For single-riveted joints in double shear:

$$\text{pitch} = 1.13\,\frac{d^2}{t} + d.$$

For double-riveted joints in single shear (mild steel):

$$\text{pitch} = 1.288\,\frac{d^2}{t} + d.$$

For double-riveted joints in double shear:

$$\text{pitch} = 2.26\,\frac{d^2}{t} + d.$$

For triple-riveted joints with rivets in single shear (mild steel):

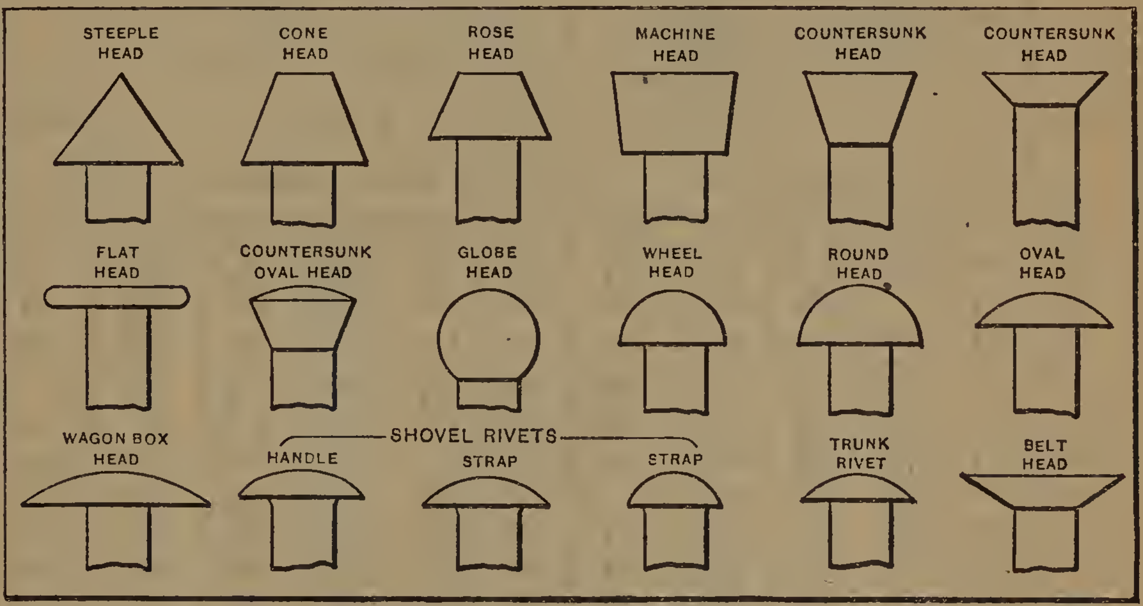
$$\text{pitch} = 1.93\,\frac{d^2}{t} + d.$$

For triple-riveted joints with rivets in double shear:

$$\text{pitch} = 3.30\,\frac{d^2}{t} + d.$$

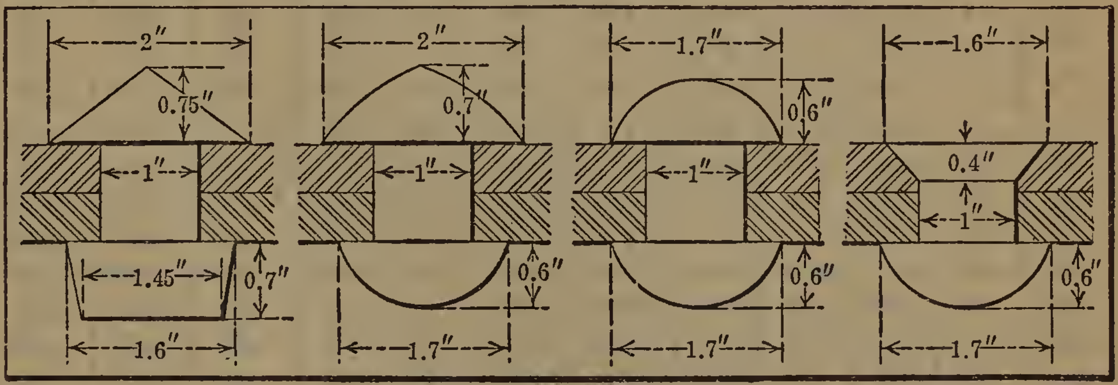
In the foregoing formulas, d = diameter of the driven rivet.

To secure a joint of maximum strength, the breadth of lap must be such as to prevent it from breaking zigzag. Tests have demonstrated that rupture is equally probable through a diagonal as through a transverse line, unless the net diagonal section exceeds the net section along the transverse line by 30 to 35 per cent. This corresponds to a diagonal pitch of $\frac{2}{3}P + \frac{d}{3}$, in which P is the straight pitch, and d, the diameter of the rivet hole. A general rule for the pitch between rows in staggered riveting is as follows: The pitch between rows should equal one-half the pitch of the rivets in a row, plus ¼ of the diameter of the rivet holes. The distance from the edge of the rivet hole to the edge of the plate should never be less than the rivet diameter.



Different Types of Rivet Heads

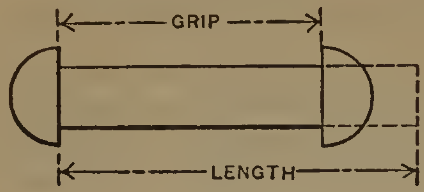
Rivet Diameter. — Rivet diameters for plates of given thickness range in practice from $d = 1.2\sqrt{t}$ to $d = 1.4\sqrt{t}$, in which d = diameter of the rivet and t = thickness of plate. The larger size is preferable for steel and single-riveted joints, and the smaller for iron and multiple-riveted joints.

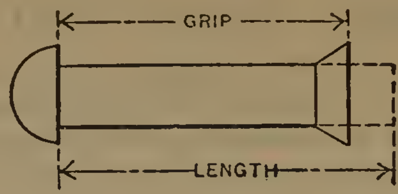


Proportions of Rivet Heads

Length and Proportions of Rivets. — In order to form the head and fill the clearance space in the rivet hole, the rivet should have a length in excess of the thickness of the plate equal to about three-fourths the diameter for countersunk head, and from 1.3 to 1.7 times the diameter for ordinary riveting. (See table, "Rivet Lengths for Forming Round and Countersunk Heads.") The proportions of rivet heads are given in the accompanying illustration (Unwin). The dimensions are given in terms of the diameter and represent the average conditions, the sizes varying more or less in different shops.

Rivet Lengths for Forming Round and Countersunk Heads

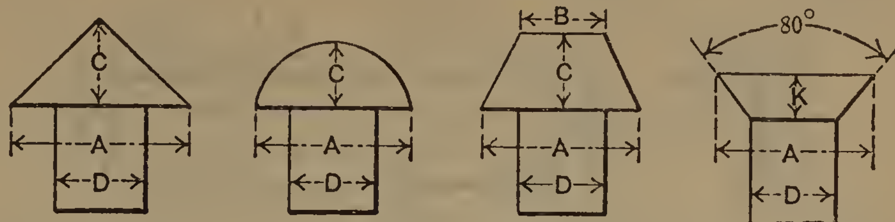




Grip in Inches	Round Head					Countersunk Head					Grip in Inches
	Diameter in Inches					Diameter in Inches					
	½	⅝	¾	⅞	1	½	⅝	¾	⅞	1	
	Length in Inches					Length in Inches					
½	1½	1¾	1⅞	2	2⅛	1⅛	1¼	1¼	1⅜	1⅜	½
⅝	1⅝	1⅞	2	2⅛	2¼	1¼	1⅜	1⅜	1½	1½	⅝
¾	1¾	2	2⅛	2¼	2⅜	1⅜	1½	1½	1⅝	1⅝	¾
⅞	1⅞	2⅛	2¼	2⅜	2½	1½	1⅝	1⅝	1¾	1¾	⅞
1	2	2¼	2⅜	2½	2⅝	1⅝	1¾	1¾	1⅞	1⅞	1
1⅛	2⅛	2⅜	2½	2⅝	2¾	1¾	1⅞	1⅞	2	2	1⅛
1¼	2¼	2½	2⅝	2¾	2⅞	1⅞	2	2	2⅛	2⅛	1¼
1⅜	2⅜	2⅝	2¾	2⅞	3	2	2⅛	2⅛	2¼	2¼	1⅜
1½	2⅝	2⅞	3	3⅛	3¼	2⅛	2¼	2⅜	2⅜	2½	1½
1⅝	2¾	3	3⅛	3¼	3⅜	2¼	2⅜	2½	2½	2⅝	1⅝
1¾	2⅞	3⅛	3¼	3⅜	3½	2⅜	2½	2⅝	2⅝	2¾	1¾
1⅞	3	3¼	3⅜	3½	3⅝	2½	2⅝	2¾	2¾	2⅞	1⅞
2	3⅛	3⅜	3½	3⅝	3¾	2⅝	2¾	2⅞	2⅞	3	2
2⅛	3¼	3½	3⅝	3¾	3⅞	2¾	2⅞	3	3	3⅛	2⅛
2¼	3⅜	3⅝	3¾	3⅞	4	2⅞	3	3⅛	3⅛	3¼	2¼
2⅜	3½	3¾	3⅞	4	4⅛	3	3⅛	3¼	3¼	3⅜	2⅜
2½	3⅝	3⅞	4	4⅛	4¼	3⅛	3¼	3⅜	3⅜	3½	2½
2⅝	3¾	4	4⅛	4¼	4⅜	3¼	3⅜	3½	3½	3⅝	2⅝
2¾	3⅞	4⅛	4¼	4⅜	4½	3⅜	3½	3⅝	3⅝	3¾	2¾
2⅞	4	4¼	4⅜	4½	4⅝	3½	3⅝	3¾	3¾	3⅞	2⅞
3	4¼	4½	4⅝	4¾	4⅞	3¾	3⅞	3⅞	4	4⅛	3
3⅛	4⅜	4⅝	4¾	4⅞	5	3⅞	4	4	4⅛	4¼	3⅛
3¼	4½	4¾	4⅞	5	5⅛	4	4⅛	4⅛	4¼	4⅜	3¼
3⅜	4⅝	4⅞	5	5⅛	5¼	4⅛	4¼	4¼	4⅜	4½	3⅜
3½	4¾	5	5⅛	5¼	5⅜	4¼	4⅜	4⅜	4½	4⅝	3½
3⅝	4⅞	5⅛	5¼	5⅜	5½	4⅜	4½	4½	4⅝	4¾	3⅝
3¾	5	5¼	5⅜	5½	5⅝	4½	4⅝	4⅝	4¾	4⅞	3¾
3⅞	5⅛	5⅜	5½	5⅝	5¾	4⅝	4¾	4¾	4⅞	5	3⅞
4	5¼	5½	5⅝	5¾	5⅞	4¾	4⅞	4⅞	5	5⅛	4
4⅛	5⅜	5⅝	5¾	5⅞	6	4⅞	5	5	5⅛	5¼	4⅛
4¼	5½	5¾	5⅞	6	6⅛	5	5⅛	5⅛	5¼	5⅜	4¼
4⅜	5⅝	5⅞	6	6⅛	6¼	5⅛	5¼	5¼	5⅜	5½	4⅜
4½	5¾	6	6¼	6⅜	6½	5¼	5⅜	5½	5½	5⅝	4½
4⅝	6	6¼	6⅜	6½	6⅝	5½	5⅝	5⅝	5⅝	5¾	4⅝
4¾	6⅛	6⅜	6½	6⅝	6¾	5⅝	5¾	5¾	5¾	5⅞	4¾
4⅞	6¼	6½	6⅝	6¾	6⅞	5¾	5⅞	5⅞	5⅞	6	4⅞
5	6⅜	6⅝	6¾	6⅞	7	5⅞	6	6	6	6⅛	5

Rivet Head Dimensions

(Adopted by American Boiler Manufacturers' Association)



$A = 2\,D$, $C = D$

$A = 1.75\,D$, $C = 0.75\,D$

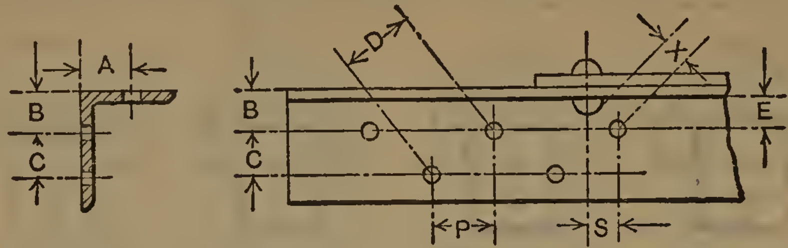
$A = 1.75\,D$, $B = 0.9375\,D$, $C = 0.875\,D$

$A = 1.839\,D$, $K = 0.5\,D$

Diameter, Ins.	Steeple		Button or Round		Cone			Countersunk	
D	*A*	*C*	*A*	*C*	*A*	*B*	*C*	*A*	*K*
1/2	1	1/2	7/8	3/8	7/8	15/32	7/16	15/16	1/4
9/16	1 1/8	9/16	63/64	27/64	63/64	17/32	31/64	1 1/32	9/32
5/8	1 1/4	5/8	1 3/32	15/32	1 3/32	19/32	35/64	1 5/32	5/16
11/16	1 3/8	11/16	1 13/64	33/64	1 13/64	41/64	39/64	1 17/64	11/32
3/4	1 1/2	3/4	1 5/16	9/16	1 5/16	45/64	21/32	1 3/8	3/8
13/16	1 5/8	13/16	1 27/64	39/64	1 27/64	49/64	23/32	1 1/2	13/32
7/8	1 3/4	7/8	1 17/32	21/32	1 17/32	13/16	49/64	1 19/32	7/16
15/16	1 7/8	15/16	1 41/64	45/64	1 41/64	7/8	13/16	1 23/32	15/32
1	2	1	1 3/4	3/4	1 3/4	15/16	7/8	1 27/32	1/2
1 1/16	2 1/8	1 1/16	1 55/64	51/64	1 55/64	1	15/16	1 61/64	17/32
1 1/8	2 1/4	1 1/8	1 31/32	27/32	1 31/32	1 1/16	63/64	2 1/16	9/16
1 3/16	2 3/8	1 3/16	2 5/64	57/64	2 5/64	1 7/64	1 1/32	2 3/16	19/32
1 1/4	2 1/2	1 1/4	2 3/16	15/16	2 3/16	1 11/64	1 3/32	2 19/64	5/8
1 3/8	2 3/4	1 3/8	2 13/32	1 1/32	2 13/32	1 9/32	1 13/64	2 17/32	11/16
1 1/2	3	1 1/2	2 5/8	1 1/8	2 5/8	1 13/32	1 5/16	2 3/4	3/4

General Rules for Rivet Spacing.—The following rules for rivet spacing (given by the Cambria Steel Co.) apply to bridge and structural work. The minimum center-to-center distance or pitch should not be less than three times the rivet diameter. In bridge work, the pitch should not exceed six inches, or sixteen times the thickness of the thinnest outside plate, except in special cases. The distance between the edge of any piece and the center of the rivet hole should not be less than 1 1/4 inch for 3/4- and 7/8-inch rivets, except in bars less than 2 1/2 inches wide; when practicable, this distance should be at least two rivet diameters for all sizes and should not exceed eight times the plate thickness. For flanges of girders and chords carrying floors, the pitch should not exceed four inches. For plates in compression, the pitch in the direction of the line of tress should not exceed sixteen times the thickness of the plate, and the pitch in a direction at right angles to the line of stress should not exceed thirty-two times the thickness, except for cover plates or top chords and end posts, in which the pitch should not exceed forty times the thickness. When rivets are adjacent to the corners of angles, etc., the space between the rivet center and the side of the adjacent leg of the angle, should not be less than one-half the diameter of the head plus 3/8 inch, for clearance. When there is a row of rivets in the adjacent side, the 3/8-inch clearance should be measured from the rivet heads.

Rivet Spacing for Angles



Length of Leg	*A*	*B*	*C*	Max. Rivet Diam.	Length of Leg	*A*	Max. Rivet Diam.
8	5	3	3	1	3	1¾	¾
7	4½	2½	3	1	2½	1⅜	¾
6	4	2½	2¼	⅞	2¼	1¼	⅝
5	3	2	1¾	⅞	2	1⅛	½
4	2½	1¾	1	⅞	1¾	1	½
3½	2	......	......	⅞	1½	⅞	⅜

Distance *D* for Varying Values of *P* and *C*

C	Pitch of Rivets = *P*											
	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½
1	1½	1⅝	1 11/16	1 13/16	1⅞	2	2⅛	2¼	2 5/16	2 7/16	2 9/16	2 11/16
1¼	1 11/16	1¾	1⅞	1 15/16	2 1/16	2⅛	2¼	2⅜	2 7/16	2 9/16	2 11/16	2 13/16
1½	1⅞	1 15/16	2	2⅛	2 3/16	2 5/16	2⅜	2½	2⅝	2 11/16	2 13/16	2 15/16
1¾	2 1/16	2⅛	2 3/16	2 5/16	2⅜	2 7/16	2 9/16	2⅝	2¾	2⅞	2 15/16	3 1/16
2	2 5/16	2⅜	2 7/16	2½	2 9/16	2⅝	2¾	2 13/16	2 15/16	3	3⅛	3 3/16
2¼	2½	2 9/16	2⅝	2 11/16	2¾	2⅞	2 15/16	3	3 1/16	3 3/16	3¼	3⅜
2½	2¾	2 13/16	2⅞	2 15/16	3	3 1/16	3⅛	3 3/16	3¼	3⅜	3 7/16	3 9/16

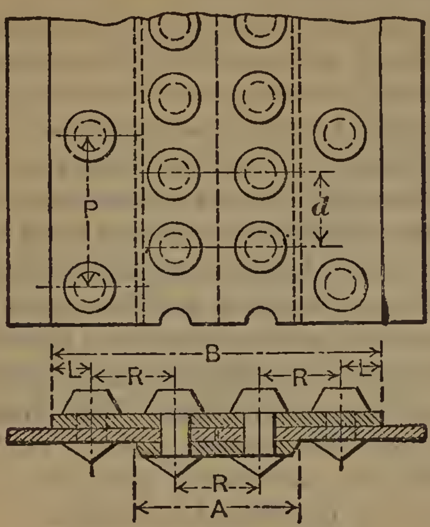
Values of *D* below or to the right of upper zigzag line are large enough for ¾-inch rivets. Values below or to the right of lower zigzag line are large enough for ⅞-inch rivets.

Required Clearance for Driving

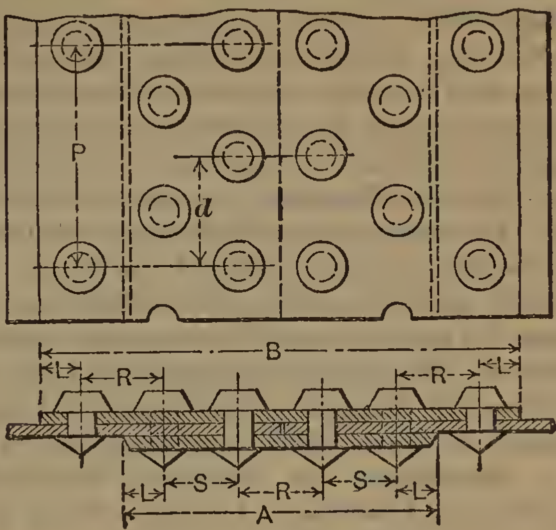
E	*S*		*E*	*S*		*E*	*S*	
	¾ Rivet	⅞ Rivet		¾ Rivet	⅞ Rivet		¾ Rivet	⅞ Rivet
1⅛	1⅛	1⅜	1⅜	¾	1⅛	1⅝	0	1 1/16
1 3/16	1 1/16	1 5/16	1 7/16	⅝	1	1 11/16	0	½
1¼	15/16	1¼	1½	7/16	15/16	1¾	0	0
1 5/16	⅞	1 3/16	1 9/16	0	13/16	......	.....	.....

X = 1 inch for ¾-inch rivets; *X* = 1 3/32 inch for ⅞-inch rivets.

Proportions of Double- and Triple-riveted Butt-joints



DOUBLE-RIVETED BUTT-JOINT



TRIPLE-RIVETED BUTT-JOINT

Double-riveted Butt-joint

Thickness of Shell Plate	Thickness of Cover Plates	Diameter of Rivets	*A*	*B*	*L*	*R*	*p*	*P*	Efficiency, Per Cent
5/16	5/16	5/8	4¼	8½	1 1/16	2⅛	1⅝	3¼	80
⅜	⅜	¾	5	10	1¼	2½	2	4	81
7/16	7/16	⅞	5¾	11½	1 7/16	2⅞	2⅜	4¾	81
½	½	15/16	6	12	1½	3	2⅝	5¼	82

Triple-riveted Butt-joint

Thickness of Shell Plate	Thickness of Cover Plates	Diameter of Rivets	*A*	*B*	*L*	*R*	*S*	*p*	*P*	Efficiency, Per Cent
5/16	¼	5/8	8	12¼	1 1/16	2⅛	1⅞	2¾	5½	88
⅜	5/16	¾	9¼	14¼	1¼	2½	2⅛	3⅛	6¼	90
7/16	⅜	⅞	10½	16¼	1 7/16	2⅞	2⅜	3½	7	90
½	7/16	15/16	11	17	1½	3	2½	3¾	7½	88

Double- and Triple-riveted Butt-joints. — The types of joints most commonly used for the longitudinal seams in boiler work are the double-riveted and triple-riveted butt-joints, with double covering strips. Quadruple-riveted joints are also used to a considerable extent for high-pressure work, these joints having a somewhat higher efficiency than triple-riveted joints. Lap-joints are no longer used in the best class of work for longitudinal seams, as experience has shown that they become weakened by continued use and are liable to fracture. Single-riveted lap-joints are sufficiently strong for the girth seams, as the pressure exerted in this direction is only one-half of that carried by the longitudinal seams. (See table "Proportions of Double- and Triple-riveted Butt-joints.")

Rivet Steel. — Boiler rivet steel, according to the standard specifications of the American Society for Testing Materials, should be made by the open-hearth process and have the following physical properties: Tensile strength, from 45,000 to 55,000 pounds per square inch; minimum yield point, one-half the tensile strength; minimum percentage of rivet elongation for a length not less than four times the rivet diameter, 1,500,000 ÷ tensile strength, but not to exceed 30 per cent. The rivet shank should bend cold through 180 degrees or flat on itself without cracking on the outside of the bend. The rivet should withstand this same test after being heated to a light cherry red as seen in the dark (not less than 1200 degrees F.) and quenched in water the temperature of which is between 80 and 90 degrees F. The rivet head should flatten, while hot, to a diameter two and one-half times the diameter of the shank without cracking at the edges. The chemical composition of the rolled bar should be as follows: Manganese, 0.30 to 0.50 per cent; phosphorus, not over 0.04; and sulphur, not over 0.045 per cent. The analysis should be from a test ingot taken during the pouring of each melt.

Efficiency and Working Strength of Riveted Joints (Unwin)

Plate Thickness	Normal Rivet Diam., Inches	Diam. Driven Rivets, Inches	Rivets in Single Shear		Rivets in Double Shear		Working Strength per Ft. of Joint, Tons	
			Pitch p, Inches	Efficiency η	Pitch p, Inches	Efficiency η	Single Shear	Double Shear
			Single-riveted Steel Joints					
5/16	11/16	0.72	1.79	0.60	2.56	0.72	14.7	17.6
3/8	3/4	0.78	1.82	0.57	2.58	0.70	16.7	20.5
7/16	13/16	0.85	1.91	0.56	2.68	0.68	19.2	23.0
1/2	7/8	0.92	2.01	0.55	2.80	0.67	21.5	26.1
5/8	15/16	0.98	2.00	0.50	2.70	0.64	24.5	31.1
3/4	1 1/8	1.17	2.35	0.50	3.24	0.64	29.3	37.4
7/8	1 3/16	1.23	2.46	0.50	3.18	0.62	34.1	42.3
1	1 5/16	1.36	2.72	0.50	3.45	0.61	39.0	47.6
			Double-riveted Joints — Mild Steel					
5/16	11/16	0.72	2.83	0.74	4.44	0.84	18.1	21.0
3/8	3/4	0.78	2.85	0.73	4.44	0.82	21.4	24.0
7/16	13/16	0.85	2.96	0.71	4.57	0.81	24.3	27.5
1/2	7/8	0.92	3.08	0.70	4.73	0.81	27.3	31.6
5/8	15/16	0.98	2.95	0.67	4.46	0.78	32.6	38.0
3/4	1 1/16	1.10	3.16	0.65	4.73	0.77	38.0	45.0
7/8	1 1/8	1.17	3.17	0.63	4.71	0.75	43.0	50.2
1	1 1/4	1.30	3.46	0.62	5.11	0.75	48.4	58.5
1 1/8	1 9/32	1.33	3.33	0.60	4.87	0.73	52.6	64.0
1 1/4	1 11/32	1.40	3.40	0.59	4.94	0.72	57.5	70.2

Let p = pitch of rivets; d = diameter of driven rivets; η = efficiency or ratio of strength of joint to strength of solid plate; then: $\eta = \frac{p-d}{p}$.

Riveting Pressures. — The pressures required for hot riveting ordinarily range from 75,000 to 150,000 pounds per square inch of rivet area, depending upon the class of work and the relation between the rivet diameter and the total plate thickness. The data in the table "Pressures for Hot Riveting" is from R. D. Wood & Co. It is assumed that the rivet passes through only two thicknesses of plate having a maximum total thickness not much in excess of the rivet diameter. As the total plate thickness increases beyond the rivet diameter, the riveting pressure increases approximately in proportion to the square root of the thickness. Thus, if the total thickness of the plate is four times the rivet diameter, twice the riveting pressure given in the table would be required in order to fill the rivet hole thoroughly and do good work. Double the thickness of plate would increase the necessary power about 40 per cent. Approximately four or five times as much power is required to drive rivets cold as to drive them hot. Thus a machine which will drive ¾-inch hot rivets will usually drive ⅜-inch cold rivets.

Pressures for Hot Riveting

Rivet Diameter	Class of Work and Pressure in Tons			Rivet Diameter	Class of Work and Pressure in Tons		
	Boiler	Tank	Girder		Boiler	Tank	Girder
½	20	15	9	1⅛	75	60	38
⅝	25	18	12	1¼	100	70	45
¾	33	22	15	1½	125	85	60
⅞	45	30	22	1¾	150	100	75
1	60	45	30	....	......	......	......

Loss of Strength when Holes are not Reamed. — When holes are punched in heavy steel plates there is considerable loss of strength unless the holes are reamed after punching. Annealing after punching also restores the strength. The loss in strength due to punching varies from about 10 per cent to 30 per cent for plates varying from ¼ to ¾ inch thick. The loss in case of thin plates is very slight. When holes are punched, instead of being drilled, usually the diameter should be increased from 1/16 to ⅛ inch by reaming, in order to remove the inferior metal around the punched hole. According to Navy Department specifications all holes in boiler plates must be drilled with the plates in place.

Loads at which Slipping Occurs in Riveted Joints. — Owing to the contraction of rivets on cooling, the plates are drawn together tightly; spaces are also left between the rivets and the holes so that it is possible for the plates to slip before shearing the rivets. The load required to overcome the frictional resistance between the plates varies (according to Bach) from 14,000 to 30,000 pounds per square inch of rivet area at each pair of surfaces in contact. It is the practice in Europe to design boiler joints with reference to the resistance to slipping, as any appreciable movement of such joints will cause leakage.

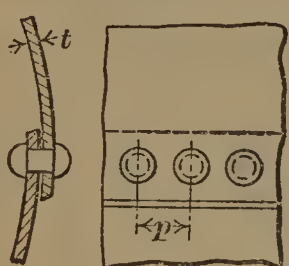
Elastic Limit of Riveted Boiler Joints. — Riveted boiler joints should cease to be steam-tight for some time before the internal pressure is equal to the elastic limit of the plate. If a boiler were stretched beyond the elastic limit of the material, the rivet holes would become stretched and the joints of the plates would be disturbed, resulting in large leakage from the rivet holes and seams. The elastic limit of riveted joints for the best quality of mild steel varies from 32,000 to 34,000 pounds per square inch and for an ordinary quality of mild steel, from 28,000 to 30,000 pounds per square inch. The elastic limit for the best quality of wrought iron varies from 24,000 to 26,000 pounds per square inch and for an ordinary quality, from 20,000 to 22,000 pounds per square inch.

Failure of Riveted Joints

A riveted joint may fail by shearing the rivets, tearing the plate between the rivets, crushing the rivets or plate, or by a combination of two or more of the foregoing causes. To determine the efficiency of a riveted joint, first calculate the breaking strength by the different ways in which it may fail. That method of failure giving the least result will show the actual strength of the joint. If this equals S_r and S equals the tensile strength of the solid plate, then the efficiency $= \frac{S_r}{S}$. In the following formulas, let,

d = diameter of rivets;
t = thickness of plate;
t_c = thickness of cover plates;
p = pitch of inner row of rivets;
P = pitch of outer row of rivets;
S = shearing strength of rivets;
T = tensile strength of plate;
C = crushing strength of rivets.

(All dimensions in inches; all stresses in pounds per square inch.)

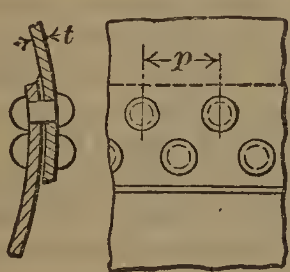


For Single-riveted Lap-joint

(1) Resistance to shearing one rivet $= \frac{\pi d^2}{4} S$

(2) Resistance to tearing plate between rivets $= (p - d)\,tT$

(3) Resistance to crushing rivet or plate $= dtC$

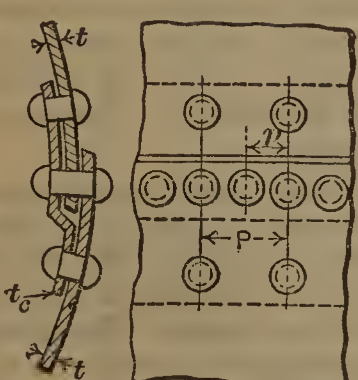


Double-riveted Lap-joint

(1) Resistance to shearing two rivets $= \frac{2\pi d^2}{4} S$

(2) Resistance to tearing between two rivets $= (p - d)\,tT$

(3) Resistance to crushing in front of two rivets $= 2\,dtC$



Single-riveted Lap-joint with Inside Cover Plate

(1) Resistance to tearing between outer row of rivets $= (P - d)\,tT$

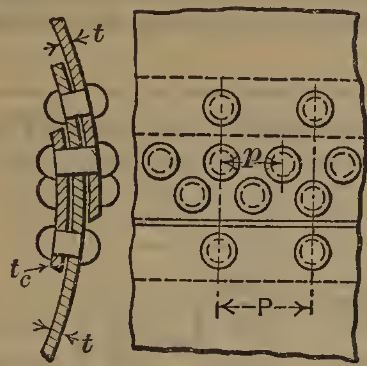
(2) Resistance to tearing between inner row of rivets, and shearing outer row of rivets $= (P - 2\,d)\,tT + \frac{\pi d^2}{4} S$

(3) Resistance to shearing three rivets $= \frac{3\,\pi d^2}{4} S$

(4) Resistance to crushing in front of three rivets $= 3\,t\,dC$

(5) Resistance to tearing at inner row of rivets, and crushing in front of one rivet in outer row $= (P - 2\,d)\,tT + t\,dC$

Failure of Riveted Joints



Double-riveted Lap-joint with Inside Cover Plate

(1) Resistance to tearing at outer row of rivets $= (P - d)\,tT$

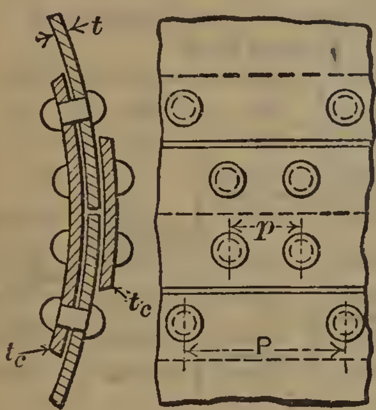
(2) Resistance to shearing four rivets $= \dfrac{4\,\pi d^2}{4} S$

(3) Resistance to tearing at inner row and shearing outer row of rivets

$$= (P - 1\tfrac{1}{2}\,d)\,tT + \frac{\pi d^2}{4} S$$

(4) Resistance to crushing in front of four rivets $= 4\,t\,dC$

(5) Resistance to tearing at inner row of rivets, and crushing in front of one rivet $= (P - 1\tfrac{1}{2}d)tT + t\,dC$



Double-riveted Butt-joint

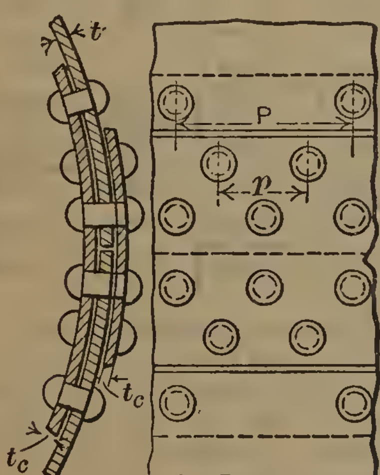
(1) Resistance to tearing at outer row of rivets $= (P - d)\,tT$

(2) Resistance to shearing two rivets in double shear and one in single shear $= \dfrac{5\,\pi d^2}{4} S$

(3) Resistance to tearing at inner row of rivets and shearing one rivet of the outer row $= (P - 2\,d)\,tT + \dfrac{\pi d^2}{4} S$

(4) Resistance to crushing in front of three rivets $= 3\,t\,dC$

(5) Resistance to tearing at inner row of rivets, and crushing in front of one rivet in outer row $= (P - 2\,d)\,tT + t\,dC$



Triple-riveted Butt-joint

(1) Resistance to tearing at outer row of rivets $= (P - d)\,tT$

(2) Resistance to shearing four rivets in double shear and one in single shear $= \dfrac{9\,\pi d^2}{4} S$

(3) Resistance to tearing at middle row of rivets and shearing one rivet

$$= (P - 2\,d)\,tT + \frac{\pi d^2}{4} S$$

(4) Resistance to crushing in front of four rivets and shearing one rivet $= 4\,dtC + \dfrac{\pi d^2}{4} S$

(5) Resistance to crushing in front of five rivets $= 4\,dtC + dt_cC$

Crushing Strength of Rivets. — The crushing strength of rivets and plates, in joints that fail by crushing, is found by experiment to be high and irregular. In some cases it has amounted to 150,000 pounds per square inch; in a few tests it has been less than 85,000 pounds per square inch. A value of 95,000 pounds may be used with safety for general calculations.

Shearing Value of Rivets — Bearing Value of Riveted Plates

All bearing values above or to the right of the upper zigzag lines are greater than double shear. Values between the upper and lower zigzag lines are less than double, and greater than single, shear. Values below and to the left of the lower zigzag lines are less than single shear.

Diam. of Rivet, Inches		Area, Square Inch	Single Shear at 6000 Pounds	Bearing Value for Different Thicknesses of Plate in Inches at 12,000 Pounds per Square Inch				
Fraction	Decimal			¼	5/16	⅜	7/16	½
⅜	0.375	0.1104	660	1130	1410	1690	.....	.....
½	0.500	0.1963	1180	1500	1880	2250	2630	3000
⅝	0.625	0.3068	1840	1880	2340	2810	3280	3750
¾	0.750	0.4418	2650	2250	2810	3380	3940	4500
⅞	0.875	0.6013	3610	2630	3280	3940	4590	5250
1	1.000	0.7854	4710	3000	3750	4500	5250	6000

Fraction	Decimal	Area, Square Inch	Single Shear at 7500 Pounds	Bearing Value at 15,000 Pounds per Square Inch				
				¼	5/16	⅜	7/16	½
⅜	0.375	0.1104	830	1410	1760	2110	.....	.....
½	0.500	0.1963	1470	1880	2340	2810	3280	3750
⅝	0.625	0.3068	2300	2340	2930	3520	4100	4690
¾	0.750	0.4418	3310	2810	3520	4220	4920	5630
⅞	0.875	0.6013	4510	3280	4100	4920	5740	6560
1	1.000	0.7854	5890	3750	4690	5620	6560	7500

Fraction	Decimal	Area, Square Inch	Single Shear at 10,000 Pounds	Bearing Value at 20,000 Pounds per Square Inch				
				¼	5/16	⅜	7/16	½
⅜	0.375	0.1104	1100	1880	2340	2810	.....	.....
½	0.500	0.1963	1960	2500	3130	3750	4380	5000
⅝	0.625	0.3068	3070	3130	3910	4690	5470	6250
¾	0.750	0.4418	4420	3750	4690	5630	6560	7500
⅞	0.875	0.6013	6010	4380	5470	6570	7660	8750
1	1.000	0.7854	7850	5000	6250	7500	8750	10000

Fraction	Decimal	Area, Square Inch	Single Shear at 12,000 Pounds	Bearing Value at 25,000 Pounds per Square Inch				
				¼	5/16	⅜	7/16	½
⅜	0.375	0.1104	1320	2350	2930	3520	.....	.....
½	0.500	0.1963	2360	3130	3910	4690	5470	6250
⅝	0.625	0.3068	3680	3910	4880	5860	6840	7810
¾	0.750	0.4418	5300	4690	5860	7030	8210	9380
⅞	0.875	0.6013	7220	5470	6840	8210	9580	10940
1	1.000	0.7854	9430	6250	7820	9380	10940	12500

Shearing Value of Rivets — Bearing Value of Riveted Plates

All bearing values above or to the right of the upper zigzag lines are greater than double shear. Values between the upper and lower zigzag lines are less than double, and greater than single, shear. Values below and to the left of the lower zigzag lines are less than single shear.

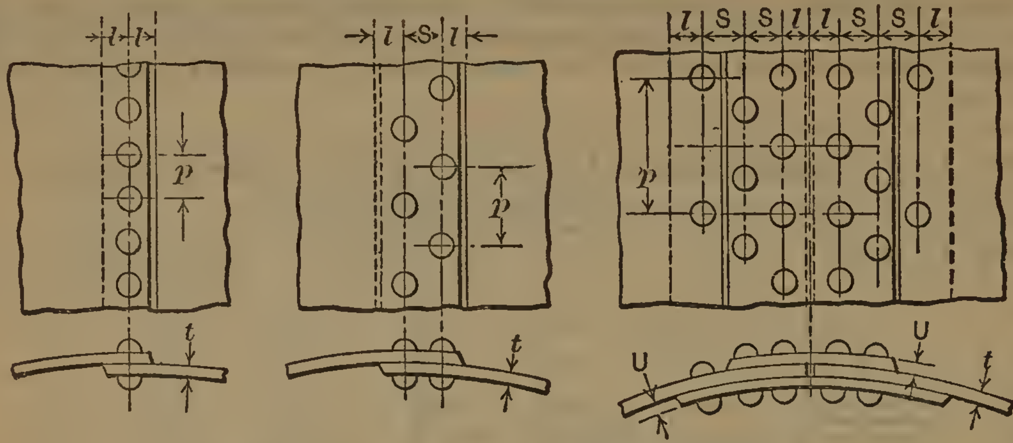
Diam. of Rivet, Inches	Bearing Value for Different Thicknesses of Plate in Inches at 12,000 Pounds per Square Inch							
	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	......	......	......	.....	.....	.....	.....	.....
1/2	......	......	......	.....	.....	.....	.....	.....
5/8	4220	4690	......	.....	.....	.....	.....	.....
3/4	5060	5630	6190	6750	.....	.....	.....	.....
7/8	5910	6560	7220	7880	8530	9190	9840	.....
1	6750	7500	8250	9000	9750	10500	11250	12000

Diam. of Rivet	Bearing Value at 15,000 Pounds per Square Inch							
	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	......	......	......	.....	.....	.....	.....	.....
1/2	......	......	......	.....	.....	.....	.....	.....
5/8	5280	5860	......	.....	.....	.....	.....	.....
3/4	6330	7030	7720	8440	.....	.....	.....	.....
7/8	7380	8200	9030	9850	10670	11480	12300	.....
1	8440	9380	10310	11250	12190	13130	14060	15000

Diam. of Rivet	Bearing Value at 20,000 Pounds per Square Inch							
	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	......	......	......	.....	.....	.....	.....	.....
1/2	......	......	......	.....	.....	.....	.....	.....
5/8	7030	7810	......	.....	.....	.....	.....	.....
3/4	8440	9380	10310	11250	.....	.....	.....	.....
7/8	9840	10940	12030	13130	14220	15310	16410	.....
1	11250	12500	13750	15000	16250	17500	18750	20000

Diam. of Rivet	Bearing Value at 25,000 Pounds per Square Inch							
	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	......	......	......	.....	.....	.....	.....	.....
1/2	......	......	......	.....	.....	.....	.....	.....
5/8	8790	9770	......	.....	.....	.....	.....	.....
3/4	10550	11720	12890	14060	.....	.....	.....	.....
7/8	12310	13670	15040	16410	17770	19140	20510	.....
1	14060	15630	17190	18750	20320	21880	23440	25000

Proportions of Riveted Joints for Pressure Tanks



t = thickness of plate,
p = pitch of rivets, center to center,
d = diameter of rivets before driving,
D = diameter of hole,
S = distance between lines of rivets,
l = distance from edge of plate to first line of rivets,
U = thickness of strap,
E = efficiency of joint in per cent.

Thickness of Plate, *t*		5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
Lap-joint, Single Riveted	*p*	2	2 1/16	2 1/8	2 1/8	2 3/16	...	...	...	...	...	...	...
	d	3/4	13/16	7/8	15/16	1	...	...	...	...	...	...	...
	D	13/16	7/8	15/16	1	1 1/16	...	...	...	...	...	...	...
	l	1 1/8	1 3/16	1 1/4	1 1/4	1 1/4	...	...	...	...	...	...	...
	E	59	56	54	53	51	...	...	...	...	...	...	...
Lap-joint, Double Riveted	*p*	3 3/16	3 3/16	3 3/16	3	3 1/16	3 1/8	3	2 7/8	...	...	...	...
	d	3/4	13/16	7/8	7/8	15/16	1	1	1	...	...	...	...
	D	13/16	7/8	15/16	15/16	1	1 1/16	1 1/16	1 1/16	...	...	...	...
	S	2	2	2	2	2	2	2	2	...	...	...	...
	l	1 1/8	1 3/16	1 1/4	1 1/4	1 1/4	1 1/4	1 1/4	1 1/4	...	...	...	...
	E	74	72	70	67	66	66	62	60	...	...	...	...
Double-strap Butt-joint, Triple Riveted	*p*	6	6	6 3/4	7	7	7	7	7	7 1/4	7	7 1/4	7
	d	5/8	3/4	7/8	7/8	7/8	7/8	15/16	15/16	1	1	1 1/8	1 1/8
	D	11/16	13/16	15/16	15/16	15/16	15/16	1	1	1 1/16	1 1/16	1 3/16	1 3/16
	l	1 1/8	1 3/16	1 1/4	1 1/4	1 1/4	1 1/4	1 1/4	1 1/4	1 1/4	1 3/8	1 3/8	1 3/8
	U	1/4	5/16	3/8	7/16	7/16	1/2	9/16	9/16	5/8	5/8	11/16	3/4
	E	88	86	86	86	86	86	85	85	85	84	83	83
	S	2 3/8 to 3 for all pitches.											

Shearing Strength of Rivets in Single Shear

Diam. of Rivet	Area of Cross-section	Shearing Strength per Square Inch				
		30,000	35,000	40,000	45,000	50,000
3/8	0.1104	3,310	3,860	4,410	4,960	5,520
1/2	0.1963	5,880	6,870	7,850	8,830	9,815
5/8	0.3068	9,200	10,730	12,270	13,800	15,340
3/4	0.4418	13,250	15,460	17,670	19,880	22,090
7/8	0.6013	18,030	21,040	24,050	27,050	30,065
1	0.7854	23,560	27,480	31,410	35,340	39,270

Shearing Strength of Rivets

Shearing resistance = NSA, in which N = number of rivets; A = cross-sectional area sheared in square inches; S = shearing strength of rivet iron = 38,000 pounds per square inch. Diameter of driven rivet equals diameter of rivet hole.

Diam. of Driven Rivet, Ins.	Area Sheared, Sq. In.	Number of Rivets Sheared = N						
		½	1	2	3	4	5	6
7/16	0.1503	2,850	5,710	11,425	17,130	22,850	28,560	34,275
½	0.1963	3,730	7,460	14,920	22,380	29,845	37,300	44,760
9/16	0.2485	4,720	9,440	18,880	28,320	37,770	47,210	56,650
⅝	0.3068	5,820	11,650	23,310	34,975	46,630	58,290	69,950
11/16	0.3712	7,050	14,100	28,210	42,310	56,425	70,530	84,630
¾	0.4418	8,390	16,780	33,570	50,360	67,150	83,940	100,720
13/16	0.5184	9,850	19,700	39,400	59,100	78,810	98,510	118,210
⅞	0.6013	11,425	22,850	45,700	68,550	91,400	114,250	137,100
15/16	0.6902	13,115	26,230	52,460	78,690	104,920	131,150	157,380
1	0.7854	14,920	29,845	59,690	89,530	119,380	149,220	179,070
1 1/16	0.8866	16,840	33,690	67,385	101,070	134,760	168,460	202,150
1⅛	0.9940	18,880	37,770	75,540	113,310	151,090	188,860	226,630
1 3/16	1.1075	21,040	42,085	84,170	126,250	168,340	210,425	252,510

Tensile Strength of Plate per 1 Inch of Width

Thickness	Tensile Strength per Square Inch				
	50,000	55,000	60,000	65,000	70,000
1/16	3,125	3,430	3,750	4,060	4,375
⅛	6,250	6,875	7,500	8,125	8,750
3/16	9,375	10,310	11,250	12,180	13,125
¼	12,500	13,750	15,000	16,250	17,500
5/16	15,625	17,180	18,750	20,310	21,875
⅜	18,750	20,625	22,500	24,375	26,250
7/16	21,875	24,060	26,250	28,430	30,625
½	25,000	27,500	30,000	32,500	35,000
9/16	28,125	30,930	33,750	36,560	39,375
⅝	31,250	34,375	37,500	40,625	43,750
11/16	34,375	37,810	41,250	44,680	48,125
¾	37,500	41,250	45,000	48,750	52,500
13/16	40,625	44,680	48,750	52,810	56,875
⅞	43,750	48,125	52,500	56,875	61,250
15/16	46,875	51,560	56,250	60,930	65,625
1	50,000	55,000	60,000	65,000	70,000

Net Areas of Structural Angles. — When using the tables of net areas of structural angles, the length of the legs of the angles are added together and the sum of the lengths is first found in the left-hand column of the tables; then the thickness of the angle is found in the second column. The other columns then give the net areas after having deducted for one or two rivets of sizes as specified at the head of

Net Areas of Standard Angles

(The size of rivet holes is 1/8 inch larger than diameter of rivet)

Sum of Length of Legs, Ins.	Thickness, Ins.	Area after Deducting for				
		One 5/8 Rivet	3/4 Rivets		7/8 Rivets	
			One	Two	One	Two
4	3/16	0.58	...	...	...	...
	1/4	0.75	...	...	...	...
	5/16	0.92	...	...	...	...
	3/8	1.08	...	...	...	...
	7/16	1.23	...	...	...	...
4½	3/16	0.67	0.65	...	...	...
	1/4	0.87	0.84	0.62	0.81	0.56
	5/16	1.08	1.04	0.77	1.00	0.69
	3/8	1.27	1.22	0.89	1.17	0.79
	7/16	1.45	1.40	1.02	1.34	0.90
	1/2	1.62	1.56	1.12	1.50	1.00
5	3/16	0.76	0.74	...	...	....
	1/4	1.00	0.97	0.75	0.94	0.69
	5/16	1.24	1.20	0.93	1.16	0.85
	3/8	1.45	1.40	1.07	1.35	0.97
	7/16	1.67	1.62	1.24	1.56	1.12
	1/2	1.87	1.81	1.37	1.75	1.25
5½	1/4	1.12	1.09	0.87	1.06	0.81
	5/16	1.39	1.35	1.08	1.31	1.00
	3/8	1.64	1.59	1.26	1.54	1.16
	7/16	1.89	1.84	1.46	1.78	1.34
	1/2	2.12	2.06	1.62	2.00	1.50
	9/16	2.36	2.29	1.80	2.22	1.66
6	1/4	1.25	1.22	1.00	1.19	0.94
	5/16	1.55	1.51	1.24	1.47	1.16
	3/8	1.83	1.78	1.45	1.73	1.35
	7/16	2.10	2.05	1.67	1.99	1.55
	1/2	2.37	2.31	1.87	2.25	1.75
	9/16	2.64	2.57	2.08	2.50	1.94
	5/8	2.89	2.81	2.26	2.73	2.10
	11/16	3.13	3.03	2.43	2.96	2.27
6½	5/16	1.70	1.66	1.39	1.62	1.31
	3/8	2.02	1.97	1.64	1.92	1.54
	7/16	2.32	2.27	1.89	2.21	1.77
	1/2	2.63	2.56	2.12	2.50	2.00
	9/16	2.92	2.85	2.36	2.78	2.22
	5/8	3.20	3.12	2.57	3.04	2.41
	11/16	3.48	3.40	2.80	3.31	2.62
7	5/16	1.86	1.82	1.55	1.78	1.47
	3/8	2.20	2.15	1.82	2.10	1.72
	7/16	2.54	2.49	2.11	2.43	1.99
	1/2	2.87	2.81	2.37	2.75	2.25
	9/16	3.20	3.13	2.64	3.06	2.50
	5/8	3.51	3.43	2.88	3.35	2.72
	11/16	3.82	3.74	3.14	3.65	2.96
	3/4	4.13	4.03	3.37	3.94	3.19

Sum of Length of Legs, Ins.	Thickness, Ins.	Area after Deducting for			
		3/4 Rivets		7/8 Rivets	
		One	Two	One	Two
7½	5/16	1.98	1.71	1.94	1.63
	3/8	2.34	2.01	2.29	1.91
	7/16	2.71	2.33	2.65	2.21
	1/2	3.06	2.62	3.00	2.50
	9/16	3.41	2.92	3.34	2.78
	5/8	3.75	3.20	3.67	3.04
	11/16	4.08	3.48	3.99	3.30
	3/4	4.40	3.74	4.31	3.56
	13/16	4.72	4.01	4.62	3.81
8	5/16	2.13	1.86	2.09	1.78
	3/8	2.53	2.20	2.48	2.10
	7/16	2.93	2.55	2.87	2.43
	1/2	3.31	2.87	3.25	2.75
	9/16	3.69	3.20	3.62	3.06
	5/8	4.06	3.51	3.98	3.35
	11/16	4.43	3.83	4.34	3.65
	3/4	4.78	4.12	4.69	3.84
	13/16	5.13	4.41	5.03	4.22
8½	5/16	2.29	2.02	2.25	1.94
	3/8	2.72	2.39	2.67	2.29
	7/16	3.15	2.77	3.09	2.65
	1/2	3.56	3.12	3.50	3.00
	9/16	3.98	3.49	3.91	3.35
	5/8	4.37	3.82	4.29	3.66
	11/16	4.77	4.17	4.68	3.99
	3/4	5.15	4.49	5.06	4.31
	13/16	5.54	4.83	5.44	4.63
	7/8	5.90	5.14	5.79	4.91
9	3/8	2.90	2.57	2.85	2.47
	7/16	3.35	2.99	3.31	2.87
	1/2	3.80	3.39	3.75	3.25
	9/16	4.26	3.77	4.19	3.63
	5/8	4.68	4.14	4.60	3.97
	11/16	5.12	4.52	5.03	4.34
	3/4	5.53	4.88	5.44	4.69
	13/16	5.94	5.23	5.84	5.03
	7/8	6.34	5.58	6.23	5.35
9½	3/8	3.09	2.76	3.04	2.66
	7/16	3.59	3.21	3.53	3.09
	1/2	4.06	3.62	4.00	3.50
	9/16	4.54	4.05	4.47	3.91
	5/8	5.00	4.45	4.92	4.29
	11/16	5.46	4.86	5.37	4.68
	3/4	5.90	5.24	5.81	5.06
	13/16	6.35	5.64	6.25	5.44
	7/8	6.78	6.02	6.67	5.79

Net Areas of Standard Angles

(The size of rivet holes is ⅛ inch larger than diameter of rivet)

Sum of Length of Legs, Ins.	Thickness, Inches	Area after Deducting for ¾ Rivets		Area after Deducting for ⅞ Rivets	
		One	Two	One	Two
10	⅜	3.28	2.95	3.23	2.85
	7/16	3.80	3.42	3.74	3.30
	½	4.31	3.87	4.25	3.75
	9/16	4.82	4.33	4.75	4.19
	⅝	5.31	4.76	5.23	4.60
	11/16	5.81	5.21	5.72	5.03
	¾	6.28	5.63	6.19	5.44
	13/16	6.76	6.05	6.66	5.85
	⅞	7.22	6.46	7.11	6.23
	15/16	7.68	6.86	7.56	6.62
	1	8.13	7.25	8.00	7.00
10½	7/16	4.02	3.64	3.96	3.53
	½	4.55	4.13	4.50	4.00
	9/16	5.10	4.61	5.03	4.47
	⅝	5.62	5.08	5.54	4.92
	11/16	6.15	5.55	6.06	5.37
	¾	6.65	6.00	6.56	5.81
	13/16	7.16	6.45	7.06	6.25
	⅞	7.65	6.89	7.54	6.67
	15/16	8.15	7.33	8.03	7.10
	1	8.63	7.75	8.50	7.50
12	⅜	4.03	3.70	3.98	3.60
	7/16	4.68	4.30	4.62	4.18
	½	5.31	4.87	5.25	4.75
	9/16	5.94	5.45	5.87	5.31
	⅝	6.56	6.01	6.48	5.85
	11/16	7.18	6.58	7.09	6.40
	¾	7.78	7.12	7.69	6.94
	13/16	8.38	7.67	8.28	7.47
	⅞	8.97	8.21	8.86	7.98
	15/16	9.55	8.73	9.43	8.49
	1	10.13	9.25	10.00	9.00
14	½	6.31	5.89	6.26	5.76
	9/16	7.16	6.67	7.09	6.53
	⅝	7.89	7.35	7.82	7.19
	11/16	8.72	8.12	8.63	7.94
	¾	9.28	8.63	9.19	8.44
	13/16	10.05	9.34	9.95	9.14
	⅞	10.85	10.09	10.74	9.87
	15/16	11.53	10.71	11.41	10.48
	1	12.19	11.31	12.06	11.06
16	½	7.30	6.88	7.25	6.75
	9/16	8.20	7.71	8.13	7.57
	⅝	9.06	8.52	8.99	8.36
	11/16	9.93	9.33	9.84	9.15
	¾	10.78	10.13	10.69	9.94
	13/16	11.63	10.93	11.52	10.72
	⅞	12.47	11.71	12.35	11.48
	15/16	13.31	12.49	13.18	12.24
	1	14.13	13.25	14.00	13.00
	1 1/16	14.95	14.01	14.81	13.75
	1⅛	15.76	14.77	15.62	14.49

Area Deducted for Various Sizes of Rivets and Thicknesses of Angles

(The size of rivet holes is ⅛ inch larger than diameter of rivet)

No. of Rivets	Size of Rivets	Thickness of Angle 3/16	¼	5/16	⅜	7/16	½	9/16	⅝	11/16	¾	13/16	⅞	15/16	1
1	⅜	0.09	0.13	0.16	0.19	0.22	0.25	0.28	0.31	0.34	0.38	0.41	0.44	0.47	0.50
1	½	0.12	0.16	0.20	0.23	0.27	0.31	0.35	0.39	0.43	0.47	0.51	0.55	0.58	0.63
1	⅝	0.14	0.19	0.23	0.28	0.33	0.38	0.42	0.47	0.52	0.56	0.61	0.66	0.70	0.75
2	⅝	0.28	0.38	0.46	0.56	0.66	0.75	0.84	0.94	1.03	1.12	1.22	1.31	1.40	1.50
1	¾	0.16	0.22	0.27	0.33	0.38	0.44	0.49	0.55	0.60	0.66	0.71	0.77	0.82	0.88
2	¾	0.33	0.44	0.54	0.66	0.76	0.87	0.98	1.09	1.20	1.31	1.42	1.53	1.64	1.75
1	⅞	0.19	0.25	0.31	0.38	0.44	0.50	0.56	0.63	0.69	0.75	0.81	0.88	0.94	1.00
2	⅞	0.37	0.50	0.62	0.75	0.87	1.00	1.12	1.25	1.38	1.50	1.62	1.75	1.87	2.00
1	1	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70	0.77	0.84	0.91	0.98	1.06	1.13

the columns. The table entitled "Area Deducted for Various Sizes of Rivets and Thicknesses of Angles" is used for determining the net area of angles of dimensions not given in the previous table, or for rivet sizes not specified.

STRENGTH AND PROPERTIES OF STEEL WIRE

Wire Gages. — A great number of different wire gages known by numbers have been in use. In order to avoid confusion, it would be well if, in general, gage numbers could be avoided and the size of the wire required given in decimals of an inch. However, when this cannot be done, care should be taken to adhere to the gage numbers which have become practically standard for certain classes of wire. Upon the recommendation of the Bureau of Standards at Washington, a number of the principal wire manufacturers and consumers have agreed that it would be well to designate the American Steel & Wire Co.'s gage, which is the same as the Washburn & Moen gage, as the "Steel Wire Gage." In cases where it becomes necessary to distinguish this from the British Imperial standard wire gage, it may be called the "U. S. Steel Wire Gage." This gage applies to all steel wire.

For copper wires and wires of other metals, the gage universally recognized in the United States is the "American Wire Gage," which is also known as the Brown & Sharpe gage. No confusion should arise between the steel wire gage and the American wire gage, because the fields covered by the two gages are distinct and different.

The piano wire gage, designated as the "American Steel & Wire Co.'s Music Wire Gage" is adopted as standard for piano wire upon the recommendation of the United States Bureau of Standards.

The trend of practice in the gaging of materials is increasingly toward the direct specification of dimensions in decimal fractions of an inch without the use of gage numbers. The United States Navy Department in 1911 ordered that all diameters and thicknesses of materials be specified directly in decimal fractions, omitting all reference to gage numbers, and the War Department issued a similar order for wires. This is similar to the practice in Europe where sizes of wire are specified directly by the diameter in millimeters.

The tariff act of 1913 provides for the use of decimal dimensions in measuring wires and rods, but the measurement of steel strips is by gage. As the particular gage was not designated in the tariff act, the Treasury Department in 1914 authorized the use of the American (B. & S.) wire gage. Prior to this the Birmingham wire gage had been employed. The Treasury Department also directed that the measurement of sheets and plates be in decimals of an inch instead of using the standard gage.

Strength of Piano and Plow-steel Wire. — The strength of wire is increased considerably by drawing. So-called piano wire has an ultimate tensile strength of from 300,000 to 340,000 pounds per square inch. The composition of this wire is as follows: Carbon, 0.57 per cent; silicon, 0.09 per cent; sulphur, 0.011 per cent; phosphorus, 0.018 per cent; manganese, 0.425 per cent. This wire is made in sizes ranging from 0.029 to 0.052 inch (music wire gage Nos. 12 to 22 inclusive). So-called "plow-steel" wire has an ultimate tensile strength of 345,000 pounds per square inch for wire 0.093 inch in diameter, and 200,000 pounds per square inch for wire 0.191 inch in diameter. The elongation is only about 1 per cent. The composition is about as follows: Carbon, 0.83 per cent; manganese, 0.59 per cent; silicon, 0.14 per cent; sulphur, 0.01 per cent; phosphorus, nil; copper, 0.03 per cent.

Wire Drawing. — Iron and steel wire from No. 3 to No. 18 Brown & Sharpe wire gage (from 0.229 to 0.040 inch in diameter) is drawn from wire about ¼ inch in diameter through holes in draw plates made of a high-grade tungsten steel. The wire is reduced at each drawing or pass by one number or step in the Brown & Sharpe wire gage scale, and at each drawing operation, the wire passes through but

Wire and Sheet Metal Gages in Approximate Decimals of an Inch

No. of Wire Gage	American or Brown & Sharpe	Birmingham or Stub's Iron Wire	Washburn & Moen, Am. Steel & Wire Co., and Roebling	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	U. S. Standard for Plate
0000000	......	......	0.4900	......	......	0.5000	0.5000
000000	0.5800	......	0.4615	......	......	0.4640	0.4688
00000	0.5165	0.500	0.4305	......	0.4500	0.4320	0.4375
0000	0.4600	0.454	0.3938	......	0.4000	0.4000	0.4063
000	0.4096	0.425	0.3625	......	0.3600	0.3720	0.3750
00	0.3648	0.380	0.3310	......	0.3300	0.3480	0.3438
0	0.3249	0.340	0.3065	......	0.3050	0.3240	0.3125
1	0.2893	0.300	0.2830	0.227	0.2850	0.3000	0.2813
2	0.2576	0.284	0.2625	0.219	0.2650	0.2760	0.2656
3	0.2294	0.259	0.2437	0.212	0.2450	0.2520	0.2500
4	0.2043	0.238	0.2253	0.207	0.2250	0.2320	0.2344
5	0.1819	0.220	0.2070	0.204	0.2050	0.2120	0.2188
6	0.1620	0.203	0.1920	0.201	0.1900	0.1920	0.2031
7	0.1443	0.180	0.1770	0.199	0.1750	0.1760	0.1875
8	0.1285	0.165	0.1620	0.197	0.1600	0.1600	0.1719
9	0.1144	0.148	0.1483	0.194	0.1450	0.1440	0.1563
10	0.1019	0.134	0.1350	0.191	0.1300	0.1280	0.1406
11	0.0907	0.120	0.1205	0.188	0.1175	0.1160	0.1250
12	0.0808	0.109	0.1055	0.185	0.1050	0.1040	0.1094
13	0.0720	0.095	0.0915	0.182	0.0925	0.0920	0.0938
14	0.0641	0.083	0.0800	0.180	0.0800	0.0800	0.0781
15	0.0571	0.072	0.0720	0.178	0.0700	0.0720	0.0703
16	0.0508	0.065	0.0625	0.175	0.0610	0.0640	0.0625
17	0.0453	0.058	0.0540	0.172	0.0525	0.0560	0.0563
18	0.0403	0.049	0.0475	0.168	0.0450	0.0480	0.0500
19	0.0359	0.042	0.0410	0.164	0.0400	0.0400	0.0438
20	0.0320	0.035	0.0348	0.161	0.0350	0.0360	0.0375
21	0.0285	0.032	0.0317	0.157	0.0310	0.0320	0.0344
22	0.0253	0.028	0.0286	0.155	0.0280	0.0280	0.0313
23	0.0226	0.025	0.0258	0.153	0.0250	0.0240	0.0281
24	0.0201	0.022	0.0230	0.151	0.0225	0.0220	0.0250
25	0.0179	0.020	0.0204	0.148	0.0200	0.0200	0.0219
26	0.0159	0.018	0.0181	0.146	0.0180	0.0180	0.0188
27	0.0142	0.016	0.0173	0.143	0.0170	0.0164	0.0172
28	0.0126	0.014	0.0162	0.139	0.0160	0.0148	0.0156
29	0.0113	0.013	0.0150	0.134	0.0150	0.0136	0.0141
30	0.0100	0.012	0.0140	0.127	0.0140	0.0124	0.0125
31	0.0089	0.010	0.0132	0.120	0.0130	0.0116	0.0109
32	0.0080	0.009	0.0128	0.115	0.0120	0.0108	0.0102
33	0.0071	0.008	0.0118	0.112	0.0110	0.0100	0.0094
34	0.0063	0.007	0.0104	0.110	0.0100	0.0092	0.0086
35	0.0056	0.005	0.0095	0.108	0.0095	0.0084	0.0078
36	0.0050	0.004	0.0090	0.106	0.0090	0.0076	0.0070
37	0.0045	......	0.0085	0.103	0.0085	0.0068	0.0066
38	0.0040	......	0.0080	0.101	0.0080	0.0060	0.0063
39	0.0035	......	0.0075	0.099	0.0075	0.0052	....
40	0.0031	......	0.0070	0.097	0.0070	0.0048	....
41	0.0028	......	0.0066	0.095	......	0.0044	....
42	0.0025	......	0.0062	0.092	......	0.0040	....
43	0.0022	......	0.0060	0.088	......	0.0036	....
44	0.0020	......	0.0058	0.085	......	0.0032	....
45	0.00176	......	0.0055	0.081	......	0.0028	....
46	0.00157	......	0.0052	0.079	......	0.0024	....
47	0.00140	......	0.0050	0.077	......	0.0020	....
48	0.00124	......	0.0048	0.075	......	0.0016	....
49	0.00099	......	0.0046	0.072	......	0.0012	....
50	0.00088	......	0.0044	0.069	......	0.0010	....

Additional Gage Numbers and Sizes of Stub's Steel Wire

Gage No.	Stub's Steel Wire	Gage No.	Stub's Steel Wire	Gage No.	Stub's Steel Wire	Gage No.	Stub's Steel Wire	Gage No.	Stub's Steel Wire
51	0.066	57	0.042	63	0.036	69	0.029	75	0.020
52	0.063	58	0.041	64	0.035	70	0.027	76	0.018
53	0.058	59	0.040	65	0.033	71	0.026	77	0.016
54	0.055	60	0.039	66	0.032	72	0.024	78	0.015
55	0.050	61	0.038	67	0.031	73	0.023	79	0.014
56	0.045	62	0.037	68	0.030	74	0.022	80	0.013

one hole in the die. In the wire gage scale compiled in 1864 by Messrs. Brown and Sharpe, the diameters of the wires of successive numbers increase according to a geometrical ratio. The diameter of each succeeding number can be found by multiplying the diameter of the preceding number by 1.123, this being the ratio of the geometrical progression. The basic size is No. 36 wire, which is 0.005 inch in diameter.

Fine sizes of iron, steel or alloy-steel wire (between 0.040 and 0.002 inch in diameter) are drawn through diamond dies, which consist of a body made from brass in which the diamond is inserted. The diamond is of the variety known as bort. In the case of these smaller sizes, the wire is passed through a succession of dies (up to ten or twelve) in a single drawing operation. The size of diamond for wire 0.040 inch in diameter is about 3 or 3½ carats, while one-half-carat stones will suffice for dies for drawing wire 0.010 inch in diameter. The life of the diamond die used for drawing steel wire averages only about three days, while for copper wire it may last for six months or a year. The speed at which the wire is drawn appears to have little effect on the life of the die, but the life depends solely on the length of wire passed through it. About 200 pounds of steel wire can be drawn through a No. 32 B. & S. diamond die, before its size is too enlarged for further use. Fifteen pounds of wire only can be drawn through holes from 0.003 to 0.005 inch in diameter. Less than one pound can be drawn through holes smaller than 0.002 inch. When the diamond dies are worn too much, they are re-drilled for a larger gage number. In drilling the diamonds, the average time for enlarging a hole 0.001 inch in diameter is about 1½ hour. For hard music wire, diamonds of comparatively large size are required; thus for holes as small as 0.005 inch in diameter, 2 to 2½ carat diamonds are used.

Copper wire is drawn through dies of chilled cast iron; the reduction for each pass is equal to one number or step on the B. & S. wire gage scale, but the wire passes through a number of successive dies at one operation, as many as ten dies often being mounted in the same wire-drawing machine.

Converting Strength of Wire into Strength in Pounds per Square Inch. — The table given for this conversion has two columns, one headed "Diam. of Wire," and the other "Factor." The strength of the wire of a given diameter should be multiplied by the factor given, in order to obtain the strength per square inch. For example, it is known that a wire 0.035 inch in diameter can sustain a load of 150 pounds before breaking. Then the strength in pounds per square inch of the wire is found as below:

$$150 \times 1040 = 156{,}000 \text{ pounds per square inch.}$$

The factor 1040 is found opposite the diameter 0.035 in the table.

Converting Strength of Wire to Strength per Square Inch

Diam. of Wire	Factor	Diam. of Wire	Factor	Diam. of Wire	Factor	Diam. of Wire	Factor	Diam. of Wire	Factor
0.001	1,256,800	0.051	489.9	0.101	124.8	0.151	55.8	0.201	31.5
0.002	314,200	0.052	470.8	0.102	122.4	0.152	55.0	0.202	31.2
0.003	141,400	0.053	452.7	0.103	120.0	0.153	54.3	0.203	30.9
0.004	78,550	0.054	436.7	0.104	117.7	0.154	53.7	0.204	30.6
0.005	50,910	0.055	421.3	0.105	115.5	0.155	53.0	0.205	30.3
0.006	35,360	0.056	406.0	0.106	113.3	0.156	52.3	0.206	30.0
0.007	25,980	0.057	392.2	0.107	111.2	0.157	51.6	0.207	29.7
0.008	19,890	0.058	378.4	0.108	109.2	0.158	51.0	0.208	29.4
0.009	16,290	0.059	368.0	0.109	107.2	0.159	50.3	0.209	29.3
0.010	12,730	0.060	357.7	0.110	105.2	0.160	49.7	0.210	28.8
0.011	10,520	0.061	344.4	0.111	103.2	0.161	49.1	0.211	28.5
0.012	8,840	0.062	331.2	0.112	101.5	0.162	48.5	0.212	28.3
0.013	7,532	0.063	321.0	0.113	99.7	0.163	47.9	0.213	28.0
0.014	6,496	0.064	310.8	0.114	98.0	0.164	47.3	0.214	27.8
0.015	5,724	0.065	301.5	0.115	96.3	0.165	46.7	0.215	27.5
0.016	4,972	0.066	292.3	0.116	94.6	0.166	46.2	0.216	27.3
0.017	4,404	0.067	283.8	0.117	93.0	0.167	45.6	0.217	27.0
0.018	4,072	0.068	275.3	0.118	91.4	0.168	45.1	0.218	26.8
0.019	3,527	0.069	262.6	0.119	89.9	0.169	44.5	0.219	26.5
0.020	3,182	0.070	260.0	0.120	88.4	0.170	44.0	0.220	26.3
0.021	2,888	0.071	252.8	0.121	86.9	0.171	43.5	0.221	26.0
0.022	2,630	0.072	245.6	0.122	85.5	0.172	43.0	0.222	25.8
0.023	2,407	0.073	239.0	0.123	84.1	0.173	42.5	0.223	25.6
0.024	2,210	0.074	232.5	0.124	82.8	0.174	42.0	0.224	25.4
0.025	2,036	0.075	226.4	0.125	81.4	0.175	41.5	0.225	25.1
0.026	1,883	0.076	220.4	0.126	80.0	0.176	41.0	0.226	24.9
0.027	1,747	0.077	214.8	0.127	78.8	0.177	40.6	0.227	24.7
0.028	1,624	0.078	209.3	0.128	77.7	0.178	40.1	0.228	24.5
0.029	1,514	0.079	204.1	0.129	76.5	0.179	39.7	0.229	24.3
0.030	1,431	0.080	198.9	0.130	75.3	0.180	39.3	0.230	24.1
0.031	1,325	0.081	194.1	0.131	74.1	0.181	38.9	0.231	23.8
0.032	1,243	0.082	189.4	0.132	73.0	0.182	38.4	0.232	23.6
0.033	1,169	0.083	184.7	0.133	71.9	0.183	38.0	0.233	23.4
0.034	1,106	0.084	180.0	0.134	70.9	0.184	37.6	0.234	23.2
0.035	1,040	0.085	176.0	0.135	69.8	0.185	37.1	0.235	23.0
0.036	982	0.086	172.1	0.136	68.8	0.186	36.7	0.236	22.8
0.037	930	0.087	168.2	0.137	67.8	0.187	36.3	0.237	22.6
0.038	882	0.088	164.4	0.138	66.9	0.188	36.0	0.238	22.5
0.039	839	0.089	160.8	0.139	65.9	0.189	35.7	0.239	22.3
0.040	796	0.090	157.2	0.140	65.0	0.190	35.3	0.240	22.1
0.041	759	0.091	153.6	0.141	64.0	0.191	34.9	0.241	21.9
0.042	722	0.092	150.0	0.142	63.0	0.192	34.5	0.242	21.7
0.043	690	0.093	147.0	0.143	62.2	0.193	34.1	0.243	21.5
0.044	658	0.094	144.1	0.144	61.4	0.194	33.8	0.244	21.4
0.045	630	0.095	141.1	0.145	60.5	0.195	33.5	0.245	21.2
0.046	602	0.096	138.2	0.146	59.7	0.196	33.1	0.246	21.0
0.047	577	0.097	135.4	0.147	58.9	0.197	32.8	0.247	20.8
0.048	552	0.098	132.6	0.148	58.1	0.198	32.5	0.248	20.7
0.049	531	0.099	129.9	0.149	57.3	0.199	32.1	0.249	20.5
0.050	509	0.100	127.3	0.150	56.6	0.200	31.8	0.250	20.4

Diameter, Strength and Weight of Steel Wire

The breaking stress of the wire is based on a tensile strength of 100,000 pounds per square inch. For wire of greater or less strength, simply multiply the values in the table by the ratio between actual strength per square inch and 100,000. Example: A No. 15 wire is made of material having a tensile strength of 150,000 pounds per square inch. Then, breaking stress of wire = (150,000 ÷ 100,000) × 407 = 610 pounds.

No. Washburn & Moen, Am. Steel & Wire Co., and Roebling Gage	Diameter, Inches	Area, Sq. Ins.	Breaking Stress of Wire, based on 100,000 Lbs. Stress per Sq. In.	Weight in Pounds		Number of Feet in 2000 Pounds.
				Per 1000 Ft.	Per Mile	
000000	0.460	0.166191	16,620.0	558.4	2948.0	3,582
00000	0.430	0.145221	14,520.0	487.9	2576.0	4,099
0000	0.393	0.121304	12,130.0	407.6	2152.0	4,907
000	0.362	0.102922	10,290.0	345.8	1826.0	5,783
00	0.331	0.086049	8,605.0	289.1	1527.0	6,917
0	0.307	0.074023	7,400.0	248.7	1313.0	8,041
1	0.283	0.062902	6,290.0	211.4	1116.0	9,463
2	0.263	0.054325	5,430.0	182.5	964.0	10,957
3	0.244	0.046760	4,680.0	157.1	830.0	12,730
4	0.225	0.039761	3,980.0	133.6	705.0	14,970
5	0.207	0.033654	3,365.0	113.1	597.0	17,687
6	0.192	0.028953	2,895.0	97.3	514.0	20,559
7	0.177	0.024606	2,460.0	82.7	437.0	24,191
8	0.162	0.020612	2,060.0	69.3	366.0	28,878
9	0.148	0.017203	1,720.0	57.8	305.0	34,600
10	0.135	0.014314	1,430.0	48.1	254.0	41,584
11	0.120	0.011310	1,130.0	38.0	201.0	52,631
12	0.105	0.008659	866.0	29.1	154.0	68,752
13	0.092	0.006648	665.0	22.3	118.0	89,525
14	0.080	0.005027	503.0	16.9	89.2	118,413
15	0.072	0.004071	407.0	13.7	72.2	146,198
16	0.063	0.003117	312.0	10.5	55.3	191,022
17	0.054	0.002290	229.0	7.70	40.6	259,909
18	0.047	0.001735	174.0	5.83	30.8	343,112
19	0.041	0.001320	132.0	4.44	23.4	450,856
20	0.035	0.000962	96.0	3.23	17.1	618,620
21	0.032	0.000804	80.0	2.70	14.3	740,193
22	0.028	0.000616	62.0	2.07	10.9	966,651
23	0.025	0.000491	49.0	1.65	8.71	......
24	0.023	0.000415	42.0	1.40	7.37	......
25	0.020	0.000314	31.0	1.06	5.58	......
26	0.018	0.000254	25.0	0.855	4.51	......
27	0.017	0.000227	23.0	0.763	4.03	......
28	0.016	0.000201	20.0	0.676	3.57	......
29	0.015	0.000177	18.0	0.594	3.14	......
30	0.014	0.000154	15.0	0.517	2.73	......
31	0.0135	0.000143	14.0	0.481	2.54	......
32	0.013	0.000133	13.0	0.446	2.36	......
33	0.011	0.000095	9.5	0.319	1.69	......
34	0.010	0.000079	7.9	0.264	1.39	......
35	0.0095	0.000071	7.1	0.238	1.26	......
36	0.009	0.000064	6.4	0.214	1.13	......

Comparison of Music Wire Gages

Gage Number	Am. Steel & Wire Co.*	Am. Screw & Wire Co.	Roebling, and Trenton Iron Co.	Wright Wire Co.	Poehlmann Music Wire	Felten & Guilleaume	Allhoff & Müller	W. N. Brunton Music Wire	English Music Wire
6/0	0.004	0.0095	......	......	......	......	......	......	......
5/0	0.005	0.010	......	......	......	......	......	......	......
4/0	0.006	0.011	0.007	......	0.006	0.0068	......	......	......
3/0	0.007	0.012	0.0075	......	0.007	0.0075	......	......	......
2/0	0.008	0.0133	0.0085	0.0085	0.008	0.0087	0.008	0.0085	......
0	0.009	0.0144	0.009	0.009	0.009	0.0093	0.009	0.009	......
1	0.010	0.0156	0.010	0.010	0.010	0.0098	0.010	0.010	......
2	0.011	0.0166	0.011	0.011	0.011	0.0106	0.011	0.011	0.0105
3	0.012	0.0178	0.012	0.012	0.012	0.0114	0.012	0.012	0.0115
4	0.013	0.0188	0.013	0.013	0.013	0.0122	0.013	0.013	0.0125
5	0.014	0.0202	0.014	0.014	0.014	0.0138	0.014	0.014	0.0145
6	0.016	0.0215	0.016	0.016	0.016	0.0157	0.016	0.016	0.015
7	0.018	0.023	0.018	0.018	0.018	0.0177	0.018	0.017	0.0175
8	0.020	0.0243	0.020	0 020	0.020	0.0197	0.020	0.019	0.019
9	0.022	0.0256	0.022	0.022	0.022	0.0216	0.022	0.022	0.022
10	0.024	0.027	0.024	0.024	0.024	0.0236	0.024	0.024	0.0245
11	0.026	0.0284	0.026	0.026	0.026	0.0260	0.026	0.027	0.027
12	0.029	0.0296	0.028	0.028	0.029	0.0283	0.028	0.029	0.0285
13	0.031	0.0314	0.030	0.0305	0.031	0.0303	0.030	0.031	0.0305
14	0.033	0.0326	0.032	0.0325	0.033	0.0323	0.032	0.032	0 032
15	0.035	0.0345	0.034	0.034	0.035	0.0342	0.034	0.034	0.035
16	0.037	0.036	0.036	0.036	0.037	0.0362	0.036	0.036	0.036
17	0.039	0.0377	0.038	0.038	0.039	0.0382	0.038	0.038	0.038
18	0.041	0.0395	0.040	0.0405	0.041	0.0400	0.040	0.040	0.040
19	0.043	0.0414	0.042	0.042	0.043	0.0420	0.042	0.042	0.042
20	0.045	0.0434	0.044	0.044	0.045	0.0440	0.044	0.044	0.043
21	0.047	0.046	0.046	0.046	0.047	0.0460	0.046	0.046	0.0445
22	0.049	0.0483	0.048	0.0485	0.049	0.0480	0.048	0.048	0.047
23	0.051	0.051	0.051	0.0505	0.051	0.0510	0.051	0.050	0.049
24	0.055	0.055	0.055	0.0545	0.055	0.0550	0.055	0.054	0.053
25	0.059	0.0586	0.059	0.0585	0.059	0.0590	0.059	0.058	0.056
26	0.063	0.0626	0.063	0.063	0.063	0.0630	0.063	0.062	0.0605
27	0.067	0.0675	0.067	0.067	0.067	0.0670	0.067	0.066	0.064
28	0.071	0.072	0.071	0.071	0.071	0.0710	0.071	0.069	0.0685
29	0.075	0.076	0.074	0.0745	0.075	0.0740	0.074	0.072	0.0715
30	0.080	0.080	0.078	0.078	0.080	0.0780	0.078	0.076	0.075
31	0.085	0.085	0.082	0.082	......	0.0820	0.082	0.080	......
32	0.090	0.092	0.086	0.086	......	0.0860	0.086	0.086	......
33	0.095	......	0.090	0.090	......	......	0.090	0.092	......
34	0.100	......	0.095	0.096	......	......	0.094	0.098	......
35	0.106	......	0.100	......	......	......	0.098	0.104	......
36	0.112	......	0.105	......	......	......	0.102	0.110	......
37	0.118	......	0.110	......	......	......	......	0.117	......
38	0.124	......	0.115	......	......	......	......	0.121	......
39	0.130	......	0.120	......	......	......	......	0.130	......
40	0.138	......	0.125	......	......	......	......	0.140	......
41	0.146	......	0.130	......	......	......	......	......	......

* The American Steel & Wire Co. also makes the following additional sizes: No. 42 — 0.154; No. 43 — 0.162; No. 44 — 0.170; No. 45 — 0.180.

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.00088	......	50							
0.00099	......	49							
0.0010	......	......	......	......	......	50			
0.0012	......	......	......	......	......	49			
0.00124	......	48							
0.00140	......	47							
0.00157	......	46							
0.0016	......	......	......	......	......	48			
0.00176	......	45							
0.00198	......	44							
0.0020	......	......	......	......	......	47			
0.00222	......	43							
0.0024	......	......	......	......	......	46			
0.00249	......	42							
0.0028	......	41	......	......	......	45			
0.00314	......	40							
0.0032	......	......	......	......	......	44			
0.00353	......	39							
0.0036	......	......	......	......	......	43			
0.0040	......	38	36	......	......	42	6/0		
0.0044	50	......	......	......	......	41			
0.0045	......	37							
0.0046	49								
0.0048	48	......	......	......	......	40			
0.0050	47	36	35	......	......	......	5/0		
0.0052	46	......	......	......	......	39			
0.0055	45								
0.0056	......	35							
0.0058	44								
0.0060	43	......	......	......	......	38	4/0		
0.0062	42								
0.0063	......	34	......	......	......	......	......	38	
0.0066	41	......	......	......	......	......	......	37	
0.0068	......	......	......	......	......	37			
0.0070	40	......	34	......	40	......	3/0	36	
0.0071	......	33							
0.0075	39	......	......	......	39				
0.0076	......	......	......	......	......	36			
0.0078	......	......	......	......	......	......	......	35	
0.0080	38	32	33	......	38	......	2/0		
0.0084	......	......	......	......	......	35	......		

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses. *(Continued)*

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.0085	37	......	......	......	37				
0.0086	......	......	......	......	......	......	......	34	
0.0089	......	31							
0.0090	36	......	32	......	36	......	0		
0.0092	......	......	......	......	......	34			
0.0094	......	......	......	......	......	......	......	33	
0.0095	35	......	......	......	35				
0.0100	......	30	31	......	34	33	1		
0.0102	......	......	......	......	......	......	......	32	
0.0104	34								
0.0108	......	......	......	......	......	32			
0.0109	......	......	......	......	......	......	......	31	
0.0110	......	......	......	......	33	......	2		
0.0113	......	29							
0.0116	......	......	......	......	......	31			
0.0118	33								
0.0120	......	......	30	......	32	......	3		
0.0124	......	......	......	......	......	30			
0.0125	......	......	......	......	......	......	......	30	
0.0126	......	28							
0.0128	32								
0.0130	......	......	29	80	31	......	4		
0.0132	31								
0.0135	......	......	......	......	......	......	......	......	80
0.0136	......	......	......	......	......	29			
0.0140	30	......	28	79	30	......	5		
0.0141	......	......	......	......	......	......	......	29	
0.0142	......	27							
0.0145	......	......	......	......	......	......	......	......	79
0.0148	......	......	......	......	......	28			
0.0150	29	......	......	78	29				
0.0156	......	......	......	......	......	......	......	28	
0.0159	......	26							
0.0160	......	......	27	77	28	......	6	......	78
0.0162	28								
0.0164	......	......	......	......	......	27			
0.0170	......	......	......	......	27				
0.0172	......	......	......	......	......	......	......	27	
0.0173	27								
0.0179	......	25							
0.0180	......	......	26	76	26	26	7	......	77

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses. (*Continued*)

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.0181	26								
0.0188								26	
0.0200			25	75	25	25	8		76
0.0201		24							
0.0204	25								
0.0210									75
0.0219								25	
0.0220			24	74		24	9		
0.0225					24				74
0.0226		23							
0.0230	24			73					
0.0240				72		23	10		73
0.0250			23		23			24	72
0.0253		22							
0.0258	23								
0.0260				71			11		71
0.0270				70					
0.0280			22		22	22			70
0.0281								23	
0.0285		21							
0.0286	22								
0.0290				69			12		
0.02925									69
0.0300				68					
0.0310				67	21		13		68
0.0313								22	
0.0317	21								
0.0320		20	21	66		21			67
0.0330				65			14		66
0.0344								21	
0.0348	20								
0.0350			20	64	20		15		65
0.0359		19							
0.0360				63		20			64
0.0370				62			16		63
0.0375								20	
0.0380				61					62
0.0390				60			17		61
0.0400				59	19	19			60
0.0403		18							
0.0410	19			58			18		59

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses. (*Continued*)

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.0420	......	......	19	57	......	......	......	......	58
0.0430	......	......	......	......	......	......	19	......	57
0.0438	......	......	......	......	......	......	......	19	
0.0450	......	......	......	56	18	......	20		
0.0453	......	17							
0.0465	......	......	......	......	......	......	......	......	56
0.0470	......	......	......	......	......	......	21		
0.0475	18								
0.0480	......	......	......	......	......	18			
0.0490	......	......	18	......	......	......	22		
0.0500	......	......	......	55	......	......	......	18	
0.0508	......	16							
0.0510	......	......	......	......	......	......	23		
0.0520	......	......	......	......	......	......	......	......	55
0.0525	......	......	......	......	17				
0.0540	17								
0.0550	......	......	......	54	......	......	24	......	54
0.0560	......	......	......	......	......	17			
0.0563	......	......	......	......	......	......	......	17	
0.0571	......	15							
0.0580	......	......	17	53					
0.0590	......	......	......	......	......	......	25		
0.0595	......	......	......	......	......	......	......	......	53
0.0610	......	......	......	......	16				
0.0625	16	......	......	......	......	......	......	16	
0.0630	......	......	......	52	......	......	26		
0.0635	......	......	......	......	......	......	......	......	52
0.0640	......	......	......	......	......	16			
0.0641	......	14							
0.0650	......	......	16						
0.0660	......	......	......	51					
0.0670	......	......	......	......	......	......	27	......	51
0.0690	......	......	......	50					
0.0700	......	......	......	......	15	......	......	......	50
0.0703	......	......	......	......	......	......	......	15	
0.0710	......	......	......	......	......	......	28		
0.0720	15	13	15	49	......	15			
0.0730	......	......	......	......	......	......	......	......	49
0.0750	......	......	......	48	......	......	29		
0.0760	......	......	......	......	......	......	......	......	48
0.0770	......	......	......	47					

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses. (*Continued*)

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.0781								14	
0.0785									47
0.0790				46					
0.0800	14				14	14	30		
0.0808		12							
0.0810				45					46
0.0820									45
0.0830			14						
0.0850				44			31		
0.0860									44
0.0880				43					
0.0890									43
0.0900							32		
0.0907		11							
0.0915	13								
0.0920				42		13			
0.0925					13				
0.0935									42
0.0938								13	
0.0950			13	41			33		
0.0960									41
0.0970				40					
0.0980									40
0.0990				39					
0.0995									39
0.1000							34		
0.1010				38					
0.1015									38
0.1019		10							
0.1030				37					
0.1040						12			37
0.1050					12				
0.1055	12								
0.1060				36			35		
0.1065									36
0.1080				35					
0.1090			12						
0.1094								12	
0.1100				34					35
0.1110									34
0.1120				33			36		

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses. *(Continued)*

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.1130									33
0.1144		9							
0.1150				32					
0.1160						11			32
0.1175					11				
0.1180							37		
0.1200			11	31					31
0.1205	11								
0.1240							38		
0.1250								11	
0.1270				30					
0.1280						10			
0.1285		8							30
0.1300					10		39		
0.1340			10	29					
0.1350	10								
0.1360									29
0.1380							40		
0.1390				28					
0.1405									28
0.1406								10	
0.1430				27					
0.1440						9			27
0.1443		7							
0.1450					9				
0.1460				26			41		
0.1470									26
0.1480			9	25					
0.1483	9								
0.1495									25
0.1510				24					
0.1520									24
0.1530				23					
0.1540							42		23
0.1550				22					
0.1563								9	
0.1570				21					22
0.1590									21
0.1600					8	8			
0.1610				20					20
0.1620	8	6					43		

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses. *(Continued)*

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.1640				19					
0.1650			8						
0.1660									19
0.1680				18					
0.1695									18
0.1700							44		
0.1719								8	
0.1720				17					
0.1730									17
0.1750				16	7				
0.1760						7			
0.1770	7								16
0.1780				15					
0.1800			7	14			45		15
0.1819		5							
0.1820				13					14
0.1850				12					13
0.1875								7	
0.1880				11					
0.1890									12
0.1900					6				
0.1910				10					11
0.1920	6					6			
0.1935									10
0.1940				9					
0.1960									9
0.1970				8					
0.1990				7					8
0.2010				6					7
0.2030			6						
0.2031								6	
0.2040				5					6
0.2043		4							
0.2050					5				
0.2055									5
0.2070	5			4					
0.2090									4
0.2120				3		5			
0.2130									3
0.2188								5	
0.2190				2					
0.2200			5						

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses. (*Continued*)

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.2210									2
0.2250					4				
0.2253	4								
0.2270				1					
0.2280									1
0.2294		3							
0.2320						4			
0.2340									A
0.2344								4	
0.2380			4						B
0.2420									C
0.2437	3								
0.2450					3				
0.2460									D
0.2500								3	E
0.2520						3			
0.2570									F
0.2576		2							
0.2590			3						
0.2610									G
0.2625	2								
0.2650					2				
0.2656								2	
0.2660									H
0.2720									I
0.2760						2			
0.2770									J
0.2810									K
0.2813								1	
0.2830	1								
0.2840			2						
0.2850					1				
0.2893		1							
0.2900									L
0.2950									M
0.3000			1			1			
0.3020									N
0.3050					0				
0.3065	0								
0.3125								0	
0.3160									O
0.3230									P

Sizes of Wire, Drills and Sheets, arranged Progressively by Diameters or Thicknesses. *(Continued)*

Diam. or Thickness	Am. Steel & Wire Co.	Am. or B. & S. (Wire or Sheets)	Birmingham or Stub's Iron Wire (or Sheets)	Stub's Steel Wire	Trenton Iron Co.	British Imperial Wire	Music Wire (Am. Steel & Wire Co.)	U. S. St'd for Plate	Drill Nos. and Letters
0.3240	......	......	......	......	......	0			
0.3249	......	0							
0.3300	......	......	......	......	2/0				
0.3310	2/0								
0.3320	......	......	......	......	......	......	......	......	Q
0.3390	......	......	......	......	......	......	......	......	R
0.3400	......	......	0						
0.3438	......	......	......	......	......	......	......	2/0	
0.3480	......	......	......	......	......	2/0	......	......	S
0.3580	......	......	......	......	......	......	......	......	T
0.3600	......	......	......	......	3/0				
0.3625	3/0								
0.3648	......	2/0							
0.3680	......	......	......	......	......	......	......	......	U
0.3720	......	......	......	......	......	3/0			
0.3750	......	......	......	......	......	......	......	3/0	
0.3770	......	......	......	......	......	......	......	......	V
0.3800	......	......	2/0						
0.3860	......	......	......	......	......	......	......	......	W
0.3938	4/0								
0.3970	......	......	......	......	......	......	......	......	X
0.4000	......	......	......	......	4/0	4/0			
0.4040	......	......	......	......	......	......	......	......	Y
0.4063	......	......	......	......	......	......	......	4/0	
0.4096	......	3/0							
0.4130	......	......	......	......	......	......	......	......	Z
0.4250	......	......	3/0						
0.4305	5/0								
0.4320	......	......	......	......	......	5/0			
0.4375	......	......	......	......	......	......	......	5/0	
0.4500	......	......	......	......	5/0				
0.4540	......	......	4/0						
0.4600	......	4/0							
0.4615	6/0								
0.4640	......	......	......	......	......	6/0			
0.4688	......	......	......	......	......	......	......	6/0	
0.4900	7/0								
0.5000	......	......	5/0	......	......	7/0	......	7/0	
0.5165	......	5/0							
0.5800	......	6/0							

It should be noted that the diameters corresponding to the numbers in Stub's steel wire gage (drill rod sizes) are not the same as the diameters that correspond to the same numbers of small drill sizes.

STRENGTH AND PROPERTIES OF WIRE ROPE

Kinds of Wire Rope. — Wire rope is made with different numbers of strands and numbers of wires to the strand, according to the purpose for which it is to be used. Hoisting rope is made of 6 strands with 19 wires to the strand. This type of rope is used for elevators of all kinds, mines, conveyors, derricks, etc. A hoisting rope known as "special flexible" is made of 6 strands with 37 wires each, and one known as "extra flexible" rope is made from 8 strands with 19 wires each. These ropes are used for cranes, counterweights, dredges and similar purposes. The standard transmission or haulage rope is made from 6 strands with 7 wires to the strand. This is much stiffer than the standard hoisting rope and will not bend around as small sheaves. It is, however, better adapted for haulage and transmission purposes, because the wires are larger and do not wear through so quickly. A greater factor of safety than is used for hoisting rope is desirable. A rope made of six strands with 12 wires each, known as "running" rope, is also made.

Wire ropes are usually made with a hemp core or center. Sometimes the hemp center is replaced by a wire strand, which adds from 7 to 10 per cent to the strength of the rope, but as the wear on the center is as great as on the outside strands, but little is gained. The tables of hoisting and haulage rope give the breaking stress in tons for ropes made from different grades of iron and steel. The working stress should, as a rule, not be made greater than one-fifth of the breaking stress. The tables are based on data furnished by the American Steel & Wire Co.

In the regular type of rope, the wires of the strands are twisted in one direction and the strands laid into the rope in the opposite direction. In the Lang's lay rope, both the wires in the strands and the strands in the rope are twisted in the same direction. This rope is more easily untwisted than the regular lay rope and it is more difficult to tuck the strands securely in a splice, but it is well adapted to withstand external wear and grip action.

Galvanized iron wire rope is used for derrick guys and also for stays aboard ship. Galvanized steel wire strands are used for smoke stack guys and similar purposes.

To preserve wire rope, linseed oil should be applied to it. The grooves of cast-iron pulleys and sheaves should be fitted with blocks of hardwood set on end, which are renewed when worn out. This will save wear and increase adhesion. When large sheaves run at high velocity, the grooves should be lined with leather set on end or with India rubber. This is done in the case of sheaves used in the transmission of power between distant points.

Stresses in Wire Ropes Due to Bending. — When wire rope is used over a pulley or sheave there is a longitudinal tension due to the weight suspended by the rope and the weight of the rope itself, and also tension and compression stresses in the part of the rope resting on the pulley due to the bending of the rope. The smaller the sheave the larger are these bending stresses. In order not to increase these stresses too much and cause undue wear in the wire rope, certain *minimum* sizes of sheaves for different sizes of ropes are recommended by the manufacturers of wire ropes and given in the accompanying tables, but whenever a larger diameter can be conveniently used, it is preferable. Various formulas have been deduced for calculating the stresses in wire rope due to the bending over the sheaves. The following, based on a formula used by a prominent wire rope manufacturer, gives safe results:

$$\text{Stress in pounds per square inch} = \frac{Ed}{D} \times 0.45$$

in which E = modulus of elasticity of the material from which the wire is made, about 30,000,000 for steel and 25,000,000 for iron wire;
d = diameter of component wire in inches;
D = diameter of sheave in inches.

In the formula given, the diameter of component wire is used. This diameter is approximately found as follows:

For 6 by 7 rope, $d = 0.106 \times$ diameter of rope;
For 6 by 19 rope, $d = 0.063 \times$ diameter of rope;
For 6 by 37 rope, $d = 0.045 \times$ diameter of rope;
For 8 by 19 rope, $d = 0.050 \times$ diameter of rope.

The total area in square inches of wires of different ropes is given in a following table.

Example: Find the stress due to bending a ½-inch, 6 by 7 extra strong crucible steel-wire rope over a sheave 24 inches in diameter.

$$\text{Stress in pounds per square inch} = \frac{30{,}000{,}000 \times 0.106 \times \frac{1}{2}}{24} \times 0.45 = 29{,}800 \text{ lbs.}$$

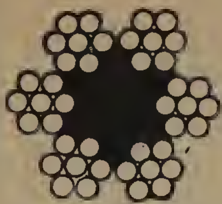
Area of rope (from table) = 0.0926 square inch.

Hence, stress due to bending = 2800 pounds, approximately, or 1.4 ton. Breaking stress of rope (from table) = 8.8 tons. Total safe stress = 8.8 ÷ 5 = 1.76 ton. Hence, in this case, the stress due to bending over the sheave is about one-sixth of the total breaking strength of the rope, which, with a factor of safety of 5, leaves a very small margin for the load. This illustrates the need of using as large sheaves as possible.

Total Area in Square Inches of Wires of Different Ropes

Diameter of Rope	6×7 Construction	6×19 Construction	6×37 Construction	8×19 Construction
2¾	......	2.6892	2.6704	......
2½	......	2.2224	2.2068	......
2¼	......	1.8000	1.7876	......
2	......	1.4216	1.4124	......
1¾	......	1.0888	1.0812	......
1⅝	......	0.9390	0.9324	......
1½	0.8334	0.7997	0.7929	0.6714
1⅜	0.7007	0.6723	0.6676	0.5643
1¼	0.5791	0.5556	0.5517	0.4664
1⅛	0.4691	0.4500	0.4469	0.3778
1	0.3706	0.3554	0.3531	0.2985
⅞	0.2840	0.2722	0.2703	0.2285
¾	0.2134	0.1999	0.1982	0.1678
⅝	0.1448	0.1389	0.1379	0.1166
9/16	0.1173	0.1125	0.1117	0.0945
½	0.0926	0.0889	0.0883	0.0746
7/16	0.0710	0.0681	0.0676	0.0571
⅜	0.0534	0.0500	0.0495	0.0419
5/16	0.0362	0.0347	0.0345	0.0291
¼	0.0231	0.0222	0.0221	0.0186

Dimensions and Strength of Haulage Rope



Standard transmission, haulage or standing rope is made of 6 strands and a hemp center, with 7 wires to the strand. The advantage of this rope is that the wires are coarse and resist abrasion and corrosion to the greatest possible extent, but the rope is not highly flexible.

Diam., Ins.	Approx. Circumference, Ins.	Weight per Ft., Lbs.	Iron			Crucible Steel		
			Breaking Strength, Tons (2000 Lbs.)	Safe Working Load, Tons	Minimum Size of Sheave, Feet	Breaking Strength, Tons (2000 Lbs.)	Safe Working Load, Tons	Minimum Size of Sheave, Feet
1½	4¾	3.55	32.0	6.40	16.0	63.0	12.6	11.0
1⅜	4¼	3.00	28.0	5.60	15.0	53.0	10.6	10.0
1¼	4	2.45	23.0	4.60	13.0	46.0	9.2	9.0
1⅛	3½	2.00	19.0	3.80	12.0	37.0	7.4	8.0
1	3	1.58	15.0	3.00	10.5	31.0	6.2	7.0
⅞	2¾	1.20	12.0	2.40	9.0	24.0	4.8	6.0
¾	2¼	0.89	8.8	1.70	7.5	18.6	3.7	5.0
11/16	2⅛	0.75	7.3	1.50	7.25	15.4	3.1	4.75
⅝	2	0.62	6.0	1.20	7.0	13.0	2.6	4.5
9/16	1¾	0.50	4.8	0.96	6.0	10.0	2.0	4.0
½	1½	0.39	3.7	0.74	5.5	7.7	1.5	3.5
7/16	1¼	0.30	2.6	0.52	4.5	5.5	1.1	3.0
⅜	1⅛	0.22	2.2	0.44	4.0	4.6	0.9	2.75
5/16	1	0.15	1.7	0.34	3.5	3.5	0.7	2.25
9/32	⅞	0.12	1.2	0.24	3.0	2.5	0.5	1.75

Diam., Ins.	Extra Strong Crucible Steel		Plow Steel		Crucible and Plow Steel, Min. Size of Sheave, Feet	Monitor Plow Steel		
	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Strength, Tons (2000 Lbs.)	Safe Working Load, Tons		Breaking Strength, Tons (2000 Lbs.)	Safe Working Load, Tons	Minimum Size of Sheave, Feet
1½	73.0	14.6	82.0	16.4	11.0	90.0	18.0	11.0
1⅜	63.0	12.6	72.0	14.4	10.0	79.0	16.0	10.0
1¼	54.0	10.8	60.0	12.0	9.0	67.0	13.0	9.0
1⅛	43.0	8.6	47.0	9.4	8.0	52.0	10.0	8.0
1	35.0	7.0	38.0	7.6	7.0	42.0	8.4	7.0
⅞	28.0	5.6	31.0	6.2	6.0	33.0	6.6	6.0
¾	21.0	4.2	23.0	4.6	5.0	25.0	5.0	5.0
11/16	16.7	3.3	18.0	3.6	4.75	20.0	4.0	4.75
⅝	14.5	2.9	16.0	3.2	4.5	17.5	3.5	4.5
9/16	11.0	2.2	12.0	2.4	4.0	13.0	2.6	4.0
½	8.8	1.8	10.0	2.0	3.5	11.0	2.2	3.5
7/16	6.2	1.2	7.0	1.4	3.0	7.75	1.5	3.0
⅜	5.2	1.0	5.9	1.2	2.75	6.5	1.3	2.5
5/16	3.9	0.8	4.4	0 9	2.25	......	......	......
9/32	2.9	0.6	3.4	0.7	1.75	......	......	......

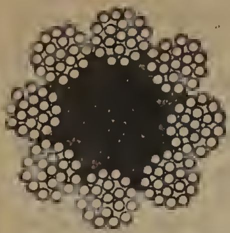
Dimensions and Strength of Standard Hoisting Rope

Standard hoisting rope is made of 6 strands and a hemp center, with 19 wires to the strand. The wires are comparatively small in diameter, and the wire rope, therefore, passes readily around sheaves and drums of moderate size.

Diam., Inches	Approx. Circumference, Ins.	Weight per Foot, Lbs.	Iron			Crucible Steel		
			Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Minimum Size of Sheave, Feet	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Minimum Size of Sheave, Feet
2¾	8⅝	11.95	111.0	22.2	17.0	211.0	42.2	11.0
2½	7⅞	9.85	92.0	18.4	15.0	170.0	34.0	10.0
2¼	7⅛	8.00	72.0	14.4	14.0	133.0	26.6	9.0
2	6¼	6.30	55.0	11.0	12.0	106.0	21.2	8.0
1⅞	5¾	5.55	50.0	10.0	12.0	96.0	19.0	8.0
1¾	5½	4.85	44.0	8.8	11.0	85.0	17.0	7.0
1⅝	5	4.15	38.0	7.6	10.0	72.0	14.4	6.5
1½	4¾	3.55	33.0	6.6	9.0	64.0	12.8	6.0
1⅜	4¼	3.00	28.0	5.6	8.5	56.0	11.2	5.5
1¼	4	2.45	22.8	4.6	7.5	47.0	9.4	5.0
1⅛	3½	2.00	18.6	3.7	7.0	38.0	7.6	4.5
1	3	1.58	14.5	2.9	6.0	30.0	6.0	4.0
⅞	2¾	1.20	11.8	2.4	5.5	23.0	4.6	3.5
¾	2¼	0.89	8.5	1.7	4.5	17.5	3.5	3.0
⅝	2	0.62	6.0	1.2	4.0	12.5	2.5	2.5
9/16	1¾	0.50	4.7	0.94	3.5	10.0	2.0	2.25
½	1½	0.39	3.9	0.78	3.0	8.4	1.68	2.00
7/16	1¼	0.30	2.9	0.58	2.75	6.5	1.30	1.75
⅜	1⅛	0.22	2.4	0.48	2.25	4.8	0.96	1.50
5/16	1	0.15	1.5	0.30	2.00	3.1	0.62	1.25
¼	¾	0.10	1.1	0.22	1.50	2.2	0.44	1.00

Diam., Inches	Extra Strong Crucible Steel		Plow Steel		Crucible and Plow Steel, Min. Size of Sheave, Feet	Monitor Plow Steel		
	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons		Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Min. Size of Sheave, Feet
2¾	243.0	48.6	275.0	55.0	11.0	315.0	63.0	11.0
2½	200.0	40.0	229.0	46.0	10.0	263.0	53.0	10.0
2¼	160.0	32.0	186.0	37.0	9.0	210.0	42.0	9.0
2	123.0	24.6	140.0	28.0	8.0	166.0	33.0	8.0
1⅞	112.0	22.4	127.0	25.0	8.0	150.0	30.0	8.0
1¾	99.0	19.8	112.0	22.0	7.0	133.0	27.0	7.0
1⅝	83.0	16.6	94.0	19.0	6.5	110.0	22.0	6.5
1½	73.0	14.6	82.0	16.0	6.0	98.0	20.0	6.0
1⅜	64.0	12.8	72.0	14.0	5.5	84.0	17.0	5.5
1¼	53.0	10.6	58.0	12.0	5.0	69.0	14.0	5.0
1⅛	43.0	8.6	47.0	9.4	4.5	56.0	11.0	4.5
1	34.0	6.8	38.0	7.6	4.0	45.0	9.0	4.0
⅞	26.0	5.2	29.0	5.8	3.5	35.0	7.0	3.5
¾	20.2	4.0	23.0	4.6	3.0	26.3	5.3	3.0
⅝	14.0	2.8	15.5	3.1	2.5	19.0	3.8	2.5
9/16	11.2	2.24	12.3	2.4	2.25	14.5	2.9	2.25
½	9.2	1.84	10.0	2.0	2.00	12.1	2.4	2.00
7/16	7.25	1.45	8.0	1.6	1.75	9.4	1.9	1.75
⅜	5.30	1.06	5.75	1.15	1.50	6.75	1.35	1.50
5/16	3.50	0.70	3.80	0.76	1.25	4.50	0.90	1.25
¼	2.43	0.49	2.65	0.53	1.00	3.15	0.63	1.00

Dimensions and Strength of Extra Flexible Hoisting Rope



Extra flexible hoisting rope is composed of 8 strands and a hemp center, with 19 wires to the strand. It is very flexible and can be used over comparatively small sheaves and drums.

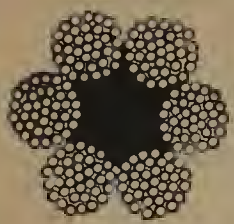
Diam., Ins.	Crucible Steel		Extra Strong Crucible Steel		Plow Steel		Monitor Plow Steel		Min. Size of Sheave, Feet
	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	
1½	58.0	11.6	66.0	13.0	74.0	14.8	80.0	16.0	3.75
1⅜	51.0	10.2	57.0	11.0	64.0	12.8	68.0	13.0	3.50
1¼	42.0	8.4	47.0	9.4	52.0	10.4	56.0	11.0	3.20
1⅛	34.0	6.8	38.0	7.6	43.0	8.6	46.0	9.2	2.83
1	26.0	5.2	29.7	5.9	33.0	6.6	36.0	7.2	2.50
⅞	20.0	4.0	23.0	4.6	26.0	5.2	28.0	5.6	2.16
¾	15.3	3.1	17.6	3.5	20.0	4.0	22.0	4.4	1.83
⅝	10.9	2.2	12.4	2.5	14.0	2.8	15.0	3.0	1.75
9/16	8.7	1.75	10.1	2.0	11.6	2.3	12.0	2.4	1.50
½	7.3	1.46	8.0	1.6	8.7	1.75	9.5	1.9	1.33
7/16	5.7	1.14	6.3	1.25	6.9	1.38	....	....	1.16
⅜	4.2	0.84	4.7	0.93	5.1	1.02	....	....	1.00
5/16	2.8	0.55	3.1	0.61	3.3	0.67	....	....	0.83
¼	1.8	0.36	2.0	0.40	2.2	0.45	....	....	0.75

Dimensions and Strength of Extra Special Flexible Hoisting Rope

(Made of 6 strands and a hemp center, with 61 wires to a strand.)

Diam., Ins.	Crucible Steel		Extra Strong Crucible Steel		Plow Steel		Monitor Plow Steel		Min. Size of Sheave, Feet
	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	
3¼	280	56	315	63	350	70	370	75	11
3	240	48	275	55	310	62	325	65	10
2¾	200	40	233	47	265	53	278	56	9
2½	160	32	187	37	214	43	225	45	8
2¼	125	25	150	30	175	35	184	37	7
2	105	21	117	23	130	26	137	27	6

Dimensions and Strength of Special Flexible Hoisting Rope



Special flexible hoisting rope is composed of 6 strands and a hemp center, with 37 wires to the strand. This is a very flexible rope and is used to a great extent on cranes and similar machinery, where the sheaves must necessarily be small in diameter. Hoisting ropes larger than 1¾ inch are usually made of 6 strands with 37 wires each, rather than of 6 strands with 19 wires.

Diam., Ins.	Crucible Steel		Extra Strong Crucible Steel		Plow Steel		Monitor Plow Steel		Min. Size of Sheave, Feet
	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	Breaking Str'th, Tons (2000 Lbs.)	Safe Working Load, Tons	
2¾	200	40.0	233.0	47.0	265.0	53.0	278.0	55.0	7.00
2½	160	32.0	187.0	37.0	214.0	43.0	225.0	45.0	6.50
2¼	125	25.0	150.0	30.0	175.0	35.0	184.0	37.0	6.00
2	105	21.0	117.0	23.0	130.0	26.0	137.0	27.0	5.50
1⅞	94	18.8	106.0	21.2	119.0	23.8	125.0	25.0	5.00
1¾	84	17.0	95.0	19.0	108.0	22.0	113.0	23.0	4.50
1⅝	71	14.2	79.0	16.0	90.0	18.0	95.0	19.0	4.00
1½	63	12.6	71.0	14.0	80.0	16.0	84.0	17.0	3.75
1⅜	55	11.0	61.0	12.0	68.0	14.0	71.0	14.0	3.50
1¼	45	9.0	50.0	10.0	55.0	11.0	58.0	11.0	3.20
1⅛	34	7.0	39.0	8.0	44.0	9.0	46.0	9.2	2.83
1	29	6.0	32.0	6.4	35.0	7.0	37.0	7.4	2.50
⅞	23	5.0	25.0	5.0	27.0	5.0	29.0	5.8	2.16
¾	17.5	3.5	19.0	3.8	21.0	4.0	23.0	4.6	1.83
⅝	11.2	2.2	12.6	2.5	14.0	3.0	16.0	3.2	1.75
9/16	9.5	1.9	10.5	2.1	11.5	2.3	12.5	2.5	1.50
½	7.2	1.5	8.25	1.65	9.25	1.85	9.75	1.9	1.33
7/16	5.5	1.1	6.35	1.27	7.20	1.40	7.50	1.5	1.15
⅜	4.2	0.8	4.65	0.93	5.10	1.00	5.30	1.16	1.00

Galvanized Steel Wire Strand

(Single Strand of 7 Wires)

Diam., Ins.	Weight per Foot, Lbs.	Breaking Stress, Lbs.	Diam., Ins.	Weight per Foot, Lbs.	Breaking Stress, Lbs.	Diam., Ins.	Weight per Foot, Lbs.	Breaking Stress, Lbs.
½	0.510	8500	5/16	0.210	3800	3/16	0.075	1400
7/16	0.415	6500	¼	0.125	2300	5/32	0.055	900
⅜	0.295	5000	7/32	0.095	1800	⅛	0.032	500

Dimensions and Strength of Galvanized Iron Wire Rope

Approximate Diameter, Inches	Circumference, Inches	Circumference of New Manila Rope of Equal Strength, Ins.	Weight per Foot, Pounds	Breaking Stress in Tons (2000 Lbs.)	Approximate Diameter, Inches	Circumference, Inches	Circumference of New Manila Rope of Equal Strength, Ins.	Weight per Foot, Pounds	Breaking Stress in Tons (2000 Lbs.)
Six strands with 7 or 12 wires each									
1¾	5½	11	4.85	44	1	3	5¾	1.44	13.0
1 11/16	5¼	10½	4.40	40	⅞	2¾	5¼	1.21	11.0
1⅝	5	10	4.00	36	13/16	2½	5	1.00	9.0
1½	4¾	9½	3.60	32	¾	2¼	4¾	0.81	7.3
1 7/16	4½	9	3.25	29	⅝	2	4½	0.64	5.8
1⅜	4¼	8½	2.90	26	9/16	1¾	3¾	0.49	4.4
1¼	4	8	2.55	23	½	1½	3	0.36	3.2
1 3/16	3¾	7½	2.25	20	7/16	1¼	2½	0.25	2.3
1⅛	3½	6½	1.95	18	⅜	1⅛	2¼	0.20	1.8
1 1/16	3¼	6	1.70	15	5/16	1	2	0.16	1.4
Five strands with 7 wires each									
9/32	⅞	1¾	0.123	1.10	7/32	⅝	1¼	0.063	0.56
¼	¾	1½	0.090	0.81	3/16	½	1⅛	0.040	0.36

Strength of Wire Rope Drums. — Usually if a drum is designed to be strong enough to resist crushing, it will be strong enough to resist bending also, except in cases of extremely long drums. To be on the safe side, however, the strength should be calculated for both crushing and bending. First consider the calculations to find the crushing stresses. Let:

T = tension in one rope, in pounds;
t = thickness of drum in inches at bottom of groove;
P = pitch of scoring, or distance between grooves in inches;
L = span in inches from center to center of bearings;
D = diameter at the bottom of the groove;
d = inside diameter of drum;
W = total load in pounds;
M_b = bending moment.

Then calculate the total crushing stresses as follows:

$$\text{Section modulus} = Z = 0.0982\left(D^3 - \frac{d^4}{D}\right);\quad M_b = \frac{WL}{4}$$

$$B = \frac{M_b}{Z};\quad C = \frac{T}{Pt}.$$

$$\text{Combined stress} = \sqrt{B^2 + C^2} \qquad (1)$$

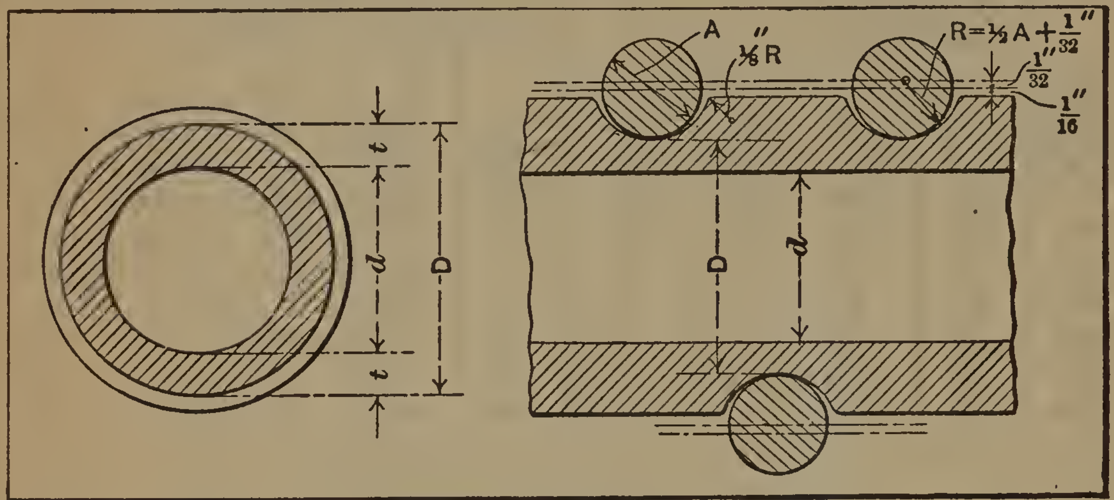
The combined stresses should not exceed 6000 pounds per square inch for cast iron.

To find the strength for bending, let S = safe fiber stress, which should not exceed 3000 pounds per square inch for cast iron.

The diameter of the hole through the drum may then be found from the following formula:

$$d = \sqrt[4]{D^4 - \frac{D \times M_b}{0.0982\,S}} \qquad (2)$$

As an example, assume that on a five-ton crane, two 7/16-inch ropes are used; that the pitch of scoring on the drum is 3/4 inch; and that the distance from center to center of the bearings on the drum is 6 feet 2 inches. The inside diameter of the



drum is 12 inches, and the diameter at the bottom of the grooves, 14 11/16 inches. From the formulas given:

$$Z = 0.0982\left(14.69^3 - \frac{12^4}{14.69}\right) = 172.6;$$

$$M_b = \frac{10{,}000 \times 74}{4} = 185{,}000;$$

$$B = \frac{185{,}000}{172.6} = 1072; \quad C = \frac{5000}{\frac{3}{4} \times 1.34} = 5000.$$

Combined stress $= \sqrt{1072^2 + 5000^2} = 5113$ pounds per square inch.

As this is less than the allowable stress of 6000 pounds per square inch, the drum is safe from crushing.

To determine if the drum will withstand bending, use Formula (2), assuming the diameter at the bottom of the grooves to be 14 11/16 inches and solving for the inside diameter:

$$d = \sqrt[4]{14.69^4 - \frac{14.69 \times 185{,}000}{0.0982 \times 3000}} = 13.9 \text{ inches.}$$

Since the inside diameter is only 12 inches instead of 13.9 inches, it follows that the drum is strong enough for its purpose.

Power Transmission by Wire Rope. — In power transmission by wire rope, a rule frequently followed is to make the minimum diameter of the sheave equal to 100 rope diameters. The radius at the bottom of the groove in the sheaves should always be greater than that of the rope itself, so that the wire rope drives simply through the friction in the bottom of the groove, which is lined with soft rubber, leather, wood or some similar comparatively soft material.

Life of Wire Rope. — The life of wire rope depends in the first place upon the proportions of the diameter of the pulley or sheave to the diameter of the rope.

In general, increasing the diameter of the pulleys by an amount equal to two circumferences of the rope, will double the life of the rope. When ropes are worked over a number of pulleys, the number of bends influences the length of life of the rope. It has also been shown that reverse bends, that is bending a rope in one direction over one pulley and then in the opposite direction over another pulley, has a more detrimental influence on the life of wire rope than two bends in the same direction. Ropes subjected to reverse bends have a life of only half of that of ropes

Transmission of Power by Wire Ropes

(Ropes of 6 strands with hemp core, each strand consisting of 7 wires)

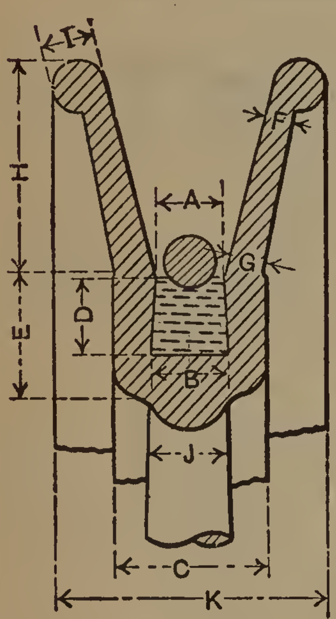
Diam. of Wheel in Feet	No. of Revolutions	Trade No. of Rope	Diam. of Rope, Inches	Horsepower	Diam. of Wheel in Feet	No. of Revolutions	Trade No. of Rope	Diam. of Rope, Inches	Horsepower
3	80	23	3/8	3	9	100	20	9/16	58
3	100	23	3/8	3½	9	100	19	5/8	60
3	120	23	3/8	4	9	120	20	9/16	69
3	140	23	3/8	4½	9	120	19	5/8	73
4	80	23	3/8	4	9	140	20	9/16	82
4	100	23	3/8	5	9	140	19	5/8	84
4	120	23	3/8	6	10	80	19	5/8	64
4	140	23	3/8	7	10	80	18	11/16	68
5	80	22	7/16	9	10	100	19	5/8	80
5	100	22	7/16	11	10	100	18	11/16	85
5	120	22	7/16	13	10	120	19	5/8	96
5	140	22	7/16	15	10	120	18	11/16	102
6	80	21	½	14	10	140	19	5/8	112
6	100	21	½	17	10	140	18	11/16	119
6	120	21	½	20	12	80	18	11/16	93
6	140	21	½	23	12	80	17	¾	99
7	80	20	9/16	20	12	100	18	11/16	116
7	100	20	9/16	25	12	100	17	¾	124
7	120	20	9/16	30	12	120	18	11/16	140
7	140	20	9/16	35	12	120	17	¾	149
8	80	19	5/8	26	12	120	16	7/8	173
8	100	19	5/8	32	14	80	8	1	141
8	120	19	5/8	39	14	80	7	1⅛	148
8	140	19	5/8	45	14	100	8	1	176
9	80	20	9/16	47	14	100	7	1⅛	185
9	80	19	5/8	48	.....	.....	.....	.....	.....

bent in one direction. If it be assumed that a rope making three bends in working, that is, one at the upper drum and one on each side of the lower pulley, as is the arrangement most frequently adopted in crane practice, has a life of 100, then the relative life of ropes with different bends is as follows:

1 single bend, 300; 3 bends in the same direction, 100; 3 bends of which one is a reverse bend, 75; 7 bends, 43; 7 bends of which one is a reverse bend, 37.

This indicates that it is important to increase the diameters of the pulleys when the rope is bent over more than one pulley, in order that the length of life of the rope may be approximately the same when bent over several pulleys, as when subjected to a single bend only. The effect of oiling ropes, upon the length of life, is to increase the life of a given rope by two to three times.

Dimensions of Wire-rope Pulleys for Power Transmission



Arms are made with elliptic cross-section.

Diam. of Rope	Diam. of Pulley, Feet	Dimensions of Pulley Rim, Inches A	B	C	D	E
½	3	⅝	¾	1¾	¾	1⅜
⅝	4	¾	⅞	2⅛	⅞	1½
¾	5	⅞	1	2⅜	1⅛	1¾
⅞	6	1	1⅛	2⅝	1¼	2
1	7	1⅛	1¼	2¾	1¼	2⅛
1⅛	8	1¼	1⅜	2⅞	1⅜	2¼
1¼	9	1⅜	1½	3⅛	1½	2½
1⅜	10	1½	1⅝	3¼	1 9/16	2⅝
1½	12	1⅝	1¾	3⅝	1¾	3

Diam. of Rope	F	G	H	I	J	K
½	¼	⅜	2	9/16	⅞	3
⅝	5/16	7/16	2½	⅝	1	3½
¾	⅜	½	3	¾	1⅛	4
⅞	⅜	½	3½	¾	1¼	4½
1	7/16	9/16	3¾	⅞	1⅜	4¾
1⅛	½	⅝	4	1	1½	5
1¼	½	⅝	4¼	1	1⅝	5¼
1⅜	9/16	11/16	4½	1⅛	1¾	5½
1½	⅝	¾	4¾	1¼	2	6

Definitions of Wire Rope Terms

Aeroplane Strand: A small 7- or 19-wire galvanized strand made from plow steel or crucible steel wire.

Arc Light Rope: Rope consisting of 9 strands of 4 or 7 galvanized wires and a hemp center.

Bicycle Cord: Small rope consisting of 19 strands of 3 wires each, made from crucible or plow steel.

Cable Laid Rope: A compound laid rope consisting of several ropes or several layers of strands laid together into one rope, as, for instance, 6 by 6 by 7.

Crane Rope: Wire rope consisting of 6 strands of 37 wires around a hemp center.

Crosby Clip: A grooved casting and U-shaped bolt with nuts for fastening wire ropes together.

Dragon Rope: A 6 by 25 triangular flattened strand rope with alternate regular and Lang lay strands, usually made with hemp center.

Elevator Rope: Wire rope usually made of iron and composed of 6 strands of 19 wires each, and a hemp core.

Extra Flexible Hoisting Rope: A rope consisting of 8 strands of 19 wires each with a large hemp center.

Ferry Rope: Rope consisting of 6 strands, 7 wires each, either bright or galvanized.

Flat Rope: A rope consisting of alternate right and left lay rope strands, each rope strand consisting of 4 strands of 7 wires, all sewed together with a number of soft iron sewing wires.

Flattened Strand Rope: A wire rope having non-cylindrical strands, usually of the oval or triangular type; the center wire of each strand is an oval or a triangular wire.

Galvanized Signal Strand: A 7-wire strand made up from single galvanized wire; sometimes made with 19 wires.

Guy Rope: Galvanized rope consisting of 6 strands, 7 wires each, and a hemp core.

Guy Strand: Galvanized 7-wire strand.

Hand Rope: Flexible rope consisting of 6 ropes, each composed of 6 strands, 7 wires each, and 7 hemp cores.

Haulage Rope: Rope usually composed of 6 strands, 7 wires each, and a hemp core.

Hawser: Wire rope usually consisting of 6 strands, 37 wires, and a hemp core, or 6 strands, 24 wires, and 7 hemp cores.

Hoisting Rope: Rope consisting of 6 strands of 19 wires each, with a hemp center.

Lang Lay Rope: Wire rope in which both the wires in the strands and the strands in the rope are twisted in the same direction.

Left Lay Rope: Wire rope, the strands of which form a left-hand helix like a left-hand screw thread.

Left Twist: Same as right lay, and corresponds to a right-hand screw thread.

Lay: The pitch or angle of the helix of the wires or strands of a rope, usually expressed by the ratio of the diameter of the strand or rope to the length required for one complete twist.

Lloyd's Hawser: Wire rope composed of 6 strands, 24 wires, and 7 hemp cores.

Messenger Strand: Seven-wire galvanized strand.

Non-spinning Rope: A wire rope consisting of 18 strands of 7 wires each in two layers; the inner layer consists of 6 strands Lang lay and left lay around a small hemp core, and the outer of 12 strands regular lay, right-hand lay. Will carry a load on a single end without untwisting.

Regular Lay: Strands twisted to the right and rope twisted to the left. Helix of strands takes the direction of a right-hand screw thread.

Reverse Laid Rope: A wire rope with alternate strands right and left lay.

Rheostat Rope: A small rope consisting of 8 strands of 7 wires each.

Right Lay: Known also as regular lay; strands twisted to the right and rope twisted to the left; corresponds to a right-hand screw thread.

Right Twist: Corresponds to left lay, or to a left-hand screw thread.

Running Rope: A flexible rope of 6 strands, 12 wires each, and 7 hemp cores.

Sand Line: A small rope composed of 6 strands of 7 wires each.

Sash Cord: Small rope consisting of 6 strands, 7 wires, and one hemp core; sizes ¼ inch and smaller; plain or galvanized.

Seizing Strand: Small galvanized 7-wire strand usually made in sizes ⅛ inch diameter and smaller.

Signal Strand: Usually consists of a 7-wire galvanized strand.

Special Flexible Hoisting Rope: A wire rope consisting of 6 strands, of 37 wires each, and a hemp core.

Standing Rope: Another term applied to galvanized guy rope which consists of 6 strands, 7 wires, and a hemp core.

Tiller Rope: Rope consisting of 6 ropes of 6 strands, each of 7 wires, and 7 hemp cores.

Towing Hawser: A large flexible wire rope made of galvanized wires. Usual construction, 6 strands of 37 wires each, or 6 strands of 24 wires each.

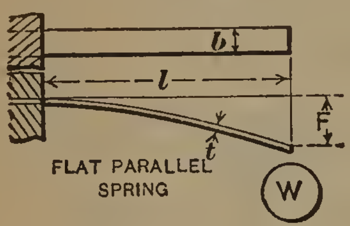
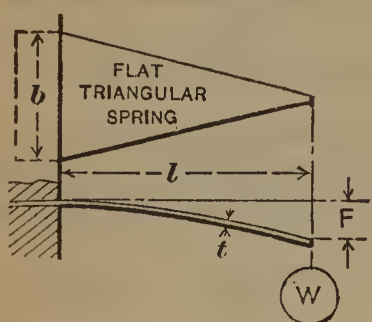
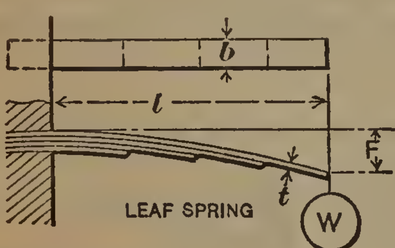
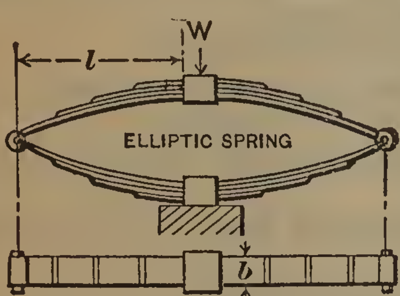
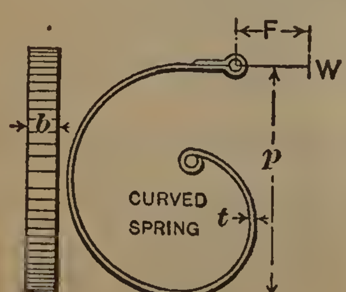
Transmission Rope: Rope composed of 6 strands, 7 wires each, and a hemp core.

Universal Lay: Another name for Lang lay.

SPRINGS

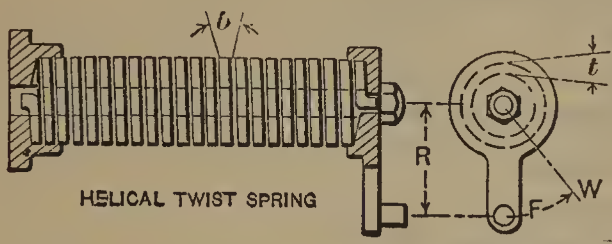
Formulas for Strength and Deflection of Springs

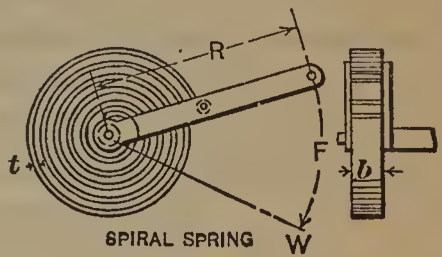
W = safe load, pull or pressure in pounds;
F = deflection at point of application of load, in inches;
S = safe tensile strength of the material, in pounds per square inch;
E = modulus of elasticity = 30,000,000 for steel.

Type of Spring	W, Safe Load	F, Deflection
	$\frac{Sbt^2}{6l}$	$\frac{4Wl^3}{Ebt^3} = \frac{2Sl^2}{3Et}$
	$\frac{Sbt^2}{6l}$	$\frac{6Wl^3}{Ebt^3} = \frac{Sl^2}{Et}$
	$\frac{SNbt^2}{6l}$ in which N = number of leaves.	$\frac{6Wl^3}{ENbt^3} = \frac{Sl^2}{Et}$ in which N = number of leaves.
	$\frac{SNbt^2}{3l}$ in which N = number of leaves in one-half of the spring; t = thickness of leaf.	$\frac{6Wl^3}{ENbt^3} = \frac{2Sl^2}{Et}$ in which N = number of leaves in one-half of the spring; t = thickness of leaf.
	$\frac{Sbt^2}{6p}$	$\frac{18Wp^3}{Ebt^3} = \frac{3Sp^2}{Et}$

Formulas for Strength and Deflection of Springs

W = safe load, pull or pressure in pounds;
F = deflection at point of application of load, in inches;
S = safe tensile strength of the material in pounds per square inch (in the torsion and helical compression and extension springs, S = torsional or shearing stress); E = modulus of elasticity = 30,000,000 for steel; G = torsional modulus of elasticity = 12,600,000 for steel.





W = safe load.

For flat or square steel: $W = \dfrac{Sbt^2}{6R}$ For round steel (d = diam. of rod): $W = \dfrac{Sd^3}{10R}$

F = deflection.

For flat or square steel (l = length of the rod or uncoiled spring): $F = \dfrac{12\,WlR^2}{Ebt^3} = \dfrac{2\,SlR}{Et}$

For round steel (d = diameter of rod; l = length, as above): $F = \dfrac{20\,WlR^2}{Ed^4} = \dfrac{2\,SlR}{Ed}$

If U = deflection expressed in revolutions of the lever:

$U = \dfrac{Sl}{3\,Et}$ for flat and square, and $\dfrac{Sl}{3\,Ed}$ for round steel, approx.

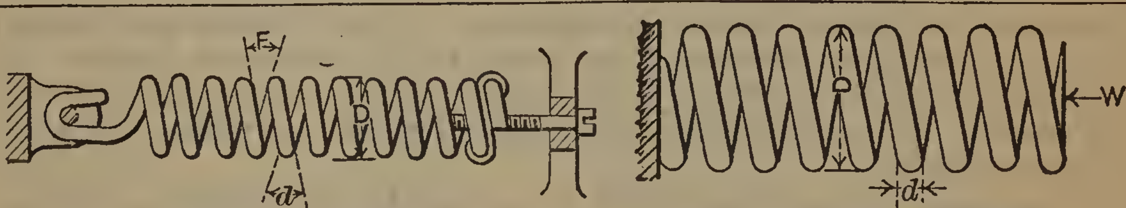
W = safe load.

For square rod: $W = \dfrac{Sd^3}{4R}$ approx.

For round rod: $W = \dfrac{Sd^3}{5R}$

F = deflection.

For square rod: $F = \dfrac{6\,WR^2l}{Gd^4} = 1.5\dfrac{SlR}{Gd}$ For round rod: $F = \dfrac{10\,WR^2l}{Gd^4} = \dfrac{2\,SlR}{Gd}$



W = safe load.

For square rod (d = side of square): $W = 0.47\dfrac{Sd^3}{D-d}$ approx.

For round rod: $W = 0.4\dfrac{Sd^3}{D-d}$

F = deflection of one coil.

For square rod (d = side of square): $F = \dfrac{4.7\,W\,(D-d)^3}{Gd^4} = \dfrac{2.2\,S\,(D-d)^2}{Gd}$

For round rod: $F = \dfrac{8\,W\,(D-d)^3}{Gd^4} = \dfrac{3.14\,S\,(D-d)^2}{Gd}$

Maximum Safe Stresses in Coil Springs. — The following values may be used for the torsional or shearing stresses in coil springs made from a good grade of steel. Assume the ratio of the mean diameter of the spring to the diameter of the bar to equal R; then:

For bars below 3/8 inch diameter:

$R = 3$ $S = 112{,}000$ pounds per square inch.
$R = 8$ $S = 85{,}000$ pounds per square inch.

For bars 7/16 to 3/4 inch in diameter:

$R = 3$ $S = 110{,}000$ pounds per square inch.
$R = 8$ $S = 80{,}000$ pounds per square inch.

For bars from 13/16 to 1 1/4 inch in diameter:

$R = 3$ $S = 105{,}000$ pounds per square inch.
$R = 8$ $S = 75{,}000$ pounds per square inch.

For bars over 1 1/4 inch in diameter a stress of more than 100,000 should not be used. Where a spring is subjected to sudden shocks, a smaller value of S is necessary.

These values are applicable to compression springs with open coils. Experience has shown that in close-coiled springs and extension springs the safe value of the stress per square inch, S, is only about two-thirds of that for open-coiled compression springs of the same dimensions. The safe torsional or shearing strength for spring brass and phosphor-bronze may be taken as 25,000 pounds per square inch. The torsional modulus of elasticity may be taken as 6,000,000 for spring brass and phosphor-bronze, and 12,600,000 for steel.

An analysis of spring steel frequently used, and known as the P.R.R. analysis, is as follows: Carbon 1.0 per cent (not under 0.90 per cent); phosphorus, 0.05 per cent (not over 0.07 per cent); manganese, 0.25 per cent (not over 0.50 per cent); silicon, not over 0.10 per cent; sulphur, not over 0.03 per cent.

The best proportions for coil springs is to use an outside diameter of the spring equal to from five to eight times the diameter of wire or bar from which the spring is made; under no circumstances should the outside diameter be made less than four times the diameter of the wire. The effective number of coils in a compression spring may be considered as 2 less than the actual number, owing to the squared ends of the spring. Springs of small diameter may be safely subjected to a higher unit stress than those of large diameter.

Formulas for Helical Round Bar Springs. — In the following are tabulated all formulas relating to helical round bar springs that are required in practice. In each case, the general formula is given, and, below, the formula is simplified for steel springs. All dimensions are in inches and all weights and stresses in pounds. In the formulas:

d = diameter of bar from which spring is made;
D = mean diameter of coil;
F = total deflection under load P; the deflection F_1 under any other load is proportionate to this deflection;
h = solid height of spring;
H = free height of spring;
L = length of bar from which spring is made;
W = weight of bar, or spring;
w = weight of material per cubic inch;
P = carrying capacity of spring or weight that will compress the spring solidly;
S = maximum fiber stress, assumed as 80,000 pounds per square inch for steel;
G = torsional modulus of elasticity, assumed as 12,600,000 for steel.

Formulas for Helical Round Bar Springs — 1

(For notation used in formulas, see previous page)

To be Found	Formulas	Examples
P, maximum carrying capacity	$P = \frac{GFd^5}{8\,hD^3}$ $= \frac{3.14\,Sd^3}{8\,D}$ or, for steel springs: $P = 1{,}575{,}000\,\frac{Fd^5}{hD^3}$ $= 31{,}400\,\frac{d^3}{D}$	Steel spring, outside diameter $= 2\frac{7}{8}$ inches; Diameter of bar $= \frac{1}{2}$ inch; Free height $= 14\frac{1}{2}$ inches; Solid height $= 10$ inches. $P = 1{,}575{,}000 \times \frac{4.5 \times 0.5^5}{10 \times 2.375^3}$ $= 1650$ pounds.
F, total deflection under load P	$F = \frac{3.14\,ShD^2}{Gd^2} = H - h$ $= \frac{H}{1 + \frac{Gd^2}{3.14\,SD^2}}$ or, for steel springs: $F = 0.02\,h\left(\frac{D}{d}\right)^2$ $= \frac{H}{1 + 50.1\left(\frac{d}{D}\right)^2}$	Steel spring, outside diameter $= 4\frac{1}{4}$ inches; Diameter of bar $= \frac{3}{4}$ inch; Solid height $= 10$ inches. $F = 0.02 \times 10 \times \left(\frac{3\frac{1}{2}}{\frac{3}{4}}\right)^2 = 4.34$ inches. Outside diameter $= 5\frac{1}{2}$ inches; Diameter of bar $= 1\frac{3}{8}$ inch; Free height $= 11\frac{3}{4}$ inches. $F = \frac{11\frac{3}{4}}{1 + 50.1 \times \left(\frac{1\frac{3}{8}}{4\frac{1}{8}}\right)^2} = 1\frac{3}{4}$ inch.
F_1, deflection under any load P_1 less than P	$F_1 = \frac{8\,P_1hD^3}{Gd^5}$ or, for steel springs: $F_1 = 0.0000006\,\frac{P_1hD^3}{d^5}$	Steel spring, outside diameter $= 4\frac{1}{4}$ inches; Diameter of bar $= \frac{3}{4}$ inch; Solid height $= 10$ inches; Load $P_1 = 1000$ pounds. $F_1 = 0.0000006\,\frac{1000 \times 10 \times (3\frac{1}{2})^3}{(\frac{3}{4})^5}$ $= 1.08$ inch.
S, maximum fiber stress	$S = \frac{8\,DP}{3.14\,d^3}$ $= \frac{GF}{3.14\,h}\left(\frac{d}{D}\right)^2$ or, for steel springs: $S = 2.55\,\frac{DP}{d^3}$ $= 4{,}000{,}000\,\frac{F}{h}\left(\frac{d}{D}\right)^2$	Steel spring, outside diameter $= 4\frac{1}{2}$ inches; Diameter of bar $= \frac{1}{2}$ inch; Free height $= 22\frac{3}{4}$ inches; Solid height $= 10$ inches. $S = 4{,}000{,}000 \times \frac{12.75}{10} \times \left(\frac{0.5}{4}\right)^2$ $= 80{,}000$ pounds, about.
H, free height	$H = h + \frac{3.14\,Sh}{G}\left(\frac{D}{d}\right)^2$ $= h + F$ or, for steel springs: $H = h + 0.02\,h\left(\frac{D}{d}\right)^2$	Steel spring, outside diameter $= 7\frac{1}{8}$ inches; Diameter of bar $= 1\frac{1}{8}$ inch; Solid height $= 10$ inches. $H = 10 + 0.02 \times 10 \times \left(\frac{6}{1\frac{1}{8}}\right)^2$ $= 15.67$ inches.

Formulas for Helical Round Bar Springs — 2

To be Found	Formulas	Examples
h, solid height	$h = \dfrac{H}{1 + \dfrac{3.14S}{G}\left(\dfrac{D}{d}\right)^2}$ or, for steel springs: $h = \dfrac{H}{1 + 0.02\left(\dfrac{D}{d}\right)^2}$	Steel spring, outside diameter = 6 inches; Diameter of bar = $1\frac{1}{8}$ inch; Free height = $13\frac{3}{4}$ inches. $h = \dfrac{13.75}{1 + 0.02\left(\dfrac{4\frac{7}{8}}{1\frac{1}{8}}\right)^2} = 10$ inches.
h, solid height	Let P_1 = any load less than P; h_1 = height under load P_1. $h = \dfrac{h_1}{1 + \dfrac{3.14S}{G}\left(\dfrac{P - P_1}{P}\right)\left(\dfrac{D}{d}\right)^2}$ or, for steel springs: $h = \dfrac{h_1}{1 + 0.02\left(\dfrac{P - P_1}{P}\right)\left(\dfrac{D}{d}\right)^2}$	Steel spring, outside diameter = $5\frac{1}{2}$ inches; Diameter of bar = $\frac{3}{4}$ inch; Free height = 18 inches. What solid height is required for carrying 1395 pounds at 14 inches? P = 2790 pounds by formula $P = \dfrac{3.14\,Sd^3}{8D}$ Then, $h = \dfrac{14}{1 + 0.02\left(\dfrac{2790 - 1395}{2790}\right)\left(\dfrac{4\frac{3}{4}}{\frac{3}{4}}\right)^2} = 10$ inches
L, length of bar	$L = \dfrac{3.14\,hD}{d}$ $= \dfrac{H}{\dfrac{SD}{Gd} + 0.32\dfrac{d}{D}}$ or, for steel springs: $L = \dfrac{H}{0.0063\dfrac{D}{d} + 0.32\dfrac{d}{D}}$	Steel spring, outside diameter = $4\frac{3}{8}$ inches; Diameter of bar = $\frac{7}{16}$ inch; Solid height = 10 inches. $L = \dfrac{3.14 \times 10 \times 3\frac{15}{16}}{\frac{7}{16}}$ = 282.7 inches.
W, weight of spring	$W = \dfrac{3.14^2\,dDhw}{4}$ $= \dfrac{2\,GwPF}{S^2}$ or, for steel springs: $W = 0.7\,dDh$ $= 0.0011\,PF$	Steel spring, outside diameter = $3\frac{3}{4}$ inches; Diameter of bar = $\frac{15}{16}$ inch; Solid height = 10 inches. $W = 0.7 \times \frac{15}{16} \times 2\frac{13}{16} \times 10$ = 18.3 pounds.
G, torsional modulus of elasticity	$G = \dfrac{3.14\,Sh}{F}\left(\dfrac{D}{d}\right)^2$ $= \dfrac{8\,PhD^3}{Fd^5}$ or, for steel springs: $G = 251{,}000\,\dfrac{h}{F}\left(\dfrac{D}{d}\right)^2$	Steel spring, outside diameter = $4\frac{7}{8}$ inches; Diameter of bar = $\frac{11}{16}$ inch; Load = 1219 pounds; Deflection = 3.7 inches; Solid height = 10 inches. $G = \dfrac{8 \times 1219 \times 10 \times (4\frac{3}{16})^3}{3.7 \times (\frac{11}{16})^5}$ = 12,600,000.

Formulas for Helical Rectangular Bar Springs

Notation: D, F, F_1, h, H, L, W, w, P, S and G denote the same quantities as for helical round bar springs; see the next preceding pages; b = width of rectangular bar; t = height of rectangular bar (parallel to axis of spring). For square bar springs $b = t$ = side of square. — These formulas give approximate results, and should be used only when the difference between b and t is not very great.

To be Found	Formulas	To be Found	Formulas
P, maximum carrying capacity	$P = \frac{Sbt\sqrt{t^2+b^2}}{3D} = \frac{GFbt^2(t^2+b^2)}{9.4\,hD^3}$ or, for steel springs: $P = \frac{26{,}700\,bt\sqrt{t^2+b^2}}{D} = \frac{1{,}337{,}000\,Fbt^2(t^2+b^2)}{hD^3}$	h, solid height	$h = \frac{H}{1+\frac{3.14\,SD^2}{Gt\sqrt{t^2+b^2}}}$ or, for steel springs: $h = \frac{H}{1+0.02\frac{D^2}{t\sqrt{t^2+b^2}}}$
F, total deflection under load P	$F = \frac{3.14\,ShD^2}{Gt\sqrt{t^2+b^2}} = \frac{H}{1+\frac{Gt\sqrt{t^2+b^2}}{3.14\,SD^2}}$ or, for steel springs: $F = 0.02\frac{hD^2}{t\sqrt{t^2+b^2}} = \frac{H}{1+50.1\frac{t\sqrt{t^2+b^2}}{D^2}}$	L, length of bar	$L = \frac{3.14\,Dh}{t} = \frac{H}{\frac{SD}{G\sqrt{t^2+b^2}}+0.32\frac{t}{D}}$ or, for steel springs: $L = \frac{H}{0.0063\frac{D}{\sqrt{t^2+b^2}}+0.32\frac{t}{D}}$
F_1, deflection under any load P_1 less than P	$F_1 = \frac{9.4\,hD^3P_1}{Gbt^2(t^2+b^2)}$ or, for steel springs: $F_1 = 0.00000075\frac{hD^3P_1}{bt^2(t^2+b^2)}$	H, free height	$H = h + \frac{3.14\,hSD^2}{Gt\sqrt{t^2+b^2}}$ or, for steel springs: $H = h + 0.02\frac{hD^2}{t\sqrt{t^2+b^2}}$
S, maximum fiber stress	$S = \frac{3\,DP}{bt\sqrt{t^2+b^2}} = \frac{GFt\sqrt{t^2+b^2}}{3.14\,hD^2}$ or, for steel springs: $S = 4{,}000{,}000\frac{Ft\sqrt{t^2+b^2}}{hD^2}$	W, weight of spring	$W = 3.14\,bDhw = \frac{3\,GwPF}{S^2}$ or, for steel springs: $W = 0.89\,bDh = 0.0017\,PF.$

Conical Coil Springs. — All dimensions in inches, all weights in pounds. Let:

F = deflection or extension, under load P;
D_1 = mean diameter of largest coil;
D_2 = mean diameter of smallest coil;
p = average horizontal pitch of coils;
d = diameter of bar or wire from which coil is made;
P = capacity of spring, or weight that will compress the spring solidly;
l = length of bar in spring;
W = weight of spring;
h = solid height of spring, assumed to equal $d \times$ number of coils.

In the following formulas the allowable stress is assumed as 80,000 pounds per square inch, and the modulus of torsional elasticity as 12,600,000.

Extension conical springs:

$$F = 0.00249 \frac{D_1^4 - D_2^4}{pdD_1} \qquad P = 31{,}400 \frac{d^3}{D_1}$$

Compression conical springs:

$$F = 0.00332 \frac{D_1^3 - D_2^3}{pd} \qquad P = 31{,}400 \frac{d^3}{D_2}$$

For all conical springs:

$$l = 1.571 \frac{h}{d}(D_1 + D_2) \qquad W = 0.35\, dh\,(D_1 + D_2)$$

If P_x is any load not exceeding P, then the corresponding deflection of a conical extension spring is:

$$F_x = \frac{P_x (D_1^4 - D_2^4)}{Gpd^4}$$

In a compression spring, the formula for the deflection under any load is the same as that just given for P_x in an extension spring, provided the load P_x is not greater than the capacity of the largest coil. If, in a compression spring, load P_x is greater than the capacity of the largest coil, but less than the capacity P of the whole spring, then some coils will be compressed down solidly, while the remainder will not be fully compressed.

Example: — Compression spring, $D_1 = 4\frac{9}{16}$ inches; $D_2 = 3\frac{9}{16}$ inches; $d = 1\frac{1}{16}$ inch; $h = 7\frac{1}{16}$ inches.

Then number of coils, $N = \frac{h}{d} = 6.65$; $\frac{D_1 - D_2}{2} = 0.5$; $p = \frac{D_1 - D_2}{2N} = 0.075$.

$$F = 0.00332 \frac{(4\frac{9}{16})^3 - (3\frac{9}{16})^3}{0.075 \times 1\frac{1}{16}} = 2\frac{1}{16} \text{ inches, approximately.}$$

Free height $= 7\frac{1}{16} + 2\frac{1}{16} = 9\frac{1}{8}$ inches.

Example: — Same spring in extension.

$$F = 0.00249 \frac{(4\frac{9}{16})^4 - (3\frac{9}{16})^4}{0.075 \times 1\frac{1}{16} \times 4\frac{9}{16}} = 1\frac{7}{8} \text{ inch, approximately.}$$

H (extended height) $= 7\frac{1}{16} + 1\frac{7}{8} = 8\frac{15}{16}$ inches.

The free height for the compression type is greater than the possible extended length for the extension type. This is because sufficient load to fully stress the smaller or stronger coils cannot be applied without distorting the extension spring, whereas the coils may all be stressed to maximum stress in the compression type, the closing of the coils solidly together protecting the spring from over-stress.

Materials Used for Springs. — Steel containing about one per cent carbon and comparatively free from phosphorus and sulphur, generally known as spring steel, is ordinarily used for springs.

For small springs, music wire is used to a great extent and is the best material obtainable for this purpose. It is especially recommended for devices where the spring is compressed frequently and suddenly. Vanadium steel has recently come into use to a considerable extent for springs. The addition of a small percentage of vanadium to steel increases the elasticity of the material, but the cost of springs made from this material is considerably higher.

Brass and phosphor-bronze should be used for springs that must resist moisture These springs, however, are much more expensive than steel springs, both on account of the higher cost of the material, and because the permissible stress is less, thus making larger sizes of these springs necessary for the same capacity.

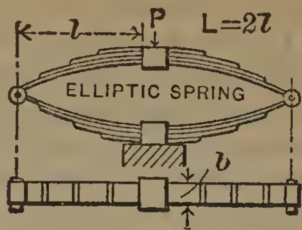
Composition of Phosphor-bronze. — One of the principal uses of phosphor-bronze is for springs. A good mixture for this purpose is as follows: Copper, 95 per cent by weight; tin, 4.5 per cent; five-per cent phosphor-tin, 0.5 per cent. — For phosphor-bronze of the highest possible strength, the following mixture is recommended: Copper, 90 per cent; tin, 9 per cent; five-per cent phosphor-tin, 1 per cent. The alloy made according to this formula is poured into ingots and then re-melted and poured into sand castings. The re-melting increases the strength. — For ordinary work, when a medium strength is required, and when the scrap is used over and over again, the following mixture is recommended: Copper, 90 per cent; tin, 8 per cent; five-per cent phosphor-tin, 2 per cent. — For phosphor-bronze bearings, the following alloy is used: Copper, 80 per cent; tin, 8 per cent; lead, 10 per cent; five-per cent phosphor-tin, 2 per cent. Zinc should never be present.

Factor of Safety in Springs Frequently Compressed. — When a spring acts only occasionally it can be safely designed to carry a load which causes a fiber stress nearly equal to the elastic limit of the spring, but when the compressions or extensions are frequent, a larger factor of safety must be used. A valve spring in an automobile motor, for example, which operates, say, 200 times a minute, should have a factor of safety of at least 4. In other words, a spring made of 1/8-inch wire, which ordinarily could be designed for a torsional stress of 100,000 pounds per square inch, should be designed to work at a stress not over 25,000 pounds per square inch when used in service of the kind mentioned.

High-class springs, such as valve springs, should have the ends squared and ground at right angles to their axis; the outside diameter should be at least one-third of the length, and it should be supported its entire length, unless it is very short, in order to prevent buckling, which introduces bending and twisting strains. High-class valve springs when placed on end on a flat plate should not vary more than 1/2 degree from the perpendicular to the plate. These springs should be protected from rusting by a good coat of japan, baked on, or by electro-galvanizing.

Elliptic Springs. — Elliptic springs are used to a great extent in automobile design, because this class of spring lends itself more readily to easy vibration, as well as to a better general design of the machine. It is possible to support a given load on a narrow-leafed elliptic spring, where there would not be room enough for a helical spring. Automobile springs call for a higher grade of steel than the ordinary spring steel. Various grades of high-carbon, nickel, chromium and vanadium steels are used. Certain alloys of the vanadium class have an elastic limit of from 180,000 to 225,000 pounds per square inch, according to the heat treatment given, and appear to be the most ideal steels for elliptic springs yet produced. Data giving the physical characteristics of various alloy steels when subjected to specific heat treatment will be found in the section "Application and Heat Treatment of S. A. E. Carbon and Alloy Steels."

Formulas for Elliptic Springs



S = fiber stress, assumed as 80,000 pounds per square inch for steel;
E = modulus of elasticity = 25,400,000;
P = maximum carrying capacity of spring;
F = deflection of spring under load P; the deflection under any load less than P is directly proportional to the deflection under full load;
n = number of plates or leaves in half-elliptic spring, or one-half total number of leaves in full-elliptic spring;
b = width of plates; h = thickness of plates; $L = 2\,l$ = effective span;
r = ratio of number of full length leaves to total number of leaves.

Type of Spring	To be Found	Formulas	Examples
Full-elliptic, with all leaves graduated	P	$P = \frac{2\,Snbh^2}{3\,L}$ or, for steel springs: $P = 53{,}000\,nbh^2 \div L$	Steel spring, number of leaves in one-half of spring = 5; Width of plates = 2 inches; Thickness of plates = $\frac{1}{4}$ inch; Effective span, $L = 33$ inches. Find safe load. $P = [53{,}000 \times 5 \times 2 \times (\frac{1}{4})^2] \div 33$ = 1000 pounds, approx.
	F	$F = \frac{SL^2}{2\,Eh}$ or, for steel springs: $F = 0.0016\,L^2 \div h$	
Full-elliptic, with portion of leaves graduated	P	$P = \frac{2\,Snbh^2}{3\,L}$ or, for steel springs: $P = 53{,}000\,nbh^2 \div L$	Same spring as above, with two leaves in each half extending full length of span. Find deflection. Here $r = 2 \div 5 = 0.4$. $F = 0.0031\,\frac{33^2}{(2 + 0.4)\,\frac{1}{4}}$ = 5.625 inches.
	F	$F = \frac{1}{2 + r} \times \frac{SL^2}{Eh}$ or, for steel springs: $F = 0.0031\,\frac{L^2}{(2 + r)\,h}$	
Half-elliptic, with all leaves graduated	P	$P = \frac{2\,Snbh^2}{3\,L}$ or, for steel springs: $P = 53{,}000\,nbh^2 \div L$	Same spring as above, but half-elliptic. Find deflection, if all leaves are fully graduated. $F = (0.00079 \times 33^2) \div \frac{1}{4}$ = 3.44 inches.
	F	$F = \frac{SL^2}{4\,Eh}$ or, for steel springs: $F = 0.00079\,L^2 \div h$	
Half-elliptic, with portion of leaves graduated	P	$P = \frac{2\,Snbh^2}{3\,L}$ or, for steel springs: $P = 53{,}000\,nbh^2 \div L$	Same spring as above, half-elliptic, with three leaves extending full length of span. Find deflection. $F = 0.0016\,\frac{33^2}{(2 + 0.6)\,\frac{1}{4}}$ = 2.68 inches.
	F	$F = \frac{1}{2\,(2 + r)} \times \frac{SL^2}{Eh}$ or, for steel springs: $F = 0.0016\,\frac{L^2}{(2 + r)\,h}$	

Simplified Elliptic Spring Calculations. — The accompanying "Semi-elliptic Spring Table," which is based on a modulus of elasticity of 25,400,000, and a fiber stress, under maximum load, of 80,000 pounds per square inch, has been prepared to facilitate the calculation of semi- or half-elliptic springs when all the leaves are fully graduated. The safe load on one leaf, one inch wide, is found by dividing the constant given in Column "P_u" by the net length or net span L (see illustration in table on preceding page for method of measuring $L = 2\,l$). The corresponding deflection is found by multiplying the constant given in the Column "F_u" by the square of the net length L.

Example: — What is the safe load on a semi- or half-elliptic fully graduated spring of five leaves, if made of ¼- by 2-inch steel; length between end bearings, 36 inches; band or seat, 3 inches?

Net length = 36 − 3 = 33 inches.

$$\text{Load on one leaf, one inch wide} = \frac{3333}{33} = 101.01 \text{ pounds.}$$

Load on one leaf two inches wide = 2 × 101.01 = 202.02 pounds.
Load on five two-inch leaves = 5 × 202.02 = 1010.1 pounds.
Corresponding deflection is:

$$0.00315 \times (33)^2 = 3.43 \text{ inches.}$$

Semi-Elliptic Spring Table

P_u = safe load for one leaf, 1 inch wide, having a net length of 1 inch. F_u is the corresponding deflection. Table is directly applicable only when all leaves are fully graduated.

Thickness of Leaf	P_u	F_u	Thickness of Leaf	P_u	F_u	Thickness of Leaf	P_u	F_u
1/32	52	0.02519	3/8	7,500	0.00210	23/32	27,550	0.00109
1/16	208	0.01260	13/32	8,800	0.00194	3/4	30,000	0.00105
3/32	469	0.00840	7/16	10,210	0.00180	25/32	32,550	0.00101
1/8	833	0.00630	15/32	11,720	0.00168	13/16	35,210	0.00097
5/32	1302	0.00504	1/2	13,330	0.00157	27/32	37,960	0.00093
3/16	1875	0.00420	17/32	15,050	0.00148	7/8	40,830	0.00090
7/32	2552	0.00360	9/16	16,875	0.00140	29/32	43,800	0.00087
1/4	3333	0.00315	19/32	18,800	0.00133	15/16	46,875	0.00084
9/32	4218	0.00280	5/8	20,830	0.00126	31/32	50,050	0.00081
5/16	5208	0.00252	21/32	22,970	0.00120	1	53,330	0.00079
11/32	6302	0.00229	11/16	25,210	0.00115	.....	.....	......

While the table is prepared for semi-elliptic springs with all leaves fully graduated, it can be used for other types of elliptic springs, if the values in the table are multiplied by certain factors.

Semi-elliptic springs with portions of the leaves graduated. — The load P remains the same as for a spring with all leaves graduated. To find the deflection, multiply the values in the table by $\frac{2}{2+r} \times L^2$, where r = the ratio of the number of full length leaves to the total number of leaves.

Full-elliptic springs with all leaves graduated. — P remains the same as for a semi-elliptic spring. To find the deflection, multiply the values in the table by $2\,L^2$.

Full-elliptic springs with a portion of the leaves graduated. — The load P remains the same as before, but to find the deflection, the values in the table must be multiplied by $\frac{4}{2+r} \times L^2$, in which r is the ratio of the number of full length leaves to the total number of leaves in one-half of the spring.

Example: — Find the load and deflection of a full-elliptic fully graduated spring having four leaves in each half; thickness of leaf = ¼ inch; effective length L = 30 inches; width of leaves, 1¾ inch. Then maximum safe load equals:

$$P = 4 \times 1\tfrac{3}{4} \times \frac{3333}{30} = 778 \text{ pounds.}$$

The deflection equals:

$$F = 2 \times 30^2 \times 0.00315 = 5.67 \text{ inches.}$$

Example: — Find the thickness and number of leaves to be used in a fully graduated full-elliptic spring which has to support 780 pounds with a deflection not exceeding 5¾ inches; the effective length L of the spring is 30 inches and the width, 1¾ inch.

The deflection equals $2\,L^2 \times$ the constant F_u in the table; hence, $F = 2\,F_uL^2$ or:

$$F_u = F \div 2\,L^2 = 5.75 \div 1800 = 0.0032 \text{ inch.}$$

The thickness of steel which corresponds to this value F_u is ¼ inch. To find the load on one leaf, 1 inch wide, the value P_u in the table is divided by the net length of the spring. Hence, load equals 3333 ÷ 30 = 111 pounds. The load on one leaf, 1¾ inch wide, is then 111 × 1.75 = 194.25 pounds. The number of leaves required is, then, equal to the total load divided by the load on one leaf, or the required number equals 780 ÷ 194.25 = 4 leaves.

The formulas given for the calculation of elliptic springs make no allowance for variation in the thickness of different leaves in a spring. When such springs are used, the deflection of the different leaves will not be uniform, and such springs must be calculated by a general formula based upon a combination of different cantilevers, thus making allowance for the different depths of the leaves. There is nothing to be gained, however, by using leaves of different thicknesses, and springs composed of leaves of but one thickness can be designed to meet all requirements.

Results obtained from fully graduated full-elliptic springs indicate that the friction between the leaves is not great enough to seriously affect the bending action. The formulas on the preceding pages give results that agree very closely with actual conditions as determined by experiments.

Tables of Powers of Numbers for Spring Calculations. — The third, fourth and fifth powers of numbers frequently are met with in spring calculations. Tables of cubes, fourth powers and fifth powers of fractional sizes, such as are likely to be used for springs, are, therefore, given in the following pages.

Fifth Powers of Numbers

Number	Fifth Power	Number	Fifth Power	Number	Fifth Power	Number	Fifth Power
1/16	0.0000010	9/16	0.056313	1 1/16	1.35408	1 9/16	9.3132
1/8	0.0000305	5/8	0.095367	1 1/8	1.80203	1 5/8	11.3310
3/16	0.0002317	11/16	0.153590	1 3/16	2.36139	1 11/16	13.6842
1/4	0.0009765	3/4	0.237305	1 1/4	3.05176	1 3/4	16.4131
5/16	0.0029802	13/16	0.354093	1 5/16	3.89490	1 13/16	19.5610
3/8	0.0074157	7/8	0.512909	1 3/8	4.91489	1 7/8	23.1743
7/16	0.0160284	15/16	0.724196	1 7/16	6.13818	1 15/16	27.3029
1/2	0.0312500	1	1.000000	1 1/2	7.59375	2	32.0000

Cubes of Fractional Numbers

Number	Cube	Number	Cube	Number	Cube	Number	Cube
1/16	0.00024	3 1/16	28.7229	6 1/16	222.8205	9 1/16	744.293
1/8	0.00195	3 1/8	30.5175	6 1/8	229.7832	9 1/8	759.798
3/16	0.00659	3 3/16	32.3854	6 3/16	236.8894	9 3/16	775.518
1/4	0.01562	3 1/4	34.3281	6 1/4	244.1406	9 1/4	791.453
5/16	0.03051	3 5/16	36.3469	6 5/16	251.5383	9 5/16	807.604
3/8	0.05273	3 3/8	38.4433	6 3/8	259.0839	9 3/8	823.974
7/16	0.08374	3 7/16	40.6188	6 7/16	266.7790	9 7/16	840.564
1/2	0.12500	3 1/2	42.8750	6 1/2	274.6250	9 1/2	857.375
9/16	0.17797	3 9/16	45.2131	6 9/16	282.6232	9 9/16	874.408
5/8	0.24414	3 5/8	47.6347	6 5/8	290.7753	9 5/8	891.666
11/16	0.32495	3 11/16	50.1413	6 11/16	299.0827	9 11/16	909.149
3/4	0.42187	3 3/4	52.7343	6 3/4	307.5468	9 3/4	926.859
13/16	0.53637	3 13/16	55.4152	6 13/16	316.1691	9 13/16	944.798
7/8	0.66992	3 7/8	58.1855	6 7/8	324.9511	9 7/8	962.966
15/16	0.82397	3 15/16	61.0466	6 15/16	333.8942	9 15/16	981.366
1	1.00000	4	64.0000	7	343.0000	10	1000.000
1 1/16	1.19946	4 1/16	67.0471	7 1/16	352.2697	10 1/16	1018.867
1 1/8	1.42382	4 1/8	70.1894	7 1/8	361.7050	10 1/8	1037.970
1 3/16	1.67456	4 3/16	73.4284	7 3/16	371.3073	10 3/16	1057.311
1 1/4	1.95312	4 1/4	76.7656	7 1/4	381.0781	10 1/4	1076.890
1 5/16	2.26098	4 5/16	80.2023	7 5/16	391.0187	10 5/16	1096.710
1 3/8	2.59960	4 3/8	83.7402	7 3/8	401.1308	10 3/8	1116.771
1 7/16	2.97045	4 7/16	87.3806	7 7/16	411.4157	10 7/16	1137.075
1 1/2	3.37500	4 1/2	91.1250	7 1/2	421.8750	10 1/2	1157.625
1 9/16	3.81469	4 9/16	94.9748	7 9/16	432.5100	10 9/16	1178.201
1 5/8	4.29101	4 5/8	98.9316	7 5/8	443.3222	10 5/8	1199.462
1 11/16	4.80542	4 11/16	102.9968	7 11/16	454.3132	10 11/16	1220.754
1 3/4	5.35937	4 3/4	107.1718	7 3/4	465.4843	10 3/4	1242.306
1 13/16	5.95434	4 13/16	111.4582	7 13/16	476.8371	10 13/16	1264.091
1 7/8	6.59179	4 7/8	115.8574	7 7/8	488.3730	10 7/8	1286.138
1 15/16	7.27319	4 15/16	120.3708	7 15/16	500.0935	10 15/16	1308.436
2	8.00000	5	125.0000	8	512.0000	11	1331.000
2 1/16	8.77368	5 1/16	129.7463	8 1/16	524.0939	11 1/16	1353.816
2 1/8	9.59570	5 1/8	134.6113	8 1/8	536.3769	11 1/8	1376.892
2 3/16	10.46752	5 3/16	139.5964	8 3/16	548.8503	11 3/16	1400.229
2 1/4	11.39062	5 1/4	144.7031	8 1/4	561.5156	11 1/4	1423.828
2 5/16	12.36645	5 5/16	149.9328	8 5/16	574.3742	11 5/16	1447.690
2 3/8	13.39648	5 3/8	155.2871	8 3/8	587.4277	11 3/8	1471.818
2 7/16	14.48217	5 7/16	160.7673	8 7/16	600.6774	11 7/16	1496.212
2 1/2	15.62500	5 1/2	166.3750	8 1/2	614.1250	11 1/2	1520.875
2 9/16	16.82641	5 9/16	172.1115	8 9/16	627.7717	11 9/16	1545.806
2 5/8	18.08789	5 5/8	177.9785	8 5/8	641.6191	11 5/8	1571.009
2 11/16	19.41088	5 11/16	183.9772	8 11/16	655.6687	11 11/16	1596.485
2 3/4	20.79687	5 3/4	190.1093	8 3/4	669.9218	11 3/4	1622.234
2 13/16	22.24731	5 13/16	196.3762	8 13/16	684.3801	11 13/16	1648.259
2 7/8	23.76367	5 7/8	202.7792	8 7/8	699.0449	11 7/8	1674.560
2 15/16	25.34741	5 15/16	209.3200	8 15/16	713.9177	11 15/16	1701.140
3	27.00000	6	216.0000	9	729.0000	12	1728.000

Fourth Powers of Fractional Numbers

Number	Fourth Power	Number	Fourth Power	Number	Fourth Power	Number	Fourth Power
1/16	0.000015	3 1/16	87.9853	6 1/16	1350.849	9 1/16	6745.15
1/8	0.000244	3 1/8	95.3674	6 1/8	1407.422	9 1/8	6933.16
3/16	0.001236	3 3/16	103.2287	6 3/16	1465.753	9 3/16	7125.07
1/4	0.003906	3 1/4	111.5664	6 1/4	1525.878	9 1/4	7320.94
5/16	0.009537	3 5/16	120.3991	6 5/16	1587.835	9 5/16	7520.81
3/8	0.019776	3 3/8	129.7463	6 3/8	1651.660	9 3/8	7724.76
7/16	0.036636	3 7/16	139.6274	6 7/16	1717.390	9 7/16	7932.82
1/2	0.062500	3 1/2	150.0625	6 1/2	1785.062	9 1/2	8145.06
9/16	0.100113	3 9/16	161.0717	6 9/16	1854.715	9 9/16	8361.53
5/8	0.152588	3 5/8	172.6760	6 5/8	1926.386	9 5/8	8582.28
11/16	0.223404	3 11/16	184.8962	6 11/16	2000.115	9 11/16	8807.38
3/4	0.316406	3 3/4	197.7539	6 3/4	2075.941	9 3/4	9036.87
13/16	0.435806	3 13/16	211.2707	6 13/16	2153.902	9 13/16	9270.83
7/8	0.586182	3 7/8	225.4689	6 7/8	2234.039	9 7/8	9509.29
15/16	0.772477	3 15/16	240.3711	6 15/16	2316.391	9 15/16	9752.33
1	1.000000	4	256.0000	7	2401.000	10	10000.00
1 1/16	1.274429	4 1/16	272.3789	7 1/16	2487.905	10 1/16	10252.35
1 1/8	1.601806	4 1/8	289.5314	7 1/8	2577.148	10 1/8	10509.45
1 3/16	1.988541	4 3/16	307.4817	7 3/16	2668.771	10 3/16	10771.35
1 1/4	2.441406	4 1/4	326.2539	7 1/4	2762.816	10 1/4	11038.12
1 5/16	2.967544	4 5/16	345.8728	7 5/16	2859.324	10 5/16	11309.82
1 3/8	3.574462	4 3/8	366.3635	7 3/8	2958.340	10 3/8	11586.50
1 7/16	4.270035	4 7/16	387.7514	7 7/16	3059.904	10 7/16	11868.22
1 1/2	5.062500	4 1/2	410.0625	7 1/2	3164.062	10 1/2	12155.06
1 9/16	5.960464	4 9/16	433.3227	7 9/16	3270.856	10 9/16	12444.75
1 5/8	6.972901	4 5/8	457.5588	7 5/8	3380.332	10 5/8	12744.29
1 11/16	8.109146	4 11/16	482.7976	7 11/16	3492.532	10 11/16	13046.81
1 3/4	9.378906	4 3/4	509.0664	7 3/4	3607.503	10 3/4	13355.79
1 13/16	10.792252	4 13/16	536.3928	7 13/16	3725.290	10 13/16	13667.98
1 7/8	12.359618	4 7/8	564.8049	7 7/8	3845.937	10 7/8	13986.75
1 15/16	14.091811	4 15/16	594.3310	7 15/16	3969.492	10 15/16	14311.02
2	16.000000	5	625.0000	8	4096.000	11	14641.00
2 1/16	18.095719	5 1/16	656.8408	8 1/16	4225.507	11 1/16	14976.59
2 1/8	20.390869	5 1/8	689.8830	8 1/8	4358.062	11 1/8	15317.93
2 3/16	22.897720	5 3/16	724.1565	8 3/16	4493.712	11 3/16	15665.06
2 1/4	25.628906	5 1/4	759.6914	8 1/4	4632.503	11 1/4	16018.06
2 5/16	28.597427	5 5/16	796.5183	8 5/16	4774.486	11 5/16	16377.00
2 3/8	31.816650	5 3/8	834.6682	8 3/8	4919.707	11 3/8	16741.93
2 7/16	35.300309	5 7/16	874.1723	8 7/16	5068.216	11 7/16	17112.93
2 1/2	39.062500	5 1/2	915.0625	8 1/2	5220.062	11 1/2	17490.06
2 9/16	43.117691	5 9/16	957.3706	8 9/16	5375.295	11 9/16	17873.39
2 5/8	47.480714	5 5/8	1001.1291	8 5/8	5533.965	11 5/8	18262.98
2 11/16	52.166764	5 11/16	1046.3708	8 11/16	5782.996	11 11/16	18658.91
2 3/4	57.191406	5 3/4	1093.1289	8 3/4	5861.816	11 3/4	19061.25
2 13/16	62.570571	5 13/16	1141.4367	8 13/16	6031.099	11 13/16	19470.05
2 7/8	68.320557	5 7/8	1191.3283	8 7/8	6204.023	11 7/8	19885.40
2 15/16	74.458023	5 15/16	1242.8379	8 15/16	6380.639	11 15/16	20307.36
3	81.000000	6	1296.0000	9	6561.000	12	20736.00

Helical Spring Tables. — These tables give the capacity of springs or the load which will compress springs solidly, and also values for determining the free height when the solid height is known. The tables are based upon the assumption that the torsional fiber stress, when the spring is compressed down solidly, is 80,000 pounds per square inch. In the tables:

D = outside diameter of coil in inches;
R = mean radius of coil in inches;
P = load when spring is compressed down solidly, in pounds;
H = free height of spring in inches;
h = solid height of spring in inches.

The use of the tables is best illustrated by an example. Find the free height and capacity of a spring made from 1¹⁄₁₆-inch steel bar, 4 inches in outside diameter, and 8 inches solid height. From the "Helical Spring Tables," we find that the capacity of this spring is 3100 pounds, and that the free height is 11.71 inches.

Find the free height of the same spring, if the solid height were 14 inches. In this case take 10 times the value of H as found in Column "1" and add to it the value of H as found in Column "4." Thus, $14.6 + 5.86 = 20.46$ inches.

Elliptic Spring Tables. — The "Elliptic Spring Tables" give the maximum static load and the deflection under this load when the length or span of the spring, the number of leaves, and the width and thickness of the leaves are known. The maximum static load as given in these tables induces a fiber strain of 80,000 pounds per square inch in the leaves, and the oscillations may increase this to 100,000 pounds. The successive leaves are supposed to be regularly shortened or "graduated" in the full-elliptic spring. In the half-elliptic spring it is assumed that one-quarter of the whole number of leaves extend to the end of the spring and that the remainder are graduated. In the tables:

L = span, or length of spring in inches, not including band.
F = deflection under load P in inches for both half- and full-elliptic springs;
P = maximum static load in pounds;
N = number of leaves in the half-elliptic spring, or number of leaves in one of the halves of a full-elliptic spring;
B = width of leaves in inches.

As an example of the use of the tables, find the maximum load and deflection of a half-elliptic spring having 5 leaves made of ⅜ by 4-inch steel and having a length of 30 inches. By referring to the tables, the deflection is found to equal 1.47 inch under a maximum static load. This load is found as follows: $N \times B = 5 \times 4 = 20$. Then the maximum static load equals 10 times the value found in the column headed "2," or $10 \times 500 = 5000$ pounds.

The table can also be used for finding the maximum static load and the deflection if all the leaves of the spring extend the full length of the span. The maximum load in this case will be the same as when the leaves are graduated, but the deflection will be less. For full-elliptic springs it will be two-thirds of the amounts given in the column under "F" headed "Full." For half-elliptic springs it will be one-third of the amounts given in the column headed "Full." As an example, find the maximum static load and the deflection of a full-elliptic spring having 7 leaves in each half, 4 inches wide, all extending the full length of the span. The leaves are made of ⅜-inch thick steel and the span is 30 inches. From the table the deflection is found as follows: $3.20 \times \frac{2}{3} = 2.14$ inches. The value of $N \times B = 4 \times 7 = 28$. The maximum load, then, equals:

The value in column "2" × 10	= 5000
The value in column "8"	= 2000
Maximum load	= 7000 pounds

Helical Spring Tables

(For explanation see page 463)

Diameter of Steel 1/8 Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h								
			1	2	3	4	5	6	7	8	9
1/2	3/16	164.0	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
9/16	7/32	140.5	1.25	2.49	3.74	4.98	6.24	7.48	8.73	9.97	11.22
5/8	1/4	123.0	1.32	2.64	3.96	5.28	6.69	7.92	9.24	10.56	11.88
11/16	9/32	109.5	1.41	2.81	4.22	5.62	7.03	8.43	9.84	11.24	12.65
3/4	5/16	98.5	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
13/16	11/32	89.5	1.61	3.21	4.82	6.42	8.03	9.63	11.24	12.84	14.45
7/8	3/8	82.0	1.72	3.44	5.16	6.88	8.60	10.32	12.04	13.76	15.48
15/16	13/32	75.5	1.85	3.69	5.54	7.38	9.24	11.08	12.93	14.77	16.63
1	7/16	70.0	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82
1 1/16	15/32	65.5	2.12	4.24	6.36	8.48	10.60	12.72	14.84	16.96	19.08
1 1/8	1/2	61.5	2.28	4.56	6.84	9.12	11.40	13.68	15.96	18.24	20.52

Diameter of Steel 3/16 Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h								
			1	2	3	4	5	6	7	8	9
3/4	9/32	368	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
13/16	5/16	332	1.22	2.45	3.67	4.89	6.12	7.34	8.56	9.78	11.01
7/8	11/32	302	1.27	2.54	3.80	5.07	6.34	7.61	8.88	10.14	11.41
15/16	3/8	276	1.32	2.64	3.96	5.28	6.69	7.92	9.24	10.56	11.88
1	13/32	255	1.38	2.75	4.13	5.50	6.88	8.25	9.68	11.00	12.38
1 1/16	7/16	237	1.44	2.87	4.31	5.74	7.18	8.61	10.05	11.48	12.92
1 1/8	15/32	221	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
1 3/16	1/2	207	1.57	3.14	4.71	6.28	7.85	9.42	10.98	12.56	14.13
1 1/4	17/32	195	1.64	3.28	4.92	6.56	8.20	9.84	11.48	13.12	14.76
1 5/16	9/16	184	1.72	3.44	5.16	6.88	8.60	10.32	12.04	13.76	15.48
1 3/8	19/32	175	1.80	3.60	5.40	7.20	9.00	10.80	12.60	14.40	16.20

Diameter of Steel 1/4 Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h								
			1	2	3	4	5	6	7	8	9
1	3/8	656	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
1 1/16	13/32	605	1.21	2.42	3.63	4.84	6.05	7.26	8.47	9.68	10.89
1 1/8	7/16	562	1.25	2.49	3.74	4.98	6.23	7.47	8.72	9.96	11.21
1 3/16	15/32	525	1.28	2.56	3.84	5.12	6.40	7.68	8.96	10.24	11.32
1 1/4	1/2	490	1.32	2.64	3.96	5.28	6.69	7.92	9.24	10.56	11.88
1 5/16	17/32	463	1.36	2.72	4.08	5.44	6.80	8.16	9.52	10.88	12.24
1 3/8	9/16	437	1.41	2.81	4.22	5.62	7.03	8.43	9.84	11.24	12.65
1 7/16	19/32	414	1.45	2.90	4.35	5.80	7.25	8.70	10.15	11.60	13.05
1 1/2	5/8	394	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
1 5/8	11/16	358	1.61	3.21	4.82	6.42	8.03	9.63	11.24	12.84	14.45
1 3/4	3/4	328	1.72	3.44	5.16	6.88	8.60	10.32	12.04	13.76	15.48
1 7/8	13/16	302	1.85	3.69	5.54	7.38	9.23	11.07	12.92	14.76	16.61
2	7/8	281	1.92	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Helical Spring Tables

Diameter of Steel 5/16 Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*: 1	2	3	4	5	6	7	8	9
1¼	15/32	1020	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
1⅜	17/32	903	1.23	2.46	3.69	4.92	6.15	7.38	8.61	9.84	11.07
1½	19/32	810	1.29	2.58	3.87	5.16	6.45	7.74	9.03	10.32	11.61
1⅝	21/32	730	1.35	2.70	4.05	5.40	6.75	8.12	9.48	10.83	12.19
1¾	23/32	668	1.42	2.85	4.27	5.70	7.12	8.54	9.97	11.39	12.82
1⅞	25/32	614	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
2	27/32	570	1.59	3.17	4.76	6.34	7.93	9.51	11.10	12.68	14.27
2⅛	29/32	530	1.68	3.36	5.03	6.71	8.39	10.07	11.75	13.32	15.00
2¼	31/32	495	1.77	3.54	5.32	7.09	8.86	10.63	12.40	14.18	15.95
2⅜	1 1/32	465	1.87	3.74	5.61	7.48	9.35	11.22	13.09	14.96	16.83
2½	1 3/32	439	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82
2⅝	1 5/32	415	2.11	4.22	6.33	8.44	10.55	12.66	14.77	16.88	18.99
2¾	1 7/32	394	2.22	4.44	6.66	8.88	11.10	13.32	15.54	17.76	19.98

Diameter of Steel ⅜ Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*: 1	2	3	4	5	6	7	8	9
1½	9/16	1470	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
1⅝	⅝	1330	1.22	2.44	3.67	4.89	6.11	7.35	8.55	9.78	11.06
1¾	11/16	1210	1.27	2.54	3.80	5.07	6.34	7.61	8.88	10.14	11.41
1⅞	¾	1100	1.32	2.64	3.96	5.28	6.69	7.92	9.24	10.56	11.88
2	13/16	1020	1.38	2.75	4.13	5.50	6.88	8.25	9.63	11.00	12.38
2⅛	⅞	948	1.44	2.87	4.31	5.74	7.18	8.61	10.05	11.48	12.92
2¼	15/16	883	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
2⅜	1	830	1.57	3.14	4.71	6.28	7.85	9.42	10.99	12.56	14.13
2½	1 1/16	780	1.64	3.28	4.92	6.56	8.20	9.84	11.48	13.12	14.76
2⅝	1⅛	736	1.72	3.44	5.16	6.88	8.60	10.32	12.04	13.76	15.48
2¾	1 3/16	698	1.80	3.60	5.40	7.20	9.00	10.80	12.60	14.40	16.20
2⅞	1¼	653	1.89	3.78	5.67	7.56	9.45	11.34	13.23	15.12	17.00
3	1 5/16	601	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel 7/16 Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*: 1	2	3	4	5	6	7	8	9
1¾	21/32	2000	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
1⅞	23/32	1830	1.22	2.43	3.65	4.86	6.08	7.29	8.51	9.72	10.94
2	25/32	1680	1.26	2.51	3.77	5.02	6.28	7.53	8.79	10.04	11.30
2⅛	27/32	1560	1.30	2.60	3.89	5.19	6.49	7.79	9.09	10.38	11.68
2¼	29/32	1450	1.35	2.70	4.04	5.38	6.73	8.07	9.42	10.76	12.11
2⅜	31/32	1360	1.39	2.78	4.18	5.57	6.96	8.35	9.74	11.14	12.53
2½	1 1/32	1270	1.44	2.89	4.33	5.77	7.22	8.66	10.10	11.54	12.99
2⅝	1 3/32	1200	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
2¾	1 5/32	1140	1.56	3.11	4.67	6.22	7.78	9.34	10.89	12.45	14.00
2⅞	1 7/32	1080	1.62	3.24	4.85	6.47	8.09	9.71	11.33	12.94	14.56
3	1 9/32	1030	1.68	3.37	5.05	6.74	8.42	10.10	11.79	13.47	15.16
3¼	1 13/32	935	1.83	3.65	5.48	7.30	9.13	10.95	12.78	14.60	16.43
3½	1 17/32	858	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Helical Spring Tables

Diameter of Steel ½ Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*								
			1	2	3	4	5	6	7	8	9
2	¾	2600	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
2⅛	13/16	2400	1.21	2.42	3.63	4.84	6.05	7.26	8.47	9.68	10.89
2¼	⅞	2250	1.25	2.49	3.74	4.98	6.24	7.48	8.72	9.97	11.21
2⅜	15/16	2100	1.28	2.56	3.85	5.13	6.41	7.69	8.97	10.26	11.54
2½	1	1970	1.32	2.64	3.96	5.28	6.69	7.92	9.24	10.56	11.88
2⅝	1 1/16	1850	1.36	2.72	4.08	5.44	6.80	8.16	9.52	10.99	12.24
2¾	1⅛	1750	1.41	2.81	4.22	5.62	7.03	8.43	9.84	11.24	12.65
2⅞	1 3/16	1650	1.45	2.90	4.35	5.80	7.25	8.70	10.15	11.60	13.05
3	1¼	1580	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
3¼	1⅜	1430	1.61	3.21	4.82	6.42	8.03	9.63	11.24	12.84	14.45
3½	1½	1310	1.72	3.44	5.16	6.88	8.60	10.32	12.04	13.76	15.48
3¾	1⅝	1210	1.85	3.69	5.54	7.38	9.23	11.07	12.92	14.76	16.61
4	1¾	1130	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel 9/16 Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*								
			1	2	3	4	5	6	7	8	9
2¼	27/32	3300	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
2⅜	29/32	3100	1.21	2.42	3.62	4.83	6.04	7.25	8.46	9.66	10.87
2½	31/32	2900	1.24	2.47	3.71	4.95	6.19	7.42	8.66	9.90	11.13
2⅝	1 1/32	2700	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43
2¾	1 3/32	2550	1.30	2.60	3.90	5.20	6.50	7.80	9.10	10.40	11.70
2⅞	1 5/32	2400	1.34	2.67	4.01	5.35	6.69	8.02	9.36	10.70	12.03
3	1 7/32	2300	1.37	2.75	4.12	5.50	6.87	8.24	9.62	10.99	12.37
3¼	1 11/32	2100	1.46	2.91	4.37	5.82	7.28	8.73	10.19	11.64	13.10
3½	1 15/32	1910	1.54	3.09	4.63	6.17	7.72	9.26	10.80	12.34	13.89
3¾	1 19/32	1760	1.64	3.28	4.92	6.56	8.20	9.84	11.48	13.12	14.76
4	1 23/32	1630	1.75	3.49	5.24	6.98	8.73	10.47	12.22	13.96	15.71
4¼	1 27/32	1520	1.86	3.72	5.58	7.44	9.30	11.16	13.02	14.88	16.74
4½	1 31/32	1420	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel ⅝ Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*								
			1	2	3	4	5	6	7	8	9
2½	15/16	4100	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
2⅝	1	3800	1.21	2.41	3.62	4.82	6.03	7.23	8.44	9.64	10.85
2¾	1 1/16	3600	1.23	2.46	3.69	4.92	6.15	7.38	8.61	9.84	11.07
2⅞	1⅛	3400	1.26	2.52	3.78	5.04	6.30	7.56	8.82	10.08	11.34
3	1 3/16	3200	1.29	2.57	3.86	5.15	6.44	7.72	9.01	10.30	11.58
3¼	1 5/16	2900	1.36	2.70	4.06	5.41	6.76	8.11	9.46	10.82	12.17
3½	1 7/16	2650	1.42	2.85	4.27	5.69	7.12	8.54	9.96	11.38	12.81
3¾	1 9/16	2450	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
4	1 11/16	2300	1.58	3.17	4.75	6.33	7.92	9.50	11.08	12.66	14.25
4¼	1 13/16	2100	1.67	3.34	5.01	6.68	8.35	10.02	11.69	13.36	15.03
4½	1 15/16	1980	1.77	3.54	5.31	7.08	8.85	10.62	12.39	14.16	15.93
4¾	2 1/16	1860	1.87	3.74	5.61	7.48	9.35	11.22	13.09	14.96	16.83
5	2 3/16	1760	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Helical Spring Tables

Diameter of Steel 11/16 Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*								
			1	2	3	4	5	6	7	8	9
2¾	1 1/32	4900	1.18	2.36	3.54	4.72	5.90	7.08	8.25	9.44	10.62
2⅞	1 3/32	4700	1.20	2.40	3.61	4.81	6.01	7.21	8.41	9.62	10.82
3	1 5/32	4400	1.23	2.45	3.68	4.90	6.13	7.35	8.58	9.80	11.03
3¼	1 9/32	4000	1.28	2.56	3.83	5.11	6.39	7.67	8.95	10.22	11.50
3½	1 13/32	3600	1.33	2.67	4.00	5.34	6.67	8.00	9.34	10.67	12.01
3¾	1 17/32	3300	1.40	2.79	4.19	5.59	6.99	8.38	9.78	11.18	12.57
4	1 21/32	3100	1.46	2.93	4.39	5.86	7.32	8.78	10.25	11.71	13.18
4¼	1 25/32	2850	1.54	3.07	4.61	6.15	7.69	9.26	10.76	12.30	13.83
4½	1 29/32	2650	1.61	3.23	4.84	6.46	8.07	9.68	11.30	12.91	14.52
4¾	2 1/32	2500	1.70	3.39	5.09	6.79	8.49	10.18	11.88	13.58	15.27
5	2 5/32	2350	1.79	3.57	5.36	7.14	8.93	10.71	12.50	14.28	16.07
5¼	2 9/32	2250	1.88	3.76	5.64	7.52	9.40	11.28	13.15	15.04	16.92
5½	2 13/32	2100	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel ¾ Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*								
			1	2	3	4	5	6	7	8	9
3	1⅛	5900	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
3¼	1¼	5300	1.22	2.44	3.67	4.89	6.11	7.33	8.55	9.78	11.00
3½	1⅜	4800	1.27	2.54	3.80	5.07	6.34	7.61	8.88	10.14	11.41
3¾	1½	4400	1.32	2.64	3.96	5.28	6.69	7.92	9.24	10.56	11.88
4	1⅝	4100	1.38	2.75	4.13	5.50	6.88	8.25	9.63	11.00	12.38
4¼	1¾	3800	1.44	2.87	4.31	5.74	7.18	8.61	10.05	11.48	12.92
4½	1⅞	3500	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
4¾	2	3300	1.57	3.14	4.71	6.28	7.85	9.42	10.99	12.56	14.13
5	2⅛	3100	1.64	3.28	4.92	6.56	8.20	9.84	11.48	13.12	14.76
5¼	2¼	2950	1.72	3.44	5.16	6.88	8.60	10.32	12.04	13.76	15.48
5½	2⅜	2800	1.80	3.60	5.40	7.20	9.00	10.80	12.60	14.40	16.20
5¾	2½	2700	1.89	3.78	5.67	7.56	9.45	11.34	13.23	15.12	17.00
6	2⅝	2550	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel 13/16 Inch

D Inches	*R* Inches	*P* Pounds	Values of *H* for Varying Values of *h*								
			1	2	3	4	5	6	7	8	9
3¼	1 7/32	6900	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
3½	1 11/32	6300	1.22	2.44	3.65	4.87	6.09	7.31	8.53	9.74	10.96
3¾	1 15/32	5800	1.26	2.52	3.78	5.04	6.30	7.56	8.82	10.08	11.34
4	1 19/32	5300	1.31	2.61	3.92	5.23	6.54	7.84	9.15	10.46	11.76
4¼	1 23/32	4900	1.36	2.71	4.07	5.42	6.78	8.14	9.49	10.85	12.20
4½	1 27/32	4600	1.41	2.82	4.23	5.64	7.05	8.46	9.87	11.28	12.69
4¾	1 31/32	4300	1.47	2.93	4.40	5.86	7.34	8.80	10.27	11.74	13.20
5	2 3/32	4000	1.53	3.06	4.59	6.12	7.65	9.18	10.71	12.24	13.77
5¼	2 7/32	3800	1.59	3.18	4.78	6.37	7.96	9.55	11.14	12.74	14.33
5½	2 11/32	3600	1.67	3.33	5.00	6.66	8.33	9.99	11.66	13.32	14.99
5¾	2 15/32	3400	1.74	3.47	5.21	6.95	8.69	10.42	12.16	13.90	15.63
6	2 19/32	3300	1.81	3.62	5.43	7.24	9.05	10.86	12.67	14.48	16.29

Helical Spring Tables

Diameter of Steel ⅞ Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h: 1	2	3	4	5	6	7	8	9
3½	1 5/16	8000	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
3¾	1 7/16	7300	1.22	2.43	3.65	4.86	6.08	7.29	8.51	9.72	10.94
4	1 9/16	6700	1.25	2.51	3.76	5.02	6.27	7.52	8.78	10.03	11.29
4¼	1 11/16	6200	1.30	2.59	3.89	5.18	6.48	7.78	9.07	10.37	11.66
4½	1 13/16	5800	1.34	2.68	4.03	5.37	6.71	8.05	9.39	10.74	12.08
4¾	1 15/16	5400	1.39	2.78	4.18	5.57	6.96	8.35	9.74	11.14	12.53
5	2 1/16	5100	1.44	2.88	4.33	5.77	7.21	8.65	10.09	11.54	12.98
5¼	2 3/16	4800	1.50	3.00	4.49	5.99	7.48	8.98	10.48	11.97	13.47
5½	2 5/16	4600	1.55	3.11	4.66	6.22	7.77	9.32	10.88	12.43	13.99
5¾	2 7/16	4300	1.62	3.23	4.85	6.46	8.08	9.69	11.31	12.92	14.54
6	2 9/16	4100	1.69	3.37	5.05	6.74	8.43	10.11	11.80	13.48	15.17
6½	2 13/16	3800	1.83	3.66	5.49	7.32	9.15	10.98	12.81	14.64	16.47

Diameter of Steel 15/16 Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h: 1	2	3	4	5	6	7	8	9
3¾	1 13/32	9200	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
4	1 17/32	8500	1.21	2.43	3.64	4.86	6.07	7.28	8.50	9.71	10.93
4¼	1 21/32	7800	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.00	11.25
4½	1 25/32	7300	1.29	2.58	3.87	5.16	6.45	7.73	9.02	10.31	11.60
4¾	1 29/32	6800	1.33	2.66	3.99	5.32	6.65	7.98	9.31	10.64	11.97
5	2 1/32	6400	1.38	2.75	4.13	5.50	6.88	8.25	9.63	11.00	12.38
5¼	2 5/32	6000	1.42	2.84	4.26	5.68	7.10	8.52	9.94	11.36	12.78
5½	2 9/32	5700	1.47	2.95	4.42	5.89	7.37	8.84	10.31	11.78	13.26
5¾	2 13/32	5400	1.53	3.06	4.58	6.11	7.64	9.17	10.70	12.22	13.75
6	2 17/32	5100	1.58	3.16	4.74	6.32	7.90	9.48	11.06	12.64	14.22
6½	2 25/32	4700	1.71	3.41	5.12	6.82	8.53	10.23	11.94	13.64	15.35
7	3 1/32	4300	1.84	3.67	5.51	7.34	9.18	11.01	12.85	14.68	16.52
7½	3 9/32	3900	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel 1 Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h: 1	2	3	4	5	6	7	8	9
4	1½	10500	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
4¼	1⅝	9700	1.21	2.42	3.63	4.84	6.05	7.26	8.47	9.68	10.89
4½	1¾	9000	1.25	2.49	3.74	4.98	6.24	7.48	8.73	9.97	11.22
4¾	1⅞	8400	1.28	2.56	3.85	5.13	6.41	7.69	8.97	10.26	11.54
5	2	7900	1.32	2.64	3.96	5.28	6.69	7.92	9.24	10.56	11.88
5¼	2⅛	7400	1.36	2.72	4.08	5.44	6.80	8.16	9.52	10.99	12.24
5½	2¼	7000	1.41	2.81	4.22	5.62	7.03	8.43	9.84	11.24	12.65
5¾	2⅜	6600	1.45	2.90	4.35	5.80	7.25	8.70	10.15	11.60	13.05
6	2½	6300	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
6½	2¾	5700	1.61	3.21	4.82	6.42	8.03	9.63	11.24	12.84	14.45
7	3	5200	1.72	3.44	5.16	6.88	8.60	10.32	12.04	13.76	15.48
7½	3¼	4800	1.85	3.69	5.54	7.38	9.23	11.07	12.92	14.76	16.61
8	3½	4500	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Helical Spring Tables

Diameter of Steel $1\frac{1}{16}$ Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h 1	2	3	4	5	6	7	8	9
4¼	1 19/32	11,800	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
4½	1 23/32	10,900	1.21	2.42	3.63	4.84	6.05	7.26	8.47	9.68	10.89
4¾	1 27/32	10,200	1.24	2.48	3.72	4.96	6.20	7.44	8.68	9.92	11.16
5	1 31/32	9,500	1.28	2.55	3.83	5.10	6.38	7.65	8.93	10.20	11.48
5¼	2 3/32	8,900	1.31	2.62	3.93	5.24	6.55	7.86	9.17	10.48	11.79
5½	2 7/32	8,400	1.35	2.70	4.04	5.39	6.74	8.09	9.44	10.78	12.13
5¾	2 11/32	8,000	1.39	2.78	4.17	5.56	6.95	8.34	9.73	11.12	12.51
6	2 15/32	7,600	1.43	2.86	4.29	5.72	7.15	8.58	10.01	11.44	12.87
6½	2 23/32	6,900	1.52	3.04	4.56	6.08	7.60	9.12	10.64	12.16	13.68
7	2 31/32	6,300	1.63	3.25	4.88	6.50	8.13	9.75	11.38	13.00	14.63
7½	3 7/32	5,800	1.73	3.46	5.19	6.92	8.65	10.38	12.11	13.84	15.57
8	3 15/32	5,400	1.85	3.70	5.55	7.40	9.25	11.10	12.95	14.80	16.65
8½	3 23/32	5,000	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel $1\frac{1}{8}$ Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h 1	2	3	4	5	6	7	8	9
4½	1 11/16	13,300	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
4¾	1 13/16	12,400	1.21	2.42	3.62	4.83	6.04	7.25	8.46	9.66	10.87
5	1 15/16	11,600	1.24	2.47	3.71	4.95	6.19	7.42	8.66	9.90	11.13
5¼	2 1/16	10,900	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43
5½	2 3/16	10,300	1.30	2.60	3.90	5.20	6.50	7.80	9.10	10.40	11.70
5¾	2 5/16	9,700	1.34	2.67	4.01	5.35	6.69	8.02	9.46	10.70	12.03
6	2 7/16	9,200	1.37	2.75	4.12	5.50	6.87	8.24	9.62	10.99	12.37
6½	2 11/16	8,300	1.46	2.91	4.37	5.82	7.28	8.73	10.19	11.64	13.10
7	2 15/16	7,600	1.54	3.09	4.63	6.17	7.72	9.26	10.80	12.34	13.89
7½	3 3/16	7,000	1.64	3.28	4.92	6.56	8.20	9.84	11.48	13.12	14.76
8	3 7/16	6,500	1.75	3.49	5.24	6.98	8.73	10.47	12.22	13.96	15.71
8½	3 11/16	6,100	1.86	3.72	5.58	7.44	9.30	11.16	13.02	14.88	16.74
9	3 15/16	5,700	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel $1\frac{3}{16}$ Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h 1	2	3	4	5	6	7	8	9
4¾	1 25/32	14,800	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
5	1 29/32	13,800	1.21	2.41	3.62	4.82	6.03	7.23	8.44	9.64	10.85
5¼	2 1/32	13,000	1.23	2.47	3.70	4.93	6.17	7.40	8.63	9.86	11.20
5½	2 5/32	12,200	1.26	2.53	3.79	5.05	6.32	7.58	8.84	10.10	11.37
5¾	2 9/32	11,500	1.30	2.59	3.89	5.18	6.48	7.77	9.07	10.36	11.66
6	2 13/32	10,900	1.33	2.66	3.98	5.31	6.64	7.97	9.30	10.62	11.95
6½	2 21/32	9,900	1.40	2.80	4.19	5.59	6.99	8.39	9.79	11.18	12.58
7	2 29/32	9,000	1.48	2.96	4.44	5.92	7.40	8.88	10.36	11.84	13.32
7½	3 5/32	8,300	1.56	3.13	4.69	6.25	7.82	9.38	10.94	12.50	14.07
8	3 13/32	7,700	1.66	3.32	4.97	6.63	8.29	9.95	11.61	13.26	14.92
8½	3 21/32	7,200	1.76	3.51	5.27	7.02	8.78	10.53	12.29	14.04	15.80
9	3 29/32	6,700	1.87	3.73	5.60	7.46	9.33	11.19	13.06	14.92	16.78
9½	4 5/32	6,300	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Helical Spring Tables

Diameter of Steel 1¼ Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h: 1	2	3	4	5	6	7	8	9
5	1⅞	16,400	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
5¼	2	15,300	1.21	2.41	3.62	4.82	6.03	7.23	8.44	9.64	10.85
5½	2⅛	14,500	1.23	2.46	3.69	4.92	6.15	7.38	8.61	9.84	11.07
5¾	2¼	13,700	1.26	2.52	3.78	5.04	6.30	7.56	8.82	10.08	11.34
6	2⅜	12,900	1.29	2.57	3.86	5.15	6.44	7.72	9.01	10.30	11.58
6½	2⅝	11,700	1.36	2.70	4.06	5.41	6.70	8.11	9.46	10.82	12.17
7	2⅞	10,700	1.42	2.85	4.27	5.69	7.12	8.54	9.96	11.38	12.81
7½	3⅛	9,800	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00	13.50
8	3⅜	9,100	1.58	3.17	4.75	6.33	7.92	9.50	11.08	12.66	14.25
8½	3⅝	8,500	1.67	3.34	5.01	6.68	8.35	10.02	11.69	13.36	15.03
9	3⅞	7,900	1.77	3.54	5.31	7.08	8.85	10.62	12.39	14.16	15.93
9½	4⅛	7,400	1.87	3.74	5.61	7.48	9.35	11.22	13.09	14.96	16.83
10	4⅜	7,000	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel 1⁵⁄₁₆ Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h: 1	2	3	4	5	6	7	8	9
5¼	1³¹⁄₃₂	18,000	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
5½	2³⁄₃₂	16,900	1.20	2.41	3.61	4.81	6.02	7.22	8.42	9.62	10.83
5¾	2⁷⁄₃₂	16,000	1.23	2.46	3.68	4.91	6.14	7.37	8.60	9.82	11.05
6	2¹¹⁄₃₂	15,100	1.26	2.51	3.77	5.02	6.28	7.53	8.79	10.04	11.30
6½	2¹⁹⁄₃₂	13,700	1.31	2.63	3.94	5.25	6.57	7.88	9.19	10.50	11.82
7	2²⁷⁄₃₂	12,500	1.38	2.75	4.13	5.51	6.89	8.26	9.64	11.02	12.39
7½	3³⁄₃₂	11,500	1.45	2.89	4.34	5.78	7.23	8.67	10.12	11.56	13.01
8	3¹¹⁄₃₂	10,600	1.52	3.04	4.56	6.08	7.60	9.12	10.64	12.16	13.68
8½	3¹⁹⁄₃₂	9,900	1.60	3.20	4.80	6.40	8.00	9.60	11.20	12.80	14.40
9	3²⁷⁄₃₂	9,200	1.69	3.37	5.06	6.74	8.43	10.11	11.80	13.48	15.17
9½	4³⁄₃₂	8,600	1.78	3.56	5.34	7.12	8.90	10.68	12.48	14.24	16.02
10	4¹¹⁄₃₂	8,200	1.88	3.75	5.63	7.50	9.38	11.25	13.13	15.00	16.88
10½	4¹⁹⁄₃₂	7,700	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Diameter of Steel 1⅜ Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h: 1	2	3	4	5	6	7	8	9
5½	2¹⁄₁₆	19,800	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
5¾	2³⁄₁₆	18,700	1.20	2.40	3.61	4.81	6.01	7.21	8.41	9.62	10.82
6	2⁵⁄₁₆	17,600	1.23	2.45	3.68	4.90	6.13	7.35	8.58	9.80	11.03
6½	2⁹⁄₁₆	16,000	1.28	2.56	3.83	5.11	6.39	7.67	8.95	10.22	11.50
7	2¹³⁄₁₆	14,600	1.33	2.67	4.00	5.34	6.67	8.00	9.34	10.67	12.01
7½	3¹⁄₁₆	13,400	1.40	2.79	4.19	5.59	6.99	8.38	9.78	11.18	12.57
8	3⁵⁄₁₆	12,400	1.46	2.93	4.39	5.86	7.32	8.78	10.25	11.71	13.18
8½	3⁹⁄₁₆	11,500	1.54	3.07	4.61	6.15	7.69	9.22	10.76	12.30	13.83
9	3¹³⁄₁₆	10,700	1.61	3.23	4.84	6.46	8.07	9.68	11.30	12.91	14.52
9½	4¹⁄₁₆	10,100	1.70	3.39	5.09	6.79	8.49	10.18	11.88	13.58	15.27
10	4⁵⁄₁₆	9,500	1.79	3.57	5.36	7.14	8.98	10.71	12.50	14.28	16.07
10½	4⁹⁄₁₆	9,000	1.88	3.76	5.64	7.52	9.40	11.28	13.16	15.04	16.92
11	4¹³⁄₁₆	8,500	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Helical Spring Tables

Diameter of Steel 1 7/16 Inch

D Inches	R Inches	P Pounds	Values of H for Varying Values of h: 1	2	3	4	5	6	7	8	9
5¾	2 5/32	21,700	1.18	2.36	3.54	4.72	5.90	7.08	8.26	9.44	10.62
6	2 9/32	20,400	1.20	2.40	3.61	4.81	6.01	7.21	8.41	9.62	10.82
6½	2 17/32	18,500	1.25	2.50	3.74	4.99	6.24	7.49	8.74	9.98	11.23
7	2 25/32	16,800	1.30	2.60	3.90	5.20	6.50	7.80	9.10	10.40	11.70
7½	3 1/32	15,400	1.36	2.71	4.07	5.42	6.78	8.14	9.49	10.85	12.20
8	3 9/32	14,200	1.42	2.84	4.25	5.67	7.09	8.51	9.93	11.34	12.76
8½	3 17/32	13,200	1.48	2.96	4.45	5.93	7.41	8.89	10.37	11.86	13.34
9	3 25/32	12,300	1.55	3.11	4.66	6.21	7.77	9.32	10.87	12.42	13.98
9½	4 1/32	11,600	1.63	3.26	4.89	6.52	8.15	9.78	11.41	13.04	14.67
10	4 9/32	10,900	1.71	3.42	5.13	6.84	8.55	10.26	11.97	13.68	15.39
10½	4 17/32	10,300	1.79	3.59	5.38	7.17	8.97	10.76	12.55	14.34	16.14
11	4 25/32	9,800	1.89	3.78	5.67	7.56	9.45	11.34	13.23	15.12	17.00
11½	5 1/32	9,300	1.98	3.96	5.94	7.92	9.90	11.88	13.86	15.84	17.82

Elliptic Spring Tables

(For explanation see page 463)

Thickness of Steel 1/16 Inch

L Inches	F Inches: Half	F Inches: Full	Values of P for Varying Values of $N \times B$: 1	2	3	4	5	6	7	8	9
3	0.09	0.19	69.4	139.0	208.0	278.0	347.9	416.0	486.0	555.0	625.0
4	0.16	0.34	52.0	104.0	156.0	208.0	260.0	312.0	364.0	416.0	468.0
5	0.25	0.53	41.7	83.4	125.0	167.0	209.0	250.0	292.0	334.0	375.0
6	0.35	0.77	34.7	69.4	104.0	139.0	174.0	208.0	243.0	278.0	312.0
7	0.48	1.04	29.8	59.6	89.4	119.0	149.0	179.0	209.0	238.0	268.0
8	0.63	1.36	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
9	0.80	1.73	23.1	46.2	69.3	92.4	116.0	139.0	162.0	185.0	208.0
10	0.98	2.13	20.8	41.6	62.4	83.2	104.0	125.0	146.0	166.0	187.0
11	1.19	2.58	18.9	37.8	56.7	75.6	94.5	113.0	132.0	151.0	170.0
12	1.42	3.07	17.4	34.8	52.2	69.6	87.0	104.0	122.0	139.0	157.0
13	1.66	3.60	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
14	1.92	4.17	14.9	29.8	44.7	59.6	74.5	89.4	104.0	119.0	134.0
15	2.21	4.79	13.9	27.8	41.7	55.6	69.5	83.4	97.3	111.0	125.0
16	2.52	5.45	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
17	2.83	6.16	12.2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	110.0
18	3.18	6.90	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.0

Thickness of Steel 1/8 Inch

L Inches	F Inches: Half	F Inches: Full	Values of P for Varying Values of $N \times B$: 1	2	3	4	5	6	7	8	9
5	0.12	0.27	167.0	334	501	668	835	1002	1169	1336	1503
6	0.18	0.39	139.0	278	417	556	695	834	973	1112	1251
7	0.24	0.52	119.0	238	357	476	595	714	833	952	1071
8	0.31	0.68	104.0	208	312	416	520	624	728	832	936
9	0.40	0.87	92.5	185	278	370	463	555	648	740	833
10	0.49	1.06	83.3	167	250	333	417	500	583	666	750
11	0.59	1.29	75.8	152	227	303	379	455	531	606	682
12	0.70	1.54	69.5	139	209	278	348	417	487	556	626

Elliptic Spring Tables

Thickness of Steel ⅛ Inch (*Continued*)

L Inches	*F* Inches Half	*F* Inches Full	Values of *P* for Varying Values of *N*×*B* 1	2	3	4	5	6	7	8	9
13	0.83	1.80	64.2	128.0	193	257	321	385	449	514	578
14	0.96	2.10	59.6	119.0	179	238	298	358	417	477	536
15	1.10	2.40	55.6	111.0	167	222	278	334	389	445	500
16	1.25	2.73	52.2	104.0	157	209	261	314	365	418	470
17	1.41	3.08	49.1	98.2	147	196	246	295	344	393	442
18	1.59	3.46	46.4	92.8	139	186	232	278	325	371	418
19	1.77	3.85	43.9	87.8	132	176	220	263	307	351	395
20	1.96	4.26	41.6	83.2	125	166	208	250	291	333	374

Thickness of Steel 3/16 Inch

L Inches	*F* Inches Half	*F* Inches Full	Values of *P* for Varying Values of *N*×*B* 1	2	3	4	5	6	7	8	9
8	0.21	0.46	234.0	468	702	936	1170	1404	1638	1872	2106
10	0.33	0.71	188.0	376	564	752	940	1128	1316	1504	1692
12	0.44	1.03	156.0	312	468	624	780	936	1092	1248	1404
14	0.64	1.40	134.0	268	402	536	670	804	938	1072	1206
16	0.83	1.82	117.0	234	351	468	585	702	819	936	1053
18	1.06	2.30	108.0	216	324	432	540	658	756	864	972
20	1.30	2.85	93.7	187	281	375	469	562	656	750	843
22	1.58	3.45	85.2	170	256	341	426	511	596	682	767
24	1.87	4.10	78.2	156	235	313	391	469	547	626	704
26	2.20	4.80	72.1	144	216	288	361	433	505	577	649
28	2.55	5.59	67.0	134	201	268	335	402	469	536	603
30	2.93	6.38	62.5	125	188	250	313	375	436	500	563
32	3.33	7.28	58.6	117	176	234	293	352	410	469	527
34	3.77	8.22	55.2	110	166	221	276	331	386	442	497
36	4.22	9.23	52.1	104	156	208	261	313	365	417	469
38	4.70	10.30	49.3	99	148	197	247	296	345	394	444

Thickness of Steel ¼ Inch

L Inches	*F* Inches Half	*F* Inches Full	Values of *P* for Varying Values of *N*×*B* 1	2	3	4	5	6	7	8	9
12	0.35	0.77	278	556	834	1112	1390	1668	1946	2224	2502
14	0.48	1.04	238	476	714	952	1190	1428	1666	1904	2142
16	0.63	1.36	209	418	627	836	1045	1254	1463	1672	1881
18	0.79	1.72	185	370	555	740	925	1110	1295	1480	1665
20	0.98	2.13	167	334	501	668	835	1002	1169	1336	1503
22	1.19	2.58	152	304	456	608	760	912	1064	1216	1368
24	1.41	3.07	139	278	417	556	695	834	973	1112	1251
26	1.66	3.60	128	256	384	512	640	768	896	1024	1152
28	1.92	4.18	119	238	357	476	595	714	833	952	1071
30	2.20	4.80	111	222	333	444	555	666	777	888	999
32	2.50	5.45	104	208	312	416	520	624	728	832	936
34	2.83	6.15	98	196	294	392	490	588	686	784	882
36	3.18	6.90	93	186	279	372	465	558	651	744	837
38	3.53	7.70	88	176	264	352	440	528	616	704	792
40	3.91	8.51	83	166	249	332	415	498	581	664	747
42	4.32	9.40	79	158	237	316	395	474	553	632	711

Elliptic Spring Tables

Thickness of Steel 5/16 Inch

L Inches	F Inches Half	F Inches Full	1	2	3	4	5	6	7	8	9
16	0.50	1.09	325	650	975	1300	1625	1950	2275	2600	2925
18	0.63	1.38	290	580	870	1160	1450	1740	2030	2320	2610
20	0.78	1.70	260	520	780	1040	1300	1560	1820	2080	2340
22	0.95	2.07	235	470	705	940	1175	1410	1645	1880	2115
24	1.13	2.45	217	434	651	868	1085	1302	1519	1736	1953
26	1.32	2.88	200	400	600	800	1000	1200	1400	1600	1800
28	1.53	3.35	186	372	558	744	930	1116	1302	1488	1674
30	1.76	3.84	173	346	519	692	865	1038	1211	1384	1657
32	2.00	4.36	163	326	489	652	815	978	1141	1304	1467
34	2.26	4.93	153	306	459	612	765	918	1071	1224	1377
36	2.53	5.52	144	288	432	576	720	864	1008	1152	1296
38	2.82	6.15	137	274	411	548	685	822	959	1096	1233
40	3.13	6.81	130	260	390	520	650	780	910	1040	1170
42	3.45	7.51	124	248	372	496	620	744	868	992	1116
44	3.78	8.25	118	236	354	472	590	708	826	944	1062
46	4.13	9.00	113	226	339	452	565	678	791	904	1017

Columns 1–9: Values of P for Varying Values of $N \times B$

Thickness of Steel 11/32 Inch

L Inches	F Inches Half	F Inches Full	1	2	3	4	5	6	7	8	9
20	0.71	1.55	315	630	945	1260	1575	1890	2205	2520	2835
22	0.86	1.87	286	571	859	1146	1432	1719	2005	2291	2578
24	1.02	2.21	262	524	785	1048	1310	1570	1831	2091	2358
26	1.20	2.62	242	484	725	967	1209	1450	1692	1935	2176
28	1.39	3.03	224	450	675	901	1125	1350	1576	1801	2026
30	1.60	3.49	210	420	630	840	1050	1260	1470	1680	1890
32	1.82	3.98	196	393	589	786	992	1179	1376	1571	1769
34	2.05	4.47	185	369	554	739	924	1109	1295	1478	1621
36	2.30	5.01	174	349	523	698	873	1047	1223	1398	1572
38	2.54	5.59	165	331	495	662	827	993	1158	1323	1489
40	2.84	6.20	156	315	472	630	786	945	1102	1260	1417
42	3.08	6.82	149	296	448	597	748	897	1046	1196	1346
44	3.44	7.48	143	286	429	572	715	859	1002	1145	1288
46	3.75	8.17	137	274	410	547	684	821	958	1095	1232
48	4.10	8.93	131	262	393	524	655	784	917	1048	1179
50	4.45	9.65	126	252	378	504	630	756	884	1008	1134

Columns 1–9: Values of P for Varying Values of $N \times B$

Thickness of Steel 3/8 Inch

L Inches	F Inches Half	F Inches Full	1	2	3	4	5	6	7	8	9
20	0.65	1.42	375	750	1125	1500	1875	2250	2625	3000	3375
22	0.79	1.72	341	682	1023	1364	1705	2046	2387	2728	3069
24	0.94	2.04	312	624	936	1248	1560	1872	2184	2496	2808
26	1.10	2.40	288	576	864	1152	1440	1728	2016	2304	2592
28	1.28	2.78	268	536	804	1072	1340	1608	1876	2144	2412
30	1.47	3.20	250	500	750	1000	1250	1500	1750	2000	2250
32	1.67	3.63	234	468	702	936	1170	1404	1638	1872	2106
34	1.88	4.10	220	440	660	880	1100	1320	1540	1760	1980

Columns 1–9: Values of P for Varying Values of $N \times B$

Elliptic Spring Tables

Thickness of Steel 3/8 Inch (*Continued*)

L Inches	*F* Inches Half	*F* Inches Full	1	2	3	4	5	6	7	8	9
36	2.12	4.58	208	416	624	832	1040	1248	1456	1664	1872
38	2.35	5.12	197	394	591	788	985	1182	1379	1576	1773
40	2.60	5.68	187	375	562	750	937	1125	1312	1500	1687
42	2.87	6.26	178	356	534	712	890	1068	1246	1424	1602
44	3.15	6.86	170	341	511	682	852	1023	1193	1364	1534
46	3.45	7.50	163	326	489	652	815	978	1141	1304	1467
48	3.75	8.16	156	312	468	624	780	936	1092	1248	1404
50	4.07	8.87	150	300	450	600	750	900	1050	1200	1350

Values of *P* for Varying Values of $N \times B$ (columns 1–9)

Thickness of Steel 7/16 Inch

L Inches	*F* Inches Half	*F* Inches Full	1	2	3	4	5	6	7	8	9
24	0.81	1.76	426	852	1278	1704	2130	2556	2982	3408	3834
26	0.95	2.06	393	786	1179	1572	1965	2358	2751	3144	3537
28	1.10	2.38	365	730	1095	1460	1825	2190	2555	2920	3285
30	1.26	2.74	341	682	1023	1364	1705	2046	2387	2728	3069
32	1.43	3.12	319	638	957	1276	1595	1914	2233	2552	2871
34	1.62	3.52	301	602	903	1204	1505	1806	2107	2408	2709
36	1.81	3.95	284	568	852	1136	1420	1704	1988	2272	2556
38	2.03	4.40	269	538	807	1076	1345	1614	1883	2152	2421
40	2.24	4.88	255	510	765	1020	1275	1530	1785	2040	2295
42	2.47	5.37	243	486	729	972	1215	1458	1701	1944	2187
44	2.71	5.90	232	464	696	928	1160	1392	1624	1856	2088
46	2.96	6.45	222	444	666	888	1110	1332	1554	1776	1998
48	3.22	7.00	213	426	639	852	1065	1278	1491	1704	1917
50	3.49	7.60	204	408	612	816	1020	1224	1428	1632	1836
52	3.78	8.25	197	394	591	788	985	1182	1379	1576	1773
54	4.08	8.90	189	378	567	756	945	1134	1323	1512	1701

Values of *P* for Varying Values of $N \times B$ (columns 1–9)

Thickness of Steel 1/2 Inch

L Inches	*F* Inches Half	*F* Inches Full	1	2	3	4	5	6	7	8	9
30	1.10	2.40	444	888	1332	1776	2220	2664	3108	3552	3996
32	1.25	2.72	416	832	1248	1664	2080	2496	2912	3328	3744
34	1.41	3.07	392	784	1176	1568	1960	2352	2744	3136	3528
36	1.58	3.45	372	744	1116	1488	1860	2232	2604	2976	3348
38	1.76	3.84	350	700	1050	1400	1750	2100	2450	2800	3150
40	1.95	4.25	333	666	999	1332	1665	1998	2331	2664	2997
42	2.16	4.68	317	634	951	1268	1585	1902	2219	2536	2853
44	2.37	5.15	303	606	909	1212	1515	1818	2121	2424	2727
46	2.58	5.62	290	580	870	1160	1450	1740	2030	2320	2610
48	2.82	6.13	277	554	831	1108	1385	1662	1939	2216	2493
50	3.06	6.65	266	532	798	1064	1330	1596	1862	2128	2394
52	3.30	7.19	256	512	768	1024	1280	1536	1792	2048	2304
54	3.57	7.75	247	494	741	988	1235	1482	1729	1976	2223
56	3.83	8.35	238	476	714	952	1190	1428	1666	1904	2142
58	4.12	8.95	230	460	690	920	1150	1380	1610	1840	2070
60	4.40	9.58	222	444	666	888	1110	1332	1554	1776	1998

Values of *P* for Varying Values of $N \times B$ (columns 1–9)

Helical Spring Tables Covering Steel Wire Gage Sizes. — The accompanying tables "Maximum Loads in Pounds and Corresponding Compressions per Coil in Inches of Helical Round Bar Springs," cover the commercial sizes of helical steel springs. These tables are calculated for a fiber stress of 100,000 pounds per square inch, with a torsional modulus of elasticity of 11,520,000, and apply to high-grade spring wire. The tables cover springs made from wire sizes varying from No. 26 (0.0181 inch) *steel wire gage* (which is the same as the American Steel & Wire Co.'s gage, the Washburn & Moen gage and the Roebling gage) to wire 5/8 inch in diameter.

Values Given in Helical Spring Tables. — The tables give in the two extreme left-hand columns the size of the wire from which the spring is made by gage number (steel wire gage) and in decimals, respectively. Along the top of the tables is given the mean or pitch diameter of the spring. To find the values relating to any one spring, first locate in the left-hand column the size of wire from which the spring is made, and then follow the horizontal line from this size to the column headed by the mean diameter of the spring. For example, assume that the values for a spring made from No. 10 gage wire, having a mean or pitch diameter of 1 1/8 inch, are to be found. The No. 10 wire is located in the left-hand column, and then the lines applying to this size of wire are followed horizontally until the column headed 1 1/8 is reached. Here three values are given one above the other as follows: 86, 0.255, and 330. The top value gives the load in pounds required to compress the spring solid, or the total capacity of the spring. In this case, 86 pounds would be required to compress the spring solid. The second value, 0.255, gives the movement per coil in inches when the spring is compressed from its free height to solid; this value is often known as the "deflection under total load, per coil." In this case then, the deflection, under a load of 86 pounds, is 0.255 inch per coil. The third value, 330, is a constant; if this constant is divided by the number of working coils in the entire spring, the load in pounds required to compress the spring one inch will be found.

Heat-treatment of the Springs — The values given in the tables apply to commercial springs which are "blued" at 600 degrees F., permitted to cool, and then compressed solid, after which they should return to the calculated free height. The values for springs of No. 3 1/2 gage and larger apply to "tempered" springs, the treatment to which they are subjected being as follows: Heat the spring to a red heat and dip it into oil, keeping it in the oil until it is quite cool; then burn off or flash off the oil, which has the same effect as drawing the temper. An exception is made in the case of car springs, which are dipped into oil and taken out again while they still show red or dark red. These springs are not flashed off.

Wire springs of the sizes varying from No. 26 to No. 4 gage, inclusive, are usually allowed to remain in the oil the time required for heating another lot of springs (about seven minutes); the temper is drawn during the last hours of the day when the heat of the furnace is going down, or during the first hour in the morning when the heat of the furnace is coming up. The reason why springs made from the larger sizes of wire require different treatment is that the wire itself has not had the same amount of work put upon it in its manufacture as the smaller sizes, and hence does not have inherent in it the same high tensile strength as the smaller sizes. A fiber stress of 100,000 pounds per square inch is permissible for a spring treated by the hardening processes previously specified, after the spring is coiled.

Relation between Spring Hardness and Movement per Coil. — This process of hardening will also show whether the spring is defective. If a compression spring is compressed solid, the coils, when released, should have a movement per coil as specified in the table. If they do not have this movement, the spring has not been hardened sufficiently. If the space between the coils exceeds that specified in the tables in the line for movement per coil, then the spring is too hard and will not last long. The temper should be drawn until the proper degree of hardness is obtained. The basis of 100,000 pounds per square inch stress, which has been used for calculating the tables, is an ample figure to use for the sizes up to No. 4 gage but may be considered the limiting value for larger sizes.

Using Spring Tables for Stresses other than 100,000 Pounds per Square Inch. — In order to apply the spring tables to stresses other than 100,000 pounds per square inch, it is necessary to take a percentage of the load, as shown in the tables, corresponding to the percentage that the stress required is of 100,000 pounds; thus, if the fiber stress is only 60,000 pounds per square inch, the loads and the movements per coil (or deflections) should be taken at 60 per cent of the values given in the table. For gas engine springs, where the compressions run from 175 to 250 per minute, a fiber stress of 60,000 pounds per square inch is as high as it is safe to assume. In that case, 60 per cent of the figures shown in the table for the load and the movement per coil should be used. The third value given for each size of wire, that is, the constant, remains the same no matter what the fiber stress to which the spring is designed, and is applied in the same manner in all cases.

Example 1: — A spring made from No. 23 spring wire has a mean diameter of ¼ inch and 18 working coils. What weight is required to compress this spring 1 inch? The constant 36, divided by the number of working coils (18), gives a weight of two pounds, which is required to compress the spring 1 inch.

Example 2: — A spring is made from No. 24 wire and has a mean diameter of ¼ inch. How many coils are required to obtain a movement of 1 inch with a load of 1 pound? By dividing the constant 25 by the load (one pound) the number of working coils required is found to equal 25. The constants in all the tables are correct regardless of the fiber stress used, provided the spring has a movement of one inch or more.

Example 3: — A spring is to be designed to work freely over a 1-inch diameter bar; it should have at least a 2-inch total movement, and should compress to a length of 2 inches without the coils touching each other. The spring should be capable of carrying a load of 44 pounds per inch of compression. To work freely over a 1-inch diameter bar, the spring should be made from No. 8½ wire and have a mean diameter of 1¼ inch. Such a spring has a full-load capacity of 120 pounds. The constant is 440, and this constant divided by the load per inch of compression (44) gives the number of working coils required, which is 10. As the diameter of the wire is 5/32 inch, the solid height of the spring with closed ends added will not be more than 1⅞ inch, so that the spring will compress to 2 inches without the coils touching each other, and will fill all requirements.

Example 4: — What load is required to compress a spring 1 inch if it is made from ¼-inch wire having 1 inch mean diameter and 14 working coils? The constant for this spring is 5625. This constant divided by 14 gives 401 pounds required to compress the spring 1 inch.

Example 5: — What would be the free height of the spring specified in the preceding example? The movement per coil or the deflection is 0.109 inch. Add to this the diameter of the wire, 0.250 inch, giving a total space or lead of spring of 0.359 inch. This value multiplied by 14 gives 5.026 inches as the free height of a spring with 14 working coils. Add to this, if the ends are ground, one diameter of wire for the end coils (dead coils) giving a total free height of 5.276 inches. If the ends are not ground, add two diameters of wire.

Navy Specifications for Spring Steel. — Spring steel for the United States Navy Department must be manufactured either by the open-hearth, crucible or electric furnace process. It must contain not less than 0.70, nor more than 1.10 per cent of carbon; not less than 0.25, nor more than 0.50 per cent of manganese; not over 0.25 per cent of silicon; not over 0.04 per cent of phosphorus; and not over 0.04 per cent of sulphur. Vanadium or other elements may be used to obtain the necessary physical requirements, in which case only the phosphorus and sulphur

Maximum Loads in Pounds and Corresponding Compressions per Coil in Inches of Helical Round Bar Springs

Size of Wire, Steel Wire Gage	Diam. of Wire, Inches	Mean or Pitch Diameter of Spring, Inches											
		1/16	3/32	1/8	5/32	3/16	7/32	1/4	9/32	5/16	3/8	7/16	1/2
26	0.0181	3.72 0.0059 630	2.48 0.0133 186	1.86 0.0236 79	1.49 0.037 40	1.24 0.053 23	1.06 0.072 15	0.940 0.094 10	0.827 0.119 7	....	....	....	....
25	0.020	5.03 0.0053 947	3.35 0.0119 280	2.51 0.0213 118	2.00 0.0328 88	1.67 0.0478 35	1.43 0.0652 21	1.25 0.0852 14	1.11 0.107 10	1.00 0.131 7.6	0.837 0.191 4.3	....	....
24	0.023	7.64 0.0046 1600	5.10 0.0104 500	3.82 0.0184 200	3.06 0.0285 100	2.55 0.0406 62	2.19 0.0571 38	1.91 0.0714 25	1.70 0.0937 18	1.52 0.114 13	1.27 0.162 8	....	....
23	0.025	9.82 0.0042 2300	6.54 0.0095 680	4.90 0.0170 280	3.92 0.026 150	3.27 0.038 85	2.70 0.052 50	2.45 0.068 36	2.18 0.086 25	1.96 0.105 18	1.63 0.152 10	1.35 0.209 6	....
22	0.028	....	9.19 0.0085 1070	6.89 0.0152 450	5.51 0.023 235	4.59 0.034 135	3.94 0.046 84	3.45 0.060 56	3.06 0.077 40	2.75 0.093 29	2.30 0.137 16.6	1.97 0.186 10.5	....
1/32″	0.031	....	12.76 0.0076 1650	9.57 0.0136 700	7.65 0.021 360	6.38 0.030 200	5.47 0.041 130	4.78 0.054 85	4.25 0.069 60	3.83 0.084 45	3.19 0.103 30	2.73 0.166 16	....
20	0.035	....	....	13.47 0.012 1100	10.77 0.019 560	8.98 0.027 330	7.69 0.037 200	6.73 0.048 138	5.98 0.061 97	5.38 0.075 70	4.49 0.109 40	3.84 0.149 25	3.36 0.193 17
19	0.041	....	....	....	....	14.43 0.023 600	12.37 0.032 380	10.82 0.041 260	9.62 0.052 180	8.62 0.064 130	7.21 0.093 77	6.18 0.127 45	5.41 0.166 32

Maximum Loads in Pounds and Corresponding Compressions per Coil in Inches of Helical Round Bar Springs

Size of Wire, Steel Wire Gage	Diam. of Wire, Inches	Mean or Pitch Diameter of Spring, Inches																
		3/16	7/32	1/4	9/32	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/8
18	0.0475	22.44 0.020 1120	19.21 0.028 680	16.83 0.036 465	14.96 0.045 330	13.46 0.056 240	11.22 0.080 140	9.61 0.110 87	8.41 0.144 58	7.48 0.182 40	6.73 0.224 30	6.12 0.273 22	5.61 0.323 17	....	....	....	....	....
17	0.054	....	28.26 0.024 1180	24.73 0.031 800	21.98 0.040 500	19.80 0.049 400	16.50 0.071 230	14.13 0.096 145	12.36 0.126 95	10.99 0.158 68	9.90 0.194 50	9.00 0.238 38	8.25 0.284 29	....	....	....	....	....
16	0.0625	....	....	38.35 0.027 1400	34.09 0.034 1000	30.67 0.042 720	25.56 0.061 410	21.91 0.083 260	19.17 0.108 170	17.04 0.138 120	15.34 0.168 90	13.91 0.206 65	12.78 0.245 52	11.80 0.288 40	10.95 0.334 30	10.22 0.383 25	9.58 0.435 21	8.52 0.552 15
15	0.072	....	....	58.23 0.0236 2460	51.72 0.031 1630	46.58 0.037 1250	38.80 0.053 730	33.27 0.069 480	29.11 0.095 300	25.86 0.119 215	23.29 0.148 155	21.17 0.178 118	19.40 0.213 90	18.05 0.250 77	16.63 0.276 60	15.63 0.333 47	14.55 0.378 38	12.93 0.479 26
14	0.080	....	....	80.42 0.0213 3700	71.48 0.027 2640	64.34 0.033 1940	53.6 0.048 1110	46.0 0.065 700	40.2 0.085 470	35.7 0.107 330	32.17 0.133 240	29.2 0.162 180	26.8 0.191 140	24.7 0.225 110	23.0 0.261 88	21.4 0.300 70	20.1 0.341 59	17.8 0.431 40
13½	0.08575	....	....	99 0.020 4950	88 0.025 3500	79 0.031 2550	66 0.046 1440	56.5 0.061 926	49.5 0.079 623	44.0 0.100 440	39.6 0.124 319	36.0 0.150 240	33.0 0.179 185	30.4 0.210 145	28.3 0.244 116	26.4 0.281 94	24.7 0.318 78	22.0 0.402 55
13	0.091	....	....	120 0.0185 6400	107 0.023 4600	92 0.029 3300	80 0.042 1900	69 0.057 1200	60 0.074 800	53.4 0.094 560	48.1 0.116 400	43.7 0.141 300	40.1 0.167 240	37.0 0.197 185	34.3 0.228 150	32.0 0.263 120	30.0 0.297 100	26.7 0.377 70
12½	0.0985	....	....	150 0.017 8600	133 0.022 6000	120 0.027 4400	100 0.039 2550	85.8 0.053 1600	75.0 0.069 1080	66.7 0.087 760	60.0 0.108 555	54.6 0.131 400	50.0 0.155 320	46.0 0.182 250	42.9 0.212 200	40.0 0.243 164	37.5 0.276 135	33.3 0.350 95

Maximum Loads in Pounds and Corresponding Compressions per Coil in Inches of Helical Round Bar Springs

Size of Wire, Steel Wire Gage	Diam. of Wire, Inches	Mean or Pitch Diameter of Spring, Inches																					
		5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/8	2 1/4	2 3/8	2 1/2
12	0.1055	145 0.025 5770	121 0.036 3300	104 0.049 2050	91 0.065 1400	81 0.082 980	73 0.101 720	66 0.122 540	61 0.146 410	52 0.198 260	45 0.260 170	40 0.328 120	36 0.406 89	33 0.488 67	30 0.584 50	28 0.686 40	26 0.795 32	24 0.913 26	22.7 1.038 20	... ...	... ...	... ...	... ...
11½	0.113	181 0.023 7700	151 0.034 4400	129 0.046 2800	113 0.060 1850	100 0.076 1300	91 0.094 960	82 0.113 780	75 0.136 550	64.7 0.185 340	56.6 0.241 234	50.3 0.305 165	45.3 0.377 120	41.1 0.454 90	37.8 0.543 69	34.8 0.637 54	32.3 0.739 43	30.1 0.848 35	28.3 0.964 29	... ...	... ...	... ...	... ...
11	0.120	... ...	180 0.032 5600	155 0.043 3600	136 0.057 2380	121 0.072 1670	108 0.089 1200	99 0.106 930	90.4 0.128 700	77.5 0.174 445	67.8 0.227 300	60.3 0.287 260	54.2 0.355 150	49.3 0.427 115	45.2 0.511 88	41.7 0.600 70	38.7 0.696 55	36.2 0.800 45	33.9 0.909 37	31.9 1.026 31	30.1 1.150 27	... ...	... ...
10½	0.127	... ...	214 0.029 7300	183 0.041 4400	161 0.053 3000	143 0.068 2000	128 0.084 1500	117 0.101 1100	107 0.121 880	92 0.164 550	80 0.214 370	71.5 0.271 260	64 0.335 190	58.5 0.404 140	53.6 0.483 110	49 0.567 85	46 0.657 70	42.9 0.755 56	40 0.858 46	38 0.970 39	36 1.084 33	... ...	... ...
10	0.135	... ...	... ...	220 0.039 5500	193 0.050 3800	172 0.064 2680	155 0.079 1960	140 0.095 1460	129 0.113 1140	110 0.154 700	96 0.202 470	86 0.255 330	77 0.315 240	70 0.380 180	64 0.454 140	59.4 0.533 110	55 0.618 88	51.5 0.710 72	48 0.808 59	45.4 0.912 49	43 1.020 42	... ...	... ...
9½	0.141	... ...	... ...	252 0.037 6700	220 0.048 4500	196 0.061 3100	176 0.075 2300	160 0.091 1700	147 0.109 1300	126 0.148 840	110 0.193 550	98 0.244 400	89 0.302 290	80 0.364 200	73 0.435 165	68 0.510 130	63 0.594 105	59 0.680 85	55 0.772 70	51.8 0.873 59	49 0.978 50	... ...	... ...
9	0.148	... ...	... ...	... ...	254 0.046 5500	224 0.058 3860	203 0.072 2800	185 0.087 2100	170 0.103 1650	146 0.141 1000	127 0.184 690	112 0.233 480	102 0.287 355	93 0.350 265	85 0.412 206	78.3 0.486 160	73 0.564 129	68 0.648 104	63.6 0.736 86	60 0.832 72	56 0.932 60	53.7 1.039 51	52 1.150 45
8½	0.1562	... ...	... ...	... ...	299 0.043 7000	266 0.055 4800	239 0.068 3500	218 0.082 2650	199 0.098 2000	171 0.134 1270	149 0.174 855	133 0.220 600	120 0.272 440	109 0.330 330	100 0.392 260	92 0.460 200	85.5 0.534 160	79.5 0.611 130	74.5 0.696 107	70.5 0.788 90	66.5 0.883 75	63 0.976 64	60 1.088 55

Maximum Loads in Pounds and Corresponding Compressions per Coil in Inches of Helical Round Bar Springs

Size of Wire, Steel Wire Gage	Diam. of Wire, Inches	Mean or Pitch Diameter of Spring, Inches																		
		9/16	5/8	11/16	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/8	2 1/4	2 3/8	2 1/2	2 5/8
8	0.162	296 0.053 5380	267 0.066 4000	243 0.080 3000	223 0.095 2300	191 0.129 1480	167 0.162 1000	148 0.213 700	133 0.263 500	121 0.320 380	111 0.378 295	102 0.444 230	95 0.515 184	89 0.591 150	83 0.673 123	78.5 0.760 103	74 0.851 86	70.3 0.949 74	66.7 1.052 63	63.6 1.165 54
7½	0.170	343 0.051 6700	309 0.063 4900	280 0.076 3680	257 0.090 2800	220 0.123 1780	193 0.160 1200	171 0.203 840	154 0.261 590	140 0.302 460	128 0.361 350	119 0.423 280	110 0.491 240	103 0.563 180	96 0.641 148	91 0.724 125	85.7 0.812 105	81 0.905 90	77 1.043 74	73.5 1.108 65
7	0.177	387 0.049 7900	348 0.060 5800	316 0.073 4300	290 0.087 3300	249 0.118 2100	217 0.154 1400	193 0.195 1000	174 0.240 725	158 0.289 540	145 0.346 418	134 0.407 329	124 0.472 262	116 0.541 214	109 0.616 175	102 0.696 146	97 0.780 124	92 0.870 105	87 0.962 90	83 1.064 78
3/16"	0.1875	....	414 0.057 7250	376 0.069 5440	345 0.081 4250	296 0.111 2650	259 0.145 1730	230 0.184 1250	207 0.227 900	188 0.276 680	172 0.327 520	159 0.384 410	148 0.445 330	139 0.511 270	129 0.582 220	122 0.657 170	115 0.736 156	109 0.800 135	103 0.908 110	98.6 1.004 98
6	0.192	....	445 0.055 8000	404 0.067 6000	370 0.080 4600	317 0.108 2900	278 0.142 1900	247 0.180 1370	222 0.222 1000	202 0.267 750	184 0.320 570	171 0.375 450	159 0.435 365	148 0.499 296	139 0.568 244	131 0.641 200	123 0.720 170	117 0.801 145	111 0.887 125	106 0.981 108
5	0.207	....	....	506 0.062 8000	464 0.074 6200	398 0.100 3900	348 0.131 2600	309 0.166 1850	278 0.206 1350	253 0.248 1000	232 0.296 780	214 0.347 610	199 0.403 490	186 0.462 400	174 0.526 330	164 0.595 275	154 0.666 230	147 0.743 200	139 0.824 168	132 0.910 145
7/32"	0.21875	....	....	... .	553 0.070 7900	473 0.095 4960	414 0.125 3330	368 0.158 2325	331 0.195 1700	301 0.234 1280	276 0.280 985	255 0.329 775	237 0.381 620	221 0.438 500	207 0.498 415	195 0.563 345	184 0.631 290	174 0.703 245	165 0.779 210	158 0.861 183
4	0.2253	....	....	....	596 0.068 8700	512 0.093 5500	447 0.121 3690	397 0.153 2580	357 0.189 1850	325 0.228 1400	298 0.272 1090	275 0.320 850	256 0.371 660	238 0.426 550	223 0.484 460	210 0.547 380	199 0.613 324	188 0.683 275	178 0.757 235	170 0.837 200

Maximum Loads in Pounds and Corresponding Compressions per Coil in Inches of Helical Round Bar Springs

Size of Wire, Steel Wire Gage	Diam. of Wire, Inches	Mean or Pitch Diameter of Spring, Inches																				
		1	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	3	3¼	3½	3¾	4	4½
3½	0.234	503	447	402	366	335	310	287	269	251	237	223	212	201	191	183	175	168	155	144	134	126	...
		0.117	0.147	0.182	0.220	0.262	0.308	0.357	0.410	0.467	0.526	0.590	0.657	0.728	0.805	0.881	0.963	1.048	1.232	1.428	1.640	1.869	...
		4260	3000	2200	1650	1270	1000	800	650	530	440	375	320	275	235	207	180	160	125	100	80	67	...
3	0.244	570	507	456	414	380	351	326	304	285	268	253	240	228	217	207	199	190	175	163	152	142	..
		0.112	0.141	0.175	0.210	0.250	0.295	0.342	0.392	0.447	0.504	0.565	0.630	0.698	0.771	0.840	0.923	1.000	1.180	1.368	1.568	1.788	...
		5080	3590	2600	1970	1500	1185	950	775	637	530	447	380	326	280	246	215	190	148	119	97	79	...
¼"	0.250	614	545	491	446	409	378	351	327	307	289	273	258	245	233	223	213	204	189	175	164	153	...
		0.109	0.138	0.170	0.205	0.245	0.288	0.334	0.383	0.436	0.492	0.552	0.615	0.680	0.753	0.820	0.901	0.983	1.152	1.336	1.533	1.744	...
		5625	3940	2880	2170	1665	1310	1050	850	700	587	494	419	360	310	270	236	205	164	130	106	87	...
2	0.263	714	635	571	519	476	440	408	381	357	336	317	301	285	272	259	249	238	220	204	190	178	...
		0.103	0.131	0.162	0.195	0.233	0.273	0.317	0.364	0.414	0.468	0.524	0.585	0.648	0.716	0.780	0.857	0.933	1.095	1.270	1.468	1.656	...
		7000	4820	3520	2660	2000	1600	1280	1045	860	720	600	510	440	380	330	290	255	193	160	128	107	...
1	0.283	...	788	710	646	592	546	507	474	444	418	398	374	355	338	323	309	296	273	254	237	222	...
		...	0.122	0.150	0.181	0.217	0.254	0.295	0.339	0.386	0.435	0.487	0.543	0.602	0.665	0.724	0.797	0.868	1.017	1.180	1.355	1.545	...
		...	6450	4750	3565	2725	2140	1718	1400	1150	960	817	688	590	508	446	387	341	268	215	175	143	...
0	0.307	...	1005	909	826	757	699	649	606	568	535	505	478	454	433	413	395	378	350	324	303	284	...
		...	0.112	0.139	0.168	0.200	0.234	0.272	0.312	0.355	0.401	0.450	0.501	0.555	0.613	0.671	0.734	0.800	0.938	1.090	1.249	1.421	...
		...	9000	6500	4900	3780	2990	2380	1940	1600	1300	1120	950	810	700	615	540	470	370	300	240	200	...
5/16"	0.3125	...	1073	966	880	805	743	690	644	604	568	537	509	483	460	439	420	403	371	345	322	302	268
		...	0.110	0.136	0.165	0.196	0.230	0.267	0.307	0.349	0.394	0.441	0.492	0.545	0.608	0.660	0.721	0.785	0.922	1.069	1.227	1.396	1.764
		...	9700	7000	5300	4100	3200	2580	2100	1860	1440	1210	1000	885	780	665	580	510	400	320	260	215	151
00	0.331	...	1266	1146	1035	949	876	814	765	712	670	633	600	570	542	518	495	474	438	407	382	356	317
		...	0.104	0.129	0.156	0.185	0.218	0.252	0.289	0.329	0.372	0.416	0.465	0.515	0.569	0.623	0.681	0.741	0.870	1.009	1.158	1.316	1.664
		...	12,170	8880	6630	5130	4010	3230	2645	2165	1800	1520	1290	1100	952	830	726	638	500	403	329	270	190
11/32"	0.34375	...	.....	1276	1160	1063	981	911	851	797	751	705	672	638	608	580	555	531	491	456	425	399	354
		...	.....	0.124	0.149	0.178	0.209	0.243	0.279	0.314	0.358	0.401	0.447	0.495	0.548	0.597	0.656	0.712	0.837	0.971	1.116	1.259	1.604
		...	.....	10,200	7785	6970	4690	3750	3048	2530	2097	1768	1500	1290	1100	970	846	760	586	469	380	317	220
000	0.362	...	.....	1490	1355	1242	1146	1064	993	931	876	828	788	745	709	677	648	621	573	532	497	465	414
		...	.....	0.116	0.140	0.168	0.196	0.231	0.265	0.301	0.337	0.377	0.421	0.467	0.520	0.562	0.620	0.678	0.784	0.923	1.059	1.204	1.508
		...	.....	12,860	9670	7390	5845	4600	3745	3100	2600	2195	1870	1638	1363	1200	1040	916	730	576	468	385	274

Maximum Loads in Pounds and Corresponding Compressions per Coil in Inches of Helical Round Bar Springs

Size of Wire, Steel Wire Gage	Diam. of Wire, Inches	Mean or Pitch Diameter of Spring, Inches																				
		1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	3	3¼	3½	3¾	4	4½	5	5½
⅜″	0.375	1506 0.137 11,000	1380 0.164 8400	1274 0.192 6600	1183 0.227 5200	1104 0.256 4300	1035 0.291 3550	974 0.328 2960	920 0.368 2500	872 0.410 2120	828 0.455 1810	789 0.504 1565	753 0.550 1365	720 0.601 1200	690 0.654 1050	637 0.768 829	591 0.891 660	553 1.022 540	518 1.164 445	460 1.572 225	... ...	... ...
13/32″	0.406	1911 0.126 15,160	1752 0.151 11,600	1617 0.177 9130	1500 0.205 7300	1402 0.236 5900	1314 0.268 4900	1237 0.306 4040	1168 0.340 3430	1106 0.380 2900	1051 0.420 2600	1000 0.465 2150	955 0.508 1870	911 0.555 1640	876 0.604 1450	808 0.709 1130	750 0.822 912	701 0.944 740	657 1.074 610	584 1.360 425	526 1.679 313	... ...
7/16″	0.4375	2384 0.118 20,100	2192 0.140 15,650	2022 0.165 12,250	1979 0.191 10,360	1754 0.219 8000	1644 0.249 6600	1547 0.281 5500	1460 0.315 4630	1384 0.351 3940	1315 0.389 3380	1252 0.430 2925	1192 0.471 2530	1144 0.515 2220	1096 0.560 1950	1011 0.658 1535	939 0.763 1230	877 0.876 1000	822 0.995 825	730 1.262 578	657 1.558 420	... ...
15/32″	0.46875	... ...	2696 0.130 20,500	2489 0.153 16,200	2311 0.178 13,000	2157 0.204 10,500	2022 0.232 8715	1903 0.263 7235	1797 0.295 6090	1703 0.328 5190	1617 0.363 4450	1541 0.401 3850	1470 0.440 3340	1407 0.480 2930	1348 0.523 2570	1244 0.614 2020	1155 0.713 1620	1078 0.818 1315	1011 0.931 1075	898 1.180 830	809 1.454 555	... ...
½″	0.500	... ...	3272 0.123 26,600	3021 0.144 21,000	2805 0.167 16,750	2617 0.192 13,600	2455 0.218 11,170	2310 0.246 9350	2181 0.276 7900	2066 0.308 6700	1967 0.341 5760	1870 0.376 4970	1785 0.413 4300	1707 0.450 3790	1636 0.491 3330	1510 0.576 2600	1402 0.668 2100	1309 0.767 1700	1227 0.873 1400	1091 1.104 975	984 1.363 720	... ...
17/32″	0.53125	... ...	3925 0.116 33,800	3623 0.135 26,800	3364 0.157 21,400	3141 0.180 17,400	2944 0.205 14,360	2770 0.232 11,940	2617 0.260 10,000	2475 0.289 8560	2355 0.320 7350	2242 0.354 6330	2144 0.388 5520	2048 0.424 4790	1962 0.462 4220	1811 0.542 3350	1682 0.628 2670	1575 0.721 2180	1472 0.820 1790	1308 1.040 1250	1177 1.283 910	... ...
9/16″	0.5625	... ...	... ...	4301 0.128 33,600	3993 0.148 27,000	3727 0.170 21,900	3495 0.194 18,000	3289 0.219 15,000	3106 0.245 12,670	2943 0.273 10,770	2796 0.304 9190	2662 0.335 7940	2542 0.365 7000	2431 0.400 6000	2330 0.436 5340	2150 0.512 4170	1996 0.594 3360	1804 0.681 2730	1747 0.774 2240	1553 0.982 1580	1398 1.212 1150	1271 1.460 870
19/32″	0.59375	... ...	... ...	... ...	4700 0.141 33,300	4383 0.161 27,200	4109 0.183 22,800	3867 0.207 19,100	3652 0.232 15,740	3460 0.259 13,300	3287 0.287 11,400	3131 0.317 9870	2988 0.347 8600	2858 0.380 7500	2739 0.413 6620	2529 0.485 5200	2348 0.562 4170	2196 0.645 3400	2054 0.734 2800	1826 0.934 1960	1643 1.158 1400	1494 1.388 1075
⅝″	0.625	... ...	... ...	... ...	... ...	5114 0.153 33,400	4793 0.175 27,300	4511 0.197 22,800	4261 0.220 19,300	4037 0.246 16,400	3835 0.272 14,000	3652 0.301 12,100	3486 0.330 10,550	3334 0.360 9200	3196 0.392 8150	2950 0.461 6350	2740 0.534 5130	2557 0.614 4160	2397 0.698 3400	2131 0.880 2420	1918 1.090 1750	1743 1.320 1319

requirements given need be adhered to. The ultimate tensile strength of the spring steel after heat-treatment must be at least 180,000 pounds per square inch, with an elastic limit of at least 75 per cent of the ultimate tensile strength. It must stand a deflection test as follows: A specimen bar, after having been heat-treated, resting upon supports 24 inches between centers, must not take a permanent set of more than 0.05 inch after the first application of a load corresponding to a fiber stress of 135,000 pounds per square inch, nor a permanent set of more than 7.5 per cent of the total deflection under a load producing a fiber stress of 160,000 pounds per square inch, nor any further set after five additional applications of a load producing a fiber stress of 150,000 pounds per square inch.

These specifications do not apply to drawn round or square wire below 7/16 inch in diameter or size, nor to flats below 3/16 inch in thickness.

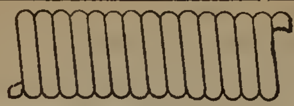
Tables for Spring Winding. — The use of the table "Data for Winding Piano Wire Tension Springs" may be best explained by an example. Assume that it is required to wind three different springs: the first to be wound from 0.035-inch wire to fit in an 11/16-inch hole, the second to be wound from 0.040-inch wire to fit in a 3/8-inch hole, and the third to be wound from 0.060-inch wire and to be a sliding fit on a 1/2-inch shaft. The tables show the proper sizes of mandrels for winding to be as follows: For the first spring, 0.562 inch; for the second spring, 0.250 inch; and for the third spring, 0.437 inch. In the latter case, 0.011 inch is allowed for play between spring and shaft. The wire sizes given in the table conform to the English music wire gage.

In all cases when the mandrel diameter is larger than 3/8 inch, the mandrel is mounted in a lathe chuck. Mandrels less than 3/8 inch in diameter are mounted in a drill chuck. In fastening the wire in a lathe chuck, one jaw is usually loosened, and when the mandrel is driven by a drill chuck, the wire is placed between the jaws and the mandrel. If a long spring is required, a mandrel of corresponding length is used, which is ground to an angle of 60 degrees at the end to fit into a female dead center for support. The wire is placed in a bench lathe boring-tool holder or a V-holder in the toolpost. A piece of brass, about 1/8 by 1/2 by 3 inches, is placed between the wire and the toolpost screw. A V-shaped groove is filed in this brass to hold the wire in place. The groove is filed in the lengthwise direction of the brass plate and is made of the proper depth for the size of wire from which the springs are being wound. This clamping arrangement is tightened up with the toolpost wrench, just enough tension being put on the wrench to keep the wire from slipping.

When springs are to be wound, the lathe is geared in the same manner as for screw cutting. The table "Gearing Lathe for Winding Wire Coil Springs" is given to indicate what gearing should be used. The figures in the body of the table give the number of threads per inch for which the lathe should be geared to wind coil springs of a given number of wire gage. The figures in the columns headed "A" are for close-wound tension springs, while the figures in the columns headed "B" are for compression springs. Assume, as an example, that it is required to wind a compression spring of No. 10 Brown & Sharpe gage wire. From the table, it will be seen that this spring should have four and one-half coils per inch, or in other words, that the lathe should be geared the same as for cutting four and one-half threads per inch.

Strength of Brass Wire. — The ultimate tensile strength of brass wire varies widely for different compositions and the strength per square inch increases as the diameter decreases. In general, the ultimate tensile strength of hard-drawn brass wire ranges from about 45,000 to 100,000 pounds per square inch, with higher values for some brass or bronze wires of special composition. Brass wire usually contains from 63 to 67 per cent of copper and from 33 to 37 per cent of zinc. The tensile strength increases with the zinc content within certain limits.

Gearing Lathe for Winding Wire Coil Springs. (See page 483.)



Tension Spring — *A*

Compression Spring — *B*

Number of Wire Gage	Brown & Sharpe		Birmingham or Stub's		Washburn & Moen Mfg. Co.		Trenton Iron Co.		Prentiss		Old English Brass Manufacturers'	
	A	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*	*A*	*B*
000000	..	...	...	...	2	1	...	...	...	...	...	...
00000	..	...	...	...	2¼	1⅛	2	1	...	...	...	...
0000	2	1	2	1	2½	1¼	2½	1¼	...	...	...	...
000	2¼	1⅛	2¼	1⅛	2¾	1⅜	2¾	1⅜	2¾	1⅜	...	...
00	2¾	1⅜	2½	1¼	3	1½	3	1½	3	1½	...	...
0	3	1½	2⅞	$1\frac{7}{16}$	3¼	1⅝	3	1½	3¼	1⅝	...	...
1	3¼	1⅝	3¼	1⅝	3½	1¾	3½	1¾	3¼	1⅝	...	...
2	3½	1¾	3½	1¾	3½	1¾	3½	1¾	3½	1¾	...	...
3	4	2	3½	1¾	4	2	4	2	4	2	...	...
4	4½	2¼	4	2	4	2	4	2	4	2	...	...
5	5½	2¾	4½	2¼	4½	2¼	4½	2¼	4½	2¼	...	...
6	6	3	4½	2¼	5	2½	5	2½	5	2½	...	...
7	6½	3¼	5½	2¾	5½	2¾	5½	2¾	5½	2¾	...	...
8	7	3½	6	3	6	3	6	3	6	3	...	...
9	8	4	6½	3¼	6½	3¼	6½	3¼	6½	3¼	...	...
10	9	4½	7	3½	7	3½	7	3½	7	3½	...	...
11	11	5½	8	4	8	4	8	4	8	4	...	...
12	12	6	9	4½	9	4½	9	4½	9	4½	...	...
13	14	7	10	5	10	5	10	5	10	5	...	...
14	14	7	12	6	12	6	12	6	12	6	12	6
15	16	8	13	6½	13	6½	14	7	13	6½	13	6½
16	18	9	14	7	14	7	16	8	14	7	14	7
17	22	11	16	8	16	8	18	9	16	8	16	8
18	24	12	20	10	20	10	22	11	20	10	20	10
19	28	14	23	11½	23	11½	24	12	23	11½	24	12
20	28	14	28	14	28	14	28	14	28	14	28	14
21	32	16	28	14	28	14	32	16	28	14	28	14
22	36	18	32	16	32	16	32	16	32	16	32	16
23	44	22	40	20	40	20	40	20	36	18	36	18
24	48	24	44	22	40	20	44	22	40	20	40	20
25	56	28	48	24	48	24	48	24	46	23	40	20
26	56	28	52	26	52	26	52	26	48	24	48	24
27	64	32	56	28	56	28	56	28	52	26	52	26
28	72	36	64	32	56	28	56	28	56	28	56	28
29	88	44	72	36	64	32	64	32	56	28	64	32
30	96	48	80	40	64	32	64	32	64	32	72	36
31	112	56	96	48	72	36	72	36	64	32	80	40
32	..	...	104	52	72	36	80	40	72	36	88	44
33	..	...	112	56	88	44	88	44	72	36	92	46
34	..	...	...	...	96	48	96	48	80	40	104	52
35	..	...	...	...	104	52	104	52	88	44	104	52

Data for Winding Piano Wire Tension Springs. (See page 483.)

Diam. of Mandrel, Inches	Inside Diam. of Spring, Inches	Outside Diam. of Spring, Inches	Number of Piano Wire	Diam. of Piano Wire, Inches	Diam. of Mandrel, Inches	Inside Diam. of Spring, Inches	Outside Diam. of Spring, Inches	Number of Piano Wire	Diam. of Piano Wire, Inches
0.125	0.130	0.150	1	0.0098	0.187	0.209	0.258	10	0.0245
0.187	0.192	0.212	1	0.0098	0.250	0.272	0.321	10	0.0245
0.250	0.255	0.275	1	0.0098	0.312	0.336	0.385	10	0.0245
0.312	0.318	0.338	1	0.0098	0.375	0.401	0.450	10	0.0245
0.375	0.382	0.402	1	0.0098	0.437	0.465	0.514	10	0.0245
0.125	0.130	0.151	2	0.0105	0.500	0.533	0.582	10	0.0245
0.187	0.192	0.213	2	0.0105	0.562	0.600	0.649	10	0.0245
0.250	0.255	0.276	2	0.0105	0.625	0.665	0.714	10	0.0245
0.312	0.318	0.339	2	0.0105	0.187	0.212	0.266	11	0.0270
0.375	0.382	0.403	2	0.0105	0.250	0.277	0.331	11	0.0270
0.125	0.130	0.152	3	0.0115	0.312	0.340	0.394	11	0.0270
0.187	0.193	0.215	3	0.0115	0.375	0.406	0.460	11	0.0270
0.250	0.256	0.278	3	0.0115	0.437	0.470	0.524	11	0.0270
0.312	0.320	0.342	3	0.0115	0.500	0.535	0.589	11	0.0270
0.375	0.382	0.404	3	0.0115	0.562	0.600	0.654	11	0.0270
0.125	0.135	0.160	4	0.0125	0.625	0.665	0.719	11	0.0270
0.187	0.197	0.222	4	0.0125	0.187	0.212	0.269	12	0.0285
0.250	0.260	0.285	4	0.0125	0.250	0.279	0.336	12	0.0285
0.312	0.322	0.347	4	0.0125	0.312	0.342	0.399	12	0.0285
0.375	0.385	0.410	4	0.0125	0.375	0.408	0.465	12	0.0285
0.125	0.135	0.164	5	0.0145	0.437	0.472	0.529	12	0.0285
0.187	0.198	0.227	5	0.0145	0.500	0.537	0.594	12	0.0285
0.250	0.261	0.290	5	0.0145	0.562	0.602	0.659	12	0.0285
0.312	0.324	0.353	5	0.0145	0.625	0.667	0.724	12	0.0285
0.375	0.389	0.418	5	0.0145	0.187	0.217	0.278	13	0.0305
0.125	0.135	0.165	6	0.0150	0.250	0.282	0.343	13	0.0305
0.187	0.198	0.228	6	0.0150	0.312	0.346	0.407	13	0.0305
0.250	0.262	0.292	6	0.0150	0.375	0.411	0.472	13	0.0305
0.312	0.325	0.355	6	0.0150	0.437	0.475	0.536	13	0.0305
0.375	0.390	0.420	6	0.0150	0.500	0.540	0.601	13	0.0305
0.125	0.137	0.172	7	0.0175	0.562	0.604	0.665	13	0.0305
0.187	0.201	0.236	7	0.0175	0.625	0.670	0.731	13	0.0305
0.250	0.266	0.301	7	0.0175	0.250	0.284	0.348	14	0.0320
0.312	0.330	0.365	7	0.0175	0.312	0.348	0.412	14	0.0320
0.375	0.395	0.430	7	0.0175	0.375	0.414	0.478	14	0.0320
0.125	0.138	0.176	8	0.0190	0.437	0.478	0.542	14	0.0320
0.187	0.202	0.240	8	0.0190	0.500	0.545	0.609	14	0.0320
0.250	0.266	0.304	8	0.0190	0.562	0.609	0.673	14	0.0320
0.312	0.330	0.368	8	0.0190	0.625	0.677	0.741	14	0.0320
0.375	0.396	0.434	8	0.0190	0.250	0.284	0.354	15	0.0350
0.125	0.145	0.189	9	0.0220	0.312	0.350	0.420	15	0.0350
0.187	0.209	0.253	9	0.0220	0.375	0.417	0.487	15	0.0350
0.250	0.271	0.315	9	0.0220	0.437	0.480	0.550	15	0.0350
0.312	0.335	0.379	9	0.0220	0.500	0.547	0.617	15	0.0350
0.375	0.400	0.444	9	0.0220	0.562	0.611	0.681	15	0.0350

Data for Winding Piano Wire Tension Springs

Diam. of Man-drel, Inches	Inside Diam. of Spring, Inches	Outside Diam. of Spring, Inches	Number of Piano Wire	Diam. of Piano Wire, Inches	Diam. of Man-drel, Inches	Inside Diam. of Spring, Inches	Outside Diam. of Spring, Inches	Number of Piano Wire	Diam. of Piano Wire, Inches
0.250	0.290	0.362	16	0.0360	0.312	0.369	0.467	23	0.0490
0.312	0.355	0.427	16	0.0360	0.375	0.436	0.534	23	0.0490
0.375	0.420	0.492	16	0.0360	0.437	0.500	0.598	23	0.0490
0.437	0.483	0.555	16	0.0360	0.500	0.565	0.663	23	0.0490
0.500	0.550	0.622	16	0.0360	0.562	0.628	0.726	23	0.0490
0.562	0.613	0.685	16	0.0360	0.625	0.700	0.798	23	0.0490
0.625	0.683	0.755	16	0.0360	0.312	0.371	0.477	24	0.0530
0.250	0.292	0.368	17	0.0380	0.375	0.438	0.544	24	0.0530
0.312	0.358	0.434	17	0.0380	0.437	0.504	0.610	24	0.0530
0.375	0.423	0.499	17	0.0380	0.500	0.568	0.674	24	0.0530
0.437	0.486	0.562	17	0.0380	0.562	0.630	0.736	24	0.0530
0.500	0.554	0.630	17	0.0380	0.625	0.702	0.808	24	0.0530
0.562	0.615	0.691	17	0.0380	0.312	0.374	0.486	25	0.0560
0.625	0.686	0.762	17	0.0380	0.375	0.441	0.553	25	0.0560
0.250	0.294	0.374	18	0.0400	0.437	0.508	0.620	25	0.0560
0.312	0.361	0.441	18	0.0400	0.500	0.571	0.683	25	0.0560
0.375	0.426	0.506	18	0.0400	0.562	0.634	0.746	25	0.0560
0.437	0.489	0.569	18	0.0400	0.625	0.706	0.818	25	0.0560
0.500	0.557	0.637	18	0.0400	0.312	0.375	0.495	26	0.0600
0.562	0.618	0.698	18	0.0400	0.375	0.442	0.562	26	0.0600
0.625	0.690	0.770	18	0.0400	0.437	0.511	0.631	26	0.0600
0.312	0.363	0.447	19	0.0420	0.500	0.573	0.693	26	0.0600
0.375	0.427	0.511	19	0.0420	0.562	0.635	0.755	26	0.0600
0.437	0.491	0.575	19	0.0420	0.625	0.710	0.830	26	0.0600
0.500	0.558	0.642	19	0.0420	0.375	0.445	0.573	27	0.0640
0.562	0.619	0.703	19	0.0420	0.437	0.513	0.641	27	0.0640
0.625	0.691	0.775	19	0.0420	0.500	0.575	0.703	27	0.0640
0.312	0.364	0.450	20	0.0430	0.562	0.637	0.765	27	0.0640
0.375	0.429	0.515	20	0.0430	0.625	0.713	0.841	27	0.0640
0.437	0.493	0.579	20	0.0430	0.375	0.446	0.583	28	0.0685
0.500	0.560	0.646	20	0.0430	0.437	0.514	0.651	28	0.0685
0.562	0.621	0.707	20	0.0430	0.500	0.575	0.712	28	0.0685
0.625	0.693	0.779	20	0.0430	0.562	0.638	0.775	28	0.0685
0.312	0.365	0.454	21	0.0445	0.625	0.714	0.851	28	0.0685
0.375	0.431	0.520	21	0.0445	0.375	0.448	0.591	29	0.0715
0.437	0.495	0.584	21	0.0445	0.437	0.516	0.659	29	0.0715
0.500	0.561	0.650	21	0.0445	0.500	0.577	0.720	29	0.0715
0.562	0.623	0.712	21	0.0445	0.562	0.640	0.783	29	0.0715
0.625	0.695	0.784	21	0.0445	0.625	0.714	0.857	29	0.0715
0.312	0.367	0.461	22	0.0470	0.375	0.451	0.603	30	0.0760
0.375	0.433	0.527	22	0.0470	0.437	0.518	0.670	30	0.0760
0.437	0.497	0.591	22	0.0470	0.500	0.580	0.732	30	0.0760
0.500	0.563	0.657	22	0.0470	0.562	0.643	0.795	30	0.0760
0.562	0.625	0.719	22	0.0470	0.625	0.717	0.869	30	0.0760
0.625	0.698	0.792	22	0.0470	0.375	0.455	0.617	31	0.0810

Data for Winding Piano Wire Tension Springs

Diam. of Mandrel, Inches	Inside Diam. of Spring, Inches	Outside Diam. of Spring, Inches	Number of Piano Wire	Diam. of Piano Wire, Inches	Diam. of Mandrel, Inches	Inside Diam. of Spring, Inches	Outside Diam. of Spring, Inches	Number of Piano Wire	Diam. of Piano Wire, Inches
0.437	0.522	0.684	31	0.081	0.375	0.480	0.682	34	0.101
0.500	0.585	0.747	31	0.081	0.437	0.550	0.752	34	0.101
0.562	0.647	0.809	31	0.081	0.500	0.610	0.812	34	0.101
0.625	0.722	0.884	31	0.081	0.562	0.673	0.875	34	0.101
0.375	0.461	0.633	32	0.086	0.625	0.750	0.952	34	0.101
0.437	0.527	0.699	32	0.086	0.375	0.490	0.708	35	0.109
0.500	0.590	0.762	32	0.086	0.437	0.560	0.778	35	0.109
0.562	0.651	0.823	32	0.086	0.500	0.622	0.840	35	0.109
0.625	0.727	0.899	32	0.086	0.562	0.686	0.904	35	0.109
0.375	0.467	0.649	33	0.091	0.625	0.765	0.983	35	0.109
0.437	0.533	0.715	33	0.091	0.375	0.500	0.736	36	0.118
0.500	0.595	0.777	33	0.091	0.437	0.572	0.808	36	0.118
0.562	0.657	0.839	33	0.091	0.500	0.637	0.873	36	0.118
0.625	0.733	0.915	33	0.091	0.562	0.702	0.938	36	0.118

Stresses Produced by Shocks

Stresses in Beams Produced by Shocks. — Any elastic structure subjected to a shock will deflect until the product of the average resistance, developed by the deflection, and the distance through which it has been overcome, has reached a value equal to the energy of the shock. It follows that for a given shock, the average resisting stresses are inversely proportional to the deflection. If the structure were perfectly rigid, the deflection would be 0, and the stress infinite. The effect of a shock is, therefore, to a great extent dependent upon the elastic property (the springiness) of the structure subjected to the impact.

The energy of a body in motion, such as a falling body, may be spent in each of four ways:

1. In deforming the body struck as a whole.
2. In deforming the falling body as a whole.
3. In partial deformation of both bodies on the surface of contact (most of this energy will be transformed into heat).
4. Part of the energy will be taken up by the supports, if these be not perfectly rigid and inelastic.

How much energy is spent in the last three ways it is in most cases difficult to determine, and for this reason it is safest to figure as if the whole amount were spent as in Case 1. In cases where a reliable judgment is possible, as to what percentage of the energy is spent in other ways than the first, a corresponding fraction of the total energy can be assumed as developing stresses in the body subjected to shocks.

From an investigation into the stresses produced by shocks (see MACHINERY, October, 1909), the following conclusions may be drawn: 1. A suddenly applied load will produce the same deflection, and, therefore, the same stress as a static load twice as great. 2. The unit stress p (see formulas in the accompanying table) for a given load producing a shock, varies directly as the square root of the modulus of elasticity E, and inversely as the square root of the length L of the beam and the area of the section. Thus, for instance, if the sectional area of a beam be increased four times, the unit stress will diminish only one-half. This is entirely different from the results produced by static loads where the stress would vary inversely with the area, and within certain limits be practically independent of the modulus of elasticity.

In the table, the expression for the approximate value of p, which is applicable whenever the deflection of the beam is small as compared with the total height h through which the body producing the shock is dropped, is always the same for beams supported at both ends and subjected to shock at *any* point between the supports. In the formulas all dimensions are in inches and weights in pounds.

Stresses Produced in Beams by Shocks

Method of Support, and Point Struck by Falling Body	Fiber (Unit) Stress p produced by Weight Q Dropped Through a Distance h	Approximate Value of p
Supported at both ends; struck in center.	$p = \frac{QaL}{4I}\left(1 + \sqrt{1 + \frac{96\,hEI}{QL^3}}\right)$	$p = a\sqrt{\frac{6\,QhE}{LI}}$
Fixed at one end; struck at the other.	$p = \frac{QaL}{I}\left(1 + \sqrt{1 + \frac{6\,hEI}{QL^3}}\right)$	$p = a\sqrt{\frac{6\,QhE}{LI}}$
Fixed at both ends; struck in center.	$p = \frac{QaL}{8I}\left(1 + \sqrt{1 + \frac{384\,hEI}{QL^3}}\right)$	$p = a\sqrt{\frac{6\,QhE}{LI}}$

I = moment of inertia of section; a = distance of extreme fiber from neutral axis: L = length of beam; E = modulus of elasticity.

Stresses in Helical Springs Produced by Shocks. — A load suddenly applied on a spring will produce the same deflection, and, therefore, also the same unit stress, as a static load twice as great. When the load drops from a height h, the stresses are as given in the accompanying table. The approximate values are applicable

Stresses Produced in Springs by Shocks

Form of Bar from Which Spring is Made	Fiber (Unit) Stress f Produced by Weight Q Dropped a Height h on a Helical Spring	Approximate Value of f
Round	$f = \frac{8\,QD}{\pi d^3}\left(1 + \sqrt{1 + \frac{Ghd^4}{4\,QD^3n}}\right)$	$f = 1.27\sqrt{\frac{QhG}{Dd^2n}}$
Square	$f = \frac{9\,QD}{4\,d^3}\left(1 + \sqrt{1 + \frac{Ghd^4}{0.9\,\pi QD^3n}}\right)$	$f = 1.34\sqrt{\frac{QhG}{Dd^2n}}$

G = modulus of elasticity for torsion; d = diameter or side of bar; D = mean diameter of spring; n = number of coils in spring.

when the deflection is small as compared with the height h. The formulas show that the fiber stress for a given shock will be greater in a spring made from a square bar, than in one made from a round bar, if the diameter of coil be the same, and the side of the square bar equals the diameter of the round bar. It is, therefore, more economical to use round stock for springs which must withstand shocks. This is due to the fact that the deflection for the same fiber stress for a square bar spring is smaller than that for a round bar spring, the ratio being as 4 to 5. The round bar spring is therefore capable of storing more energy than a square bar spring for the same stress.

Shocks from Bodies in Motion. — The formulas given can be applied, in general, to shocks from bodies in motion. A body of the weight W moving horizontally with the velocity of v feet per second, has a stored-up energy:

$$A = \frac{1}{2} \times \frac{Wv^2}{g} \text{ foot-pounds, or } \frac{6\,Wv^2}{g} \text{ inch-pounds.}$$

This expression may be substituted for Qh in the tables in the equations for unit stresses containing this quantity, and the stresses produced by the energy of the moving body thereby determined.

The formulas in the tables give the maximum value of the stresses, providing the designer with something definite to guide him even in cases where he may be justified in assuming that only a part of the energy of the shock is taken up by the member under stress.

TORSIONAL STRENGTH — SHAFTING

General Formula for Torsional Strength. — In the following formulas:

P = load applied at end of lever arm R, in pounds;
R = length of lever arm in inches;
M_t = torsional or twisting moment in inch-pounds;
S = permissible working stress in pounds per square inch;
I_p = polar moment of inertia of cross-section;
y = distance from center of gravity to most remote fiber;
Z_p = section modulus for torsion, or polar section modulus.

Then, for all cross-sections:

$$P \times R = M_t = S \times Z_p.$$

For circular cross-sections:

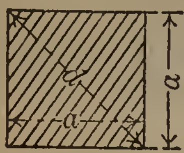
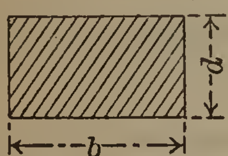
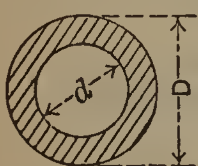
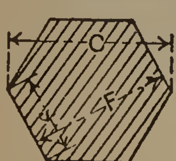
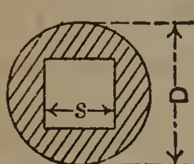
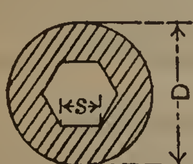
$$P \times R = M_t = \frac{S \times I_p}{y} = S \times Z_p.$$

(Compare also page 336, "General Formulas for Strength of Materials.")

Polar Moment of Inertia and Polar Section Modulus. — The polar moment of inertia of a surface is the moment of inertia with respect to an axis through the center of gravity, at right angles to the plane of the surface. The polar moment of inertia equals the sum of two moments of inertia taken with respect to two gravity-axes in the plane of the surface at right angles to each other. Thus, for example, the polar moment of inertia of a circle or a square is equal to two times the moment of inertia with respect to an axis in the plane of the surface through the center of gravity.

The polar section modulus or section modulus of torsion for *circular* sections equals the polar moment of inertia divided by the distance from the center of gravity to the most remote fiber. This method of obtaining the polar section modulus may also be applied with fair accuracy to sections that are nearly circular. For other cross-sections, the section modulus of torsion does not equal the polar moment of inertia divided by the distance from the center of gravity to the most remote fiber. The accompanying table gives formulas for the polar section modulus for a number of sections, some of which are not circular. In the latter case, the formulas are approximate.

Polar Moment of Inertia and Polar Section Modulus

Section	Polar Moment of Inertia I_p	Polar Section Modulus Z_p
	$\frac{a^4}{6} = 0.1667\,a^4$	$\frac{2}{9}\,a^3 = 0.22\,a^3 = 0.08\,d^3$
	$\frac{bd\,(b^2 + d^2)}{12}$	$\frac{2}{9}\,bd^2 = 0.22\,bd^2$ (d is the shorter side)
	$\frac{\pi D^4}{32} = 0.098\,D^4$	$\frac{\pi D^3}{16} = 0.196\,D^3$
	$\frac{\pi}{32}\,(D^4 - d^4) = 0.098\,(D^4 - d^4)$	$\frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right) = 0.196\left(\frac{D^4 - d^4}{D}\right)$
	$\frac{5\sqrt{3}}{8}\,s^4 = 1.0825\,s^4$	$0.92\,s^3 = 0.115\,C^3 = 0.178\,F^3$
	$\frac{\pi D^4}{32} - \frac{s^4}{6} = 0.098\,D^4 - 0.167\,s^4$	$\frac{\pi D^3}{16} - \frac{s^4}{3\,D} = 0.196\,D^3 - 0.333\,\frac{s^4}{D}$
	$\frac{\pi D^4}{32} - \frac{5\sqrt{3}}{8}\,s^4 = 0.098\,D^4 - 1.0825\,s^4$	$\frac{\pi D^3}{16} - \frac{5\sqrt{3}\,s^4}{4\,D} = 0.196\,D^3 - 2.165\,\frac{s^4}{D}$
	$\frac{\sqrt{3}\,s^4}{48} = 0.036\,s^4$	$\frac{s^3}{20} = 0.05\,s^3$

Angle of Torsional Deflection of a Cylindrical Shaft. — Let,

L = length of shaft being twisted, in inches;
D = diameter of shaft, in inches;
T = twisting moment, in inch-pounds;
G = torsional modulus of elasticity, generally assumed as 12,000,000 for steel shafting;
α = angle of torsional deflection in degrees.

Then:

$$\alpha = \frac{583.6\,TL}{D^4G}$$

Example: — Find the torsional deflection for a shaft 4 inches in diameter and 48 inches long, subjected to a twisting moment of 24,000 inch-pounds.

$$\alpha = \frac{583.6 \times 24{,}000 \times 48}{4^4 \times 12{,}000{,}000} = 0.22 \text{ degree, or } 13 \text{ minutes.}$$

Strength of Shafting. — The twisting strength of a shaft is determined from the formula:

$$T = PR = \frac{3.14\,d^3S}{16} = \frac{d^3S}{5.1}$$

or

$$d = \sqrt[3]{\frac{5.1\,PR}{S}} = \sqrt[3]{\frac{321{,}000 \text{ H.P.}}{nS}}$$

in which T = twisting moment in inch-pounds; P = force acting on the shaft, producing rotation, in pounds; R = length of lever arm of force P, in inches; d = diameter of shaft in inches; S = allowable torsional shearing stress in pounds per square inch; n = number of revolutions per minute; and H.P. = horsepower to be transmitted.

The allowable stress for ordinary shafting may be assumed as 4000 pounds per square inch for main power-transmitting shafts; 6000 pounds per square inch for lineshafts carrying pulleys; and 8500 pounds per square inch for small, short shafts, countershafts, etc. The horsepower transmitted using these allowable stresses is as follows:

For main power-transmitting shafts:

$$\text{H.P.} = \frac{d^3n}{80}\text{; or } d = \sqrt[3]{\frac{80 \text{ H.P.}}{n}}$$

For lineshafts carrying pulleys:

$$\text{H.P.} = \frac{d^3n}{53.5}\text{; or } d = \sqrt[3]{\frac{53.5 \text{ H.P.}}{n}}$$

For small, short shafts:

$$\text{H.P.} = \frac{d^3n}{38}\text{; or } d = \sqrt[3]{\frac{38 \text{ H.P.}}{n}}$$

Shafting which is subjected to shocks, sudden starting and stopping, etc., should be given a greater factor of safety than is indicated by the allowable stresses just mentioned.

Example: — What would be the diameter of a lineshaft to transmit 10 horsepower? The shaft makes 150 revolutions per minute.

$$d = \sqrt[3]{\frac{53.5 \times 10}{150}} = 1.53\text{, or, say, } 1\tfrac{9}{16} \text{ inch.}$$

Example: — What horsepower would a short shaft, 2 inches in diameter, carrying but two pulleys close to the bearings transmit? The shaft makes 300 revolutions per minute.

$$\text{H.P.} = \frac{2^3 \times 300}{38} = 63.$$

Torsional Deflection of Shafting. — The shafting must be proportioned not only so that it has the required strength for transmitting a given power, but so that it cannot be twisted through a greater angle than has been found satisfactory by experience. The allowable twist in degrees should, according to some authorities, not be over 5 minutes or about 0.08 degree per foot length of the shaft.

If G = the torsional modulus of elasticity of the material (= 12,000,000); L = the length of the shaft in feet; α = the angle of torsional deflection; and the other letters denote the same quantities as in the formulas just given for the strength of shafting, then the diameter, as determined from the torsional deflection, will be:

$$d = \sqrt[4]{\frac{32 \times 12 \times L \times 360 \times PR}{3.14 \times G \times \alpha \times 2 \times 3.14}}$$

For an angle of deflection equal to 0.08 degree per foot length of the shaft, or a total angle α of 0.08 L,

$$d = 0.29\sqrt[4]{PR} = 4.6\sqrt[4]{\frac{\text{H.P.}}{n}}$$

Example: — Find the diameter of a lineshaft to transmit 10 horsepower at 150 revolutions per minute with a torsional deflection not exceeding 0.08 degree per foot of length.

$$d = 4.6\sqrt[4]{\frac{10}{150}} = 2.35 \text{ inches.}$$

It will be seen, by comparing with the section, "Strength of Shafting," that a larger diameter is required, in this case, to prevent excessive torsional deflection than is required by mere considerations of strength. For short shafts, it is unnecessary to calculate for the angular deflection. It is only in the case of long shafts that this is necessary, and even then only if the torsional deflection would be objectionable.

Linear Deflection of Shafting. — For lineshafting, it is considered good practice to limit the deflection to a maximum of 0.010 inch per foot of length. The maximum distance in feet between bearings, for average conditions, in order to avoid excessive linear deflection, is determined by the formulas:

$$L = \sqrt[3]{720\, d^2} \text{ for bare shafts;}$$

$$L = \sqrt[3]{140\, d^2} \text{ for shafts carrying pulleys, etc.,}$$

in which d = diameter of shaft in inches; L = maximum distance between bearings in feet. Pulleys should be placed as close to the bearings as possible.

Tables of Horsepower Transmitted by Shafting. — The accompanying table, "Horsepower Transmitted by Shafting made from Medium Steel" gives the relation between the diameter of shaft, revolutions per minute, horsepower transmitted, and maximum distance in feet between bearings. Assume, for example, that it is required to find the diameter of a shaft for transmitting 40 horsepower at a speed of 250 revolutions per minute. The shaft is not subjected to any bending action except its own weight. From the table, it is found, by locating "40" in the column under 250 revolutions per minute, that the diameter of the shaft required is 2 inches. The maximum permissible distance between the shaft bearings is slightly more than 14 feet. When the exact horsepower cannot be found in the table, it is advisable to take the nearest larger value listed in the table and find the diameter of shafting required to transmit this horsepower. Tables are also given for the horsepower which can be safely transmitted by cold-rolled and turned steel lineshafting. The table for cold-rolled steel shafting is carried up to 5 inches only, because this diameter is the largest which is cold-rolled at the present time. These tables are used by the transmission department of the Jones & Laughlin Steel Co., and are based on the assumption that bearings are placed at intervals of from 8 to 10 feet and all

Horsepower Transmitted by Shafting made from Medium Steel

Diam. of Shaft, Inches	Transmitting Power, but Subject to No Bending Action Except Its Own Weight						Transmitting Power, and Subject to Bending Action of Pulleys, Belting, Etc.					
	Revolutions per Minute					Max. Distance in Feet Between Bearings	Revolutions per Minute					Max. Distance in Feet Between Bearings
	100 H.P.	150 H.P.	200 H.P.	250 H.P.	300 H.P.		100 H.P.	150 H.P.	200 H.P.	250 H.P.	300 H.P.	
1½	7	10	14	17	20	11.7	5	7	10	12	14	6.8
1⅝	9	13	17	21	26	12.4	6	9	12	15	18	7.2
1¾	11	16	21	26	32	13.0	8	11	15	18	22	7.5
1⅞	13	20	26	33	40	13.6	9	14	19	23	28	7.9
2	16	24	32	40	48	14.2	11	17	23	28	34	8.2
2⅛	19	29	38	48	58	14.8	14	21	27	34	42	8.6
2¼	23	34	46	57	68	15.4	16	24	33	41	48	8.9
2⅜	27	40	54	67	80	16.0	19	29	38	48	58	9.2
2½	31	47	63	78	94	16.5	22	33	45	55	66	9.6
2¾	42	62	83	102	124	17.6	30	44	59	74	89	10.2
3	54	81	108	134	162	18.6	39	58	77	96	116	10.8
3¼	69	103	137	172	206	19.7	49	74	98	123	148	11.4
3½	86	129	172	215	258	20.7	61	92	123	153	184	12.0
3¾	105	158	211	264	316	21.6	75	113	151	188	226	12.5
4	128	192	256	320	384	22.6	91	137	183	228	274	13.1

pulleys are located as near to the bearings as possible. In these tables, the body part in each gives the number of horsepower to be transmitted. For example, assume that a 3-inch cold-rolled steel lineshaft revolves at a speed of 400 revolutions per minute. Find the power that this shaft can safely transmit. By locating 3 inches in the left-hand column and 400 at the top of the vertical columns, and following the vertical column downward until opposite 3 inches, it is found that under the given conditions 154 horsepower may be safely transmitted.

In general, shafting up to three inches in diameter is almost always made from cold-rolled steel. This shafting is true and straight and needs no turning, but if keyways are cut in the shaft, it must, as a rule, be straightened afterwards, as the cutting of the keyways relieves the tension on the surface of the shaft due to the cold-rolling process. Sizes of shafting from three to five inches in diameter may be either cold-rolled or turned, more frequently the latter, while all larger sizes of shafting must be turned, because cold-rolled shafting is not available in diameters larger than five inches.

Horsepower Transmitted by Turned Steel Lineshafting

Diam. of Shaft	Number of Revolutions per Minute												
	100	125	150	175	200	225	250	300	350	400	450	500	600
1½	3.7	4.7	5.6	6.6	7.5	8.4	9.4	11.2	13.1	15.0	16.9	18.8	22
1 9/16	4.2	5.3	6.4	7.4	8.5	9.5	10.6	12.7	14.8	17.0	19.0	21	25
1⅝	4.8	5.9	7.1	8.3	9.5	10.7	11.9	14.3	16.6	19.0	21	24	28
1 11/16	5.3	6.7	8.0	9.3	10.7	12.0	13.4	16.0	18.7	21	24	27	32
1¾	5.9	7.4	8.9	10.4	11.9	13.4	14.9	17.9	21	24	27	30	36
1 13/16	6.6	8.2	9.9	11.5	13.2	14.8	16.5	19.8	23	26	30	33	40
1⅞	7.3	9.1	11.0	12.8	14.7	16.5	18.3	22	26	29	33	37	44
1 15/16	8.1	10.0	12.1	14.1	16.1	18.2	20	24	28	32	36	40	48
2	8.9	11.1	13.3	15.6	17.8	20	22	27	31	35	40	44	53
2 1/16	9.8	12.3	14.7	17.2	19.6	22	24	29	34	39	44	49	59
2⅛	10.6	13.3	16.0	18.6	21	24	27	32	37	43	48	53	64
2 3/16	11.6	14.6	17.5	20.0	23	26	29	35	41	47	52	58	70
2¼	12.6	15.8	19.0	22.0	25	28	32	38	44	51	57	63	76
2 5/16	13.7	17.2	21	24	27	31	34	41	48	55	62	69	82
2⅜	14.9	18.6	22	26	30	33	37	45	52	60	67	74	89
2 7/16	16.0	20	24	28	32	36	40	48	56	64	72	80	96
2½	17.4	22	26	30	35	39	43	52	61	69	78	87	104
2 9/16	18.7	23	28	33	37	42	47	56	66	75	84	94	112
2⅝	20	25	30	35	40	45	50	60	71	80	90	100	120
2 11/16	21	27	32	38	43	48	54	65	76	86	97	108	129
2¾	23	29	35	40	46	52	58	69	81	92	104	115	138
2 13/16	25	31	37	43	49	56	62	74	87	99	111	124	148
2⅞	26	33	40	46	53	59	66	79	92	105	119	132	158
2 15/16	28	35	42	49	56	63	70	84	99	113	127	141	169
3	30	37	45	52	60	67	75	90	105	120	135	150	180
3⅛	34	42	51	59	68	76	85	102	119	136	152	170	203
3¼	38	48	57	67	76	86	95	114	134	153	172	191	229
3⅜	43	53	64	75	85	96	107	128	150	171	192	213	256
3½	48	60	72	83	95	107	119	143	167	190	214	238	286
3⅝	53	66	79	93	106	119	132	159	185	211	238	265	317
3¾	59	73	88	103	117	132	146	176	205	234	264	293	351
3⅞	65	81	97	113	129	145	161	194	226	258	291	322	387
4	71	89	107	125	142	160	178	213	249	284	320	356	427
4⅛	78	98	117	136	156	176	195	235	273	312	351	390	468
4¼	85	107	128	149	170	192	213	256	298	341	385	426	511
4⅜	93	116	139	163	186	210	233	279	326	372	419	466	559
4½	102	127	152	178	203	228	253	305	356	405	456	507	610
4⅝	110	138	165	193	220	247	275	330	385	440	495	550	660
4¾	119	149	179	209	238	268	298	357	416	476	537	595	714
4⅞	129	161	193	226	258	290	322	387	452	516	581	646	775
5	139	174	208	244	278	313	347	417	486	557	625	695	835

Horsepower Transmitted by Turned Steel Lineshafting

Diam. of Shaft	Number of Revolutions per Minute									
	100	125	150	175	200	250	300	400	500	600
5⅛	150	187	225	262	300	375	450	600	750	900
5¼	161	201	242	281	322	403	483	644	805	966
5⅜	172	215	259	301	344	430	516	689	861	1033
5½	184	230	277	322	369	461	553	738	922	1106
5⅝	197	247	297	345	395	495	593	791	989	1186
5¾	211	264	317	369	422	528	633	844	1055	1260
5⅞	225	282	339	394	451	564	677	902	1128	1353
6	240	300	360	419	480	600	720	960	1200	1440
6⅛	255	320	384	446	511	639	766	1022	1278	1533
6¼	271	339	407	473	542	678	813	1084	1355	1626
6⅜	287	360	432	502	575	720	863	1151	1439	1726
6½	305	382	459	535	611	764	917	1222	1528	1833
6⅝	322	403	484	564	644	806	966	1289	1611	1933
6¾	341	427	513	598	682	853	1023	1364	1705	2046
6⅞	361	452	543	632	722	903	1083	1444	1805	2160
7	381	476	573	667	762	953	1143	1524	1905	2286
7⅛	401	502	603	702	802	1003	1203	1604	2005	2406
7¼	423	529	636	742	847	1059	1270	1693	2116	2540
7⅜	445	557	670	782	891	1115	1336	1782	2228	2673
7½	468	586	704	822	938	1173	1406	1875	2344	2814
7⅝	492	616	740	864	987	1231	1477	1969	2461	2953
7¾	516	646	776	904	1033	1293	1550	2066	2583	3100
7⅞	545	682	820	957	1091	1365	1637	2182	2728	3273
8	568	712	855	998	1138	1423	1707	2275	2844	3413
8⅛	593	742	892	1041	1187	1484	1780	2373	2966	3560
8¼	623	780	937	1094	1247	1559	1870	2493	3116	....
8⅜	651	816	980	1145	1305	1632	1957	2608	3261	....
8½	681	853	1025	1197	1364	1707	2047	2728	3411	....
8⅝	713	892	1072	1252	1427	1785	2140	2855	3566	....
8¾	744	931	1119	1306	1489	1863	2233	2977	3722	....
8⅞	766	972	1167	1363	1553	1943	2330	3106	3883	....
9	809	1013	1217	1421	1620	2027	2430	3240	....	....
9⅛	844	1056	1269	1482	1689	2113	2533	3377	....	....
9¼	878	1099	1321	1542	1758	2198	2637	3515	....	....
9⅜	915	1145	1376	1606	1831	2291	2747	3662	....	....
9½	951	1191	1431	1671	1904	2382	2858	3808	....	....
9⅝	989	1238	1488	1737	1980	2477	2972	3960	....	....
9¾	1029	1288	1548	1808	2060	2577	3090	....	....	....
9⅞	1069	1338	1608	1878	2140	2677	3210	....	....	....
10	1111	1388	1666	1944	2222	2778	3333	....	....	....
10¼	1195	1497	1798	2100	2393	2994	3590	....	....	....
10½	1285	1608	1934	2258	2573	3219	3860	....	....	....
10¾	1379	1726	2074	2422	2760	3453	4140	....	....	....
11	1477	1850	2223	2595	2958	3700	4437	....	....	....
11½	1688	2114	2540	2966	3380	4247	5070	....	....	....
12	1918	2402	2886	3369	3840	4804	5760	....	....	....

Horsepower Transmitted by Cold-rolled Steel Lineshafting

Diam. of Shaft	Number of Revolutions per Minute												
	100	125	150	175	200	225	250	300	350	400	450	500	600
1½	4.8	6.0	7.2	8.4	9.6	10.8	12.0	14.4	16.9	19.2	22	24	29
1 9/16	5.5	6.8	8.2	9.5	10.9	12.2	13.6	16.4	19.0	22	25	27	33
1⅝	6.1	7.6	9.2	10.7	12.2	13.8	15.3	18.4	21	24	28	31	37
1 11/16	6.9	8.6	10.3	12.0	13.7	15.4	17.1	21	24	27	31	34	41
1¾	7.7	9.6	11.5	13.4	15.3	17.2	19.1	23	27	31	34	38	46
1 13/16	8.5	10.6	12.7	14.8	16.9	19.0	21	25	30	34	38	42	51
1⅞	9.4	11.7	14.1	16.4	18.8	21	23	28	33	38	42	47	57
1 15/16	10.4	13.0	15.6	18.2	21	23	26	31	36	42	47	52	62
2	11.4	14.3	17.2	20	23	26	29	34	40	46	51	57	69
2 1/16	12.6	15.7	18.9	22	25	28	31	38	44	50	56	63	76
2⅛	13.7	17.1	21	24	27	31	34	41	48	55	61	68	82
2 3/16	15.0	18.7	22	26	30	34	37	45	52	60	67	75	90
2¼	16.3	20	24	29	33	37	41	49	57	65	73	81	98
2 5/16	17.7	22	27	31	35	40	44	53	62	71	80	88	106
2⅜	19.2	24	29	34	38	43	48	57	67	76	86	96	115
2 7/16	20	25	30	36	41	46	51	61	72	81	91	102	122
2½	22	28	33	39	45	50	56	67	78	89	100	112	133
2 9/16	24	30	36	42	48	54	60	72	84	96	108	120	144
2⅝	26	32	39	45	52	58	64	77	90	104	116	129	155
2 11/16	28	35	42	48	55	62	69	83	97	111	124	138	166
2¾	30	37	44	52	59	67	74	89	104	119	133	148	178
2 13/16	32	40	47	55	63	71	79	95	111	127	143	159	190
2⅞	34	42	51	59	68	76	85	101	119	135	152	169	203
2 15/16	36	45	54	63	72	81	90	108	127	144	162	181	217
3	39	48	58	67	77	87	96	116	135	154	173	192	231
3⅛	44	54	65	76	87	98	109	131	152	174	196	218	261
3¼	49	61	73	86	98	110	122	147	172	196	221	245	294
3⅜	55	69	83	96	110	124	137	165	192	220	247	275	330
3½	61	77	92	107	123	138	153	184	214	245	276	307	367
3⅝	68	85	102	119	136	153	170	204	238	272	306	340	408
3¾	75	94	113	132	151	170	189	226	264	301	340	377	452
3⅞	83	104	125	145	166	187	207	249	291	332	379	415	498
4	92	114	137	160	183	206	229	274	320	366	411	457	549
4⅛	101	125	150	175	201	226	251	300	351	401	451	501	601
4¼	110	137	164	192	219	246	273	328	383	438	492	547	657
4⅜	120	150	180	210	239	268	298	358	418	478	538	597	717
4½	130	163	195	228	261	293	326	391	455	521	586	651	781
4⅝	141	177	212	247	283	318	354	425	495	566	636	707	848
4¾	153	191	230	268	307	344	382	459	537	613	688	765	919
4⅞	166	207	249	290	331	372	413	496	580	662	745	827	994
5	179	224	268	313	358	402	447	537	625	715	805	895	1074

Shafting Subjected to Unusual or Severe Stresses. — The formulas given on the preceding pages relate to shafting used under normal conditions. Sometimes the distance between bearings must be abnormally great or the shafts are subjected to very severe stresses between the bearings, due to the gears, pulleys, etc., mounted on it. In such cases, it is necessary to calculate the stresses in the shafting and consider both the weight of the shafting itself and that of the pulleys, gears or other machine parts mounted on it. In calculating the stresses caused by the weight of the shaft itself, the total weight between the bearings is considered as uniformly distributed along the whole shaft, the shaft being considered as a beam freely supported at the bearings. The bending moments caused by pulleys, gears, etc., are then determined and added to find the total bending moment. This, in turn, is then combined with the torsional moment in the manner indicated below.

Combined Bending and Torsional Moments. — One of the most familiar examples of combined stresses in shafting is that of torsion and bending, the torsional stresses being shearing stresses and the bending stresses being tension and compression stresses. As stated under the head "Formulas for Combined Bending and Torsion," where this subject is dealt with in greater detail, the combined moment for ductile materials such as are used for steel shafting, may be written:

$$\text{Combined moment} = \sqrt{M_b^2 + M_t^2} = SZ$$

in which S = the permissible torsional stress in pounds per square inch;

Z = the section modulus for torsion;

M_b = maximum bending moment in inch-pounds;

M_t = torsional moment in inch-pounds.

The accompanying tables "Combined Bending and Torsional Moments" give the combined moment for a number of combinations of bending and torsional moments. For example, a shaft 3½ inches in diameter is subjected to a torsional moment of 36,000 inch-pounds and a bending moment of 35,000 inch-pounds. What is the combined moment and the stresses due to it?

Referring to the tables, and noting that all values may be multiplied by 10, it is found by locating the torsional moment 3600 (instead of 36,000) at the top, and the bending moment 3500 (instead of 35,000) in the left-hand column that the maximum combined moment in this case is 50,210 inch-pounds. Having found the maximum moment, the maximum stress can readily be determined. The section modulus Z for a 3½-inch shaft is 4.2; then the stress $S = 50{,}210 \div 4.2 = 11{,}900$ pounds, approximately.

It should be noted that in the case of shafting, the location and direction of the tooth loads, belt pulls, etc., which produce bending, remain fixed while the shaft rotates. The bending stresses are thus constantly varying in direction, and a greater factor of safety should be used than for a beam subjected to a load in one direction only.

Diameters of Shafts for Combined Torsional and Bending Stresses. — The tables "Diameters of Shafts subjected to Combined Torsional and Bending Stresses" are arranged for fiber stresses of 7500, 10,000 and 12,500 pounds per square inch. As an example, find the diameter of a shaft to sustain a bending moment of 80,000 inch-pounds and a torsional moment of 100,000 inch-pounds, if a fiber-stress of 10,000 pounds per square inch is allowable. By locating the torsional moment, as given in thousands of inch-pounds, at the top of the table, and the bending moment at the left-hand side (in the line and column corresponding to a fiber stress of 10,000 pounds per square inch), the diameter of the shaft corresponding to these moments is found in the body of the table to be 4¾ inches.

Combined Bending and Torsional Moments

All values in the table may be multiplied by 10, 100, 1000, etc., as required, but when any one value is thus multiplied, *all* the other values involved in the same problem must be multiplied by the same factor. The values at the right of the heavy zigzag line across the upper right-hand corner are for bending moments 2000 to 4500, given in the extreme right-hand column.

Bending Moments, In.-lbs. M_b	Torsional Moments in Inch-pounds (M_t)																Bending Moments, In.-lbs. M_b
	50	100	150	200	250	300	400	500	600	800	900	1000	1200	1400	1600	1800	
50	70.71	111.8	158.1	206.2	255.0	304.1	403.1	502.5	602	2154	2193	2236	2332	2441	2561	2691	2000
75	90.14	125.0	167.7	213.6	261.0	309.2	407.0	505.6	605	803	2377	2417	2506	2608	2720	2843	2200
100	111.8	141.4	180.3	223.6	269.3	316.2	412.3	509.9	608	806	2563	2600	2683	2778	2884	3000	2400
125	134.6	160.1	195.3	235.9	279.5	325.0	419.1	515.4	613	810	908	2786	2864	2953	3053	3162	2600
150	158.1	180.3	212.1	250.0	291.5	335.4	427.2	522.0	618	814	912	2973	3046	3131	3225	3329	2800
200	206.2	223.6	250.0	282.8	320.2	360.6	447.2	538.5	632	825	922	1020	3231	3311	3400	3499	3000
250	255.0	269.3	291.5	320.2	353.6	390.5	471.7	559.0	650	838	934	1031	1226	3770	3848	3936	3500
300	304.2	316.2	335.4	360.6	390.5	424.3	500.0	583.1	671	854	948	1044	1237	1432	4308	4386	4000
350	353.6	364.0	380.8	403.1	430.1	461.0	531.5	610.3	695	873	965	1059	1250	1443	1638	4847	4500
400	403.1	412.3	427.2	447.2	471.7	500.0	565.7	640.3	721	894	984	1077	1265	1456	1649	1844	400
450	452.8	461.0	474.3	492.4	514.8	540.8	602.1	672.7	750	918	1006	1097	1282	1471	1662	1855	450
500	....	....	522.0	538.5	559.0	583.1	640.3	707.1	781	943	1030	1118	1300	1487	1676	1868	500
600	....	....	618.5	632.5	650.0	670.8	721.1	781.0	848	1000	1082	1166	1342	1523	1709	1897	600
700	....	....	....	728.0	743.3	761.6	806.2	860.2	922	1063	1140	1221	1389	1565	1746	1931	700
800	....	....	....	824.6	838.2	854.4	894.4	943.3	1000	1131	1204	1281	1442	1612	1789	1970	800
900	....	....	....	....	934.1	948.7	984.9	1030.0	1082	1204	1273	1345	1500	1664	1836	2012	900
1000	....	....	....	....	1031.0	1044.0	1077.0	1118.0	1166	1281	1345	1414	1562	1720	1887	2059	1000
1100	....	....	....	....	....	1140.0	1170.0	1208.0	1253	1360	1421	1487	1628	1780	1942	2110	1100
1200	....	....	....	....	....	1237.0	1265.0	1300.0	1342	1442	1500	1562	1697	1844	2000	2163	1200
1400	....	....	....	....	....	....	1456.0	1487.0	1523	1612	1664	1720	1844	1980	2126	2280	1400
1600	....	....	....	....	....	....	1649.0	1676.0	1709	1789	1836	1887	2000	2126	2263	2408	1600
1800	....	....	....	....	....	....	....	1868.0	1897	1970	2012	2059	2163	2280	2408	2546	1800

Combined Bending and Torsional Moments

All values in the table may be multiplied by 10, 100, 1000, etc., as required, but when any one value is thus multiplied, *all* the other values involved in the same problem must be multiplied by the same factor.

Bending Moments, In.-lbs., M_b	Torsional Moments in Inch-pounds (M_t)													
	2000	2400	2800	3200	3600	4000	4400	4800	5200	6000	7000	8000	10,000	12,000
600	2088	2474	2864	3256	3650	4045	4441	4837	5235	6030	7026	8023	10,018	12,015
700	2119	2500	2886	3276	3667	4061	4455	4851	5247	6041	7035	8031	10,025	12,020
800	2154	2530	2912	3298	3688	4076	4472	4866	5261	6053	7046	8040	10,032	12,027
900	2193	2563	2941	3324	3711	4100	4491	4884	5277	6067	7058	8051	10,040	12,034
1000	2236	2600	2973	3353	3736	4123	4512	4903	5295	6083	7071	8062	10,050	12,042
1100	2283	2640	3008	3384	3764	4149	4535	4924	5315	6100	7086	8075	10,060	12,050
1200	2332	2683	3046	3418	3795	4176	4561	4948	5337	6119	7102	8090	10,072	12,060
1400	2441	2778	3131	3493	3863	4238	4617	5000	5385	6161	7139	8122	10,097	12,081
1600	2561	2884	3225	3578	3940	4308	4682	5060	5441	6210	7181	8158	10,127	12,106
1800	2691	3000	3329	3672	4025	4386	4754	5126	5503	6264	7228	8200	10,161	12,134
2000	2828	3124	3441	3774	4119	4472	4837	5200	5571	6325	7280	8246	10,198	12,166
2200	2973	3256	3561	3883	4219	4565	4919	5280	5646	6391	7338	8297	10,239	12,200
2400	3124	3394	3688	4000	4327	4665	5012	5367	5727	6462	7400	8352	10,284	12,238
2600	3280	3538	3821	4123	4441	4771	5111	5459	5814	6532	7467	8412	10,333	12,278
2800	3441	3688	3960	4252	4561	4883	5216	5557	5906	6621	7533	8476	10,385	12,322
3000	3601	3842	4104	4386	4868	5000	5325	5660	6003	6708	7616	8544	10,440	12,364
3500	4031	4244	4482	4742	5021	5315	5622	5941	6268	6946	7826	8732	10,609	12,500
4000	4472	4665	4883	5123	5381	5657	5946	6248	6561	7211	8062	8944	10,770	12,649
4500	4924	5100	5300	5522	5763	6021	6294	6580	6877	7500	8322	9179	10,966	12,816
5000	5385	5546	5731	5936	6161	6403	6660	6931	7214	7810	8602	9434	11,180	13,000
5500	5852	6001	6172	6363	6573	6801	7043	7300	7569	8139	8902	9708	11,413	13,200
6000	6325	6462	6621	6800	6987	7211	7440	7684	7940	8485	9220	10,000	11,662	13,416

Diameters of Shafts for Combined Torsional and Bending Stresses

Bending Moments in Thousands of Inch-pounds (left and right columns)

Fiber Stress in Pounds per Square Inch			Torsional Moments in Thousands of Inch-pounds																		Fiber Stress in Pounds per Square Inch		
7500			0	3.75	7.5	11.25	15	18.75	22.5	30	37.5	45	52.5	60	67.5	75	90	105	120	135			7500
	10,000		0	5	10	15	20	25	30	40	50	60	70	80	90	100	120	140	160	180		10,000	
		12,500	0	6.25	12.5	18.75	25	31.25	37.5	50	62.5	75	87.5	100	112.5	125	150	175	200	225	12,500		
0	0	0	0	1 3/8	1 23/32	1 31/32	2 3/16	2 11/32	2 1/2	2 3/4	2 15/16	3 1/8	3 5/16	3 7/16	3 19/32	3 23/32	3 15/16	4 5/32	4 11/32	4 1/2	0	0	0
3.75	5	6.25	1 23/32	1 27/32	2 1/32	2 7/32	2 3/8	2 1/2	2 5/8	2 27/32	3 1/16	3 7/32	5 15/16	5 31/32	5 31/32	6	6 1/16	6 1/16	6 1/8	6 3/16	250	200	150
5.625	7.5	9.375	1 31/32	2 1/32	2 3/16	2 5/16	2 15/32	2 19/32	2 11/16	2 29/32	3 3/32	3 1/4	3 13/32	3 9/16	6 3/16	6 3/16	6 1/4	6 1/4	6 5/16	6 3/8	275	220	165
7.5	10	12.5	2 3/16	2 7/32	2 5/16	2 7/16	2 9/16	2 21/32	2 25/32	2 31/32	3 5/32	3 5/16	3 7/16	3 19/32	6 5/16	6 3/8	6 3/8	6 7/16	6 7/16	6 1/2	300	240	180
9.375	12.5	15.63	2 11/32	2 3/8	2 7/16	2 17/32	2 21/32	2 3/4	2 27/32	3 1/32	3 3/16	3 11/32	3 1/2	3 5/8	3 3/4	6 1/2	6 9/16	6 9/16	6 5/8	6 5/8	325	260	195
11.25	15	18.75	2 1/2	2 1/2	2 9/16	2 21/32	2 3/4	2 13/16	2 29/32	3 3/32	3 1/4	3 13/32	3 17/32	3 21/32	3 25/32	6 5/8	6 11/16	6 11/16	6 3/4	6 13/16	350	280	210
15	20	25	2 3/4	2 3/4	2 25/32	2 27/32	2 29/32	3	3 1/16	3 7/32	3 11/32	3 1/2	3 5/8	3 3/4	3 27/32	3 31/32	6 13/16	6 7/8	6 7/8	6 15/16	375	300	225
18.75	25	31.25	2 15/16	2 15/16	2 31/32	3 1/32	3 3/32	3 1/8	3 3/16	3 11/32	3 15/32	3 19/32	3 23/32	3 13/16	3 15/16	4 1/32	4 7/32	7 3/16	7 3/16	7 1/4	437.5	350	262.5
22.5	30	37.5	3 1/8	3 5/32	3 5/32	3 3/16	3 1/4	3 9/32	3 11/32	3 7/16	3 9/16	3 11/16	3 25/32	3 7/8	4	4 3/32	4 9/32	4 15/32	7 1/2	7 1/2	500	400	300
26.25	35	43.75	3 9/32	3 9/32	3 5/16	3 11/32	3 3/8	3 13/32	3 15/32	3 9/16	3 21/32	3 3/4	3 7/8	3 31/32	4 1/16	4 5/32	4 11/32	4 1/2	4 21/32	7 13/16	562.5	450	337.5
30	40	50	3 7/16	...	...	3 15/32	3 1/2	3 17/32	3 19/32	3 21/32	3 3/4	3 27/32	3 15/16	4 1/16	4 1/8	4 7/32	4 13/32	4 9/16	4 23/32	4 27/32	50	40	30
33.75	45	56.25	3 19/32	...	...	...	3 5/8	3 21/32	3 11/16	3 25/32	3 27/32	3 15/16	4 1/32	4 1/8	4 7/32	4 9/32	4 7/16	4 5/8	4 3/4	4 29/32	56.25	45	33.75
37.5	50	62.5	3 23/32	...	...	...	3 3/4	3 25/32	3 13/16	3 7/8	3 15/16	4 1/32	4 1/8	4 3/16	4 9/32	4 11/32	4 1/2	4 21/32	4 13/16	4 15/16	62.5	50	37.5
45	60	75	3 15/16	...	...	...	3 31/32	4	4 1/32	4 1/16	4 1/8	4 3/16	4 9/32	4 11/32	4 13/32	4 1/2	4 5/8	4 25/32	4 29/32	5 1/32	75	60	45
52.5	70	87.5	4 5/32	...	...	...	4 3/16	4 7/16	4 7/32	4 1/4	4 5/16	4 3/8	4 13/32	4 15/32	4 17/32	4 5/8	4 3/4	4 7/8	5	5 1/8	87.5	70	52.5
60	80	100	4 11/32	...	...	...	...	4 3/8	4 3/8	4 13/32	4 15/32	4 1/2	4 17/32	4 19/32	4 21/32	4 3/4	4 27/32	4 31/32	5 3/32	5 7/32	100	80	60
67.5	90	112.5	4 1/2	...	...	...	...	...	4 17/32	4 19/32	4 5/8	4 21/32	4 23/32	4 3/4	4 13/16	4 7/8	4 31/32	5 1/16	5 3/16	5 5/16	112.5	90	67.5
75	100	125	4 11/16	...	...	...	...	...	4 23/32	4 23/32	4 3/4	4 25/32	4 13/16	4 7/8	4 15/16	4 31/32	5 3/32	5 3/16	5 9/16	5 3/8	125	100	75
82.5	110	137.5	4 13/16	...	...	...	...	...	4 27/32	4 7/8	4 29/32	4 15/16	4 31/32	5	5 1/32	5 3/32	5 3/16	5 9/32	5 3/8	5 15/32	137.5	110	82.5
90	120	150	4 31/32	...	...	...	...	...	...	5	5 1/32	5 1/16	5 3/32	5 1/8	5 5/32	5 7/32	5 9/32	5 3/8	5 15/32	5 9/16	150	120	90
105	140	175	5 7/32	...	...	...	...	...	...	5 1/4	5 9/32	5 5/16	5 11/32	5 11/32	5 3/8	5 7/16	5 1/2	5 9/16	5 21/32	5 23/32	175	140	105
120	160	200	5 15/32	...	...	...	...	...	...	...	5 1/2	5 17/32	5 17/32	5 9/16	5 19/32	5 5/8	5 11/16	5 3/4	5 13/16	6	200	160	120
135	180	225	5 11/16	...	...	...	...	...	...	...	5 23/32	5 3/4	5 3/4	5 25/32	5 25/32	5 13/16	5 7/8	5 15/16	6	6 1/16	225	180	135

Values at right of heavy zigzag line across upper right-hand corner are for the bending moments in upper part of three right-hand columns.

Diameters of Shafts for Combined Torsional and Bending Stresses

Fiber Stress in Pounds per Square Inch			Torsional Moments in Thousands of Inch-pounds																		
7500			0	150	165	180	210	240	270	300	330	360	390	420	450	525	600	675	750	825	900
	10,000		0	200	220	240	280	320	360	400	440	480	520	560	600	700	800	900	1000	1100	1200
		12,500	0	250	275	300	350	400	450	500	550	600	650	700	750	875	1000	1125	1250	1375	1500
Bending Moments in Thousands of Inch-pounds																					
0	0	0	0	4 11/16	4 13/16	4 31/32	5 7/32	5 15/32	5 11/16	5 7/8	6 1/16	6 1/4	6 7/16	6 9/16	6 3/4	7 1/16	7 7/16	7 11/16	8	8 1/4	8 1/2
45	60	75	3 15/16	5 5/32	5 9/32	5 3/8	5 19/32	5 13/16	6	6 3/16	6 3/8	6 1/2	6 11/16	6 13/16	6 15/16	7 5/16	7 5/8	7 7/8	8 1/8	8 3/8	8 5/8
52.5	70	87.5	4 5/32	5 1/4	5 11/32	5 15/32	5 11/16	5 7/8	6 1/16	6 1/4	6 7/16	6 9/16	6 3/4	6 7/8	7	7 5/16	7 5/8	7 15/16	8 3/16	8 7/16	8 5/8
60	80	100	4 11/32	5 5/16	5 7/16	5 17/32	5 3/4	5 15/16	6 1/8	6 5/16	6 7/16	6 5/8	6 3/4	6 7/8	7 1/16	7 3/8	7 11/16	7 15/16	8 3/16	8 7/16	8 11/16
67.5	90	112.5	4 1/2	5 13/32	5 1/2	5 5/8	5 13/16	6	6 3/16	6 5/16	6 1/2	6 5/8	6 13/16	6 15/16	7 1/16	7 3/8	7 11/16	8	8 1/4	8 1/2	8 11/16
75	100	125	4 11/16	5 15/32	5 19/32	5 11/16	5 7/8	6 1/16	6 1/4	6 3/8	6 9/16	6 11/16	6 7/8	7	7 1/8	7 7/16	7 3/4	8	8 1/4	8 1/2	8 3/4
82.5	110	137.5	4 13/16	5 9/16	5 21/32	5 3/4	5 15/16	6 1/8	6 5/16	6 7/16	6 5/8	6 3/4	6 7/8	7	7 1/8	7 1/2	7 3/4	8 1/16	8 5/16	8 1/2	8 3/4
90	120	150	4 31/32	5 21/32	5 3/4	5 27/32	6	6 3/16	6 3/8	6 1/2	6 5/8	6 3/4	6 15/16	7 1/16	7 3/16	7 1/2	7 13/16	8 1/16	8 5/16	8 9/16	8 3/4
105	140	175	5 7/32	5 13/16	5 7/8	5 31/32	6 1/8	6 5/16	6 7/16	6 5/8	6 3/4	6 7/8	7	7 1/8	7 1/4	7 9/16	7 7/8	8 1/8	8 3/8	8 5/8	8 13/16
120	160	200	5 15/32	5 31/32	6 1/16	6 1/8	6 5/16	6 7/16	6 9/16	6 11/16	6 7/8	7	7 1/8	7 1/4	7 3/8	7 5/8	7 15/16	8 3/16	8 7/16	8 11/16	8 7/8
135	180	225	5 11/16	6 1/8	6 3/16	6 1/4	6 3/8	6 9/16	6 11/16	6 13/16	6 15/16	7 1/16	7 3/16	7 5/16	7 7/16	7 11/16	8	8 1/4	8 1/2	8 11/16	8 15/16
150	200	250	5 7/8	6 1/4	6 5/16	6 3/8	6 1/2	6 5/8	6 13/16	6 15/16	7 1/16	7 3/16	7 1/4	7 3/8	7 1/2	7 3/4	8 1/16	8 5/16	8 9/16	8 3/4	8 15/16
165	220	275	6 1/16	6 7/16	6 1/2	6 9/16	6 5/8	6 3/4	6 15/16	7	7 1/8	7 1/4	7 3/8	7 1/2	7 9/16	7 7/8	8 1/8	8 3/8	8 9/16	8 13/16	9
180	240	300	6 1/4	6 9/16	6 5/8	6 11/16	6 3/4	6 7/8	7	7 1/8	7 1/4	7 5/16	7 7/16	7 9/16	7 11/16	7 15/16	8 3/16	8 7/16	8 5/8	8 7/8	9 1/16
195	260	325	6 7/16	6 11/16	6 3/4	6 13/16	6 7/8	7	7 1/8	7 3/16	7 5/16	7 7/16	7 9/16	7 5/8	7 3/4	8	8 1/4	8 1/2	8 11/16	8 15/16	9 1/8
210	280	350	6 9/16	6 13/16	6 7/8	6 15/16	7	7 1/8	7 3/16	7 5/16	7 7/16	7 1/2	7 5/8	7 3/4	7 13/16	8 1/16	8 5/16	8 9/16	8 3/4	8 15/16	9 3/16
225	300	375	6 3/4	6 15/16	7	7 1/16	7 1/8	7 1/4	7 3/8	7 7/16	7 1/2	7 5/8	7 11/16	7 13/16	7 15/16	8 1/8	8 3/8	8 5/8	8 13/16	9	9 1/4
262.5	350	437.5	7 1/16	7 1/4	7 5/16	7 3/8	7 7/16	7 1/2	7 9/16	7 5/8	7 3/4	7 13/16	7 15/16	8	8 1/8	8 5/16	8 9/16	8 3/4	8 15/16	9 1/8	9 3/8
300	400	500	7 7/16	7 9/16	7 9/16	7 5/8	7 11/16	7 3/4	7 13/16	7 7/8	8	8 1/16	8 1/8	8 3/16	8 5/16	8 1/2	8 11/16	8 7/8	9 1/8	9 5/16	9 7/16
337.5	450	562.5	7 11/16	7 13/16	7 7/8	7 7/8	7 15/16	8	8 1/16	8 1/8	8 3/16	8 1/4	8 5/16	8 7/16	8 1/2	8 11/16	8 7/8	9 1/16	9 1/4	9 7/16	9 5/8
375	500	625	8	8 1/16	8 1/8	8 1/8	8 3/16	8 1/4	8 5/16	8 3/8	8 7/16	8 1/2	8 9/16	8 5/8	8 11/16	8 7/8	9	9 3/16	9 3/8	9 9/16	9 3/4
412.5	550	687.5	8 1/4	8 5/16	8 3/8	8 3/8	8 7/16	8 7/16	8 1/2	8 9/16	8 5/8	8 11/16	8 3/4	8 13/16	8 7/8	9	9 3/16	9 3/8	9 1/2	9 11/16	9 7/8
450	600	750	8 1/2	8 9/16	8 9/16	8 5/8	8 5/8	8 11/16	8 3/4	8 3/4	8 13/16	8 7/8	8 15/16	9	9 1/16	9 3/16	9 3/8	9 1/2	9 5/8	9 13/16	10

Table Giving Comparative Torsional Strength and Weight of Hollow and Solid Shafting with Same Outside Diameter

(Upper figures in each line give number of per cent decrease in strength; lower figures give per cent decrease in weight.)

Example: — A 6-inch shaft, with a 4-inch hole through it, has a weight 44.44 per cent less than a solid 6-inch shaft, but its strength is decreased only 19.76 per cent.

Diam. of Solid and Hollow Shaft, Inches	Diameter of Axial Hole in Hollow Shaft, Inches									
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼
1½	19.76 44.44	48.23 69.44	....	....	....	....	....	....	....	....
1¾	10.67 32.66	26.04 51.02	53.98 73.49	....	....	....	....	....	....	....
2	6.25 25.00	15.26 39.07	31.65 56.25	58.62 76.54	....	....	....	....	....	....
2¼	3.91 19.75	9.53 30.87	19.76 44.44	36.60 60.49	62.43 79.00	....	....	....	....	....
2½	2.56 16.00	6.25 25.00	12.96 36.00	24.01 49.00	40.96 64.00	65.61 81.00	....	....	....	....
2¾	1.75 13.22	4.28 20.66	8.86 29.74	16.40 40.48	27.98 52.89	44.82 66.92	68.30 82.63	....	....	....
3	1.24 11.11	3.01 17.36	6.25 25.00	11.58 34.01	19.76 44.44	31.65 56.25	48.23 69.44	70.61 84.03	....	....
3¼	0.87 9.46	2.19 14.80	4.54 21.30	8.41 29.00	14.35 37.87	23.00 47.90	35.02 59.17	51.27 71.61	72.61 85.22	....
3½	0.67 8.16	1.63 12.76	3.38 18.36	6.25 25.00	10.67 32.66	17.08 41.33	26.04 51.02	38.12 61.75	53.98 73.49	74.35 86.23
3¾	0.51 7.11	1.24 11.11	2.56 16.00	4.75 21.77	8.09 28.45	12.96 36.00	19.76 44.44	28.93 53.79	40.96 64.00	56.42 75.11
4	0.40 6.25	0.96 9.77	1.98 14.06	3.68 19.14	6.25 25.00	10.02 31.64	15.26 39.07	22.34 47.28	31.65 56.25	43.58 66.02
4¼	0.31 5.54	0.74 8.65	1.56 12.45	2.89 16.95	4.91 22.15	7.86 28.03	11.99 34.61	17.53 41.89	24.83 49.85	34.20 58.49
4½	0.25 4.94	0.70 7.72	1.24 11.11	2.29 15.12	3.91 19.75	6.25 25.00	9.53 30.87	13.87 37.36	19.76 44.44	27.20 52.16
4¾	0.20 4.43	0.50 6.93	1.00 9.97	1.85 13.57	3.15 17.73	5.04 22.44	7.68 27.70	11.24 33.53	15.92 39.90	21.92 46.82
5	0.16 4.00	0.40 6.25	0.81 8.10	1.51 12.25	2.56 16.00	4.11 20.25	6.25 25.00	9.16 30.26	12.96 36.00	17.86 42.25
5½	0.11 3.30	0.27 5.17	0.55 7.43	1.03 10.12	1.75 13.22	2.81 16.73	4.27 20.66	6.25 25.00	8.86 29.76	12.13 34.92
6	0.09 2.77	0.19 4.34	0.40 6.25	0.73 8.50	1.24 11.11	1.98 14.06	3.02 17.36	4.42 21.01	6.25 25.00	8.61 29.34
6½	0.06 2.36	0.14 3.70	0.29 5.32	0.59 7.24	0.90 9.47	1.45 11.98	2.19 14.79	3.21 17.90	4.54 21.30	6.25 25.00
7	0.05 2.04	0.11 3.19	0.22 4.59	0.40 6.25	0.67 8.16	1.07 10.33	1.63 12.76	2.39 15.44	3.38 18.36	4.65 21.55
7½	0.04 1.77	0.08 2.77	0.16 4.00	0.30 5.44	0.51 7.11	0.81 9.00	1.24 11.11	1.81 13.44	2.56 16.00	3.53 18.77
8	0.03 1.56	0.06 2.44	0.13 3.51	0.23 4.78	0.40 6.25	0.63 7.91	0.96 9.77	1.40 11.82	1.98 14.06	2.73 16.50

Table Giving Comparative Torsional Strength and Weight of Hollow and Solid Shafting with Same Outside Diameter. (*Continued*)

Diam. of Solid and Hollow Shaft, Inches	Diameter of Axial Hole in Hollow Shaft, Inches									
	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	6
3¾	75.89 87.10	....	....	....	....	....	....	....	....	....
4	58.62 76.56	77.25 87.90	....	....	....	....	....	....	....	....
4¼	46.00 67.83	60.62 77.87	78.47 88.59	....	....	....	....	....	....	....
4½	36.60 60.49	48.23 69.44	62.43 79.00	79.57 89.31	....	....	....	....	....	....
4¾	29.48 54.29	38.85 62.33	50.29 70.91	64.09 80.04	80.56 89.75	....	....	....	....	....
5	24.01 49.00	31.65 56.25	40.96 64.00	52.21 72.23	65.61 81.00	81.46 90.24	....	....	....	....
5½	16.40 40.48	21.62 46.48	27.98 52.89	35.66 59.69	44.82 66.94	55.64 74.58	68.31 82.64	83.03 91.11	....	....
6	11.58 34.02	15.26 39.06	19.76 44.44	25.18 50.16	31.65 56.25	39.28 62.67	48.23 69.44	58.62 76.56	70.61 84.02	....
6½	8.41 28.99	11.08 33.28	14.35 37.87	18.28 42.74	23.98 47.93	28.52 53.40	35.02 59.17	42.56 65.24	51.27 71.60	72.61 85.21
7	6.25 25.00	8.24 28.70	10.67 32.66	13.59 36.85	17.08 41.33	21.21 46.04	26.04 51.02	31.65 56.25	38.12 61.75	53.98 73.49
7½	4.75 21.77	6.25 25.00	8.09 28.45	10.32 32.10	12.96 36.00	16.09 40.11	19.76 44.44	24.01 49.00	28.93 53.79	40.96 64.00
8	3.68 19.14	4.83 21.97	6.25 25.00	7.97 28.21	10.02 31.64	12.43 35.25	15.26 39.06	18.55 43.94	22.35 47.28	31.65 56.25

Hollow Shafts. — The following is a simple method for finding the dimensions of a hollow shaft which can be substituted for a solid shaft of equal strength to resist bending or torsion.

Let D_1 = diameter of solid shaft;
D = outside diameter of hollow shaft;
d = inside diameter of hollow shaft;
$t = \frac{1}{2}(D - d)$ = thickness of metal of hollow shaft;
$k = d \div D$ = ratio of diameters of hollow shaft.

As the hollow shaft is to have the same strength to resist bending as the solid shaft, the moment of resistance of both must be equal. Hence:

$$\frac{\pi\,(D^4 - d^4)}{32\,D} = \frac{\pi D_1^3}{32}, \text{ from which } D^3 - \frac{d^4}{D} = D_1^3 \qquad (1)$$

If kD is substituted for d in Equation (1):

$$D^3 - D^3k^4 = D_1^3, \text{ from which } \frac{D}{D_1} = \sqrt[3]{\frac{1}{1 - k^4}} \qquad (2)$$

In a similar manner, by substituting $\frac{d}{k}$ for D in Equation (1):

$$\frac{d}{D_1} = k\sqrt[3]{\frac{1}{1-k^4}} \tag{3}$$

Further, as $t = \frac{1}{2}(D - d)$, Formula (4) is found by substitution and simplification:

$$\frac{t}{D_1} = \frac{1-k}{2}\sqrt[3]{\frac{1}{1-k^4}} \tag{4}$$

In the accompanying table the values of the factors containing k in Equations (2), (3), and (4) are calculated for certain values of k. The bottom line of the table gives the weight of the hollow shaft in per cent of that of the solid.

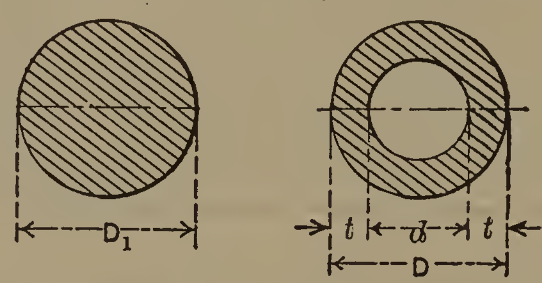
It is evident that Equation (1) would be the same, if it were derived under the assumption that the hollow shaft had the same torsional strength as the solid one, instead of having the same strength against bending, as assumed. The table will therefore hold true for shafts subjected to bending or torsion, or both.

Assume, as an example, that a solid shaft 3 inches in diameter is to be replaced by a hollow shaft, ratio k being 0.5. Then, by inserting the value found from the table in Equation (2):

$$\frac{D}{D_1} = 1.021 \text{ and } D = 3 \times 1.021 = 3.063 \text{ inches,}$$

$$d = 0.5\,D = 1.532 \text{ inch.}$$

Table of Factors for Finding Dimensions of Hollow Shafts to Replace Solid Shafts



Ratio of	Ratio $d \div D$								
	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
$D \div D_1 =$	1.021	1.032	1.047	1.067	1.095	1.135	1.192	1.279	1.427
$d \div D_1 =$	0.510	0.567	0.628	0.694	0.767	0.851	0.951	1.087	1.284
$t \div D_1 =$	0.257	0.232	0.209	0.186	0.164	0.141	0.119	0.096	0.071
Weight of hollow shaft*...	78.3	74.35	70.2	65.8	61.3	56.4	51.6	45.4	38.7

* Weight of hollow shaft is given in per cent of weight of solid shaft.

The use of hollow shafts not only reduces the weight of a shaft for a given strength, but increases the reliability of the shafting, on account of the removal of the metal from the core of the shaft. This applies especially to shafts of large diameters, as in large steel ingots the central core is likely to be less dense than the outer portion and to show shrinkage cavities near the center. If the ingot is bored out, the spongy or "piped" portion will be removed, and the metal remaining will be superior in quality to that in a solid shaft. Ingots for shafting should, however, not be cast hollow, but be bored out after having been cast solid.

FRICTION

Friction is the resistance to motion which takes place when one body is moved upon another, and is generally defined as "that force which acts between two bodies at their surface of contact, so as to resist their sliding on each other." The force of friction, F, bears — according to the conditions under which sliding occurs — a certain relation to the pressure between the two bodies; this pressure is called the normal pressure N. The relation between force of friction and normal pressure is given by the *coefficient of friction*, generally denoted by the Greek letter μ. Thus:

$$F = \mu \times N, \quad \text{and} \quad \mu = \frac{F}{N}$$

Example: — A body weighing 28 pounds rests on a horizontal surface. The force required to keep it in motion along the surface is 7 pounds. Find the coefficient of friction.

$$\mu = \frac{F}{N} = \frac{7}{28} = 0.25$$

If a body is placed on an inclined plane, the friction between the body and the plane will prevent it from sliding down the inclined surface, provided the angle of the plane with the horizontal is not too great. There will be a certain angle, however, at which the body will just barely be able to remain stationary, the frictional resistance being very nearly overcome by the tendency of the body to slide down. This angle is termed the angle of repose, and the tangent of this angle equals the coefficient of friction. The angle of repose is frequently denoted by the Greek letter θ. Thus, $\mu = \tan \theta$.

A greater force is required to start a body from a state of rest than to merely keep it in motion, because the *friction of rest* is greater than the *friction of motion*.

Rolling Friction. — When a body rolls on a surface, the force resisting the motion is termed *rolling friction*. This has a different value from that of the ordinary, or sliding, friction. Let W = total weight of rolling body or load on wheel, in pounds; r = radius of wheel, in feet; f = coefficient of rolling friction. Then:

$$\text{Resistance to rolling, in pounds} = \frac{W \times f}{r}$$

The coefficient of rolling friction varies with the conditions. For wood on wood it may be assumed as 0.005; for iron on iron, from 0.002 to 0.005; iron on granite, 0.007; iron on asphalt, 0.012; iron on wood, 0.018.

Laws of Friction. — The earliest experiments made on friction, which led to the establishment of definite laws, were undertaken by Morin and Rennie about 1830. The laws laid down by these early investigators, however, have been considerably modified by later investigations. The following may be considered as a correct statement of the laws of friction in their modified form, for unlubricated or dry surfaces.

1. For low pressures the friction is directly proportional to the normal pressure between the two surfaces. As the pressure increases to a high value the friction does not rise as rapidly; but when the pressure becomes abnormally high, the friction increases at a rapid rate until seizing takes place.

2. The friction both in its total amount and its coefficient is independent of the areas in contact, so long as the total pressure remains the same. This is true for moderate pressures only. For high pressures, this law is modified in the same way as in the first case.

3. At very low velocities the friction is independent of the velocity of rubbing. As the velocities increase, the friction decreases.

Lubricated Surfaces. — For well lubricated surfaces, the laws of friction are considerably different from those governing dry or poorly lubricated surfaces.

1. The frictional resistance is almost independent of the pressure per square inch, if the surfaces are flooded with oil.

2. The friction varies directly as the speed, at low pressures; but for high pressures the friction is very great at low velocities, approaching a minimum at about two feet per second linear velocity, and afterwards increasing approximately as the square root of the speed.

3. For well lubricated surfaces the frictional resistance depends, to a very great extent, on the temperature, partly because of the change in the viscosity of the oil and partly because the diameter of the bearing increases with the rise of temperature more rapidly than the diameter of the shaft, thus relieving the bearing of side pressure.

4. If the bearing surfaces are flooded with oil, the friction is almost independent of the nature of the material of the surfaces in contact. As the lubrication becomes less ample, the coefficient of friction becomes more dependent upon the material of the surfaces.

Experiments made by Tower indicate that the oil bath is very much superior to any other method of lubrication. Under conditions when with an oil bath the coefficient of friction was 0.0014, the value of this coefficient with a syphon lubricator was 0.0098, and with a pad under the journal, 0.009. This indicates that the friction when using an oil bath is only about one-seventh of the friction when using other methods of lubrication.

The comparative value of different lubricants is indicated by the same experiments. If the friction of a journal in its bearing, using a sperm oil bath, is assumed equal to 1, the friction using rape oil is 1.06; mineral oil, 1.29; lard oil, 1.35; olive oil, 1.35; and mineral grease, 2.17.

Lubricants. — The value of an oil as a lubricant depends mainly upon its film-forming capacity; that is, its capability of maintaining a film of oil between the bearing surfaces. The film-forming capacity depends to a large extent on the viscosity of the oil, but this should not be understood to mean that the oil of the highest viscosity is in every case the most suitable lubricant. On the other hand, an oil of the lowest viscosity which will retain an unbroken oil film between the bearing surfaces is the most suitable for purposes of lubrication, because a higher viscosity than that necessary to retain the oil film results in a waste of power, due to the expenditure of energy necessary to overcome the internal friction of the oil itself. For internal lubrication, only mineral oil of good quality should be used. Vegetable or animal oils are unsuitable owing to the fact that they decompose at high temperatures, forming acids which are injurious to metals. (See section on "Bearing Lubricants," page 533.) It is of importance to note that the following oils have a tendency to corrode the metals mentioned: Tallow oil, iron and copper; seal oil, copper; whale oil, lead; lard oil, lead; sperm oil, lead and zinc; rape oil, copper; cottonseed oil, tin.

Tables of Coefficients of Friction. — As the coefficient of friction depends on so many variable factors, it is impossible to give data that can be depended upon to meet all conditions, except in a very general way. The accompanying tables, however, give coefficients of friction as determined by various experimenters with different materials, conditions of loading and kinds of lubricants. The surfaces in contact, especially when poorly lubricated or dry, largely influence the coefficient of friction, which, of course, diminishes with the increasing smoothness of the surface.

Experiments by Thurston, to determine the coefficients of friction of motion and of rest of a journal, show that with a rubbing speed of 2.5 feet per second, using sperm oil as a lubricant, the coefficients at the starting were about five times the coefficients of motion, for a pressure of 50 pounds per square inch. As the pressure was increased to 500 pounds per square inch, the coefficient at starting was about thirty times greater than that of motion. With lard oil, the ratio between the two coefficients at 50 pounds pressure per square inch was about 3.5, and at 500 pounds pressure per square inch, about 20.

Coefficients of Friction

Low pressures (14 to 20 pounds per square inch). Sliding friction.

Bronze on bronze, dry	0.20
Bronze on cast iron, dry	0.21
Bronze on wrought iron, slightly lubricated	0.16
Cast iron on cast iron, slightly lubricated	0.15
Cast iron on wrought iron, dry	0.18
Wrought iron on wrought iron, dry	0.44
Cast iron on hard wood, dry	0.49
Cast iron on hard wood, slightly lubricated	0.19
Wrought iron on hard wood, well lubricated	0.08
Brass on hard wood, dry	0.48
Hard wood on hard wood, dry	0.48
Hard wood on hard wood, well lubricated	0.16
Leather on hard wood, dry	0.33
Leather on cast iron, dry	0.56

(In all cases where wood is mentioned, the motion is in the direction of the fibers of the wood.)

Coefficients of Sliding Friction

(Surfaces slightly lubricated)

Pressure, Lbs. per Sq. In.	Wrought Iron on Wrought Iron	Cast Iron on Wrought Iron	Steel on Cast Iron	Brass on Cast Iron	Pressure, Lbs. per Sq. In.	Wrought Iron on Wrought Iron	Cast Iron on Wrought Iron	Steel on Cast Iron	Brass on Cast Iron
125	0.14	0.17	0.17	0.16	485	0.40	0.37	0.36	0.22
185	0.25	0.27	0.30	0.22	525	0.41	0.37	0.36	0.22
225	0.27	0.29	0.33	0.22	565	Seized	0.37	0.36	0.23
260	0.28	0.32	0.34	0.21	600	.....	0.37	0.36	0.23
300	0.30	0.33	0.34	0.21	635	.....	0.37	0.37	0.23
335	0.31	0.33	0.35	0.21	670	.....	0.38	0.40	0.23
370	0.35	0.35	0.35	0.21	710	.....	0.43	Seized	0.23
390	0.38	0.36	0.35	0.20	780	.....	Seized	.....	0.23
450	0.39	0.36	0.35	0.21	820	.....	.....	.....	0.27

Influence of Friction on the Efficiency of Small Machine Elements. — The friction between machine parts lowers the efficiency, or the ratio of the power possible to take out of a machine element and that put into it. In the following are given average values of the efficiency, in per cent, of the most common machine elements when carefully made. Ordinary bearings, 95 to 98; roller bearings, 98; ball bearings, 99; spur gears with cast teeth, including bearings, 93; spur gears with cut teeth, including bearings, 96; bevel gears with cast teeth, including bearings, 92; bevel gears with cut teeth, including bearings, 95; belting, from 96 to 98; high-class silent power transmission chain, 97 to 99; roller chains, 95 to 97.

Friction of Journals. — The general laws of friction, as applied to unlubricated surfaces, do not hold true when a bearing is properly lubricated. From experiments by Thurston and Tower, the friction of a journal per square inch of bearing surface for any given speed and velocity of rubbing is equal to:

$$F = kp^n v^m$$

in which F = force of friction at the rubbing surface, in pounds per square inch;
k = a constant which varies with the quality of the lubricant, being 0.02 for the best and 0.04 for the lower grades;
p = normal pressure in pounds per square inch of bearing surface;
n = exponent depending upon the manner of oiling; its value is 1, in the case of dry surfaces; 0.50, in the case of drop-feed lubrication; 0.40, in the case of ring and pad lubrication; and 0, in case oil is forced into the bearing under sufficient pressure to float the shaft.
v = the velocity of rubbing, in feet per second;
m = exponent varying from 0, in the case of dry surfaces, to 0.20, in the case of drop-feed, and 0.50, in the case of an oil bath.

From this formula, it will be seen that the friction increases with the load on the bearing and also with the velocity of rubbing, but at a much slower rate than either.

Coefficients of Journal Friction

Cast iron journal in bronze bearing. Velocity of rubbing surface, 12 feet per second. Intermittent oil feed through an oil-hole.

Kind of Oil	Pressure, Pounds per Square Inch			
	8	16	32	48
Sperm or lard oil..	0.159 to 0.250	0.138 to 0.192	0.086 to 0.141	0.077 to 0.144
Olive or cotton-seed oil.........	0.160 to 0.283	0.107 to 0.245	0.101 to 0.168	0.079 to 0.131
Mineral oil........	0.154 to 0.261	0.145 to 0.233	0.086 to 0.178	0.094 to 0.222

Coefficients of Journal Friction

Method of lubrication: oil-bath

Kind of Oil	Velocity, Feet per Minute	Pressure, Pounds per Square Inch						
		100	153	205	310	415	520	625
Lard oil......	157	0.0042	0.0027	0.0020	0.0014	0.0012	0.0009	......
	471	0.0090	0.0052	0.0042	0.0029	0.0021	0.0017	......
Mineral grease	157	0.0076	0.0038	0.0034	0.0022	0.0016	0.0014	0.001
	471	0.0151	0.0083	0.0066	0.0040	0.0027	0.0022	0.002
Sperm oil.....	157	0.0030	0.0019	0.0016	0.0011	0.0015	Seized	......
	471	0.0064	0.0037	0.0027	0.0019	0.0021	......	......
Rape oil......	157	0.0040	0.0020	0.0014	0.0008	0.0009	0.0010	0.001*
	471	0.0070	0.0040	0.0024	0.0016	0.0016	0.0015	......
Mineral oil...	157	0.0040	......	0.0021	0.0014	0.0012	0.0012	0.0013
	471	0.0070	......	0.0035	0.0024	0.0020	0.0018	......

* Pressure 573 pounds per square inch.

BEARINGS

Generation of Heat in Bearings. — The rise in temperature of a bearing above that of the surrounding air, due to the friction of the surfaces, varies from less than 10 to nearly 100 degrees F., and is commonly about 30 degrees F. In order to limit the amount of heat in the bearing, it is necessary to limit the work of friction to a certain definite value per square inch of bearing area. As the work of friction is due to the bearing pressure per square inch and the velocity of rubbing, the product of these two quantities should not exceed a certain value.

The limiting value for the product of pressure and velocity is determined by Thurston's formula:

$$pv = C$$

in which p = bearing pressure, in pounds per square inch;
v = velocity of rubbing, in feet per second;
C = constant having values varying from 800 foot-pounds per second, in the case of iron or low-carbon steel shafts, to 2600 foot-pounds, in the case of high-carbon steel crankpins.

This formula is satisfactory for bearings running at ordinary speeds, although it must be modified for extreme cases.

Influence of Quality of Oil. — The thicker and less free-flowing an oil is, the greater the unit pressure it will stand in a bearing without squeezing out. A watch oil or a light spindle oil can only be used under a very small unit pressure. Sometimes they are squeezed out of the bearing when the pressure does not exceed 50 pounds per square inch. A cylinder oil of good body will stand a pressure of over 2000 pounds per square inch in the same bearing. It is important to determine, if possible, in each case, what quality of oil is best adapted to each particular bearing.

The allowance commonly made for the running fit of the box and shaft is about $0.0005\ (D + 1)$, in which D is the nominal diameter of the shaft in inches. Some manufacturers of fast-running machinery make the diameter of the box exceed that of the shaft by nearly twice this amount. The oil should be introduced at that point where the forces acting tend to separate the shaft and the box. At this point, grooves must be cut in the surface of the box so as to distribute the lubricant evenly over the entire length of the journal. Additional oil grooves should not be used, as these merely tend to rupture the oil film. Their only purpose is to distribute the supply of lubricant over the whole length of the journal surface and to collect the oil which would otherwise run out at the ends of the bearing and return it to some point where it may again be used.

Calculating Dimensions of Bearings. —The first consideration is the allowable pressure p in pounds per square inch of projected area of bearing. The amount of pressure that commercial oils will endure at low speeds without rupture of the oil film varies from 500 to 1000 pounds per square inch, when the load is steady. It is not safe, however, to load a bearing to this extent, since it is only under favorable circumstances that the film will stand this pressure. On this account, journal bearings should not be required to stand more than two-thirds of this pressure at slow speeds and the pressure should be reduced when the speed increases, except in cases of very perfect lubrication. The approximate unit pressure which a bearing will endure without seizing is:

$$p = \frac{PK}{DN + K} \qquad (1)$$

in which p = allowable pressure in pounds per square inch of projected area;
D = diameter of bearing in inches;

N = number of revolutions of journal per minute;

P = maximum safe unit pressure under given circumstances at slow speed. Ordinarily the value of P is 200 for collar thrust bearings; 400 for shaft bearings; 800 for car journals; 1200 for crankpins; and 1600 for wrist-pins. Under exceptional circumstances, these values may be increased by as much as 50 per cent, but only when the workmanship is the best and the care of the bearing the most skillful; in addition, a bearing should be readily accessible and the oil of the best quality and unusually viscous. Only in the case of very large machinery, which will have the most expert supervision, can these higher values be safely adopted.

K = quantity depending on method of oiling, etc. Its value may be assumed for ordinary work, drop-feed lubrication, as 700; first-class care, drop-feed lubrication, 1000; for force-feed lubrication or ring oiling, 1200 to 1500; extreme limit for perfect lubrication and air-cooled bearings, 2000. The value of 2000 is seldom used except in locomotive work where the rapid circulation of the air cools the journals. Higher values than 2000 may be used only in the case of water-cooled bearings.

Formula (1) is in a convenient form for calculating journals. In case the bearing is some form of sliding shoe, the quantity 240 V should be substituted for the quantity DN in the equation, V being the velocity of rubbing in feet per second.

There are a few cases where a unit pressure sufficient to break down the oil film is allowable. Such cases are the pins of punching and shearing machines, pivots of swing bridges and similar constructions, where the motion is slow and heating cannot well result. In such cases, pressures up to 4000 pounds per square inch are permissible.

Diameter of Shaft or Pin. — The diameter of a shaft or pin must be such that it will be strong and stiff enough to carry the load. In order to design it for strength and stiffness, the approximate length must be known. This will be assumed from the following equation:

$$L = \frac{20\,W\sqrt{N}}{PK} \tag{2}$$

in which L = length of bearing in inches;
W = total load upon bearing in pounds;
N = number of revolutions of journal per minute;
P and K = same quantities as in Equation (1).

When the approximate length has been found by the use of this equation, the diameter of the shaft or pin may be found by the general formulas for the strength of materials. The length of the journal must then be recomputed by the formula given in the next paragraph.

Length of Bearing. — Having obtained the proper diameter, a bearing length must be selected long enough so that the unit pressure shall not exceed the required value. This length may be found directly from the equation:

$$L = \frac{W}{PK}\left(N + \frac{K}{D}\right) \tag{3}$$

in which L = length of bearing in inches;
W = total load upon bearing in pounds;
P, K, N and D = same quantities as in Equation (1).

Should the length obtained by this formula not give practical dimensions for proper proportions, the diameter and length of the bearing must be adjusted to meet the conditions. A good rule for the length of the journal, after the requirements with relation to the bearing pressures have been met, is to make the ratio of the length to the diameter about equal to 1/8 of the square root of the number of revolutions per minute. This quantity may be decreased from 10 to 20 per cent in the case of crankpins and increased in the same proportion in the case of shaft bearings, but should not be departed from too widely. In the case of an engine making 100 revolutions per minute, the length of the bearings would, by this rule, be from 1¼ to 1½ times the diameter. In the case of a motor running at 1000 R.P.M., the bearings would be about 4 diameters long. This rule, while it cannot be adhered to on all occasions, is an excellent guide.

Example: — Design a collar thrust bearing for a 10-inch propeller shaft running at 150 revolutions per minute, and with a thrust of 60,000 pounds. Assuming that the thrust rings will be 2 inches wide, their mean diameter will be 12 inches. From Equation (1) we will have for the allowable bearing pressure $\frac{200 \times 700}{12 \times 150 + 700}$, or 56 pounds per square inch. This will require a bearing of 60,000 ÷ 56, or 1070 square inches area. Since each ring has an area of 0.7854 ($14^2 - 10^2$), or about 75 square inches, the number of rings needed will be 1070 ÷ 75, or 14. In case it were desirable to keep down the size of this bearing, the constant K might have been given values as high as 1000 instead of 700.

Example: — Design a main bearing for a horizontal engine. Assume that the diameter of the shaft is 15 inches and that the weight of the shaft, flywheel, crankpin, one-half of the connecting-rod, and any other moving parts that may be supported by the bearings, is 120,000 pounds, and that two-thirds of this weight comes on the main bearing, the remainder coming on the outboard bearing. The engine runs at 100 revolutions per minute. In this case, $W = 80{,}000$ pounds, $P = 400$ pounds per square inch, and K depends upon the care and method of lubrication. Assuming that the bearing will be flushed with oil by some gravity system, and that, since the engine is large, the care will be excellent, take $K = 1500$. This gives the length of the bearing from Formula (3):

$$L = \frac{80{,}000}{400 \times 1500}\left(100 + \frac{1500}{15}\right) = 26\tfrac{1}{2} \text{ inches (about).}$$

In computing the length of this bearing, the pressure of the steam is not considered since it is not a steady pressure; but the projected area of the main bearing must be greater than the projected area of the crankpin.

Example: — Find the dimensions for the bearings of a 100,000-pound hopper car weighing 40,000 pounds, having eight 33-inch wheels. The journals are 5½ inches in diameter, and the car is to run at 30 miles per hour. The wheels will make 307 revolutions per minute when running at this speed, and the load on each journal will be 140,000 ÷ 8, or 17,500 pounds. Although the journal will be well lubricated by means of an oil pad, it will receive but indifferent care, so the value of K will be taken as 1200. The length of the journal will then be:

$$L = \frac{17{,}500}{800 \times 1200}\left(307 + \frac{1200}{5.5}\right) = 9\tfrac{5}{8} \text{ inches (about).}$$

Example: — Design a crankpin for an engine with a 20-inch steam cylinder running at 80 revolutions per minute, and having a maximum unbalanced steam pressure of 100 pounds per square inch. The maximum and not the mean steam pressure should be taken in the case of crank- and wrist-pins. The total steam load on the piston is 31,400 pounds. P will be taken as 1200, and K as 1000. By Formula (2):

$$L = \frac{20 \times 31,400 \times \sqrt{80}}{1200 \times 1000} = 4.7, \text{ or, say, } 4\tfrac{3}{4} \text{ inches.}$$

In order that the deflection of the pin shall not be sufficient to destroy the lubricating film:

$$D = 0.09 \sqrt[4]{WL^3}$$

which limits the deflection to 0.003 inch. Substituting in this equation, the diameter becomes 3.85, or, say, $3\frac{7}{8}$ inches. With this diameter, obtain the length of the bearing, by using Formula (3):

$$L = \frac{31,400}{1200 \times 1000}\left(80 + \frac{1000}{3\frac{7}{8}}\right) = 8.85, \text{ say, } 9 \text{ inches.}$$

The mean of this value and the one obtained before is about 7 inches. Substituting this in the equation for the diameter, $D = 5\frac{1}{4}$ inches. Substituting this new diameter in Equation (3):

$$L = \frac{31,400}{1200 \times 1000}\left(80 + \frac{1000}{5\frac{1}{4}}\right) = 7.1, \text{ say, } 7 \text{ inches.}$$

It would now be preferable to take about half an inch off the length of this pin, and add it to the diameter, making it $5\frac{3}{4} \times 6\frac{1}{2}$ inches, and this will be found to bring the ratio of the length to the diameter nearer to one-eighth of the square root of the number of revolutions.

Bearing Pressures for Various Classes of Bearings (Kimball and Barr)

Class of Bearing and Condition of Operation		Allowable Bearing Pressure, Lbs. per Sq. In.
Bearings for very slow speed as in turntables in bridge work		7000 to 9000
Bearings for slow speed and intermittent load as in punch presses		3000 to 4000
Locomotive Wrist-pins		3000 to 4000
Locomotive Crankpins		1500 to 1700
Locomotive Driving Journals		190 to 220
Railway Car Axles		300 to 325
Marine Engine Main Bearings	naval practice	275 to 400
	merchant practice	400 to 500
Marine Engine Crankpins		400 to 500
Stationary Engine Main Bearings (high speed)	for dead load*	60 to 120
	for steam load	150 to 250
Stationary Engine Crankpins (high speed)	overhung crank	900 to 1500
	center crank	400 to 600
Stationary Engine Wrist-pins (high speed)		1000 to 1800
Stationary Engine Main Bearings (slow speed)	for dead load*	80 to 140
	for steam load	200 to 400
Stationary Engine Crankpins (slow speed)		800 to 1300
Stationary Engine Wrist-pins (slow speed)		1000 to 1500
Gas Engines, Main Bearings		500 to 700
Gas Engines, Crankpins		1500 to 1800
Gas Engines, Wrist-pins		1500 to 2000
Heavy Lineshaft Brass or Babbitt Lining		100 to 150
Light Lineshaft Cast-iron Bearing Surfaces		15 to 25
Generator and Dynamo Bearings		30 to 80

* Weight of shaft, flywheels, etc.

Relation of Length l to Diameter d of Journals

Type of Bearing	Values of $\frac{l}{d}$
Marine Engine Main Bearings	1 to 1.5
Marine Engine Crankpins	1 to 1.5
Stationary Engine Main Journals	1½ to 2.5
Stationary Engine Crankpins	1
Stationary Engine Crosshead Pins	1 to 1.5
Ordinary Heavy Shafting with Fixed Bearings	2 to 3
Ordinary Shafting with Self-adjusting Bearings	3 to 4
Generator Bearings	3

High-speed Bearings. — In carefully lubricated high-speed bearings, very high unit pressures are permissible. The following figures represent the practice of a large concern building electrical machinery in large units:

Velocity in feet per second of rubbing surfaces	20	30	40	60	75
Permissible pressure in pounds per square inch	165	190	205	225	230

It will be seen that the permissible pressures increase with the speed. The reason for this is that the higher the surface speed, the more effectively is the lubricant dragged in against the hydraulic pressure due to the load carried, the high-speed bearing acting in a measure as a pump for its lubricant.

Allowance for Oil between Shaft and Journal in Medium and High-speed Bearings

Diameter of Journal	Allowance	Diameter of Journal	Allowance	Diameter of Journal	Allowance
⅜ to 1	0.002	6	0.009	12	0.016
1⅛ to 2½	0.003	7	0.011	13	0.017
2⅝ to 3½	0.004	8	0.012	14	0.018
3⅝ to 4½	0.005	9	0.013	15	0.019
5	0.006	10	0.014	16 to 24	0.020
5½	0.007	11	0.015	....	....

Thrust Bearings. — Flat thrust bearings should be made of an annular form having an inside diameter one-half of the external diameter. Experiments carried out by Schiele to determine the wear in pivot bearings show that the wear is theoretically along a line called the *tractrix*, and that an end-thrust bearing made of this form will have the wear in the direction of the axis of a thrust shaft uniform at all points, but it has been shown in practice that little is to be gained by the use of bearings having this complicated shape. Experiments made on flat pivot thrust bearings, three inches in diameter, indicate that the coefficient of friction between a steel pivot and a manganese-bronze bearing, properly lubricated, using two radial oil-grooves only, varies from 0.018 at 50 revolutions per minute, to an average of 0.011 at 350 revolutions per minute. If four radial oil-grooves are used instead of two, the friction is approximately doubled, due to rupture of the oil film.

The load that may be safely carried by a thrust bearing varies with the velocity of the rubbing surfaces. The table following may be used as a guide in designing bearings in which the shaft is made from wrought iron or steel and the bearing from bronze or brass, and which have ample lubrication. In general, it is possible to use bath lubrication for thrust bearings, that is, the running surfaces are submerged constantly in a bath of oil. If the shaft is made from cast iron running on bronze or brass bearings, the values in the table for allowable pressure should be only one-half of those given.

Allowable Pressure, in Pounds per Square Inch, on Thrust Bearings

Average Velocity of Rubbing Surface, Feet per Minute	Safe Pressure, Lbs. per Sq. In.	Average Velocity of Rubbing Surface, Feet per Minute	Safe Pressure, Lbs. per Sq. In.
Slow and intermittent	1500	100 to 150	75
50	200	150 to 200	60
50 to 100	100	Over 200	50

Collar Thrust Bearings. — In collar thrust bearings, the thrust is taken by projections or shoulders on the shaft, often at some distance from its end. This type of bearing is used when a greater thrust than can be conveniently placed on a single flat or step bearing is to be taken care of. In a well-made bearing, each of the collar surfaces takes its proportionate part of the load, and it is thus possible, without using excessive diameters, to properly distribute a very great thrust on a number of collars formed solidly with the shaft, by cutting a number of grooves in the latter. One advantage of the collar bearing is that the difference between the outer and inner diameters of the bearing surface is not very great, and hence the velocities at the outer and inner edges do not vary appreciably; this, again, eliminates unequal wear on the thrust collar surfaces. The safe load that may be placed on collar thrust bearings varies between 60 to 100 pounds per square inch.

Composition of Bearing Metals

Alloys	Lead	Tin	Antimony	Copper	Zinc	Other Constituents
Babbitt 1	80.0	20.0	....	....	....	....
Babbitt 2	72.0	21.0	7.0	....	....	....
Babbitt 3	70.0	10.0	20.0	....	....	....
Babbitt 4	80.5	11.5	7.5	0.5	....	....
Babbitt 5	0.5	68.0	....	1.0	30.5	....
Babbitt 6	....	20.0	....	....	80.0	....
Babbitt 7	....	86.0	10.0	4.0	....	....
White metal	82.0	....	12.0	6.0	....	....
White brass	....	64.0	....	2.00	34.0	....
Magnolia metal	80.0	4.75	15.0	trace	....	$Bi = 0.25$
Car brass lining	80.5	11.5	7.5	0.5	....	....
Ajax plastic bronze	30.0	5.0	....	65.0	....	....
Ajax metal	11.5	11.5	....	77.0	....	....
P. R. R. car brass, B	15.0	8.0	....	76.2	....	$P = 0.80$
S bearing metal	9.5	10.0	....	80.5	....	....
Delta metal	....	3.0	....	60.0	36.0	$Fe = 1.0$
Camelia metal	14.8	4.3	....	70.2	10.2	$Fe = 0.5$
Tempered lead	98.5	0.08	....	0.11	....	$Na = 1.30$

Bi = bismuth; P = phosphorus; Fe = iron; Na = sodium.

Hydraulically Supported Step Bearings. — The type of thrust bearing which is hydraulically supported has very little frictional resistance and is adapted to heavy pressures and high speeds. Bearings of this type which have been applied to Curtis vertical steam turbines are so designed that oil (water may also be used with this type of bearing), under sufficient pressure to sustain the load, is forced between recessed plates at the bottom of the shaft and then passes out radially in the form of a thin film and up through a cylindrical guide bearing located just above the bottom plates, thus floating the shaft upon the oil film.

Thrust Bearing Design Based on Principle of Wedge-shaped Oil Film. — The investigations of Professor Osborne Reynolds, following the experiments of Tower on well fitted car journals and brasses flooded with oil, showed that the oil, because of its viscosity and adhesion to the journal, is, by the journal rotation, dragged into a wedge-shaped space between the journal and brass. This action sets up pressure in the oil film which, in turn, supports the load, thus separating the bearing surfaces. The design of the Kingsbury thrust bearing is based on this principle, the bearing floating the load on wedge-shaped oil films which form automatically and without employing a high pressure oil pump. There is usually a flat, annular, revolving plate with the bearing face immersed in oil and supported on one or more shoes which are mounted to tilt as required by running conditions. These bearings are made for both horizontal and vertical shafts. The low-speed bearings may be loaded to 1000 pounds or more per square inch when using heavy oil, and high-speed bearings with light oils regularly carry loads up to 500 pounds per square inch. The friction loss in this bearing is very low. According to an approximate rule for vertical bearings having six shoes with the inside diameter one-half the outside diameter and loaded to 350 pounds per square inch of shoe area, the mean coefficient of friction is 0.00009 times the square root of the revolutions per minute and varies inversely as the square root of the unit pressure, when using dynamo oil having a temperature of about 40 degrees C. The coefficient of friction has been found by a large number of tests to vary between 0.0008 and 0.003.

Frictional Power Losses in Bearings. — In the following formulas for determining power losses due to friction, W = work expended in friction in foot-pounds per minute; μ = coefficient of friction; L = total load in pounds on bearing; D = diameter (cylindrical bearing); R = maximum radius (step or thrust bearing); r = minimum radius; N = revolutions per minute; α = one-half included angle (conical pivot bearing); M = moment of friction in inch-pounds. Then, $W = \frac{2\pi MN}{12}$. The values of M and W for different bearings are as follows: *Shafts and journals:* $M = \frac{1}{2}\mu LD$ and $W = \frac{2\pi\mu LDN}{24} = 0.2618\,\mu LDN$. *Flat pivot or step bearing:* $M = \frac{2}{3}\mu LR$ and $W = 0.349\,\mu LRN$. *Collar thrust bearing:* $M = \frac{2}{3}\mu L\frac{R^3 - r^3}{R^2 - r^2}$ and $W = 0.349\,\mu LN\frac{R^3 - r^3}{R^2 - r^2}$. *Conical pivot bearing:* $M = \frac{2}{3}\mu LR$ cosec α and $W = 0.349\,\mu LRN$ cosec α. *Truncated conical pivot:* $M = \frac{2}{3}\mu L\frac{R^3 - r^3}{R\sin\alpha}$ and $W = 0.349\,\mu LN\frac{R^3 - r^3}{R\sin\alpha}$. *Hemispherical step bearing and Schiele's tractrix pivot:* $M = \mu LR$ and $W = 0.5236\,\mu LRN$. For flat bearing surfaces in general $W = \mu LS$ in which S = rate, in feet per minute, at which sliding action occurs.

Knife-edge Bearings. — The knife-edge bearings of weighing and testing apparatus should be made of steel having 0.90 to 1.00 per cent carbon. An angle of 90 degrees is recommended for knife-edge bearings. The seats for supporting the pivots should be drawn to a light straw color and the pivots slightly darker. The bearing edge may be left sharp for loads up to 1000 pounds per inch of length; for heavier loads the edge is dulled slightly with an oilstone.

Bearing Metals. — Bearing metals are usually composed of alloys of copper, lead, tin, antimony and zinc, and are known as babbitt metal, white metal, brass, phosphor-bronze, and by various trade names. The price of these bearing metals depends largely upon the constituents. Lead and zinc are cheapest, with antimony, copper, and tin increasing progressively in price in the order named, tin being the most expensive. The more lead is used in a bearing, the cheaper it will be. Lead, however, is too soft to be used alone and must be alloyed with one of the other metals. Antimony added to lead increases the hardness and brittleness; with tin added, a tougher alloy is obtained. Nearly all the various babbitt metals are alloys of lead, tin and antimony. In such babbitts, the wear generally increases with an increase of antimony and the price with an increase of tin. High antimony babbitts are used in heavy machinery because they are harder, while those low in antimony are used in high-speed machinery. Soft babbitts do not have sufficient strength to sustain the weight and shocks incident to heavy machinery bearings.

The table "Composition of Bearing Metals" gives the constituents of some of the metals on the market. Wide deviation in the composition of babbitt is shown in the table. The first babbitt in the table is a fairly good alloy for high-speed machinery, but is not very hard. Its melting point is about 500 degrees F. The second babbitt is somewhat harder and melts at a higher temperature. Both of these are largely used for lining purposes. The fourth babbitt is widely used for heavy machinery and the sixth has good wearing properties, but cannot be used for high speeds. The white metals are used extensively for bearings in generators, motors, electric cars, etc. Alloys containing sodium have the peculiar property of producing, by oxidation, material which will saponify with the oil used in the bearing and thus assist lubrication. The alloys of lead, copper and tin usually are provided with a thin lining of lead or of a soft babbitt; this combination wears better than an entire bearing of the soft metal. The "P. R. R. car brass, B" is considered one of the best bearing bronzes that can be obtained. It contains approximately the smallest quantity of tin that will hold the lead alloyed with the copper. The table "Composition of Bronzes" gives a list of alloys used by the U. S. Navy Department.

Composition of Bronzes

	Parts
White Metal:	
Tin	7.6
Copper	2.3
Zinc	83.3
Antimony	3.8
Lead	3.0
Hard Bronze for Piston Rings:	
Tin	22.0
Copper	78.0
Brazing Metal:	
Copper	85.0
Zinc	15.0

	Parts
Naval Brass:	
Copper	62.0
Tin	1.0
Zinc	37.0
Bearings — Wearing Surfaces, etc.:	
Copper	6
Tin	1
Zinc	¼
Anti-friction Metal:	
Copper — (best refined)	3.7
Banca tin	88.8
Regulus of antimony	7.5

Well fluxed with borax and rosin in mixing.

The exact composition of the original babbitt metal is not known. The ingredients were copper, tin and antimony in approximately the following proportions: 89.3 per cent tin; 3.6 per cent copper; 7.1 per cent antimony. This metal possesses great anti-frictional qualities, but the high percentage of tin makes it expensive and has led to the substitution of other metals which are marketed under the name of babbitt metal. These cheaper grades, when properly made, are for some purposes superior to the original babbitt metal. The composition of metals for different classes of bearings frequently used is given in the following table:

Babbitt Metal Compositions

Class of Service Adapted for	Composition of Metal			
	Tin	Antimony	Copper	Lead
High-pressure bearings	90	7	3	....
High pressure and fast speed	86	12	2	....
Medium pressure and high speed	30	20	....	50
Medium pressure and medium speed	15	25	....	60
Low pressure and medium speed	8	20	....	72
Principally for shaftings, etc.	....	10	....	90

American Society for Testing Materials Specifications for Babbitt Metals.— A sub-committee of the American Society for Testing Materials has proposed to reduce the large number of babbitt metals in use to five, a number which the committee think will be ample for every class of work. The following table gives the composition of the series which it is believed covers the range for all requirements:

Number	Tin, Per cent	Antimony, Per cent	Copper, Per cent	Lead, Per cent
1	83.33	8.33	8.33	....
2	89.00	7.00	4.00	....
3	50.00	15.00	2.00	33.00
4	5.00	15.00	....	80.00
5	....	10.00	....	90.00

High-grade Bearing Metal. — A high-class bearing metal is prepared as follows: Melt 7 parts of copper at as low a heat as possible; then add 25 parts of antimony and 200 parts of tin. This mixture is cast in iron ingot molds. It is then re-melted and to each five pounds of the ingots is added eight pounds of tin, this second alloy being cast in bars to suit the requirements.

Babbitting. — Babbitt metal is extensively used as a lining for bearings, not only for its anti-frictional qualities, but because it is much cheaper than a machined box. It is easily melted, and by the use of a babbitting mandrel, the diameter of which is equal to, or slightly greater than, the journal, the operation of machining the bearing can often be eliminated. In bearings which are comparatively large and important, the mandrel is sometimes made smaller than the finished bearing and the latter is bored after the metal has been expanded and compressed. The lining of a two-part bearing is commonly expanded by the peen of a hammer, whereas, for small solid bearings, the same practical result is obtained by forcing through a tapered arbor in a broaching press or by other means, the size of the arbor being such as to leave sufficient metal for reaming or boring.

Prior to pouring the babbitt, the bearing should be heated to prevent the molten metal from becoming chilled and sluggish. Bronze shells which are to have babbitt linings should first be tinned by immersing in a pot of molten solder. Use solder of "half and half" composition, and zinc chloride as a flux. The shell should be babbitted immediately after tinning. Babbitt should not be used for tinning, because it has a much higher melting point, which makes it difficult to maintain a molten film on the surface to be tinned. Cast-iron shells are rarely, if ever, tinned.

To avoid blow-holes and defects in babbitt linings, it is the practice of the Westinghouse Electric & Mfg. Co. to cover the mandrel with a thin coating of clay wash by plunging it, while heated, into a pail of water containing, in solution, one or two pounds of Jersey red clay. When the mandrel is dry, the thin clay coating prevents the formation of bubbles, and the lining has a smooth surface. Oil causes the babbitt to blister. When babbitting solid bearings, it is the practice in some shops to smoke the mandrel or cover it with paper to facilitate its removal from the bear-

Miscellaneous Bearing Metals

The following alloys are stated to have given complete satisfaction for the purposes mentioned:

Used for	Tin	Lead	Zinc	Antimony	Copper	Bismuth
Dynamos — high speed........	88	....	....	8	3.5	0.5
Marine engines...............	77	17	....	3	3	....
Eccentrics....................	5	77.75	....	15	2	0.25
Submerged bearings...........	40	48	....	10	2	....
Main bearings................	34	44	....	16	6	....
Slides, thrust bearings.........	65	....	30	2.5	2.5	....
Railway trucks...............	42	....	56	....	2	....
Axle-boxes...................	74.55	13.50	1.80	6.55	3.6	....
Plastic metal................	80	10	....	1	8	1
Genuine babbitt (hard)........	80	....	....	10	10	....
Genuine babbitt (No. 2).......	83	....	....	9	8	....
Universal bearing metal........	6	77.75	....	16	....	0.25

ing. For two-part bearings, this is not necessary. Before pouring, babbitt should be stirred thoroughly to insure a lining of uniform composition. Whenever practicable, the bearing should be in a vertical position while pouring. The ladle should have a rounded spout rather than one which is sharp or broad. A broad thin stream, or one that is intermittent, tends to produce porous areas or blow-holes. Putty is preferable to clay for luting or sealing the ends of the bearings, as the moisture in the clay tends to cause sputtering.

Ball and Roller Bearings

Ball bearings are used in preference to sliding bearings principally for the following reasons: There is less loss of power on account of the lower coefficient of friction; the friction of a ball bearing is independent of the viscosity of the lubricant or its temperature; the frictional resistance at starting is very much less than in a sliding bearing; ball bearings are much shorter and more compact than sliding bearings; the scraping and fitting of bearing linings is not necessary; the danger of heated bearings is practically eliminated; a bearing of proper construction can adjust itself to deflections of the shaft; the wear is practically negligible.

Steel for Balls. — In order to determine the best steel for ball bearings, one of the largest elevator companies in the United States tested, by actual use, 432 different samples of steel obtained in this country and abroad. These tests indicated that the two grades of carbon and alloy steels given herewith are especially adapted for making steel balls:

Carbon, 1.12; silicon, 0.015; phosphorus, 0.017; manganese, 0.19; sulphur, 0.019; chromium, 0.25 per cent.

Carbon, 0.95; silicon, 0.014; sulphur, 0.019; phosphorus, 0.018; manganese, 0.025; chromium, 1.25; tungsten, 0.25 per cent.

Testing Strength of Steel Balls. — The testing of a steel ball for crushing strength can be done between hardened plates by placing three balls into a rather close-fitting tube. The center ball is the one that will be tested. When the pressure is applied, the upper and lower balls will sink into the plate somewhat, which will give them a greater surface bearing than the middle ball, the latter bearing only at two points between the upper and lower balls; hence, the middle ball will ordinarily break first. If a ball is properly hardened, it will break into several pieces. Uniformity of hardness throughout the entire cross-section and a fine grain are important qualities, both of which can be determined by the appearance of the fracture. Of course, the heat-treatment must be governed by the nature of the material. The accompanying table gives the crushing loads ordinarily required by ball manufacturers for regular tool-steel balls. The figures in this table are the result of repeated tests and are considered safe for use in calculations. In selecting balls for a bearing, a factor of safety of 10 should be adopted unless the bearing is required in an extremely narrow space. The loads listed by makers for ball bearings vary considerably and when the bearings are much alike in regard to size, design and material, this difference is due to the factor of safety adopted. Alloy steel balls will usually carry from 25 to 50 per cent more load than ordinary tool-steel balls.

Crushing Loads for Tool-steel Balls

Size of Ball, Inches	Ultimate Strength, Pounds	Size of Ball, Inches	Ultimate Strength, Pounds	Size of Ball, Inches	Ultimate Strength, Pounds	Size of Ball, Inches	Ultimate Strength, Pounds
1/16	390	1/4	6,215	5/8	39,000	1 1/8	125,000
3/32	875	5/16	9,940	3/4	56,250	1 1/4	156,000
7/64	1562	3/8	14,000	13/16	66,000	1 1/2	225,000
1/8	2450	7/16	19,100	7/8	76,000	1 5/8	263,000
3/16	3496	1/2	25,000	15/16	88,000	1 3/4	306,000
7/32	4780	9/16	31,500	1	100,000	2	400,000

Test of Heat-Treatment. — In order to determine whether a ball has been properly heat-treated, enclose the finished ball in a piece of waste, place it on an anvil and break it open with a heavy blow. (The waste prevents the pieces from flying about.) If the ball has been properly heat-treated, the surface of the fracture will have a fine even grain. If it has not been properly heat-treated, the surface will appear coarse and granular. Tests have shown that the average crushing load of balls that were annealed before hardening is approximately 30 per cent greater than of balls which were not so heat-treated. The heat-treatment not only strengthens the balls but prevents them from flaking off, crumbling, etc. If during the test the ball should break in half, it would indicate that the steel had not been properly drawn after hardening, but was still subjected to internal stresses. If a ball has been properly drawn, it can be "touched" with a fine Swiss file; and under the blows of a heavy hammer, it will break into several pieces, as mentioned.

Quality of Steel Balls. — Nothing but tool or alloy steel balls should be used for high-grade work, and it is very important that they be accurate as to size and shape and be properly heat-treated to obtain a fine grain. The material of which they are made should have as high an elastic limit as possible. Case-hardened machine steel balls should not be used when heavy duty is required. While a ball can be case-hardened very deeply, the process does not remove the injurious elements, phosphorus, sulphur and silicon, which the cheaper steels contain in comparatively large percentages. Case-hardened balls have, however, proved satisfactory for many purposes, where the duty is not too severe.

Lubrication of Ball Bearings. — Ball bearings require lubrication and should never be run dry. The actual lubricating property of the lubricant used is not as important as its freedom from acid and its effectiveness as a preventative of rust. A very small quantity is sufficient for lubrication, but it is advisable to use a comparatively large supply to prevent rust. If an acid oil is used, the finely finished surfaces of the balls and races will be etched or pitted and the efficiency of the bearing seriously affected. Free acid is not found in properly refined mineral or hydrocarbon oils; hence mineral oil should be used instead of vegetable oils, such as castor, cotton-seed, rape and linseed oils, which tend to develop acid and become gummy and rancid. Animal oils are objectionable for the same reasons. A mixture of vaseline and vaseline oil, or a good mineral oil is recommended. To test the neutral quality of a lubricant, soak some threads of waste in it and wrap them around a clean piece of polished steel; then expose directly to the sun's rays, in a warm place. A good oil or grease will show no signs of etching after months of exposure, but an acid oil will discolor the steel and show signs of impairing its finished surface within a few days. A period of one or two weeks is long enough to determine definitely the nature of the lubricant. In general, greases are recommended for speeds up to 2000 or 2500 revolutions per minute. When a high speed and resulting increase in temperature causes the grease to thin out, oil should be substituted. Ordinarily, grease is preferable because it can be more easily retained in the bearing case. A grease should be used that does not contain free alkali, as the latter will pit or etch the polished steel surfaces the same as acids in oils.

Mounting Ball Bearings. — In the mounting of ball bearings, there are certain requirements which apply generally, although the exact arrangement must be governed to some extent by the conditions. If the bearing is to carry a radial load without thrust, the inner race should have a light driving fit on the shaft and be securely clamped against a shoulder by a nut or clamping device which is proof against jarring loose. The outer race of a bearing subjected to a radial load only, should fit closely in its retaining box or housing, but be free to "float" or shift in an endwise direction. When the outer race is mounted in this way, it will align itself with reference to the inner race and will tend to have a slow intermittent creeping movement, insuring a proper distribution of the load over the entire surface of the outer race. Even when it is desired to prevent end motion of the shaft, a slight lateral clearance of 0.010 or 0.015 inch should be provided, to permit the creeping movement and prevent the load from always bearing in one position on the race. When mounting for combined radial and thrust loads, the outer race of one bearing must be secured against endwise movement. If there are several radial bearings on the same shaft, the end thrust in both directions should be taken by the same bearing, and the outer races of the other bearings should be free to locate themselves. It is considered good practice when two bearings are mounted on one shaft, to prevent axial thrust by making the inner race of each bearing a light driving fit on the shaft. The outer race of one bearing has a sliding fit in its seat and is given a slight amount of axial play (say, from 0.010 to 0.020 inch); the outer race of the other bearing is also made a sliding fit, but is allowed considerable axial play. The first bearing takes the radial load and end thrust, and the second bearing, a radial load only. In selecting a bearing, it is well to remember that the rated capacity is usually for steady loads and speeds, and variations from these conditions require a reduction of the listed capacity. It is important to mount bearings so that they will be free from grit and moisture. If it is necessary to take ball bearings apart, the balls from different bearings should not be mixed, as they may vary in size more than is permissible for the individual bearing.

Thrust in Radial Bearings. — It is common practice to use the radial type of ball bearing when there is an axial thrust in addition to the normal radial load. Radial bearings, however, should not be subjected to end thrust when the speed of rotation is low, especially if the end thrust is great, as in automobile front wheels, etc. When ball bearings are used for such purposes, make the outer rims of the two bearings a sliding fit and allow them considerable axial play, so that they cannot possibly take end thrust. The thrust is then taken by a thrust bearing, usually placed between the radial bearings. If the speed of rotation is above 1200 or 1500 R.P.M., the thrust can be taken by a radial bearing, since at high speeds the radial type is superior to a regular thrust bearing. The following information regarding thrust loads on radial bearings is given by Mr. Henry Hess (see also *Proceedings of A.S.M.E.*, June, 1907). Theoretically, the radial bearing seems to be incapable of carrying a thrust load, owing to the wedging of the balls between the races. Actual running tests, however, as well as considerable experience, have demonstrated that radial bearings will carry much more thrust load than theoretical calculations indicate. It has been experimentally determined that the thrust carrying capacity of the uninterrupted type of radial bearings is to the radial capacity as 0.1 is to 1 and may be as great as 0.25 to 1, the variation depending upon the ball diameter, race curvature and number of balls. Tests have also shown that for speeds above 1500 revolutions per minute, radial bearings are more efficient thrust bearings than the collar type. In selecting a radial ball bearing to take both thrust and radial loads, the Hess-Bright Mfg. Co. advises that 10 pounds of rated radial capacity should be allowed for each pound of thrust load. For thrust and radial loads of 150 and 1500 pounds, respectively, select a bearing rated at $150 \times 10 = 1500$ pounds, for the thrust, and 1500 for the radial load, or a total rated radial capacity of 3000 pounds. For speeds less than 1500 revolutions per minute, and for heavy duty, a thrust and radial bearing combination is preferable.

The S. K. F. radial bearings are designed to carry approximately 25 per cent of their rated radial load as pure thrust. For combined radial and thrust loads on a single radial bearing, the allowable thrust may be calculated in accordance with the following formula, in which P = total load; T = thrust load; R = radial load.

$$P = 4T + R$$

Under these conditions, P should, in no case, exceed the allowable rating for the bearing. In many instances, exact load conditions are indeterminate and due allowance should be made for shocks or rapidly changing load conditions. It is well to obtain information regarding the form of bearing, mounting, capacity, etc., directly from the manufacturers, especially when the conditions are unusual.

Ball Thrust Bearings. — Thrust bearings are adapted for heavy axial loads. They are made with both races flat on the bearing side; with one race flat and the other grooved; and with both races grooved. A form that is commonly used has two grooved races of circular section, and a spherical seat for one of the races or collars, so that the latter will align itself in order to distribute the load evenly over the entire number of balls. Some bearings of the thrust type have both of the outer thrust surfaces parallel. With this design, the collars should be exactly parallel, so that the load will be uniformly distributed on the balls. In fact, when two flat disks without the spherical seat are used, the capacity of the bearing depends largely upon the accuracy of the alignment, because, with even a slight distortion of the shaft, the load will not be sustained by the total number of the balls. When mounting single thrust bearings, the flat disk should properly have a tight fit on the shaft and rotate with it, the spherical seat remaining stationary, except for adjustments to compensate for changes in shaft alignment.

Capacity of Ball Thrust Bearings. — The speed at which thrust bearings are run, decidedly affects the carrying capacity, the latter rapidly diminishing as the speed increases. The centrifugal force at comparatively high speeds is the limiting feature in a bearing of this type. A general idea of the maximum loads which thrust bearings will carry at different speeds may be obtained from the accompanying tables.

Frictional Resistance. — Extensive tests for determining the coefficients of friction for various types of ball bearings show that the coefficient varies between close limits at different loads and speeds, whereas, for sliding bearings, it extends over a wide range for even small changes in speed or load. The frictional resistance of ball bearings has by actual experiment been found to vary between 0.0011 to 0.0095. These are the coefficients of friction referred to the shaft diameter, thus permitting direct comparison with coefficients of sliding friction. Ball bearings having a coefficient of friction much above 0.0015 under the greatest allowable load should not be recommended, because they are too short-lived. A coefficient of 0.0015 for a good ball bearing under its maximum load (independent of the speed, within limits) will, however, rise to approximately 0.0030, under a reduction of the load to about one-tenth of the maximum.

Loads on Ball Bearings. — The permissible load that a two-point annular ball bearing will carry can be determined approximately by the following formula (based upon the investigations of Prof. Stribeck), after a suitable value has been determined for constant K:

$$P = 0.44\,Kd^2n$$

in which K is a factor, depending upon the material in the balls, the form of the ball races and the operating speed of the bearings; n equals the number of balls; and d their diameter, taking ⅛ inch as the unit; for example, if the actual diameter is ¼ inch, d equals 2; if the diameter is ⅝ inch, d equals 5, etc. The better the material and the more careful the heat-treatment, the greater the value of factor K. When the diameter of the balls is increased, the value of K is decreased, as it is more difficult to harden a large ball uniformly, than a small one. With regard to the rotary speed, K should be diminished as the number of revolutions increases. For ball bearings made of high-grade material and accurately machined, K has the following approximate values for steady loads and uniform speeds:

Revolutions per Minute......	10	150	300	500	1000	1500
Values of K................	20	18	15	10	7.5	5

The preceding formula should be sufficiently accurate for bearings in which the balls vary from 10 to 20 in a set, provided the balls and races are properly proportioned. If a bearing had a comparatively large number of balls of such diameter that the strength of the races was impaired, the formula would indicate a greater load capacity than the bearing could safely carry. Even when the formula is applicable, which, as has been indicated, is only within certain limitations, it will sometimes be found that a bearing having a larger number of smaller diameter balls may be stronger than one with a less number of a larger diameter, or *vice versa*. The difference between the relative capacities may also be greater than the formula indicates, as the conditions of loading are more advantageous in a bearing having a greater number of smaller balls.

Load Capacities of Radial Ball Bearings *

Series or Class	Bore Diam., Inches	Outside Diam., Inches	Width, Inches	R.P.M. and Load in Pounds				Ball Diam.	Bearing Weight, Pounds
				200	800	1200	1500		
	0.3937	1.1811	0.3543	165	120	100	90	3/16	0.07
	0.4724	1.2598	0.3937	190	135	115	105	3/16	0.09
	0.5906	1.3780	0.4331	255	185	155	140	7/32	0.11
	0.6693	1.5748	0.4724	290	210	175	160	7/32	0.15
	0.7874	1.8504	0.5512	375	270	230	205	1/4	0.25
	0.9843	2.0473	0.5906	475	345	290	260	9/32	0.29
	1.1811	2.4410	0.6299	645	470	395	355	5/16	0.45
	1.3780	2.8347	0.6693	925	675	565	505	3/8	0.66
	1.5748	3.1496	0.7087	1,085	790	665	590	13/32	0.85
	1.9685	3.5433	0.7874	1,445	1,045	875	780	15/32	1.00
Light	2.1654	3.9370	0.8268	1,845	1,335	1,120	995	17/32	1.34
	2.3622	4.3307	0.8661	2,065	1,490	1,250	1110	9/16	1.75
	2.5591	4.7244	0.9055	2,540	1,830	1,535	1360	5/8	2.22
	2.7559	4.9213	0.9449	2,790	2,015	1,690	1495	21/32	2.41
	2.9528	5.1181	0.9843	3,055	2,205	1,845	1635	11/16	2.59
	3.1496	5.5118	1.0236	3,620	2,615	2,185	1930	3/4	3.13
	3.3465	5.9055	1.1024	3,910	2,820	2,350	2080	25/32	3.94
	3.7402	6.6929	1.2598	4,880	3,510	2,920	2580	7/8	5.90
	3.9370	7.0866	1.3386	5,740	4,125	3,430	3015	29/32	7.20
	4.1339	7.4804	1.4173	5,960	4,265	3,555	3115	31/32	8.47
	4.3307	7.8741	1.4961	6,330	4,530	3,780	3300	1	10.20
	0.3937	1.3780	0.4331	250	180	155	140	1/4	0.13
	0.4724	1.4567	0.4724	290	210	180	160	1/4	0.14
	0.5906	1.6536	0.5118	335	240	205	185	1/4	0.19
	0.6693	1.8504	0.5512	455	330	280	250	5/16	0.25
	0.7874	2.0473	0.5906	520	380	320	285	5/16	0.33
	0.9843	2.4410	0.6693	740	540	455	405	3/8	0.55
	1.1811	2.8347	0.7480	1,010	730	615	550	7/16	0.80
	1.3780	3.1496	0.8268	1,310	945	795	710	1/2	1.03
	1.5748	3.5433	0.9055	1,655	1,190	1,000	890	9/16	1.41
	1.9685	4.3307	1.0630	2,445	1,760	1,480	1310	11/16	2.48
Medium	2.1654	4.7244	1.1417	2,900	2,090	1,745	1545	3/4	3.09
	2.3622	5.1181	1.2205	3,380	2,435	2,030	1790	13/16	3.88
	2.5591	5.5118	1.2992	3,905	2,810	2,335	2065	7/8	4.75
	2.7559	5.9055	1.3780	4,465	3,195	2,665	2340	15/16	5.60
	2.9528	6.2992	1.4567	5,070	3,625	3,020	2640	1	6.82
	3.1496	6.6929	1.5354	5,685	4,080	3,385	2960	1 1/16	8.25
	3.3465	7.0866	1.6142	6,340	4,560	3,780	3290	1 1/8	10.25
	3.7402	7.8741	1.7717	7,790	5,570	4,610	4015	1 1/4	13.80
	3.9370	8.4646	1.8504	8,550	6,120	5,065	4390	1 5/16	17.20
	4.1339	8.8583	1.9291	9,350	6,700	5,530	4780	1 3/8	19.30
	4.3307	9.4489	1.9685	11,080	7,900	6,525	5620	1 1/2	22.20
	0.6693	2.4410	0.6693	985	710	600	530	1/2	0.59
	0.7874	2.8347	0.7480	1,240	895	750	665	9/16	0.75
	0.9843	3.1496	0.8268	1,520	1,100	920	820	5/8	1.16
	1.1811	3.5433	0.9055	1,835	1,320	1,110	980	11/16	1.56
	1.3780	3.9370	0.9843	2,175	1,570	1,310	1160	3/4	2.06
	1.5748	4.3307	1.0630	2,535	1,825	1,520	1345	13/16	2.34
	1.7717	4.7244	1.1417	3,420	2,460	2,045	1805	7/8	3.44
	1.9685	5.1181	1.2205	3,910	2,795	2,330	2050	15/16	4.25
	2.1654	5.5118	1.2992	4,435	3,175	2,645	2310	1	5.10
Heavy	2.3622	5.9055	1.3780	4,970	3,570	2,960	2590	1 1/16	6.19
	2.5591	6.2992	1.4567	5,560	3,995	3,315	2880	1 1/8	7.41
	2.7559	7.0866	1.6536	6,820	4,870	4,030	3515	1 1/4	10.75
	2.9528	7.4804	1.7717	7,490	5,350	4,430	3845	1 5/16	14.00
	3.1496	7.8741	1.8898	8,180	5,860	4,840	4180	1 3/8	15.90
	3.3465	8.2678	2.0473	9,700	6,920	5,700	4920	1 1/2	18.90
	3.5433	8.8583	2.1260	11,300	8,040	6,625	5675	1 5/8	22.80
	3.7402	9.8426	2.1654	13,020	9,250	7,600	6500	1 3/4	30.10
	3.9370	10.4331	2.3622	14,830	10,520	8,630	7370	1 7/8	38.10
	4.1339	11.4174	2.5591	16,800	11,900	9,700	8250	2	52.60
	4.3307	12.5985	2.7559	18,830	13,270	10,840	9540	2 1/8	70.60

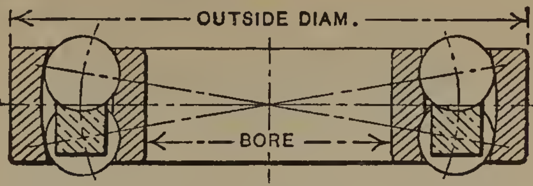
* The Hess-Bright Mfg. Co.

Load Capacities of Radial Ball Bearings *

Bore Diam., Inches	Outside Diam., Inches	Single Row				Double Row			
		Width, Inches	Balls		Load, 600 R.P.M., Pounds	Width, Inches	Balls per Row		Load, 600 R.P.M., Pounds
			Diam.	No.			Diam.	No.	
0.3937	1.1811	0.3543	7/32	7	135	1/2	3/16	8	140
	1.3780	0.4331	1/4	6	205	3/4	1/4	8	255
0.4724	1.2598	0.3937	7/32	7	175	1/2	3/16	10	200
	1.4567	0.4724	9/32	6	290	3/4	1/4	8	360
0.5906	1.3780	0.4331	7/32	8	215	1/2	3/16	11	245
	1.6536	0.5118	5/16	6	360	3/4	1/4	10	440
0.6693	1.5748	0.4724	1/4	8	280	1/2	3/16	12	320
	1.8504	0.5512	11/32	9	435	7/8	5/16	8	515
0.7874	1.8504	0.5512	9/32	8	390	3/4	1/4	11	415
	2.0473	0.5906	11/32	10	485	7/8	5/16	9	575
	2.8347	0.7480	9/16	8	1,035	1 3/8	1/2	8	1,500
0.9843	2.0473	0.5906	5/16	12	480	3/4	1/4	12	500
	2.4410	0.6693	13/32	11	745	1	3/8	9	1,000
	3.1496	0.8268	5/8	8	1,280	1 3/8	1/2	8	1,760
1.1811	2.4410	0.6299	11/32	13	630	3/4	1/4	12	670
	2.8347	0.7480	15/32	11	990	1 3/16	7/16	9	1,385
	3.5433	0.9055	11/16	9	1,745	1 9/16	9/16	10	2,320
1.3780	2.8347	0.6693	19/32	13	880	7/8	5/16	14	1,035
	3.1496	0.8268	17/32	11	1,275	1 3/8	1/2	10	1,815
	3.9370	0.9843	3/4	9	2,075	1 3/4	5/8	10	2,670
1.5748	3.1496	0.7087	7/16	14	1,095	1	3/8	12	1,325
	3.5433	0.9055	19/32	11	1,590	1 7/16	17/32	11	2,245
	4.3307	1.0630	13/16	9	2,440	1 15/16	11/16	10	3,030
1.7717	3.3465	0.7480	7/16	15	1,175	1	3/8	15	1,415
	3.9370	0.9843	21/32	12	2,120	1 9/16	9/16	11	2,850
	4.7244	1.1417	7/8	10	3,140	2 1/8	3/4	10	3,630
1.9685	3.5433	0.7874	15/32	15	1,350	1	3/8	16	1,620
	4.3307	1.0630	23/32	12	2,540	1 3/4	5/8	11	3,275
	5.1181	1.2205	15/16	10	3,585	2 5/16	13/16	10	3,950
2.1654	3.9370	0.8268	1/2	16	1,640	1 3/16	7/16	15	1,960
	4.7244	1.1417	25/32	12	3,000	1 15/16	11/16	11	3,690
	5.5118	1.2992	1	10	4,105	2 1/2	7/8	11	4,360
2.3622	4.3307	0.8661	17/32	16	1,850	1 3/8	1/2	14	2,190
	5.1181	1.2204	27/32	12	3,500	2 1/8	3/4	11	4,200
2.5591	4.7244	0.9055	9/16	17	2,215	1 3/8	1/2	16	2,635
	5.5118	1.2992	29/32	12	4,045	2 5/16	13/16	11	4,825
2.7559	4.9213	0.9449	19/32	17	2,455	1 7/16	1/2	16	2,900
	5.9055	1.3780	31/32	12	4,620	2 1/2	7/8	11	5,550
2.9528	5.1181	0.9843	5/8	17	2,720	1 7/16	1/2	17	3,200
	6.2992	1.4567	1	13	5,335	2 11/16	15/16	11	6,400
3.1496	5.5118	1.0236	11/16	17	3,295	1 5/8	9/16	18	3,700
	6.6929	1.5354	1 1/16	13	6,015	2 11/16	1	12	7,200
3.3465	5.9055	1.1024	3/4	16	3,685	1 3/4	9/16	18	4,000
	7.0866	1.6142	1 1/8	13	6,750	2 7/8	1 1/16	12	8,200
3.5433	6.2992	1.1811	13/16	16	4,335	2	11/16	17	4,700
	7.4804	1.6929	1 3/16	13	7,515	2 7/8	1 1/8	12	9,100
3.7402	6.6929	1.2598	7/8	16	5,030	2 3/16	3/4	16	5,400
	7.8741	1.7717	1 1/4	13	8,335	3 1/16	1 3/16	12	10,000
3.9370	7.0866	1.3386	15/16	16	5,765	2 3/8	13/16	16	6,500
	8.4646	1.8504	1 3/8	12	9,300	3 1/4	1 1/4	12	12,000
4.1339	8.8583	1.9291	1 7/16	12	10,155	3 7/16	1 5/16	12	13,400
4.3307	9.4489	1.9685	1 1/2	12	11,070	3 5/8	1 3/8	13	16,525

* The New Departure Mfg. Co.

Load Capacities of Radial Ball Bearings — 1 *



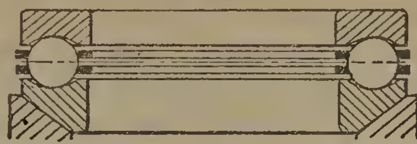
Bore Diam., Inches	Outside Diam., Inches	Width, Inches	Revolutions per Minute and Load Rating in Pounds						
			150	300	500	1000	2000	3000	5000
			Extra-heavy Type						
0.5906	2.0472	0.7087	860	770	595	500	365	280	220
0.6693	2.4409	0.7874	1,175	1,050	815	680	500	380	300
0.7874	2.8346	0.9055	1,925	1,725	1,350	1,125	825	630	500
0.9843	3.1496	0.9843	2,650	2,375	1,875	1,575	1150	875	695
1.1811	3.5433	1.1024	3,200	2,875	2,275	1,900	1400	1050	840
1.3780	3.9370	1.1811	3,775	3,400	2,700	2,250	1650	1250	995
1.5748	4.3307	1.2992	4,400	3,975	3,175	2,650	1950	1475	....
1.7717	4.7244	1.3780	5,100	4,550	3,675	3,075	2250	1700	....
1.9685	5.1181	1.4567	5,850	5,200	4,225	3,525	2575	1965	....
2.1654	5.5118	1.5748	7,050	6,400	5,150	4,300	3175	2400	....
2.3622	5.9055	1.6535	8,050	7,150	5,850	4,900	3600	....	....
2.5591	6.2992	1.7717	9,050	8,050	6,600	5,525	4050	....	....
2.7559	7.0866	1.9685	11,100	9,900	8,275	6,900	5075	....	....
2.9528	7.4803	2.0866	12,200	10,900	9,175	7,675	....	....	....
3.1496	7.8740	2.2441	13,100	11,600	9,900	8,275	....	....	....
3.3465	8.2677	2.3622	14,100	12,600	10,775	9,000	....	....	....
3.5433	8.8583	2.4803	17,000	15,100	13,000	10,875	....	....	....
3.7402	9.8425	2.6378	18,800	16,600	14,675	12,275	....	....	....
3.9370	10.4331	2.7559	21,400	18,800	16,850	.....	....	....	....
			Heavy Type						
0.4724	1.8504	0.5512	680	540	465	390	285	215	175
0.5906	2.0472	0.5906	740	590	510	425	310	235	185
0.6693	2.4409	0.6693	1,100	880	760	630	465	350	280
0.7874	2.8346	0.7480	1,400	1,150	980	815	600	455	360
0.9843	3.1496	0.8268	1,775	1,450	1,250	1,025	755	575	455
1.1811	3.5433	0.9055	2,200	1,800	1,550	1,275	940	715	565
1.3780	3.9370	0.9843	2,875	2,325	2,000	1,675	1225	925	740
1.5748	4.3307	1.0630	3,250	2,525	2,200	1,825	1350	1025	810
1.7717	4.7244	1.1417	4,000	3,275	2,850	2,350	1725	1300	1050
1.9685	5.1181	1.2205	4,750	3,875	3,350	2,775	2050	1550	1250
2.1654	5.5118	1.2992	5,200	4,275	3,700	3,075	2250	1700	1350
2.3622	5.9055	1.3780	6,150	5,125	4,425	3,675	2700	2050	....
2.5591	6.2992	1.4567	6,700	5,575	4,825	4,000	2950	2250	....
2.7559	7.0866	1.6535	8,500	7,075	6,100	5,075	3725	2825	....
2.9528	7.4803	1.7717	9,700	8,175	7,050	5,875	4300	....	....
3.1496	7.8740	1.8898	10,900	9,300	8,025	6,675	4900	....	....
3.3465	8.2677	2.0472	12,200	10,500	9,075	7,550	5550	....	....
3.7402	9.8425	2.1654	15,500	13,350	11,450	9,575	....	....	....
3.9370	10.4331	2.3622	16,700	14,500	12,475	10,425	....	....	....

* S K F Ball Bearing Co.

Load Capacities of Radial Ball Bearings — 2

Bore Diam., Inches	Outside Diam., Inches	Width, Inches	Revolutions per Minute and Load Rating in Pounds						
			150	300	500	1000	2000	3000	5000
Medium Type									
0.3937	1.3780	0.4331	330	270	230	190	140	105	85
0.4724	1.4567	0.4724	440	350	300	250	180	140	110
0.5906	1.6535	0.5118	490	390	335	280	200	155	125
0.6693	1.8504	0.5512	680	540	465	390	285	215	175
0.7874	2.0472	0.5906	740	590	510	425	310	235	185
0.9843	2.4409	0.6693	1,100	880	760	630	465	350	280
1.1811	2.8346	0.7480	1,400	1,150	980	815	600	455	360
1.3780	3.1496	0.8268	1,775	1,450	1,250	1,025	755	575	455
1.5748	3.5433	0.9055	2,200	1,800	1,550	1,275	940	715	565
1.7717	3.9370	0.9843	2,875	2,325	2,000	1,675	1225	925	740
1.9685	4.3307	1.0630	3,250	2,550	2,200	1,825	1350	1025	810
2.1654	4.7244	1.1417	4,000	3,275	2,850	2,350	1725	1300	1050
2.3622	5.1181	1.2205	4,750	3,875	3,350	2,775	2050	1550	1250
2.5591	5.5118	1.2992	5,200	4,275	3,700	3,075	2250	1700	1350
2.7559	5.9055	1.3780	6,150	5,125	4,425	3,675	2700	2050	....
2.9528	6.2992	1.4567	6,700	5,575	4,825	4,000	2950	2250	....
3.1496	6.6929	1.5354	7,400	6,150	5,300	4,400	3250	2450	....
3.3465	7.0866	1.6142	8,500	7,075	6,100	5,075	3725	2825	....
3.5433	7.4803	1.6929	9,700	8,175	7,050	5,875	4300	....	....
3.7402	7.8740	1.7717	10,900	9,300	8,025	6,675	4900	....	....
4.1339	8.8583	1.9291	13,650	13,550	11,700	9,750	....	....	....
4.3307	9.4488	1.9685	15,500	14,950	12,900	10,725	....	....	....
Light Type									
0.3937	1.1811	0.3543	240	195	170	140	105	80	65
0.4724	1.2598	0.3937	275	215	190	155	115	90	70
0.5906	1.3780	0.4331	375	295	255	215	155	120	95
0.6693	1.5748	0.4724	450	310	265	220	160	125	100
0.7874	1.8504	0.5512	580	465	395	335	245	185	150
0.9843	2.0472	0.5906	750	540	465	385	285	215	175
1.1811	2.4409	0.6299	1050	850	725	610	445	335	270
1.3780	2.8346	0.6693	1200	970	830	695	510	390	310
1.5748	3.1496	0.7087	1550	1250	1075	895	655	500	395
1.7717	3.3465	0.7480	1775	1400	1200	1000	730	555	440
1.9685	3.5433	0.7874	1950	1575	1350	1125	825	625	495
2.1654	3.9370	0.8268	2425	1950	1675	1400	1025	775	615
2.3622	4.3307	0.8661	2800	2250	1925	1625	1200	900	715
2.5591	4.7244	0.9055	3075	2500	2150	1775	1300	995	795
2.7559	4.9213	0.9449	3350	2725	2350	1950	1450	1100	865
2.9528	5.1181	0.9843	3825	3100	2650	2225	1625	1250	985
3.1496	5.5118	1.0236	4200	3400	2925	2450	1800	1350	1085
3.3465	5.9055	1.1024	5050	4100	3550	2950	2175	1650	1300
3.5433	6.2992	1.1811	5600	4600	3950	3300	2425	1850	1465
3.7402	6.6929	1.2598	6500	5350	4600	3825	2825	2150	....
4.1339	7.4803	1.4173	7700	6400	5500	4600	3375	2550	....
4.3307	7.8740	1.4961	9050	7575	6500	5450	4000	....	....

Loads for Thrust Collar Bearings *



Number of Balls	Size of Balls	Revolutions per Minute								
		1500	1000	500	300	150	100	50	25	10
		Medium Weight, Load in Pounds								
8	¼	145	190	245	285	330	395	540	660	1,100
10	¼	185	245	310	365	430	505	660	825	1,320
13	¼	240	320	395	485	550	650	870	1,080	1,760
16	¼	295	395	495	585	680	770	1,045	1,355	2,200
18	¼	330	440	550	660	770	880	1,175	1,520	2,420
17	5/16	440	550	660	880	990	1,285	1,725	2,245	3,300
18	5/16	550	660	770	990	1,210	1,395	1,905	2,400	3,520
17	⅜	660	770	880	1210	1,540	1,890	2,585	3,255	4,620
19	⅜	770	880	1100	1430	1,760	2,110	2,880	3,650	5,060
18	7/16	880	1100	1320	1650	2,200	2,465	3,255	4,355	6,380
19	7/16	990	1210	1540	1870	2,420	2,605	3,430	4,620	6,820
18	½	1210	1430	1760	2200	2,640	3,235	4,235	5,720	8,360
19	½	1320	1540	1980	2420	3,080	3,300	4,335	6,070	8,800
20	½	1430	1650	2090	2530	3,300	3,485	4,555	6,380	9,240
19	9/16	1540	1760	2420	2640	3,740	4,180	5,455	8,790	11,000
19	⅝	1870	2090	2860	3300	4,400	4,950	6,600	9,295	13,200
20	⅝	1980	2200	2970	3520	4,620	5,225	6,950	9,790	13,860
19	11/16	2200	2530	3520	4180	5,280	5,995	7,965	11,255	15,400
20	11/16	2420	2640	3740	4400	5,500	6,510	8,745	11,440	16,280
19	¾	2640	3080	3960	4840	5,940	7,370	9,845	12,640	17,600
19	13/16	2860	3520	4840	5500	7,040	8,635	11,605	14,830	22,000
19	⅞	3080	4180	5280	6380	8,140	10,340	13,970	17,600	24,200
19	1	3740	4840	6600	8140	10,560	13,510	18,215	23,010	28,600
		Light Weight, Load in Pounds								
21	5/16	640	770	900	1155	1410	1630	2200	2970	4,180
23	5/16	705	860	990	1265	1540	1760	2420	3255	4,620
21	⅜	860	990	1210	1595	1980	2245	3035	4005	5,500
22	⅜	905	1045	1265	1675	2090	2355	3170	4180	5,940
21	7/16	1100	1320	1705	2090	2530	2970	4025	5280	7,480
22	7/16	1155	1385	1785	2200	2775	3125	4225	5500	7,920
23	7/16	1210	1455	1870	2310	2860	3255	4400	5785	8,140
25	7/16	1320	1585	2035	2530	3080	3520	4775	6270	9,020
27	7/16	1430	1715	2200	2750	3300	3830	5170	6765	9,680
28	7/16	1485	1785	2310	2860	3410	3960	5370	7040	10,120
29	7/16	1540	1850	2420	2970	3520	4115	5545	7260	10,340
30	7/16	1595	1915	2530	3080	3630	4270	5720	7525	10,780

* The Hess-Bright Mfg. Co.

Allowable Limits for Mounting Ball Bearings on Shafts and in Housings *

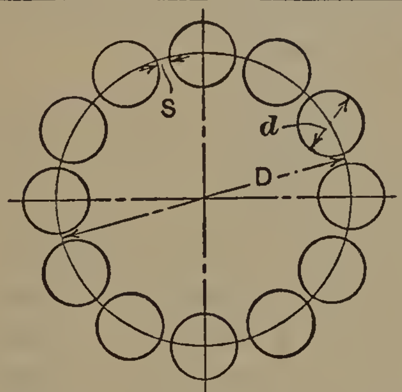
Bore Diam., Inches			Shaft Diam., Inches			Fit Allowance	
Nominal	Max.	Min.	Standard	Max.	Min.	Max.	Min.
1.9685	1.9687	1.9681	1.9692	1.9695	1.9689	0.0014	0.0002
2.1654	2.1656	2.1650	2.1661	2.1664	2.1658	0.0014	0.0002
2.3622	2.3624	2.3618	2.3629	2.3632	2.3626	0.0014	0.0002
2.5591	2.5593	2.5587	2.5599	2.5603	2.5595	0.0016	0.0002
2.7559	2.7561	2.7555	2.7567	2.7571	2.7563	0.0016	0.0002
3.1496	3.1498	3.1492	3.1504	3.1508	3.1500	0.0016	0.0002
3.3465	3.3467	3.3461	3.3473	3.3477	3.3469	0.0016	0.0002
3.5433	3.5435	3.5429	3.5441	3.5445	3.5437	0.0016	0.0002
3.7402	3.7404	3.7398	3.7410	3.7414	3.7406	0.0016	0.0002
4.1339	4.1341	4.1335	4.1347	4.1351	4.1343	0.0016	0.0002
4.3307	4.3309	4.3303	4.3315	4.3319	4.3311	0.0016	0.0002
4.5276	4.5278	4.5272	4.5285	4.5290	4.5280	0.0018	0.0002
4.7244	4.7246	4.7240	4.7253	4.7258	4.7248	0.0018	0.0002
5.1181	5.1183	5.1177	5.1190	5.1195	5.1185	0.0018	0.0002
5.3150	5.3152	5.3146	5.3159	5.3164	5.3154	0.0018	0.0002
5.5118	5.5120	5.5114	5.5127	5.5132	5.5122	0.0018	0.0002
5.7087	5.7089	5.7083	5.7096	5.7101	5.7091	0.0018	0.0002
6.1024	6.1026	6.1020	6.1038	6.1044	6.1032	0.0024	0.0006
6.2992	6.2994	6.2988	6.3006	6.3012	6.3000	0.0024	0.0006
6.4961	6.4963	6.4957	6.4975	6.4981	6.4969	0.0024	0.0006
6.6929	6.6931	6.6925	6.6943	6.6949	6.6937	0.0024	0.0006
Outside Diam., Bearing			**Housing Bore Diam.**			**Fit Allowance**	
Nominal	Max.	Min.	Standard	Max.	Min.	Max.	Min.
1.5748	1.5748	1.5742	1.5756	1.5762	1.5750	0.0020	0.0002
1.8504	1.8504	1.8498	1.8512	1.8518	1.8506	0.0020	0.0002
2.4409	2.4409	2.4401	2.4421	2.4429	2.4413	0.0028	0.0004
3.1496	3.1496	3.1488	3.1508	3.1516	3.1500	0.0028	0.0004
3.5433	3.5433	3.5425	3.5445	3.5453	3.5437	0.0028	0.0004
4.3307	4.3307	4.3299	4.3327	4.3339	4.3315	0.0040	0.0008
4.9213	4.9213	4.9205	4.9233	4.9245	4.9221	0.0040	0.0008
5.5118	5.5118	5.5110	5.5138	5.5150	5.5126	0.0040	0.0008
6.2992	6.2992	6.2980	6.3016	6.3028	6.3004	0.0048	0.0012
7.0866	7.0866	7.0854	7.0890	7.0902	7.0878	0.0048	0.0012
7.8740	7.8740	7.8728	7.8764	7.8776	7.8752	0.0048	0.0012
8.4646	8.4646	8.4634	8.4670	8.4682	8.4658	0.0048	0.0012
8.8583	8.8583	8.8571	8.8607	8.8619	8.8595	0.0048	0.0012
9.4488	9.4488	9.4476	9.4512	9.4524	9.4500	0.0048	0.0012
10.2362	10.2362	10.2350	10.2386	10.2398	10.2374	0.0048	0.0012
10.6299	10.6299	10.6287	10.6327	10.6338	10.6315	0.0051	0.0016
11.4173	11.4173	11.4161	11.4201	11.4212	11.4189	0.0051	0.0016
12.2047	12.2047	12.2035	12.2075	12.2086	12.2063	0.0051	0.0016
12.9921	12.9921	12.9909	12.9949	12.9960	12.9937	0.0051	0.0016
13.7795	13.7795	13.7783	13.7823	13.7834	13.7811	0.0051	0.0016
14.5669	14.5669	14.5657	14.5697	14.5708	14.5685	0.0051	0.0016

* S K F Ball Bearing Co.

Dimensions and Approximate Load per Bearing for Ball Thrust Bearings

Diameter of Bearing, Inches		Diam. of Balls, Inches	Load in Pounds at					
Inside	Outside		1500 R.P.M.	1000 R.P.M.	500 R.P.M.	300 R.P.M.	150 R.P.M.	10 R.P.M.
1¼	2¹¹⁄₃₂	¼	350	500	600	750	800	2,750
1⁷⁄₁₆	2¹⁵⁄₃₂	¼	500	600	750	1000	1100	3,750
1⅝	2³¹⁄₃₂	⁵⁄₁₆	600	750	850	1100	1350	4,000
1¾	3³⁄₃₂	⁵⁄₁₆	750	850	1000	1350	1750	5,000
2	3¹¹⁄₃₂	⁵⁄₁₆	850	1000	1250	1600	2000	5,500
2⅛	3¹⁹⁄₃₂	⁵⁄₁₆	1000	1250	1500	1850	2500	7,000
2¼	3²³⁄₃₂	⁵⁄₁₆	1100	1350	1750	2100	2750	7,500
2½	3³¹⁄₃₂	⁵⁄₁₆	1250	1600	2000	2500	3000	9,500
2⅝	4¹¹⁄₃₂	⅜	1500	1750	2250	2750	3500	10,000
2¾	4¹⁵⁄₃₂	⅜	1750	2000	2750	3000	4200	12,500
3¼	4³¹⁄₃₂	⅜	2100	2350	3200	3750	5000	15,000
3½	5⁷⁄₃₂	⅜	2500	2850	4000	4750	6000	17,500

Constants for Finding Ball-circle Diameters with Space between Balls



No. of Balls	C	No. of Balls	C	No. of Balls	C	No. of Balls	C	No. of Balls	C
6	2.0000	16	5.1258	26	8.2963	48	15.290	70	22.292
7	2.3048	17	5.4422	27	8.6138	49	15.608	71	22.609
8	2.6131	18	5.7588	28	8.9314	50	15.926	72	22.925
9	2.9238	19	6.0755	29	9.2491	51	16.244	73	23.245
10	3.2360	20	6.3925	30	9.5668	52	16.562	74	23.563
11	3.5495	21	6.7095	31	9.8844	53	16.880	75	23.883
12	3.8637	22	7.0267	32	10.203	54	17.200	76	24.201
13	4.1786	23	7.3439	33	10.520	55	17.516	77	24.516
14	4.4940	24	7.6613	34	10.840	56	17.835	78	24.838
15	4.8097	25	7.9787	35	11.157	57	18.152	79	25.151
				36	11.473	58	18.471	80	25.471
				37	11.791	59	18.790	81	25.786
				38	12.110	60	19.106	82	26.103
				39	12.427	61	19.429	83	26.427
				40	12.745	62	19.743	84	26.738
				41	13.063	63	20.064	85	27.064
				42	13.381	64	20.379	86	27.382
				43	13.699	65	20.700	87	27.701
				44	14.017	66	21.017	88	28.019
				45	14.335	67	21.336	89	28.337
				46	14.652	68	21.654	90	28.653
				47	14.972	69	21.973	..	....

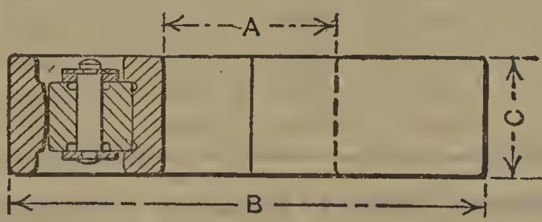
Ball-circle Diameter for Ball Bearings. — When the number and size of the balls and the space required between consecutive balls are known, the ball-circle diameter can be found by the formula: $D = C(d + S)$, in which D = ball-circle diameter; d = ball diameter; S = space between balls; and C = a constant taken from the accompanying table. (The distance S is measured on a straight line joining the centers of adjacent balls.)

Example: — If the required number of balls is 17; the diameter, 5/16 inch; and the space between the balls, 1/8 inch, what is the ball-circle diameter D?

Referring to the table, opposite 17, in the column headed "No. of Balls," the value of constant C is found to be 5.4422. Substituting known values in the formula, $D = 5.4422\ (5/16 + 1/8) = 2.381$ inches = diameter of ball circle.

Roller Bearings. — A long series of tests with various types of roller bearings indicate that while each type has its peculiarities, there are certain principles which apply quite generally within narrow limits. The coefficient of friction of roller bearings is greater at low than at high speeds, but is much more nearly constant than in plain lubricated bearings. The coefficient of friction in bearings in which there is pure rolling is nearly constant at all speeds, but when there is end thrust, the friction decreases as the speed increases. The coefficient of friction is indepen-

Load Capacities of Roller Bearings — Medium Service Series*



Dimensions in Inches			Revolutions per Minute							
			10	100	300	500	1000	1500	2000	3000
A	*B*	*C*	Load Capacity in Pounds							
0.9842	2.4410	0.67	1700	1450	1300	1200	850	800	750	700
1.1811	2.8346	0.75	2100	1900	1700	1600	1100	1000	950	900
1.3779	3.1496	0.83	2700	2200	2100	1900	1300	1200	1100	1000
1.5748	3.5433	0.91	3300	2900	2700	2400	1700	1550	1450	1300
1.7716	3.9370	0.98	4400	3900	3500	3200	2200	2000	1900	1750
1.9685	4.3307	1.06	5100	4400	4000	3600	2600	2300	2100	....
2.1653	4.7244	1.14	6400	5500	5100	4600	3200	2900	2700	....
2.3622	5.1181	1.22	8000	6900	6200	5700	4000	3600	3300	....
2.5590	5.5118	1.30	8600	7300	6800	6200	4200	3900	3500	....
2.7559	5.9055	1.38	10,100	8,800	7,900	7,300	5100	4600	4200	....
2.9527	6.2992	1.46	11,500	9,700	8,800	8,200	5700	5100	4600	....
3.1496	6.6929	1.54	12,500	10,600	9,700	8,800	6200	5300	....	....
3.3464	7.0866	1.61	14,000	11,900	11,000	9,900	6800	6200	....	....
3.5433	7.4803	1.69	15,500	13,200	12,100	11,000	7700	....	....	....
3.7401	7.8740	1.77	16,300	14,100	12,800	11,700	8200	....	....	....
3.9370	8.4645	1.85	20,000	16,500	15,200	13,900	9500	....	....	....

The above rating is for steady loads and speeds, and fifty per cent temporary overload capacity can be added. For varying loads and speeds the relative value of load and overload must be modified. Both races of Norma Roller Bearings must be rigidly held endwise. The rollers are cylindrical; the inner race is ground cylindrical, and the outer race, instead of being flat, is ground to a slightly convex surface.

* The Norma Company of America

dent of the temperature of the bearings, unless the end thrust is excessive. The friction in a well-designed bearing is not greatly affected by lubrication. The wear of the rollers is often excessive, if the rotating parts and the casing are not hardened and well finished, especially when the bearing is subjected to end thrust. The end thrust on the rollers varies almost directly as the load on the bearing and usually diminishes as the speed increases. The direction of the thrust is usually reversed when the direction of rotation is reversed.

Machine Tool Roller Bearing Load Table — 1 *

Shaft Diam., Inches	Bearing Length, Inches	Roller Diam., Inches	Outside Diam. Bearing	Type of Outer Race	Speeds in R.P.M. and Load Capacities				
					25	150	300	500	1000
½	1	¼	1 1/16	Split	65	58	52	45	33
½	2	¼	1 1/16	Split	156	140	124	107	80
½	1	¼	1 3/16	Solid	260	234	207	180	140
½	2	¼	1 3/16	Solid	625	560	495	430	315
¾	1	5/16	1 7/16	Split	95	83	71	61	48
¾	2	5/16	1 7/16	Split	226	197	169	146	115
¾	1	5/16	1 5/8	Solid	380	330	284	243	190
¾	2	5/16	1 5/8	Solid	900	785	675	580	450
1	2	3/8	1 15/16	Split	304	260	227	195	....
1	3	3/8	1 15/16	Split	490	420	367	314	....
1	1	½	2 ¼	Solid	540	480	420	365	275
1	2	½	2 ¼	Solid	1370	1220	1085	935	700
1 ¼	2	½	2 7/16	Split	370	314	274	232	....
1 ¼	4	½	2 7/16	Split	830	705	614	520	....
1 ¼	1 1/8	5/8	2 ¾	Solid	790	700	615	530	390
1 ¼	2 ¼	5/8	2 ¾	Solid	1940	1725	1500	1300	955
1 ½	2	9/16	2 13/16	Split	440	370	320	266	....
1 ½	4	9/16	2 13/16	Split	990	830	715	595	....
1 ½	1 ¼	¾	3 3/8	Solid	1100	965	835	710	525
1 ½	2 ½	¾	3 3/8	Solid	2620	2300	2000	1700	1250
1 ¾	2	9/16	3 1/16	Split	507	421	360	295	....
1 ¾	1 ¼	¾	3 5/8	Solid	1260	1100	945	805	582
1 ¾	2 ½	¾	3 5/8	Solid	3020	2640	2260	1925	1400
2	2	5/8	3 ½	Split	575	477	400	325	....
2	4	5/8	3 ½	Split	1285	1070	900	730	....
2	2 ¾	7/8	4 1/8	Solid	3800	3350	2810	2400	1710
2 ¼	3	5/8	3 ¾	Split	1030	850	710	575	....
2 ¼	2 ¾	7/8	4 3/8	Solid	4250	3670	3120	2620	1850
2 ½	5	11/16	4 1/8	Split	2010	1650	1360	1090	....
2 ½	3	15/16	4 ¾	Solid	5150	4450	3730	3100	2170
2 ¾	3	11/16	4 3/8	Split	1250	1000	825	650	....
2 ¾	3	15/16	5	Solid	5600	4800	4000	3350	2280

* This table gives the load capacities of Hyatt roller bearings with rollers operating directly on shafts. The loads are based on the use of steel containing from 0.40 to 0.50 per cent carbon with a hardness of 165 Brinell or 25 scleroscope for split outer races, and 600 Brinell for solid outer races. For other degrees of hardness the loads given in the table should be multiplied by a suitable hardness factor (see end of table on following page). When the required shaft hardness cannot be obtained a bearing having a hardened solid inner race is used.

Machine Tool Roller Bearing Load Table — 2

Shaft Diam., Inches	Bearing Length, Inches	Roller Diam., Inches	Outside Diam. Bearing	Type of Outer Race	Speeds in R.P.M. and Load Capacities				
					25	150	300	500	1000
3	3	¾	4¾	Split	1,350	1,080	875	700	....
3	1¾	1	5⅜	Solid	3,200	2,730	2,270	1880	1280
3	3½	1	5⅜	Solid	7,200	6,150	5,100	4230	2880
3½	3	1⅛	6⅛	Split	1,730	1,450	1,200	1000	....
3½	4	1⅛	6⅜	Solid	9,050	7,650	6,300	5250	....
4	3	1⅛	6⅝	Split	1,935	1,610	1,320	1080	....
4	4	1⅛	6⅞	Solid	10,100	8,450	6,900	5620	....
4½	4	1¼	7⅜	Split	2,930	2,420	1,970	1610	....
5	4	1¼	7⅞	Split	3,150	2,600	2,090	1690	....
5	7	1¼	7⅞	Split	5,770	4,750	3,830	3100	....
5½	7	1¼	8⅜	Split	6,250	5,050	4,100	3230	....
6	4	1¼	8⅞	Split	3,610	2,900	2,330	1810	....
6	7	1¼	8⅞	Split	6,620	5,300	4,280	3330	....
7	5	1⅜	10⅛	Split	5,150	4,025	3,150	2410	....
7	5	1⅜	10¾	Solid	20,600	16,000	12,500	9650	....

Journal Hardness Factors

Journal Material	Split Outer Race			Solid Outer Race		
	Brinell Reading	Scleroscope	Factor	Brinell Reading	Scleroscope	Factor
Steel containing about 0.10% C.....	110	16	0.66	600	88	1.00
Steel containing about 0.20% C.....	130	19	0.78	500	75	0.85
Steel containing about 0.30% C.....	150	23	0.90	400	60	0.70
Steel containing about 0.40–0.45% C.	165	25	1.00	300	45	0.50
Steel containing about 0.50–0.55% C.	175	27	1.10	....	....	....
Steel heat-treated to.............	200	30	1.20	....	....	....
Steel heat-treated to.............	250	38	1.50	....	....	....
Steel heat-treated to.............	300	45	1.80	....	....	....
Inner races and split outer races....	....	....	1.80	....	....	....

Example: — Determine the load capacity of a 2- by 4-inch bearing having a *split* outer race, assuming the speed is 300 R.P.M. and the shaft hardness 250 Brinell.

The load capacity given in the preceding table is 900 pounds which is based on a hardness of 165 Brinell. The hardness factor for 250 Brinell is 1.5 for a split outer race; hence the load capacity equals 900 × 1.5 = 1350 pounds.

Loads on Roller Bearings. — The safe load diminishes as the speed of rotation of the rollers increases. If the speed is not excessive, the safe load may be determined approximately by the following formula: $P = \frac{Klnd^2}{ND + 2000\,d}$ in which P = safe load, in pounds; l = length of each roller, in inches; n = number of rollers; d = diameter of rollers, in inches; N = revolutions per minute of shaft; D = diameter of sleeve or roller path, in inches; K = a constant. For first-class workmanship and solid steel rollers with hardened and ground surfaces, K = 1,200,000 to 2,000,000, for rollers having a length equal to one diameter.

Bearing Lubricants

Classes of Lubricants. — Lubricants are derived from mineral, vegetable and animal sources. *Mineral Oils:* Nearly all commercial lubricating oils are obtained from petroleum, although some mineral oils are also derived from shale. Mineral oils are classed commercially as "pale" and "dark." The pale oils are somewhat transparent and are tinged with a variety of yellow and red shades. The dark oils are opaque and are either greenish- or brownish-black. The specific gravity of mineral oils usually varies from about 0.860 to 0.940, and the flashing point, from 300 to 600 degrees F. The oils obtained from petroleum have a much wider range of viscosity than the "fixed oils." The thinnest are more fluid than sperm oil, and the thickest more viscous than castor oil. The shale lubricating oils are of low viscosity. *Fixed Oils and Fats:* Fixed oils are so named because they are not volatile without decomposition. They are obtained from the seeds or fruits of plants and the tissues of animals. All fixed oils become fats at low temperatures and, inversely, all fats become oils at 150 degrees F. The most common lubricants among animal oils are tallow, lard, neat's foot and sperm oil, and among vegetable oils, olive, rape and castor oil. Ordinarily, animal oils are either colorless or yellow, whereas vegetable oils have various shades of yellow and green. The specific gravity of fixed oils varies from about 0.879 to 0.968 at 60 degrees F. Sperm oil has the lowest viscosity and castor oil the highest. *Blown or Thickened Oils:* Lubricants of this class are fixed oils (usually rape or cotton-seed) which are artificially thickened by forcing a current of air through the heated oil. This process increases the density and viscosity. Blown oils mixed with mineral oils are very largely used as lubricants. The mineral oils used for this purpose are usually of rather low viscosity. *Rosin Oil:* This oil is obtained from common rosin by distillation, and is not suitable for the lubrication of machinery, but is used to adulterate mineral and other lubricating oils. *Soap and Grease Lubricants:* Some mineral oils are thickened artificially by the addition of a substance such as aluminum soap. This increases the viscosity, but the latter rapidly diminishes when the oil is heated. Mineral oils are also thickened with sufficient soap (such as lime, soda or lead soap) to form a grease at ordinary temperatures. These greases may also contain some solid lubricant, such as graphite, talc, etc. *Adulterants:* Many of the more expensive oils, such as sperm, olive and lard, are adulterated with cheaper fixed oils and mineral oils. Cotton-seed is often mixed with lard and olive oils, and is sometimes substituted for the latter. The use of adulterants in order to increase the viscosity is usually resorted to in the case of mineral oils. The presence of these adulterants can sometimes be detected by comparatively simple tests. (See "Testing Lubricating Oils.")

How Lubricants are Affected by Use. — Experiments have been made to determine whether or not lubricating oil "wears out" in long continued service. These tests were made at Cornell University and indicated that oil gains in specific gravity by continued use, on account of the loss of volatile constituents. The used oil has a higher viscosity than new oil, indicating that it gains in "body" with use. Friction tests showed that new oil has a slightly lower coefficient of friction at low bearing pressures, but the reverse is true for high pressures. The differences, however, are so small that the coefficient of friction may be considered equal for new and old oils.

Qualities and Properties of Lubricants. — Although animal and vegetable lubricants differ from mineral lubricants chemically, many of the physical properties are similar. Of these properties, viscosity and "oiliness" are the most valuable. The viscosity of a particular animal or vegetable oil does not vary greatly in different samples, but mineral oils have a wide range of viscosity, and the required degree of fluidity must be considered in selecting a mineral lubricant.

The viscosity of all lubricants varies considerably with changes of temperature. For instance, an oil might be sufficiently viscous to carry a given load under normal conditions, but become so fluid, due to a rise in temperature, as to cause the bearing surfaces to cut and abrade each other. The viscosity of mineral oils varies more with the temperature than the viscosity of fixed oils; hence a bearing lubricated with a mineral oil needs more attention than one lubricated with a fatty oil.

Blended Oils: Mineral oils possess valuable properties which are conferred upon other lubricants with which they are mixed; as they are also a great deal cheaper than good fixed oils, blended or mixed oils are commonly used. The percentage of mineral oil that should be added varies according to the load, speed, etc., because many mineral lubricants are deficient in the property known as "oiliness" and can seldom be used pure, except when the bearing is kept flooded with oil by bath, ring or forced lubrication. Pure mineral oils are quite satisfactory for forced lubrication, and must be employed for lubricating the cylinders of condensing engines, because fixed oils would be likely to enter the boilers and cause damage. In general, fixed oils (if of sufficient viscosity) and fats are adapted for comparatively high bearing pressures, and mineral oils, for light pressures. The speed should also be considered, viscous oils giving the best results at low speeds, and thin oils, at high speeds. The viscosity, however, even at low speed, should be as low as possible to prevent excessive friction. Pressures below 70 pounds per square inch may be regarded as light, and speeds below 100 feet per minute, as low speeds. Pure mineral oils may be used for pressures below 70 pounds per square inch, with free lubrication, provided the bearing metals are such as do not readily "seize" each other. As the bearing pressure increases, the proportion of fixed oil should also increase.

Fats and Greases: It may be said, in general, that the lubricating properties of a mineral grease like vaseline, and an animal fat such as tallow, differ just as in the case of mineral and fixed oils, the animal fat possessing in a much greater degree the property of greasiness. Many lubricating greases are made by incorporating or emulsifying animal and vegetable fats with soap and water, or by thickening mineral lubricating oils with soap. These lubricants are often used with good results for railway car axles and the bearings of slow-moving machinery. If the bearing loads are exceptionally heavy, solids such as plumbago and soap-stone are sometimes added to the grease. Except in special cases, grease should not be used for fast-running journals, because the increased frictional resistance results in great loss of power. It is only when the surface speeds of bearings are low that economy results from the use of greases. Very satisfactory results, however, have been obtained by the use of grease for locomotive lubrication.

Gumming: Many oils when exposed to the atmosphere in thin films (as on bearings) form gummy masses, which clog the moving parts and cease to act as lubricants. A high temperature increases this tendency; hence gumming oils such as rape should not be used in warm places. (To clean surfaces which are covered with a gummy oxidized lubricant, the parts should be well moistened with paraffin oil.)

Foreign Substances: When bearings become heated, first ascertain definitely whether the fault really lies with the lubricant or is due to impurities accidentally produced. Bearings sometimes become heated when an excellent quality of lubricant is being used, and this is often due to the presence of foreign elements such as sand, vegetable fiber, etc.

It is well to remember, when selecting a lubricant, that a cheap oil will often result in greatly increasing other expenses; hence quality and adaptability for the surface required are the most important considerations.

Lubricants for Heavy Loads and Low Speeds. — The relative carrying capacity of oils, greases and solid lubricants can be determined only approximately, and the selection must finally be governed by actual trial, in order to secure a lubricant which gives a minimum amount of heating and friction. Four classes of lubricants for low speeds and comparatively high pressures are referred to in the following: 1. Graphite, soap-stone and other solid lubricants used dry. This class has the greatest carrying power, and the efficiency of the lubricant depends largely upon the nature of the bearing surface. Graphite, for example, gives the best results on cast-iron surfaces. 2. Solid lubricants mixed with animal fats, greases, vaseline, etc., or rosin grease. These mixtures are adapted for heavy work, especially when metal works against wood. 3. Axle grease composed entirely of animal and vegetable fats, or mineral oils emulsified with water, soap and enough alkali to make them neutral. These greases are excellent for slow-moving shafts and journals subjected to considerable loads. 4. Fixed oils, mineral oils and mixtures of the two. In selecting oils for low speeds and high pressures, viscosity must be the first consideration and, next to that, oiliness. If the viscosity is high enough, mineral oil may be as good as or even better than a fixed oil; or a mixture may be preferable to either.

Lubricants for Moderate Speeds. — Mineral oils are excellent for bearings subjected to comparatively light loads and running at moderate speeds. In general, the viscosity should increase as the speed of rotation diminishes. This applies particularly to mineral oils. As the loads increase, wear and overheating may result unless fixed oils be added, although it is rarely necessary to use pure fixed oils. For locomotives, 75 per cent mineral oil and 25 per cent rape oil has been found satisfactory. The necessity for using expensive fixed oils instead of the cheaper mineral oils is sometimes due to imperfect lubricating devices. When selecting a lubricant for bearings operating at ordinary speeds, viscosity is the most important property to consider, provided the loads do not exceed 200 to 250 pounds per square inch.

Lubricants for High Speeds. — High-speed bearings for armature shafts, etc., require thin lubricating oils. If ample bearing surface has been provided, thus reducing the unit pressure, pure mineral oils can be used to advantage. When good lubrication is impossible, it may be necessary to use some fixed oil. A mineral oil containing about 10 per cent fixed oil, gives little frictional resistance and minimizes wear.

Lubricants for Different Classes of Machinery. — A mixture of mineral oil and from 10 to 20 per cent neutral animal or vegetable oil is suitable for bearings of *machine tools*, *shafting*, and all machinery of medium weight and speed. Animal oils are usually preferable to vegetable, as they are less liable to gum. The heavier the machine and the slower the speed, the greater the viscosity required. For *dynamos* and *motors* with bath or ring lubrication, use mineral oil (preferably pure) having from two-thirds to about three times the viscosity of rape oil at 60 degrees F., depending upon the size, weight and speed of the machine. For *ball bearings* and *chains* for motor vehicles, etc., a neutral animal oil or fat, either pure or mixed with a fairly viscous mineral lubricating oil, is the most suitable. For the *chain* of a bicycle or motor, use a good grease or fatty oil. Good results are obtained by running the chain through a bath of sperm oil, or a mixture of good mineral oil and a little refined animal oil. Except when an oil bath is used, a thick oil or grease is necessary, as a thin lubricant runs off rapidly. For *light, delicate machinery* running at fairly high speeds, use mineral lubricating oil having about the same viscosity as sperm oil, and preferably mixed with 10 to 20 per cent sperm oil. For *pneumatic hammers*, use a good quality of light mineral oil, as a heavy oil tends to clog the parts. The ring spindles of *textile machinery*, running in an oil bath at speeds as high as 10,000 revolutions per minute, should be lubricated with a mineral oil of low viscosity not exceeding that of sperm oil at 60 deg. F. For *turbine bearings* having forced or circulating-pump lubrication, use pure mineral oil having a viscosity of from one to five times that of refined rape oil at 60 degrees F., depending upon the bearing pressure, speed and temperature conditions. For *high-speed engine bearings* having forced lubrication, use pure mineral oil of about twice the viscosity of rape oil at 60 degrees F. For *steam engine cylinders*, use heavy mineral cylinder oils mixed with from 5 to 25 per cent of rape or other fixed oil, the proportion of the latter being reduced as low as possible without impairing the lubricating quality. Sometimes pure mineral oil must be used in the cylinders of marine and other engines provided with surface condensers, to prevent fixed oils from entering the boiler. Vegetable and animal oils are unsuitable for cylinder lubrication, because when subjected to high temperatures they undergo a chemical change, resulting in the formation of free, fatty acids which may cause serious corrosion. Fatty oils, however, when mixed with mineral oils in quantities not exceeding from 5 to 20 per cent, do not seem to produce these objectionable results. For *locomotive axles*, use mineral oil having from two to four times the viscosity of rape oil at 60 degrees F., mixed with refined rape oil in the proportion of three parts mineral to one part rape. For very *heavy locomotives*, especially in warm climates, it is desirable to increase the viscosity of the mixture by adding some good mineral cylinder oil. For *bearings working in hot places*, use mineral oil to which is added from 20 to 33 per cent blown or thickened vegetable oil (usually rape). For *gas engine cylinders*, a mixture of 90 per cent mineral oil and 10 per cent neutral fixed oil is largely used. The viscosity is about the same as that of rape oil at 60 degrees F. As carbonaceous deposits are liable to form in the cylinder by partial combustion of the lubricant, especially when used in excess, the mineral oil selected should have undergone very careful rectification, and have little tendency to decompose and deposit carbon when heated. They should also be oils of low volatility, not being affected appreciably by evaporation at working temperatures.

Over-heated Bearings. — An over-heated bearing may be due to a number of different causes, such as incorrect design of bearing, use of an unsuitable lubricant, or improper method of applying the lubricant. When a bearing suddenly becomes heated, it is advisable to immediately apply a fatty oil of good quality, as the viscosity of an oil of this kind does not diminish with the rise of temperature to the same extent as with a mineral oil; moreover, the fixed oils are more "oily." Rape or olive oils are good for such emergencies. Sometimes a bearing runs hot because it has been without oil for a short time. When it is not feasible to stop the rotating part (as in the case of marine engine bearings) it is common to turn a stream of clear or soapy water upon the heated part. Sometimes serious damage can be prevented by throwing plumbago upon the journal or by applying the plumbago mixed with oil. When a bearing which has run cool (thus showing that the design is not at fault) repeatedly becomes heated, it should be taken apart and be examined carefully. The boxes may be adjusted too closely, the journal may be out of true, or the bearing surfaces be scored or grooved. Sometimes a surface crack in a bearing will cause heating, because it intercepts the oil film and allows the lubricant to escape. If the bearing is apparently in good order, the lubricant may be the cause of the trouble. If the viscosity is too low, the oil film does not form properly and there is direct contact between the rubbing surfaces. A bearing also tends to become heated if the viscosity of the oil is too high.

Testing Lubricating Oils. — The quality and properties of some lubricating oils can be determined approximately without the use of special testing apparatus by the following simple methods: To determine the presence of solid impurities in the oil, kerosene is added to half a tumbler of oil until the whole becomes quite thin. The mixture is then passed through filter paper or ordinary white blotting paper, and after all the oil has passed through, the paper is washed with kerosene; the residue on the paper, if any, will show if the oil had any solid impurities. Impurities may also be roughly detected by smearing a piece of writing paper with oil and holding it against the light; if the oil is free from solid impurities the blot will be equally transparent throughout — otherwise, the solid particles will show. The oil must not resinify: To test it in this respect, pour into a shallow dish and leave in a warm place about a week. There must not be the slightest crust at the end of that time.

Another way to test oil is by mixing it with fumes of nitric acid: If the oil is pure, a thick mass will form in a few hours, while resinifying oil will remain thin. Acids are very injurious impurities in lubricating oil, since, in time, they attack the machine parts lubricated. To test for acids, copper oxide or copper ash is added to the oil in a glass vessel; acid-free oil retains its original color, while, if it contains acid, it becomes greenish or bluish. Another test is to drop the oil on a sheet of copper or brass and leave it there for a week; if the oil contains acid there will be a green spot on the metal. A good oil must be greasy in order to have good lubricating qualities; to find which of several oils is best in this respect, place a few drops of the different oils on a smooth, slightly inclined metal or glass sheet; the drop of the best oil will travel furthest in a given time.

Cylinder oil may be tested by heating it and noting its color. A good cylinder oil will not change color to any noticeable degree, when heated to 480 degrees F., or to a temperature higher than that existing in a high-pressure engine cylinder. Low-grade oils, however, will darken when heated to this temperature. Another method of testing cylinder oils which gives good results is as follows: Heat the oil in a current of air for one hour, at a temperature corresponding to that of the required steam pressure. The loss in weight should not exceed 0.5 per cent.

Lubricants for Lathe Centers. — The following lubricants are recommended for lathe centers to prevent cutting or abrasion: 1. Dry or powdered red lead mixed with a good grade of mineral oil to the consistency of cream. 2. White lead mixed with sperm oil with enough graphite added to give the mixture a dark lead color (when necessary, thin by adding more oil). 3. Graphite one part and tallow four parts, the two ingredients being thoroughly mixed.

Rust Resisting Grease for Nuts. — When nuts, wash-out plugs, etc., are lubricated to prevent corrosion and facilitate their removal after a long period, a lubricant should be used which does not evaporate, and which does not corrode or otherwise injure the metallic surfaces. Use a lubricant consisting of thick mineral cylinder oil, or petroleum jelly mixed with black lead. Do not use tallow or any fat oil or grease, as fat soon undergoes a change and allows the thread surfaces to corrode. A thin mineral oil is also unsuitable as it soon flows out of the joint and evaporates. Sometimes there should be no lubricant, especially if the nuts are subjected to vibration which tends to loosen them, as the nuts of rail fish-plates, etc.

Protection of Finished Surfaces. — The finished or polished surfaces of machinery can be protected from rust by applying (while warm) a mixture of white lead and tallow, this being very commonly used. Neutral petroleum jelly is also a good rust preventive for steel, provided the surfaces to which it is applied are perfectly dry. To prevent the rusting of tools stored in the stock-room, etc., a coating of what is known as "cosmolubric oil No. 1" will be found very effective. If the oil becomes too thick, heat it slightly. Another good rust preventing lubricant is made from equal parts of turpentine and linseed oil. Machinery that may be exposed to rain or considerable moisture while being exported can be protected by coating the surfaces with black enamel or japan.

KEYS AND KEYWAYS

Dimensions of Sunk Keys. — Keys are generally proportioned with relation to the shaft diameter, instead of considering the torsional load in each case, because of practical reasons, such as standardization and interchangeability. There are, however, no fixed standards among manufacturers, although a fair idea of the average practice can be obtained from the following rules which apply to sunk keys:

Rule 1. The key width equals $\frac{1}{4}$ of the shaft diameter; the thickness, $\frac{1}{6}$ of the shaft diameter; the minimum length, $1\frac{1}{2}$ times the shaft diameter. Expressing these rules as formulas:

$$W = \frac{D}{4} \qquad T = \frac{1}{6}D \qquad L = 1.5\,D$$

in which W = key width; T = key thickness; L = key length; and D = shaft diameter. This notation is also used in the following formulas:

Rule 2. $W = \frac{3}{16}D + \frac{1}{16}$ inch; $T = \frac{1}{8}D + \frac{1}{8}$ inch; $L = \frac{3}{10}D^2 \div T$

For splines or feather keys, interchange the dimensions for width and thickness.

Rule 3. (Unwin.) $W = \frac{1}{4}D + \frac{1}{8}$ inch; $T = \frac{1}{8}D + \frac{1}{8}$ inch.

When gears or pulleys transmitting only a small amount of power are keyed to large shafts, these dimensions are excessive. In such cases, if P = horsepower transmitted by the gear or pulley; N = revolutions per minute; F = force in pounds acting at circumference; and R = radius of pulley in inches, then:

$$D = \sqrt[3]{\frac{100\,P}{N}} \quad \text{or} \quad \sqrt[3]{\frac{FR}{630}}$$

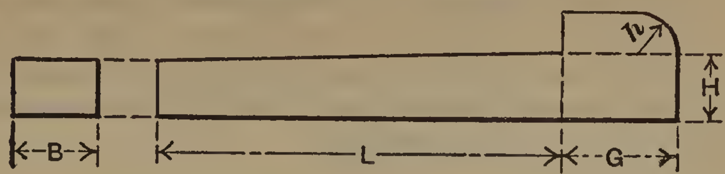
The taper of sunk keys is usually about $\frac{1}{8}$ or $\frac{3}{16}$ inch per foot. The depth of a taper keyseat at the deep end should be $\frac{3}{5}$ of the key thickness.

Proportions of Sunk Keys (U. S. Navy Standard)

Diam. of Shaft	Width of Key	Thickness of Key	Diam. of Shaft	Width of Key	Thickness of Key	Diam. of Shaft	Width of Key	Thickness of Key	Diam. of Shaft	Width of Key	Thickness of Key
1/2	7/32	3/16	2 1/4	9/16	5/16	5	1 1/16	5/8	7 3/4	1 9/16	7/8
5/8	1/4	3/16	2 1/2	5/8	3/8	5 1/4	1 1/8	5/8	8	1 5/8	7/8
3/4	9/32	3/16	2 3/4	5/8	3/8	5 1/2	1 3/16	5/8	8 1/4	1 5/8	15/16
7/8	9/32	7/32	3	11/16	7/16	5 3/4	1 3/16	11/16	8 1/2	1 3/4	15/16
1	5/16	7/32	3 1/4	3/4	7/16	6	1 1/4	11/16	8 3/4	1 3/4	15/16
1 1/8	11/32	1/4	3 1/2	13/16	7/16	6 1/4	1 5/16	3/4	9	1 3/4	1
1 1/4	3/8	1/4	3 3/4	13/16	1/2	6 1/2	1 3/8	3/4	9 1/4	1 7/8	1
1 3/8	3/8	1/4	4	7/8	1/2	6 3/4	1 3/8	3/4	9 1/2	1 7/8	1 1/16
1 1/2	13/32	1/4	4 1/4	15/16	9/16	7	1 7/16	13/16	9 3/4	2	1 1/16
1 3/4	7/16	5/16	4 1/2	1	9/16	7 1/4	1 1/2	13/16	10	2	1 1/16
2	1/2	5/16	4 3/4	1	9/16	7 1/2	1 9/16	7/8	...	...	....

Propeller Keys. — The practice of the U. S. Navy in proportioning keys for propellers is to make the width of the key about 1½ times its thickness. The thickness, in turn, is so determined that the side pressure on the propeller hub, calculated

Dimensions of Gib Keys



Keys of proportions given below are weakest in shear.
The safe twisting moment per inch of length of keys = $R \times B \times S$

- R = radius of shaft;
- B = breadth of key;
- S = safe shearing strength of material in key;
- B = ¼ times diam. of bore, up to 6 inches; for larger sizes, $B = 0.211 \times$ bore, approximately;
- $G = B$, approximately;
- H = ⅙ times diam. of bore up to 6 inches; for larger sizes, H = ⅛ $\times$ bore;
- h = radius = ⅛ times diam. of bore, approx., but minimum value = 3/16 inch;
- L = length of hub + ½ inch.

Taper ⅛ inch per foot.

Bore and Shaft Diameter	Width of Key B	Height of Key H	Depth of Key-way $\frac{H}{2}$	Rad. h	G	Safe Twisting Moment on Key per Inch of Length f or S = 5000	7500	10,000
15/16 to 1⅛	¼	3/16	3/32	3/16	¼	630	940	1,250
1 3/16 to 1⅜	⅜	¼	⅛	3/16	⅜	1,170	1,760	2,340
1 7/16 to 1⅝	⅜	¼	⅛	3/16	⅜	1,410	2,110	2,810
1 11/16 to 1⅞	½	5/16	5/32	¼	½	2,190	3,280	4,380
1 15/16 to 2⅛	½	⅜	3/16	¼	½	2,500	3,750	5,000
2 3/16 to 2⅜	⅝	⅜	3/16	⅜	⅝	3,520	5,270	7,030
2 7/16 to 2⅝	⅝	7/16	7/32	⅜	⅝	3,910	5,860	7,810
2 11/16 to 2⅞	¾	½	¼	⅜	¾	5,160	7,730	10,313
2 15/16 to 3⅛	¾	½	¼	⅜	¾	5,620	8,420	11,250
3 3/16 to 3⅜	⅞	9/16	9/32	½	⅞	7,110	10,660	14,220
3 7/16 to 3⅝	⅞	⅝	5/16	½	⅞	7,660	11,480	15,310
3 11/16 to 3⅞	1	⅝	5/16	½	1	9,380	14,060	18,750
3 15/16 to 4⅛	1	11/16	11/32	½	1	10,000	15,000	20,000
4 3/16 to 4⅜	1⅛	11/16	11/32	⅝	1	11,950	17,930	23,910
4 7/16 to 4¾	1⅛	¾	⅜	⅝	1	12,660	18,980	25,310
4 13/16 to 5¼	1¼	⅞	7/16	⅝	1¼	15,620	23,440	31,250
5 5/16 to 5¾	1⅜	⅞	7/16	¾	1¼	18,910	28,360	37,810
5 13/16 to 6¼	1½	1	½	¾	1½	22,500	33,750	45,000
6 5/16 to 6¾	1½	1	½	⅞	1½	24,380	36,560	48,750
6 13/16 to 7¼	1½	1	½	⅞	1¾	26,250	39,380	52,500
7 5/16 to 7¾	1⅝	1	½	1	1¾	30,470	45,700	60,940
7 13/16 to 8¾	1¾	1⅛	9/16	1	2	36,090	54,140	72,190
8 13/16 to 9¾	2	1¼	⅝	1⅛	2	46,250	69,380	92,500
9 13/16 to 10¾	2¼	1½	¾	1¼	2	57,660	86,480	115,320
10 13/16 to 11¾	2½	1½	¾	1¼	2	70,310	105,470	140,630
11 13/16 to 12¾	2½	1½	¾	1¼	2	76,560	114,840	153,130

from the maximum turning moment on the shaft, does not exceed 25,000 pounds per square inch. Ordinarily, the key is so proportioned that the pressure on the keyway will not exceed 22,000 pounds per square inch. With this pressure, if the key thickness is over $\frac{1}{8}$ of the shaft diameter, two keys set opposite are preferred. The hub of the propeller is bored, tapered and fitted to a corresponding taper on the shaft, which is provided with a retaining nut.

Dimensions of Flat Keys

Diam. of Shaft	1	1¼	1½	1¾	2	2½	3	3½	4	5	6	7	8	9	10
Width of Key	¼	5/16	⅜	7/16	½	⅝	¾	⅞	1	1⅛	1⅜	1½	1¾	2	2¼
Thickness of Key	5/32	3/16	¼	9/32	5/16	⅜	7/16	½	⅝	11/16	13/16	⅞	1	1¼	1½

Dimensions of Feather Keys or Splines

Diam. of Shaft	1	1¼	1½	1¾	2	2½	3	3½	4	5	6	7	8	9	10
Width of Feather	¼	5/16	⅜	7/16	½	⅝	¾	⅞	1	1⅛	1⅜	1½	1¾	2	2¼
Thickness of Feather	⅜	7/16	½	9/16	⅝	¾	⅞	1	1¼	1⅜	1⅝	1¾	2	2½	2¾

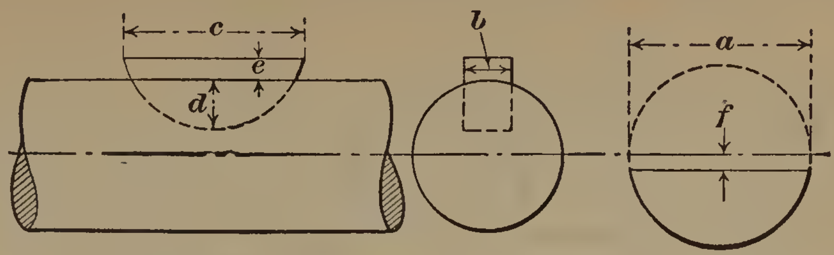
Woodruff Keys. — Woodruff keys are used extensively in machine tool construction. They should project above the shaft a distance equal to one-half of the thickness. The accompanying tables give the proportions of the regular standard and also the special Woodruff keys. When using the table which indicates the sizes of keys that are best adapted to different shaft diameters, where more than two keys are given for a certain diameter, the medium size key should be used in ordinary practice.

When milling the shaft to receive a Woodruff key, the shaft is brought against the cutter until a flat of the same width as the thickness of the cutter is formed; the cutter is then fed into the shaft to the required depth. If the keyseating cutter should cut a little large, the size of the cutter may be reduced slightly by holding an oilstone against the face of the revolving cutter. Care should be taken to eliminate all end-play in the spindle of the machine. When fitting a Woodruff key, the edges and corners should be "brushed" slightly with a file to insure the key entering readily.

Standard Woodruff Keys to Use for Different Shaft Diameters

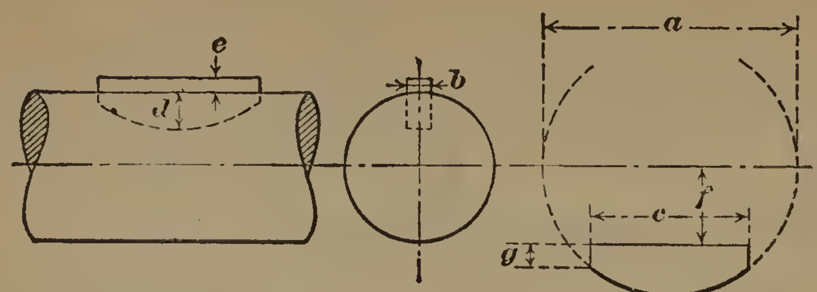
Diameter of Shaft	Number of Keys	Diameter of Shaft	Number of Keys	Diameter of Shaft	Number of Keys
5/16 –⅜	1	⅞ – 15/16	6, 8, 10	1⅜ –1 7/16	14, 17, 20
7/16 –½	2, 4	1	9, 11, 13	1½ –1⅝	15, 18, 21, 24
9/16 –⅝	3, 5	1 1/16–1⅛	9, 11, 13, 16	1 11/16–1¾	18, 21, 24
11/16–¾	3, 5, 7	1 3/16	11, 13, 16	1 13/16–2	23, 25
13/16	6, 8	1¼ –1 5/16	12, 14, 17, 20	2 1/16 –2½	25

Dimensions of Woodruff Standard Keys



No. of Key and Cutter	Diameter of Cutter, a	Thickness of Key and Cutter, in Common and Decimal Fractions, b		Length of Key Approx., c	Depth to be Cut in Shaft, in Common and Decimal Fractions, d		Height of Key Above Shaft, e	Center of Stock to Top of Key, f
1	1/2	1/16	0.0625	1/2	11/64	0.1718	1/32	3/64
2	1/2	3/32	0.0937	1/2	5/32	0.1562	3/64	3/64
3	1/2	1/8	0.1250	1/2	9/64	0.1406	1/16	3/64
4	5/8	3/32	0.0937	5/8	13/64	0.2031	3/64	1/16
5	5/8	1/8	0.1250	5/8	3/16	0.1875	1/16	1/16
6	5/8	5/32	0.1562	5/8	11/64	0.1718	5/64	1/16
61	5/8	3/16	0.1875	5/8	5/32	0.1562	3/32	1/16
7	3/4	1/8	0.1250	3/4	1/4	0.2500	1/16	1/16
8	3/4	5/32	0.1562	3/4	15/64	0.2343	5/64	1/16
9	3/4	3/16	0.1875	3/4	7/32	0.2187	3/32	1/16
91	3/4	1/4	0.2500	3/4	3/16	0.1875	1/8	1/16
10	7/8	5/32	0.1562	7/8	19/64	0.2968	5/64	1/16
11	7/8	3/16	0.1875	7/8	9/32	0.2812	3/32	1/16
12	7/8	7/32	0.2187	7/8	17/64	0.2656	7/64	1/16
A	7/8	1/4	0.2500	7/8	1/4	0.2500	1/8	1/16
13	1	3/16	0.1875	1	11/32	0.3437	3/32	1/16
14	1	7/32	0.2187	1	21/64	0.3281	7/64	1/16
15	1	1/4	0.2500	1	5/16	0.3125	1/8	1/16
B	1	5/16	0.3125	1	9/32	0.2812	5/32	1/16
152	1	3/8	0.3750	1	1/4	0.2500	3/16	1/16
16	1 1/8	3/16	0.1875	1 1/8	25/64	0.3906	3/32	5/64
17	1 1/8	7/32	0.2187	1 1/8	3/8	0.3750	7/64	5/64
18	1 1/8	1/4	0.2500	1 1/8	23/64	0.3593	1/8	5/64
C	1 1/8	5/16	0.3125	1 1/8	21/64	0.3281	5/32	5/64
19	1 1/4	3/16	0.1875	1 1/4	29/64	0.4531	3/32	5/64
20	1 1/4	7/32	0.2187	1 1/4	19/32	0.5937	7/64	5/64
21	1 1/4	1/4	0.2500	1 1/4	27/64	0.4218	1/8	5/64
D	1 1/4	5/16	0.3125	1 1/4	25/64	0.3906	5/32	5/64
E	1 1/4	3/8	0.3750	1 1/4	23/64	0.3593	3/16	5/64
22	1 3/8	1/4	0.2500	1 3/8	15/32	0.4687	1/8	3/32
23	1 3/8	5/16	0.3125	1 3/8	7/16	0.4375	5/32	3/32
F	1 3/8	3/8	0.3750	1 3/8	13/32	0.4062	3/16	3/32
24	1 1/2	1/4	0.2500	1 1/2	33/64	0.5156	1/8	7/64
25	1 1/2	5/16	0.3125	1 1/2	31/64	0.4843	5/32	7/64
G	1 1/2	3/8	0.3750	1 1/2	29/64	0.4531	3/16	7/64

Dimensions of Woodruff Special Keys

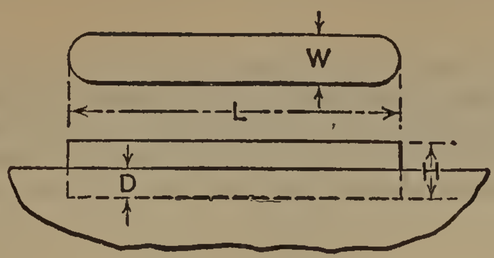


No. of Key and Cutter *	Diameter of Cutter, a	Thickness of Key and Cutter, in Common and Decimal Fractions, b		Length of Key, c	Depth to be Cut in Shaft, d	Depth of Keyway, e	Center of Stock to Top of Key, f	Width of Flat, g
126	2⅛	3/16	0.1875	1⅜	0.313	3/32	21/32	5/32
127	2⅛	¼	0.2500	1⅜	0.281	⅛	21/32	5/32
128	2⅛	5/16	0.3125	1⅜	0.250	5/32	21/32	5/32
129	2⅛	⅜	0.3750	1⅜	0.219	3/16	21/32	5/32
26	2⅛	3/16	0.1875	1 23/32	0.437	3/32	17/32	3/32
27	2⅛	¼	0.2500	1 23/32	0.406	⅛	17/32	3/32
28	2⅛	5/16	0.3125	1 23/32	0.375	5/32	17/32	3/32
29	2⅛	⅜	0.3750	1 23/32	0.344	3/16	17/32	3/32
Rx	2¾	¼	0.2500	2	0.469	⅛	25/32	0.1625
Sx	2¾	5/16	0.3125	2	0.437	5/32	25/32	0.1625
Tx	2¾	⅜	0.3750	2	0.406	3/16	25/32	0.1625
Ux	2¾	7/16	0.4375	2	0.375	7/32	25/32	0.1625
Vx	2¾	½	0.5000	2	0.344	¼	25/32	0.1625
R	2¾	¼	0.2500	2 5/16	0.625	⅛	⅝	⅛
S	2¾	5/16	0.3125	2 5/16	0.594	5/32	⅝	⅛
T	2¾	⅜	0.3750	2 5/16	0.562	3/16	⅝	⅛
U	2¾	7/16	0.4375	2 5/16	0.532	7/32	⅝	⅛
V	2¾	½	0.5000	2 5/16	0.500	¼	⅝	⅛
30	3½	⅜	0.3750	2⅞	0.750	3/16	13/16	3/16
31	3½	7/16	0.4375	2⅞	0.718	7/32	13/16	3/16
32	3½	½	0.5000	2⅞	0.687	¼	13/16	3/16
33	3½	9/16	0.5625	2⅞	0.656	9/32	13/16	3/16
34	3½	⅝	0.6250	2⅞	0.625	5/16	13/16	3/16
35	3½	11/16	0.6875	2⅞	0.593	11/32	13/16	3/16
36	3½	¾	0.7500	2⅞	0.562	⅜	13/16	3/16

* Use cutters Nos. 26, 27 and 28 for keys Nos. 126, 127 and 128.

Cotters. — A cotter is a form of key that is used to connect rods, etc., that are subjected either to tension or compression or both, the cotter being subjected to shearing stresses at two transverse cross-sections. When taper cotters are used for drawing and holding parts together, if the cotter is held in place by the friction between the bearing surfaces, the taper should not be too great. Ordinarily a taper varying from ¼ to ½ inch per foot is used for plain cotters. When a set-screw or other device is used to prevent the cotter from backing out of its slot, the taper may vary from 1½ to 2 inches per foot.

The Pratt & Whitney System of Keys

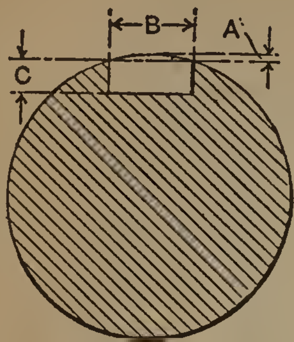


No. of Key	L	W	H	D
1	1/2	1/16	3/32	1/16
2	1/2	3/32	9/64	3/32
3	1/2	1/8	3/16	1/8
4	5/8	3/32	9/64	3/32
5	5/8	1/8	3/16	1/8
6	5/8	5/32	15/64	5/32
7	3/4	1/8	3/16	1/8
8	3/4	5/32	15/64	5/32
9	3/4	3/16	9/32	3/16
10	7/8	5/32	15/64	5/32
11	7/8	3/16	9/32	3/16
12	7/8	7/32	21/64	7/32
A	7/8	1/4	3/8	1/4
13	1	3/16	9/32	3/16
14	1	7/32	21/64	7/32
15	1	1/4	3/8	1/4
B	1	5/16	15/32	5/16
16	1 1/8	3/16	9/32	3/16
17	1 1/8	7/32	21/64	7/32
18	1 1/8	1/4	3/8	1/4
C	1 1/8	5/16	15/32	5/16
19	1 1/4	3/16	9/32	3/16
20	1 1/4	7/32	21/64	7/32
21	1 1/4	1/4	3/8	1/4
D	1 1/4	5/16	15/32	5/16
E	1 1/4	3/8	9/16	3/8
22	1 3/8	1/4	3/8	1/4
23	1 3/8	5/16	15/32	5/16
F	1 3/8	3/8	9/16	3/8
24	1 1/2	1/4	3/8	1/4
25	1 1/2	5/16	15/32	5/16
G	1 1/2	3/8	9/16	3/8
51	1 3/4	1/4	3/8	1/4
52	1 3/4	5/16	15/32	5/16
53	1 3/4	3/8	9/16	3/8
26	2	3/16	9/32	3/16
27	2	1/4	3/8	1/4
28	2	5/16	15/32	5/16
29	2	3/8	9/16	3/8
54	2 1/4	1/4	3/8	1/4
55	2 1/4	5/16	15/32	5/16
56	2 1/4	3/8	9/16	3/8
57	2 1/4	7/16	21/32	7/16
58	2 1/2	5/16	15/32	5/16
59	2 1/2	3/8	9/16	3/8
60	2 1/2	7/16	21/32	7/16
61	2 1/2	1/2	3/4	1/2
30	3	3/8	9/16	3/8
31	3	7/16	21/32	7/16
32	3	1/2	3/4	1/2
33	3	9/16	27/32	9/16
34	3	5/8	15/16	5/8

Pratt & Whitney Co.'s System of Keys. — A system of round-end feather keys has been developed by the Pratt & Whitney Co., and is used in their plant. The system is also being adopted by many other manufacturers. The keyways are milled by means of a spline milling machine made by the company. The dimensions given in the table are those used in the Pratt & Whitney works. The numbers of the keys correspond to the numbers used for designating keys in the Woodruff system, and the dimensions are proportionately the same. The length *L* of the key, however, may vary from the length given in the table, but it should be equal to at least two times the width *W* of the key. The keys are made 0.001 inch over standard size in width to insure proper fitting in the keyway.

Friction of Sliding Keyways. — When a clutch moves along a shaft to which it is keyed, it is always preferable to use two keyways, one on each side of the shaft, instead of a keyway only on one side. Experiments have shown that when two keys are used, the frictional resistance of a clutch to sliding on the shaft is only one-half of what it is with one key.

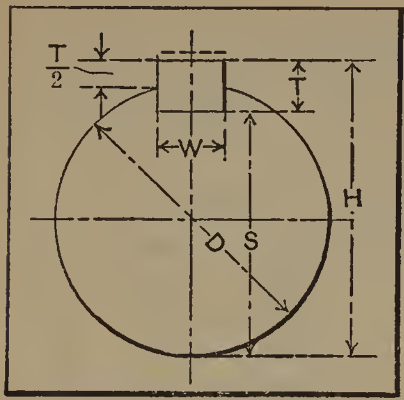
Dimensions for Obtaining Depths of Keyseats



The values in the body of the table give the dimension *A*, which should be added to the depth *C* of the keyway in order to find the total depth from the outside of the shaft to the bottom of the keyway. When milling keyways, the cutter can be fed down this total depth, and no further measuring is necessary.

Size of Shaft	Width of Keyway *B*					Size of Shaft	Width of Keyway *B*				
	1/4	5/16	3/8	7/16	1/2		1/4	5/16	3/8	7/16	1/2
1/2	0.032	...	...	....	....	2 5/16	0.006	0.010	0.015	0.020	0.027
9/16	0.028	...	...	....	....	2 3/8	0.006	0.010	0.015	0.020	0.026
5/8	0.025	0.041	...	....	....	2 7/16	0.006	0.010	0.014	0.019	0.026
11/16	0.023	0.037	...	....	....	2 1/2	0.006	0.009	0.014	0.019	0.025
3/4	0.022	0.034	0.051	....	....	2 9/16	0.006	0.009	0.014	0.018	0.024
13/16	0.019	0.031	0.046	....	....	2 5/8	0.006	0.009	0.013	0.018	0.024
7/8	0.017	0.028	0.042	0.058	....	2 11/16	0.005	0.008	0.013	0.018	0.023
15/16	0.016	0.026	0.039	0.054	....	2 3/4	0.005	0.008	0.013	0.017	0.023
1	0.015	0.024	0.036	0.050	0.067	2 13/16	0.005	0.008	0.012	0.017	0.022
1 1/16	0.014	0.022	0.034	0.047	0.062	2 7/8	0.005	0.008	0.012	0.016	0.022
1 1/8	0.013	0.021	0.032	0.044	0.058	2 15/16	0.005	0.008	0.012	0.016	0.021
1 3/16	0.013	0.020	0.030	0.042	0.055	3	0.005	0.008	0.011	0.016	0.021
1 1/4	0.012	0.019	0.029	0.039	0.052	3 1/16	0.005	0.008	0.011	0.015	0.020
1 5/16	0.012	0.019	0.027	0.038	0.049	3 1/8	0.005	0.007	0.011	0.015	0.020
1 3/8	0.012	0.018	0.026	0.036	0.047	3 3/16	0.005	0.007	0.011	0.015	0.019
1 7/16	0.011	0.017	0.025	0.034	0.045	3 1/4	0.004	0.007	0.011	0.014	0.019
1 1/2	0.011	0.016	0.024	0.032	0.042	3 5/16	0.004	0.007	0.010	0.014	0.019
1 9/16	0.010	0.015	0.023	0.030	0.041	3 3/8	0.004	0.007	0.010	0.014	0.018
1 5/8	0.010	0.015	0.022	0.029	0.039	3 7/16	0.004	0.007	0.010	0.014	0.018
1 11/16	0.010	0.014	0.021	0.028	0.038	3 1/2	0.004	0.007	0.010	0.013	0.018
1 3/4	0.009	0.014	0.020	0.027	0.037	3 9/16	0.004	0.006	0.010	0.013	0.017
1 13/16	0.009	0.013	0.019	0.026	0.035	3 5/8	0.004	0.006	0.010	0.013	0.017
1 7/8	0.009	0.013	0.019	0.025	0.033	3 11/16	0.004	0.006	0.009	0.013	0.017
1 15/16	0.009	0.012	0.018	0.025	0.032	3 3/4	0.004	0.006	0.009	0.012	0.016
2	0.008	0.012	0.017	0.024	0.031	3 13/16	0.004	0.006	0.009	0.012	0.016
2 1/16	0.008	0.011	0.017	0.023	0.030	3 7/8	0.004	0.006	0.009	0.012	0.016
2 1/8	0.007	0.011	0.016	0.022	0.029	3 15/16	0.004	0.006	0.009	0.012	0.016
2 3/16	0.007	0.010	0.016	0.022	0.029	4	0.004	0.006	0.009	0.012	0.016
2 1/4	0.007	0.010	0.015	0.021	0.028	...	...	...	....	....	....

Gaging Depths of Keyseats. — The table, "Dimensions for Obtaining Depths of Keyseats" was compiled to facilitate the accurate milling of keyseats. This table gives the distance *A* (see illustration accompanying table) between the top of the shaft and a line passing through the upper corners or edges of the keyseat. Dimension *A* is added to the side depth of the keyseat to get the total depth from the outside of the shaft. To mill a keyseat, adjust the cutter until it just touches the top of the shaft, and set the dial of the elevating screw to zero; then sink the cutter to the total depth, which equals depth C at the side, plus distance A obtained from the table. For example, if the diameter of the shaft is 3 inches and the key width is ½ inch, 0.021 inch should be added to the depth C in order to get the total depth from the top of the shaft. The height A of the arc can be calculated by the following formula:



$$A = R - \sqrt{R^2 - (\tfrac{1}{2}B)^2}$$

in which A = height of arc; R = radius of shaft; B = width of keyseat.

Example: — If the shaft radius is 5 inches and the width of the keyseat, 2 inches, then:

$$A = 5 - \sqrt{5^2 - 1^2} = 5 - \sqrt{24} = 0.101 \text{ inch.}$$

A method of measuring keyseats, which reduces the amount of fitting to a minimum, is indicated by the accompanying illustration. The keyway in the shaft is measured for depth as indicated by S, and the keyseat in the hub is measured across the bore, as at H. The advantage of this method is that the space T between the shaft and the hub does not vary (provided the keyseats are accurately cut and gaged) even though there be slight variations between the shaft diameter and the bore of the hub. The dimensions H and S for various diameters of shafts are given in the table, "Dimensions for Measuring Keyseats Across Shaft and Bore of Hub." In compiling this table, the following rules were used: The key width equals the fractional size nearest ¼ of the shaft diameter, the fractions varying by 1/16 inch increments up to ¾ inch widths, then by ⅛ inch up to 1¼ inch, after which the advance is made by ¼ inch increments. The key thickness at the thin end equals ¾ of the width, and the taper is ⅛ inch per foot. Keys 1 inch wide and wider are made thinner than the foregoing rule calls for. The keyway depth in the hub and shaft are made approximately equal at the thin end of the key, and the depth in the hub is measured at the shallow end of the keyway. With this system, there is no necessity of varying the standard for an extra long hub, as the depth simply increases with the length.

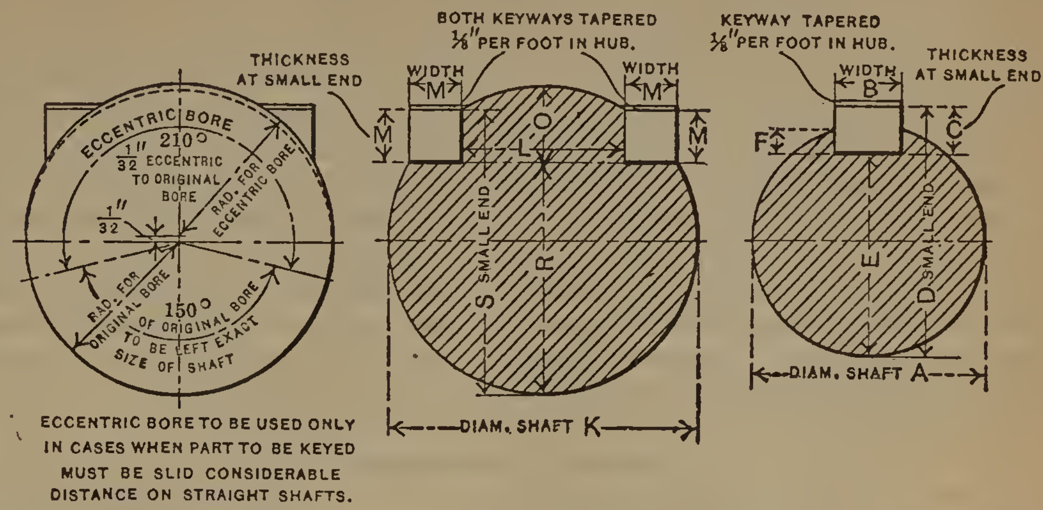
Key Fitting. — The proper method of fitting keys depends somewhat upon the type of key used. The tapered sunk key not only acts as a driver for the keyed part, but holds it against axial or endwise movement, and should have a bearing on all sides. The straight sunk key should have a good bearing on the sides, and ordinarily there is either a slight clearance at the top or a light bearing. When a straight key is to resist endwise, as well as rotary, movement, it is made to bear all over, but, in any case, the principal bearing should be on the sides. The saddle key, which is simply concaved on one side to fit the shaft and is tapered on the top, should be so fitted that it bears lightly on the sides and heavily between the shaft and hub throughout its entire length. As the drive with this type of key is not positive, it is only used when there is little power to transmit. The flat key, which is of rectangular section and bears against a flat surface on the shaft, is fitted practically in the same manner as the saddle key. The corners of all keys should be filed off slightly before driving, to prevent a heavy bearing at these points. When fitting Woodruff keys, it is good practice to bevel the circular edges somewhat to insure the key entering easily.

Dimensions for Measuring Keyseats Across Shaft and Bore of Hub

Diam. of Shaft, Inches D	Width of Key, Inches W	Thickness of Key, Inches T	Hub Dimension for Keyway at Thin End, Inches H*	Shaft Dimension for Keyway, Inches S*	Diam. of Shaft, Inches D	Width of Key, Inches W	Thickness of Key, Inches T	Hub Dimension for Keyway at Thin End, Inches H*	Shaft Dimension for Keyway, Inches S*
3/4	3/16	9/64	13/16	43/64	3 1/2	7/8	21/32	3 25/32	3 1/8
13/16	3/16	9/64	7/8	47/64	3 9/16	7/8	21/32	3 27/32	3 3/16
7/8	3/16	9/64	15/16	51/64	3 5/8	7/8	21/32	3 29/32	3 1/4
15/16	1/4	3/16	1 1/64	53/64	3 11/16	7/8	21/32	3 31/32	3 5/16
1	1/4	3/16	1 5/64	57/64	3 3/4	7/8	21/32	4 1/32	3 3/8
1 1/16	1/4	3/16	1 9/64	61/64	3 13/16	7/8	21/32	4 3/32	3 7/16
1 1/8	1/4	3/16	1 13/64	1 1/64	3 7/8	1	11/16	4 5/32	3 15/32
1 3/16	5/16	15/64	1 9/32	1 3/64	3 15/16	1	11/16	4 7/32	3 17/32
1 1/4	5/16	15/64	1 11/32	1 7/64	4	1	11/16	4 9/32	3 19/32
1 5/16	5/16	15/64	1 13/32	1 11/64	4 1/16	1	11/16	4 11/32	3 21/32
1 3/8	5/16	15/64	1 15/32	1 15/64	4 1/8	1	11/16	4 13/32	3 23/32
1 7/16	3/8	9/32	1 9/16	1 9/32	4 3/16	1	11/16	4 15/32	3 25/32
1 1/2	3/8	9/32	1 5/8	1 11/32	4 1/4	1	11/16	4 17/32	3 27/32
1 9/16	3/8	9/32	1 11/16	1 13/32	4 5/16	1	11/16	4 19/32	3 29/32
1 5/8	3/8	9/32	1 3/4	1 15/32	4 3/8	1 1/8	3/4	4 11/16	3 15/16
1 11/16	7/16	21/64	1 53/64	1 1/2	4 7/16	1 1/8	3/4	4 3/4	4
1 3/4	7/16	21/64	1 57/64	1 9/16	4 1/2	1 1/8	3/4	4 13/16	4 1/16
1 13/16	7/16	21/64	1 61/64	1 5/8	4 9/16	1 1/8	3/4	4 7/8	4 1/8
1 7/8	7/16	21/64	2 1/64	1 11/16	4 5/8	1 1/8	3/4	4 15/16	4 3/16
1 15/16	1/2	3/8	2 3/32	1 23/32	4 11/16	1 1/8	3/4	5	4 1/4
2	1/2	3/8	2 5/32	1 25/32	4 3/4	1 1/8	3/4	5 1/16	4 5/16
2 1/16	1/2	3/8	2 7/32	1 27/32	4 7/8	1 1/4	13/16	5 1/4	4 7/16
2 1/8	1/2	3/8	2 9/32	1 29/32	4 15/16	1 1/4	13/16	5 5/16	4 1/2
2 3/16	9/16	27/64	2 23/64	1 15/16	5	1 1/4	13/16	5 3/8	4 9/16
2 1/4	9/16	27/64	2 27/64	2	5 1/8	1 1/4	13/16	5 1/2	4 11/16
2 5/16	9/16	27/64	2 31/64	2 1/16	5 3/16	1 1/4	13/16	5 9/16	4 3/4
2 3/8	9/16	27/64	2 35/64	2 1/8	5 1/4	1 1/4	13/16	5 5/8	4 13/16
2 7/16	5/8	15/32	2 5/8	2 5/32	5 3/8	1 1/4	13/16	5 3/4	4 15/16
2 1/2	5/8	15/32	2 11/16	2 7/32	5 7/16	1 1/4	13/16	5 13/16	5
2 9/16	5/8	15/32	2 3/4	2 9/32	5 1/2	1 1/4	13/16	5 7/8	5 1/16
2 5/8	5/8	15/32	2 13/16	2 11/32	5 5/8	1 1/4	13/16	6	5 3/16
2 11/16	11/16	33/64	2 59/64	2 13/32	5 11/16	1 1/4	13/16	6 1/16	5 1/4
2 3/4	11/16	33/64	2 63/64	2 15/32	5 3/4	1 1/4	13/16	6 1/8	5 5/16
2 13/16	11/16	33/64	3 3/64	2 17/32	5 7/8	1 1/2	7/8	6 1/4	5 3/8
2 7/8	11/16	33/64	3 7/64	2 19/32	6	1 1/2	7/8	6 3/8	5 1/2
2 15/16	3/4	9/16	3 11/64	2 39/64	6 1/8	1 1/2	7/8	6 1/2	5 5/8
3	3/4	9/16	3 15/64	2 43/64	6 1/4	1 1/2	7/8	6 5/8	5 3/4
3 1/16	3/4	9/16	3 19/64	2 47/64	6 3/8	1 1/2	7/8	6 3/4	5 7/8
3 1/8	3/4	9/16	3 23/64	2 51/64	6 1/2	1 1/2	7/8	6 7/8	6
3 3/16	3/4	9/16	3 27/64	2 55/64	6 5/8	1 1/2	7/8	7	6 1/8
3 1/4	3/4	9/16	3 31/64	2 59/64	6 7/8	1 3/4	1	7 1/4	6 1/4
3 5/16	3/4	9/16	3 35/64	2 63/64	7 1/8	1 3/4	1	7 1/2	6 1/2
3 3/8	7/8	21/32	3 21/32	3	7 3/8	1 3/4	1	7 3/4	6 3/4
3 7/16	7/8	21/32	3 23/32	3 1/16	7 5/8	1 3/4	1	8	7

* For notation see illustration accompanying paragraph, "Gaging Depths of Keyseats."

Kennedy Double and Single Keys



For shafts above 6 inches in diameter, double keys should be used. If the torque is intermittent and power is transmitted alternately in opposite directions, double keys should be used for diameters down to and including 4 inches. Single keys may be used for sizes up to and including 6 inches, provided the torque is quite constant and the power transmission is always in one direction.

The sides and bottom of the keyway in a shaft should be straight and parallel. In the hub, the sides should be straight and parallel and the bottom tapered ⅛ inch per foot.

Double Keys

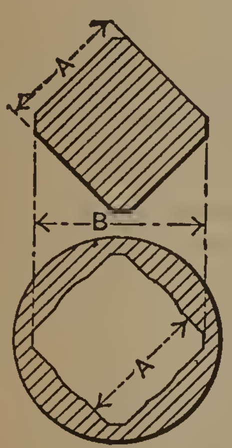
K	L	M	O	R	S	K	L	M	O	R	S
4	2⅛	1 11/16	1	3	3 11/16	16	8	3	4⅛	11⅞	14⅞
4½	2⅜	¾	1⅛	3⅜	4⅛	18	9	3⅜	4⅝	13⅜	16¾
5	2 9/16	⅞	1¼	3¾	4⅝	20	10	3¾	5⅛	14⅞	18⅝
6	3 1/16	1 1/16	1½	4½	5 9/16	22	11	4⅛	5⅝	16⅜	20½
7	3 9/16	1¼	1¾	5¼	6½	24	12	4½	6⅛	17⅞	22⅜
8	4 1/16	1 7/16	2	6	7 7/16	26	13	4⅞	6⅝	19⅜	24¼
9	4 9/16	1⅝	2¼	6¾	8⅜	28	14	5¼	7⅛	20⅞	26⅛
10	5 1/16	1 13/16	2½	7½	9 5/16	30	15	5⅝	7⅝	22⅜	28
11	5½	2	2¾	8¼	10¼	32	16	6	8⅛	23⅞	29⅞
12	6	2¼	3⅛	8⅞	11⅛	34	17	6⅜	8⅝	25⅜	31¾
14	7	2⅝	3⅝	10⅜	13	36	18	6¾	9⅛	26⅞	33⅝

Single Keys

A	B	C	D	E	F	A	B	C	D	E	F
1	⅜	7/32	1 3/32	⅞	0.088	3	⅞	⅝	3¼	2⅝	0.309
1¼	7/16	5/16	1⅜	1 1/16	0.148	3½	1	11/16	3 13/16	3⅛	0.302
1½	½	11/32	1 21/32	1 5/16	0.144	4	1¼	13/16	4 5/16	3½	0.399
1¾	9/16	7/16	1 15/16	1½	0.203	4½	1⅜	⅞	4 13/16	3 15/16	0.454
2	⅝	½	2 3/16	1 11/16	0.262	5	1½	1	5⅜	4⅜	0.509
2½	¾	9/16	2¾	2 3/16	0.254	6	1¾	1¼	6½	5¼	0.619

The Kennedy Key. — The Kennedy or double key system is used in rolling mills and is adapted to the transmission of heavy loads, especially where the torque is intermittent and the direction of rotation periodically reversed. Each key is so located that a diagonal line passing through two corners of the key approximately intersects the shaft axis. The keys have a taper of $\frac{1}{8}$ inch per foot on the hub side and the sides fit closely between the shaft and hub. The keys are driven in from opposite sides of the hub. When the part to be keyed must be moved some distance along a straight shaft before attaching it, the hub is first bored to the exact size of the shaft, and then one side is rebored eccentrically with the original bore, the eccentricity being 1/32 inch, as indicated by the diagram accompanying the table of Kennedy keys. The keys are located on the eccentric side, and when a load is applied, they are in compression instead of being subjected to a shearing stress, as with the ordinary key. These keys should be made of 0.50 carbon steel, and whenever possible heads should be provided for driving and withdrawing.

Standard Square Shafts



Fixed and Sliding Fits — 0.80 Ratio				Heavily Loaded Sliding Fits — 0.73 Ratio			
Width *A*	Diam. *B*	Width *A*	Diam. *B*	Width *A*	Diam. *B*	Width *A*	Diam. *B*
1/4	5/16	1 1/2	1 7/8	1/4	11/32	1 1/2	2 1/16
3/8	15/32	1 3/4	2 3/16	3/8	33/64	1 3/4	2 3/8
1/2	5/8	2	2 1/2	1/2	11/16	2	2 3/4
5/8	25/32	2 1/4	2 13/16	5/8	27/32	2 1/4	3 1/16
3/4	15/16	2 1/2	3 1/8	3/4	1 1/32	2 1/2	3 7/16
7/8	1 1/16	2 3/4	3 3/8	7/8	1 3/16	2 3/4	3 3/4
1	1 1/4	3	3 3/4	1	1 3/8	3	4 1/8
1 1/8	1 3/8	3 1/2	4 3/8	1 1/8	1 9/16	3 1/2	4 3/4
1 1/4	1 5/8	4	5	1 1/4	1 11/16	4	5 1/2
1 3/8	1 11/16	.....	.....	1 3/8	1 7/8	....	....

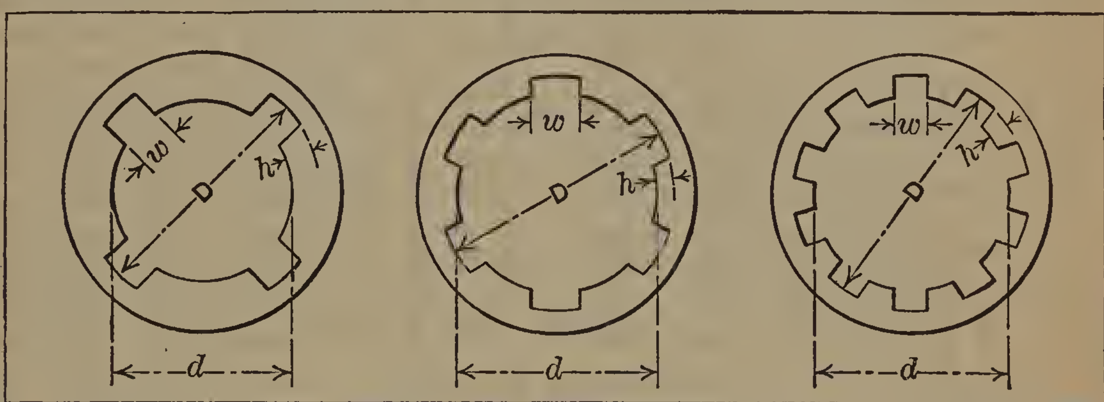
Square Shafts. — When square shafts are used in geared transmissions, etc., it is recommended that the holes be made to a standard size, for both sliding and fixed fits, and that the size of the shaft be varied to give the required fit. The relation between the diameter *B* (see engraving accompanying table, "Standard Square Shafts") across the corners and the width *A* of the square is in the ratio of 0.8 for fixed or permanent fits and also for non-loaded sliding fits, such as are employed for transmission gears; a ratio of 0.73 is used for fits that are required to slide while subjected to load. The table, "Proportions of Square Shafts and Fit Allowances," is based on the assumption that the parts containing the holes are not hardened; the shafts may be either soft or hardened.

Multiple Splines. — Multiple or "integral shaft" splines are used in automobile transmissions and the sliding-gear transmissions of machine tools, etc. The square shaft has been largely replaced by the multiple-spline shaft which is lighter for the same strength and provides a greater key-bearing area. The spline shafts may be produced quite rapidly by hobbing, the operation being the same, in principle, as hobbing spur gears; the grooves in the hubs or fittings which receive the shafts are broached. The S. A. E. standards for broached fittings include four-, six-, ten- and sixteen-spline fittings. The standard dimensions apply only to the soft broached hole and allowance must be made on the shaft to secure the required fit. The range of diameters of the different fittings will be given, with formulas for determining the sizes of the spline grooves.

Four-spline Fittings. — The maximum diameters D (see accompanying illustration) vary from 3/4 to 3 inches, inclusive, advancing by 1/8-inch increments up to 1 3/4 inch, and then by 1/4-inch increments up to 2 1/2 inches. For a permanent fitting, $d = 0.850\,D$; $h = 0.075\,D$; $w = 0.241\,D$. If the splined part is to slide when not under load, $d = 0.750\,D$; $h = 0.125\,D$; $w = 0.241\,D$. These formulas and those which follow for other classes of fittings give maximum dimensions. The minus tolerance for diameters D, d and for depth h is 0.001 inch for diameters up to and including 1 3/4 inch, and 0.002 for larger sizes. The tolerance for width w is 0.002 inch for diameters up to and including 1 3/4 inch, and 0.003 for larger sizes.

Six-spline Fittings. — The maximum diameters D vary from 3/4 to 3 inches inclusive, advancing by 1/8-inch increments up to 1 3/4 inch, and by 1/4-inch increments up to 2 1/2 inches. For a permanent fitting, $d = 0.900\,D$; $h = 0.050\,D$; $w = 0.250\,D$. For fittings that are to slide when not under load, $d = 0.850\,D$; $h = 0.075\,D$; $w = 0.250\,D$. If fittings are to slide when under load, $d = 0.800\,D$; $h = 0.100\,D$; $w = 0.250\,D$. All the tolerances are the same as previously given for four-spline fittings.

Ten-spline Fittings. — The maximum diameters D vary from 3/4 inch to 6 inches inclusive, advancing by 1/8-inch increments up to 1 3/4 inch; by 1/4-inch increments up to 2 1/2 inches and by 1/2-inch increments for larger sizes. For a permanent fit, $d = 0.910\,D$; $h = 0.045\,D$; $w = 0.156\,D$. For a fit to slide when not under load, $d = 0.860\,D$; $h = 0.070\,D$; $w = 0.156\,D$. For a fit to slide when under load, $d = 0.810\,D$; $h = 0.095\,D$; $w = 0.156\,D$. The tolerance for diameters D and d is 0.001 inch for sizes up to and including 1 3/4 inch, 0.002 inch for sizes up to and including 3 inches, and 0.003 inch for larger sizes. The tolerance for width w is 0.002 inch for diameters up to and including 1 3/4 inch and 0.003 inch for larger sizes.

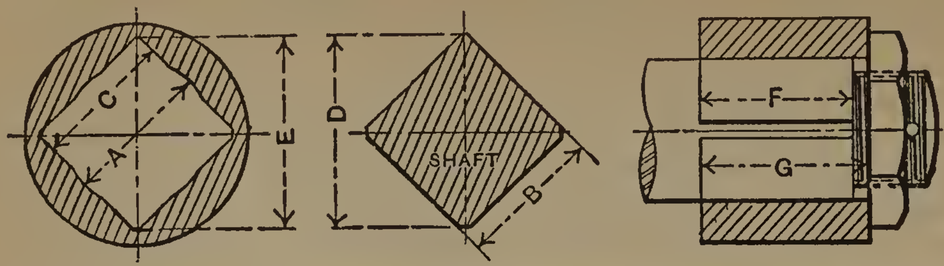


Sixteen-spline Fittings. — The maximum diameters D vary from 2 to 6 inches inclusive, advancing by 1/2-inch increments. The formulas for dimensions d and h are the same as for ten-spline fittings. The width $w = 0.098\,D$ for a permanent fit and when the fitting is to slide either under load or not under load. The tolerance for dimensions D, d and w is 0.003 inch.

Torque Capacity of Spline Fittings. — The torque capacity of spline fittings, per inch of bearing length at 1000 pounds pressure per square inch on the sides of the spline, may be determined by the following formula, in which T = torque capacity in inch-pounds per inch of length, N = number of splines, R = mean radius or radial distance from center of hole to center of spline, h = depth of spline:

$$T = 1000\,NRh$$

Proportions of Square Shafts and Fit Allowances



Nominal Diam.	Permanent Fit $\frac{B}{D} = 0.80$							Sliding Fit $\frac{B}{D} = 0.73$				
	A	*B*	*C*	*D*	*E*	*F*	*G*	*A*	*B*	*C*	*D*	*E*
1/4	0.193	0.189 0.188	0.187 0.186	0.250 0.245	0.260 0.252	5/16	3/8	0.257	0.248 0.247	0.250 0.249	0.344 0.339	0.354 0.346
3/8	0.290	0.283 0.282	0.281 0.280	0.375 0.370	0.385 0.377	7/16	1/2	0.386	0.373 0.372	0.375 0.374	0.516 0.511	0.526 0.518
1/2	0.386	0.377 0.376	0.375 0.374	0.500 0.495	0.510 0.502	11/16	3/4	33/64	0.498 0.497	0.500 0.499	0.687 0.682	0.697 0.689
5/8	33/64	0.502 0.501	0.500 0.499	0.625 0.620	0.635 0.627	11/16	3/4	41/64	0.623 0.622	0.625 0.624	0.844 0.839	0.854 0.846
3/4	37/64	0.564 0.563	0.562 0.561	0.750 0.745	0.760 0.752	15/16	1	49/64	0.748 0.747	0.750 0.749	1.031 1.026	1.051 1.036
7/8	45/64	0.689 0.688	0.687 0.686	0.875 0.870	0.885 0.877	1 1/8	1 1/4	29/32	0.873 0.872	0.875 0.874	1.187 1.182	1.207 1.192
1	27/32	0.815 0.814	0.812 0.811	1.000 0.995	1.020 1.005	1 3/8	1 1/2	1 1/32	0.998 0.997	1.000 0.999	1.375 1.370	1.395 1.380
1 1/8	29/32	0.878 0.877	0.875 0.874	1.125 1.120	1.145 1.130	1 3/8	1 1/2	1 5/32	1.123 1.122	1.125 1.124	1.562 1.557	1.582 1.567
1 1/4	1 1/32	1.103 1.102	1.000 0.999	1.250 1.245	1.270 1.255	1 3/8	1 1/2	1 9/32	1.248 1.247	1.250 1.249	1.687 1.682	1.707 1.692
1 3/8	1 5/32	1.128 1.127	1.125 1.124	1.375 1.370	1.395 1.380	1 7/8	2	1 27/64	1.373 1.372	1.375 1.374	1.875 1.870	1.895 1.880
1 1/2	1 5/32	1.128 1.127	1.125 1.124	1.500 1.495	1.520 1.505	1 7/8	2	1 35/64	1.498 1.497	1.500 1.499	2.062 2.057	2.082 2.067
1 3/4	1 27/64	1.378 1.377	1.375 1.374	1.750 1.745	1.770 1.755	2 1/8	2 1/4	1 13/16	1.748 1.747	1.750 1.749	2.375 2.370	2.395 2.380
2	1 35/64	1.504 1.503	1.500 1.498	2.000 1.995	2.020 2.005	2 7/8	3	2 1/16	1.997 1.996	2.000 1.998	2.750 2.745	2.770 2.755
2 1/4	1 13/16	1.754 1.753	1.750 1.748	2.250 2.245	2.270 2.255	2 7/8	3	2 5/16	2.247 2.246	2.250 2.248	3.062 3.057	3.082 3.067
2 1/2	2 1/16	2.004 2.003	2.000 1.998	2.500 2.495	2.520 2.505	3 3/8	3 1/2	2 37/64	2.497 2.496	2.500 2.498	3.437 3.432	3.457 3.442
2 3/4	2 5/16	2.254 2.253	2.250 2.248	2.750 2.745	2.770 2.755	3 3/8	3 1/2	2 55/64	2.747 2.746	2.750 2.748	3.750 3.745	3.770 3.755
3	2 37/64	2.504 2.503	2.500 2.498	3.000 2.995	3.020 3.005	3 7/8	4	3 3/32	2.997 2.996	3.000 2.998	4.125 4.120	4.145 4.130
3 1/2	2 55/64	2.754 2.753	2.750 2.748	3.500 3.495	3.520 3.505	4 3/8	4 1/2	3 39/64	3.497 3.496	3.500 3.498	4.750 4.745	4.770 4.755
4	3 23/64	3.254 3.253	3.250 3.248	4.000 3.995	4.020 4.005	5 3/8	5 1/2	4 1/8	3.997 3.996	4.000 3.998	5.500 5.495	5.520 5.505

CLUTCHES AND COUPLINGS

Positive Clutches. — When the driving and driven members of a clutch are connected by the engagement of interlocking teeth or projecting lugs, the clutch is said to be "positive" to distinguish it from the type in which the power is transmitted by frictional contact. The positive clutch is employed when a sudden starting action is not objectionable and when the inertia of the driven parts is relatively small. The various forms of positive clutches differ merely in the angle or shape of the engaging surfaces. The least positive form is one having planes of engagement which incline backward, with respect to the direction of motion. The tendency of such a clutch is to disengage under load, in which case it must be held in

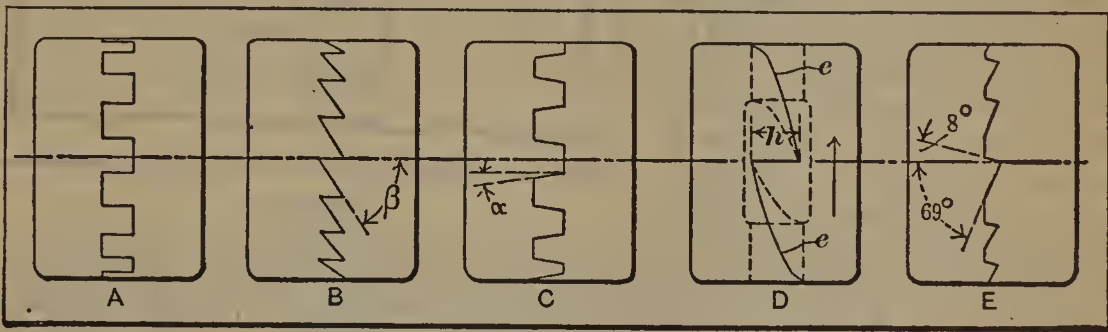


Fig. 1.— Types of Clutch Teeth

position by axial pressure. This pressure may be regulated to perform normal duty, permitting the clutch to slip and disengage when over-loaded. Positive clutches, with the engaging planes parallel to the axis of rotation, are held together to obviate the tendency to jar out of engagement, but they provide no safety feature against over-load. So-called "under-cut" clutches engage more tightly the heavier the load, and are designed to be disengaged only when free from load. The teeth of positive clutches are made in a variety of forms, a few of the more common styles being shown in Fig. 1. Clutch *A* is a straight-toothed type, and *B* has angular

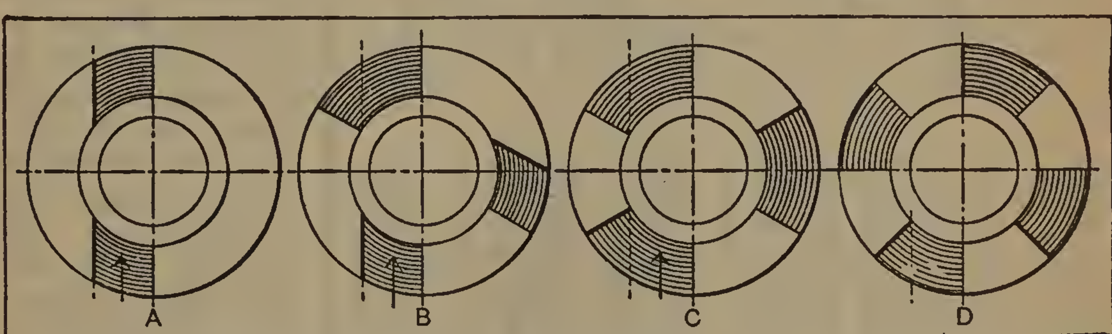


Fig. 2.— Diagrammatical View Showing Method of Cutting Clutch Teeth

or saw-shaped teeth. The driving member of the former can be rotated in either direction; the latter is adapted to the transmission of motion in one direction only, but is more readily engaged. The angle β of the cutter for a saw-tooth clutch *B* is ordinarily 60 degrees. Clutch *C* is similar to *A*, except that the sides of the teeth are inclined to facilitate engagement and disengagement. Teeth of this shape are sometimes used when a clutch is required to run in either direction without backlash. Angle α is varied to suit requirements and should not exceed 8 or 9 degrees. The straight-tooth clutch *A* is also modified to make the teeth engage more readily, by rounding the corners of the teeth at the top and bottom. Clutch *D* (commonly called a "spiral-jaw" clutch) differs from *B* in that the surfaces *e* are helicoidal. The driving member of this clutch can only transmit motion in one direction.

Clutches of this type are known as right- and left-hand, the former driving when turning to the right, as indicated by the arrow in the illustration. Clutch *E* is the form used on the back-shaft of the Brown & Sharpe automatic screw machines. The faces of the teeth are radial and incline at an angle of 8 degrees with the axis, so that the clutch can readily be disengaged. This type of clutch is easily operated, with little jar or noise. The 2-inch diameter size has 10 teeth. Height of working face, 1/8 inch.

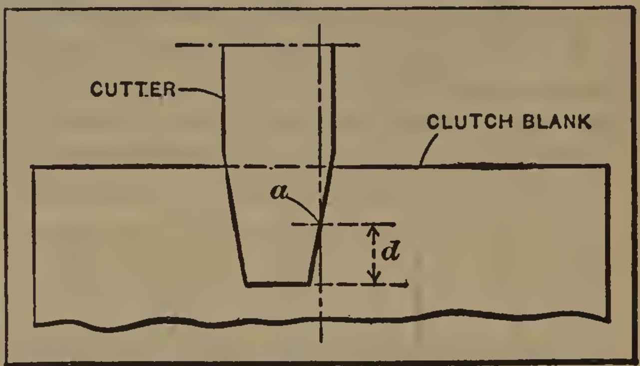


Fig. 3

Cutting Clutch Teeth. — A common method of cutting a straight-tooth clutch is indicated by the diagrams *A*, *B* and *C*, Fig. 2, which show the first, second and third cuts required for forming the three teeth. The work is held in the chuck of a dividing-head, the latter being set at right angles to the table. A plain milling cutter may be used (unless the corners of the teeth are rounded), the side of the cutter being set to exactly coincide with the center-line. When the number of teeth in the clutch is odd, the cut can be taken clear across the blank as shown, thus finishing the sides of two teeth with one passage of the cutter. When the number of teeth is even, as at *D*, it is necessary to mill all the teeth on one side and then set the cutter for finishing the opposite side. Therefore, clutches of this type commonly have an odd number of teeth. The maximum width of the cutter depends upon the width of the space at the narrow ends of the teeth If the cutter must be quite narrow in order to pass the narrow ends, some stock may be left in the tooth spaces, which must be removed by a separate cut. If the tooth is of the modified form shown at *C*, Fig. 1, the cutter should be set as indicated in Fig. 3; that is, so that a point *a* on the cutter at a radial distance *d* equal to one-half the depth of the clutch teeth lies in a radial plane. When it is important to eliminate all backlash, point *a* is sometimes located at a radial distance *d* equal to six-tenths of the depth of the tooth, in order to leave clearance spaces at the bottoms of the teeth; the two clutch members will then fit together tightly. Clutches of this type must be held in mesh.

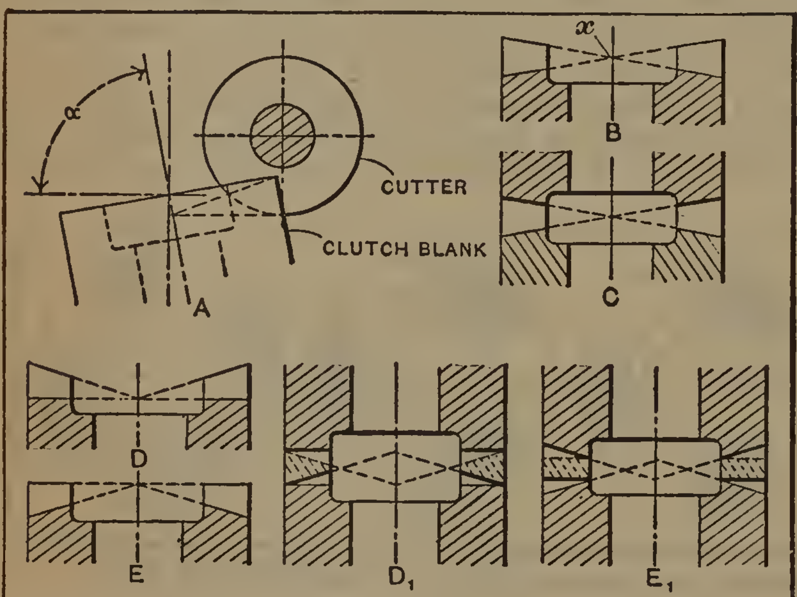


Fig. 4

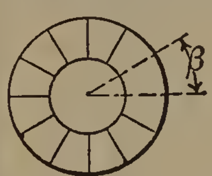
Cutting Saw-tooth Clutches. — When milling clutches having angular teeth as shown at *B*, Fig. 1, the axis of the clutch blank should be inclined a certain angle α from the vertical, as shown at *A* in Fig. 4. If the teeth were milled with the blank vertical, the tops of the teeth would incline towards the center as at D, whereas, if the blank were set to such an angle that the tops of the teeth were square with the axis, the bottoms would incline upwards as at E. In either case, the two clutch members would not mesh completely; the engagement of the teeth cut as shown at D and E would be as indicated at D_1 and E_1 respectively. As will be seen, when the outer points of the teeth at D_1 are at the bottom of the grooves in the opposite member, the inner ends are not together, the contact area being represented by the dotted lines. At E_1 the inner ends of the teeth strike first and spaces are left between the teeth around the outside of the clutch. To overcome this objectionable feature, the clutch teeth should be cut as indicated at B, or so that the bottoms and tops of the teeth have the same inclination, converging at a central point x. The teeth of both members will then engage across the entire width as shown at C. The angle α required for cutting a clutch as at B can be determined by the following formula in which α equals the required angle, and N, the number of teeth;

$$\cos \alpha = \tan \frac{180 \text{ deg.}}{N} \times \cot \text{ cutter angle.}$$

Expressing this formula as a rule: To determine the cosine of angle α (see diagram A, Fig. 4) find the tangent of the angle obtained by dividing 180 degrees by the number of teeth, and multiply this tangent by the cotangent of the cutter angle.

These angles for various numbers of teeth and for either a 60-, 70- or 80-degree cutter are given in the following table:

Angles for Setting Dividing-head when Cutting Clutches with Angular Cutters

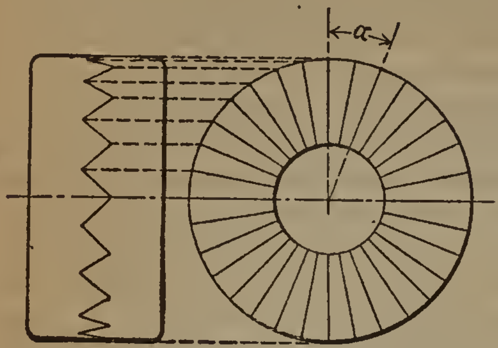


The cosine of the angle to which the milling machine index head is set equals the sine of angle β (see illustration) multiplied by the cotangent of the cutter angle, divided by the cosine of angle β plus 1.

No. of Teeth	Angle of Cutter 60°	70°	80°	No. of Teeth	Angle of Cutter 60°	70°	80°
5	....	....	82° 12′	18	84° 9′	86° 19′	88° 13′
6	....	77° 52′	84 9	19	84 30	86 31	88 19
7	73° 50′	79 54	85 10	20	84 46	86 42	88 24
8	76 10	81 20	85 48	21	85 1	86 51	88 29
9	77 52	82 23	86 19	22	85 13	87 0	88 33
10	79 12	83 13	86 43	23	85 27	87 8	88 37
11	80 14	83 54	87 4	24	85 38	87 15	88 40
12	81 6	84 24	87 18	25	85 49	87 22	88 43
13	81 49	84 51	87 30	26	85 59	87 28	88 46
14	82 26	85 12	87 42	27	86 8	87 34	88 50
15	82 57	85 34	87 51	28	86 16	87 39	88 52
16	83 24	85 51	87 59	29	86 24	87 44	88 54
17	83 48	86 6	88 7	30	86 31	87 48	88 56

Friction Clutches. — Clutches which transmit motion from the driving to the driven member by the friction between the engaging surfaces are built in many different designs, although practically all of them can be classified under four general types, namely, conical clutches; radially-expanding clutches; contracting-

Angles for Setting Index Head when Milling V-shaped Grooves



The cosine of the angle to which to set the index head equals the tangent of one-quarter of angle α (α = 360 ÷ number of teeth) multiplied by the cotangent of one-half the cutter angle. Thus:

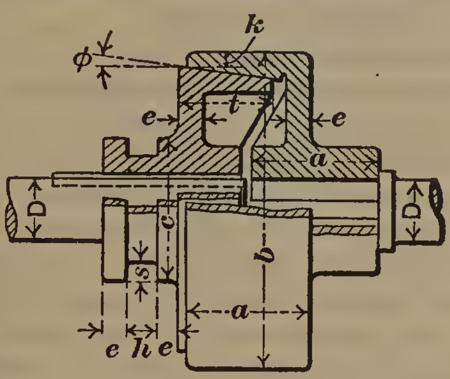
$$\cos \text{index-head angle} = \tan \frac{90}{N} \times \cot \frac{\text{cutter angle}}{2}$$

No. of Teeth	Included Angle of Cutter		No. of Teeth	Included Angle of Cutter	
	60°	90°		60°	90°
10	74° 5′	80° 53′	31	84° 57′	87° 5′
11	75 35	81 53	32	85 6	87 11
12	76 50	82 26	33	85 16	87 16
13	77 52	83 2	34	85 25	87 21
14	78 45	83 32	35	85 32	87 26
15	79 31	83 58	36	85 40	87 30
16	80 11	84 21	37	85 47	87 34
17	80 46	84 41	38	85 54	87 38
18	81 17	84 59	39	86 0	87 42
19	81 45	85 15	40	86 6	87 45
20	82 10	85 29	41	86 12	87 48
21	82 34	85 42	42	86 17	87 51
22	82 53	85 54	43	86 22	87 54
23	83 12	86 5	44	86 27	87 57
24	83 29	86 15	45	86 32	88 0
25	83 45	86 24	46	86 37	88 3
26	84 1	86 32	47	86 41	88 5
27	84 13	86 39	48	86 45	88 8
28	84 25	86 46	49	86 49	88 10
29	84 37	86 53	50	86 53	88 12
30	84 47	86 59	..	.. ..	.. ...

The angles given in the table above are applicable to the milling of V-shaped grooves in brackets, etc., which must have toothed surfaces to prevent the two members from turning relative to each other, except when unclamped for angular adjustment.

band clutches; and friction disk clutches in single and multiple types. There are many modifications of these general classes, some of which combine the features of different types. The proportions of various sizes of cone clutches are given in the table "Cast-iron Friction Clutches." The multi-cone friction clutch is a further development of the cone clutch. Instead of having a single cone-shaped surface, there is a series of concentric conical rings which engage annular grooves formed by corresponding rings on the opposite clutch member. The internal-expanding type is provided with shoes which are forced outward against an enclosing drum by the action of levers connecting with a collar free to slide along the shaft. The engaging shoes are commonly lined with wood to increase the coefficient of friction. The well-known Weston disk clutch is based on the principle of multiple-plane friction. It consists of a series of alternating plates or disks so arranged that one set engages with an outside cylindrical case and the other set with the shaft. When these plates are pressed together by spring pressure, or by other means, motion is transmitted from the driving to the driven members connected to the clutch. Some disk clutches have a few rather heavy or thick plates and others a relatively large number of thinner plates. Clutches of the latter type are common in automobile construction. One set of disks may be of soft steel and the other set of phosphor-bronze, or some other combination may be employed. For instance, disks are sometimes provided with cork inserts, or one set or series of disks may be faced with a special friction material such as asbestos-wire fabric, as in the case of "dry plate" clutches, the disks of which are not lubricated like the disks of a clutch having, for example, the steel and phosphor-bronze combination. It is common practice to hold the driving and driven members of friction clutches into engagement by means of spring pressure, although pneumatic and hydraulic pressure is sometimes employed.

Cast-iron Friction Clutches



For sizes not given below:

$a = 2\,D$
$b = 4$ to $8\,D$
$c = 2\frac{1}{4}\,D$
$t = 1\frac{1}{2}\,D$
$e = \frac{3}{8}\,D$
$h = \frac{1}{2}\,D$
$s = \frac{5}{16}\,D$, nearly
$k = \frac{1}{4}\,D$

Note:— The angle ϕ of the cone may be from 4 to 10 degrees.

D	a	b	c	t	e	h	s	k
1	2	4–8	2¼	1½	⅜	½	5/16	¼
1¼	2½	5–10	2⅞	1⅞	½	⅝	⅜	5/16
1½	3	6–12	3⅜	2¼	⅝	¾	½	⅜
1¾	3½	7–14	4	2⅝	⅝	⅞	⅝	7/16
2	4	8–16	4½	3	¾	1	⅝	½
2¼	4½	9–18	5	3⅜	⅞	1⅛	⅝	9/16
2½	5	10–20	5⅝	3¾	1	1¼	¾	⅝
2¾	5½	11–22	6¼	4⅛	1	1⅜	⅞	11/16
3	6	12–24	6¾	4½	1⅛	1½	⅞	¾
3¼	6½	13–26	7⅜	4⅞	1¼	1⅝	1	13/16
3½	7	14–28	7⅞	5¼	1⅜	1¾	1	⅞
3¾	7½	15–30	8½	5⅝	1⅜	1⅞	1¼	15/16
4	8	16–32	9	6	1½	2	1¼	1
4¼	8½	17–34	9½	6⅜	1⅝	2⅛	1⅜	1 1/16
4½	9	18–36	10¼	6¾	1¾	2¼	1⅜	1⅛
4¾	9½	19–38	10¾	7⅛	1¾	2⅜	1½	1 3/16
5	10	20–40	11¼	7½	1⅞	2½	1½	1¼
5¼	10½	21–42	11¾	7⅞	2	2⅝	1⅝	1 5/16
5½	11	22–44	12⅜	8¼	2	2¾	1¾	1⅜
5¾	11½	23–46	13	8⅝	2¼	2⅞	1¾	1 7/16
6	12	24–48	13½	9	2¼	3	1⅞	1½

Power Transmitting Capacity of Friction Clutches. — When selecting a clutch for a given class of service, it is advisable to consider any overloads that may be encountered and base the power transmitting capacity of the clutch upon such overloads. When the load varies or is subject to frequent release or engagement, the clutch capacity should be greater than the actual amount of power transmitted. If the power is derived from a gas or gasoline engine, the horsepower rating of the clutch should be 75 or 100 per cent greater than that of the engine.

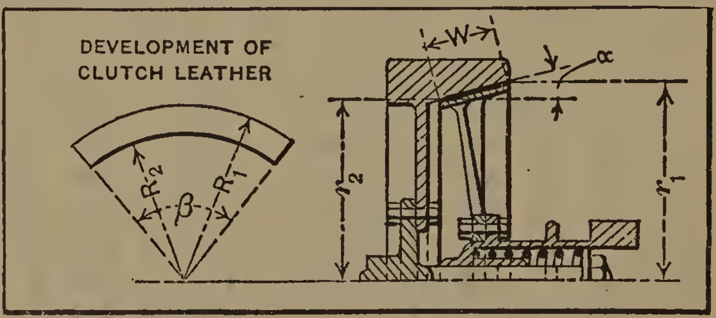
Formulas for Cone Clutches. — In cone clutch design, different formulas have been developed for determining the horsepower transmitted. These formulas, at first sight, do not seem to agree, there being a variation due to the fact that in some of the formulas the friction clutch surfaces are assumed to engage without slip, whereas, in others, some allowance is made for slip. The following formulas include both of these conditions:

H.P. = horsepower transmitted;
N = revolutions per minute;
r = mean radius of friction cone, in inches;
r_1 = large radius of friction cone, in inches;
r_2 = small radius of friction cone, in inches;
R_1 = outside radius of leather band, in inches;
R_2 = inside radius of leather band, in inches;
V = velocity of a point at distance r from the center, in feet per minute;
F = tangential force acting at radius r, in pounds;
P_n = total normal pressure between cone surfaces, in pounds;
P_s = spring pressure, in pounds;
α = angle of clutch surface with axis of shaft = 7 to 13 degrees;
β = included angle of clutch leather, when developed, in degrees;
f = coefficient of friction = 0.20 to 0.25 for greasy leather on iron;
p = allowable pressure per square inch of leather band = 7 to 8 pounds;
W = width of clutch leather, in inches.

$$R_1 = \frac{r_1}{\sin\alpha} \qquad R_2 = \frac{r_2}{\sin\alpha}$$

$$\beta = \sin\alpha \times 360 \qquad r = \frac{r_1 + r_2}{2}$$

$$V = \frac{2\pi r N}{12}$$



$$F = \frac{\text{H.P.} \times 33{,}000}{V} \qquad W = \frac{P_n}{2\pi r p} \qquad \text{H.P.} = \frac{P_n f r N}{63{,}025}$$

For engagement with some slip:

$$P_n = \frac{P_s}{\sin\alpha} \qquad P_s = \frac{\text{H.P.} \times 63{,}025 \sin\alpha}{frN}$$

For engagement without slip:

$$P_n = \frac{P_s}{\sin\alpha + f\cos\alpha} \qquad P_s = \frac{\text{H.P.} \times 63{,}025\,(\sin\alpha + f\cos\alpha)}{frN}$$

Angle of Cone. — If the angle of the conical surface of the cone type of clutch is too small, it may be difficult to release the clutch on account of the wedging effect, whereas, if the angle is too large, excessive spring pressure will be required to prevent slipping. The minimum angle for a leather-faced cone is about 8 or 9 degrees and the maximum angle about 13 degrees. An angle of 12½ degrees appears to be the most common and is generally considered good practice. These angles are given with relation to the clutch axis and are one-half the included angle.

Power Transmitted by Disk Clutches. — The approximate amount of power that a disk clutch will transmit may be determined from the following formula, in which H = horsepower transmitted by the clutch: μ = coefficient of friction; r = mean radius of engaging surfaces; F = axial force in pounds (spring pressure) holding disks in contact; N = number of frictional surfaces; S = speed of shaft in revolutions per minute:

$$H = \frac{\mu r F N S}{63{,}000}$$

Frictional Coefficients for Clutch Calculations. — While the frictional coefficients used by designers of clutches differ somewhat and depend upon variable factors, the following values may be used in clutch calculations: For greasy leather on cast iron about 0.20 or 0.25; leather on metal that is quite oily 0.15; metal and cork on oily metal 0.32; the same on dry metal 0.35; metal on dry metal 0.15; disk clutches having lubricated surfaces 0.10.

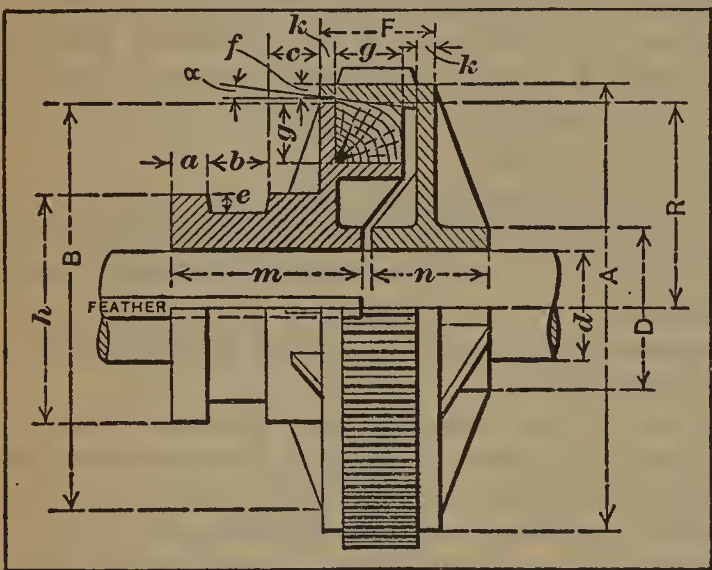
Magnetic Clutches. — Clutches of the magnetic type, like other electrical apparatus, are adapted to remote and automatic control. They are especially applicable for high-speed drives; for heavy duty; for use with motors that cannot start heavy loads; and for stopping machinery quickly, in which case a brake is used in combination with the clutch. The Cutler-Hammer magnetic clutch has a field or driving member and an armature or driven member. Each of these parts is carried by a flexible spring steel plate so that when current passes through the winding of the field, the armature is attracted to it and the friction surfaces come into engagement. The turning power of the clutch depends entirely upon the friction surfaces which are held together by magnetic attraction. Current is conducted to the magnetizing winding of the field through two collector rings and graphite brushes. These clutches are operated by direct current. The ratings of some of the different sizes are given in the accompanying table.

Magnetic Clutch Ratings *

Nominal Size, Inches	Maximum Speed, R.P.M.	Ratings Type H-30 Clutches			Ratings Type H-60 Clutches		
		Maximum Torque, Lbs. at 1 Ft. Radius	Safe H.P. at 100 R.P.M.	Current Consumption, Watts	Maximum Torque, Lbs. at 1 Ft. Radius	Safe H.P. at 100 R.P.M.	Current Consumption, Watts
10	2000	89	1.1	78	....	....	......
12	1680	154	2.0	93	....	....	......
14	1440	245	3.0	115	490	6	130
16	1260	366	4.5	133	732	9	160
20	1000	714	9.0	177	1,428	18	200
24	840	1233	15.5	260	2,466	31	247
28	725	1960	25.0	280	3,920	49	253
32	635	2920	37.0	315	5,840	74	250
40	500	5710	72.0	380	10,420	132	341
48	420	9860	124.0	460	19,720	250	400
60	340	....	......	......	38,600	485	645

* Cutler-Hammer Mfg. Co.

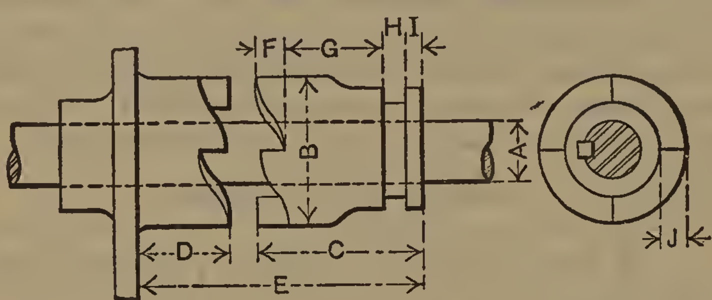
Friction Clutches for Hoisting Machinery. — In the operation of hoisting machinery, it is important that certain motions be engaged without stopping the motor. This is done by friction clutches in which one portion is keyed to the shaft while the other is keyed to the member to be driven. Arrangement is made to thrust the two portions of a clutch together and lock them in frictional engagement. The dimensions below give proportions for couplings for hoisting machinery.



$A = 4d$ to $8d$;
$f = 0.2d + 0.1$ inch;
$B = A - (2f + ¼$ inch);
$h = 2d + 1$ inch;
$c = 0.5d$;
$a = 0.3d + 0.3$ inch;
$b = 0.4d + 0.4$ inch;
$e = 0.3d + 0.1$ inch;
$k = 0.2d + 0.3$ inch;
$g = d$ to $1.4d$;
$D = 1.8d + 0.5$ inch;
$F = (1.4d$ to $1.8d)$ + 1 inch;
$m = 2d$;
$n = d$ to $1.5d$.

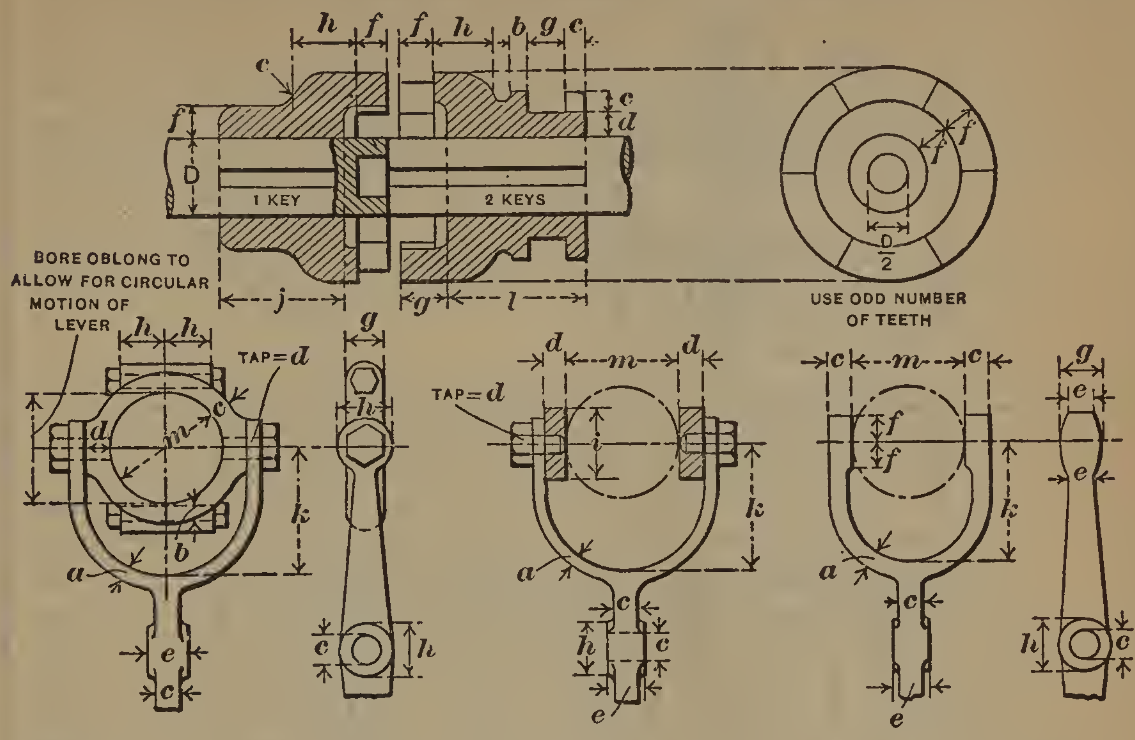
Authorities differ as to the angle α, making it all the way from 2 to 10 degrees. A good average angle for general purposes is 7 degrees.

Dimensions of Spiral-jaw Clutches



A	B	C	D	E	F	G	H	I	J
15/16	3	3 3/4	1 3/4	5 3/4	3/4	1 15/16	11/16	3/8	5/8
1 3/16	3 1/2	4 1/8	2 3/8	6 3/4	13/16	2 1/8	11/16	1/2	11/16
1 7/16	4	4 1/2	2 1/2	7 1/4	7/8	2 7/16	11/16	1/2	3/4
1 11/16	4 1/2	5	2 5/8	8	1	2 13/16	11/16	1/2	7/8
1 15/16	5	5 1/2	2 7/8	8 3/4	1	3 5/16	11/16	1/2	7/8
2 3/16	5 1/2	5 7/8	3	9 1/4	1	3 11/16	11/16	1/2	7/8
2 7/16	6	6 1/4	3	9 1/2	1 1/8	3 11/16	11/16	3/4	1
2 11/16	6 1/2	6 7/8	3 1/2	10 3/4	1 1/4	4 3/16	11/16	3/4	1 1/8
2 15/16	7	7 3/8	3 5/8	11 1/4	1 3/8	4 5/16	15/16	3/4	1 1/4
3 3/16	7 1/2	7 3/4	3 3/4	11 3/4	1 3/8	4 11/16	15/16	3/4	1 1/4
3 9/16	8	8 1/4	4 1/8	12 3/4	1 1/2	4 13/16	15/16	1	1 3/8
3 11/16	8 1/2	8 3/4	4 1/8	13 1/4	1 1/2	4 15/16	1 5/16	1	1 3/8
3 15/16	9	9 1/4	4 1/8	13 3/4	1 1/2	5 7/16	1 5/16	1	1 3/8
4 7/16	10	10 3/8	4 7/8	15 1/2	1 5/8	6 7/16	1 5/16	1	1 1/2
5 7/16	12	11 1/8	5	16 1/2	1 3/4	7 1/16	1 5/16	1	1 3/4

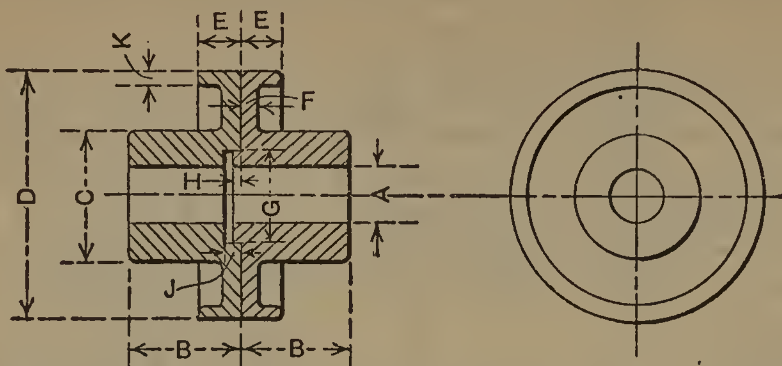
Proportions of Clutch Couplings and Shifting Levers



D	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
1	1/4	5/16	5/16	3/8	7/16	5/8	3/4	7/8	1 1/2	1 7/8	2 3/8	2 1/2	1 15/16
1 1/4	5/16	5/16	3/8	7/16	1/2	3/4	7/8	1	1 5/8	2 1/8	2 3/4	3	2 3/16
1 1/2	3/8	3/8	7/16	1/2	5/8	3/4	7/8	1 1/8	1 7/8	2 1/2	3	3 1/2	2 1/2
1 3/4	3/8	1/2	1/2	9/16	5/8	7/8	1	1 1/4	2	2 3/4	3 1/2	3 3/4	2 7/8
2	3/8	1/2	5/8	5/8	3/4	7/8	1 1/8	1 3/8	2 1/4	3	3 3/4	4	3 1/4
2 1/4	7/16	1/2	5/8	3/4	3/4	1	1 1/4	1 1/2	2 1/2	3 1/4	4	4 1/2	3 5/8
2 1/2	7/16	9/16	5/8	3/4	7/8	1 1/8	1 3/8	1 5/8	2 3/4	3 1/2	4 1/2	5	4
2 3/4	1/2	5/8	3/4	7/8	1	1 1/8	1 3/8	1 3/4	2 7/8	3 3/4	4 3/4	5 1/4	4 3/8
3	1/2	5/8	3/4	7/8	1	1 1/4	1 1/2	1 7/8	3 1/8	4	5	5 1/2	4 3/4
3 1/4	1/2	3/4	3/4	1	1	1 3/8	1 5/8	2	3 1/4	4 1/4	5 1/2	6	5 1/8
3 1/2	5/8	3/4	7/8	1	1 1/8	1 1/2	1 3/4	2 1/8	3 1/2	4 1/2	5 3/4	6 1/2	5 1/2
3 3/4	5/8	3/4	7/8	1 1/8	1 1/4	1 1/2	1 3/4	2 1/4	3 3/4	4 3/4	6	7	5 3/4
4	5/8	7/8	1	1 1/8	1 1/4	1 5/8	1 7/8	2 3/8	4	5	6 1/2	7	6 1/4
4 1/4	3/4	7/8	1	1 1/4	1 3/8	1 3/4	2	2 1/2	4	5 1/4	6 3/4	7 1/2	6 1/2
4 1/2	3/4	7/8	1	1 1/4	1 3/8	1 3/4	2	2 5/8	4 1/2	5 1/2	7	8	7
4 3/4	3/4	7/8	1 1/8	1 1/4	1 1/2	1 7/8	2 1/4	2 3/4	4 1/2	5 3/4	7 1/2	8	7 1/4
5	3/4	1	1 1/8	1 1/4	1 1/2	2	2 1/4	2 7/8	4 1/2	6	7 3/4	8 1/2	7 5/8
5 1/2	7/8	1	1 1/4	1 1/2	1 5/8	2 1/8	2 1/2	3 1/8	5	6 3/4	8 1/2	9 1/2	8 1/2
6	7/8	1	1 3/8	1 1/2	1 7/8	2 1/4	2 3/4	3 3/8	5 1/2	7 1/4	9	10	9
6 1/2	1	1 1/4	1 1/2	1 3/4	2	2 1/2	3	3 5/8	6	7 3/4	9 3/4	11	10
7	1	1 1/4	1 1/2	1 3/4	2 1/8	2 5/8	3	3 7/8	6 1/2	8 1/4	10 1/2	12	10 1/2

Clutches with Cork Inserts. — The friction surfaces of some clutches have cork inserts, owing to the fact that cork has a high coefficient of friction — probably double that of wood or leather on iron. As a rule, the cork, which has previously been boiled and thereby softened, is forced into suitable cavities formed in one of the metallic friction surfaces. When so inserted, it normally protrudes above

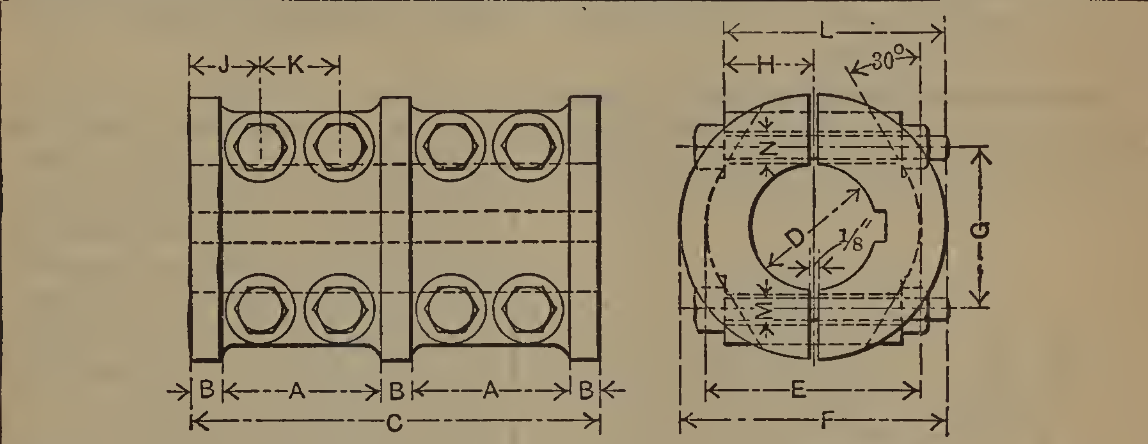
Safety Flange Couplings



A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*J*	*K*	Bolts	
										No.	Diam.
1	1¾	2¼	4	11/16	5/16	1½	¼	9/32	¼	5	⅜
1¼	2 3/16	2¾	5	13/16	⅜	1⅞	¼	9/32	¼	5	7/16
1½	2⅝	3⅜	6	15/16	7/16	2¼	¼	9/32	¼	5	½
1¾	3 1/16	4	7	1 1/16	½	2⅝	¼	9/32	¼	5	9/16
2	3½	4½	8	1 3/16	9/16	3	¼	9/32	5/16	5	⅝
2¼	3 15/16	5⅛	9	1 5/16	⅝	3⅜	¼	9/32	5/16	5	11/16
2½	4⅜	5⅝	10	1 7/16	11/16	3¾	¼	9/32	5/16	5	¾
2¾	4 13/16	6¼	11	1 9/16	¾	4⅛	¼	9/32	5/16	5	13/16
3	5¼	6¾	12	1 11/16	13/16	4½	¼	9/32	⅜	5	⅞
3¼	5 11/16	7⅜	13	1 13/16	⅞	4⅞	¼	9/32	⅜	5	15/16
3½	6⅛	8	14	1 15/16	15/16	5¼	¼	9/32	⅜	5	1
3¾	6 9/16	8½	15	2 1/16	1	5⅝	¼	9/32	⅜	5	1 1/16
4	7	9	16	2¼	1⅛	6	¼	9/32	7/16	5	1⅛
4½	7⅞	10¼	18	2½	1¼	6¾	¼	9/32	7/16	5	1¼
5	8¾	11¼	20	2¾	1⅜	7½	¼	9/32	7/16	5	1⅜
5½	8¾	11¼	20	2¾	1⅜	7½	¼	9/32	7/16	5	1⅜
6	10½	12⅜	22	2 15/16	1½	8¼	5/16	11/32	½	5	1 7/16
6½	11⅜	13½	24	3⅛	1⅝	9	5/16	11/32	½	5	1½
7	12¼	14⅝	26	3¼	1¾	9¾	5/16	11/32	9/16	6	1½
7½	13⅛	15¾	28	3 7/16	1⅞	10½	5/16	11/32	9/16	6	1 9/16
8	14	16⅞	28	3½	2	10⅞	5/16	11/32	⅝	7	1½
8½	14⅞	18	30	3 11/16	2⅛	11¼	5/16	11/32	⅝	7	1 9/16
9	15¾	19⅛	31	3¾	2¼	11⅝	5/16	11/32	11/16	8	1½
9½	16⅝	20¼	32	3 15/16	2⅜	12	5/16	11/32	11/16	8	1 9/16
10	17½	21⅜	34	4⅛	2½	12¾	5/16	11/32	¾	8	1⅝
10½	18⅜	22½	35	4¼	2⅝	13⅛	5/16	11/32	¾	10	1⅝
11	19¼	23⅝	36	4 7/16	2¾	13½	5/16	11/32	⅞	10	1 11/16
11½	20⅛	24¾	37	4⅝	2⅞	13⅞	5/16	11/32	⅞	10	1¾
12	21	25⅞	38	4 13/16	3	14¼	5/16	11/32	1	10	1 13/16

the surrounding surface and engages first. If sufficient pressure is applied to the clutch, the cork is pressed down flush with the metal surface and acts with it in carrying the load. Two forms of cork are used, one being cork in its natural condition and the other prepared as follows: Small pieces are compressed into sheets and blocks of any desired shape, under great pressure and while subjected to enough heat to cause the natural gums of the cork to exude and act as a binder. This prepared cork is more enduring than the natural; it is stronger and firmer, and yet possesses considerable elasticity. It is expensive, however, and for that reason has not been widely used. The coefficient of friction with cork-insert surfaces averages about 0.34, whereas the average coefficient of friction for cast iron on cast iron is about 0.16, and for bronze on cast iron, about 0.14. It is claimed that the coefficient of friction of cork is not lessened very much by lubrication. Cork is also affected very little by moisture.

Clamp Couplings



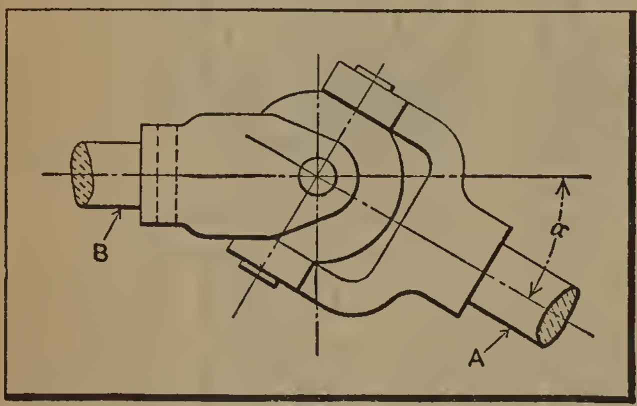
Bore D	A	B	C	E	F	G	H	J	K	L	M	N	Key
1 1/4	2 17/32	5/16	6	2 1/8	3	1 7/8	13/16	15/16	1 5/16	2 5/8	1/2	9/16	3/8×1/4
1 1/2	2 17/32	5/16	6	2 1/2	3 3/8	2 1/8	7/8	15/16	1 5/16	2 3/4	1/2	9/16	1/2×11/32
1 3/4	3 11/32	7/16	8	2 7/8	4	2 1/2	1 1/16	1 5/16	1 5/8	3	5/8	11/16	1/2×11/32
2	3 11/32	7/16	8	3 3/8	4 3/8	2 3/4	1 3/16	1 5/16	1 5/8	3 1/2	5/8	11/16	5/8×13/32
2 1/4	3 21/32	9/16	9	3 7/8	5 1/4	3 1/8	1 5/16	1 7/16	1 7/8	3 3/4	3/4	13/16	5/8×13/32
2 1/2	4 1/16	5/8	10	4 1/4	5 9/16	3 3/8	1 7/16	1 9/16	2 1/8	4	3/4	13/16	5/8×13/32
2 3/4	4 15/32	11/16	11	4 5/8	6	3 3/4	1 9/16	1 3/4	2 5/16	4 1/2	7/8	15/16	3/4×1/2
3	4 7/8	3/4	12	5	6 5/16	4	1 11/16	1 15/16	2 1/2	4 3/4	7/8	15/16	3/4×1/2
3 1/4	5 1/32	13/16	12 1/2	5 7/16	6 3/4	4 1/4	2 1/2	2 1/16	2 17/32	6 1/4	7/8	15/16	7/8×19/32
3 1/2	5 3/16	7/8	13	5 11/16	7	4 1/2	2 5/8	2 1/8	2 11/16	6 1/2	7/8	15/16	7/8×19/32
3 3/4	5 7/16	7/8	13 1/2	6 1/4	7 3/4	4 7/8	2 11/16	2 5/16	2 11/16	7	1	1 1/16	1 ×5/8
4	5 11/16	7/8	14	6 3/4	8 1/4	5 1/4	2 7/8	2 3/8	2 3/4	7 1/4	1	1 1/16	1 ×5/8
4 1/4	5 3/4	1	14 1/2	7 1/8	8 3/4	5 1/2	3	2 7/16	2 13/16	7 1/2	1	1 1/16	1 1/8×3/4
4 1/2	6	1	15	7 1/2	9 1/4	5 3/4	3 1/8	2 9/16	2 7/8	7 3/4	1	1 1/16	1 1/8×3/4
4 3/4	6 1/16	1 1/8	15 1/2	8	9 3/4	6 1/8	3 1/4	2 11/16	2 7/8	8	1 1/8	1 3/16	1 1/4×3/4
5	6 5/16	1 1/8	16	8 3/8	10 1/4	6 3/8	3 3/8	2 3/4	3 1/16	8 3/4	1 1/8	1 3/16	1 1/4×3/4

The Universal Joint. — This form of coupling, originally known as Hooke's coupling, is used for connecting two shafts the axes of which are not in line with each other, but which merely intersect at a point. There are many different designs of universal joints or couplings, which are based on the principle embodied in the original design. One well-known type is shown by the accompanying diagram. A fork- or U-shaped member is attached to each shaft and is pivoted to a connecting piece. The axes of the four bearings should lie in the same plane as shown by the diagram, although some universal joints are so made that two of the bearings are offset and not in the same plane as the other two. With this construction, which has only a very limited use, only one of the connected shafts can be held rigidly and permit rotary motion.

As a rule, a universal joint does not work well if the angle α (see illustration) is more than 45 degrees, and the angle should preferably be limited to about 20 degrees or 25 degrees, excepting when the speed of rotation is slow and little power is transmitted.

Variation in Angular Velocity of Driven Shaft. — Owing to the angularity between two shafts connected by a universal joint, there is a variation in the angular velocity of one shaft during a single revolution, and because of this, the use of universal couplings is sometimes prohibited. Thus, the angular velocity of the driven shaft will not be the same at all points of the revolution as the angular velocity of the driving shaft. In other words, if the driving shaft moves with a uniform motion, then the driven shaft will have a variable motion and, therefore, the universal joint should not be used when absolute uniformity of motion is essential for the driven shaft.

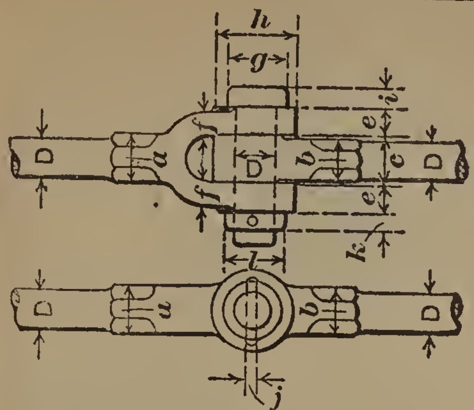
Determining Maximum and Minimum Velocities. — If shaft A (see diagram) runs at a constant speed, shaft B revolves at maximum speed when shaft A occupies the position shown in the illustration, and the minimum speed of shaft B occurs when the fork of the driving shaft A has turned 90 degrees from the position illustrated. The maximum speed of the driven shaft may be obtained by multiplying the speed of the driving shaft by the secant of angle α. The minimum speed of the driven shaft equals the speed of the driver multiplied by cosine α. Thus, if the driver rotates at a constant speed of 100 revolutions per minute and the shaft angle is 25 degrees, the maximum speed of the driven shaft is at a rate equal to $1.1034 \times 100 = 110.34$ R.P.M. The minimum speed rate equals $0.9063 \times 100 = 90.63$; hence, the extreme variation equals $110.34 - 90.63 = 19.71$ R.P.M.



Use of Intermediate Shaft between Two Universal Joints. — The lack of uniformity in the speed of the driven shaft resulting from the use of a universal coupling, as previously explained, is objectionable for some forms of mechanisms. This variation may be avoided if the two shafts are connected with an intermediate shaft and two universal joints, provided the latter are properly arranged or located. Two conditions are necessary to obtain a constant speed ratio between the driving and driven shafts. First, the shafts must make the same angle with the intermediate shaft; second, the universal joint forks (assuming that the fork design is employed) on the intermediate shaft must be placed relatively so that when the plane of the fork at the left end coincides with the center lines of the intermediate shaft and the shaft attached to the left-hand coupling, the plane of the right-hand fork must also coincide with the center lines of the intermediate shaft and the shaft attached to the right-hand coupling; therefore the driving and the driven shafts may be placed in a variety of positions. One of the most common arrangements, however, is with the driving and driven shafts parallel. In this case, the forks on the intermediate shafts should be placed in the same plane.

This intermediate connecting shaft is frequently made telescoping, and then the driving and driven shafts can be moved independently of each other within certain limits in longitudinal and lateral directions. The telescoping intermediate shaft consists of a rod which enters a sleeve and is provided with a suitable spline, to prevent rotation between the rod and sleeve and permit a sliding movement. This arrangement is applied to various machine tools.

Proportions of Knuckle Joints



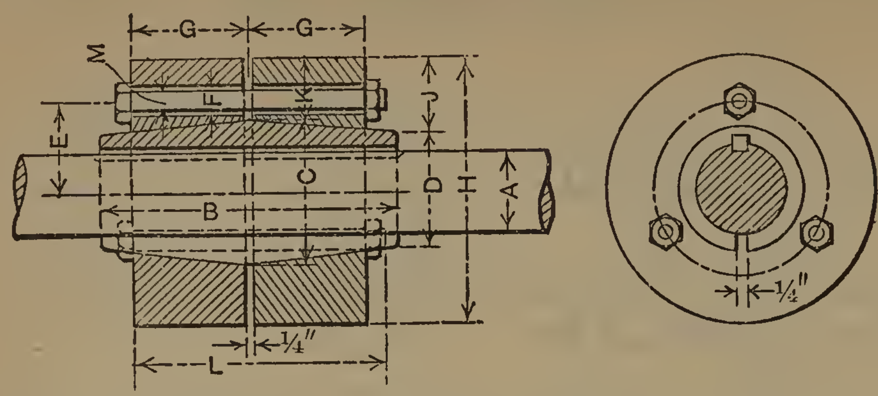
For sizes not given below:

$a = 1.2\,D$	$h = 2\,D$
$b = 1.1\,D$	$i = 0.5\,D$
$c = 1.2\,D$	$j = 0.25\,D$
$e = 0.75\,D$	$k = 0.5\,D$
$f = 0.6\,D$	$l = 1.5\,D$
$g = 1.5\,D$	

D	*a*	*b*	*c*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*
½	⅝	9/16	⅝	⅜	5/16	¾	1	¼	⅛	¼	¾
¾	⅞	¾	⅞	9/16	7/16	1⅛	1½	⅜	3/16	⅜	1⅛
1	1¼	1⅛	1¼	¾	⅝	1½	2	½	¼	½	1½
1¼	1½	1⅜	1½	15/16	¾	1⅞	2½	⅝	5/16	⅝	1⅞
1½	1¾	1⅝	1¾	1⅛	⅞	2¼	3	¾	⅜	¾	2¼
1¾	2⅛	2	2⅛	1 5/16	1 1/16	2⅝	3½	⅞	7/16	⅞	2⅝
2	2⅜	2¼	2⅜	1½	1 3/16	3	4	1	½	1	3
2¼	2¾	2½	2¾	1 11/16	1⅜	3⅜	4½	1⅛	9/16	1⅛	3⅜
2½	3	2¾	3	1⅞	1½	3¾	5	1¼	⅝	1¼	3¾
2¾	3¼	3	3¼	2 1/16	1⅝	4⅛	5½	1⅜	11/16	1⅜	4⅛
3	3⅝	3¼	3⅝	2¼	1 13/16	4½	6	1½	¾	1½	4½
3¼	4	3⅝	4	2 7/16	2	4⅞	6½	1⅝	13/16	1⅝	4⅞
3½	4¼	3⅞	4¼	2⅝	2⅛	5¼	7	1¾	⅞	1¾	5¼
3¾	4½	4⅛	4½	2 13/16	2¼	5⅝	7½	1⅞	15/16	1⅞	5⅝
4	4¾	4⅜	4¾	3	2⅜	6	8	2	1	2	6
4¼	5⅛	4¾	5⅛	3 3/16	2 9/16	6⅜	8½	2⅛	1 1/16	2⅛	6⅜
4½	5½	5	5½	3⅜	2¾	6¾	9	2¼	1⅛	2¼	6¾
4¾	5¾	5¼	5¾	3 9/16	2⅞	7⅛	9½	2⅜	1 3/16	2⅜	7⅛
5	6	5½	6	3¾	3	7½	10	2½	1¼	2½	7½

Flexible Couplings. — Flexible couplings are used mostly for coupling together electrical machinery or for coupling electrical to other machinery. The general types of flexible couplings include the leather link coupling, the endless belt coupling, and the rubber buffer coupling. The leather link coupling consists of two iron castings with flanges which are connected by leather links and bolts. The bolts are generally six in number and each alternate bolt is tightly fitted in the flange of one casting, but has considerable play in the other. The leather links are placed around pairs of adjacent bolts and provide a slight flexibility for the drive. This coupling is adapted for shafts up to 3½ inches in diameter. The endless belt flexible coupling is adapted for shafts of larger diameter. It consists of two steel rings, one outer and one inner, in which slots are formed and through which two endless leather belts are interwoven. The rubber buffer coupling is formed of two disks; the driving side transmits motion to the driven side by means of studs, bolts or interlocking arms surrounded by heavy rubber bushings which give the necessary flexibility. The "mill type" flexible coupling, which is used chiefly in steel mills, is formed of three steel castings and is adapted to severe service.

Double-cone Clamping Coupling



A	B	C	D	E	F	G	H	J	K	L	M	No. of Bolts	No. of Keys
1 7/16	5¼	2¾	2⅛	1⅝	⅝	2⅛	4¾	1⅛	1	5	½	3	1
1 15/16	7	3½	2⅞	2⅛	⅝	2¾	6¼	1⅛	1⅜	6¼	½	3	1
2 7/16	8¾	4 5/16	3⅝	3	¾	3½	7 13/16	1⅞	1¾	7⅞	⅝	3	1
3	10½	5½	4 3/32	3½	¾	4 3/16	9	2¼	2	9½	⅝	3	1
3½	12¼	7	5⅜	4⅜	⅞	5 1/16	11¼	2⅝	2⅛	11¼	¾	4	1
4	14	7	5½	4¾	⅞	5½	12	3¾	2½	12	¾	4	1
4½	15½	8	6⅞	5¼	⅞	6¾	13½	3¾	2¾	14½	¾	4	1
5	17	9	7¼	5¾	⅞	7	15	3¾	3	15¼	¾	4	1
5½	17½	9½	7¾	6¼	1	7	15½	3¾	3	15¼	⅞	4	1
6	18	10	8¼	6¾	1	7	16	3¾	3	15¼	⅞	4	2

FRICTION BRAKES

Formulas for Band Brakes. — In any band brake, such as shown in Fig. 1, in the tabulation of the formulas, where the brake wheel rotates in a clockwise direction, the tension in that part of the band marked x equals $P\,\dfrac{1}{e^{\mu\theta} - 1}$

The tension in that part marked y equals $P\,\dfrac{e^{\mu\theta}}{e^{\mu\theta} - 1}$

P = tangential force in pounds at rim of brake wheel;
e = base of natural logarithms = 2.71828;
μ = coefficient of friction between the brake band and the brake wheel;
θ = angle of contact of the brake band with the brake wheel expressed in radians (one radian $= \dfrac{180 \text{ deg.}}{\pi} = 57.296$ degrees).

For simplicity in the formulas presented, the tensions at x and y (Fig. 1) are denoted by T_1 and T_2 respectively, for clockwise rotation. When the direction of the rotation is reversed, the tension in x equals T_2, and the tension in y equals T_1, which is the reverse of the tension in the clockwise direction.

The value of the expression $e^{\mu\theta}$ in these formulas may be most easily solved by means of logarithms. The value of $e^{\mu\theta}$ is found by multiplying the logarithm of e by the product of the numerical values of μ and θ, and finding the number whose logarithm is equal to the result of this multiplication. The procedure may be best illustrated by an example.

Formulas for Simple and Differential Band Brakes

F = force in pounds at end of brake handle; P = tangential force in pounds at rim of brake wheel; e = base of natural logarithms = 2.71828; μ = coefficient of friction between the brake band and the brake wheel; θ = angle of contact of the brake band with the brake wheel, expressed in radians (one radian = $\frac{180°}{\pi}$ = 57.296 degrees).

$$T_1 = P\frac{1}{e^{\mu\theta}-1} \qquad T_2 = P\frac{e^{\mu\theta}}{e^{\mu\theta}-1}$$

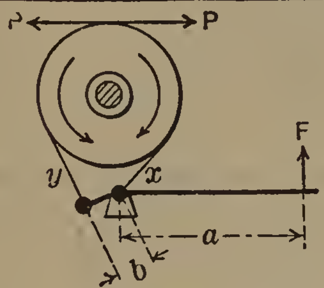


Fig. 1

Simple band brake.
For clockwise rotation:

$$F = \frac{bT_2}{a} = \frac{Pb}{a}\left(\frac{e^{\mu\theta}}{e^{\mu\theta}-1}\right)$$

For counter clockwise rotation:

$$F = \frac{bT_1}{a} = \frac{Pb}{a}\left(\frac{1}{e^{\mu\theta}-1}\right)$$

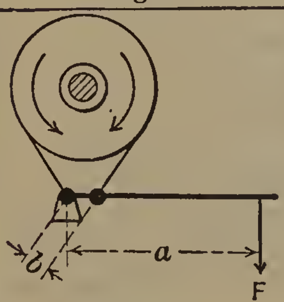


Fig. 2

Simple band brake.
For clockwise rotation:

$$F = \frac{bT_1}{a} = \frac{Pb}{a}\left(\frac{1}{e^{\mu\theta}-1}\right)$$

For counter clockwise rotation:

$$F = \frac{bT_2}{a} = \frac{Pb}{a}\left(\frac{e^{\mu\theta}}{e^{\mu\theta}-1}\right)$$

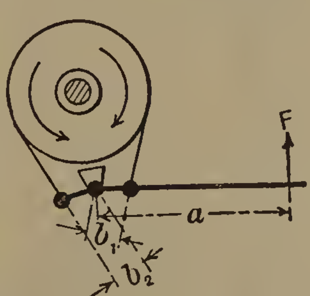


Fig. 3

Differential band brake.
For clockwise rotation:

$$F = \frac{b_2T_2 - b_1T_1}{a} = \frac{P}{a}\left(\frac{b_2e^{\mu\theta}-b_1}{e^{\mu\theta}-1}\right)$$

For counter clockwise rotation:

$$F = \frac{b_2T_1 - b_1T_2}{a} = \frac{P}{a}\left(\frac{b_2-b_1e^{\mu\theta}}{e^{\mu\theta}-1}\right)$$

In this case, if b_2 is equal to, or less than, $b_1e^{\mu\theta}$, the force F will be 0 or negative and the band brake works automatically.

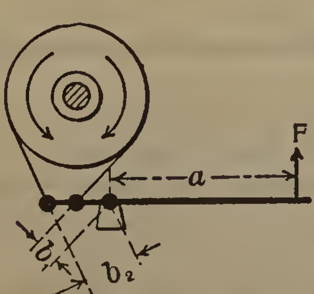


Fig. 4

Differential band brake.
For clockwise rotation:

$$F = \frac{b_2T_2 + b_1T_1}{a} = \frac{P}{a}\left(\frac{b_2e^{\mu\theta}+b_1}{e^{\mu\theta}-1}\right)$$

For counter clockwise rotation:

$$F = \frac{b_1T_2 + b_2T_1}{a} = \frac{P}{a}\left(\frac{b_1e^{\mu\theta}+b_2}{e^{\mu\theta}-1}\right)$$

If $b_2 = b_1$, both of the above formulas reduce to $F = \frac{Pb_1}{a}\left(\frac{e^{\mu\theta}+1}{e^{\mu\theta}-1}\right)$. In this case, the same force F is required for rotation in either direction.

In a band brake of the type in Fig. 1, dimension $a = 24$ inches, and $b = 4$ inches; force $P = 100$ pounds; coefficient $\mu = 0.2$, and angle of contact = 240 degrees, or

$$\theta = \frac{240}{180} \times \pi = 4.18.$$

The rotation is clockwise. Find force F required.

$$F = \frac{Pb}{a}\left(\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right) = \frac{100 \times 4}{24}\left(\frac{2.71828^{0.2\times 4.18}}{2.71828^{0.2\times 4.18} - 1}\right)$$

$$= \frac{400}{24} \times \frac{2.71828^{0.836}}{2.71828^{0.836} - 1} = 16.66 \times \frac{2.31}{2.31 - 1} = 29.4.$$

The calculations for determining the value of $e^{\mu\theta}$ are rather cumbersome, and the accompanying table will save calculations.

Table of Values of $e^{\mu\theta}$

Proportion of Contact to Whole Circumference, $\frac{\theta}{2\pi}$	Steel Band on Cast Iron, $\mu = 0.18$	Leather Belt on			
		Wood	Cast Iron		
		Slightly Greasy; $\mu = 0.47$	Very Greasy; $\mu = 0.12$	Slightly Greasy; $\mu = 0.28$	Damp; $\mu = 0.38$
0.1	1.12	1.34	1.01	1.19	1.27
0.2	1.25	1.81	1.16	1.42	1.61
0.3	1.40	2.43	1.25	1.69	2.05
0.4	1.57	3.26	1.35	2.02	2.60
0.425	1.62	3.51	1.38	2.11	2.76
0.45	1.66	3.78	1.40	2.21	2.93
0.475	1.71	4.07	1.43	2.31	3.11
0.5	1.76	4.38	1.46	2.41	3.30
0.525	1.81	4.71	1.49	2.52	3.50
0.55	1.86	5.63	1.51	2.63	3.72
0.6	1.97	5.88	1.57	2.81	4.19
0.7	2.21	7.90	1.66	3.43	5.32
0.8	2.47	10.60	1.83	4.09	6.75
0.9	2.77	14.30	1.97	4.87	8.57
1.0	3.10	19.20	2.12	5.81	10.90

Coefficient of Friction in Brakes. — The coefficients of friction that may be assumed for friction brake calculations are as follows: Iron on iron, 0.25 to 0.3; leather on iron, 0.3; cork on iron, 0.35. Values somewhat lower than these should be assumed when the velocities exceed 400 feet per minute at the beginning of the braking operation.

For brakes where wooden brake blocks are used on iron drums, poplar has proved the best brake-block material. The best material for the brake drum is wrought iron. Poplar gives a high coefficient of friction, and is little affected by oil. The average coefficient of friction for poplar brake blocks and wrought-iron drums is

0.6; for poplar on cast iron, 0.35; for oak on wrought iron, 0.5; for oak on cast iron, 0.3; for beech on wrought iron, 0.5; for beech on cast iron, 0.3; for elm on wrought iron, 0.6; and for elm on cast iron, 0.35. The objection to elm is that the friction decreases rapidly if the friction surfaces are oily. The coefficient of friction for elm and wrought iron, if oily, is less than 0.4.

Formulas for Block Brakes

F = force in pounds at end of brake handle;
P = tangential force in pounds at rim of brake wheel;
μ = coefficient of friction between the brake block and brake wheel.

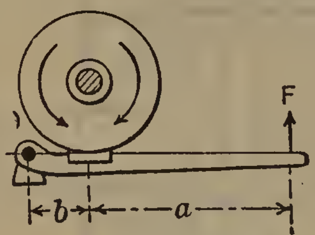


Fig. 1

Block brake.
For rotation in either direction:

$$F = P\frac{b}{a+b} \times \frac{1}{\mu} = \frac{Pb}{a+b}\left(\frac{1}{\mu}\right)$$

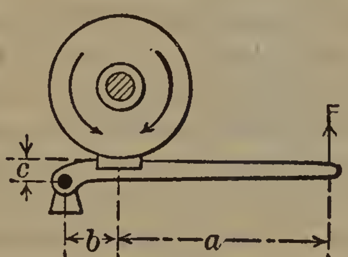


Fig. 2

Block brake.
For clockwise rotation:

$$F = \frac{\frac{Pb}{\mu} - Pc}{a+b} = \frac{Pb}{a+b}\left(\frac{1}{\mu} - \frac{c}{b}\right)$$

For counter clockwise rotation:

$$F = \frac{\frac{Pb}{\mu} + Pc}{a+b} = \frac{Pb}{a+b}\left(\frac{1}{\mu} + \frac{c}{b}\right)$$

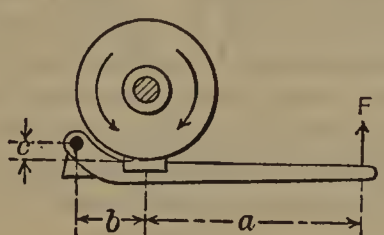


Fig. 3

Block brake.
For clockwise rotation:

$$F = \frac{\frac{Pb}{\mu} + Pc}{a+b} = \frac{Pb}{a+b}\left(\frac{1}{\mu} + \frac{c}{b}\right)$$

For counter clockwise rotation:

$$F = \frac{\frac{Pb}{\mu} - Pc}{a+b} = \frac{Pb}{a+b}\left(\frac{1}{\mu} - \frac{c}{b}\right)$$

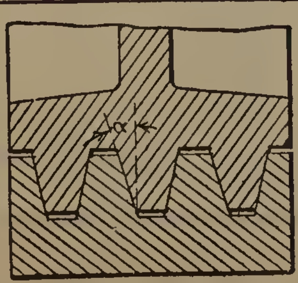


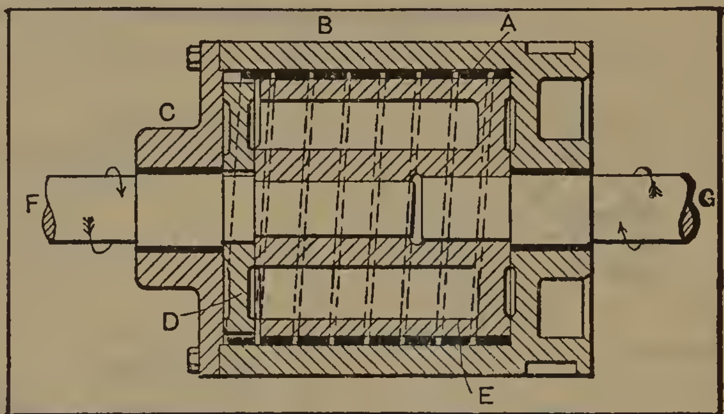
Fig. 4

The brake wheel and friction block of the block brake are often grooved as shown in Fig. 4. In this case, substitute for μ in the above equations the value $\frac{\mu}{\sin\alpha + \mu\cos\alpha}$ where α is one-half the angle included by the faces of the grooves.

Coil Brakes. — The coil brake, as shown in the accompanying illustration, is used as an automatic brake. It is, in effect, an internal band brake. The ratio between the tensions at the ends of the strap equals k, the value of k being obtained by the formula:

$$\text{hyp. log } k = \mu \times \frac{l}{r}$$

in which μ = coefficient of friction; l = length of the circular arc in contact with the strap; and r = radius of drum. The accompanying table gives the value of k figured for a coefficient of friction equal to 0.33 for various arcs of contact.



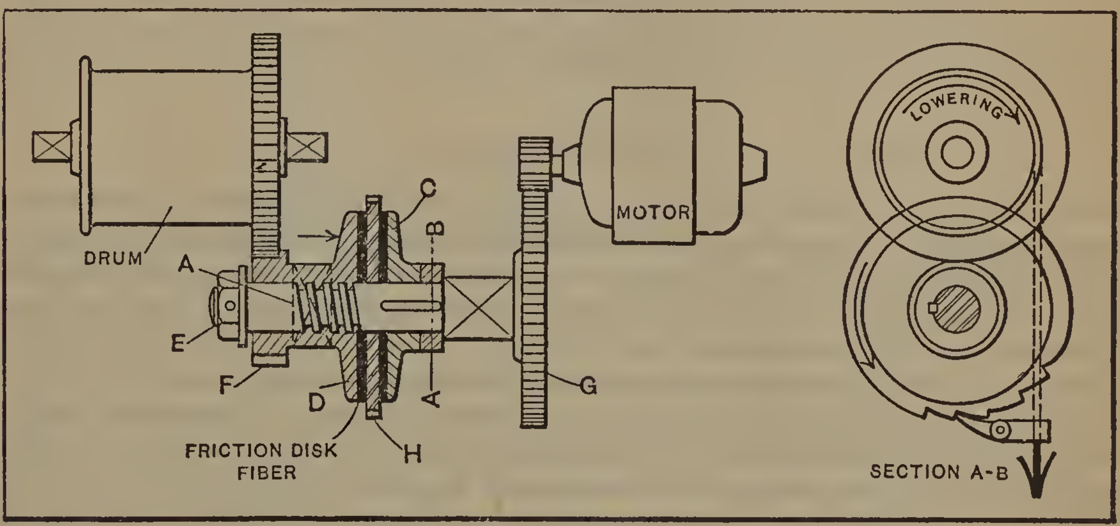
In the illustration, drum E is mounted on the ends of the motor shaft F and pinion shaft G, the former being loose and the latter keyed to the drum. Coil A is made of flat rod wound into a helix and ground on the external and internal surfaces, the diameter of the former being made slightly greater than the internal diameter of casing B, so that the coil must be slightly compressed when assembling the brake. One end of the coil is fastened to flange D, keyed to shaft F, and the other end is fastened to drum E. The coils may be made of steel or bronze bars. When properly designed with ample friction surfaces, and many coils with sufficient heat radiating surface, it is one of the most satisfactory types of brakes and seems especially adapted to the electric crane. In hoisting a load, shaft F turns in the direction of the arrow at the left, thus expanding the coil against the casing and causing drum E and shaft G to revolve with the casing. When holding a load suspended, shaft G and drum E tend to turn in the direction of the arrow at the right under the action of the load, thus expanding the coil against the casing, which is prevented from turning by pawls engaging the ratchet teeth on the end of the casing. When lowering the load, the motor turns in a direction opposite to that of the arrow at the left, thus releasing coil after coil from contact with the casing and winding them on drum E. When sufficient coils have been thus released, shaft G will turn under the influence of the load in the direction of the right-hand arrow, thus again expanding the coil sufficiently to hold the load. This cycle of events is repeated an infinite number of times in a unit of time, thus causing a uniform descent of the load.

Table of Values of k for Different Arcs of Contact

Arc of Contact, Degrees	Number of Revolutions	k	Arc of Contact, Degrees	Number of Revolutions	k
360	1	8.1	2880	8	18,913,800
720	2	65.9	3600	10	1,247,380,000
1440	4	4350.0	....	......	....

Formulas for Calculating Friction Disk Brakes. — In the following are given a set of formulas for calculations relating to the friction disk brake generally used on electric hoists. The object of the brake is to permit the lowering of the load at a constant speed with the power reversed, and the holding of the load suspended when the current in the motor is shut off. When the load is lowered it rotates, by means of the drum and gearing, the flange D against the ratchet disk H; the flange D is mounted as a nut on screw A; flange C is keyed to the shaft. The pawl engaging with the ratchet disk does not permit it to rotate when the load is lowered. As the ratchet disk thus is stationary, the work of the motor and of the lowered load must equal the work absorbed by the friction surfaces in the brake. When hoisting, the motor rotates flange D against flange C, thereby gripping friction plates and the ratchet H; the motor is thus free to hoist the load without any frictional resistance.

The amount of opening between the friction disks should be as small as possible, and the number of teeth in the ratchet disk as great as possible, consistent with the required strength. In this way excessive pressure on the ratchet teeth, due to the sudden dropping of the load when the current in the motor is shut off after



hoisting, may be avoided. The drop of the load is proportional to the opening between the friction disks and the amount the pawl allows the ratchet disk to rotate before engaging a tooth and stopping it positively.

In the formulas:

E = energy absorbed by friction disks in foot-pounds per minute;
E_1 = energy of the lowered load in foot-pounds;
E_2 = energy of the motor in foot-pounds;
e_1 = total efficiency of mechanism between load and flange D;
e_2 = total efficiency of the mechanism between the motor and flange C;
T = torque in inch-pounds on pinion F;
r = mean radius of screw in inches;
r_1 = inside radius of friction disks in inches;
r_2 = outside radius of friction disks in inches;
n = number of friction surfaces;
N = number of revolutions per minute of flanges C and D;
A = area of each friction disk in square inches;
W = total pressure in pounds on friction disk;
f = coefficient of friction between the flanges and the friction disks;
P = lead of the screw in inches;
ϕ = angle of repose of screw in degrees;
α = helix angle of the thread of screw in degrees;

y = number of thermal units of heat conducted away per square inch of bearing surface per minute; y may be considered to be from 4 to 7, when the mechanism is exposed to a current of cold air and in intermittent service, and as equal to 0.75 to 1 in tolerably cool places with intermittent service.

When a sufficient number of quantities are known, the energy absorbed by friction, the pressure on the disks, and the lead P of the screw in the brake may be found by the following formulas:

$$E = E_1e_1 + E_2e_2 \tag{1}$$

$$E = 0.349\,nfWN\,\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \tag{2}$$

$$W = \frac{T}{r\tan(\alpha + \phi)} \tag{3}$$

$$P = 2\pi r\tan\alpha \tag{4}$$

$$y = \frac{E}{778 \times 2A} \tag{5}$$

Example: — Assume the following data: At 230 volts and a speed of 1000 revolutions per minute, 25 amperes are required for hoisting the load. At a speed of 1500 revolutions per minute, 9 amperes are required for lowering the load. The motor efficiency is 80 per cent, the drum is 12 inches in diameter, the drum gear ratio is 1 to 7, and the motor gear ratio is 6 to 1. The efficiency of each set of gearing is 90 per cent. The outside radius of the friction disks is 7 inches, the inside radius, 3 inches, and the mean radius of the screw, $1\frac{1}{4}$ inch. The number of friction disks is 2, the number of revolutions of the friction disks is $1500 \div 6 = 250$, the coefficient of friction between the flanges and the friction disks is 0.07, and the angle of repose of the screw, 8 degrees 30 minutes. From these data it will be found that the maximum load of 2200 pounds may be hoisted at a speed of 75 feet per minute and lowered at a speed of 112.5 feet per minute:

$$\frac{1000 \times 12 \times 3.14}{6 \times 7 \times 12} = 75 \text{ (hoisting speed);}$$

$$75 \times 1.5 = 112.5 \text{ (lowering speed);}$$

$$\frac{230 \times 25 \times 44.2 \times 80 \times 90 \times 90}{75 \times 100 \times 100 \times 100} = 2200 \text{ pounds, nearly (max. load).}$$

Now find the lead P of a screw which will give a lowering speed of 112.5 feet per minute with the given conditions.

$$E_1e_1 = 2200 \times 0.90 \times 112.5 = 222{,}750 \text{ foot-pounds.}$$

$$E_2e_2 = 230 \times 9 \times 44.2 \times 0.90 \times 0.80 = 65{,}900 \text{ foot-pounds.}$$

From Formula (1):

$$E = E_1e_1 + E_2e_2 = 222{,}750 + 65{,}900 = 288{,}650 \text{ foot-pounds.}$$

From Formula (2):

$$W = \frac{E}{0.349\,nfN\,\dfrac{r_2^3 - r_1^3}{r_2^2 - r_1^2}} = \frac{288{,}650}{0.349 \times 2 \times 0.07 \times 250 \times 7.9} = 3000, \text{ approx.}$$

From Formula (3):

$$\tan(\alpha+\phi) = \frac{T}{Wr} = \frac{[(2200 \times 6) \div 7] \times 0.90}{3000 \times 1.25} = 0.453.$$

Hence, $\alpha+\phi = 24° \, 20'$, and $\alpha = 24° \, 20' - 8° \, 30' = 15° \, 50'$.

From Formula (4):

$$P = 2 \times 3.14 \times 1.25 \times \tan 15° \, 50' = 2.23 \text{ inches.}$$

This gives a screw of practically 2¼-inch lead. If a double thread is used the pitch will be 1⅛ inch.

From Formula (5):

$$y = \frac{288{,}650}{778 \times 2 \times 125.6} = 1.48,$$

which would be satisfactory when the brake is well exposed to the air.

The maximum allowable pressure per square inch on the friction disks should be limited to 200 pounds. In the example, the pressure equals:

$$\frac{W}{A} = \frac{3000}{125.6} = 24 \text{ pounds per square inch, approximately.}$$

Heat Radiation in Brakes. — The friction in brakes must be distributed over a sufficiently large area to prevent undue heating of the parts in contact. Brakes with wooden friction blocks on iron drums give satisfactory service if one square inch of friction surface is allowed for each 200 or 250 foot-pounds of energy absorbed per minute. Car brakes running with iron on iron, under conditions favorable to cooling, often absorb as much as 10,000 to 15,000 foot-pounds of energy per square inch of friction surface. Figures for other types of brakes will lie between these extremes. To facilitate radiation of heat, a ribbed exterior of the casing may be used for block brakes, and also thin ribs or vanes on the brake drum. All radiating surfaces should be left rough and black, because a finished or polished surface confines the heat instead of radiating it into the atmosphere.

Friction Wheels for Power Transmission

When a rotating member is driven intermittently and the rate of driving does not need to be positive, friction wheels are frequently used, especially when the amount of power to be transmitted is comparatively small. The driven wheels in a pair of friction disks should always be made of a harder material than the driving wheels, so that if the driven wheel should be held stationary by the load, while the driving wheel revolves under its own pressure, a flat spot may not be rapidly worn on the driven wheel. The driven wheels, therefore, are usually made of iron, while the driving wheels are made of or covered with, rubber, paper, leather, wood or fiber. The safe working pressures per inch of contact for various materials are as follows: Straw fiber, 150; leather fiber, 240; tarred fiber, 240; leather, 150; wood, 100 to 150; paper, 150.

The working values in the table for coefficients of friction for different combinations may be used. A smaller coefficient of friction should be employed when the linear velocity of the driver is very great, or when the driver is starting the driven wheel under load.

Horsepower of Friction Wheels. — Let D = diameter of friction wheel in inches; N = number of revolutions per minute; W = width of face in inches; f =

Working Values of Coefficient of Friction

Materials	Coefficient of Friction	Materials	Coefficient of Friction
Straw fiber and cast iron......	0.26	Tarred fiber and aluminum...	0.18
Straw fiber and aluminum....	0.27	Leather and cast iron.........	0.14
Leather fiber and cast iron....	0.31	Leather and aluminum........	0.22
Leather fiber and aluminum...	0.30	Leather and typemetal........	0.25
Tarred fiber and cast iron.....	0.15	Wood and metal...............	0.25
Paper and cast iron	0.20		

coefficient of friction; P = pressure in pounds, per inch width of face. Then:

$$\text{H.P.} = \frac{3.1416 \times D \times N \times P \times W \times f}{33{,}000 \times 12}$$

Assume

$$\frac{3.1416 \times P \times f}{33{,}000 \times 12} = C;$$

then,

$$\text{for } P = 100 \text{ and } f = 0.20, \quad C = 0.00016;$$
$$\text{for } P = 150 \text{ and } f = 0.20, \quad C = 0.00024;$$
$$\text{for } P = 200 \text{ and } f = 0.20, \quad C = 0.00032.$$

The horsepower transmitted is then:

$$\text{H.P.} = D \times N \times W \times C.$$

Example: — Find the horsepower transmitted by a pair of friction wheels; the diameter of the driving wheel is 10 inches, and it revolves at 200 revolutions per minute. The width of the wheel is 2 inches. The pressure per inch width of face is 150 pounds, and the coefficient of friction 0.20.

$$\text{H.P.} = 10 \times 200 \times 2 \times 0.00024 = 0.96 \text{ horsepower.}$$

Horsepower Which May be Transmitted by Means of a Clean Paper Friction Wheel of One-inch Face when Run Under a Pressure of 150 Pounds

(Rockwood Mfg. Co.)

Diameter of Friction	Revolutions per Minute										
	25	50	75	100	150	200	300	400	600	800	1000
4	0.023	0.047	0.071	0.095	0.142	0.190	0.285	0.380	0.571	0.76	0.95
6	0.035	0.071	0.107	0.142	0.214	0.285	0.428	0.571	0.856	1.14	1.42
8	0.047	0.095	0.142	0.190	0.285	0.380	0.571	0.761	1.142	1.52	1.90
10	0.059	0.119	0.178	0.238	0.357	0.476	0.714	0.952	1.428	1.90	2.38
14	0.083	0.166	0.249	0.333	0.499	0.666	0.999	1.332	1.999	2.66	3.33
16	0.095	0.190	0.285	0.380	0.571	0.761	1.142	1.523	2.284	3.04	3.80
18	0.107	0.214	0.321	0.428	0.642	0.856	1.285	1.713	2.570	3.42	4.28
24	0.142	0.285	0.428	0.571	0.856	1.142	1.713	2.284	3.427	4.56	5.71
30	0.178	0.357	0.535	0.714	1.071	1.428	2.142	2.856	4.284	5.71	7.14
36	0.214	0.428	0.642	0.856	1.285	1.713	2.570	3.427	5.140	6.85	8.56
42	0.249	0.499	0.749	0.999	1.499	1.999	2.998	3.998	5.997	7.99	9.99
48	0.285	0.571	0.856	1.142	1.713	2.284	3.427	4.569	6.854	9.13	11.42
50	0.297	0.595	0.892	1.190	1.785	2.380	3.570	4.760	7.140	9.52	11.90

It is necessary that friction wheels be kept clean, and if foreign substances are likely to enter between the friction surfaces they should be protected by suitable guards Rigid supports should be provided for shafts as close to the wheels as possible so that a proper driving contact is provided across the full face of the wheel. The wider the face, the more vital is this point, and the more difficult is it to accomplish. Hence, the faces of friction wheels should be kept as narrow as the design in general will permit.

Dynamometers

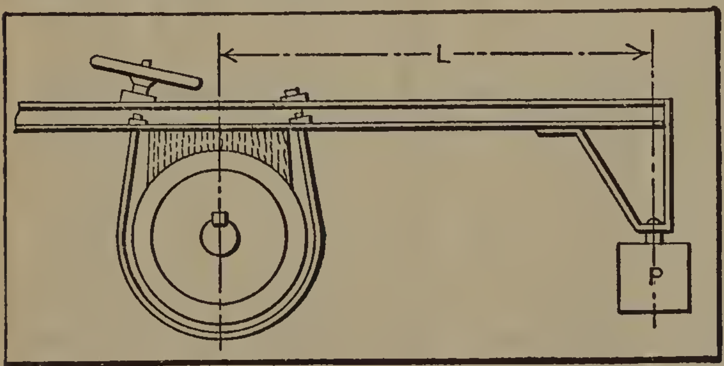
Dynamometers. — Dynamometers are instruments used for measuring power. One of the most common types is the Prony brake, which consists primarily of a lever connected to jaws which grip a revolving shaft or pulley so that the friction between the surfaces in contact tends to rotate the arm in the direction in which the shaft revolves. The tendency to rotation is overcome by weights supported at the end of the lever. The power for a given number of revolutions is measured by the weights required to balance the lever in a horizontal direction when the clamps are screwed up against the shaft so as to induce enough friction to raise the lever. The rubbing surfaces between the wooden blocks and the brake pulley are lubricated with heavy grease. Soft woods such as beech, poplar or maple should be used for brake blocks in preference to the harder woods.

Calculating Horsepower from Dynamometer Tests. — When a dynamometer is arranged for obtaining the horsepower transmitted by a shaft, as indicated by the diagrammatic view in the accompanying illustration, the horsepower may be obtained by the formula:

$$\text{H.P.} = \frac{2\pi LPN}{33{,}000}$$

in which H.P. = horsepower transmitted; N = number of revolutions per minute; L = distance (as shown in illustration) from center of pulley to point of action of weight P, in feet; P = weight hung on brake arm or read on scale.

By adopting a length of brake arm equal to 5 feet 3 inches, the formula may be reduced to the simple form:



$$\text{H.P.} = \frac{NP}{1000}$$

If a length of brake arm equal to 2 feet 7½ inches is adopted as a standard, the formula takes the form:

$$\text{H.P.} = \frac{NP}{2000}$$

The accompanying tables are based upon these formulas. They give the brake horsepower for velocities varying from 120 to 800 revolutions per minute. For example, assume that we wish to find the horsepower in a case where the shaft makes 300 revolutions per minute, a 60-pound weight being attached to a lever arm 5 feet 3 inches. Then, according to the simplified formula:

$$\text{H.P.} = \frac{NP}{1000} = \frac{300 \times 60}{1000} = 18$$

This value may be found directly from the tables by locating P = 60 in the extreme left-hand column, and N = 300 at the top of the columns. At the place where the two rows of figures intersect, the horsepower — in this case 18 — may be read off.

Table Giving Horsepower in Dynamometer Tests

Load arm L (see illustration in explanatory paragraph) equals 5 feet 3 inches

Load on Brake Arm, P	Revolutions per Minute, N									
	240	260	280	300	320	340	360	380	400	420
12	2.88	3.12	3.36	3.6	3.84	4.08	4.32	4.56	4.8	5.04
14	3.36	3.64	3.92	4.2	4.48	4.76	5.04	5.32	5.6	5.88
16	3.84	4.16	4.48	4.8	5.12	5.44	5.76	6.08	6.4	6.72
18	4.32	4.68	5.08	5.4	5.76	6.12	6.50	6.85	7.2	7.56
20	4.80	5.20	5.60	6.0	6.40	6.80	7.20	7.60	8.0	8.40
22	5.28	5.72	6.16	6.6	7.04	7.48	7.92	8.36	8.8	9.25
24	5.76	6.24	6.83	7.2	7.68	8.16	8.65	9.13	9.6	10.08
26	6.24	6.76	7.28	7.8	8.32	8.84	9.36	9.90	10.4	10.92
28	6.73	7.30	7.84	8.4	8.96	9.53	10.10	10.65	11.2	11.78
30	6.90	7.80	8.40	9.0	9.60	10.20	10.80	11.40	12.0	12.60
32	7.68	8.33	8.96	9.6	10.25	10.80	11.50	12.15	12.8	13.45
36	8.65	9.36	10.2	10.8	11.50	12.25	12.95	13.7	14.4	15.1
40	9.60	10.40	11.2	12.0	12.80	13.60	14.40	15.2	16.0	16.8
44	10.56	11.45	12.3	13.2	14.08	14.96	15.85	16.7	17.6	18.5
48	11.5	12.5	13.4	14.4	15.3	16.3	17.3	18.2	19.2	20.1
50	12.0	13.0	14.0	15.0	16.0	17.0	18.0	19.0	20.0	21.0
52	12.5	13.5	14.5	15.6	16.6	17.7	18.7	19.7	20.8	21.8
56	13.4	14.5	15.7	16.8	17.9	19.0	20.1	21.3	22.4	23.5
60	14.4	15.6	16.8	18.0	19.2	20.4	21.6	22.8	24.0	25.2
65	15.6	16.9	18.2	19.5	20.8	22.1	23.4	24.7	26.0	27.3
70	16.8	18.2	19.6	21.0	22.4	23.8	25.2	26.6	28.0	29.4
75	18.0	19.5	21.0	22.5	24.0	25.5	27.0	28.5	30.0	31.5
80	19.2	20.8	22.4	24.0	25.6	27.2	28.8	30.4	32.0	33.6
85	20.4	22.1	23.8	25.5	27.2	28.9	30.6	32.3	34.0	35.7
90	21.6	23.4	25.2	27.0	28.8	30.6	32.4	34.2	36.0	37.8
95	22.8	24.7	26.6	28.5	30.4	32.3	34.2	36.1	38.0	39.9
100	24.0	26.0	28.0	30.0	32.0	34.0	36.0	38.0	40.0	42.0
105	25.2	27.3	29.4	31.5	33.6	35.7	37.8	39.9	42.0	44.1
110	26.4	28.6	30.8	33.0	35.2	37.4	39.6	41.8	44.0	46.2
115	27.6	29.9	32.2	34.5	36.8	39.1	41.4	43.7	46.0	48.3
120	28.8	31.2	33.6	36.0	38.4	40.8	43.2	45.6	48.0	50.4
125	30.0	32.5	35.0	37.5	40.0	42.5	45.0	47.5	50.0	52.5
130	31.2	33.8	36.4	39.0	41.6	45.2	47.8	49.4	52.0	54.6
135	32.4	35.1	37.8	40.5	43.2	45.8	48.6	51.4	54.0	56.8
140	33.6	36.4	39.2	42.0	44.8	47.6	50.4	53.2	56.0	58.8
145	34.8	37.7	40.6	43.5	46.4	49.3	52.2	55.1	58.0	60.9
150	36.0	39.0	42.0	45.0	48.0	51.0	54.0	57.0	60.0	63.0
160	38.4	41.6	44.8	48.0	51.2	54.4	57.6	60.8	64.0	67.2
170	40.8	44.2	47.6	51.0	53.4	57.8	61.2	64.6	68.0	71.4
180	43.2	46.8	50.4	54.0	57.6	61.2	64.8	68.4	72.0	75.6
190	45.6	49.4	53.2	57.0	60.8	64.6	68.4	72.2	76.0	79.8
200	48.0	52.0	56.0	60.0	64.0	68.0	72.0	76.0	80.0	84.0
210	50.4	54.6	58.8	63.0	67.2	71.4	75.6	80.0	84.0	87.2
220	52.8	57.2	61.6	66.0	70.4	74.8	79.2	83.6	88.0	92.4
230	55.2	59.8	64.4	69.0	73.6	78.4	82.8	87.5	92.0	96.6
240	57.6	62.4	67.2	72.0	76.8	81.6	86.4	91.2	96.0	101.0

Table Giving Horsepower in Dynamometer Tests

Load arm L (see illustration in explanatory paragraph) equals 5 feet 3 inches

Load on Brake Arm, P	Revolutions per Minute, N									
	440	460	480	500	520	540	560	600	660	700
12	5.28	5.52	5.76	6.0	6.24	6.48	6.72	7.2	7.92	8.4
14	6.16	6.45	6.73	7.0	7.28	7.56	7.84	8.4	9.24	9.8
16	7.04	7.36	7.68	8.0	8.32	8.64	8.96	9.6	10.56	11.2
18	7.92	8.28	8.65	9.0	9.36	9.70	10.08	10.8	11.88	12.6
20	8.80	9.20	9.60	10.0	10.40	10.80	11.20	12.0	13.20	14.0
22	9.70	10.12	10.57	11.0	11.40	11.90	12.30	13.2	14.50	15.4
24	10.57	11.05	11.52	12.0	12.40	12.90	13.40	14.4	15.80	16.8
26	11.45	11.97	12.50	13.0	13.50	14.00	14.60	15.6	17.10	18.2
28	12.33	12.90	13.40	14.0	14.50	15.10	15.70	16.8	18.50	19.6
30	13.20	13.80	14.40	15.0	15.60	16.20	16.80	18.0	19.80	21.0
32	14.10	14.72	15.36	16.0	16.60	17.20	17.90	19.2	21.10	22.4
36	15.85	16.50	17.40	18.0	18.70	19.40	20.10	21.6	23.80	25.2
40	17.60	18.40	19.20	20.0	20.80	21.60	22.40	24.0	26.40	28.0
44	19.35	20.20	21.10	22.0	22.90	23.75	24.60	26.4	29.00	30.3
48	21.10	22.10	23.00	24.0	25.00	25.90	26.90	28.8	31.50	33.6
50	22.00	23.00	24.00	25.0	26.00	27.00	28.00	30.0	33.00	35.0
52	22.80	23.90	25.00	26.0	27.00	28.10	29.10	31.2	34.30	36.4
56	24.60	25.70	26.90	28.0	29.10	30.20	31.40	33.6	37.00	39.2
60	26.40	27.60	28.80	30.0	31.20	32.40	33.60	36.0	39.60	42.0

Load on Brake Arm, P	Revolutions per Minute, N									
	120	130	140	150	160	170	180	190	200	220
60	7.2	7.80	8.4	9.00	9.6	10.20	10.8	11.40	12.0	13.2
65	7.8	8.45	9.1	9.75	10.4	11.05	11.7	12.35	13.0	14.3
70	8.4	9.10	9.8	10.50	11.2	11.90	12.6	13.30	14.0	15.4
75	9.0	9.75	10.5	11.25	12.0	12.75	13.5	14.25	15.0	16.5
80	9.8	10.40	11.2	12.00	12.8	13.60	14.4	15.20	16.0	17.6
85	10.4	11.00	11.9	12.70	13.6	14.40	15.3	16.10	17.0	18.7
90	10.8	11.70	12.6	13.50	14.4	15.30	16.2	17.10	18.0	19.8
95	11.4	12.30	13.3	14.25	15.2	16.15	17.1	18.05	19.0	20.9
100	12.0	13.00	14.0	15.00	16.0	17.00	18.0	19.00	20.0	22.0
105	12.6	13.70	14.7	15.75	16.8	17.80	18.9	19.90	21.0	23.1
110	13.2	14.30	15.4	16.50	17.6	18.70	19.8	20.90	22.0	24.2
115	13.8	14.90	16.1	17.20	18.9	19.55	20.7	21.70	23.0	25.3
120	14.4	15.60	16.8	18.00	19.2	20.40	21.6	22.80	24.0	26.4
125	15.0	16.20	17.5	18.70	20.0	21.20	22.5	23.70	25.0	27.5
130	15.6	16.90	18.2	19.50	20.8	22.10	23.4	24.70	26.0	28.6
140	16.8	18.40	19.6	21.00	22.4	23.80	25.2	26.60	28.0	30.8
150	18.0	19.50	21.0	22.50	24.0	25.50	27.0	28.50	30.0	33.0
160	19.4	20.80	22.4	24.00	25.6	27.20	28.8	30.40	32.0	35.2
170	20.4	22.10	23.8	25.50	27.2	28.90	30.6	32.30	34.0	37.4
180	21.6	23.40	25.2	27.00	28.8	30.60	32.4	34.20	36.0	39 6
190	22.8	24.70	26.6	28.50	30.4	32.30	34.2	36.10	38.0	41.8
200	24.0	26.00	28.0	30.00	32.0	34.00	36.0	38.00	40.0	44.0
220	26.4	28.60	30.8	33.00	35.2	37.40	39.6	41.80	44.0	48.4
240	28.8	31.20	33.8	36.00	38.4	40.80	43.2	45.60	48.0	52.8

Table Giving Horsepower in Dynamometer Tests

Load arm L (see illustration in explanatory paragraph) equals 2 feet 7½ inches

Load on Brake Arm, P	Revolutions per Minute, N										
	340	360	380	400	420	440	460	480	500	520	540
3	0 51	0.54	0.57	0.60	0.63	0.66	0.69	0.72	0.75	0.78	0.81
4	0.68	0.72	0.76	0.80	0.84	0.88	0.92	0.96	1.00	1.04	1.08
5	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.35
6	1.02	1.08	1.14	1.20	1.26	1.32	1.38	1.44	1.50	1.56	1.62
7	1.19	1.26	1.33	1.40	1.47	1.54	1.61	1.68	1.75	1.82	1.89
8	1.36	1.44	1.52	1.60	1.68	1.76	1.84	1.92	2.00	2.08	2.16
9	1.54	1.63	1.72	1.80	1.89	1.98	2.07	2.16	2.25	2.34	2.43
10	1.70	1.80	1.90	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70
11	1.87	1.98	2.09	2.20	2.31	2.42	2.53	2.64	2.75	2.86	2.97
12	2.04	2.16	2.28	2.40	2.52	2.64	2.76	2.88	3.00	3.12	3.24
13	2.21	2.34	2.47	2.60	2.73	2.86	2.99	3.12	3.25	3.38	3.51
14	2.38	2.52	2.66	2 80	2.94	3.08	3.22	3.36	3.50	3.64	3.78
15	2.55	2.70	2.85	3.00	3.15	3.30	3.45	3.60	3.75	3.90	4.05
16	2.72	2.88	3.04	3.20	3.36	3.52	3.68	3.84	4.00	4.16	4.32
18	3.06	3.24	3.42	3.60	3.78	3.96	4.14	4.32	4.50	4.68	4.86
20	3.40	3.60	3.80	4.00	4.20	4.40	4.60	4.80	5.00	5.20	5.40
22	3.73	3.96	4.18	4.40	4.62	4.84	5.06	5.28	5.50	5.72	5.94
24	4.08	4.32	4.56	4.80	5.04	5.28	5.52	5.76	6.00	6.24	6.48
25	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50	6.75
26	4.42	4.68	4.94	5.20	5.46	5.72	5.98	6.24	6.50	6.76	7.02
28	4.76	5.04	5.32	5.60	5.88	6.16	6.44	6.72	7.00	7.28	7.56
30	5.10	5.40	5.70	6.00	6.30	6.60	6.90	7.20	7.50	7.80	8.10

Load on Brake Arm, P	Revolutions per Minute, N										
	560	580	600	620	640	660	680	700	720	760	800
3	0.84	0.87	0.9	0.93	0.96	0.99	1.02	1.05	1.08	1.14	1.2
4	1.12	1.16	1.2	1.24	1.28	1.32	1.36	1.40	1.44	1.52	1.6
5	1.40	1.45	1.5	1.55	1.60	1.65	1.70	1.75	1.80	1.90	2.0
6	1.68	1.74	1.8	1.86	1.92	1.98	2.04	2.10	2.16	2.28	2.4
7	1.96	2.03	2.1	2.17	2.24	2.31	2.38	2.45	2.52	2.66	2.8
8	2.24	2.32	2.4	2.48	2.56	2.64	2.72	2.80	2.88	3.04	3.2
9	2.52	2.61	2.7	2.79	2.88	2.97	3.06	3.15	3.24	3.42	3.6
10	2.80	2.90	3.0	3.10	3.20	3.30	3.40	3.50	3.60	3.80	4.0
11	3.08	3.19	3.3	3.41	3.52	3.63	3.74	3.85	3.96	4.18	4.4
12	3.36	3.48	3.6	3.72	3.84	3.96	4.08	4.20	4.32	4.56	4.8
13	3.64	3.77	3.9	4.03	4.16	4.29	4.42	4.55	4.68	4.94	5.2
14	3.92	4.06	4.2	4.34	4.48	4.62	4.76	4.90	5.04	5.32	5.6
15	4.20	4.35	4.5	4.65	4.80	4.95	5.10	5.25	5.40	5.70	6.0
16	4.48	4.64	4.8	4.96	5.12	5.28	5.44	5.60	5.76	6.08	6.4
18	5.04	5.22	5.4	5.58	5.76	5.94	6.12	6.30	6.48	6.84	7.2
20	5.60	5.80	6.0	6.20	6.40	6.60	6.80	7.00	7.20	7.60	8.0
22	6.16	6.38	6.6	6.82	7.04	7.26	7.48	7.70	7.92	8.36	8.8
24	6.72	6.96	7.2	7.44	7.68	7.92	8.16	8.40	8.64	9.12	9.6
25	7.00	7.25	7.5	7.75	8.00	8.25	8.50	8.75	9.00	9.50	10.0
26	7.28	7.54	7.8	8.06	8.32	8.58	8.84	9.10	9.36	9.88	10.4
28	7.84	8.12	8.4	8.68	8.96	9.24	9.52	9.80	10.08	10.60	11.2
30	8.40	8.70	9.0	9.30	9.60	9.90	10.20	10.50	10.80	11.40	12.0

CAMS AND CAM DESIGN

Classes of Cams. — Cams may, in general, be divided into two classes: uniform motion cams and uniformly accelerated motion cams. The uniform motion cam moves the follower at the same rate of speed from the beginning to the end of the stroke; but as the movement is started from zero to the full speed of the uniform motion and stops in the same abrupt way, there is a distinct shock at the beginning and end of the stroke, if the movement is at all rapid. In machinery working at a high rate of speed, therefore, it is important that cams are so constructed that sudden shocks are avoided when starting the motion or when reversing the direction of motion of the follower. The cam best suited for high speeds is one where the speed at first is slow and then accelerated at a uniform rate until the maximum speed is reached. The speed is then again uniformly retarded until the rate of motion of the follower is zero or nearly zero when the reversal takes place. A cam constructed along these lines is called a uniformly accelerated motion cam. The distances

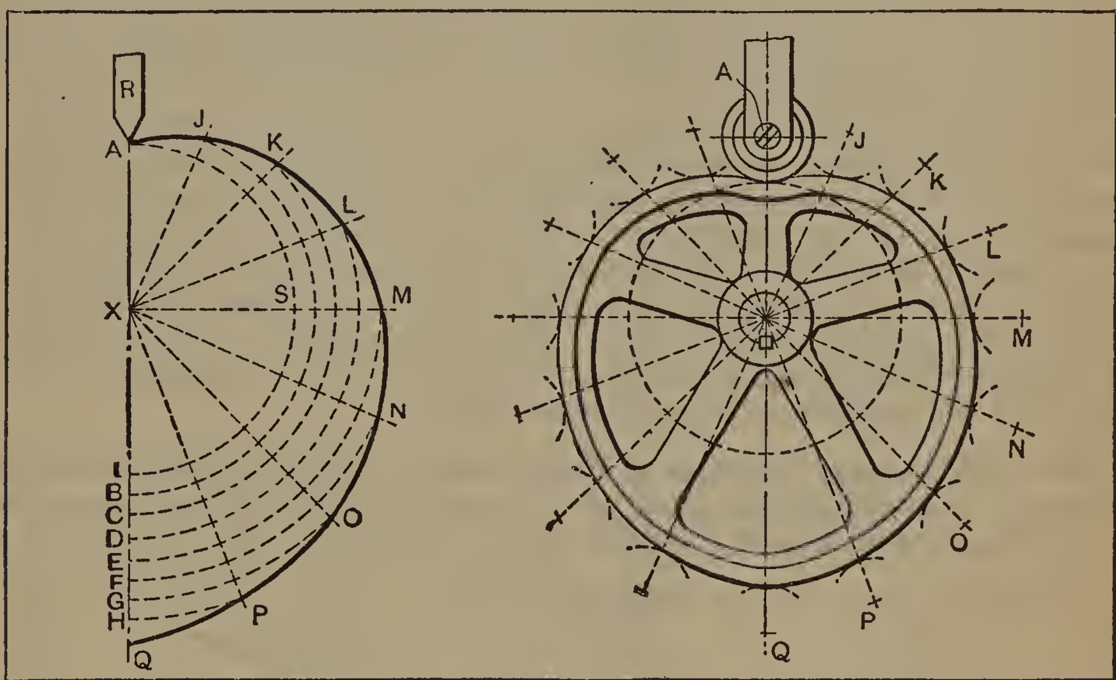


Fig. 1 Fig. 2

which the follower passes through during equal periods of time increase in the same ratio as the distances passed through in consecutive seconds by a freely falling body acted upon by gravity alone. A cam constructed on these lines, therefore, is often called a gravity-curve cam.

Laying-out a Uniform Motion Cam. — The laying-out of a heart-shaped cam will serve as an illustration of the general method. In Fig. 1, the pointed follower *R* is to be given a reciprocating motion. The throw is assumed to be 1½ inch. Let *X* be the center of the cam. Let *A* be the point at which the follower is at the lower end of the stroke. Draw semi-circle *ASI*, and extend the diameter at the side opposite *A*, a distance *IQ*, equal to the required throw. Divide *IQ* into any number of equal parts, as at *B*, *C*, *D*, etc., and divide the semi-circle by the same number of radii, equally distributed. With *X* as a center and a radius equal to *XB*, describe an arc intersecting *XJ* at *J*. With the same center and a radius equal to *XC*, describe an arc intersecting *XK* at *K*. Continue this process through the points *D*, *E*, *F*, etc., thus obtaining the points *L*, *M*, *N*, etc. The latter are points on the required curve. The other half of the cam is laid out in the same manner.

The excessive friction of a pointed follower such as that shown at *R* necessitates the use of a follower that will reduce the amount of friction to a minimum. A small roller meets this requirement. If a roller is employed as a follower, the problem of laying out the cam curve becomes modified. A roller traveling along the curve shown in Fig. 1 would not impart to the follower-rod the desired uniform rise and fall. The variation would be but slight, yet sufficient to merit consideration where accuracy is desired.

Cams with Roller Followers. — Fig. 2 represents a heart-shaped cam of the same dimensions as that in Fig. 1, but with a roller follower. It is the path of the center of this roller that requires consideration, as the position of this center regulates the throw. Therefore, the position of the center of the roller at various intervals in the rotation of the cam must be determined. This may be done by adding to each of the distances *XJ*, *XK*, etc., in Fig. 1, the radius of the roller, and thus obtaining the points *J*, *K*, *L*, etc., Fig. 2. With these points as centers and with radii equal to that of the roller, describe arcs. A curve drawn tangent to these arcs is the required cam curve.

This cam depends upon the action of gravity, or a spring, to keep the follower in contact with the driver. It can be made positive in action by the use of two followers placed at the extremities of the diameter of the cam, or by drawing curves tangent to both the top and bottom of the follower roller in its various positions, and taking the two curves as the boundaries of a groove cut into the metal. A familiar application of the use of a heart-shaped cam may be found in the bobbin-winder of the domestic sewing machine. The thread is fed to and fro at a uniform rate, the follower of the cam acting as a guide for the thread. The action is made positive by the employment of two follower rollers.

Effect of Changing Location of Cam Roller. — When the line of motion of a follower passes through the center of rotation of the cam, and the angle of the curve causes it to work hard, the curve may be modified, and the same motion of

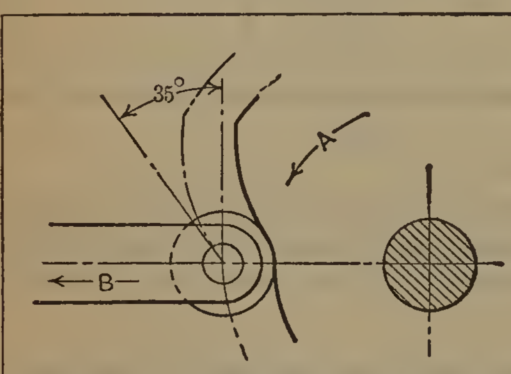


Fig. 3

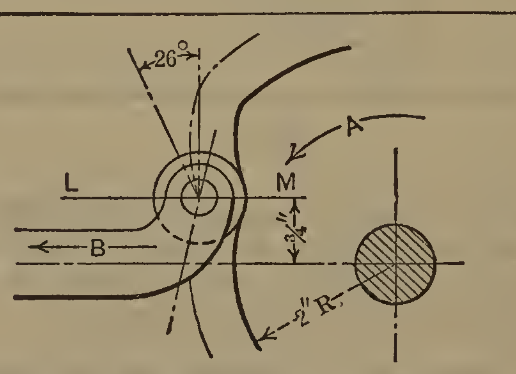


Fig. 4

follower obtained by placing the follower with its line of action parallel to its original position and not passing through the center of the cam. A condition may be assumed, as shown in Fig. 3. Here the cam rotates in the direction indicated by arrow *A*. It moves the follower ¾ inch in the direction indicated by arrow *B*, during a 30-degree angle of motion of the cam-shaft. The angle of the cam presented to the follower at the beginning of the stroke would be 35 degrees, as determined by the tangent to the curve. Should the cam curve work hard at the required speed, the cam would be made of greater diameter, if possible, which would reduce the angle of the cam. The design of the machine, however, might make this change impossible. Another way consists in changing the location of the cam roller. In Fig. 4 all conditions are the same as in Fig. 3, except that the roller has been placed ¾ inch above the line passing through the center of the cam. The center of the roller will now pass along the line *LM*, or parallel to the line of motion in Fig. 3. The angle of the curve presented to the roller in this case is 26 degrees — much less than the angle in Fig. 3 — and the angle decreases as the roller moves away from the center of rotation. There is, of course, a limit to the distance the roller may be changed, for if placed too far away from the center line, the thrust in the direction at right angles to the direction of motion of the follower would be so great as to offset the advantage gained. Even without the aid of an illustration it may be seen that to place the cam roller on the other side of the center would cause the angle of the cam curve to increase, thus making conditions worse. The offset of the roller should be in the direction opposed to the direction of motion of the cam.

Laying-out a Uniformly Accelerated Motion Cam. — When a uniformly accelerated motion is used, the distances passed through by the follower during equal periods of time increase uniformly, so that if, for instance, the follower moves a distance equal to 1 length unit during the first second, and 3 during the second, it will move 5 length units during the third second, 7 during the fourth, 9 during

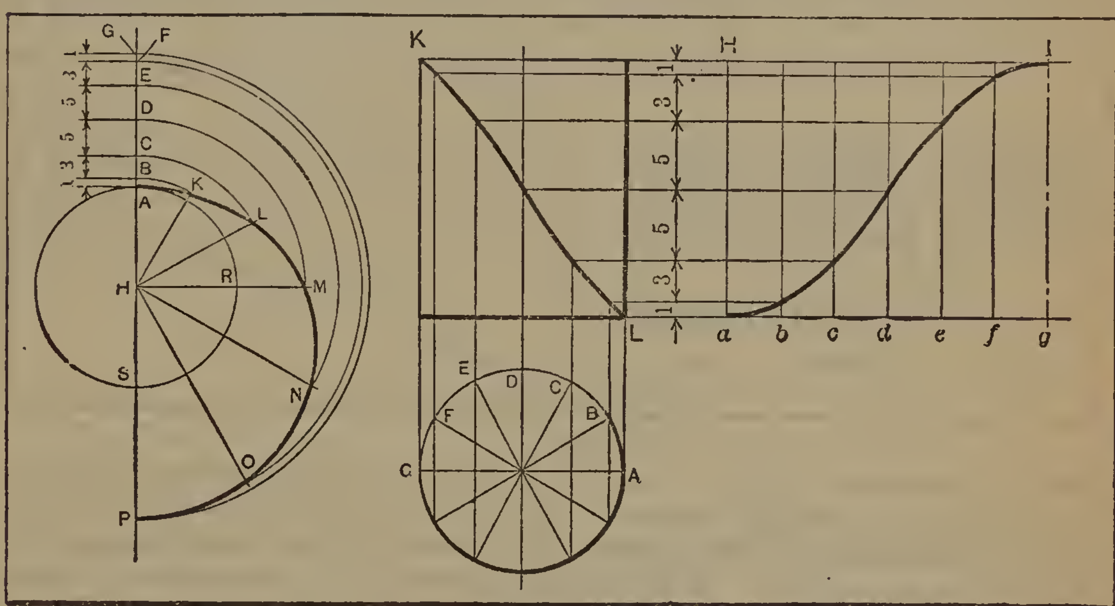


Fig. 5 Fig. 6

the fifth, and so on. When the motion is retarded, it will move 9, 7, 5, 3 and 1 length units during successive seconds. In Fig. 5 is shown a layout of a uniformly accelerated motion plate cam. The motion of the follower is back and forth from *A* to *G*. The rise takes place during 180 degrees and the fall during an equal period, the two halves of the cam being alike. To construct the cam, divide the half circle *ARS* into six equal parts, and draw radii *HK*, *HL*, etc. Divide *AG* into two equal parts, *AD* and *DG*, and then divide each of these parts into three divisions, the lengths of which are to each other as 1 : 3 : 5. With *H* as a center, draw circular arcs from *B*, *C*, *D*, etc., to *K*, *L*, *M*, etc. The intersections between the circles and the radii are points on the cam curve. If the half circle *ARS* should be divided into eight equal parts instead of six, line *AG* would also have to be divided into eight parts in the proportions 1 : 3 : 5 : 7 : 7 : 5 : 3 : 1. With a cam constructed in this manner, the follower starts at A with a velocity of zero; it reaches its maximum velocity at D, and at G, where the motion is reversed, the velocity is again zero.

Development and Layout of a Uniformly Accelerated Motion Cylindrical Cam. — To the right in Fig. 6 is shown the development of a uniformly accelerated motion cam curve laid out on the surface of a cylindrical cam. This development is necessary for finding the projection on the cylindrical surface, as shown at KL. To construct the developed curve, first divide the base circle of the cylinder into, say, twelve equal parts. Set off these parts along line ag. Only one-half of the layout has been shown, as the other half is constructed in the same manner, except that the curve is here falling instead of rising. Divide line aH into the same number of divisions as the half circle, the divisions being in the proportion 1 : 3 : 5 : 5 : 3 : 1. Draw horizontal lines from these division points and vertical lines from a, b, c, etc. The intersections between the two sets of lines are points on the developed cam curve. These points are transferred to the cylindrical surface at the left by projection in the usual manner.

Shape of Rolls for Cylinder Cams. — The rolls for cylindrical cams working in a groove in the cam should be conical rather than cylindrical in shape, in order that they may rotate freely and without excessive friction. Fig. 1 shows a straight roll and groove, the action of which is faulty because of the varying surface speed at the top and bottom of the groove. Fig. 2 shows a roll with curved surface. For heavy work, however, the small bearing area is quickly worn down and the roll

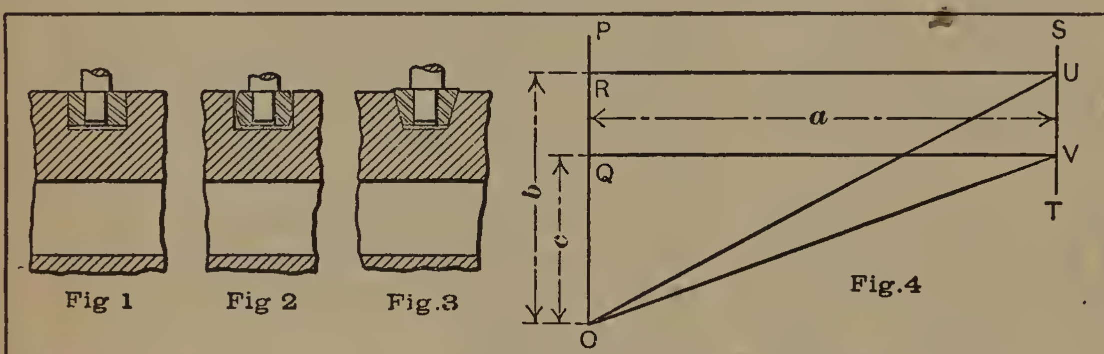


Fig 1 Fig 2 Fig.3 Fig.4

presses a groove into the side of the cam as well, thus destroying the accuracy of the movement and creating backlash. Fig. 3 shows the conical shape which permits a true rolling action in the groove. The amount of taper depends on the angle of spiral of the cam groove. As this angle, as a rule, is not constant for the whole movement, the roll and groove should be designed to meet the requirements on that section of the cam where the heaviest duty is performed. Frequently the cam groove is of a nearly even spiral angle for a considerable length. The method for determining the angle of the roll and groove to work correctly during the important part of the cycle is as follows:

In Fig. 4, b is the circumferential distance on the surface of the cam that includes the section of the groove for which correct rolling action is required. The throw of the cam for this circumferential movement is a. Line OU is the development of the movement of the cam roll during the given part of the cycle, and c is the movement corresponding to b, but on a circle the diameter of which is equal to that of the cam at the bottom of the groove. With the same throw a as before, the line OV will be the development of the cam at the bottom of the groove. OU then is the length of the helix traveled by the top of the roll, while OV is the travel at the bottom of the groove. If then the top width and bottom width of the groove be made proportional to OU and OV, the groove will be properly proportioned.

Cam Rolls and Roll Studs. — It is important that the cam roll and roll stud be ground all over after hardening. The end of the roller should be cut back or recessed 1/64 of an inch or thereabouts on the sides for some distance, beginning at the periphery, so as to avoid undue friction against the collar of the stud or the part in which it is mounted. On account of the warping that takes place in hardening, rolls that are not ground both on the inside and on the outside often will stop under heavy load, until in time flat spots are worn on the face, and then the working surface of the cam will begin to wear or is roughed up. Roll studs that are out of parallel with the working surface of the cam, even to a very small degree, also cause trouble. The same difficulty is met with on cylinder or barrel cams if the milling cutter is set below or above the center of the cam when cutting it. The roll will then bear at one end only at the most important time — when the throw takes place.

There is a great deal of end pressure on the conical rolls used in barrel cams, and this must be taken care of by thrust collars on the studs on which the rolls are mounted, or, better still, by a ball race scored in the collar and the large end of the roll, so as to provide for a ball thrust bearing. The end pressure on a conical roll, however, reduces the side pressure on the stud to a considerable extent, so that the stud may be made shorter or smaller in diameter than when a roll with parallel sides is used.

Cam Milling. — Plate cams having a constant rise, such as are used on automatic screw machines, can be cut in a universal milling machine, with the spiral head either in a vertical position or set at an angle α, as shown by the illustration. When the spiral head is set vertical, the "lead" of the cam (or its rise for one complete revolution) is the same as the lead for which the machine is geared; but when the spiral head and cutter are inclined, any lead or rise of the cam can be obtained, provided it is less than the lead for which the machine is geared, that is, less than the forward feed of the table for one turn of the spiral-head spindle. The cam lead, then, can be varied within certain limits by simply changing the inclination α of the spiral head and cutter. In the following formulas for determining this angle of inclination, for a given rise of cam and with the machine geared for a certain lead, let

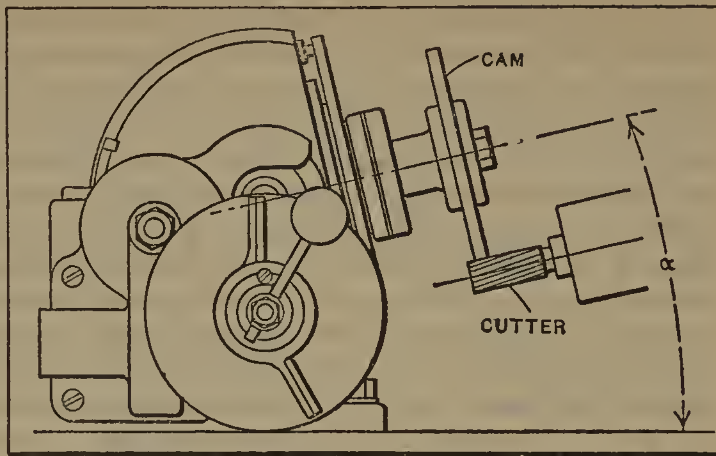
α = angle to which index head and milling attachment are set;
r = rise of cam in given part of circumference;
R = "lead" of cam, or rise if latter were continued at given rate for one complete revolution;
L = spiral lead for which milling machine is geared;
N = part of circumference in which rise is required, expressed as a decimal in hundredths of cam circumference.

$$\sin\alpha = \frac{R}{L}, \text{ and } R = \frac{r}{N}; \text{ hence, } \sin\alpha = \frac{r}{N \times L}$$

For example, suppose a cam is to be milled having a rise of 0.125 inch in 300 degrees or in 0.83 of the circumference, and that the machine is geared for the smallest possible lead, or 0.67 inch; then:

$$\sin\alpha = \frac{r}{N \times L} = \frac{0.125}{0.83 \times 0.67} = 0.2247,$$

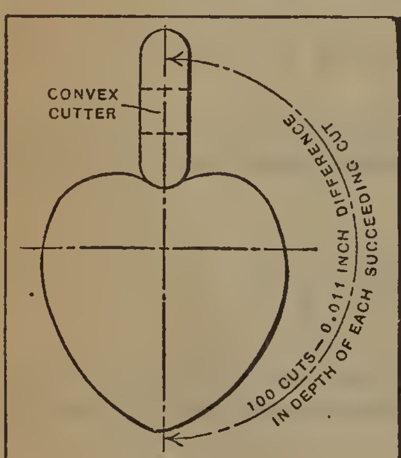
which is approximately the sine of 13 degrees. Therefore, to secure a rise of 0.125 inch with the machine geared for 0.67 inch lead, the spiral head is elevated to an angle of 13 degrees and the vertical milling attachment is also swiveled around to locate the cutter in line with the spiral-head spindle, so that the edge of the finished cam will be parallel to its axis of rotation. When there are several lobes on a cam, having different leads, the machine can be geared for a lead somewhat in excess of the greatest lead on the cam, and then all the lobes can be milled without changing the spiral head gearing, by simply varying the angle of the spiral head and cutter to suit the different cam leads. Whenever possible, it is advisable to mill on the under side of the cam, as there is less interference from chips; moreover, it is easier to see any lines that may be laid out on the cam face. To set the cam for a new cut, it is first turned back by operating the handle of the table feed screw, after which the index crank is disengaged from the plate and turned the required amount.



The accompanying tables give the combinations of change gears and the angular setting required for cutting a cam of any lead likely to be met with in practice. The figures in the column headed "Lead of Cam," represent the rise for one complete revolution. Set the vertical attachment to the angle given in the table. For the dividing head, subtract the angle in the table from 90 degrees; the difference is the angle to which the spindle must be raised from the horizontal position.

Example: — If the angle is 39½ degrees, set the spindle of the vertical attachment 39½ degrees from the vertical. Set the dividing head 50½ degrees from the horizontal position (90 − 39½ = 50½). These tables were compiled by the Cincinnati Milling Machine Co.

Simple Method for Cutting Uniform Motion Cams. — Cams are generally laid out with dividers, machined and filed to the line; but for a cam that must advance a certain number of thousandths per revolution of spindle this method is not accurate. Cams are easily and accurately cut in the following manner. Let it be required to make the heart cam shown in the illustration. The throw of this cam is 1.1 inch. Now, by setting the index on the milling machine to cut 200 teeth and also dividing 1.1 inch by 100, we find that we have 0.011 inch to recede from or advance towards the cam center for each cut across the cam. Placing the cam securely on an arbor, and the latter between the centers of the milling machine, and using a convex cutter set the proper distance from the center of the arbor, make the first cut across the cam. Then, by lowering the milling machine knee 0.011 inch and turning the index pin the proper number of holes on the index plate, take the next cut and so on. Each cut should be marked on paper so that there will be no mistake as to the number of cuts taken; when 100 cuts have been made, the knee must be raised in order to complete the opposite side of the cam.



This method can also be used to advantage for milling uniform motion cam lobes extending only over a portion of the cam circumference. After the milling has been completed, the surface of the cam must be smoothed off by means of filing.

Change Gears and Angles for Cam Milling — 1

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
0.600	24	86	24	100	26½	0.650	24	86	24	100	14	0.700	24	72	24	100	29
0.601	24	86	24	100	26	0.651	24	86	28	100	33½	0.701	24	72	24	86	41
0.602	24	86	24	100	26	0.652	24	86	24	100	13½	0.702	24	86	28	100	26
0.603	24	86	28	100	39½	0.653	24	86	32	100	43	0.703	24	72	24	100	28½
0.604	24	72	24	100	41	0.654	24	86	24	100	12½	0.704	24	86	32	100	38
0.605	24	86	24	100	25½	0.655	24	86	24	100	12	0.705	24	86	28	100	25½
0.606	24	86	28	100	39	0.656	24	86	24	100	11½	0.706	24	72	24	100	28
0.607	24	86	24	100	25	0.657	24	86	24	100	11	0.707	24	72	24	86	40½
0.608	24	72	24	100	40½	0.658	24	86	32	100	42½	0.708	24	86	28	100	25
0.609	24	86	24	100	24½	0.659	24	72	24	100	34½	0.709	24	72	28	100	40½
0.610	24	86	24	100	24½	0.660	24	86	24	100	10	0.710	24	72	24	100	27½
0.611	24	86	28	100	38½	0.661	24	86	28	100	32	0.711	24	86	28	100	24½
0.612	24	86	24	100	24	0.662	24	86	28	100	32	0.712	24	72	24	86	40
0.613	24	72	24	100	40	0.663	24	72	24	100	34	0.713	24	72	24	100	27
0.614	24	86	24	100	23½	0.664	24	86	32	100	42	0.714	24	64	24	100	37½
0.615	24	86	28	100	38	0.665	24	72	28	100	44½	0.715	24	72	28	100	40
0.616	24	86	24	100	23	0.666	24	86	28	100	31½	0.716	24	86	28	100	23½
0.617	24	72	24	100	39½	0.667	24	72	24	100	33½	0.717	24	72	24	86	39½
0.618	24	72	24	100	39½	0.668	24	64	24	100	42	0.718	24	86	32	100	36½
0.619	24	86	24	100	22½	0.669	24	72	24	86	44	0.719	24	86	28	100	23
0.620	24	86	28	100	37½	0.670	24	86	28	100	31	0.720	24	72	28	100	39½
0.621	24	86	24	100	22	0.671	24	72	24	100	33	0.721	24	86	28	100	22½
0.622	24	72	24	100	39	0.672	24	86	28	100	30½	0.722	24	72	24	100	25½
0.623	24	86	24	100	21½	0.673	24	86	28	100	30½	0.723	24	64	24	100	36½
0.624	24	86	28	100	37	0.674	24	64	24	100	41½	0.724	24	86	28	100	22
0.625	24	86	24	100	21	0.675	24	72	24	100	32½	0.725	24	72	24	100	25
0.626	24	86	32	100	45½	0.676	24	86	28	100	30	0.726	24	86	28	100	21½
0.627	24	86	24	100	20½	0.677	24	72	28	100	43½	0.727	24	86	32	100	35½
0.628	24	86	28	100	36½	0.678	24	72	24	100	32	0.728	24	72	24	100	24½
0.629	24	86	24	100	20	0.679	24	86	32	100	40½	0.729	24	86	28	100	21
0.630	24	72	24	100	38	0.680	24	72	24	86	43	0.730	24	72	28	100	38½
0.631	24	86	32	100	45	0.681	24	72	24	100	31½	0.731	24	72	24	100	24
0.632	24	86	28	100	36	0.682	24	72	28	100	43	0.732	24	86	28	100	20½
0.633	24	86	24	100	19	0.683	24	86	28	100	29	0.733	24	72	24	86	38
0.634	24	72	24	100	37½	0.684	24	86	32	100	40	0.734	24	86	28	100	20
0.635	24	86	24	100	18½	0.685	24	72	24	86	42½	0.735	24	72	28	100	38
0.636	24	86	28	100	35½	0.686	24	86	28	100	28½	0.736	24	86	28	100	19½
0.637	24	86	32	100	44½	0.687	24	72	28	100	42½	0.737	24	64	24	100	35
0.638	24	72	24	100	37	0.688	24	72	28	100	42½	0.738	24	72	24	86	37½
0.639	24	86	24	100	17½	0.689	24	86	32	100	39½	0.739	24	72	24	100	22½
0.640	24	86	28	100	35	0.690	24	86	28	100	28	0.740	24	72	28	100	37½
0.641	24	86	24	100	17	0.691	24	72	24	86	42	0.741	24	86	28	100	18½
0.642	24	86	32	100	44	0.692	24	86	28	100	27½	0.742	24	72	24	100	22
0.643	24	86	28	100	34½	0.693	24	72	28	100	42	0.743	24	86	28	100	18
0.644	24	86	24	100	16	0.694	24	86	32	100	39	0.744	24	72	24	100	21½
0.645	24	86	24	100	15½	0.695	24	64	24	100	39½	0.745	24	72	28	100	37
0.646	24	86	24	100	15½	0.696	24	86	28	100	27	0.746	24	64	24	100	34
0.647	24	86	24	100	15	0.697	24	72	24	86	41½	0.747	24	86	28	100	17
0.648	24	86	32	100	43½	0.698	24	72	28	100	41½	0.748	24	72	24	86	36½
0.649	24	86	24	100	14½	0.699	24	86	32	100	38½	0.749	24	86	28	100	16½

Change Gears and Angles for Cam Milling — 2

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
0.750	24	72	28	100	36½	0.800	24	72	28	100	31	0.850	24	72	24	86	24
0.751	24	86	28	100	16	0.801	24	72	24	86	30½	0.851	24	64	24	100	19
0.752	24	72	24	100	20	0.802	24	64	24	86	40	0.852	24	72	28	100	24
0.753	24	86	32	100	32½	0.803	24	86	32	100	26	0.853	24	72	24	86	23½
0.754	24	72	24	100	19½	0.804	28	86	32	100	39½	0.854	24	86	32	100	17
0.755	24	72	28	100	36	0.805	24	72	32	100	41	0.855	24	64	28	100	35½
0.756	24	86	28	100	14½	0.806	24	86	32	100	25½	0.856	24	86	32	100	16½
0.757	24	72	24	86	35½	0.807	24	64	24	86	39½	0.857	24	64	24	86	35
0.758	24	86	28	100	14	0.808	24	72	28	100	30	0.858	24	64	24	100	17½
0.759	24	64	24	100	32½	0.809	24	64	24	100	26	0.859	24	72	24	86	22½
0.760	24	72	28	100	35½	0.810	28	86	32	100	39	0.860	24	64	28	100	35
0.761	24	86	28	100	13	0.811	24	72	32	100	40½	0.861	24	72	28	86	37½
0.762	24	72	24	86	35	0.812	24	72	28	100	29½	0.862	24	72	28	100	22½
0.763	24	72	24	100	17½	0.813	24	72	24	86	29	0.863	24	64	24	100	16½
0.764	24	86	28	100	12	0.814	24	64	24	86	39	0.864	28	86	32	100	34
0.765	24	72	24	100	17	0.815	28	86	32	100	38½	0.865	24	64	24	100	16
0.766	24	72	24	86	34½	0.816	24	72	28	100	29	0.866	24	86	32	100	14
0.767	24	72	24	100	16½	0.817	24	72	24	86	28½	0.867	24	64	24	100	15½
0.768	24	86	28	100	10½	0.818	24	72	28	86	41	0.868	24	72	28	100	21½
0.769	24	86	28	100	10	0.819	24	86	32	100	23½	0.869	24	64	24	100	15
0.770	24	86	32	100	30½	0.820	24	72	28	100	28½	0.870	24	86	32	100	13
0.771	24	72	24	86	34	0.821	24	72	24	86	28	0.871	24	64	24	100	14½
0.772	24	72	24	100	15	0.822	24	86	32	100	23	0.872	24	86	32	100	12½
0.773	24	86	32	100	30	0.823	24	72	32	100	39½	0.873	24	64	24	100	14
0.774	24	72	24	100	14½	0.824	24	72	28	100	28	0.874	24	72	24	86	20
0.775	24	64	24	100	30½	0.825	24	72	24	86	27½	0.875	24	86	32	100	11½
0.776	24	72	24	100	14	0.826	28	86	32	100	37½	0.876	24	64	28	100	33½
0.777	24	86	32	100	29½	0.827	24	72	28	100	27½	0.877	24	64	24	100	13
0.778	24	72	28	100	33½	0.828	24	86	32	100	22	0.878	24	86	32	100	10½
0.779	24	72	24	100	13	0.829	24	86	40	100	42	0.879	24	64	24	100	12½
0.780	24	72	24	86	33	0.830	24	64	24	86	37½	0.880	24	64	24	100	12
0.781	24	72	24	100	12½	0.831	24	72	28	86	40	0.881	24	64	28	100	33
0.782	24	72	28	100	33	0.832	24	72	24	86	26½	0.882	24	64	24	100	11½
0.783	24	64	24	100	29½	0.833	24	56	24	100	36	0.883	24	64	24	86	32½
0.784	24	72	24	100	11½	0.834	24	86	32	100	21	0.884	28	86	32	100	32
0.785	24	86	32	100	28½	0.835	24	72	32	100	38½	0.885	24	64	24	100	10½
0.786	28	86	32	100	41	0.836	24	72	24	86	26	0.886	24	64	24	100	10
0.787	24	64	24	100	29	0.837	24	72	28	86	39½	0.887	24	72	24	86	17½
0.788	24	72	24	100	10	0.838	24	56	24	100	35½	0.888	24	64	24	86	32
0.789	24	72	24	86	32	0.839	24	86	32	100	20	0.889	24	72	24	86	17
0.790	24	64	24	86	41	0.840	24	64	24	100	21	0.890	24	72	28	100	17½
0.791	24	64	24	100	28½	0.841	24	72	32	100	38	0.891	24	56	24	100	30
0.792	24	86	32	100	27½	0.842	24	86	32	100	19½	0.892	24	72	24	86	16½
0.793	24	72	24	86	31½	0.843	24	72	28	86	39	0.893	28	86	32	100	31
0.794	24	72	28	86	43	0.844	24	86	32	100	19	0.894	24	72	24	86	16
0.795	24	72	28	100	31½	0.845	24	72	28	100	25	0.895	24	64	28	100	31½
0.796	24	64	24	86	40½	0.846	24	64	24	100	20	0.896	24	72	24	86	15½
0.797	24	72	24	86	31	0.847	24	86	32	100	18½	0.897	24	72	28	100	16
0.798	28	86	32	100	40	0.848	24	64	24	100	19½	0.898	24	72	24	86	15
0.799	24	72	32	100	41½	0.849	24	86	32	100	18	0.899	24	72	28	100	15½

Change Gears and Angles for Cam Milling—3

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
0.900	24	56	24	100	29	0.950	24	72	32	86	40	1.000	24	86	44	100	35½
0.901	24	72	28	100	15	0.951	24	56	24	100	22½	1.001	24	56	24	100	13½
0.902	24	72	24	86	14	0.952	28	86	32	100	24	1.002	28	86	32	100	16
0.903	24	72	28	100	14½	0.953	24	64	24	86	24½	1.003	24	56	24	100	13
0.904	24	72	24	86	13½	0.954	24	56	24	100	22	1.004	28	86	32	100	15½
0.905	24	72	28	100	14	0.955	24	72	32	100	26½	1.005	24	56	24	100	12½
0.906	24	72	24	86	13	0.956	24	64	28	86	38½	1.006	24	56	24	100	12
0.907	24	72	28	100	13½	0.957	24	56	24	100	21½	1.007	24	64	24	86	16
0.908	24	72	24	86	12½	0.958	24	72	28	86	28	1.008	24	56	24	100	11½
0.909	24	72	28	100	13	0.959	24	72	32	100	26	1.009	28	86	32	100	14½
0.910	24	72	32	100	31½	0.960	24	64	24	86	23½	1.010	24	56	24	100	11
0.911	24	72	28	100	12½	0.961	24	86	44	100	38½	1.011	28	86	32	100	14
0.912	24	72	28	100	12	0.962	24	72	28	86	27½	1.012	24	56	24	100	10½
0.913	24	72	24	86	11	0.963	28	86	32	100	22½	1.013	24	56	24	100	10
0.914	24	72	28	100	11½	0.964	24	56	24	100	20½	1.014	24	64	24	86	14½
0.915	24	72	32	100	31	0.965	24	64	32	100	36½	1.015	28	86	32	100	13
0.916	24	72	24	86	10	0.966	28	86	32	100	22	1.016	24	64	24	86	14
0.917	24	72	28	100	10½	0.967	24	56	24	100	20	1.017	28	86	32	100	12½
0.918	24	64	28	100	29	0.968	24	56	24	86	36	1.018	24	64	24	86	13½
0.919	24	72	28	100	10	0.969	24	64	28	86	37½	1.019	28	86	32	100	12
0.920	28	86	32	100	28	0.970	24	56	24	100	19½	1.020	24	64	24	86	13
0.921	24	56	24	100	26½	0.971	24	72	28	86	26½	1.021	28	86	32	100	11½
0.922	24	64	28	86	41	0.972	86	44	32	64	6	1.022	24	64	24	86	12½
0.923	24	64	28	100	28½	0.973	24	56	24	100	19	1.023	28	86	32	100	11
0.924	28	86	32	100	27½	0.974	24	64	24	86	21½	1.024	24	64	24	86	12
0.925	24	56	24	100	26	0.975	24	72	32	100	24	1.025	24	64	28	100	12½
0.926	24	64	32	100	39½	0.976	28	86	32	100	20½	1.026	24	64	24	86	11½
0.927	24	64	28	100	28	0.977	24	64	28	100	21½	1.027	24	64	28	100	12
0.928	24	64	28	86	40½	0.978	24	56	24	100	18	1.028	24	64	24	86	11
0.929	24	56	24	100	25½	0.979	28	86	32	100	20	1.029	24	64	28	100	11½
0.930	24	72	28	86	31	0.980	24	64	28	100	21	1.030	24	64	24	86	10½
0.931	24	64	28	100	27½	0.981	24	64	24	86	20½	1.031	24	64	28	100	11
0.932	28	72	32	100	41½	0.982	28	86	32	100	19½	1.032	24	64	28	100	10½
0.933	24	64	24	86	27	0.983	24	72	28	86	25	1.033	24	72	32	100	14½
0.934	24	86	44	100	40½	0.984	24	56	24	100	17	1.034	24	64	28	100	10
0.935	24	72	28	86	30½	0.985	28	86	32	100	19	1.035	24	72	32	100	14
0.936	24	56	24	100	24½	0.986	24	72	32	100	22½	1.036	24	56	24	86	30
0.937	24	64	24	86	26½	0.987	24	64	24	86	19½	1.037	24	72	32	100	13½
0.938	24	72	32	100	28½	0.988	28	86	32	100	18½	1.038	24	72	28	86	17
0.939	24	64	32	100	38½	0.989	24	56	24	100	16	1.039	24	64	32	100	30
0.940	24	56	24	100	24	0.990	24	64	24	86	19	1.040	24	72	32	100	13
0.941	24	64	24	86	26	0.991	28	86	32	100	18	1.041	24	56	24	86	29½
0.942	24	72	32	100	28	0.992	24	56	24	100	15½	1.042	24	72	32	100	12½
0.943	24	72	32	86	40½	0.993	24	64	24	86	18½	1.043	24	72	28	86	16
0.944	24	56	24	100	23½	0.994	24	56	24	100	15	1.044	24	72	32	100	12
0.945	24	64	24	86	25½	0.995	24	72	28	86	23½	1.045	24	86	40	100	20½
0.946	24	72	32	100	27½	0.996	24	56	24	100	14½	1.046	24	72	32	100	11½
0.947	24	56	24	100	23	0.997	24	56	24	86	33½	1.047	24	72	32	100	11
0.948	28	86	32	100	24½	0.998	24	56	24	100	14	1.048	24	72	28	86	15
0.949	24	64	24	86	25	0.999	28	86	32	100	16½	1.049	24	72	32	100	10½

Change Gears and Angles for Cam Milling — 4

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
1.050	24	72	28	86	14½	1.100	28	72	32	86	40½	1.150	24	56	24	86	16
1.051	24	72	32	100	10	1.101	24	56	24	86	23	1.151	24	64	32	100	16½
1.052	24	86	40	100	19½	1.102	24	64	28	86	25½	1.152	28	86	44	100	36½
1.053	24	72	28	86	14	1.103	28	72	32	100	27½	1.153	24	56	24	86	15½
1.054	24	72	28	86	14	1.104	24	86	44	100	26	1.154	24	64	28	86	19
1.055	24	72	28	86	13½	1.105	24	56	24	86	22½	1.155	24	56	24	86	15
1.056	24	56	24	86	28	1.106	40	64	24	100	42½	1.156	24	64	32	100	15½
1.057	24	72	28	86	13	1.107	24	64	28	86	25	1.157	28	72	32	100	21½
1.058	24	86	40	100	18½	1.108	24	86	44	100	25½	1.158	24	56	24	86	14½
1.059	24	72	28	86	12½	1.109	24	56	24	86	22	1.159	24	64	32	100	15
1.060	28	86	40	100	35½	1.110	24	72	32	86	26½	1.160	24	56	24	86	14
1.061	24	72	28	86	12	1.111	24	64	28	86	24½	1.161	24	64	28	86	18
1.062	24	72	28	86	12	1.112	24	72	40	100	33½	1.162	24	64	32	100	14½
1.063	24	72	28	86	11½	1.113	24	56	24	86	21½	1.163	24	56	24	86	13½
1.064	24	86	40	100	17½	1.114	24	64	32	86	37	1.164	24	64	32	100	14
1.065	24	72	28	86	11	1.115	24	64	28	86	24	1.165	24	56	24	86	13
1.066	24	56	24	86	27	1.116	24	56	24	86	21	1.166	24	72	40	100	29
1.067	24	72	28	86	10½	1.117	24	86	44	100	24½	1.167	24	64	32	100	13½
1.068	24	64	28	86	29	1.118	28	72	32	100	26	1.168	24	56	24	86	12½
1.069	24	72	28	86	10	1.119	24	72	32	86	25½	1.169	24	64	32	100	13
1.070	24	86	40	100	16½	1.120	24	56	24	86	20½	1.170	24	56	24	86	12
1.071	32	56	24	100	38½	1.121	24	64	32	86	36½	1.171	24	64	28	86	16½
1.072	28	72	32	100	30½	1.122	24	86	44	100	24	1.172	24	56	24	86	11½
1.073	24	86	40	100	16	1.123	28	72	32	100	25½	1.173	28	72	32	100	19½
1.074	24	64	32	100	26½	1.124	24	56	24	86	20	1.174	24	56	24	86	11
1.075	24	86	40	100	15½	1.125	28	64	32	100	36½	1.175	28	86	40	100	25½
1.076	24	64	32	86	39½	1.126	24	86	44	100	23½	1.176	24	56	24	86	10½
1.077	28	72	32	100	30	1.127	24	56	24	86	19½	1.177	24	64	28	86	15½
1.078	24	86	40	100	15	1.128	24	64	32	100	20	1.178	24	56	24	86	10
1.079	24	56	24	86	25½	1.129	24	64	32	86	36	1.179	24	64	28	86	15
1.080	24	86	40	100	14½	1.130	24	72	40	100	32	1.180	24	64	32	100	10½
1.081	28	64	32	100	39½	1.131	24	56	24	86	19	1.181	32	56	24	100	30½
1.082	28	86	44	100	41	1.132	24	64	28	86	22	1.182	24	64	32	100	10
1.083	24	86	40	100	14	1.133	24	72	32	86	24	1.183	24	86	44	100	15½
1.084	24	56	24	86	25	1.134	24	56	24	86	18½	1.184	24	64	32	100	9½
1.085	24	86	40	100	13½	1.135	24	64	32	100	19	1.185	24	64	28	86	14
1.086	28	86	40	100	33½	1.136	24	64	28	86	21½	1.186	24	86	44	100	15
1.087	24	86	40	100	13	1.137	24	56	24	86	18	1.187	24	64	28	86	13½
1.088	24	56	24	86	24½	1.138	24	64	32	100	18½	1.188	24	72	40	100	27
1.089	24	86	40	100	12½	1.139	28	86	40	100	29	1.189	24	86	44	100	14½
1.090	24	72	32	86	28½	1.140	24	64	28	86	21	1.190	24	64	28	86	13
1.091	24	86	48	100	35½	1.141	24	56	24	86	17½	1.191	24	86	44	100	14
1.092	24	86	40	100	12	1.142	24	64	32	86	35	1.192	24	64	28	86	12½
1.093	24	56	24	86	24	1.143	24	86	44	100	21½	1.193	28	72	32	100	16½
1.094	24	86	40	100	11½	1.144	24	56	24	86	17	1.194	24	64	28	86	12
1.095	24	72	32	86	28	1.145	28	72	32	100	23	1.195	24	72	32	86	15½
1.096	24	86	40	100	11	1.146	24	86	44	100	21	1.196	28	72	32	100	16
1.097	24	86	40	100	10½	1.147	24	56	24	86	16½	1.197	24	64	28	86	11½
1.098	28	72	32	100	28	1.148	24	64	32	100	17	1.198	24	72	32	86	15
1.099	24	86	40	100	10	1.149	28	72	32	100	22½	1.199	24	64	28	86	11

Change Gears and Angles for Cam Milling — 5

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
1.200	24	72	32	86	14½	1.250	24	64	28	72	31	1.300	24	86	48	100	14
1.201	24	64	28	86	10½	1.251	24	86	48	100	21	1.301	24	72	40	100	12½
1.202	24	64	28	86	10	1.252	28	86	40	100	16	1.302	24	64	32	86	21
1.203	24	86	44	100	11½	1.253	24	72	40	100	20	1.303	24	86	48	100	13½
1.204	28	72	32	100	14½	1.254	24	64	32	86	26	1.304	24	72	40	100	12
1.205	24	86	44	100	11	1.255	28	86	40	100	15½	1.305	24	64	28	72	26½
1.206	24	72	32	86	13½	1.256	24	64	28	72	30½	1.306	24	72	40	100	11½
1.207	24	86	44	100	10½	1.257	24	72	40	100	19½	1.307	32	56	24	100	17½
1.208	24	72	32	86	13	1.258	28	86	40	100	15	1.308	24	72	40	100	11
1.209	24	86	44	100	10	1.259	24	86	48	100	20	1.309	28	86	44	100	24
1.210	28	72	32	100	13½	1.260	28	86	40	100	14½	1.310	24	64	28	72	26
1.211	24	72	32	86	12½	1.261	28	86	40	100	14½	1.311	24	72	40	100	10½
1.212	28	72	32	100	13	1.262	32	56	24	100	23	1.312	40	64	24	100	29
1.213	24	72	32	86	12	1.263	28	86	40	100	14	1.313	24	72	40	100	10
1.214	24	86	48	100	25	1.264	24	72	40	100	18½	1.314	28	86	44	100	23½
1.215	24	72	32	86	11½	1.265	28	86	44	100	28	1.315	24	86	48	100	11
1.216	32	56	24	100	27½	1.266	28	86	40	100	13½	1.316	28	64	32	100	20
1.217	24	72	32	86	11	1.267	24	86	48	100	19	1.317	28	72	32	86	24½
1.218	24	72	40	100	24	1.268	24	72	40	100	18	1.318	24	86	48	100	10½
1.219	24	72	32	86	10½	1.269	28	86	40	100	13	1.319	24	64	32	86	19
1.220	28	86	40	100	20½	1.270	24	72	44	100	30	1.320	24	86	48	100	10
1.221	24	72	32	86	10	1.271	28	86	40	100	12½	1.321	32	56	24	100	15½
1.222	24	72	40	100	23½	1.272	28	72	32	86	28½	1.322	28	72	32	86	24
1.223	28	72	32	100	10½	1.273	28	86	40	100	12	1.323	24	64	32	86	18½
1.224	28	86	40	100	20	1.274	28	86	40	100	12	1.324	32	56	24	100	15
1.225	28	72	32	100	10	1.275	24	72	40	100	17	1.325	28	86	48	100	32
1.226	24	72	48	100	40	1.276	28	86	40	100	11½	1.326	32	86	40	100	27
1.227	28	86	40	100	19½	1.277	28	86	44	100	27	1.327	32	56	24	100	14½
1.228	28	86	44	100	31	1.278	28	86	40	100	11	1.328	28	64	32	100	18½
1.229	24	86	48	100	23½	1.279	24	64	32	86	23½	1.329	28	86	44	100	22
1.230	28	64	32	100	28½	1.280	28	86	40	100	10½	1.330	32	56	24	100	14
1.231	28	86	40	100	19	1.281	24	72	40	100	16	1.331	28	64	32	100	18
1.232	24	72	40	100	22½	1.282	28	86	40	100	10	1.332	28	64	32	100	18
1.233	32	86	40	100	34	1.283	28	72	32	86	27½	1.333	32	56	24	100	13½
1.234	24	86	48	100	23	1.284	24	72	40	100	15½	1.334	24	64	32	86	17
1.235	28	86	40	100	18½	1.285	24	72	40	100	15½	1.335	28	64	32	100	17½
1.236	24	72	40	100	22	1.286	40	64	24	100	31	1.336	32	56	24	100	13
1.237	32	56	24	100	25½	1.287	24	72	40	100	15	1.337	28	72	32	86	22½
1.238	28	86	40	100	18	1.288	24	72	40	100	15	1.338	32	56	24	100	12½
1.239	24	64	32	72	42	1.289	24	64	32	86	22½	1.339	32	56	24	100	12½
1.240	24	72	40	100	21½	1.290	24	72	40	100	14½	1.340	24	72	44	100	24
1.241	28	86	44	100	30	1.291	24	72	40	100	14½	1.341	32	56	24	100	12
1.242	28	86	40	100	17½	1.292	32	56	24	100	19½	1.342	28	64	32	100	16½
1.243	32	56	24	100	25	1.293	24	72	40	100	14	1.343	32	56	24	100	11½
1.244	24	72	40	100	21	1.294	24	86	48	100	15	1.344	24	64	32	86	15½
1.245	28	86	40	100	17	1.295	28	72	32	86	26½	1.345	24	72	44	100	23½
1.246	32	72	40	100	45½	1.296	24	72	40	100	13½	1.346	32	56	24	100	11
1.247	24	86	48	100	21½	1.297	24	86	48	100	14½	1.347	24	64	32	86	15
1.248	28	86	40	100	16½	1.298	24	64	32	86	21½	1.348	32	56	24	100	10½
1.249	24	72	40	100	20½	1.299	24	72	40	100	13	1.349	28	64	32	100	15½

Change Gears and Angles for Cam Milling — 6

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
1.350	32	56	24	100	10	1.400	40	64	24	100	21	1.450	32	86	40	100	13
1.351	24	64	32	86	14½	1.401	28	72	32	86	14½	1.451	28	64	32	86	27
1.352	28	64	32	100	15	1.402	28	86	44	100	12	1.452	40	64	24	100	14½
1.353	24	72	44	86	37½	1.403	24	72	44	100	17	1.453	32	86	40	100	12½
1.354	24	64	32	86	14	1.404	28	72	32	86	14	1.454	28	86	48	100	21½
1.355	28	64	32	100	14½	1.405	24	64	28	72	15½	1.455	32	86	40	100	12
1.356	24	64	32	86	13½	1.406	28	86	44	100	11	1.456	24	72	40	86	20
1.357	24	64	28	72	21½	1.407	28	72	32	86	13½	1.457	24	72	40	86	20
1.358	28	64	32	100	14	1.408	24	64	28	72	15	1.458	32	86	40	100	11½
1.359	24	64	32	86	13	1.409	28	86	44	100	10½	1.459	40	64	24	100	13½
1.360	28	72	32	86	20	1.410	28	72	32	86	13	1.460	24	44	32	86	44
1.361	28	64	32	100	13½	1.411	28	86	48	100	25½	1.461	32	86	40	100	11
1.362	24	64	32	86	12½	1.412	24	64	28	72	14½	1.462	40	64	24	100	13
1.363	28	86	44	100	18	1.413	28	72	32	86	12½	1.463	32	86	40	100	10½
1.364	24	64	32	86	12	1.414	24	72	44	100	15½	1.464	40	64	24	100	12½
1.365	24	64	32	86	12	1.415	28	72	32	86	12	1.465	32	86	40	100	10
1.366	24	64	28	72	20½	1.416	24	64	44	86	42½	1.466	24	72	40	86	19
1.367	24	64	32	86	11½	1.417	24	72	44	100	15	1.467	40	64	24	100	12
1.368	28	72	32	86	19	1.418	28	72	32	86	11½	1.468	28	64	40	100	33
1.369	24	64	32	86	11	1.419	32	86	40	100	17½	1.469	28	86	48	100	20
1.370	28	86	44	100	17	1.420	28	72	32	86	11	1.470	40	64	24	100	11½
1.371	24	64	32	86	10½	1.421	24	64	28	72	13	1.471	28	72	40	100	19
1.372	24	64	32	86	10½	1.422	40	64	24	100	18½	1.472	40	64	24	100	11
1.373	28	64	44	100	44½	1.423	28	72	32	86	10½	1.473	40	64	24	100	11
1.374	24	64	32	86	10	1.424	28	64	32	86	29	1.474	24	72	40	86	18
1.375	32	86	40	100	22½	1.425	28	72	32	86	10	1.475	40	64	24	100	10½
1.376	28	72	32	86	18	1.426	24	64	28	72	12	1.476	28	72	40	100	18½
1.377	28	64	32	100	10½	1.427	32	86	40	100	16½	1.477	40	64	24	100	10
1.378	28	86	44	100	16	1.428	28	86	48	100	24	1.478	24	72	40	86	17½
1.379	28	64	32	100	10	1.429	24	64	28	72	11½	1.479	24	64	32	72	27½
1.380	28	72	32	86	17½	1.430	32	86	40	100	16	1.480	28	72	40	100	18
1.381	28	86	44	100	15½	1.431	24	64	28	72	11	1.481	28	64	32	86	24½
1.382	24	64	32	72	34	1.432	24	72	44	100	12½	1.482	24	72	40	86	17
1.383	24	64	28	72	18½	1.433	28	86	48	100	23½	1.483	44	64	24	100	26
1.384	28	86	44	100	15	1.434	24	64	28	72	10½	1.484	28	72	40	100	17½
1.385	28	64	44	100	44	1.435	24	72	44	100	12	1.485	24	64	32	72	27
1.386	40	64	24	100	22½	1.436	24	64	28	72	10	1.486	24	72	40	86	16½
1.387	28	86	44	100	14½	1.437	32	86	40	100	15	1.487	28	86	48	100	18
1.388	28	64	32	86	31½	1.438	24	72	44	100	11½	1.488	28	72	40	100	17
1.389	32	86	40	100	21	1.439	28	86	48	100	23	1.489	24	72	48	100	21½
1.390	28	86	44	100	14	1.440	24	72	44	100	11	1.490	24	72	40	86	16
1.391	28	72	32	86	16	1.441	32	86	40	100	14½	1.491	28	86	48	100	17½
1.392	44	64	24	100	32½	1.442	24	72	44	100	10½	1.492	28	72	40	100	16½
1.393	28	86	44	100	13½	1.443	24	72	44	100	10½	1.493	28	64	32	86	23½
1.394	28	72	32	86	15½	1.444	32	86	40	100	14	1.494	24	72	40	86	15½
1.395	24	72	44	100	18	1.445	24	72	44	100	10	1.495	28	86	48	100	17
1.396	28	86	44	100	13	1.446	24	72	44	86	32	1.496	28	72	40	100	16
1.397	24	72	44	86	35	1.447	32	56	40	100	13½	1.497	24	72	40	86	15
1.398	28	72	32	86	15	1.448	28	72	40	100	21½	1.498	32	64	40	100	41½
1.399	28	86	44	100	12½	1.449	40	64	24	100	15	1.499	28	72	40	100	15½

Change Gears and Angles for Cam Milling — 7

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
1.500	28	64	40	100	31	1.550	44	64	24	100	20	1.600	44	56	24	100	21
1.501	32	86	44	100	23½	1.551	44	64	24	100	20	1.601	28	64	32	86	10½
1.502	28	86	48	100	16	1.552	24	72	48	100	14	1.602	24	64	32	72	16
1.503	28	72	40	100	15	1.553	28	64	32	72	37	1.603	28	64	32	86	10
1.504	24	72	40	86	14	1.554	24	64	40	86	27	1.604	32	86	44	100	11½
1.505	24	64	32	72	25½	1.555	44	64	24	100	19½	1.605	40	56	24	100	20½
1.506	28	72	40	100	14½	1.556	24	64	32	72	21	1.606	24	64	32	72	15½
1.507	24	72	40	86	13½	1.557	28	64	32	86	17	1.607	32	86	44	100	11
1.508	24	72	48	100	19½	1.558	24	72	44	86	24	1.608	44	64	24	100	13
1.509	28	64	32	86	22	1.559	24	72	48	100	13	1.609	28	72	48	100	30½
1.510	24	72	40	86	13	1.560	44	64	24	100	19	1.610	32	86	44	100	10½
1.511	24	64	32	72	25	1.561	28	64	32	86	16½	1.611	44	64	24	100	12½
1.512	24	64	44	86	38	1.562	24	72	48	100	12½	1.612	32	86	44	100	10
1.513	24	72	40	86	12½	1.563	28	72	44	100	24	1.613	28	72	44	100	19½
1.514	24	72	48	86	35½	1.564	24	72	44	86	23½	1.614	44	64	24	100	12
1.515	28	64	32	86	21½	1.565	24	72	48	100	12	1.615	24	64	48	86	39½
1.516	24	72	40	86	12	1.566	32	86	44	100	17	1.616	40	56	24	100	19½
1.517	24	72	48	100	18½	1.567	28	64	44	100	35½	1.617	44	64	24	100	11½
1.518	32	86	44	100	22	1.568	24	72	48	100	11½	1.618	24	64	32	72	14
1.519	24	72	40	86	11½	1.569	28	64	32	86	15½	1.619	32	86	48	100	25
1.520	28	86	48	100	13½	1.570	32	72	40	100	28	1.620	44	64	24	100	11
1.521	24	72	40	86	11	1.571	24	72	48	100	11	1.621	24	64	32	72	13½
1.522	24	72	40	86	11	1.572	28	64	32	86	15	1.622	44	64	24	100	10½
1.523	28	86	48	100	13	1.573	24	72	48	100	10½	1.623	28	72	44	100	18½
1.524	24	72	40	86	10½	1.574	32	86	44	100	16	1.624	24	64	32	72	13
1.525	28	72	40	100	11½	1.575	24	72	44	86	22½	1.625	44	64	24	100	10
1.526	24	72	40	86	10	1.576	24	72	48	100	10	1.626	24	72	44	86	17½
1.527	28	72	40	100	11	1.577	32	86	44	100	15½	1.627	24	64	32	72	12½
1.528	32	86	44	100	21	1.578	44	64	24	100	17	1.628	24	64	32	72	12½
1.529	28	86	48	100	12	1.579	28	100	56	86	30	1.629	32	72	40	86	38
1.530	28	72	40	100	10½	1.580	28	64	32	86	14	1.630	24	64	32	72	12
1.531	28	72	44	100	26½	1.581	32	86	44	100	15	1.631	24	64	32	72	12
1.532	28	72	40	100	10	1.582	44	64	24	100	16½	1.632	28	72	44	100	17½
1.533	32	86	44	100	20½	1.583	28	64	32	86	13½	1.633	28	72	40	86	25½
1.534	28	86	48	100	11	1.584	40	56	24	100	22½	1.634	24	64	32	72	11½
1.535	28	64	32	86	19½	1.585	32	86	44	100	14½	1.635	24	72	44	86	16½
1.536	32	72	40	86	42	1.586	28	64	32	86	13	1.636	24	64	32	72	11
1.537	28	86	48	100	10½	1.587	24	64	40	86	24½	1.637	32	72	40	100	23
1.538	24	72	48	100	16	1.588	32	86	44	100	14	1.638	32	86	48	100	23½
1.539	28	86	48	100	10	1.589	28	64	32	86	12½	1.639	24	64	32	72	10½
1.540	28	100	56	72	45	1.590	44	64	24	100	15½	1.640	28	72	40	86	25
1.541	40	56	24	100	26	1.591	32	72	40	100	26½	1.641	28	72	44	100	16½
1.542	24	72	48	100	15½	1.592	28	64	32	86	12	1.642	24	64	32	72	10
1.543	32	86	44	100	19½	1.593	24	64	40	86	24	1.643	24	72	44	86	15½
1.544	28	64	32	86	18½	1.594	44	64	24	100	15	1.644	24	64	40	86	19½
1.545	24	72	48	100	15	1.595	28	64	32	86	11½	1.645	28	72	44	100	16
1.546	24	72	48	100	15	1.596	28	64	44	100	34	1.646	28	72	40	86	24½
1.547	40	56	24	100	25½	1.597	44	64	24	100	14½	1.647	24	72	44	86	15
1.548	28	64	32	86	18	1.598	28	64	32	86	11	1.648	40	56	24	100	16
1.549	24	72	48	100	14½	1.599	24	64	40	86	23½	1.649	28	72	44	100	15½

Change Gears and Angles for Cam Milling — 8

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
1.650	28	64	40	100	19½	1.700	32	72	40	100	17	1.750	32	86	48	100	11½
1.651	24	72	44	86	14½	1.701	32	64	40	86	43	1.751	32	72	40	100	10
1.652	40	56	24	100	15½	1.702	28	64	40	100	13½	1.752	28	100	56	86	16
1.653	28	72	44	100	15	1.703	24	64	40	86	12½	1.753	32	86	48	100	11
1.654	24	72	44	86	14	1.704	28	64	40	72	45½	1.754	28	72	48	100	20
1.655	28	64	40	100	19	1.705	28	64	40	100	13	1.755	28	72	40	86	14
1.656	28	72	44	100	14½	1.706	24	64	40	86	12	1.756	32	86	48	100	10½
1.657	28	72	44	100	14½	1.707	40	86	44	100	33½	1.757	28	100	56	86	15½
1.658	24	72	44	86	13½	1.708	28	64	40	100	12½	1.758	32	72	44	100	26
1.659	24	64	40	86	18	1.709	24	64	40	86	11½	1.759	32	86	48	100	10
1.660	28	72	44	100	14	1.710	28	72	40	86	19	1.760	28	72	48	100	19½
1.661	24	72	44	86	13	1.711	32	72	44	100	29	1.761	28	100	56	86	15
1.662	32	86	48	100	21½	1.712	24	64	40	86	11	1.762	28	64	32	72	25
1.663	40	56	24	100	14	1.713	32	72	40	100	15½	1.763	28	72	40	86	13
1.664	28	72	44	100	13½	1.714	32	64	40	100	31	1.764	24	72	48	86	18½
1.665	24	72	44	86	12½	1.715	24	64	40	86	10½	1.765	28	100	56	86	14½
1.666	28	64	32	72	31	1.716	28	72	40	86	18½	1.766	28	72	40	86	12½
1.667	28	72	44	100	13	1.717	32	72	40	100	15	1.767	44	56	24	100	20½
1.668	24	72	44	86	12	1.718	24	64	40	86	10	1.768	32	72	48	100	34
1.669	28	64	40	100	17½	1.719	28	72	48	100	23	1.769	28	72	40	86	12
1.670	28	72	44	100	12½	1.720	28	72	40	86	18	1.770	28	72	48	100	18½
1.671	24	72	44	86	11½	1.721	28	64	40	100	10½	1.771	28	72	48	100	18½
1.672	24	64	40	86	16½	1.722	24	44	32	86	32	1.772	44	56	24	100	20
1.673	40	56	24	100	12½	1.723	28	64	40	100	10	1.773	28	72	40	86	11½
1.674	24	72	44	86	11	1.724	28	100	56	86	19	1.774	24	72	48	86	17½
1.675	28	64	44	100	29½	1.725	32	72	40	100	14	1.775	24	44	32	86	29
1.676	24	72	44	86	10½	1.726	24	56	40	86	30	1.776	28	72	40	86	11
1.677	28	72	44	100	11½	1.727	32	72	44	100	28	1.777	32	56	40	100	39
1.678	28	64	40	100	16½	1.728	32	72	40	100	13½	1.778	44	56	24	100	19½
1.679	24	72	44	86	10	1.729	32	72	40	100	13½	1.779	28	72	40	86	10½
1.680	28	72	44	100	11	1.730	28	72	40	86	17	1.780	28	100	56	86	12½
1.681	24	64	40	86	15½	1.731	24	72	48	86	21½	1.781	28	72	40	86	10
1.682	28	72	44	100	10½	1.732	32	72	40	100	13	1.782	32	64	40	100	27
1.683	40	56	24	100	11	1.733	32	86	48	100	14	1.783	28	100	56	86	12
1.684	32	86	48	100	19½	1.734	28	72	40	86	16½	1.784	24	56	40	86	26½
1.685	28	72	44	100	10	1.735	28	72	40	86	16½	1.785	28	72	48	100	17
1.686	28	64	40	100	15½	1.736	32	72	40	100	12½	1.786	28	100	56	86	11½
1.687	32	64	40	100	32½	1.737	32	86	48	100	13½	1.787	32	72	44	100	24
1.688	40	56	24	100	10	1.738	32	56	40	100	40½	1.788	24	72	48	86	16
1.689	32	86	48	100	19	1.739	32	72	40	100	12	1.789	28	100	56	86	11
1.690	28	64	40	100	15	1.740	32	86	48	100	13	1.790	28	100	56	86	11
1.691	32	72	40	100	18	1.741	28	72	44	86	29	1.791	24	64	44	86	21
1.692	24	64	40	86	14	1.742	32	72	40	100	11½	1.792	28	100	56	86	10½
1.693	24	72	48	86	24½	1.743	28	72	40	86	15½	1.793	28	100	56	86	10½
1.694	32	72	44	100	30	1.744	32	86	48	100	12½	1.794	44	56	24	100	18
1.695	44	56	24	100	26	1.745	32	72	40	100	11	1.795	28	100	56	86	10
1.696	24	64	40	86	13½	1.746	24	64	44	86	24½	1.796	28	64	32	72	22½
1.697	28	72	44	86	31½	1.747	32	86	48	100	12	1.797	24	72	48	86	15
1.698	28	64	40	100	14	1.748	32	72	40	100	10½	1.798	24	64	44	86	20½
1.699	24	64	40	86	13	1.749	28	72	48	100	20½	1.799	28	72	48	100	15½

Change Gears and Angles for Cam Milling — 9

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
1.800	32	72	44	100	23	1.850	28	64	44	100	16	1.900	28	64	40	86	21
1.801	24	72	48	86	14½	1.851	44	56	24	100	11	1.901	28	64	32	72	12
1.802	28	64	32	72	22	1.852	28	72	44	86	21½	1.902	28	64	32	72	12
1.803	28	72	48	100	15	1.853	28	56	32	72	33½	1.903	28	72	44	86	17
1.804	32	72	44	86	37½	1.854	44	56	24	100	10½	1.904	24	64	48	86	24½
1.805	24	72	48	86	14	1.855	28	64	44	100	15½	1.905	28	64	32	72	11½
1.806	24	56	40	86	25	1.856	24	64	40	72	27	1.906	32	72	44	100	13
1.807	28	72	48	100	14½	1.857	44	56	24	100	10	1.907	32	64	40	100	17½
1.808	28	72	48	100	14½	1.858	24	64	44	86	14½	1.908	28	64	32	72	11
1.809	24	72	48	86	13½	1.859	28	64	44	100	15	1.909	32	72	48	100	26½
1.810	28	72	48	86	33½	1.860	32	72	44	100	18	1.910	32	72	44	100	12½
1.811	28	72	48	100	14	1.861	24	56	40	86	21	1.911	24	56	40	86	16½
1.812	24	72	48	86	13	1.862	24	64	44	86	14	1.912	28	64	32	72	10½
1.813	44	56	24	100	16	1.863	24	56	40	86	20½	1.913	32	72	44	100	12
1.814	24	64	44	86	19	1.864	28	64	44	100	14½	1.914	28	64	32	72	10
1.815	28	72	48	100	13½	1.865	32	72	44	100	17½	1.915	28	64	32	72	10
1.816	24	72	48	86	12½	1.866	24	64	44	86	13½	1.916	24	56	40	86	16
1.817	44	56	24	100	15½	1.867	32	64	40	100	21	1.917	32	72	44	100	11½
1.818	28	72	44	86	24	1.868	28	64	44	100	14	1.918	28	72	44	86	15½
1.819	24	72	48	86	12	1.869	28	64	32	72	16	1.919	24	44	32	86	19
1.820	24	64	44	86	18½	1.870	24	64	44	86	13	1.920	32	72	44	100	11
1.821	28	64	32	72	20½	1.871	32	72	44	100	17	1.921	24	56	40	86	15½
1.822	44	56	24	100	15	1.872	28	64	44	100	13½	1.922	28	72	44	86	15
1.823	24	72	48	86	11½	1.873	24	64	44	86	12½	1.923	32	72	44	100	10½
1.824	40	86	44	100	27	1.874	24	64	44	86	12½	1.924	28	64	40	86	19
1.825	24	64	44	86	18	1.875	32	72	44	100	16½	1.925	24	56	40	86	15
1.826	24	72	48	86	11	1.876	28	64	44	100	13	1.926	32	72	44	100	10
1.827	28	64	32	72	20	1.877	24	64	44	86	12	1.927	28	72	44	86	14½
1.828	24	56	40	86	23½	1.878	28	64	32	72	15	1.928	28	64	44	86	30½
1.829	24	72	48	86	10½	1.879	28	64	44	100	12½	1.929	24	56	40	86	14½
1.830	28	72	48	100	11½	1.880	24	64	44	86	11½	1.930	24	56	40	86	14½
1.831	28	64	44	100	18	1.881	32	72	40	86	24½	1.931	28	72	44	86	14
1.832	24	72	48	86	10	1.882	28	64	32	72	14½	1.932	32	64	40	100	15
1.833	28	72	48	100	11	1.883	28	64	44	100	12	1.933	48	56	24	100	20
1.834	44	56	24	100	13½	1.884	24	64	44	86	11	1.934	24	56	40	86	14
1.835	24	64	44	86	17	1.885	32	72	44	100	15½	1.935	28	72	44	86	13½
1.836	28	72	48	100	10½	1.886	28	64	44	100	11½	1.936	32	64	40	100	14½
1.837	28	64	40	86	25½	1.887	24	64	44	86	10½	1.937	28	64	48	86	37½
1.838	44	56	24	100	13	1.888	28	72	44	86	18½	1.938	24	56	40	86	13½
1.839	28	72	48	100	10	1.889	32	72	44	100	15	1.939	28	72	44	86	13
1.840	24	64	44	86	16½	1.890	24	64	44	86	10	1.940	28	64	48	100	22½
1.841	44	56	24	100	12½	1.891	32	64	40	100	19	1.941	32	64	40	100	14
1.842	32	72	40	86	27	1.892	32	72	48	100	27½	1.942	24	56	40	86	13
1.843	28	64	48	86	41	1.893	28	64	44	100	10½	1.943	28	72	44	86	12½
1.844	28	64	32	72	18½	1.894	28	64	32	72	13	1.944	32	56	40	86	43
1.845	44	56	24	100	12	1.895	24	56	40	86	18	1.945	48	56	24	100	19
1.846	28	64	44	100	16½	1.896	28	64	44	100	10	1.946	28	72	44	86	12
1.847	24	44	32	86	24½	1.897	32	64	40	100	18½	1.947	28	72	44	86	12
1.848	44	56	24	100	11½	1.898	28	64	32	72	12½	1.948	32	72	40	86	19½
1.849	24	64	44	86	15½	1.899	28	72	48	86	29	1.949	24	56	40	86	12

Change Gears and Angles for Cam Milling — 10

Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
1.950	28	72	44	86	11½	2.200	24	56	40	72	22½	2.450	24	64	48	72	11½
1.955	32	72	48	86	38	2.205	48	56	32	100	36½	2.455	40	72	48	100	23
1.960	28	72	44	86	10	2.210	48	100	56	86	45	2.460	28	64	48	72	32½
1.965	24	44	32	86	14½	2.215	24	56	40	72	21½	2.465	32	64	44	86	15½
1.970	32	64	40	100	10	2.220	32	72	44	86	12½	2.470	28	40	32	86	18½
1.975	28	64	40	86	14	2.225	28	44	32	86	20	2.475	32	64	40	72	27
1.980	28	64	48	100	19½	2.230	32	64	40	86	16½	2.480	44	48	28	100	15
1.985	24	64	48	86	18½	2.235	44	86	48	100	24½	2.485	28	72	56	86	11
1.990	28	64	40	86	12	2.240	32	56	40	100	11½	2.490	28	72	56	86	10½
1.995	40	86	44	100	13	2.245	28	64	56	86	38	2.495	24	44	40	86	10½
2.000	48	56	24	100	13½	2.250	24	64	44	72	11	2.500	28	64	48	72	31
2.005	28	100	56	72	23	2.255	32	64	48	100	20	2.505	24	56	44	72	17
2.010	32	72	40	86	13½	2.260	44	56	32	100	26	2.510	28	40	44	86	45½
2.015	40	86	48	100	25½	2.265	28	44	32	86	17	2.515	32	64	44	86	10½
2.020	28	72	48	86	21½	2.270	28	44	32	86	16½	2.520	44	48	28	100	11
2.025	32	72	40	86	11½	2.275	32	64	40	86	12	2.525	48	56	32	100	23
2.030	24	64	40	72	13	2.280	28	64	44	72	31½	2.530	24	56	44	72	15
2.035	24	64	48	86	13½	2.285	44	86	48	100	21½	2.535	32	56	40	86	17½
2.040	32	72	48	100	17	2.290	24	44	40	86	25½	2.540	32	64	48	86	24½
2.045	24	64	40	72	11	2.295	32	64	48	100	17	2.545	32	56	44	86	29½
2.050	28	64	48	100	12½	2.300	24	56	40	72	15	2.550	28	64	44	72	17½
2.055	24	64	48	86	11	2.305	24	56	40	72	14½	2.555	32	56	40	86	16
.060	32	72	48	100	15	2.310	24	56	40	72	14	2.560	32	64	48	86	23½
2.065	28	64	48	100	10½	2.315	24	56	40	72	13½	2.565	28	40	32	86	10
2.070	32	72	48	100	14	2.320	28	44	32	86	11½	2.570	44	48	40	100	45½
2.075	40	44	24	100	18	2.325	28	44	32	86	11	2.575	24	56	44	72	10½
2.080	32	64	44	100	19	2.330	40	100	56	72	41½	2.580	40	72	56	86	44½
2.085	24	64	48	72	33½	2.335	28	64	48	86	17	2.585	32	56	40	86	13½
2.090	32	72	48	100	11½	2.340	24	56	48	72	35	2.590	32	56	40	86	13
2.095	28	56	32	72	19½	2.345	24	56	40	72	10	2.595	44	40	32	100	42½
2.100	28	44	32	86	27½	2.350	28	64	44	72	28½	2.600	32	56	40	86	12
2.105	40	86	48	100	19½	2.355	44	86	48	100	16½	2.605	32	56	40	86	11½
2.110	28	64	44	86	19½	2.360	32	64	48	100	10½	2.610	32	64	40	72	20
2.115	28	72	48	86	13	2.365	24	56	48	86	8½	2.615	44	48	40	100	44½
2.120	28	72	48	86	12½	2.370	44	56	32	100	19½	2.620	28	64	44	72	11½
2.125	32	64	44	100	15	2.375	28	64	48	86	13½	2.625	44	56	24	64	27
2.130	28	100	56	72	12	2.380	32	100	56	72	17	2.630	48	56	32	100	16½
2.135	28	72	48	86	10½	2.385	32	72	56	86	34½	2.635	40	72	44	86	22
2.140	24	56	40	72	26	2.390	28	64	40	72	10½	2.640	48	100	56	72	45
2.145	28	100	56	72	10	2.395	40	72	44	100	11½	2.645	24	40	44	86	30½
2.150	32	72	44	86	19	2.400	56	64	32	100	31	2.650	40	56	44	86	43½
2.155	44	56	32	100	31	2.405	28	64	48	86	10	2.655	56	64	32	100	18½
2.160	32	64	44	100	11	2.410	32	100	56	72	14½	2.660	44	48	40	100	43½
2.165	28	56	32	72	13	2.415	44	86	48	100	10½	2.665	28	64	48	72	24
2.170	32	72	48	86	29	2.420	32	100	56	72	13½	2.670	28	48	44	72	41½
2.175	32	72	44	86	17	2.425	32	100	56	72	13	2.675	48	64	28	56	44½
2.180	40	86	48	100	12½	2.430	32	100	56	72	12½	2.680	28	44	48	86	41
2.185	28	56	32	72	10½	2.435	32	72	48	86	11	2.685	48	100	56	72	44
2.190	32	56	40	86	34½	2.440	32	72	48	86	10½	2.690	40	64	44	100	12
2.195	28	64	48	86	26	2.445	44	56	32	100	13½	2.695	40	64	44	100	11½

Change Gears and Angles for Cam Milling — 11

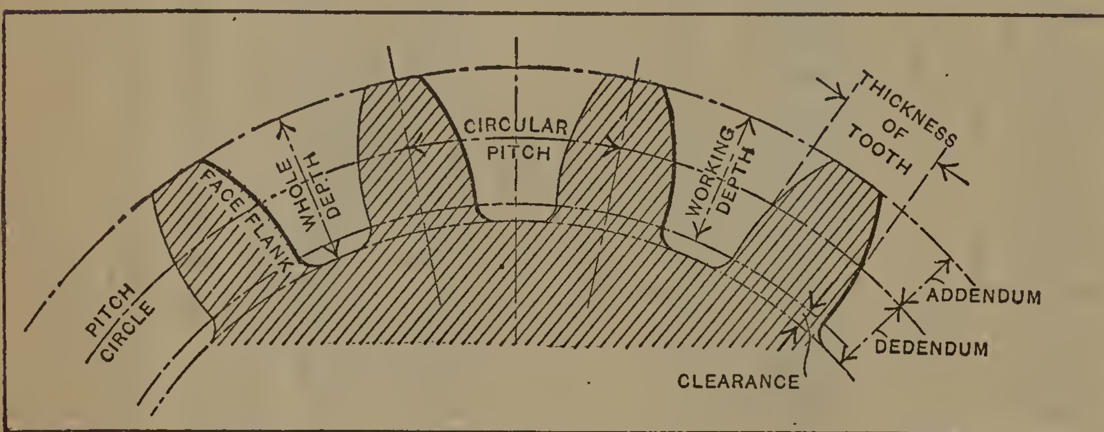
Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle	Lead of Cam	Gear on Worm	First Intermediate	Second Intermediate	Gear on Screw	Angle
2.700	28	44	48	86	40½	2.950	40	64	48	100	10½	3.200	48	100	56	72	31
2.705	56	64	32	100	15	2.955	48	64	28	56	38	3.205	28	40	44	86	26½
2.710	40	72	56	86	41½	2.960	24	44	48	72	35½	3.210	24	44	48	72	28
2.715	40	56	44	86	42	2.965	32	64	44	72	14	3.215	40	64	48	72	39½
2.720	48	64	28	56	43½	2.970	32	64	48	72	27	3.220	56	44	28	86	39
2.725	44	48	40	100	42	2.975	40	56	44	86	35½	3.225	24	44	48	72	27½
2.730	48	100	56	72	43	2.980	48	40	28	100	27½	3.230	48	40	28	100	16
2.735	44	40	32	100	39	2.985	44	48	40	100	35½	3.235	32	72	64	86	12
2.740	28	44	48	86	39½	2.990	28	48	44	72	33	3.240	24	44	48	72	27
2.745	40	72	44	86	15	2.995	48	64	28	56	37	3.245	28	40	44	86	25
2.750	28	64	48	72	19½	3.000	40	100	56	64	31	3.250	44	64	48	100	10
2.755	44	40	32	100	38½	3.005	40	64	48	86	30½	3.255	32	48	40	72	28½
2 760	28	44	48	86	39	3.010	28	56	64	86	36	3.260	32	56	44	72	21
2.765	48	64	28	56	42½	3.015	48	64	28	56	36½	3.265	48	100	56	72	29
2.770	28	48	44	72	39	3.020	48	100	56	72	36	3.270	40	56	44	86	26½
2.775	40	72	44	86	12½	3.025	40	100	56	72	13½	3.275	44	40	32	100	21½
2.780	40	72	44	86	12	3.030	40	64	44	72	37½	3.280	48	64	28	56	29
2.785	24	44	48	72	40	3.035	24	40	48	86	25	3.285	32	48	40	72	27½
2.790	28	48	44	72	38½	3.040	44	48	40	100	34	3.290	32	44	40	86	13½
2.795	32	48	40	72	41	3.045	32	64	48	72	24	3.295	24	44	48	72	25
2.800	24	56	48	72	11½	3.050	40	56	44	100	14	3.300	32	48	40	72	27
2.805	24	56	48	72	11	3.055	56	44	28	86	42½	3.305	40	72	56	86	24
2.810	44	56	24	64	17½	3.060	28	44	48	86	30½	3.310	44	48	40	100	25½
2.815	28	44	40	86	18	3.065	40	56	44	86	33	3.315	32	48	40	72	26½
2.820	40	56	44	86	39½	3.070	28	40	44	86	31	3.320	28	40	44	86	22
2.825	32	56	44	72	36	3.075	44	48	40	100	33	3.325	40	56	44	86	24½
2.830	48	64	28	56	41	3.080	40	64	48	86	28	3.330	28	56	64	86	26½
2.835	28	48	40	72	29	3.085	28	56	64	86	34	3.335	28	64	56	72	11½
2.840	40	56	44	86	39	3.090	48	64	28	56	34½	3.340	40	64	44	72	29
2.845	28	44	40	86	16	3.095	48	100	56	72	34	3.345	32	44	48	86	34½
2.850	28	56	64	86	40	3.100	24	44	48	72	31½	3.350	44	48	40	100	24
2.855	28	44	48	86	36½	3.105	40	100	56	64	27½	3.355	48	100	56	72	26
2.860	40	56	44	86	38½	3.110	44	48	40	100	32	3.360	40	56	48	100	11½
2.865	24	44	48	72	38	3.115	28	48	40	72	16	3.365	28	40	44	86	20
2.870	44	48	40	100	38½	3.120	44	64	48	100	19	3.370	48	64	28	56	26
2.875	40	64	48	86	34½	3.125	32	56	44	72	26½	3.375	44	48	40	100	23
2.880	48	100	56	72	39½	3.130	32	56	48	86	11	3.380	32	56	48	72	27½
2.885	24	44	48	72	37½	3.135	28	44	48	86	28	3.385	48	64	28	56	25½
2.890	44	48	40	100	38	3.140	32	56	48	86	10	3.390	48	40	32	100	28
2.895	32	56	44	72	34	3.145	48	64	28	56	33	3.395	32	56	44	72	13½
2.900	28	44	40	86	11½	3.150	28	44	48	86	27½	3.400	40	56	44	86	21½
2.905	40	72	48	86	20½	3.155	28	64	56	72	22	3.405	28	44	48	86	16½
2.910	28	44	40	86	10½	3.160	44	48	40	100	30½	3.410	32	48	40	72	23
2.915	28	40	44	86	35½	3.165	24	44	48	72	29½	3.415	28	40	44	86	17½
2.920	28	48	44	72	35	3.170	28	48	40	72	12	3.420	32	40	48	86	40
2.925	40	64	48	86	33	3.175	32	48	40	72	31	3.425	28	56	64	86	23
2.930	32	64	44	72	16½	3.180	40	56	44	86	29½	3.430	28	44	48	86	15
2.935	48	64	28	56	38½	3.185	40	100	56	64	24½	3.435	44	48	40	100	20½
2.940	40	64	48	100	11½	3.190	28	56	64	86	31	3.440	48	100	56	64	35
2.945	40	72	56	86	35½	3.195	44	72	48	86	20½	3.445	28	44	48	86	14

GEARING

Spur Gearing

Systems of Spur Gearing. — Definitions. — Two systems of gear teeth are used for spur gearing: the *cycloidal* and the *involute*. Of these the involute system is the one most commonly used, especially for cut gearing. The standard involute gear tooth has a 14½-degree pressure angle; hence, the rack meshing with gears cut according to this standard has straight sides inclined 14½ degrees from the vertical. In the table of "Rules and Formulas for Dimensions of Spur Gears," the notation below is used:

P = diametral pitch;
P' = circular pitch;
N = number of teeth; (if the number of teeth in both gear and pinion are referred to, N_g = number of teeth in gear, and N_p = number of teeth in pinion);
D = pitch diameter;
C = center distance;
S = addendum;
F = clearance;
W = whole depth of tooth;
T = thickness of tooth;
O = outside diameter of gear.



Gear Tooth Parts

The *circular pitch* is defined as the distance from center to center of two adjacent teeth along the pitch circle. The *diametral pitch* is a number found by dividing the number of teeth by the pitch diameter. In other words, it gives the number of teeth for each inch of pitch diameter.

Cutters for Cutting Gear Teeth

Involute System		Cycloidal System					
No. of Cutter	Number of Teeth	Letter of Cutter	Number of Teeth	Letter of Cutter	Number of Teeth	Letter of Cutter	Number of Teeth
1	135 to rack	*A*	12	*I*	20	*Q*	43 to 49
2	55 to 134	*B*	13	*J*	21 to 22	*R*	50 to 59
3	35 to 54	*C*	14	*K*	23 to 24	*S*	60 to 74
4	26 to 34	*D*	15	*L*	25 to 26	*T*	75 to 99
5	21 to 25	*E*	16	*M*	27 to 29	*U*	100 to 149
6	17 to 20	*F*	17	*N*	30 to 33	*V*	150 to 249
7	14 to 16	*G*	18	*O*	34 to 37	*W*	250 or more
8	12 to 13	*H*	19	*P*	38 to 42	*X*	Rack

Rules and Formulas for Dimensions of Spur Gears

No. of Rule	To Find	Rule	Formula
1	Diametral Pitch	Divide 3.1416 by circular pitch.	$P = \frac{3.1416}{P'}$
2	Circular Pitch	Divide 3.1416 by diametral pitch.	$P' = \frac{3.1416}{P}$
3	Pitch Diameter	Divide number of teeth by diametral pitch.	$D = \frac{N}{P}$
4	Pitch Diameter	Multiply number of teeth by circular pitch and divide the product by 3.1416.	$D = \frac{NP'}{3.1416}$
5	Center Distance	Add the number of teeth in both gears and divide the sum by two times the diametral pitch.	$C = \frac{N_g + N_p}{2P}$
6	Center Distance	Multiply the sum of the number of teeth in both gears by circular pitch and divide the product by 6.2832.	$C = \frac{(N_g + N_p)P'}{6.2832}$
7	Addendum	Divide 1 by diametral pitch.	$S = \frac{1}{P}$
8	Addendum	Divide circular pitch by 3.1416.	$S = \frac{P'}{3.1416}$
9	Clearance	Divide 0.157 by diametral pitch.	$F = \frac{0.157}{P}$
10	Clearance	Divide circular pitch by 20.	$F = \frac{P'}{20}$
11	Whole Depth of Tooth	Divide 2.157 by diametral pitch.	$W = \frac{2.157}{P}$
12	Whole Depth of Tooth	Multiply 0.6866 by circular pitch.	$W = 0.6866\,P'$
13	Thickness of Tooth	Divide 1.5708 by diametral pitch.	$T = \frac{1.5708}{P}$
14	Thickness of Tooth	Divide circular pitch by 2.	$T = \frac{P'}{2}$
15	Outside Diameter	Add 2 to the number of teeth and divide the sum by diametral pitch.	$O = \frac{N+2}{P}$
16	Outside Diameter	Multiply the sum of the number of teeth plus 2 by circular pitch and divide the product by 3.1416.	$O = \frac{(N+2)P'}{3.1416}$
17	Diametral Pitch	Divide number of teeth by pitch diameter.	$P = \frac{N}{D}$
18	Circular Pitch	Multiply pitch diameter by 3.1416 and divide by number of teeth.	$P' = \frac{3.1416\,D}{N}$
19	Pitch Diameter	Subtract two times the addendum from outside diameter.	$D = O - 2S$
20	Number of Teeth	Multiply pitch diameter by diametral pitch.	$N = P \times D$
21	Number of Teeth	Multiply pitch diameter by 3.1416 and divide the product by circular pitch.	$N = \frac{3.1416\,D}{P'}$
22	Outside Diameter	Add two times the addendum to the pitch diameter.	$O = D + 2S$
23	Length of Rack	Multiply number of teeth in rack by 3.1416 and divide by diametral pitch.	$L = \frac{3.1416\,N}{P}$
24	Length of Rack	Multiply the number of teeth in the rack by circular pitch.	$L = NP'$

GEARING

According to the system for cutting gear teeth adopted by the Brown & Sharpe Mfg. Co., any gear of one pitch will mesh with any other gear or with a rack of the same pitch. For cutting involute gear teeth, eight cutters are required for each pitch. These cutters are adapted to cut from a pinion of 12 teeth to a rack, and are numbered as shown in the table of "Cutters for Cutting Gear Teeth."

Cutters for the cycloidal form of teeth are also made so that a gear of any pitch will mesh into any other gear or into a rack of the same pitch, but twenty-four cutters are required for each pitch. In order that gears with this form of tooth shall run well together, they must be cut accurately to the exact depth. To secure the proper depth of tooth, the cutters are made with a shoulder determining the required depth. The twenty-four cutters for the cycloidal form of teeth are designated by letters A, B, C, etc. A list of these cutters is also given in the table.

Internal Spur Gears. — The dimensions for internal spur gears may be found by the same formulas as those for external spur gears, except for the modifications made necessary by the fact that the center distance in internal gearing is equal to the *difference* between the two pitch radii, instead of their *sum*. In addition, the term "inside diameter" (I in the table of dimensions of internal spur gears) takes the place of the outside diameter of external spur gearing. This diameter, of course, is the diameter of the hole in the blank before the teeth are cut. In laying out the shape of teeth for internal gearing, interferences are almost sure to be met with. The points of internal gear teeth must, therefore, be relieved to avoid interference with the flanks of the meshing teeth. Interference occurs also when the pinion has too nearly the same number of teeth as the gear. In this case, there is a tendency for the points of the pinion and the gear teeth to strike as they roll into and out of engagement. To avoid this interference, the teeth must be cut by specially made cutters or shaped on a gear shaping machine.

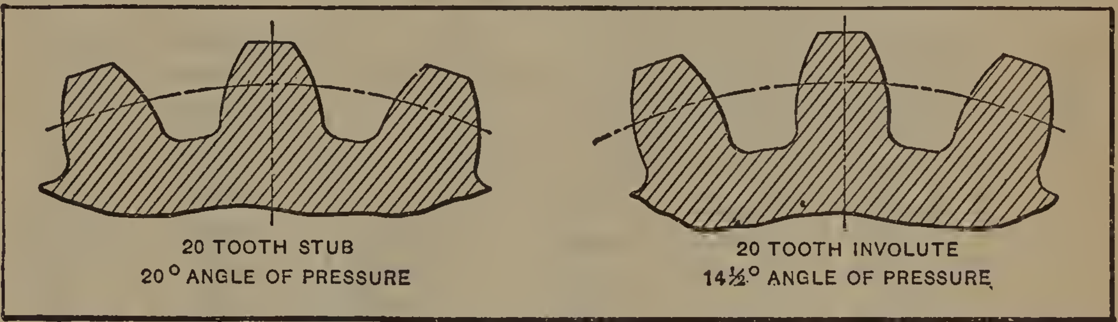
Rules and Formulas for Internal Spur Gears

(Where rules and formulas are not given, they are the same as for external gears)

No. of Rule	To Find	Rule	Formula
5	Center Distance	Subtract the number of teeth in the pinion from the number of teeth in the gear and divide the remainder by 2 times the diametral pitch.	$C = \frac{N_g - N_p}{2P}$
6	Center Distance	Multiply the difference of the numbers of teeth in the gear and pinion by the circular pitch and divide the product by 6.2832.	$C = \frac{(N_g - N_p)\,P'}{6.2832}$
15	Inside Diameter	Subtract 2 from the number of teeth and divide the remainder by the diametral pitch.	$I = \frac{N - 2}{P}$
16	Inside Diameter	Subtract 2 from the number of teeth, multiply the remainder by the circular pitch, and divide the product by 3.1416.	$I = \frac{(N - 2)\,P'}{3.1416}$
19	Pitch Diameter	Add twice the addendum to the inside diameter.	$D = I + 2S$
22	Inside Diameter	Subtract twice the addendum from the pitch diameter.	$I = D - 2S$

The Stub Gear Tooth. — The stub gear tooth has been applied successfully, particularly to automobile drives. This gear tooth has a shorter addendum and dedendum than the ordinary involute tooth, and the pressure angle is 20 degrees instead of 14½, as in the ordinary standard. There are two systems of these teeth in use, tables for both of which are given herewith. (See "Dimensions of Stub Gear Teeth" and the following tables arranged according to both diametral and circular pitches.)

The proportions of the stub gear teeth introduced by Fellows Gear Shaper Co. are based on the use of two diametral pitches. One diametral pitch, say 8, is used as the basis for obtaining the dimensions for the addendum and dedendum,



Comparison between Stub Gear Teeth and Standard Involute Teeth

while another diametral pitch, say 6, is used for obtaining the dimensions of the thickness of the tooth, the number of teeth, and the pitch diameter. Teeth made according to this system are designated as 6/8 pitch, 12/14 pitch, etc., the numerator in this fraction indicating the pitch determining the thickness of the tooth and the number of teeth, and the denominator, the pitch determining the depth of the tooth. The clearance is made greater than in the ordinary gear-tooth system and equals 0.25 ÷ the denominator of the diametral pitch.

Dimensions of Stub Gear Teeth (Fellows Gear Shaper Co.'s System)

Diametral Pitch	Thickness of Tooth	Addendum	Working Depth	Depth of Space below Pitch Line	Clearance	Whole Depth of Tooth
4/5	0.3927	0.2000	0.4000	0.2500	0.0500	0.4500
5/7	0.3142	0.1429	0.2858	0.1786	0.0357	0.3214
6/8	0.2618	0.1250	0.2500	0.1562	0.0312	0.2812
7/9	0.2244	0.1111	0.2222	0.1389	0.0278	0.2500
8/10	0.1963	0.1000	0.2000	0.1250	0.0250	0.2250
9/11	0.1745	0.0909	0.1818	0.1136	0.0227	0.2045
10/12	0.1571	0.0833	0.1667	0.1041	0.0208	0.1875
12/14	0.1309	0.0714	0.1429	0.0893	0.0179	0.1607

In the other system for stub gear teeth (originated by the R. D. Nuttall Co.), the tooth dimensions are based directly upon the circular pitch. The addendum is made 0.250 × the circular pitch, and the dedendum, 0.300 × the circular pitch.

The advantages of the stub gear tooth may be stated as follows: 1. Greater strength. 2. Equal arc of rolling contact as in the 14½-degree involute gear. 3. Extreme sliding contact avoided. 4. More even wearing contact. If the teeth are milled a greater number of rotary cutters are required to cover a given range of teeth. While only eight cutters are required for each pitch in the standard involute system, fifteen cutters are recommended for stub-tooth gears of equal accuracy.

When a generating machine, such as the Fellows gear shaper, is used, only one cutter is required for each pitch, on account of the cutting action of this machine.

Strength of Stub Tooth. — As a general rule, stub-tooth pinions with less than 25 teeth are about 25 per cent stronger than the standard 20-degree involute, and 40 per cent stronger than the standard 14½-degree involute tooth. For larger gears, the difference is in favor of the stub tooth, but is not quite so marked.

Diametral Pitch Gear Tooth Parts for Stub Teeth (Nuttall System)

(Based on: addendum = 0.250 × circular pitch; dedendum = 0.300 × circular pitch.)

Diametral Pitch	Equivalent Circular Pitch	Thickness of Tooth on Pitch Line	Addendum	Working Depth	Depth of Space below Pitch Line	Whole Depth
½	6.2832	3.1416	1.5708	3.1416	1.8849	3.4557
¾	4.1888	2.0944	1.0472	2.0944	1.2566	2.3038
1	3.1416	1.5708	0.7854	1.5708	0.9424	1.7278
1¼	2.5133	1.2566	0.6283	1.2566	0.7539	1.3822
1½	2.0944	1.0472	0.5236	1.0472	0.6283	1.1519
1¾	1.7952	0.8976	0.4488	0.8976	0.5385	0.9873
2	1.5708	0.7854	0.3927	0.7854	0.4712	0.8639
2¼	1.3963	0.6981	0.3490	0.6981	0.4188	0.7678
2½	1.2566	0.6283	0.3141	0.6283	0.3769	0.6910
2¾	1.1424	0.5712	0.2856	0.5712	0.3427	0.6283
3	1.0472	0.5236	0.2618	0.5236	0.3141	0.5759
3½	0.8976	0.4488	0.2244	0.4488	0.2692	0.4936
4	0.7854	0.3927	0.1963	0.3927	0.2355	0.4318
5	0.6283	0.3141	0.1570	0.3142	0.1884	0.3454
6	0.5236	0.2618	0.1309	0.2618	0.1571	0.2880
7	0.4488	0.2244	0.1122	0.2244	0.1346	0.2468
8	0.3927	0.1963	0.0981	0.1963	0.1177	0.2158
9	0.3491	0.1745	0.0872	0.1745	0.1046	0.1918
10	0.3142	0.1571	0.0785	0.1571	0.0942	0.1727
11	0.2856	0.1428	0.0714	0.1428	0.0857	0.1571
12	0.2618	0.1309	0.0654	0.1309	0.0785	0.1439
13	0.2417	0.1208	0.0604	0.1208	0.0725	0.1329
14	0.2244	0.1122	0.0561	0.1122	0.0673	0.1234
15	0.2094	0.1047	0.0523	0.1047	0.0627	0.1150
16	0.1963	0.0982	0.0491	0.0982	0.0589	0.1080
17	0.1848	0.0924	0.0462	0.0924	0.0554	0.1016
18	0.1745	0.0873	0.0436	0.0873	0.0523	0.0959
19	0.1653	0.0827	0.0413	0.0827	0.0495	0.0908
20	0.1571	0.0785	0.0392	0.0785	0.0470	0.0862
22	0.1428	0.0714	0.0357	0.0714	0.0423	0.0785
24	0.1309	0.0654	0.0327	0.0654	0.0392	0.0719
26	0.1208	0.0604	0.0302	0.0604	0.0362	0.0664
28	0.1122	0.0561	0.0280	0.0561	0.0336	0.0616
30	0.1047	0.0524	0.0262	0.0524	0.0313	0.0575
32	0.0982	0.0491	0.0245	0.0491	0.0294	0.0539
34	0.0924	0.0462	0.0231	0.0462	0.0277	0.0508
36	0.0873	0.0436	0.0218	0.0436	0.0261	0.0479

Circular Pitch Gear Tooth Parts for Stub Teeth (Nuttall System)

(Based on: addendum = 0.250 × circular pitch; dedendum = 0.300 × circular pitch.)

Circular Pitch	Circular Pitch (Decimal Equivalent)	Diametral Pitch	Thickness of Tooth on Pitch Line	Addendum	Working Depth	Depth of Space below Pitch Line	Whole Depth
2	2.0000	1.5708	1.0000	0.5000	1.0000	0.6000	1.1000
1⅞	1.8750	1.6755	0.9375	0.4687	0.9375	0.5624	1.0311
1¾	1.7500	1.7952	0.8750	0.4375	0.8750	0.5250	0.9625
1⅝	1.6250	1.9333	0.8125	0.4062	0.8125	0.4874	0.8936
1½	1.5000	2.0944	0.7500	0.3750	0.7500	0.4500	0.8250
1 7/16	1.4375	2.1855	0.7187	0.3593	0.7187	0.4311	0.7904
1⅜	1.3750	2.2848	0.6875	0.3437	0.6875	0.4124	0.7561
1⅓	1.3333	2.3562	0.6666	0.3333	0.6666	0.3999	0.7332
1 5/16	1.3125	2.3936	0.6562	0.3281	0.6562	0.3937	0.7218
1¼	1.2500	2.5133	0.6250	0.3125	0.6250	0.3750	0.6875
1 3/16	1.1875	2.6456	0.5937	0.2968	0.5937	0.3561	0.6529
1⅛	1.1250	2.7925	0.5625	0.2812	0.5625	0.3374	0.6186
1 1/16	1.0625	2.9568	0.5312	0.2656	0.5312	0.3187	0.5843
1	1.0000	3.1416	0.5000	0.2500	0.5000	0.3000	0.5500
15/16	0.9375	3.3510	0.4687	0.2343	0.4687	0.2811	0.5154
⅞	0.8750	3.5904	0.4375	0.2187	0.4375	0.2624	0.4811
13/16	0.8125	3.8666	0.4062	0.2031	0.4062	0.2437	0.4468
¾	0.7500	4.1888	0.3750	0.1875	0.3750	0.2250	0.4125
11/16	0.6875	4.5696	0.3437	0.1718	0.3437	0.2061	0.3779
⅔	0.6666	4.7124	0.3333	0.1666	0.3333	0.1999	0.3666
⅝	0.6250	5.0265	0.3125	0.1562	0.3125	0.1874	0.3436
9/16	0.5625	5.5851	0.2812	0.1400	0.2812	0.1687	0.3093
½	0.5000	6.2832	0.2500	0.1250	0.2500	0.1500	0.2750
7/16	0.4375	7.1808	0.2187	0.1093	0.2187	0.1311	0.2404
⅜	0.3750	8.3776	0.1875	0.0937	0.1875	0.1124	0.2061
⅓	0.3333	9.4248	0.1666	0.0833	0.1666	0.0999	0.1832
5/16	0.3125	10.0531	0.1562	0.0781	0.1562	0.0937	0.1718
¼	0.2500	12.5664	0.1250	0.0625	0.1250	0.0750	0.1375
⅕	0.2000	15.7080	0.1000	0.0500	0.1000	0.0600	0.1100
3/16	0.1875	16.7552	0.0937	0.0468	0.0937	0.0561	0.1029
⅙	0.1666	18.8496	0.0833	0.0416	0.0833	0.0499	0.0915
⅛	0.1250	25.1327	0.0625	0.0312	0.0625	0.0374	0.0686
1/9	0.1111	28.2743	0.0555	0.0277	0.0555	0.0332	0.0609
1/10	0.1000	31.4159	0.0500	0.0250	0.0500	0.0300	0.0550
1/16	0.0625	50.2655	0.0312	0.0156	0.0312	0.0187	0.0343
1/20	0.0500	62.8318	0.0250	0.0125	0.0250	0.0150	0.0275

Clearance of Gears Cut on the Gear Shaper. — When gears are cut on the gear shaper, the clearance is made equal to 0.25 ÷ diametral pitch. Hence the root diameter of these gears is smaller than the root diameter of ordinary milled gears. A table is given (page 624) of root diameters of gears cut on the gear shaper. The pitch and outside (blank) diameters are the same as for ordinary milled gears, and the tables, "Pitch Diameters of Diametral Pitch Gears," and "Outside Diameters of Diametral Pitch Gears," apply to gears cut on the gear shaper also.

Gear Tooth Parts

(Diametral Pitch Gears)

Diametral Pitch	Circular Pitch	Thickness of Tooth on Pitch Line	Addendum	Working Depth of Tooth	Depth of Space below Pitch Line	Whole Depth of Tooth
P	P'	T	S	W'	$S+F$	W
½	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
¾	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
1¼	2.5133	1.2566	0.8000	1.6000	0.9257	1.7257
1½	2.0944	1.0472	0.6666	1.3333	0.7714	1.4381
1¾	1.7952	0.8976	0.5714	1.1429	0.6612	1.2326
2	1.5708	0.7854	0.5000	1.0000	0.5785	1.0785
2¼	1.3963	0.6981	0.4444	0.8888	0.5143	0.9587
2½	1.2566	0.6283	0.4000	0.8000	0.4628	0.8628
2¾	1.1424	0.5712	0.3636	0.7273	0.4208	0.7844
3	1.0472	0.5236	0.3333	0.6666	0.3857	0.7190
3½	0.8976	0.4488	0.2857	0.5714	0.3306	0.6163
4	0.7854	0.3927	0.2500	0.5000	0.2893	0.5393
5	0.6283	0.3142	0.2000	0.4000	0.2314	0.4314
6	0.5236	0.2618	0.1666	0.3333	0.1928	0.3595
7	0.4488	0.2244	0.1429	0.2857	0.1653	0.3081
8	0.3927	0.1963	0.1250	0.2500	0.1446	0.2696
9	0.3491	0.1745	0.1111	0.2222	0.1286	0.2397
10	0.3142	0.1571	0.1000	0.2000	0.1157	0.2157
11	0.2856	0.1428	0.0909	0.1818	0.1052	0.1961
12	0.2618	0.1309	0.0833	0.1666	0.0964	0.1798
13	0.2417	0.1208	0.0769	0.1538	0.0890	0.1659
14	0.2244	0.1122	0.0714	0.1429	0.0826	0.1541
15	0.2094	0.1047	0.0666	0.1333	0.0771	0.1438
16	0.1963	0.0982	0.0625	0.1250	0.0723	0.1348
17	0.1848	0.0924	0.0588	0.1176	0.0681	0.1269
18	0.1745	0.0873	0.0555	0.1111	0.0643	0.1198
19	0.1653	0.0827	0.0526	0.1053	0.0609	0.1135
20	0.1571	0.0785	0.0500	0.1000	0.0579	0.1079
22	0.1428	0.0714	0.0455	0.0909	0.0526	0.0980
24	0.1309	0.0654	0.0417	0.0833	0.0482	0.0898
26	0.1208	0.0604	0.0385	0.0769	0.0445	0.0829
28	0.1122	0.0561	0.0357	0.0714	0.0413	0.0770
30	0.1047	0.0524	0.0333	0.0666	0.0386	0.0719
32	0.0982	0.0491	0.0312	0.0625	0.0362	0.0674
34	0.0924	0.0462	0.0294	0.0588	0.0340	0.0634
36	0.0873	0.0436	0.0278	0.0555	0.0321	0.0599
38	0.0827	0.0413	0.0263	0.0526	0.0304	0.0568
40	0.0785	0.0393	0.0250	0.0500	0.0289	0.0539
42	0.0748	0.0374	0.0238	0.0476	0.0275	0.0514
44	0.0714	0.0357	0.0227	0.0455	0.0263	0.0490
46	0.0683	0.0341	0.0217	0.0435	0.0252	0.0469
48	0.0654	0.0327	0.0208	0.0417	0.0241	0.0449
50	0.0628	0.0314	0.0200	0.0400	0.0231	0.0431

Gear Tooth Parts

(Circular Pitch Gears)

Circular Pitch	Diametral Pitch	Thickness of Tooth on Pitch Line	Addendum	Working Depth of Tooth	Depth of Space below Pitch Line	Whole Depth of Tooth
P'	P	T	S	W'	$S+F$	W
4	0.7854	2.0000	1.2732	2.5464	1.4732	2.7464
3½	0.8976	1.7500	1.1140	2.2281	1.2890	2.4031
3	1.0472	1.5000	0.9549	1.9098	1.1049	2.0598
2¾	1.1424	1.3750	0.8753	1.7506	1.0128	1.8881
2½	1.2566	1.2500	0.7957	1.5915	0.9207	1.7165
2¼	1.3963	1.1250	0.7162	1.4323	0.8287	1.5448
2	1.5708	1.0000	0.6366	1.2732	0.7366	1.3732
1⅞	1.6755	0.9375	0.5968	1.1937	0.6906	1.2874
1¾	1.7952	0.8750	0.5570	1.1141	0.6445	1.2016
1⅝	1.9333	0.8125	0.5173	1.0345	0.5985	1.1158
1½	2.0944	0.7500	0.4775	0.9549	0.5525	1.0299
1 7/16	2.1855	0.7187	0.4576	0.9151	0.5294	0.9870
1⅜	2.2848	0.6875	0.4377	0.8754	0.5064	0.9441
1 5/16	2.3936	0.6562	0.4178	0.8356	0.4834	0.9012
1¼	2.5133	0.6250	0.3979	0.7958	0.4604	0.8583
1 3/16	2.6456	0.5937	0.3780	0.7560	0.4374	0.8154
1⅛	2.7925	0.5625	0.3581	0.7162	0.4143	0.7724
1 1/16	2.9568	0.5312	0.3382	0.6764	0.3913	0.7295
1	3.1416	0.5000	0.3183	0.6366	0.3683	0.6866
15/16	3.3510	0.4687	0.2984	0.5968	0.3453	0.6437
⅞	3.5904	0.4375	0.2785	0.5570	0.3223	0.6007
13/16	3.8666	0.4062	0.2586	0.5173	0.2993	0.5579
¾	4.1888	0.3750	0.2387	0.4775	0.2762	0.5150
11/16	4.5696	0.3437	0.2189	0.4377	0.2532	0.4720
⅔	4.7124	0.3333	0.2122	0.4244	0.2455	0.4577
⅝	5.0265	0.3125	0.1989	0.3979	0.2301	0.4291
9/16	5.5851	0.2812	0.1790	0.3581	0.2071	0.3862
½	6.2832	0.2500	0.1592	0.3183	0.1842	0.3433
7/16	7.1808	0.2187	0.1393	0.2785	0.1611	0.3003
⅖	7.8540	0.2000	0.1273	0.2546	0.1473	0.2746
⅜	8.3776	0.1875	0.1194	0.2387	0.1381	0.2575
⅓	9.4248	0.1666	0.1061	0.2122	0.1228	0.2289
5/16	10.0531	0.1562	0.0995	0.1989	0.1151	0.2146
2/7	10.9956	0.1429	0.0909	0.1819	0.1052	0.1962
¼	12.5664	0.1250	0.0796	0.1591	0.0921	0.1716
2/9	14.1372	0.1111	0.0707	0.1415	0.0818	0.1526
⅕	15.7080	0.1000	0.0637	0.1273	0.0737	0.1373
3/16	16.7552	0.0937	0.0597	0.1194	0.0690	0.1287
⅙	18.8496	0.0833	0.0531	0.1061	0.0614	0.1144
1/7	21.9911	0.0714	0.0455	0.0910	0.0526	0.0981
⅛	25.1327	0.0625	0.0398	0.0796	0.0460	0.0858
1/9	28.2743	0.0555	0.0354	0.0707	0.0409	0.0763
1/10	31.4159	0.0500	0.0318	0.0637	0.0368	0.0687
1/16	50.2655	0.0312	0.0199	0.0398	0.0230	0.0429

Pitch Diameters of Diametral Pitch Gears

No. of Teeth	Diametral Pitch								
	3 *P*	4 *P*	5 *P*	6 *P*	8 *P*	10 *P*	12 *P*	14 *P*	16 *P*
10	3.333	2.500	2.000	1.667	1.250	1.000	0.833	0.714	0.625
11	3.667	2.750	2.200	1.833	1.375	1.100	0.917	0.786	0.687
12	4.000	3.000	2.400	2.000	1.500	1.200	1.000	0.857	0.750
13	4.333	3.250	2.600	2.167	1.625	1.300	1.083	0.929	0.812
14	4.667	3.500	2.800	2.333	1.750	1.400	1.167	1.000	0.875
15	5.000	3.750	3.000	2.500	1.875	1.500	1.250	1.071	0.937
16	5.333	4.000	3.200	2.667	2.000	1.600	1.333	1.143	1.000
17	5.667	4.250	3.400	2.833	2.125	1.700	1.417	1.214	1.062
18	6.000	4.500	3.600	3.000	2.250	1.800	1.500	1.286	1.125
19	6.333	4.750	3.800	3.167	2.375	1.900	1.583	1.357	1.187
20	6.667	5.000	4.000	3.333	2.500	2.000	1.667	1.429	1.250
21	7.000	5.250	4.200	3.500	2.625	2.100	1.750	1.500	1.312
22	7.333	5.500	4.400	3.667	2.750	2.200	1.833	1.571	1.375
23	7.667	5.750	4.600	3.833	2.875	2.300	1.917	1.643	1.437
24	8.000	6.000	4.800	4.000	3.000	2.400	2.000	1.714	1.500
25	8.333	6.250	5.000	4.167	3.125	2.500	2.083	1.786	1.562
26	8.667	6.500	5.200	4.333	3.250	2.600	2.167	1.857	1.625
27	9.000	6.750	5.400	4.500	3.375	2.700	2.250	1.929	1.687
28	9.333	7.000	5.600	4.667	3.500	2.800	2.333	2.000	1.750
29	9.667	7.250	5.800	4.833	3.625	2.900	2.417	2.071	1.812
30	10.000	7.500	6.000	5.000	3.750	3.000	2.500	2.143	1.875
31	10.333	7.750	6.200	5.167	3.875	3.100	2.583	2.214	1.937
32	10.667	8.000	6.400	5.333	4.000	3.200	2.667	2.286	2.000
33	11.000	8.250	6.600	5.500	4.125	3.300	2.750	2.357	2.062
34	11.333	8.500	6.800	5.667	4.250	3.400	2.833	2.429	2.125
35	11.667	8.750	7.000	5.833	4.375	3.500	2.917	2.500	2.187
36	12.000	9.000	7.200	6.000	4.500	3.600	3.000	2.571	2.250
37	12.333	9.250	7.400	6.167	4.625	3.700	3.083	2.643	2.312
38	12.667	9.500	7.600	6.333	4.750	3.800	3.167	2.714	2.375
39	13.000	9.750	7.800	6.500	4.875	3.900	3.250	2.786	2.437
40	13.333	10.000	8.000	6.667	5.000	4.000	3.333	2.857	2.500
41	13.667	10.250	8.200	6.833	5.125	4.100	3.417	2.929	2.562
42	14.000	10.500	8.400	7.000	5.250	4.200	3.500	3.000	2.625
43	14.333	10.750	8.600	7.167	5.375	4.300	3.583	3.071	2.687
44	14.667	11.000	8.800	7.333	5.500	4.400	3.667	3.143	2.750
45	15.000	11.250	9.000	7.500	5.625	4.500	3.750	3.214	2.812
46	15.333	11.500	9.200	7.667	5.750	4.600	3.833	3.286	2.875
47	15.667	11.750	9.400	7.833	5.875	4.700	3.917	3.357	2.937
48	16.000	12.000	9.600	8.000	6.000	4.800	4.000	3.429	3.000
49	16.333	12.250	9.800	8.167	6.125	4.900	4.083	3.500	3.062
50	16.667	12.500	10.000	8.333	6.250	5.000	4.167	3.571	3.125
51	17.000	12.750	10.200	8.500	6.375	5.100	4.250	3.643	3.187
52	17.333	13.000	10.400	8.667	6.500	5.200	4.333	3.714	3.250
53	17.667	13.250	10.600	8.833	6.625	5.300	4.417	3.786	3.312
54	18.000	13.500	10.800	9.000	6.750	5.400	4.500	3.857	3.375
55	18.333	13.750	11.000	9.167	6.875	5.500	4.583	3.929	3.437

Pitch Diameters of Diametral Pitch Gears

No. of Teeth	Diametral Pitch								
	3 *P*	4 *P*	5 *P*	6 *P*	8 *P*	10 *P*	12 *P*	14 *P*	16 *P*
56	18.667	14.000	11.200	9.333	7.000	5.600	4.667	4.000	3.500
57	19.000	14.250	11.400	9.500	7.125	5.700	4.750	4.071	3.562
58	19.333	14.500	11.600	9.667	7.250	5.800	4.833	4.143	3.625
59	19.667	14.750	11.800	9.833	7.375	5.900	4.917	4.214	3.687
60	20.000	15.000	12.000	10.000	7.500	6.000	5.000	4.286	3.750
61	20.333	15.250	12.200	10.167	7.625	6.100	5.083	4.357	3.812
62	20.667	15.500	12.400	10.333	7.750	6.200	5.167	4.429	3.875
63	21.000	15.750	12.600	10.500	7.875	6.300	5.250	4.500	3.937
64	21.333	16.000	12.800	10.667	8.000	6.400	5.333	4.571	4.000
65	21.667	16.250	13.000	10.833	8.125	6.500	5.417	4.643	4.062
66	22.000	16.500	13.200	11.000	8.250	6.600	5.500	4.714	4.125
67	22.333	16.750	13.400	11.167	8.375	6.700	5.583	4.786	4.187
68	22.667	17.000	13.600	11.333	8.500	6.800	5.667	4.857	4.250
69	23.000	17.250	13.800	11.500	8.625	6.900	5.750	4.929	4.312
70	23.333	17.500	14.000	11.667	8.750	7.000	5.833	5.000	4.375
71	23.667	17.750	14.200	11.833	8.875	7.100	5.917	5.071	4.437
72	24.000	18.000	14.400	12.000	9.000	7.200	6.000	5.143	4.500
73	24.333	18.250	14.600	12.167	9.125	7.300	6.083	5.214	4.562
74	24.667	18.500	14.800	12.333	9.250	7.400	6.167	5.286	4.625
75	25.000	18.750	15.000	12.500	9.375	7.500	6.250	5.357	4.687
76	25.333	19.000	15.200	12.667	9.500	7.600	6.333	5.429	4.750
77	25.667	19.250	15.400	12.833	9.625	7.700	6.417	5.500	4.812
78	26.000	19.500	15.600	13.000	9.750	7.800	6.500	5.571	4.875
79	26.333	19.750	15.800	13.167	9.875	7.900	6.583	5.643	4.937
80	26.667	20.000	16.000	13.333	10.000	8.000	6.667	5.714	5.000
81	27.000	20.250	16.200	13.500	10.125	8.100	6.750	5.786	5.062
82	27.333	20.500	16.400	13.667	10.250	8.200	6.833	5.857	5.125
83	27.667	20.750	16.600	13.833	10.375	8.300	6.917	5.929	5.187
84	28.000	21.000	16.800	14.000	10.500	8.400	7.000	6.000	5.250
85	28.333	21.250	17.000	14.167	10.625	8.500	7.083	6.071	5.312
86	28.667	21.500	17.200	14.333	10.750	8.600	7.167	6.143	5.375
87	29.000	21.750	17.400	14.500	10.875	8.700	7.250	6.214	5.437
88	29.333	22.000	17.600	14.667	11.000	8.800	7.333	6.286	5.500
89	29.667	22.250	17.800	14.833	11.125	8.900	7.417	6.357	5.562
90	30.000	22.500	18.000	15.000	11.250	9.000	7.500	6.429	5.625
91	30.333	22.750	18.200	15.167	11.375	9.100	7.583	6.500	5.687
92	30.667	23.000	18.400	15.333	11.500	9.200	7.667	6.571	5.750
93	31.000	23.250	18.600	15.500	11.625	9.300	7.750	6.643	5.812
94	31.333	23.500	18.800	15.667	11.750	9.400	7.833	6.714	5.875
95	31.667	23.750	19.000	15.833	11.875	9.500	7.917	6.786	5.937
96	32.000	24.000	19.200	16.000	12.000	9.600	8.000	6.857	6.000
97	32.333	24.250	19.400	16.167	12.125	9.700	8.083	6.929	6.062
98	32.667	24.500	19.600	16.333	12.250	9.800	8.167	7.000	6.125
99	33.000	24.750	19.800	16.500	12.375	9.900	8.250	7.071	6.187
100	33.333	25.000	20.000	16.667	12.500	10.000	8.333	7.143	6.250

Outside Diameters of Diametral Pitch Gears

No. of Teeth	Diametral Pitch								
	3 *P*	4 *P*	5 *P*	6 *P*	8 *P*	10 *P*	12 *P*	14 *P*	16 *P*
10	4.000	3.000	2.400	2.000	1.500	1.200	1.000	0.857	0.750
11	4.333	3.250	2.600	2.167	1.625	1.300	1.083	0.929	0.812
12	4.667	3.500	2.800	2.333	1.750	1.400	1.167	1.000	0.875
13	5.000	3.750	3.000	2.500	1.875	1.500	1.250	1.071	0.937
14	5.333	4.000	3.200	2.667	2.000	1.600	1.333	1.143	1.000
15	5.667	4.250	3.400	2.833	2.125	1.700	1.417	1.214	1.062
16	6.000	4.500	3.600	3.000	2.250	1.800	1.500	1.286	1.125
17	6.333	4.750	3.800	3.167	2.375	1.900	1.583	1.357	1.187
18	6.667	5.000	4.000	3.333	2.500	2.000	1.667	1.429	1.250
19	7.000	5.250	4.200	3.500	2.625	2.100	1.750	1.500	1.312
20	7.333	5.500	4.400	3.667	2.750	2.200	1.833	1.571	1.375
21	7.667	5.750	4.600	3.833	2.875	2.300	1.917	1.643	1.437
22	8.000	6.000	4.800	4.000	3.000	2.400	2.000	1.714	1.500
23	8.333	6.250	5.000	4.167	3.125	2.500	2.083	1.786	1.562
24	8.667	6.500	5.200	4.333	3.250	2.600	2.167	1.857	1.625
25	9.000	6.750	5.400	4.500	3.375	2.700	2.250	1.929	1.687
26	9.333	7.000	5.600	4.667	3.500	2.800	2.333	2.000	1.750
27	9.667	7.250	5.800	4.833	3.625	2.900	2.417	2.071	1.812
28	10.000	7.500	6.000	5.000	3.750	3.000	2.500	2.143	1.875
29	10.333	7.750	6.200	5.167	3.875	3.100	2.583	2.214	1.937
30	10.667	8.000	6.400	5.333	4.000	3.200	2.667	2.286	2.000
31	11.000	8.250	6.600	5.500	4.125	3.300	2.750	2.357	2.062
32	11.333	8.500	6.800	5.667	4.250	3.400	2.833	2.429	2.125
33	11.667	8.750	7.000	5.833	4.375	3.500	2.917	2.500	2.187
34	12.000	9.000	7.200	6.000	4.500	3.600	3.000	2.571	2.250
35	12.333	9.250	7.400	6.167	4.625	3.700	3.083	2.643	2.312
36	12.667	9.500	7.600	6.333	4.750	3.800	3.167	2.714	2.375
37	13.000	9.750	7.800	6.500	4.875	3.900	3.250	2.786	2.437
38	13.333	10.000	8.000	6.667	5.000	4.000	3.333	2.857	2.500
39	13.667	10.250	8.200	6.833	5.125	4.100	3.417	2.929	2.562
40	14.000	10.500	8.400	7.000	5.250	4.200	3.500	3.000	2.625
41	14.333	10.750	8.600	7.167	5.375	4.300	3.583	3.071	2.687
42	14.667	11.000	8.800	7.333	5.500	4.400	3.667	3.143	2.750
43	15.000	11.250	9.000	7.500	5.625	4.500	3.750	3.214	2.812
44	15.333	11.500	9.200	7.667	5.750	4.600	3.833	3.286	2.875
45	15.667	11.750	9.400	7.833	5.875	4.700	3.917	3.357	2.937
46	16.000	12.000	9.600	8.000	6.000	4.800	4.000	3.429	3.000
47	16.333	12.250	9.800	8.167	6.125	4.900	4.083	3.500	3.062
48	16.667	12.500	10.000	8.333	6.250	5.000	4.167	3.571	3.125
49	17.000	12.750	10.200	8.500	6.375	5.100	4.250	3.643	3.187
50	17.333	13.000	10.400	8.667	6.500	5.200	4.333	3.714	3.250
51	17.667	13.250	10.600	8.833	6.625	5.300	4.417	3.786	3.312
52	18.000	13.500	10.800	9.000	6.750	5.400	4.500	3.857	3.375
53	18.333	13.750	11.000	9.167	6.875	5.500	4.583	3.929	3.437
54	18.667	14.000	11.200	9.333	7.000	5.600	4.667	4.000	3.500
55	19.000	14.250	11.400	9.500	7.125	5.700	4.750	4.071	3.562

Outside Diameters of Diametral Pitch Gears

No. of Teeth	Diametral Pitch								
	3 P	4 P	5 P	6 P	8 P	10 P	12 P	14 P	16 P
56	19.333	14.500	11.600	9.667	7.250	5.800	4.833	4.143	3.625
57	19.667	14.750	11.800	9.833	7.375	5.900	4.917	4.214	3.687
58	20.000	15.000	12.000	10.000	7.500	6.000	5.000	4.286	3.750
59	20.333	15.250	12.200	10.167	7.625	6.100	5.083	4.357	3.812
60	20.667	15.500	12.400	10.333	7.750	6.200	5.167	4.429	3.875
61	21.000	15.750	12.600	10.500	7.875	6.300	5.250	4.500	3.937
62	21.333	16.000	12.800	10.667	8.000	6.400	5.333	4.571	4.000
63	21.667	16.250	13.000	10.833	8.125	6.500	5.417	4.643	4.062
64	22.000	16.500	13.200	11.000	8.250	6.600	5.500	4.714	4.125
65	22.333	16.750	13.400	11.167	8.375	6.700	5.583	4.786	4.187
66	22.667	17.000	13.600	11.333	8.500	6.800	5.667	4.857	4.250
67	23.000	17.250	13.800	11.500	8.625	6.900	5.750	4.929	4.312
68	23.333	17.500	14.000	11.667	8.750	7.000	5.833	5.000	4.375
69	23.667	17.750	14.200	11.833	8.875	7.100	5.917	5.071	4.437
70	24.000	18.000	14.400	12.000	9.000	7.200	6.000	5.143	4.500
71	24.333	18.250	14.600	12.167	9.125	7.300	6.083	5.214	4.562
72	24.667	18.500	14.800	12.333	9.250	7.400	6.167	5.286	4.625
73	25.000	18.750	15.000	12.500	9.375	7.500	6.250	5.357	4.687
74	25.333	19.000	15.200	12.667	9.500	7.600	6.333	5.429	4.750
75	25.667	19.250	15.400	12.833	9.625	7.700	6.417	5.500	4.812
76	26.000	19.500	15.600	13.000	9.750	7.800	6.500	5.571	4.875
77	26.333	19.750	15.800	13.167	9.875	7.900	6.583	5.643	4.937
78	26.667	20.000	16.000	13.333	10.000	8.000	6.667	5.714	5.000
79	27.000	20.250	16.200	13.500	10.125	8.100	6.750	5.786	5.062
80	27.333	20.500	16.400	13.667	10.250	8.200	6.833	5.857	5.125
81	27.667	20.750	16.600	13.833	10.375	8.300	6.917	5.929	5.187
82	28.000	21.000	16.800	14.000	10.500	8.400	7.000	6.000	5.250
83	28.333	21.250	17.000	14.167	10.625	8.500	7.083	6.071	5.312
84	28.667	21.500	17.200	14.333	10.750	8.600	7.167	6.143	5.375
85	29.000	21.750	17.400	14.500	10.875	8.700	7.250	6.214	5.437
86	29.333	22.000	17.600	14.667	11.000	8.800	7.333	6.286	5.500
87	29.667	22.250	17.800	14.833	11.125	8.900	7.417	6.357	5.562
88	30.000	22.500	18.000	15.000	11.250	9.000	7.500	6.429	5.625
89	30.333	22.750	18.200	15.167	11.375	9.100	7.583	6.500	5.687
90	30.667	23.000	18.400	15.333	11.500	9.200	7.667	6.571	5.750
91	31.000	23.250	18.600	15.500	11.625	9.300	7.750	6.643	5.812
92	31.333	23.500	18.800	15.667	11.750	9.400	7.833	6.714	5.875
93	31.667	23.750	19.000	15.833	11.875	9.500	7.917	6.786	5.937
94	32.000	24.000	19.200	16.000	12.000	9.600	8.000	6.857	6.000
95	32.333	24.250	19.400	16.167	12.125	9.700	8.083	6.929	6.062
96	32.667	24.500	19.600	16.333	12.250	9.800	8.167	7.000	6.125
97	33.000	24.750	19.800	16.500	12.375	9.900	8.250	7.071	6.187
98	33.333	25.000	20.000	16.667	12.500	10.000	8.333	7.143	6.250
99	33.667	25.250	20.200	16.833	12.625	10.100	8.417	7.214	6.312
100	34.000	25.500	20.400	17.000	12.750	10.200	8.500	7.286	6.375

Root Diameters of Diametral Pitch Gears

No. of Teeth	Diametral Pitch								
	3 *P*	4 *P*	5 *P*	6 *P*	8 *P*	10 *P*	12 *P*	14 *P*	16 *P*
10	2.562	1.921	1.537	1.281	0.961	0.769	0.640	0.549	0.480
11	2.895	2.171	1.737	1.448	1.086	0.869	0.724	0.621	0.543
12	3.229	2.421	1.937	1.614	1.211	0.969	0.807	0.692	0.605
13	3.562	2.671	2.137	1.781	1.336	1.069	0.890	0.763	0.668
14	3.895	2.921	2.337	1.948	1.461	1.169	0.974	0.835	0.730
15	4.229	3.171	2.537	2.114	1.586	1.269	1.057	0.906	0.793
16	4.562	3.421	2.737	2.281	1.711	1.369	1.140	0.978	0.855
17	4.895	3.671	2.937	2.448	1.836	1.469	1.224	1.049	0.918
18	5.229	3.921	3.137	2.614	1.961	1.569	1.307	1.121	0.980
19	5.562	4.171	3.337	2.781	2.086	1.669	1.390	1.192	1.043
20	5.895	4.421	3.537	2.948	2.211	1.769	1.474	1.263	1.105
21	6.229	4.671	3.737	3.114	2.336	1.869	1.557	1.335	1.168
22	6.562	4.921	3.937	3.281	2.461	1.969	1.640	1.406	1.230
23	6.895	5.171	4.137	3.448	2.586	2.069	1.724	1.478	1.293
24	7.229	5.421	4.337	3.614	2.711	2.169	1.807	1.549	1.355
25	7.562	5.671	4.537	3.781	2.836	2.269	1.890	1.621	1.418
26	7.895	5.921	4.737	3.948	2.961	2.369	1.974	1.692	1.480
27	8.229	6.171	4.937	4.114	3.086	2.469	2.057	1.763	1.543
28	8.562	6.421	5.137	4.281	3.211	2.569	2.140	1.835	1.605
29	8.895	6.671	5.337	4.448	3.336	2.669	2.224	1.906	1.668
30	9.229	6.921	5.537	4.614	3.461	2.769	2.307	1.978	1.730
31	9.562	7.171	5.737	4.781	3.586	2.869	2.390	2.049	1.793
32	9.895	7.421	5.937	4.948	3.711	2.969	2.474	2.121	1.855
33	10.229	7.671	6.137	5.114	3.836	3.069	2.557	2.192	1.918
34	10.562	7.921	6.337	5.281	3.961	3.169	2.640	2.263	1.980
35	10.895	8.171	6.537	5.448	4.086	3.269	2.724	2.335	2.043
36	11.229	8.421	6.737	5.614	4.211	3.369	2.807	2.406	2.105
37	11.562	8.671	6.937	5.781	4.336	3.469	2.890	2.478	2.168
38	11.895	8.921	7.137	5.948	4.461	3.569	2.974	2.549	2.230
39	12.229	9.171	7.337	6.114	4.586	3.669	3.057	2.621	2.293
40	12.562	9.421	7.537	6.281	4.711	3.769	3.140	2.692	2.355
41	12.895	9.671	7.737	6.448	4.836	3.869	3.224	2.763	2.418
42	13.229	9.921	7.937	6.614	4.961	3.969	3.307	2.835	2.480
43	13.562	10.171	8.137	6.781	5.086	4.069	3.390	2.906	2.543
44	13.895	10.421	8.337	6.948	5.211	4.169	3.474	2.978	2.605
45	14.229	10.671	8.537	7.114	5.336	4.269	3.557	3.049	2.668
46	14.562	10.921	8.737	7.281	5.461	4.369	3.640	3.121	2.730
47	14.895	11.171	8.937	7.448	5.586	4.469	3.724	3.192	2.793
48	15.229	11.421	9.137	7.614	5.711	4.569	3.807	3.263	2.855
49	15.562	11.671	9.337	7.781	5.836	4.669	3.890	3.335	2.918
50	15.895	11.921	9.537	7.948	5.961	4.769	3.974	3.406	2.980
51	16.229	12.171	9.737	8.114	6.086	4.869	4.057	3.478	3.043
52	16.562	12.421	9.937	8.281	6.211	4.969	4.140	3.549	3.105
53	16.895	12.671	10.137	8.448	6.336	5.069	4.224	3.621	3.168
54	17.229	12.921	10.337	8.614	6.461	5.169	4.307	3.692	3.230
55	17.562	13.171	10.537	8.781	6.586	5.269	4.390	3.763	3.293

Root Diameters of Diametral Pitch Gears

No. of Teeth	Diametral Pitch								
	3 *P*	4 *P*	5 *P*	6 *P*	8 *P*	10 *P*	12 *P*	14 *P*	16 *P*
56	17.895	13.421	10.737	8.948	6.711	5.369	4.474	3.835	3.355
57	18.229	13.671	10.937	9.114	6.836	5.469	4.557	3.906	3.418
58	18.562	13.921	11.137	9.281	6.961	5.569	4.640	3.978	3.480
59	18.895	14.171	11.337	9.448	7.086	5.669	4.724	4.049	3.543
60	19.229	14.421	11.537	9.614	7.211	5.769	4.807	4.121	3.605
61	19.562	14.671	11.737	9.781	7.336	5.869	4.890	4.192	3.668
62	19.895	14.921	11.937	9.948	7.461	5.969	4.974	4.263	3.730
63	20.229	15.171	12.137	10.114	7.586	6.069	5.057	4.335	3.793
64	20.562	15.421	12.337	10.281	7.711	6.169	5.140	4.406	3.855
65	20.895	15.671	12.537	10.448	7.836	6.269	5.224	4.478	3.918
66	21.229	15.921	12.737	10.614	7.961	6.369	5.307	4.549	3.980
67	21.562	16.171	12.937	10.781	8.086	6.469	5.390	4.621	4.043
68	21.895	16.421	13.137	10.948	8.211	6.569	5.474	4.692	4.105
69	22.229	16.671	13.337	11.114	8.336	6.669	5.557	4.763	4.168
70	22.562	16.921	13.537	11.281	8.461	6.769	5.640	4.835	4.230
71	22.895	17.171	13.737	11.448	8.586	6.869	5.724	4.906	4.293
72	23.229	17.421	13.937	11.614	8.711	6.969	5.807	4.978	4.355
73	23.562	17.671	14.137	11.781	8.836	7.069	5.890	5.049	4.418
74	23.895	17.921	14.337	11.948	8.961	7.169	5.974	5.121	4.480
75	24.229	18.171	14.537	12.114	9.086	7.269	6.057	5.192	4.543
76	24.562	18.421	14.737	12.281	9.211	7.369	6.140	5.263	4.605
77	24.895	18.671	14.937	12.448	9.336	7.469	6.224	5.335	4.668
78	25.229	18.921	15.137	12.614	9.461	7.569	6.307	5.406	4.730
79	25.562	19.171	15.337	12.781	9.586	7.669	6.390	5.478	4.793
80	25.895	19.421	15.537	12.948	9.711	7.769	6.474	5.549	4.855
81	26.229	19.671	15.737	13.114	9.836	7.869	6.557	5.621	4.918
82	26.562	19.921	15.937	13.281	9.961	7.969	6.640	5.692	4.980
83	26.895	20.171	16.137	13.448	10.086	8.069	6.724	5.763	5.043
84	27.229	20.421	16.337	13.614	10.211	8.169	6.807	5.835	5.105
85	27.562	20.671	16.537	13.781	10.336	8.269	6.890	5.906	5.168
86	27.895	20.921	16.737	13.948	10.461	8.369	6.974	5.978	5.230
87	28.229	21.171	16.937	14.114	10.586	8.469	7.057	6.049	5.293
88	28.562	21.421	17.137	14.281	10.711	8.569	7.140	6.121	5.355
89	28.895	21.671	17.337	14.448	10.836	8.669	7.224	6.192	5.418
90	29.229	21.921	17.537	14.614	10.961	8.769	7.307	6.263	5.480
91	29.562	22.171	17.737	14.781	11.086	8.869	7.390	6.335	5.543
92	29.895	22.421	17.937	14.948	11.211	8.969	7.474	6.406	5.605
93	30.229	22.671	18.137	15.114	11.336	9.069	7.557	6.478	5.668
94	30.562	22.921	18.337	15.281	11.461	9.169	7.640	6.549	5.730
95	30.895	23.171	18.537	15.448	11.586	9.269	7.724	6.621	5.793
96	31.229	23.421	18.737	15.614	11.711	9.369	7.807	6.692	5.855
97	31.562	23.671	18.937	15.781	11.836	9.469	7.890	6.763	5.918
98	31.895	23.921	19.137	15.948	11.961	9.569	7.974	6.835	5.980
99	32.229	24.171	19.337	16.114	12.086	9.669	8.057	6.906	6.043
100	32.562	24.421	19.537	16.281	12.211	9.769	8.140	6.978	6.105

Pitch Diameters for Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches								
	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
12	1.910	2.387	2.865	3.342	3.820	4.297	4.775	5.252	5.730
13	2.069	2.586	3.104	3.621	4.138	4.655	5.173	5.689	6.207
14	2.228	2.785	3.342	3.900	4.456	5.013	5.570	6.127	6.684
15	2.387	2.984	3.581	4.178	4.775	5.371	5.968	6.565	7.162
16	2.546	3.183	3.820	4.456	5.093	5.730	6.366	7.003	7.639
17	2.705	3.382	4.059	4.735	5.411	6.088	6.764	7.440	8.117
18	2.865	3.581	4.297	5.013	5.730	6.446	7.162	7.878	8.594
19	3.024	3.780	4.536	5.292	6.048	6.804	7.560	8.316	9.072
20	3.183	3.979	4.775	5.570	6.366	7.162	7.958	8.753	9.549
21	3.342	4.178	5.014	5.849	6.684	7.520	8.356	9.191	10.027
22	3.501	4.377	5.252	6.128	7.003	7.878	8.754	9.629	10.504
23	3.660	4.576	5.491	6.406	7.321	8.236	9.152	10.067	10.981
24	3.820	4.775	5.730	6.684	7.639	8.594	9.550	10.504	11.459
25	3.979	4.974	5.969	6.962	7.958	8.953	9.948	10.941	11.936
26	4.138	5.173	6.207	7.241	8.276	9.311	10.345	11.379	12.414
27	4.297	5.372	6.446	7.520	8.594	9.669	10.743	11.817	12.891
28	4.456	5.571	6.684	7.798	8.913	10.027	11.141	12.255	13.369
29	4.615	5.770	6.923	8.077	9.231	10.385	11.539	12.692	13.846
30	4.775	5.969	7.162	8.356	9.549	10.743	11.937	13.130	14.324
31	4.934	6.168	7.401	8.634	9.868	11.101	12.335	13.568	14.801
32	5.093	6.366	7.639	8.913	10.186	11.459	12.733	14.006	15.279
33	5.252	6.565	7.878	9.191	10.504	11.817	13.131	14.443	15.756
34	5.411	6.764	8.117	9.470	10.822	12.175	13.528	14.881	16.234
35	5.570	6.963	8.356	9.748	11.141	12.533	13.926	15.319	16.711
36	5.730	7.162	8.594	10.026	11.459	12.891	14.324	15.756	17.189
37	5.889	7.361	8.833	10.305	11.778	13.250	14.722	16.193	17.666
38	6.048	7.560	9.072	10.584	12.096	13.608	15.120	16.631	18.144
39	6.207	7.759	9.311	10.862	12.414	13.966	15.518	17.069	18.621
40	6.366	7.958	9.549	11.141	12.732	14.324	15.916	17.507	19.099
41	6.525	8.157	9.788	11.419	13.051	14.682	16.314	17.944	19.576
42	6.684	8.356	10.027	11.698	13.369	15.040	16.711	18.382	20.053
43	6.844	8.554	10.266	11.976	13.687	15.398	17.109	18.820	20.531
44	7.003	8.753	10.504	12.255	14.006	15.756	17.507	19.258	21.008
45	7.162	8.952	10.743	12.533	14.324	16.114	17.905	19.695	21.486
46	7.321	9.151	10.981	12.812	14.642	16.472	18.303	20.133	21.963
47	7.480	9.350	11.220	13.091	14.961	16.830	18.701	20.571	22.441
48	7.639	9.549	11.459	13.369	15.279	17.189	19.099	21.008	22.918
49	7.798	9.748	11.698	13.647	15.597	17.547	19.497	21.446	23.396
50	7.958	9.947	11.936	13.926	15.916	17.905	19.894	21.884	23.873
51	8.117	10.146	12.175	14.205	16.234	18.263	20.292	22.322	24.351
52	8.276	10.345	12.414	14.483	16.552	18.621	20.690	22.759	24.828
53	8.435	10.544	12.653	14.761	16.870	18.979	21.088	23.196	25.305
54	8.594	10.743	12.891	15.040	17.189	19.337	21.486	23.634	25.783
55	8.753	10.942	13.130	15.319	17.507	19.695	21.884	24.072	26.260
56	8.913	11.141	13.369	15.597	17.825	20.053	22.282	24.510	26.738

Pitch Diameters for Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches								
	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
57	9.072	11.340	13.608	15.875	18.144	20.412	22.680	24.947	27.215
58	9.231	11.539	13.847	16.154	18.462	20.770	23.078	25.385	27.693
59	9.390	11.738	14.086	16.433	18.780	21.128	23.476	25.823	28.170
60	9.549	11.937	14.324	16.711	19.099	21.486	23.873	26.260	28.648
61	9.708	12.136	14.563	16.989	19.417	21.844	24.271	26.698	29.125
62	9.868	12.335	14.802	17.268	19.735	22.202	24.669	27.136	29.603
63	10.027	12.533	15.041	17.547	20.054	22.560	25.067	27.574	30.080
64	10.186	12.732	15.279	17.825	20.372	22.918	25.465	28.011	30.558
65	10.345	12.931	15.518	18.103	20.690	23.277	25.863	28.449	31.035
66	10.504	13.130	15.756	18.382	21.008	23.635	26.260	28.887	31.513
67	10.663	13.329	15.995	18.661	21.327	23.993	26.658	29.324	31.991
68	10.822	13.528	16.234	18.939	21.645	24.351	27.056	29.762	32.468
69	10.981	13.727	16.473	19.218	21.963	24.709	27.454	30.199	32.945
70	11.141	13.926	16.711	19.497	22.282	25.067	27.852	30.637	33.422
71	11.300	14.125	16.950	19.775	22.600	25.425	28.250	31.075	33.900
72	11.459	14.324	17.189	20.053	22.918	25.783	28.648	31.513	34.377
73	11.618	14.523	17.428	20.331	23.237	26.141	29.046	31.950	34.855
74	11.777	14.722	17.666	20.610	23.555	26.499	29.443	32.388	35.332
75	11.936	14.921	17.905	20.889	23.873	26.857	29.841	32.826	35.810
76	12.096	15.120	18.144	21.167	24.192	27.215	30.239	33.263	36.287
77	12.255	15.319	18.383	21.446	24.510	27.574	30.637	33.701	36.765
78	12.414	15.518	18.621	21.725	24.828	27.932	31.035	34.139	37.242
79	12.573	15.716	18.860	22.003	25.146	28.290	31.433	34.576	37.720
80	12.732	15.915	19.099	22.282	25.465	28.648	31.831	35.014	38.197
81	12.891	16.114	19.338	22.560	25.783	29.006	32.229	35.452	38.674
82	13.051	16.313	19.576	22.839	26.101	29.364	32.627	35.890	39.152
83	13.210	16.512	19.815	23.117	26.420	29.722	33.025	36.327	39.629
84	13.369	16.711	20.053	23.396	26.738	30.080	33.423	36.765	40.107
85	13.528	16.910	20.292	23.674	27.056	30.438	33.821	37.202	40.584
86	13.687	17.109	20.531	23.953	27.375	30.796	34.218	37.640	41.062
87	13.846	17.308	20.770	24.232	27.693	31.154	34.616	38.078	41.539
88	14.006	17.507	21.008	24.510	28.011	31.513	35.014	38.515	42.017
89	14.165	17.706	21.247	24.788	28.330	31.871	35.412	38.953	42.494
90	14.324	17.905	21.486	25.067	28.648	32.229	35.810	39.391	42.972
91	14.483	18.104	21.725	25.345	28.966	32.587	36.208	39.829	43.449
92	14.642	18.303	21.963	25.624	29.284	32.945	36.606	40.266	43.927
93	14.801	18.502	22.202	25.902	29.603	33.303	37.004	40.704	44.404
94	14.961	18.701	22.441	26.181	29.921	33.661	37.402	41.142	44.882
95	15.120	18.900	22.680	26.460	30.239	34.019	37.800	41.579	45.359
96	15.279	19.099	22.918	26.738	30.558	34.377	38.197	42.017	45.837
97	15.438	19.298	23.157	27.016	30.876	34.736	38.595	42.454	46.314
98	15.597	19.497	23.396	27.295	31.194	35.094	38.993	42.892	46.792
99	15.756	19.695	23.635	27.574	31.513	35.452	39.391	43.330	47.269
100	15.915	19.894	23.873	27.852	31.831	35.810	39.789	43.767	47.746

Pitch Diameters for Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches									
	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2¾	3
12	6.20	6.68	7.16	7.64	8.12	8.59	9.07	9.55	10.50	11.46
13	6.72	7.24	7.76	8.28	8.79	9.31	9.83	10.34	11.38	12.41
14	7.24	7.80	8.35	8.91	9.47	10.03	10.59	11.14	12.25	13.36
15	7.76	8.35	8.95	9.54	10.15	10.74	11.34	11.93	13.13	14.32
16	8.27	8.91	9.55	10.18	10.82	11.46	12.10	12.73	14.00	15.28
17	8.79	9.47	10.14	10.82	11.50	12.17	12.85	13.53	14.88	16.23
18	9.31	10.03	10.74	11.46	12.17	12.89	13.61	14.32	15.76	17.19
19	9.83	10.58	11.34	12.10	12.85	13.61	14.36	15.12	16.63	18.14
20	10.34	11.14	11.93	12.73	13.53	14.32	15.12	15.92	17.51	19.10
21	10.86	11.70	12.53	13.37	14.20	15.04	15.87	16.71	18.38	20.05
22	11.38	12.25	13.13	14.00	14.88	15.76	16.63	17.50	19.26	21.01
23	11.90	12.81	13.73	14.64	15.56	16.47	17.39	18.30	20.13	21.96
24	12.41	13.37	14.32	15.27	16.23	17.19	18.14	19.10	21.01	22.92
25	12.93	13.93	14.92	15.91	16.91	17.90	18.90	19.89	21.88	23.87
26	13.45	14.48	15.52	16.55	17.59	18.62	19.65	20.69	22.76	24.83
27	13.96	15.04	16.11	17.19	18.26	19.34	20.41	21.48	23.63	25.78
28	14.48	15.60	16.71	17.82	18.94	20.05	21.17	22.28	24.51	26.74
29	15.00	16.15	17.31	18.46	19.61	20.77	21.93	23.07	25.38	27.69
30	15.52	16.70	17.90	19.10	20.29	21.48	22.69	23.87	26.26	28.64
31	16.03	17.26	18.50	19.73	20.96	22.20	23.44	24.66	27.14	29.60
32	16.55	17.82	19.10	20.37	21.64	22.92	24.19	25.46	28.01	30.55
33	17.07	18.38	19.69	21.00	22.32	23.63	24.94	26.25	28.88	31.50
34	17.59	18.94	20.29	21.64	23.00	24.34	25.70	27.05	29.76	32.46
35	18.10	19.49	20.89	22.27	23.67	25.06	26.46	27.85	30.64	33.42
36	18.61	20.05	21.48	22.91	24.35	25.78	27.21	28.65	31.51	34.38
37	19.13	20.61	22.08	23.55	25.03	26.50	27.97	29.44	32.39	35.33
38	19.65	21.17	22.68	24.19	25.70	27.21	28.73	30.24	33.26	36.29
39	20.17	21.73	23.28	24.83	26.38	27.93	29.48	31.03	34.13	37.24
40	20.69	22.28	23.87	25.46	27.06	28.65	30.24	31.83	35.01	38.19
41	21.21	22.84	24.47	26.09	27.73	29.36	30.99	32.62	35.89	39.15
42	21.72	23.39	25.07	26.73	28.41	30.08	31.75	33.42	36.76	40.11
43	22.24	23.95	25.66	27.37	29.09	30.80	32.50	34.21	37.64	41.06
44	22.76	24.51	26.26	28.01	29.77	31.51	33.26	35.01	38.51	42.01
45	23.28	25.07	26.86	28.65	30.44	32.23	34.02	35.81	39.39	42.97
46	23.79	25.62	27.45	29.28	31.11	32.94	34.77	36.60	40.26	43.93
47	24.31	26.18	28.05	29.92	31.79	33.66	35.53	37.40	41.14	44.88
48	24.82	26.74	28.64	30.55	32.47	34.38	36.28	38.19	42.02	45.84
49	25.34	27.29	29.24	31.19	33.14	35.09	37.04	38.99	42.89	46.79
50	25.86	27.85	29.84	31.83	33.82	35.81	37.80	39.79	43.77	47.75
51	26.38	28.41	30.44	32.47	34.49	36.52	38.55	40.58	44.64	48.70
52	26.90	28.97	31.04	33.10	35.17	37.24	39.31	41.38	45.51	49.65
53	27.41	29.52	31.63	33.74	35.85	37.96	40.07	42.18	46.39	50.61
54	27.93	30.08	32.23	34.38	36.53	38.68	40.83	42.97	47.27	51.56
55	28.45	30.64	32.82	35.01	37.20	39.39	41.58	43.76	48.14	52.52
56	28.96	31.19	33.42	35.65	37.88	40.11	42.33	44.56	49.02	53.47

Pitch Diameters for Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches									
	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2¾	3
57	29.48	31.75	34.02	36.28	38.55	40.82	43.09	45.36	49.89	54.43
58	30.00	32.31	34.62	36.92	39.23	41.54	43.85	46.15	50.77	55.38
59	30.52	32.87	35.21	37.56	39.91	42.25	44.60	46.95	51.64	56.34
60	31.03	33.42	35.80	38.20	40.58	42.97	45.36	47.74	52.52	57.30
61	31.55	33.98	36.40	38.83	41.26	43.69	46.11	48.54	53.39	58.25
62	32.07	34.54	37.00	39.47	41.93	44.40	46.87	49.34	54.27	59.20
63	32.58	35.09	37.60	40.10	42.61	45.12	47.62	50.13	55.14	60.16
64	33.10	35.65	38.19	40.74	43.29	45.84	48.38	50.93	56.02	61.12
65	33.62	36.21	38.79	41.38	43.97	46.55	49.14	51.72	56.90	62.07
66	34.14	36.76	39.39	42.02	44.64	47.27	49.89	52.52	57.77	63.02
67	34.66	37.32	39.99	42.65	45.32	47.98	50.65	53.32	58.65	63.98
68	35.17	37.88	40.58	43.29	45.99	48.70	51.41	54.11	59.52	64.94
69	35.69	38.44	41.18	43.92	46.67	49.42	52.16	54.91	60.40	65.89
70	36.21	38.99	41.78	44.56	47.35	50.13	52.92	55.70	61.27	66.84
71	36.72	39.55	42.37	45.20	48.02	50.85	53.67	56.50	62.15	67.80
72	37.24	40.11	42.97	45.84	48.70	51.56	54.43	57.30	63.02	68.76
73	37.76	40.66	43.57	46.47	49.38	52.28	55.19	58.09	63.90	69.71
74	38.28	41.22	44.16	47.11	50.05	53.00	55.94	58.89	64.77	70.66
75	38.79	41.78	44.76	47.74	50.73	53.71	56.70	59.68	65.65	71.62
76	39.31	42.33	45.36	48.38	51.41	54.43	57.45	60.48	66.52	72.57
77	39.83	42.89	45.95	49.02	52.08	55.14	58.21	61.27	67.40	73.53
78	40.34	43.45	46.55	49.66	52.76	55.86	58.96	62.07	68.28	74.48
79	40.86	44.01	47.15	50.29	53.43	56.58	59.72	62.86	69.15	75.44
80	41.38	44.56	47.74	50.93	54.11	57.29	60.48	63.66	70.03	76.39
81	41.90	45.12	48.34	51.56	54.79	58.01	61.23	64.46	70.90	77.35
82	42.41	45.68	48.94	52.20	55.46	58.73	61.99	65.25	71.78	78.30
83	42.93	46.23	49.54	52.84	56.14	59.44	62.75	66.05	72.65	79.26
84	43.45	46.79	50.14	53.47	56.81	60.16	63.50	66.84	73.52	80.21
85	43.97	47.35	50.73	54.11	57.49	60.88	64.26	67.64	74.40	81.17
86	44.48	47.90	51.33	54.75	58.17	61.59	65.01	68.44	75.28	82.12
87	45.00	48.46	51.92	55.38	58.85	62.31	65.77	69.23	76.15	83.08
88	45.52	49.02	52.52	56.02	59.52	63.02	66.53	70.03	77.03	84.03
89	46.03	49.58	53.12	56.66	60.20	63.74	67.28	70.82	77.91	84.99
90	46.55	50.13	53.71	57.30	60.88	64.46	68.04	71.62	78.78	85.94
91	47.07	50.69	54.31	57.93	61.55	65.17	68.79	72.41	79.65	86.90
92	47.59	51.25	54.91	58.57	62.23	65.89	69.55	73.21	80.53	87.85
93	48.10	51.80	55.50	59.20	62.90	66.60	70.30	74.00	81.41	88.81
94	48.62	52.36	56.10	59.84	63.58	67.32	71.06	74.80	82.28	89.76
95	49.14	52.92	56.70	60.48	64.26	68.04	71.81	75.60	83.16	90.72
96	49.65	53.47	57.29	61.11	64.93	68.75	72.57	76.39	84.03	91.67
97	50.17	54.03	57.89	61.75	65.61	69.47	73.33	77.19	84.91	92.62
98	50.69	54.59	58.49	62.39	66.29	70.19	74.08	77.98	85.78	93.58
99	51.21	55.15	59.08	63.02	66.96	70.90	74.84	78.78	86.66	94.54
100	51.72	55.70	59.68	63.66	67.64	71.62	75.59	79.57	87.53	95.49

Outside Diameters of Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches								
	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
12	2.228	2.785	3.342	3.900	4.456	5.013	5.570	6.127	6.684
13	2.387	2.984	3.581	4.178	4.775	5.371	5.968	6.565	7.162
14	2.546	3.183	3.820	4.456	5.093	5.730	6.366	7.003	7.639
15	2.705	3.382	4.059	4.735	5.411	6.088	6.764	7.440	8.117
16	2.865	3.581	4.297	5.013	5.730	6.446	7.162	7.878	8.594
17	3.024	3.780	4.536	5.292	6.048	6.804	7.560	8.316	9.072
18	3.183	3.979	4.775	5.570	6.366	7.162	7.958	8.753	9.549
19	3.342	4.178	5.014	5.849	6.684	7.520	8.356	9.191	10.027
20	3.501	4.377	5.252	6.128	7.003	7.878	8.754	9.629	10.504
21	3.660	4.576	5.491	6.406	7.321	8.236	9.152	10.067	10.981
22	3.820	4.775	5.730	6.684	7.639	8.594	9.550	10.504	11.459
23	3.979	4.974	5.969	6.962	7.958	8.953	9.948	10.941	11.936
24	4.138	5.173	6.207	7.241	8.276	9.311	10.345	11.379	12.414
25	4.297	5.372	6.446	7.520	8.594	9.669	10.743	11.817	12.891
26	4.456	5.571	6.684	7.798	8.913	10.027	11.141	12.255	13.369
27	4.615	5.770	6.923	8.077	9.231	10.385	11.539	12.692	13.846
28	4.775	5.969	7.162	8.356	9.549	10.743	11.937	13.130	14.324
29	4.934	6.168	7.401	8.634	9.868	11.101	12.335	13.568	14.801
30	5.093	6.366	7.639	8.913	10.186	11.459	12.733	14.006	15.279
31	5.252	6.565	7.878	9.191	10.504	11.817	13.131	14.443	15.756
32	5.411	6.764	8.117	9.470	10.822	12.175	13.528	14.881	16.234
33	5.570	6.963	8.356	9.748	11.141	12.533	13.926	15.319	16.711
34	5.730	7.162	8.594	10.026	11.459	12.891	14.324	15.756	17.189
35	5.889	7.361	8.833	10.305	11.778	13.250	14.722	16.193	17.666
36	6.048	7.560	9.072	10.584	12:096	13.608	15.120	16.631	18.144
37	6.207	7.759	9.311	10.862	12.414	13.966	15.518	17.069	18.621
38	6.366	7.958	9.549	11.141	12.732	14.324	15.916	17.507	19.099
39	6.525	8.157	9.788	11.419	13.051	14.682	16.314	17.944	19.576
40	6.684	8.356	10.027	11.698	13.369	15.040	16.711	18.382	20.053
41	6.844	8.554	10.266	11.976	13.687	15.398	17.109	18.820	20.531
42	7.003	8.753	10.504	12.255	14.006	15.756	17.507	19.258	21.008
43	7.162	8.952	10.743	12.533	14.324	16.114	17.905	19.695	21.486
44	7.321	9.151	10.981	12.812	14.642	16.472	18.303	20.133	21.963
45	7.480	9.350	11.220	13.091	14.961	16.830	18.701	20.571	22.441
46	7.639	9.549	11.459	13.369	15.279	17.189	19.099	21.008	22.918
47	7.798	9.748	11.698	13.647	15.597	17.547	19.497	21.446	23.396
48	7.958	9.947	11.936	13.926	15.916	17.905	19.894	21.884	23.873
49	8.117	10.146	12.175	14.205	16.234	18.263	20.292	22.322	24.351
50	8.276	10.345	12.414	14.483	16.552	18.621	20.690	22.759	24.828
51	8.435	10.544	12.653	14.761	16.870	18.979	21.088	23.196	25.305
52	8.594	10.743	12.891	15.040	17.189	19.337	21.486	23.634	25.783
53	8.753	10.942	13.130	15.319	17.507	19.695	21.884	24.072	26.260
54	8.913	11.141	13.369	15.597	17.825	20.053	22.282	24.510	26.738
55	9.072	11.340	13.608	15.875	18.144	20.412	22.680	24.947	27.215
56	9.231	11.539	13.847	16.154	18.462	20.770	23.078	25.385	27.693

Outside Diameters of Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches								
	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
57	9.390	11.738	14.086	16.433	18.780	21.128	23.476	25.823	28.170
58	9.549	11.937	14.324	16.711	19.099	21.486	23.873	26.260	28.648
59	9.708	12.136	14.563	16.989	19.417	21.844	24.271	26.698	29.125
60	9.868	12.335	14.802	17.268	19.735	22.202	24.669	27.136	29.603
61	10.027	12.533	15.041	17.547	20.054	22.560	25.067	27.574	30.080
62	10.186	12.732	15.279	17.825	20.372	22.918	25.465	28.011	30.558
63	10.345	12.931	15.518	18.103	20.690	23.277	25.863	28.449	31.035
64	10.504	13.130	15.756	18.382	21.008	23.635	26.260	28.887	31.513
65	10.663	13.329	15.995	18.661	21.327	23.993	26.658	29.324	31.991
66	10.822	13.528	16.234	18.939	21.645	24.351	27.056	29.762	32.468
67	10.981	13.727	16.473	19.218	21.963	24.709	27.454	30.199	32.945
68	11.141	13.926	16.711	19.497	22.282	25.067	27.852	30.637	33.422
69	11.300	14.125	16.950	19.775	22.600	25.425	28.250	31.075	33.900
70	11.459	14.324	17.189	20.053	22.918	25.783	28.648	31.513	34.377
71	11.618	14.523	17.428	20.331	23.237	26.141	29.046	31.950	34.855
72	11.777	14.722	17.666	20.610	23.555	26.499	29.443	32.388	35.332
73	11.936	14.921	17.905	20.889	23.873	26.857	29.841	32.826	35.810
74	12.096	15.120	18.144	21.167	24.192	27.215	30.239	33.263	36.287
75	12.255	15.319	18.383	21.446	24.510	27.574	30.637	33.701	36.765
76	12.414	15.518	18.621	21.725	24.828	27.932	31.035	34.139	37.242
77	12.573	15.716	18.860	22.003	25.146	28.290	31.433	34.576	37.720
78	12.732	15.915	19.099	22.282	25.465	28.648	31.831	35.014	38.197
79	12.891	16.114	19.338	22.560	25.783	29.006	32.229	35.452	38.674
80	13.051	16.313	19.576	22.839	26.101	29.364	32.627	35.890	39.152
81	13.210	16.512	19.815	23.117	26.420	29.722	33.025	36.327	39.629
82	13.369	16.711	20.053	23.396	26.738	30.080	33.423	36.765	40.107
83	13.528	16.910	20.292	23.674	27.056	30.438	33.821	37.202	40.584
84	13.687	17.109	20.531	23.953	27.375	30.796	34.218	37.640	41.062
85	13.846	17.308	20.770	24.232	27.693	31.154	34.616	38.078	41.539
86	14.006	17.507	21.008	24.510	28.011	31.513	35.014	38.515	42.017
87	14.165	17.706	21.247	24.788	28.330	31.871	35.412	38.953	42.494
88	14.324	17.905	21.486	25.067	28.648	32.229	35.810	39.391	42.972
89	14.483	18.104	21.725	25.345	28.966	32.587	36.208	39.829	43.449
90	14.642	18.303	21.963	25.624	29.284	32.945	36.606	40.266	43.927
91	14.801	18.502	22.202	25.902	29.603	33.303	37.004	40.704	44.404
92	14.961	18.701	22.441	26.181	29.921	33.661	37.402	41.142	44.882
93	15.120	18.900	22.680	26.460	30.239	34.019	37.800	41.579	45.359
94	15.279	19.099	22.918	26.738	30.558	34.377	38.197	42.017	45.837
95	15.438	19.298	23.157	27.016	30.876	34.736	38.595	42.454	46.314
96	15.597	19.497	23.396	27.295	31.194	35.094	38.993	42.892	46.792
97	15.756	19.695	23.635	27.574	31.513	35.452	39.391	43.330	47.269
98	15.915	19.894	23.873	27.852	31.831	35.810	39.789	43.767	47.746
99	16.074	20.093	24.112	28.131	32.149	36.168	40.187	44.205	48.224
100	16.233	20.292	24.350	28.409	32.467	36.526	40.585	44.643	48.701

Outside Diameters of Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches									
	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2¾	3
12	7.24	7.80	8.35	8.91	9.47	10.03	10.59	11.14	12.25	13.36
13	7.76	8.35	8.95	9.54	10.15	10.74	11.35	11.93	13.13	14.32
14	8.27	8.91	9.55	10.18	10.82	11.46	12.10	12.73	14.00	15.28
15	8.79	9.47	10.14	10.82	11.50	12.17	12.85	13.53	14.88	16.23
16	9.31	10.03	10.74	11.46	12.17	12.89	13.61	14.32	15.76	17.19
17	9.83	10.58	11.34	12.10	12.85	13.61	14.36	15.12	16.63	18.14
18	10.34	11.14	11.93	12.73	13.53	14.32	15.12	15.92	17.51	19.10
19	10.86	11.70	12.53	13.37	14.20	15.04	15.87	16.71	18.38	20.05
20	11.38	12.25	13.13	14.00	14.88	15.76	16.63	17.50	19.26	21.01
21	11.90	12.81	13.73	14.64	15.56	16.47	17.39	18.30	20.13	21.96
22	12.41	13.37	14.32	15.27	16.23	17.19	18.14	19.10	21.01	22.92
23	12.93	13.93	14.92	15.91	16.91	17.90	18.90	19.89	21.88	23.87
24	13.45	14.48	15.52	16.55	17.59	18.62	19.65	20.69	22.76	24.83
25	13.96	15.04	16.11	17.19	18.26	19.34	20.41	21.48	23.63	25.78
26	14.48	15.60	16.71	17.82	18.94	20.05	21.17	22.28	24.51	26.74
27	15.00	16.15	17.31	18.46	19.61	20.77	21.93	23.07	25.38	27.69
28	15.52	16.70	17.90	19.10	20.29	21.48	22.69	23.87	26.26	28.64
29	16.03	17.26	18.50	19.73	20.96	22.20	23.44	24.66	27.14	29.60
30	16.55	17.82	19.10	20.37	21.64	22.92	24.19	25.46	28.01	30.55
31	17.07	18.38	19.69	21.00	22.32	23.63	24.94	26.25	28.88	31.50
32	17.59	18.94	20.29	21.64	23.00	24.34	25.70	27.05	29.76	32.46
33	18.10	19.49	20.89	22.27	23.67	25.06	26.46	27.85	30.64	33.42
34	18.61	20.05	21.48	22.91	24.35	25.78	27.21	28.65	31.51	34.38
35	19.13	20.61	22.08	23.55	25.03	26.50	27.97	29.44	32.39	35.33
36	19.65	21.17	22.68	24.19	25.70	27.21	28.73	30.24	33.26	36.29
37	20.17	21.73	23.28	24.83	26.38	27.93	29.48	31.03	34.13	37.24
38	20.69	22.28	23.87	25.46	27.06	28.65	30.24	31.83	35.01	38.19
39	21.21	22.84	24.47	26.09	27.73	29.36	30.99	32.62	35.89	39.15
40	21.72	23.39	25.07	26.73	28.41	30.08	31.75	33.42	36.76	40.11
41	22.24	23.95	25.66	27.37	29.09	30.80	32.50	34.21	37.64	41.06
42	22.76	24.51	26.26	28.01	29.77	31.51	33.26	35.01	38.51	42.01
43	23.28	25.07	26.86	28.65	30.44	32.23	34.02	35.81	39.39	42.97
44	23.79	25.62	27.45	29.28	31.11	32.94	34.77	36.60	40.26	43.93
45	24.31	26.18	28.05	29.92	31.79	33.66	35.53	37.40	41.14	44.88
46	24.82	26.74	28.64	30.55	32.47	34.38	36.28	38.19	42.02	45.84
47	25.34	27.29	29.24	31.19	33.14	35.09	37.04	38.99	42.89	46.79
48	25.86	27.85	29.84	31.83	33.82	35.81	37.80	39.79	43.77	47.75
49	26.38	28.41	30.44	32.47	34.49	36.52	38.55	40.58	44.64	48.70
50	26.90	28.97	31.04	33.10	35.17	37.24	39.31	41.38	45.51	49.65
51	27.41	29.52	31.63	33.74	35.85	37.96	40.07	42.18	46.39	50.61
52	27.93	30.08	32.23	34.38	36.53	38.68	40.83	42.97	47.27	51.56
53	28.45	30.64	32.82	35.01	37.20	39.39	41.58	43.76	48.14	52.52
54	28.96	31.19	33.42	35.65	37.88	40.11	42.33	44.56	49.02	53.47
55	29.48	31.75	34.02	36.28	38.55	40.82	43.09	45.36	49.89	54.43
56	30.00	32.31	34.62	36.92	39.23	41.54	43.85	46.15	50.77	55.38

Outside Diameters of Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches									
	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2¾	3
57	30.52	32.87	35.21	37.56	39.91	42.25	44.60	46.95	51.64	56.34
58	31.03	33.42	35.80	38.20	40.58	42.97	45.36	47.74	52.52	57.30
59	31.55	33.98	36.40	38.83	41.26	43.69	46.11	48.54	53.39	58.25
60	32.07	34.54	37.00	39.47	41.93	44.40	46.87	49.34	54.27	59.20
61	32.58	35.09	37.60	40.10	42.61	45.12	47.62	50.13	55.14	60.16
62	33.10	35.65	38.19	40.74	43.29	45.84	48.38	50.93	56.02	61.12
63	33.62	36.21	38.79	41.38	43.97	46.55	49.14	51.72	56.90	62.07
64	34.14	36.76	39.39	42.02	44.64	47.27	49.89	52.52	57.77	63.02
65	34.66	37.32	39.99	42.65	45.32	47.98	50.65	53.32	58.65	63.98
66	35.17	37.88	40.58	43.29	45.99	48.70	51.41	54.11	59.52	64.94
67	35.69	38.44	41.18	43.92	46.67	49.42	52.16	54.91	60.40	65.89
68	36.21	38.99	41.78	44.56	47.35	50.13	52.92	55.70	61.27	66.84
69	36.72	39.55	42.37	45.20	48.02	50.85	53.67	56.50	62.15	67.80
70	37.24	40.11	42.97	45.84	48.70	51.56	54.43	57.30	63.02	68.76
71	37.76	40.66	43.57	46.47	49.38	52.28	55.19	58.09	63.90	69.71
72	38.28	41.22	44.16	47.11	50.05	53.00	55.94	58.89	64.77	70.66
73	38.79	41.78	44.76	47.74	50.73	53.71	56.70	59.68	65.65	71.62
74	39.31	42.33	45.36	48.38	51.41	54.43	57.45	60.48	66.52	72.57
75	39.83	42.89	45.95	49.02	52.08	55.14	58.21	61.27	67.40	73.53
76	40.34	43.45	46.55	49.66	52.76	55.86	58.96	62.07	68.28	74.48
77	40.86	44.01	47.15	50.29	53.43	56.58	59.72	62.86	69.15	75.44
78	41.38	44.56	47.74	50.93	54.11	57.29	60.48	63.66	70.03	76.39
79	41.90	45.12	48.34	51.56	54.79	58.01	61.23	64.46	70.90	77.35
80	42.41	45.68	48.94	52.20	55.46	58.73	61.99	65.25	71.78	78.30
81	42.93	46.23	49.54	52.84	56.14	59.44	62.75	66.05	72.65	79.26
82	43.45	46.79	50.14	53.47	56.81	60.16	63.50	66.84	73.52	80.21
83	43.97	47.35	50.73	54.11	57.49	60.88	64.26	67.64	74.40	81.17
84	44.48	47.90	51.33	54.75	58.17	61.59	65.01	68.44	75.28	82.12
85	45.00	48.46	51.92	55.38	58.85	62.31	65.77	69.23	76.15	83.08
86	45.52	49.02	52.52	56.02	59.52	63.02	66.53	70.03	77.03	84.03
87	46.03	49.58	53.12	56.66	60.20	63.74	67.28	70.82	77.91	84.99
88	46.55	50.13	53.71	57.30	60.88	64.46	68.04	71.62	78.78	85.94
89	47.07	50.69	54.31	57.93	61.55	65.17	68.79	72.41	79.65	86.90
90	47.59	51.25	54.91	58.57	62.23	65.89	69.55	73.21	80.53	87.85
91	48.10	51.80	55.50	59.20	62.90	66.60	70.30	74.00	81.41	88.81
92	48.62	52.36	56.10	59.84	63.58	67.32	71.06	74.80	82.28	89.76
93	49.14	52.92	56.70	60.48	64.26	68.04	71.81	75.60	83.16	90.72
94	49.65	53.47	57.29	61.11	64.93	68.75	72.57	76.39	84.03	91.67
95	50.17	54.03	57.89	61.75	65.61	69.47	73.33	77.19	84.91	92.62
96	50.69	54.59	58.49	62.39	66.29	70.19	74.08	77.98	85.78	93.58
97	51.21	55.15	59.08	63.02	66.96	70.90	74.84	78.78	86.66	94.54
98	51.72	55.70	59.68	63.66	67.64	71.62	75.59	79.57	87.53	95.49
99	52.24	56.25	60.28	64.30	68.32	72.34	76.35	80.37	88.41	96.45
100	52.76	56.81	60.88	64.94	68.99	73.05	77.11	81.16	89.28	97.40

Root Diameters of Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches ½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
12	1.542	1.927	2.313	2.698	3.083	3.468	3.855	4.240	4.626
13	1.701	2.126	2.552	2.977	3.401	3.826	4.253	4.677	5.103
14	1.860	2.325	2.790	3.256	3.719	4.184	4.650	5.115	5.580
15	2.019	2.524	3.029	3.534	4.038	4.542	5.048	5.553	6.058
16	2.178	2.723	3.268	3.812	4.356	4.901	5.446	5.991	6.535
17	2.337	2.922	3.507	4.091	4.674	5.259	5.844	6.428	7.013
18	2.497	3.121	3.745	4.369	4.993	5.617	6.242	6.866	7.490
19	2.656	3.320	3.984	4.648	5.311	5.975	6.640	7.304	7.968
20	2.815	3.519	4.223	4.926	5.629	6.333	7.038	7.741	8.445
21	2.974	3.718	4.462	5.205	5.947	6.691	7.436	8.179	8.923
22	3.133	3.917	4.700	5.484	6.266	7.049	7.834	8.617	9.400
23	3.292	4.116	4.939	5.762	6.584	7.407	8.232	9.055	9.877
24	3.452	4.315	5.178	6.040	6.902	7.765	8.630	9.492	10.355
25	3.611	4.514	5.417	6.318	7.221	8.124	9.028	9.929	10.832
26	3.770	4.713	5.655	6.597	7.539	8.482	9.425	10.367	11.310
27	3.929	4.912	5.894	6.876	7.857	8.840	9.823	10.805	11.787
28	4.088	5.111	6.132	7.154	8.176	9.198	10.221	11.243	12.265
29	4.247	5.310	6.371	7.433	8.494	9.556	10.619	11.680	12.742
30	4.407	5.509	6.610	7.712	8.812	9.914	11.017	12.118	13.220
31	4.566	5.708	6.849	7.990	9.131	10.272	11.415	12.556	13.697
32	4.725	5.906	7.087	8.269	9.449	10.630	11.813	12.994	14.175
33	4.884	6.105	7.326	8.547	9.767	10.988	12.211	13.431	14.652
34	5.043	6.304	7.565	8.826	10.085	11.346	12.608	13.869	15.130
35	5.202	6.503	7.804	9.104	10.404	11.704	13.006	14.307	15.607
36	5.362	6.702	8.042	9.382	10.722	12.062	13.404	14.744	16.085
37	5.521	6.901	8.281	9.661	11.041	12.421	13.802	15.181	16.562
38	5.680	7.100	8.520	9.940	11.359	12.779	14.200	15.619	17.040
39	5.839	7.299	8.759	10.218	11.677	13.137	14.598	16.057	17.517
40	5.998	7.498	8.997	10.497	11.995	13.495	14.996	16.495	17.995
41	6.157	7.697	9.236	10.775	12.314	13.853	15.394	16.932	18.472
42	6.316	7.896	9.475	11.054	12.632	14.211	15.791	17.370	18.949
43	6.476	8.094	9.714	11.332	12.950	14.569	16.189	17.808	19.427
44	6.635	8.293	9.952	11.611	13.269	14.927	16.587	18.246	19.904
45	6.794	8.492	10.191	11.889	13.587	15.285	16.985	18.683	20.382
46	6.953	8.691	10.429	12.168	13.905	15.643	17.383	19.121	20.859
47	7.112	8.890	10.668	12.447	14.224	16.001	17.781	19.559	21.337
48	7.271	9.089	10.907	12.725	14.542	16.360	18.179	19.996	21.814
49	7.430	9.288	11.146	13.003	14.860	16.718	18.577	20.434	22.292
50	7.590	9.487	11.384	13.282	15.179	17.076	18.974	20.872	22.769
51	7.749	9.686	11.623	13.561	15.497	17.434	19.372	21.310	23.247
52	7.908	9.885	11.862	13.839	15.815	17.792	19.770	21.747	23.724
53	8.067	10.084	12.101	14.117	16.133	18.150	20.168	22.184	24.201
54	8.226	10.283	12.339	14.396	16.452	18.508	20.566	22.622	24.679
55	8.385	10.482	12.578	14.675	16.770	18.866	20.964	23.060	25.156
56	8.545	10.681	12.817	14.953	17.088	19.224	21.362	23.498	25.634

Root Diameters of Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches								
	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
57	8.704	10.880	13.056	15.231	17.407	19.583	21.760	23.935	26.111
58	8.863	11.079	13.295	15.510	17.725	19.941	22.158	24.373	26.589
59	9.022	11.278	13.534	15.789	18.043	20.299	22.556	24.811	27.066
60	9.181	11.477	13.772	16.067	18.362	20.657	22.953	25.248	27.544
61	9.340	11.676	14.011	16.345	18.680	21.015	23.351	25.686	28.021
62	9.500	11.875	14.250	16.624	18.998	21.373	23.749	26.124	28.499
63	9.659	12.073	14.489	16.903	19.317	21.731	24.147	26.562	28.976
64	9.818	12.272	14.727	17.181	19.635	22.089	24.545	26.999	29.454
65	9.977	12.471	14.966	17.459	19.953	22.448	24.943	27.437	29.931
66	10.136	12.670	15.204	17.738	20.271	22.806	25.340	27.875	30.409
67	10.295	12.869	15.443	18.017	20.590	23.164	25.738	28.312	30.886
68	10.454	13.068	15.682	18.295	20.908	23.522	26.136	28.750	31.364
69	10.613	13.267	15.921	18.574	21.226	23.880	26.534	29.187	31.841
70	10.773	13.466	16.159	18.853	21.545	24.238	26.932	29.625	32.318
71	10.932	13.665	16.398	19.131	21.863	24.596	27.330	30.063	32.796
72	11.091	13.864	16.637	19.409	22.181	24.954	27.728	30.501	33.273
73	11.250	14.063	16.876	19.687	22.500	25.312	28.126	30.938	33.751
74	11.409	14.262	17.114	19.966	22.818	25.670	28.523	31.376	34.228
75	11.568	14.461	17.353	20.245	23.136	26.028	28.921	31.814	34.706
76	11.728	14.660	17.592	20.523	23.455	26.386	29.319	32.251	35.183
77	11.887	14.859	17.831	20.802	23.773	26.745	29.717	32.689	35.661
78	12.046	15.058	18.069	21.081	24.091	27.103	30.115	33.127	36.138
79	12.205	15.256	18.308	21.359	24.409	27.461	30.513	33.564	36.616
80	12.364	15.455	18.547	21.638	24.728	27.819	30.911	34.002	37.093
81	12.523	15.654	18.786	21.916	25.046	28.177	31.309	34.440	37.570
82	12.683	15.853	19.024	22.195	25.364	28.535	31.707	34.878	38.048
83	12.842	16.052	19.263	22.473	25.683	28.893	32.105	35.315	38.525
84	13.001	16.251	19.501	22.752	26.001	29.251	32.503	35.753	39.003
85	13.160	16.450	19.740	23.030	26.319	29.609	32.901	36.190	39.480
86	13.319	16.649	19.979	23.309	26.638	29.967	33.298	36.628	39.958
87	13.478	16.848	20.218	23.588	26.956	30.325	33.696	37.066	40.435
88	13.638	17.047	20.456	23.866	27.274	30.684	34.094	37.503	40.913
89	13.797	17.246	20.695	24.144	27.593	31.042	34.492	37.941	41.390
90	13.956	17.445	20.934	24.423	27.911	31.400	34.890	38.379	41.868
91	14.115	17.644	21.173	24.701	28.229	31.758	35.288	38.817	42.345
92	14.274	17.843	21.411	24.980	28.547	32.116	35.686	39.254	42.823
93	14.433	18.042	21.650	25.258	28.866	32.474	36.084	39.692	43.300
94	14.593	18.241	21.889	25.537	29.184	32.832	36.482	40.130	43.778
95	14.752	18.440	22.128	25.816	29.502	33.190	36.880	40.567	44.255
96	14.911	18.639	22.366	26.094	29.821	33.548	37.277	41.005	44.733
97	15.070	18.838	22.605	26.372	30.139	33.907	37.675	41.442	45.210
98	15.229	19.037	22.844	26.651	30.457	34.265	38.073	41.880	45.688
99	15.388	19.235	23.083	26.930	30.776	34.623	38.471	42.318	46.165
100	15.547	19.434	23.321	27.208	31.094	34.981	38.869	42.755	46.642

Root Diameters of Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches									
	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2¾	3
12	5.00	5.39	5.78	6.16	6.55	6.93	7.32	7.70	8.47	9.24
13	5.52	5.95	6.38	6.80	7.22	7.65	8.08	8.50	9.35	10.20
14	6.04	6.51	6.97	7.43	7.90	8.37	8.84	9.30	10.22	11.15
15	6.56	7.06	7.57	8.06	8.58	9.08	9.60	10.09	11.10	12.11
16	7.07	7.62	8.17	8.70	9.25	9.80	10.35	10.89	11.97	13.07
17	7.59	8.18	8.76	9.34	9.93	10.51	11.10	11.69	12.85	14.02
18	8.11	8.74	9.36	9.98	10.60	11.23	11.86	12.48	13.73	14.98
19	8.63	9.29	9.96	10.62	11.28	11.95	12.61	13.28	14.60	15.93
20	9.14	9.85	10.55	11.25	11.96	12.66	13.37	14.08	15.48	16.89
21	9.66	10.41	11.15	11.89	12.63	13.38	14.12	14.87	16.35	17.84
22	10.18	10.96	11.75	12.52	13.31	14.10	14.88	15.66	17.23	18.80
23	10.70	11.52	12.35	13.16	13.99	14.81	15.64	16.46	18.10	19.75
24	11.21	12.08	12.94	13.79	14.66	15.53	16.39	17.26	18.98	20.71
25	11.73	12.64	13.54	14.43	15.34	16.24	17.15	18.05	19.85	21.66
26	12.25	13.19	14.14	15.07	16.02	16.96	17.90	18.85	20.73	22.62
27	12.76	13.75	14.73	15.71	16.69	17.68	18.66	19.64	21.60	23.57
28	13.28	14.31	15.33	16.34	17.37	18.39	19.42	20.44	22.48	24.53
29	13.80	14.86	15.93	16.98	18.04	19.11	20.18	21.23	23.35	25.48
30	14.32	15.41	16.52	17.62	18.72	19.82	20.94	22.03	24.23	26.43
31	14.83	15.97	17.12	18.25	19.39	20.54	21.69	22.82	25.11	27.39
32	15.35	16.53	17.72	18.89	20.07	21.26	22.44	23.62	25.98	28.34
33	15.87	17.09	18.31	19.52	20.75	21.97	23.19	24.41	26.85	29.29
34	16.39	17.65	18.91	20.16	21.43	22.68	23.95	25.21	27.73	30.25
35	16.90	18.20	19.51	20.79	22.10	23.40	24.71	26.01	28.61	31.21
36	17.41	18.76	20.10	21.43	22.78	24.12	25.46	26.81	29.48	32.17
37	17.93	19.32	20.70	22.07	23.46	24.84	26.22	27.60	30.36	33.12
38	18.45	19.88	21.30	22.71	24.13	25.55	26.98	28.40	31.23	34.08
39	18.97	20.44	21.90	23.35	24.81	26.27	27.73	29.19	32.10	35.03
40	19.49	20.99	22.49	23.98	25.49	26.99	28.49	29.99	32.98	35.98
41	20.01	21.55	23.09	24.61	26.16	27.70	29.24	30.78	33.86	36.94
42	20.52	22.10	23.69	25.26	26.84	28.42	30.00	31.58	34.73	37.90
43	21.04	22.66	24.28	25.89	27.52	29.14	30.75	32.37	35.61	38.85
44	21.56	23.22	24.88	26.53	28.20	29.85	31.51	33.17	36.48	39.80
45	22.08	23.78	25.48	27.17	28.87	30.57	32.27	33.97	37.36	40.76
46	22.59	24.33	26.07	27.80	29.54	31.28	33.02	34.76	38.23	41.72
47	23.11	24.89	26.67	28.44	30.22	32.00	33.78	35.56	39.11	42.67
48	23.62	25.45	27.26	29.07	30.90	32.72	34.53	36.35	39.99	43.63
49	24.14	26.00	27.86	29.71	31.57	33.43	35.29	37.15	40.86	44.58
50	24.66	26.56	28.46	30.35	32.25	34.15	36.04	37.95	41.74	45.54
51	25.18	27.12	29.06	30.99	32.92	34.86	36.80	38.74	42.61	46.49
52	25.70	27.68	29.66	31.62	33.60	35.58	37.56	39.54	43.48	47.44
53	26.21	28.23	30.25	32.26	34.28	36.30	38.32	40.34	44.36	48.40
54	26.73	28.79	30.85	32.90	34.96	37.02	39.08	41.13	45.24	49.35
55	27.25	29.35	31.44	33.53	35.63	37.73	39.83	41.92	46.11	50.31
56	27.76	29.90	32.04	34.17	36.31	38.45	40.58	42.72	46.99	51.26

Root Diameters of Circular Pitch Gears

No. of Teeth	Circular Pitch in Inches									
	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2¾	3
57	28.28	30.46	32.64	34.81	36.98	39.16	41.34	43.52	47.86	52.22
58	28.80	31.02	33.24	35.45	37.66	39.88	42.10	44.31	48.74	53.17
59	29.32	31.58	33.83	36.09	38.34	40.59	42.85	45.11	49.61	54.13
60	29.83	32.13	34.42	36.73	39.01	41.31	43.61	45.90	50.49	55.09
61	30.35	32.69	35.02	37.36	39.69	42.03	44.36	46.70	51.36	56.04
62	30.87	33.25	35.62	38.00	40.36	42.74	45.12	47.50	52.24	56.99
63	31.38	33.80	36.22	38.63	41.04	43.46	45.87	48.29	53.11	57.95
64	31.90	34.36	36.81	39.27	41.72	44.18	46.63	49.09	53.99	58.91
65	32.42	34.92	37.41	39.91	42.40	44.89	47.39	49.88	54.87	59.86
66	32.94	35.47	38.01	40.55	43.07	45.61	48.14	50.68	55.74	60.81
67	33.46	36.03	38.61	41.18	43.75	46.32	48.90	51.48	56.62	61.77
68	33.97	36.59	39.20	41.82	44.42	47.04	49.66	52.27	57.49	62.73
69	34.49	37.15	39.80	42.45	45.10	47.76	50.41	53.07	58.37	63.68
70	35.01	37.70	40.40	43.09	45.78	48.47	51.17	53.86	59.24	64.63
71	35.52	38.26	40.99	43.73	46.45	49.18	51.92	54.66	60.12	65.59
72	36.04	38.82	41.59	44.37	47.13	49.90	52.68	55.46	60.99	66.55
73	36.56	39.37	42.19	45.00	47.81	50.62	53.44	56.25	61.87	67.50
74	37.08	39.93	42.78	45.64	48.48	51.34	54.19	57.05	62.74	68.45
75	37.59	40.49	43.38	46.27	49.16	52.05	54.95	57.84	63.62	69.41
76	38.11	41.04	43.98	46.91	49.84	52.77	55.70	58.64	64.49	70.36
77	38.63	41.60	44.57	47.55	50.51	53.48	56.46	59.43	65.37	71.32
78	39.14	42.16	45.17	48.19	51.19	54.20	57.21	60.23	66.25	72.27
79	39.66	42.72	45.77	48.82	51.86	54.92	57.97	61.02	67.12	73.23
80	40.18	43.27	46.36	49.46	52.54	55.63	58.73	61.82	68.00	74.18
81	40.70	43.83	46.96	50.09	53.22	56.35	59.48	62.62	68.87	75.14
82	41.21	44.39	47.56	50.73	53.89	57.07	60.24	63.41	69.75	76.09
83	41.73	44.94	48.16	51.37	54.57	57.78	61.00	64.21	70.62	77.05
84	42.25	45.50	48.76	52.00	55.24	58.50	61.75	65.00	71.49	78.00
85	42.77	46.06	49.35	52.64	55.92	59.22	62.51	65.80	72.37	78.96
86	43.28	46.61	49.95	53.28	56.60	59.93	63.26	66.60	73.25	79.91
87	43.80	47.17	50.54	53.91	57.28	60.65	64.02	67.39	74.12	80.87
88	44.32	47.73	51.14	54.55	57.95	61.36	64.78	68.19	75.00	81.82
89	44.83	48.29	51.74	55.19	58.63	62.08	65.53	68.98	75.88	82.78
90	45.35	48.84	52.33	55.83	59.31	62.80	66.29	69.78	76.75	83.73
91	45.87	49.40	52.93	56.46	59.98	63.51	67.04	70.57	77.62	84.69
92	46.39	49.96	53.53	57.10	60.66	64.23	67.80	71.37	78.50	85.64
93	46.90	50.51	54.12	57.73	61.33	64.94	68.55	72.16	79.38	86.60
94	47.42	51.07	54.72	58.37	62.01	65.66	69.31	72.96	80.25	87.55
95	47.94	51.63	55.32	59.01	62.69	66.38	70.06	73.76	81.13	88.51
96	48.45	52.18	55.91	59.64	63.36	67.09	70.82	74.55	82.00	89.46
97	48.97	52.74	56.51	60.28	64.04	67.81	71.58	75.35	82.88	90.41
98	49.49	53.30	57.11	60.92	64.72	68.53	72.33	76.14	83.75	91.37
99	50.01	53.86	57.70	61.55	65.39	69.24	73.09	76.94	84.63	92.33
100	50.52	54.41	58.30	62.19	66.07	69.96	73.84	77.73	85.50	93.28

Outside Diameters of Stub-tooth Gears

(Fellows Gear Shaper Co.'s System)

No. of Teeth	Diametral Pitch					
	4/5	5/7	6/8	8/10	10/12	12/14
10	2.900	2.286	1.917	1.450	1.167	0.976
11	3.150	2.486	2.083	1.575	1.267	1.060
12	3.400	2.686	2.250	1.700	1.367	1.143
13	3.650	2.886	2.417	1.825	1.467	1.226
14	3.900	3.086	2.583	1.950	1.567	1.310
15	4.150	3.286	2.750	2.075	1.667	1.393
16	4.400	3.486	2.917	2.200	1.767	1.476
17	4.650	3.686	3.083	2.325	1.867	1.560
18	4.900	3.886	3.250	2.450	1.967	1.643
19	5.150	4.086	3.417	2.575	2.067	1.726
20	5.400	4.286	3.583	2.700	2.167	1.810
21	5.650	4.486	3.750	2.825	2.267	1.893
22	5.900	4.686	3.917	2.950	2.367	1.976
23	6.150	4.886	4.083	3.075	2.467	2.060
24	6.400	5.086	4.250	3.200	2.567	2.143
25	6.650	5.286	4.417	3.325	2.667	2.226
26	6.900	5.486	4.583	3.450	2.767	2.310
27	7.150	5.686	4.750	3.575	2.867	2.393
28	7.400	5.886	4.917	3.700	2.967	2.476
29	7.650	6.086	5.083	3.825	3.067	2.560
30	7.900	6.286	5.250	3.950	3.167	2.643
31	8.150	6.486	5.417	4.075	3.267	2.726
32	8.400	6.686	5.583	4.200	3.367	2.810
33	8.650	6.886	5.750	4.325	3.467	2.893
34	8.900	7.086	5.917	4.450	3.567	2.976
35	9.150	7.286	6.083	4.575	3.667	3.060
36	9.400	7.486	6.250	4.700	3.767	3.143
37	9.650	7.686	6.417	4.825	3.867	3.226
38	9.900	7.886	6.583	4.950	3.967	3.310
39	10.150	8.086	6.750	5.075	4.067	3.393
40	10.400	8.286	6.917	5.200	4.167	3.476
41	10.650	8.486	7.083	5.325	4.267	3.560
42	10.900	8.686	7.250	5.450	4.367	3.643
43	11.150	8.886	7.417	5.575	4.467	3.726
44	11.400	9.086	7.583	5.700	4.567	3.810
45	11.650	9.286	7.750	5.825	4.667	3.893
46	11.900	9.486	7.917	5.950	4.767	3.976
47	12.150	9.686	8.083	6.075	4.867	4.060
48	12.400	9.886	8.250	6.200	4.967	4.143
49	12.650	10.086	8.417	6.325	5.067	4.226
50	12.900	10.286	8.583	6.450	5.167	4.310
51	13.150	10.486	8.750	6.575	5.267	4.393
52	13.400	10.686	8.917	6.700	5.367	4.476
53	13.650	10.886	9.083	6.825	5.467	4.560
54	13.900	11.086	9.250	6.950	5.567	4.643
55	14.150	11.286	9.417	7.075	5.667	4.726

Outside Diameters of Stub-tooth Gears

(Fellows Gear Shaper Co.'s System)

No. of Teeth	Diametral Pitch					
	4/5	5/7	6/8	8/10	10/12	12/14
56	14.400	11.486	9.583	7.200	5.767	4.810
57	14.650	11.686	9.750	7.325	5.867	4.893
58	14.900	11.886	9.917	7.450	5.967	4.976
59	15.150	12.086	10.083	7.575	6.067	5.060
60	15.400	12.286	10.250	7.700	6.167	5.143
61	15.650	12.486	10.417	7.825	6.267	5.226
62	15.900	12.686	10.583	7.950	6.367	5.310
63	16.150	12.886	10.750	8.075	6.467	5.393
64	16.400	13.086	10.917	8.200	6.567	5.476
65	16.650	13.286	11.083	8.325	6.667	5.560
66	16.900	13.486	11.250	8.450	6.767	5.643
67	17.150	13.686	11.417	8.575	6.867	5.726
68	17.400	13.886	11.583	8.700	6.967	5.810
69	17.650	14.086	11.750	8.825	7.067	5.893
70	17.900	14.286	11.917	8.950	7.167	5.976
71	18.150	14.486	12.083	9.075	7.267	6.060
72	18.400	14.686	12.250	9.200	7.367	6.143
73	18.650	14.886	12.417	9.325	7.467	6.226
74	18.900	15.086	12.583	9.450	7.567	6.310
75	19.150	15.286	12.750	9.575	7.667	6.393
76	19.400	15.486	12.917	9.700	7.767	6.476
77	19.650	15.686	13.083	9.825	7.867	6.560
78	19.900	15.886	13.250	9.950	7.967	6.643
79	20.150	16.086	13.417	10.075	8.067	6.726
80	20.400	16.286	13.583	10.200	8.167	6.810
81	20.650	16.486	13.750	10.325	8.267	6.893
82	20.900	16.686	13.917	10.450	8.367	6.976
83	21.150	16.886	14.083	10.575	8.467	7.060
84	21.400	17.086	14.250	10.700	8.567	7.143
85	21.650	17.286	14.417	10.825	8.667	7.226
86	21.900	17.486	14.583	10.950	8.767	7.310
87	22.150	17.686	14.750	11.075	8.867	7.393
88	22.400	17.886	14.917	11.200	8.967	7.476
89	22.650	18.086	15.083	11.325	9.067	7.560
90	22.900	18.286	15.250	11.450	9.167	7.643
91	23.150	18.486	15.417	11.575	9.267	7.726
92	23.400	18.686	15.583	11.700	9.367	7.810
93	23.650	18.886	15.750	11.825	9.467	7.893
94	23.900	19.086	15.917	11.950	9.567	7.976
95	24.150	19.286	16.083	12.075	9.667	8.060
96	24.400	19.486	16.250	12.200	9.767	8.143
97	24.650	19.686	16.417	12.325	9.867	8.226
98	24.900	19.886	16.583	12.450	9.967	8.310
99	25.150	20.086	16.750	12.575	10.067	8.393
100	25.400	20.286	16.917	12.700	10.167	8.476

Root Diameters of Stub-tooth Gears

(Fellows Gear Shaper Co.'s System)

No. of Teeth	Diametral Pitch					
	4/5	5/7	6/8	8/10	10/12	12/14
10	2.000	1.643	1.355	1.000	0.792	0.655
11	2.250	1.843	1.521	1.125	0.892	0.739
12	2.500	2.043	1.688	1.250	0.992	0.822
13	2.750	2.243	1.855	1.375	1.092	0.905
14	3.000	2.443	2.021	1.500	1.192	0.989
15	3.250	2.643	2.188	1.625	1.292	1.072
16	3.500	2.843	2.355	1.750	1.392	1.155
17	3.750	3.043	2.521	1.875	1.492	1.239
18	4.000	3.243	2.688	2.000	1.592	1.322
19	4.250	3.443	2.855	2.125	1.692	1.405
20	4.500	3.643	3.021	2.250	1.792	1.489
21	4.750	3.843	3.188	2.375	1.892	1.572
22	5.000	4.043	3.355	2.500	1.992	1.655
23	5.250	4.243	3.521	2.625	2.092	1.739
24	5.500	4.443	3.688	2.750	2.192	1.822
25	5.750	4.643	3.855	2.875	2.292	1.905
26	6.000	4.843	4.021	3.000	2.392	1.989
27	6.250	5.043	4.188	3.125	2.492	2.072
28	6.500	5.243	4.355	3.250	2.592	2.155
29	6.750	5.443	4.521	3.375	2.692	2.239
30	7.000	5.643	4.688	3.500	2.792	2.322
31	7.250	5.843	4.855	3.625	2.892	2.405
32	7.500	6.043	5.021	3.750	2.992	2.489
33	7.750	6.243	5.188	3.875	3.092	2.572
34	8.000	6.443	5.355	4.000	3.192	2.655
35	8.250	6.643	5.521	4.125	3.292	2.739
36	8.500	6.843	5.688	4.250	3.392	2.822
37	8.750	7.043	5.855	4.375	3.492	2.905
38	9.000	7.243	6.021	4.500	3.592	2.989
39	9.250	7.443	6.188	4.625	3.692	3.072
40	9.500	7.643	6.355	4.750	3.792	3.155
41	9.750	7.843	6.521	4.875	3.892	3.239
42	10.000	8.043	6.688	5.000	3.992	3.322
43	10.250	8.243	6.855	5.125	4.092	3.405
44	10.500	8.443	7.021	5.250	4.192	3.489
45	10.750	8.643	7.188	5.375	4.292	3.572
46	11.000	8.843	7.355	5.500	4.392	3.655
47	11.250	9.043	7.521	5.625	4.492	3.739
48	11.500	9.243	7.688	5.750	4.592	3.822
49	11.750	9.443	7.855	5.875	4.692	3.905
50	12.000	9.643	8.021	6.000	4.792	3.989
51	12.250	9.843	8.188	6.125	4.892	4.072
52	12.500	10.043	8.355	6.250	4.992	4.155
53	12.750	10.243	8.521	6.375	5.092	4.239
54	13.000	10.443	8.688	6.500	5.192	4.322
55	13.250	10.643	8.855	6.625	5.292	4.405

Root Diameters of Stub-tooth Gears

(Fellows Gear Shaper Co.'s System)

No. of Teeth	Diametral Pitch					
	4/5	5/7	6/8	8/10	10/12	12/14
56	13.500	10.843	9.021	6.750	5.392	4.489
57	13.750	11.043	9.188	6.875	5.492	4.572
58	14.000	11.243	9.355	7.000	5.592	4.655
59	14.250	11.443	9.521	7.125	5.692	4.739
60	14.500	11.643	9.688	7.250	5.792	4.822
61	14.750	11.843	9.855	7.375	5.892	4.905
62	15.000	12.043	10.021	7.500	5.992	4.989
63	15.250	12.243	10.188	7.625	6.092	5.072
64	15.500	12.443	10.355	7.750	6.192	5.155
65	15.750	12.643	10.521	7.875	6.292	5.239
66	16.000	12.843	10.688	8.000	6.392	5.322
67	16.250	13.043	10.855	8.125	6.492	5.405
68	16.500	13.243	11.021	8.250	6.592	5.489
69	16.750	13.443	11.188	8.375	6.692	5.572
70	17.000	13.643	11.355	8.500	6.792	5.655
71	17.250	13.843	11.521	8.625	6.892	5.739
72	17.500	14.043	11.688	8.750	6.992	5.822
73	17.750	14.243	11.855	8.875	7.092	5.905
74	18.000	14.443	12.021	9.000	7.192	5.989
75	18.250	14.643	12.188	9.125	7.292	6.072
76	18.500	14.843	12.355	9.250	7.392	6.155
77	18.750	15.043	12.521	9.375	7.492	6.239
78	19.000	15.243	12.688	9.500	7.592	6.322
79	19.250	15.443	12.855	9.625	7.692	6.405
80	19.500	15.643	13.021	9.750	7.792	6.489
81	19.750	15.843	13.188	9.875	7.892	6.572
82	20.000	16.043	13.355	10.000	7.992	6.655
83	20.250	16.243	13.521	10.125	8.092	6.739
84	20.500	16.443	13.688	10.250	8.192	6.822
85	20.750	16.643	13.855	10.375	8.292	6.905
86	21.000	16.843	14.021	10.500	8.392	6.989
87	21.250	17.043	14.188	10.625	8.492	7.072
88	21.500	17.243	14.355	10.750	8.592	7.155
89	21.750	17.443	14.521	10.875	8.692	7.239
90	22.000	17.643	14.688	11.000	8.792	7.322
91	22.250	17.843	14.855	11.125	8.892	7.405
92	22.500	18.043	15.021	11.250	8.992	7.489
93	22.750	18.243	15.188	11.375	9.092	7.572
94	23.000	18.443	15.355	11.500	9.192	7.655
95	23.250	18.643	15.521	11.625	9.292	7.739
96	23.500	18.843	15.688	11.750	9.392	7.822
97	23.750	19.043	15.855	11.875	9.492	7.905
98	24.000	19.243	16.021	12.000	9.592	7.989
99	24.250	19.443	16.188	12.125	9.692	8.072
100	24.500	19.643	16.355	12.250	9.792	8.155

Root Diameters of Gears Cut on Gear Shapers

No. of Teeth	Diametral Pitch								
	3 *P*	4 *P*	5 *P*	6 *P*	8 *P*	10 *P*	12 *P*	14 *P*	16 *P*
10	2.500	1.875	1.500	1.250	0.938	0.750	0.625	0.536	0.469
11	2.833	2.125	1.700	1.417	1.063	0.850	0.708	0.607	0.531
12	3.167	2.375	1.900	1.583	1.188	0.950	0.792	0.679	0.594
13	3.500	2.625	2.100	1.750	1.313	1.050	0.875	0.750	0.656
14	3.833	2.875	2.300	1.917	1.438	1.150	0.958	0.822	0.719
15	4.167	3.125	2.500	2.083	1.563	1.250	1.042	0.893	0.781
16	4.500	3.375	2.700	2.250	1.688	1.350	1.125	0.965	0.844
17	4.833	3.625	2.900	2.417	1.813	1.450	1.208	1.036	0.906
18	5.167	3.875	3.100	2.583	1.938	1.550	1.292	1.107	0.969
19	5.500	4.125	3.300	2.750	2.063	1.650	1.375	1.179	1.031
20	5.833	4.375	3.500	2.917	2.188	1.750	1.458	1.250	1.094
21	6.167	4.625	3.700	3.083	2.313	1.850	1.542	1.322	1.156
22	6.500	4.875	3.900	3.250	2.438	1.950	1.625	1.393	1.219
23	6.833	5.125	4.100	3.417	2.563	2.050	1.708	1.465	1.281
24	7.167	5.375	4.300	3.583	2.688	2.150	1.792	1.536	1.344
25	7.500	5.625	4.500	3.750	2.813	2.250	1.875	1.607	1.406
26	7.833	5.875	4.700	3.917	2.938	2.350	1.958	1.679	1.469
27	8.167	6.125	4.900	4.083	3.063	2.450	2.042	1.750	1.531
28	8.500	6.375	5.100	4.250	3.188	2.550	2.125	1.822	1.594
29	8.833	6.625	5.300	4.417	3.313	2.650	2.208	1.893	1.656
30	9.167	6.875	5.500	4.583	3.438	2.750	2.292	1.965	1.719
31	9.500	7.125	5.700	4.750	3.563	2.850	2.375	2.036	1.781
32	9.833	7.375	5.900	4.917	3.688	2.950	2.458	2.107	1.844
33	10.167	7.625	6.100	5.083	3.813	3.050	2.542	2.179	1.906
34	10.500	7.875	6.300	5.250	3.938	3.150	2.625	2.250	1.969
35	10.833	8.125	6.500	5.417	4.063	3.250	2.708	2.322	2.031
36	11.167	8.375	6.700	5.583	4.188	3.350	2.792	2.393	2.094
37	11.500	8.625	6.900	5.750	4.313	3.450	2.875	2.465	2.156
38	11.833	8.875	7.100	5.917	4.438	3.550	2.958	2.536	2.219
39	12.167	9.125	7.300	6.083	4.563	3.650	3.042	2.607	2.281
40	12.500	9.375	7.500	6.250	4.688	3.750	3.125	2.679	2.344
41	12.833	9.625	7.700	6.417	4.813	3.850	3.208	2.750	2.406
42	13.167	9.875	7.900	6.583	4.938	3.950	3.292	2.822	2.469
43	13.500	10.125	8.100	6.750	5.063	4.050	3.375	2.893	2.531
44	13.833	10.375	8.300	6.917	5.188	4.150	3.458	2.965	2.594
45	14.167	10.625	8.500	7.083	5.313	4.250	3.542	3.036	2.656
46	14.500	10.875	8.700	7.250	5.438	4.350	3.625	3.107	2.719
47	14.833	11.125	8.900	7.417	5.563	4.450	3.708	3.179	2.781
48	15.167	11.375	9.100	7.583	5.688	4.550	3.792	3.250	2.844
49	15.500	11.625	9.300	7.750	5.813	4.650	3.875	3.322	2.906
50	15.833	11.875	9.500	7.917	5.938	4.750	3.958	3.393	2.969
51	16.167	12.125	9.700	8.083	6.063	4.850	4.042	3.465	3.031
52	16.500	12.375	9.900	8.250	6.188	4.950	4.125	3.536	3.094
53	16.833	12.625	10.100	8.417	6.313	5.050	4.208	3.607	3.156
54	17.167	12.875	10.300	8.583	6.438	5.150	4.292	3.679	3.219
55	17.500	13.125	10.500	8.750	6.563	5.250	4.375	3.750	3.281

Root Diameters of Gears Cut on Gear Shapers

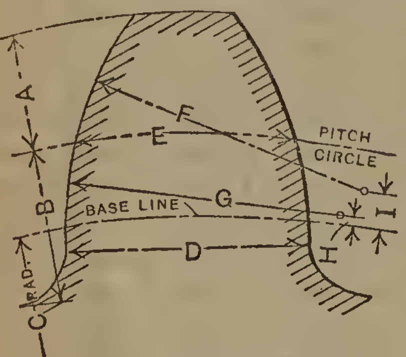
No. of Teeth	Diametral Pitch								
	3 *P*	4 *P*	5 *P*	6 *P*	8 *P*	10 *P*	12 *P*	14 *P*	16 *P*
56	17.833	13.375	10.700	8.917	6.688	5.350	4.458	3.822	3.344
57	18.167	13.625	10.900	9.083	6.813	5.450	4.542	3.893	3.406
58	18.500	13.875	11.100	9.250	6.938	5.550	4.625	3.965	3.469
59	18.833	14.125	11.300	9.417	7.063	5.650	4.708	4.036	3.531
60	19.167	14.375	11.500	9.583	7.188	5.750	4.792	4.107	3.594
61	19.500	14.625	11.700	9.750	7.313	5.850	4.875	4.179	3.656
62	19.833	14.875	11.900	9.917	7.438	5.950	4.958	4.250	3.719
63	20.167	15.125	12.100	10.083	7.563	6.050	5.042	4.322	3.781
64	20.500	15.375	12.300	10.250	7.688	6.150	5.125	4.393	3.844
65	20.833	15.625	12.500	10.417	7.813	6.250	5.208	4.465	3.906
66	21.167	15.875	12.700	10.583	7.938	6.350	5.292	4.536	3.969
67	21.500	16.125	12.900	10.750	8.063	6.450	5.375	4.607	4.031
68	21.833	16.375	13.100	10.917	8.188	6.550	5.458	4.679	4.094
69	22.167	16.625	13.300	11.083	8.313	6.650	5.542	4.750	4.156
70	22.500	16.875	13.500	11.250	8.438	6.750	5.625	4.822	4.219
71	22.833	17.125	13.700	11.417	8.563	6.850	5.708	4.893	4.281
72	23.167	17.375	13.900	11.583	8.688	6.950	5.792	4.965	4.344
73	23.500	17.625	14.100	11.750	8.813	7.050	5.875	5.036	4.406
74	23.833	17.875	14.300	11.917	8.938	7.150	5.958	5.107	4.469
75	24.167	18.125	14.500	12.083	9.063	7.250	6.042	5.179	4.531
76	24.500	18.375	14.700	12.250	9.188	7.350	6.125	5.250	4.594
77	24.833	18.625	14.900	12.417	9.313	7.450	6.208	5.322	4.656
78	25.167	18.875	15.100	12.583	9.438	7.550	6.292	5.393	4.719
79	25.500	19.125	15.300	12.750	9.563	7.650	6.375	5.465	4.781
80	25.833	19.375	15.500	12.917	9.688	7.750	6.458	5.536	4.844
81	26.167	19.625	15.700	13.083	9.813	7.850	6.542	5.607	4.906
82	26.500	19.875	15.900	13.250	9.938	7.950	6.625	5.679	4.969
83	26.833	20.125	16.100	13.417	10.063	8.050	6.708	5.750	5.031
84	27.167	20.375	16.300	13.583	10.188	8.150	6.792	5.822	5.094
85	27.500	20.625	16.500	13.750	10.313	8.250	6.875	5.893	5.156
86	27.833	20.875	16.700	13.917	10.438	8.350	6.958	5.965	5.219
87	28.167	21.125	16.900	14.083	10.563	8.450	7.042	6.036	5.281
88	28.500	21.375	17.100	14.250	10.688	8.550	7.125	6.107	5.344
89	28.833	21.625	17.300	14.417	10.813	8.650	7.208	6.179	5.406
90	29.167	21.875	17.500	14.583	10.938	8.750	7.292	6.250	5.469
91	29.500	22.125	17.700	14.750	11.063	8.850	7.375	6.322	5.531
92	29.833	22.375	17.900	14.917	11.188	8.950	7.458	6.393	5.594
93	30.167	22.625	18.100	15.083	11.313	9.050	7.542	6.465	5.656
94	30.500	22.875	18.300	15.250	11.438	9.150	7.625	6.536	5.719
95	30.833	23.125	18.500	15.417	11.563	9.250	7.708	6.607	5.781
96	31.167	23.375	18.700	15.583	11.688	9.350	7.792	6.679	5.844
97	31.500	23.625	18.900	15.750	11.813	9.450	7.875	6.750	5.906
98	31.833	23.875	19.100	15.917	11.938	9.550	7.958	6.822	5.969
99	32.167	24.125	19.300	16.083	12.063	9.650	8.042	6.893	6.031
100	32.500	24.375	19.500	16.250	12.188	9.750	8.125	6.965	6.094

Special Tooth Shape for Rolling Mill Gears. — The illustration and table below give data for laying out special 22½-degree involute gears for rolling mill service. These teeth vary somewhat from the standard shape, and liberal fillets are provided at the root of the teeth to prevent breakage due to sharp corners at these points. The dimensions given in the table are for 1-inch circular pitch. To find the dimensions for any other pitch, multiply the dimension given by the circular pitch. The length of the face of rolling mill pinions is made about equal to the pitch diameter. The angle of the tooth with a line parallel to the axis of the gear is usually made 30 degrees. The figures given in the column headed "Strength for 1-inch Pitch, 1-inch Face" are based on a working stress of 1000 pounds per square inch. To find the strength of teeth for other dimensions and other working stresses, multiply the figures given by: face of gear × circular pitch × $\frac{\text{working stress}}{1000}$.

Example: — Find allowable pressure on teeth with a working stress of 4000 pounds per square inch, for a 2-inch circular pitch gear of 4-inch face having 20 teeth.

$$136 \times 4 \times 2 \times \frac{4000}{1000} = 4352 \text{ pounds.}$$

Rolling Mill Gear Teeth



Formulas:

$A = 0.275 \times$ circular pitch;
$B = 0.325 \times$ circular pitch;
$C = 0.462 \times$ pitch diameter;
$E = 0.49 \times$ circular pitch;
$F = 0.251 \times$ pitch diameter;
$G = 0.136 \times$ pitch diameter;
Pressure angle = 22½ degrees.

No. of Teeth	Pitch Diam.	Base Radius, *C*	Thickness, *D*	Face Radius, *F*	Flank Radius, *G*	Distance, *H*	Distance, *I*	Strength for 1" Pitch, 1" Face, Lbs. at Stress of 1000 Lbs. per Sq. In.
10	3.183	1.470	0.480	0.800	0.433	0.010	0.010	105
11	3.501	1.617	0.496	0.879	0.476	0.014	0.014	115
12	3.820	1.764	0.505	0.959	0.520	0.016	0.016	117
13	4.138	1.912	0.515	1.04	0.563	0.005	0.005	121
14	4.456	2.059	0.526	1.12	0.606	0.010	0.006	126
15	4.775	2.206	0.545	1.20	0.650	0.000	0.000	134
16	5.093	2.353	0.550	1.28	0.693	0.005	0.000	137
17	5.411	2.500	0.570	1.36	0.736	0.006	0.006	146
18	5.730	2.647	0.572	1.44	0.779	0.000	0.000	145
19	6.048	2.794	0.575	1.52	0.823	0.006	0.006	140
20	6.366	2.941	0.580	1.60	0.866	0.014	0.007	136
21	6.684	3.088	0.585	1.68	0.899	0.015	0.000	136

Grant's Odontograph. — The table entitled "Grant's Odontograph" provides a simple means for laying out accurately shaped gear teeth by means of circular arcs which very closely approximate the exact tooth curves. The method was devised by Mr. George B. Grant, and differs for cycloidal and involute teeth.

Odontograph for Cycloidal System. — First draw the pitch, addendum, root and clearance circles and space off the pitch of the teeth on the pitch circle in the usual way. Then draw the circle marked "line of flank centers" at the distance d, as given in the table, outside of the pitch line and draw the "line of face centers" at the distance D inside of it. With the face radius R in the dividers, draw in all the face curves from centers on the "line of face centers"; then with the flank radius r draw all the flank curves from centers on the "line of flank centers."

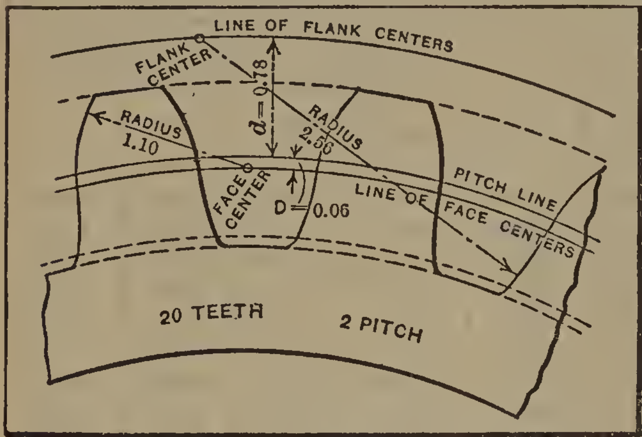


Fig. 1

The table gives the distances D and d and radii R and r, for pitches either exactly 1 diametral or 1 inch circular pitch. For any other pitch, divide or multiply as directed in the table. The illustration, Fig. 1, shows the method applied to laying out a two diametral pitch gear. The odontograph may also be applied to laying out teeth for internal gears.

Odontograph Table for Involute System. — Lay off the pitch, addendum, root and clearance circles and space off the teeth on the pitch line, as indicated in Fig. 2. Draw the "base line" 1/60 of the pitch diameter inside the pitch line. Use the face radius as given in the table for involute teeth for drawing all the faces from the pitch line to the addendum line, the centers being on the "base line." If the pitch is any other than 1 diametral or 1-inch circular pitch, divide or multiply the values given in the table as directed. To draw the flanks of the teeth from the pitch line to the "base line," use the flank radius given, with the center on the "base line." Then draw straight radial flanks from the "base line" to the root line, and round them into the clearance line. The illustration, Fig. 2, shows the method applied to laying out a two diametral pitch gear. The odontograph table for involute teeth can be used for internal gears in the same way as for external gears, but care must be taken that the point of the tooth of the gear is cut off to avoid interference. No correction for interference, however, is needed on the points of the pinion teeth or on the flanks of the gear teeth.

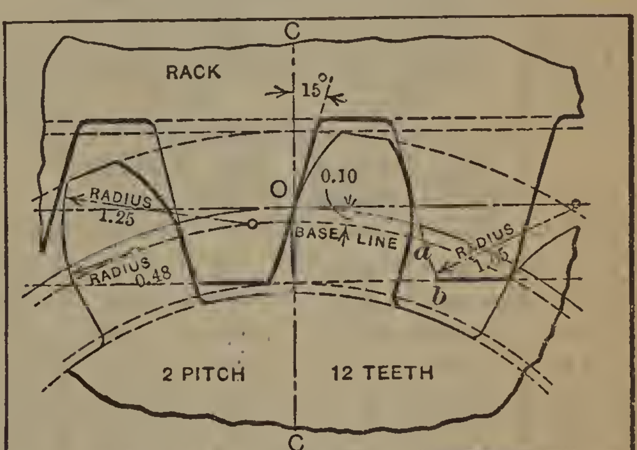


Fig. 2

Special Rule for Involute Rack. — Draw the sides of the rack teeth as straight lines inclined 15 degrees to the center line COC. Draw the outer half ab of the face by means of a circular arc having a radii of 2.10 inches divided by the diametral pitch, or 0.67 inch multiplied by the circular pitch, the center for this arc being on the pitch line of the rack.

Grant's Odontograph

Table for Cycloidal Teeth

Number of Teeth in the Gear		R, r, D and d for One Diametral Pitch; for any other Pitch divide Values given by that Pitch				R, r, D and d for One Inch Circular Pitch; for any other Pitch multiply Values given by that Pitch			
		Faces		Flanks		Faces		Flanks	
Exact	Also Used for	R	D	r	d	R	D	r	d
10	10	1.99	0.02	−8.00	4.00	0.62	0.01	−2.55	1.27
11	11	2.00	0.04	−11.05	6.50	0.63	0.01	−3.34	2.07
12	12	2.01	0.06	∞	∞	0.64	0.02	∞	∞
13½	13- 14	2.04	0.07	15.10	9.43	0.65	0.02	4.80	3.00
15½	15- 16	2.10	0.09	7.86	3.46	0.67	0.03	2.50	1.10
17½	17- 18	2.14	0.11	6.13	2.20	0.68	0.04	1.95	0.70
20	19- 21	2.20	0.13	5.12	1.57	0.70	0.04	1.63	0.50
23	22- 24	2.26	0.15	4.50	1.13	0.72	0.05	1.43	0.36
27	25- 29	2.33	0.16	4.10	0.96	0.74	0.05	1.30	0.29
33	30- 36	2.40	0.19	3.80	0.72	0.76	0.06	1.20	0.23
42	37- 48	2.48	0.22	3.52	0.63	0.79	0.07	1.12	0.20
58	49- 72	2.60	0.25	3.33	0.54	0.83	0.08	1.06	0.17
97	73-144	2.83	0.28	3.14	0.44	0.90	0.09	1.00	0.14
290	145-300	2.92	0.31	3.00	0.38	0.93	0.10	0.95	0.12
∞	Rack	2.96	0.34	2.96	0.34	0.94	0.11	0.94	0.11

Table for Involute Teeth

No. of Teeth in the Gear	Radii for One Diametral Pitch; for any other Pitch divide Values given by that Pitch		Radii for One Inch Circular Pitch; for any other Pitch multiply Values given by that Pitch		No. of Teeth in the Gear	Radii for One Diametral Pitch; for any other Pitch divide Values given by that Pitch		Radii for One Inch Circular Pitch; for any other Pitch multiply Values given by that Pitch	
	Face Radius	Flank Radius	Face Radius	Flank Radius		Face Radius	Flank Radius	Face Radius	Flank Radius
10	2.28	0.69	0.73	0.22	28	3.92	2.59	1.25	0.82
11	2.40	0.83	0.76	0.27	29	3.99	2.67	1.27	0.85
12	2.51	0.96	0.80	0.31	30	4.06	2.76	1.29	0.88
13	2.62	1.09	0.83	0.34	31	4.13	2.85	1.31	0.91
14	2.72	1.22	0.87	0.39	32	4.20	2.93	1.34	0.93
15	2.82	1.34	0.90	0.43	33	4.27	3.01	1.36	0.96
16	2.92	1.46	0.93	0.47	34	4.33	3.09	1.38	0.99
17	3.02	1.58	0.96	0.50	35	4.39	3.16	1.39	1.01
18	3.12	1.69	0.99	0.54	36	4.45	3.23	1.41	1.03
19	3.22	1.79	1.03	0.57	37- 40	4.20		1.34	
20	3.32	1.89	1.06	0.60	41- 45	4.63		1.48	
21	3.41	1.98	1.09	0.63	46- 51	5.06		1.61	
22	3.49	2.06	1.11	0.66	52- 60	5.74		1.83	
23	3.57	2.15	1.13	0.69	61- 70	6.52		2.07	
24	3.64	2.24	1.16	0.71	71- 90	7.72		2.46	
25	3.71	2.33	1.18	0.74	91-120	9.78		3.11	
26	3.78	2.42	1.20	0.77	121-180	13.38		4.26	
27	3 85	2.50	1.23	0.80	181-360	21.62		6.88	

Metric or Module System of Gear Teeth. — In the metric system, the diametral pitch is not used, but instead, the dimensions of gear teeth are expressed by reference to the *module* of the gear. The module is equal to the pitch diameter in millimeters divided by the number of teeth in the gear. For example, if the pitch diameter of a gear is 50 millimeters and the number of teeth, 25, then the module equals $50 \div 25 = 2$. The accompanying table gives a comparison between the module and the corresponding diametral pitch of gears.

The module is also equal to the circular pitch in millimeters divided by 3.1416. Either rule gives the same result.

Module and Corresponding Diametral Pitch

Module	Corresponding English Diametral Pitch	Module	Corresponding English Diametral Pitch	Module	Corresponding English Diametral Pitch	Module	Corresponding English Diametral Pitch
0.5	50.800	2.25	11.288	5.0	5.080	11	2.309
0.75	33.867	2.5	10.160	5.5	4.618	12	2.117
1.0	25.400	2.75	9.236	6.0	4.233	14	1.814
1.25	20.320	3.0	8.466	7.0	3.628	16	1.587
1.5	16.933	3.5	7.257	8.0	3.175	18	1.411
1.75	14.514	4.0	6.350	9.0	2.822	20	1.270
2.0	12.700	4.5	5.644	10.0	2.540	24	1.058

Selecting the Number of Teeth for Gears and Sprockets. — The tables "Gear Ratios and Their Decimal Equivalents" contain the decimal equivalents of all fractions with denominators up to 60. In machine design it is frequently necessary to determine gears or sprockets with low numbers of teeth to give approximately such ratios as can be expressed exactly only with very high numbers. For example, it may be required to have the speeds of the driving and driven gears as nearly as possible to 1149 and 473 revolutions per minute. It may be stipulated, however, that the number of teeth in the larger gear must not exceed 60. Dividing 473 by 1149, we find that the ratio is 0.4117. By referring to the tables, the nearest fractional value to this ratio, with a denominator less than 60, is found to be 7/17; thus, the nearest number of teeth in the gears can be 14 and 34, or 21 and 51. This will give speeds of 1149 and 473.118 revolutions per minute, which introduces a very small error. In the absence of such tables, the method of obtaining the approximate fraction 7/17 would be very cumbersome.

As another example, suppose it is desired to feed stock to a punch press through rolls of 4-inch diameter, the rolls being turned by a ratchet and pawl at the end of each stroke of the punch. The feed is to be as near as possible to 2¼ inches, and the number of teeth in the rachet to be as low as possible. To find the answer to this problem with the aid of the tables, proceed as follows: The feed for one revolution of the rolls is $4\pi = 12.5664$. To feed 2¼ inches, the rolls must make $2.25 \div 12.5664 = 0.1790$ of a revolution. Referring to the table, the nearest fraction to this ratio is 5/28; hence we choose a ratchet gear of 28 teeth, feeding five teeth at a stroke. The feed will be 2.244 inches instead of 2.25 inches, an error of only 0.006 inch. If we should choose the next higher fraction to the ratio 0.1790, which is 7/39, the ratchet would have 39 teeth, and a feed of seven teeth in this ratchet would be equivalent to a feed of 2.256 inches, which also involves an error of 0.006 inch.

Gear Ratios and Their Decimal Equivalents

Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio
0.0167	1/60	0.0455	1/22	0.0862	5/58	0.1277	6/47	0.1698	9/53	0.2105	4/19
0.0169	1/59	0.0465	2/43	0.0870	2/23	0.1282	5/39	0.1702	8/47	0.2115	11/52
0.0172	1/58	0.0476	1/21	0.0877	5/57	0.1290	4/31	0.1707	7/41	0.2121	7/33
0.0175	1/57	0.0488	2/41	0.0882	3/34	0.1296	7/54	0.1714	6/35	0.2128	10/47
0.0178	1/56	0.0500	1/20	0.0889	4/45	0.1304	3/23	0.1724	5/29	0.2143	3/14
0.0182	1/55	0.0508	3/59	0.0893	5/56	0.1316	5/38	0.1731	9/52	0.2157	11/51
0.0185	1/54	0.0513	2/39	0.0909	1/11	0.1321	7/53	0.1739	4/23	0.2162	8/37
0.0189	1/53	0.0517	3/58	0.0926	5/54	0.1333	2/15	0.1750	7/40	0.2167	13/60
0.0192	1/52	0.0526	1/19	0.0930	4/43	0.1346	7/52	0.1754	10/57	0.2174	5/23
0.0196	1/51	0.0536	3/56	0.0937	3/32	0.1351	5/37	0.1765	3/17	0.2182	12/55
0.0200	1/50	0.0541	2/37	0.0943	5/53	0.1356	8/59	0.1778	8/45	0.2187	7/32
0.0204	1/49	0.0545	3/55	0.0952	2/21	0.1364	3/22	0.1786	5/28	0.2195	9/41
0.0208	1/48	0.0555	1/18	0.0962	5/52	0.1373	7/51	0.1795	7/39	0.2200	11/50
0.0213	1/47	0.0566	3/53	0.0968	3/31	0.1379	4/29	0.1800	9/50	0.2203	13/59
0.0217	1/46	0.0571	2/35	0.0976	4/41	0.1389	5/36	0.1818	2/11	0.2222	2/9
0.0222	1/45	0.0577	3/52	0.0980	5/51	0.1395	6/43	0.1833	11/60	0.2241	13/58
0.0227	1/44	0.0588	1/17	0.1000	1/10	0.1400	7/50	0.1837	9/49	0.2245	11/49
0.0233	1/43	0.0600	3/50	0.1017	6/59	0.1404	8/57	0.1842	7/38	0.2250	9/40
0.0238	1/42	0.0606	2/33	0.1020	5/49	0.1429	1/7	0.1852	5/27	0.2258	7/31
0.0244	1/41	0.0612	3/49	0.1026	4/39	0.1455	8/55	0.1860	8/43	0.2264	12/53
0.0250	1/40	0.0625	1/16	0.1034	3/29	0.1458	7/48	0.1864	11/59	0.2273	5/22
0.0256	1/39	0.0638	3/47	0.1042	5/48	0.1463	6/41	0.1875	3/16	0.2281	13/57
0.0263	1/38	0.0645	2/31	0.1053	2/19	0.1471	5/34	0.1887	10/53	0.2286	8/35
0.0270	1/37	0.0652	3/46	0.1064	5/47	0.1481	4/27	0.1892	7/37	0.2292	11/48
0.0278	1/36	0.0667	1/15	0.1071	3/28	0.1489	7/47	0.1897	11/58	0.2308	3/13
0.0286	1/35	0.0678	4/59	0.1081	4/37	0.1500	3/20	0.1904	4/21	0.2321	13/56
0.0294	1/34	0.0682	3/44	0.1087	5/46	0.1509	8/53	0.1915	9/47	0.2326	10/43
0.0303	1/33	0.0690	2/29	0.1091	6/55	0.1515	5/33	0.1923	5/26	0.2333	7/30
0.0312	1/32	0.0698	3/43	0.1111	1/9	0.1522	7/46	0.1930	11/57	0.2340	11/47
0.0323	1/31	0.0702	4/57	0.1132	6/53	0.1525	9/59	0.1935	6/31	0.2353	4/17
0.0333	1/30	0.0714	1/14	0.1136	5/44	0.1538	2/13	0.1944	7/36	0.2364	13/55
0.0339	2/59	0.0727	4/55	0.1143	4/35	0.1552	9/58	0.1951	8/41	0.2368	9/38
0.0345	1/29	0.0732	3/41	0.1154	3/26	0.1556	7/45	0.1956	9/46	0.2373	14/59
0.0351	2/57	0.0741	2/27	0.1163	5/43	0.1562	5/32	0.1961	10/51	0.2381	5/21
0.0357	1/28	0.0750	3/40	0.1167	7/60	0.1569	8/51	0.1964	11/56	0.2391	11/46
0.0364	2/55	0.0755	4/53	0.1176	2/17	0.1579	3/19	0.2000	1/5	0.2400	6/25
0.0370	1/27	0.0769	1/13	0.1186	7/59	0.1591	7/44	0.2034	12/59	0.2407	13/54
0.0377	2/53	0.0784	4/51	0.1190	5/42	0.1600	4/25	0.2037	11/54	0.2414	7/29
0.0385	1/26	0.0789	3/38	0.1200	3/25	0.1607	9/56	0.2040	10/49	0.2424	8/33
0.0392	2/51	0.0800	2/25	0.1207	7/58	0.1613	5/31	0.2045	9/44	0.2432	9/37
0.0400	1/25	0.0811	3/37	0.1212	4/33	0.1622	6/37	0.2051	8/39	0.2439	10/41
0.0408	2/49	0.0816	4/49	0.1220	5/41	0.1628	7/43	0.2059	7/34	0.2444	11/45
0.0417	1/24	0.0833	1/12	0.1224	6/49	0.1633	8/49	0.2069	6/29	0.2449	12/49
0.0426	2/47	0.0847	5/59	0.1228	7/57	0.1636	9/55	0.2075	11/53	0.2453	13/53
0.0435	1/23	0.0851	4/47	0.1250	1/8	0.1667	1/6	0.2083	5/24	0.2456	14/57
0.0444	2/45	0.0857	3/35	0.1273	7/55	0.1695	10/59	0.2093	9/43	0.2500	1/4

Gear Ratios and Their Decimal Equivalents *(Continued)*

Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio
0.2542	15/59	0.2927	12/41	0.3390	20/59	0.3778	17/45	0.4186	18/43	0.4596	17/37
0.2545	14/55	0.2931	17/58	0.3393	19/56	0.3784	14/37	0.4194	13/31	0.4600	23/50
0.2549	13/51	0.2941	5/17	0.3396	18/53	0.3793	11/29	0.4200	21/50	0.4615	6/13
0.2553	12/47	0.2955	13/44	0.3400	17/50	0.3800	19/50	0.4211	8/19	0.4630	25/54
0.2558	11/43	0.2963	8/27	0.3404	16/47	0.3810	8/21	0.4222	19/45	0.4634	19/41
0.2564	10/39	0.2973	11/37	0.3409	15/44	0.3818	21/55	0.4231	11/26	0.4643	13/28
0.2571	9/35	0.2979	14/47	0.3415	14/41	0.3824	13/34	0.4237	25/59	0.4651	20/43
0.2581	8/31	0.2982	17/57	0.3421	13/38	0.3830	18/47	0.4242	14/33	0.4655	27/58
0.2586	15/58	0.3000	3/10	0.3429	12/35	0.3833	23/60	0.4250	17/40	0.4667	7/15
0.2593	7/27	0.3019	16/53	0.3437	11/32	0.3846	5/13	0.4255	20/47	0.4681	22/47
0.2600	13/50	0.3023	13/43	0.3448	10/29	0.3860	22/57	0.4259	23/54	0.4687	15/32
0.2609	6/23	0.3030	10/33	0.3455	19/55	0.3864	17/44	0.4286	3/7	0.4694	23/49
0.2619	11/42	0.3036	17/56	0.3462	9/26	0.3871	12/31	0.4310	25/58	0.4706	8/17
0.2632	5/19	0.3043	7/23	0.3469	17/49	0.3878	19/49	0.4314	22/51	0.4717	25/53
0.2642	14/53	0.3051	18/59	0.3478	8/23	0.3889	7/18	0.4318	19/44	0.4722	17/36
0.2647	9/34	0.3056	11/36	0.3488	15/43	0.3898	23/59	0.4324	16/37	0.4727	26/55
0.2653	13/49	0.3061	15/49	0.3500	7/20	0.3902	16/41	0.4333	13/30	0.4737	9/19
0.2667	4/15	0.3077	4/13	0.3509	20/57	0.3913	9/23	0.4340	23/53	0.4746	28/59
0.2679	15/56	0.3091	17/55	0.3514	13/37	0.3922	20/51	0.4348	10/23	0.4750	19/40
0.2683	11/41	0.3095	13/42	0.3519	19/54	0.3929	11/28	0.4359	17/39	0.4762	10/21
0.2692	7/26	0.3103	9/29	0.3529	6/17	0.3939	13/33	0.4364	24/55	0.4773	21/44
0.2703	10/37	0.3111	14/45	0.3542	17/48	0.3947	15/38	0.4375	7/16	0.4783	11/23
0.2708	13/48	0.3125	5/16	0.3548	11/31	0.3953	17/43	0.4386	25/57	0.4792	23/48
0.2712	16/59	0.3137	16/51	0.3556	16/45	0.3958	19/48	0.4390	18/41	0.4800	12/25
0.2727	3/11	0.3143	11/35	0.3559	21/59	0.3962	21/53	0.4400	11/25	0.4808	25/52
0.2745	14/51	0.3148	17/54	0.3571	5/14	0.3966	23/58	0.4407	26/59	0.4815	13/27
0.2750	11/40	0.3158	6/19	0.3585	19/53	0.4000	2/5	0.4412	15/34	0.4821	27/56
0.2759	8/29	0.3166	19/60	0.3590	14/39	0.4035	23/57	0.4419	19/43	0.4827	14/29
0.2766	13/47	0.3171	13/41	0.3600	9/25	0.4038	21/52	0.4423	23/52	0.4833	29/60
0.2778	5/18	0.3182	7/22	0.3611	13/36	0.4043	19/47	0.4444	4/9	0.4839	15/31
0.2791	12/43	0.3191	15/47	0.3617	17/47	0.4048	17/42	0.4464	25/56	0.4848	16/33
0.2800	7/25	0.3200	8/25	0.3621	21/58	0.4054	15/37	0.4468	21/47	0.4857	17/35
0.2807	16/57	0.3208	17/53	0.3636	4/11	0.4062	13/32	0.4474	17/38	0.4865	18/37
0.2812	9/32	0.3214	9/28	0.3654	19/52	0.4068	24/59	0.4483	13/29	0.4872	19/39
0.2821	11/39	0.3220	19/59	0.3659	15/41	0.4074	11/27	0.4490	22/49	0.4878	20/41
0.2826	13/46	0.3226	10/31	0.3667	11/30	0.4082	20/49	0.4500	9/20	0.4884	21/43
0.2830	15/53	0.3235	11/34	0.3673	18/49	0.4091	9/22	0.4510	23/51	0.4889	22/45
0.2833	17/60	0.3243	12/37	0.3684	7/19	0.4103	16/39	0.4516	14/31	0.4894	23/47
0.2857	2/7	0.3250	13/40	0.3696	17/46	0.4107	23/56	0.4524	19/42	0.4898	24/49
0.2881	17/59	0.3256	14/43	0.3703	10/27	0.4118	7/17	0.4528	24/53	0.4902	25/51
0.2885	15/52	0.3261	15/46	0.3714	13/35	0.4130	19/46	0.4545	5/11	0.4906	26/53
0.2889	13/45	0.3265	16/49	0.3721	16/43	0.4138	12/29	0.4561	26/57	0.4909	27/55
0.2895	11/38	0.3269	17/52	0.3725	19/51	0.4146	17/41	0.4565	21/46	0.4912	28/57
0.2903	9/31	0.3273	18/55	0.3729	22/59	0.4151	22/53	0.4571	16/35	0.4915	29/59
0.2909	16/55	0.3276	19/58	0.3750	3/8	0.4167	5/12	0.4576	27/59	0.5000	1/2
0.2917	7/24	0.3333	1/3	0.3774	20/53	0.4182	23/55	0.4583	11/24	0.5085	30/59

Gear Ratios and Their Decimal Equivalents *(Continued)*

Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio
0.5088	29/57	0.5435	25/46	0.5854	24/41	0.6275	32/51	0.6731	35/52	0.7111	32/45
0.5091	28/55	0.5439	31/57	0.5862	17/29	0.6279	27/43	0.6735	33/49	0.7115	37/52
0.5094	27/53	0.5455	6/11	0.5870	27/46	0.6286	22/35	0.6739	31/46	0.7119	42/59
0.5098	26/51	0.5472	29/53	0.5882	10/17	0.6296	17/27	0.6744	29/43	0.7143	5/7
0.5102	25/49	0.5476	23/42	0.5893	33/56	0.6304	29/46	0.6750	27/40	0.7167	43/60
0.5106	24/47	0.5484	17/31	0.5897	23/39	0.6316	12/19	0.6757	25/37	0.7170	38/53
0.5111	23/45	0.5490	28/51	0.5909	13/22	0.6326	31/49	0.6765	23/34	0.7174	33/46
0.5116	22/43	0.5500	11/20	0.5918	29/49	0.6333	19/30	0.6774	21/31	0.7179	28/39
0.5122	21/41	0.5510	27/49	0.5926	16/27	0.6341	26/41	0.6780	40/59	0.7187	23/32
0.5128	20/39	0.5517	16/29	0.5932	35/59	0.6346	33/52	0.6786	19/28	0.7193	41/57
0.5135	19/37	0.5526	21/38	0.5937	19/32	0.6364	7/11	0.6792	36/53	0.7200	18/25
0.5142	18/35	0.5532	26/47	0.5946	22/37	0.6379	37/58	0.6800	17/25	0.7209	31/43
0.5151	17/33	0.5536	31/56	0.5952	25/42	0.6383	30/47	0.6809	32/47	0.7222	13/18
0.5161	16/31	0.5556	5/9	0.5957	28/47	0.6389	23/36	0.6818	15/22	0.7234	34/47
0.5167	31/60	0.5577	29/52	0.5962	31/52	0.6400	16/25	0.6829	28/41	0.7241	21/29
0.5172	15/29	0.5581	24/43	0.5965	34/57	0.6410	25/39	0.6833	41/60	0.7250	29/40
0.5179	29/56	0.5588	19/34	0.6000	3/5	0.6415	34/53	0.6842	13/19	0.7255	37/51
0.5185	14/27	0.5593	33/59	0.6034	35/58	0.6429	9/14	0.6852	37/54	0.7273	8/11
0.5192	27/52	0.5600	14/25	0.6038	32/53	0.6441	38/59	0.6857	24/35	0.7288	43/59
0.5200	13/25	0.5610	23/41	0.6042	29/48	0.6444	29/45	0.6863	35/51	0.7292	35/48
0.5208	25/48	0.5614	32/57	0.6047	26/43	0.6452	20/31	0.6875	11/16	0.7297	27/37
0.5217	12/23	0.5625	9/16	0.6053	23/38	0.6458	31/48	0.6889	31/45	0.7308	19/26
0.5227	23/44	0.5636	31/55	0.6060	20/33	0.6471	11/17	0.6897	20/29	0.7317	30/41
0.5238	11/21	0.5641	22/39	0.6071	17/28	0.6481	35/54	0.6905	29/42	0.7321	41/56
0.5250	21/40	0.5652	13/23	0.6078	31/51	0.6486	24/37	0.6909	38/55	0.7333	11/15
0.5254	31/59	0.5660	30/53	0.6087	14/23	0.6491	37/57	0.6923	9/13	0.7347	36/49
0.5263	10/19	0.5667	17/30	0.6098	25/41	0.6500	13/20	0.6939	34/49	0.7353	25/34
0.5273	29/55	0.5676	21/37	0.6102	36/59	0.6512	28/43	0.6949	41/59	0.7358	39/53
0.5278	19/36	0.5682	25/44	0.6111	11/18	0.6522	15/23	0.6957	16/23	0.7368	14/19
0.5283	28/53	0.5686	29/51	0.6122	30/49	0.6531	32/49	0.6964	39/56	0.7381	31/42
0.5294	9/17	0.5690	33/58	0.6129	19/31	0.6538	17/26	0.6970	23/33	0.7391	17/23
0.5306	26/49	0.5714	4/7	0.6136	27/44	0.6545	36/55	0.6977	30/43	0.7400	37/50
0.5312	17/32	0.5741	31/54	0.6140	35/57	0.6552	19/29	0.6981	37/53	0.7407	20/27
0.5319	25/47	0.5745	27/47	0.6154	8/13	0.6562	21/32	0.7000	7/10	0.7414	43/58
0.5333	8/15	0.5750	23/40	0.6167	37/60	0.6571	23/35	0.7018	40/57	0.7419	23/31
0.5345	31/58	0.5757	19/33	0.6170	29/47	0.6579	25/38	0.7021	33/47	0.7429	26/35
0.5349	23/43	0.5763	34/59	0.6176	21/34	0.6585	27/41	0.7027	26/37	0.7436	29/39
0.5357	15/28	0.5769	15/26	0.6182	34/55	0.6591	29/44	0.7037	19/27	0.7442	32/43
0.5366	22/41	0.5778	26/45	0.6190	13/21	0.6596	31/47	0.7045	31/44	0.7447	35/47
0.5370	29/54	0.5789	11/19	0.6200	31/50	0.6600	33/50	0.7059	12/17	0.7451	38/51
0.5385	7/13	0.5800	29/50	0.6207	18/29	0.6604	35/53	0.7069	41/58	0.7455	41/55
0.5400	27/50	0.5806	18/31	0.6216	23/37	0.6607	37/56	0.7073	29/41	0.7458	44/59
0.5405	20/37	0.5814	25/43	0.6222	28/45	0.6610	39/59	0.7083	17/24	0.7500	3/4
0.5417	13/24	0.5818	32/55	0.6226	33/53	0.6666	2/3	0.7091	39/55	0.7544	43/57
0.5424	32/59	0.5833	7/12	0.6250	5/8	0.6724	39/58	0.7097	22/31	0.7547	40/53
0.5429	19/35	0.5849	31/53	0.6271	37/59	0.6727	37/55	0.7105	27/38	0.7551	37/49

Gear Ratios and Their Decimal Equivalents (*Continued*)

Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio	Decimal Equivalent	Gear Ratio
0.7561	31/41	0.7949	31/39	0.8378	31/37	0.8788	29/33	0.9189	34/37	0.9600	24/25
0.7568	28/37	0.7955	35/44	0.8387	26/31	0.8793	51/58	0.9200	23/25	0.9608	49/51
0.7576	25/33	0.7959	39/49	0.8393	47/56	0.8800	22/25	0.9211	35/38	0.9615	25/26
0.7586	22/29	0.7963	43/54	0.8400	21/25	0.8810	37/42	0.9216	47/51	0.9623	51/53
0.7593	41/54	0.7966	47/59	0.8409	37/44	0.8814	52/59	0.9231	12/13	0.9630	26/27
0.7600	19/25	0.8000	4/5	0.8421	16/19	0.8824	15/17	0.9245	49/53	0.9636	53/55
0.7609	35/46	0.8036	45/56	0.8431	43/51	0.8833	53/60	0.9250	37/40	0.9643	27/28
0.7619	16/21	0.8039	41/51	0.8437	27/32	0.8837	38/43	0.9259	25/27	0.9649	55/57
0.7627	45/59	0.8043	37/46	0.8444	38/45	0.8846	23/26	0.9268	38/41	0.9655	28/29
0.7632	29/38	0.8049	33/41	0.8448	49/58	0.8857	31/35	0.9273	51/55	0.9661	57/59
0.7636	42/55	0.8056	29/36	0.8462	11/13	0.8864	39/44	0.9286	13/14	0.9667	29/30
0.7647	13/17	0.8065	25/31	0.8475	50/59	0.8868	47/53	0.9298	53/57	0.9677	30/31
0.7660	36/47	0.8070	46/57	0.8478	39/46	0.8889	8/9	0.9302	40/43	0.9687	31/32
0.7667	23/30	0.8077	21/26	0.8485	28/33	0.8909	49/55	0.9310	27/29	0.9697	32/33
0.7674	33/43	0.8085	38/47	0.8491	45/53	0.8913	41/46	0.9318	41/44	0.9706	33/34
0.7679	43/56	0.8095	17/21	0.8500	17/20	0.8919	33/37	0.9322	55/59	0.9714	34/35
0.7692	10/13	0.8103	47/58	0.8511	40/47	0.8929	25/28	0.9333	14/15	0.9722	35/36
0.7708	37/48	0.8108	30/37	0.8519	23/27	0.8936	42/47	0.9348	43/46	0.9730	36/37
0.7714	27/35	0.8113	43/53	0.8529	29/34	0.8947	17/19	0.9355	29/31	0.9737	37/38
0.7719	44/57	0.8125	13/16	0.8537	35/41	0.8958	43/48	0.9362	44/47	0.9743	38/39
0.7727	17/22	0.8136	48/59	0.8542	41/48	0.8966	26/29	0.9375	15/16	0.9750	39/40
0.7736	41/53	0.8140	35/43	0.8545	47/55	0.8974	35/39	0.9388	46/49	0.9756	40/41
0.7742	24/31	0.8148	22/27	0.8571	6/7	0.8980	44/49	0.9394	31/33	0.9762	41/42
0.7750	31/40	0.8158	31/38	0.8596	49/57	0.8983	53/59	0.9400	47/50	0.9767	42/43
0.7755	38/49	0.8163	40/49	0.8600	43/50	0.9000	9/10	0.9412	16/17	0.9773	43/44
0.7759	45/58	0.8167	49/60	0.8604	37/43	0.9020	46/51	0.9423	49/52	0.9778	44/45
0.7778	7/9	0.8182	9/11	0.8611	31/36	0.9024	37/41	0.9429	33/35	0.9783	45/46
0.7797	46/59	0.8200	41/50	0.8621	25/29	0.9032	28/31	0.9434	50/53	0.9787	46/47
0.7800	39/50	0.8205	32/39	0.8627	44/51	0.9038	47/52	0.9444	17/18	0.9792	47/48
0.7805	32/41	0.8214	23/28	0.8636	19/22	0.9048	19/21	0.9455	52/55	0.9796	48/49
0.7812	25/32	0.8222	37/45	0.8644	51/59	0.9057	48/53	0.9459	35/37	0.9800	49/50
0.7818	43/55	0.8235	14/17	0.8649	32/37	0.9062	29/32	0.9464	53/56	0.9804	50/51
0.7826	18/23	0.8246	47/57	0.8654	45/52	0.9070	39/43	0.9474	18/19	0.9808	51/52
0.7833	47/60	0.8250	33/40	0.8667	13/15	0.9074	49/54	0.9483	55/58	0.9811	52/53
0.7838	29/37	0.8261	19/23	0.8679	46/53	0.9091	10/11	0.9487	37/39	0.9815	53/54
0.7843	40/51	0.8269	43/52	0.8684	33/38	0.9107	51/56	0.9492	56/59	0.9818	54/55
0.7857	11/14	0.8276	24/29	0.8696	20/23	0.9111	41/45	0.9500	19/20	0.9821	55/56
0.7872	37/47	0.8286	29/35	0.8704	47/54	0.9118	31/34	0.9512	39/41	0.9825	56/57
0.7879	26/33	0.8293	34/41	0.8710	27/31	0.9123	52/57	0.9524	20/21	0.9828	57/58
0.7885	41/52	0.8298	39/47	0.8718	34/39	0.9130	21/23	0.9535	41/43	0.9831	58/59
0.7895	15/19	0.8302	44/53	0.8723	41/47	0.9138	53/58	0.9545	21/22	0.9833	59/60
0.7907	34/43	0.8305	49/59	0.8727	48/55	0.9143	32/35	0.9555	43/45	....	...
0.7917	19/24	0.8333	5/6	0.8750	7/8	0.9149	43/47	0.9565	22/23	....	...
0.7926	42/53	0.8364	46/55	0.8772	50/57	0.9153	54/59	0.9574	45/47	....	...
0.7931	23/29	0.8367	41/49	0.8775	43/49	0.9167	11/12	0.9583	23/24	....	...
0.7941	27/34	0.8372	36/43	0.8780	36/41	0.9184	45/49	0.9592	47/49	....	...

Chordal Thicknesses of Gear Teeth at Pitch Line

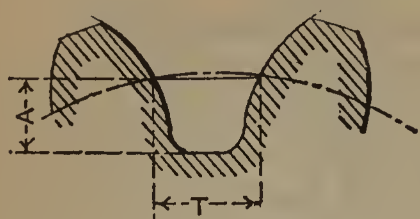
T = chordal thickness of tooth at pitch line;
H = perpendicular distance from chord to outside circumference of gear (corrected addendum).

Diametral Pitch	Dimension	No. 1 135 Teeth	No. 2 55 Teeth	No. 3 35 Teeth	No. 4 26 Teeth	No. 5 21 Teeth	No. 6 17 Teeth	No. 7 14 Teeth	No. 8 12 Teeth
1	T	1.5707	1.5706	1.5702	1.5698	1.5694	1.5686	1.5675	1.5663
	H	1.0047	1.0112	1.0176	1.0237	1.0294	1.0362	1.0440	1.0514
1½	T	1.0471	1.0470	1.0468	1.0465	1.0462	1.0457	1.0450	1.0442
	H	0.6698	0.6741	0.6784	0.6824	0.6862	0.6908	0.6960	0.7009
2	T	0.7853	0.7853	0.7851	0.7849	0.7847	0.7843	0.7837	0.7831
	H	0.5023	0.5056	0.5088	0.5118	0.5147	0.5181	0.5220	0.5257
2½	T	0.6283	0.6282	0.6281	0.6279	0.6277	0.6274	0.6270	0.6265
	H	0.4018	0.4044	0.4070	0.4094	0.4117	0.4144	0.4176	0.4205
3	T	0.5235	0.5235	0.5234	0.5232	0.5231	0.5228	0.5225	0.5221
	H	0.3349	0.3370	0.3392	0.3412	0.3431	0.3454	0.3480	0.3504
3½	T	0.4487	0.4487	0.4486	0.4485	0.4484	0.4481	0.4478	0.4475
	H	0.2870	0.2889	0.2916	0.2919	0.2935	0.2954	0.2977	0.3004
4	T	0.3926	0.3926	0.3926	0.3924	0.3923	0.3921	0.3919	0.3915
	H	0.2511	0.2528	0.2544	0.2559	0.2573	0.2590	0.2610	0.2628
5	T	0.3141	0.3141	0.3140	0.3139	0.3138	0.3137	0.3135	0.3132
	H	0.2009	0.2022	0.2035	0.2047	0.2058	0.2072	0.2088	0.2102
6	T	0.2618	0.2617	0.2617	0.2616	0.2615	0.2614	0.2612	0.2610
	H	0.1674	0.1685	0.1696	0.1706	0.1715	0.1727	0.1740	0.1752
7	T	0.2244	0.2243	0.2243	0.2242	0.2242	0.2240	0.2239	0.2237
	H	0.1435	0.1444	0.1453	0.1462	0.1470	0.1480	0.1491	0.1502
8	T	0.1963	0.1963	0.1962	0.1962	0.1961	0.1960	0.1959	0.1958
	H	0.1255	0.1264	0.1272	0.1279	0.1286	0.1295	0.1305	0.1314
9	T	0.1745	0.1745	0.1744	0.1744	0.1743	0.1743	0.1741	0.1740
	H	0.1116	0.1123	0.1130	0.1137	0.1143	0.1151	0.1160	0.1168
10	T	0.1570	0.1570	0.1570	0.1569	0.1569	0.1568	0.1567	0.1566
	H	0.1004	0.1011	0.1017	0.1023	0.1029	0.1036	0.1044	0.1051
11	T	0.1428	0.1428	0.1427	0.1427	0.1426	0.1426	0.1425	0.1424
	H	0.0913	0.0919	0.0925	0.0930	0.0935	0.0942	0.0949	0.0955
12	T	0.1309	0.1309	0.1308	0.1308	0.1308	0.1307	0.1306	0.1305
	H	0.0837	0.0842	0.0848	0.0853	0.0857	0.0863	0.0870	0.0876
14	T	0.1122	0.1122	0.1121	0.1121	0.1121	0.1120	0.1119	0.1118
	H	0.0717	0.0722	0.0726	0.0731	0.0735	0.0740	0.0745	0.0751
16	T	0.0981	0.0981	0.0981	0.0981	0.0980	0.0980	0.0979	0.0979
	H	0.0628	0.0632	0.0636	0.0639	0.0643	0.0647	0.0652	0.0657
18	T	0.0872	0.0872	0.0872	0.0872	0.0872	0.0871	0.0870	0.0870
	H	0.0558	0.0561	0.0565	0.0568	0.0571	0.0575	0.0580	0.0584
20	T	0.0785	0.0785	0.0785	0.0785	0.0784	0.0784	0.0783	0.0783
	H	0.0502	0.0505	0.0508	0.0511	0.0514	0.0518	0.0522	0.0525

Chordal Thicknesses of Gear Teeth at Pitch Line

Circular Pitch	Dimension	Number of Gear Cutter, and Corresponding Number of Teeth							
		No. 1 135 Teeth	No. 2 55 Teeth	No. 3 35 Teeth	No. 4 26 Teeth	No. 5 21 Teeth	No. 6 17 Teeth	No. 7 14 Teeth	No. 8 12 Teeth
¼	T	0.1250	0.1250	0.1249	0.1249	0.1249	0.1248	0.1247	0.1246
	H	0.0799	0.0804	0.0809	0.0814	0.0819	0.0824	0.0830	0.0836
5/16	T	0.1562	0.1562	0.1562	0.1561	0.1561	0.1560	0.1559	0.1558
	H	0.0999	0.1006	0.1012	0.1018	0.1023	0.1030	0.1038	0 1045
⅜	T	0.1875	0.1875	0.1874	0.1873	0.1873	0.1872	0.1871	0.1870
	H	0.1199	0.1207	0.1214	0.1221	0.1228	0.1236	0.1245	0.1254
7/16	T	0.2187	0.2187	0.2186	0.2186	0.2185	0.2184	0.2183	0.2181
	H	0.1399	0.1408	0.1416	0.1425	0.1433	0.1443	0.1453	0.1464
½	T	0.2500	0.2500	0.2499	0.2498	0.2498	0.2496	0.2495	0.2493
	H	0.1599	0.1609	0.1619	0.1629	0.1638	0.1649	0.1661	0.1673
9/16	T	0.2812	0.2812	0.2811	0.2810	0.2810	0.2808	0.2806	0.2804
	H	0.1799	0.1810	0.1821	0.1832	0.1842	0.1855	0.1868	0.1882
⅝	T	0.3125	0.3125	0.3123	0.3123	0.3122	0.3120	0.3118	0.3116
	H	0.1998	0.2012	0.2023	0.2036	0.2047	0.2061	0.2076	0.2091
11/16	T	0.3437	0.3437	0.3436	0.3435	0.3434	0.3432	0.3430	0.3427
	H	0.2198	0.2213	0.2226	0.2239	0.2252	0.2267	0.2283	0.2300
¾	T	0.3750	0.3750	0.3748	0.3747	0.3747	0.3744	0.3742	0.3740
	H	0.2398	0.2414	0.2428	0.2443	0.2457	0.2473	0.2491	0.2509
13/16	T	0.4062	0.4062	0.4060	0.4059	0.4059	0.4056	0.4054	0.4050
	H	0.2598	0.2615	0.2631	0.2647	0.2661	0.2679	0.2699	0.2718
⅞	T	0.4375	0.4375	0.4373	0.4372	0.4371	0.4368	0.4366	0.4362
	H	0.2798	0.2816	0.2833	0.2850	0.2866	0.2885	0.2906	0.2927
15/16	T	0.4687	0.4687	0.4685	0.4684	0.4683	0.4680	0.4678	0.4674
	H	0.2998	0.3018	0.3035	0.3054	0.3071	0.3092	0.3114	0.3137
1	T	0.5000	0.5000	0.4998	0.4997	0.4996	0.4993	0.4990	0.4986
	H	0.3198	0.3219	0.3238	0.3258	0.3276	0.3298	0.3322	0.3346
1⅛	T	0.5625	0.5625	0.5623	0.5621	0.5620	0.5617	0.5613	0.5610
	H	0.3597	0.3621	0.3642	0.3665	0.3685	0.3710	0.3737	0.3764
1¼	T	0.6250	0.6250	0.6247	0.6246	0.6245	0.6241	0.6237	0.6232
	H	0.3997	0.4023	0.4047	0.4072	0.4095	0.4122	0.4152	0.4182
1⅜	T	0.6875	0.6875	0.6872	0.6870	0.6869	0.6865	0.6861	0.6856
	H	0.4397	0.4426	0.4452	0.4479	0.4504	0.4534	0.4567	0.4600
1½	T	0.7500	0.7500	0.7497	0.7495	0.7494	0.7489	0.7485	0.7480
	H	0.4797	0.4828	0.4857	0.4887	0.4914	0.4947	0.4983	0.5019
1¾	T	0.8750	0.8750	0.8746	0.8744	0.8743	0.8737	0.8732	0.8726
	H	0.5596	0.5633	0.5666	0.5701	0.5733	0.5771	0.5813	0.5855
2	T	1.0000	1.0000	0.9996	0.9994	0.9992	0.9986	0.9980	0.9972
	H	0.6396	0.6438	0.6476	0.6516	0.6552	0.6596	0.6644	0.6692
2¼	T	1.1250	1.1250	1.1246	1.1242	1.1240	1.1234	1.1226	1.1220
	H	0.7195	0.7242	0.7285	0.7330	0.7371	0.7420	0.7474	0.7528
2½	T	1.2500	1.2500	1.2494	1.2492	1.2490	1.2482	1.2474	1.2464
	H	0.7995	0.8047	0.8095	0.8145	0.8190	0.8245	0.8305	0.8365
3	T	1.5000	1.5000	1.4994	1.4990	1.4990	1.4978	1.4970	1.4960
	H	0.9594	0.9657	0.9714	0.9774	0.9828	0.9894	0.9966	1.0038

Chordal Thicknesses and Addenda for Gear Cutters



T = chordal thickness of cutter at pitch line;
A = perpendicular distance from chord to outside circumference of cutter (corrected addendum).

Diametral Pitch	Dimension	No. 1 135 Teeth	No. 2 55 Teeth	No. 3 35 Teeth	No. 4 26 Teeth	No. 5 21 Teeth	No. 6 17 Teeth	No. 7 14 Teeth	No. 8 12 Teeth
1	T	1.5707	1.5706	1.5702	1.5698	1.5694	1.5686	1.5675	1.5663
	A	1.1525	1.1459	1.1395	1.1334	1.1277	1.1209	1.1131	1.1057
1½	T	1.0471	1.0470	1.0468	1.0465	1.0462	1.0457	1.0450	1.0442
	A	0.7683	0.7639	0.7596	0.7556	0.7518	0.7472	0.7420	0.7371
2	T	0.7853	0.7853	0.7851	0.7849	0.7847	0.7843	0.7837	0.7831
	A	0.5762	0.5729	0.5697	0.5667	0.5638	0.5604	0.5565	0.5528
2½	T	0.6283	0.6282	0.6281	0.6279	0.6277	0.6274	0.6270	0.6265
	A	0.4610	0.4583	0.4558	0.4533	0.4511	0.4483	0.4452	0.4423
3	T	0.5235	0.5235	0.5234	0.5232	0.5231	0.5228	0.5225	0.5221
	A	0.3841	0.3819	0.3798	0.3778	0.3759	0.3736	0.3710	0.3685
3½	T	0.4487	0.4487	0.4486	0.4485	0.4484	0.4481	0.4478	0.4475
	A	0.3292	0.3274	0.3255	0.3238	0.3222	0.3202	0.3180	0.3159
4	T	0.3926	0.3926	0.3925	0.3924	0.3923	0.3921	0.3919	0.3915
	A	0.2881	0.2864	0.2848	0.2833	0.2819	0.2802	0.2783	0.2764
5	T	0.3141	0.3141	0.3140	0.3139	0.3138	0.3137	0.3135	0.3132
	A	0.2305	0.2291	0.2279	0.2266	0.2255	0.2242	0.2226	0.2211
6	T	0.2617	0.2617	0.2617	0.2616	0.2615	0.2614	0.2612	0.2610
	A	0.1921	0.1910	0.1899	0.1889	0.1879	0.1868	0.1855	0.1842
7	T	0.2243	0.2243	0.2243	0.2242	0.2242	0.2240	0.2239	0.2237
	A	0.1646	0.1637	0.1627	0.1619	0.1611	0.1601	0.1590	0.1579
8	T	0.1963	0.1963	0.1962	0.1962	0.1961	0.1960	0.1959	0.1957
	A	0.1440	0.1432	0.1424	0.1416	0.1409	0.1401	0.1391	0.1382
9	T	0.1745	0.1745	0.1744	0.1744	0.1743	0.1742	0.1741	0.1740
	A	0.1280	0.1273	0.1266	0.1259	0.1253	0.1245	0.1236	0.1228
10	T	0.1570	0.1570	0.1570	0.1569	0.1569	0.1568	0.1567	0.1566
	A	0.1152	0.1144	0.1139	0.1133	0.1127	0.1120	0.1113	0.1105
11	T	0.1427	0.1427	0.1427	0.1427	0.1426	0.1426	0.1425	0.1423
	A	0.1047	0.1041	0.1035	0.1030	0.1025	0.1019	0.1012	0.1005
12	T	0.1308	0.1308	0.1308	0.1308	0.1308	0.1307	0.1306	0.1305
	A	0.0960	0.0954	0.0949	0.0944	0.0939	0.0934	0.0927	0.0921
14	T	0.1122	0.1122	0.1121	0.1121	0.1121	0.1120	0.1119	0.1118
	A	0.0823	0.0818	0.0814	0.0809	0.0805	0.0800	0.0795	0.0789
16	T	0.0981	0.0981	0.0981	0.0981	0.0981	0.0980	0.0979	0.0979
	A	0.0720	0.0716	0.0712	0.0708	0.0705	0.0700	0.0695	0.0691
18	T	0.0872	0.0872	0.0872	0.0872	0.0872	0.0871	0.0871	0.0870
	A	0.0640	0.0636	0.0633	0.0629	0.0626	0.0622	0.0618	0.0614
20	T	0.0785	0.0785	0.0785	0.0785	0.0784	0.0784	0.0783	0.0783
	A	0.0576	0.0573	0.0569	0.0566	0.0564	0.0560	0.0556	0.0553

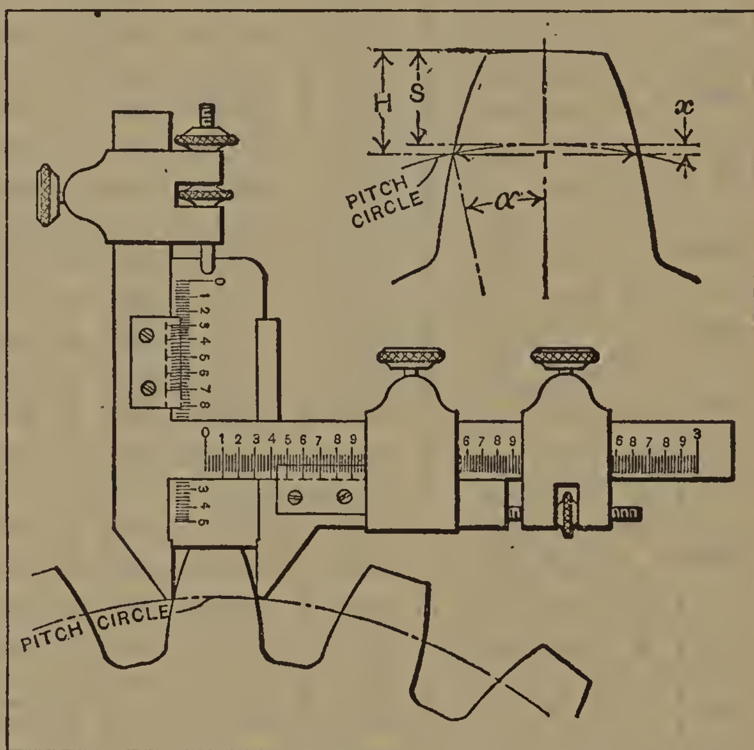
Chordal Thicknesses and Addenda for Gear Cutters

Circular Pitch	Dimension	No. 1 135 Teeth	No. 2 55 Teeth	No. 3 35 Teeth	No. 4 26 Teeth	No. 5 21 Teeth	No. 6 17 Teeth	No. 7 14 Teeth	No. 8 12 Teeth
1/4	T	0.1250	0.1250	0.1249	0.1249	0.1249	0.1248	0.1247	0.1246
	A	0.0917	0.0911	0.0906	0.0902	0.0897	0.0892	0.0885	0.0879
5/16	T	0.1562	0.1562	0.1562	0.1561	0.1561	0.1560	0.1559	0.1558
	A	0.1146	0.1139	0.1133	0.1127	0.1121	0.1115	0.1107	0.1099
3/8	T	0.1875	0.1875	0.1874	0.1873	0.1873	0.1872	0.1871	0.1870
	A	0.1375	0.1367	0.1360	0.1353	0.1345	0.1338	0.1328	0.1319
7/16	T	0.2187	0.2187	0.2186	0.2186	0.2185	0.2184	0.2183	0.2181
	A	0.1604	0.1595	0.1586	0.1578	0.1565	0.1561	0.1550	0.1539
1/2	T	0.2500	0.2500	0.2499	0.2498	0.2498	0.2496	0.2495	0.2493
	A	0.1834	0.1823	0.1813	0.1804	0.1794	0.1784	0.1771	0.1760
9/16	T	0.2812	0.2812	0.2811	0.2810	0.2810	0.2808	0.2806	0.2804
	A	0.2063	0.2050	0.2039	0.2029	0.2018	0.2007	0.1992	0.1978
5/8	T	0.3125	0.3125	0.3123	0.3123	0.3122	0.3120	0.3118	0.3116
	A	0.2292	0.2279	0.2266	0.2255	0.2243	0.2230	0.2214	0.2199
11/16	T	0.3437	0.3437	0.3436	0.3435	0.3434	0.3432	0.3430	0.3427
	A	0.2522	0.2507	0.2493	0.2480	0.2462	0.2453	0.2436	0.2419
3/4	T	0.3750	0.3750	0.3748	0.3747	0.3747	0.3744	0.3742	0.3740
	A	0.2751	0.2735	0.2720	0.2706	0.2691	0.2676	0.2657	0.2639
13/16	T	0.4062	0.4062	0.4060	0.4059	0.4059	0.4056	0.4054	0.4050
	A	0.2980	0.2961	0.2946	0.2931	0.2916	0.2898	0.2878	0.2857
7/8	T	0.4375	0.4375	0.4373	0.4372	0.4371	0.4368	0.4366	0.4362
	A	0.3209	0.3191	0.3173	0.3157	0.3140	0.3122	0.3100	0.3079
15/16	T	0.4687	0.4687	0.4685	0.4684	0.4683	0.4680	0.4678	0.4674
	A	0.3438	0.3418	0.3399	0.3382	0.3359	0.3345	0.3321	0.3299
1	T	0.5000	0.5000	0.4998	0.4997	0.4996	0.4993	0.4990	0.4986
	A	0.3668	0.3647	0.3627	0.3608	0.3589	0.3568	0.3543	0.3519
1 1/8	T	0.5625	0.5625	0.5623	0.5621	0.5620	0.5617	0.5613	0.5610
	A	0.4127	0.4103	0.4080	0.4058	0.4037	0.4013	0.3986	0.3959
1 1/4	T	0.6250	0.6250	0.6247	0.6246	0.6245	0.6241	0.6237	0.6232
	A	0.4585	0.4559	0.4533	0.4510	0.4486	0.4460	0.4428	0.4398
1 3/8	T	0.6875	0.6875	0.6872	0.6870	0.6869	0.6865	0.6861	0.6856
	A	0.5043	0.5014	0.4987	0.4961	0.4934	0.4906	0.4871	0.4838
1 1/2	T	0.7500	0.7500	0.7497	0.7495	0.7494	0.7489	0.7485	0.7480
	A	0.5502	0.5470	0.5440	0.5412	0.5383	0.5352	0.5314	0.5279
1 3/4	T	0.8750	0.8750	0.8746	0.8744	0.8743	0.8737	0.8732	0.8726
	A	0.6419	0.6382	0.6347	0.6314	0.6280	0.6244	0.6200	0.6158
2	T	1.0000	1.0000	0.9996	0.9994	0.9992	0.9986	0.9980	0.9972
	A	0.7336	0.7294	0.7254	0.7216	0.7178	0.7136	0.7086	0.7038
2 1/4	T	1.1250	1.1250	1.1246	1.1242	1.1240	1.1234	1.1226	1.1220
	A	0.8254	0.8206	0.8160	0.8116	0.8074	0.8026	0.7972	0.7918
2 1/2	T	1.2500	1.2500	1.2494	1.2492	1.2490	1.2482	1.2474	1.2464
	A	0.9170	0.9118	0.9066	0.9020	0.8972	0.8920	0.8856	0.8796
3	T	1.5000	1.5000	1.4994	1.4990	1.4988	1.4978	1.4970	1.4960
	A	1.1004	1.0940	1.0880	1.0824	1.0766	1.0704	1.0628	1.0558

Chordal Thicknesses and Addenda for Gear Teeth and Gear Cutters. — In measuring the thickness of gear teeth, it is necessary to make allowance for the curve of the pitch circle of the gear. The tables of chordal thicknesses for gear cutters and gear teeth give the straight-line distance across the tooth at the pitch circle, and also what is called the "corrected addendum," that is, the perpendicular distance from the chord at the pitch circle to the outside of the gear.

In the case of the gear cutter, the chordal thickness is the same as that for the gear, but the corrected addendum for the gear cutter is different from the corrected addendum for the gear. The two dimensions added together give the total depth of the gear tooth. The tables given are calculated for both diametral and circular pitches and for all commonly used pitches in either system.

Testing the Tooth Thickness when Milling Gear Teeth. — The special vernier gear-tooth caliper illustrated is sometimes used for testing the thickness of the first tooth milled. This test is especially desirable if there is any doubt about the accuracy of the blank diameter. (The outside diameter of a gear blank can be found by adding 2 to the number of teeth and dividing by the diametral pitch.) To test the tooth thickness, a trial cut is taken for a very short distance at one side of the blank; then the work is indexed for the next space, after which another trial cut is taken part way across the gear. The vertical scale of the caliper is set so that when it rests on top of the tooth, as shown, the lower ends of the caliper jaws will be at the height of the pitch circle. The horizontal scale then shows the chordal thickness of the tooth at this point. When a gear tooth is measured in this way, it is the chordal thickness T (see detail of tooth) that is obtained, instead of the thickness along the pitch circle. Hence, when measuring teeth of coarse pitch, especially if the diameter of the gear is quite small, dimension T should be obtained. It is also necessary to find the height x of the arc and add it to the addendum S to obtain the corrected height H, in order to measure the chordal thickness T at the proper point on the sides of the tooth. The table "Chordal Thicknesses of Gear Teeth at Pitch Line" gives this dimension T and corrected height H for various diametral and circular pitches and numbers of teeth.



If α = one-half of the angle subtended from the center of the gear by one gear tooth (see illustration); N = number of teeth in gear; T = chordal thickness of tooth at pitch line; and R = pitch radius of gear; then:

$$\alpha = 90° \div N; \qquad T = 2R \times \sin \alpha.$$

The height x of the arc equals 1 minus the cosine of angle α, multiplied by the pitch radius of the gear, or, expressed as a formula, $x = R\,(1 - \cos\,\alpha)$. The vertical scale of the caliper is set to dimension H or x + addendum S.

Depth of Cut for Milling Spur-gear Teeth. — The whole depth of cut can be found by dividing the constant 2.157 by the diametral pitch of the gear. For example, if the diametral pitch is 12, the depth of the tooth space equals $\frac{2.157}{12}$ = 0.180 inch. The depth of the cut also equals 0.6866 multiplied by the circular pitch. When milling gear teeth that are coarser than 6 or 7 diametral pitch, it is advisable to first rough mill all the teeth, and then take finishing cuts. Special "stocking cutters" are often used for rough milling coarse pitch gears preparatory to finishing by an accurately formed cutter.

Limits for Gearing. — The limits for center distance, pitch diameter and outside diameter of blanks, given in the table below, are applicable to spur gearing used under ordinary conditions. The + sign indicates dimensions over, and the − sign, dimensions under, the actual theoretical dimension.

Manufacturing Limits for Gearing

Diametral Pitch	Center Distance	Pitch Diameter	Blanks, Outside Diameter
16	±0.002	−0.003 to −0.005	0.000 to −0.005
14	±0.003	−0.004 to −0.006	0.000 to −0.005
12	±0.0035	−0.0045 to −0.007	0.000 to −0.006
10	±0.004	−0.005 to −0.008	0.000 to −0.006
8	±0.005	−0.006 to −0.009	0.000 to −0.007
6	±0.006	−0.007 to −0.010	0.000 to −0.008
5	±0.007	−0.008 to −0.011	0.000 to −0.010
4	±0.008	−0.009 to −0.012	0.000 to −0.015

Strength of Gear Teeth. — The method in most common use for determining the strength of gears is the one introduced by Mr. Wilfred Lewis and described by him in a paper read before the Engineers' Club of Philadelphia, October 15, 1892, and published in the proceedings of the club January, 1893. The Lewis formula is directly applicable to cut gears. For cast gears, the best method is to use the Lewis formula so modified as to give a factor of safety two or three times that required by this formula, depending upon the severity of the conditions under which the gears are used. A general rule for cast gears used under varying conditions can, however, hardly be devised.

The accompanying tables of "Rules and Formulas for the Strength of Gear Teeth," "Factors for Calculating the Strength of Gear Teeth" and "Working Stresses used in the Lewis Formula for the Strength of Gear Teeth" make it possible to quickly calculate the strength of spur gears. The formulas and factors given are based on the use of the diametral pitch, and the constants Y given in the factor table are valid only when the diametral pitch is used. If the circular pitch is given, it should be transformed into diametral pitch by dividing 3.1416 by the circular pitch. By means of the formulas given, the horsepower which can be transmitted by a gear of a given pitch diameter and diametral pitch, running at a given number of revolutions per minute, can be found by using Formulas (1) to (4) in the order given. The allowable static unit stress for the material in the gear is selected from the first line (velocity = 0) in the table of working stresses; the stress at any given velocity may also be found directly from the table.

As an example, assume that it is required to find the horsepower which it is permissible to transmit by a spur gear having 15-inch pitch diameter, 4 diametral pitch, making 100 revolutions per minute, and having a width of face of 1½ inch, if made according to the 14½-degree involute system. The gear is made of steel and the allowable static unit stress for the material may, therefore, be assumed to be 15,000 pounds per square inch. First insert the values of the revolutions per

Factors for Calculating Strength of Gear Teeth

No. of Teeth	Outline Factor = Y		No. of Teeth	Outline Factor = Y		No. of Teeth	Outline Factor = Y	
	14½° Involute and Cycloidal	20° Involute		14½° Involute and Cycloidal	20° Involute		14½° Involute and Cycloidal	20° Involute
12	0.210	0.245	20	0.283	0.320	43	0.346	0.396
13	0.220	0.261	21	0.289	0.327	50	0.352	0.408
14	0.226	0.276	23	0.295	0.333	60	0.358	0.421
15	0.236	0.289	25	0.305	0.339	75	0.364	0.434
16	0.242	0.295	27	0.314	0.349	100	0.371	0.446
17	0.251	0.302	30	0.320	0.358	150	0.377	0.459
18	0.261	0.308	34	0.327	0.371	300	0.383	0.471
19	0.273	0.314	38	0.336	0.383	Rack	0.390	0.484

Rules and Formulas for the Strength of Gear Teeth

(Based on the Lewis Formula)

D = pitch diameter of gear in ins.;
R = revolutions per minute;
V = velocity in ft. per min. at pitch diameter;
S_s = allowable static unit stress for material;
S = allowable unit stress for material at given velocity;
A = width of face in inches;
Y = outline factor (see table above);
P = diametral pitch (if circular pitch is given, divide 3.1416 by circular pitch to obtain diametral pitch);
W = maximum safe tangential load in pounds at pitch diameter;
H.P. = maximum safe horsepower.

Use rules and Formulas (1) to (4) in the order given.

No.	To Find	Rule	Formula
1	Velocity in feet per min. at the pitch diameter.	Multiply the product of the diameter in inches and the number of revolutions per minute, by 0.262.	$V = 0.262\, DR$
2	Allowable unit stress at given velocity.	Multiply the allowable static stress by 600 and divide the result by the velocity in feet per minute plus 600.	$S = S_s \times \frac{600}{600 + V}$
3	Maximum safe tangential load at pitch diameter.	Multiply together the allowable stress for the given velocity, the width of face, and the tooth outline factor; divide the result by the diametral pitch.	$W = \frac{SAY}{P}$
4	Maximum safe horsepower.	Multiply the safe load at the pitch line by the velocity in feet per minute, and divide the result by 33,000.	$\text{H.P.} = \frac{WV}{33{,}000}$

minute and the pitch diameter in Formula (1), and thus find the velocity in feet per minute at the pitch diameter. This velocity, as found in Formula (1), together with the allowable static unit stress, is then inserted in Formula (2), and the allowable unit stress at the given velocity is then found. This unit stress is now inserted in Formula (3), together with the width of face, the outline factor Y (which is found from the table to be 0.358 for 60 teeth), and the diametral pitch, and in this way the maximum safe tangential load W is found. Finally, by inserting the value of W just found, and the value of V found from Formula (1), in Formula (4), determine the maximum safe horsepower which can be transmitted by the gear.

Working Stresses used in the Lewis Formula for the Strength of Gear Teeth

Velocity in Feet per Minute = V	Strength Factors	Safe Working Unit Stress = S, in Pounds per Square Inch					
		Cast Iron		Phosphor Bronze		Steel	
		Ordinary Workmanship	High-grade Workmanship	Ordinary Workmanship	High-grade Workmanship	Ordinary Workmanship	High-grade Workmanship
0	1.000	6000	8000	9000	12,000	15,000	20,000
100	0.857	5150	6850	7700	10,300	12,800	17,100
200	0.750	4500	6000	6750	9,000	11,200	15,000
300	0.666	4000	5350	6000	8,000	10,000	13,300
450	0.571	3400	4550	5150	6,850	8,550	11,400
600	0.500	3000	4000	4500	6,000	7,500	10,000
900	0.400	2400	3200	3600	4,800	6,000	8,000
1200	0.333	2000	2650	3000	4,000	5,000	6,650
1800	0.250	1500	2000	2250	3,000	3,750	5,000
2400	0.200	1200	1600	1800	2,400	3,000	4,000

Simple Formula for Determining Power-transmitting Capacity of Spur Gears. — As electric motors are used more widely for machine and mill drives, the problem of calculating the size of gears required arises more frequently; and as motors are rated according to horsepower and speed in revolutions per minute, it is desirable to rate or calculate gears the same way, rather than to figure the breaking strength of the gears at loads in pounds.

The following formula, which is based on the Lewis formula, has been arranged to simplify this work, especially for those who frequently have to calculate the power-transmitting capacity of gearing. In this formula the notation is the same as given previously under "Rules and Formulas for the Strength of Gear Teeth." The outline factor Y for given number of teeth and pressure angle may be obtained from the table "Factors for Calculating Strength of Gear Teeth."

$$H = \frac{S_S A Y V}{P\ 55\quad (600 + V)}$$

The velocity in feet per minute at pitch line of gear = 0.262 × pitch diameter in inches × R.P.M. In using this formula, the values of S for accurately cut gears made of different materials are as follows: cast iron, 8000; semi-steel, 9000; U. S. Government bronze, 10,000; 0.30 per cent carbon steel, 15,000; 0.50 per cent carbon steel, 25,000; rawhide, 5000; bakelite, condensite, formica, textoil, etc., 5000.

Example: Find the horsepower that can be transmitted safely by a cast-iron spur gear which is to have 40 teeth of 4 diametral pitch, a 2-inch face width, and a speed of 80 R.P.M. It is assumed that the gear has accurately cut teeth, and the pressure angle is 14½ degrees. Then,

$$H = \frac{8000 \times 2 \times 0.34 \times 210}{4 \times 55 \times (600 + 210)} = \frac{80 \times 2 \times 34 \times 210}{4 \times 55 \times 810} = 6.4 \text{ horsepower}$$

It will be noted that the arithmetical work is reduced somewhat simply by dropping the decimal point from the outline factor 0.34 and at the same time cancelling two ciphers from 8000 representing the static unit stress.

Pitch of Gear for Given Power-transmitting Capacity. — The following formulas make it possible to determine directly the diametral or circular pitch of a spur gear when the horsepower to be transmitted and the velocity in feet per minute at the pitch diameter are known. The notation used is the same as given under "Rules and Formulas for the Strength of Gear Teeth."

Let P_1 = circular pitch $= \dfrac{3.1416}{P}$, or $P = \dfrac{3.1416}{P_1}$

By inserting this value in Formula (3) for determining the value of W, we have:

$$W = \frac{P_1 SAY}{3.1416}$$

But A (the width of face) is usually a certain number of times the circular pitch; so that we may write $A = kP_1$, where k generally is from 3 to 4 for gears having cut teeth and from 2 to 3 for cast-teeth gears. Then

$$W = \frac{P_1 SkP_1 Y}{3.1416} = \frac{P_1^2 SkY}{3.1416}$$

From this it follows that

$$P_1 = \sqrt{\frac{3.1416\, W}{SkY}}$$

But $W = \dfrac{33{,}000 \text{ H.P.}}{V}$; Hence, $P_1 = \sqrt{\dfrac{3.1416 \times 33{,}000 \text{ H.P.}}{SkYV}}$

To obtain the diametral pitch P, substitute $\dfrac{3.1416}{P}$ for P_1 and solve for P:

$$P = \sqrt{\frac{3.1416\, SkYV}{33{,}000 \text{ H.P.}}}$$

Rawhide Gears. — Comparatively little information is available as to the strength of rawhide pinions. One prominent maker of rawhide gears makes a regular practice of replacing high-speed cast-iron pinions with those made from rawhide of the same dimensions, providing the peripheral velocity exceeds 1600 feet per minute. When a rawhide pinion is to replace a steel gear, working under severe conditions, a special construction is used in which the weaker material is strengthened by a bronze reinforcement. Rawhide pinions will stand a load of 150 pounds per inch of face for a 1-inch circular pitch gear, with other pitches in proportion, and the load may be increased to a maximum of 250 pounds per inch of face, but it is undesirable to go beyond this limit, owing to the compression of rawhide under heavy loads. Rawhide gears are especially fitted for service where the gear teeth must withstand shocks, and for this reason are suitable for high speeds, as the decrease of the strength of metal gears with increasing velocity is due to the effect of impact.

Horsepower Transmitted by Rawhide Pinions. — The table "Horsepower Transmitted by Rawhide Pinions" gives the power that will be transmitted by a pinion having one-inch face. For gears having other than one-inch face, the value given in the table should be multiplied by the width of face in inches. As an example of the use of the table, assume that a rawhide gear having 20 teeth, 4 diametral pitch, and making 200 revolutions per minute is required to transmit 2 horsepower. What width of face should this gear have in order to safely transmit the power? From the table it is found that a gear of the given dimensions with one inch width of face would safely transmit 0.93 horsepower. A gear $2\frac{1}{4}$ inches wide would then transmit $2\frac{1}{4} \times 0.93 = 2.09$ horsepower. A width of face of $2\frac{1}{4}$ inches would therefore be the correct width.

Horsepower Transmitted by Cast-iron Pinions. — The tables "Horsepower Transmitted by Cast-iron Pinions" are calculated for pinions of one-inch face. For gears having other than one-inch face, the value given in the table should be multiplied by the width of face in inches. As an example of the use of the tables, assume that a cast-iron gear having 20 teeth, 4 diametral pitch, and making 200 revolutions per minute is required to transmit 2 horsepower. What width of face should this gear have in order to safely transmit the power? From the tables it is found that a gear of the given dimensions with one inch width of face would safely transmit 1.48 horsepower per inch width of face. Hence, the gear should be made $1\frac{3}{8}$ inch wide, and will then transmit $1\frac{3}{8} \times 1.48 = 2.035$ horsepower.

Allowance for wear has not been provided in the tables, but has been left to the discretion of the designer. According to Reuleaux the coefficient of wear, C, equals the pressure p on the teeth at the pitch circle in pounds, multiplied by the number of revolutions per minute, N, this product being divided by the width of face A, or, $C = \frac{pN}{A}$. The value of this fraction should never exceed 28,000.

In a pair of cast-iron gears the greatest wear comes on the smallest gear, and while the coefficient of wear should not exceed 28,000 for the largest gear, it may be taken as low as 12,000, if possible without obtaining too large dimensions. To avoid excessive wear on the smaller gear, steel or bronze may be substituted for the pinion.

Tables of Relation between Velocity, Load and Diametral Pitch. — These tables are given for determining the diametral pitches for $14\frac{1}{2}$-degree involute and cycloidal cast-iron gear teeth when the load in pounds per inch width of face, the circumferential velocity, and the number of teeth are given. The speed or circumferential velocity of the teeth, as indicated in the headings of the various tables, is first located; then the load in pounds per inch width of face is found in the column to the left, and the number of teeth located at the top of the tables. The diametral pitch is then obtained from the figures in the body of the tables, opposite the load in pounds and in the column under the teeth. For example, assume that it is required to find the diametral pitch of the teeth in a pinion having 15 teeth, running at a speed of 150 feet per minute, and subjected to a load of 500 pounds per inch width of face at the pitch line. As the speed of the teeth is between 100 and 200 feet per minute, the answer is found in the section thus headed, opposite 500 in the left-hand column, and beneath the heading "14–16 Teeth" at the top of the column. The required diametral pitch of the teeth then is $2\frac{3}{4}$. If the gear is 3 inches wide and subjected at the pitch line to a load of 600 pounds, the load per inch width of face is, of course, 200 pounds, and the diametral pitch for 15 teeth and 150 feet speed would be 7.

The tables can also be used for finding the permissible load in pounds per inch width of face when the diametral pitch and the other factors mentioned are given.

In this case, speed, diametral pitch, and number of teeth are first located and the load is read off in the left-hand column opposite the line in which the given diametral pitch is found. For example, assume that a gear of 8 diametral pitch runs at a speed of 75 feet per minute, the gear having 24 teeth. Locate 8 diametral pitch in the part of the table for 0 to 100 feet per minute, in the column headed "21–25 Teeth," and follow the line in which 8 diametral pitch is given, to the left; the permissible tooth load is 300 pounds per inch width of face. If the gear were only $\frac{3}{4}$ inch wide, then the permissible load, of course, would be $300 \times \frac{3}{4} = 225$ pounds.

General Rules Governing Power-transmitting Capacity of Spur Gears. — In determining the power-transmitting capacity of spur gears having cut teeth, the following general rules should be considered:

1. The horsepower a gear will transmit depends upon the quality of the material, accuracy of the teeth, rigidity of shafts, bearings, and housings, and the accuracy of the shaft alignment.

2. A gear supported on only one side should be made with a comparatively narrow face, a good rule being to make the face width in inches equal to 10 ÷ diametral pitch.

3. Gears should always be made *strong* enough; that is, they should be capable of transmitting at least 25 per cent more than the rated horsepower. The reason for this is that most gearing is subject to overload from one cause or another. If the drive is subject to a known overload, the gears should be made strong enough for the full overload, plus about 25 per cent for safety. Gears for crushers, rolls, and reciprocating pumps must often be made capable of carrying 250 per cent of the normal or rated load.

4. While it is possible to make spur gears with as few as three teeth, small numbers of teeth are not satisfactory for high-speed or hard work. As a general rule, it is not practical to use a pinion with less than 18 teeth, when the pitch-line velocity exceeds 800 feet per minute.

5. When the gears are of rigid construction, mounted on rigid shafts supported on both sides of the gear face, and run in oil, a pitch-line velocity of 2000 feet per minute is permissible in some cases. Usually, however, it is well to keep below 800 feet per minute for metal gears to be used under ordinary conditions. Above 800 feet per minute it is well to use rawhide, bakelite, or some approved type of nonmetallic gear in conjunction with a mating gear made of metal. If noise is not objectionable, both gears may be made of metal. Strength and quietness can be obtained by making the pinion of laminated construction, using alternate plates of steel and bakelite, riveted together.

6. The pitch should be as fine as possible without requiring too wide a face to transmit the power, since the finer pitches tend toward greater efficiency for the gearing.

Proportions of Cast Spur Gears. — The table "Dimensions of Spur Gears" gives the formulas for proportioning cast gears with arms of different cross-sections. This table will prove satisfactory for general conditions, but in individual cases modifications must be made to suit particular designs. The oval arm is the one best adapted for small and medium size gears and gives the best appearance. It requires somewhat more metal for the same strength than the designs shown in the lower part of the table, but it is very easily molded. For large size gears, however, arms of the +, T- and H-sections are largely used. In these designs the metal is so distributed as to give a high degree of rigidity in proportion to the weight; but these forms are more difficult to mold than the oval sections. The H-section, however, is the most desirable for gears, the faces of which are very wide in proportion to their pitch. The draft for removing the pattern from the sand in molding is not shown in the illustrations in the table, but it should be provided for liberally, and should be added to the dimensions given, rather than taken off.

Horsepower Transmitted by Rawhide Pinions, 1-inch Face. (See page 643.)

Pitch		R.P.M.		100	150	200	250	300	350	400	450
		Teeth	Pitch Diam.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
5 D.P.	0.6283 C.P.	17	3.40	0.25	0.38	0.50	0.63	0.76	0.88	1.02	1.14
		18	3.60	0.27	0.40	0.53	0.67	0.80	0.94	1.07	1.21
		19	3.80	0.28	0.42	0.56	0.71	0.85	0.99	1.13	1.28
		20	4.00	0.30	0.44	0.59	0.74	0.89	1.04	1.19	1.34
		21	4.20	0.31	0.47	0.62	0.78	0.94	1.10	1.25	1.41
4 D.P.	0.7854 C.P.	17	4.25	0.40	0.59	0.79	0.99	1.19	1.39	1.59	1.79
		18	4.50	0.42	0.63	0.84	1.05	1.26	1.47	1.68	1.89
		19	4.75	0.44	0.66	0.88	1.04	1.33	1.55	1.77	1.99
		20	5.00	0.46	0.70	0.93	1.17	1.40	1.63	1.87	2.10
		21	5.25	0.49	0.73	0.98	1.23	1.47	1.72	1.96	2.21
3½ D.P.	0.8976 C.P.	17	4.86	0.51	0.77	1.04	1.29	1.55	1.81	2.07	2.33
		18	5.14	0.54	0.82	1.10	1.37	1.65	1.92	2.19	2.47
		19	5.43	0.58	0.87	1.16	1.45	1.74	2.03	2.32	2.61
		20	5.72	0.61	0.91	1.22	1.52	1.83	2.13	2.44	2.74
		21	6.00	0.64	0.96	1.28	1.60	1.92	2.24	2.56	2.88
3 D.P.	1.0472 C.P.	17	5.66	0.70	1.06	1.41	1.76	2.12	2.47	2.82	3.17
		18	6.00	0.74	1.12	1.50	1.87	2.24	2.62	2.99	3.36
		19	6.33	0.78	1.18	1.58	1.97	2.37	2.76	3.16	3.55
		20	6.66	0.83	1.25	1.66	2.08	2.49	2.91	3.32	3.74
		21	7.00	0.87	1.31	1.74	2.18	2.62	3.05	3.49	3.93
2½ D.P.	1.2566 C.P.	17	6.80	1.02	1.52	2.03	2.54	3.05	3.56	4.06	4.57
		18	7.20	1.08	1.61	2.15	2.69	3.23	3.76	4.30	4.84
		19	7.60	1.14	1.70	2.27	2.84	3.41	3.97	4.54	5.11
		20	8.00	1.20	1.79	2.39	2.99	3.59	4.18	4.78	5.38
		21	8.40	1.26	1.88	2.51	3.14	3.77	4.39	5.02	5.65

Pitch		R.P.M.		100	150	200	250	300	350	400	450
		Teeth	Pitch Diam.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
2 D.P.	1.5708 C.P.	17	8.50	1.59	2.38	3.17	3.97	4.76	5.56	6.35	7.15
		18	9.00	1.68	2.52	3.36	4.20	5.04	5.88	6.72	7.57
		19	9.50	1.78	2.66	3.55	4.44	5.32	6.21	7.10	7.99
		20	10.00	1.87	2.80	3.74	4.67	5.60	6.54	7.47	8.40
		21	10.50	1.96	2.94	3.92	4.90	5.89	6.87	7.85	8.83
1¾ D.P.	1.7952 C.P.	17	9.71	1.93	2.89	3.85	4.82	5.78	6.74	7.71	8.67
		18	10.30	2.04	3.06	4.08	5.10	6.12	7.14	8.16	9.18
		19	10.86	2.15	3.23	4.30	5.38	6.46	7.53	8.61	9.68
		20	11.43	2.27	3.40	4.54	5.67	6.80	7.94	9.07	10.20
		21	12.00	2.38	3.57	4.76	5.95	7.14	8.33	9.52	10.71
1½ D.P.	2.0944 C.P	17	11.33	2.25	3.38	4.51	5.63	6.76	7.89	9.01	10.14
		18	12.00	2.39	3.58	4.77	5.96	7.16	8.35	9.54	10.73
		19	12.67	2.52	3.77	5.03	6.29	7.55	8.81	10.07	11.32
		20	13.33	2.65	3.98	5.30	6.63	7.96	9.28	10.61	11.94
		21	14.00	2.78	4.17	5.56	6.96	8.35	9.74	11.13	12.52
1¼ D.P.	2.5133 C.P.	17	13.60	2.70	4.04	5.39	6.74	8.09	9.43	10.78	12.13
		18	14.40	2.85	4.28	5.71	7.13	8.56	9.98	11.42	12.84
		19	15.20	3.01	4.52	6.02	7.53	9.04	10.54	12.05	13.55
		20	16.00	3.17	4.76	6.34	7.93	9.51	11.10	12.68	14.27
		21	16.80	3.33	4.99	6.66	8.32	9.99	11.65	13.32	14.98
1 D.P.	3.1416 C.P.	17	17.00	3.38	5.07	6.76	8.45	10.14	11.83	13.52	15.21
		18	18.00	3.58	5.37	7.16	8.95	10.73	12.52	14.31	16.10
		19	19.00	3.78	5.66	7.55	9.44	11.33	13.22	15.10	....
		20	20.00	3.98	5.96	7.95	9.94	11.93	13.91	15.90	....
		21	21.00	4.17	6.26	8.35	10.44	12.53	14.61	.....	....

Horsepower Transmitted by Cast-iron Pinions, 1-inch Face. (See page 643.)

Pitch		R.P.M.		100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	950	1050	1150
		Teeth	Pitch Diam.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
5 D.P.	0.6283 C.P.	17	3.40	0.424	0.636	0.827	1.01	1.19	1.38	1.54	1.70	1.85	2.00	2.14	2.28	2.45	2.58	2.70	2.83	3.05	3.26	3.51
		18	3.60	0.453	0.665	0.866	1.07	1.26	1.44	1.61	1.80	1.96	2.11	2.27	2.41	2.55	2.68	2.82	2.94	3.18	3.40	3.60
		19	3.80	0.478	0.702	0.915	1.12	1.32	1.51	1.69	1.86	2.03	2.19	2.35	2.50	2.64	2.79	2.92	3.05	3.30	3.53	3.70
		20	4.00	0.503	0.739	0.964	1.18	1.39	1.59	1.78	1.96	2.14	2.31	2.43	2.58	2.74	2.88	3.02	3.16	3.42	3.66	3.89
		21	4.20	0.528	0.775	1.00	1.23	1.44	1.65	1.83	2.02	2.21	2.38	2.55	2.71	2.88	2.97	3.12	3.26	3.53	3.73	3.97
4 D.P.	0.7854 C.P.	17	4.25	0.668	0.969	1.26	1.62	1.82	2.07	2.31	2.55	2.79	3.01	3.22	3.43	3.57	3.76	3.95	4.13	4.46	4.72	5.02
		18	4.50	0.701	1.03	1.34	1.70	1.91	2.19	2.45	2.66	2.85	3.13	3.35	3.58	3.78	3.92	4.11	4.30	4.66	5.00	5.18
		19	4.75	0.740	1.08	1.40	1.72	2.00	2.26	2.54	2.81	3.06	3.24	3.49	3.90	3.92	4.07	4.27	4.46	4.77	5.12	5.47
		20	5.00	0.779	1.13	1.48	1.78	2.09	2.36	2.67	2.90	3.16	3.36	3.60	3.83	4.06	4.21	4.42	4.63	5.02	5.26	5.59
		21	5.25	0.808	1.18	1.53	1.86	2.19	2.45	2.76	3.04	3.26	3.54	3.78	3.96	4.13	4.42	4.58	4.72	5.12	5.52	5.71
3½ D.P.	0.8976 C.P.	17	4.86	0.863	1.25	1.64	2.00	2.32	2.64	2.96	3.27	3.51	3.79	4.07	4.25	4.50	4.75	4.98	5.14	5.57	5.98	6.38
		18	5.14	0.915	1.33	1.72	2.08	2.45	2.80	3.08	3.40	3.71	3.95	4.23	4.50	4.70	4.95	5.12	5.45	5.73	6.18	6.56
		19	5.43	0.955	1.39	1.79	2.20	2.53	2.90	3.26	3.53	3.85	4.10	4.39	4.68	4.88	5.07	5.41	5.57	5.90	6.33	6.75
		20	5.72	1.00	1.46	1.89	2.31	2.67	3.05	3.36	3.65	3.99	4.31	4.55	4.77	5.05	5.34	5.52	5.70	6.21	6.48	6.90
		21	6.00	1.06	1.53	1.98	2.38	2.75	3.15	3.53	3.83	4.12	4.45	4.70	5.01	5.23	5.44	5.63	5.99	6.33	6.62	7.08
3 D.P.	1.0472 C.P.	17	5.66	1.16	1.69	2.18	2.67	3.09	3.47	3.88	4.29	4.62	4.99	5.26	5.60	5.94	6.17	6.38	6.59	7.18	7.82	7.98
		18	6.00	1.23	1.77	2.31	2.78	3.21	3.68	4.05	4.47	4.81	5.19	5.48	5.85	6.11	6.34	6.58	6.98	7.39	7.94	8.26
		19	6.33	1.28	1.87	2.41	2.94	3.38	3.80	4.27	4.65	4.98	5.39	5.69	5.98	6.25	6.69	6.94	7.19	7.59	8.15	8.50
		20	6.66	1.35	1.97	2.52	3.02	3.56	4.00	4.40	4.79	5.16	5.42	5.73	6.30	6.58	6.85	7.10	7.55	8.00	8.38	8.72
		21	7.00	1.42	2.05	2.64	3.17	3.68	4.13	4.57	5.05	5.42	5.77	6.11	6.41	6.71	7.01	7.26	7.71	8.16	8.58	9.16
2½ D.P.	1.2566 C.P.	17	6.80	1.65	2.38	3.08	3.70	4.29	4.90	5.40	5.88	6.31	6.72	7.11	7.47	8.05	8.38	8.71	8.98	9.50	10.00	10.67
		18	7.20	1.73	2.52	3.23	3.85	4.44	5.10	5.64	6.11	6.58	7.00	7.53	7.91	8.30	8.65	8.95	9.25	9.81	10.30	11.05
		19	7.60	1.83	2.63	3.38	4.06	4.69	5.28	5.84	6.35	6.83	7.29	7.71	8.11	8.52	8.86	9.19	9.50	10.10	10.88	....
		20	8.00	1.93	2.74	3.56	4.27	4.85	5.48	6.03	6.57	7.07	7.67	7.89	8.32	8.70	9.07	9.42	9.76	10.37	10.90	....
		21	8.40	2.00	2.88	3.67	4.41	5.09	5.75	6.33	6.79	7.32	7.81	8.28	8.75	9.14	9.26	9.64	9.99	10.88	....	....

Horsepower Transmitted by Cast-iron Pinions, 1-inch Face

Pitch		R.P.M.		100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	950
		Teeth	Pitch Diam.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
2 D.P.	1.5708 C.P.	17	8.50	2.53	3.61	4.64	5.58	6.44	7.15	7.89	8.59	9.26	9.88	10.47	10.73	10.94	11.43	11.90	12.64	13.72
		18	9.00	2.66	3.82	4.80	5.79	6.71	7.57	8.22	8.95	9.51	10.16	10.81	11.36	11.92	12.10	12.60	13.05	....
		19	9.50	2.80	4.00	5.07	6.11	6.97	7.84	8.54	9.32	10.04	10.40	11.07	11.67	12.23	12.47	12.96	....	....
		20	10.00	2.95	4.17	5.34	6.32	7.21	8.12	8.84	9.51	10.27	11.01	11.36	11.95	12.56	12.80	13.64	....	....
		21	10.50	3.07	4.38	5.51	6.52	7.42	8.39	9.15	9.98	10.51	11.21	11.58	12.24	12.86	13.43	...	....	....
1¾ D.P.	1.7952 C.P.	17	9.71	3.28	4.67	5.93	7.02	8.15	9.02	9.98	10.90	11.40	12.23	12.95	13.28	13.95	14.57	...	....	....
		18	10.30	3.43	4.90	6.17	7.43	8.47	9.39	10.25	11.17	12.07	12.55	13.33	14.05	14.40	15.03	...	....	....
		19	10.86	3.48	5.06	6.51	7.73	8.77	9.75	10.82	11.46	12.41	12.90	13.69	14.46	14.80	...	...	....	....
		20	11.43	3.78	5.33	6.72	7.99	9.10	10.10	11.05	12.08	12.70	13.59	14.07	14.86	....	...	...	....	....
		21	12.00	3.91	5.60	6.94	8.25	9.40	10.42	11.27	12.36	12.97	14.27	14.40	15.20	....	...	...	....	....
1½ D.P.	2.0944 C.P.	17	11.33	4.37	6.17	7.39	9.24	10.52	11.69	12.78	13.62	14.68	15.72	16.27	17.20	....	...	...	....	....
		18	12.00	4.57	6.53	8.08	9.61	10.96	12.21	13.15	14.41	15.14	16.18	16.80	17.76	....	...	...	....	....
		19	12.67	4.82	6.76	8.54	10.14	11.42	12.50	13.87	14.76	15.65	16.66	17.28	....	....	...	...	....	....
		20	13.33	5.04	7.13	8.84	10.49	11.80	13.16	14.25	15.14	16.36	17.13	18.21	....	....	...	...	....	....
		21	14.00	5.23	7.35	9.14	10.83	12.21	13.42	14.50	15.45	16.75	17.50	....	....	....	...	...	....	....
1¼ D.P.	2.5133 C.P.	17	13.60	6.17	8.57	10.81	12.63	14.23	16.10	17.43	18.51	19.53	20.42	....	....	....	...	...	....	....
		18	14.40	6.46	9.23	11.28	13.16	15.07	16.57	17.91	19.07	20.17	21.62	....	....	....	...	...	....	....
		19	15.20	6.76	9.36	11.69	13.66	15.42	17.05	18.40	19.65	20.75	....	....	....	....	...	...	....	....
		20	16.00	7.12	9.74	12.30	14.38	15.78	17.41	18.83	20.08	21.86	....	....	....	....	...	...	....	....
		21	16.80	7.32	10.20	12.48	14.66	16.57	18.29	19.30	20.64	....	....	....	....	....	...	...	....	....
1 D.P.	3.1416 C.P.	17	17.00	9.26	12.90	15.79	18.68	20.96	22.50	24.40	26.10	....	....	....	....	....	...	...	....	....
		18	18.00	9.80	13.42	16.45	19.02	21.63	23.82	25.20	27.86	....	....	....	....	....	...	...	....	....
		19	19.00	10.34	13.95	17.08	20.08	22.15	24.46	25.90	....	....	....	....	....	....	...	...	....	....
		20	20.00	10.68	14.42	17.68	20.55	22.70	25.13	27.30	....	....	....	....	....	....	...	...	....	....
		21	21.00	11.03	14.86	18.32	21.03	23.83	25.73	....	....	....	....	....	....	....	...	...	....	....

Relation Between Velocity, Load and Diametral Pitch. (See page 643.)

Diametral Pitches for 14½° Involute and Cycloidal Cast-iron Gear Teeth, per Inch Width of Face, for Continuous Service in One Direction

Based on Wilfred Lewis' constants for speed and form of tooth. For ordinary steel castings, double the pitch for corresponding load, or double the load for corresponding pitch.

Load in Pounds	Diametral Pitch							
	12–13 Teeth	14–16 Teeth	17–20 Teeth	21–25 Teeth	26–34 Teeth	35–54 Teeth	55–134 Teeth	135 Teeth Rack
Speed of Teeth, 0 to 100 Feet per Minute								
200	8	9	10	12	12	14	14	14
300	5½	6	7	8	8	9	9	10
400	4	4½	6	6	6	7	7	7
500	3¼	3½	4	4½	5	5	5½	6
600	3	3	3½	4	4	4¼	4½	5
700	2½	2½	3	3½	3½	3¾	4	4
800	2	2¼	2½	3	3	3¼	3½	3½
900	2	2	2¼	2½	2¾	3	3	3¼
1000	.....	2	2	2¼	2½	2½	2¾	3
1100	.....	.....	2	2	2¼	2¼	2½	2¾
1200	.....	.....	.....	2	2	2¼	2¼	2½
1300	.....	.....	.....	.....	2	2	2¼	2¼
1400	.....	.....	.....	.....	.....	2	2	2
1500	.....	.....	.....	.....	.....	.....	2	2
Speed of Teeth, 100 to 200 Feet per Minute								
100	12	14	16	.....	.....	.....	.....	.....
200	6	7	8	9	9	10	11	12
300	4	4½	5	6	6	6	7	7
400	3	3½	4	4½	4½	5	5	5½
500	2½	2¾	3	3½	3½	4	4	4½
600	2	2¼	2½	2¾	3	3¼	3½	3¾
700	2	2	2¼	2½	2½	3	3	3¼
800	.....	.....	2	2¼	2¼	2½	2½	2¾
900	.....	.....	.....	2	2	2¼	2¼	2½
1000	.....	.....	.....	.....	2	2	2	2¼
1100	.....	.....	.....	.....	.....	.....	2	2
1200	.....	.....	.....	.....	.....	.....	.....	2
Speed of Teeth, 200 to 300 Feet per Minute								
100	10	11	12	14	14	16	.....	.....
200	5	5½	6	7	7	8	8	9
300	3¼	3½	4	4½	5	5	5½	6
400	2½	2¾	3	3½	3½	4	4	4½
500	2	2¼	2½	2¾	3	3	3¼	3½
600	.....	.....	2	2¼	2½	2½	3	3
700	.....	.....	.....	2	2	2¼	2½	2½
800	.....	.....	.....	.....	2	2	2	2¼
900	.....	.....	.....	.....	.....	.....	2	2

Relation Between Velocity, Load and Diametral Pitch

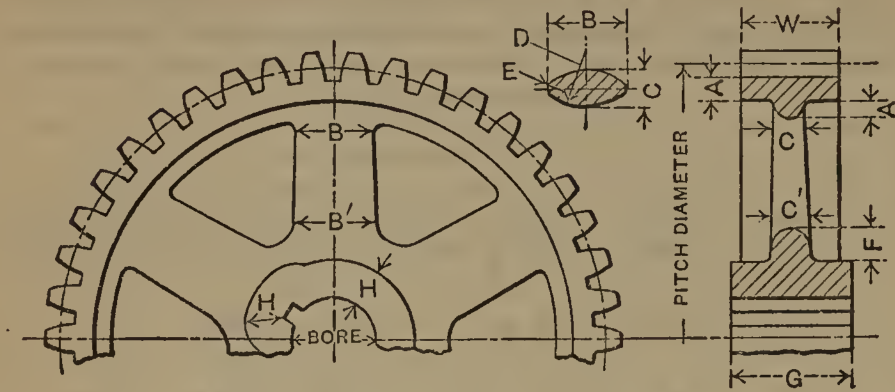
Diametral Pitches for 14½° Involute and Cycloidal Cast-iron Gear Teeth, per Inch Width of Face, for Continuous Service in One Direction

Based on Wilfred Lewis' constants for speed and form of tooth. For ordinary steel castings, double the pitch for corresponding load, or double the load for corresponding pitch.

Load in Pounds	Diametral Pitch							
	12–13 Teeth	14–16 Teeth	17–20 Teeth	21–25 Teeth	26–34 Teeth	35–54 Teeth	55–134 Teeth	135 Teeth Rack
Speed of Teeth, 300 to 600 Feet per Minute								
100	8	9	10	12	12	14	14	14
200	4	4½	5	6	6	7	7	7
300	2½	3	3¼	3¾	4	4	4½	5
400	2	2¼	2½	3	3	3¼	3½	3¾
500	.....	2	2	2¼	2½	2¾	3	3
600	.....	.....	.....	2	2	2¼	2¼	2½
700	.....	.....	.....	.....	.....	2	2	2
800	.....	.....	.....	.....	.....	.....	.....	2
Speed of Teeth, 600 to 900 Feet per Minute								
100	6	7	8	9	9	10	10	12
200	3	3½	3¾	4½	4½	5	5¼	5½
300	2	2¼	2½	3	3	3¼	3½	3¾
400	.....	.....	2	2¼	2¼	2½	2½	3
500	.....	.....	.....	.....	2	2	2	2¼
Speed of Teeth, 900 to 1200 Feet per Minute								
100	5	5½	6	7	7	8	8	9
200	2½	2¾	3	3½	3½	4	4	4½
300	.....	2	2	2¼	2½	2½	2¾	3
400	.....	.....	.....	.....	2	2	2	2¼
500	.....	.....	.....	.....	.....	.....	.....	2
Speed of Teeth, 1200 to 1800 Feet per Minute								
100	4	4½	5	5½	6	7	7	7
200	2	2¼	2½	2¾	3	3¼	3½	3¾
300	.....	.....	.....	2	2	2¼	2½	2½
400	.....	.....	.....	.....	.....	.....	2	2
Speed of Teeth, 1800 to 2400 Feet per Minute								
100	3½	3¾	4¼	5	5	5½	6	6
200	2	2	2¼	2½	2½	2¾	3	3
300	.....	.....	.....	.....	.....	2	2	2

Dimensions of Spur Gears. (See page 644.)

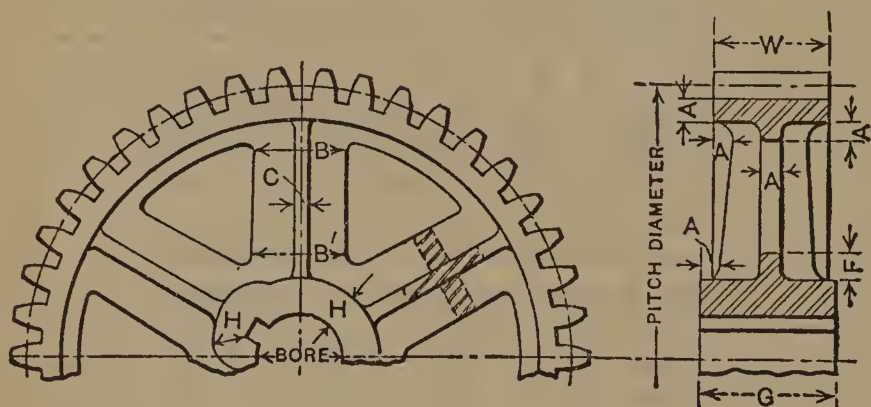
Dimensions of Spur Gears with Oval Arms

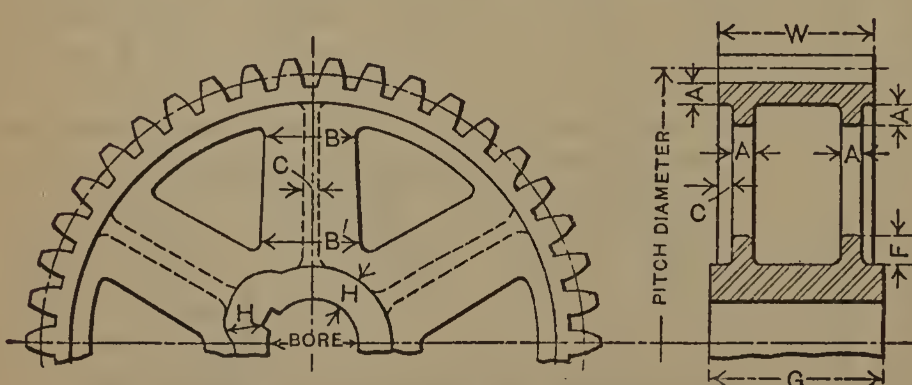


P = diametral pitch, P' = circular pitch.

$A = 1.57 \div P = 0.5\,P'$;
$B = 6.28 \div P = 2.0\,P'$;
$C = 3.14 \div P = P'$;
$D = 4.71 \div P = 1.5\,P'$;
$E = 0.79 \div P = 0.25\,P'$;
$F = 2.00 \div P = 0.65\,P'$;
$G = W + 0.025$ pitch diameter;
$H = 0.44 \times$ bore;
$B' = B + \frac{3}{4}$ inch per foot;
$C' = C + \frac{3}{4}$ inch per foot.

Dimensions of Spur Gears with Ribbed Arms or Arms of H-section

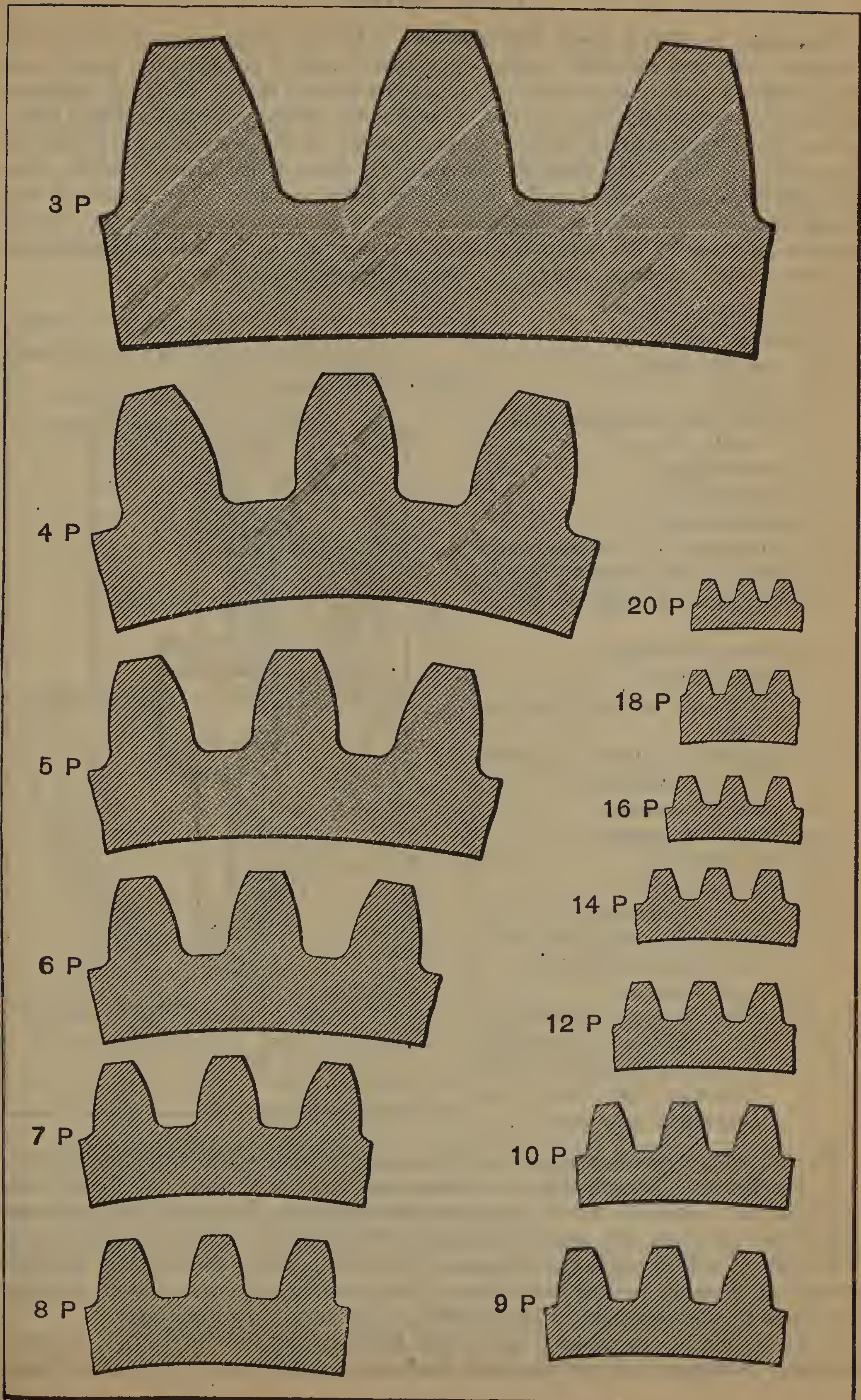




P = diametral pitch, P' = circular pitch.

$A = 1.57 \div P = 0.5\,P'$;
$B = 7.85 \div P = 2.5\,P'$;
$C = 0.94 \div P = 0.3\,P'$;
$F = 2.20 \div P = 0.7\,P'$;
$G = W + 0.025$ pitch diameter;
$H = 0.44 \times$ bore;
$B' = B + \frac{3}{4}$ inch per foot.

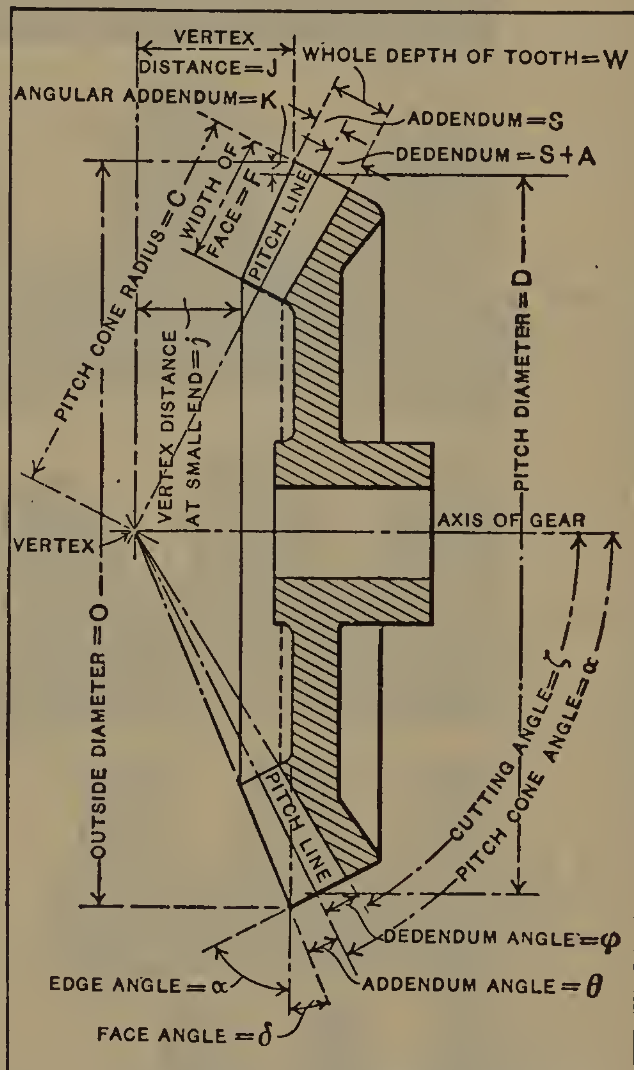
Gear Teeth of Different Diametral Pitch, Full Size



Bevel Gearing

Formulas for Bevel Gear Calculations. — On the following pages are given complete rules and formulas for the calculation of bevel gearing with shafts at a right angle, an acute angle, and an obtuse angle with each other. Separate formulas for miter bevel gearing are also given, as well as for crown gears and internal bevel gears. The numbers given in the left-hand column are for convenient reference to any particular rule. The rules and formulas are given in the order in which they would ordinarily be used by the designer of bevel gearing. The names of the various angles and dimensions referred to in bevel gearing are given in the accompanying engraving. The notation used in the formulas, which is easily understood by comparing the formula with the corresponding rule, is as follows:

N = number of teeth;
P = diametral pitch;
P' = circular pitch;
π = 3.1416;
α = pitch cone angle and edge angle;
γ = center angle;
D = pitch diameter;
S = addendum;
$S+A$ = dedendum (A = clearance);
W = whole depth of tooth space;
T = thickness of tooth at pitch line;
C = pitch cone radius;
F = width of face;
s = addendum at small end of tooth;
t = thickness of tooth at pitch line at small end;
θ = addendum angle;
ϕ = dedendum angle;
δ = face angle;
ζ = cutting angle;
K = angular addendum;
O = outside diameter (edge diameter for internal gears);
J = vertex distance;
j = vertex distance at small end;
N' = number of teeth for which to select cutter, also called "number of teeth in equivalent spur gear."

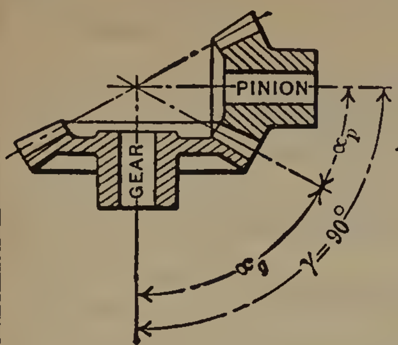


The following exceptions to, and modifications of, the rules given should be noted:

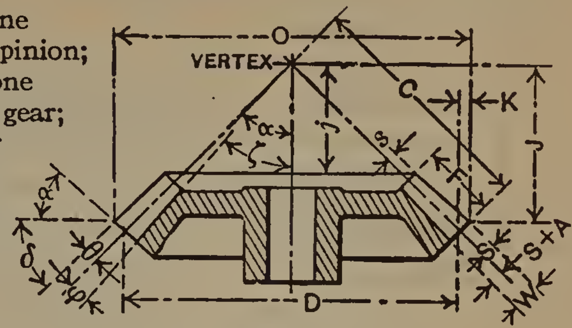
1. The Brown & Sharpe Mfg. Co. recommends that for shaping bevel gear teeth with a formed cutter, the cutting angle be determined by subtracting the *addendum* angle from the pitch cone angle, instead of subtracting the dedendum angle, as in Rule (15). In other words, the clearance at the bottom of the tooth is made uniform instead of tapering toward the vertex. This gives a somewhat closer approximation to the desired shape. This applies, of course, also to Rule (25).

2. In generating machines (such as the Bilgram and the Gleason) it is often advisable to depart from the standard dimensions of gear teeth as given by Rules

Rules and Formulas for Calculating Bevel Gears with Shafts at Right Angles



α_p = Pitch cone angle of pinion;
α_g = Pitch cone angle of gear;
N_p = Number of teeth in pinion, etc.



Use Rules and Formulas Nos. 1 to 21 in the order given.

No.		To Find	Rule	Formula
1		Pitch Cone Angle (or Edge Angle) of Pinion.	Divide the number of teeth in the pinion by the number of teeth in the gear to get the tangent.	$\tan \alpha_p = \frac{N_p}{N_g}$
2		Pitch Cone Angle (or Edge Angle) of Gear.	Divide the number of teeth in the gear by the number of teeth in the pinion to get the tangent.	$\tan \alpha_g = \frac{N_g}{N_p}$
3		Proof of Calculations for Pitch Cone Angles.	The sum of the pitch cone angles of the pinion and gear equals 90 degrees.	$\alpha_p + \alpha_g = 90°$
4		Pitch Diameter.	Divide the number of teeth by the diametral pitch; or multiply the number of teeth by the circular pitch and divide by 3.1416.	$D = \frac{N}{P} = \frac{NP'}{\pi}$
5	These dimensions are the same for both gear and pinion.	Addendum.	Divide 1.0 by the diametral pitch; or multiply the circular pitch by 0.318.	$S = \frac{1.0}{P} = 0.318\,P'$
6		Dedendum.	Divide 1.157 by the diametral pitch; or multiply the circular pitch by 0.368.	$S + A = \frac{1.157}{P} = 0.368\,P'$
7		Whole Depth of Tooth Space.	Divide 2.157 by the diametral pitch; or multiply the circular pitch by 0.687.	$W = \frac{2.157}{P} = 0.687\,P'$
8		Thickness of Tooth at Pitch Line.	Divide 1.571 by the diametral pitch; or divide the circular pitch by 2.	$T = \frac{1.571}{P} = \frac{P'}{2}$
9		Pitch Cone Radius.	Divide the pitch diameter by twice the sine of the pitch cone angle.	$C = \frac{D}{2 \times \sin \alpha}$
10		Addendum of Small End of Tooth.	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the addendum.	$s = S \times \frac{C - F}{C}$
11		Thickness of Tooth at Pitch Line at Small End.	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the thickness of the tooth at the pitch line.	$t = T \times \frac{C - F}{C}$
12		Addendum Angle.	Divide the addendum by the pitch cone radius to get the tangent.	$\tan \theta = \frac{S}{C}$
13		Dedendum Angle.	Divide the dedendum by the pitch cone radius to get the tangent.	$\tan \phi = \frac{S + A}{C}$

Rules and Formulas for Calculating Bevel Gears with Shafts at Right Angles

No.	To Find	Rule	Formula
14	Face Angle.	Subtract the sum of the pitch cone and addendum angles from 90 degrees.	$\delta = 90° - (\alpha + \theta)$
15	Cutting Angle.*	Subtract the dedendum angle from the pitch cone angle	$\zeta = \alpha - \phi$
16	Angular Addendum.	Multiply the addendum by the cosine of the pitch cone angle.	$K = S \times \cos \alpha$
17	Outside Diameter.	Add twice the angular addendum to the pitch diameter.	$O = D + 2K$
18	Apex Distance.	Multiply one-half the outside diameter by the tangent of the face angle.	$J = \frac{O}{2} \times \tan \delta$
19	Apex Distance at Small End of Tooth.	Subtract the width of face from the pitch cone radius; divide the remainder by the pitch cone radius and multiply by the apex distance.	$j = J \times \frac{C - F}{C}$
20	Number of Teeth for which to Select Cutter.	Divide the number of teeth by the cosine of the pitch cone angle.	$N' = \frac{N}{\cos \alpha}$
21	Proof of Calculations by Rules Nos. 9, 12, 14, 16 and 17.	The outside diameter equals twice the pitch cone radius multiplied by the cosine of the face angle and divided by the cosine of the addendum angle.	$O = \frac{2C \times \cos \delta}{\cos \theta}$

* See paragraph "Formulas for Bevel Gear Calculations."

and Formulas (1) to (44). For instance, where the pinion is made of bronze and the gear of steel, the teeth of the former can be made wider and those of the latter correspondingly thinner, so as to nearly equalize the strength of the two. Again, where the pinion has few teeth and the gear many, it may be advisable to make the addendum on the pinion larger and the dedendum correspondingly smaller, reversing this on the gear, making the addendum smaller and the dedendum larger. This is done to avoid interference and consequent undercut on the flanks of pinions having a small number of teeth. Such changes are easily effected on generating machines, and instructions for doing this for any case will usually be furnished by the makers of the various machines.

Internal bevel gearing should be avoided except in cases where cast gears would be satisfactory, because it is practically impossible to cut internal bevel gearing. It may be possible to produce internal bevel gears on some forms of templet planing machines, if the pitch cone angle is not too great, but it is impossible on any form of generating machine. Internal bevel gearing can usually be avoided, and be replaced by external bevel gearing, by extending one of the shafts between which motion is to be transmitted, and mounting the gears in such a position that the required motion can be transmitted by a pair of ordinary bevel gears.

Examples of Bevel Gear Calculations. — In the following are given a number of examples of the use of the formulas for bevel gear calculations. While it is not necessary in practice to have the dimensions accurate within 0.0001 inch, it is well to carry out the calculations to four decimal places. This permits accurate checking of the results by Formulas (3) and (21).

Shafts at Right Angles. — Let it be required to make the necessary calculations for a pair of bevel gears in which the shafts are at right angles; diametral pitch = 3, number of teeth in gear = 60, number of teeth in pinion = 15, and width of face = 4 inches.

$$\tan \alpha_p = 15 \div 60 = 0.25000 = \tan 14^\circ\ 2' \tag{1}$$

$$\tan \alpha_g = 60 \div 15 = 4.00000 = \tan 75^\circ\ 58' \tag{2}$$

$$\gamma = 14^\circ\ 2' + 75^\circ\ 58' = 90^\circ \tag{3}$$

$$D_p = 15 \div 3 = 5.000'' \tag{4}$$

$$S = 1 \div 3 = 0.3333'' \tag{5}$$

$$S + A = \frac{1.157}{3} = 0.3856'' \tag{6}$$

$$W = \frac{2.157}{3} = 0.7190'' \tag{7}$$

$$T = \frac{1.571}{3} = 0.5236'' \tag{8}$$

$$C = \frac{5}{2 \times 0.24249} = 10.3097'' \tag{9}$$

$$s = 0.3333 \times \frac{6.31}{10.31} = 0.2040'' \tag{10}$$

$$t = 0.5236 \times \frac{6.31}{10.31} = 0.3204'' \tag{11}$$

$$\tan \theta = \frac{0.3333}{10.3097} = 0.03233 = \tan 1^\circ\ 51' \tag{12}$$

$$\tan \phi = \frac{0.3856}{10.3097} = 0.03740 = \tan 2^\circ\ 9' \tag{13}$$

$$\delta = 90^\circ - (14^\circ\ 2' + 1^\circ\ 51') = 74^\circ\ 7' \tag{14}$$

$$\zeta = 14^\circ\ 2' - 2^\circ\ 9' = 11^\circ\ 53' \tag{15}$$

$$K = 0.3333 \times 0.97015 = 0.3234'' \tag{16}$$

$$O = 5.000 + 2 \times 0.3234 = 5.6468'' \tag{17}$$

$$J = \frac{5.6468}{2} \times 3.51441 = 9.9225'' \tag{18}$$

$$j = 9.9225 \times \frac{6.31}{10.31} = 6.0726'' \tag{19}$$

$$N' = \frac{15}{0.97015} = 15.4 \tag{20}$$

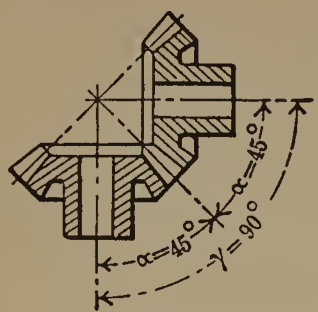
$$5.6468'' = \frac{20.6194 \times 0.27368}{0.99948} = 5.6461'' \tag{21}$$

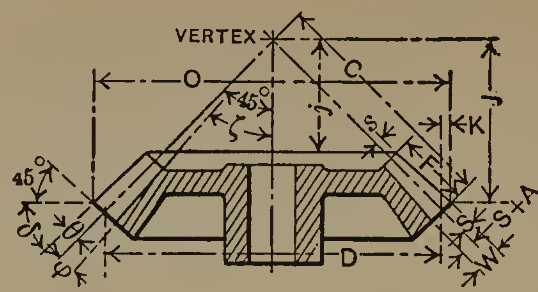
This gives all the data required for the pinion. Rules (5) to (13), inclusive, apply equally to the gear and the pinion, so that only calculations by Rules and Formulas (4) and (14) to (21) need be made for the gear, although it is well to calculate Formula (9) a second time as a check for the same calculation for the pinion.

$$D = \frac{60}{3} = 20.000'' \tag{4}$$

$$C = \frac{20}{2 \times 0.97015} = 10.3077'' \tag{9}$$

Rules and Formulas for Calculating Miter Bevel Gearing





Use Rules and Formulas Nos. 22, 4–8, 23, 10–13, 24–26, 17–19, 27 and 21 in the order given. All dimensions thus obtained are the same for both gears of a pair.

No.	To Find	Rule	Formula
22	Pitch Cone Angle.	Pitch cone angle equals 45 degrees.	$\alpha = 45°$
23	Pitch Cone Radius.	Multiply the pitch diameter by 0.707.	$C = 0.707\ D$
24	Face*Angle.	Subtract the addendum angle from 45°.	$\delta = 45° - \theta$
25	Cutting Angle.*	Subtract the dedendum angle from 45°.	$\zeta = 45° - \phi$
26	Angular Addendum.	Multiply the addendum by 0.707.	$K = 0.707\ S$
27	Number of Teeth for which to Select Cutter.	Multiply the number of teeth by 1.41.	$N' = 1.41\ N$

* See paragraph "Formulas for Bevel Gear Calculations."

$$\delta = 90 - (75°\ 58' + 1°\ 51') = 12°\ 11' \tag{14}$$

$$\zeta = 75°\ 58' - 2°\ 9' = 73°\ 49' \tag{15}$$

$$K = 0.3333 \times 0.24249 = 0.0808'' \tag{16}$$

$$O = 20 + 2 \times 0.0808 = 20.1616'' \tag{17}$$

$$J = \frac{20.1616}{2} \times 0.2159 = 2.1764'' \tag{18}$$

$$j = 2.1764 \times \frac{6.31}{10.31} = 1.3320'' \tag{19}$$

$$N' = \frac{60}{0.24249} = 247 \tag{20}$$

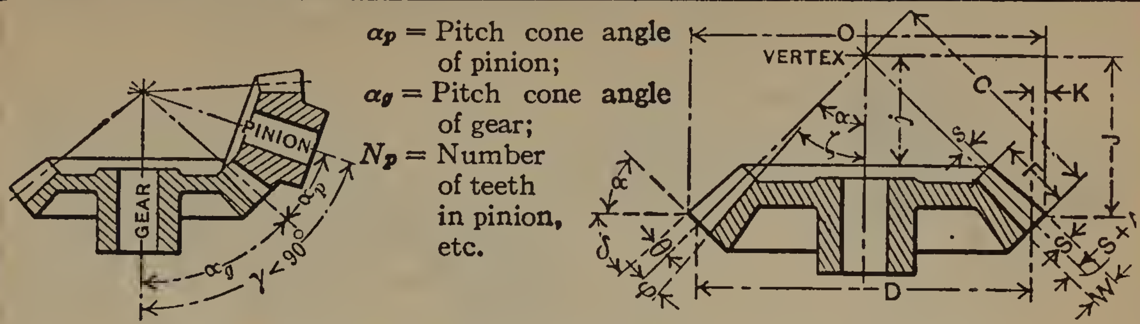
$$20.1616'' \cong \frac{20.6154 \times 0.97748}{0.99948} = 20.1615'' \tag{21}$$

This gives the calculations necessary for this pair of gears.

Acute Angle Bevel Gearing.— Let it next be required to calculate the dimensions of a pair of bevel gears the center angle of which is 75 degrees, the number of teeth in the pinion 15, the number of teeth in the gear 60, the diametral pitch 3, and the width of face 4 inches. Following the directions given in the tables of rules and formulas for bevel gear calculations:

$$\tan \alpha_p = \frac{0.96593}{\frac{60}{15} + 0.25882} = 0.22681 = \tan 12°\ 47' \tag{28}$$

Rules and Formulas for Calculating Bevel Gears with Shafts at an Acute Angle



α_p = Pitch cone angle of pinion; α_g = Pitch cone angle of gear; N_p = Number of teeth in pinion, etc.

Use Rules and Formulas Nos. 28–30, and 4–21 in the order given.

No.	To Find	Rule	Formula
28	Pitch Cone Angle (or Edge Angle) of Pinion.	Divide the sine of the center angle by the sum of the cosine of the center angle and the quotient of number of teeth in the gear divided by the number of teeth in the pinion; this gives the tangent.	$\tan \alpha_p = \dfrac{\sin y}{\dfrac{N_g}{N_p} + \cos y}$
29	Pitch Cone Angle (or Edge Angle) of Gear.	Divide the sine of the center angle by the sum of the cosine of the center angle and the quotient of the number of teeth in the pinion divided by the number of teeth in the gear; this gives the tangent.	$\tan \alpha_g = \dfrac{\sin y}{\dfrac{N_p}{N_g} + \cos y}$
30	Proof of Calculations for Pitch Cone Angles.	The sum of the pitch cone angles of the pinion and gear equals the center angle.	$\alpha_p + \alpha_g = y$

$$\tan \alpha_g = \frac{0.96593}{\frac{15}{60} + 0.25882} = 1.89837 = \tan 62° \, 13' \qquad (29)$$

$$\gamma = 12° \, 47' + 62° \, 13' = 75° \qquad (30)$$

Formulas (4) to (8) as in first example; also, $C = 11.2989''$, $s = 0.2154''$, $t = 0.3382''$, $\theta = 1° \, 41'$, $\phi = 1° \, 57'$, $\delta = 75° \, 32'$, $\zeta = 10° \, 50'$, $K = 0.3251''$, $O = 5.6502''$, $J = 10.9501''$, $j = 7.0748''$, and $N' = 15.3$, also,

$$5.6502'' \cong \frac{22.598 \times 0.24982}{0.99957} = 5.6483'' \qquad (21)$$

For the gear, the additional calculations give: $C = 11.303''$, $\delta = 26° \, 6'$, $\zeta = 60° \, 16'$, $K = 0.1553''$, $O = 20.3106''$, $J = 4.9748''$, $j = 3.2142''$, $N' = 129$.

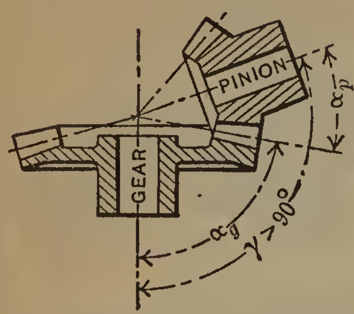
$$20.3106'' \cong \frac{22.606 \times 0.89803}{0.99957} = 20.3096'' \qquad (21)$$

The above calculations are not all given in full, as most of them are merely duplications of formulas previously used.

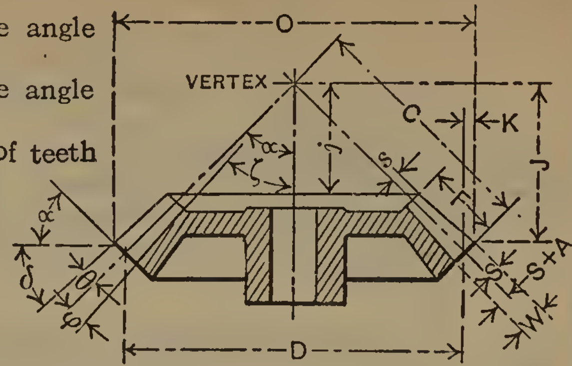
Obtuse Angle Bevel Gearing. — Let it be required to calculate the dimensions of the same set of gears with a center angle of 100 degrees. This being an example of obtuse angle gearing, apply Formula (31) as follows:

$$\tan \alpha_p = \frac{0.98481}{\frac{60}{15} - 0.17365} = 0.25738 = \tan 14° \, 26' \qquad \mathbf{(31)}$$

Rules and Formulas for Calculating Bevel Gears with Shafts at an Obtuse Angle



α_p = Pitch cone angle of pinion;
α_g = Pitch cone angle of gear;
N_p = Number of teeth in pinion, etc.



Use Rules and Formulas Nos. 31 and 32 as directed below.

No.	To Find	Rule	Formula
31	Pitch Cone Angle (or Edge Angle) of Pinion.	Divide the sine of 180 degrees minus the center angle by the difference between the quotient of the number of teeth in the gear divided by the number of teeth in the pinion and the cosine of 180 degrees minus the center angle; this gives the tangent.	$\tan \alpha_p = \dfrac{\sin (180^\circ - \gamma)}{\dfrac{N_g}{N_p} - \cos (180^\circ - \gamma)}$
32	Whether Gear is a Regular Bevel Gear, a Crown Gear, or an Internal Bevel Gear.	Add 90 degrees to the pitch cone angle of the pinion. If the sum is greater than the center angle use Rules and Formulas Nos. 33, 30 and 4–21 in the order given. If the sum equals the center angle see rules and formulas for crown gear. If the sum is less than the center angle see rules and formulas for internal bevel gear.	
33	Pitch Cone Angle (or Edge Angle) of Gear.	Divide the sine of 180 degrees minus the center angle by the difference between the quotient of the number of teeth in the pinion divided by the number of teeth in the gear and the cosine of 180 degrees minus the center angle; this gives the tangent.	$\tan \alpha_g = \dfrac{\sin (180^\circ - \gamma)}{\dfrac{N_p}{N_g} - \cos (180^\circ - \gamma)}$

and thus discover that it is an example of regular obtuse angle gearing, since

$$14^\circ\ 26' + 90^\circ = 104^\circ\ 26' > 100^\circ \tag{32}$$

The remaining calculations for the angles are as follows:

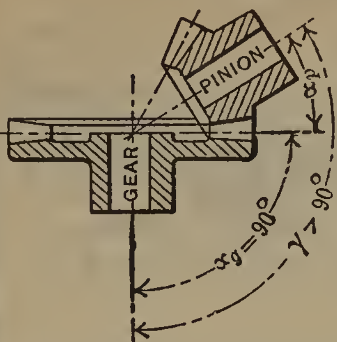
$$\tan \alpha = \frac{0.98481}{\frac{15}{60} - 0.17365} = 12.8986 = \tan 85^\circ\ 34' \tag{33}$$

$$\gamma = 14^\circ\ 26' + 85^\circ\ 34' = 100^\circ \tag{30}$$

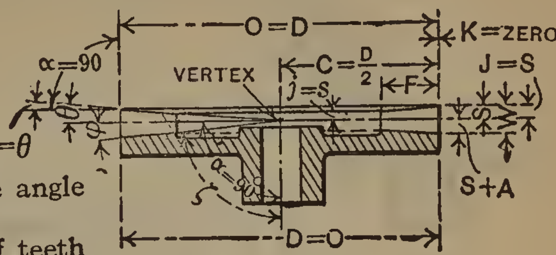
and the calculations for the other dimensions as per the table.

Crown Gear. — Suppose it is required to make a crown gear and a pinion for the same number of teeth, pitch and face as in the previous examples. What are

Rules and Formulas for Calculating Crown Gears



α_p = Pitch cone angle of pinion;
N_p = Number of teeth in pinion;
N_g = Number of teeth in gear, etc.



Use Rules Nos. 31 and 4–21 in the order given, for the pinion; use Rules Nos. 30, 4–8, 36, 10–13, 37, 15 and 38 in the order given for the crown gear; if dimensions for crown gear are known, to find center angle and dimensions of pinion, use Rules and Formulas Nos. 34, 35 and 4–21 in the order given.

No.	To Find	Rule	Formula
34	Pitch Cone Angle (or Edge Angle) of Pinion.	Divide the number of teeth in the pinion by the number of teeth in the gear, to get the sine.	$\sin\alpha_p = \frac{N_p}{N_g}$
35	Center Angle.	Add 90 degrees to the pitch cone angle of the pinion.	$\gamma = 90^\circ + \alpha_p$
36	Pitch Cone Radius.	Divide the pitch diameter by 2.	$C = \frac{D}{2}$
37	Face Angle of Gear.	The face cone angle of the gear equals the addendum angle.	$\delta_g = \theta$
38	Number of Teeth for which to Select Cutter.	The teeth are equivalent in form to rack teeth.	$N_g' =$ infinity

the additional calculations necessary? Following the proper formulas in the order given:

$$\sin\alpha_p = \frac{15}{60} = 0.25000 = \sin 14^\circ\ 29' \tag{34}$$

$$\gamma = 90^\circ + 14^\circ\ 29' = 104^\circ\ 29' \tag{35}$$

The other calculations are similar to those already given.

Internal Bevel Gear. — Let it be required to design a pair of bevel gears of the same number of teeth, pitch and face, in which the center angle is 115 degrees. This being an example of obtuse angle gearing, use Formula (31).

$$\tan\alpha_p = \frac{0.90631}{\frac{60}{15} - 0.42262} = 0.25334 = \tan 14^\circ\ 13' \tag{31}$$

Then, according to Rule (32):

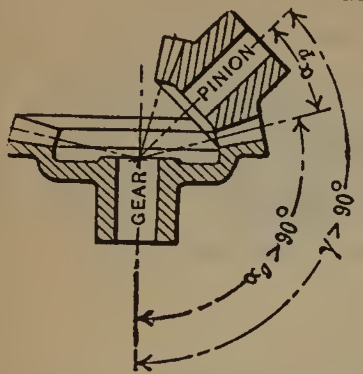
$$14^\circ\ 13' + 90^\circ = 104^\circ\ 13' < 115^\circ \tag{32}$$

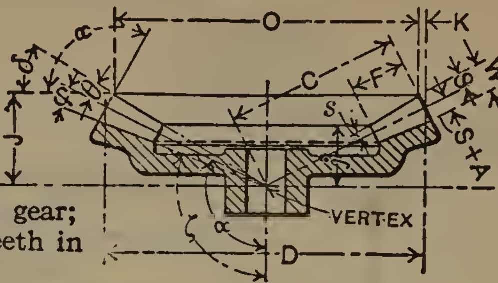
showing that the gear is an internal bevel gear. Applying the rules and formulas for internal bevel gearing:

$$\tan\alpha_a = \frac{0.90631}{0.42262 - \frac{15}{60}} = 5.25032 = \tan 79^\circ\ 13'$$

$$180^\circ - 79^\circ\ 13' = 100^\circ\ 47' \tag{39}$$

Rules and Formulas for Calculating Internal Bevel Gears





δ_g = Face angle of gear;
N_p = Number of teeth in pinion;
N_g = Number of teeth in gear, etc.

Use Rules and Formulas Nos. 31 and 4–21 inclusive for the pinion; use Rules and Formulas Nos. 39, 30, 40, 41, 15, 42, 43, 18, 19, 44 and 21 in the order given for the gear.

No.	To Find	Rule	Formula
39	Pitch Cone Angle (or Edge Angle) of Gear.	Divide the sine of 180 degrees minus the center angle by the difference between the cosine of 180 degrees minus the center angle and the quotient of the number of teeth in the pinion divided by the number of teeth in the gear; subtract the angle whose tangent is thus found from 180 degrees.	$\tan \alpha_a = \dfrac{\sin(180 - \gamma)}{\cos(180 - \gamma) - \dfrac{N_p}{N_g}}$ $\alpha_g = 180 - \alpha_a$
40	Pitch Cone Radius.	Divide the pitch diameter by twice the sine of 180 degrees minus the pitch cone angle.	$C = \dfrac{D_g}{2 \sin(180 - \alpha_g)}$
41	Face Angle of Gear.	Subtract 90 degrees from the sum of the pitch cone angle and the addendum angle.	$\delta_g = \alpha_g + \theta - 90°$
42	Angular Addendum of Gear.	Multiply the addendum by the cosine of 180 degrees minus the pitch cone angle.	$K_g = S \times \cos(180 - \alpha_g)$
43	Outside (or Edge) Diameter of Gear.	Subtract twice the angular addendum from the pitch diameter.	$O_g = D_g - 2K_g$
44	Number of Teeth for which to Select Cutter.	Divide the number of teeth by the cosine of 180 degrees minus the pitch cone angle.	$N_g' = \dfrac{N_g}{\cos(180 - \alpha_g)}$

$$\gamma = 100° \, 47' + 14° \, 13' = 115° \qquad (30)$$

$$C = \frac{20}{2 \times 0.98234} = 10.1797'' \qquad (40)$$

$$\delta = 100° \, 47' + 1° \, 53' - 90° = 12° \, 40' \qquad (41)$$

$$\zeta = 98° \, 37', \text{ and } K = 0.0624''$$

$$O = 20 - 2 \times 0.0624 = 19.8752'' \qquad (43)$$

$$N' = \frac{60}{0.1871} = 320 \text{ (internal)} \qquad (44)$$

The calculations for the pinion and the other calculations for the gear are similar to those already given.

Forms of Bevel Gear Teeth. — Five systems of tooth outlines are used for bevel gearing. They are the cycloid, the standard 14½-degree involute, the 20-degree involute, and the 15- and 20-degree octoid. The cycloidal form of tooth is obsolete for cut bevel gears, and is also rarely met with nowadays for cast gears. It requires very careful workmanship, and is difficult or impossible to generate. Most bevel gears are, therefore, made on the involute system, with either the standard 14½-degree or the 20-degree pressure angle. The 20-degree tooth is broader at the base and stronger in form than the 14½-degree tooth, but most bevel gears that are milled with form cutters are made to the 14½-degree standard. Planed gears made by the templet or generating processes are nowadays often made to the 20-degree pressure angle.

When planing a pair of bevel gears on the Bilgram, Gleason or other similar generating machines, a form of tooth named the "octoid" is used. In generating machines the teeth of the gears are shaped by a tool which represents the side of the tooth of an imaginary crown gear. The cutting edge of the tool is a straight line, since the imaginary crown gear has teeth whose sides are plane surfaces. It can be shown that the teeth of a true involute crown gear have sides which are very slightly curved. The minute difference between the tooth shapes produced by a plane crown tooth and a slightly curved crown tooth is the minute difference between the octoid and involute forms. Both give theoretically correct action.

Tables for Determining the Outside Diameter of Bevel Gears. — These tables are used for finding the outside diameter of bevel gears when the pitch diameter is known, and apply only to bevel gears with axes at right angles. The figures given in the table (in the column giving the number of teeth in the gear and opposite the number of teeth in the pinion) are for 1 diametral pitch; they are, therefore, to be divided by the diametral pitch of the gears, and the quotient thus obtained is added to the pitch diameter. For each combination of teeth, two figures are given. The upper value is for the gear and the lower for the pinion.

Example. — Find the outside diameter of a pair of bevel gears, 10 diametral pitch, with 35 and 23 teeth.

In the tables the diameter increments are found to be 1.10 for the gear and 1.67 for the pinion, for 1 diametral pitch. For 10 diametral pitch, then, $1.10 \div 10 = 0.110$, and $1.67 \div 10 = 0.167$. The pitch diameter of the gear is 3.5 inches; thus, $3.5 + 0.110 = 3.610$ inches is the outside diameter of the gear. The pitch diameter of the pinion is 2.3 inches; hence, $2.3 + 0.167 = 2.467$ inches is the outside diameter of the pinion.

The table "Diameter Increments for Bevel Gears" can be used in cases where the required number of teeth is not given in the tables just referred to. The object of the table is the same — that of obtaining the outside diameter when the pitch diameter is known. In these tables the column headed "Ratio" gives the ratio of the number of teeth in a pair of gears, and the two columns headed "Gear" and "Pinion" give the amount to add to the pitch diameter to obtain the outside diameter for 1 diametral pitch. Thus, to find the diameter increments, first divide the number of teeth in the gear by the number of teeth in the pinion, and find this ratio in the table. Then divide the values given in the table opposite this ratio by the given pitch. The table is applicable to bevel gears with axes at right angles only.

Example. — Find the outside diameter of a pair of bevel gears, 75 and 20 teeth, 6 diametral pitch.

Dividing the number of teeth in the gear by the number of teeth in the pinion gives a ratio of 3.75. From the table the diameter increment is then found to be 0.515 for the gear, and 1.933 for the pinion. Dividing each by 6 gives 0.086 and 0.322. The pitch diameter of the gear is 12.5 inches and of the pinion 3.333 inches. Adding the diameter increments just found gives 12.586 inches for the outside diameter of the gear, and 3.655 inches for the outside diameter of the pinion.

Diameter Increments for Bevel Gears

Ratio	Gear	Pinion	Ratio	Gear	Pinion	Ratio	Gear	Pinion
1.000	1.414	1.414	2.250	0.812	1.827	4.70	0.416	1.956
1.025	1.396	1.431	2.275	0.805	1.831	4.75	0.411	1.957
1.050	1.380	1.448	2.30	0.797	1.834	4.80	0.407	1.958
1.075	1.362	1.464	2.35	0.783	1.840	4.85	0.404	1.959
1.100	1.345	1.480	2.40	0.770	1.846	4.90	0.400	1.960
1.125	1.328	1.494	2.45	0.755	1.851	5.00	0.392	1.961
1.150	1.312	1.509	2.50	0.743	1.857	5.10	0.385	1.962
1.175	1.296	1.523	2.55	0.730	1.862	5.20	0.377	1.964
1.200	1.280	1.536	2.60	0.718	1.867	5.30	0.370	1.965
1.225	1.264	1.549	2.65	0.706	1.871	5.40	0.364	1.966
1.250	1.249	1.561	2.70	0.694	1.875	5.50	0.357	1.967
1.275	1.234	1.573	2.75	0.683	1.880	5.60	0.351	1.968
1.300	1.219	1.585	2.80	0.672	1.883	5.70	0.345	1.970
1.325	1.204	1.596	2.85	0.662	1.887	5.80	0.340	1.971
1.350	1.190	1.607	2.90	0.652	1.890	5.90	0.334	1.972
1.375	1.176	1.617	2.95	0.642	1.894	6.00	0.329	1.973
1.400	1.162	1.627	3.00	0.632	1.897	6.10	0.323	1.974
1.425	1.149	1.636	3.05	0.623	1.900	6.20	0.318	1.975
1.450	1.135	1.646	3.10	0.614	1.903	6.30	0.313	1.975
1.475	1.122	1.655	3.15	0.605	1.906	6.40	0.309	1.976
1.500	1.109	1.664	3.20	0.596	1.909	6.50	0.304	1.977
1.525	1.096	1.672	3.25	0.588	1.911	6.60	0.299	1.977
1.550	1.084	1.680	3.30	0.580	1.914	6.70	0.295	1.978
1.575	1.072	1.688	3.35	0.571	1.916	6.80	0.291	1.979
1.600	1.060	1.696	3.40	0.564	1.918	6.90	0.287	1.979
1.625	1.047	1.703	3.45	0.557	1.921	7.00	0.283	1.980
1.650	1.036	1.710	3.50	0.550	1.923	7.10	0.279	1.980
1.675	1.025	1.717	3.55	0.542	1.925	7.20	0.275	1.981
1.700	1.014	1.723	3.60	0.535	1.927	7.30	0.271	1.981
1.725	1.003	1.730	3.65	0.528	1.929	7.40	0.268	1.982
1.750	0.992	1.736	3.70	0.521	1.931	7.50	0.264	1.982
1.775	0.982	1.742	3.75	0.515	1.933	7.60	0.261	1.983
1.800	0.971	1.748	3.80	0.509	1.934	7.70	0.257	1.983
1.825	0.961	1.754	3.85	0.503	1.936	7.80	0.254	1.983
1.850	0.951	1.760	3.90	0.497	1.937	7.90	0.251	1.984
1.875	0.941	1.765	3.95	0.491	1.939	8.00	0.248	1.984
1.900	0.931	1.770	4.00	0.485	1.940	8.10	0.245	1.985
1.925	0.922	1.775	4.05	0.479	1.941	8.20	0.242	1.985
1.950	0.912	1.780	4.10	0.473	1.943	8.30	0.239	1.985
1.975	0.903	1.784	4.15	0.468	1.944	8.40	0.236	1.986
2.000	0.894	1.790	4.20	0.463	1.945	8.50	0.234	1.986
2.025	0.885	1.793	4.25	0.458	1.947	8.60	0.231	1.986
2.050	0.876	1.797	4.30	0.453	1.948	8.70	0.228	1.986
2.075	0.868	1.801	4.35	0.448	1.949	8.80	0.225	1.987
2.100	0.860	1.805	4.40	0.443	1.950	8.90	0.223	1.987
2.125	0.851	1.809	4.45	0.438	1.951	9.00	0.220	1.987
2.150	0.843	1.813	4.50	0.434	1.952	9.20	0.216	1.988
2.175	0.835	1.817	4.55	0.429	1.953	9.40	0.211	1.988
2.200	0.828	1.821	4.60	0.425	1.954	9.50	0.209	1.989
2.225	0.820	1.824	4.65	0.420	1.955	10.00	0.198	1.990

Amount to Add to Pitch Diameter to Obtain Outside Diameter of Bevel Gears

Number of Teeth in Pinion	Number of Teeth in Gear														
	72	71	70	69	68	67	66	65	64	63	62	61	60	59	58
12	0.33 1.97	0.33 1.97	0.34 1.97	0.34 1.97	0.35 1.97	0.35 1.97	0.36 1.97	0.36 1.97	0.37 1.97	0.37 1.96	0.38 1.96	0.39 1.96	0.39 1.96	0.40 1.96	0.41 1.96
13	0.36 1.97	0.36 1.97	0.37 1.97	0.37 1.97	0.38 1.96	0.38 1.96	0.39 1.96	0.39 1.96	0.40 1.96	0.40 1.96	0.41 1.96	0.42 1.96	0.42 1.95	0.43 1.95	0.44 1.95
14	0.38 1.96	0.39 1.96	0.39 1.96	0.40 1.96	0.40 1.96	0.41 1.96	0.42 1.96	0.42 1.96	0.43 1.95	0.43 1.95	0.44 1.95	0.45 1.95	0.45 1.95	0.46 1.95	0.47 1.94
15	0.41 1.96	0.41 1.96	0.42 1.96	0.42 1.95	0.43 1.95	0.44 1.95	0.44 1.95	0.45 1.95	0.46 1.95	0.46 1.95	0.47 1.94	0.48 1.94	0.48 1.94	0.49 1.94	0.50 1.94
16	0.43 1.95	0.44 1.95	0.45 1.95	0.45 1.95	0.46 1.95	0.46 1.95	0.47 1.94	0.48 1.94	0.48 1.94	0.49 1.94	0.50 1.94	0.51 1.93	0.52 1.93	0.52 1.93	0.53 1.93
17	0.46 1.95	0.47 1.95	0.47 1.94	0.48 1.94	0.48 1.94	0.49 1.94	0.50 1.94	0.51 1.93	0.51 1.93	0.52 1.93	0.53 1.93	0.54 1.93	0.55 1.92	0.55 1.92	0.56 1.92
18	0.48 1.94	0.49 1.94	0.50 1.94	0.50 1.94	0.51 1.93	0.52 1.93	0.53 1.93	0.53 1.93	0.54 1.93	0.55 1.92	0.56 1.92	0.57 1.92	0.57 1.92	0.58 1.91	0.59 1.91
19	0.51 1.93	0.52 1.93	0.52 1.93	0.53 1.93	0.54 1.93	0.55 1.92	0.55 1.92	0.56 1.92	0.57 1.92	0.58 1.91	0.59 1.91	0.59 1.91	0.60 1.91	0.61 1.90	0.62 1.90
20	0.54 1.93	0.54 1.93	0.55 1.92	0.56 1.92	0.56 1.92	0.57 1.92	0.58 1.91	0.59 1.91	0.60 1.91	0.61 1.91	0.61 1.90	0.62 1.90	0.63 1.90	0.64 1.89	0.65 1.89
21	0.56 1.92	0.57 1.92	0.57 1.92	0.58 1.91	0.59 1.91	0.60 1.91	0.61 1.91	0.61 1.90	0.62 1.90	0.63 1.90	0.64 1.89	0.65 1.89	0.66 1.89	0.67 1.88	0.68 1.88
22	0.58 1.91	0.59 1.91	0.60 1.91	0.61 1.91	0.62 1.90	0.62 1.90	0.63 1.90	0.64 1.89	0.65 1.89	0.66 1.89	0.67 1.88	0.68 1.88	0.69 1.88	0.70 1.87	0.71 1.87
23	0.61 1.91	0.62 1.90	0.62 1.90	0.63 1.90	0.64 1.89	0.65 1.89	0.66 1.89	0.67 1.89	0.68 1.88	0.69 1.88	0.70 1.88	0.71 1.87	0.72 1.87	0.73 1.86	0.74 1.86
24	0.63 1.90	0.64 1.89	0.65 1.89	0.66 1.89	0.67 1.89	0.67 1.88	0.68 1.88	0.69 1.88	0.70 1.87	0.71 1.87	0.72 1.87	0.73 1.86	0.74 1.86	0.75 1.85	0.76 1.85
25	0.66 1.89	0.67 1.88	0.67 1.88	0.68 1.88	0.69 1.88	0.70 1.87	0.71 1.87	0.72 1.87	0.73 1.86	0.74 1.86	0.75 1.86	0.76 1.85	0.77 1.85	0.78 1.84	0.79 1.84
26	0.68 1.88	0.69 1.88	0.70 1.87	0.71 1.87	0.71 1.87	0.72 1.86	0.73 1.86	0.74 1.86	0.75 1.85	0.76 1.85	0.77 1.84	0.78 1.84	0.80 1.84	0.81 1.83	0.82 1.82
27	0.70 1.87	0.71 1.87	0.72 1.87	0.73 1.86	0.74 1.86	0.75 1.86	0.76 1.85	0.77 1.85	0.78 1.84	0.79 1.84	0.80 1.83	0.81 1.83	0.82 1.82	0.83 1.82	0.84 1.81
28	0.72 1.86	0.73 1.86	0.74 1.86	0.75 1.85	0.76 1.85	0.77 1.85	0.78 1.84	0.79 1.84	0.80 1.83	0.81 1.83	0.82 1.82	0.83 1.82	0.85 1.81	0.86 1.81	0.87 1.80
29	0.75 1.86	0.76 1.85	0.77 1.85	0.78 1.84	0.78 1.84	0.79 1.84	0.80 1.83	0.82 1.83	0.83 1.82	0.84 1.82	0.85 1.81	0.86 1.81	0.87 1.80	0.88 1.80	0.89 1.79
30	0.77 1.85	0.78 1.84	0.79 1.84	0.80 1.83	0.81 1.83	0.82 1.83	0.83 1.82	0.84 1.82	0.85 1.81	0.86 1.81	0.87 1.80	0.88 1.79	0.89 1.79	0.91 1.78	0.92 1.78
31	0.79 1.84	0.80 1.83	0.81 1.83	0.82 1.82	0.83 1.82	0.84 1.82	0.85 1.81	0.86 1.81	0.87 1.80	0.88 1.79	0.89 1.79	0.91 1.78	0.92 1.78	0.93 1.77	0.94 1.76
32	0.81 1.83	0.82 1.82	0.83 1.82	0.84 1.81	0.85 1.81	0.86 1.80	0.87 1.80	0.88 1.79	0.89 1.79	0.91 1.78	0.92 1.78	0.93 1.77	0.94 1.76	0.95 1.76	0.97 1.75
33	0.83 1.82	0.84 1.81	0.85 1.81	0.86 1.80	0.87 1.80	0.88 1.79	0.89 1.79	0.91 1.78	0.92 1.78	0.93 1.77	0.94 1.77	0.95 1.76	0.96 1.75	0.98 1.75	0.99 1.74
34	0.85 1.81	0.86 1.80	0.87 1.80	0.88 1.79	0.89 1.79	0.91 1.78	0.92 1.78	0.93 1.77	0.94 1.77	0.95 1.76	0.96 1.75	0.97 1.75	0.99 1.74	1.00 1.73	1.01 1.73
35	0.87 1.80	0.88 1.79	0.89 1.79	0.90 1.78	0.92 1.78	0.93 1.77	0.94 1.77	0.95 1.76	0.96 1.75	0.97 1.75	0.98 1.74	1.00 1.73	1.01 1.73	1.02 1.72	1.03 1.71
36	0.89 1.79	0.90 1.78	0.91 1.78	0.93 1.77	0.94 1.77	0.95 1.76	0.96 1.76	0.97 1.75	0.98 1.74	0.99 1.74	1.00 1.73	1.02 1.72	1.03 1.71	1.04 1.71	1.05 1.70
37	0.91 1.78	0.92 1.77	0.93 1.77	0.95 1.76	0.96 1.76	0.97 1.75	0.98 1.74	0.99 1.74	1.00 1.73	1.01 1.72	1.03 1.72	1.04 1.71	1.05 1.70	1.06 1.69	1.08 1.69
38	0.93 1.77	0.94 1.76	0.95 1.76	0.97 1.75	0.98 1.75	0.99 1.74	1.00 1.73	1.01 1.73	1.02 1.72	1.03 1.71	1.05 1.71	1.06 1.70	1.07 1.69	1.08 1.68	1.10 1.67

Of the two values given, the upper is for the gear, the lower for the pinion.

Amount to Add to Pitch Diameter to Obtain Outside Diameter of Bevel Gears *(Continued)*

Number of Teeth in Pinion	Number of Teeth in Gear: 57	56	55	54	53	52	51	50	49	48	47	46	45	44	43
12	0.41 1.96	0.42 1.96	0.43 1.95	0.43 1.95	0.44 1.95	0.45 1.95	0.46 1.95	0.47 1.94	0.48 1.94	0.48 1.94	0.49 1.94	0.50 1.94	0.52 1.93	0.53 1.93	0.54 1.93
13	0.44 1.95	0.45 1.95	0.46 1.95	0.47 1.94	0.48 1.94	0.48 1.94	0.49 1.94	0.50 1.94	0.51 1.93	0.52 1.93	0.53 1.93	0.54 1.92	0.56 1.92	0.57 1.92	0.58 1.91
14	0.48 1.94	0.48 1.94	0.49 1.94	0.50 1.94	0.51 1.93	0.52 1.93	0.53 1.93	0.54 1.93	0.55 1.92	0.56 1.92	0.57 1.92	0.58 1.91	0.59 1.91	0.61 1.91	0.62 1.90
15	0.51 1.94	0.52 1.93	0.53 1.93	0.54 1.93	0.54 1.92	0.55 1.92	0.56 1.92	0.57 1.92	0.59 1.91	0.60 1.91	0.61 1.91	0.62 1.90	0.63 1.90	0.65 1.89	0.66 1.89
16	0.54 1.93	0.55 1.92	0.56 1.92	0.57 1.92	0.58 1.91	0.59 1.91	0.60 1.91	0.61 1.90	0.62 1.90	0.63 1.90	0.64 1.89	0.66 1.89	0.67 1.88	0.68 1.88	0.70 1.87
17	0.57 1.92	0.58 1.91	0.59 1.91	0.60 1.91	0.61 1.90	0.62 1.90	0.63 1.90	0.64 1.89	0.66 1.89	0.67 1.89	0.68 1.88	0.69 1.88	0.71 1.87	0.72 1.87	0.74 1.86
18	0.60 1.91	0.61 1.90	0.62 1.90	0.63 1.90	0.64 1.89	0.65 1.89	0.67 1.89	0.68 1.88	0.69 1.88	0.70 1.87	0.72 1.87	0.73 1.86	0.74 1.86	0.76 1.85	0.77 1.84
19	0.63 1.90	0.64 1.89	0.65 1.89	0.66 1.89	0.67 1.88	0.69 1.88	0.70 1.87	0.71 1.87	0.72 1.86	0.74 1.86	0.75 1.85	0.76 1.85	0.78 1.84	0.79 1.84	0.81 1.83
20	0.66 1.89	0.67 1.88	0.68 1.88	0.69 1.88	0.71 1.87	0.72 1.87	0.73 1.86	0.74 1.86	0.76 1.85	0.77 1.85	0.78 1.84	0.80 1.83	0.81 1.83	0.83 1.82	0.84 1.81
21	0.70 1.87	0.70 1.87	0.71 1.87	0.72 1.86	0.74 1.86	0.75 1.85	0.76 1.85	0.77 1.84	0.79 1.84	0.80 1.83	0.82 1.83	0.83 1.82	0.85 1.81	0.86 1.80	0.88 1.80
22	0.72 1.87	0.73 1.86	0.74 1.86	0.75 1.85	0.77 1.85	0.78 1.84	0.79 1.84	0.81 1.83	0.82 1.82	0.83 1.82	0.85 1.81	0.86 1.80	0.88 1.80	0.89 1.79	0.91 1.78
23	0.75 1.85	0.76 1.85	0.77 1.85	0.78 1.84	0.80 1.83	0.81 1.83	0.82 1.82	0.84 1.82	0.85 1.81	0.86 1.80	0.88 1.80	0.89 1.79	0.91 1.78	0.93 1.77	0.94 1.76
24	0.78 1.84	0.79 1.84	0.80 1.83	0.81 1.83	0.83 1.82	0.84 1.82	0.85 1.81	0.87 1.80	0.88 1.80	0.89 1.79	0.91 1.78	0.93 1.77	0.94 1.76	0.96 1.76	0.97 1.75
25	0.80 1.83	0.82 1.83	0.83 1.82	0.84 1.81	0.85 1.81	0.87 1.80	0.88 1.80	0.89 1.79	0.91 1.78	0.92 1.77	0.94 1.77	0.95 1.76	0.97 1.75	0.99 1.74	1.01 1.73
26	0.83 1.82	0.84 1.81	0.85 1.81	0.87 1.80	0.88 1.80	0.89 1.79	0.91 1.78	0.92 1.77	0.94 1.77	0.95 1.76	0.97 1.75	0.98 1.74	1.00 1.73	1.02 1.72	1.04 1.71
27	0.86 1.81	0.87 1.80	0.88 1.80	0.89 1.79	0.91 1.78	0.92 1.78	0.94 1.77	0.95 1.76	0.97 1.75	0.98 1.74	1.00 1.73	1.01 1.72	1.03 1.71	1.05 1.70	1.06 1.69
28	0.88 1.80	0.89 1.79	0.91 1.78	0.92 1.78	0.93 1.77	0.95 1.76	0.96 1.75	0.98 1.75	0.99 1.74	1.01 1.73	1.02 1.72	1.04 1.71	1.06 1.70	1.07 1.69	1.09 1.68
29	0.91 1.78	0.92 1.78	0.93 1.77	0.95 1.76	0.96 1.75	0.97 1.75	0.99 1.74	1.00 1.73	1.02 1.72	1.03 1.71	1.05 1.70	1.07 1.69	1.08 1.68	1.10 1.67	1.12 1.66
30	0.93 1.77	0.94 1.76	0.96 1.76	0.98 1.74	0.99 1.74	1.00 1.73	1.01 1.72	1.03 1.71	1.04 1.71	1.06 1.70	1.08 1.69	1.09 1.68	1.11 1.66	1.13 1.65	1.14 1.64
31	0.96 1.76	0.97 1.75	0.98 1.74	1.00 1.73	1.01 1.73	1.02 1.72	1.04 1.71	1.05 1.70	1.07 1.69	1.09 1.68	1.10 1.67	1.12 1.66	1.13 1.65	1.15 1.63	1.17 1.62
32	0.98 1.74	0.99 1.74	1.01 1.73	1.02 1.72	1.03 1.71	1.04 1.71	1.06 1.69	1.08 1.68	1.09 1.67	1.11 1.66	1.13 1.65	1.14 1.64	1.16 1.63	1.18 1.62	1.19 1.60
33	1.00 1.73	1.02 1.72	1.03 1.71	1.04 1.71	1.06 1.70	1.08 1.69	1.09 1.68	1.10 1.67	1.12 1.66	1.13 1.65	1.15 1.64	1.17 1.63	1.18 1.61	1.20 1.60	1.22 1.59
34	1.02 1.72	1.04 1.71	1.05 1.70	1.07 1.69	1.08 1.68	1.09 1.67	1.11 1.66	1.12 1.65	1.14 1.64	1.16 1.63	1.17 1.62	1.19 1.61	1.21 1.59	1.22 1.58	1.24 1.57
35	1.05 1.70	1.06 1.70	1.07 1.69	1.09 1.68	1.10 1.67	1.12 1.66	1.13 1.65	1.15 1.64	1.17 1.63	1.18 1.62	1.19 1.60	1.21 1.59	1.23 1.58	1.25 1.57	1.26 1.55
36	1.07 1.69	1.08 1.68	1.10 1.67	1.11 1.66	1.12 1.65	1.14 1.64	1.15 1.63	1.17 1.62	1.18 1.61	1.19 1.60	1.21 1.59	1.23 1.57	1.25 1.56	1.27 1.55	1.28 1.53
37	1.09 1.68	1.10 1.67	1.12 1.66	1.13 1.65	1.14 1.64	1.16 1.63	1.17 1.62	1.19 1.61	1.21 1.60	1.22 1.58	1.24 1.57	1.25 1.56	1.27 1.55	1.29 1.53	1.30 1.52

Of the two values given, the upper is for the gear, the lower for the pinion.

Amount to Add to Pitch Diameter to Obtain Outside Diameter of Bevel Gears (*Continued*)

Number of Teeth in Pinion	Number of Teeth in Gear: 42	41	40	39	38	37	36	35	34	33	32	31	30	29	28
12	0.55	0.56	0.58	0.59	0.61	0.63	0.63	0.65	0.67	0.68	0.70	0.72	0.74	0.76	0.79
	1.92	1.92	1.92	1.91	1.91	1.90	1.90	1.89	1.88	1.88	1.87	1.87	1.86	1.85	1.84
13	0.59	0.60	0.61	0.63	0.65	0.66	0.68	0.70	0.71	0.73	0.75	0.77	0.80	0.82	0.84
	1.91	1.91	1.90	1.90	1.89	1.89	1.88	1.87	1.87	1.86	1.85	1.84	1.83	1.82	1.81
14	0.63	0.65	0.66	0.67	0.69	0.71	0.72	0.74	0.76	0.78	0.80	0.82	0.85	0.87	0.89
	1.90	1.89	1.89	1.88	1.88	1.87	1.86	1.86	1.85	1.84	1.83	1.82	1.81	1.80	1.79
15	0.67	0.69	0.70	0.72	0.74	0.75	0.77	0.79	0.81	0.83	0.85	0.87	0.89	0.92	0.94
	1.88	1.88	1.87	1.87	1.86	1.85	1.85	1.84	1.83	1.82	1.81	1.80	1.79	1.78	1.76
16	0.71	0.73	0.74	0.76	0.77	0.79	0.81	0.83	0.85	0.88	0.89	0.91	0.94	0.97	0.99
	1.87	1.86	1.86	1.85	1.85	1.84	1.83	1.82	1.81	1.80	1.79	1.77	1.75	1.75	1.74
17	0.75	0.77	0.78	0.79	0.81	0.83	0.86	0.88	0.89	0.91	0.94	0.96	0.99	1.01	1.04
	1.85	1.85	1.84	1.83	1.83	1.82	1.81	1.80	1.79	1.77	1.76	1.75	1.74	1.73	1.71
18	0.79	0.80	0.82	0.84	0.86	0.88	0.89	0.91	0.93	0.94	0.98	1.01	1.03	1.06	1.08
	1.84	1.83	1.82	1.81	1.81	1.80	1.79	1.78	1.77	1.76	1.74	1.73	1.72	1.70	1.68
19	0.82	0.84	0.86	0.88	0.89	0.91	0.93	0.95	0.97	0.99	1.02	1.04	1.07	1.10	1.12
	1.82	1.81	1.81	1.80	1.79	1.78	1.77	1.76	1.75	1.73	1.72	1.70	1.69	1.67	1.66
20	0.86	0.88	0.89	0.91	0.93	0.95	0.97	0.99	1.01	1.04	1.06	1.08	1.11	1.14	1.16
	1.81	1.80	1.79	1.78	1.77	1.76	1.75	1.74	1.72	1.71	1.70	1.68	1.66	1.64	1.63
21	0.89	0.91	0.93	0.94	0.97	0.99	1.01	1.03	1.05	1.07	1.10	1.12	1.14	1.17	1.20
	1.79	1.78	1.77	1.76	1.75	1.74	1.73	1.72	1.70	1.69	1.67	1.65	1.64	1.62	1.60
22	0.93	0.95	0.96	0.98	1.00	1.02	1.04	1.06	1.09	1.11	1.13	1.16	1.18	1.21	1.24
	1.77	1.76	1.75	1.74	1.73	1.72	1.71	1.69	1.68	1.66	1.65	1.63	1.61	1.59	1.57
23	0.96	0.98	1.00	1.01	1.04	1.06	1.08	1.10	1.12	1.14	1.17	1.19	1.21	1.24	1.27
	1.75	1.74	1.73	1.72	1.71	1.70	1.68	1.67	1.66	1.64	1.62	1.61	1.59	1.57	1.55
24	0.99	1.01	1.03	1.05	1.07	1.08	1.11	1.13	1.15	1.17	1.20	1.23	1.25	1.28	1.30
	1.74	1.72	1.71	1.70	1.69	1.68	1.66	1.65	1.63	1.62	1.60	1.58	1.56	1.54	1.52
25	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.23	1.26	1.28	1.31	1.33
	1.72	1.71	1.70	1.68	1.67	1.65	1.64	1.63	1.61	1.59	1.58	1.56	1.54	1.52	1.49
26	1.05	1.07	1.09	1.11	1.13	1.15	1.17	1.19	1.21	1.24	1.26	1.28	1.31	1.34	1.36
	1.70	1.69	1.68	1.66	1.65	1.64	1.62	1.61	1.59	1.57	1.55	1.53	1.51	1.49	1.47
27	1.08	1.10	1.12	1.14	1.15	1.18	1.20	1.22	1.24	1.27	1.29	1.31	1.34	1.36	1.39
	1.68	1.67	1.66	1.64	1.63	1.62	1.60	1.58	1.57	1.54	1.53	1.51	1.49	1.46	1.44
28	1.11	1.13	1.14	1.16	1.19	1.21	1.23	1.25	1.27	1.29	1.32	1.34	1.36	1.39	1.41
	1.66	1.65	1.64	1.62	1.61	1.59	1.58	1.56	1.54	1.53	1.51	1.48	1.46	1.44	1.41
29	1.14	1.15	1.17	1.19	1.21	1.23	1.26	1.28	1.30	1.32	1.34	1.37	1.39	1.41	....
	1.65	1.63	1.62	1.60	1.59	1.57	1.56	1.54	1.52	1.50	1.48	1.46	1.44	1.41	....
30	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.35	1.37	1.39	1.41	....	....
	1.63	1.61	1.60	1.59	1.57	1.55	1.54	1.52	1.50	1.48	1.46	1.44	1.41	....	....
31	1.19	1.21	1.23	1.25	1.26	1.28	1.31	1.33	1.35	1.37	1.39	1.41	....	....	....
	1.61	1.59	1.58	1.57	1.55	1.53	1.51	1.50	1.48	1.46	1.44	1.41	....	....	....
32	1.21	1.23	1.25	1.27	1.29	1.31	1.33	1.35	1.37	1.39	1.41	....	....	....	....
	1.59	1.58	1.56	1.54	1.53	1.51	1.50	1.48	1.46	1.44	1.41	....	....	....	....
33	1.24	1.25	1.27	1.29	1.31	1.33	1.35	1.37	1.39	1.41	....	....	....	....	....
	1.57	1.56	1.54	1.53	1.51	1.49	1.48	1.45	1.43	1.41	....	....	....	....	....
34	1.26	1.28	1.30	1.31	1.33	1.35	1.37	1.39	1.41	....	....	....	....	....	....
	1.55	1.54	1.52	1.51	1.49	1.48	1.45	1.43	1.41	....	....	....	....	....	....
35	1.28	1.30	1.32	1.34	1.35	1.38	1.39	1.41	....	....	....	....	....	....	....
	1.54	1.52	1.50	1.49	1.48	1.45	1.43	1.41	....	....	....	....	....	....	....
36	1.30	1.32	1.34	1.36	1.38	1.40	1.41	....	....	....	....	....	....	....	....
	1.52	1.50	1.49	1.47	1.45	1.43	1.41	....	....	....	....	....	....	....	....
37	1.32	1.34	1.36	1.38	1.40	1.41	....	....	....	....	....	....	....	....	....
	1.50	1.49	1.47	1.45	1.43	1.41	....	....	....	....	....	....	....	....	....

Of the two values given, the upper is for the gear, the lower for the pinion.

Amount to Add to Pitch Diameter to Obtain Outside Diameter of Bevel Gears (*Continued*)

Number of Teeth in Pinion	Number of Teeth in Gear														
	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13
12	0.81 1.83	0.84 1.82	0.87 1.80	0.89 1.79	0.93 1.77	0.96 1.76	0.99 1.74	1.03 1.71	1.07 1.69	1.11 1.66	1.15 1.63	1.20 1.60	1.25 1.56	1.30 1.52	1.36 1.47
13	0.87 1.80	0.89 1.79	0.92 1.77	0.95 1.76	0.98 1.74	1.02 1.72	1.05 1.70	1.09 1.68	1.13 1.65	1.17 1.62	1.21 1.59	1.26 1.55	1.31 1.51	1.36 1.47	1.41 1.41
14	0.92 1.78	0.95 1.76	0.98 1.75	1.01 1.73	1.04 1.71	1.07 1.69	1.11 1.66	1.15 1.64	1.19 1.61	1.23 1.58	1.27 1.54	1.32 1.50	1.36 1.46	1.41 1.41	....
15	0.97 1.75	1.00 1.73	1.03 1.71	1.06 1.70	1.09 1.68	1.13 1.65	1.16 1.63	1.20 1.60	1.24 1.57	1.28 1.54	1.32 1.50	1.37 1.46	1.41 1.41	....	....
16	1.02 1.72	1.05 1.70	1.08 1.68	1.11 1.66	1.14 1.64	1.18 1.62	1.21 1.59	1.25 1.56	1.29 1.53	1.33 1.49	1.37 1.46	1.41 1.41	....	....	....
17	1.07 1.69	1.09 1.67	1.12 1.65	1.16 1.63	1.19 1.61	1.22 1.58	1.26 1.55	1.30 1.52	1.33 1.49	1.37 1.45	1.41 1.41	....	....	....	....
18	1.11 1.66	1.14 1.64	1.17 1.62	1.20 1.60	1.23 1.57	1.27 1.55	1.30 1.52	1.34 1.49	1.38 1.45	1.41 1.41	....	....	....	....	....
19	1.15 1.64	1.18 1.61	1.21 1.59	1.24 1.57	1.27 1.54	1.31 1.51	1.34 1.48	1.38 1.45	1.41 1.41	....	....	....	....	....	....
20	1.19 1.61	1.22 1.59	1.25 1.56	1.28 1.54	1.31 1.51	1.35 1.48	1.38 1.45	1.41 1.41	....	....	....	....	....	....	....
21	1.23 1.58	1.26 1.56	1.29 1.53	1.32 1.50	1.35 1.48	1.38 1.45	1.41 1.41	....	....	....	....	....	....	....	....
22	1.26 1.55	1.29 1.53	1.32 1.50	1.35 1.47	1.38 1.45	1.41 1.41	....	....	....	....	....	....	....	....	....
23	1.30 1.52	1.33 1.50	1.35 1.47	1.38 1.44	1.41 1.41	....	....	....	....	....	....	....	....	....	....
24	1.33 1.49	1.36 1.47	1.39 1.44	1.41 1.41	....	....	....	....	....	....	....	....	....	....	....
25	1.36 1.47	1.39 1.44	1.41 1.41	....	....	....	....	....	....	....	....	....	....	....	....
26	1.39 1.44	1.41 1.41	....	....	....	....	....	....	....	....	....	....	....	....	....
27	1.41 1.41	....	....	....	....	....	....	....	....	....	....	....	....	....	....

Of the two values given, the upper is for the gear, the lower for the pinion.

Edge or Pitch Cone Angles, Face Angles and Cutting Angles of Bevel Gears. — These tables have been computed by the Brown & Sharpe Mfg. Co. for convenience in calculating the data for bevel gears with axes at right angles. They do not apply to bevel gears with axes at any other angle. To use the tables, the number of teeth in the gear and pinion must be known. Locate the number of teeth in the gear on the horizontal line at the top of the table and the number of teeth in the pinion in the vertical column on the left-hand side. At the intersection of the lines or columns for these numbers of teeth, two angles are given. The angle applying to the gear is always placed above the angle applying to the pinion.

The tables "Face Angles of Bevel Gears" are arranged in the same manner as the tables for "Edge or Pitch Cone Angles." Of the two angles that are given for each combination of number of teeth, the upper is for the gear, and the lower for the pinion. The cutting angle for a gear or pinion, when cut by rotary milling cutters, is equal to the angle of face of its mate.

As an example of the use of the tables, find the cutting angles for a pair of bevel gears with 15 and 60 teeth. From the table on page 673, these angles are found to be 12 deg. 11 min., and 74 deg. 7 min., if the gears are cut with rotary milling cutters.

Edge or Pitch Cone Angles of Bevel Gears

Number of Teeth in Pinion	Number of Teeth in Gear: 72	71	70	69	68	67	66	65	64	63	62	61
12	80°33′ 9°27′	80°25′ 9°35′	80°16′ 9°44′	80°8′ 9°52′	79°59′ 10°1′	79°51′ 10°9′	79°42′ 10°18′	79°32′ 10°28′	79°23′ 10°37′	79°13′ 10°47′	79°3′ 10°57′	78°52′ 11°8′
13	79°46′ 10°14′	79°37′ 10°23′	79°29′ 10°31′	79°20′ 10°40′	79°11′ 10°49′	79°1′ 10°59′	78°51′ 11°9′	78°41′ 11°19′	78°31′ 11°29′	78°20′ 11°40′	78°9′ 11°51′	77°58′ 12°2′
14	79°0′ 11°0′	78°51′ 11°9′	78°41′ 11°19′	78°32′ 11°28′	78°22′ 11°38′	78°11′ 11°49′	78°1′ 11°59′	77°51′ 12°9′	77°40′ 12°20′	77°28′ 12°32′	77°17′ 12°43′	77°5′ 12°55′
15	78°14′ 11°46′	78°4′ 11°56′	77°54′ 12°6′	77°44′ 12°16′	77°34′ 12°26′	77°23′ 12°37′	77°12′ 12°48′	77°0′ 13°0′	76°48′ 13°12′	76°36′ 13°24′	76°24′ 13°36′	76°11′ 13°49′
16	77°28′ 12°32′	77°18′ 12°42′	77°7′ 12°53′	76°57′ 13°3′	76°45′ 13°15′	76°34′ 13°26′	76°22′ 13°38′	76°10′ 13°50′	75°58′ 14°2′	75°45′ 14°15′	75°32′ 14°28′	75°18′ 14°42′
17	76°43′ 13°17′	76°32′ 13°28′	76°21′ 13°39′	76°10′ 13°50′	75°58′ 14°2′	75°45′ 14°14′	75°33′ 14°27′	75°21′ 14°39′	75°8′ 14°52′	74°54′ 15°6′	74°40′ 15°20′	74°25′ 15°35′
18	75°58′ 14°2′	75°46′ 14°14′	75°35′ 14°25′	75°23′ 14°37′	75°10′ 14°50′	74°58′ 15°2′	74°45′ 15°15′	74°31′ 15°29′	74°17′ 15°43′	74°3′ 15°57′	73°49′ 16°11′	73°33′ 16°27′
19	75°13′ 14°47′	75°1′ 14°59′	74°49′ 15°11′	74°36′ 15°24′	74°23′ 15°37′	74°10′ 15°50′	73°56′ 16°4′	73°42′ 16°18′	73°28′ 16°32′	73°13′ 16°47′	72°58′ 17°2′	72°42′ 17°18′
20	74°29′ 15°31′	74°16′ 15°44′	74°3′ 15°57′	73°50′ 16°10′	73°37′ 16°23′	73°23′ 16°42′	73°9′ 16°51′	72°54′ 17°6′	72°39′ 17°21′	72°23′ 17°37′	72°7′ 17°53′	71°51′ 18°9′
21	73°45′ 16°15′	73°32′ 16°28′	73°18′ 16°42′	73°4′ 16°56′	72°50′ 17°10′	72°36′ 17°24′	72°21′ 17°39′	72°6′ 17°54′	71°50′ 18°10′	71°34′ 18°26′	71°17′ 18°43′	71°0′ 19°0′
22	73°1′ 16°59′	72°47′ 17°13′	72°33′ 17°27′	72°19′ 17°41′	72°4′ 17°56′	71°49′ 18°11′	71°34′ 18°26′	71°18′ 18°42′	71°2′ 18°58′	70°45′ 19°15′	70°28′ 19°34′	70°10′ 19°50′
23	72°17′ 17°43′	72°3′ 17°57′	71°49′ 18°11′	71°34′ 18°26′	71°19′ 18°41′	71°3′ 18°57′	70°47′ 19°13′	70°30′ 19°30′	70°14′ 19°46′	69°57′ 20°3′	69°39′ 20°21′	69°20′ 20°40′
24	71°34′ 18°26′	71°19′ 18°41′	71°5′ 18°55′	70°49′ 19°11′	70°34′ 19°26′	70°17′ 19°43′	70°1′ 19°59′	69°44′ 20°16′	69°26′ 20°34′	69°9′ 20°51′	68°50′ 21°10′	68°31′ 21°29′
25	70°51′ 19°9′	70°36′ 19°24′	70°21′ 19°39′	70°5′ 19°55′	69°49′ 20°11′	69°32′ 20°28′	69°15′ 20°45′	68°57′ 21°3′	68°40′ 21°20′	68°21′ 21°39′	68°3′ 21°57′	67°43′ 22°17′
26	70°9′ 19°51′	69°53′ 20°7′	69°37′ 20°23′	69°21′ 20°39′	69°4′ 20°56′	68°48′ 21°12′	68°30′ 21°30′	68°12′ 21°48′	67°54′ 22°6′	67°34′ 22°26′	67°15′ 22°45′	66°55′ 23°5′
27	69°27′ 20°33′	69°10′ 20°50′	68°54′ 21°6′	68°38′ 21°22′	68°20′ 21°40′	68°3′ 21°57′	67°45′ 22°15′	67°26′ 22°34′	67°8′ 22°52′	66°48′ 23°12′	66°28′ 23°32′	66°7′ 23°53′
28	68°45′ 21°15′	68°29′ 21°31′	68°12′ 21°48′	67°55′ 22°5′	67°37′ 22°23′	67°19′ 22°41′	67°1′ 22°59′	66°42′ 23°18′	66°22′ 23°38′	66°2′ 23°58′	65°42′ 24°18′	65°21′ 24°39′
29	68°4′ 21°56′	67°47′ 22°13′	67°30′ 22°30′	67°12′ 22°48′	66°54′ 23°6′	66°36′ 23°24′	66°17′ 23°43′	65°57′ 24°3′	65°37′ 24°23′	65°16′ 24°44′	64°55′ 25°5′	64°34′ 25°26′
30	67°23′ 22°37′	67°6′ 22°54′	66°48′ 23°12′	66°30′ 23°30′	66°12′ 23°48′	65°52′ 24°8′	65°33′ 24°27′	65°14′ 24°46′	64°53′ 25°7′	64°32′ 25°28′	64°10′ 25°50′	63°49′ 26°11′
31	66°42′ 23°18′	66°25′ 23°35′	66°6′ 23°54′	65°48′ 24°12′	65°29′ 24°31′	65°10′ 24°50′	64°50′ 25°10′	64°30′ 25°30′	64°9′ 25°51′	63°48′ 26°12′	63°26′ 26°34′	63°3′ 26°57′
32	66°2′ 23°58′	65°44′ 24°16′	65°26′ 24°34′	65°7′ 24°53′	64°48′ 25°12′	64°28′ 25°32′	64°8′ 25°52′	63°47′ 26°13′	63°26′ 26°34′	63°4′ 26°56′	62°42′ 27°18′	62°19′ 27°41′
33	65°23′ 24°37′	65°4′ 24°56′	64°45′ 25°15′	64°26′ 25°34′	64°7′ 25°53′	63°47′ 26°13′	63°26′ 26°34′	63°5′ 26°55′	62°43′ 27°17′	62°21′ 27°39′	61°58′ 28°2′	61°35′ 28°25′
34	64°43′ 25°17′	64°25′ 25°35′	64°5′ 25°55′	63°46′ 26°14′	63°26′ 26°34′	63°5′ 26°55′	62°45′ 27°15′	62°23′ 27°37′	62°1′ 27°59′	61°38′ 28°22′	61°15′ 28°45′	60°52′ 29°8′
35	64°5′ 25°55′	63°45′ 26°15′	63°26′ 26°34′	63°6′ 26°54′	62°46′ 27°14′	62°25′ 27°35′	62°4′ 27°56′	61°42′ 28°18′	61°19′ 28°41′	60°57′ 29°3′	60°33′ 29°27′	60°9′ 29°51′
36	63°26′ 26°34′	63°7′ 26°53′	62°47′ 27°13′	62°27′ 27°33′	62°6′ 27°54′	61°45′ 28°15′	61°23′ 28°37′	61°1′ 28°59′	60°38′ 29°22′	60°15′ 29°45′	59°51′ 30°9′	59°27′ 30°33′
37	62°48′ 27°12′	62°28′ 27°32′	62°8′ 27°52′	61°48′ 28°12′	61°27′ 28°33′	61°5′ 28°55′	60°44′ 29°16′	60°21′ 29°39′	59°58′ 30°2′	59°35′ 30°25′	59°10′ 30°50′	58°46′ 31°14′
38	62°11′ 27°49′	61°51′ 28°9′	61°30′ 28°30′	61°9′ 28°51′	60°48′ 29°12′	60°26′ 29°34′	60°4′ 29°56′	59°41′ 30°19′	59°18′ 30°42′	58°54′ 31°6′	58°30′ 31°30′	58°5′ 31°55′

Of the two angles given, the upper is for the gear, the lower for the pinion.

Edge or Pitch Cone Angles of Bevel Gears (*Continued*)

Number of Teeth in Pinion	Number of Teeth in Gear: 60	59	58	57	56	55	54	53	52	51	50	49
12	78°41′ 11°19′	78°30′ 11°30′	78°19′ 11°41′	78°7′ 11°53′	77°54′ 12°6′	77°42′ 12°18′	77°28′ 12°32′	77°15′ 12°45′	77°0′ 13°0′	76°46′ 13°14′	76°30′ 13°30′	76°14′ 13°46′
13	77°46′ 12°14′	77°34′ 12°26′	77°22′ 12°38′	77°9′ 12°51′	76°56′ 13°4′	76°42′ 13°18′	76°28′ 13°32′	76°13′ 13°47′	75°58′ 14°2′	75°42′ 14°18′	75°26′ 14°34′	75°8′ 14°52′
14	76°52′ 13°8′	76°39′ 13°21′	76°26′ 13°34′	76°12′ 13°48′	75°58′ 14°2′	75°43′ 14°17′	75°28′ 14°32′	75°12′ 14°48′	74°56′ 15°4′	74°39′ 15°21′	74°21′ 15°39′	74°3′ 15°57′
15	75°58′ 14°2′	75°44′ 14°16′	75°30′ 14°30′	75°15′ 14°45′	75°0′ 15°0′	74°44′ 15°16′	74°29′ 15°31′	74°12′ 15°48′	73°55′ 16°5′	73°37′ 16°23′	73°18′ 16°42′	72°59′ 17°1′
16	75°4′ 14°56′	74°49′ 15°11′	74°35′ 15°25′	74°19′ 15°41′	74°3′ 15°57′	73°47′ 16°13′	73°30′ 16°30′	73°12′ 16°48′	72°54′ 17°6′	72°35′ 17°25′	72°15′ 17°45′	71°55′ 18°5′
17	74°11′ 15°49′	73°56′ 16°4′	73°40′ 16°20′	73°24′ 16°36′	73°7′ 16°53′	72°49′ 17°11′	72°31′ 17°29′	72°13′ 17°47′	71°54′ 18°6′	71°34′ 18°26′	71°13′ 18°47′	70°52′ 19°8′
18	73°18′ 16°42′	73°2′ 16°58′	72°45′ 17°15′	72°29′ 17°31′	72°11′ 17°49′	71°53′ 18°7′	71°34′ 18°26′	71°15′ 18°45′	70°54′ 19°6′	70°33′ 19°27′	70°12′ 19°48′	69°50′ 20°10′
19	72°20′ 17°34′	72°9′ 17°51′	71°52′ 18°8′	71°34′ 18°26′	71°15′ 18°45′	70°57′ 19°1′	70°37′ 19°23′	70°17′ 19°43′	69°56′ 20°4′	69°34′ 20°26′	69°12′ 20°48′	68°48′ 21°12′
20	71°34′ 18°26′	71°16′ 18°44′	70°59′ 19°1′	70°40′ 19°20′	70°21′ 19°39′	70°1′ 19°59′	69°41′ 20°19′	69°19′ 20°41′	68°57′ 21°3′	68°35′ 21°25′	68°12′ 21°48′	67°48′ 22°12′
21	70°43′ 19°17′	70°24′ 19°36′	70°6′ 19°54′	69°46′ 20°14′	69°26′ 20°34′	69°6′ 20°54′	68°45′ 21°15′	68°23′ 21°37′	68°0′ 22°0′	67°37′ 22°23′	67°13′ 22°47′	66°48′ 23°12′
22	69°52′ 20°8′	69°33′ 20°27′	69°13′ 20°47′	68°54′ 21°6′	68°33′ 21°27′	68°12′ 21°48′	67°50′ 22°10′	67°27′ 22°33′	67°4′ 22°56′	66°40′ 23°20′	66°15′ 23°45′	65°49′ 24°11′
23	69°2′ 20°58′	68°42′ 21°18′	68°22′ 21°38′	68°2′ 21°58′	67°41′ 22°19′	67°18′ 22°42′	66°55′ 23°5′	66°32′ 23°28′	66°8′ 23°52′	65°44′ 24°16′	65°18′ 24°42′	64°51′ 25°9′
24	68°12′ 21°48′	67°52′ 22°8′	67°31′ 22°29′	67°10′ 22°50′	66°48′ 23°12′	66°26′ 23°34′	66°2′ 23°58′	65°38′ 24°22′	65°14′ 24°46′	64°48′ 25°12′	64°22′ 25°38′	63°54′ 26°6′
25	67°23′ 22°37′	67°2′ 22°58′	66°41′ 23°19′	66°19′ 23°41′	65°57′ 24°3′	65°33′ 24°27′	65°9′ 24°51′	64°45′ 25°15′	64°20′ 25°40′	63°53′ 26°7′	63°26′ 26°34′	62°58′ 27°2′
26	66°34′ 23°26′	66°13′ 23°47′	65°51′ 24°9′	65°29′ 24°31′	65°6′ 24°54′	64°42′ 25°18′	64°18′ 25°42′	63°52′ 26°8′	63°26′ 26°34′	62°59′ 27°1′	62°31′ 27°29′	62°3′ 27°57′
27	65°46′ 24°14′	65°25′ 24°35′	65°2′ 24°58′	64°39′ 25°21′	64°16′ 25°44′	63°51′ 26°9′	63°26′ 26°34′	63°0′ 27°0′	62°34′ 27°26′	62°6′ 27°54′	61°38′ 28°22′	61°8′ 28°52′
28	64°59′ 25°1′	64°37′ 25°23′	64°14′ 25°46′	63°50′ 26°10′	63°26′ 26°34′	63°1′ 26°59′	62°36′ 27°24′	62°9′ 27°51′	61°42′ 28°18′	61°14′ 28°46′	60°45′ 29°15′	60°15′ 29°45′
29	64°12′ 25°48′	63°50′ 26°10′	63°26′ 26°34′	63°2′ 26°58′	62°37′ 27°23′	62°12′ 27°48′	61°45′ 28°15′	61°19′ 28°41′	60°51′ 29°9′	60°23′ 29°37′	59°53′ 30°7′	59°23′ 30°37′
30	63°26′ 26°34′	63°3′ 26°57′	62°39′ 27°21′	62°14′ 27°46′	61°49′ 28°11′	61°23′ 28°37′	60°57′ 29°3′	60°29′ 29°31′	60°1′ 29°59′	59°32′ 30°28′	59°2′ 30°58′	58°32′ 31°28′
31	62°40′ 27°20′	62°18′ 27°42′	61°53′ 28°7′	61°28′ 28°32′	61°2′ 28°58′	60°36′ 29°24′	60°6′ 29°54′	59°41′ 30°19′	59°12′ 30°48′	58°42′ 31°18′	58°12′ 31°48′	57°41′ 32°19′
32	61°56′ 28°4′	61°32′ 28°28′	61°7′ 28°53′	60°41′ 29°19′	60°15′ 29°45′	59°48′ 30°12′	59°21′ 30°39′	58°52′ 31°8′	58°34′ 31°26	57°54′ 32°6′	57°23′ 32°37′	56°52′ 33°8′
33	61°11′ 28°49′	60°47′ 29°13′	60°21′ 29°39′	59°56′ 30°4′	59°29′ 30°31′	59°2′ 30°58′	58°34′ 31°26′	58°5′ 31°55′	57°36′ 32°24′	57°6′ 32°54′	56°34′ 33°26′	56°2′ 33°58′
34	60°28′ 29°32′	60°3′ 29°57′	59°37′ 30°23′	59°11′ 30°49′	58°44′ 31°16′	58°16′ 31°44′	57°48′ 32°12′	57°19′ 32°41′	56°49′ 33°11′	56°19′ 33°41′	55°47′ 34°13′	55°15′ 34°45′
35	59°45′ 30°15′	59°19′ 30°41′	58°53′ 31°7′	58°27′ 31°33′	58°0′ 32°0′	57°32′ 32°28′	57°3′ 32°57′	56°33′ 33°27′	56°3′ 33°57′	55°32′ 34°28′	55°0′ 35°0′	54°28′ 35°32′
36	59°2′ 30°58′	58°37′ 31°23′	58°10′ 31°50′	57°43′ 32°17′	57°16′ 32°44′	56°48′ 33°12′	56°19′ 33°41′	55°49′ 34°11′	55°18′ 34°42′	54°47′ 35°13′	54°15′ 35°45′	53°42′ 36°18′
37	58°20′ 31°40′	57°54′ 32°6′	57°28′ 32°32′	57°1′ 32°59′	56°32′ 33°28′	56°4′ 33°56′	55°35′ 34°25′	55°5′ 34°55′	54°34′ 35°26′	54°2′ 35°58′	53°30′ 36°30′	52°56′ 37°4′
38	57°39′ 32°21′	57°13′ 32°47′	56°46′ 33°14′	56°19′ 33°41′	55°51′ 34°9′	55°21′ 34°39′	54°52′ 35°8′	54°23′ 35°37′	53°51′ 36°9′	53°18′ 36°42′	52°46′ 37°14′	52°12′ 37°48′

Of the two angles given, the upper is for the gear, the lower for the pinion.

Edge or Pitch Cone Angles of Bevel Gears (*Continued*)

Number of Teeth in Pinion	Number of Teeth in Gear 48	47	46	45	44	43	42	41	40	39	38	37
12	75°58′ 14°2′	75°41′ 14°19′	75°23′ 14°37′	75°4′ 14°56′	74°45′ 15°15′	74°25′ 15°35′	74°3′ 15°57′	73°41′ 16°19′	73°18′ 16°42′	72°54′ 17°6′	72°28′ 17°32′	72°2′ 17°58′
13	74°51′ 15°9′	74°32′ 15°28′	74°13′ 15°47′	73°53′ 16°7′	73°32′ 16°28′	73°11′ 16°49′	72°48′ 17°12′	72°25′ 17°35′	71°59′ 18°1′	71°34′ 18°26′	71°7′ 18°53′	70°39′ 19°21′
14	73°44′ 16°16′	73°25′ 16°35′	73°4′ 16°56′	72°43′ 17°17′	72°21′ 17°39′	71°58′ 18°2′	71°34′ 18°26′	71°9′ 18°51′	70°43′ 19°17′	70°15′ 19°45′	69°46′ 20°14′	69°16′ 20°44′
15	72°39′ 17°21′	72°18′ 17°42′	71°56′ 18°4′	71°34′ 18°26′	71°10′ 18°50′	70°46′ 19°14′	70°21′ 19°39′	69°54′ 20°6′	69°26′ 20°34′	68°58′ 21°2′	68°28′ 21°32′	67°56′ 22°4′
16	71°34′ 18°26′	71°12′ 18°48′	70°49′ 19°11′	70°26′ 19°34′	70°1′ 19°59′	69°35′ 20°25′	69°9′ 20°51′	68°41′ 21°19′	68°12′ 21°48′	67°42′ 22°18′	67°10′ 22°50′	66°37′ 23°23′
17	70°30′ 19°30′	70°7′ 19°53′	69°43′ 20°17′	69°17′ 20°43′	68°52′ 21°8′	68°26′ 21°34′	67°58′ 22°2′	67°29′ 22°31′	66°58′ 23°2′	66°27′ 23°33′	65°54′ 24°6′	65°19′ 24°41′
18	69°26′ 20°34′	69°3′ 20°57′	68°38′ 21°22′	68°12′ 21°48′	67°45′ 22°15′	67°17′ 22°43′	66°48′ 23°12′	66°18′ 23°42′	65°46′ 24°14′	65°14′ 24°46′	64°39′ 25°21′	64°4′ 25°56′
19	68°25′ 21°35′	67°59′ 22°1′	67°34′ 22°26′	67°6′ 22°54′	66°38′ 23°22′	66°10′ 23°50′	65°39′ 24°21′	65°8′ 24°52′	64°36′ 25°24′	64°2′ 25°58′	63°26′ 26°34′	62°49′ 27°11′
20	67°23′ 22°37′	66°57′ 23°3′	66°30′ 23°30′	66°2′ 23°58′	65°33′ 24°27′	65°3′ 24°57′	64°32′ 25°28′	64°0′ 26°0′	63°26′ 26°34′	62°51′ 27°9′	62°14′ 27°46′	61°37′ 28°23′
21	66°22′ 23°38′	65°55′ 24°5′	65°28′ 24°32′	64°59′ 25°1′	64°29′ 25°31′	63°58′ 26°2′	63°26′ 26°34′	62°53′ 27°7′	62°18′ 27°42′	61°42′ 28°18′	61°4′ 28°56′	60°25′ 29°35′
22	65°23′ 24°37′	64°55′ 25°5′	64°26′ 25°34′	63°57′ 26°3′	63°26′ 26°34′	62°54′ 27°6′	62°21′ 27°39′	61°47′ 28°13′	61°11′ 28°49′	60°34′ 29°26′	59°56′ 30°4′	59°15′ 30°45′
23	64°24′ 25°36′	63°55′ 26°5′	63°26′ 26°34′	62°56′ 27°4′	62°24′ 27°36′	61°52′ 28°8′	61°18′ 28°42′	60°42′ 29°18′	60°6′ 29°54′	59°28′ 30°32′	58°49′ 31°11′	58°8′ 31°52′
24	63°26′ 26°34′	62°57′ 27°3′	62°27′ 27°33′	61°56′ 28°4′	61°23′ 28°37′	60°50′ 29°10′	60°15′ 29°45′	59°39′ 30°21′	59°2′ 30°58′	58°23′ 31°37′	57°44′ 32°16′	57°2′ 32°58′
25	62°29′ 27°31′	61°59′ 28°1′	61°29′ 28°31′	60°57′ 29°3′	60°24′ 29°36′	59°50′ 30°10′	59°14′ 30°46′	58°38′ 31°22′	58°0′ 32°0′	57°20′ 32°40′	56°40′ 33°20′	55°57′ 34°3′
26	61°33′ 28°27′	61°3′ 28°57′	60°31′ 29°29′	59°59′ 30°1′	59°25′ 30°35′	58°50′ 31°10′	58°14′ 31°46′	57°37′ 32°23′	56°58′ 33°2′	56°19′ 33°41′	55°37′ 34°23′	54°54′ 35°6′
27	60°38′ 29°22′	60°7′ 29°53′	59°35′ 30°25′	59°2′ 30°58′	58°28′ 31°32′	57°53′ 32°7′	57°16′ 32°44′	56°38′ 33°22′	55°59′ 34°1′	55°18′ 34°42′	54°36′ 35°24′	53°53′ 36°7′
28	59°45′ 30°15′	59°13′ 30°47′	58°40′ 31°20′	58°7′ 31°53′	57°32′ 32°28′	56°56′ 33°4′	56°19′ 33°41′	55°40′ 34°20′	55°0′ 35°0′	54°19′ 35°41′	53°37′ 36°23′	52°53′ 37°7′
29	58°52′ 31°8′	58°19′ 31°41′	57°46′ 32°14′	57°12′ 32°48′	56°37′ 33°23′	56°0′ 34°0′	55°23′ 34°37′	54°44′ 35°16′	54°3′ 35°57′	53°22′ 36°38′	52°39′ 37°21′	51°55′ 38°5′
30	58°0′ 32°0′	57°27′ 32°33′	56°53′ 33°7′	56°19′ 33°41′	55°43′ 34°17′	55°5′ 34°55′	54°28′ 35°32′	53°48′ 36°12′	53°7′ 36°53′	52°26′ 37°34′	51°42′ 38°18′	50°58′ 39°2′
31	57°8′ 32°52′	56°36′ 33°24′	56°1′ 33°59′	55°26′ 34°34′	54°50′ 35°10′	54°12′ 35°48′	53°34′ 36°26′	52°54′ 37°6′	52°13′ 37°47′	51°31′ 38°29′	50°48′ 39°12′	50°2′ 39°58′
32	56°19′ 33°41′	55°45′ 34°15′	55°11′ 34°49′	54°35′ 35°25′	53°58′ 36°2′	53°21′ 36°39′	52°42′ 37°18′	52°2′ 37°58′	51°20′ 38°40′	50°38′ 39°22′	49°54′ 40°6′	49°9′ 40°51′
33	55°30′ 34°30′	54°56′ 35°4′	54°21′ 35°39′	53°45′ 36°15′	53°8′ 36°52′	52°29′ 37°31′	51°50′ 38°9′	51°10′ 38°50′	50°29′ 39°31′	49°46′ 40°14′	49°2′ 40°58′	48°16′ 41°44′
34	54°41′ 35°19′	54°7′ 35°53′	53°32′ 36°28′	52°52′ 37°8′	52°18′ 37°42′	51°40′ 38°20′	51°0′ 39°0′	50°20′ 39°40′	49°38′ 40°22′	48°55′ 41°5′	48°11′ 41°49′	47°25′ 42°35′
35	53°54′ 36°6′	53°20′ 36°40′	52°44′ 37°16′	52°8′ 37°52′	51°30′ 38°30′	50°51′ 39°9′	50°12′ 39°48′	49°31′ 40°29′	48°48′ 41°12′	48°5′ 41°55′	47°21′ 42°39′	46°35′ 43°25′
36	53°8′ 36°52′	52°33′ 37°27′	51°57′ 38°3′	51°20′ 38°40′	50°43′ 39°17′	50°4′ 39°56′	49°24′ 40°36′	48°43′ 41°17′	48°0′ 42°0′	47°17′ 42°43′	46°33′ 43°27′	45°47′ 44°13′
37	52°23′ 37°37′	51°47′ 38°13′	51°12′ 38°48′	50°35′ 39°25′	49°56′ 40°4′	49°17′ 40°43′	48°37′ 41°23′	47°56′ 42°4′	47°14′ 42°46′	46°30′ 43°30′	45°46′ 44°14′	45°
38	51°38′ 38°22′	51°3′ 38°57′	50°27′ 39°33′	49°49′ 40°11′	49°11′ 40°49′	48°32′ 41°28′	47°52′ 42°8′	47°10′ 42°50′	46°28′ 43°32′	45°45′ 44°15′	45°	......

Of the two angles given, the upper is for the gear, the lower for the pinion.

Edge or Pitch Cone Angles of Bevel Gears (*Continued*)

Number of Teeth in Pinion	Number of Teeth in Gear: 36	35	34	33	32	31	30	29	28	27	26	25
12	71°34′ 18°26′	71°5′ 18°55′	70°34′ 19°26′	70°1′ 19°59′	69°26′ 20°34′	68°50′ 21°10′	68°12′ 21°48′	67°31′ 22°29′	66°48′ 23°12′	66°2′ 23°58′	65°14′ 24°46′	64°22′ 25°38′
13	70°9′ 19°51′	69°37′ 20°23′	69°5′ 20°55′	68°30′ 21°30′	67°53′ 22°7′	67°15′ 22°45′	66°34′ 23°26′	65°51′ 24°9′	65°6′ 24°54′	64°17′ 25°43′	63°26′ 26°34′	62°31′ 27°29′
14	68°45′ 21°15′	68°12′ 21°48′	67°37′ 22°23′	67°0′ 23°0′	66°23′ 23°37′	65°42′ 24°18′	64°59′ 25°1′	64°14′ 25°46′	63°26′ 26°34′	62°36′ 27°24′	61°42′ 28°18′	60°45′ 29°15′
15	67°23′ 22°37′	66°48′ 23°12′	66°12′ 23°48′	65°33′ 24°27′	64°53′ 25°7′	64°10′ 25°50′	63°26′ 26°34′	62°39′ 27°21′	61°49′ 28°11′	60°57′ 29°3′	60°1′ 29°59′	59°2′ 30°58′
16	66°2′ 23°58′	65°26′ 24°34′	64°48′ 25°12′	64°8′ 25°52′	63°26′ 26°34′	62°42′ 27°18′	61°56′ 28°4′	61°7′ 28°53′	60°15′ 29°45′	59°21′ 30°39′	58°23′ 31°37′	57°23′ 32°37′
17	64°43′ 25°17′	64°6′ 25°54′	63°26′ 26°34′	62°45′ 27°15′	62°1′ 27°59′	61°15′ 28°45′	60°28′ 29°32′	59°37′ 30°23′	58°44′ 31°16′	57°48′ 32°12′	56°49′ 33°11′	55°47′ 34°13′
18	63°26′ 26°34′	62°47′ 27°13′	62°6′ 27°54′	61°23′ 28°37′	60°38′ 29°22′	59°51′ 30°9′	59°2′ 30°58′	58°10′ 31°50′	57°16′ 32°44′	56°19′ 33°41′	55°18′ 34°42′	54°15′ 35°45′
19	62°10′ 27°50′	61°30′ 28°30′	60°48′ 29°12′	60°4′ 29°56′	59°18′ 30°42′	58°30′ 31°30′	57°39′ 32°21′	56°46′ 33°14′	55°51′ 34°9′	54°52′ 35°8′	53°51′ 36°9′	52°46′ 37°14′
20	60°57′ 29°3′	60°15′ 29°45′	59°32′ 30°28′	58°47′ 31°13′	58°0′ 32°0′	57°10′ 32°50′	56°19′ 33°41′	55°24′ 34°36′	54°28′ 35°32′	53°28′ 36°32′	52°26′ 37°34′	51°20′ 38°40′
21	59°45′ 30°15′	59°2′ 30°58′	58°18′ 31°42′	57°32′ 32°28′	56°43′ 33°17′	55°53′ 34°7′	55°0′ 35°0′	54°5′ 35°55′	53°7′ 36°53′	52°8′ 37°52′	51°4′ 38°56′	49°58′ 40°2′
22	58°34′ 31°26′	57°51′ 32°9′	57°6′ 32°54′	56°19′ 33°41′	55°29′ 34°31′	54°38′ 35°22′	53°45′ 36°15′	52°49′ 37°11′	51°50′ 38°10′	50°49′ 39°11′	49°46′ 40°14′	48°39′ 41°21′
23	57°25′ 32°35′	56°41′ 33°19′	55°55′ 34°5′	55°7′ 34°53′	54°18′ 35°42′	53°26′ 36°34′	52°31′ 37°29′	51°35′ 38°25′	50°36′ 39°24′	49°34′ 40°26′	48°30′ 41°30′	47°23′ 42°37′
24	56°19′ 33°41′	55°33′ 34°27′	54°47′ 35°13′	53°58′ 36°2′	53°8′ 36°52′	52°15′ 37°45′	51°20′ 38°40′	50°23′ 39°37′	49°24′ 40°36′	48°22′ 41°38′	47°17′ 42°43′	46°10′ 43°50′
25	55°13′ 34°47′	54°28′ 35°32′	53°40′ 36°20′	52°51′ 37°9′	52°0′ 38°0′	51°7′ 38°53′	50°12′ 39°48′	49°14′ 40°46′	48°14′ 41°46′	47°12′ 42°48′	46°7′ 43°53′	45°
26	54°10′ 35°50′	53°24′ 36°36′	52°36′ 37°24′	51°46′ 38°14′	50°54′ 39°6′	50°1′ 39°59′	49°5′ 40°55′	48°7′ 41°53′	47°7′ 42°53′	46°5′ 43°55′	45°	......
27	53°7′ 36°53′	52°21′ 37°39′	51°33′ 38°27′	50°43′ 39°17′	49°51′ 40°9′	48°57′ 41°3′	48°0′ 42°0′	47°3′ 42°57′	46°2′ 43°58′	45°	......	......
28	52°8′ 37°52′	51°20′ 38°40′	50°32′ 39°28′	49°41′ 40°19′	48°49′ 41°11′	47°55′ 42°5′	46°58′ 43°2′	46°0′ 44°0′	45°	......	......	......
29	51°9′ 38°51′	50°21′ 39°39′	49°32′ 40°28′	48°41′ 41°19′	47°49′ 42°11′	46°54′ 43°6′	45°58′ 44°2′	45°	......	......	......	......
30	50°12′ 39°48′	49°24′ 40°36′	48°35′ 41°25′	47°43′ 42°17′	46°51′ 43°9′	45°56′ 44°4′	45°	......	......	......	......	......
31	49°16′ 40°44′	48°28′ 41°32′	47°39′ 42°21′	46°47′ 43°13′	45°54′ 44°6′	45°	......	......	......	......	......	......
32	48°22′ 41°38′	47°34′ 42°26′	46°44′ 43°16′	45°53′ 44°7′	45°	......	......	......	......	......	......	......
33	47°29′ 42°31′	46°41′ 43°19′	45°51 44°9′	45°	......	......	......	......	......	......	......	......
34	46°38′ 43°22′	45°50′ 44°10′	45°	......	......	......	......	......	......	......	......	......
35	45°48′ 44°12′	45°	......	......	......	......	......	......	......	......	......	......
36	45°	......	......	......	......	......	......	......	......	......	......	......

Of the two angles given, the upper is for the gear, the lower for the pinion.

Edge or Pitch Cone Angles of Bevel Gears (*Continued*)

Number of Teeth in Pinion	Number of Teeth in Gear: 24	23	22	21	20	19	18	17	16	15	14	13
12	63°26′ 26°34′	62°27′ 27°33′	61°23′ 28°37′	60°15′ 29°45′	59°2′ 30°58′	57°44′ 32°16′	56°19′ 33°41′	54°47′ 35°13′	53°7′ 36°53′	51°20′ 38°40′	49°24′ 40°36′	47°17′ 42°43′
13	61°33′ 28°27′	60°31′ 29°29′	59°25′ 30°35′	58°14′ 31°46′	56°58′ 33°2′	55°37′ 34°23′	54°10′ 35°50′	52°36′ 37°24′	50°54′ 39°6′	49°5′ 40°55′	47°7′ 42°53′	45°
14	59°45′ 30°15′	58°40′ 31°20′	57°32′ 32°28′	56°19′ 33°41′	55°0′ 35°0′	53°37′ 36°23′	52°8′ 37°52′	50°32′ 39°28′	48°48′ 41°12′	46°58′ 43°2′	45°	.....
15	58°0′ 32°0′	56°53′ 33°7′	55°43′ 34°17′	54°28′ 35°32′	53°7′ 36°53′	51°42′ 38°18′	50°12′ 39°48′	48°35′ 41°25′	46°51′ 43°9′	45°	.....	.....
16	56°19′ 33°41′	55°11′ 34°49′	53°58′ 36°2′	52°42′ 37°18′	51°20′ 38°40′	49°54′ 40°6′	48°22′ 41°38′	46°44′ 43°16′	45°	.....	.....	.....
17	54°41′ 35°19′	53°32′ 36°28′	52°18′ 37°42′	51°0′ 39°0′	49°38′ 40°22′	48°11′ 41°49′	46°38′ 43°22′	45°	.....	.....	.....	.....
18	53°7′ 36°53′	51°57′ 38°3′	50°43′ 39°17′	49°24′ 40°36′	48°0′ 42°0′	46°33′ 43°27′	45°	.....	.....	.....	.....	.....
19	51°38′ 38°22′	50°26′ 39°34′	49°11′ 40°49′	47°52′ 42°8′	46°28′ 43°32′	45°	.....	.....	.....	.....	.....	.....
20	50°12′ 39°48′	48°59′ 41°1′	47°43′ 42°17′	46°24′ 43°36′	45°	.....	.....	.....	.....	.....	.....	.....
21	48°48′ 41°12′	47°36′ 42°24′	46°20′ 43°40′	45°	.....	.....	.....	.....	.....	.....	.....	.....
22	47°29′ 42°31′	46°16′ 43°44′	45°	.....	.....	.....	.....	.....	.....	.....	.....	.....
23	46°13′ 43°47′	45°	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
24	45°	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

Of the two angles given, the upper is for the gear, the lower for the pinion.

Materials Used for Making Bevel Gears. — Cast iron is used for the largest work, and for smaller work which is not to be subjected to heavy duty. In cases where great working stress or a sudden shock is liable to come on the teeth, steel is ordinarily used. Such gears are made from bar stock for the smallest work, from drop forgings for intermediate sizes made on a manufacturing basis and from steel castings for heavy work. The softer grades of steel are not suitable for high-speed service, as this material abrades more rapidly than cast iron. This objection does not apply to hardened steel, such as is used in automobile transmission gears.

It is quite common to make the gear and pinion of different materials. This is advantageous from the standpoint of both efficiency and durability. Cast iron and steel, and steel and bronze are common combinations. In general, the pinion should be made of the stronger material, since it is of weak form; and it should be made of the more durable material, as it revolves more rapidly and each tooth comes into working contact more times per minute than do those of the larger mating gear. In a steel and cast iron combination, then, the pinion should be of steel, while the gear is of cast iron.

Rawhide and fiber are quite largely used for pinion blanks in cases where it is desired to run gearing at a very high speed and with as little noise as possible. Fiber, has the merit of convenience and comparative inexpensiveness, as it may be purchased in a variety of sizes of bars, rods, tubes, etc., ready to be worked up into pinion blanks; but it is not so strong as rawhide, and is difficult to machine owing to its gritty composition. It is satisfactory for light duty at high speed.

Face Angles of Bevel Gears

(Table is also used for finding *cutting angles* of gears cut with rotary milling cutters. See explanatory paragraph, page 666.)

Number of Teeth in Pinion	Number of Teeth in Gear: 72	71	70	69	68	67	66	65	64	63	62	61
12	7°53′ 78°59′	8° 78°50′	8°7′ 78°39′	8°14′ 78°30′	8°21′ 78°19′	8°28′ 78°10′	8°35′ 77°59′	8°43′ 77°47′	8°51′ 77°37′	8°59′ 77°25′	9°7′ 77°13′	9°17′ 77°1′
13	8°40′ 78°12′	8°48′ 78°2′	8°54′ 77°52′	9°2′ 77°42′	9°9′ 77°31′	9°18′ 77°20′	9°26′ 77°8′	9°35′ 76°56′	9°43′ 76°45′	9°52′ 76°32′	10°1′ 76°19′	10°11′ 76°7′
14	9°26′ 77°26′	9°34′ 77°16′	9°42′ 77°4′	9°50′ 76°54′	9°59′ 76°43′	10°8′ 76°30′	10°16′ 76°18′	10°25′ 76°7′	10°35′ 75°55′	10°45′ 75°41′	10°54′ 75°28′	11°5′ 75°15′
15	10°12′ 76°40′	10°21′ 76°29′	10°30′ 76°18′	10°38′ 76°6′	10°47′ 75°55′	10°57′ 75°43′	11°6′ 75°30′	11°16′ 75°16′	11°27′ 75°3′	11°37′ 74°49′	11°47′ 74°35′	11°59′ 74°21′
16	10°59′ 75°55′	11°7′ 75°43′	11°17′ 75°31′	11°26′ 75°20′	11°37′ 75°7′	11°46′ 74°54′	11°56′ 74°40′	12°7′ 74°27′	12°17′ 74°13′	12°29′ 73°59′	12°40′ 73°44′	12°52′ 73°28′
17	11°44′ 75°10′	11°54′ 74°58′	12°4′ 74°46′	12°13′ 74°33′	12°24′ 74°20′	12°34′ 74°5′	12°46′ 73°52′	12°56′ 73°38′	13°7′ 73°23′	13°21′ 73°9′	13°32′ 72°52′	13°45′ 72°35′
18	12°29′ 74°25′	12°40′ 74°12′	12°50′ 74°	13° 73°46′	13°12′ 73°32′	13°23′ 73°19′	13°34′ 73°4′	13°47′ 72°49′	13°59′ 72°33′	14°12′ 72°18′	14°24′ 72°2′	14°38′ 71°44′
19	13°14′ 73°40′	13°25′ 73°27′	13°36′ 73°14′	13°48′ 73°	14° 72°46′	14°11′ 72°31′	14°24′ 72°16′	14°36′ 72°	14°49′ 71°45′	15°2′ 71°28′	15°15′ 71°11′	15°30′ 70°54′
20	13°59′ 72°57′	14°11′ 72°43′	14°23′ 72°29′	14°34′ 72°14′	14°46′ 72°	15°4′ 71°45′	15°11′ 71°29′	15°25′ 71°13′	15°39′ 70°56′	15°52′ 70°38′	16°7′ 70°21′	16°21′ 70°3′
21	14°43′ 72°13′	14°55′ 71°59′	15°8′ 71°44′	15°21′ 71°29′	15°33′ 71°13′	15°46′ 70°58′	15°59′ 70°41′	16°13′ 70°25′	16°28′ 70°8′	16°42′ 69°50′	16°58′ 69°32′	17°13′ 69°13′
22	15°27′ 71°29′	15°40′ 71°14′	15°53′ 70°59′	16°6′ 70°44′	16°20′ 70°28′	16°33′ 70°11′	16°47′ 69°55′	17°2′ 69°38′	17°16′ 69°20′	17°31′ 69°1′	17°49′ 68°43′	18°3′ 68°23′
23	16°12′ 70°46′	16°24′ 70°30′	16°38′ 70°16′	16°51′ 69°59′	17°5′ 69°43′	17°20′ 69°26′	17°34′ 69°8′	17°50′ 68°50′	18°5′ 68°33′	18°20′ 68°14′	18°36′ 67°54′	18°54′ 67°34′
24	16°55′ 70°3′	17°9′ 69°47′	17°22′ 69°32′	17°37′ 69°15′	17°51′ 68°59′	18°6′ 68°40′	18°21′ 68°23′	18°37′ 68°5′	18°53′ 67°45′	19°9′ 67°27′	19°26′ 67°6′	19°44′ 66°46′
25	17°39′ 69°21′	17°52′ 69°4′	18°6′ 68°48′	18°21′ 68°31′	18°36′ 68°14′	18°52′ 67°56′	19°7′ 67°37′	19°24′ 67°18′	19°40′ 67°	19°57′ 66°39′	20°14′ 66°20′	20°32′ 65°58′
26	18°21′ 68°39′	18°36′ 68°22′	18°51′ 68°5′	19°6′ 67°48′	19°22′ 67°30′	19°37′ 67°13′	19°53′ 66°53′	20°10′ 66°34′	20°26′ 66°14′	20°45′ 65°53′	21°2′ 65°32′	21°21′ 65°11′
27	19°3′ 67°57′	19°19′ 67°39′	19°34′ 67°22′	19°49′ 67°5′	20°6′ 66°46′	20°22′ 66°28′	20°38′ 66°8′	20°56′ 65°48′	21°13′ 65°29′	21°32′ 65°8′	21°50′ 64°46′	22°10′ 64°24′
28	19°46′ 67°16′	20°1′ 66°59′	20°17′ 66°41′	20°32′ 66°22′	20°50′ 66°4′	21°6′ 65°44′	21°23′ 65°25′	21°41′ 65°5′	22° 64°44′	22°18′ 64°22′	22°37′ 64°1′	22°56′ 63°38′
29	20°27′ 66°35′	20°43′ 66°17′	20°59′ 65°59′	21°16′ 65°40′	21°33′ 65°21′	21°50′ 65°2′	22°8′ 64°42′	22°27′ 64°21′	22°45′ 63°59′	23°5′ 63°37′	23°25′ 63°15′	23°44′ 62°52′
30	21°9′ 65°55′	21°25′ 65°37′	21°42′ 65°18′	21°58′ 64°58′	22°15′ 64°39′	22°34′ 64°18′	22°52′ 63°58′	23°10′ 63°38′	23°30′ 63°16′	23°50′ 62°54′	24°10′ 62'30′	24°30′ 62°7′
31	21°50′ 65°14′	22°6′ 64°56′	22°24′ 64°36′	22°41′ 64°17′	22°59′ 63°57′	23°17′ 63°37′	23°35′ 63°15′	23°55′ 62°55′	24°14′ 62°32′	24°34′ 62°10′	24°54′ 61°46′	25°17′ 61°23′
32	22°31′ 64°35′	22°48′ 64°16′	23°4′ 63°56′	23°23′ 63°37′	23°40′ 63°16′	23°59′ 62°55′	24°18′ 62°34′	24°38′ 62°12′	24°58′ 61°50′	25°18′ 61°26′	25°39′ 61°3′	26°1′ 60°39′
33	23°10′ 63°56′	23°28′ 63°36′	23°46′ 63°16′	24°4′ 62°56′	24°22′ 62°36′	24°41′ 62°15′	25°1′ 61°53′	25°21′ 61°31′	25°42′ 61°8′	26°2′ 60°44′	26°24′ 60°20′	26°45′ 59°55′
34	23°51′ 63°17′	24°8′ 62°58′	24°27′ 62°37′	24°44′ 62°16′	25°4′ 61°56′	25°23′ 61°33′	25°42′ 61°12′	26°3′ 60°49′	26°24′ 60°26′	26°46′ 60°2′	27°7′ 59°37′	27°29′ 59°13′
35	24°29′ 62°39′	24°48′ 62°18′	25°6′ 61°58′	25°25′ 61°37′	25°44′ 61°16′	26°4′ 60°54′	26°24′ 60°32′	26°45′ 60°9′	27°6′ 59°44′	27°28′ 59°22′	27°50′ 58°56′	28°13′ 58°31′
36	25°9′ 62°1′	25°27′ 61°41′	25°45′ 61°20′	26°5′ 60°59′	26°24′ 60°36′	26°45′ 60°15′	27°5′ 59°51′	27°26′ 59°28′	27°48′ 59°4′	28°10′ 58°40′	28°33′ 58°15′	28°56′ 57°50′
37	25°47′ 61°23′	26°6′ 61°2′	26°25′ 60°41′	26°44′ 60°20′	27°4′ 59°58′	27°25′ 59°35′	27°45′ 59°13′	28°7′ 58°49′	28°29′ 58°25′	28°51′ 58°1′	29°15′ 57°35′	29°38′ 57°10′

Of the two angles given, the upper is for the gear, the lower for the pinion.

Face Angles of Bevel Gears (*Continued*)

Number of Teeth in Pinion	Number of Teeth in Gear: 60	59	58	57	56	55	54	53	52	51	50	49
12	9°26′ 76°48′	9°35′ 76°35′	9°45′ 76°23′	9°55′ 76°8′	10°6′ 75°54′	10°16′ 75°40′	10°28′ 75°24′	10°39′ 75°9′	10°52′ 74°52′	11°3′ 74°37′	11°15′ 74°15′	11°30′ 73°58′
13	10°21′ 75°53′	10°31′ 75°39′	10°42′ 75°26′	10°53′ 75°11′	11°4′ 74°56′	11°16′ 74°40′	11°28′ 74°24′	11°42′ 74°8′	11°54′ 73°50′	12°8′ 73°32′	12°20′ 73°12′	12°37′ 72°53′
14	11°16′ 75°	11°27′ 74°45′	11°39′ 74°31′	11°50′ 74°14′	12°2′ 73°58′	12°16′ 73°42′	12°29′ 73°25′	12°43′ 73°7′	12°57′ 72°49′	13°11′ 72°29′	13°26′ 72°8′	13°42′ 71°48′
15	12°11′ 74°7′	12°22′ 73°50′	12°35′ 73°35′	12°48′ 73°18′	13°1′ 73°1′	13°16′ 72°44′	13°28′ 72°26′	13°43′ 72°7′	13°59′ 71°49′	14°14′ 71°28′	14°30′ 71°6′	14°47′ 70°45′
16	13°5′ 73°13′	13°18′ 72°56′	13°30′ 72°40′	13°45′ 72°23′	13°59′ 72°5′	14°13′ 71°47′	14°28′ 71°28′	14°44′ 71°8′	15°1′ 70°49′	15°17′ 70°27′	15°35′ 70°5′	15°52′ 69°42′
17	13°59′ 72°21′	14°11′ 72°3′	14°26′ 71°45′	14°40′ 71°28′	14°57′ 71°9′	15°11′ 70°49′	15°28′ 70°30′	15°44′ 70°10′	16°1′ 69°49′	16°18′ 69°26′	16°37′ 69°3′	16°55′ 68°39′
18	14°52′ 71°28′	15°6′ 71°10′	15°21′ 70°51′	15°36′ 70°34′	15°52′ 70°14′	16°7′ 69°53′	16°26′ 69°34′	16°42′ 69°12′	17°1′ 68°49′	17°20′ 68°26′	17°39′ 68°3′	17°58′ 67°38′
19	15°44′ 70°30′	15°59′ 70°17′	16°15′ 69°59′	16°31′ 69°39′	16°49′ 69°19′	17°2′ 68°58′	17°23′ 68°37′	17°41′ 68°15′	18° 67°52′	18°21′ 67°29′	18°40′ 67°4′	19°1′ 66°37′
20	16°37′ 69°45′	16°53′ 69°25′	17°8′ 69°6′	17°26′ 68°46′	17°44′ 68°26′	18°1′ 68°3′	18°19′ 67°41′	18°40′ 67°18′	19° 66°54′	19°20′ 66°30′	19°41′ 66°5′	20°2′ 65°38′
21	17°28′ 68°54′	17°46′ 68°34′	18°2′ 68°14′	18°20′ 67°52′	18°39′ 67°31′	18°57′ 67°9′	19°16′ 66°46′	19°37′ 66°23′	19°58′ 65°58′	20°19′ 65°33′	20°41′ 65°7′	21°3′ 64°39′
22	18°20′ 68°4′	18°37′ 67°43′	18°56′ 67°22′	19°13′ 67°1′	19°32′ 66°38′	19°52′ 66°16′	20°12′ 65°52′	20°33′ 65°27′	20°55′ 65°3′	21°17′ 64°37′	21°40′ 64°10′	22°3′ 63°41′
23	19°10′ 67°14′	19°28′ 66°52′	19°48′ 66°32′	20°5′ 66°9′	20°25′ 65°47′	20°47′ 65°23′	21°8′ 64°58′	21°29′ 64°33′	21°52′ 64°8′	22°13′ 63°41′	22°37′ 63°13′	23°2′ 62°44′
24	20°1′ 66°25′	20°19′ 66°3′	20°39′ 65°41′	20°58′ 65°18′	21°19′ 64°55′	21°39′ 64°31′	22°1′ 64°5′	22°24′ 63°40′	22°46′ 63°14′	23°10′ 62°46′	23°36′ 62°19′	24° 61°48′
25	20°51′ 65°37′	21°10′ 65°14′	21°29′ 64°51′	21°50′ 64°28′	22°11′ 64°5′	22°33′ 63°39′	22°56′ 63°14′	23°18′ 62°48′	23°41′ 62°21′	24°7′ 61°53′	24°32′ 61°24′	24°57′ 60°53′
26	21°41′ 64°49′	22° 64°26′	22°20′ 64°2′	22°41′ 63°39′	23°3′ 63°15′	23°25′ 62°49′	23°47′ 62°23′	24°13′ 61°56′	24°36′ 61°28′	25°1′ 60°59′	25°28′ 60°30′	25°53′ 59°59′
27	22°29′ 64°1′	22°49′ 63°39′	23°10′ 63°14′	23°31′ 62°49′	23°53′ 62°25′	24°16′ 61°58′	24°40′ 61°32′	25°5′ 61°5′	25°29′ 60°37′	25°55′ 60°7′	26°22′ 59°38′	26°48′ 59°5′
28	23°17′ 63°15′	23°38′ 62°52′	23°59′ 62°27′	24°21′ 62°1′	24°44′ 61°36′	25°7′ 61°9′	25°31′ 60°43′	25°56′ 60°14′	26°22′ 59°46′	26°48′ 59°16′	27°15′ 58°45′	27°43′ 58°13′
29	24°4′ 62°28′	24°25′ 62°5′	24°48′ 61°40′	25°10′ 61°14′	25°33′ 60°47′	25°57′ 60°21′	26°22′ 59°52′	26°47′ 59°25′	27°14′ 58°56′	27°40′ 58°26′	28°8′ 57°54′	28°36′ 57°22′
30	24°51′ 61°43′	25°12′ 61°18′	25°36′ 60°54′	25°59′ 60°27′	26°22′ 60°	26°47′ 59°33′	27°12′ 59°6′	27°38′ 58°36′	28°4′ 58°6′	28°32′ 57°36′	29° 57°4′	29°28′ 56°32′
31	25°38′ 60°58′	25°58′ 60°34′	26°22′ 60°8′	26°46′ 59°42′	27°10′ 59°14′	27°34′ 58°46′	28°3′ 58°15′	28°27′ 57°49′	28°54′ 57°18′	29°23′ 56°47′	29°51′ 56°15′	30°20′ 55°42′
32	26°23′ 60°15′	26°45′ 59°49′	27°9′ 59°23′	27°34′ 58°56′	27°58′ 58°28′	28°23′ 57°59′	28°49′ 57°31′	29°17′ 57°1′	29°33′ 56°41′	30°12′ 56°	30°42′ 55°28′	31°10′ 54°54′
33	27°9′ 59°31′	27°31′ 59°5′	27°56′ 58°38′	28°19′ 58°11′	28°45′ 57°43′	29°10′ 57°14′	29°37′ 56°45′	30°5′ 56°15′	30°32′ 55°49′	31°1′ 55°13′	31°31′ 54°39′	32°1′ 54°5′
34	27°52′ 58°48′	28°16′ 58°22′	28°40′ 57°54′	29°5′ 57°27′	29°31′ 56°59′	29°57′ 56°29′	30°24′ 56°	30°51′ 55°29′	31°20′ 54°58′	31°49′ 54°27′	32°19′ 53°53′	32°50′ 53°20′
35	28°36′ 58°6′	29°1′ 57°39′	29°25′ 57°11′	29°50′ 56°44′	30°15′ 56°15′	30°42′ 55°46′	31°10′ 55°16′	31°38′ 54°44′	32°7′ 54°13′	32°36′ 53°40′	33°7′ 53°7′	33°38′ 52°34′
36	29°20′ 57°24′	29°43′ 56°57′	30°9′ 56°29′	30°35′ 56°1′	31° 55°32′	31°27′ 55°3′	31°55′ 54°33′	32°23′ 54°1′	32°53′ 53°28′	33°23′ 52°57′	33°53′ 52°23′	34°25′ 51°40′
37	30°2′ 56°42′	30°27′ 56°15′	30°52′ 55°48′	31°18′ 55°20′	31°45′ 54°49′	32°12′ 54°20′	32°40′ 53°50′	33°8′ 53°18′	33°38′ 52°46′	34°9′ 52°13′	34°40′ 51°40′	35°12′ 51°4′
38	30°44′ 56°2′	31°9′ 55°35′	31°35′ 55°7′	32°1′ 54°39′	32°27′ 54°9′	32°56′ 53°38′	33°24′ 53°8′	33°52′ 52°38′	34°22′ 52°4′	34°54′ 51°30′	35°24′ 50°56′	35°57′ 50°21′

Of the two angles given, the upper is for the gear, the lower for the pinion.

Face Angles of Bevel Gears *(Continued)*

Number of Teeth in Pinion	Number of Teeth in Gear: 48	47	46	45	44	43	42	41	40	39	38	37
12	11°43′ 73°39′	11°58′ 73°20′	12°13′ 72°59′	12°29′ 72°37′	12°45′ 72°15′	13°1′ 71°51′	13°19′ 71°25′	13°37′ 70°59′	13°57′ 70°33′	14°18′ 70°6′	14°39′ 69°35′	15°1′ 69°5′
13	12°51′ 72°33′	13°7′ 72°11′	13°23′ 71°49′	13°40′ 71°26′	13°58′ 71°2′	14°16′ 70°38′	14°35′ 70°11′	14°55′ 69°45′	15°17′ 69°15′	15°39′ 68°47′	16°1′ 68°15′	16°25′ 67°43′
14	13°59′ 71°27′	14°15′ 71°5′	14°33′ 70°41′	14°51′ 70°17′	15°10′ 69°52′	15°30′ 69°26′	15°51′ 68°59′	16°13′ 68°31′	16°34′ 68°0′	16°59′ 67°29′	17°24′ 66°56′	17°50′ 66°22′
15	15°5′ 70°23′	15°23′ 69°59′	15°42′ 69°34′	16°1′ 69°9′	16°22′ 68°42′	16°43′ 68°15′	17°5′ 67°47′	17°28′ 67°16′	17°53′ 66°45′	18°18′ 66°14′	18°44′ 65°40′	19°11′ 65°3′
16	16°11′ 69°19′	16°30′ 68°54′	16°50′ 68°28′	17°10′ 68°2′	17°32′ 67°34′	17°56′ 67°6′	18°18′ 66°36′	18°42′ 66°4′	19°9′ 65°33′	19°35′ 64°59′	20°3′ 64°23′	20°32′ 63°46′
17	17°15′ 68°15′	17°36′ 67°50′	17°57′ 67°23′	18°20′ 66°54′	18°43′ 66°27′	19°6′ 65°58′	19°31′ 65°27′	19°56′ 64°54′	20°24′ 64°20′	20°51′ 63°45′	21°21′ 63°9′	21°53′ 62°31′
18	18°20′ 67°12′	18°41′ 66°47′	19°3′ 66°19′	19°27′ 65°51′	19°50′ 65°20′	20°18′ 64°50′	20°42′ 64°18′	21°9′ 63°45′	21°37′ 63°9′	22°6′ 62°34′	22°38′ 61°56′	23°9′ 61°17′
19	19°22′ 66°12′	19°46′ 65°44′	20°8′ 65°16′	20°34′ 64°46′	20°59′ 64°15′	21°24′ 63°44′	21°52′ 63°10′	22°20′ 62°36′	22°49′ 62°1′	23°20′ 61°24′	23°52′ 60°44′	24°26′ 60°4′
20	20°25′ 65°11′	20°49′ 64°43′	21°13′ 64°13′	21°39′ 63°43′	22°5′ 63°11′	22°32′ 62°38′	23°0′ 62°4′	23°30′ 61°30′	24°1′ 60°53′	24°32′ 60°14′	25°6′ 59°34′	25°40′ 58°54′
21	21°27′ 64°11′	21°52′ 63°42′	22°17′ 63°13′	22°43′ 62°41′	23°10′ 62°8′	23°38′ 61°34′	24°8′ 61°0′	24°39′ 60°25′	25°10′ 59°46′	25°43′ 59°7′	26°18′ 58°26′	26°53′ 57°43′
22	22°17′ 63°13′	22°53′ 62°43′	23°19′ 62°11′	23°46′ 61°40′	24°15′ 61°7′	24°44′ 60°32′	25°14′ 59°56′	25°46′ 59°20′	26°19′ 58°41′	26°53′ 58°1′	27°27′ 57°19′	28°5′ 56°35′
23	23°27′ 62°15′	23°54′ 61°44′	24°21′ 61°13′	24°49′ 60°41′	25°18′ 60°6′	25°47′ 59°31′	26°18′ 58°54′	26°52′ 58°16′	27°26′ 57°38′	28°0′ 56°56′	28°36′ 56°14′	29°14′ 55°30′
24	24°26′ 61°18′	24°53′ 60°47′	25°21′ 60°15′	25°49′ 59°41′	26°20′ 59°6′	26°51′ 58°31′	27°23′ 57°53′	27°57′ 57°15′	28°31′ 56°35′	29°7′ 55°53′	29°43 55°11′	30°22′ 54°26′
25	25°24′ 60°22′	25°52′ 59°50′	26°20′ 59°18′	26°50′ 58°44′	27°21′ 58°9′	27°52′ 57°32′	28°26′ 56°54′	28°59′ 56°15′	29°34′ 55°34′	30°12′ 54°52′	30°49′ 54°9′	31°29′ 53°23′
26	26°21′ 59°27′	26°49′ 58°55′	27°19′ 58°21′	27°49′ 57°47′	28°21′ 57°11′	28°54′ 56°34′	29°27′ 55°55′	30°1′ 55°15′	30°38′ 54°34′	31°14′ 53°52′	31°54′ 53°8′	32°34′ 52°22′
27	27°17′ 58°33′	27°46′ 58°0′	28°16′ 57°26′	28°47′ 56°51′	29°19′ 56°15′	29°52′ 55°38′	30°27′ 54°59′	31°3′ 54°19′	31°39′ 53°37′	32°18′ 52°54′	32°57′ 52°9′	33°37′ 51°23′
28	28°12′ 57°42′	28°42′ 57°8′	29°12′ 56°32′	29°43′ 55°57′	30°16′ 55°20′	30°50′ 54°42′	31°25′ 54°3′	32°2′ 53°22′	32°39′ 52°39′	33°18′ 51°56′	33°57′ 51°11′	34°39′ 50°25′
29	29°5′ 56°49′	29°37′ 56°15′	30°8′ 55°40′	30°40′ 55°4′	31°13′ 54°27′	31°48′ 53°48′	32°23′ 53°9′	32°59′ 52°27′	33°38′ 51°44′	34°17′ 51°1′	34°58′ 50°16′	35°39′ 49°29′
30	29°58′ 55°58′	30°30′ 55°24′	31°2′ 54°48′	31°34′ 54°12′	32°8′ 53°34′	32°44′ 52°54′	33°19′ 52°15′	33°57′ 51°33′	34°36′ 50°50′	35°15′ 50°7′	35°56′ 49°20′	36°38′ 48°34′
31	30°52′ 55°8′	31°22′ 54°34′	31°55′ 53°57′	32°29′ 53°21′	33°2′ 52°42′	33°39′ 52°3′	34°15′ 51°23′	34°53′ 50°41′	35°31′ 49°57′	36°11′ 49°13′	36°52′ 48°28′	37°35′ 47°39′
32	31°42′ 54°20′	32°14′ 53°44′	32°46′ 53°8′	33°21′ 52°31′	33°56′ 51°52′	34°31′ 51°13′	35°8′ 50°32′	35°46′ 49°50′	36°27′ 49°7′	37°6′ 48°22′	37°48′ 47°36′	38°31′ 46°49′
33	32°32′ 53°32′	33°4′ 52°56′	33°38′ 52°20′	34°12′ 51°42′	34°47′ 51°3′	35°24′ 50°22′	36°0′ 49°41′	36°39′ 48°59′	37°19′ 48°17′	38°0′ 47°32′	38°42′ 46°46′	39°26′ 45°58′
34	33°22′ 52°44′	33°54′ 52°8′	34°28′ 51°32′	35°6′ 50°50′	35°38′ 50°14′	36°15′ 49°35′	36°53′ 48°53′	37°32′ 48°12′	38°11′ 47°27′	38°53′ 46°43′	39′35° 45°57′	40°18′ 45°8′
35	34°10′ 51°58′	34°42′ 51°22′	35°17′ 50°45′	35°51′ 50°7′	36°27′ 49°27′	37°5′ 48°47′	37°42′ 48°6′	38°22′ 47°24′	39°3′ 46°39′	39°44′ 45°54′	40°26′ 45°8′	41°10′ 44°20′
36	34°57′ 51°13′	35°31′ 50°37′	36°5′ 49°59′	36°41′ 49°21′	37°16′ 48°42′	37°53′ 48°1′	38°32′ 47°20′	39°11′ 46°37′	39°52′ 45°52′	40°34′ 45°8′	41°15′ 44°21′	42°0′ 43°34′
37	35°43′ 50°29′	36°18′ 49°52′	36°51′ 49°15′	37°27′ 48°37′	38°4′ 47°56′	38°42′ 47°16′	39°20′ 46°34′	40°0′ 45°52′	40°40′ 45°8′	41°22′ 44°22′	42°5′ 43°37′	42°48′
38	36°29′ 49°45′	37°3′ 49°9′	37°38′ 48°32′	38°14′ 47°52′	38°51′ 47°13′	39°28′ 46°32′	40°7′ 45°51′	40°47′ 45°7′	41°28′ 44°24′	42°9′ 43°39′	42°52′	.....

Of the two angles given, the upper is for the gear, the lower for the pinion.

Face Angles of Bevel Gears *(Continued)*

Number of Teeth in Pinion	Number of Teeth in Gear											
	36	35	34	33	32	31	30	29	28	27	26	25
12	15°24′ 68°32′	15°49′ 67°59′	16°15′ 67°23′	16°43′ 66°45′	17°13′ 66°5′	17°43′ 65°23′	18°15′ 64°39′	18°51′ 63°53′	19°27′ 63°3′	20°5′ 62°9′	20°46′ 61°14′	21°31′ 60°15′
13	16°51′ 67°9′	17°19′ 66°33′	17°46′ 65°56′	18°16′ 65°16′	18°48′ 64°34′	19°21′ 63°51′	19°57′ 63°5′	20°32′ 62°14′	21°11′ 61°23′	21°54′ 60°28′	22°37′ 59°29′	23°26′ 58°28′
14	18°17′ 65°47′	18°45′ 65°9′	19°16′ 64°30′	19°48′ 63°48′	20°20′ 63°6′	20°56′ 62°20′	21°34′ 61°32′	22°13′ 60°41′	22°55′ 59°47′	23°38′ 58°50′	24°25′ 57°49′	25°16′ 56°46′
15	19°40′ 64°26′	20°11′ 63°49′	20°44′ 63°8′	21°18′ 62°24′	21°53′ 61°39′	22°31′ 60°51′	23°10′ 60°2′	23°51′ 59°9′	24°35′ 58°13′	25°20′ 57°14′	26°11′ 56°13′	27°3′ 55°7′
16	21°3′ 63°7′	21°36′ 62°28′	22°9′ 61°45′	22°45′ 61°1′	23°22′ 60°14′	24°1′ 59°25′	24°42′ 58°34′	25°26′ 57°40′	26°12′ 56°42′	27°1′ 55°43′	27°52′ 54°38′	28°45′ 53°31′
17	22°24′ 61°50′	22°57′ 61°9′	23°33′ 60°25′	24°10′ 59°40′	24°50′ 58°52′	25°31′ 58°1′	26°14′ 57°10′	26°59′ 56°13′	27°47′ 55°15′	28°37′ 54°13′	29°30′ 53°8′	30°26′ 52°0′
18	23°43′ 60°35′	24°18′ 59°52′	24°56′ 59°8′	25°34′ 58°20′	26°15′ 57°31′	26°57′ 56°39′	27°42′ 55°46′	28°29′ 54°49′	29°18′ 53°50′	30°9′ 52°47′	31°5′ 51°41′	32°2′ 50°32′
19	25°1′ 59°21′	25°37′ 58°37′	26°15′ 57°51′	26°56′ 57°4′	27°38′ 56°14′	28°22′ 55°22′	29°8′ 54°26′	29°56′ 53°28′	30°43′ 52°28′	31°40′ 51°24′	32°36′ 50°18′	33°36′ 49°8′
20	26°16′ 58°10′	26°55′ 57°25′	27°34′ 56°38′	28°15′ 55°49′	28°58′ 54°58′	29°44′ 54°4′	30°31′ 53°9′	31°21′ 52°9′	32°13′ 51°9′	33°8′ 50°4′	34°5′ 48°57′	35°6′ 47°46′
21	27°30′ 57°0′	28°10′ 56°14′	28°50′ 55°26′	29°32′ 54°36′	30°17′ 53°43′	31°4′ 52°50′	31°52′ 51°52′	32°43′ 50°53′	33°36′ 49°50′	34°31′ 48°47′	35°31′ 47°39′	36°32′ 46°28′
22	28°43′ 55°51′	29°22′ 55°4′	30°5′ 54°17′	30°48′ 53°26′	31°34′ 52°32′	32°22′ 51°38′	33°11′ 50°41′	34°3′ 49°41′	34°57′ 48°37′	35°54′ 47°32′	36°52′ 46°24′	37°55′ 45°13′
23	29°53′ 54°43′	30°35′ 53°57′	31°18′ 53°8′	32°1′ 52°15′	32°48′ 51°24′	33°36′ 50°28′	34°27′ 49°29′	35°20′ 48°30′	36°15′ 47°27′	37°12′ 46°20′	38°12′ 45°12′	39°15′ 44°1′
24	31°2′ 53°40′	31°45′ 52°51′	32°28′ 52°2′	33°14′ 51°10′	34°1′ 50°17′	34°50′ 49°20′	35°42′ 48°22′	36°35′ 47°21′	37°30′ 46°18′	38°28′ 45°12′	39°29′ 44°3′	40°32′ 42°52′
25	32°10′ 52°36′	32°52′ 51°48′	33°37′ 50°57′	34°23′ 50°5′	35°11′ 49°11′	36°0′ 48°14′	36°52′ 47°16′	37°47′ 46°15′	38°43′ 45°11′	39°41′ 44°5′	40°43′ 42°57′	41°46′
26	33°15′ 51°35′	33°58′ 50°46′	34°45′ 49°55′	35°31′ 49°3′	36°19′ 48°7′	37°10′ 47°12′	38°2′ 46°12′	38°56′ 45°10′	39°53′ 44°7′	40°52′ 43°2′	41°53′	.....
27	34°20′ 50°34′	35°3′ 49°45′	35°49′ 48°55′	36°36′ 48°2′	37°25′ 47°7′	38°16′ 46°10′	39°10′ 45°10′	40°4′ 44°10′	41°1′ 43°5′	42°	.....	.....
28	35°21′ 49°31′	36°7′ 48°47′	36°52′ 47°56′	37°40′ 47°2′	38°29′ 46°7′	39°21′ 45°11′	40°15′ 44°11′	41°9′ 43°9′	42°7′	.....	.....	.....
29	36°23′ 48°41′	37°8′ 47°50′	37°54′ 46°58′	38°42′ 46°4′	39°32′ 45°10′	40°24′ 44°12′	41°18′ 43°14′	42°13′	.....	.....	.....	.....
30	37°21′ 47°45′	38°7′ 46°55′	38°53′ 46°3′	39°43′ 45°9′	40°32′ 44°14′	41°25′ 43°17′	42°18′	.....	.....	.....	.....	.....
31	38°20′ 46°52′	39°5′ 46°1′	39°52′ 45°10′	40°41′ 44°15′	41°32′ 43°20′	42°23′	.....	.....	.....	.....	.....	.....
32	39°15′ 45°59′	40°1′ 45°9′	40°49′ 44°17′	41°38′ 43°24′	42°28′	.....	.....	.....	.....	.....	.....	.....
33	40°0′ 45°8′	40°56′ 44°18′	41°44′ 43°26′	42°33′	.....	.....	.....	.....	.....	.....	.....	.....
34	41°4′ 44°20′	41°49′ 43°29′	42°37′	.....	.....	.....	.....	.....	.....	.....	.....	.....
35	41°55′ 43°31′	42°41′	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
36	42°45′	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

Of the two angles given, the upper is for the gear, the lower for the pinion.

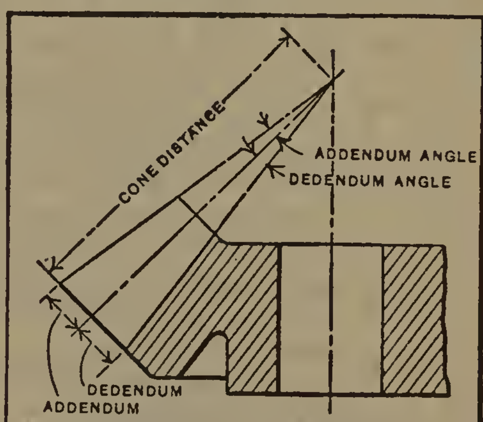
Face Angles of Bevel Gears *(Continued)*

Number of Teeth in Pinion	Number of Teeth in Gear: 24	23	22	21	20	19	18	17	16	15	14	13
12	22°18′ 59°10′	23°8′ 58°2′	24°3′ 56°49′	25°2′ 55°32′	26°3′ 54°7′	27°11′ 52°39′	28°25′ 51°3′	29°43′ 49°17′	31°11′ 47°25′	32°44′ 45°24′	34°26′ 43°14′	36°16′ 40°50′
13	24°15′ 57°21′	25°9′ 56°11′	26°6′ 54°56′	27°8′ 53°36′	28°14′ 52°10′	29°25′ 50°39′	30°42′ 49°2′	32°4′ 47°16′	33°34′ 45°22′	35°10′ 43°20′	36°55′ 41°9′	38°48′
14	26°8′ 55°38′	27°5′ 54°25′	28°4′ 53°8′	29°9′ 51°47′	30°20′ 50°20′	31°33′ 48°47′	32°52′ 47°8′	34°18′ 45°12′	35°50′ 43°26′	37°28′ 41°24′	39°15′	.....
15	27°58′ 53°58′	28°58′ 52°44′	30°0′ 51°26′	31°6′ 50°2′	32°19′ 48°33′	33°36′ 47°0′	34°56′ 45°20′	36°23′ 43°33′	37°57′ 41°39′	39°38′	.....	.....
16	29°43′ 52°21′	30°44′ 51°6′	31°50′ 49°46′	32°58′ 48°22′	34°12′ 46°52′	35°31′ 45°19′	36°54′ 43°38′	38°23′ 41°51′	39°57′	.....	.....	.....
17	31°26′ 50°48′	32°28′ 49°32′	33°35′ 48°11′	34°47′ 46°47′	36°0′ 45°16′	37°21′ 43°43′	38°45′ 42°1′	40°15′	.....	.....	.....	.....
18	33°4′ 49°18′	34°8′ 48°2′	35°15′ 46°41′	36°28′ 45°16′	37°45′ 43°45′	39°5′ 42°11′	40°31′	.....	.....	.....	.....	.....
19	34°38′ 47°54′	35°49′ 46°36′	36°53′ 45°15′	38°6′ 43°50′	39°24′ 42°20′	40°45′	.....	.....	.....	.....	.....	.....
20	36°8′ 46°32′	37°16′ 45°14′	38°26′ 43°52′	39°39′ 42°27′	40°57′	.....	.....	.....	.....	.....	.....	.....
21	37°37′ 45°13′	38°44′ 43°56′	39°54′ 42°34′	41°8′	.....	.....	.....	.....	.....	.....	.....	.....
22	39°0′ 43°58′	40°8′ 42°40′	41°19′	.....	.....	.....	.....	.....	.....	.....	.....	.....
23	40°20′ 42°46′	41°28′	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
24	41°38′	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

Of the two angles given, the upper is for the gear, the lower for the pinion.

Dedendum Angles for Different Addendum Angles. — When the shafts of bevel gears are not at right angles or when the depth of the tooth is greater or smaller than the standard depth for the thickness of tooth used, the table "Dedendum Angles for Different Addendum Angles" may be employed. The addendum angle is first calculated from the formula:

Addendum ÷ cone distance
= tangent of addendum angle.



Then the dedendum angle may be found directly from the table. When the table is used for teeth that are not of standard depth, the *ratio* of the dedendum to the addendum must still remain the same as for standard involute teeth.

As an example of the use of the table, find the dedendum angle for an addendum angle of 3 degrees 43 minutes. At the intersection of the vertical column under 3 degrees and the horizontal line opposite 43 minutes, the answer, 4 degrees 18 minutes, is found. This difference between addendum and dedendum angles does not apply to gears cut by rotary cutters (see page 652).

Dedendum Angles for Different Addendum Angles

Minutes	Degrees 0	Degrees 1	Degrees 2	Degrees 3	Degrees 4	Degrees 5	Minutes	Degrees 0	Degrees 1	Degrees 2	Degrees 3	Degrees 4	Degrees 5
0	0° 0′	1° 9′	2° 19′	3° 28′	4° 38′	5° 47′	30	0° 35′	1° 44′	2° 54′	4° 3′	5° 12′	6° 22′
1	0 1	1 11	2 20	3 29	4 39	5 48	31	0 36	1 45	2 55	4 4	5 14	6 23
2	0 2	1 12	2 21	3 31	4 40	5 49	32	0 37	1 46	2 56	4 5	5 15	6 24
3	0 3	1 13	2 22	3 32	4 41	5 51	33	0 38	1 48	2 57	4 6	5 16	6 25
4	0 5	1 14	2 23	3 33	4 42	5 52	34	0 39	1 49	2 58	4 8	5 17	6 26
5	0 6	1 15	2 25	3 34	4 44	5 53	35	0 41	1 50	2 59	4 9	5 18	6 28
6	0 7	1 16	2 26	3 35	4 45	5 54	36	0 42	1 51	3 1	4 10	5 19	6 29
7	0 8	1 18	2 27	3 36	4 46	5 55	37	0 43	1 52	3 2	4 11	5 21	6 30
8	0 9	1 19	2 28	3 38	4 47	5 56	38	0 44	1 53	3 3	4 12	5 22	6 31
9	0 10	1 20	2 29	3 39	4 48	5 58	39	0 45	1 55	3 4	4 13	5 23	6 32
10	0 12	1 21	2 30	3 40	4 49	5 59	40	0 46	1 56	3 5	4 15	5 24	6 33
11	0 13	1 22	2 32	3 41	4 50	6 0	41	0 47	1 57	3 6	4 16	5 25	6 35
12	0 14	1 23	2 33	3 42	4 52	6 1	42	0 49	1 58	3 7	4 17	5 26	6 36
13	0 15	1 24	2 34	3 43	4 53	6 2	43	0 50	1 59	3 9	4 18	5 27	6 37
14	0 16	1 26	2 35	3 44	4 54	6 3	44	0 51	2 0	3 10	4 19	5 29	6 38
15	0 17	1 27	2 36	3 46	4 55	6 5	45	0 52	2 2	3 11	4 20	5 30	6 39
16	0 19	1 28	2 37	3 47	4 56	6 6	46	0 53	2 3	3 12	4 22	5 31	6 40
17	0 20	1 29	2 39	3 48	4 57	6 7	47	0 54	2 4	3 13	4 23	5 32	6 42
18	0 21	1 30	2 40	3 49	4 59	6 8	48	0 56	2 5	3 14	4 24	5 33	6 43
19	0 22	1 31	2 41	3 50	5 0	6 9	49	0 57	2 6	3 16	4 25	5 34	6 44
20	0 23	1 33	2 42	3 51	5 1	6 10	50	0 58	2 7	3 17	4 26	5 36	6 45
21	0 24	1 34	2 43	3 53	5 2	6 11	51	0 59	2 8	3 18	4 27	5 37	6 46
22	0 25	1 35	2 44	3 54	5 3	6 13	52	1 0	2 10	3 19	4 28	5 38	6 47
23	0 27	1 36	2 45	3 55	5 4	6 14	53	1 1	2 11	3 20	4 30	5 39	6 48
24	0 28	1 37	2 47	3 56	5 6	6 15	54	1 2	2 12	3 21	4 31	5 40	6 50
25	0 29	1 38	2 48	3 57	5 7	6 16	55	1 4	2 13	3 23	4 32	5 41	6 51
26	0 30	1 40	2 49	3 58	5 8	6 17	56	1 5	2 14	3 24	4 33	5 43	6 52
27	0 31	1 41	2 50	4 0	5 9	6 18	57	1 6	2 15	3 25	4 34	5 44	6 53
28	0 33	1 42	2 51	4 1	5 10	6 20	58	1 7	2 17	3 26	4 35	5 45	6 54
29	0 34	1 43	2 52	4 2	5 11	6 21	59	1 8	2 18	3 27	4 37	5 46	6 55

Number of Cutters for Cutting Bevel Gears

Number of Teeth in Gear	Number of Teeth in Pinion																
	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
12	7–7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	....
13	6–7	6–6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	....
14	5–7	6–6	6–6	...	...	...	...	...	...	...	...	...	...	...	...	...	....
15	5–7	5–6	5–6	5–5	...	...	...	...	...	...	...	...	...	...	...	...	....
16	4–7	5–7	5–6	5–6	5–5	...	...	...	...	...	...	...	...	...	...	...	....
17	4–7	4–7	4–6	5–6	5–5	5–5	...	...	...	...	...	...	...	...	...	...	....
18	4–7	4–7	4–6	4–6	4–5	4–5	5–5	...	...	...	...	...	...	...	...	...	....
19	3–7	4–7	4–6	4–6	4–6	4–5	4–5	4–4	...	...	...	...	...	...	...	...	....
20	3–7	3–7	4–6	4–6	4–6	4–5	4–5	4–4	4–4	...	...	...	...	...	...	...	....
21	3–8	3–7	3–7	3–6	4–6	4–5	4–5	4–5	4–4	4–4	...	...	...	...	...	...	....
22	3–8	3–7	3–7	3–6	3–6	3–5	4–5	4–5	4–4	4–4	4–4	...	...	...	...	...	....
23	3–8	3–7	3–7	3–6	3–6	3–5	3–5	3–5	3–4	4–4	4–4	4–4	...	...	...	...	....
24	3–8	3–7	3–7	3–6	3–6	3–6	3–5	3–5	3–4	3–4	3–4	4–4	4–4	...	...	...	....
25	2–8	2–7	3–7	3–6	3–6	3–6	3–5	3–5	3–5	3–4	3–4	3–4	4–4	3–3	...	...	....
26	2–8	2–7	3–7	3–6	3–6	3–6	3–5	3–5	3–5	3–4	3–4	3–4	3–4	3–3	3–3	...	....
27	2–8	2–7	2–7	2–6	3–6	3–6	3–5	3–5	3–5	3–4	3–4	3–4	3–4	3–4	3–3	3–3	....
28	2–8	2–7	2–7	2–6	2–6	3–6	3–5	3–5	3–5	3–4	3–4	3–4	3–4	3–4	3–3	3–3	3–3
29	2–8	2–7	2–7	2–7	2–6	2–6	3–5	3–5	3–5	3–4	3–4	3–4	3–4	3–4	3–3	3–3	3–3
30	2–8	2–7	2–7	2–7	2–6	2–6	2–5	2–5	3–5	3–5	3–4	3–4	3–4	3–4	3–4	3–3	3–3
31	2–8	2–7	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	3–4	3–4	3–4	3–4	3–4	3–3	3–3
32	2–8	2–7	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–4	2–4	3–4	3–4	3–4	3–3	3–3
33	2–8	2–8	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–4	2–4	2–4	3–4	3–4	3–4	3–3
34	2–8	2–8	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4	3–4	3–3
35	2–8	2–8	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4	2–4	2–3
36	2–8	2–8	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4	2–3
37	2–8	2–8	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4	2–3
38	2–8	2–8	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4	2–4
39	2–8	2–8	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4	2–4
40	1–8	2–8	2–7	2–7	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4	2–4
41	1–8	1–8	2–7	2–7	2–6	2–6	2–6	2–6	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4	2–4
42	1–8	1–8	2–7	2–7	2–6	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
43	1–8	1–8	1–7	2–7	2–6	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
44	1–8	1–8	1–7	1–7	2–6	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
45	1–8	1–8	1–7	1–7	1–6	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
46	1–8	1–8	1–7	1–7	1–7	2–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
47	1–8	1–8	1–7	1–7	1–7	1–6	2–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
48	1–8	1–8	1–7	1–7	1–7	1–6	1–6	2–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
49	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
50	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	2–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
51	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–5	2–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
52	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–5	1–5	2–5	2–5	2–4	2–4	2–4	2–4	2–4
53	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–5	1–5	1–5	2–5	2–4	2–4	2–4	2–4	2–4
54	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–5	1–5	1–5	1–5	2–4	2–4	2–4	2–4	2–4
55	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	2–4	2–4	2–4	2–4

For bevel gears with axes at right angles.
Number of cutter for gear given first, followed by number for pinion.

Number of Cutters for Cutting Bevel Gears

Number of Teeth in Gear	Number of Teeth in Pinion																
	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
56	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	2–4	2–4	2–4
57	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	2–4	2–4
58	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4	2–4
59	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
60	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
61	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
62	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
63	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
64	1–8	1–8	1–7	1–7	1–6	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
65	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
66	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
67	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
68	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
69	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
70	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
71	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
72	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
73	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
74	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
75	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
76	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
77	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
78	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
79	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
80	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
81	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
82	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
83	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
84	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
85	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
86	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
87	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
88	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
89	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
90	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
91	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
92	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
93	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
94	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
95	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
96	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
97	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
98	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
99	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4
100	1–8	1–8	1–7	1–7	1–7	1–6	1–6	1–6	1–6	1–5	1–5	1–5	1–5	1–4	1–4	1–4	1–4

For bevel gears with axes at right angles.
Number of cutter for gear given first, followed by number for pinion.

General Considerations Relating to Design of Bevel Gears. — The performance of the most carefully designed and made bevel gears depends to a considerable extent on the design of the machine in which they are used. When the shafts on which a pair of bevel gears are mounted are poorly supported or poorly fitted in their bearings, the pressure of the driving gear on the driven causes it to climb upon the latter, throwing the shafts out of alignment. This in turn causes the teeth to bear with a greater pressure at one end of the face (usually at the outer end) than at the other, thus making the tooth more liable to break than is the case when the pressure is more evenly distributed. It is important, therefore, to provide rigid shafts and bearings and careful workmanship for bevel gearing.

Proportions of Bevel Gears. — Various forms may be given to the blanks or wheels on which bevel gear teeth are cut, depending on the size, material, service, etc., to be provided for. The pinion type of blank is mostly used for gears of a small number of teeth and small pitch cone angle. When the diameter of the bore comes too near to the bottoms of the teeth at the small end, it is customary to omit the usual recess in the front face. For gears of a larger number of teeth, the web type is appropriate. This does not require to be finished all over, as the sides of the web, the outside of the hub, and the under side of the rim may be rough.

A gear suitable for very heavy work should have the web reinforced by ribs. The web may be cut out so that the rim is supported by T-shaped arms. This makes a very stiff wheel and at the same time a very light one, in proportion to its strength.

The question of alignment of the shafts should be considered in deciding upon the width of face of the gear. Making the width of the face more than one-third of the pitch cone radius adds practically nothing to the strength of the gear, since the added portion is progressively weaker as the tooth is lengthened. In addition to this, there is the danger that through springing of the shafts or poor workmanship, the load will be thrown onto the weak end of the tooth, thus fracturing it. For this reason it may be laid down as a definite rule that there is nothing to be gained by making the face of the bevel gear more than one-third of the pitch cone radius.

The Brown & Sharpe Mfg. Co., in one of its publications, gives a rule for the maximum width of face allowable for a given pitch. The width of face should not exceed 2½ times the circular pitch, or 8 divided by the diametral pitch. This rule is rational, since the danger to the teeth from the misalignment of the shafts increases both with the width of face and with the decrease of the size of the tooth, so that both of these should be reckoned with. In designing gearing it is well to check the width of face from the rule relating to the pitch cone radius and that relating to the pitch as well, to see that it does not exceed the maximum allowed by either.

Table for Selecting Cutters for Bevel Gears. — This table gives the number of cutter to use for various numbers of teeth in the gear and pinion. It applies to bevel gears with axes at right angles only. The number of the cutter for the gear is given first, followed by the number for the pinion.

Example. — Required the cutters for a pair of bevel gears where the gear has 24 teeth and the pinion, 12 teeth. The table shows that the numbers of the cutters to be used are Nos. 3 and 8, No. 3 being for the gear, and No. 8 for the pinion.

For 14½-degree involute teeth, the standard cutter series furnished by the makers of formed gear cutters is commonly used. There are 8 cutters in the series, to cover the full range from the 12-tooth pinion to a crown gear. The standard bevel gear cutter is made thinner than the standard spur gear cutter, as it must pass through the narrow tooth space at the inner end of the face. As usually kept in stock, these cutters are thin enough for bevel gears in which the width of face is not more than one-third the pitch cone radius.

Offset of Cutter for Milling Bevel Gears. — When milling bevel gears with a rotary formed cutter, it is necessary to take two cuts through each tooth space with the gear blank slightly off center, first on one side and then on the other, to obtain a tooth of approximately the correct form. The gear blank is also rotated proportionately to obtain the proper tooth thickness at the large and small ends. The amount that the gear blank or cutter should be offset from the central position can be determined quite accurately by the use of the table "Factors for Obtaining Offset for Cutting Bevel Gears," in conjunction with the following rule: Find the factor in the table corresponding to the number of cutter used and to the ratio of the pitch cone radius to the face width; then divide this factor by the diametral pitch and subtract the result from half the thickness of the cutter at the pitch line.

Factors for Obtaining Offset for Cutting Bevel Gears

No. of Cutter	Ratio of Pitch Cone Radius to Width of Face $\left(\frac{C}{F}\right)$												
	$\frac{3}{1}$	$\frac{3\frac{1}{4}}{1}$	$\frac{3\frac{1}{2}}{1}$	$\frac{3\frac{3}{4}}{1}$	$\frac{4}{1}$	$\frac{4\frac{1}{4}}{1}$	$\frac{4\frac{1}{2}}{1}$	$\frac{4\frac{3}{4}}{1}$	$\frac{5}{1}$	$\frac{5\frac{1}{2}}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$
1	0.254	0.254	0.255	0.256	0.257	0.257	0.257	0.258	0.258	0.259	0.260	0.262	0.264
2	0.266	0.268	0.271	0.272	0.273	0.274	0.274	0.275	0.277	0.279	0.280	0.283	0.284
3	0.266	0.268	0.271	0.273	0.275	0.278	0.280	0.282	0.283	0.286	0.287	0.290	0.292
4	0.275	0.280	0.285	0.287	0.291	0.293	0.296	0.298	0.298	0.302	0.305	0.308	0.311
5	0.280	0.285	0.290	0.293	0.295	0.296	0.298	0.300	0.302	0.307	0.309	0.313	0.315
6	0.311	0.318	0.323	0.328	0.330	0.334	0.337	0.340	0.343	0.348	0.352	0.356	0.362
7	0.289	0.298	0.308	0.316	0.324	0.329	0.334	0.338	0.343	0.350	0.360	0.370	0.376
8	0.275	0.286	0.296	0.309	0.319	0.331	0.338	0.344	0.352	0.361	0.368	0.380	0.386

Note. — For obtaining offset by above table, use formula:

$$\text{Offset} = \frac{T}{2} - \frac{\text{factor from table}}{P}$$

P = diametral pitch of gear to be cut;

T = thickness of cutter used, measured at pitch line.

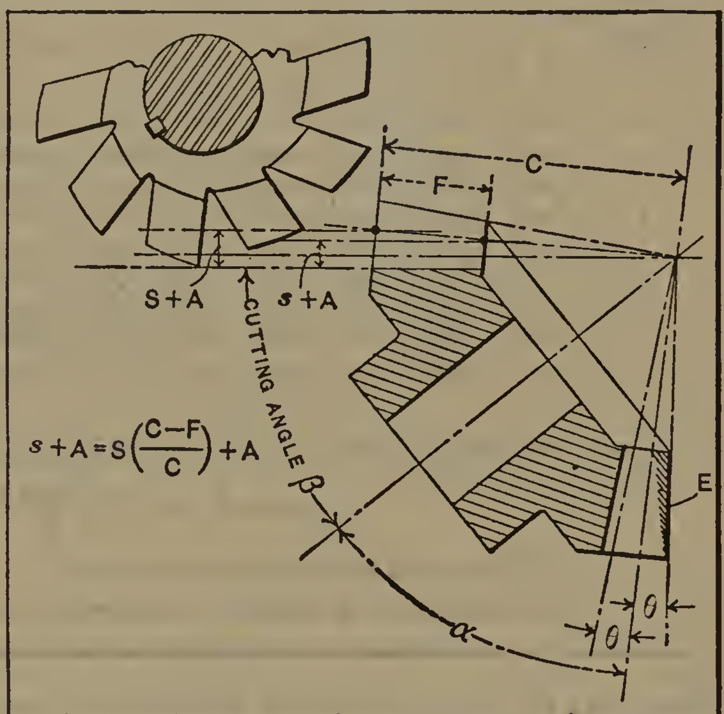
Expressing this rule as a formula, in which O = amount of offset; T = thickness of cutter at pitch line; P = diametral pitch of gear; F = factor from table, we have

$$O = \frac{T}{2} - \frac{F}{P}$$

To illustrate, what would be the amount of offset for a bevel gear having 24 teeth, 6 pitch, 30-degree pitch cone angle and 1¼ inch face or tooth length? In order to obtain a factor from the table, the ratio of the pitch cone radius to the face width must be determined. The pitch cone radius equals the pitch diameter divided by twice the sine of the pitch cone angle $= \frac{4}{2 \times 0.5} = 4$ inches. As the face width is 1.25, the ratio is $\frac{4}{1.25} = \frac{3.2}{1}$ or about 3¼. The factor in the table for this ratio is 0.280 with a No. 4 cutter, which would be the cutter number for this particular gear. The thickness of the cutter at the pitch line is measured by using a vernier gear tooth caliper. The depth $S + A$ (see illustration; S = addendum; A = clearance) at which to take the measurement equals 1.157 divided by the diametral pitch; thus, $1.157 \div 6 = 0.1928$ inch. The cutter thickness at this depth will vary with different cutters and even with the same cutter as it is ground away, because formed bevel gear cutters are commonly provided with side relief. Assuming that the thickness is 0.1745 inch, and substituting the values in the formula given, we have:

$$\text{Offset} = \frac{0.1745}{2} - \frac{0.280}{6} = 0.0406.$$

Adjusting the Gear Blank for Milling. — After the offset is determined, the blank is adjusted laterally this amount, and the tooth spaces are milled around the blank. After having milled one side of each tooth to the proper dimensions, the blank is set over in the opposite direction the same amount from a position central with the cutter, and is rotated to line up the cutter with a tooth space at the small end. A trial cut is then taken, which will leave the tooth being milled a little too thick, provided the cutter is thin enough — as it should be — to pass through the small end of the tooth space of the finished gear. This trial tooth is made the proper thickness by rotating the blank toward the cutter. To test the amount of offset measure the tooth thickness (with a vernier caliper) at the large and small ends. The caliper should be set so that the addendum at the small end is in proper proportion to the addendum at the large end; that is, in the ratio, $\frac{C-F}{C}$ (see illustration). In taking these measurements, if the thicknesses at both ends (which should be in this same ratio) are too great, rotate the tooth toward the cutter and take trial cuts until the proper thickness at either the large or small end is obtained. If the large end of the tooth is the right thickness and the small end too thick, the blank was offset too much; inversely, if the small end is correct and the large end too thick, the blank was not set enough off center, and, in either case, its position should be changed accordingly. The formula and table previously referred to will enable a properly turned blank to be set accurate enough for general work. The dividing head should be set to the cutting angle β (see illustration), which is found by subtracting the addendum angle θ from the pitch cone angle α. After cutting a bevel gear by the method described, the sides of the teeth at the small end should be filed as indicated by the shade lines at E; that is, by filing off a triangular area from the point of the tooth at the large end to the point at the small end, thence down to the pitch line and back diagonally to a point at the large end.



Long Addendum Bevel Gears. — In order to obtain more quietly running gears and greater strength and durability, especially in automobile rear axle bevel gear transmissions, the common practice has been to lengthen the addendum of the pinion tooth and shorten the addendum of the gear tooth a corresponding amount, the whole depth of the tooth remaining the same as for standard involute teeth. With this system, a smaller number of pinion teeth can be used than would be possible with teeth of standard proportions. The addendum of the pinion tooth equals 0.7 of the working depth and the gear tooth addendum equals 0.3 of the working depth both for 14½- and 20-degree pressure angles. (This Gleason 0.7 and 0.3 standard has been supplemented by another system which is briefly described in the following paragraph.) The circular thickness of the tooth at the pitch line, using these dimensions, is 0.566 times the circular pitch for the pinion, and 0.434 times the circular pitch for the gear, for the 14½-degree pressure angle. For the 20-degree pressure angle, the circular thickness is 0.5927 times the circular pitch for the pinion, and 0.4073 times the circular pitch for the gear. For gears of this type the 20-degree pressure angle is extensively used. The long addendum pinion must always be the driver.

Gleason Works System of Bevel Gears. — The system of bevel gears introduced by the Gleason Works, Rochester, N. Y., is designed to give the quietest form of tooth consistent with strength and wearing qualities. With this system the gear addendum is decreased and the pinion addendum is increased as the ratios of the numbers of teeth in the gear and pinion become greater. The basis of the system is in using the lowest pressure angle that can be employed without sacrificing strength by introducing excessive under-cut. The use of a low pressure angle in preference to a higher one does not reduce the effective strength to the extent ordinarily supposed, because the stronger tooth section of the higher pressure angle is offset by the greater arc of action with the lower angle. The Gleason system has three pressure angles of 14½, 17½, and 20 degrees for all straight-tooth bevel gears having ten or more teeth in the pinion, and one angle of 14½ degrees for spiral bevel gears, except in a few special cases. This system is applicable to any pair of generated spiral- or straight-tooth bevel gears operating at right angles, where the pinion is the driver and, in the case of straight tooth gearing, has ten or more teeth; spiral bevel gears having less than ten teeth are included in the system.

Corrected Pitch Depth for Bevel Gears. — In order to measure the theoretically correct thicknesses of a bevel gear tooth, the "corrected pitch depth" or "corrected addendum" and the chordal thickness must be determined. The chordal thickness is the same as for a spur gear having the same number of teeth and pitch (see formula on page 638). The corrected pitch depth for a bevel gear is obtained by the following formula in which H = corrected pitch depth; S = addendum; R = pitch radius; α = pitch cone angle; b = angle subtended by lines from the apex of the back cone, which intersect one side and the center of the tooth respectively, at the pitch line.

$$H = S + \frac{R}{\cos\alpha}(1 - \cos b)$$

The sine of angle b is found by the following formula in which N = the number of teeth:

$$\text{Sin}\, b = \sin\left(\frac{90}{N}\right)\cos\alpha$$

Recommended Practice for Bevel Gearing. — The American Gear Manufacturers' Association adopted as recommended practice the following rules:

The maximum length of face of bevel gears should not be over one-third of the cone distance for gears up to 3 inches pitch diameter and not over one-quarter of the cone distance for gears from 3 to 20 inches pitch diameter, assuming that the pitch in every case will be in proper proportion to the size of the gears. A safe rule is to make the face from 1½ to 2½ times the circular pitch.

The minimum length of bearing along the face is to be at least one-half the length of the face when the gears are held in correct alignment. Gear users should test the strength of the gear housings as follows: Block the driven gear shaft and then apply a lever to the shaft of the driving gear and measure the displacement of both gears when a dead load is applied through the lever. This measurement can be made by inserting feelers or shims on the driving side of both ends of the teeth before and after the load is applied.

Bevel gears with generated involute teeth of standard addendum, having a pressure angle of 14½ degrees, may be used according to the following rule:

Ratio	Number of Teeth
1 to 1	14 and over
1½ to 1	18 and over
2 to 1	19 and over
3 to 1 and over	21 and over

The rules as given apply mainly to gears up to 20 inches pitch diameter and to average machine design as distinguished from gears for automobiles.

Strength of Bevel Gears. — The method for calculating the strength of bevel gearing is based on the Lewis formula for the strength of gear teeth, and is practically the same as that used for spur gears. The accompanying tables of "Rules and Formulas for the Strength of Bevel Gears," and "Factors for Calculating the Strength of Bevel Gears," in combination with the table "Working Stresses used in the Lewis Formula for the Strength of Gear Teeth," in the section on spur gearing, give all the necessary data for calculating the strength of bevel gears. The formulas and factors given are based on the use of the diametral pitch of the gears, and constants Y given in the factor table are figured for diametral pitch. If the circular pitch is given, it should be transformed into diametral pitch by dividing 3.1416 by the circular pitch. By means of the Formulas (1) to (4), the horsepower which can be transmitted by a gear of given pitch diameter and diametral pitch, running at a given number of revolutions per minute, can be found. The formulas should be used in the order given. The allowable static unit stress S_s is found from the first line (velocity = 0) in the table of "Working Stresses used in the Lewis Formula for the Strength of Gear Teeth" given in the section on spur gearing. The allowable working stress S at any velocity may also be found directly from the table.

As an example, find the horsepower which can be safely transmitted by a bevel gear having 15 inch pitch diameter, 60-degree pitch cone angle, 4 diametral pitch, making 100 revolutions per minute and having a width of face of 1½ inch. The teeth are cut according to the 14½-degree involute system. The gear is made of steel and the allowable static unit stress for the material, as found from the table of working stresses in the section on spur gearing, may be assumed to be 15,000 pounds per square inch. First insert the values of the pitch diameter and the revolutions per minute in Formula (1). This gives the velocity in feet per minute at the pitch diameter. Then insert this velocity, as found in Formula (1), together with the allowable static unit stress, in Formula (2), thus finding the allowable unit stress at the given velocity. This unit stress is now inserted, together with the width of face, the outline factor Y (found from the factor table to be about 0.373 for $N' = 60 \div \cos 60° = 120$), the factor $\frac{C-F}{C}$ and the diametral pitch, in Formula (3). This will then give the maximum safe tangential load W. Finally, by inserting the value of W just found and the value of V found from Formula (1), in Formula (4), determine the maximum safe horsepower which can be transmitted by the gear.

The factor $\frac{C-F}{C}$ approximately expresses the ratio of the strength of a bevel gear to that of a spur gear of the same pitch and number of teeth. The smaller strength of the bevel gear is due to the fact that the pitch grows finer towards the vertex of the gear. The factor is approximate only, and should not be used for cases where the width of face is more than one-third of the pitch cone radius C;

but since no bevel gears should be made with width F more than one-third of radius C (see illustration with table), the rule is of universal application for good practice.

As an example showing the application of the rules and formulas for the strength of bevel gears, calculate the maximum load at the pitch line which can be safely carried by a pair of bevel gears of 3 diametral pitch, 20-degree involute teeth, the pitch diameter of the pinion being 5 inches, and of the gear, 20 inches. The maximum allowable static stress for the pinion (made of machine steel) is 20,000 pounds,

Rules and Formulas for the Strength of Bevel Gears

D = pitch diameter of gear in inches;
R = revolutions per minute;
V = velocity in ft. per min. at pitch diam.;
S_s = allowable static unit stress for material;
S = allowable unit stress for material at given velocity;
F = width of face;
N' = No. of teeth in equivalent spur gear (see diagram in table, page 641);
Y = outline factor (see table, page 641);
P = diametral pitch (if circular pitch is given, divide 3.1416 by circular pitch to obtain diametral pitch);
C = pitch cone radius;
W = maximum safe tangential load in pounds at pitch diameter;
H.P. = maximum safe horsepower.

Use Rules and Formulas Nos. 1 to 4 in the order given.

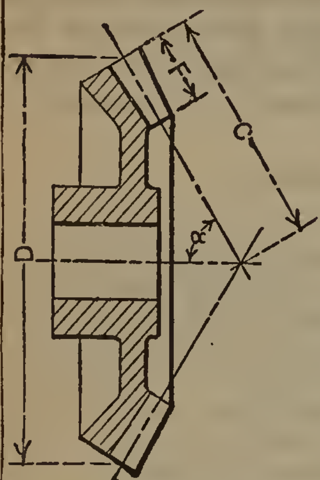
No.	To Find	Rule	Formula
1	Velocity in Feet per Minute at the Pitch Diameter.	Multiply the product of the diameter in inches and the number of revolutions per minute by 0.262.	$V = 0.262\,DR$
2	Allowable Unit Stress at given Velocity.	Multiply the allowable static stress by 600 and divide the result by the velocity in feet per minute plus 600.	$S = S_s \times \frac{600}{600 + V}$
3	Maximum Safe Tangential Load at Pitch Diameter.	Multiply together the allowable stress for the given velocity, the width of face, the tooth outline factor and the difference between the pitch cone radius and the width of face; divide the result by the product of the diametral pitch and the pitch cone radius.	$W = \frac{SFY\,(C-F)}{PC}$
4	Maximum Safe Horsepower.	Multiply the safe load at the pitch line by the velocity in feet per minute, and divide the result by 33,000.	$\text{H.P.} = \frac{WV}{33{,}000}$

and for the gear (made of cast iron), 8000 pounds per square inch. The pinion runs at 300 revolutions per minute. The width of face is 4 inches, and the pitch cone radius, 10.3 inches. Pitch cone angle of pinion, 14 degrees, and of gear 76 degrees. The calculations for the pinion are as follows:

$$N' = \frac{15}{\cos 14^\circ} = 15.5, \text{ approx.}$$

$$V = 0.262 \times 5 \times 300 = 400 \text{ feet per minute (about)} \qquad (1)$$

Factors for Calculating Strength of Bevel Gears



$$N' = \frac{\text{Number of teeth}}{\cos \alpha}$$

Table of Outline Factors (Y) for 14½° and 20° Involute

N'	Outline Factor = Y 14½° Involute (Std.)	Outline Factor = Y 20° Involute	N'	Outline Factor = Y 14½° Involute (Std.)	Outline Factor = Y 20° Involute
12	0.210	0.245	27	0.314	0.349
13	0.220	0.261	30	0.320	0.358
14	0.226	0.276	34	0.327	0.371
15	0.236	0.289	38	0.336	0.383
16	0.242	0.295	43	0.346	0.396
17	0.251	0.302	50	0.352	0.408
18	0.261	0.308	60	0.358	0.421
19	0.273	0.314	75	0.364	0.434
20	0.283	0.320	100	0.371	0.446
21	0.289	0.327	150	0.377	0.459
23	0.295	0.333	300	0.383	0.471
25	0.305	0.339	Rack	0.390	0.484

$$S = 20{,}000 \times \frac{600}{600 + 400} = 12{,}000 \text{ pounds per square inch} \qquad (2)$$

$$W = \frac{12{,}000 \times 4 \times 0.292 \times 6.3}{3 \times 10.3} = 2850 \text{ pounds} \qquad (3)$$

For the gear, the velocity is the same as for the pinion. The necessary calculations are as follows:

$$N' = \frac{60}{\cos 76^\circ} = 250, \text{ approx.}$$

$$S = 8000 \times \frac{600}{600 + 400} = 4800 \text{ pounds per square inch} \qquad (2)$$

$$W = \frac{4800 \times 4 \times 0.467 \times 6.3}{3 \times 10.3} = 1830 \text{ pounds} \qquad (3)$$

The gear is, therefore, the weaker of the two, and thus limits the allowable tooth pressure. The maximum horsepower this gearing will transmit safely is found as follows:

$$\text{H.P.} = \frac{1830 \times 400}{33{,}000} = 22, \text{ approx.} \qquad (4)$$

Durability is practically of as much importance as strength in proportioning bevel gears, but unfortunately no data are as yet available for making satisfactory comparisons of durability.

Simplified Formulas for the Strength of Bevel Gears. — In the section on spur gearing, simplified formulas for the strength of these gears were given by means of which the circular pitch to transmit a given horsepower at a given speed could be found. Formulas similar in form to the spur gear formulas are given in the following. These formulas are based on the assumption that the pinion (for which the strength usually is calculated) has 15 teeth. If the number of teeth in the pinion is other than 15, multiply the horsepower, as given in the formulas below, by $(0.027\,N + 0.6)$, in which N = number of teeth. In the formulas:

P' = circular pitch;
H.P. = horsepower to be transmitted;
R.P.M. = revolutions per minute of the pinion;
D = pitch diameter of 15-tooth pinion.

	Stress according to the Lewis Tables	Stress ⅔ of that given in the Lewis Tables
Cast-iron Bevel Gear	$P' = \sqrt[5]{\dfrac{5.0\,\text{H.P.}^2}{\text{R.P.M.}}}$	$\sqrt[5]{\dfrac{11.0\,\text{H.P.}^2}{\text{R.P.M.}}}$
Cast-steel Bevel Gear	$P' = \sqrt[5]{\dfrac{0.8\,\text{H.P.}^2}{\text{R.P.M.}}}$	$\sqrt[5]{\dfrac{1.8\,\text{H.P.}^2}{\text{R.P.M.}}}$

$$D = 4.77\,P'$$

The formula in the column headed "Stress According to the Lewis Tables" is used for ordinary conditions and steady drive. The column headed "Stress Two-thirds of that given in Lewis Tables" should be used for gears subjected to rapidly varying loads or where the drive is often started and stopped.

Spiral Type Bevel Gears. — This type of gearing is similar to ordinary bevel gearing except that the teeth are curved and are at an angle to the pitch cone radius of the gear. The shape of the tooth which is actually produced is not a spiral or curve in one plane, but, when projected on a plane, is close to a true spiral; hence the name "spiral type bevel gear." The tooth curve, if continued, would intersect the cone center of the gear, which distinguishes the spiral type bevel gear from the "skew bevel gear" in which the tooth elements are straight and do not intersect the cone center, if extended. The spiral type bevel gear is adapted for high-ratio drives; 10-, 11-, and 12-tooth pinions are used in ratios up to 6 to 1 with satisfactory results.

Angle of Spiral. — An angle of spiral which gives a lead of about 1¼ to 1½ times the circular pitch within the width of face has given good results and quiet operation. For 5 diametral pitch and coarser, the angle should be about 30 degrees. For finer pitches, the angle should be between 25 and 30 degrees. The angles may be increased somewhat for very long faces and decreased for very short faces. An angle greater than 35 degrees, however, is not recommended in any case, nor should an angle less than 20 degrees be used.

Right- and Left-hand Gears. — Spiral type bevel gears are either right-hand or left-hand according to the direction of the pinion's spiral. For instance, if the pinion is cut with a right-hand, and the mating gear with a left-hand, spiral, the pair is called "right-hand." The direction of spiral of the pinion is named the same as if it were a threaded screw.

Cutting Spiral Type Bevel Gears. — The cutter used on the Gleason spiral-type bevel gear generator is in the form of a face mill, which holds quite a number of blades or cutters which pass through the blank along a curved path. The same cutter is used for both sides of the tooth and for both pinion and gear. The gear and the cutter roll together, giving correctly generated teeth of involute form. The long and short addendum teeth are used for spiral type bevel gears in order to avoid undercutting in the pinion. The pitch depth of the pinion teeth is made

0.7 of the working depth, and the pitch depth of the gear teeth, 0.3 of the working depth. This determines the face angles and outside diameters of the blanks. On the gear blank, the flank depth will be increased, and care must be taken that there is sufficient metal under the teeth. The same values are used for pitch (either diametral or circular), pitch angle, face angle, root angle, pitch diameter, and outside diameter, as for straight-tooth bevel gearing.

Load Capacity and Wear. — The load-carrying capacity of spiral type bevel gears is practically the same as for straight-tooth gears, and there is no greater tooth wear than in a straight-tooth gear. The only feature tending to change the efficiency of the spiral type bevel gear, as compared with straight bevel gears, is the increased end-thrust, and if this is properly taken care of in the bearings, the decrease in efficiency is negligible. On the other hand, the smoothness of the gears tends to increase their efficiency, offsetting any loss due to the thrust.

End Thrust of Spiral Type Bevel Gears. — When mounting spiral bevel gears, the additional end-thrust caused by the spiral tooth must be considered. This end-thrust is the most important in connection with the pinion and should be provided for in both directions. If the transmitted tooth load (torque) is known, the thrust can be determined approximately by multiplying the transmitted tooth load by a factor C obtained from the accompanying table, "Factors for End-thrust Calculations." These constants are average results for ratios between 3 to 1 and 7 to 1.

If E=end-thrust of pinion; H=number of horsepower to be transmitted; D=pitch diameter, in feet; S=speed of pinion in revolutions per minute; C=factor obtained from table; then:

$$E = \frac{H \times 33{,}000 \times C}{\pi \times D \times S}$$

Values of C with a negative sign indicate pull toward the cone center, as in the case of a pinion having a right-hand spiral and revolving clockwise, as seen from the large end; values with a positive sign indicate thrust away from the gear. The value used for transmitted tooth load or torque is to be figured for the full pitch diameter of the pinion, not the mean diameter.

Example. — Assume that 25 horsepower is to be transmitted; the pinion makes 500 revolutions per minute and has 15 teeth of 5 diametral pitch and a 30-degree angle of spiral. Find the thrust for a right-hand spiral forward drive.

The pitch diameter equals $15 \div 5 = 3$ inches or 0.25 feet; factor C for a right-hand spiral forward drive and 30-degree angle, equals 0.56, and the minus sign preceding it in the table indicates that the thrust will be toward the cone center. Therefore:

$$E = \frac{25 \times 33{,}000 \times 0.56}{3.1416 \times 0.25 \times 500} = 1176 \text{ pounds.}$$

Factors for End-thrust Calculations

Direction of Spiral	Angle of Spiral and Factors C			
	20°	25°	30°	35°
Right-hand Spiral — forward	−0.29	−0.44	−0.56	−0.69
Right-hand Spiral — reverse	+0.41	+0.59	+0.72	+0.85
Left-hand Spiral — forward	+0.41	+0.59	+0.72	+0.85
Left-hand Spiral — reverse	−0.29	−0.44	−0.56	−0.69

Advantages of Spiral Type Bevel Gears. — Spiral type bevel gears are quiet when running, as the engagement is gradual. The teeth, instead of striking a full-line contact at once, roll on each other with an action which suggests the smoothness of a worm and worm-wheel; nevertheless, the action is purely a rolling action, as in the case of straight-tooth gears. Another point of advantage is the range of endwise adjustment of the pinion, which is possible without causing noise or spoiling the bearing of the teeth. This range of adjustment is greater than that obtained with straight-tooth bevel gears.

Skew Bevel Gears. — Skew bevel gears are used instead of regular bevel gears to connect two shafts which are offset or not in the same plane, and which are so close together that spiral or worm gearing cannot be satisfactorily applied. Skew bevel gears have straight teeth which bear on each other along a straight line, but these teeth do not converge to a common center or apex as in the case of ordinary bevel gears. In place of a single apex, there is a circle of apexes, so that accurate mathematical calculations of the various angles are complicated and involved. However, close approximations of the true angles and diameters can be arrived at by the use of simple formulas (see Machinery's Encyclopedia, Vol. V., page 367).

Types of Skew Bevel Gears. — There are two general types of skew bevel gears. One type has an ordinary bevel gear pinion, the oblique teeth being confined to the gear. The teeth of the other type are oblique in both the pinion and the gear. In each type, the pitch surface of the pinion is the frustum of a figure generated by the revolution of a straight line about the axis of the figure, the generating line lying in a plane parallel to a plane through the axis, and being neither parallel nor at right angles to the axis, *i.e.*, the figure is a hyperboloid of revolution.

Worm Gearing

Formulas for Worm Gearing. — Worm gearing is employed for two purposes: first, when power is to be transmitted under conditions making smoothness of action and a great reduction in velocity desirable, and in the second place, when a great increase in the effective power is required. In such cases, advantage is generally taken of the possibility of making the gearing self-locking. The service in the latter case is usually intermittent, and the waste of power due to the decreased efficiency of self-locking worm gearing is of less importance than for a steady drive. The table, "Rules and Formulas for Worm Gearing," gives all the directions necessary for calculating the dimensions. The notation used in the formulas is as follows:

P = circular pitch of wheel and linear pitch of worm;
l = lead of worm;
n = number of teeth or threads in worm;
S = addendum, or height of worm tooth above pitch line;
d = pitch diameter of worm;
D = pitch diameter of worm-wheel;
o = outside diameter of worm;
O = throat diameter of worm-wheel;
O' = outside diameter of worm-wheel (to sharp corners);
b = bottom or root diameter of worm;
N = number of teeth in worm-wheel;
W = whole depth of worm tooth;
T = width of thread tool at end;
α = face angle of worm-wheel;
β = helix angle of worm and gashing angle of wheel;
U = radius of curvature of worm-wheel throat;
C = distance between centers;
x = threaded length of worm.

When the number of threads n in the worm is spoken of, the number of threads per inch is not referred to, but the number of threads in the whole worm; that is, one, if it is single-threaded, four, if it is quadruple-threaded, etc.

Rules and Formulas for Worm Gearing

No.	To Find	Rule	Formula
1	Linear Pitch.	Divide the lead by the number of threads. — It is understood that by the number of threads is meant, not number of threads per inch, but the number of threads in the whole worm — one, if it is single-threaded, four, if it is quadruple-threaded, etc.	$P = \frac{l}{n}$
2	Addendum of Worm Tooth.	Multiply the linear pitch by 0.3183.	$S = 0.3183\,P$
3	Pitch Diameter of Worm.	Subtract twice the addendum from the outside diameter.	$d = o - 2\,S$
4	Pitch Diameter of Worm-wheel.	Multiply the number of teeth in the wheel by the linear pitch of the worm, and divide the product by 3.1416.	$D = \frac{NP}{3.1416}$
5	Center Distance between Worm and Gear.	Add together the pitch diameter of the worm and the pitch diameter of the worm-wheel, and divide the sum by 2.	$C = \frac{D + d}{2}$
6	Whole Depth of Worm Tooth	Multiply the linear pitch by 0.6866.	$W = 0.6866\,P$
7	Bottom Diameter of Worm.	Subtract twice the whole depth of tooth from the outside diameter.	$b = o - 2\,W$
8	Helix Angle of Worm.	Multiply the pitch diameter of the worm by 3.1416, and divide the product by the lead; the quotient is the cotangent of the helix angle of the worm.	$\cot \beta = \frac{3.1416\,d}{l}$
9	Width of Thread Tool at End.	Multiply the linear pitch by 0.31.	$T = 0.31\,P$
10	Throat Diameter of Worm-wheel.	Add twice the addendum of the worm tooth to the pitch diameter of the worm-wheel.	$O = D + 2\,S$
11	Radius of Worm-wheel Throat.	Subtract twice the addendum of the worm tooth from half the outside diameter of the worm.	$U = \frac{o}{2} - 2\,S$
12	Diameter of Worm-wheel to Sharp Corners.	Multiply the radius of curvature of the worm-wheel throat by the cosine of half the face angle, subtract this quantity from the radius of curvature, multiply the remainder by 2, and add the product to the throat diameter of the worm-wheel.	$O' = 2\left(U - U \times \cos\frac{\alpha}{2}\right) + O$
13	Minimum Length of Worm for Complete Action.	Subtract four times the addendum of the worm thread from the throat diameter of the wheel, square the remainder, and subtract the result from the square of the throat diameter of the wheel. The square root of the result is the minimum length of worm advisable.	$x = \sqrt{O^2 - (O - 4S)^2}$
14	Outside Diameter of Worm.	Add together the pitch diameter and twice the addendum.	$o = d + 2\,S$
15	Pitch Diameter of Worm.	Subtract the pitch diameter of the worm-wheel from twice the center distance.	$d = 2\,C - D$

Careful distinction must be made between the terms "pitch" and "lead." The term lead is defined as the distance which any one thread advances in one revolution of the worm, while the pitch, or, more correctly, the linear pitch, is the distance between the centers of two adjacent threads. The lead and pitch are equal for a single-threaded worm. For a double-threaded worm the lead is twice the linear pitch, and for a triple-threaded worm, three times the linear pitch.

The rules and formulas given are sometimes departed from. The throat diameter of the wheel and the center distance may have to be altered in some cases. For example, if worm-wheels with small numbers of teeth are made to the dimensions found from the rules and formulas, it will be found that the flanks of the teeth will be partly cut away by the tops of the hob teeth, so that a full bearing area is not available. This latter affects the worm-wheel drive seriously when there are less than twenty-five teeth in the worm-wheel. There are two ways of avoiding this difficulty. One is to increase the included angle of the sides of the worm thread and the angle of the hob. This departure from the standard form, however, may be avoided by an increase in the throat diameter of the wheel, and consequently in the center distance. (See paragraph, "Data for Turning Worm-wheel.") Some designers, again, claim to obtain better results in efficiency and durability by making the throat diameter of the worm-wheel smaller than the standard, when it is possible to do so without too much under-cutting. In no case, however, should the throat diameter be made so small as to produce more interference than is met with in a standard 25-tooth worm-wheel.

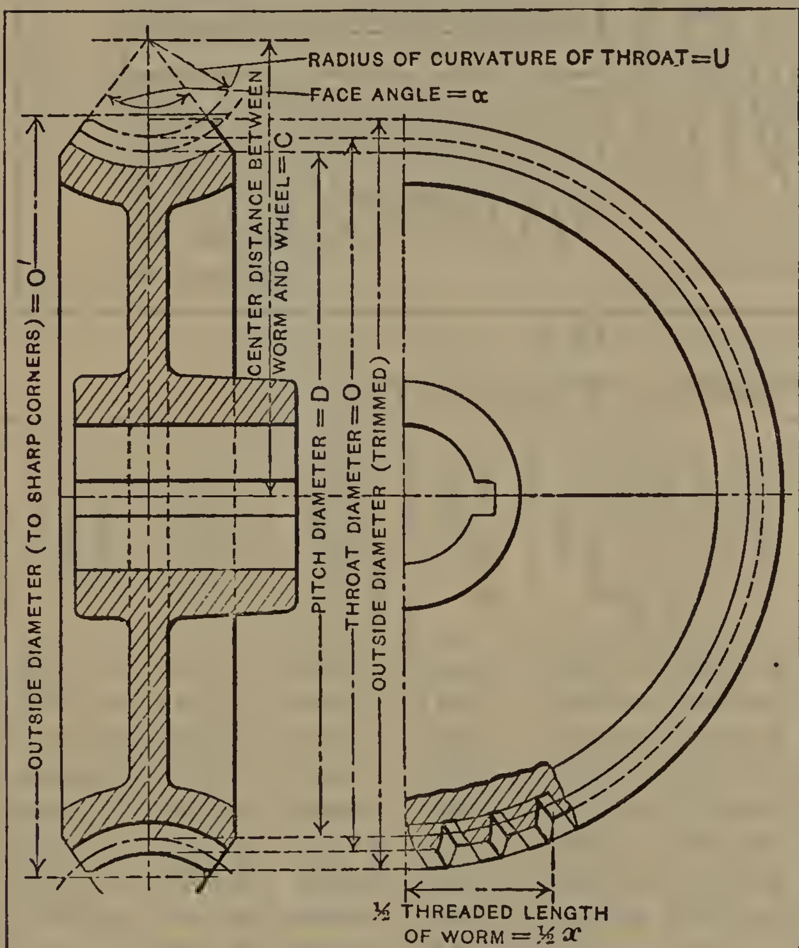


Fig. 1

The standard form of worm thread, measured in an axial section, as indicated in the illustration, Fig. 2, has the same dimensions as the standard form of involute rack tooth of the same linear pitch. It is not exactly of the same shape, however, as it is not rounded at the top, nor provided with fillets at the bottom. The thread is cut with a straight-sided tool having a square, flat end. The sides have an inclination with each other of 29 degrees, or 14½ degrees with the center line.

Rule (13) in the table gives the minimum length of the worm for complete action, or the *least* that should be used, and ordinarily the worm should be longer than the dimension found by the formula.

Worm hobs, in particular, should be long enough for the largest wheel that they are ever likely to have to cut. The face width of the worm-gear should not exceed one-half of the diameter of the worm.

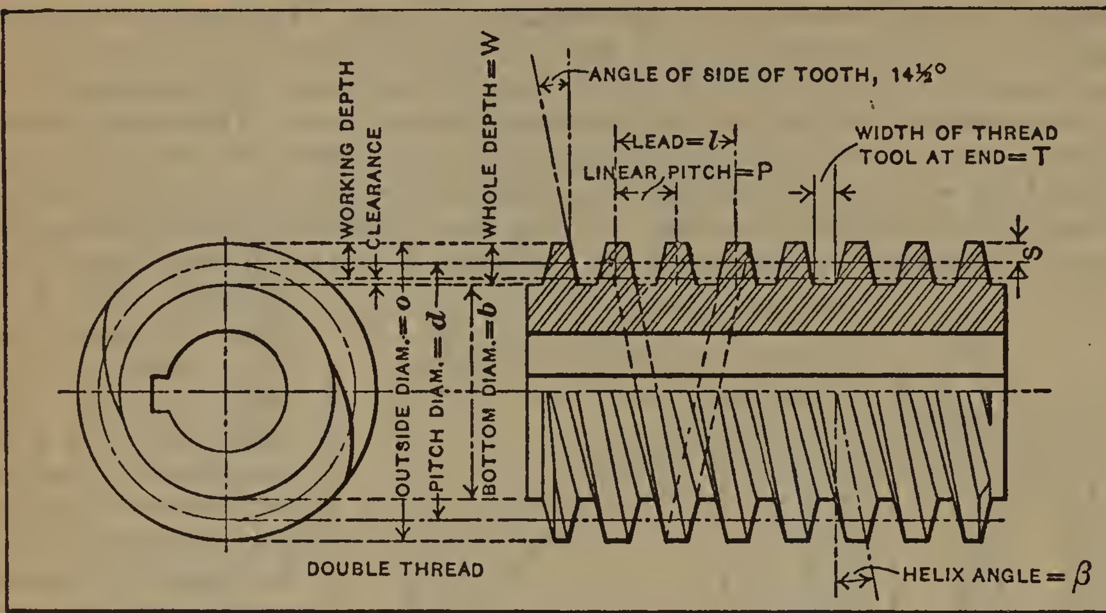


Fig. 2

Dimensions of Worm-thread Parts

Number of Threads per Inch	Circular or Linear Pitch, Inches	Circ. or Lin. Pitch, Decimal Equivalents	Height of Tooth above Pitch Line	Depth of Space below Pitch Line	Whole Depth of Tooth	Thickness of Tooth on Pitch Line	Width of Thread Tool at End	Width of Thread at Top
½	2	2.0000	0.6366	0.7366	1.3732	1.0000	0.6200	0.6708
4/7	1¾	1.7500	0.5570	0.6445	1.2015	0.8750	0.5425	0.5869
⅔	1½	1.5000	0.4775	0.5524	1.0299	0.7500	0.4650	0.5031
⅘	1¼	1.2500	0.3979	0.4603	0.8582	0.6250	0.3875	0.4192
1	1	1.0000	0.3183	0.3683	0.6866	0.5000	0.3100	0.3354
1⅓	¾	0.7500	0.2387	0.2762	0.5149	0.3750	0.2325	0.2515
1½	⅔	0.6667	0.2122	0.2455	0.4577	0.3333	0.2066	0.2236
2	½	0.5000	0.1592	0.1841	0.3433	0.2500	0.1550	0.1677
2½	⅖	0.4000	0.1273	0.1473	0.2746	0.2000	0.1240	0.1341
3	⅓	0.3333	0.1061	0.1228	0.2289	0.1667	0.1033	0.1118
3½	2/7	0.2857	0.0909	0.1053	0.1962	0.1429	0.0886	0.0958
4	¼	0.2500	0.0796	0.0920	0.1716	0.1250	0.0775	0.0838
4½	2/9	0.2222	0.0707	0.0819	0.1526	0.1111	0.0689	0.0745
5	⅕	0.2000	0.0637	0.0736	0.1373	0.1000	0.0620	0.0670
6	⅙	0.1667	0.0531	0.0613	0.1144	0.0833	0.0516	0.0559
7	1/7	0.1429	0.0455	0.0526	0.0981	0.0714	0.0443	0.0479
8	⅛	0.1250	0.0398	0.0460	0.0858	0.0625	0.0387	0.0419
9	1/9	0.1111	0.0354	0.0409	0.0763	0.0556	0.0344	0.0373
10	1/10	0.1000	0.0318	0.0369	0.0687	0.0500	0.0310	0.0335
12	1/12	0.0833	0.0265	0.0307	0.0572	0.0416	0.0258	0.0279
14	1/14	0.0714	0.0227	0.0263	0.0490	0.0357	0.0221	0.0239
16	1/16	0.0625	0.0199	0.0230	0.0429	0.0312	0.0194	0.0209
18	1/18	0.0556	0.0177	0.0205	0.0382	0.0278	0.0172	0.0186

Worms with Large Helix Angle. — When worms have a large helix angle (15 degrees or more) the dimensions of the thread should be measured at right angles to the helix. In such cases, the following changes should be made in the formulas in the table. Let:

$$P_n = \text{normal circular pitch} = P \times \cos \beta.$$

Then Formulas (2), (6) and (9) will be written as follows:

$$S = 0.3183\ P_n; \qquad W = 0.6866\ P_n; \qquad T = 0.31\ P_n.$$

When these changes are made all the other formulas will give correct results when used in their original form.

Example of Worm-gear Calculation. — Design a worm and wheel so as to utilize a hob already in stock for cutting the worm gear. The hob is double-threaded, ½ inch linear pitch, 2½ inches in diameter; the center distance is immaterial, but the worm-wheel ought to have about forty-five teeth, to obtain the required ratio. The calculations are as follows:

$$l = P \times n = 0.5 \times 2 = 1 \text{ inch};$$
$$S = 0.3183 \times 0.5 = 0.1591 \text{ inch};$$
$$d = 2.5 - 2 \times 0.1591 = 2.1818 \text{ inches};$$
$$D = (45 \times 0.5) \div 3.1416 = 7.1620 \text{ inches};$$
$$C = (2.1818 + 7.1620) \div 2 = 4.6719 \text{ inches};$$
$$W = 0.6866 \times 0.5 = 0.3433 \text{ inch};$$
$$b = 2.5 - 2 \times 0.3433 = 1.8134 \text{ inch};$$
$$\cot \beta = (3.14 \times 2.18) \div 1 = 6.845; \text{ hence } \beta = 8 \text{ degrees } 20 \text{ minutes, about};$$
$$T = 0.31 \times 0.5 = 0.155;$$
$$O = 7.1620 + 2 \times 0.1591 = 7.4802 \text{ inches};$$
$$U = (2.5 \div 2) - (2 \times 0.1591) = 0.9318 \text{ inch};$$
$$x = \sqrt{7.4802^2 - (7.4802 - 4 \times 0.1591)^2} = 3 \text{ inches, approximately}.$$

It is seldom necessary to calculate the diameter of the worm-wheel to sharp corners, if an accurate drawing is made of the worm-wheel. In the present case, the angle of face may be set at, say, 75 degrees, and the "trimmed diameter" may be scaled and will be found to be 7¾ inches. The face angle may, in general, be arbitrarily selected; 75 degrees is a good average angle, but as acute an angle as 60 degrees is frequently used, and sometimes the angle is made as large as 80 or even 90 degrees, although there is little advantage in carrying the gear around so great a portion of the circumference of the worm, especially when steep pitches are used. The width of the worm-wheel at the root of the teeth may be made from three-fifths to two-thirds of the outside diameter of the worm. In this case, then, the width at the root of the teeth may be made about 1½ inch, making the total width of the worm-gear blank about 1¾ inch.

Table for Calculating the Outside Diameter of Worm-wheels. — The regular formula for calculating the outside diameter (to sharp corners) of a worm-wheel is:

$$O' = 2\left(U - U \cos \frac{\alpha}{2}\right) + O,$$

in which O' = the outside diameter of worm-wheel to sharp corners; U = the radius of the curvature of worm-wheel throat; α = face angle of worm-wheel; O = throat diameter of worm-wheel.

By writing this formula in the form:

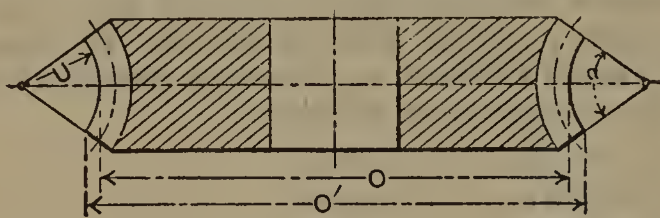
$$O' = 2\,U\left(1 - \cos\frac{\alpha}{2}\right) + O$$

it will be seen that the expression within the parentheses can be tabulated for various face angles, and such a table is given herewith. By using this table and calling the values found in the table for various angles, C, the formula takes the simple form:

$$O' = 2\,U \times C + O,$$

in which C can be found in the table for any angle from 30 to 90 degrees.

Table of Factors C Used in Worm-gear Formula



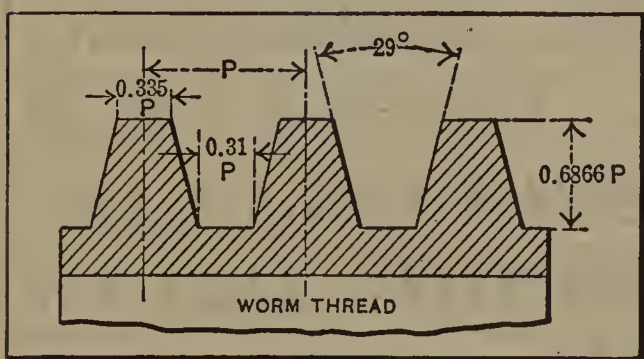
Angle α, Degrees	Factor C	Angle α, Degrees	Factor C	Angle α, Degrees	Factor C	Angle α, Degrees	Factor C
30	0.034	46	0.080	62	0.143	78	0.223
31	0.036	47	0.083	63	0.147	79	0.228
32	0.039	48	0.086	64	0.152	80	0.234
33	0.041	49	0.090	65	0.157	81	0.240
34	0.044	50	0.094	66	0.161	82	0.245
35	0.046	51	0.097	67	0.166	83	0.251
36	0.049	52	0.101	68	0.171	84	0.257
37	0.052	53	0.105	69	0.176	85	0.263
38	0.054	54	0.109	70	0.181	86	0.269
39	0.057	55	0.113	71	0.186	87	0.275
40	0.060	56	0.117	72	0.191	88	0.281
41	0.063	57	0.121	73	0.196	89	0.287
42	0.066	58	0.125	74	0.201	90	0.293
43	0.070	59	0.130	75	0.207	......	......
44	0.073	60	0.134	76	0.212	......	......
45	0.076	61	0.138	77	0.217	......	......

Worm-thread Helix Angles. — In the body of the tables, "Worm-thread Helix Angles," are found the approximate helix angles of worm threads, when the lead and the pitch-line diameter of the worm are given. For example, if the lead of the worm is 1 inch and its pitch-line diameter is 2 inches, then the angle of helix is approximately 9 degrees, as found from the tables. These tables can, of course, be used in a reverse order. If the thread angle in degrees and the pitch-line diameter are known, the approximate lead may be found, and if the angle and lead are known, the approximate diameter may be determined.

Assume, as an example, that it is required to find the diameter of a worm, the lead of which should be ½ inch, and the helix angle of which should not exceed 5 degrees. What would be the minimum pitch-line diameter of a worm that would meet these conditions? From the table it is found that the minimum pitch-line diameter would be 1⅞ inch.

Data for Turning Worm-wheel. — The throat diameter of a worm-wheel blank is the same as the outside diameter of a spur gear having the same number of teeth and pitch, and equals the pitch diameter plus twice the addendum; but if worm-wheels having less than 30 teeth are made according to this rule, the flanks of the teeth will be undercut by the hob and the worm will have a poor bearing below the pitch line. One method of avoiding this undercutting is to increase the throat diameter of the wheel blank in accordance with the following rule: To obtain the throat diameter, multiply the pitch diameter of the wheel by 0.937 and add to the product 4 times the addendum of the worm-wheel tooth. This diameter can also be obtained as follows: Multiply the product of the circular pitch and number of teeth in the worm-wheel by 0.298; then add 1.273 times the circular pitch. If it is necessary to keep the original center-to-center distance, the outside diameter of the worm must be reduced the same amount that the throat diameter is increased. When turning blanks, it is the general practice to simply reduce the central part of the throat to the required diameter, the remainder being left somewhat over size so that the tops of the teeth will be finished to the proper radius by the hob.

Data for Threading the Worm. — Worm threads have an included angle between the sides of 29 degrees, as shown by the sectional view. The width of the worm-thread tool at the end equals the linear pitch P of the worm (or circular pitch of the gear) multiplied by 0.31. To obtain the total depth of the worm thread, multiply the linear pitch P by 0.6866. When the thread is cut to this depth, it should have a width at the top equal to 0.335 of the linear pitch. To obtain the outside diameter of the worm blank, add twice the addendum to the pitch diameter of the worm. The addendum, or height of the worm tooth above the pitch line, equals the linear pitch multiplied by 0.3183. The threaded part of the worm should be long enough to make the action of the worm-wheel teeth upon the threads as complete as possible. To find the minimum length, use Rule (13) in the table "Rules and Formulas for Worm Gearing." Another common rule is to make the length of the worm equal to 6 times the circular pitch.



Threading Worms of Large Lead. — It is difficult to thread worms having a large lead or "quick pitch" on an ordinary lathe, because the lead-screw must be geared to run several times faster than the spindle, thus imposing excessive strains on the gearing. A common method of overcoming this difficulty is to mount a belt pulley on the lead-screw, beside the change gear and belt it to the countershaft; the spindle is then driven through the change gearing.

Gashing a Worm-wheel. — The teeth of worm-wheels, when cut in milling machines, are commonly formed by two operations; namely, gashing and hobbing. Gashing consists in cutting teeth around the periphery of the blank, which are approximately the shape of the finished teeth. This is done preferably by an involute gear cutter of a number and pitch corresponding to the worm-wheel. If a gear cutter is not available, a plain milling cutter, the thickness of which should not exceed 0.3 of the circular pitch, may be used. The corners of the cutter should be rounded to prevent removing the fillets of the teeth. The blank to be gashed is mounted on an arbor placed between the centers of the dividing head and tailstock, with the driving dog secured to prevent rotation. The table is adjusted laterally

Worm-thread Helix Angles in Degrees

Lead of Worm, Inches	Pitch-line Diameter of Worm, Inches																				
	1	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾	2⅞	3	3⅛	3¼	3⅜	3½
¼	4½	4	3½	3¼	3	2¾	2½	2½	2¼	2¼	2	2	2	1¾	1¾	1½	1½	1½	1½	1½	1½
⅜	6¾	6	5½	5	4½	4¼	4	3¾	3½	3¼	3	3	2¾	2½	2½	2½	2¼	2¼	2¼	2	2
½	9	8	7¼	6¾	6	5½	5¼	5	4½	4¼	4¼	4	3¾	3½	3¼	3¼	3	3	2¾	2¾	2½
⅝	11¼	10	9	8¼	7½	7	6½	6	5¾	5¼	5	4¾	4½	4½	4¼	4	3¾	3¾	3½	3½	3¼
¾	13½	12	10¾	10	9	8½	7¾	7¼	6¾	6½	6	5¾	5½	5¼	5	4¾	4½	4½	4¼	4¼	4
⅞	15½	14	12½	11½	10½	9¾	9	8½	8	7½	7	6¾	6½	6	5¾	5½	5¼	5	5	4¾	4½
1	17¾	16	14¼	13	12	11¼	10¼	9¾	9	8½	8	7¾	7¼	7	6¾	6¼	6	5¾	5½	5½	5¼
1⅛	19¾	18	16	14¾	13½	12½	11½	11	10¼	9½	9	8½	8	7¾	7½	7¼	6¾	6½	6¼	6	6
1¼	21¾	19½	17¾	16¼	15	13¾	12¾	12	11¼	10¾	10	9½	9	8¾	8¼	8	7¾	7½	7	6¾	6½
1⅜	23¾	21¼	19¼	17¾	16¼	15	14	13	12½	11¾	11	10½	10	9½	9	8¾	8¼	8	7¾	7½	7
1½	25½	23	21	19¼	17½	16½	15¼	14¼	13½	12¾	12	11½	11	10¼	10	9½	9	8¾	8½	8	7¾
1⅝	27½	24¾	22½	20¾	19	17¾	16½	15½	14½	13¾	13	12¼	11¾	11¼	10¾	10¼	9¾	9½	9	8¾	8½
1¾	29¼	26½	24	22	20½	19	17¾	16½	15½	14¾	14	13¼	12½	12	11½	11	10½	10	9¾	9½	9
1⅞	30¾	28	25½	23½	21¾	20¼	19	17½	16¾	15¾	15	14¼	13½	13	12¼	11¾	11¼	11	10½	10	9¾
2	32½	29¾	27	25	23	21½	20	19	17¾	16¾	15¾	15	14¼	13¾	13¼	12½	12	11½	11	10¾	10½
2¼	35¾	32¾	30	27½	25½	24	22¼	21	19¾	18¾	17¾	17	16	15¼	14¾	14	13½	13	12½	12	11½
2½	38½	35½	32½	30	28	26¼	24½	23	21¾	20¾	19½	18½	17¾	17	16¼	15½	15	14¼	13¾	13½	12¾
2¾	41¼	38	35	32½	30¼	28½	26½	25	23¾	22½	21¼	20¼	19¼	18½	17¾	17	16¼	15¾	15	14¾	14
3	43¾	40½	37½	35	32½	30½	28¾	27	25½	24¼	23	22	21	20	19¼	18½	17¾	17	16½	16	15¼
3¼	46	42¾	39¾	37	34½	32½	30¾	29	27½	26	24¼	23½	22½	21½	20¾	19¾	19	18½	17¾	17	16½
3½	48¼	45	41¾	39¼	36¾	34½	32½	30¾	29¼	27¾	26¼	25¼	24	23	22	21¼	20½	19½	19	18¼	17¾
3¾	50¼	47	43¾	41	38¾	36½	34¼	32½	31	29½	28	26¾	25½	24½	23½	22½	21¾	21	20¼	19½	19
4	51¾	48½	45¾	43	40½	38¼	36	34¼	32¾	31	29½	28¼	27	26	25	24	23	22	21½	20¾	20
4¼	53½	50½	47½	44¾	42¼	39¾	37¾	36	34¼	32½	31	29¾	28½	27¼	26¼	25¼	24¼	23½	22½	22	21¼
4½	55	42	49	46½	43¾	41½	39½	37½	35¾	34	32½	31¼	30	28¾	27½	26½	25½	24½	23¾	23	22½
4¾	56½	53½	50½	48	45¼	43	40¾	39	37¼	35½	34	32½	31¼	30	29	27¾	26¾	25¾	25	24	23½
5	57¾	54¾	51¾	49¼	46¾	44½	42¼	40½	38¾	37	35¼	34	32½	31¼	30¼	29	28	27	26	25¼	24½
5¼	59¼	56¼	53¼	50¾	48	46	43¾	41¾	40	38¼	36½	35¼	34	32½	31½	30¼	29	28¼	27	26½	25½
5½	60¼	57¼	54¾	52	49½	47¼	45	43	41¼	39½	38	36½	35	33¾	32¾	31½	30¼	29¼	28¼	27½	26½
5¾	61¼	58½	55¾	53	50¾	48½	46¼	44¼	42½	40¾	39¼	37¾	36¼	35	33¾	32½	31½	30¼	29½	28½	27¾
6	62¼	59¾	57	54¼	52	49½	47½	45¾	43¾	42	40¼	39	37½	36¼	35	33¾	32½	31½	30½	29½	28¾

Worm-thread Helix Angles in Degrees

Lead of Worm, Inches	Pitch-line Diameter of Worm, Inches																			
	3⅝	3¾	3⅞	4	4⅛	4¼	4⅜	4½	4⅝	4¾	4⅞	5	5⅛	5¼	5⅜	5½	5⅝	5¾	5⅞	6
¼	1¼	1¼	1¼	1¼	1¼	1¼	1	1	1	1	1	1	1	1	1	1	1	1	1	1
⅜	2	2	2	1¾	1¾	1¾	1½	1½	1½	1½	1½	1½	1½	1½	1¼	1¼	1¼	1¼	1¼	1¼
½	2½	2½	2½	2¼	2¼	2¼	2	2	2	2	2	2	2	2	1¾	1¾	1¾	1¾	1¾	1½
⅝	3¼	3	3	2¾	2¾	2¾	2½	2½	2½	2½	2½	2¼	2¼	2¼	2¼	2¼	2¼	2	2	2
¾	3¾	3¾	3½	3¼	3¼	3¼	3¼	3	3	3	3	2¾	2¾	2¾	2¾	2½	2½	2¼	2¼	2¼
⅞	4½	4¼	4¼	4	4	4	3¾	3½	3½	3½	3¼	3¼	3¼	3	3	3	2¾	2¾	2¾	2¾
1	5	5	4¾	4½	4½	4½	4¼	4	4	3¾	3¾	3¾	3½	3½	3½	3¼	3¼	3¼	3¼	3
1⅛	5¾	5½	5½	5¼	5	5	4¾	4½	4½	4¼	4¼	4¼	4	4	3¾	3¾	3¾	3¾	3½	3½
1¼	6¼	6	6	5¾	5½	5½	5¼	5	5	4¾	4¾	4½	4½	4½	4¼	4¼	4¼	4	4	3¾
1⅜	7	6¾	6½	6¼	6	6	5¾	5½	5½	5¼	5¼	5	5	4¾	4¾	4½	4½	4½	4¼	4¼
1½	7½	7¼	7	6¾	6¾	6½	6¼	6	6	5¾	5½	5½	5¼	5¼	5	5	4¾	4¾	4¾	4½
1⅝	8¼	8	7¾	7½	7¼	7	6¾	6½	6½	6¼	6	6	5¾	5¾	5½	5½	5¼	5¼	5	5
1¾	8¾	8½	8¼	8	7¾	7½	7¼	7	7	6¾	6½	6½	6¼	6	6	5¾	5¾	5½	5½	5½
1⅞	9½	9	8¾	8½	8¼	8	7¾	7½	7½	7¼	7	7	6¾	6½	6½	6¼	6	6	5¾	5¾
2	10	9¾	9½	9	8¾	8½	8½	8¼	8	7¾	7½	7¼	7¼	7	6¾	6½	6½	6½	6¼	6
2¼	11¼	11	10½	10¼	10	9½	9½	9	9	8¾	8½	8¼	8	8	7¾	7½	7½	7¼	7	7
2½	12½	12	11¾	11¼	11	10¾	10½	10	10	9½	9¼	9¼	9	8¾	8½	8¼	8¼	8	7¾	7½
2¾	13¾	13¼	12¾	12¼	12	11¾	11½	11	10¾	10½	10¼	10	9¾	9½	9¼	9¼	9	8¾	8½	8½
3	15	14½	14	13½	13	13	12¼	12	11½	11½	11	11	10¾	10½	10¼	10	9¾	9½	9½	9¼
3¼	16	15½	15	14½	14¼	14	13¼	13	12½	12¼	12	11¾	11½	11	11	10¾	10½	10¼	10¼	10
3½	17¼	16¾	16	15½	15½	15	14¼	14	13½	13¼	13	12½	12¼	12	11¾	11½	11¼	11	10¾	10½
3¾	18½	18	17¼	16½	16¼	15¾	15¼	15	14½	14¼	13¾	13½	13	13	12½	12¼	12	11¾	11½	11¼
4	19½	19	18½	17½	17¼	16¾	16¼	16	15½	15	14½	14¼	14	13¾	13¼	13	13	12½	12¼	12
4¼	20½	20	19½	18½	18½	17¾	17¼	16¾	16½	16	15½	15¼	15	14½	14	14	13¾	13½	13	12¾
4½	21½	21	20½	19¾	19½	18½	18¼	17½	17¼	17	16½	16	15¾	15¼	15	14¾	14½	14¼	13¾	13½
4¾	22¾	22	21½	20¾	20½	19½	19¼	18½	18¼	18	17¼	17	16½	16	16	15½	15¼	15	14½	14¼
5	23¾	23	22½	21¾	21¼	20½	20	19½	19	18½	18¼	18	17¼	17	16½	16¼	16	15½	15¼	15
5¼	24¾	24	23½	22½	22	21½	21	20½	20	19½	19	18½	18¼	17¾	17½	17	16¾	16¼	16	15¾
5½	25¾	25¼	24½	23½	23	22½	22	21½	21	20½	20	19½	19	18½	18¼	18	17½	17	16¾	16½
5¾	27	26¼	25½	24½	24	23½	23	22	21¾	21	20½	20¼	19¾	19½	19	18½	18¼	17¾	17½	17
6	27¾	27	26½	25½	25	24¼	23¾	23	22½	22	21½	21	20½	20	19½	19¼	19	18½	18	17¾

Angles for Gashing Worm-wheels

Pitch Diam. of Worm	Lead of Worm Thread in Inches																							
	1/10	1/9	1/8	2/15	1/7	2/13	1/6	2/11	1/5	2/9	1/4	2/7	1/3	4/11	3/8	2/5	3/7	4/9	1/2	4/7	3/5	2/3	3/4	4/5
	Number of Threads per Inch in Worm																							
	10	9	8	7½	7	6½	6	5½	5	4½	4	3½	3	2¾	2⅔	2½	2⅓	2¼	2	1¾	1⅔	1½	1⅓	1¼
5/8	2°55'	3°14'	3°38'	3°51'	4°10'	4°29'	4°51'	5°18'	5°49'	6°28'	7°16'	8°17'	9°38'	10°30'	10°49'	11°31'	....	....	....	....	....	....	....	....
3/4	2°26'	2°42'	3°2'	3°14'	3°28'	3°44'	4°3'	4°25'	4°51'	5°23'	6°4'	6°55'	8°3'	8°46'	9°3'	9°38'	10°18'	....	....	....	....	....	....	....
7/8	2°5'	2°19'	2°36'	2°47'	2°58'	3°12'	3°28'	3°47'	4°10'	4°37'	5°12'	5°56'	6°55'	7°32'	7°46'	8°17'	8°52'	....	....	....	....	....	....	....
1	1°49'	2°1'	2°17'	2°26'	2°36'	2°48'	3°2'	3°19'	3°39'	4°3'	4°33'	5°12'	6°3'	6°36'	6°48'	7°15'	7°47'	8°3'	....	....	....	....	....	....
1⅛	1°37'	1°48'	2°2'	2°10'	2°19'	2°30'	2°42'	2°57'	3°14'	3°36'	4°3'	4°37'	5°23'	5°52'	6°4'	6°27'	6°55'	7°10'	....	....	....	....	....	....
1¼	1°28'	1°37'	1°49'	1°57'	2°5'	2°15'	2°26'	2°39'	2°55'	3°14'	3°39'	4°10'	4°51'	5°17'	5°27'	5°49'	6°14'	6°27'	7°15'	....	....	....	....	....
1⅜	1°20'	1°28'	1°39'	1°46'	1°54'	2°2'	2°13'	2°25'	2°39'	2°57'	3°19'	3°47'	4°25'	4°49'	4°58'	5°17'	5°40'	5°52'	6°36'	....	...	....	....	....
1½	1°13'	1°21'	1°31'	1°37'	1°44'	1°52'	2°1'	2°13'	2°26'	2°42'	3°2'	3°28'	4°3'	4°25'	4°33'	4°51'	5°12'	5°23'	6°3'	6°55'	....	....	....	....
1⅝	1°7'	1°15'	1°24'	1°30'	1°36'	1°44'	1°52'	2°2'	2°15'	2°30'	2°48'	3°12'	3°44'	4°4'	4°12'	4°29'	4°48'	4°59'	5°36'	6°23'	6°42'	....	....	....
1¾	1°2'	1°9'	1°18'	1°23'	1°29'	1°36'	1°44'	1°54'	2°5'	2°19'	2°36'	2°58'	3°28'	3°47'	3°54'	4°10'	4°27'	4°37'	5°12'	5°56'	6°14'	....	....	....
1⅞	58'	1°5'	1°13'	1°18'	1°23'	1°30'	1°37'	1°46'	1°57'	2°10'	2°26'	2°47'	3°14'	3°32'	3°39'	3°53'	4°10'	4°19'	4°51'	5°32'	5°49'	6°27'	....	....
2	55'	1°1'	1°8'	1°13'	1°18'	1°24'	1°31'	1°39'	1°49'	2°2'	2°17'	2°36'	3°2'	3°19'	3°25'	3°39'	3°54'	4°3'	4°33'	5°12'	5°27'	6°3'	....	....
2⅛	52'	57'	1°4'	1°9'	1°14'	1°19'	1°26'	1°34'	1°43'	1°54'	2°9'	2°27'	2°52'	3°7'	3°13'	3°26'	3°40'	3°46'	4°17'	4°54'	5°8'	5°42'	6°25'	....
2¼	49'	54'	1°1'	1°5'	1°9'	1°15'	1°21'	1°28'	1°37'	1°48'	2°2'	2°19'	2°42'	2°57'	3°2'	3°14'	3°28'	3°36'	4°3'	4°37'	4°51'	5°23'	6°3'	....
2⅜	46'	51'	58'	1°1'	1°6'	1°11'	1°17'	1°24'	1°32'	1°42'	1°55'	2°12'	2°33'	2°47'	2°53'	3°4'	3°17'	3°25'	3°50'	4°23'	4°36'	5°6'	5°44'	6°7'
2½	44'	49'	54'	58'	1°3'	1°7'	1°13'	1°20'	1°27'	1°37'	1°49'	2°5'	2°26'	2°39'	2°44'	2°55'	3°7'	3°14'	3°39'	4°10'	4°22'	4°51'	5°27'	5°49'

Angles for Gashing Worm-wheels

Pitch Diam. of Worm	Lead of Worm Thread in Inches: 1/8	1/7	2/13	1/6	2/11	1/5	2/9	1/4	2/7	1/3	4/11	3/8	2/5	3/7	4/9	1/2	4/7	3/5	2/3	3/4	4/5	1	1 1/3	1 1/2
	Number of Threads per Inch in Worm: 8	7	6 1/2	6	5 1/2	5	4 1/2	4	3 1/2	3	2 3/4	2 2/3	2 1/2	2 1/3	2 1/4	2	1 3/4	1 2/3	1 1/2	1 1/3	1 1/4	1	3/4	2/3
2 5/8	52′	1°	1°4′	1°9′	1°16′	1°23′	1°33′	1°44′	1°59′	2°19′	2°31′	2°36′	2°47′	2°59′	3°5′	3°28′	3°58′	4°10′	4°37′	5°12′	5°32′	6°55′	....	....
2 3/4	50′	57′	1°1′	1°6′	1°12′	1°20′	1°28′	1°39′	1°54′	2°13′	2°25′	2°29′	2°39′	2°50′	2°57′	3°19′	3°47′	3°58′	4°25′	4°58′	5°17′	6°36′	....	....
2 7/8	48′	54′	58′	1°3′	1°9′	1°16′	1°25′	1°35′	1°49′	2°7′	2°18′	2°23′	2°32′	2°43′	2°49′	3°10′	3°37′	3°48′	4°13′	4°45′	5°4′	6°19′	8°24′	....
3	46′	52′	56′	1°1′	1°6′	1°13′	1°21′	1°31′	1°44′	2°2′	2°13′	2°17′	2°26′	2°36′	2°42′	3°2′	3°28′	3°39′	4°3′	4°33′	4°51′	6°3′	8°3′	9°3′
3 1/4	42′	48′	52′	56′	1°1′	1°7′	1°15′	1°24′	1°36′	1°52′	2°2′	2°6′	2°14′	2°24′	2°30′	2°48′	3°12′	3°22′	3°44′	4°12′	4°29′	5°36′	7°26′	8°22′
3 1/2	39′	45′	48′	52′	57′	1°3′	1°9′	1°18′	1°29′	1°44′	1°54′	1°57′	2°5′	2°14′	2°19′	2°36′	2°59′	3°7′	3°28′	3°54′	4°10′	5°12′	6°54′	7°46′
3 3/4	36′	42′	45′	48′	53′	58′	1°5′	1°13′	1°23′	1°37′	1°46′	1°49′	1°57′	2°5′	2°10′	2°26′	2°47′	2°55′	3°14′	3°39′	3°53′	4°51′	6°27′	7°15′
4	34′	39′	42′	46′	50′	55′	1°1′	1°8′	1°18′	1°31′	1°39′	1°43′	1°49′	1°55′	2°2′	2°17′	2°36′	2°44′	3°2′	3°25′	3°39′	4°33′	6°4′	6°49′
4 1/4	32′	37′	40′	43′	47′	52′	57′	1°4′	1°14′	1°26′	1°34′	1°37′	1°43′	1°50′	1°54′	2°9′	2°27′	2°34′	2°52′	3°13′	3°26′	4°17′	5°42′	6°26′
4 1/2	....	35′	37′	40′	44′	49′	54′	1°1′	1°9′	1°21′	1°28′	1°31′	1°37′	1°44′	1°48′	2°2′	2°19′	2°26′	2°42′	3°2′	3°14′	4°3′	5°23′	6°4′
4 3/4	....	33′	35′	38′	42′	46′	51′	58′	1°6′	1°17′	1°24′	1°26′	1°32′	1°37′	1°42′	1°55′	2°12′	2°18′	2°33′	2°53′	3°4′	3°50′	5°6′	6°44′
5	....	....	34′	36′	40′	44′	49′	55′	1°3′	1°13′	1°20′	1°22′	1°28′	1°34′	1°37′	1°49′	2°5′	2°11′	2°26′	2°44′	2°55′	3°39′	4°51′	5°27′
5 1/4	....	....	....	35′	38′	42′	46′	52′	1°1′	1°9′	1°16′	1°18′	1°23′	1°29′	1°33′	1°44′	1°59′	2°5′	2°19′	2°36′	2°47′	3°28′	4°37′	5°12′
5 1/2	....	....	....	....	36′	40′	44′	50′	57′	1°6′	1°12′	1°15′	1°20′	1°25′	1°28′	1°39′	1°54′	2°59′	2°13′	2°29′	2°39′	3°19′	4°25′	4°58′
5 3/4	....	....	....	....	....	38′	42′	48′	54′	1°3′	1°9′	1°11′	1°16′	1°22′	1°24′	1°35′	1°49′	1°54′	2°7′	2°23′	2°32′	3°10′	4°13′	4°45′
6	....	....	....	....	....	....	40′	46′	52′	1°1′	1°6′	1°8′	1°13′	1°18′	1°21′	1°31′	1°44′	1°49′	2°1′	2°17′	2°26′	3°2′	4°2′	4°33′

and longitudinally to center the blank with the cutter, after which it is set to the proper angle for gashing the teeth. This angle may be found by dividing the lead of the worm thread by the circumference of the pitch circle of the worm, which gives the tangent of the desired angle. When milling the gashes, if the diameter of the cutter is no larger than that of the hob to be used, the depth of the gashes should be slightly less than the whole depth of the teeth, which equals the linear pitch multiplied by 0.6866. If the cutter is larger than the hob, the whole depth of tooth should be laid off on the side of the blank and the gashes cut to this line.

Hobbing a Worm-wheel. — After gashing a blank, the table is set at right angles with the machine spindle and the cutter is replaced with a hob. The outside diameter of the hob and the diameter at the bottom of the teeth should be slightly greater than the corresponding dimensions of the worm, to provide clearance between the worm and worm-wheel. (See paragraph on "Hobs for Worm Gears.") Before hobbing, the dog is removed from the arbor to permit the latter to turn freely upon its centers. As the gear and hob revolve together, the latter is sunk to the proper depth, thus finishing the teeth to conform with the worm. When worm-wheels are cut in machines especially designed for this purpose, the wheel blanks, instead of being mounted on a free running arbor, are driven by gearing at the proper speed. Gashing the blanks preparatory to hobbing is then unnecessary, as the change gears insure a correct spacing of the worm-wheel teeth.

Tables of Angles for Gashing Worm-wheels. — These tables give the angle with the axis of the worm-wheel to which the cutter should be set for gashing teeth of worm-wheels when the pitch diameter and the lead of the worm are known. If the worm has a pitch diameter which is not given exactly in the table, the angle can be approximated from the nearest sizes given. For example, find the angle to which the cutter is to be set with the axis of the worm-wheel for gashing a wheel having a pitch diameter of 2 inches and a lead of 1/4 inch. From the table this angle is found to be 2 degrees 17 minutes.

Hindley Worm Gearing. — The Hindley type of worm gear was first used in the Hindley dividing engine. In the Hindley worm gear the worm, instead of being cylindrical in outline, and the projection of the pitch line being a straight line, has a curved outline, the projection of the pitch line of the worm being a circular arc corresponding to the pitch line of the gear. The following claims are made for the contact of the Hindley worm and gear: — 1. The contact is purely sliding contact. 2. The nature of the contact is linear, closely resembling surface contact. 3. Linear contact extends from the top to the root of the teeth. 4. The contact is on the axial section. 5. The thread section fills the tooth space on the axial section only. 6. The mid-portion of the hob has little or no effect in shaping the teeth of the gear. 7. Surface contact exists on opposite sides of the axial plane at the end of the worm thread, and is intermittent in nature, because the end of the thread passes out of contact with the tooth in the revolving of the worm. This contact is on a plane normal with the thread angle.

There is, however, considerable difference of opinion as to the actual contact between the worm and wheel in the Hindley worm gear. In practice, it is usual to allow considerable back-lash between the thread and the teeth of the worm gear. This play tends to counteract faults in the workmanship, either in construction or erection. The worm of Hindley gearing must be centered in both planes, with relation to the wheel, and be carefully secured axially, as endwise movement would seriously affect the bearing of the teeth and greatly increase the friction. The length of the worm should not exceed one-third of the wheel diameter. The face of the wheel at the root of the tooth should not exceed, in width, one-half of the worm diameter at the center.

Change Gears for Cutting Diametral Pitch Worms*

Diametral Pitch to be Cut	Width of Tool Point	Threads per Inch on Lead Screw							
		2	3	4	5	6	7	8	10
2	0.487	22/7	33/7	44/7	55/7	66/7	77/7	88/7	110/7
2¼	0.433	176/63	88/21	352/63	440/63	176/21	88/9	704/63	880/63
2½	0.390	88/35	132/35	176/35	44/7	264/35	308/35	352/35	440/35
2¾	0.354	16/7	24/7	32/7	40/7	48/7	56/7	64/7	72/7
3	0.325	44/21	22/7	88/21	110/21	44/7	22/3	176/21	220/21
3½	0.278	88/49	132/49	176/49	220/49	264/49	44/7	352/49	440/49
4	0.243	11/7	33/14	22/7	55/14	33/7	11/2	44/7	55/7
4½	0.217	88/63	44/21	176/63	220/63	88/21	44/9	352/63	440/63
5	0.195	44/35	66/35	88/35	22/7	132/35	22/5	176/35	44/7
6	0.162	22/21	11/7	44/21	55/21	22/7	11/3	88/21	110/21
7	0.139	44/49	66/49	88/49	110/49	132/49	22/7	176/49	220/49
8	0.122	11/14	33/28	11/7	55/28	33/14	11/4	22/7	55/14
9	0.108	44/63	22/21	88/63	110/63	44/21	22/9	176/63	220/63
10	0.097	22/35	33/35	44/35	11/7	66/35	11/5	88/35	22/7
11	0.088	4/7	6/7	8/7	10/7	12/7	14/7	16/7	20/7
12	0.081	11/21	11/14	22/21	55/42	11/7	11/6	44/21	55/21
14	0.069	22/49	33/49	44/49	55/49	66/49	11/7	88/49	110/49
16	0.061	11/28	33/56	22/28	55/56	33/28	77/56	11/7	55/28
18	0.054	22/63	11/21	44/63	55/63	22/21	11/9	88/63	110/63
20	0.049	11/35	33/70	22/35	11/14	33/35	77/70	44/35	11/7
22	0.044	2/7	3/7	4/7	5/7	6/7	7/7	8/7	10/7
24	0.040	11/42	33/84	11/21	55/84	33/42	77/84	22/21	55/42
26	0.037	22/91	33/91	44/91	55/91	66/91	77/91	88/91	110/91
28	0.035	11/49	33/98	22/49	55/98	33/49	11/14	44/49	55/49
30	0.032	22/105	11/35	44/105	11/21	22/35	77/105	88/105	22/21
32	0.030	11/56	33/112	11/28	55/112	33/56	77/112	11/14	55/56
40	0.024	11/70	33/140	11/35	11/28	33/70	77/140	22/35	11/14
48	0.020	11/84	33/168	11/42	55/168	33/84	77/168	11/21	55/84

* The ratio of change gears for cutting diametral pitch worms is as 22 times the threads per inch on lead-screw is to 7 times the diametral pitch to be cut. Thus,

$$\frac{22 \times \text{Threads per Inch}}{7 \times \text{Diametral Pitch}} = \text{Ratio of Change Gears}$$

Efficiency of Worm Gearing. — The following table gives the theoretical efficiency of worm gearing for a number of different coefficients of friction. Practical experiments carried out by the Oerlikon Company, Oerlikon by Zürich, Switzerland, agree closely with the results from theoretical calculations given in the table. These experiments indicate that the efficiency increases with the angle of inclination, up to a certain point. They also show that for larger angles of inclination than 25 degrees to 30 degrees the efficiency increases very little, especially if the coefficient of friction is small, and this fact is of importance in practice, because, for reasons of gear ratio and conditions of a constructive nature, an angle greater than 30 degrees cannot be employed. The coefficient of friction increases with the load and diminishes to a certain extent with increase of speed. Besides the friction between the worm and the wheel teeth, there is also the friction of the spindle bearings and the ball bearings for taking the axial thrust. To obtain the best results, there must be very careful choice of dimensions of teeth, of the stress between them, and the angle of inclination.

To show what can be done, the following are the results of a test with an Oerlikon worm gear for a colliery winding engine: The motor gave 30 brake horsepower to 40 brake horsepower at 780 revolutions. The normal load was 25 brake horsepower, but at starting it could develop 40 brake horsepower. The worm-gear ratio was 13.6 to 1, the helicoidal bronze wheel having 68 teeth on a pitch circle of 7.283 inches, and the worm 5 threads. The power required at no load for the whole mechanism was 520 watts, corresponding to 2.8 per cent of the normal. The efficiency at one-third normal load gave 90 per cent, at full load 94½, and at 50 per cent overload 93 per cent. The efficiency of the *worm and wheel* alone is higher, and knowing the no-load power, is calculated to be 97½ per cent. According to

Table Giving Theoretical Efficiency of Worm Gearing

Coefficient of Friction	Angle of Inclination of Thread								
	5 Deg.	10 Deg.	15 Deg.	20 Deg.	25 Deg.	30 Deg.	35 Deg.	40 Deg.	45 Deg.
0.01	89.7	94.5	96.1	97.0	97.4	97.7	97.9	98.0	98.0
0.02	81.3	89.5	92.6	94.1	95.0	95.5	95.9	96.0	96.1
0.03	74.3	85.0	89.2	91.4	92.7	93.4	93.9	94.1	94.2
0.04	68.4	80.9	86.1	88.8	90.4	91.4	92.0	92.2	92.3
0.05	63.4	77.2	83.1	86.3	88.2	89.4	90.1	90.4	90.5
0.06	59.0	73.8	80.4	84.0	86.1	87.5	88.2	88.6	88.7
0.07	55.2	70.7	77.8	81.7	84.1	85.6	86.4	86.9	86.9
0.08	51.9	67.8	75.4	79.6	82.2	83.8	84.7	85.2	85.2
0.09	48.9	65.2	73.1	77.6	80.3	82.0	83.0	83.5	83.5
0.10	46.3	62.7	70.9	75.6	78.5	80.3	81.4	81.9	81.8

the table of theoretical efficiencies given, this gives the coefficient of friction as 0.01. To obtain a reduction of 13.6 to 1 with spur gears would have necessitated two pinions and two wheels with their spindles and bearings, and if the bearing friction was taken into consideration, the efficiency of such gearing would not have reached the figure of 94½ per cent at full load.

Allowable Load and Average Efficiency of Worm Gearing. — In the following formulas, let:

P = pressure of worm-wheel on the worm parallel to the worm-shaft;
F = force which must be applied at the pitch radius of the worm at right angles to the worm-shaft to overcome P;
α = angle of thread with a line at right angles to the axis of the worm;
f = coefficient of friction;
E = efficiency of worm gearing in per cent.

Then:

$$F = P \times \frac{f + \tan \alpha}{1 - f \tan \alpha}$$

$$E = \frac{\tan \alpha \, (1 - f \tan \alpha)}{f + \tan \alpha}$$

The efficiency of worm gearing increases very rapidly with an increase in the thread angle α, for small angles. After a thread angle of about 15 degrees has been reached, there is a comparatively small change in the efficiency for a wide range of angles. It is, therefore, essential for high efficiency to use thread angles that are not too small. The value of the coefficient of friction for low speeds is given in the accompanying table of "Safe Load on Worm Gear Teeth." The coefficient of friction has not been determined by accurate tests for higher speeds than 80 feet per minute, but there is a general tendency of the value of the coefficient f to decrease as the speed increases.

The allowable load on worm gearing may be determined by the equation:

$$P = C \times p \times b,$$

in which p = circular pitch; b = width of gear teeth; C = a constant for the given speed; and P = pressure of the worm-wheel on the worm parallel to the worm-shaft. The value of the factor C varies with the speed and must be determined by experiments. Safe values for this factor are given in the accompanying table. These loads will, under ordinary circumstances, not cause a heating of the oil above

Safe Load on Worm-gear Teeth

Load per unit of the product (pitch × width of tooth), for 90-degree F. temperature difference between oil and surrounding air. More than 1000 pounds per unit of product (pitch × width of tooth) should not be allowed under ordinary circumstances. Cut bronze-gear, cut steel-worm.

Velocity in Feet per Minute	Load in Pounds per Unit of (Pitch × Width of Tooth)	Coefficient of Friction	Velocity in Feet per Minute	Load in Pounds per Unit of (Pitch × Width of Tooth)	Coefficient of Friction
5	1000	0.146	200	403	....
10	1000	0.116	250	371	....
20	956	0.090	300	341	....
30	790	....	350	315	....
40	703	0.070	400	292	....
50	646	....	500	257	....
75	564	....	600	228	....
80	554	0.054	700	206	....
100	514	....	800	185	....
125	477	....	900	167	....
150	448	....	1000	151	....
175	424	....	1200	128	....

90 degrees F. over that of the surrounding air. The loads given are for continuous service, and as it will take several minutes, and, in some cases, hours, before a constant temperature of 90 degrees F. above the room temperature is reached, a higher load is justified for intermittent service, where the oil has time to cool down. The danger of abrasion, however, depends on the actual temperature of the oil and not on the temperature difference. Therefore, the danger of abrasion increases when the gearing is installed in a place where the surrounding temperature is high. The danger of abrasion will also, to a large extent, depend upon the character of the lubricant. Obviously, a very viscous oil will offer greater resistance to being squeezed out between the rubbing surfaces than one that is less viscous.

A gear with many teeth has a better contact with the worm both on account of the flatter curve of the engaging segment and on account of the larger average radii of curvature of its teeth.

The angle of the thread (the helix angle) has no direct bearing on the allowable load and speed of the rubbing surfaces, although it has a direct influence on the efficiency of the gearing.

With the allowable load decreasing as the speed increases, as apparent from the table, a speed of rubbing surfaces as high as 1000 feet per minute, or even higher, can be used with success for cut gearing. The loads given represent tangential loads at right angles to the worm-gear shaft. The actual pressure between the rubbing surfaces will be more, and will increase with the angle of thread, but the increase for gears in common use (less than 20-degree thread angle) is not very great.

Except for hand-operated worm gearing or for machinery which is only operated occasionally and for a very short time, the worm and gear should be enclosed in an oil casing and the worm should always be placed below the gear to insure the submersion of the rubbing surfaces in oil. The best combination of materials for worm gearing is hardened steel worms meshing with phosphor-bronze wheels. This combination wears longer than any other commercial combination.

Relation Between Velocity at Pitch Line, Angle of Thread and Efficiency

Velocity at Pitch Line, Feet per Minute	Angle of Thread, Degrees					
	5	10	20	30	40	45
	Efficiency, Per Cent					
5	40	56	69	76	79	80
10	47	62	74	79	82	82
20	52	67	78	83	85	86
30	56	71	81	85	87	87
40	60	74	83	87	88	88
50	63	76	85	88	89	89
75	67	80	87	90	90	90
100	70	82	88	91	91	91
150	74	84	90	92	92	92
200	76	85	91	92	92	92

Self-locking Worm Gearing. — A set of worm gearing will be self-locking when the thread angle is equal to, or smaller than, the angle of friction. In other words, the thread angle must be so selected that its tangent is equal to or smaller than the coefficient of friction. It is impossible to obtain an efficiency greater than 0.5 if the gears are to be self-locking independently of the remainder of the mechanism. Of course, there is always some friction in the worm-shaft bearings and other parts of the machinery which may prevent the pressure on the worm-gear from actually turning the machinery, as a whole, backwards, even if the angle of thread is larger than that of the angle of friction. This, in connection with the fact that the efficiency for backward movement is low, is probably the reason why many worm-gear drives applied as self-locking have angles of thread in excess of the friction angle and yet seem to work satisfactorily.

Example Involving Allowable Load and Efficiency of Worm Gearing. — Assume that a worm-gear drive is used for driving a freight elevator. The worm-gear shaft runs at a speed of 5 R.P.M.; the required turning moment is 42,000 inch-pounds; the worm gearing must be self-locking to prevent the elevator from running backwards in case the driving belt breaks or runs off while hoisting.

As the elevator must come to a stop before it can begin to run backward, it is sufficient if the tangent of the thread angle is equal to or smaller than the coefficient of friction for rest. Assuming the coefficient of friction to be equal to 0.15 at rest, then the thread angle will equal about 8½ degrees. Using a double-threaded worm of 1¾-inch pitch, the pitch diameter of the worm will be $\frac{1\frac{3}{4} \times 2}{0.15 \times 3.14} = 7.43$ inches. The face of the worm-gear may be made two-thirds of the pitch diameter of the worm or $\frac{2}{3} \times 7.43 = 4.95$, or say 5 inches. Assume that the worm-gear has a 28-inch pitch diameter, 1¾-inch pitch and 50 teeth; then the worm-shaft will run at $5 \times \frac{50}{2} = 125$ revolutions per minute, and the rubbing speed will be:

$$\frac{3.14 \times 7.43 \times 125}{12 \times \cos 8^\circ\ 30'} = 247 \text{ feet per minute.}$$

From the table "Safe Load on Worm-gear Teeth," it is found that for a speed of 250 feet per minute the allowable load is 371 pounds per unit of the product (pitch × width of tooth); hence the total allowable load in this case is: $371 \times 1\frac{3}{4} \times 5 = 3246$ pounds. This load at a radius of 14 inches gives a turning moment of $14 \times 3246 = 45{,}444$ inch-pounds, which is somewhat in excess of the 42,000 inch-pounds required.

It has been assumed that the self-locking gearing will come into play only when the elevator is running upward. If the gearing is expected to lock while running at full speed, or 247 feet per minute, downwards, at which speed the friction coefficient would not be more than 0.05, the thread angle would, approximately, be only 2 degrees 50 minutes, if expected to be self-locking under these conditions; and a thread angle of 8½ degrees could not be expected to lock.

To find the efficiency of the worm gearing when running at full speed, assume a coefficient of friction of 0.05, then:

$$E = \frac{0.15\ (1 - 0.05 \times 0.15)}{0.05 + 0.15} = 74 \text{ per cent.}$$

This is the efficiency of the worm gearing only, and does not allow for the friction lost in the worm-gear shaft or any other frictional loss in other parts of the machine.

To find the force F which must be exerted at the pitch radius of the worm to turn the worm-shaft with its load ($f = 0.15$, when starting):

$$P = \frac{42{,}000}{14} = 3000; \qquad F = 3000 \times \frac{0.15 + 0.15}{1 - 0.15 \times 0.15} = 920 \text{ pounds.}$$

Horsepower Transmitted by Worm Gearing. — In worm drives it is important to keep the diameter of the worm as small as possible to reduce the velocity; if the worm diameter is too large the gearing may become hot and start to cut. If there is a choice between using worms of single, double or triple threads for the same drive, it is preferable to use either double- or triple-threaded types, in order to increase the efficiency. In calculating the horsepower which may be transmitted by worm gearing, the Foote Bros. Gear & Machine Co. recommends that the strength of the worm-wheel be dealt with rather than that of the worm and that, for all practical purposes, the worm-wheel may be considered as an ordinary spur gear so far as the strength of the teeth is concerned.

The accompanying table, which is based upon the well-known Lewis formula for finding the strength of spur gear teeth gives the horsepower that may be transmitted by worm-wheels of 1 diametral pitch and 1-inch width of face, running at various speeds. These values may be used for drives having worms of single, double, triple or quadruple threads, the slight difference due to the number of threads having been disregarded. The table gives the number of revolutions per minute of a worm-wheel corresponding to its pitch-line velocity in feet per minute, so that the horsepower may be determined when either of these two factors is known.

Values for worm-wheels of other diametral pitches and widths of face may be obtained readily from the table. Thus, to obtain the horsepower, divide the horsepower value given in the table by the required diametral pitch and then multiply the quotient by the required width of the worm-wheel face. To determine this face width, measured along the pitch line, multiply the pitch radius of the worm by the value L (see table p. 72) corresponding to the face angle of the worm wheel. (This face angle should not exceed 75 degrees ordinarily.) To obtain the number of revolutions the gear makes to transmit the required horsepower, multiply the number of revolutions per minute tabulated for a worm-wheel of the same number of teeth, by the required diametral pitch of the worm-wheel. The values given are for cast-iron worm-wheels of high-grade workmanship and are based upon an allowable load of 8000 pounds per square inch along the pitch circle of the teeth of a stationary worm-wheel. For phosphor-bronze worm-wheels of high-grade workmanship, the horse power values should be multiplied by 1.5 and for high-grade steel worm-wheels, these values should be multiplied by 2.5.

Power-transmitting Capacity of Cast-iron Worm-wheels. — Assume that it is desired to determine the horsepower which may be transmitted by a cast-iron worm-wheel of high-grade workmanship, having 20 teeth of 1 diametral pitch, a face width of 1 inch and running 100 feet per minute pitch-line velocity; also determine the number of revolutions per minute. From the table it will be seen that a worm-wheel of 20 teeth running 100 feet per minute will transmit 5.9 horsepower and revolve at the rate of 19.1 revolutions per minute. Now, assume that the worm-wheel in this example is of 4 diametral pitch and the face width 2 inches. Such a worm-wheel is capable of transmitting $(5.9 \div 4) \times 2 = 2.9$ horsepower and the number of revolutions per minute will equal 19.1×4 or 76.4 R.P.M.

When the number of revolutions per minute of a worm-wheel is known, the method of determining the horsepower from the table is as follows: Assume that a 25-tooth worm-wheel of 5 diametral pitch and having a face width of 1½ inches runs at the rate of 150 revolutions per minute. As the table is based upon worm-wheels of 1 diametral pitch, the revolutions per minute of the worm-wheel under consideration must be divided by its diametral pitch in order to use the table. Thus, $150 \div 5 = 30$ revolutions per minute. Reference to the table shows that a 25-tooth worm-wheel of 1 diametral pitch and running 30.5 revolutions per minute, has a speed of 200 feet per minute and that the horsepower is 11.1 per inch of face. The latter must be divided and multiplied, respectively, by the diametral pitch and width of face of the worm-wheel being considered. Thus, $(11.1 \div 5) \times 1.5 = 3.3$ horsepower.

Phosphor-bronze and Steel Worm-wheels. — Assume that a high-grade phosphor-bronze worm-wheel having 60 teeth of 5 diametral pitch and a face width of 3½ inches, runs at the rate of 1200 feet per minute. According to the table, a cast-iron worm-wheel having 60 teeth of 1 diametral pitch and 1 inch face width transmits 34.7 horsepower when running 1200 feet per minute and rotates at the rate of 76.4 revolutions per minute. Therefore, a cast-iron worm-wheel having 60 teeth of 5 diametral pitch, a face width of 3½ inches and running at the specified speed would transmit $(34.7 \div 5) \times 3.5 = 24.3$ horsepower and the number of revolutions per minute would equal 76.4×5 or 382. As the worm-wheel under consideration is made of phosphor-bronze instead of cast iron, however, the horsepower value obtained must be multiplied by 1.5. Thus, $1.5 \times 24.3 = 36.4$ horsepower may be transmitted by a phosphor-bronze worm-wheel. If the worm-wheel were made of steel, the horsepower value for the cast-iron worm-wheel would be multiplied by 2.5. Thus, $2.5 \times 24.3 = 60.7$ horsepower could be transmitted by a steel

Horsepower of Cast-iron Worm-wheels of 1 Diametral Pitch, per Inch of Face Width

No. of Teeth for R.P.M. and H.P.		Pitch-line Velocity of Worm-wheel, in Feet per Minute							
		100	200	300	600	900	1200	1800	2400
12	R.P.M.	31.8	63.7	95.5	191.0	286.5	382.0	572.9	763.9
	H.P.	4.4	7.6	10.2	15.3	18.3	20.4	22.9	24.4
15	R.P.M.	25.5	50.9	76.4	152.8	229.2	305.6	458.4	611.1
	H.P.	4.9	8.6	11.4	17.2	20.6	22.9	25.7	27.5
17	R.P.M.	22.5	44.9	67.4	134.8	202.2	269.6	404.4	539.2
	H.P.	5.2	9.1	12.2	18.2	21.9	24.3	27.4	29.2
20	R.P.M.	19.1	38.2	57.3	114.6	171.9	229.2	343.8	458.4
	H.P.	5.9	10.3	13.7	20.6	24.7	27.4	30.9	32.9
22	R.P.M.	17.4	34.7	52.1	104.2	156.3	208.3	312.5	416.7
	H.P.	6.1	10.6	14.1	21.2	25.5	28.3	31.8	34.0
25	R.P.M.	15.3	30.5	45.8	91.7	137.5	183.3	275.0	366.7
	H.P.	6.3	11.1	14.8	22.2	26.6	29.6	33.3	35.5
27	R.P.M.	14.1	28.3	42.4	84.9	127.3	169.7	254.6	339.5
	H.P.	6.5	11.4	15.2	22.8	27.4	30.4	34.2	36.5
30	R.P.M.	12.7	25.5	38.2	76.4	114.6	152.8	229.2	305.6
	H.P.	6.6	11.6	15.5	23.3	27.9	31.0	34.9	37.2
32	R.P.M.	11.9	23.9	35.8	71.6	107.4	143.2	214.8	286.5
	H.P.	6.7	11.8	15.7	23.5	28.2	31.4	35.3	37.6
35	R.P.M.	10.9	21.8	32.7	65.5	98.2	131.0	196.4	261.9
	H.P.	6.8	12.0	16.0	23.9	28.7	31.9	35.9	38.3
37	R.P.M.	10.3	20.6	31.0	61.9	92.9	123.9	185.8	247.8
	H.P.	6.9	12.1	16.2	24.3	29.1	32.4	36.4	38.8
40	R.P.M.	9.5	19.1	28.6	57.3	85.9	114.6	171.9	229.2
	H.P.	7.1	12.4	16.5	24.7	29.6	33.0	37.1	39.6
42	R.P.M.	9.1	18.2	27.3	54.6	81.8	109.1	163.7	218.3
	H.P.	7.1	12.5	16.7	25.0	30.0	33.3	37.5	40.0
45	R.P.M.	8.5	17.0	25.5	50.9	76.4	101.8	152.8	203.7
	H.P.	7.2	12.6	16.9	25.3	30.3	33.7	37.9	40.5
47	R.P.M.	8.1	16.2	24.4	48.8	73.1	97.5	146.3	195.0
	H.P.	7.2	12.7	16.9	25.4	30.5	33.9	38.1	40.6
50	R.P.M.	7.6	15.3	22.9	45.8	68.7	91.7	137.5	183.3
	H.P.	7.3	12.8	17.1	25.6	30.7	34.1	38.4	41.0
55	R.P.M.	6.9	13.9	20.8	41.7	62.5	83.3	125.0	166.7
	H.P.	7.4	12.9	17.2	25.8	31.0	34.4	38.7	41.3
60	R.P.M.	6.4	12.7	19.1	38.2	57.3	76.4	114.6	152.8
	H.P.	7.4	13.0	17.3	26.0	31.2	34.7	39.0	41.6
70	R.P.M.	5.4	10.9	16.4	32.7	49.1	65.5	98.2	131.0
	H.P.	7.5	13.2	17.5	26.3	31.6	35.1	39.5	42.1
80	R.P.M.	4.8	9.5	14.3	28.6	43.0	57.3	85.9	114.6
	H.P.	7.6	13.3	17.7	26.6	31.9	35.4	39.8	42.5
90	R.P.M.	4.2	8.5	12.7	25.5	38.2	50.9	76.4	101.8
	H.P.	7.6	13.4	17.8	26.8	32.1	35.7	40.2	42.8
100	R.P.M	3.8	7.6	11.4	22.9	34.4	45.8	68.7	91.7
	H.P.	7.7	13.5	18.0	27.0	32.4	36.0	40.5	43.2
115	R.P.M.	3.3	6.6	10.0	19.9	29.9	39.8	59.8	79.7
	H.P.	7.7	13.5	18.1	27.1	32.5	36.1	40.7	43.4
130	R.P.M.	2.9	5.9	8.8	17.6	26.4	35.2	52.9	70.5
	H.P.	7.8	13.6	18.2	27.2	32.7	36.3	40.9	43.6
150	R.P.M.	2.5	5.1	7.6	15.3	22.9	30.5	45.8	61.1
	H.P.	7.8	13.7	18.3	27.4	32.9	36.5	41.1	43.9

worm-wheel of the specifications. When it is desired to find the strength of a worm-wheel having a number of teeth not given in the table, an approximate value sufficiently close for practical purposes may be secured by interpolation.

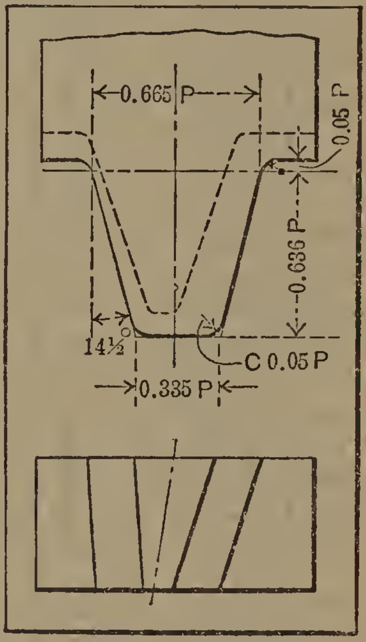
Calculating the Safe Working Load on the Teeth. — In worm-wheel calculations frequently the safe working load on the teeth is desired rather than the horsepower which may be transmitted. This safe load can be determined by using the horsepower values given in the table in connection with the formula

$$W = \frac{\text{H.P.} \times 33{,}000}{V}$$

in which W = the safe working load on the worm-wheel teeth in pounds per inch of face; H.P. = the horsepower which may be transmitted by the worm-wheel; and V = the velocity of the worm-wheel in feet per minute. In order to illustrate the application of this formula, assume that 6.7 horsepower may be transmitted by a certain drive in which the velocity of the worm-wheel is 100 feet per minute. By inserting the known values in the formula, the safe working load on the worm-wheel teeth may be calculated. Thus,

$$W = \frac{6.7 \times 33{,}000}{100} = 2211 \text{ pounds}$$

Hobs for Worm Gears. — The hob for worm gears should not be made an exact duplicate of the worm. The corners at the top of the hob teeth should be rounded and fillets should also be provided at the root of the thread. The radii of these rounded corners and fillets should be as large as the clearance allows, or about 1/20 of the circular pitch of the thread. The dimensions of the hob which are different from the dimensions of the worm are as follows:



Outside diameter of hob = outside diameter of worm + 0.1 × linear pitch of worm.

Root diameter of hob = outside diameter of worm − 1.2732 × linear pitch of worm.

Dimensions for proportioning the tool for cutting the hob thread are given in the accompanying illustration. When threading with such a tool the depth of the thread is gaged by the shoulder on the tool, and accuracy is insured if the blank diameter is correct. The tool ought to be provided with side clearance of from 5 to 10 degrees from the angle of the thread. Grinding a tool of this kind, of course, changes its form so that it must not be used indefinitely in making a large number of similar hobs.

For finding the approximate number of flutes in a hob, the following rule may be used: Multiply the diameter of the hob by 3, and divide this product by twice the linear pitch. This rule gives suitable results for hobs for general purposes. Certain modifications, however, are necessary as explained in the following paragraph "Number of Flutes or Gashes in Hobs." The cutter used in gashing hobs should be from 1/8 to 1/4 inch thick at the periphery, according to the pitch of the thread of the hob. The width of the gash at the periphery of the hob should be about 0.4 times the pitch of the flutes. The cutter should be sunk into the blank so that it reaches from 3/16 to 1/4 inch below the root of the thread. If a hob is to be used a great deal and is subjected to much wear, it is advisable to increase the outside diameter above the dimensions given from 0.010 to 0.030 inch, according to its diameter and pitch, to allow for the decrease in diameter due to the relief and repeated grindings.

Number of Flutes or Gashes in Hobs. — It is important that the number of flutes or gashes in hobs bear a certain relation to the number of threads in the hob and the number of teeth in the worm-wheel to be hobbed. In the first place, avoid having a common factor between the number of threads in the hob and the number of flutes; that is, if the worm is double-threaded, the number of gashes should be, say, 7 or 9, rather than 8. If it is triple-threaded, the number of gashes should be 7 or 11, rather than 6 or 9. The second requirement is to avoid having a common factor between the number of threads in the hob and the number of teeth in the worm-wheel. For example, if the number of teeth in the wheel is 28, it would be best to have the hob triple-threaded, as 3 is not a factor of 28. Again, if there were to be 36 threads in the worm-gear, it would be preferable to have 5 threads in the hob.

Spiral Fluted Hobs. — Hobs are generally fluted parallel with the axis, but it is obvious that the cutting action will be better if they are fluted on a spiral at right angles with the thread helix. The difficulty of relieving the teeth with the ordinary backing-off attachment is the cause for using a flute parallel with the axis. Flutes cut at right angles to the direction of the thread can, however, also be relieved, if the angle of the flutes is slightly modified. In order to relieve hobs with a regular relieving attachment, it is necessary that the number of teeth in one revolution along the thread helix be such that the relieving attachment can be geared to suit it. The following method makes it possible to select an angle of flute that will make the flute come *approximately* at right angles to the thread, and at the same time the angle is so selected that the relieving attachment can be properly geared for relieving the hob.

Let C = pitch circumference;
T = developed length of thread in one turn;
N = number of teeth in one turn along thread helix;
F = number of flutes;
α = angle of thread helix.

Then (see illustration on the following page):

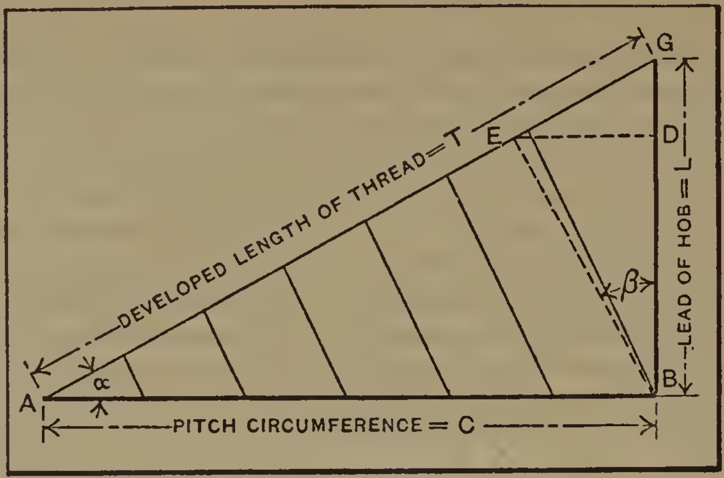
$C \div F$ = length of each small division on pitch circumference;
$(C \div F) \times \cos \alpha$ = length of division on developed thread;
$C \div \cos \alpha = T$.

Hence
$$\frac{T}{(C \div F)\cos \alpha} = N = \frac{F}{\cos^2 \alpha}$$

Now, if $\alpha = 30$ degrees, $N = 1\frac{1}{3} F$;
$\alpha = 45$ degrees, $N = 2F$;
$\alpha = 60$ degrees, $N = 4F$.

In most cases, however, such simple relations are not obtained. Suppose for example that $F = 7$, and $\alpha = 35$ degrees. Then $N = 10.432$, and no gears could be selected that would relieve this hob. By a very slight change in the spiral angle of the flute, however, we can change N to 10 or $10\frac{1}{2}$; in either case we can find suitable gears for the relieving attachment.

The rule for finding the modified spiral lead of the flute is: Multiply the lead of the hob by F, and divide the product by the difference between the desired values of N and F.



Hence, the lead of flute required to make $N = 10$ is:

Lead of hob $\times (7 \div 3)$.

To make $N = 10\frac{1}{2}$, we have:

Lead of flute = lead of hob $\times (7 \div 3.5)$.

From this the angle of the flute can easily be found.

That the rule given is correct will be understood from the following consideration. Change the angle of the flute helix β so that AG contains the required number of parts N desired. Then EG contains $N - F$ parts. But $\cot \beta = BD \div ED$ and by the law of similar triangles,

$$BD = \frac{F}{N} \times BG, \text{ and } ED = \frac{N-F}{N} C$$

The lead of the spiral of the flute, however, is $C \times \cot \beta$.

Hence, the required lead of spiral of the flute:

$$C \times \cot \beta = \frac{F}{N-F} L$$

This formula makes it possible always to flute hobs so that they can be conveniently relieved, and at the same time have the flutes at approximately right angles to the thread.

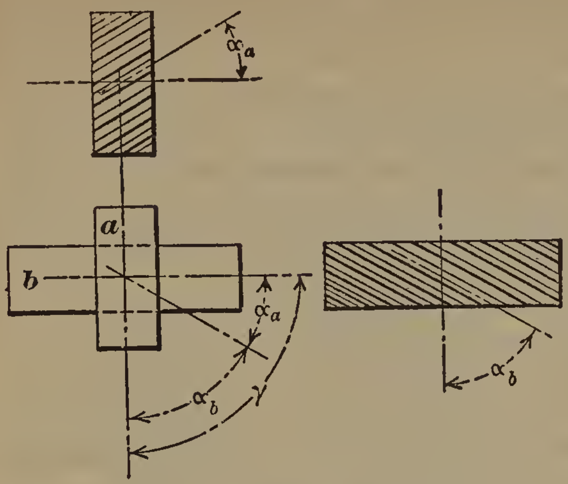
Spiral Gearing

Basic Rules and Formulas for Spiral Gear Calculations. — The ten rules and formulas in the accompanying table may be called the basic rules for spiral gear calculations. The following definitions should be clearly understood in order to avoid misunderstandings. The *center angle* of a pair of spiral gears is the angle made by the two center lines or axes of the gears. The *tooth angle* is the angle which the direction of the tooth makes with the axis of the gear. The *normal diametral pitch* is the diametral pitch of the cutter used for cutting the teeth in spiral gears. In the formulas in the table of "Basic Rules and Formulas for Spiral Gear Calculations" the following notation is used:

P_n = normal diametral pitch (pitch of cutter);
D = pitch diameter;
N = number of teeth;
α = spiral angle;
γ = center angle, or angle between shafts;
C = center distance;
N' = number of teeth for which to select cutter;
L = lead of tooth helix;
S = addendum;
W = whole depth of tooth;
T_n = normal tooth thickness at pitch line;
O = outside diameter.

The rules and formulas are given in the same order as they would ordinarily be used by the designer when calculating a pair of spiral gears. The formulas, how-

Basic Rules and Formulas for Spiral Gear Calculations



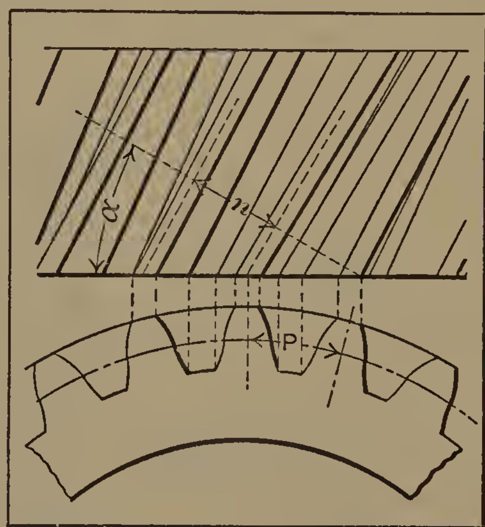
In the formulas, N, α, etc., are the numbers of teeth, spiral angle, etc., for *either* gear or pinion; the notations N_a, N_b, α_a, α_b, etc., refer to the teeth or angles in the pinion or gear, respectively, in a pair of gears a and b.

No.	To Find	Rule	Formula
1	Relation between Shaft and Tooth Angles.	The sum of the tooth angles of a pair of mating helical gears is equal to the shaft angle.	$\gamma = \alpha_a + \alpha_b$
2	Pitch Diameter.	Divide the number of teeth by the product of the normal pitch and the cosine of the tooth angle.	$D = \frac{N}{P_n \cos\alpha}$
3	Center Distance.	Add together the pitch diameters of the two gears and divide by 2.	$C = \frac{D_a + D_b}{2}$
4	Checking Calculations in (2) and (3).	To prove the calculations for pitch diameters and center distance, multiply the number of teeth in the first gear by the tangent of the tooth angle of that gear, and add the number of teeth in the second gear to the product; the sum should equal twice the product of the center distance multiplied by the normal diametral pitch, multiplied by the sine of the tooth angle of the first gear.	$N_b + (N_a \times \tan\alpha_a) = 2CP_n \times \sin\alpha_a$
5	Number of Teeth for which to Select Cutter.	Divide the number of teeth in the gear by the cube of the cosine of the tooth angle.	$N' = \frac{N}{(\cos\alpha)^3}$
6	Lead of Tooth Helix.	Multiply the pitch diameter by 3.1416 times the cotangent of the tooth angle.	$L = \pi D \times \cot\alpha$
7	Addendum.	Divide 1 by the normal diametral pitch.	$S = \frac{1}{P_n}$
8	Whole Depth of Tooth.	Divide 2.157 by the normal diametral pitch.	$W = \frac{2.157}{P_n}$
9	Normal Tooth Thickness at Pitch Line.	Divide 1.571 by the normal diametral pitch.	$T_n = \frac{1.571}{P_n}$
10	Outside Diameter.	Add twice the addendum to the pitch diameter.	$O = D + 2S$

ever, cannot be directly applied to all cases of spiral gear problems, and a complete set of formulas for each of the sixteen different cases which are frequently met with is, therefore, given in the following, together with an example for each case. These sixteen cases are:

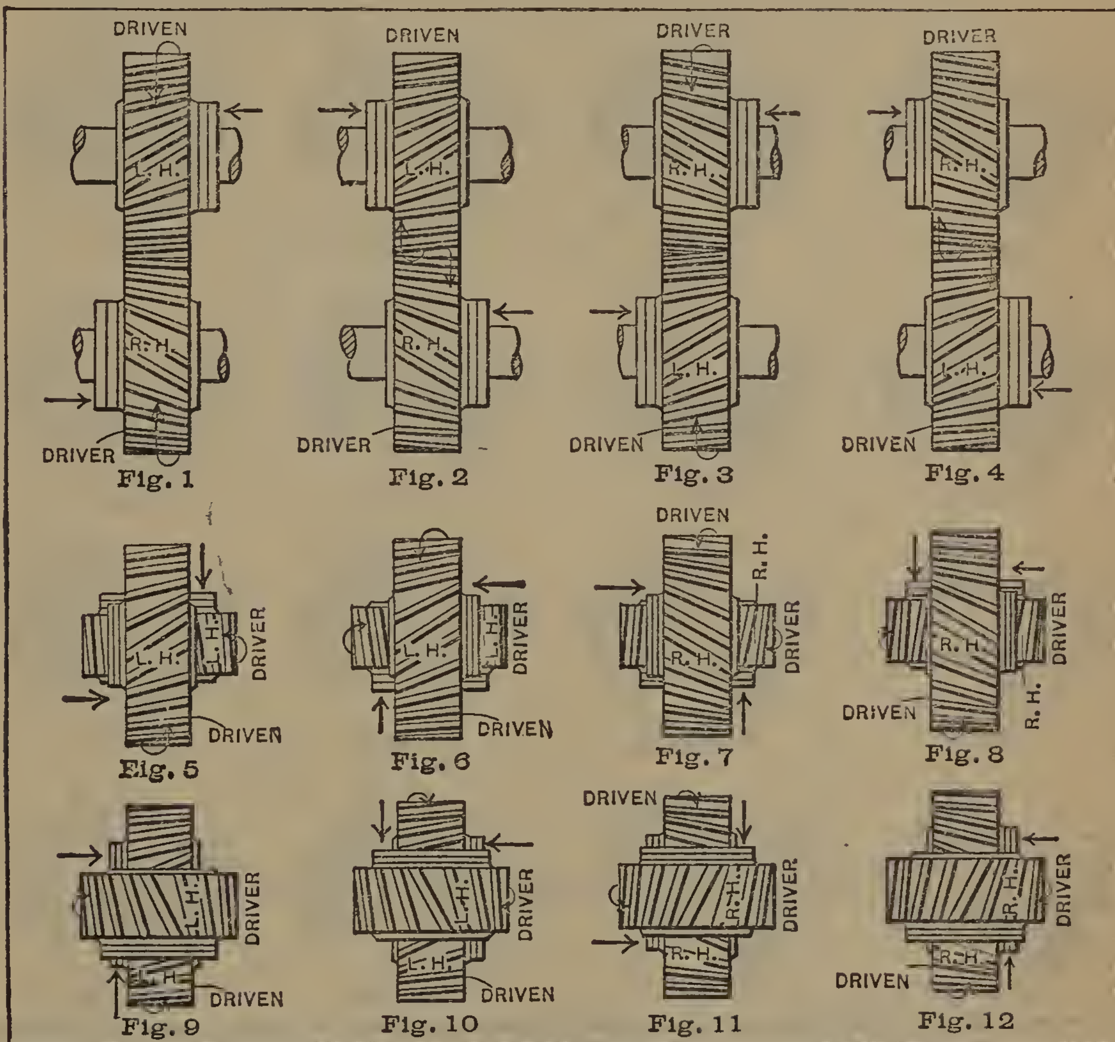
1. Shafts parallel, ratio equal, and center distance approximate.
2. Shafts parallel, ratio equal, and center distance exact.
3. Shafts parallel, ratio unequal, and center distance approximate.
4. Shafts parallel, ratio unequal, and center distance exact.
5. Shafts at right angles, ratio equal, and center distance approximate.
6. Shafts at right angles, ratio equal, and center distance exact.
7. Shafts at right angles, ratio unequal, and center distance approximate.
8. Shafts at right angles, ratio unequal, and center distance exact.
9. Shafts at 45-degree angle, ratio equal, and center distance approximate.
10. Shafts at 45-degree angle, ratio equal, and center distance exact.
11. Shafts at 45-degree angle, ratio unequal, and center distance approximate.
12. Shafts at 45-degree angle, ratio unequal, and center distance exact.
13. Shafts at any angle, ratio equal, and center distance approximate.
14. Shafts at any angle, ratio equal, and center distance exact.
15. Shafts at any angle, ratio unequal, and center distance approximate.
16. Shafts at any angle, ratio unequal, and center distance exact.

Pitch of Cutter for Spiral Gears. — The thickness of the cutter at the pitch line for milling spiral gears should equal one-half the normal circular pitch n (see illustration). If a cutter were used having a thickness, at the pitch line, equal to one-half the circular pitch P, as for spur gearing, the spaces between the teeth would be cut too wide, thus producing thin teeth. The normal pitch varies with the angle α of the spiral; hence, the spiral angle must be considered when selecting a cutter. The cutter should be of the same pitch as the *normal* diametral pitch of the gear and this normal pitch is found by dividing the "real" diametral pitch by the cosine of the spiral angle. To illustrate, if the pitch diameter of a spiral gear is 6.718 and there are 38 teeth having a spiral angle of 45 degrees, the "real" diametral pitch equals $38 \div 6.718 = 5.656$; then, the normal diametral pitch equals 5.656 divided by the cosine of 45 degrees or $5.656 \div 0.707 = 8$. A cutter, then, of 8 diametral pitch is the one to use for this particular gear. This same result could also be obtained as follows: If the circular pitch P is 0.5554, the normal circular pitch n can be found by multiplying the circular pitch P by the cosine of the spiral angle. For example, $0.5554 \times 0.707 = 0.3927$. The normal diametral pitch is then found by dividing 3.1416 by the normal circular pitch. Thus $\frac{3.1416}{0.3927} = 8$, which is the diametral pitch of the cutter.



According to the Brown & Sharpe system of cutters for spur gears having involute teeth, eight different shapes of cutters (marked by numbers) are used for various numbers of teeth in gears of any one pitch. When the diametral pitch is known, the number of cutter for that particular pitch must, therefore, be determined. The method of doing this for spiral gears is explained in the paragraph, "Number of Cutter for Spiral Gears," page 734.

Procedure in Calculating Spiral Gears. — One of the first steps necessary in spiral gear design is to determine the direction of the thrust, if the thrust is to be taken in one direction only. When the direction of the thrust has been determined and the relative position of the driver and driven gear is known, the direction of spiral (right- or left-hand) may be found. The thrust diagrams, Figs. 1 to 28, are used for finding the direction of spiral. The arrows at the end bearings of the gears indicate the direction of the reaction against the thrust caused by the tooth pressure. The direction of the thrust depends on the direction of spiral, the relative positions of driver and driven gear, and the direction of rotation. If the

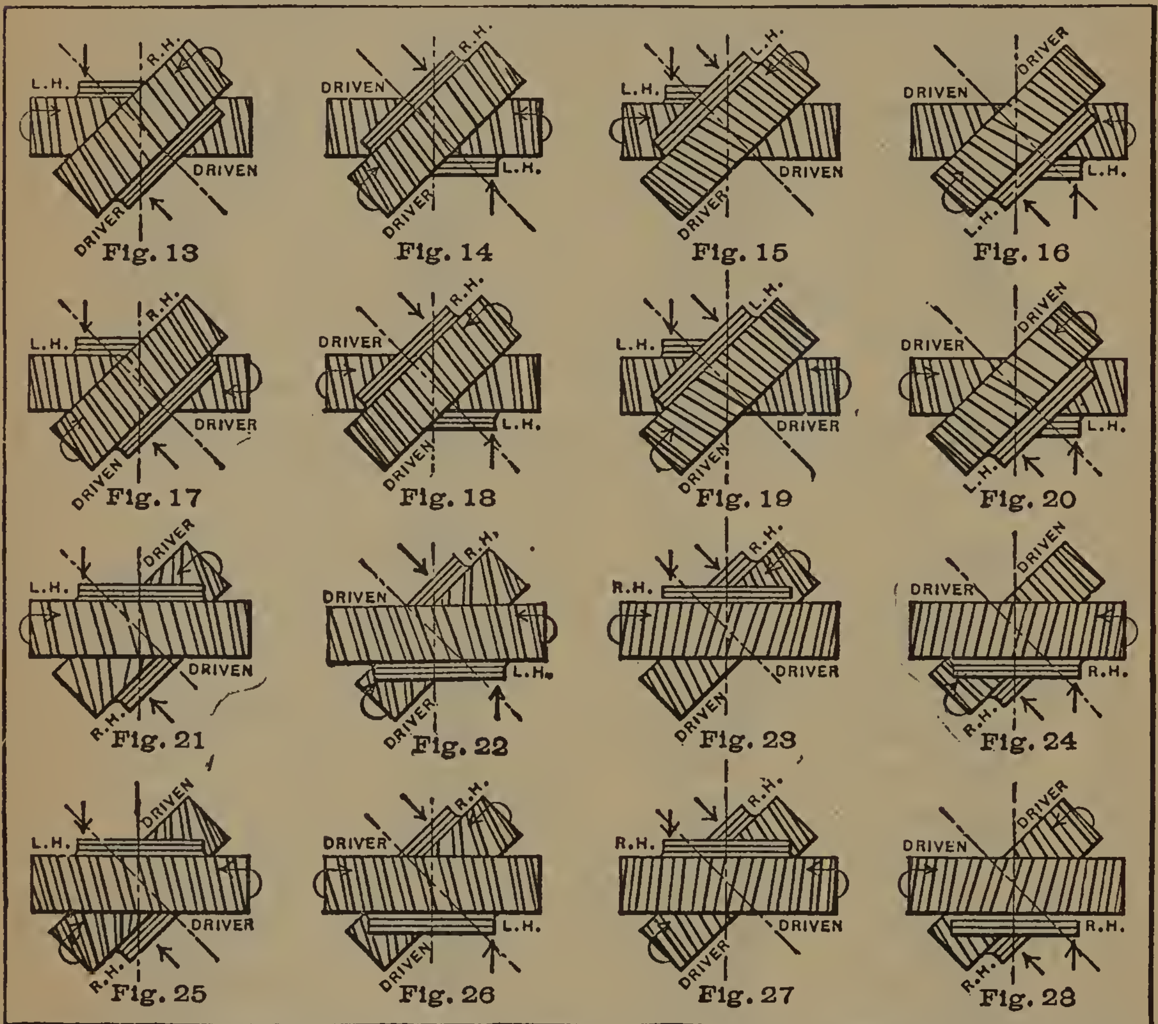


Figs. 1 to 12. Thrust Diagrams for Spiral Gears — Direction of Thrust depends upon Direction of Rotation, Relative Position of Driver and Driven Gear, and Direction of Spiral

exact condition with regard to thrust is not found in the diagrams, it may be obtained by changing any one of these three conditions; that is, in Fig. 1 the thrust may be changed to the opposite direction by interchanging driver or driven gear, by reversing the direction of rotation or by changing the direction of spiral. Any one of these alterations will produce a thrust in the opposite direction.

The conditions of the design determine the nature of the center distance, whether it must be exact or approximate. The number of teeth in each gear is, of course, determined by the required speed ratios of the shafts. The angle of spiral depends upon the conditions of the design and the relative position of the shafts. For parallel shafts the spiral angle should not exceed 20 degrees in order to avoid excessive end thrust. In order to obtain smooth running gears, the spiral angle should be such that one end of the tooth remains in contact until the opposite end of the following tooth has found a bearing.

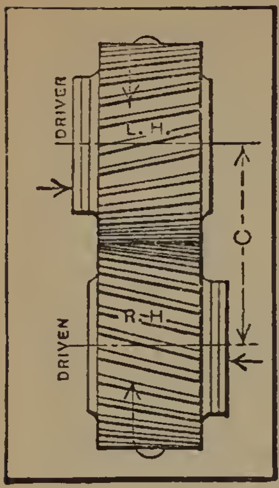
As far as the calculations are concerned, the formulas are the same for a 135-degree shaft angle as for an angle of 45 degrees. The following general rule relative to gears having a shaft angle of 45 degrees should be observed: When the spiral angle of each gear is less than 45 degrees, then the spiral angles are of the same hand and one spiral angle is 45 degrees minus the other. When the spiral angle of either gear is greater than 45 degrees, then the spiral angles are of opposite hand, and the spiral angle of one gear is 45 degrees plus the spiral angle of the other.



Figs. 13 to 28. Thrust Diagrams for Spiral Gears — Direction of Thrust depends upon Direction of Rotation, Relative Position of Driver and Driven Gear, and Direction of Spiral

When designing spiral gears with shafts at an angle other than 90 degrees to each other, it is of considerable advantage to draw the outline of one gear on a piece of drawing paper tacked to the drawing board, and the outline of the other on a piece of tracing paper. In this way the gear drawn on the tracing paper can be moved about to the correct angle with relation to the gear beneath, and the conditions of thrust, direction of rotation and hand of spiral can be more easily determined. The following rules should be observed for spiral gears with shafts at any given angle. If each spiral angle is less than the shaft angle, then the sum of the spiral angles of the two gears will equal the angle between the shafts, and the spiral is of the same hand in both gears; if the spiral angle of one of the gears is greater than the shaft angle, then the difference between the spiral angles of the two gears will be equal to the shaft angle, and the gears will be of opposite hand.

1. Shafts Parallel, Ratio Equal, Center Distance Approximate.—



Given or assumed:

1. Hand of spiral on driver or driven gear depending on rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. N = number of teeth.
5. α = angle of spiral (usually less than 20 degrees to avoid excessive end thrust).

To find:

1. D = pitch diameter = $\dfrac{N}{P_n \cos \alpha}$
2. O = outside diameter = $D + \dfrac{2}{P_n}$
3. T = number of teeth marked on cutter = $\dfrac{N}{\cos^3 \alpha}$
4. L = lead of spiral = $\pi D \cot \alpha$.

Example

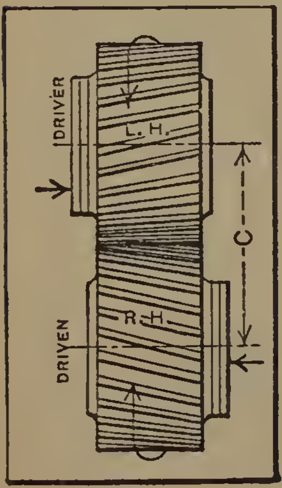
Given or assumed:

1. See illustration.
2. $C_a = 3$ inches
3. $P_n = 8$.
4. $N = 24$.
5. $\alpha = 15$ degrees.

To find:

1. $D = \dfrac{N}{P_n \cos \alpha} = \dfrac{24}{8 \times 0.9659} = 3.106$ inches.
2. $O = 3.106 + \dfrac{2}{8} = 3.356$ inches.
3. $T = \dfrac{N}{\cos^3 \alpha} = \dfrac{24}{0.9} = 26.6$, say 27 teeth.
4. $L = \pi D \cot 15° = 3.1416 \times 3.106 \times 3.732 = 36.416$ inches.

2. Shafts Parallel, Ratio Equal, Center Distance Exact.



Given or assumed:

1. Position of gear having right- or left-hand spiral, depending on rotation and direction in which thrust is to be received.
2. C = exact center distance = pitch diameter D.
3. P_n = normal pitch (pitch of cutter).
4. N = number of teeth in each gear.

To find:

1. $\cos \alpha = \dfrac{N}{P_n D}$
2. O = outside diameter = $D + \dfrac{2}{P_n}$
3. T = number of teeth marked on cutter = $\dfrac{N}{\cos^3 \alpha}$
4. L = lead of spiral = $\pi D \cot \alpha$.

α is usually less than 20 degrees to avoid excessive end thrust.

Example

Given or assumed:

1. See illustration.
2. $C = 3$ inches.
3. $P_n = 8$.
4. $N = 22$.

To find:

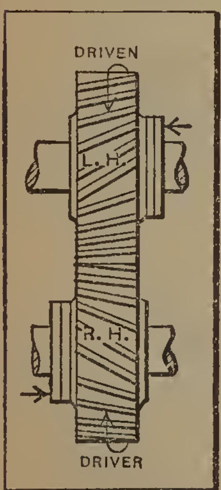
1. $\cos\alpha = \dfrac{N}{P_nD} = \dfrac{22}{8\times 3} = 0.9166$, or $\alpha = 23°\ 34'$.

2. $O = D + \dfrac{2}{P_n} = 3 + \dfrac{2}{8} = 3\frac{1}{4}$ inches.

3. $T = \dfrac{N}{\cos^3\alpha} = \dfrac{22}{(0.92)^3} = 28.2$, say 28 teeth.

4. $L = \pi D \cot\alpha = 3.1416 \times 3 \times 2.29 = 21.58$ inches.

3. Shafts Parallel, Ratio Unequal, Center Distance Approximate. — Given or assumed:



1. Position of gear having right- or left-hand spiral, depending upon rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch.
4. N = number of teeth in large gear.
5. n = number of teeth in small gear.
6. α = angle of spiral.

To find:

1. D = pitch diameter of large gear $= \dfrac{N}{P_n \cos\alpha}$
2. d = pitch diameter of small gear $= \dfrac{n}{P_n \cos\alpha}$
3. O = outside diameter of large gear $= D + \dfrac{2}{P_n}$
4. o = outside diameter of small gear $= d + \dfrac{2}{P_n}$
5. T = number of teeth marked on cutter (large gear) $= \dfrac{N}{\cos^3\alpha}$
6. t = number of teeth marked on cutter (small gear) $= \dfrac{n}{\cos^3\alpha}$
7. L = lead of spiral on large gear $= \pi D \cot\alpha$.
8. l = lead of spiral on small gear $= \pi d \cot\alpha$.
9. C = center distance (if not right vary α) $= \frac{1}{2}(D + d)$.

Example

Given or assumed:

1. See illustration.
2. $C_a = 17$ inches.
3. $P_n = 2$.
4. $N = 48$.
5. $n = 20$.
6. $\alpha = 20$ degrees.

To find:

1. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{48}{2 \times 0.9397} = 25.541$ inches.

2. $d = \dfrac{n}{P_n \cos\alpha} = \dfrac{20}{2 \times 0.9397} = 10.642$ inches.

3. $O = D + \frac{2}{P_n} = 25.541 + \frac{2}{2} = 26.541$ inches.

4. $o = d + \frac{2}{P_n} = 10.642 + \frac{2}{2} = 11.642$ inches.

5. $T = \frac{N}{\cos^3 \alpha} = \frac{48}{(0.9397)^3} = 57.8$, say 58 teeth.

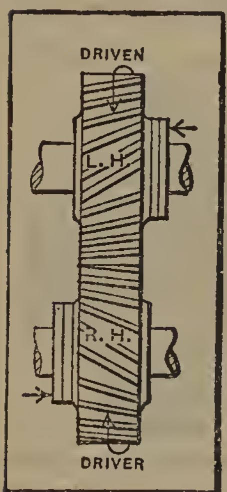
6. $t = \frac{n}{\cos^3 \alpha} = \frac{20}{(0.9397)^3} = 24.1$, say 24 teeth.

7. $L = \pi D \cot \alpha = 3.1416 \times 25.541 \times 2.747 = 220.42$ inches.

8. $l = \pi d \cot \alpha = 3.1416 \times 10.642 \times 2.747 = 91.84$ inches.

9. $C = \frac{1}{2}(D + d) = \frac{1}{2}(25.541 + 10.642) = 18.091$ inches.

4. Shafts Parallel, Ratio Unequal, Center Distance Exact. —



Given or assumed:

1. Position of gear having right- or left-hand spiral, depending upon rotation and direction in which thrust is to be received.
2. C = exact center distance.
3. P_n = normal pitch (pitch of cutter).
4. N = number of teeth in large gear.
5. n = number of teeth in small gear.

To find:

1. $\cos \alpha = \frac{N + n}{2 P_n C}$
2. D = pitch diameter of large gear $= \frac{N}{P_n \cos \alpha}$
3. d = pitch diameter of small gear $= \frac{n}{P_n \cos \alpha}$
4. O = outside diameter of large gear $= D + \frac{2}{P_n}$
5. o = outside diameter of small gear $= d + \frac{2}{P_n}$
6. T = number of teeth marked on cutter (large gear) $= \frac{N}{\cos^3 \alpha}$
7. t = number of teeth marked on cutter (small gear) $= \frac{n}{\cos^3 \alpha}$
8. L = lead of spiral (large gear) $= \pi D \cot \alpha$
9. l = lead of spiral (small gear) $= \pi d \cot \alpha$

Example

Given or assumed:

1. See illustration. 2. $C = 18.75$ inches. 3. $P_n = 4$. 4. $N = 96$. 5. $n = 48$.

To find:

1. $\cos \alpha = \frac{N + n}{2 P_n C} = \frac{96 + 48}{2 \times 4 \times 18.75} = 0.96$, or $\alpha = 16° 16'$.

2. $D = \frac{N}{P_n \cos \alpha} = \frac{96}{4 \times 0.96} = 25$ inches.

3. $d = \dfrac{n}{P_n \cos\alpha} = \dfrac{48}{4 \times 0.96} = 12.5$ inches.

4. $O = D + \dfrac{2}{P_n} = 25 + \dfrac{2}{4} = 25.5$ inches.

5. $o = d + \dfrac{2}{P_n} = 12.5 + \dfrac{2}{4} = 13$ inches.

6. $T = \dfrac{N}{\cos^3\alpha} = \dfrac{96}{(0.96)^3} = 108$ teeth.

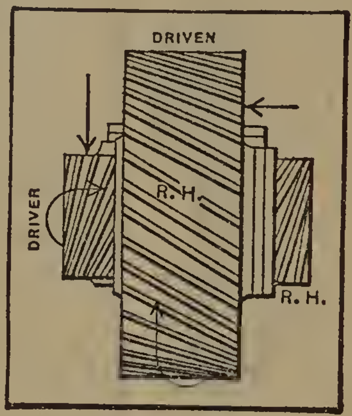
7. $t = \dfrac{n}{\cos^3\alpha} = \dfrac{48}{(0.96)^3} = 54$ teeth.

8. $L = \pi D \cot\alpha = 3.1416 \times 25 \times 3.427 = 269.15$ inches.

9. $l = \pi d \cot\alpha = 3.1416 \times 12.5 \times 3.427 = 134.57$ inches.

5. Shafts at Right Angles, Ratio Equal, Center Distance Approximate. —

When the spiral angles are 45 degrees, the gears are exactly alike; when other than 45 degrees, the sum of the spiral angles must equal 90 degrees.



Given or assumed:

1. Position of gear having right- or left-hand spiral, depending on the rotation and direction in which the thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. N = number of teeth.
5. α = angle of spiral.

To find:

(*a*) When spiral angles are 45 degrees.

1. D = pitch diameter $= \dfrac{N}{0.70711\, P_n}$ 2. O = outside diameter $= D + \dfrac{2}{P_n}$

3. T = number of teeth marked on cutter $= \dfrac{N}{0.353}$

4. L = lead of spiral $= \pi D$. 5. C = center distance $= D$.

(*b*) When spiral angles are other than 45 degrees.

1. D = pitch diameter $= \dfrac{N}{P_n \cos\alpha}$

2. T = number of teeth marked on cutter $= \dfrac{N}{\cos^3\alpha}$

3. C = center distance = sum of pitch radii.

4. L = lead of spiral $= \pi D \cot\alpha$.

Example

Given or assumed:

1. See illustration. 2. $C_a = 2.5$ inches. 3. $P_n = 10$. 4. $N = 18$ teeth. 5. $\alpha = 45$ degrees.

To find:

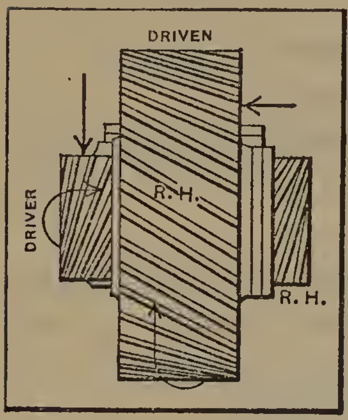
1. $D = \dfrac{N}{0.70711\, P_n} = \dfrac{18}{0.70711 \times 10} = 2.546$ inches.

2. $O = D + \dfrac{2}{P_n} = 2.546 + \dfrac{2}{10} = 2.746$ inches.

3. $T = \dfrac{N}{\cos^3 \alpha} = \dfrac{18}{0.353} = 51$ teeth.

4. $L = \pi D \times 1 = 3.1416 \times 2.546 = 7.999$ inches.

6. Shafts at Right Angles, Ratio Equal, Center Distance Exact. —



Gears have same direction of spiral but probably different pitch diameters and spiral angles; the sum of the latter must be 90 degrees.

Given or assumed:

1. Position of gear having right- and left-hand spiral depending on rotation and direction in which thrust is to be received.
2. P_n = normal pitch (pitch of cutter).
3. ϕ = approximate spiral angle of one gear.
4. C = center distance.
5. N = number of teeth = nearest whole number to $CP_n \times \cos \phi$.

To find:

1. α = spiral angle of one gear

$$\sin\ 2\alpha = \frac{N^2}{2\,C^2P_n^2} \pm \sqrt{\frac{N^2}{C^2P_n^2} + \left(\frac{N^2}{2\,C^2P_n^2}\right)^2}$$

2. β = spiral angle of other gear $= 90° - \alpha$.

3. D = pitch diameter of one gear $= \dfrac{N}{P_n \cos \alpha}$

4. d = pitch diameter of other gear $= \dfrac{N}{P_n \cos \beta}$

5. O = outside diameter of one gear $= D + \dfrac{2}{P_n}$

6. o = outside diameter of other gear $= d + \dfrac{2}{P_n}$

7. T = number of teeth marked on cutter for one gear $= \dfrac{N}{\cos^3 \alpha}$

8. t = number of teeth marked on cutter for other gear $= \dfrac{N}{\cos^3 \beta}$

9. L = lead of spiral for one gear $= \pi D \cot \alpha$.

10. l = lead of spiral for other gear $= \pi d \cot \beta$.

Example

Given or assumed:

1. See illustration. 2. $P_n = 10$. 3. $\phi = 45$ degrees. 4. $C = 4$ inches.

5. $N = CP_n \cos \phi = 4 \times 10 \times 0.70711 = 28.28$, say 28 teeth.

To find:

1. $\sin 2\alpha = 0.98664$, or $\alpha = 40° 19'$. 2. $\beta = 90° - \alpha = 49° 41'$.

3. $D = \dfrac{N}{P_n \cos \alpha} = \dfrac{28}{10 \times 0.76248} = 3.672$ inches.

4. $d = \dfrac{N}{P_n \cos \beta} = \dfrac{28}{10 \times 0.64701} = 4.328$ inches.

5. $O = 3.672 + 0.2 = 3.872$ inches.

6. $o = 4.328 + 0.2 = 4.528$ inches.

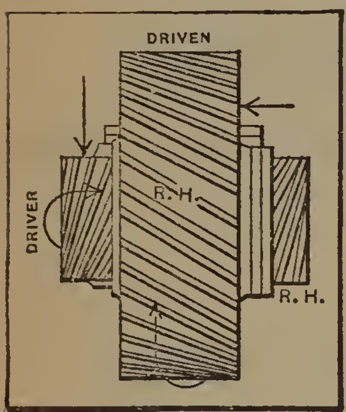
7. $T = \dfrac{N}{\cos^3 \alpha} = \dfrac{28}{(0.762)^3} = 63.6$, say 64 teeth.

8. $t = \dfrac{N}{\cos^3 \beta} = \dfrac{28}{(0.647)^3} = 103.8$, say 104 teeth.

9. $L = \pi D \cot \alpha = 3.1416 \times 3.672 \times 1.1787 = 13.597$ inches.

10. $l = \pi d \cot \beta = 3.1416 \times 4.328 \times 0.84841 = 11.536$ inches.

7. **Shafts at Right Angles, Ratio Unequal, Center Distance Approximate.** — Sum of spiral angles of gear and pinion must equal 90 degrees.



Given or assumed:

1. Position of gear having right- or left-hand spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. R = ratio of gear to pinion.
5. n = number of teeth in pinion = $\dfrac{1.41\, C_a P_n}{R+1}$ for 45 degrees; and $\dfrac{2\, C_a P_n \cos\alpha \cos\beta}{R\cos\beta + \cos\alpha}$ for any angle.
6. N = number of teeth in gear = nR.
7. α = angle of spiral on gear.
8. β = angle of spiral on pinion.

To find:

(*a*) When spiral angles are 45 degrees.

1. D = pitch diameter of gear = $\dfrac{N}{0.70711\, P_n}$
2. d = pitch diameter of pinion = $\dfrac{n}{0.70711\, P_n}$
3. O = outside diameter of gear = $D + \dfrac{2}{P_n}$
4. o = outside diameter of pinion = $d + \dfrac{2}{P_n}$
5. T = number of cutter (gear) = $\dfrac{N}{0.353}$
6. t = number of cutter (pinion) = $\dfrac{n}{0.353}$.
7. L = lead of spiral on gear = πD.
8. l = lead of spiral on pinion = πd.
9. C = center distance (exact) = $\dfrac{D+d}{2}$

(*b*) When spiral angles are other than 45 degrees.

1. $D = \dfrac{N}{P_n \cos\alpha}$ 2. $d = \dfrac{n}{P_n \cos\beta}$ 3. $T = \dfrac{N}{\cos^3\alpha}$

4. $t = \dfrac{n}{\cos^3 \beta}$ 5. $L = \pi D \cot \alpha$ 6. $l = \pi d \cot \beta$

Example

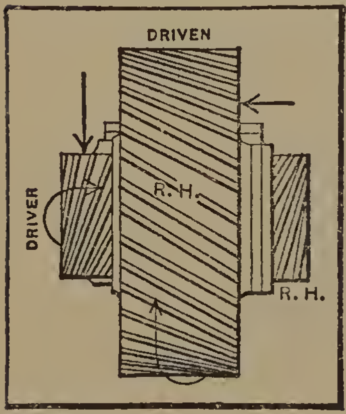
Given or assumed:

1. See illustration. 2. $C_a = 3.2$ inches. 3. $P_n = 10$. 4. $R = 1.5$.
5. $n = \dfrac{1.41\, C_a P_n}{R+1} = \dfrac{1.41 \times 3.2 \times 10}{1.5+1}$ = say 18 teeth.
6. $N = nR = 18 \times 1.5 = 27$ teeth.
7. $\alpha = 45$ degrees. 8. $\beta = 45$ degrees.

To find:

1. $D = \dfrac{N}{0.70711\, P_n} = \dfrac{27}{0.70711 \times 10} = 3.818$ inches.
2. $d = \dfrac{n}{0.70711\, P_n} = \dfrac{18}{0.70711 \times 10} = 2.545$ inches.
3. $O = D + \dfrac{2}{P_n} = 3.818 + \dfrac{2}{10} = 4.018$ inches.
4 $o = d + \dfrac{2}{P_n} = 2.545 + \dfrac{2}{10} = 2.745$ inches.
5. $T = \dfrac{N}{0.353} = \dfrac{27}{0.353} = 76.5$, say 76 teeth.
6. $t = \dfrac{n}{0.353} = \dfrac{18}{0.353} = 51$ teeth.
7. $L = \pi D = 3.1416 \times 3.818 = 12$ inches.
8. $l = \pi d = 3.1416 \times 2.545 = 8$ inches.
9. $C = \dfrac{D+d}{2} = \dfrac{3.818 + 2.545}{2} = 3.182$ inches.

8. Shafts at Right Angles, Ratios Unequal, Center Distance Exact. — Gears have same direction of spiral. The sum of the spiral angles will equal 90 degrees.



Given or assumed:

1. Position of gear having right- or left-hand spiral depending on rotation and direction in which thrust is to be received.
2. P_n = normal pitch (pitch of cutter).
3. R = ratio of number of teeth in large gear to number of teeth in small gear.
4. α_a = approximate spiral angle of large gear.
5. C = exact center distance.

To find:

1. n = number of teeth in small gear nearest

$$\frac{2\,CP_n \sin \alpha_a}{1 + R \tan \alpha_a}$$

2. N = number of teeth in large gear = Rn.
3. α = exact spiral angle of large gear, found by trial from

$$R \sec \alpha + \operatorname{cosec} \alpha = \frac{2\,CP_n}{n}$$

4. β = exact spiral angle of small gear = $90° - \alpha$.

5. D = pitch diameter of large gear = $\dfrac{N}{P_n \cos\alpha}$

6. d = pitch diameter of small gear = $\dfrac{n}{P_n \cos\beta}$

7. O = outside diameter of large gear = $D + \dfrac{2}{P_n}$

8. o = outside diameter of small gear = $d + \dfrac{2}{P_n}$

9. T = number of teeth marked on cutter for large gear = $\dfrac{N}{\cos^3\alpha}$

10. t = number of teeth marked on cutter for small gear = $\dfrac{n}{\cos^3\beta}$

11. L = lead of spiral on large gear = $\pi D \cot\alpha$.

12. l = lead of spiral on small gear = $\pi d \cot\beta$.

Example

Given or assumed:

1. See illustration. 2. $P_n = 8$. 3. $R = 3$. 4. $\alpha_a = 45$ degrees.
5. $C = 10$ inches.

To find:

1. $n = \dfrac{2CP_n \sin\alpha_a}{1 + R\tan\alpha_a} = \dfrac{2 \times 10 \times 8 \times 0.70711}{1+3} = 28.25$, say 28 teeth.

2. $N = Rn = 3 \times 28 = 84$ teeth.

3. $R\sec\alpha + \operatorname{cosec}\alpha = \dfrac{2CP_n}{n} = \dfrac{2 \times 10 \times 8}{28} = 5.714$, or $\alpha = 46° 6'$.

4. $\beta = 90° - \alpha = 90° - 46° 6' = 43° 54'$.

5. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{84}{8 \times 0.6934} = 15.143$ inches.

6. $d = \dfrac{n}{P_n \cos\beta} = \dfrac{28}{8 \times 0.72055} = 4.857$ inches.

7. $O = D + \dfrac{2}{P_n} = 15.143 + 0.25 = 15.393$ inches.

8. $o = d + \dfrac{2}{P_n} = 4.857 + 0.25 = 5.107$ inches.

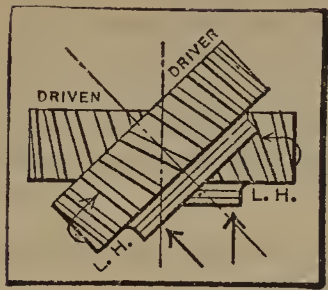
9. $T = \dfrac{N}{\cos^3\alpha} = \dfrac{84}{0.333}$, say 252 teeth.

10. $t = \dfrac{n}{\cos^3\beta} = \dfrac{28}{0.374}$, say 75 teeth.

11. $L = \pi D \cot\alpha = 3.1416 \times 15.143 \times 0.96232 = 45.78$ inches.

12. $l = \pi d \cot\beta = 3.1416 \times 4.857 \times 1.0392 = 15.857$ inches.

9. Shafts at 45-Degree Angle, Ratio Equal, Center Distance Approximate. —



The sum of the spiral angles of the two gears equals 45 degrees, and the gears are of the same hand, if each angle is less than 45 degrees. The difference between the spiral angles equals 45 degrees, and the gears are of opposite hand, if either angle is greater than 45 degrees.

Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. α = angle of spiral of driving gear.
5. β = angle of spiral of driven gear.
6. N = number of teeth nearest $\dfrac{2\,C_a P_n \cos\alpha\cos\beta}{\cos\alpha + \cos\beta}$

To find:

(*a*) When spiral angles are 22½ degrees.

1. D = pitch diameter $= \dfrac{N}{0.9239\,P_n}$ 2. O = outside diameter $= D + \dfrac{2}{P_n}$
3. T = number of teeth marked on cutter $= N \div 0.788$.
4. L = lead of spiral $= 7.584\,D$.
5. C = center distance $= D$.

(*b*) When spiral angles are other than 22½ degrees.

1. D = pitch diameter of driver $= \dfrac{N}{P_n \cos\alpha}$
2. d = pitch diameter of driven gear $= \dfrac{N}{P_n \cos\beta}$
3. O = outside diameter of driver $= D + \dfrac{2}{P_n}$
4. o = outside diameter of driven gear $= d + \dfrac{2}{P_n}$
5. T = number of teeth marked on cutter for driver $= N \div \cos^3\alpha$.
6. t = number of teeth marked on cutter for driven gear $= N \div \cos^3\beta$.
7. L = lead of spiral for driver $= \pi D \cot\alpha$.
8. l = lead of spiral for driven gear $= \pi d \cot\beta$.
9. C = actual center distance = sum of pitch radii.

Example

Given or assumed:

1. See illustration. 2. $C_a = 4$ inches. 3. $P_n = 10$.
4 and 5. $\alpha = \beta = 22½$ deg. 6. $N = 37$.

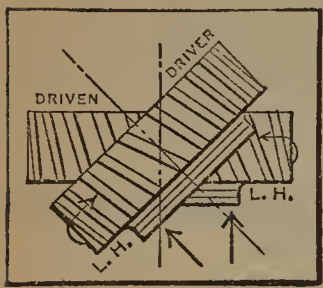
To find:

1. $D = \dfrac{N}{0.9239\,P_n} = \dfrac{37}{0.9239 \times 10} = 4.005$ inches.

2. $O = D + \dfrac{2}{P_n} = 4.005 + \dfrac{2}{10} = 4.205$ inches.

3. $T = N \div 0.788 = 37 \div 0.788 = 47$ teeth.
4. $L = 7.584\,D = 7.584 \times 4.005 = 30.374$ inches.

10. Shafts at 45-Degree Angle, Ratio Equal, Center Distance Exact. — The sum of the spiral angles of the two gears equals 45 degrees, and the gears are of the same hand, if each angle is less than 45 degrees. The difference between the spiral angles equals 45 degrees, and the gears are of opposite hand, if either angle is greater than 45 degrees.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. P_n = normal pitch (pitch of cutter).
3. C = center distance.
4. α_a = approximate spiral angle of one gear.
5. β_a = approximate spiral angle of the other gear.
6. N = number of teeth nearest $\dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{\cos\alpha_a + \cos\beta_a}$

To find:

1. α and β = exact spiral angles found by trial from $\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{N}$
2. D = pitch diameter of one gear $= \dfrac{N}{P_n \cos\alpha}$
3. d = pitch diameter of the other gear $= \dfrac{N}{P_n \cos\beta}$
4. O = outside diameter of one gear $= D + \dfrac{2}{P_n}$
5. o = outside diameter of other gear $= d + \dfrac{2}{P_n}$
6. T = number of teeth marked on cutter for one gear $= N \div \cos^3\alpha$.
7. t = number of teeth marked on cutter for other gear $= N \div \cos^3\beta$.
8. L = lead of spiral for one gear $= \pi D \cot\alpha$.
9. l = lead of spiral for other gear $= \pi d \cot\beta$.

Example

Given or assumed:

1. See illustration. 2. $P_n = 8$. 3. $C = 10$ inches. 4. $\alpha_a = 15°$. 5. $\beta_a = 30°$.

6. $N = \dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{\cos\alpha_a + \cos\beta_a} = \dfrac{2 \times 10 \times 8 \times 0.96593 \times 0.86603}{0.96593 + 0.86603} = 73$ teeth.

To find:

1. α and β from $\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{N} = \dfrac{2 \times 10 \times 8}{73} = 2.1918$; by trial α and β, respectively, $= 14° 44'$ and $30° 16'$.

2. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{73}{8 \times 0.96712} = 9.435$ inches.

3. $d = \dfrac{N}{P_n \cos\beta} = \dfrac{73}{8 \times 0.86369} = 10.565$ inches.

4. $O = D + \dfrac{2}{P_n} = 9.435 + \dfrac{2}{8} = 9.685$ inches.

5. $o = d + \frac{2}{P_n} = 10.565 + \frac{2}{8} = 10.815$ inches.

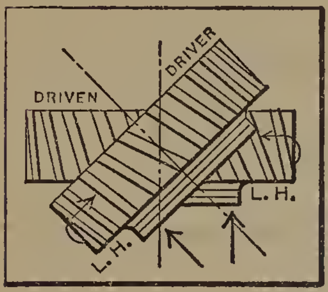
6. $T = N \div \cos^3 \alpha = 73 \div 0.904 = 81$ teeth.

7. $t = N \div \cos^3 \beta = 73 \div 0.645 = 113$ teeth.

8. $L = \pi D \cot \alpha = \pi \times 9.435 \times 3.803 = 112.72$ inches.

9. $l = \pi d \cot \beta = \pi \times 10.565 \times 1.714 = 56.889$ inches.

11. Shafts at 45-Degree Angle, Ratio Unequal, Center Distance Approximate. —



The sum of the spiral angles of the two gears equals 45 degrees, and the gears are of the same hand, if each angle is less than 45 degrees. The difference between the spiral angles equals 45 degrees, and the gears are of opposite hand, if either angle is greater than 45 degrees.

Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = center distance.
3. P_n = normal pitch (pitch of cutter).
4. R = ratio of gear to pinion, $N \div n$.
5. α = angle of spiral on gear.
6. β = angle of spiral on pinion.
7. n = number of teeth in pinion nearest $\frac{2\, C_a P_n \cos \alpha \cos \beta}{R \cos \beta + \cos \alpha}$
8. N = number of teeth in gear = Rn.

To find:

(*a*) When $\alpha = \beta = 22\frac{1}{2}$ degrees.

1. D = pitch diameter of gear = $\frac{N}{0.9239\, P_n}$
2. d = pitch diameter of pinion = $\frac{n}{0.9239\, P_n}$
3. O = outside diameter of gear = $D + \frac{2}{P_n}$
4. o = outside diameter of pinion = $d + \frac{2}{P_n}$
5. T = number of teeth marked on cutter for gear = $N \div 0.788$.
6. t = number of teeth marked on cutter for pinion = $n \div 0.788$.
7. L = lead of spiral on gear = $7.584\, D$.
8. l = lead of spiral on pinion = $7.584\, d$.
9. C = actual center distance = $\frac{D + d}{2}$.

(*b*) When α and β are any angles.

1. D = pitch diameter of gear = $\frac{N}{P_n \cos \alpha}$
2. d = pitch diameter of pinion = $\frac{n}{P_n \cos \beta}$
3. O = outside diameter of gear = $D + \frac{2}{P_n}$

4. o = outside diameter of pinion $= d + \frac{2}{P_n}$
5. T = number of teeth marked on cutter for gear $= N \div \cos^3 \alpha$
6. t = number of teeth marked on cutter for pinion $= n \div \cos^3 \beta$
7. L = lead of spiral on gear $= \pi D \cot \alpha$.
8. l = lead of spiral on pinion $= \pi d \cot \beta$.
9. C = actual center distance $= \frac{D+d}{2}$

Example

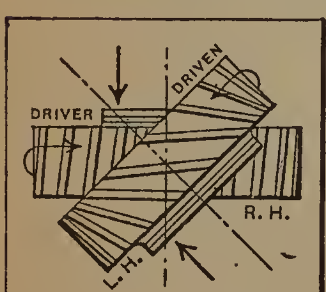
Given or assumed:

1. See illustration. 2. $C = 12$ inches. 3. $P_n = 6$. 4. $R = 3$.
5. $\alpha = 20$ deg. 6. $\beta = 25$ deg.
7. $n = \frac{2\,CP_n \cos\alpha \cos\beta}{R\cos\beta + \cos\alpha} = \frac{2 \times 12 \times 6 \times 0.93969 \times 0.90631}{(3 \times 0.90631) + 0.93969} = 34$ teeth, approx.
8. $N = Rn = 3 \times 34 = 102$ teeth.

To find:

1. $D = \frac{N}{P_n \cos\alpha} = \frac{102}{6 \times 0.93969} = 18.091$ inches.
2. $d = \frac{n}{P_n \cos\beta} = \frac{34}{6 \times 0.90631} = 6.252$ inches.
3. $O = D + \frac{2}{P_n} = 18.091 + \frac{2}{6} = 18.424$ inches.
4. $o = d + \frac{2}{P_n} = 6.252 + \frac{2}{6} = 6.585$ inches.
5. $T = N \div \cos^3\alpha = 102 \div 0.83 = 123$ teeth.
6. $t = n \div \cos^3\beta = 34 \div 0.744 = 46$ teeth.
7. $L = \pi D \cot\alpha = \pi \times 18.091 \times 2.747 = 156.12$ inches.
8. $l = \pi d \cot\beta = \pi \times 6.252 \times 2.145 = 42.13$ inches.
9. $C = \frac{D+d}{2} = \frac{18.091 + 6.252}{2} = 12.1715$ inches.

12. Shafts at 45-Degree Angle, Ratio Unequal, Center Distance Exact. — The sum of the spiral angles of the two gears equals 45 degrees, and the gears are of the same hand, if each angle is less than 45 degrees. The difference between the spiral angles equals 45 degrees, and the gears are of opposite hand, if either angle is greater than 45 degrees.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. P_n = normal pitch (pitch of cutter).
3. R = ratio of large to small gear $= N \div n$.
4. α_a = approximate spiral angle of large gear.
5. β_a = approximate spiral angle of small gear.

6. C = center distance.

7. n = number of teeth in small gear nearest $\dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{R\cos\beta_a + \cos\alpha_a}$

8. N = number of teeth in large gear = Rn.

To find:

1. α and β, exact spiral angles, by trial from $R\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{n}$

2. D = pitch diameter of large gear = $\dfrac{N}{P_n \cos\alpha}$

3. d = pitch diameter of small gear = $\dfrac{n}{P_n \cos\beta}$

4. O = outside diameter of large gear = $D + \dfrac{2}{P_n}$

5. o = outside diameter of small gear = $d + \dfrac{2}{P_n}$

6. T = number of teeth marked on cutter for large gear = $N \div \cos^3\alpha$.

7. t = number of teeth marked on cutter for small gear = $n \div \cos^3\beta$.

8. L = lead of spiral for large gear = $\pi D \cot\alpha$.

9. l = lead of spiral for small gear = $\pi d \cot\beta$.

Example

Given or assumed:

1. See illustration.
2. $P_n = 4$.
3. $R = 4$.
4. $\alpha_a = 50$ degrees.
5. $\beta_a = 5$ degrees.
6. $C = 30$ inches.
7. $n = \dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{R\cos\beta_a + \cos\alpha_a} = \dfrac{2 \times 30 \times 4 \times 0.643 \times 0.996}{(4 \times 0.996) + 0.643} = 33$ teeth.
8. $N = Rn = 4 \times 33 = 132$ teeth.

To find:

1. α and β from $R\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{n} = \dfrac{2 \times 30 \times 4}{33} = 7.273$. By trial $\alpha = 50^\circ\,21'$, and $\beta = 5^\circ\,21'$.

2. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{132}{4 \times 0.63810} = 51.716$ inches.

3. $d = \dfrac{n}{P_n \cos\beta} = \dfrac{33}{4 \times 0.99564} = 8.286$ inches.

4. $O = D + \dfrac{2}{P_n} = 51.716 + \dfrac{2}{4} = 52.216$ inches.

5. $o = d + \dfrac{2}{P_n} = 8.286 + \dfrac{2}{4} = 8.786$ inches.

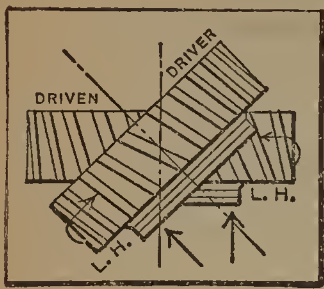
6. $T = N \div \cos^3\alpha = 132 \div 0.26 = 508$ teeth.

7. $t = n \div \cos^3\beta = 33 \div 0.987 = 33$ teeth.

8. $L = \pi D \cot\alpha = \pi \times 51.716 \times 0.82874 = 134.6$ inches.

9. $l = \pi d \cot\beta = \pi \times 8.286 \times 10.678 = 278$ inches.

13. Shafts at any Angle, Ratio Equal, Center Distance Approximate. — The sum of the spiral angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the spiral angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = approximate center distance.
3. P_n = normal pitch (pitch of cutter).
4. α = angle of spiral of one gear.
5. β = angle of spiral of other gear.
6. N = number of teeth nearest $\dfrac{2\,C_a P_n \cos\alpha\cos\beta}{\cos\alpha+\cos\beta}$

To find:

1. D = pitch diameter of one gear $= \dfrac{N}{P_n \cos\alpha}$
2. d = pitch diameter of other gear $= \dfrac{N}{P_n \cos\beta}$
3. O = outside diameter of one gear $= D + \dfrac{2}{P_n}$
4. o = outside diameter of other gear $= d + \dfrac{2}{P_n}$
5. T = number of teeth marked on cutter for one gear $= N \div \cos^3\alpha$.
6. t = number of teeth marked on cutter for other gear $= N \div \cos^3\beta$.
7. L = lead of spiral for one gear $= \pi D \cot\alpha$.
8. l = lead of spiral for other gear $= \pi d \cot\beta$.
9. C = actual center distance $= \dfrac{D+d}{2}$

Example

Given or assumed (angle of shafts, 30 degrees):

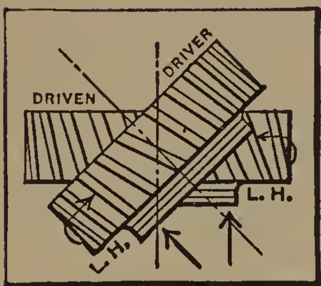
1. See illustration. 2. C_a = 5 inches. 3. P_n = 10.
4. α = 20 degrees. 5. β = 10 degrees. 6. N = 48.

To find:

1. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{48}{10 \times 0.9397} = 5.108$ inches.
2. $d = \dfrac{N}{P_n \cos\beta} = \dfrac{48}{10 \times 0.9848} = 4.874$ inches.
3. $O = D + \dfrac{2}{P_n} = 5.108 + \dfrac{2}{10} = 5.308$ inches.
4. $o = d + \dfrac{2}{P_n} = 4.874 + \dfrac{2}{10} = 5.074$ inches.
5. $T = N \div \cos^3\alpha = 48 \div 0.83 = 58$ teeth.
6. $t = N \div \cos^3\beta = 48 \div 0.96 = 50$ teeth.

7. $L = \pi D \cot \alpha = \pi \times 5.108 \times 2.747 = 44.08$ inches.
8. $l = \pi d \cot \beta = \pi \times 4.874 \times 5.671 = 86.84$ inches.
9. $C = \dfrac{D+d}{2} = \dfrac{5.108 + 4.874}{2} = 4.991$ inches.

14. Shafts at Any Angle, Ratio Equal, Center Distance Exact. — The sum of the spiral angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the spiral angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C = center distance.
3. P_n = normal pitch (pitch of cutter).
4. α_a = approximate spiral angle of one gear.
5. β_a = approximate spiral angle of other gear.
6. N = number of teeth nearest $\dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{\cos\alpha_a + \cos\beta_a}$

To find:

1. α and β = exact spiral angles, found by trial from $\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{N}$
2. D = pitch diameter of one gear $= \dfrac{N}{P_n \cos\alpha}$
3. d = pitch diameter of other gear $= \dfrac{N}{P_n \cos\beta}$
4. O = outside diameter of one gear $= D + \dfrac{2}{P_n}$
5. o = outside diameter of other gear $= d + \dfrac{2}{P_n}$
6. T = number of teeth marked on cutter for one gear $= N \div \cos^3\alpha$.
7. t = number of teeth marked on cutter for other gear $= N \div \cos^3\beta$.
8. L = lead of spiral for one gear $= \pi D \cot\alpha$.
9. l = lead of spiral for other gear $= \pi d \cot\beta$.

Example

Given or assumed (angle of shafts, 50 degrees):

1. See illustration. 2. $C = 10$ inches. 3. $P_n = 10$. 4. $\alpha_a = 20$ deg.
5. $\beta_a = 30$ deg.
6. $N = \dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{\cos\alpha_a + \cos\beta_a} = \dfrac{2 \times 10 \times 10 \times 0.93969 \times 0.86603}{0.93969 + 0.86603} = 90$ teeth.

To find:

1. α and β from $\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{N} = \dfrac{2 \times 10 \times 10}{90} = 2.222$. By trial α and β, respectively, $= 19°\,20'$ and $30°\,40'$.

2. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{90}{10 \times 0.94361} = 9.537$ inches.

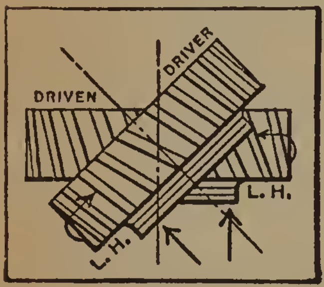
3. $d = \dfrac{N}{P_n \cos\beta} = \dfrac{90}{10 \times 0.86015} = 10.463$ inches.

4. $O = D + \dfrac{2}{P_n} = 9.537 + \dfrac{2}{10} = 9.737$ inches.

5. $o = d + \dfrac{2}{P_n} = 10.463 + \dfrac{2}{10} = 10.663$ inches.

6. $T = N \div \cos^3\alpha = 90 \div 0.84 = 107$ teeth.
7. $t = N \div \cos^3\beta = 90 \div 0.64 = 141$ teeth.
8. $L = \pi D \cot\alpha = \pi \times 9.537 \times 2.85 = 85.39$ inches.
9. $l = \pi d \cot\beta = \pi \times 10.463 \times 1.686 = 55.42$ inches.

15. Shafts at Any Angle, Ratio Unequal, Center Distance Approximate. — The sum of the spiral angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the spiral angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C_a = center distance.
3. P_n = normal pitch (pitch of cutter).
4. R = ratio of gear to pinion = $N \div n$.
5. α = angle of spiral on gear.
6. β = angle of spiral on pinion.
7. n = number of teeth in pinion nearest $\dfrac{2\,C_a P_n \cos\alpha \cos\beta}{R\cos\beta + \cos\alpha}$ for any angle, and $\dfrac{2\,C_a P_n \cos\alpha}{R + 1}$ when both angles are equal.
8. N = number of teeth in gear = Rn.

To find:

1. D = pitch diameter of gear = $\dfrac{N}{P_n \cos\alpha}$
2. d = pitch diameter of pinion = $\dfrac{n}{P_n \cos\beta}$
3. O = outside diameter of gear = $D + \dfrac{2}{P_n}$
4. o = outside diameter of pinion = $d + \dfrac{2}{P_n}$
5. T = number of teeth marked on cutter for gear = $N \div \cos^3\alpha$.
6. t = number of teeth marked on cutter for pinion = $n \div \cos^3\beta$.
7. L = lead of spiral on gear = $\pi D \cot\alpha$.
8. l = lead of spiral on pinion = $\pi d \cot\beta$.
9. C = actual center distance = $\dfrac{D + d}{2}$

Example

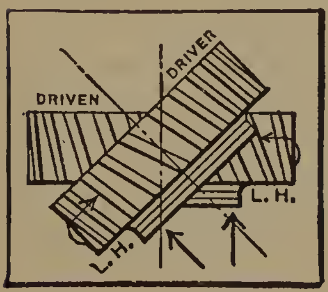
Given or assumed (angle of shafts, 60 degrees):

1. See illustration. 2. $C_a = 12$ inches. 3. $P_n = 8$.
4. $R = 4$. 5. $\alpha = 30$ degrees. 6. $\beta = 30$ degrees.
7. $n = \dfrac{2\,C_a P_n \cos\alpha}{R+1} = \dfrac{2 \times 12 \times 8 \times 0.86603}{4+1} = 33$ teeth.
8. $N = 4 \times 33 = 132$ teeth.

To find:

1. $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{132}{8 \times 0.86603} = 19.052$ inches.
2. $d = \dfrac{n}{P_n \cos\beta} = \dfrac{33}{8 \times 0.86603} = 4.763$ inches.
3. $O = D + \dfrac{2}{P_n} = 19.052 + \dfrac{2}{8} = 19.302$ inches.
4. $o = d + \dfrac{2}{P_n} = 4.763 + \dfrac{2}{8} = 5.013$ inches.
5. $T = N \div \cos^3\alpha = 132 \div 0.65 = 203$ teeth.
6. $t = n \div \cos^3\beta = 33 \div 0.65 = 51$ teeth.
7. $L = \pi D \cot\alpha = \pi \times 19.052 \times 1.732 = 103.66$ inches.
8. $l = \pi d \cot\beta = \pi \times 4.763 \times 1.732 = 25.92$ inches.
9. $C = \dfrac{D+d}{2} = \dfrac{19.052 + 4.763}{2} = 11.9075$ inches.

16. Shafts at Any Angle, Ratio Unequal, Center Distance Exact. — The sum of the spiral angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the spiral angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1. Hand of spiral, depending on rotation and direction in which thrust is to be received.
2. C = center distance.
3. P_n = normal pitch (pitch of cutter).
4. α_a = approximate spiral angle of gear.
5. β_a = approximate spiral angle of pinion.
6. R = ratio of gear to pinion = $N \div n$.
7. n = number of teeth in pinion nearest $\dfrac{2\,CP_n \cos\alpha_a \cos\beta_a}{R\cos\beta_a + \cos\alpha_a}$
8. N = number of teeth in gear = Rn.

To find:

1. α and β, exact spiral angles, found by trial from $R\sec\alpha + \sec\beta = \dfrac{2\,CP_n}{n}$
2. D = pitch diameter of gear = $\dfrac{N}{P_n \cos\alpha}$
3. d = pitch diameter of pinion = $\dfrac{n}{P_n \cos\beta}$

4. O = outside diameter of gear $= D + \dfrac{2}{P_n}$

5. o = outside diameter of pinion $= d + \dfrac{2}{P_n}$

6. T = number of teeth marked on cutter for gear $= N \div \cos^3 \alpha$.
7. t = number of teeth marked on cutter for pinion $= n \div \cos^3 \beta$.
8. L = lead of spiral on gear $= \pi D \cot \alpha$.
9. l = lead of spiral on pinion $= \pi d \cot \beta$.

Example

Given or assumed (angle of shafts, 60 degrees):

1. See illustration. 2. $C = 40$ inches. 3. $P_n = 4$.
4. $\alpha_a = 20$ degrees. 5. $\beta_a = 40$ degrees. 6. $R = 3$.

7. $$n = \frac{2\,CP_n \cos \alpha_a \cos \beta_a}{R \cos \beta_a + \cos \alpha_a} = \frac{2 \times 40 \times 4 \times 0.9397 \times 0.766}{(3 \times 0.766) + 0.9397} = 71 \text{ teeth.}$$

8. $N = Rn = 3 \times 71 = 213$ teeth.

To find:

1. α and β from $R \sec \alpha + \sec \beta = \dfrac{2\,CP_n}{n} = \dfrac{2 \times 40 \times 4}{71} = 4.507$.

   By trial $\alpha = 22° \, 24' \, 30''$ and $\beta = 37° \, 35' \, 30''$.

2. $$D = \frac{N}{P_n \cos \alpha} = \frac{213}{4 \times 0.92449} = 57.599 \text{ inches.}$$

3. $$d = \frac{n}{P_n \cos \beta} = \frac{71}{4 \times 0.79238} = 22.401 \text{ inches.}$$

4. $$O = D + \frac{2}{P_n} = 57.599 + \frac{2}{4} = 58.099 \text{ inches.}$$

5. $$o = d + \frac{2}{P_n} = 22.401 + \frac{2}{4} = 22.901 \text{ inches.}$$

6. $T = N \div \cos^3 \alpha = 213 \div 0.79 = 270$ teeth.
7. $t = n \div \cos^3 \beta = 71 \div 0.497 = 143$ teeth.
8. $L = \pi D \cot \alpha = \pi \times 57.599 \times 2.4252 = 438.8$ inches.
9. $l = \pi d \cot \beta = \pi \times 22.401 \times 1.2989 = 91.41$ inches.

Special Case of Spiral Gear Design. — The following method is used when the distance between the centers of the shafts, the speed ratio and an approximate ratio of the pitch diameters of the gears are given. (Shafts at 90 degrees angle.) In the formulas, let:

D = diameter of driver;
d = diameter of driven gear;
S = speed of driver;
s = speed of driven gear;
P_n = normal diametral pitch;
α = angle of teeth in driver with its axis;
N = number of teeth in driver;
n = number of teeth in driven gear;
C = center distance.

Assume trial values for D and d; then an approximate angle α is derived from the formula:

$$\frac{ds}{DS} = \cot \alpha \qquad (1)$$

Then find by trial the number of teeth for each of the gears which, with the given speed ratio, will most nearly satisfy the equation:

$$2\,C = \frac{N}{P_n \cos \alpha} + \frac{n}{P_n \sin \alpha} \qquad (2)$$

Then make corrections of the angle α until a value is found which exactly satisfies the last equation. This being done, the pitch diameters are:

$$D = \frac{N}{P_n \cos \alpha} \qquad d = \frac{n}{P_n \sin \alpha}$$

Example: — Find the diameters and angles of teeth of two spiral gears with shafts at right angles; the distance between the centers is 4⅛ inches, the speed ratio of the driver to the follower is 2 to 1, and the ratio of D to d is about 9 to 8.

Following the method outlined:

$$\frac{ds}{DS} = \frac{8 \times 1}{9 \times 2} = 0.444 = \cot 66°, \text{ approx.}$$

By trial, it will be found that 14 and 28 teeth will nearly satisfy Equation (2), making $2\,C = 8.134$.

Subtracting, $8.250 - 8.134 = 0.116$; thus an angle of 66 degrees introduces an error of 0.116 inch for twice the distance between the centers of the shafts. By repeated trials it is found that 66 degrees 48 minutes gives:

$$\frac{14}{8 \times 0.3939} + \frac{28}{8 \times 9191} = 8.25 \text{ inches.}$$

Hence this angle is the tooth angle of the driver, and:

$$D = \frac{14}{8 \times 0.3939} = 4.442 \text{ inches.} \qquad d = \frac{28}{8 \times 0.9191} = 3.808 \text{ inches.}$$

Constants for Calculating Spiral Gears. — The tables entitled "Constants for Calculating Spiral Gears" give factors which can frequently be employed in spiral gear calculations to reduce the time necessary for the computation of the angles and dimensions. The body of the table gives constants C_t (= center distance of shafts per tooth of pinion) for each speed ratio given, for *shafts at right angles only;* factors U, F and L are applicable to gears with shafts at any angle. The constants for unit diameter of gear per tooth, U, and for unit center distance per tooth of fastest running gear, C_t, are calculated for gears cut with spur gear cutters of one diametral pitch. For any other pitch, divide the constant by the diametral pitch of the cutter used. The tables are calculated for each degree from 12 to 78 degrees helix angle. For fractional parts of degrees sufficient accuracy for practical purposes is obtained by simply proportioning the value between those given. The notation used in the tables and in the following examples is as follows:

U = unit diameter per tooth, for 1 diametral pitch;
F = cutter factor;
L = lead of spiral per inch pitch diameter;
P_n = normal diametral pitch;
α = spiral angle;
D = pitch diameter;
N = number of teeth (N_a = number of teeth in pinion);
C = center distance;
C_t = center distance per tooth of pinion for 1 diametral pitch.

For finding the pitch diameter, the center distance, the lead of the helix and the number of teeth for which to select the cutter, the following formulas are used:

$$D = \frac{U \times N}{P_n} \qquad C = \frac{C_t}{P_n} \times N_a$$

Lead of helix $= L \times D$.

Number of teeth for which to select cutter $= F \times N$.

Example: — Find the number of teeth, diameters and center distances for a pair of gears where the helix angle of the pinion is 60 degrees; of the gear, 30 degrees; speed ratio, 2 to 5; diametral pitch, 6.

From the table, $C_t = 2.4435$; then:

$$C = \frac{2.4435}{6} \times N_a = 0.40725\ N_a$$

Assume a required center distance of approximately 5 inches, and make $N_a = 12$, then:

$$C = 0.40725 \times 12 = 4.887 \text{ inches.}$$

Number of teeth in gear: $12 \times 5/2 = 30$.

Pitch diameter of pinion: $D = \dfrac{U \times N}{P_n} = \dfrac{2 \times 12}{6} = 4$ inches.

Lead of spiral of pinion: $L \times D = 1.814 \times 4 = 7.256$ inches.

Number of teeth for which to select cutter $= F \times N = 8 \times 12 = 96$.

The formulas are used in the same manner for finding the pitch diameter, lead and cutter for the gear.

Example: — Shaft angle, 65 degrees; speed ratio, 1 to 4; 8 diametral pitch.

In this case, factors C_t do not apply, because the shafts are not at right angles.

Assume pinion to have 8 teeth, 30-degree helix angle; the gear, 32 teeth, 35-degree helix angle, then:

Diameter of pinion: $D = \dfrac{1.1547 \times 8}{8} = 1.155$ inch.

Diameter of gear: $D = \dfrac{1.2208 \times 32}{8} = 4.883$ inches.

Center distance: $C = \dfrac{1.155 + 4.883}{2} = 3.019$ inches.

In this case the constants for both gear and pinion are found at the top of the tables, in the section designated "Gear."

Number of Cutter for Spiral Gears. — The proper cutter to use for *spur* gears depends upon the pitch of the teeth and also upon the number of teeth, but a cutter for *spiral* gears is not selected with reference to the actual number of teeth in the gear, as in spur gearing. If the actual number of teeth in a spiral gear is divided by the cube of the cosine of the tooth angle, the quotient will represent the number of teeth for which the cutter should be selected, according to the system for spur gear cutters. Suppose a spiral gear is to have 38 teeth cut at an angle of 45 degrees; then the cutter to use would be determined as follows: The cosine of 45 degrees is 0.7071, and $38 \div 0.7071^3 = \dfrac{38}{0.3535} = 107$. The table in the section on Spur Gearing, "Cutters for Cutting Gear Teeth," calls for a No. 2 cutter for spur gears having any number of teeth between 55 and 134; hence, that is the cutter to use for a spiral gear having 38 teeth and a tooth angle of 45 degrees. It will be understood that this "No. 2" has nothing to do with the pitch of the cutter, which is determined as explained on page 712.

Constants for Calculating Spiral Gears

Gear	L	14.780	13.6077	12.6002	11.7246	10.9560	10.2757	9.6688	9.1238	8.6315
	F	1.07	1.08	1.09	1.11	1.12	1.14	1.16	1.18	1.20
	U	1.0223	1.0263	1.0306	1.0353	1.0403	1.0457	1.0515	1.0576	1.0642
	α	12	13	14	15	16	17	18	19	20
Speed Ratio		C_t = Center Distance per Tooth of Pinion. Shaft Angle, 90°								
1 to 10		7.5166	7.3543	7.2199	7.1083	7.0156	6.9385	6.8754	6.8240	6.7830
1 to 9		7.0054	6.8412	6.7047	6.5907	6.4954	6.4157	6.3497	6.2951	6.2509
1 to 8		6.4942	6.3280	6.1892	6.0730	5.9753	5.8928	5.8239	5.7663	5.7188
1 to 7		5.9831	5.8148	5.6739	5.5554	5.4551	5.3700	5.2982	5.2375	5.1867
1 to 6		5.4719	5.3017	5.1586	5.0377	4.9350	4.8472	4.7725	4.7087	4.6545
1 to 5		4.9607	4.7885	4.6433	4.5201	4.4148	4.3243	4.2467	4.1799	4.1224
2 to 9		4.7051	4.5319	4.3857	4.2613	4.1547	4.0629	3.9839	3.9155	3.8564
1 to 4		4.4495	4.2753	4.1281	4.0024	3.8946	3.8015	3.7210	3.6510	3.5903
2 to 7		4.1939	4.0187	3.8706	3.7436	3.6346	3.5401	3.4581	3.3866	3.3243
3 to 10		4.1087	3.9332	3.7845	3.6573	3.5479	3.4529	3.3705	3.2985	3.2356
1 to 3		3.9388	3.7622	3.6127	3.4848	3.3745	3.2787	3.1953	3.1222	3.0582
3 to 8		3.7680	3.5911	3.4409	3.3123	3.2011	3.1044	3.0200	2.9460	2.8809
2 to 5		3.6828	3.5056	3.3551	3.2260	3.1144	3.0173	2.9324	2.8579	2.7922
3 to 7		3.5976	3.4201	3.2692	3.1397	3.0277	2.9301	2.8448	2.7697	2.7035
4 to 9		3.5550	3.3773	3.2262	3.0966	2.9844	2.8866	2.8010	2.7256	2.6592
1 to 2		3.4272	3.2490	3.0974	2.9672	2.8543	2.7558	2.6695	2.5934	2.5261
5 to 9		3.3249	3.1465	2.9943	2.8636	2.7503	2.6513	2.5644	2.4877	2.4197
4 to 7		3.2994	3.1207	2.9686	2.8372	2.7243	2.6251	2.5381	2.4612	2.3931
3 to 5		3.2568	3.0779	2.9256	2.7946	2.6810	2.5816	2.4943	2.4171	2.3488
5 to 8		3.2237	3.0437	2.8912	2.7601	2.6463	2.5467	2.4593	2.3819	2.3133
2 to 3		3.1713	2.9924	2.8397	2.7083	2.5943	2.4944	2.4067	2.3290	2.2601
7 to 10		3.1422	2.9558	2.8028	2.6713	2.5571	2.4571	2.3691	2.2912	2.2221
5 to 7		3.1205	2.9411	2.7882	2.6566	2.5422	2.4421	2.3541	2.2761	2.2069
3 to 4		3.0864	2.9069	2.7539	2.6221	2.5076	2.4073	2.3190	2.2409	2.1714
7 to 9		3.0621	2.8825	2.7293	2.5974	2.4828	2.3824	2.2940	2.2157	2.1461
4 to 5		3.0438	2.8641	2.7109	2.5789	2.4642	2.3637	2.2752	2.1968	2.1271
5 to 6		3.0183	2.8385	2.6836	2.5532	2.4382	2.3376	2.2490	2.1704	2.1005
6 to 7		3.0013	2.8214	2.6680	2.5358	2.4209	2.3201	2.2314	2.1527	2.0827
7 to 8		2.9891	2.8091	2.6557	2.5235	2.4085	2.3077	2.2189	2.1401	2.0701
8 to 9		2.9799	2.8000	2.6465	2.5142	2.3992	2.2984	2.2095	2.1307	2.0605
9 to 10		2.9728	2.7929	2.6393	2.5070	2.3920	2.2911	2.2022	2.1234	2.0532
1 to 1		2.9160	2.7358	2.5821	2.4495	2.3342	2.2330	2.1438	2.0646	1.9940
Pinion	α	78	77	76	75	74	73	72	71	70
	U	4.8097	4.4454	4.1336	3.8637	3.6280	3.4203	3.2361	3.0715	2.9238
	F	111.0	87.9	70.6	57.8	47.8	40.0	33.9	28.9	25.0
	L	0.6678	0.7253	0.7833	0.8418	0.9008	0.9605	1.0208	1.0817	1.1434

Constants for Calculating Spiral Gears

Gear	L	8.1841	7.7757	7.4011	7.0561	6.7372	6.4412	6.1657	5.9085	5.6676
	F	1.23	1.25	1.28	1.31	1.34	1.37	1.41	1.45	1.49
	U	1.0711	1.0785	1.0864	1.0946	1.1034	1.1126	1.1223	1.1326	1.1433
	α	21	22	23	24	25	26	27	28	29
Speed Ratio		C_t = Center Distance per Tooth of Pinion. Shaft Angle, 90°								
1 to 10		6.7511	6.7275	6.7115	6.7025	6.6999	6.7036	6.7130	6.7278	6.7481
1 to 9		6.2155	6.1882	6.1683	6.1552	6.1482	6.1473	6.1518	6.1616	6.1764
1 to 8		5.6799	5.6490	5.6251	5.6078	5.5965	5.5910	5.5907	5.5953	5.6048
1 to 7		5.1443	5.1097	5.0819	5.0605	5.0449	5.0347	5.0295	5.0290	5.0331
1 to 6		4.6087	4.5704	4.5388	4.5132	4.4932	4.4784	4.4683	4.4627	4.4614
1 to 5		4.0731	4.0311	3.9956	3.9659	3.9415	3.9221	3.9072	3.8964	3.8897
2 to 9		3.8054	3.7615	3.7240	3.6922	3.6657	3.6440	3.6266	3.6133	3.6039
1 to 4		3.5376	3.4919	3.4524	3.4186	3.3898	3.3658	3.3460	3.3302	3.3181
2 to 7		3.2698	3.2222	3.1808	3.1449	3.1140	3.0877	3.0654	3.0470	3.0322
3 to 10		3.1805	3.1324	3.0903	3.0537	3.0220	2.9949	2.9719	2.9526	2.9370
1 to 3		3.0020	2.9526	2.9092	2.8713	2.8382	2.8095	2.7849	2.7639	2.7464
3 to 8		2.8234	2.7728	2.7282	2.6888	2.6543	2.6241	2.5978	2.5751	2.5558
2 to 5		2.7342	2.6830	2.6376	2.5976	2.5623	2.5314	2.5043	2.4807	2.4606
3 to 7		2.6449	2.5931	2.5471	2.5064	2.4707	2.4387	2.4108	2.3864	2.3653
4 to 9		2.6003	2.5481	2.5018	2.4608	2.4244	2.3923	2.3640	2.3392	2.3176
1 to 2		2.4664	2.4133	2.3660	2.3239	2.2865	2.2532	2.2237	2.1976	2.1747
5 to 9		2.3593	2.3055	2.2574	2.2145	2.1762	2.1420	2.1115	2.0843	2.0604
4 to 7		2.3325	2.2785	2.2302	2.1871	2.1486	2.1142	2.0834	2.0560	2.0318
3 to 5		2.2879	2.2336	2.1850	2.1415	2.1026	2.0678	2.0367	2.0088	1.9842
5 to 8		2.2522	2.1976	2.1488	2.1050	2.0658	2.0307	1.9992	1.9711	1.9461
2 to 3		2.1986	2.1437	2.0945	2.0502	2.0107	1.9751	1.9431	1.9145	1.8889
7 to 10		2.1603	2.1052	2.0556	2.0112	1.9712	1.9353	1.9030	1.8740	1.8481
5 to 7		2.1450	2.0898	2.0401	1.9956	1.9555	1.9195	1.8870	1.8578	1.8317
3 to 4		2.1093	2.0538	2.0039	1.9591	1.9187	1.8824	1.8496	1.8201	1.7936
7 to 9		2.0838	2.0281	1.9781	1.9330	1.8924	1.8559	1.8229	1.7931	1.7664
4 to 5		2.0647	2.0089	1.9587	1.9135	1.8727	1.8360	1.8028	1.7729	1.7460
5 to 6		2.0379	1.9819	1.9315	1.8861	1.8451	1.8082	1.7748	1.7446	1.7174
6 to 7		2.0201	1.9639	1.9134	1.8679	1.8268	1.7897	1.7561	1.7257	1.6983
7 to 8		2.0073	1.9511	1.9004	1.8548	1.8136	1.7764	1.7427	1.7122	1.6847
8 to 9		1.9978	1.9415	1.8908	1.8451	1.8038	1.7665	1.7327	1.7021	1.6745
9 to 10		1.9903	1.9340	1.8832	1.8375	1.7961	1.7587	1.7249	1.6942	1.6666
1 to 1		1.9308	1.8740	1.8229	1.7766	1.7348	1.6969	1.6625	1.6313	1.6031
Pinion	α	69	68	67	66	65	64	63	62	61
	U	2.7904	2.6695	2.5593	2.4586	2.3662	2.2812	2.2027	2.1300	2.0627
	F	21.7	19.1	16.8	14.9	13.3	11.9	10.7	9.71	8.79
	L	1.2059	1.2693	1.3335	1.3987	1.4649	1.5322	1.6007	1.6704	1.7414

Constants for Calculating Spiral Gears

Gear	L	5.4414	5.2282	5.0276	4.8376	4.6576	4.4867	4.3240	4.1690
	F	1.54	1.59	1.64	1.69	1.75	1.81	1.88	1.96
	U	1.1547	1.1666	1.1792	1.1924	1.2062	1.2208	1.2361	1.2521
	α	30	31	32	33	34	35	36	37
Speed Ratio		C_t = Center Distance per Tooth of Pinion. Shaft Angle, 90°							
1 to 10		6.7738	6.8040	6.8395	6.8799	6.9252	6.9755	7.0311	7.0916
1 to 9		6.1964	6.2207	6.2499	6.2837	6.3221	6.3651	6.4131	6.4655
1 to 8		5.6190	5.6373	5.6603	5.6875	5.7190	5.7547	5.7950	5.8394
1 to 7		5.0416	5.0540	5.0707	5.0914	5.1159	5.1444	5.1770	5.2133
1 to 6		4.4643	4.4707	4.4811	4.4952	4.5128	4.5340	4.5589	4.5873
1 to 5		3.8869	3.8874	3.8915	3.8990	3.9097	3.9236	3.9409	3.9612
2 to 9		3.5982	3.5958	3.5967	3.6009	3.6081	3.6184	3.6319	3.6482
1 to 4		3.3095	3.3041	3.3019	3.3028	3.3066	3.3132	3.3228	3.3351
2 to 7		3.0208	3.0124	3.0071	3.0047	3.0050	3.0081	3.0138	3.0221
3 to 10		2.9246	2.9152	2.9089	2.9054	2.9045	2.9063	2.9108	2.9177
1 to 3		2.7321	2.7208	2.7123	2.7066	2.7035	2.7029	2.7048	2.7090
3 to 8		2.5397	2.5263	2.5158	2.5079	2.5024	2.4994	2.4988	2.5004
2 to 5		2.4435	2.4291	2.4176	2.4085	2.4019	2.3977	2.3958	2.3960
3 to 7		2.3472	2.3319	2.3193	2.3092	2.3014	2.2959	2.2928	2.2917
4 to 9		2.2991	2.2833	2.2702	2.2595	2.2512	2.2451	2.2413	2.2395
1 to 2		2.1548	2.1374	2.1227	2.1104	2.1004	2.0925	2.0868	2.0830
5 to 9		2.0393	2.0208	2.0048	1.9912	1.9798	1.9704	1.9632	1.9578
4 to 7		2.0104	1.9916	1.9754	1.9614	1.9496	1.9399	1.9322	1.9265
3 to 5		1.9623	1.9430	1.9262	1.9117	1.8993	1.8890	1.8807	1.8743
5 to 8		1.9238	1.9041	1.8869	1.8720	1.8591	1.8483	1.8396	1.8326
2 to 3		1.8661	1.8458	1.8280	1.8124	1.7988	1.7873	1.7778	1.7699
7 to 10		1.8249	1.8041	1.7859	1.7698	1.7557	1.7437	1.7336	1.7252
5 to 7		1.8084	1.7875	1.7690	1.7527	1.7385	1.7263	1.7159	1.7073
3 to 4		1.7699	1.7486	1.7297	1.7130	1.6983	1.6856	1.6747	1.6656
7 to 9		1.7424	1.7208	1.7016	1.6846	1.6696	1.6565	1.6453	1.6358
4 to 5		1.7217	1.6999	1.6806	1.6633	1.6481	1.6347	1.6232	1.6134
5 to 6		1.6929	1.6708	1.6511	1.6335	1.6179	1.6042	1.5923	1.5821
6 to 7		1.6736	1.6514	1.6314	1.6136	1.5978	1.5838	1.5717	1.5612
7 to 8		1.6599	1.6375	1.6174	1.5994	1.5834	1.5693	1.5570	1.5463
8 to 9		1.6496	1.6271	1.6069	1.5888	1.5727	1.5584	1.5460	1.5352
9 to 10		1.6416	1.6190	1.5987	1.5805	1.5643	1.5499	1.5374	1.5265
1 to 1		1.5774	1.5541	1.5332	1.5143	1.4973	1.4821	1.4687	1.4569
Pinion	α	60	59	58	57	56	55	54	53
	U	2.0000	1.9416	1.8871	1.8361	1.7883	1.7434	1.7013	1.6616
	F	8.00	7.31	6.72	6.18	5.72	5.30	4.93	4.59
	L	1.8138	1.8877	1.9631	2.0402	2.1190	2.1997	2.2825	2.3673

Constants for Calculating Spiral Gears

Gear	L	4.0211	3.8795	3.7439	3.6139	3.4891	3.3689	3.2532	3.1416
	F	2.04	2.13	2.23	2.33	2.44	2.56	2.69	2.83
	U	1.2690	1.2868	1.3054	1.3250	1.3456	1.3673	1.3902	1.4142
	α	38	39	40	41	42	43	44	45
Speed Ratio		C_t = Center Distance per Tooth of Pinion. Shaft Angle, 90°							
1 to 10		7.1573	7.2283	7.3050	7.3872	7.4755	7.5699	7.6706	7.7782
1 to 9		6.5228	6.5849	6.6522	6.7247	6.8027	6.8862	6.9755	7.0711
1 to 8		5.8882	5.9415	5.9995	6.0622	6.1298	6.2025	6.2805	6.3640
1 to 7		5.2537	5.2981	5.3468	5.3997	5.4570	5.5189	5.5854	5.6569
1 to 6		4.6192	4.6548	4.6941	4.7372	4.7842	4.8352	4.8903	4.9497
1 to 5		3.9847	4.0114	4.0414	4.0747	4.1114	4.1515	4.1952	4.2426
2 to 9		3.6675	3.6897	3.7151	3.7434	3.7750	3.8097	3.8477	3.8891
1 to 4		3.3502	3.3680	3.3887	3.4121	3.4385	3.4678	3.5001	3.5355
2 to 7		3.0330	3.0463	3.0624	3.0809	3.1022	3.1260	3.1526	3.1820
3 to 10		2.9272	2.9391	2.9536	2.9705	2.9900	3.0121	3.0368	3.0641
1 to 3		2.7157	2.7246	2.7360	2.7496	2.7657	2.7842	2.8051	2.8284
3 to 8		2.5042	2.5102	2.5184	2.5288	2.5415	2.5563	2.5734	2.5927
2 to 5		2.3985	2.4030	2.4097	2.4184	2.4293	2.4424	2.4575	2.4749
3 to 7		2.2927	2.2957	2.3009	2.3080	2.3172	2.3284	2.3417	2.3570
4 to 9		2.2398	2.2421	2.2465	2.2528	2.2611	2.2714	2.2838	2.2981
1 to 2		2.0812	2.0813	2.0833	2.0871	2.0929	2.1005	2.1100	2.1213
5 to 9		1.9543	1.9526	1.9528	1.9546	1.9584	1.9638	1.9710	1.9799
4 to 7		1.9226	1.9204	1.9201	1.9215	1.9247	1.9296	1.9362	1.9446
3 to 5		1.8697	1.8668	1.8657	1.8662	1.8687	1.8726	1.8783	1.8856
5 to 8		1.8274	1.8239	1.8222	1.8221	1.8238	1.8271	1.8320	1.8385
2 to 3		1.7639	1.7596	1.7569	1.7559	1.7565	1.7587	1.7625	1.7678
7 to 10		1.7186	1.7136	1.7103	1.7086	1.7085	1.7099	1.7128	1.7173
5 to 7		1.7005	1.6953	1.6917	1.6896	1.6892	1.6903	1.6929	1.6971
3 to 4		1.6582	1.6524	1.6482	1.6455	1.6444	1.6447	1.6466	1.6499
7 to 9		1.6280	1.6217	1.6171	1.6139	1.6123	1.6122	1.6135	1.6163
4 to 5		1.6053	1.5987	1.5938	1.5903	1.5883	1.5878	1.5887	1.5910
5 to 6		1.5736	1.5666	1.5611	1.5571	1.5547	1.5536	1.5539	1.5556
6 to 7		1.5524	1.5451	1.5394	1.5351	1.5322	1.5308	1.5308	1.5321
7 to 8		1.5373	1.5298	1.5238	1.5193	1.5162	1.5145	1.5142	1.5153
8 to 9		1.5260	1.5183	1.5122	1.5075	1.5042	1.5023	1.5018	1.5026
9 to 10		1.5172	1.5094	1.5031	1.4983	1.4949	1.4928	1.4921	1.4928
1 to 1		1.4467	1.4379	1.4306	1.4246	1.4201	1.4168	1.4149	1.4142
Pinion	α	52	51	50	49	48	47	46	45
	U	1.6243	1.5890	1.5557	1.5242	1.4945	1.4663	1.4396	1.4142
	F	4.29	4.01	3.77	3.54	3.34	3.15	2.98	2.83
	L	2.4545	2.5440	2.6361	2.7302	2.8287	2.9296	3.0338	3.1416

Two to One Spiral Gears—Shafts at 90-Degree Angle—I

Angle of driver *D* is 47 degrees 34½ minutes; angle of driven gear *d* is 42 degrees 25½ minutes. — See foot-note below Table 2.

Gear	Number of Teeth	14 Diametral Pitch			12 Diametral Pitch			10 Diametral Pitch			8 Diametral Pitch		
		Pitch Diam.	Outside Diam.	Center Distance	Pitch Diam.	Outside Diam.	Center Distance	Pitch Diam.	Outside Diam.	Center Distance	Pitch Diam.	Outside Diam.	Center Distance
D	6	0.6352	0.7780	0.8982	0.7411	0.9077	1.0479	0.8893	1.0893	1.2575	1.1117	1.3617	1.5718
d	12	1.1611	1.3039		1.3547	1.5213		1.6256	1.8256		2.0320	2.2820	
D	7	0.7411	0.8839	1.0479	0.8646	1.0312	1.2225	1.0376	1.2376	1.4671	1.2970	1.5470	1.8338
d	14	1.3547	1.4975		1.5804	1.7470		1.8965	2.0965		2.3707	2.6207	
D	8	0.8470	0.9898	1.1976	0.9882	1.1548	1.3972	1.1858	1.3858	1.6766	1.4823	1.7323	2.0958
d	16	1.5482	1.6910		1.8062	1.9728		2.1675	2.3675		2.7094	2.9594	
D	9	0.9529	1.0957	1.3473	1.1117	1.2783	1.5718	1.3340	1.5340	1.8862	1.6675	1.9175	2.3578
d	18	1.7417	1.8845		2.0320	2.1986		2.4384	2.6384		3.0481	3.2981	
D	10	1.0587	1.2015	1.4970	1.2352	1.4018	1.7465	1.4823	1.6823	2.0958	1.8528	2.1028	2.6198
d	20	1.9353	2.0781		2.2578	2.4244		2.7094	2.9094		3.3867	3.6367	
D	11	1.1646	1.3074	1.6467	1.3587	1.5253	1.9211	1.6305	1.8305	2.3054	2.0381	2.2881	2.8817
d	22	2.1288	2.2716		2.4836	2.6502		2.9803	3.1803		3.7254	3.9754	
D	12	1.2705	1.4133	1.7964	1.4823	1.6489	2.0958	1.7787	1.9787	2.5150	2.2234	2.4734	3.1437
d	24	2.3223	2.4651		2.7094	2.8760		3.2513	3.4513		4.0641	4.3141	
D	13	1.3764	1.5192	1.9461	1.6058	1.7724	2.2704	1.9269	2.1269	2.7246	2.4087	2.6587	3.4057
d	26	2.5158	2.6586		2.9351	3.1017		3.5222	3.7222		4.4028	4.6528	
D	14	1.4823	1.6251	2.0958	1.7293	1.8959	2.4451	2.0752	2.2752	2.9342	2.5940	2.8440	3.6677
d	28	2.7094	2.8522		3.1609	3.3275		3.7931	3.9931		4.7414	4.9914	
D	15	1.5881	1.7309	2.2455	1.8528	2.0194	2.6197	2.2234	2.4234	3.1438	2.7793	3.0293	3.9297
d	30	2.9029	3.0457		3.3867	3.5533		4.0641	4.2641		5.0801	5.3301	
D	16	1.6940	1.8368	2.3952	1.9764	2.1430	2.7944	2.3716	2.5716	3.3534	2.9646	3.2146	4.1916
d	32	3.0964	3.2392		3.6125	3.7791		4.3350	4.5350		5.4188	5.6688	
D	17	1.7999	1.9427	2.5449	2.0999	2.2665	2.9690	2.5199	2.7199	3.5630	3.1498	3.3998	4.4536
d	34	3.2900	3.4328		3.8383	4.0049		4.6060	4.8060		5.7575	6.0075	
D	18	1.9058	2.0486	2.6946	2.2234	2.3900	3.1437	2.6681	2.8681	3.7725	3.3351	3.5851	4.7156
d	36	3.4835	3.6263		4.0641	4.2307		4.8769	5.0769		6.0962	6.3462	
D	19	2.0117	2.1545	2.8443	2.3469	2.5135	3.3183	2.8163	3.0163	3.9821	3.5204	3.7504	4.9776
d	38	3.6770	3.8198		4.2898	4.4564		5.1478	5.3478		6.4348	6.6848	
D	20	2.1175	2.2603	2.9940	2.4705	2.6371	3.4930	2.9646	3.1646	4.1917	3.7057	3.9557	5.2396
d	40	3.8706	4.0134		4.5156	4.6822		5.4188	5.6188		6.7735	7.0235	

Two to One Spiral Gears — Shafts at 90-Degree Angle — 2 *

Angle of driver D is 47 degrees 34½ minutes; angle of driven gear d is 42 degrees 25½ minutes.

Gear	Number of Teeth	7 Diametral Pitch			6 Diametral Pitch			5 Diametral Pitch			4 Diametral Pitch		
		Pitch Diam.	Outside Diam.	Center Distance	Pitch Diam.	Outside Diam.	Center Distance	Pitch Diam.	Outside Diam.	Center Distance	Pitch Diam.	Outside Diam.	Center Distance
D	9	1.9058	2.1916	2.6946	2.2234	2.5567	3.1437	2.6681	3.0681	3.7725	3.3351	3.8351	4.7156
d	18	3.4835	3.7693		4.0641	4.3974		4.8769	5.2769		6.0961	6.5961	
D	10	2.1175	2.4033	2.9940	2.4705	2.8038	3.4931	2.9646	3.3646	4.1917	3.7057	4.2057	5.2396
d	20	3.8706	4.1564		4.5157	4.8490		5.4188	5.8188		6.7735	7.2735	
D	11	2.3293	2.6151	3.2935	2.7175	3.0508	3.8424	3.2610	3.6610	4.6109	4.0763	4.5763	5.7636
d	22	4.2576	4.5434		4.9672	5.3005		5.9607	6.3607		7.4508	7.9508	
D	12	2.5410	2.8268	3.5929	2.9646	3.2979	4.1917	3.5575	3.9575	5.0300	4.4469	4.9469	6.2875
d	24	4.6447	4.9305		5.4188	5.7521		6.5026	6.9026		8.1282	8.6282	
D	13	2.7528	3.0386	3.8923	3.2116	3.5449	4.5410	3.8540	4.2540	5.4492	4.8175	5.3175	6.8115
d	26	5.0317	5.3175		5.8704	6.2037		7.0444	7.4444		8.8056	9.3056	
D	14	2.9646	3.2504	4.1917	3.4587	3.7920	4.8903	4.1504	4.5504	5.8684	5.1880	5.6880	7.3355
d	28	5.4188	5.7046		6.3219	6.6552		7.5863	7.9863		9.4829	9.9829	
D	15	3.1763	3.4621	4.4911	3.7057	4.0390	5.2396	4.4469	4.8469	6.2875	5.5586	6.0586	7.8594
d	30	5.8059	6.0917		6.7735	7.1068		8.1282	8.5282		10.1603	10.6603	
D	16	3.3881	3.6739	4.7905	3.9528	4.2861	5.5889	4.7433	5.1433	6.7067	5.9292	6.4292	8.3834
d	32	6.1929	6.4787		7.2251	7.5584		8.6701	9.0701		10.8376	11.3376	
D	17	3.5998	3.8856	5.0899	4.1998	4.5331	5.9382	5.0398	5.4398	7.1259	6.2998	6.7998	8.9074
d	34	6.5800	6.8658		7.6766	8.0099		9.2120	9.6120		11.5150	12.0150	
D	18	3.8116	4.0974	5.3893	4.4469	4.7802	6.2875	5.3363	5.7363	7.5451	6.6703	7.1703	9.4313
d	36	6.9670	7.2528		8.1282	8.4615		9.7539	10.1539		12.1923	12.6923	
D	19	4.0234	4.3092	5.6887	4.6939	5.0272	6.6368	5.6327	6.0327	7.9642	7.0409	7.5409	9.9553
d	38	7.3541	7.6399		8.5798	8.9131		10.2957	10.6957		12.8697	13.3697	
D	20	4.2351	4.5209	5.9881	4.9410	5.2743	6.9862	5.9292	6.3292	8.3834	7.4115	7.9115	10.4793
d	40	7.7412	8.0270		9.0314	9.3647		10.8376	11.2376		13.5470	14.0470	

* These tables are especally applicable to the design of timing gears for gas and gasoline engines, or wherever a 2 to 1 ratio is required. If a greater number of teeth or a larger pitch is required than given in the tables, multiples may be used.

Example: — Determine the dimensions of two spiral gears having a ratio of 2 to 1, and 26 and 52 teeth of 6 diametral pitch. Find the pitch diameters for 13 and 26 teeth, in the table, and multiply by 2, thus obtaining the pitch diameters for 26 and 52 teeth. $3.2116 \times 2 = 6.4232$, and $5.8704 \times 2 = 11.7408$. To find the outside diameter add twice the addendum to the pitch diameters just found. Dimensions for diametral pitches not given in the table may be found by proportion. For example, the dimensions for 3 diametral pitch may be found by multiplying the pitch diameters found in the 6-diametral pitch column by 2.

Diagram for Finding Cutter for Milling Spiral Gears. — A diagram has been prepared, giving directly the number of cutter to be used for a given number of teeth and a given spiral angle. The heavy lines drawn in the diagram are division lines between the fields to which each cutter applies. For example, suppose the angle of the teeth of a gear is 37 degrees with its axis, and the number of teeth is

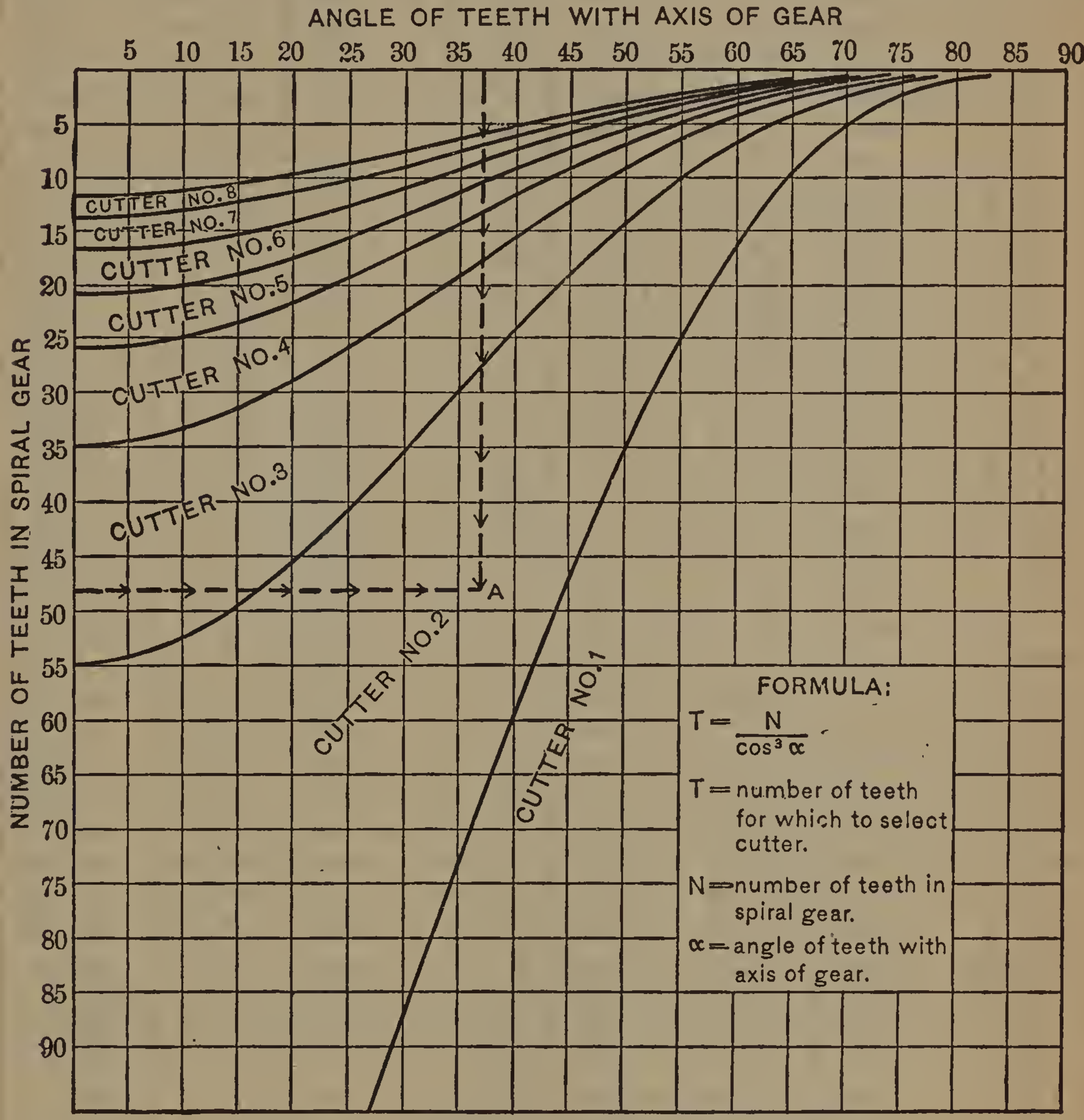


Diagram for Finding Cutter for Milling Spiral Gears

48. The point A at which the horizontal line representing the number of teeth and the vertical line representing the angle intersect, falls within the area marked cutter No. 2; therefore, a No. 2 cutter is required for cutting a 48-tooth spiral gear having a spiral angle of 37 degrees.

Table for Selecting Cutter for Milling Spiral Gears. — The "Table for Selecting Cutter for Milling Spiral Gears" gives the value of the factor $K = \frac{1}{\cos^3 \alpha}$ which enters in the formula for finding the number of teeth for which to select the cutter for milling spiral gears. The table is used as follows: Multiply the actual number of teeth in the spiral gear to be cut by the factor K, as given in the table opposite the angle of spiral. The product gives the number of teeth for which to select the cutter.

Table for Selecting Cutter for Milling Spiral Gears

Angle of Spiral, α	K	Angle of Spiral, α	K	Angle of Spiral, α	K	Angle of Spiral, α	K
0° 0′	1.000	21° 0′	1.228	42° 0′	2.436	63° 0′	10.69
0° 30′	1.000	21° 30′	1.241	42° 30′	2.495	63° 30′	11.27
1° 0′	1.001	22° 0′	1.254	43° 0′	2.557	64° 0′	11.87
1° 30′	1.001	22° 30′	1.268	43° 30′	2.621	64° 30′	12.55
2° 0′	1.002	23° 0′	1.282	44° 0′	2.687	65° 0′	13.25
2° 30′	1.003	23° 30′	1.297	44° 30′	2.756	65° 30′	14.03
3° 0′	1.004	24° 0′	1.312	45° 0′	2.828	66° 0′	14.86
3° 30′	1.005	24° 30′	1.328	45° 30′	2.902	66° 30′	15.80
4° 0′	1.007	25° 0′	1.344	46° 0′	2.983	67° 0′	16.76
4° 30′	1.009	25° 30′	1.360	46° 30′	3.066	67° 30′	17.85
5° 0′	1.011	26° 0′	1.377	47° 0′	3.152	68° 0′	18.98
5° 30′	1.013	26° 30′	1.395	47° 30′	3.242	68° 30′	20.33
6° 0′	1.016	27° 0′	1.414	48° 0′	3.336	69° 0′	21.72
6° 30′	1.019	27° 30′	1.434	48° 30′	3.436	69° 30′	23.33
7° 0′	1.022	28° 0′	1.454	49° 0′	3.540	70° 0′	25.00
7° 30′	1.026	28° 30′	1.474	49° 30′	3.650	70° 30′	26.97
8° 0′	1.030	29° 0′	1.495	50° 0′	3.767	71° 0′	28.97
8° 30′	1.034	29° 30′	1.517	50° 30′	3.887	71° 30′	31.40
9° 0′	1.038	30° 0′	1.540	51° 0′	4.012	72° 0′	33.88
9° 30′	1.042	30° 30′	1.563	51° 30′	4.144	72° 30′	36.92
10° 0′	1.047	31° 0′	1.588	52° 0′	4.284	73° 0′	40.00
10° 30′	1.052	31° 30′	1.613	52° 30′	4.433	73° 30′	43.88
11° 0′	1.057	32° 0′	1.640	53° 0′	4.586	74° 0′	47.79
11° 30′	1.062	32° 30′	1.667	53° 30′	4.752	74° 30′	54.72
12° 0′	1.068	33° 0′	1.695	54° 0′	4.925	75° 0′	57.68
12° 30′	1.074	33° 30′	1.724	54° 30′	5.101	75° 30′	64.15
13° 0′	1.080	34° 0′	1.755	55° 0′	5.295	76° 0′	70.65
13° 30′	1.087	34° 30′	1.787	55° 30′	5.497	76° 30′	79.20
14° 0′	1.094	35° 0′	1.819	56° 0′	5.710	77° 0′	87.78
14° 30′	1.102	35° 30′	1.853	56° 30′	5.940	77° 30′	99.50
15° 0′	1.110	36° 0′	1.889	57° 0′	6.190	78° 0′	111.3
15° 30′	1.118	36° 30′	1.926	57° 30′	6.435	79° 0′	144.0
16° 0′	1.127	37° 0′	1.963	58° 0′	6.720	80° 0′	191.2
16° 30′	1.136	37° 30′	2.003	58° 30′	7.010	81° 0′	261.4
17° 0′	1.145	38° 0′	2.044	59° 0′	7.321	82° 0′	370.6
17° 30′	1.154	38° 30′	2.086	59° 30′	7.650	83° 0′	552.1
18° 0′	1.163	39° 0′	2.130	60° 0′	8.000	84° 0′	876.4
18° 30′	1.172	39° 30′	2.176	60° 30′	8.380	85° 0′	1509.0
19° 0′	1.182	40° 0′	2.225	61° 0′	8.780	86° 0′	2940.0
19° 30′	1.193	40° 30′	2.275	61° 30′	9.209	87° 0′	6990.0
20° 0′	1.204	41° 0′	2.326	62° 0′	9.658	......	....
20° 30′	1.216	41° 30′	2.380	62° 30′	10.160	......	....

Example: — Angle of spiral = 30 degrees; number of teeth in spiral gear = 18. Factor K for 30 degrees, as found from the table, equals 1.540. Then, number of teeth for which to select the cutter = $18 \times 1.540 = 28$, approximately. Hence, use spur gear cutter for 28 teeth, or cutter No. 4.

Angular Position of Table. — When cutting spiral gears in a milling machine it is common practice to set the table to the angle of the teeth at the pitch line, although some contend that if the angle is taken at some point below the pitch line, teeth of a better shape will be obtained. In any case, the angle is determined by first getting the tangent of the angle, and then finding the corresponding angle from a table of tangents. For example, if the pitch diameter of the gear is 4.46 and the lead of the spiral, 20 inches, the tangent equals $\frac{4.46 \times 3.1416}{20} = 0.700$, which is the tangent of 35 degrees; therefore the table should be swiveled 35 degrees from its position at right angles to the spindle.

Milling the Spiral Teeth. — The teeth of a spiral gear are proportioned from the normal pitch and not the circular pitch. The whole depth of the tooth can be found by dividing 2.157 by the normal diametral pitch of the gear, which corresponds to the pitch of the cutter. The thickness of the tooth at the pitch line equals 1.571 divided by the normal diametral pitch. After a tooth space has been milled, the cutter should be prevented from dragging through it when being returned for another cut. This can be done by lowering the blank slightly, or by stopping the machine and turning the cutter to such a position that the teeth will not touch the work. If the gear has teeth coarser than 10 or 12 diametral pitch, it is well to take a roughing and a finishing cut. When pressing a spiral gear blank on the arbor, it should be remembered that it is more likely to slip when being milled than a spur gear, because the pressure of the cut, being at an angle, tends to rotate the blank on the arbor.

General Remarks on Spiral Gear Hobbing. — In cutting teeth having large angles, it is desirable to have the direction of spiral of the hob the same as the direction of spiral of the gear, or in other words, the gear and the hob of the same "hand." Then the direction of the cut will come against the movement of the blank. At ordinary angles, however, one hob will cut both right- and left-hand gears. In setting up the hobbing machine for spiral gears, care should be taken to see that the vertical feed does not trip until the machine has been stopped or the hob has fed down past the finished gear. Should the feed stop while the hob is still in mesh with the gear, and revolving at the ratio required to generate a spiral, the hob will cut into the teeth and spoil the gear.

Change Gears for Spiral Gear-hobbing Machines. — The change gears to be used for generating spiral gears on gear-hobbing machines may be found by the following formula:

$$\frac{L \div F}{(L \div F) \pm 1} \times \frac{P}{p} = \frac{S}{s}$$

in which.

L = lead of spiral;
F = feed per revolution;
P = product of driving gears for cutting spur gears with the same number of teeth;
p = product of driven gears for cutting spur gears with the same number of teeth;
S = product of driving gears for cutting spiral gears;
s = product of driven gears for cutting spiral gears.

In the formula, use + sign when gear and hob are of opposite "hand", and − sign when they are of same "hand."

Example: — Two spiral gears are to be cut on a gear-hobbing machine. Gear No. 1 has 30 teeth, 24.549-inch lead, and a feed of 1/24 inch. The change gears used on the machine for cutting a spur gear with 30 teeth have 48 (driving gear) and 60 (driven gear) teeth, respectively. The hob and gear are of the same "hand."

Gear No. 2 has 60 teeth, 49.098-inch lead and is cut with a feed of 1/16 inch. The change gears used to cut a spur gear with 60 teeth, on this machine, have 48 and 40 teeth, for the driving gears, and 60 and 80 teeth, for the driven gears. The hob and gear are of the same "hand."

In the problems given the data are as follows:

	30-tooth Gear		60-tooth Gear
L	24.549	L	49.098
F	1/24	F	1/16
P	48	P	40×48
p	60	p	60×80

Calculations for Thirty-tooth Gear

By inserting the values given:

$$\frac{L \div F}{(L \div F) - 1} = \frac{589.176}{588.176}$$

The ratio written above can be simplified to the form $\frac{589}{588}$. Factoring:

$$\frac{589}{588} = \frac{19 \times 31}{12 \times 49}$$

Now, multiply this value with the ratio of the gears for a 30-tooth spur gear:

$$\frac{19 \times 31}{12 \times 49} \times \frac{48}{60} = \frac{76 \times 31}{60 \times 49}$$

Having obtained the gears that should be used, investigate what lead these gears will give. Apparently they will not give the exact lead desired, as an approximate ratio has been used instead of the exact one.

To prove, assume $F = 1/24$ and solve for L.

$$\frac{L \div F}{(L \div F) - 1} = \frac{589}{588}$$

From this, $L = 24.541$, which is very nearly equal to the required lead.

Calculations for Sixty-tooth Gear

By proceeding in the same way for the 60-tooth gear:

$$\frac{L \div F}{(L \div F) - 1} = \frac{785.568}{784.568}$$

Then factor the fraction $\frac{785}{784}$, thus: $\frac{785}{784} = \frac{5 \times 157}{4 \times 196}$.

As 157 is a prime number, and gives too large a number of teeth for any of the gears in the train, try $\frac{784}{783}$ which ratio is very nearly equivalent to that required.

$$\frac{784}{783} = \frac{49 \times 16}{29 \times 27}$$

Multiply this value with the ratio of the gears for a 60 tooth spur gear:

$$\frac{49 \times 16}{29 \times 27} \times \frac{40 \times 48}{60 \times 80} = \frac{49 \times 32}{29 \times 135} \quad \text{or} \quad \frac{49 \times 32}{87 \times 45}$$

Possibly the 135-tooth gear is impracticable, on account of being too large, in which case the other combination must be tried.

If the lead resulting from the gears found is calculated in the same manner as in the previous case, $L = 49.001$.

Influence of Small Changes in the Ratio on the Lead. — It is of importance to note that a comparatively slight change in the ratio $\frac{L \div F}{(L \div F) - 1}$ makes a very decided change in the lead obtained. To illustrate, assume that in the first example given the ratio $\frac{589}{588} = 1.001701$ were changed to 1.002. What effect has this change on the lead obtained ($F = 1/24$)?

$$\frac{L \div F}{(L \div F) - 1} = 1.002$$

Solve for L; then $L = 20.875$, which is a very different lead from the one to be obtained.

Herringbone Gears

Herringbone Gears.—The advantages of herringbone gears may be summarized as follows: Their action is continuous and smooth; there is no shock when the load is transferred from tooth to tooth, and therefore wear is practically eliminated; the bending action of the load on the teeth is less than with straight spur gearing; the gears work silently and without vibration; back-lash is practically absent and herringbone gears can be used for very high ratios and for great velocities. In the Wuest system of herringbone gears, the right- and left-hand sides of the gears are stepped half a space apart and do not meet at a common apex at the center of the face, as in the usual type of herringbone gearing. This stepped form wears more evenly under extreme loads than the ordinary type. The following rules and formulas are especially applicable to Wuest herringbone gearing. The spiral angle of the teeth is made about 23 degrees with the axis. The face of the gear may ordinarily be made about six times the circular pitch, for pinions having not less than 25 teeth. The width of face for high ratio gears with small pinions may be from six to twelve times the circular pitch. As the nature of the action eliminates shock, the pitch required for any given conditions can be much finer than would be chosen for spur gears. The pressure angle adopted for these gears is 20 degrees. The teeth are shorter than the usual standards, because the high ratios for which these gears are used call for an average pinion diameter which is less than is used with straight spur gears for similar duties. The tooth shape is the involute. In the following formulas:

D = pitch diameter;	S = addendum;
N = number of teeth;	$S + A$ = dedendum (A = clearance);
P = diametral pitch;	W = full depth of tooth;
d = diameter of blank;	W_1 = working depth.

For 20 teeth and over:

$$D = \frac{N}{P} \qquad d = \frac{N + 1.6}{P}$$

For gears with less than 20 teeth:

$$D = \frac{0.95\,N + 1}{P} \qquad d = \frac{0.95\,N + 2.6}{P}$$

For all herringbone gears:

$$S = \frac{0.8}{P} \qquad S + A = \frac{1}{P} \qquad W = \frac{1.8}{P} \qquad W_1 = \frac{1.6}{P}$$

When a pinion of less than 20 teeth is used with a standard gear, the center distance must be slightly increased to suit the enlargement of the pinion. If it is desired to keep the center distance to the standard dimensions, the gear diameter must be reduced by the amount of the enlargement given to the pinion.

Example: — A pinion of 10 teeth, 5 diametral pitch, is to mesh with a gear of 90 teeth, the center distance being 10 inches.

$$\text{Pitch diameter of pinion} = \frac{0.95 \times 10 + 1}{5} = 2.1 \text{ inches.}$$

Enlargement over standard pinion = 0.1 inch.

$$\text{Pitch diameter of standard gear} = \frac{90}{5} = 18.0 \text{ inches.}$$

Reduced pitch diameter of gear = 18.0 − 0.1 = 17.9 inches.

$$\text{Center distance} = \frac{17.9 + 2.1}{2} = 10 \text{ inches.}$$

Power Transmitted by Herringbone Gears. — The accompanying table gives the allowable shearing stress K in pounds per square inch at various velocities in feet per minute at the pitch diameter. These values are entirely empirical, but are based on results of extended experience and lead to dimensions which are safe and reliable. When the pinion and gear are of different materials, it is necessary to use the values for the lowest rate of material in the combination. In the formulas:

H.P. = horsepower transmitted;
N = revolutions per minute;
D = pitch diameter in inches;
p = circular pitch in inches;
F = total width of face in inches;
V = pitch line velocity in feet per minute;
W = total tooth pressure at pitch line in pounds;
K = stress factor as obtained from table.

Then:

$$V = \frac{3.14\,DN}{12} \qquad W = \frac{\text{H.P.} \times 33{,}000}{V} = \frac{pFK}{2.5}$$

In gears with moderate ratio (not exceeding 1 to 6), and face width equivalent to 6 times the circular pitch, make:

$$W = 2.4\,p^2K, \qquad \text{or} \qquad p = \sqrt{\frac{W}{2.4\,K}}$$

For higher ratios, make $F = Rp$ (where R = ratio of gears) up to a maximum of $F = 10\,p$. The circular pitch for high ratios is found from:

$$p = \sqrt{\frac{2.5\,W}{RK}}$$

When the face width is equivalent to 8 times the circular pitch, $W = 3.2\,p^2K$, and when the face width is equivalent to 10 times the circular pitch, $W = 4\,p^2K$.

For ordinary service, pitch line velocities between 1000 and 2000 feet per minute, with 1500 feet as a fair average, may be considered safe. Cast iron is preferable to steel castings for gears of large diameters and moderate powers, but steel castings will be found more economical for high tooth pressures. Pinions are usually made of steel forgings containing from 0.40 to 0.50 per cent carbon. Very soft steel pinions should never be used for herringbone gears.

The ordinary methods of calculation just given should not be used for rolling mill gears which are often subjected to stresses which are so far in excess of the average working load that it is necessary to consider the strength of the teeth in regard to the extreme overloads. Gears running at a very high velocity, such as are used for steam turbines, require additional working surface, and are characterized by extreme width of face combined with very fine pitch.

Table of Safe Shearing Stresses K, in Pounds per Square Inch, for Herringbone Gears

Velocity in Feet per Minute	Factor *K* for					
	Brass	Cast Iron	Gun Metal	Phosphor Bronze	Steel Castings	High-carbon Steel Forgings
100	600	800	1000	1150	1325	1800
200	575	750	950	1100	1275	1750
300	550	700	900	1060	1250	1700
400	525	675	860	1030	1200	1660
500	500	650	830	1000	1175	1630
600	475	625	800	975	1150	1600
800	425	575	750	925	1100	1550
1000	400	525	700	875	1050	1500
1200	380	500	650	825	1000	1450
1500	360	475	600	775	925	1350
1800	350	450	550	725	875	1275
2100	340	425	525	675	825	1200
2400	325	400	500	650	775	1125
3000	300	375	475	600	700	1050

Tables of Horsepower for Herringbone Gears. — The accompanying tables give the horsepower transmitted by herringbone gears in which the pinion has 21 teeth, and the width of face corresponds to 8 and 10 times the circular pitch. To find the horsepower for any other number of teeth, ascertain the pitch line velocity, and under the given diametral pitch, find the horsepower corresponding to this velocity. To find the horsepower transmitted by a brass gear, multiply that found for a cast-iron gear by 0.8; for gun metal, multiply by 1.25; and for phosphor bronze, by 1.63. For high-carbon steel forgings, multiply the horsepower transmitted by steel-casting gears by 1.45.

In the design of herringbone gears, a pinion with 21 teeth will be found satisfactory for average conditions, although it is possible to have a pinion with a smaller number of teeth; pinions with as few as thirteen teeth have been used with satisfactory results. When such small pinions are used, however, they are made solid with the shaft. To illustrate the method of using the tables for horsepower transmitted by herringbone gears, assume that a herringbone gear drive is required to transmit 100 horsepower from a motor running at 650 revolutions per minute with a speed reduction of 12 to 1. In the table, 101 horsepower and 637 revolutions per minute will be found under 2½ diametral pitch and 10-inch face cast-iron gear. This gear will run at a pitch line velocity of 1400 feet per minute. The pitch diameter of the gear would be 100.8 inches to mesh with a 21-tooth, 2½ diametral

Horsepower Transmitted by Herringbone Gears

Velocity at Pitch Circle, Feet per Minute	1½ Diametral Pitch, 21 Teeth, 14-inch Pitch Diameter					1¾ Diametral Pitch, 21 Teeth, 12-inch Pitch Diameter				
	Revolutions per Minute	16-inch Face		20-inch Face		Revolutions per Minute	14½-inch Face		18-inch Face	
		Cast Iron	Steel Casting	Cast Iron	Steel Casting		Cast Iron	Steel Casting	Cast Iron	Steel Casting
400	110	110	199	137	249	127	86	155	106	192
600	160	152	280	190	350	190	118	218	146	271
800	220	187	358	234	447	254	146	279	180	345
1000	270	215	426	269	534	318	166	331	206	413
1200	330	244	488	305	610	382	190	378	234	470
1400	380	270	540	338	675	445	210	420	261	521
1500	410	282	564	343	707	477	219	438	272	545
1600	435	292	585	366	732	510	228	455	282	562
1800	490	311	640	388	800	572	241	496	300	610
2000	540	333	671	406	838	636	258	522	320	710

Velocity at Pitch Circle, Feet per Minute	2 Diametral Pitch, 21 Teeth, 10½-inch Pitch Diameter					2½ Diametral Pitch, 21 Teeth, 8.4-inch Pitch Diameter				
	Revolutions per Minute	12½-inch Face		15½-inch Face		Revolutions per Minute	10-inch Face		12½-inch Face	
		Cast Iron	Steel Casting	Cast Iron	Steel Casting		Cast Iron	Steel Casting	Cast Iron	Steel Casting
400	145	64	116	79	144	182	41	74	51	93
600	218	89	164	111	204	273	57	105	71	130
800	291	109	207	136	259	364	70	134	87	167
1000	364	125	250	154	309	455	80	160	100	200
1200	437	143	285	178	355	546	91	185	114	228
1400	510	158	316	196	393	637	101	202	126	253
1500	546	166	329	206	410	683	106	211	132	264
1600	580	171	342	213	425	729	109	220	137	274
1800	655	182	375	226	465	820	116	240	145	300
2000	730	195	392	242	488	910	122	251	152	314

Velocity at Pitch Circle, Feet per Minute	3 Diamctral Pitch, 21 Teeth, 7-inch Pitch Diameter					3½-inch Diametral Pitch, 21 Teeth, 6-inch Pitch Diameter				
	Revolutions per Minute	8¼-inch Face		10½-inch Face		Revolutions per Minute	7¼-inch Face		9-inch Face	
		Cast Iron	Steel Casting	Cast Iron	Steel Casting		Cast Iron	Steel Casting	Cast Iron	Steel Casting
400	218	28	51	36	65	255	21	39	26	48
600	327	39	72	50	92	382	29	55	36	68
800	436	48	92	61	117	510	36	70	45	87
1000	546	55	110	70	140	637	42	83	52	103
1200	655	63	126	80	160	765	47	95	58	118
1400	765	69	139	89	177	892	52	105	65	130
1500	820	72	146	93	184	955	55	110	68	136
1600	875	75	151	96	192	1020	57	114	71	141
1800	983	80	165	102	210	1150	61	123	76	154
2000	1090	84	173	107	220	1275	65	131	81	162

Horsepower Transmitted by Herringbone Gears

Velocity at Pitch Circle, Feet per Minute	4 Diametral Pitch, 21 Teeth, 5¼-inch Pitch Diameter					5 Diametral Pitch, 21 Teeth, 4.2-inch Pitch Diameter					6 Diametral Pitch, 21 Teeth, 3½-inch Pitch Diameter				
	R.P.M.	6¼-inch Face		7¾-inch Face		R.P.M.	5-inch Face		6¼-inch Face		R.P.M.	4-inch Face		5¼-inch Face	
		C.I.	S.C.	C.I.	S.C.		C.I.	S.C.	C.I.	S.C.		C.I.	S.C.	C.I.	S.C.
400	292	16	29	20	36	364	10	19	13	24	436	7	13	9	17
600	437	22	41	27	51	545	14	27	18	34	655	10	18	13	24
800	584	27	53	34	66	725	17	34	21	43	873	12	23	16	30
1000	730	31	62	39	77	910	20	40	25	50	1090	14	27	18	35
1200	875	36	71	44	88	1090	23	46	29	58	1310	15	31	20	41
1400	1020	40	80	50	99	1270	25	51	31	64	1525	17	35	22	46
1500	1090	41	83	51	103	1360	26	53	32	66	1635	18	36	24	47
1600	1160	43	86	53	107	1450	27	55	34	69	1745	19	37	25	49
1800	1310	46	93	57	116	1630	29	59	36	74	1960	20	40	26	53
2000	1460	49	99	61	122	1820	31	63	39	79	2180	21	42	28	55

pitch pinion. If a 17-tooth pinion were used, the pitch line velocity would be 1150 (or, say 1200) feet per minute, and in the 1200 feet velocity line it will be seen that a gear running at this speed would transmit 91 horsepower. Hence, a gear of wider face must be used if a 17-tooth pinion is applied, the width of the gear required being $(100 \times 10) \div 91 = 11$ inches. In giving the material as cast iron, this refers to the gear only. The pinion, as has already been stated, should be made of steel forgings.

Comparison of Action of Spur and Herringbone Gearing. — There are three distinct phases in the engagement between spur gears. 1. The root of the pinion tooth engages the point of the gear tooth. 2. The teeth are engaged near the pitch line. 3. The point of the pinion tooth engages the root of the gear tooth. Assume that the teeth are accurately cut to involute form, so that if the pinion moves with even angular velocity it will produce a correspondingly even motion in the gear. Assume also that the pinion has a sufficient number of teeth to allow the engagement of successive teeth to overlap. At the beginning of the first phase while the load is carried near the point of the gear tooth, that tooth is subjected to a maximum bending stress along its whole length. The portion of the pinion tooth near the root is sliding over the outer portion of the gear tooth; that is to say, two metallic surfaces of small area are sliding under heavy compression.

The action during the second phase more nearly approaches ideal conditions. The teeth are engaged near their respective pitch lines and very little sliding takes place. During the third and final phase, the pinion tooth is subjected to a maximum bending stress, while the tooth surfaces again slide over each other, this time with the outer portion of the pinion tooth engaging the gear tooth near its root. The result is that the points and roots of all the teeth tend to wear away more rapidly than the portions near the pitch lines. The sliding action can be partially eliminated by shortening the teeth so that they engage only during the phase of rolling contact, and this has been tried with a certain measure of success in the stub-tooth gear.

Herringbone gears, when accurately cut, practically overcome all these difficulties. The fundamental principle of the action of herringbone gear teeth lies in the fact that all phases of engagement take place simultaneously for every position of pinion and gear, providing the relationship between pitch, width of face, and angle of spiral is such that it insures a complete overlap of engagement. Since all the phases of engagement occur simultaneously, it is evident that the load is partly carried by tooth surfaces in sliding contact and partly by surfaces in rolling contact. The portions of the teeth farthest from the pitch line, which engage with sliding action, tend to wear away more rapidly than the portions nearest to the pitch line. The pitch line portion, however, is always carrying part of the load and the effect of wear on the ends of the teeth merely is to throw more load on the center portions; or in other words, there is a constant tendency to concentrate the load near the pitch line.

Epicyclic Gearing

Speed Ratio Tables. — The tables "Simple Epicyclic Gearing" and "Compound Epicyclic Gearing" make it possible to rapidly find the number of revolutions of the various members of an epicyclic gear train. By means of the tables, the revolutions of any of the gears indicated in the diagrammatical illustration may be found for one revolution of any of the other gears or movable arms. For example,

Simple Epicyclic Gearing

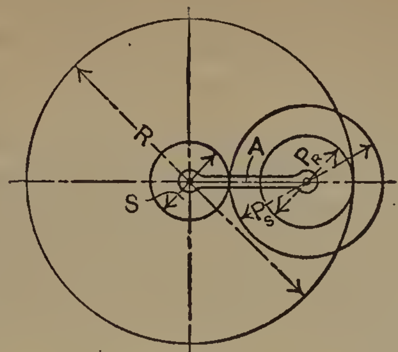
S = Sun Pinion;
P = Planet Pinion;
R = Internal Gear;
A = Arm Carrying P.

Table gives the relative speeds of each element for one revolution of driving member.

Letters, in formulas, denote pitch diameter or number of teeth.

Stationary Member	Driving Member	Driven Member	Revolutions of S	Revolutions of A	Revolutions of P, around its own Center	Revolutions of R
A	S	R	1	0	$\frac{S}{P}$	$\frac{S}{R}$
A	R	S	$\frac{R}{S}$	0	$\frac{R}{P}$	1
R	A	S	$\frac{R+S}{S}$	1	$\frac{R}{P}$	0
S	A	R	0	1	$\frac{S}{P}$	$\frac{R+S}{R}$
R	S	A	1	$\frac{S}{R+S}$	$\frac{S}{R+S} \times \frac{R}{P}$	0
S	R	A	0	$\frac{R}{R+S}$	$\frac{R}{R+S} \times \frac{S}{P}$	1
P	A	S and R	1	1	0	1

Compound Epicyclic Gearing



S = Sun Pinion;
R = Internal Gear;
A = Arm Carrying P_s and P_r;
P_r = Planet Wheel Meshing with R;
P_s = Planet Wheel Meshing with S;
P_r is keyed to P_s.

Table gives the relative speeds of each element for one revolution of driving member. Letters, in formulas, denote pitch diameter or number of teeth.

Stationary Member	Driving Member	Driven Member	Revolutions of S	Revolutions of A	Revolutions of P_r and P_s around their own Centers	Revolutions of R
A	S	R	1	0	$\frac{S}{P_s}$	$\frac{S}{P_s} \times \frac{P_r}{R}$
A	R	S	$\frac{R}{P_r} \times \frac{P_s}{S}$	0	$\frac{R}{P_r}$	1
R	A	S	$\frac{P_rS + P_sR}{P_rS}$	1	$\frac{R}{P_r}$	0
S	A	R	0	1	$\frac{S}{P_s}$	$\frac{P_rS + P_sR}{P_sR}$
R	S	A	1	$\frac{P_rS}{P_rS + P_sR}$	$\frac{P_rS}{P_rS + P_sR} \times \frac{R}{P_r}$	0
S	R	A	0	$\frac{P_sR}{P_rS + P_sR}$	$\frac{P_sR}{P_rS + P_sR} \times \frac{S}{P_s}$	1
P_r and P_s	A	S and R	1	1	0	1

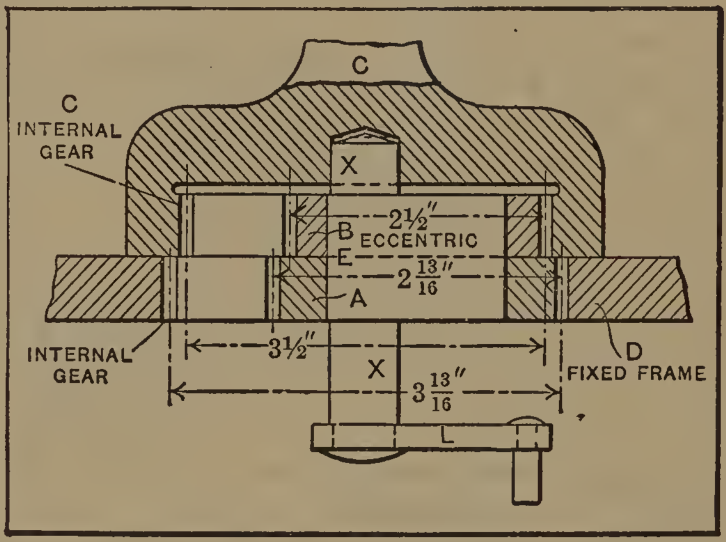
referring to the table of "Simple Epicyclic Gearing," assume that it is required to find how many revolutions a "planet" pinion P makes while the "sun" pinion S makes one revolution; S is the driving member; R, the driven member; and arm A, the stationary member. Look down the first three columns in the table and find the condition where A is the stationary member, S, the driving member and R, the driven member. This, it will be seen, is in the first line from the top. In the same horizontal line it is then found that for one revolution of S the number of revolutions of P around its own center is $S \div P$. In the formulas, S and P are the pitch diameters of the gears. If the revolutions of R are required while S makes one revolution, this is found in the same horizontal line under "Revolutions of R." It will be seen that while S makes one revolution, R makes $S \div R$ revolutions.

Assume as another example that the sun pinion S is the stationary member, arm A is the driving member, and internal pinion R is the driven member; then find the number of revolutions of driven member R for one revolution of arm A. Look down the first three columns and find the condition where S is the stationary member, A, the driving member, and R, the driven member. This is found to be in the fourth line from the top. For one revolution of A, the number of revolutions of R equals

$\frac{R+S}{R}$. The use of the table for "Compound Epicyclic Gearing" is exactly the same as for the simple gearing, except that the formulas are somewhat more complicated.

Analyzing Epicyclic Gearing Problems. — The simplest method for analyzing epicyclic gearing is to first consider all of the gears locked together and imagine the entire combination turned one turn. Then, as one member is fixed, this member must be imagined as turned back one turn in order to bring it to its original position. While turning back this member, the member which in the actual gear train is the driving member, is held stationary, and the motion of the other parts is noted. The results are tabulated as shown in the following.

As an example, analyze the motion of the gearing in the accompanying illustration. The construction of the mechanism is as follows: An eccentric E is driven by the crank L and is free to turn in the gears A and B; stud X is free to turn in casting C. Gear A meshes with the internal gear of the fixed frame D and is bolted to gear B which, in turn, meshes with the internal gear C. When the eccentric is turned by crank L, gear A is moved around inside of D and carries B with it rolling inside of C. A has 45 teeth, 16 pitch; B, 45 teeth, 18 pitch; C, 63 teeth, 18 pitch and D, 61 teeth, 16 pitch.



First consider all of the gears locked together and imagine the entire combination turned to the right one turn. Then, since the internal gear D is fixed, it must be turned back one turn in order to bring it to its original position. While performing this last operation, consider the eccentric E as being held stationary and then calculate the total turns made by C during the entire proceeding. The result is the number of turns made by C for one turn of E or L. Forward movements are designated + and backward movements −. With E held stationary and D turned −1 times, A and B will each turn $-\frac{61}{45}$ times. C will make $-(\frac{61}{45} \times \frac{45}{63})$ turns (in the same direction as D). Tabulating the results obtained gives the following number of revolutions and portions of revolutions for each of the gears:

	E	A	B	C	D
Wheels locked.........	+1	+1	+1	+1	+1
E stationary..........	0	$-\frac{61}{45}$	$-\frac{61}{45}$	$-(\frac{61}{45} \times \frac{45}{63})$	−1
	+1	$-\frac{16}{45}$	$-\frac{16}{45}$	$+\frac{2}{63}$	0

From this, it will be seen that for one turn of the crank in a right-hand direction, A and B make each $\frac{16}{45}$ turns in a left-hand direction and C makes $\frac{2}{63}$ turns in a right-hand direction. In other words, C makes one complete turn for each 31.5 turns of handle L. This method of analysis may be used for any gear train, no matter how complicated.

Ratchet Gearing

Ratchet gearing may be used to transmit intermittent motion, or its only function may be to prevent the ratchet wheel from rotating backward. Ratchet gearing of this latter form is commonly used in connection with hoisting mechanisms of various kinds, to prevent the hoisting drum or shaft from rotating in a reverse direction under the action of the load.

Use of Multiple Pawls. — Some ratchet gearing is equipped with multiple pawls so that the relative motion between the pawl and ratchet wheel for engaging the pawl with successive teeth will be less than the pitch of the teeth. While the motion of the pawl lever that is necessary for engaging the pawl with successive teeth on the ratchet wheel may be reduced by making the pitch of the ratchet wheel teeth smaller, this weakens the teeth and is not always desirable. The effect of a smaller pitch may be obtained by using two or more pawls which are placed side by side and are pivoted on the same pin but are of different lengths. For example, if there are two pawls, one of these pawls is longer than the other by an amount equal to one-half the pitch of the ratchet wheel teeth, so that the practical effect is that of reducing the pitch one-half. By placing a number of pawls side by side and proportioning their lengths according to the pitch of the teeth, a very fine feed can be obtained with a ratchet wheel of comparatively coarse pitch.

Double-action Ratchet Mechanism. — Ratchet gearing is sometimes arranged so as to impart a rotary movement to the ratchet wheel for both the forward and backward motions of a lever to which two pawls are attached. The pawl lever is fulcrumed between the two pawls; consequently a movement of the pawl lever to the left causes one pawl to rotate the wheel while the other pawl is withdrawing for engaging the next successive tooth. As the pawl lever is moved in the opposite direction, this second pawl becomes the driving member.

Shape of Ratchet Wheel Teeth. — When designing ratchet gearing, it is important to so shape the teeth that the pawl will remain in engagement when a load is applied. The faces of the teeth which engage the end of the pawl should be in such relation with the center of the pawl pivot that a line perpendicular to the face of the engaging tooth will pass somewhere between the center of the ratchet wheel and the center of the pivot about which the pawl swings. If a line perpendicular to the face of the engaging tooth is beyond the pawl pivot, any load or pressure between the wheel and pawl would tend to disengage the latter, this tendency depending upon the inclination of the teeth and the amount of load which is applied. Ratchet teeth may be either cut by a milling cutter having the correct angle, or hobbed in a gear-hobbing machine by the use of a special hob.

Pitch of Ratchet Wheel Teeth. — The pitch of ratchet wheels used for holding suspended loads may be calculated by the following formula, in which P = circular pitch, in inches, measured at the outside circumference; M = turning moment acting upon the ratchet wheel shaft, in inch-pounds; L = length of tooth face, in inches (thickness of ratchet gear); S = safe stress (for steel, 2500 pounds per square inch when subjected to shock, and 4000 pounds per square inch when not subjected to shock); N = number of teeth in ratchet wheel; F = a factor the value of which is 50 for ratchet gears with 12 teeth or less, 35 for gears having from 12 to 20 teeth, and 20 for gears having over 20 teeth:

$$P = \sqrt{\frac{FM}{LSN}}$$

This formula has been used in the calculation of ratchet gears for crane design, and will give ample safety.

BELTS AND PULLEYS

Selection of Belting. — Oak-tanned leather is usually considered the best for belting, although many high-grade belts are no longer tanned by the use of oak bark. Assuming that a good grade of leather is used, uniformity in the material is of first importance; that is, the different sections of which the belt is made should all be of the same grade. The belts should also be thoroughly stretched so that they do not have to be "taken up" every few days. The leather for the best grades of belting is taken from the central part of the hide along the back of the animal. That part of the hide extending along the spine and for some distance down the sides is firm and close in texture and the strongest for a belt. If the leather is taken too far down the side, it will be flexible and lack strength and closeness of texture. If the strips are cut too long, the ends will be taken from the neck of the animal, which is also inferior stock. A "short lap" belt is one made entirely from that part of the hide which comes from the back of the animal and the strips are not long enough to include any portion of the neck stock.

Navy Department Specifications for Leather Belting. — *Hides:* All leather belting to be made from strictly No. 1 native packer steer hides, or their equal. *Tanning:* All hides to be tanned with white or chestnut oak by the slow process (6 or 8 months); chemical processes must not be used. *Currying:* Leather to be thoroughly curried by hand, and not to be stuffed or loaded for artificial weight. Leather must not crack open on the grain side when doubled strongly by hand with the grain side out. *Cutting:* Belting to be cut from the central part of the hide, not farther than 15 inches from backbone or more than 48 inches from the tail toward the shoulder. Belts 8 inches and over must be cut to include the backbone. *Stretching:* All leather to be stretched 6 inches in the lengthwise direction of the butt, and not to exceed 54 inches after stretching. Centers and sides to be stretched 6 inches separately; that is, all the side leather from which widths under 8 inches are to be cut, must be stretched after being removed from the backbone center section; center sections to be stretched in exactly the same size for which they are used. *Laps:* For single belts up to 6 inches, laps must not exceed 6 inches, or be less than 3½ inches in length; for single belts above 6 inches, laps must not be more than 1 inch larger than width of belt. For double belts, laps must not exceed 5½ inches, nor be less than 3½ inches. No filling straps will be permitted. *Cement.* All laps to be held securely at every part, with best quality of belt cement, and when pulled apart shall show no resinous, vitreous, oily, or watery condition. Belting to be stretched again after manufacture. *Weights:* Belting to weigh for all sizes of single belts, 16 ounces to the square foot; double belts as follows: 1 to 2 inches, 26 ounces per square foot; 2½ to 4 inches, 28 ounces per square foot; 4½ to 5½ inches, 30 ounces per square foot, 6 inches and above, 32 ounces per square foot. *Rawhide Lacing Leather:* Only hand-cut green slaughter hides of very best quality to be used. Rawhide laces to be in the following sizes: ¼, 5/16, ⅜, 7/16, ½, ⅝ and ¾ inch. They must be cut lengthwise from the hide and have an average ultimate tensile strength not less than:

Width...........	¼	5/16	⅜	7/16	½	⅝	¾ inch.
Tensile strength...	95	125	155	165	180	205	230 pounds.

Application of Belts. — Whenever practicable, belts should be installed so that the slack side is above and the driving side below the pulleys. If this condition is reversed and the slack side is below, the arc of contact is materially lessened. Belts should also be placed on the pulleys with the hair or "grain" side next to the pulley rims.

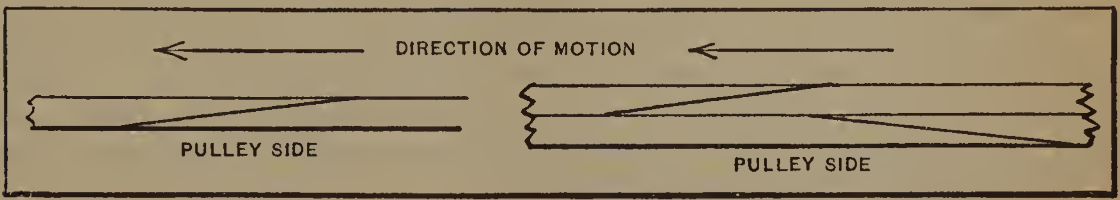
Cemented Belt Joints. — The most satisfactory method of joining the ends of a belt is by making a cemented lap joint. The belt ends should be tapered to a smooth even surface, square with the edges, and the length of the lap should vary with the belt width approximately as given below:

Belt width in inches.........	1	2	3	4	5	6	7	8
Length of lap..............	5	5	6	6	7	8	8	9

For belting varying from 9 to 18 inches in width, the length of the lap is made equal to the belt width, and for widths above 18 inches, the lap need not be greater than 18 inches. Before cementing a joint, the belt should be placed over the pulleys and the proper tension obtained by means of clamps. These clamps are fastened across the belt on each side of the joint and are drawn together by threaded rods extending along the sides. Each clamp should be square with the belt and centrally located. After drawing the belt a little tighter than is desired, to allow for slack, bevel the ends to form the lap, and apply the cement to both surfaces while hot. If the belt is large, cement a few inches at a time, then rub with a "rub stone" and hammer down the cemented section thoroughly, proceeding this way until the entire lap has been cemented. The joint should then be carefully hammered, especially along the edges. The clamps should remain in position for about an hour, or longer if convenient. Small and medium-sized belts are often joined by clamping the cemented lap between boards to which the belt has been tacked in the proper position.

The following preparation can be used for cementing leather belts: Place equal parts of glue and isinglass in a glue pot. Add enough water to cover the two ingredients and let them soak ten hours. Then bring to boiling point and add pure tannin until the mixture appears like the white of an egg. Apply the cement while warm. For rubber belts use 16 parts gutta-percha, four parts India rubber, 2 parts caulker's pitch and one part linseed oil. These ingredients should be melted together and used while hot. This cement can also be applied to leather.

Inclination of Lap-joint. — The direction that the lap-joint of a belt should incline relative to the direction of the belt's motion is shown by the accompanying illustration. For a single-ply belt, the leading end or point of the lap is on the



pulley side. The lap is inclined in this way to prevent the end from opening; when the leading end is on the outside, it tends to open up slightly, especially if the belt is operated at high speed, owing to the resistance of the air. As soon as there is a very slight opening, the atmospheric resistance tends to increase it, but when the leading end is next to the pulley, any tendency of the point to raise is overcome by frequent contact with the pulleys. To the right in the illustration is shown how the lap-joints of a double or two-ply belt should be inclined. In this case, the lap of the outer ply is in the same relation to the direction of motion as for a single belt, but the lap of the inner ply inclines in the opposite direction. With this arrangement, the leading ends of the laps in both plies will be inside and protected, and the outer ends are to the rear and not subjected to the atmospheric resistance. Opinions on this point, however, differ.

Laced Belt Joints. — When making a laced joint, cut the ends of the belt perfectly square and punch the holes exactly opposite one another in the two ends. In each end there should be two rows of staggered holes. The recommended number of holes for various widths is given in the table, "Belt Laces and Holes for Laced Joints." Begin to lace in the center of the belt and be careful to keep the belt ends exactly in line and to lace both sides with equal tension. The lacing should not be crossed on that side of the belt which runs next to the pulley.

Belt Laces and Holes for Laced Joints

Width of Belt, Inches	Width of Lace, Inches	No. of Holes	Distance of Holes from End: First Row, Inches	Distance of Holes from End: Second Row, Inches	Width of Belt, Inches	Width of Lace, Inches	No. of Holes	Distance of Holes from End: First Row, Inches	Distance of Holes from End: Second Row, Inches
1 –1¾	¼	2 or 3	⅜	...	6	⅜	9	¾	1¼
2 –2½	5/16	3	⅜	¾	8	½	11	¾	1⅜
2¾–3¼	5/16	5	½	1	10	½	13	1	1¾
3½–4½	⅜	5	⅝	1⅛	12	½	15	1	1¾
5	⅜	7	⅝	1⅛	14	½	17	1¼	2

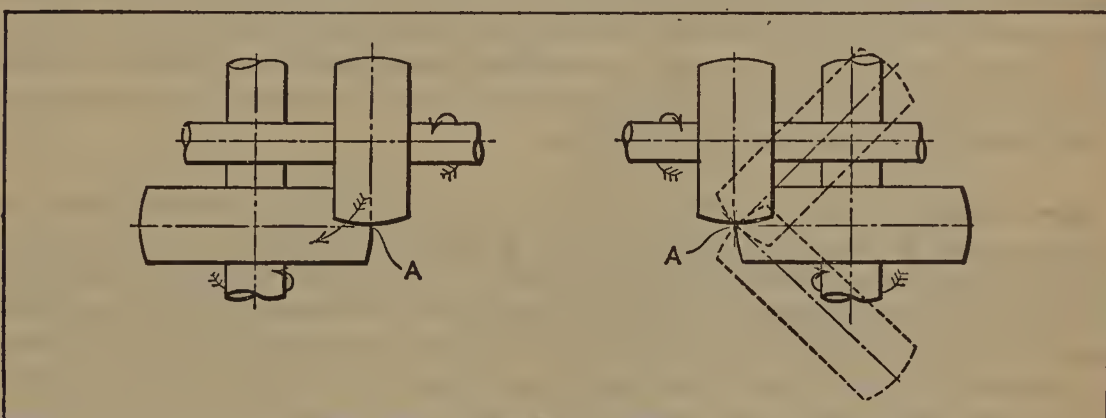
Belt Dressings. — Belts should be cleaned and greased every five or six months to give the grain side a soft adherent surface. The following mixtures are recommended: Take two parts of beef tallow to one part of cod liver oil (by weight); melt the tallow and allow it to cool until the finger can be inserted without burning; then add the cod liver oil and stir until cooled. A light coat of this mixture should be applied to the driving side of the belt after it has been cleaned. Rosin or rosinous mixtures should never be used to prevent belts from slipping. They will cause temporary adhesion, but the belt soon becomes glazed and slips more than before the rosin was applied. Lubricating oils should not be permitted to drop onto belts. If a belt has become saturated with oil, scrape it and pack it in dry sawdust or some other absorbent material for three or four days. When belting becomes dry, all surface dirt should first be removed before applying the dressing; this usually can be done by rubbing the belt with a cloth dampened with kerosene. If necessary, use a wooden or metal scraper. A dressing recommended for rubber belts consists of equal parts of red lead, black lead, French yellow and litharge, mixed with boiled linseed oil and enough japan to make it dry quickly. Animal oil or grease should never be used on rubber belts.

Thickness and Width of Belts. — Narrow, thick belts are more desirable and work more satisfactorily than wide and thin belts. According to the experiments of Mr. Fred W. Taylor, it is advisable to use double belts on pulleys 12 inches in diameter or larger; triple belts on pulleys 20 inches in diameter or larger, and quadruple belts on pulleys 30 inches in diameter or larger. If thin belts are operated at high speed, they tend to run in waves on the slack side and travel laterally, especially if there are sudden load changes. This waving and snapping wears the belt rapidly and can be practically eliminated by having the thickness in proper proportion to the width. The speed at which belting runs has comparatively little effect upon its life until the velocity is higher than 2500 to 3000 feet per minute. The life is affected principally by the power transmitted, the method of fastening the ends, and the care of the belting.

Leather-covered Pulleys. — As the friction of leather on leather is much greater than of leather on iron or steel, leather-covered pulleys will transmit considerably more power than plain uncovered pulleys for the same belt tension. Before covering a pulley, clean the surface of all grease by washing it with naphtha or gasoline. Make the cover endless and about 1/8 inch to the foot shorter than the circumference of the pulley. Then place the endless cover on the pulley for a distance of about one inch. Next cover with glue the exposed inside surface of the cover and the exposed surface of the pulley. After applying the glue, drive the cover on by lifting the pulley and striking the cover edges against the floor or bench. This should be done quickly and by striking lightly to prevent bending the cover. When the cover is in place, rub the edges with a round stick or handle to secure a good contact. A properly covered pulley does not need rivets, although it is customary to insert a few copper rivets. Pulleys should be allowed to set two or three hours before using.

Direction of Belt Creep. — Belts connecting parallel shafts tend to run toward that part of the pulley which is largest in diameter; hence pulleys are crowned to keep the belt in the center of the rim. If the shafts are not parallel and the pulleys are cylindrical, the belt will run toward the "low" side of the pulley or the side where the centers of the shafts are closest.

Angular Belt Drives. — A general rule for aligning belt pulleys connecting shafts which are not parallel is as follows: The center of the face of the *driven* pulley must be aligned with the center of that face of the *driving* pulley from which



the belt leaves, as at *A* in the illustrations. The manner in which the belt passes over the pulleys is indicated by the arrow heads showing the direction of rotation. The driven pulley can be set at any angle in relation to the driving pulley, provided it is turned about point *A*, as indicated by the dotted lines. The direction of rotation should not be reversed, unless the relative positions of the pulleys are changed in accordance with the foregoing rule.

Belt Speed and Belt Thickness. — In tests made by French engineers to determine the most favorable conditions under which belting should be run, it was found that the greatest efficiency of transmission was obtained when oak-tanned belts ran at a speed of from 65 to 80 feet per second. What is termed "chromium-treated" leather belts ran most favorably at about 100 feet per second. The most satisfactory working tension was from 575 to 850 pounds per square inch of section. The tests indicated that the thickness of the belt should be from 1/20 to 1/30 of the radius of the pulley. When chromium-treated belts are used, which are more elastic, a thickness of about 1/15 of the radius is permissible. In fact, a chromium-treated leather belt, 0.4 inch thick, gave good results on a pulley 9 3/8 inches in diameter.

Rubber Belting. — Rubber belts are used in places exposed to the weather or the action of steam, as they do not absorb moisture or stretch as readily as leather belts, under like conditions. The quality of rubber belting depends on the mixture (containing more or less rubber) that forms the coating, the cotton duck that gives strength to the belt and the method of manufacture. As to the rubber mixture, there are, in general, two kinds, one composed entirely of new rubber, and the other containing some, if not all, "re-worked" rubber. The latter is derived from discarded rubber articles such as rubber shoes, etc., and has lost much of its life. The best grades of rubber belting contain nothing but new rubber; the cheapest grades are composed largely of reclaimed rubber. The weight of the cotton duck is an important consideration. High-grade belts contain what is known as a 32-ounce cotton duck, and the cheaper grades have either a 30-ounce or 28-ounce duck. If the proper weight of duck is used, a 3- or 4-ply rubber belt is equal in strength to a single leather belt; a 5- or 6-ply rubber belt is equal to a double leather belt, and a 7- or 8-ply rubber belt is equal to a triple leather belt. A test commonly made to determine what is known as the "friction" of the belt, or the tenacity with which the different plies are held together by the rubber mixture, is as follows: The belt is cut so that the different plies of cotton duck can be pulled apart, and the amount of pull necessary to separate the plies determines the frictional value.

Canvas Belting. — Canvas stitched belting is made of several laps or plies of cotton duck stitched lengthwise and the belt is afterwards treated with a compound made principally of linseed oil. This oil saturates the cotton duck, which is thus protected from dampness, and the belt is not easily injured by heat, cold, steam, gas or acid fumes. Canvas stitched belting is often used where the material coming in contact with the belt or the surrounding atmosphere would ruin an ordinary leather, cotton or rubber belt. It is applicable to belt conveyors, when the material to be handled will not cut the cotton fiber.

Steel Belts. — Thin, flat steel belts have been used to some extent abroad, and tests are said to have demonstrated the following advantages: Steel belts can transmit much more power for the same width; they are not affected to any appreciable extent by temperature changes or changes in the humidity of the air, which adapts them for use in damp places; there is little stretching or slipping, and it is claimed that the initial cost is less than that of leather or rubber belts. The thickness of steel belts varies from 0.008 to 0.035 inch, and the width from ⅞ to 8 inches. One of the first difficulties met with in the production of steel belts was that of securing a perfectly homogeneous metal. The steel now used is of special manufacture and high tensile strength. Steel belts run fairly well on smooth cast-iron pulleys, but after a time there is a tendency to polish the surface of the pulley; hence it has been found desirable to cover the rim with thin cork, which is glued to a piece of canvas cemented to the pulley. There is practically no slip on pulleys treated in this way. Tests at the Charlottenburg Polytechnical Institute indicate that it is the best practice to have an initial tension of 10,000 pounds per square inch of belt section. Belts were run in these tests at velocities ranging from 3000 to 7000 feet per minute and the percentage of slip varied from 0.036 per cent to 2.2 per cent, the average being about 0.55 per cent. According to statements of the steel belt manufacturers, a steel belt four inches wide will transmit as much power as a leather belt 19 inches wide. The weight of the driving pulleys as compared with those for leather belting, is but one-half, and as compared with those for rope driving, approximately one-fourth. The total cost of steel belting, when installed, is said to be 60 per cent less than for leather belting, and 33 per cent less than for rope driving. Steel belts can be run at speeds as high as 10,000 feet per minute. They must be carefully installed, however, to insure that the power is evenly distributed over the full width of the steel band, as otherwise one edge might be stressed beyond its breaking strength, with the result that the entire belt would fail. The pulleys for steel belts must be cylindrical and not crowned.

Horsepower Transmitted by Belting. — The horsepower which a belt of given size should transmit, depends principally upon the speed of the belt and the working stress or pull per inch of width to which it should be subjected. There is considerable difference of opinion regarding the proper working stress, owing to the fact that experiments have not been made upon the same basis. If the problem is to determine the maximum amount of power a given belt will transmit, naturally the working stress will be higher than when the durability and the cost of repairs are considered as well as the power to be transmitted. A commonly used value for the effective pull is 35 pounds per inch of width for single belts, and from 55 to 65 pounds for double belts. Extensive tests conducted by Mr. Fred W. Taylor indicate that these values are excessive when the life of the belt and the expense incident to belt failures are considered, and that a pull of 35 pounds per inch width for oak-tanned and fulled double belts will give the most satisfactory results. The most economical speed for belting was found to be between 4000 and 4500 feet per minute. To determine the horsepower that can be transmitted by a belt of given width, let

D = diameter of driving pulley in inches;
V = velocity of belt in feet per minute;
N = number of revolutions of pulley per minute;
S = effective pull of belt per inch of width, in pounds;
W = width of belt in inches.

Then:

$$V = \frac{\pi DN}{12} = 0.2618\,DN; \qquad \text{H.P.} = \frac{SVW}{33{,}000} = \frac{0.2618\,SDNW}{33{,}000}$$

As the effective pull is an assumed quantity of uncertain value, it is, of course, not necessary to retain in the formula so exact a quantity as 0.2618. If this number is given in round figures as 0.25 or ¼, the formula is simplified as follows:

$$\text{H.P.} = \frac{SDNW}{4 \times 33{,}000}$$

Example: — What horsepower can be transmitted by a 2½-inch belt, assuming that the driving pulley is 12 inches in diameter, the speed of pulley 200 revolutions per minute, and the effective pull 33 pounds per inch of width? Inserting these values in the formula:

$$\text{H.P.} = \frac{33 \times 12 \times 200 \times 2.5}{4 \times 33{,}000} = 1.5$$

If the horsepower to be transmitted is known, the required width of belt can be found by transposing the given formula as follows:

$$W = \frac{\text{H.P.} \times 33{,}000 \times 4}{SDN}$$

A simple rule for determining approximately the horsepower that should be transmitted by rubber belting is as follows: Multiply the number of plies in the belt, its width in inches, the pulley diameter in inches, and the speed in revolutions per minute; then divide this product by 12,000. Expressing this rule as a formula:

$$\text{H.P.} = PWDN \div 12{,}000,$$

in which H.P. = horsepower; P = number of plies in the belt; W = width of belt in inches; D = diameter of pulley in inches; N = revolutions of pulley per minute.

Horsepower Transmitted by Leather Belting*

Table giving number of horsepower transmitted by belts one inch wide, considering the effects of centrifugal force, so that the tension on belt is constant at all speeds.

Speed in Feet per Minute	Thickness of Belt: Single	Double	Triple	Four-ply	Speed in Feet per Minute	Thickness of Belt: Single	Double	Triple	Four-ply
100	0.14	0.24	0.33	0.44	3400	3.89	6.74	9.10	11.96
200	0.27	0.48	0.67	0.88	3600	4.03	6.95	9.35	12.28
300	0.41	0.73	1.00	1.32	3800	4.14	7.12	9.55	12.57
400	0.54	0.96	1.33	1.75	4000	4.24	7.26	9.70	12.73
500	0.68	1.21	1.66	2.19	4200	4.33	7.36	9.79	12.84
600	0.81	1.44	1.99	2.62	4400	4.39	7.42	9.83	12.88
700	0.95	1.68	2.31	3.05	4600	4.43	7.44	9.80	12.84
800	1.08	1.93	2.64	3.48	4800	4.45	7.42	9.72	12.71
900	1.21	2.15	2.96	3.90	5000	4.45	7.37	9.56	12.50
1000	1.34	2.38	3.28	4.32	5200	4.43	7.26	9.34	12.20
1100	1.47	2.61	3.59	4.73	5400	4.38	7.10	9.05	11.80
1200	1.60	2.85	3.90	5.14	5600	4.31	6.92	8.69	11.30
1300	1.73	3.07	4.21	5.55	5800	4.21	6.65	8.25	10.70
1400	1.86	3.30	4.51	5.94	6000	4.09	6.35	7.73	10.00
1500	1.98	3.53	4.81	6.34	6200	3.94	6.01	7.13	9.19
1600	2.10	3.73	5.10	6.72	6400	3.76	5.58	6.44	8.26
1700	2.23	3.94	5.39	7.10	6600	3.56	5.11	5.67	7.22
1800	2.34	4.15	5.67	7.47	6800	3.32	4.57	4.80	6.06
1900	2.46	4.35	5.94	7.83	7000	3.05	3.98	3.84	4.77
2000	2.58	4.56	6.21	8.18	7200	2.75	3.31	2.79	3.36
2200	2.80	4.94	6.73	8.85	7400	2.42	2.60	1.64	1.82
2400	3.01	5.30	7.21	9.51	7600	2.05	1.82	0.39	0.14
2600	3.21	5.65	7.67	10.09	7800	1.65	0.95	....	....
2800	3.40	5.97	8.09	10.64	8000	1.21	....	....	....
3000	3.58	6.25	8.47	11.14	8200	0.74	....	....	....
3200	3.74	6.52	8.80	11.58	8400	0.23	....	....	....

In all the above data it is assumed that the arc of contact of the belt is not less than 180 degrees.

If this arc is...........	90°	112½°	120°	135°	150°	157½°
Divide H.P. given by..	2.21	1.72	1.6	1.4	1.24	1.17

* Note. — This table is based on an effective pull of 45 pounds per inch of width for single belts 3/16 inch thick; 80 pounds for double belts 3/8 inch thick; 110 pounds for triple belts 9/16 inch thick; and 145 pounds for 4-ply belts 3/4 inch thick. The table shows that there is no advantage in running belts faster than 4400 to 4800 feet per minute, due to the action of the centrifugal force.

Horsepower Transmitted by Leather Belting

The body of the table below gives the value of F in the equations:

$$\text{H.P.} = \frac{V \times W}{F} \quad \text{and} \quad W = \frac{\text{H.P.} \times F}{V}$$

in which H.P. = horsepower transmitted; V = belt velocity in feet per minute; W = width of belt in inches.

Example: — How wide should a single belt be in order to transmit 2 H.P. at 600 feet per minute over a 4-inch pulley with 140 degrees wrap?

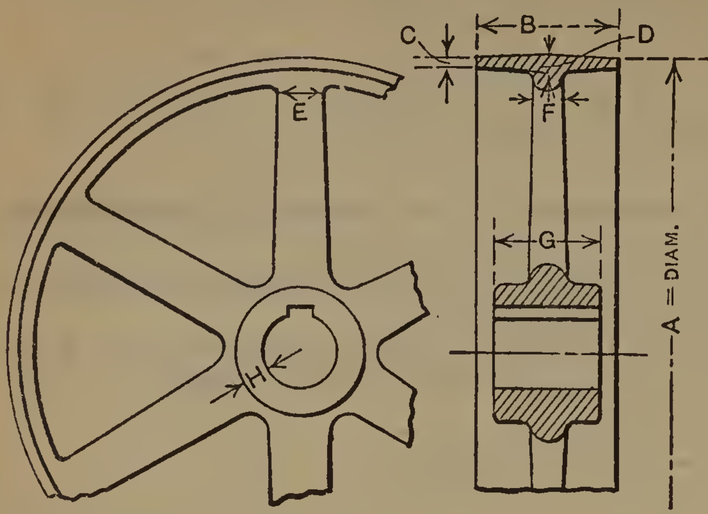
From the table we find that the value of F for the given conditions is 1270. Inserting this and the known values in the formula for belt width we have:

$$W = \frac{2 \times 1270}{600} = 4.23 \text{ inches.}$$

Belt Thickness	Diam. Small Pulley	Arc of Contact								
		210°	200°	190°	180°	170°	160°	150°	140°	130°
Single	Up to 8 inches	1010	1040	1070	1100	1140	1180	1220	1270	1330
	8 to 36 inches	830	860	890	920	950	990	1040	1100	1170
	Over 36 inches	750	770	800	830	860	890	930	980	1030
Double	Up to 14 inches	570	590	610	630	650	670	700	730	760
	14 to 60 inches	470	480	500	520	540	570	600	630	660
	Over 60 inches	430	440	450	470	490	510	530	560	590
Triple	Up to 21 inches	400	410	420	440	460	480	500	520	540
	21 to 84 inches	330	340	350	370	390	410	430	450	470
	Over 84 inches	300	310	320	330	340	360	380	400	420

The Crowning of the Face of Pulleys. — The amount of "crowning" that should be given to a pulley differs with the conditions under which it works. The amount should be greater for leather belting than for cotton belting and also greater for low speeds than for high speeds. Different authorities recommend very different amounts of crown. One recommends a crown of (or height at center) 1/20 of the width of the pulley in the case of leather belting and 1/150 of the width for cotton belting. Another recommends 1/16 to 1/8 inch per foot width of crown for high speeds and 1/4 inch for low speeds, but these figures must be modified to meet individual conditions. The crowning of a pulley tends to keep the belt on only when the belt as a whole does not slip. A slipping belt will run off a crowned-face pulley quicker than from a straight-faced one.

Dimensions of Pulleys



In all cases, the number of arms is 6. The arms increase in size towards the hub, the taper being ½ inch per foot. It is not safe to run cast-iron pulleys at a higher rim speed than 100 feet per second, and, in general, it is best to limit the speed to about 85 feet per second.

Diam. *A*	Face *B*	*C*	*D*	*E*	*F*	*G*	*H*
6	4	1/8	3/16	3/4	7/16	3	3/8
6	6	1/8	3/16	3/4	7/16	3 1/2	1/2
6	8	1/8	3/16	3/4	7/16	3 1/2	1/2
6	12	1/8	3/16	3/4	7/16	4	1/2
8	4	1/8	3/16	13/16	7/16	3	3/8
8	6	1/8	3/16	13/16	7/16	3 1/2	1/2
8	8	5/32	1/4	1 1/16	9/16	4 1/2	1/2
8	12	5/32	1/4	1 1/16	9/16	5 1/2	1/2
10	4	1/8	3/16	15/16	9/16	3	1/2
10	6	5/32	1/4	1 1/16	9/16	3 1/2	1/2
10	8	5/32	1/4	1 1/16	9/16	4 1/2	1/2
10	12	5/32	1/4	1 5/16	5/8	5 1/2	5/8
12	4	5/32	1/4	1	7/16	3 1/4	1/2
12	6	5/32	1/4	1 3/4	1/2	4	1/2
12	8	5/32	1/4	1 3/4	1/2	5	5/8
12	12	3/16	5/16	1 1/2	3/4	6 1/2	5/8
14	4	5/32	1/4	1 1/8	1/2	3 1/2	1/2
14	6	5/32	1/4	1 1/8	1/2	4 1/2	5/8
14	8	3/16	5/16	1 5/16	9/16	5	5/8
14	12	3/16	5/16	1 11/16	13/16	6 1/2	5/8
16	4	5/32	1/4	1 3/8	9/16	3 1/2	1/2
16	8	3/16	5/16	1 7/16	5/8	5	5/8
16	12	7/32	11/32	1 7/16	5/8	6 1/2	3/4
16	16	7/32	11/32	1 7/8	15/16	8 1/4	7/8
18	4	3/16	5/16	1 5/16	9/16	4	5/8
18	8	7/32	11/32	1 1/2	11/16	5 1/2	3/4
18	12	7/32	11/32	1 1/2	11/16	7 1/4	7/8
18	20	1/4	3/8	2 1/4	1 1/4	9	7/8
20	4	3/16	5/16	1 3/8	5/8	4	5/8
20	8	3/16	5/16	1 3/8	5/8	5	3/4
20	12	7/32	11/32	1 5/8	3/4	7	3/4
20	20	9/32	7/16	2 1/4	1 1/8	10	1
22	4	3/16	5/16	1 1/2	5/8	4	5/8
22	8	3/16	5/16	1 1/2	5/8	5	3/4
22	12	7/32	11/32	1 3/4	13/16	6 1/2	7/8
22	20	9/32	7/16	2 1/2	1 1/4	11	1 1/8
24	4	7/32	11/32	1 9/16	11/16	4	5/8
24	8	7/32	11/32	1 9/16	11/16	5 1/2	3/4

Dimensions of Pulleys

Diam. *A*	Face *B*	*C*	*D*	*E*	*F*	*G*	*H*
24	12	1/4	3/8	1 7/8	11/16	7	7/8
24	16	1/4	3/8	1 7/8	11/16	9 1/2	1
24	24	5/16	15/32	2 3/4	1 3/8	11	1 1/8
26	4	7/32	11/32	1 11/32	3/4	4 1/4	3/4
26	8	7/32	11/32	1 11/32	3/4	6	7/8
26	12	1/4	3/8	2	7/8	7 1/2	7/8
26	16	1/4	3/8	2	7/8	10	1 1/8
26	24	5/16	15/32	2 15/16	1 7/16	11	1 1/8
28	4	7/32	15/32	1 3/4	3/4	4 1/2	3/4
28	8	7/32	15/32	1 3/4	3/4	7	7/8
28	12	1/4	3/8	2 1/8	13/16	8	1
28	16	1/4	3/8	2 1/8	13/16	10	1
28	24	11/32	1/2	3 1/2	1 1/2	11	1 1/8
30	4	7/32	11/32	1 7/8	13/16	4 1/2	3/4
30	8	7/32	11/32	1 7/8	13/16	6 1/4	7/8
30	12	9/32	7/16	2 1/4	1	8	1
30	16	9/32	7/16	2 1/4	1	8 1/2	1
30	24	3/8	9/16	3 5/16	1 5/8	13	1 1/4
32	4	1/4	3/8	2 1/8	15/16	4 1/2	7/8
32	8	1/4	3/8	2 1/8	15/16	6 1/2	1
32	12	5/16	15/32	2 7/16	1 1/16	8	1 1/8
32	16	5/16	15/32	2 7/16	1 1/16	9 1/2	1 1/8
32	24	5/16	15/32	2 7/16	1 1/16	13	1 1/4
36	4	1/4	3/8	2 3/16	15/16	4 1/2	7/8
36	8	1/4	3/8	2 3/16	15/16	6 3/4	7/8
36	12	5/16	15/32	2 3/16	15/16	7 3/4	1 1/8
36	16	5/16	15/32	2 9/16	1 1/8	10 1/4	1 1/4
36	24	5/16	15/32	2 9/16	1 1/8	13 1/2	1 3/8
40	8	5/16	15/32	2 5/16	1	6 3/4	1
40	12	5/16	15/32	2 5/16	1	7 3/4	1 1/8
40	16	11/32	1/2	2 3/4	1 1/4	10	1 1/4
40	24	11/32	1/2	2 3/4	1 1/4	15 1/4	1 1/2
44	8	9/32	7/16	2 1/2	1 1/4	6 3/4	1 1/8
44	16	11/32	1/2	3	1 5/16	10	1 1/4
44	24	11/32	1/2	3 1/2	1 3/4	15	1 1/2
48	8	9/32	7/16	2 3/4	1 3/4	7 1/2	1 1/8
48	16	3/8	9/16	2 3/4	1 7/16	10	1 3/8
48	24	3/8	9/16	3 1/4	1 7/16	15	1 1/2
54	12	5/16	15/32	3	1 5/16	9 3/4	1 3/8
54	16	5/16	15/32	3	1 5/16	11 1/4	1 1/2
54	24	13/32	19/32	3 5/8	1 5/8	15	1 3/4
60	12	11/32	1/2	3 5/16	1 7/16	10	1 3/8
60	16	11/32	1/2	3 5/16	1 7/16	11 1/4	1 1/2
60	24	7/16	5/8	3 13/16	1 3/4	15	1 3/4
66	12	11/32	1/2	3 9/16	1 9/16	10	1 1/2
66	16	11/32	1/2	3 9/16	1 9/16	11 1/2	1 5/8
66	24	1/2	3/4	4 1/4	1 15/16	15	1 7/8
72	12	3/8	9/16	3 7/8	1 11/16	10 1/2	1 5/8
72	16	3/8	9/16	3 7/8	1 11/16	12 1/2	1 3/4
72	24	9/16	13/16	4 5/8	2 1/16	15	2

Cast-iron Pulleys. — Cast-iron pulleys formed of one solid casting may or may not have a split or divided hub. The solid-hub pulley is held to its shaft either by a key, a key and one or two set-screws, or by simply using one or more set-screws without a key as in the case of small pulleys, especially on low-grade machinery where there is little power to transmit. When the hub is split or divided, it is provided with clamping bolts, and when these are tightened, the split hub grips the shaft tightly. In addition to clamping bolts, a key or a key and set-screws may be used. Pulleys of this kind are known as the clamp-hub type.

Most pulleys have six arms. For diameters less than 15 or 20 inches, there may be four arms, and pulleys 5 feet or larger in diameter often have eight arms.

Split Cast-iron Pulleys. — The split pulley which is formed of two separate sections bolted together both at the hub and on opposite sides of the rim, can be placed between other pulleys on a shaft without removing either the pulleys or the shaft. These pulleys often have interchangeable hub bushings to fit shafts of different diameter. It is good practice to make pulleys having a face width of 10 inches or over, either of the clamp-hub or split form, because shrinkage strains are either greatly reduced or practically eliminated, and the hub of the pulley can be firmly clamped to a shaft even though the bore is not an accurate fit. If the face width of a cast-iron pulley is greater than from 20 to 24 inches, there should be two sets of arms to provide better support for the rim.

Wood Pulleys. — Wood pulleys are not only much lighter than cast-iron pulleys but they are superior as transmitters of power; in fact it is claimed that they will transmit from 35 to 50 per cent more power for the same belt tension. Wood pulleys should not be used where they are exposed to excessive moisture. Ordinarily the rims are built up of segments, and the arrangement of the arms varies on different sizes and makes. Some wood pulleys intended for unusually severe duty have a rim which is joined to an iron center or hub by a solid web of wood. Other pulleys of the iron-center type have cast-iron hubs and arms and a wood rim. Internal shrinkage strains are thus eliminated and the pulleys are adapted to unusually high speeds. Well-seasoned maple is adapted to wood pulleys. Wood bushings are often inserted in the hubs to permit using the pulleys on shafts of different size.

Steel Pulleys. — Pulleys formed of sheet steel combine lightness with strength and they are free from the initial stresses which are such an uncertain factor in many cast-iron pulleys. The weight is ordinarily from 45 to 55 per cent less than the weight of a cast-iron pulley of equal power-transmitting capacity, which lessens the weight on the lineshaft and reduces the frictional losses. A series of tests showed that the percentage of slip was from 2.35 to 2.70 per cent less for steel pulleys than for cast-iron pulleys Steel pulleys are ordinarily of the split type.

Safe Speeds for Pulleys. — The maximum safe rim speeds for solid cast-iron pulleys is as a general rule about 5000 feet per minute. If the pulley is split or formed of separate sections which are bolted together at the rim, the maximum speed should be limited to about 55 or 60 per cent of the maximum speed for solid pulleys. While the safe speeds of built-up steel pulleys are subject to some variation on account of differences in design or construction, in general such pulleys may be run at about 6000 feet per minute. The safe speeds recommended for wood pulleys vary considerably according to the type; thus, the maximum speeds recommended may be 5000 feet per minute for some pulleys and 10,000 feet per minute for others of different construction. A pulley having a cast-iron hub and arms, with a wood rim, has been operated under test at a rim speed of five and one-half miles per minute. For additional information on speeds see "Safe Speeds for Flywheels."

Belt Velocity or Circumferential Speed of Pulleys

Pulley Diam. in Inches	Revolutions per Minute																				
	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	250	260	280	300
	Velocity in Feet per Minute																				
6	110	126	141	157	173	188	204	220	235	251	267	282	298	314	330	346	361	392	408	440	471
7	128	146	165	183	201	220	238	256	275	293	312	330	348	367	385	403	421	458	477	513	550
8	146	167	188	210	230	251	272	293	314	335	356	377	398	419	440	461	481	523	545	586	628
9	165	188	212	236	259	282	306	330	353	377	400	424	447	471	495	518	542	588	613	660	707
10	183	209	235	262	288	314	340	366	392	419	445	471	497	524	549	576	602	654	681	733	785
12	220	252	282	314	346	377	408	440	471	502	534	565	597	628	659	691	722	785	817	880	942
14	256	293	330	366	403	440	476	513	550	586	623	659	696	733	769	806	843	916	953	1026	1100
16	293	335	377	419	460	502	544	586	628	670	712	754	796	838	879	921	963	1046	1089	1173	1257
18	330	377	424	471	518	565	612	659	707	754	801	848	895	942	989	1037	1084	1178	1225	1319	1414
20	366	419	471	524	576	628	681	733	785	838	890	942	995	1047	1099	1152	1204	1309	1361	1466	1571
22	403	460	518	576	634	691	749	806	864	921	979	1037	1094	1152	1209	1267	1325	1440	1497	1612	1728
24	440	502	565	628	691	754	817	880	942	1005	1068	1131	1194	1257	1319	1382	1445	1571	1633	1759	1885
26	476	545	612	681	749	817	885	953	1021	1089	1157	1225	1293	1361	1429	1497	1565	1701	1770	1906	2042
28	513	586	659	733	806	880	953	1026	1100	1173	1246	1319	1393	1466	1539	1613	1686	1832	1906	2052	2199
30	550	628	706	785	864	942	1022	1100	1178	1256	1335	1413	1492	1571	1649	1728	1806	1963	2042	2199	2356
32	586	670	754	838	921	1005	1089	1173	1257	1340	1424	1508	1592	1675	1759	1843	1927	2094	2178	2345	2513
34	623	712	801	890	979	1068	1157	1246	1335	1424	1513	1602	1691	1780	1869	1958	2047	2225	2314	2492	2670
36	659	754	848	942	1037	1131	1225	1319	1414	1508	1602	1696	1791	1885	1978	2073	2168	2356	2450	2639	2827
40	733	837	942	1047	1152	1256	1361	1466	1571	1675	1780	1885	1989	2094	2199	2304	2408	2618	2723	2932	3141
48	879	1005	1131	1257	1382	1508	1633	1759	1885	2010	2136	2262	2387	2513	2639	2765	2890	3142	3267	3518	3769
54	989	1131	1272	1414	1555	1696	1838	1979	2120	2262	2403	2545	2686	2827	2969	3110	3251	3534	3676	3959	4240
60	1099	1256	1414	1571	1728	1885	2042	2199	2356	2513	2670	2827	2984	3141	3298	3456	3613	3927	4084	4398	4712
66	1209	1382	1550	1728	1900	2073	2246	2419	2592	2764	2937	3110	3283	3455	3628	3801	3974	4319	4492	4838	5183
72	1319	1508	1696	1885	2073	2262	2450	2639	2827	3016	3204	3392	3581	3770	3958	4147	4335	4713	4900	5278	5654
78	1429	1633	1838	2042	2245	2450	2655	2859	3063	3267	3472	3676	3880	4084	4288	4492	4696	5105	5309	5717	6125
84	1539	1754	1978	2199	2419	2639	2859	3079	3298	3518	3738	3958	4178	4398	4618	4838	5058	5497	5717	6157	6597

Rules for Calculating Diameters and Speeds of Pulleys

Speed of Driven Pulley Required. — Diameter and speed of driving pulley, and diameter of driven pulley are known. *Rule:* Multiply the diameter of the driving pulley by its speed in revolutions per minute, and divide the product by the diameter of the driven pulley.

Example: — If the diameter of the driving pulley is 15 inches and its speed, 180 revolutions per minute, and the diameter of the driven pulley, 9 inches, then the speed of the driven pulley $= \frac{15 \times 180}{9} = 300$ revolutions per minute.

Diameter of Driven Pulley Required. — Diameter and speed of driving pulley, and revolutions per minute of driven pulley are known. *Rule:* Multiply the diameter of the driving pulley by its speed in revolutions per minute, and divide the product by the required speed of the driven pulley.

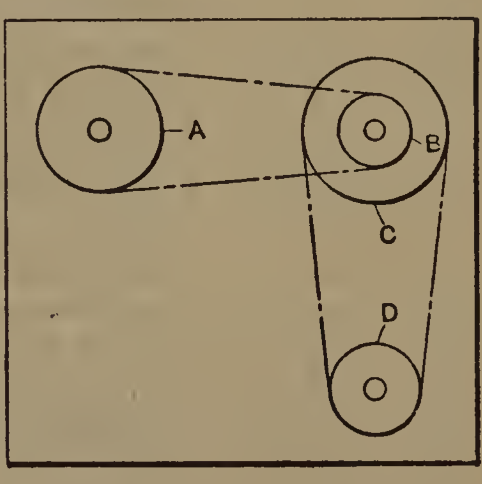
Example: — If the diameter of the driving pulley is 24 inches and its speed, 100 revolutions per minute, and the driven pulley is to rotate 600 revolutions per minute, then the diameter of the driven pulley $= \frac{24 \times 100}{600} = 4$ inches.

Diameter of Driving Pulley Required. — Diameter and speed of driven pulley, and speed of driving pulley are known. *Rule:* Multiply the diameter of the driven pulley by its speed in revolutions per minute, and divide the product by the speed of the driving pulley.

Example: — If the diameter of the driven pulley is 36 inches and its required speed, 150 revolutions per minute, and the speed of the driving pulley is 600 revolutions per minute, then the diameter of the driving pulley $= \frac{36 \times 150}{600} = 9$ inches.

Speed of Driving Pulley Required. — Diameters of driving and driven pulleys, and speed of driven pulley are known. *Rule:* Multiply the diameter of the driven pulley by its speed, and divide the product by the diameter of the driving pulley.

Example: — If the diameter of driven pulley is 4 inches, its required speed, 800 revolutions per minute, and the diameter of the driver, 26 inches, then the required speed of the driver $= \frac{4 \times 800}{26} = 123$ revolutions per minute, approximately.



Speed of Driven Pulley in Compound Drive Required. — Diameters of pulleys *A*, *B*, *C* and *D* (see illustration), and speed of pulley *A* are known; find speed of pulley *D*. *Rule:* Divide product of diameters of driving pulleys by product of diameters of driven pulleys, and multiply quotient by speed of first driving pulley.

Example: — If the diameters of the driving pulleys *A* and *C* are 18 and 24 inches; the diameters of the driven pulleys *B* and *D*, 12 and 13 inches; and the speed of the driver *A*, 260 revolutions per minute; then the speed of the driven pulley $D = \frac{18 \times 24}{12 \times 13} \times 260 = 720$ revolutions per minute.

Pulley Diameters in Compound Drive Required. — Speeds of driving and driven pulleys are known; find diameters of the four pulleys A, B, C and D. *Rule:* Place the speed of the driving pulley as the numerator of a fraction, and the speed of driven pulley as the denominator, and reduce this fraction to its lowest terms; then resolve both the numerator and denominator into two factors, and multiply each "pair" of factors (a pair being one factor in the numerator and one in the denominator) by a trial number which will give pulleys of suitable diameters.

Example: — If the speed of pulley A is 260 revolutions per minute, and the required speed of pulley D is 720 revolutions per minute, find the diameters of the four pulleys The fraction $\frac{260}{720}$ reduced to its lowest terms is $\frac{13}{36}$, which represents the required speed ratio. Resolve $\frac{13}{36}$ into two factors; $\frac{13}{36} = \frac{1 \times 13}{2 \times 18}$. Multiply by trial numbers 12 and 1:

$$\frac{(1 \times 12) \times (13 \times 1)}{(2 \times 12) \times (18 \times 1)} = \frac{12 \times 13}{24 \times 18}$$

The values 12 and 13 in the numerator represent the diameters of the *driven* pulleys B and D and values 24 and 18 in the denominator, the diameters of the *driving* pulleys.

Lengths of Open and Crossed Belts. — The following formulas for determining the lengths of belts on pulleys are accurate enough for practical purposes. No allowance is made for a lap joint. In these formulas, L = length of open belt; L_c = length of cross belt; D, C and R = diameter, circumference and radius, respectively, of large pulley; d, c and r = diameter, circumference and radius, respectively, of small pulley; x = center-to-center distance.

$$L = \frac{D+d}{2} \times 3\tfrac{1}{4} + 2x; \quad L_c = \frac{C}{2} + \frac{c}{2} + 2\sqrt{x^2 + (R+r)^2}$$

Rules for Calculating Speeds of Gearing

The relative speeds of shafts connected by spur or bevel gearing can be determined by the foregoing rules for pulley-and-belt drives, provided the *pitch diameter* or the number of teeth in the gear is substituted for the pulley diameter, in each case, as shown by the following examples:

Speed of Driven Gear Required. — Number of teeth in driving gear, its speed, and number of teeth in driven gear are known. *Rule:* Multiply the number of teeth in the driving gear by its speed in revolutions per minute, and divide by the number of teeth in the driven gear.

Example: — If the driving gear has 20 teeth and rotates 80 revolutions per minute, and the driven gear has 40 teeth, then the speed of the driven gear $= \frac{20 \times 80}{40}$ = 40 revolutions per minute.

If one or more intermediate gears are placed in a direct train between the driving and driven gears, the speed ratio will remain the same.

Pitch Diameter of Driven Gear Required. — The pitch diameter of the driving gear, its speed, and speed required for driven gear are known. *Rule:* Multiply the pitch diameter of the driving gear by its speed in revolutions per minute, and divide by the required speed of the driven gear.

Example: — If the pitch diameter of the driver is 8 inches, its speed, 75 revolutions per minute, and the speed required for the driven gear, 20 revolutions per minute, then the pitch diameter of the driven gear $= \frac{8 \times 75}{20}$ = 30 inches.

Machine Tool Drives

Machine Tool Drives with Speeds in Geometrical Ratio. — When designing a machine tool drive, the speeds obtainable on the machine should form a geometrical progression. Let n be the total number of required speeds; a, the slowest speed, in number of revolutions per minute; b, the fastest speed, in number of revolutions per minute; and r, the ratio of the geometrical progression, or the factor with which to multiply any speed to get the next higher speed. If the maximum and minimum speeds and the number of speed changes are known, the ratio of the progression may be found by the formula:

$$r = \sqrt[n-1]{\frac{b}{a}}$$

In most cases, this formula requires logarithms for its solution and can be written:

$$\text{Log } r = \frac{\log b - \log a}{n - 1}$$

As an example, assume that a drive is to be designed to give a range of 18 spindle speeds from 10 to 223 revolutions per minute. By means of the formula, ratio r is found to be 1.20, and by continued multiplication, the speeds are found to be:

10	12	14.4	17.25	20.7	24.85
29.8	35.8	43	51.6	62	74.4
89.4	107	129	155	186	223

Assume the use of a three-step cone, double back-gears, and two countershaft speeds. There are two methods of arranging the countershaft speeds; first, by shifting the machine belt over the entire range of the cone before changing the countershaft speed; and second, by changing the countershaft speed after each shifting of the machine belt. The design of the cone will be considerably different, according to which method is used. In the first case, there will be very small differences in the diameters of the steps, while in the second case these differences will be large, producing a cone with a steep incline. For the first arrangement, the following table may be made:

Cone	Open Belt		Small Ratio Back Gears in		Large Ratio Back Gears in	
	Fast Counter	Slow Counter	Fast Counter	Slow Counter	Fast Counter	Slow Counter
Step 1......	223	129.0	74.4	43.0	24.85	14.4
Step 2......	186	107.0	62.0	35.8	20.70	12.0
Step 3......	155	89.4	51.6	29.8	17.25	10.0
	1	2	3	4	5	6

From the above table, the ratio of the two sets of back-gears, the countershaft speeds, and the speeds off each step of the cone may be obtained. The ratio of the large ratio back-gears is found by dividing a term in column 2 by a corresponding term in column 6. The ratio of the small ratio gears is found by dividing a term in column 2 by a corresponding term in column 4. The ratio of countershaft speeds is obtained by dividing a term in column 5 by a corresponding term in column 6, and the ratio of speeds off each step of the cone, by dividing the term corresponding to step 1 in any column by a term corresponding to steps 2 or 3, as desired, in the same column. The results in the example given would be:

Ratio of large ratio gears is	8.94 to 1
Ratio of small ratio gears is	2.98 to 1
Ratio of countershaft speeds is	1.725 to 1
Ratio of speeds off step 1 to those off step 2	1.2 to 1
Ratio of speeds off step 1 to those off step 3	1.44 to 1

The diameter of the largest step of the cone is assumed to be 15 inches. The ratio of the speeds off step 1 and step 3 is 1.44 to 1; with equal pulleys on countershaft and on machine, this ratio also equals $(D \times D) \div (d \times d)$, where D is the diameter of the largest step and d is the diameter of the smallest step. Hence, $D^2 \div d^2 = 1.44$, and when D is 15 inches, $d = 12.5$ inches.

In the second case, where the countershaft speed is changed after each shifting of the machine belt, the speeds may be tabulated as follows:

Cone	Open Belt		Small Ratio Gears in		Large Ratio Gears in	
	Fast Counter Speed	Slow Counter Speed	Fast Counter Speed	Slow Counter Speed	Fast Counter Speed	Slow Counter Speed
Step 1	223	186.0	74.4	62.0	24.85	20.7
Step 2	155	129.0	51.6	43.0	17.25	14.4
Step 3	107	89.4	35.8	29.8	12.00	10.0
	1	2	3	4	5	6

The various ratios are:

Large ratio gears	8.94 to 1
Small ratio gears	2.98 to 1
Countershaft speeds	1.2 to 1
Speeds off step 1 to those off step 2	1.44 to 1
Speeds off step 1 to those off step 3	2.07 to 1

In this case, if the large step is 15 inches in diameter, the smallest step will be 10.4 inches in diameter.

Ratio of Speed Changes for Machine Tools. — It is of little practical advantage to reduce the speed ratio below 1.2, and in the case of machine tools of ordinary type, a ratio of 1.3 is as small as is advisable. On the other hand, it is inadvisable to let the speed ratio be greater than 1.5, except in the case of cheap machinery, when ratios up to 1.7 may be permissible. In other words, if the lowest speed is 20 revolutions per minute, the next speed for ordinary machine tools should, as a rule, not be less than $20 \times 1.3 = 26$ revolutions per minute, and not more than $20 \times 1.5 = 30$ revolutions per minute. Succeeding speeds are found by multiplying each previous speed by the same factor.

The ratio between successive feeds should always be less than 1.3, and in the case of high-class machinery a value of 1.2 or less is preferable.

Table of Geometrical Progression. — The table of geometrical progression will be found useful in calculating speed ranges for machine tools. This table gives the values of consecutive terms in a geometrical progression when the increase of successive terms in per cent varies from 10 to 100, or when the ratio varies from 1.10 to 2. The columns 2, 3, 4, etc., give the values of the respective terms in the progression, when the first term is 1. For example, if the speed ratio is 1.15 and the first term in the progression is 1, then the eighth term in this geometrical progression would be 2.66. If the first term is any other value than 1, the value of the other terms may be found by multiplying the various tabulated values by the value of the first term. For example, if the speed ratio is 1.15 and the first speed is 20 revolutions per minute, then the eighth speed would be $20 \times 2.66 = 53.2$ revolutions per minute.

Table for Simplifying the Calculation of Cone Drive and Back-gear Designs. — The table "Geometrical Progressions for Spindle Speeds" gives the consecutive speeds in a properly arranged machine tool drive for ratios more finely subdivided than in the case of the previous table. The use of the table will be best shown by a practical example. Assume that the spindle of a lathe requires 18 speed changes, varying from 6 to 250 R.P.M., that the cone has three steps, the largest step being 15 inches in diameter, that the lathe is double back-geared, and that a two-speed countershaft is provided. The questions to be answered are then: What are the intermediate speeds between 6 and 250 R.P.M.? What are the diameters of the two remaining cone steps? What are the back-gear ratios and what should be the countershaft speeds?

In the table the maximum speed in every case is given as 1000, which, in the present case is four times greater than the maximum speed of the spindle. In order to reduce the figures given in the table to correspond with those of this example, divide the speeds given in the table by 4; $(1000 \div 4 = 250)$; the slowest speed, given as 6 R.P.M., will then correspond to a speed given in the table equal to 24 R.P.M.; $(24 \div 4 = 6)$. It will be seen that 24 and 1000 are in exactly the same ratio as 6 and 250. The number of spindle speeds being 18, follow the horizontal line from the figure 18 in the left-hand column of the table, until reaching the number nearest to 24, the number in this case being 22.5 in the 20 per cent column. The figures in this column, divided by 4, will give the range of the speeds desired, these speeds being as follows:

Range of Speeds

250	128	65.5	33.5	17.17	8.8
200	102.5	52.5	26.75	13.7	7.04
160	82	42	21.5	11	5.62

The speed ratio for the first back-gears is obtained by dividing 250, the fastest open belt speed, by 65.5, the fastest first back-gear speed; and the second back-gear ratio is found by dividing 250, the fastest open belt speed, by 17.17, the fastest second back-gear speed. These two ratios are then found to be 3.82 to 1, and 14.6 to 1, respectively. The countershaft speeds will be found to be 200 and 102.5 (the speeds of the middle cone steps), if consecutive spindle speeds are obtained by moving the belt from one step on the cone to another; but if consecutive speeds are obtained by shifting the countershaft, then this latter would be required to run at 160 and 128 R.P.M., which would then be the speeds of the middle cone steps. The diameter of the smallest cone step, if consecutive speeds are obtained by changing the countershaft speed, will equal $\frac{15 \times 160}{250} = 9.6$ inches. The diameter of the middle step would then be 12.3 inches approximately.

Table of Geometrical Progression

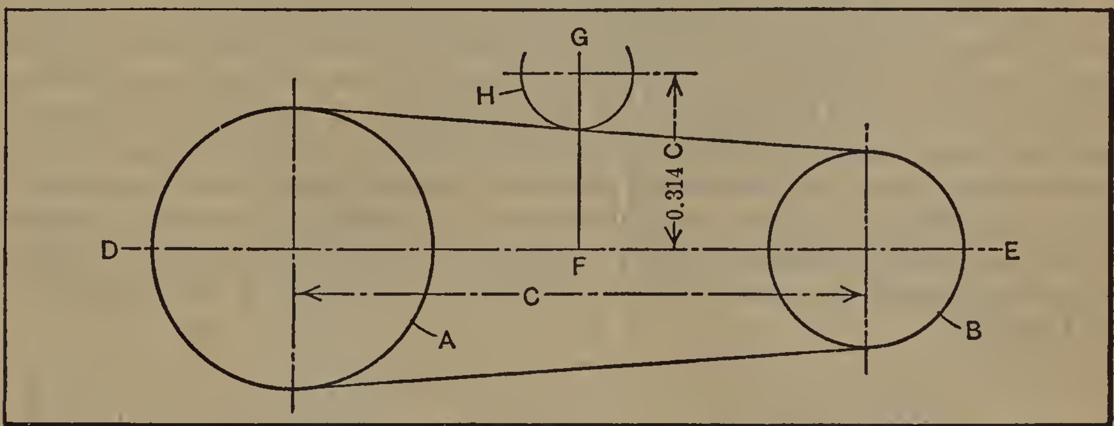
Ratio	Increase in Per cent	1	2	3	4	5	6	7	8	9	10	11	12
1.10	10	1	1.10	1.21	1.33	1.46	1.61	1.77	1.95	2.14	2.36	2.59	2.85
1.15	15	1	1.15	1.32	1.52	1.75	2.01	2.31	2.66	3.06	3.52	4.05	4.65
1.20	20	1	1.20	1.44	1.73	2.07	2.49	2.99	3.58	4.30	5.16	6.19	7.43
1.25	25	1	1.25	1.56	1.95	2.44	3.05	3.81	4.77	5.96	7.45	9.31	11.64
1.30	30	1	1.30	1.69	2.20	2.86	3.71	4.83	6.27	8.16	10.61	13.79	17.92
1.35	35	1	1.35	1.82	2.46	3.32	4.48	6.05	8.17	11.03	14.88	20.09	27.14
1.40	40	1	1.40	1.96	2.74	3.84	5.38	7.53	10.54	14.76	20.66	28.93	40.50
1.45	45	1	1.45	2.10	3.05	4.42	6.41	9.29	13.48	19.54	28.33	41.09	59.57
1.50	50	1	1.50	2.25	3.38	5.06	7.59	11.39	17.09	25.63	38.44	57.67	86.50
1.55	55	1	1.55	2.40	3.72	5.77	8.95	13.87	21.49	33.32	51.64	80.04	124.10
1.60	60	1	1.60	2.56	4.10	6.55	10.49	16.78	26.84	42.95	68.72	110.00	....
1.65	65	1	1.65	2.72	4.49	7.41	12.23	20.18	33.30	54.94	90.65	149.60	....
1.70	70	1	1.70	2.89	4.91	8.35	14.20	24.14	41.03	69.75	118.60	....	....
1.75	75	1	1.75	3.06	5.36	9.38	16.41	28.72	50.27	87.96	153.90	....	....
1.80	80	1	1.80	3.24	5.83	10.50	18.90	34.01	61.22	110.20	....	....	....
1.85	85	1	1.85	3.42	6.33	11.71	21.67	40.09	74.17	137.20	....	....	....
1.90	90	1	1.90	3.61	6.86	13.03	24.76	47.05	89.39	169.80	....	....	....
1.95	95	1	1.95	3.80	7.41	14.46	28.20	54.98	107.20	....	....	....	....
2.00	100	1	2.00	4.00	8.00	16.00	32.00	64.00	128.00	....	....	....	....

Ratio	Per cent	13	14	15	16	17	18	19	20	21	22	23	24
1.10	10	3.14	3.45	3.80	4.18	4.60	5.05	5.56	6.11	6.73	7.40	8.14	8.95
1.15	15	5.35	6.15	7.08	8.14	9.36	10.76	12.38	14.23	16.37	18.82	21.65	24.89
1.20	20	8.92	10.70	12.84	15.41	18.49	22.19	26.62	31.95	38.34	46.01	55.21	66.25
1.25	25	14.55	18.19	22.74	28.42	35.53	44.41	55.51	69.39	86.74	108.40	....	....
1.30	30	23.30	30.29	39.37	51.19	66.54	86.50	112.50	....	....	....	....	....
1.35	35	36.64	49.47	66.78	90.37	121.70	....	....	....	....	....	....	....
1.40	40	56.69	79.37	111.10	....	....	....	....	....	....	....	....	....
1.45	45	86.38	125.30	....	....	....	....	....	....	....	....	....	....
1.50	50	129.80	....	....	....	....	....	....	....	....	....	....	....

Geometrical Progressions for Spindle Speeds. (See page 770.)

Number of Speed Changes Required	Percentage of Decrease of Consecutive Speeds																			
	15	16	17	18	19	20	22	24	25	26	28	30	32	34	36	38	40	44	48	50
1	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
2	850	840	830	820	810	800	780	760	750	740	720	700	680	660	640	620	600	560	520	500
3	722	706	689	672	656	640	608	578	562	548	518	490	462	436	410	385	360	314	270	250
4	614	593	572	551	531	512	474	439	421	405	373	343	315	288	263	238	216	176	141	125
5	522	498	474	452	430	410	370	333	316	300	269	240	214	190	169	148	129	98	73.3	62.5
6	444	418	394	371	349	328	289	253	237	222	193	168	146	126	108	91.7	77.5	54.8	38.2	31.2
7	377	351	327	304	282	262	225	193	178	163	139	118	99	83	69.2	56.8	46.5	30.7	19.9	15.6
8	320	295	271	249	229	210	176	146	133	121	100	82.3	68.3	54.8	44.4	35.2	27.9	17.2	10.3	7.81
9	272	248	225	204	185	168	137	111	100	89.4	72.2	57.6	46.5	36.2	28.4	21.8	16.8	9.6	5.37	3.91
10	231	208	187	167	150	134	107	84.5	75	66.1	52.4	40.3	31.5	23.8	18.2	13.5	10.0	5.38	2.8	1.95
11	197	175	155	137	121	107	83.3	64.2	56.2	48.9	37.4	28.2	21.4	15.8	11.63	8.38	6.0	3.0	1.45	0.97
12	167	147	128	113	98.4	86	65	48.8	42.2	36.2	26.9	19.8	14.5	10.4	7.46	5.2	3.6	1.68	0.75	0.47
13	142	123	107	92.4	79.7	68.7	50.7	37.1	31.6	26.8	19.4	13.8	9.85	6.9	4.78	3.22	2.16	0.94	....	....
14	121	103	88.5	75.7	64.5	55	39.5	28.2	23.7	19.8	13.9	9.7	6.7	4.55	3.06	2.0	1.29	....	....	....
15	103	87	73.4	62.1	52.3	44	30.8	21.4	17.8	14.6	10	6.8	4.55	3.0	1.96	1.24	0.77	....	....	....
16	87.3	73	60.9	50.9	42.3	35.2	24	16.3	13.3	10.8	7.23	4.7	3.09	1.99	1.25	0.77	....	....	....	....
17	74.2	61.3	50.5	41.7	34.2	28.16	18.7	12.4	10	8.0	5.21	3.32	2.1	1.32	0.8	....	....	....	....	....
18	63	51.5	41.5	34.2	27.7	22.5	14.6	9.4	7.5	5.9	3.75	2.32	1.43	0.87	....	....	....	....	....	....
19	53.6	43.2	34.8	28	22.4	18	11.4	7.14	5.6	4.4	2.7	1.63	0.97	....	....	....	....	....	....	....
20	45.5	36.3	28.9	23	18.1	14.9	8.9	5.42	4.2	3.25	1.94	1.14	....	....	....	....	....	....	....	....
21	38.7	30.5	24	18.9	14.7	11.5	6.94	4.12	3.2	2.4	1.39	....	....	....	....	....	....	....	....	....
22	32.9	25.6	19.9	15.5	11.9	9.2	5.4	3.13	2.4	1.78	1.00	....	....	....	....	....	....	....	....	....
23	27.9	21.5	16.5	12.7	9.6	7.37	4.22	2.38	1.8	1.31	....	....	....	....	....	....	....	....	....	....
24	23.7	18	13.7	10.4	7.8	5.9	3.29	1.81	1.3	0.97	....	....	....	....	....	....	....	....	....	....

Cone Pulley Design. — When designing a pair of cone pulleys for belt power transmission, it is not possible to merely design the two pulleys with equal differences between the steps, because the length of belt required on the largest and smallest steps would be different from the length required on the two middle steps. If a crossed belt is used, all that is necessary is that the sum of the diameters of any pair of steps shall be equal to the sum of the diameters of any other pair of steps, but when an open belt is used, as is usually the case, the sum of the diameters of the steps at or near the middle of the cones must be somewhat greater than the sum of the diameters of the steps at or near the ends. The following method (Transactions of the American Society of Mechanical Engineers, Volume X, page 269) will be found a close approximation for determining the sizes of the steps of cone pulleys:



Let distance C be the distance between the centers of the shafts (see illustration) and draw circles A and B to represent a known pair of steps on the cones; at a point midway between the shaft centers erect a perpendicular FG. Then with a center on FG located at a distance from center-line DE equal to $0.314 \times C$, draw a circular arc H tangent to the belt line of the given pair of steps. The belt line of any other pair of steps should also be tangent to this arc.

Should the angle which the belt makes with the center-line DE exceed 18 degrees, a slight modification is made as follows: Draw a line tangent to the arc H at an angle of 18 degrees with DE, and with a center on FG, located at a distance from line DE equal to $0.298 \times C$, draw an arc tangent to this 18-degree line. All belt lines which make an angle with DE greater than 18 degrees should be made tangent to this new arc.

In all cases, 0.8 of the thickness of the belt should be subtracted from the diameters of the pulleys found by the graphical method, in order to obtain the actual diameter of the pulley. This is done because the length of the belt drawn tight around the pulleys is longer than the length of a tape line measured around them.

Length of Belt on Pulleys. — The following formula is given by Rankin for the length of belt required to pass over two pulleys:

$$\text{Length of belt} = 2\,C + \frac{11\,D + 11\,d}{7} + \frac{(D-d)^2}{4\,C}$$

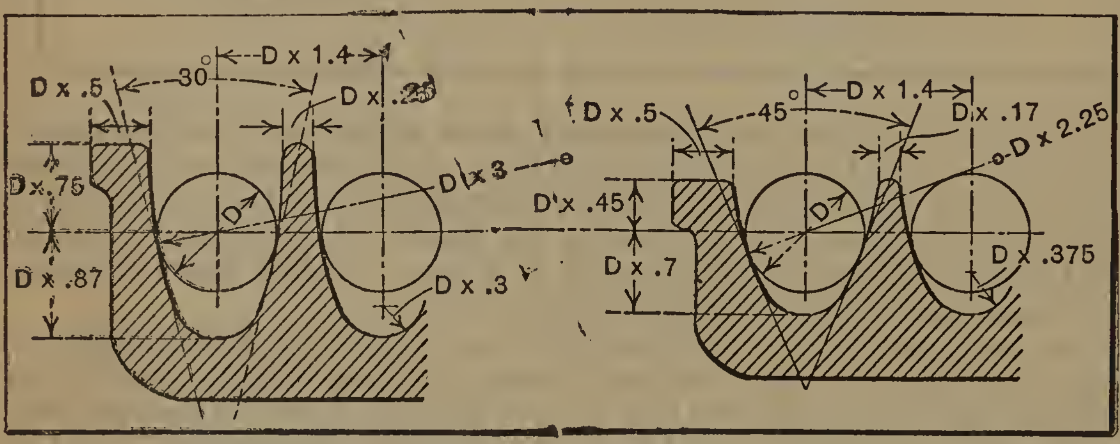
in which C = center distance between pulleys;
D = diameter of large pulley;
d = diameter of small pulley.

This formula, while not theoretically correct, is accurate enough for all practical purposes, and may be used as a check for cone pulley diameters; by means of this formula it can quickly be determined if the belt length is approximately correct for the various steps, as determined by a graphical method.

ROPE TRANSMISSION

American System of Rope Transmission. — In the continuous or American system of rope driving, one long rope is wound around the driving and driven pulleys as many tines as there are grooves to fill; the rope is then conducted from the last groove back to the first one, by means of an idler pulley held at the required angle. This idler is mounted on an adjustable carriage to which is attached a weight for taking up the slack of the rope in order to secure the necessary frictional contact in the pulley grooves. The continuous system is adapted to either vertical or angular drives.

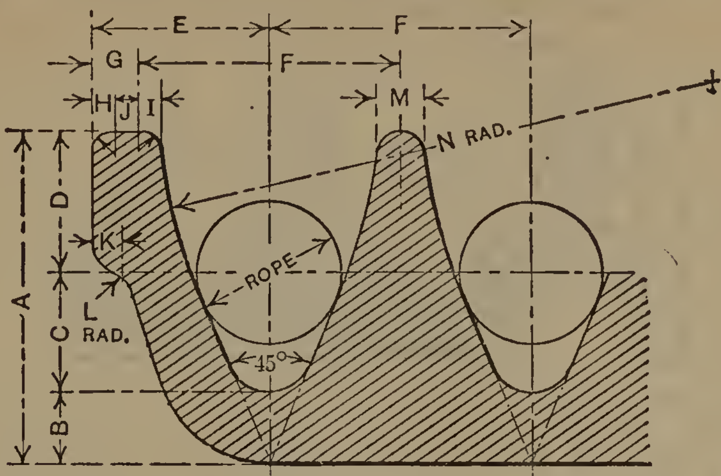
Multiple System of Rope Transmission. — The English or multiple system differs from the American system in that a number of parallel ropes, one in each groove in the pulleys, are used instead of a single rope wound continuously. The required tension is obtained by the weight of the ropes, no tension carriage being employed; hence, in the multiple system, the driving and driven sheaves should be located far enough apart to obtain the required tension. One advantage of the multiple system is that in case one of the ropes breaks, the drive can be operated until it is convenient to replace the broken rope. It is difficult, however, to obtain a uniform load on the different ropes; consequently those that are more heavily loaded are subjected to excessive wear. The multiple system is not adapted to vertical drives.



Grooves for the English or Multiple System of Rope Transmission — Grooves for the American or Continuous System of Rope Transmission

Grooves of Rope Pulleys. — The most important feature of rope driving pulleys is the adoption of the proper angle for the grooves, in order to secure the necessary adhesion. An included angle of 45 degrees is recommended for the American system and an included angle of 30 degrees for the multiple system; angles up to 60 degrees, however, are sometimes employed. The sides of the grooves are either made straight, or with a slight curvature so that the rope will revolve to insure uniform wear. The curved form is considered objectionable by some authorities who claim that the rolling action shortens the life of the rope and that the straight-sided grooves are preferable as they tend to hold the rope in a fixed position. In any case, the driving rope should be large enough to rest upon the sides of the grooves and not come in contact with the bottom. Grooves for idler pulleys having no power to transmit may be formed so that the rope rests on the bottom of the groove, in order to minimize the loss of power. Grooves for the American and English systems may be proportioned as shown by the accompanying illustrations. The sides of the grooves should be finished smooth, as the life of the rope will be materially shortened if there are rough spots on the sheaves. Sand and blowholes should also be carefully avoided as they will rapidly cut the rope fiber.

Grooves for Manila Rope Sheaves



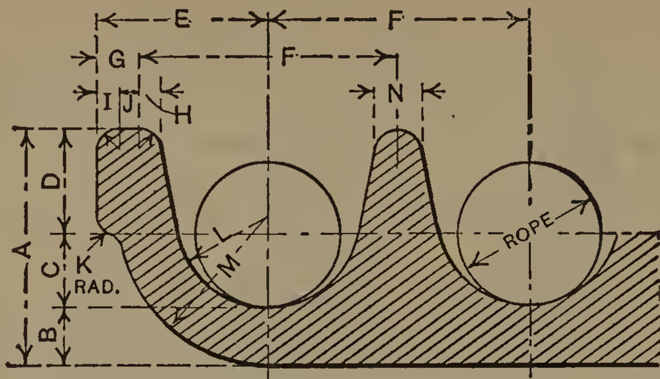
Rope Diam.	A	B	C	D	E	F	G	H	I	J	K	L	M	N
3/8	15/16	1/4	5/16	3/8	17/32	13/16	1/8	1/8	1/8	....	1/8	3/16	1/4	1 3/8
1/2	1 1/8	1/4	3/8	1/2	5/8	1	1/8	1/8	1/8	....	1/8	3/16	1/4	1 15/16
5/8	1 7/16	5/16	1/2	5/8	13/16	1 1/4	3/16	1/8	1/8	1/16	3/16	3/16	1/4	2 3/8
3/4	1 3/4	3/8	5/8	3/4	15/16	1 3/8	1/4	1/8	1/8	1/8	3/16	3/16	1/4	2 15/16
7/8	2 1/16	7/16	3/4	7/8	1 1/16	1 1/2	5/16	3/16	1/8	1/8	3/16	3/16	1/4	2 15/16
1	2 5/16	1/2	13/16	1	1 1/4	1 7/8	5/16	3/16	3/16	1/8	3/16	1/4	3/8	3 7/8
1 1/8	2 5/8	9/16	15/16	1 1/8	1 3/8	2	3/8	1/4	3/16	1/8	3/16	1/4	3/8	3 7/8
1 1/4	2 7/8	5/8	1	1 1/4	1 1/2	2 1/8	7/16	1/4	3/16	3/16	3/16	1/4	3/8	4
1 3/8	3 3/16	11/16	1 1/8	1 3/8	1 9/16	2 1/8	1/2	5/16	3/16	3/16	3/16	1/4	3/8	4 1/8
1 1/2	3 1/2	3/4	1 1/4	1 1/2	1 11/16	2 1/4	9/16	3/8	3/16	3/16	3/16	1/4	3/8	4 5/16
1 3/4	4 1/16	7/8	1 7/16	1 3/4	1 15/16	2 1/2	11/16	7/16	3/16	1/4	3/16	1/4	3/8	4 3/8
2	4 5/8	1	1 5/8	2	2 1/8	2 3/4	3/4	1/2	3/16	1/4	3/16	1/4	3/8	4 9/16

Horsepower Transmitted by Manila Rope

Conditions assumed: Working strain = 200 × diameter^2; angle of groove, 45°; contact on smaller sheave, 165°; coefficient of friction, 0.31

Diam. of Rope, Inches	Velocity of Rope in Feet per Minute											
	1000	2000	2500	3000	3500	4000	4500	5000	5500	6000	6500	7000
	Horsepower Transmitted per Rope											
5/8	1.2	2.7	3.3	3.8	4.3	4.7	5.0	5.2	5.3	5.0	4.7	4.1
3/4	2.2	3.8	4.7	5.4	6.2	6.8	7.2	7.5	7.6	7.3	6.8	5.9
1	3.6	6.8	8.4	9.8	11.1	12.1	12.9	13.3	13.5	13.1	12.1	10.5
1 1/4	5.6	10.7	13.1	15.4	17.3	19.0	20.1	20.7	21.1	20.3	19.0	16.4
1 1/2	8.0	15.4	18.8	21.8	24.9	27.3	29.0	29.9	30.4	29.3	27.3	23.7
1 3/4	10.8	20.9	25.6	29.7	34.0	37.2	39.4	40.6	41.4	39.8	37.2	32.2
2	14.2	27.3	33.5	38.9	44.3	48.6	51.6	53.1	54.1	52.1	48.6	42.2

Grooves for Manila Rope Idler Sheaves



Rope Diam.	A	B	C	D	E	F	G	H	I	J	K	L	M	N
3/8	11/16	3/16	3/16	5/16	17/32	13/16	1/8	1/8	1/8	. . .	1/8	1/4	7/16	1/4
1/2	7/8	1/4	1/4	3/8	5/8	1	1/8	1/8	1/8	. . .	1/8	11/32	19/32	1/4
5/8	1 1/16	1/4	5/16	1/2	3/4	1 1/4	1/8	1/8	1/8	. . .	1/8	13/32	21/32	1/4
3/4	1 1/4	5/16	3/8	9/16	7/8	1 3/8	3/16	1/8	3/16	. . .	1/8	15/32	25/32	1/4
7/8	1 7/16	5/16	7/16	11/16	15/16	1 1/2	3/16	1/8	3/16	. . .	1/8	17/32	27/32	1/4
1	1 5/8	3/8	1/2	3/4	1 3/16	1 7/8	1/4	3/16	3/16	1/16	3/16	19/32	31/32	3/8
1 1/8	1 7/8	7/16	9/16	7/8	1 7/16	2	5/16	3/16	3/16	1/8	3/16	21/32	1 3/32	3/8
1 1/4	2 1/16	1/2	5/8	15/16	1 7/16	2 1/8	3/8	3/16	1/4	1/8	3/16	3/4	1 1/4	3/8
1 3/8	2 1/4	1/2	11/16	1 1/16	1 7/16	2 1/8	3/8	3/16	1/4	1/8	3/16	13/16	1 5/16	3/8
1 1/2	2 7/16	9/16	3/4	1 1/8	1 9/16	2 1/4	7/16	3/16	1/4	3/16	3/16	7/8	1 7/16	3/8
1 3/4	2 13/16	5/8	7/8	1 5/16	1 13/16	2 1/2	9/16	3/16	5/16	1/4	3/16	1	1 5/8	3/8
2	3 1/4	3/4	1	1 1/2	2	2 3/4	5/8	3/16	3/8	1/4	3/16	1 1/8	1 7/8	3/8

Manila Transmission Rope

Diam. of Rope	Approx. Weight per Foot, Pounds	Breaking Strength, Pounds	Maximum Allowable Tension, Pounds	Length of Splice, Feet			Smallest Diam. of Sheaves, Inches	Maximum Number of Revolutions per Minute
				3 Strands	4 Strands	6 Strands		
3/4	0.20	3,950	112	6	8	....	28	760
7/8	0.26	5,400	153	6	8	....	32	650
1	0.34	7,000	200	7	10	14	36	570
1 1/8	0.43	8,900	253	7	10	16	40	510
1 1/4	0.53	10,900	312	7	10	16	46	460
1 3/8	0.65	13,200	378	8	12	16	50	415
1 1/2	0.77	15,700	450	8	12	18	54	380
1 5/8	0.90	18,500	528	8	12	18	60	344
1 3/4	1.04	21,400	612	8	12	18	64	330
2	1.36	28,000	800	9	14	20	72	290
2 1/4	1.73	35,400	1012	9	14	20	82	255
2 1/2	2.13	43,700	1250	10	16	22	90	230

Horsepower Transmitted by Cotton Driving Ropes

Velocity in Feet per Min.	Diameter of Ropes in Inches									
	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	2
	Horsepower Transmitted by Each Rope									
2000	5.1	7.0	9.1	11.0	14.2	17.3	20.5	24.1	28.0	36.5
2100	5.3	7.3	9.5	12.1	14.9	18.0	21.4	25.1	29.2	38.1
2200	5.6	7.6	9.9	12.6	15.5	18.8	22.3	26.2	30.4	39.7
2300	5.8	7.9	10.2	13.0	16.1	19.4	23.1	27.1	31.5	41.1
2400	6.0	8.1	10.6	13.4	16.6	20.1	23.9	28.1	32.6	42.5
2500	6.2	8.4	11.0	13.9	17.2	20.8	24.7	29.0	33.7	44.0
2600	6.4	8.7	11.3	14.4	17.8	21.5	25.5	30.0	34.8	45.4
2700	6.6	8.9	11.7	14.8	18.3	22.1	26.3	30.9	35.9	46.8
2800	6.8	9.2	12.0	15.2	18.8	22.8	27.1	31.8	36.8	48.1
2900	6.9	9.4	12.3	15.6	19.3	23.3	27.8	32.5	37.8	49.3
3000	7.1	9.6	12.6	16.0	19.8	23.9	28.4	33.3	38.7	50.4
3100	7.3	9.9	12.9	16.3	20.2	24.4	29.0	34.1	39.6	51.6
3200	7.4	10.1	13.1	16.6	20.6	24.9	29.6	34.8	40.4	52.7
3300	7.6	10.3	13.4	17.0	21.0	25.4	30.2	35.4	41.2	53.8
3400	7.7	10.6	13.7	17.3	21.5	26.0	30.8	36.2	42.0	54.6
3500	7.8	10.7	13.9	17.6	21.8	26.4	31.4	36.8	42.8	55.8
3600	8.0	10.8	14.1	17.9	22.1	26.8	31.8	37.4	43.3	56.5
3700	8.1	11.0	14.3	18.2	22.4	27.1	32.3	37.9	44.0	57.3
3800	8.2	11.1	14.5	18.4	22.7	27.5	32.7	38.4	44.5	58.2
3900	8.3	11.3	14.7	18.5	23.0	27.8	33.1	38.8	45.0	58.8
4000	8.4	11.4	14.8	18.7	23.2	28.1	33.4	39.2	45.5	59.4
4100	8.4	11.5	15.0	19.0	23.5	28.4	33.7	39.6	46.0	60.0
4200	8.5	11.5	15.1	19.1	23.7	28.6	34.0	39.9	46.3	60.4
4300	8.6	11.6	15.2	19.2	23.8	28.8	34.2	40.2	46.6	60.8
4400	8.6	11.7	15.3	19.3	23.9	28.9	34.4	40.4	46.8	61.2
4500	8.7	11.7	15.3	19.4	24.0	29.0	34.5	40.5	47.0	61.4
4600	8.7	11.8	15.4	19.4	24.1	29.1	34.6	40.6	47.1	61.5
4700	8.7	11.8	15.4	19.5	24.1	29.2	34.7	40.7	47.2	61.6
4800	8.7	11.8	15.4	19.5	24.2	29.2	34.7	40.8	47.3	61.7
4900	8.7	11.8	15.4	19.5	24.1	29.2	34.7	40.7	47.2	61.6
5000	8.7	11.8	15.4	19.4	24.1	29.1	34.6	40.6	47.1	61.5
5100	8.7	11.7	15.3	19.4	24.0	28.9	34.5	40.5	47.0	61.2
5200	8.6	11.7	15.2	19.3	23.9	28.8	34.3	40.2	46.7	61.0
5300	8.5	11.6	15.1	19.2	23.7	28.7	34.1	40.0	46.4	60.6
5400	8.5	11.5	15.0	19.0	23.5	28.4	33.8	39.6	46.0	60.0
5500	8.4	11.4	14.8	18.8	23.2	28.1	33.4	39.2	45.5	59.4
5600	8.3	11.2	14.7	18.6	23.1	27.8	33.1	38.8	45.2	58.8
5700	8.2	11.1	14.4	18.3	22.6	27.3	32.5	38.1	44.3	57.8
5800	8.1	10.9	14.2	18.0	22.3	27.0	32.1	37.6	43.7	57.1
5900	7.9	10.7	14.0	17.8	22.0	26.6	31.6	37.0	43.0	56.2
6000	7.7	10.6	13.9	17.6	21.7	26.3	31.2	36.7	42.5	55.5
6500	6.8	9.2	12.0	15.2	18.8	22.8	27.1	31.8	36.9	48.1
7000	5.4	7.4	9.6	12.2	15.0	18.2	21.4	25.4	29.5	38.4

Ropes for Power Transmission. — Cotton ropes are generally considered the best, because they will transmit more power than Manila ropes, and will wear longer. The initial cost, however, is greater. Cotton ropes are used extensively in England, and Manila ropes in the United States. The life of driving ropes depends upon their size and the conditions under which they work. The most economical diameters for cotton ropes range from 1½ to 1¾ inch, the larger size being commonly used for transmitting considerable power. Regarding the relative merits of the three- and four-strand ropes, the former is superior in that it will transmit the same power with a fewer number of ropes. The four-strand rope, however, has the advantage of stretching less and there is a larger surface in contact with the pulley grooves.

Rope Pulley Diameters. — The diameter of a rope driving pulley should not be less than thirty times the diameter of the rope, and the ratio between the pulleys should not be greater than 5 to 1. If the pulleys are made smaller than the diameter obtained by the "thirty-rope diameter" rule, a considerable loss in power will result. When large pulleys are impracticable, instead of adding a number of heavy ropes to make up for the loss of power transmitted, it is better to divide the total power to be transmitted between the necessary number of smaller ropes.

Rope Pulley Center Distances. — The center-to-center distance for under-driven installations is governed by the size of the pulleys, which should allow sufficient clearance between the slack of the trailing span and the tight portion of the ropes. This clearance should not be less than one-tenth the distance between the centers. With over-driven installations, a center distance of 100 feet or over is practicable, although it is necessary to provide against the extra strain imposed upon the shafts and bearings by the weight of the ropes. In ordinary mill practice, the center distance seldom exceeds 90 feet. If possible, the distances between the pulley faces should not be less than 25 feet.

Sag and Stretch of Ropes. — With the multiple system, the drive should, if possible, be horizontal and with the slack side of the rope on top. The sag of ropes has nearly the form of a parabola. The sag on the tight side can be determined quite closely if the power to be transmitted is known. The amount can be approximated by multiplying the weight of one foot of rope by the square of the distance from sheave to sheave at the point where the rope leaves, and dividing this product by eight times the tension on the rope. The sag on the slack side can be ascertained by the same method, by adding to the initial tension the tension from centrifugal force. A new rope will stretch considerably for a short time, but after the strands are firmly bedded together there is little additional stretch. The total stretch should not exceed 5 per cent under average conditions.

Lubrication of Driving Ropes. — New driving ropes are lubricated to provide a smooth coating and prevent friction between the twisted surfaces of the strands in contact with each other. A good lubricant is compounded from saponified tallow, wax and plumbago. Greasy compounds likely to penetrate the fiber should be avoided, as they increase the tendency to slip. If ropes have been so treated, a liberal application of whiting should be used, as this absorbs the grease and falls away in flakes. Re-lubrication is seldom necessary unless the ropes are working in an unusually dry atmosphere.

Speed for Rope Transmission. — The speed of driving ropes usually varies from 2000 to 5000 feet per minute. The most economical speed is between 4500 and 5000 feet per minute. If the speed is too low, the ropes are likely to slip, and if it is too high, the action due to centrifugal force affects the efficiency. As the ropes operate in V-shaped grooves, the loss due to centrifugal force is less than with belt drives; hence ropes are adapted to higher velocities. It is important when installing rope drives to encase the sides of the pulleys with sheet metal or boards, as otherwise the displacement of air by the arms of high-speed pulleys adds materially to the load. Some firms advocate encasing all pulleys, while others do not consider it necessary unless the pulleys are at least 7 feet in diameter and have a peripheral velocity of 4000 feet or over.

Power Transmitted by Rope Drives. — The power that can economically be transmitted by ropes depends upon the speed and the allowable working tension or strain to which the ropes are subjected. To secure the best results in regard to efficiency and durability, the working tension must be a small percentage of the ultimate strength of the rope. In the following formulas, T = safe working tension in pounds, per rope; F = centrifugal tension in pounds, per rope; H.P. = horsepower transmitted per rope; V = velocity of rope in feet per second; W = weight of rope per foot; and D = nominal diameter of rope in inches. Then:

$$\text{H.P.} = \frac{2\,V\,(T-F)}{3 \times 550} \qquad T = 160\,D^2 \qquad F = \frac{WV^2}{32}$$

As the above formula gives the power per rope, the total power transmitted equals H.P. $\times$ N, in which N equals the number of ropes in the drive. The tension available for the transmission of power is considerably reduced at high speeds by the centrifugal force. By referring to the accompanying table "Horsepower Transmitted by Cotton Driving Ropes," it will be seen that 4800 feet per minute is the maximum speed advisable. When higher speeds are employed, the centrifugal tension increases at a greater rate than the power due to the increased rope velocity, thus causing a reduction in the amount of power that can be transmitted. This table is based on the assumption that the safe working tension $T = 160\,D^2$ (corresponding to a stress of about 200 pounds per square inch of section) and that the weight of the rope per foot W equals $0.27\,D^2$, in which formulas D equals diameter of rope in inches. Some engineers make the safe allowable tension equal to 1/36 of the breaking strength of the rope. The weight of Manila transmission rope equals $0.34\,D^2$ and the breaking strength approximately $7000\,D^2$.

CHAIN TRANSMISSION

Chain drives are especially applicable when the distance between the driving and driven shafts is too short for belting and too long for gearing. The chain drive is positive, compact, and without the initial tension required for a belt drive. Chain drives frequently are substituted for belting when the driving and driven shafts are comparatively close, and, if correctly installed, the chain system of transmission is much more durable than belting. The initial cost is somewhat higher, but for many classes of service this extra cost amounts to little when compared with the effectiveness of the chain drive. A chain drive is more efficient than a belt drive, and a uniform turning movement is obtained. The chain may, in some cases, reduce impulses from the driving end. The slip which often occurs with non-positive drives seriously affects the uniformity of driving, in many cases. Frequently, a considerable saving in space may be effected, because the distance between the shafts may be much less with a chain than with a belt drive.

Various forms or types of chains have been developed for power transmission. The *block chain*, which consists of steel blocks connected by side links or plates, is adapted for light machine drives. The double width or twin type is used when the amount of power to be transmitted is relatively high. *Roller chains* differ from block chains in that bushings and rollers are inserted between the links instead of solid blocks. Roller chains are stronger than block chains and are used in preference when the speeds and amount of power to be transmitted are relatively high. The *bushing chain* resembles a roller chain somewhat, but differs from the latter in that the bushings between the side links are not provided with rollers. Bushing chains are used when considerable power is to be transmitted at relatively low speeds.

Number of Teeth in Sprockets. — The number of teeth in chain sprockets should be as large as is consistent with other conditions. A large number of teeth will reduce the noise, the wear of the chain on the sprockets, and the loss of energy from friction. The best results are obtained with sprockets having 16 teeth or more. Eight- or nine-tooth sprockets will almost invariably ruin a roller chain, and those having ten or eleven teeth are adapted for only medium and slow speeds, with other conditions favorable. The ratio between the driven and the driving sprocket should not be over 1 to 8 for block chains of the bicycle type, nor over 1 to 5 for roller chains of ¾-inch pitch and larger. The number of teeth in sprockets for "silent" chain drives should vary from 15 as a minimum to about 125 as a maximum and the driving pinions should have an odd number of teeth. This feature causes the chain to change its bearing on the wheel with every complete revolution, the result being an increase in the life of the pinion.

Chain Pitch and Sprocket Speed. — The maximum speed of the smaller sprocket should be considered when determining the pitch of the chain. A higher sprocket speed is allowable with a chain of short pitch than with a long pitch chain. Some chain manufacturers recommend maximum chain speeds of 1000 to 1200 feet per minute for roller chains and 700 to 800 feet per minute for block chains, but according to the Diamond Chain & Mfg. Co., if the conditions regarding sprocket speed and chain pull are favorable, chains may be run much faster than the maximum speeds which are based on chain speeds only. In fact, it is claimed that roller chains will operate under loads of from 1 to 150 horsepower at speeds up to 4000 feet per minute. A long series of observations and experiments demonstrated that the chain speed alone has very little to do with the destructive action between the chain and sprocket, but high sprocket speed combined with long pitch was found to be very destructive as well as noisy, because of the impact between the chain link and sprocket as the roller seats itself. The force of this impact increases in proportion to the weight of the chain and the square of the velocity with which the roller strikes the sprocket. The angular velocity of a chain link about its stud is the same as the angular velocity of the sprocket about its bearing, and the velocity of impact of the roller is equal to the linear velocity of a point on a sprocket at a distance from the center equal to the pitch. Therefore, the destructive action between the chain and sprocket (other conditions being equal) increases as the square of the pitch and also as the square of the velocity. Hence, the pitch of the chain should not be greater than the speed allowed by the smaller sprocket. If P = maximum allowable pitch of chain and S = maximum speed of sprocket in revolutions per minute, then the following formulas, which are approximate, may be used to determine the values of P and S:

$$P = \left(\frac{900}{S}\right)^{2/3}; \qquad S = \frac{900}{\sqrt{P^3}}$$

When the sprocket speed is high and there is more than one weight of chain of the same pitch and width to choose from, the lighter chain should be selected, provided the rivet area and ultimate strength are sufficient, because the force of the impact increases directly as the weight of the chain link.

Width of Chain. — A short pitch and a wide chain is, as a general rule, preferable to a narrow chain of longer pitch. There is less frictional loss in chains having rivets of small diameters than for rivets of larger diameters; consequently, if the small rivet has sufficient strength in shear for the service required, it is preferable to increase the length of the rivet instead of its diameter in order to obtain the necessary rivet area.

Tension on Driving Chains for Given Speed and Power Transmission *
(Baldwin Chain & Mfg. Co.)

H.P. Transmitted	Speed of Chain in Feet per Minute											
	350 Feet	400 Feet	450 Feet	500 Feet	550 Feet	600 Feet	650 Feet	700 Feet	800 Feet	900 Feet	1000 Feet	1100 Feet
10	943	825	733	660	600	550	508	471	413	367	330	300
15	1414	1238	1100	990	900	825	762	707	614	550	495	450
20	1885	1650	1466	1320	1200	1100	1015	942	825	733	660	600
25	2357	2062	1833	1650	1500	1375	1270	1178	1031	916	825	750
30	2828	2475	2200	1980	1800	1650	1523	1414	1227	1100	990	900
35	3300	2887	2566	2310	2100	1925	1777	1650	1443	1283	1125	1050
40	3771	3300	2933	2640	2400	2200	2030	1885	1650	1466	1320	1200
45	4242	3712	3300	2970	2700	2475	2284	2121	1856	1650	1485	1350
50	4714	4125	3666	3300	3000	2750	2537	2357	2062	1833	1650	1500
55	5185	4537	4033	3630	3300	3025	2791	2592	2268	2016	1815	1650
60	5657	4950	4400	3960	3600	3300	3045	2828	2475	2200	1980	1800

* For actual service, a chain should be selected having an ultimate strength from 6 to 20 times greater than the working strain given in the table in order to make allowance for shocks. This factor of safety will depend upon the steadiness of the load and the seriousness of the consequences that would follow a break.

Ultimate Strength of Roller Chains
(Baldwin Chain & Mfg. Co.)

Chain No.*	Pitch, Inch	Roll Width, Inch	Roll Diam., Inch	Ultimate Strength, Pounds	Chain No.	Pitch, Inch	Roll Width, Inch	Roll Diam., Inch	Ultimate Strength, Pounds
111	1/2	3/16	0.306	1,200	36	1 1/4	5/8	5/8	12,000
112	1/2	1/8	0.306	1,200	18	1 1/4	5/8	3/4	15,000
113	1/2	1/4	0.306	1,200	58	1 1/4	5/8	3/4	20,000
108	5/8	3/8	0.400	3,500	19	1 1/4	3/4	3/4	20,000
109	5/8	1/4	0.400	3,500	38	1 1/4	3/4	3/4	20,000
110	5/8	1/4	0.400	3,500	20	1 1/2	5/8	3/4	12,000
8	3/4	3/8	15/32	4,000	62	1 1/2	3/4	3/4	24,000
9	3/4	1/2	15/32	4,000	22	1 1/2	3/4	7/8	24,000
95	3/4	5/8	15/32	7,500	30	1 1/2	1	7/8	30,000
10	15/16	3/8	9/16	7,500	51	1 1/2	1	7/8	36,000
11	1	3/8	9/16	6,000	23	1 3/4	1	1	36,000
12	1	3/8	5/8	6,000	52	1 3/4	1	1	36,000
13	1	1/2	9/16	6,000	33	2	1 1/4	1 1/8	42,000
46	1	1/2	9/16	6,000	53	2	1 1/4	1 1/8	42,000
14	1	1/2	5/8	9,000	61	2 1/2	1 1/8	1 9/16	95,000
27	1	5/8	5/8	12,000	63	2 1/2	1 1/2	1 9/16	95,000

* Chains Nos. 58, 38, 62, 51, 52 and 53 are extra heavy and designed for commercial car use.

Chain Pull or Working Load. — The chain pull or working load for roller chain transmissions should not exceed 1/10 of the ultimate tensile strength of the chain, and sometimes the load is only 1/30 or 1/40 of the ultimate strength, especially when the loads are suddenly applied. The working load varies according to the amount of rivet bearing surface, the steadiness of the load, the speed of the driving sprocket, the quality of the chain and the care it receives as regards lubrication and cleanliness. The chain pull should not, as a rule, exceed 1000 pounds per square inch of projected rivet area, although a transmission operating at a slow speed may be fairly satisfactory with a chain pull of 2000 or even 3000 pounds per square inch of projected rivet area. If C = the chain pull in pounds; H = the number of horsepower to be transmitted; V = the chain velocity in feet per minute, then $C = \frac{33{,}000 \times H}{V}$. The values given in the accompanying table, "Tension on Driving Chains for a Given Speed and Power Transmission," are based on this formula.

Length of Driving Chain. — The total length of a block chain should be given in multiples of the pitch, whereas for a roller chain, the length should be in multiples of twice the pitch, because the ends must be connected with an outside and inside link. The length of a chain can be calculated accurately enough for ordinary practice by the use of the following formula, in which C = center distance in pitches; N = number of teeth in large sprocket; n = number of teeth in small sprocket:

$$\text{Chain length in pitches} = 2\,C + \frac{N}{2} + \frac{n}{2} + \frac{\left(\frac{N-n}{2\pi}\right)^2}{C}$$

To the length obtained by this formula, add enough to make a whole number (and for a roller chain, an even number) of pitches. If a roller chain has an odd number of pitches, it will be necessary to use an offset connecting link.

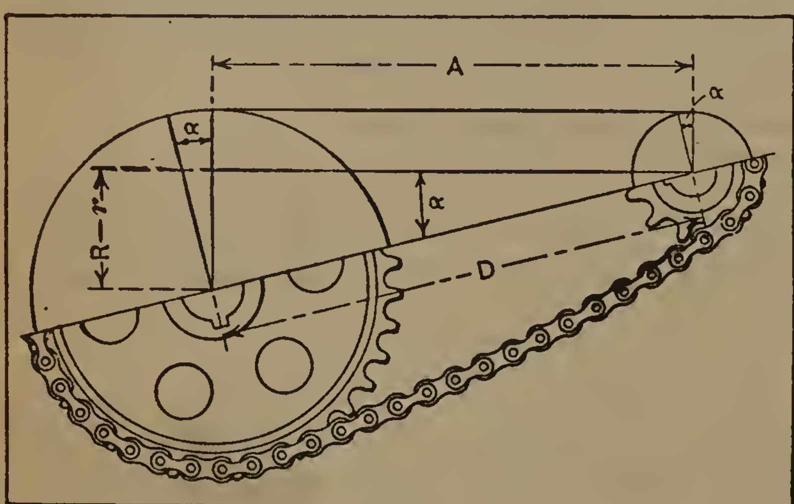


Fig. 1

Following is another formula for obtaining the exact chain length, all dimensions being in inches: D = distance between centers of shafts; R = pitch radius of large sprocket; r = pitch radius of small sprocket; N = number of teeth in large sprocket; n = number of teeth in small sprocket; P = pitch of chain and sprockets; A = distance between limiting points of contact (see Fig. 1); L = required chain length in inches:

$$L = \frac{180 + 2\,\alpha}{360} NP + \frac{180 - 2\,\alpha}{360} nP + 2\,D \cos\alpha;$$

$$\sin\alpha = \frac{R - r}{D}; \quad A = D\cos\alpha$$

Angle of contact on large sprocket = $180° + 2\,\alpha$;
Angle of contact on small sprocket = $180° - 2\,\alpha$.

Center Distance between Sprockets. — The center-to-center distance between sprockets, as a general rule, should not be less than 1½ times the diameter of the larger sprocket nor more than about 60 times the pitch, although much depends upon the speed and other conditions. If possible, the center distance should be adjustable in order to take care of slack due to elongation from wear. An adjustment equal to the pitch of the chain is sufficient. A little slack is desirable as it allows the chain links to take the best position on the sprocket teeth and reduces the wear on the bearings. In automobile drives, too much sag or an excessive distance between the sprockets, causes the chain to whip up and down — a condition detrimental to smooth running and very destructive to the chain. For transmissions of this class, the best results are obtained with center distances of approximately three feet or less and a distance of four feet should not be exceeded. The sprockets for machine-made chain should run in a vertical plane, the

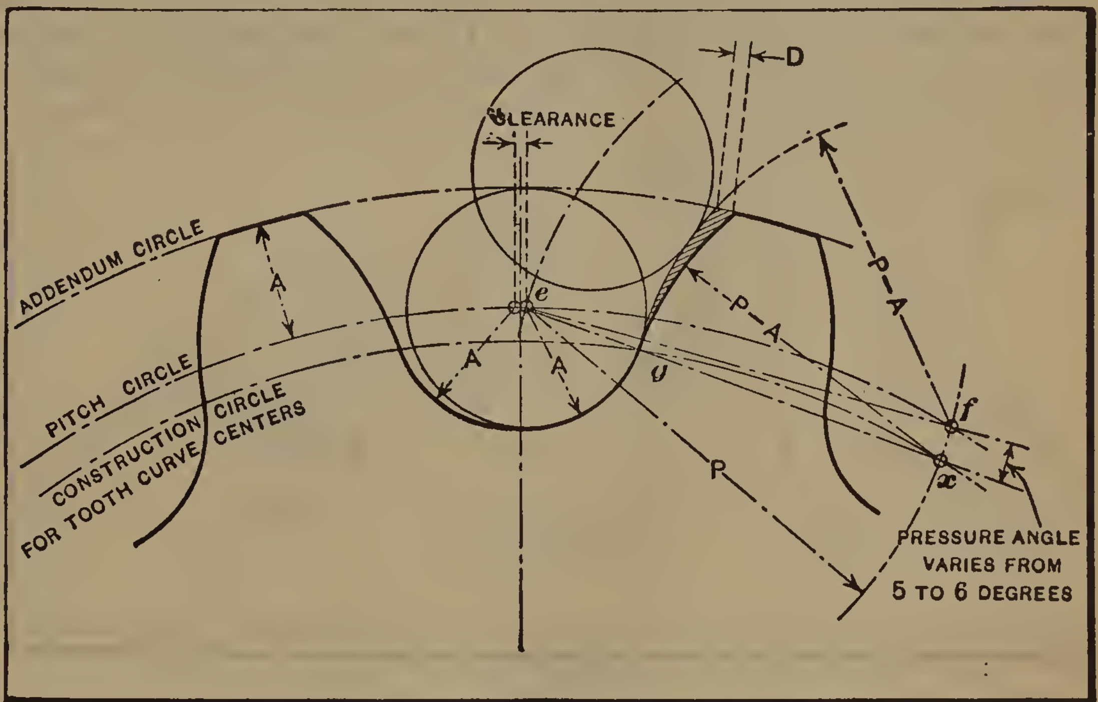


Fig. 2

sprocket axes being approximately horizontal, unless an idler is used on the slack side to keep the chain in position. More satisfactory results are obtained when the slack side of the chain is on the bottom.

Center Distance for a Given Chain Length. — When the distance between the driving and driven sprockets can be varied to suit the length of the chain, this center distance for a tight chain may be determined by the following formula, in which C = center-to-center distance in inches; L = chain length in pitches; and the other notation is the same as for the preceding formulas for determining chain lengths:

$$C = \frac{P}{8}\left[2L - N - n + \sqrt{(2L - N - n)^2 - 0.824\,(N - n)^2}\right]$$

This formula is approximate, but the error is less than the variation in the length of the best chains. The length L in pitches should be an even number for a roller chain, so that the use of an offset connecting link will not be necessary.

Idler Sprockets. — When sprockets have a fixed center distance or are non-adjustable, it may be advisable to use an idler sprocket for taking up the slack. The idler should preferably be placed against the slack side between the two strands of the chain. When a sprocket is applied to the tight side of the chain to reduce vibration, it should be on the lower side and so located that the chain will run in a straight line between the two main sprockets. A sprocket will wear excessively if the number of teeth is too small and the speed too high, because there is impact between the teeth and rollers even though the idler carries practically no load.

Sprockets having Pitch Line Clearance. — The chains used for power transmission elongate more or less due to wear of the rivets and bushings or to the stretch of the material, especially if the chain is subjected to suddenly applied loads. There are two general methods of providing for this elongation. One method is to cut the sprocket teeth so that there is a certain amount of pitch line clearance, to provide room for the chain as it lengthens. Thus, as the chain gradually increases in length, the rollers shift back along the pitch circle, and while there is sufficient pitch-line clearance, interference between the rollers and teeth is avoided, and the teeth do not exert a wedging effect on the chain and subject it to excessive strains. A form of tooth having pitch line clearance is illustrated by the diagram, Fig. 2. The radius of the tooth curve is equal to the pitch P minus radius A of

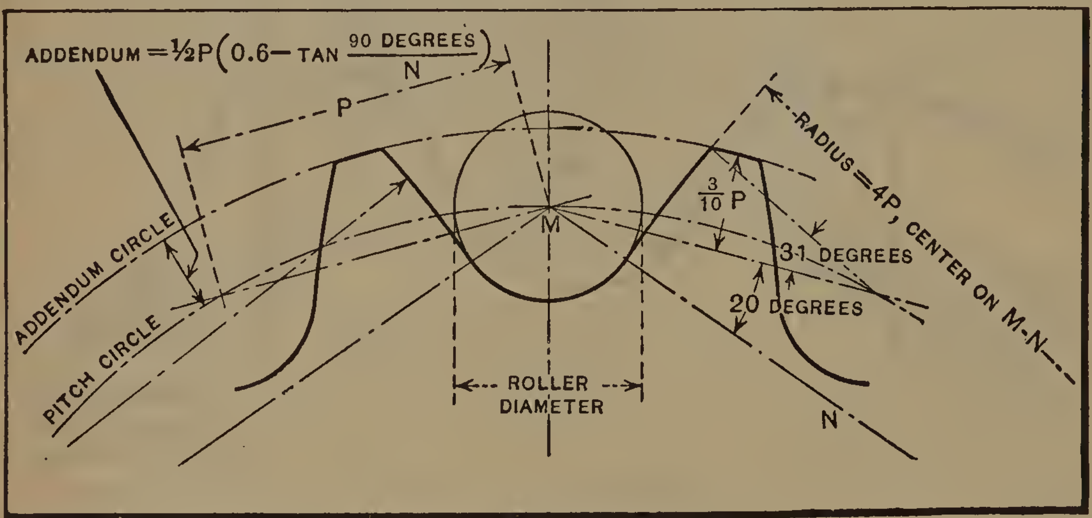


Fig. 3

the chain roller, and center x of the tooth curve is so located that clearance D at the end of the tooth is one-tenth of the roller diameter. The chain pull is in the direction of line e–x and the *pressure angle* is the angle included between lines e–f and e–x. This angle is about five degrees on a nine-tooth sprocket and about six degrees on a thirty-five-tooth sprocket. The pitch line clearance is usually about 1/20 of the roller diameter.

Sprockets having Constant Pressure Angle. Another method for providing for chain elongation is to use sprockets which have only a slight amount of pitch line clearance but an increased pressure angle so that, as the chain elongates, the rollers ride at higher points on the teeth as the chain adapts itself to the proper pitch circle, according to the amount of stretch. This method would not be practical when using sprockets having small pressure angles of five or six degrees but it is applicable to the form of sprocket tooth (illustrated in diagram, Fig. 3) developed by the Diamond Chain & Mfg. Co. The teeth of all sprockets have a constant pressure angle of about twenty degrees, which is the angle between the direction in which the chain is pulling and a normal to the tooth outline at the point of contact. The advantages claimed for this form of sprocket tooth are as follows. (1) The chain passes on and off the sprocket tooth more easily and with less noise and wear. (2) The pressure is distributed among all of the teeth in mesh. (3) As the chain elongates and adapts itself to a proportionately larger pitch circle, new parts of the tooth are subjected to wear, thus reducing the tendency to form hook-shaped teeth. (4) The pressure angle is the same for all numbers of teeth and an elongated chain will run as well on a 100-tooth sprocket as on one having twenty teeth. (5) Any number of teeth for a given pitch and roll diameter can be cut with a single cutter. (6) The length of the tooth is such that the angular motion of the link, between the pitch circle and addendum circle, is always 31 degrees.

Sprockets having Constant Space Angle. — The Renold form of sprocket tooth illustrated by the diagram, Fig. 4. is another form which is designed to allow a chain gradually to ascend the tooth as it elongates and to accommodate itself to a larger pitch circle This tooth has a variable pressure angle but a uniform *space angle* and no pitch line clearance The pressure angle for a seven-tooth sprocket is 4¼ degrees; for an eight-tooth sprocket, 7½ degrees; and the angle increases until for an eighty-tooth sprocket it is 27¾ degrees. This angle is reduced to zero for a six-tooth sprocket. The included angle of the tooth space, or the cutter

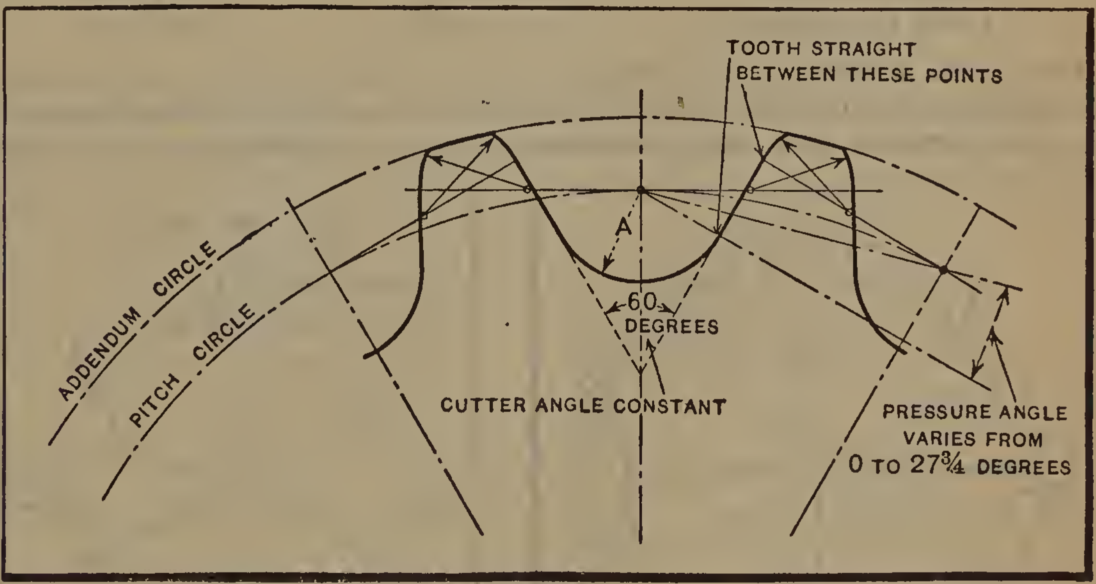


Fig. 4

angle, is sixty degrees for all sprockets. A series of five cutters is required for cutting sprockets ranging from six to eighty teeth. If P = pitch of chain and N = number of teeth in sprocket, then the height x of the addendum may be determined approximately by the following formula:

$$\text{Addendum} = \tfrac{1}{2} P \left(0.6 - \tan \frac{90°}{N} \right)$$

Pitch Diameters of Chain Sprockets. — The pitch diameter of a sprocket for block-center and twin-roller chains may be obtained by the following formulas, in which N = number of teeth; b = diameter of round part of chain block; B = center to center of holes in chain block; A = center to center of holes in side links; α and β = the angles illustrated in Fig. 5.

$$\text{Pitch diameter} = \frac{A}{\sin \beta}$$

$$\alpha = \frac{180°}{N}; \quad \tan \beta = \frac{\sin \alpha}{\frac{B}{A} + \cos \alpha}$$

For roller chains, when P = pitch of chain; D = diameter of roller; α = the angle illustrated in Fig. 6:

$$\text{Pitch diameter} = \frac{P}{\sin \alpha}; \quad \alpha = \frac{180°}{N}$$

Outside and Base Diameters of Roller Chain Sprockets. — The base or bottom diameter of a roller chain sprocket equals the pitch diameter minus diameter D (see Fig. 6) of the roller. The outside diameter, according to common rule, equals the pitch diameter plus the roller diameter for sprockets of medium and large size. The outside diameters given in the accompanying tables, "Roller Chain Sprocket Diameters" conform to this rule, provided the sprockets have 17 teeth or more. The outside diameters of smaller sprockets are reduced as follows, to avoid interference with the chain:

$$\text{Outside diameter} = \text{pitch diameter} + D - E.$$

The values of E are given in the following table:

Pitch	Values of E	
	8 to 12 teeth	13 to 16 teeth
½ inch to ¾ inch	0.062 inch	0.031 inch
1 inch to 2 inches	0.125 inch	0.062 inch

The outside diameters of the Diamond Chain & Mfg. Co.'s roller chain sprockets are somewhat less than the diameters obtained by adding the roller diameter to the pitch diameter, and may be determined by the following formula in which

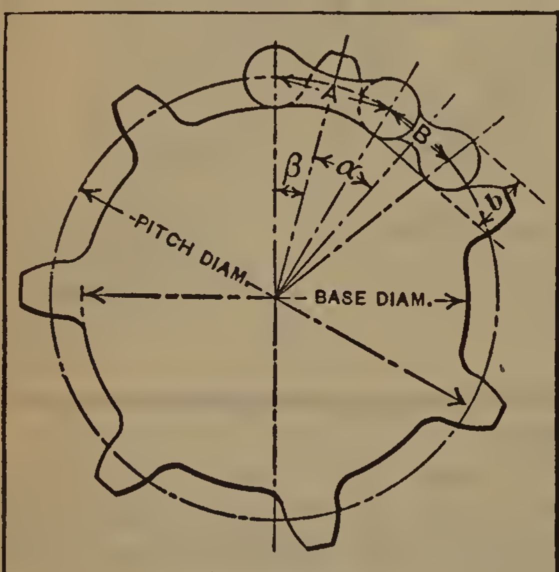


Fig. 5

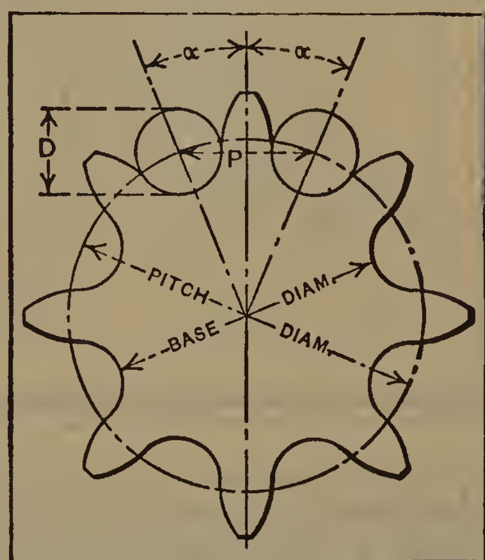


Fig. 6

O = outside diameter; P = pitch diameter; p = pitch of chain; N = number of teeth in sprocket.

$$O = P + p\left(0.6 - \tan\frac{90^\circ}{N}\right)$$

The bottom diameter equals the pitch diameter minus the roller diameter, but when sprockets are cut with the "Diamond universal cutter" referred to later, the bottom diameter should be reduced from 0.003 to 0.005 inch. If a sprocket has an odd number of teeth, the modified base diameter B, for calipering the sprocket, may be determined by the following formula, in which D = the roller diameter:

$$B = P \times \cos\frac{90^\circ}{N} - D$$

Outside and Base Diameters of Block Chain Sprockets. — The base diameter of a sprocket for either block-center or twin-roller chains is equal to the pitch diameter minus dimension b (see Fig. 5). The outside diameters given in the accompanying table "Block Chain Sprocket Diameters" are equal to the pitch diameter plus dimension b. The formulas of the Diamond Chain & Mfg. Co. give somewhat smaller outside diameters. In these formulas O = outside diameter; P = pitch diameter; N = number of teeth; A = center-to-center distance (Fig. 5) of holes in side bars; β = angle shown in Fig. 5.

$$O = P + A\left(0.6 - \tan\frac{\beta}{2}\right)$$

The value of β may be determined by the formula previously given in paragraph headed "Pitch Diameters of Chain Sprockets." The following formulas will give the outside diameters accurate enough for practical purposes:

$$O = P + \frac{45\,N - 72}{125\,N} \text{ for 1-inch pitch block chains.}$$

$$O = P + \frac{9\,N - 15}{16\,N} \text{ for 1½-inch pitch block chains.}$$

Cutting Sprocket Teeth. — Sprockets which have pitch line clearance may be milled with cutters that provide the clearance, but if the cutter has little or no allowance for clearance, the sprocket can be milled as follows: First cut the teeth in the usual manner; then revolve the sprocket a distance C (see Fig. 7) and take an additional cut around the sprocket. This pitch line clearance C may be obtained from the accompanying table.

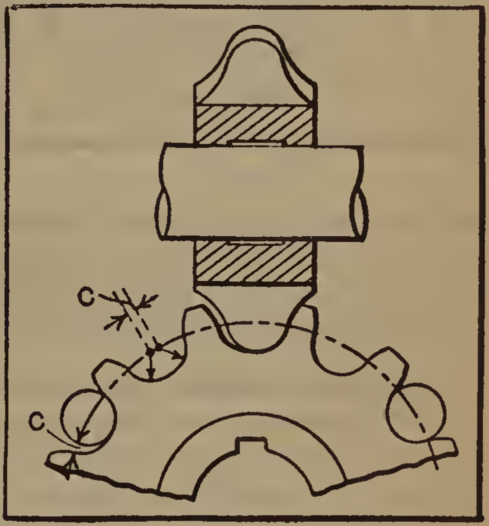


Fig. 7

A number of sprocket cutters are made for each pitch, the same as for spur gears, and in order to obtain a tooth adapted for high speeds, it is common practice to use cutters made for fewer teeth than the number in the sprocket to be cut. For example, to cut a 10-tooth sprocket a cutter marked "9 teeth" would be used. In this way, the teeth are made more rounding, to facilitate engagement and disengagement of the rollers. When using Brown & Sharpe cutters, the cutter numbers for different numbers of teeth are as follows:

To cut a 9-tooth sprocket, use cutter marked 8 teeth; to cut a 10-tooth sprocket, use cutter marked 9 teeth; to cut an 11-tooth sprocket, use cutter marked 10–11 teeth; to cut a 12-tooth sprocket, use cutter marked 10–11 teeth; to cut a 13-tooth sprocket, use cutter marked 12–13 teeth; to cut a 14- and 15-tooth sprocket, use cutter marked 12–13 teeth; to cut a 16-, 17- and 18-tooth sprocket, use cutter marked 14 to 16 teeth; to cut a 19- to 25-tooth sprocket, use cutter marked 17 to 20 teeth; to cut a 26-tooth sprocket and over, use cutter marked 21 to 34 teeth.

Pitch Line Clearance for Roller Chain Sprockets

Pitch	Roller Diam.	Clearance, C
½ and ⅝ inch	0.325 inch	1/32 inch
¾ inch	15/32 inch	1/16 inch
1 inch	9/16 inch	3/32 inch
1¼ inch	⅝ inch	5/32 inch
1½ inch	¾ inch	3/16 inch
1¾ inch	1 inch	⅛ inch
2 inch	1⅛ inch	5/32 inch

Any number of teeth for a given pitch and diameter of roller may be cut with a Diamond universal sprocket cutter, which is used for the form of teeth illustrated by the diagram, Fig. 3. When using this type of cutter, it is recommended that the machine be indexed so that every other tooth will be cut at first and then the teeth between, the object being to eliminate any side-thrust tendency. If the number of teeth is *even*, every other tooth space is first milled and then the sprocket is indexed one tooth space, after which the work is again indexed every other tooth space. If there is an *odd* number of teeth, the teeth will be finished in two revolutions of the sprocket, provided the indexing mechanism is set for every other tooth space.

Cross-sectional Profile of Sprocket Teeth. — A chain will not run well unless the sprockets have sidewise clearance and the teeth are rounded at the ends. In the following formulas, P = pitch of chain; W = length of the roller (see Fig. 8); T = width of sprocket teeth; R = radius of side of tooth at top; C = distance from top of tooth to line from which radius R is drawn. The profile recommended by the Society of Automotive Engineers is as follows: $T = 0.93\ W - 0.006$; $C = 0.3\ P$; $E = \frac{P}{8}$; minimum radius $R = 0.43\ P$; limits for $T = \pm\ (0.01\ W + 0.002)$.

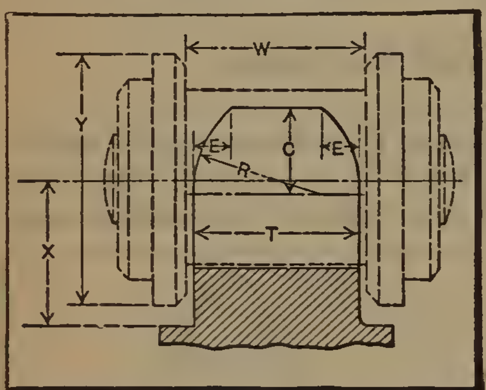


Fig. 8

The chamfering or rounding of the tooth from the pitch line outward is important, because if the chain sways while running and the sprockets are not accurately aligned, the chamfered ends easily engage the chain and prevent noise. This is especially true of motor cars, owing to the fact that one side of the car is frequently lower than the other, thus throwing the sprockets temporarily out of perpendicular alignment.

Clearance between Flange and Side Links. — When sprockets have flanges or hubs, clearance must be provided so that the chain can seat itself properly without coming into contact with the flange or hub. The following values of x (see Fig. 8) provide sufficient clearance for chains of different pitch.

Pitch of Chain	x not less than
1-inch pitch block or ½-inch and ⅝-inch pitch roller chain	¼ inch
1½-inch pitch block and ¾-inch pitch roller chain	11/32 inch
1-inch pitch roller chain	½ inch
1¼-inch pitch roller chain	⅝ inch
1½-inch pitch roller chain	¾ inch
1¾-inch pitch roller chain	15/16 inch
2-inch pitch roller chain	1 inch

Another rule is as follows: The diameter of the flange or hub should not exceed the pitch diameter of the sprocket minus 1.1 times width y (see Fig. 8).

Standard Roller Chain Dimensions. — The dimensions of heavy roller transmission chains given in the accompanying table are recommended by the Society of Automotive Engineers as the best practice for adoption both as national and international standard. The roll diameters equal approximately $0.625\ P$ (P = pitch), the pin diameters equal approximately ½ of the roll diameters; the thicknesses of inside plates equal $0.125\ P$; the widths of narrow chains equal approximately $0.41\ P$, and of wide chains $0.625\ P$. The sprocket thickness =

$0.93\,W - 0.006 \pm (0.01\,W + 0.002)$, in which W = width of roller. The test load equals approximately $3300\,P^2$ or ⅓ the ultimate strength. It is recommended that the dimensions of the medium series of roller transmission chains of any pitch conform to the roller and pin dimensions for the next shorter pitch of the heavy series. The light series of any given pitch are to have the roller and pin dimensions for the second shorter pitch of the heavy series.

Heavy Roller Transmission Chain Dimensions

Pitch	Roll Diam.	Pin Diam.	Thickness of Inside Plates	Chain Widths Minimum		Sprocket Thickness		Measuring Load, Lbs.	Approx. Test Load, Lbs.
				Narrow	Wide	Narrow	Wide		
*⅜	0.250	0.125	0.050	0.156	0.250	0.139±0.003	0.227±0.004	14	464
*½	0.3125	0.156	0.060	0.219	0.3125	0.198±0.004	0.284±0.005	25	825
*⅝	*0.400	*0.200	*0.080	*0.250	*0.375	0.226±0.005	0.343±0.006	39	1,280
*¾	*0.469	0.234	0.094	*0.3125	*0.500	0.284±0.005	0.459±0.007	56	1,850
*1	*0.625	*0.3125	0.125	0.4375	*0.625	0.401±0.006	0.575±0.008	100	3,300
*1¼	0.781	*0.375	*0.156	0.500	0.781	0.459±0.007	0.720±0.010	156	5,150
*1½	0.937	0.469	0.1875	0.625	*1.000	0.575±0.008	0.924±0.012	225	7,430
*1¾	1.100	0.547	*0.220	0.750	1.125	0.692±0.010	1.040±0.013	325	10,700
*2	1.250	0.625	*0.250	0.875	*1.250	0.808±0.011	1.156±0.015	400	13,200
*2½	1.550	0.781	0.3125	.....	1.5625	....	1.389±0.017	625	20,600
3	1.900	0.9375	0.375	.....	1.875	....	1.738±0.021	900	29,700
4	2.500	1.250	0.500	.....	2.500	....	2.319±0.027	1600	52,800
5	3.000	1.500	0.625	.....	3.000	....	2.784±0.032	2500	82,500

* Denotes manufacturers' standards in present use. The use of wide chains for all pitches is considered preferable. The dimensions in the above table are recommended by the Society of Automotive Engineers.

Lubricants for Driving Chains. — The life of chains for power transmission may be lengthened by proper care. If the chain is subjected to dust and grit, it should occasionally be thoroughly cleaned by washing in gasoline, kerosene or in hot soda water. When the chain is dry it should be immersed in molten tallow from 10 to 20 minutes and then be hung up to drain and cool. Always replace the chain to run in the same direction and with the same side up, and never attempt to use a new chain on a sprocket that is considerably worn. For chains of the silent type, the following method of cleaning and lubricating is recommended: Immerse the chain in a bath of paraffin or turpentine and work the dirt out of the joints by the use of a scrubbing brush. Remove the paraffin from the chain by thoroughly draining or washing in hot soda water, and then dip it in a bath of fairly thick lubricating oil. Have the oil bath warm and, if convenient, leave the chain in it for several hours. This treatment should be applied three or four times a year and more frequently when the conditions are severe.

The lubricant to be used for general lubrication of driving chains should be determined by the nature of the drive. If the temperature is much above normal, use a thick oil having considerable body. In other cases, use a thinner oil, unless it cannot be retained owing to the speed. For high speeds, use a thin oil in order to penetrate the joints, and then apply a thicker oil which will not be thrown off by centrifugal force. Lubricant is best applied with a brush to the inner side of the chain, while running slowly. For ordinary drives, lubricate once a week, and where the conditions are severe, as in the case of continuous operation, fluctuating loads, hot or dirty location, lubricate every day. For detachable link-belts or chains, lubricate the joints with a heavy oil or good grease, and also apply lubricant to the rims and teeth of the sprockets at points of contact. Where link-belting is exposed to gritty dust, use plumbago or graphite grease.

The life of a driving chain can be greatly lengthened by enclosing it in a dust-proof case. Many chain installations are lacking in this respect. Transmission gears on automobiles, etc., are ordinarily protected, but driving chains, in many cases, are exposed to dust and grit. Obviously, when a chain is properly protected, it will not only wear much longer, but will also require less attention.

Horsepower Transmitted by Chain Drives

This table gives values for ideal conditions. For ordinary duty deduct from 30 to 40 per cent.

Detachable Link-belt				Jeffrey Mey-Oborn Chain			
Chain or Sprocket Number	Working Strain, Pounds	Horsepower 250 Feet	Horsepower 500 Feet	Chain or Sprocket Number	Working Strain, Pounds	Horsepower 250 Feet	Horsepower 500 Feet
25	75	0.56	1.12	25	100	0.75	1.50
32	150	1.12	2.25	....	......	......	......
33	200	1.50	3.00	33	250	1.12	2.25
34	225	1.68	3.37	34	250	1.12	2.25
35	250	1.87	3.75	....	......	......	......
42	300	2.25	4.50	42	400	3.00	6.00
45	350	2.50	5.02	45	400	3.00	6.00
51	375	2.81	5.62	50	250	1.12	2.25
52	500	3.75	7.50	52	600	4.50	9.00
52½–55	450	3.37	6.75	55	600	4.50	9.00
57	600	4.50	9.00	52 spec.	800	6.00	12.00
62	650	4.87	9.75	57	700	5.25	10.50
66–67	700	5.25	10.50	62–67	750	5.62	11.25
75	750	5.62	11.25	75	1000	7.50	15.00
77	800	6.00	12.00	77	900	6.75	13.50
78	1000	7.50	15.00	78	1100	8.25	16.50
83–88	1200	9.00	18.00	83	1500	11.12	22.25
85	1300	9.75	19.50	85	1600	12.00	24.00
95	1600	12.00	24.00	88	1300	9.75	19.50
103–108	1800	13.50	27.00	103–121	2000	15.00	30.00
114	2000	15.00	30.00	108	2200	16.50	33.00
122	2200	16.50	33.00	122–146	3000	22.50	45.00
124	2250	16.87	33.75	124	4000	30.00	60.00

To find horsepower at any other speed up to 1000 feet per minute, proceed as follows: Find horsepower in table for chain at 500 feet. Then multiply this horsepower by speed required and divide result by 500. *Example:* Required horsepower of M. O. chain No. 124 at 700 feet. Capacity of No. 124 M. O. chain at 500 feet = 60 horsepower. Then 60 × 700 ÷ 500 = 84 horsepower at 700 feet.

Roller-Chain Sprocket Diameters — 1

½-inch Pitch Diameter of Roller, 0.306 Inch				⅝-inch Pitch Diameter of Roller, 0.325 Inch			
No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter	No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter
8	1.307	1.550	1.001	8	1.633	1.896	1.308
9	1.462	1.705	1.156	9	1.827	2.090	1.502
10	1.618	1.862	1.312	10	2.023	2.285	1.698
11	1.775	2.018	1.469	11	2.218	2.481	1.893
12	1.932	2.175	1.626	12	2.415	2.677	2.090
13	2.089	2.364	1.783	13	2.612	2.905	2.287
14	2.247	2.522	1.941	14	2.809	3.102	2.484
15	2.405	2.680	2.099	15	3.006	3.300	2.681
16	2.563	2.838	2.257	16	3.204	3.497	2.879
17	2.721	3.027	2.415	17	3.401	3.726	3.076
18	2.879	3.185	2.573	18	3.599	3.924	3.274
19	3.038	3.344	2.732	19	3.797	4.122	3.472
20	3.196	3.502	2.890	20	3.995	4.320	3.670
21	3.355	3.661	3.049	21	4.194	4.519	3.869
22	3.513	3.819	3.207	22	4.392	4.717	4.067
23	3.672	3.978	3.366	23	4.590	4.915	4.265
24	3.831	4.137	3.525	24	4.788	5.113	4.463
25	3.989	4.295	3.683	25	4.987	5.312	4.662
26	4.148	4.454	3.842	26	5.185	5.510	4.860
27	4.307	4.613	4.001	27	5.384	5.709	5.059
28	4.466	4.772	4.160	28	5.582	5.907	5.257
29	4.625	4.931	4.319	29	5.781	6.106	5.456
30	4.783	5.089	4.477	30	5.979	6.304	5.654
31	4.942	5.248	4.636	31	6.178	6.503	5.853
32	5.101	5.407	4.795	32	6.376	6.701	6.051
33	5.260	5.566	4.954	33	6.575	6.900	6.250
34	5.419	5.725	5.113	34	6.774	7.099	6.449
35	5.578	5.884	5.272	35	6.972	7.297	6.647
36	5.737	6.043	5.431	36	7.171	7.496	6.846
37	5.896	6.202	5.590	37	7.370	7.695	7.045
38	6.055	6.361	5.749	38	7.569	7.894	7.244
39	6.214	6.520	5.908	39	7.767	8.092	7.442
40	6.373	6.679	6.067	40	7.966	8.291	7.641
41	6.532	6.838	6.226	41	8.165	8.490	7.840
42	6.691	6.997	6.385	42	8.363	8.688	8.038
43	6.850	7.156	6.544	43	8.562	8.887	8.237
44	7.009	7.315	6.703	44	8.761	9.086	8.436
45	7.168	7.474	6.862	45	8.960	9.285	8.635
46	7.327	7.633	7.021	46	9.159	9.484	8.834
47	7.486	7.792	7.180	47	9.357	9.682	9.032
48	7.645	7.951	7.339	48	9.556	9.881	9.231
49	7.804	8.110	7.498	49	9.755	10.080	9.430
50	7.963	8.269	7.657	50	9.954	10.279	9.629
51	8.122	8.428	7.816	51	10.153	10.478	9.828
52	8.281	8.587	7.975	52	10.351	10.676	10.026
53	8.440	8.746	8.134	53	10.550	10.875	10.225

Roller-Chain Sprocket Diameters — 2

⅝-inch Pitch, Diameter of Roller, 0.400 Inch				¾-inch Pitch, Diameter of Roller, 15/32 Inch			
No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter	No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter
8	1.633	1.953	1.233	9	2.193	2.661	1.725
9	1.827	2.147	1.427	10	2.427	2.895	1.959
10	2.023	2.342	1.623	11	2.662	3.130	2.194
11	2.218	2.538	1.818	12	2.898	3.360	2.430
12	2.415	2.734	2.015	13	3.134	3.602	2.666
13	2.612	2.971	2.212	14	3.370	3.838	2.902
14	2.809	3.168	2.409	15	3.607	4.075	3.139
15	3.006	3.366	2.606	16	3.844	4.312	3.376
16	3.204	3.563	2.804	17	4.081	4.549	3.613
17	3.401	3.801	3.001	18	4.319	4.787	3.851
18	3.599	3.999	3.199	19	4.557	5.025	4.089
19	3.797	4.197	3.397	20	4.795	5.263	4.327
20	3.995	4.395	3.595	21	5.033	5.501	4.565
21	4.194	4.594	3.794	22	5.271	5.739	4.803
22	4.392	4.792	3.992	23	5.508	5.976	5.040
23	4.590	4.990	4.190	24	5.746	6.214	5.278
24	4.788	5.188	4.388	25	5.984	6.452	5.516
25	4.987	5.387	4.587	26	6.222	6.690	5.754
26	5.185	5.585	4.785	27	6.460	6.928	5.992
27	5.384	5.784	4.984	28	6.698	7.166	6.230
28	5.582	5.982	5.182	29	6.936	7.404	6.468
29	5.781	6.181	5.381	30	7.175	7.643	6.707
30	5.979	6.379	5.579	31	7.413	7.881	6.945
31	6.178	6.578	5.778	32	7.652	8.120	7.184
32	6.376	6.776	5.976	33	7.890	8.358	7.422
33	6.575	6.975	6.175	34	8.129	8.597	7.661
34	6.774	7.174	6.374	35	8.367	8.835	7.899
35	6.972	7.372	6.572	36	8.605	9.073	8.137
36	7.171	7.571	6.771	37	8.844	9.312	8.376
37	7.370	7.770	6.970	38	9.083	9.551	8.615
38	7.569	7.969	7.169	39	9.321	9.789	8.853
39	7.767	8.167	7.367	40	9.560	10.028	9.092
40	7.966	8.366	7.566	41	9.798	10.266	9.330
41	8.165	8.565	7.765	42	10.037	10.505	9.569
42	8.363	8.763	7.963	43	10.274	10.742	9.806
43	8.562	8.962	8.162	44	10.512	10.980	10.044
44	8.761	9.161	8.361	45	10.751	11.219	10.283
45	8.960	9.360	8.560	46	10.990	11.458	10.422
46	9.159	9.559	8.759	47	11.229	11.697	10.861
47	9.357	9.757	8.957	48	11.468	11.936	11.000
48	9.556	9.956	9.156	49	11.706	12.174	11.238
49	9.755	10.155	9.355	50	11.945	12.413	11.477
50	9.954	10.354	9.554	51	12.184	12.652	11.716
51	10.153	10.553	9.753	52	12.422	12.890	11.954
52	10.351	10.751	9.951	53	12.661	13.129	12.193
53	10.550	10.950	10.150	54	12.900	13.368	12.432

Roller-Chain Sprocket Diameters — 3

1-inch Pitch Diameter of Roller, 9/16 Inch				1-inch Pitch Diameter of Roller, 5/8 Inch			
No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter	No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter
9	2.924	3.361	2.361	9	2.924	3.423	2.299
10	3.236	3.674	2.674	10	3.236	3.736	2.611
11	3.549	3.987	2.986	11	3.549	4.049	2.924
12	3.864	4.301	3.301	12	3.864	4.363	3.239
13	4.179	4.679	3.616	13	4.179	4.741	3.554
14	4.494	4.994	3.932	14	4.494	5.056	3.869
15	4.810	5.310	4.247	15	4.810	5.372	4.185
16	5.126	5.626	4.563	16	5.126	5.688	4.501
17	5.442	6.005	4.880	17	5.442	6.067	4.817
18	5.759	6.321	5.196	18	5.759	6.384	5.134
19	6.076	6.638	5.513	19	6.076	6.701	5.451
20	6.393	6.955	5.830	20	6.393	7.018	5.768
21	6.710	7.272	6.147	21	6.710	7.335	6.085
22	7.027	7.589	6.464	22	7.027	7.652	6.402
23	7.344	7.906	6.781	23	7.344	7.969	6.719
24	7.661	8.124	7.099	24	7.661	8.286	7.036
25	7.979	8.541	7.416	25	7.979	8.604	7.354
26	8.296	8.859	7.734	26	8.296	8.921	7.671
27	8.614	9.176	8.051	27	8.614	9.239	7.989
28	8.932	9.494	8.369	28	8.932	9.557	8.307
29	9.249	9.812	8.687	29	9.249	9.874	8.624
30	9.567	10.129	9.004	30	9.567	10.192	8.942
31	9.885	10.447	9.322	31	9.885	10.510	9.260
32	10.202	10.765	9.640	32	10.202	10.827	9.577
33	10.520	11.083	9.958	33	10.520	11.145	9.895
34	10.838	11.401	10.276	34	10.838	11.463	10.213
35	11.156	11.718	10.593	35	11.156	11.781	10.531
36	11.474	12.036	10.911	36	11.474	12.099	10.849
37	11.792	12.354	11.229	37	11.792	12.417	11.167
38	12.110	12.672	11.547	38	12.110	12.735	11.485
39	12.428	12.990	11.865	39	12.428	13.053	11.803
40	12.746	13.308	12.183	40	12.746	13.371	12.121
41	13.064	13.626	12.501	41	13.064	13.689	12.439
42	13.382	13.944	12.819	42	13.382	14.007	12.757
43	13.700	14.262	13.137	43	13.700	14.325	13.075
44	14.018	14.580	13.455	44	14.018	14.643	13.393
45	14.336	14.898	13.773	45	14.336	14.961	13.711
46	14.654	15.216	14.091	46	14.654	15.279	14.029
47	14.972	15.534	14.409	47	14.972	15.597	14.347
48	15.290	15.852	14.727	48	15.290	15.915	14.665
49	15.608	16.170	15.045	49	15.608	16.233	14.983
50	15.926	16.489	15.364	50	15.926	16.551	15.301
51	16.244	16.807	15.682	51	16.244	16.869	15.619
52	16.562	17.124	15.999	52	16.562	17.187	15.937
53	16.880	17.443	16.318	53	16.880	17.505	16.255
54	17.198	17.761	16.636	54	17.198	17.823	16.573

Roller-Chain Sprocket Diameters — 4

1¼-inch Pitch Diameter of Roller, ⅝ Inch				1¼-inch Pitch Diameter of Roller, ¾ Inch			
No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter	No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter
9	3.655	4.155	3.030	9	3.654	4.404	2.904
10	4.045	4.545	3.420	10	4.045	4.795	3.295
11	4.438	5.063	3.813	11	4.436	5.186	3.686
12	4.830	5.330	4.205	12	4.829	5.579	4.079
13	5.224	5.786	4.599	13	5.223	5.973	4.473
14	5.618	6.180	4.993	14	5.617	6.367	4.867
15	6.013	6.575	5.388	15	6.012	6.762	5.262
16	6.408	6.970	5.783	16	6.407	7.157	5.657
17	6.803	7.428	6.178	17	6.802	7.552	6.052
18	7.199	7.824	6.574	18	7.198	7.948	6.448
19	7.595	8.220	6.970	19	7.594	8.344	6.844
20	7.991	8.616	7.366	20	7.990	8.740	7.240
21	8.388	9.013	7.763	21	8.386	9.136	7.636
22	8.784	9.409	8.159	22	8.783	9.533	8.033
23	9.180	9.805	8.555	23	9.180	9.930	8.430
24	9.576	10.201	8.951	24	9.576	10.326	8.826
25	9.974	10.599	9.349	25	9.973	10.723	9.223
26	10.371	10.996	9.746	26	10.370	11.120	9.620
27	10.768	11.393	10.143	27	10.767	11.517	10.017
28	11.164	11.789	10.539	28	11.164	11.914	10.414
29	11.561	12.186	10.936	29	11.561	12.311	10.811
30	11.958	12.583	11.333	30	11.958	12.708	11.208
31	12.355	12.980	11.730	31	12.355	13.105	11.605
32	12.753	13.388	12.128	32	12.752	13.502	12.002
33	13.150	13.775	12.525	33	13.150	13.900	12.400
34	13.548	14.173	12.923	34	13.547	14.297	12.797
35	13.945	14.570	13.320	35	13.944	14.694	13.194
36	14.343	14.968	13.718	36	14.342	15.092	13.592
37	14.739	15.364	14.114	37	14.739	15.489	13.989
38	15.138	15.763	14.513	38	15.137	15.887	14.387
39	15.534	16.159	14.909	39	15.534	16.284	14.784
40	15.933	16.558	15.308	40	15.931	16.681	15.181
41	16.330	16.955	15.705	41	16.329	17.079	15.579
42	16.728	17.353	16.103	42	16.726	17.476	15.976
43	17.124	17.749	16.499	43	17.124	17.874	16.374
44	17.523	18.148	16.898	44	17.521	18.271	16.771
45	17.919	18.544	17.294	45	17.919	18.669	17.169
46	18.316	18.941	17.691	46	18.317	19.067	17.567
47	18.715	19.340	18.090	47	18.714	19.464	17.964
48	19.114	19.739	18.489	48	19.112	19.862	18.362
49	19.510	20.135	18.885	49	19.509	20.259	18.759
50	19.909	20.534	19.284	50	19.907	20.657	19.157
51	20.305	20.930	19.680	51	20.305	21.055	19.555
52	20.703	21.328	20.078	52	20.702	21.452	19.952
53	21.100	21.725	20.475	53	21.100	21.850	20.350
54	21.500	22.125	20.875	54	21.498	22.248	20.748

Roller-Chain Sprocket Diameters — 5

1½-inch Pitch Diameter of Roller, ¾ Inch				1½-inch Pitch Diameter of Roller, ⅞ Inch			
No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter	No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter
9	4.386	5.011	3.636	9	4.386	5.085	3.511
10	4.854	5.479	4.104	10	4.854	5.554	3.979
11	5.324	5.949	4.574	11	5.324	6.024	4.449
12	5.796	6.421	5.046	12	5.796	6.495	4.921
13	6.268	6.955	5.518	13	6.268	7.055	5.393
14	6.741	7.428	5.991	14	6.741	7.528	5.866
15	7.215	7.902	6.465	15	7.215	8.002	6.340
16	7.689	8.376	6.939	16	7.689	8.476	6.814
17	8.163	8.913	7.413	17	8.163	9.038	7.288
18	8.638	9.388	7.888	18	8.638	9.513	7.763
19	9.113	9.863	8.363	19	9.113	9.988	8.238
20	9.589	10.339	8.839	20	9.589	10.464	8.714
21	10.064	10.814	9.314	21	10.064	10.939	9.189
22	10.540	11.290	9.790	22	10.540	11.415	9.665
23	11.016	11.766	10.266	23	11.016	11.891	10.141
24	11.492	12.242	10.742	24	11.492	12.367	10.617
25	11.968	12.718	11.218	25	11.968	12.843	11.093
26	12.444	13.194	11.694	26	12.444	13.319	11.569
27	12.921	13.671	12.171	27	12.921	13.796	12.046
28	13.397	14.147	12.647	28	13.397	14.272	12.522
29	13.874	14.624	13.124	29	13.874	14.749	12.999
30	14.350	15.100	13.600	30	14.350	15.225	13.475
31	14.827	15.577	14.077	31	14.827	15.702	13.952
32	15.303	16.053	14.553	32	15.303	16.178	14.428
33	15.780	16.530	15.030	33	15.780	16.655	14.905
34	16.257	17.007	15.507	34	16.257	17.132	15.382
35	16.734	17.484	15.984	35	16.734	17.609	15.859
36	17.211	17.961	16.461	36	17.211	18.086	16.336
37	17.687	18.437	16.937	37	17.687	18.562	16.812
38	18.164	18.914	17.414	38	18.164	19.039	17.289
39	18.641	19.391	17.891	39	18.641	19.516	17.766
40	19.118	19.868	18.368	40	19.118	19.993	18.243
41	19.595	20.345	18.845	41	19.595	20.470	18.720
42	20.072	20.822	19.322	42	20.072	20.947	19.197
43	20.549	21.299	19.799	43	20.549	21.424	19.674
44	21.026	21.776	20.276	44	21.026	21.901	20.151
45	21.503	22.253	20.753	45	21.503	22.378	20.628
46	21.980	22.730	21.230	46	21.980	22.855	21.105
47	22.458	23.208	21.708	47	22.458	23.333	21.583
48	22.935	23.685	22.185	48	22.935	23.810	22.060
49	23.412	24.162	22.662	49	23.412	24.289	22.537
50	23.889	24.639	23.139	50	23.889	24.764	23.014
51	24.366	25.116	23.616	51	24.366	25.241	23.491
52	24.843	25.593	24.093	52	24.843	25.718	23.968
53	25.320	26.070	24.570	53	25.320	26.195	24.445
54	25.798	26.548	25.048	54	25.798	26.673	24.923

Roller-Chain Sprocket Diameters — 6

1¾-inch Pitch Diameter of Roller, 1 Inch				2-inch Pitch Diameter of Roller, 1⅛ Inch			
No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter	No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter
9	5.116	6.116	4.116	10	6.472	7.597	5.377
10	5.663	6.663	4.663	11	7.100	8.225	5.975
11	6.211	7.211	5.211	12	7.728	8.853	6.603
12	6.761	7.761	5.761	13	8.358	9.483	7.233
13	7.312	8.312	6.312	14	8.988	10.113	7.863
14	7.864	8.864	6.864	15	9.620	10.745	8.495
15	8.417	9.417	7.417	16	10.252	11.377	9.127
16	8.970	9.970	7.970	17	10.884	12.009	9.759
17	9.524	10.524	8.524	18	11.518	12.643	10.393
18	10.078	11.078	9.078	19	12.152	13.277	11.027
19	10.632	11.632	9.632	20	12.786	13.911	11.661
20	11.186	12.186	10.186	21	13.420	14.545	12.295
21	11.741	12.741	10.741	22	14.054	15.179	12.929
22	12.296	13.296	11.296	23	14.688	15.813	13.563
23	12.852	13.852	11.852	24	15.322	16.447	14.197
24	13.407	14.407	12.407	25	15.958	17.083	14.833
25	13.962	14.962	12.962	26	16.594	17.719	15.469
26	14.518	15.518	13.518	27	17.228	18.353	16.103
27	15.074	16.074	14.074	28	17.862	18.907	16.737
28	15.630	16.630	14.630	29	18.498	19.623	17.373
29	16.186	17.186	15.186	30	19.132	20.257	18.007
30	16.742	17.742	15.742	31	19.768	20.893	18.643
31	17.298	18.298	16.298	32	20.404	21.529	19.279
32	17.854	18.854	16.854	33	21.040	22.165	19.915
33	18.410	19.410	17.410	34	21.676	22.801	20.551
34	18.967	19.967	17.967	35	22.312	23.437	21.187
35	19.523	20.523	18.523	36	22.948	24.073	21.823
36	20.079	21.079	19.079	37	23.581	24.706	22.456
37	20.635	21.635	19.635	38	24.220	25.345	23.095
38	21.191	22.191	20.191	39	24.854	25.979	23.729
39	21.748	22.748	20.748	40	25.492	26.617	24.367
40	22.304	23.304	21.304	41	26.128	27.253	25.003
41	22.861	23.861	21.861	42	26.764	27.889	25.639
42	23.417	24.417	22.417	43	27.398	28.523	26.273
43	23.974	24.974	22.974	44	28.036	29.161	26.911
44	24.530	25.530	23.530	45	28.670	29.795	27.545
45	25.087	26.087	24.087	46	29.306	30.431	28.181
46	25.643	26.643	24.643	47	29.944	31.069	28.819
47	26.200	27.200	25.200	48	30.582	31.707	29.457
48	26.757	27.757	25.757	49	31.216	32.341	30.091
49	27.313	28.313	26.313	50	31.854	32.979	30.729
50	27.870	28.870	26.870	51	32.488	33.613	31.363
51	28.427	29.427	27.427	52	33.124	34.249	31.999
52	28.983	29.983	27.983	53	33.760	34.885	32.635
53	29.540	30.540	28.540	54	34.400	35.525	33.275
54	30.097	31.097	29 097	55	35.032	36.157	33.907

Block-Chain Sprocket Diameters

1-inch Pitch Diameter of Block, 0.325 Inch				1½-inch Pitch Diameter of Block, 0.532 Inch			
No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter	No. Teeth	Pitch Diameter	Outside Diameter	Base Diameter
6	1.935	2.260	1.610	9	4.322	4.729	3.790
7	2.249	2.574	1.924	10	4.797	5.204	4.265
8	2.565	2.890	2.240	11	5.272	5.679	4.740
9	2.881	3.206	2.556	12	5.748	6.155	5.216
10	3.198	3.523	2.873	13	6.224	6.694	5.692
11	3.515	3.840	3.190	14	6.700	7.170	6.168
12	3.832	4.157	3.507	15	7.177	7.646	6.645
13	4.149	4.474	3.824	16	7.653	8.123	7.121
14	4.467	4.792	4.142	17	8.130	8.662	7.598
15	4.785	5.110	4.460	18	8.607	9.139	8.075
16	5.102	5.427	4.777	19	9.084	9.616	8.552
17	5.420	5.745	5.095	20	9.560	10.092	9.028
18	5.738	6.063	5.413	21	10.037	10.569	9.505
19	6.056	6.381	5.731	22	10.514	11.046	9.982
20	6.374	6.699	6.049	23	10.991	11.523	10.459
21	6.692	7.017	6.367	24	11.468	12.000	10.936
22	7.010	7.335	6.685	25	11.946	12.478	11.414
23	7.328	7.653	7.003	26	12.423	12.955	11.891
24	7.646	7.971	7.321	27	12.900	13.432	12.368
25	7.964	8.289	7.639	28	13.377	13.909	12.845
26	8.282	8.607	7.957	29	13.854	14.386	13.322
27	8.600	8.925	8.275	30	14.331	14.863	13.799
28	8.918	9.243	8.593	31	14.809	15.341	14.277
29	9.236	9.561	8.911	32	15.286	15.818	14.754
30	9.554	9.879	9.229	33	15.763	16.295	15.231
31	9.872	10.197	9.547	34	16.240	16.772	15.708
32	10.191	10.516	9.866	35	16.718	17.250	16.186
33	10.509	10.834	10.184	36	17.195	17.727	16.663
34	10.827	11.152	10.502	37	17.672	18.204	17.140
35	11.145	11.470	10.820	38	18.149	18.681	17.617
36	11.463	11.788	11.138	39	18.627	19.159	18.095
37	11.781	12.106	11.456	40	19.104	19.636	18.572
38	12.100	12.425	11.775	41	19.581	20.113	19.049
39	12.418	12.743	12.093	42	20.059	20.591	19.527
40	12.736	13.061	12.411	43	20.536	21.068	20.004
41	13.054	13.379	12.729	44	21.013	21.545	20.481
42	13.373	13.698	13.048	45	21.491	22.023	20.959
43	13.691	14.016	13.366	46	21.968	22.500	21.436
44	14.009	14.334	13.684	47	22.446	22.978	21.914
45	14.327	14.652	14.002	48	22.923	23.455	22.391
46	14.645	14.970	14.320	49	23.400	23.932	22.868
47	14.964	15.289	14.639	50	23.878	24.410	23.346
48	15.282	15.607	14.957	51	24.355	24.887	23.823
49	15.600	15.925	15.275	52	24.832	25.364	24.300
50	15.918	16.243	15.593	53	25.310	25.842	24.778
51	16.237	16.562	15.912	54	25.787	26.319	25.255

Link-belt Driving Chains. — The working load recommended for detachable link-belting is determined as follows:

For a speed of 200 feet per minute and under, divide average ultimate strength by 6.

For a speed of 300 feet per minute and under, divide average ultimate strength by 8.

For a speed of 400 feet per minute and under, divide average ultimate strength by 10.

For a speed of 500 feet per minute and under, divide average ultimate strength by 12.

For a speed of 600 feet per minute and under, divide average ultimate strength by 16.

For a speed of 700 feet per minute and under, divide average ultimate strength by 20.

Average Ultimate Strength of Link-belts

Chain No.	Approx. Number of Links in 10 Feet	Average Ultimate Strength, Pounds	Chain No.	Approx. Number of Links in 10 Feet	Average Ultimate Strength, Pounds	Chain No.	Approx. Number of Links in 10 Feet	Average Ultimate Strength, Pounds
25	133	700	57	52	2800	93	30	7,500
32	104	1100	62	73	3100	95	30	8,700
33	86	1190	66	60	2600	103	39	9,600
34	86	1300	67	52	3300	108	25½	9,900
35	74	1200	75	46	4000	110	25½	12,700
42	88	1500	77	52	3600	114	37	11,000
45	74	1600	78	46	4900	122	20	15,000
51	104	1900	83	30	4950	124	30	12,700
52	80	2300	85	30	7600	146	20	14,000
55	74	2200	88	46	5750	....	......	......

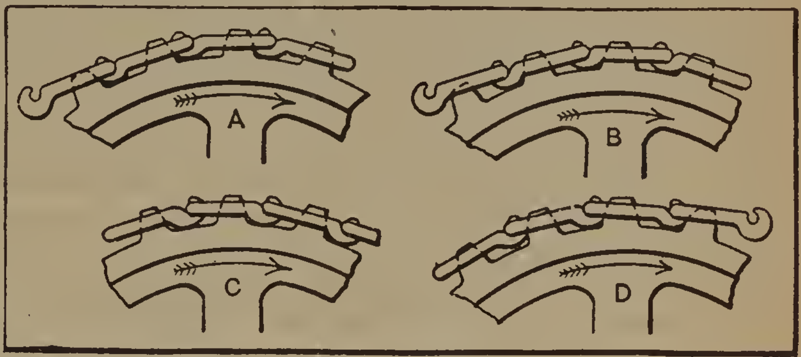
Example: — The average ultimate strength of a No. 35 chain is 1200 pounds, as shown by the table "Average Ultimate Strength of Link-belts." Therefore, the working strength at a speed of 200 feet per minute equals $1200 \div 6 = 200$ pounds.

In transmitting power, the engagement of each chain link with the sprocket wheel teeth is attended by a certain amount of shock and as this is intensified as the speed is increased, the working load of the chain should be reduced in a compensating ratio. If the load to be transmitted is irregular or subject to sudden variation, the working load should be reduced below what would be obtained by the foregoing figures.

To obtain the horsepower that can be transmitted by a link-belt, multiply the working strain (ascertained as in the foregoing) by the number of feet the chain travels per minute, and divide the result by 33,000.

Link-belt Size Numbers. — The sizes of detachable link-belting are designated by numbers. The numbers of Ewart link-belting and the average ultimate strength, in pounds, are given in the accompanying table. These chains are made in a number of different types or patterns suitable for different purposes. Some of these patterns are adapted for conveyor chains but are not suitable for power transmission, and *vice versa*. For ordinary transmission purposes, a chain of medium pitch is desirable. Typical chain numbers for power transmission are Nos. 35, 45, 67, 75, 77, 78, 88, 103, 114 and 124. Patterns such as Nos. 51, 52 and 62, which have a comparatively small tooth space are used for driving where back-lash or lost motion is objectionable. Sprocket wheels having a large number of teeth will not work well with these short-link chains, for a long period, because the small clearance between the teeth and chain links allows only a small amount of stretch before the chain tends to climb on top of the sprocket teeth.

Applying Link-belt Chains. — Whenever possible, a link-belt should run with the back of the coupling hook to the sprocket wheel. If it is necessary to run a sprocket in contact with the face side of a link-belt or the side on which the coupling hook is open, use "face wheels." The accompanying illustrations *A* and *C* represent wheels which drive the belts, and *B* and *D*, wheels which are driven by the belts. A long series of experiments based on conditions of wear on belts and sprockets at the points where they engage and disengage under strain, has demonstrated that for driving purposes or the transmission of power, the belt should run with the hook end of the link foremost, as at *C* and *D*; for ordinary elevator and conveyor work, the straight end-bar should run foremost, as at *A* and *B*, unless an elevator is handling fine gritty material which would have less tendency to work into the belt joints when running with the hook end foremost, as at *C* and *D*.



Sprockets for Detachable Chains. — In the design of sprockets for malleable and pressed steel chains of the detachable type, the ideal condition is to have the chordal pitch of the sprocket teeth exactly the same as the pitch of the chain, so that the load is equally distributed over all the teeth; but this is impracticable because the pitch of the chain lengthens as the result of the wear and stretching of the chain, which begins as soon as the chain is put into service. When the pitch of the chain has become greater than the chordal pitch of the sprocket, it is necessary for the chain to assume a larger pitch circle, and consequently it does not rest on the root circle, so that all the wear is on the faces of the teeth. In practice, this causes excessive wear on both the chain and the sprocket, and imparts a jerky, uneven motion to the drive. Therefore, it is considered good practice to anticipate this increase in the chain pitch and vary the sprocket sizes instead of making them to the dimensions which are theoretically correct.

Variations in Sizes of Sprockets. — To compensate for the elongation of the chain it is common practice to make the pitch of the driving sprocket larger than the theoretically correct pitch; the result is that the last tooth engaging the chain will carry the load and as it leaves the chain, the latter slips back until the next tooth takes the load. If the tooth curves are correct, this backward slipping or creeping movement of the chain will occur smoothly, the chain seating itself against the next tooth without shock. The entering tooth in this case has clearance and the chain comes quietly into contact with the root diameter of the sprocket. On the contrary, if the driving sprocket is smaller than the theoretical size, the entering tooth must "pick up" the load as the link slides into place on it. This means that the chain must be pushed ahead as the following tooth, in turn, picks up the load. As the tooth picks up the load while the chain is in the act of seating, this tends to prevent the link from seating to the full depth at once, which causes a jerking action of the chain and results in noisy operation.

The foregoing conditions are reversed when considering the driven sprocket. For example, if the driven sprocket is smaller than the theoretical diameter, the last tooth engaging the chain carries the load and, if properly shaped, it allows the chain to slip smoothly as it withdraws and as the succeeding link takes the load. The entering link also has plenty of clearance for seating to the full depth. If the driven sprocket, however, is made larger than the theoretical size, the entering tooth must pick up the load; hence there are the same objections previously mentioned in connection with the driving sprocket that is smaller than the theoretical size. For these reasons, the pitch and diameter of the driving sprocket should be larger than the theoretically correct dimensions, so that the *releasing* teeth are the working teeth and the chain can seat quietly and take the load gradually as the sprocket revolves. Some authorities advise making the driven sprocket smaller than the theoretical size, but this is not essential, because as soon as the chain stretches, its action on a driven sprocket of normal size will be the same as on an enlarged driving sprocket, and by making the driven sprocket the normal or theoretical size instead of under size, more space is left to provide for chain elongation. The diameter of the pattern for the driven sprocket (as measured with a "shrink rule") should be a little smaller than the theoretical size, however, to allow for

Average Pitches of Standard Link-belts

Chain No.	Average Pitch, Inches	Chain No.	Average Pitch, Inches	Chain No.	Average Pitch, Inches	Chain No.	Average Pitch, Inches	Chain No.	Average Pitch, Inches
25	0.902	45	1.630	66	2.013	85	4.000	110	4.720
32	1.154	51	1.155	67	2.308	88	2.609	114	3.250
33	1.394	52	1.506	75	2.609	93	4.033	122	6.050
34	1.398	55	1.631	77	2.293	95	3.967	124	4.063
35	1.630	57	2.308	78	2.609	103	3.075	146	6.150
42	1.375	62	1.654	83	4.000	108	4.720	....	.....

whatever increase there may be in the size of the casting as the result of the molding process, because over-size driven sprockets are as objectionable as under-size driving sprockets.

Pitch Diameter of Sprocket. — The pitch circle of a sprocket for malleable or detachable chains should intersect or pass through the pivot points or centers of the hook-shaped ends of the links. These chain links represent the sides of a polygon which is inscribed within the pitch circle. The following formulas, in which P = pitch diameter, p = pitch of chain, and N = number of teeth, give a theoretically correct pitch diameter:

$$P = \frac{p}{\text{Sin}\left(\frac{180}{N}\right)^\circ} = p \times \text{cosec}\left(\frac{180}{N}\right)^\circ$$

The theoretical pitch diameter obtained with one of these formulas may be increased in the case of a driver, or possibly decreased in the case of a driven sprocket, by adding or subtracting a fixed amount to compensate for chain elongation. The practice of the J. I. Case Threshing Machine Co., when making sprocket patterns, is first to obtain the correct or theoretical pitch diameter and then add a fixed amount to the driver and subtract a fixed amount from the driven sprocket so that the casting for the driver will be about 1/16 inch over size and the driven sprocket casting slightly under size. This practice is followed regardless of the size or number of the teeth in the sprockets. The exact amount to be allowed for variations in molding, shrinkage, etc., will depend somewhat upon the foundry doing the work. At the plant referred to, a driving sprocket pattern which is to be machine molded, is made 1/32 inch larger than the calculated diameter, which results in a casting that is about 1/16 inch over size. The driven sprocket pattern is made 1/16 inch smaller than the calculated size because the castings tend to "grow" or increase in size in the foundry. In the case of driven sprockets, the closer the castings are to the theoretical size, the better, provided none of the castings are over size as a result of too small an allowance. Patterns for driving sprockets which are to be hand molded are made to the calculated or theoretical size and the driven sprockets 3/32 inch under size. The patterns are, of course, measured with a shrink-rule.

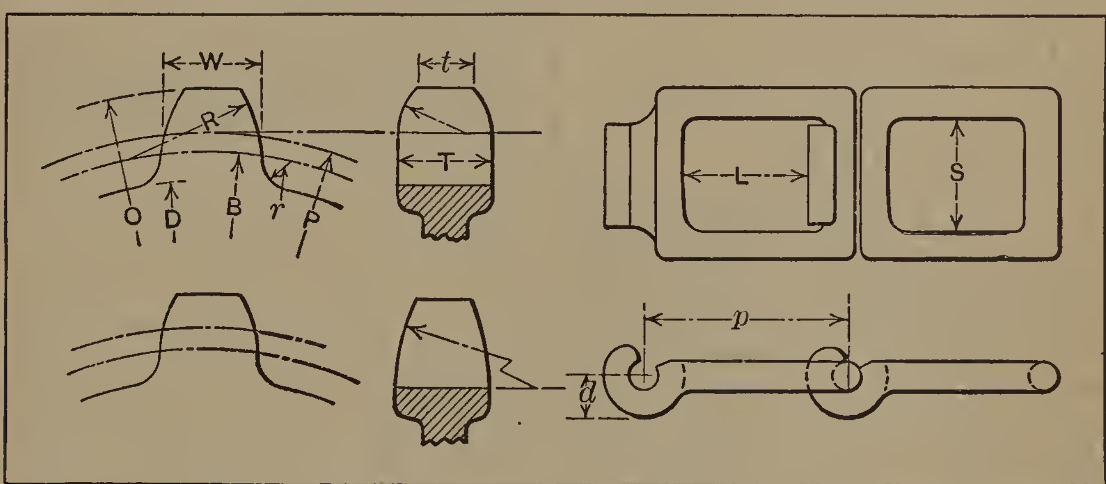
Example: Find the pitch diameter of a machine molded pattern for a driving sprocket having 20 teeth and intended for a No. 52 malleable chain.

The pitch (see table "Average Pitches of Standard Link Belts") is 1.506 inch; hence,

$$P = \frac{1.506}{\sin\left(\frac{180}{20}\right)^\circ} = 9.627 \text{ inches.}$$

$9.627 + 0.031 = 9.658$ or $9\frac{21}{32}$ inches nearly, which is the pattern diameter as measured with a shrink-rule.

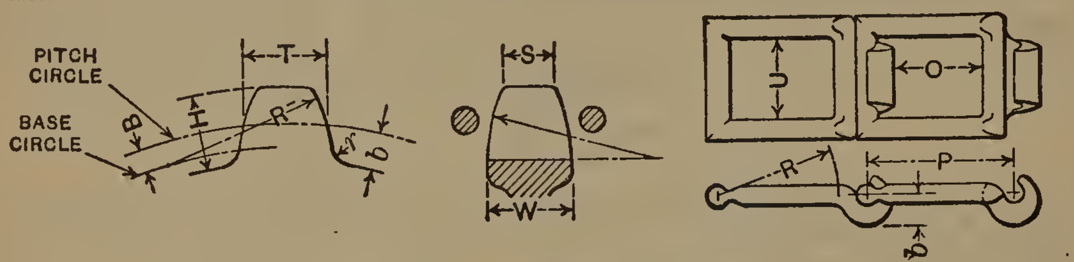
Root and Outside Diameters of Sprockets. — Root diameter D (see accompanying illustration) of a sprocket for detachable chain is found by subtracting



twice the dedendum d of the chain (or the distance from the center of the chain to the back of the hook-shaped end) from the pitch diameter. The outside diameter O is equal to the pitch diameter P plus twice the dedendum d of the chain, or the outside diameter may be reduced somewhat by adding to the pitch diameter from 1.6 to 1.8 times the dedendum instead of twice the dedendum. If the sprockets are originally designed as *driving* and *driven* sprockets, the shorter teeth or smaller outside diameter may be preferable, but a *combination sprocket* which is designed for use either as driver or driven, should have the longer teeth, as otherwise the chain might become caught on top of a tooth. The cross-sectional shape of the shorter tooth should be modified by locating the center of the arc on the root line instead of the pitch line, as shown by the lower sectional view in the illustration.

Design of Sprocket Teeth. — The arcs forming the faces of the sprocket teeth have a radius R and their centers are located on a base circle B (see illustration). Radius R may be taken as 3/4 of the chain pitch or it may be based upon the pitch diameter. According to one rule, R should equal 0.17 of the pitch diameter, but the minimum radius must not be less than the pitch p of the chain minus dedendum d. The radius of base circle B should be from 0.47 to 0.48 of the pitch circle diameter. The root radius r is usually from 0.6 to 0.7 of the dedendum d. The width W generally varies from 0.5 to 0.7 of the space L between the assembled links and may be as high as 0.8 L; but this will give, in most cases, a tooth that is wider than necessary, and in order to secure more clearance space to allow for chain elongation, the width W may be reduced. It is good practice to proportion width W with reference to the number of teeth in the sprocket, W being increased as the size of the sprocket or number of teeth decreases. If the width W is not increased in smaller sprockets, a tooth having the necessary clearance on the face may be either very narrow on top or entirely cut off below the circle representing the outside

Dimensions of Sprocket Teeth for Pressed Steel Chain *



All dimensions in inches

Number of Chain	Pitch of Chain	Face Radius	Pitch Circle to Base Circle	Backing b		Fillet Radius	Length of Opening	Thickness of Tooth	Height of Tooth	Width of Point	Width of Tooth	Width of Opening
	P	R	B	Mall.	Steel	r	O	T	H	S	W	U
25	0.902	1 1/16	1/8	0.203	0.177	3/32	9/16	5/16	3/8	3/16	11/32	7/16
32	1.154	7/8	1/8	0.245	0.227	1/8	1 1/16	7/16	7/16	1/4	1/2	19/32
34	1.398	1 1/16	1/8	0.262	0.257	5/32	27/32	1/2	1/2	5/16	9/16	21/32
35	1.630	1 1/4	3/16	0.262	0.300	3/16	1 1/8	5/8	9/16	3/8	11/16	25/32
42	1.375	1 1/16	1/8	0.282	0.262	5/32	27/32	9/16	9/16	11/32	5/8	3/4
45	1.630	1 1/4	3/16	0.293	0.300	3/16	1 1/16	5/8	9/16	3/8	11/16	25/32
52	1.506	1 1/8	3/16	0.346	0.300	3/16	7/8	5/8	5/8	13/32	3/4	13/16
55	1.631	1 1/4	3/16	0.353	0.317	3/16	1	5/8	9/16	3/8	11/16	25/32
62	1.654	1 1/4	3/16	0.410	0.332	3/16	1	5/8	5/8	1/2	7/8	1

* J. I. Case Threshing Machine Co.'s standard.

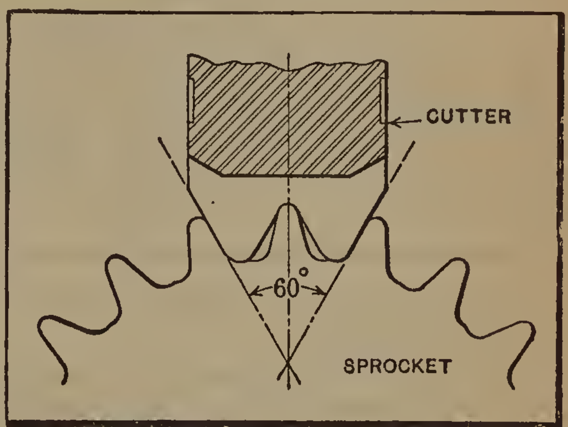
diameter. The tooth should have a flat on the top of at least 1/8 inch in width. The thickness T of the tooth should be from 0.85 to 0.90 of the space S between the sides of the links. The thickness t at the top usually varies from 0.55 to 0.60 of T.

Silent Chain Sprockets. — The outside and pitch diameters of Whitney silent chain sprockets are the same regardless of the number of teeth. The sprockets for Morse silent chains are without an addendum when the number of teeth in the sprocket is 32 or less. For sprockets having 33 teeth or more, the outside diameter equals the pitch diameter plus one-fifth of the pitch; the addendum = pitch ÷ 10; hence, the outside diameter also equals the pitch diameter + (2 × addendum). If D = pitch diameter; P = pitch; and N = number of teeth, then the Morse formula for pitch diameter is as follows: $D = \frac{P \times N}{3.1416}$. The pitch diameters of sprockets for the Whitney and Renold silent chains are determined as follows:

$$D = \frac{P}{\sin\left(\frac{180}{N}\right)^\circ} = P \times \operatorname{cosec}\left(\frac{180}{N}\right)^\circ$$

Tooth Forms for Silent Chains. — The three elements relating to the design of teeth for silent chains are: 1. The pitch or distance between the centers of the joints. 2. The face angle, or the angle between the working faces. 3. The radius of the tangency circle, or the distance from the center of the bearing to the working face of the link.

The Pitch. — The pitch, or distance between the centers of the bearings, depends upon the power that the chain is required to transmit, and the speed at which it is required to run. A chain of a given pitch can be built up to transmit more or less power by making it wider or narrower, that is by using longer or shorter studs or bearings, and thus giving a greater or less resistance to wear. There will, however, be a certain speed beyond which it is not wise to run a chain of any given pitch.



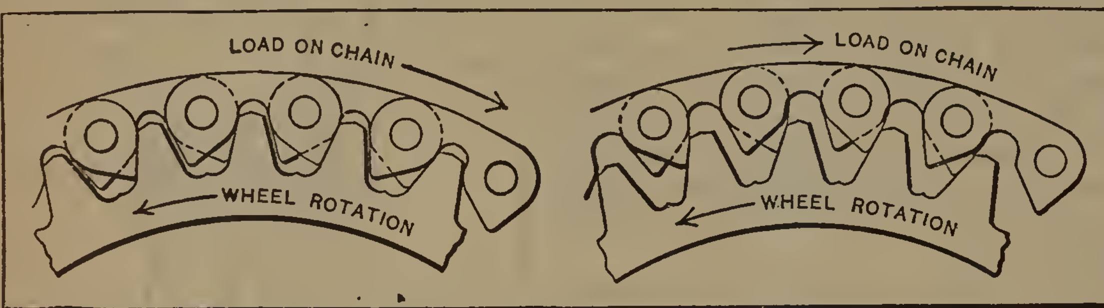
Face Angle. — The angle between the working faces that has been accepted by most chain manufacturers is 60 degrees, and this has been found by experiments with larger and smaller angles to be the most generally useful. (See accompanying illustration showing part of sprocket and cutter.) The effect of varying the angle is to raise or lower the maximum and minimum numbers of teeth that can be used in the wheels.

Tangency Radius. — In any given case the effects of using a larger tangency radius are: 1. To permit the use of a larger pin and, consequently, to increase the bearing area and durability of the chain. 2. To reduce the size of the tooth on the wheel, including its height. Standards for this element are desirable as they would have the effect of making it possible for any make of chain to run on any make of wheel of the same pitch.

The remainder of the outline of the link form is fixed more or less arbitrarily. The end of the link and the point of the tooth must be made of such form, and at such a distance from the center of the pin, that no fouling of the tooth at entry or on leaving can take place.

Characteristics of Silent Chain Drives. — The silent or "inverted-tooth" driving chain has the following characteristics: The chain passes over the face of the wheel like a belt and the wheel teeth do not project through it; the chain engages the wheel by means of teeth extending across the full width of the under side, with the exception of those chains having a central guide link; the chain teeth and wheel teeth are of such a shape that as the chain pitch increases through wear at the joints, the chain shifts outward upon the teeth, thus engaging the wheel on a pitch circle of increasing diameter; the result of this action is that the pitch of the wheel teeth increases at the same rate as the chain pitch. The accompanying illustration shows an unworn chain to the left, and a worn chain to the right, which has moved outward as the result of wear. Another distinguishing feature of the silent chain is that the power is transmitted by and to all the teeth in the arc of contact, irrespective of the increasing pitch due to elongation. The links have no sliding action either on or off the teeth, which results in a smooth and practically noiseless action, the chain being originally designed for the transmission of power at higher speeds than are suitable for roller chains.

The efficiency of the silent chain itself may be as high as 99 per cent, and for the complete drive, from 96 to 97 per cent, under favorable conditions; from 94 to 96 per cent can be secured with well-designed drives under average conditions. Silent chains are manufactured in pitches varying from 8 millimeters (0.315 inch) up to 3 inches. While the name "silent chain" is derived from the fact that the operation is practically noiseless, the term is not applicable to other types which may run silently, but is used to designate the inverted-tooth form of chain. The distinguishing feature of different makes of silent chain is in the joint, the other characteristics



being practically the same, except for variations in regard to accuracy and manufacturing methods.

The life and upkeep of silent chains depends largely upon the design of the entire drive, and especially upon the provision for adjustment. If there is much slack, the whipping of the chain will greatly increase the wear, and means of adjustment may double the life of the chain. A slight amount of play is necessary for satisfactory operation. The minimum amount of sag should be about ⅛ inch. Although the silent chain shifts outward from the teeth and adjusts itself for an increase of pitch, it cannot take up the increased pitch in that portion of the chain between the wheels; therefore, the wheel must lag to the extent of the increased pitch in the straight portion of the chain. For a given length, the angularity of this lag diminishes as the wheel size increases; hence it is advisable to use as large a wheel as is practicable.

Design of Silent Chain Drives. — The principal points to be considered when designing a silent chain transmission are the speed of the chain, the ratio of wheel diameters, the position of the drive, the efficiency of the lubricant, the center distance, the proportions of the chain and wheel teeth, whether the load is continuous or fluctuating, the strength of the chain relative to the power transmitted, and the design of the bearings.

Speed: The normal maximum speed is about 1300 feet per minute, but if the chain is provided with an oil bath, the speed may be increased to 1600 or even 1800 feet per minute on small sizes.

Ratio between Wheels: According to Renold, the maximum normal ratio desirable is 1 to 6, although 1 to 7½ may be operated. If possible, 17 or more teeth should be used in the pinion, the number being odd to secure a "hunting-tooth" effect, thus increasing the life of the chain and wheel.

Position of Drive: It is advisable to locate the drive as nearly horizontal as is consistent with other conditions. When in an inclined position, the line joining the wheel centers should preferably not exceed 60 degrees from the horizontal, and vertical drives should be avoided if practicable.

Lubrication: The life of a silent chain subjected to conditions such as are common to automobile drives, depends largely upon the wear of the joints. On account of the high speed and whipping action, it is important to have the chains well oiled. When splash lubrication is employed, the supply pipe should be placed so that the oil will be directed against the inside of the chain. It will then work through the links by the action of the centrifugal force.

Factor of Safety: The proportion between the breaking strength of the chain and the working load may vary from 30, for 1.75-inch pitch chains, to 50, for 0.5-inch pitch chains, at the maximum speed. These high factors are not needed to insure safety, but rather to obtain a moderate bearing pressure on the studs, to prolong the life of the chain.

Center Distance: According to Renold, the normal minimum distance between the shafts should equal 50 pitches of the chain used. The distance may, however, be

Data for "Whitney" Chains, Silent Type

Pitch, Inches	Widths, Inches: Between Guide Plates	Widths, Inches: Over Regular Rivets	Joint Wearing Area, Sq. In.	Weight Per Foot, Pounds
3/8	1/2	1 3/16	0.171	0.56
	3/4	1 1/16	0.247	0.74
	1	1 5/16	0.322	0.92
	1 1/4	1 9/16	0.398	1.10
	1 1/2	1 13/16	0.474	1.28
1/2	1/2	7/8	0.245	0.83
	3/4	1 1/8	0.349	1.08
	1	1 3/8	0.453	1.33
	1 1/4	1 5/8	0.557	1.58
	1 1/2	1 7/8	0.661	1.83
	1 3/4	2 1/8	0.765	2.08
	2	2 3/8	0.869	2.33
5/8	3/4	1 3/16	0.454	1.35
	1	1 7/16	0.588	1.66
	1 1/4	1 11/16	0.722	1.97
	1 1/2	1 15/16	0.855	2.28
	1 3/4	2 3/16	0.989	2.59
	2	2 7/16	1.123	2.90
	2 1/4	2 11/16	1.257	3.21
	2 1/2	2 15/16	1.390	3.52
	2 3/4	3 3/16	1.524	3.83
	3	3 7/16	1.658	4.14
	3 1/4	3 11/16	1.791	4.45
	3 1/2	3 15/16	1.925	4.76
3/4	3/4	1 1/4	0.558	1.74
	1	1 1/2	0.720	2.11

Pitch, Inches	Widths, Inches: Between Guide Plates	Widths, Inches: Over Regular Rivets	Joint Wearing Area, Sq. In.	Weight Per Foot, Pounds
3/4	1 1/4	1 3/4	0.881	2.48
	1 1/2	2	1.043	2.85
	1 3/4	2 1/4	1.204	3.22
	2	2 1/2	1.366	3.59
	2 1/4	2 3/4	1.528	3.96
	2 1/2	3	1.689	4.33
	2 3/4	3 1/4	1.851	4.70
	3	3 1/2	2.012	5.07
	3 1/4	3 3/4	2.174	5.44
	3 1/2	4	2.335	5.81
	3 3/4	4 1/4	2.497	6.18
	4	4 1/2	2.659	6.55
	4 1/2	5	2.982	7.29
1	1	1 9/16	0.955	3.24
	1 1/2	2 1/16	1.371	4.25
	2	2 9/16	1.788	5.26
	2 1/2	3 1/16	2.204	6.27
	3	3 9/16	2.620	7.28
	3 1/2	4 1/16	3.036	8.29
	4	4 9/16	3.453	9.30
	4 1/2	5 1/16	3.869	10.31
	5	5 9/16	4.285	11.32
	5 1/2	6 1/16	4.701	12.33
	6	6 9/16	5.118	13.34
	6 1/2	7 1/16	5.534	14.35
	7	7 9/16	5.950	15.36
	7 1/2	8 1/16	6.367	16.37
	8	8 9/16	6.783	17.38

Data for Design of Morse Silent Chain Drives

Pitch of chain (inches)	1/2	5/8	3/4	9/10	1 3/10	1 1/2	2	3
Minimum no. teeth: Small sprocket, driver	13	13	13	15	15	17	17	17
Small sprocket, driven	17	17	21	25	29	29	31	35
Desirable no. teeth in driving sprocket	15 17	17 21	17 21	17 23	17 23	17 27	17 31	19 31
Maximum no. teeth in sprockets	99	109	115	125	129	129	129	131
Desirable no. teeth in driven sprocket	55 75	55 75	55 85	55 95	55 105	55 115	55 115	55 115
To find pitch diam. multiply no. teeth by	0.159	0.199	0.239	0.2865	0.382	0.477	0.636	0.955
Addendum	0.05	0.06	0.075	0.09	0.12	0.15	0.20	0.30
Maximum R.P.M.	2400	1800	1200	1100	800	600	400	250
Tension per inch of chain width, pounds: Small sprocket, driver	80	100	120	150	200	270	450	750
Small sprocket, driven	65	80	95	120	160	210	350	600
Radial clearance beyond tooth, inches	0.50	0.62	0.75	0.90	1.2	1.5	2.0	3.0
Approx. weight of chain per ft., 1 inch wide	1.00	1.20	1.50	1.80	2.50	3.00	4.00	6.00

Number of teeth = T: exact outside diam. = D. When T is less than 33, D = pitch diam. When T is more than 32, D = pitch diam. + (2 × addendum). Thickness of sprocket rim, including teeth, should be at least 1.2 times chain pitch. Width of sprocket should be 1/8 to 1/4 inch greater than nominal width of chain. An even number of links in chain, and an odd number of teeth in sprockets is desirable. Maximum linear velocity for commercial service, 1200 to 1600 feet per minute. Adjustable sprocket centers are desirable for horizontal drives and essential for vertical drives.

reduced until the wheels just clear, but the life of the chain also decreases out of all proportion to the decrease in the center distance. According to the Coventry Chain Co., when the ratio of the drive is less than 3 to 1, the center distance should agree with the following formula, which applies only to two-wheel drives:

$$\text{Center distance} = \frac{D + d + p}{2}$$

in which D = pitch diameter of large wheel; d = diameter of small wheel; p = length of pitch in inches.

Number of Links: When the chain is designed to run over sprockets having a fixed center distance, the drive should be so proportioned that there will be an even number of links. This will permit the chain ends to match, whereas, if an odd number is used, a "cranked link" will have to be employed, which materially weakens the chain.

Silent Chain Sprocket Diameters
(Link-Belt Co.)

No. Teeth	Pitch Diam.	No. Teeth	Pitch Diam.	No. Teeth	Pitch Diam.	No. Teeth	Pitch Diam.	No. Teeth	Pitch Diam.
3/8-inch Pitch									
13	1.567	34	4.064	55	6.569	76	9.075	97	11.581
14	1.685	35	4.183	56	6.688	77	9.194	98	11.700
15	1.803	36	4.303	57	6.807	78	9.313	99	11.820
16	1.922	37	4.422	58	6.927	79	9.432	100	11.939
17	2.041	38	4.541	59	7.046	80	9.552	101	12.058
18	2.160	39	4.660	60	7.165	81	9.671	102	12.178
19	2.278	40	4.780	61	7.284	82	9.791	103	12.300
20	2.397	41	4.899	62	7.404	83	9.910	104	12.416
21	2.516	42	5.018	63	7.523	84	10.029	105	12.536
22	2.635	43	5.137	64	7.643	85	10.149	106	12.655
23	2.754	44	5.257	65	7.762	86	10.268	107	12.775
24	2.872	45	5.376	66	7.881	87	10.387	108	12.893
25	2.992	46	5.495	67	8.001	88	10.507	109	13.013
26	3.111	47	5.614	68	8.120	89	10.626	110	13.132
27	3.230	48	5.734	69	8.239	90	10.745	111	13.252
28	3.349	49	5.853	70	8.359	91	10.865	112	13.371
29	3.468	50	5.972	71	8.478	92	10.984	113	13.491
30	3.587	51	6.092	72	8.597	93	11.103	114	13.610
31	3.707	52	6.211	73	8.717	94	11.223	115	13.730
32	3.826	53	6.330	74	8.836	95	11.342	116	13.849
33	3.945	54	6.449	75	8.955	96	11.461	117	13.968
1/2-inch Pitch									
15	2.424	36	5.783	57	9.149	78	12.517	99	15.885
16	2.583	37	5.943	58	9.310	79	12.677	100	16.045
17	2.743	38	6.103	59	9.470	80	12.838	101	16.206
18	2.902	39	6.264	60	9.630	81	12.998	102	16.366
19	3.062	40	6.424	61	9.791	82	13.158	103	16.527
20	3.222	41	6.584	62	9.951	83	13.319	104	16.687
21	3.382	42	6.744	63	10.111	84	13.479	105	16.848
22	3.542	43	6.905	64	10.272	85	13.640	106	17.008
23	3.701	44	7.065	65	10.432	86	13.800	107	17.168
24	3.861	45	7.225	66	10.592	87	13.960	108	17.329
25	4.021	46	7.385	67	10.753	88	14.121	109	17.489
26	4.181	47	7.546	68	10.913	89	14.281	110	17.650
27	4.341	48	7.706	69	11.073	90	14.442	111	17.810
28	4.501	49	7.866	70	11.234	91	14.602	112	17.970
29	4.662	50	8.027	71	11.394	92	14.762	113	18.131
30	4.822	51	8.187	72	11.555	93	14.923	114	18.291
31	4.982	52	8.347	73	11.715	94	15.083	115	18.452
32	5.142	53	8.508	74	11.875	95	15.244	116	18.612
33	5.302	54	8.668	75	12.036	96	15.404	117	18.772
34	5.462	55	8.828	76	12.196	97	15.564	118	18.933
35	5.623	56	8.989	77	12.356	98	15.725	119	19.093

Center Distances, 3/8-Inch Pitch Silent Chain*

(Chain of even number of links)

Number of Pitches in Chain Strand	Number of Teeth in Sprockets						
	15–30	17–34	19–38	21–42	23–46	25–50	27–54
50	5.076	4.476	3.864	....	....	....	....
52	5.456	4.861	4.254	....	....	....	....
54	5.837	5.244	4.643	4.022	....	....	....
56	6.216	5.627	5.028	4.414	....	....	....
58	6.595	6.008	5.412	4.804	....	....	....
60	6.974	6.387	5.795	5.190	4.573	....	....
62	7.352	6.766	6.177	5.575	4.965	....	....
64	7.730	7.145	6.557	5.959	5.353	4.731	....
66	8.107	7.523	6.937	6.343	5.740	5.125	....
68	8.484	7.901	7.317	6.725	6.126	5.515	....
70	8.861	8.278	7.696	7.107	6.511	5.903	5.282
72	9.238	8.655	8.076	7.488	6.893	6.290	5.675
74	9.614	9.032	8.454	7.868	7.276	6.676	6.064
76	9.990	9.410	8.833	8.247	7.657	7.061	6.452
78	10.367	9.788	9.211	8.627	8.038	7.444	6.840
80	10.744	10.165	9.590	9.006	8.418	7.826	7.226
82	11.120	10.542	9.967	9.384	8.799	8.208	7.611
84	11.497	10.919	10.344	9.763	9.178	8.589	7.994
86	11.873	11.296	10.721	10.141	9.557	8.969	8.376
88	12.249	11.674	11.098	10.519	9.936	9.349	8.758
90	12.625	12.051	11.475	10.896	10.314	9.729	9.139
92	13.000	12.427	11.851	11.273	10.692	10.109	9.520
94	13.376	12.804	12.228	11.651	11.070	10.488	9.900
96	13.752	13.180	12.604	12.028	11.448	10.868	10.280
100	14.503	13.931	13.357	12.782	12.203	11.625	11.038

Number of Pitches in Chain Strand	Number of Teeth in Sprockets						
	15–15	17–17	19–19	21–21	23–23	25–25	27–27
50	6.562	6.187	5.812	5.437	5.062	4.687	4.312
52	6.937	6.562	6.187	5.812	5.437	5.062	4.687
54	7.312	6.937	6.562	6.187	5.812	5.437	5.062
56	7.687	7.312	6.937	6.562	6.187	5.812	5.437
58	8.062	7.687	7.312	6.937	6.562	6.187	5.812
60	8.437	8.062	7.687	7.312	6.937	6.562	6.187
62	8.812	8.437	8.062	7.687	7.312	6.937	6.562
64	9.187	8.812	8.437	8.062	7.687	7.312	6.937
66	9.562	9.187	8.812	8.437	8.062	7.687	7.312
68	9.937	9.562	9.187	8.812	8.437	8.062	7.687
70	10.312	9.937	9.562	9.187	8.812	8.437	8.062
72	10.687	10.312	9.937	9.562	9.187	8.812	8.437
74	11.062	10.687	10.312	9.937	9.562	9.187	8.812
76	11.437	11.062	10.687	10.312	9.937	9.562	9.187
78	11.812	11.437	11.062	10.687	10.312	9.937	9.562
80	12.187	11.812	11.437	11.062	10.687	10.312	9.937
82	12.562	12.187	11.812	11.437	11.062	10.687	10.312
84	12.937	12.562	12.187	11.812	11.437	11.062	10.687
86	13.312	12.937	12.562	12.187	11.812	11.437	11.062
88	13.687	13.312	12.937	12.562	12.187	11.812	11.437
90	14.062	13.687	13.312	12.937	12.562	12.187	11.812
92	14.437	14.062	13.687	13.312	12.937	12.562	12.187
94	14.812	14.437	14.062	13.687	13.312	12.937	12.562
96	15.187	14.812	14.437	14.062	13.687	13.312	12.937
100	15.937	15.562	15.187	14.812	14.437	14.062	13.687

* Link-Belt Co.

Center Distances, ½-Inch Pitch Silent Chain*

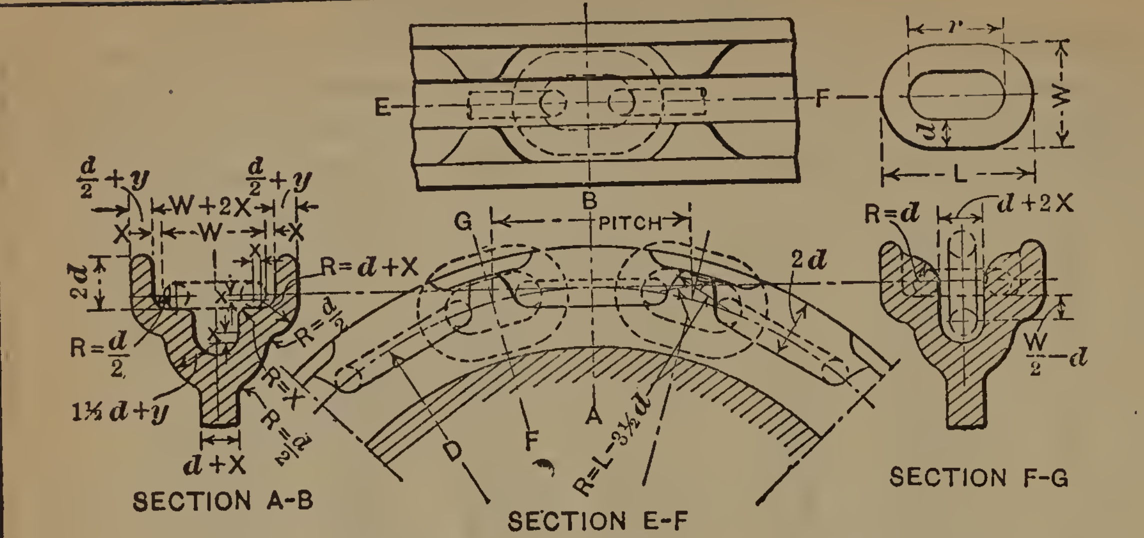
(Chain of even number of links)

Number of Pitches in Chain Strand	Number of Teeth in Sprockets						
	15-30	17-34	19-38	21-42	23-46	25-50	27-54
50	6.823	6.016	5.193	....	....	....	....
52	7.334	6.533	5.718	....	....	....	....
54	7.845	7.049	6.240	5.406	....	....	....
56	8.355	7.563	6.758	5.933	....	....	....
58	8.864	8.075	7.274	6.457	....	....	....
60	9.373	8.585	7.789	6.976	6.147	....	....
62	9.881	9.094	8.302	7.493	6.673	....	....
64	10.389	9.603	8.813	8.009	7.195	6.359	....
66	10.896	10.111	9.324	8.525	7.715	6.888	....
68	11.403	10.619	9.834	9.039	8.234	7.412	....
70	11.910	11.126	10.344	9.552	8.751	7.934	7.100
72	12.416	11.633	10.854	10.064	9.265	8.454	7.628
74	12.922	12.140	11.363	10.575	9.779	8.973	8.151
76	13.428	12.647	11.872	11.085	10.292	9.490	8.672
78	13.934	13.155	12.380	11.595	10.804	10.005	9.193
80	14.440	13.662	12.888	12.104	11.315	10.519	9.712
82	14.946	14.169	13.396	12.613	11.826	11.032	10.229
84	15.452	14.676	13.903	13.122	12.336	11.544	10.744
86	15.958	15.183	14.410	13.630	12.845	12.055	11.258
88	16.463	15.690	14.917	14.138	13.354	12.566	11.771
90	16.968	16.197	15.423	14.645	13.863	13.077	12.283
92	17.473	16.703	15.929	15.152	14.371	13.587	12.795
94	17.978	17.209	16.435	15.659	14.879	14.097	13.306
96	18.483	17.714	16.941	16.166	15.387	14.607	13.817
98	18.988	18.219	17.447	16.673	15.895	15.116	14.327
100	19.493	18.724	17.953	17.180	16.402	15.625	14.836

Number of Pitches in Chain Strand	Number of Teeth in Sprockets						
	15-15	17-17	19-19	21-21	23-23	25-25	27-27
50	8.820	8.316	7.812	7.308	6.804	6.300	5.796
52	9.324	8.820	8.316	7.812	7.308	6.804	6.300
54	9.828	9.324	8.820	8.316	7.812	7.308	6.804
56	10.332	9.828	9.324	8.820	8.316	7.812	7.308
58	10.836	10.332	9.828	9.324	8.820	8.316	7.812
60	11.340	10.836	10.332	9.828	9.324	8.820	8.316
62	11.844	11.340	10.836	10.332	9.828	9.324	8.820
64	12.348	11.844	11.340	10.836	10.332	9.828	9.324
66	12.852	12.348	11.844	11.340	10.836	10.332	9.828
68	13.356	12.852	12.348	11.844	11.340	10.836	10.332
70	13.860	13.356	12.852	12.348	11.844	11.340	10.836
72	14.364	13.860	13.356	12.852	12.348	11.844	11.340
74	14.868	14.364	13.860	13.356	12.852	12.348	11.844
76	15.372	14.868	14.364	13.860	13.356	12.852	12.348
78	15.876	15.372	14.868	14.364	13.860	13.356	12.852
80	16.380	15.876	15.372	14.868	14.364	13.860	13.356
82	16.884	16.380	15.876	15.372	14.868	14.364	13.860
84	17.388	16.884	16.380	15.876	15.372	14.868	14.364
86	17.892	17.388	16.884	16.380	15.876	15.372	14.868
88	18.396	17.892	17.388	16.884	16.380	15.876	15.372
90	18.900	18.396	17.892	17.388	16.884	16.380	15.876
92	19.404	18.900	18.396	17.892	17.388	16.884	16.380
94	19.908	19.404	18.900	18.396	17.892	17.388	16.884
96	20.412	19.908	19.404	18.900	18.396	17.892	17.388
98	20.916	20.412	19.908	19.404	18.900	18.396	17.892
100	21.420	20.916	20.412	19.908	19.404	18.900	18.396

* Link-Belt Co.

Sprocket Wheels for Ordinary Link Chain



$$D=\sqrt{\left(\frac{r}{\sin\alpha}\right)^2+\left(\frac{d}{\cos\alpha}\right)^2}\qquad \alpha=\frac{90}{N}\qquad N=\text{number of teeth.}$$

d = Size of Chain	L = Length of Link	W = Width of Link	No. of Teeth N = 7	8	9	10	11	12	13	14	15	16	17	X	y
		Angle α =	12°51′	11°15′	10°0′	9°0′	8°11′	7°30′	6°55′	6°25′	6°0′	5°37′	5°17′		
			D = Pitch Diameter in Inches												
3/16	1 3/8	1 3/16	4.50	5.13	5.76	6.40	7.03	7.66	8.29	8.93	9.57	10.20	10.84	1/16	3/32
1/4	1 1/2	1	4.50	5.13	5.76	6.40	7.03	7.66	8.29	8.93	9.57	10.20	10.84	3/32	3/32
5/16	1 3/4	1 3/16	5.06	5.77	6.48	7.18	7.91	8.62	9.33	10.05	10.76	11.47	12.19	3/32	3/32
3/8	2	1 3/8	5.63	6.42	7.21	8.00	8.79	9.59	10.38	11.17	11.96	12.76	13.56	3/32	3/32
7/16	2 1/4	1 9/16	6.18	7.06	7.74	8.79	9.67	10.53	11.41	12.28	13.16	14.03	14.90	3/32	3/32
1/2	2 1/2	1 3/4	6.76	7.71	8.65	9.61	10.55	11.49	12.45	13.40	14.35	15.30	16.26	3/32	1/16
9/16	2 7/8	1 15/16	7.88	8.97	10.08	11.19	12.30	13.41	14.52	15.63	16.74	17.85	18.97	1/8	1/16
5/8	3 1/4	2 1/8	9.01	10.27	11.53	12.80	14.07	15.33	16.60	17.90	19.14	20.41	21.68	1/8	1/16
11/16	3 1/2	2 5/16	9.58	10.91	12.26	13.61	14.95	16.29	17.65	18.99	20.34	21.69	23.04	1/8	1/16
3/4	3 3/4	2 1/2	10.14	11.56	12.98	14.40	15.83	17.26	18.68	20.06	21.54	22.97	24.40	1/8	1/16
13/16	4	2 11/16	10.71	12.20	13.72	15.21	16.71	18.20	19.72	21.23	22.74	24.24	25.75	1/8	1/16
7/8	4 1/4	3	11.27	12.85	14.43	16.01	17.55	19.17	20.76	22.35	23.93	25.52	27.11	1/8	1/16
15/16	4 1/2	3 1/4	11.84	13.50	15.15	16.81	18.47	20.13	21.80	23.46	25.13	26.80	28.47	1/8	1/16
1	4 3/4	3 1/2	12.40	14.13	15.87	17.61	19.35	21.09	22.84	24.58	26.33	28.08	29.83	1/8	...

d	L	W	No. of Teeth N = 18	19	20	21	22	23	24	25	26	27	28	X	y
		Angle α =	5°0′	4°44′	4°30′	4°17′	4°6′	3°55′	3°45′	3°36′	3°28′	3°20′	3°13′		
3/16	1 3/8	1 3/16	11.47	12.11	12.75	13.38	14.02	14.66	15.29	15.93	16.56	17.20	17.84	1/16	3/32
1/4	1 1/2	1	11.47	12.11	12.75	13.38	14.02	14.66	15.29	15.93	16.56	17.20	17.84	3/32	3/32
5/16	1 3/4	1 3/16	12.91	13.62	14.34	15.05	15.77	16.49	17.20	17.92	18.62	19.34	20.06	3/32	3/32
3/8	2	1 3/8	14.36	15.16	15.96	16.74	17.53	18.32	19.11	19.90	20.70	21.50	22.29	3/32	3/32
7/16	2 1/4	1 9/16	15.78	16.65	17.53	18.40	19.27	20.15	21.02	21.90	22.77	23.65	24.52	3/32	3/32
1/2	2 1/2	1 3/4	17.21	18.16	19.12	20.07	21.03	21.98	22.94	23.89	24.85	25.80	26.75	3/32	1/16
9/16	2 7/8	1 15/16	20.08	21.19	22.30	23.42	24.53	25.64	26.76	27.87	28.98	30.10	31.21	1/8	1/16
5/8	3 1/4	2 1/8	22.95	24.22	25.50	26.77	28.03	29.31	30.58	31.85	33.13	34.40	35.67	1/8	1/16
11/16	3 1/2	2 5/16	24.34	25.73	27.09	28.44	29.79	31.14	32.49	33.84	35.20	36.55	37.90	1/8	1/16
3/4	3 3/4	2 1/2	25.83	27.26	28.69	30.12	31.55	32.97	34.41	35.84	37.27	38.70	40.04	1/8	1/16
13/16	4	2 11/16	27.26	28.77	30.28	31.79	33.30	34.81	36.32	37.83	39.34	40.85	...	1/8	1/16
7/8	4 1/4	3	28.70	30.29	31.88	33.46	35.04	36.63	38.23	39.82	41.41	...	...	1/8	1/16
15/16	4 1/2	3 1/4	30.14	31.80	33.46	35.13	36.83	38.48	40.15	...	...	...	...	1/8	1/16
1	4 3/4	3 1/2	31.57	33.31	35.06	36.81	38.56	40.30	...	...	...	...	...	1/8	...

CRANE CHAIN AND HOOKS

Material for Crane Chains. — The best material for crane and hoisting chains is a good grade of wrought iron, in which the percentage of phosphorus, sulphur, silicon and other impurities is comparatively low. The tensile strength of the best grades of wrought iron does not exceed 46,000 pounds per square inch, whereas mild steel with about 0.15 per cent carbon has a tensile strength nearly double this amount. The ductility and toughness of wrought iron, however, is greater than that of ordinary commercial steel, and for this reason it is preferable for chains subjected to heavy intermittent strains, because wrought iron will always give warning by bending or stretching, before breaking. Another important reason for using wrought iron in preference to steel is that a perfect weld can be effected more easily.

Strength of Chains. — When calculating the strength of chains, it should be observed that the strength of a link subjected to tensile stresses is not equal to twice the strength of an iron bar of the same diameter as the link stock, but is a certain amount less, owing to the bending action caused by the manner in which the load is applied to the link. The strength is also reduced somewhat by the weld. The following empirical formula is commonly used for calculating the breaking load, in pounds, of wrought-iron crane chains:

$$W = 54{,}000\,D^2$$

in which W = breaking load in pounds, and D = diameter of bar (in inches) from which links are made. The working load for chains should not exceed one-third the value of W, and, in many cases, it is one-fourth or one-fifth of the breaking load. When a chain is wound around a casting and severe bending stresses are introduced, a greater factor of safety should be used.

Safe Loads in Tons for Ropes, Chains and Cables

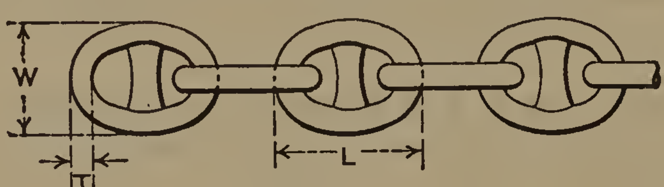
Manila Rope*				Chains				Wire Cable			
Rope Diam.	Single Rope	Two Part	Four Part	Diam. Link Stock	Single Chain	Two Part	Four Part	Cable Diam.	Single Cable	Two Part	Four Part
½	⅛	¼	½	¼	½	⅞	1½	½	1	2	3½
⅝	¼	½	¾	⅜	1	1¾	3	⅝	1¾	3¼	6½
¾	⅜	¾	1¼	½	2	3½	6	¾	2½	4½	9
⅞	½	1	2	⅝	3	5	9	⅞	3¼	6	12
1	¾	1½	2½	¾	5	9	15	1	4	8	16
1¼	1	2	3	⅞	6	10½	18	1¼	6	12	24
1½	1¼	2½	4	1	8	14	24	1½	10	19	36
1¾	2	4	6	1⅛	11	19	33	1¾	13	25	48
2	2½	5	8	1¼	13	23	39	2	16	32	60
2¼	3½	6½	11	1½	18	32	54	...	...	...	...
2½	4½	8	13	....	...	...	...	...	...	...	...

* These figures apply only to a rope in fairly good condition.

Care of Hoisting and Crane Chains. — All chains used for hoisting heavy loads are subject to deterioration, both apparent and invisible. The links wear, and there is also an alteration in the nature or fiber of the material, owing to the strains, shocks, etc., producing crystallization. Chain wear can be reduced considerably by occasional lubrication. The life of a chain can also be prolonged by frequent annealing, as this restores the fibrous quality of the material to some extent, although there may be a slight decrease in the tensile strength. To anneal a chain, heat it to cherry-red and allow it to cool slowly. This should be done every six months, and oftener if the chain is subjected to unusually severe service. Chains should be examined periodically for twists, as a twisted chain will wear rapidly. Any links which have worn excessively should be replaced with new ones, so that every link will do its full share of work during the life of the chain, without exceeding the limit of safety. Chains for hoisting purposes should be made with short links, so that they will wrap closely around the sheaves or drums without bending. The diameter of the winding drums should be not less than 25 or 30 times the diameter of the iron used for the links.

Studded Chains. — Tests have demonstrated that the ultimate breaking strength of a chain with studded links is less than that of an unstudded chain. This is probably due to the fact that the open links of an unstudded chain collapse until the sides are approximately parallel, so that the stresses are lower than in the studded links, the sides of which are prevented from collapsing by the studs. The principal function of the stud is to prevent the chain from kinking and catching, so that it will run free from chain lockers, etc. The stud also prevents the chain from becoming rigid under heavy strains.

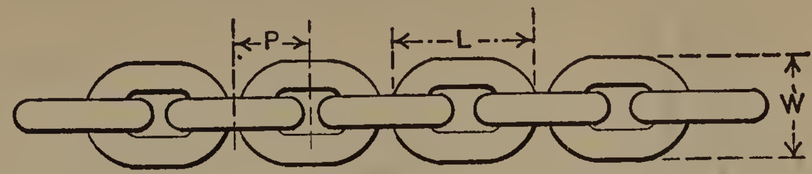
Studded Cable Chain



Size of Chain in Inches	Length of Link in Inches	Width of Link in Inches	Weight per Foot of Chain	Proof Test	Size of Chain in Inches	Length of Link in Inches	Width of Link in Inches	Weight per Foot of Chain	Proof Test
T	*L*	*W*	Pounds	Tons	*T*	*L*	*W*	Pounds	Tons
3/4	4 3/8	2 3/4	5.5	10.1	1 1/2	8 1/2	5 3/8	21.2	40.5
13/16	4 3/4	3	6.3	12.0	1 9/16	8 7/8	5 5/8	23.8	44.0
7/8	5	3 1/4	8.2	13.7	1 5/8	9 1/4	5 7/8	25.0	47.5
15/16	5 3/8	3 1/2	9.2	15.7	1 11/16	9 5/8	6	26.2	51.2
1	5 7/8	3 3/4	10.2	18.0	1 3/4	10	6 1/4	28.8	55.2
1 1/16	6 1/4	3 7/8	11.5	20.3	1 7/8	10 1/2	6 3/4	33.8	63.3
1 1/8	6 1/2	4 1/8	12.3	22.8	1 15/16	10 3/4	7	35.8	67.5
1 3/16	6 3/4	4 1/4	13.5	25.5	2	11 1/8	7 1/4	38.8	72.0
1 1/4	7 1/8	4 1/2	15.0	28.1	2 1/16	11 1/2	7 1/2	42.3	76.5
1 5/16	7 3/8	4 5/8	16.2	31.0	2 1/8	12	7 3/4	46.0	81.2
1 3/8	7 3/4	4 7/8	18.3	34.0	2 3/16	12 1/2	8	48.3	86.1
1 7/16	8 1/8	5 1/8	18.8	37.2	2 1/4	13	8 1/4	50.0	91.0

Note: Safe working loads are one-half of proof test loads.

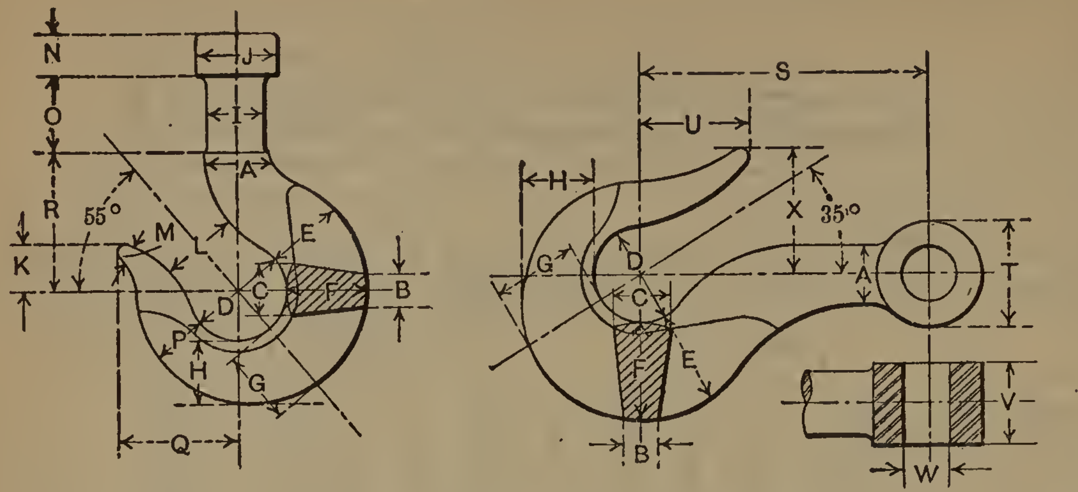
Close-link Hoisting, Sling and Crane Chain



Size	Standard Pitch, *P*	Average Weight per Foot in Pounds	Outside Length, *L*	Outside Width, *W*	Average Safe Working Load in Pounds	Proof Test in Pounds*	Approximate Breaking Strain in Pounds
1/4	25/32	3/4	1 5/16	7/8	1,200	2,500	5,000
5/16	27/32	1	1 1/2	1 1/16	1,700	3,500	7,000
3/8	31/32	1 1/2	1 3/4	1 1/4	2,500	5,000	10,000
7/16	1 5/32	2	2 1/16	1 3/8	3,500	7,000	14,000
1/2	1 11/32	2 1/2	2 3/8	1 11/16	4,500	9,000	18,000
9/16	1 15/32	3 1/4	2 5/8	1 7/8	5,500	11,000	22,000
5/8	1 23/32	4	3	2 1/16	6,700	14,000	27,000
11/16	1 13/16	5	3 1/4	2 1/4	8,100	17,000	32,500
3/4	1 15/16	6 1/4	3 1/2	2 1/2	10,000	20,000	40,000
13/16	2 1/16	7	3 3/4	2 11/16	10,500	23,000	42,000
7/8	2 3/16	8	4	2 7/8	12,000	26,000	48,000
15/16	2 7/16	9	4 3/8	3 1/16	13,500	29,000	54,000
1	2 1/2	10	4 5/8	3 1/4	15,200	32,000	61,000
1 1/16	2 5/8	12	4 7/8	3 5/16	17,200	35,000	69,000
1 1/8	2 3/4	13	5 1/8	3 3/4	19,500	40,000	78,000
1 3/16	3 1/16	14 1/2	5 9/16	3 7/8	22,000	46,000	88,000
1 1/4	3 1/8	16	5 3/4	4 1/8	23,700	51,000	95,000
1 5/16	3 3/8	17 1/2	6 1/8	4 1/4	26,000	54,000	104,000
1 3/8	3 9/16	19	6 7/16	4 9/16	28,500	58,000	114,000
1 7/16	3 11/16	21 1/2	6 11/16	4 3/4	30,500	62,000	122,000
1 1/2	3 7/8	23	7	5	33,500	67,000	134,000
1 9/16	4	25	7 3/8	5 5/16	35,500	70,500	142,000
1 5/8	4 1/4	28	7 3/4	5 1/2	38,500	77,000	154,000
1 11/16	4 1/2	30	8 1/8	5 11/16	39,500	79,000	158,000
1 3/4	4 3/4	31	8 1/2	5 7/8	41,500	83,000	166,000
1 13/16	5	33	8 7/8	6 1/16	44,500	89,000	178,000
1 7/8	5 1/4	35	9 1/4	6 3/8	47,500	95,000	190,000
1 15/16	5 1/2	38	9 5/8	6 9/16	50,500	101,000	202,000
2	5 3/4	40	10	6 3/4	54,000	108,000	216,000
2 1/16	6	43	10 3/8	6 15/16	57,500	115,000	230,000
2 1/8	6 1/4	47	10 3/4	7 1/8	61,000	122,000	244,000
2 3/16	6 1/2	50	11 1/8	7 5/16	64,500	129,000	258,000
2 1/4	6 3/4	53	11 1/2	7 5/8	68,200	136,500	273,000
2 3/8	6 7/8	58 1/2	11 7/8	8	76,000	152,000	304,000
2 1/2	7	65	12 1/4	8 3/8	84,200	168,500	337,000
2 5/8	7 1/8	70	12 5/8	8 3/4	90,500	181,000	362,000
2 3/4	7 1/4	73	13	9 1/8	96,700	193,500	387,000
2 7/8	7 1/2	76	13 1/2	9 1/2	103,000	206,000	412,000
3	7 3/4	86	14	9 7/8	109,000	218,000	436,000

* Chains tested to U. S. Government and American Bureau of Shipping requirements.

Dimensions of Crane Hooks — 1



	Capacity of Hook in Tons											
	1/8	1/4	1/2	1	1 1/2	2	3	4	5	6	8	10
	Dimensions Common to Plain and Swivel Hooks											
A	5/8	11/16	3/4	1 1/16	1 1/4	1 3/8	1 3/4	2	2 1/4	2 1/2	2 7/8	3 1/4
B	5/32	11/64	3/16	17/64	5/16	11/32	7/16	1/2	9/16	5/8	23/32	13/16
C	35/64	5/8	21/32	15/16	1 3/32	1 7/32	1 17/32	1 3/4	1 31/32	2 3/16	2 1/2	2 13/16
D	1 5/16	1 3/8	1 1/2	1 3/4	2	2 1/4	2 3/4	3 1/4	3 3/4	4 1/4	5 1/4	6 1/4
E	11/16	3/4	13/16	1 5/32	1 23/64	1 1/2	1 57/64	2 11/64	2 7/16	2 45/64	3 1/8	3 1/2
F	27/32	59/64	1	1 27/64	1 43/64	1 27/32	2 11/32	2 43/64	3	3 11/32	3 27/32	4 11/32
G	3/4	53/64	29/32	1 9/32	1 1/2	1 21/32	2 7/64	2 13/32	2 45/64	3	3 29/64	3 29/32
H	23/32	25/32	55/64	1 13/64	1 27/64	1 9/16	1 63/64	2 17/64	2 35/64	2 27/32	3 1/4	3 11/16
	Additional Dimensions for Swivel Hooks											
I	9/16	5/8	11/16	15/16	1 1/8	1 1/4	1 9/16	1 3/4	2	2 3/16	2 1/2	2 7/8
J	13/16	7/8	15/16	1 3/8	1 9/16	1 3/4	2 3/16	2 1/2	2 13/16	3 1/8	3 5/8	4
K	15/16	31/32	1 1/16	1 7/32	1 3/8	1 9/16	1 7/8	2 1/4	2 9/16	2 7/8	3 9/16	4 1/4
L	1	1 1/16	1 1/8	1 5/16	1 1/2	1 11/16	2	2 7/16	2 13/16	3 3/16	3 15/16	4 11/16
M	5/16	11/32	3/8	17/32	5/8	11/16	7/8	1	1 1/8	1 1/4	1 7/16	1 5/8
N	21/64	3/8	7/16	41/64	51/64	29/32	1 3/16	1 21/64	1 35/64	1 45/64	1 31/32	2 9/32
O	9/16	5/8	11/16	7/8	1 1/16	1 1/4	1 5/8	2	2 3/8	2 3/4	3 1/2	4 1/4
P	21/32	47/64	51/64	1 1/8	1 5/16	1 29/64	1 27/32	2 7/64	2 3/8	2 5/8	3 1/32	3 7/16
Q	1 11/16	1 3/4	1 15/16	2 1/4	2 9/16	2 15/16	3 9/16	4 3/16	4 13/16	5 1/2	6 3/4	8
R	1 11/16	1 3/4	1 15/16	2 1/4	2 9/16	2 15/16	3 9/16	4 3/16	4 13/16	5 1/2	6 3/4	8
	Additional Dimensions for Plain Hooks											
S	3	3 1/4	3 9/16	5	6	6 1/2	8 3/8	9 1/2	10	10 1/4	10 3/8	10 1/2
T	1 1/8	1 1/4	1 3/8	1 7/8	2 1/4	2 1/2	3 1/8	3 1/2	4	4 3/8	5	5 3/4
U	1 1/8	1 1/4	1 3/8	1 7/8	2 1/4	2 1/2	3 1/8	3 1/2	4	4 3/8	5	5 3/4
V	13/16	7/8	15/16	1 3/8	1 9/16	1 3/4	2 3/16	2 1/2	2 13/16	3 1/8	3 5/8	4
W	9/16	5/8	11/16	15/16	1 1/8	1 1/4	1 9/16	1 3/4	2	2 3/16	2 1/2	2 7/8
X	1 1/4	1 3/8	1 1/2	2 1/8	2 1/2	2 3/4	3 1/2	4	4 1/2	5	5 3/4	6 1/2

Dimensions based on formulas from Unwin's " Elements of Machine Design "

Loads Lifted by Crane Chains. — To find the approximate weight a chain will lift when rove as a tackle, multiply the safe load given in the table "Close-link Hoisting, Sling and Crane Chain" by the number of parts or chains at the movable block, and subtract one-quarter for frictional resistance. To find the size of chain required for lifting a given weight, divide the weight by the number of chains at the movable block, and add one-third for friction; next find in the column headed "Average Safe Working Load" the corresponding load, and then the corresponding size of chain in the column headed "Size." In case of heavy chain or where chain is unusually long, the weight of the chain itself should also be considered.

Dimensions of Crane Hooks. — The shape of the cross-section of a crane hook is such that it does not lend itself readily to exact mathematical treatment, but approximations can be made which are fairly accurate. The following formulas are for the design of large crane hooks, and the results obtained by these formulas have been thoroughly tested in practice. In the formulas:

A = area of section shown in the illustration (Table 2), in square inches;
f = allowable fiber stress in pounds per square inch;
P = load in pounds;
R^2 = square of the radius of gyration in inches.

For b, b_1, d, r, y_0 and y_1, see Fig. 1, Table 2.

$$\frac{P}{f} = \frac{A}{1 + \frac{y_1 y_0}{R^2}}; \qquad A = \frac{b + b_1}{2} \times d; \qquad y_1 = \frac{b + 2b_1}{b + b_1} \times \frac{d}{3};$$

$$y_0 = \frac{b + 2b_1}{b + b_1} \times \frac{d}{3} + r; \qquad R^2 = \frac{(b^2 + 4bb_1 + b_1^2)\,d^2}{18\,(b^2 + 2bb_1 + b_1^2)};$$

Assume $b = 0.65\,d$, and $b_1 = 0.3\,d$; then

$$\frac{P}{f} = \frac{d^3}{7.2\,d + 11.615\,r} = \text{constant in table.}$$

When using Table 2 for designing crane hooks, the load P, in pounds, which the hook will be required to carry, is first determined. Then the allowable fiber stress f is assumed and the quotient $\frac{P}{f}$ is obtained. (For ordinary service a fiber stress of from 16,000 to 25,000 pounds per square inch may be safely used.) This quotient is found in the body of the table. When the nearest value to $\frac{P}{f}$ has been located in the vertical column under the required radius r, follow the line horizontally to the left-hand column headed "d." The figure in this column gives the required width d (Fig. 1) of the hook and all other dimensions are proportioned from dimension d, as shown by the illustration Fig. 2.

As an example, assume that a crane hook is to be designed for a 50-ton crane, that the radius r is to be 3 inches, and that the allowable unit fiber stress is 20,000 pounds Expressed in pounds, P = 100,000 pounds. This number divided by 20,000 gives the quotient 5, the nearest value of which is found in the table in the vertical column under the 3-inch radius. It will be seen that the nearest value is 4.75 and by following the horizontal line in which 4.75 is located, to the left-hand column for d, we find that d = 7.5 inches. All other dimensions can now be found by inserting this value of d in the formulas given in connection with Fig. 2, "Dimensions of Crane Hooks — 2."

Dimensions of Crane Hooks — 2

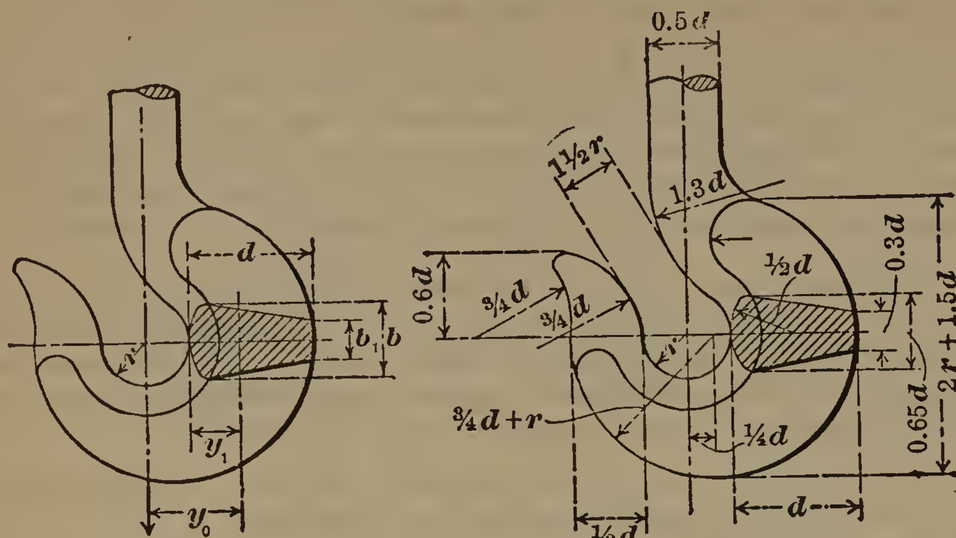
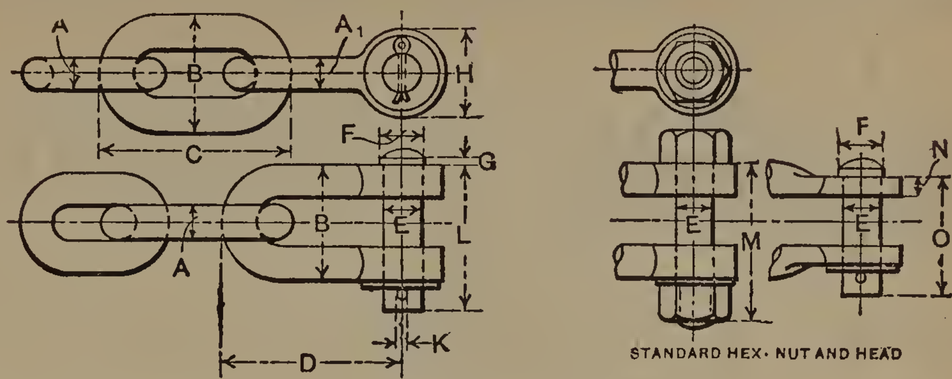


Fig.1. Fig.2

Divide load in pounds by allowable fiber stress in pounds per square inch and find the quotient in the column headed by the required radius *r* in the table. Then follow the horizontal line from this quotient to the left-hand column headed *d*. The figure in this column gives the required width *d* of the crane hook, and all the other dimensions are found from Fig. 2 as functions either of *d* or *r*. (See preceding page.)

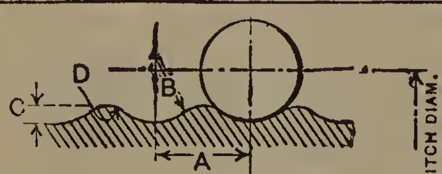
d	Values of *r*											
	0.75	1.00	1.25	1.50	1.75	2.00	2.50	3.00	3.50	4.00	4.50	5.00
2.00	0.34	0.30	0.27	0.25	0.23	0.21	0.18	0.16	0.14	0.13	...	...
2.25	0.45	0.40	0.37	0.33	0.31	0.28	0.25	0.22	0.20	0.18	...	...
2.50	0.58	0.52	0.48	0.44	0.40	0.37	0.33	0.29	0.26	0.24	0.22	...
2.75	0.73	0.66	0.60	0.55	0.51	0.48	0.42	0.38	0.34	0.30	0.28	...
3.00	...	0.81	0.74	0.69	0.64	0.60	0.53	0.47	0.43	0.39	0.36	0.34
3.50	...	1.16	1.08	1.00	0.94	0.88	0.79	0.71	0.65	0.60	0.55	0.51
4.00	...	...	1.47	1.38	1.30	1.23	1.10	1.00	0.92	0.85	0.79	0.73
4.50	...	...	1.94	1.82	1.72	1.63	1.48	1.35	1.24	1.15	1.07	1.00
5.00	...	...	...	2.34	2.22	2.11	1.92	1.76	1.63	1.51	1.41	1.33
5.50	...	...	...	2.92	2.78	2.65	2.42	2.23	2.08	1.93	1.81	1.70
6.00	...	...	...	...	3.40	3.25	2.99	2.77	2.57	2.41	2.26	2.13
6.50	...	...	...	...	4.08	3.91	3.61	3.36	3.13	2.94	2.76	2.61
7.00	...	...	...	...	...	4.66	4.32	4.03	3.77	3.54	3.34	3.16
7.50	...	...	...	...	...	5.46	5.08	4.75	4.46	4.20	3.96	3.76
8.00	...	...	...	...	...	...	5.91	5.54	5.21	4.92	4.76	4.43
8.50	...	...	...	...	...	...	6.80	6.40	6.03	5.68	5.41	5.14
9.00	...	...	...	...	...	...	...	7.32	6.92	6.55	6.24	5.94
9.50	Figures in body of table				...	...	...	8.30	7.86	7.47	7.11	6.77
10.00	are values of $\frac{P}{f}$			...	...	...	...	...	8.88	8.44	8.05	7.68
10.50				...	...	...	...	...	9.96	9.48	9.06	8.66
11.00	...	...	...	...	...	...	...	...	...	10.60	10.10	9.70
11.50	...	...	...	...	...	...	...	...	...	11.70	11.20	10.80
12.00	...	...	...	...	...	...	...	...	...	...	13.70	13.10

Dimensions of Chain End-Link and Narrow Shackle — U. S. Navy Standard



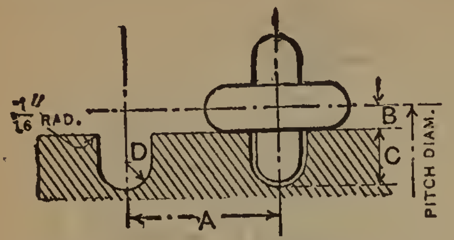
A	*A₁*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	*L*	*M*	*N*	*O*
1/2	9/16	1 7/8	3 1/2	2 1/2	3/4	1 1/16	3/16	1 1/2	3/16	2 3/8	2 3/4	..	..
9/16	5/8	2 1/16	3 5/8	2 3/4	3/4	1 1/16	3/16	1 1/2	3/16	2 5/8	2 7/8	..	..
5/8	11/16	2 1/4	4 3/8	3	1	1 7/16	1/4	2	3/16	3	3 3/8	..	..
11/16	3/4	2 1/2	4 5/8	3 1/4	1	1 7/16	1/4	2	3/16	3 1/4	3 5/8	..	..
3/4	13/16	2 11/16	5	3 1/2	1 1/8	1 9/16	1/4	2 1/4	3/16	3 1/2	4	..	..
13/16	7/8	2 7/8	5 1/4	3 3/4	1 1/8	1 9/16	5/16	2 1/4	3/16	3 3/4	4 1/8	..	..
7/8	1	3 1/4	5 3/4	4 1/8	1 1/4	1 11/16	5/16	2 1/2	1/4	4 1/8	4 5/8	5/8	3 1/2
15/16	1 1/16	3 9/16	6	4 3/8	1 1/4	1 11/16	5/16	2 1/2	1/4	4 1/2	5	5/8	3 5/8
1	1 1/8	3 3/4	6 5/8	4 5/8	1 1/2	2 1/16	3/8	3	1/4	4 7/8	5 3/8	5/8	3 7/8
1 1/16	1 3/16	3 7/8	6 7/8	4 7/8	1 1/2	2 1/16	3/8	3	1/4	5	5 5/8	5/8	4
1 1/8	1 1/4	4 1/8	7 1/2	5 1/8	1 3/4	2 5/16	7/16	3 1/2	5/16	5 1/2	6	3/4	4 1/2
1 3/16	1 5/16	4 3/8	7 3/4	5 3/8	1 3/4	2 5/16	7/16	3 1/2	5/16	5 3/4	6 1/4	3/4	4 3/8
1 1/4	1 3/8	4 9/16	8 1/8	5 3/4	1 7/8	2 7/16	7/16	3 3/4	5/16	6	6 5/8	3/4	4 3/4
1 5/16	1 7/16	4 3/4	8 3/8	5 7/8	1 7/8	2 7/16	1/2	3 3/4	5/16	6 1/4	6 3/4	3/4	4 7/8
1 3/8	1 1/2	5	8 3/4	6 1/4	2	2 11/16	1/2	4	5/16	6 1/2	7 1/8	7/8	5 1/4
1 7/16	1 9/16	5 3/16	9	6 1/2	2	2 11/16	1/2	4	5/16	6 3/4	7 3/8	7/8	5 3/8
1 1/2	1 5/8	5 3/8	9 5/8	6 7/8	2 1/4	3 1/16	9/16	4 1/2	3/8	7	7 3/4	7/8	5 1/2
1 9/16	1 11/16	5 5/8	9 7/8	7 1/8	2 1/4	3 1/16	9/16	4 1/2	3/8	7 1/4	8	7/8	5 5/8
1 5/8	1 3/4	5 13/16	10 1/2	7 3/8	2 1/2	3 7/16	5/8	5	3/8	7 1/2	8 1/2	1	6 1/8
1 11/16	1 13/16	6	10 3/4	7 3/4	2 1/2	3 7/16	5/8	5	3/8	7 3/4	8 3/4	1	6 1/4
1 3/4	1 7/8	6 1/4	11 3/8	8	2 3/4	3 11/16	11/16	5 1/2	7/16	8	9 1/8	1	6 3/8
1 13/16	1 15/16	6 7/16	11 5/8	8 1/4	2 3/4	3 11/16	11/16	5 1/2	7/16	8 1/4	9 3/8	1	6 1/2
1 7/8	2	6 11/16	11 3/4	8 1/2	2 3/4	3 11/16	11/16	5 1/2	7/16	8 1/2	9 5/8	1 1/8	6 3/4
1 15/16	2 1/16	6 7/8	12	8 3/4	2 3/4	3 11/16	11/16	5 1/2	7/16	8 3/4	9 7/8	1 1/8	6 7/[illegible]

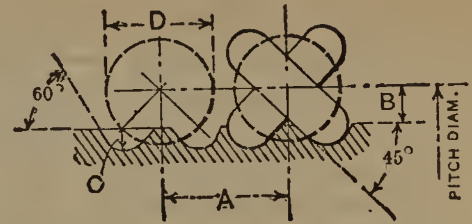
Winding Drum Score for Wire Rope



Rope Diam.	*A*	*B*	*C*	*D*
3/8	7/16	7/32	3/32	3/32
7/16	1/2	1/4	7/64	3/32
1/2	9/16	9/32	1/8	3/32
9/16	5/8	5/16	9/64	1/8
5/8	11/16	11/32	5/32	1/8
11/16	3/4	3/8	11/64	1/8
3/4	13/16	13/32	3/16	5/32
13/16	7/8	7/16	13/64	5/32
7/8	15/16	15/32	7/32	5/32
15/16	1	1/2	15/64	3/16
1	1 1/16	17/32	1/4	3/16

Winding Drum Scores for Chain





Chain Size	A	B	C	D	Chain Size	A	B	C	D
3/8	1 1/2	3/16	9/16	3/16	3/8	1 1/4	11/32	3/16	1
7/16	1 11/16	7/32	5/8	9/32	7/16	1 7/16	3/8	7/32	1 1/8
1/2	1 7/8	1/4	11/16	5/16	1/2	1 9/16	7/16	1/4	1 1/4
9/16	2 1/16	9/32	3/4	11/32	9/16	1 3/4	15/32	9/32	1 3/8
5/8	2 5/16	5/16	13/16	3/8	5/8	1 7/8	17/32	5/16	1 1/2
11/16	2 1/2	11/32	7/8	13/32	11/16	2 1/16	9/16	11/32	1 5/8
3/4	2 11/16	3/8	15/16	7/16	3/4	2 3/16	5/8	3/8	1 3/4
13/16	2 7/8	13/32	1	15/32	13/16	2 3/8	21/32	13/32	1 7/8
7/8	3 1/8	7/16	1 1/16	1/2	7/8	2 1/2	23/32	7/16	2
15/16	3 5/16	15/32	1 1/8	17/32	15/16	2 11/16	3/4	15/32	2 1/8
1	3 1/2	1/2	1 3/16	9/16	1	2 13/16	13/16	1/2	2 1/4

Strength of Manila Rope

Approximate Diameter, Inches	Circumference, Inches	Weight of 100 Feet of Rope, Pounds	Ultimate Tensile Strength of Rope, Pounds	Working Load, Total Area of Rope, in Lbs.			Minimum Diameter of Sheaves, in Inches		
				Rapid*	Medium†	Slow†	Rapid*	Medium†	Slow‡
3/16	9/16	2	230	10	20	40	8	3	1 1/2
5/16	1	4	630	20	40	100	13	5	2 1/2
3/8	1 1/8	5	900	30	60	140	15	6	3
1/2	1 1/2	7 2/3	1,620	50	100	250	20	8	4
5/8	2	13 1/3	2,880	90	180	450	25	10	5
13/16	2 1/2	20	4,500	150	300	650	35	11	7
1	3	28 1/3	6,480	200	400	1,000	40	12	8
1 1/8	3 1/2	38	8,820	250	500	1,250	45	13	9
1 5/16	4	52	11,500	350	700	1,700	55	15	11
1 1/2	4 1/2	65	14,600	450	900	2,200	60	16	12
1 5/8	5	80	18,000	550	1100	2,600	65	17	13
2	6	113	25,900	750	1500	3,700	80	22	16
2 1/4	7	153	35,300	....	2100	5,100	.....	24	18
2 5/8	8	211	46,100	....	2700	6,600	.....	28	21
3	9	262	58,300	....	3400	8,400	.....	32	24
3 1/4	10	325	72,000	....	4200	10,300	.....	35	26

* Speed from 400 to 800 feet per minute. † Hoisting speed from 150 to 300 feet per minute. ‡ Hoisting speed from 50 to 100 feet per minute.

Lifting Magnets. — Lifting magnets are especially adapted for handling pig iron, metal plates, billets, scrap, iron and steel castings, rails, "skull crackers," etc. The magnet is energized by a direct current of any common voltage. The weight that a magnet of given size can lift depends upon the form of material and the evenness of the surfaces which must be gripped by the magnet. It might be possible to lift 20,000 pounds under favorable conditions and only 1000 pounds, or less, under adverse conditions. For example, a much greater weight can be lifted when there is a solid mass of steel or iron, than when there are a number of pieces which not only cling to the magnet but to each other. The table below represents the performance of the Electric Controller & Mfg. Co.'s magnets. The weights of billets, etc., that can be lifted, depend to some extent upon the dimensions and whether the material is in an irregular pile or stacked evenly. Brass and copper, being non-magnetic, cannot be handled with lifting magnets.

Average Weights Lifted by Magnets

Material Lifted by Magnet	Magnet Diam. 36 Ins.	Magnet Diam. 43 Ins.	Magnet Diam. 52 Ins.	Magnet Diam. 61 Ins.
	Average Weight in Pounds			
"Skull Cracker" Balls up to	12,000	20,000	20,000	20,000
Billets and Slabs up to	20,000	32,000	42,000	50,000
Machine-cast Pig Iron	720*	1,200*	1,800*	2,300*
	800†	1,350†	1,950†	2,400†
Broken, Sand-cast Pig Iron	660*	1,100*	1,700*	2,200*
	720†	1,200†	1,800†	2,300†
Crop Ends of Billets, Rails or Structural Shapes	750	1,250	1,700	2,200
Boiler Plate Scrap	600	1,100	1,500	2,100
Small Risers from Castings	900	1,600	2,200	2,900
Fine Wire Scrap	400	500	700	900
Busheling Scrap	600	1,200	1,600	2,300
Scrap Pipe Tubing not over 3 feet long	200	500	700	900
Loose Tin or Lamination Scrap	200	500	700	900
Miscellaneous Junk Dealers' Scrap	250–500	400–800	550–1150	750–1500

* Average weights when unloading railway cars, including the "lean" lifts when cleaning up. † Average weights when lifting from stock pile.

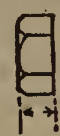
Current Required for Lifting Magnets. — The average current consumption of magnets listed in the table "Average Weights Lifted by Magnets" is as follows: With a 36-inch magnet, the average current at 220 volts is 11 amperes; 43-inch magnet, average current at 220 volts, 27 amperes; 52-inch magnet, average current at 220 volts, 38 amperes; 61-inch magnet, average current at 220 volts, 47 amperes.

The efficiency resulting from the use of lifting magnets is indicated by the following performance in unloading a cargo of 4,000,000 pounds of pig iron. This work was done by two men in eleven hours using two 62-inch Cutler-Hammer magnets, of the high-duty type. These magnets have an average lifting capacity of about 4200 pounds and require current of 72 amperes at 220 volts. The 52-inch magnet, which is extensively used for general work, has an average lifting capacity of 2500 to 2800 pounds and requires 50 amperes at 220 volts. The 36-inch magnet has an average lifting capacity of 1000 to 1400 pounds as applied to general service in handling pig iron, scrap, etc., and the current required is 26 amperes at 220 volts.

BOLTS, NUTS AND MACHINE DETAILS

In the following is given general information relating to the dimensions of bolts, nuts and screws used in machine construction. For detailed data relating to the proportions of various forms of screw threads, see the section on "Screw Thread Systems."

Whitworth Standard Threads, Bolts and Nuts

Diameter	No. of Threads per Inch	Diameter at Root of Thread	Diameter of Tap Drill	Area at Root of Threads, Sq. Ins.							
					Max.	Min.		Max.	Min.	Max.	Min.
¼	20	0.1860	3/16	0.0272	0.525	0.520	0.61	0.26	0.25	0.23	0.22
5/16	18	0.2414	¼	0.0458	0.600	0.595	0.69	0.32	0.31	0.28	0.27
⅜	16	0.2950	19/64	0.0683	0.710	0.705	0.82	0.39	0.38	0.34	0.33
7/16	14	0.3460	23/64	0.0940	0.820	0.815	0.95	0.45	0.44	0.39	0.38
½	12	0.3933	13/32	0.1215	0.920	0.915	1.06	0.51	0.50	0.45	0.44
9/16	12	0.4558	15/32	0.1632	1.010	1.002	1.17	0.57	0.56	0.50	0.49
⅝	11	0.5086	33/64	0.2032	1.100	1.092	1.27	0.64	0.63	0.56	0.55
11/16	11	0.5711	37/64	0.2562	1.200	1.192	1.39	0.70	0.69	0.61	0.60
¾	10	0.6219	⅝	0.3038	1.300	1.292	1.50	0.76	0.75	0.67	0.66
13/16	10	0.6844	11/16	0.3679	1.390	1.382	1.61	0.82	0.81	0.72	0.71
⅞	9	0.7327	47/64	0.4216	1.480	1.472	1.71	0.89	0.88	0.78	0.77
1	8	0.8399	27/32	0.5540	1.670	1.662	1.93	1.01	1.00	0.89	0.88
1⅛	7	0.9420	61/64	0.6969	1.860	1.850	2.15	1.15	1.13	1.00	0.98
1¼	7	1.0670	1 1/16	0.8942	2.050	2.040	2.37	1.27	1.25	1.11	1.09
1⅜	6	1.1616	1 11/64	1.0597	2.220	2.210	2.56	1.40	1.38	1.22	1.20
1½	6	1.2866	1 19/64	1.3001	2.410	2.400	2.78	1.52	1.50	1.33	1.31
1⅝	5	1.3689	1⅜	1.4718	2.580	2.570	2.98	1.65	1.63	1.44	1.42
1¾	5	1.4939	1½	1.7528	2.760	2.750	3.19	1.77	1.75	1.55	1.53
2	4½	1.7154	1 23/32	2.3111	3.150	3.140	3.64	2.02	2.00	1.77	1.75
2¼	4	1.9298	1 15/16	2.9249	3.550	3.535	4.10	2.27	2.25	1.99	1.97
2½	4	2.1798	2 3/16	3.7318	3.890	3.875	4.49	2.52	2.50	2.21	2.19
2¾	3½	2.3841	2 13/32	4.4641	4.180	4.165	4.83	2.77	2.75	2.43	2.41
3	3½	2.6341	2 41/64	5.4496	4.530	4.515	5.23	3.02	3.00	2.65	2.63
3¼	3¼	2.8560	2 55/64	6.4063	4.850	4.830	5.60	3.27	3.25	2.86	2.84
3½	3¼	3.1060	3⅛	7.5769	5.180	5.160	5.98	3.52	3.50	3.08	3.06
3¾	3	3.3231	3 5/16	8.6732	5.550	5.530	6.41	3.77	3.75	3.30	3.28
4	3	3.5731	3 9/16	10.0272	5.950	5.930	6.87	4.02	4.00	3.52	3.50
4½	2⅞	4.0546	4 1/16	12.9118	6.820	6.795	7.88	4.53	4.50	3.97	3.94
5	2¾	4.5343	4 9/16	16.1477	7.800	7.775	9.01	5.03	5.00	4.41	4.38
5½	2⅝	5.0121	5	19.7301	8.850	8.820	10.22	5.53	5.50	4.84	4.81
6	2½	5.4877	5½	23.6521	10.000	9.970	11.55	6.03	6.00	5.28	5.25

U. S. Standard Threads, Bolts and Nuts

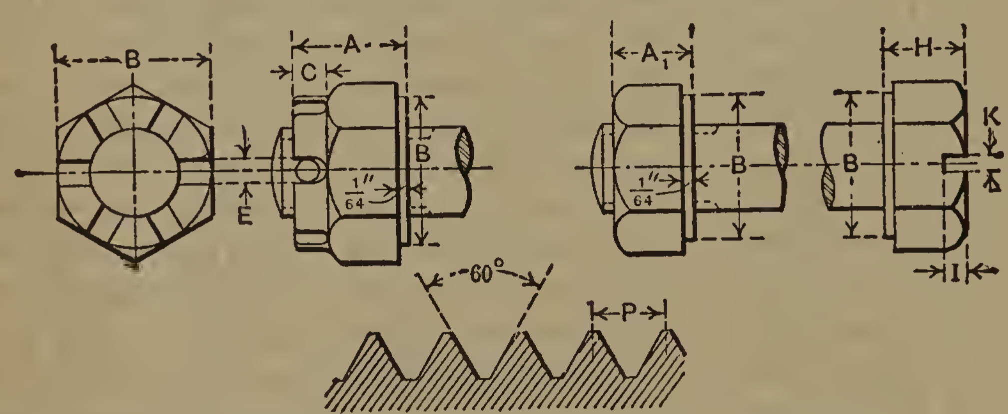
The tap drill diameters in the table provide for a slight clearance at the root of the thread, in order to facilitate tapping and reduce tap breakages. If full threads are required, use the diameters at the root of the threads for the tap drill diameters

Diameter	No. of Threads per Inch	Diameter at Root of Thread	Diameter of Tap Drill	Area in Sq. Inches		Tensile Strength at Stress of 6000 Pounds per Sq. Inch	Dimensions of Nuts and Bolt Heads				
				Of Bolt	At Root of Thread						
1/4	20	0.185	13/64	0.049	0.026	160	1/2	0.578	0.707	1/4	1/4
5/16	18	0.240	1/4	0.076	0.045	270	19/32	0.686	0.840	5/16	19/64
3/8	16	0.294	5/16	0.110	0.068	410	11/16	0.794	0.972	3/8	11/32
7/16	14	0.345	23/64	0.150	0.093	560	25/32	0.902	1.105	7/16	25/64
1/2	13	0.400	27/64	0.196	0.126	760	7/8	1.011	1.237	1/2	7/16
9/16	12	0.454	15/32	0.248	0.162	1,000	31/32	1.119	1.370	9/16	31/64
5/8	11	0.507	17/32	0.307	0.202	1,210	1 1/16	1.227	1.502	5/8	17/32
3/4	10	0.620	41/64	0.442	0.302	1,810	1 1/4	1.444	1.768	3/4	5/8
7/8	9	0.731	3/4	0.601	0.419	2,520	1 7/16	1.660	2.033	7/8	23/32
1	8	0.838	55/64	0.785	0.551	3,300	1 5/8	1.877	2.298	1	13/16
1 1/8	7	0.939	31/32	0.994	0.694	4,160	1 13/16	2.093	2.563	1 1/8	29/32
1 1/4	7	1.064	1 3/32	1.227	0.893	5,350	2	2.310	2.828	1 1/4	1
1 3/8	6	1.158	1 7/32	1.485	1.057	6,340	2 3/16	2.527	3.093	1 3/8	1 3/32
1 1/2	6	1.283	1 11/32	1.767	1.295	7,770	2 3/8	2.743	3.358	1 1/2	1 3/16
1 5/8	5 1/2	1.389	1 27/64	2.074	1.515	9,090	2 9/16	2.960	3.623	1 5/8	1 9/32
1 3/4	5	1.490	1 17/32	2.405	1.746	10,470	2 3/4	3.176	3.889	1 3/4	1 3/8
1 7/8	5	1.615	1 21/32	2.761	2.051	12,300	2 15/16	3.393	4.154	1 7/8	1 15/32
2	4 1/2	1.711	1 49/64	3.142	2.302	13,800	3 1/8	3.609	4.419	2	1 9/16
2 1/4	4 1/2	1.961	2 1/64	3.976	3.023	18,100	3 1/2	4.043	4.949	2 1/4	1 3/4
2 1/2	4	2.175	2 15/64	4.909	3.719	22,300	3 7/8	4.476	5.479	2 1/2	1 15/16
2 3/4	4	2.425	2 31/64	5.940	4.620	27,700	4 1/4	4.909	6.010	2 3/4	2 1/8
3	3 1/2	2.629	2 11/16	7.069	5.428	32,500	4 5/8	5.342	6.540	3	2 5/16
3 1/4	3 1/2	2.879	2 15/16	8.296	6.510	39,000	5	5.775	7.070	3 1/4	2 1/2
3 1/2	3 1/4	3.100	3 11/64	9.621	7.548	45,300	5 3/8	6.208	7.600	3 1/2	2 11/16
3 3/4	3	3.317	3 3/8	11.045	8.641	51,800	5 3/4	6.641	8.131	3 3/4	2 7/8
4	3	3.567	3 5/8	12.566	9.963	59,700	6 1/8	7.074	8.661	4	3 1/16
4 1/4	2 7/8	3.798	3 27/32	14.186	11.340	68,000	6 1/2	7.508	9.191	4 1/4	3 1/4
4 1/2	2 3/4	4.028	4 3/32	15.904	12.750	76,500	6 7/8	7.941	9.721	4 1/2	3 7/16
4 3/4	2 5/8	4.255	4 5/16	17.721	14.215	85,500	7 1/4	8.374	10.252	4 3/4	3 5/8
5	2 1/2	4.480	4 9/16	19.635	15.760	94,000	7 5/8	8.807	10.782	5	3 13/16
5 1/4	2 1/2	4.730	4 13/16	21.648	17.570	105,500	8	9.240	11.312	5 1/4	4
5 1/2	2 3/8	4.953	5 1/32	23.758	19.260	116,000	8 3/8	9.673	11.842	5 1/2	4 3/16
5 3/4	2 3/8	5.203	5 9/32	25.967	21.250	127,000	8 3/4	10.106	12.373	5 3/4	4 3/8
6	2 1/4	5.423	5 1/2	28.274	23.090	138,000	9 1/8	10.539	12.903	6	4 9/16

S. A. E. Standard Screws and Nuts. — The proportions of standard screw threads adopted by the Society of Automotive Engineers are given in the table "S. A. E. Standard Screws and Nuts." The material from which the screws and nuts are made is steel having a tensile strength of not less than 100,000 pounds per square inch and an elastic limit of 60,000 pounds per square inch. The screws, as well as the screw heads and plain nuts, are to be left soft; the castle-nuts are to be casehardened. The shape of the thread is the same as the U. S. standard, but the number of threads per inch is greater. The length of the threaded portion equals one and one-half times the diameter + ¼″. The clearance between the top of the bolt thread and the bottom of the thread in the nut is proportionately the same as that adopted by machine-screw makers, the top being between 0.002 and 0.003 inch large. The body diameter of the screw is 0.001 inch less than the nominal diameter, and should be made within such limits that it is not larger than the nominal diameter, nor smaller than 0.002 inch less than the nominal diameter. It is assumed that when screws are to be used in soft materials, such as aluminum, brass, etc., the pitch of the threads will be made to conform to the U. S. standard, in order to provide for adequate proportions of the thread.

The S. A. E. screw standard supplants the A. L. A. M. standard adopted by the Association of Licensed Automobile Manufacturers in April, 1906.

S. A. E. Standard Screws and Nuts

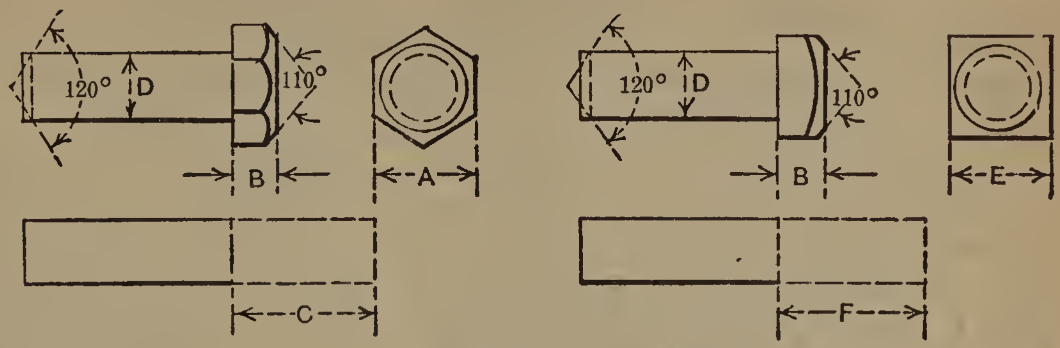


D (in table) = Diameter of screw;
N = Number of threads per inch;
B applies to all nuts and screw heads;
d = Diameter of cotter pin;

S = Tap drill size;
$D \times 1.5 + ¼'' =$ Length of thread;
$\frac{P}{8} =$ Width of flat.

D	¼	5/16	⅜	7/16	½	9/16	⅝	11/16	¾	⅞	1	1⅛	1¼	1⅜	1½
N	28	24	24	20	20	18	18	16	16	14	14	12	12	12	12
A	9/32	21/64	13/32	29/64	9/16	39/64	23/32	49/64	13/16	29/32	1	1 5/32	1¼	1 13/32	1½
A_1	7/32	17/64	21/64	⅜	7/16	31/64	35/64	19/32	21/32	49/64	⅞	63/64	1 3/32	1 13/64	1 5/16
B	7/16	½	9/16	⅝	¾	⅞	15/16	1	1 1/16	1¼	1 7/16	1⅝	1 13/16	2	2 3/16
C	3/32	3/32	⅛	⅛	3/16	3/16	¼	¼	¼	¼	¼	5/16	5/16	⅜	⅜
E	5/64	5/64	⅛	⅛	⅛	5/32	5/32	5/32	5/32	5/32	5/32	7/32	7/32	¼	¼
H	3/16	15/64	9/32	21/64	⅜	27/64	15/32	33/64	9/16	21/32	¾	27/32	15/16	1 1/32	1⅛
I	3/32	7/64	⅛	⅛	⅛	⅛	⅛	⅛	⅛	⅛	⅛	7/32	7/32	¼	¼
K	1/16	1/16	3/32	3/32	3/32	3/32	3/32	3/32	3/32	3/32	3/32	5/32	5/32	3/16	3/16
d	1/16	1/16	3/32	3/32	3/32	⅛	⅛	⅛	⅛	⅛	⅛	11/64	11/64	13/64	13/64
S	7/32	17/64	21/64	⅜	7/16	½	9/16	39/64	43/64	25/32	29/32	1 1/64	1 9/64	1 17/64	1 25/64

U. S. and Manufacturers' Standard Hexagon and Square Head Bolts and Approximate Amount of Stock Required to Form Heads



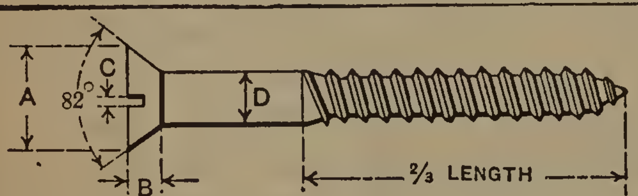
United States Standard						Manufacturers' Standard					
D	*A*	*B*	*C*	*E*	*F*	*D*	*A*	*B*	*C*	*E*	*F*
1/4	1/2	1/4	1 3/32	1/2	1 9/32	1/4	7/16	3/16	23/32	3/8	17/32
5/16	19/32	19/64	1 3/16	19/32	1 3/8	5/16	17/32	1/4	3/4	15/32	11/16
3/8	11/16	11/32	1 1/4	11/16	1 1/2	3/8	5/8	9/32	15/16	9/16	13/16
7/16	25/32	25/64	1 3/8	25/32	1 5/8	7/16	23/32	21/64	1	21/32	31/32
1/2	7/8	7/16	1 1/2	7/8	1 23/32	1/2	13/16	3/8	1 3/32	3/4	1 1/16
9/16	31/32	31/64	1 5/8	31/32	1 13/16	9/16	29/32	27/64	1 1/4	27/32	1 7/32
5/8	1 1/16	17/32	1 11/16	1 1/16	2	5/8	1	17/32	1 13/32	15/16	1 11/32
3/4	1 1/4	5/8	1 15/16	1 1/4	2 7/32	3/4	1 3/16	9/16	1 9/16	1 1/8	1 5/8
7/8	1 7/16	23/32	2 5/32	1 7/16	2 1/2	7/8	1 3/8	21/32	1 7/8	1 5/16	1 7/8
1	1 5/8	13/16	2 3/8	1 5/8	2 3/4	1	1 9/16	3/4	2 1/32	1 1/2	2 5/32
1 1/8	1 13/16	29/32	2 19/32	1 13/16	3	1 1/8	1 3/4	27/32	2 7/32	1 11/16	2 13/32
1 1/4	2	1	2 27/32	2	3 1/4	1 1/4	1 15/16	15/16	2 7/16	1 7/8	2 11/16
1 3/8	2 3/16	1 3/32	3	2 3/16	3 9/16	1 3/8	2 1/8	1 1/16	2 27/32	2 1/16	3 1/32
1 1/2	2 3/8	1 3/16	3 5/16	2 3/8	3 13/16	1 1/2	2 5/16	1 1/8	3	2 1/4	3 7/32
1 5/8	2 9/16	1 9/32	3 1/2	2 9/16	4 1/16	1 5/8	2 1/2	1 7/32	3 1/4	2 7/16	3 19/32
1 3/4	2 3/4	1 3/8	3 3/4	2 3/4	4 3/8	1 3/4	2 11/16	1 5/16	3 15/32	2 5/8	3 3/4
1 7/8	2 15/16	1 15/32	4	2 15/16	4 5/8	1 7/8	2 7/8	1 13/32	3 23/32	2 13/16	4 3/32
2	3 1/8	1 9/16	4 1/4	3 1/8	4 7/8	2	3 1/16	1 1/2	4	3	4 5/16
2 1/4	3 1/2	1 3/4	4 25/32	3 1/2	5 7/16	2 1/4	3 7/16	1 11/16	4 9/16	3 3/8	5
2 1/2	3 7/8	1 15/16	5 3/16	3 7/8	5 15/16	2 1/2	3 13/16	1 7/8	4 13/16	3 3/4	5 3/8
2 3/4	4 1/4	2 1/8	5 1/2	4 1/4	6 7/16	2 3/4	4 3/16	2 1/16	5 5/8	4 1/8	5 29/32
3	4 5/8	2 5/16	6 1/16	4 5/8	7	3	4 9/16	2 1/4	5 11/16	4 1/2	6 7/16
3 1/4	5	2 1/2	6 1/2	5	7 9/16	...	...	...	...	...	...
3 1/2	5 3/8	2 11/16	7	5 3/8	8 1/8	...	...	...	...	...	...
3 3/4	5 3/4	2 7/8	7 1/2	5 3/4	8 11/16	...	...	...	...	...	...
4	6 1/8	3 1/16	8	6 1/8	9 5/16	...	...	...	...	...	...

Extra Stock Required for Upsetting Screw Ends

Figures in body of table give extra length of rod (in inches) required for upsetting screw ends.

Diameters before and after Upsetting		Required Length of Upset End					Diameters before and after Upsetting		Required Length of Upset End				
From	To	1 In.	2 In.	3 In.	4 In.	5 In.	From	To	1 In.	2 In.	3 In.	4 In.	5 In.
¾	⅞	0.36	0.72	1.08	1.44	1.81	1¼	1½	0.44	0.88	1.32	1.76	2.20
¾	1	0.78	1.56	2.33	3.11	3.89	1¼	1⅝	0.69	1.38	2.07	2.76	3.45
¾	1⅛	1.25	2.50	3.75	5.00	6.25	1⅜	1½	0.19	0.38	0.57	0.76	0.95
⅞	1	0.31	0.61	0.92	1.22	1.53	1⅜	1⅝	0.40	0.79	1.19	1.59	1.98
⅞	1⅛	0.65	1.31	1.96	2.61	3.27	1⅜	1¾	0.62	1.24	1.86	2.48	3.10
⅞	1¼	1.04	2.08	3.12	4.16	5.20	1½	1⅝	0.17	0.35	0.52	0.69	0.87
1	1⅛	0.27	0.53	0.80	1.06	1.33	1½	1¾	0.36	0.72	1.08	1.44	1.81
1	1¼	0.56	1.13	1.69	2.25	2.81	1½	1⅞	0.56	1.13	1.69	2.25	2.81
1	1⅜	0.89	1.78	2.67	3.56	4.45	1⅝	1¾	0.16	0.32	0.48	0.64	0.80
1⅛	1¼	0.23	0.47	0.70	0.94	1.17	1⅝	1⅞	0.33	0.66	0.99	1.33	1.66
1⅛	1⅜	0.49	0.99	1.48	1.98	2.47	1⅝	2	0.51	1.03	1.54	2.06	2.57
1⅛	1½	0.78	1.56	2.33	3.11	3.89	1¾	1⅞	0.15	0.30	0.44	0.59	0.74
1¼	1⅜	0.21	0.42	0.63	0.84	1.05	1¾	2	0.31	0.61	0.92	1.22	1.53

Wood Screws*

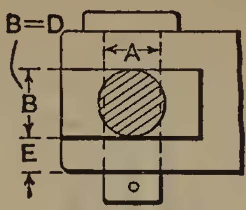


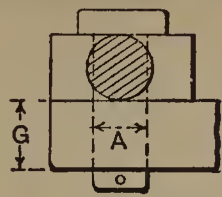
Approximate Formula:
$D = 0.01325\,N + 0.056$;
D = Diameter of screw;
N = Number of screw.

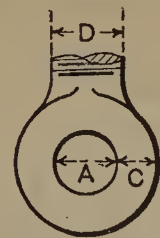
Screw Numbers	*A*	*B*	*C*	*D*	Number of Threads per Inch	Screw Numbers	*A*	*B*	*C*	*D*	Number of Threads per Inch
0	....	....	...	0.0578	30	13	0.4427	0.1286	0.055	0.2289	11
1	....	....	...	0.0710	28	14	0.4790	0.1362	0.057	0.2421	10
2	0.1631	0.0454	0.030	0.0841	26	15	0.5053	0.1437	0.059	0.2552	9.5
3	0.1894	0.0530	0.032	0.0973	24	16	0.5316	0.1513	0.061	0.2684	9
4	0.2158	0.0605	0.034	0.1105	22	17	0.5579	0.1589	0.064	0.2815	8.5
5	0.2421	0.0681	0.036	0.1236	20	18	0.5842	0.1665	0.066	0.2947	8
6	0.2684	0.0757	0.039	0.1368	18	20	0.6368	0.1816	0.070	0.3210	7.5
7	0.2947	0.0832	0.041	0.1500	17	22	0.6895	0.1967	0.075	0.3474	7.5
8	0.3210	0.0908	0.043	0.1631	15	24	0.7421	0.1967	0.079	0.3737	7
9	0.3474	0.0984	0.045	0.1763	14	26	0.7421	0.2118	0.084	0.4000	6.5
10	0.3737	0.1059	0.048	0.1894	13	28	0.7948	0.2118	0.088	0.4263	6.5
11	0.4000	0.1134	0.050	0.2026	12.5	30	0.8474	0.2270	0.093	0.4546	6
12	0.4263	0.1210	0.052	0.2158	12	...	....	....	....	....	...

* Asa S. Cook Co.'s Standard.

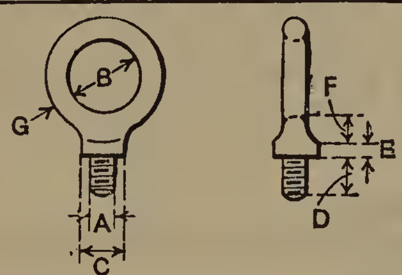
Proportions of Eyebolts





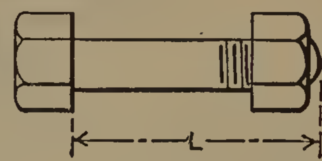


Diameter of Bolt, *D*	U. S. Threads per Inch	*A*, in Double Shear	*A*, in Single Shear	*C*	*E*	*G*	Safe Load, Pounds	Diameter of Bolt, *D*	U. S. Threads per Inch	*A*, in Double Shear	*A*, in Single Shear	*C*	*E*	*G*	Safe Load, Pounds
1/2	13	13/32	9/16	1/4	5/16	5/8	1,890	1 3/4	5	1 13/32	1 29/32	7/8	1 1/4	2 1/2	26,000
5/8	11	1/2	11/16	5/16	7/16	13/16	3,030	1 7/8	5	1 1/2	2 1/16	15/16	1 3/8	2 11/16	31,000
3/4	10	19/32	13/16	3/8	1/2	1	4,530	2	4 1/2	1 5/8	2 7/32	1	1 7/16	2 7/8	35,000
7/8	9	11/16	31/32	7/16	5/8	1 3/16	6,300	2 1/8	4 1/2	1 23/32	2 11/32	1 1/16	1 9/16	3 1/16	40,000
1	8	13/16	1 3/32	1/2	11/16	1 3/8	8,250	2 1/4	4 1/2	1 13/16	2 15/32	1 1/8	1 5/8	3 1/4	45,000
1 1/8	7	29/32	1 1/4	9/16	13/16	1 9/16	10,410	2 3/8	4 1/2	1 29/32	2 5/8	1 3/16	1 3/4	3 7/16	50,500
1 1/4	7	1	1 3/8	5/8	7/8	1 3/4	13,400	2 1/2	4	2	2 3/4	1 1/4	1 13/16	3 5/8	56,000
1 3/8	6	1 3/32	1 1/2	11/16	1	1 15/16	15,800	2 5/8	4	2 1/8	2 7/8	1 5/16	1 15/16	3 13/16	61,500
1 1/2	6	1 1/4	1 21/32	3/4	1 1/16	2 1/8	19,400	2 3/4	4	2 7/32	3 1/32	1 3/8	2	4	66,000
1 5/8	5 1/2	1 5/16	1 25/32	13/16	1 3/16	2 5/16	23,000	3	3 1/2	2 13/32	3 5/16	1 1/2	2 3/16	4 3/8	78,000



A	*B*	*C*	*D*	*E*	*F*	*G*	Number of Threads per Inch	Strength at Bottom of Thread, at 10,000 Pounds per Sq. In. Tensile Strength	Safe Loads for Chain Made from Bar of Diam. *G*
3/8	2	3/4	5/8	3/16	3/8	1/4	16	675	1,200
1/2	2 1/8	1	3/4	1/4	1/2	5/16	13	1,250	1,700
5/8	2 1/4	1 1/4	1	5/16	5/8	7/16	11	2,020	3,500
3/4	2 3/8	1 7/16	1 1/8	5/16	11/16	1/2	10	3,020	4,500
7/8	2 1/2	1 11/16	1 3/8	3/8	3/4	5/8	9	4,195	6,700
1	2 3/4	1 7/8	1 1/2	1/2	7/8	3/4	8	5,510	10,000
1 1/8	2 7/8	2 1/8	1 5/8	1/2	1	13/16	7	6,930	10,500
1 1/4	3	2 3/8	1 3/4	1/2	1 1/8	7/8	7	8,900	12,000
1 3/8	3 1/8	2 5/8	1 7/8	9/16	1 3/16	1	6	10,540	15,200
1 1/2	3 1/4	2 3/4	2	5/8	1 1/4	1 1/16	6	12,940	17,200
1 5/8	3 3/8	3	2 1/8	11/16	1 3/8	1 1/8	5 1/2	15,150	19,500
1 3/4	3 1/2	3 1/4	2 1/4	3/4	1 1/2	1 1/4	5	17,440	23,700
1 7/8	3 5/8	3 1/2	2 3/8	13/16	1 5/8	1 5/16	5	20,490	26,000
2	3 3/4	3 3/4	2 1/2	7/8	1 3/4	1 3/8	4 1/2	23,000	28,500

Weights in Pounds of Steel Bolts and Nuts, and Steel Bars*



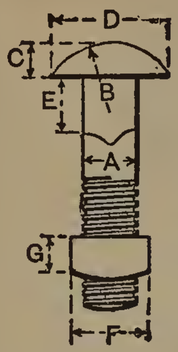
Length L, Inches	Diameter of Bolt												
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1¾
¾	0.033	0.085	0.175	....	....	...	...	...					
⅞	0.034	0.089	0.182	....	....	...	...	...					
1	0.036	0.093	0.189	0.335	....	...	...	...					
1⅛	0.038	0.097	0.196	0.346	0.553	...	...	...					
1¼	0.040	0.101	0.203	0.357	0.568	0.849	...	...					
1⅜	0.041	0.105	0.210	0.367	0.584	0.870	1.23	...					
1½	0.041	0.116	0.230	0.391	0.612	0.899	1.26	1.61					
1¾	0.044	0.125	0.244	0.414	0.643	0.941	1.32	1.68	2.33	...	...	...	...
2	0.048	0.132	0.258	0.435	0.675	0.984	1.37	1.75	2.43	3.10	...	...	...
2¼	0.052	0.140	0.272	0.457	0.706	1.02	1.43	1.82	2.52	3.20	4.02	...	...
2½	0.055	0.147	0.286	0.478	0.737	1.06	1.48	1.89	2.60	3.31	4.12	4.77	...
2¾	0.058	0.156	0.300	0.501	0.768	1.11	1.54	1.96	2.68	3.41	4.25	4.91	5.87
3	0.062	0.163	0.314	0.522	0.800	1.15	1.59	2.03	2.78	3.52	4.40	5.06	6.04
3½	0.069	0.178	0.342	0.565	0.862	1.23	1.71	2.17	2.95	3.73	4.65	5.35	6.38
4	0.076	0.194	0.370	0.609	0.925	1.32	1.82	2.31	3.13	3.94	4.90	5.65	6.72
4½	0.083	0.209	0.398	0.652	0.987	1.40	1.93	2.45	3.30	4.15	5.12	5.94	7.06
5	0.090	0.225	0.426	0.696	1.05	1.49	2.04	2.60	3.47	4.36	5.40	6.24	7.40
5½	0.097	0.240	0.454	0.739	1.11	1.57	2.15	2.74	3.65	4.57	5.62	6.53	7.74
6	0.104	0.256	0.482	0.783	1.17	1.66	2.26	2.88	3.82	4.78	5.90	6.82	8.08
6½	0.111	0.271	0.510	0.826	1.23	1.74	2.37	3.02	4.00	4.99	6.15	7.12	8.42
7	0.118	0.287	0.538	0.870	1.30	1.83	2.48	3.16	4.17	5.20	6.40	7.41	8.76
7½	0.125	0.302	0.566	0.913	1.36	1.91	2.59	3.30	4.34	5.41	6.62	7.71	9.10
8	0.132	0.318	0.594	0.957	1.42	2.00	2.70	3.44	4.52	5.62	6.90	8.00	9.44
8½	0.139	0.333	0.622	1.000	1.48	2.08	2.82	3.58	4.69	5.83	7.15	8.29	9.78
9	0.146	0.349	0.650	1.04	1.55	2.17	2.93	3.72	4.87	6.04	7.40	8.59	10.12
9½	0.157	0.364	0.678	1.08	1.61	2.25	3.04	3.86	5.04	6.25	7.62	8.88	10.46
10	0.160	0.380	0.706	1.13	1.67	2.34	3.15	4.00	5.21	6.46	7.90	9.18	10.81
11	0.174	0.411	0.762	1.21	1.80	2.51	3.37	4.28	5.56	6.88	8.40	9.76	11.49
12	0.188	0.442	0.818	1.30	1.92	2.68	3.61	4.56	5.91	7.30	8.90	10.35	12.17

Weight of Steel Bars

Length of Bar, Feet	Diameter of Bar												
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1¾
1	0.167	0.376	0.668	1.04	1.50	2.04	2.67	3.38	4.17	5.04	6.00	7.05	8.17
2	0.334	0.752	1.336	2.08	3.00	4.08	5.34	6.76	8.34	10.08	12.00	14.10	16.34
3	0.501	1.128	2.004	3.12	4.50	6.12	8.01	10.14	12.51	15.12	18.00	21.15	24.51
4	0.668	1.504	2.672	4.16	6.00	8.16	10.68	13.52	16.68	20.16	24.00	28.20	32.68
5	0.835	1.880	3.340	5.20	7.50	10.20	13.35	16.90	20.85	25.20	30.00	35.25	40.85
6	1.002	2.256	4.008	6.24	9.00	12.24	16.02	20.28	25.02	30.24	36.00	42.30	49.02
7	1.169	2.632	4.676	7.28	10.50	14.28	18.69	23.66	29.19	35.28	42.00	49.35	57.19
8	1.336	3.008	5.344	8.32	12.00	16.32	21.36	27.04	33.36	40.32	48.00	56.40	65.36
9	1.503	3.384	6.012	9.36	13.50	18.36	24.03	30.42	37.53	45.36	54.00	63.45	73.53
10	1.670	3.760	6.680	10.40	15.00	20.40	26.70	33.80	41.70	50.40	60.00	70.50	81.70

* When the length L of bolt is over 12 inches, compute by using table of weight of steel bars. Example: Weight of 1 inch bolt, 18 inches long = weight of 6 inches long bolt + weight of 1 foot long bar.

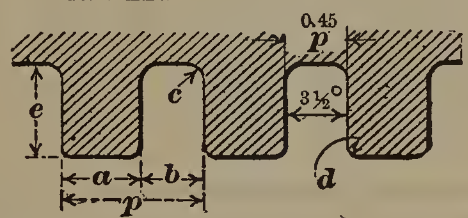
Carriage Bolts and Threads



Carriage-bolt Heads, Shanks and Screw Threads

A	B	C	D	E	Number of Threads	F	G
3/16	5/16	3/32	7/16	3/16	24	3/8	3/16
1/4	13/32	1/8	9/16	1/4	20	7/16	7/32
5/16	7/16	3/16	11/16	5/16	18	1/2	1/4
3/8	9/16	7/32	7/8	3/8	16	5/8	11/32
7/16	21/32	1/4	1	7/16	14	11/16	3/8
1/2	3/4	5/16	1 3/16	1/2	13	13/16	7/16
5/8	31/32	11/32	1 7/16	9/16	11	1	9/16
3/4	1 1/16	13/32	1 5/8	5/8	10	1 3/16	5/8

Philadelphia Carriage-bolt Screw Threads

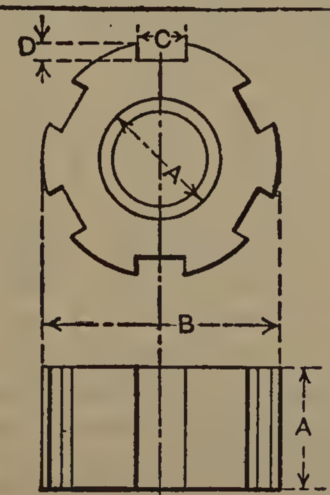


p = pitch; $a = 0.53\,p$; $b = 0.47\,p$;

$c = 0.18\,p$; $d = 0.10\,p$; $e = 0.625\,p$.

Threads per Inch	e	p	a	b	c	d
26	0.024	0.03846	0.0204	0.0181	0.0069	0.0038
24	0.026	0.04167	0.0221	0.0196	0.0075	0.0042
20	0.031	0.0500	0.0265	0.0235	0.0090	0.0050
18	0.0347	0.05556	0.0294	0.0262	0.0100	0.0056
16	0.039	0.06250	0.0331	0.0294	0.0113	0.0062
14	0.0445	0.07142	0.0379	0.0335	0.0129	0.0071
12	0.052	0.08333	0.0442	0.0391	0.0150	0.0083
11	0.0568	0.09091	0.0482	0.0427	0.0164	0.0091
10	0.0625	0.1000	0.0530	0.0470	0.0180	0.0100
9	0.0695	0.11111	0.0589	0.0522	0.0200	0.0111
8	0.0780	0.1250	0.0663	0.0587	0.0225	0.0125

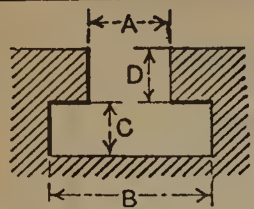
Round Slotted Nuts *



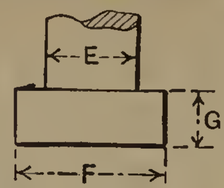
A	B	C	D	A	B	C	D
3/4	1 1/2	5/16	1/8	2 3/4	4 7/8	5/8	1/4
7/8	1 3/4	5/16	1/8	3	5 1/4	11/16	1/4
1	2	5/16	1/8	3 1/4	5 5/8	3/4	5/16
1 1/8	2 1/8	3/8	1/8	3 1/2	6 1/8	3/4	5/16
1 1/4	2 3/8	3/8	1/8	3 3/4	6 1/2	13/16	5/16
1 3/8	2 5/8	3/8	3/16	4	6 7/8	7/8	3/8
1 1/2	2 3/4	7/16	3/16	4 1/4	7 3/8	7/8	3/8
1 5/8	3	7/16	3/16	4 1/2	7 3/4	15/16	3/8
1 3/4	3 1/8	7/16	3/16	4 3/4	8 1/8	1	3/8
1 7/8	3 3/8	1/2	3/16	5	8 1/2	1	7/16
2	3 5/8	1/2	3/16	5 1/4	9	1 1/8	7/16
2 1/4	4	9/16	1/4	5 1/2	9 3/8	1 1/8	7/16
2 1/2	4 3/8	9/16	1/4	5 3/4	9 3/4	1 1/8	1/2

* Number of slots to vary from four up.

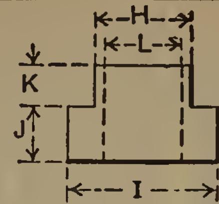
T-Slots, T-Bolts and T-Nuts
(Pratt & Whitney Co. St'd.)



Slot



Bolt-head

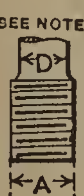


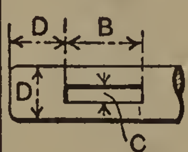
T-nut

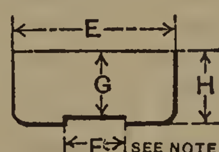
Slot				Bolt-head			T-nut				
A	*B*	*C*	*D**	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
1/4	1/2	5/32	3/16	3/16	7/16	1/8	3/16	7/16	1/8	3/32	1/8
5/16	5/8	5/32	3/16	1/4	9/16	1/8	1/4	9/16	1/8	1/8	1/8 3/16
3/8	11/16	7/32	1/4	5/16	5/8	3/16	5/16	5/8	3/16	1/8	3/16 1/4
7/16	13/16	7/32	9/32	3/8	3/4	3/16	3/8	3/4	3/16	5/32	1/4 5/16
1/2	15/16	9/32	5/16	7/16	7/8	1/4	7/16	7/8	1/4	3/16	5/16 3/8
5/8	1 3/16	13/32	3/8	9/16	1 1/8	11/32	9/16	1 1/8	11/32	3/16	7/16 1/2
3/4	1 5/16	17/32	1/2	11/16	1 1/4	15/32	11/16	1 1/4	15/32	1/4	9/16 5/8
7/8	1 5/8	11/16	9/16	3/4	1 1/2	9/16	3/4	1 1/2	9/16	5/16	5/8 3/4
1	1 7/8	13/16	5/8	7/8	1 3/4	11/16	7/8	1 3/4	11/16	5/16	3/4 7/8

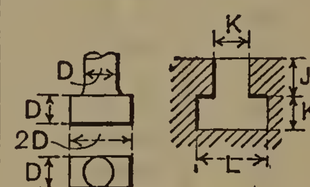
* Minimum thickness permissible. Maximum thickness equals *A* + 1/16 for bolts up to 5/8-inch diameter; 1 inch for 11/16-inch bolt; 1 1/16 for 3/4-inch bolt; and 1 3/16 for 7/8-inch bolt.

Upset, Cottered and T-Bolt Ends







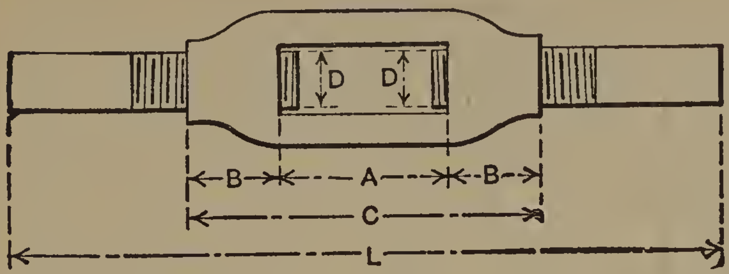


Diam. of Bolt		Extra Length of Bar for 1 Inch of Upset									
D	*A*		*B*	*C*	*E*	*F*	*G*	*H*	*J*	*K*	*L*
1/2	3/4	1 1/4	3/4	1/8	1 1/2	9/16	5/8	11/16	3/4	9/16	1 1/8
5/8	7/8	1	1	3/16	1 7/8	11/16	7/8	15/16	15/16	11/16	1 3/8
3/4	1	3/4	1 1/8	3/16	2 1/4	13/16	1	1 1/16	1 1/8	13/16	1 5/8
7/8	1 1/4	1	1 1/4	1/4	2 5/8	15/16	1 1/8	1 3/16	1 5/16	1	2
1	1 3/8	7/8	1 3/8	1/4	3	1 1/16	1 1/4	1 5/16	1 1/2	1 1/8	2 1/4
1 1/8	1 1/2	3/4	1 5/8	5/16	3 3/8	1 1/4	1 3/8	1 1/2	1 11/16	1 1/4	2 1/2
1 1/4	1 5/8	5/8	1 7/8	5/16	3 3/4	1 3/8	1 5/8	1 3/4	1 7/8	1 3/8	2 3/4
1 3/8	1 3/4	5/8	2	3/8	4 1/8	1 1/2	1 3/4	1 7/8	2 1/16	1 1/2	3
1 1/2	2	3/4	2 1/8	3/8	4 1/2	1 5/8	1 7/8	2	2 1/4	1 5/8	3 1/4
1 5/8	2	1/2	2 1/4	7/16	4 7/8	1 3/4	2	2 1/8	2 7/16	1 3/4	3 1/2
1 3/4	2 1/4	5/8	2 1/2	7/16	5 1/4	1 7/8	2 1/4	2 3/8	2 5/8	1 7/8	3 3/4
1 7/8	2 1/4	1/2	2 5/8	1/2	5 5/8	2	2 3/8	2 1/2	2 13/16	2	4
2	2 1/2	1/2	2 3/4	1/2	6	2 1/8	2 1/2	2 5/8	3	2 1/8	4 1/4

Note. — The upset diameter *A* gives an area at the root of the thread about 20 per cent in excess of *D*, to allow for the reduced strength caused by upsetting.

Note. — Up to 2-inch diameter the cottered ends have a tensile strength practically equal to the bolts. For larger sizes the ends given have from 10 to 20 per cent less strength than the bolts, and upset ends must be used for full strength.

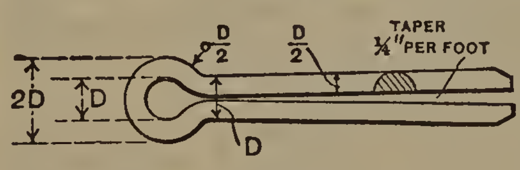
Dimensions of Turnbuckles



D = Nominal size = outside diameter of screws;
A = Length in clear between heads = 6 inches for all sizes;
B = Length of tapped heads = 1½ D;
C = Total length of buckle without bolt ends = 6 inches + 3 D;
L = Total length of buckle and stub ends when open.

Size D, Inches	Length L, Inches	Weight of Buckle, Pounds	Weight of Buckle and Bolt Ends, Pounds	Size D, Inches	Length L, Inches	Weight of Buckle, Pounds	Weight of Buckle and Bolt Ends, Pounds	Size D, Inches	Length L, Inches	Weight of Buckle, Pounds	Weight of Buckle and Bolt Ends, Pounds
⅜	22	1	1½	1⅜	27	7	16	2⅝	32	30	70
7/16	22	1	1¾	1½	27	8	19	2¾	33	33	78
½	22	1	2	1⅝	28	10	23	2⅞	33	36	86
9/16	22	1¼	2½	1¾	28	11	26	3	34	40	96
⅝	22	1½	3	1⅞	29	12	30	3⅛	36	45	108
¾	23	2	4	2	29	14	35	3¼	36	50	120
⅞	24	3	6	2⅛	29	17	41	3⅜	37	57	134
1	25	4	8	2¼	30	20	47	3½	37	65	150
1⅛	25	5	11	2⅜	31	22	53	3¾	39	74	168
1¼	26	6	13	2½	32	25	61	4	41	84	188

Commercial Sizes and Lengths of Cotter Pins



Diam. D Wire Gage No.	Diam. D Inches	Lengths, Inches
13	3/32	½ to 2, by ¼ ths
12	7/64	½ to 2, by ¼ ths
11	⅛	½ to 2½, by ¼ ths
10	9/64	½ to 2½, by ¼ ths
9	5/32	½ to 2½, by ¼ ths
8	11/64	½ to 2½, by ¼ ths
7	3/16	¾ to 3, by ¼ ths
6	13/64	¾ to 3, by ¼ ths
5	7/32	1 to 3, by ¼ ths
4	¼	1 to 4, by ¼ ths
1	5/16	1 to 4, by ¼ ths
.....	⅜	1½ to 4, by ¼ ths
.....	7/16	1¾ to 4, by ¼ ths
.....	½	2 to 4, by ¼ ths
.....	⅝	3 to 4, by ¼ ths

Table 1. S. A. E. Standard Sizes of Cotter Pins *

Length of Shank, Inches	B. W. Gage (Upper Line) and Nominal Diameter in Inches (Lower Line)						
		13	12	11	10	8	6
	1/16	3/32	7/64	1/8	9/64	11/64	13/64
5/16	x	......	......	......	......	......	......
7/16	x	......	......	......	......	......	......
1/2	x	x	......	......	......	......	......
5/8	x	x	......	......	......	......	......
3/4	x^1	x	......	......	......	......	......
7/8	......	x	A	x	......	......	......
1	......	x^1	x	x	......	......	......
1 1/8	......	x^1	x	x	......	......	......
1 1/4	......	......	x^1	x	A	......	......
1 3/8	......	......	x^1	x	x	......	......
1 1/2	......	......	......	x	A	......	......
1 5/8	......	......	......	A	x	x	......
1 3/4	......	......	......	x^1	x^1	x	......
2	......	......	......	......	x^1	x	x
2 1/4	......	......	......	......	A	x^1	x
2 1/2	......	......	......	......	......	x^1	x^1
2 3/4	......	......	......	......	......	......	x^1
3	......	......	......	......	......	......	x^1

* Note. — x, short series; x^1, long series; A, arbitrary sizes.

Table 2. Applications of S. A. E. Standard Cotter Pins

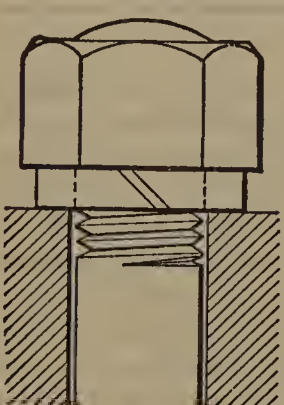
Body Size of Bolt or Pin	Cotter Pins for Yoke and Rod Ends				Cotter Pins for S. A. E. Bolts				Cotter Pins for U. S. S. Bolts			
	Pin Dia.	Pin Length		Drill No.	Pin Dia.	Pin Length		Drill No.	Pin Dia.	Pin Length		Drill No.
		Short	Long			Short	Long			Short	Long	
3/16	1/16	5/16	1/2	48	.....	.....	.....	.....	.....	.....	.....	.....
1/4	1/16	7/16	5/8	48	1/16	1/2	5/8	48	1/16	1/2	5/8	48
5/16	3/32	1/2	5/8	36	1/16	5/8	3/4	48	1/16	5/8	3/4	48
3/8	3/32	5/8	3/4	36	3/32	5/8	3/4	36	3/32	3/4	7/8	36
7/16	3/32	3/4	7/8	36	3/32	5/8	3/4	36	3/32	3/4	1	36
1/2	1/8	7/8	1	28	3/32	3/4	7/8	36	3/32	7/8	1 1/8	36
9/16	.....	.....	.....	.....	1/8	7/8	1 1/8	28	7/64	1	1 1/4	30
5/8	.....	.....	.....	.....	1/8	1	1 1/4	28	7/64	1 1/8	1 3/8	30
11/16	.....	.....	.....	.....	.....	.....	.....	.....	7/64	1 1/8	1 3/8	30
3/4	.....	.....	.....	.....	1/8	1 1/8	1 3/8	28	1/8	1 1/4	1 1/2	28
7/8	.....	.....	.....	.....	1/8	1 1/4	1 1/2	28	9/64	1 3/8	1 3/4	21
1	.....	.....	.....	.....	1/8	1 1/2	1 3/4	28	9/64	1 5/8	2	21
1 1/8	.....	.....	.....	.....	11/64	1 5/8	2	11	11/64	1 3/4	2 1/4	11
1 1/4	.....	.....	.....	.....	11/64	1 3/4	2 1/4	11	11/64	2	2 1/2	11
1 3/8	.....	.....	.....	.....	13/64	2	2 1/2	2	13/64	2 1/4	2 3/4	2
1 1/2	.....	.....	.....	.....	13/64	2 1/4	2 3/4	2	13/64	2 1/4	3	2

S. A. E. Standard Cotter Pins. — The accompanying table of S. A. E. standard cotter pins contains 41 recommended standard sizes which should replace between 250 and 300 different sizes used heretofore. Table 1 shows the range of sizes, while Table 2 shows where they may be applied. The sizes marked "x" are the ones in the short series of Table 1; those marked "x¹" are the sizes required to complete the long series, and those indicated by "A" are extra sizes not called for on Table 2, which seem desirable in order to fill out the line.

Lock Washers. — The S. A. E. standard lock washers listed in the accompanying table are, before compression, similar to one turn of a helical spring formed of square steel stock. When the washer is compressed it exerts a pressure on the nut which tends to prevent the nut from turning. The washers for S. A. E. and U. S. standard bolts are of two classes; namely, the heavy section for general use and the light section for optional use against soft metal. According to the S. A. E. specifications, all lock washers are to have parallel-faced sections, bulging or malformed ends being avoided. The temper of a washer is tested by compressing it until flat. The reaction should then be sufficient to indicate the necessary spring power, and when the washer is again compressed until flat, there should be no appreciable loss in reaction. To test the toughness, about forty-five per cent of the washer including one end should be firmly secured in a vise and then forty-five per cent including the other end is to be secured firmly between the parallel jaws of a wrench. The washer should then withstand twisting through an angle of 45 degrees without sign of fracture.

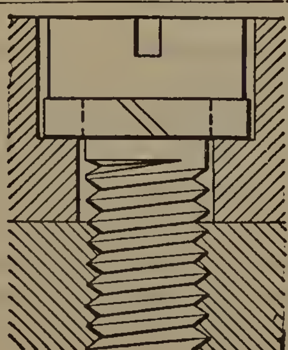
Lock Washers — S. A. E. Standard

Sections for S. A. E. and U. S. Standard Bolts *



Bolt Diameter, Inches	Washer: Heavy Section, Inches	Washer: Light Section, Inches	Bolt Diameter, Inches	Washer: Heavy Section, Inches	Washer: Light Section, Inches
3/16	1/16 × 1/16	1/16 × 3/64	11/16	1/4 × 1/4	1/4 × 3/16
1/4	5/64 × 5/64	5/64 × 1/16	3/4	1/4 × 1/4	1/4 × 3/16
5/16	1/8 × 1/8	1/8 × 3/32	7/8	17/64 × 17/64	17/64 × 3/16
3/8	1/8 × 1/8	1/8 × 3/32	1	5/16 × 5/16	5/16 × 1/4
7/16	11/64 × 11/64	11/64 × 1/8	1 1/8	5/16 × 5/16	....
1/2	11/64 × 11/64	11/64 × 1/8	1 1/4	3/8 × 3/8	....
9/16	13/64 × 13/64	13/64 × 5/32	1 3/8	3/8 × 3/8	....
5/8	13/64 × 13/64	13/64 × 5/32	1 1/2	7/16 × 7/16	....

Sections for Machine Screws with Counterbores



Number of Screw	Diameter of Screw	Washer: Inside Diameter	Washer: Heavy Section	Washer: Light Section	Counterbore Diameter
4	0.112	0.114	1/32 × 1/32	1/32 × 0.022	3/16
6	0.138	9/64	3/64 × 3/64	3/64 × 1/32	1/4
8	0.164	11/64	3/64 × 3/64	3/64 × 1/32	9/32
10	0.190	13/64	1/16 × 1/16	1/16 × 3/64	11/32
12	0.216	7/32	1/16 × 1/16	1/16 × 3/64	3/8
14	0.242	1/4	5/64 × 5/64	5/64 × 1/16	7/16

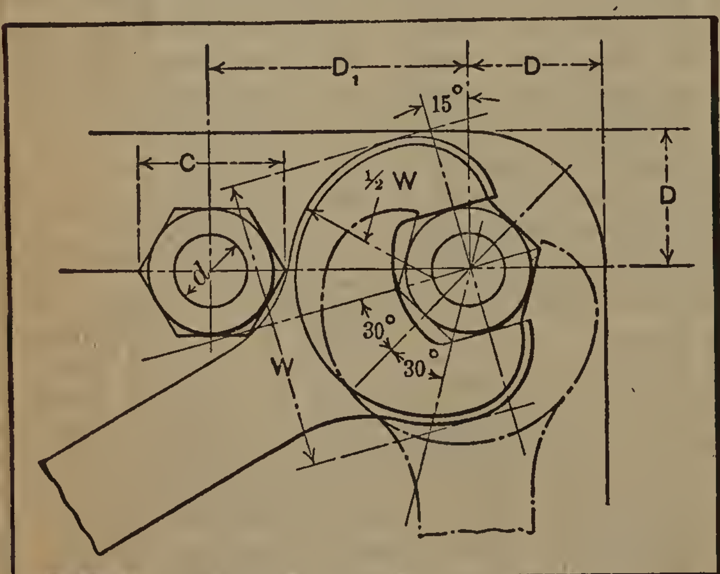
* Inside diameters of the washers for S. A. E. and U. S. standard bolts shall be from 1/64 to 1/32 inch greater than bolt diameters. Outside diameters shall coincide practically with "long diameters" of S. A. E. standard nuts and "short diameters" of U. S. standard nuts.

Dimensions and Weight of Wrought Iron and Steel Washers

(Manufacturers' Standard Sizes)

Size of Bolt, Inches	Outside Diam., Inches	Size of Hole, Inches	Thickness, U. S. S. Plate Gage	Approximate Thickness, Inches	Approximate Number in 100 Pounds	Approximate Weight per 100, in Pounds	Size of Bolt, Inches	Outside Diam., Inches	Size of Hole, Inches	Thickness, U. S. S. Plate Gage	Approximate Thickness, Inches	Approximate Number in 100 Pounds	Approximate Weight per 100, in Pounds
3/16	9/16	1/4	18	1/20	44,075	0.2	1	2 1/2	1 1/16	9	5/32	617	16
1/4	3/4	5/16	16	1/16	13,845	0.7	1 1/8	2 3/4	1 1/4	9	5/32	516	20
5/16	7/8	3/8	16	1/16	11,220	0.9	1 1/4	3	1 3/8	8	11/64	403	25
3/8	1	7/16	14	5/64	6,573	1.5	1 3/8	3 1/4	1 1/2	8	11/64	320	30
7/16	1 1/4	1/2	14	5/64	4,261	3.9	1 1/2	3 1/2	1 5/8	8	11/64	278	36
1/2	1 3/8	9/16	12	7/64	2,683	5	1 5/8	3 3/4	1 3/4	8	11/64	247	40
9/16	1 1/2	5/8	12	7/64	2,249	6	1 3/4	4	1 7/8	8	11/64	224	45
5/8	1 3/4	11/16	10	9/64	1,315	8	1 7/8	4 1/4	2	8	11/64	200	50
3/4	2	13/16	10	9/64	1,013	10	2	4 1/2	2 1/8	8	11/64	180	60
7/8	2 1/4	15/16	9	5/32	858	12	..	..	..	..	...	...	...

Spacing of Bolts for Wrench Clearance. — The spacing required for bolts, so as to obtain sufficient room between adjacent nuts for wrench clearance, may be determined by the following formulas, in which D = distance from center of bolt to nearest side of corner (see accompanying illustration); D_1 = distance between centers of adjacent bolts; d = diameter of bolt; C = distance across corners of nut; W = maximum width of head of wrench.



$$D = 0.48\ W$$

The value of W may be obtained from page 848, "Dimensions of Wrenches," or it may be taken as $3\frac{1}{2}\ d + \frac{1}{8}$ inch which will be found ample for the wrenches on the market. The slight clearance at the sides, obtained with the formula for D, may need to be increased in some cases. The following expression gives the distance D_1 between centers of adjacent bolts:

$$D_1 = \tfrac{1}{2}\ W + \tfrac{1}{2}\ C + \tfrac{1}{2}\ d$$

The distance C across the corners of the nuts may be found in table "U. S. Standard Threads, Bolts and Nuts."

Length of Thread Cut on Bolts *

Diameter of Bolt, Inches	Length of Bolt, Inches							
	1 to 1½	1⅝ to 2	2⅛ to 2½	2⅝ to 3	3⅛ to 4	4⅛ to 8	8⅛ to 12	12⅛ to 20
¼	¾	⅞	1	1	1⅛	1¼	.....	.....
5/16	¾	⅞	1	1	1⅛	1¼	.....	.....
⅜	⅞	1	1⅛	1⅛	1¼	1⅜	1½	.....
7/16	⅞	1	1⅛	1⅛	1¼	1⅜	1½	.....
½	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1¾
9/16	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1¾
⅝	1	1⅛	1¼	1⅜	1½	1⅝	1¾	2
¾	.....	1¼	1⅜	1½	1⅝	1¾	2	2¼
⅞	.....	1¼	1½	1¾	1⅞	2	2¼	2½
1	.....	.....	1¾	2	2¼	2½	2¾	3
1⅛	.....	.....	.....	2¼	2½	2¾	3	3¼
1¼	.....	.....	.....	.....	2½	3	3¼	3½

* Bolts larger in diameter and longer than given in the table are threaded a length equal to three times their diameter.

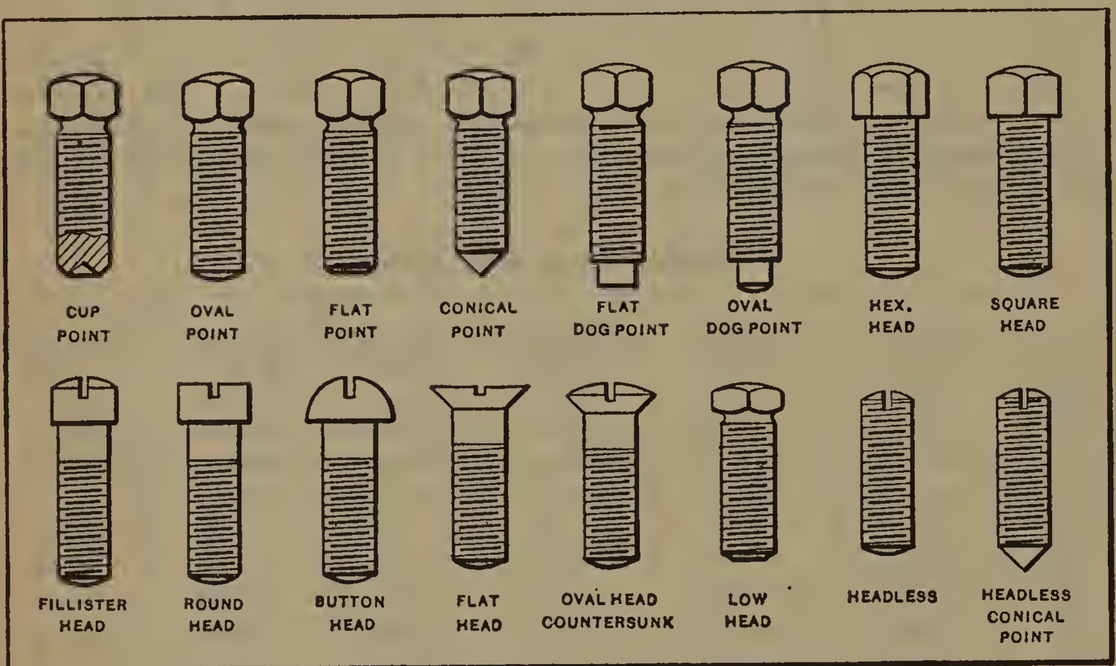
Working Strength of Bolts *

Diam. of Bolt, Inches	Strength of Bolt, 5000 Pounds Stress	Strength of Bolt, 6000 Pounds Stress	Strength of Bolt, 7000 Pounds Stress	Strength of Bolt, 8000 Pounds Stress	Strength of Bolt, 10,000 Pounds Stress	Strength of Bolt, 12,000 Pounds Stress
⅝	220	265	300	350	440	525
¾	565	680	790	900	1,130	1,355
⅞	1,000	1,200	1,400	1,600	2,000	2,400
1	1,490	1,790	2,085	2,385	2,980	3,475
1⅛	2,055	2,465	2,875	3,290	4,110	4,930
1¼	2,890	3,470	4,045	4,625	5,780	6,935
1⅜	3,550	4,260	4,970	5,680	7,100	8,520
1½	4,585	5,500	6,420	7,335	9,170	10,500
1⅝	5,525	6,630	7,735	8,840	11,050	13,260
1¾	6,525	7,830	9,135	10,440	13,050	15,660
1⅞	7,890	9,470	11,045	12,625	15,780	18,935
2	8,990	10,790	12,585	14,385	17,980	21,575
2¼	12,280	14,735	17,190	19,650	24,560	29,470
2½	15,445	18,535	21,620	24,710	30,890	37,070
2¾	19,635	23,560	27,490	31,415	39,270	47,125
3	23,360	28,030	32,700	37,375	46,720	56,065
3¼	28,450	34,140	39,830	45,520	56,900	68,280
3½	33,330	39,995	46,665	53,330	66,660	79,990

* The figures given in the table above show the stress to which it is safe to subject the bolt when due allowance has been made for the stresses in the bolt caused by tightening the nut. The table refers specifically to bolts for cylinder heads, receivers containing fluids under pressure, etc. For work of this character, bolts less than ⅝ inch in diameter should not be employed.

Working Strength of Bolts. — Whether or not the initial tension on a tightened bolt or stud holding a part subjected to pressure is increased by that pressure before the initial tension of the bolt is exceeded, depends upon the following conditions: When a bolt is more elastic than the material compressed, the stress in it equals either *the initial stress* (due to tightening the nut) or *the force applied*, depending upon which is greater. If the material compressed is more elastic than the bolt, the stress in the bolt equals the *initial stress plus the force applied.*

Bolts subjected to tension are used for two different classes of service: They either hold two heavy and rigid flanges together, metal to metal, or they serve to compress a comparatively elastic packing in order to make a tight joint. In either case, the bolt is under considerable initial tension due to the strain of tightening the nut. When flanges are held together, metal to metal, they are much more unyielding than the bolts; hence, when the bolts are tightened they are stretched a great deal more than the flanges are compressed. If we assume that the flanges cannot be compressed at all, each bolt is practically a spring, and in order to produce in it a



Different Types of Points and Heads on Set-screws and Cap-screws

stress greater than the initial stress, the flanges must be subjected to a load which exceeds that initial stress. If a gasket that is more elastic than the bolts is placed between the flanges, the total stress on the bolts would equal the initial tension (due to tightening the nuts) plus the force applied.

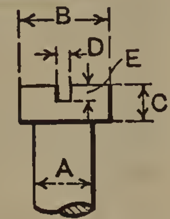
Some experiments made at Cornell University to determine the initial stress due to tightening bolts sufficiently to make a packed joint steam-tight, showed that the stress is often sufficient to break off a ½-inch bolt, but not larger sizes, assuming that the nut is tightened by an experienced mechanic; hence, bolts smaller than ⅝ inch should not be used for holding cylinder heads or other parts requiring a tight joint. The following rule may be used for determining the size of bolts or studs to be used for joints: Make the "working section" of the bolt equal to its area at the root of the thread, less 0.25 times its diameter in inches. This "working section" must be sufficient to sustain, with a liberal factor of safety, the stress due to the steam load or other force tending to separate the flanges.

Example: — The area of a 1-inch bolt at the root of the thread is 0.551 square inch. Then $0.551 - 0.25 \times 1 = 0.301$ square inch, which is the "working section" of a 1-inch bolt. At 10,000 pounds to the square inch, this bolt will sustain a stress of slightly more than 3000 pounds, in addition to the stress due to screwing up nut.

A.S.M.E. Standard Proportions for Machine Screw Heads

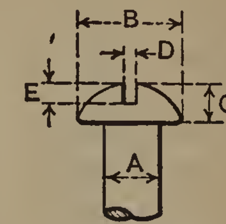
Flat Fillister Head Screws

A = Diameter of body;
$B = 1.64\,A - 0.009$ = Diameter of head;
$C = 0.66\,A - 0.002$ = Height of head;
$D = 0.173\,A + 0.015$ = Width of slot;
$E = \frac{1}{2}C$ = Depth of slot.



Round Head Screws

A = Diameter of body;
$B = 1.85\,A - 0.005$ = Diameter of head;
$C = 0.7\,A$ = Height of head;
$D = 0.173\,A + 0.015$ = Width of slot;
$E = \frac{1}{2}C + 0.01$ = Depth of slot.

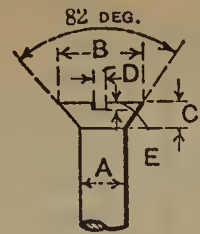


Flat Fillister Head Screws					Round Head Screws				
A	*B*	*C*	*D*	*E*	*A*	*B*	*C*	*D*	*E*
0.060	0.0894	0.0376	0.025	0.019	0.060	0.106	0.042	0.025	0.031
0.073	0.1107	0.0461	0.028	0.023	0.073	0.130	0.051	0.028	0.035
0.086	0.1320	0.0548	0.030	0.027	0.086	0.154	0.060	0.030	0.040
0.099	0.1530	0.0633	0.032	0.032	0.099	0.178	0.069	0.032	0.044
0.112	0.1747	0.0719	0.034	0.036	0.112	0.202	0.078	0.034	0.049
0.125	0.1960	0.0805	0.037	0.040	0.125	0.226	0.087	0.037	0.053
0.138	0.2170	0.0890	0.039	0.044	0.138	0.250	0.096	0.039	0.058
0.151	0.2386	0.0976	0.041	0.049	0.151	0.274	0.105	0.041	0.062
0.164	0.2599	0.1062	0.043	0.053	0.164	0.298	0.114	0.043	0.067
0.177	0.2813	0.1148	0.046	0.057	0.177	0.322	0.123	0.046	0.071
0.190	0.3026	0.1234	0.048	0.062	0.190	0.346	0.133	0.048	0.076
0.216	0.3452	0.1405	0.052	0.070	0.216	0.394	0.151	0.052	0.085
0.242	0.3879	0.1577	0.057	0.079	0.242	0.443	0.169	0.057	0.094
0.268	0.4305	0.1748	0.061	0.087	0.268	0.491	0.187	0.061	0.103
0.294	0.4731	0.1920	0.066	0.096	0.294	0.539	0.205	0.066	0.112
0.320	0.5158	0.2092	0.070	0.104	0.320	0.587	0.224	0.070	0.122
0.346	0.5584	0.2263	0.075	0.113	0.346	0.635	0.242	0.075	0.131
0.372	0.6010	0.2435	0.079	0.122	0.372	0.683	0.260	0.079	0.140
0.398	0.6437	0.2606	0.084	0.130	0.398	0.731	0.278	0.084	0.149
0.424	0.6863	0.2778	0.088	0.139	0.424	0.779	0.296	0.088	0.158
0.450	0.7270	0.2950	0.093	0.147	0.450	0.827	0.315	0.093	0.167

A.S.M.E. Standard Proportions for Machine Screw Heads

Flat Head Screws

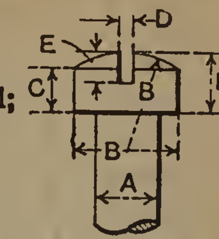
A = Diameter of body
$B = 2A - 0.008$ = Diameter of head;
$C = \dfrac{A - 0.008}{1.739}$ = Thickness of head;
$D = 0.173A + 0.015$ = Width of slot;
$E = \frac{1}{3}C$ = Depth of slot.



A	B	C	D	E
0.060	0.112	0.029	0.025	0.010
0.073	0.138	0.037	0.028	0.012
0.086	0.164	0.045	0.030	0.015
0.099	0.190	0.052	0.032	0.017
0.112	0.216	0.060	0.034	0.020
0.125	0.242	0.067	0.037	0.022
0.138	0.268	0.075	0.039	0.025
0.151	0.294	0.082	0.041	0.027
0.164	0.320	0.090	0.043	0.030
0.177	0.346	0.097	0.046	0.032
0.190	0.372	0.105	0.048	0.035
0.216	0.424	0.120	0.052	0.040
0.242	0.476	0.135	0.057	0.045
0.268	0.528	0.150	0.061	0.050
0.294	0.580	0.164	0.066	0.055
0.320	0.632	0.179	0.070	0.060
0.346	0.684	0.194	0.075	0.065
0.372	0.736	0.209	0.079	0.070
0.398	0.788	0.224	0.084	0.075
0.424	0.840	0.239	0.088	0.080
0.450	0.892	0.254	0.093	0.085

Oval Fillister Head Screws

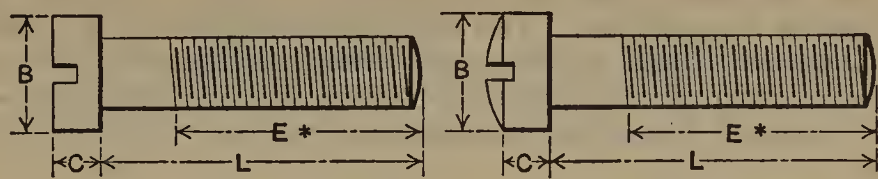
A = Diameter of body;
$B = 1.64A - 0.009$ = Diameter of head and radius for oval;
$C = 0.66A - 0.002$ = Height of side;
$D = 0.173A + 0.015$ = Width of slot;
$E = \frac{1}{2}F$ = Depth of slot;
$F = 0.134B + C$ = Height of head.



A	B	C	D	E	F
0.060	0.0894	0.0376	0.025	0.025	0.0496
0.073	0.1107	0.0461	0.028	0.030	0.0609
0.086	0.1320	0.0548	0.030	0.036	0.0725
0.099	0.1530	0.0633	0.032	0.042	0.0838
0.112	0.1747	0.0719	0.034	0.048	0.0953
0.125	0.1960	0.0805	0.037	0.053	0.1068
0.138	0.2170	0.0890	0.039	0.059	0.1180
0.151	0.2386	0.0976	0.041	0.065	0.1296
0.164	0.2599	0.1062	0.043	0.071	0.1410
0.177	0.2813	0.1148	0.046	0.076	0.1524
0.190	0.3026	0.1234	0.048	0.082	0.1639
0.216	0.3452	0.1405	0.052	0.093	0.1868
0.242	0.3879	0.1577	0.057	0.105	0.2097
0.268	0.4305	0.1748	0.061	0.116	0.2325
0.294	0.4731	0.1920	0.066	0.128	0.2554
0.320	0.5158	0.2092	0.070	0.140	0.2783
0.346	0.5584	0.2263	0.075	0.150	0.3011
0.372	0.6010	0.2435	0.079	0.162	0.3240
0.398	0.6437	0.2606	0.084	0.173	0.3469
0.424	0.6863	0.2778	0.088	0.185	0.3698
0.450	0.7270	0.2950	0.093	0.201	0.4024

Flat Fillister- and Oval Fillister-head Machine Screws

(A.S.M.E. Standard and Special)



FLAT FILLISTER HEAD   OVAL FILLISTER HEAD

Screw Number	Outside Diam., Inch	Size of Head to Nearest 1/64 Inch		Threads per Inch			Shortest Lengths Listed †	Longest Lengths Commonly Listed †
		B ‡	*C* ‡	A.S.M.E. Standard	A.S.M.E. Special	Other Pitches		
0	0.060	3/32	1/32	80	......	......	.....	....
1	0.073	7/64	3/64	72	64	......	.....	....
2	0.086	1/8	3/64	64	56	48	1/8	7/8
3	0.099	5/32	1/16	56	48	......	1/8	7/8, 15/16
4	0.112	11/64	5/64	48	36, 40	32	1/8	2
5	0.125	13/64	5/64	44	36, 40	32	1/8	2 1/4, 2 1/2
6	0.138	7/32	3/32	40	32, 36	30	1/8	2 1/4, 2 1/2
7	0.151	15/64	3/32	36	30, 32	......	1/8	3
8	0.164	17/64	7/64	36	30, 32	......	1/8	2 1/2, 3
9	0.177	9/32	7/64	32	24, 30	......	3/16	3 1/2, 4
10	0.190	19/64	1/8	30	24, 32	......	3/16	3 1/2, 4
12	0.216	11/32	9/64	28	24	20	1/4	3 1/2, 4
14	0.242	25/64	5/32	24	20	18	1/4	4
16	0.268	7/16	11/64	22	20	16, 18	5/16	4
18	0.294	15/32	3/16	20	18	16	3/8	4
20	0.320	33/64	13/64	20	18	16	3/8	4
22	0.346	9/16	7/32	18	16	......	3/8	4
24	0.372	19/32	1/4	16	18	14	3/8	4
26	0.398	41/64	17/64	16	14	......	3/4, 7/8	4
28	0.424	11/16	9/32	14	16	......	3/4, 7/8	4
30	0.450	47/64	19/64	14	16	......	3/4, 1	4

* The length *E* of the thread according to the rule of the Scovill Mfg. Co. is as follows: Screws up to 1 inch long inclusive are threaded up to the head; longer screws are threaded 3/4 of the length. According to the rule of the Corbin Screw Corporation, machine screws having cut threads are threaded up to the head for lengths up to 1/2 inch inclusive; longer sizes are threaded 2/3 of the length.

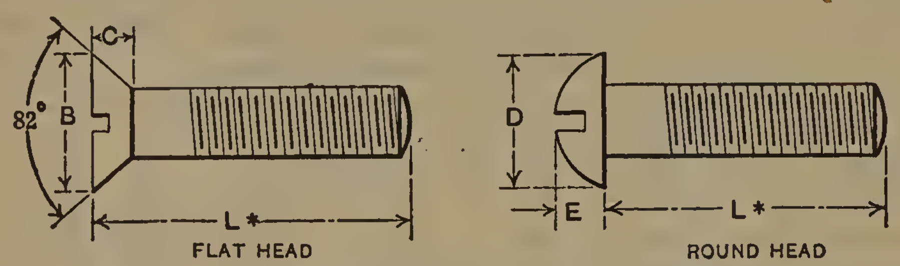
† The amounts by which the lengths increase from minimum to maximum are as follows according to the practice of several prominent screw manufacturers: Lengths *L* up to 1 inch inclusive, increase either by 1/16 inch increments, or by 1/16ths up to 5/8 and then by 1/8ths up to 1; lengths *L* from 1 to 2 inches inclusive, increase either by 1/8 or 1/4 inch increments; lengths *L* from 2 to 4 inches inclusive, increase by 1/4 inch increments.

‡ The exact dimensions may be obtained from the tables "A.S.M.E. Standard Proportions for Machine Screw Heads."

Cap-screws. — While there is no generally recognized standard for cap-screws, the most important dimensions of cap-screws made by screw manufacturers in the United States are either the same or differ only slightly. The accompanying tables give the proportions of cap-screws made by several prominent manufacturers and show to what extent the dimensions vary. The cap-screws made by other manufacturers either conform to the dimensions given, or differ only slightly. The lengths L (see the illustrations accompanying the various tables) indicate the nominal length of the cap-screw. This length is usually measured to the extreme point. The range of lengths in which cap-screws of different diameters are made varies somewhat, although the lengths given in the tables "Commercial Lengths of Cap-screws" are listed by several manufacturers and conform closely to the general practice.

Flat-head and Round-head Machine Screws *

(A.S.M.E. Standard and Special)



Screw Number	Outside Diam., Inch	Size Flat Head to Nearest 1/64 Inch		Size Round Head to Nearest 1/64 Inch		Threads per Inch		
		B †	*C* †	*D* †	*E* †	A.S.M.E. Standard	A.S.M.E. Special	Other Pitches
0	0.060	7/64	1/32	7/64	3/64	80	......	......
1	0.073	9/64	1/32	1/8	3/64	72	64	......
2	0.086	11/64	3/64	5/32	1/16	64	56	48
3	0.099	3/16	3/64	11/64	1/16	56	48	......
4	0.112	7/32	1/16	13/64	5/64	48	36, 40	32
5	0.125	1/4	1/16	7/32	3/32	44	36, 40	32
6	0.138	17/64	5/64	1/4	3/32	40	32, 36	30
7	0.151	19/64	5/64	9/32	7/64	36	30, 32	......
8	0.164	5/16	3/32	19/64	7/64	36	30, 32	......
9	0.177	11/32	3/32	21/64	1/8	32	24, 30	......
10	0.190	3/8	7/64	11/32	9/64	30	24, 32	......
12	0.216	27/64	1/8	25/64	5/32	28	24	20
14	0.242	15/32	9/64	7/16	11/64	24	20	18
16	0.268	17/32	5/32	31/64	3/16	22	20	16, 18
18	0.294	37/64	11/64	35/64	13/64	20	18	16
20	0.320	5/8	11/64	19/32	7/32	20	18	16
22	0.346	11/16	3/16	41/64	15/64	18	16	......
24	0.372	47/64	13/64	11/16	17/64	16	18	14
26	0.398	25/32	7/32	47/64	9/32	16	14	......
28	0.424	27/32	15/64	25/32	19/64	14	16	......
30	0.450	57/64	1/4	53/64	5/16	14	16	......

* The minimum and maximum commercial lengths, and the amounts by which they increase from the shortest to the longest sizes listed, are given in the table "Flat Fillister- and Oval Fillister-head Machine Screws."

† The exact dimensions may be obtained from the tables "A.S.M.E Standard Proportions for Machine Screw Heads."

S.A.E. Standard Nuts for A.S.M.E. Standard Machine Screws

Size of Screw		Width Across Flats, Hexagonal or Square	Normal Thickness of Nut, Inch	Size of screw		Width Across Flats, Hexagonal or Square	Normal Thickness of Nut, Inch
Number	Outside Diam., Inch			Number	Outside Diam., Inch		
0*	0.060	1/8	0.045	12	0.216	7/16	5/32
1*	0.073	5/32	0.055	14	0.242	1/2	3/16
2	0.086	3/16	0.065	16†	0.268	1/2	3/16
3	0.099	7/32‡	0.074‡	18	0.294	9/16	1/4
4	0.112	1/4	0.084	20	0.320	19/32	1/4
5	0.125	9/32‡	0.094‡	22†	0.346	19/32	1/4
6	0.138	5/16	0.104	24	0.372	11/16	5/16
7‡	0.151	5/16	0.104	26†	0.398	11/16	5/16
8	0.164	11/32	1/8	28†	0.424	11/16	5/16
9‡	0.177	11/32	1/8	30	0.450	25/32	3/8
10	0.190	3/8	9/64	......	......	....	....

* Made from bar stock only.

† Nuts for these sizes are special.

‡ Unless made from bar stock, nuts for Nos. 3 and 5 will be made the same dimensions as Nos. 2 and 4 respectively.

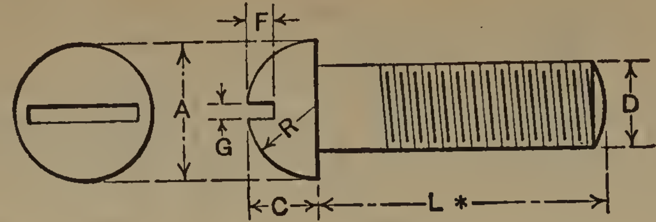
The above table gives all stock sizes for square and hexagon machine screw nuts of brass, iron and steel. Hexagon nuts have 30-degree chamfer on one end.

Commercial Lengths of Cap-screws *

Cap-screw Diameter	Lengths of Hexagon and Square-head Cap-screws		Lengths of Fillister- and Round-head Cap-screws		Lengths of Flat- and Button-head Cap-screws	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
1/8	....	....	3/4	2 1/2	3/4	1 3/4
3/16	....	....	3/4	2 3/4	3/4	2
1/4	3/4	3	3/4	3	3/4	2 1/4
5/16	3/4	3	3/4	3 1/4	3/4	2 1/2, 2 3/4
3/8	3/4	3	3/4	3 1/2	3/4	2 3/4, 3
7/16	3/4	4	3/4	4 1/4	3/4, 1	3
1/2	3/4	4	3/4	4 1/2	1, 1 1/4	3
9/16	1	4 1/2	1	4 1/2	1 1/4, 1 1/2	3
5/8	1	4 1/2	1 1/4	4 1/2	1 1/2, 1 3/4	3
3/4	1 1/4	4 3/4	1 1/2	4 3/4	1 3/4, 2	3
7/8	1 1/2	5	1 3/4	5	....	....
1	1 3/4	5	2	5	....	....
1 1/8	2	5	....	....	....	....
1 1/4	2	5	....	....	....	....

* The range of lengths in which cap-screws of different diameters are made, varies somewhat, although the lengths given in the above table conform either exactly, or quite closely, to the lengths commonly listed by screw manufacturers. The amounts by which the lengths increase from minimum to maximum are as follows, according to the practice of several prominent screw manufacturers: Lengths up to 1 inch inclusive, increase by 1/8 inch increments; lengths from 1 to 5 inches increase by 1/4 inch increments.

Button-head Cap-Screws — 1

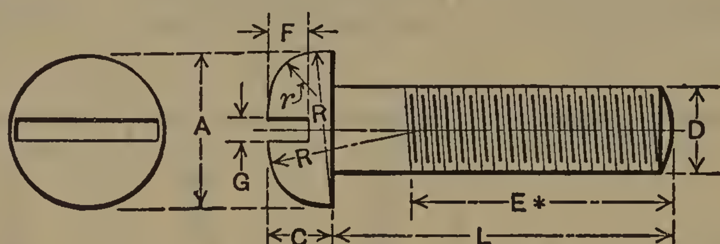


Outside Diameter of Screw, D	Number of Threads per Inch	National Acme Co. Hartford Mach. Screw Co.			National Acme Co.		Hartford Machine Screw Co.	
		A	C	R	F	G	F	G
1/8	40	7/32	7/64	7/64	0.037	0.025	1/16	0.028
3/16	24	5/16	5/32	5/32	0.056	0.037	3/32	0.035
1/4	20	7/16	7/32	7/32	0.075	0.050	1/8	0.049
5/16	18	9/16	9/32	9/32	0.094	0.062	5/32	0.065
3/8	16	5/8	5/16	5/16	0.112	0.075	3/16	0.065
7/16	14	3/4	3/8	3/8	0.131	0.087	7/32	0.083
1/2	12 or 13	13/16	13/32	13/32	0.150	0.100	1/4	0.083
9/16	12	15/16	15/32	15/32	0.169	0.112	9/32	0.083
5/8	11	1	1/2	1/2	0.187	0.125	5/16	0.109
3/4	10	1 1/4	5/8	5/8	0.225	0.150	3/8	0.134

* For minimum and maximum lengths L, see table "Commercial Lengths of Cap-screws."

Button-head Cap-screws — 2 *

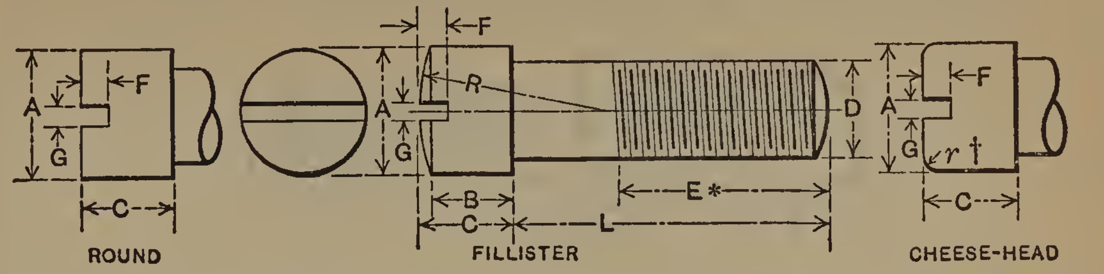
(Scovill Manufacturing Co.)



Outside Diameter of Screw, D	Number of Threads per Inch	Diameter of Head, A	Thickness of Head, C	Depth of Slot, F	Width of Slot, G	Radius, R	Radius, r
1/4	20	7/16	3/16	1/8	1/16	7/16	1/8
5/16	18	35/64	15/64	5/32	5/64	35/64	5/32
3/8	16	21/32	9/32	3/16	3/32	21/32	3/16
7/16	14	49/64	21/64	7/32	7/64	49/64	7/32
1/2	13	7/8	3/8	1/4	1/8	7/8	1/4
9/16	12	63/64	27/64	9/32	9/64	63/64	9/32
5/8	11	1 3/32	15/32	5/16	5/32	1 3/32	5/16
3/4	10	1 5/16	9/16	3/8	3/16	1 5/16	3/8

* Cap-screws having a length L of 1 inch or less are threaded up to the head. For lengths greater than 1 inch, see note below Scovill Mfg. Co.'s table of "Square-head Cap-screws."

Round-head, Fillister-head and "Cheese-head" Cap-screws



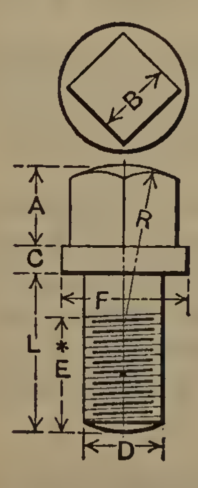
Outside Diameter of Screw, D	Threads per Inch	Nat. Acme Co.; Hartford Mach. Screw Co.		National Acme Co.			Hartford Machine Screw Co.			Nat. Acme Co.,	Hartford Mach. Screw Co.,
		A	C	B	F	G	B	F	G	R	R
1/8	40	3/16	1/8	7/64	0.037	0.025	3/32	1/16	0.028	9/32	3/16
3/16	24	1/4	3/16	11/64	0.056	0.037	5/32	3/32	0.035	3/8	1/4
1/4	20	3/8	1/4	7/32	0.075	0.050	13/64	1/8	0.049	9/16	3/8
5/16	18	7/16	5/16	9/32	0.094	0.062	1/4	5/32	0.065	21/32	7/16
3/8	16	9/16	3/8	21/64	0.112	0.075	19/64	3/16	0.065	27/32	9/16
7/16	14	5/8	7/16	25/64	0.131	0.087	23/64	3/16	0.083	15/16	5/8
1/2	12 or 13	3/4	1/2	7/16	0.150	0.100	13/32	7/32	0.083	1 1/8	3/4
9/16	12	13/16	9/16	1/2	0.169	0.112	29/64	1/4	0.083	1 7/32	13/16
5/8	11	7/8	5/8	35/64	0.187	0.125	1/2	9/32	0.109	1 5/16	7/8
3/4	10	1	3/4	21/32	0.225	0.150	39/64	5/16	0.134	1 1/2	1
7/8	9	1 1/8	7/8	25/32	0.262	0.175	23/32	11/32	0.134	1 11/16	1 1/8
1	8	1 1/4	1	57/64	0.300	0.200	53/64	3/8	0.165	1 7/8	1 1/4

* For lengths L of 4 inches or less, E = 3/4 of L; for lengths greater than 4 inches, E = 1/2 of L.

† Radius r of "cheese head" is 1/32 inch for diameters up to 1/4 inch inclusive; 3/64 inch for diameters from 5/16 inch up to 9/16 inch inclusive; and 1/16 inch for larger diameters.

Collar Screws

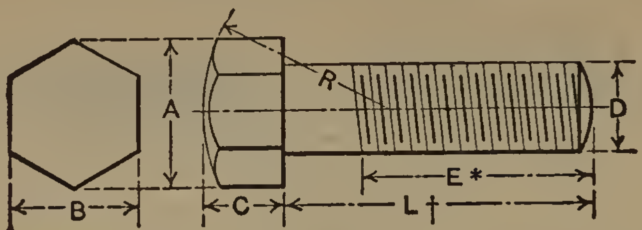
(Hartford Machine Screw Co.)



Outside Diameter of Screw, D	Number of Threads per Inch	Length of Head, A	Size of Square, B	Thickness of Collar, C	Diameter of Collar, F	Radius of Head, R
1/8	40	1/8	1/8	1/16	1/4	1/4
3/16	24	3/16	3/16	5/64	11/32	3/8
1/4	20	1/4	1/4	5/64	7/16	1/2
5/16	18	5/16	5/16	7/64	1/2	5/8
3/8	16	3/8	3/8	1/8	5/8	3/4
7/16	14	7/16	7/16	1/8	11/16	7/8
1/2	12 or 13	1/2	1/2	3/16	13/16	1
9/16	12	9/16	9/16	7/32	15/16	1 1/8
5/8	11	5/8	5/8	15/64	1	1 1/4
3/4	10	3/4	3/4	9/32	1 1/4	1 1/2

* On all screws four inches long and under, threads are cut 3/4 of the length L; longer than four inches, threads are cut half of the length L.

Hexagon-head Cap-screws



Outside Diameter of Screw, D	Number of Threads per Inch	National Acme Co.; Scovill Mfg. Co.;‡ Hartford Mach. Sc. Co.; Nat. Screw & Tack Co.; Corbin Screw Corporation A	B	Nat. Acme Co.; Scovill Mfg. Co.;‡ Nat. Sc. & Tack Co.; Corbin Sc. Corp. C	Hartford Machine Screw Co. C	National Acme Co. R	Hartford Machine Screw Co. R	National Screw & Tack Co.; Scovill Mfg. Co.‡ R
¼	20	½	7/16	¼	7/32	¾	21/32	½
5/16	18	37/64	½	5/16	9/32	⅞	¾	⅝
⅜	16	21/32	9/16	⅜	5/16	31/32	27/32	¾
7/16	14	23/32	⅝	7/16	⅜	1 1/16	15/16	⅞
½	12 or 13	55/64	¾	½	7/16	1 9/32	1⅛	1
9/16	12	15/16	13/16	9/16	½	1 13/32	1 7/32	1⅛
⅝	11	1	⅞	⅝	9/16	1½	1 5/16	1¼
¾	10	1 5/32	1	¾	11/16	1 23/32	1½	1½
⅞	9	1 19/64	1⅛	⅞	¾	1 15/16	1 11/16	1¾
1	8	1 7/16	1¼	1	⅞	2 5/32	1⅞	2
1⅛	7	1 19/32	1⅜	1⅛	1	2⅜	2 1/16	2¼
1¼	7	1 47/64	1½	1¼	1⅛	2 19/32	2¼	2½

* According to a common rule, the length E of the thread is determined as follows: For lengths L of 4 inches or less, E = ¾ of length L; for lengths L greater than 4 inches, E = ½ of length L. Length L is the nominal length of the cap-screw and is measured to the extreme point.

† For minimum and maximum lengths L see table "Commercial Lengths of Cap-screws."

‡ The cap-screws listed by the Scovill Mfg. Co. vary from ¼ to ⅞ inch in diameter but do not include the larger diameters given in the above table. According to this company's rule, lengths L of 1 inch or less are threaded up to the head. For lengths greater than 1 inch, see note below Scovill Mfg. Co.'s table of Square-head Cap-screws.

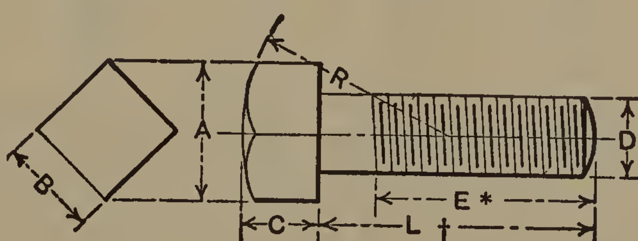
Cap-screw Threads. — Most cap-screws have the U. S. standard thread, with 13 threads per inch for ½-inch diameters, although 12 or 13 threads per inch is usually listed in the catalogues of screw manufacturers, indicating that cap-screws of ½-inch size will be cut to whichever pitch is ordered. Dimensions for the S.A.E. cap-screws may be obtained from the table "S.A.E. Standard Screws and Nuts." These cap-screws have the U. S. form of thread, but a finer pitch for the same diameter than the U. S. standard.

Lengths of Cap-screw Threads. — According to a common rule, the length E of the thread is determined as follows: For lengths L of 4 inches or less, the length of thread E equals ¾ of length L; for lengths L greater than 4 inches, E equals ½ of L.

Names of Machine Screws and Cap-screws. — Machine screws and cap-screws which have heads of the same shape, are not given the same names by most screw manufacturers. The names given in the tables "A.S.M.E. Standard Proportions for Machine Screw Heads" are in common use as applied to machine screws. The common names for cap-screws are given in connection with the illustration of set-screws and cap-screws on page 834. By comparing these names it will be noted that a "flat fillister head" machine screw has a head of the same shape as a "round" cap-screw; a "round-head" machine screw has the same shape as a "button-head" cap-screw; and an "oval fillister head" machine screw is similar to a "fillister head" cap-screw. Machine and cap-screws having a countersunk head are both known as "flat" screws. The oval or countersunk cap-screw (shown in the illustration referred to) is often known as a "French head" cap-screw.

Studs vs. Cap-screws. — Studs are preferred to cap-screws, especially on heavy machinery, for the following reasons: (1) A stud and nut can generally be screwed tighter than a cap-screw because of better alignment and reduced effect of torsional elasticity. (2) A stud and nut is less likely to break than a cap-screw when making repairs. In case a nut is rusted fast on a stud it can be split with a cold chisel; but a cap-screw "seized" in the casting is frequently twisted off. The loss of time and extra labor incident to breakages of cap-screws are important disad-

Square-head Cap-screws — 1



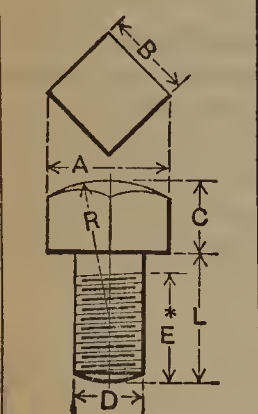
Outside Diameter of Screw,	Number of Threads per Inch	National Acme Co.; Hartford Mach. Sc. Co.; Nat. Screw & Tack Co.		Nat. Acme Co.; Nat. Screw & Tack Co.	Hartford Machine Screw Co.	National Acme Co.	Hartford Machine Screw Co.	National Screw & Tack Co.
D		*A*	*B*	*C*	*C*	*R*	*R*	*R*
1/4	20	17/32	3/8	1/4	7/32	25/32	3/4	1/2
5/16	18	5/8	7/16	5/16	9/32	15/16	7/8	5/8
3/8	16	45/64	1/2	3/8	5/16	1 1/16	1	3/4
7/16	14	51/64	9/16	7/16	3/8	1 3/16	1 1/8	7/8
1/2	12 or 13	57/64	5/8	1/2	7/16	1 5/16	1 1/4	1
9/16	12	31/32	11/16	9/16	1/2	1 15/32	1 3/8	1 1/8
5/8	11	1 1/16	3/4	5/8	9/16	1 19/32	1 1/2	1 1/4
3/4	10	1 15/64	7/8	3/4	11/16	1 27/32	1 3/4	1 1/2
7/8	9	1 19/32	1 1/8	7/8	3/4	2 3/8	2 1/4	1 3/4
1	8	1 49/64	1 1/4	1	7/8	2 21/32	2 1/2	2
1 1/8	7	1 15/16	1 3/8	1 1/8	1	2 29/32	2 3/4	2 1/4
1 1/4	7	2 1/8	1 1/2	1 1/4	1 1/8	3 3/16	3	2 1/2

* For lengths L of 4 inches or less, E = 3/4 of length L; for lengths greater than 4 inches E = 1/2 of length L, according to a common rule.

† For minimum and maximum lengths L, see table, "Commercial Lengths of Cap-screws."

vantages when making repairs. (3) Covers secured with nuts and studs can be loosened and tightened without serious deterioration of the fastening means. The nuts on studs can be loosened many times without appreciable wear of the threads, but cap-screws soon wear the cast-iron threads. (4) Studs have the advantage of holding gaskets in place while a cover is being applied. (5) A stud made from a round bar is stronger than a cap-screw turned down from a hexagon bar. The latter uses the weakest part of the metal at and near the center for the body.

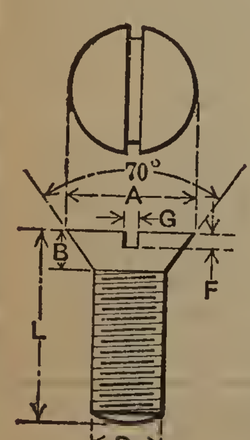
Square-head Cap-screws — 2 *
(Scovill Manufacturing Co.)



Outside Diameter of Screw, *D*	Number of Threads per Inch	Distance Across Corners, *A*	Width of Head, *B*	Thickness of Head, *C*	Radius of Head, *R*
1/4	20	27/64	19/64	1/4	1/2
5/16	18	9/16	13/32	5/16	5/8
3/8	16	21/32	15/32	3/8	3/4
7/16	14	47/64	17/32	7/16	7/8
1/2	13	7/8	5/8	1/2	1
9/16	12	15/16	43/64	9/16	1 1/8
5/8	11	1	23/32	5/8	1 1/4
3/4	10	1 5/32	53/64	3/4	1 1/2
7/8	9	1 11/32	61/64	7/8	1 3/4

* Cap-screws having a length L of one inch or less are threaded up to the head. Cap-screws longer than one inch and in the following diameters and lengths, have an unthreaded portion 3/16 inch long: 1/4 inch diameter up to and including 1 1/8 inch long; 5/16 inch diameter up to and including 1 1/4 inch long; 3/8 inch diameter up to and including 1 1/4 inch long; 7/16 inch diameter up to and including 1 3/8 inch long; 1/2 inch diameter up to and including 1 1/2 inch long; 9/16 inch diameter up to and including 1 3/4 inch long; 5/8 inch diameter up to and including 2 inches long; 3/4 inch diameter up to and including 2 1/2 inches long; and 7/8 inch diameter up to and including 3 inches long. When the cap-screws are longer than specified for the various diameters, the length E of the threaded portion is made one diameter less than the length L. The diameter of the plain portion is the same as that of the threaded section.

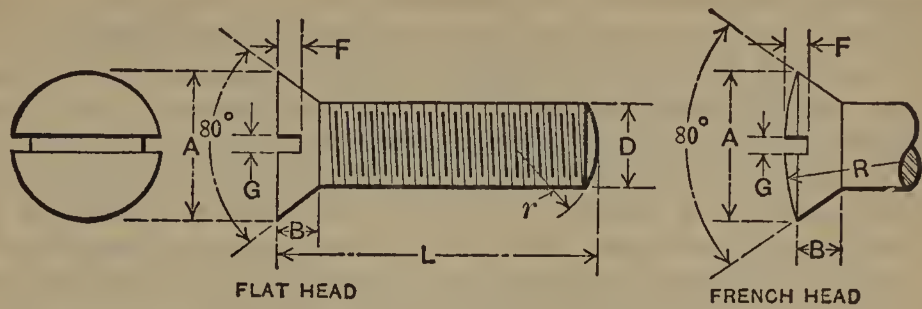
Flat-head Cap-screws
(Hartford Machine Screw Co.)



Outside Diameter of Screw, *D*	Number of Threads per Inch	Diameter of Head, *A*	Depth of Head, *B*	Depth of Slot, *F*	Width of Slot, *G*
1/8	40	1/4	3/32	3/64	0.028
3/16	24	3/8	1/8	1/16	0.035
1/4	20	15/32	5/32	1/16	0.049
5/16	18	5/8	7/32	3/32	0.065
3/8	16	3/4	17/64	3/32	0.065
7/16	14	13/16	17/64	3/32	0.083
1/2	12 or 13	7/8	17/64	1/8	0.083
9/16	12	1	5/16	1/8	0.083
5/8	11	1 1/8	23/64	5/32	0.109
3/4	10	1 3/8	29/64	5/32	0.134

Flat- and French-head Cap-screws

(National Acme Co.)



Outside Diameter of Screw, *D*	Number of Threads per Inch	Diameter of Head, *A*	Depth of Head, *B*	Depth of Slot, *F*	Width of Slot, *G*	Radius of French Head, *R*	Radius of Point, *r*
1/8	40	1/4	5/64	0.037	0.025	3/8	1/8
3/16	24	3/8	7/64	0.056	0.037	9/16	3/16
1/4	20	15/32	1/8	0.075	0.050	45/64	1/4
5/16	18	5/8	3/16	0.094	0.062	15/16	5/16
3/8	16	3/4	7/32	0.112	0.075	1 1/8	3/8
7/16	14	13/16	7/32	0.131	0.087	1 7/32	7/16
1/2	12 or 13	7/8	7/32	0.150	0.100	1 5/16	1/2
9/16	12	1	17/64	0.169	0.112	1 1/2	9/16
5/8	11	1 1/8	19/64	0.187	0.125	1 11/16	5/8
3/4	10	1 3/8	3/8	0.225	0.150	2 1/16	3/4

Holding Power of Set-screws. — While the amount of power a set-screw of given size will transmit without slipping (when used for holding a pulley, gear, or other part from turning relative to a shaft) varies somewhat according to the physical properties of both set-screw and shaft and other variable factors, experiments have shown that the safe holding power in pounds for different diameters of set-screws should be approximately as follows: For 1/4-inch diameter, 100 pounds; 3/8-inch diameter, 250 pounds; 1/2-inch diameter, 500 pounds; 3/4-inch diameter, 1250 pounds; 1-inch diameter, 2500 pounds; with values for other sizes in the same proportion.

If S = pressure factor representing the holding power of the set-screw; r = shaft radius in inches; N = speed of shaft in revolutions per minute; H = number of horsepower to be transmitted; 396,000 = equivalent of 1 horsepower expressed in inch-pounds; then:

$$H = \frac{Sr\,2\pi N}{396{,}000} = \frac{SrN}{63{,}000}$$

$$S = \frac{63{,}000\,H}{rN}$$

After determining the value of S, the diameter of the set-screw may be selected by referring to the values previously given for different diameters.

Example: — What diameter of set-screw should be used to transmit 3 horsepower safely when the shaft radius is one inch and the speed 375 revolutions per minute?

$S = \frac{63,000 \times 3}{1 \times 375} = 504$. Hence, either a ½-inch set-screw should be used or two of ⅜-inch diameter.

Professor Lanza's experiments for determining the load at which set-screws slip were made with ⅝-inch wrought-iron set-screws having 10 threads per inch. The pulley was provided with two set-screws. Factors representing the maximum holding power were obtained by multiplying the load required at the pulley rim to cause slippage, by the radius of the pulley plus the radius of the wire rope, and then dividing the product by twice the shaft radius, twice the radius being used because there were two set-screws. The shaft was keyed to holders to prevent rotation, and it was of steel and rather hard. The set-screws were tightened by applying a load of 75 pounds at the end of a 10-inch wrench. Four different kinds of set-screw ends were used, and the following average results obtained:

Ends perfectly flat and 9/16 inch in diameter; average holding power 2064 pounds.

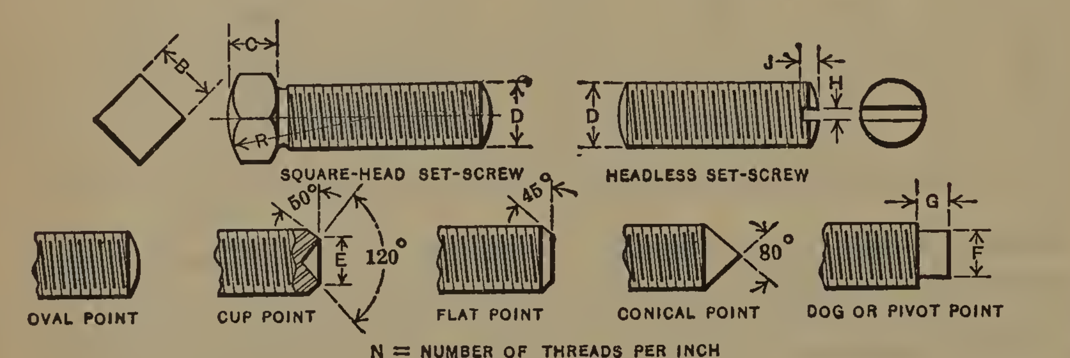
Rounded ends with a radius of about ½ inch; average holding power 2912 pounds.

Rounded ends with a radius of about ¼ inch; average holding power 2573 pounds.

Cup-shaped ends, case-hardened; average holding power 2470 pounds.

These results indicate that the values previously given are far enough below the load at which slipping occurs to provide a safe margin.

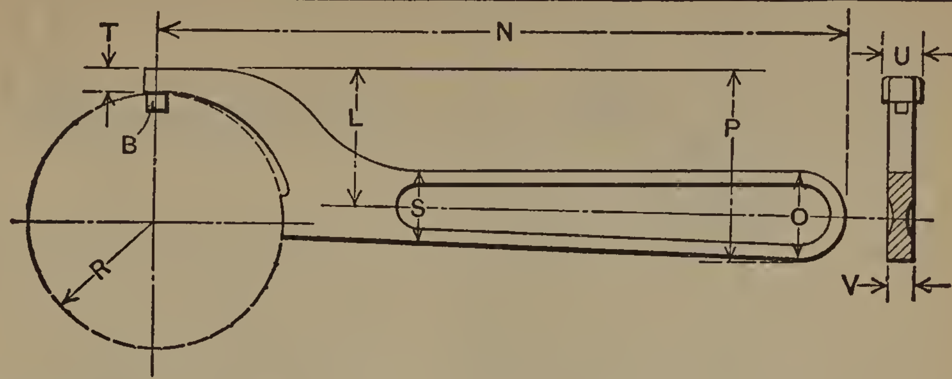
Square-head and Headless Set-screws *



D	N	B	C	R	E	F	G	H	J
¼	20	¼	3/16	½	0.166	0.185	⅛	0.049	1/16
5/16	18	5/16	15/64	⅝	0.208	0.240	5/32	0.065	3/32
⅜	16	⅜	9/32	¾	0.250	0.293	3/16	0.065	3/32
7/16	14	7/16	21/64	⅞	0.291	0.344	7/32	0.083	⅛
½	12 or 13	½	⅜	1	0.333	0.391	¼	0.083	⅛
9/16	12	9/16	27/64	1⅛	0.375	0.454	9/32	0.083	5/32
⅝	11	⅝	15/32	1¼	0.416	0.506	5/16	0.109	3/16
¾	10	¾	9/16	1½	0.500	0.620	⅜	0.134	7/32
⅞	9	⅞	21/32	1¾	0.583	0.730	7/16	0.134	¼
1	8	1	¾	2	0.666	0.837	½	0.165	9/32
1⅛	7	1⅛	27/32	2¼	0.750	0.939	9/16	0.165	9/32
1¼	7	1¼	15/16	2½	0.833	1.064	⅝	0.165	5/16

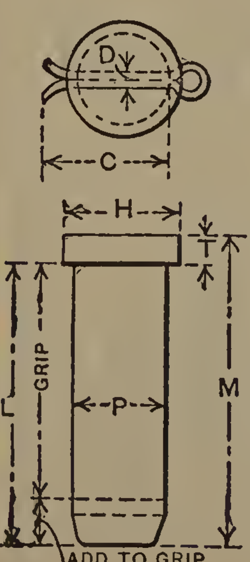
* The above table is the standard of the Hartford Machine Screw Co. The widths *B* of square-head set-screws made by other manufacturers are commonly made equal to the screw diameter as in this table.

General Dimensions of Spanner Wrenches



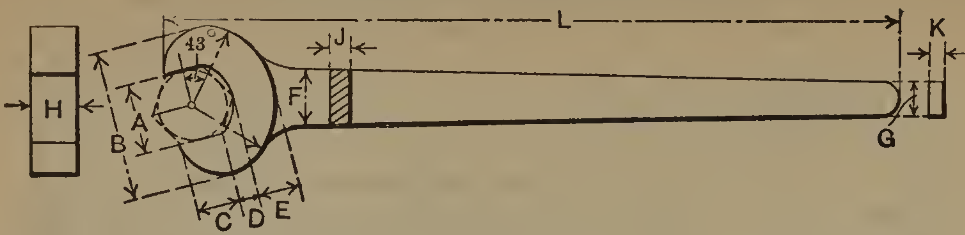
Dimensions of Nut: Diam.	Dimensions of Nut: Min. Width	*R*	Max. Diam. of Pin, *B*	*N*	*L*	*P*	*O*	*S*	*T*	*U*	*V*
1 1/8	1/2	9/16	7/32	5	7/8	1 3/16	5/8	7/16	3/16	9/32	3/16
1 1/4	1/2	5/8	7/32	5	7/8	1 3/16	5/8	7/16	3/16	9/32	3/16
1 1/2	9/16	3/4	15/64	5	7/8	1 3/16	5/8	7/16	3/16	9/32	3/16
1 3/4	9/16	7/8	15/64	6	1 1/4	1 5/8	3/4	9/16	7/32	5/16	7/32
2	5/8	1	1/4	6	1 1/4	1 5/8	3/4	9/16	7/32	5/16	7/32
2 1/4	5/8	1 1/8	1/4	6	1 1/4	1 5/8	3/4	9/16	7/32	5/16	7/32
2 1/2	5/8	1 1/4	1/4	6 3/4	1 1/2	1 15/16	7/8	5/8	1/4	11/32	1/4
2 3/4	11/16	1 3/8	17/64	6 3/4	1 1/2	1 15/16	7/8	5/8	1/4	11/32	1/4
3	11/16	1 1/2	17/64	6 3/4	1 1/2	1 15/16	7/8	5/8	1/4	11/32	1/4
3 1/4	3/4	1 5/8	9/32	7 3/4	1 7/8	2 3/8	1	3/4	9/32	3/8	9/32
3 1/2	3/4	1 3/4	9/32	7 3/4	1 7/8	2 3/8	1	3/4	9/32	3/8	9/32
3 3/4	3/4	1 7/8	9/32	7 3/4	1 7/8	2 3/8	1	3/4	9/32	3/8	9/32

Dimensions of Pins With Cotters

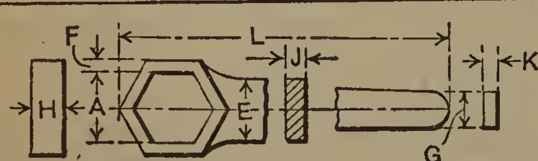


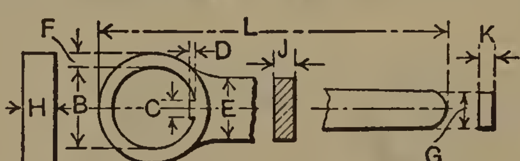
Diam. of Pin *P*	Diam. of Pin Hole	Head: Diam. *H*	Head: Thickness *T*	Cotter: Length *C*	Cotter: Diam. *D*	Add to Grip: To Obtain Length *M*, over All	Add to Grip: To Obtain Length *L*, under Head
1	1 1/32	1 1/4	1/4	1 3/4	1/4	7/8	5/8
1 1/4	1 9/32	1 1/2	1/4	2	1/4	7/8	5/8
1 1/2	1 17/32	1 3/4	1/4	2 1/2	5/16	1 1/8	7/8
1 3/4	1 25/32	2	1/4	2 3/4	5/16	1 1/8	7/8
2	2 1/32	2 3/8	3/8	3	3/8	1 3/8	1
2 1/4	2 9/32	2 5/8	3/8	3 1/4	3/8	1 3/8	1
2 1/2	2 17/32	2 7/8	3/8	3 3/4	7/16	1 1/2	1 1/8
2 3/4	2 25/32	3 1/8	3/8	4	7/16	1 1/2	1 1/8
3	3 1/32	3 1/2	1/2	5	1/2	1 7/8	1 3/8
3 1/4	3 9/32	3 3/4	1/2	5	1/2	1 7/8	1 3/8
3 1/2	3 17/32	4	1/2	6	5/8	2 1/8	1 5/8
3 3/4	3 25/32	4 1/4	1/2	6	5/8	2 1/8	1 5/8

Dimensions of Wrenches



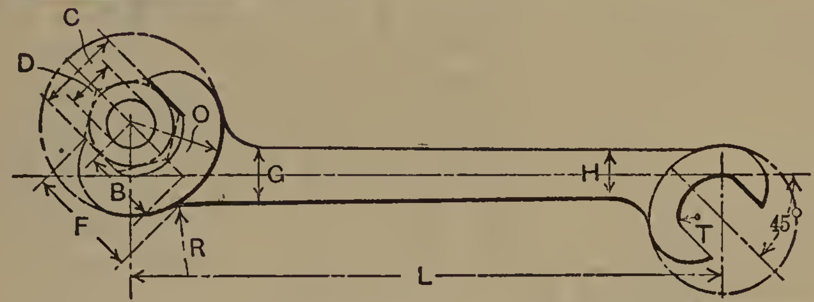
Bolt Diam.	A	B	C	D	E	F	G	H	J	K	L
1/4	1/2	1	1/4	1/8	1/4	1/2	3/4	1/4	1/4	1/4	5 1/2
5/16	19/32	1 3/16	5/16	5/32	5/16	9/16	3/4	5/16	5/16	5/16	6
3/8	11/16	1 3/8	3/8	3/16	3/8	5/8	3/4	3/8	5/16	5/16	7
7/16	25/32	1 9/16	7/16	7/32	7/16	11/16	3/4	7/16	5/16	5/16	8
1/2	7/8	1 3/4	1/2	1/4	1/2	3/4	7/8	1/2	5/16	5/16	9
9/16	31/32	1 15/16	9/16	9/32	9/16	7/8	7/8	9/16	5/16	5/16	10
5/8	1 1/16	2 1/8	5/8	5/16	5/8	1	7/8	5/8	3/8	3/8	11
3/4	1 1/4	2 1/2	3/4	3/8	3/4	1 1/8	1	11/16	3/8	3/8	12 1/2
7/8	1 7/16	2 15/16	13/16	7/16	13/16	1 5/16	1	3/4	7/16	3/8	14 1/2
1	1 5/8	3 5/16	15/16	1/2	15/16	1 1/2	1	7/8	1/2	3/8	16 1/2
1 1/8	1 13/16	3 11/16	1 1/16	9/16	1 1/16	1 5/8	1 1/16	15/16	1/2	7/16	18
1 1/4	2	4 1/16	1 1/8	9/16	1 1/8	1 3/4	1 1/16	1	9/16	7/16	20
1 3/8	2 3/16	4 7/16	1 1/4	5/8	1 1/4	1 7/8	1 1/8	1 1/8	5/8	7/16	22
1 1/2	2 3/8	4 13/16	1 3/8	11/16	1 3/8	2	1 1/8	1 1/4	5/8	1/2	24
1 5/8	2 9/16	5 3/16	1 1/2	3/4	1 1/2	2 1/4	1 3/16	1 1/4	11/16	1/2	26
1 3/4	2 3/4	5 9/16	1 5/8	13/16	1 5/8	2 3/8	1 3/16	1 3/8	11/16	1/2	28
1 7/8	2 15/16	6	1 11/16	7/8	1 11/16	2 1/2	1 1/4	1 3/8	3/4	9/16	30
2	3 1/8	6 3/8	1 13/16	15/16	1 13/16	2 5/8	1 1/4	1 1/2	3/4	9/16	33
2 1/4	3 1/2	7 1/8	2	1	2	3	1 5/16	1 5/8	7/8	5/8	35
2 1/2	3 7/8	7 7/8	2 1/4	1 1/8	2 1/4	3 1/4	1 3/8	1 3/4	1	5/8	39
3	4 5/8	9 3/8	2 11/16	1 3/8	2 11/16	3 7/8	1 1/2	2	1 1/8	11/16	46
3 1/2	5 3/8	10 7/8	3 1/8	1 9/16	3 1/8	4 1/2	1 5/8	2 1/4	1 1/4	3/4	53

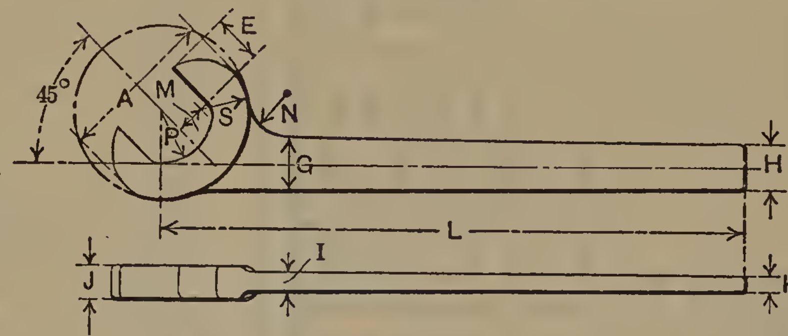




Bolt Diam.	A	B	C	D	E	F	G	H	J	K	L
1 1/2	2 3/8	2 3/4	7/16	3/16	2 1/8	3/8	1 1/8	1 1/4	3/4	1/2	26
1 5/8	2 9/16	3	7/16	3/16	2 1/4	7/16	1 3/16	1 1/4	3/4	1/2	28
1 3/4	2 3/4	3 1/8	7/16	3/16	2 3/8	7/16	1 3/16	1 3/8	13/16	1/2	30
1 7/8	2 15/16	3 3/8	1/2	3/16	2 1/2	1/2	1 1/4	1 3/8	13/16	9/16	32
2	3 1/8	3 5/8	1/2	3/16	2 5/8	1/2	1 1/4	1 1/2	7/8	9/16	33
2 1/4	3 1/2	4	9/16	1/4	2 3/4	9/16	1 5/16	1 5/8	15/16	5/8	38
2 1/2	3 7/8	4 3/8	9/16	1/4	3	5/8	1 3/8	1 3/4	1	5/8	42
2 3/4	4 1/4	4 7/8	5/8	1/4	3 1/4	5/8	1 7/16	1 7/8	1	11/16	45
3	4 5/8	5 1/4	11/16	1/4	3 1/2	11/16	1 1/2	2	1 1/8	11/16	49
3 1/4	5	5 5/8	3/4	5/16	3 3/4	3/4	1 9/16	2 1/8	1 1/8	3/4	54
3 1/2	5 3/8	6 1/8	3/4	5/16	4	3/4	1 5/8	2 1/4	1 1/4	3/4	57

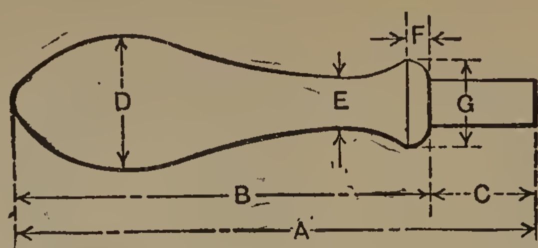
Dimensions of Wrenches





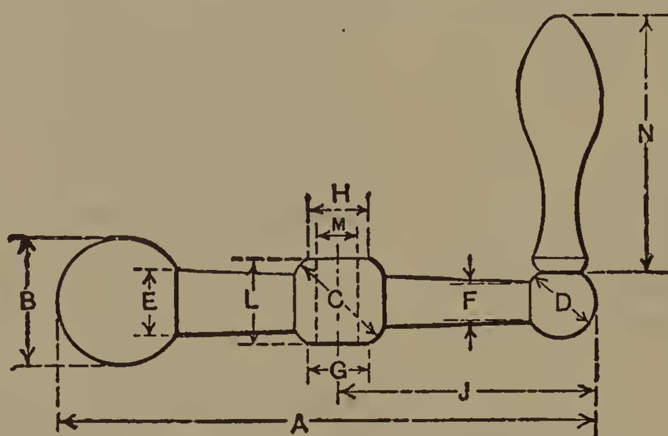
Diam. of Bolt, D	Width of Jaws, C Max.	Width of Jaws, C Min.	A	B	E	F	G	H	I	J	K	L	M	N	O	P	R	S	T
1/4	0.530	0.527	1.06	0.47	0.32	0.74	0.39	0.34	0.14	0.21	0.12	5	0.19	0.33	0.59	0.33	0.94	0.33	0.06
5/16	0.605	0.602	1.21	0.54	0.37	0.85	0.45	0.39	0.16	0.24	0.14	5	0.22	0.37	0.68	0.37	1.08	0.37	0.07
3/8	0.715	0.712	1.43	0.64	0.43	1.00	0.53	0.47	0.19	0.28	0.17	6	0.26	0.44	0.80	0.44	1.28	0.44	0.08
7/16	0.825	0.822	1.65	0.73	0.50	1.16	0.61	0.54	0.22	0.33	0.19	6	0.30	0.51	0.92	0.51	1.47	0.51	0.10
1/2	0.925	0.922	1.85	0.82	0.56	1.30	0.69	0.60	0.24	0.37	0.22	8	0.34	0.57	1.04	0.57	1.65	0.57	0.11
9/16	1.015	1.012	2.03	0.90	0.62	1.43	0.75	0.66	0.27	0.40	0.24	8	0.38	0.63	1.14	0.63	1.81	0.63	0.12
5/8	1.105	1.102	2.21	0.99	0.67	1.55	0.82	0.72	0.29	0.44	0.26	9	0.41	0.69	1.24	0.69	1.97	0.69	0.13
11/16	1.205	1.202	2.41	1.08	0.74	1.69	0.89	0.79	0.32	0.48	0.28	9	0.45	0.75	1.35	0.75	2.15	0.75	0.15
3/4	1.305	1.302	2.61	1.16	0.80	1.84	0.97	0.85	0.35	0.52	0.31	10	0.48	0.81	1.46	0.81	2.33	0.81	0.16
13/16	1.395	1.392	2.79	1.25	0.85	1.96	1.04	0.91	0.37	0.55	0.33	10	0.52	0.87	1.56	0.87	2.49	0.87	0.17
7/8	1.485	1.482	2.97	1.33	0.91	2.09	1.10	0.97	0.39	0.59	0.35	12	0.55	0.92	1.67	0.92	2.65	0.93	0.18
15/16	1.585	1.582	3.17	1.42	0.97	2.23	1.18	1.04	0.42	0.63	0.37	12	0.59	0.99	1.78	0.99	2.83	0.99	0.19
1	1.675	1.672	3.35	1.50	1.02	2.36	1.25	1.10	0.44	0.67	0.39	15	0.62	1.04	1.88	1.04	2.99	1.04	0.20
1 1/8	1.870	1.864	3.74	1.67	1.14	2.63	1.39	1.22	0.50	0.74	0.44	15	0.69	1.16	2.10	1.16	3.34	1.17	0.23
1 1/4	2.060	2.054	4.12	1.84	1.26	2.90	1.53	1.35	0.55	0.82	0.49	18	0.77	1.28	2.31	1.28	3.68	1.29	0.25
1 3/8	2.230	2.224	4.46	1.99	1.36	3.14	1.66	1.46	0.59	0.89	0.53	18	0.83	1.39	2.50	1.39	3.99	1.39	0.27
1 1/2	2.420	2.414	4.84	2.16	1.48	3.41	1.80	1.59	0.64	0.96	0.57	21	0.90	1.51	2.72	1.51	4.33	1.51	0.30
1 5/8	2.590	2.584	5.18	2.32	1.59	3.65	1.93	1.70	0.69	1.03	0.61	21	0.96	1.61	2.91	1.61	4.63	1.62	0.32
1 3/4	2.770	2.764	5.54	2.48	1.70	3.90	2.06	1.82	0.74	1.10	0.65	23	1.03	1.73	3.11	1.73	4.95	1.73	0.34
1 7/8	3.030	3.024	6.06	2.71	1.86	4.27	2.26	1.99	0.81	1.21	0.72	23	1.13	1.89	3.40	1.89	5.42	1.89	0.37
2	3.160	3.154	6.32	2.83	1.94	4.45	2.35	2.07	0.84	1.26	0.75	26	1.18	1.97	3.55	1.97	5.65	1.97	0.39

Machine Handles



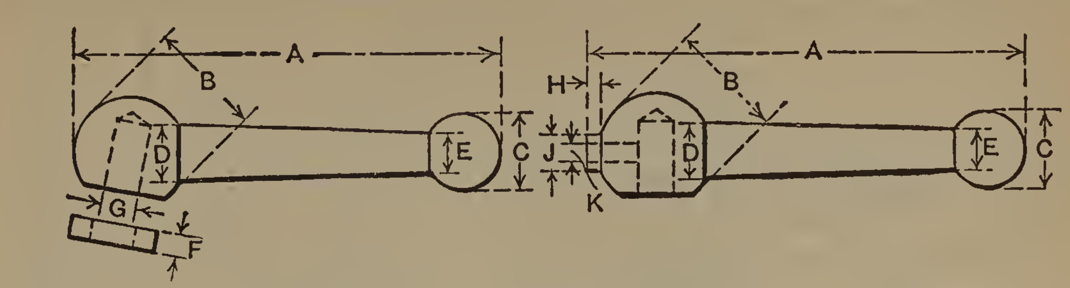
A	B	C	D	E	F	G	A	B	C	D	E	F	G
1 13/16	1 5/16	1/2	7/16	7/32	3/32	9/32	4	3 1/4	3/4	1 1/16	17/32	3/16	11/16
2	1 1/2	1/2	1/2	1/4	3/32	5/16	4 3/8	3 1/2	7/8	1 1/8	9/16	3/16	23/32
2 1/4	1 3/4	1/2	9/16	9/32	1/8	3/8	4 5/8	3 5/8	1	1 3/16	19/32	7/32	13/16
2 3/4	2 1/8	5/8	11/16	11/32	1/8	7/16	5 1/8	4 1/8	1	1 3/8	11/16	7/32	7/8
3 1/8	2 3/8	3/4	3/4	3/8	5/32	1/2	5 3/4	4 1/2	1 1/4	1 1/2	3/4	1/4	31/32
3 1/2	2 3/4	3/4	7/8	7/16	5/32	9/16	6	4 3/4	1 1/4	1 9/16	25/32	1/4	1 1/32

Ball-crank Machine Handles



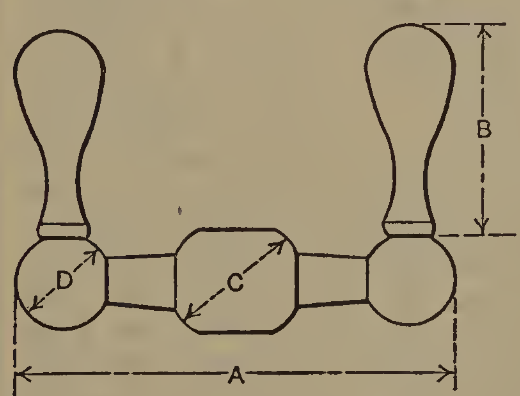
A	B	C	D	E	F	G	H	J	L	M	N
3	7/8	11/16	9/16	3/8	5/16	1/2	7/16	1 1/2	1/2	1/4	1 1/2
3 1/4	15/16	25/32	5/8	7/16	5/16	9/16	1/2	1 5/8	9/16	5/16	1 5/8
3 1/2	1 1/16	29/32	3/4	15/32	11/32	5/8	9/16	1 3/4	11/16	3/8	1 3/4
4	1 1/4	1	3/4	17/32	13/32	3/4	11/16	2	3/4	7/16	1 7/8
4 1/2	1 5/16	1 3/32	27/32	19/32	7/16	3/4	11/16	2 1/4	13/16	1/2	2
5	1 1/2	1 5/16	1	3/4	1/2	7/8	13/16	2 1/2	1	1/2	2 3/16
5 1/2	1 1/2	1 5/16	1	3/4	1/2	7/8	13/16	2 3/4	1	1/2	2 3/8
6	1 5/8	1 3/8	1	3/4	1/2	1	15/16	3	1	5/8	2 9/16
6 1/2	1 5/8	1 3/8	1	3/4	1/2	1	15/16	3 1/4	1	5/8	2 3/4
7	1 3/4	1 7/16	1	3/4	9/16	1	15/16	3 1/2	1 1/16	5/8	2 15/16
7 1/2	1 3/4	1 1/2	1	3/4	9/16	1	15/16	3 3/4	1 3/32	5/8	3 1/8
8	1 3/4	1 1/2	1 1/16	3/4	9/16	1	15/16	4	1 1/8	5/8	3 5/16
8 1/2	1 3/4	1 9/16	1 1/8	3/4	5/8	1 1/8	1 1/16	4 1/4	1 3/16	3/4	3 1/2
9	1 3/4	1 5/8	1 3/16	3/4	5/8	1 1/8	1 1/16	4 1/2	1 1/4	3/4	3 3/4

Two-ball Clamping Levers



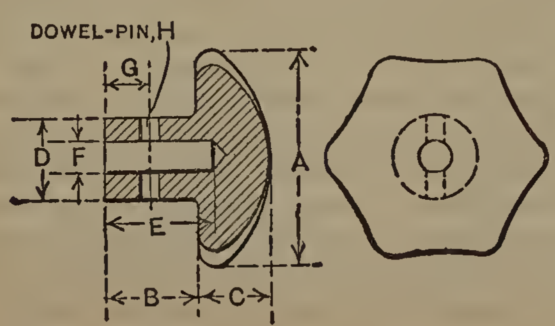
A	B	C	D	E	F	G	H	J	K
4 1/2	1 3/8	7/8	5/8	7/16	1/4	1/2	1/8	5/8	5/16
5 1/2	1 1/2	1	11/16	1/2	1/4	1/2	1/8	5/8	3/8
6 1/2	1 3/4	1	3/4	9/16	1/4	1/2	3/16	5/8	3/8
7 1/2	1 3/4	1	3/4	9/16	1/4	5/8	3/16	5/8	7/16
8 1/2	1 3/4	1 1/8	3/4	5/8	1/4	5/8	1/4	11/16	7/16
9	1 3/4	1 3/16	13/16	5/8	1/4	5/8	1/4	11/16	7/16

Compound-rest Handles



A	B	C	D
2 1/2	1 1/4	1 1/16	3/4
2 1/2	1 1/4	1 1/16	3/4
2 1/2	1 1/4	1 1/16	3/4
3	1 1/2	1 1/16	3/4
3	1 1/2	1 1/16	3/4
3	1 1/2	1 1/16	3/4
3 1/2	1 3/4	1 1/16	3/4
3 1/2	1 3/4	1 1/16	3/4
3 1/2	1 3/4	1 1/16	3/4
4	2	1 3/16	7/8
4	2	1 3/16	7/8
4	2	1 3/16	7/8

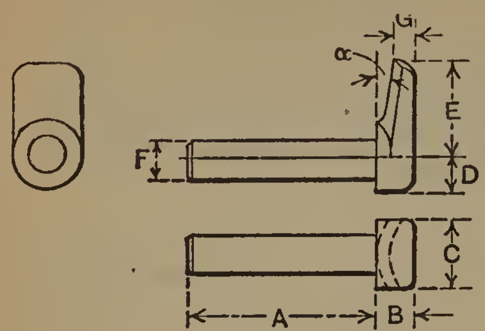
Knobs for Machine Doors



A	B	C	D
2 1/4	1 1/16	1 1/16	7/8
2 3/4	1 1/16	3/4	1
3 1/4	1 1/16	15/16	1

E	F	G	H
1 1/4	7/16	1/2	3/16 by 15/16
1 1/4	1/2	1/2	3/16 by 1 1/16
1 1/4	1/2	1/2	3/16 by 1 1/16

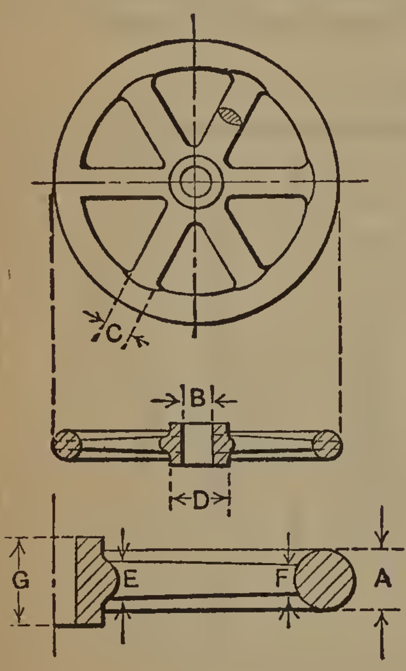
Latches for Machine Doors



A	B	C	D
2	7/16	3/4	3/8
2	7/16	3/4	3/8
2	1/2	7/8	7/16

E	F	G	α
1	7/16	1/4	8°
1	1/2	1/4	8°
1 1/8	1/2	5/16	8°

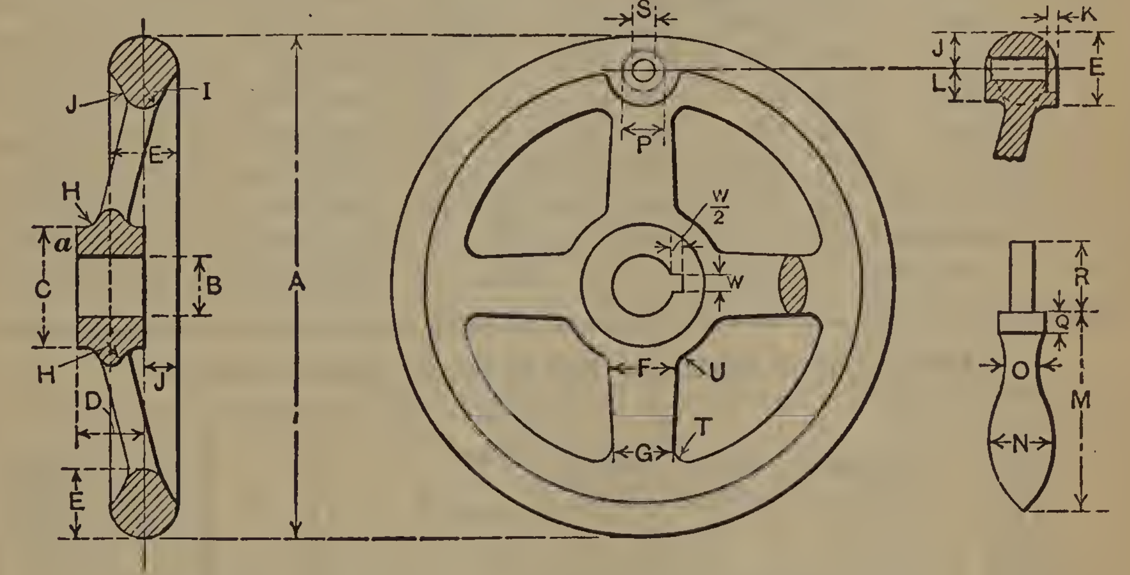
Proportions of Large Handwheels



Diam.	A	B	C	D	E	F	G
8	3/4	3/4	5/8	1 1/2	3/8	5/16	1 1/8
9	13/16	13/16	11/16	1 5/8	13/32	11/32	1 1/4
10	7/8	7/8	3/4	1 3/4	7/16	3/8	1 5/16
11	15/16	15/16	3/4	1 7/8	15/32	3/8	1 3/8
12	1	1	13/16	2	1/2	13/32	1 1/2
13	1 1/16	1 1/16	13/16	2 1/8	17/32	13/32	1 5/8
14	1 1/8	1 1/8	7/8	2 1/4	9/16	7/16	1 11/16
15	1 3/16	1 3/16	15/16	2 3/8	19/32	15/32	1 3/4
16	1 1/4	1 1/4	1	2 1/2	5/8	1/2	1 7/8
17	1 5/16	1 5/16	1	2 5/8	21/32	1/2	1 15/16
18	1 3/8	1 3/8	1 1/16	2 3/4	11/16	17/32	2 1/16
19	1 7/16	1 7/16	1 1/8	2 7/8	23/32	9/16	2 1/8
20	1 1/2	1 1/2	1 3/16	3	3/4	19/32	2 1/4
21	1 9/16	1 9/16	1 1/4	3 1/8	25/32	5/8	2 5/16
22	1 5/8	1 5/8	1 1/4	3 1/4	13/16	5/8	2 7/16
23	1 11/16	1 11/16	1 5/16	3 3/8	27/32	21/32	2 1/2
24	1 3/4	1 3/4	1 3/8	3 1/2	7/8	11/16	2 5/8
27	1 15/16	1 15/16	1 1/2	3 7/8	31/32	3/4	2 7/8
30	2 1/8	2 1/8	1 5/8	4 1/4	1 1/16	13/16	3 3/16
33	2 5/16	2 5/16	1 3/4	4 5/8	1 5/32	7/8	3 7/16
36	2 1/2	2 1/2	2	5	1 1/4	1	3 3/4

Machine Handwheels. — The accompanying table gives complete dimensions for "dished-arm" machine handwheels. The following remarks relating to hubs, keyways, arms and rims apply to handwheels of the design illustrated. *Hubs.* In Column *D*, the minimum dimension that should be used is given. The hub may be increased in length on side *a*, if necessary. Length *D* is sufficient for Woodruff keys. *Keyway.* This is designed so that either a straight key or a Woodruff key may be used, without changing the dimensions. *Arms.* These incline to form a "dished" wheel. There are two reasons for this: First, it often happens that it is convenient, or necessary, to fasten the handwheel with a nut on the end of the shaft, and if the wheel is dished, there is a recess for this nut, so that the operator does not strike his arm when turning the handwheel with a handle. Second, when casting handwheels with straight arms, the arms often break due to strains, but if dished, the strains are taken up by the arms and hub. The arms are oval in section, as indicated; the taper from F to G is 1 inch to the foot for all sizes. In Column U for a 14-inch handwheel, it will be noted that the radius is less than for a 12-inch size. This is because the 14-inch wheel has six arms, whereas the 12-inch size has four arms. *Rims.* The inner half of the rim is reduced to permit finishing the outside half and to provide an even stopping place for the machined surface. This eliminates the filing that is otherwise required. Note that when a handle is to be used, a boss is cast on the outside of the rim at the end of one of the arms, and a smaller boss on the inside for the handle shank. The counterbore P receives the straight section Q on the handle, so that the latter will not project. Length R on the handle allows 1/16 inch for riveting.

Machine Handwheels

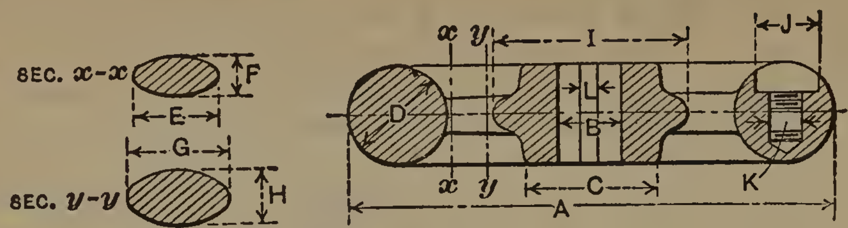


No. of Arms: 4 for sizes up to and including 12 inches; 6 for larger sizes.

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
6	¾	1¾	⅞	⅞	⅞	25/32	7/32	3/16	7/16	3/32	5/16
8	⅞	1⅞	1	1⅛	1	27/32	¼	13/64	½	3/32	⅜
10	1	2⅛	1⅛	1¼	1¼	1 1/16	9/32	7/32	9/16	5/32	½
12	1⅛	2¼	1¼	1 5/16	1⅜	1 3/32	5/16	¼	⅝	¼	9/16
14	1¼	2½	1⅜	1⅜	1⅜	1 1/16	11/32	9/32	11/16	¼	9/16
16	1⅜	2¾	1½	1½	1½	1⅛	⅜	5/16	¾	...	....
18	1½	3	1⅝	1⅝	1⅝	1 3/16	13/32	⅜	13/16	...	....

A	*M*	*N*	*O*	*P*	*Q*	*R*	*S*	*T*	*U*	*W*	Size of Woodruff Key
6	2½	1	7/16	½	⅜	⅞	¼	¼	½	5/32	6
8	2¾	1⅛	½	⅝	⅜	1 1/16	5/16	9/32	9/16	3/16	11
10	3⅛	1¼	⅝	¾	⅜	1 3/16	⅜	5/16	⅝	¼	15
12	3⅜	1⅜	¾	1	½	1⅛	½	11/32	11/16	¼	18
14	3⅝	1⅜	¾	1	½	1 3/16	½	⅜	¼	5/16	*D*
16	...	...	...	...	...	...	...	7/16	5/16	5/16	23
18	...	...	...	...	...	...	...	½	⅜	⅜	*F*

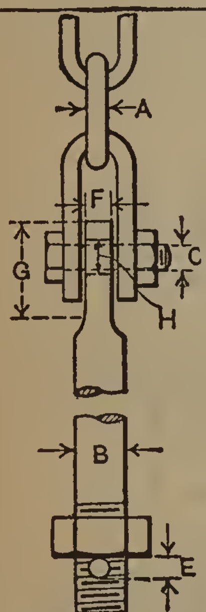
Handwheels with Straight Arms



No. of Arms: 4 up to the 10-inch size; 5 in the 11-inch size; 6 in the 12-inch size.

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
4	3/8	7/8	11/16	1/2	1/4	5/8	5/16	1 3/8	15/32	5/16	1/16 by 1/32
5	7/16	1	13/16	9/16	9/32	11/16	11/32	1 5/8	15/32	3/8	3/32 by 3/64
6	9/16	1 3/16	15/16	5/8	5/16	3/4	3/8	1 7/8	1/2	3/8	3/32 by 3/64
7	11/16	1 3/8	1	11/16	11/32	7/8	7/16	2 1/8	19/32	3/8	1/8 by 1/16
8	3/4 to 7/8	1 1/2	1 1/8	3/4	3/8	15/16	15/32	2 3/8	19/32	3/8	1/8 by 1/16
9	13/16 to 1	1 5/8	1 3/16	13/16	13/32	1	1/2	2 5/8	19/32	3/8	1/8 by 1/16
10	7/8 to 1	1 3/4	1 5/16	7/8	7/16	1 1/8	9/16	2 7/8	21/32	7/16	3/16 by 3/32
11	15/16 to 1 1/8	1 7/8	1 3/8	15/16	15/32	1 3/16	9/16	3 1/8	21/32	7/16	3/16 by 3/32
12	1 to 1 1/4	2	1 1/2	1	1/2	1 1/4	5/8	3 3/8	3/4	1/2	1/4 by 1/8

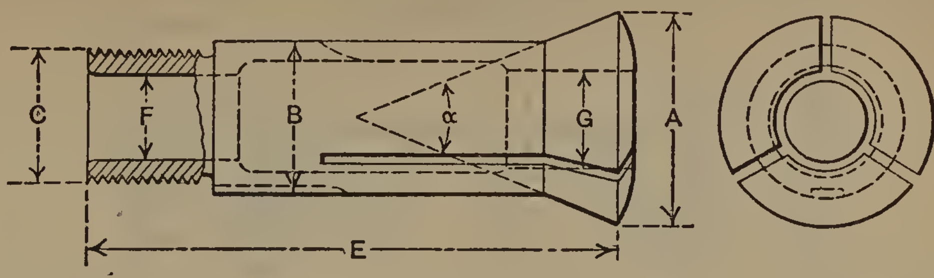
Dimensions of Chain and Rod to Support Counter-weights



Size of Chain, *A*	Diam. of Rod, *B*	Size of Bolt, *C*	Diam. of Cotter, *E*	*F*	*G*	Diam. of Hole, *H*	Max. Safe Load
3/16	3/8	5/16	3/32	3/16	1	3/8	350
1/4	1/2	3/8	1/8	1/4	1 1/4	7/16	650
5/16	5/8	1/2	1/8	1/4	1 1/2	9/16	1300
3/8	3/4	5/8	3/16	5/16	1 3/4	11/16	1900
1/2	7/8	3/4	3/16	5/16	2 1/4	13/16	3300
1/2	1	7/8	3/16	3/8	2 1/2	15/16	3400

Material for Bolts and Nuts. — The Navy Department specifications for steel bolts and nuts are, in part, as follows: Bolts must be made of a good quality of medium steel and conform to the United States standard for both heads and threads. Test bolts are to be bent cold on the unthreaded portion, through 180 degrees, around a diameter equal to one-half the bolt diameter, without breaking; only a slight fracture of the "skin" on one side will be allowed. When practicable, a tensile test of bolts and nuts combined shall be made. Bolts so tested must fracture at the threads and not at the juncture with the head; they must withstand a tensile stress of at least 58,000 pounds and have an elastic limit of not less than 30,000 pounds per square inch of sectional area.

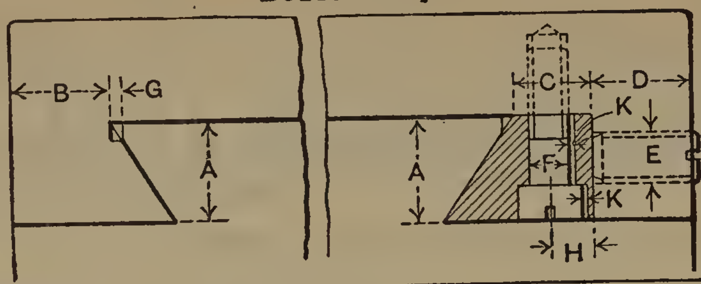
Draw-back Chucks for Bench Lathes



Name of Chuck	Diam. *A* of Head	Diam. *B* of Body	Diam. *C* of Thread	No. of Threads per Inch	One-half Included Angle $=\frac{1}{2}\alpha$, Degrees	Length *E*	Largest Hole *F*	Largest Hole *G*
Cataract No. 1	0.500	0.335	0.325	40	15	1.437	0.250	0.250
Cataract No. 2	0.625	0.450	0.445	30	15	1.812	0.343	0.343
Cataract No. 3	0.850	0.650	0.645	26	12	2.687	0.500	0.500
Cataract No. 4	1.156	0.950	0.945	20	10	3.000	0.750	0.750
Cataract No. 5	1.465	1.250	1.245	20	10	3.281	1.000	1.000
Cataract No. 6	1.875	1.625	1.5625	20	8	3.8125	1.375	1.375
Cataract No. 7	2.312	2.000	1.9375	20	8	5.000	1.687	1.687
Dale No. 1	0.500	0.335	0.295	40	15	1.437	0.218	0.250
Dale No. 2	0.625	0.450	0.365	30	15	1.812	0.297	0.343
Dale No. 3	0.890	0.650	0.560	24	15	1.250	0.406	0.500
Dale No. 4	1.125	0.8265	0.700	20	15	3.125	0.500	0.625
Rivett No. 1	0.500	0.300	0.265	40	20	1.250	0.200	0.235
Rivett No. 3	0.825	0.590	0.525	26	20	2.125	0.375	0.500
Rivett No. 4	1.025	0.750	0.665	20	20	2.750	0.500	0.562
Rivett No. 5	1.325	1.0625	0.945	18	20	3.437	0.718	0.875
Hopkins or Van Norman No. 1	0.453	0.2285	0.187	48	25	1.031	0.109	0.165
Hopkins or Van Norman No. 2	0.530	0.325	0.250	36	25	1.187	0.171	0.250
Hopkins or Van Norman No. 3	0.460	0.264	0.220	40	25	1.000	0.156	0.200
Hopkins or Van Norman No. 4	0.850	0.605	0.545	24	20	2.437	0.375	0.500
Hopkins or Van Norman No. 5	1.120	0.850	0.765	18	15	3.125	0.625	0.678
Hendey No. 2	1.125	0.826	0.810	20	10	4.250	0.625	0.625
Hendey No. 3	1.375	1.125	1.062	20	10	4.437	1.000	1.000
Hendey No. 5	1.325	1.0625	0.945	18	20	3.437	0.718	0.875
Hendey No. 6	1.625	1.375	1.312	20	10	4.750	1.125	1.125
Pratt & Whitney No. 3	0.850	0.600	0.500	24	20	2.062	0.375	0.500
P. & W. New Style No. 3	0.925	0.650	0.645	24	20	2.125	0.500	0.500
P. & W. Tool Lathe	1.104	0.8125	0.687	28	12½	2.906	0.500	0.562
Stark No. 1	0.435	0.1875	0.165	48	22½	1.109	0.109	0.140
Stark No. 2	0.500	0.2205	0.185	48	22½	1.250	0.115	0.165
Stark Watchmakers No. 3	0.500	0.245	0.185	48	20	1.218	0.115	0.175
Stark Bench Lathe No. 3	0.875	0.590	0.508	26	20	2.125	0.375	0.500
Stark Bench Lathe No. 4	1.375	0.998	0.990	20	15	3.250	0.781	0.781
Sloan & Chase No. 5	0.900	0.600	0.500	26	20	2.437	0.375	0.500
Sloan & Chase No. 5½	1.050	0.800	0.800	20	12	3.670	0.650	0.650
Tarrant Bench Lathe	0.800	0.550	0.475	32	25	2.375	0.375	0.437
Springfield No. 2	0.600	0.320	0.260	40	25	1.563	0.187	0.250
Springfield No. 3	0.725	0.440	0.365	32	25	1.812	0.250	0.312
Springfield No. 4	0.800	0.500	0.425	32	25	1.875	0.312	0.406
Ide Bench Lathe	0.800	0.500	0.425	32	20	2.000	0.312	0.406
Precision Watch Lathe No. 1	0.500	0.375	0.270	40	25	1.187	0.218	0.250

Dimensions of Machine Slides

Bedded Strips



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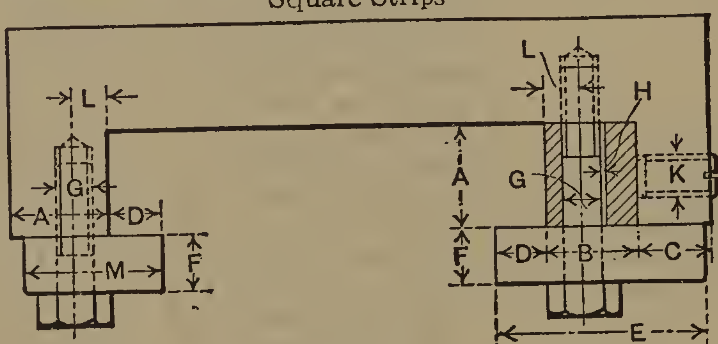
1/2	5/8	5/8	3/8	1/4	5/16	1/32	3/8	1/32
5/8	3/4	3/4	1/2	5/16	3/8	1/32	1/2	1/32
3/4	7/8	7/8	5/8	5/16	3/8	3/64	5/8	1/32
7/8	1	1	3/4	3/8	1/2	3/64	11/16	1/16
1	1 1/4	1 1/8	7/8	3/8	1/2	1/16	13/16	1/16
1 1/4	1 3/8	1 1/4	1	1/2	5/8	5/64	7/8	1/16
1 1/2	1 5/8	1 3/8	1 1/4	5/8	5/8	3/32	1	1/16
1 3/4	1 7/8	1 1/2	1 3/8	5/8	3/4	7/64	1	1/16
2	2 1/4	1 3/4	1 1/2	3/4	7/8	1/8	1 1/4	1/16
2 1/4	2 1/2	2	1 3/4	3/4	1	1/8	1 3/8	1/8
2 1/2	2 3/4	2 1/4	2	7/8	1	5/32	1 1/2	1/8
2 3/4	3	2 1/2	2 1/4	7/8	1 1/8	5/32	1 3/4	1/8
3	3 1/4	2 3/4	2 1/2	1	1 1/8	3/16	2	1/8

Square Strips



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3/8	1/2	5/16	5/16	1 3/32	1/4	1/4	1/32	3/16	7/32	21/32
1/2	5/8	3/8	3/8	1 5/16	5/16	5/16	1/32	1/4	9/32	13/16
5/8	3/4	1/2	1/2	1 11/16	3/8	3/8	1/32	5/16	5/16	1 1/16
3/4	7/8	5/8	5/8	2 1/16	1/2	3/8	1/32	5/16	5/16	1 5/16
7/8	1	3/4	3/4	2 7/16	5/8	1/2	1/16	3/8	3/8	1 9/16
1	1 1/8	7/8	7/8	2 3/4	3/4	1/2	1/16	3/8	7/16	1 3/4
1 1/4	1 1/4	1	1	3 1/8	7/8	5/8	1/16	1/2	7/16	2 1/8
1 1/2	1 3/8	1 1/8	1 1/8	3 1/2	1	5/8	1/16	1/2	1/2	2 1/2
1 3/4	1 1/2	1 1/4	1 1/4	3 7/8	1 1/8	3/4	1/16	5/8	9/16	2 7/8
2	1 3/4	1 1/2	1 1/2	4 5/8	1 1/4	7/8	1/16	3/4	5/8	3 3/8
2 1/4	2	1 5/8	1 5/8	5	1 3/8	7/8	1/8	3/4	11/16	3 5/8
2 1/2	2 1/4	1 3/4	1 3/4	5 1/2	1 1/2	1	1/8	7/8	3/4	4
2 3/4	2 1/2	1 7/8	1 7/8	6	1 5/8	1	1/8	7/8	13/16	4 3/8
3	2 3/4	2	2	6 1/2	1 3/4	1 1/8	1/8	1	7/8	4 3/4
3 1/2	3 1/8	2 1/4	2 1/4	7 1/4	1 7/8	1 1/4	1/8	1 1/8	1	5 3/8
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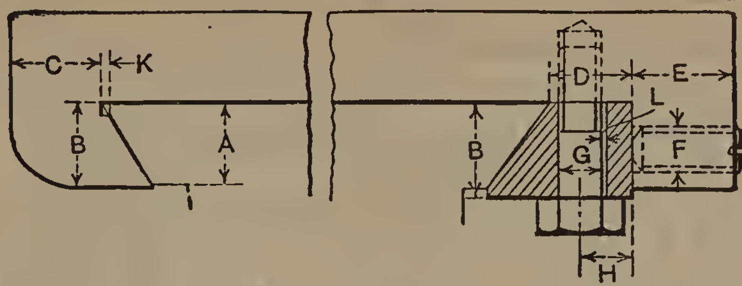
Dimensions of Machine Slides

Overhung Strips



A	B	C	D	E	F	G	H	K	L
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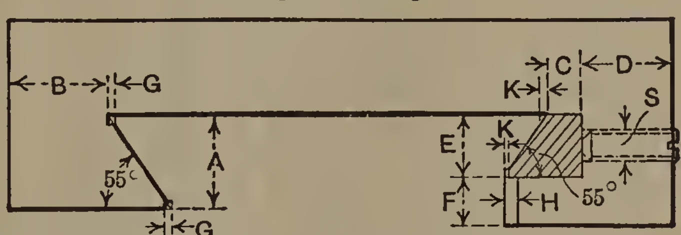
3/8	13/32	7/16	1/2	5/16	3/16	1/4	5/16	3/128	1/32
1/2	9/16	5/8	5/8	3/8	1/4	5/16	3/8	1/32	1/32
5/8	11/16	3/4	3/4	1/2	5/16	3/8	1/2	1/32	1/32
3/4	13/16	3/4	3/4	5/8	5/16	1/2	5/8	3/64	1/32
7/8	15/16	1	1	3/4	3/8	1/2	1 1/16	3/64	1/16
1	1 1/8	1 1/4	1	7/8	3/8	1/2	1 1/16	1/16	1/16
1 1/4	1 3/8	1 1/2	1 1/8	1	1/2	5/8	3/4	5/64	1/16
1 1/2	1 5/8	1 3/4	1 1/4	1 1/8	1/2	5/8	7/8	3/32	1/16
1 3/4	1 7/8	2	1 1/2	1 1/4	5/8	3/4	1	3/32	1/16
2	2 3/16	2 1/4	1 3/4	1 1/2	3/4	7/8	1 1/4	1/8	1/16
2 1/4	2 1/2	2 1/2	2	1 5/8	3/4	7/8	1 1/2	1/8	1/8
2 1/2	2 3/4	2 3/4	2 1/4	1 3/4	7/8	1	1 7/8	5/32	1/8
2 3/4	3	3	2 1/2	1 7/8	7/8	1	1 7/8	5/32	1/8
3	3 1/4	3 1/4	2 3/4	2	1	1 1/8	2	3/16	1/8

Special Strips



A	B	C	D	E	F	G	H	K	S
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1 1/8	1 3/16	9/16	11/16	7/8	9/16	1/16	3/16	3/64	3/8
1 1/4	1 5/16	5/8	13/16	15/16	5/8	1/16	3/16	3/64	3/8
1 3/8	1 7/16	11/16	7/8	1 1/16	11/16	3/32	1/4	3/64	1/2
1 1/2	1 9/16	3/4	15/16	1 1/8	3/4	3/32	1/4	3/64	1/2
1 5/8	1 11/16	13/16	1	1 1/4	13/16	3/32	1/4	3/64	1/2
1 3/4	1 13/16	7/8	1 1/8	1 5/16	7/8	1/8	3/8	3/32	5/8
1 7/8	1 15/16	15/16	1 3/16	1 7/16	15/16	1/8	3/8	3/32	5/8
2	2 1/8	1	1 1/4	1 1/2	1	1/8	3/8	3/32	3/4
2 1/4	2 3/8	1 1/8	1 3/8	1 11/16	1 1/8	1/8	1/2	3/32	3/4
2 1/2	2 5/8	1 1/4	1 9/16	1 7/8	1 1/4	3/16	1/2	1/8	7/8
2 3/4	2 7/8	1 3/8	1 3/4	2 1/16	1 3/8	3/16	9/16	1/8	7/8
3	3 3/16	1 1/2	1 7/8	2 1/4	1 1/2	3/16	9/16	1/8	1
3 1/4	3 7/16	1 5/8	2	2 7/16	1 5/8	3/16	5/8	1/8	1 1/8
3 1/2	3 11/16	1 3/4	2 3/16	2 5/8	1 3/4	1/4	5/8	3/16	1 1/4
3 3/4	3 15/16	1 7/8	2 3/8	2 13/16	1 7/8	1/4	3/4	3/16	1 1/4
4	4 1/4	2	2 1/2	3	2	1/4	3/4	3/16	1 1/2

Standard Wire Nails and Spikes

(Size, Length and Approximate Number to Pound)

Size of Nail	Length, Inches	Common Wire Nails and Brads: Gage	No. to Lb.	Flooring Brads: Gage	No. to Lb.	Fence Nails: Gage	No. to Lb.	Casing, Smooth and Barbed Box: Gage	No. to Lb.	Finishing Nails: Gage	No. to Lb.
2 *d*	1	15	876	.....	....	.....	....	15½	1010	16½	1351
3 *d*	1¼	14	568	.....	....	.....	....	14½	635	15½	807
4 *d*	1½	12½	316	.....	....	.....	....	14	473	15	584
5 *d*	1¾	12½	271	.....	....	10	142	14	406	15	500
6 *d*	2	11½	181	11	157	10	124	12½	236	13½	309
7 *d*	2¼	11½	161	11	139	9	92	12½	210	13	238
8 *d*	2½	10¼	106	10	99	9	82	11½	145	12½	189
9 *d*	2¾	10¼	96	10	90	8	62	11½	132	12½	172
10 *d*	3	9	69	9	69	7	50	10½	94	11½	121
12 *d*	3¼	9	64	8	54	6	40	10½	87	11½	113
16 *d*	3½	8	49	7	43	5	30	10	71	11	90
20 *d*	4	6	31	6	31	4	23	9	52	10	62
30 *d*	4½	5	24	.....	....	.....	....	9	46	.....	....
40 *d*	5	4	18	.....	....	.....	....	8	35	.....	....
50 *d*	5½	3	16	.....	....	.....	....	.....	....	.....	....
60 *d*	6	2	11	.....	....	.....	....	.....	....	.....	....

Size and Length		Hinge Nails, Heavy		Hinge Nails, Light		Clinch Nails		Barbed Car Nails, Heavy		Barbed Car Nails, Light	
2 *d*	1	.....	....	.....	.....	14	710	.....	....	.....	....
3 *d*	1¼	.....	....	.....	.....	13	429	.....	....	.....	....
4 *d*	1½	3	50	6	82	12	274	10	165	12	274
5 *d*	1¾	3	38	6	62	12	235	9	118	10	142
6 *d*	2	3	30	6	50	11	157	9	103	10	124
7 *d*	2¼	00	12	3	25	11	139	8	76	9	92
8 *d*	2½	00	11	3	23	10	99	8	69	9	82
9 *d*	2¾	00	10	3	22	10	90	7	54	8	62
10 *d*	3	00	9	3	19	9	69	7	50	8	57
12 *d*	3¼	.....	....	.....	.....	9	62	6	42	7	50
16 *d*	3½	.....	....	.....	.....	8	49	6	35	7	43
20 *d*	4	.....	....	.....	.....	7	37	5	26	6	31
30 *d*	4½	.....	....	.....	.....	.....	....	5	24	6	28
40 *d*	5	.....	....	.....	....	.....	....	4	18	5	21
50 *d*	5½	.....	....	.....	....	.....	....	3	15	4	17
60 *d*	6	.....	....	.....	....	.....	....	3	13	4	15

Size and Length		Boat Nails, Heavy		Boat Nails, Light		Slating Nails	
2 *d*	1	.....	....	.....	....	12	411
3 *d*	1¼	.....	....	.....	....	10½	225
4 *d*	1½	¼	44	3/16	82	10½	187
5 *d*	1¾	.....	....	.....	....	10	142
6 *d*	2	¼	32	3/16	62	9	103
7 *d*	2¼	.....	....	.....	....	.....	....
8 *d*	2½	¼	26	3/16	50	.....	....
9 *d*	2¾	.....	....	.....	....	.....	....
10 *d*	3	⅜	14	¼	22	.....	....
12 *d*	3¼	⅜	13	¼	20	.....	....
16 *d*	3½	⅜	12	¼	18	.....	....
20 *d*	4	⅜	10	¼	16	.....	....
30 *d*	4½	.....	....	.....	....	.....	....
40 *d*	5	.....	....	.....	....	.....	....
50 *d*	5½	.....	....	.....	....	.....	....
60 *d*	6	.....	....	.....	....	.....	....

Spikes

Size and Length		Gage	No. to Lb.
10 *d*	3	6	41
12 *d*	3¼	6	38
16 *d*	3½	5	30
20 *d*	4	4	23
30 *d*	4½	3	17
40 *d*	5	2	13
50 *d*	5½	1	10
60 *d*	6	1	8
.....	7	0	7
.....	8	00	6
.....	9	00	5
.....	10	⅜	4
.....	12	⅜	3

CUTTING SPEEDS AND FEEDS

Cutting Speeds for Turning. — The cutting speed is governed principally by the hardness of the metal to be turned; the kind of steel of which the turning tool is made; the shape of the tool and its heat-treatment; the feed and depth of cut; the cooling medium used, if any; the power of the machine; and its design and condition. The table "Cutting Speeds and Feeds for Turning Tools" will be found useful in determining the most economical speeds for a given depth of cut and feed when turning hard, medium and soft steel or cast iron. These tables are based upon the results of the experiments conducted by Mr. F. W. Taylor, and it is assumed that a tool made of a good grade of high-speed steel, properly heat-treated and correctly ground, is used. It will be noted that the cutting speed is much slower for cast iron than for steel. Cast iron is cut with less pressure or resistance than soft steel, but the slower speed required for cast iron is probably due to the fact that the pressure of the chip is concentrated closer to the cutting edge, combined with the fact that cast iron wears the tool faster than steel. The speeds given are higher than those ordinarily used, and, in many cases, a slower rate would be necessary to prevent chattering, or because of some other limiting condition. Ordinary machine steel is generally turned at a speed varying between 60 and 80 feet per minute. For ordinary gray cast iron, the speed usually varies from 50 to 60 feet per minute; for annealed tool steel, from 35 to 45 feet per minute; for soft yellow brass, from 150 to 200 feet per minute; for hard bronze, from 35 to 80 feet per minute, the speed depending upon the composition of the alloy. While these latter speeds correspond closely to general practice, they can be exceeded for many machining operations.

The cutting speed is limited by the durability of the turning tool or the length of time it will turn effectively without re-grinding. The hardness of the metal being turned, combined with the quality of the tool, are the two factors, aside from the speed, which largely govern the time that the tool can be used before re-grinding is necessary. The experiments of Mr. Taylor led to the conclusion that, as a rule, it is not economical to use roughing tools at a speed so slow as to cause them to last more than 1½ hour, without being re-ground; hence, the speeds given in the table previously referred to are based upon this length of time between grindings. Sometimes the work speed cannot be as high as the tool will permit, because of the chattering that often results when a machine is old or not massive enough to absorb the vibrations. The radius of the tool point or "nose" also affects the cutting speed and durability. In order to determine how the durability is affected by the nose radius, tests were made in cast iron and steel with tools having a cutting angle of 75 degrees. The radius of the tools was gradually increased without changing the speed, feed or depth of cut, and in all cases, the durability of the tools increased as the radius became larger.

The United States Navy Department requires the following tests to be made on high-speed steel tools. The lathe tool, known as ⅞-inch standard, must be able to take a cut 3/16 inch deep with 1/16 inch feed and a surface speed of 60 feet per minute, for 20 minutes, without re-grinding, on a steel forging of open-hearth steel having a minimum tensile strength of 80,000 pounds per square inch, elastic limit of 50,000 pounds per square inch, and elongation in 2 inches of 25 per cent. This steel forging is annealed prior to the test.

Effect of Cooling Water. — It has been estimated that the cutting speed, when using high-speed steel tools for turning steel, can be increased about 40 per cent by throwing a large stream of cooling water (supersaturated with soda to prevent rusting) directly upon the chip at the point where it is being removed by the tool. For ordinary carbon steel tools, the gain is about 25 per cent. The

Cutting Speeds and Feeds for Turning Tools*

Steel — Standard 7/8-inch Tool

Depth of Cut in Inches	Feed in Inches	Speed in Feet per Minute for a Tool which is to last 1½ Hour before Re-grinding: Soft Steel	Medium Steel	Hard Steel
3/32	1/64	476	238	108
	1/32	325	162	73.8
	1/16	222	111	50.4
	3/32	177	88.4	40.2
1/8	1/64	420	210	95.5
	1/32	286	143	65.0
	1/16	195	97.6	44.4
	1/8	133	66.4	30.2
3/16	1/64	352	176	80.0
	1/32	240	120	54.5
	1/16	164	82	37.3
	1/8	112	56	25.5
1/4	1/64	312	156	70.9
	1/32	213	107	48.4
	1/16	145	72.6	33.0
	3/32	116	58.1	26.4
3/8	1/64	264	132	60.0
	1/32	180	90.2	41.0
	1/16	122	61.1	27.8
1/2	1/64	237	118	53.8
	1/32	162	80.8	36.7

Cast Iron — Standard 7/8-inch Tool

Depth of Cut in Inches	Feed in Inches	Speed in Feet per Minute for a Tool which is to last 1½ Hour before Re-grinding: Soft Cast Iron	Medium Cast Iron	Hard Cast Iron
3/32	1/16	122	61.2	35.7
	1/8	86.4	43.2	25.2
	3/16	70.1	35.1	20.5
1/8	1/32	156	77.8	45.4
	1/16	112	56.2	32.8
	1/8	79.3	39.7	23.2
	3/16	64.3	32.2	18.8
3/16	1/32	137	68.6	40.1
	1/16	99.4	49.7	29.0
	1/8	70.1	35.0	20.5
	3/16	56.8	28.4	16.6
1/4	1/32	126	62.9	36.7
	1/16	90.8	45.4	26.5
	1/8	64.1	32.0	18.7
	3/16	52	26.0	15.2
3/8	1/32	111	55.4	32.3
	1/16	80	40.0	23.4
	1/8	56.4	28.2	16.5
1/2	1/32	104	52.1	30.4
	1/16	75.2	37.6	22.0
	1/8	43.1	21.6	12.6

Steel — Standard 5/8-inch Tool

Depth of Cut	Feed	Soft Steel	Medium Steel	Hard Steel
1/16	1/64	548	274	125
	1/32	358	179	81.6
	1/16	235	117	53.3
3/32	1/64	467	234	106
	1/32	306	153	69.5
	1/16	200	100	45.5
	3/32	156	78	35.5
1/8	1/64	417	209	94.8
	1/32	273	136	62.0
	1/16	179	89.3	40.6
	3/32	140	69.8	31.7
3/16	1/64	362	181	82.2
	1/32	236	118	53.8
	1/16	155	77.4	35.2
1/4	1/64	328	164	74.5
	1/32	215	107	48.8
3/8	1/64	286	143	65.0

Cast Iron — Standard 5/8-inch Tool

Depth of Cut	Feed	Soft Cast Iron	Medium Cast Iron	Hard Cast Iron
3/32	1/32	160	80.0	46.6
	1/16	110	55.0	32.2
	1/8	75.4	37.7	22.0
1/8	1/32	148	74.0	43.3
	1/16	104	51.8	32.0
	1/8	69.6	34.8	20.3
3/16	1/64	183	91.6	68.0
	1/32	135	67.5	39.4
	1/16	94	47.0	27.4
	1/8	64.3	32.2	18.8
1/4	1/64	171	85.7	50.1
	1/32	126	63.2	36.9
	1/16	87.8	43.9	25.6
	3/32	70.4	35.2	20.6
3/8	1/64	156	77.8	45.4
	1/32	116	57.8	33.8
	1/16	79.7	39.9	23.3

* Cutting speeds for tools of a good grade of high-speed steel, properly ground and heat-treated.

most satisfactory results are obtained from a stream of water falling at a rather slow velocity, but in large volume. The gain in cutting speed by the use of water or other fluid is practically the same for all qualities of steel from the softest to the hardest.

Feed and Depth of Cut for Turning. — Ordinarily, coarser feeds and a greater depth of cut can be used for cast iron than for soft steel. In general, with a given depth of cut, metal can be removed more quickly by using a coarse feed and a slower speed, than by using a fine feed and a proportionate increase in speed. When the turning operation is simply to remove metal, the feed should be coarse and the cut as deep as practicable. Sometimes the cut must be comparatively light, either because the work is too fragile and springy to withstand the strain of a heavy cut, or because the lathe has not sufficient power. The effect of the feed and the depth of cut on the cutting speed is shown by the table "Cutting Speeds and Feeds for Turning Tools."

Influence of Temperature on Durability of Tool. — By testing various samples of carbon steel in a special testing machine, it has been found that carbon tool steels have a very low durability at a low cutting speed; that there is an increase of durability as the cutting speed increases; and that a maximum durability is obtained at a cutting speed of about 50 to 80 feet per minute. There is then a decline of durability to a very low value if the cutting speed is further increased. These general characteristics are common to all tool steels, whether of the carbon or high-speed steel type (tungsten or tungsten-vanadium varieties). All of these steels show, when the durability is recorded in diagrammatic form, a single- or double-peaked curve, according to the heat-treatment they have received. All show a low durability at low cutting speed, this characteristic being especially marked in the case of some high-speed steels, which latter often retain their durability at very high speeds. One of these double-peaked curves is shown in the illustration. With this particular steel, the durability increased up to a cutting speed of 50 feet per minute; it then diminished until the cutting speed reached 70 feet per minute, when the durability again increased up to 80 feet per minute. Hence, the tool, in this instance, would work longer between grindings with a cutting speed of 80 feet per minute than with a speed of 70 feet per minute. The durability, however, might be affected by chattering, as the result of the higher speed.

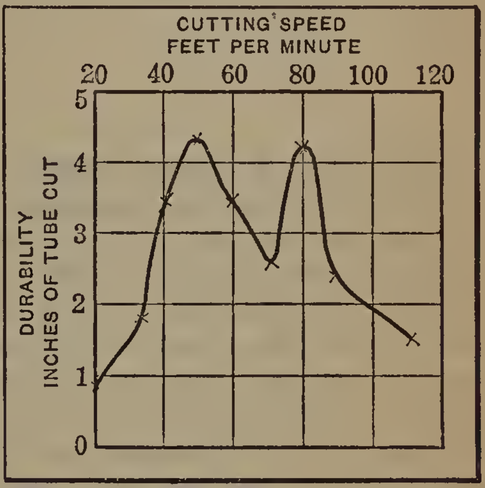


Diagram showing Relation between Cutting Speed and Durability, the latter measured by Length of Test Tube cut

The changes in the durability of cutting tools are mainly caused by the changes in the temperature of the cutting edge, due to the heat generated at different cutting speeds. This heat theory has been confirmed by experiments showing that changes of durability corresponding to those which occur under varying cutting speeds can be produced by varying the temperature of the tool in other ways, while the cutting speed remains constant — for instance, by varying the temperature of the water with which the tool is flooded, by varying the depth of the cut or by dispensing entirely with the cooling water.

Many cases are known to have occurred in ordinary machine shop practice, where an increase in cutting speed has actually resulted in increased durability of the tool. Low durability at low cutting temperatures (for example, on finishing cuts) is a familiar characteristic of high-speed steels, and is most marked in tools which have been suitably hardened for very high temperature work. High-speed steel can be so hardened as to retain its durability at fairly low temperatures, and there are now on the market tungsten steels specially adapted for low temperature work, such as finishing heavy forgings; but all of the steels tested proved to be less durable if the cutting temperature was low enough.

Rules for Calculating Cutting Speeds. — To find the number of revolutions required for a given cutting speed, in feet per minute: *Multiply the given cutting speed by 12, and divide the product by the circumference (in inches) of the turned part.*

To find the cutting speed in feet per minute for a given number of revolutions and diameter: *Multiply the revolutions per minute by the circumference, and divide the product by 12.*

Expressing these rules as formulas:

$$N = \frac{C \times 12}{3.1416\, D} \qquad C = \frac{3.1416\, DN}{12}$$

in which N = revolutions per minute; C = cutting speed in feet per minute; and D = diameter in inches.

To find the time, in minutes, required to take one complete cut over a part to be turned, when the feed per revolution, the total length of the cut, and the number of revolutions per minute are given: *Divide the total length of the cut by the product of the number of revolutions per minute and the feed per revolution.*

If L = total length of cut in inches; N = revolutions per minute; F = feed per revolution in inches; and T = time required to take one complete cut, in minutes; then:

$$T = \frac{L}{N \times F}$$

Speeds and Feeds for Milling. — A general idea of the speeds that are feasible when using carbon-steel cutters can be obtained from the following figures, which represent the velocity (in feet per minute) at the circumference of the cutter: For roughing cuts in cast iron, 40 feet per minute; in machine steel, 60 feet per minute; in tool steel (annealed), 25 feet per minute; in brass, 75 feet per minute. For finishing cuts, the speeds vary from 50 to 55 feet for cast iron; from 75 to 80 feet for machine steel; from 30 to 35 feet for annealed tool steel; from 95 to 100 feet for brass. These figures represent a fair average for carbon-steel cutters; high-speed steel cutters of good quality can usually be operated at approximately twice the speeds given. The speed of a milling cutter, in revolutions per minute, may be obtained by means of the following formulas which give results representing ordinary shop practice: Let N = revolutions per minute; D = diameter of cutter; then,

For Cast Iron	For Machine Steel	For Tool Steel	For Brass
$N = \frac{150}{D}$	$N = \frac{230}{D}$	$N = \frac{95}{D}$	$N = \frac{285}{D}$

The rate of feed, aside from the cutter design, depends upon the width and depth of the cut, the kind of material being milled, the quality of finish desired, the rigidity of the work, the power of the machine, etc. As a general rule, a relatively low cutting speed and a heavy feed is used for roughing; for finishing, the speed is increased and the feed diminished. The data given in connection with the examples of milling shown by the tables "Speeds and Feeds for Milling" indicate, in a general way, what speeds and feeds are practicable for different classes of work, when using a rigid machine and modern cutters.

Time Required for Cutting Tool to Travel 1 Inch, When the Feed is 1/32 Inch per Revolution

Diam., Inches	Surface Speeds in Feet per Minute															
	20 Feet		25 Feet		30 Feet		35 Feet		40 Feet		45 Feet		50 Feet		60 Feet	
	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
1	0	25	0	20	0	17	0	14	0	12	0	11	0	10	0	8
2	0	50	0	40	0	33	0	29	0	25	0	22	0	20	0	17
3	1	15	1	0	0	50	0	43	0	38	0	33	0	30	0	25
4	1	40	1	20	1	7	0	57	0	50	0	45	0	40	0	33
5	2	6	1	41	1	24	1	12	1	3	0	56	0	50	0	42
6	2	31	2	1	1	41	1	26	1	15	1	7	1	0	0	50
7	2	56	2	21	1	57	1	41	1	28	1	18	1	10	0	58
8	3	21	2	41	2	14	1	35	1	40	1	29	1	20	1	7
9	3	46	3	1	2	31	2	9	1	53	1	41	1	30	1	15
10	4	11	3	21	2	48	2	24	2	6	1	52	1	41	1	24
11	4	36	3	41	3	4	2	38	2	18	2	3	1	51	1	32
12	5	2	4	1	3	21	2	52	2	31	2	14	2	1	1	41
13	5	27	4	21	3	38	3	7	2	43	2	25	2	11	1	49
14	5	52	4	41	3	55	3	21	2	56	2	36	2	21	1	57
15	6	17	5	2	4	11	3	35	3	8	2	48	2	31	2	6
16	6	42	5	22	4	28	3	50	3	21	2	59	2	41	2	14
17	7	7	5	42	4	45	4	4	3	34	3	10	2	51	2	22
18	7	32	6	2	5	2	4	18	3	46	3	21	3	1	2	31
19	7	57	6	22	5	18	4	33	3	59	3	32	3	11	2	39
20	8	22	6	42	5	35	4	47	4	11	3	43	3	21	2	48
21	8	48	7	2	5	52	5	2	4	24	3	55	3	31	2	56
22	9	13	7	22	6	9	5	16	4	36	4	6	3	41	3	4
23	9	38	7	42	6	25	5	30	4	49	4	17	3	51	3	12
24	10	3	8	3	6	42	5	45	5	2	4	28	4	1	3	21
25	10	28	8	23	6	59	5	59	5	15	4	39	4	11	3	30
26	10	53	8	43	7	16	6	13	5	27	4	50	4	21	3	38
27	11	19	9	3	7	32	6	28	5	39	5	2	4	31	3	46
28	11	44	9	23	7	49	6	42	5	52	5	13	4	41	3	55
29	12	9	9	43	8	6	6	56	6	4	5	24	4	51	4	3
30	12	34	10	3	8	23	7	11	6	17	5	35	5	2	4	11
31	12	59	10	23	8	39	7	25	6	30	5	46	5	12	4	19
32	13	24	10	43	8	56	7	40	6	42	5	57	5	22	4	28
33	13	49	11	3	9	13	7	54	6	55	6	9	5	32	4	36
34	14	15	11	24	9	30	8	8	7	7	6	20	5	42	4	45
35	14	40	11	44	9	46	8	23	7	20	6	31	5	52	4	53
36	15	5	12	4	10	3	8	37	7	32	6	42	6	2	5	2
37	15	30	12	24	10	20	8	51	7	45	6	53	6	12	5	10
38	15	55	12	44	10	37	9	6	7	57	7	4	6	22	5	18
39	16	20	13	4	10	53	9	20	8	10	7	16	6	32	5	27
40	16	45	13	24	11	11	9	34	8	22	7	27	6	42	5	35

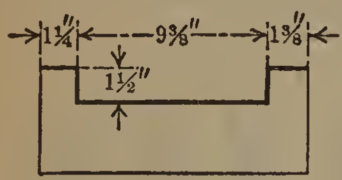
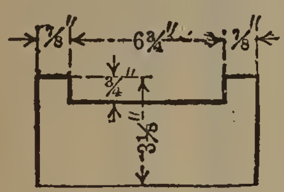
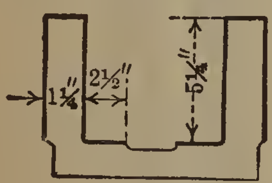
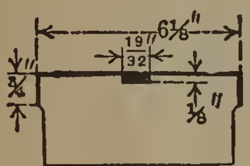
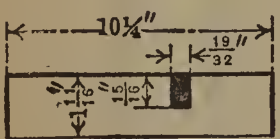
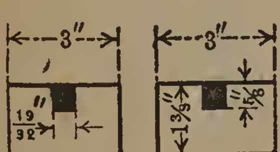
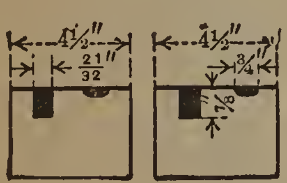
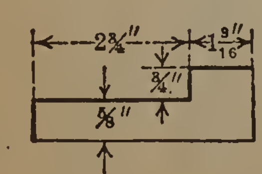
Revolutions per Minute for Various Cutting Speeds and Diameters

Diameter, Inches	Cutting Speed, Feet per Minute											
	25	30	35	40	45	50	55	60	65	70	75	80
	Revolutions per Minute											
¼	382	458	535	611	688	764	851	917	994	1070	1147	1222
5/16	306	367	428	489	550	611	672	733	794	856	917	978
⅜	255	306	357	408	458	509	560	611	662	713	764	815
7/16	218	262	306	349	393	437	481	524	568	611	656	699
½	191	229	268	306	344	382	420	459	497	535	573	611
9/16	170	204	238	272	306	340	373	407	441	475	509	543
⅝	153	184	214	245	276	306	337	367	398	428	459	489
11/16	138	167	194	222	249	273	300	333	360	389	416	444
¾	127	153	178	203	229	254	279	306	330	357	381	408
13/16	119	142	166	190	213	237	261	284	308	332	356	379
⅞	109	131	153	175	196	219	241	262	285	306	329	349
15/16	101	122	142	163	183	204	224	244	265	285	305	326
1	95.5	115	134	153	172	191	210	229	258	267	287	306
1 1/16	89.8	108	126	144	162	180	197	215	233	251	269	287
1⅛	85.0	102	119	136	153	170	187	204	221	238	255	272
1 3/16	80.5	96.6	113	129	145	161	177	193	209	225	242	258
1¼	76.3	91.8	107	123	137	153	168	183	199	214	230	245
1 5/16	72.8	87.3	102	116	131	146	160	175	189	204	218	233
1⅜	69.5	83.3	97.2	111	125	139	153	167	180	195	208	222
1 7/16	66.3	79.5	92.8	106	119	133	146	159	172	186	199	212
1½	63.7	76.3	89.2	102	115	127	140	153	165	178	191	204
1 9/16	61.0	73.2	85.4	97.6	110	122	134	146	159	171	183	195
1⅝	58.8	70.5	82.2	93.9	106	117	129	141	152	165	176	188
1 11/16	56.5	67.8	79.1	90.4	102	113	124	136	147	158	170	181
1¾	54.5	65.5	76.4	87.3	98.2	109	120	131	142	153	164	175
1⅞	50.9	61.1	71.3	81.5	91.9	102	112	122	133	143	153	163
2	47.8	57.3	66.9	76.4	86.0	95.5	105	115	124	134	143	153
2⅛	45.0	54.0	63.0	72.0	81.0	90.0	99.0	108	117	126	135	144
2¼	42.4	51.0	59.4	68.0	76.2	85.5	93.5	102	111	119	128	136
2⅜	40.3	48.3	56.4	64.4	72.5	80.5	88.6	96.6	105	113	121	129
2½	38.2	45.8	53.5	61.2	68.8	76.3	84.2	91.7	99.5	107	114	122
2⅝	36.3	43.5	50.8	58.0	65.3	72.5	79.8	87.0	94.3	102	109	116
2¾	34.7	41.7	48.6	55.6	62.5	69.5	76.5	83.4	90.4	97.2	104	111
2⅞	33.0	39.6	46.2	52.8	59.4	66.0	72.6	79.2	85.8	92.4	99.0	106
3	31.8	38.2	44.6	51.0	57.3	63.7	69.9	76.4	82.6	89.1	95.3	102
3⅛	30.5	36.6	42.7	48.8	54.9	61.0	67.1	73.2	79.3	85.4	91.5	97.6
3¼	29.3	35.1	41.0	46.8	52.7	58.5	64.4	70.2	76.1	81.9	87.8	93.6
3⅜	28.3	33.9	39.6	45.2	50.9	56.5	62.2	67.8	73.5	79.1	84.8	90.4
3½	27.3	32.7	38.2	43.6	49.1	54.5	60.0	65.5	70.8	76.4	81.8	87.4
3⅝	26.3	31.5	36.8	42.0	47.3	52.5	57.8	63.0	68.3	73.5	78.8	84.0
3¾	25.5	30.6	35.7	40.8	45.9	51.0	56.1	61.2	66.3	71.4	76.5	81.6
3⅞	24.6	29.6	34.5	39.4	44.3	49.3	54.2	59.1	64.0	69.0	73.8	78.8
4	23.9	28.7	33.4	38.2	43.0	47.8	52.6	57.3	62.1	66.9	71.7	76.4
4¼	22.5	26.9	31.4	35.9	40.4	44.9	49.4	53.9	58.4	62.9	67.4	71.8
4½	21.2	25.4	29.6	34.0	38.2	42.4	46.6	51.0	55.1	59.4	63.6	67.9
4¾	20.1	24.1	28.1	32.2	36.2	40.2	44.2	48.2	52.3	56.3	60.3	64.3
5	19.1	22.9	26.7	30.6	34.4	38.2	42.0	45.9	49.7	53.5	57.3	61.1
5¼	18.2	21.8	25.4	29.1	32.7	36.4	40.0	43.6	47.3	50.9	54.5	58.2
5½	17.4	20.8	24.3	27.8	31.3	34.7	38.2	41.7	45.1	48.6	52.0	55.6
5¾	16.6	19.9	23.2	26.6	29.9	33.2	36.5	39.8	43.2	46.5	49.8	53.1
6	15.9	19.1	22.3	25.5	28.7	31.8	35.0	38.2	41.3	44.6	47.7	51.0
6¼	15.3	18.3	21.4	24.4	27.5	30.6	33.6	36.7	39.7	42.8	45.8	48.9
6½	14.7	17.6	20.5	23.5	26.4	29.4	32.3	35.2	38.2	41.1	44.0	47.0
6¾	14.2	17.0	19.8	22.6	25.5	28.3	31.1	34.0	36.8	39.6	42.5	45.3
7	13.6	16.4	19.1	21.8	24.6	27.3	30.0	32.7	35.5	38.2	41.0	43.7
7¼	13.2	15.8	18.4	21.1	23.7	26.4	29.0	31.6	34.3	36.9	39.5	42.2
7½	12.7	15.3	17.8	20.4	22.9	25.4	28.0	30.5	33.1	35.6	38.2	40.7
7¾	12.3	14.8	17.2	19.7	22.1	24.6	27.1	29.5	32.0	34.4	36.9	39.4
8	11.9	14.3	16.7	19.1	21.1	23.9	26.3	28.7	31.0	33.4	35.9	38.2

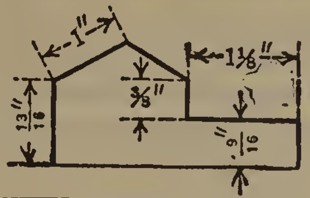
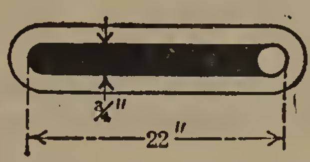
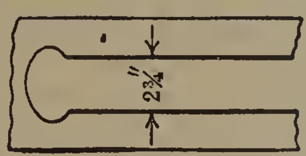
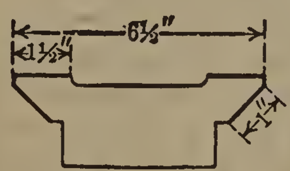
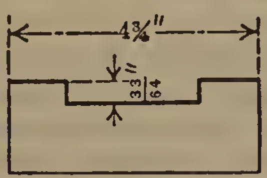
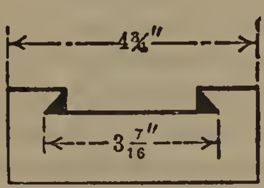
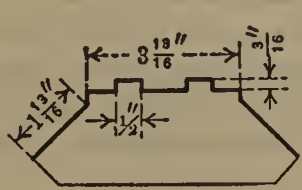
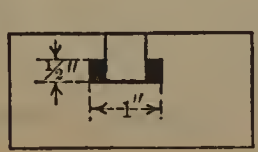
Revolutions per Minute for Various Cutting Speeds and Diameters

Diameter, Inches	Cutting Speed, Feet per Minute										
	90	100	110	120	130	140	150	160	170	180	200
	Revolutions per Minute										
1/4	1376	1528	1681	1834	1986	2139	2292	2445	2598	2750	3056
5/16	1100	1222	1344	1466	1589	1711	1833	1955	2077	2200	2444
3/8	916	1018	1121	1222	1323	1425	1527	1629	1731	1832	2036
7/16	786	874	961	1049	1136	1224	1311	1398	1486	1573	1748
1/2	688	764	840	917	993	1070	1146	1222	1299	1375	1528
9/16	611	679	747	813	883	951	1019	1086	1154	1222	1358
5/8	552	612	673	736	796	857	918	979	1040	1102	1224
11/16	500	555	611	666	722	770	833	888	944	999	1110
3/4	458	508	559	610	661	711	762	813	864	914	1016
13/16	427	474	521	569	616	664	711	758	806	853	948
7/8	392	438	482	526	569	613	657	701	745	788	876
15/16	366	407	448	488	529	570	611	651	692	733	814
1	344	382	420	458	497	535	573	611	649	688	764
1 1/16	323	359	395	431	467	503	539	579	610	646	718
1 1/8	306	340	374	408	442	476	510	544	578	612	680
1 3/16	290	322	354	386	419	451	483	515	547	580	644
1 1/4	274	306	337	367	398	428	459	490	520	551	612
1 5/16	262	291	320	349	378	407	437	466	495	524	582
1 3/8	250	278	306	334	361	389	417	445	472	500	556
1 7/16	239	265	292	318	345	371	398	424	451	477	530
1 1/2	230	254	279	305	330	356	381	406	432	457	508
1 9/16	220	244	268	293	317	342	366	390	415	439	488
1 5/8	212	234	257	281	304	328	351	374	398	421	468
1 11/16	203	226	249	271	294	316	339	362	384	407	452
1 3/4	196	218	240	262	283	305	327	349	371	392	436
1 7/8	184	204	224	244	265	286	306	326	347	367	408
2	172	191	210	229	248	267	287	306	325	344	382
2 1/8	162	180	198	216	234	252	270	288	306	324	360
2 1/4	153	170	187	204	221	238	255	272	289	306	340
2 3/8	145	161	177	193	209	225	242	258	274	290	322
2 1/2	138	153	168	184	199	213	230	245	260	275	306
2 5/8	131	145	160	174	189	203	218	232	247	261	290
2 3/4	125	139	153	167	181	195	209	222	236	250	278
2 7/8	119	132	145	158	172	185	198	211	224	238	264
3	114	127	140	152	165	178	191	203	216	228	254
3 1/8	110	122	134	146	159	171	183	195	207	219	244
3 1/4	105	117	129	140	152	164	176	188	199	211	234
3 3/8	102	113	124	136	147	158	170	181	192	203	226
3 1/2	98.1	109	120	131	142	153	164	174	186	196	218
3 5/8	94.5	105	116	126	137	147	158	168	179	189	210
3 3/4	91.8	102	112	122	133	143	153	163	175	184	205
3 7/8	88.6	98.5	108	118	128	138	148	158	167	177	197
4	86.0	95.6	105	115	124	134	143	153	163	172	191
4 1/4	80.8	89.8	98.8	108	117	126	135	144	153	162	180
4 1/2	76.3	84.8	93.3	102	110	119	127	136	144	153	170
4 3/4	72.4	80.4	88.4	96.9	105	113	121	129	137	145	161
5	68.8	76.4	84.0	91.7	99.3	107	115	122	130	138	153
5 1/4	65.4	72.7	80.0	87.2	94.5	102	109	116	124	131	145
5 1/2	62.5	69.4	76.3	83.3	90.2	97.2	104	111	118	125	139
5 3/4	59.8	66.4	73.0	80.0	86.3	93.0	99.6	106	113	120	133
6	57.2	63.6	70.0	76.3	82.7	89.0	95.4	102	108	114	127
6 1/4	55.0	61.1	67.2	73.3	79.4	85.5	91.7	97.7	104	110	122
6 1/2	52.8	58.7	64.6	70.4	76.3	82.2	88.1	93.9	100	106	117
6 3/4	50.9	56.6	62.3	67.9	73.6	79.2	84.9	90.6	96.2	102	113
7	49.1	54.6	60.1	65.5	71.0	76.4	81.9	87.4	92.8	98.3	109
7 1/4	47.4	52.7	58.0	63.2	68.5	73.8	79.1	84.3	89.6	94.9	105
7 1/2	45.8	50.9	56.0	61.1	66.2	71.0	76.4	81.4	86.5	91.6	102
7 3/4	44.3	49.2	54.1	59.0	64.0	68.9	73.8	78.7	83.6	88.6	98.4
8	43.0	47.8	52.6	57.4	62.1	66.9	71.7	76.5	81.3	86.0	95.6

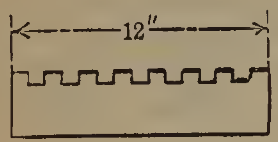
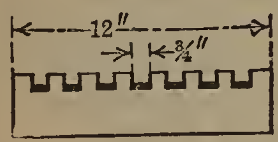
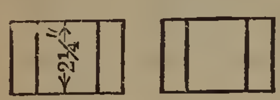
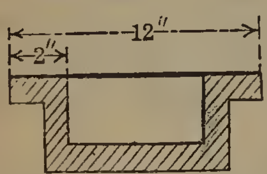
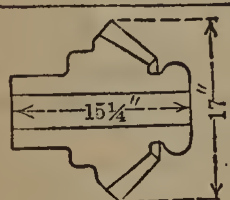
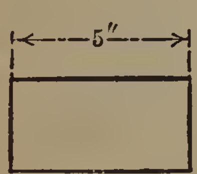
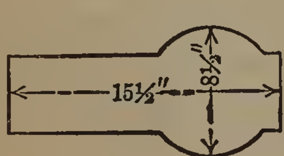
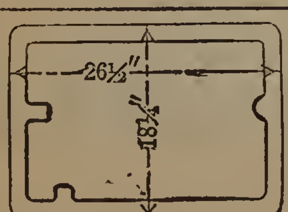
Speeds and Feeds for Milling — From Practice — 1
(Cincinnati Milling Machine Co.)

Milled Surface Indicated by Heavy Line	Material Milled, Width and Depth of Cut, Feed of Table and Speed of Cutters
	Material, cast iron. Total width of cut, 15 inches. Maximum depth, 3/16 inch. Feed, 4¾ inches per minute. Largest cutter of gang, 6 inches diameter, 9⅜-inch face. Speed, 32 revolutions per minute.
	Material, cast iron. Total width of milled surface, 16⅜ inches. Depth of cut, 3/16 inch. Feed, 6.3 inches per minute. Diameter of side mills for finishing sides of casting, 10½ inches. Speed of cutter, 21 revolutions per minute.
	Material, cast iron. Total width of milled surface, 23 inches. Depth of cut, ⅛ inch. Feed of table, 4¾ inches per minute. Diameter of side mills, 13½ inches. Diameter of intermediate cutters for top surfaces, 3 inches. Speed, 14 revolutions per minute.
	Material, cast iron. Total width of cut, 7⅝ inches. Size of milled groove, 19/32 inch wide, ⅛ inch deep. Feed of table, 9.9 inches per minute. Depth of cut, ⅛ inch.
	Material, cast iron. Total width of cut, 13⅝ inches. Size of slot milled from solid, 15/16 by 19/32 inch. Depth of cut on top and sides, 3/16 inch. Feed, 4 inches per minute. Cutter diameters, 8, 3½ and 5¾ inches. Speed, 36 revolutions per minute.
	Material, close-grained cast iron. Operation, roughing two castings simultaneously, removing 3/16 inch stock and milling from solid, two ⅝-inch slots. Feed, 4 inches per minute. Speed of gang cutter, 36 revolutions per minute.
	Material, 50-point carbon steel bars. Operation, milling two bars simultaneously. Travel of table, 1 1/16 inch per minute. Depth of cut, ¼ inch. High-speed steel cutters, 4 inches and 5¾ inches in diameter. Peripheral speed of large cutter, 30 feet per minute.
	Material, cast iron. Total width of finished surface, 4¾ inches. Depth of cut, ⅛ inch. Feed per minute, 1⅞ inch. Cutter diameters, 4½ and 3 inches, respectively. Limit of accuracy, 0.001 inch.

Speeds and Feeds for Milling — From Practice — 2

Milled Surface Indicated by Heavy Line	Material Milled, Width and Depth of Cut, Feed of Table and Speed of Cutters
	Material, cast iron. Total width of surface finished, 5 inches. Depth of cut, 5/32 inch. Feed of table, 2.9 inches per minute. Largest cutters in gang, 5½ inches; smallest, 2 inches. Speed, 53 revolutions per minute.
	Operation, milling slots in cast-iron bars 1 inch thick, with a single cut, using a ¾-inch, 3-fluted, high-speed steel end mill. Feed of table, 3⅝ inches per minute. Speed of cutter, 365 revolutions per minute, giving a surface speed of 72 feet.
	Material, close-grained cast iron. Operation, milling both sides of a slot. Diameter of cutter, 2¾ inches. Width of cut, 2¾ inches. Depth of cut at top and bottom, 1/16 inch. Feed, 3 inches per minute.
	Material, close-grained cast iron. Operation, roughing dovetail bearings. Depth of cut, 3/16 inch. Feed, 7¾ inches per minute. Width of surface milled by side mill, 1½ inch; by angular cutter, 1¼ inch. Speed, 36 revolutions per minute. Machine, vertical type.
	Material, steel castings. First operation, roughing out channel and top surface. Depth of cut, ⅛ inch; ¼ inch on the sides of channel. Feed, 7¾ inches per minute. Cutters, high-speed steel. (Succeeding operation follows.)
	Second operation, in vertical machine. Cutters, 6-inch side mill, 3-inch angular mill, mounted as a gang, but used independently. First cut, truing top surface. Feed, 2¼ inches per minute. Dovetailed sides finished in two cuts. Feed, 1 1/16 inch per minute, feed being slow to insure accuracy.
	Material, gray iron. Total width of cut, 9.4 inches. Average depth, 3/16 inch. Feed, 6⅛ inches per minute. Diameter of angular cutters for sloping sides, 7½ inches. Diameter of cutters for top surfaces, 3½ and 3⅛ inches. Speed, 33 revolutions per minute.
	Operation, undercutting T-slots in cast iron. Cutter, high-speed steel, 1 inch in diameter, ½ inch wide. Speed, 286 revolutions per minute. Feed of table, 15¾ inches per minute.

Speeds and Feeds for Milling — From Practice — 3

Milled Surface Indicated by Heavy Line	Material Milled, Width and Depth of Cut, Feed of Table and Speed of Cutters
	Material, cast-iron plates. First operation, milling full width of plate. Depth of cut, ⅛ inch. Feed of table, 4.9 inches per minute. Cutter, 4 inches in diameter. Speed, 66 revolutions per minute. (Succeeding operation follows.)
	Second operation, finishing slots and sides. Width of slots, ¾ inch. Feed of table, 1¾ inch per minute. Cutter diameters, 4 and 8 inches, respectively. Speed, 66 revolutions per minute. Limit of accuracy, 0.001 inch.
	Material, steel. Operation, finishing simultaneously the four sides of two connecting-rod straps. Width of each milled surface, 2¼ inches. Depth of cut, ⅛ to 3⁄16 inch. Feed of table, 1 inch per minute. Cutters, high-speed steel, 8½ inches in diameter. Cutting speed, 50 feet per minute.
	Material, gray iron. Diameter of face mill, 12⅝ inches. Surface is rough milled by one passage of cutter, then feed is reversed and a finishing cut 0.010 inch deep is taken. Rate of feed, 20 inches per minute. Machine, vertical type.
	Material, steel castings. Operation, facing flat surface with a 9½-inch face mill. Depth of roughing cut, 3⁄16 inch. Feed, 3 11⁄16 inches per minute. Speed, 21 revolutions per minute. Depth of finishing cut, 0.010 inch. Machine, vertical type.
	Material, machine steel bars. Operation, milling flat surface 5 inches wide, with 12-inch inserted-tooth cutter. Depth of cut, ⅛ inch. Feed, 16 inches per minute. Speed, 17 revolutions per minute. Machine, vertical type.
	Material, cast iron. Maximum width of cut, 8½ inches. First operation, roughing cut 3⁄16 inch deep; feed, 7¾ inches per minute. Second operation, finishing cut; feed, 20 inches per minute. Machine, vertical type.
	Material, cast iron. Roughing cut, 3⁄16 inch deep; feed, 20 inches per minute. Cutter, 10-inch face mill. Rectangular surface is covered by using longitudinal and cros. feeds. Machine, vertical type.

Power Required for Milling. — It has been shown by experiments that a proportionately greater amount of power is required for light cuts than for heavy or deep cuts. The depth of the cut does not increase the power required in the same proportion as does the width of the cut. When milling with a coarse feed and taking a deep, but comparatively narrow cut, much less power is required for the amount of metal removed, than when the feed is slow and the cut of moderate depth, but wide. Ordinarily, a slow feed on the milling machine is not economical. The power required for milling and the influence of the depth and width of the cut on the power consumption is shown by the table "Power Required for Milling." The experiments of which the results are here recorded were made on a special milling machine, and the results tabulated are of tests in which the power was definitely determined.

Power Required for Milling

(Cutters used made from high-speed steel)

Revolutions of Cutter per Minute	Feed		Cutting Speed of Cutter per Minute, Feet	Depth of Cut, Inches	Width of Cut, Inches	Power Required, H.P.	Metal Removed per Hour, Pounds	Power Required per Pound-hour, H.P.
	Per Minute, Inches	Per Revolution, Inches						
24	2.46	0.10	37.0	0.26	23.6	25	245	0.102
24	3.50	0.15	37.0	0.26	10.2	17	150	0.113
24	4.35	0.18	37.0	0.14	9.8	17	97	0.175
24	3.50	0.15	37.0	0.49	9.8	27	490	0.055
19	4.33	0.23	29.5	0.28	9.3	17	331	0.051
23	4.17	0.18	36.0	0.28	20.5	27	386	0.070
23	4.17	0.18	36.0	0.28	9.8	20	183	0.109
40	1.89	0.05	64.0	0.24	10.2	17	74	0.230
40	3.94	0.10	64.0	0.37	13.8	21	331	0.063
40	5.79	0.14	64.0	0.16	16.5	17	123	0.138

Speeds and Feeds for Drilling. — Drill speeds, in revolutions per minute, decrease as the drill diameter increases, but the peripheral speed should be practically constant for all diameters and for a given material. The feed should increase as the drill diameter increases. The speeds given in the table, "Drilling Speeds — Carbon Steel Drills," will suit average conditions. The feeds for these drills vary, ordinarily, from 0.005 to 0.010 inch per revolution, but the feed must sometimes be varied to suit conditions. An approximate idea of the feed to use for various drill diameters can be obtained from the following figures: A ¼-inch drill should have a feed of about 0.005 inch; a ½-inch drill, 0.007 inch; a ¾-inch drill, 0.010 inch, per revolution. The most efficient speed and feed can be determined only by actual test with the drilling machine to be used and on the material to be drilled. In general, the cutting speed for high-speed steel drills can be at least twice as great as for drills made of carbon steel. The tables of speeds and feeds for high-speed drills are intended as a general guide for average conditions. The data given on page 871 represent the results of a long series of tests made to determine the most efficient speeds and feeds. The speeds are comparatively high and are only recommended for drilling machines of the high-speed type. The speed is governed not only by the composition of the material to be drilled, but by the design and condition of the drilling machine, the shape and degree of sharpness of the drill point, and the quality of steel in the drill itself. There is no general agreement among the makers of high-speed twist drills as to what the cutting speed should be for ordinary shop practice. Some decrease the peripheral speed with the increase of drill diameter, others recommend the reverse, but most manufacturers advise a constant peripheral speed. The average speed is about 60 feet per minute, with a feed per revolution of 0.01 $D^{0.33}$ inch for machine steel, D being the drill diameter. For cast iron, it is usual to decrease this speed about 20 per cent and increase the feed a similar amount. An estimate of the net horsepower required for drilling is given on page 872; this table shows the influence of the speed, feed and drill diameter on the power consumption.

High Speed and Light Feed Recommended. — The remedy for properly ground drills which chip out at the cutting edges is to decrease the feed and increase the speed. Although 50-point carbon steel has been drilled with a 2-inch carbon-steel drill having a peripheral speed of 60 feet per minute and a feed of 0.065 inch per revolution, this is not recommended as good practice. In most cases, better results are obtained with a comparatively light feed and a high speed, increased to the point where the outside corners of the drill begin to wear away. For automatic machines, when the holes do not exceed two drill diameters in depth, and a flood of lard oil is used, high speeds and light feeds are especially recommended. For deeper holes, slower speeds and heavier feeds should be used in order to facilitate getting rid of the chips. Drills for automatic machines should, if possible, be ground so as to sever the chip in a small compact roll.

Drilling Speeds — Carbon Steel Drills

Drill Diam., Inches	Iron, Steel, 30 Feet	Cast Iron, 35 Feet	Brass, 60 Feet	Drill Diam., Inches	Iron, Steel, 30 Feet	Cast Iron, 35 Feet	Brass, 60 Feet
	Revolutions per Minute				Revolutions per Minute		
1/16	1830	2140	3665	1 1/2	75	90	155
1/8	915	1070	1835	1 9/16	75	85	145
3/16	610	715	1220	1 5/8	70	80	140
1/4	460	535	915	1 11/16	70	80	135
5/16	365	430	735	1 3/4	65	75	130
3/8	305	355	610	1 13/16	65	75	125
7/16	260	305	525	1 7/8	60	70	120
1/2	230	265	460	1 15/16	60	70	120
9/16	205	240	405	2	55	65	115
5/8	185	215	365	2 1/16	55	65	110
11/16	165	195	335	2 1/8	55	65	110
3/4	155	180	305	2 3/16	50	60	105
13/16	140	165	280	2 1/4	50	60	100
7/8	130	155	260	2 5/16	50	60	100
15/16	120	145	245	2 3/8	50	55	95
1	115	135	230	2 7/16	45	55	95
1 1/16	110	125	215	2 1/2	45	55	90
1 1/8	100	120	205	2 9/16	45	50	90
1 3/16	95	115	195	2 5/8	45	50	85
1 1/4	90	105	185	2 3/4	40	50	85
1 5/16	85	100	175	2 7/8	40	45	80
1 3/8	85	95	165	3	40	45	75
1 7/16	80	95	160	....	......	......	......

Speeds and Feeds for Drilling*

High-speed Steel Drills

Size of Drill	Feed per Rev.	Bronze, Brass, 300 Feet	Cast Iron, Annealed, 170 Feet	Cast Iron, Hard, 80 Feet	Mild Steel, 120 Feet	Drop Forg., 60 Feet	Mal. Iron, 90 Feet	Tool Steel, 60 Feet	Cast Steel, 40 Feet
Inches	Inches	R.P.M.	R.P.M.	R.P.M.	R.P.M.	R.P.M.	R.P.M.	R.P M.	R.P.M.
1/16	0.003	.....	.....	4880	.....	3660	....	3660	2440
1/8	0.004	.....	5185	2440	3660	1830	2745	1830	1220
3/16	0.005	.....	3456	1626	2440	1210	1830	1220	807
1/4	0.006	4575	2593	1220	1830	915	1375	915	610
5/16	0.007	3660	2074	976	1464	732	1138	732	490
3/8	0.008	3050	1728	813	1220	610	915	610	407
7/16	0.009	2614	1482	698	1046	522	784	522	348
1/2	0 010	2287	1296	610	915	458	636	458	305
5/8	0.011	1830	1037	488	732	366	569	366	245
3/4	0.012	1525	864	407	610	305	458	305	203
7/8	0.013	1307	741	349	523	261	392	261	174
1	0.014	1143	648	305	458	229	349	229	153
1 1/4	0.016	915	519	244	366	183	275	183	122
1 1/2	0.016	762	432	204	305	153	212	153	102
1 3/4	0.016	654	371	175	262	131	196	131	87
2	0.016	571	323	153	229	115	172	115	77

Carbon Steel Drills

Size of Drill	Feed per Rev.	Bronze, Brass, 150 Feet	Cast Iron, Annealed, 85 Feet	Cast Iron, Hard, 40 Feet	Mild Steel, 60 Feet	Drop Forg., 30 Feet	Mal. Iron, 45 Feet	Tool Steel, 30 Feet	Cast Steel, 20 Feet
Inches	Inches	R.P.M.	R.P.M.	R.P.M.	R.P.M.	R.P.M.	R.P.M.	R.P.M.	R.P.M.
1/16	0.003	.....	5185	2440	3660	1830	2745	1830	1220
1/8	0.004	4575	2593	1220	1840	915	1375	915	610
3/16	0.005	3050	1728	813	1220	610	915	610	407
1/4	0.006	2287	1296	610	915	458	636	458	305
5/16	0.007	1830	1037	488	732	366	569	366	245
3/8	0.008	1525	864	407	610	305	458	305	203
7/16	0.009	1307	741	349	523	261	392	261	174
1/2	0.010	1143	648	305	458	229	343	229	153
5/8	0.011	915	519	244	366	183	275	183	122
3/4	0.012	762	432	204	305	153	212	153	102
7/8	0.013	654	371	175	262	131	196	131	87
1	0.014	571	323	153	229	115	172	115	77
1 1/4	0.016	458	260	122	183	92	138	92	61
1 1/2	0.016	381	216	102	153	77	106	77	51
1 3/4	0.016	327	186	88	131	66	98	66	44
2	0.016	286	162	77	115	58	86	58	39

Speeds and Feeds for Drilling — High-speed Steel Drills

Drilling speeds, feeds, cubic inches removed per minute and horsepower required for various drill diameters, no lubricant being used.

Drill Diam., Inches	Revolutions per Minute	Feed per Revolution	Cu. Ins. Removed per Minute	Cutting Horse-power	Feeding Horse-power	Total Horse-power	Horse-power per Cu. In. Removed
Cast Iron							
0.25	735.0	0.0075	0.270	0.29	0.0050	0.2950	1.092
0.375	490.0	0.0086	0.462	0.435	0.0055	0.4405	0.954
0.50	368.0	0.0094	0.682	0.58	0.0059	0.5860	0.862
0.75	245.0	0.0109	1.170	0.87	0.0066	0.8766	0.748
1.00	184.0	0.0119	1.715	1.16	0.0070	1.1670	0.681
1.25	147.0	0.0129	2.320	1.45	0.0073	1.4570	0.628
1.50	122.0	0.0136	2.920	1.74	0.0078	1.7480	0.598
1.75	105.0	0.0144	3.630	2.03	0.0081	2.0380	0.563
2.00	92.0	0.0150	4.320	2.32	0.0084	2.3280	0.539
2.25	81.7	0.0156	5.050	2.61	0.0086	2.6190	0.519
2.50	73.5	0.0162	5.820	2.90	0.0089	2.9090	0.500
2.75	66.75	0.0167	6.600	3.19	0.0091	3.1990	0.486
3.00	61.3	0.0172	7.400	3.48	0.0093	3.4890	0.472
3.25	56.5	0.0176	8.220	3.77	0.0095	3.7800	0.460
3.50	52.5	0.0181	9.050	4.06	0.0096	4.0700	0.450
3.75	49.0	0.0185	10.000	4.35	0.0098	4.3600	0.436
4.00	46.0	0.0190	10.800	4.64	0.00995	4.6500	0.431
Steel — Medium Hard							
0.25	920.0	0.0063	0.284	0.712	0.0092	0.721	2.540
0.375	614.0	0.0072	0.485	1.068	0.0102	1.078	2.220
0.50	460.0	0.0079	0.716	1.425	0.0109	1.426	1.990
0.75	306.0	0.0091	1.230	2.140	0.0121	2.152	1.750
1.00	230.0	0.0100	1.800	2.850	0.0130	2.863	1.590
1.25	184.0	0.0108	2.440	3.560	0.0138	3.574	1.470
1.50	153.0	0.0114	3.080	4.270	0.0145	4.285	1.390
1.75	131.0	0.0121	3.810	4.990	0.0150	5.005	1.310
2.00	115.0	0.0126	4.540	5.700	0.0155	5.715	1.260
2.25	102.0	0.0131	5.300	6.420	0.0159	6.436	1.210
2.50	92.0	0.0136	6.120	7.120	0.0163	7.136	1.165
2.75	83.5	0.0140	6.920	7.840	0.0167	7.857	1.135
3.00	76.5	0.0144	7.760	8.550	0.0171	8.567	1.105
3.25	70.5	0.0148	8.660	9.250	0.0175	9.267	1.070
3.50	65.6	0.0151	9.500	9.980	0.0178	9.998	1.050
3.75	61.25	0.0155	10.480	10.700	0.0181	10.718	1.024
4.00	57.5	0.0158	11.400	11.400	0.0184	11.420	1.000

Drilling Speeds and Feeds for Different Materials. — The Cleveland Twist Drill Co. recommends the following speeds and feeds as a guide, until more definite data can be obtained in actual practice: Start at a moderate speed and feed. For carbon-steel twist drills, when drilling soft tool or machine steel, use a peripheral speed of 30 feet per minute; for cast iron, 35 feet; for brass, 60 feet; use a feed of from 0.004 to 0.007 inch per revolution for drills ½ inch and smaller, and from 0.005 to 0.015 inch per revolution for drills larger than ½ inch. A comparatively heavy feed should be used in drilling brass, especially in automatic machines, to insure the chips working out. High speeds in cast iron tend to wear away the "land" of the drill, and 35 feet per minute peripheral speed should not be exceeded for carbon-steel drills. In the case of high-speed drills, the foregoing feeds should remain unchanged, but the speeds should be increased from two to two and one-half times. With these speeds and feeds as a starting point, the maximum results should be obtained by noting the condition of the drill in connection with the following suggestions:

If the drill chips out at the cutting edges, the feed is too great or the drill has been ground with an excessive lip clearance. A drill split up along the web is evidence of too heavy feed or of improper grinding. Insufficient lip clearance at the center of the drill will almost always cause it to split up along the web. When the extreme outer corners of the cutting edges wear away too rapidly, it is evidence of excessive speed. The best performance of the drill may be obtained when the effect of the work on its cutting end is somewhere between these two extremes.

The Standard Tool Co. recommends the following speeds and feeds: Start high-speed steel drills at peripheral speeds of from 50 to 70 feet per minute, for wrought iron and steel; from 60 to 80 feet per minute, for cast iron; and 140 feet per minute, for brass. After a few holes have been drilled, these speeds may be increased as much as conditions will permit, better results being obtained by allowing the drills to reach the maximum speed gradually. The feeds recommended are as follows: For wrought iron and steel, 0.004 inch per revolution for 1/16 inch drill; 0.005 inch for ¼ inch drill; 0.008 inch for ½ inch drill; 0.010 inch for 1 inch drill; 0.015 inch for 2 inch drill.

Speeds for Tapping. — The speeds for tapping cast iron, tough alloy steels, and drop-forgings usually vary from 18 to 20 feet per minute. The softer grades of steel, such as Bessemer, open-hearth, and screw stock may be tapped at speeds of 25 or 30 feet per minute under suitable conditions. Owing to the numerous alloys designated as brass, it is difficult to give speed data for them, but, in general, the speeds for tapping brass may be several times as fast as those for cast iron or steel. The speeds for taps are usually somewhat less than for dies, as the former do not cut as freely nor discharge chips as readily; cooling compounds may also be applied to dies more effectively.

Speeds and Feeds for Thread Milling. — When milling screw threads, the surface speed of the blank may not exceed 2 or 3 inches per minute if the thread is of coarse pitch, and especially if the material is tough, whereas, when the pitches are finer and the material softer, the surface speed may be increased to 6 or 8 inches per minute for steel, and be two or three times faster for softer materials. The speed of the milling cutter usually varies from 100 to 125 feet per minute, with slower and faster speeds according to conditions. For instance, a speed of 100 feet per minute might be satisfactory for machine steel, whereas for tool steel, the speed might be reduced to about 70 feet per minute. The design of the machine and the general type may affect the speeds and feeds somewhat, and the steel used for the cutters is another important factor.

Speeds and Feeds For Milling, Drilling and Turning *

Material to be Machined. For materials marked (†) use a cooling compound. For those marked (‡) use lard oil or similar coolant.	Speeds in Feet per Minute at Largest Diameter — Figures given in Body of Table															
	Milling						Drilling		Turning							
	Carbon Steel Cutter		High-speed Steel Cutter		Stellite No. 3		Carbon Steel Drill	High-speed Steel	Carbon Steel Tool		High-speed Steel Tool		Stellite Tools			
	Rough.	Fin.	Rough.	Fin.	Rough.	Fin.			Rough.	Fin.	Rough.	Fin.	Stell. No.	Rough.	Fin.
Cast Iron — Malleable	60	75	90	100	135	170	50	70	90	100	130	150	2	200	240
Cast Iron — Machine Castings	40	50	60	70	90	110	35	50	60	75	90	110	2	135	160
Cast Iron — Med. Hard Castings	20	25	30	40	45	55	16	25	30	35	45	50	3	70	85
Cast Iron — Hard Castings	10	12	15	20	22	27	8	12	15	18	22	25	3	35	42
Steel Castings †	30	35	45	55	65	80	25	35	45	55	65	80	2	100	120
Steel up to 0.30 % C. — Soft. Mach. St. †	60	75	90	110	135	170	50	70	90	110	130	160	1	200	240
Steel up to 0.30 % C. — Medium †	40	50	60	70	90	100	35	50	60	70	90	100	1	135	160
Steel up to 0.30 % C. — Fairly Hard †	25	35	40	50	55	80	20	30	35	45	50	65	1	80	95
Tool Steel, well annealed ‡	25	35	40	50	55	80	20	30	35	45	50	65	2	80	95
Cr. Ni. Steel, annealed ‡	25	35	40	50	55	80	20	30	35	45	50	65	2	80	95
Brass — Yellow	75	95	110	150	170	200	60	90	110	150	160	220	3	250	300
Bronze	40	50	60	70	90	100	35	50	60	70	90	100	2	135	160
Aluminum	460	550	700	900	1000	1200	350	550	700	900	1000	1300	3	1500	1800
Feeds	The feed for rough-milling should be all that the machine, work, fixture and tool will withstand — the feed for roughing is usually too low. The feeds for finishing are, with spiral mills, about 0.005 inch per tooth per revolution; with face mills, about 0.010 inch per tooth per revolution. The drilling feeds per revolution are, for 1/8 inch diameter, 0.002 inch; 1/4 inch diameter, 0.006 inch; 1/2 inch diameter, 0.007 inch; 3/4 inch diameter, 0.010 inch. The feeds for rough-turning should be all that machine and work will withstand. Finishing feed depends upon finish required and kind of material and tool.															

* These speeds and feeds were compiled by the Kempsmith Mfg. Co. as representing conservative, modern practice. They are intended only as a general guide, and when all conditions are favorable, may often be increased considerably; or, in the case of hard materials, they must sometimes be less than those given. The speeds for tools of simple form which are easy to grind and replace, such as lathe tools, etc., should be much higher than the speeds for tools, such as milling cutters, which are more difficult to grind and must last longer between the sharpening periods to secure greater economy. The speeds should be reduced for tools used in machines of the automatic class, because such tools, as a rule, require more time for resetting in the working position.

Cutting Speeds and Feeds for Turret Lathes. — The accompanying table "Average Cutting Speeds for Turret Lathes" and the following data on feeds were obtained from the Warner & Swasey Co. These figures are intended only as a general guide, since both speeds and feeds depend upon variable factors which should be considered. Thus the speeds listed can often be increased 50 per cent when conditions are favorable. The speeds in the table are for high-speed steel cutters operating on material that is clean and free from sand or scale. For carbon steel cutters the speeds should be decreased about 50 per cent.

The feeds of turret lathe tools are commonly specified by "feeds per inch," or the number of revolutions of spindle to one-inch travel of the tool. The following feeds are expressed in "feeds per inch" followed by the decimal equivalent or "feed per revolution."

The turning feeds generally used on turret lathes for machine steel or tool steel, range from 125 (0.008 inch per revolution) to 31 (0.032 inch) depending on the grade of finish wanted and the depth of cut. When a smooth finish is desired, or on light machines, feeds from 125 (0.008 inch) to 66 (0.015 inch) are generally used. When the turned work is to be finished afterward by grinding, turning feeds from

Average Cutting Speeds for Turret Lathes

Material	Surface Speed, Feet per Minute	Material	Surface Speed, Feet per Minute
Soft cast iron roughing...	50 to 60	Soft machine steel.........	80 to 100
Soft cast iron finishing...	60 to 80	Medium hard machine steel	60 to 80
Hard cast iron roughing .	35 to 50	Hard machine steel........	40 to 60
Hard cast iron finishing..	60 to 80	Tool steel annealed........	60 to 80
Malleable cast iron......	80 to 90	Tool steel unannealed......	25 to 35
Steel casting............	50 to 60	Alloy steel annealed........	50 to 60
Brass.................	150 to 250	Alloy steel treated.........	30 to 40
Bronze................	100 to 150	Cutting threads on brass. . .	60 to 150
Hard bronze............	80 to 100	Cutting threads on steel and cast iron................	25 to 40
Copper................	150 to 200		
Aluminum.............	250 to 400		

66 (0.015 inch) to 31 (0.032 inch) can be used. For light cuts these feeds can often be increased. For forming and cutting off, fine feeds are used, ranging from 1000 (0.001 inch) to 100 (0.010 inch) depending upon the width of cut. For roughing cast iron, feeds from 31 (0.032 inch) to 16 (0.062 inch) can be used. For machining brass, coarse feeds are used, ranging from 20 (0.050 inch) to 4 (0.250 inch). In order to obtain very coarse feeds on brass, hand feed is used.

The following feed data from the Foster Machine Co. is given as representing a good average for a turret lathe like the No. 5 size, which has a 16-inch swing over the bed. The feed per revolution for box tools, when turning machine steel, varies from about 0.017 to 0.020 inch; when turning ordinary yellow brass, from 0.017 to 0.40 inch; when turning annealed tool steel, from 0.010 to 0.020 inch. The feed per revolution for drills when drilling machine steel varies from 0.007 to 0.012 inch; for ordinary yellow brass from 0.007 to 0.015 inch; for annealed tool steel from 0.007 to 0.012 inch. The feed per revolution for reamers when reaming machine steel, varies from 0.040 to 0.070 inch; for ordinary yellow brass from 0.070 to 0.125 inch; for annealed tool steel from 0.032 to 0.062 inch.

The Gisholt Machine Co. gives the following average figures for Gisholt turret lathes ranging from 13- to 34-inch sizes: Feed per revolution for box tools turning machine steel or yellow brass, about 1/32 inch; feed per revolution for reamers reaming machine steel, 1/16 inch; reaming yellow brass, 1/8 inch; reaming annealed tool steel, 1/32 inch.

Cutting Speeds for Threading Dies. — When conducting tests to determine the most economical speed for threading dies, it is preferable to begin with a relatively slow speed and increase it as much as the die will stand without excessive dulling. As dies are not sharpened so easily as some other forms of cutting tools, this should be considered when determining the speed. The following speeds from the National Acme Co. are based upon considerable experience in screw cutting. A die equipped with carbon-steel chasers should, under normal conditions, have a cutting speed of 30 feet per minute (which represents the surface speed at the pitch circumference) when threading Bessemer and open-hearth steel, cold-rolled screw stock, 3- to 5-per cent nickel steel, malleable iron, brass, bronze, and similar alloys. When using high-speed or semi-high-speed steel chasers, a cutting speed of 20 feet per minute is recommended for threading chrome-vanadium, tough alloy steels, cast iron, drop-forgings, and all heat-treated steels. These speeds, given both for carbon and high-speed steel chasers, may be increased about 20 per cent for cutting threads of fine pitch.

Speeds for Thread-cutting Dies, Based on Amount of Metal Removed per Minute

Diam. Screw Thread	Threads per Inch	Rev. per Min.	Surface Speed, Outside Diam. Ft. per Min.	Cu. In. Removed per Min.	Diam. Screw Thread	Threads per Inch	Rev. per Min.	Surface Speed, Outside Diam. Ft. per Min.	Cu. In. Removed per Min.
1/4	20	1062.0	69.5	0.5896	11/16	11	117.0	21.0	0.6201
5/16	18	691.5	56.5	0.6004	3/4	10	88.5	17.0	0.6177
3/8	16	453.0	44.5	0.6025	13/16	10	81.7	17.0	0.6225
7/16	14	297.5	34.0	0.6009	7/8	9	61.7	14.0	0.6170
1/2	13	224.5	29.0	0.6084	15/16	9	57.6	14.0	0.6276
9/16	12	169.5	25.0	0.6085	1	8	40.6	10.6	0.5948
5/8	11	128.7	21.0	0.6147	....	....	.....	.....	.....

Cutting speeds have been determined on the basis that the die should remove the same amount of metal in a given time for all pitches. The accompanying table is based on the removal of about 0.6 cubic inch of metal per minute and represents actual practice in cutting threads on screw stock with self-opening dies, at the plant of the Greenfield Tap & Die Corporation. As will be seen, the speeds vary approximately from 10 to 30 feet per minute for screw thread diameters, ranging from 1 down to 1/2 inch, and then increase rapidly for the smaller sizes.

Hacksaw Speeds. — The following hacksaw speeds are given on the authority of a leading manufacturer: The total amount of travel of the cutting blade in feet per minute, including forward and return strokes, should be as follows: For mild steel, 130; for annealed tool steel, 90; and for unannealed tool steel, 60 feet per minute. Thus in the case of a 6-inch stroke, for example, the revolutions per minute of the driving crank should be 130 for mild steel, 90 for annealed tool steel, and 60 for unannealed tool steel. All of these steels are cut with the use of a cutting compound. Bronze can ordinarily be cut at the same speed as mild steel when a suitable compound is used. Brass heats the blade very rapidly if cut dry, and must be cut with a cooling compound adapted to brass. It also fills up the teeth of the saw if not used with the right kind of compound, but with a suitable compound, it may be cut at the same speed as machine steel.

Speeds for Tapping Nuts

Peripheral Speed, 10 Feet per Minute				Peripheral Speed, 15 Feet per Minute				Peripheral Speed, 20 Feet per Minute			
Tap Diam.	Rev. per Min.	Tap Diam.	Rev. per Min.	Tap Diam.	Rev. per Min.	Tap Diam.	Rev. per Min.	Tap Diam.	Rev. per Min.	Tap Diam.	Rev. per Min.
⅛	Faster than 10 ft. per minute	1½	25	⅛	460	1½	38	⅛	612	1½	Slower than 20 ft. per minute
¼		1⅝	23	¼	230	1⅝	35	¼	306	1⅝	
5/16		1¾	22	5/16	188	1¾	32	5/16	244	1¾	
⅜		1⅞	20	⅜	153	1⅞	30	⅜	204	1⅞	
7/16		2	19	7/16	131	2	28	7/16	176	2	
½		2¼	17	½	115	2¼	25	½	153	2¼	
9/16		2½	15	9/16	102	2½	22	9/16	136	2½	
⅝		2¾	14	⅝	93	2¾	20	⅝	122	2¾	
¾		3	12	¾	75	3	18	¾	102	3	
⅞		3¼	11	⅞	65	...	...	⅞	88	3¼	
1	38	3½	10	1	55	...	...	1	76	3½	
1⅛	34	3¾	9	1⅛	50	...	...	1⅛	...	3¾	
1¼	30	4	8	1¼	45	...	...	1¼	...	4	
1⅜	28	...	...	1⅜	40	...	...	1⅜	...	...	

Planing Speeds. — The speeds for planing usually vary from 30 to 50 feet per minute on the cutting stroke, with a return speed three to four times as great. A general idea of planer speeds may be obtained from the following figures, given by the Cincinnati Planer Co., and representing the practice in some of the best machine shops: Cast iron, roughing, 40 to 50 feet per minute; cast iron, finishing, 20 to 25 feet per minute; steel castings, roughing, 30 to 35 feet per minute; wrought iron, roughing, 30 to 45 feet per minute; steel castings, finishing, 20 feet per minute; wrought iron, finishing, 20 feet per minute; bronze and brass, 50 to 60 feet per minute; machinery steel, 30 to 35 feet per minute. When high-speed steel tools are used, a speed of 55 feet per minute is given as about the maximum that can ordinarily be used to advantage. The net or actual cutting speeds for various combinations of forward and return speeds are given in the table, "Actual Cutting Speeds of Planers." The upper half of this table shows how many feet per minute the planer actually cuts, and the lower half gives the same data in feet per hour. A slight increase in the forward speed has a much greater effect on the net cutting speed and rate of production than a comparatively high increase of the return speed. To illustrate, when the cutting speed is 30 feet per minute and the return speed, 90 feet per minute, a planer tool actually cuts 22.5 feet per minute. If the return speed is increased to 150 feet per minute, the net cutting speed is increased to 25 feet — a gain of 2.5 feet for a return-speed increase of approximately 66 per cent. If the cutting speed is increased to 35 feet (the return speed remaining at 90), the net cutting speed is increased to approximately 25 feet per minute, as before, but with a cutting speed increase of only about 16 per cent. Hence, it is important to have the cutting speed as high as conditions will permit.

Feeds for Planing. — The feed of a planing tool varies widely for different kinds of material and classes of work; it is also governed by the depth of cut, the nature of the cut (whether roughing or finishing), and by the rigidity of the work when clamped in position for planing. Feeds ordinarily vary from 1/16 to ⅛ inch for rough-planing steel, and from ⅛ to 3/16 inch for roughing cast iron. When taking light finishing cuts in cast iron, a broad tool having a flat edge is commonly used and the feed ordinarily varies from ¼ to ½ inch per stroke. When planing large rigid castings, a feed as coarse as ¾ or 1 inch per stroke is often employed.

Actual Cutting Speeds of Planers

Cutting Speed, Feet per Minute	Return Speed, Feet per Minute							
	50	60	70	80	90	100	120	150
	Actual Number of Feet Traversed on Cutting Strokes per Minute							
20	14.3	15.0	15.5	16.0	16.4	16.7	17.1	17.6
25	16.7	17.6	18.4	19.0	19.6	20.0	20.7	21.4
30	18.7	20.0	21.0	21.8	22.5	23.1	24.0	25.0
35	20.6	22.0	23.3	24.3	25.2	25.9	27.1	28.4
40	22.2	24.0	25.4	26.7	27.7	28.6	30.0	31.6
45	23.7	25.7	27.4	28.8	30.0	31.0	31.1	34.6
50	25.0	27.3	29.2	30.8	32.1	33.3	35.3	37.5
	Actual Number of Feet Traversed on Cutting Strokes per Hour							
20	857	900	933	960	981	1000	1028	1058
25	1000	1058	1105	1142	1173	1200	1241	1285
30	1125	1200	1260	1309	1350	1384	1440	1500
35	1235	1321	1400	1460	1512	1555	1625	1702
40	1333	1440	1527	1600	1661	1714	1800	1894
45	1421	1542	1643	1728	1800	1862	1863	2076
50	1500	1636	1750	1846	1928	2000	2117	2250

Speeds and Feeds for Gear Cutting. — The variations in speed and feed, owing to limiting conditions, are probably greater for gear cutting than for most other machining operations. The important factors governing the speed and feed, aside from the design and condition of the machine, are the pitch of the gear to be cut, the hardness of the metal, the kind of steel used for making the cutter, the form and condition of the cutter, the rigidity of the gear blank when mounted for cutting, the accuracy and finish required for the gear, and the cutting lubricant used in the case of steel gears. The following speeds are given by Gould & Eberhardt as a fair average for cutting medium grades of cast iron and steel, when using carbon and high-speed steel cutters:

Material	Carbon Steel Cutters			High-speed Steel Cutters		
	Min.	Average	Max.	Min.	Average	Max.
Cast Iron, Feet per Minute	35	45	60	60	70	80
Steel, Feet per Minute	25	30	40	45	50	55

While these figures are based upon gear-cutting practice, they are merely intended to serve as a guide until the most economical speed for a given case can be determined by actual tests. More specific data on speeds and feeds are given in the tables "Speeds and Feeds for Gear Cutting." These tables apply to machines of the rotary-cutter type and cover diametral pitches varying from 1 to 24. The feeds and speeds listed are for cast iron and steel gears of medium grade, with suitable variations for carbon or high-speed steel cutters. The first column gives the maximum capacity of the machine, in diametral pitch; it will be noted that the values vary somewhat for a gear of given pitch, for machines of different capacity. For example, when a cast-iron gear of 6 diametral pitch is cut on a machine having a capacity of 1¾ diametral pitch, the feed per minute is given as 8.5 inches; but when the same pitch is cut on a larger machine of 1 diametral pitch capacity, the feed can be increased to 11 inches per minute, high-speed steel cutters being used in each case.

Feeds for Given Pitch and Material. — The tables "Feeds for Gear Cutting" contain practical data on the feeds to use for different materials and pitches. Table 1 is for carbon-steel cutters and Table 2 for high-speed steel cutters. The feeds listed vary according to diametral pitch, kind of material and nature of cut. The shearing-tooth cutter for roughing, referred to in the tables, is an ordinary gear cutter having faces of alternate teeth ground at an angle of 11 degrees, so that the teeth have rake and cut similar to a side turning tool. The following conditions which affect the time required for gear cutting are given by the Cincinnati Gear Cutting Machine Company:

1. The material of which the cutter is made: This may be carbon steel, poor high-speed steel, or good high-speed steel. The feed must be varied according to the speed the cutter will stand.
2. The material in the gears to be cut: This may be hard or soft cast iron, low-carbon or high-carbon steel, nickel-steel alloy, etc.
3. The shape of the gear: It may be light in design and large in diameter, so that high speeds and feeds cannot be taken, or the design may be such that heavy feeds and fast speeds can be used, even though the material is comparatively hard.
4. Accuracy of finish: The gears to be cut may not require the refinement and accuracy that would necessitate two cuts, one for roughing and one for finishing.
5. The type of cutter: It may not be possible to take heavy cuts, either because of the shape of the cutter and lack of strength in the teeth, or because the latter have not the proper angle to enable the chips to free themselves.
6. Quality of lubricant: The speed and feed may be affected by the kind of lubricant used; that is, whether a good lard oil, a fair grade of oil cutting compound or simply a water cutting compound is used.
7. The gib adjustment and rigidity of the machine may make a great difference in the amount of feed.

In the feed tables, the figures are based upon the assumption that the machine gibs are properly adjusted, that the belts are in good condition, and that lard oil is used as a lubricant when cutting steel gears.

Number of Cuts required. — Cast-iron gears up to 2½ diametral pitch can usually be finished in one cut with a rotary formed cutter, provided the machine used is rated for this pitch. Cast-iron gears coarser than 2 diametrai pitch should, as a rule, have the teeth roughed or "blocked out" prior to finishing, although the necessity for taking two cuts depends, to some extent, upon the accuracy and finish required. Steel or hard bronze gears of 4 diametral pitch, and coarser, should also be blocked out and then be finished by a second cut. The stepped type of stocking cutter is very efficient for roughing, as it breaks up the chips and makes possible faster speeds and feeds. To secure best results, gear cutters must be kept well sharpened. When the cutter becomes dull, it not only is very inefficient, but tends to crowd to one side, thus springing the gear and producing thick and thin teeth. When cutting large gears, it is very important to support the rim against the thrust of the cut, and prevent springing.

Speeds and Feeds for Cutting Bevel Gears

The data given below are especially applicable to No. 13 B. & S. Gear Cutting Machine

Diametral Pitch	Material*	Steel for Cutter*	Diameter of Cutter	Speed, R.P.M.	Feed, Inches per Min.	Diametral Pitch	Material*	Steel for Cutter*	Diameter of Cutter	Speed, R.P.M.	Feed, Inches per Min.
6	C. I.	H. S.	2¾	105	5½	9	ST.	H. S.	2⅜	130	7
6	ST.	H. S.	2¾	105	5½	9	ST.	H. S.	2⅜	165	12
7	C. I.	H. S.	2⅝	130	9	10	C. I.	H. S.	2⅜	165	9
7	C. I.	C. S.	2⅝	105	5½	10	M. S.	H. S.	2⅜	130	5½
7	M. S.	H. S.	2⅝	105	5½	10	ST.	H. S.	2⅜	130	7
7	ST.	H. S.	2⅝	130	5½	12	C. I.	H. S.	2	165	12
8	C. I.	H. S.	2½	165	12	12	ST.	H. S.	2	165	9
8	ST.	H. S.	2½	130	5½	...	....	....	...	...	...

* Meaning of abbreviations: C. I. (cast iron); ST. (steel); M. S. (machine steel); H. S. (high-speed steel); C. S. (carbon steel).

Feeds for Hobbing. — The feeds for hobbing usually vary from about 0.020 to 0.200 inch per revolution of the gear. Gould & Eberhardt recommend the following feeds for hobbing spur gears. The figures give the movement of the hob per revolution of the gear.

Material	Feed, Inches	Material	Feed, Inches	Material	Feed, Inches
Tool Steel......	0.010	Soft Steel.......	0.040	Roughing Soft Steel...........	0.080
Tough Steel....	0.020	Average Cast Iron..........	0.050	Roughing Average Cast Iron...	0.100
Average Steel...	0.030	Soft Cast Iron ..	0.060	Roughing Soft Cast Iron.......	0.150

When hobbing helical gears, the feed depends somewhat upon the angle of the teeth. For angles varying from 0 to 36 degrees the feed should be about the same as for spur gears; for angles from 36 to 48 degrees, the feed should equal ⅘ of that for spur gears; from 48 to 60 degrees, ⅔ of the spur gear feed; from 60 to 70 degrees, ½ of the spur gear feed; for angles above 70 degrees, ⅓ of the spur gear feed. The feed is diminished in the case of spiral gears, because when the hob is set to conform to the helix angle, the feed of the cutter, per revolution, is increased by the turning movement of the gear blank necessary for generating the helical teeth.

Feeds for Gear Cutting — 1

Compiled by the Cincinnati Gear Cutting Machine Co. The gears are assumed to be of substantial design, and the belts on the machine to be in good condition. For steel gears, a lard oil lubricant is used. For teeth coarser than 1¾ diametral pitch, feeds are not given for finishing in one cut, because satisfactory results cannot be obtained with a single cut on the No. 7 machine.

For *Carbon-steel Cutters* with a peripheral speed on cast iron of 35 feet per minute, and on steel of 30 feet per minute.

Diametral Pitch of Gear	Cast-iron Gears	Soft Steel Gears	High-carbon Steel	Nickel-steel Alloy	Cast-iron Gears	Soft Steel Gears	High-carbon Steel	Nickel-steel Alloy	Arrows Indicate Range of Different Sizes of Machines
	Feed, Inches per Min., when Finishing in One Cut				Feed, Inches per Min. for Roughing Cut, leaving 0.010 to 0.030 Inch for Finishing				
1	.....	.....	.....	.....	1 5/8	1 1/4	1	1	
1¼	.....	.....	.....	.....	1 5/8	1 1/4	1	1	
1½	.....	.....	.....	.....	2 1/16	1 5/8	1 1/4	1	
1¾	2 1/16	1 11/16	1 5/16	1	3 3/8	2 11/16	2 1/16	1 11/16	
2	2 1/2	2	1 11/16	1 1/4	4 1/16	3 1/8	2 1/2	2	
2½	2 1/2	2	1 11/16	1 1/4	4 1/16	3 1/8	2 1/2	2	
3	3 7/16	2 1/2	1 5/8	1 1/4	4 1/16	3 7/16	2 1/2	2	
4	3 7/16	2 1/2	2	1 5/8	4 1/4	3 7/16	2 1/2	2	
5	3 7/16	2 1/2	2	1 5/8	4 1/4	3 7/16	2 1/2	2	
6	3 7/16	2 1/2	2	1 5/8	4 1/4	3 7/16	2 1/2	2	No. 3 No. 4 No. 5 No. 6 No. 7
7	3 7/16	2 1/2	2	1 5/8	4 1/4	3 7/16	2 1/2	2	
8	3 7/16	2 1/2	2	1 5/8	5 7/16	4 1/4	3 7/16	2 1/2	
9	4 1/4	3 7/16	2 1/2	2	5 7/16	4 1/4	3 7/16	2 1/2	
10	4 1/4	3 7/16	2 1/2	2	5 7/16	4 1/4	3 7/16	2 1/2	
	When using Shearing Tooth Cutter for Roughing Cut				Finishing Cut				
1	2 1/16	1 5/8	1 1/4	1 1/4	2 1/16	1 5/8	1 5/8	1 1/4	
1¼	2 1/16	1 5/8	1 1/4	1 1/4	2 1/16	1 5/8	1 5/8	1 1/4	
1½	2 11/16	2 11/16	1 5/8	1 1/4	2 11/16	2 1/16	2 1/16	1 5/8	
1¾	4 3/8	4 3/8	3 3/8	2 11/16	4 3/8	3 3/8	2 11/16	2 1/16	
2	5 1/8	4 1/16	3 1/8	2 1/2	4 1/16	3 1/8	2 1/2	2	
2½	5 1/8	4 1/16	3 1/8	2 1/2	4 1/16	3 1/8	2 1/2	2	
3	5 7/16	4 1/4	3 7/16	2 1/2	4 1/4	3 7/16	2 1/2	2	
4	5 7/16	4 1/4	3 7/16	2 1/2	4 1/4	3 7/16	2 1/2	2	
5	5 7/16	4 1/4	3 7/16	2 1/2	4 1/4	3 7/16	2 1/2	2	
6	5 7/16	4 1/4	3 7/16	2 1/2	4 1/4	3 7/16	2 1/2	2	No. 3 No. 4 No. 5 No. 6 No. 7
7	5 7/16	4 1/4	3 7/16	2 1/2	4 1/4	3 7/16	2 1/2	2	
8	6 13/16	5 7/16	4 1/4	3 7/16	5 7/16	4 1/4	3 7/16	2 1/2	
9	6 13/16	5 7/16	4 1/4	3 7/16	5 7/16	4 1/4	3 7/16	2 1/2	
10	6 13/16	5 7/16	4 1/4	3 7/16	5 7/16	4 1/4	3 7/16	2 1/2	

Feeds for Gear Cutting — 2

Compiled by the Cincinnati Gear Cutting Machine Co. The gears are assumed to be of substantial design, and the belts on the machine to be in good condition. For steel gears, a lard oil lubricant is used. For teeth coarser than 1¾ diametral pitch, feeds are not given for finishing in one cut, because satisfactory results cannot be obtained with a single cut on the No. 7 machine.

For *High-speed Steel Cutters* with a peripheral speed on cast iron of 55 feet per minute and 80 feet per minute on steel.

Diametral Pitch of Gear	Cast-iron Gears	Soft Steel Gears	High-carbon Steel	Nickel-steel Alloy	Cast-iron Gears	Soft Steel Gears	High-carbon Steel	Nickel-steel Alloy	Arrows Indicate Range of Different Sizes of Machines
	Feed, Inches per Min., when Finishing in One Cut				Feed, Inches per Min. for Roughing Cut, leaving 0.010 to 0.030 Inch for Finishing				
1	.....	.....	.....	.....	2	1⅝	1⅝	1¼	
1¼	.....	.....	.....	.....	2½	2	2	1⅝	
1½	.....	.....	.....	.....	3⅜	2 1/16	2 1/16	2 1/16	
1¾	2 11/16	2 1/16	2 1/16	1 11/16	4⅜	3⅜	3⅜	2 11/16	
2	3⅛	2½	2½	1 15/16	5⅛	4	4	3⅛	
2½	3⅛	2½	2½	1 15/16	5⅛	4	4	3⅛	
3	3 7/16	2½	2½	2	5 7/16	4¼	4¼	3 7/16	
4	3 7/16	2½	2½	2	5 7/16	4¼	4¼	3 7/16	
5	3 7/16	2½	2½	2	5 7/16	4¼	4¼	3 7/16	
6	4¼	3 7/16	3 7/16	2½	5 7/16	4¼	4¼	3 7/16	No. 3 No. 4 No. 5 No. 6 No. 7
7	4¼	3 7/16	3 7/16	2½	5 7/16	4¼	4¼	3 7/16	
8	4¼	3 7/16	3 7/16	2½	5 7/16	4¼	4¼	3 7/16	
9	5 7/16	4¼	4¼	3 7/16	6 13/16	5 7/16	5 7/16	4¼	
10	5 7/16	4¼	4¼	3 7/16	6 13/16	5 7/16	5 7/16	4¼	
	When using Shearing Tooth Cutter for Roughing Cut				Finishing Cut				
1	2½	2	2	1⅝	2 11/16	2 11/16	2 11/16	2 1/16	
1¼	3 5/16	2½	2½	2	3 5/16	3 5/16	3 5/16	2 11/16	
1½	4⅜	4⅜	3⅜	3⅜	4⅜	4⅜	4⅜	3⅜	
1¾	5 9/16	5 9/16	4⅜	4 1/16	5 9/16	5 9/16	5 9/16	4⅜	
2	6 9/16	6 9/16	5⅛	4 1/16	6 9/16	6 9/16	6 9/16	5⅛	
2½	6 9/16	6 9/16	5⅛	4 1/16	6 9/16	6 9/16	6 9/16	5⅛	
3	8½	8½	6 13/16	5 7/16	6 13/16	6 13/16	6 13/16	5 7/16	
4	8½	8½	6 13/16	5 7/16	6 13/16	6 13/16	6 13/16	6 13/16	
5	8½	8½	6 13/16	5 7/16	6 13/16	6 13/16	6 13/16	6 13/16	
6	8½	8½	6 13/16	5 7/16	6 13/16	6 13/16	6 13/16	6 13/16	No. 3 No. 4 No. 5 No. 6 No. 7
7	8½	8½	6 13/16	5 7/16	6 13/16	6 13/16	6 13/16	6 13/16	
8	10⅝	10⅝	8½	8½	6 13/16	6 13/16	6 13/16	6 13/16	
9	10⅝	10⅝	8½	8½	8½	8½	8½	8½	
10	10⅝	10⅝	8½	8½	8½	8½	8½	8½	

Speeds and Feeds for Gear Cutting — 1*

Capacity of Machine	Diam. Pitch	Cutter: Diam.	Cutter: No. of Teeth	R.P.M.: Cast Iron	R.P.M.: Steel	Cutting Speed, Feet per Min.: Cast Iron	Cutting Speed, Feet per Min.: Steel	Feed per Min.: Cast Iron	Feed per Min.: Steel	No. of Cuts: Cast Iron	No. of Cuts: Steel
	Carbon Steel Cutters										
1 Diametral Pitch for Cast Iron; 1 1/4 D. P. for Steel.	1.00	8.00	13	18.5	15	39	32	2.3	1.9	2	2
	1.25	7.25	14	18.5	15	35	29	2.8	1.9	2	2
	1.50	7.25	14	18.5	15	35	29	2.8	1.9	2	2
	1.75	6.25	10	22.5	18.5	37	30.75	2.8	1.9	2	2
	2.00	6.25	11	22.5	18.5	37	30.75	2.8	2.3	2	2
	2.25	6.25	11	22.5	18.5	37	30.75	2.8	2.3	1	2
	2.50	6.25	14	22.5	18.5	37	30.75	3.5	2.3	1	2
	2.75	5.25	12	27.5	22.5	38	31.5	4.5	2.8	1	2
	3.00	5.25	12	27.5	22.5	38	31.5	4.5	2.8	1	2
	4.00	5.25	16	27.5	22.5	38	31.5	5.6	3.5	1	2
	5.00	5.25	18	27.5	22.5	38	31.5	6.9	4.5	1	2
	6.00	4.25	16	33.5	27.5	37	31	6.9	4.5	1	2
	High-speed Steel Cutters										
	1.00	8.00	13	27.5	22.5	58	47.5	3.5	2.8	2	2
	1.25	7.25	14	33.5	22.5	64	43.5	4.5	2.8	2	2
	1.50	7.25	14	33.5	22.5	64	43.5	4.5	2.8	2	2
	1.75	6.25	10	40.5	27.5	67	45.5	4.5	2.8	2	2
	2.00	6.25	11	40.5	27.5	67	45.5	4.5	3.5	2	2
	2.25	6.25	11	40.5	27.5	67	45.5	4.5	3.5	1	2
	2.50	6.25	14	40.5	27.5	67	45.5	5.6	3.5	1	2
	2.75	5.25	12	49.5	33.5	69	46.5	6.9	4.5	1	2
	3.00	5.25	12	49.5	33.5	69	46.5	6.9	4.5	1	2
	4.00	5.25	16	49.5	33.5	69	46.5	8.5	5.6	1	2
	5.00	5.25	18	49.5	33.5	69	46.5	11.0	6.9	1	2
	6.00	4.25	16	60.5	40.5	68	45.5	11.0	6.9	1	2
	Carbon Steel Cutters										
1 3/4 D. P. for Cast Iron; 2 D. P. for Steel.	1.75	6.25	10	22.5	18.5	37	30.75	2.3	1.4	2	3
	2.00	6.25	11	22.5	18.5	37	30.75	2.8	1.9	2	2
	2.25	6.25	11	22.5	18.5	37	30.75	2.8	1.9	2	2
	2.50	6.25	14	22.5	18.5	37	30.75	3.5	2.3	2	2
	2.75	5.25	12	27.5	22.5	38	31.5	3.5	2.3	1	2
	3.00	5.25	12	27.5	22.5	38	31.5	3.5	2.3	1	2
	4.00	5.25	16	27.5	22.5	38	31.5	4.5	3.5	1	2
	5.00	5.25	18	27.5	22.5	38	31.5	5.6	3.5	1	2
	6.00	4.25	16	33.5	27.5	37	31	5.6	3.5	1	2
	7.00	4.25	18	33.5	27.5	37	31	6.9	4.5	1	2
	8.00	4.25	20	33.5	27.5	37	31	6.9	4.5	1	2
	10.00	4.25	22	33.5	27.5	37	31	8.5	5.6	1	1

* Compiled by Gould & Eberhardt.

Speeds and Feeds for Gear Cutting — 2

Capacity of Machine	Diam. Pitch	Cutter		R.P.M.		Cutting Speed, Feet per Min.		Feed per Min.		No. of Cuts	
		Diam.	No. of Teeth	Cast Iron	Steel	Cast Iron	Steel	Cast Iron	Steel	Cast Iron	Steel
	High-speed Steel Cutters										
1¾ D. P. for Cast Iron; 2 D. P. for Steel.	1.75	6.25	10	40.5	27.5	67	45.5	3.5	2.3	2	3
	2.00	6.25	11	40.5	27.5	67	45.5	3.5	2.8	2	2
	2.25	6.25	11	40.5	27.5	67	45.5	3.5	2.8	2	2
	2.50	6.25	14	40.5	27.5	67	45.5	4.5	3.5	2	2
	2.75	5.25	12	49.5	33.5	69	46.5	5.6	3.5	1	2
	3.00	5.25	12	49.5	33.5	69	46.5	5.6	3.5	1	2
	4.00	5.25	16	49.5	33.5	69	46.5	6.9	4.5	1	2
	5.00	5.25	18	49.5	33.5	69	46.5	8.5	5.6	1	2
	6.00	4.25	16	60.5	40.5	68	45.5	8.5	5.6	1	2
	7.00	4.25	18	60.5	40.5	68	45.5	11.0	6.9	1	2
	8.00	4.25	20	60.5	40.5	68	45.5	11.0	8.5	1	2
	10.00	4.25	22	60.5	40.5	68	45.5	13.6	8.5	1	1
2 Diametral Pitch for Cast Iron; 3 D. P. for Steel.	Carbon Steel Cutters										
	2.00	6.25	11	21	18	34.5	30	2.7	2.2	2	3
	2.25	6.25	11	21	18	34.5	30	2.7	2.2	2	2
	2.50	6.25	14	21	18	34.5	30	2.7	2.2	2	2
	2.75	5.25	12	26.5	21	36.5	29.5	3.3	2.7	1	2
	3.00	5.25	12	26.5	21	36.5	29.5	3.3	2.7	1	2
	4.00	5.25	16	26.5	21	36.5	29.5	4	3.3	1	2
	5.00	5.25	18	26.5	21	36.5	29.5	5	4	1	2
	6.00	4.25	16	33	26.5	37	30	6	4	1	2
	7.00	4.25	18	33	26.5	37	30	6	5	1	2
	8.00	4.25	20	33	26.5	37	30	7.3	6	1	2
	10.00	4.25	22	33	26.5	37	30	7.3	6	1	1
	12.00	4.25	24	33	26.5	37	30	9.1	7.3	1	1
	High-speed Steel Cutters										
	2.00	6.25	11	38	26.5	63	44	4	2.7	2	3
	2.25	6.25	11	38	26.5	63	44	4	2.7	2	2
	2.50	6.25	14	38	26.5	63	44	6	4	2	2
	2.75	5.25	12	48	33	67	46	6	4	1	2
	3.00	5.25	12	48	33	67	46	6	4	1	2
	4.00	5.25	16	48	33	67	46	7.3	6	1	2
	5.00	5.25	18	48	33	67	46	9	6	1	2
	6.00	4.25	16	60.5	38	68	43	11.5	7.3	1	2
	7.00	4.25	18	60.5	38	68	43	11.5	7.3	1	2
	8.00	4.25	20	60.5	38	68	43	14.5	9.1	1	2
	10.00	4.25	22	60.5	38	68	43	14.5	9.1	1	1
	12.00	4.25	24	60.5	38	68	43	14.5	9.1	1	1

Speeds and Feeds for Gear Cutting — 3

Capacity of Machine	Diam. Pitch	Cutter		R.P.M.		Cutting Speed, Feet per Min.		Feed per Min.		No. of Cuts	
		Diam.	No. of Teeth	Cast Iron	Steel	Cast Iron	Steel	Cast Iron	Steel	Cast Iron	Steel
	Carbon Steel Cutters										
3 Diametral Pitch for Cast Iron; 4 D. P. for Steel.	3.00	4.75	11	27.5	20	35.5	25	2.6	2.0	1	2
	4.00	4.00	12	39	27.5	41.5	29.25	3.5	2.6	1	2
	5.00	3.50	12	46.5	39	43	36	4.5	3.5	1	2
	6.00	3.50	13	46.5	39	43	36	4.5	3.5	1	2
	7.00	3.50	14	46.5	39	43	36	4.5	3.5	1	2
	8.00	3.50	16	46.5	39	43	36	5.8	4.5	1	2
	10.00	3.50	18	46.5	39	43	36	5.8	4.5	1	1
	12.00	3.50	18	46.5	39	43	36	5.8	4.5	1	1
	14.00	2.875	16	55.5	46.5	42	35.5	5.8	4.5	1	1
	16.00	2.875	18	55.5	46.5	42	35.5	7.6	5.8	1	1
	18.00	2.875	18	55.5	46.5	42	35.5	7.6	5.8	1	1
	20.00	2.875	20	55.5	46.5	42	35.5	7.6	5.8	1	1
	High-speed Steel Cutters										
	3.00	4.75	11	46.5	39	59	49	3.5	2.6	1	2
	4.00	4.00	12	55.5	46.5	59	49	4.5	3.5	1	2
	5.00	3.50	12	55.5	46.5	51.5	43	4.5	3.5	1	2
	6.00	3.50	13	55.5	46.5	51.5	43	4.5	3.5	1	2
	7.00	3.50	14	55.5	46.5	51.5	43	5.8	4.5	1	2
	8.00	3.50	16	55.5	46.5	51.5	43	5.8	4.5	1	2
	10.00	3.50	18	55.5	46.5	51.5	43	7.6	5.8	1	1
	12.00	3.50	18	55.5	46.5	51.5	43	7.6	5.8	1	1
	14.00	2.875	16	79	55.5	60	40.5	7.6	5.8	1	1
	16.00	2.875	18	79	55.5	60	40.5	10.0	7.6	1	1
	18.00	2.875	18	79	55.5	60	40.5	10.0	7.6	1	1
	20.00	2.875	20	79	55.5	60	40.5	10.0	7.6	1	1
	Carbon Steel Cutters										
4 D. P. for Cast Iron; 5 D. P. for Steel.	4.00	4.00	12	39	27.5	41.5	29.25	2.6	1.5	1	2
	5.00	3.50	12	46.5	39	43	36	3.5	2.6	1	2
	6.00	3.50	13	46.5	39	43	36	3.5	2.6	1	2
	7.00	3.50	14	46.5	39	43	36	3.5	2.6	1	2
	8.00	3.50	16	46.5	39	43	36	4.5	3.5	1	2
	10.00	3.50	18	46.5	39	43	36	4.5	3.5	1	1
	12.00	3.50	18	46.5	39	43	36	4.5	3.5	1	1
	14.00	2.875	16	55.5	46.5	42	35.5	5.8	4.5	1	1
	16.00	2.875	18	55.5	46.5	42	35.5	5.8	4.5	1	1
	18.00	2.875	18	55.5	46.5	42	35.5	5.8	4.5	1	1
	20.00	2.875	20	55.5	46.5	42	35.5	7.6	5.8	1	1
	24.00	2.875	20	55.5	46.5	42	35.5	7.6	5.8	1	1

Speeds and Feeds for Gear Cutting — 4

Capacity of Machine	Diam. Pitch	Cutter		R.P.M.		Cutting Speed, Feet per Min.		Feed per Min.		No. of Cuts	
		Diam.	No. of Teeth	Cast Iron	Steel	Cast Iron	Steel	Cast Iron	Steel	Cast Iron	Steel
	High-speed Steel Cutters										
4 D. P. for Cast Iron; 5 D. P. for Steel.	4.00	4.00	12	55.5	46.5	59	49	4.5	3.5	1	2
	5.00	3.50	12	55.5	46.5	51.5	43	4.5	3.5	1	2
	6.00	3.50	13	55.5	46.5	51.5	43	4.5	3.5	1	2
	7.00	3.50	14	55.5	46.5	51.5	43	4.5	3.5	1	2
	8.00	3.50	16	55.5	46.5	51.5	43	5.8	4.5	1	2
	10.00	3.50	18	55.5	46.5	51.5	43	5.8	4.5	1	1
	12.00	3.50	18	55.5	46.5	51.5	43	5.8	4.5	1	1
	14.00	2.875	16	79	55.5	60	40.5	7.6	5.8	1	1
	16.00	2.875	18	79	55.5	60	40.5	7.6	5.8	1	1
	18.00	2.875	18	79	55.5	60	40.5	7.6	5.8	1	1
	20.00	2.875	20	79	55.5	60	40.5	10.0	7.6	1	1
	24.00	2.875	20	79	55.5	60	40.5	10.0	7.6	1	1

Speeds and Feeds for Cold Sawing. — Only general figures can be given, as the speeds and feeds depend upon the grade of material to be cut as well as upon the design and size of the machine and saw. The speeds and feeds should be regulated to give an economical output for each grinding of the saw. One manufacturer of cold saw cutting-off machines gives the following data: For 0.30 per cent carbon, open-hearth machine steel bars about 5 inches in diameter, use a feed of 1 inch per minute and a peripheral saw speed of 45 feet per minute. This speed and feed is considered economical when using high-carbon vanadium-steel blades. For some classes of work the feed can be increased to 2 inches per minute, but for average shop conditions this is considered excessive. It has been found that some of the special alloy steels can be cut quite freely; for example, 3½ per cent nickel steel can be sawed as easily as 0.30 per cent open-hearth carbon steel. On the other hand, some of the softer grades of stock, such as open-hearth machine steel having about 0.15 per cent carbon, does not cut as freely as stock with a higher percentage of carbon. This is because the softer steel is inclined to roll up into the spaces between the saw teeth. As to phosphor-bronze, some grades can be cut very easily, whereas some of the bronzes used for gears, etc., are very hard to cut, and the edge of the saw is inclined to glaze over; hence it is difficult to give even a general idea of the speeds and feeds to be used.

Another concern gives the following time data for various materials: Time for sawing steel forging, 6⅛ inches in diameter containing 0.74 carbon, 1.12 manganese, 0.30 nickel, 7½ minutes; tool steel bar, 2½ inches in diameter, 4 minutes; steel bar, 0.50 per cent carbon, 3½ inches in diameter, 3½ minutes; Krupp chrome-nickel steel bar, 4½ inches in diameter, 5 minutes; Krupp chrome-nickel steel bar, 6 inches in diameter, 8 to 10 minutes; 6½-inch bar, 0.20 per cent carbon, 5 minutes; 6-inch bar, 0.30 per cent carbon, 4½ to 6 minutes; 8-inch bar of high-carbon steel, 13 minutes; 8-inch diameter hard bronze ingot, 6 minutes. All these data represent actual practice and are given as a general guide, although it is necessary, of course, to make considerable allowance for the design of the saw as well as the size and power of the machine.

Speeds for Turning Unusual Materials. — *Slate*, on account of its peculiarly stratified formation, is rather difficult to turn, but if handled carefully, can be machined in an ordinary lathe. The cutting speed should be about the same as for cast iron. A sheet of fiber or pressed paper should be interposed between the chuck or steadyrest jaws and the slate, to protect the latter. Slate rolls must not be centered and run on the tailstock. A satisfactory method of supporting a slate roll having journals at the ends is to bore a piece of lignum vitæ to receive the turned end of the roll, and center it for the tailstock spindle.

Rubber can be turned at a peripheral speed of 200 feet per minute, although it is much easier to grind it with an abrasive wheel that is porous and soft. For cutting a rubber roll in two, the ordinary parting tool should not be used, but a tool shaped like a knife; such a tool severs the rubber without removing any material.

Gutta percha can be turned as easily as wood, but the tools must be sharp and a good soap-and-water lubricant used.

Copper can be turned easily at 200 feet per minute. When machining commutators that is the speed commonly used. Light cuts are taken with a pointed tool and the surface is finished with a wooden buff lined with glass paper. This buff is in the form of a 90-degree segment and is approximately 1¾ inch thick. It is held against the commutator with the point of an ordinary tool and is fed back and forth across the surface. In this way the segments are gradually ground down until there is not the slightest gap or break between the copper segments and the mica insulation.

Lime-stone such as is used in the construction of pillars for balconies, etc., can be turned at 150 feet per minute, and the formation of ornamental contours is quite easy. *Marble* is a treacherous material to turn. It should be cut with a tool such as would be used for brass, but at a speed suitable for cast iron. It must be handled very carefully to prevent flaws in the surface.

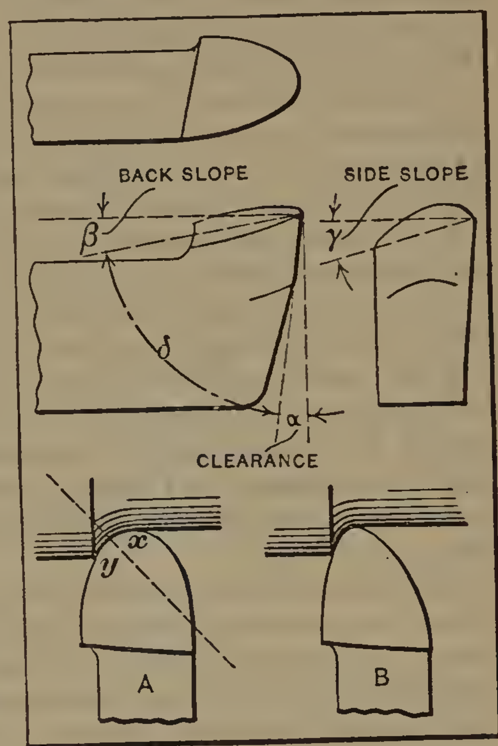
Tool Grinding

While turning or planing tools must of necessity be varied considerably in shape to adapt them to different purposes, there are certain principles governing the form which apply generally. When grinding lathe or other turning tools, there are three things of importance to be considered: First, the cutting edge of the tool, as viewed from the top, must have a certain shape, or contour; second, there must be a certain amount of clearance below or back of the cutting edge; and third, tools (with certain exceptions) are given a backward slope, a side slope or a combination of these two slopes, on that part against which the chip bears when the tool is in use.

Experiments have shown that a tool *A* (see illustration) having a cutting edge of large radius is adapted to roughing and is capable of retaining its edge under higher cutting speeds than a tool like *B*, with a smaller point; but if tool *A* were used for turning a flexible part, chattering might result; consequently, in that case, tool *B*, having a smaller radius, would be preferable, if not absolutely necessary. In some cases, therefore, it may be impracticable to use a tool that is theoretically correct, owing to the flexibility of the work or lack of rigidity in the machine.

The shape of a tool (as viewed from the top), which is intended for a more specific purpose than plain cylindrical turning, can be determined largely by considering the tool under working conditions. This point may be illustrated by the "parting tool," which is used for cutting grooves, etc. Evidently this tool must be widest at the cutting edge to prevent it from binding when fed into a narrow groove.

Direction of Slope. — Aside from the contour of the cutting edge, there remains to be determined the amount of clearance and also the amount and direction of the slope for the top of the tool, the word top being used to mean that surface against which the chip bears while it is being severed. Generally speaking, the top of the tool should slope away from what is to be the *working part* of the cutting edge. For example, the working edge of a roughing tool *A* (see illustration) is between points *x* and *y*, because most of the work is done by this part of the cutting edge; the top should slope back from this part of the edge, in the direction indicated by the dotted line. Obviously, a tool ground in this way has both a back and side slope. When most of the work is done at the point or "nose" of the tool, as in the case of a broad finishing tool, the slope should be straight back from the cutting edge. The direction of the slope is important, because, when it is in the right direction, less power is required for cutting. Tools for certain classes of work, such as thread tools or those for turning brass or chilled iron, are ground flat on the top, or without slope.



The Clearance Angle. — In order that the cutting edge may work without interference, the edge must be ground with a certain clearance angle α which should be just enough to permit the tool to cut freely. A clearance angle of 8 or 10 degrees is about right for lathe turning tools intended for general work. Tools for turning brass or other soft metals, especially when considerable hand manipulation is required, should have a clearance of 12 or 14 degrees, so that the tool can easily be fed into the metal. Excessive clearance weakens the cutting edge and may cause it to crumble under the pressure of the cut.

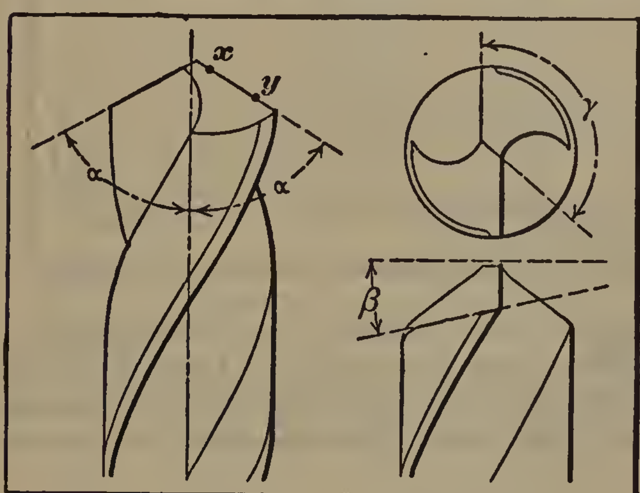
Amount of Slope and Lip Angle. — The efficiency of the tool largely depends upon the lip angle or angle of keenness, δ. As the illustration shows, this angle is governed by the clearance α and slope β, and as the clearance remains practically the same, it is the slope which is varied to meet different conditions. The amount of slope depends upon the kind of material which is to be cut. A turning tool for roughing medium or soft steel should have a back slope β of 8 degrees and a side slope γ ranging from 14 to 20 degrees; a tool for cutting very hard steel should have a back slope of 5 degrees and a side slope of 9 degrees. The reason for decreasing the slope and thus increasing the lip angle δ for harder metals is to increase the strength of the cutting edge to prevent it from crumbling under the pressure of the cut. The lip angle, as a general rule, should be as small as possible, without weakening the tool so that it cannot do the required work. In order to secure a strong, well-supported cutting edge, tools used for turning very hard metals, such as chilled rolls, etc., are ground with practically no slope and little clearance. Brass tools, while given considerable clearance, are ground flat on top or without slope. This, however, is not done to give strength to the cutting edge, but rather to prevent the tool from gouging into the work, as it is likely to do if there is top slope and the part being turned is at all flexible.

Experiments for determining the most efficient form of lathe roughing tools have shown that the nearer the lip angle approached 61 degrees, the higher the cutting speed. This, however, does not apply to tools for turning cast iron, as the latter will work more efficiently with a lip angle of about 68 degrees. This is because the chip pressure, when turning cast iron, comes closer to the cutting edge, which should, therefore, be more blunt to withstand the abrasive action and heat. The foregoing remarks concerning lip angles apply more particularly to tools used for roughing. Inserted-cutter turning tools should, of course, be ground so as to have the right slope when in position in the holder.

Planing Tools. — Many of the principles which govern the shape of turning tools also apply in the grinding of tools for planing. The amount of slope depends upon the hardness of the material, and the direction of the slope should be away from the *working part* of the cutting edge. The angle of clearance should be about 4 or 5 degrees for planer tools, which is much less than for lathe tools. This small clearance is allowable because a planer tool is held about square with the platen, whereas a lathe tool, the height and inclination of which can be varied, may not always be clamped in the same position.

When grinding any kind of tool, it should not be forced too hard against the emery wheel or grindstone, as is often done in an attempt to grind quickly. The tool should be ground with a moderate pressure and it should be withdrawn frequently, especially when forming a broad flat surface, to prevent excessive heating and "burning" of the edge. Grinding wheels should always be supplied with plenty of cooling water.

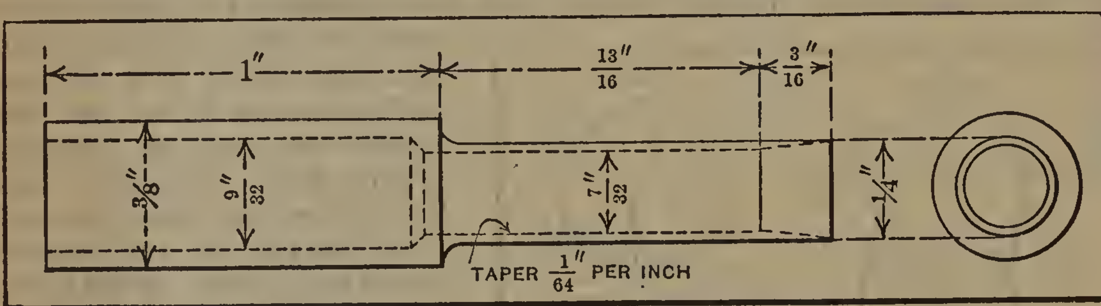
Twist Drill Grinding. — The cutting end of a drill should conform to the following requirements: The two cutting edges should incline at the proper angle α with the axis (see illustration); each edge should have the *same* inclination and be of the same length; the angle of clearance, β, back of the cutting edge should be sufficient to permit the drill to cut freely; and the clearance should be the same on both sides and increase toward the point or center of the drill.



Angle of Cutting Edge. — The angle α of the cutting edge should be about 59 degrees, as measured from the axis, although there is a difference of opinion regarding this angle. As the angle is decreased, the pressure required for feeding a drill through the metal diminishes, but the length of each cutting edge is increased with the result that more power is required to turn the drill. An included angle of 118 degrees (59 degrees between the cutting edge and axis) is thought by some authorities to equalize the thrust and torsion to best advantage, while others advocate more acute angles. A spotting drill should have an included angle of 100 degrees, so that the body of the drill following will be properly supported before the point begins to cut.

Clearance Angle. — The clearance angle β is very important, and the splitting of drills through the web is often an indication of insufficient clearance, especially at the drill point. If the end of a drill conforms exactly to the conical shape of the bottom of a hole, it will not cut, because the lack of clearance prevents the cutting edges from sinking into the metal. Theoretically, the clearance should be just enough to permit the drill to cut freely, because excessive clearance weakens the cutting edges. One prominent drill concern advocates an angle β of 12 degrees at the periphery of the drill, with a gradual increase towards the center until the straight line formed by the end or point of the drill is at an angle γ of approximately 135 degrees, as shown by the plan view. When soft metal is to be drilled, and heavier feeds are possible, the angle of clearance may be increased to 15 degrees, whereas, for hard material, such as tool steel, the amount of clearance is diminished, because a finer feed must be used and a strong cutting edge is required. As previously stated, the clearance angle should gradually increase toward the drill point. The reason for this will be apparent by considering the movements of two points x and y on the cutting edge as the drill is fed down, one point being much nearer the center than the other. Assuming that the feed is constant, each of these points will follow a helical path, and as the vertical distance that each point feeds downward, per revolution of the drill, will be the same, the angle of the smaller helix will be greater than that of the larger one. Hence, the clearance angle should increase toward the drill point, because the helix angle in each case indicates the minimum clearance necessary at that particular point.

Drill for Paper. — The tubular drill shown in the illustration is adapted for drilling holes in paper leaves, the particular size shown being used for loose-leaf ledger work in preference to punching. The outside diameter, which corresponds to the size of hole required, has a total taper of 0.001 inch per inch, and the inside or bore tapers 1/64 inch per inch. To provide clearance, the bore of the drill shank



is enlarged 1/16 inch to allow the scrap or paper cuttings to pass through freely. The cutting end is beveled on the inside to form a sharp edge. The drill should be hardened in oil and drawn to a dark brown at the cutting edge, this color running off gradually to a blue at the back end of the tube. This drill should run about 1500 revolutions per minute.

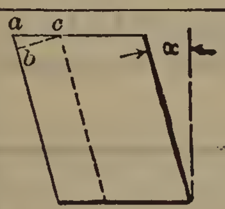
Drilling Holes in Glass. — There are several methods of drilling holes in glass. For holes of medium and large size, use brass or copper tubing, having an outside diameter equal to the size of hole required. Revolve the tube at a peripheral speed of about 100 feet per minute, and use carborundum (80 to 100 grit) and light machine oil between the end of the pipe and the glass. Insert the abrasive under the drill with a thin piece of soft wood, to avoid scratching the glass. The glass should be supported by a felt or rubber cushion, not much larger than the hole to be drilled. If practicable, it is well to drill about halfway through and then turn the glass over and drill down to meet the first cut. Any fin that may be left in the hole can be removed with a round second-cut file wet with turpentine. For comparatively small holes, a solid drill is often used. Use steel rod or an old three-cornered file, grinding the end to a long tapering triangular shaped point. Grip the drill in a chuck and rotate rapidly. Use a mixture of turpentine and camphor as a lubricant. Holes up to 1/2 inch in diameter can be drilled in glass with a flat drill which has been hardened in sulphurous acid, a mixture of turpentine and camphor being used as a lubricant.

AUTOMATIC SCREW MACHINES

Forming Tools

Forming tools are made either flat or circular. Forming tools for the lathe or planer are ordinarily made flat. The latter type is also used for backing off formed milling cutter teeth. For automatic screw machine work, forming tools are made either flat or circular; the circular type is most commonly used. Flat tools are made solid or with cutter and shank separate. If but one tool is required, the solid form is preferable, but when a number of different tools are needed, it is more economical to make one shank and attach separate cutters of the required form. A forming tool of the straight type, which is extensively used, consists of a heavy block or holder which is bolted to the cross-slide. The front face of this block has a dovetail slot into which the cutter is clamped by a suitable bolt, the rear side of the cutter being dovetailed to fit the holder. To reduce the tendency to chatter when heavy cuts are taken, it is sometimes advisable to make a spring holder similar to the "goose-neck" planer tool. Circular forming tools are easily duplicated after a master tool is once made, and they can be ground repeatedly without changing the shape, which is also true of the straight type which has the same cross-sectional shape throughout its length.

Dimensions of Straight Forming Tools



The table gives depth *bc* on steps of forming tool for various dimensions *ac* along cutting face. First locate the depth *ac* required on work and then find depth *bc* on tool in column headed by clearance angle α, at which tool is held in machine.

ac = Actual Cutting Distance	*bc* for Angle $\alpha = 10°$	*bc* for Angle $\alpha = 15°$	*bc* for Angle $\alpha = 20°$	*ac* = Actual Cutting Distance	*bc* for Angle $\alpha = 10°$	*bc* for Angle $\alpha = 15°$	*bc* for Angle $\alpha = 20°$
0.001	0.00098	0.00096	0.00094	0.040	0.03939	0.03863	0.03758
0.002	0.00197	0.00193	0.00187	0.050	0.04924	0.04829	0.04698
0.003	0.00295	0.00289	0.00281	0.060	0.05908	0.05795	0.05638
0.004	0.00393	0.00386	0.00375	0.070	0.06893	0.06761	0.06577
0.005	0.00492	0.00483	0.00469	0.080	0.07878	0.07727	0.07517
0.006	0.00590	0.00579	0.00563	0.090	0.08863	0.08693	0.08457
0.007	0.00689	0.00676	0.00657	0.100	0.09848	0.09659	0.09396
0.008	0.00787	0.00772	0.00751	0.200	0.19696	0.19318	0.18793
0.009	0.00886	0.00869	0.00845	0.300	0.29544	0.28977	0.28190
0.010	0.00984	0.00965	0.00939	0.400	0.39392	0.38637	0.37587
0.020	0.01969	0.01931	0.01879	0.500	0.49240	0.48296	0.46984
0.030	0.02954	0.02897	0.02819	......	......	......	......

Straight Forming Tools. — The table "Dimensions of Straight Forming Tools" was compiled to facilitate the obtaining of the distance from one step to another, at right angles to the front face of the tool. By referring to the illustration accompanying this table, it will be seen that line *ac*, along the cutting face, is longer than *bc* at right angles to the formed surface; *ac* must equal the difference between the two radii on the work, and *bc* will equal *ac* × the cosine of clearance angle α.

In the first column of the table, the distance *ac* or the actual cutting distance is given; the second, third and fourth columns give corresponding distances *bc*, when the clearance angles are 10, 15 and 20 degrees, respectively. To illustrate the use of the table, suppose a tool is required for turning the piece shown in Fig. 1, having diameters of 0.75, 1.25 and 1.75 inch, respectively. Reducing these diameters to their respective radii, we have 0.375, 0.625 and 0.875. The difference between the largest and smallest radii is 0.5, which would equal the actual cutting edge of the tool or dimension *ac*. Assume that the tool is to be set at an angle of 15 degrees. The distance *ac*, or 0.5, is located in the column headed "Actual Cutting Distance," and then the corresponding value for *bc* in the 15-degree column. As will be seen, $bc = 0.48296$. In the same way, the depth of the second step on the tool is determined. As the difference in radii equals 0.25, length *bc* for a 15-degree angle equals $0.19318 + 0.04829 = 0.24147$. If the forming tool is made so that the steps measure 0.4830 and 0.2415, respectively, along the line *bc*, the diameters turned will correspond to those given, provided the cutting face of the tool is central with the work, and the front face held so that the clearance angle is 15 degrees.

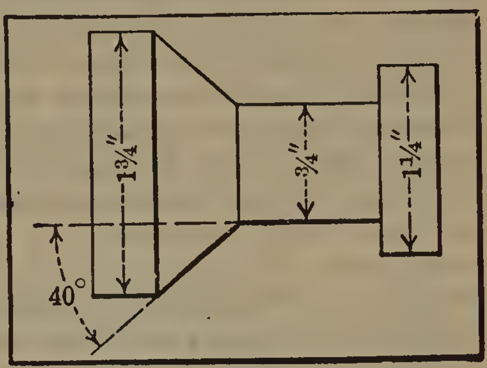
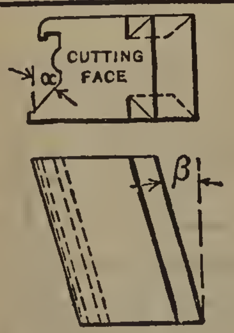


Fig. 1

Sometimes it is necessary to turn an angular surface. The table "Angles for Straight Forming Tools" gives the required angle for the tool face for turning

Angles for Straight Forming Tools



The table gives angles of forming surface on tool which coincide with angle α on cutting face when tool is made with clearance angles of 10, 15 or 20 degrees. Thus, to turn to an angle of 5 degrees, when the clearance angle β is 10 degrees, the tool should be made to an angle of 4 degrees, 55 minutes.

α = Angle on Cutting Face of Forming Tool	β = 10 Degrees	β = 15 Degrees	β = 20 Degrees	α = Angle on Cutting Face of Forming Tool	β = 10 Degrees	β = 15 Degrees	β = 20 Degrees
5°	4° 55′	4° 50′	4° 42′	50°	49° 34′	49° 1′	48° 14′
10	9 51	9 40	9 24	55	54 35	54 4	53 18
15	14 47	14 31	14 8	60	59 37	59 8	58 26
20	19 43	19 22	18 53	65	64 40	64 14	63 36
25	24 40	24 15	23 40	70	69 43	69 21	68 50
30	29 37	29 9	28 29	75	74 47	74 30	74 5
35	34 35	34 4	33 20	80	79 51	79 39	79 22
40	39 34	39 1	38 15	85	84 55	84 49	84 41
45	44 34	44 0	43 13	......	......	......	......

the work to a given angle. These angles are not alike, because of the clearance angle of the tool. They are measured from the center-line of the piece or, what is equivalent, from the formed face of the tool. For example, what angle α (see illustration in table) should a forming tool have to turn a surface to an angle of 40 degrees (see Fig. 1), assuming that the clearance angle of the forming tool is 15 degrees? Find angle α in the first column of the table and then the corrected angle in the column headed "$\beta = 15$ degrees." The corrected angle in this case is 39 degrees 1 minute.

Circular Forming Tools. — To provide sufficient periphery clearance on circular forming tools, the cutting face is off-set with relation to the center of the tool a distance C as shown in Fig. 2. Whenever a circular tool has two or more diameters, the difference in the radii of the steps on the tool will, therefore, not correspond exactly to the difference in the steps on the work. The form produced with the tool also changes, although the change is very slight, unless the amount of off-set C is considerable. Assume that a circular tool is required to produce the piece A having two diameters as shown. If the difference D_1 between the large and small radii of the tool were made equal to dimension D required on the work, D would be a certain amount over-size, depending upon the off-set C of the cutting edge. The following formulas can be used to determine the radii of circular forming tools for turning parts to different diameters:

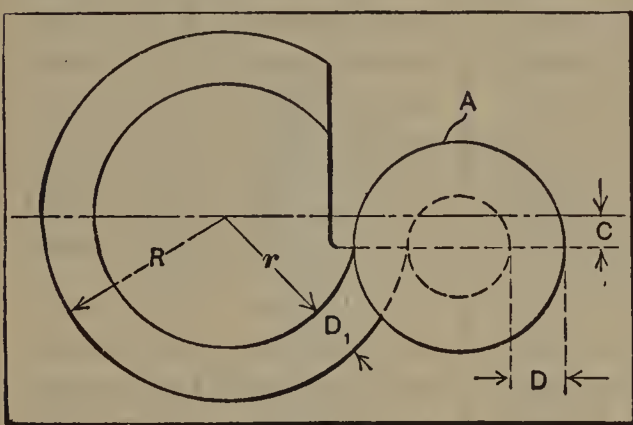


Fig. 2

Let R = largest radius of tool in inches;
D = difference in radii of steps on work;
C = amount cutting edge is off-set from center of tool;
r = required radius in inches;

then:

$$r = \sqrt{(\sqrt{R^2 - C^2} - D)^2 + C^2}. \qquad (1)$$

If the small radius r is given and the large radius R is required, then:

$$R = \sqrt{(\sqrt{r^2 - C^2} + D)^2 + C^2}. \qquad (2)$$

To illustrate, if D (Fig. 2) is to be ⅛ inch, the large radius R is 1⅛ inch, and C is 5/32 inch, what radius r would be required to compensate for the off-set C of the cutting edge? Inserting these values in Formula (1):

$$r = \sqrt{(\sqrt{(1\tfrac{1}{8})^2 - (\tfrac{5}{32})^2} - \tfrac{1}{8})^2 + (\tfrac{5}{32})^2} = 1.0014 \text{ inch.}$$

The value of r is thus found to be 1.0014 inch; hence the diameter = 2 × 1.0014 = 2.0028 inches instead of 2 inches, as would have been the case if the cutting edge had been exactly on the center-line. Formulas for circular tools used on different makes of screw machines can be simplified when the values R and C are constant for each size of machine. The accompanying table "Formulas for Circular Form-

Formulas for Circular Forming Tools

(For notation, see Fig. 2)

Make of Machine	Size of Machine	Radius R, Inches	Offset C, Inches	Radius r, Incnes
Brown & Sharpe	No. 00	0.875	0.125	$r=\sqrt{(0.8660-D)^2+0.0156}$
	No. 0	1.125	0.15625	$r=\sqrt{(1.1141-D)^2+0.0244}$
	No. 2	1.50	0.250	$r=\sqrt{(1.4790-D)^2+0.0625}$
	No. 6	2.00	0.3125	$r=\sqrt{(1.975\ -D)^2+0.0976}$
Acme	No. 51	0.75	0.09375	$r=\sqrt{(0.7441-D)^2+0.0088}$
	No. 515	0.75	0.09375	$r=\sqrt{(0.7441-D)^2+0.0088}$
	No. 52	1.0	0.09375	$r=\sqrt{(0.9956-D)^2+0.0088}$
	No. 53	1.1875	0.125	$r=\sqrt{(1.1809-D)^2+0.0156}$
	No. 54	1.250	0.15625	$r=\sqrt{(1.2402-D)^2+0.0244}$
	No. 55	1.250	0.15625	$r=\sqrt{(1.2402-D)^2+0.0244}$
	No. 56	1.50	0.1875	$r=\sqrt{(1.4882-D)^2+0.0352}$
Cleveland	¼″	0.625	0.03125	$r=\sqrt{(0.6242-D)^2+0.0010}$
	⅜″	0.84375	0.0625	$r=\sqrt{(0.8414-D)^2+0.0039}$
	⅝″	1.15625	0.0625	$r=\sqrt{(1.1546-D)^2+0.0039}$
	⅞″	1.1875	0.0625	$r=\sqrt{(1.1859-D)^2+0.0039}$
	1¼″	1.375	0.0625	$r=\sqrt{(1.3736-D)^2+0.0039}$
	2″	1.375	0.0625	$r=\sqrt{(1.3736-D)^2+0.0039}$
	2¼″	1.625	0.125	$r=\sqrt{(1.6202-D)^2+0.0156}$
	2¾″	1.875	0.15625	$r=\sqrt{(1.8685-D)^2+0.0244}$
	3¼″	1.875	0.15625	$r=\sqrt{(1.8685-D)^2+0.0244}$
	4¼″	2.50	0.250	$r=\sqrt{(2.4875-D)^2+0.0625}$
	6″	2.625	0.250	$r=\sqrt{(2.6131-D)^2+0.0625}$

ing Tools" gives the standard values of R and C for circular tools used on different automatics. The formulas for determining the radius r (see column at right-hand side of table) contain a constant which represents the value of the expression $\sqrt{R^2-C^2}$ in formula (1) on page 893.

The table "Constants for Determining Diameters of Circular Forming Tools" has been compiled to facilitate proportioning tools of this type. It gives constants for computing the various diameters of forming tools, when the cutting face of the tool is either ⅛, $^3/_{16}$, ¼ or $^5/_{16}$ inch below the horizontal center-line. As there is no standard distance for the location of the cutting face, the table has been prepared to correspond with distances commonly used. As an example, suppose the tool is required for a part having three diameters of 1.75, 0.75 and 1.25 inch, respectively, as shown in Fig. 1, and that the largest diameter of the tool is 3 inches and its cutting face is ¼ inch below the horizontal center-line. The first step would be to determine approximately the respective diameters of the forming tool and then correct these diameters by the use of the table. To produce the three diameters shown in Fig. 1, with a 3-inch forming tool, the tool diameters would be approximately 2, 3 and 2.5 inches, respectively. The first dimension (2 inches) is 1 inch less in diameter than that of the tool, and the necessary correction should be given in the column "Correction for Difference in Diameter"; but as the table is only extended to half-inch differences, it will be necessary to obtain this particular correction in two steps. On the line for 3-inch diameter and under corrections for ½ inch, we find 0.0085; then in line with 2½ and under the same heading, we find 0.0129; hence the total correction would be 0.0085 + 0.0129 = 0.0214 inch. This correction is added to the approximate diameter, making the exact diameter of the first step 2 + 0.0214 = 2.0214 inches. The next step would be computed in the same way, by noting on the 3-inch line the correction for ½ inch and adding it to the approximate diameter of the second step, giving an exact diameter of 2.5 + 0.0085 = 2.5085 inches. Therefore, to produce the part shown in Fig. 1, the tool should have three steps of 3, 2.0214 and 2.5085 inches, respectively, provided the cutting face is ¼ inch below the center. All diameters are computed in this way, from the largest diameter of the tool.

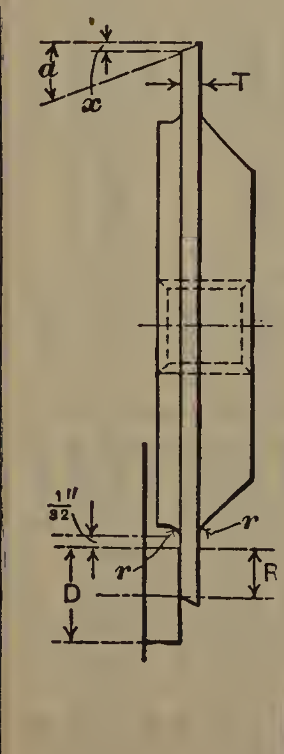
The tables "Corrected Diameters of Circular Forming Tools" are especially applicable to tools used on Brown & Sharpe automatic screw machines. Directions for using these tables are given at the end of Table 4.

Circular Tools Having Top Rake. — Circular forming tools without top rake are satisfactory for brass, but tools for steel or other tough metals cut better when there is a rake angle of 10 or 12 degrees. For such tools the small radius r (see Fig. 2) for an outside radius R may be found by the formula:

$$r = \sqrt{P^2 + R^2 - 2\,PR\cos\theta}$$

To find the value of P proceed as follows: $\sin\phi$ = small rad. on work × sin rake angle ÷ large rad. on work. Angle β = rake angle − ϕ. P = large rad. on work × $\sin\beta$ ÷ sin rake angle. Angle θ = rake angle + δ. Sin δ = vertical height C from center of tool to center of work ÷ R. It is assumed that the point of tool is to be set at same height as the work center.

Dimensions for Circular Cut-off Tools



Diam. of Stock	Soft Brass, Copper a = 23 Deg. T	Soft Brass, Copper a = 23 Deg. x	Norway Iron, Machine Steel a = 15 Deg. T	Norway Iron, Machine Steel a = 15 Deg. x	Drill Rod, Tool Steel a = 12 Deg. T	Drill Rod, Tool Steel a = 12 Deg. x
1/16	0.031	0.013	0.039	0.010	0.043	0.009
⅛	0.044	0.019	0.055	0.015	0.062	0.013
3/16	0.052	0.022	0.068	0.018	0.076	0.016
¼	0.062	0.026	0.078	0.021	0.088	0.019
5/16	0.069	0.029	0.087	0.023	0.098	0.021
⅜	0.076	0.032	0.095	0.025	0.107	0.023
7/16	0.082	0.035	0.103	0.028	0.116	0.025
½	0.088	0.037	0.110	0.029	0.124	0.026
9/16	0.093	0.039	0.117	0.031	0.131	0.028
⅝	0.098	0.042	0.123	0.033	0.137	0.029
11/16	0.103	0.044	0.129	0.035	0.145	0.031
¾	0.107	0.045	0.134	0.036	0.152	0.032
13/16	0.112	0.047	0.141	0.038	0.158	0.033
⅞	0.116	0.049	0.146	0.039	0.164	0.035
15/16	0.120	0.051	0.151	0.040	0.170	0.036
1	0.124	0.053	0.156	0.042	0.175	0.037

The length of the blade equals radius of stock $R + x + r$ + 1/32 inch (for notation see illustration above): r = 1/16 inch for ⅜- to ¾-inch stock, and 3/32 inch for ¾- to 1-inch stock.

Constants for Determining Diameters of Circular Forming Tools

Diam. of Tool	Radius of Tool	Cutting Face ⅛ Inch Below Center			Cutting Face 3/16 Inch Below Center			Cutting Face ¼ Inch Below Center			Cutting Face 5/16 Inch Below Center		
		Correction for ⅛ Inch Difference in Diam.	Correction for ¼ Inch Difference in Diam.	Correction for ½ Inch Difference in Diam.	Correction for ⅛ Inch Difference in Diam.	Correction for ¼ Inch Difference in Diam.	Correction for ½ Inch Difference in Diam.	Correction for ⅛ Inch Difference in Diam.	Correction for ¼ Inch Difference in Diam.	Correction for ½ Inch Difference in Diam.	Correction for ⅛ Inch Difference in Diam.	Correction for ¼ Inch Difference in Diam.	Correction for ½ Inch Difference in Diam.
1	0.500	......	......	......	......	......	......	......	......	......	......	......	......
1⅛	0.5625	0.0036	......	......	0.0086	......	......	0.0167	......	......	0.0298	......	......
1¼	0.625	0.0028	0.0065	......	0.0067	0.0154	......	0.0128	0.0296	......	0.0221	0.0519	......
1⅜	0.6875	0.0023	......	......	0.0054	......	......	0.0102	......	......	0.0172	......	......
1½	0.750	0.0019	0.0042	0.0107	0.0045	0.0099	0.0253	0.0083	0.0185	0.0481	0.0138	0.0310	0.0829
1⅝	0.8125	0.0016	......	......	0.0037	......	......	0.0069	......	......	0.0114	......	......
1¾	0.875	0.0014	0.0030	......	0.0032	0.0069	......	0.0058	0.0128	......	0.0095	0.0210	......
1⅞	0.9375	0.0012	......	......	0.0027	......	......	0.0050	......	......	0.0081	......	......
2	1.000	0.0010	0.0022	0.0052	0.0024	0.0051	0.0121	0.0044	0.0094	0.0223	0.0070	0.0152	0.0362
2⅛	1.0625	0.0009	......	......	0.0021	......	......	0.0038	......	......	0.0061	......	......
2¼	1.125	0.0008	0.0017	......	0.0018	0.0040	......	0.0034	0.0072	......	0.0054	0.0116	......
2⅜	1.1875	0.0007	......	......	0.0016	......	......	0.0029	......	......	0.0048	......	......
2½	1.250	0.0006	0.0014	0.0031	0.0015	0.0031	0.0071	0.0027	0.0057	0.0129	0.0043	0.0092	0.0208
2⅝	1.3125	0.0006	......	......	0.0013	......	......	0.0024	......	......	0.0038	......	......
2¾	1.375	0.0005	0.0011	......	0.0012	0.0026	......	0.0022	0.0046	......	0.0035	0.0073	......
2⅞	1.4375	0.0005	......	......	0.0011	......	......	0.0020	......	......	0.0032	......	......
3	1.500	0.0004	0.0009	0.0021	0.0010	0.0021	0.0047	0.0018	0.0038	0.0085	0.0029	0.0061	0.0135
3⅛	1.5625	0.0004	......	......	0.0009	......	......	0.0017	......	......	0.0027	......	......
3¼	1.625	0.0003	0.0008	......	0.0008	0.0018	......	0.0015	0.0032	......	0.0024	0.0051	......
3⅜	1.6875	0.0003	......	......	0.0008	......	......	0.0014	......	......	0.0023	......	......
3½	1.750	0.0003	0.0007	0.0015	0.0007	0.0015	0.0033	0.0013	0.0028	0.0060	0.0021	0.0044	0.0095
3⅝	1.8125	0.0003	......	......	0.0007	......	......	0.0012	......	......	0.0019	......	......
3¾	1.875	0.0002	0.0006	......	0.0006	0.0013	......	0.0011	0.0024	......	0.0018	0.0038	......

Corrected Diameters of Circular Forming Tools — 1

Length c on Tool	Number of B. & S. Automatic Screw Machine			Length c on Tool	Number of B. & S. Automatic Screw Machine		
	No. 00	No. 0	No. 2		No. 00	No. 0	No. 2
0.001	1.7480	2.2480	2.9980	0.058	1.6353	2.1352	2.8857
0.002	1.7460	2.2460	2.9961	0.059	1.6333	2.1332	2.8837
0.003	1.7441	2.2441	2.9941	0.060	1.6313	2.1312	2.8818
0.004	1.7421	2.2421	2.9921	0.061	1.6294	2.1293	2.8798
0.005	1.7401	2.2401	2.9901	0.062	1.6274	2.1273	2.8778
0.006	1.7381	2.2381	2.9882	1/16	1.6264	2.1263	2.8768
0.007	1.7362	2.2361	2.9862	0.063	1.6254	2.1253	2.8759
0.008	1.7342	2.2341	2.9842	0.064	1.6234	2.1233	2.8739
0.009	1.7322	2.2321	2.9823	0.065	1.6215	2.1213	2.8719
0.010	1.7302	2.2302	2.9803	0.066	1.6195	2.1194	2.8699
0.011	1.7282	2.2282	2.9783	0.067	1.6175	2.1174	2.8680
0.012	1.7263	2.2262	2.9763	0.068	1.6155	2.1154	2.8660
0.013	1.7243	2.2243	2.9744	0.069	1.6136	2.1134	2.8640
0.014	1.7223	2.2222	2.9724	0.070	1.6116	2.1115	2.8621
0.015	1.7203	2.2203	2.9704	0.071	1.6096	2.1095	2.8601
1/64	1.7191	2.2191	2.9692	0.072	1.6076	2.1075	2.8581
0.016	1.7184	2.2183	2.9685	0.073	1.6057	2.1055	2.8561
0.017	1.7164	2.2163	2.9665	0.074	1.6037	2.1035	2.8542
0.018	1.7144	2.2143	2.9645	0.075	1.6017	2.1016	2.8522
0.019	1.7124	2.2123	2.9625	0.076	1.5997	2.0996	2.8503
0.020	1.7104	2.2104	2.9606	0.077	1.5978	2.0976	2.8483
0.021	1.7085	2.2084	2.9586	0.078	1.5958	2.0956	2.8463
0.022	1.7065	2.2064	2.9566	5/64	1.5955	2.0954	2.8461
0.023	1.7045	2.2045	2.9547	0.079	1.5938	2.0937	2.8443
0.024	1.7025	2.2025	2.9527	0.080	1.5918	2.0917	2.8424
0.025	1.7005	2.2005	2.9507	0.081	1.5899	2.0897	2.8404
0.026	1.6986	2.1985	2.9488	0.082	1.5879	2.0877	2.8384
0.027	1.6966	2.1965	2.9468	0.083	1.5859	2.0857	2.8365
0.028	1.6946	2.1945	2.9448	0.084	1.5839	2.0838	2.8345
0.029	1.6926	2.1925	2.9428	0.085	1.5820	2.0818	2.8325
0.030	1.6907	2.1906	2.9409	0.086	1.5800	2.0798	2.8306
0.031	1.6887	2.1886	2.9389	0.087	1.5780	2.0778	2.8286
1/32	1.6882	2.1881	2.9384	0.088	1.5760	2.0759	2.8266
0.032	1.6867	2.1866	2.9369	0.089	1.5740	2.0739	2.8247
0.033	1.6847	2.1847	2.9350	0.090	1.5721	2.0719	2.8227
0.034	1.6827	2.1827	2.9330	0.091	1.5701	2.0699	2.8207
0.035	1.6808	2.1807	2.9310	0.092	1.5681	2.0679	2.8187
0.036	1.6788	2.1787	2.9290	0.093	1.5661	2.0660	2.8168
0.037	1.6768	2.1767	2.9271	3/32	1.5647	2.0645	2.8153
0.038	1.6748	2.1747	2.9251	0.094	1.5642	2.0640	2.8148
0.039	1.6729	2.1727	2.9231	0.095	1.5622	2.0620	2.8128
0.040	1.6709	2.1708	2.9211	0.096	1.5602	2.0600	2.8109
0.041	1.6689	2.1688	2.9192	0.097	1.5582	2.0581	2.8089
0.042	1.6669	2.1668	2.9172	0.098	1.5563	2.0561	2.8069
0.043	1.6649	2.1649	2.9152	0.099	1.5543	2.0541	2.8050
0.044	1.6630	2.1629	2.9133	0.100	1.5523	2.0521	2.8030
0.045	1.6610	2.1609	2.9113	0.101	1.5503	2.0502	2.8010
0.046	1.6590	2.1589	2.9093	0.102	1.5484	2.0482	2.7991
3/64	1.6573	2.1572	2.9076	0.103	1.5464	2.0462	2.7971
0.047	1.6570	2.1569	2.9073	0.104	1.5444	2.0442	2.7951
0.048	1.6550	2.1549	2.9054	0.105	1.5425	2.0422	2.7932
0.049	1.6531	2.1529	2.9034	0.106	1.5405	2.0403	2.7912
0.050	1.6511	2.1510	2.9014	0.107	1.5385	2.0383	2.7892
0.051	1.6491	2.1490	2.8995	0.108	1.5365	2.0363	2.7873
0.052	1.6471	2.1470	2.8975	0.109	1.5346	2.0343	2.7853
0.053	1.6452	2.1451	2.8955	7/64	1.5338	2.0336	2.7846
0.054	1.6432	2.1431	2.8936	0.110	1.5326	2.0324	2.7833
0.055	1.6412	2.1411	2.8916	0.111	1.5306	2.0304	2.7814
0.056	1.6392	2.1391	2.8896	0.112	1.5287	2.0284	2.7794
0.057	1.6373	2.1372	2.8877	0.113	1.5267	2.0264	2.7774

Corrected Diameters of Circular Forming Tools — 2

Length c on Tool	Number of B. & S. Automatic Screw Machine			Length c on Tool	Number of B. & S. Automatic Screw Machine		
	No. 00	No. 0	No. 2		No. 00	No. 0	No. 2
0.113	1.5267	2.0264	2.7774	0.171	1.4124	1.9119	2.6634
0.114	1.5247	2.0245	2.7755	11/64	1.4107	1.9103	2.6617
0.115	1.5227	2.0225	2.7735	0.172	1.4104	1.9099	2.6614
0.116	1.5208	2.0205	2.7715	0.173	1.4084	1.9080	2.6595
0.117	1.5188	2.0185	2.7696	0.174	1.4065	1.9060	2.6575
0.118	1.5168	2.0166	2.7676	0.175	1.4045	1.9040	2.6556
0.119	1.5148	2.0146	2.7656	0.176	1.4025	1.9021	2.6536
0.120	1.5129	2.0126	2.7637	0.177	1.4006	1.9001	2.6516
0.121	1.5109	2.0106	2.7617	0.178	1.3986	1.8981	2.6497
0.122	1.5089	2.0087	2.7597	0.179	1.3966	1.8961	2.6477
0.123	1.5070	2.0067	2.7578	0.180	1.3947	1.8942	2.6457
0.124	1.5050	2.0047	2.7558	0.181	1.3927	1.8922	2.6438
0.125	1.5030	2.0027	2.7538	0.182	1.3907	1.8902	2.6418
0.126	1.5010	2.0008	2.7519	0.183	1.3888	1.8882	2.6398
0.127	1.4991	1.9988	2.7499	0.184	1.3868	1.8863	2.6379
0.128	1.4971	1.9968	2.7479	0.185	1.3848	1.8843	2.6359
0.129	1.4951	1.9948	2.7460	0.186	1.3829	1.8823	2.6339
0.130	1.4932	1.9929	2.7440	0.187	1.3809	1.8804	2.6320
0.131	1.4912	1.9909	2.7420	3/16	1.3799	1.8794	2.6310
0.132	1.4892	1.9889	2.7401	0.188	1.3789	1.8784	2.6300
0.133	1.4872	1.9869	2.7381	0.189	1.3770	1.8764	2.6281
0.134	1.4853	1.9850	2.7361	0.190	1.3750	1.8744	2.6261
0.135	1.4833	1.9830	2.7342	0.191	1.3730	1.8725	2.6241
0.136	1.4813	1.9810	2.7322	0.192	1.3711	1.8705	2.6222
0.137	1.4794	1.9790	2.7302	0.193	1.3691	1.8685	2.6202
0.138	1.4774	1.9771	2.7282	0.194	1.3671	1.8665	2.6182
0.139	1.4754	1.9751	2.7263	0.195	1.3652	1.8646	2.6163
0.140	1.4734	1.9731	2.7243	0.196	1.3632	1.8626	2.6143
9/64	1.4722	1.9719	2.7231	0.197	1.3612	1.8606	2.6123
0.141	1.4715	1.9711	2.7224	0.198	1.3592	1.8587	2.6104
0.142	1.4695	1.9692	2.7204	0.199	1.3573	1.8567	2.6084
0.143	1.4675	1.9672	2.7184	0.200	1.3553	1.8547	2.6064
0.144	1.4655	1.9652	2.7165	0.201	....	1.8527	2.6045
0.145	1.4636	1.9632	2.7145	0.202	....	1.8508	2.6025
0.146	1.4616	1.9613	2.7125	0.203	....	1.8488	2.6006
0.147	1.4596	1.9593	2.7106	13/64	....	1.8486	2.6003
0.148	1.4577	1.9573	2.7086	0.204	....	1.8468	2.5986
0.149	1.4557	1.9553	2.7066	0.205	....	1.8449	2.5966
0.150	1.4537	1.9534	2.7047	0.206	....	1.8429	2.5947
0.151	1.4517	1.9514	2.7027	0.207	....	1.8409	2.5927
0.152	1.4498	1.9494	2.7007	0.208	....	1.8390	2.5908
0.153	1.4478	1.9474	2.6988	0.209	....	1.8370	2.5888
0.154	1.4458	1.9455	2.6968	0.210	....	1.8350	2.5868
0.155	1.4439	1.9435	2.6948	0.211	....	1.8330	2.5849
0.156	1.4419	1.9415	2.6929	0.212	....	1.8311	2.5829
5/32	1.4414	1.9410	2.6924	0.213	....	1.8291	2.5809
0.157	1.4399	1.9395	2.6909	0.214	....	1.8271	2.5790
0.158	1.4380	1.9376	2.6889	0.215	....	1.8252	2.5770
0.159	1.4360	1.9356	2.6870	0.216	....	1.8232	2.5751
0.160	1.4340	1.9336	2.6850	0.217	....	1.8212	2.5731
0.161	1.4321	1.9317	2.6830	0.218	....	1.8193	2.5711
0.162	1.4301	1.9297	2.6811	7/32	....	1.8178	2.5697
0.163	1.4281	1.9277	2.6791	0.219	....	1.8173	2.5692
0.164	1.4262	1.9257	2.6772	0.220	....	1.8153	2.5672
0.165	1.4242	1.9238	2.6752	0.221	....	1.8133	2.5653
0.166	1.4222	1.9218	2.6732	0.222	....	1.8114	2.5633
0.167	1.4203	1.9198	2.6713	0.223	....	1.8094	2.5613
0.168	1.4183	1.9178	2.6693	0.224	....	1.8074	2.5594
0.169	1.4163	1.9159	2.6673	0.225	....	1.8055	2.5574
0.170	1.4144	1.9139	2.6654	0.226	....	1.8035	2.5555

Corrected Diameters of Circular Forming Tools — 3

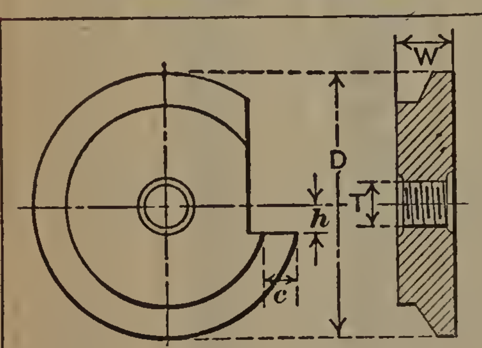
Length c on Tool	No. of B. & S. Machine		Length c on Tool	No. of B. & S. Machine		Length c on Tool	No. 2 B. & S Machine
	No. 0	No. 2		No. 0	No. 2		
0.227	1.8015	2.5535	0.284	1.6894	2.4418	0.341	2.3303
0.228	1.7996	2.5515	0.285	1.6874	2.4398	0.342	2.3284
0.229	1.7976	2.5496	0.286	1.6854	2.4378	0.343	2.3264
0.230	1.7956	2.5476	0.287	1.6835	2.4359	11/32	2.3250
0.231	1.7936	2.5456	0.288	1.6815	2.4340	0.344	2.3245
0.232	1.7917	2.5437	0.289	1.6795	2.4320	0.345	2.3225
0.233	1.7897	2.5417	0.290	1.6776	2.4300	0.346	2.3206
0.234	1.7877	2.5398	0.291	1.6756	2.4281	0.347	2.3186
15/64	1.7870	2.5390	0.292	1.6736	2.4261	0.348	2.3166
0.235	1.7858	2.5378	0.293	1.6717	2.4242	0.349	2.3147
0.236	1.7838	2.5358	0.294	1.6697	2.4222	0.350	2.3127
0.237	1.7818	2.5339	0.295	1.6677	2.4203	0.351	2.3108
0.238	1.7799	2.5319	0.296	1.6658	2.4183	0.352	2.3088
0.239	1.7779	2.5300	19/64	1.6641	2.4166	0.353	2.3069
0.240	1.7759	2.5280	0.297	1.6638	2.4163	0.354	2.3049
0.241	1.7739	2.5260	0.298	1.6618	2.4144	0.355	2.3030
0.242	1.7720	2.5241	0.299	1.6599	2.4124	0.356	2.3010
0.243	1.7700	2.5221	0.300	1.6579	2.4105	0.357	2.2991
0.244	1.7680	2.5201	0.301	....	2.4085	0.358	2.2971
0.245	1.7661	2.5182	0.302	....	2.4066	0.359	2.2952
0.246	1.7641	2.5162	0.303	....	2.4046	23/64	2.2945
0.247	1.7621	2.5143	0.304	....	2.4026	0.360	2.2932
0.248	1.7602	2.5123	0.305	....	2.4007	0.361	2.2913
0.249	1.7582	2.5104	0.306	....	2.3987	0.362	2.2893
0.250	1.7562	2.5084	0.307	....	2.3968	0.363	2.2874
0.251	1.7543	2.5064	0.308	....	2.3948	0.364	2.2854
0.252	1.7523	2.5045	0.309	....	2.3929	0.365	2.2835
0.253	1.7503	2.5025	0.310	....	2.3909	0.366	2.2815
0.254	1.7484	2.5005	0.311	....	2.3890	0.367	2.2796
0.255	1.7464	2.4986	0.312	....	2.3870	0.368	2.2776
0.256	1.7444	2.4966	5/16	....	2.3860	0.369	2.2757
0.257	1.7425	2.4947	0.313	....	2.3851	0.370	2.2737
0.258	1.7405	2.4927	0.314	....	2.3831	0.371	2.2718
0.259	1.7385	2.4908	0.315	....	2.3811	0.372	2.2698
0.260	1.7366	2.4888	0.316	....	2.3792	0.373	2.2679
0.261	1.7346	2.4868	0.317	....	2.3772	0.374	2.2659
0.262	1.7326	2.4849	0.318	....	2.3753	0.375	2.2640
0.263	1.7306	2.4829	0.319	....	2.3733	0.376	2.2620
0.264	1.7287	2.4810	0.320	....	2.3714	0.377	2.2601
0.265	1.7267	2.4790	0.321	....	2.3694	0.378	2.2581
17/64	1.7255	2.4778	0.322	....	2.3675	0.379	2.2562
0.266	1.7248	2.4770	0.323	....	2.3655	0.380	2.2542
0.267	1.7228	2.4751	0.324	....	2.3636	0.381	2.2523
0.268	1.7208	2.4731	0.325	....	2.3616	0.382	2.2503
0.269	1.7189	2.4712	0.326	....	2.3596	0.383	2.2484
0.270	1.7169	2.4692	0.327	....	2.3577	0.384	2.2464
0.271	1.7149	2.4673	0.328	....	2.3557	0.385	2.2445
0.272	1.7130	2.4653	21/64	....	2.3555	0.386	2.2425
0.273	1.7110	2.4633	0.329	....	2.3538	0.387	2.2406
0.274	1.7090	2.4614	0.330	....	2.3518	0.388	2.2386
0.275	1.7071	2.4594	0.331	....	2.3499	0.389	2.2367
0.276	1.7051	2.4575	0.332	....	2.3479	0.390	2.2347
0.277	1.7031	2.4555	0.333	....	2.3460	25/64	2.2335
0.278	1.7012	2.4535	0.334	....	2.3440	0.391	2.2328
0.279	1.6992	2.4516	0.335	....	2.3421	0.392	2.2308
0.280	1.6972	2.4496	0.336	....	2.3401	0.393	2.2289
0.281	1.6953	2.4477	0.337	....	2.3381	0.394	2.2269
9/32	1.6948	2.4472	0.338	....	2.3362	0.395	2.2250
0.282	1.6933	2.4457	0.339	....	2.3342	0.396	2.2230
0.283	1.6913	2.4438	0.340	....	2.3323	0.397	2.2211

Corrected Diameters of Circular Forming Tools — 4

Length c on Tool	No. 2 B. & S. Machine	Length c on Tool	No. 2 B. & S. Machine	Length c on Tool	No. 2 B. & S. Machine	Length c on Tool	No. 2 B. & S. Machine
0.398	2.2191	0.423	2.1704	0.449	2.1199	0.474	2.0713
0.399	2.2172	0.424	2.1685	0.450	2.1179	0.475	2.0694
0.400	2.2152	0.425	2.1666	0.451	2.1160	0.476	2.0674
0.401	2.2133	0.426	2.1646	0.452	2.1140	0.477	2.0655
0.402	2.2113	0.427	2.1627	0.453	2.1121	0.478	2.0636
0.403	2.2094	0.428	2.1607	29/64	2.1118	0.479	2.0616
0.404	2.2074	0.429	2.1588	0.454	2.1101	0.480	2.0597
0.405	2.2055	0.430	2.1568	0.455	2.1082	0.481	2.0577
0.406	2.2035	0.431	2.1549	0.456	2.1063	0.482	2.0558
13/32	2.2030	0.432	2.1529	0.457	2.1043	0.483	2.0538
0.407	2.2016	0.433	2.1510	0.458	2.1024	0.484	2.0519
0.408	2.1996	0.434	2.1490	0.459	2.1004	0.485	2.0500
0.409	2.1977	0.435	2.1471	0.460	2.0985	0.486	2.0480
0.410	2.1957	0.436	2.1452	0.461	2.0966	0.487	2.0461
0.411	2.1938	0.437	2.1432	0.462	2.0946	0.488	2.0441
0.412	2.1919	7/16	2.1422	0.463	2.0927	0.489	2.0422
0.413	2.1899	0.438	2.1413	0.464	2.0907	0.490	2.0403
0.414	2.1880	0.439	2.1393	0.465	2.0888	0.491	2.0383
0.415	2.1860	0.440	2.1374	0.466	2.0868	0.492	2.0364
0.416	2.1841	0.441	2.1354	0.467	2.0849	0.493	2.0344
0.417	2.1821	0.442	2.1335	0.468	2.0830	0.494	2.0325
0.418	2.1802	0.443	2.1315	15/32	2.0815	0.495	2.0306
0.419	2.1782	0.444	2.1296	0.469	2.0810	0.496	2.0286
0.420	2.1763	0.445	2.1276	0.470	2.0791	0.497	2.0267
0.421	2.1743	0.446	2.1257	0.471	2.0771	0.498	2.0247
27/64	2.1726	0.447	2.1237	0.472	2.0752	0.499	2.0228
0.422	2.1724	0.448	2.1218	0.473	2.0733	0.500	2.0209

Method of Using Tables for "Corrected Diameters of Circular Forming Tools." — These tables are especially applicable to the Brown & Sharpe automatic screw machines. The maximum diameter D of forming tools for these machines should be as follows: For No. 00 machine, 1¾ inch; for No. 0 machine, 2¼ inches; for No. 2 machine, 3 inches. To find the other diameters of the tool for any piece

Dimensions of Forming Tools for B. & S. Automatic Screw Machines

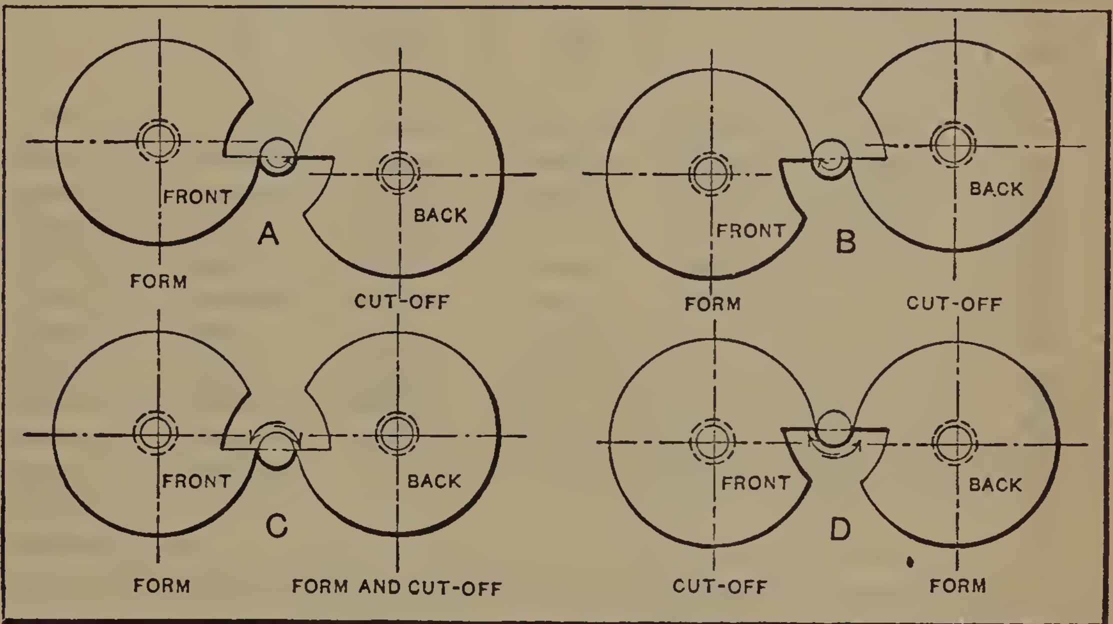


No. of Machine	Max. Diam., D	h	T	W
00	1¾	⅛	⅜–16	¼
0	2¼	5/32	½–14	5/16
2	3	¼	⅝–12	⅜
6	4	5/16	¾–12	⅜

to be formed, proceed as follows: Subtract the smallest diameter of the work from that diameter of the work which is to be formed by the required tool diameter; divide the remainder by 2; locate the quotient obtained in the column headed "Length c on Tool," and opposite the figure thus located and in the column headed by the number of the machine used, read off directly the diameter to which the tool is to be made. The quotient obtained, which is located in the column headed "Length c on Tool," is the length c as shown in the illustration above.

Example: — A piece of work is to be formed on a No. 0 machine to two diameters, one being ¼ inch and one 0.550 inch; find the diameters of the tool. The maximum tool diameter is 2¼ inches. This will be the diameter which will cut the ¼ inch diameter of the work. To find the other diameter, proceed according to the rule given: 0.550 − ¼ = 0.300; 0.300 ÷ 2 = 0.150. In Table 2, opposite 0.150, we find that the required tool diameter is 1.9534 inch. These tables are for tools without rake

Arrangement of Circular Tools. — When applying circular tools to automatic screw machines, their arrangement has an important bearing on the results obtained. The various ways of arranging the circular tools, with relation to the rotation of the spindle, are shown at *A*, *B*, *C* and *D*, in the illustration. These diagrams represent the view obtained when looking towards the chuck. The arrangement at *A* gives good results for long forming on brass, steel or gun-screw iron, for the reason that the pressure of the cut on the front tool is downward; the support is more rigid than when the forming tool is turned upside down on the front slide as shown at *B*; here the stock, turning up towards the tool, has a tendency to lift



the cross-slide, causing chattering; therefore, the arrangement shown at *A* is recommended when a high finish is desired. The arrangement at *B* works satisfactorily for short steel pieces which do not require a high finish; it allows the chips to drop clear of the work, and is especially advantageous when making screws, when the forming and cut-off tools operate after the die, as no time is lost in reversing the spindle. The arrangement at *C* is recommended for heavy cutting on large work, when both tools are used for forming the piece; a rigid support is then necessary for both tools and a good supply of oil is also required. The arrangement at *D* is objectionable and should be avoided; it is used only when a left-hand thread is cut on the piece and when the cut-off tool is used on the front slide, leaving the heavy cutting to be performed from the rear slide. In all "cross-forming" work, it is essential that the spindle be kept in good condition, and that the collet or chuck have a parallel contact upon the bar which is being formed.

Speeds for Forming Tools. — Surface speeds for turning different materials with forming tools are given in the following table. These speeds correspond approximately to general practice, and are intended to serve as a guide. An ample supply of lard oil should be used and the tools kept in good condition.

Surface Speeds for Forming Tools

Material	Surface Speed, Feet per Minute	
	Carbon Steel	High-speed Steel
Brass rod	175–200	225–300
Gun-screw iron	75– 95	100–125
Norway iron and machine steel	55– 75	80–100
Drill rod and tool steel	40– 55	60– 75

Feeds per Revolution for Forming Tools

Width of Forming Tool	Smallest Diameter of Part Formed					
	3/32	1/8	5/32	3/16	7/32	1/4
1/16	0.00075	0.0008	0.0009	0.001	0.0011	0.0012
3/32	0.0007	0.0008	0.0009	0.001	0.001	0.0013
1/8	0.00055	0.0007	0.0008	0.001	0.0009	0.0012
3/16	0.0003	0.0007	0.0008	0.0009	0.0009	0.0011
7/32	....	0.0005	0.0007	0.0009	0.0009	0.0011
1/4	....	0.0002	0.0007	0.0009	0.0009	0.001
5/16	....	....	0.0005	0.0008	0.0008	0.001
3/8	....	....	0.0002	0.0008	0.0008	0.0009
1/2	....	....	....	0.0008	0.0008	0.0009
5/8	....	....	....	0.0003	0.0005	0.0009
3/4	....	....	....	....	0.0002	0.0008
7/8	....	....	....	....	....	0.0005
1	....	....	....	....	....	0.0002

Width of Forming Tool	Smallest Diameter of Part Formed					
	5/16	3/8	1/2	5/8	3/4	7/8
1/16	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012
3/32	0.0014	0.0015	0.0017	0.0019	0.002	0.0021
1/8	0.0014	0.0016	0.002	0.0023	0.0025	0.0025
3/16	0.0015	0.0016	0.0018	0.0019	0.0021	0.0022
7/32	0.0013	0.0015	0.0017	0.0018	0.002	0.0021
1/4	0.0012	0.0015	0.0016	0.0017	0.0018	0.002
5/16	0.0011	0.0013	0.0015	0.0016	0.0017	0.0018
3/8	0.001	0.0012	0.0012	0.0014	0.0016	0.0017
1/2	0.0009	0.001	0.0011	0.0013	0.0015	0.0016
5/8	0.0009	0.0009	0.001	0.0012	0.0014	0.0015
3/4	0.0008	0.0009	0.0009	0.0011	0.0013	0.0014
7/8	0.0006	0.0008	0.0009	0.001	0.0012	0.0013
1	0.0005	0.0008	0.0008	0.0009	0.001	0.0012

Hollow Milling, Drilling, Reaming, Threading and Knurling

Hollow Mills. — For roughing cuts, especially in brass, a hollow mill gives satisfactory results. The accompanying table gives the proportions of hollow mills and cutting angles for various materials. The sizes varying from 0.065 to 0.462 inch given in Column *A* are for roughing mills for the A. S. M. E. standard and special machine screw sizes, there being an allowance of from 0.005 to 0.015 inch for finishing. In making these hollow mills, they should be reamed out tapering from the rear to give clearance to the blades. A taper of from 1/8 to 3/16 inch per foot is usually satisfactory. For steel, the cutting edge is set about 1/10 of the diameter ahead of the center, but for brass, it should be on the center-line.

Speeds for External Cutting Tools. — The following speeds are applicable to external cutting tools, such as box-tool cutters, hollow mills, etc. These speeds are intended for average conditions on the materials specified.

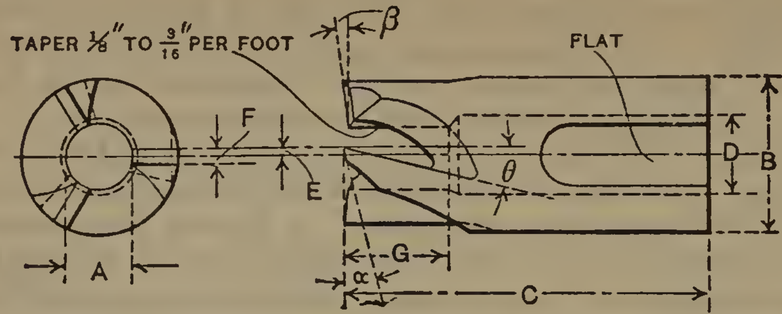
Material	Surface Speed, Feet per Minute	
	Carbon Steel	High-speed Steel
Brass (ordinary quality)	170–180	250–270
Gun-screw iron	70– 80	100–120
Norway iron and machine steel	60– 70	90–100
Drill rod and tool steel	35– 40	50– 60

Centering Tools. — When drilling holes which are less than 3/16 inch in diameter in the automatic screw machine, it is always advisable, especially if the hole passes through the work, to use a starting or centering tool. The included angle of the cutting edges on the centering tool should be less than the angle of the drill which is to follow, as otherwise the point of the drill will start to cut before the body of the drill is properly supported, and the drill may not start concentric with the work. The included angle of the point which has been found suitable for centering tools varies from 90 to 100 degrees, 90 degrees being preferable for brass and 100 degrees for steel. When the material is quite hard, a centering tool having a double angle is sometimes used. The point has an included angle of 118 degrees, while the remaining part of the cutting edge has an included angle of 90 degrees. This strengthens the point of the tool and, at the same time, a center is formed that supports the body of the drill before the point begins to cut. The table "Length of Point on Twist Drills and Centering Tools" was compiled to facilitate determining the rise on the cam necessary for the centering operation. Feeds which have been found satisfactory for general work are given in the table "Feeds for Centering Tools."

Feeds for Centering Tools

Diameter in Inches	Feed in Inches per Revolution		
	Brass Rod	Machine Steel	Tool Steel
1/4	0.004	0.003	0.002
5/16	0.004	0.004	0.003
3/8	0.005	0.0045	0.004
1/2	0.0055	0.005	0.0045
3/4	0.006	0.005	0.005
1	0.0065	0.005	0.0055

Proportions of Hollow Mills



A	B	C	D	E	F	G
0.065	½	1¼	¼	0.007	0.020	¼
0.078	½	1¼	¼	0.008	0.020	¼
0.091	½	1¼	¼	0.009	0.025	¼
0.105	½	1¼	¼	0.011	0.025	¼
0.120	½	1¼	¼	0.012	0.025	¼
0.135	½	1¼	¼	0.014	0.030	¼
0.148	½	1¼	¼	0.015	0.030	¼
0.161	½	1¼	¼	0.016	0.035	5/16
0.174	½	1¼	¼	0.017	0.035	5/16
0.187	½	1¼	¼	0.019	0.040	5/16
0.200	⅝	1½	⅜	0.020	0.040	5/16
0.226	⅝	1½	⅜	0.023	0.045	5/16
0.252	⅝	1½	⅜	0.025	0.045	5/16
0.280	⅝	1½	⅜	0.028	0.050	⅜
0.305	⅝	1½	⅜	0.031	0.060	⅜
0.332	⅝	1½	⅜	0.033	0.065	⅜
0.358	⅝	1½	⅜	0.036	0.070	7/16
0.385	1	2¼	¾	0.039	0.075	7/16
0.410	1	2¼	¾	0.041	0.080	7/16
0.436	1	2¼	¾	0.044	0.085	½
0.462	1	2¼	¾	0.046	0.090	½
0.480	1	2¼	¾	0.048	0.095	⅝
0.515	1	2¼	¾	0.052	0.100	⅝
0.578	1	2¼	¾	0.058	0.105	¾
0.640	1	2¼	¾	0.064	0.110	¾
0.703	1	2¼	13/16	0.070	0.115	⅞
0.765	1	2¼	13/16	0.077	0.120	⅞

Cutting Angles for Hollow Mills

Angle	Brass Rod	Machine Steel	Tool Steel
α	8 degrees	15 degrees	10 degrees
β	6 degrees	5 degrees	3 degrees
θ	....	15 degrees	10 degrees

Feeds for Swing Tools and Hollow Mills
(High-speed or Carbon Steel Tools)

Smallest Diam. of Stock	Brass Rod, Feed per Revolution	Machine Steel, Feed per Revolution	Tool Steel, Feed per Revolution	Smallest Diam. of Stock	Brass Rod, Feed per Revolution	Machine Steel, Feed per Revolution	Tool Steel, Feed per Revolution
Feeds for Swing Tools							
1/32-inch Chip				1/8-inch Chip			
1/16	0.0010	0.0008	0.0005	3/8	0.0020	0.0015	0.0010
1/8	0.0015	0.0010	0.0008	7/16	0.0025	0.0018	0.0015
3/16	0.0020	0.0015	0.0010	1/2	0.0030	0.0020	0.0018
1/4	0.0030	0.0020	0.0015	9/16	0.0035	0.0025	0.0020
5/16	0.0035	0.0025	0.0018	5/8	0.0038	0.0028	0.0022
3/8	0.0040	0.0030	0.0020	11/16	0.0042	0.0030	0.0025
1/16-inch Chip				3/16-inch Chip			
1/4	0.0025	0.0020	0.0010	1/2	0.0020	0.0010	0.0008
5/16	0.0030	0.0022	0.0013	9/16	0.0025	0.0013	0.0010
3/8	0.0035	0.0025	0.0015	5/8	0.0028	0.0015	0.0012
7/16	0.0040	0.0028	0.0018	11/16	0.0030	0.0018	0.0015
1/2	0.0045	0.0030	0.0020	3/4	0.0035	0.0020	0.0018
9/16	0.0050	0.0032	0.0025	13/16	0.0038	0.0022	0.0020
5/8	0.0060	0.0035	0.0028	7/8	0.0040	0.0025	0.0020
Feeds for Hollow Mills							
1/16-inch Chip				3/16-inch Chip			
3/16	0.0045	0.0030	0.0015	1/2	0.0060	0.0045	0.0020
1/4	0.0050	0.0040	0.0018	9/16	0.0065	0.0050	0.0023
5/16	0.0055	0.0045	0.0020	5/8	0.0070	0.0055	0.0025
3/8	0.0060	0.0050	0.0025	11/16	0.0080	0.0060	0.0030
7/16	0.0070	0.0050	0.0028	3/4	0.0090	0.0065	0.0035
1/2	0.0080	0.0060	0.0030	13/16	0.0100	0.0070	0.0040
1/8-inch Chip				1/4-inch Chip			
3/8	0.0070	0.0050	0.0030	5/8	0.0050	0.0035	0.0015
7/16	0.0075	0.0055	0.0035	11/16	0.0055	0.0040	0.0018
1/2	0.0080	0.0060	0.0040	3/4	0.0060	0.0050	0.0020
9/16	0.0090	0.0065	0.0050	13/16	0.0070	0.0055	0.0025
5/8	0.0110	0.0075	0.0060	7/8	0.0080	0.0060	0.0030
11/16	0.0130	0.0090	0.0070	......	......	......	......
3/4	0.0150	0.0110	0.0080	......	......	......	......

Length of Point on Twist Drills and Centering Tools

Size of Drill	Decimal Equivalent	Length of Point when Included Angle = 90°	Length of Point when Included Angle = 118°	Size of Drill	Decimal Equivalent	Length of Point when Included Angle = 90°	Length of Point when Included Angle = 118°	Size or Diam. of Drill	Decimal Equivalent	Length of Point when Included Angle = 90°	Length of Point when Included Angle = 118°	Diam. of Drill	Decimal Equivalent	Length of Point when Included Angle = 90°	Length of Point when Included Angle = 118°
60	0.0400	0.020	0.012	37	0.1040	0.052	0.031	14	0.1820	0.091	0.055	3/8	0.3750	0.188	0.113
59	0.0410	0.021	0.012	36	0.1065	0.054	0.032	13	0.1850	0.093	0.056	25/64	0.3906	0.195	0.117
58	0.0420	0.021	0.013	35	0.1100	0.055	0.033	12	0.1890	0.095	0.057	13/32	0.4063	0.203	0.122
57	0.0430	0.022	0.013	34	0.1110	0.056	0.033	11	0.1910	0.096	0.057	27/64	0.4219	0.211	0.127
56	0.0465	0.023	0.014	33	0.1130	0.057	0.034	10	0.1935	0.097	0.058	7/16	0.4375	0.219	0.131
55	0.0520	0.026	0.016	32	0.1160	0.058	0.035	9	0.1960	0.098	0.059	29/64	0.4531	0.227	0.136
54	0.0550	0.028	0.017	31	0.1200	0.060	0.036	8	0.1990	0.100	0.060	15/32	0.4688	0.234	0.141
53	0.0595	0.030	0.018	30	0.1285	0.065	0.039	7	0.2010	0.101	0.060	31/64	0.4844	0.242	0.145
52	0.0635	0.032	0.019	29	0.1360	0.068	0.041	6	0.2040	0.102	0.061	1/2	0.5000	0.250	0.150
51	0.0670	0.034	0.020	28	0.1405	0.070	0.042	5	0.2055	0.103	0.062	33/64	0.5156	0.258	0.155
50	0.0700	0.035	0.021	27	0.1440	0.072	0.043	4	0.2090	0.105	0.063	17/32	0.5313	0.266	0.159
49	0.0730	0.037	0.022	26	0.1470	0.074	0.044	3	0.2130	0.107	0.064	35/64	0.5469	0.273	0.164
48	0.0760	0.038	0.023	25	0.1495	0.075	0.045	2	0.2210	0.111	0.067	9/16	0.5625	0.281	0.169
47	0.0785	0.040	0.024	24	0.1520	0.076	0.046	1	0.2280	0.114	0.068	37/64	0.5781	0.289	0.173
46	0.0810	0.041	0.024	23	0.1540	0.077	0.046	15/64	0.2344	0.117	0.070	19/32	0.5938	0.297	0.178
45	0.0820	0.041	0.025	22	0.1570	0.079	0.047	1/4	0.2500	0.125	0.075	39/64	0.6094	0.305	0.183
44	0.0860	0.043	0.026	21	0.1590	0.080	0.048	17/64	0.2656	0.133	0.080	5/8	0.6250	0.313	0.188
43	0.0890	0.045	0.027	20	0.1610	0.081	0.048	9/32	0.2813	0.141	0.084	41/64	0.6406	0.320	0.192
42	0.0935	0.047	0.028	19	0.1660	0.083	0.050	19/64	0.2969	0.148	0.089	21/32	0.6563	0.328	0.197
41	0.0960	0.048	0.029	18	0.1695	0.085	0.051	5/16	0.3125	0.156	0.094	43/64	0.6719	0.336	0.202
40	0.0980	0.049	0.029	17	0.1730	0.087	0.052	21/64	0.3281	0.164	0.098	11/16	0.6875	0.344	0.206
39	0.0995	0.050	0.030	16	0.1770	0.089	0.053	11/32	0.3438	0.171	0.103	23/32	0.7188	0.359	0.216
38	0.1015	0.051	0.030	15	0.1800	0.090	0.054	23/64	0.3594	0.180	0.108	3/4	0.7500	0.375	0.225

Speeds for Drilling. — When drilling in automatic screw machines designed for high speeds, the best results are generally obtained by giving the drills light feeds and high peripheral velocities. High-speed steel drills are adapted to drilling Norway iron, machine steel, tool steel, etc., and ordinary carbon steel drills are used for brass and similar materials, when the surface speed does not exceed that given in the following:

Material	Surface Speed, Feet per Minute	
	Carbon Steel	High-speed Steel
Brass (ordinary quality)	160–180	....
Gun-screw iron	60– 70	100–125
Norway iron and machine steel	50– 60	80–100
Drill rod and tool steel	30– 40	50– 60

Feeds for Drilling. — Feeds for high-speed and ordinary carbon steel twist drills are given in the table "Feeds for Twist Drills used in Automatic Screw Machines." These feeds are for general work and can be increased in some cases. For general practice, it is more satisfactory to use rather light feeds, as a straighter hole can be produced than when the drill is forced. Drills from 1/8 inch to 3/16 inch in diameter are capable of standing the heaviest feeds in proportion to their diameter, and when a hole does not pass through the work, a 1/8-inch drill has been fed as much as 0.016 inch per revolution when drilling brass. Feeds as heavy as this are not recommended, because concentric holes cannot be produced when the drill is forced to such an extent.

Feeds for Twist Drills Used in Automatic Screw Machines
(High-speed and Carbon Steel Drills)

Drill Size	Feed per Revolution			Drill Size	Feed per Revolution		
	Brass Rod	Machine Steel	Tool Steel		Brass Rod	Machine Steel	Tool Steel
70	0.00070	0.00060	0.00050	9/32	0.0058	0.0052	0.0042
65	0.00075	0.00065	0.00055	5/16	0.0059	0.0055	0.0043
60	0.00080	0.00070	0.00060	11/32	0.0060	0.0058	0.0045
56	0.00120	0.00100	0.00080	3/8	0.0062	0.0060	0.0048
1/16	0.00180	0.00150	0.00100	13/32	0.0065	0.0062	0.0050
5/64	0.00250	0.00200	0.00120	7/16	0.0068	0.0065	0.0052
3/32	0.00250	0.00230	0.00150	15/32	0.0070	0.0068	0.0055
7/64	0.00300	0.00250	0.00180	1/2	0.0070	0.0070	0.0058
1/8	0.00320	0.00280	0.00200	17/32	0.0072	0.0072	0.0059
9/64	0.00350	0.00300	0.00230	9/16	0.0075	0.0075	0.0060
5/32	0.00380	0.00320	0.00250	19/32	0.0078	0.0078	0.0062
11/64	0.00400	0.00350	0.00280	5/8	0.0080	0.0079	0.0063
3/16	0.00420	0.00400	0.00300	21/32	0.0082	0.0080	0.0064
13/64	0.00450	0.00420	0.00320	11/16	0.0085	0.0082	0.0065
7/32	0.00480	0.00450	0.00350	3/4	0.0090	0.0083	0.0068
15/64	0.00500	0.00480	0.00380	13/16	0.0095	0.0085	0.0069
1/4	0.00550	0.00500	0.00400	7/8	0.0100	0.0088	0.0070

Counterboring. — As a rule, more trouble is experienced in using counterbores on automatic machines than with any other cutting tool. This is probably due to the fact that counterbores are generally improperly made for the work on which they are to operate. Generally speaking, there are several reasons for the unsuccessful working of counterbores, some of which may be summed up as follows: 1. Too many cutting edges, not allowing enough chip space and also not providing for sufficient lubrication. 2. Too much cutting surface in contact with the work. 3. Insufficient clearance on the periphery of the teeth. 4. Improper location of the cutting edges relative to the center. 5. Improper method of holding the counterbore. 6. Improper grinding of the cutting edges. 7. Too weak a cross-section. 8. The use of a feed and speed in excess of what the tool will stand.

For work in automatic machines, where the counterbore cannot be withdrawn when it plugs up with chips and seizes in the work, the tool should not have more than three cutting teeth. The periphery of the teeth should be backed off eccentrically, and the body of the counterbore should taper towards the back. The amount of taper generally varies from 0.020 to 0.040 inch per foot. The relation of the cutting edge to the center has an important bearing on the efficiency of the tool. For deep counterboring, where the difference between the diameter of the teat and the body of the counterbore is great, the cutting edge should never be located ahead of the center; often, if it is located a little behind the center, better results are obtained; but this rule is only general, as the material to a considerable extent governs the location of the cutting edges. It is advisable to have the cutting edge ahead of the center when the counterbore is to be used as a facing tool, or for counterboring brass, provided it is not required to enter the work to a depth greater than its diameter. For general work, the cutting edges should be radial. Straight flutes are suitable for either brass or steel, but for steel, it is better to have the teeth cut spirally, the spiral being sufficient to give a rake of from 10 to 15 degrees. If the difference between the diameter of the pilot and the body of the counterbore is not very great, and if the counterbore must extend into the work to a depth greater than its diameter, the cutting edge should be back of the center, that is, to the rear of the radial line parallel to the cutting face. When the counterbore has to remove considerable material or enter the work to a depth greater than its diameter, it is generally advisable to rough out the hole to the diameter of the body of the counterbore with a three-fluted drill; then the counterbore is used only for squaring up the bottom of the hole. This method is especially advisable when counterboring machine or tool steel.

Speeds for Counterbores. — The surface speed at which a counterbore can be worked is slightly less than the surface speed used for drilling. The surface speeds given below are recommended for counterbores made from carbon and high-speed steel.

Material	Surface Speed, Feet per Minute	
	Carbon Steel	High-speed Steel
Brass (ordinary quality)	150–160	180–200
Gun-screw iron	50– 60	80– 90
Norway iron and machine steel	40– 50	70– 80
Drill rod and tool steel	30– 35	45– 50

Feeds for Counterbores. — The method of holding a counterbore when applying it to the work, and the strength of the cross-section in proportion to the width of the chip being removed, governs, to a considerable extent, the amount of feed. The material being cut and the depth to which the counterbore penetrates into the work also have an important bearing on the rate of feed. These conditions should be taken into consideration when using the feeds given in the table "Feeds for Counterboring in Automatic Screw Machines." These feeds are for counterbores having three cutting edges; if there is but one cutting edge, the feed should be decreased from 40 to 50 per cent, and for two cutting edges, from 15 to 20 per cent. The feeds given in this table apply only when the counterbore penetrates from one-half to three-quarters of its diameter into the work. When the counterbore penetrates to a greater distance, the feed should be decreased from 15 to 25 per cent. It is good practice to always drop the counterbore back after it has penetrated to a depth equal to half its diameter, to remove the chips and to cool and lubricate it.

Feeds for Counterboring in Automatic Screw Machines
(High-speed and Carbon Steel Tools)

Diam. of Counterbore	Feed per Revolution			Diam. of Counterbore	Feed per Revolution		
	Brass Rod	Machine Steel	Tool Steel		Brass Rod	Machine Steel	Tool Steel
	Chip Width, 1/32 Inch				Chip Width, 1/8 Inch		
1/8	0.0025	0.0018	0.0015	1/2	0.0050	0.0042	0.0025
3/16	0.0030	0.0023	0.0020	9/16	0.0052	0.0045	0.0030
1/4	0.0035	0.0030	0.0025	5/8	0.0055	0.0048	0.0032
5/16	0.0045	0.0040	0.0030	11/16	0.0058	0.0050	0.0035
3/8	0.0050	0.0045	0.0035	3/4	0.0060	0.0055	0.0040
7/16	0.0060	0.0050	0.0038	13/16	0.0065	0.0058	0.0045
1/2	0.0075	0.0052	0.0040	7/8	0.0070	0.0060	0.0050
Diam.	Chip Width, 1/16 Inch			Diam.	Chip Width, 3/16 Inch		
1/4	0.0030	0.0028	0.0020	9/16	0.0045	0.0035	0.0025
5/16	0.0035	0.0030	0.0025	5/8	0.0048	0.0038	0.0028
3/8	0.0040	0.0035	0.0028	11/16	0.0050	0.0040	0.0030
7/16	0.0045	0.0038	0.0030	3/4	0.0055	0.0043	0.0032
1/2	0.0050	0.0040	0.0035	13/16	0.0060	0.0045	0.0035
9/16	0.0055	0.0045	0.0038	7/8	0.0065	0.0048	0.0038
5/8	0.0060	0.0050	0.0040	15/16	0.0070	0.0050	0.0040
Diam.	Chip Width, 3/32 Inch			Diam.	Chip Width, 1/4 Inch		
3/8	0.0040	0.0032	0.0020	11/16	0.0040	0.0030	0.0020
7/16	0.0045	0.0035	0.0025	3/4	0.0042	0.0032	0.0022
1/2	0.0050	0.0040	0.0030	13/16	0.0045	0.0035	0.0025
9/16	0.0055	0.0045	0.0035	7/8	0.0048	0.0038	0.0030
5/8	0.0060	0.0050	0.0040	15/16	0.0050	0.0040	0.0032
11/16	0.0070	0.0055	0.0045	1	0.0050	0.0045	0.0035
7/8	0.0075	0.0060	0.0050	......	......	......	......

Reaming. — When reaming holes in automatic screw machines, it is advisable not to leave any more material to be removed by the reamer than is absolutely necessary. For general work, the following allowances will give good results for reamers ranging in diameter from 1/8 to 3/8 inch. For reamers over 3/8 inch diameter, a drill 1/64 inch less in diameter is generally used; this would leave from 0.012 to 0.015 inch to remove, as the drill will cut slightly larger than its nominal size.

Diameter of reamer in inches	1/8	3/16	1/4	5/16	3/8
Diameter of hole before reaming	0.120	0.182	0.242	0.302	0.368

Various reasons for the inefficient working of reamers are as follows: 1. Chattering, which results when the teeth are evenly spaced. 2. Chips clinging to the teeth, owing to high peripheral velocities and insufficient clearance. 3. Enlarged and tapered holes, due to holding the reamer rigidly instead of "floating." The floating type of holder should always be used when reaming deep holes. There are various methods adopted to prevent reamers from chattering, but the unequal spacing of the teeth has been found the most satisfactory and inexpensive. For machine reamers varying from 1/8 to 1/4 inch, three cutting edges are sometimes used, but the difficulty of measuring the diameter limits their use. As a general rule, four and six cutting edges are used on reamers varying from 1/8 inch to 3/8 inch, and eight to twelve cutting edges on reamers varying from 3/8 inch to 7/8 inch.

Speeds for Reaming. — The surface speeds used for reaming should be slightly less than for counterboring, as the reamer generally penetrates to a greater depth and has more cutting edges in contact with the work. When a good supply of lard oil is used, the following surface speeds will be found satisfactory:

Material	Surface Speed, Feet per Minute	
	Carbon Steel	High-speed Steel
Brass (ordinary quality)	120–125	150–160
Gun-screw iron	35– 40	65– 75
Norway iron and machine steel	30– 35	50– 60
Drill rod and tool steel	20– 25	30– 40

Feeds for Reaming in Automatic Screw Machines
(High-speed and Carbon Steel Tools)

Diam. of Reamer	Feed per Revolution			Diam. of Reamer	Feed per Revolution		
	Brass Rod	Machine Steel	Tool Steel		Brass Rod	Machine Steel	Tool Steel
1/8	0.007	0.004	0.002	9/16	0.014	0.010	0.009
3/16	0.008	0.004	0.003	5/8	0.015	0.011	0.010
1/4	0.009	0.005	0.004	11/16	0.016	0.012	0.011
5/16	0.010	0.006	0.005	3/4	0.017	0.013	0.011
3/8	0.011	0.007	0.006	13/16	0.018	0.014	0.012
7/16	0.012	0.008	0.007	7/8	0.020	0.015	0.012
1/2	0.013	0.009	0.008	......	......	......	......

Feeds for Reaming. — The feeds for reaming, given in the table "Feeds for Reaming in Automatic Screw Machines" will be found suitable when the amount of material removed does not exceed that given in the preceding paragraph on "Reaming." When reaming thin tubing, especially of brass, the feed should be decreased somewhat.

Speeds for Recessing Tools. — The surface speeds of recessing tools can be slightly greater than those used for counterboring, on account of the light feeds and small amount of cutting surface in contact with the work. As a rule, the following surface speeds can be used with satisfactory results:

Material	Surface Speed, Feet per Minute	
	Carbon Steel	High-speed Steel
Brass (ordinary quality)	170–180	200–225
Gun-screw iron	60– 70	90–100
Norway iron and machine steel	45– 55	75– 85
Drill rod and tool steel	35– 40	50– 60

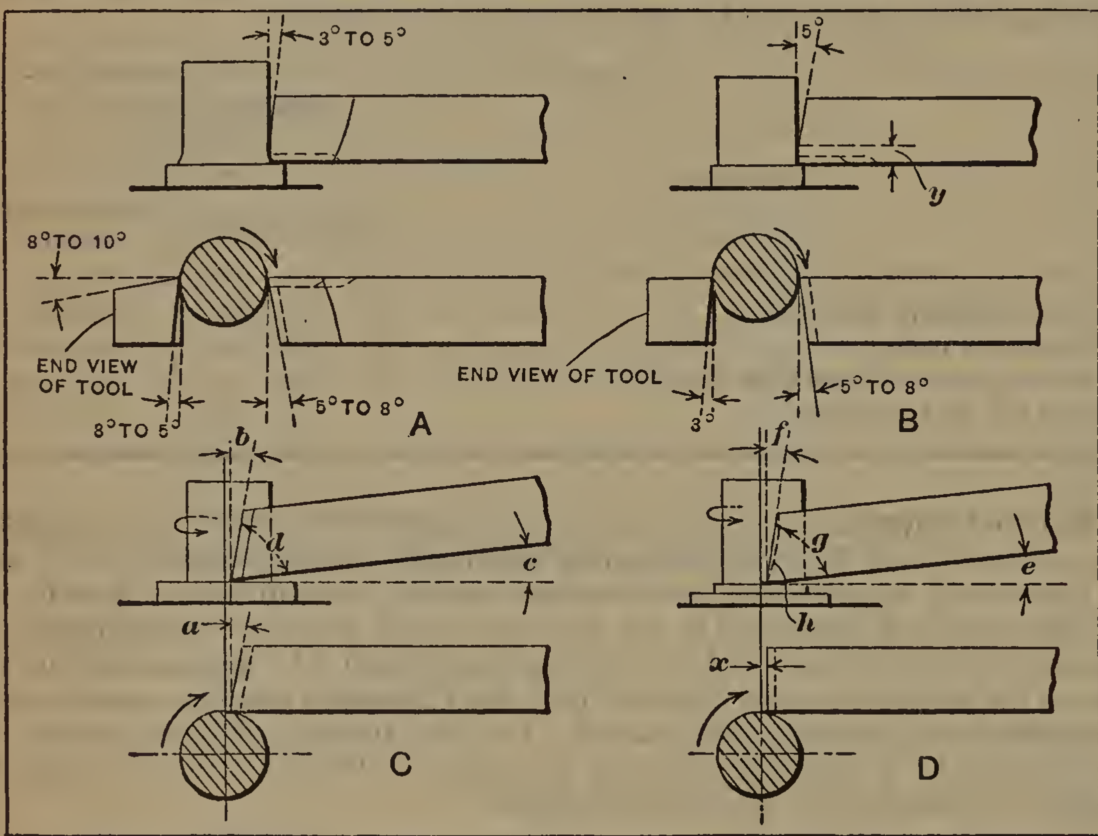
Box-tool Cutters. — Box-tool cutters are applied to the work either radially, as shown at *A* and *B* (see accompanying illustration), or tangentially, as at *C* and *D*. Generally, in automatic screw machine practice, the cutter is set radially for turning brass and, when held in this way, the cutting angles are approximately as given in the illustration. Tool *A* is for roughing and tool *B* for finishing, the cutting face of the latter being ground parallel for a short distance *y* equal to approximately one-fifth of the diameter being turned. For steel turning, the cutter should be set tangentially to the work as shown at *C* and *D*. The end of tool *C* should be ground to approximately the following angles:

Cutting Angles for Machine Steel	Cutting Angles for Tool Steel
a = 10 degrees;	a = 8 degrees;
b = 10 degrees;	b = 8 degrees;
c = 8 to 10 degrees;	c = 8 to 10 degrees;
d = 70 to 72 degrees.	d = 72 to 74 degrees.

The form of tool shown at *C* is commonly used for roughing cuts, but will not produce an absolutely square shoulder. For finishing cuts, the tool is ground as shown at *D*, which produces a square shoulder. The cutting angles for tool *D* are as follows:

Cutting Angles for Machine Steel	Cutting Angles for Tool Steel
e = from 10 to 12 degrees;	e = from 8 to 10 degrees;
f = from 15 to 18 degrees;	f = from 8 to 10 degrees;
g = from 60 to 65 degrees.	g = from 70 to 74 degrees.

While the cutting face on the tool shown at *D* is straight, it is usually advisable, especially when cutting machine steel and Norway iron, to give more "lip" to the tool as shown by the dotted line *h*. The cutting edge of a radial cutter for rough turning brass rod is set above the horizontal center-line of the work, an amount equal to about 0.02 times the diameter being turned. If the stock is rough or of irregular shape, the cutter should precede the support by an amount equal to from 0.010 to 0.020 inch, but when the bar is cylindrical and has a finished surface, the support for roughing cuts should precede the tool. The face of a tangent cutter should be set back a distance x (see illustration at D) equal to about 1/8 the diameter being turned, for tool steel, and 1/10 the diameter, for machine steel. Sometimes, it is also advisable, especially when cutting machine steel, to elevate the tool from the horizontal an angle of from 1 to 2 degrees, to increase the clearance.



Diagrammatical Views of Different Methods of Application of Box-tool Cutters in Automatic Screw Machine Practice

Size of Steel for Box-tool Cutters. — For special conditions, the tool is sometimes made of rectangular section but ordinarily square stock is used. The square sections recommended for box-tool cutters are as follows:

Largest diameter of work, in inches:	1/4	3/8	1/2	3/4	1
Square section of tool, in inches:	3/16	1/4	5/16	3/8	7/16

Automatic Screw Machine Tapping. — Tapping on the automatic screw machine presents certain difficulties. There is a tendency for the chips to clog and break the tap at the moment of reversal, as the chips then lodge back of the cutting edges and tend to prevent the tap from reversing. Taps for screw machine work should have a liberal chip space, the flutes being as deep and the lands as narrow as possible. The cutting edges are, in general, radial. The tap drill diameters listed in the table "Tap Drills for A. S. M. E. Standard and Special Machine Screws" have given good results in practice. These sizes may be used for all classes of work and material. The thread obtained varies from 5/8 to 3/4 of a full thread.

Feeds for Box-tools
(High-speed or Carbon Steel Tools)

Roughing Box-tools							
Smallest Diam. of Stock	Brass Rod, Feed per Revolution	Machine Steel, Feed per Revolution	Tool Steel, Feed per Revolution	Smallest Diam. of Stock	Brass Rod, Feed per Revolution	Machine Steel, Feed per Revolution	Tool Steel, Feed per Revolution
1/32-inch Chip				1/8-inch Chip			
1/16	0.0020	0.0015	0.0010	3/8	0.0045	0.0030	0.0020
1/8	0.0030	0.0020	0.0015	7/16	0.0050	0.0035	0.0025
3/16	0.0040	0.0030	0.0020	1/2	0.0060	0.0040	0.0030
1/4	0.0050	0.0040	0.0025	9/16	0.0070	0.0050	0.0035
5/16	0.0060	0.0045	0.0030	5/8	0.0085	0.0060	0.0040
3/8	0.0075	0.0050	0.0035	3/4	0.0100	0.0070	0.0050
1/16-inch Chip				3/16-inch Chip			
1/4	0.0045	0.0030	0.0020	1/2	0.0040	0.0025	0.0015
5/16	0.0060	0.0040	0.0025	9/16	0.0045	0.0030	0.0018
3/8	0.0090	0.0060	0.0030	5/8	0.0050	0.0032	0.0020
7/16	0.0105	0.0070	0.0040	11/16	0.0055	0.0035	0.0023
1/2	0.0120	0.0080	0.0050	3/4	0.0060	0.0040	0.0025
9/16	0.0135	0.0090	0.0060	13/16	0.0070	0.0045	0.0028
5/8	0.0150	0.0100	0.0075	7/8	0.0075	0.0050	0.0030

Finishing Box-tools							
0.005-inch Chip				0.020-inch Chip			
1/32	0.0020	0.0020	0.0018	3/8	0.0040	0.0040	0.0025
1/16	0.0030	0.0030	0.0020	7/16	0.0045	0.0045	0.0030
3/32	0.0045	0.0045	0.0025	1/2	0.0050	0.0050	0.0035
1/8	0.0060	0.0060	0.0030	9/16	0.0060	0.0060	0.0035
3/16	0.0070	0.0070	0.0040	5/8	0.0070	0.0070	0.0040
1/4	0.0080	0.0080	0.0050	11/16	0.0075	0.0075	0.0045
5/16	0.0100	0.0100	0.0060	3/4	0.0080	0.0080	0.0050
3/8	0.0120	0.0120	0.0080	13/16	0.0090	0.0090	0.0050
0.010-inch Chip				0.030-inch Chip			
1/4	0.0070	0.0070	0.0035	1/2	0.0040	0.0040	0.0020
5/16	0.0080	0.0080	0.0040	9/16	0.0045	0.0045	0.0022
3/8	0.0085	0.0085	0.0045	5/8	0.0050	0.0050	0.0025
7/16	0.0090	0.0090	0.0050	11/16	0.0055	0.0055	0.0028
1/2	0.0095	0.0095	0.0055	3/4	0.0060	0.0060	0.0030
9/16	0.0100	0.0100	0.0060	13/16	0.0070	0.0070	0.0035
5/8	0.0100	0.0100	0.0065	7/8	0.0080	0.0080	0.0040

Speeds for Dies and Taps. — As a general rule, a die can be operated at a higher rate of speed than a tap, because a die can be left harder than a tap, and it can be supplied with oil much easier. The following surface speeds have been found suitable for taps and dies made from ordinary carbon steel:

Material	Surface Speeds, Feet per Minute	
	For Dies	For Taps
Brass (ordinary quality)	190–200	150–160
Norway iron and machine steel	30– 40	25– 30
Drill rod and tool steel	20– 30	15– 20

Tap Drills for A. S. M. E. Standard and Special Machine Screws

Northern Electric Co.'s Practice for Screw Machine Work

(Special sizes are marked *)

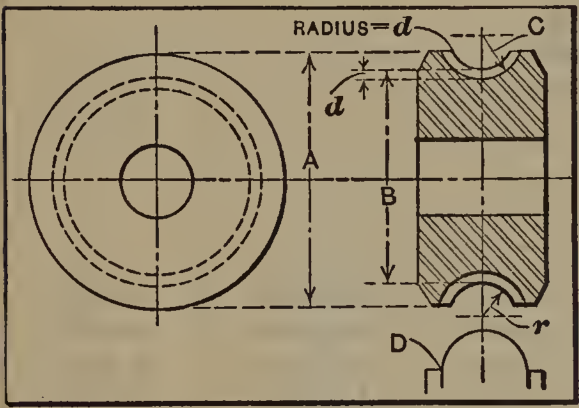
Size of Screw	Threads per Inch	Size of Tap Drill	Decimal Equivalent of Tap Drill	Size of Screw	Threads per Inch	Size of Tap Drill	Decimal Equivalent of Tap Drill
0.060	80	56	0.0465	*0.177	24	27	0.1440
0.073	72	53	0.0595	*0.190	32	19	0.1660
*0.073	64	53	0.0595	0.190	30	20	0.1610
0.086	64	49	0.0730	*0.190	24	21	0.1590
*0.086	56	50	0.0700	0.216	28	13	0.1850
0.099	56	45	0.0820	*0.216	24	14	0.1820
*0.099	48	46	0.0810	0.242	24	5	0.2055
0.112	48	42	0.0935	*0.242	20	7	0.2010
*0.112	40	43	0.0890	0.268	22	1	0.2280
*0.112	36	43	0.0890	*0.268	20	1	0.2280
0.125	44	37	0.1040	0.294	20	1/4	0.2500
*0.125	40	37	0.1040	*0.294	18	1/4	0.2500
*0.125	36	38	0.1015	0.320	20	J	0.2770
0.138	40	32	0.1160	*0.320	18	J	0.2770
*0.138	36	33	0.1130	0.346	18	19/64	0.2968
*0.138	32	32	0.1160	*0.346	16	19/64	0.2968
0.151	36	30	0.1285	*0.372	18	21/64	0.3281
*0.151	32	1/8	0.1250	0.372	16	P	0.3230
*0.151	30	1/8	0.1250	0.398	16	11/32	0.3437
0.164	36	28	0.1405	*0.398	14	11/32	0.3437
*0.164	32	28	0.1405	*0.424	16	3/8	0.3750
*0.164	30	28	0.1405	0.424	14	U	0.3680
0.177	32	24	0.1520	*0.450	16	13/32	0.4062
*0.177	30	24	0.1520	0.450	14	25/64	0.3906

Straight Knurls. — It is important to select a suitable angle for the teeth for knurling different materials. A blunt knurl will work better on soft materials than one with a more acute angle. The following angles are satisfactory:

Brass and hard copper .....90 deg. Norway iron and machine steel ..70 deg.
Gun-screw iron ....80 deg. Drill rod and tool steel ....60 deg.

When laying out a set of cams for knurling operations, it is necessary to know the depth of the knurl teeth. This depth can be obtained direct from the table, "Depth of Teeth in Knurls," which covers pitches ranging from 16 to 62 teeth per inch of circumference.

Concave Knurls. — The radius of a concave knurl should not be the same as the radius of the piece to be knurled. If the knurl and the work are of the same radius, the material compressed by the knurl will be forced down on the shoulder D and spoil the appearance of the work. A design of concave knurl is shown in the accompanying illustration, and all the important dimensions are designated by letters. To find these dimensions, the pitch of the knurl required must be known, and also, approximately, the throat diameter B. This diameter must suit the knurl holder used, and be such that the circumference contains an even number of teeth with the required pitch. When these dimensions have been decided upon, all the other unknown factors can be found by the following formula: Let R = radius of piece to be knurled; r = radius of concave part of knurl; C = radius of cutter or hob for cutting the teeth in the knurl; B = diameter over concave part of knurl (throat diameter); A = outside diameter of knurl; d = depth of tooth in knurl; P = pitch of knurl (number of teeth per inch circumference); p = circular pitch of knurl; then, $r = R + \frac{1}{2} d$; $C = r + d$; $A = B + 2r - (3d + 0.010 \text{ inch})$.



As the depth of the tooth is usually very slight, the throat diameter B will be accurate enough for all practical purposes for calculating the pitch, and it is not necessary to take into consideration the pitch circle. For example, assume that the pitch of a knurl is 32, that the throat diameter B is 0.5561 inch, that the radius R of the piece to be knurled is $\frac{1}{16}$ inch, and that the angle of the teeth is 90 degrees; find the dimensions of the knurl. Using the notation given:

$$p = \frac{1}{P} = \frac{1}{32} = 0.03125 \text{ inch;}$$

$d = 0.0156$ inch (see table "Depth of Teeth in Knurls");

$$r = \frac{1}{16} + \frac{0.0156}{2} = 0.0703 \text{ inch;}$$

$$C = 0.0703 + 0.0156 = 0.0859 \text{ inch;}$$

$$A = 0.5561 + 0.1406 - (0.0468 + 0.010) = 0.6399 \text{ inch.}$$

Speeds and Feeds for Knurling. — When the knurl has been designed, the next thing to consider, before laying out the cams, is the speed and feed for knurling. As a general rule, a knurl can be worked at the same speed as the circular forming and cut-off tools. It is good practice to feed the knurl gradually to the center of the work, starting to feed when the knurl touches the work and then pass off the center of the work with a quick rise on the cam. The knurl should also dwell for a certain number of revolutions, depending on its pitch and the nature of the material being worked upon. Some advocate bringing the knurl into position on the center of the work on the quick rise of the cam, and then allowing it to dwell for a certain number of revolutions; but for general conditions it will be found that gradually feeding the knurl to the center of the work is preferable.

The feed required for knurling is governed by the nature of the material being knurled, the diameter of the material, and the width and pitch of the knurl. The surest and most practical way to find the feed required is by experimenting. The results of different experiments are given in the table "Feeds for Cross-slide Knurling." These feeds are applicable only when knurling from the cross-slide.

Depth of Teeth in Knurls

P = number of teeth in one inch of circumference;
p = circular pitch;
α = included angle of teeth;
d = depth of teeth in knurl.

P	p	$\alpha = 90°$	$\alpha = 80°$	$\alpha = 70°$	$\alpha = 60°$
		d	d	d	d
16	0.0625	0.0312	0.0371	0.0445	0.0540
18	0.0555	0.0277	0.0330	0.0395	0.0480
20	0.0500	0.0250	0.0297	0.0357	0.0433
22	0.0454	0.0227	0.0260	0.0324	0.0393
24	0.0416	0.0208	0.0247	0.0297	0.0360
26	0.0384	0.0192	0.0228	0.0274	0.0332
28	0.0357	0.0178	0.0212	0.0254	0.0308
30	0.0333	0.0166	0.0199	0.0237	0.0287
32	0.0312	0.0156	0.0185	0.0222	0.0270
34	0.0294	0.0147	0.0175	0.0209	0.0254
36	0.0277	0.0138	0.0164	0.0197	0.0239
38	0.0263	0.0131	0.0156	0.0187	0.0226
40	0.0250	0.0125	0.0148	0.0178	0.0216
42	0.0238	0.0119	0.0142	0.0169	0.0206
44	0.0227	0.0113	0.0134	0.0161	0.0195
46	0.0217	0.0108	0.0128	0.0154	0.0187
48	0.0208	0.0104	0.0124	0.0148	0.0180
50	0.0200	0.0100	0.0119	0.0142	0.0173
52	0.0192	0.0096	0.0114	0.0137	0.0166
54	0.0185	0.0092	0.0109	0.0131	0.0159
56	0.0178	0.0089	0.0106	0.0127	0.0154
58	0.0172	0.0086	0.0102	0.0122	0.0148
60	0.0166	0.0083	0.0099	0.0118	0.0143
62	0.0161	0.0080	0.0096	0.0114	0.0138

Turret Knurling. — Definite information cannot be given for feeds for knurling from the turret, as it is impossible to take into consideration all the various conditions under which a knurl will be operated. When two knurls are employed for spiral and diamond knurling, the knurls can be operated at a higher rate of feed for producing a spiral than they can for producing a diamond knurl. The reason for this is that in the first case the two knurls work in the same groove, whereas in the latter case they work independently of each other. For end knurling, when the knurl only has to be fed in to the depth of the tooth, the feed varies from that used for spiral or diamond knurling. The diameter of the work is also a determining factor. The table "Feeds for Turret Knurling" is applicable particularly to spiral and diamond knurling, but can also be used as a guide for bevel or end knurling. The diameter of the work or its strength to resist the torsional stress resulting from the knurling operation is not taken into consideration, and allowance should be made for this when using the feeds given. The feeds to be used for backing the knurls off the work should be as follows: For brass, screw stock and machine steel, twice the feeds given in the table; for tool steel, three times the feeds given in the table.

Feeds for Cross-slide Knurling

Diameter of Stock, Inches	Width of Knurl, Inches							
	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2
	Feed per Revolution, Inches							
1/16	0.0010	0.0005	.....	.....	.....	.....	.....	.....
1/8	0.0014	0.0009	0.0005	.....	.....	.....	.....	.....
3/16	0.0018	0.0012	0.0010	0.0005	.....	.....	.....	.....
1/4	0.0022	0.0016	0.0014	0.0010	0.0005	.....	.....	.....
5/16	0.0026	0.0020	0.0018	0.0013	0.0010	0.0005	.....	.....
3/8	0.0030	0.0025	0.0022	0.0017	0.0015	0.0010	0.0005	.....
7/16	0.0034	0.0029	0.0026	0.0021	0.0018	0.0015	0.0010	0.0005
1/2	0.0039	0.0032	0.0030	0.0025	0.0022	0.0020	0.0014	0.0008
9/16	0.0042	0.0036	0.0034	0.0029	0.0028	0.0024	0.0017	0.0012
5/8	0.0046	0.0040	0.0038	0.0033	0.0031	0.0028	0.0020	0.0016
11/16	0.0050	0.0045	0.0042	0.0037	0.0034	0.0031	0.0023	0.0020
3/4	0.0054	0.0049	0.0048	0.0041	0.0038	0.0034	0.0026	0.0023
13/16	0.0059	0.0052	0.0052	0.0045	0.0042	0.0037	0.0029	0.0026
7/8	0.0062	0.0058	0.0055	0.0049	0.0045	0.0040	0.0033	0.0029
15/16	0.0068	0.0062	0.0058	0.0052	0.0048	0.0042	0.0037	0.0032
1	0.0070	0.0065	0.0060	0.0055	0.0050	0.0045	0.0040	0.0035

Revolutions Required for Knurling. — The depth of the tooth and the feed per revolution govern the number of revolutions required for knurling. If R is the radius of the stock, d is the depth of the tooth, c is the distance the knurl travels from the point of contact to the center of the work, at the feed required for knurling (the knurl is removed from the central position by a quick rise on the cam); then

$$c = \sqrt{R^2 - (R - d)^2}.$$

Let $R = 0.125$ inch, and $d = 0.0164$ inch; then $c = \sqrt{0.125^2 - 0.1086^2} = 0.062$ inch = rise required. Assume that it is required to find the number of revolutions to knurl a piece of gun-screw iron, 1/4 inch in diameter, with a knurl 1/8 inch wide of 36 pitch. The included angle of the tooth for gun-screw iron is 80 degrees. The circular pitch is 0.0277, and, referring to the table "Depth of Teeth in Knurls," the depth is found to be 0.0164. The distance c (as worked out in the previous example) is 0.062 inch. Then, referring to the table "Feeds for Cross-slide Knurling," the feed per revolution for a knurl 1/8 inch wide, knurling on 1/4-inch stock, is 0.0016 inch. Therefore, total revolutions required = $0.062 \div 0.0016 = 38.7$ or, approximately, 39 revolutions. In some cases, the feeds given in this table can be increased 50 per cent and still give good results.

Knurls for Knurling in the Lathe. — The knurls commonly used for lathe work have spiral teeth and ordinarily there are three classes known as coarse, medium and fine. The medium pitch is generally used. The teeth of coarse knurls have a spiral angle of 36 degrees and the pitch of the knurled cut (measured parallel to the axis of the work) should be about 8 per inch. For medium knurls, the spiral angle is 29½ degrees and the pitch, measured as before, is 12 per inch. For fine knurls, the spiral angle is 25¾ degrees and the pitch, 20 per inch. The knurls should be about ¾ inch in diameter and ⅜ inch wide; when made to these dimensions, coarse knurls have 34 teeth, medium, 50 teeth, and fine knurls 80 teeth. To prevent forming a double set of projections when knurling, feed the knurl in with considerable pressure at the start, and then partially relieve the pressure before engaging the power feed. Use oil when knurling.

Feeds for Turret Knurling

Pitch of Knurl	Feed per Revolution				Pitch of Knurl	Feed per Revolution			
	Brass Rod	Gun-screw Iron	Machine Steel	Tool Steel		Brass Rod	Gun-screw Iron	Machine Steel	Tool Steel
16	0.0100	0.0080	0.0060	0.0040	40	0.0158	0.0128	0.0086	0.0058
18	0.0105	0.0084	0.0063	0.0042	42	0.0164	0.0132	0.0088	0.0059
20	0.0110	0.0088	0.0065	0.0044	44	0.0168	0.0136	0.0090	0.0061
22	0.0115	0.0092	0.0068	0.0046	46	0.0173	0.0140	0.0092	0.0062
24	0.0118	0.0096	0.0070	0.0048	48	0.0178	0.0143	0.0094	0.0063
26	0.0123	0.0100	0.0072	0.0050	50	0.0182	0.0145	0.0098	0.0064
28	0.0128	0.0103	0.0074	0.0051	52	0.0185	0.0148	0.0103	0.0065
30	0.0135	0.0106	0.0076	0.0052	54	0.0189	0.0150	0.0108	0.0066
32	0.0140	0.0110	0.0078	0.0053	56	0.0193	0.0153	0.0111	0.0067
34	0.0145	0.0115	0.0080	0.0054	58	0.0195	0.0156	0.0115	0.0068
36	0.0150	0.0120	0.0082	0.0056	60	0.0198	0.0158	0.0118	0.0069
38	0.0153	0.0125	0.0084	0.0057	62	0.0200	0.0160	0.0120	0.0070

Cams for Threading. — The tables "Spindle Revolutions and Cam Rise for Threading" give the revolutions required for threading various lengths and pitches and the corresponding rise for the cam lobe. To illustrate the use of these tables, suppose a set of cams is required for threading a screw to the length of ⅜ inch in a B. & S. machine. Assume that the spindle speed is 2400 revolutions per minute; the number of revolutions to complete one piece, 400; time required to make one piece, 10 seconds; pitch of the thread, 1/32 inch or 32 threads per inch. By referring to the table, under 32 threads per inch, and opposite ⅜ inch (length of threaded part), the number of revolutions required is found to be 15 and the rise required for the cam, 0.413 inch.

Cams of this type are often cut on a circular milling attachment. When this method is employed, the number of minutes the attachment should be revolved for each 0.001 inch rise, is first determined. As 15 revolutions are required for threading and 400 for completing one piece, that part of the cam surface required for the actual threading operation equals $15 \div 400 = 0.0375$, which is equivalent to 810 minutes of the circumference. As the total rise, in this case, through an arc of 810 minutes is 0.413 inch, the number of minutes for each 0.001 inch rise equals $810 \div 413 = 1.96$, or, approximately, two minutes. If the attachment is graduated to read to five minutes, the cam will be fed laterally 0.0025 inch each time it is turned five minutes

Spindle Revolutions and Cam Rise for Threading — 1

Length of Threaded Portion	Number of Threads per Inch														
	80	72	64	56	48	40	36	32	30	28	24	20	18	16	14
	First Line: Revolutions of Spindle for Threading. Second Line: Rise on Cam for Threading														
1/32	7.00	7.00	6.50	6.50	4.50	4.50	4.00	4.00	4.00	4.00	.....	....	....	....	....
	0.079	0.088	0.091	0.104	0.082	0.099	0.098	0.110	0.117	0.126	.....	....	....	....	....
1/16	9.50	9.00	8.50	8.00	6.00	5.50	5.50	5.00	5.00	5.00	3.00	....	....	....	....
	0.107	0.113	0.120	0.129	0.110	0.121	0.134	0.138	0.147	0.157	0.106	....	....	....	....
3/32	12.00	11.50	10.50	10.00	7.50	7.00	6.50	6.00	6.00	5.50	4.00	3.50	....	....	....
	0.135	0.144	0.148	0.161	0.137	0.154	0.159	0.165	0.176	0.173	0.142	0.149	....	....	....
1/8	14.50	13.50	12.50	11.50	9.00	8.00	7.00	7.00	7.00	6.50	4.50	4.00	3.50	3.50	....
	0.163	0.169	0.176	0.185	0.165	0.176	0.171	0.193	0.205	0.204	0.159	0.170	0.165	0.186	....
5/32	17.00	16.00	14.50	13.50	10.50	9.50	8.50	8.00	7.50	7.50	5.50	4.50	4.00	4.00	3.50
	0.191	0.200	0.204	0.217	0.192	0.209	0.208	0.220	0.220	0.236	0.195	0.191	0.189	0.212	0.212
3/16	19.50	18.00	16.50	15.00	12.00	10.50	10.00	9.00	8.50	8.50	6.00	5.50	5.00	4.50	4.00
	0.219	0.225	0.232	0.241	0.220	0.231	0.244	0.248	0.249	0.267	0.213	0.234	0.236	0.239	0.243
7/32	22.00	20.50	18.50	17.00	13.50	12.00	11.00	10.00	9.50	9.00	7.00	6.00	5.50	5.00	4.50
	0.248	0.256	0.260	0.273	0.247	0.264	0.269	0.275	0.279	0.283	0.248	0.255	0.260	0.266	0.273
1/4	24.50	23.50	20.50	18.50	15.00	13.00	12.00	11.00	10.50	10.00	7.50	6.50	6.00	5.50	5.00
	0.276	0.294	0.288	0.297	0.275	0.286	0.293	0.303	0.308	0.314	0.266	0.276	0.283	0.292	0.304
9/32	27.00	25.00	22.50	20.50	16.50	14.50	13.00	12.00	11.50	11.00	8.50	7.00	6.50	6.00	5.50
	0.304	0.313	0.316	0.329	0.302	0.319	0.318	0.330	0.337	0.346	0.301	0.298	0.307	0.319	0.334
5/16	29.50	27.00	24.50	22.00	18.00	15.50	14.50	13.00	12.50	12.00	9.00	8.00	7.00	6.50	6.00
	0.332	0.338	0.345	0.354	0.340	0.341	0.354	0.358	0.367	0.377	0.319	0.340	0.330	0.345	0.364
11/32	32.00	29.50	26.50	24.00	19.50	17.00	15.50	14.00	13.50	12.50	10.00	8.50	7.50	7.00	6.50
	0.360	0.369	0.373	0.386	0.357	0.374	0.379	0.385	0.396	0.393	0.354	0.361	0.354	0.372	0.395
3/8	34.50	31.50	28.50	25.50	21.00	18.00	16.50	15.00	14.50	13.50	10.50	9.00	8.50	7.50	7.00
	0.388	0.394	0.401	0.410	0.385	0.396	0.403	0.413	0.425	0.424	0.372	0.383	0.401	0.398	0.425

Spindle Revolutions and Cam Rise for Threading — 2

First Line: Revolutions of Spindle for Threading. Second Line: Rise on Cam for Threading

Length of Threaded Portion	Number of Threads per Inch: 80	72	64	56	48	40	36	32	30	28	24	20	18	16	14
13/32	37.00	34.00	30.50	27.50	22.50	19.50	17.50	16.00	15.00	14.50	11.50	9.50	9.00	8.00	7.00
	0.416	0.425	0.429	0.442	0.412	0.429	0.428	0.440	0.440	0.456	0.407	0.404	0.425	0.425	0.425
7/16	39.50	36.00	32.50	29.00	24.00	20.50	19.00	17.00	16.00	15.50	12.00	10.50	9.50	8.50	7.50
	0.444	0.450	0.457	0.466	0.440	0.451	0.464	0.468	0.469	0.487	0.425	0.446	0.448	0.451	0.455
15/32	42.00	38.50	34.50	31.00	25.50	22.00	20.00	18.00	17.00	16.00	13.00	11.00	10.50	9.00	8.00
	0.473	0.481	0.484	0.498	0.477	0.484	0.489	0.495	0.499	0.503	0.460	0.468	0.496	0.478	0.486
1/2	44.50	40.50	36.50	32.50	27.00	23.00	21.00	19.00	18.00	17.00	13.50	11.50	10.50	9.50	8.50
	0.501	0.506	0.513	0.522	0.495	0.506	0.513	0.523	0.528	0.534	0.478	0.489	0.496	0.504	0.516
17/32	47.00	43.00	38.50	34.50	28.50	24.50	22.00	20,00	19.00	18.00	14.50	12.00	11.00	10.00	9.00
	0.529	0.538	0.541	0.554	0.522	0.539	0.538	0.550	0.557	0.566	0.514	0.510	0.519	0.531	0.546
9/16	49.50	45.00	40.50	36.00	30.00	25.50	23.50	21.00	20.00	19.00	15.00	13.00	11.50	10.50	9.50
	0.559	0.563	0.570	0.579	0.550	0.561	0.574	0.578	0.587	0.597	0.531	0.553	0.543	0.558	0.577
19/32	52.00	47.50	42.50	38.00	31.50	27.00	24.50	22.00	21.00	19.50	16.00	13.50	12.00	11.00	10.00
	0.585	0.594	0.598	0.611	0.577	0.594	0.599	0.605	0.616	0.613	0.567	0.574	0.566	0.584	0.607
5/8	54.50	49.50	44.50	39.50	33.00	28.00	25.50	23.00	22.00	20.50	16.50	14.00	13.00	11.50	10.50
	0.613	0.619	0.626	0.635	0.605	0.616	0.623	0.633	0.645	0.644	0.584	0.595	0.614	0.611	0.637
21/32	57.00	52.00	46.50	41.50	34.50	29.50	26.50	24.00	23.00	21.50	17.50	14.50	13.50	12.00	10.50
	0.641	0.650	0.654	0.667	0.622	0.649	0.648	0.660	0.675	0.676	0.620	0.616	0.637	0.637	0.637
11/16	59.50	54.00	48.50	43.00	36.00	30.50	28.00	25.00	23.50	22.50	18.00	15.50	14.00	12.50	11.00
	0.679	0.675	0.682	0.691	0.660	0.671	0.684	0.688	0.689	0.707	0.638	0.659	0.661	0.664	0.668
23/32	62.00	56.50	50.50	45.00	37.50	32.00	29.00	26.00	24.50	23.00	19.00	16.00	14.50	13.00	11.50
	0.698	0.706	0.710	0.723	0.677	0.704	0.709	0.715	0.719	0.723	0.673	0.680	0.684	0.690	0.698
3/4	64.50	58.50	52.50	46.50	39.00	33.00	30.00	27.00	25.50	24.00	19.50	16.50	15.00	13.50	12.00
	0.726	0.731	0.738	0.747	0.715	0.726	0.733	0.743	0.748	0.754	0.691	0.701	0.708	0.717	0.728

Spindle Revolutions and Cam Rise for Threading — 3

Length of Threaded Portion	Number of Threads per Inch														
	80	72	64	56	48	40	36	32	30	28	24	20	18	16	14
	First Line: Revolutions of Spindle for Threading. Second Line: Rise on Cam for Threading														
25/32	67.00 0.754	61.00 0.763	54.50 0.767	48.50 0.779	40.50 0.742	34.50 0.759	31.00 0.758	28.00 0.770	26.50 0.777	25.00 0.786	20.50 0.726	17.00 0.723	15.50 0.732	14.00 0.743	12.50 0.759
13/16	69.50 0.782	63.00 0.788	56.50 0.795	50.00 0.804	42.00 0.770	35.50 0.781	32.50 0.794	29.00 0.798	27.50 0.807	26.00 0.817	21.00 0.744	18.00 0.765	16.00 0.755	14.50 0.770	13.00 0.789
27/32	72.00 0.810	65.50 0.819	58.50 0.823	52.00 0.836	43.50 0.797	37.00 0.814	33.50 0.819	30.00 0.825	28.50 0.836	26.50 0.833	22.00 0.779	18.50 0.786	16.50 0.779	15.00 0.797	13.50 0.819
7/8	74.50 0.838	67.50 0.844	60.50 0.851	53.50 0.860	45.00 0.825	38.00 0.836	34.50 0.843	31.00 0.853	29.50 0.865	27.50 0.864	22.50 0.797	19.00 0.808	17.50 0.826	15.50 0.823	14.00 0.850
29/32	77.00 0.866	70.00 0.875	62.50 0.879	55.50 0.892	46.50 0.842	39.50 0.869	35.50 0.868	32.00 0.880	30.00 0.880	28.50 0.895	23.50 0.832	19.50 0.829	18.00 0.850	16.00 0.850	14.00 0.850
15/16	79.50 0.894	72.00 0.900	64.50 0.907	57.00 0.916	48.00 0.880	40.50 0.891	37.00 0.904	33.00 0.908	31.00 0.909	29.50 0.927	24.00 0.850	20.50 0.871	18.50 0.873	16.50 0.876	14.50 0.880
31/32	82.00 0.923	74.50 0.931	66.50 0.935	59.00 0.948	49.50 0.907	42.00 0.924	38.00 0.929	34.00 0.935	32.00 0.939	30.00 0.943	25.00 0.885	21.00 0.893	19.00 0.897	17.00 0.903	15.00 0.911
1	84.50 0.951	76.50 0.956	68.50 0.963	60.50 0.972	51.00 0.918	43.00 0.946	39.00 0.953	35.00 0.963	33.00 0.968	31.00 0.974	25.50 0.903	21.50 0.914	19.50 0.920	17.50 0.929	15.50 0.941
1 1/16	89.50 1.007	81.00 1.013	72.50 1.019	64.00 1.028	54.00 0.990	45.50 1.001	41.50 1.013	37.00 1.018	35.00 1.026	32.00 1.005	27.00 0.956	23.00 0.978	20.50 0.968	18.50 0.982	16.50 1.002
1 1/8	94.50 1.063	85.50 1.069	76.50 1.076	67.50 1.084	57.00 1.045	48.00 1.056	43.50 1.061	39.00 1.073	37.00 1.084	34.50 1.083	28.50 1.009	24.00 1.020	22.00 1.038	19.50 1.035	17.50 1.062
1 3/16	99.50 1.119	90.00 1.125	80.50 1.126	71.00 1.141	60.00 1.100	50.50 1.111	46.00 1.122	41.00 1.128	38.50 1.128	36.50 1.146	30.00 1.062	25.50 1.084	23.00 1.086	20.50 1.089	18.00 1.093
1 1/4	104.5 1.176	94.50 1.181	84.50 1.188	74.50 1.197	63.00 1.155	53.00 1.166	48.00 1.171	43.00 1.183	40.50 1.187	38.00 1.193	31.50 1.115	26.50 1.126	24.00 1.133	21.50 1.142	19.00 1.153

Spindle Revolutions and Cam Rise for Threading — 4

Length of Threaded Portion	Number of Threads per Inch														
	80	72	64	56	48	40	36	32	30	28	24	20	18	16	14
	First Line: Revolutions of Spindle for Threading. Second Line: Rise on Cam for Threading														
1 5/16	.....	99.00	88.50	78.00	66.00	55.50	50.50	45.00	42.50	40.00	33.00	28.00	25.00	22.50	20.00
	.....	1.238	1.244	1.253	1.210	1.221	1.232	1.238	1.245	1.256	1.168	1.190	1.180	1.195	1.214
1 3/8	.....	103.5	92.50	81.50	69.00	58.00	52.50	47.00	44.50	41.50	34.50	29.00	26.50	23.50	21.00
	.....	1.294	1.301	1.310	1.265	1.276	1.281	1.293	1.304	1.303	1.211	1.233	1.251	1.248	1.275
1 7/16	.....	.....	96.50	85.00	72.00	60.50	55.00	49.00	46.00	43.50	36.00	30.50	27.50	24.50	21.50
	.....	.....	1.357	1.366	1.320	1.331	1.342	1.348	1.348	1.366	1.274	1.296	1.298	1.301	1.305
1 1/2	.....	.....	100.5	88.50	75.00	63.00	57.00	51.00	48.00	45.00	37.50	31.50	28.50	25.50	22.50
	.....	.....	1.413	1.422	1.375	1.386	1.391	1.403	1.406	1.413	1.328	1.339	1.345	1.354	1.366
1 9/16	.....	.....	104.5	92.00	78.00	65.50	59.50	53.00	50.00	47.00	39.00	33.00	29.50	26.50	23.50
	.....	.....	1.469	1.478	1.430	1.441	1.452	1.458	1.465	1.476	1.381	1.403	1.392	1.407	1.426
1 5/8	.....	.....	.....	95.50	81.00	68.00	61.50	55.00	52.00	48.50	40.50	34.00	31.00	27.50	24.50
	.....	.....	.....	1.535	1.485	1.496	1.501	1.513	1.524	1.523	1.434	1.445	1.463	1.460	1.487
1 11/16	.....	.....	.....	99.00	84.00	70.50	64.00	57.00	53.50	50.50	42.00	35.50	32.00	28.50	25.00
	.....	.....	.....	1.591	1.540	1.551	1.562	1.568	1.568	1.586	1.487	1.509	1.510	1.513	1.518
1 3/4	.....	.....	.....	102.5	87.00	73.00	66.00	59.00	55.50	52.00	43.50	36.50	33.00	29.50	26.00
	.....	.....	.....	1.647	1.595	1.606	1.610	1.623	1.626	1.633	1.540	1.551	1.558	1.566	1.578
1 13/16	.....	.....	.....	106.0	90.00	75.50	68.50	61.00	57.50	54.00	45.00	38.00	34.00	30.50	27.00
	.....	.....	.....	1.703	1.650	1.661	1.671	1.678	1.685	1.696	1.593	1.615	1.605	1.620	1.639
1 7/8	.....	.....	.....	.....	93.00	78.00	70.50	63.00	59.50	55.50	46.50	39.00	35.50	31.50	28.00
	.....	.....	.....	.....	1.705	1.716	1.720	1.733	1.743	1.743	1.646	1.658	1.676	1.673	1.700
1 15/16	.....	.....	.....	.....	96.00	80.50	73.00	65.00	61.00	57.50	48.00	40.50	36.50	32.50	28.50
	.....	.....	.....	.....	1.760	1.771	1.781	1.788	1.787	1.806	1.700	1.721	1.723	1.726	1.730
2	.....	.....	.....	.....	99.00	83.00	75.00	67.00	63.00	59.00	49.50	41.50	37.50	33.50	29.50
	.....	.....	.....	.....	1.815	1.826	1.830	1.843	1.846	1.853	1.752	1.764	1.770	1.779	1.791

Surface Speeds of Screw Machine Stock

Diam. of Stock	Decimal Equiv-alent	Revolutions per Minute (B. & S. Machines)							
		2400	2048	1800	1748	1492	1474	1273	1200
		Surface Speed in Feet per Minute							
1/16	0.062	39.3	33.5	29.4	28.6	24.4	24.1	20.8	19.6
5/64	0.078	49.1	41.9	36.8	35.7	30.5	30.1	26.0	24.5
3/32	0.094	61.4	52.4	46.1	44.7	38.1	37.7	32.6	30.7
1/8	0.125	78.5	67.0	58.9	57.2	48.8	48.2	41.6	39.3
5/32	0.156	98.0	83.6	73.5	71.4	60.9	60.2	52.0	49.0
3/16	0.187	117.8	100.5	88.3	85.8	73.2	72.3	62.5	58.9
7/32	0.219	137.6	117.3	103.1	100.1	85.4	84.4	72.9	68.7
1/4	0.250	157.1	134.0	117.8	114.4	97.6	96.5	83.3	78.5
9/32	0.281	176.7	150.8	132.5	128.7	109.8	108.5	93.7	88.3
5/16	0.312	196.3	167.5	147.3	143.0	122.0	120.6	104.1	98.2
11/32	0.344	215.9	184.3	161.9	157.3	134.2	132.6	114.6	107.9
3/8	0.375	235.6	201.0	176.7	171.6	146.4	144.7	125.0	117.8
13/32	0.406	255.2	217.8	191.4	185.9	158.5	156.8	135.4	127.6
7/16	0.437	274.9	234.5	206.2	200.2	170.7	168.8	145.8	137.4
15/32	0.469	294.5	251.3	220.9	214.5	183.0	180.9	156.2	147.3
1/2	0.500	314.2	268.1	235.6	228.8	195.1	192.9	166.6	157.1
17/32	0.531	333.8	284.8	250.3	243.1	207.3	205.0	177.0	166.9
9/16	0.562	353.4	301.6	265.1	257.4	219.5	217.1	187.5	176.7
19/32	0.594	373.1	318.3	279.8	271.7	231.7	229.1	197.9	186.5

Diam. of Stock	Decimal Equiv-alent	Revolutions per Minute							
		1087	988	973	927	810	792	675	640
		Surface Speed in Feet per Minute							
1/16	0.062	17.9	16.2	15.9	15.2	13.2	12.9	11.0	10.5
5/64	0.078	22.2	20.2	19.9	18.9	16.9	16.2	13.8	13.1
3/32	0.094	27.8	24.2	24.9	22.7	20.7	20.3	16.6	16.4
1/8	0.125	35.6	32.3	31.8	30.3	26.5	25.9	22.1	20.9
5/32	0.156	44.4	40.4	39.7	37.9	33.1	32.4	27.6	26.1
3/16	0.187	53.3	48.5	47.8	45.5	39.8	38.9	33.1	31.4
7/32	0.219	62.2	56.6	55.7	53.1	46.4	45.3	38.6	36.6
1/4	0.250	71.1	64.7	63.7	60.7	53.0	51.8	44.2	41.9
9/32	0.281	80.0	72.7	71.6	68.2	59.7	58.2	49.7	47.1
5/16	0.312	88.9	80.8	79.6	75.8	66.3	64.7	55.2	52.4
11/32	0.344	97.8	88.9	87.6	83.4	73.0	71.2	60.7	57.6
3/8	0.375	106.7	97.0	95.5	91.0	79.6	77.6	66.3	62.9
13/32	0.406	115.6	105.1	103.5	98.6	86.3	84.1	71.8	68.1
7/16	0.437	124.5	113.2	111.4	106.2	92.9	90.6	77.3	73.3
15/32	0.469	133.4	121.2	119.4	113.8	99.6	97.0	82.8	78.5
1/2	0.500	142.3	129.3	127.4	121.3	106.2	103.5	88.3	83.8
17/32	0.531	151.2	137.4	135.3	128.9	112.9	109.9	93.9	89.0
9/16	0.562	160.1	145.5	143.3	136.5	119.5	116.4	99.4	94.2
19/32	0.594	168.9	153.6	151.2	144.1	126.2	122.9	104.9	99.5

Surface Speeds of Screw Machine Stock

Diam. of Stock	Decimal Equivalent	Revolutions per Minute (B. & S. Machines)							
		576	543	519	492	445	420	364	342
		Surface Speed in Feet per Minute							
9/32	0.281	42.6	40.0	38.2	36.2	32.7	31.7	26.8	25.2
5/16	0.312	47.1	44.4	42.4	40.2	36.4	34.4	29.8	28.0
11/32	0.344	51.8	48.9	46.7	44.3	40.0	37.8	32.7	30.8
3/8	0.375	56.5	53.3	50.9	48.3	43.8	41.2	35.7	33.6
13/32	0.406	61.2	57.7	55.2	52.3	47.3	44.7	38.7	36.4
7/16	0.437	66.0	62.2	59.4	56.4	51.0	48.1	42.6	39.2
15/32	0.469	70.7	66.6	63.7	60.4	54.6	51.5	44.6	42.0
1/2	0.500	75.4	71.1	67.9	64.4	58.2	55.0	47.6	44.8
17/32	0.531	80.1	75.5	72.2	68.4	61.9	58.4	50.6	47.6
9/16	0.562	84.8	80.0	76.4	72.5	65.5	61.8	53.6	50.4
19/32	0.594	89.5	84.4	80.7	76.5	69.2	65.3	56.6	53.2
5/8	0.625	94.2	88.8	84.9	80.5	72.8	68.7	59.5	55.9
21/32	0.656	98.9	93.3	89.2	84.5	76.4	72.1	62.5	58.7
11/16	0.687	103.7	97.7	93.4	88.5	80.1	75.6	65.5	61.5
3/4	0.750	113.1	106.6	101.9	96.6	87.3	82.4	71.4	67.1
13/16	0.812	122.5	115.5	110.4	104.6	94.6	89.3	77.4	72.7
7/8	0.875	131.9	124.3	118.8	112.7	101.9	96.2	83.3	78.3
15/16	0.937	141.3	133.2	127.3	120.7	109.2	103.0	89.3	83.9
1	1.000	150.7	142.1	135.8	128.7	116.4	109.9	95.2	89.5

Diam. of Stock	Decimal Equivalent	Revolutions per Minute							
		298	277	244	225	200	182	148	120
		Surface Speed in Feet per Minute							
9/32	0.281	21.9	20.4	18.0	16.9	14.7	13.4	10.8	8.8
5/16	0.312	24.9	22.7	20.1	18.4	16.4	14.5	12.1	9.8
11/32	0.344	26.8	24.9	21.9	20.2	18.0	16.4	13.3	10.8
3/8	0.375	29.3	27.2	23.9	22.1	19.6	17.9	14.5	11.8
13/32	0.406	31.7	29.5	25.9	23.9	21.3	19.4	15.7	12.8
7/16	0.437	34.1	31.7	27.9	25.8	23.0	20.8	16.9	13.7
15/32	0.469	36.6	34.0	29.9	27.6	24.5	22.3	18.6	14.7
1/2	0.500	39.0	36.3	31.9	29.5	26.2	23.8	19.4	15.7
17/32	0.531	41.4	38.5	33.9	31.3	27.8	25.3	20.6	16.7
9/16	0.562	43.9	40.8	35.9	33.1	29.5	26.8	21.8	17.7
19/32	0.594	46.3	43.1	37.9	35.0	31.1	28.3	23.0	18.6
5/8	0.625	48.7	45.3	39.9	36.8	32.7	29.8	24.2	19.6
21/32	0.656	51.2	47.6	41.9	38.7	34.4	31.3	25.4	20.6
11/16	0.687	53.6	49.9	43.9	40.5	36.0	32.8	26.6	21.6
3/4	0.750	58.5	54.4	48.0	44.2	39.3	35.7	29.0	23.6
13/16	0.812	63.4	58.9	52.0	47.9	42.5	38.7	31.5	25.5
7/8	0.875	68.2	63.5	56.0	51.6	45.8	41.7	33.9	27.5
15/16	0.937	73.1	68.0	60.0	55.2	49.1	44.7	36.3	29.4
1	1.000	78.0	72.6	64.0	58.9	52.4	47.7	38.7	31.4

Practical Points on Cam and Tool Design. — The following general rules are given to aid in designing cams and special tools for automatic screw machines, and apply particularly to B. & S. machines:

1. Use the highest spindle speeds that the various tools will stand.

2. Use the arrangement of circular tools best suited for the class of work. (See paragraph: "Arrangement of Circular Tools.")

3. Decide on the quickest and best method of arranging the operations before designing the cams.

4. Do not use turret tools for forming when the cross-slide tools can be used to better advantage.

5. Do not use a circular cut-off tool without top rake when cutting Norway iron, machine steel, etc.

6. Make the shoulder on the circular cut-off tool large enough so that the clamping screw will grip firmly.

7. When chips clinging to the work are objectionable, the circular forming tool should be turned upside down and placed on the rear cross-slide.

8. Do not use too narrow a cut-off blade.

9. Allow 0.005 to 0.010 inch for the circular tools to approach the work and 0.003 to 0.005 inch for the cut-off tool to pass the center.

10. When cutting off work large in diameter, the feed of the cut-off tool should be increased until near the end of the cut where the piece breaks off. After it breaks off, the feed should again be increased until the tool has passed the center.

11. When a thread is cut up to a shoulder, the piece should be grooved or necked to make allowance for the lead on the die. This requires an extra projection on the forming tool and also an extra amount of rise on the cam.

12. Use circular forming and cut-off tools made from high-speed steel when cutting Norway iron, machine steel, etc.

13. Use a fine feed and high spindle speed for all cutting tools.

14. Allow sufficient clearance for tools to pass one another.

15. Always make a diagram of the cross-slide tools in position on the work when difficult operations are to be performed; it is also necessary to make a diagram of the tools held in the turret.

16. Do not drill a hole the depth of which is more than 2½ times the diameter of the drill, but use two or more drills as required. If there are not sufficient holes in the turret, drop the drill back clear of the hole, and advance it into the hole again.

17. Do not run a drill at a slow speed.

18. When the turret tools operate farther in than the face of the chuck, see that they will clear the chute when revolving the turret.

19. See that the body of all turret tools will clear the side of the chute when revolving the turret.

20. Do not use a box-tool for a roughing cut. Use a hollow mill.

21. Do not use a box-tool with solid supports. Use solid supports only on cold-drawn or finished stock.

22. The rise on the thread lobe should be reduced so that the spindle will reverse when the die or tap holder is drawn out.

23. When threading Norway iron, machine steel, etc., if the spindle speed used for the other tools is too high for threading, use a special threading attachment.

24. When bringing another tool into position after a threading operation, allow clearance before revolving the turret.

25. Make provision to revolve the turret rapidly, especially when pieces are being made in from three to five seconds and when only a few tools are used in the turret. It is sometimes convenient to use two sets of tools.

26. When using a belt-shifting attachment for threading, clearance should be allowed, as it requires extra time to shift the belt.

27. When laying out a set of cams for operating on a piece which requires to be slotted, cross-drilled or burred, allowance should be made on the lead cam so that the transferring arm can descend and ascend to and from the work without coming in contact with any of the turret tools.

28. Always allow a vacant hole in the turret when it is necessary to use the transferring arm.

29. Use standard tools whenever possible.

30. When designing special tools allow as much clearance as possible. Do not make them so that they will just clear each other, as a slight inaccuracy in the dimensions will then often cause trouble.

31. When designing special tools having intricate movements, avoid springs as much as possible, and use positive actions.

Stock for Screw Machine Products. — The amount of stock required for the production of 1000 pieces on the automatic screw machine can be obtained directly from the table "Stock required for Screw Machine Products." To use this table, add to the length of the work the width of the cut-off tool blade; then the number of feet of material required for 1000 pieces can be found opposite the figure thus obtained, in the column headed "Feet per 1000 Parts." Screw machine stock usually comes in bars 10 feet long, and in compiling this table an allowance was made for chucking on each bar.

The table can be extended by using the following formula, in which F = number of feet required for 1000 pieces; L = length of piece in inches; W = width of cut-off tool blade in inches.

$$F = (L + W) \times 84.$$

The amount to add to the length of the work, or the width of the cut-off tool, is given in the following, which is standard in a number of machine shops:

Diameter of Stock, Inches	Width of Cut-off Tool Blade, Inches
0.000–0.250	0.045
0.251–0.375	0.062
0.376–0.625	0.093
0.626–1.000	0.125
1.000–1.500	0.156

It is sometimes convenient to know the weight of a certain number of pieces, when estimating the price. The weight of round bar stock can be found by means of the following formulas, in which W = weight in pounds; D = diameter of stock in inches; F = length in feet:

For brass stock: $W = D^2 \times 2.86 \times F.$
For steel stock: $W = D^2 \times 2.675 \times F.$
For iron stock: $W = D^2 \times 2.65 \times F.$

Stock Required for Screw Machine Products

The table gives the amount of stock, in feet, required for 1000 pieces, when the length of the finished part plus the thickness of the cut-off tool blade is known. Allowance has been made for chucking. To illustrate, if length of cut-off tool and work equals 0.140 inch, 11.8 feet of stock is required for the production of 1000 parts.

Length of Piece and Cut-off Tool	Feet per 1000 Parts	Length of Piece and Cut-off Tool	Feet per 1000 Parts	Length of Piece and Cut-off Tool	Feet per 1000 Parts	Length of Piece and Cut-off Tool	Feet per 1000 Parts
0.050	4.2	0.240	20.2	0.430	36.1	0.620	52.1
0.055	4.6	0.245	20.6	0.435	36.6	0.625	52.5
0.060	5.0	0.250	21.0	0.440	37.0	0.630	52.9
0.065	5.5	0.255	21.4	0.445	37.4	0.635	53.4
0.070	5.9	0.260	21.8	0.450	37.8	0.640	53.8
0.075	6.3	0.265	22.3	0.455	38.2	0.645	54.2
0.080	6.7	0.270	22.7	0.460	38.7	0.650	54.6
0.085	7.1	0.275	23.1	0.465	39.1	0.655	55.0
0.090	7.6	0.280	23.5	0.470	39.5	0.660	55.5
0.095	8.0	0.285	23.9	0.475	39.9	0.665	55.9
0.100	8.4	0.290	24.4	0.480	40.3	0.670	56.3
0.105	8.8	0.295	24.8	0.485	40.8	0.675	56.7
0.110	9.2	0.300	25.2	0.490	41.2	0.680	57.1
0.115	9.7	0.305	25.6	0.495	41.6	0.685	57.6
0.120	10.1	0.310	26.1	0.500	42.0	0.690	58.0
0.125	10.5	0.315	26.5	0.505	42.4	0.695	58.4
0.130	10.9	0.320	26.9	0.510	42.9	0.700	58.8
0.135	11.3	0.325	27.3	0.515	43.3	0.705	59.2
0.140	11.8	0.330	27.7	0.520	43.7	0.710	59.7
0.145	12.2	0.335	28.2	0.525	44.1	0.715	60.1
0.150	12.6	0.340	28.6	0.530	44.5	0.720	60.5
0.155	13.0	0.345	29.0	0.535	45.0	0.725	61.0
0.160	13.4	0.350	29.4	0.540	45.4	0.730	61.3
0.165	13.9	0.355	29.8	0.545	45.8	0.735	61.8
0.170	14.3	0.360	30.3	0.550	46.2	0.740	62.2
0.175	14.7	0.365	30.7	0.555	46.6	0.745	62.6
0.180	15.1	0.370	31.1	0.560	47.1	0.750	63.0
0.185	15.5	0.375	31.5	0.565	47.5	0.755	63.4
0.190	16.0	0.380	31.9	0.570	47.9	0.760	63.9
0.195	16.4	0.385	32.4	0.575	48.3	0.765	64.3
0.200	16.8	0.390	32.8	0.580	48.7	0.770	64.7
0.205	17.2	0.395	33.2	0.585	49.2	0.775	65.1
0.210	17.6	0.400	33.6	0.590	49.6	0.780	65.5
0.215	18.1	0.405	34.0	0.595	50.0	0.785	66.0
0.220	18.5	0.410	34.5	0.600	50.4	0.790	66.4
0.225	19.0	0.415	34.9	0.605	50.8	0.795	66.8
0.230	19.3	0.420	35.3	0.610	51.3	0.800	67.2
0.235	19.7	0.425	35.7	0.615	51.7	0.805	67.6

Stock Required for Screw Machine Products

The table gives the amount of stock, in feet, required for 1000 pieces, when the length of the finished part plus the thickness of the cut-off tool blade is known. Allowance has been made for chucking. To illustrate, if length of cut-off tool and work equals 0.9 inch, 75.6 feet of stock is required for the production of 1000 parts.

Length of Piece and Cut-off Tool	Feet per 1000 Parts	Length of Piece and Cut-off Tool	Feet per 1000 Parts	Length of Piece and Cut-off Tool	Feet per 1000 Parts	Length of Piece and Cut-off Tool	Feet per 1000 Parts
0.810	68.1	1.000	84.0	1.380	116.0	1.760	147.9
0.815	68.5	1.010	84.9	1.390	116.8	1.770	148.7
0.820	68.9	1.020	85.7	1.400	117.6	1.780	149.6
0.825	69.3	1.030	86.6	1.410	118.5	1.790	150.4
0.830	69.7	1.040	87.4	1.420	119.3	1.800	151.3
0.835	70.2	1.050	88.2	1.430	120.2	1.810	152.1
0.840	70.6	1.060	89.1	1.440	121.0	1.820	152.9
0.845	71.0	1.070	89.9	1.450	121.8	1.830	153.8
0.850	71.4	1.080	90.8	1.460	122.7	1.840	154.6
0.855	71.8	1.090	91.6	1.470	123.5	1.850	155.5
0.860	72.3	1.100	92.4	1.480	124.4	1.860	156.3
0.865	72.7	1.110	93.3	1.490	125.2	1.870	157.1
0.870	73.1	1.120	94.1	1.500	126.1	1.880	158.0
0.875	73.5	1.130	95.0	1.510	126.9	1.890	158.8
0.880	73.9	1.140	95.8	1.520	127.7	1.900	159.7
0.885	74.4	1.150	96.6	1.530	128.6	1.910	160.5
0.890	74.8	1.160	97.5	1.540	129.4	1.920	161.3
0.895	75.2	1.170	98.3	1.550	130.3	1.930	162.2
0.900	75.6	1.180	99.2	1.560	131.1	1.940	163.0
0.905	76.0	1.190	100.0	1.570	131.9	1.950	163.9
0.910	76.5	1.200	100.8	1.580	132.8	1.960	164.7
0.915	76.9	1.210	101.7	1.590	133.6	1.970	165.5
0.920	77.3	1.220	102.5	1.600	134.5	1.980	166.4
0.925	77.7	1.230	103.4	1.610	135.3	1.990	167.2
0.930	78.2	1.240	104.2	1.620	136.1	2.000	168.1
0.935	78.6	1.250	105.0	1.630	137.0	2.050	172.3
0.940	79.0	1.260	105.9	1.640	137.8	2.100	176.5
0.945	79.4	1.270	106.7	1.650	138.7	2.150	180.7
0.950	79.8	1.280	107.6	1.660	139.5	2.200	184.9
0.955	80.3	1.290	108.4	1.670	140.3	2.250	189.1
0.960	80.7	1.300	109.2	1.680	141.2	2.300	193.3
0.965	81.1	1.310	110.1	1.690	142.0	2.350	197.5
0.970	81.5	1.320	110.9	1.700	142.9	2.400	201.7
0.975	81.9	1.330	111.8	1.710	143.7	2.450	205.9
0.980	82.4	1.340	112.6	1.720	144.5	2.500	210.1
0.985	82.8	1.350	113.4	1.730	145.4	2.550	214.3
0.990	83.2	1.360	114.3	1.740	146.2	2.600	218.5
0.995	83.6	1.370	115.1	1.750	147.1	2.650	222.7

THREAD ROLLING

Thread-rolling Process. — The formation of screw threads by rolling is effected by means of hardened rolls or dies having threads or ridges which roll grooves into a blank and raise enough metal above the surface of the blank to form a thread. This process may be defined as an impression or displacement method, since the thread grooves are not cut, but are formed entirely by the displacement of the metal. Screw threads may be rolled (1) by using a circular disk or roll having a threaded periphery, or (2) by rolling the blank between dies which may be either flat or circular in form. Most rolled screw threads are rolled between flat dies and in special thread-rolling machines. The important commercial application of the thread-rolling process is found in shops and factories where bolts, screws, studs, threaded rods, etc., are required in large quantities. Screw threads on bolts and screws that are within the range of the rolling process may be produced more rapidly by this method than in any other way.

Types of Thread-rolling Machines. — Most of the machines designed exclusively for rolling screw threads are equipped with flat dies. One die is stationary and the other has a reciprocating movement when the machine is in use. The ridges on these dies, which form the screw thread, incline at an angle equal to the helix angle of the thread. The thread is formed in one passage of the work, which is inserted at one end of the dies, either by hand or automatically, and then rolls between the die faces until it is ejected at the opposite end. The relation between the position of the dies and a screw thread being rolled is such that the top of the thread-shaped ridge of one die, at the point of contact with the screw thread, is directly opposite the bottom of the thread groove in the other die at the point of contact. Thread-rolling machines are equipped with some form of mechanism that insures starting the blank at the right time and also square with the dies. Thread-rolling machines of the flat-die type are made in both horizontal and vertical designs.

The rotary type of thread-rolling machine is equipped with a die which revolves continuously in one direction. This revolving die is cylindrical, forming a complete circle, whereas the other die is the segment of a circle and remains stationary.

Advantages of Thread-rolling Process.— The chief advantage of the thread-rolling process is that it is the most rapid method of forming screw threads, assuming that the work is suitable for a thread-rolling machine. Another advantage claimed for rolled screw threads is their strength as compared with cut threads. Extensive experiments conducted at one of the leading universities showed that the average rolled thread tested had an elastic limit 13 per cent higher than the elastic limit of corresponding cut threads. Another advantage of the rolling process is that no stock is wasted in forming the thread, and the surface of a rolled thread is harder than that of a cut thread and better able to withstand wear. The rolling process has been applied extensively for threading machine screws, various forms of special screws and bolts, wood-screws, lag screws, slender wires, and a great variety of small- and medium-sized screws especially if required in large quantities.

Capacities of Thread-rolling Machines. — The thread-rolling machines listed by several different manufacturers have nominal rated capacities for screw thread diameters varying from 1/16 inch up to 2 inches. The rated capacity, however, does not necessarily represent the maximum diameter which can be rolled with the machine. For instance, a 1-inch machine is sometimes used for rolling 1 1/4-inch diameters or even larger sizes, the maximum diameter depending somewhat upon the depth of the dies used. The rated capacity represents the size which the manufacturers believe to be the maximum, if the best grade of work is desired.

Rate of Production. — The rate at which screw threads can be rolled varies greatly according to the size of the machine. For instance, a certain machine which has a rated capacity of 1/16 inch is capable of threading 125 screws per minute, whereas a 1-inch machine of the same make will roll about 30 screw threads per minute. The production rate for a 1/4-inch machine may be 70 or 80 screw threads per minute, and for a 1/2-inch machine, 45 or 50 per minute. These figures indicate, in a general way, the time required for rolling screw threads and indicate the superiority of the thread-rolling process for work within its range.

Stock for Thread Rolling. — Soft steel is the stock that is adapted for thread rolling, and it may contain from 0.07 to 0.12 per cent of carbon. Iron of ordinary quality is not suitable, because of its fibrous structure, which makes it liable to split or fracture as the result of the pressure due to rolling the thread.

Flat Thread-rolling Dies. — The ridges on flat thread-rolling dies represent a development of the screw thread, and they incline in the same direction when viewed from the rolling sides or faces, the angle corresponding to the helix angle of the thread. The grooves which form these ridges on the dies may be either planed or milled. The milling cutter or "hob" used has teeth which are relieved like a formed milling cutter, but are not helical, the annular parallel rows of teeth being perpendicular to the cutter axis. The teeth of dies for rolling United States standard threads, V-threads, or Whitworth threads have a uniform cross-sectional shape from one end of the die to the other, with the possible exception of a short length near the ends, which is relieved so that the work will roll off the dies without being marked by the corners. The teeth of dies used for rolling square threads are made V-shaped at the "starting" end so that the thread is formed more gradually. The included angle of the ridges of a die for rolling U. S. standard or V-threads should be about 58 1/2 degrees instead of 60 degrees, in order to produce a 60-degree thread. This reduction in the angle is necessary on account of the elasticity of the stock, which causes the lower part of a rolled thread to spring back slightly after leaving the die. Aside from this modification, the ridges on thread-rolling dies are of the same cross-sectional shape as the screw thread.

Diameter of Blank. — The plain blanks upon which threads are to be rolled are somewhat smaller in diameter than the finished thread, because the rolling process displaces a certain amount of metal which is forced up above the original surface of the blank. This increase in diameter is approximately equal to the depth of one thread, and the blank should be just small enough to compensate for the metal that is forced upward by the rolling process. While there are rules and formulas for determining blank diameters, it may be necessary to make slight changes in the calculated size in order to secure a well-formed thread. The calculated blank diameter should be verified by trial, especially when rolling accurate screw threads. The blank diameter is affected somewhat by the nature of the material and the condition of the surface. Some stock offers greater resistance to displacement than other stock, owing to the greater hardness or tenacity of the metal.

There are three general classes of blank sizes, according to the practice in different plants where thread rolling is done. Some blanks are made a little larger than the pitch diameter of the screw thread, others are approximately equal to the pitch diameter, and in some cases they are slightly less than the pitch diameter. The blanks are made a little larger than the pitch diameter for threads which are to be rolled as accurately as possible. According to the average practice, as near as this can be determined, the blank diameters for screws varying from 1/4 to 1/2 inch are from 0.002 to 0.0025 inch larger than the pitch diameter, and for screws varying from 1/2 to 1 inch or larger, the blank diameters are from 0.0025 to 0.003 inch larger than the pitch diameter. Blanks which are slightly less than the pitch diameter are intended for bolts, screws, etc., which are to have a comparatively free fit. Blanks for this class of work may vary from 0.002 to 0.003 inch less than the pitch diameter for screw thread sizes varying from 1/4 to 1/2 inch, and from 0.003 to 0.005 inch less than the pitch diameter for sizes above ½ inch. If the screw threads are smaller than ¼ inch, the blanks are usually from 0.001 to 0.0015 inch less than the pitch diameter for ordinary grades of work.

Thread Rolling in Automatic Screw Machines. — Screw threads are sometimes rolled in automatic screw machines and turret lathes when the thread is behind a shoulder so that it cannot be cut with a die. In such cases, the advantage of rolling the thread is that a second operation is avoided. A circular roll is used for rolling threads in screw machines. The roll may be presented to the work either in a tangential direction or radially, either method producing a satisfactory thread. In the former case, the roll gradually comes into contact with the periphery of the work and completes the thread as it passes across the surface to be threaded. When the roll is held in a radial position, it is simply forced against one side until a complete thread is formed. The method of applying the roll may depend upon the relation between the threading operation and other machining operations. Thread rolling in automatic screw machines is generally applied only to brass and other relatively soft metals, owing to the difficulty of rolling threads in steel. Thread rolls made of chrome-nickel steel containing from 0.15 to 0.20 per cent of carbon have given fairly good results, however, when applied to steel. A 3 per cent nickel steel containing about 0.12 per cent carbon has also proved satisfactory for threading brass.

Factors Governing the Diameter of Threading Roll. — The threading roll used in screw machines may be about the same diameter as the screw thread, but for sizes smaller than, say, ¾ inch, the roll diameter is some multiple of the thread diameter minus a slight amount to obtain a better rolling action. When the diameters of the thread and roll are practically the same, a single-threaded roll is used to form a single thread on the screw. If the diameter of the roll is made double that of the screw, in order to avoid using a small roll, then the roll must have a double thread. If the thread roll is three times the size of the screw thread, a triple thread is used, and so on. These multiple threads are necessary when the roll diameter is some multiple of the work, in order to obtain corresponding helix angles on the roll and work.

Calculating Diameter of Threading Roll. — The outside diameter of a threading roll may be determined by the following formula, in which R = outside diameter of roll; N = number of separate threads or "starts" on roll; S = outside diameter of finished screw thread; D = depth of a single thread.

$$R = N\,(S - 1.25\,D)$$

Example. — Find the diameter of a threading roll for a ⅜-inch U. S. standard screw thread. In this example,

$$S = 0.375 \quad \text{and} \quad D = 0.0406$$

If a roll three times the diameter of the thread is considered large enough, then N equals 3; hence

$$R = 3\,(0.375 - 1.25 \times 0.0406) = 0.972 \text{ inch.}$$

The following formula for determining the size of a threading roll gives the pitch diameter, and in this formula,

P = pitch diameter of threading roll;
N = number of single threads or "starts" on the roll. (This number is selected with reference to the diameter of roll desired);
D = pitch diameter of screw thread;
T = single depth of thread.

$$P = N\left(D - \frac{T}{3}\right)$$

Kind of Thread on Roll and Its Shape. — The thread (or threads) on the roll should be left hand for rolling a right-hand thread, and *vice versa*. The roll should be wide enough to overlap the part to be threaded, provided there are clearance spaces at the ends, which should be formed if possible. The thread on the roll should be sharp on top for rolling a U. S. standard thread as well as for a V-thread, so that less pressure will be required to displace the metal when rolling the thread. The bottom of the thread groove on the roll may also be left sharp or it may have a flat. If the bottom is sharp, the roll is sunk only far enough into the blank to form a thread having a flat top, assuming that the U. S. standard form is being rolled. The number of threads on the roll (whether double, triple, quadruple, etc.) is selected, as a rule, so that the diameter of the thread roll will be somewhere between 1¼ and 2¼ inches. In making a thread roll, the ends are beveled at an angle of 45 degrees, to prevent the threads on the ends of the roll from chipping. Precautions should be taken in hardening, because if the sharp edges are burnt, the roll will be useless. Thread rolls, as a rule, are lapped after hardening. This is done by holding them on an arbor in the lathe and using emery and oil on a piece of hard wood. A thread roll, to give good results, should fit closely in the holder. If the roll is made to fit loosely, it will mar the threads.

Application of Thread Roll. — The shape of the work, and the character of the operations necessary to produce it, govern, to a large extent, the method employed in applying the thread roll. Some of the points to consider are as follows: 1. Diameter of the part to be threaded. 2. Location of the part to be threaded. 3. Length of the part to be threaded. 4. Relation that the thread rolling operation bears to the other operations. 5. Shape of the part to be threaded, whether straight, tapered or otherwise. 6. Method of applying the support. When the diameter to be rolled is much smaller than the diameter of the shoulder preceding it, a cross-slide knurl-holder should be used. If the part to be threaded is not behind a shoulder, a holder on the swing principle should be used. When the work is long (greater in length than two-and-one-half times its diameter) a swing roll-holder should be employed, carrying a support. When the work can be cut off after the thread is rolled, a cross-slide roll-holder should be used. The method of applying the support to the work also governs to some extent the method of applying the thread roll. When no other tool is working at the same time as the thread roll, and when there is freedom from chips, the roll can be held more rigidly by passing it under instead of over the work. When passing the roll over the work, it has a tendency to raise the cross-slide. Where the part to be threaded is tapered, the roll can best be presented to the work by holding it in a cross-slide roll-holder.

Speeds and Feeds for Thread Rolling. — When the thread roll is made from high-carbon steel and used on brass, a surface speed as high as 200 feet per minute can be used. Better results, however, are obtained by using a lower speed than this. When the roll is held in a holder attached to the cross-slide, and is presented either tangentially or radially to the work, a considerably higher speed can be used than if it is held in a swing tool. This is due to the lack of rigidity in a holder of the swing type. The feeds to be used when a cross-slide roll-holder is used are given in the upper half of the table "Feeds for Thread Rolling"; the lower half of the table gives the feeds for thread rolling with swing tools. These feeds are applicable for rolling threads without a support, when the root diameter of the blank is not less than five times the double depth of the thread. When the root diameter is less than this, a support should be used. A support should also be used when the width of the roll is more than two-and-one-half times the smallest diameter of the piece to be rolled, irrespective of the pitch of the thread. When the smallest diameter of the piece to be rolled is much less than the root diameter of the thread, the smallest diameter should be taken as the deciding factor for the feed to be used.

Feeds for Thread Rolling

Root Diam. of Blank	72	64	56	48	44	40	36	32	28	24	22	20	18	14
	Number of Threads per Inch													
	Cross-slide Holders — Feed per Revolution in Inches													
1/8	0.0045	0.0040	0.0035	0.0030	0.0025	0.0020	0.0015	0.0010	.....	.....	.....	.....	.....	.....
3/16	0.0050	0.0045	0.0040	0.0035	0.0030	0.0025	0.0020	0.0015	0.0005	.....	.....	.....	.....	.....
1/4	0.0055	0.0050	0.0045	0.0040	0.0035	0.0030	0.0025	0.0020	0.0010	0.0005	0.0005	.....	.....	.....
5/16	0.0060	0.0055	0.0050	0.0045	0.0040	0.0035	0.0030	0.0025	0.0015	0.0010	0.0010	0.0005	0.0005	.....
3/8	0.0065	0.0060	0.0055	0.0050	0.0045	0.0040	0.0035	0.0030	0.0020	0.0015	0.0015	0.0010	0.0010	0.0005
7/16	0.0070	0.0065	0.0060	0.0055	0.0050	0.0045	0.0040	0.0035	0.0025	0.0020	0.0020	0.0015	0.0015	0.0010
1/2	0.0075	0.0070	0.0065	0.0060	0.0055	0.0050	0.0045	0.0040	0.0030	0.0025	0.0025	0.0020	0.0020	0.0015
5/8	0.0080	0.0075	0.0070	0.0065	0.0060	0.0055	0.0050	0.0045	0.0035	0.0030	0.0030	0.0025	0.0025	0.0020
3/4	0.0085	0.0080	0.0075	0.0070	0.0065	0.0060	0.0055	0.0050	0.0040	0.0035	0.0035	0.0030	0.0030	0.0025
7/8	0.0090	0.0085	0.0080	0.0075	0.0070	0.0065	0.0060	0.0055	0.0045	0.0040	0.0040	0.0035	0.0035	0.0030
1	0.0095	0.0090	0.0085	0.0080	0.0075	0.0070	0.0065	0.0060	0.0050	0.0045	0.0045	0.0040	0.0040	0.0035
Root Diam.	Swing Holders — Feed per Revolution in Inches													
1/8	0.0025	0.0020	0.0015	0.0010	0.0005	.....	.....	.....	.....	.....	.....	.....	.....	.....
3/16	0.0028	0.0025	0.0020	0.0015	0.0008	0.0005	.....	.....	.....	.....	.....	.....	.....	.....
1/4	0.0030	0.0030	0.0025	0.0020	0.0010	0.0010	0.0005	0.0005	0.0005	.....	.....	.....	.....	.....
5/16	0.0035	0.0035	0.0030	0.0025	0.0015	0.0015	0.0010	0.0010	0.0010	0.0005	.....	.....	.....	.....
3/8	0.0040	0.0040	0.0035	0.0030	0.0020	0.0020	0.0015	0.0015	0.0015	0.0010	0.0005	0.0005	0.0005	.....
7/16	0.0045	0.0045	0.0040	0.0035	0.0030	0.0025	0.0020	0.0020	0.0020	0.0015	0.0010	0.0010	0.0010	.....
1/2	0.0048	0.0048	0.0045	0.0040	0.0035	0.0030	0.0025	0.0025	0.0025	0.0020	0.0015	0.0015	0.0015	0.0005
5/8	0.0050	0.0050	0.0048	0.0043	0.0040	0.0035	0.0030	0.0030	0.0028	0.0025	0.0020	0.0020	0.0018	0.0010
3/4	0.0055	0.0052	0.0050	0.0045	0.0043	0.0040	0.0035	0.0035	0.0030	0.0028	0.0025	0.0022	0.0020	0.0013
7/8	0.0058	0.0055	0.0052	0.0048	0.0045	0.0043	0.0040	0.0038	0.0032	0.0030	0.0028	0.0025	0.0022	0.0015
1	0.0060	0.0058	0.0054	0.0050	0.0048	0.0047	0.0043	0.0040	0.0035	0.0032	0.0030	0.0028	0.0025	0.0018

TAPPING AND THREAD CUTTING

Diameter of Tap Drill. — Tapping troubles are often caused by using tap drills that are too small in diameter. For ordinary manufacturing, not more than 75 or 80 per cent of the standard thread depth is necessary, and for some classes of work, not more than 50 per cent is required. Tap drill sizes, especially for machine screws, should be varied according to the material to be tapped and the depth of the tapped hole. In general, if the screws enter more than one and one-half times the diameter, one-half of the full thread is sufficient. Soft tough material, such as copper, Norway iron, drawn aluminum, etc., should have a larger hole for the tap than hard crystalline materials, such as cast metals. When tapping soft materials, if the hole is too small, the threads will be torn off to some extent, thus actually decreasing the effective thread depth as compared to what it would be if the tap drill had been of larger diameter; but if the hole is drilled rather large, when tapping tenacious materials, the metal at the top of the thread is drawn somewhat, thereby increasing the depth of the threads. This is more likely to occur after the keen edge of the tap has been slightly dulled by use.

The diameters of tap drills can be found by the formula, $D = T - 0.75 \times 2d$, in which D = drill diameter, T = diameter of tap or thread and d = depth of thread. Values of the factor "$2d$" (double depth of thread) for various numbers of threads are given in the tables in the section "Screw Thread Systems." The diameters obtained by this formula allow for a thread having 75 per cent of the standard depth which is sufficient for general work. The formula applies to either U. S. standard or V-threads. The diameter of the tap drill should not be smaller than is necessary to give the required strength of thread, as every decrease of even 0.001 inch in diameter of the tap drill materially increases the power required for tapping and the percentage of broken taps.

The practice of a prominent manufacturer is to make a distinction in the sizes of tap drills according to whether the hole is to be hand or machine tapped. The table below gives the amount to subtract from the outside diameter of the screw to find the diameter of the tap drill for both machine and hand tapping for different numbers of threads per inch of U. S. standard form. In this case the tap drills for machine tapping will give 0.8 of a full thread and those for hand tapping 0.9 of a full thread.

Threads per Inch	Amount to Subtract from Outside Diameter to find Diameter of Tap Drill		Threads per Inch	Amount to Subtract from Outside Diameter to find Diameter of Tap Drill		Threads per Inch	Amount to Subtract from Outside Diameter to find Diameter of Tap Drill	
	Machine Tapping	Hand Tapping		Machine Tapping	Hand Tapping		Machine Tapping	Hand Tapping
64	0.0162	0.0183	26	0.0400	0.0450	9	0.1155	0.1299
60	0.0173	0.0195	24	0.0433	0.0487	8	0 1299	0.1461
56	0.0186	0.0209	22	0.0472	0.0531	7	0.1485	0.1670
50	0.0208	0.0234	20	0.0520	0.0584	6	0.1732	0.1949
48	0.0216	0.0243	18	0.0577	0.0649	5½	0.1889	0.2126
44	0.0236	0.0266	16	0.0649	0.0731	5	0.2078	0.2338
40	0.0260	0.0292	14	0.0742	0.0835	4½	0.2309	0.2598
36	0.0289	0.0325	13	0.0799	0.0899	4	0.2598	0.2923
32	0.0325	0.0365	12	0.0866	0.0974	3½	0.2969	0.3340
30	0.0346	0.0390	11½	0.0904	0.1017	3	0.3464	0.3897
28	0.0371	0.0417	11	0.0945	0.1063	.....	....	....
27	0.0385	0.0433	10	0.1039	0.1169	.....	....	....

Tap Drill Sizes for U. S. Standard Threads

These tap drill diameters allow approximately 75 per cent of a full thread

Thread Diam.	No. of Threads	Diam. Tap Drill	Thread Diam.	No. of Threads	Diam. Tap Drill	Thread Diam.	No. of Threads	Diam. Tap Drill
1/4	20	13/64	13/16	10	45/64	1 3/8	6	1 7/32
5/16	18	1/4	7/8	9	49/64	1 1/2	6	1 11/32
3/8	16	5/16	15/16	9	53/64	1 5/8	5 1/2	1 7/16
7/16	14	23/64	1	8	7/8	1 3/4	5	1 35/64
1/2	13	27/64	1 1/16	8	15/16	1 7/8	5	1 43/64
9/16	12	15/32	1 1/8	7	63/64	2	4 1/2	1 25/32
5/8	11	17/32	1 3/16	7	1 3/64	2 1/4	4 1/2	2 1/32
11/16	11	19/32	1 1/4	7	1 7/64	2 1/2	4	2 1/4
3/4	10	41/64	1 5/16	7	1 11/64	2 3/4	4	2 1/2

Tap Drill Sizes for V-threads

These tap drill diameters allow approximately 75 per cent of a full thread

Thread Diam.	No. of Threads	Diam. Tap Drill	Thread Diam.	No. of Threads	Diam. Tap Drill	Thread Diam.	No. of Threads	Diam. Tap Drill
1/4	20	3/16	13/16	10	43/64	1 3/8	6	1 5/32
5/16	18	15/64	7/8	9	23/32	1 1/2	6	1 9/32
3/8	16	19/64	15/16	9	25/32	1 5/8	5	1 23/64
7/16	14	11/32	1	8	53/64	1 3/4	5	1 31/64
1/2	12	25/64	1 1/16	8	57/64	1 7/8	4 1/2	1 37/64
9/16	12	29/64	1 1/8	7	15/16	2	4 1/2	1 45/64
5/8	11	1/2	1 3/16	7	1	2 1/4	4 1/2	1 31/32
11/16	11	9/16	1 1/4	7	1 1/16	2 1/2	4	2 11/64
3/4	10	39/64	1 5/16	7	1 1/8	2 3/4	4	2 27/64

Tap Drills for Pipe Taps *

Size of Tap	Drills for Briggs Pipe Taps	Drills for Whitworth Pipe Taps	Size of Tap	Drills for Briggs Pipe Taps	Drills for Whitworth Pipe Taps	Size of Tap	Drills for Briggs Pipe Taps	Drills for Whitworth Pipe Taps
1/8	11/32	5/16	1 1/4	1 1/2	1 15/32	3 1/4	.....	3 1/2
1/4	7/16	27/64	1 1/2	1 23/32	1 25/32	3 1/2	3 13/16	3 3/4
3/8	19/32	9/16	1 3/4	.....	1 15/16	3 3/4	.....	4
1/2	23/32	11/16	2	2 3/16	2 5/32	4	4 1/4	4 1/4
5/8	.....	25/32	2 1/4	.....	2 13/32	4 1/2	4 3/4	4 3/4
3/4	15/16	29/32	2 1/2	2 11/16	2 25/32	5	5 5/16	5 1/4
7/8	.....	1 1/16	2 3/4	.....	3 1/32	5 1/2	.....	5 3/4
1	1 5/32	1 1/8	3	3 5/16	3 9/32	6	6 3/8	6 1/4

* To secure the best results, the hole should be reamed before tapping with a reamer having a taper of 3/4 inch per foot.

Tap Drill Sizes for A. S. M. E. Standard Machine Screws

These tap drill diameters allow approximately 75 per cent of a full thread.

No. of Screw	Diam. of Thread	Threads per Inch	Tap Drill Diam.	Drill No. or Letter	No. of Screw	Diam. of Thread	Threads per Inch	Tap Drill Diam.	Drill No. or Letter
Standard Machine Screws									
0	0.060	80	0.0465	56	12	0.216	28	0.180	15
1	0.073	72	0.0595	53	14	0.242	24	0.201	7
2	0.086	64	0.070	50	16	0.268	22	0.221	2
3	0.099	56	0.081	46	18	0.294	20	0.242	C
4	0.112	48	0.089	43	20	0.320	20	0.266	H
5	0.125	44	0.1015	38	22	0.346	18	0.290	L
6	0.138	40	0.113	33	24	0.372	16	0.302	N
7	0.151	36	0.120	31	26	0.398	16	0.332	Q
8	0.164	36	0.136	29	28	0.424	14	0.348	S
9	0.177	32	0.147	26	30	0.450	14	0.377	V
10	0.190	30	0.157	22	.....	.....	.....	....	....
Special Machine Screws									
1	0.073	64	0.055	54	9	0.177	24	0.136	29
2	0.086	56	0.067	51	10	0.190	32	0.159	21
3	0.099	48	0.0785	47	10	0.190	24	0.147	26
4	0.112	40	0.086	44	12	0.216	24	0.173	17
4	0.112	36	0.082	45	14	0.242	20	0.191	11
5	0.125	40	0.0995	39	16	0.268	20	0.213	3
5	0.125	36	0.098	40	18	0.294	18	0.238	B
6	0.138	36	0.111	34	20	0.320	18	0.266	H
6	0.138	32	0.1065	36	22	0.346	16	0.281	K
7	0.151	32	0.120	31	24	0.372	18	0.316	O
7	0.151	30	0.116	32	26	0.398	14	0.323	P
8	0.164	32	0.1285	30	28	0.424	16	0.358	T
8	0.164	30	0.1285	30	30	0.450	16	0.386	W
9	0.177	30	0.144	27	.....	.....	.....	....	....

Tap Drill Sizes for S. A. E. Standard Threads

Diam. of Tap	No. of Threads	Diam. of Tap Drill	Diam. of Tap	No. of Threads	Diam. of Tap Drill	Diam. of Tap	No. of Threads	Diam. of Tap Drill
1/4	28	0.213	9/16	18	0.500	1	14	0.921
5/16	24	0.272	5/8	18	0.562	1 1/8	12	1.031
3/8	24	0.332	11/16	16	0.625	1 1/4	12	1.156
7/16	20	0.386	3/4	16	0.687	1 3/8	12	1.281
1/2	20	0.437	7/8	14	0.796	1 1/2	12	1.406

Tap Drills for Small V-thread Taps

Size of Tap	No. of Threads	Drill No.	Size of Tap	No. of Threads	Drill No.	Size of Tap	No. of Threads	Drill No.
1/16	60	55	9/64	32	32	13/64	28	20
5/64	60	52	9/64	36	35	13/64	32	20
3/32	48	47	9/64	40	33	7/32	22	19
3/32	56	46	5/32	30	31	7/32	24	18
3/32	60	46	5/32	32	30	7/32	28	17
7/64	32	45	5/32	36	29	7/32	30	15
7/64	36	44	5/32	40	29	7/32	32	13
7/64	40	43	11/64	32	30	15/64	22	10
7/64	44	43	11/64	36	29	15/64	24	10
7/64	48	42	11/64	40	28	15/64	28	9
1/8	32	40	3/16	24	27	15/64	32	9
1/8	36	38	3/16	28	26	1/4	20	7
1/8	40	37	3/16	30	23	1/4	22	5
1/8	44	36	3/16	32	23	1/4	24	2
9/64	30	35	13/64	24	21	1/4	32	2

Power Required for Tapping. — The power required for tapping depends upon the diameter of the tap drill hole; the kind of lubricant used; the shape of the tap flutes; amount that the end of the tap is ground back, that is, the "chamfer"; and the condition of the tap. The accompanying table, "Average Torque in Inch-pounds for Tapping Different Materials," contains figures which are the average of a large number of tests. The maximum and minimum torque in inch-pounds is given. The taps were ½ inch U. S. standard; depth of tapped hole, ½ inch; and diameter of tap drill, 0.420 inch. The table, "Effect of Lubricants and Tap Drill Diameters when Tapping," shows the variation in power, the number of breakages and the quality of thread resulting from the use of different lubricants and three different sizes of tap drills. These data represent a long series of tests. For comparative purposes, the breaking strength is taken as 100, and the power required for tapping is given as a percentage of this number. The test pieces were common hexagon, cold-punched nuts accurately reamed to the sizes specified, and regular ½ inch U. S. standard taps were used. The torque required to break a properly made ½ inch U. S. standard tap is approximately 1000 inch-pounds; hence, by multiplying the percentage given in the table, by 10, the average actual torque in inch-pounds can be obtained. (From paper by Mr. F. O. Wells, read before the American Society of Mechanical Engineers.)

Average Torque in Inch-pounds for Tapping Different Materials
(One-half inch U. S. tap; diameter of tap drill, 0.420 inch.)

Material Tapped	Torque in Inch-pounds	
	Maximum	Minimum
Hexagon Drawn Brass	64	63
Crucible Tool Steel	261	258
Cold Punched Hexagon Steel Nuts	182	176
Hexagon Screw Stock	205	189
Drawn Hexagon Phosphor Bronze	234	221

Effect of Lubricants and Tap Drill Diameters when Tapping

Lubricant	Diameter Tap Hole, 0.425 inch (75 per cent Thread)			Diameter Tap Hole, 0.410 inch (90 per cent Thread)			Diameter Tap Hole, 0.400 inch (Full Thread)		
	Power Required*	Per cent of Breakages	Quality of Thread	Power Required*	Per cent of Breakages	Quality of Thread	Power Required*	Per cent of Breakages	Quality of Thread
Animal Lard Oil	15.9	0	Smooth	.....	....	....	.....	....	....
Sperm Oil	16.5	0	Smooth	23	0	Smooth	35.5	0	Smooth, Taps Torn
Graphite, 10 per cent; Tallow, 90 per cent	16.9	0	Smooth	.....	.....	....	.....	....	....
Cataract Soap Compound	18.9	0	Smooth	25.1	0	Smooth	41	0	Slightly Rough, Taps Torn
Mineral Lard Oil	19.9	0	Smooth	36.5	0	Smooth	57.5	0	Smooth, Taps Torn
Tapped Without Oil	29.9	14	Rough	60.2	50	Rough	71.8	66	Torn, Partly Stripped
Machine Oil	34.2	15	Torn	62.5	71.5	Badly Torn	100	100	Torn, Chips Wedged

* In per cent of breaking strength of tap.

Power for Pipe Taps. — The power required for driving pipe taps is given in the following table, which includes nominal pipe tap sizes from 2 to 8 inches.

Power Required for Pipe Taps

Nominal Tap Size	Rev. per Min.	Net H.P.	Thickness of Metal	Nominal Tap Size	Rev. per Min.	Net H.P.	Thickness of Metal
2	40	4.24	1⅛	3½	25.6	7.20	1¾
2½	40	5.15	1⅛	4	18	6.60	2
*2½	38.5	9.14	1⅛	5	18	7.70	2
3	40	5.75	1⅛	6	17.8	8.80	2
*3	38.5	9.70	1⅛	8	14	7.96	2½

* Tapping steel casting; other tests in cast iron.

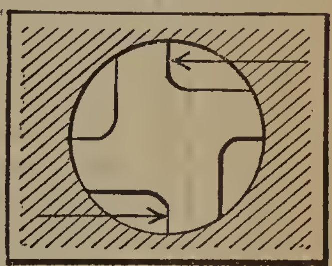
The holes to be tapped were reamed with standard pipe tap reamers before tapping. The horsepower recorded was read off just before the tap was reversed.

The table gives the net horsepower, deductions being made for the power required to run the machine without a load. The material tapped was cast iron, except in two instances, where steel casting was tapped. It will be seen that nearly double the power is required for tapping steel casting. The power varies, of course, with the conditions. More power than that indicated in the table will be required if the cast iron is of a harder quality or if the taps are not properly relieved. The taps used in these experiments were of the inserted-blade type, the blades being made of high-speed steel.

Speeds for Standard Taps in Revolutions per Minute

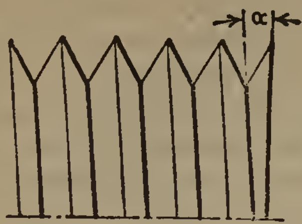
Diam. of Tap	Cast Iron	Wrought Iron	Diam. of Tap	Cast Iron	Wrought Iron	Diam. of Tap	Cast Iron	Wrought Iron
3/16	340	265	5/8	117	91	1 3/8	51	41
1/4	295	230	3/4	96	76	1 1/2	46	38
5/16	240	190	7/8	84	65	2	34	28
3/8	197	152	1	72	57	2 1/4	30	26
7/16	170	122	1 1/8	63	50	2 1/2	26	23
1/2	145	114	1 1/4	57	45	.....	.....	......

Removing a Broken Tap. — When a tap is broken near the surface, it can easily be removed by driving on both sides (as indicated by the arrows in the illustration), using drifts or blunt cape chisels. Two men should start at the same time, with light blows, and when they are striking in unison, the force of the blows should be increased, as may be required. By driving on both sides, the tap is not wedged against one side of the hole, as when using a single drift, but is forced to rotate. This is an old, but very effective method. Another method of removing broken taps, which has proved very effective in some cases, is to inject into the hole a little nitric acid, diluted in the proportion of about one part acid to five parts water. The action of the acid upon the steel loosens the tap so that it usually can be removed readily. The remaining acid should afterwards be washed out of the hole so that it will not continue to eat the threads.



Lubricants for Tapping. — The breakage of taps can be reduced greatly by using the proper lubricant. A good grade of animal lard oil, sperm oil, and graphite and tallow mixtures (10 per cent graphite, 90 per cent tallow) are the best lubricants to use when tapping steel or iron. A good soap compound is better than "mineral lard oil." Machine oil is a poor tapping lubricant. By referring to the column "Power Required," in the table "Effect of Lubricants and Tap Drill Diameters when Tapping," it will be seen that the power required when using sperm oil is 16.5, as compared with 34.2 when machine oil is used. Incidentally, this increase is almost as great as that due to decreasing the diameter of the tap drill from 0.425 to 0.400 inch when using sperm oil, the increase being from 16.5 to 35.5. This shows that a poor lubricant may increase the power for tapping as much as would a considerable reduction in the diameter of the hole to be tapped. For cast iron, soap compounds give excellent results, and lard oil is also used. Oil for cast iron, however, has the disadvantage of causing the chips to stick in the tap flutes, thus preventing the lubricant from reaching the cutting edges; hence a thinner lubricant is preferable. When tapping long holes in cast iron, a very small amount of kerosene will facilitate the work.

Thread Angles for Different Diameters and Threads per Inch

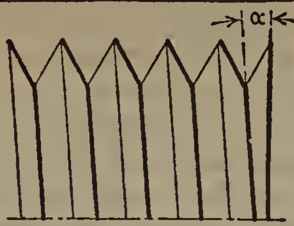


$$\tan \alpha = \frac{L}{3.1416D}$$

L = lead of thread;
D = diameter of thread.

No. of Threads per Inch	Diameters and Corresponding Angles								
	⅛	3/16	¼	½	¾	1	1¼	1½	1¾
100	1° 27′	58′	44′	22′	15′	11′	9′	7′	6′
80	1 49	1° 13	55	27	18	14	11	9	8
64	2 17	1 31	1° 8	34	23	17	14	12	10
56	2 36	1 44	1 18	39	26	19	16	13	11
48	3 2	2 2	1 31	46	30	23	18	15	13
40	3 40	2 31	1 49	55	37	27	22	18	16
36	4 2	2 42	2 2	1° 0	40	30	24	20	17
32	4 33	3 2	2 17	1 8	46	34	27	23	19
30	4 51	3 14	2 27	1 13	49	37	29	24	21
28	5 12	3 28	2 36	1 18	52	39	31	26	22
26	5 36	3 44	2 48	1 24	56	42	34	28	24
24	6 4	4 3	3 2	1 31	1° 0	46	37	30	26
22	6 38	4 25	3 19	1 39	1 6	50	40	33	28
20	7 16	4 52	3 38	1 49	1 13	55	44	37	31
18	8 8	5 24	4 3	2 2	1 21	1° 0	49	40	35
16	9 8	6 4	4 33	2 17	1 31	1 8	55	46	39
14	....	6 56	5 12	2 36	1 44	1 18	1° 1	52	44
13	....	7 28	5 36	2 48	1 52	1 24	1 8	56	48
12	....	8 4	6 4	3 2	2 1	1 31	1 13	1° 0	52
11	....	8 48	6 38	3 19	2 12	1 39	1 20	1 6	56
10	....	....	7 16	3 38	2 26	1 49	1 27	1 13	1° 2
9	....	....	8 8	4 4	2 42	2 2	1 38	1 21	1 10
8	....	....	9 8	4 34	3 2	2 17	1 49	1 31	1 18
7	....	....	....	5 12	3 28	2 36	2 1	1 44	1 28
6	....	....	....	6 4	4 2	3 2	2 26	2 1	1 44
5½	....	....	....	6 38	4 24	3 19	2 40	2 12	1 52
5	....	....	....	7 16	4 52	3 38	2 54	2 26	2 4
4½	....	....	....	8 8	5 24	4 4	3 16	2 42	2 20
4	....	....	....	9 8	6 4	4 34	3 38	3 2	2 36
3½	....	....	....	....	6 56	5 12	4 2	3 28	2 56
3¼	....	....	....	....	7 28	5 36	4 32	3 44	3 12
3	....	....	....	....	8 4	6 4	4 52	4 2	3 28
2⅞	....	....	....	....	8 26	6 20	5 6	4 13	3 37
2¾	....	....	....	....	8 48	6 38	5 20	4 24	3 44
2⅝	....	....	....	....	9 20	6 56	5 34	4 40	3 58
2½	....	....	....	....	.....	7 16	5 48	4 52	4 8
2⅜	....	....	....	....	.....	7 42	6 10	5 8	4 24
2¼	....	....	....	....	.....	8 8	6 32	5 24	4 40
2	....	....	....	....	.....	9 8	7 8	6 4	5 12

Thread Angles for Different Diameters and Threads per Inch



$$\text{Tan } \alpha = \frac{L}{3.1416\,D}$$

L = lead of thread;
D = diameter of thread.

No. of Threads per Inch	Diameters and Corresponding Angles								
	2	2½	3	3½	4	4½	5	5½	6
100	5′	....	....	....	.....	.....	.....	.....	.....
80	7	5′	....	....	.....	.....	.....	.....	.....
64	8	7	6′	5′	.....	.....	.....	.....	.....
56	10	8	7	6	5′	.....	.....	.....	.....
48	12	9	8	7	6	5′	.....	.....	.....
40	14	11	9	8	7	6	5′	.....	.....
36	15	12	10	9	8	7	6	6′	5′
32	17	14	12	10	9	8	7	6	6
30	18	15	12	11	9	8	7	7	6
28	19	16	13	11	10	9	8	7	7
26	21	17	14	12	11	9	8	8	7
24	23	18	15	13	11	10	9	8	8
22	25	20	17	14	13	11	10	9	8
20	27	22	18	16	14	12	11	10	9
18	30	24	20	17	15	14	12	11	10
16	34	27	23	20	17	15	14	12	11
14	39	31	26	22	19	17	15	14	13
13	42	34	28	24	21	18	17	15	14
12	46	37	30	26	23	20	18	16	15
11	50	40	33	28	25	22	20	18	17
10	55	44	37	31	27	24	22	20	18
9	1° 0	49	40	35	30	27	24	22	20
8	1 8	55	46	39	34	30	27	25	23
7	1 18	1° 1	52	44	39	35	31	28	26
6	1 31	1 13	1° 0	52	46	41	37	33	30
5½	1 39	1 20	1 6	56	50	45	40	36	33
5	1 49	1 27	1 13	1° 2	55	49	44	40	37
4½	2 2	1 38	1 21	1 10	1° 0	54	49	44	40
4	2 17	1 49	1 31	1 18	1 8	1° 1	55	50	46
3½	2 36	2 1	1 44	1 28	1 18	1 8	1° 1	56	52
3¼	2 48	2 16	1 52	1 36	1 24	1 16	1 8	1° 2	56
3	3 2	2 26	2 1	1 44	1 31	1 22	1 13	1 6	1° 0
2⅞	3 10	2 33	2 7	1 48	1 35	1 25	1 16	1 9	1 3
2¾	3 19	2 40	2 12	1 52	1 39	1 29	1 20	1 13	1 6
2⅝	3 28	2 47	2 20	1 59	1 44	1 33	1 24	1 16	1 10
2½	3 38	2 54	2 26	2 4	1 49	1 38	1 27	1 20	1 13
2⅜	3 51	3 5	2 34	2 12	1 56	1 44	1 33	1 25	1 17
2¼	4 4	3 16	2 42	2 20	2 2	1 50	1 38	1 29	1 21
2	4 34	3 38	3 2	2 36	2 17	2 3	1 49	1 40	1 31

Planing Clearance on Threading Tools.— In the following are given formulas for finding the angle to which the planer or shaper head should be set when planing threading tools with both side and front clearance. The expression "leading" side indicates the side of the tool which first enters the work when a thread is cut; the "following" side is that which enters the work last. In the formulas, a = depth of thread; b = width of flat on offset tool; c = actual width of flat; d = outside diameter of screw; v = front clearance angle; w = one-half angle of thread; y = angle of thread helix; and x = angle to which to set the planer head when planing the tool on the side. Then, for tools with side clearance:

$$\tan y = \frac{\text{lead of thread}}{3.1416\,(d-a)}$$

and angle x is found from:

$$\tan x = \frac{\cos y \pm (\cot w \times \sin v \times \sin y)}{\cot w \times \cos v}$$

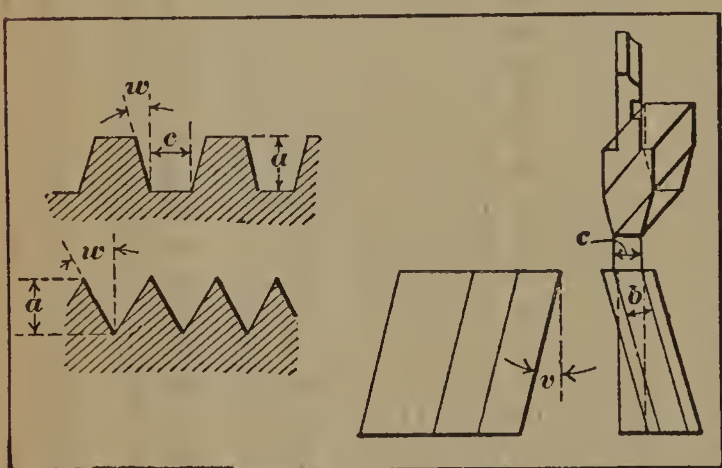
Use + for leading side and – for following side.

For Acme (29-degree) thread and 15 degrees clearance angle, the formula can, for all practical purposes, be written:

$$\tan x = \frac{\cos y \pm \sin y}{3.735}$$

The width of flat on the offset tool is figured from the formula:

$$b = c \times \cos y$$



If the tool has no side clearance, the angle of helix can be considered equal to 0 degrees, and the formula reduces itself to $\tan x = \dfrac{\tan w}{\cos v}$

For a 60-degree screw thread, U. S. standard, the formula will then be:

$$\tan x = \frac{\tan 30^\circ}{\cos 15^\circ} = 0.5977;$$

$$x = 30 \text{ deg. } 52 \text{ min.}$$

Example: Find the angles to which to set the planer heads when planing the sides of an Acme thread tool for a screw 2 inches in diameter having 2 threads per inch.

$$\tan y = \frac{0.5}{3.1416\,(2-0.26)} = 0.0915; \quad y = 5^\circ\ 14'.$$

$$\tan x = \frac{\cos 5^\circ\ 14' \pm \sin 5^\circ\ 14'}{3.735} = \frac{0.9958 \pm 0.0912}{3.735}$$

$$\tan x \text{ (for "leading" side)} = 0.291; \quad x = 16^\circ\ 14'.$$

$$\tan x \text{ (for "following" side)} = 0.242; \quad x = 13^\circ\ 37'$$

Lathe Change Gears

Change Gears for Thread Cutting. — To determine the change gears to use for cutting a thread of given pitch, first find what number of threads per inch will be cut when gears of the same size are placed on the lead-screw and spindle stud, either by actual trial or by referring to the index plate; then multiply this number, called the "lathe screw constant," by some trial number to obtain the number of teeth in the gear for the spindle stud, and multiply the threads per inch to be cut by the *same* trial number to obtain the number of teeth in the gear for the lead-screw. Expressing this rule as a formula:

$$\frac{\text{Trial number} \times \text{lathe screw constant}}{\text{Trial number} \times \text{threads per inch to be cut}} = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear on lead-screw}}.$$

For example, suppose the available change gears supplied with the lathe have 24, 28, 32, 36 teeth, etc., the number increasing by four up to one hundred, and that 10 threads per inch are to be cut in a lathe having a lathe screw constant of 6; then, if the screw constant is written as the numerator, and the number of threads per inch to be cut, as the denominator of a fraction, and both numerator and denominator are multiplied by some trial number, say 4, it is found that gears having 24 and 40 teeth can be used. Thus:

$$\frac{6}{10} = \frac{6 \times 4}{10 \times 4} = \frac{24}{40}$$

The 24-tooth gear goes on the spindle stud and the 40-tooth gear on the lead-screw.

The lathe screw constant is, of course, equal to the number of threads per inch on the lead-screw, provided the spindle stud and spindle are geared in the ratio of 1 to 1, which, however, is not always the case.

Compound Gearing. — To find the change gears used in compound gearing, place the screw constant as the numerator, and the number of threads per inch to be cut as the denominator of a fraction; resolve both numerator and denominator into two factors each, and multiply each "pair" of factors by the same number, until values are obtained representing suitable numbers of teeth for the change gears. (One factor in the numerator and one in the denominator make a "pair" of factors.)

Example: — $1\frac{3}{4}$ threads per inch are to be cut in a lathe having a screw constant of 8; the available gears have 24, 28, 32, 36, 40 teeth, etc., increasing by four up to one hundred. Following the rule:

$$\frac{8}{1\frac{3}{4}} = \frac{2 \times 4}{1 \times 1\frac{3}{4}} = \frac{(2 \times 36) \times (4 \times 16)}{(1 \times 36) \times (1\frac{3}{4} \times 16)} = \frac{72 \times 64}{36 \times 28}$$

The gears having 72 and 64 teeth are the *driving* gears and those with 36 and 28 teeth are the *driven* gears.

Fractional Threads. — Sometimes the lead of a thread is given as a fraction of an inch instead of stating the number of threads per inch. For example, a thread may be required to be cut, having $\frac{3}{8}$ inch lead. The expression "$\frac{3}{8}$ inch lead" should first be transformed to "number of threads per inch." The number of threads per inch (the thread being single) equals:

$$\frac{1}{\frac{3}{8}} = 1 \div \frac{3}{8} = \frac{8}{3} = 2\frac{2}{3}$$

To find the change gears to cut $2\frac{2}{3}$ threads per inch in a lathe having a screw constant 8 and change gears running from 24 to 100 teeth, increasing by 4, proceed as below:

$$\frac{8}{2\frac{2}{3}} = \frac{2 \times 4}{1 \times 2\frac{2}{3}} = \frac{(2 \times 36) \times (4 \times 24)}{(1 \times 36) \times (2\frac{2}{3} \times 24)} = \frac{72 \times 96}{36 \times 64}$$

Change Gears for Metric Pitches. — When screws are cut in accordance with the metric system, it is the usual practice to give the lead of the thread in millimeters, instead of the number of threads per unit of measurement. To find the change gears for cutting metric threads, when using a lathe having an English lead-screw, first determine the number of threads per inch corresponding to the given lead in millimeters. Suppose a thread of 3 millimeters lead is to be cut in a lathe having an English lead-screw and a screw constant of 6. As there are 25.4 millimeters per inch, the number of threads per inch will equal $25.4 \div 3$. Place the screw constant as the numerator, and the number of threads per inch to be cut as the denominator:

$$\frac{6}{\frac{25.4}{3}} = 6 \div \frac{25.4}{3} = \frac{6 \times 3}{25.4}$$

The numerator and denominator of this fractional expression of the change gear ratio is next multiplied by some trial number to determine the size of the gears. The first whole number by which 25.4 can be multiplied so as to get a whole number as the result is 5. Thus, $25.4 \times 5 = 127$. Hence, one gear having 127 teeth is always used when cutting metric threads with an English lead-screw. The other gear required in this case has 90 teeth. Thus:

$$\frac{6 \times 3 \times 5}{25.4 \times 5} = \frac{90}{127}$$

Therefore, the following rule can be used to find the change gears for cutting metric pitches with an English lead-screw:

Rule: Place the lathe screw constant multiplied by the lead of the required thread in millimeters multiplied by 5, as the numerator of the fraction, and 127 as the denominator. The product of the numbers in the numerator equals the number of teeth for the spindle-stud gear, and 127 is the number of teeth for the lead-screw gear.

If the lathe has a metric pitch lead-screw, and a screw having a given number of threads per inch is to be cut, first find the "metric screw constant" of the lathe or the lead of thread in millimeters that would be cut with change gears of equal size on the lead-screw and spindle stud; then the method of determining the change gears is simply the reverse of the one already explained for cutting a metric thread with an English lead-screw.

Rule: To find the change gears for cutting English threads with a metric lead-screw, place 127 in the numerator and the threads per inch to be cut, multiplied by the metric screw constant multiplied by 5, in the denominator; 127 is the number of teeth on the spindle-stud gear and the product of the numbers in the denominator equals the number of teeth in the lead-screw gear.

Threads per Inch Obtained with a Given Combination of Gears. — To determine the number of threads per inch that will be obtained with a given combination of gearing, multiply the lathe screw constant by the number of teeth in the *driven* gear (or by the product of the numbers of teeth in both driven gears of compound gearing), and divide the product thus obtained by the number of teeth in the *driving* gear (or by the product of the two driving gears of a compound train). The quotient equals the number of threads per inch.

Change Gears for Fractional Ratios. — When gear ratios cannot be expressed exactly in whole numbers which are within the range of ordinary gearing, the combination of gearing required for the fractional ratio may be determined quite easily, in some cases, by the "cancellation method." To illustrate this method, assume that the speeds of two gears are to be in the ratio of 3.423 to 1. The number 3.423 is first changed to $\frac{3423}{1000}$ to clear it of decimals. Then, in order to secure a fraction that can be reduced, 3423 is changed to 3420;

$$\frac{3420}{1000} = \frac{342}{100} = \frac{3 \times 2 \times 57}{2 \times 50} = \frac{3 \times 57}{1 \times 50}$$

Then, multiplying $\frac{3}{1}$ by some trial number, say, 24, the following gear combination is obtained:

$$\frac{72}{24} \times \frac{57}{50} = \frac{4104}{1200} = \frac{3.42}{1}$$

As the desired ratio is 3.423 to 1, there is an error of 0.003. When the ratios are comparatively simple, the cancellation method is not difficult and is frequently used; but by the logarithmic method to be described, more accurate results are possible in most cases.

Logarithms of Change-gear Ratios. — Change-gear problems can be solved readily by the use of the accompanying tables which contain the six-place logarithms of the ratios of all gear combinations between 16 and 120 teeth, inclusive, excepting the 1 to 1 ratios. To illustrate how these logarithms of ratios are obtained, take as an example gears having 72 and 41 teeth, respectively; the ratio equals 72 divided by 41, and to divide by means of logarithms, the logarithm of one number is subtracted from the logarithm of the other, thus:

$$\begin{aligned} \log 72 &= 1.857333 \\ \log 41 &= \underline{1.612784} \\ \text{ratio } \log &= 0.244549 \end{aligned}$$

The logarithms for ratios of gear combinations between 16 and 120 have been arranged in numerical order in the tables. In a number of cases, more than one combination gives the same logarithm, so that the different gears that equal the logarithm have been repeated. In some simple cases, only the ratio has been given in order to shorten the table; for instance, all the gear combinations that equal a 2 to 1 ratio have been omitted and only the ratio is given.

There are nearly 5000 different ratios represented in the gear tables between the extremes 1.0084+ to 1 (120 : 119) and 7½ to 1 (120 : 16). As the sum of any two two-gear logarithms equals a four-gear logarithm, the tables represent over 12,000,000 four-gear combinations; and by using three pairs of gears in a train, there are over 20,000,000,000 six-gear combinations available.

Solving Change-gear Problems by Use of Logarithms. — To show how the tables of logarithms for different gear ratios are used, suppose that gears having the ratio 3.423 : 1 are desired. Log 3.423 = 0.534407. From the table, log 89 : 26 = 0.534417; therefore, the gears having 89 and 26 teeth are the nearest to the ratio 3.423 to 1, and as 89 ÷ 26 = 3.423077, the ratio error is only 0.000077.

When solving gear problems the ratio should be reduced to terms of 1. For example, what two gears will drive two shafts at a speed ratio of 7.182 to 3.902? $\frac{7.182}{3.902} = \frac{1.84059}{1}$; the log of 1.84059 is 0.264957. From the table, 81 : 44 = log 0.265032. As 81 ÷ 44 = 1.84091, the error is only 0.00032.

A more rapid solution of the same problem, which may be used by those familiar with logarithms, is:

$$\begin{aligned}\log 7.182 &= 0.856245\\ \log 3.902 &= \underline{0.591287}\\ \text{ratio } \log &= 0.264958\end{aligned}$$

From the table, log 81 : 44 = 0.265032. To find the error, proceed thus:

$$\begin{aligned}&\log 0.265032\\ &\log \underline{0.264958}\\ \text{log of ratio error } &0.000074\end{aligned}$$

Finding Four-gear Ratios. — When four gears must be used, the gear logarithms make it possible to obtain more accurate results than any other method. For example, suppose it is desired to find four gears that will have a ratio of 2.105399 to 1. Log 2.105399 = 0.323334. To keep the reduction about equal in each pair of gears, it is necessary to select from the table that set of gears the logarithm of which is equal to about one-half the ratio logarithm, as log 57 : 37 = 0.187673. By subtracting this from log 0.323334, the other logarithm is found to be 0.135661. From the table, log 41 : 30 = 0.135663, so the error is log 0.000002. Thus this result is obtained:

$$\frac{57}{37} \times \frac{41}{30} = \frac{2337}{1110} = 2.105405$$

As the ratio of the gears is 2.105405 and the desired ratio is 2.105399, the error in the ratio is 0.000006. The two largest gears will be drivers or driven, whichever the case may be.

In case no combination can be found that nearly equals the logarithm of the ratio, a suitable four-gear combination may be found by reversing the second ratio selected from the table. For example, what gears will drive two shafts at a ratio of 595 to 594? Taking the logarithms from a six-place table,

$$\begin{aligned}\log 595 &= 2.774517\\ \log 594 &= \underline{2.773786}\\ \log \text{ratio} &= 0.000731\end{aligned}$$

From the table of logarithms for gear ratios, select any ratio, say log 72 : 70 = 0.012235, and add the logarithm of the ratio 595 : 594, or 0.000731; the sum is 0.012966. Select the logarithm nearest the sum from the table; this is found to be log 68 : 66 = 0.012965. Now by reversing this pair the difference of the ratios, or 594 : 595, will be obtained. Reversing 68 : 66 gives 66 : 68; therefore, the gears required are $\frac{72}{70} \times \frac{66}{68}$. The proof of this is: $\frac{72}{70} \times \frac{66}{68} = \frac{4752}{4760} = \frac{594}{595}$.

Driving and Driven Gears. — Gears for the ratio 7.32 : 4.17 are selected in the same manner as gears for the ratio 4.17 : 7.32. The logarithm of the smaller number is subtracted from the logarithm of the larger, giving the logarithm of the ratio. The first figure or term of a gear ratio is usually considered to be the driver. For example, if two shafts are to run at a ratio of 3 to 1, it is implied that 3 is the driver and the gear with the largest number of teeth will be placed on the driving shaft. In so far as the use of the gear logarithm tables is concerned, it is immaterial which is the driver and which is the driven gear, and by comparing the gears selected with the ratio, no confusion should result.

Lathe Change-gears. — For calculating the change-gears to cut any lead on a lathe, the "constant" of the machine must be known. For any lathe, $C : L$ = driver : driven gear, in which C = constant of machine and L = lead desired.

For example, what change-gears are required to cut 1.7345 threads per inch on a lathe having a constant of 4?

$$C : L = 4 : 1.7345$$

$$\log 4 = 0.602060$$

$$\log 1.7345 = \underline{0.239174}$$

$$\text{ratio log} = 0.362886$$

From the table, $\log 113 : 49 = \underline{0.362882}$

$$\text{log of ratio error} = 0.000004$$

Therefore, the driver has 113 teeth, and the driven gear, 49 teeth.

Relieving Spiral Fluted Hobs. — The problem of relieving hobs that have been fluted at right angles to the thread is an example of the special application of the gear logarithms to difficult problems. The usual method is to alter the angle of the spiral flutes to agree with previously calculated change-gears for the relieving attachment. The ratio between the hob and the relieving attachment cam is expressed by the following terms:

$$(N + \sin^2 \alpha) : C = \text{drivers} : \text{driven gears}$$

$$\text{Tan}\ \alpha = \frac{P'}{H_c}$$

in which
N = number of flutes in hob;
α = helix angle of thread from plane perpendicular to axis;
C = constant of relieving attachment;
P' = circular pitch, corresponding to pitch of hob;
H_c = hob circumference, or $3.1416 \times$ outside diameter.

The constant of a relieving attachment can be found on its index-plate, and is determined by the number of flutes that require equal gears on the change-gear studs. This will vary with different makes of lathes, some relieving attachments having cams with different numbers of risers.

Example. — What four change-gears must be used to relieve a spiral fluted hob, 10 diametral pitch, 2½ inches in diameter, 2 degrees, 17 minutes, 30 seconds angle of thread, with six spiral flutes, assuming that a relieving attachment having a constant of 4 is to be used?

$$\text{Sin 2 degrees, 17 minutes, 30 seconds} = 0.03998;$$

$$0.03998^2 = 0.001598;$$

$$\frac{6 + 0.001598}{4} = \frac{6.001598}{4}$$

$$\log 6.001598 = 0.778267$$

$$\log 4 = \underline{0.602060}$$

$$\log \text{ratio} = 0.176207$$

From the tables, $\log 79 : 61 = \underline{0.112297}$

Subtracting from the log ratio $= 0.063910$

From table, $\log 95 : 82 = 0.063910$

Therefore, the gears are $\frac{79}{61} \times \frac{95}{82} = \frac{\text{drivers}}{\text{driven}}$

The accuracy of the gear-logarithm method of determining the change-gear ratios shown in the foregoing example is indicated by the fact that when using seven-place logs the ratio error is log 0.0000002 in the gears selected. It is hardly necessary to say that backlash in the lathe gears, spring of the relieving tool and inaccuracy of the lead-screw are items of greater inaccuracy than this.

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
120 : 16	0.875061	99 : 16	0.791515	101 : 18	0.749049	115 : 22	0.718275
119 : 16	.871427	105 : 17	.790740	112 : 20	.748188	94 : 18	.717855
118 : 16	.867762	111 : 18	.790051	95 : 17	.747275	120 : 23	.717453
117 : 16	.864065	98 : 16	.787106	106 : 19	.746552	99 : 19	.716882
116 : 16	.860338	104 : 17	.786584	117 : 21	.745967	109 : 21	.715207
115 : 16	.856578	110 : 18	.786120	89 : 16	.745270	83 : 16	.714958
114 : 16	.852785	116 : 19	.785704	100 : 18	.744728	114 : 22	.714482
113 : 16	.848958	97 : 16	.782652	111 : 20	.744293	88 : 17	.714034
120 : 17	0.848732	103 : 17	0.782388	94 : 17	0.742680	119 : 23	0.713819
119 : 17	.845098	109 : 18	.782154	105 : 19	.742436	93 : 18	.713210
112 : 16	.845098	115 : 19	.781944	116 : 21	.742239	98 : 19	.712473
118 : 17	.841433	114 : 19	.778151	110 : 20	.740363	103 : 20	.711807
111 : 16	.841203	108 : 18	.778151	99 : 18	.740363	108 : 21	.711205
117 : 17	.837737	102 : 17	.778151	88 : 16	.740363	113 : 22	.710656
110 : 16	.837272	96 : 16	.778151	115 : 21	.738479	118 : 23	.710154
116 : 17	.834009	119 : 20	.774517	104 : 19	.738280	82 : 16	.709694
109 : 16	0.833307	113 : 19	0.774325	93 : 17	0.738034	87 : 17	0.709070
115 : 17	.830249	107 : 18	.774111	120 : 22	.736759	92 : 18	.708515
108 : 16	.829304	101 : 17	.773873	109 : 20	.736397	97 : 19	.708018
114 : 17	.826456	95 : 16	.773604	98 : 18	.735954	102 : 20	.707570
107 : 16	.825264	112 : 19	.770464	87 : 16	.735400	107 : 21	.707165
120 : 18	.823909	106 : 18	.770033	114 : 21	.734686	112 : 22	.706795
113 : 17	.822629	100 : 17	.769551	103 : 19	.734084	117 : 23	.706458
106 : 16	.821186	94 : 16	.769008	92 : 17	.733339	81 : 16	.704365
119 : 18	0.820275	117 : 20	0.767156	119 : 22	0.733124	86 : 17	0.704050
112 : 17	.818769	111 : 19	.766570	108 : 20	.732394	91 : 18	.703769
105 : 16	.817069	105 : 18	.765917	97 : 18	.731499	96 : 19	.703518
118 : 18	.816609	99 : 17	.765186	113 : 21	.730859	101 : 20	.703291
111 : 17	.814874	93 : 16	.764363	86 : 16	.730379	106 : 21	.703087
117 : 18	.812913	116 : 20	.763429	102 : 19	.729847	111 : 22	.702900
104 : 16	.812913	110 : 19	.762639	118 : 22	.729459	116 : 23	.702730
110 : 17	.810944	104 : 18	.761761	91 : 17	.728593	120 : 24	.698970
116 : 18	0.809186	98 : 17	0.760777	107 : 20	0.728354	115 : 23	0.698970
103 : 16	.808717	115 : 20	.759668	112 : 21	.726999	110 : 22	.698970
109 : 17	.806978	92 : 16	.759668	96 : 18	.726999	105 : 21	.698970
115 : 18	.805425	109 : 19	.758673	117 : 22	.725763	100 : 20	.698970
102 : 16	.804480	103 : 18	.757565	101 : 19	.725568	95 : 19	.698970
108 : 17	.802975	120 : 21	.756962	85 : 16	.725299	90 : 18	.698970
114 : 18	.801632	114 : 20	.755875	106 : 20	.724276	85 : 17	.698970
120 : 19	.800428	91 : 16	.754921	90 : 17	.723794	80 : 16	.698970
101 : 16	0.800201	108 : 19	0.754670	111 : 21	0.723104	119 : 24	0.695336
107 : 17	.798936	119 : 21	.753328	95 : 18	.722451	114 : 23	.695171
113 : 18	.797806	102 : 18	.753328	116 : 22	.722035	109 : 22	.695004
119 : 19	.796793	113 : 20	.752048	100 : 19	.721246	104 : 21	.694814
100 : 16	.795880	96 : 17	.751822	105 : 20	.720159	99 : 20	.694605
106 : 17	.794857	107 : 19	.750630	84 : 16	.720159	94 : 19	.694374
112 : 18	.793946	90 : 16	.750123	110 : 21	.719173	89 : 18	0.694118
118 : 19	.793128	118 : 21	.749663	89 : 17	.718941	84 : 17	.693830

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
79 : 16	0.693507	103 : 22	0.670415	76 : 17	0.650365	90 : 21	0.632023
118 : 24	.691671	117 : 25	.670246	116 : 26	.649485	107 : 25	.631444
113 : 23	.691351	112 : 24	.669007	107 : 24	.649173	77 : 18	.631218
108 : 22	.691001	98 : 21	.669007	98 : 22	.648803	94 : 22	.630705
103 : 21	.690618	84 : 18	.669007	89 : 20	.648360	111 : 26	.630350
98 : 20	.690196	107 : 23	.667656	80 : 18	.647818	81 : 19	.629731
93 : 19	.689729	93 : 20	.667453	120 : 27	.647817	98 : 23	.629498
88 : 18	.689210	79 : 17	.667178	111 : 25	.647383	115 : 27	.629334
83 : 17	0.688629	116 : 25	0.666518	71 : 16	0.647138	119 : 28	0.628389
117 : 24	.687975	102 : 22	.666178	102 : 23	.646872	85 : 20	.628389
78 : 16	.687975	88 : 19	.665730	93 : 21	.646264	68 : 16	.628389
112 : 23	.687490	111 : 24	.665112	115 : 26	.645725	106 : 25	.627366
107 : 22	.686961	74 : 16	.665112	84 : 19	.645526	89 : 21	.627171
102 : 21	.686381	97 : 21	.664552	106 : 24	.645095	72 : 17	.626884
97 : 20	.685742	120 : 26	.664208	75 : 17	.644612	110 : 26	.626419
92 : 19	.685034	83 : 18	.663806	97 : 22	.644349	93 : 22	.626060
116 : 24	0.684247	106 : 23	0.663578	119 : 27	0.644183	114 : 27	0.625541
87 : 18	.684247	115 : 25	.662758	110 : 25	.643453	76 : 18	.625541
111 : 23	.683595	92 : 20	662758	88 : 20	.643453	97 : 23	.625044
82 : 17	.683365	101 : 22	.661899	101 : 23	.642594	118 : 28	.624724
106 : 22	.682883	78 : 17	.661645	79 : 18	.642355	80 : 19	.624336
77 : 16	.682371	110 : 24	.661182	114 : 26	.641932	101 : 24	.624110
101 : 21	.682102	87 : 19	.660766	92 : 21	.641569	105 : 25	.623249
120 : 25	.681241	119 : 26	.660574	105 : 24	.640978	84 : 20	.623249
96 : 20	0.681241	96 : 21	0.660052	70 : 16	0.640978	109 : 26	0.622453
115 : 24	.680487	105 : 23	.659462	118 : 27	.640518	88 : 21	.622263
91 : 19	.680288	73 : 16	.659203	83 : 19	.640324	67 : 16	.621955
110 : 23	.679665	114 : 25	.658965	96 : 22	.639849	113 : 27	.621715
86 : 18	.679226	82 : 18	.658541	109 : 25	.639487	92 : 22	.621365
105 : 22	.678767	91 : 20	.658011	74 : 17	.638783	117 : 28	.621028
81 : 17	.678036	100 : 22	.657577	87 : 20	.638489	71 : 17	.620809
100 : 21	.677781	109 : 24	.657215	100 : 23	.638272	96 : 23	.620543
119 : 25	0.677607	118 : 26	0.656909	113 : 26	0.638105	100 : 24	0.619789
114 : 24	.676694	77 : 17	.656042	117 : 27	.636822	75 : 18	.619789
95 : 20	.676694	86 : 19	.655745	104 : 24	.636822	104 : 25	.619093
76 : 16	.676694	95 : 21	.655504	91 : 21	.636822	79 : 19	.618874
109 : 23	.675699	104 : 23	.655306	78 : 18	.636822	108 : 26	.618451
90 : 19	.675489	113 : 25	.655138	108 : 25	.635484	83 : 20	.618048
104 : 22	.674611	117 : 26	.653213	95 : 22	.635301	112 : 27	.617854
85 : 18	.674146	108 : 24	.653213	82 : 19	.635060	116 : 28	.617300
118 : 25	0.673942	99 : 22	0.653213	69 : 16	0.634729	87 : 21	0.617300
99 : 21	.673416	90 : 20	.653213	112 : 26	.634245	120 : 29	.616783
113 : 24	.672867	81 : 18	.653213	99 : 23	.633907	91 : 22	.616619
80 : 17	.672641	72 : 16	.653213	86 : 20	.633469	95 : 23	.615996
94 : 20	.672098	112 : 25	.651278	116 : 27	.633094	99 : 24	.615424
108 : 23	.671696	103 : 23	.651109	73 : 17	.632874	66 : 16	.615424
75 : 16	.670941	94 : 21	.650909	103 : 24	.632626	103 : 25	.614897
89 : 19	.670636	85 : 19	.650665	120 : 28	.632023	70 : 17	.614649

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
107 : 26	0.614411	71 : 18	0.595986	99 : 26	0.580662	81 : 22	0.566062
111 : 27	.613959	67 : 17	.595626	118 : 31	.580520	92 : 25	.565848
74 : 18	.613959	63 : 16	.595221	114 : 30	.579784	103 : 28	.565679
115 : 28	.613540	118 : 30	.594761	95 : 25	.579784	114 : 31	.565543
78 : 19	.613341	114 : 29	.594507	76 : 20	.579784	88 : 24	.564272
119 : 29	.613149	110 : 28	.594235	110 : 29	.578995	110 : 30	.564271
82 : 20	.612784	106 : 27	.593942	91 : 24	.578830	99 : 27	.564271
86 : 21	.612279	102 : 26	.593627	72 : 19	.578579	77 : 21	.564271
90 : 22	0.611820	98 : 25	0.593286	106 : 28	0.578148	66 : 18	0.564271
94 : 23	.611400	94 : 24	.592917	87 : 23	.577792	117 : 32	.563036
98 : 24	.611015	90 : 23	.592515	102 : 27	.577236	106 : 29	.562908
102 : 25	.610660	86 : 22	.592076	68 : 18	.577236	73 : 20	.562293
106 : 26	.610333	82 : 21	.591595	117 : 31	.576824	95 : 26	.562750
110 : 27	.610029	117 : 30	.591065	83 : 22	.576655	84 : 23	.562552
114 : 28	.609747	78 : 20	.591065	98 : 26	.576253	62 : 17	.561943
112 : 29	.609484	113 : 29	.590680	113 : 30	.575957	113 : 31	.561717
65 : 16	0.608793	74 : 19	0.590478	64 : 17	0.575731	102 : 28	0.561442
69 : 17	.608400	109 : 28	.590269	79 : 21	.575408	91 : 25	.561101
73 : 18	.608050	105 : 27	.589826	94 : 25	.575188	120 : 33	.560667
77 : 19	.607737	70 : 18	.589826	109 : 29	.575029	80 : 22	.560667
81 : 20	.607455	101 : 26	.589348	120 : 32	.574031	109 : 30	.560305
85 : 21	.607200	66 : 17	.589105	105 : 28	.574031	69 : 19	.560096
89 : 22	.606967	97 : 25	.588832	90 : 24	.574031	98 : 27	.559862
93 : 23	.606755	62 : 16	.588272	75 : 20	.574031	116 : 32	.559308
97 : 24	0.606561	120 : 31	0.587820	60 : 16	0.574031	87 : 24	0.559308
101 : 25	.606381	89 : 23	.587662	116 : 31	.573096	105 : 29	.558791
105 : 26	.606216	116 : 30	.587337	86 : 23	.572770	76 : 21	.558594
109 : 27	.606063	85 : 22	.586996	71 : 19	.572505	94 : 26	.558155
113 : 28	.605920	112 : 29	.586820	112 : 30	.572097	112 : 31	.557856
117 : 29	.605788	108 : 28	.586266	97 : 26	.571798	65 : 18	.557641
116 : 23	.602730	81 : 21	.586266	82 : 22	.571391	83 : 23	.557350
64 : 16		104 : 27	.585670	108 : 29	.571026	101 : 28	.557163
or		77 : 20	0.585461	67 : 18	0.570802	119 : 33	0.557033
any	0.602060	100 : 26	.585027	93 : 25	.570543	108 : 30	.556303
4 to 1		73 : 19	.584569	119 : 32	.570397	90 : 25	.556303
ratio		96 : 25	.584331	104 : 28	.569875	72 : 20	.556303
119 : 30	.598426	119 : 31	.584185	78 : 21	.569875	115 : 32	.555548
115 : 29	.598300	115 : 30	.583577	115 : 31	.569336	97 : 27	.555408
111 : 28	.598165	92 : 24	.583577	89 : 24	.569179	79 : 22	.555204
107 : 27	.598020	69 : 18	.583577	63 : 17	.568892	61 : 17	.554881
103 : 26	0.597864	111 : 29	0.582925	100 : 27	0.568636	104 : 29	0.554635
99 : 25	.597695	88 : 23	.582755	111 : 30	.568202	86 : 24	.554287
95 : 24	.597512	65 : 17	.582465	74 : 20	.568202	111 : 31	.553961
91 : 23	.597314	107 : 28	.582226	85 : 23	.567691	68 : 19	.553755
87 : 22	.597097	84 : 22	.581857	96 : 26	.567298	93 : 26	.553504
83 : 21	.596859	103 : 27	.581473	107 : 29	.566986	118 : 33	.553368
79 : 20	.596597	61 : 16	.581210	118 : 32	.566732	100 : 28	.552842
75 : 19	.596308	80 : 21	.580871	70 : 19	.566344	75 : 21	.552842

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
107 : 30	0.552263	76 : 22	0.538391	104 : 31	0.525672	101 : 31	0.512960
82 : 23	.552086	107 : 31	.538022	114 : 34	.525426	114 : 35	.512837
114 : 32	.551755	69 : 20	.537819	67 : 20	.525045	117 : 36	.511883
89 : 25	.551450	100 : 29	.537602	77 : 23	.524763	104 : 32	.511883
96 : 27	.550907	93 : 27	.537119	87 : 26	.524546	91 : 28	.511883
64 : 18	.550907	62 : 18	.537119	97 : 29	.524374	65 : 20	.511883
103 : 29	.550439	117 : 34	.536707	107 : 32	.524234	120 : 37	.510980
71 : 20	.550228	86 : 25	.536559	117 : 35	.524118	107 : 33	.510870
110 : 31	0.550031	110 : 32	0.536243	120 : 36	0.522879	94 : 29	0.510730
117 : 33	.549672	55 : 16	.536243	110 : 33	.522879	81 : 25	.510545
78 : 22	.549672	79 : 23	.535899	100 : 30	.522879	68 : 21	.510290
85 : 24	.549208	103 : 30	.535716	90 : 27	.522879	110 : 34	.509914
92 : 26	.548815	120 : 35	.535113	80 : 24	.522879	97 : 30	.509650
99 : 28	.548477	96 : 28	.535113	70 : 21	.522879	84 : 26	.509306
106 : 30	.548185	72 : 21	.535113	60 : 18	.522879	113 : 35	.509010
113 : 32	.547928	113 : 33	.534565	113 : 34	.521600	71 : 22	.508836
120 : 34	0.547702	89 : 26	0.534417	103 : 31	0.521476	100 : 31	0.508638
67 : 19	.547321	65 : 19	.534160	93 : 28	.521325	116 : 36	.508156
74 : 21	.547012	106 : 31	.533944	83 : 25	.521138	87 : 27	.508156
81 : 23	.546757	82 : 24	.533603	73 : 22	.520900	103 : 32	.507687
88 : 25	.546543	99 : 29	.533237	63 : 19	.520587	74 : 23	.507504
95 : 27	.546360	116 : 34	.532980	116 : 35	.520390	119 : 37	.507345
102 : 29	.546202	75 : 22	.532639	106 : 32	.520156	90 : 28	.507085
109 : 31	.546065	92 : 27	.532424	96 : 29	.519873	106 : 33	.506792
119 : 34	0.544068	109 : 32	0.532277	86 : 26	0.519525	61 : 19	0.506576
112 : 32	.544068	119 : 35	.531479	119 : 36	.519245	77 : 24	.506280
105 : 30	.544068	102 : 30	.531479	76 : 23	.519086	93 : 29	.506085
98 : 28	.544068	85 : 25	.531479	109 : 33	.518913	109 : 34	.505948
91 : 26	.544068	68 : 20	.531479	99 : 30	.518514	112 : 35	.505150
84 : 24	.544068	112 : 33	.530704	89 : 27	.518026	96 : 30	.505150
77 : 22	.544068	95 : 28	.530566	112 : 34	.517739	80 : 25	.505150
70 : 20	.544068	78 : 23	.530367	79 : 24	.517416	64 : 20	.505150
63 : 18	0.544068	61 : 18	0.530057	102 : 31	0.517239	115 : 36	0.504395
56 : 16	.544068	105 : 31	.529828	115 : 35	.516630	99 : 31	.504274
115 : 33	.542184	88 : 26	.529509	69 : 21	.516630	83 : 26	.504105
108 : 31	.542062	115 : 34	.529219	92 : 28	.516629	67 : 21	.503856
101 : 29	.541923	71 : 21	.529039	105 : 32	.516039	118 : 37	.503680
87 : 25	.541579	98 : 29	.528828	82 : 25	.515874	102 : 32	.503450
80 : 23	.541362	108 : 32	.528274	118 : 36	.515580	86 : 27	.503135
73 : 21	.541104	81 : 24	.528274	95 : 29	.515326	105 : 33	.502675
66 : 19	0.540790	118 : 35	0.527814	108 : 33	0.514910	70 : 22	0.502675
118 : 34	.540403	91 : 27	.527678	77 : 22	.514910	89 : 28	.502232
111 : 32	.540173	64 : 19	.527426	85 : 26	.514446	108 : 34	.501945
104 : 30	.539912	101 : 30	.527200	98 : 30	.514105	73 : 23	.501595
97 : 28	.539614	111 : 33	.526809	111 : 34	.513844	92 : 29	.501390
90 : 26	.539269	74 : 22	.526809	62 : 19	.513638	111 : 35	.501255
83 : 24	.538867	84 : 25	.526339	75 : 23	.513334	95 : 30	.500602
114 : 33	.538391	94 : 28	.525970	88 : 27	.513119	76 : 24	.500602

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
117 : 37	0.499984	120 : 39	0.488117	92 : 31	0.472426	107 : 37	0.461182
98 : 31	.499864	80 : 26	.488117	89 : 30	.472269	69 : 25	.460909
79 : 25	.499687	83 : 27	.487714	86 : 29	.472101	104 : 36	.460731
120 : 38	.499398	86 : 28	.487341	83 : 28	.471920	78 : 27	.460731
101 : 32	.499171	89 : 29	.486992	80 : 27	.471726	101 : 35	.460253
82 : 26	.498841	92 : 30	.486667	77 : 26	.471517	75 : 26	.460088
104 : 33	.498519	95 : 31	.486362	74 : 25	.471291	98 : 34	.459747
63 : 20	.498311	98 : 32	.486076	71 : 24	.471047	72 : 25	.459393
85 : 27	0.498055	101 : 33	0.485808	68 : 23	0.470781	95 : 33	0.459210
107 : 34	.497905	104 : 34	.485554	65 : 22	.470491	118 : 41	.459098
110 : 35	.497325	107 : 35	.485316	62 : 21	.470172	115 : 40	.458638
88 : 28	.497325	110 : 36	.485090	118 : 40	.469822	92 : 32	.458638
113 : 36	.496776	113 : 37	.484877	115 : 39	.469633	69 : 24	.458638
91 : 29	.496643	116 : 38	.484674	112 : 38	.469434	112 : 39	.458153
69 : 22	.496426	119 : 39	.484482	109 : 37	.469225	89 : 31	.458028
116 : 37	.496256	61 : 20	.484300	106 : 36	.469003	66 : 23	.457816
94 : 30	0.496007	64 : 21	0.483961	103 : 35	0.468769	109 : 38	0.457643
119 : 38	.495763	67 : 22	.483652	100 : 34	.468521	86 : 30	.457377
72 : 23	.495605	70 : 23	.483370	97 : 33	.468258	106 : 37	.457104
97 : 31	.495410	73 : 24	.483112	94 : 32	.467978	63 : 22	.456918
100 : 32	.494850	76 : 25	.482874	91 : 31	.467680	83 : 29	.456680
75 : 24	.494850	79 : 26	.482654	88 : 30	.467361	103 : 36	.456535
103 : 33	.494323	82 : 27	.482450	85 : 29	.467021	120 : 42	.455932
78 : 25	.494155	85 : 28	.482261	82 : 28	.466656	100 : 35	.455932
106 : 34	0.493827	88 : 29	0.482085	120 : 41	0.466397	80 : 28	0.455932
81 : 26	.493511	91 : 30	.481920	79 : 27	.466263	60 : 21	.455932
109 : 35	.493359	94 : 31	.481766	117 : 40	.466126	117 : 41	.455402
112 : 36	.492916	97 : 32	.481622	114 : 39	.465840	97 : 34	.455293
84 : 27	.492916	100 : 33	.481486	76 : 26	.465840	77 : 27	.455127
115 : 37	.492496	103 : 34	.481358	111 : 38	.465539	114 : 40	.454845
87 : 28	.492361	106 : 35	.481237	73 : 25	.465383	94 : 33	.454614
118 : 38	.492098	109 : 36	.481124	108 : 37	.465222	111 : 39	.454258
90 : 29	0.491845	112 : 37	0.481016	70 : 24	0.464887	74 : 26	0.454258
93 : 30	.491362	115 : 38	.480914	102 : 35	.464532	91 : 32	.453891
62 : 20	.491362	118 : 39	.480817	67 : 23	.464347	71 : 25	.453318
96 : 31	.490910	48 : 16		99 : 34	.464156	88 : 31	.453121
65 : 21	.490694	or any	.477121	96 : 33	.463757	105 : 37	.452988
99 : 32	.490485	3 to 1		64 : 22	.463757	119 : 42	.452298
102 : 33	.490086	ratio		93 : 32	.463333	102 : 36	.452298
68 : 22	.490086	119 : 40	.473487	90 : 31	.462881	85 : 30	.452298
105 : 34	0.489710	116 : 39	0.473393	61 : 21	0.463111	68 : 24	0.452298
71 : 23	.489531	113 : 38	.473295	119 : 41	.462763	116 : 41	.451674
108 : 35	.489356	110 : 37	.473191	116 : 40	.462398	99 : 35	.451567
111 : 36	.489021	107 : 36	.473081	87 : 30	.462398	82 : 29	.451416
74 : 24	.489021	104 : 35	.472965	58 : 20	.462398	65 : 23	.451186
114 : 37	.488703	101 : 34	.472843	84 : 29	.461881	113 : 40	.451018
77 : 25	.488551	98 : 33	.472712	110 : 38	.461609	96 : 34	.450792
117 : 38	.488402	95 : 32	.472574	81 : 28	.461327	79 : 28	.450469

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
110 : 39	0.450328	88 : 32	0.439333	94 : 35	0.429060	84 : 32	0.419129
93 : 33	.449969	77 : 28	.439333	102 : 38	.428817	118 : 45	.418670
62 : 22	.449969	118 : 43	.438414	110 : 41	.428609	97 : 37	.418570
107 : 38	.449600	107 : 39	.438319	59 : 22	.428429	76 : 29	.418416
76 : 27	.449450	96 : 35	.438203	67 : 25	.428135	55 : 21	.418143
90 : 32	.449092	85 : 31	.438057	75 : 28	.427903	89 : 34	.417911
104 : 37	.448832	74 : 27	.437868	83 : 31	.427716	102 : 39	.417536
118 : 42	.448633	63 : 23	.437613	91 : 34	.427563	68 : 26	.417536
73 : 26	0.448350	115 : 42	0.437449	99 : 37	0.427434	115 : 44	0.417245
87 : 31	.448158	104 : 38	.437250	107 : 40	.427324	81 : 31	.417123
101 : 36	.448019	93 : 34	.437004	115 : 43	.427229	94 : 36	.416825
115 : 41	.447914	82 : 30	.436693	120 : 45	.425969	107 : 41	.416600
98 : 35	.447158	112 : 41	.436434	112 : 42	.425969	60 : 23	.416424
84 : 30	.447158	71 : 26	.436285	104 : 39	.425969	73 : 28	.416165
112 : 40	.447158	101 : 37	.436120	96 : 36	.425969	86 : 33	.415985
70 : 25	.447158	90 : 33	.435729	88 : 33	.425969	99 : 38	.415852
109 : 39	0.446362	60 : 22	0.435729	80 : 30	0.425969	112 : 43	0.415750
95 : 34	.446245	109 : 40	.435367	72 : 27	.425969	117 : 45	.414973
81 : 29	.446087	79 : 29	.435229	64 : 24	.425969	91 : 35	.414973
67 : 24	.445864	98 : 36	.434924	56 : 21	.425969	78 : 30	.414973
120 : 43	.445713	117 : 43	.434717	117 : 44	.424733	52 : 20	.414973
106 : 38	.445522	68 : 25	.434569	109 : 41	.424643	109 : 42	.414177
92 : 33	.445274	87 : 32	.434369	101 : 38	.424538	96 : 37	.414070
117 : 42	.444937	106 : 39	.434241	93 : 35	.424415	83 : 32	.413928
78 : 28	0.444937	114 : 42	0.433656	85 : 32	0.424269	70 : 27	0.413734
103 : 37	.444636	95 : 35	.433656	77 : 29	.424093	57 : 22	.413452
64 : 23	.444452	76 : 28	.433656	69 : 26	.423876	101 : 39	.413257
89 : 32	.444240	57 : 21	.433656	61 : 23	.423602	88 : 34	.413004
114 : 41	.444121	103 : 38	.433054	114 : 43	.423436	119 : 46	.412789
100 : 36	.443698	84 : 31	.432918	106 : 40	.423246	75 : 29	.412663
75 : 27	.443698	65 : 24	.432702	98 : 37	.423024	106 : 41	.412522
111 : 40	.443263	111 : 41	.432539	90 : 34	.422764	62 : 24	.412181
86 : 31	0.443137	92 : 34	0.432309	82 : 31	0.422452	93 : 36	0.412180
61 : 22	.442907	119 : 44	.432094	119 : 45	.422335	111 : 43	.411855
97 : 35	.442704	73 : 27	.431959	111 : 42	.422074	80 : 31	.411728
72 : 26	.442359	100 : 37	.431798	74 : 28	.422074	98 : 38	.411443
108 : 39	.442359	108 : 40	.431364	103 : 39	.421773	116 : 45	.411246
119 : 43	.442079	81 : 30	.431364	95 : 36	.421421	67 : 26	.411102
83 : 30	.441957	54 : 20	.431364	87 : 33	.421005	85 : 33	.410905
94 : 34	.441649	116 : 43	.430990	58 : 22	.421005	103 : 40	.410777
105 : 38	0.441406	89 : 33	0.430876	108 : 41	0.420640	90 : 35	0.410175
58 : 21	.441209	62 : 23	.430664	79 : 30	.420506	72 : 28	.410175
80 : 29	.440692	97 : 36	.430469	100 : 38	.420216	54 : 21	.410175
91 : 33	.440528	105 : 39	.430125	71 : 27	.419895	113 : 44	.409626
102 : 37	.440399	70 : 26	.430125	92 : 35	.419720	95 : 37	.409522
113 : 41	.440295	113 : 42	.429829	113 : 43	.419610	77 : 30	.409369
110 : 40	.439333	78 : 29	.429697	105 : 40	.419130	59 : 23	.409124
99 : 36	.439333	86 : 32	.429349	63 : 24	.419130	100 : 39	.408935

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
82 : 32	0.408664	40 : 16		110 : 45	0.388180	48 : 20	0.380211
105 : 41	.408405	or any	0.397940	88 : 36	.388180	115 : 48	.379457
64 : 25	.408240	2½ to 1		66 : 27	.388180	103 : 43	.379369
87 : 34	.408040	ratio		105 : 43	.387721	91 : 38	.379258
110 : 43	.407924	117 : 47	.396088	83 : 34	.387599	79 : 33	.379113
115 : 45	.407485	112 : 45	.396006	61 : 25	.387390	67 : 28	.378916
92 : 36	.407485	107 : 43	.395915	100 : 41	.387216	55 : 23	.378635
69 : 27	.407485	102 : 41	.395816	117 : 48	.386945	98 : 41	.378442
120 : 47	0.407083	97 : 39	0.395707	78 : 32	0.386945	86 : 36	0.378196
97 : 38	.406988	92 : 37	.395586	95 : 39	.386659	117 : 49	.377990
74 : 29	.406834	87 : 35	.395451	112 : 46	.386460	74 : 31	.377870
51 : 20	.406540	82 : 33	.395300	56 : 23	.386460	105 : 44	.377737
79 : 31	.406265	77 : 31	.395129	73 : 30	.386202	93 : 39	.377418
107 : 42	.406135	72 : 29	.394935	90 : 37	.386041	62 : 26	.377418
84 : 33	.405765	67 : 27	.394711	107 : 44	.385931	112 : 47	.377120
56 : 22	.405765	62 : 25	.394452	102 : 42	.385351	81 : 34	.377006
117 : 46	0.405428	119 : 48	0.394306	119 : 49	0.385351	100 : 42	0.376751
89 : 35	.405322	57 : 23	.394147	85 : 35	.385351	50 : 21	.376751
61 : 24	.405119	109 : 44	.393974	68 : 28	.385351	119 : 50	.376577
94 : 37	.404926	52 : 21	.393784	51 : 21	.385351	69 : 29	.376451
99 : 39	.404571	99 : 40	.393575	114 : 47	.384807	88 : 37	.376281
66 : 26	.404571	94 : 38	.393344	97 : 40	.384712	107 : 45	.376171
104 : 41	.404249	89 : 36	.393088	80 : 33	.384576	95 : 40	.375664
71 : 28	.404100	84 : 34	.392800	63 : 26	.384367	76 : 32	.375664
109 : 43	0.403958	79 : 32	0.392477	109 : 45	0.384214	57 : 24	0.375664
114 : 45	.403692	116 : 47	.392360	92 : 38	.384004	102 : 43	.375132
76 : 30	.403692	111 : 45	.392111	75 : 31	.383700	83 : 35	.375010
119 : 47	.403449	74 : 30	.392116	104 : 43	.383565	64 : 27	.374816
81 : 32	.403335	106 : 43	.391837	87 : 36	.383217	109 : 46	.374669
86 : 34	.403020	69 : 28	.391691	58 : 24	.383217	90 : 38	.374459
91 : 36	.402739	101 : 41	.391538	99 : 41	.382851	116 : 49	.374262
96 : 38	.402488	96 : 39	.391207	70 : 29	.382700	71 : 30	.374137
101 : 40	0.402261	64 : 26	0.391207	111 : 46	0.382565	97 : 41	0.373988
53 : 21	.402057	91 : 37	.390840	82 : 34	.382335	78 : 33	.373581
111 : 44	.401870	59 : 24	.390641	94 : 39	.382063	52 : 22	.373581
58 : 23	.401700	86 : 35	.390431	53 : 22	.381853	111 : 47	.373225
63 : 25	.401401	113 : 46	.390321	118 : 49	.381686	85 : 36	.373116
68 : 27	.401145	108 : 44	.389971	65 : 27	.381550	59 : 25	.372912
73 : 29	.400925	81 : 33	.389971	77 : 32	.381341	92 : 39	.372723
78 : 31	.400733	54 : 22	.389971	89 : 37	.381188	66 : 28	.372386
83 : 33	0.400564	103 : 42	0.389588	101 : 42	0.381072	99 : 42	0.372386
88 : 35	.400415	76 : 31	.389452	113 : 47	.380981	106 : 45	.372093
93 : 37	.400281	98 : 40	.389166	120 : 50	.380211	73 : 31	.371961
98 : 39	.400162	49 : 20	.389166	108 : 45	.380211	113 : 48	.371837
103 : 41	.400053	120 : 49	.388985	96 : 40	.380211	120 : 51	.371611
108 : 43	.399955	71 : 29	.388860	84 : 35	.380211	80 : 34	.371611
113 : 45	.399866	93 : 38	.388699	72 : 30	.380211	87 : 37	.371318
118 : 47	.399784	115 : 47	.388600	60 : 25	.380211	94 : 40	.371068

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
47 : 20	0.371068	115 : 50	0.361728	70 : 31	0.353736	62 : 28	0.345234
101 : 43	.370853	92 : 40	.361728	79 : 35	.353559	104 : 47	.344935
54 : 23	.370666	69 : 30	.361728	88 : 39	.353418	73 : 33	.344809
115 : 49	.370502	46 : 20	.361728	97 : 43	.353303	115 : 52	.344695
61 : 26	.370357	108 : 47	.361326	106 : 47	.353208	84 : 38	.344496
68 : 29	.370111	85 : 37	.361217	115 : 51	.353128	95 : 43	.344255
75 : 32	.369911	62 : 27	.361028	117 : 52	.352183	53 : 24	.344065
82 : 35	.369746	101 : 44	.360869	108 : 48	.352183	117 : 53	.343910
89 : 38	0.369606	117 : 51	0.360616	99 : 44	0.352183	64 : 29	0.343782
96 : 41	.369487	78 : 34	.360616	90 : 40	.352183	75 : 34	.343582
103 : 44	.369385	94 : 41	.360344	81 : 36	.352183	86 : 39	.343434
110 : 47	.369295	55 : 24	.360152	72 : 32	.352183	97 : 44	.343319
117 : 50	.369216	71 : 31	.359897	63 : 28	.352183	108 : 49	.343228
119 : 51	.367977	87 : 38	.359736	45 : 20	.352183	119 : 54	.343153
105 : 45	.367977	103 : 45	.359625	119 : 53	.351271	99 : 45	.342423
98 : 42	.367977	119 : 52	.359544	110 : 49	.351197	88 : 40	.342423
91 : 39	0.367977	112 : 49	0.359022	101 : 45	0.351109	77 : 35	0.342423
84 : 36	.367977	96 : 42	.359022	92 : 41	.351004	66 : 30	.342423
77 : 33	.367977	80 : 35	.359022	83 : 37	.350876	55 : 25	.342423
70 : 30	.367977	64 : 28	.359022	74 : 33	.350718	112 : 51	.341648
63 : 27	.367977	48 : 21	.359022	65 : 29	.350515	101 : 46	.341564
56 : 24	.367977	105 : 46	.358432	56 : 25	.350248	90 : 41	.341459
49 : 21	.367977	89 : 39	.358325	103 : 46	.350079	79 : 36	.341325
114 : 49	.366709	73 : 32	.358173	94 : 42	.349879	57 : 26	.340902
107 : 46	0.366626	57 : 25	0.357935	85 : 38	0.349635	103 : 47	0.340739
100 : 43	.366532	98 : 43	.357758	114 : 51	.349335	92 : 42	.340539
93 : 40	.366423	82 : 36	.357511	76 : 34	.349335	46 : 21	.340539
86 : 37	.366297	107 : 47	.357286	105 : 47	.349091	81 : 37	.340283
79 : 34	.366148	66 : 29	.357146	67 : 30	.348954	116 : 53	.340182
72 : 31	.365971	91 : 40	.356981	96 : 43	.348803	105 : 48	.339948
65 : 28	.365755	116 : 51	.356888	87 : 39	.348455	70 : 32	.339948
58 : 25	.365488	75 : 33	.356547	58 : 26	.348455	93 : 43	.339659
109 : 47	0.365329	50 : 22	0.356547	107 : 48	0.348143	118 : 54	0.339488
102 : 44	.365148	100 : 44	.356547	78 : 35	.348027	59 : 27	.339488
95 : 41	.364940	109 : 48	.356185	98 : 44	.347773	83 : 38	.339295
88 : 38	.364700	84 : 37	.356078	49 : 22	.347773	107 : 49	.339188
81 : 35	.364417	59 : 26	.355879	118 : 53	.347606	120 : 55	.338819
118 : 51	.364312	93 : 41	.355699	69 : 31	.347487	96 : 44	.338819
111 : 48	.364083	102 : 45	.355388	89 : 40	.347330	72 : 33	.338819
74 : 32	.364082	68 : 30	.355388	109 : 49	.347230	48 : 22	.338819
104 : 45	0.363821	111 : 49	0.355127	100 : 45	0.346788	109 : 50	0.338457
67 : 29	.363677	77 : 34	.355012	80 : 36	.346788	85 : 39	.338354
90 : 39	.363178	120 : 53	.354905	60 : 27	.346788	61 : 28	.338172
60 : 26	.363178	86 : 38	.354715	111 : 50	.346353	98 : 45	.338014
113 : 49	.362882	95 : 42	.354474	91 : 41	.346258	111 : 51	.337753
53 : 23	.362548	52 : 23	.354276	51 : 23	.345842	74 : 34	.337753
76 : 33	.362300	113 : 50	.354108	113 : 51	.345508	87 : 40	.337459
99 : 43	.362167	61 : 27	.353966	93 : 42	.345234	100 : 46	.337242

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
113 : 52	0.337075	111 : 52	0.329320	88 : 42	0.321233	107 : 52	0.313381
63 : 29	.336943	96 : 45	.329059	44 : 21	.321233	72 : 35	.313265
76 : 35	.336746	64 : 30	.329059	111 : 53	.321047	109 : 53	.313151
89 : 41	.336606	113 : 53	.328803	67 : 32	.320925	74 : 36	.312929
102 : 47	.336502	81 : 38	.328701	90 : 43	.320774	111 : 54	.312929
115 : 53	.336412	98 : 46	.328468	113 : 54	.320685	113 : 55	.312716
117 : 54	.335792	115 : 54	.328304	115 : 55	.320335	76 : 37	.312612
91 : 42	.335792	66 : 31	.328182	92 : 44	.320335	115 : 56	.312510
78 : 36	0.335792	83 : 39	0.328014	69 : 33	0.320335	117 : 57	0.312311
65 : 30	.335792	100 : 47	.327902	46 : 22	.320335	78 : 38	.312311
52 : 24	.335792	117 : 55	.327823	117 : 56	.319998	119 : 58	.312119
119 : 55	.335184	119 : 56	.327359	71 : 34	.319779	80 : 39	.312025
106 : 49	.335110	85 : 40	.327359	119 : 57	.319672	82 : 40	.311754
93 : 43	.335014	68 : 42	.327359	96 : 46	.319513	41 : 20	.311754
80 : 37	.334888	51 : 24	.327359	73 : 35	.319255	84 : 41	.311495
67 : 31	.334713	104 : 49	.326837	98 : 47	.319128	86 : 42	.311249
54 : 25	0.334454	87 : 41	0.326735	100 : 48	0.318759	43 : 21	0.311249
95 : 44	.334271	70 : 33	.326584	75 : 36	.318759	88 : 43	.311014
82 : 38	.334030	89 : 42	.326141	102 : 49	.318404	90 : 44	.310790
110 : 51	.333823	108 : 51	.325854	77 : 37	.318289	45 : 22	.310790
69 : 32	.333699	72 : 34	.325854	52 : 25	.318063	92 : 45	.310575
97 : 45	.333559	91 : 43	.325573	79 : 38	.317844	94 : 46	.310370
84 : 39	.333215	55 : 26	.325389	106 : 51	.317736	96 : 47	.310173
56 : 26	.333215	74 : 35	.325164	81 : 39	.317420	98 : 48	.309985
99 : 46	0.332877	93 : 44	0.325030	110 : 53	0.317117	100 : 49	0.309804
71 : 33	.332744	112 : 53	.324942	83 : 40	.317018	102 : 50	.309630
114 : 53	.332629	95 : 45	.324511	112 : 54	.316824	51 : 25	.309630
86 : 40	.332439	76 : 36	.324511	85 : 41	.316635	104 : 51	.309463
43 : 20	.332439	57 : 27	.324511	114 : 55	.316542	53 : 26	.309303
101 : 47	.332224	116 : 55	.324095	87 : 42	.316270	108 : 53	.309148
58 : 27	.332064	97 : 46	.324014	58 : 28	.316270	55 : 27	.308999
73 : 34	.331844	78 : 37	.323893	118 : 57	.316007	112 : 55	.308855
88 : 41	0.331699	59 : 28	0.323694	89 : 43	0.315922	57 : 28	0.308717
103 : 48	.331596	99 : 47	.323537	120 : 58	.315753	116 : 57	.308583
118 : 55	.331519	120 : 57	.323306	60 : 29	.315753	59 : 29	.308454
105 : 49	.330993	80 : 38	.323306	91 : 44	.315589	120 : 59	.308329
90 : 42	.330993	101 : 48	.323080	93 : 45	.315270	61 : 30	.308209
75 : 35	.330993	61 : 29	.322932	62 : 30	.315270	63 : 31	.307979
60 : 28	.330993	82 : 39	.322749	95 : 46	.314966	65 : 32	.307763
45 : 21	.330993	103 : 49	.322641	64 : 31	.314818	67 : 33	.307561
107 : 50	0.330414	105 : 50	0.322219	97 : 47	0.314674	118 : 58	0.307454
92 : 43	.330319	84 : 40	.322219	99 : 48	.314394	69 : 34	.307370
77 : 36	.330188	63 : 30	.322219	66 : 32	.314394	71 : 35	.307190
62 : 29	.329994	42 : 20	.322219	101 : 49	.314125	73 : 36	.307020
109 : 51	.329855	107 : 51	.321814	68 : 33	.313995	75 : 37	.306860
94 : 44	.329675	86 : 41	.321715	103 : 50	.313867	77 : 38	.306707
47 : 22	.329675	65 : 31	.321552	105 : 51	.313619	79 : 39	.306563
79 : 37	.329425	109 : 52	.321423	70 : 34	.313619	81 : 40	.306425

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
83 : 41	0.306294	71 : 36	0.294956	93 : 48	0.287242	99 : 52	0.279632
85 : 42	.306170	69 : 35	.294781	62 : 32	.287242	59 : 31	.279490
87 : 43	.306051	67 : 34	.294596	91 : 47	.286944	78 : 41	.279311
89 : 44	.305937	65 : 33	.294400	120 : 62	.286790	97 : 51	.279202
91 : 45	.305829	63 : 32	.294191	89 : 46	.286632	116 : 61	.279128
93 : 46	.305725	61 : 31	.293968	118 : 61	.286552	95 : 50	.278754
95 : 47	.305626	120 : 61	.293851	116 : 60	.286307	76 : 40	.278754
97 : 48	.305531	59 : 30	.293731	56 : 30	.286307	57 : 30	.278754
99 : 49	0.305439	116 : 59	0.293606	114 : 59	0.286053	38 : 20	0.278754
101 : 50	.305351	57 : 29	.293477	85 : 44	.285966	112 : 59	.278366
103 : 51	.305267	112 : 57	.293343	56 : 29	.285790	93 : 49	.278287
105 : 52	.305186	55 : 28	.293205	83 : 43	.285610	74 : 39	.278167
107 : 53	.305108	108 : 55	.293061	110 : 57	.285518	55 : 29	.277965
109 : 54	.305033	53 : 27	.292912	81 : 42	.285236	91 : 48	.277800
111 : 55	.304960	104 : 53	.292757	54 : 28	.285235	108 : 57	.277549
113 : 56	.304890	51 : 26	.292597	106 : 55	.284943	72 : 38	.277549
115 : 57	0.304823	100 : 51	0.292430	79 : 41	0.284843	89 : 47	0.277292
117 : 58	.304758	98 : 50	.292256	52 : 27	.284640	53 : 28	.277118
119 : 59	.304695	96 : 49	.292075	77 : 40	.284431	87 : 46	.276762
32 : 16		94 : 48	.291887	102 : 53	.284324	104 : 55	.276671
or		92 : 47	.291690	100 : 52	.283997	119 : 63	.276207
any	.301030	90 : 46	.291485	75 : 39	.283997	85 : 45	.276207
2 to 1		88 : 45	.291270	98 : 51	.283656	68 : 36	.276206
ratio		86 : 44	.291046	73 : 38	.283540	117 : 62	.275794
119 : 60	0.297396	84 : 43	0.290811	96 : 50	0.283301	100 : 53	0.275724
117 : 59	.297334	82 : 42	.290565	119 : 62	.283155	83 : 44	.275625
115 : 58	.297270	41 : 21	.290565	71 : 37	.283057	66 : 35	.275476
113 : 57	.297204	80 : 41	.290306	94 : 49	.282932	115 : 61	.275368
111 : 56	.297135	119 : 61	.290217	117 : 61	.282856	98 : 52	.275223
109 : 55	.297064	78 : 40	.290035	92 : 48	.282547	81 : 43	.275017
107 : 54	.296990	39 : 20	.290035	69 : 36	.282547	113 : 60	.274927
105 : 53	.296913	115 : 59	.289846	113 : 59	.282226	96 : 51	.274701
103 : 52	0.296833	76 : 39	0.289749	90 : 47	0.282145	64 : 34	0.274701
101 : 51	.296751	113 : 58	.289650	67 : 35	.282007	111 : 59	.274471
99 : 50	.296665	111 : 57	.289448	111 : 58	.281895	79 : 42	.274378
97 : 49	.296576	74 : 38	.289448	88 : 46	.281725	94 : 50	.274158
95 : 48	.296482	109 : 56	.289239	109 : 57	.281552	109 : 58	.273999
93 : 47	.296385	72 : 37	.289131	65 : 34	.281435	62 : 33	.273878
91 : 46	.296284	107 : 55	.289021	86 : 45	.281286	77 : 41	.273707
89 : 45	.296178	70 : 36	.288796	107 : 56	.281196	92 : 49	.273592
87 : 44	0.296067	103 : 53	0.288561	84 : 44	0.280827	107 : 57	0.273509
85 : 43	.295950	68 : 35	.288441	63 : 33	.280827	120 : 64	.273001
83 : 42	.295829	101 : 52	.288318	103 : 54	.280443	105 : 56	.273001
81 : 41	.295701	66 : 34	.288065	82 : 43	.280345	90 : 48	.273001
79 : 40	.295567	99 : 51	.288065	61 : 32	.280180	75 : 40	.273001
77 : 39	.295426	97 : 50	.287802	101 : 53	.280046	60 : 32	.273001
75 : 38	.295278	64 : 33	.287666	80 : 42	.279841	118 : 63	.272542
73 : 37	.295121	95 : 49	.287528	40 : 21	.279841	103 : 55	.272475

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
88 : 47	0.272385	94 : 51	0.265558	87 : 48	0.258278	116 : 65	0.251545
73 : 39	.272258	70 : 38	.265314	58 : 32	.258278	91 : 51	.251471
58 : 31	.272066	116 : 63	.265118	96 : 53	.257995	66 : 37	.251342
101 : 54	.271928	81 : 44	.265032	67 : 37	.257873	107 : 60	.251233
86 : 46	.271741	92 : 50	.264818	105 : 58	.257761	82 : 46	.251056
114 : 61	.271575	103 : 56	.264649	38 : 21	.257564	98 : 55	.250863
99 : 53	.271360	68 : 37	.264307	76 : 42	.257564	57 : 32	.250725
112 : 60	.271067	79 : 43	.264159	85 : 47	.257321	73 : 41	.250539
84 : 45	0.271067	90 : 49	0.264046	94 : 52	0.257125	89 : 50	0.250420
56 : 30	.271067	101 : 55	.263959	103 : 57	.256962	105 : 59	.250337
97 : 52	.270768	112 : 61	.263888	112 : 62	.256826	112 : 63	.249878
69 : 37	.270647	110 : 60	.263241	56 : 31	.256826	96 : 54	.249878
110 : 59	.270541	99 : 54	.263241	65 : 36	.256611	80 : 45	.249878
82 : 44	.270361	88 : 48	.263241	74 : 41	.256448	64 : 36	.249878
41 : 22	.270361	77 : 42	.263241	83 : 46	.256320	119 : 67	.249472
95 : 51	.270153	66 : 36	.263241	92 : 51	.256218	103 : 58	.249409
54 : 29	0.269996	55 : 30	0.263241	101 : 56	0.256133	87 : 49	0.249323
67 : 36	.269772	119 : 65	.262634	110 : 61	.256063	71 : 40	.249198
80 : 43	.269622	108 : 59	.262572	119 : 66	.256003	110 : 62	.249001
93 : 50	.269513	97 : 53	.262496	99 : 55	.255273	55 : 31	.249001
106 : 57	.269431	86 : 47	.262401	90 : 50	.255273	94 : 53	.248852
119 : 64	.269367	75 : 41	.262277	81 : 45	.255273	117 : 66	.248642
117 : 63	.268845	64 : 35	.262112	72 : 40	.255273	78 : 44	.248642
91 : 49	.268845	53 : 29	.261878	63 : 35	.255273	101 : 57	.248447
78 : 42	0.268845	95 : 52	0.261720	54 : 30	0.255273	62 : 35	0.248324
65 : 35	.268845	84 : 46	.261522	115 : 64	.254518	85 : 48	.248178
52 : 28	.268845	115 : 63	.261357	106 : 59	.254454	108 : 61	.248094
39 : 21	.268845	73 : 40	.261262	97 : 54	.254378	115 : 65	.247785
115 : 62	.268306	104 : 57	.261158	88 : 49	.254287	92 : 52	.247785
102 : 55	.268238	62 : 34	.260913	79 : 44	.254174	69 : 39	.247785
89 : 48	.268149	93 : 51	.260912	70 : 39	.254033	99 : 56	.247447
76 : 41	.268030	113 : 62	.260687	61 : 34	.253851	76 : 43	.247345
63 : 34	0.267862	82 : 45	0.260601	113 : 63	0.253738	106 : 60	0.247155
113 : 61	.267749	51 : 28	.260412	52 : 29	.253605	53 : 30	.247155
100 : 54	.267606	71 : 39	.260194	95 : 53	.253448	83 : 47	.246980
87 : 47	.267421	91 : 50	.260071	86 : 48	.253257	113 : 64	.246898
111 : 60	.267172	111 : 61	.259993	120 : 67	.253106	120 : 68	.246672
74 : 40	.267172	100 : 55	.259637	77 : 43	.253022	90 : 51	.246672
37 : 20	.267172	80 : 44	.259637	111 : 62	.252931	60 : 34	.246672
98 : 53	.266950	40 : 22	.259637	102 : 57	.252725	97 : 55	.246409
61 : 33	0.266816	109 : 60	0.259275	68 : 38	0.252725	67 : 38	0.246291
85 : 46	.266661	89 : 49	.259194	93 : 52	.252480	104 : 59	.246181
109 : 59	.266575	69 : 38	.259066	118 : 66	.252338	111 : 63	.245982
96 : 52	.266268	118 : 65	.258969	59 : 33	.252338	74 : 42	.245982
72 : 39	.266268	98 : 54	.258832	84 : 47	.252181	37 : 21	.245982
107 : 58	.265956	78 : 43	.258626	109 : 61	.252097	118 : 67	.245807
118 : 64	.265702	107 : 59	.258532	100 : 56	.251812	81 : 46	.245727
59 : 32	.265702	116 : 64	.258278	75 : 42	.251812	88 : 50	.245513

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
95 : 54	0.245330	52 : 30	0.238882	111 : 65	0.232410	111 : 66	0.225779
51 : 29	.245172	97 : 56	.238584	70 : 41	.232314	74 : 44	.225779
109 : 62	.245035	71 : 41	.238474	87 : 51	.231949	116 : 69	.225609
58 : 33	.244914	116 : 67	.238383	58 : 34	.231949	79 : 47	.225529
65 : 37	.244712	90 : 52	.238239	104 : 61	.231704	84 : 50	.225309
72 : 41	.244549	109 : 63	.238086	75 : 44	.231608	89 : 53	.225114
79 : 45	.244415	64 : 37	.237978	92 : 54	.231394	94 : 56	.224940
86 : 49	.244302	83 : 48	.237837	109 : 64	.231247	99 : 59	.224783
93 : 53	0.244207	102 : 59	0.237748	63 : 37	0.231139	52 : 31	0.224642
100 : 57	.244125	95 : 55	.237361	80 : 47	.230992	109 : 65	.224513
107 : 61	.244054	76 : 44	.237361	97 : 57	.230897	114 : 68	.224396
114 : 65	.243992	57 : 33	.237361	114 : 67	.230830	57 : 34	.224396
119 : 68	.243038	38 : 22	.237361	85 : 50	.230449	119 : 71	.224289
112 : 64	.243038	107 : 62	.236992	68 : 40	.230449	62 : 37	.224190
98 : 56	.243038	88 : 51	.236913	51 : 30	.230449	67 : 40	.224015
91 : 52	.243038	69 : 40	.236789	107 : 63	.230043	72 : 43	.223864
84 : 48	0.243038	119 : 69	0.236698	90 : 53	0.229967	77 : 46	0.223733
77 : 44	.243038	100 : 58	.236572	73 : 43	.229854	82 : 49	.223618
70 : 40	.243038	50 : 29	.236572	112 : 66	.229674	87 : 52	.223516
63 : 36	.243038	81 : 47	.236387	56 : 33	.229674	92 : 55	.223425
35 : 20	.243038	112 : 65	.236305	95 : 56	.229536	97 : 58	.223344
117 : 67	.242111	62 : 36	.236089	117 : 69	.229337	102 : 61	.223270
110 : 63	.242052	93 : 54	.236089	78 : 46	.229337	107 : 64	.223204
103 : 59	.241985	105 : 61	.235860	100 : 59	.229148	112 : 67	.223143
96 : 55	0.241909	74 : 43	0.235763	61 : 36	0.229027	117 : 70	0.223088
89 : 51	.241820	117 : 68	.235677	83 : 49	.228882	100 : 60	.221849
82 : 47	.241716	86 : 50	.235529	105 : 62	.228798	95 : 57	.221849
75 : 43	.241593	98 : 57	.235351	110 : 65	.228479	90 : 54	.221849
68 : 39	.241444	110 : 64	.235213	88 : 52	.228479	85 : 51	.221849
61 : 35	.241262	55 : 32	.235213	66 : 39	.228479	80 : 48	.221849
115 : 66	.241154	67 : 39	.235010	115 : 68	.228187	75 : 45	.221849
108 : 62	.241032	79 : 46	.234869	93 : 55	.228120	70 : 42	.221849
54 : 31	0.241032	91 : 53	0.234766	71 : 42	0.228009	65 : 39	0.221849
101 : 58	.240893	103 : 60	.234686	120 : 71	.227923	60 : 36	.221849
94 : 54	.240734	115 : 67	.234623	98 : 58	.227798	55 : 33	.221849
87 : 50	.240549	96 : 56	.234083	76 : 45	.227601	50 : 30	.221849
120 : 69	.240332	84 : 49	.234083	103 : 61	.227507	118 : 71	.220624
80 : 46	.240332	72 : 42	.234083	81 : 48	.227244	113 : 68	.220570
113 : 65	.240165	60 : 35	.234083	54 : 32	.227244	108 : 65	.220510
73 : 42	.240074	113 : 66	.233535	113 : 67	.227004	103 : 62	.220446
106 : 61	0.239976	101 : 59	0.233469	86 : 51	0.226928	98 : 59	0.220374
99 : 57	.239760	89 : 52	.233388	59 : 35	.226784	93 : 56	.220295
66 : 38	.239760	77 : 45	.233278	91 : 54	.226648	88 : 53	.220207
92 : 53	.239512	65 : 38	.233130	64 : 38	.226396	83 : 50	.220109
59 : 34	.239373	118 : 69	.233033	96 : 57	.226396	78 : 47	.219997
85 : 49	.239223	53 : 31	.232914	101 : 60	.226170	73 : 44	.219870
111 : 64	.239143	94 : 55	.232765	69 : 41	.226065	68 : 41	.219725
78 : 45	.238882	82 : 48	.232573	106 : 63	.225965	63 : 38	.219557

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
58 : 35	0.219360	116 : 71	0.213200	29 : 18	0.207126	81 : 51	0.200915
111 : 67	.219248	98 : 60	.213075	95 : 59	.206872	54 : 34	.200915
53 : 32	.219126	49 : 30	.213075	66 : 41	.206760	100 : 63	.200660
101 : 61	.218992	80 : 49	.212894	103 : 64	.206651	73 : 46	.200565
96 : 58	.218843	111 : 68	.212814	111 : 69	.206474	119 : 75	.200486
91 : 55	.218679	93 : 57	.212608	74 : 46	.206474	92 : 58	.200360
86 : 52	.218495	62 : 38	.212608	119 : 74	.206315	111 : 70	.200225
81 : 49	.218289	106 : 65	.212393	82 : 51	.206244	65 : 41	.200130
119 : 72	0.218215	75 : 46	0.212304	90 : 56	0.206055	84 : 53	0.200003
76 : 46	.218056	119 : 73	.212224	98 : 61	.205896	103 : 65	.199924
109 : 66	.217883	88 : 54	.212089	53 : 33	.205762	114 : 72	.199572
71 : 43	.217790	101 : 62	.211930	114 : 71	.205647	95 : 60	.199572
104 : 63	.217693	57 : 35	.211807	61 : 38	.205546	76 : 48	.199572
66 : 40	.217484	70 : 43	.211630	69 : 43	.205380	57 : 36	.199572
94 : 57	.217253	96 : 59	.211419	77 : 48	.205250	106 : 67	.199231
61 : 37	.217128	109 : 67	.211352	85 : 53	.205143	87 : 55	.199157
89 : 54	0.216996	117 : 72	0.210853	93 : 58	0.205055	68 : 43	0.199040
117 : 71	.216928	91 : 56	.210853	101 : 63	.204981	117 : 74	.198954
112 : 68	.216709	78 : 48	.210853	109 : 68	.204918	98 : 62	.198834
84 : 51	.216709	65 : 40	.210853	117 : 73	.204863	49 : 31	.198834
56 : 34	.216709	112 : 69	.210369	96 : 30	.204120	79 : 50	.198657
107 : 65	.216470	99 : 61	.210305	88 : 55	.204120	109 : 69	.198577
79 : 48	.216386	86 : 53	.210223	80 : 50	.204120	90 : 57	.198368
102 : 62	.216209	73 : 45	.210110	72 : 45	.204120	60 : 38	.198368
51 : 31	0.216209	120 : 74	0.209950	64 : 40	0.204120	101 : 64	0.198141
74 : 45	.216019	60 : 37	.209950	56 : 35	.204120	71 : 45	.198046
97 : 59	.215920	107 : 66	.209840	32 : 20	.204120	112 : 71	.197960
120 : 73	.215858	94 : 58	.209700	115 : 72	.203365	82 : 52	.197811
115 : 70	.215600	81 : 50	.209515	107 : 67	.203309	93 : 59	.197631
92 : 56	.215600	115 : 71	.209440	99 : 62	.203244	52 : 33	.197489
69 : 42	.215600	68 : 42	.209260	91 : 57	.203167	115 : 73	.197375
110 : 67	.215318	34 : 21	.209260	83 : 52	.203075	63 : 40	.197281
87 : 53	0.215243	89 : 55	0.209027	75 : 47	0.202963	74 : 47	0.197134
64 : 39	.215115	55 : 34	.208884	67 : 42	.202826	85 : 54	.197025
105 : 64	.215009	76 : 47	.208716	59 : 37	.202650	96 : 61	.196941
82 : 50	.214844	97 : 60	.208620	110 : 69	.202544	107 : 68	.196875
100 : 61	.214670	118 : 73	.208559	51 : 32	.202420	118 : 75	.196821
118 : 72	.214550	84 : 52	.208276	94 : 59	.202276	110 : 70	.196295
59 : 36	.214550	63 : 39	.208276	86 : 54	.202105	99 : 63	.196295
77 : 47	.214393	113 : 70	.207980	78 : 49	.201899	88 : 56	.196295
95 : 58	0.214296	92 : 57	0.207913	113 : 71	0.201820	77 : 49	0.196295
113 : 69	.214229	71 : 44	.207806	116 : 73	.201645	66 : 42	.196295
90 : 55	.213878	100 : 62	.207608	70 : 44	.201645	55 : 35	.196295
72 : 44	.213878	50 : 31	.207608	97 : 61	.201442	113 : 72	.195746
54 : 33	.213878	79 : 49	.207431	62 : 39	.201327	91 : 58	.195613
36 : 22	.213878	108 : 67	.207349	89 : 56	.201202	102 : 65	.195607
103 : 63	.213497	87 : 54	.207126	116 : 73	.201135	69 : 44	.195396
67 : 41	.213291	58 : 36	.207126	108 : 68	.200915	58 : 37	.195226

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
105 : 67	0.195115	99 : 64	0.189455	110 : 72	0.184060	68 : 45	0.179296
94 : 60	.194977	116 : 75	.189397	55 : 36	.184060	71 : 47	.179160
47 : 30	.194977	119 : 77	.189056	84 : 55	.183917	74 : 49	.179036
83 : 53	.194802	102 : 66	.189056	113 : 74	.183847	77 : 51	.178921
119 : 76	.194733	85 : 55	.189056	116 : 76	.183644	80 : 53	.178811
108 : 69	.194575	68 : 44	.189056	87 : 57	.183644	83 : 55	.178715
97 : 62	.194380	51 : 33	.189056	58 : 38	.183664	86 : 57	.178624
61 : 39	.194265	34 : 22	.189056	119 : 78	.183452	89 : 59	.178538
86 : 55	0.194136	105 : 68	0.188680	90 : 59	0.183391	92 : 61	0.178458
111 : 71	.194065	88 : 57	.188608	61 : 40	.183270	95 : 63	.178383
100 : 64	.193820	71 : 46	.188501	93 : 61	.183153	101 : 67	.178257
75 : 48	.193820	54 : 35	.188326	96 : 63	.182931	104 : 69	.178184
50 : 32	.193820	91 : 59	.188190	64 : 42	.182931	107 : 71	.178126
114 : 73	.193582	111 : 72	.187991	32 : 21	.182931	110 : 73	.178070
89 : 57	.193515	74 : 48	.187991	99 : 65	.182722	113 : 75	.178017
64 : 41	.193396	94 : 61	.187798	67 : 44	.182622	116 : 77	.177967
103 : 66	0.193293	57 : 37	0.187673	102 : 67	0.182525	119 : 79	0.177910
117 : 75	.193125	77 : 50	.187521	105 : 69	.182340	24 : 16	
78 : 50	.193125	97 : 63	.187431	70 : 46	.182340	or	
92 : 59	.192936	117 : 76	.187372	108 : 71	.182166	any	.176091
53 : 34	.192797	100 : 65	.187087	73 : 48	.182082	3 to 2	
120 : 77	.192691	80 : 52	.187087	111 : 73	.182000	ratio	
67 : 43	.192606	60 : 39	.187087	114 . 75	.181844	118 : 79	.174255
81 : 52	.192482	103 : 67	.186762	76 : 50	.181844	115 : 77	.174207
95 : 61	0.192394	83 : 54	0.186684	117 : 77	0.181695	112 : 75	0.174157
109 : 70	.192329	63 : 41	.186557	79 : 52	.181624	109 : 73	.174104
112 : 72	.191886	106 : 69	.186457	120 : 79	.181554	106 : 71	.174048
98 : 63	.191886	86 : 56	.186311	82 : 54	.181420	103 : 69	.173988
94 : 54	.191886	109 : 71	.186168	85 : 56	.181231	100 : 67	.173925
70 : 45	.191886	66 : 43	.186075	88 : 58	.181055	97 : 65	.173858
56 : 36	.191886	89 : 58	.185962	91 : 60	.180890	94 : 63	.173787
115 : 74	.191466	112 : 73	.185895	94 : 62	.180736	91 : 61	.173712
101 : 65	0.191408	115 : 75	0.185637	47 : 31	0.180736	88 : 59	0.173631
87 : 56	.191331	92 : 60	.185637	97 : 64	.180592	85 : 57	.173544
73 : 47	.191225	69 : 45	.185637	100 : 66	.180456	82 : 55	.173451
59 : 38	.191068	46 : 30	.185637	50 : 33	.180456	79 : 53	.173351
104 : 67	.190959	95 : 62	.185332	103 : 68	.180328	76 : 51	.173243
90 : 58	.190815	72 : 47	.185235	106 : 70	.180208	73 : 49	.173127
76 : 49	.190618	98 : 64	.185046	53 : 35	.180208	70 : 47	.173000
107 : 69	.190535	49 : 32	.185046	109 : 72	.180094	67 : 45	.172862
93 : 60	0.190332	75 : 49	0.184865	112 : 74	0.179986	64 : 43	0.172712
62 : 40	.190332	101 : 66	.184778	53 : 35	.179986	61 : 41	.172546
110 : 71	.190134	104 : 68	.184524	115 : 76	.179884	119 : 80	.172457
79 : 51	.190057	78 : 51	.184524	118 : 78	.179787	58 : 39	.172363
96 : 62	.189880	52 : 34	.184524	59 : 39	.179787	113 : 76	.172265
113 : 73	.189756	118 : 77	.184391	102 : 69	.179751	55 : 37	.172161
65 : 42	.189664	107 : 70	.184286	62 : 41	.179608	107 : 72	.172051
82 : 53	.189538	81 : 53	.184209	65 : 43	.179445	104 : 70	.171935

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
52 : 35	0.171935	47 : 32	0.166948	77 : 53	0.162215	102 : 71	0.157342
101 : 68	.171813	116 : 79	.166831	61 : 42	.162081	79 : 55	.157264
98 : 66	.171682	69 : 47	.166751	106 : 73	.161983	112 : 78	.157123
49 : 33	.171682	91 : 62	.166650	90 : 62	.161851	56 : 39	.157123
95 : 64	.171544	113 : 77	.166588	45 : 31	.161851	89 : 62	.156998
92 : 62	.171396	110 : 75	.166331	119 : 82	.161733	99 : 69	.156786
46 : 31	.171396	88 : 60	.166331	74 : 51	.161662	66 : 46	.156786
89 : 60	.171239	66 : 45	.166331	103 : 71	.161579	109 : 76	.156613
86 : 58	0.171071	44 : 30	0.166331	116 : 80	0.161368	76 : 53	0.156538
83 : 56	.170890	107 : 73	.166061	87 : 60	.161368	119 : 83	.156469
120 : 81	.170696	85 : 58	.165991	58 : 40	.161368	86 : 60	.156347
80 : 54	.170696	63 : 43	.165872	100 : 69	.161151	43 : 30	.156347
117 : 79	.170559	104 : 71	.165775	71 : 49	.161062	96 : 67	.156196
77 : 52	.170487	82 : 56	.165626	113 : 78	.160984	106 : 74	.156074
114 : 77	.170414	101 : 69	.165472	84 : 58	.160851	53 : 37	.156074
111 : 75	.170262	120 : 82	.165367	97 : 67	.160697	116 : 81	.155973
74 : 50	0.170262	60 : 41	0.165367	110 : 76	0.160579	63 : 44	0.155888
108 : 73	.170101	79 : 54	.165233	55 : 38	.160579	73 : 51	.155753
71 : 48	.170017	98 : 67	.165151	68 : 47	.160411	93 : 65	.155570
105 : 71	.169931	117 : 80	.165096	81 : 56	.160297	103 : 72	.155505
68 : 46	.169751	114 : 78	.164810	94 : 65	.160215	113 : 79	.155451
99 : 67	.169560	95 : 65	.164810	107 : 74	.160152	120 : 84	.154902
65 : 44	.169461	76 : 52	.164810	120 : 83	.160103	110 : 77	.154902
96 : 65	.169358	57 : 39	.164810	117 : 81	.159701	100 : 70	.154902
93 : 63	0.169142	111 : 76	0.164509	104 : 72	0.159701	90 : 63	0.154902
62 : 42	.169142	92 : 63	.164447	91 : 63	.159701	80 : 56	.154902
31 : 21	.169142	73 : 50	.164353	78 : 54	.159701	70 : 49	.154902
90 : 61	.168913	108 : 74	.164192	65 : 45	.159701	60 : 42	.154902
59 : 40	.168792	54 : 37	.164192	52 : 36	.159701	30 : 21	.154902
118 : 80	.168792	89 : 61	.164060	114 : 79	.159278	117 : 82	.154372
87 : 59	.168667	105 : 72	.163857	101 : 70	.159223	107 : 75	.154323
115 : 78	.168603	70 : 48	.163857	88 : 61	.159153	97 : 68	.154263
112 : 76	0.168404	86 : 59	0.163647	75 : 52	0.159058	87 : 61	0.154180
84 : 57	.168404	102 : 70	.163502	62 : 43	.158923	77 : 54	.154097
56 : 38	.168404	51 : 35	.163502	111 : 77	.158832	67 : 47	.153977
109 : 74	.168196	118 : 81	.163397	98 : 68	.158717	114 : 80	.153815
81 : 55	.168122	67 : 46	.163317	49 : 34	.158717	57 : 40	.153815
106 : 72	.167973	83 : 57	.163203	85 : 59	.158567	104 : 73	.153710
53 : 36	.167973	99 : 68	.163126	108 : 75	.158363	94 : 66	.153584
78 : 53	.167819	115 : 79	.163071	72 : 50	.158363	47 : 33	.153584
103 : 70	0.167739	112 : 77	0.162727	95 : 66	0.158180	84 : 59	0.153427
100 : 68	.167491	96 : 66	.162727	59 : 41	.158068	111 : 78	.153228
75 : 51	.167491	80 : 55	.162727	82 : 57	.157939	74 : 52	.153228
50 : 34	.167491	64 : 44	.162727	105 : 73	.157866	101 : 71	.153063
97 : 66	.167228	48 : 33	.162727	115 : 80	.157608	64 : 45	.152968
72 : 49	.167136	32 : 22	.162727	92 : 64	.157608	91 : 64	.152861
119 : 81	.167062	109 : 75	.162365	69 : 48	.157608	118 : 83	.152804
94 : 64	.166948	93 : 64	.162303	46 : 32	.157608	108 : 76	.152610

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
81 : 57	0.152610	45 : 32	0.148063	114 : 82	0.143091	106 : 77	0.138815
54 : 38	.152610	97 : 69	.147923	57 : 41	.143091	117 : 85	.138767
98 : 69	.152377	104 : 74	.147802	82 : 59	.142962	118 : 80	.138303
71 : 50	.152288	52 : 37	.147802	107 : 77	.142893	99 : 72	.138303
115 : 81	.152213	111 : 79	.147696	100 : 72	.142668	88 : 64	.138303
88 : 62	.152091	118 : 84	.147603	75 : 54	.142668	77 : 56	.138303
44 : 31	.152091	66 : 47	.147446	50 : 36	.142668	66 : 48	.138303
105 : 74	.151958	73 : 52	.147320	118 : 85	.142463	55 : 40	.138303
61 : 43	0.151861	80 : 57	0.147215	93 : 67	0.142408	44 : 32	0.138303
78 : 55	.151732	87 : 62	.147128	68 : 49	.142313	114 : 83	.137827
95 : 67	.151649	94 : 67	.147053	111 : 80	.142233	103 : 75	.137776
112 : 79	.151591	101 : 72	.146989	86 : 62	.142107	92 : 67	.137713
119 : 84	.151268	108 : 77	.146933	43 : 31	.142107	81 : 59	.137633
102 : 72	.151268	115 : 82	.146884	104 : 75	.141972	70 : 51	.137528
85 : 60	.151268	98 : 70	.146128	61 : 44	.141877	118 : 86	.137384
68 : 48	.151268	91 : 65	.146128	79 : 57	.141752	59 : 43	.137384
51 : 36	0.151268	84 : 60	0.146128	97 : 70	0.141674	107 : 78	0.137289
109 : 77	.150936	77 : 55	.146128	115 : 83	.141620	96 : 70	.137173
92 : 65	.150874	70 : 50	.146128	108 : 78	.141329	48 : 35	.137173
75 : 53	.150785	63 : 45	.146128	90 : 65	.141329	85 : 62	.137027
116 : 82	.150644	56 : 40	.146128	72 : 52	.141329	111 : 81	.136838
58 : 41	.150644	49 : 35	.146128	54 : 39	.141329	74 : 54	.136838
99 : 70	.150537	42 : 30	.146128	119 : 86	.141049	100 : 73	.136677
82 : 58	.150386	35 : 25	.146128	101 : 73	.140999	63 : 46	.136583
106 : 75	0.150245	116 : 83	0.145380	83 : 60	0.140927	89 : 65	0.136477
65 : 46	.150156	109 : 78	.145332	65 : 47	.140816	115 : 84	.136419
89 : 63	.150050	102 : 73	.145277	112 : 81	.140733	104 : 76	.136220
113 : 80	.149988	88 : 63	.145142	94 : 68	.140619	78 : 57	.136220
120 : 85	.149762	81 : 58	.145057	47 : 34	.140619	52 : 38	.136220
96 : 68	.149762	74 : 53	.144956	76 : 55	.140451	119 : 87	.136028
72 : 51	.149762	67 : 48	.144834	105 : 76	.140376	93 : 68	.135974
48 : 34	.149762	120 : 86	.144683	116 : 84	.140179	67 : 49	.135879
103 : 73	0.149514	60 : 43	0.144683	87 : 63	0.140179	108 : 79	0.135797
79 : 56	.149439	113 : 81	.144593	58 : 42	.140179	82 : 60	.135663
110 : 78	.149298	106 : 76	.144492	29 : 21	.140179	41 : 30	.135663
55 : 39	.149298	53 : 38	.144492	98 : 71	.139968	97 : 71	.135513
86 : 61	.149169	99 : 71	.144377	69 : 50	.139879	112 : 82	.135404
117 : 83	.149108	92 : 66	.144244	109 : 79	.139799	56 : 41	.135404
93 : 66	.148939	46 : 33	.144244	120 : 87	.139662	71 : 52	.135255
62 : 44	.148939	85 : 61	.144089	80 : 58	.139662	86 : 63	.135158
31 : 22	0.148939	117 : 84	0.143907	91 : 66	0.139498	101 : 74	0.135090
100 : 71	.148742	78 : 56	.143907	102 : 74	.139369	116 : 85	.135039
69 : 49	.148653	110 : 79	.143766	51 : 37	.139369	105 : 77	.134699
107 : 76	.148570	71 : 51	.143688	113 : 82	.139265	90 : 66	.134699
114 : 81	.148420	103 : 74	.143606	62 : 45	.139179	75 : 55	.134699
76 : 54	.148420	96 : 69	.143422	73 : 53	.139047	60 : 44	.134699
83 : 59	.148226	64 : 46	.143422	84 : 61	.138950	45 : 33	.134699
90 : 64	.148063	89 : 64	.143210	95 : 69	.138875	30 : 22	.134699

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
109 : 80	0.134337	58 : 43	0.129960	104 : 78	0.124939	95 : 72	0.120391
94 : 69	.134279	89 : 66	.129846	88 : 66	.124939	62 : 47	.120294
79 : 58	.134199	120 : 89	.129791	84 : 63	.124939	91 : 69	.120192
64 : 47	.134082	93 : 69	.129634	64 : 48	.124939	120 : 91	.120140
113 : 83	.134000	62 : 46	.129634	60 : 45	.124939	116 : 88	.119975
98 : 72	.133894	97 : 72	.129439	44 : 33	.124939	87 : 66	.119975
49 : 36	.133894	66 : 49	.129348	40 : 30	.124939	58 : 44	.119975
83 : 61	.133748	101 : 75	.129260	28 : 21	.124939	29 : 22	.119975
117 : 86	0.133687	105 : 78	0.129095	117 : 88	0.123703	112 : 85	0.119799
102 : 75	.133539	70 : 52	.129095	113 : 85	.123660	83 : 63	.119738
68 : 50	.133539	109 : 81	.128942	109 : 82	.123613	108 : 82	.119610
87 : 64	.133339	74 : 55	.128869	105 : 79	.123562	54 : 41	.119610
106 : 78	.133211	113 : 84	.128799	101 : 76	.123508	79 : 60	.119476
72 : 53	.133057	117 : 87	.128667	97 : 73	.123449	104 : 79	.119406
91 : 67	.132967	78 : 58	.128667	93 : 70	.123385	100 : 76	.119186
110 : 81	.132908	82 : 61	.128484	89 : 67	.123315	75 : 57	.119186
114 : 84	0.132626	86 : 64	0.128319	85 : 64	0.123238	50 : 38	0.119186
95 : 70	.132626	43 : 32	.128319	81 : 61	.123155	96 : 73	.118948
76 : 56	.132626	90 : 67	.128168	77 : 58	.123063	71 : 54	.118865
57 : 42	.132626	94 : 70	.128030	73 : 55	.122960	117 : 89	.118796
118 : 87	.132363	47 : 35	.128030	69 : 52	.122846	92 : 70	.118690
99 : 73	.132312	98 : 73	.127903	65 : 49	.122717	46 : 35	.118690
80 : 59	.132238	102 : 76	.127787	61 : 46	.122572	113 : 86	.118580
61 : 45	.132117	51 : 38	.127787	118 : 89	.122492	67 : 51	.118505
103 : 76	0.132024	106 : 79	0.127679	114 : 86	0.122406	88 : 67	0.118408
84 : 62	.131888	110 : 82	.127579	57 : 43	.122406	109 : 83	.118348
42 : 31	.131888	55 : 41	.127579	110 : 83	.122315	84 : 64	.118099
107 : 79	.131757	114 : 85	.127486	53 : 40	.122216	63 : 48	.118099
65 : 48	.131672	118 : 88	.127399	106 : 80	.122216	42 : 32	.118099
88 : 65	.131570	59 : 44	.127399	102 : 77	.122110	101 : 77	.117831
111 : 82	.131509	63 : 47	.127243	98 : 74	.121994	80 : 61	.117760
115 : 85	.131279	67 : 50	.127105	49 : 37	.121994	118 : 90	.117640
92 : 68	0.131279	71 : 53	0.126982	94 : 71	0.121870	59 : 45	0.117640
69 : 51	.131279	75 : 56	.126873	90 : 68	.121734	97 : 74	.117540
46 : 34	.131279	79 : 59	.126775	45 : 34	.121734	114 : 87	.117386
119 : 88	.131064	83 : 62	.126686	86 : 65	.121585	76 : 58	.117386
96 : 71	.131013	87 : 65	.126606	82 : 62	.121422	93 : 71	.117225
73 : 54	.130929	91 : 68	.126533	41 : 31	.121422	110 : 84	.117113
100 : 74	.130768	95 : 71	.126465	119 : 90	.121305	55 : 42	.117113
50 : 37	.130768	99 : 74	.126404	78 : 59	.121243	105 : 80	.117099
77 : 57	0.130616	103 : 77	0.126347	115 : 87	0.121179	72 : 55	0.116970
104 : 77	.130543	107 : 80	.126294	111 : 84	.121044	89 : 68	.116881
81 : 60	.130334	111 : 83	.126245	74 : 56	.121044	106 : 81	.116821
54 : 40	.130334	115 : 86	.126199	107 : 81	.120899	119 : 91	.116506
27 : 20	.130334	119 : 89	.126157	70 : 53	.120822	102 : 78	.116506
112 : 83	.130140	120 : 90	.124939	103 : 78	.120743	85 : 65	.116506
85 : 63	.130078	112 : 84	.124939	99 : 75	.120574	68 : 52	.116506
116 : 86	.129960	108 : 81	.124939	66 : 50	.120574	51 : 39	.116506

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
34 : 26	0.116506	101 : 78	0.112227	118 : 92	0.108094	75 : 59	0.104209
115 : 88	.116215	110 : 85	.111974	59 : 46	.108094	61 : 48	.104089
98 : 75	.116165	88 : 68	.111974	109 : 85	.108008	108 : 85	.104005
81 : 62	.116093	44 : 34	.111974	100 : 78	.107905	94 : 74	.103896
64 : 49	.115984	66 : 51	.111974	50 : 39	.107905	47 : 37	.103896
111 : 85	.115904	119 : 92	.111759	91 : 71	.107783	80 : 63	.103750
94 : 72	.115795	97 : 75	.111710	82 : 64	.107634	113 : 89	.103688
47 : 36	.115795	75 : 58	.111633	41 : 32	.107634	99 : 78	.103541
77 : 59	0.115639	106 : 82	0.111492	114 : 89	0.107515	66 : 52	0.103541
107 : 82	.115570	53 : 41	.111492	73 : 57	.107448	118 : 93	.103399
120 : 92	.115393	84 : 65	.111366	105 : 82	.107375	85 : 67	.103344
90 : 69	.115393	115 : 89	.111308	96 : 75	.107210	104 : 82	.103219
60 : 46	.115393	93 : 72	.111151	64 : 50	.107210	52 : 41	.103219
30 : 23	.115393	62 : 48	.111151	119 : 93	.107064	90 : 71	.102984
103 : 79	.115210	102 : 79	.110973	87 : 68	.107010	109 : 86	.102928
73 : 56	.115135	71 : 55	.110896	110 : 86	.106894	95 : 75	.102662
116 : 89	0.115068	111 : 86	0.110825	55 : 43	0.106894	76 : 60	0.102662
86 : 66	.114955	120 : 93	.110698	78 : 61	.106765	57 : 45	.102662
43 : 33	.114955	80 : 62	.110698	101 : 79	.106694	38 : 30	.102662
99 : 76	.114822	40 : 31	.110698	115 : 90	.106455	119 : 94	.102419
112 : 86	.114720	89 : 69	.110541	92 : 72	.106455	100 : 79	.102373
56 : 43	.114720	98 : 76	.110413	69 : 54	.106455	81 : 64	.102305
69 : 53	.114573	49 : 38	.110413	46 : 36	.106455	62 : 49	.102196
82 : 63	.114473	107 : 83	.110306	106 : 83	.106228	105 : 83	.102111
95 : 73	0.114401	116 : 90	0.110216	83 : 65	0.106165	86 : 68	0.101990
108 : 83	.114346	58 : 45	.110216	120 : 94	.106053	43 : 34	.101990
117 : 90	.113943	67 : 52	.110072	60 : 47	.106053	110 : 87	.101873
104 : 80	.113943	76 : 59	.109962	97 : 76	.105958	67 : 53	.101799
91 : 70	.113943	85 : 66	.109875	111 : 87	.105804	91 : 72	.101709
78 : 60	.113943	94 : 73	.109805	74 : 58	.105804	115 : 91	.101656
65 : 50	.113943	103 : 80	.109747	88 : 69	.105634	120 : 95	.101458
52 : 40	.113943	112 : 87	.109699	102 : 80	.105510	96 : 76	.101458
39 : 30	0.113943	117 : 91	0.109145	51 : 40	0.105510	72 : 57	0.101458
26 : 20	.113943	108 : 84	.109145	116 : 91	.105417	48 : 38	.101458
113 : 87	.113559	99 : 77	.109145	65 : 51	.105343	101 : 80	.101231
100 : 77	.113509	90 : 70	.109145	79 : 62	.105235	77 : 61	.101161
87 : 67	.113445	81 : 63	.109145	93 : 73	.105160	106 : 84	.101027
74 : 57	.113357	72 : 56	.109145	107 : 84	.105105	53 : 42	.101027
61 : 47	.113232	63 : 49	.109145	112 : 88	.104735	82 : 65	.100901
109 : 84	.113147	54 : 42	.109145	98 : 77	.104735	111 : 88	.100840
96 : 74	0.113040	45 : 35	0.109145	84 : 66	0.104735	116 : 92	0.100670
48 : 37	.113040	27 : 21	.109145	70 : 55	.104735	87 : 69	.100670
83 : 64	.112898	113 : 88	.108596	56 : 44	.104735	58 : 46	.100670
118 : 41	.112841	104 : 81	.108548	42 : 33	.104735	29 : 23	.100670
70 : 54	.112704	95 : 74	.108492	28 : 22	.104735	92 : 73	.100465
92 : 71	.112530	86 : 67	.108424	117 : 92	.104398	63 : 50	.100371
57 : 44	.112422	77 : 60	.108339	103 : 81	.104352	97 : 77	.100281
79 : 61	.112297	68 : 53	.108233	89 : 70	.104292	102 : 81	.100115

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
68 : 54	0.100115	117 : 94	0.095058	74 : 60	0.091080	88 : 72	0.087150
34 : 27	.100115	56 : 45	.094976	37 : 30	.091080	77 : 63	.087150
107 : 85	.099964	107 : 86	.094885	90 : 73	.090920	66 : 54	.087150
73 : 58	.099894	102 : 82	.094786	106 : 86	.090807	55 : 45	.087150
112 : 89	.099828	51 : 41	.094786	53 : 43	.090807	44 : 36	.087150
78 : 62	.099703	97 : 78	.094677	69 : 56	.090661	116 : 95	.086734
39 : 31	.099703	92 : 74	.094556	85 : 69	.090570	105 : 86	.086691
83 : 66	.099534	46 : 37	.094556	101 : 82	.090508	94 : 77	.086637
88 : 70	0.099385	87 : 70	0.094421	117 : 95	0.090462	83 : 68	0.086569
44 : 35	.099385	82 : 66	.094270	112 : 91	.090177	72 : 59	.086481
93 : 74	.099251	41 : 33	.094270	96 : 78	.090177	61 : 50	.086360
98 : 78	.099132	118 : 95	.094158	80 : 65	.090177	111 : 91	.086282
49 : 39	.099132	77 : 62	.094099	64 : 52	.090177	100 : 82	.086186
103 : 82	.099023	113 : 91	.094037	48 : 39	.090177	50 : 41	.086186
108 : 86	.098925	108 : 87	.093905	107 : 87	.089865	89 : 73	.086067
54 : 43	.098925	72 : 58	.093905	91 : 74	.089810	117 : 96	.085915
113 : 90	0.098836	103 : 83	0.093759	75 . 61	0.089732	78 : 64	0.085915
118 : 94	.098754	67 : 54	.093681	118 : 96	.089611	39 : 32	.085915
59 : 47	.098754	98 : 79	.093599	59 : 48	.089611	106 : 87	.085787
64 : 51	.098610	93 : 75	.093422	102 : 83	.089522	67 : 55	.085712
69 : 55	.098486	62 : 50	.093422	86 : 70	.089401	95 : 78	.085629
74 : 59	.098380	119 : 96	.093276	43 : 35	.089401	112 : 92	.085430
79 : 63	.098287	88 : 71	.093224	113 : 92	.089291	84 : 69	.085430
84 : 67	.098205	114 : 92	.093117	70 : 57	.089223	56 : 46	.085430
89 : 71	0.098132	57 : 46	0.093117	97 : 79	0.089145	28 : 23	0.085430
94 : 75	.098067	83 : 67	.093003	108 : 88	.088941	101 : 83	.085243
99 : 79	.098008	109 : 88	.092944	81 : 66	.088941	118 : 97	.085110
104 : 83	.097955	104 : 84	.092754	54 : 44	.088941	90 : 74	.085011
109 : 87	.097907	78 : 63	.092754	27 : 22	.088941	45 : 37	.085011
114 : 91	.097864	52 : 42	.092754	119 : 97	.088775	107 : 88	.084901
119 : 95	.097823	26 : 21	.092754	65 : 53	.088638	62 : 51	.084822
20 : 16		99 : 80	.092545	103 : 84	.088558	79 : 65	.084714
or		73 : 59	0.092471	114 : 93	0.088422	96 : 79	0.084644
any	0.096910	120 : 97	.092410	76 : 62	.088422	113 : 93	.084596
5 to 4		94 : 76	.092314	38 : 31	.088422	119 : 98	.084321
ratio		47 : 38	.092314	87 : 71	.088261	102 : 84	.084321
116 : 93	.095975	68 : 55	.092146	98 : 80	.088136	85 : 70	.084321
111 : 89	.095933	115 : 93	.092215	49 : 40	.088136	68 : 56	.084321
106 : 85	.095887	89 : 72	.092058	109 : 89	.088037	51 : 42	.084321
101 : 81	.095836	110 : 89	.092003	120 : 98	.087955	108 : 89	.084034
96 : 77	0.095781	105 : 85	0.091770	60 : 49	0.087955	91 : 75	0.083980
91 : 73	.095719	84 : 68	.091770	71 : 58	.087830	74 : 61	.083902
86 : 69	.095649	63 : 51	.091770	82 : 67	.087739	114 : 94	.083777
81 : 65	.095572	42 : 34	.091770	93 : 76	.087669	57 : 47	.083777
76 : 61	.095484	116 : 94	.091330	104 : 85	.087614	97 : 80	.083682
71 : 57	.095383	58 : 47	.091330	115 : 94	.087570	120 : 99	.083546
66 : 53	.095268	95 : 77	0.091233	110 : 90	.087150	80 : 66	.083546
61 : 49	.095134	111 : 90	.091080	99 : 81	.087150	40 : 33	.083546

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
103 : 85	0.083418	115 : 96	0.078427	114 : 96	0.074634	73 : 62	0.070931
63 : 52	.083337	109 : 91	.078385	95 : 80	.074634	93 : 79	.070856
86 : 71	.083240	103 : 86	.078339	76 : 64	.074634	113 : 96	.070808
109 : 90	.083184	97 : 81	.078287	57 : 48	.074634	100 : 85	.070581
115 : 95	.082974	91 : 76	.078228	38 : 32	.074634	80 : 68	.070581
92 : 76	.082974	85 : 71	.078161	108 : 91	.074382	60 : 51	.070581
69 : 57	.082974	79 : 66	.078083	89 : 75	.074329	40 : 34	.070581
46 : 38	.082974	73 : 61	.077993	70 : 59	.074246	107 : 91	.070342
98 : 81	0.082741	67 : 56	0.077887	102 : 86	0.074102	87 : 74	0.070288
75 : 62	.082670	61 : 51	.077760	51 : 43	.074102	67 : 57	.070200
52 : 43	.082535	116 : 97	.077686	83 : 70	.073980	114 : 97	.070133
81 : 67	.082410	55 : 46	.077605	115 : 97	.073927	94 : 80	.070038
110 : 91	.082351	104 : 87	.077514	96 : 81	.073786	47 : 40	.070038
116 : 96	.082187	98 : 82	.077412	64 : 54	.073786	74 : 63	.069891
87 : 72	.082187	49 : 41	.077412	109 : 92	.073639	101 : 86	.069823
58 : 48	.082187	92 : 77	.077297	77 : 65	.073577	81 : 69	.069636
29 : 24	0.082187	86 : 72	0.077166	90 : 76	0.073429	54 : 46	0.069636
93 : 77	.081992	43 : 36	.077166	45 : 38	.073429	115 : 98	.069472
64 : 53	.081904	80 : 67	.077015	103 : 87	.073318	88 : 75	.069421
99 : 82	.081821	117 : 98	.076960	116 : 98	.073232	61 : 52	.069327
105 : 87	.081670	111 : 93	.076840	58 : 49	.073232	95 : 81	.069239
70 : 58	.081670	74 : 62	.076840	71 : 60	.073107	68 : 58	.069081
76 : 63	.081473	37 : 31	.076840	84 : 71	.073021	109 : 93	.068944
117 : 97	.081414	105 : 88	.076707	97 : 82	.072958	75 : 64	.068881
82 : 68	0.081305	68 : 57	0.076634	110 : 93	0.072910	116 : 99	0.068823
41 : 34	.081305	99 : 83	.076557	117 : 99	.072551	82 : 70	.068716
88 : 73	.081160	93 : 78	.076388	104 : 88	.072551	41 : 35	.068716
94 : 78	.081033	62 : 52	.076388	91 : 77	.072551	89 : 76	.068576
47 : 39	.081033	118 : 99	.076247	78 : 66	.072551	96 : 82	.068457
100 : 83	.080922	87 : 73	.076196	65 : 55	.072551	48 : 41	.068457
106 : 88	.080823	112 : 94	.076090	39 : 33	.072551	103 : 88	.068355
53 : 44	.080823	56 : 47	.076090	52 : 34	.072551	55 : 47	.068265
118 : 98	0.080656	81 : 68	0.075976	111 : 94	0.072195	117 : 100	0.068186
59 : 48	.080656	106 : 89	.075916	98 : 83	.072148	62 : 53	.068116
65 : 54	.080510	100 : 84	.075721	85 : 72	.072086	69 : 59	.067997
71 : 59	.080406	75 : 63	.075721	72 : 61	.072003	76 : 65	.067900
77 : 64	.080311	50 : 42	.075721	118 : 100	.071882	83 : 71	.067820
83 : 69	.080229	25 : 21	.075721	59 : 50	.071882	90 : 77	.067752
89 : 74	.080158	119 : 100	0.075547	105 : 89	.071797	97 : 83	.067694
95 : 79	.080097	94 : 79	.075501	92 : 78	.071693	104 : 89	.067643
107 : 89	0.079994	69 : 58	0.075421	46 : 39	.071693	111 : 95	0.067599
113 : 94	.079951	113 : 95	.075355	79 : 67	0.071552	98 : 84	.066947
119 : 99	.079912	88 : 74	.075251	112 : 95	.071494	91 : 78	.066947
30 : 25		44 : 37	.075251	99 : 84	.071356	84 : 72	.066947
or		107 : 90	.075141	66 : 56	.071356	77 : 66	.066947
any	.079181	63 : 53	.075065	86 : 73	.071176	70 : 60	.066947
6 to 5		82 : 69	.074965	106 : 90	.071063	63 : 54	.066947
ratio		101 : 85	.074903	53 : 45	.071063	56 : 48	.066947

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
49 : 42	0.066947	75 : 65	0.062148	113 : 99	0.057443	69 : 61	0.053519
42 : 36	.066947	60 : 52	.062148	105 : 92	.057402	95 : 84	.053444
35 : 30	.066947	45 : 39	.062148	97 : 85	.057353	78 : 69	.053246
113 : 97	.066307	113 : 98	.061852	89 : 78	.057295	52 : 46	.053246
106 : 91	.066265	98 : 85	.061807	81 : 71	.057227	113 : 100	.053078
99 : 85	.066216	83 : 72	.061746	73 : 64	.057143	87 : 77	.053029
92 : 79	.066161	68 : 59	.061657	65 : 57	.057039	61 : 54	.052936
85 : 73	.066096	53 : 46	.061518	57 : 50	.056905	96 : 85	.052852
78 : 67	0.066020	91 : 79	0.061414	106 : 93	0.056823	70 : 62	0.052706
71 : 61	.065929	76 : 66	.061270	98 : 86	.056728	35 : 31	.052706
64 : 55	.065817	38 : 33	.061270	49 : 43	.056728	79 : 70	.052529
57 : 49	.065679	99 : 86	.061137	90 : 79	.056615	88 : 78	.052388
107 : 92	.065596	61 : 53	.061054	82 : 72	.056481	44 : 39	.052388
100 : 86	.065502	84 : 73	.060956	41 : 36	.056481	97 : 86	.052273
50 : 43	.065502	107 : 93	.060901	74 : 65	.056318	53 : 47	.052178
93 : 80	.065393	92 : 80	.060698	107 : 94	.056256	62 : 55	.052029
86 : 74	0.065267	69 : 60	0.060698	99 : 87	0.056116	71 : 63	0.051918
43 : 37	.065267	46 : 40	.060698	66 : 58	.056116	80 : 71	.051832
79 : 68	.065118	100 : 87	.060481	91 : 80	.055951	89 : 79	.051763
115 : 99	.065063	77 : 67	.060416	58 : 51	.055858	98 : 87	.051707
72 : 62	.064941	54 : 47	.060296	83 : 73	.055755	107 : 95	.051660
36 : 31	.064941	85 : 74	.060187	108 : 95	.055700	99 : 88	.051153
101 : 87	.064802	93 : 81	.059998	100 : 88	.055517	90 : 80	.051153
65 : 56	.064725	62 : 54	.059998	75 : 66	.055517	81 : 72	.051153
94 : 81	0.064643	101 : 88	0.059839	50 : 44	0.055517	72 : 64	0.051153
87 : 75	.064458	70 : 61	.059768	92 : 81	.055303	63 : 56	.051153
109 : 94	.064299	109 : 95	.059703	67 : 59	.055223	54 : 48	.051153
80 : 69	.064241	78 : 68	.059586	109 : 96	.055155	45 : 40	.051153
102 : 88	.064118	39 : 34	.059586	84 : 74	.055048	36 : 32	.051153
51 : 44	.064118	86 : 75	.059437	42 : 37	.055048	109 : 97	.050655
73 : 63	.063982	97 : 41	.059314	101 : 89	.054931	100 : 89	.050610
95 : 82	.063910	94 : 82	.059314	59 : 52	.054849	91 : 81	.050556
88 : 76	0.063669	102 : 89	0.059210	76 : 67	0.054739	82 : 73	0.050491
66 : 57	.063669	55 : 48	.059122	93 : 82	.054669	73 : 65	.050410
44 : 38	.063669	63 : 55	.058978	110 : 97	.054621	64 : 57	.050305
103 : 89	.063447	71 : 62	.058867	102 : 90	.054358	55 : 49	.050167
81 : 70	.063387	79 : 69	.058778	85 : 75	.054358	101 : 90	.050079
59 : 51	.063282	87 : 76	.058706	68 : 60	.054358	92 : 82	.049974
96 : 83	.063193	95 : 83	.058646	51 : 45	.054358	46 : 41	.049974
74 : 64	.063052	103 : 90	.058595	34 : 30	.054358	83 : 74	.049846
37 : 32	0.063052	96 : 84	0.057992	111 : 98	0.054097	74 : 66	0.049688
89 : 77	.062899	88 : 77	.057992	94 : 83	.054050	37 : 33	.049688
52 : 45	.062791	80 : 70	.057992	77 : 68	.053982	102 : 91	.049558
67 : 58	.062647	72 : 63	.057992	60 : 53	.053875	65 : 58	.049485
82 : 71	.062556	64 : 56	.057992	103 : 91	.053796	93 : 83	.049405
97 : 84	.062493	56 : 49	.057992	86 : 76	.053685	56 : 50	.049218
112 : 97	.062446	48 : 42	.057992	43 : 38	.053685	84 : 75	.049218
90 : 78	.062148	40 : 35	.057992	112 : 99	.053583	103 : 92	.049049

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
75 : 67	0.048987	41 : 37	0.044582	57 : 52	0.039872	38 : 35	0.035716
94 : 84	.048849	72 : 65	.044419	80 : 73	.039767	89 : 82	.035576
47 : 42	.048849	103 : 93	.044354	103 : 94	.039709	51 : 47	.035472
66 : 59	.048692	93 : 84	.044204	92 : 84	.039509	64 : 59	.035328
85 : 76	.048605	62 : 56	.044204	69 : 63	.039509	77 : 71	.035232
104 : 93	.048550	83 : 75	.044017	46 : 42	.039509	90 : 83	.035164
95 : 85	.048305	52 : 47	.043905	104 : 95	.039310	103 : 95	.035114
76 : 68	.048305	73 : 66	.043779	81 : 74	.039253	91 : 84	.034762
57 : 51	0.048305	94 : 85	0.043709	58 : 53	0.039152	78 : 72	0.034762
38 : 34	.048305	84 : 76	.043466	93 : 85	.039064	65 : 60	.034762
105 : 94	.048061	63 : 57	.043466	105 : 96	.038918	52 : 48	.034762
86 : 77	.048008	42 : 38	.043466	70 : 64	.038918	39 : 36	.034762
67 : 60	.047924	95 : 86	.043225	35 : 32	.038918	105 : 97	.034418
96 : 86	.047773	74 : 67	.043157	82 : 75	.038753	92 : 85	.034369
48 : 43	.047773	53 : 48	.043035	94 : 86	.038629	79 : 73	.034304
77 : 69	.047642	85 : 77	.042928	47 : 32	.038629	66 : 61	.034214
106 : 95	0.047582	96 : 87	0.042752	106 : 97	0.038534	53 : 49	0.034080
87 : 78	.047425	64 : 58	.042752	59 : 54	.038458	93 : 86	.033984
58 : 52	.047425	107 : 97	.042612	71 : 65	.038345	80 : 74	.033858
29 : 26	.047425	75 : 68	.042552	83 : 76	.038265	40 : 37	.033858
97 : 87	.047252	86 : 78	.042404	95 : 87	.038204	107 : 99	.033749
39 : 35	.046997	43 : 39	.042404	107 : 98	.038158	67 : 62	.033683
88 : 79	.046856	97 : 88	.042289	96 : 88	.037789	94 : 87	.033609
98 : 88	.046743	54 : 49	.042198	84 : 77	0.037789	81 : 75	.033424
49 : 44	0.046743	65 : 59	0.042061	72 : 66	.037789	54 : 50	0.033424
108 : 97	.046652	76 : 69	.041965	60 : 55	.037789	95 : 88	.033241
59 : 53	.046576	87 : 79	.041892	48 : 44	.037789	68 : 63	.033168
69 : 62	.046457	98 : 89	.041836	36 : 33	.037789	82 : 76	.033000
79 : 71	.046369	109 : 99	.041791	109 : 100	.037427	41 : 38	.033000
89 : 80	.046300	99 : 90	.041393	97 : 89	.037382	96 : 89	.032881
99 : 89	.046245	88 : 80	.041393	85 : 79	.037324	55 : 51	.032793
109 : 98	.046200	77 : 70	.041393	73 : 67	.037248	69 : 64	.032669
100 : 90	0.045758	66 : 60	0.041393	61 : 56	0.037142	83 : 77	0.032587
90 : 81	.045758	55 : 50	.041393	98 : 90	.036984	97 : 90	.032529
80 : 72	.045758	44 : 40	.041393	49 : 45	.036984	98 : 91	.032185
70 : 63	.045758	33 : 30	.041393	86 : 79	.036871	84 : 78	.032185
60 : 54	.045758	100 : 91	.040959	74 : 68	.036723	70 : 65	.032185
50 : 45	.045758	89 : 81	.040905	37 : 34	.036723	56 : 52	.032185
40 : 36	.045758	78 : 71	.040836	99 : 91	.036594	42 : 39	.032185
111 : 100	.045323	67 : 61	.040745	62 : 57	.036517	99 : 92	.031847
101 : 91	0.045280	56 : 51	0.040618	87 : 80	0.036429	85 : 79	0.031792
91 : 82	.045228	101 : 92	.040534	100 : 92	.036212	71 : 66	.031714
81 : 73	.045162	90 : 82	.040429	75 : 69	.036212	57 : 53	.031599
71 : 64	.045078	45 : 41	.040429	50 : 46	.036212	100 : 93	.031517
61 : 55	.044967	79 : 72	.040295	88 : 81	.035998	86 : 80	.031409
51 : 46	.044812	68 : 62	.040117	63 : 58	.035913	43 : 40	.031409
92 : 83	.044710	34 : 31	.040117	101 : 93	.035839	72 : 67	.031258
82 : 74	.044582	91 : 83	.039963	76 : 70	.035716	101 : 94	.031194

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
87 : 81	0.031034	50 : 47	0.026872	80 : 76	0.022276	100 : 96	0.017729
58 : 54	.031034	67 : 63	.026734	60 : 57	.022276	75 : 72	.017729
102 : 95	.030877	84 : 79	.026652	40 : 38	.022276	50 : 48	.017729
73 : 68	.030814	101 : 95	.026598	28 : 26	.022185	101 : 97	.017550
88 : 82	.030669	85 : 80	.026329	101 : 96	.022050	76 : 73	.017491
44 : 41	.030669	68 : 64	.026329	81 : 77	.021994	51 : 49	.017374
103 : 96	.030566	51 : 48	.026329	61 : 58	.021902	77 : 74	.017259
59 : 55	.030489	34 : 32	.026329	103 : 98	.021611	103 : 99	.017202
74 : 69	0.030383	103 : 97	0.026066	62 : 59	0.021540	78 : 75	0.017033
89 : 83	.030312	86 : 81	.026014	83 : 79	.021451	52 : 50	.017033
104 : 97	.030262	69 : 65	.025936	104 : 99	.021398	79 : 76	.016814
105 : 98	.029963	52 : 49	.025807	84 : 80	.021189	53 : 51	.016706
90 : 84	.029963	87 : 82	.025705	63 : 60	.021189	80 : 77	.016599
75 : 70	.029963	70 : 66	.025554	42 : 40	.021189	81 : 78	.016391
60 : 56	.029963	35 : 33	.025554	85 : 81	.020934	54 : 52	.016391
45 : 42	.029963	88 : 83	.025405	64 : 61	.020850	82 : 79	.016187
106 : 99	0.029671	53 : 50	0.025306	86 : 82	0.020685	55 : 53	0.016087
91 : 85	.029623	71 : 67	.025184	43 : 41	.020685	83 : 80	.015988
76 : 71	.029555	89 : 84	.025111	65 : 62	.020522	84 : 81	.015794
61 : 57	.029455	90 : 85	.024824	87 : 83	.020441	56 : 54	.015794
107 : 100	.029384	72 : 68	.024824	88 : 84	.020203	85 : 82	.015605
92 : 84	.029289	54 : 51	.024824	66 : 63	.020203	57 : 55	.015512
46 : 43	.029289	36 : 34	.024824	44 : 42	.020203	86 : 83	.015420
77 : 72	.029158	91 : 86	.024543	89 : 85	.019971	87 : 84	.015240
93 : 87	0.028964	73 : 69	0.024474	67 : 64	0.019895	58 : 56	0.015240
62 : 58	.028964	55 : 52	.024359	90 : 86	.019744	88 : 85	.015064
78 : 73	.028772	92 : 87	.024269	45 : 43	.019744	59 : 57	.014977
97 : 44	.028645	74 : 70	.024134	68 : 65	.019596	89 : 86	.014892
94 : 88	.028645	37 : 35	.024134	91 : 87	.019522	90 : 87	.014723
63 : 59	.028489	93 : 88	.024000	92 : 88	.019305	60 : 58	.014723
79 : 74	.028395	56 : 53	.023912	69 : 66	.019305	91 : 88	.014559
95 : 89	.028334	75 : 71	.023803	46 : 44	.019305	61 : 59	.014478
96 : 90	0.028029	94 : 89	0.023738	93 : 89	0.019093	92 : 89	0.014398
80 : 75	.028029	95 : 90	.023481	70 : 67	.019023	93 : 90	.014240
64 : 60	.028029	76 : 72	.023481	94 : 90	.018885	62 : 60	.014240
48 : 45	.028029	57 : 54	.023481	47 : 45	.018885	31 : 30	.014240
32 : 30	.028029	38 : 36	.023481	71 : 68	.018749	94 : 91	.014087
97 : 91	.027730	96 : 91	.023230	95 : 91	.018682	63 : 61	.014011
81 : 76	.027671	77 : 73	.023168	96 : 92	.018483	95 : 92	.013936
65 : 61	.027584	58 : 55	.023065	72 : 69	.018483	96 : 93	.013788
98 : 92	0.027438	97 : 92	0.022984	48 : 46	0.018483	64 : 62	0.013788
82 : 77	.027438	78 : 74	.022863	24 : 23	.018483	32 : 31	.013788
49 : 46	.027438	39 : 37	.022863	97 : 93	.018289	97 : 94	.013644
99 : 93	.027152	98 : 93	.022743	73 : 70	.018225	65 : 63	.013573
66 : 62	.027152	59 : 56	.022664	98 : 94	.018098	98 : 95	.013503
33 : 31	.027152	79 : 75	.022566	49 : 47	.018098	99 : 96	.013364
83 : 78	.026984	99 : 94	.022507	74 : 71	.017973	66 : 64	.013364
100 : 94	.026872	100 : 95	.022276	99 : 95	.017912	33 : 32	.013364

Logarithms of Gear Ratios from 1.0084+ to 7.5

Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio	Numbers of Teeth	Logarithm of Ratio
100 : 97	0.013228	87 : 85	0.010100	61 : 60	0.007179	93 : 92	0.004695
67 : 65	.013161	88 : 86	.009984	62 : 61	.007062	94 : 93	.004645
101 : 98	.013095	44 : 43	.009984	63 : 62	.006949	95 : 94	.004596
68 : 66	.012965	89 : 87	.009871	64 : 63	.006840	96 : 95	.004548
34 : 33	.012965	90 : 88	.009760	65 : 64	.006733	97 : 96	.004501
103 : 100	.012837	45 : 44	.009760	66 : 65	.006631	98 : 97	.004454
69 : 67	.012774	91 : 89	.009651	67 : 66	.006531	99 : 98	.004409
70 : 68	.012589	92 : 90	.009545	68 : 67	.006434	100 : 99	.004365
35 : 34	0.012589	46 : 45	0.009545	69 : 68	0.006340	101 : 100	0.004321
72 : 70	.012235	93 : 91	.009442	70 : 69	.006249	102 : 101	.004279
36 : 35	.012235	94 : 92	.009340	71 : 70	.006160	103 : 102	.004237
73 : 71	.012065	47 : 46	.009340	72 : 71	.006074	104 : 103	.004196
74 : 72	.011899	95 : 93	.009241	73 : 72	.005990	105 : 104	.004156
37 : 36	.011899	96 : 94	.009143	74 : 73	.005909	106 : 105	.004117
75 : 73	.011738	48 : 47	.009143	75 : 74	.005830	107 : 106	.004078
76 : 74	.011582	97 : 95	.009048	76 : 75	.005752	108 : 107	.004040
38 : 37	0.011582	98 : 96	0.008955	77 : 76	0.005677	109 : 108	0.004002
77 : 75	.011429	49 : 48	.008955	78 : 77	.005604	110 : 109	.003967
78 : 76	.011281	99 : 97	.008864	79 : 78	.005533	111 : 110	.003930
39 : 38	.011281	100 : 98	.008774	80 : 79	.005463	112 : 111	.003895
79 : 77	.011136	50 : 49	.008774	81 : 80	.005395	113 : 112	.003860
80 : 78	.010995	101 : 99	.008686	82 : 81	.005329	114 : 113	.003827
40 : 39	.010995	51 : 50	.008600	83 : 82	.005264	115 : 114	.003793
81 : 79	.010858	52 : 51	.008433	84 : 83	.005201	116 : 115	.003760
82 : 80	0.010724	53 : 52	0.008273	85 : 84	0.005140	117 : 116	0.003728
41 : 40	.010724	54 : 53	.008118	86 : 85	.005080	118 : 117	.003696
83 : 81	.010593	55 : 54	.007969	87 : 86	.005021	119 : 118	.003665
84 : 82	.010465	56 : 55	.007825	88 : 87	.004963	120 : 119	.003634
42 : 41	.010465	57 : 56	.007687	89 : 88	.004907	......	......
85 : 83	.010341	58 : 57	.007553	90 : 89	.004853	......	......
86 : 84	.010219	59 : 58	.007424	91 : 90	.004799	......	......
43 : 42	.010219	60 : 59	.007299	92 : 91	.004746	......	......

Thread Milling. — Screw threads may be milled by using either a single cutter or a multiple cutter. When a single cutter is used, the thread is milled as either the cutter or the work is traversed in a lengthwise direction, the method varying on different machines. Thus, on some designs, the cutter revolves in one position while the screw blank moves in a lengthwise direction as it slowly rotates, whereas, on other machines, the cutter is mounted on a carriage and is traversed parallel to the axis of the work. This single-cutter process is especially applicable to milling large screw threads of coarse pitch, long threads, and the heavier classes of work. For fine pitches and short threads, the multiple-cutter method is usually preferable as it is more rapid. The multiple-cutter has annular rows of teeth which do not lie in a helical path but are perpendicular to the cutter axis. When such a cutter is wide enough to cover the entire surface to be threaded, the thread is completed in approximately one revolution of the work, the cutter being fed in the full depth and then either the cutter or screw blank being moved axially a distance equal to the lead of the thread. If an exceptionally smooth thread were required, the work might be revolved two revolutions, the cutter being traversed a distance equal to twice the lead of the thread; during the first revolution, the thread would be rough-milled, and a light finishing cut taken during the second revolution.

Classes of Work for Thread Milling Machines. — Thread milling machines are used in preference to lathes or taps and dies for certain threading operations. There are four general reasons why a thread milling machine may be preferred: (1) Because the pitch of the thread is too coarse for cutting with a die; (2) because the milling process is more efficient than using a single-point tool in a lathe; (3) to secure a smoother and more accurate thread than would be obtained with a tap or die; (4) because the thread is so located relative to a shoulder or other surface that the milling method is superior, if not the only practicable way. A thread milling machine having a single cutter is especially adapted for coarse pitches, multiple-threaded screws, or any form or size of thread requiring the removal of a relatively large amount of metal, particularly if the pitch of the thread is large in proportion to the screw diameter, since the torsional strain due to the milling process is relatively small. While thread milling has little, if any, advantage over the lathe in regard to accuracy of lead, it gives a higher rate of production, and a thread is usually finished by means of a single passage of the cutter. The multiple-cutter type of thread milling machine frequently comes into competition with dies and taps, and especially self-opening dies and collapsing taps. The use of a multiple cutter is desirable when a thread must be cut close to a shoulder or to the bottom of a shallow recess, although the usefulness of the multiple cutter is not confined to shoulder work and "blind" holes.

Maximum Pitches of Die-cut Threads. — Dies of special design could be constructed for practically any pitch, if the screw blank were strong enough to resist the cutting strains and the size and cost of the die were immaterial; but, as a general rule, when the pitch is coarser than four or five threads per inch, the difficulty of cutting threads with dies increases rapidly, although in a few cases some dies are used successfully on screw threads having two or three threads per inch or less. Much depends upon the design of the die, the finish or smoothness required, and the relation between the pitch of the thread and the diameter of the screw. When the screw diameter is relatively small in proportion to the pitch, there may be considerable distortion due to the twisting strains set up when the thread is being cut. If the number of threads per inch is only one or two less than the standard number for a given diameter, a screw blank ordinarily will be strong enough to permit the use of a die.

Changing Pitch of Screw Thread Slightly. — A very slight change in the pitch of a screw thread may be necessary as, for example, when the pitch of a tap is increased a small amount to compensate for shrinkage in hardening. One method of obtaining slight variations in pitch is by means of a taper attachment. This attachment is set at an angle and the work is located at the same angle by adjusting the tailstock center. The result is that the tool follows an angular path relative to the movement of the carriage and, consequently, the pitch of the thread is increased slightly, the amount depending upon the angle to which the work and taper attachment are set. The cosine of this angle, for obtaining a given increase in pitch, equals the standard pitch (which would be obtained with the lathe used in the regular way) divided by the increased pitch necessary to compensate for shrinkage.

Example: — If the pitch of a ¾-inch U. S. standard screw is to be increased from 0.100 to 0.1005, the cosine of the angle to which the taper attachment and work should be set is found as follows:

$$\text{Cosine of required angle} = \frac{0.100}{0.1005} = 0.9950$$

which is the cosine of 5 degrees 45 minutes, nearly.

Oils and Compounds for Machining Operations

The principal functions of an oil or compound, as applied to metal-cutting tools, may be briefly summarized as follows: (1) To carry off the heat developed in separating the chip from the work, and thereby prevent a dangerous rise in temperature through the accumulation of such heat; (2) to lubricate the chip as it slides over the tool or the work, and thereby reduce the generation of frictional heat. (This applies more especially to the cutting of steel); (3) to improve the finish of the work and guard against rusting; (4) to increase the durability of the tool; (5) to flush out the "cutting area" and wash out the small chips.

There is a great diversity of practice in the use of oils and compounds for machining operations. Pure lard oil, mineral lard oil, pure mineral oil and soluble cutting compounds are the four general classes of "lubricants" used on the majority of the cutting tools. A careful investigation of practice in representative manufacturing plants goes to show that there are particular classes of work in which each of the commonly used classes of oils and compounds, gives good service.

Use of Lard Oil. — Pure lard oil is one of the most efficient lubricants available for use on metal-cutting tools in general, but owing to its high price, the use of undiluted lard oil is not generally recommended.

For some classes of automatic screw machine work some manufacturers use pure lard oil, but the so called "mineral lard oil" mixtures, ranging from 30 per cent of lard oil and 70 per cent of medium petroleum oil up to equal parts of lard oil and petroleum oil, have been found to give practically as good results as pure lard oil, and the mineral lard oil is cheaper. Furthermore, mineral lard oil has an advantage over pure lard oil in that it is more fluid and thus runs more freely to the tool and work; also, this mixed oil is not so likely to give trouble from gumming.

Prime lard oil is nearly colorless, having a pale yellow or greenish tinge. The solidifying point and other characteristics of the oil depend upon the temperature at which it was expressed, winter-pressed lard oil containing less solid constituents of the lard than that expressed in warm weather. The specific gravity should not exceed 0.916; it is sometimes increased by adulterants, such as cotton-seed and maize oils.

Navy Department Specifications for Lard Oil. — Lard oil must be of a good commercial quality, and must be purchased and inspected by weight; the number of pounds per gallon is to be determined by the specific gravity of the oil at 60 degrees F. multiplied by 8.33 pounds (the weight of a gallon of distilled water at the same temperature). Oil will not be accepted which contains a mixture of any mineral oil (10 per cent vegetable or fish oil is allowed); nor must the oil contain more acidity than the equivalent of 5 per cent of oleic acid, nor show a cold test above 55 degrees F. The specific gravity must not be above 0.92, nor below 0.90. With the view of reducing the cost of lubricants, some manufacturers have resorted to the use of pure petroleum oil on such machining operations as milling and turning.

Soluble Oils and Compounds. — The so called soluble oils and compounds which are used with water to form the white emulsions, are relatively cheap, the price per gallon varying according to the degree of dilution. Opinion is divided in regard to the advisability of using these water emulsions, but the following seems to be representative of experience in shops where the question has received most careful consideration. For milling, drilling, grinding and other operations where short chips are produced, these water emulsions give very satisfactory results. They flow freely, and as water has a higher specific heat than any of the oils, these emulsions are more efficient than oil for cooling. In cases where lubrication of the tool is also important, it is good practice to add more of the soluble oil or paste compound in mixing the emulsion than where cooling only is necessary; but under the most favorable conditions these emulsions have but a slight lubricating action, so that where long chips are produced, some kind of oil will give more satisfactory results.

When cutting emulsions are made by mixing paste or soluble oil with water, thorough mixing is important and the emulsion should be stable at all temperatures under which it is likely to be used. It is advisable to place in tall glass bottles test samples mixed with water in the proportions recommended, and observe whether there is any tendency for the constituents to settle out in layers. Any compounds which show this tendency should not be used. To secure the best results with mixtures of oil or emulsions which the consumer mixes in his own shop, care should be taken to follow the instructions given by the firm from which oils are purchased, as even slight deviations from the recommended practice will often seriously reduce the efficiency in cooling or lubricating.

Soda-water Mixtures. — Soda-water mixtures are still used to a considerable extent. While plain water is very effective as a cooling medium it has little lubricating value, and will rust any but the non-corroding metals. Soda-water mixtures were developed to avoid some of the objections to the use of plain water. These usually consist of a mixture of sal-soda (carbonate of soda) and water, to which is added some ingredient such as lard oil or soft soap to thicken or give body to the mixture and increase its lubricating value. A cheap mixture for turning, milling, etc., and one that has been extensively used, is made in the following proportions: 1 pound of sal-soda (carbonate of soda), 1 quart of lard oil, 1 quart of soft soap, and enough water to make 10 or 12 gallons. This mixture is boiled for one-half hour, preferably by passing a steam coil through it. If the solution should have an objectionable smell, this can be eliminated by adding about 2 pounds of unslaked lime.

For general use in drilling, milling and other operations for which a cutting compound may be used, the following formula produces a coolant that will be found to give satisfactory results: Take two galvanized iron buckets and fill one two-thirds full with No. 1 lard oil and the other two-thirds full with No. 1 screw cutting oil. To one pail add a pint measure of Proctor & Gamble's white soap chips and to the other pail add one-half pound of powdered soda. The contents of these two pails are then poured into a wooden barrel and thoroughly boiled with live steam which results in dissolving the soap and soda and thoroughly mixing it with the oil. After this has been done, the barrel is filled with cold water and thoroughly stirred to secure a uniform mixture, after which the contents are run into the storage tank of the central distributing station, from which pumps deliver it to circulating pipes leading to the machines in the factory.

Mineral Lard Oil Mixtures. — Mineral lard oil mixtures are used for automatic screw machine work and for numerous other machining operations, and the following mixtures have been found satisfactory: (1) Equal parts of lard oil and petroleum machine oil. (2) Lard oil, 30 per cent, and mineral oil, 70 per cent. (3) On Cleveland automatic screw machines for cutting steel of different grades, from 10 to 12 per cent pure lard oil and 88 to 90 per cent neutral mineral oil of about 32 degrees Bé. gravity. The fluidity of this mixture permits it to reach the extreme cutting point of the tool and it possesses sufficient viscosity to form the required film on the work. (4) One part lard oil and three parts Pennsylvania petroleum oil. (5) Mineral lard oil reduced with from 33⅓ to 66⅔ per cent kerosene or paraffin. (6) Ten gallons lard oil to one gallon kerosene. (7) For drilling, reaming and gear planing, 30 per cent lard oil and 70 per cent petroleum.

Oils and Compounds for Different Machining Operations. — A careful investigation of the oils and compounds used in representative machine building plants showed that the use of different oils and mixtures for the same classes of work is very common.

The lubricants listed in the following, for different machining operations and materials, have proved satisfactory. Where the use of "compound" is recommended, this means one of the emulsions made by mixing soluble oil or paste with water. Attention is called to the fact that no recommendations are made in the case of such materials as cast-iron, etc., where it is good practice to conduct the machining operation dry.

Automatic Screw Machine Work. — High-carbon and alloy steel: lard oil, mineral lard oil consisting of ten parts mineral lard oil and one part kerosene. Low-carbon steel: mineral lard oil, asphaltic base petroleum oil, paraffin oil and mineral lard oil in equal proportions. Wrought iron: mineral lard oil, asphaltic base petroleum oil. Brass: mineral lard oil, light paraffin oil. Bronze: mineral lard oil, asphaltic base petroleum oil. Copper: mineral lard oil. Aluminum: mineral lard oil, compound. Hard rubber: dry, compressed air. Fiber: dry. In all cases the mineral lard oil mixture may vary from equal parts of mineral oil and lard oil, to 70 per cent mineral oil and 30 per cent lard oil.

Turning. — High-carbon and alloy steel: mineral lard oil, lard oil, compound, paraffin oil of 28 degrees Bé. gravity, signal oil. Low-carbon steel: mineral lard oil, signal oil, petroleum oil, compound. Wrought iron: petroleum oil, compound. Brass: compound. Bronze: compound. Copper: lard oil or kerosene. Aluminum: kerosene. Monel metal: compound. Hard rubber: cold water.

Broaching. — High-carbon steel: neat's foot oil, compound. Low-carbon steel: neat's foot oil, compound. Wrought iron: neat's foot oil, compound. Malleable iron: neat's foot oil, compound. Bronze: neat's foot oil, compound.

A mixture which has been recommended for broaching steel contains 2½ pounds of soda ash and 3 gallons of mineral lard oil to 50 gallons of water. The soda ash and lard oil is mixed with 10 gallons of water, and then the remaining 40 gallons of water are added. When holes to be broached are of exceptional length, a good grade of oil is better than soda water or similar cutting lubricants, as the oil will cling to the cutting edges of the broach for a longer time.

Cutting off with Cold-saws. — High-carbon and alloy steel: mineral lard oil, mixture of two parts kerosene and one part signal oil, petroleum oil. Low-carbon steel: mineral lard oil, petroleum oil, compound, mixture of two parts kerosene and one part signal oil. Wrought iron: mineral lard oil, petroleum oil, compound. Bronze: compound.

Cutting off with Hacksaw Machines. — On all metals use soda-water mixture of two pounds soda to three gallons water.

Drilling. — High-carbon and alloy steel: mineral lard oil, lard oil. Low-carbon steel: mineral lard oil, petroleum oil, compound. Very hard steel: turpentine or mixture of turpentine and spirits of camphor, kerosene. Cast iron: compressed air. Wrought iron: mineral lard oil, petroleum oil, compound. Malleable iron: compound, petroleum oil. Brass: compound. Bronze: compound. Copper: lard oil, mineral lard oil, kerosene. Aluminum: kerosene, beeswax or tallow (rubbed on rotating drill after cutting two or three holes). Monel metal: compound. Glass: turpentine, turpentine and spirits of camphor, kerosene.

For drilling and milling operations, a good cutting compound is obtained by the following formula: Dissolve 2½ pounds soda ash in water and mix with 2 or 3 gallons lubricating oil. These constituents are thoroughly stirred to secure a uniform mixture and are then added to 40 gallons of water.

Gear Cutting. — High-carbon and alloy steel: lard oil, mineral lard oil. Low-carbon steel: mineral lard oil, petroleum oil, compound. Bronze: lard oil, mineral lard oil, compound. To make a lubricant for gear cutting with rotary cutters or hobs, the following formula produces an emulsion that gives excellent results: Stir together 3½ gallons mineral lard oil and 2¾ pounds sal-soda, and when thoroughly mixed add to one barrel of soft water. This compound does not thicken or leave a gummy residue.

Grinding. — High-carbon and alloy steel: compound. Low-carbon steel: compound. Cast iron: compound. Wrought iron: compound. Brass: compound. Bronze: compound. To make a grinding compound, the following formula is recommended: Dissolve 75 pounds soft soap and 30 pounds sal-soda in 15 gallons boiling water. Keep the mixture boiling and stir in 10 gallons lard oil. To this mixture add 1 ounce creosote oil as a disinfectant. When cool, mix 1 gallon of this stock solution with 3 gallons water to make the compound delivered to the wheel and work.

Milling. — High-carbon and alloy steel: mineral lard oil, compound, petroleum oil, paraffin oil, lard oil. Low-carbon steel: compound, mineral lard oil, petroleum oil. Cast iron: compressed air. Wrought iron: mineral lard oil, compound, petroleum oil, soda water. Brass: compound. Bronze: compound, kerosene. Copper: mineral lard oil, lard oil, kerosene. Aluminum: kerosene.

Reaming. — High-carbon and alloy steel: lard oil, mineral lard oil, sperm oil, mixture of lard oil and white lead of about the consistency of glue. Low-carbon steel: lard oil, mineral lard oil, compound, lard oil and white lead. Wrought iron: lard oil, mineral lard oil. Brass: compound. Copper: mineral lard oil, lard oil, kerosene. Aluminum: lard oil, kerosene. For reaming and tapping holes for stay-bolts, one of the largest locomotive shops in the country uses a compound made up according to the following formula: Mix 18 gallons good grade lard oil, 60 pounds tallow and 100 pounds white lead.

Tapping. — High-carbon and alloy steel: lard oil, mineral lard oil, mixture of 90 per cent tallow and 10 per cent graphite, cottonseed oil. Low-carbon steel: lard oil, mineral lard oil, tallow and graphite, lard oil and white lead mixed to the consistency of glue, cottonseed oil. Cast iron: lard oil, compound, white lead. Wrought iron: lard oil, compound. Malleable iron: lard oil, compound. Brass: lard oil, lard oil and white lead mixed to the consistency of glue. Copper: lard oil. Aluminum: lard oil, kerosene, beeswax or tallow (rubbed on rotating tap after each operation). Babbitt: lard oil, soap (packed into hole before tapping). Nuts: mineral lard oil compound.

Thread Cutting. — High-carbon and alloy steel: lard oil, mineral lard oil, cottonseed oil, grapeseed oil. Low-carbon steel: mineral lard oil, mixture of mineral lard oil and 25 to 50 per cent kerosene, turpentine and white lead, lard oil, cottonseed oil, grapeseed oil. Very hard steel: turpentine. Wrought iron: mineral lard oil, compound. Brass: compound. Bronze: mineral lard oil. Copper: mineral lard oil. Aluminum: kerosene. Monel metal: mixture of lard oil and white lead reduced to the consistency of glue.

Thread Milling. — High-carbon and alloy steel: mineral lard oil, paraffin oil. Low-carbon steel: mineral lard oil, paraffin oil of 28 degrees Bé. gravity.

Threading with Dies. — High-carbon and alloy steel: lard oil, sperm oil. Low-carbon steel: lard oil, mineral lard oil, compound. Wrought iron: lard oil, mineral lard oil. Brass: mineral lard oil, compound. Copper: lard oil, mineral lard oil.

Machining Aluminum. — Tools for turning, drilling, or planing aluminum should have acute cutting angles. After rough-grinding the tool, it is advisable to finish-sharpening the cutting edge on a grindstone or with an oilstone for fine work, as a keen edge is very essential. High speeds and comparatively light cuts are recommended. For drilling, a straight-fluted drill gives good results. If a twist drill is used, the cutting edges should be ground without front rake. The principal difficulty in the machining of aluminum and aluminum alloys is caused by the clogging of the chips, which become so firmly wedged between the teeth of milling cutters, counterbores and similar tools, that they cannot be removed with a brush. This difficulty can be largely avoided by the use of the right kind of cutting lubricant. Soap-water and kerosene are commonly employed. The latter enables a fine finish to be obtained, provided the cutting tool is properly ground. For milling flat surfaces, it is preferable to use end mills rather than cylindrical cutters. The cutting edges or corners of the cutter should be sharp instead of rounded. The mill will cut better if a high cutting speed and moderate feed is employed. The depth and width of the cut are of minor importance. A cutting speed of 325 feet per minute is practicable, and from 2½ to 4 cubic inches of aluminum can be removed per minute.

The following information on this subject was obtained from the Brown-Lipe Gear Co., where aluminum parts are machined in large quantities: For finishing bored holes, a bar equipped with cutters has been found more practicable than reamers. The cutters used for machining 4-inch holes have a clearance of from 20 to 22 degrees and no rake or slope on the front faces against which the chips bear. The roughing cutters for this work have a rather sharp nose, being ground on the point to a radius of about 3/32 inch, but for securing a smooth surface, the finishing tools are rounded to a radius of about ¾ inch. When machining aluminum on the profiler, a milling cutter about 3 or 4 inches in diameter is used, having spiral teeth, which are sharp-cornered on the end of the mill. The sharp-cornered tooth has given much better service at high speed than a mill having round-cornered teeth. When aluminum is machined with an inserted-tooth milling cutter, the teeth are inclined at a slight angle, not exceeding two degrees. The cutting speed, as well as the feed, for machining aluminum, is from 50 to 60 per cent faster than the speeds and feeds for cast iron. The lubricant used by this company is composed of one part "aqualine," and 20 parts water. This lubricant not only gives a smooth finish but preserves a keen cutting edge and enables tools to be used much longer without grinding. Formerly, a lubricant composed of one part of high-grade lard oil and one part of kerosene was used. This mixture costs approximately 30 cents per gallon, whereas the aqualine and water mixture now being used costs less than 4 cents per gallon, and has proved more effective than the lubricant formerly employed.

RUNNING, PRESSED AND SHRINKAGE FITS

Classes of Fits. — In ordinary machine construction, five classes of fits are commonly used: running fit; push fit; driving fit; forced fit; and shrinkage fit. The running fit, as the name implies, is employed when the parts must rotate; a push fit is not sufficiently free to rotate; the other classes referred to are used in assembling parts which must be held in fixed positions. The limits recommended by the Engineering Standards Committee of Great Britain for running fits are given in the table "Allowances and Tolerances for Running Fits." When the allowance is smaller than for a running fit, and a moderate pressure is required in assembling the parts, the term "push fit" is sometimes used. The tables "Allowances for Different Classes of Fits" are intended to cover average machine work. (See also "Grinding Limits for Cylindrical Parts" and "Forced Fit Allowances.") As the factors which determine the proper allowances vary considerably, the dimensions given in these tables may sometimes have to be increased or decreased. For example, the allowances for forced fits usually increase with the diameter to secure greater pressure, but in some shops the allowance is made practically the same for all diameters, the increased surface area of the larger sizes giving sufficient increase in pressure. For running fits, the allowances are also increased with the diameter, but may be varied according to the length of the bearing surface.

Allowances for Different Classes of Fits — 1

(Newall Engineering Co.)

Class	Nominal Diameters	Up to ½″	9⁄16″–1″	1 1⁄16″–2″	2 1⁄16″–3″	3 1⁄16″–4″	4 1⁄16″–5″
	Tolerances in Standard Holes*						
A	High Limit	+0.0002	+0.0005	+0.0007	+0.0010	+0.0010	+0.0010
	Low Limit	−0.0002	−0.0002	−0.0002	−0.0005	−0.0005	−0.0005
	Tolerance	0.0004	0.0007	0.0009	0.0015	0.0015	0.0015
B	High Limit	+0.0005	+0.0007	+0.0010	+0.0012	+0.0015	+0.0017
	Low Limit	−0.0005	−0.0005	−0.0005	−0.0007	−0.0007	−0.0007
	Tolerance	0.0010	0.0012	0.0015	0.0019	0.0022	0.0024
	Allowances for Forced Fits						
F	High Limit	+0.0010	+0.0020	+0.0040	+0.0060	+0.0080	+0.0100
	Low Limit	+0.0005	+0.0015	+0.0030	+0.0045	+0.0060	+0.0080
	Tolerance	0.0005	0.0005	0.0010	0.0015	0.0020	0.0020
	Allowances for Driving Fits						
D	High Limit	+0.0005	+0.0010	+0.0015	+0.0025	+0.0030	+0.0035
	Low Limit	+0.0002	+0.0007	+0.0010	+0.0015	+0.0020	+0.0025
	Tolerance	0.0003	0.0003	0.0005	0.0010	0.0010	0.0010
	Allowances for Push Fits						
P	High Limit	−0.0002	−0.0002	−0.0002	−0.0005	−0.0005	−0.0005
	Low Limit	−0.0007	−0.0007	−0.0007	−0.0010	−0.0010	−0.0010
	Tolerance	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
	Allowances for Running Fits†						
X	High Limit	−0.0010	−0.0012	−0.0017	−0.0020	−0.0025	−0.0030
	Low Limit	−0.0020	−0.0027	−0.0035	−0.0042	−0.0050	−0.0057
	Tolerance	0.0010	0.0015	0.0018	0.0022	0.0025	0.0027
Y	High Limit	−0.0007	−0.0010	−0.0012	−0.0015	−0.0020	−0.0022
	Low Limit	−0.0012	−0.0020	−0.0025	−0.0030	−0.0035	−0.0040
	Tolerance	0.0005	0.0010	0.0013	0.0015	0.0015	0.0018
Z	High Limit	−0.0005	−0.0007	−0.0007	−0.0010	−0.0010	−0.0012
	Low Limit	−0.0007	−0.0012	−0.0015	−0.0020	−0.0022	−0.0025
	Tolerance	0.0002	0.0005	0.0008	0.0010	0.0012	0.0013

Formulas for Determining Allowances

Class	High Limit	Low Limit	Class	High Limit	Low Limit
A	$+\sqrt{D}\times 0.0006$	$-\sqrt{D}\times 0.0003$	X	$-\sqrt{D}\times 0.00125$	$-\sqrt{D}\times 0.0025$
B	$+\sqrt{D}\times 0.0008$	$-\sqrt{D}\times 0.0004$	Y	$-\sqrt{D}\times 0.001$	$-\sqrt{D}\times 0.0018$
P	$-\sqrt{D}\times 0.0002$	$-\sqrt{D}\times 0.0006$	Z	$-\sqrt{D}\times 0.0005$	$-\sqrt{D}\times 0.001$

* Tolerance is provided for holes, which ordinary standard reamers can produce, in two grades, Classes A and B, the selection of which is a question for the user's decision and dependent upon the quality of the work required; some prefer to use Class A as working limits and Class B as inspection limits.

† Running fits, which are the most commonly required, are divided into three grades: Class X for engine and other work where easy fits are wanted; Class Y for high speeds and good average machine work; Class Z for fine tool work.

Allowances for Different Classes of Fits — 2*

Diameter, Inches	Running Fits	Push Fits
Up to ½	− 0.00075 to − 0.0015	− 0.00025 to − 0.00075
½ to 1	− 0.001 to − 0.002	− 0.0005 to − 0.001
1 to 2	− 0.0015 to − 0.0025	− 0.0005 to − 0.0015
2 to 3	− 0.0015 to − 0.003	− 0.0005 to − 0.0015
3 to 4	− 0.002 to − 0.0035	− 0.00075 to − 0.002
4 to 5	− 0.0025 to − 0.004	− 0.00075 to − 0.002
5 to 6	− 0.0025 to − 0.0045	− 0.00075 to − 0.002
Diameter, Inches	**Driving Fits**	**Forced Fits**
Up to ½	+ 0.0004 to + 0.0006	+ 0.0005 to + 0.001
½ to 1	+ 0.0005 to + 0.001	+ 0.001 to + 0.003
1 to 2	+ 0.00075 to + 0.002	+ 0.002 to + 0.004
2 to 3	+ 0.0015 to + 0.003	+ 0.003 to + 0.006
3 to 4	+ 0.002 to + 0.004	+ 0.005 to + 0.008
4 to 5	+ 0.002 to + 0.0045	+ 0.006 to + 0.010
5 to 6	+ 0.003 to + 0.005	+ 0.008 to + 0.012

* These allowances are intended for average machine work. If the bearings are long, the allowances for running fits may have to be increased.

Allowances and Tolerances for Running Fits

Nominal Diameter	Shaft			Allowance (Minimum Difference between Shaft and Hole)	Hole		
	Minimum Diameter	Tolerance (Difference)	Maximum Diameter		Minimum Diameter	Tolerance (Difference)	Maximum Diameter
¼	0.2495	0.0005	0.25	0.0005	0.2505	0.0003	0.2508
½	0.4993	0.0007	0.50	0.0007	0.5007	0.0007	0.5014
¾	0.7491	0.0009	0.75	0.0008	0.7508	0.0009	0.7517
1	0.9990	0.0010	1.00	0.0010	1.0010	0.0010	1.0020
1½	1.4988	0.0012	1.50	0.0012	1.5012	0.0013	1.5025
2	1.9985	0.0015	2.00	0.0015	2.0015	0.0015	2.0030
3	2.9982	0.0018	3.00	0.0018	3.0018	0.0017	3.0035
4	3.9980	0.0020	4.00	0.0020	4.0020	0.0020	4.0040
5	4.9980	0.0020	5.00	0.0020	5.0020	0.0020	5.0040
6	5.9975	0.0025	6.00	0.0025	6.0025	0.0025	6.0050
7	6.9975	0.0025	7.00	0.0025	7.0025	0.0025	7.0050
8	7.9975	0.0025	8.00	0.0025	8.0025	0.0025	8.0050
9	8.9970	0.0030	9.00	0.0030	9.0030	0.0030	9.0060
10	9.9970	0.0030	10.00	0.0030	10.0030	0.0030	10.0060
11	10.9970	0.0030	11.00	0.0030	11.0030	0.0030	11.0060
12	11.9970	0.0030	12.00	0.0030	12.0030	0.0030	12.0060

The above allowances and tolerances for running fits are recommended by the Engineering Standards Committee of Great Britain, for first-class work. For second- and third-class work, multiply the tolerances by 2 and 3, respectively. For extra fine quality of work, about ⅖ the above allowances for first-class work are recommended. The maximum diameter of the shaft is the nominal diameter in all grades of work.

Grinding Limits for Cylindrical Parts*

Running Fits for Shafts — Speeds Under 600 R.P.M. — Ordinary Working Conditions

Diameter, Inches	Limits, Inches
Up to ½	− 0.0005 to − 0.001
½ to 1	− 0.00075 to − 0.0015
1 to 2	− 0.0015 to − 0.0025
2 to 3½	− 0.002 to − 0.003
3½ to 6	− 0.0025 to − 0.004

Running Fits for Shafts — Speeds Over 600 R.P.M. — Heavy Pressure — Working Conditions Severe

Diameter, Inches	Limits, Inches
Up to ½	− 0.0005 to − 0.001
½ to 1	− 0.001 to − 0.002
1 to 2	− 0.002 to − 0.003
2 to 3½	− 0.003 to − 0.004
3½ to 6	− 0.004 to − 0.005

Sliding Fits for Shafts with Gears, Clutches, or Similar Parts which must be Free to Slide

Diameter, Inches	Limits, Inches
Up to ½	− 0.0005 to − 0.001
½ to 1	− 0.00075 to − 0.0015
1 to 2	− 0.0015 to − 0.0025
2 to 3½	− 0.002 to − 0.003
3½ to 6	− 0.0025 to − 0.004

Standard Fits for Light Service where Part is Keyed to Shaft and Clamped Endwise — No Fitting

Diameter, Inches	Limits, Inches
Up to ½	Standard to − 0.00025
½ to 3½	Standard to − 0.0005
3½ to 6	Standard to − 0.00075

Standard Fits with Play Eliminated — Parts should Assemble Readily — Some Fitting and Selecting may be Required

Diameter, Inches	Limits, Inches
Up to ½	Standard to + 0.00025
½ to 3½	Standard to + 0.0005
3½ to 6	Standard to + 0.00075

Driving Fits for Permanent Assembly of Parts so Located that Driving cannot be Done readily

Diameter, Inches	Limits, Inches
Up to ½	Standard to 0.00025
½ to 1	+ 0.00025 to + 0.0005
1 to 2	+ 0.0005 to + 0.00075
2 to 6	+ 0.0005 to + 0.001

Driving Fits for Permanent Assembly and Severe Duty and where there is Ample Room for Driving

Diameter, Inches	Limits, Inches
Up to 2	+ 0.0005 to + 0.001
2 to 3½	+ 0.00075 to + 0.00125
3½ to 6	+ 0.001 to + 0.0015

Forced Fits for Permanent Assembly and Very Severe Service — Hydraulic Press Used for Larger Parts

Diameter, Inches	Limits, Inches
Up to ½	+ 0.00075 to + 0.001
½ to 1	+ 0.001 to + 0.002
1 to 2	+ 0.002 to + 0.003
2 to 3½	+ 0.003 to + 0.004
3½ to 6	+ 0.004 to + 0.005

Shrinkage Fits — For Pieces to take Hardened Shells having a Thickness of ⅜ Inch or Less

Diameter, Inches	Limits, Inches
Up to 1	+ 0.00025 to + 0.0005
1 to 2	+ 0.0005 to + 0.00075
2 to 3½	+ 0.0005 to + 0.001
3½ to 6	+ 0.001 to + 0.0015

Shrinkage Fits — For Pieces to take Shells, etc., having a Thickness of More than ⅜ Inch

Diameter, Inches	Limits, Inches
Up to ½	+ 0.0005 to + 0.001
½ to 1	+ 0.001 to + 0.002
1 to 2	+ 0.002 to + 0.003
2 to 3½	+ 0.003 to + 0.004
3½ to 6	+ 0.004 to + 0.005

* Recommended by Brown & Sharpe Mfg. Co. for use under ordinary conditions. The hole is considered standard and the limits given are based upon the standard hole. The grinding limits for holes in hardened pieces are: Up to 2 inches diameter inclusive, standard to 0.0005 inch large; to 3½ inches diameter inclusive, standard to 0.00075 inch large; to 6 inches diameter inclusive, standard to 0.001 inch large.

Forced Fit Allowances — From Practice

For mild steel pins and shafts, pressed into cast-iron crank disks, except size marked * which had a cast steel crank disk.

Mean Diam. of Pin, Inches	Length of Fit, Inches	Mean Diam. of Hole, Inches	Total Allowance	Allowance per Inch of Diam.	Area of Fitted Surface, Square Inches	Volume within Fitted Surface, Cubic Inches	Pressure at Mid-position, Tons	Maximum Pressure, Tons
1.8798	6.125	1.8767	0.0031	0.0017	36.0	16.7	10	20
1.8819	6.125	1.8770	0.0042	0.0022	36.0	16.7	15	23
1.8774	4.375	1.8764	0.0010	0.0005	24.4	13.7	1	1
2.7455	4.500	2.7387	0.0068	0.0024	38.7	26.5	12	25
2.7465	4.500	2.7437	0.0028	0.0010	38.7	26.5	12	23
3.2610	5.000	3.2542	0.0068	0.0021	51.0	41.5	20	45
3.2625	5.000	3.2555	0.0070	0.0020	51.0	41.5	15	30
3.2670	5.000	3.2610	0.0060	0.0018	51.0	41.5	15	20
4.2505	6.000	4.2402	0.0103	0.0024	79.8	85.1	22	44
4.2388	6.625	4.2478	0.0091	0.0021	78.1	93.4	30	60
*4.2303	6.500	4.2224	0.0079	0.0019	95.8	91.0	60	125
5.9343	4.062	5.9216	0.0127	0.0022	75.7	112.2	16	25
5.9381	4.000	5.9252	0.0129	0.0022	74.4	110.4	18	35
5.9294	4.125	5.9194	0.0100	0.0017	76.7	113.8	15	25
6.8829	5.125	6.8697	0.0132	0.0020	110.7	190.1	20	42
6.8890	5.000	6.8785	0.0105	0.0015	108.0	185.9	22	45
6.8692	4.875	6.8550	0.0142	0.0021	104.8	180.4	35	65
7.8884	5.500	7.8730	0.0154	0.0020	135.9	267.3	32	64
7.8715	6.500	7.8575	0.0140	0.0018	160.5	315.9	25	50
7.8620	5.625	7.8460	0.0160	0.0020	138.2	272.8	40	80
8.9240	6.125	8.9050	0.0190	0.0021	170.8	378.9	45	68
8.9000	6.750	8.8848	0.0152	0.0017	188.4	419.9	47	96
8.8780	6.500	8.8669	0.0112	0.0013	180.7	401.0	45	92

Forced Fit Allowances — From Practice

For engine cranks of open-hearth steel castings, bored smooth and keyseated

Diameter of Hub	Diameter of Bore	Length of Hub	Allowance for Fit	Allowance per Inch Diameter	Pressure in Tons	Diameter of Hub	Diameter of Bore	Length of Hub	Allowance for Fit	Allowance per Inch Diameter	Pressure in Tons
4¼	2	2⅜	0.006	0.0030	3	8½	4½	5	0.012	0.0033	40
4½	2⅜	2⅜	0.007	0.0029	6	9½	4¾	5¼	0.015	0.0034	45
4½	2⅜	2½	0.008	0.0033	8	11	5½	5 9/16	0.015	0.0024	45
5½	2¾	2¾	0.008	0.0029	12	12	5¾	5¾	0.015	0.0026	55
6¼	3	3⅛	0.010	0.0033	18	14	6	6	0.015	0.0025	70
7¾	3½	3⅜	0.010	0.0028	25	14½	6¾	6¾	0.015	0.0022	75
8	3¾	3⅝	0.012	0.0034	35	15	7	7	0.015	0.0020	85
8½	4¼	4½	0.012	0.0028	35	...	...	...	....	....	...

Forced Fits. — This is the term used when a pin, shaft or other cylindrical part is forced into a hole of slightly smaller diameter by the use of a hydraulic press or other means. As a rule, forced fits are restricted to parts of small and medium size, while shrinkage fits have no such limitations and are especially applicable when a maximum "grip" is desired, or when (as in the construction of ordnance) accurate results as to the intensity of stresses produced in the parts united, are required. The proper allowance for a forced fit depends upon the mass of metal surrounding the hole, the size of the work, the kind and quality of the material of which the parts are composed and the smoothness and accuracy of the pin and bore. When a pin or other part is pressed into a hole a second time, the allowance for a given tonnage should be diminished somewhat, because the surface of the bore is smoother and the metal more compact. The pressure required in assembling a forced fit will also vary for cast hubs of the same size, if they are not uniform in hardness. Then there is the personal factor which is much in evidence in work of this kind; hence, data and formulas for forced fit allowances must be general in their application.

Allowance for Forced Fits. — The allowance per inch of diameter usually ranges from 0.001 inch to 0.0025 inch, 0.0015 being a fair average. Ordinarily the allowance per inch decreases as the diameter increases; thus the total allowance for a diameter of 2 inches might be 0.004 inch, whereas for a diameter of 8 inches the total allowance might not be over 0.009 or 0.010 inch. The parts to be assembled by forced fits are usually made cylindrical, although sometimes they are slightly tapered. The advantages of the taper form are that the possibility of abrasion of the fitted surfaces is reduced; that less pressure is required in assembling; and that the parts are more readily separated when renewal is required. On the other hand, the taper fit is less reliable, because if it loosens, the entire fit is free with but little axial movement. Some lubricant, such as white lead and lard oil mixed to the consistency of paint, should be applied to the pin and bore before assembling, to reduce the tendency of abrasion.

Pressure for Forced Fits. — The pressure required for assembling cylindrical parts depends not only upon the allowance for the fit, but also upon the area of the fitted surfaces, the pressure increasing in proportion to the distance that the inner member is forced in. The approximate ultimate pressure in pounds can be determined by the use of the following formula in conjunction with the accompanying

Pressure Factors

Diameter, Inches	Pressure Factor	Diameter, Inches	Pressure Factor	Diameter, Inches	Pressure Factor	Diameter, Inches	Pressure Factor	Diameter, Inches	Pressure Factor
1	500	3½	132	6	75	9	48.7	14	30.5
1¼	395	3¾	123	6¼	72	9½	46.0	14½	29.4
1½	325	4	115	6½	69	10	43.5	15	28.3
1¾	276	4¼	108	6¾	66	10½	41.3	15½	27.4
2	240	4½	101	7	64	11	39.3	16	26.5
2¼	212	4¾	96	7¼	61	11½	37.5	16½	25.6
2½	189	5	91	7½	59	12	35.9	17	24.8
2¾	171	5¼	86	7¾	57	12½	34.4	17½	24.1
3	156	5½	82	8	55	13	33.0	18	23.4
3¼	143	5¾	78	8½	52	13½	31.7	....	....

table of "Pressure Factors." Assuming that A = area of surface in contact in "fit"; a = total allowance in inches; P = ultimate pressure required, in tons; F = pressure factor based upon assumption that the diameter of the hub is twice the diameter of the bore, that the shaft is of machine steel, and that the hub is of cast iron:

$$P = \frac{A \times a \times F}{2}$$

Example: — What will be the approximate pressure required for forcing a 4-inch machine steel shaft having an allowance of 0.0085 inch, into a cast-iron hub 6 inches long?

$A = 4 \times 3.1416 \times 6 = 75.39$ square inches;
F for a diameter of 4 inches = 115 (see table of "Pressure Factors").

Then:

$$P = \frac{75.39 \times 0.0085 \times 115}{2} = 37 \text{ tons, approximately.}$$

Allowance for Given Pressure. — By transposing the preceding formula, the approximate allowance for a required ultimate tonnage can be determined. Thus, $a = \frac{2P}{AF}$. The average ultimate pressure in tons commonly used ranges from 7 to 10 times the diameter in inches. Assuming that the diameter of a machine steel shaft is 4 inches and an ultimate pressure of about 30 tons is desired for forcing it into a cast-iron hub having a length of 5½ inches, what should be the allowance?

$A = 4 \times 3.1416 \times 5\frac{1}{2} = 69$ square inches;
F for a diameter of 4 inches = 115.

Then:

$$a = \frac{2 \times 30}{69 \times 115} = 0.0075 \text{ inch.}$$

Shrinkage Fits. — General practice seems to favor a smaller allowance for shrinkage fits than for forced fits, although in many shops the allowances are practically the same in each case, and for some classes of work, shrinkage allowances exceed those for forced fits. In any case, the shrinkage allowance varies to a great extent with the form and construction of the part which has to be shrunk into place. The thickness or amount of metal around the hole is the most important factor. The way in which the metal is distributed also has an influence on the results. Shrinkage allowances for locomotive driving wheel tires adopted by the American Railway Master Mechanics Association are as follows:

Center diameter, inches..........	38	44	50	56	62	66
Allowance, inches............ ...	0.040	0.047	0.053	0.060	0.066	0.070

Whether parts are to be assembled by forced or shrinkage fits depends upon conditions. For example, to press a tire over its wheel center, without heating, would ordinarily be a rather awkward and difficult job. On the other hand, pins, etc., are easily and quickly forced into place with a hydraulic press and there is the additional advantage of knowing the exact pressure required in assembling, whereas there is more or less uncertainty connected with a shrinkage fit, unless the stresses are calculated. Tests to determine the difference in the quality of shrinkage and forced fits showed that the resistance of a shrinkage fit to slippage was, for an axial pull, 3.66 times greater than that of a forced fit, and in rotation or torsion, 3.2 times greater. In each comparative test, the dimensions and allowances were the same.

Allowances for Shrinkage Fits. — The most important point to consider when calculating shrinkage fits is the stress in the hub at the bore, which depends chiefly upon the shrinkage allowance. If the allowance is excessive, the elastic limit of the material will be exceeded and permanent set will occur, or, in extreme cases, the ultimate strength of the metal will be exceeded and the hub will burst. The intensity of the grip of the fit and the resistance to slippage depends mainly upon the thickness of the hub; the greater the thickness, the stronger the grip, and *vice versa*. Assuming the modulus of elasticity for steel to be 30,000,000, and for cast iron, 15,000,000, the shrinkage allowance per inch of nominal diameter can be determined by the following formula, in which A = allowance per inch of diameter; T = true tangential tensile stress at inner surface of outer member; C = factor taken from one of the accompanying tables, "Factors for Calculating Shrinkage Fit Allowances." For a cast-iron hub and steel shaft:

$$A = \frac{T(2+C)}{30,000,000} \tag{1}$$

When both hub and shaft are of steel:

$$A = \frac{T(1+C)}{30,000,000} \tag{2}$$

If the shaft is solid, the factor C is taken from Table 1; if it is hollow and the hub is of steel, factor C is taken from Table 2; if it is hollow and the hub is of cast iron, the factor is taken from Table 3.

Table 1. Factors for Calculating Shrinkage Fit Allowances

Values of Ratio C for solid steel shafts of nominal diameter D_1, and hubs of steel or cast iron of nominal external and internal diameters D_2 and D_1, respectively.

Ratio of Diameters $\frac{D_2}{D_1}$	Steel Hub	Cast-iron Hub	Ratio of Diameters $\frac{D_2}{D_1}$	Steel Hub	Cast-iron Hub
1.5	0.227	0.234	2.8	0.410	0.432
1.6	0.255	0.263	3.0	0.421	0.444
1.8	0.299	0.311	3.2	0.430	0.455
2.0	0.333	0.348	3.4	0.438	0.463
2.2	0.359	0.377	3.6	0.444	0.471
2.4	0.380	0.399	3.8	0.450	0.477
2.6	0.397	0.417	4.0	0.455	0.482

Example 1: A steel crank web 15 inches outside diameter is to be shrunk on a 10-inch solid steel shaft. Required the allowance per inch of shaft diameter to produce a maximum tensile stress in the crank of 25,000 pounds per square inch, assuming the stresses in the crank to be equivalent to those in a ring of the diameter given.

The ratio of the external to the internal diameters equals $15 \div 10 = 1.5$; $T = 25,000$ pounds; from Table 1, $C = 0.227$. Substituting in Formula (2):

$$A = \frac{25,000 \times (1 + 0.227)}{30,000,000} = 0.001 \text{ inch.}$$

Example 2: Find the allowance per inch of diameter for a 10-inch shaft having a 5-inch axial hole through it, other conditions being the same as in Example 1.

The ratio of external to internal diameters of the hub equals 15 ÷ 10 = 1.5, as before, and the ratio of external to internal diameters of the shaft equals 10 ÷ 5 = 2. From Table 2, we find that factor $C = 0.455$; $T = 25{,}000$ pounds. Substituting these values in Formula (2):

$$A = \frac{25{,}000 \times (1 + 0.455)}{30{,}000{,}000} = 0.0012 \text{ inch.}$$

The increase in allowance, as compared with Example 1, is due to the fact that the hollow shaft is more compressible.

Table 2. Factors for Calculating Shrinkage Fit Allowances

Values of Ratio C for hollow steel shafts of external and internal diameters D_1 and D_0, respectively, and steel hubs of nominal external diameter D_2.

$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C	$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C	$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C
	2.0	0.455		2.0	0.760		2.0	0.876
1.5	2.5	0.357	2.4	2.5	0.597	3.4	2.5	0.689
	3.0	0.313		3.0	0.523		3.0	0.602
	3.5	0.288		3.5	0.481		3.5	0.555
	2.0	0.509		2.0	0.793		2.0	0.888
1.6	2.5	0.400	2.6	2.5	0.624	3.6	2.5	0.698
	3.0	0.350		3.0	0.546		3.0	0.611
	3.5	0.322		3.5	0.502		3.5	0.562
	2.0	0.599		2.0	0.820		2.0	0.900
1.8	2.5	0.471	2.8	2.5	0.645	3.8	2.5	0.707
	3.0	0.412		3.0	0.564		3.0	0.619
	3.5	0.379		3.5	0.519		3.5	0.570
	2.0	0.667		2.0	0.842		2.0	0.909
2.0	2.5	0.524	3.0	2.5	0.662	4.0	2.5	0.715
	3.0	0.459		3.0	0.580		3.0	0.625
	3.5	0.422		3.5	0.533		3.5	0.576
	2.0	0.718		2.0	0.860		.....	......
2.2	2.5	0.565	3.2	2.5	0.676	.....	.....	......
	3.0	0.494		3.0	0.591		.....	......
	3.5	0.455		3.5	0.544		.....	......

Example 3: If the crank web in Example 1 is of cast iron and 4000 pounds per square inch is the maximum tensile stress in the hub, what is the allowance per inch of diameter?

$$\frac{D_2}{D_1} = 1.5; \quad T = 4000.$$

In Table 1, we find that $C = 0.234$. Substituting in Formula (1), for cast-iron hubs, $A = 0.0003$ inch, which, owing to the lower tensile strength of cast iron, is about one-third the shrinkage allowance in Example 1, although the stress is two-thirds of the elastic limit.

Temperatures for Shrinkage Fits. — The temperature to which the outer member in a shrinkage fit should be heated for clearance in assembling the parts depends on the total expansion required and on the coefficient α of linear expansion of the metal (that is, the increase in length of any section of the metal in any direction for an increase in temperature of 1 degree F.). The total expansion in diameter which is required consists of the total allowance for shrinkage and an added amount for clearance. The value of the coefficient α is, for nickel-steel, 0.000007; for steel

Table 3. Factors for Calculating Shrinkage Fit Allowances

Values of Ratio C for hollow steel shafts and cast-iron hubs. Notation as in Table 2.

$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C	$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C	$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C
1.5	2.0	0.468	2.4	2.0	0.798	3.4	2.0	0.926
	2.5	0.368		2.5	0.628		2.5	0.728
	3.0	0.322		3.0	0.549		3.0	0.637
	3.5	0.296		3.5	0.506		3.5	0.587
1.6	2.0	0.527	2.6	2.0	0.834	3.6	2.0	0.941
	2.5	0.414		2.5	0.656		2.5	0.740
	3.0	0.362		3.0	0.574		3.0	0.647
	3.5	0.333		3.5	0.528		3.5	0.596
1.8	2.0	0.621	2.8	2.0	0.864	3.8	2.0	0.953
	2.5	0.488		2.5	0.679		2.5	0.749
	3.0	0.427		3.0	0.594		3.0	0.656
	3.5	0.393		3.5	0.547		3.5	0.603
2.0	2.0	0.696	3.0	2.0	0.888	4.0	2.0	0.964
	2.5	0.547		2.5	0.698		2.5	0.758
	3.0	0.479		3.0	0.611		3.0	0.663
	3.5	0.441		3.5	0.562		3.5	0.610
2.2	2.0	0.753	3.2	2.0	0.909	.....	.....	......
	2.5	0.592		2.5	0.715		.....	......
	3.0	0.518		3.0	0.625		.....	......
	3.5	0.477		3.5	0.576		.....	......

in general, 0.0000065; for cast iron, 0.0000062. As an example, take an outer member of steel to be expanded 0.005 inch per inch of internal diameter, 0.001 being the shrinkage allowance and the remainder for clearance. Then:

$$\alpha \times t^\circ = 0.005$$

$$t = \frac{0.005}{0.0000065} = 769 \text{ degrees F.}$$

The value t is the number of degrees F. which the temperature of the member must be raised above that of the room temperature.

Wheel and Axle Press Fits

Axle						Wheel							Assembling Pressure, Tons
Size of Journal, Inches	Diameter Wheel Fit		Cutting Speed, Ft. per Minute	Depth of Cut	Feed, Inches	Diam., Inches	Material	Diameter of Bore		Cutting Speed, Ft. per Minute	Feed, Inches	Cuts per Wheel	
	Outside End	Inside End						Outside End	Inside End				
4¼ × 8	5.708 5.704	5.706 5.704	40	1/32	1/8	33	C. I.	5.700 5.702	5.700 5.702	25	3/16	2	50
3¾ × 7	5.341 5.341	5.345 5.345	40	1/32	1/8	33	C. I.	5.332 5.332	5.336 5.331	25	5/16	2	65
3¾ × 7	5.339 5.339	5.338 5.338	40	1/32	1/8	33	C. I.	5.333 5.333	5.333 5.333	25	5/16	2	40
5½ × 10	5.480 5.480	5.480 5.480	25	1/32	1/8	30	S. T.	5.469 5.467	5.465 5.470	25	3/16	2	75*
4¼ × 8	5.763 5.763	5.761 5.763	50	1/32	1/16	33	C. I.	5.7445 5.7445	5.743 5.743	18	1/8	2	61
4¼ × 8	5.769 5.775	5.770 5.769	50	1/32	1/16	33	C. I.	5.756 5.756	5.7545 5.7545	18	1/8	2	62
5½ × 10	6.873 6.8715	6.872 6.869	50	1/32	1/16	33	C. I.	6.851 6.851	6.851 6.851	18	1/8	2	75
5½ × 10	6.969 6.969	6.972 6.971	50	1/32	1/16	33	C. I.	6.956 6.956	6.9545 6.9545	18	1/8	2	75
4¼ × 8	5.748 5.743	5.749 5.749	25	1/32	3/32	36	C. I.	5.740 5.740	5.740 5.740	16	9/16	2	57
5 × 9	6.371 6.371	6.380 6.382	25	1/32	3/32	33	C. I.	6.362 6.362	6.362 6.362	18	9/16	2	65
3¾ × 7	5.355 5.355	5.356 5.359	25	1/32	3/32	33	C. I.	5.352 5.352	5.348 5.348	18	3/32	1	60
3¾ × 7	5.388 5.390	5.395 5.398	25	1/32	3/32	33	C. I.	5.383 5.383	5.383 5.383	18	9/16	2	60
4¼ × 8	5.787 5.783	5.797 5.792	35	1/32	5/32	36	S.	5.785 5.785	5.785 5.785	18	1/16	2	119*
4¼ × 8	5.774 5.774	5.777 5.776	35	1/32	5/32	36	S.	5.767 5.771	5.775 5.776	18	1/16	2	51
5 × 9	6.436 6.438	6.432 6.433	35	1/32	5/32	33	C. I.	6.401 6.404	6.412 6.410	24	1/8	2	85
3¾ × 7	5.336 5.345	5.341 5.345	35	1/32	5/32	33	C. I.	5.333 5.333	5.337 5.337	24	1/8	2	64

* Wheel removed–pressure excessive. C. I. = cast iron; S. T. = steel tired; S. = steel. (The table represents practice in four railroad shops. Two dimensions are given for each axle or bore diameter, these dimensions being measured at right angles to each other.)

ALLOWANCES AND TOLERANCES FOR SCREW THREADS AND GAGES

Thread Standards of National Screw Thread Commission. — The National Screw Thread Commission was authorized by Congress July, 1918, for the purpose of ascertaining and establishing screw thread standards for the use of manufacturers and various branches of the Federal Government. The aim of the Commission in establishing thread systems has been to eliminate all unnecessary sizes and to utilize, as far as possible, present predominating sizes.

Definitions of Terms. — Some of the terms used in the report of the Commission will be defined to avoid any misunderstanding, especially in regard to certain of the terms which have not been generally used heretofore in connection with screw threads.

Major Diameter: The largest diameter of the thread on the screw or nut. The term "major diameter" replaces the term "outside diameter" as applied to the thread of a screw and also the term "full diameter" as applied to the thread of a nut.

Minor Diameter: The smallest diameter of the thread on the screw or nut. The term "minor diameter" replaces the term "core diameter" as applied to the thread of a screw and also the term "inside diameter" as applied to the thread of a nut.

Pitch Diameter: On a straight screw thread, the diameter of an imaginary cylinder which would pass through the threads at such points as to make the width of the threads and the width of the spaces cut by the surface of the cylinder equal.

Angle of Thread: The angle included between the sides of the thread measured in an axial plane.

Helix Angle: The angle made by the helix of the thread at the pitch diameter, with a plane perpendicular to the axis.

Crest: The top surface joining the two sides of a thread.

Root: The bottom surface joining the sides of two adjacent threads.

Base of Thread: The bottom section of the thread or the greatest section between the two adjacent roots.

Length of Engagement: The length of contact between two mating parts, measured axially.

Depth of Engagement: The depth of thread in contact between two mating parts, measured radially.

Tolerance: A definite difference in the dimensions prescribed in order to permit of variations in manufacture. The "extreme tolerance" is the maximum and minimum tolerance permitted by the designer, the limits of which are to be placed on the drawings; it is the net tolerance as affected by the master gage tolerance. The "net tolerance" is the tolerance limits within which the product is ordinarily passed by the master gages; it is the extreme tolerance as affected by the master gage increment.

Basic: The theoretical or nominal standard size from which all variations are made.

Crest Clearance: Defined on a screw form as the space between the top of a thread and the root of its mating thread.

Neutral Zone (Allowance): A space between the mating parts which must not be encroached upon.

Gage Increment: A predetermined allowance by which the net tolerance of the product is increased for gaging purposes.

Limits: Dimensions, the extremes of which are prescribed, to provide for variations in fit and workmanship.

Table 1. National Coarse Thread Series

Numbered and Fractional Sizes	Number of Threads per Inch	Basic Diameters			Metric Equivalent of Major Diam., mm.	Pitch, Inches	Depth of Thread, Inches
		Major Diam., Inches	Pitch Diam., Inches	Minor Diam., Inches			
1	64	0.073	0.0629	0.0527	1.854	0.01562	0.0101
2	56	0.086	0.0744	0.0628	2.184	0.01785	0.0116
3	48	0.099	0.0855	0.0719	2.515	0.02083	0.0135
4	40	0.112	0.0958	0.0795	2.845	0.02500	0.0162
5	40	0.125	0.1088	0.0925	3.175	0.02500	0.0162
6	32	0.138	0.1177	0.0974	3.505	0.03125	0.0203
8	32	0.164	0.1437	0.1234	4.166	0.03125	0.0203
10	24	0.190	0.1629	0.1359	4.826	0.04166	0.0271
12	24	0.216	0.1889	0.1619	5.486	0.04166	0.0271
1/4	20	0.2500	0.2175	0.1850	6.350	0.05000	0.0325
5/16	18	0.3125	0.2764	0.2403	7.938	0.05555	0.0361
3/8	16	0.3750	0.3344	0.2938	9.525	0.06250	0.0406
7/16	14	0.4375	0.3911	0.3447	11.113	0.07142	0.0464
1/2	13	0.5000	0.4500	0.4001	12.700	0.07692	0.0500
9/16	12	0.5625	0.5084	0.4542	14.288	0.08333	0.0541
5/8	11	0.6250	0.5660	0.5069	15.875	0.09090	0.0590
3/4	10	0.7500	0.6850	0.6201	19.050	0.10000	0.0650
7/8	9	0.8750	0.8028	0.7307	22.225	0.11111	0.0722
1	8	1.0000	0.9188	0.8376	25.400	0.12500	0.0812
1 1/8	7	1.1250	1.0322	0.9394	28.575	0.14285	0.0928
1 1/4	7	1.2500	1.1572	1.0644	31.750	0.14285	0.0928
1 1/2	6	1.5000	1.3917	1.2835	38.100	0.16666	0.1083
1 3/4	5	1.7500	1.6201	1.4902	44.450	0.20000	0.1299
2	4 1/2	2.0000	1.8557	1.7113	50.800	0.22222	0.1443
2 1/4	4 1/2	2.2500	2.1057	1.9613	57.150	0.22222	0.1443
2 1/2	4	2.5000	2.3376	2.1752	63.500	0.25000	0.1624
2 3/4	4	2.7500	2.5876	2.4252	69.850	0.25000	0.1624
3	4	3.0000	2.8376	2.6752	76.200	0.25000	0.1624

National Form of Thread. — The form of thread profile recommended by the Commission is known as the "National Form" and is the same as the U. S. standard or Sellers profile. The National form is intended for all screw thread work except when otherwise specified for special purposes. A clearance is to be provided at the minor diameter of the nut by removing the thread form at the crest by an amount equal to from 1/8 to 1/4 of the basic thread depth. A clearance at the major diameter of the nut is to be provided by decreasing the depth of the truncation triangle by an amount equal to from 1/3 to 2/3 of its theoretical value.

National Coarse Thread Series. — Specifications for what is known as the National Coarse Thread Series are given in Table 1, which contains the numbered and fractional sizes and the basic diameters. This series contains certain sizes known previously as the U. S. standard threads and also certain sizes known as the A. S. M. E. machine screw threads. There are included in the National Coarse

Table 2. National Fine Thread Series

Numbered and Fractional Sizes	Number of Threads per Inch	Basic Diameters			Metric Equivalent of Major Diam., mm.	Pitch, Inches	Depth of Thread, Inches
		Major Diam., Inches	Pitch Diam., Inches	Minor Diam., Inches			
0	80	0.060	0.0519	0.0438	1.524	0.01250	0.00812
1	72	0.073	0.0640	0.0550	1.854	0.01388	0.00902
2	64	0.086	0.0759	0.0657	2.184	0.01562	0.01014
3	56	0.099	0.0874	0.0758	2.515	0.01785	0.01160
4	48	0.112	0.0985	0.0849	2.845	0.02083	0.01353
5	44	0.125	0.1102	0.0955	3.175	0.02272	0.01476
6	40	0.138	0.1218	0.1055	3.506	0.02500	0.01624
8	36	0.164	0.1460	0.1279	4.166	0.02777	0.01804
10	32	0.190	0.1697	0.1494	4.826	0.03125	0.02030
12	28	0.216	0.1928	0.1696	5.486	0.03571	0.02319
1/4	28	0.2500	0.2268	0.2036	6.350	0.03571	0.02319
5/16	24	0.3125	0.2854	0.2584	7.938	0.04166	0.02706
3/8	24	0.3750	0.3479	0.3209	9.525	0.04166	0.02706
7/16	20	0.4375	0.4050	0.3725	11.113	0.05000	0.03248
1/2	20	0.5000	0.4675	0.4350	12.700	0.05000	0.03248
9/16	18	0.5625	0.5264	0.4903	14.288	0.05555	0.03608
5/8	18	0.6250	0.5889	0.5528	15.875	0.05555	0.03608
3/4	16	0.7500	0.7094	0.6688	19.050	0.06250	0.04060
7/8	14	0.8750	0.8286	0.7822	22.225	0.07142	0.04640
1	14	1.0000	0.9536	0.9072	25.400	0.07142	0.04640
1 1/8	12	1.1250	1.0709	1.0167	28.575	0.08333	0.05413
1 1/4	12	1.2500	1.1959	1.1417	31.750	0.08333	0.05413
1 1/2	12	1.5000	1.4459	1.3917	38.100	0.08333	0.05413
1 3/4	12	1.7500	1.6959	1.6417	44.450	0.08333	0.05413
2	12	2.0000	1.9459	1.8917	50.800	0.08333	0.05413
2 1/4	12	2.2500	2.1959	2.1417	57.150	0.08333	0.05413
2 1/2	12	2.5000	2.4459	2.3917	63.500	0.08333	0.05413
2 3/4	12	2.7500	2.6959	2.6417	69.850	0.08333	0.05413
3	10	3.0000	2.9350	2.8701	76.200	0.10000	0.06495

Thread Series only those sizes which are essential. The National coarse threads are recommended for general use in engineering work, in machine construction where conditions are favorable to the use of bolts, screws and other threaded components where quick and easy assembly of the parts is desired, and for all work where conditions do not require the use of fine pitch threads. The National (U. S. standard) form of thread profile is used for the coarse series.

National Fine Thread Series. — The National Fine Thread Series contains certain sizes known previously as the S. A. E. threads and also certain sizes known as the A. S. M. E. machine screw sizes. The fine thread series is recommended for general use in automotive and aircraft work, for use where the design requires both strength and reduction in weight, and where special conditions require a fine thread, such as, for instance, on large sizes where sufficient force cannot be secured to set properly a screw or bolt of coarse pitch, by exerting on an ordinary wrench the strength of a man. The specifications for the fine thread series are given in Table 2. The National (U. S. standard) form of thread profile is used for the fine series.

National Fire Hose Coupling Threads. — The National Fire Hose Threads are intended for all couplings and hydrant connections for fire protection systems and for all other purposes where hose couplings and connections are required in sizes between 2½ inches and 4½ inches in diameter. The basic sizes and dimensions and the form of thread profile correspond to those recommended by the National Fire Protection Association and by the Bureau of Standards. The specifications are similar to those given in the table, page 1165, "Fire Hose Connections — National Standard." Specifications for the special form of thread for fire hose couplings are given in the table referred to.

National Hose Coupling Threads. — The National Hose Coupling Thread is intended for all couplings and connections for sizes between ¾ inch and 2 inches in diameter. The form of thread profile is the same as the National (U. S. standard) form. The sizes and basic dimensions are specified in Table 3.

Manufacturing Specifications for National Screw Threads. — The National coarse threads, fine threads, fire hose coupling threads, and hose coupling threads are to be produced in accordance with the following specifications covering the classification of screw thread fits and the tolerances. It is recommended that all specifications be so written that the qualities in the product desired shall be stated in definite terms of known measurable standards and correctly defined by the largest tolerance limits compatible with the satisfactory operation or performance of the articles or material for the purpose intended. To this end every factor involved in the acceptability of the manufactured product required should be comparable within specified limits with a known measurable standard. Every specification should be so concise that no dispute regarding the limiting lines of acceptance can arise.

The specifications previously referred to, covering classification and tolerances, are intended for the sole purpose of establishing the physical dimensions of screw thread products. While tolerances are given for various grades of fits or work-

Table 3. National Hose Coupling Threads

Nominal Size, Inches	Threads per Inch	Pitch, Inches	Depth of Thread, Inches	Major Diameter		Pitch Diam., Inches	Minor Diam., Inches	Allowance, Inches
				mm.	Inches			
Basic Minimum Coupling Dimensions								
¾	11½	0.08696	0.0565	27.242	1.0725	1.0160	0.9595	0.01
1	11½	0.08696	0.0565	33.150	1.3051	1.2486	1.1922	0.01
1¼	11½	0.08696	0.0565	41.908	1.6499	1.5934	1.5369	0.01
1½	11½	0.08696	0.0565	47.976	1.8888	1.8323	1.7759	0.01
2	11½	0.08696	0.0565	60.015	2.3628	2.3063	2.2498	0.01
Basic Maximum Nipple Dimensions								
¾	11½	0.08696	0.0565	26.988	1.0625	1.0060	0.9495	0.01
1	11½	0.08696	0.0565	32.896	1.2951	1.2386	1.1822	0.01
1¼	11½	0.08696	0.0565	41.654	1.6399	1.5834	1.5269	0.01
1½	11½	0.08696	0.0565	47.722	1.8788	1.8223	1.7659	0.01
2	11½	0.08696	0.0565	59.761	2.3528	2.2963	2.2398	0.01

manship, it is not intended in any way to specify or limit the material or physical qualities required by the user. These specifications as to material and physical qualities must be established according to individual needs. Here again the importance of stating these requirements on concise and definite specifications is emphasized.

Classification of Screw Thread Fits. — The National Screw Thread Commission established for general use, unless otherwise specified, four distinct classes of screw thread fits (with certain subdivisions) for the purpose of insuring the interchangeable manufacture of screw thread parts throughout the country. The examples referred to in connection with each of the following classes of fits are for purposes of illustration only, as it was not the intention of the Commission arbitrarily to place a general class or grade of work in a specific class of fit. The tolerances and dimensions for each class of fit are given in Tables 4 to 15, inclusive.

Table 4. Allowances and Tolerances for Loose Fit
(Screws, Nuts and Gages)

No. of Threads per Inch	Allowances, Inches	Extreme or Drawing Pitch Diameter Tolerances, Inches	Master Gage Tolerances			Net Pitch Diameter Tolerances, Inches
			Diameter, Inches	Lead,* Inches	½ Angle	
80	0.0007	0.0024	0.0002	±0.0002	±0° 30′	0.0020
72	0.0007	0.0025	0.0002	±0.0002	±0 30	0.0021
64	0.0007	0.0026	0.0002	±0.0002	±0 30	0.0022
56	0.0008	0.0028	0.0002	±0.0002	±0 30	0.0024
48	0.0009	0.0031	0.0002	±0.0002	±0 30	0.0027
44	0.0009	0.0032	0.0002	±0.0002	±0 30	0.0028
40	0.0010	0.0034	0.0002	±0.0002	±0 20	0.0030
36	0.0011	0.0036	0.0002	±0.0002	±0 20	0.0032
32	0.0011	0.0038	0.0002	±0.0002	±0 20	0.0034
28	0.0012	0.0043	0.0003	±0.0002	±0 15	0.0037
24	0.0013	0.0046	0.0003	±0.0002	±0 15	0.0040
20	0.0015	0.0051	0.0003	±0.0002	±0 15	0.0045
18	0.0016	0.0057	0.0004	±0.0003	±0 10	0.0049
16	0.0018	0.0063	0.0004	±0.0003	±0 10	0.0055
14	0.0021	0.0070	0.0004	±0.0003	±0 10	0.0062
13	0.0022	0.0074	0.0004	±0.0003	±0 10	0.0066
12	0.0024	0.0079	0.0004	±0.0003	±0 10	0.0071
11	0.0026	0.0085	0.0004	±0.0003	±0 10	0.0077
10	0.0028	0.0092	0.0004	±0.0004	±0 5	0.0084
9	0.0031	0.0100	0.0004	±0.0004	±0 5	0.0092
8	0.0034	0.0111	0.0004	±0.0004	±0 5	0.0103
7	0.0039	0.0124	0.0004	±0.0004	±0 5	0.0116
6	0.0044	0.0145	0.0006	±0.0005	±0 5	0.0133
5	0.0052	0.0169	0.0006	±0.0005	±0 5	0.0157
4½	0.0057	0.0184	0.0006	±0.0005	±0 5	0.0172
4	0.0064	0.0204	0.0006	±0.0005	±0 5	0.0192

* Allowable variation in lead between any two threads not farther apart than the length of engagement.

Loose Fit. — This class includes screw threads of a rough commercial quality which must assemble readily, a certain amount of looseness or play not being objectionable. *Example:* stove bolts, carriage bolts, hose couplings, threads for artillery ammunition, and other threaded work of a similar nature.

Medium Fit. — This class is sub-divided into "regular" and "special" fits. The "regular" subdivision includes the great bulk of screw thread work of ordinary quality, such as finished and semi-finished bolts and nuts, machine screws, cap screws, most of the fastening screws for instruments, small arms and other ordnance, screws for sewing machines, typewriters, etc. The "special" subdivision includes the better grade of interchangeable screw thread work, such as high-grade automobile and aircraft bolts and nuts.

Close Fit. — This class includes screw thread work requiring a fine snug fit, which is somewhat closer than the "special" medium fit. Selective assembly of parts may be required for screw threads of the "close-fit" class.

Wrench Fit. — This class applies to threaded parts of ¼-inch diameter or larger, which are to be set or assembled with a wrench. As the material in this case is an important factor in determining the fit between the threaded members, there are two subdivisions for this class of fit, which differ mainly in the amount of allowance (interference) values for the different pitches. The first subdivision includes screw threads used in light sections with moderate stresses, such as aircraft and automobile engine work. The second subdivision includes screw threads used in heavy sections and for higher stresses as, for example, in steam engine and heavy hydraulic work. On account of the lack of data, tolerances and allowances are not specified for wrench fits.

General Specifications. — The following general specifications apply to the different classes of screw thread fits referred to.

The Basic Diameter. — The minimum threaded hole or nut corresponds to the basic size; that is, the pitch diameter of the minimum nut is basic for all classes of fit. This condition permits the use of taps which, when new, are over size and which are discarded when the hole cut is at the basic size. In order to secure the desired fit the screw size is varied. Thus, the maximum screw corresponds to the basic size for the "medium fit" class; it is slightly above basic size for the "close fit" class; it is considerably above the basic size in the "wrench fit" class; and it is below the basic size for the "loose fit" class.

Length of Engagement. — The maximum length of engagement for screw threads manufactured in accordance with any of the classes of fit specified shall not exceed the quantity as determined in the following formula, in which L = length of engagement and D = basic major diameter of thread: $L = 1.5\,D$.

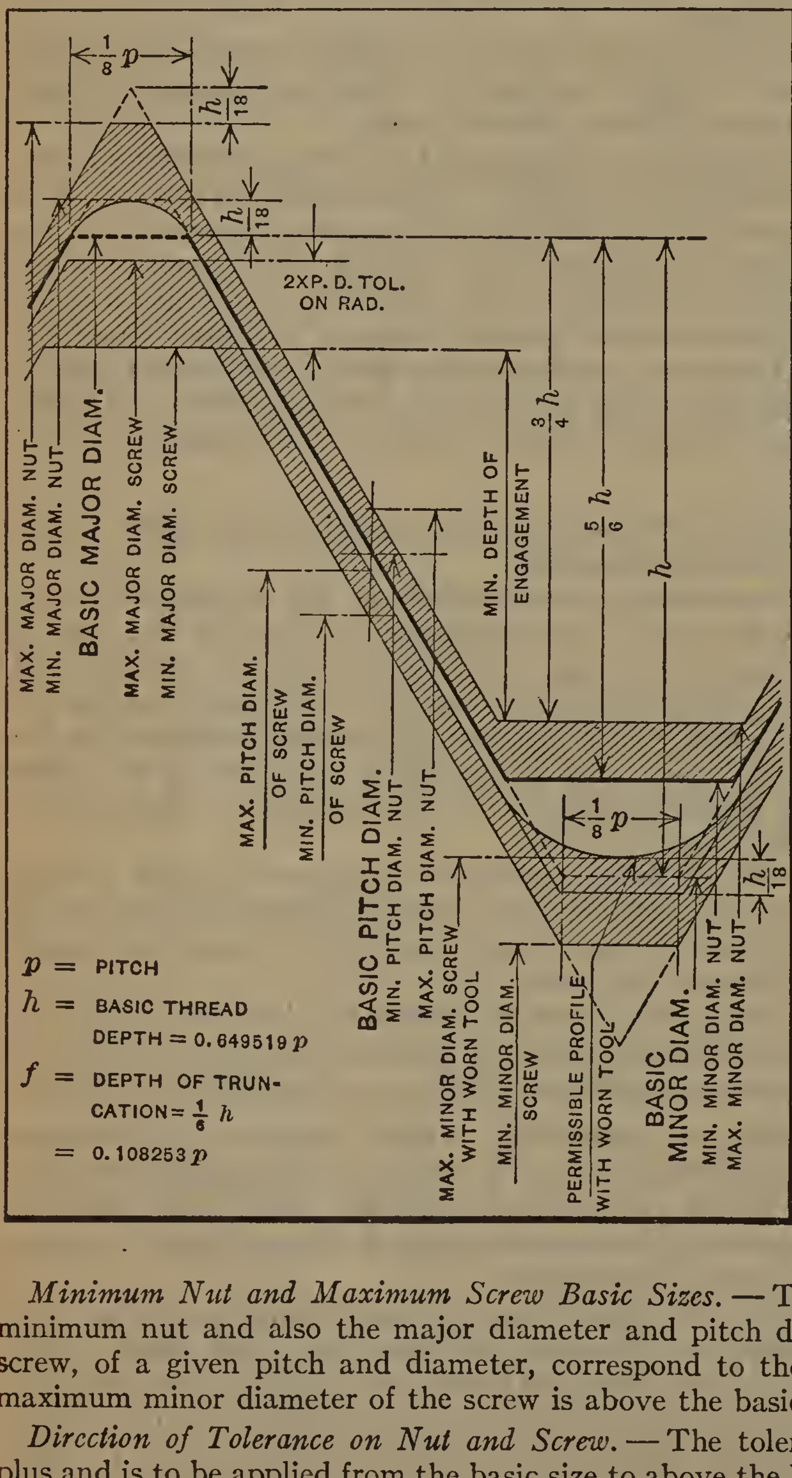
Scope of Classification. — The specifications established for the various classes of fit are applicable to the National coarse threads, fine threads, hose threads, and to any special thread required in manufacture which is not intentionally tapered.

Tolerances and Diameters for Loose Fits. — The "loose fit" class of screw threads is intended to cover the manufacture of strictly interchangeable threaded parts which are produced in two or more manufacturing plants and this class applies to work of such a nature that rapid assembly is necessary and a certain amount of looseness or play is not objectionable.

Minimum Nut Basic Size. — The pitch diameter of the minimum nut of a given diameter and pitch corresponds to the basic pitch diameter. For example, the minimum pitch diameter of a nut for a nominal screw thread size of ¼ inch, is 0.2175 inch as given in the right-hand half of Table 5, which corresponds to the basic pitch diameter for the National Coarse Thread Series as given in Table 1. The accompanying diagram illustrates the tolerances, allowances and clearances for a 1-inch screw thread (8 threads per inch) of the "loose fit" class.

Maximum Screw below Basic Size. — The major and pitch diameters of a maximum screw of given pitch and diameter are below the basic dimensions specified in Table 1, by the amount of allowance given in Table 4. For example, the maximum pitch diameter of a ¼-inch screw thread (see Table 5) is 0.2160. The basic pitch diameter (see Table 1) is 0.2175. Hence, the difference equals 0.2175 − 0.2160 = 0.0015, which equals the allowance given in Table 4 for 20 threads per inch. The maximum minor diameter of the screw is above the basic minor diameter.

Direction of Tolerances. — The tolerance on a nut will be plus, and is to be applied from the basic size to above the basic size. The tolerance on the screw will be minus, and is applied from the maximum screw dimension to below the maximum screw dimension.



Allowance and Tolerance Values. — The allowance provided between the size of the minimum nut, which is basic, and the size of the maximum screw for a screw thread of given pitch, will be as specified in Table 4. The tolerance allowed on a screw or nut of a given pitch will also be as specified in this table.

Tolerances and Diameters for Medium Fits. — As previously explained, screw threads of the medium fit class are subdivided into "regular" and "special" fits. The "regular" fits are intended to apply to interchangeable manufacture where the threaded members are to be assembled nearly, or entirely, with the fingers and where a moderate amount of play between the assembled threaded members is not objectionable.

Minimum Nut and Maximum Screw Basic Sizes. — The pitch diameter of the minimum nut and also the major diameter and pitch diameter of the maximum screw, of a given pitch and diameter, correspond to the basic dimensions. The maximum minor diameter of the screw is above the basic minor diameter.

Direction of Tolerance on Nut and Screw. — The tolerance on the nut will be plus and is to be applied from the basic size to above the basic size. The tolerance on the screw will be minus and is applied from the basic size to below the basic size.

Table 5. Loose Fit — National Coarse Thread Series

Numbered and Fractional Sizes	No. of Threads per Inch	Screw Sizes: Major Diam. Max. Inches	Screw Sizes: Major Diam. Min. Inches	Screw Sizes: Pitch Diam. Max. Inches	Screw Sizes: Pitch Diam. Min. Inches	Screw Sizes: Minor Diam. Max.* Inches	Screw Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Max. Inches	Nut Sizes: Pitch Diam. Min. Inches	Nut Sizes: Pitch Diam. Max. Inches	Nut Sizes: Major Diam. Min.* Inches	Nut Sizes: Major Diam. Max. Inches	Basic Major Diam., Inches	Numbered and Fractional Sizes
1	64	0.0723	0.0671	0.0622	0.0596	0.0531	0.0494	0.0561	0.0578	0.0629	0.0655	0.0741	0.0779	0.073	1
2	56	0.0852	0.0796	0.0736	0.0708	0.0633	0.0592	0.0667	0.0686	0.0744	0.0772	0.0873	0.0914	0.086	2
3	48	0.0981	0.0919	0.0846	0.0815	0.0725	0.0679	0.0764	0.0787	0.0855	0.0886	0.1005	0.1051	0.099	3
4	40	0.1110	0.1042	0.0948	0.0914	0.0803	0.0751	0.0849	0.0876	0.0958	0.0992	0.1138	0.1190	0.112	4
5	40	0.1240	0.1172	0.1078	0.1044	0.0933	0.0881	0.0979	0.1006	0.1088	0.1122	0.1268	0.1320	0.125	5
6	32	0.1369	0.1293	0.1166	0.1128	0.0986	0.0925	0.1042	0.1076	0.1177	0.1215	0.1403	0.1463	0.138	6
8	32	0.1629	0.1553	0.1426	0.1388	0.1246	0.1186	0.1302	0.1336	0.1437	0.1475	0.1663	0.1723	0.164	8
10	24	0.1887	0.1795	0.1616	0.1570	0.1376	0.1300	0.1449	0.1494	0.1629	0.1675	0.1930	0.2006	0.190	10
12	24	0.2147	0.2055	0.1876	0.1830	0.1636	0.1560	0.1709	0.1754	0.1889	0.1935	0.2190	0.2266	0.216	12
1/4	20	0.2485	0.2383	0.2160	0.2109	0.1872	0.1784	0.1959	0.2013	0.2175	0.2226	0.2536	0.2623	0.250	1/4
5/16	18	0.3109	0.2995	0.2748	0.2691	0.2427	0.2330	0.2524	0.2584	0.2764	0.2821	0.3165	0.3262	0.3125	5/16
3/8	16	0.3732	0.3606	0.3326	0.3263	0.2965	0.2857	0.3073	0.3141	0.3344	0.3407	0.3795	0.3903	0.3750	3/8
7/16	14	0.4354	0.4214	0.3890	0.3820	0.3478	0.3356	0.3602	0.3679	0.3911	0.3981	0.4427	0.4548	0.4375	7/16
1/2	13	0.4978	0.4830	0.4478	0.4404	0.4034	0.3905	0.4167	0.4251	0.4500	0.4574	0.5056	0.5185	0.5000	1/2
9/16	12	0.5601	0.5443	0.5060	0.4981	0.4579	0.4439	0.4723	0.4813	0.5084	0.5163	0.5685	0.5824	0.5625	9/16
5/8	11	0.6224	0.6054	0.5634	0.5549	0.5109	0.4958	0.5266	0.5364	0.5660	0.5745	0.6316	0.6466	0.6250	5/8
3/4	10	0.7472	0.7288	0.6822	0.6730	0.6245	0.6081	0.6417	0.6526	0.6850	0.6942	0.7572	0.7736	0.7500	3/4
7/8	9	0.8719	0.8519	0.7997	0.7897	0.7356	0.7176	0.7547	0.7667	0.8028	0.8128	0.8830	0.9010	0.8750	7/8
1	8	0.9966	0.9744	0.9154	0.9043	0.8432	0.8231	0.8647	0.8782	0.9188	0.9299	1.0090	1.0291	1.0000	1
1 1/8	7	1.1211	1.0963	1.0283	1.0159	0.9458	0.9231	0.9704	0.9858	1.0322	1.0446	1.1353	1.1580	1.1250	1 1/8
1 1/4	7	1.2461	1.2213	1.1533	1.1409	1.0708	1.0481	1.0954	1.1108	1.1572	1.1696	1.2603	1.2830	1.2500	1 1/4
1 1/2	6	1.4956	1.4666	1.3873	1.3728	1.2911	1.2646	1.3196	1.3376	1.3917	1.4062	1.5120	1.5386	1.5000	1 1/2
1 3/4	5	1.7448	1.7110	1.6149	1.5980	1.4994	1.4681	1.5335	1.5551	1.6201	1.6370	1.7644	1.7958	1.7500	1 3/4
2	4 1/2	1.9943	1.9575	1.8500	1.8316	1.7217	1.6872	1.7594	1.7835	1.8557	1.8741	2.0160	2.0505	2.0000	2
2 1/4	4 1/2	2.2443	2.2075	2.1000	2.0816	1.9717	1.9372	2.0094	2.0335	2.1057	2.1241	2.2660	2.3005	2.2500	2 1/4
2 1/2	4	2.4936	2.4528	2.3312	2.3108	2.1869	2.1484	2.2294	2.2564	2.3376	2.3580	2.5180	2.5565	2.5000	2 1/2
2 3/4	4	2.7436	2.7028	2.5812	2.5608	2.4369	2.3984	2.4794	2.5064	2.5876	2.6080	2.7680	2.8065	2.7500	2 3/4
3	4	2.9936	2.9528	2.8312	2.8108	2.6869	2.6484	2.7294	2.7564	2.8376	2.8580	3.0180	3.0565	3.0000	3

* Dimensions given are figured to the intersection of the worn tool arc with a center line through crest and root (see accompanying diagram). The dimensions given in the tables of screw and nut sizes are the dimensions of the work and not the gages.

Table 6. Loose Fit — National Fine Thread Series

Numbered and Fractional Sizes	No. of Threads per Inch	Screw Sizes						Nut Sizes						Basic Major Diam., Inches	Numbered and Fractional Sizes
		Major Diam.		Pitch Diam.		Minor Diam.		Minor Diam.		Pitch Diam.		Major Diam.			
		Max. Inches	Min. Inches	Max. Inches	Min. Inches	Max.* Inches	Min. Inches	Min. Inches	Max. Inches	Min. Inches	Max. Inches	Min.* Inches	Max. Inches		
0	80	0.0593	0.0545	0.0512	0.0488	0.0440	0.0407	0.0465	0.0478	0.0519	0.0543	0.0609	0.0642	0.060	0
1	72	0.0723	0.0673	0.0633	0.0608	0.0553	0.0518	0.0580	0.0595	0.0640	0.0665	0.0740	0.0775	0.073	1
2	64	0.0853	0.0801	0.0752	0.0726	0.0661	0.0624	0.0691	0.0708	0.0759	0.0785	0.0871	0.0909	0.086	2
3	56	0.0982	0.0926	0.0866	0.0838	0.0763	0.0722	0.0797	0.0816	0.0874	0.0902	0.1003	0.1044	0.099	3
4	48	0.1111	0.1049	0.0976	0.0945	0.0855	0.0809	0.0894	0.0917	0.0985	0.1016	0.1135	0.1181	0.112	4
5	44	0.1241	0.1177	0.1093	0.1061	0.0962	0.0914	0.1004	0.1029	0.1102	0.1134	0.1266	0.1315	0.125	5
6	40	0.1370	0.1302	0.1208	0.1174	0.1063	0.1011	0.1109	0.1136	0.1218	0.1252	0.1398	0.1450	0.138	6
8	36	0.1629	0.1557	0.1449	0.1413	0.1288	0.1232	0.1339	0.1369	0.1460	0.1496	0.1660	0.1716	0.164	8
10	32	0.1889	0.1813	0.1686	0.1648	0.1506	0.1445	0.1562	0.1596	0.1697	0.1735	0.1923	0.1983	0.190	10
12	28	0.2148	0.2062	0.1916	0.1873	0.1710	0.1641	0.1773	0.1812	0.1928	0.1971	0.2186	0.2255	0.216	12
1/4	28	0.2488	0.2402	0.2256	0.2213	0.2050	0.1981	0.2113	0.2152	0.2268	0.2311	0.2526	0.2595	0.2500	1/4
5/16	24	0.3112	0.3020	0.2841	0.2795	0.2601	0.2525	0.2674	0.2719	0.2854	0.2900	0.3155	0.3231	0.3125	5/16
3/8	24	0.3737	0.3645	0.3466	0.3420	0.3226	0.3150	0.3299	0.3344	0.3479	0.3525	0.3780	0.3856	0.3750	3/8
7/16	20	0.4360	0.4258	0.4035	0.3984	0.3747	0.3659	0.3834	0.3888	0.4050	0.4101	0.4411	0.4498	0.4375	7/16
1/2	20	0.4985	0.4883	0.4660	0.4609	0.4372	0.4284	0.4459	0.4513	0.4675	0.4726	0.5036	0.5123	0.5000	1/2
9/16	18	0.5609	0.5495	0.5248	0.5191	0.4927	0.4830	0.5024	0.5084	0.5264	0.5321	0.5665	0.5762	0.5625	9/16
5/8	18	0.6234	0.6120	0.5873	0.5816	0.5552	0.5455	0.5649	0.5709	0.5889	0.5946	0.6290	0.6387	0.6250	5/8
3/4	16	0.7482	0.7356	0.7076	0.7013	0.6715	0.6607	0.6823	0.6891	0.7094	0.7157	0.7545	0.7653	0.7500	3/4
7/8	14	0.8729	0.8589	0.8265	0.8195	0.7853	0.7731	0.7977	0.8054	0.8286	0.8356	0.8802	0.8923	0.8750	7/8
1	14	0.9979	0.9839	0.9515	0.9445	0.9103	0.8981	0.9227	0.9304	0.9536	0.9606	1.0052	1.0173	1.0000	1
1 1/8	12	1.1226	1.1068	1.0685	1.0606	1.0204	1.0064	1.0348	1.0438	1.0709	1.0788	1.1310	1.1449	1.1250	1 1/8
1 1/4	12	1.2476	1.2318	1.1935	1.1856	1.1454	1.1314	1.1598	1.1688	1.1959	1.2038	1.2560	1.2699	1.2500	1 1/4
1 1/2	12	1.4976	1.4818	1.4435	1.4356	1.3954	1.3814	1.4098	1.4188	1.4459	1.4538	1.5060	1.5199	1.5000	1 1/2
1 3/4	12	1.7476	1.7318	1.6935	1.6856	1.6454	1.6314	1.6598	1.6688	1.6959	1.7038	1.7560	1.7699	1.7500	1 3/4
2	12	1.9976	1.9818	1.9435	1.9356	1.8954	1.8814	1.9098	1.9188	1.9459	1.9538	2.0060	2.0199	2.0000	2
2 1/4	12	2.2476	2.2318	2.1935	2.1856	2.1454	2.1314	2.1598	2.1688	2.1959	2.2038	2.2560	2.2699	2.2500	2 1/4
2 1/2	12	2.4976	2.4818	2.4435	2.4356	2.3954	2.3814	2.4098	2.4188	2.4459	2.4538	2.5060	2.5199	2.5000	2 1/2
2 3/4	12	2.7476	2.7318	2.6935	2.6856	2.6454	2.6314	2.6598	2.6688	2.6959	2.7038	2.7560	2.7699	2.7500	2 3/4
3	10	2.9972	2.9788	2.9322	2.9230	2.8745	2.8581	2.8917	2.9026	2.9350	2.9442	3.0072	3.0236	3.0000	3

* Dimensions given are figured to the intersection of the worn tool arc with a center line through crest and root.

Allowance and Tolerance. — The allowance between the size of the maximum screw and minimum nut will be zero for all pitches and all diameters. The tolerance for a screw or nut of a given pitch will be as specified in Table 7.

The "special" fits of the medium class apply particularly to high grades of automobile screw thread work. This "special" subdivision of the medium fit class is the same as the "regular" subdivision, except that the tolerances are smaller as shown by Table 10.

Tolerances and Diameters for Close Fits. — Screw threads of the "close fit" class are intended for threaded work of the finest commercial quality (where the thread has practically no backlash) and for light screwdriver fits. In the manufacture of screw thread products belonging in this class, it will be necessary to use precision tools, selected master gages, and many other refinements. This quality of work should, therefore, be used only in cases where requirements of the mechanism

Table 7. Allowances and Tolerances for Medium Fit (Regular)
(Screws, Nuts and Gages)

No. of Threads per Inch	Allowances, Inches	Extreme or Drawing Pitch Diameter Tolerances, Inches	Master Gage Tolerances			Net Pitch Diameter Tolerances, Inches
			Diameter, Inches	Lead,* Inches	½ Angle	
80	0.0000	0.0017	0.0002	±0.0002	±0° 30′	0.0013
72	0.0000	0.0018	0.0002	±0.0002	±0 30	0.0014
64	0.0000	0.0019	0.0002	±0.0002	±0 30	0.0015
56	0.0000	0.0020	0.0002	±0.0002	±0 30	0.0016
48	0.0000	0.0022	0.0002	±0.0002	±0 30	0.0018
44	0.0000	0.0023	0.0002	±0.0002	±0 30	0.0019
40	0.0000	0.0024	0.0002	±0.0002	±0 20	0.0020
36	0.0000	0.0025	0.0002	±0.0002	±0 20	0.0021
32	0.0000	0.0027	0.0002	±0.0002	±0 20	0.0023
28	0.0000	0.0031	0.0003	±0.0002	±0 15	0.0025
24	0.0000	0.0033	0.0003	±0.0002	±0 15	0.0027
20	0.0000	0.0036	0.0003	±0.0002	±0 15	0.0030
18	0.0000	0.0041	0.0004	±0.0003	±0 10	0.0033
16	0.0000	0.0045	0.0004	±0.0003	±0 10	0.0037
14	0.0000	0.0049	0.0004	±0.0003	±0 10	0.0041
13	0.0000	0.0052	0.0004	±0.0003	±0 10	0.0044
12	0.0000	0.0056	0.0004	±0.0003	±0 10	0.0048
11	0.0000	0.0059	0.0004	±0.0003	±0 10	0.0051
10	0.0000	0.0064	0.0004	±0.0004	±0 5	0.0056
9	0.0000	0.0070	0.0004	±0.0004	±0 5	0.0062
8	0.0000	0.0076	0.0004	±0.0004	±0 5	0.0068
7	0.0000	0.0085	0.0004	±0.0004	±0 5	0.0077
6	0.0000	0.0101	0.0006	±0.0005	±0 5	0.0089
5	0.0000	0.0116	0.0006	±0.0005	±0 5	0.0104
4½	0.0000	0.0127	0.0006	±0.0005	±0 5	0.0115
4	0.0000	0.0140	0.0006	±0.0005	±0 5	0.0128

* Allowable variation in lead between any two threads not farther apart than the length of engagement.

being produced are exacting, or where special conditions require screws having a precision fit. In order to secure the fit desired, it may be necessary, in some cases, to select the parts when the product is being assembled.

Maximum Screw above Basic Size. — The major diameter and the pitch diameter of the maximum screw of a given diameter and pitch, will be above the basic dimensions by the amount of the allowance (interference) specified in Table 13.

Direction of Tolerance on Nut and Screw. — The tolerance on the nut will be plus and is to be applied from the basic size to above basic size. The tolerance on the screw will be minus and is to be applied from the maximum screw dimensions to below the maximum screw dimensions.

Allowance and Tolerance Values. — The allowance (interference) provided between the size of the minimum nut, which is basic, and the size of the maximum screw, which is above basic, will be as specified in Table 13. The tolerance for a screw or nut of given pitch will be as specified in this same table.

Application of Tolerances. — Three different sets of tolerances are given in Tables 4, 7, 10 and 13, for use in connection with the various classes of screw thread fits previously mentioned. The tolerance limits established represent, in reality, the sizes of the "go" and "not go" master gages. Errors in lead and angle which occur on the threaded work, can be offset by a suitable alteration of the pitch diameter of the work. If the "go" gage passes the threaded work, interchangeability is secured and the thread profile may differ from that of the "go" gage in either pitch diameter, lead or angle. The "not go" gage checks pitch diameter only, and thus insures that the pitch diameter is such that the fit will not be too loose.

The pitch diameter tolerances provided for a screw of a given class of fit are the same as the pitch diameter tolerances provided for a nut corresponding to the same class of fit. The tolerances established for loose and medium fits permit the use of commercial taps. For close fits, when it is desired to produce a hole close to basic size, the use of a selected tap is recommended.

Tolerances on Major Diameter of Screw. — The allowable tolerances on the major diameter of screws of a given classification, will be twice the tolerance values allowed on the pitch diameters of screws of the same class.

Tolerances on Minor Diameter of Screw. — The minimum minor diameter of a screw of a given pitch will be such as to result in a basic flat ($\frac{1}{8} \times$ pitch) at the root when the pitch diameter of the screw is at its minimum value. (*Note:* When the maximum screw is basic, the minimum minor diameter of the screw will be below the *basic* minor diameter by the amount of the specified pitch diameter tolerance.) The maximum minor diameter may be such as results from the use of a worn or rounded threading tool, when the pitch diameter is at its maximum value. In no case, however, should the form of the screw, as results from tool wear, be such as to cause the screw to be rejected on the maximum minor diameter by a "go" ring gage, the minor diameter of which is equal to the minimum minor diameter of the nut.

Tolerances on Major Diameter of Nut. — The maximum major diameter of the nut of a given pitch will be such as to result in a flat $\frac{1}{3}$ of the basic flat ($\frac{1}{24} \times$ pitch) when the pitch diameter of the nut is at its maximum value. (*Note:* When the minimum nut is basic, the maximum major diameter will be above the *basic* major diameter by the amount of the specified pitch diameter tolerance plus $\frac{2}{9}$ of the basic thread depth.)

The nominal minimum major diameter of a nut will be above the basic major diameter by an amount equal to $\frac{1}{6}$ of the basic thread depth plus the neutral space. This results in a clearance which is provided to facilitate manufacture by permitting a slight rounding or wear at the crest of the tap. In no case, however, should the

Table 8. Medium Fit (Regular) — National Coarse Thread Series

Numbered and Fractional Sizes	No. of Threads per Inch	Screw Sizes: Major Diam. Max. Inches	Screw Sizes: Major Diam. Min. Inches	Screw Sizes: Pitch Diam. Max. Inches	Screw Sizes: Pitch Diam. Min. Inches	Screw Sizes: Minor Diam. Max.* Inches	Screw Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Max. Inches	Nut Sizes: Pitch Diam. Min. Inches	Nut Sizes: Pitch Diam. Max. Inches	Nut Sizes: Major Diam. Min.* Inches	Nut Sizes: Major Diam. Max. Inches	Basic Major Diam., Inches	Numbered and Fractional Sizes
1	64	0.073	0.0692	0.0629	0.0610	0.0538	0.0508	0.0561	0.0578	0.0629	0.0648	0.0741	0.0772	0.073	1
2	56	0.086	0.0820	0.0744	0.0724	0.0641	0.0608	0.0667	0.0686	0.0744	0.0764	0.0873	0.0906	0.086	2
3	48	0.099	0.0946	0.0855	0.0833	0.0734	0.0697	0.0764	0.0787	0.0855	0.0877	0.1005	0.1042	0.099	3
4	40	0.112	0.1072	0.0958	0.0934	0.0813	0.0771	0.0849	0.0876	0.0958	0.0982	0.1138	0.1180	0.112	4
5	40	0.125	0.1202	0.1088	0.1064	0.0943	0.0901	0.0979	0.1006	0.1088	0.1112	0.1268	0.1310	0.125	5
6	32	0.138	0.1326	0.1177	0.1150	0.0997	0.0947	0.1042	0.1076	0.1177	0.1204	0.1403	0.1452	0.138	6
8	32	0.164	0.1586	0.1437	0.1410	0.1257	0.1207	0.1302	0.1336	0.1437	0.1464	0.1663	0.1712	0.164	8
10	24	0.190	0.1834	0.1629	0.1596	0.1389	0.1326	0.1449	0.1494	0.1629	0.1662	0.1930	0.1992	0.190	10
12	24	0.216	0.2094	0.1889	0.1856	0.1649	0.1586	0.1709	0.1754	0.1889	0.1922	0.2190	0.2253	0.216	12
1/4	20	0.250	0.2428	0.2175	0.2139	0.1887	0.1814	0.1959	0.2013	0.2175	0.2211	0.2536	0.2608	0.250	1/4
5/16	18	0.3125	0.3043	0.2764	0.2723	0.2443	0.2362	0.2524	0.2584	0.2764	0.2805	0.3165	0.3246	0.3125	5/16
3/8	16	0.3750	0.3660	0.3344	0.3299	0.2983	0.2893	0.3073	0.3141	0.3344	0.3389	0.3795	0.3885	0.3750	3/8
7/16	14	0.4375	0.4277	0.3911	0.3862	0.3499	0.3398	0.3602	0.3679	0.3911	0.3960	0.4427	0.4527	0.4375	7/16
1/2	13	0.5000	0.4896	0.4500	0.4448	0.4056	0.3949	0.4167	0.4251	0.4500	0.4552	0.5056	0.5163	0.5000	1/2
9/16	12	0.5625	0.5513	0.5084	0.5028	0.4603	0.4486	0.4723	0.4813	0.5084	0.5140	0.5685	0.5800	0.5625	9/16
5/8	11	0.6250	0.6132	0.5660	0.5601	0.5135	0.5010	0.5266	0.5364	0.5660	0.5719	0.6316	0.6440	0.6250	5/8
3/4	10	0.7500	0.7372	0.6850	0.6786	0.6273	0.6137	0.6417	0.6526	0.6850	0.6914	0.7572	0.7708	0.7500	3/4
7/8	9	0.8750	0.8610	0.8028	0.7958	0.7387	0.7237	0.7547	0.7667	0.8028	0.8098	0.8830	0.8980	0.8750	7/8
1	8	1.0000	0.9848	0.9188	0.9112	0.8466	0.8300	0.8647	0.8782	0.9188	0.9264	1.0090	1.0256	1.0000	1
1 1/8	7	1.1250	1.1080	1.0322	1.0237	0.9497	0.9309	0.9704	0.9858	1.0322	1.0407	1.1353	1.1541	1.1250	1 1/8
1 1/4	7	1.2500	1.2330	1.1572	1.1487	1.0747	1.0559	1.0954	1.1108	1.1572	1.1657	1.2603	1.2791	1.2500	1 1/4
1 1/2	6	1.5000	1.4798	1.3917	1.3816	1.2955	1.2734	1.3196	1.3376	1.3917	1.4018	1.5120	1.5342	1.5000	1 1/2
1 3/4	5	1.7500	1.7268	1.6201	1.6085	1.5046	1.4786	1.5335	1.5551	1.6201	1.6317	1.7644	1.7905	1.7500	1 3/4
2	4 1/2	2.0000	1.9746	1.8557	1.8430	1.7274	1.6986	1.7594	1.7835	1.8557	1.8684	2.0160	2.0448	2.0000	2
2 1/4	4 1/2	2.2500	2.2246	2.1057	2.0930	1.9774	1.9486	2.0094	2.0335	2.1057	2.1184	2.2660	2.2948	2.2500	2 1/4
2 1/2	4	2.5000	2.4720	2.3376	2.3236	2.1933	2.1612	2.2294	2.2564	2.3376	2.3516	2.5180	2.5501	2.5000	2 1/2
2 3/4	4	2.7500	2.7220	2.5876	2.5736	2.4433	2.4112	2.4794	2.5064	2.5876	2.6016	2.7680	2.8001	2.7500	2 3/4
3	4	3.0000	2.9720	2.8376	2.8236	2.6933	2.6612	2.7294	2.7564	2.8376	2.8516	3.0180	3.0501	3.0000	3

* Dimensions given are figured to the intersection of the worn tool arc with a center line through crest and root.

Table 9. Medium Fit (Regular) — National Fine Thread Series

Numbered and Fractional Sizes	No. of Threads per Inch	Screw Sizes: Major Diam. Max. Inches	Screw Sizes: Major Diam. Min. Inches	Screw Sizes: Pitch Diam. Max. Inches	Screw Sizes: Pitch Diam. Min. Inches	Screw Sizes: Minor Diam. Max.* Inches	Screw Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Max. Inches	Nut Sizes: Pitch Diam. Min. Inches	Nut Sizes: Pitch Diam. Max. Inches	Nut Sizes: Major Diam. Min.* Inches	Nut Sizes: Major Diam. Max. Inches	Basic Major Diam., Inches	Numbered and Fractional Sizes
0	80	0.060	0.0566	0.0519	0.0502	0.0447	0.0421	0.0465	0.0478	0.0519	0.0536	0.0609	0.0635	0.060	0
1	72	0.073	0.0694	0.0640	0.0622	0.0560	0.0532	0.0580	0.0595	0.0640	0.0658	0.0740	0.0768	0.073	1
2	64	0.086	0.0822	0.0759	0.0740	0.0668	0.0638	0.0691	0.0708	0.0759	0.0778	0.0871	0.0902	0.086	2
3	56	0.099	0.0950	0.0874	0.0854	0.0771	0.0738	0.0797	0.0816	0.0874	0.0894	0.1003	0.1036	0.099	3
4	48	0.112	0.1076	0.0985	0.0963	0.0864	0.0827	0.0894	0.0917	0.0985	0.1007	0.1135	0.1172	0.112	4
5	44	0.125	0.1204	0.1102	0.1079	0.0971	0.0932	0.1004	0.1029	0.1102	0.1125	0.1266	0.1306	0.125	5
6	40	0.138	0.1332	0.1218	0.1194	0.1073	0.1031	0.1109	0.1136	0.1218	0.1242	0.1398	0.1440	0.138	6
8	36	0.164	0.1590	0.1460	0.1435	0.1299	0.1254	0.1339	0.1369	0.1460	0.1485	0.1660	0.1705	0.164	8
10	32	0.190	0.1846	0.1697	0.1670	0.1517	0.1467	0.1562	0.1596	0.1697	0.1724	0.1923	0.1972	0.190	10
12	28	0.216	0.2098	0.1928	0.1897	0.1722	0.1665	0.1773	0.1812	0.1928	0.1959	0.2186	0.2243	0.216	12
1/4	28	0.2500	0.2438	0.2268	0.2237	0.2062	0.2005	0.2113	0.2152	0.2268	0.2299	0.2526	0.2583	0.2500	1/4
5/16	24	0.3125	0.3059	0.2854	0.2821	0.2614	0.2551	0.2674	0.2719	0.2854	0.2887	0.3155	0.3218	0.3125	5/16
3/8	24	0.3750	0.3684	0.3479	0.3446	0.3239	0.3176	0.3299	0.3344	0.3479	0.3512	0.3780	0.3843	0.3750	3/8
7/16	20	0.4375	0.4303	0.4050	0.4014	0.3762	0.3689	0.3834	0.3888	0.4050	0.4086	0.4411	0.4483	0.4375	7/16
1/2	20	0.5000	0.4928	0.4675	0.4639	0.4387	0.4314	0.4459	0.4513	0.4675	0.4711	0.5036	0.5108	0.5000	1/2
9/16	18	0.5625	0.5543	0.5264	0.5223	0.4943	0.4862	0.5024	0.5084	0.5264	0.5305	0.5665	0.5746	0.5625	9/16
5/8	18	0.6250	0.6168	0.5889	0.5848	0.5568	0.5487	0.5649	0.5709	0.5889	0.5930	0.6290	0.6371	0.6250	5/8
3/4	16	0.7500	0.7410	0.7094	0.7049	0.6733	0.6643	0.6823	0.6891	0.7094	0.7139	0.7545	0.7635	0.7500	3/4
7/8	14	0.8750	0.8652	0.8286	0.8237	0.7874	0.7773	0.7977	0.8054	0.8286	0.8335	0.8802	0.8902	0.8750	7/8
1	14	1.0000	0.9902	0.9536	0.9487	0.9124	0.9023	0.9227	0.9304	0.9536	0.9585	1.0052	1.0152	1.0000	1
1 1/8	12	1.1250	1.1138	1.0709	1.0653	1.0228	1.0111	1.0348	1.0438	1.0709	1.0765	1.1310	1.1426	1.1250	1 1/8
1 1/4	12	1.2500	1.2388	1.1959	1.1903	1.1478	1.1361	1.1598	1.1688	1.1959	1.2015	1.2560	1.2676	1.2500	1 1/4
1 1/2	12	1.5000	1.4888	1.4459	1.4403	1.3978	1.3861	1.4098	1.4188	1.4459	1.4515	1.5060	1.5176	1.5000	1 1/2
1 3/4	12	1.7500	1.7388	1.6959	1.6903	1.6478	1.6361	1.6598	1.6688	1.6959	1.7015	1.7560	1.7676	1.7500	1 3/4
2	12	2.0000	1.9888	1.9459	1.9403	1.8978	1.8861	1.9098	1.9188	1.9459	1.9515	2.0060	2.0176	2.0000	2
2 1/4	12	2.2500	2.2388	2.1959	2.1903	2.1478	2.1361	2.1598	2.1688	2.1959	2.2015	2.2560	2.2676	2.2500	2 1/4
2 1/2	12	2.5000	2.4888	2.4459	2.4403	2.3978	2.3861	2.4098	2.4188	2.4459	2.4515	2.5060	2.5176	2.5000	2 1/2
2 3/4	12	2.7500	2.7388	2.6959	2.6903	2.6478	2.6361	2.6598	2.6688	2.6959	2.7015	2.7560	2.7676	2.7500	2 3/4
3	10	3.0000	2.9872	2.9350	2.9286	2.8773	2.8637	2.8917	2.9026	2.9350	2.9414	3.0072	3.0208	3.0000	3

* Dimensions given are figured to the intersection of the worn tool arc with a center line through crest and root.

minimum major diameter of the nut, as results from a worn tap or cutting tool, be such as to cause the nut to be rejected on the minimum major diameter by a "go" plug gage made to the standard form at the crest.

Tolerances on Minor Diameter of Nut. — The tolerances on minor diameter of a nut of a given pitch will be ⅙ of the basic thread depth regardless of the class of fit being produced.

Scope of Tolerance Specifications. — The specifications establishing the various sets of tolerances for the different classes of screw thread fits, will apply to the manufacture of National coarse threads, National fine threads, National hose coupling threads, National fire hose coupling threads, straight pipe threads, and wherever applicable to the production of all special threads.

Where tolerances are desired for a special thread and the pitch is not listed in the tables given, the tolerance values should be chosen corresponding to the num-

Table 10. Allowances and Tolerances for Medium Fit (Special)
(Screws, Nuts and Gages)

No. of Threads per Inch	Allowances, Inches	Extreme or Drawing Pitch Diameter Tolerances, Inches	Master Gage Tolerances			Net Pitch Diameter Tolerances, Inches
			Diameter, Inches	Lead,* Inches	½ Angle	
80	0.0000	0.0013	0.0002	±0.0002	±0° 30′	0.0009
72	0.0000	0.0013	0.0002	±0.0002	±0 30	0.0009
64	0.0000	0.0014	0.0002	±0.0002	±0 30	0.0010
56	0.0000	0.0015	0.0002	±0.0002	±0 30	0.0011
48	0.0000	0.0016	0.0002	±0.0002	±0 30	0.0012
44	0.0000	0.0016	0.0002	±0.0002	±0 30	0.0012
40	0.0000	0.0017	0.0002	±0.0002	±0 20	0.0013
36	0.0000	0.0018	0.0002	±0.0002	±0 20	0.0014
32	0.0000	0.0019	0.0002	±0.0002	±0 20	0.0015
28	0.0000	0.0022	0.0003	±0.0002	±0 15	0.0016
24	0.0000	0.0024	0.0003	±0.0002	±0 15	0.0018
20	0.0000	0.0026	0.0003	±0.0002	±0 15	0.0020
18	0.0000	0.0030	0.0004	±0.0003	±0 10	0.0022
16	0.0000	0.0032	0.0004	±0.0003	±0 10	0.0024
14	0.0000	0.0036	0.0004	±0.0003	±0 10	0.0028
13	0.0000	0.0037	0.0004	±0.0003	±0 10	0.0029
12	0.0000	0.0040	0.0004	±0.0003	±0 10	0.0032
11	0.0000	0.0042	0.0004	±0.0003	±0 10	0.0034
10	0.0000	0.0045	0.0004	±0.0004	±0 5	0.0037
9	0.0000	0.0049	0.0004	±0.0004	±0 5	0.0041
8	0.0000	0.0054	0.0004	±0.0004	±0 5	0.0046
7	0.0000	0.0059	0.0004	±0.0004	±0 5	0.0051
6	0.0000	0.0071	0.0006	±0.0005	±0 5	0.0059
5	0.0000	0.0082	0.0006	±0.0005	±0 5	0.0070
4½	0.0000	0.0089	0.0006	±0.0005	±0 5	0.0077
4	0.0000	0.0097	0.0006	±0.0005	±0 5	0.0085

* Allowable variation in lead between any two threads not farther apart than the length of engagement.

Table 11. Medium Fit (Special) — National Coarse Thread Series

Numbered and Fractional Sizes	No. of Threads per Inch	Screw Sizes: Major Diam. Max. Inches	Screw Sizes: Major Diam. Min. Inches	Screw Sizes: Pitch Diam. Max. Inches	Screw Sizes: Pitch Diam. Min. Inches	Screw Sizes: Minor Diam. Max.* Inches	Screw Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Max. Inches	Nut Sizes: Pitch Diam. Min. Inches	Nut Sizes: Pitch Diam. Max. Inches	Nut Sizes: Major Diam. Min.* Inches	Nut Sizes: Major Diam. Max. Inches	Basic Major Diam., Inches	Numbered and Fractional Sizes
1	64	0.073	0.0702	0.0629	0.0615	0.0538	0.0513	0.0561	0.0578	0.0629	0.0643	0.0741	0.0767	0.073	1
2	56	0.086	0.0830	0.0744	0.0729	0.0641	0.0613	0.0667	0.0686	0.0744	0.0759	0.0873	0.0901	0.086	2
3	48	0.099	0.0958	0.0855	0.0839	0.0734	0.0703	0.0764	0.0787	0.0855	0.0871	0.1005	0.1036	0.099	3
4	40	0.112	0.1086	0.0958	0.0941	0.0813	0.0778	0.0849	0.0876	0.0958	0.0975	0.1138	0.1173	0.112	4
5	40	0.125	0.1216	0.1088	0.1071	0.0943	0.0908	0.0979	0.1006	0.1088	0.1105	0.1268	0.1303	0.125	5
6	32	0.138	0.1342	0.1177	0.1158	0.0997	0.0955	0.1042	0.1076	0.1177	0.1196	0.1403	0.1444	0.138	6
8	32	0.164	0.1602	0.1437	0.1418	0.1257	0.1215	0.1302	0.1336	0.1437	0.1456	0.1663	0.1704	0.164	8
10	24	0.190	0.1852	0.1629	0.1605	0.1389	0.1335	0.1449	0.1494	0.1629	0.1653	0.1930	0.1984	0.190	10
12	24	0.216	0.2112	0.1889	0.1865	0.1649	0.1595	0.1709	0.1754	0.1889	0.1913	0.2190	0.2244	0.216	12
1/4	20	0.250	0.2448	0.2175	0.2149	0.1887	0.1824	0.1959	0.2013	0.2175	0.2201	0.2536	0.2598	0.250	1/4
5/16	18	0.3125	0.3065	0.2764	0.2734	0.2443	0.2373	0.2524	0.2584	0.2764	0.2794	0.3165	0.3235	0.3125	5/16
3/8	16	0.3750	0.3686	0.3344	0.3312	0.2983	0.2906	0.3073	0.3141	0.3344	0.3376	0.3795	0.3872	0.3750	3/8
7/16	14	0.4375	0.4303	0.3911	0.3875	0.3499	0.3411	0.3602	0.3679	0.3911	0.3947	0.4427	0.4514	0.4375	7/16
1/2	13	0.5000	0.4926	0.4500	0.4463	0.4056	0.3964	0.4167	0.4251	0.4500	0.4537	0.5056	0.5148	0.5000	1/2
9/16	12	0.5625	0.5545	0.5084	0.5044	0.4603	0.4502	0.4723	0.4813	0.5084	0.5124	0.5685	0.5785	0.5625	9/16
5/8	11	0.6250	0.6166	0.5660	0.5618	0.5135	0.5027	0.5266	0.5364	0.5660	0.5702	0.6316	0.6423	0.6250	5/8
3/4	10	0.7500	0.7410	0.6850	0.6805	0.6273	0.6156	0.6417	0.6526	0.6850	0.6895	0.7572	0.7689	0.7500	3/4
7/8	9	0.8750	0.8652	0.8028	0.7979	0.7387	0.7258	0.7547	0.7667	0.8028	0.8077	0.8830	0.8959	0.8750	7/8
1	8	1.0000	0.9892	0.9188	0.9134	0.8466	0.8322	0.8647	0.8782	0.9188	0.9242	1.0090	1.0234	1.0000	1
1 1/8	7	1.1250	1.1132	1.0322	1.0263	0.9497	0.9335	0.9704	0.9858	1.0322	1.0381	1.1353	1.1515	1.1250	1 1/8
1 1/4	7	1.2500	1.2382	1.1572	1.1513	1.0747	1.0585	1.0954	1.1108	1.1572	1.1631	1.2603	1.2765	1.2500	1 1/4
1 1/2	6	1.5000	1.4858	1.3917	1.3846	1.2955	1.2764	1.3196	1.3376	1.3917	1.3988	1.5120	1.5312	1.5000	1 1/2
1 3/4	5	1.7500	1.7336	1.6201	1.6119	1.5046	1.4820	1.5335	1.5551	1.6201	1.6283	1.7644	1.7871	1.7500	1 3/4
2	4 1/2	2.0000	1.9822	1.8557	1.8468	1.7274	1.7024	1.7594	1.7835	1.8557	1.8646	2.0160	2.0410	2.0000	2
2 1/4	4 1/2	2.2500	2.2322	2.1057	2.0968	1.9774	1.9524	2.0094	2.0335	2.1057	2.1146	2.2660	2.2910	2.2500	2 1/4
2 1/2	4	2.5000	2.4806	2.3376	2.3279	2.1933	2.1655	2.2294	2.2564	2.3376	2.3473	2.5180	2.5458	2.5000	2 1/2
2 3/4	4	2.7500	2.7306	2.5876	2.5779	2.4433	2.4155	2.4794	2.5064	2.5876	2.5973	2.7680	2.7958	2.7500	2 3/4
3	4	3.0000	2.9806	2.8376	2.8279	2.6933	2.6655	2.7294	2.7564	2.8376	2.8473	3.0180	3.0458	3.0000	3

* Dimensions given are figured to the intersection of the worn tool arc with a center line through crest and root.

Table 12. Medium Fit (Special) — National Fine Thread Series

Numbered and Fractional Sizes	No. of Threads per Inch	Screw Sizes: Major Diam. Max. Inches	Screw Sizes: Major Diam. Min. Inches	Screw Sizes: Pitch Diam. Max. Inches	Screw Sizes: Pitch Diam. Min. Inches	Screw Sizes: Minor Diam. Max.* Inches	Screw Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Min. Inches	Nut Sizes: Minor Diam. Max. Inches	Nut Sizes: Pitch Diam. Min. Inches	Nut Sizes: Pitch Diam. Max. Inches	Nut Sizes: Major Diam. Min.* Inches	Nut Sizes: Major Diam. Max. Inches	Basic Major Diam., Inches	Numbered and Fractional Sizes
0	80	0.060	0.0574	0.0519	0.0506	0.0447	0.0425	0.0465	0.0478	0.0519	0.0532	0.0609	0.0631	0.060	0
1	72	0.073	0.0704	0.0640	0.0627	0.0560	0.0537	0.0580	0.0595	0.0640	0.0653	0.0740	0.0763	0.073	1
2	64	0.086	0.0832	0.0759	0.0745	0.0668	0.0643	0.0691	0.0708	0.0759	0.0773	0.0871	0.0897	0.086	2
3	56	0.099	0.0960	0.0874	0.0859	0.0771	0.0743	0.0797	0.0816	0.0874	0.0889	0.1003	0.1031	0.099	3
4	48	0.112	0.1088	0.0985	0.0969	0.0864	0.0833	0.0894	0.0917	0.0985	0.1001	0.1135	0.1166	0.112	4
5	44	0.125	0.1218	0.1102	0.1086	0.0971	0.0939	0.1004	0.1029	0.1102	0.1118	0.1266	0.1299	0.125	5
6	40	0.138	0.1346	0.1218	0.1201	0.1073	0.1038	0.1109	0.1136	0.1218	0.1235	0.1398	0.1433	0.138	6
8	36	0.164	0.1604	0.1460	0.1442	0.1299	0.1261	0.1339	0.1369	0.1460	0.1478	0.1660	0.1698	0.164	8
10	32	0.190	0.1862	0.1697	0.1678	0.1517	0.1475	0.1562	0.1596	0.1697	0.1716	0.1923	0.1964	0.190	10
12	28	0.216	0.2116	0.1928	0.1906	0.1722	0.1674	0.1773	0.1812	0.1928	0.1950	0.2186	0.2234	0.216	12
1/4	28	0.2500	0.2456	0.2268	0.2246	0.2062	0.2014	0.2113	0.2152	0.2268	0.2290	0.2526	0.2574	0.2500	1/4
5/16	24	0.3125	0.3077	0.2854	0.2830	0.2614	0.2560	0.2674	0.2719	0.2854	0.2878	0.3155	0.3209	0.3125	5/16
3/8	24	0.3750	0.3702	0.3479	0.3455	0.3239	0.3185	0.3299	0.3344	0.3479	0.3503	0.3780	0.3834	0.3750	3/8
7/16	20	0.4375	0.4323	0.4050	0.4024	0.3762	0.3699	0.3834	0.3888	0.4050	0.4076	0.4411	0.4473	0.4375	7/16
1/2	20	0.5000	0.4948	0.4675	0.4649	0.4387	0.4324	0.4459	0.4513	0.4675	0.4701	0.5036	0.5098	0.5000	1/2
9/16	18	0.5625	0.5565	0.5264	0.5234	0.4943	0.4873	0.5024	0.5084	0.5264	0.5294	0.5665	0.5735	0.5625	9/16
5/8	18	0.6250	0.6190	0.5889	0.5859	0.5568	0.5498	0.5649	0.5709	0.5889	0.5919	0.6290	0.6360	0.6250	5/8
3/4	16	0.7500	0.7436	0.7094	0.7062	0.6733	0.6656	0.6823	0.6891	0.7094	0.7126	0.7545	0.7622	0.7500	3/4
7/8	14	0.8750	0.8678	0.8286	0.8250	0.7874	0.7786	0.7977	0.8054	0.8286	0.8322	0.8802	0.8889	0.8750	7/8
1	14	1.0000	0.9928	0.9536	0.9500	0.9124	0.9036	0.9227	0.9304	0.9536	0.9572	1.0052	1.0139	1.0000	1
1 1/8	12	1.1250	1.1170	1.0709	1.0669	1.0228	1.0127	1.0348	1.0438	1.0709	1.0749	1.1310	1.1410	1.1250	1 1/8
1 1/4	12	1.2500	1.2420	1.1959	1.1919	1.1478	1.1377	1.1598	1.1688	1.1959	1.1999	1.2560	1.2660	1.2500	1 1/4
1 1/2	12	1.5000	1.4920	1.4459	1.4419	1.3978	1.3877	1.4098	1.4188	1.4459	1.4499	1.5060	1.5160	1.5000	1 1/2
1 3/4	12	1.7500	1.7420	1.6959	1.6919	1.6478	1.6377	1.6598	1.6688	1.6959	1.6999	1.7560	1.7660	1.7500	1 3/4
2	12	2.0000	1.9920	1.9459	1.9419	1.8978	1.8877	1.9098	1.9188	1.9459	1.9499	2.0060	2.0160	2.0000	2
2 1/4	12	2.2500	2.2420	2.1959	2.1919	2.1478	2.1377	2.1598	2.1688	2.1959	2.1999	2.2560	2.2660	2.2500	2 1/4
2 1/2	12	2.5000	2.4920	2.4459	2.4419	2.3978	2.3877	2.4098	2.4188	2.4459	2.4499	2.5060	2.5160	2.5000	2 1/2
2 3/4	12	2.7500	2.7420	2.6959	2.6919	2.6478	2.6377	2.6598	2.6688	2.6959	2.6999	2.7560	2.7660	2.7500	2 3/4
3	10	3.0000	2.9910	2.9350	2.9305	2.8773	2.8656	2.8917	2.9026	2.9350	2.9395	3.0072	3.0189	3.0000	3

* Dimensions given are figured to the intersection of the worn tool arc with a center line through crest and root.

ber of threads per inch nearest to that of the special thread being produced. Where the number of threads per inch is midway between two of the pitches listed, the tolerance corresponding to the coarser pitch should be used. For instance, the tolerance on a screw having 11½ threads per inch would correspond to the tolerances specified for a screw of 11 threads per inch.

Gaging Interchangeable Threaded Parts. — In order to produce interchangeable threaded parts in large quantities and in accordance with specifications of the National Screw Thread Commission, an adequate system of measuring or gaging is necessary. In the report of the Commission no attempt was made to cover fully the gaging of screw threads but rather to deal with the fundamental principles involved. Inasmuch as the threaded plug and ring limit gages are the types most generally used, the report deals with what is considered the best practice in the production and application of such gages, but it is not the intention of the Com-

Table 13. Allowances and Tolerances for Close Fit
(Screws, Nuts and Gages)

No. of Threads per Inch	Interference or Negative Allowances, Inches	Extreme or Drawing Pitch Diameter Tolerances, Inches	Master Gage Tolerances			Net Pitch Diameter Tolerances, Inches
			Diameter, Inches	Lead,* Inches	½ Angle	
80	0.0001	0.0006	0.0001	±0.0001	±15′ 00″	0.0004
72	0.0001	0.0007	0.0001	±0.0001	±15 00	0.0005
64	0.0001	0.0007	0.0001	±0.0001	±15 00	0.0005
56	0.0002	0.0007	0.0001	±0.0001	±15 00	0.0005
48	0.0002	0.0008	0.0001	±0.0001	±15 00	0.0006
44	0.0002	0.0008	0.0001	±0.0001	±15 00	0.0006
40	0.0002	0.0009	0.0001	±0.0001	±10 00	0.0007
36	0.0002	0.0009	0.0001	±0.0001	±10 00	0.0007
32	0.0002	0.0010	0.0001	±0.0001	±10 00	0.0008
28	0.0002	0.0011	0.00015	±0.0001	± 7 30	0.0008
24	0.0003	0.0012	0.00015	±0.0001	± 7 30	0.0009
20	0.0003	0.0013	0.00015	±0.0001	± 7 30	0.0010
18	0.0003	0.0015	0.0002	±0.00015	± 5 00	0.0011
16	0.0004	0.0016	0.0002	±0.00015	± 5 00	0.0012
14	0.0004	0.0018	0.0002	±0.00015	± 5 00	0.0014
13	0.0004	0.0019	0.0002	±0.00015	± 5 00	0.0015
12	0.0005	0.0020	0.0002	±0.00015	± 5 00	0.0016
11	0.0005	0.0021	0.0002	±0.00015	± 5 00	0.0017
10	0.0006	0.0023	0.0002	±0.0002	± 2 30	0.0019
9	0.0006	0.0024	0.0002	±0.0002	± 2 30	0.0020
8	0.0007	0.0027	0.0002	±0.0002	± 2 30	0.0023
7	0.0008	0.0030	0.0002	±0.0002	± 2 30	0.0026
6	0.0009	0.0036	0.0003	±0.00025	± 2 30	0.0030
5	0.0010	0.0041	0.0003	±0.00025	± 2 30	0.0035
4½	0.0011	0.0044	0.0003	±0.00025	± 2 30	0.0038
4	0.0013	0.0048	0.0003	±0.00025	± 2 30	0.0042

* Allowable variation in lead between any two threads not farther apart than the length of engagement.

Table 14. Close Fit — National Coarse Thread Series

Numbered and Fractional Sizes	No. of Threads per Inch	Screw Sizes						Nut Sizes						Basic Major Diam., Inches	Numbered and Fractional Sizes
		Major Diam.		Pitch Diam.		Minor Diam.		Minor Diam.		Pitch Diam.		Major Diam.			
		Max. Inches	Min. Inches	Max. Inches	Min. Inches	Max.* Inches	Min. Inches	Min. Inches	Max. Inches	Min. Inches	Max. Inches	Min.* Inches	Max. Inches		
1	64	0.0731	0.0717	0.0630	0.0623	0.0539	0.0521	0.0561	0.0578	0.0629	0.0636	0.0741	0.0760	0.073	1
2	56	0.0862	0.0848	0.0746	0.0739	0.0643	0.0623	0.0667	0.0686	0.0744	0.0751	0.0873	0.0893	0.086	2
3	48	0.0992	0.0976	0.0857	0.0849	0.0736	0.0713	0.0764	0.0787	0.0855	0.0863	0.1005	0.1028	0.099	3
4	40	0.1122	0.1104	0.0960	0.0951	0.0815	0.0788	0.0849	0.0876	0.0958	0.0967	0.1138	0.1165	0.112	4
5	40	0.1252	0.1234	0.1090	0.1081	0.0945	0.0918	0.0979	0.1006	0.1088	0.1097	0.1268	0.1295	0.125	5
6	32	0.1382	0.1362	0.1179	0.1169	0.0999	0.0966	0.1042	0.1076	0.1177	0.1187	0.1403	0.1435	0.138	6
8	32	0.1642	0.1622	0.1439	0.1429	0.1259	0.1226	0.1302	0.1336	0.1437	0.1447	0.1663	0.1695	0.164	8
10	24	0.1903	0.1879	0.1632	0.1620	0.1392	0.1350	0.1449	0.1494	0.1629	0.1641	0.1930	0.1972	0.190	10
12	24	0.2163	0.2139	0.1892	0.1880	0.1652	0.1610	0.1709	0.1754	0.1889	0.1901	0.2190	0.2232	0.216	12
1/4	20	0.2503	0.2477	0.2178	0.2165	0.1890	0.1840	0.1959	0.2013	0.2175	0.2188	0.2536	0.2585	0.250	1/4
5/16	18	0.3128	0.3098	0.2767	0.2752	0.2446	0.2391	0.2524	0.2584	0.2764	0.2779	0.3165	0.3220	0.3125	5/16
3/8	16	0.3754	0.3722	0.3348	0.3332	0.2987	0.2926	0.3073	0.3141	0.3344	0.3360	0.3795	0.3856	0.3750	3/8
7/16	14	0.4379	0.4343	0.3915	0.3897	0.3503	0.3433	0.3602	0.3679	0.3911	0.3929	0.4427	0.4496	0.4375	7/16
1/2	13	0.5004	0.4966	0.4504	0.4485	0.4059	0.3986	0.4167	0.4251	0.4500	0.4519	0.5056	0.5131	0.5000	1/2
9/16	12	0.5630	0.5590	0.5089	0.5069	0.4608	0.4527	0.4723	0.4813	0.5084	0.5104	0.5685	0.5765	0.5625	9/16
5/8	11	0.6255	0.6213	0.5665	0.5644	0.5140	0.5053	0.5266	0.5364	0.5660	0.5681	0.6316	0.6403	0.6250	5/8
3/4	10	0.7506	0.7460	0.6856	0.6833	0.6279	0.6184	0.6417	0.6526	0.6850	0.6873	0.7572	0.7667	0.7500	3/4
7/8	9	0.8756	0.8708	0.8034	0.8010	0.7393	0.7289	0.7547	0.7667	0.8028	0.8052	0.8830	0.8934	0.8750	7/8
1	8	1.0007	0.9953	0.9195	0.9168	0.8473	0.8356	0.8647	0.8782	0.9188	0.9215	1.0090	1.0207	1.0000	1
1 1/8	7	1.1258	1.1198	1.0330	1.0300	0.9505	0.9372	0.9704	0.9858	1.0322	1.0352	1.1353	1.1486	1.1250	1 1/8
1 1/4	7	1.2508	1.2448	1.1580	1.1550	1.0755	1.0622	1.0954	1.1108	1.1572	1.1602	1.2603	1.2736	1.2500	1 1/4
1 1/2	6	1.5009	1.4937	1.3926	1.3890	1.2964	1.2808	1.3196	1.3376	1.3917	1.3953	1.5120	1.5277	1.5000	1 1/2
1 3/4	5	1.7510	1.7428	1.6211	1.6170	1.5056	1.4871	1.5335	1.5551	1.6201	1.6242	1.7644	1.7830	1.7500	1 3/4
2	4 1/2	2.0011	1.9923	1.8568	1.8524	1.7285	1.7080	1.7594	1.7835	1.8557	1.8601	2.0160	2.0365	2.0000	2
2 1/4	4 1/2	2.2511	2.2423	2.1068	2.1024	1.9785	1.9580	2.0094	2.0335	2.1057	2.1101	2.2660	2.2865	2.2500	2 1/4
2 1/2	4	2.5013	2.4917	2.3389	2.3341	2.1946	2.1717	2.2294	2.2564	2.3376	2.3424	2.5180	2.5409	2.5000	2 1/2
2 3/4	4	2.7513	2.7417	2.5889	2.5841	2.4446	2.4217	2.4794	2.5064	2.5876	2.5924	2.7680	2.7909	2.7500	2 3/4
3	4	3.0013	2.9917	2.8389	2.8341	2.6946	2.6717	2.7294	2.7564	2.8376	2.8424	3.0180	3.0409	3.0000	3

* Dimensions given are figured to the intersection of the worn tool arc with a center line through crest and root.

Table 15. Close Fit — National Fine Thread Series

Numbered and Fractional Sizes	No. of Threads per Inch	Screw Sizes Major Diam. Max. Inches	Screw Sizes Major Diam. Min. Inches	Screw Sizes Pitch Diam. Max. Inches	Screw Sizes Pitch Diam. Min. Inches	Screw Sizes Minor Diam. Max.* Inches	Screw Sizes Minor Diam. Min. Inches	Nut Sizes Minor Diam. Min. Inches	Nut Sizes Minor Diam. Max. Inches	Nut Sizes Pitch Diam. Min. Inches	Nut Sizes Pitch Diam. Max. Inches	Nut Sizes Major Diam. Min.* Inches	Nut Sizes Major Diam. Max. Inches	Basic Major Diam., Inches	Numbered and Fractional Sizes
0	80	0.0601	0.0589	0.0520	0.0514	0.0448	0.0433	0.0465	0.0478	0.0519	0.0525	0.0609	0.0624	0.060	0
1	72	0.0731	0.0717	0.0641	0.0634	0.0561	0.0544	0.0580	0.0595	0.0640	0.0647	0.0740	0.0757	0.073	1
2	64	0.0861	0.0847	0.0760	0.0753	0.0669	0.0651	0.0691	0.0708	0.0759	0.0766	0.0871	0.0890	0.086	2
3	56	0.0992	0.0978	0.0876	0.0869	0.0773	0.0753	0.0797	0.0816	0.0874	0.0881	0.1003	0.1023	0.099	3
4	48	0.1122	0.1106	0.0987	0.0979	0.0866	0.0843	0.0894	0.0917	0.0985	0.0993	0.1135	0.1158	0.112	4
5	44	0.1252	0.1236	0.1104	0.1096	0.0973	0.0949	0.1004	0.1029	0.1102	0.1110	0.1266	0.1291	0.125	5
6	40	0.1382	0.1364	0.1220	0.1211	0.1075	0.1048	0.1109	0.1136	0.1218	0.1227	0.1398	0.1425	0.138	6
8	36	0.1642	0.1624	0.1462	0.1453	0.1301	0.1272	0.1339	0.1369	0.1460	0.1469	0.1660	0.1689	0.164	8
10	32	0.1902	0.1882	0.1699	0.1689	0.1519	0.1486	0.1562	0.1596	0.1697	0.1707	0.1923	0.1955	0.190	10
12	28	0.2162	0.2140	0.1930	0.1919	0.1724	0.1687	0.1773	0.1812	0.1928	0.1939	0.2186	0.2223	0.216	12
1/4	28	0.2502	0.2480	0.2270	0.2259	0.2064	0.2027	0.2113	0.2152	0.2268	0.2279	0.2526	0.2563	0.2500	1/4
5/16	24	0.3128	0.3104	0.2857	0.2845	0.2617	0.2575	0.2674	0.2719	0.2854	0.2866	0.3155	0.3197	0.3125	5/16
3/8	24	0.3753	0.3729	0.3482	0.3470	0.3242	0.3200	0.3299	0.3344	0.3479	0.3491	0.3780	0.3822	0.3750	3/8
7/16	20	0.4378	0.4352	0.4053	0.4040	0.3765	0.3715	0.3834	0.3888	0.4050	0.4063	0.4411	0.4460	0.4375	7/16
1/2	20	0.5003	0.4977	0.4678	0.4665	0.4390	0.4340	0.4459	0.4513	0.4675	0.4688	0.5036	0.5085	0.5000	1/2
9/16	18	0.5628	0.5598	0.5267	0.5252	0.4946	0.4891	0.5024	0.5084	0.5264	0.5279	0.5665	0.5720	0.5625	9/16
5/8	18	0.6253	0.6223	0.5892	0.5877	0.5571	0.5516	0.5649	0.5709	0.5889	0.5904	0.6290	0.6345	0.6250	5/8
3/4	16	0.7504	0.7472	0.7098	0.7082	0.6737	0.6676	0.6823	0.6891	0.7094	0.7110	0.7545	0.7606	0.7500	3/4
7/8	14	0.8754	0.8718	0.8290	0.8272	0.7878	0.7807	0.7977	0.8054	0.8286	0.8304	0.8802	0.8872	0.8750	7/8
1	14	1.0004	0.9968	0.9540	0.9522	0.9128	0.9058	0.9227	0.9304	0.9536	0.9554	1.0052	1.0121	1.0000	1
1 1/8	12	1.1255	1.1215	1.0714	1.0694	1.0233	1.0152	1.0348	1.0438	1.0709	1.0729	1.1310	1.1390	1.1250	1 1/8
1 1/4	12	1.2505	1.2465	1.1964	1.1944	1.1483	1.1402	1.1598	1.1688	1.1959	1.1979	1.2560	1.2640	1.2500	1 1/4
1 1/2	12	1.5005	1.4965	1.4464	1.4444	1.3983	1.3902	1.4098	1.4188	1.4459	1.4479	1.5060	1.5140	1.5000	1 1/2
1 3/4	12	1.7505	1.7465	1.6964	1.6944	1.6483	1.6402	1.6598	1.6688	1.6959	1.6979	1.7560	1.7640	1.7500	1 3/4
2	12	2.0005	1.9965	1.9464	1.9444	1.8983	1.8902	1.9098	1.9188	1.9459	1.9479	2.0060	2.0140	2.0000	2
2 1/4	12	2.2505	2.2465	2.1964	2.1944	2.1483	2.1402	2.1598	2.1688	2.1959	2.1979	2.2560	2.2640	2.2500	2 1/4
2 1/2	12	2.5005	2.4965	2.4465	2.4444	2.3983	2.3902	2.4098	2.4188	2.4459	2.4479	2.5060	2.5140	2.5000	2 1/2
2 3/4	12	2.7505	2.7465	2.6964	2.6944	2.6483	2.6402	2.6598	2.6688	2.6959	2.6979	2.7560	2.7640	2.7500	2 3/4
3	10	3.0006	2.9960	2.9356	2.9333	2.8779	2.8684	2.8917	2.9026	2.9350	2.9373	3.0072	3.0167	3.0000	3

* Dimensions given are figured to the intersection of the worn tool arc with a center line through crest and root.

mission to confine manufacturers to any particular method of gaging nor to limit them to any particular methods of testing the accuracy either of the manufactured article or of the gages. However, when the ordinary forms of thread gages are used, the following specifications apply.

Fundamental Features of Gaging Systems.— Specifications for gages are based upon the following fundamental assumptions:

1. Approved limit master gages do not reduce the net working tolerance.
2. Permissible errors in angle of thread specified for "go" gages tend to reduce the net working tolerance, while similar permissible errors on the "not go" gage tend to increase the net working tolerance. These two factors, therefore, balance each other.
3. Permissible lead errors specified for the "go" gage reduce the net working tolerance, while permissible lead errors on the "not go" gage tend to increase the net working tolerance.
4. In order to realize the full net working tolerance, the permissible diametrical variation specified for both "go" and "not go" gages (gage increment) is placed outside of the net tolerance limits. The extreme tolerance equals the net tolerance plus gage increment.
5. The "go" gage should check simultaneously all elements of the thread (all diameters, lead, angle, etc.).
6. The "not go" gage should check separately the elements of the thread.

General Specifications for Gaging System. — The following general specifications refer in particular to gaging systems which have been found satisfactory by the Army and Navy for the production of interchangeable threaded parts as previously specified for the various classes of screw threads and the tolerances recommended by the National Screw Thread Commission. These specifications are included for the use of manufacturers where definite information is lacking. They are not to be considered mandatory.

Gage Classification. — Thread gages may be included in one of four classes; namely, standard master gages, limit master gages, inspection gages, and working gages.

Standard Master Gage. — The standard master gage is a threaded plug representing as exactly as possible all physical dimensions of the nominal or basic size of the threaded component. In order that the standard master gage be authentic, the deviations of this gage from the exact standard should be ascertained by the National Bureau of Standards and the gage should be used with knowledge of these deviations or corrections.

Limit Master Gages. — Limit master gages are for reference only. They represent the extreme upper and lower tolerance limits allowed on the dimensions of the part being produced. They are often of the same design as inspection gages. In many cases, however, the design of the master gage is that of a check which can be used to verify the inspection or working gage.

Inspection Gages. — Inspection gages are for the use of the purchaser in accepting the product. They are generally of the same design as the working gages and the dimensions are such that they represent nearly the net tolerance limits on the parts being produced. Inasmuch as a certain amount of wear must be provided for on an inspection gage, the latter cannot represent the net tolerance limit until it is worn to master gage size.

Working Gages. — Working gages are used in manufacturing to check the parts produced as they are machined. It is recommended that the working gages be made to represent limits considerably inside of the net limits in order that sufficient

Table 16. Manufacturing Tolerances on Plain Gages

Manufacturing Tolerance Allowed on Work	Allowable Tolerance for Master Gages		Allowable Tolerance for Inspection Gages		Suggested Tolerance for Working Gages	
	Minimum Gage	Maximum Gage	Minimum Gage	Maximum Gage	Minimum Gage	Maximum Gage
0.002	+0.0000 +0.0001	−0.0000 −0.0001	+0.0001 +0.0003	−0.0001 −0.0003	+0.0003 +0.0005	−0.0003 −0.0005
0.002 to 0.004	+0.0000 +0.0002	−0.0000 −0.0002	+0.0002 +0.0004	−0.0002 −0.0004	+0.0004 +0.0007	−0.0004 −0.0007
0.004 to 0.006	+0.0000 +0.0003	−0.0000 −0.0003	+0.0004 +0.0007	−0.0004 −0.0007	+0.0007 +0.0011	−0.0007 −0.0011
0.006 to 0.010	+0.0000 +0.0004	−0.0000 −0.0004	+0.0006 +0.0010	−0.0006 −0.0010	+0.0010 +0.0015	−0.0010 −0.0015
0.010 to 0.020	+0.0000 +0.0005	−0.0000 −0.0005	+0.0010 +0.0015	−0.0010 −0.0015	+0.0015 +0.0021	−0.0015 −0.0021
0.020 to 0.050	+0.0000 +0.0006	−0.0000 −0.0006	+0.0020 +0.0026	−0.0020 −0.0026	+0.0026 +0.0033	−0.0026 −0.0033

wear will be provided for the working gages, and in order that the product passed by the working gages will be passed by the inspection gages.

Inspection and Working Gage Sets for Screws. — The following inspection and working gages are for producing strictly interchangeable screws as specified for National coarse threads, National fine threads, or other straight threads.

1. A maximum or "go" ring thread gage, preferably adjustable, having the required pitch diameter and minor diameter. The major diameter may be "cleared" to facilitate grinding and lapping.
2. A minimum or "not go" ring thread gage, preferably adjustable, to check only the pitch diameter of the threaded work.
3. A maximum or "go" plain ring gage to check the major diameter of the threaded work.
4. A minimum or "not go" snap gage to check the major diameter of the threaded work.

Inspection and Working Gage Sets for Nuts. — The following inspection and working gages are for producing strictly interchangeable nuts, as specified for National coarse threads, National fine threads, or other straight threads.

1. A minimum or "go" thread plug gage of the required pitch diameter and major diameter. The minor diameter of the thread plug gage may be cleared to facilitate grinding and lapping.
2. A maximum or "not go" thread plug gage to check only the pitch diameter of the threaded work.
3. A "go" plain plug gage to check the minor diameter of the threaded work.
4. A "not go" plain plug gage to check the minor diameter of the threaded work.

Limit Master Gages for Checking Working or Inspection Gages. — The following limit master gages are required for checking the working or inspection gages previously listed for checking the screw.

1. A set plug or check for the maximum "go" thread ring gage, having the same dimensions as the largest permissible screw.

2. A set plug or check for the minimum or "not go" thread ring gage having the same dimensions as the smallest permissible screw.

3. A maximum plain plug for checking the minor diameter of both the "go" and "not go" inspection thread ring gage.

The limit master gages required for checking the working or inspection gages previously listed for checking the nut, are as follows:

1. A minimum or "go" threaded plug to be used as a reference for comparative measurements; this plug should correspond to the basic dimension or standard master gage.

2. A maximum or "not go" threaded plug to be used as a reference for comparative measurements; this plug should correspond to the largest permissible threaded hole.

3. A minimum plain ring gage to check the major diameter of the "go" and "not go" master threaded plug, unless suitable measuring facilities are available for this purpose.

Gage Tolerances. — The accompanying tables (16 to 18 inclusive) of gage manufacturing tolerances are for use in the production of gages for National coarse threads, fine threads, hose coupling threads and other straight threads, and also for pipe threads.

The tolerances in Table 16 apply to all plain plug, ring and snap gages used in connection with a measurement of screw thread diameters. In addition to the master gage tolerances, suggested tolerances for inspection and working gages are also given. The left-hand half of Table 17 covers both standard and limit master

Table 17. Tolerances for Master Thread Gages and Inspection Gages for Loose and Medium Fits

Tolerances on Master Thread Gages, for Both Standard and Limit Master Gages					Suggested Manufacturing Tolerances for Inspection Gages			
Number of Threads per Inch	Allowable Variation in Lead *	Allowable Variation in One-half Angle of Thread	Allowable Tolerances on Diameters: Minimum Thread Gages	Allowable Tolerances on Diameters: Maximum Thread Gages	Allowable Variation in Lead*	Allowable Variation in One-half Angle of Thread	Allowable Tolerances on Diameters: Minimum Thread Gages	Allowable Tolerances on Diameters: Maximum Thread Gages
4 to 6	±0.0005	±0° 5′	+0.0000 +0.0006	−0.0000 −0.0006	±0.0006	±0° 5′	+0.0006 +0.0015	−0.0006 −0.0015
7 to 10	±0.0004	±0° 5′	+0.0000 +0.0004	−0.0000 −0.0004	±0.0005	±0° 10′	+0.0004 +0.0010	−0.0004 −0.0010
11 to 18	±0.0003	±0° 10′	+0.0000 +0.0004	−0.0000 −0.0004	±0.0004	±0° 15′	+0.0004 +0.0008	−0.0004 −0.0008
20 to 28	±0.0002	±0° 15′	+0.0000 +0.0003	−0.0000 −0.0003	±0.0003	±0° 20′	+0.0003 +0.0006	−0.0003 −0.0006
30 to 40	±0.0002	±0° 20′	+0.0000 +0.0002	−0.0000 −0.0002	±0.0002	±0° 30′	+0.0002 +0.0005	−0.0002 −0.0005
44 to 80	±0.0002	±0° 30′	+0.0000 +0.0002	−0.0000 −0.0002	±0.0002	±0° 45′	+0.0002 +0.0004	−0.0002 −0.0004

* This variation is between any two threads not farther apart than the length of engagement.

thread gages for work threaded in accordance with the manufacturing tolerances for loose and medium fits as given in Tables 4, 7 and 10. The right-hand half of Table 17 contains suggested manufacturing tolerances for inspection thread gages (with a small allowance for wear) for use in the quantity production of loose and medium fits.

The left-hand half of Table 18 contains suggested manufacturing tolerances for working thread gages (with a small allowance for wear) for use in quantity production of loose and medium fits. The right-hand half of Table 18 contains the tolerances suggested for both standard and limit master thread gages for work designed in accordance with manufacturing tolerances for close fit screw threads made to Table 13. As the component tolerances for this class are relatively small, it is believed that the working gages will be required to be held within the gage tolerances shown in the right-hand half of Table 18.

Tolerances for Plain Gages. — For plain plug gages, plain ring gages and plain snap gages required for measuring diameters of screw threads, the gage tolerances specified in Tables 17 and 18 should be used. The tolerances on thread diameters vary in accordance with the number of threads per inch on the screw or nut being manufactured. When making a plain plug, ring or snap gage, without knowing the number of threads per inch of the screw to be made, or for gage dimensions other than thread diameters, the tolerances for plain gages given in Table 16 may be used.

Tolerances on Lead. — The tolerances on lead are specified as an allowable variation between any two threads not farther apart than the length of thread

Table 18. Tolerances for Working Gages and Master Gages

Number of Threads per Inch	Suggested Manufacturing Tolerances for Working Gages for Loose and Medium Fits				Standard and Limit Master Gage Tolerances for Close Fits			
	Allowable Variation in Lead*	Allowable Variation in One-half Angle of Thread	Allowable Tolerances on Diameters		Allowable Variation in Lead*	Allowable Variation in One-half Angle of Thread	Allowable Tolerances on Diameters	
			Minimum Thread Gages	Maximum Thread Gages			Minimum Thread Gages	Maximum Thread Gages
4 to 6	±0.0006	±0° 5′	+0.0015 +0.0025	−0.0015 −0.0025	±0.00025	± 2′ 30″	+0.0000 +0.0003	−0.0000 −0.0003
7 to 10	±0.0005	±0° 10′	+0.0010 +0.0020	−0.0010 −0.0020	±0.0002	± 2′ 30″	+0.0000 +0.0002	−0.0000 −0.0002
11 to 18	±0.0004	±0° 15′	+0.0008 +0.0015	−0.0008 −0.0015	±0.00015	± 5′ 00″	+0.0000 +0.0002	−0.0000 −0.0002
20 to 28	±0.0003	±0° 20′	+0.0006 +0.0012	−0.0006 −0.0012	±0.0001	± 7′ 30″	+0.0000 +0.00015	−0.0000 −0.00015
30 to 40	±0.0002	±0° 30′	+0.0005 +0.0010	−0.0005 −0.0010	±0.0001	±10′ 00″	+0.0000 +0.0001	−0.0000 −0.0001
44 to 80	±0.0002	±0° 45′	+0.0004 +0.0006	−0.0004 −0.0006	±0.0001	±15′ 00″	+0.0000 +0.0001	−0.0000 −0.0001

* This variation is between any two threads not farther apart than the length of engagement.

engagement as determined by the formula: $L = 1.5\,D$; in which L = length of thread engagement and D = basic major diameter of thread.

Tolerances on Angle of Thread. — The tolerances on angle of thread as specified for the various pitches are tolerances on one-half of the included angle. This insures that the bisector of the included angle will be perpendicular to the axis of the thread within proper limits. The equivalent deviation from the true thread form caused by such irregularities as convex or concave sides of thread, rounded crests, or slight projections on the thread form, should not exceed the tolerances allowable on angle of thread.

Tolerances on Diameters. — The tolerances given for thread diameters in Tables 17 and 18, are applied in such a manner that the tolerances permitted on the inspection and working gages occupy part of the extreme tolerance. This insures that all work passed by the gages will be within the tolerance limits specified on the part drawing as represented by the limit master gages. The tolerances given also permit the classification and selection of gages so that if a gage is not suitable for a master gage it may be classified and used as an inspection or working gage, provided that the errors do not pass outside of the net tolerance limits. The application of the tolerances on diameters of thread gages is exactly the same as explained for plain gages.

Relation between Tolerances of Gage and Work. — The tolerances given in the Tables 4, 7, 10 and 13 are the net tolerances which are in no way reduced by permissible manufacturing tolerances provided for master gages. These master gage tolerances are provided for by being added to the net tolerances. Thus the extreme or drawing tolerances are the net working tolerances increased by the master gage increment or equivalent diametrical space required to provide for the master gage tolerances. The limits established for the extreme tolerances should in no case be exceeded. The application of gage tolerances in relation to tolerances allowed on the work, can be best understood by considering that the extreme tolerances represent the absolute limits over which variations of the work must not pass. The manufacturing tolerances required for master gages are then deducted from the extreme working tolerances producing the figures specified as net tolerances. Further reduction of the extreme tolerances is caused by the manufacturing tolerances required for the inspection gages and working gages.

It is essential that the proportion of the tolerance used by the workmen producing the work at the machine, be well within the net tolerance limits. The result will be that a very large percentage of the work will be accepted, and spoiled or rejected work will be reduced to practically nothing. If the net tolerance limits are used as working limits at the machine, there will be a larger percentage of rejections due to differences in gages and wear of both tools and gages.

Gage Tolerance Based upon Work Tolerance. — According to the plan to be described (which represents the practice of a prominent manufacturer of gages), a tolerance equal to 10 per cent of the tolerance on the work is generally allowed on ordinary working and inspection gages. Thus, if the work tolerance is 0.005 inch, the gage tolerance equals 0.0005 inch for both the working and inspection gages. There is a difference, however, between the maximum and minimum dimensions of the working and inspection gages. The minimum size of the working gage is made 10 per cent of the tolerance *larger* than the minimum size of the inspection gage, and the maximum size of the working gage is made 10 per cent of the tolerance *smaller* than the maximum size of the inspection gage.

Assume that the minimum and maximum diameters of a shaft are 1 and 1.005 inches respectively, the tolerance being 0.005 inch. The "not go" working gage will then measure 1.0005 inches, 10 per cent of the 0.005-inch tolerance being added to the minimum dimension of the shaft (1.000 + 10 per cent of 0.005 = 1.0005). The "go" working gage will measure 1.0045 inch, since 10 per cent of the tolerance, or 0.0005, is subtracted from the maximum size of the shaft (1.005 − 0.0005 = 1.0045).

When working gages are within the minimum and maximum limits allowed on the work and on the inspection gages, it is evident that all parts which pass the working gages will also pass the inspection gages. When the working gages are made the same size as the inspection gages, disputes are liable to arise and, moreover, working gages may wear faster and become larger than the inspection gages, provided they were both made the same size originally.

The tolerance of the gage itself should be properly applied to avoid any overlapping of dimensions between the working and inspection gages. This gage tolerance should be *minus* on "go" female gages and "not go" male gages, with the exception of the outside and root diameters of "not go" thread gages. The tolerance should be *plus* on "go" male gages and "not go" female gages, again excepting the outside and root diameters of "not go" thread gages. The dimensions on thread gages should be the same as corresponding diameters of "go" gages.

Limits for Holes in Gears. — The plus and minus limits given in the table "Limits for Holes in Gears" are recommended by the American Gear Manufacturers' Association. Holes in gears are divided into three groups or classes: Class 1 applies to precision gears, such as used in aircraft, printing machinery, etc.; Class 2 applies to gears for automobiles, machine tools, etc.; and Class 3 applies to pumps, hoisting machinery, and general jobbing gears. The recommendations cover hole diameters ranging from ½ to 12 inches. It is believed that these limits, if adopted as representative of standard practice in gear-making and used by all makers of gears, will produce a standard quality. It will be noted that the tolerance of Class 2 is about twice that of Class 1 and that of Class 3 about twice that of Class 2.

Limits for Holes in Gears

Diameter, Inches	Class 1 Precision Machines		Class 2 Automobile Machine Tools, etc.		Class 3 Standard Jobbing Gears	
	+ Limit "Not Go"	− Limit "Go"	+ Limit "Not Go"	− Limit "Go"	+ Limit "Not Go"	− Limit "Go"
0–½	0.000	0.00025	0.00025	0.00025	0.0005	0.0005
½–1	0.000	0.0005	0.0005	0.0005	0.00075	0.00075
1–2	0.000	0.00075	0.00075	0.00075	0.001	0.001
2–3	0.00025	0.00075	0.001	0.001	0.00125	0.00125
3–4	0.0005	0.00075	0.00125	0.001	0.0015	0.0015
4–5	0.0005	0.001	0.0013	0.0012	0.00175	0.00175
5–6	0.0005	0.001	0.00175	0.00125	0.002	0.002
8–9	0.00075	0.001	0.002	0.002	0.003	0.003
11–12	0.001	0.001	0.0025	0.0025	0.004	0.004

Tolerances for Cold-drawn Shafting. — The tolerances for cold-drawn shafting are usually on the minus side only and vary from 0.002 to 0.004 or 0.005 inch, depending upon the diameter of the shafting. According to the practice of several manufacturers of cold-drawn shafting, the tolerances are as follows: For shafting diameters smaller than 2 or 2½ inches, the tolerance is plus 0 and minus from 0.002 to 0.0025 inch. For diameters larger than 2 or 2½ inches, the tolerance is plus 0 and minus from 0.001 to 0.0015 inch per inch of diameter. For example, the minus tolerance for a 3-inch shaft would be 0.003 inch or possibly 0.0045 inch.

Johansson System of Tolerances with Diameter of Hole as Basic Size

Diameter of Hole, Inches	Tolerances for Plug Gage used in Hole *				Shaft Tolerances	
	Class *A*, Inch		Class *B*, Inch		Light Running Fit, Inch	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
1/32 – 1/8	−0.00016	+0.00016	−0.00008	+0.00008	−0.00083	−0.00043
1/8 – 1/4	−0.00024	+0.00024	−0.00012	+0.00012	−0.00122	−0.00063
1/4 – 13/32	−0.00031	+0.00031	−0.00016	+0.00016	−0.00165	−0.00087
13/32 – 23/32	−0.00043	+0.00043	−0.00024	+0.00020	−0.00217	−0.00118
23/32 – 1 1/8	−0.00055	+0.00055	−0.00028	+0.00028	−0.00276	−0.00157
1 1/8 – 1 7/8	−0.00067	+0.00067	−0.00035	+0.00031	−0.00335	−0.00197
1 7/8 – 2 15/16	−0.00083	+0.00083	−0.00043	+0.00039	−0.00402	−0.00236
2 15/16 – 4 17/32	−0.00098	+0.00098	−0.00051	+0.00047	−0.00473	−0.00276
4 17/32 – 6 7/8	−0.00118	+0.00118	−0.00059	+0.00059	−0.00551	−0.00315
6 7/8 – 10 7/16	−0.00138	+0.00138	−0.00071	+0.00067	−0.00630	−0.00354
10 7/16 – 15 3/4	−0.00157	+0.00157	−0.00079	+0.00079	−0.00709	−0.00394

Diameter of Hole, Inches	Shaft Tolerances					
	Running Fit, Inch		Sliding Fit, Inch		Push Fit, Inch	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
1/32 – 1/8	−0.00043	−0.00020	−0.00020	−0.00008	−0.00008	+0.00012
1/8 – 1/4	−0.00063	−0.00031	−0.00031	−0.00012	−0.00012	+0.00020
1/4 – 13/32	−0.00087	−0.00043	−0.00043	−0.00016	−0.00016	+0.00028
13/32 – 23/32	−0.00118	−0.00059	−0.00059	−0.00020	−0.00020	+0.00031
23/32 – 1 1/8	−0.00157	−0.00079	−0.00079	−0.00024	−0.00024	+0.00031
1 1/8 – 1 7/8	−0.00197	−0.00098	−0.00098	−0.00031	−0.00031	+0.00031
1 7/8 – 2 15/16	−0.00236	−0.00118	−0.00118	−0.00039	−0.00039	+0.00028
2 15/16 – 4 17/32	−0.00276	−0.00138	−0.00138	−0.00047	−0.00047	+0.00024
4 17/32 – 6 7/8	−0.00315	−0.00157	−0.00157	−0.00055	−0.00055	+0.00020
6 7/8 – 10 7/16	−0.00354	−0.00177	−0.00177	−0.00067	−0.00067	+0.00020
10 7/16 – 15 3/4	−0.00394	−0.00197	−0.00197	−0.00075	−0.00075	+0.00020

Diameter of Hole, Inches	Easy Driving Fit, Inch		Close Driving Fit, Inch		Forced Fit, Inch	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
1/32 – 1/8	+0.00012	+0.00024	+0.00024	+0.00039	+0.00039	+0.00059
1/8 – 1/4	+0.00020	+0.00035	+0.00035	+0.00059	+0.00059	+0.00098
1/4 – 13/32	+0.00028	+0.00047	+0.00047	+0.00083	+0.00083	+0.00146
13/32 – 23/32	+0.00031	+0.00059	+0.00059	+0.00110	+0.00110	+0.00197
23/32 – 1 1/8	+0.00031	+0.00071	+0.00071	+0.00142	+0.00142	+0.00252
1 1/8 – 1 7/8	+0.00031	+0.00087	+0.00087	+0.00177	+0.00177	+0.00319
1 7/8 – 2 15/16	+0.00028	+0.00102	+0.00102	+0.00213	+0.00213	+0.00394
2 15/16 – 4 17/32	+0.00024	+0.00118	+0.00118	+0.00256	+0.00256	+0.00481
4 17/32 – 6 7/8	+0.00020	+0.00138	+0.00138	+0.00303	+0.00303	+0.00579
6 7/8 – 10 7/16	+0.00020	+0.00157	+0.00157	+0.00354	+0.00354	+0.00689
10 7/16 – 15 3/4	+0.00020	+0.00177	+0.00177	+0.00414	+0.00414	+0.00808

* Use column *A* for ordinary work, where greater tolerances are allowable. Use column *B* for more accurate work, where smaller tolerances are required. The values, in inches, in the table above have been given to five decimals to give the exact value of the dimensions in millimeters in the original tables.

Tolerances for Cold-drawn Tool Steel. — The tolerances for cold-drawn tool steel or *drill rod* usually vary from 0.0005 to 0.001 inch, the tolerance increasing for the larger sizes. Some drill manufacturers want the tolerance divided equally plus and minus and others prefer the tolerance either on the plus or minus side. The tolerances for round stock, flats and other shapes are given in the accompanying table which covers both carbon and high-speed steel rods and bars.

S. A. E. Standard Tolerances for Bronze and Brass Sheets and Strips. — The thickness tolerances for cold-rolled sheets and strips of either brass or bronze are given in the accompanying table, which covers thicknesses from No. 0000 to No. 38, American or B. & S. gage and widths from 5 to 14 inches. The thickness of hot-rolled bronze sheets up to and including 48 inches wide, may vary 5 per cent under or over the gage. For widths over 48 inches up to and including 60 inches, the variation may be 7 per cent under or over the gage.

Tolerances for Brass and Copper Tubing. — The S. A. E. standard tolerances for non-ferrous tubing, such as copper, brass and bronze, are given in the accompanying table which covers the tolerances for outside and inside diameters and the wall thickness tolerance. No combination of variations on the same tube is allowed

Tolerances for Brass and Bronze Tubing
(S. A. E. Standard)

Diameters, Outside and Inside	Tolerance, Inch	Thickness of Wall	Tolerance, Inch
Up to ½ inch, inc.	±0.002	Up to 1/64 inch, inc.	±0.001
Over ½ to ¾ in., inc.	±0.0025	Over 1/64 to 1/32 in., inc.	±0.002
Over ¾ to 1 in., inc.	±0.003	Over 1/32 to 1/16 in., inc.	±0.003
Over 1 to 1¼ in., inc.	±0.0035	Over 1/16 to ⅛ in., inc.	±0.005
Over 1¼ to 1½ in., inc.	±0.004	Over ⅛ to ¼ in., inc.	±0.008
Over 1½ to 1¾ in., inc.	±0.0045	Over ¼ to 5/16 in., inc.	±0.0125
Over 1¾ to 2 in., inc.	±0.005	Over 5/16 to ⅜ in., inc.	±0.015
Over 2 in.	±¼ of 1 per cent		

to make the thickness of the wall vary from the nominal size by more than the amounts given in that part of the table headed "Thickness of Wall." When the tolerances given in this table are not permissible, the tolerances should be specified in the order.

Tolerances for Cold-drawn Screw Stock. — The tolerance specifications for screw stock given in the accompanying table have been adopted by the American Society for Testing Materials. Tolerances are given both for the permissible diameter variations and the allowable eccentricity.

Tolerances for Machining Operations

The following tolerances for different classes of machine work are based upon the experience and practice of the Pratt & Whitney Co. in making equipment for rifle manufacture. These figures are subject to variation, and are given as a guide. It is assumed that the machines in each case are in good condition. The figures are also intended to apply only to the manufacture of duplicate parts on an interchangeable basis, and are not given as representing the greatest degree of accuracy obtainable.

Lathe Work: — Rough turning: Minimum tolerance of 0.005 inch for diameters from ¼ to ½ inch; 0.007 inch for diameters from ½ to 1 inch; 0.010 inch for diam-

Tolerances for Cold-drawn Tool Steel Rods and Bars *

Grade of Steel	Kind of Rods or Bars	Finish	Size of Stock	Tolerance, Inch	Hardness Test: Brinell Maximum	Hardness Test: Scleroscope
Carbon	Drill rod	Polished	Under ¼″	−0.00025	200	33
Carbon	Drill rod	Polished	¼″ to ½″	−0.0005	200	33
Carbon	Drill rod	Polished	½″ and over	±0.0005	220	36
Carbon	Drill rod	Lime Drawn	½″ and under	±0.0005	200	33
Carbon	Drill rod	Lime Drawn	Over ½″	±0.001	220	36
Carbon	Round	Rough Drawn	½″ and under	±0.001	210	35
Carbon	Round	Rough Drawn	Over ½″	±0.0015	228	38
Carbon	Round	Rough Drawn Annealed	½″ and under	±0.001	190	32
Carbon	Round	Rough Drawn Annealed	Over ½″	±0.0015	210	35
Carbon	Flats or Shapes	Lime Drawn	All dimensions	±0.001	220	36
Carbon	Flats or Shapes	Rough Drawn	All dimensions	±0.0015	228	38
High Speed	Drill Rod	Polished	All dimensions	±0.0005	250	42
High Speed	Drill Rod	Lime Drawn	All dimensions	±0.0005	250	42
High Speed	Rounds	Rough Drawn	All dimensions	±0.001	250	42
High Speed	Flats or Shapes	Lime Drawn	All dimensions	±0.001	250	42
High Speed	Flats or Shapes	Rough Drawn	All dimensions	±0.0015	250	42

* Atlas Crucible Steel Co.

Thickness Tolerances for Brass and Bronze Sheets and Strips

(S. A. E. Standard)

Thickness of Stock, American or Brown & Sharpe Gage	Tolerances for Different Widths: Up to 5 in., inc.	5 to 8 in., inc.	8 to 11 in., inc.	11 to 14 in., inc.
No. 0000 to No. 0 inc. (0.4600–0.3249)	±0.0044″	±0.0048″	±0.0051″	±0.0055″
Below No. 0 to No. 4 inc. (0.3249–0.2043)	±0.0039	±0.0043	±0.0046	±0.0050
Below No. 4 to No. 8 inc. (0.2043–0.1285)	±0.0034	±0.0038	±0.0041	±0.0045
Below No. 8 to No. 14 inc. (0.1285–0.0641)	±0.0029	±0.0033	±0.0036	±0.0040
Below No. 14 to No. 18 inc. (0.0641–0.0403)	±0.0025	±0.0029	±0.0033	±0.0037
Below No. 18 to No. 24 inc. (0.0403–0.0201)	±0.0020	±0.0024	±0.0028	±0.0032
Below No. 24 to No. 28 inc. (0.0201–0.0126)	±0.0016	±0.0020	±0.0024	±0.0028
Below No. 28 to No. 32 inc. (0.0126–0.0080)	±0.0013	±0.0017	±0.0020	±0.0024
Below No. 32 to No. 35 inc. (0.0080–0.0056)	±0.0010	±0.0014	±0.0017	±0.0022
Below No. 35 to No. 38 inc. (0.0056–0.0040)	±0.0008	±0.0012	±0.0015	±0.0019

Screw-stock Tolerances

(Adopted by American Society for Testing Materials)

Diameter of Screw Stock	Over Size	Under Size	Eccentricity
Up to and including 0.3 inch	0	1% of diam.	0.5% of diam.
Over 0.3 inch to and including 1 inch	0	0.003 inch	0.0015 inch
Over 1 inch to and including 2½ inches	0	0.004 inch	0.0020 inch
Over 2½ inches	0	0.005 inch	0.0025 inch

eters from 1 inch to 2 inches; and 0.015 inch for larger diameters. Finish turning: Tolerance of 0.002 inch for diameters from ¼ to ½ inch; 0.003 inch for diameters from ½ to 1 inch; 0.005 inch for diameters from 1 inch to 2 inches; and 0.007 inch for larger diameters. The most accurate as well as most economical method of finishing many classes of cylindrical work is by grinding, so that accurate lathe work prior to grinding is not necessary.

Automatic Screw Machine Work: — Tolerance of 0.003 inch for turning with box tools; tolerance of 0.003 inch for forming tools less than ¾ inch wide; and 0.004 inch for widths between ¾ inch and 1½ inches. Tolerance of 0.006 inch for hollow milling from 3/16 to ½ inch diameter; 0.008 inch from ½ to ¾ inch diameter; and 0.010 inch from ¾ to 1 inch diameter. For drilling, tolerances range from 0.002 inch for drills from No. 60 to No. 30, to about 0.007 inch for drills from ¾ to 1 inch in diameter. Reaming enables the tolerance to be reduced to 0.001 for sizes up to ½ inch diameter and to 0.0015 for sizes from ½ to 1 inch.

Milling Operations: — While a tolerance of 0.002 inch is feasible, it should, if possible, be increased to 0.004 or 0.005 inch to secure greater economy in manufacturing. If a single surface is to be milled, the tolerance may be from 0.002 to 0.003 inch; but when there are two or more surfaces to be milled, all but the most important one should be given tolerances of about 0.005 inch, if practicable. The tolerance for straddle-milling may be 0.003 inch; for form-milling, 0.005 inch; for end-milling, when the depth of the slot is little deeper than the mill diameter, 0.004 inch for widths from ¼ to ½ inch, 0.006 inch for widths from ½ to ¾ inch, and 0.008 inch for widths from ¾ to 1 inch. Somewhat greater tolerances should be allowed for hand-milling than for power-milling, because the feeding motion is not so even.

Drilling: — For drills from Nos. 60 to 30, 0.002 inch tolerance; for Nos. 30 to 1, 0.003 inch tolerance; for drill diameters from ¼ to ½ inch, 0.004 inch tolerance; for diameters from ½ to ¾ inch, 0.005 inch tolerance; for diameters from ¾ to 1 inch, 0.007 inch tolerance; and for diameters from 1 inch to 2 inches, 0.010 inch tolerance.

Grinding: — Cylindrical and surface grinding, 0.0005 inch tolerance. Vertical surface-grinding machine, 0.002 inch, which tolerance may be reduced to 0.001 inch under favorable conditions.

Planing Operations: — Tolerances varying from 0.005 to 0.010 inch may be maintained in planing comparatively large parts, such as machine tool slides, etc.

Thread-cutting: — When screw threads are cut in a lathe, tolerances on the pitch diameter of from 0.0015 to 0.002 inch may be maintained. When milling screw threads, it is possible, with a machine extremely well taken care of and when using a very accurate form of cutter, to maintain a tolerance of 0.001 inch for short pieces on the pitch diameter, and a tolerance of 0.002 inch on the outside and bottom diameter; but it is impracticable to give tolerances for interchangeable manufacture more accurate than 0.002 inch for the pitch diameter and 0.004 inch for the outside and bottom diameter. Tolerances on the outside diameter refer only to Whitworth or other threads with a formed top of thread. The larger tolerance given for the outside diameter does not affect the accuracy or working of the thread, because the apex of the thread is of little value and the important dimension is the pitch diameter.

Hand- and Machine-reaming: — For diameters up to 1 inch, a tolerance of 0.0004 inch may be maintained for hand-reaming; for diameters above 1 inch, the tolerance may be 0.0006 inch. These tolerances are increased somewhat for machine-reaming. For diameters up to ½ inch, the tolerance may be about 0.0005 inch; for diameters from ½ to 1 inch, from 0.00075 inch to 0.001 inch; and for diameters above 1 inch, 0.0015 inch.

MEASURING INSTRUMENTS AND GAGING METHODS

Reading a Vernier. — A general rule for taking readings with a vernier scale is as follows: Note the number of inches and sub-divisions of an inch that the zero mark of the vernier scale has moved along the true scale, and then add to this reading as many thousandths, or hundredths, or whatever fractional part of an inch the vernier reads to, as there are spaces between the vernier zero and that line on the vernier which coincides with one on the true scale. For example, if the zero line of a vernier which reads to thousandths is slightly beyond the 0.5 inch division on the main or true scale, as shown in Fig. 1, and graduation line 10 on the vernier exactly coincides with one on the true scale, the reading is 0.5 + 0.010 or 0.510 inch. In order to determine the reading or fractional part of an inch that can be obtained by a vernier, multiply the denominator of the finest sub-division given on the true scale by the total number of divisions on the vernier. For example, if one inch on the true scale is divided into 40 parts or fortieths (as in Fig. 1), and the vernier into twenty-five parts, the vernier will read to thousandths of an inch, as 25 × 40 = 1000. Similarly, if there are sixteen divisions to the inch on the true scale and a total of eight on the vernier, the latter will enable readings within one-hundred-twenty-eighths of an inch to be taken, as 8 × 16 = 128.

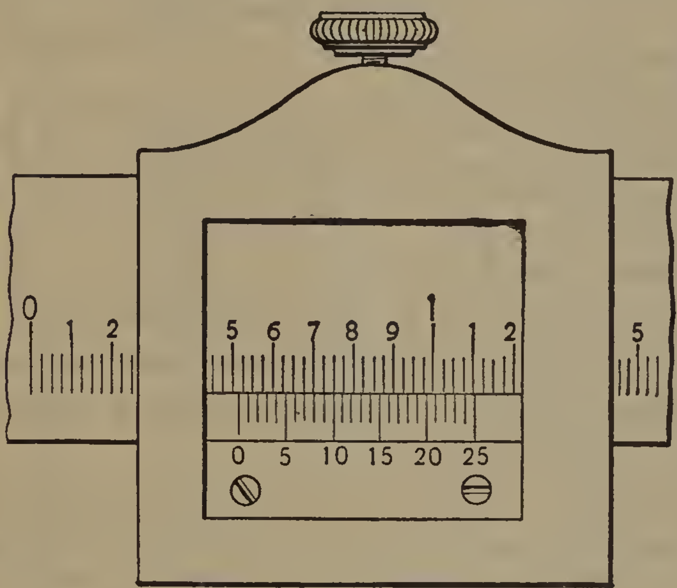


Fig.1

Fig.2

If the vernier is on a protractor, note the whole number of degrees passed by the vernier zero mark and then count the spaces between the vernier zero and that line which coincides with a graduation on the protractor scale. If the vernier indicates angles within five minutes or one-twelfth degree (as in Fig. 2), the number of spaces multiplied by 5 will, of course, give the number of minutes to be added to the whole number of degrees. The reading of the protractor set as illustrated would be 14 whole degrees (the number passed by the zero mark on the vernier) plus 30 minutes, as the graduation 30 on the vernier is the only one to the right of the vernier zero which exactly coincides with a line on the protractor scale. It will be noted that there are duplicate scales on the vernier, one being to the right and the other to the left of zero. The left-hand scale is used when the vernier zero is moved to the left of the zero of the protractor scale, whereas the right-hand graduations are used when the movement is to the right.

Reading a Micrometer. — To read a micrometer, count the number of whole divisions that are visible on the scale of the frame, multiply this number by 25 (the number of thousandths of an inch that each division represents) and add to the product the number of that division on the thimble which coincides with the axial zero line on the frame. The result will be the diameter expressed in thousandths of an inch. As the numbers 1, 2, 3, etc., opposite every fourth sub-division on the frame, indicate hundreds of thousandths, the reading can easily be taken mentally. Suppose the thimble were screwed out so that graduation 2, and three additional sub-divisions, were visible (as shown in Fig. 1), and that graduation 10 on the thimble coincided with the axial line on the frame. The reading then would be 0.200 + 0.075 + 0.010, or 0.285 inch.

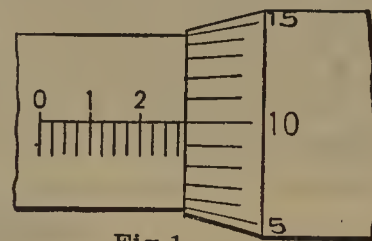


Fig. 1

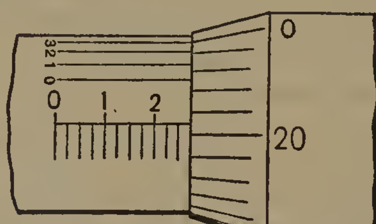
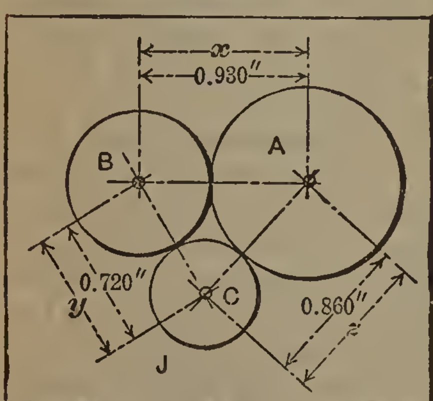


Fig. 2

Some micrometers have a vernier scale on the frame in addition to the regular graduations, so that measurements within 0.0001 part of an inch can be taken. Micrometers of this type are read as follows: First determine the number of thousandths, as with an ordinary micrometer, and then find a line on the vernier scale that exactly coincides with one on the thimble; the number of this line represents the number of ten-thousandths to be added to the number of thousandths obtained by the regular graduations. The reading shown in the illustration, Fig. 2, is 0.270 + 0.0003 = 0.2703 inch.

Micrometers graduated according to the English system of measurement ordinarily have a table of decimal equivalents stamped on the sides of the frame, so that fractions such as sixty-fourths, thirty-seconds, etc., can readily be converted into decimals. The decimal equivalent table is omitted on micrometers graduated according to the metric system, since all divisions in this system are decimal.

Locating Holes by the Disk Method.— When machining holes in comparatively small precision work, three carefully centered disks are sometimes used to align the respective holes with the lathe spindle. These disks are made to such diameters that when their peripheries are in contact, each disk center will coincide with the position of the hole to be machined; the centers are then used for locating the work. The diameters of the disks can be found as follows: Subtract dimension y from x, thus obtaining the difference between the radii of disks C and A; add this difference to dimension z to obtain the diameter of disk A; subtract the radius of disk A from the center distance x to obtain the radius of B; subtract the radius of B from dimension y to obtain the radius of C.



Example: $0.930 - 0.720 = 0.210$, or the difference between the radii of disks C and A. The diameter of A equals $0.210 + 0.860 = 1.070$ inch, and the radius equals $1.070 \div 2 = 0.535$ inch. The radius of $B = 0.930 - 0.535 = 0.395$ inch. The radius of $C = 0.720 - 0.395 = 0.325$.

The center-to-center distance between two disks is determined accurately by first measuring the over-all distance with a micrometer or vernier caliper, and then deducting the radius of each disk from the over-all dimension.

Constants for Setting a 5-inch Sine-bar

Min.	0°	1°	2°	3°	4°	5°	6°	7°
0	0.00000	0.08725	0.17450	0.26170	0.34880	0.43580	0.52265	0.60935
1	.00145	.08870	.17595	.26315	.35025	.43725	.52410	.61080
2	.00290	.09015	.17740	.26460	.35170	.43870	.52555	.61225
3	.00435	.09160	.17885	.26605	.35315	.44015	.52700	.61370
4	.00580	.09310	.18030	.26750	.35460	.44155	.52845	.61510
5	0.00725	0.09455	0.18175	0.26895	0.35605	0.44300	0.52985	0.61655
6	.00875	.09600	.18320	.27040	.35750	.44445	.53130	.61800
7	.01020	.09745	.18465	.27185	.35895	.44590	.53275	.61945
8	.01165	.09890	.18615	.27330	.36040	.44735	.53420	.62090
9	.01310	.10035	.18760	.27475	.36185	.44880	.53565	.62235
10	0.01455	0.10180	0.18905	0.27620	0.36330	0.45025	0.53710	0.62380
11	.01600	.10325	.19050	.27765	.36475	.45170	.53855	.62520
12	.01745	.10470	.19195	.27910	.36620	.45315	.54000	.62665
13	.01890	.10615	.19340	.28055	.36765	.45460	.54145	.62810
14	.02035	.10760	.19485	.28200	.36910	.45605	.54290	.62955
15	0.02180	0.10905	0.19630	0.28345	0.37055	0.45750	0.54435	0.63100
16	.02325	.11055	.19775	.28490	.37200	.45895	.54580	.63245
17	.02475	.11200	.19920	.28635	.37345	.46040	.54725	.63390
18	.02620	.11345	.20065	.28780	.37490	.46185	.54865	.63530
19	.02765	.11490	.20210	.28925	.37635	.46330	.55010	.63675
20	0.02910	0.11635	0.20355	0.29070	0.37780	0.46475	0.55155	0.63820
21	.03055	.11780	.20500	.29220	.37925	.46620	.55300	.63965
22	.03200	.11925	.20645	.29365	.38070	.46765	.55445	.64110
23	.03345	.12070	.20795	.29510	.38215	.46910	.55590	.64255
24	.03490	.12215	.20940	.29655	.38360	.47055	.55735	.64400
25	0.03635	0.12360	0.21085	0.29800	0.38505	0.47200	0.55880	0.64540
26	.03780	.12505	.21230	.29945	.38650	.47345	.56025	.64685
27	.03925	.12650	.21375	.30090	.38795	.47490	.56170	.64830
28	.04070	.12800	.21520	.30235	.38940	.47635	.56315	.64975
29	.04220	.12945	.21665	.30380	.39085	.47780	.56455	.65120
30	0.04365	0.13090	0.21810	0.30525	0.39230	0.47925	0.56600	0.65265
31	.04510	.13235	.21955	.30670	.39375	.48070	.56745	.65405
32	.04655	.13380	.22100	.30815	.39520	.48210	.56890	.65550
33	.04800	.13525	.22245	.30960	.39665	.48355	.57035	.65695
34	.04945	.13670	.22390	.31105	.39810	.48500	.57180	.65840
35	0.05090	0.13815	0.22535	0.31250	0.39955	0.48645	0.57325	0.65985
36	.05235	.13960	.22680	.31395	.40100	.48790	.57470	.66130
37	.05380	.14105	.22825	.31540	.40245	.48935	.57615	.66270
38	.05525	.14250	.22970	.31685	.40390	.49080	.57760	.66415
39	.05670	.14395	.23115	.31830	.40535	.49225	.57900	.66560
40	0.05820	0.14540	0.23265	0.31975	0.40680	0.49370	0.58045	0.66705
41	.05965	.14690	.23410	.32120	.40825	.49515	.58190	.66850
42	.06110	.14835	.23555	.32265	.40970	.49660	.58335	.66995
43	.06255	.14980	.23700	.32410	.41115	.49805	.58480	.67135
44	.06400	.15125	.23845	.32555	.41260	.49950	.58625	.67280
45	0.06545	0.15270	0.23990	0.32700	0.41405	0.50095	0.58770	0.67425
46	.06690	.15415	.24135	.32845	.41550	.50240	.58915	.67570
47	.06835	.15560	.24280	.32990	.41695	.50385	.59060	.67715
48	.06980	.15705	.24425	.33135	.41840	.50530	.59200	.67860
49	.07125	.15850	.24570	.33280	.41985	.50675	.59345	.68000
50	0.07270	0.15995	0.24715	0.33425	0.42130	0.50820	0.59490	0.68145
51	.07415	.16140	.24860	.33570	.42275	.50960	.59635	.68290
52	.07565	.16285	.25005	.33715	.42420	.51105	.59780	.68435
53	.07710	.16430	.25150	.33865	.42565	.51250	.59925	.68580
54	.07855	.16580	.25295	.34010	.42710	.51395	.60070	.68720
55	0.08000	0.16725	0.25440	0.34155	0.42855	0.51540	0.60215	0.68865
56	.08145	.16870	.25585	.34300	.43000	.51685	.60355	.69010
57	.08290	.17015	.25730	.34445	.43145	.51830	.60500	.69155
58	.08435	.17160	.25875	.34590	.43290	.51975	.60645	.69300
59	.08580	.17305	.26028	.34735	.43435	.52120	.60790	.69445
60	0.08725	0.17450	0.26170	0.34880	0.43580	0.52265	0.60935	0.69585

Constants for Setting a 5-inch Sine-bar

Min.	8°	9°	10°	11°	12°	13°	14°	15°
0	0.69585	0.78215	0.86825	0.95405	1.0395	1.1247	1.2096	1.2941
1	.69730	.78360	.86965	.95545	.0410	.1261	.2110	.2955
2	.69875	.78505	.87110	.95690	.0424	.1276	.2124	.2969
3	.70020	.78650	.87255	.95835	.0438	.1290	.2138	.2983
4	.70165	.78790	.87395	.95975	.0452	.1304	.2152	.2997
5	0.70305	0.78935	0.87540	0.96120	1.0466	1.1318	1.2166	1.3011
6	.70450	.79080	.87685	.96260	0481	.1332	.2181	.3025
7	.70595	.79225	.87825	.96405	.0495	.1346	.2195	.3039
8	.70740	.79365	.87970	.96545	.0509	.1361	.2209	.3053
9	.70885	.79510	.88115	.96690	.0523	.1375	.2223	.3067
10	0.71025	0.79655	0.88255	0.96830	1.0538	1.1389	1.2237	1.3081
11	.71170	.79795	.88400	.96975	.0552	.1403	.2251	.3095
12	.71315	.79940	.88540	.97115	.0566	.1417	.2265	.3109
13	.71460	.80085	.88685	.97260	.0580	.1431	.2279	.3123
14	.71600	.80230	.88830	.97405	.0594	.1446	.2293	.3137
15	0.71745	0.80370	0.88970	0.97545	1.0609	1 1460	1.2307	1.3151
16	.71890	.80515	.89115	.97690	.0623	.1474	.2322	.3165
17	.72035	.80660	.89260	.97830	.0637	.1488	.2336	.3179
18	.72180	.80800	.89400	.97975	.0651	.1502	.2350	.3193
19	.72320	.80945	.89545	.98115	.0665	.1516	.2364	.3207
20	0.72465	0.81090	0.89685	0.98260	1.0680	1.1531	1.2378	1.3221
21	.72610	.81230	.89830	.98400	.0694	.1545	.2392	.3235
22	.72755	.81375	.89975	.98545	.0708	.1559	.2406	.3250
23	.72900	.81520	.90115	.98685	.0722	.1573	.2420	.3264
24	.73040	.81665	.90260	.98830	.0737	.1587	.2434	.3278
25	0.73185	0.81805	0.90405	0.98970	1.0751	1.1601	1.2448	1.3292
26	.73330	.81950	.90545	.99115	.0765	.1615	.2462	.3306
27	.73475	.82095	.90690	.99255	.0779	.1630	.2477	.3320
28	.73615	.82235	.90830	.99400	.0793	.1644	.2491	.3334
29	.73760	.82380	.90975	.99540	.0808	.1658	.2505	.3348
30	0.73905	0.82525	0.91120	0.99685	1.0822	1.1672	1.2519	1.3362
31	.74050	.82665	.91260	.99825	.0836	.1686	.2533	.3376
32	.74190	.82810	.91405	.99970	.0850	.1700	.2547	.3390
33	.74335	.82955	.91545	1.0011	.0864	.1714	.2561	.3404
34	.74480	.83100	.91690	.0016	.0879	.1729	.2575	.3418
35	0.74625	0.83240	0.91835	1.0039	1.0893	1.1743	1.2589	1.3432
36	.74770	.83385	.91975	.0054	.0907	.1757	.2603	.3446
37	.74910	.83530	.92120	.0068	.0921	.1771	.2617	.3460
38	.75055	.83670	.92260	.0082	.0935	.1785	.2631	.3474
39	.75200	.83815	.92405	.0096	.0949	.1799	.2645	.3488
40	0.75345	0.83960	0.92545	1.0110	1.0964	1.1813	1.2660	1.3502
41	.75485	.84100	.92690	.0125	.0978	.1828	.2674	.3516
42	.75630	.84245	.92835	.0139	.0992	.1842	.2688	.3530
43	.75775	.84390	.92975	.0153	.1006	.1856	.2702	.3544
44	.75920	.84530	.93120	.0168	.1020	.1870	.2716	.3558
45	0.76060	0.84675	0.93260	1.0182	1.1035	1.1884	1.2730	1.3572
46	.76205	.84820	.93405	.0196	.1049	.1898	.2744	.3586
47	.76350	.84960	.93550	.0210	.1063	.1912	.2758	.3600
48	.76495	.85105	.93690	.0225	.1077	.1926	.2772	.3614
49	.76635	.85250	.93835	.0239	.1091	.1941	.2786	.3628
50	0.76780	0.85390	0.93975	1.0253	1.1106	1.1955	1.2800	1.3642
51	.76925	.85535	.94120	.0267	.1120	.1969	.2814	.3656
52	.77070	.85680	.94260	.0281	.1134	.1983	.2828	.3670
53	.77210	.85820	.94405	.0296	.1148	.1997	.2842	.3684
54	.77355	.85965	.94550	.0310	.1162	.2011	.2856	.3698
55	0.77500	0.86110	0.94690	1.0324	1.1176	1.2025	1.2870	1.3712
56	.77645	.86250	.94835	.0338	.1191	.2039	.2884	.3726
57	.77785	.86395	.94975	.0353	.1205	.2054	.2899	.3740
58	.77930	.86540	.95120	.0367	.1219	.2068	.2913	.3754
59	.78075	.86680	.95260	.0381	.1233	.2082	.2927	.3768
60	0.78215	0.86825	0.95405	1.0395	1.1247	1.2096	1.2941	1.3782

Constants for Setting a 5-inch Sine-bar

Min.	16°	17°	18°	19°	20°	21°	22°	23°
0	1.3782	1.4618	1.5451	1.6278	1.7101	1.7918	1.8730	1.9536
1	.3796	.4632	.5464	.6292	.7114	.7932	.8744	.9550
2	.3810	.4646	.5478	.6306	.7128	.7945	.8757	.9563
3	.3824	.4660	.5492	.6319	.7142	.7959	.8771	.9576
4	.3838	.4674	.5506	.6333	.7155	.7972	.8784	.9590
5	1.3852	1.4688	1.5520	1.6347	1.7169	1.7986	1.8797	1.9603
6	.3865	.4702	.5534	.6361	.7183	.8000	.8811	.9617
7	.3879	.4716	.5547	.6374	.7196	.8013	.8824	.9630
8	.3893	.4730	.5561	.6388	.7210	.8027	.8838	.9643
9	.3907	.4743	.5575	.6402	.7224	.8040	.8851	.9657
10	1.3921	1.4757	1.5589	1.6416	1.7237	1.8054	1.8865	1.9670
11	.3935	.4771	.5603	.6429	.7251	.8067	.8878	.9683
12	.3949	.4785	.5616	.6443	.7265	.8081	.8892	.9697
13	.3963	.4799	.5630	.6457	.7278	.8094	.8905	.9710
14	.3977	.4813	.5644	.6471	.7292	.8108	.8919	.9724
15	1.3991	1.4827	1.5658	1.6484	1.7306	1.8122	1.8932	1.9737
16	.4005	.4841	.5672	.6498	.7319	.8135	.8946	.9750
17	.4019	.4855	.5686	.6512	.7333	.8149	.8959	.9764
18	.4033	.4868	.5699	.6525	.7347	.8162	.8973	.9777
19	.4047	.4882	.5713	.6539	.7360	.8176	.8986	.9790
20	1.4061	1.4896	1.5727	1.6553	1.7374	1.8189	1.8999	1.9804
21	.4075	.4910	.5741	.6567	.7387	.8203	.9013	.9817
22	.4089	.4924	.5755	.6580	.7401	.8217	.9026	.9830
23	.4103	.4938	.5768	.6594	.7415	.8230	.9040	.9844
24	.4117	.4952	.5782	.6608	.7428	.8244	.9053	.9857
25	1.4131	1.4966	1.5796	1.6622	1.7442	1.8257	1.9067	1.9870
26	.4145	.4980	.5810	.6635	.7456	.8271	.9080	.9884
27	.4159	.4993	.5824	.6649	.7469	.8284	.9094	.9897
28	.4173	.5007	.5837	.6663	.7483	.8298	.9107	.9911
29	.4187	.5021	.5851	.6676	.7496	.8311	.9120	.9924
30	1.4201	1.5035	1.5865	1.6690	1.7510	1.8325	1.9134	1.9937
31	.4214	.5049	.5879	.6704	.7524	.8338	.9147	.9951
32	.4228	.5063	.5893	.6718	.7537	.8352	.9161	.9964
33	.4242	.5077	.5906	.6731	.7551	.8365	.9174	.9977
34	.4256	.5091	.5920	.6745	.7565	.8379	.9188	.9991
35	1.4270	1.5104	1.5934	1.6759	1.7578	1.8392	1.9201	2.0004
36	.4284	.5118	.5948	.6772	.7592	.8406	.9215	.0017
37	.4298	.5132	.5961	.6786	.7605	.8419	.9228	.0031
38	.4312	.5146	.5975	.6800	.7619	.8433	.9241	.0044
39	.4326	.5160	.5989	.6813	.7633	.8447	.9255	.0057
40	1.4340	1.5174	1.6003	1.6827	1.7646	1.8460	1.9268	2.0070
41	.4354	.5188	.6017	.6841	.7660	.8474	.9282	.0084
42	.4368	.5201	.6030	.6855	.7673	.8487	.9295	.0097
43	.4382	.5215	.6044	.6868	.7687	.8501	.9308	.0110
44	.4396	.5229	.6058	.6882	.7701	.8514	.9322	.0124
45	1.4410	1.5243	1.6072	1.6896	1.7714	1.8528	1.9335	2.0137
46	.4423	.5257	.6085	.6909	.7728	.8541	.9349	.0150
47	.4437	.5271	.6099	.6923	.7742	.8555	.9362	.0164
48	.4451	.5285	.6113	.6937	.7755	.8568	.9376	.0177
49	.4465	.5298	.6127	.6950	.7769	.8582	.9389	.0190
50	1.4479	1.5312	1.6141	1.6964	1.7782	1.8595	1.9402	2.0204
51	.4493	.5326	.6154	.6978	.7796	.8609	.9416	.0217
52	.4507	.5340	.6168	.6991	.7809	.8622	.9429	.0230
53	.4521	.5354	.6182	.7005	.7823	.8636	.9443	.0244
54	.4535	.5368	.6196	.7019	.7837	.8649	.9456	.0257
55	1.4549	1.5381	1.6209	1.7032	1.7850	1.8663	1.9469	2.0270
56	.4563	.5395	.6223	.7046	.7864	.8676	.9483	.0283
57	.4577	.5409	.6237	.7060	.7877	.8690	.9496	.0297
58	.4591	.5423	.6251	.7073	.7891	.8703	.9510	.0310
59	.4604	.5437	.6264	.7087	.7905	.8717	.9523	.0323
60	1.4618	1.5451	1.6278	1.7101	1.7918	1.8730	1.9536	2.0337

Constants for Setting a 5-inch Sine-bar

Min.	24°	25°	26°	27°	28°	29°	30°	31°
0	2.0337	2.1131	2.1918	2.2699	2.3473	2.4240	2.5000	2.5752
1	.0350	.1144	.1931	.2712	.3486	.4253	.5012	.5764
2	.0363	.1157	.1944	.2725	.3499	.4266	.5025	.5777
3	.0376	.1170	.1958	.2738	.3512	.4278	.5038	.5789
4	.0390	.1183	.1971	.2751	.3525	.4291	.5050	.5802
5	2.0403	2.1197	2.1984	2.2764	2.3538	2.4304	2.5063	2.5814
6	.0416	.1210	.1997	.2777	.3550	.4317	.5075	.5826
7	.0430	.1223	.2010	.2790	.3563	.4329	.5088	.5839
8	.0443	.1236	.2023	.2803	.3576	.4342	.5100	.5851
9	.0456	.1249	.2036	.2816	.3589	.4355	.5113	.5864
10	2.0469	2.1262	2.2049	2.2829	2.3602	2.4367	2.5126	2.5876
11	.0483	.1276	.2062	.2842	.3614	.4380	.5138	.5889
12	.0496	.1289	.2075	.2855	.3627	.4393	.5151	.5901
13	.0509	.1302	.2088	.2868	.3640	.4405	.5163	.5914
14	.0522	.1315	.2101	.2881	.3653	.4418	.5176	.5926
15	2.0536	2.1328	2.2114	2.2893	2.3666	2.4431	2.5188	2.5938
16	.0549	.1341	.2127	.2906	.3679	.4444	.5201	.5951
17	.0562	.1354	.2140	.2919	.3691	.4456	.5214	.5963
18	.0575	.1368	.2153	.2932	.3704	.4469	.5226	.5976
19	.0589	.1381	.2166	.2945	.3717	.4482	.5239	.5988
20	2.0602	2.1394	2.2179	2.2958	2.3730	2.4494	2.5251	2.6001
21	.0615	.1407	.2192	.2971	.3743	.4507	.5264	.6013
22	.0628	.1420	.2205	.2984	.3755	.4520	.5276	.6025
23	.0642	.1433	.2218	.2997	.3768	.4532	.5289	.6038
24	.0655	.1447	.2232	.3010	.3781	.4545	.5301	.6050
25	2.0668	2.1460	2.2245	2.3023	2.3794	2.4558	2.5314	2.6063
26	.0681	.1473	.2258	.3036	.3807	.4570	.5327	.6075
27	.0695	.1486	.2271	.3048	.3819	.4583	.5339	.6087
28	.0708	.1499	.2284	.3061	.3832	.4596	.5352	.6100
29	.0721	.1512	.2297	.3074	.3845	.4608	.5364	.6112
30	2.0734	2.1525	2.2310	2.3087	2.3858	2.4621	2.5377	2.6125
31	.0748	.1538	.2323	.3100	.3870	.4634	.5389	.6137
32	.0761	.1552	.2336	.3113	.3883	.4646	.5402	.6149
33	.0774	.1565	.2349	.3126	.3896	.4659	.5414	.6162
34	.0787	.1578	.2362	.3139	.3909	.4672	.5427	.6174
35	2.0801	2.1591	2.2375	2.3152	2.3922	2.4684	2.5439	2.6187
36	.0814	.1604	.2388	.3165	.3934	.4697	.5452	.6199
37	.0827	.1617	.2401	.3177	.3947	.4709	.5464	.6211
38	.0840	.1630	.2414	.3190	.3960	.4722	.5477	.6224
39	.0853	.1643	.2427	.3203	.3973	.4735	.5489	.6236
40	2.0867	2.1656	2.2440	2.3216	2.3985	2.4747	2.5502	2.6249
41	.0880	.1670	.2453	.3229	.3998	.4760	.5514	.6261
42	.0893	.1683	.2466	.3242	.4011	.4773	.5527	.6273
43	.0906	.1696	.2479	.3255	.4024	.4785	.5539	.6286
44	.0920	.1709	.2492	.3268	.4036	.4798	.5552	.6298
45	2.0933	2.1722	2.2505	2.3280	2.4049	2.4811	2.5564	2.6310
46	.0946	.1735	.2518	.3293	.4062	.4823	.5577	.6323
47	.0959	.1748	.2531	.3306	.4075	.4836	.5589	.6335
48	.0972	.1761	.2544	.3319	.4087	.4848	.5602	.6348
49	.0986	.1774	.2557	.3332	.4100	.4861	.5614	.6360
50	2.0999	2.1787	2.2570	2.3345	2.4113	2.4874	2.5627	2.6372
51	.1012	.1801	.2583	.3358	.4126	.4886	.5639	.6385
52	.1025	.1814	.2596	.3371	.4138	.4899	.5652	.6397
53	.1038	.1827	.2609	.3383	.4151	.4912	.5664	.6409
54	.1052	.1840	.2621	.3396	.4164	.4924	.5677	.6422
55	2.1065	2.1853	2.2634	2.3409	2.4177	2.4937	2.5689	2.6434
56	.1078	.1866	.2647	.3422	.4189	.4949	.5702	.6446
57	.1091	.1879	.2660	.3435	.4202	.4962	.5714	.6459
58	.1104	.1892	.2673	.3448	.4215	.4975	.5727	.6471
59	.1117	.1905	.2686	.3460	.4228	.4987	.5739	.6483
60	2.1131	2.1918	2.2699	2.3473	2.4240	2.5000	2.5752	2.6496

Constants for Setting a 5-inch Sine-bar

Min.	32°	33°	34°	35°	36°	37°	38°	39°
0	2.6496	2.7232	2.7959	2.8679	2.9389	3.0091	3.0783	3.1466
1	.6508	.7244	.7971	.8690	.9401	.0102	.0794	.1477
2	.6520	.7256	.7984	.8702	.9413	.0114	.0806	.1488
3	.6533	.7268	.7996	.8714	.9424	.0125	.0817	.1500
4	.6545	.7280	.8008	.8726	.9436	.0137	.0829	.1511
5	2.6557	2.7293	2.8020	2.8738	2.9448	3.0149	3.0840	3.1522
6	.6570	.7305	.8032	.8750	.9460	.0160	.0852	.1534
7	.6582	.7317	.8044	.8762	.9471	.0172	.0863	.1545
8	.6594	.7329	.8056	.8774	.9483	.0183	.0874	.1556
9	.6607	.7341	.8068	.8786	.9495	.0195	.0886	.1567
10	2.6619	2.7354	2.8080	2.8798	2.9507	3.0207	3.0897	3.1579
11	.6631	.7366	.8092	.8809	.9518	.0218	.0909	.1590
12	.6644	.7378	.8104	.8821	.9530	.0230	.0920	.1601
13	.6656	.7390	.8116	.8833	.9542	.0241	.0932	.1612
14	.6668	.7402	.8128	.8845	.9554	.0253	.0943	.1624
15	2.6680	2.7414	2.8140	2.8857	2.9565	3.0264	3.0954	3.1635
16	.6693	.7427	.8152	.8869	.9577	.0276	.0966	.1646
17	.6705	.7439	.8164	.8881	.9589	.0288	.0977	.1658
18	.6717	.7451	.8176	.8893	.9600	.0299	.0989	.1669
19	.6730	.7463	.8188	.8905	.9612	.0311	.1000	.1680
20	2.6742	2.7475	2.8200	2.8916	2.9624	3.0322	3.1012	3.1691
21	.6754	.7487	.8212	.8928	.9636	.0334	.1023	.1703
22	.6767	.7499	.8224	.8940	.9647	.0345	.1034	.1714
23	.6779	.7512	.8236	.8952	.9659	.0357	.1046	.1725
24	.6791	.7524	.8248	.8964	.9671	.0369	.1057	.1736
25	2.6803	2.7536	2.8260	2.8976	2.9682	3.0380	3.1069	3.1748
26	.6816	.7548	.8272	.8988	.9694	.0392	.1080	.1759
27	.6828	.7560	.8284	.8999	.9706	.0403	.1091	.1770
28	.6840	.7572	.8296	.9011	.9718	.0415	.1103	.1781
29	.6852	.7584	.8308	.9023	.9729	.0426	.1114	.1792
30	2.6865	2.7597	2.8320	2.9035	2.9741	3.0438	3.1125	3.1804
31	.6877	.7609	.8332	.9047	.9753	.0449	.1137	.1815
32	.6889	.7621	.8344	.9059	.9764	.0461	.1148	.1826
33	.6902	.7633	.8356	.9070	.9776	.0472	.1160	.1837
34	.6914	.7645	.8368	.9082	.9788	.0484	.1171	.1849
35	2.6926	2.7657	2.8380	2.9094	2.9799	3.0495	3.1182	3.1860
36	.6938	.7669	.8392	.9106	.9811	.0507	.1194	.1871
37	.6951	.7681	.8404	.9118	.9823	.0519	.1205	.1882
38	.6963	.7694	.8416	.9130	.9834	.0530	.1216	.1893
39	.6975	.7706	.8428	.9141	.9846	.0542	.1228	.1905
40	2.6987	2.7718	2.8440	2.9153	2.9858	3.0553	3.1239	3.1916
41	.7000	.7730	.8452	.9165	.9869	.0565	.1251	.1927
42	.7012	.7742	.8464	.9177	.9881	.0576	.1262	.1938
43	.7024	.7754	.8476	.9189	.9893	.0588	.1273	.1949
44	.7036	.7766	.8488	.9200	.9904	.0599	.1285	.1961
45	2.7048	2.7778	2.8500	2.9212	2.9916	3.0611	3.1296	3.1972
46	.7061	.7790	.8512	.9224	.9928	.0622	.1307	.1983
47	.7073	.7802	.8523	.9236	.9939	.0634	.1319	.1994
48	.7085	.7815	.8535	.9248	.9951	.0645	.1330	.2005
49	.7097	.7827	.8547	.9259	.9963	.0657	.1341	.2016
50	2.7110	2.7839	2.8559	2.9271	2.9974	3.0668	3.1353	3.2028
51	.7122	.7851	.8571	.9283	.9986	.0680	.1364	.2039
52	.7134	.7863	.8583	.9295	.9997	.0691	.1375	.2050
53	.7146	.7875	.8595	.9307	3.0009	.0703	.1387	.2061
54	.7158	.7887	.8607	.9318	.0021	.0714	.1398	.2072
55	2.7171	2.7899	2.8619	2.9330	3.0032	3.0725	3.1409	3.2083
56	.7183	.7911	.8631	.9342	.0044	.0737	.1421	.2095
57	.7195	.7923	.8643	.9354	.0056	.0748	.1432	.2106
58	.7207	.7935	.8655	.9365	.0067	.0760	.1443	.2117
59	.7220	.7947	.8667	.9377	.0079	.0771	.1454	.2128
60	2.7232	2.7959	2.8679	2.9389	3.0091	3.0783	3.1466	3.2139

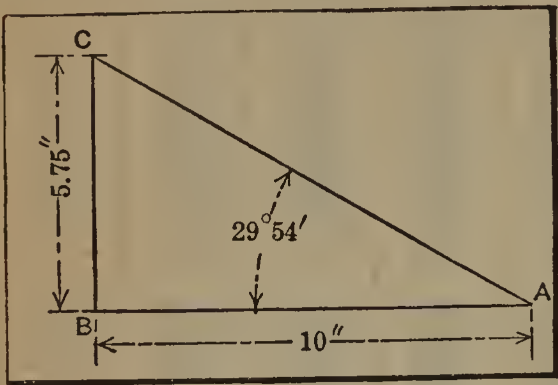
Constants for Setting a 5-inch Sine-bar

Min.	40°	41°	42°	43°	44°	45°	46°	47°
0	3.2139	3.2803	3.3456	3.4100	3.4733	3.5355	3.5967	3.6567
1	.2150	.2814	.3467	.4110	.4743	.5365	.5977	.6577
2	.2161	.2825	.3478	.4121	.4754	.5376	.5987	.6587
3	.2173	.2836	.3489	.4132	.4764	.5386	.5997	.6597
4	.2184	.2847	.3499	.4142	.4774	.5396	.6007	.6607
5	3.2195	3.2858	3.3510	3.4153	3.4785	3.5406	3.6017	3.6617
6	.2206	.2869	.3521	.4163	.4795	.5417	.6027	.6627
7	.2217	.2879	.3532	.4174	.4806	.5427	.6037	.6637
8	.2228	.2890	.3543	.4185	.4816	.5437	.6047	.6647
9	.2239	.2901	.3553	.4195	.4827	.5448	.6058	.6657
10	3.2250	3.2912	3.3564	3.4206	3.4837	3.5458	3.6068	3.6666
11	.2262	.2923	.3575	.4217	.4848	.5468	.6078	.6676
12	.2273	.2934	.3586	.4227	.4858	.5478	.6088	.6686
13	.2284	.2945	.3597	.4238	.4868	.5489	.6098	.6696
14	.2295	.2956	.3607	.4248	.4879	.5499	.6108	.6706
15	3.2306	3.2967	3.3618	3.4259	3.4889	3.5509	3.6118	3.6716
16	.2317	.2978	.3629	.4269	.4900	.5519	.6128	.6726
17	.2328	.2989	.3640	.4280	.4910	.5529	.6138	.6736
18	.2339	.3000	.3650	.4291	.4921	.5540	.6148	.6745
19	.2350	.3011	.3661	.4301	.4931	.5550	.6158	.6755
20	3.2361	3.3022	3.3672	3.4312	3.4941	3.5560	3.6168	3.6765
21	.2373	.3033	.3683	.4322	.4952	.5570	.6178	.6775
22	.2384	.3044	.3693	.4333	.4962	.5581	.6188	.6785
23	.2395	.3054	.3704	.4344	.4973	.5591	.6198	.6795
24	.2406	.3065	.3715	.4354	.4983	.5601	.6208	.6805
25	3.2417	3.3076	3.3726	3.4365	3.4993	3.5611	3.6218	3.6814
26	.2428	.3087	.3736	.4375	.5004	.5621	.6228	.6824
27	.2439	.3098	.3747	.4386	.5014	.5632	.6238	.6834
28	.2450	.3109	.3758	.4396	.5024	.5642	.6248	.6844
29	.2461	.3120	.3769	.4407	.5035	.5652	.6258	.6854
30	3.2472	3.3131	3.3779	3.4417	3.5045	3.5662	3.6268	3.6864
31	.2483	.3142	.3790	.4428	.5056	.5672	.6278	.6873
32	.2494	.3153	.3801	.4439	.5066	.5683	.6288	.6883
33	.2505	.3163	.3811	.4449	.5076	.5693	.6298	.6893
34	.2516	.3174	.3822	.4460	.5087	.5703	.6308	.6903
35	3.2527	3.3185	3.3833	3.4470	3.5097	3.5713	3.6318	3.6913
36	.2538	.3196	.3844	.4481	.5107	.5723	.6328	.6923
37	.2550	.3207	.3854	.4491	.5118	.5734	.6338	.6932
38	.2561	.3218	.3865	.4502	.5128	.5744	.6348	.6942
39	.2572	.3229	.3876	.4512	.5138	.5754	.6358	.6952
40	3.2583	3.3240	3.3886	3.4523	3.5149	3.5764	3.6368	3.6962
41	.2594	.3250	.3897	.4533	.5159	.5774	.6378	.6972
42	.2605	.3261	.3908	.4544	.5169	.5784	.6388	.6981
43	.2616	.3272	.3918	.4554	.5180	.5795	.6398	.6991
44	.2627	.3283	.3929	.4565	.5190	.5805	.6408	.7001
45	3.2638	3.3294	3.3940	3.4575	3.5200	3.5815	3.6418	3.7011
46	.2649	.3305	.3950	.4586	.5211	.5825	.6428	.7020
47	.2660	.3316	.3961	.4596	.5221	.5835	.6438	.7030
48	.2671	.3326	.3972	.4607	.5231	.5845	.6448	.7040
49	.2682	.3337	.3982	.4617	.5242	.5855	.6458	.7050
50	3.2693	3.3348	3.3993	3.4628	3.5252	3.5866	3.6468	3.7060
51	.2704	.3359	.4004	.4638	.5262	.5876	.6478	.7069
52	.2715	.3370	.4014	.4649	.5273	.5886	.6488	.7079
53	.2726	.3381	.4025	.4659	.5283	.5896	.6498	.7089
54	.2737	.3391	.4036	.4670	.5293	.5906	.6508	.7099
55	3.2748	3.3402	3.4046	3.4680	3.5304	3.5916	3.6518	3.7108
56	.2759	.3413	.4057	.4691	.5314	.5926	.6528	.7118
57	.2770	.3424	.4068	.4701	.5324	.5936	.6538	.7128
58	.2781	.3435	.4078	.4712	.5335	.5947	.6548	.7138
59	.2792	.3445	.4089	.4722	.5345	.5957	.6558	.7147
60	3.2803	3.3456	3.4100	3.4733	3.5355	3.5967	3.6567	3.7157

Constants for Setting a 5-inch Sine-bar

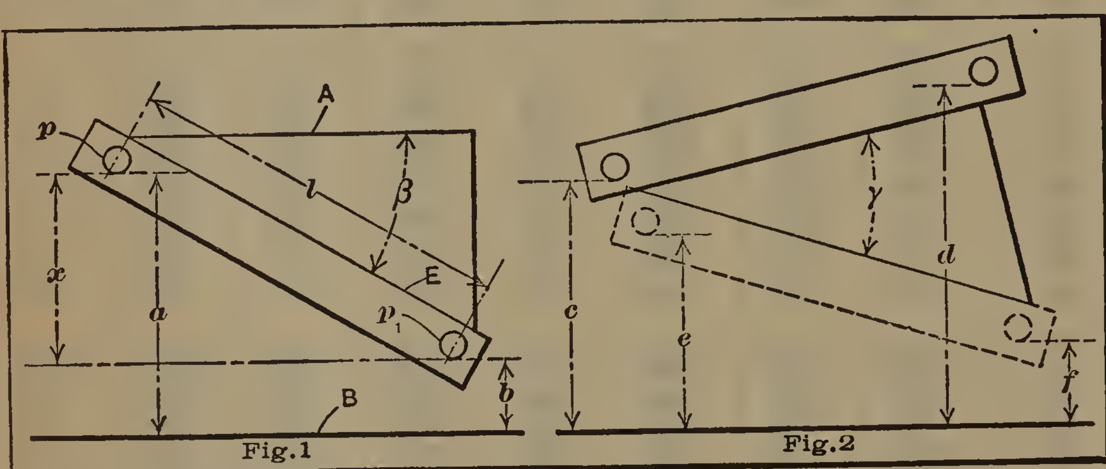
Min.	48°	49°	50°	51°	52°	53°	54°	55°
0	3.7157	3.7735	3.8302	3.8857	3.9400	3.9932	4.0451	4.0957
1	.7167	.7745	.8311	.8866	.9409	.9940	.0459	.0966
2	.7176	.7754	.8321	.8875	.9418	.9949	.0468	.0974
3	.7186	.7764	.8330	.8884	.9427	.9958	.0476	.0982
4	.7196	.7773	.8339	.8894	.9436	.9967	.0485	.0991
5	3.7206	3.7783	3.8349	3.8903	3.9445	3.9975	4.0493	4.0999
6	.7215	.7792	.8358	.8912	.9454	.9984	.0502	.1007
7	.7225	.7802	.8367	.8921	.9463	.9993	.0510	.1016
8	.7235	.7811	.8377	.8930	.9472	4.0001	.0519	.1024
9	.7244	.7821	.8386	.8939	.9481	.0010	.0527	.1032
10	3.7254	3.7830	3.8395	3.8948	3.9490	4.0019	4.0536	4.1041
11	.7264	.7840	.8405	.8958	.9499	.0028	.0544	.1049
12	.7274	.7850	.8414	.8967	.9508	.0036	.0553	.1057
13	.7283	.7859	.8423	.8976	.9516	.0045	.0561	.1066
14	.7293	.7869	.8433	.8985	.9525	.0054	.0570	.1074
15	3.7303	3.7878	3.8442	3.8994	3.9534	4.0062	4.0578	4.1082
16	.7312	.7887	.8451	.9003	.9543	.0071	.0587	.1090
17	.7322	.7897	.8460	.9012	.9552	.0080	.0595	.1099
18	.7332	.7906	.8470	.9021	.9561	.0089	.0604	.1107
19	.7341	.7916	.8479	.9030	.9570	.0097	.0612	.1115
20	3.7351	3.7925	3.8488	3.9039	3.9579	4.0106	4.0621	4.1124
21	.7361	.7935	.8498	.9049	.9588	.0115	.0629	.1132
22	.7370	.7944	.8507	.9058	.9596	.0123	.0638	.1140
23	.7380	.7954	.8516	.9067	.9605	.0132	.0646	.1148
24	.7390	.7963	.8525	.9076	.9614	.0141	.0655	.1157
25	3.7399	3.7973	3.8535	3.9085	3.9623	4.0149	4.0663	4.1165
26	.7409	.7982	.8544	.9094	.9632	.0158	.0672	.1173
27	.7419	.7992	.8553	.9103	.9641	.0167	.0680	.1181
28	.7428	.8001	.8562	.9112	.9650	.0175	.0689	.1190
29	.7438	.8011	.8572	.9121	.9659	.0184	.0697	.1198
30	3.7448	3.8020	3.8581	3.9130	3.9667	4.0193	4.0706	4.1206
31	.7457	.8029	.8590	.9139	.9676	.0201	.0714	.1214
32	.7467	.8039	.8599	.9148	.9685	.0210	.0722	.1223
33	.7476	.8048	.8609	.9157	.9694	.0219	.0731	.1231
34	.7486	.8058	.8618	.9166	.9703	.0227	.0739	.1239
35	3.7496	3.8067	3.8627	3.9175	3.9712	4.0236	4.0748	4.1247
36	.7505	.8077	.8636	.9184	.9720	.0244	.0756	.1255
37	.7515	.8086	.8646	.9193	.9729	.0253	.0765	.1264
38	.7525	.8096	.8655	.9202	.9738	.0262	.0773	.1272
39	.7534	.8105	.8664	.9212	.9747	.0270	.0781	.1280
40	3.7544	3.8114	3.8673	3.9221	3.9756	4.0279	4.0790	4.1288
41	.7553	.8124	.8683	.9230	.9765	.0288	.0798	.1296
42	.7563	.8133	.8692	.9239	.9773	.0296	.0807	.1305
43	.7573	.8143	.8701	.9248	.9782	.0305	.0815	.1313
44	.7582	.8152	.8710	.9257	.9791	.0313	.0823	.1321
45	3.7592	3.8161	3.8719	3.9266	3.9800	4.0322	4.0832	4.1329
46	.7601	.8171	.8729	.9275	.9809	.0331	.0840	.1337
47	.7611	.8180	.8738	.9284	.9817	.0339	.0849	.1346
48	.7620	.8190	.8747	.9293	.9826	.0348	.0857	.1354
49	.7630	.8199	.8756	.9302	.9835	.0356	.0865	.1362
50	3.7640	3.8208	3.8765	3.9311	3.9844	4.0365	4.0874	4.1370
51	.7649	.8218	.8775	.9320	.9853	.0374	.0882	.1378
52	.7659	.8227	.8784	.9329	.9861	.0382	.0891	.1386
53	.7668	.8236	.8793	.9338	.9870	.0391	.0899	.1395
54	.7678	.8246	.8802	.9347	.9879	.0399	.0907	.1403
55	3.7687	3.8255	3.8811	3.9355	3.9888	4.0408	4.0916	4.1411
56	.7697	.8265	.8820	.9364	.9896	.0416	.0924	.1419
57	.7707	.8274	.8830	.9373	.9905	.0425	.0932	.1427
58	.7716	.8283	.8839	.9382	.9914	.0433	.0941	.1435
59	.7726	.8293	.8848	.9391	.9923	.0442	.0949	.1443
60	3.7735	3.8302	3.8857	3.9400	3.9932	4.0451	4.0957	4.1452

To Lay Out Angles Accurately. — Angles can be laid out accurately without the use of a protractor, provided a table of natural tangents is available. *Example:* A line is to be drawn at an angle of 29 degrees 54 minutes with another line, as shown in the illustration. First draw a line AB of indefinite length, and then erect a perpendicular BC at a distance from A of, say 10 inches; then find in a table the tangent of 29 degrees 54 minutes, which equals 0.57503 for a radius of 1; hence, BC equals $0.57503 \times 10 = 5.7503$, or $5\frac{3}{4}$ inches. Measure $5\frac{3}{4}$ inches from B to C, at right angles to line AB, and draw a line from A to C, thus obtaining the required angle. Conversely, the angularity of two lines can be determined by measuring line BC and dividing by length AB; the quotient equals the tangent; then find the corresponding angle.



This method of laying out or measuring angles will give more accurate results than an ordinary protractor.

Sine-bar for Measuring Angles. — The sine-bar is used either for measuring angles accurately or for locating work to a given angle. It consists of an accurate straight-edge to which are attached two hardened and ground plugs p and p_1 (see illustration). These plugs must be of the same diameter, and the distance l between their centers should, preferably, be an even dimension, to facilitate calculations The edges of the straight-edge must be parallel with a line through the plug centers. The sine-bar is always used in conjunction with some true surface B

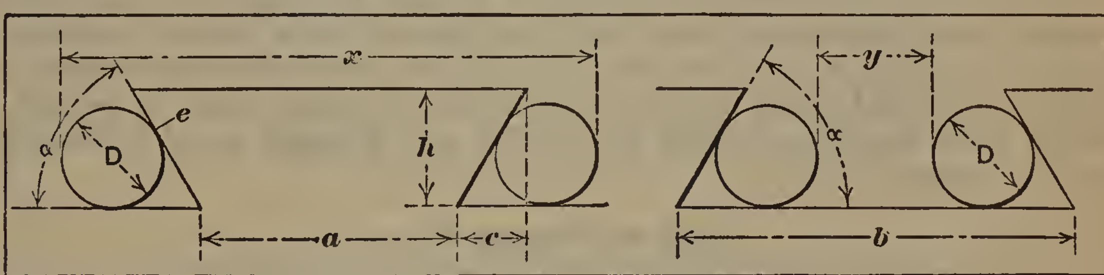


from which measurements can be taken. Two methods of measuring an angle are illustrated in Figs. 1 and 2. Referring to Fig. 1, the upper edge A of the part to be measured is set parallel with surface plate B. The heights a and b from the surface plate to the plugs p and p_1 are carefully measured either by using a micrometer gage or a vernier height gage. The difference between a and b is determined, and this difference, divided by the length l between the plugs of the sine-bar, equals the sine of the required angle β. The angle is then found by referring to a table of sines. For example, suppose length l is 10 inches, height a, 7.256 inches and height b, 2.14 inches; then the sine of the required angle equals $(7.256 - 2.14) \div 10 = 0.5116$, which is the sine of 30 degrees 46 minutes. A 10-inch sine-bar is convenient to use, as division can be performed mentally by simply moving the decimal one point to the left.

Fig. 2 illustrates a method of measuring an angle without first setting one edge parallel to surface B, the angle of each edge being measured separately. Suppose the height d equals 8.75 inches and c equals 6.5 inches. Subtracting c from d: $8.75 - 6.5 = 2.25$. Next shift the sine-bar to the position shown by the dotted lines. Assuming that $e = 5$ inches and $f = 2.15$, then $e - f = 5 - 2.15 = 2.85$. Dividing 2.25 and 2.85 by 10 (or the center distance between the sine-bar plugs) we get 0.225 and 0.285 as the sines of the angles; 0.225 is the sine of 13 degrees 1 minute, and 0.285 is the sine of 16 degrees 34 minutes. The sum of these angles or $(13° 1') + (16° 34') = 29$ degrees 35 minutes or the required angle γ.

When the sine-bar is to be set to a given angle for locating some part with reference to it, it is first set approximately. The sine of the required angle is then found and this sine is multiplied by the distance l between the plug centers, to obtain the vertical distance x (see Fig. 1) for that particular angle. The bar is then adjusted until the vertical distance x coincides with the dimension found. The vertical distance x for any given angle up to 56 degrees may be obtained directly from the accompanying tables which are for sine-bars having a center-to-center distance l between the plugs of 5 inches. If edge A is to be ground to an angle of 30 degrees and 46 minutes from edge E, the vertical distance x for a 5-inch sine-bar is, according to the fourth table, 2.5577 inches.

Measuring Dovetail Slides. — Dovetail slides which must be machined accurately to a given width are commonly gaged by using pieces of cylindrical rod or wire and measuring as indicated by the dimensions x and y of the accompanying



illustrations. To obtain dimension x for measuring male dovetails, add 1 to the cotangent of one-half the dovetail angle α, multiply by diameter D of the rods used, and add the product to dimension a. To obtain dimension y for measuring a female dovetail, add 1 to the cotangent of one-half the dovetail angle α, multiply by diameter D of the rod used, and subtract the result from dimension b. Expressing these rules as formulas:

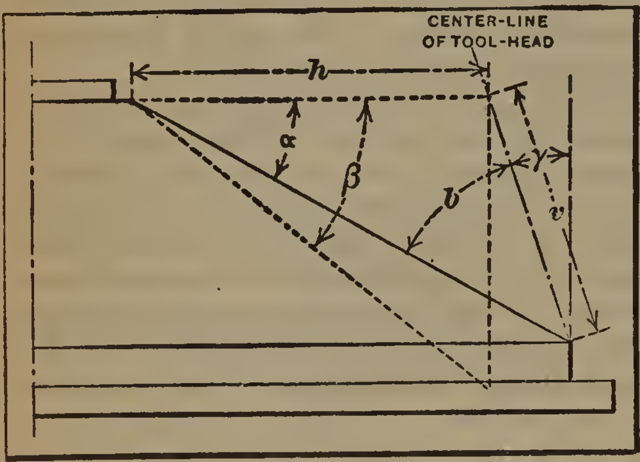
$$x = D\,(1 + \cot \tfrac{1}{2}\alpha) + a.$$

$$y = b - D\,(1 + \cot \tfrac{1}{2}\alpha).$$

Dimension c equals $h \times \cot \alpha$.

The rod or wire used should be small enough so that the point of contact e is somewhat below the corner or edge of the dovetail.

Taper Turning with Combined Feeds. — When it is necessary to machine, on the boring mill, a conical surface which has such a large included angle that the tool-bar cannot be swiveled far enough to permit turning by the usual method, the combined vertical and horizontal feeds are sometimes used to obtain the required taper. Suppose a conical casting is to be turned to an angle α of 30 degrees (see illustration), and that the tool-head of the boring mill feeds horizontally ¼ inch per turn of the screw and has a vertical movement of 3/16 inch per turn of the vertical feed shaft. If the two feeds are used simultaneously with the tool-bar at right angles to the table, the tool will move a distance h of eight inches, while it moves downward a distance of six inches, thus turning the surface to an angle β. This angle β is greater than the required angle α, but if the tool-bar is swiveled to an angle γ, the tool, as it moves downward, will be advanced horizontally in addition to the regular horizontal feeding movement. Hence, if the tool-bar is set over to the proper angle γ, the surface can be turned to an angle α. The problem, then, is to determine what the angle γ should be for turning to a given angle α.



Angle γ can be calculated as follows: $\text{Sin } b = \frac{\sin\alpha \times h}{v}$ in which h represents the rate of horizontal feed and v the rate of vertical feed. Having angles α and b, the desired angle γ is obtained by subtracting the sum of the former angles from 90 degrees. To illustrate (using the values given in the foregoing) the sine of 30 degrees is 0.5; then, $\sin b = \frac{0.5 \times \frac{1}{4}}{\frac{3}{16}} = 0.6666$. Hence, angle $b = 41$ degrees 48 minutes and $\gamma = 90° - (30° + 41° 48') = 18$ degrees 12 minutes. If angle α were greater than angle β obtained from the combined feeds with the tool-bar in the vertical position, it would be necessary to swing the lower end of the bar to the left rather than to the right of the vertical plane; that is, the lower end of the bar would be inclined to the left of the vertical an amount equal to the sum of angles α and b subtracted from 90 degrees.

Rules for Figuring Tapers

Given	To Find	Rule
The taper per foot.	The taper per inch.	Divide the taper per foot by 12.
The taper per inch.	The taper per foot.	Multiply the taper per inch by 12.
End diameters and length of taper in inches.	The taper per foot.	Subtract small diameter from large; divide by length of taper, and multiply quotient by 12.
Large diameter and length of taper in inches, and taper per foot.	Diameter at small end in inches.	Divide taper per foot by 12; multiply by length of taper, and subtract result from large diameter.
Small diameter and length of taper in inches, and taper per foot.	Diameter at large end in inches.	Divide taper per foot by 12; multiply by length of taper, and add result to small diameter.
The taper per foot and two diameters in inches.	Distance between two given diameters in inches.	Subtract small diameter from large; divide remainder by taper per foot, and multiply quotient by 12.
The taper per foot.	Amount of taper in a certain length given in inches.	Divide taper per foot by 12; multiply by given length of tapered part.

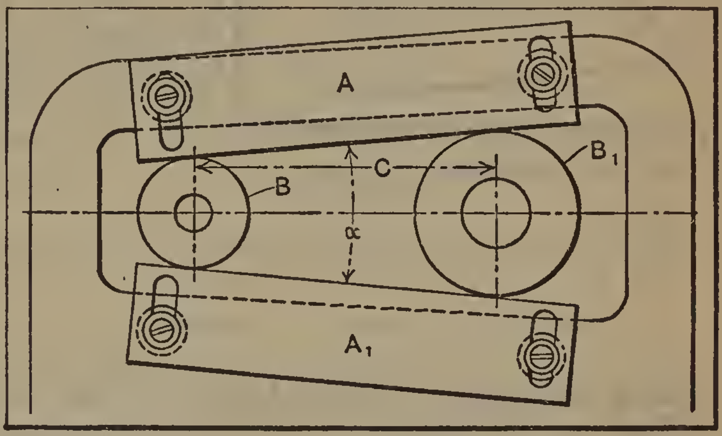
Amount of Taper in a Given Length, When the Taper per Foot is Known

Length of Taper	Taper per Foot										
	1/16	3/32	1/8	1/4	3/8	1/2	0.600	5/8	3/4	1	1 1/4
1/32	0.0002	0.0002	0.0003	0.0007	0.0010	0.0013	0.0016	0.0016	0.0020	0.0026	0.0033
1/16	0.0003	0.0005	0.0007	0.0013	0.0020	0.0026	0.0031	0.0033	0.0039	0.0052	0.0065
1/8	0.0007	0.0010	0.0013	0.0026	0.0039	0.0052	0.0062	0.0065	0.0078	0.0104	0.0130
3/16	0.0010	0.0015	0.0020	0.0039	0.0059	0.0078	0.0094	0.0098	0.0117	0.0156	0.0195
1/4	0.0013	0.0020	0.0026	0.0052	0.0078	0.0104	0.0125	0.0130	0.0156	0.0208	0.0260
5/16	0.0016	0.0024	0.0033	0.0065	0.0098	0.0130	0.0156	0.0163	0.0195	0.0260	0.0326
3/8	0.0020	0.0029	0.0039	0.0078	0.0117	0.0156	0.0187	0.0195	0.0234	0.0312	0.0391
7/16	0.0023	0.0034	0.0046	0.0091	0.0137	0.0182	0.0219	0.0228	0.0273	0.0365	0.0456
1/2	0.0026	0.0039	0.0052	0.0104	0.0156	0.0208	0.0250	0.0260	0.0312	0.0417	0.0521
9/16	0.0029	0.0044	0.0059	0.0117	0.0176	0.0234	0.0281	0.0293	0.0352	0.0469	0.0586
5/8	0.0033	0.0049	0.0065	0.0130	0.0195	0.0260	0.0312	0.0326	0.0391	0.0521	0.0651
11/16	0.0036	0.0054	0.0072	0.0143	0.0215	0.0286	0.0344	0.0358	0.0430	0.0573	0.0716
3/4	0.0039	0.0059	0.0078	0.0156	0.0234	0.0312	0.0375	0.0391	0.0469	0.0625	0.0781
13/16	0.0042	0.0063	0.0085	0.0169	0.0254	0.0339	0.0406	0.0423	0.0508	0.0677	0.0846
7/8	0.0046	0.0068	0.0091	0.0182	0.0273	0.0365	0.0437	0.0456	0.0547	0.0729	0.0911
15/16	0.0049	0.0073	0.0098	0.0195	0.0293	0.0391	0.0469	0.0488	0.0586	0.0781	0.0977
1	0.0052	0.0078	0.0104	0.0208	0.0312	0.0417	0.0500	0.0521	0.0625	0.0833	0.1042
2	0.0104	0.0156	0.0208	0.0417	0.0625	0.0833	0.1000	0.1042	0.1250	0.1667	0.2083
3	0.0156	0.0234	0.0312	0.0625	0.0937	0.1250	0.1500	0.1562	0.1875	0.2500	0.3125
4	0.0208	0.0312	0.0417	0.0833	0.1250	0.1667	0.2000	0.2083	0.2500	0.3333	0.4167
5	0.0260	0.0391	0.0521	0.1042	0.1562	0.2083	0.2500	0.2604	0.3125	0.4167	0.5208
6	0.0312	0.0469	0.0625	0.1250	0.1875	0.2500	0.3000	0.3125	0.3750	0.5000	0.6250
7	0.0365	0.0547	0.0729	0.1458	0.2187	0.2917	0.3500	0.3646	0.4375	0.5833	0.7292
8	0.0417	0.0625	0.0833	0.1667	0.2500	0.3333	0.4000	0.4167	0.5000	0.6667	0.8333
9	0.0469	0.0703	0.0937	0.1875	0.2812	0.3750	0.4500	0.4687	0.5625	0.7500	0.9375
10	0.0521	0.0781	0.1042	0.2083	0.3125	0.4167	0.5000	0.5208	0.6250	0.8333	1.0417
11	0.0573	0.0859	0.1146	0.2292	0.3437	0.4583	0.5500	0.5729	0.6875	0.9167	1.1458
12	0.0625	0.0937	0.1250	0.2500	0.3750	0.5000	0.6000	0.6250	0.7500	1.0000	1.2500

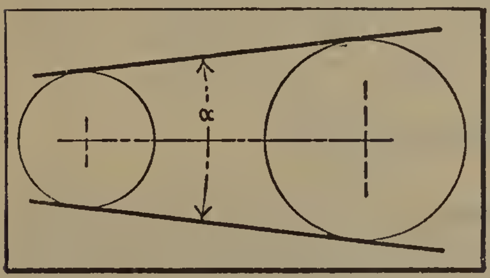
Tapers per Foot and Corresponding Angles

Taper per Foot	Included Angle	Angle with Center Line	Taper per Foot	Included Angle	Angle with Center Line
1/64	0° 4′ 28″	0° 2′ 14″	1 7/8	8° 56′ 2″	4° 28′ 1″
1/32	0 8 58	0 4 29	1 15/16	9 13 51	4 36 56
1/16	0 17 53	0 8 57	2	9 31 37	4 45 49
3/32	0 26 52	0 13 26	2 1/8	10 7 11	5 3 35
1/8	0 35 47	0 17 54	2 1/4	10 42 41	5 21 21
5/32	0 44 45	0 22 23	2 3/8	11 18 12	5 39 6
3/16	0 53 44	0 26 52	2 1/2	11 53 38	5 56 49
7/32	1 2 39	0 31 20	2 5/8	12 29 2	6 14 31
1/4	1 11 38	0 35 49	2 3/4	13 4 25	6 32 13
9/32	1 20 33	0 40 16	2 7/8	13 39 44	6 49 52
5/16	1 29 31	0 44 46	3	14 15 0	7 7 30
11/32	1 38 30	0 49 15	3 1/8	14 50 15	7 25 8
3/8	1 47 25	0 53 42	3 1/4	15 25 27	7 42 43
13/32	1 56 24	0 58 12	3 3/8	16 0 34	8 0 17
7/16	2 5 18	1 2 39	3 1/2	16 35 41	8 17 50
15/32	2 14 17	1 7 8	3 5/8	17 10 42	8 35 21
1/2	2 23 12	1 11 36	3 3/4	17 45 40	8 52 50
17/32	2 32 10	1 16 5	3 7/8	18 20 35	9 10 18
9/16	2 41 7	1 20 34	4	18 55 31	9 27 45
19/32	2 50 4	1 25 2	4 1/8	19 30 18	9 45 9
5/8	2 59 3	1 29 31	4 1/4	20 5 1	10 2 31
21/32	3 7 57	1 33 59	4 3/8	20 39 44	10 19 52
11/16	3 16 56	1 38 28	4 1/2	21 14 20	10 37 10
23/32	3 25 51	1 42 55	4 5/8	21 48 55	10 54 28
3/4	3 34 48	1 47 24	4 3/4	22 23 27	11 11 43
25/32	3 43 44	1 51 52	4 7/8	22 57 50	11 28 55
13/16	3 52 42	1 56 21	5	23 32 12	11 46 6
27/32	4 1 38	2 0 49	5 1/8	24 6 28	12 3 14
7/8	4 10 32	2 5 16	5 1/4	24 40 43	12 20 21
29/32	4 19 31	2 9 46	5 3/8	25 14 50	12 37 25
15/16	4 28 26	2 14 13	5 1/2	25 48 53	12 54 27
31/32	4 37 25	2 18 42	5 5/8	26 22 52	13 11 26
1	4 46 19	2 23 10	5 3/4	26 56 48	13 28 24
1 1/16	5 4 12	2 32 6	5 7/8	27 30 35	13 45 18
1 1/8	5 22 2	2 41 1	6	28 4 20	14 2 10
1 3/16	5 39 55	2 49 58	6 1/8	28 37 59	14 19 0
1 1/4	5 57 45	2 58 53	6 1/4	29 11 36	14 35 48
1 5/16	6 15 38	3 7 49	6 3/8	29 45 4	14 52 32
1 3/8	6 33 29	3 16 44	6 1/2	30 18 28	15 9 14
1 7/16	6 51 21	3 25 41	6 5/8	30 51 49	15 25 55
1 1/2	7 9 10	3 34 35	6 3/4	31 25 2	15 42 31
1 9/16	7 27 0	3 43 30	6 7/8	31 58 11	15 59 5
1 5/8	7 44 49	3 52 24	7	32 31 14	16 15 37
1 11/16	8 2 38	4 1 19	7 1/8	33 4 10	16 32 5
1 3/4	8 20 28	4 10 14	7 1/4	33 37 3	16 48 32
1 13/16	8 38 17	4 19 8	7 3/8	34 9 49	17 4 55

Accurate Measurement of Angles and Tapers. — When great accuracy is required in the measurement of angles, or when originating tapers, disks are commonly used. The principle of the disk method of taper measurement is that if two disks of unequal diameters are placed either in contact or a certain distance apart, lines tangent to their peripheries will represent an angle or taper, the degree of which depends upon the diameters of the two disks and the distance between them. The gage shown in the accompanying illustration, which is a form commonly used for originating tapers or measuring angles accurately, is set by means of disks. This gage consists of two adjustable straight-edges A and A_1, which are in contact with disks B and B_1. The angle α or the taper between the straight-edges depends, of course, upon the diameters of the disks and the center distance C, and as these three dimensions can be measured accurately, it is possible to set the gage to a given angle within very close limits. Moreover, if a record of the three dimensions is kept, the exact setting of the gage can be reproduced quickly at any time. The following rules may be used for adjusting a gage of this type, and cover all problems likely to arise in practice. Disks are also occasionally used for the setting of parts in angular positions for accurately machining them to a given angle; the rules will be found applicable to these conditions also.



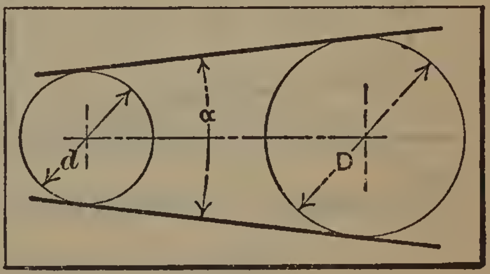
To Find Angle for Given Taper per Foot. — When the taper in inches per foot is known, and the corresponding angle α is required. *Rule:* Divide the taper in inches per foot by 24; find the angle corresponding to the quotient, in a table of tangents, and double this angle.



Example: What angle α is equivalent to a taper of 1½ inch per foot?

$\frac{1.5}{24} = 0.0625$. The angle whose tangent is 0.0625 equals 3 degrees 35 minutes, nearly; then, 3 deg. 35 min. $\times$ 2 = 7 deg. 10 min.

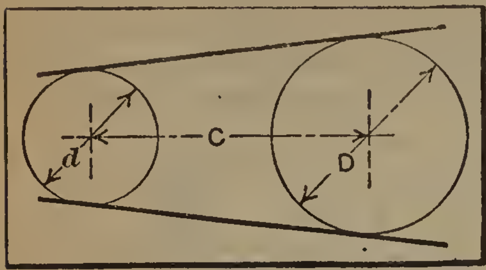
To Find Angle for Given Disk Dimensions. — When the diameters D and d of the large and small disks and the center distance are given, to determine the angle α. *Rule:* Divide the difference between the disk diameters by twice the center distance; find the angle corresponding to the quotient, in a table of sines, and double the angle.



Example: If the disk diameters are 1 and 1.5 inch, respectively, and the center distance is 5 inches, find the included angle α.

$\frac{1.5 - 1}{2 \times 5} = 0.05$. The angle whose sine is 0.05 equals 2 degrees 52 minutes; then, 2 deg. 52 min. $\times$ 2 = 5 deg. 44 min. = angle α.

To Find the Taper per Foot. — When the diameters D and d of the large and small disks and the center distance C are given, to determine the taper per foot (measured at right angles to line through disk centers). *Rule:* Divide the difference between the disk diameters by twice the center distance; find the angle corresponding to the quotient, in a table of sines; then find the tangent corresponding to this angle, and multiply the tangent by 24.

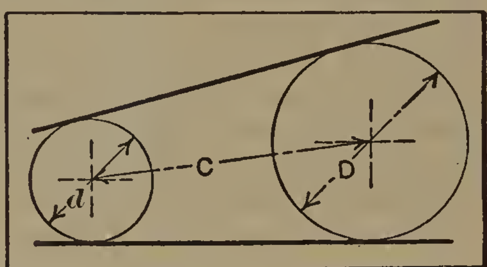


Example: If disk diameters are 1 and 1.5 inch, respectively, and center distance is 5 inches, find the taper per foot.

$\frac{1.5-1}{2\times 5}=0.05$. The angle whose sine is 0.05 equals 2 degrees 52 minutes;

$\tan 2^\circ 52' = 0.05007$; $0.05007 \times 24 = 1.2017$ inch taper per foot.

Taper Measured at Right Angles to One Side. — When one side is taken as a base line, and the taper is measured at right angles to that side, use the following rule for determining the taper per foot. *Rule:* Divide the difference between the disk diameters D and d by twice the center distance C; find the angle corresponding to the quotient, in a table of sines; double this angle and find the corresponding tangent; then multiply the tangent by 12.



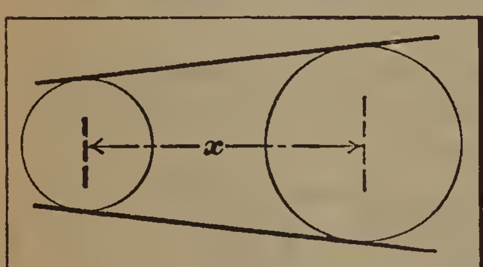
Example: If the disk diameters are 2 and 3 inches, respectively, and the center distance is 5 inches, what is the taper per foot measured at right angles to one side?

$\frac{3-2}{2\times 5}=0.1$. The angle whose sine is 0.1 equals 5 degrees 45 minutes, nearly;

then, 2×5 deg. 45 min. = 11 deg. 30 min.; $\tan 11^\circ 30' = 0.20345$;

$0.20345 \times 12 = 2.4414$ inches taper per foot.

To Find Center Distance for a Given Taper. — When the taper, in inches per foot, is given, to determine center distance x. *Rule:* Divide the taper by 24 and find the angle corresponding to the quotient in a table of tangents; then find the sine corresponding to this angle and divide the difference between the disk diameters by twice the sine.



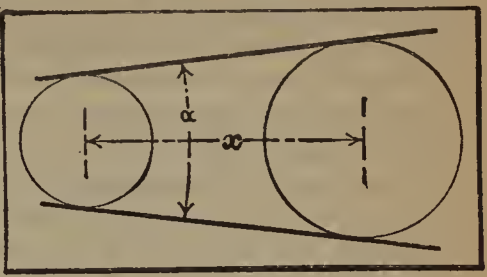
Example: Gage is to be set to ¾ inch per foot, and disk diameters are 1.25 and 1.5 inch, respectively. Find the required center distance for the disks.

$\frac{0.75}{24}=0.03125$. The angle whose tangent is 0.03125 equals 1 degree 47.4 minutes;

$\sin 1^\circ 47.4' = 0.03123$; $1.50 - 1.25 = 0.25$ inch;

$\frac{0.25}{2\times 0.03123} = 4.002$ inches = center distance x.

To Find Center Distance for a Given Angle. — When straight-edges must be set to a given angle α, to determine center distance x between disks of known diameter. *Rule:* Find the sine of half the angle α in a table of sines; divide the difference between the disk diameters by double this sine.

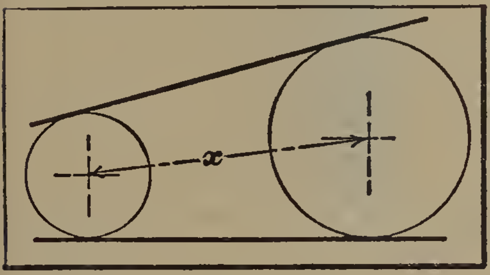


Example: If an angle α of 20 degrees is required, and the disks are 1 and 3 inches in diameter, respectively, find the required center distance x.

$$\frac{20}{2} = 10 \text{ degrees}; \qquad \sin 10° = 0.17365;$$

$$\frac{3-1}{2 \times 0.17365} = 5.759 \text{ inches} = \text{center distance } x.$$

Center Distance when Taper is Measured from One Side. — When taper is measured at right angles to one side, use the following rule for determining the center distance x. *Rule:* Divide the taper in inches per foot by 12; find the angle corresponding to the quotient, in a table of tangents; find the sine of one-half this angle, and then divide the difference between the disk diameters by double the sine.



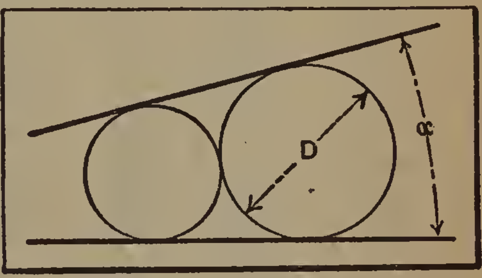
Example: If taper measured at right angles to one side is 6.9 inches per foot, and the disks are 2 and 5 inches in diameter, respectively, what is center distance x?

$\frac{6.9}{12} = 0.575$. The angle whose tangent is 0.575 equals 29 degrees 54 minutes;

then, $$\frac{29 \text{ deg. } 54 \text{ min.}}{2} = 14 \text{ deg. } 57 \text{ min}; \qquad \sin 14° 57' = 0.25798;$$

$$\frac{5-2}{2 \times 0.25798} = 5.814 \text{ inches, center distance.}$$

Angular Measurements with Disks in Contact. — When the two disks are to be in contact and the diameter of the small disk is known, the diameter D of the large disk for a given angle α can be obtained as follows. *Rule:* Multiply twice the diameter of the small disk by the sine of one-half the required angle; divide this product by 1 minus the sine of one-half the required angle; add the quotient to the diameter of the small disk to obtain the diameter of the large disk.



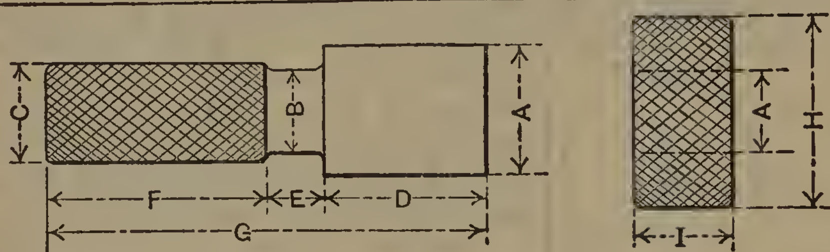
Example: The required angle α is 15 degrees. Find diameter of large disk, to be in contact with a standard 1-inch reference disk.

$$\sin 7° 30' = 0.13053. \qquad 2 \times 1 \times 0.13053 = 0.26106;$$

$$\frac{0.26106}{1 - 0.13053} = 0.3002. \qquad 1 + 0.3002 = 1.3002 = \text{diameter of large disk.}$$

Plug and Ring Gages. — The cheapest method by which plug and ring gages can be made is to use gages inserted in machine steel handles. The best form adapted to plug gages from 0.075 inch in diameter and upwards is to make the gage with a taper shank, the taper being 1/2 inch per foot. Gages smaller than 0.075 inch in diameter should be made from straight hardened wire and inserted into the handle, which is then drilled to suit the diameter of the gage. For these small sizes, grinding of the plugs is unnecessary, as they can simply be lapped to size from a wire 0.001 inch larger in diameter than the finished gage. When the blanks are made from bar steel they should be at least 0.080 inch larger in diameter than the finished size, so that the decarbonized surface of the bar may be turned off and the uniform structure beneath reached. When gages are made from drill rod only one-half of this allowance is necessary. The plugs are turned to within 0.005 or 0.010 inch of the finished size, hardened, and ground to within 0.001 or 0.0015 inch of the finished size, after which they are lapped to size. The ring gages are inserted in circular holders made from machine steel.

Dimensions of Solid Plug and Ring Gages



A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
1/4	7/32	7/16	15/16	5/16	1 7/8	3 1/8	1	1/2
5/16	9/32	1/2	1	5/16	2	3 5/16	1 1/16	1/2
3/8	5/16	1/2	1 1/16	5/16	2 1/8	3 1/2	1 1/8	9/16
7/16	3/8	9/16	1 1/8	5/16	2 1/4	3 11/16	1 3/16	9/16
1/2	7/16	9/16	1 3/16	3/8	2 3/8	3 15/16	1 1/4	5/8
9/16	1/2	5/8	1 1/4	3/8	2 1/2	4 1/8	1 5/16	5/8
5/8	9/16	5/8	1 5/16	3/8	2 5/8	4 5/16	1 7/16	11/16
11/16	5/8	11/16	1 3/8	3/8	2 3/4	4 1/2	1 9/16	11/16
3/4	5/8	11/16	1 7/16	7/16	2 7/8	4 3/4	1 11/16	3/4
13/16	11/16	3/4	1 1/2	7/16	3	4 15/16	1 13/16	13/16
7/8	11/16	3/4	1 9/16	7/16	3 1/8	5 1/8	1 15/16	7/8
15/16	3/4	13/16	1 9/16	7/16	3 1/8	5 1/8	2 1/16	15/16
1	13/16	7/8	1 5/8	1/2	3 1/4	5 3/8	2 3/16	1
1 1/8	7/8	1	1 5/8	1/2	3 1/4	5 3/8	2 3/8	1 1/8
1 1/4	1	1 1/8	1 3/4	1/2	3 1/4	5 1/2	2 1/2	1 1/8
1 3/8	1 1/8	1 1/4	1 3/4	1/2	3 3/8	5 5/8	2 5/8	1 1/4
1 1/2	1 1/4	1 3/8	1 7/8	5/8	3 3/8	5 7/8	2 3/4	1 1/4
1 5/8	1 3/8	1 1/2	1 7/8	5/8	3 1/2	6	2 7/8	1 1/2
1 3/4	1 3/8	1 1/2	2	3/4	3 1/2	6 1/4	3	1 1/2
1 7/8	1 1/2	1 5/8	2	3/4	3 1/2	6 1/4	3 1/8	1 1/2
2	1 1/2	1 5/8	2 1/8	3/4	3 1/2	6 3/8	3 1/4	1 5/8
2 1/4	1 5/8	1 3/4	2 1/8	3/4	3 5/8	6 1/2	3 1/2	1 5/8
2 1/2	1 5/8	1 3/4	2 1/4	3/4	3 5/8	6 5/8	3 3/4	1 5/8
2 3/4	1 5/8	1 3/4	2 1/4	3/4	3 5/8	6 5/8	4 1/8	1 3/4
3	1 5/8	1 3/4	2 1/2	3/4	3 5/8	6 7/8	4 1/2	1 3/4

Sectional Gages. — A sectional snap gage formed of four parts is shown in Fig. 1. The measuring jaws, instead of being integral with the gage body, are attached to a central block by screws, as shown. The width *a* of one end of this central block equals the size of the "go" end of the gage; width *b* equals the size of the "not go" end. The gage jaws are made flat. The advantage of this design, as compared with a solid snap gage, is that when the accuracy is impaired as the result of wear, the gage can be restored to its original accuracy by simply removing the gage jaws and truing them by grinding and lapping.

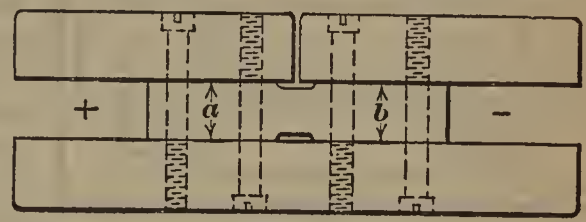


Fig. 1

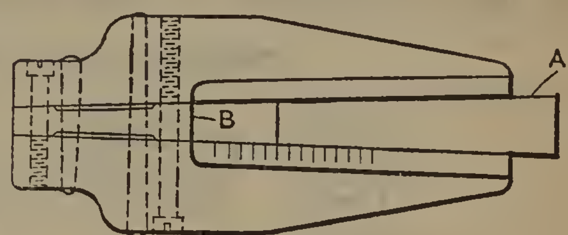
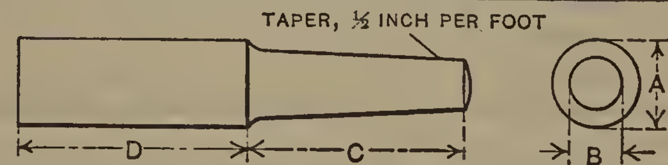


Fig. 2

The same principle can also be applied to an angular taper gage, as shown in Fig. 2. The gage jaws are attached to a central block *B* finished accurately to the required taper, and the size of the work *A* is tested by pushing it between the jaws and noting the position of the end relative to a standard graduation mark. When the gage becomes inaccurate, as the result of wear, the jaws are removed and trued. A master plug should be used, occasionally, for testing the accuracy of the gage. By having one jaw graduated, as shown, the amount of inaccuracy in size of end of tapered piece may also be gaged, by noting how far the end of the work comes short of or projects beyond the standard dimension mark.

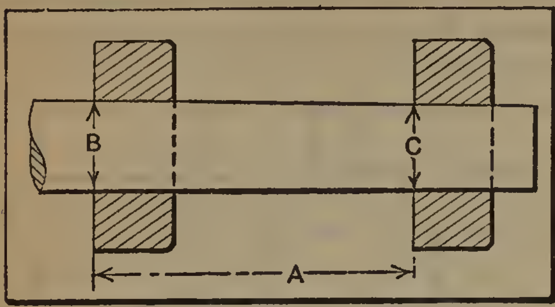
Dimensions of Plug Gage Blanks



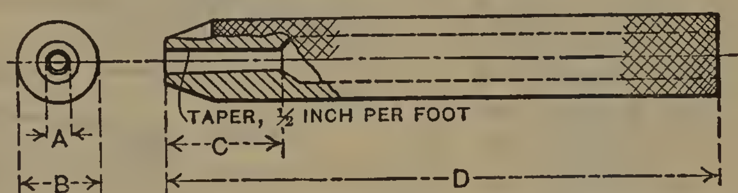
No.	*A*	*B*	*C*	*D*	No. of Handle
1	⅛	0.100	⅝	¾ to 1¼	1
2	3/16	0.150	⅞	1 to 1⅝	2
3	¼	0.200	1¼	1 to 2	3
4	⅜	0.300	1¼	1⅛ to 2	4
5	½	0.300	1¼	1⅛ to 2	4
6	¾	0.400	1¼	1¼ to 2	5
7	⅞	0.400	1¼	1¼ to 2	5
8	1⅛	0.400	1¼	1½ to 2¼	5
9	1⅜	0.500	1⅜	1½ to 2¼	6
10	1⅝	0.500	1⅜	1½ to 2¼	6
11	1⅞	0.500	1⅜	1¾ to 2½	6

Testing Slight Tapers. — The ring method of testing taper reamers or any part having a slight taper is shown in the illustration. The test rings are ground to diameters *B* and *C*, varying just enough to locate the rings a convenient distance

A apart, when on the piece to be gaged. After determining the distance *A* for a given taper, the testing of duplicate tapers is effected by simply sliding each ring along the taper until it fits tightly, and then measuring the distance *A*, which varies for even a slight change in the taper. This method of gaging is applied to taper reamers such as are used for locomotive work. The standard taper of these reamers is 3/32 inch per foot; hence, if diameters *B* and *C* of the rings vary 3/64 inch, distance *A* equals 6 inches. The bore of these rings is usually ground cylindrical or straight; if tapered, the taper should be slightly less than that to be measured, to insure a bearing at points *B* and *C*.



Dimensions of Plug Gage Handles



No.	*A*	*B*	*C*	*D*
0	solid	5/16	½	2
1	0.120	3/8	½	2¾
2	0.205	½	¾	3
3	0.245	½	1	3¼
4	0.345	5/8	1	3½
5	0.445	¾	1	4
6	0.545	7/8	1⅛	4

Steel for Gages. — Machine steel, plain carbon steel and special alloy steels are used for making various classes of gages. Machine steel is used extensively for gage work. The carbon content of machine steel for gages usually ranges from 0.15 to 0.25 per cent, although it may be as high as 0.50 per cent, especially for ring or plug gages. A 0.20-per cent carbon steel, containing from 0.90 to 1.10 per cent manganese and about 0.05 per cent of phosphorus and sulphur is considered very satisfactory for gages. This steel should not contain silicon as this causes warping in hardening. The carbon in the so-called 0.20-per cent carbon steel may vary from 0.15 to 0.25 per cent and many gage-makers prefer the steel having the smaller amount. This general class of steel is extensively used for making drop-forged snap gages. Plug gages, ring gages or other forms which may be ground easily after hardening are often made of steel containing about 0.50 per cent carbon.

A high-carbon or tool steel is sometimes preferred to machine steel because it can be hardened in much less time than is required for carburizing and hardening machine steel gages. High-carbon steel for gages often contains about 0.90 per cent carbon, about 0.30 per cent manganese, a phosphorus and sulphur content not exceeding 0.025 per cent, and about 0.15 per cent silicon. Special alloy steels have been developed which are adapted to fine gage work partly because changes due to hardening are exceedingly small.

Specifications for Thread Gages. — The following specifications of the National Screw Thread Commission will be helpful in the design and construction of gages used for producing threaded work. (For specific information on the different types of gages recommended by the commission, see "Gaging Interchangeable Threaded Parts," page 1004, and the following list of gages under "Gage Classification.")

Gage Steel. — Gages may be made of a good grade of machine steel pack-hardened, or of straight carbon steel of not less than 1 per cent carbon; or preferably of an oil-hardening steel of approximately 1.10 per cent carbon and 1.40 per cent chromium. The handles should be made of a good grade of machine steel plainly marked to identify the gage.

Plug Gages. — All plug gages, whether plain or threaded, should be single-ended. Plug gages of 2 inches and less in diameter should be made with a plug inserted in the handle and fastened thereto by means of a pin. Plug gages of more than 2 inches in diameter should have the gaging blank so made as to be reversible. This can be accomplished by having a finished hole in the gage blank fitting a shouldered projection on the end of the handle, the gage blank being held on with a nut and keyed in the case of a threaded plug gage. The "go" plug gage should be noticeably longer than the "not go" gage, or some distinguishing feature in the design of the handle should be used to serve as a ready means of identification, such as a chamfer on the handle of the "go" gage.

Plain Ring Gages. — Both the "go" and "not go" gages should have their outside diameters knurled if made circular. The "go" gage should have a decided chamfer in order to provide a ready means of identification for distinguishing the "go" from the "not go" gage.

Snap Gages. — Snap gages may be either adjustable or nonadjustable. It is recommended that all snap gages up to and including 1/8 inch be of the built-up type. For larger snap gages, forged blanks, flat plate stock or other suitable construction may be used. Sufficient clearance beyond the mouth of the gage should be provided to permit the gaging of cylindrical work. Snap gages for measuring lengths and diameters may have one gaging dimension only, or may have a maximum and minimum gaging dimension, both on one end, or maximum and minimum gaging dimension on opposite ends of the gage. When the maximum and minimum gaging dimensions are placed on opposite ends of the gage, the maximum or "go" end of the snap gage should be distinguished from the minimum or "not go" end by having the corners of the gage on the "go" end decidedly chamfered.

Plug Thread Gages. — End threads on plug thread gages should not be chamfered, but the first half turn of the end thread should be flattened to avoid a feather edge.

Dirt Grooves. — Inspection and working thread plug gages should be provided with dirt grooves which extend into the gage for a depth of from one to four threads.

Length of Thread. — The length of thread parallel to the axis of the gage should, for all standard "go" thread plug and ring gages, be at least as much as the quantity expressed in the following formula, in which L = length of thread and D = basic major diameter of thread: $L = 1.5\,D$.

For threaded work of shorter length of engagement than $1.5\,D$, the length of thread on the "go" gage may be correspondingly shorter.

"Not Go" Gage for Pitch Diameter only. — All "not go" thread plug gages should be made to check the pitch diameter only. This necessitates removal of the crest of the thread so that the dimension of the major diameter is never greater than that specified for the "go" gage, and also removing the portion of the thread at the root of the standard thread form.

Ring Thread Gages. — All ring thread gages should be made adjustable. The "go" gage should be distinguished from the "not go" gage by having a decided chamfer, and both gages should have their outside diameter knurled if made circular. The end threads on ring thread gages should not be chamfered but the first half turn of the end thread should be flattened to avoid a feather edge.

"Not go" ring thread gages should be made to check the pitch diameter only. This necessitates removal of the crest of the thread (so that the dimension of the minor diameter is never less than that specified for the maximum or "go" gage) and also removing the portion of the thread at the root of the standard form.

The Marking of Gages. — The maximum and minimum limits or sizes of gages may be marked in different ways. In the case of a plug gage, for example, the larger end may be marked either "max." (maximum), "high," "+" (plus), or "not go," while the small end would be marked "min." (minimum), "low," "–" (minus), or "go." In the case of a snap gage, the maximum dimension would be marked "max.," "high," "+," or "go," and the minimum, "min.," "low," "–," or "not go." The markings "max.," "min.," "high," and "low" refer to the dimension, while the markings "go" and "not go" refer to the use of the gage. When plug gages are marked "max." and "min.," it is evident that the "min." size is intended to pass into the hole while the "max." size is not supposed to enter. With a snap gage, however, the conditions are reversed: the "max." size should pass over the shaft, while the "min." size should not. Were the gages marked "go" and "not go," the meaning of these words would, in both cases, be the same, which is an advantage. That part of a gage marked "go" would pass over or into the work, while the part marked "not go" would not pass over or into the work.

Working and inspection double-ended plug gages should have the "go" end longer than the "not go" end. Working and inspection double-ended snap gages should have the "go" end rounded to a radius of about 1/8 inch, while the "not go" end should be beveled for a distance of about 1/8 inch. This makes it possible to see at a glance which is the "go" and which the "not go" end.

In marking the sizes on gages, the marking, when expressed in inches, should always be carried to at least three decimals, whether the last decimal is a 0 or not; for example, 0.370 and 0.200, etc. When the exact size requires more than three decimals, as, for example, 0.5798, the required number of decimals should, of course, be stamped on the gage. When the size is expressed in millimeters, the marking should be carried to at least two decimals. For example, 6.00 and 7.40; and if more than two decimals are required to express the exact size, the required number of decimals will, of course, be given.

Allowance for Lapping Thread Gages. — The allowance for lapping usually varies from 0.0002 to 0.0005 inch, although, in some cases, the allowance may be as high as 0.001 inch or more, the amount left for lapping increasing with the size of the gage.

As to the material for laps, some gage-makers prefer cast iron and others, soft steel. It is essential to use laps which are accurate as to lead and thread form, although some laps intended for correcting errors have thread angles which are slightly greater or less than the standard, the object being to change the angle of the gage thread more readily.

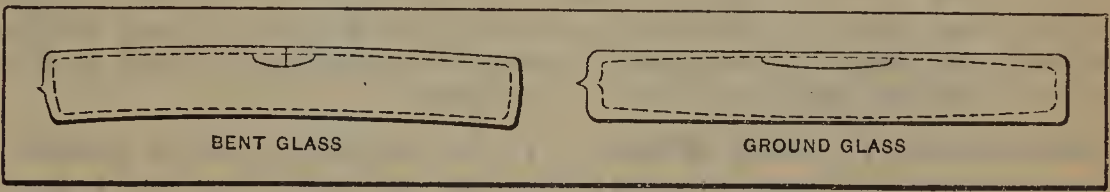
Abrasives for Lapping Gages. — Flour of emery is extensively used for lapping gages, and artificial abrasives are also used. The abrasive is mixed with some oil such as lard oil, sperm oil, or possibly kerosene oil. When a very slow cutting abrasive is required and the amount to be removed by lapping is small, rouge and lard oil may be used. Information regarding the different kinds of artificial abrasives adapted for lapping may be obtained from the manufacturers. After lapping a gage, it should be washed in gasoline before measuring it. If the gage has been heated appreciably as the result of lapping, it should be cooled in water down to the room temperature before measuring.

Spirit Levels.— The accuracy of a spirit level depends entirely upon the curvature of the glass tube. This tube is ground on the inside to a barrel shape, except in cheap levels which simply have a glass tube bent to the approximate curve. Bent and ground glasses are shown in the illustration. The bent tube type is not to be recommended except for work which does not require great accuracy. The tube is nearly filled with spirits of wine, ether, or some similar fluid and is hermetically sealed at each end. The larger radius of curvature the glass has, the more sensitive will be the level. The following table gives the curvature of levels of various degrees of sensitiveness, the divisions of graduations on the level being in tenths of an inch:

	Seconds					Min.	Deg.
Angular Value of Each Graduation	2	5	10	20	30	1	1
Corresponding Diameter of Curvature in Feet	1718	687	343	171	114	57	0.95

The air space in a ground glass is much longer than in a bent one, being ordinarily from ¼ to ⅓ the length of the tube.

Modern levels are graduated to tenths and twentieths of an inch, except when they are divided according to the metric system. The angular value of a division



may be determined roughly as follows: Support the level upon a piece of metal, the lower surface of which has been filed or cut away so that it bears on two points exactly 12 inches apart. Insert packing under one of the bearing points to bring the air space near the center. Note carefully where the air space is and then put a "feeler gage," say, 0.002 inch thick, under one of the bearing points; then if the air space moves, say, one-tenth inch, the angular value in seconds for one division of the level is found as follows: The distance from the bearing point to the feeler gage is 12 inches, which is the radius of a circle the circumference of which is 75.3984; hence 75.3984 inches is equivalent to 1,296,000 seconds angular measurement. Therefore, 0.002 inch equals 34.3 seconds and each one-tenth inch on the level also equals 34.3 seconds. If the user of a level does not know the angular value of the graduations, he can, in this way, ascertain definitely the amount a surface varies from the horizontal.

A good level is a very sensitive instrument and should be carefully used. The leveling glass or "bubble" is generally fixed in a brass tube with plaster-of-paris. This method is satisfactory for all levels having an accuracy of about five seconds angular measurement to each one-tenth inch graduation. For finer levels, it is better to fix one end only with plaster-of-paris and the other end with cork, for if the glass is fixed rigidly at both ends with plaster-of-paris, there will be a strain on the level due to temperature changes, and as the expansion of glass and brass is different, a slight inaccuracy is liable to result. It is also advisable to have an extra glass tube surrounding the leveling tube for very accurate levels, in order to provide insulation from the heat of the hand. A level of one minute angular measurement to one-tenth inch graduation is the most serviceable for general use. One having an accuracy of 30 seconds to one-tenth inch should be used on a floor free from vibration. Finer levels are used mostly on surveying and astronomical instruments.

CHANGE GEARS FOR MILLING SPIRALS

Lead of a Milling Machine. — If gears with an equal number of teeth are placed on the table feed-screw and the worm-gear stud, then the *lead of the milling machine* is the distance the table will travel while the index spindle makes one complete revolution. This distance is a constant used in figuring the change gears.

The lead of a helix or spiral is the distance, measured along the axis of the work, in which the spiral makes one full turn around the work. The lead of the milling machine may, therefore, also be expressed as the lead of the spiral that will be cut when gears with an equal number of teeth are placed on the feed-screw and the worm-gear stud, and an idler of suitable size is interposed between the gears.

Rule: — To find the lead of a milling machine, place equal gears on the worm-gear stud and on the feed-screw, and multiply the number of revolutions made by the feed-screw to produce one revolution of the index head spindle, by the lead of the thread on the feed-screw.

Expressing the rule given as a formula:

$$\begin{array}{c}\text{lead of milling}\\ \text{machine}\end{array} = \begin{array}{c}\text{rev. of feed-screw for one}\\ \text{revolution of index spindle}\\ \text{with equal gears}\end{array} \times \begin{array}{c}\text{lead of}\\ \text{feed-screw.}\end{array}$$

Assume that it is necessary to make 40 revolutions of the feed-screw to turn the index head spindle one complete revolution, when the gears are equal, and that the lead of the thread on the feed-screw of the milling machine is ¼ inch; then the lead of the machine equals $40 \times \frac{1}{4}$ inch = 10 inches.

Change Gears for Spiral Milling. — To find the change gears to be used in the compound train of gears for spiral milling, place the lead of the spiral to be cut in the numerator and the lead of the milling machine in the denominator of a fraction; divide numerator and denominator into two factors each; and multiply each "pair" of factors by the *same* number until suitable numbers of teeth for the change gears are obtained. (One factor in the numerator and one in the denominator are considered as one "pair" in this calculation.)

Example. — Assume that the lead of a machine is 10 inches, and that a spiral having a 48-inch lead is to be cut. Following the method explained:

$$\frac{48}{10} = \frac{6 \times 8}{2 \times 5} = \frac{(6 \times 12) \times (8 \times 8)}{(2 \times 12) \times (5 \times 8)} = \frac{72 \times 64}{24 \times 40}$$

The gear having 72 teeth is placed on the worm-gear stud and meshes with the 24-tooth gear on the intermediate stud. On the same intermediate stud is then placed the gear having 64 teeth, which is driven by the gear having 40 teeth placed on the feed-screw. This makes the gears having 72 and 64 teeth the driven gears, and the gears having 24 and 40 teeth the driving gears. In general, for compound gearing, the following formula may be used:

$$\frac{\text{lead of spiral to be cut}}{\text{lead of machine}} = \frac{\text{product of driven gears}}{\text{product of driving gears}}$$

Lead of Spiral for Given Change-gear Combination. — Inasmuch as the lead of the spiral divided by the lead of the machine equals the product of the driven gears divided by the product of the driving gears, the lead of spiral obtained with any combination of change gears can be determined as follows: Multiply the product of the driven gears by the lead of the machine, and divide by the product of the driving gears. As the common lead of universal milling machines is 10 inches, the rule can be simplified as follows: Divide 10 times the product of the driven gears by the product of the drivers, and the quotient will be the lead of the spiral.

Change Gears for Different Leads — 0.670 Inch to 2.658 Inches

Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw
0.670	24	86	24	100	1.711	28	72	44	100	2.182	24	44	40	100
0.781	24	86	28	100	1.714	24	56	40	100	2.188	24	48	28	64
0.800	24	72	24	100	1.744	24	64	40	86	2.193	24	56	44	86
0.893	24	86	32	100	1.745	24	44	32	100	2.200	24	48	44	100
0.930	24	72	24	86	1.750	28	64	40	100	2.222	24	48	32	72
1.029	24	56	24	100	1.776	24	44	28	86	2.233	40	86	48	100
1.042	28	86	32	100	1.778	32	72	40	100	2.238	28	64	44	86
1.047	24	64	24	86	1.786	24	86	64	100	2.240	28	40	32	100
1.050	24	64	28	100	1.800	24	64	48	100	2.250	24	40	24	64
1.067	24	72	32	100	1.809	28	72	40	86	2.274	32	72	44	86
1.085	24	72	28	86	1.818	24	44	24	72	2.286	32	56	40	100
1.116	24	86	40	100	1.823	28	86	56	100	2.292	24	64	44	72
1.196	24	56	24	86	1.860	28	56	32	86	2.326	32	64	40	86
1.200	24	48	24	100	1.861	24	72	48	86	2.333	28	48	40	100
1.221	24	64	28	86	1.867	28	48	32	100	2.338	24	44	24	56
1.228	24	86	44	100	1.875	24	48	24	64	2.344	28	86	72	100
1.240	24	72	32	86	1.886	24	56	44	100	2.368	28	44	32	86
1.250	24	64	24	72	1.905	24	56	32	72	2.381	32	86	64	100
1.302	28	86	40	100	1.919	24	64	44	86	2.386	24	44	28	64
1.309	24	44	24	100	1.920	24	40	32	100	2.392	24	56	48	86
1.333	24	72	40	100	1.925	28	64	44	100	2.400	28	56	48	100
1.340	24	86	48	100	1.944	24	48	28	72	2.424	24	44	32	72
1.371	24	56	32	100	1.954	24	40	28	86	2.431	28	64	40	72
1.395	24	48	24	86	1.956	32	72	44	100	2.442	24	32	28	86
1.400	24	48	28	100	1.990	28	72	44	86	2.445	40	72	44	100
1.429	24	56	24	72	1.993	24	56	40	86	2.450	28	64	56	100
1.440	24	40	24	100	2.000	24	40	24	72	2.456	44	86	48	100
1.458	24	64	28	72	2.009	24	86	72	100	2.481	32	72	48	86
1.467	24	72	44	100	2.030	24	44	32	86	2.489	32	72	56	100
1.488	32	86	40	100	2.035	28	64	40	86	2.500	24	48	28	56
1.500	24	64	40	100	2.036	28	44	32	100	2.514	32	56	44	100
1.522	24	44	24	86	2.045	24	44	24	64	2.532	28	72	56	86
1.550	24	72	40	86	2.047	40	86	44	100	2.537	24	44	40	86
1.563	24	86	56	100	2.057	24	28	24	100	2.546	28	44	40	100
1.595	24	56	32	86	2.067	32	72	40	86	2.558	32	64	44	86
1 600	24	48	32	100	2.083	24	64	40	72	2.567	28	48	44	100
1.607	24	56	24	64	2.084	28	86	64	100	2.571	24	40	24	56
1.628	24	48	28	86	2.093	24	64	48	86	2.593	28	48	32	72
1.637	32	86	44	100	2.100	24	64	56	100	2.605	28	40	32	86
1.650	24	64	44	100	2.121	24	44	28	72	2.618	24	44	48	100
1.667	24	56	28	72	2.133	24	72	64	100	2.619	24	56	44	72
1.674	24	40	24	86	2.143	24	56	32	64	2.625	24	40	28	64
1.680	24	40	28	100	2.171	24	72	56	86	2.640	24	40	44	100
1.706	24	72	44	86	2.178	28	72	56	100	2.658	32	56	40	86

Change Gears for Different Leads — 2.667 Inches to 4.040 Inches

Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw
2.667	40	72	48	100	3.140	24	86	72	64	3.588	72	56	24	86
2.674	28	64	44	72	3.143	40	56	44	100	3.600	72	48	24	100
2.678	24	56	40	64	3.150	28	100	72	64	3.618	56	72	40	86
2.679	32	86	72	100	3.175	32	56	40	72	3.636	24	44	32	48
2.700	24	64	72	100	3.182	28	44	32	64	3.637	48	44	24	72
2.713	28	48	40	86	3.189	32	56	48	86	3.646	40	48	28	64
2.727	24	44	32	64	3.190	24	86	64	56	3.655	40	56	44	86
2.743	24	56	64	100	3.198	40	64	44	86	3.657	64	56	32	100
2.750	40	64	44	100	3.200	28	100	64	56	3.663	72	64	28	86
2.778	32	64	40	72	3.214	24	56	48	64	3.667	40	48	44	100
2.791	28	56	48	86	3.225	24	100	86	64	3.673	24	28	24	56
2.800	24	24	28	100	3.241	28	48	40	72	3.684	44	86	72	100
2.812	24	32	24	64	3.256	24	24	28	86	3.686	86	56	24	100
2.828	28	44	32	72	3.267	28	48	56	100	3.704	32	48	40	72
2.843	40	72	44	86	3.273	24	40	24	44	3.721	24	24	32	86
2.845	32	72	64	100	3.275	44	86	64	100	3.733	48	72	56	100
2.849	28	64	56	86	3.281	24	32	28	64	3.750	24	32	24	48
2.857	24	48	32	56	3.300	44	64	48	100	3.763	86	64	28	100
2.865	44	86	56	100	3.308	32	72	64	86	3.771	44	56	48	100
2.867	86	72	24	100	3.333	32	64	48	72	3.772	24	28	44	100
2.880	24	40	48	100	3.345	28	100	86	72	3.799	56	48	28	86
2.894	28	72	64	86	3.349	40	86	72	100	3.809	24	28	32	72
2.909	32	44	40	100	3.360	56	40	24	100	3.810	64	56	24	72
2.917	24	64	56	72	3.383	32	44	40	86	3.818	24	40	28	44
2.924	32	56	44	86	3.403	28	64	56	72	3.819	40	64	44	72
2.933	44	72	48	100	3.409	24	44	40	64	3.822	86	72	32	100
2.934	32	48	44	100	3.411	32	48	44	86	3.837	24	32	44	86
2.946	24	56	44	64	3.422	44	72	56	100	3.840	64	40	24	100
2.950	28	44	40	86	3.428	24	40	32	56	3.850	44	64	56	100
2.977	40	86	64	100	3.429	40	28	24	100	3.876	24	72	100	86
2.984	28	48	44	86	3.438	24	48	44	64	3.889	32	64	56	72
3.000	24	40	28	56	3.488	40	64	48	86	3.896	24	44	40	56
3.030	24	44	40	72	3.491	64	44	24	100	3.907	56	40	24	86
3.044	24	44	48	86	3.492	32	56	44	72	3.911	44	72	64	100
3.055	28	44	48	100	3.500	40	64	56	100	3.920	28	40	56	100
3.056	32	64	44	72	3.520	32	40	44	100	3.927	72	44	24	100
3.070	24	40	44	86	3.535	28	44	40	72	3.929	32	56	44	64
3.080	28	40	44	100	3.552	56	44	24	86	3.977	28	44	40	64
3.086	24	56	72	100	3.556	40	72	64	100	3.979	44	72	56	86
3.101	40	72	48	86	3.564	56	44	28	100	3.987	24	28	40	86
3.111	28	40	32	72	3.565	28	48	44	72	4.000	24	40	32	48
3.117	24	44	32	56	3.571	24	48	40	56	4.011	28	48	44	64
3.125	28	56	40	64	3.572	48	86	64	100	4.019	72	86	48	100
3.126	48	86	56	100	3.582	44	40	28	86	4.040	32	44	40	72

Change Gears for Different Leads — 4.059 Inches to 5.568 Inches

Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw
4.059	32	44	48	86	4.567	72	44	24	86	5.105	28	48	56	64
4.060	64	44	24	86	4.572	40	56	64	100	5.116	44	24	24	86
4.070	28	32	40	86	4.582	72	44	28	100	5.119	86	56	24	72
4.073	64	44	28	100	4.583	44	64	48	72	5.120	64	40	32	100
4.074	32	48	44	72	4.584	32	48	44	64	5.133	56	48	44	100
4.091	24	44	48	64	4.651	40	24	24	86	5.134	44	24	28	100
4.093	32	40	44	86	4.655	64	44	32	100	5.142	72	56	40	100
4.114	48	28	24	100	4.667	28	40	32	48	5.143	24	28	24	40
4.125	24	40	44	64	4.675	24	28	24	44	5.156	44	32	24	64
4.135	40	72	64	86	4.687	40	32	24	64	5.160	86	40	24	100
4.144	56	44	28	86	4.688	56	86	72	100	5.168	100	72	32	86
4.167	28	48	40	56	4.691	86	44	24	100	5.185	28	24	32	72
4.186	72	64	32	86	4.714	44	40	24	56	5.186	64	48	28	72
4.200	48	64	56	100	4.736	64	44	28	86	5.195	32	44	40	56
4.242	28	44	32	48	4.762	40	28	24	72	5.209	100	64	24	72
4.253	64	56	32	86	4.773	24	32	28	44	5.210	64	40	28	86
4.264	40	48	44	86	4.778	86	72	40	100	5.226	86	64	28	72
4.267	64	48	32	100	4.784	72	56	32	86	5.233	72	64	40	86
4.278	28	40	44	72	4.785	48	28	24	86	5.236	72	44	32	100
4.286	24	28	24	48	4.800	48	24	24	100	5.238	44	28	24	72
4.300	86	56	28	100	4.813	44	40	28	64	5.250	24	32	28	40
4.320	72	40	24	100	4.821	72	56	24	64	5.256	86	72	44	100
4.341	48	72	56	86	4.849	32	44	48	72	5.280	48	40	44	100
4.342	64	48	28	86	4.861	40	32	28	72	5.303	28	44	40	48
4.361	100	64	24	86	4.884	48	64	56	86	5.316	40	28	32	86
4.363	24	40	32	44	4.889	32	40	44	72	5.328	72	44	28	86
4.364	40	44	48	100	4.898	24	28	32	56	5.333	40	24	32	100
4.365	40	56	44	72	4.900	56	32	28	100	5.347	44	64	56	72
4.375	24	24	28	64	4.911	40	56	44	64	5.348	44	32	28	72
4.386	24	28	44	86	4.914	86	56	32	100	5.357	40	28	24	64
4.400	24	24	44	100	4.950	56	44	28	72	5.358	64	86	72	100
4.444	64	56	28	72	4.961	64	48	32	86	5.375	86	64	40	100
4.465	64	40	24	86	4.978	56	72	64	100	5.400	72	32	24	100
4.466	48	40	32	86	4.984	100	56	24	86	5.413	64	44	32	86
4.477	44	32	28	86	5.000	24	24	28	56	5.426	40	24	28	86
4.479	86	64	24	72	5.017	86	48	28	100	5.427	40	48	56	86
4.480	56	40	32	100	5.023	72	40	24	86	5.444	56	40	28	72
4.500	72	64	40	100	5.029	44	28	32	100	5.455	48	44	28	56
4.522	100	72	28	86	5.040	72	40	28	100	5.469	40	32	28	64
4.537	56	48	28	72	5.074	40	44	48	86	5.473	86	44	28	100
4.545	24	44	40	48	5.080	64	56	32	72	5.486	64	28	24	100
4.546	28	44	40	56	5.088	100	64	28	86	5.500	44	40	24	48
4.548	44	72	64	86	5.091	56	44	40	100	5.556	40	24	24	72
4.558	56	40	28	86	5.093	40	48	44	72	5.568	56	44	28	64

Change Gears for Different Leads — 5.581 Inches to 7.500 Inches

Lead in Inches	Driven Gear on Worm	Driver First Gear on Stud	Driven Second Gear on Stud	Driver Gear on Screw	Lead in Inches	Driven Gear on Worm	Driver First Gear on Stud	Driven Second Gear on Stud	Driver Gear on Screw	Lead in Inches	Driven Gear on Worm	Driver First Gear on Stud	Driven Second Gear on Stud	Driver Gear on Screw
5.581	64	32	24	86	6.172	72	28	24	100	6.825	86	56	32	72
5.582	48	24	24	86	6.202	40	24	32	86	6.857	32	28	24	40
5.600	56	24	24	100	6.222	64	40	28	72	6.875	44	24	24	64
5.625	48	32	24	64	6.234	32	28	24	44	6.880	86	40	32	100
5.657	56	44	32	72	6.250	24	24	40	64	6.944	100	48	24	72
5.698	56	32	28	86	6.255	86	44	32	100	6.945	100	56	28	72
5.714	48	28	24	72	6.279	72	64	48	86	6.968	86	48	28	72
5.730	40	48	44	64	6.286	44	40	32	56	6.977	48	32	40	86
5.733	86	48	32	100	6.300	72	32	28	100	6.982	64	44	48	100
5.756	72	64	44	86	6.343	100	44	24	86	6.984	44	28	32	72
5.759	86	56	24	64	6.350	40	28	32	72	7.000	28	24	24	40
5.760	72	40	32	100	6.364	56	44	24	48	7.013	72	44	24	56
5.788	64	72	56	86	6.379	64	28	24	86	7.040	64	40	44	100
5.814	100	64	32	86	6.396	44	32	40	86	7.071	56	44	40	72
5.818	64	44	40	100	6.400	64	24	24	100	7.104	56	44	48	86
5.833	28	24	24	48	6.417	44	40	28	48	7.106	100	72	44	86
5.847	64	56	44	86	6.429	24	28	24	32	7.111	64	40	32	72
5.848	44	28	32	86	6.450	86	64	48	100	7.130	44	24	28	72
5.861	72	40	28	86	6.460	100	72	40	86	7.143	40	28	32	64
5.867	44	24	32	100	6.465	64	44	32	72	7.159	72	44	28	64
5.893	44	32	24	56	6.482	56	48	40	72	7.163	56	40	44	86
5.912	86	64	44	100	6.512	56	24	24	86	7.167	86	40	24	72
5.920	56	44	40	86	6.515	86	44	24	72	7.176	72	28	24	86
5.926	64	48	32	72	6.534	56	24	28	100	7.200	72	24	24	100
5.952	100	56	24	72	6.545	48	40	24	44	7.268	100	64	40	86
5.954	64	40	32	86	6.548	44	48	40	56	7.272	64	44	28	56
5.969	44	24	28	86	6.563	56	32	24	64	7.273	32	24	24	44
5.972	86	48	24	72	6.578	72	56	44	86	7.292	56	48	40	64
5.980	72	56	40	86	6.600	48	32	44	100	7.310	44	28	40	86
6.000	48	40	28	56	6.645	100	56	32	86	7.314	64	28	32	100
6.016	44	32	28	64	6.667	64	48	28	56	7.326	72	32	28	86
6.020	86	40	28	100	6.689	86	72	56	100	7.330	86	44	24	64
6.061	40	44	32	48	6.697	100	56	24	64	7.333	44	24	40	100
6.077	100	64	28	72	6.698	72	40	32	86	7.334	44	40	32	48
6.089	72	44	32	86	6.719	86	48	24	64	7.347	48	28	24	56
6.109	56	44	48	100	6.720	56	40	48	100	7.371	86	56	48	100
6.112	24	24	44	72	6.735	44	28	24	56	7.372	86	28	24	100
6.122	40	28	24	56	6.750	72	40	24	64	7.400	100	44	28	86
6.125	56	40	28	64	6.757	86	56	44	100	7.408	40	24	32	72
6.137	72	44	24	64	6.766	64	44	40	86	7.424	56	44	28	48
6.140	48	40	44	86	6.784	100	48	28	86	7.442	64	24	24	86
6.143	86	56	40	100	6.806	56	32	28	72	7.465	86	64	40	72
6.160	56	40	44	100	6.818	40	32	24	44	7.467	64	24	28	100
6.171	72	56	48	100	6.822	44	24	32	86	7.500	48	24	24	64

Change Gears for Different Leads — 7.525 Inches to 9.598 Inches

Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw
7.525	86	32	28	100	8.140	56	32	40	86	8.800	48	24	44	100
7.543	48	28	44	100	8.145	64	44	56	100	8.838	100	44	28	72
7.576	100	44	24	72	8.148	64	48	44	72	8.839	72	56	44	64
7.597	56	24	28	86	8.149	44	24	32	72	8.909	56	40	28	44
7.601	86	44	28	72	8.163	40	28	32	56	8.929	100	48	24	56
7.611	72	44	40	86	8.167	56	40	28	48	8.930	64	40	48	86
7.619	64	48	32	56	8.182	48	32	24	44	8.953	56	32	44	86
7.020	64	28	24	72	8.186	64	40	44	86	8.959	86	48	28	56
7.636	56	40	24	44	8.212	86	64	44	72	8.960	64	40	56	100
7.639	44	32	40	72	8.229	72	28	32	100	8.980	44	28	32	56
7.644	86	72	64	100	8.250	44	32	24	40	9.000	48	32	24	40
7.657	56	32	28	64	8.306	100	56	40	86	9.044	100	72	56	86
7.674	72	48	44	86	8.312	64	44	32	56	9.074	56	24	28	72
7.675	48	32	44	86	8.333	40	24	24	48	9.091	40	24	24	44
7.679	86	48	24	56	8.334	40	24	28	56	9.115	100	48	28	64
7.680	64	40	48	100	8.361	86	40	28	72	9.134	72	44	48	86
7.700	56	32	44	100	8.372	72	24	24	86	9.137	100	56	44	86
7.714	72	40	24	56	8.377	86	44	24	56	9.143	64	40	32	56
7.752	100	48	32	86	8.400	72	24	28	100	9.164	72	44	56	100
7.778	32	24	28	48	8.437	72	32	24	64	9.167	44	24	24	48
7.792	40	28	24	44	8.457	100	44	32	86	9.210	72	40	44	86
7.813	100	48	24	64	8.484	32	24	28	44	9.214	86	40	24	56
7.815	56	40	48	86	8.485	64	44	28	48	9.260	100	48	32	72
7.818	86	44	40	100	8.485	56	44	32	48	9.302	48	24	40	86
7.838	86	48	28	64	8.506	64	28	32	86	9.303	56	28	40	86
7.855	72	44	48	100	8.523	100	44	24	64	9.333	64	40	28	48
7.857	44	24	24	56	8.527	44	24	40	86	9.334	32	24	28	40
7.872	44	28	32	64	8.532	86	56	40	72	9.351	48	28	24	44
7.875	72	40	28	64	8.534	64	24	32	100	9.375	48	32	40	64
7.883	86	48	44	100	8.552	86	44	28	64	9.382	86	44	48	100
7.920	72	40	44	100	8.556	56	40	44	72	9.385	86	56	44	72
7.936	100	56	32	72	8.572	64	32	24	56	9.406	86	40	28	64
7.954	40	32	28	44	8.572	48	24	24	56	9.428	44	28	24	40
7.955	56	44	40	64	8.594	44	32	40	64	9.429	48	40	44	56
7.963	86	48	32	72	8.600	86	24	24	100	9.460	86	40	44	100
7.974	48	28	40	86	8.640	72	40	48	100	9.472	64	44	56	86
7.994	100	64	44	86	8.681	100	64	40	72	9.524	40	28	32	48
8.000	64	32	40	100	8.682	64	24	28	86	9.545	72	44	28	48
8 021	44	32	28	48	8.687	86	44	32	72	9.546	56	32	24	44
8.035	72	56	40	64	8.721	100	32	24	86	9.547	56	44	48	64
8.063	86	40	24	64	8.727	48	40	32	44	9.549	100	64	44	72
8.081	64	44	40	72	8.730	44	28	40	72	9.556	86	40	32	72
8.102	100	48	28	72	8.750	28	24	24	32	9.569	72	28	32	86
8.119	64	44	48	86	8.772	48	28	44	86	9.598	86	56	40	64

Change Gears for Different Leads — 9.600 Inches to 12.375 Inches

Lead in Inches	Driven — Gear on Worm	Driver — First Gear on Stud	Driven — Second Gear on Stud	Driver — Gear on Screw	Lead in Inches	Driven — Gear on Worm	Driver — First Gear on Stud	Driven — Second Gear on Stud	Driver — Gear on Screw	Lead in Inches	Driven — Gear on Worm	Driver — First Gear on Stud	Driven — Second Gear on Stud	Driver — Gear on Screw
9.600	72	24	32	100	10.370	64	24	28	72	11.314	72	28	44	100
9.625	44	32	28	40	10.371	64	48	56	72	11.363	100	44	24	48
9.643	72	32	24	56	10.390	40	28	32	44	11.401	86	44	28	48
9.675	86	64	72	100	10.417	100	32	24	72	11.429	32	24	24	28
9.690	100	48	40	86	10.419	64	40	56	86	11.454	72	40	28	44
9.697	64	48	32	44	10.451	86	32	28	72	11.459	44	24	40	64
9.723	40	24	28	48	10.467	72	32	40	86	11.467	86	24	32	100
9.741	100	44	24	56	10.473	72	44	64	100	11.512	72	32	44	86
9.768	72	48	56	86	10.476	44	24	32	56	11.518	86	28	24	64
9.773	86	44	24	48	10.477	48	28	44	72	11.520	72	40	64	100
9.778	64	40	44	72	10.500	56	32	24	40	11.574	100	48	40	72
9.796	64	28	24	56	10.558	86	56	44	64	11.629	100	24	24	86
9.818	72	40	24	44	10.571	100	44	40	86	11.638	64	40	32	44
9.822	44	32	40	56	10.606	56	44	40	48	11.667	56	24	24	48
9.828	86	28	32	100	10.631	64	28	40	86	11.688	72	44	40	56
9.844	72	32	28	64	10.655	72	44	56	86	11.695	64	28	44	86
9.900	72	32	44	100	10.659	100	48	44	86	11.719	100	32	24	64
9.921	100	56	40	72	10.667	64	40	48	72	11.721	72	40	56	86
9.923	64	24	32	86	10.694	44	24	28	48	11.728	86	40	24	44
9.943	100	44	28	64	10.713	40	28	24	32	11.733	64	24	44	100
9.954	86	48	40	72	10.714	48	32	40	56	11.757	86	32	28	64
9.967	100	56	48	86	10.750	86	40	24	48	11.785	72	48	44	56
9.968	100	28	24	86	10.800	72	32	48	100	11.786	44	28	24	32
10.000	56	28	24	48	10.853	56	24	40	86	11.825	86	32	44	100
10.033	86	24	28	100	10.859	86	44	40	72	11.905	100	28	24	72
10.046	72	40	48	86	10.909	72	44	32	48	11.938	56	24	44	86
10.057	64	28	44	100	10.913	100	56	44	72	11.944	86	24	24	72
10.078	86	32	24	64	10.937	56	32	40	64	11.960	72	28	40	86
10.080	72	40	56	100	10.945	86	44	56	100	12.000	48	24	24	40
10.101	100	44	32	72	10.949	86	48	44	72	12.031	56	32	44	64
10.159	64	28	32	72	10.972	64	28	48	100	12.040	86	40	56	100
10.175	100	32	28	86	11.000	44	24	24	40	12.121	40	24	32	44
10.182	64	40	28	44	11.021	72	28	24	56	12.153	100	32	28	72
10.186	44	24	40	72	11.057	86	56	72	100	12.178	72	44	64	86
10.209	56	24	28	64	11.111	40	24	32	48	12.216	86	44	40	64
10.228	72	44	40	64	11.137	56	32	28	44	12.222	44	24	32	48
10.233	48	24	44	86	11.160	100	56	40	64	12.245	48	28	40	56
10.238	86	28	24	72	11.163	72	24	32	86	12.250	56	32	28	40
10.267	56	24	44	100	11.169	86	44	32	56	12.272	72	32	24	44
10.286	48	28	24	40	11.198	86	48	40	64	12.277	100	56	44	64
10.312	48	32	44	64	11.200	56	24	48	100	12.286	86	28	40	100
10.313	72	48	44	64	11.225	44	28	40	56	12.318	86	48	44	64
10.320	86	40	48	100	11.250	72	24	24	64	12.343	72	28	48	100
10.336	100	72	64	86	11.313	64	44	56	72	12.375	72	40	44	64

Change Gears for Different Leads — 12.403 Inches to 16.000 Inches

Lead in Inches	Driven	Driver	Driven	Driver	Lead in Inches	Driven	Driver	Driven	Driver	Lead in Inches	Driven	Driver	Driven	Driver
	Gear on Worm	First Gear on Stud	Second Gear on Stud	Gear on Screw		Gear on Worm	First Gear on Stud	Second Gear on Stud	Gear on Screw		Gear on Worm	First Gear on Stud	Second Gear on Stud	Gear on Screw
12.403	64	24	40	86	13.438	86	24	24	64	14.668	44	24	32	40
12.444	64	40	56	72	13.469	48	28	44	56	14.694	72	28	32	56
12.468	64	28	24	44	13.500	72	32	24	40	14.743	86	28	48	100
12.500	40	24	24	32	13.514	86	28	44	100	14.780	86	40	44	64
12.542	86	40	28	48	13.566	100	24	28	86	14.800	100	44	56	86
12.508	86	44	64	100	13.611	56	24	28	48	14.815	64	24	40	72
12.558	72	32	48	86	13.636	48	32	40	44	14.849	56	24	28	44
12.571	64	40	44	56	13.643	64	24	44	86	14.880	100	48	40	56
12.572	44	28	32	40	13.650	86	28	32	72	14.884	64	28	56	86
12.600	72	32	56	100	13.672	100	32	28	64	14.931	86	32	40	72
12.627	100	44	40	72	13.682	86	40	28	44	14.933	64	24	56	100
12.686	100	44	48	86	13.713	64	40	48	56	14.950	100	56	72	86
12.698	64	28	40	72	13.715	64	28	24	40	15.000	48	24	24	32
12.727	64	32	28	44	13.750	44	24	24	32	15.050	86	32	56	100
12.728	56	24	24	44	13.760	86	40	64	100	15.150	100	44	32	48
12.732	100	48	44	72	13.889	100	24	24	72	15.151	100	44	48	72
12.758	64	28	48	86	13.933	86	48	56	72	15.202	86	44	56	72
12.791	100	40	44	86	13.935	86	24	28	72	15.238	64	28	48	72
12.798	86	48	40	56	13.953	72	24	40	86	15.239	64	28	32	48
12.800	64	28	56	100	13.960	86	44	40	56	15.272	56	40	48	44
12.834	56	40	44	48	13.968	64	28	44	72	15.278	44	24	40	48
12.857	72	28	32	64	14.000	56	24	24	40	15.279	100	40	44	72
12.858	48	28	24	32	14.025	72	44	48	56	15.306	100	28	24	56
12.900	86	32	48	100	14.026	72	28	24	44	15.349	72	24	44	86
12.963	56	24	40	72	14.063	72	32	40	64	15.357	86	28	24	48
12.987	100	44	32	56	14.071	86	44	72	100	15.429	72	40	48	56
13.020	100	48	40	64	14.078	86	48	44	56	15.469	72	32	44	64
13.024	56	24	48	86	14.142	72	40	44	56	15.480	86	40	72	100
13.030	86	44	32	48	14.204	100	44	40	64	15.504	100	48	64	86
13.062	64	28	32	56	14.260	56	24	44	72	15.556	64	32	56	72
13.082	100	64	72	86	14.286	40	24	24	28	15.584	48	28	40	44
13.090	72	40	32	44	14.318	72	32	28	44	15.625	100	24	24	64
13.096	44	28	40	48	14.319	72	44	56	64	15.636	86	40	32	44
13.125	72	32	28	48	14.322	100	48	44	64	15.677	86	32	28	48
13.139	86	40	44	72	14.333	86	40	32	48	15.714	44	24	24	28
13.157	72	28	44	86	14.352	72	28	48	86	15.750	72	32	28	40
13.163	86	28	24	56	14.400	72	24	48	100	15.767	86	24	44	100
13.200	72	24	44	100	14.536	100	32	40	86	15.873	100	56	64	72
13.258	100	44	28	48	14.545	64	24	24	44	15.874	100	28	32	72
13.289	100	28	32	86	14.583	56	32	40	48	15.909	100	40	28	44
13.333	64	24	24	48	14.584	40	24	28	32	15.925	86	48	64	72
13.393	100	56	48	64	14.651	72	32	56	86	15.926	86	24	32	72
13.396	72	40	64	86	14.659	86	44	48	64	15.989	100	32	44	86
13.437	86	32	28	56	14.667	64	40	44	48	16.000	64	24	24	40

Change Gears for Different Leads — 16.042 Inches to 21.39 Inches

Lead in Inches	Gear on Worm (Driven)	First Gear on Stud (Driver)	Second Gear on Stud (Driven)	Gear on Screw (Driver)	Lead in Inches	Gear on Worm (Driven)	First Gear on Stud (Driver)	Second Gear on Stud (Driven)	Gear on Screw (Driver)	Lead in Inches	Gear on Worm (Driven)	First Gear on Stud (Driver)	Second Gear on Stud (Driven)	Gear on Screw (Driver)
16.042	56	24	44	64	17.442	100	32	48	86	19.350	86	32	72	100
16.043	44	24	28	32	17.454	64	40	48	44	19.380	100	24	40	86
16.071	72	32	40	56	17.500	56	24	24	32	19.394	64	24	32	44
16.125	86	32	24	40	17.550	86	28	32	56	19.444	40	24	28	24
16.204	100	24	28	72	17.677	100	44	56	72	19.480	100	28	24	44
16.233	100	44	40	56	17.679	72	32	44	56	19.531	100	32	40	64
16.280	100	40	56	86	17.778	64	24	32	48	19.535	72	24	56	86
16.288	86	44	40	48	17.858	100	24	24	56	19.545	86	24	24	44
16.296	64	24	44	72	17.917	86	24	32	64	19.590	64	28	48	56
16.327	64	28	40	56	17.918	86	24	24	48	19.635	72	40	48	44
16.333	56	24	28	40	17.959	64	28	44	56	19.642	100	40	44	56
16.364	72	24	24	44	18.000	72	24	24	40	19.643	44	28	40	32
16.370	100	48	44	56	18.181	56	28	40	44	19.656	86	28	64	100
16.423	86	32	44	72	18.182	48	24	40	44	19.687	72	32	56	64
16.456	72	28	64	100	18.229	100	32	28	48	19.710	86	40	44	48
16.500	72	40	44	48	18.273	100	28	44	86	19.840	100	28	40	72
16.612	100	28	40	86	18.285	64	28	32	40	19.886	100	44	56	64
16.623	64	28	32	44	18.333	56	28	44	48	19.887	100	32	28	44
16.667	56	28	40	48	18.367	72	28	40	56	19.908	86	24	40	72
16.722	86	40	56	72	18.428	86	28	24	40	19.934	100	28	48	86
16.744	72	24	48	86	18.476	86	32	44	64	20.00	72	24	32	48
16.752	86	44	48	56	18.519	100	24	32	72	20.07	86	24	56	100
16.753	86	28	24	44	18.605	100	40	64	86	20.09	100	56	72	64
16.797	86	32	40	64	18.663	100	64	86	72	20.16	86	48	72	64
16.800	72	24	56	100	18.667	64	24	28	40	20.20	100	44	64	72
16.875	72	32	48	64	18.700	72	44	64	56	20.35	100	32	56	86
16.892	86	40	44	56	18.750	100	32	24	40	20.36	64	40	56	44
16.914	100	44	64	86	18.750	72	32	40	48	20.41	100	28	32	56
16.969	64	44	56	48	18.770	86	28	44	72	20.42	56	24	28	32
16.970	64	24	28	44	18.812	86	32	28	40	20.45	72	32	40	44
17.045	100	32	24	44	18.858	48	28	44	40	20.48	86	48	64	56
17.046	100	44	48	64	18.939	100	44	40	48	20.57	72	40	64	56
17.062	86	28	40	72	19.029	100	44	72	86	20.63	72	32	44	48
17.101	86	44	56	64	19.048	40	24	32	28	20.74	64	24	56	72
17.102	86	32	28	44	19.090	56	32	48	44	20.78	64	28	40	44
17.141	64	32	48	56	19.091	72	24	28	44	20.83	100	32	48	72
17.143	64	28	24	32	19.096	100	32	44	72	20.90	86	32	56	72
17.144	48	24	24	28	19.111	86	40	64	72	20.93	100	40	72	86
17.188	100	40	44	64	19.136	72	28	64	86	20.95	64	28	44	48
17.200	86	32	64	100	19.197	86	32	40	56	21.00	56	32	48	40
17.275	86	56	72	64	19.200	72	24	64	100	21.12	86	32	44	56
17.361	100	32	40	72	19.250	56	32	44	40	21.32	100	24	44	86
17.364	64	24	56	86	19.285	72	32	48	56	21.33	100	56	86	72
17.373	86	44	64	72	19.286	72	28	24	32	21.39	44	24	28	24

Change Gears for Different Leads — 21.43 Inches to 32.09 Inches

Lead in Inches	Gear on Worm (Driven)	First Gear on Stud (Driver)	Second Gear on Stud (Driven)	Gear on Screw (Driver)	Lead in Inches	Gear on Worm (Driven)	First Gear on Stud (Driver)	Second Gear on Stud (Driven)	Gear on Screw (Driver)	Lead in Inches	Gear on Worm (Driven)	First Gear on Stud (Driver)	Second Gear on Stud (Driven)	Gear on Screw (Driver)
21.43	100	40	48	56	24.88	100	72	86	48	28.05	72	28	48	44
21.48	100	32	44	64	24.93	64	28	48	44	28.06	100	28	44	56
21.50	86	24	24	40	25.00	72	24	40	48	28.13	100	40	72	64
21.82	72	44	64	48	25.08	86	24	28	40	28.15	86	28	44	48
21.88	100	40	56	64	25.09	86	40	56	48	28.29	72	28	44	40
21.90	86	24	44	72	25.13	86	44	72	56	28.41	100	32	40	44
21.94	86	28	40	56	25.14	64	28	44	40	28.57	100	56	64	40
21.99	86	44	72	64	25.45	64	44	56	32	28.64	72	44	56	32
22.00	64	32	44	40	25.46	100	24	44	72	28.65	100	32	44	48
22.04	72	28	48	56	25.51	100	28	40	56	28.67	86	40	64	48
22.11	86	28	72	100	25.57	100	64	72	44	29.09	64	24	48	44
22.22	100	40	64	72	25.60	86	28	40	48	29.17	100	40	56	48
22.34	86	44	64	56	25.67	56	24	44	40	29.22	100	56	72	44
22.40	86	32	40	48	25.71	72	24	48	56	29.32	86	48	72	44
22.50	72	24	48	64	25.72	72	24	24	28	29.34	64	24	44	40
22.73	100	24	24	44	25.80	86	24	72	100	29.39	72	28	64	56
22.80	86	48	56	44	25.97	100	44	64	56	29.56	86	32	44	40
22.86	64	24	24	28	26.04	100	32	40	48	29.76	100	28	40	48
22.91	72	44	56	40	26.06	86	44	64	48	29.86	100	40	86	72
22.92	100	40	44	48	26.16	100	32	72	86	29.90	100	28	72	86
22.93	86	24	64	100	26.18	72	40	64	44	30.00	56	28	48	32
23.04	86	56	72	48	26.19	44	24	40	28	30.23	86	32	72	64
23.14	100	24	40	72	26.25	72	32	56	48	30.30	100	48	64	44
23.26	100	32	64	86	26.33	86	28	48	56	30.48	64	24	32	28
23.33	64	32	56	48	26.52	100	44	56	48	30.54	100	44	86	64
23.38	72	28	40	44	26.58	100	28	64	86	30.56	44	24	40	24
23.44	100	48	72	64	26.67	64	28	56	48	30.61	100	28	48	56
23.45	86	40	48	44	26.79	100	48	72	56	30.71	86	24	48	56
23.52	86	32	56	64	26.88	86	28	56	64	30.72	86	24	24	28
23.57	72	28	44	48	27.00	72	32	48	40	30.86	72	28	48	40
23.81	100	48	64	56	27.13	100	24	56	86	31.01	100	24	64	86
23.89	86	32	64	72	27.15	100	44	86	72	31.11	64	24	56	48
24.00	64	40	72	48	27.22	56	24	28	24	31.25	100	28	56	64
24.13	86	28	44	56	27.27	100	40	48	44	31.27	86	40	64	44
24.19	86	40	72	64	27.30	86	28	64	72	31.35	86	32	56	48
24.24	64	24	40	44	27.34	100	32	56	64	31.36	86	24	28	32
24.31	100	32	56	72	27.36	86	40	56	44	31.43	64	28	44	32
24.43	86	32	40	44	27.43	64	28	48	40	31.50	72	32	56	40
24.44	44	24	32	24	27.50	56	32	44	28	31.75	100	72	64	28
24.54	72	32	48	44	27.64	86	40	72	56	31.82	100	44	56	40
24.55	100	32	44	56	27.78	100	32	64	72	31.85	86	24	64	72
24.57	86	40	64	56	27.87	86	24	56	72	31.99	100	56	86	48
24.64	86	24	44	64	27.92	86	28	40	44	32.00	64	28	56	40
24.75	72	32	44	40	28.00	100	64	86	48	32.09	56	24	44	32

Change Gears for Different Leads — 32.14 Inches to 60.00 Inches

Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw	Lead in Inches	Driven: Gear on Worm	Driver: First Gear on Stud	Driven: Second Gear on Stud	Driver: Gear on Screw
32.14	100	56	72	40	38.20	100	24	44	48	46.07	86	28	72	48
32.25	86	48	72	40	38.39	100	40	86	56	46.67	64	24	56	32
32.41	100	24	56	72	38.57	72	28	48	32	46.88	100	32	72	48
32.47	100	28	40	44	38.89	56	24	40	24	47.15	72	24	44	28
32.58	86	24	40	44	38.96	100	28	48	44	47.62	100	28	64	48
32.73	72	32	64	44	39.09	86	32	64	44	47.78	86	24	64	48
32.74	100	28	44	48	39.29	100	28	44	40	47.99	100	32	86	56
32.85	86	24	44	48	39.42	86	24	44	40	48.00	72	24	64	40
33.00	72	24	44	40	39.49	86	28	72	56	48.38	86	32	72	40
33.33	100	24	32	40	39.77	100	32	56	44	48.61	100	24	56	48
33.51	86	28	48	44	40.00	72	24	64	48	48.86	100	40	86	44
33.59	100	64	86	40	40.18	100	32	72	56	48.89	64	24	44	24
33.79	86	28	44	40	40.31	86	32	72	48	49.11	100	28	44	32
33.94	64	24	56	44	40.72	100	44	86	48	49.14	86	28	64	40
34.09	100	48	72	44	40.82	100	28	64	56	49.27	86	24	44	32
34.20	86	44	56	32	40.91	100	40	72	44	49.77	100	24	86	72
34.29	72	48	64	28	40.95	86	28	64	48	50.00	100	28	56	40
34.38	100	32	44	40	40.96	86	24	32	28	50.17	86	24	56	40
34.55	86	32	72	56	41.14	72	28	64	40	50.26	86	28	72	44
34.72	100	24	40	48	41.25	72	24	44	32	51.14	100	32	72	44
34.88	100	24	72	86	41.67	100	32	64	48	51.19	86	24	40	28
34.90	100	56	86	44	41.81	86	24	56	48	51.43	72	28	64	32
35.00	72	24	56	48	41.91	64	24	44	28	51.95	100	28	64	44
35.10	86	28	64	56	41.99	100	32	86	64	52.12	86	24	64	44
35.16	100	32	72	64	42.00	72	24	56	40	52.50	72	24	56	32
35.18	86	44	72	40	42.23	86	28	44	32	53.03	100	24	56	44
35.36	72	32	44	28	42.66	100	28	86	72	53.33	64	24	56	28
35.56	64	24	32	24	42.78	56	24	44	24	53.57	100	28	72	48
35.71	100	32	64	56	42.86	100	28	48	40	53.75	86	24	48	32
35.72	100	24	24	28	43.00	86	32	64	40	54.85	100	28	86	56
35.83	86	32	64	48	43.64	72	24	64	44	55.00	72	24	44	24
36.00	72	32	64	40	43.75	100	32	56	40	55.28	86	28	72	40
36.36	100	44	64	40	43.98	86	32	72	44	55.56	100	24	32	24
36.46	100	48	56	32	44.44	64	24	40	24	55.99	100	24	86	64
36.67	48	24	44	24	44.64	100	28	40	32	56.25	100	32	72	40
36.86	86	28	48	40	44.68	86	28	64	44	56.31	86	24	44	28
37.04	100	24	64	72	44.79	100	40	86	48	57.14	100	28	64	40
37.33	100	32	86	72	45.00	72	28	56	32	57.30	100	24	44	32
37.40	72	28	64	44	45.45	100	32	64	44	57.33	86	24	64	40
37.50	100	48	72	40	45.46	100	28	56	44	58.33	100	24	56	40
37.63	86	32	56	40	45.61	86	24	56	44	58.44	100	28	72	44
37.88	100	24	40	44	45.72	64	24	48	28	58.64	86	24	72	44
38.10	64	24	40	28	45.84	100	24	44	40	59.53	100	24	40	28
38.18	72	24	56	44	45.92	100	28	72	56	60.00	72	24	64	32

Lead of Helix for Given Angle When Diameter = 1

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	60′
0	Infin.	1800.001	899.997	599.994	449.993	359.992	299.990	257.130	224.986	199.983	179.982
1	179.982	163.616	149.978	138.438	128.545	119.973	112.471	105.851	99.967	94.702	89.964
2	89.964	85.676	81.778	78.219	74.956	71.954	69.183	66.617	64.235	62.016	59.945
3	59.945	58.008	56.191	54.485	52.879	51.365	49.934	48.581	47.299	46.082	44.927
4	44.927	43.827	42.780	41.782	40.829	39.918	39.046	38.212	37.412	36.645	35.909
5	35.909	35.201	34.520	33.866	33.235	32.627	32.040	31.475	30.928	30.400	29.890
6	29.890	29.397	28.919	28.456	28.008	27.573	27.152	26.743	26.346	25.961	25.586
7	25.586	25.222	24.868	24.524	24.189	23.863	23.545	23.236	22.934	22.640	22.354
8	22.354	22.074	21.801	21.535	21.275	21.021	20.773	20.530	20.293	20.062	19.835
9	19.835	19.614	19.397	19.185	18.977	18.773	18.574	18.379	18.188	18.000	17.817
10	17.817	17.637	17.460	17.287	17.117	16.950	16.787	16.626	16.469	16.314	16.162
11	16.162	16.013	15.866	15.722	15.581	15.441	15.305	15.170	15.038	14.908	14.780
12	14.780	14.654	14.530	14.409	14.289	14.171	14.055	13.940	13.828	13.717	13.608
13	13.608	13.500	13.394	13.290	13.187	13.086	12.986	12.887	12.790	12.695	12.600
14	12.600	12.507	12.415	12.325	12.237	12.148	12.061	11.975	11.890	11.807	11.725
15	11.725	11.643	11.563	11.484	11.405	11.328	11.252	11.177	11.102	11.029	10.956
16	10.956	10.884	10.813	10.743	10.674	10.606	10.538	10.471	10.405	10.340	10.276
17	10.276	10.212	10.149	10.086	10.025	9.964	9.904	9.844	9.785	9.727	9.669
18	9.669	9.612	9.555	9.499	9.444	9.389	9.335	9.281	9.228	9.176	9.124
19	9.124	9.072	9.021	8.971	8.921	8.872	8.823	8.774	8.726	8.679	8.631
20	8.631	8.585	8.539	8.493	8.447	8.403	8.358	8.314	8.270	8.227	8.184
21	8.184	8.142	8.099	8.058	8.016	7.975	7.935	7.894	7.855	7.815	7.776
22	7.776	7.737	7.698	7.660	7.622	7.584	7.547	7.510	7.474	7.437	7.401
23	7.401	7.365	7.330	7.295	7.260	7.225	7.191	7.157	7.123	7.089	7.056
24	7.056	7.023	6.990	6.958	6.926	6.894	6.862	6.830	6.799	6.768	6.737
25	6.737	6.707	6.676	6.646	6.617	6.586	6.557	6.528	6.499	6.470	6.441
26	6.441	6.413	6.385	6.357	6.329	6.300	6.274	6.246	6.219	6.192	6.166
27	6.166	6.139	6.113	6.087	6.061	6.035	6.009	5.984	5.959	5.933	5.908
28	5.908	5.884	5.859	5.835	5.810	5.786	5.762	5.738	5.715	5.691	5.668
29	5.668	5.644	5.621	5.598	5.575	5.553	5.530	5.508	5.486	5.463	5.441

Lead of Helix for Given Angle When Diameter = 1

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	60′
30	5.441	5.420	5.398	5.376	5.355	5.333	5.312	5.291	5.270	5.249	5.228
31	5.228	5.208	5.187	5.167	5.147	5.127	5.107	5.087	5.067	5.047	5.028
32	5.028	5.008	4.989	4.969	4.950	4.931	4.912	4.894	4.875	4.856	4.838
33	4.838	4.819	4.801	4.783	4.764	4.746	4.728	4.711	4.693	4.675	4.658
34	4.658	4.640	4.623	4.605	4.588	4.571	4.554	4.537	4.520	4.503	4.487
35	4.487	4.470	4.453	4.437	4.421	4.404	4.388	4.372	4.356	4.340	4.324
36	4.324	4.308	4.292	4.277	4.261	4.246	4.230	4.215	4.199	4.184	4.169
37	4.169	4.154	4.139	4.124	4.109	4.094	4.079	4.065	4.050	4.036	4.021
38	4.021	4.007	3.992	3.978	3.964	3.950	3.935	3.921	3.907	3.893	3.880
39	3.880	3.866	3.852	3.838	3.825	3.811	3.798	3.784	3.771	3.757	3.744
40	3.744	3.731	3.718	3.704	3.691	3.678	3.665	3.652	3.640	3.627	3.614
41	3.614	3.601	3.589	3.576	3.563	3.551	3.538	3.526	3.514	3.501	3.489
42	3.489	3.477	3.465	3.453	3.440	3.428	3.416	3.405	3.393	3.381	3.369
43	3.369	3.358	3.346	3.334	3.322	3.311	3.299	3.287	3.276	3.265	3.253
44	3.253	3.242	3.231	3.219	3.208	3.197	3.186	3.175	3.164	3.153	3.142
45	3.142	3.131	3.120	3.109	3.098	3.087	3.076	3.066	3.055	3.044	3.034
46	3.034	3.023	3.013	3.002	2.992	2.981	2.971	2.960	2.950	2.940	2.930
47	2.930	2.919	2.909	2.899	2.889	2.879	2.869	2.859	2.849	2.839	2.829
48	2.829	2.819	2.809	2.799	2.789	2.779	2.770	2.760	2.750	2.741	2.731
49	2.731	2.721	2.712	2.702	2.693	2.683	2.674	2.664	2.655	2.645	2.636
50	2.636	2.627	2.617	2.608	2.599	2.590	2.581	2.571	2.562	2.553	2.544
51	2.544	2.535	2.526	2.517	2.508	2.499	2.490	2.481	2.472	2.463	2.454
52	2.454	2.446	2.437	2.428	2.419	2.411	2.402	2.393	2.385	2.376	2.367
53	2.367	2.359	2.350	2.342	2.333	2.325	2.316	2.308	2.299	2.291	2.282
54	2.282	2.274	2.266	2.257	2.249	2.241	2.233	2.224	2.216	2.208	2.200
55	2.200	2.192	2.183	2.175	2.167	2.159	2.151	2.143	2.135	2.127	2.119
56	2.119	2.111	2.103	2.095	2.087	2.079	2.072	2.064	2.056	2.048	2.040
57	2.040	2.032	2.025	2.017	2.009	2.001	1.994	1.986	1.978	1.971	1.963
58	1.963	1.955	1.948	1.940	1.933	1.925	1.918	1.910	1.903	1.895	1.888
59	1.888	1.880	1.873	1.865	1.858	1.851	1.843	1.836	1.828	1.821	1.814

Lead of Helix for Given Angle When Diameter = 1

Deg.	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	60′
60	1.814	1.806	1.799	1.792	1.785	1.777	1.770	1.763	1.756	1.749	1.741
61	1.741	1.734	1.727	1.720	1.713	1.706	1.699	1.692	1.685	1.677	1.670
62	1.670	1.663	1.656	1.649	1.642	1.635	1.628	1.621	1.615	1.608	1.601
63	1.601	1.594	1.587	1.580	1.573	1.566	1.559	1.553	1.546	1.539	1.532
64	1.532	1.525	1.519	1.512	1.505	1.498	1.492	1.485	1.478	1.472	1.465
65	1.465	1.458	1.452	1.445	1.438	1.432	1.425	1.418	1.412	1.405	1.399
66	1.399	1.392	1.386	1.379	1.372	1.366	1.359	1.353	1.346	1.340	1.334
67	1.334	1.327	1.321	1.314	1.308	1.301	1.295	1.288	1.282	1.276	1.269
68	1.269	1.263	1.257	1.250	1.244	1.237	1.231	1.225	1.219	1.212	1.206
69	1.206	1.200	1.193	1.187	1.181	1.175	1.168	1.162	1.156	1.150	1.143
70	1.143	1.137	1.131	1.125	1.119	1.112	1.106	1.100	1.094	1.088	1.082
71	1.082	1.076	1.069	1.063	1.057	1.051	1.045	1.039	1.033	1.027	1.021
72	1.021	1.015	1.009	1.003	0.997	0.991	0.985	0.978	0.972	0.966	0.960
73	0.960	0.954	0.948	0.943	0.937	0.931	0.925	0.919	0.913	0.907	0.901
74	0.901	0.895	0.889	0.883	0.877	0.871	0.865	0.859	0.854	0.848	0.842
75	0.842	0.836	0.830	0.824	0.818	0.812	0.807	0.801	0.795	0.789	0.783
76	0.783	0.777	0.772	0.766	0.760	0.754	0.748	0.743	0.737	0.731	0.725
77	0.725	0.720	0.714	0.708	0.702	0.696	0.691	0.685	0.679	0.673	0.668
78	0.668	0.662	0.656	0.651	0.645	0.639	0.633	0.628	0.622	0.616	0.611
79	0.611	0.605	0.599	0.594	0.588	0.582	0.577	0.571	0.565	0.560	0.554
80	0.554	0.548	0.543	0.537	0.531	0.526	0.520	0.514	0.509	0.503	0.498
81	0.498	0.492	0.486	0.481	0.475	0.469	0.464	0.458	0.453	0.447	0.441
82	0.441	0.436	0.430	0.425	0.419	0.414	0.408	0.402	0.397	0.391	0.386
83	0.386	0.380	0.375	0.369	0.363	0.358	0.352	0.347	0.341	0.336	0.330
84	0.330	0.325	0.319	0.314	0.308	0.302	0.297	0.291	0.286	0.280	0.275
85	0.275	0.269	0.264	0.258	0.253	0.247	0.242	0.236	0.231	0.225	0.220
86	0.220	0.214	0.209	0.203	0.198	0.192	0.187	0.181	0.176	0.170	0.165
87	0.165	0.159	0.154	0.148	0.143	0.137	0.132	0.126	0.121	0.115	0.110
88	0.110	0.104	0.099	0.093	0.088	0.082	0.077	0.071	0.066	0.060	0.055
89	0.055	0.049	0.044	0.038	0.033	0.027	0.022	0.016	0.011	0.005	0.000

Leads, Change Gears and Angles for Cutting Spirals

Lead of Spiral, Inches	Gear on Worm	1st Intermediate Gear	2d Intermediate Gear	Gear on Screw	Diameter of Work, Inches									
					⅛	¼	⅜	½	⅝	¾	⅞	1	1¼	1½
0.67	24	86	24	100	30¼	...	...	...	Approximate Angles for Milling Machine Table					
0.78	24	86	28	100	26	44½	...	...						
0.89	24	86	32	100	23½	41	...	...						
1.12	24	86	40	100	19	34½	...	...						
1.34	24	86	48	100	16	30¼	41½	...	...	...	...	...	...	...
1.46	24	64	28	72	14¾	28	38½	...	...	...	...	...	...	...
1.56	24	86	56	100	13¾	26½	37	...	...	...	...	...	...	...
1.67	24	64	32	72	12¾	25	34¾	43¼	...	...	...	...	...	...
1.94	32	64	28	72	11¼	21¾	31	39	45	...	...	...	...	...
2.08	24	64	40	72	10¼	20½	29½	37	43¼	...	...	...	...	...
2.22	32	56	28	72	9¾	19¼	27½	35	41¼	...	...	...	...	...
2.50	24	64	48	72	8¾	17	25	32	38	43¼	...	...	...	...
2.78	40	56	28	72	8	15½	23	29½	35¼	40½	44¾	...	...	...
2.92	24	64	56	72	7½	15	21¾	28¼	34	39	43¼	...	...	...
3.24	40	48	28	72	6¾	13¼	19¾	25¾	31¼	36	40½	44¼	...	...
3.70	40	48	32	72	6	11¾	17½	23	28	32½	36½	40½	...	...
3.89	56	48	24	72	5½	11¼	16¾	22	26¾	31¼	35¼	39	...	...
4.17	40	72	48	64	5¼	10½	15¾	20½	25¼	29½	33½	37	43¼	...
4.46	48	40	32	86	4¾	9¾	14¾	19¼	23¾	27¾	31½	35	41½	...
4.86	40	64	56	72	4½	9	13½	17¾	22	25¾	29½	33	39	44¼
5.33	48	40	32	72	4	8¼	12¼	16½	20¼	23¾	27¼	30½	36½	41½
5.44	56	40	28	72	4	8	12	16	20	23½	26¾	30	36	41
6.12	56	40	28	64	3½	7¼	11	14½	17¾	21	24¼	27	33	37¾
6.22	56	40	32	72	3½	7	10¾	14¼	17½	20¾	23¾	26¾	32½	37¼
6.48	56	48	40	72	3¼	6¾	10¼	13½	16¾	20	23	25¾	31½	36¼
6.67	64	48	28	56	3¼	6½	10	13¼	16½	19½	22½	25¼	30¾	35¼
7.29	56	48	40	64	3	6¼	9¼	12¼	15	18	20½	23½	23½	33
7.41	64	48	40	72	3	6	9	12	14¾	17¾	20¼	22¾	28¼	32½
7.62	64	48	32	56	2¾	5¾	8¾	11½	14½	17¼	19¾	22¼	27½	32
8.33	48	32	40	72	2½	5¼	8	10½	13¼	15¾	18¼	20½	25½	29½
8.95	86	48	28	56	2½	5	7½	10	12½	14¾	17	19¼	24	28
9.33	56	40	48	72	2¼	4¾	7¼	9½	11¾	14	16¼	18½	23	27
9.52	64	48	40	56	2¼	4½	7	9¼	11½	13¾	16	18¼	22½	26½
10.29	72	40	32	56	2	4¼	6½	8¾	10¾	12¾	15	17¼	21	24¾
10.37	64	48	56	72	2	4¼	6½	8½	10½	12¾	14¾	17	20¾	24½
10.50	48	40	56	64	2	4¼	6¼	8½	10½	12½	14½	16¾	20½	24¼
10.67	64	40	48	72	2	4	6¼	8¼	10¼	12¼	14¼	16½	20¼	24
10.94	56	32	40	64	2	4	6	8¼	10¼	12	14	16¼	20	23½
11.11	64	32	40	72	2	4	6	8	10	11¾	13¾	16	19¾	23
11.66	56	32	48	72	1¾	3¾	5¾	7½	9½	11¼	13¼	15¼	18¾	22
12.00	72	40	32	48	1¾	3¾	5½	7¼	9¼	11	12¾	15	18¼	21½
13.12	56	32	48	64	1½	3½	5¼	6¾	8½	10¼	11¾	13½	16¾	20
13.33	56	28	48	72	1½	3¼	5	6½	8¼	10	11½	13¼	16½	19½
13.71	64	40	48	56	1½	3¼	4¾	6½	8	9¾	11¼	13	16	19
15.24	64	28	48	72	1½	3	4½	5¾	7¼	8¾	10¼	11¾	14½	17¼
15.56	64	32	56	72	1¼	2¾	4¼	5¾	7¼	8¾	10	11½	14¼	17
15.75	56	64	72	40	1¼	2¾	4¼	5½	7	8½	9¾	11¼	14	16¾
16.87	72	32	48	64	1¼	2½	4	5¼	6¾	7¾	9¼	10½	13¼	15¾
17.14	64	32	48	56	1¼	2½	4	5¼	6½	7¾	9	10¼	13	15½
18.75	72	32	40	48	1	2¼	3½	4¾	6	7¼	8¼	9½	12	14¼
19.29	72	32	48	56	1	2¼	3½	4½	5¾	7	8	9¼	11½	13¾
19.59	64	28	48	56	1	2¼	3¼	4½	5¾	6¾	8	9¼	11½	13½
19.69	72	32	56	64	1	2¼	3¼	4½	5¾	6¾	8	9	11½	13½
21.43	72	24	40	56	1	2	3¼	4¼	5¼	6¼	7½	8½	10½	12½
22.50	72	28	56	64	1	2	3	4	5	6	7	8	10	12
23.33	64	32	56	48	1	2	3	4	5	5¾	6¾	7¾	9¾	11½
26.25	72	24	56	64	1	1¾	2¾	3½	4¼	5	6	7	8½	10¼
26.67	64	28	56	48	¾	1¾	2¾	3½	4¼	5	6	6¾	8½	10
28.00	64	32	56	40	¾	1¾	2½	3¼	4	4¾	5¾	6½	8	9½
30.86	72	28	48	40	¾	1½	2¼	3	3¾	4½	5	5¾	7¼	8¾

Leads, Change Gears and Angles for Cutting Spirals

Lead of Spiral, Inches	Gear on Worm	1st Intermediate Gear	2d Intermediate Gear	Gear on Screw	Diameter of Work, Inches									
					1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4
6.12	56	40	28	64	42	...	...	...	Approximate Angles for Milling Machine Table					
6.22	56	40	32	72	41½	...	...	...						
6.48	56	48	40	72	40¼	44¼	...	...						
6.67	64	48	28	56	39½	43½	...	...	...	...	...	...	...	...
7.29	56	48	40	64	37	41	44¼	...	...	...	...	...	...	...
7.41	64	48	40	72	36½	40¼	43¾	...	...	...	...	...	...	...
7.62	64	48	32	56	36	39½	43	...	...	...	...	...	...	...
8.33	48	32	40	72	33½	37	40½	43½	...	...	...	...	...	...
8.95	86	48	28	56	31¾	35¼	38½	41¼	44	...	...	...	...	...
9.33	56	40	48	72	30½	34	37¼	40¼	43	...	...	...	...	...
9.52	64	48	40	56	30	33½	36½	39½	42¼	45	...	...	...	...
10.29	72	40	32	56	28¼	31½	34½	37½	40	42½	45	...	...	...
10.37	64	48	56	72	28	31¼	34¼	37¼	39¾	42¼	44¾	...	...	...
10.50	48	40	56	64	27¾	31	34	36¾	39½	42	44¼	...	...	...
10.67	64	40	48	72	27¼	30½	33½	36½	39	41½	43¾	...	...	...
10.94	56	32	40	64	26¾	30	33	35¾	38¼	40¾	43	...	...	...
11.11	64	32	40	72	26½	29½	32½	35¼	38	40¼	42½	44¾	...	...
11.66	56	32	48	72	25¼	28½	31¼	34	36½	39	41¼	43½	...	...
12.00	72	40	32	48	24¾	27¾	30½	33¼	35¾	38	40¼	42½	44¾	...
13.12	56	32	48	64	22¾	25¾	28¼	31	33¼	35¾	37¾	40	42	43¾
13.33	56	28	48	72	22½	25½	28	30½	33	35¼	37½	39½	41½	43¼
13.71	64	40	48	56	22	24¾	27¼	30	32¼	34½	36½	38¾	40¾	42½
15.24	64	28	48	72	20	22½	25	27¼	29½	31¾	34	35¾	37¾	39½
15.56	64	32	56	72	19½	22	24½	27	29	31¼	33¼	35¼	37	39
15.75	56	64	72	40	19¼	21¾	24¼	26½	28¾	31	33	35	36¾	38½
16.87	72	32	48	64	18¼	20½	22¾	25	27	29¼	31¼	33¼	35	36½
17.14	64	32	48	56	17¾	20¼	22¼	24¾	26¾	29	30¾	32¾	34½	36
18.75	72	32	40	48	16¼	18½	20¾	22¾	25	26¾	28½	30¼	32	33¾
19.29	72	32	48	56	16	18¼	20¼	22¼	24	26	28	29¾	31½	33
19.59	64	28	48	56	15¾	18	20	22	23¾	25¾	27½	29¼	31	32¾
19.69	72	32	56	64	15¾	17¾	20	21¾	23¾	25½	27½	29¼	31	32½
21.43	72	24	40	56	14½	16½	18½	20¼	22	23¾	25½	27¼	29	30¼
22.50	72	28	56	64	13¾	15¾	17½	19¼	21	22¾	24½	26	27¾	29¼
23.33	64	32	56	48	13¼	15¼	17	18¾	20¼	22	23½	25¼	27	28¼
26.25	72	24	56	64	12	13½	15	16¾	18¼	19¾	21¼	22¾	24¼	25½
26.67	64	28	56	48	11¾	13¼	14¾	16½	18	19½	21	22¼	23¾	25¼
28.00	64	32	56	40	11¼	12¾	14¼	15¾	17¼	18¾	20	21½	22¾	24
30.86	72	28	48	40	10	11½	13	14¼	15½	17	18½	19½	21	22
31.50	72	32	56	40	10	11¼	12¾	14	15¼	16½	18	19¼	20½	21¾
36.00	72	32	64	40	8¾	10	11	12¼	13½	14¾	16	17	18¼	19¼
41.14	72	28	64	40	7¾	8¾	9¾	10¾	11¾	13	14	15	16	17
45.00	72	28	56	32	7	8	9	10	11	11¾	12¾	13¾	14¾	15½
48.00	72	24	64	40	6½	7½	8½	9¼	10¼	11¼	12	13	13¾	14½
51.43	72	28	64	32	6	7	7¾	8¾	9½	10½	11¼	12	12¾	13¾
60.00	72	24	64	32	5¼	6	6¾	7½	8¼	9	9½	10¼	11	11¾
68.57	72	24	64	28	4¼	5¼	5¾	6½	7¼	8	8½	9	9¾	10¼

Angle for Given Lead and Diameter. — To obtain the angle for any lead and diameter, divide the circumference of the work by the lead of the spiral, and the quotient will be the tangent of the angle which the spiral makes with its axis. For milling a right-hand spiral, turn the right-hand end of the machine table toward the rear, and, inversely, for a left-hand spiral, turn the left-hand end of the table toward the rear.

Ratios for Determining Helix Angles

Ratio of Lead to Diam.	Helix Angle	Ratio of Lead to Diam.	Helix Angle	Ratio of Lead to Diam.	Helix Angle	Ratio of Lead to Diam.	Helix Angle
0.10	88° 11′	4.90	32° 40′	9.70	17° 57′	21.00	8° 32′
0.20	86 21	5.00	32 8	9.80	17 47	22.00	8 9
0.30	84 32	5.10	31 38	9.90	17 37	23.00	7 47
0.40	82 45	5.20	31 8	10.00	17 27	24.00	7 28
0.50	80 57	5.30	30 39	10.20	17 6	25.00	7 10
0.60	79 11	5.40	30 11	10.40	16 47	26.00	6 54
0.70	77 26	5.50	29 44	10.60	16 29	27.00	6 38
0.80	75 43	5.60	29 18	10.80	16 12	28.00	6 23
0.90	74 1	5.70	28 52	11.00	15 56	29.00	6 10
1.00	72 21	5.80	28 27	11.20	15 40	30.00	5 59
1.10	70 42	5.90	28 2	11.40	15 24	31.00	5 48
1.20	69 7	6.00	27 38	11.60	15 9	32.00	5 38
1.30	67 33	6.10	27 15	11.80	14 54	33.00	5 28
1.40	65 0	6.20	26 52	12.00	14 40	34.00	5 18
1.50	64 32	6.30	26 30	12.20	14 27	35.00	5 8
1.60	63 4	6.40	26 9	12.40	14 14	36.00	5 0
1.70	61 37	6.50	25 48	12.60	14 2	37.00	4 52
1.80	60 12	6.60	25 27	12.80	13 50	38.00	4 44
1.90	58 51	6.70	25 7	13.00	13 38	39.00	4 36
2.00	57 31	6.80	24 48	13.20	13 26	40.00	4 29
2.10	56 15	6.90	24 29	13.40	13 14	42.00	4 17
2.20	55 0	7.00	24 10	13.60	13 2	44.00	4 5
2.30	53 48	7.10	23 52	13.80	12 50	46.00	3 55
2.40	52 37	7.20	23 34	14.00	12 39	48.00	3 45
2.50	51 30	7.30	23 17	14.20	12 29	50.00	3 36
2.60	50 23	7.40	23 0	14.40	12 19	55.00	3 18
2.70	49 19	7.50	22 44	14.60	12 9	60.00	3 0
2.80	48 16	7.60	22 28	14.80	11 59	65.00	2 46
2.90	47 17	7.70	22 12	15.00	11 50	70.00	2 34
3.00	46 19	7.80	21 56	15.20	11 41	75.00	2 24
3.10	45 23	7.90	21 41	15.40	11 32	80.00	2 15
3.20	44 28	8.00	21 26	15.60	11 23	85.00	2 7
3.30	43 36	8.10	21 12	15.80	11 14	90.00	2 0
3.40	42 44	8.20	20 58	16.00	11 6	95.00	1 54
3.50	41 55	8.30	20 44	16.20	10 58	100.00	1 48
3.60	41 7	8.40	20 30	16.40	10 50	110.00	1 38
3.70	40 21	8.50	20 17	16.60	10 42	120.00	1 30
3.80	39 35	8.60	20 4	16.80	10 35	130.00	1 23
3.90	38 52	8.70	19 52	17.00	10 28	140.00	1 17
4.00	38 10	8.80	19 40	17.20	10 21	150.00	1 12
4.10	37 29	8.90	19 27	17.40	10 14	160.00	1 7
4.20	36 48	9.00	19 15	17.60	10 7	170.00	1 3
4.30	36 9	9.10	19 2	17.80	10 0	180.00	1 0
4.40	35 30	9.20	18 50	18.00	9 54	190.00	0 57
4.50	34 54	9.30	18 39	18.20	9 47	200.00	0 54
4.60	34 19	9.40	18 28	18.60	9 35	300.00	0 36
4.70	33 45	9.50	18 18	19.00	9 23	400.00	0 27
4.80	33 12	9.60	18 7	20.00	8 56	500.00	0 22

Lead of Helix for Given Angle. — The lead of a helix or "spiral" for given angles measured with the axis of the work is given in the table, pages 1051–1053, for a diameter of 1. For other diameters, the lead equals the value found in the table multiplied by the given diameter. Suppose the angle is 55 degrees, and the diameter 5 inches, what would be the lead? By referring to the table (Part 2), it is found that the lead for a diameter of 1 and an angle of 55 degrees 0 minutes equals 2.200. Multiply this value by 5; $5 \times 2.200 = 11$ inches, which is the required lead. If the lead and diameter are given, and the angle is wanted, divide the given lead by the given diameter, thus obtaining the lead for a diameter equal to 1; then find the angle corresponding to this lead in the table. If the lead and angle are given, and the diameter is wanted, divide the lead by the value in the table for the angle.

Helix Angles. — The table, "Ratios for Determining Helix Angles," gives the values of helix angles (α) for values of the ratio of the lead (L) to the diameter (D), so that the helix angle can be determined without using the formula, $\tan \alpha = (3.1416\ D) \div L$, or a table of trigonometrical functions. To find the helix angle for a value of $L \div D$ not included in the table, a result sufficiently accurate for practical purposes can be obtained by the simple proportion method. Suppose $L \div D = 3.75$. By referring to the table, it is found that when $L \div D = 3.70$, $\alpha = 40$ degrees 21 minutes; and when $L \div D = 3.80$, $\alpha = 39$ degrees 35 minutes. Therefore, when $L \div D = 3.75$, $\alpha = \dfrac{40^\circ\ 21' + 39^\circ\ 35'}{2} = 39$ deg. 58 min.

SIMPLE, COMPOUND, DIFFERENTIAL AND BLOCK INDEXING

Simple Indexing. — A general rule for determining the number of turns the crank of a dividing head must make, to obtain a given number of divisions, is as follows: Divide the number of turns required for one revolution of the dividing-head spindle by the number of divisions into which the periphery of the work is to be divided.

Example: — If 40 turns of the index crank are required for one revolution of the spindle, and 12 divisions are required, the number of turns of the index crank for each indexing would equal $40 \div 12 = 3\tfrac{1}{3}$ turns.

Compound Indexing. — This method is sometimes used to obtain divisions which are beyond the range of those secured by the simple method. The crank is first turned a definite amount in the regular way, and then the index plate is also turned either in the same or opposite direction, in order to locate the index crank in the proper position. Thus, there are two separate movements which are, in reality, two simple indexing operations. The following rule is for determining what circles of holes can be used for indexing by the compound method.

Rule: Resolve into its factors the number of divisions required; then choose at random two circles of holes, subtract one from the other, and factor the difference; place the two sets of factors thus obtained above a horizontal line. Next factor the number of turns of the crank required for one revolution of the spindle, and also the number of holes in each of the chosen circles; place the three sets of factors thus obtained below the horizontal line. If all the factors *above* the line can be canceled by those below, the two circles chosen will give the required number of divisions; if not, other circles must be chosen and another trial made.

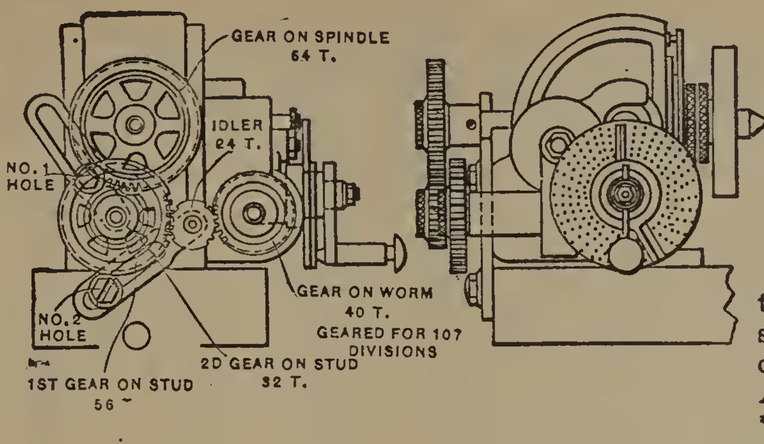
Example: — Assume that 69 divisions are required, and that circles having 33 and 23 holes are chosen for the first trial. Then, by applying the foregoing rule, it is found that all the factors above the line cancel:

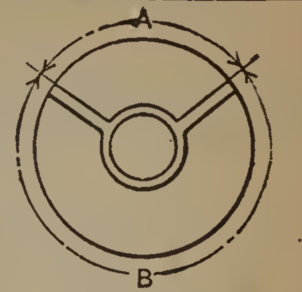
$$\frac{3 \times 23 \times 2 \times 5}{2 \times 2 \times 2 \times 5 \times 3 \times 11 \times 23} = \frac{1}{2 \times 2 \times 11}$$

This shows that these circles can be used. The factors 2, 2 and 11 remain uncanceled below the line. The amount the crank and index plate must be moved in their respective circles is next determined by multiplying together all these uncanceled factors. Thus $2 \times 2 \times 11 = 44$. This means that we can index $\frac{1}{69}$ revolution by turning the crank forward 44 holes in the 23-hole circle, and the index plate backward 44 holes in the 33-hole circle. The movement could also be forward 44 holes in the 33-hole circle and backward 44 holes in the 23-hole circle, without affecting the result. The movements obtained by the foregoing rule are expressed in compound indexing tables in the form of fractions, as, for example: $+\frac{44}{23}-\frac{44}{33}$. The numerators represent the number of holes indexed and the denominators the circles used; the + and − signs show that the movements of the crank and index plate are opposite in direction. These fractions can often be reduced and simplified, so that it will not be necessary to move so many holes, by adding some number to them algebraically. The number is chosen by trial, and its sign should be opposite that of the fraction to which it is added. Suppose, for example, a fraction is added representing one complete turn, to each of the fractions referred to; then there will be a movement of 21 holes in the 23-hole circle, and a movement of 11 holes in the opposite direction, in the 33-hole circle.

Differential Indexing. — This method is the same, in principle, as compound indexing, but differs from the latter in that the index plate is rotated by suitable gearing which connects it to the spiral-head spindle. This rotation or differential motion of the index plate takes place when the crank is turned, the plate moving either in the same direction as the crank or opposite to it, as may be required. The result is that the *actual* movement of the crank, at every indexing, is either greater or less than its movement with relation to the index plate. The differential method makes it possible to obtain almost any division, by using only one circle of holes for that division and turning the index crank in one direction, the same as for plain indexing. The gears to use for moving the index plate the required amount (when gears are required) are shown by the tables, "Simple and Differential Indexing." This table shows what divisions can be obtained by plain indexing, and also when it is necessary to use gears and the differential system. For example, if 50 divisions are required, the 20-hole index circle is used and the crank is moved 16 holes, but no gears are required. For 51 divisions, a 24-tooth gear is placed on the worm-shaft and a 48-tooth gear is mounted on the spindle. These two gears are connected by two idler gears having 24 and 44 teeth, respectively. To illustrate the principle of differential indexing, suppose a dividing head is to be geared for 271 divisions. The table calls for a gear on the worm-shaft having 56 teeth; a spindle gear with 72 teeth; and a 24-toothed idler which serves to rotate the index plate in the same direction as the crank. The sector should be set for giving the crank a movement of 3 holes in the 21-hole circle. If the spindle and index plate were not connected through gearing, 280 divisions would be obtained by successively moving the crank 3 holes in the 21-hole circle, but the gears cause the index plate to turn in the same direction as the crank at such a rate that, when 271 indexings have been made, the work is turned one complete revolution; therefore, we have 271 divisions instead of 280, the number being reduced because the total movement of the crank, for each indexing, is equal to its movement relative to the index plate, *plus* the movement of the plate itself when (as in this case) the crank and plate rotate in the same direction. If they were rotated in opposite directions, the crank would have a total movement equal to the amount it turned relative to the plate, *minus* the plate's movement. Sometimes it is necessary to use compound gearing, in order to move the index plate the required amount for each turn of the crank. The differential method cannot be used in connection with helical or spiral milling, because the spiral head is then geared to the lead-screw of the machine.

Simple and Differential Indexing — Brown & Sharpe Milling Machines





Note: Graduations in table indicate setting for sector arms when index crank moves through arc *A*, except figures marked *, when crank moves through arc *B*.

Number of Divisions	Index Circle	Number of Turns of Crank	Graduation on Sector	Number of Divisions	Index Circle	Number of Turns of Crank	Graduation on Sector	Gear on Worm	No. 1 Hole		Gear on Spindle	Idlers	
									First Gear on Stud	Second Gear on Stud		No. 1 Hole	No. 2 Hole†
2	Any	20	....	33	33	1 7/33	41	...	...	...	...	...	...
3	39	13 13/39	65	34	17	1 3/17	33	...	...	...	...	...	...
4	Any	10	....	35	49	1 7/49	26	...	...	...	...	...	...
5	Any	8	....	36	27	1 3/27	21	...	...	...	...	...	...
6	39	6 26/39	132	37	37	1 3/37	15	...	...	...	...	...	...
7	49	5 35/49	140	38	19	1 1/19	9	...	...	...	...	...	...
8	Any	5	....	39	39	1 1/39	3	...	...	...	...	...	...
9	27	4 12/27	88	40	Any	1	...	...	...	...	...	...	...
10	Any	4	....	41	41	40/41	3*	...	...	...	...	...	...
11	33	3 21/33	126	42	21	20/21	9*	...	...	...	...	...	...
12	39	3 13/39	65	43	43	40/43	12*	...	...	...	...	...	...
13	39	3 3/39	14	44	33	30/33	17*	...	...	...	...	...	...
14	49	2 42/49	169	45	27	24/27	21*	...	...	...	...	...	...
15	39	2 26/39	132	46	23	20/23	172	...	...	...	...	...	...
16	20	2 10/20	98	47	47	40/47	168	...	...	...	...	...	...
17	17	2 6/17	69	48	18	15/18	165	...	...	...	...	...	...
18	27	2 6/27	43	49	49	40/49	161	...	...	...	...	...	...
19	19	2 2/19	19	50	20	16/20	158	...	...	...	...	...	...
20	Any	2	....	51	17	14/17	33*	24	...	...	48	24	44
21	21	1 19/21	18*	52	39	30/39	152	...	...	...	...	...	...
22	33	1 27/33	161	53	49	35/49	140	56	40	24	72	...	...
23	23	1 17/23	147	54	27	20/27	147	...	...	...	...	...	...
24	39	1 26/39	132	55	33	24/33	144	...	...	...	...	...	...
25	20	1 12/20	118	56	49	35/49	140	...	...	...	...	...	...
26	39	1 21/39	106	57	21	15/21	142	56	...	...	40	24	44
27	27	1 13/27	95	58	29	20/29	136	...	...	...	...	...	...
28	49	1 21/49	83	59	39	26/39	132	48	...	...	32	44	...
29	29	1 11/29	75	60	39	26/39	132	...	...	...	...	...	...
30	39	1 13/39	65	61	39	26/39	132	48	...	...	32	24	44
31	31	1 9/31	56	62	31	20/31	127	...	...	...	...	...	...
32	20	1 5/20	48	63	39	26/39	132	24	...	...	48	24	44

† On Nos. 1, 1½ and 2 machines, No. 2 hole is in machine table. On Nos. 3 and 4 machines, No. 2 hole is in head.

Simple and Differential Indexing

No. of Divisions	Index Circle	No. of Turns of Crank	Graduation on Sector	Gear on Worm	No. 1 Hole		Gear on Spindle	Idlers	
					First Gear on Stud	Second Gear on Stud		No. 1 Hole	No. 2 Hole
64	16	10/16	123	....	....	....	....	....	....
65	39	24/39	121	....	....	....	....	....	....
66	33	20/33	120	....	....	....	....	....	....
67	21	12/21	113	28	....	....	48	44	....
68	17	10/17	116	....	....	....	....	....	....
69	20	12/20	118	40	....	....	56	24	44
70	49	28/49	112	....	....	....	....	....	....
71	18	10/18	109	72	....	....	40	24	....
72	27	15/27	110	....	....	....	....	....	....
73	21	12/21	113	28	....	....	48	24	44
74	37	20/37	107	....	....	....	....	....	....
75	15	8/15	105	....	....	....	....	....	....
76	19	10/19	103	....	....	....	....	....	....
77	20	10/20	98	32	....	....	48	44	....
78	39	20/39	101	....	....	....	....	....	....
79	20	10/20	98	48	....	....	24	44	....
80	20	10/20	98	....	....	....	....	....	....
81	20	10/20	98	48	....	....	24	24	44
82	41	20/41	96	....	....	....	....	....	....
83	20	10/20	98	32	....	....	48	24	44
84	21	10/21	94	....	....	....	....	....	....
85	17	8/17	92	....	....	....	....	....	....
86	43	20/43	91	....	....	....	....	....	....
87	15	7/15	92	40	....	....	24	24	44
88	33	15/33	89	....	....	....	....	....	....
89	18	8/18	87	72	....	....	32	44	....
90	27	12/27	88	....	....	....	....	....	....
91	39	18/39	91	24	....	....	48	24	44
92	23	10/23	86	....	....	....	....	....	....
93	18	8/18	87	24	....	....	32	24	44
94	47	20/47	83	....	....	....	....	....	....
95	19	8/19	82	....	....	....	....	....	....
96	21	9/21	85	28	....	....	32	24	44
97	20	8/20	78	40	....	....	48	44	....
98	49	20/49	79	....	....	....	....	....	....
99	20	8/20	78	56	28	40	32	....	....
100	20	8/20	78	....	....	....	....	....	....
101	20	8/20	78	72	24	40	48	....	24
102	20	8/20	78	40	....	....	32	24	44
103	20	8/20	78	40	....	....	48	24	44
104	39	15/39	75	....	....	....	....	....	....
105	21	8/21	75	....	....	....	....	....	....
106	43	16/43	73	86	24	24	48	....	....

Simple and Differential Indexing

No. of Divisions	Index Circle	No. of Turns of Crank	Graduation on Sector	Gear on Worm	No. 1 Hole: First Gear on Stud	No. 1 Hole: Second Gear on Stud	Gear on Spindle	Idlers: No. 1 Hole	Idlers: No. 2 Hole
107	20	8/20	78	40	56	32	64	....	24
108	27	10/27	73	....	....	....	....	....	....
109	16	6/16	73	32	....	....	28	24	44
110	33	12/33	71	....	....	....	....	....	....
111	39	13/39	65	24	....	....	72	32	....
112	39	13/39	65	24	....	....	64	44	....
113	39	13/39	65	24	....	....	56	44	....
114	39	13/39	65	24	....	....	48	44	....
115	23	8/23	68	....	....	....	....	....	....
116	29	10/29	68	....	....	....	....	....	....
117	39	13/39	65	24	....	....	24	56	....
118	39	13/39	65	48	....	....	32	44	....
119	39	13/39	65	72	....	....	24	44	....
120	39	13/39	65	....	....	....	....	....	....
121	39	13/39	65	72	....	....	24	24	44
122	39	13/39	65	48	....	....	32	24	44
123	39	13/39	65	24	....	....	24	24	44
124	31	10/31	63	....	....	....	....	....	....
125	39	13/39	65	24	....	....	40	24	44
126	39	13/39	65	24	....	....	48	24	44
127	39	13/39	65	24	....	....	56	24	44
128	16	5/16	61	....	....	....	....	....	....
129	39	13/39	65	24	....	....	72	24	44
130	39	12/39	60	....	....	....	....	....	....
131	20	6/20	58	40	....	....	28	44	....
132	33	10/33	59	....	....	....	....	....	....
133	21	6/21	56	24	....	....	48	44	....
134	21	6/21	56	28	....	....	48	44	....
135	27	8/27	58	....	....	....	....	....	....
136	17	5/17	57	....	....	....	....	....	....
137	21	6/21	56	28	....	....	24	56	....
138	21	6/21	56	56	....	....	32	44	....
139	21	6/21	56	56	32	48	24	....	....
140	49	14/49	55	....	....	....	....	....	....
141	18	5/18	54	48	....	....	40	44	....
142	21	6/21	56	56	....	....	32	24	44
143	21	6/21	56	28	....	....	24	24	44
144	18	5/18	54	....	....	....	....	....	....
145	29	8/29	54	....	....	....	....	....	....
146	21	6/21	56	28	....	....	48	24	44
147	21	6/21	56	24	....	....	48	24	44
148	37	10/37	53	....	....	....	....	....	....
149	21	6/21	56	28	....	....	72	24	44

Simple and Differential Indexing

No of Divisions	Index Circle	No. of Turns of Crank	Graduation on Sector	Gear on Worm	No. 1 Hole: First Gear on Stud	No. 1 Hole: Second Gear on Stud	Gear on Spindle	Idlers: No. 1 Hole	Idlers: No. 2 Hole
150	15	4/15	52	....	....	....	....	....	....
151	20	5/20	48	32	....	....	72	44	....
152	19	5/19	51	....	....	....	....	....	....
153	20	5/20	48	32	....	....	56	44	....
154	20	5/20	48	32	....	....	48	44	....
155	31	8/31	50	....	....	....	....	....	....
156	39	10/39	50	....	....	....	....	....	....
157	20	5/20	48	32	....	....	24	56	....
158	20	5/20	48	48	....	....	24	44	....
159	20	5/20	48	64	32	56	28	....	....
160	20	5/20	48	....	....	....	....	....	....
161	20	5/20	48	64	32	56	28	....	24
162	20	5/20	48	48	....	....	24	24	44
163	20	5/20	48	32	....	....	24	24	44
164	41	10/41	47	....	....	....	....	....	....
165	33	8/33	47	....	....	....	....	....	....
166	20	5/20	48	32	....	....	48	24	44
167	20	5/20	48	32	....	....	56	24	44
168	21	5/21	47	....	....	....	....	....	....
169	20	5/20	48	32	....	....	72	24	44
170	17	4/17	45	....	....	....	....	....	....
171	21	5/21	47	56	....	....	40	24	44
172	43	10/43	44	....	....	....	....	....	....
173	18	4/18	43	72	56	32	64	....	....
174	18	4/18	43	24	....	....	32	56	....
175	18	4/18	43	72	40	32	64	....	....
176	18	4/18	43	72	24	24	64	....	....
177	18	4/18	43	72	....	....	48	24	....
178	18	4/18	43	72	....	....	32	44	....
179	18	4/18	43	72	24	48	32	....	....
180	18	4/18	43	....	....	....	....	....	....
181	18	4/18	43	72	24	48	32	....	24
182	18	4/18	43	72	....	....	32	24	44
183	18	4/18	43	48	....	....	32	24	44
184	23	5/23	42	....	....	....	....	....	....
185	37	8/37	42	....	....	....	....	....	....
186	18	4/18	43	48	....	....	64	24	44
187	18	4/18	43	72	48	24	56	....	24
188	47	10/47	40	....	....	....	....	....	....
189	18	4/18	43	32	....	....	64	24	44
190	19	4/19	40	....	....	....	....	....	....
191	20	4/20	38	40	....	....	72	24	....
192	20	4/20	38	40	....	....	64	44	....

Simple and Differential Indexing

No. of Divisions	Index Circle	No. of Turns of Crank	Graduation on Sector	Gear on Worm	No. 1 Hole: First Gear on Stud	No. 1 Hole: Second Gear on Stud	Gear on Spindle	Idlers: No. 1 Hole	Idlers: No. 2 Hole
193	20	4/20	38	40	....	....	56	44	....
194	20	4/20	38	40	....	....	48	44	....
195	39	8/39	39	....	....	....	....	....	....
196	49	10/49	38	....	....	....	....	....	....
197	20	4/20	38	40	....	....	24	56	....
198	20	4/20	38	56	28	40	32	....	....
199	20	4/20	38	100	40	64	32	....	....
200	20	4/20	38	....	....	....	....	....	....
201	20	4/20	38	72	24	40	24	....	24
202	20	4/20	38	72	24	40	48	....	24
203	20	4/20	38	40	....	....	24	24	44
204	20	4/20	38	40	....	....	32	24	44
205	41	8/41	37	....	....	....	....	....	....
206	20	4/20	38	40	....	....	48	24	44
207	20	4/20	38	40	....	....	56	24	44
208	20	4/20	38	40	....	....	64	24	44
209	20	4/20	38	40	....	....	72	24	44
210	21	4/21	37	....	....	....	....	....	....
211	16	3/16	36	64	....	....	28	44	....
212	43	8/43	35	86	24	24	48	....	....
213	27	5/27	36	72	....	....	40	44	....
214	20	4/20	38	40	56	32	64	....	24
215	43	8/43	35	....	....	....	....	....	....
216	27	5/27	36	....	....	....	....	....	....
217	21	4/21	37	48	....	....	64	24	44
218	16	3/16	36	64	....	....	56	24	44
219	21	4/21	37	28	....	....	48	24	44
220	33	6/33	35	....	....	....	....	....	....
221	17	3/17	33	24	....	....	24	56	....
222	18	3/18	32	24	....	....	72	44	....
223	43	8/43	35	86	48	24	64	....	24
224	18	3/18	32	24	....	....	64	44	....
225	27	5/27	36	24	....	....	40	24	44
226	18	3/18	32	24	....	....	56	44	....
227	49	8/49	30	56	64	28	72	....	....
228	18	3/18	32	24	....	....	48	44	....
229	18	3/18	32	24	....	....	44	48	....
230	23	4/23	34	....	....	....	....	....	....
231	18	3/18	32	32	....	....	48	44	....
232	29	5/29	33	....	....	....	....	....	....
233	18	3/18	32	48	....	....	56	44	....
234	18	3/18	32	24	....	....	24	56	....
235	47	8/47	32	....	....	....	....	....	....

Simple and Differential Indexing

No. of Divisions	Index Circle	No. of Turns of Crank	Graduation on Sector	Gear on Worm	No. 1 Hole		Gear on Spindle	Idlers	
					First Gear on Stud	Second Gear on Stud		No. 1 Hole	No. 2 Hole
236	18	3/18	32	48	....	....	32	44	....
237	18	3/18	32	48	....	....	24	44	....
238	18	3/18	32	72	....	....	24	44	....
239	18	3/18	32	72	24	64	32	....	....
240	18	3/18	32	....	....	....	....	....	....
241	18	3/18	32	72	24	64	32	....	24
242	18	3/18	32	72	....	....	24	24	44
243	18	3/18	32	64	....	....	32	24	44
244	18	3/18	32	48	....	....	32	24	44
245	49	8/49	30	....	....	....	....	....	....
246	18	3/18	32	24	....	....	24	24	44
247	18	3/18	32	48	....	....	56	24	44
248	31	5/31	31	....	....	....	....	....	....
249	18	3/18	32	32	....	....	48	24	44
250	18	3/18	32	24	....	....	40	24	44
251	18	3/18	32	48	44	32	64	....	24
252	18	3/18	32	24	....	....	48	24	44
253	33	5/33	29	24	....	....	40	56	....
254	18	3/18	32	24	....	....	56	24	44
255	18	3/18	32	48	40	24	72	....	24
256	18	3/18	32	24	....	....	64	24	44
257	49	8/49	30	56	48	28	64	....	24
258	43	7/43	31	32	....	....	64	24	44
259	21	3/21	28	24	....	....	72	44	....
260	39	6/39	29	....	....	....	....	....	....
261	29	4/29	26	48	64	24	72	....	....
262	20	3/20	28	40	....	....	28	44	....
263	49	8/49	30	56	64	28	72	....	24
264	33	5/33	29	....	....	....	....	....	....
265	21	3/21	28	56	40	24	72	....	....
266	21	3/21	28	32	....	....	64	44	....
267	27	4/27	28	72	....	....	32	44	....
268	21	3/21	28	28	....	....	48	44	....
269	20	3/20	28	64	32	40	28	....	24
270	27	4/27	28	....	....	....	....	....	....
271	21	3/21	28	56	24	24	72	....	....
272	21	3/21	28	56	....	....	64	24	....
273	21	3/21	28	24	....	....	24	56	....
274	21	3/21	28	56	....	....	48	44	....
275	21	3/21	28	56	....	....	40	44	....
276	21	3/21	28	56	....	....	32	44	....
277	21	3/21	28	56	....	....	24	44	....
278	21	3/21	28	56	32	48	24	....	....

Simple and Differential Indexing

No. of Divisions	Index Circle	No. of Turns of Crank	Graduation on Sector	Gear on Worm	No. 1 Hole: First Gear on Stud	No. 1 Hole: Second Gear on Stud	Gear on Spindle	Idlers: No. 1 Hole	Idlers: No. 2 Hole
279	27	4/27	28	24	....	....	32	24	44
280	49	7/49	26	....	....	....	....	....	....
281	21	3/21	28	72	24	56	24	....	24
282	43	6/43	26	86	24	24	56	....	....
283	21	3/21	28	56	....	....	24	24	44
284	21	3/21	28	56	....	....	32	24	44
285	21	3/21	28	56	....	....	40	24	44
286	21	3/21	28	56	....	....	48	24	44
287	21	3/21	28	24	....	....	24	24	44
288	21	3/21	28	28	....	....	32	24	44
289	21	3/21	28	56	24	24	72	....	24
290	29	4/29	26	....	....	....	....	....	....
291	15	2/15	25	40	....	....	48	44	....
292	21	3/21	28	28	....	....	48	24	44
293	15	2/15	25	48	32	40	56	....	....
294	21	3/21	28	24	....	....	48	24	44
295	15	2/15	25	48	....	....	32	44	....
296	37	5/37	26	....	....	....	....	....	....
297	33	4/33	23	28	48	24	56	....	....
298	21	3/21	28	28	....	....	72	24	44
299	23	3/23	25	24	....	....	24	56	....
300	15	2/15	25	....	....	....	....	....	....
301	43	6/43	26	24	....	....	48	24	44
302	16	2/16	24	32	....	....	72	24	....
303	15	2/15	25	72	24	40	48	....	24
304	16	2/16	24	24	....	....	48	44	....
305	15	2/15	25	48	....	....	32	24	44
306	15	2/15	25	40	....	....	32	24	44
307	15	2/15	25	72	48	40	56	....	24
308	16	2/16	24	32	....	....	48	44	....
309	15	2/15	25	40	....	....	48	24	44
310	31	4/31	24	....	....	....	....	....	....
311	16	2/16	24	64	24	24	72	....	....
312	39	5/39	24	....	....	....	....	....	....
313	16	2/16	24	32	....	....	28	56	....
314	16	2/16	24	32	....	....	24	56	....
315	16	2/16	24	64	....	....	40	24	....
316	16	2/16	24	64	....	....	32	44	....
317	16	2/16	24	64	....	....	24	44	....
318	16	2/16	24	56	28	48	24	....	....
319	29	4/29	26	48	64	24	72	....	24
320	16	2/16	24	....	....	....	....	....	....
321	16	2/16	24	72	24	64	24	....	24

Simple and Differential Indexing

No. of Divisions	Index Circle	No. of Turns of Crank	Graduation on Sector	Gear on Worm	No. 1 Hole: First Gear on Stud	No. 1 Hole: Second Gear on Stud	Gear on Spindle	Idlers: No. 1 Hole	Idlers: No. 2 Hole
322	23	8/23	25	32	....	....	64	24	44
323	16	2/16	24	64	....	....	24	24	44
324	16	2/16	24	64	....	....	32	24	44
325	16	2/16	24	64	....	....	40	24	44
326	16	2/16	24	32	....	....	24	24	44
327	16	2/16	24	32	....	....	28	24	44
328	41	5/41	23	....	....	....	....	....	....
329	16	2/16	24	64	24	24	72	....	24
330	33	4/33	23	....	....	....	....	....	....
331	16	2/16	24	64	44	24	48	....	24
332	16	2/16	24	32	....	....	48	24	44
333	18	2/18	21	24	....	....	72	44	....
334	16	2/16	24	32	....	....	56	24	44
335	33	4/33	23	72	48	44	40	....	24
336	16	2/16	24	32	....	....	64	24	44
337	43	5/43	21	86	40	32	56	....	....
338	16	2/16	24	32	....	....	72	24	44
339	18	2/18	21	24	....	....	56	44	....
340	17	2/17	22	....	....	....	....	....	....
341	43	5/43	21	86	24	32	40	....	....
342	18	2/18	21	32	....	....	64	44	....
343	15	2/15	25	40	64	24	86	....	24
344	43	5/43	21	....	....	....	....	....	....
345	18	2/18	21	24	....	....	40	56	....
346	18	2/18	21	72	56	32	64	....	....
347	43	5/43	21	86	24	32	40	....	24
348	18	2/18	21	24	....	....	32	56	....
349	18	2/18	21	72	44	24	48	....	....
350	18	2/18	21	72	40	32	64	....	....
351	18	2/18	21	24	....	....	24	56	....
352	18	2/18	21	72	24	24	64	....	....
353	18	2/18	21	72	24	24	56	....	....
354	18	2/18	21	72	....	....	48	24	....
355	18	2/18	21	72	....	....	40	24	....
356	18	2/18	21	72	....	....	32	24	....
357	18	2/18	21	72	....	....	24	44	....
358	18	2/18	21	72	32	48	24	....	....
359	43	5/43	21	86	48	32	100	....	24
360	18	2/18	21	....	....	....	....	....	....
361	19	2/19	19	32	....	....	64	44	....
362	18	2/18	21	72	28	56	32	....	24
363	18	2/18	21	72	....	....	24	24	44
364	18	2/18	21	72	....	....	32	24	44

Indexing Movements for Standard Index Plate — Cincinnati Milling Machine

The standard index plate indexes all numbers up to and including 60; all even numbers and those divisible by 5 up to 120; and all divisions listed below up to 400. This plate is drilled on both sides, and has holes as follows:

First side: 24, 25, 28, 30, 34, 37, 38, 39, 41, 42, 43.
Second side: 46, 47, 49, 51, 53, 54, 57, 58, 59, 62, 66.

No. of Divisions	Circle	Turns	Holes	No. of Divisions	Circle	Holes	No. of Divisions	Circle	Holes	No. of Divisions	Circle	Holes
2	Any	20	...	44	66	60	104	39	15	205	41	8
3	24	13	8	45	54	48	105	42	16	210	42	8
4	Any	10	...	46	46	40	106	53	20	212	53	10
5	Any	8	...	47	47	40	108	54	20	215	43	8
6	24	6	16	48	24	20	110	66	24	216	54	10
7	28	5	20	49	49	40	112	28	10	220	66	12
8	Any	5	...	50	25	20	114	57	20	224	28	5
9	54	4	24	51	51	40	115	46	16	228	57	10
10	Any	4	...	52	39	30	116	58	20	230	46	8
11	66	3	42	53	53	40	118	59	20	232	58	10
12	24	3	8	54	54	40	120	66	22	235	47	8
13	39	3	3	55	66	48	124	62	20	236	59	10
14	49	2	42	56	28	20	125	25	8	240	66	11
15	24	2	16	57	57	40	130	39	12	245	49	8
16	24	2	12	58	58	40	132	66	20	248	62	10
17	34	2	12	59	59	40	135	54	16	250	25	4
18	54	2	12	60	42	28	136	34	10	255	51	8
19	38	2	4	62	62	40	140	28	8	260	39	6
20	Any	2	...	64	24	15	144	54	15	264	66	10
21	42	1	38	65	39	24	145	58	16	270	54	8
22	66	1	54	66	66	40	148	37	10	272	34	5
23	46	1	34	68	34	20	150	30	8	280	28	4
24	24	1	16	70	28	16	152	38	10	290	58	8
25	25	1	15	72	54	30	155	62	16	296	37	5
26	39	1	21	74	37	20	156	39	10	300	30	4
27	54	1	26	75	30	16	160	28	7	304	38	5
28	42	1	18	76	38	20	164	41	10	310	62	8
29	58	1	22	78	39	20	165	66	16	312	39	5
30	24	1	8	80	34	17	168	42	10	320	24	3
31	62	1	18	82	41	20	170	34	8	328	41	5
32	28	1	7	84	42	20	172	43	10	330	66	8
33	66	1	14	85	34	16	176	66	15	336	42	5
34	34	1	6	86	43	20	180	54	12	340	34	4
35	28	1	4	88	66	30	184	46	10	344	43	5
36	54	1	6	90	54	24	185	37	8	360	54	6
37	37	1	3	92	46	20	188	47	10	368	46	5
38	38	1	2	94	47	20	190	38	8	370	37	4
39	39	1	1	95	38	16	192	24	5	376	47	5
40	Any	1	...	96	24	10	195	39	8	380	38	4
41	41	...	40	98	49	20	196	49	10	390	39	4
42	42	...	40	100	25	10	200	30	6	392	49	5
43	43	...	40	102	51	20	204	51	10	400	30	3

Indexing Movements for High Numbers — Cincinnati Milling Machine

This set of 3 index plates indexes all numbers up to and including 200; all even numbers and those divisible by 5 up to and including 400. The plates are drilled on each side, making six sides *A*, *B*, *C*, *D*, *E* and *F*.

Example: — It is required to index 35 divisions. The preferred side is *F*, since this requires the least number of holes; but should one of plates *D*, *A* or *E* be in place, either can be used, thus avoiding the changing of plates.

No. of Divisions	Side	Circle	Turns	Holes	No. of Divisions	Side	Circle	Turns	Holes	No. of Divisions	Side	Circle	Turns	Holes
2	Any	Any	20	....	15	*C*	93	2	62	28	*D*	77	1	33
3	*A*	30	13	10	15	*F*	159	2	106	28	*A*	91	1	39
3	*B*	36	13	12	16	*E*	26	2	13	29	*E*	87	1	33
3	*E*	42	13	14	16	*F*	28	2	14	30	*A*	30	1	10
3	*C*	93	13	31	16	*A*	30	2	15	30	*B*	36	1	12
3	*F*	159	13	53	16	*D*	32	2	16	30	*E*	42	1	14
4	Any	Any	10	....	16	*C*	34	2	17	30	*C*	93	1	31
5	Any	Any	8	....	16	*B*	36	2	18	30	*F*	159	1	53
6	*A*	30	6	20	17	*C*	34	2	12	31	*C*	93	1	27
6	*B*	36	6	24	17	*E*	119	2	42	32	*F*	28	1	7
6	*E*	42	6	28	17	*C*	153	2	54	32	*D*	32	1	8
6	*C*	93	6	62	17	*F*	187	2	66	32	*B*	36	1	9
6	*F*	159	6	106	18	*B*	36	2	8	32	*A*	48	1	12
7	*F*	28	5	20	18	*A*	99	2	22	33	*A*	99	1	21
7	*E*	42	5	30	18	*C*	153	2	34	34	*C*	34	1	6
7	*D*	77	5	55	19	*F*	38	2	4	34	*E*	119	1	21
7	*A*	91	5	65	19	*E*	133	2	14	34	*F*	187	1	33
8	Any	Any	5	....	19	*A*	171	2	18	35	*F*	28	1	4
9	*B*	36	4	16	20	Any	Any	2	....	35	*D*	77	1	11
9	*A*	99	4	44	21	*E*	42	1	38	35	*A*	91	1	13
9	*C*	153	4	68	21	*A*	147	1	133	35	*E*	119	1	17
10	Any	Any	4	....	22	*D*	44	1	36	36	*B*	36	1	4
11	*D*	44	3	28	22	*A*	99	1	81	36	*A*	99	1	11
11	*A*	99	3	63	22	*F*	143	1	117	36	*C*	153	1	17
11	*F*	143	3	91	23	*C*	46	1	34	37	*B*	111	1	9
12	*A*	30	3	10	23	*A*	69	1	51	38	*F*	38	1	2
12	*B*	36	3	12	23	*E*	161	1	119	38	*E*	133	1	7
12	*E*	42	3	14	24	*A*	30	1	20	38	*A*	171	1	9
12	*C*	93	3	31	24	*B*	36	1	24	39	*A*	117	1	3
12	*F*	159	3	53	24	*E*	42	1	28	40	Any	Any	1	....
13	*E*	26	3	2	24	*C*	93	1	62	41	*C*	123	....	120
13	*A*	91	3	7	24	*F*	159	1	106	42	*E*	42	....	40
13	*F*	143	3	11	25	*A*	30	1	18	42	*A*	147	....	140
13	*B*	169	3	13	25	*E*	175	1	105	43	*A*	129	....	120
14	*F*	28	2	24	26	*F*	26	1	14	44	*D*	44	....	40
14	*E*	42	2	36	26	*A*	91	1	49	44	*A*	99	....	90
14	*D*	77	2	66	26	*B*	169	1	91	44	*F*	143	....	130
14	*A*	91	2	78	27	*B*	81	1	39	45	*B*	36	....	32
15	*A*	30	2	20	27	*A*	189	1	91	45	*A*	99	....	88
15	*B*	36	2	24	28	*F*	28	1	12	45	*C*	153	....	136
15	*E*	42	2	28	28	*E*	42	1	18	46	*C*	46	....	40

Indexing Movements for High Numbers — Cincinnati Milling Machine

No. of Divisions	Side	Circle	Holes	No. of Divisions	Side	Circle	Holes	No. of Divisions	Side	Circle	Holes
46	A	69	60	70	E	119	68	96	B	36	15
46	E	161	140	71	F	71	40	96	A	48	20
47	B	141	120	72	B	36	20	97	B	97	40
48	A	30	25	72	A	117	65	98	A	147	60
48	B	36	30	72	C	153	85	99	A	99	40
49	A	147	120	73	E	73	40	100	A	30	12
50	A	30	24	74	B	111	60	100	E	175	70
50	E	175	140	75	A	30	16	101	F	101	40
51	C	153	120	76	F	38	20	102	C	153	60
52	E	26	20	76	E	133	70	103	E	103	40
52	A	91	70	76	A	171	90	104	E	26	10
52	F	143	110	77	D	77	40	104	A	91	35
52	B	169	130	78	A	117	60	104	F	143	55
53	F	159	120	79	C	79	40	104	B	169	65
54	B	81	60	80	E	26	13	105	E	42	16
54	A	189	140	80	F	28	14	105	A	147	56
55	D	44	32	80	A	30	15	106	F	159	60
55	F	143	104	80	D	32	16	107	D	107	40
56	F	28	20	80	C	34	17	108	B	81	30
56	E	42	30	80	B	36	18	108	A	189	70
56	D	77	55	80	E	42	21	109	C	109	40
56	A	91	65	81	B	81	40	110	D	44	16
57	A	171	120	82	C	123	60	110	A	99	36
58	E	87	60	83	F	83	40	110	F	143	52
59	A	177	120	84	E	42	20	111	B	111	40
60	A	30	20	84	A	147	70	112	F	28	10
60	B	36	24	85	C	34	16	112	E	42	15
60	E	42	28	85	E	119	56	113	F	113	40
60	F	159	106	85	F	187	88	114	A	171	60
61	B	183	120	86	A	129	60	115	C	46	16
62	C	93	60	87	E	87	40	115	A	69	24
63	A	189	120	88	D	44	20	115	E	161	56
64	D	32	20	88	A	99	45	116	E	87	30
64	A	48	30	88	F	143	65	117	A	117	40
65	E	26	16	89	D	89	40	118	A	177	60
65	A	91	56	90	B	36	16	119	E	119	40
65	F	143	88	90	A	99	44	120	A	30	10
65	B	169	104	90	C	153	68	120	B	36	12
66	A	99	60	91	A	91	40	120	E	42	14
67	B	67	40	92	C	46	20	120	C	93	31
68	C	34	20	92	A	69	30	120	F	159	53
68	E	119	70	92	E	161	70	121	D	121	40
68	F	187	110	93	C	93	40	122	B	183	60
69	A	69	40	94	B	141	60	123	C	123	40
70	F	28	16	95	F	38	16	124	C	93	30
70	D	42	24	95	E	133	56	125	E	175	56
70	A	91	52	95	A	171	72	126	A	189	60

Indexing Movements for High Numbers — Cincinnati Milling Machine

No. of Divisions	Side	Circle	Holes	No. of Divisions	Side	Circle	Holes	No. of Divisions	Side	Circle	Holes
127	*B*	127	40	160	*A*	48	12	198	*A*	99	20
128	*D*	32	10	161	*E*	161	40	199	*B*	199	40
128	*A*	48	15	162	*B*	81	20	200	*A*	30	6
129	*A*	129	40	163	*D*	163	40	200	*E*	175	35
130	*E*	26	8	164	*C*	123	30	202	*F*	101	20
130	*A*	91	28	165	*A*	99	24	204	*C*	153	30
130	*F*	143	44	166	*F*	83	20	205	*C*	123	24
130	*B*	169	52	167	*C*	167	40	206	*E*	103	20
131	*F*	131	40	168	*E*	42	10	208	*E*	26	5
132	*A*	99	30	168	*A*	147	35	210	*E*	42	8
133	*E*	133	40	169	*B*	169	40	210	*A*	147	28
134	*B*	67	20	170	*C*	34	8	212	*F*	159	30
135	*B*	81	24	170	*E*	119	28	214	*D*	107	20
135	*A*	189	56	170	*F*	187	44	215	*A*	129	24
136	*C*	34	10	171	*A*	171	40	216	*B*	81	15
136	*E*	119	35	172	*A*	129	30	216	*A*	189	35
137	*D*	137	40	173	*F*	173	40	218	*C*	109	20
138	*A*	69	20	174	*E*	87	20	220	*D*	44	8
139	*C*	139	40	175	*E*	175	40	220	*A*	99	18
140	*F*	28	8	176	*D*	44	10	220	*F*	143	26
140	*E*	42	12	177	*A*	177	40	222	*B*	111	20
140	*D*	77	22	178	*D*	89	20	224	*F*	28	5
140	*A*	91	26	179	*D*	179	40	226	*F*	113	20
141	*B*	141	40	180	*B*	36	8	228	*A*	171	30
142	*F*	71	20	180	*A*	99	22	230	*C*	46	8
143	*F*	143	40	180	*C*	153	34	230	*A*	69	12
144	*B*	36	10	181	*C*	181	40	230	*E*	161	28
145	*E*	87	24	182	*A*	91	20	232	*E*	87	15
146	*E*	73	20	183	*B*	183	40	234	*A*	117	20
147	*A*	147	40	184	*C*	46	10	235	*B*	141	24
148	*B*	111	30	184	*A*	69	15	236	*A*	177	30
149	*E*	149	40	184	*E*	161	35	238	*E*	119	20
150	*A*	30	8	185	*B*	111	24	240	*A*	30	5
151	*D*	151	40	186	*C*	93	20	240	*B*	36	6
152	*F*	38	10	187	*F*	187	40	240	*E*	42	7
152	*E*	133	35	188	*B*	141	30	240	*A*	48	8
152	*A*	171	45	189	*A*	189	40	242	*D*	121	20
153	*C*	153	40	190	*F*	38	8	244	*B*	183	30
154	*D*	77	20	190	*E*	133	28	245	*A*	147	24
155	*C*	93	24	190	*A*	171	36	246	*C*	123	20
156	*A*	117	30	191	*E*	191	40	248	*C*	93	15
157	*B*	157	40	192	*A*	48	10	250	*E*	175	28
158	*C*	79	20	193	*D*	193	40	252	*A*	189	30
159	*F*	159	40	194	*B*	97	20	254	*B*	127	20
160	*F*	28	7	195	*A*	117	24	255	*C*	153	24
160	*D*	32	8	196	*A*	147	30	256	*D*	32	5
160	*B*	36	9	197	*C*	197	40	258	*A*	129	20

Indexing Movements for High Numbers — Cincinnati Milling Machine

No. of Divisions	Side	Circle	Holes	No. of Divisions	Side	Circle	Holes	No. of Divisions	Side	Circle	Holes
260	*E*	26	4	304	*F*	38	5	354	*A*	177	20
260	*A*	91	14	305	*B*	183	24	355	*F*	71	8
260	*F*	143	22	306	*C*	153	20	356	*D*	89	10
260	*B*	169	26	308	*D*	77	10	358	*D*	179	20
262	*F*	131	20	310	*C*	93	12	360	*B*	36	4
264	*A*	99	15	312	*A*	117	15	360	*A*	99	11
265	*F*	159	24	314	*B*	157	20	360	*C*	153	17
266	*E*	133	20	315	*A*	189	24	362	*C*	181	20
268	*B*	67	10	316	*C*	79	10	364	*A*	91	10
270	*B*	81	12	318	*F*	159	20	365	*E*	73	8
270	*A*	189	28	320	*D*	32	4	366	*B*	183	20
272	*C*	34	5	320	*A*	48	6	368	*C*	46	5
274	*D*	137	20	322	*E*	161	20	370	*B*	111	12
276	*A*	69	10	324	*B*	81	10	372	*C*	93	10
278	*C*	139	20	326	*D*	163	20	374	*F*	187	20
280	*F*	28	4	328	*C*	123	15	376	*B*	141	15
280	*E*	42	6	330	*A*	99	12	378	*A*	189	20
280	*D*	77	11	332	*F*	83	10	380	*F*	38	4
280	*A*	91	13	334	*C*	167	20	380	*E*	133	14
282	*B*	141	20	335	*B*	67	8	380	*A*	171	18
284	*F*	71	10	336	*E*	42	5	382	*E*	191	20
285	*A*	171	24	338	*B*	169	20	384	*A*	48	5
286	*F*	143	20	340	*C*	34	4	385	*D*	77	8
288	*B*	36	5	340	*E*	119	14	386	*D*	193	20
290	*E*	87	12	340	*F*	187	22	388	*B*	97	10
292	*E*	73	10	342	*A*	171	20	390	*A*	117	12
294	*A*	147	20	344	*A*	129	15	392	*A*	147	15
295	*A*	177	24	345	*A*	69	8	394	*C*	197	20
296	*B*	111	15	346	*F*	173	20	395	*C*	79	8
298	*E*	149	20	348	*E*	87	10	396	*A*	99	10
300	*A*	30	4	350	*E*	175	20	398	*B*	199	20
302	*D*	151	20	352	*D*	44	5	400	*A*	30	3

Angular Indexing. — With the ordinary indexing head, in which 40 turns of the index crank are required for one revolution of the work, one turn of the index crank equals 9 degrees. Hence, when one complete turn of the index crank equals 9 degrees, two holes in the 18-hole circle, or 3 holes in the 27-hole circle, must correspond to one degree. The first principle or rule for indexing for angles is therefore that two holes in the 18-hole circle or 3 holes in the 27-hole circle equals a movement of one degree of the index head spindle and the work.

Assume that an indexing movement of 35 degrees is required. One complete turn of the index crank equals 9 degrees; therefore, first divide the number of degrees for which to index, by 9, in order to find how many complete turns the index crank should make. The number of degrees left to turn after having completed the full turns are indexed by taking two holes in the 18-hole circle for each degree. In this case, $\frac{35}{9} = 3\frac{8}{9}$, which indicates that the index crank must be turned three full revolutions, and then 8 degrees more are indexed by moving 16 holes in the 18-hole circle.

To index for 11½ degrees, for example, first turn the index crank one revolution, this being a 9-degree movement. Then to index 2½ degrees, move the index crank 5 holes in the 18-hole circle (4 holes for the two whole degrees and one hole for the ½ degree equals the total movement of 5 holes).

Below is shown how this calculation may be carried out to plainly indicate the movement required for this angle:

11½ deg. = 9 deg. + 2 deg. + ½ deg.
1 turn + 4 holes + 1 hole in the 18-hole circle.

Should it be required to index only ⅓ degree, this may be done by using the 27-hole circle. In this circle a three-hole movement equals one degree, and a one-hole movement in that circle thus equals ⅓ degree, or 20 minutes. Assume that it is required to index the work through an angle of 48 degrees 40 minutes. Below is plainly shown how this calculation may be carried out:

48 deg. 40 min. = 45 deg. + 3 deg. + 40 min.
5 turns + 9 holes + 2 holes in the 27-hole circle.

Angular Values of One-Hole Moves — B. & S. Index Plates

15-hole circle = 36 minutes	29-hole circle = 18.620 minutes
16-hole circle = 33.750 minutes	31-hole circle = 17.419 minutes
17-hole circle = 31.764 minutes	33-hole circle = 16.363 minutes
18-hole circle = 30 minutes	37-hole circle = 14.594 minutes
19-hole circle = 28.421 minutes	39-hole circle = 13.846 minutes
20-hole circle = 27 minutes	41-hole circle = 13.170 minutes
21-hole circle = 25.714 minutes	47-hole circle = 11.489 minutes
23-hole circle = 23.478 minutes	49-hole circle = 11.020 minutes
27-hole circle = 20 minutes	....

Approximate Indexing for Angles.— The following general rule for *approximate* indexing of small angles is applicable to any index head requiring 40 revolutions of the index crank for one revolution of the work.

Rule: Divide 540 by the total number of minutes to be indexed. If the quotient is approximately equal to the number of holes in any index circle available, the angular movement is obtained by moving the crank one hole in this index circle; but if the quotient is not approximately equal, multiply it by any trial number which will give a product equal to the number of holes in an available index circle and move the index crank as many holes as are indicated by the trial number. (If the quotient of 540 divided by the total number of minutes is greater than the number of holes in any of the index circles, it is not possible to obtain the required movement for the angle by simple indexing.)

Example: — Assume that it is required to index to an angle of 2 degrees 46 minutes. Changing this to minutes gives a total of 166 minutes. Dividing 540 by 166 we have 540 ÷ 166 = 3.253. This quotient is next multiplied by some trial number to obtain a product which equals the number of holes in an available index circle. Multiplying by 12, we have 3.253 × 12 = 39.036. Therefore, for indexing 2 degrees 46 minutes, the 39-hole circle can be used and the index crank would be moved 12 holes.

Tables for Angular Indexing. — The table, "Angular Indexing," gives the number of turns of the index crank for indexing various angles. In the column headed, "Turns of Index Crank," the whole number (where given) indicates the number of full revolutions; the numerator of the fraction, the number of holes additional; and the denominator, the number of holes in the index circle to be used. The angular movement obtained for a movement of one hole, in various index plates is given in the table, "Angular Values of One-Hole Moves."

Angular Indexing

Angle in Degs.	Turns of Index Crank	Angle in Degs.	Turns of Index Crank	Angle in Degs.	Turns of Index Crank	Angle in Degs.	Turns of Index Crank	Angle in Degs.	Turns of Index Crank
1	2/18	10	1 2/18	19	2 2/18	28	3 2/18	37	4 2/18
1 1/3	4/27	10 1/3	1 4/27	19 1/3	2 4/27	28 1/3	3 4/27	37 1/3	4 4/27
1 1/2	3/18	10 1/2	1 3/18	19 1/2	2 3/18	28 1/2	3 3/18	37 1/2	4 3/18
1 2/3	5/27	10 2/3	1 5/27	19 2/3	2 5/27	28 2/3	3 5/27	37 2/3	4 5/27
2	4/18	11	1 4/18	20	2 4/18	29	3 4/18	38	4 4/18
2 1/3	7/27	11 1/3	1 7/27	20 1/3	2 7/27	29 1/3	3 7/27	38 1/3	4 7/27
2 1/2	5/18	11 1/2	1 5/18	20 1/2	2 5/18	29 1/2	3 5/18	38 1/2	4 5/18
2 2/3	8/27	11 2/3	1 8/27	20 2/3	2 8/27	29 2/3	3 8/27	38 2/3	4 8/27
3	6/18	12	1 6/18	21	2 6/18	30	3 6/18	39	4 6/18
3 1/3	10/27	12 1/3	1 10/27	21 1/3	2 10/27	30 1/3	3 10/27	39 1/3	4 10/27
3 1/2	7/18	12 1/2	1 7/18	21 1/2	2 7/18	30 1/2	3 7/18	39 1/2	4 7/18
3 2/3	11/27	12 2/3	1 11/27	21 2/3	2 11/27	30 2/3	3 11/27	39 2/3	4 11/27
4	8/18	13	1 8/18	22	2 8/18	31	3 8/18	40	4 8/18
4 1/3	13/27	13 1/3	1 13/27	22 1/3	2 13/27	31 1/3	3 13/27	40 1/3	4 13/27
4 1/2	9/18	13 1/2	1 9/18	22 1/2	2 9/18	31 1/2	3 9/18	40 1/2	4 9/18
4 2/3	14/27	13 2/3	1 14/27	22 2/3	2 14/27	31 2/3	3 14/27	40 2/3	4 14/27
5	10/18	14	1 10/18	23	2 10/18	32	3 10/18	41	4 10/18
5 1/3	16/27	14 1/3	1 16/27	23 1/3	2 16/27	32 1/3	3 16/27	41 1/3	4 16/27
5 1/2	11/18	14 1/2	1 11/18	23 1/2	2 11/18	32 1/2	3 11/18	41 1/2	4 11/18
5 2/3	17/27	14 2/3	1 17/27	23 2/3	2 17/27	32 2/3	3 17/27	41 2/3	4 17/27
6	12/18	15	1 12/18	24	2 12/18	33	3 12/18	42	4 12/18
6 1/3	19/27	15 1/3	1 19/27	24 1/3	2 19/27	33 1/3	3 19/27	42 1/3	4 19/27
6 1/2	13/18	15 1/2	1 13/18	24 1/2	2 13/18	33 1/2	3 13/18	42 1/2	4 13/18
6 2/3	20/27	15 2/3	1 20/27	24 2/3	2 20/27	33 2/3	3 20/27	42 2/3	4 20/27
7	14/18	16	1 14/18	25	2 14/18	34	3 14/18	43	4 14/18
7 1/3	22/27	16 1/3	1 22/27	25 1/3	2 22/27	34 1/3	3 22/27	43 1/3	4 22/27
7 1/2	15/18	16 1/2	1 15/18	25 1/2	2 15/18	34 1/2	3 15/18	43 1/2	4 15/18
7 2/3	23/27	16 2/3	1 23/27	25 2/3	2 23/27	34 2/3	3 23/27	43 2/3	4 23/27
8	16/18	17	1 16/18	26	2 16/18	35	3 16/18	44	4 16/18
8 1/3	25/27	17 1/3	1 25/27	26 1/3	2 25/27	35 1/3	3 25/27	44 1/3	4 25/27
8 1/2	17/18	17 1/2	1 17/18	26 1/2	2 17/18	35 1/2	3 17/18	44 1/2	4 17/18
8 2/3	26/27	17 2/3	1 26/27	26 2/3	2 26/27	35 2/3	3 26/27	44 2/3	4 26/27
9	1	18	2	27	3	36	4	45	5
9 1/3	1 1/27	18 1/3	2 1/27	27 1/3	3 1/27	36 1/3	4 1/27	45 1/3	5 1/27
9 1/2	1 1/18	18 1/2	2 1/18	27 1/2	3 1/18	36 1/2	4 1/18	45 1/2	5 1/18
9 2/3	1 2/27	18 2/3	2 2/27	27 2/3	3 2/27	36 2/3	4 2/27	45 2/3	5 2/27

Accurate Angular Indexing Movements — 1 *

Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *
0.0152	.....	1/66	0.0541	2/37	2/37	0.1000	2/20	3/30
0.0161	.....	1/62	0.0556	1/18	3/54	0.1017	.....	6/59
0.0169	.....	1/59	0.0566	.....	3/53	0.1020	5/49	5/49
0.0172	.....	1/58	0.0588	1/17	2/34	0.1026	4/39	4/39
0.0175	.....	1/57	0.0588	.....	3/51	0.1034	3/29	6/58
0.0185	.....	1/54	0.0606	2/33	4/66	0.1053	2/19	4/38
0.0189	.....	1/53	0.0612	3/49	3/49	0.1053	.....	6/57
0.0196	.....	1/51	0.0625	1/16	.....	0.1061	.....	7/66
0.0204	1/49	1/49	0.0638	3/47	3/47	0.1064	5/47	5/47
0.0213	1/47	1/47	0.0645	2/31	4/62	0.1071	.....	3/28
0.0217	.....	1/46	0.0652	.....	3/46	0.1081	4/37	4/37
0.0233	1/43	1/43	0.0667	.....	2/30	0.1087	.....	5/46
0.0238	.....	1/42	0.0678	.....	4/59	0.1111	2/18	.....
0.0244	1/41	1/41	0.0690	2/29	4/58	0.1111	3/27	6/54
0.0256	1/39	1/39	0.0698	3/43	3/43	0.1129	.....	7/62
0.0263	.....	1/38	0.0702	.....	4/57	0.1132	.....	6/53
0.0270	1/37	1/37	0.0714	.....	2/28	0.1163	5/43	5/43
0.0294	.....	1/34	0.0714	.....	3/42	0.1176	2/17	4/34
0.0303	1/33	2/66	0.0732	3/41	3/41	0.1176	.....	6/51
0.0323	1/31	2/62	0.0741	2/27	4/54	0.1186	.....	7/59
0.0333	.....	1/30	0.0755	.....	4/53	0.1190	.....	5/42
0.0338	.....	2/59	0.0758	.....	5/66	0.1200	.....	3/25
0.0345	1/29	2/58	0.0769	3/39	3/39	0.1207	.....	7/58
0.0351	.....	2/57	0.0784	.....	4/51	0.1212	4/33	8/66
0.0357	.....	1/28	0.0789	.....	3/38	0.1220	5/41	5/41
0.0370	1/27	2/54	0.0800	.....	2/25	0.1224	6/49	6/49
0.0377	.....	2/53	0.0806	.....	5/62	0.1228	.....	7/57
0.0392	.....	2/51	0.0811	3/37	3/37	0.1250	2/16	3/24
0.0400	.....	1/25	0.0816	4/49	4/49	0.1277	6/47	6/47
0.0408	2/49	2/49	0.0833	.....	2/24	0.1282	5/39	5/39
0.0417	.....	1/24	0.0847	.....	5/59	0.1290	4/31	8/62
0.0426	2/47	2/47	0.0851	4/47	4/47	0.1296	.....	7/54
0.0435	1/23	2/46	0.0862	.....	5/58	0.1304	3/23	6/46
0.0454	.....	3/66	0.0870	2/23	4/46	0.1316	.....	5/38
0.0465	2/43	2/43	0.0877	.....	5/57	0.1321	.....	7/53
0.0476	1/21	2/42	0.0882	.....	3/34	0.1333	2/15	4/30
0.0484	.....	3/62	0.0909	3/33	6/66	0.1351	5/37	5/37
0.0488	2/41	2/41	0.0926	.....	5/54	0.1356	.....	8/59
0.0500	1/20	.....	0.0930	4/43	4/43	0.1364	.....	9/66
0.0508	.....	3/59	0.0943	.....	5/53	0.1372	.....	7/51
0.0513	2/39	2/39	0.0952	2/21	4/42	0.1379	4/29	8/58
0.0517	.....	3/58	0.0968	3/31	6/62	0.1395	6/43	6/43
0.0526	1/19	2/38	0.0976	4/41	4/41	0.1404	.....	8/57
0.0526	.....	3/57	0.0980	.....	5/51	0.1429	.....	4/28

* See explanatory note below Table 8.

Accurate Angular Indexing Movements — 2 *

Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *
0.1429	3/21	6/42	0.1864	.....	11/59	0.2308	9/39	9/39
0.1429	7/49	7/49	0.1875	3/16	.....	0.2326	10/43	10/43
0.1452	.....	9/62	0.1887	.....	10/53	0.2333	.....	7/30
0.1463	6/41	6/41	0.1892	7/37	7/37	0.2340	11/47	11/47
0.1471	.....	5/34	0.1897	.....	11/58	0.2353	4/17	8/34
0.1481	4/27	8/54	0.1905	4/21	8/42	0.2353	.....	12/51
0.1489	7/47	7/47	0.1915	9/47	9/47	0.2368	.....	9/38
0.1500	3/20	.....	0.1930	.....	11/57	0.2373	.....	14/59
0.1509	.....	8/53	0.1935	6/31	12/62	0.2381	5/21	10/42
0.1515	5/33	10/66	0.1951	8/41	8/41	0.2391	.....	11/46
0.1522	.....	7/46	0.1957	.....	9/46	0.2400	.....	6/25
0.1525	.....	9/59	0.1961	.....	10/51	0.2407	.....	13/54
0.1538	6/39	6/39	0.1970	.....	13/66	0.2414	7/29	14/58
0.1552	.....	9/58	0.2000	3/15	5/25	0.2419	.....	15/62
0.1569	.....	8/51	0.2000	4/20	6/30	0.2424	8/33	16/66
0.1579	3/19	6/38	0.2034	.....	12/59	0.2432	9/37	9/37
0.1579	.....	9/57	0.2037	.....	11/54	0.2439	10/41	10/41
0.1600	.....	4/25	0.2041	10/49	10/49	0.2449	12/49	12/49
0.1613	5/31	10/62	0.2051	8/39	8/39	0.2453	.....	13/53
0.1622	6/37	6/37	0.2059	.....	7/34	0.2456	.....	14/57
0.1628	7/43	7/43	0.2069	6/29	12/58	0.2500	4/16	6/24
0.1633	8/49	8/49	0.2075	.....	11/53	0.2500	5/20	7/28
0.1667	3/18	11/66	0.2083	.....	5/24	0.2542	.....	15/59
0.1667	.....	9/54	0.2093	9/43	9/43	0.2549	.....	13/51
0.1667	.....	7/42	0.2097	.....	13/62	0.2553	12/47	12/47
0.1667	.....	5/30	0.2105	4/19	8/38	0.2558	11/43	11/43
0.1667	.....	4/24	0.2105	.....	12/57	0.2564	10/39	10/39
0.1695	.....	10/59	0.2121	7/33	14/66	0.2576	.....	17/66
0.1698	.....	9/53	0.2128	10/47	10/47	0.2581	8/31	16/62
0.1702	8/47	8/47	0.2143	.....	6/28	0.2586	.....	15/58
0.1707	7/41	7/41	0.2143	.....	9/42	0.2593	7/27	14/54
0.1724	5/29	10/58	0.2157	.....	11/51	0.2609	6/23	12/46
0.1739	4/23	8/46	0.2162	8/37	8/37	0.2619	.....	11/42
0.1754	.....	10/57	0.2174	5/23	10/46	0.2632	5/19	10/38
0.1765	3/17	6/34	0.2195	9/41	9/41	0.2632	.....	15/57
0.1765	.....	9/51	0.2203	.....	13/59	0.2642	.....	14/53
0.1774	.....	11/62	0.2222	4/18	.....	0.2647	.....	9/34
0.1786	.....	5/28	0.2222	6/27	12/54	0.2653	13/49	13/49
0.1795	7/39	7/39	0.2241	.....	13/58	0.2667	4/15	8/30
0.1818	6/33	12/66	0.2245	11/49	11/49	0.2683	11/41	11/41
0.1839	9/49	9/49	0.2258	7/31	14/62	0.2703	10/37	10/37
0.1842	.....	7/38	0.2264	.....	12/53	0.2712	.....	16/59
0.1852	5/27	10/54	0.2273	.....	15/66	0.2727	9/33	18/66
0.1860	8/43	8/43	0.2281	.....	13/57	0.2742	.....	17/62

* See explanatory note below Table 8.

Accurate Angular Indexing Movements — 3 *

Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *
0.2745	.....	14/51	0.3191	15/47	15/47	0.3617	17/47	17/47
0.2759	8/29	16/58	0.3200	.....	8/25	0.3621	.....	21/58
0.2766	13/47	13/47	0.3208	.....	17/53	0.3636	12/33	24/66
0.2778	5/18	15/54	0.3214	.....	9/28	0.3659	15/41	15/41
0.2791	12/43	12/43	0.3220	.....	19/59	0.3667	.....	11/30
0.2800	.....	7/25	0.3226	10/31	20/62	0.3673	18/49	18/49
0.2807	.....	16/57	0.3235	.....	11/34	0.3684	7/19	14/38
0.2821	11/39	11/39	0.3243	12/37	12/37	0.3684	.....	21/57
0.2826	.....	13/46	0.3256	14/43	14/43	0.3696	.....	17/46
0.2830	.....	15/53	0.3261	.....	15/46	0.3704	10/27	20/54
0.2857	.....	8/28	0.3265	16/49	16/49	0.3710	.....	23/62
0.2857	14/49	14/49	0.3276	.....	19/58	0.3721	16/43	16/43
0.2857	6/21	12/42	0.3333	6/18	8/24	0.3725	.....	19/51
0.2879	.....	19/66	0.3333	5/15	10/30	0.3729	.....	22/59
0.2881	.....	17/59	0.3333	13/39	13/39	0.3750	6/16	9/24
0.2895	.....	11/38	0.3333	7/21	14/42	0.3774	.....	20/53
0.2903	9/31	18/62	0.3333	.....	17/51	0.3784	14/37	14/37
0.2917	.....	7/24	0.3333	9/27	18/54	0.3788	.....	25/66
0.2927	12/41	12/41	0.3333	.....	19/57	0.3793	11/29	22/58
0.2931	.....	17/58	0.3333	11/33	22/66	0.3810	8/21	16/42
0.2941	.....	15/51	0.3387	.....	21/62	0.3824	.....	13/34
0.2941	5/17	10/34	0.3390	.....	20/59	0.3830	18/47	18/47
0.2963	8/27	16/54	0.3396	.....	18/53	0.3846	15/39	15/39
0.2973	11/37	11/37	0.3404	16/47	16/47	0.3860	.....	22/57
0.2979	14/47	14/47	0.3415	14/41	14/41	0.3871	12/31	24/62
0.2982	.....	17/57	0.3421	.....	13/38	0.3878	19/49	19/49
0.3000	6/20	9/30	0.3448	10/29	20/58	0.3889	7/18	21/54
0.3019	.....	16/53	0.3469	17/49	17/49	0.3898	.....	23/59
0.3023	13/43	13/43	0.3478	8/23	16/46	0.3902	16/41	16/41
0.3030	10/33	20/66	0.3485	.....	23/66	0.3913	9/23	18/46
0.3043	7/23	14/46	0.3488	15/43	15/43	0.3922	.....	20/51
0.3051	.....	18/59	0.3500	7/20	.....	0.3929	.....	11/28
0.3061	15/49	15/49	0.3509	.....	20/57	0.3939	13/33	26/66
0.3065	.....	19/62	0.3514	13/37	13/37	0.3947	.....	15/38
0.3077	12/39	12/39	0.3519	.....	19/54	0.3953	17/43	17/43
0.3095	.....	13/42	0.3529	6/17	12/34	0.3962	.....	21/53
0.3103	9/29	18/58	0.3529	.....	18/51	0.3966	.....	23/58
0.3125	5/16	.....	0.3548	11/31	22/62	0.4000	6/15	10/25
0.3137	.....	16/51	0.3559	.....	21/59	0.4000	8/20	12/30
0.3148	.....	17/54	0.3571	.....	10/28	0.4032	.....	25/62
0.3158	6/19	12/38	0.3571	.....	15/42	0.4035	.....	23/57
0.3158	.....	18/57	0.3585	.....	19/53	0.4043	19/47	19/47
0.3171	13/41	13/41	0.3590	14/39	14/39	0.4048	.....	17/42
0.3182	.....	21/66	0.3600	.....	9/25	0.4054	15/37	15/37

* See explanatory note below Table 8.

Accurate Angular Indexing Movements — 4 *

Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *
0.4068	.....	24/59	0.4490	22/49	22/49	0.4912	.....	28/57
0.4074	11/27	22/54	0.4500	9/20	.....	0.4915	.....	29/59
0.4082	20/49	20/49	0.4510	.....	23/51	0.5000	8/16	12/24
0.4091	.....	27/66	0.4516	14/31	28/62	0.5000	9/18	14/28
0.4103	16/39	16/39	0.4524	.....	19/42	0.5000	10/20	15/30
0.4118	7/17	14/34	0.4528	.....	24/53	0.5000	.....	17/34
0.4118	.....	21/51	0.4545	15/33	30/66	0.5000	.....	19/38
0.4130	.....	19/46	0.4561	.....	26/57	0.5000	.....	21/42
0.4138	12/29	24/58	0.4565	.....	21/46	0.5000	.....	23/46
0.4146	17/41	17/41	0.4576	.....	27/59	0.5000	.....	27/54
0.4151	.....	22/53	0.4583	.....	11/24	0.5000	.....	29/58
0.4167	.....	10/24	0.4595	17/37	17/37	0.5000	.....	31/62
0.4186	18/43	18/43	0.4615	18/39	18/39	0.5000	.....	33/66
0.4194	13/31	26/62	0.4630	.....	25/54	0.5085	.....	30/59
0.4211	8/19	16/38	0.4634	19/41	19/41	0.5088	.....	29/57
0.4211	.....	24/57	0.4643	.....	13/28	0.5094	.....	27/53
0.4237	.....	25/59	0.4651	20/43	20/43	0.5098	.....	26/51
0.4242	14/33	28/66	0.4655	.....	27/58	0.5102	25/49	25/49
0.4255	20/47	20/47	0.4667	7/15	14/30	0.5106	24/47	24/47
0.4259	.....	23/54	0.4677	.....	29/62	0.5116	22/43	22/43
0.4286	.....	12/28	0.4681	22/47	22/47	0.5122	21/41	21/41
0.4286	9/21	18/42	0.4694	23/49	23/49	0.5128	20/39	20/39
0.4286	21/49	21/49	0.4697	.....	31/66	0.5135	19/37	19/37
0.4310	.....	25/58	0.4706	8/17	16/34	0.5152	17/33	34/66
0.4314	.....	22/51	0.4706	.....	24/51	0.5161	16/31	32/62
0.4324	16/37	16/37	0.4717	.....	25/53	0.5172	15/29	30/58
0.4333	.....	13/30	0.4737	9/19	27/57	0.5185	14/27	28/54
0.4340	.....	23/53	0.4746	.....	28/59	0.5200	.....	13/25
0.4348	10/23	20/46	0.4762	10/21	20/42	0.5217	12/23	24/46
0.4355	.....	27/62	0.4783	11/23	22/46	0.5238	11/21	22/42
0.4359	17/39	17/39	0.4800	.....	12/25	0.5254	.....	31/59
0.4375	7/16	.....	0.4814	.....	26/54	0.5263	10/19	20/38
0.4386	.....	25/57	0.4815	13/27	.....	0.5263	.....	30/57
0.4390	18/41	18/41	0.4828	14/29	28/58	0.5283	.....	28/53
0.4394	.....	29/66	0.4839	15/31	30/62	0.5294	9/17	18/34
0.4400	.....	11/25	0.4848	16/33	32/66	0.5294	.....	27/51
0.4407	.....	26/59	0.4865	18/37	18/37	0.5303	.....	35/66
0.4412	.....	15/34	0.4872	19/39	19/39	0.5306	26/49	26/49
0.4419	19/43	19/43	0.4878	20/41	20/41	0.5319	25/47	25/47
0.4444	8/18	.....	0.4884	21/43	21/43	0.5323	.....	33/62
0.4444	12/27	24/54	0.4894	23/47	23/47	0.5333	8/15	16/30
0.4468	21/47	21/47	0.4898	24/49	24/49	0.5345	.....	31/58
0.4474	.....	17/38	0.4902	.....	25/51	0.5349	23/43	23/43
0.4483	13/29	26/58	0.4906	.....	26/53	0.5357	.....	15/28

* See explanatory note below Table 8.

Accurate Angular Indexing Movements — 5 *

Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *
0.5366	22/41	22/41	0.5789	.....	33/57	0.6250	10/16	15/24
0.5370	.....	29/54	0.5806	18/31	36/62	0.6271	.....	37/59
0.5385	21/39	21/39	0.5814	25/43	25/43	0.6275	.....	32/51
0.5405	20/37	20/37	0.5833	.....	14/24	0.6279	27/43	27/43
0.5417	.....	13/24	0.5849	.....	31/53	0.6290	.....	39/62
0.5424	.....	32/59	0.5854	24/41	24/41	0.6296	17/27	34/54
0.5435	.....	25/46	0.5862	17/29	34/58	0.6304	.....	29/46
0.5439	.....	31/57	0.5870	.....	27/46	0.6316	12/19	24/38
0.5455	18/33	36/66	0.5882	10/17	20/34	0.6316	.....	36/57
0.5472	.....	29/53	0.5882	.....	30/51	0.6327	31/49	31/49
0.5476	.....	23/42	0.5897	23/39	23/39	0.6333	.....	19/30
0.5484	17/31	34/62	0.5909	.....	39/66	0.6341	26/41	26/41
0.5490	.....	28/51	0.5918	29/49	29/49	0.6364	21/33	42/66
0.5500	11/20	.....	0.5926	16/27	32/54	0.6379	.....	37/58
0.5510	27/49	27/49	0.5932	.....	35/59	0.6383	30/47	30/47
0.5517	16/29	32/58	0.5946	22/37	22/37	0.6400	.....	16/25
0.5526	.....	21/38	0.5952	.....	25/42	0.6410	25/39	25/39
0.5532	26/47	26/47	0.5957	28/47	28/47	0.6415	.....	34/53
0.5556	10/18	.....	0.5965	.....	34/57	0.6429	.....	18/28
0.5556	15/27	30/54	0.5968	.....	37/62	0.6429	.....	27/42
0.5581	24/43	24/43	0.6000	9/15	15/25	0.6441	.....	38/59
0.5588	.....	19/34	0.6000	12/20	18/30	0.6452	20/31	40/62
0.5593	.....	33/59	0.6034	.....	35/58	0.6471	11/17	22/34
0.5600	.....	14/25	0.6038	.....	32/53	0.6471	.....	33/51
0.5606	.....	37/66	0.6047	26/43	26/43	0.6481	.....	35/54
0.5610	23/41	23/41	0.6053	.....	23/38	0.6486	24/37	24/37
0.5614	.....	32/57	0.6061	20/33	40/66	0.6491	.....	37/57
0.5625	9/16	.....	0.6071	.....	17/28	0.6500	13/20	.....
0.5641	22/39	22/39	0.6078	.....	31/51	0.6512	28/43	28/43
0.5645	.....	35/62	0.6087	14/23	28/46	0.6515	.....	43/66
0.5652	13/23	26/46	0.6098	25/41	25/41	0.6522	15/23	30/46
0.5660	.....	30/53	0.6102	.....	36/59	0.6531	32/49	32/49
0.5667	.....	17/30	0.6111	11/18	33/54	0.6552	19/29	38/58
0.5676	21/37	21/37	0.6122	30/49	30/49	0.6579	.....	25/38
0.5686	.....	29/51	0.6129	19/31	38/62	0.6585	27/41	27/41
0.5690	.....	33/58	0.6140	.....	35/57	0.6596	31/47	31/47
0.5714	.....	16/28	0.6154	24/39	24/39	0.6604	.....	35/53
0.5714	12/21	24/42	0.6170	29/47	29/47	0.6610	.....	39/59
0.5714	28/49	28/49	0.6176	.....	21/34	0.6613	.....	41/62
0.5741	.....	31/54	0.6190	13/21	26/42	0.6667	12/18	16/24
0.5745	27/47	27/47	0.6207	18/29	36/58	0.6667	10/15	20/30
0.5758	19/33	38/66	0.6212	.....	41/66	0.6667	26/39	26/39
0.5763	.....	34/59	0.6216	23/37	23/37	0.6667	14/21	28/42
0.5789	11/19	22/38	0.6226	.....	33/53	0.6667	.....	34/51

* See explanatory note below Table 8.

Accurate Angular Indexing Movements — 6 *

Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *
0.6667	18/27	36/54	0.7119	.....	42/59	0.7576	25/33	50/66
0.6667	.....	38/57	0.7121	.....	47/66	0.7581	.....	47/62
0.6667	22/33	44/66	0.7143	.....	20/28	0.7586	22/29	44/58
0.6724	.....	39/58	0.7143	15/21	30/42	0.7593	.....	41/54
0.6735	33/49	33/49	0.7143	35/49	35/49	0.7600	.....	19/25
0.6739	.....	31/46	0.7170	.....	38/53	0.7609	.....	35/46
0.6744	29/43	29/43	0.7174	.....	33/46	0.7619	16/21	32/42
0.6757	25/37	25/37	0.7179	28/39	28/39	0.7627	.....	45/59
0.6765	.....	23/34	0.7193	.....	41/57	0.7632	.....	29/38
0.6774	21/31	42/62	0.7200	.....	18/25	0.7647	13/17	26/34
0.6780	.....	40/59	0.7209	31/43	31/43	0.7647	.....	39/51
0.6786	.....	19/28	0.7222	13/18	39/54	0.7660	36/47	36/47
0.6792	.....	36/53	0.7234	34/47	34/47	0.7667	.....	23/30
0.6800	.....	17/25	0.7241	21/29	42/58	0.7674	33/43	33/43
0.6809	32/47	32/47	0.7255	.....	37/51	0.7692	30/39	30/39
0.6818	.....	45/66	0.7258	.....	45/62	0.7719	.....	44/57
0.6829	28/41	28/41	0.7273	24/33	48/66	0.7727	.....	51/66
0.6842	13/19	26/38	0.7288	.....	43/59	0.7736	.....	41/53
0.6842	.....	39/57	0.7297	27/37	27/37	0.7742	24/31	48/62
0.6852	.....	37/54	0.7317	30/41	30/41	0.7755	38/49	38/49
0.6863	.....	35/51	0.7333	11/15	22/30	0.7759	.....	45/58
0.6875	11/16	.....	0.7347	36/49	36/49	0.7778	14/18	.....
0.6897	20/29	40/58	0.7353	.....	25/34	0.7778	21/27	42/54
0.6905	.....	29/42	0.7358	.....	39/53	0.7797	.....	46/59
0.6923	27/39	27/39	0.7368	14/19	28/38	0.7805	32/41	32/41
0.6935	.....	43/62	0.7368	.....	42/57	0.7826	18/23	36/46
0.6939	34/49	34/49	0.7381	.....	31/42	0.7838	29/37	29/37
0.6949	.....	41/59	0.7391	17/23	34/46	0.7843	.....	40/51
0.6957	16/23	32/46	0.7407	20/27	40/54	0.7857	.....	22/28
0.6970	23/33	46/66	0.7414	.....	43/58	0.7857	.....	33/42
0.6977	30/43	30/43	0.7419	23/31	46/62	0.7872	37/47	37/47
0.6981	.....	37/53	0.7424	.....	49/66	0.7879	26/33	52/66
0.7000	14/20	21/30	0.7436	29/39	29/39	0.7895	15/19	30/38
0.7018	.....	40/57	0.7442	32/43	32/43	0.7895	.....	45/57
0.7021	33/47	33/47	0.7447	35/47	35/47	0.7903	.....	49/62
0.7027	26/37	26/37	0.7451	.....	38/51	0.7907	34/43	34/43
0.7037	19/27	38/54	0.7458	.....	44/59	0.7917	.....	19/24
0.7059	12/17	24/34	0.7500	12/16	18/24	0.7925	.....	42/53
0.7059	.....	36/51	0.7500	15/20	21/28	0.7931	23/29	46/58
0.7069	.....	41/58	0.7544	.....	43/57	0.7941	.....	27/34
0.7073	29/41	29/41	0.7547	.....	40/53	0.7949	31/39	31/39
0.7083	.....	17/24	0.7551	37/49	37/49	0.7959	39/49	39/49
0.7097	22/31	44/62	0.7561	31/41	31/41	0.7963	.....	43/54
0.7105	.....	27/38	0.7568	28/37	28/37	0.7966	.....	47/59

* See explanatory note below Table 8.

Accurate Angular Indexing Movements — 7 *

Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *
0.8000	16/20	20/25	0.8431	.....	43/51	0.8871	.....	55/62
0.8000	12/15	24/30	0.8448	.....	49/58	0.8889	16/18	.....
0.8030	.....	53/66	0.8462	33/39	33/39	0.8889	24/27	48/54
0.8039	.....	41/51	0.8475	.....	50/59	0.8913	.....	41/46
0.8043	.....	37/46	0.8478	.....	39/46	0.8919	33/37	33/37
0.8049	33/41	33/41	0.8485	28/33	56/66	0.8929	.....	25/28
0.8065	25/31	50/62	0.8491	.....	45/53	0.8936	42/47	42/47
0.8070	.....	46/57	0.8500	17/20	.....	0.8939	.....	59/66
0.8085	38/47	38/47	0.8511	40/47	40/47	0.8947	17/19	34/38
0.8095	17/21	34/42	0.8519	23/27	46/54	0.8947	.....	51/57
0.8103	.....	47/58	0.8529	.....	29/34	0.8966	26/29	52/58
0.8108	30/37	30/37	0.8537	35/41	35/41	0.8974	35/39	35/39
0.8113	.....	43/53	0.8548	.....	53/62	0.8980	44/49	44/49
0.8125	13/16	.....	0.8571	.....	24/28	0.8983	.....	53/59
0.8136	.....	48/59	0.8571	18/21	36/42	0.9000	18/20	27/30
0.8140	.....	35/43	0.8571	42/49	42/49	0.9020	.....	46/51
0.8148	.....	44/54	0.8596	.....	49/57	0.9024	37/41	37/41
0.8158	.....	31/38	0.8605	37/43	37/43	0.9032	28/31	56/62
0.8163	40/49	40/49	0.8621	25/29	50/58	0.9048	19/21	38/42
0.8182	27/33	54/66	0.8627	.....	44/51	0.9057	.....	48/53
0.8205	32/39	32/39	0.8636	.....	57/66	0.9070	39/43	39/43
0.8214	.....	23/28	0.8644	.....	51/59	0.9074	.....	49/54
0.8226	.....	51/62	0.8649	32/37	32/37	0.9090	30/33	60/66
0.8235	14/17	28/34	0.8667	13/15	26/30	0.9118	.....	31/34
0.8235	.....	42/51	0.8679	.....	46/53	0.9123	.....	52/57
0.8246	.....	47/57	0.8684	.....	33/38	0.9130	21/23	42/46
0.8261	19/23	38/46	0.8696	20/23	40/46	0.9138	.....	53/58
0.8276	24/29	48/58	0.8704	.....	47/54	0.9149	43/47	43/47
0.8293	34/41	34/41	0.8710	27/31	54/62	0.9153	.....	54/59
0.8298	39/47	39/47	0.8718	34/39	34/39	0.9167	.....	22/24
0.8302	.....	44/53	0.8723	41/47	41/47	0.9184	45/49	45/49
0.8305	.....	49/59	0.8750	14/16	21/24	0.9189	34/37	34/37
0.8333	15/18	20/24	0.8772	.....	50/57	0.9194	.....	57/62
0.8333	.....	25/30	0.8776	43/49	43/49	0.9200	.....	23/25
0.8333	.....	35/42	0.8780	36/41	36/41	0.9211	.....	35/38
0.8333	.....	45/54	0.8788	29/33	58/66	0.9216	.....	47/51
0.8333	.....	55/66	0.8793	.....	51/58	0.9231	36/39	36/39
0.8367	41/49	41/49	0.8800	.....	22/25	0.9242	.....	61/66
0.8372	36/43	36/43	0.8810	.....	37/42	0.9245	.....	49/53
0.8378	31/37	31/37	0.8814	.....	52/59	0.9259	25/27	50/54
0.8387	26/31	52/62	0.8824	15/17	30/34	0.9268	38/41	38/41
0.8400	.....	21/25	0.8824	.....	45/51	0.9286	.....	26/28
0.8421	.....	32/38	0.8837	38/43	38/43	0.9286	.....	39/42
0.8421	16/19	48/57	0.8868	.....	47/53	0.9298	.....	53/57

* See explanatory note below Table 8.

Accurate Angular Indexing Movements — 8 *

Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *	Fractional Indexing Movement	B. & S., Becker, Hendey, K. & T. and Rockford	Cincinnati and LeBlond *
0.9302	40/43	40/43	0.9500	19/20	.....	0.9697	32/33	64/66
0.9310	27/29	54/58	0.9512	39/41	39/41	0.9706	.....	33/34
0.9322	.....	55/59	0.9516	.....	59/62	0.9730	36/37	36/37
0.9333	14/15	28/30	0.9524	20/21	40/42	0.9737	.....	37/38
0.9348	.....	43/46	0.9535	41/43	41/43	0.9744	38/39	38/39
0.9355	29/31	58/62	0.9545	.....	63/66	0.9756	40/41	40/41
0.9362	44/47	44/47	0.9565	22/23	44/46	0.9762	.....	41/42
0.9375	15/16	.....	0.9574	45/47	45/47	0.9767	42/43	42/43
0.9388	46/49	46/49	0.9583	.....	23/24	0.9783	.....	45/46
0.9394	31/33	62/66	0.9592	47/49	47/49	0.9787	46/47	46/47
0.9412	16/17	32/34	0.9600	.....	24/25	0.9796	48/49	48/49
0.9412	.....	48/51	0.9608	.....	49/51	0.9804	.....	50/51
0.9434	.....	50/53	0.9623	.....	51/53	0.9811	.....	52/53
0.9444	17/18	51/54	0.9630	26/27	52/54	0.9815	.....	53/54
0.9459	35/37	35/37	0.9643	.....	27/28	0.9825	.....	56/57
0.9474	18/19	36/38	0.9649	.....	55/57	0.9828	.....	57/58
0.9474	.....	54/57	0.9655	28/29	56/58	0.9831	.....	58/59
0.9483	.....	55/58	0.9661	.....	57/59	0.9839	.....	61/62
0.9487	37/39	37/39	0.9667	.....	29/30	0.9848	.....	65/66
0.9492	.....	56/59	0.9677	30/31	60/62	......	.....	.....

* The foregoing tables may be used when indexing for angles in degrees, minutes or seconds. The tables are used as follows: Reduce the angle to seconds and divide the value thus obtained by 32,400. The quotient gives the number of complete turns and decimal fraction of a turn required. Then find the decimal (or nearest decimal) to this decimal fraction in the tables. Opposite this decimal will be found the fractional number indicating the indexing movement.

Example: — Assume that an angle of 10 degrees, 32 minutes, 12 seconds is to be indexed. Then, $10° 32' 12'' = 37,932$ seconds and $37,932 \div 32,400 = 1.1707$; therefore this indexing can be made by one complete turn and 0.1707 part of a turn. The second table shows that 0.1707 part of a turn is obtained by moving 7 holes in the 41-hole circle.

Example: — Two slots are to be milled in the edge of a disk and the angle between their center-lines is 58 degrees 51 minutes and 53 seconds. Determine the indexing movement.

The angle $58° 51' 53''$ reduced to seconds = 211,913 seconds and $211,913 \div 32,400 = 6.5405$; therefore this indexing movement requires six complete turns and 0.5405 part of a turn. The fifth table shows that 0.5405 part of a turn is obtained by moving 20 holes in the 37-hole circle.

The number of holes in the index circles of the indexing-heads made by the Brown & Sharpe Mfg. Co., Becker Milling Machine Co., Hendey Machine Co., Kearney & Trecker Co., and the Rockford Milling Machine Co. are the same. The index circles of the Cincinnati Milling Machine Co. differ from these; hence, a separate column is given in the table for the "Cincinnati" index-head. The R. K. LeBlond Machine Tool Co.'s dividing head has the same index circles as that of the Cincinnati Milling Machine Co., except that the former does not have the 24-, 25-, 28-, and 30-hole circles, but has, instead, 36-, 48-, and 56-hole circles. The movements in the 24- and 28-hole circles of the Cincinnati index-head may be made on the LeBlond index-head by taking double the number of holes in the 48-hole and 56-hole circles, respectively. In this way, the table can be used for practically all movements with LeBlond milling machines.

Block or Multiple Indexing for Gear Cutting

Teeth to be Cut	Number Indexed at Once	First Driver	First Follower	Second Driver	Second Follower	Turns of Locking Disk	Teeth to be Cut	Number Indexed at Once	First Driver	First Follower	Second Driver	Second Follower	Turns of Locking Disk
25	4	100	50	72	30	4	77	4	100	70	96	44	2
26	3	100	50	90	52	4	78	5	100	30	90	78	2
27	2	100	50	60	54	4	80	3	100	50	90	80	2
28	3	100	50	90	56	4	81	7	100	30	84	52	2
29	3	100	50	90	58	4	82	5	100	30	90	82	2
30	1	100	50	60	60	2	84	5	100	30	90	84	2
31	3	100	50	90	62	4	85	4	100	50	96	68	2
32	3	100	50	90	64	4	86	5	100	30	90	86	2
33	4	100	50	80	44	4	87	7	100	30	84	58	2
34	3	100	50	90	68	4	88	5	100	30	90	88	2
35	4	100	50	96	56	4	90	7	100	30	70	50	2
36	5	100	48	80	40	4	91	3	100	70	72	52	2
37	5	100	30	90	74	4	92	5	100	30	90	92	2
38	5	100	30	90	76	4	93	7	100	30	84	62	2
39	5	100	30	90	78	4	94	5	100	30	90	94	2
40	3	100	50	90	80	4	95	4	100	50	96	76	2
41	5	100	30	90	82	4	96	5	100	30	90	96	2
42	5	100	30	90	84	4	98	5	100	30	90	98	2
43	5	100	30	90	86	4	99	10	100	30	80	44	2
44	5	100	30	90	88	4	100	7	100	50	84	40	2
45	7	100	50	70	30	4	102	5	100	30	60	68	2
46	5	100	30	90	92	4	104	5	100	60	90	52	2
47	5	100	30	90	94	4	105	4	100	70	96	60	2
48	5	100	30	90	96	4	108	7	100	30	70	60	2
49	5	100	30	90	98	4	110	7	100	50	84	44	2
50	7	100	50	84	40	4	111	5	100	74	80	40	2
51	4	100	30	96	68	2	112	5	100	60	90	56	2
52	5	100	30	90	52	2	114	7	100	30	84	76	2
54	5	100	30	90	54	2	115	8	100	50	96	46	2
55	4	100	50	96	44	2	116	5	100	60	90	58	2
56	5	100	30	90	56	2	117	8	100	30	96	78	2
57	4	100	30	96	76	2	119	3	100	70	72	68	2
58	5	100	30	90	58	2	120	7	100	50	70	40	2
60	7	100	30	84	40	2	121	4	60	66	96	44	2
62	5	100	30	90	62	2	123	7	100	30	84	82	2
63	5	100	30	80	56	2	124	5	100	60	90	62	2
64	5	100	30	90	64	2	125	7	100	50	84	50	2
65	4	100	50	96	52	2	126	5	100	50	50	42	2
66	5	100	44	80	40	2	128	5	100	60	90	64	2
67	5	100	30	90	67	2	129	7	100	30	84	86	2
68	5	100	30	90	68	2	130	7	100	50	84	52	2
69	5	100	46	80	40	2	132	5	100	88	80	40	2
70	3	100	50	90	70	2	133	4	100	70	96	76	2
72	5	100	30	90	72	2	134	5	100	60	90	67	2
74	5	100	30	90	74	2	135	7	100	50	84	54	2
75	7	100	30	84	50	2	136	5	100	60	90	68	2
76	5	100	30	90	76	2	138	5	100	92	80	40	2

Block or Multiple Indexing for Gear Cutting

Teeth to be Cut	Number Indexed at Once	First Driver	First Follower	Second Driver	Second Follower	Turns of Locking Disk	Teeth to be Cut	Number Indexed at Once	First Driver	First Follower	Second Driver	Second Follower	Turns of Locking Disk
140	3	50	50	90	70	2	170	7	100	50	84	68	2
141	5	100	94	80	40	2	171	5	70	42	80	76	2
143	6	90	66	96	52	2	172	5	100	60	90	86	2
144	5	100	60	90	72	2	174	7	100	60	84	58	2
145	6	100	50	72	58	2	175	8	100	50	96	70	2
147	5	100	98	80	40	2	176	5	100	60	90	88	2
148	5	100	60	90	74	2	180	7	100	60	70	50	2
150	7	100	60	84	50	2	182	9	90	56	96	52	2
152	5	100	60	90	76	2	184	5	100	60	90	92	2
153	5	100	68	80	60	2	185	6	100	50	72	74	2
154	5	100	56	72	66	2	186	7	100	60	84	62	2
155	6	100	50	72	62	2	187	5	100	44	48	68	2
156	5	100	60	90	78	2	188	5	100	60	90	94	2
160	7	100	50	84	64	2	189	5	100	60	80	84	2
161	5	100	70	60	46	2	190	7	100	50	84	76	2
162	7	100	60	84	52	2	192	5	100	60	90	96	2
164	5	100	60	90	82	2	195	7	100	50	84	78	2
165	7	100	50	84	66	2	196	5	100	60	90	98	2
168	5	100	60	90	84	2	198	7	100	50	70	66	2
169	6	96	52	90	78	2	200	7	60	60	84	40	2

Block or Multiple Indexing for Gear Cutting. — With the block system of indexing, a number of teeth are indexed at one time, instead of cutting the teeth consecutively, and the gear is revolved several times before the teeth are all finished. For example, when cutting a gear having 25 teeth, the indexing mechanism is geared to index four teeth at once (see table) and the first time around, six widely separated tooth spaces are cut. The second time around, the cutter is one tooth behind the spaces previously milled. On the third indexing, the cutter has dropped back another tooth, thus finishing the gear (in this case) by indexing it around four times. The various combinations of change gears to use for block or multiple indexing are given in the accompanying table. The advantage claimed for block indexing is that the heat generated by the cutter (especially when cutting cast-iron gears of coarse pitch) is distributed more evenly about the rim and dissipated to a greater extent, thus avoiding distortion due to local heating and permitting higher speeds and feeds. The table given is intended for use with Brown & Sharpe automatic gear-cutting machines, but the gears for any other machine equipped with a similar indexing mechanism can be calculated. Assume, for example, that a gear cutter requires the following change gears for indexing a certain number of teeth: Driving gears having 20 and 30 teeth, respectively, and driven gears having 50 and 60 teeth. Then if it is desired to cut, say, every fifth tooth, multiply the fractions $\frac{20}{60}$ and $\frac{30}{50}$ by 5. Then, $\frac{20}{60} \times \frac{30}{50} \times \frac{5}{1} = \frac{1}{1}$. In this particular instance, then, the blank could be divided so that every fifth space would be cut, by using gears of equal size. The number of teeth in the gear, in any case, must not be a multiple of the number of teeth indexed.

Indexing Movements for 60-Tooth Worm-Wheel Dividing Head

Divi-sions	Index Circle	No. of Turns	No. of Holes	Divi-sions	Index Circle	No. of Turns	No. of Holes	Divi-sions	Index Circle	No. of Holes	Divi-sions	Index Circle	No. of Holes
2	Any	30	..	50	60	1	12	98	49	30	146	73	30
3	Any	20	..	51	17	1	3	99	33	20	147	49	20
4	Any	15	..	52	26	1	4	100	60	36	148	37	15
5	Any	12	..	53	53	1	7	101	101	60	149	149	60
6	Any	10	..	54	27	1	3	102	17	10	150	60	24
7	21	8	12	55	33	1	3	103	103	60	151	151	60
8	26	7	13	56	28	1	2	104	26	15	152	76	30
9	21	6	14	57	19	1	1	105	21	12	153	51	20
10	Any	6	..	58	29	1	1	106	53	30	154	77	30
11	33	5	15	59	59	1	1	107	107	60	155	31	12
12	Any	5	..	60	Any	1	..	108	27	15	156	26	10
13	26	4	16	61	61	..	60	109	109	60	157	157	60
14	21	4	6	62	31	..	30	110	33	18	158	79	30
15	Any	4	..	63	21	..	20	111	37	20	159	53	20
16	28	3	21	64	32	..	32	112	28	15	160	32	12
17	17	3	9	65	26	..	24	113	113	60	161	161	60
18	21	3	7	66	33	..	30	114	19	10	162	27	10
19	19	3	3	67	67	..	60	115	23	12	163	163	60
20	Any	3	..	68	17	..	15	116	29	15	164	41	15
21	21	2	18	69	23	..	20	117	39	20	165	33	12
22	33	2	24	70	21	..	18	118	59	30	166	83	30
23	23	2	14	71	71	..	60	119	119	60	167	167	60
24	26	2	13	72	60	..	50	120	26	13	168	28	10
25	60	2	24	73	73	..	60	121	121	60	169	169	60
26	26	2	8	74	37	..	30	122	61	30	170	17	6
27	27	2	6	75	60	..	48	123	41	20	171	57	20
28	21	2	3	76	19	..	15	124	31	15	172	43	15
29	29	2	2	77	77	..	60	125	100	48	173	173	60
30	Any	2	..	78	26	..	20	126	21	10	174	29	10
31	31	1	29	79	79	..	60	127	127	60	175	35	12
32	32	1	28	80	28	..	21	128	32	15	176	44	15
33	33	1	27	81	27	..	20	129	43	20	177	59	20
34	17	1	13	82	41	..	30	130	26	12	178	89	30
35	21	1	15	83	83	..	60	131	131	60	179	179	60
36	21	1	14	84	21	..	15	132	33	15	180	21	7
37	37	1	23	85	17	..	12	133	133	60	181	181	60
38	19	1	11	86	43	..	30	134	67	30	182	91	30
39	26	1	14	87	29	..	20	135	27	12	183	61	20
40	26	1	13	88	44	..	30	136	68	30	184	46	15
41	41	1	19	89	89	..	60	137	137	60	185	37	12
42	21	1	9	90	21	..	14	138	23	10	186	31	10
43	43	1	17	91	91	..	60	139	139	60	187	187	60
44	33	1	12	92	23	..	15	140	21	9	188	47	15
45	21	1	7	93	31	..	20	141	47	20	189	63	20
46	23	1	7	94	47	..	30	142	71	30	190	19	6
47	47	1	13	95	19	..	12	143	143	60	191	191	60
48	28	1	7	96	32	..	20	144	60	25	192	32	10
49	49	1	11	97	97	..	60	145	29	12	193	193	60

Indexing Movements for 60-Tooth Worm-Wheel Dividing Head

Divisions	Index Circle	No. of Holes	Divisions	Index Circle	No. of Holes	Divisions	Index Circle	No. of Holes	Divisions	Index Circle	No. of Holes
194	97	30	236	59	15	278	139	30	320	32	6
195	39	12	237	79	20	279	93	20	321	107	20
196	49	15	238	119	30	280	28	6	322	161	30
197	197	60	239	239	60	281	281	60	323	323	60
198	33	10	240	24	6	282	47	10	324	27	5
199	199	60	241	241	60	283	283	60	325	65	12
200	60	18	242	121	30	284	71	15	326	163	30
201	67	20	243	81	20	285	19	4	327	109	20
202	101	30	244	61	15	286	143	30	328	82	15
203	203	60	245	49	12	287	287	60	329	329	60
204	17	5	246	41	10	288	24	5	330	33	6
205	41	12	247	247	60	289	289	60	331	331	60
206	103	30	248	62	15	290	29	6	332	83	15
207	69	20	249	83	20	291	97	20	333	111	20
208	52	15	250	100	24	292	73	15	334	167	30
209	209	60	251	251	60	293	293	60	335	67	12
210	21	6	252	21	5	294	39	10	336	28	5
211	211	60	253	253	60	295	59	12	337	337	60
212	53	15	254	127	30	296	74	15	338	169	30
213	71	20	255	17	4	297	99	20	339	113	20
214	107	30	256	64	15	298	149	30	340	17	3
215	43	12	257	257	60	299	299	60	341	341	60
216	18	5	258	43	10	300	60	12	342	57	10
217	217	60	259	259	60	301	301	60	343	343	60
218	109	30	260	26	6	302	151	30	344	86	15
219	73	20	261	87	20	303	101	20	345	23	4
220	33	9	262	131	30	304	76	15	346	173	30
221	221	60	263	263	60	305	61	12	347	347	60
222	37	10	264	44	10	306	51	10	348	29	5
223	223	60	265	53	12	307	307	60	349	349	60
224	56	15	266	133	30	308	77	15	350	35	6
225	60	16	267	89	20	309	103	20	351	117	20
226	113	30	268	67	15	310	31	6	352	88	15
227	227	60	269	269	60	311	311	60	353	353	60
228	19	5	270	27	6	312	26	5	354	59	10
229	229	60	271	271	60	313	313	60	355	71	12
230	23	6	272	68	15	314	157	30	356	89	15
231	77	20	273	91	20	315	21	4	357	119	20
232	58	15	274	137	30	316	79	15	358	179	30
233	233	60	275	55	12	317	317	60	359	359	60
234	39	16	276	23	5	318	53	10	360	60	10
235	47	12	277	277	60	319	319	60	....	....	...

The dividing heads of all modern milling machines built in this country have 40-tooth worm-wheels (40 turns of the index crank being required for one revolution of the spindle), but formerly some dividing heads were equipped with 60-tooth worm-wheels, and many of these are still in use.

Compound Indexing *

No. of Divisions	Indexing Movements	No. of Times Around	No. of Divisions	Indexing Movements	No. of Times Around	No. of Divisions	Indexing Movements	No. of Times Around
51	8 41/47 − 12/49	11	133	3 23/29 − 16/33	11	198*	3/27 + 3/33	...
53	6 43/47 − 6/49	9	134	3 27/47 + 15/49	13	199	2 12/41 − 4/49	11
57	4 40/47 + 3/49	7	137	3 17/43 − 9/49	11	201	2 18/47 + 10/49	13
59	7 10/47 + 12/49	11	138*	11/33 − 1/23	...	202	3 10/41 + 6/49	17
61	3 42/47 + 2/49	6	139	2 25/37 + 24/49	11	203	1 23/39 + 9/49	9
63	4 19/29 + 14/33	8	141	1 32/39 + 22/49	8	204	2 20/41 + 3/49	13
67	2 27/41 + 16/49	5	142	4 1/47 + 10/49	15	206	2 34/39 + 2/49	15
69*	21/23 − 11/33	...	143	1 36/47 − 18/49	5	207	3 8/41 − 24/49	14
71	3 34/41 − 22/49	6	146	2 3/37 − 8/49	7	208	1 19/47 + 16/49	9
73	6 28/47 − 1/49	12	147*	13/39 − 3/49	...	209	8/49 + 9/41	2
77*	9/21 + 3/33	...	149	3 5/43 − 8/49	11	211	1 28/39 + 18/49	11
79	2 42/43 + 3/49	6	151	1 42/43 − 6/49	7	212	3 4/47 + 6/49	17
81	5 5/41 − 9/49	10	153	2 45/47 − 4/49	11	213	1 18/39 + 2/49	8
83	3 45/47 − 5/49	8	154*	8/21 − 4/33	...	214	3 9/47 − 19/49	15
87*	23/29 − 11/33	...	157	2 23/31 + 2/33	11	217	2 3/43 + 16/49	13
89	3 28/39 − 6/49	8	158	5 5/43 − 15/49	19	218	1 22/47 − 9/49	7
91*	6/39 + 14/49	...	159	2 7/37 + 16/49	10	219	3 29/43 − 10/49	19
93*	3/31 + 11/33	...	161	2 10/39 − 1/49	9	221	1 5/47 − 1/49	6
96*	3/18 + 5/20	...	162	1 30/39 − 2/49	7	222	2 8/43 − 10/49	11
97	4 27/41 − 6/49	11	163	3 7/37 − 24/49	11	223	2 26/43 + 13/49	16
99*	15/27 − 5/33	...	166	1 19/43 + 12/49	7	224	2 6/23 + 2/33	13
101	4 32/43 − 19/49	11	167	2 1/29 + 4/33	9	225*	5/18 − 2/20	...
102	4 17/43 − 4/49	11	169	1 32/37 + 13/49	9	226	1 38/39 + 16/49	13
103	1 8/43 + 18/49	4	171	1 29/47 + 1/49	7	227	3 3/43 + 5/49	18
106	2 38/41 + 23/49	9	173	1 7/43 + 11/49	6	228	2 8/41 − 13/49	11
107	2 21/31 − 2/33	7	174*	11/33 − 3/29	...	229	2 19/41 − 18/49	12
109	2 19/39 + 4/49	7	175	1 4/31 + 8/33	6	231*	3/21 + 1/33	...
111	3 29/47 + 17/49	11	176	1 14/43 + 13/49	7	233	1 36/47 + 6/49	11
112	4 10/31 − 13/33	11	177	2 19/47 + 4/49	11	234	2 21/29 + 6/33	17
113	3 26/47 − 18/49	9	178	3 28/47 + 11/49	17	236	2 30/43 + 9/49	17
114	1 35/37 + 25/49	7	179	2 34/47 − 13/49	11	237	2 12/47 − 3/49	13
117	7 1/47 − 9/49	20	181	2 8/43 + 12/49	11	238	2 3/31 + 14/33	15
118	1 8/39 + 24/49	5	182*	3/39 + 7/49	...	239	1 23/43 + 15/49	11
119	3 4/23 − 16/33	8	183	1 24/41 + 8/49	8	241	1 1/41 + 23/49	9
121	1 14/47 − 15/49	3	186*	17/31 − 11/33	...	242	2 23/41 − 4/49	15
122	3 41/43 − 17/49	11	187	1 20/47 + 14/49	8	243	1 29/41 − 3/49	10
123	1 12/43 + 17/49	5	189	2 26/41 − 15/49	11	244	2 15/31 + 10/33	17
125	2 33/41 − 12/49	8	191	1 38/47 + 14/49	10	246	1 6/43 − 16/49	5
126	3 16/19 − 7/20	11	192	2 22/41 − 12/49	11	247	2 15/43 − 4/49	14
127	2 23/39 + 12/49	9	193	1 5/37 − 15/49	4	249	3 4/43 − 2/49	19
129	5 24/41 + 15/49	19	194	2 22/37 − 16/49	11	250	2 9/37 − 8/49	13
131	2 40/43 + 21/49	11	197	1 39/43 + 16/49	11	...	....	...

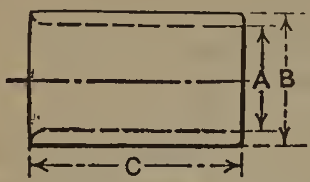
* The indexing movements are exact for the divisions marked with an asterisk (*); the errors of the other divisions are so slight as to be negligible for all ordinary classes of work, such as gear-cutting, etc.

JIGS AND FIXTURES

Material for Jig Bushings. — Bushings are generally made of a good grade of tool steel to insure hardening at a fairly low temperature and to lessen the danger of fire cracking. They can also be made from machine steel, which will answer all practical purposes, provided the bushings are properly casehardened to a depth of about 1/16 inch. Sometimes bushings for guiding tools may be made of cast iron, but only when the cutting tool is of such a design that no cutting edges come within the bushing itself. For example, bushings used simply to support the smooth surface of a boring-bar or the shank of a reamer might, in some instances, be made of cast iron, but hardened steel bushings should always be used for guiding drills, reamers, taps, etc., when the cutting edges come in direct contact with the guiding surfaces. If the outside diameter of the bushing is very large, as compared with the diameter of the cutting tool, the cost of the bushing can sometimes be reduced by using an outer cast-iron body and inserting a hardened tool steel bushing. Occasionally a bushing having a large outside diameter is required as, for example, when a large counterbore must be used in a small hole, which makes it necessary to have a large opening in the jig body.

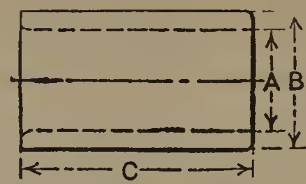
Dimensions of Jig Bushings. — There are no standard dimensions for jig bushings, as the sizes depend more or less upon the conditions under which the bushing is to be used. As a rule, the length of a stationary drill bushing is made twice the diameter of the hole, but for very small bushings (1/4 inch in diameter, or less) the length should be longer than twice the diameter. Inversely, for very large bushings, the length may be made somewhat less than twice the diameter. Table 1 gives the proportions of stationary drill bushings. The dimensions listed in this table will be found suitable except when special conditions make it necessary to deviate from ordinary practice. Table 2 gives dimensions for lining bushings. These are not intended to guide the drill or other tool directly, but to hold removable bushings which guide the cutting tools. The dimensions given in Tables 1 and 2 are for bushings made from either tool or machine steel. While, in some cases, it may be difficult to draw a distinct line between stationary and lining bushings, it may be said, in general, that the bushings in Table 1 are used when drilling holes directly either with a full size drill (when a smooth accurate hole is not required) or for guiding a close-fitting spotting drill, which is followed with a so-called "reamer drill" and then a finishing reamer. These bushings, therefore, are generally used when no tapping or counterboring is required. The lining bushings listed in Table 2 may be used to guide tools directly, or removable bushings may be inserted. Removable bushings are frequently employed for work which must be drilled, reamed and tapped, in which case there is one bushing for each of the cutting tools. They are also used when different parts of the same hole must be drilled to different diameters, or when the upper part of the hole must be counterbored or a lug faced off. For such work, each tool, of course, has its own guiding bushing. In Table 3 are given dimensions for removable bushings. The style illustrated in connection with this table is one that is commonly used. The bushing is turned down under the head to form a step, which can be used for prying the bushing out of the hole. A convenient method of preventing this style of bushing from turning is to file or mill a half-round slot in the periphery of the head and insert a pin or screw in the jig body. Table 4 gives the sizes of bushings for holes which are to be reamed with a rose chucking reamer after having first been drilled somewhat smaller than the finished diameter. The bushing shown to the extreme right is a lining bushing and is made of casehardened machine steel. The bushing in the center is for the drill and is made of either tool steel or machine steel. The bushing to the extreme left is for the rose chucking reamer and is made of cast iron. Sometimes removable

Table 1. Dimensions of Stationary Drill Bushings



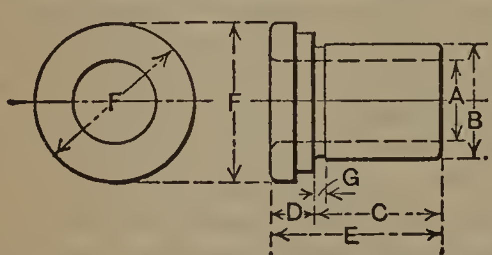
A	B	C	A	B	C
1/16	3/16	3/8	1 1/16	1 7/16	2
1/8	1/4	1/2	1 1/8	1 1/2	2 1/8
3/16	5/16	1/2	1 3/16	1 9/16	2 1/8
1/4	3/8	5/8	1 1/4	1 5/8	2 1/4
5/16	1/2	5/8	1 5/16	1 11/16	2 1/4
3/8	9/16	3/4	1 3/8	1 3/4	2 3/8
7/16	5/8	7/8	1 7/16	1 13/16	2 1/2
1/2	11/16	1	1 1/2	1 7/8	2 5/8
9/16	13/16	1 1/8	1 9/16	1 15/16	2 3/4
5/8	7/8	1 1/4	1 5/8	2	2 7/8
11/16	15/16	1 3/8	1 11/16	2 1/8	2 7/8
3/4	1	1 1/2	1 3/4	2 1/4	3
13/16	1 1/8	1 1/2	1 13/16	2 5/16	3 1/8
7/8	1 3/16	1 5/8	1 7/8	2 3/8	3 1/4
15/16	1 1/4	1 3/4	1 15/16	2 7/16	3 3/8
1	1 3/8	1 7/8	2	2 5/8	3 1/2

Table 2. Dimensions of Lining Bushings



A	B	C	A	B	C
5/16	1/2	1/2	1 9/16	1 15/16	2 1/8
3/8	9/16	1/2	1 5/8	2	2 1/8
7/16	5/8	5/8	1 3/4	2 1/4	2 1/4
9/16	13/16	5/8	1 13/16	2 5/16	2 1/4
5/8	7/8	3/4	1 7/8	2 3/8	2 3/8
11/16	15/16	7/8	1 15/16	2 7/16	2 1/2
3/4	1	1	2 1/8	2 5/8	2 5/8
13/16	1 1/8	1 1/8	2 3/16	2 11/16	2 3/4
15/16	1 1/4	1 1/4	2 1/4	2 3/4	2 7/8
1	1 3/8	1 3/8	2 5/16	2 7/8	2 7/8
1 1/16	1 7/16	1 1/2	2 3/8	3	3
1 1/8	1 1/2	1 1/2	2 9/16	3 3/16	3 1/8
1 1/4	1 5/8	1 5/8	2 5/8	3 1/4	3 1/4
1 5/16	1 11/16	1 3/4	2 11/16	3 5/16	3 3/8
1 3/8	1 3/4	1 7/8	2 3/4	3 3/8	3 1/2
1 7/16	1 13/16	2	....	...	...

Table 3. Dimensions of Removable Drill Bushings

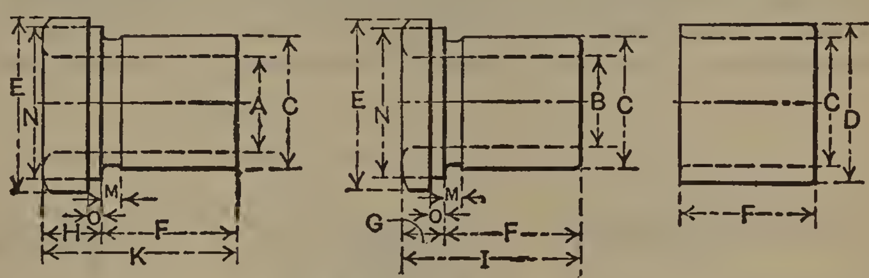


A	B	C	D	E	F	G
3/16	3/8	1/2	1/8	5/8	5/8	1/8
1/4	7/16	5/8	1/8	3/4	3/4	1/8
5/16	9/16	5/8	1/8	3/4	7/8	1/8
3/8	5/8	3/4	1/8	7/8	7/8	1/8
7/16	11/16	7/8	1/8	1	1	1/8
1/2	3/4	1	1/8	1 1/8	1	1/8
9/16	13/16	1 1/8	1/8	1 1/4	1 1/8	1/8
5/8	15/16	1 1/4	3/16	1 7/16	1 1/4	1/8
11/16	1	1 3/8	3/16	1 9/16	1 1/4	1/8
3/4	1 1/16	1 1/2	3/16	1 11/16	1 3/8	1/8
13/16	1 1/8	1 1/2	3/16	1 11/16	1 3/8	1/8

A	B	C	D	E	F	G
7/8	1 1/4	1 5/8	3/16	1 13/16	1 1/2	1/8
15/16	1 5/16	1 3/4	3/16	1 15/16	1 5/8	1/8
1	1 3/8	1 7/8	1/4	2 1/8	1 3/4	3/16
1 1/16	1 7/16	2	1/4	2 1/4	1 7/8	3/16
1 1/8	1 9/16	2 1/8	1/4	2 3/8	2	3/16
1 3/16	1 5/8	2 1/8	1/4	2 3/8	2	3/16
1 1/4	1 3/4	2 1/4	5/16	2 9/16	2 1/8	3/16
1 5/16	1 13/16	2 1/4	5/16	2 9/16	2 1/4	3/16
1 3/8	1 7/8	2 3/8	5/16	2 11/16	2 1/4	3/16
1 7/16	1 15/16	2 1/2	5/16	2 13/16	2 3/8	3/16
1 1/2	2 1/8	2 5/8	3/8	3	2 1/2	3/16
1 9/16	2 3/16	2 3/4	3/8	3 1/8	2 5/8	3/16
1 5/8	2 1/4	2 7/8	3/8	3 1/4	2 5/8	3/16
1 11/16	2 5/16	2 7/8	3/8	3 1/4	2 3/4	3/16
1 3/4	2 3/8	3	3/8	3 3/8	2 3/4	3/16
1 13/16	2 9/16	3 1/8	1/2	3 5/8	3	1/4
1 7/8	2 5/8	3 1/4	1/2	3 3/4	3 1/8	1/4
1 15/16	2 11/16	3 3/8	1/2	3 7/8	3 1/8	1/4
2	2 3/4	3 1/2	1/2	4	3 1/4	1/4

bushings are threaded on the outside and made to fit a tapped hole in the jig. The lower part of a screw bushing is usually turned straight and ground, in order to center the bushing in the hole in the jig, and the head is either knurled or milled hexagon for a wrench. Ordinarily, screw bushings are not only used to guide the cutting tools but also serve to locate and clamp the work.

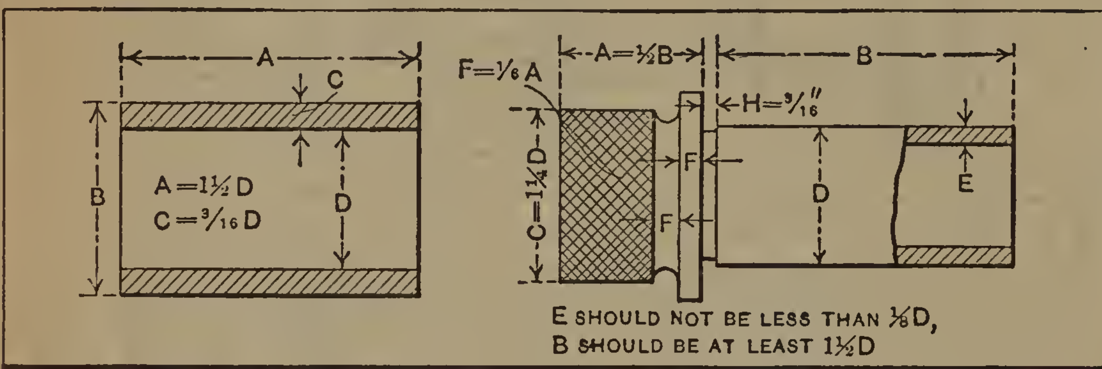
Table 4. Bushings for Holes Reamed with Rose Chucking Reamers



A	B	C	D	E	F	G	H	I	K	M	N	O
3/4	11/16	1 1/4	1 5/8	1 1/2	1 1/4	3/16	1/4	1 7/16	1 1/2	1/8	1 3/8	1/16
13/16	3/4	1 5/16	1 11/16	1 5/8	1 5/16	3/16	1/4	1 1/2	1 9/16	1/8	1 1/2	1/16
7/8	13/16	1 3/8	1 3/4	1 5/8	1 3/8	3/16	1/4	1 9/16	1 5/8	1/8	1 1/2	1/16
15/16	7/8	1 7/16	1 13/16	1 3/4	1 7/16	3/16	1/4	1 5/8	1 11/16	1/8	1 5/8	1/16
1	15/16	1 1/2	1 7/8	1 3/4	1 1/2	1/4	5/16	1 3/4	1 13/16	3/16	1 5/8	1/8
1 1/16	1	1 9/16	1 15/16	1 7/8	1 5/8	1/4	5/16	1 7/8	1 15/16	3/16	1 3/4	1/8
1 1/8	1 1/16	1 5/8	2	1 7/8	1 11/16	1/4	5/16	1 15/16	2	3/16	1 3/4	1/8
1 3/16	1 1/8	1 11/16	2 1/8	2	1 3/4	1/4	5/16	2	2 1/16	3/16	1 7/8	1/8
1 1/4	1 3/16	1 13/16	2 5/16	2 1/8	1 7/8	5/16	3/8	2 3/16	2 1/4	3/16	2	1/8
1 5/16	1 1/4	1 15/16	2 7/16	2 1/4	2	5/16	3/8	2 5/16	2 3/8	3/16	2 1/8	1/8
1 3/8	1 5/16	2	2 1/2	2 1/4	2 1/8	5/16	3/8	2 7/16	2 1/2	3/16	2 1/8	1/8
1 7/16	1 3/8	2 1/16	2 9/16	2 3/8	2 1/4	5/16	3/8	2 9/16	2 5/8	3/16	2 1/4	1/8
1 1/2	1 7/16	2 1/8	2 5/8	2 3/8	2 3/8	5/16	3/8	2 11/16	2 3/4	3/16	2 1/4	1/8
1 9/16	1 1/2	2 3/16	2 11/16	2 1/2	2 1/2	3/8	7/16	2 7/8	2 15/16	3/16	2 3/8	1/8
1 5/8	1 9/16	2 3/8	3	2 5/8	2 5/8	3/8	7/16	3	3 1/16	3/16	2 1/2	1/8
1 11/16	1 5/8	2 7/16	3 1/16	2 3/4	2 3/4	3/8	7/16	3 1/8	3 3/16	3/16	2 5/8	1/8
1 3/4	1 11/16	2 1/2	3 1/8	2 7/8	2 7/8	3/8	7/16	3 1/4	3 5/16	3/16	2 3/4	1/8
1 13/16	1 3/4	2 9/16	3 3/16	3	3	7/16	1/2	3 7/16	3 1/2	1/4	2 13/16	3/16
1 7/8	1 13/16	2 5/8	3 1/4	3	3 1/8	7/16	1/2	3 9/16	3 5/8	1/4	2 13/16	3/16
1 15/16	1 7/8	2 11/16	3 5/16	3 1/8	3 1/4	7/16	1/2	3 11/16	3 3/4	1/4	2 15/16	3/16
2	1 15/16	2 3/4	3 3/8	3 1/8	3 3/8	7/16	1/2	3 13/16	3 7/8	1/4	2 15/16	3/16
2 1/16	2	2 15/16	3 9/16	3 1/2	3 1/2	1/2	5/8	4	4 1/8	1/4	3 1/4	3/16
2 1/8	2 1/16	3	3 5/8	3 1/2	3 1/2	1/2	5/8	4	4 1/8	1/4	3 1/4	3/16
2 3/16	2 1/8	3 1/16	3 13/16	3 5/8	3 5/8	1/2	5/8	4 1/8	4 1/4	1/4	3 3/8	3/16
2 1/4	2 3/16	3 1/8	3 7/8	3 5/8	3 3/4	1/2	5/8	4 1/4	4 3/8	1/4	3 3/8	3/16
2 3/8	2 5/16	3 3/8	4 1/8	3 7/8	3 7/8	1/2	5/8	4 3/8	4 1/2	1/4	3 5/8	3/16
2 1/2	2 7/16	3 1/2	4 1/4	4	4	1/2	5/8	4 1/2	4 5/8	1/4	3 3/4	3/16
2 5/8	2 9/16	3 5/8	4 3/8	4 1/8	4 1/8	1/2	5/8	4 5/8	4 3/4	1/4	3 7/8	3/16
2 3/4	2 11/16	3 3/4	4 1/2	4 1/4	4 1/4	1/2	5/8	4 3/4	4 7/8	1/4	4	3/16

Hardening Jig Bushings. — When hardening bushings made of tool steel they should be brought to an even red heat in a clean fire; the heating should never be hurried. When bushings are heated quickly, they are apt to heat unevenly, which results in warping or distortion that makes it impossible to finish them to the required size. Gas furnaces are excellent for heating, but a clean charcoal fire will answer the purpose. As soon as the bushing has been brought to an even red heat, it should be dipped in water just warm enough to take off the chill. The bushing should then be heated to a "sizzling" heat, after which it is left in the air to cool.

Allowances for Grinding Jig Bushings. — In making allowances for grinding and lapping, many toolmakers leave so little stock that the bushing will not true up. On the other hand, the allowance is often too liberal, thus causing unnecessary labor. The allowances given in the table "Allowances for Grinding and Lapping



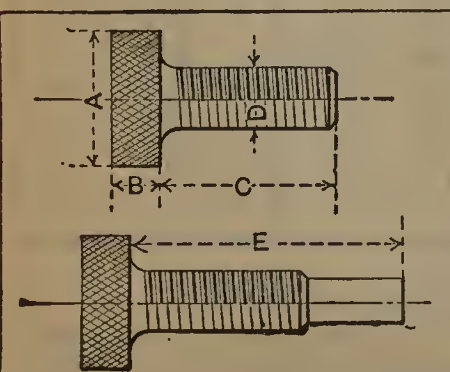
Jig Bushings" can be safely used when the bushings are made somewhere near the proportions given in the accompanying illustration. For extra long bushings more liberal allowances should be made.

Allowances for Grinding and Lapping Jig Bushings

Operation	Diameter of Bushings in Inches					
	½	1	1½	2	2½	3
A	0.008	0.010	0.013	0.016	0.020	0.025
B	0.0005	0.0005	0.0007	0.0008	0.0009	0.001
C	0.008	0.010	0.013	0.016	0.020	0.025
D	0.0005	0.0005	0.0007	0.0008	0.0009	0.001

A — Grind outside; *B* — Lap outside after grinding; *C* — Grind inside; *D* — Lap inside after grinding.

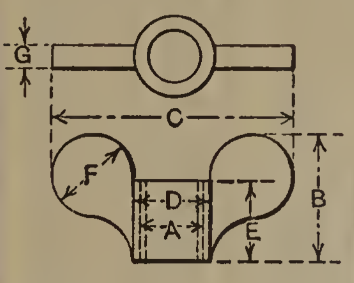
Knurled-Head Thumb-Screws



A	*B*	*C*	*D*	*E*
½	5/32	½	⅛	¾
9/16	5/32	⅝	5/32	15/16
⅝	3/16	¾	3/16	1⅛
11/16	7/32	⅞	¼	1 5/16
¾	¼	1	5/16	1½
⅞	5/16	1⅛	⅜	1¾

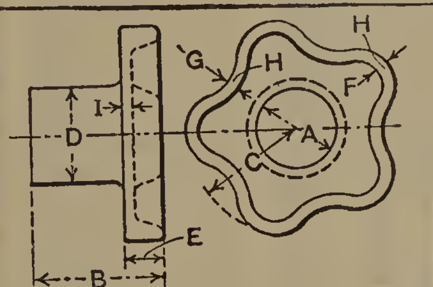
Wing-nuts for Jigs. — Star Handwheels. — Wing-nuts are used on hook bolts or swiveling eye-bolts, when a comparatively light pressure is required. The thumb- or wing-nut is preferable to a knurled nut, as it gives a better grip and makes it possible to tighten the bolt more firmly. The dimensions of an excellent design of handwheel for use on jigs, etc., are given in an accompanying table. These wheels have a rather long stem or hub which provides a good length of thread and brings the grip or handle far enough from the jig body to prevent the fingers or knuckles from striking it. The "star" design of handle also permits a good grip. By having the casting solid, these handwheels can be tapped out for any size thread, or a plain hole can be drilled when it is desired to attach the handles to round stock.

Dimensions of Wing or Thumb Nuts



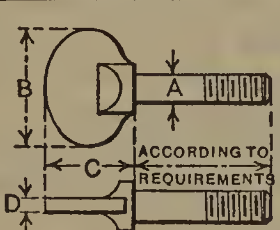
A	B	C	D	E	F	G
3/16	5/8	1 1/8	5/16	3/8	7/16	1/8
1/4	3/4	1 3/8	15/32	1/2	17/32	5/32
5/16	3/4	1 3/8	15/32	1/2	17/32	5/32
3/8	13/16	1 11/16	17/32	9/16	5/8	5/32
7/16	7/8	2	21/32	5/8	11/16	3/16
1/2	1 1/16	2 1/4	3/4	13/16	7/8	3/16

Star Handwheels for Jigs



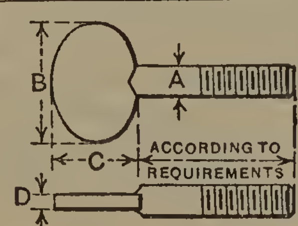
A	B	C	D	E	F	G	H	I
3/4	1 3/4	1	1	3/8	3/16	5/16	1/8	1/8
1	1 7/8	1 1/4	1 1/8	7/16	5/16	7/16	1/8	1/8
1 1/8	2	1 1/2	1 3/8	1/2	3/8	9/16	3/16	3/16
1 1/2	2 1/8	2	1 5/8	9/16	1/2	11/16	3/16	3/16
1 5/8	2 1/4	2 1/2	1 3/4	5/8	7/8	7/8	3/16	1/4

Shoulder Thumb-Screws



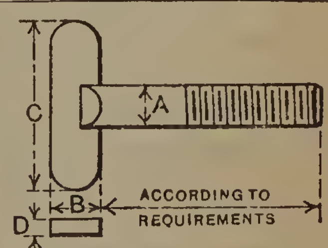
A	B	C	D
3/16	9/16	1/2	3/32
1/4	1	11/16	1/8
5/16	1 1/8	3/4	5/32
3/8	1 1/4	13/16	5/32
7/16	1 1/2	15/16	3/16
1/2	1 5/8	1 1/8	3/16

Regular Thumb-Screws



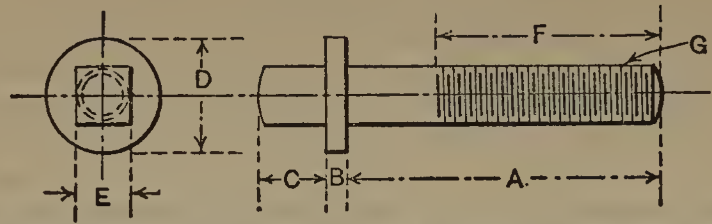
A	B	C	D
3/16	3/4	5/8	3/32
1/4	15/16	3/4	1/8
5/16	1 1/8	7/8	5/32
3/8	1 1/4	15/16	5/32
7/16	1 1/2	1 1/16	3/16
1/2	1 5/8	1 3/16	3/16

Thumb-Screws with Wide Grip



A	B	C	D
3/16	1/4	7/8	3/32
1/4	5/16	1	1/8
3/8	7/16	1 1/2	5/32
7/16	1/2	1 3/4	3/16
1/2	9/16	2	3/16
5/8	11/16	2 1/2	7/32

Collar-head Screws



A	*B*	*C*	*D*	*E*	*F*	Diameter *G* and Threads per Inch
5/8	1/16	3/16	5/16	3/16	1/2	No. 10 — 32
7/8	1/16	3/16	5/16	3/16	5/8	No. 10 — 32
1 1/8	1/16	3/16	5/16	3/16	7/8	No. 10 — 32
1 3/8	1/16	3/16	5/16	3/16	1	No. 10 — 32
1 5/8	1/16	3/16	5/16	3/16	1 1/4	No. 10 — 32
7/8	3/32	1/4	7/16	1/4	5/8	No. 14 — 24
1 1/8	3/32	1/4	7/16	1/4	7/8	No. 14 — 24
1 3/8	3/32	1/4	7/16	1/4	1	No. 14 — 24
1 5/8	3/32	1/4	7/16	1/4	1 1/4	No. 14 — 24
1 7/8	3/32	1/4	7/16	1/4	1 7/16	No. 14 — 24
7/8	1/8	5/16	9/16	5/16	5/8	5/16 — 18
1 1/4	1/8	5/16	9/16	5/16	1	5/16 — 18
1 5/8	1/8	5/16	9/16	5/16	1 1/4	5/16 — 18
2	1/8	5/16	9/16	5/16	1 1/2	5/16 — 18
1	1/8	3/8	1 1/16	3/8	3/4	3/8 — 16
1 3/4	1/8	3/8	1 1/16	3/8	1 5/16	3/8 — 16
2 1/2	1/8	3/8	1 1/16	3/8	1 7/8	3/8 — 16
1 3/8	3/16	7/16	3/4	7/16	1	7/16 — 14
2 1/8	3/16	7/16	3/4	7/16	1 5/8	7/16 — 14
2 1/2	3/16	7/16	3/4	7/16	1 7/8	7/16 — 14
1 3/4	3/16	1/2	7/8	1/2	1 5/16	1/2 — 13
2 1/2	3/16	1/2	7/8	1/2	1 7/8	1/2 — 13
3 1/4	3/16	1/2	7/8	1/2	2 3/8	1/2 — 13

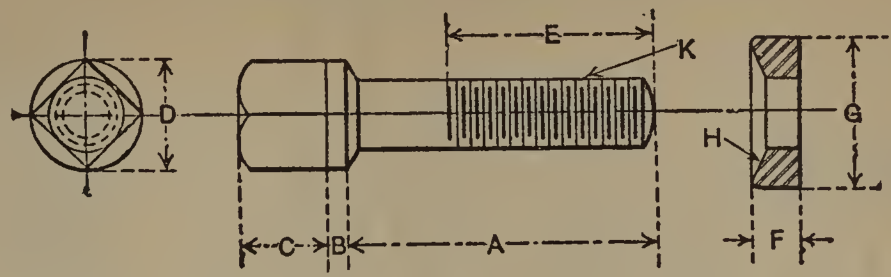
Clamping Screws, Screw Bushings and Studs. — Collar-head screws are used on jigs and fixtures in conjunction with clamps, straps and latches for clamping purposes (see table for dimensions). Rocking collar-screws are used with clamps for rough work, since they adapt themselves to any irregularities of the work and give a full bearing on the clamps in any position the work may assume.

Shoulder-screws are used for fastening clamping blocks, latches, or any parts that must move through a limited distance while still remaining permanently fastened to the tool.

Quarter-turn thumb-screws may be rapidly manipulated and are especially of use in box jigs. An objection to this type of screw is that the wear takes place on the boss on which it acts. Half-turn thumb-screws are also used in box jigs when the quarter-turn thumb-screw cannot be used on account of the work or bushing protruding through the end of the jig. These screws are used in pairs, one on each side of the jig cover.

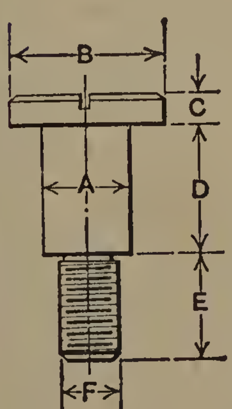
Screw bushings are generally avoided when accurate work is required, as a threaded bushing is likely to be out of true. Sometimes, however, no other type of bushing is adapted for the work in hand. (See table of "Aligning Screw Bushings.") Studs are used in jig design for locating work with holes in it. The accompanying table shows a recommended form of collar stud.

Rocking Collar-screws



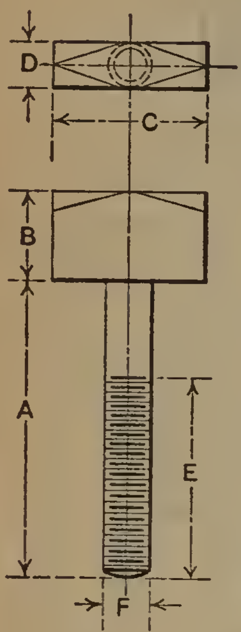
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	Diameter *K* and Threads per Inch
5/8	3/32	3/16	11/32	1/2	1/8	7/16	1/4	No. 10 — 32
1 1/8	3/32	3/16	11/32	7/8	1/8	7/16	1/4	No. 10 — 32
1 5/8	3/32	3/16	11/32	1 1/4	1/8	7/16	1/4	No. 10 — 32
7/8	3/32	1/4	7/16	5/8	3/16	5/8	1/2	No. 14 — 24
1 3/8	3/32	1/4	7/16	1	3/16	5/8	1/2	No. 14 — 24
1 7/8	3/32	1/4	7/16	1 7/16	3/16	5/8	1/2	No. 14 — 24
1 5/8	1/8	5/16	1/2	1 1/4	3/16	11/16	1/2	5/16 — 18
2	1/8	5/16	1/2	1 1/2	3/16	11/16	1/2	5/16 — 18
1	1/8	3/8	5/8	3/4	3/16	13/16	1/2	3/8 — 16
1 3/4	1/8	3/8	5/8	1 5/16	3/16	13/16	1/2	3/8 — 16
2 1/2	1/8	3/8	5/8	1 7/8	3/16	13/16	1/2	3/8 — 16
1 3/4	1/8	7/16	11/16	1 5/16	1/4	15/16	5/8	7/16 — 14
2 1/2	1/8	7/16	11/16	1 7/8	1/4	15/16	5/8	7/16 — 14
1 3/4	1/8	1/2	13/16	1 5/16	1/4	1 1/16	5/8	1/2 — 13
2 1/2	1/8	1/2	13/1	1 7/8	1/4	1 1/16	5/8	1/2 — 13
3 1/4	1/8	1/2	13/16	2 3/8	1/4	1 1/16	5/8	1/2 — 13

Shoulder-screws



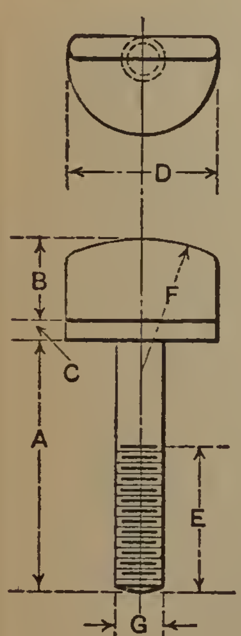
A	*B*	*C*	*D*	*E*	Diameter *F* and Threads per inch
0.249	1/2	1/8	1/2 to 3/4	7/16	No. 10 — 32
0.249	1/2	1/8	3/16 to 7/16	5/16	No. 10 — 32
0.3115	5/8	5/32	3/16 to 1/2	3/8	No. 14 — 24
0.3115	5/8	5/32	9/16 to 7/8	1/2	No. 14 — 24
0.374	3/4	5/32	1/4 to 7/16	3/8	No. 14 — 24
0.374	3/4	5/32	1/2 to 11/16	1/2	No. 14 — 24
0.4365	13/16	5/32	3/8 to 9/16	1/2	5/16 — 18
0.4365	13/16	5/32	7/8 to 1 1/8	3/4	5/16 — 18
0.499	7/8	3/16	3/8 to 9/16	1/2	3/8 — 16
0.499	7/8	3/16	15/16 to 1 1/4	3/4	3/8 — 16
0.5615	1	3/16	1/2 to 3/4	5/8	7/16 — 14
0.5615	1	3/16	1 1/4 to 1 1/2	7/8	7/16 — 14
0.6235	1 1/8	1/4	1/2 to 5/8	5/8	1/2 — 13
0.6235	1 1/8	1/4	1 1/8 to 1 5/8	7/8	1/2 — 13
0.686	1 1/4	1/4	1 1/8 to 1 3/4	1	1/2 — 13
0.7485	1 3/8	5/16	3/4 to 1 1/8	7/8	5/8 — 11
0.7485	1 3/8	5/16	1 1/4 to 2	1 1/8	5/8 — 11

Quarter-turn Thumb-screws



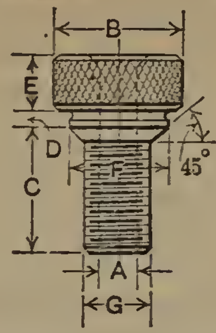
A	*B*	*C*	*D*	*E*	Diameter *F* and Threads per Inch
7/16	1/2	1 1/16	3/16	5/16	No. 10 — 32
5/8	1/2	1 1/16	3/16	1/2	No. 10 — 32
7/8	1/2	1 1/16	3/16	5/8	No. 10 — 32
1 3/8	1/2	1 1/16	3/16	1	No. 10 — 32
5/8	9/16	1 3/16	1/4	1/2	No. 14 — 24
1 1/8	9/16	1 3/16	1/4	7/8	No. 14 — 24
1 5/8	9/16	1 3/16	1/4	1 1/4	No. 14 — 24
5/8	5/8	1	5/16	1/2	5/16 — 18
1 1/4	5/8	1	5/16	1	5/16 — 18
2	5/8	1	5/16	1 1/2	5/16 — 18
3/4	1 1/16	1 3/16	3/8	9/16	3/8 — 16
1	1 1/16	1 3/16	3/8	3/4	3/8 — 16
1 3/4	1 1/16	1 3/16	3/8	1 5/16	3/8 — 16
2 1/2	1 1/16	1 3/16	3/8	1 7/8	3/8 — 16
1	3/4	1 5/16	7/16	3/4	7/16 — 14
1 3/4	3/4	1 5/16	7/16	1 5/16	7/16 — 14
2 1/2	3/4	1 5/16	7/16	1 1/4	7/16 — 14
1 3/4	3/4	1 5/16	1/2	1 5/16	1/2 — 13
2 1/2	3/4	1 5/16	1/2	1 1/4	1/2 — 13
3 1/4	3/4	1 5/16	1/2	1 5/8	1/2 — 13

Half-turn Thumb-screws



A	*B*	*C*	*D*	*E*	*F*	Diameter *G* and Threads per Inch
7/16	1/2	3/16	1 1/16	5/16	3/4	No. 10 — 32
5/8	1/2	3/16	1 1/16	1/2	3/4	No. 10 — 32
7/8	1/2	3/16	1 1/16	5/8	3/4	No. 10 — 32
1 3/8	1/2	3/16	1 1/16	1	3/4	No. 10 — 32
5/8	9/16	3/16	1 3/16	1/2	7/8	No. 14 — 24
1 1/8	9/16	3/16	1 3/16	7/8	7/8	No. 14 — 24
1 5/8	9/16	3/16	1 3/16	1 1/4	7/8	No. 14 — 24
5/8	5/8	3/16	1	1/2	1	5/16 — 18
1 1/4	5/8	3/16	1	1	1	5/16 — 18
2	5/8	3/16	1	1 1/2	1	5/16 — 18
3/4	1 1/16	1/4	1 3/16	9/16	1 1/4	3/8 — 16
1	1 1/16	1/4	1 3/16	3/4	1 1/4	3/8 — 16
1 3/4	1 1/16	1/4	1 3/16	1 5/16	1 1/4	3/8 — 16
2 1/2	1 1/16	1/4	1 3/16	1 7/8	1 1/4	3/8 — 16
1	3/4	1/4	1 5/16	3/4	1 3/8	7/16 — 14
1 3/4	3/4	1/4	1 5/16	1 5/16	1 3/8	7/16 — 14
2 1/2	3/4	1/4	1 5/16	1 1/4	1 3/8	7/16 — 14
1 3/4	3/4	1/4	1 5/16	1 5/16	1 1/2	1/2 — 13
2 1/2	3/4	1/4	1 5/16	1 1/4	1 1/2	1/2 — 13
3 1/4	3/4	1/4	1 5/16	1 5/8	1 1/2	1/2 — 13

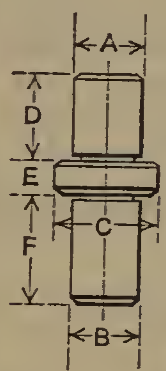
Aligning Screw Bushings



A and *C*, according to requirements

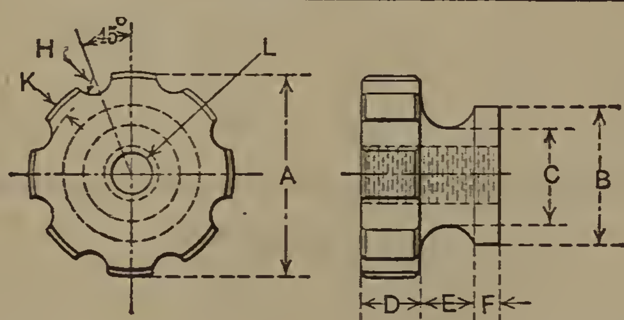
B	*D*	*E*	*F*	Diameter *G* and Threads per Inch
7/8	1/8	5/16	11/16	1/2 — 13
1	5/32	5/16	7/8	5/8 — 11
1 1/4	3/16	5/16	1	3/4 — 10
1 1/2	3/16	7/16	1 1/4	1 — 14
1 7/8	1/4	1/2	1 5/8	1 1/4 — 12
2 1/4	1/4	5/8	2	1 1/2 — 12
2 3/4	1/4	3/4	2 3/8	1 3/4 — 8
3 1/4	1/4	7/8	2 3/4	2 — 8

Collar Studs — Hardened and Ground



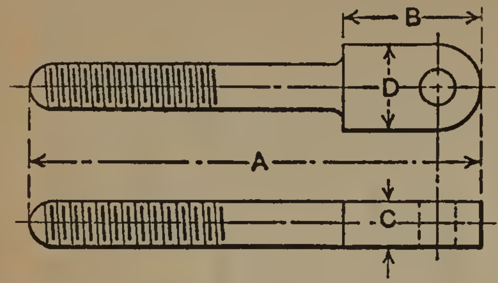
A	*B*	*C*	*D*	*E*	*F*
0.251	0.249	1/2	1/2	3/16	1/4 — 5/8
0.3135	0.3115	9/16	1/2	3/16	5/16 — 11/16
0.376	0.374	5/8	5/8	3/16	3/8 — 3/4
0.4385	0.4365	11/16	5/8	3/16	7/16 — 13/16
0.501	0.499	3/4	5/8	1/4	1/2 — 7/8
0.5635	0.5615	7/8	5/8	1/4	1/2 — 7/8
0.626	0.624	1	3/4	1/4	1/2 — 1 1/16
0.6885	0.6865	1 1/8	3/4	1/4	5/8 — 1 3/16
0.751	0.749	1 1/4	7/8	1/4	5/8 — 1 3/8

Hand Nuts



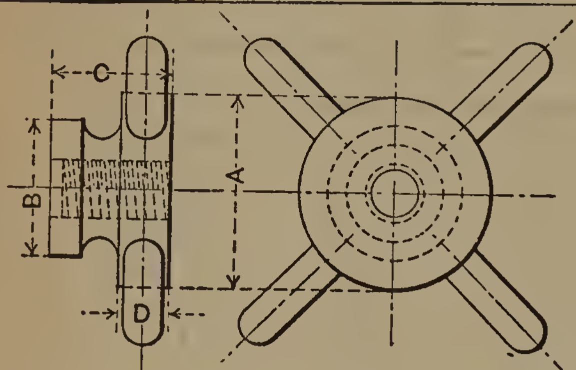
A	*B*	*C*	*D*	*E*	*F*	*H*	*K*	Diameter *L* and Threads per Inch
1 5/8	1 1/4	15/16	9/16	5/16	1/8	1/4	1/8	1/2 — 13
2 1/8	1 1/2	1 1/16	11/16	1/2	3/16	5/16	3/16	5/8 — 11
2 1/8	1 1/2	1 1/16	11/16	1/2	3/16	5/16	3/16	3/4 — 10
2 1/2	1 3/4	1 1/4	3/4	11/16	3/16	3/8	1/4	7/8 — 9
2 3/4	2	1 1/2	13/16	11/16	1/4	3/8	1/4	1 — 8
2 7/8	2 1/8	1 5/8	7/8	3/4	1/4	3/8	1/4	1 1/8 — 7
3 1/8	2 1/4	1 3/4	1	3/4	5/16	7/16	5/16	1 1/4 — 7

Dimensions of Jig-Screw Latches



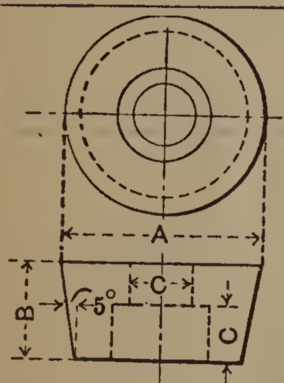
A	B	C	D
1¼	⅜	⅛	5/16
1¾	⅝	5/32	⅜
2⅜	¾	3/16	7/16
2⅞	⅞	¼	½
3½	1	5/16	⅝
4⅛	1¼	⅜	¾

Dimensions of Latch Nuts



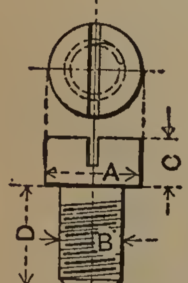
A	B	C	D
⅝	7/16	5/16	5/32
¾	½	⅜	5/32
⅞	9/16	7/16	3/16
1	⅝	½	3/16
1⅛	¾	⅝	¼
1¼	⅞	¾	5/16

Standard Jig Feet



A	B	C	A	B	C
⅜	3/16	⅛	13/16	13/32	7/32
7/16	7/32	9/64	¾	⅜	¼
½	¼	5/32	⅞	7/16	9/32
9/16	9/32	11/64	1	½	5/16
⅝	5/16	3/16	......	......	......

Screws for Jig Feet



A	B	C	D	A	B	C	D
0.160	⅛	0.110	9/32	0.299	7/32	0.192	7/16
0.191	9/64	0.123	5/16	0.343	¼	0.219	15/32
0.213	5/32	0.137	11/32	0.386	9/32	0.246	½
0.233	11/64	0.150	⅜	0.426	5/16	0.273	17/32
0.256	3/16	0.164	13/32	.....	....	.....	....

Definition of Jig and Fixture. — The distinction between a jig and fixture is not easy to define, but, as a general rule, it is as follows: A jig either holds or is held on the work, and, at the same time, contains guides for the various cutting tools, whereas a fixture holds the work while the cutting tools are in operation, but does not contain any special arrangements for guiding the tools. A fixture, therefore, must be securely held or fixed to the machine on which the operation is performed — hence the name. A fixture is sometimes provided with a number of gages and stops, but not with bushings or other devices for guiding and supporting the cutting tools.

Pointers on Jig and Fixture Design. — Before designing a jig or fixture, compare the cost of production when using present equipment with the expected cost when using the proposed equipment, and see that the cost of the fixture is not in excess of the expected gain.

Before laying out a jig or fixture, decide upon the locating points and outline a clamping arrangement. Make the clamping and binding devices as quick-acting as possible. For rough castings, make some of the locating points adjustable.

Arrange the jig so that the work can only be inserted in the correct way. Provide handles whenever they will make it more convenient to manipulate the jig.

Locate clamps so that they will be in the best position to resist the pressure of the cutting tools. If possible, make all clamps integral parts of the jig or fixture, and place the clamps as nearly as possible opposite bearing points on the work, to avoid springing. Avoid complicated clamping arrangements.

Provide feet opposite all surfaces containing guide bushings. Place all bushings inside of the geometrical figure formed by lines connecting the supporting feet.

If possible, design the jig so that all locating points are visible to the operator when placing the work in position. Provide holes for the escape of chips. Provide tongues on all milling and planing fixtures for engaging table slots, and clamping lugs on jigs or fixtures which must be securely held to the machine while in use.

GRINDING, POLISHING AND LAPPING

Cylindrical Grinding

Grade and Grain of Grinding Wheels. — The term "grade," as applied to a grinding wheel, refers to the tenacity with which the bond holds the cutting particles or abrasive grains in place, and not to the hardness of the abrasive. A wheel from which the abrasive grains can easily be dislodged is called "soft," or of "soft grade," and one which holds the grains securely is referred to as a "hard wheel." By varying the amount and composition of the bond, wheels of different grades are obtained. The grade is designated either by letters of the alphabet or numbers. According to the system employed by several manufacturers, the letter M represents a medium grade, and the successive order of letters preceding and following M denote softer and harder wheels, or *vice versa.* The grain or coarseness of a wheel is designated by numbers which indicate the number of meshes to the inch through which the kernels of the abrasive material will pass. For example, a 36 grain means that the abrasive will pass through a sieve having 36 meshes to the linear inch. Some grain and grade symbols consist of additional letters or numbers which may be used to indicate either different bond mixtures or the "temper" of the abrasive.

Selection of Wheels for Grinding. — The grade and grain to use depend upon the kind of material to be ground, its degree of hardness and the surface area in contact with the wheel. Theoretically, a wheel is of the proper grade when the bond is just hard enough to hold the abrasive until it becomes too dull to cut effectively; then, because of the increased friction, the dull grains are torn out and new points come into action, so that the wheel automatically sharpens itself. The harder the stock being ground, the more quickly the grains are dulled; hence, as a general rule, the harder the material, the softer the wheel, and *vice versa*, although some very soft materials, such as brass, are ground with a soft wheel which crumbles easily and does not become "loaded" or clogged with metal. When a hard wheel is used for grinding hard material, the grains become dulled, but are not dislodged as rapidly as they should be; consequently, the periphery of the wheel is worn smooth and becomes glazed, and excessive pressure is required to make the wheel cut. Any undue pressure tends to distort the work, and this tendency is increased by the heat generated. If the surface of the wheel becomes loaded with chips and burns the work, even when plenty of water is used, it is too hard. A highly polished surface is sometimes obtained at the expense of accuracy by using hard wheels that require so much pressure to make them grind that the work is distorted. In order to secure accuracy, as well as the most economical results, the wheel must cut freely and without perceptible pressure.

The area of the surface in contact with the wheel also affects the grade. For a given material the wheel should be softer, as the area increases. To illustrate, grade N might be suitable for grinding a diameter of two inches, but not suitable for a diameter of four inches, because of the increase in contact area. As the conditions under which wheels are used vary widely, no definite rule can be given for selecting the proper grade and grain. Definite information on the selection of grinding wheels suitable for different classes of work may be obtained from various grinding wheel manufacturers.

The grain or coarseness of the wheel depends upon the hardness of the material, its composition and the finish required. Generally speaking, coarse wheels are better adapted to most work, because the larger grains permit deeper cuts to be taken. The quality of the finish (except when grinding brass or other soft metals) depends more on the depth of cut and condition of the wheel face than on the fineness of the abrasive material. In fact, very fine surfaces can be obtained with a comparatively coarse wheel, provided there is the proper relation between the grade and surface speeds of the wheel and work. When rough grinding, the cutting particles are constantly worn away or dislodged, so that the face of the wheel is kept rough or sharp, and the ground surface is also comparatively rough; after the wheel face has been trued with a diamond, light finishing cuts in conjunction with a reduced work speed will give a smooth finish. When grinding brass or soft bronze, the grain of the wheel must be as fine as the finish desired; that is, it is not practicable to use a coarse wheel for finishing these metals. Bronzes containing manganese or phosphor permit the use of coarser wheels.

Mounting Grinding Wheels. — Grinding wheels should fit freely on their spindles but without unnecessary play. If a wheel is forced on the spindle, there is danger of starting cracks. The diameter of the flanges should be one-half the wheel diameter (never less than one-third), and the flanges should be relieved or recessed to secure an annular bearing at their circumference. The inner flange should be keyed or shrunk on the spindle. Compressible washers of blotting paper or rubber should be placed between the wheel and the flanges, to distribute the clamping pressure evenly. The flanges should be clamped just tight enough to hold the wheel firmly. Wheels should be carefully inspected, and be tapped lightly before mounting, as new wheels occasionally burst when first brought up to speed, because of hidden cracks resulting from rough handling in transit.

Glazed and Loaded Wheels. — A wheel is glazed when the cutting particles have become dull, or worn down even with the bond, which latter is so hard that the abrasive grains are not dislodged when too dull to cut effectively. Glazing may indicate either that the wheel is too hard for the work, or that the wheel speed is too high. The remedy, then, for glazing is to decrease the speed or use a softer wheel.

A wheel is "loaded" when the pores or interstices between the cutting particles are partly or entirely clogged with the material being ground. Loading prevents the wheel from cutting and causes excessive heat to be generated. If a wheel becomes loaded, the bond may be too hard or the speed too slow. The remedy for loading is to increase the speed or use a softer wheel.

Speed of Grinding Wheel and Work. — The peripheral speed of a grinding wheel is usually somewhere between 5500 and 6000 feet per minute, but speeds varying from 5000 to 7000 feet per minute are employed. As the wheel diminishes in size, it appears to get softer. This is because the grit of a small wheel is in contact with the work oftener, owing to the increased number of revolutions necessary for the same surface speed. There are a number of factors, such as the kind of material, finish desired, etc., which determine the proper *work* speed; hence, the speed must be varied to suit conditions. A surface speed of 25 feet per minute might be suitable for rough grinding a certain piece of steel, but it would not give the best results for another steel part of different composition. Twenty-five feet per minute is a fair average for rough grinding soft steel when using comparatively soft, free-cutting wheels; for finishing with the same wheel, the speed would ordinarily be reduced about 25 per cent. When harder and more compact wheels are used, the speed is increased for finishing. It is the modern practice, when rough grinding, to use a fairly coarse wheel of soft enough grade to cut freely, and a comparatively slow work speed in conjunction with a coarse side feed of the wheel or work. This method of grinding is employed when using large machines which have sufficient driving power to enable broad cuts to be taken and are rigid enough to prevent excessive vibration. With small light grinders, it is not always feasible to use a coarse side feed, owing to the lack of rigidity and driving power. The depth of the cut, or amount that the wheel is fed inward at each reversal, must also be governed, to some extent, by the power and rigidity of the machine.

Table of Grinding Wheel Speeds

Diameter of Wheel, Inches	Rev. per Min. for Surface Speed of 4000 Feet	Rev. per Min. for Surface Speed of 5000 Feet	Rev. per Min. for Surface Speed of 6000 Feet	Diameter of Wheel, Inches	Rev. per Min. for Surface Speed of 4000 Feet	Rev. per Min. for Surface Speed of 5000 Feet	Rev. per Min. for Surface Speed of 6000 Feet
1	15279	19099	22918	28	546	683	819
2	7639	9549	11459	30	509	637	764
3	5093	6366	7639	32	477	596	716
4	3820	4775	5730	34	449	561	674
5	3056	3820	4584	36	424	531	637
6	2546	3183	3820	38	402	503	603
7	2183	2728	3274	40	382	478	573
8	1910	2387	2865	42	364	455	546
10	1528	1910	2292	44	347	434	521
12	1273	1592	1910	46	332	415	498
14	1091	1364	1637	48	318	397	477
16	955	1194	1432	50	306	383	459
18	849	1061	1273	52	294	369	441
20	764	955	1146	54	283	354	425
22	694	868	1042	56	273	341	410
24	637	796	955	58	264	330	396
26	586	733	879	60	255	319	383

Allowance for Grinding. — The amount of stock that can be economically removed by grinding depends primarily upon the size and power of the grinding machine. For ordinary commercial grinding, the work is generally reduced in a lathe to somewhere between 1/64 and 1/32 inch of the required diameter. When the diameter has been rough turned to this dimension, it is more economical to remove the stock by grinding than by taking a light finishing cut in the lathe, provided a grinder of sufficient power is available. If the work is turned too close to the finished size, the cost of turning becomes excessive. On the other hand, too large an allowance means that stock is being ground that could be removed to better advantage by rough turning. The modern practice is to simply rough turn in the lathe, using a round-nosed tool and very coarse feeds, thus leaving a rough ridged surface on the work. The stock is turned to somewhere near the allowance and the grinder is used to correct all errors and give the required size and finish. This practice applies especially to the turning and grinding of machine parts in connection with manufacturing operations. When work is rough turned in this way, the time for grinding is often increased considerably, but this is much more than offset by the decrease in time required for turning, thus reducing the total time necessary for both operations. The table, "Allowances for Grinding" (compiled by the Landis Tool Co.), gives allowances for various diameters and lengths under average conditions.

Allowances for Grinding

Diameter, Inches	Length, Inches										
	3	6	9	12	15	18	24	30	36	42	48
	Allowance, Inches										
½	0.010	0.010	0.010	0.010	0.015	0.015	0.015	0.020	0.020	0.020	0.020
¾	0.010	0.010	0.010	0.010	0.015	0.015	0.015	0.020	0.020	0.020	0.020
1	0.010	0.010	0.010	0.015	0.015	0.015	0.015	0.020	0.020	0.020	0.020
1¼	0.010	0.010	0.015	0.015	0.015	0.015	0.015	0.020	0.020	0.020	0.020
1½	0.010	0.015	0.015	0.015	0.015	0.015	0.020	0.020	0.020	0.020	0.020
2	0.015	0.015	0.015	0.015	0.015	0.020	0.020	0.020	0.020	0.020	0.025
2¼	0.015	0.015	0.015	0.015	0.020	0.020	0.020	0.020	0.020	0.025	0.025
2½	0.015	0.015	0.015	0.020	0.020	0.020	0.020	0.020	0.025	0.025	0.025
3	0.015	0.015	0.020	0.020	0.020	0.020	0.020	0.025	0.025	0.025	0.025
3½	0.015	0.020	0.020	0.020	0.020	0.020	0.025	0.025	0.025	0.025	0.025
4	0.020	0.020	0.020	0.020	0.020	0.025	0.025	0.025	0.025	0.025	0.030
4½	0.020	0.020	0.020	0.020	0.025	0.025	0.025	0.025	0.025	0.030	0.030
5	0.020	0.020	0.020	0.025	0.025	0.025	0.025	0.025	0.030	0.030	0.030
6	0.020	0.020	0.025	0.025	0.025	0.025	0.025	0.030	0.030	0.030	0.030
7	0.020	0.025	0.025	0.025	0.025	0.025	0.030	0.030	0.030	0.030	0.030
8	0.025	0.025	0.025	0.025	0.025	0.030	0.030	0.030	0.030	0.030	0.030
9	0.025	0.025	0.025	0.025	0.030	0.030	0.030	0.030	0.030	0.030	0.030
10	0.025	0.025	0.025	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
11	0.025	0.025	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
12	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030

Abrasives for Grinding. — The commercial abrasive materials for grinding wheels are both natural and artificial. Emery and corundum are natural abrasives; materials like alundum, carborundum, crystolon, aloxite, adamite and carbolite are produced artificially. *Emery* is a very tough and durable abrasive, but contains iron and other non-cutting elements, and is little used in automatic grinding machines. *Corundum* is purer than emery and contains a much larger percentage of crystalline alumina, which is the element in both abrasives that does the cutting. *Carborundum*, which is a trade name for carbide of silicon, is a product of the electric furnace. The principal materials used in the manufacture of carborundum are coke and sand. The coke is used to supply the carbon, and the sand, the silicon. *Alundum* is also made in the electric furnace by the fusion of a mineral called bauxite, which was considered infusible until the invention of the electric process. Bauxite is a soft earth and somewhat resembles light yellow clay. Chemically, it is the purest form of aluminum oxide found in nature. *Aloxite* consists essentially of aluminum oxide. It is a product of the electric furnace, and is made by heating an ore containing this oxide with certain ingredients which are added to remove the impurities.

Vitrified Grinding Wheels. — The wheels most generally used in automatic grinding machines are bonded by the vitrified process. Vitrified wheels are porous and free cutting and are not affected by water, acids, oils, heat or cold. The bond is composed of suitable clays and fluxes, which are mixed with the abrasive in power-driven mixing kettles. The wheels, after being molded, are baked or burned continuously for a period of 100 hours or more, the time depending upon the size of the wheels. During this baking process the temperature is gradually raised until the clay is partially melted and vitrified. The wheels are then allowed to cool slowly for a week, care being taken to prevent sudden temperature changes. As the cooling takes place, the clay crystallizes and binds the abrasive grains firmly together.

The Silicate Process. — Silicate of soda is the principal ingredient in the bond of silicate grinding wheels or those made by the silicate process. The abrasive grains are first mixed with the bond in special machines, and the mixture is then tamped into molds. After the wheels are molded, they are dried and baked in special ovens from which all fire gases are excluded. This causes a chemical reaction which hardens or sets the bond. The temperature of the ovens is much lower than is required in connection with the vitrified process. Some shapes of silicate wheels are molded under hydraulic pressure. This method is employed for disk wheels and very hard wheels. Silicate wheels can be made in large sizes and can be produced in a comparatively short time. Vitrified wheels are rarely made larger than 36 inches in diameter, 30 inches being the maximum size with some manufacturers; silicate wheels are made as large as 60 inches in diameter. Silicate wheels are especially adapted for grinding operations in which it is important to have the lowest possible wheel wear compatible with cool cutting.

Elastic Wheels. — Very thin grinding wheels are made by the elastic process. Shellac is the principal ingredient in the bond and the wheels are baked at a low temperature to set the shellac. Wheels made by this process are strong and have considerable elasticity, so that very thin elastic wheels can be safely used; wheels 1/32 inch thick are manufactured. These wheels are used principally for fine grinding, cutting off stock, or wherever a thin wheel is necessary.

Vulcanite, Celluloid, and Oil Processes. — Vulcanite wheels are bonded with vulcanized rubber. Very hard, tough, thin wheels can be produced by this process, but they are expensive. Wheels made by the celluloid process have a bond of celluloid, as the name implies. The abrasive grains are mixed with the celluloid, and this mixture is rolled into sheets from which the wheels are cut. After seasoning for several months, the wheels are ready to finish. With the oil process, an oxidizing oil is mixed with the grains. This mixture is then formed into wheels by compressing it into molds with a hydraulic press. Oil wheels are similar in action to elastic wheels but are less dependable as to grade and uniformity. These three processes are only used to a limited extent.

Truing a Grinding Wheel. — The only satisfactory method of truing the wheels used in automatic grinding machines is by the use of a diamond tool. This tool should be rigidly clamped to the machine and the wheel should revolve at the speed required for grinding. The diamond tool should be held with the point quite close to the supporting clamp, in order to reduce vibration and give a smooth accurate wheel surface. Diamond tools usually have round shanks to permit clamping them in different positions, so that the wear on the diamond will not be confined to one or two points. When truing the wheel, light cuts should be taken and water be used to keep the diamond cool.

Diamond Tools. — The diamonds used in tools for truing grinding wheels are of two kinds, the carbon or black diamond and bort. The black diamond rarely has any visible crystallization; its color varies, but is often a very dark purple-brown. Bort is a semi-transparent stone — an imperfect "brilliant." It is not as hard as a black diamond, and is considerably lower in price. For truing soft wheels, bort may be more economical than the more expensive stones, but, as a general rule, the black diamond is cheaper in the long run.

Setting Diamonds. — The diamonds used for truing grinding wheels are usually set in the end of a soft-steel rod. A hole is drilled in the end, just a little deeper than the length of the stone and of the same diameter as the thickest part. The diamond is then fixed in place by carefully peening the metal over it, by using a small set. The end of the rod is then ground away to expose part of the diamond. Diamonds are also brazed in position: First drill a hole a little deeper than the greatest dimension of the diamond. The drilling should be done without lubricant, as oil of any kind tends to prevent the spelter from flowing smoothly. This being done, the hole should be closed in slightly — just enough to make it out of round. The molten spelter is now poured into the hole, filling it completely, and the diamond, held in a pair of tweezers, is pushed into the liquid spelter in the hole, until it strikes the bottom. After the spelter has cooled, the end of the rod in which the diamond is located can be shaped in the customary manner. The fact that the hole is closed in slightly, prevents the core of spelter from working out of the end of the rod.

Wheels for Surface Grinding. — Comparatively soft wheels are used on vertical spindle surface grinders and other types using cup wheels. This is because of the relatively large contact area between the wheel and work, and also because there is not the same clearance as in other grinding operations. Owing to the large contact area between the wheel and work, the selection of wheels of the proper grade for surface grinding is of particular importance, for if a wheel is a little too soft it will wear rapidly, and one that is a little too hard will fill and glaze.

Wheels for Internal Grinding. — When selecting wheels of the proper grain and grade for internal grinding, the following points should be considered: Diameter of hole; speed of wheel-spindle; kind of work; whether the hole is plain or keyseated; nature of material; stiffness of machine; rigidity of wheel-spindle; and whether the cut is for roughing or finishing. As regards the diameter of the hole, there are several points to consider. If the hole is below ¾ or 1 inch in diameter, the wheel should be as large as possible so that it will last longer. The wheel is usually ordered of the same diameter or larger than the hole to be ground and then is trued until it enters the hole. By using a wheel which is only slightly smaller than the hole, the arc of contact of the wheel and work is large, and, consequently, a much softer wheel must be used than if the wheel were small in relation to the diameter of the hole. When a hole is plain, a softer wheel should be used than if the hole were keyseated. Slots or keyseats have a shaving action on the wheel-face and quickly tear out the grains; hence, for keyseated work, a harder wheel should be used than on plain hole work, and it should also have a wider face. The greater the rigidity of the machine, the softer the grade and coarser the grain of the wheel should be. The rigid machine also has the advantage of removing the stock with rapidity and without chatter marks. In commercial grinding a wheel is generally selected which will be fairly suitable for both roughing and finishing to avoid changing wheels.

Disk Grinding

Abrasives for Disk Grinding. — Many of the abrasive materials used in grinding wheels are also employed for disk grinding. The abrasive is attached to cloth or paper disks, which are glued or cemented to the sides or faces of steel disk-wheels. The coarseness of the grains is designated by numbers. For example, a No. 36 disk is one having abrasive grains that will just pass through a sieve having 36 meshes to the linear inch. The principal grains used in modern disk grinding practice are 12, 16, 24, 36, 46, 60, 90 and 120. The numbers generally used are 12, 16, 24 and 36. Most disks or "circles" have cloth backs. Paper-back circles can be employed to advantage on many light "smoothing" jobs. Paper-back circles should not be used for a coarser grain than No. 24.

Selection of Abrasives for Disk Grinding. — The following general information regarding the selection of abrasives for disk grinding different metals is given by the Gardner Machine Co.: Use nothing coarser than grain No. 16 for grinding a surface of less than 3 square inches area. Use nothing coarser than No. 16 on soft steel or brass, even if the area is large. Use nothing coarser than No. 24 for hardened steel, whatever the size of piece, and seldom use coarser than No. 24 on brass, unless there is a large amount of stock to be removed from rough castings. For brass, grain No. 36 is a good number, as it cuts fast and leaves a fairly fine finish which can easily be polished by buffing or with a very fine disk. No. 46 is used considerably for grinding small surfaces on brass, and for finish grinding soft steel and cast iron, after rough grinding with a No. 16 or No. 12 disk. For grinding cast iron, when the area of the surface is more than 3 or 4 square inches, use from No. 16 to 12. It is a mistake to use fine numbers on cast iron scale. If a fine surface is required, it is much better and more economical to first rough grind close to size and then finish with finer numbers.

The following suggestions are given by Charles H. Besly & Co. as a guide in selecting grain numbers for various disk grinding operations: The best "circles" for general grinding are grain Nos. 16 and 24. For rough grinding cast iron, malleable iron, bronze, etc., use nothing finer than grain 16 on areas larger than 2 square inches. On smaller areas, use nothing coarser than No. 24. For rough grinding cast brass, use nothing coarser than No. 24, except on extremely large scaly surfaces, in which case a grain as coarse as No. 16 can sometimes be used to advantage. Most rough grinding on brass castings is done with circles or disks of No. 36 grain, as they are coarse enough to remove stock rapidly, and fine enough to leave a finish that is satisfactory for many classes of work. When a fine finish is required on brass, circles of No. 60 grain are used extensively, No. 36 being used first for roughing and No. 60 for finishing. No. 46 circles are often satisfactory for roughing and finishing small brass parts in one operation. On extremely hard material, it is a mistake to use very coarse grains. For hardened steel, chilled iron, and very hard unannealed tool steel, use nothing coarser than No. 24. Circles of No. 36 grain are employed for grinding hard forging dies and casehardened valve gear parts in locomotive shops.

Other suggestions based on modern practice are as follows: For cleaning and finishing the sides of steel hexagon nuts or bolt heads on the double-spindle grinder, use grain No. 16 for large work, and No. 24 for small work. For roughing brass hexagon nuts on the double-spindle grinder, use grain No. 36. For securing a very fine finish on small surfaces of cold-drawn mild steel parts in one operation, use grain No. 90. For finishing the sides of cap-screw heads, use grain No. 24 for hot-rolled stock, and No. 36 for cold-drawn stock. For facing composition packing rings, such as are used for seats in globe valves, use grain No. 46 and a double-spindle grinder. For rough grinding the sides of automobile engine piston rings on the double-spindle grinder, use grain No. 24. For grinding water jacket covers and seats on automobile engine cylinders from the rough, use grain No. 16. For polishing the faces of casehardened valve seats, circles of No. 36 grain are used to remove the tool-marks left by the screw machine and to give the surface a polish. For finishing brass that is to be nickel-plated, it is recommended that leather disks be used. These should be charged with No. 120 (or finer) abrasive grains. Leather disks are also used to advantage for finishing small tools and hardware specialties, such as padlock parts, etc., as they leave a fine finish closely resembling handwork. For producing this "hand-finish" on soft brass parts, felt disks about 7/16 inch thick can be used. These felt disks should be charged with very fine grains of a suitable abrasive material.

Allowances for Disk Grinding. — The amount of stock to be removed, the area of the ground surface, and its distribution are important factors in disk grinding. The removal of from 0.005 inch to 0.050 inch of stock will usually "clean up" a surface. The following figures taken from actual practice represent allowances used in connection with Besly disk grinders: Drop-forged wrenches, 0.008 to 0.015 inch; brass hexagon nuts up to 2 inches diameter, 0.015 inch; larger sizes, up to 0.030 inch; steel punchings, 0.005 to 0.015 inch; cast-iron machine parts, 1/32 to 1/16 inch; cast-brass machine parts, 1/64 to 3/32 inch. The amount of stock that can economically be removed by disk grinding depends largely upon the nature of the material being ground. Cast metal is more easily ground than rolled or wrought material, and small thin castings are usually harder to grind than larger and thicker castings, owing to the greater density of the metal. When castings have a hard scale, it is often desirable to partially remove it before disk grinding. The hard scale is "broken up" either by grinding on vitrified wheels or by tumbling, sand blasting or pickling. The latter method is the best for forgings or hot rolled material that has considerable scale.

Speeds for Disk Grinding. — Speeds recommended for disk grinding must necessarily be a compromise, because the speed at any given point on the side of a disk wheel depends upon the radius; hence if the speed near the circumference is correct for grinding a certain class of work, the central part of the disk will be running too slowly. Therefore, it is important to consider the average speed. The speeds given in the table, "Speeds of Wheels for Disk Grinders," are recommended by Charles H. Besly & Co. They are given in revolutions per minute for disk wheels of different diameters, and are based on the following averages: For grinding steel, cast iron, bronze, etc., an average abrasive speed of 5000 feet per minute; for grinding brass or for burring and finishing, 6500 feet per minute; for grinding aluminum alloy and very soft material like artificial rubber, 7500 feet per minute; and for grinding wood, 4000 feet per minute. When grinding hard metals, an excessive speed will dull and glaze the disks; too slow a speed for soft metals tends to fill and clog the disks. Too high a speed for wood tends to burn the work.

The Gardner Machine Co. recommends the following speeds which represent velocities at three-fourths the radius of the disk wheel: For grinding cast iron or steel, 4000 to 4500 feet per minute; for grinding brass, 5000 to 5500 feet per minute. A speed of 5000 feet per minute is given as the maximum for cast iron and steel. If the grinder is to be used for both steel and brass and is without means for changing the speed, slower speeds, ranging from 4000 to 4500 feet per minute, are recommended. The speeds given in the foregoing represent average practice and variations may be necessary for certain classes of work.

Speeds of Wheels for Disk Grinders

Diameter of Disk Wheel, Inches	Speed in Revolutions per Minute			
	For Cast Iron and Steel	For Brass and for Burring and Finishing	For Aluminum Alloys	For Wood Grinding
12	2000	2600	3000	1600
18	1400	1800	2100	1100
20	1250	1600	1900	1000
23	1100	1300	1650	850
26	1000	1250	1500	800
30	900	1150	1350	750

Feeding Pressure for Disk Grinding. — The feeding pressure that will give the most accurate results for disk grinding depends upon the nature of the work, the area of the surface being ground, and the grade and condition of the disk. According to the Gardner Machine Co., a pressure of less than 10 pounds per square inch of ground surface is ordinarily not sufficient, if very much stock is to be removed A feeding pressure of from 20 to 30 pounds per square inch will be found a fair average, and 40 pounds is practically the limit of pressure. While the life of an abrasive disk depends largely upon the kind of material being ground, the length of time a disk will cut effectively can be greatly increased by proper feeding. Any sudden jamming of the work against the disk, either by hand or with a feed lever, will soon destroy its cutting qualities, whereas, too light a pressure tends to dull the disk before a sufficient number of parts have been ground. Charles H. Besly & Co. give the following information regarding feeding pressures: On most work, the feeding pressure for rapid rough grinding varies as follows: Cast gray iron, 60 to 80 pounds per square inch; soft malleable iron, 20 to 40 pounds per square inch; yellow brass castings, 20 to 30 pounds per square inch; hot rolled mild steel, pickled, 25 to 50 pounds per square inch. In exceptional cases, a feeding pressure of 100 pounds per square inch of ground surface area has been used without destructive effect on the circle.

Attaching Abrasive Disks. — Before a new abrasive disk is applied to a disk wheel, the latter should be clean and dry. If the disk wheel is greasy, it should be cleaned before applying the cement, by washing in a rather strong and hot solution of potash or concentrated lye, rinsing with hot water. Do not apply the glue or cement until the wheel is dry. The disk wheel should remain in the press from 15 minutes to an hour, depending upon whether glue or cement is used. It may be left in the press for a much longer period, if convenient. The wheel should be allowed to dry thoroughly after being taken from the press and before using. Artificial heat should not be used for drying. Ordinary patternmakers' glue is often used for attaching disks, but the special disk wheel cements sold by disk grinder manufacturers are generally considered more satisfactory. When hot glue is used, the steel disk should first be warmed, but cement may be applied to a cold wheel. To remove worn-out disks or circles, immerse the wheel in water. Cold water can be used, but hot water will loosen the disks much quicker. The disks should be stored in a cool, dry place. They should be kept free from oil, grease or water, and not be subjected to dry artificial heat, especially heat from direct steam radiation, as this will injure the bond, causing the cutting particles to shell off when in service.

Polishing and Buffing

The terms "polishing" and "buffing" are sometimes applied to similar classes of work in different plants, but according to approved usage of the terms, there is the following distinction: Polishing is any operation performed with wheels having abrasive glued to the working surfaces, whereas, buffing is done with wheels having the abrasive applied loosely instead of imbedding it into glue; moreover, buffing is not so harsh an operation as ordinary polishing, and it is commonly utilized for obtaining very fine surfaces having a "grainless finish." In general, polishing includes all classes of work performed with glued-abrasive wheels and ranges from the "flexible grinding" operations performed on the rough forgings to the production of a bright luster, such as is given to surgical instruments, high quality scissors, and other kinds of general hardware. The abrasives which are glued to a polishing wheel are intended to grind away the roughness left by a grinding wheel or a steel cutting tool, although, as just intimated, the polishing type of wheel may also be used for obtaining very fine finish, by selecting the right kind of wheel and abrasive. Buffing, however, is primarily for the purpose of producing a very fine finish and use is made of such soft cutting materials as tripoli, lime, crocus, or rouge prepared in cake form, with tallow and other greases as a body; this cake is applied to the cloth buff by hand, from time to time, so that the surfaces of the buff are coated with the composition. Some metals like German silver and white metal are buffed before plating. Steel parts to be plated are usually prepared for plating by polishing, and buffing is employed to give a luster to the plated surface. Pocket-knife blades are polished with emery and then highly finished (colored) by what is known as "crocus polishing" for which a wheel similar to a leather-faced wood polishing wheel is used for buffing.

Polishing Wheels. — The principal materials from which polishing wheels are made are wood, leather, canvas, cotton cloth, felt, paper, walrus or sea-horse hide, sheepskin, impregnated rubber, canvas composition, and wool. Leather and canvas are the materials most commonly used in polishing wheel construction. Bullneck leather wheels are made of oak tanned bullneck leather cut into disks of uniform thickness and cemented together. Wooden wheels covered with leather to which emery or some other abrasive is glued, are employed extensively for polishing flat surfaces, especially when good edges must be maintained. Canvas wheels are made in various ways; wheels having disks that are cemented together are very hard and used for rough, coarse work, whereas those having sewed disks are made of varying densities by sewing together a larger or smaller number of disks into sections and gluing them. Wheels in which the disks are held together by sewing and which are not stiffened by the use of glue, usually require metal side plates to support the canvas disks. Muslin wheels are made from sewed buffs glued together, but the outer edges of a wheel frequently are left open or free from glue to provide an open face of any desired depth. Wool felt wheels are flexible and resilient, and the density may be varied by sewing two or more disks together and then cementing these to form a wheel. Solid wheels made of Spanish or Mexican felt are quite popular for fine finishing but have little value as general utility wheels. Paper wheels are made from strawboard paper disks and are cemented together under pressure to form a very hard wheel for rough work. Softer wheels are similarly made from felt paper. Walrus leather or sea-horse hide may be used for fine polishing, but these wheels are expensive. The "compress" canvas wheel is commonly used in place of walrus wheels. This compress type of wheel has a cushion of polishing material formed by pieces of leather, canvas, felt, or whatever material is used, which are held in a crosswise radial position by two side plates attached to the wheel hub. This cushion of polishing material may be varied in density to suit the requirements; it may readily be shaped to conform to the curvature of the work and this shape can be maintained. Sheepskin polishing wheels and also paper wheels are used very little at the present time.

Polishing Operations and Abrasives. — Polishing operations on such parts as chisels, hammers, screwdrivers, wrenches, and other parts which are given a fine finish but are not plated, usually require four operations which are "roughing," "dry fining," "greasing" and "coloring." The roughing is frequently regarded as a solid grinding wheel job. Sometimes there are two steps to the greasing operation — rough and fine greasing. For some hardware, such as the cheaper screwdrivers, wrenches, etc., the operations of roughing and dry fining are considered sufficient. For knife blades and cutlery the roughing operation is performed with solid grinding wheels and the polishing is known as fine or blue glazing, but these terms are never used when referring to the polishing of hardware parts, plumbers' supplies, etc. A term used in finishing German silver, white metal, and similar materials is "sand-buffing," which, in distinction from the ordinary buffing operation that is used only to produce a very high finish, actually removes considerable metal, as in rough polishing or flexible grinding. For sand-buffing, rotten-stone and pumice are loosely applied.

For the finer finishing and coloring work, emery is employed quite generally in preference to artificial abrasives. The abrasive numbers used for roughing ordinarily range from 60 to 80; for dry fining from 90 to 120; and for finishing from 150 to one of the "flour" grades.

Buffing Wheels. — Buffing wheels, as defined by the Metal Finishers' Equipment Association, are wheels manufactured from disks (either whole or pieced) of bleached or unbleached cotton or woolen cloth, and they are used as the agent for carrying abrasive powders, such as tripoli, crocus, rouge, lime, etc., which are mixed with waxes or greases as a bond. There are two main classes of buffs known as the "pieced-sewed" buffs, which are made from various weaves and weights of cloths, and the "full disk" buffs which are made from the best sheeting and shirting. Bleached cloth is harder and stiffer than unbleached cloth, and is used for the faster cutting buffs. Coarsely woven unbleached cloth is recommended for highly colored work on soft metals, while the finer woven unbleached cloths are better adapted for the harder metals. A stiff buff when working at the usual speed is not suitable for "cutting down" soft metal or for use on light plated ware, but is used on the harder metals and for heavy nickel-plated articles.

Speed of Polishing Wheels. — The proper speed for polishing is governed to some extent by the nature of the work, but for ordinary operations the polishing wheel, according to one manufacturer, should have a peripheral speed of about 7500 feet per minute. If run at a lower rate of speed, the work tends to tear the polishing material from the wheel too readily, and the work is not as good in quality. Another manufacturer recommends the following speeds: Muslin, felt or sea-horse polishing wheels having wood or iron centers should be run at peripheral speeds varying from 3000 to 7000 feet per minute. It is rarely necessary to exceed 6000 feet per minute, and for most purposes 4000 feet per minute is sufficient. If the wheels are kept in good condition, in perfect balance, and are suitably mounted on substantial buffing lathes, they are safe for speeds within the limits given.

Grain Numbers of Emery. — The numbers commonly used in designating the different grains of emery, corundum and other abrasives are 10, 12, 14, 16, 18, 20, 24, 30, 36, 40, 46, 54, 60, 70, 80, 90, 100, 120, 150, 180 and 200, ranging from coarse to fine. These numbers represent the number of meshes per linear inch in the grading sieve. An abrasive finer than No. 200 is known as "flour" and the degree of fineness is designated by the letters CF, F, FF, FFF, FFFF and PCF or SF, ranging from coarse to fine. The methods of grading flour-emery adopted by different manufacturers do not exactly agree, the letters differing somewhat for the finer grades.

Grades of Emery Cloth. — The coarseness of emery cloth is indicated by letters and numbers corresponding to the grain number of loose emery. The letters and numbers for grits ranging from fine to coarse are as follows: FF, F, 120, 100, 90, 80, 70, 60, 54, 46, 40. For large work roughly filed, use coarse cloth such as Nos. 46 or 54, and then finer grades to obtain the required polish. If the work has been carefully filed, a good polish can be obtained with Nos. 60 and 90 cloth, and a brilliant polish by finishing with No. 120 and flour-emery.

Mixture for Cementing Emery Cloth to a Lapping Wheel. — Use 4½ pounds of rosin; 3 pounds of paraffine; 9 ounces of vaseline; melt the ingredients and mix them thoroughly. Heat the surface of the lapping wheel and spread on the mixture; then rub the emery cloth down so as to exclude all air from between the surface of the wheel and cloth. The surface of the lapping wheel should be clean before the cement is applied.

Exhaust Systems for Grinding, Polishing and Buffing Wheels

Defects in Exhaust Systems. — The principal defects in the exhaust systems for grinding, polishing and buffing wheels are as follows: 1. Making the suction duct too small and, not infrequently, of the same size throughout its length. 2. Running the branch pipes into the main suction pipe at right angles, and sometimes at the bottom of the main. 3. Providing a fan too small for the service. 4. Using a discharge pipe too small for the fan. 5. Using a cyclone separator or dust separator too small for the system. The result of such mistakes in the design of exhaust systems is that the suction is entirely inadequate for carrying off the dust, which then clogs the pipes and spreads about the room.

Branch Pipe Specifications. — The following specifications for the design, construction and operation of exhaust systems conform to Section 81 of the New York State Labor Law: The diameter of branch pipes leading from the wheel hoods to the main suction duct must conform to the sizes given in the accompanying table, "Diameters of Branch Pipes for Grinding and Polishing Wheel Exhaust Systems." In case the grinding wheel is thicker than is given in this table, or if a disk instead of a regular wheel is used, it must have a branch pipe not smaller than is required for the grinding surface given. Buffing wheels six inches or less in diameter, used for jewelry work, may have a three-inch branch pipe. The thickness given for buffing wheels applies to the thickness of the wheel at the center. In case the wheel is thicker than is given in this table, it must have a branch pipe no smaller than is called for by its grinding surface. Branch pipes must not be smaller than the sizes specified, throughout their entire length. All branch pipes must enter the main suction duct at an angle not exceeding forty-five degrees and should be inclined in the direction of the air flow at the junction with the main. Branch pipes must not project into the main duct. All laps in piping must be made in the direction of the air flow. All bends, turns or elbows, whether in main or branch pipes, must be made with a radius in the throat equal to at least one and one-half times the diameter of the pipe to which they are connected.

Diameters of Suction Ducts for Exhaust Systems

The table gives the diameter in inches of the main suction duct at any point for any number of uniform-size branch pipes when the area of the main at any point is made equal to the combined areas of the branch pipes preceding that point, plus twenty per cent.

Number of Branch Pipes	Diameter of Branch Pipes, Inches								
	3	3½	4	4½	5	5½	6	6½	7
	Area of each Branch Pipe in Square Inches								
	7.07	9.62	12.566	15.9	19.635	23.758	28.274	33.183	38.485
	Area of each Branch Pipe plus 20 per cent (Square Inches)								
	8.484	11.544	15.08	19.08	23.562	28.51	33.93	39.82	46.182
1	3⅜	3⅞	4⅜	5	5½	6	6⅝	7⅛	7¾
2	4¾	5½	6¼	7	7¾	8⅝	9¼	10⅛	10⅞
3	5¾	6⅝	7⅝	8⅝	9½	10½	11½	12⅜	13¼
4	6⅝	7¾	8¾	9⅞	11	12⅛	13⅛	14¼	15⅜
5	7⅜	8⅝	9⅞	11	12¼	13½	14¾	16	17⅛
6	8⅛	9½	10¾	12⅛	13½	14¾	16⅛	17½	18¾
7	8¾	10¼	11⅝	13⅛	14½	16	17½	18⅞	20¼
8	9⅜	10⅞	12⅜	14	15½	17⅛	18⅝	20⅛	21¾
9	9⅞	11½	13⅛	14⅞	16½	18⅛	19¾	21⅜	23
10	10½	12⅛	13⅞	15⅝	17⅜	19⅛	20¾	22½	24¼
11	11	12¾	14⅝	16⅜	18¼	20	21⅞	23⅝	25½
12	11½	13⅜	15¼	17⅛	19	20⅞	22¾	24¾	26⅝
13	11⅞	13⅞	15⅞	17⅞	19¾	21¾	23¾	25¾	27¾
14	12⅜	14⅜	16½	18½	20½	22⅝	24⅝	26¾	28¾
15	12¾	14⅞	17	19⅛	21¼	23⅜	25½	27⅝	29¾
16	13¼	15⅜	17⅝	19¾	22	24⅛	26⅜	28½	30¾
17	13⅝	15⅞	18⅛	20⅜	22⅝	24⅞	27⅛	29⅜	31⅝
18	14	16⅜	18⅝	21	23¼	25⅝	27⅞	30¼	32⅝
19	14⅜	16¾	19⅛	21½	23⅞	26¼	28¾	31⅛	33½
20	14¾	17⅛	19⅝	22⅛	24½	27	29½	31⅞	34⅜
21	15⅛	17⅝	20⅛	22⅝	25⅛	27⅝	30⅛	32¾	35⅛
22	15½	18	20⅝	23⅛	25¾	28⅜	30⅞	33½	36
23	15¾	18½	21⅛	23¾	26⅜	29	31½	34¼	36¾
24	16⅛	18⅞	21½	24¼	26⅞	29⅝	32¼	34⅞	37⅝
25	16½	19¼	22	24¾	27½	30⅛	32⅞	35⅝	38⅜
26	16¾	19⅝	22⅜	25⅛	28	30¾	33½	36⅜	39⅛
27	17⅛	20	22⅞	25⅝	28½	31⅜	34⅛	37	39⅞
28	17½	20⅜	23¼	26⅛	29	32	34¾	37¾	40⅝
29	17¾	20¾	23⅝	26⅝	29½	32½	35½	38⅜	41⅜
30	18	21	24	27	30	33	36	39	42

Area of Fan Inlet and Main Suction Duct. — The inlet of the fan or exhauster should be at least twenty per cent greater in area than the sum of the areas of all the branch pipes, and such increase should be carried proportionately throughout the entire length of the main suction duct; that is, the area of the main at any given point should be at least twenty per cent greater than the combined areas of all branch pipes entering it between that point and the closed end of the system. If such increase is made greater than twenty per cent, the area of the main at any point (except that proportion of it between the branch pipe nearest the fan, and the fan) should bear approximately the same ratio to the combined areas of the branches between that point and the closed end of the system, as the area of the main at the branch nearest the fan bears to the combined areas of all the branches. (This provision is made to permit the use of a fan having a larger inlet area than the area of the main at the branch pipe nearest to the fan, if desired.) The table, "Diameters of Suction Ducts for Exhaust Systems," gives the sizes of main suction ducts:

Diameters of Branch Pipes for Grinding and Polishing Wheel Exhaust Systems

Emery or Other Grinding Wheels				Buffing, Polishing or Rag Wheels			
Diameter of Wheel, Inches	Max. Thickness of Wheel	Max. Grinding Surface, Square Inches	Min. Diameter of Branch Pipe, Inches	Diameter of Wheel, Inches	Max. Thickness of Wheel	Max. Grinding Surface, Square Inches	Min. Diameter of Branch Pipe, Inches
Up to 6	1	19	3	Up to 6	1	19	3½
7–9	1½	43	3½	7–12	1½	57	4
10–16	2	101	4	13–16	2	101	4½
17–19	3	180	4½	17–20	3	189	5
20–24	4	302	5	21–24	4	302	5½
25–30	5	472	6	25–30	5	472	6½

Suction and Discharge Pipe Sizes. — The area of the discharge pipe from the fan should be as large or larger than the area of the fan inlet, throughout its entire length. In the main trunk lines, both suction and discharge should be provided with suitable "clean-out doors" not over ten feet apart, and the end of the main suction duct should be blanked off with a removable cap.

Suction Head. — Sufficient static suction head should be maintained in each branch pipe within one foot of the hood, to produce a difference in level of two inches of water between the two sides of a U-shaped tube. The test is to be made by placing one end of a rubber tube over a small hole made in the pipe, the other end of the tube being connected to one side of the U-shaped water-gage. All branch pipes must be open and unobstructed while test is made.

In addition to the foregoing specifications, which are compulsory under the New York law, a number of recommendations are given in the following which are designed to make exhaust systems more efficient and durable.

Arrangement of Suction and Discharge Pipes. — In the case of undershot wheels (the top of the wheel running towards the operator, which is almost invariably the direction of rotation of both emery and buffing wheels), the main suction duct should be back of and below the wheels, and as close to them as practicable. Sometimes it is preferable to fasten the suction duct to the ceiling of the floor below. Both the main suction and discharge pipes should be short and have as few bends as possible to avoid frictional losses. If one or the other must be of considerable length, it is better to place the fan quite close to the nearest branch pipe which enters the large end of the main, as a long discharge pipe is preferable to a long suction pipe. Avoid any pockets or low places in the ducts where dust might accumulate. The main suction duct should be enlarged between every branch pipe entering it, and in no case should it receive more than two branches in a section of uniform area. All enlargements in the size of the main suction duct should be made by tapering it and not by an abrupt change of diameter.

Arrangement of Branch Pipes. — Branch pipes should enter the main at the top or sides — never at the bottom. Two branches should never enter a main directly opposite each other. Each branch pipe should be equipped with a shut-off damper or "blast gate" which may be closed, if desired, when the wheel is not in use. Not more than twenty-five per cent of the blast gates should be closed at one time, because the air velocity in the main duct may drop too low and let the dust accumulate on the bottom. The lower part of the wheel hood should extend far enough beneath the front of the wheel, so that the dust will enter it and not fall outside. This should be done even though it is necessary to leave considerable space between the wheel and the lower part of the hood to prevent interference with the work. Branch pipes should join with the hood as near as possible at the point where the dust will naturally be thrown into them by the wheels; this is very important. A screen across the mouth of the branch pipe, where it enters the hood, is objectionable, because it obstructs the passage of material and the ravelings from buffing wheels, with the result that within a short time the draft is entirely cut off. It is good practice to use a trap at the junction of the hood and branch pipe, provided it is cleaned out regularly and not allowed to fill with dust. It will catch the heavier particles and so take some wear off the fan. All bends, turns or elbows, whether in main or branch pipes, should be made with a radius in the throat equal to twice the diameter of the pipe to which they are connected, wherever space permits. Elbows should be made of metal one or two gages heavier than the pipe to which they are connected, as the wear on them is much greater.

The Cyclone Separator. — The size of the cyclone separator or dust collector is governed by the operating conditions, light dusts requiring a larger separator than heavy dusts. The separator should have an inlet area at least as large as the area of the fan discharge pipe. For light buffing dusts, lint, etc., the air outlet from the top of the separator should be large enough so that the velocity of the discharge will not exceed 300 to 480 feet per minute. A separator should then be selected having other dimensions in proportion. The air outlet should be provided with a canopy or elbow to exclude the weather, but should be otherwise unobstructed. There should be ample clearance in the separator for the accumulation or storage of dust, which should never be allowed to pile up as high as the bottom of the separator.

Emery and buffing wheel exhaust systems should be kept separate, owing to the danger of sparks from the former setting fire to the lint dust in the latter. The withdrawal of air from a room by an exhaust system naturally tends to create a slight vacuum, and for this reason, inlets for air, at least equal to the sum of the areas of the branch pipes, should be left open.

Grindstones and Oilstones

Grindstones. — Most of the grindstones used in this country come from Huron, Mich.; Berea, Ohio; or from Grindstone Island, Nova Scotia. All of these localities produce several grades. Most Berea stones are rather coarse; those from Nova Scotia are of all grades. Grindstones are natural sand-stones, and the cutting material is oxide of silicon (SiO_2), or quartz sand, as it is commonly called. Grindstones are softer when wet than when dry, and they should never be left standing with one side in the water, because when the stone is again used, this side will be worn away faster than the other. The large, rapidly revolving stones used in connection with some manufacturing operations, for producing smooth surfaces, are hacked around the periphery to make them cut faster. A hack, which somewhat resembles an adz, should have an edge that is a little longer than one-half the width of the stone. When hacking, the tool is held at a slight angle while cutting around one-half the face of the stone; the angle of the hack is then changed for finishing the other half, the cuts being about 1¼ inch apart. As the stone wears away, this operation is repeated. The high spots are hacked closely to increase the wear and keep the stone true.

Speeds for Grindstones. — The proper speed for a grindstone depends upon its use. For grinding machinists' tools, the peripheral speed should range from 800 to 1000 feet per minute, and for carpenters' tools, from 500 to 600 feet per minute. When grindstones are used for smoothing surfaces preliminary to polishing operations, they are run at much higher speeds. One prominent cutlery concern operates Huron stones at 4300 feet per minute, and soft stones at approximately 3600 feet per minute. The maximum speed is, of course, limited by the strength of the stone. It is difficult to determine what the safe maximum speed is, because stones from the same quarry vary in strength. Some Sheffield grinders run their stones at 4500 feet per minute, and others limit the speed to 2500 feet per minute. According to some authorities, the speeds given in the table "Maximum Speeds for Grindstones" should not be exceeded unless the stone is very hard and strong. The number of revolutions given in this table, for various diameters, corresponds to a peripheral speed of approximately 3400 feet per minute.

Maximum Speeds for Grindstones

Diameter of Stone	Revolutions per Minute	Diameter of Stone	Revolutions per Minute	Diameter of Stone	Revolutions per Minute
3 feet	365	4 feet 6 inches	240	6 feet	180
3 feet 6 inches	308	5 feet	216	6 feet 6 inches	166
4 feet	270	5 feet 6 inches	196	7 feet	154

Tests made in the Sheffield district show that the strength of a wet grindstone is considerably less than one in a dry condition. The water which soaks into the stone not only reduces its tensile strength, but also increases the weight and centrifugal stress for the same peripheral speed. This reduction of strength in the case of a wet grindstone was found in some cases to be as much as 40 or 50 per cent. For example, a stone of one-square-inch section broke under a stress of 146 pounds when dry, but when soaked in water over night, another piece of the same stone broke at only 80 pounds per square inch. In another test, the figures were 186 and 116 pounds for the dry and wet stones, respectively.

Doubtless some of the speeds given as permissible for grindstones are excessive, except for very strong stones. The following figures are the result of a careful investigation, and are given as the *safe* maximum speed for Ohio and Huron grindstones, which are the two varieties most widely used. The velocity for Ohio stones should never exceed 3000 feet per minute and, ordinarily, should not be higher than 2500 feet per minute. Huron stones may be run at 3500 feet per minute, but it is safer to limit them to 3000 feet per minute. If the variety of the stone is not known, the speed should be limited to 2500 feet per minute.

Mounting Grindstones. — Many grindstone failures have resulted from improper mounting, rather than from too high a speed. The use of wooden wedges has been the principal source of trouble. These are either driven in too tight or become wet after being inserted, so that cracks are started in the corners of the square hole in the stone. These cracks extend outward and so weaken the stone that rupture is frequently the result. The tendency for cracks to start can be overcome by a proper method of mounting. It is good practice to fill the central space around the arbor with cement or lead after the stone is centered. Wooden wedges should never be used. The stone should be supported by flanges of generous proportions, and wooden washers from ½ to 1 inch in thickness (or a double thickness of leather or rubber) should be inserted between the flanges and the stone to compensate for surface inequalities.

Oilstones. — The natural oilstones commonly used are the Washita and Arkansas. The Washita is a coarser and more rapidly cutting stone then the Arkansas, and is generally considered the most satisfactory for sharpening woodworkers' tools. There are various grades of Washita rock, varying from the perfect crystallized and porous whetstone grit, to vitreous flint and hard sandstone. The sharpness of the grit of any Washita stone depends entirely upon the character of its crystallization. The best whetstones are porous and uniform in texture and are composed entirely of silica crystals. The poorer grades are less porous, making them vitreous or "glassy." They may also have hard spots or sand holes, or contain grains of sand among the crystals. For general work, a soft, free-grit, quick-cutting stone is required, although a fine-grit medium-hard stone is sometimes preferable. Washita stones are sometimes white in color, but frequently streaked more or less with a yellow or red tinge. They are found in the spurs of the Ozark mountains of Arkansas.

The Arkansas stones are of finer grain and appear like white marble. They are excellent for sharpening delicate instruments, producing keen, smooth edges. The Arkansas stone is harder, more transparent and more compact than the Washita, has an exceedingly sharp grit, and will both cut and polish very hard metals. This rock is quarried with difficulty, for it is so badly cracked, seamed and streaked with quartz that only a small portion of sound, pure rock can be obtained of sufficient size and quality for whetstones. The Arkansas stone is used more frequently in machine shops than the Washita. The "soft Arkansas" is a grade between the regular Arkansas and the Washita stone. It is not quite so fine grained and hard as the regular Arkansas, but cuts faster and is better for some purposes. The soft grade is especially adapted for sharpening the tools of wood-carvers, pattern-makers, etc.

The Turkey oilstone is well-known throughout the world, and until the introduction of the Washita was the leading oilstone for sharpening mechanics' tools. It is quarried in the interior of Asia Minor, and is a very fine close-grained stone, containing from 70 to 75 per cent silica, closely blended with from 20 to 25 per cent calcite. It has exceptional abrading qualities, but is found in small pieces and contains so many seams and flaws as to make it difficult to procure a real good stone of sufficient size to be serviceable.

The Hindostan is a fine-gritted sandstone quarried in Indiana. It is much softer than the Washita or other oilstones commonly used, and wears away much faster. It is very sharp-gritted, however, and cuts steel rapidly, imparting a medium coarse edge. This kind of stone may be used with oil or water.

Many artificial oilstones are now used for various classes of work. These are commonly furnished in three grits: *viz.*, fine, medium and coarse, and in all required shapes. Coarse stones are used for sharpening large and very dull tools, nicked tools, machine knives, and for general use where fast cutting is required without regard to fine finish. Medium stones are used for sharpening mechanics' tools in general especially those used by carpenters and in wood-working establishments. The medium grit gives a medium fine edge, well suited for working soft wood, cloth, leather, rubber and paper cutting machine or hand knives. Fine stones are used by machinists and engravers, die workers, instrument makers, cabinet makers, and all users of tools requiring a very fine, keen edge. Some artificial stones have one medium and one coarse face, thus combining two stones in one.

Truing Surfaces of Oilstones. — Oilstones which have uneven surfaces can be trued by the following method: Secure a cast-iron block having a true surface, and cover the surface with loose emery mixed with water; then place the oilstone upon the cast-iron block and grind it true. This method is applicable to either coarse oilstones or fine razor hones. Stones of special shape may be formed by planing a groove of corresponding shape in the cast-iron block and drawing the stone through the groove, using emery and water as an abrasive.

Care of Oilstones. — Oilstones should be properly cared for: first, in order to retain the original life and sharpness of the grit; second, to keep the surface flat and even; third, to prevent glazing. The following instructions are given by the Pike Mfg. Co.: An oilstone should be kept clean and moist; allowing it to remain dry a long time, or exposing it to the air, tends to harden it. A new stone should be soaked in oil for several days before using (with the exception of Pike India and Pike Crystolon). If the stone is kept in a dry place, it should be placed in a box having a closed cover, and a few drops of fresh, clean oil should be placed on it. To restore an even flat surface on an oilstone, grind it on the side of a grindstone, or rub it down with sand-stone or an emery brick.

An oilstone can be prevented from glazing by the proper use of oil or water. Either oil or water will prevent the particles of steel that are cut away from the tool being sharpened from filling the surface of the stone. Plenty of water should be used on all coarse-grained natural stones; on medium- or fine-grained natural stones, such as Arkansas or Washita, as well as on all artificial stones, oil should be used invariably, as water is not thick enough to keep the steel particles out of the pores. To further prevent glazing, dirty oil should always be wiped off the stone as soon as possible after using. This is very important, for if the oil is left on the stone, it dries in, carrying steel dust with it. Cotton waste is one of the best things for cleaning a stone. If a stone does become glazed or gummed up, cleaning with gasoline or ammonia will usually restore its cutting qualities; but if this treatment is not effective, scour the stone with loose emery or a piece of sand-paper fastened to a flat board.

Laps and Lapping

Material for Laps. — Laps are usually made of soft cast iron, copper, brass or lead. In general, the best material for laps to be used on very accurate work is soft, close-grained cast iron. If the grinding, prior to lapping, is of inferior quality, or an excessive allowance has been left for lapping, copper laps may be preferable. They can be charged more easily and cut more rapidly than cast iron, but do not produce as good a finish. Whatever material is used, the lap should be softer than the work, as, otherwise, the latter will become charged with the abrasive and cut the lap, the order of the operation being reversed. A common and inexpensive form of lap for holes is made of lead which is cast around a tapering steel arbor. The arbor usually has a groove or keyway extending lengthwise, into which the lead flows, thus forming a key that prevents the lap from turning. When the lap has worn slightly smaller than the hole and ceases to cut, the lead is expanded or stretched a little by the driving in of the arbor. When this expanding operation has been repeated two or three times, the lap usually must be trued or replaced with a new one, owing to distortion.

The tendency of lead laps to lose their form is an objectionable feature. They are, however, easily molded, inexpensive, and quickly charged with the cutting abrasive. A more elaborate form for holes is composed of a steel arbor and a split cast-iron or copper shell which is sometimes prevented from turning by a small dowel pin. The lap is split so that it can be expanded to accurately fit the hole being operated upon. For hardened work, some toolmakers prefer copper to either cast iron or lead. For holes varying from 1/4 to 1/2 inch in diameter, copper or brass is sometimes used; cast iron is used for holes larger than 1/2 inch in diameter. The arbors for these laps should have a taper of about 1/4 or 3/8 inch per foot. The length of the lap should be somewhat greater than the length of the hole, and the thickness of the shell or lap proper should be from 1/8 to 1/6 its diameter.

External laps are commonly made in the form of a ring, there being an outer ring or holder and an inner shell which forms the lap proper. This inner shell is made of cast iron, copper, brass or lead. Ordinarily the lap is split and screws are provided in the holder for adjustment. The length of an external lap should at least equal the diameter of the work, and might well be longer. Large ring laps usually have a handle for moving them across the work.

Laps for Flat Surfaces. — Laps for producing plane surfaces are made of cast iron. In order to secure accurate results, the lapping surface must be a true plane. A flat lap that is used for roughing or "blocking down" will cut better if the surface is scored by narrow grooves. These are usually located about 1/2 inch apart and extend both lengthwise and crosswise, thus forming a series of squares similar to those on a checker-board. An abrasive of No. 100 or 120 emery and lard oil can be used for charging the roughing lap. For finer work, a lap having an unscored surface is used, and the lap is charged with a finer abrasive. After a lap is charged, all loose abrasive should be washed off with gasoline, for fine work, and when lapping, the surface should be kept moist, preferably with kerosene. Gasoline will cause the lap to cut a little faster, but it evaporates so rapidly that the lap soon becomes dry and the surface caked and glossy in spots. Loose emery should not be applied while lapping, for if the lap is well charged with abrasive in the beginning, is kept well moistened and not crowded too hard, it will cut for a considerable time. The pressure upon the work should be just enough to insure constant contact. The lap can be made to cut only so fast, and if excessive pressure is applied it will become "stripped" in places. The causes of scratches are: Loose abrasive on the lap; too much pressure on the work, and poorly graded abrasive. To produce a perfectly smooth surface free from scratches, the lap should be charged with a very fine abrasive.

Grading Abrasives for Lapping. — For high-grade lapping, abrasives can be evenly graded as follows: A quantity of flour-emery or other abrasive is placed in a heavy cloth bag, which is gently tapped, causing very fine particles to be sifted through. When a sufficient quantity has been obtained in this way, it is placed in a dish of lard or sperm oil. The largest particles will then sink to the bottom and in about one hour the oil should be poured into another dish, care being taken not to disturb the sediment at the bottom. The oil is then allowed to stand for several hours, after which it is poured again, and so on, until the desired grade is obtained.

Charging Laps. — To charge a flat cast-iron lap, spread a very thin coating of the prepared abrasive over the surface and press the small cutting particles into the lap with a hard steel block. There should be as little rubbing as possible. When the entire surface is apparently charged, clean and examine for bright spots; if any are visible, continue charging until the entire surface has a uniform gray appearance. When the lap is once charged, it should be used without applying more abrasive until it ceases to cut. If a lap is over-charged and an excessive amount of abrasive is used, there is a rolling action between the work and lap which results in inaccuracy. The surface of a flat lap is usually finished true, prior to charging, by scraping and testing with a standard surface-plate, or by the well-known method of scraping-in three plates together, in order to secure a plane surface. In any case, the bearing marks or spots should be uniform and close together. These spots can be blended by covering the plates evenly with a fine abrasive and rubbing them together. While the plates are being ground in, they should be carefully tested and any high spots which may form should be reduced by rubbing them down with a smaller block.

To charge cylindrical laps for internal work, spread a thin coating of prepared abrasive over the surface of a hard steel block, preferably by rubbing lightly with a cast-iron or copper block; then insert an arbor through the lap and roll the latter over the steel block, pressing it down firmly to imbed the abrasive into the surface of the lap. For external cylindrical laps, the inner surface can be charged by rolling-in the abrasive with a hard steel roller that is somewhat smaller in diameter than the lap. The taper cast-iron blocks which are sometimes used for lapping taper holes can also be charged by rolling-in the abrasive, as previously described; there is usually one roughing and one finishing lap, and when charging the former, it may be necessary to vary the charge in accordance with any error which might exist in the taper.

Rotary Diamond Lap. — This style of lap is used for accurately finishing very small holes, which, because of their size, cannot be ground. While the operation is referred to as lapping, it is, in reality, a grinding process, the lap being used the same as a grinding wheel. Laps employed for this work are made of mild steel, soft material being desirable because it can be charged readily. Charging is usually done by rolling the lap between two hardened steel plates. The diamond dust and a little oil is placed on the lower plate, and as the lap revolves, the diamond is forced into its surface. After charging, the lap should be washed in benzine. The rolling plates should also be cleaned before charging with dust of a finer grade. It is very important not to force the lap when in use, especially if it is a small size. The lap should just make contact with the high spots and gradually grind them off. If a diamond lap is lubricated with kerosene, it will cut freer and faster. These small laps are run at very high speeds, the rate depending upon the lap diameter. Soft work should never be ground with diamond dust because the dust will leave the lap and charge the work.

When using a diamond lap, it should be remembered that such a lap will not produce sparks like a regular grinding wheel; hence, it is easy to crowd the lap and "strip" some of the diamond dust. To prevent this, a sound intensifier or "harker" should be used. This is placed against some stationary part of the grinder spindle, and indicates when the lap touches the work, the sound produced by the slightest contact being intensified.

Grading Diamond Dust. — The grades of diamond dust used for charging laps are designated by numbers, the fineness of the dust increasing as the numbers increase. The diamond, after being crushed to powder in a mortar, is thoroughly mixed with high-grade olive oil. This mixture is allowed to stand five minutes and then the oil is poured into another receptacle. The coarse sediment which is left is removed and labeled No. 0, according to one system. The oil poured from No. 0 is again stirred and allowed to stand ten minutes, after which it is poured into another receptacle and the sediment remaining is labeled No. 1. This operation is repeated until practically all of the dust has been recovered from the oil, the time that the oil is allowed to stand being increased as shown by the following table, in order to obtain the smaller particles that require a longer time for precipitation:

To obtain No. 1 — 10 minutes.	To obtain No. 4 — 2 hours.
To obtain No. 2 — 30 minutes.	To obtain No. 5 — 10 hours.
To obtain No. 3 — 1 hour.	To obtain No. 6 — until oil is clear.

The No. 0 or coarse diamond which is obtained from the first settling is usually washed in benzine, and re-crushed unless very coarse dust is required. This No. 0 grade is sometimes known as "ungraded" dust. In some places the time for settling, in order to obtain the various numbers, is greater than that given in the table.

Cutting Properties of Laps and Abrasives. — In order to determine the cutting properties of abrasives when used with different lapping materials and lubricants, a series of tests was conducted, the results of which were given in a paper by W. A. Knight and A. A. Case, presented before the American Society of Mechanical Engineers. In connection with these tests, a special machine was used, the construction being such that quantitative results could be obtained with various combinations of abrasive, lubricant, and lap material. These tests were confined to surface lapping.

It was not the intention to test a large variety of abrasives, three being selected as representative; namely, Naxos emery, carborundum, and alundum. Abrasive No. 150 was used in each case, and seven different lubricants, five different pressures, and three different lap materials were employed. The lubricants were lard oil, machine oil, kerosene, gasoline, turpentine, alcohol, and soda water.

These tests indicated throughout that there is, for each different combination of lap and lubricant, a definite size of grain that will give the maximum amount of cutting. With all the tests, except when using the two heavier lubricants, some reduction in the size of the grain below that used in the tests (No. 150) seemed necessary before the maximum rate of cutting was reached. This reduction, however, was continuous and soon passed below that which gave the maximum cutting rate.

Cutting Qualities with Different Laps. — The surfaces of the steel and cast-iron laps were finished by grinding. The hardness of the different laps, as determined by the scleroscope was, for cast-iron, 28; steel, 18; copper, 5. The total amount ground from the test-pieces with each of the three laps showed that, taking the whole number of tests as a standard, there is scarcely any difference between the steel and cast iron, but that copper has somewhat better cutting qualities, although, when comparing the laps on the basis of the highest and lowest values obtained with each lap, steel and cast iron are as good for all practical purposes as copper, when the proper abrasive and lubricant are used.

Wear of Laps. — The wear of laps depends upon the material from which they are made and the abrasive used. The wear on all laps was about twice as fast with carborundum as with emery, while with alundum the wear was about one and one-fourth times that with emery. On an average, the wear of the copper lap was about three times that of the cast-iron lap. This is not absolute wear, but wear in proportion to the amount ground from the test-pieces.

Lapping Abrasives. — As to the qualities of the three abrasives tested, it was found that carborundum usually began at a lower rate than the other abrasives, but, when once started, its rate was better maintained. The performance gave a curve that was more nearly a straight line. The charge or residue as the grinding proceeded remained cleaner and sharper and did not tend to become pasty or muck-like, as is so frequently the case with emery. When using a copper lap, carborundum shows but little gain over the cast-iron and steel laps, whereas, with emery and alundum, the gain is considerable.

Effect of Different Lapping Lubricants. — The action of the different lubricants was found to depend upon the kind of abrasive and the lap material.

Lard and Machine Oil. — The test showed that lard oil, without exception, gave the higher rate of cutting, and that, in general, the initial rate of cutting is higher with the lighter lubricants, but falls off more rapidly as the test continues. The lowest results were obtained with machine oil, when using an emery-charged, cast-iron lap. When using lard oil and a carborundum-charged steel lap, the highest results were obtained.

Gasoline and Kerosene. — On the cast-iron lap, gasoline was superior to any of the lubricants tested. Considering all three abrasives, the relative value of gasoline, when applied to the different laps, is as follows: Cast iron, 127; copper, 115; steel, 106. Kerosene, like gasoline, gives the best results on cast iron and the poorest on steel. The values obtained by carborundum were invariably higher than those obtained with emery, except when using gasoline and kerosene on a copper lap.

Turpentine and Alcohol. — Turpentine was found to do good work with carborundum on any lap. With emery, turpentine did fair work on the copper lap, but, with the emery on cast-iron and steel laps, it was distinctly inferior. Alcohol gives the lowest results with emery on the cast-iron and steel laps.

Soda Water. — Soda water gives medium results with almost any combination of lap and abrasives, the best work being on the copper lap and the poorest, on the steel lap. On the cast-iron lap, soda water is better than machine or lard oil, but not so good as gasoline or kerosene. Soda water when used with alundum on the copper lap, gave the highest results of any of the lubricants used with that particular combination.

Lapping Pressures. — Within the limits of the pressures used, that is, up to 25 pounds per square inch, the rate of cutting was found to be practically proportional to the pressure. The higher pressures of 20 and 25 pounds per square inch are not so effective on the copper lap as on the other materials.

Wet and Dry Lapping. — With the "wet method" of using a surface lap, there is a surplus of oil and abrasive on the surface of the lap. As the specimen being lapped is moved over it, there is more or less movement or shifting of the abrasive particles. With the "dry method," the lap is first charged by rubbing or rolling the abrasive into its surface. All surplus oil and abrasive are then washed off, leaving a clean surface, but one that has embedded uniformly over it small particles of the abrasive. It is then like the surface of a very fine oilstone and will cut away hardened steel that is rubbed over it. While this has been termed the dry method, in practice, the lap surface is kept moistened with kerosene or gasoline.

Experiments on dry lapping were carried out on the cast-iron, steel, and copper laps used in the previous tests, and also on one of tin made expressly for the purpose. Carborundum alone was used as the abrasive and a uniform pressure of 15 pounds per square inch was applied to the specimen throughout the tests. In dry lapping, much depends upon the manner of charging the lap. The rate of cutting decreased much more rapidly after the first 100 revolutions than with the wet method. Considering the amounts ground off during the first 100 revolutions, and the best result obtained with each lap taken as the basis of comparison, it was found that with a tin lap, charged by rolling No. 150 carborundum into the surface, the rate of cutting, when dry, approached that obtained with the wet method. With the other lap materials, the rate with the dry method was about one-half that of the wet method.

Summary of Lapping Tests. — In order to present the results of the tests referred to in the foregoing in a more usable form, the principal facts are here summarized:

The initial rate of cutting does not greatly differ for different abrasives. There is no advantage in using an abrasive coarser than No. 150. The rate of cutting is practically proportional to the pressure. The wear of the laps is in the following proportions: cast iron, 1.00; steel, 1.27; copper, 2.62. In general, copper and steel cut faster than cast iron, but, where permanence of form is a consideration, cast iron is the superior metal. Gasoline and kerosene are the best lubricants to use with a cast-iron lap. Machine and lard oil are the best lubricants with copper or steel laps. They are least effective on a cast-iron lap.

High-speed Balancing

Static Balancing. — There are several methods of testing the standing or static balance of a circular part. A simple method that is sometimes used for flywheels, etc., is illustrated by the diagram, Fig. 1. An accurate shaft is inserted through the bore of the finished wheel, which is then mounted on carefully leveled "parallels" *A*. If the wheel is in an unbalanced state, it will turn until the heavy side is downward. When it will stand in any position as the result of counter-balancing and reducing the heavy portions, it is said to be in standing or static balance.

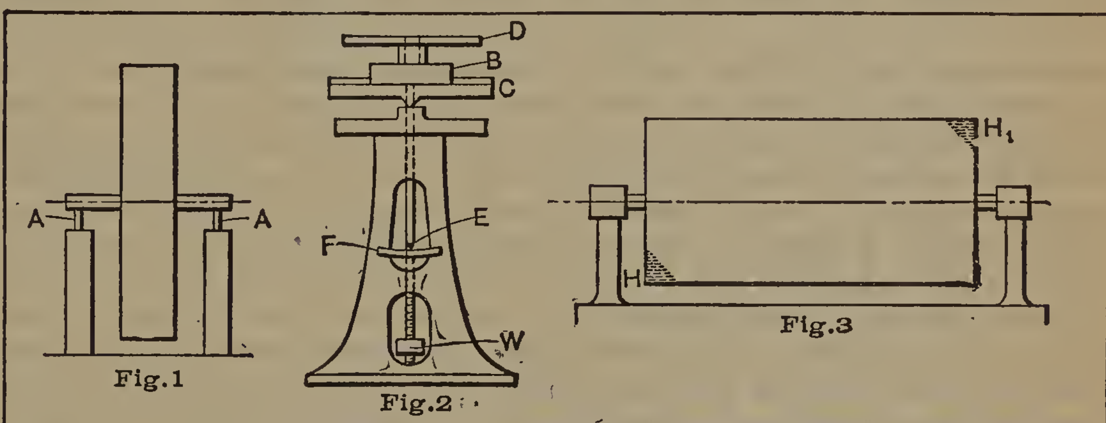


Fig. 1 Fig. 2 Fig. 3

Another test which is used for disk-shaped parts is shown in Fig. 2. The disk *D* is mounted on a vertical arbor attached to an adjustable cross-slide *B*. The latter is carried by a table *C*, which is supported by a knife-edged bearing. A pendulum having an adjustable screw-weight *W* at the lower end, is suspended from cross-slide *B*. To test the static balance of disk *D*, slide *B* is adjusted until pointer *E* of the pendulum coincides with the center of a stationary scale *F*. Disk *D* is then turned halfway around without moving the slide, and if the indicator remains stationary, it shows that the disk is in balance for this particular position. The test is then repeated for ten or twelve other positions, and the heavy sides are reduced, usually by drilling out the required amount of metal. There are several other devices for testing the static balance which are designed on this same principle.

Running Balance. — A cylindrical body may be in perfect static balance and not be in a balanced state when rotating at high speed. If the rotating part is in the form of a thin disk, static balancing, if carefully done, may be accurate enough for high speeds, but if the rotating part is long in proportion to its diameter, and the unbalanced portions are at opposite ends or in different planes, the balancing must be done so as to counteract the centrifugal force of these heavy parts when they are rotating rapidly. This is known as a running balance or dynamic balancing. To illustrate, if a heavy section is located at H (Fig. 3), and another correspondingly heavy section at H_1, one may exactly counter-balance the other when the cylinder is stationary, and this static balance may be sufficient for a part rigidly mounted and rotating at a comparatively slow speed; but when the speed is very high, as in the case of turbine rotors, etc., the heavy masses H and H_1, being in different planes, are in an unbalanced state owing to the effect of centrifugal force, which results in excessive strains and injurious vibrations. Theoretically, to obtain a perfect running balance, the exact position of the heavy sections should be located and the balancing effected either by reducing their weight or by adding counter-weights opposite each section and in the same plane at the proper radius; but if the rotating part is rigidly mounted on a stiff shaft, a running balance that is sufficiently accurate for practical purposes can be obtained by means of comparatively few counter-balancing weights located with reference to the unbalanced parts. A convenient method is to locate the weights in circumferential slots near the ends of the part to be balanced, with intermediate weights (if practicable), in case the rotating part is somewhat flexible. The balancing, then, consists in locating the position of the counter-balancing weights, the amount of the weight, their radii from the axis of rotation and their angular positions.

Locating the Heavy Side. — The first step in dynamic balancing is to locate the heavy sides. This may be done by mounting the cylindrical part in bearings which are spring supported and free to move in any direction. The cylinder is rotated at the normal speed and the "high sides" of the unbalanced body are marked on each end by holding chalk just near enough to touch them. The direction of rotation is then reversed and other chalk marks are made. If the two marks on either end are in different positions, they show that the heavy unbalanced section lies midway between them, but on which side of the cylinder can only be determined by trial. Attach counter-balancing weights at both ends (in circumferential pockets, if these have been provided) midway between the first and second marks. These weights should be heavy enough to more than counter-balance the heavy sections. If the weights are on the same side as the heavy sections, another set of marks made on the rotating cylinder will coincide with those previously made, when the cylinder is again rotated in the same direction. But if the weights are opposite the heavy spots and are heavy enough to more than counter-balance them, the new marks will be opposite the original ones, thus indicating that the balancing weights are in the proper location. The balancing is then completed by reducing the amount of first one weight and then the other, until the cylinder will rotate at the normal speed without vibration. The temporary weights should then be made permanent.

When an unbalanced body is rotated at high speed, the heavy side may or may not be farthest from the axis of rotation, depending upon whether the "critical speed" (see "Critical Speed of Rotating Body") has been exceeded or not. That is, when a pointer or piece of chalk is held close to an unbalanced rotating part, the mark indicating the "high" side may be directly over the heavy spot, directly opposite it, or at some intermediate point. This is due to the fact that the relation between the high side and the heavy side may be governed by different factors; hence, when testing the running balance, the direction of rotation is reversed, as previously explained, to obtain a central position, between the two sets of marks, which bears a relation to all the factors.

If a comparatively long cylindrical body intended for high rotative speeds can be designed so as to be formed of a series of disk-shaped sections, a practical way of obtaining a running balance is to balance each section statically. This laminated construction, however, is often impracticable.

PUNCHES — DIES — PRESS WORK

Clearance between Punches and Dies. — The amount of clearance between a punch and die for blanking and perforating, or the difference between the size of the punch and die opening, is governed largely by the thickness of the stock to be operated upon. For thin material such as tin, for example, the punch should be a close sliding fit, as, otherwise, the punching will have ragged edges, but for heavier stock there should be some clearance, the amount depending upon the thickness of the material. The clearance between the punch and die when working heavy material lessens the danger of breaking the punch, and reduces the pressure required for the punching operation. To obtain the clearance between the punch and die, divide the thickness of the stock by a number or constant selected according to the following rules which apply to different materials: For soft steel and brass, divide the thickness of the stock by the constant 20; for medium rolled steel, divide by 16; for hard rolled steel, divide by 14.

Example: What would be the clearance between a punch and die to be used for perforating or blanking soft steel 0.050 inch thick?

$$\frac{\text{Thickness of stock}}{20} = \frac{0.050}{20} = 0.0025 \text{ inch.}$$

Whether this clearance is deducted from the diameter of the punch or added to the diameter of the die depends upon the nature of the work. If a blank of given size is required, the die is made to that size and the punch is made smaller. Inversely, when holes of a given size are required, the punch is made to correspond with the diameter wanted and the die is made larger. Therefore, for blanking to a given size, the clearance is deducted from the diameter of the punch, and for perforating, the clearance is added to the diameter of the die. To illustrate, suppose we want to blank hard rolled steel having a thickness of 0.0625 inch (No. 16 gage) to a diameter of 1 inch. What would be the sizes for the punch and die? The clearance equals $\frac{0.0625}{14}$ = 0.0044 inch. As this is a blanking operation, the die is made 1 inch, and the punch diameter equals 1 − 0.0044 = 0.9956 inch.

Clearances Between Punches and Dies for Different Materials

Thickness of Stock, Inches	Clearance, Inches			Thickness of Stock, Inches	Clearance, Inches		
	Brass, Soft Steel	Medium Rolled Steel	Hard Rolled Steel		Brass, Soft Steel	Medium Rolled Steel	Hard Rolled Steel
0.010	0.0005	0.0006	0.0007	0.120	0.0065	0.0072	0.0084
0.020	0.0010	0.0012	0.0014	0.140	0.0075	0.0084	0.0098
0.030	0.0015	0.0018	0.0021	0.160	0.0085	0.0096	0.0112
0.040	0.0020	0.0024	0.0028	0.180	0.0095	0.0108	0.0126
0.050	0.0025	0.0030	0.0035	0.200	0.0105	0.0120	0.0140
0.060	0.0030	0.0036	0.0042	0.220	0.0115	0.0132	0.0164
0.070	0.0035	0.0042	0.0049	0.240	0.0125	0.0144	0.0178
0.080	0.0045	0.0048	0.0056	0.260	0.0135	0.0156	0.0192
0.090	0.0050	0.0054	0.0063	0.280	0.0145	0.0168	0.0206
0.100	0.0055	0.0060	0.0070	0.300	0.0155	0.0178	0.0220

Angular Clearance for Dies. — The amount of angular clearance ordinarily given a blanking die varies from one to two degrees, although dies that are to be used for producing a comparatively small number of blanks are sometimes given a clearance angle of four or five degrees to facilitate making the die quickly. When a large number of blanks are required, a clearance of about one degree is used. There are two methods of giving clearance to dies: In one case the clearance extends to the top face of the die; in the other, there is a space about 1/8 inch below the cutting edge which is left practically straight, or having a very small amount of clearance. For very soft metal, such as soft, thin brass, the first method is employed, but for harder material, such as hard brass, steel, etc., it is better to have a very shallow clearance for a short distance below the cutting edge. When a die is made in this way, thousands of blanks can be cut with little variation in their size, as grinding the die face will not enlarge the hole to any appreciable extent.

Lubricants for Press Work. — Dies are often run without lubrication, but they will last longer if oiled slightly. The oil is applied to the stock either from a saturated felt-roller, brush or pad, or by coating one sheet thickly and then feeding it through the rolls. By the latter method, the rolls are coated with sufficient lubricant for a number of sheets, and a very thin coat is applied to the material so that the work does not have to be cleaned, as is sometimes necessary when a felt-roller or pad is used. Lard or sperm oil is used when punching iron, steel or copper. For drawing steel, the following mixture is recommended: 25 per cent flaked graphite; 25 per cent beef tallow; and 50 per cent lard oil. This mixture should be heated and the work dipped into it. Oildag mixed with heavy grease is also used for steel, and a thin mixture of grease (preferably tallow) and white lead has proved satisfactory. The following compound is also used for drawing sheet steel of a mild grade: Mix one pound of white lead, one quart of fish oil, three ounces of black lead, and one pint of water. These ingredients should be boiled until thoroughly mixed. For drawing brass and copper, a solution obtained by dissolving soap in hot water is often used. (Ivory soap has given good results.) The quantity of soap to use depends upon the thickness of the metal, a thin solution being preferable for thin stock. For cutting aluminum, use kerosene, and for drawing aluminum, use kerosene or vaseline of a cheap grade. Lard oil is also applied to aluminum when drawing deep shells. Aluminum should never be worked without a lubricant. For many classes of die work, no lubricant is required, especially when the metal is of a "greasy" nature, like tin plate, for instance.

Annealing Drawn Shells. — When drawing steel, iron, brass or copper, annealing is necessary after two or three draws have been made, as the metal is hardened by the drawing process. For steel and brass, anneal between every other reduction, at least. Tin plate or stock that cannot be annealed without spoiling the finish must ordinarily be drawn to size in one or two operations. Aluminum can be drawn deeper and with less annealing than the other commercial metals, provided the proper grade is used. In case it is necessary to anneal aluminum, this can be done by heating it in a muffle furnace, care being taken to see that the temperature does not exceed 700 degrees F.

Drawing Brass. — When drawing brass shells or cup-shaped articles, it is usually possible to make the depth of the first draw equal to the diameter of the shell. By heating brass to a temperature just below what would show a dull red in a dark room, it is possible to draw difficult shapes, otherwise almost impossible, and to get shapes with square corners.

Drawing Rectangular Shapes. — When square or rectangular shapes are to be drawn, the radius of the corners should be as large as possible, because it is in the corners that defects occur when drawing. Moreover, the smaller the radius, the less the depth which can be obtained in the first draw. The maximum depths which can be drawn with corners of a given radii are approximately as follows: With a radius of $\frac{3}{32}$ to $\frac{3}{16}$ inch, depth of draw, 1 inch; radius $\frac{3}{16}$ to $\frac{3}{8}$ inch, depth $1\frac{1}{2}$ inch; radius $\frac{3}{8}$ to $\frac{1}{2}$ inch, depth, 2 inches; radius $\frac{1}{2}$ to $\frac{3}{4}$ inch, depth, 3 inches. These figures are taken from actual practice and can doubtless be exceeded slightly when using extra good metal. If the box needs to be quite deep and the radius is quite small, two or more drawing operations will be necessary.

When Punch and Die should be Hardened. — The blanking or cutting dies used on comparatively thin stock, such as tin, brass, aluminum, iron, steel, copper, zinc, etc., are ordinarily hardened and tempered to suit the work, and the punch is left quite soft, so that it can be "hammered up" to fit the die when worn. This practice is followed in some plants for all metals less than $\frac{1}{16}$ inch thick which are not harder than iron or very mild steel. After the end of the punch has been upset by hammering, the punch and the die are oiled and forced together, which causes the hard die to shave the punch to a close fit. If the die is dull, it should be sharpened prior to this shearing operation. For some classes of work, the punch is made hard and the die soft. Both the punch and die should be hardened when they are to be used for blanking thick iron, steel, brass or other heavy metals.

Speeds and Pressures for Presses. — The speeds for presses equipped with cutting dies depend largely upon the kind of material being worked, and its thickness. For punching and shearing ordinary metals not over $\frac{1}{4}$ inch thick, the speeds usually range between 50 and 200 strokes per minute, 100 strokes per minute being a fair average. For punching metal over $\frac{1}{4}$ inch thick, geared presses with speeds ranging from 25 to 75 strokes per minute are commonly employed.

The cutting pressures required depend upon the shearing strength of the material, and the actual area of the surface being severed. For round holes the pressure required equals the circumference of the hole $\times$ the thickness of the stock $\times$ the shearing strength. To allow for some excess pressure, the tensile strength may be substituted for the shearing strength; the tensile strength for these calculations may be roughly assumed as follows: Mild steel, 60,000 pounds per square inch; wrought iron, 50,000 pounds; bronze, 40,000 pounds; copper, 30,000 pounds; aluminum, 20,000 pounds; zinc, 10,000 pounds; tin and lead, 5,000 pounds.

Pressure required for Punching. — The following approximate rule may be used for rapidly finding the pressure in tons required for punching circular holes in sheet steel: Multiply the diameter of the hole in inches by the thickness of the sheet steel and multiply this product by 80. The result is the pressure in tons required. To find the pressure required for punching holes in brass, multiply the diameter of the hole by the thickness, and multiply this product by 65.

Example: — What pressure is required for punching a hole 2 inches in diameter through $\frac{1}{4}$-inch steel stock? According to the rule, $2 \times \frac{1}{4} \times 80 = 40$ tons.

If a hole is not circular, use as a factor, instead of the diameter of the hole, one-third of the total length of the outline of the hole to be punched. For example, if a hole 1-inch square is to be punched through $\frac{1}{4}$-inch metal, the total outline (the four sides added together) is 4 inches. One-third of this is $1\frac{1}{3}$, and the pressure in tons required equals $1\frac{1}{3} \times \frac{1}{4} \times 80 = 26\frac{2}{3}$ tons.

Dinking Dies. — A dinking die is practically a hollow punch or cutter having a sharp cutting edge shaped to correspond with the contour of the part to be cut. Dies of this class are used for cutting forms from leather, cloth or paper. They are either operated in a press or are driven through the material by a mallet. The

Pressures Required for Punching

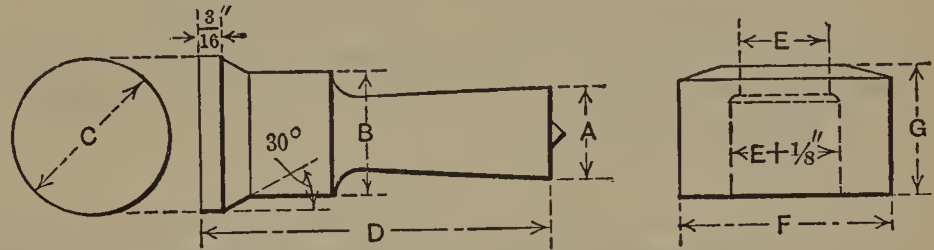
Pressure required to punch 0.25 per cent carbon steel of 65,000 pounds per square inch tensile strength. Circumference of hole × thickness of plate × 50,000 = pressure required in pounds, approximately.

Thickness of Plate, Inches	Diameter of Punch, Inches						
	1¼	1⅛	1	15/16	⅞	13/16	¾
	Pressure in Pounds						
⅛	24,600	22,100	19,600	18,400	17,200	16,000	14,700
3/16	36,800	33,100	29,500	27,600	25,800	24,000	22,100
¼	49,100	44,200	39,300	36,800	34,400	32,000	29,500
5/16	61,400	55,200	49,100	46,000	42,900	39,900	36,800
⅜	73,600	66,300	58,900	55,200	51,500	47,900	44,200
7/16	85,900	77,300	68,700	64,400	60,100	55,900	51,500
½	98,200	88,400	78,500	73,600	68,700	63,800	58,900
9/16	110,400	99,400	88,400	82,800	77,300	71,800	66,200
⅝	122,700	110,400	98,200	92,000	85,900	79,800	73,600
11/16	135,000	121,400	108,000	101,200	94,500	87,800	81,000
¾	147,300	132,500	117,800	110,400	103,100	95,800	88,400
13/16	159,500	143,500	127,600	119,600	111,700	103,700	95,700
⅞	171,800	154,600	137,400	128,800	120,300	111,700	103,100
15/16	184,100	165,600	147,200	138,000	128,800	119,700	......
1	196,400	176,700	157,100	147,200	137,400	......	......
1⅛	220,900	198,800	176,700	165,600	......	......	......
1¼	245,400	220,900	196,400	....	......	......	......

Thickness of Plate, Inches	Diameter of Punch, Inches							
	11/16	⅝	9/16	½	7/16	⅜	5/16	¼
	Pressure in Pounds							
⅛	13,500	12,300	11,100	9,800	8,600	7,400	6,100	4,900
3/16	20,300	18,400	16,600	14,700	12,900	11,000	9,200	7,300
¼	27,000	24,500	22,100	19,600	17,200	14,700	12,300	9,800
5/16	33,800	30,700	27,600	24,500	21,500	18,400	15,300	12,300
⅜	40,500	36,800	33,200	29,500	25,800	22,100	18,400	14,700
7/16	47,300	42,900	38,600	34,400	30,100	25,800	21,400	.....
½	54,000	49,100	44,200	39,300	34,400	29,600	......	.....
9/16	60,800	55,200	49,700	44,200	38,700	.....	.....	.....
⅝	67,500	61,400	55,300	49,100	.....	.....	.....	.....
11/16	74,300	67,500	60,900	.....	.....	.....	.....	.....
¾	81,000	73,600	......	.....	.....	.....	.....	.....
13/16	87,800	......	......	.....	.....	.....	.....	.....

body of a dinking die is usually made of Swedish iron and the cutting edge, which should be of high-grade tool steel, is welded to the body. The outside bevel which forms the sharp cutting edge should have an angle of about 20 degrees. A good block for the cutting edge of the die to strike against can be made of seasoned rock maple. This block is laminated or built up of small strips which are glued or bolted together with the grain endwise. A block of this kind will give better results if kept damp by covering it with a wet cloth when not in use.

Standard Punches and Dies



Pratt & Whitney Co.

No. of Punch	Diam. *A*	*B*	*C*	*D*
2	1/8 to 3/8	13/32	17/32	1 9/32
3	1/8 to 9/16	19/32	23/32	1 15/32
4	1/4 to 13/16	25/32	29/32	1 1/2
5	3/8 to 1	1 1/32	1 5/32	1 15/16
6	9/16 to 1 1/4	1 7/32	1 3/8	2 3/8
7	11/16 to 1 9/16	1 17/32	1 11/16	2 11/16
8	1 to 1 3/4	1 25/32	2 1/16	3
9	1 1/2 to 2 1/4	2 9/32	2 9/16	3
10	2 to 2 5/8	2 21/32	2 31/32	3

No. of Die	Diam. Hole *E*	*F*	*G*
2	1/8 to 5/16	3/4	5/8
3	1/8 to 9/16	1	3/4
4	1/4 to 3/4	1 1/2	1
5	1/4 to 1	2	1
6	1/2 to 1 1/4	2 3/8	1 1/4
7	3/4 to 1 9/16	2 7/8	1 1/4

Richards

No. of Punch	Diam. *A*	*B*	*C*	*D*
0	5/32 to 13/32	13/32	9/16	1 1/4
1	9/32 to 9/16	9/16	3/4	1 1/2
2	9/16 to 13/16	13/16	1	1 1/2
3	11/16 to 1 1/16	1 1/16	1 1/4	1 7/8
3 S	9/32 to 9/16	13/16	1	1 7/8
4	15/16 to 1 5/16	1 5/16	1 1/2	2 1/4
4 S	9/16 to 1 1/16	1 1/16	1 1/4	2 1/4
5	1 1/16 to 1 9/16	1 9/16	1 7/8	2 5/8
5 S	11/16 to 1 5/16	1 5/16	1 1/2	2 5/8

No. of Die	Diam. Hole *E*	*F*	*G*
1	1/8 to 9/16	1 1/4	3/4
2	1/4 to 13/16	1 5/8	7/8
3	1/4 to 1 1/16	2	1
5	15/16 to 1 9/16	2 7/8	1 1/8
....	....	.....	.....
....	....	.....	.....

Cleveland Punch and Shear Works Co.

No. of Punch	Diam. *A*	*B*	*C*	*D*	No. of Die	Diam. Hole *E*	*F*	*G*
6	1/8 to 7/16	9/16	3/4	2 1/8	42	1/8 to 9/16	1	7/8
11	15/32 to 11/16	3/4	1	2 1/8	43	1/8 to 5/8	1 1/4	7/8
16	23/32 to 1 1/16	1 1/16	1 7/32	2 1/8	44	1/8 to 5/8	1 5/16	3/4
23	1 1/16 to 1 5/16	1 5/16	1 1/2	2 1/8	45	1/4 to 15/16	1 1/2	7/8
28	1 5/16 to 1 9/16	1 9/16	1 7/8	2 5/8	46	1/4 to 15/16	1 1/2	1
29	1 5/16 to 1 9/16	1 9/16	1 7/8	3 3/8	48	1/4 to 15/16	1 9/16	27/32
30	1 5/16 to 2 1/16	2 1/16	2 1/4	2 5/8	55	1/4 to 1 1/16	1 13/16	1 1/8
31	1 5/16 to 2 1/16	2 1/16	2 1/4	3 3/8	60	1/2 to 1 1/4	2	1 1/8
32	2 1/16 to 2 9/16	2 9/16	2 13/16	3 3/8	65	5/8 to 1 5/8	2 1/2	1 1/4

Diameters of Shell Blanks. — The diameters of blanks for drawing plain cylindrical shells can be obtained from the accompanying table, which gives a very close approximation for thin stock. The blank diameters given in this table are for sharp cornered shells and are found by the following formula:

$$D = \sqrt{d^2 + 4\,dh}, \qquad (1)$$

in which D = diameter of flat blank; d = diameter of finished shell; h = height of finished shell.

Example: — If the diameter of the finished shell is to be 1.5 inch, and the height, 2 inches, the trial diameter of the blank would be found as follows:

$$D = \sqrt{1.5^2 + 4 \times 1.5 \times 2} = \sqrt{14.25} = 3.78 \text{ inches.}$$

For a round-cornered cup, the following formula, in which r equals the radius of the corner, will give fairly accurate diameters, provided the radius does not exceed, say, ¼ the height of the shell:

$$D = \sqrt{d^2 + 4\,dh} - r. \qquad (2)$$

These formulas are based on the assumption that the thickness of the drawn shell is the same as the original thickness of the stock, and that the blank is so proportioned that its area will equal the area of the drawn shell. This method of calculating the blank diameter is quite accurate for thin material, when there is only a slight reduction in the thickness of the metal incident to drawing; but when heavy stock is drawn and the thickness of the finished shell is much less than the original thickness of the stock, the blank diameter obtained from Formulas (1) or (2) will be too large, because when the stock is drawn thinner, there is an increase in area. When an appreciable reduction in thickness is to be made, the blank diameter can be obtained by first determining the "mean height" of the drawn shell by the following formula. This formula is only approximately correct, but will give results sufficiently accurate for most work:

$$M = \frac{ht}{T} \qquad (3)$$

in which M = approximate mean height of drawn shell; h = height of drawn shell; t = thickness of shell; T = thickness of metal before drawing.

After determining the mean height, the blank diameter for the required shell diameter is obtained from the table previously referred to, the mean height being used instead of the actual height.

Example: — Suppose a shell 2 inches in diameter and 3¾ inches high is to be drawn, and that the original thickness of the stock is 0.050 inch, and thickness of drawn shell, 0.040 inch. To what diameter should the blank be cut? Using Formula (3) to obtain the mean height:

$$M = \frac{ht}{T} = \frac{3.75 \times 0.040}{0.050} = 3 \text{ inches.}$$

According to the table, the blank diameter for a shell 2 inches in diameter and 3 inches high is 5.29 inches. This formula is accurate enough for all practical purposes, unless the reduction in the thickness of the metal is greater than about one-fifth the original thickness. When there is considerable reduction, a blank calculated by this formula produces a shell that is too long. This, however, is an error in the right direction, as the edges of drawn shells are ordinarily trimmed. If the shell has a rounded corner, the radius of the corner should be deducted from the figures given in the table. For example, if the shell referred to in the foregoing example had a corner of ¼-inch radius, the blank diameter would equal 5.29 − 0.25 = 5.04 inches.

Diameters of Blanks for Drawn Shells

Diam. of Shell	Height of Shell ¼	½	¾	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4½	5	5½	6
¼	0.56	0.75	0.90	1.03	1.14	1.25	1.35	1.44	1.52	1.60	1.68	1.75	1.82	1.89	1.95	2.01	2.14	2.25	2.36	2.46
½	0.87	1.12	1.32	1.50	1.66	1.80	1.94	2.06	2.18	2.29	2.40	2.50	2.60	2.69	2.78	2.87	3.04	3.21	3.36	3.50
¾	1.14	1.44	1.68	1.89	2.08	2.25	2.41	2.56	2.70	2.84	2.97	3.09	3.21	3.33	3.44	3.54	3.75	3.95	4.13	4.31
1	1.41	1.73	2.00	2.24	2.45	2.65	2.83	3.00	3.16	3.32	3.46	3.61	3.74	3.87	4.00	4.12	4.36	4.58	4.80	5.00
1¼	1.68	2.01	2.30	2.56	2.79	3.01	3.21	3.40	3.58	3.75	3.91	4.07	4.22	4.37	4.51	4.64	4.91	5.15	5.39	5.62
1½	1.94	2.29	2.60	2.87	3.12	3.36	3.57	3.78	3.97	4.15	4.33	4.50	4.66	4.82	4.98	5.12	5.41	5.68	5.94	6.18
1¾	2.19	2.56	2.88	3.17	3.44	3.68	3.91	4.13	4.34	4.53	4.72	4.91	5.08	5.26	5.41	5.58	5.88	6.17	6.45	6.71
2	2.45	2.83	3.16	3.46	3.74	4.00	4.24	4.47	4.69	4.90	5.10	5.29	5.48	5.66	5.83	6.00	6.32	6.63	6.93	7.21
2¼	2.70	3.09	3.44	3.75	4.04	4.31	4.56	4.80	5.03	5.25	5.46	5.66	5.86	6.05	6.23	6.41	6.75	7.07	7.39	7.69
2½	2.96	3.36	3.71	4.03	4.33	4.61	4.87	5.12	5.36	5.59	5.81	6.02	6.22	6.42	6.61	6.80	7.16	7.50	7.82	8.14
2¾	3.21	3.61	3.98	4.31	4.62	4.91	5.18	5.44	5.68	5.92	6.15	6.37	6.58	6.79	6.99	7.18	7.55	7.91	8.25	8.58
3	3.46	3.87	4.24	4.58	4.90	5.20	5.48	5.74	6.00	6.25	6.48	6.71	6.93	7.14	7.35	7.55	7.94	8.31	8.66	9.00
3¼	3.71	4.13	4.51	4.85	5.18	5.48	5.77	6.04	6.31	6.56	6.80	7.04	7.27	7.49	7.70	7.91	8.31	8.69	9.06	9.41
3½	3.97	4.39	4.77	5.12	5.45	5.77	6.06	6.34	6.61	6.87	7.12	7.36	7.60	7.83	8.05	8.26	8.67	9.07	9.45	9.81
3¾	4.22	4.64	5.03	5.39	5.73	6.05	6.35	6.64	6.91	7.18	7.44	7.69	7.92	8.16	8.38	8.61	9.03	9.44	9.83	10.20
4	4.47	4.90	5.29	5.66	6.00	6.32	6.63	6.93	7.21	7.48	7.75	8.00	8.25	8.49	8.72	8.94	9.38	9.80	10.20	10.58
4¼	4.72	5.15	5.55	5.92	6.27	6.60	6.91	7.22	7.50	7.78	8.05	8.31	8.56	8.81	9.04	9.28	9.72	10.15	10.56	10.96
4½	4.98	5.41	5.81	6.19	6.54	6.87	7.19	7.50	7.79	8.08	8.35	8.62	8.87	9.12	9.37	9.60	10.06	10.50	10.92	11.32
4¾	5.22	5.66	6.07	6.45	6.80	7.15	7.47	7.78	8.08	8.37	8.65	8.92	9.18	9.44	9.69	9.93	10.40	10.84	11.27	11.69
5	5.48	5.92	6.32	6.71	7.07	7.42	7.75	8.06	8.37	8.66	8.94	9.22	9.49	9.75	10.00	10.25	10.72	11.18	11.62	12.04
5¼	5.73	6.17	6.58	6.97	7.33	7.68	8.02	8.34	8.65	8.95	9.24	9.52	9.79	10.05	10.31	10.56	11.05	11.51	11.96	12.39
5½	5.98	6.42	6.84	7.23	7.60	7.95	8.29	8.62	8.93	9.23	9.53	9.81	10.08	10.36	10.62	10.87	11.37	11.84	12.30	12.74
5¾	6.23	6.68	7.09	7.49	7.86	8.22	8.56	8.89	9.21	9.52	9.81	10.10	10.38	10.66	10.92	11.18	11.69	12.17	12.63	13.08
6	6.48	6.93	7.35	7.75	8.12	8.49	8.83	9.17	9.49	9.80	10.10	10.39	10.68	10.95	11.23	11.49	12.00	12.49	12.96	13.42

Another formula which is sometimes used for obtaining blank diameters for shells, when there is a reduction in the thickness of the stock, is as follows:

$$D = \sqrt{a^2 + \left(a^2 - b^2\right)\frac{h}{t}} \qquad (4)$$

In this formula D = blank diameter; a = outside diameter; b = inside diameter; t = thickness of shell at bottom; h = depth of shell. This formula is based on the cubic contents of the drawn shell. It is assumed that the shells are cylindrical, and no allowance is made for a rounded corner at the bottom, or for trimming the shell after drawing. To allow for trimming, add the required amount to depth h. When a shell is of irregular cross-section, if its weight is known, the blank diameter can be determined by the following formula:

$$D = 1.1284\sqrt{\frac{W}{wt}} \qquad (5)$$

in which D = blank diameter in inches; W = weight of shell; w = weight of metal per cubic inch; t = thickness of the shell.

In the construction of dies for producing shells, especially of irregular form, a common method of procedure is to make the drawing parts first. The actual blank diameter can then be determined by trial. One method is to cut a trial blank as near to size as can be estimated. The outline of this blank is then scribed on a flat sheet, after which the blank is drawn. If the finished shell shows that the blank is not of the right diameter, a new trial blank is cut either larger or smaller than the size indicated by the line previously scribed, this line acting as a guide. If a model shell is available, the blank diameter can also be determined as follows: First cut a blank somewhat large, and from the same material used for making the model; then, reduce the size of the blank until its weight equals the weight of the model.

Depth and Diameter Reductions of Drawn Shells.—The depth to which metal can be drawn in one operation depends upon the quality and kind of material, its thickness, the slant or angle of the dies, and the amount that the stock is thinned or "ironed" in drawing. A general rule for determining the depth to which cylindrical shells can be drawn in one operation is as follows: The depth or length of the first draw should never be greater than the diameter of the shell. If the shell is to have a flange at the top, it may not be practicable to draw as deeply as is indicated by this rule, unless the metal is extra good, because the stock is subjected to a higher tensile stress, owing to the larger blank which is necessary for forming the flange. According to another rule, the depth given the shell on the first draw should equal one-third the diameter of the blank. Ordinarily, it is possible to draw sheet steel of any thickness up to ¼ inch, so that the diameter of the first shell equals about six-tenths of the blank diameter. When drawing plain shells, the amount that the diameter is reduced for each draw must be governed by the quality of the metal and its susceptibility to drawing. The reduction for various thicknesses of metal is about as follows:

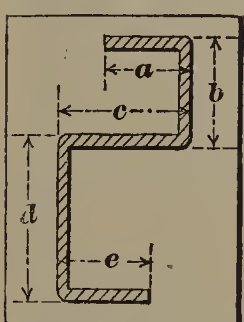
Approximate thickness of sheet steel	1/16	1/8	3/16	1/4	5/16
Possible reduction in diameter for each succeeding step, per cent	20	15	12	10	8

For example, if a shell made of 1/16 inch stock is 3 inches in diameter after the first draw, it can be reduced 20 per cent on the next draw, and so on until the required diameter is obtained. These figures are based upon the assumption that the shell is annealed after the first drawing operation, and at least between every two of the following operations. Necking operations — that is, the drawing out of a short portion of the lower part of the cup into a long neck — may be done without such frequent annealings. In double-action presses, where the inside of the cup is supported by a bushing during drawing, the reductions possible may be increased to 30, 24, 18, 15 and 12 per cent, respectively. (The latter figures may also be used for brass in single-action presses.).

When a hole is to be pierced at the bottom of a cup and the remaining metal is to be drawn after the hole has been pierced or punched, always pierce from the opposite direction to that in which the stock is to be drawn after piercing. In extreme cases, it is necessary to machine the metal around the pierced hole in order to prevent the starting of cracks or flaws in the subsequent drawing operations.

The foregoing figures represent conservative practice and it is often possible to make greater reductions than are indicated by these figures, especially when using a good drawing metal. Taper shells require smaller reductions than cylindrical shells, because the metal tends to wrinkle if the shell to be drawn is much larger than the punch. The amount that the stock is "ironed" or thinned out while being drawn must also be considered, because a reduction in gage or thickness means greater pressure of the punch against the bottom of the shell; hence the amount that the shell diameter is reduced for each drawing operation must be lessened when much ironing is necessary. The extent to which a shell can be ironed in one drawing operation ranges between 0.002 and 0.004 inch per side, and should not exceed 0.001 inch on the final draw, if a good finish is required.

Allowances for Bending Sheet-Metal. — When bending sheet steel or brass, add from 1/3 to 1/2 of the thickness of the stock, for *each bend*, to the sum of the inside dimensions of the finished piece, to get the length of the straight blank. The harder the material the greater the allowance (1/3 of the thickness is added for soft stock and 1/2 of the thickness for hard material). The data given in the table, "Allowances for Bends in Sheet Metal," refer more particularly to the bending of sheet metal for counters, bank fittings and general office fixtures, for which purpose it is not absolutely essential to have the sections of the bends within very close limits. Absolutely accurate data for this work cannot be deduced, as the stock varies considerably as to hardness, etc. The figures given apply to sheet steel, aluminum, brass and bronze. Experience has demonstrated that for the semisquare corners, such as are formed in a V-die, the amount to be deducted from the sum of the outside bend dimensions, as shown in the accompanying illustration by the sum of the letters from a to e, is as follows: $X = 1.67\ BG$, where X = the amount to be deducted; B = the number of bends; and G = the decimal equivalent of the gage. The values of X for different gages and numbers of bends are given in the table. Its application may be illustrated by an example: A strip having two bends is to have outside dimensions of 2, 1½ and 2 inches, and is made of stock 0.125 inch thick. The sum of the outside dimensions is thus 5½ inches, and from the table the amount to be deducted is found to be 0.416; hence the blank will be $5.5 - 0.416 = 5.084$ inches long.



The lower part of the table applies to square bends which are either drawn through a block of steel made to the required shape, or else drawn through rollers in a draw-bench. The pressure applied not only gives a much sharper corner, but it also elongates the material more than in the V-die process. In this case, the deduction is $X = 1.33\ BG$.

Allowances for Bends in Sheet Metal

Square Bends	Gage	Thickness, Inches	Amount to be Deducted from the Sum of the Outside Bend Dimensions, Inches						
			1 Bend	2 Bends	3 Bends	4 Bends	5 Bends	6 Bends	7 Bends
Formed in a Press by a V-die	18	0.0500	0.083	0.166	0.250	0.333	0.416	0.500	0.583
	16	0.0625	0.104	0.208	0.312	0.416	0.520	0.625	0.729
	14	0.0781	0.130	0.260	0.390	0.520	0.651	0.781	0.911
	13	0.0937	0.156	0.312	0.468	0.625	0.781	0.937	1.093
	12	0.1093	0.182	0.364	0.546	0.729	0.911	1.093	1.276
	11	0.1250	0.208	0.416	0.625	0.833	1.041	1.250	1.458
	10	0.1406	0.234	0.468	0.703	0.937	1.171	1.406	1.643
Rolled or Drawn in a Draw-bench	18	0.0500	0.066	0.133	0.200	0.266	0.333	0.400	0.466
	16	0.0625	0.083	0.166	0.250	0.333	0.416	0.500	0.583
	14	0.0781	0.104	0.208	0.312	0.416	0.521	0.625	0.729
	13	0.0937	0.125	0.250	0.375	0.500	0.625	0.750	0.875
	12	0.1093	0.145	0.291	0.437	0.583	0.729	0.875	1.020
	11	0.1250	0.166	0.333	0.500	0.666	0.833	1.000	1.166
	10	0.1406	0.187	0.375	0.562	0.750	0.937	1.125	1.312

Drop-Forging Dies

Steel for Drop-forging Dies. — Practically all drop-forging dies are made of high-grade open-hearth steel. A 60-point carbon steel is mostly used, although steel as low as 40-point and as high as 85-point carbon is employed in some cases. A special hardening treatment is required for the low-carbon steel, which more than offsets the saving in price, and, except in special cases, there is no advantage in using high-carbon steels, owing to the expense. The average 60-point carbon steel die, if properly hardened, should last for from 15,000 to 40,000 forgings, and sometimes as many as 70,000 forgings can be made from one set of dies. When making dies for large forgings, it is often thought advisable to use 80-point carbon steel, and not harden the dies. This obviates the danger from "checking" or cracking in hardening, and the un-hardened steel is hard enough to resist the tendency to stretch. A steel that is quite high in carbon should always be used for dies that are intended for making forgings from tool steel or any other hard steel.

Allowance for Shrinkage. — When making dies for small cold-trimmed steel forgings, the proper allowance for shrinkage is 3/16 inch to the foot, or 0.015 inch to the inch. Such forgings are finished at a bright red heat and the rate of shrinkage is considerable. When making dies for hot-trimmed steel forgings of medium and large sizes, the shrinkage allowance is 1/8 inch to the foot, or 0.010 inch to the inch. Hot-trimmed forgings receive the finishing blow while comparatively cold, and shrink a smaller amount than the cold-trimmed forgings. The foregoing allowances are used for all dimensions of the die impression, such as depth, width or length. The shrinkage allowance for dies to be used in forging bronze or copper is practically the same as that for steel.

Draft Allowance. — The amount of draft in a drop-forging die varies from 3 to 10 degrees. If the die is for a thin forging of uniform section, 3 degrees is ample, but if the forging is deep and has narrow ribs which are apt to stick, at least 7 degrees is necessary. If a die is used for forging a piece that is ring-shaped or has an annular part, the central plug that forms the interior of the ring should have a draft of 10 degrees, because, as the forging cools while being worked, it tends to shrink around the plug and if the draft is insufficient, it will stick in the die. With the foregoing exception, most drop-forging dies have a 7-degree draft. For convenience in laying out, it is well to remember that a 7-degree taper is approximately equal to a 1/8-inch taper to the inch, and a 10-degree taper, 3/16 inch to the inch.

Locating Impression in the Die. — When laying out a drop-forging die, the impression should be located so that the heaviest end of the forging will be at the front of the die-block. This makes the forging easier to handle and also permits the use of a fairly large sprue. There should be at least 1½ inch left all around between the impression and the outside edge of the block. This also holds true for any part of the die, such as the edger, anvil or forming impression. If the forging has a hub or other projection that extends some distance from the main part on one side, the upper or top die should contain this deeper impression.

Obtaining Weight of Forging from Lead Proof. — After the upper and lower dies have been completed, shrinkage allowances and the general finish of the impressions are ordinarily tested by taking a "lead proof," and by weighing the lead, an approximate idea of the weight of the finished forging can be obtained. Roughly speaking, the finished forging will weigh two-thirds as much as the lead proof. The shrinkage of lead is practically the same as that of steel, so that the finished forging will also measure about the same as the one made of lead. In case of dies for eye-bolts and similar work, this rule must be disregarded, because the plugs that form the central opening will prevent the lead from shrinking naturally. When taking the lead proof, the die impressions are dusted with powdered chalk, and after the dies are clamped together, the molten lead is poured.

Amount of Flash for Drop-forging Dies. — Theoretically, there should be just enough forging metal in a die to fill the impression, and no more, but this is, of course, not practicable, as there is always some stock that must be disposed of after the impression is filled. To take care of this excess metal, dies are relieved all around the impression by milling a flat shallow recess about 1/64 inch deep and 5/8 inch wide. These dimensions are for dies of average size; in comparatively large dies this recess or "flash" would be a little deeper and wider. Both the upper and lower dies are flashed in this way. In addition, the upper die is "back-flashed," which means that there is a deeper recess, sometimes called the "gutter," milled around the impression at a distance of 1/4 inch from the impression at every point. This back-flash is 3/64 inch deep and acts as a relief for the excess metal after it has been squeezed from the flash proper. Only the finishing impression is provided with a flash and back-flash.

The Break-down of Drop-forging Dies. — The width of section used as a break-down (also known as the edger or side cut) should be enough wider than the forging to give plenty of room for the work of forging. A forging 1 inch thick should have a break-down 1½ inch wide, and about the same proportions should be followed for forgings of other widths. The break-down should have a section corresponding with the gate and sprue of the die impression, but it should be made slightly longer, so that the forging will not be stretched when struck in the impression.

Hardening Drop-forging Dies. — Dies to be carbonized should always be packed for hardening in cast-iron or sheet-iron boxes containing a mixture of fresh bone and charcoal. The ordinary mixture is half bone and half charcoal. More bone gives greater hardness and more charcoal, less hardness, for a given heat; hence, the proportions should be varied according to requirements. The die should be packed face down on a one- or two-inch layer of this mixture and be settled so that the impression is filled. Sometimes the face is coated, before packing, with a thick paste of linseed oil and powdered bone-black, to protect the delicate edges from oxidation when in contact with the air. Fill the space between the sides of the die and the box with the bone and charcoal mixture, and cover over with a thick layer of wet clay paste to prevent the charcoal from burning out. Dies made of steel having less than 60-point carbon content should always be carbonized. Open-hearth steel dies containing 60-point carbon or over can be hardened without carbonizing.

Heating the Die. — An oil or gas furnace is recommended for heating, although a coal or coke-fired muffle furnace, capable of maintaining a temperature of at least 1600 degrees F., may be used, provided the temperature can be held constant. A temperature indicating device is necessary. The die should be put into the furnace as soon as the latter is lighted. If the correct quenching temperature for the steel is, say, 1500 degrees F., the furnace should be checked when the pyrometer indicates 1400 degrees, the die being allowed to "soak" at that heat for three or four hours. Then the heat should be slowly raised to 1500 degrees and held at that point one or two hours longer, according to the size of the die. Five hours is the minimum total time for heating, and seven or eight hours is much safer. A 60-point carbon die should be quenched between 1425 and 1450 degrees F.

Cooling the Heated Die. — When cooling, the face of the die should receive a sufficient flow of cold water to cause it to harden to the greatest possible depth. The back of the die should, at the same time, be cooled to make the shrinkage of the face and back equal, and to prevent warping. A good form of cooling tank is one having a large supply pipe extending up through the bottom for cooling the die face, and a smaller pipe above the tank to cool the back. Unless a jet of water under pressure is applied to the face of the die, the sunken parts of the impression will not harden equally with the face. Dies should not be cooled in a tank of still water, because steam forms in the die cavity which prevents the water from entering, thus causing the formation of soft spots. To overcome this, the water must be forced into the impression by pressure sufficient to overcome the resistance of the steam thus formed. Oil should not be used for hardening hammer dies, as its cooling action is not great enough to produce a sufficient depth of hardening. Hammer dies which are simply surface hardened will not withstand the heavy blows received in service. To secure a greater hardening effect, brine of about 40 per cent solution is used by some die-makers.

Tempering Dies. — Dies should be tempered and drawn as soon as they are cool enough to remove from the tank. The dies should be heated in an oil bath, and quenched in water or cool oil. Any high-grade cylinder oil of high flash-point is suitable. Low-grade oils smoke unpleasantly and will not stand high temperatures. The drawing temperature of die steels is about 450 degrees F., for average conditions. The corners of the die and the cut-off should be drawn to a purple color with the aid of a blow torch.

Dies for Bronze and Copper Forgings. — Dies for producing drop-forgings from bronze or copper differ from those used for steel or iron forgings principally in the matter of finish. Owing to the softness of copper and bronze, the metal is driven into very minute impressions in the surface of the dies; hence, these surfaces must be perfectly free from scratches, in order to insure a smooth finish on the work. Even though these metals are soft, the hammering necessary when forging is very hard on the dies, and to prevent them from dishing or spreading, tool steel is ordinarily used, unless the forgings are extra large and heavy. The shrinkage, draft and finish allowances on this class of drop-forging dies are practically the same as on dies for steel and iron.

Trimming Dies for Drop-forgings. — Hot-trimming dies are made of a special grade of steel known as hot-trimming die-stock. The objection to using ordinary tool steel for hot-trimming dies is that the edges of a hardened die check badly after the die has been used for a short time, and this checking is followed by a breaking away of the steel around the edges, thus rendering the die unfit for use. This special steel requires no hardening, and after the die is in use, the edges toughen and give better service than the best hardened tool steel. The usual form of punch for hot-trimming dies merely supports the forging while it is being pushed through. If the forging has a broad, flat top face, the punch need only be a little more than a flat piece that covers the forging and acts as a pusher. Such punches are commonly made of cast iron. Cold-trimming dies are made from good tool steel of from 1.00 to 1.25 per cent carbon, and hardened and drawn to a dark straw color. The punches for cold trimmers are also made of tool steel and are hardened and drawn to a very dark straw color. These punches are hardened to prevent them from upsetting at the edges. As with hot-trimming punches, the punch should fit the die loosely, but it should support the forging at every point while it is being pushed through the die. There are two instances in which trimming punches should fit the dies as closely as the average punching die for sheet metal work; first, when trimming forgings on which the fin comes at the corner of the forging; second, forgings that are formed all in one die, the other die being flat. In these two cases, unless the dies fit very well, there will be burrs at the trimmed edges.

BROACHES AND BROACHING

The Broaching Process. — Broaching is especially adapted to the finishing of square, rectangular or irregular-shaped holes. It is also applicable to a wide variety of miscellaneous work, such as the cutting of single or multiple keyways in hubs, forming splines, cutting teeth in small internal gears and ratchets, etc. There are two general methods of broaching: One is by pushing comparatively short broaches through the work, usually by means of a hand press, a hydraulically-operated press, or an ordinary punch press. With the other method, a special broaching machine is used, and the broach, which is usually much longer than a "push broach," is pulled through the work by means of a screw forming part of the machine. Push broaches must necessarily be quite short to prevent excessive deflection; consequently it is often necessary to force several broaches through the work. The longer broaches which are pulled through in regular broaching machines commonly finish parts in one passage, although a series of two or more broaches are often used for long holes, or when considerable stock must be removed. The number of broaches ordinarily used varies from one to four. Comparatively short broaches are sometimes used, because they are easier to make, are not warped excessively in hardening and are easier to handle. Two or more parts can frequently be finished simultaneously on a regular broaching machine, the pieces being placed one against the other, in tandem.

Types of Broaches. — A number of typical broaches and the operations for which they are intended are shown by the diagrams, Fig. 1. Broach *A* produces a round-cornered, square hole. Prior to broaching square holes, it is usually the practice to drill a round hole having a diameter *d* somewhat larger than the width of the square. Hence, the sides are not completely finished, but this unfinished part is not objectionable in most cases. In fact, this clearance space is an advantage during the broaching operation in that it serves as a channel for the broaching lubricant; moreover, the broach has less metal to remove. Broach *B* is for finishing round holes. Broaching is superior to reaming for some classes of work, because the broach will hold its size for a much longer period, thus insuring greater accuracy. Broaches *C* and *D* are for cutting single and double keyways, respectively. The former is of rectangular section and, when in use, slides through a guiding bushing which is inserted in the hole. Broach *E* is for forming four integral splines in a hub. The broach at *F* is for producing hexagonal holes. Rectangular holes are

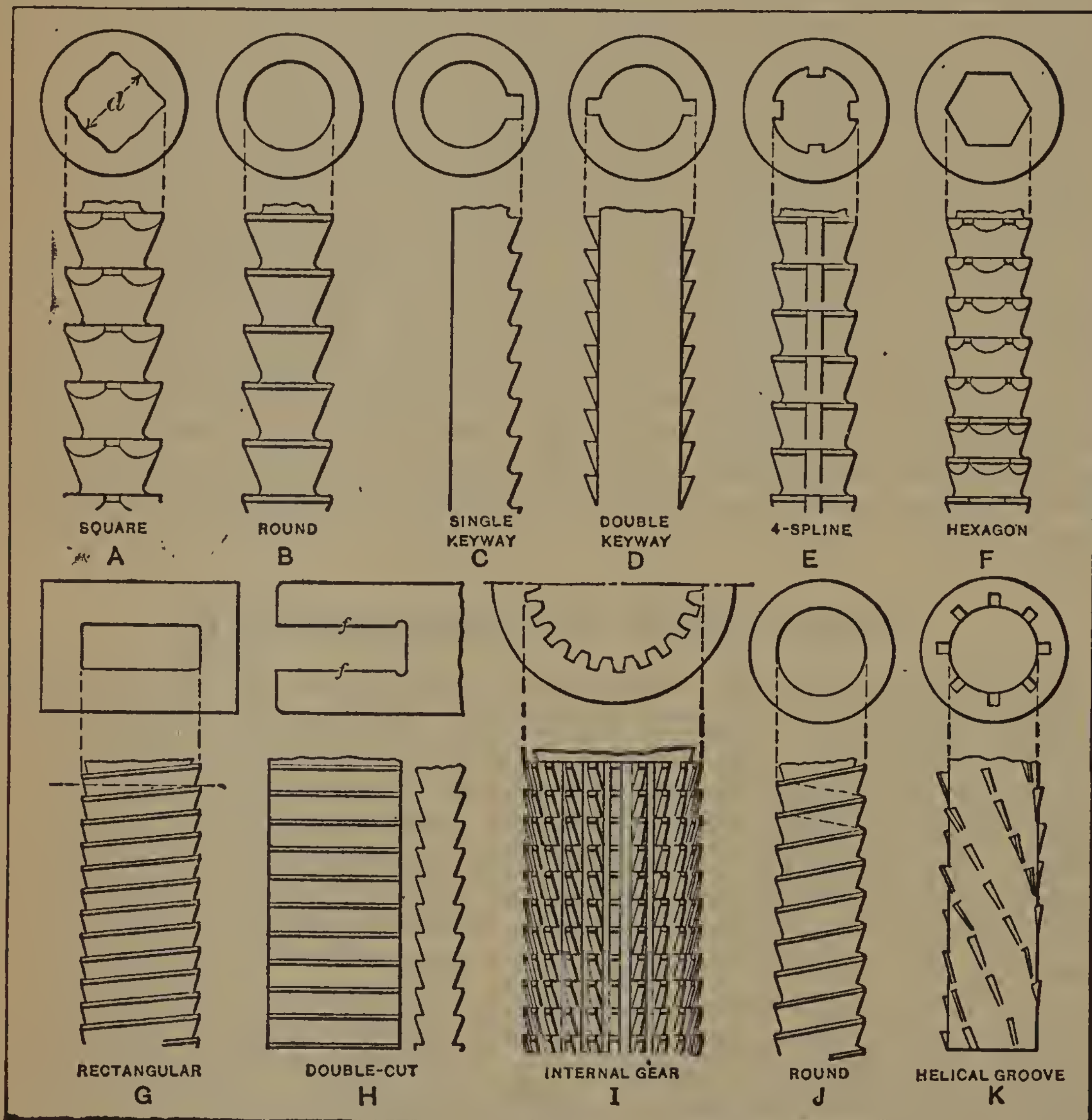


Fig. 1. Types of Broaches

finished by broach *G*. The teeth on the sides of this broach are inclined in opposite directions, which has the following advantages: The broach is stronger than it would be if the teeth were opposite and parallel to each other; thin work cannot drop between the inclined teeth, as it tends to do when the teeth are at right angles, because at least two teeth are always cutting; the inclination in opposite directions neutralizes the lateral thrust. The teeth on the edges are staggered, the teeth on one side being midway between the teeth on the other edge, as shown by the dotted line. A double cut broach is shown at *H*. This type is for finishing, simultaneously, both sides *f* of a slot, and for similar work. Broach *I* is the style used for forming the teeth in internal gears. It is practically a series of gear-shaped cutters, the outside diameters of which gradually increase toward the finishing end of the broach. Broach *J* is for round holes but differs from style *B* in that it has a continuous helical cutting edge. Some prefer this form because it gives a shearing cut. Broach *K* is for cutting a series of helical grooves in a hub or bushing. The work rests against a special rotating support, and revolves to form the helical grooves, as the broach is pulled through.

In addition to the typical broaches shown in Fig. 1, many special designs are now in use for performing more complex operations. Two surfaces on opposite sides of a casting or forging are sometimes machined simultaneously by twin broaches and, in other cases, three or four broaches are drawn through a part at the same time, for finishing as many duplicate holes or surfaces. Special work-holding and broach-guiding fixtures are commonly used for multiple broaching.

Pitch of Broach Teeth. — The pitch of the teeth (or the distance from one tooth to the next) and the increase in size of successive teeth, both depend upon the nature of the work. As a general rule, the pitch *P* (see Fig. 2) should increase as the length of the hole increases, to provide sufficient space between the teeth

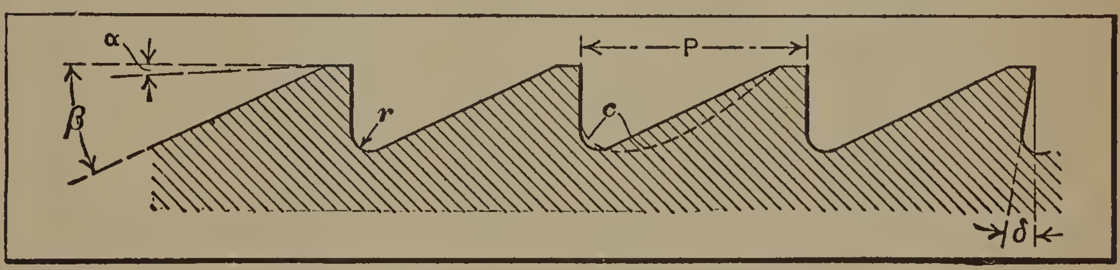


Fig. 2. Shape of Broach Teeth

for the chips. The pitch of the broach teeth for *average* conditions can be determined approximately by the following formula, in which P = pitch of teeth, and L = length of hole to be broached:

$$P = \sqrt{L} \times 0.35$$

For example, if a broach is required for a square hole, 3 inches long, the pitch of the teeth would equal $\sqrt{3} \times 0.35 = 0.6$ inch, approximately. Of course, a given pitch will cover quite a range of lengths, the maximum being the length in which the chip space will be completely filled. The constant given in the preceding formula may be as low as 0.3 for some broaches and as high as 0.4 for others, although the pitch obtained with the value 0.35 corresponds closely to average practice. When a broach is quite large in diameter, thus permitting deep chip spaces in front of the teeth, the pitch might be decreased in order to reduce the total length of the broach. Inversely, if the work is very hard and tough, a coarser pitch might be advisable in order to reduce the power required to force the broach through the hole. If the pitch is too fine in proportion to the size of the broach, there may be difficulty in hardening, owing to the fact that the fine teeth will cool much more rapidly than the broach body, thus producing severe strains which tend to crack the teeth, especially at the corners. In general, the pitch should be as coarse as possible, without weakening the broach too much, but at least two teeth should be in contact when broaching work of minimum length. If the teeth are closely spaced, so much power may be required for drawing the broach through the work that there is danger of pulling the broach apart.

Depth of Cut per Tooth. — The amount of metal that the successive teeth of a broach should remove, or the increase in size per tooth, depends largely upon the hardness or toughness of the material to be broached. The size of the hole in proportion to its length also affects the depth of cut, so that it is impossible to give more than a general idea of the increase in size per tooth. Medium-sized broaches for round or square holes usually have an increase of from 0.001 to 0.003 inch per tooth for broaching steel, and approximately double these amounts for soft cast iron or brass. Large broaches up to 2- or 3-inch sizes may have an increase of from 0.005 to 0.010 inch per tooth. Obviously, the depth of cut is governed almost entirely by the nature of the work. For example, a small broach for use on brass or other soft material might have a larger increase per tooth than a much larger broach for cutting steel. If the amount of metal to be removed is comparatively small, and the broach is used principally for finishing, the increase per tooth may not be over 0.001 inch, even for large broaches.

The diagrams *A* and *B*, Fig. 3, show a common method of broaching square holes in the hubs of automobile transmission gears, etc. Prior to broaching, a hole is drilled slightly larger in diameter than the square width. The first tooth on the broach is rounded considerably and cuts a long circular chip, as indicated at *a*, and the following teeth form the square corners by removing successive chips (as shown by the dotted lines) until the square is finished as at *B*. As will be seen, the first tooth has the widest cut, the chip width *a* decreasing towards the finishing end of the broach; hence, if this hole is finished with a single broach, it is advisable to vary the sizes of the teeth so that the depth of cut gradually increases as width *a* decreases. It is good practice to nick some of the wide teeth (as indicated at *n*) in order to break up the chips, as a broad curved chip does not bend or curl easily. In case two or more broaches are required, the first broach of the set might have a uniform variation in the radii *r* of different teeth, but the depth of cut should be less than for the following broaches which remove comparatively narrow chips from the corners of the square. Several end teeth, especially on the last broach of a set, are made to the finished size. This feature, which is common to broaches in general, aids the broach in retaining its size and tends to produce a more accurately finished hole.

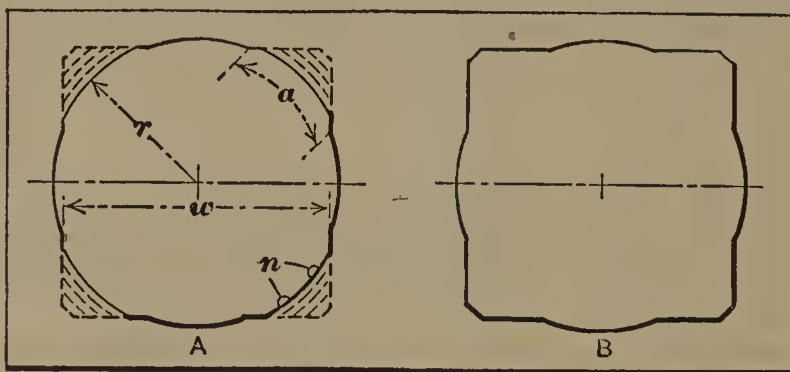


Fig. 3

Testing Uniformity of Teeth. — When testing a broach to determine if all of the teeth cut equally, first use a test piece not longer than two times the pitch of the teeth. Pull the broach through and note the amount of chips removed by each tooth; then stone down the high teeth and test by drawing through a somewhat longer piece, and so on, until the broach is finally pulled through the full length required. If a broach is warped much or is otherwise inaccurate, some teeth may take such deep cuts that the broach would break, if an attempt were made to pull it through a long hole on the first trial.

Clearance Angles for Broach Teeth. — The clearance angle α (Fig. 2) for the teeth of broaches is usually very small, and some broaches are made with practically no clearance. Ordinarily, there should be a clearance angle varying from 1 to 3 degrees, 2 degrees being a fair average. A common method of providing the necessary clearance is as follows: All the lands of the hardened broach are first ground parallel, and then they are "backed off" slightly by means of an oilstone.

Just back of the narrow land (which may not be over 1/32 inch wide) there is a clearance of 2 or 3 degrees machined prior to hardening. The angle β and the resulting clearance space for the chips depend upon the length of the hole and the depth of the cut. When the cut is light and especially if the material to be broached is tough, thus making it necessary to use as strong a broach as possible, the clearance space should be proportionately small. The fillet at the base of each tooth should have as large a radius r (Fig. 2) as practicable, and the grooves between the teeth should be smooth so that the chips will curl easily. A curved clearance space, similar to that indicated by the dotted line c, is superior to the straight slope, although not so easily machined. The front faces of the teeth are sometimes given a rake δ of from 5 to 8 degrees, so that the broach will cut easier and require less pressure to force it through the work.

Steel for Broaches. — Three kinds of steel are used for making broaches, alloy steel, carbon steel and, to some extent, casehardened machine steel. Carbon-vanadium tool steel is especially adapted for making broaches. This type of steel differs from the high-speed steels in which vanadium is also used in that it does not contain tungsten or chromium, but is simply a high-grade carbon steel containing a certain percentage of vanadium. The addition of vanadium to carbon steel imparts certain qualities, the most important of which are, first, the higher temperature to which the steel can be heated without coarsening of the grain (thus permitting a greater range in temperature for hardening without spoiling the tool), and, second, a tough core which makes the broach stronger and more durable than one made of regular high-carbon steel. The makers recommend hardening carbon-vanadium steel at a temperature varying from 1350 to 1425 degrees F., the temperature depending somewhat upon the size of the tool. The steel is then drawn like any other tool steel to suit conditions, the drawing temperature generally being about 460 degrees F. This particular brand of steel will not harden in oil.

Regular carbon steel that is used for broaches should have from 1.00 to 1.10 per cent carbon. To prevent steel from warping excessively, the broach should be annealed after the teeth have been roughed out. A successful method of hardening to prevent excessive warping is as follows: After machining the broach and before hardening, heat to a dark red and allow the broach to cool while lying on a flat plate; then heat to the hardening temperature and harden in the usual manner. This method, which is applicable to all tool steels, reduces warping to a minimum and is of especial value when hardening slender broaches.

Open-hearth steel having about 0.25 per cent carbon has proved satisfactory for some broaching operations. These broaches are casehardened by packing them in a tube containing some carbonaceous material such as slightly charred, granulated bone. After heating to a temperature of about 1700 degrees F., the tube and broach should be allowed to cool slowly; then re-heat to about 1450 degrees prior to quenching, in order to refine the steel. The temper is then drawn at about 375 degrees F. (See "Casehardening.")

Straightening Hardened Broaches. — Broaches that have been warped by hardening can best be straightened at the time the temper is drawn. Place the broach on two wooden blocks on the table of a drill press equipped with a lever feed, and insert a wooden block in the end of the drill press spindle. Heat the broach with a Bunsen burner until the hand can barely touch it; then apply pressure to the high side. Continue heating (as uniformly as possible) and bending, until the broach is straight, but complete the straightening operation before the broach has reached a temperature of about 350 degrees F., so that the drawing temperature will not be exceeded. With this method, the heat required for straightening is also used for drawing the temper.

Examples of Broaching. — The following examples of broaching taken from actual practice indicate, in a general way, the proportions of broaches for various operations:

Operation 1. — Broaching 15/16-inch square holes in alloy steel gears having hubs 3 inches long. Broaches used: The first or No. 1 broach in the set of three has teeth which increase in diameter from the starting end 0.002 inch; the teeth on No. 2 broach increase 0.003 inch, and those on No. 3, 0.004 inch. The leading ends or shanks of the three broaches are 0.005 inch less in diameter than the 1-inch hole drilled prior to the broaching operation. The pitch of the teeth is ½ inch; the width of the lands, ⅛ inch; the last two teeth on broaches Nos. 1 and 2 are made the finished size; six teeth of the finished size are left on broach No. 3. When more than one broach is used, it is common practice to make the last tooth on one broach and the first tooth on the following broach of the same size.

Operation 2. — Broaching a ⅝-inch square hole, 1½ inch long, in carbon steel. Broaches used: Set of three push broaches (for use under a press), 10¼ inches long; pitch of teeth, 5/16 inch; increase in size per tooth, 0.003 inch (0.0015 on each side). A 21/32-inch hole is drilled prior to broaching.

Operation 3. — Broaching a 9/16-inch hexagon hole, ⅞ inch long, in high-grade carbon steel. Broaches used: Set of four push broaches, 6 inches long; pitch of teeth, ¼ inch; increase in size per tooth, 0.010 inch (0.005 on each side) for first six teeth (because of small corner cuts taken by leading teeth), and 0.003 inch for remaining eight teeth. The last six teeth on broach No. 4 are made the same size.

Operation 4. — Finishing babbitted or bronze bearings, 1½-inch diameter, 3 inches long. Broaches used: Pitch of teeth, 7/16 inch; length of toothed section, 4 inches; increase in size per tooth, 0.001 inch; number of uniformly sized finishing teeth, 3; width of lands, 1/32 inch; size of pilot, 1.495 inch; length, 1¾ inch; size of plain cylindrical section following finishing teeth for producing hard and compact surface, 1.505 inch.

Operation 5. — Broaching the teeth in machine steel internal gears of 3.3-inch pitch diameter; 20 diametral pitch, with teeth ½ inch long. Broaches used: Pitch of teeth (distance between centers of successive rows), ⅜ inch; increase in outside diameter for each annular row of teeth, 0.006 inch; number of rows of uniform diameter, last three. This type of broach is illustrated at *I*, in Fig. 1, and is made as follows: After roughing out the blank, anneal the steel; then mill the teeth the same as if making a long gear; harden and grind the front faces of the teeth to produce sharp edges. The cutting ends of the teeth require little or no clearance.

Time required for Broaching Operations. — Some typical broaching operations are illustrated in Fig. 4. The dimensions of these parts and the number broached per hour are given in the following. (Data compiled by Lapointe Machine Tool Co.).

Sample A: ¾-inch square hole; sharp corners; 1⅜ inch long; 50 per hour.
Sample B: 15/16-inch square hole; sharp corners; 1½ inch long; 50 per hour.
Sample C: 1⅜-inch square hole; sharp corners; 4 inches long; 15 per hour.
Sample D: 13/32-inch square hole; round corners; 2 inches long; 45 per hour.
Sample E: 1⅜-inch square hole; round corners; distance across corners, 1¾ inch; 1½ inch long; 45 per hour.
Sample F: ⅝-inch hexagon hole; 1⅜ inch long; 60 per hour.
Sample G: 1⅝-inch hexagon hole; 2 inches long; 45 per hour.
Sample H: 1¾-inch hexagon hole; 1½ inch long; 45 per hour.
Sample I: 1-inch hole; ¼ by ⅛ inch keyway; ½ inch long; 250 per hour.

Sample J: Two-spline hole; 1½-inch diameter; 5/16 by 5/32 inch splines; 1½ inch long; 90 per hour.
Sample K: Two ½ by ¼ inch keyways in 1½-inch hole; 3 inches long; 45 per hour.
Sample L: 2-inch hole; ¾ by 5/16 inch keyways; 3 inches long; 45 per hour.
Sample M: Three-spline dovetail hole; 1⅛-inch diameter; outside diameter, 1⅜ inch; ½ inch long; 135 per hour.
Sample N: Four-spline 15/16-inch hole; splines ¼ by ⅛ inch wide; 1 inch long; 100 per hour.
Sample O: Four-spline dovetail hole; 29/32 inch; 2 inches long; 45 per hour.
Sample P: Four-spline hole; 1¼-inch diameter; 1⅝-inch outside diameter; width of spline, 5/16 inch; 3 inches long; 45 per hour.
Sample Q: Four-spline hole; 1¾-inch diameter; outside diameter, 2⅜ inches; splines 9/16 inch wide; 4 inches long; 15 per hour.

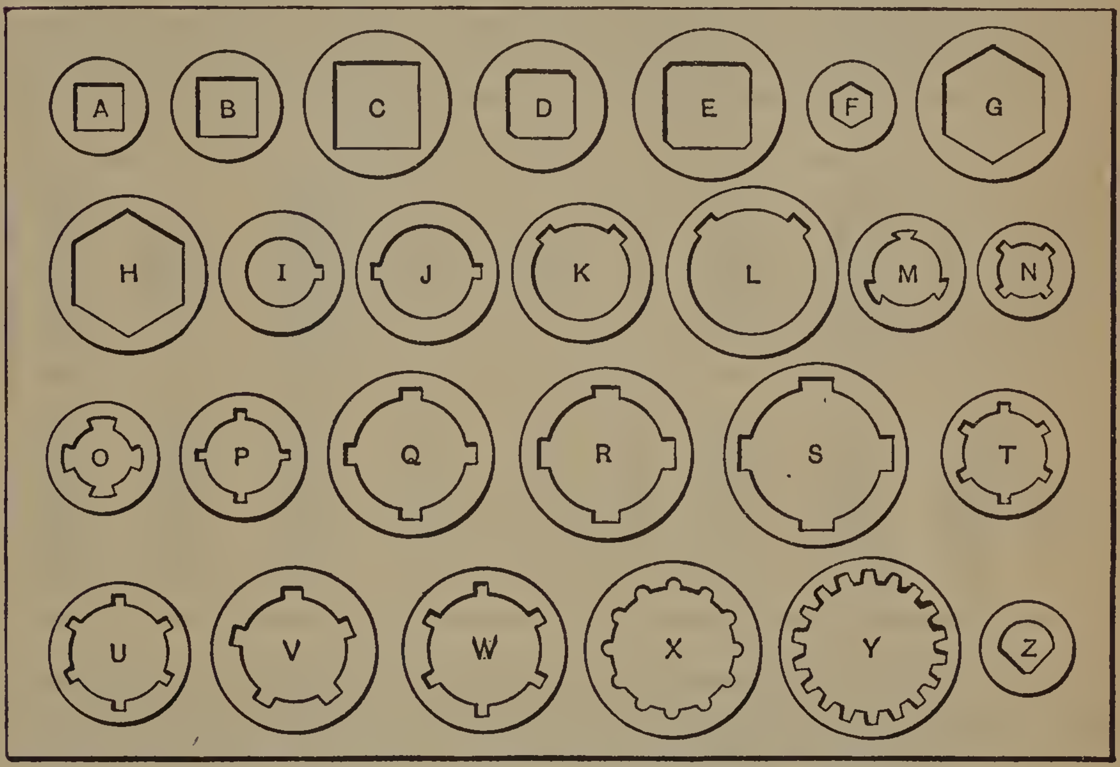
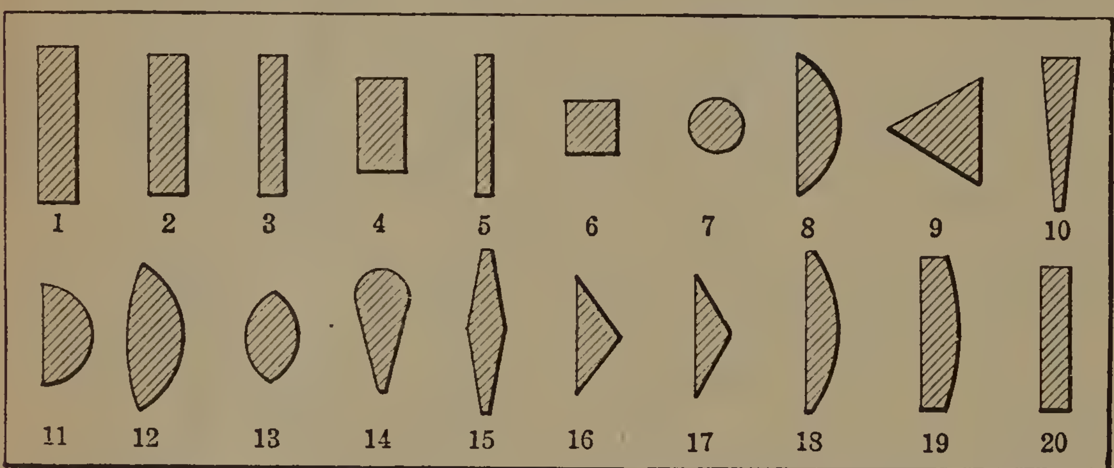


Fig. 4. Typical Examples of Broaching Operations

Sample R: Four-spline hole, 1⅞ inch; keyways ¾ by 3/16 inch; 3 inches long; 20 per hour.
Sample S: Four-spline hole; 2⅛-inch diameter; outside diameter, 2½ inches; splines ⅞ inch wide; 2 inches long; 45 per hour.
Sample T: Six-spline, 17/16-inch hole; outside diameter, 1 11/16 inch; width of spline ⅜ inch; 4 inches long; 25 per hour.
Sample U: Six-spline hole; 1½-inch diameter; splines ⅜ by 3/16 inch; 1½ inch long; 45 per hour.
Sample V: Five-spline hole, 1 47/64 inch; outside diameter, 2 3/16 inch; width of spline, 7/16 inch; 3¾ inches long; 15 per hour.
Sample W: Six-spline hole; 1 13/16-inch diameter; splines ⅜ inch wide; outside diameter, 2 1/16 inches; 4 inches long; 15 per hour.
Sample X: Twelve-spline; 2-inch diameter; grooves ⅛-inch radius; ½ inch long; 180 per hour.
Sample Y: Internal gear; 18 teeth; 2⅛-inch hole; ½-inch face; 120 per hour.
Sample Z: ¾-inch semi-square; 1-inch corner diameter; 1 inch long; 40 per hour.

FILES

Classes of Files. — Files are classified according to their shape or cross-section, and according to the pitch or spacing of the teeth and the nature of the cut. The cross-sections of file steel for all shapes in general use are shown in the illustration. The names of files made from steel of these sections are, referring to the numbers in the illustration: (1) hand; (2) flat; (3) mill; (4) pillar; (5) warding; (6) square; (7) round or "rat-tail"; (8) half-round; (9) three-square or triangular; (10) knife; (11) pit-saw; (12) crossing; (13) tumbler; (14) cross-cut; (15) feather-edge; (16) cant-saw; (17) cant-file; (18) cabinet; (19) shoe-rasp; (20) rasp. The blank for a "hand" file is parallel in thickness from the heel to the middle and tapered from the middle to the point, the latter being about one-half the thickness of the stock. The edges of the blank are usually parallel, but are sometimes drawn in slightly at the point. The "flat" file blank is parallel in both of its longitudinal sections from the heel to the middle and is tapered in both sections from the middle to the point, the thickness of the point being about two-thirds, and the width about one-half that of the stock. The "mill" file blank is parallel in thickness from the heel to the point and is usually tapered to about three-fourths the width of stock.



The mill file is also made blunt or of equal width and thickness throughout its length. The "warding" file is tapered in width from the heel to the point and is of uniform thickness. Aside from width, the "pillar" file is similar to the hand file; it is also made in narrow and extra-narrow patterns. The "three-square," "square" and "round" files are also made in slim and blunt forms. The slim form is of regular length but of smaller cross-section, and the blunt is of equal cross-section from the heel to the point, being either slim or regular. The "half-round" is not a full semi-circle, the arc being about one-third of a full circle. The "pit-saw" is a full half circle in section. The "three-square," "cant-saw" and "cant-file" differ as to their angles, the former having equal angles (60 degrees) and equal sides; the cant-saw, 35-, 35- and 110-degree angles; and the cant-file, 30-, 30- and 120-degree angles. The well-known hand-saw file has the same section as the three-square type, but differs in that the edges are given the proper bluntness to insure durability; in the three-square files, the edges are left sharp, thus making them entirely unfit for filing saws.

File Teeth. — There are three general classifications of files according to the cut — single-cut, double-cut and rasp. The single-cut file (or "float," as the coarser cuts are sometimes called) has single rows of parallel teeth extending across the face at an angle of from 65 to 85 degrees with the axis of the file. This angle depends upon the form of the file and the nature of the work it is intended for. The double-cut file has two rows of teeth crossing each other. The angle of the first row is, for general work, from 40 to 45 degrees, and the second row, from 70 to 80 degrees. The angle of the first cut for double-cut finishing files is about 30 degrees and the second cut, from 80 to 87 degrees. The double cut gives a broken tooth, the surface of the file having a large number of small teeth inclining toward the point and resembling, in shape, the end of a diamond-pointed cold chisel. The second or "up-cut" is usually a little finer and not as deep as the first or "over-cut." Rasp teeth are round on top and disconnected, being formed by raising, with a punch, small portions of stock from the surface of the blank.

Coarseness of Cut. — Single- and double-cut files are further classified according to the spacing of the teeth. The names commonly used to designate the different grades of cut are "rough," "coarse," "bastard," "second-cut," "smooth," "dead-smooth" or "super-smooth." "Rough" files are usually single-cut, and the "dead-smooth," double-cut. The other grades are made in both double- and single-cuts. These degrees of coarseness are only comparable when files of the same length are considered, the number of teeth per inch of length decreasing as the length or size of the file increases. Some makers use a series of numbers to designate the cut or coarseness instead of names. The number of teeth per inch varies considerably for different sizes and shapes and on files of different makes. A fair average for ordinary machinists' files made in this country is as follows: Bastard-cut, from 20 to 25 teeth per inch; second-cut, 30 to 40 per inch; smooth, 50 to 60 per inch; dead-smooth, 70 to 80 per inch. The spacing of finer Grobet Swiss files is as follows, the grade being indicated by numbers: No. 0, from 40 to 70 teeth per inch; No. 1, 75 to 88; No. 2, 58 to 104; No. 3, 100 to 130; No. 4, 120 to 160; No. 6, 200 to 220. For American-made files of similar shapes and sizes: No. 0, 35 to 60; No. 1, 55 to 75; No. 2, 80 to 95; No. 3, 90 to 120; No. 4, 125 to 135; No. 6, 160 to 200.

Application of Different Grades and Shapes. — The files most commonly used in general practice are 12- and 14-inch, "flat" and "half-round" double- and single-cut files in bastard, second-cut and smooth grades. The coarse and bastard cuts are generally used on coarser grades of work, and the second-cut and smooth, on comparatively fine work. The coarse and dead-smooth cuts are not often used in ordinary practice, although a rough single-cut is sometimes needed for soft material. The dead-smooth double-cut file is occasionally required for producing very fine surfaces. Single-cut mill files are commonly employed for lathe work. The mill-bastard is adapted to a large variety of lathe filing, and a mill second-cut is used on finer classes of work. The double-cut hand file with one "safe-edge" is used for finishing flat surfaces. The grade of cut is mostly bastard, although many second-cut and smooth are employed. The double-cut square-bastard in both taper and blunt forms is widely used for enlarging or truing rectangular slots, etc. Flat files are mostly double-cut bastard types, second-cut and smooth being used less frequently. Pillar files with one or both edges safe are useful on narrow work. Sharper files are required for cast iron and brass than for steel and wrought iron. Broad surfaces also require sharper files than narrow ones.

File Definitions. — The length of a file means the distance from the point to the heel and does not include the tang. The *heel* is that end of the file body adjacent to the handle. A *blunt* file is one having the same sectional shape from the point to the tang. An *equaling* file is similar in appearance to the blunt form, but has a very slight curvature from the point to the tang. The *back* is the convex side of the half-round, cabinet, pit-saw and similar forms. *Bellied* is the term used to describe a file having a fullness in the center. The coarse grades of single-cut files are sometimes called *floats*. *Safe-edge* means that the edge or side is smooth and without teeth, and may be presented to a surface that does not require filing. *Over-cut* is a term used to describe the first series of teeth on a double-cut file. *Up-cut* means the series of teeth superimposed on the over-cut series of a double-cut file. The term *superfine* (or *super*) cut is used by Lancashire file makers to designate the grade of cut known in this country as "dead-smooth." *Taper* is used to distinguish a file having tapering sides from one that is blunt or straight. Custom has also established this term as a short name for "three-square" or triangular hand-saw files.

Increment-cut Files. — The teeth of an "increment-cut" file are irregularly spaced, the method followed by a prominent concern being as follows: The rows of teeth are spaced progressively wider from the point towards the middle of the file, by regular increments of spacing, and progressively narrower from the middle towards the heel, by regular decrements of spacing. This spacing of the teeth is modified by introducing, while the teeth are being cut, a controllable irregularity as to the spacing, which is confined within maximum and minimum limits but is not a regular progressive increment or decrement. The successive rows are not exactly parallel, but are cut slightly angular with respect to each other, the angle or inclination being reversed during the operation of cutting, as may be required. It will be understood that the increments of spacing are very small. The theory is that if the teeth are equally spaced and have a uniform height and outline, the file will not cut so readily, the reason being that when so many teeth are in contact, considerable pressure is required to make them cut; but when the teeth are slightly irregular in height, a smaller number will be in contact and the pressure required will be correspondingly light. As the longer teeth wear down, the shorter ones will begin to work, although then the file will not cut so freely. File teeth are also formed by the increment-cut, so that they will produce smoother surfaces. If the teeth follow each other at regular intervals, they tend to drop into the cuts or furrows made by the preceding ones, causing chattering; the uneven teeth of hand and increment-cut files tend to prevent this chattering. The opinions of file makers differ regarding the foregoing advantages and objections.

File Testing. — The quality of files can be tested by a special machine (Herbert file-testing machine) which records the endurance and capacity for removing metal, by producing a curve or diagram on sectioned paper wound about a cylindrical drum connected with the file reciprocating mechanism, so as to make one revolution to 120,000 strokes of the file. On these diagrams, the horizontal distances represent the number of strokes made by the file being tested and the vertical distances, the number of cubic inches of metal removed. Tests show a remarkable difference in the quality of files, some being worn out after removing less than one cubic inch of iron, and cutting at the rate of only one cubic inch per 10,000 strokes; whereas, files of good quality remove 12½ cubic inches and cut at the rate of 5 cubic inches per 10,000 strokes. The difference in quality of two sides of the same file also varies greatly; in fact, the two sides are seldom equal in efficiency and durability. The files are tested until they slip over the surface of the test-bar without cutting, this condition being indicated by the curve taking a horizontal course. Tests which are stopped before this point is reached may give a false impression as to the relative merits of files. Thus, it may happen that two files cut equally well during the first 50,000 strokes, and if the tests were stopped at this point, the files would be considered equal in quality, but if they were continued, one file might cease cutting at 60,000 strokes and the other continue for 400,000 strokes, there being a great difference in the durability.

Application of Different Types of Files

Type of File	Character of Cut	Ordinary Use
Mill file, tapered or blunt	Single-cut, mostly bastard	Sharpening mill saws, mowing machine knives, etc. For lathe work, draw filing and, to some extent, for finishing brass and bronze.
Equaling file, mill sections, blunt	Double-cut, mostly bastard	General machine shop work (seldom used).
Flat file, taper	Double-cut, mostly bastard, but also second-cut and smooth	One of the most common files in use; not confined to any specific kind of work, but employed for a great variety of purposes.
Hand file, parallel sides, tapered in thickness	Double-cut, mostly bastard, but also second-cut, smooth and dead-smooth	Preferred by machinists for finishing flat surfaces. Its shape and safe-edge adapt it where a flat file could not be used.
Pillar file, parallel sides, tapered in thickness	Double-cut, same as "hand file" in coarseness	For general machine shop use on narrow work.
Square file, taper	Double-cut, bastard	Principally for enlarging openings of square or rectangular shape.
Square blunt file	Double-cut, bastard	For the rougher work in finishing or enlarging mortises or keyways when of considerable length.
Warding file, parallel in thickness, much taper on sides	Double-cut, mostly bastard	Used by jewelers and machinists, but more especially by locksmiths, for filing "wardnotches" in keys.
Round file, taper and blunt	Mostly bastard	For enlarging round holes and shaping curved surfaces (sometimes called "rat-tail" file).
Half-round, taper	Double-cut, mostly bastard, but also second-cut and smooth	Extensively used in machine shops on curved surfaces, etc.
Hook-tooth file, blunt	Single-cut, bastard	Principally for sharpening the teeth of cross-cut saws, called "hook-tooth" saws.

Application of Different Types of Files

Type of File	Character of Cut	Ordinary Use
Pit-saw file, blunt	Single-cut, second-cut	For filing teeth of what are known as pit and frame saws.
Three-square file, taper	Double-cut, mostly bastard	For filing acute angles, finishing corners, etc.
Hand-saw taper, tapered to a point	Single-cut, second-cut	Principally for sharpening hand saws.
Hand-saw taper	Double-cut, second-cut	Preferred by some for filing fine-toothed hand and hacksaws.
Slim hand-saw taper, three-square section	Single-cut, second-cut	Has largely superseded the regular hand-saw file, the advantage being a greater stroke.
Cant-saw file	Single-cut, bastard	Principally for filing cross-cut saws having M-shaped teeth.
Knife file, taper	Double-cut, mostly bastard	Quite generally used for various purposes to which the knife-shape is adapted.
Crossing file, double oval	Bastard, second-cut and smooth	Used as an engineer file.
Feather-edge file, blunt	Double-cut, bastard, second-cut and smooth	Used for practically the same purposes as knife file (seldom used).
Reaper file, several sections, all blunt	Single-cut, bastard	Principally for sharpening the knives of mowing and reaping machines.
Drill file	Cut only upon the edges	Especially adapted to extending or rounding the ends of slots, when a round file would be too frail.
Half-round wood-file	Double-cut, coarse	Ordinarily used by wood-workers, and sometimes on coarser kinds of brass work.
Cabinet file, wider and thinner than half-round	Double-cut, coarse bastard	For cabinet makers and wood-workers generally.

Results of File Tests. — The file testing machine has shown that the shape of the teeth has much more influence on the rate of cutting and the durability than the quality of the steel of which the file is made. The variations in the rate of cutting are more marked on cast iron than on steel. Of five files tested, all of which were worn out after making about 110,000 strokes, the following results were obtained: When filing cast iron, the best file, when new, cut at the rate of 14 cubic inches per 10,000 strokes, while the poorest file only cut a little over one-half cubic inch during the same number of strokes. When filing steel, a rate of 6 cubic inches per 10,000 strokes is rarely exceeded. The chief factors which affect the cutting efficiency of the file are the sharpness of the teeth; slope of front faces of the teeth, or rake; slope of the back faces of the teeth, or clearance; the temper; the angles of the two cuts relative to the axis of the file; the pitch or coarseness of the cut; and the ratio between the pitch or number of cuts per inch in the "up cut" and in the "over cut." The pitch or coarseness of the cut does not seem to influence the efficiency to any great extent. Very coarse files, however, are almost always inefficient, probably because of the difficulty of producing very large teeth that are, at the same time, sufficiently thin and sharp. On the other hand, very smooth files cut slower and do less work than those of somewhat coarser cut, although in some cases surprising results have been obtained from smooth files.

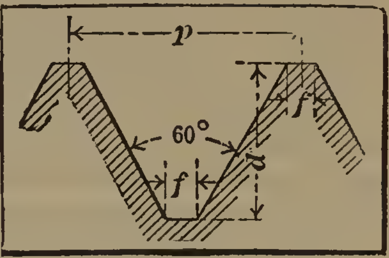
Toolmakers' Files. — Toolmakers' files can be obtained in sizes from 2 to 12 inches in length and in eleven cuts, designated by numbers. The first five numbers correspond approximately in fineness (number of teeth to the inch) to ordinary domestic files as follows: No. 000, same as "rough"; No. 00, same as "bastard"; No. 0, same as "second-cut"; No. 1, same as "smooth"; No. 2, same as "super-smooth." Toolmakers' files from No. 2 to No. 8 have no equivalent in ordinary files. The toolmakers' file is distinguished by its sharp outline, the teeth extending to the extreme points and edges. In width and thickness, these files are also more slender than common files, and somewhat lighter. The exact number of teeth per inch varies in both classes of files and with different makes of the same class. When ordering files of precision, the order should invariably specify toolmakers' files.

Height of Work for Filing. — For filing in a vise, the vise jaws should be level with the workman's elbows, which height varies from 40 to 44 inches from the floor; hence 42 inches is a good average height for a vise that is fixed permanently. If the work to be filed is small and delicate, requiring simply a movement of the arms, the vise should be higher so that the workman can stand erect and see the work to better advantage. If the parts to be filed are heavy, thus requiring considerable effort, the surface should preferably be below the elbow joint, as the operator stands further away from the work, and in a slightly stooping posture. Moreover, in this class of work, it is desirable to throw the weight of the body upon the file to make it cut.

Work Benches. — The height of work benches usually varies from 32 to 36 inches from the floor to the top of the bench, the height depending somewhat upon the nature of the work, lighter work being done on higher benches. For general purposes, the height should be about 34 inches. The width should be about 30 inches, and the top is ordinarily composed of heavy planks, 2 or 3 inches thick, in the front, and lighter 1-inch boards in the back. The thickness of the front planks is varied in accordance with the weight of the work for which the bench is intended. Maple and ash are considered the best woods for bench planking. The preferable position for benches, especially if used for fine accurate work, is the north side of the building, because the light on that side is more even throughout the day. The clearance space or gangway between the bench and the end of any projecting machine handles, handwheels, etc., should not be less than 2 feet 10 inches.

SCREW THREAD SYSTEMS

United States Standard Thread (U.S.S.), also known as the Sellers thread, is the most commonly used screw thread in the United States. When merely the form of thread of the U. S. standard, but not the number of threads per inch corresponding to a certain diameter, is referred to, the abbreviation U. S. F. (United States form) is employed. The sides of the thread form an angle of 60 degrees with each other. The thread is flattened at top and bottom; the width of the flat equals one-eighth of the pitch. If p = pitch of thread, d = depth of thread, and f = width of flat at top and bottom of thread, then:



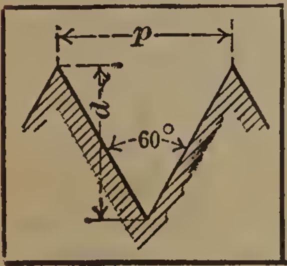
$$p = \frac{1}{\text{number of threads per inch}}$$

$$d = \tfrac{3}{4} \times p \times \cos 30 \text{ deg.} = 0.6495\, p = \frac{0.6495}{\text{no. of threads per inch}}$$

$$f = \frac{p}{8} = \frac{1}{8 \times \text{number of threads per inch}}$$

Tables of number of threads per inch for certain diameters, and of thread parts, are given in the following pages.

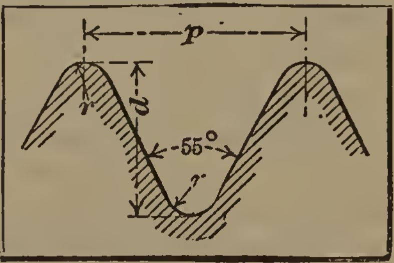
Sharp V-thread (V). — The sides of the thread form an angle of 60 degrees with each other. The top and bottom of the thread are, theoretically, sharp, but in practice it is necessary to make the thread with a slight flat. There is no standard adopted for this flat, but it is usually made about one-twenty-fifth of the pitch. If p = pitch of thread, and d = depth of thread, then:



$$d = p \times \cos 30 \text{ deg.} = 0.866\, p = \frac{0.866}{\text{no. of threads per inch}}$$

Tables for the sharp V-thread are given in the following pages.

Whitworth Standard Thread, also known as British Standard Whitworth (B. S. W.), is used principally in Great Britain, but also to a large extent for stay-bolts in the United States. The form of the thread is shown by the engraving. If p = pitch of thread, d = depth of thread, and r = radius at top and bottom of thread, then:



$$d = \frac{2}{3} \times \frac{p}{2} \times \cot 27 \text{ deg. } 30 \text{ min.} = 0.6403\, p$$

$$= \frac{0.6403}{\text{no. of threads per inch}}$$

$$r = 0.1373\, p = \frac{0.1373}{\text{number of threads per inch}}$$

Tables for the Whitworth standard thread are given in the following pages.

Lloyd & Lloyd threads, sometimes referred to, are simply regular Whitworth threads.

Pitch Diameters of U. S. Thread Screws — 1

Threads per Inch	1/64	1/32	1/16	3/32	1/8	5/32	3/16	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	Threads per Inch
64	.0055	.0211	.0524	.0836	.1149	.1461	.1774	.2086	.2399	.2711	.3024	.3336	.3649	.3961	.4274	.4586	64
62	.0051	.0207	.0520	.0832	.1145	.1457	.1770	.2082	.2395	.2707	.3020	.3332	.3645	.3957	.4270	.4582	62
60	.0048	.0204	.0517	.0829	.1142	.1454	.1767	.2079	.2392	.2704	.3017	.3329	.3642	.3954	.4267	.4579	60
58	.0044	.0200	.0513	.0825	.1138	.1450	.1763	.2075	.2388	.2700	.3013	.3325	.3638	.3950	.4263	.4575	58
56	.0040	.0196	.0509	.0821	.1134	.1446	.1759	.2071	.2384	.2696	.3009	.3321	.3634	.3946	.4259	.4571	56
54	.0036	.0192	.0505	.0817	.1130	.1442	.1755	.2067	.2380	.2692	.3005	.3317	.3630	.3942	.4255	.4567	54
52	.0031	.0187	.0500	.0812	.1125	.1437	.1750	.2062	.2375	.2687	.3000	.3312	.3625	.3937	.4250	.4562	52
50	.0026	.0182	.0495	.0807	.1120	.1432	.1745	.2057	.2370	.2682	.2995	.3307	.3620	.3932	.4245	.4557	50
48	.0021	.0177	.0490	.0802	.1115	.1427	.1740	.2052	.2365	.2677	.2990	.3302	.3615	.3927	.4240	.4552	48
46	.0015	.0171	.0484	.0796	.1109	.1421	.1734	.2046	.2359	.2671	.2984	.3296	.3609	.3921	.4234	.4546	46
44	.0008	.0164	.0477	.0789	.1102	.1414	.1727	.2039	.2352	.2664	.2977	.3289	.3602	.3914	.4227	.4539	44
42	.0001	.0157	.0470	.0782	.1095	.1407	.1720	.2032	.2345	.2657	.2970	.3282	.3595	.3907	.4220	.4532	42
40	.9994	.0150	.0463	.0775	.1088	.1400	.1713	.2025	.2338	.2650	.2963	.3275	.3588	.3900	.4213	.4525	40
38	.9985	.0141	.0454	.0766	.1079	.1391	.1704	.2016	.2329	.2641	.2954	.3266	.3579	.3891	.4204	.4516	38
36	.9976	.0132	.0445	.0757	.1070	.1382	.1695	.2007	.2320	.2632	.2945	.3257	.3570	.3882	.4195	.4507	36
34	.9965	.0121	.0434	.0746	.1059	.1371	.1684	.1996	.2309	.2621	.2934	.3246	.3559	.3871	.4184	.4496	34
32	.9953	.0109	.0422	.0734	.1047	.1359	.1672	.1984	.2297	.2609	.2922	.3234	.3547	.3859	.4172	.4484	32
30	.9939	.0095	.0408	.0720	.1033	.1345	.1658	.1970	.2283	.2595	.2908	.3220	.3533	.3845	.4158	.4470	30
28	.9924	.0080	.0393	.0705	.1018	.1330	.1643	.1955	.2268	.2580	.2893	.3205	.3518	.3830	.4143	.4455	28
27	.9915	.0071	.0384	.0696	.1009	.1321	.1634	.1946	.2259	.2571	.2884	.3196	.3509	.3821	.4134	.4446	27
26	.9906	.0062	.0375	.0687	.1000	.1312	.1625	.1937	.2250	.2562	.2875	.3187	.3500	.3812	.4125	.4437	26
24	.9885	.0041	.0354	.0666	.0979	.1291	.1604	.1916	.2229	.2541	.2854	.3166	.3479	.3791	.4104	.4416	24
22	.9861	.0017	.0330	.0642	.0955	.1267	.1580	.1892	.2205	.2517	.2830	.3142	.3455	.3767	.4080	.4392	22
20	.9832	.9988	.0301	.0613	.0926	.1238	.1551	.1863	.2176	.2488	.2801	.3113	.3426	.3738	.4051	.4363	20
18	.9796	.9952	.0265	.0577	.0890	.1202	.1515	.1827	.2140	.2452	.2765	.3077	.3390	.3702	.4015	.4327	18
16	.9750	.9906	.0219	.0531	.0844	.1156	.1469	.1781	.2094	.2406	.2719	.3031	.3344	.3656	.3969	.4281	16
14	.9692	.9848	.0161	.0473	.0786	.1098	.1411	.1723	.2036	.2348	.2661	.2973	.3286	.3598	.3911	.4223	14
13	.9657	.9813	.0126	.0438	.0751	.1063	.1376	.1688	.2001	.2313	.2626	.2938	.3251	.3563	.3876	.4188	13
12	.9615	.9771	.0084	.0396	.0709	.1021	.1334	.1646	.1959	.2271	.2584	.2896	.3209	.3521	.3834	.4146	12
11	.9566	.9722	.0035	.0347	.0660	.0972	.1285	.1597	.1910	.2222	.2535	.2847	.3160	.3472	.3785	.4097	11
10	.9507	.9663	.9976	.0288	.0601	.0913	.1226	.1538	.1851	.2163	.2476	.2788	.3101	.3413	.3726	.4038	10
9	.9435	.9591	.9904	.0216	.0529	.0841	.1154	.1466	.1779	.2091	.2404	.2716	.3029	.3341	.3654	.3966	9
8	.9344	.9500	.9813	.0125	.0438	.0750	.1063	.1375	.1688	.2000	.2313	.2625	.2938	.3250	.3563	.3875	8
7	.9228	.9384	.9697	.0009	.0322	.0634	.0947	.1259	.1572	.1884	.2197	.2509	.2822	.3134	.3447	.3759	7
6	.9074	.9230	.9543	.9855	.0168	.0480	.0793	.1105	.1418	.1730	.2043	.2355	.2668	.2980	.3293	.3605	6

When the outside diameter and the number of threads per inch are known, the pitch diameter is found in the table above. Decimals only are given, so that whatever the whole number is that precedes the fraction at the head of the columns, giving the outside diameter, if the same number is placed before the decimal, the correct pitch diameter, is obtained, except in that portion of the table below the heavy line, where the whole number preceding the decimal should be one less than the whole number preceding the fraction at the top the column.

Pitch Diameters of U. S. Thread Screws — 2

Threads per Inch	½	17/32	9/16	19/32	⅝	21/32	11/16	23/32	¾	25/32	13/16	27/32	⅞	29/32	15/16	31/32	1	Threads per Inch
64	.4899	.5211	.5524	.5836	.6149	.6461	.6774	.7086	.7399	.7711	.8024	.8336	.8649	.8961	.9274	.9586	.9899	64
62	.4895	.5207	.5520	.5832	.6145	.6457	.6770	.7082	.7395	.7707	.8020	.8332	.8645	.8957	.9270	.9582	.9895	62
60	.4892	.5204	.5517	.5829	.6142	.6454	.6767	.7079	.7392	.7704	.8017	.8329	.8642	.8954	.9267	.9579	.9892	60
58	.4888	.5200	.5513	.5825	.6138	.6450	.6763	.7075	.7388	.7700	.8013	.8325	.8638	.8950	.9263	.9575	.9888	58
56	.4884	.5196	.5509	.5821	.6134	.6446	.6759	.7071	.7384	.7696	.8009	.8321	.8634	.8946	.9259	.9571	.9884	56
54	.4880	.5192	.5505	.5817	.6130	.6442	.6755	.7067	.7380	.7692	.8005	.8317	.8630	.8942	.9255	.9567	.9880	54
52	.4875	.5187	.5500	.5812	.6125	.6437	.6750	.7062	.7375	.7687	.8000	.8312	.8625	.8937	.9250	.9562	.9875	52
50	.4870	.5182	.5495	.5807	.6120	.6432	.6745	.7057	.7370	.7682	.7995	.8307	.8620	.8932	.9245	.9557	.9870	50
48	.4865	.5177	.5490	.5802	.6115	.6427	.6740	.7052	.7365	.7677	.7990	.8302	.8615	.8927	.9240	.9552	.9865	48
46	.4859	.5171	.5484	.5796	.6109	.6421	.6734	.7046	.7359	.7671	.7984	.8296	.8609	.8921	.9234	.9546	.9859	46
44	.4852	.5164	.5477	.5789	.6102	.6414	.6727	.7039	.7352	.7664	.7977	.8289	.8602	.8914	.9227	.9539	.9852	44
42	.4845	.5157	.5470	.5782	.6095	.6407	.6720	.7032	.7345	.7657	.7970	.8282	.8595	.8907	.9220	.9532	.9845	42
40	.4838	.5150	.5463	.5775	.6088	.6400	.6713	.7025	.7338	.7650	.7963	.8275	.8588	.8900	.9213	.9525	.9838	40
38	.4829	.5141	.5454	.5766	.6079	.6391	.6704	.7016	.7329	.7641	.7954	.8266	.8579	.8891	.9204	.9516	.9829	38
36	.4820	.5132	.5445	.5757	.6070	.6382	.6695	.7007	.7320	.7632	.7945	.8257	.8570	.8882	.9195	.9507	.9820	36
34	.4809	.5121	.5434	.5746	.6059	.6371	.6684	.6996	.7309	.7621	.7934	.8246	.8559	.8871	.9184	.9496	.9809	34
32	.4797	.5109	.5422	.5734	.6047	.6359	.6672	.6984	.7297	.7609	.7922	.8234	.8547	.8859	.9172	.9484	.9797	32
30	.4783	.5095	.5408	.5720	.6033	.6345	.6658	.6970	.7283	.7595	.7908	.8220	.8533	.8845	.9158	.9470	.9783	30
28	.4768	.5080	.5393	.5705	.6018	.6330	.6643	.6955	.7268	.7580	.7893	.8205	.8518	.8830	.9143	.9455	.9768	28
27	.4759	.5071	.5384	.5696	.6009	.6321	.6634	.6946	.7259	.7571	.7884	.8196	.8509	.8821	.9134	.9446	.9759	27
26	.4750	.5062	.5375	.5687	.6000	.6312	.6625	.6937	.7250	.7562	.7875	.8187	.8500	.8812	.9125	.9437	.9750	26
24	.4729	.5041	.5354	.5666	.5979	.6291	.6604	.6916	.7229	.7541	.7854	.8166	.8479	.8791	.9104	.9416	.9729	24
22	.4705	.5017	.5330	.5642	.5955	.6267	.6580	.6892	.7205	.7517	.7830	.8142	.8455	.8767	.9080	.9392	.9705	22
20	.4676	.4988	.5301	.5613	.5926	.6238	.6551	.6863	.7176	.7488	.7801	.8113	.8426	.8738	.9051	.9363	.9676	20
18	.4640	.4952	.5265	.5577	.5890	.6202	.6515	.6827	.7140	.7452	.7765	.8077	.8390	.8702	.9015	.9327	.9640	18
16	.4594	.4906	.5219	.5531	.5844	.6156	.6469	.6781	.7094	.7406	.7719	.8031	.8344	.8656	.8969	.9281	.9594	16
14	.4536	.4848	.5161	.5473	.5786	.6098	.6411	.6723	.7036	.7348	.7661	.7973	.8286	.8598	.8911	.9223	.9536	14
13	.4501	.4813	.5126	.5438	.5751	.6063	.6376	.6688	.7001	.7313	.7626	.7938	.8251	.8563	.8876	.9188	.9501	13
12	.4459	.4771	.5084	.5396	.5709	.6021	.6334	.6646	.6959	.7271	.7584	.7896	.8209	.8521	.8834	.9146	.9459	12
11	.4410	.4722	.5035	.5347	.5660	.5972	.6285	.6597	.6910	.7222	.7535	.7847	.8160	.8472	.8785	.9097	.9410	11
10	.4351	.4663	.4976	.5288	.5601	.5913	.6226	.6538	.6851	.7163	.7476	.7788	.8101	.8413	.8726	.9038	.9351	10
9	.4279	.4591	.4904	.5216	.5529	.5841	.6154	.6466	.6779	.7091	.7404	.7716	.8029	.8341	.8654	.8966	.9279	9
8	.4188	.4500	.4813	.5125	.5438	.5750	.6063	.6375	.6688	.7000	.7313	.7625	.7938	.8250	.8563	.8875	.9188	8
7	.4072	.4384	.4697	.5009	.5322	.5634	.5947	.6259	.6572	.6884	.7197	.7509	.7822	.8134	.8447	.8759	.9072	7
6	.3918	.4230	.4543	.4855	.5168	.5480	.5793	.6105	.6418	.6730	.7043	.7355	.7668	.7980	.8293	.8605	.8918	6

When the outside diameter and the number of threads per inch are known, the pitch diameter is found in the table above. Decimals only are given, so that whatever the whole number is that precedes the fraction at the head of the columns, giving the outside diameter, if the same number is placed before the decimal, the correct pitch diameter is obtained.

United States Standard Thread — Number of Threads per Inch Corresponding to a Given Diameter

Diameter	Threads per Inch	Diameter at Root of Thread	Diameter	Threads per Inch	Diameter at Root of Thread	Diameter	Threads per Inch	Diameter at Root of Thread
1/16	64	0.0422	1 1/8	7	0.9394	2 3/4	4	2.4252
3/32	50	0.0678	1 3/16	7	1.0019	2 7/8	3 1/2	2.5038
1/8	40	0.0925	1 1/4	7	1.0644	3	3 1/2	2.6288
5/32	36	0.1202	1 5/16	6	1.0960	3 1/8	3 1/2	2.7538
3/16	32	0.1469	1 3/8	6	1.1585	3 1/4	3 1/2	2.8788
7/32	28	0.1724	1 7/16	6	1.2210	3 3/8	3 1/4	2.9753
1/4	20	0.1850	1 1/2	6	1.2835	3 1/2	3 1/4	3.1003
5/16	18	0.2403	1 9/16	5 1/2	1.3263	3 5/8	3 1/4	3.2253
3/8	16	0.2938	1 5/8	5 1/2	1.3888	3 3/4	3	3.3170
7/16	14	0.3447	1 11/16	5 1/2	1.4513	3 7/8	3	3.4420
1/2	13	0.4001	1 3/4	5	1.4902	4	3	3.5670
9/16	12	0.4542	1 13/16	5	1.5527	4 1/4	2 7/8	3.7982
5/8	11	0.5069	1 7/8	5	1.6152	4 1/2	2 3/4	4.0276
11/16	11	0.5694	1 15/16	5	1.6777	4 3/4	2 5/8	4.2551
3/4	10	0.6201	2	4 1/2	1.7113	5	2 1/2	4.4804
13/16	10	0.6826	2 1/8	4 1/2	1.8363	5 1/4	2 1/2	4.7304
7/8	9	0.7307	2 1/4	4 1/2	1.9613	5 1/2	2 3/8	4.9530
15/16	9	0.7932	2 3/8	4	2.0502	5 3/4	2 3/8	5.2030
1	8	0.8376	2 1/2	4	2.1752	6	2 1/4	5.4226
1 1/16	7	0.8769	2 5/8	4	2.3002	......	....	....

Elements of the United States Standard Thread

Threads per Inch	Depth of Thread	Width of Flat	Double Depth of Thread	Threads per Inch	Depth of Thread	Width of Flat	Double Depth of Thread
2 1/4	0.2887	0.0556	0.5774	18	0.0361	0.0069	0.0722
2 3/8	0.2735	0.0526	0.5470	20	0.0325	0.0062	0.0650
2 1/2	0.2598	0.0500	0.5196	22	0.0295	0.0057	0.0590
2 5/8	0.2474	0.0476	0.4949	24	0.0271	0.0052	0.0541
2 3/4	0.2362	0.0455	0.4724	26	0.0250	0.0048	0.0500
2 7/8	0.2259	0.0435	0.4518	28	0.0232	0.0045	0.0464
3	0.2165	0.0417	0.4330	30	0.0217	0.0042	0.0433
3 1/4	0.1998	0.0385	0.3997	32	0.0203	0.0039	0.0406
3 1/2	0.1856	0.0357	0.3712	34	0.0191	0.0037	0.0382
4	0.1624	0 0312	0.3248	36	0.0180	0.0035	0.0361
4 1/2	0.1443	0.0278	0.2887	38	0.0171	0.0033	0.0342
5	0.1299	0.0250	0.2598	40	0.0162	0.0031	0.0325
5 1/2	0.1181	0.0227	0.2362	42	0.0155	0.0030	0.0309
6	0.1083	0.0208	0.2165	44	0.0148	0.0028	0.0295
7	0.0928	0.0179	0.1856	46	0.0141	0.0027	0.0282
8	0.0812	0.0156	0.1624	48	0.0135	0.0026	0.0271
9	0.0722	0.0139	0.1443	50	0.0130	0.0025	0.0260
10	0.0650	0.0125	0.1299	52	0.0125	0.0024	0.0250
11	0.0590	0.0114	0.1181	56	0.0116	0.0022	0.0232
12	0.0541	0.0104	0.1083	60	0.0108	0.0021	0.0217
13	0.0500	0.0096	0.0999	64	0.0101	0.0020	0.0203
14	0.0464	0.0089	0.0928	68	0.0096	0.0018	0.0191
15	0.0433	0.0083	0.0866	72	0.0090	0.0017	0.0180
16	0.0406	0.0078	0.0812	80	0.0081	0.0016	0.0162

Sharp V-thread — Number of Threads per Inch Corresponding to a Given Diameter

Diameter	Threads per Inch	Diameter at Root of Thread	Diameter	Threads per Inch	Diameter at Root of Thread	Diameter	Threads per Inch	Diameter at Root of Thread
1/16	72	0.0384	1 1/8	7	0.8776	2 3/4	4	2.3170
3/32	56	0.0628	1 3/16	7	0.9401	2 7/8	4	2.4420
1/8	40	0.0817	1 1/4	7	1.0026	3	3 1/2	2.5051
5/32	32	0.1021	1 5/16	7	1.0651	3 1/8	3 1/2	2.6301
3/16	24	0.1153	1 3/8	6	1.0863	3 1/4	3 1/2	2.7551
7/32	24	0.1465	1 7/16	6	1.1488	3 3/8	3 1/4	2.8421
1/4	20	0.1634	1 1/2	6	1.2113	3 1/2	3 1/4	2.9671
5/16	18	0.2163	1 9/16	6	1.2738	3 5/8	3 1/4	3.0921
3/8	16	0.2667	1 5/8	5	1.2786	3 3/4	3	3.1726
7/16	14	0.3138	1 11/16	5	1.3411	3 7/8	3	3.2976
1/2	12	0.3557	1 3/4	5	1.4036	4	3	3.4226
9/16	12	0.4182	1 13/16	5	1.4661	4 1/4	2 7/8	3.6475
5/8	11	0.4675	1 7/8	4 1/2	1.4901	4 1/2	2 3/4	3.8702
11/16	11	0.5300	1 15/16	4 1/2	1.5526	4 3/4	2 5/8	4.0902
3/4	10	0.5768	2	4 1/2	1.6151	5	2 1/2	4.3072
13/16	10	0.6393	2 1/8	4 1/2	1.7401	5 1/4	2 1/2	4.5572
7/8	9	0.6825	2 1/4	4 1/2	1.8651	5 1/2	2 3/8	4.7707
15/16	9	0.7450	2 3/8	4 1/2	1.9901	5 3/4	2 3/8	5.0207
1	8	0.7835	2 1/2	4	2.0670	6	2 1/4	5.2302
1 1/16	8	0.8460	2 5/8	4	2.1920	......	....	....

Elements of the Sharp V-thread

Threads per Inch	Depth of Thread	Width of Flat*	Double Depth of Thread	Threads per Inch	Depth of Thread	Width of Flat*	Double Depth of Thread
2 1/4	0.3849	0.0178	0.7698	18	0.0481	0.0022	0.0962
2 3/8	0.3646	0.0168	0.7293	20	0.0433	0.0020	0.0866
2 1/2	0.3464	0.0160	0.6928	22	0.0394	0.0018	0.0787
2 5/8	0.3299	0.0152	0.6598	24	0.0361	0.0017	0.0722
2 3/4	0.3149	0.0145	0.6298	26	0.0333	0.0015	0.0666
2 7/8	0.3012	0.0139	0.6025	28	0.0309	0.0014	0.0619
3	0.2887	0.0133	0.5774	30	0.0289	0.0013	0.0577
3 1/4	0.2665	0.0123	0.5329	32	0.0271	0.0012	0.0541
3 1/2	0.2474	0.0114	0.4949	34	0.0255	0.0012	0.0509
4	0.2165	0.0100	0.4330	36	0.0241	0.0011	0.0481
4 1/2	0.1925	0.0089	0.3849	38	0.0228	0.0011	0.0456
5	0.1732	0.0080	0.3464	40	0.0217	0.0010	0.0433
5 1/2	0.1575	0.0073	0.3149	42	0.0206	0.0010	0.0412
6	0.1443	0.0067	0.2887	44	0.0197	0.0009	0.0394
7	0.1237	0.0057	0.2474	46	0.0188	0.0009	0.0377
8	0.1083	0.0050	0.2165	48	0.0180	0.0008	0.0361
9	0.0962	0.0044	0.1925	50	0.0173	0.0008	0.0346
10	0.0866	0.0040	0.1732	52	0.0167	0.0008	0.0333
11	0.0787	0.0036	0.1575	56	0.0155	0.0007	0.0309
12	0.0722	0.0033	0.1443	60	0.0144	0.0007	0.0289
13	0.0666	0.0031	0.1332	64	0.0135	0.0006	0.0271
14	0.0619	0.0029	0.1237	68	0.0127	0.0006	0.0255
15	0.0577	0.0027	0.1155	72	0.0120	0.0006	0.0241
16	0.0541	0.0025	0.1083	80	0.0108	0.0005	0.0217

* Flat for avoiding extreme sharp corner usually equals 1/25 of the pitch.

Whitworth Standard Thread — Number of Threads per Inch Corresponding to a Given Diameter

Diameter	Threads per Inch	Diameter at Root of Thread	Diameter	Threads per Inch	Diameter at Root of Thread	Diameter	Threads per Inch	Diameter at Root of Thread
1/16	60	0.0412	1 1/8	7	0.9420	2 3/4	3 1/2	2.3841
3/32	48	0.0670	1 3/16	7	1.0045	2 7/8	3 1/2	2.5091
1/8	40	0.0930	1 1/4	7	1.0670	3	3 1/2	2.6341
5/32	32	0.1162	1 5/16	7	1.1295	3 1/8	3 1/2	2.7591
3/16	24	0.1341	1 3/8	6	1.1616	3 1/4	3 1/4	2.8560
7/32	24	0.1653	1 7/16	6	1.2241	3 3/8	3 1/4	2.9810
1/4	20	0.1860	1 1/2	6	1.2866	3 1/2	3 1/4	3.1060
5/16	18	0.2414	1 9/16	6	1.3491	3 5/8	3 1/4	3.2310
3/8	16	0.2950	1 5/8	5	1.3689	3 3/4	3	3.3231
7/16	14	0.3460	1 11/16	5	1.4314	3 7/8	3	3.4481
1/2	12	0.3933	1 3/4	5	1.4939	4	3	3.5731
9/16	12	0.4558	1 13/16	5	1.5564	4 1/4	2 7/8	3.8046
5/8	11	0.5086	1 7/8	4 1/2	1.5904	4 1/2	2 7/8	4.0546
11/16	11	0.5711	1 15/16	4 1/2	1.6529	4 3/4	2 3/4	4.2843
3/4	10	0.6219	2	4 1/2	1.7154	5	2 3/4	4.5343
13/16	10	0.6844	2 1/8	4 1/2	1.8404	5 1/4	2 5/8	4.7621
7/8	9	0.7327	2 1/4	4	1.9298	5 1/2	2 5/8	5.0121
15/16	9	0.7952	2 3/8	4	2.0548	5 3/4	2 1/2	5.2377
1	8	0.8399	2 1/2	4	2.1798	6	2 1/2	5.4877
1 1/16	8	0.9024	2 5/8	4	2.3048	......	....	....

Elements of the Whitworth Standard Thread

Threads per Inch	Depth of Thread	Radius	Double Depth of Thread	Threads per Inch	Depth of Thread	Radius	Double Depth of Thread
2 1/4	0.2846	0.0610	0.5692	18	0.0356	0.0076	0.0711
2 3/8	0.2696	0.0578	0.5392	20	0.0320	0.0069	0.0640
2 1/2	0.2561	0.0549	0.5123	22	0.0291	0.0062	0.0582
2 5/8	0.2439	0.0523	0.4879	24	0.0267	0.0057	0.0534
2 3/4	0.2328	0.0499	0.4657	26	0.0246	0.0053	0.0493
2 7/8	0.2227	0.0478	0.4454	28	0.0229	0.0049	0.0457
3	0.2134	0.0458	0.4269	30	0.0213	0.0046	0.0427
3 1/4	0.1970	0.0422	0.3940	32	0.0200	0.0043	0.0400
3 1/2	0.1830	0.0392	0.3659	34	0.0188	0.0040	0.0377
4	0.1601	0.0343	0.3202	36	0.0178	0.0038	0.0356
4 1/2	0.1423	0.0305	0.2846	38	0.0169	0.0036	0.0337
5	0.1281	0.0275	0.2561	40	0.0160	0.0034	0.0320
5 1/2	0.1164	0.0250	0.2328	42	0.0152	0.0033	0.0305
6	0.1067	0.0229	0.2134	44	0.0146	0.0031	0.0291
7	0.0915	0.0196	0.1830	46	0.0139	0.0030	0.0278
8	0.0800	0.0172	0.1601	48	0.0133	0.0029	0.0267
9	0.0711	0.0153	0.1423	50	0.0128	0.0027	0.0256
10	0.0640	0.0137	0.1281	52	0.0123	0.0026	0.0246
11	0.0582	0.0125	0.1164	56	0.0114	0.0025	0.0229
12	0.0534	0.0114	0.1067	60	0.0107	0.0023	0.0213
13	0.0493	0.0106	0.0985	64	0.0100	0.0021	0.0200
14	0.0457	0.0098	0.0915	68	0.0094	0.0020	0.0188
15	0.0427	0.0092	0.0854	72	0.0089	0.0019	0.0178
16	0.0400	0.0086	0.0800	80	0.0080	0.0017	0.0160

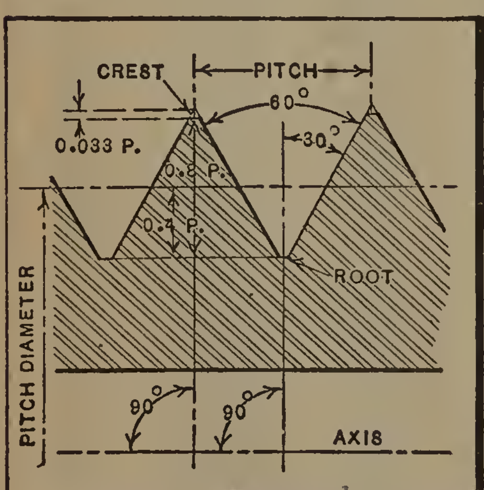
Society of Automotive Engineers' Standard Screw Threads
(S. A. E. Standard)

Screw Diam., Inches	Threads per Inch		Screw Diam., Inches	Threads per Inch		Screw Diam., Inches	Threads per Inch	
	Regular	Fine *		Regular	Fine		Regular	Fine
1/4	28	36	1 7/8	12	16	4	10	16
5/16	24	32	2	12	16	4 1/8	10	16
3/8	24	32	2 1/8	12	16	4 1/4	10	16
7/16	20	28	2 1/4	12	16	4 3/8	10	16
1/2	20	28	2 3/8	12	16	4 1/2	10	16
9/16	18	24	2 1/2	12	16	4 5/8	10	16
5/8	18	24	2 5/8	12	16	4 3/4	10	16
11/16	16	24	2 3/4	12	16	4 7/8	10	16
3/4	16	20	2 7/8	12	16	5	10	16
7/8	14	20	3	10	16	5 1/8	10	16
1	14	20	3 1/8	10	16	5 1/4	10	16
1 1/8	12	18	3 1/4	10	16	5 3/8	10	16
1 1/4	12	18	3 3/8	10	16	5 1/2	10	16
1 3/8	12	18	3 1/2	10	16	5 5/8	10	16
1 1/2	12	18	3 5/8	10	16	5 3/4	10	16
1 5/8	12	16	3 3/4	10	16	5 7/8	10	16
1 3/4	12	16	3 7/8	10	16	6 and over	8	16

Note: — All threads to be U. S. standard form. The maximum screw size equals the nominal or basic screw size for all except wrench fits. In the minimum gage for internal threads and the maximum gage for external threads, the profile of the thread shall be such as not to encroach on that of the true U. S. standard thread. The above note is given to insure interchangeability, and does not pertain to dimensions or clearances.

* Fine pitches for 1/4 to 1 1/2 inch diameter inclusive, adopted for aeronautic standard, March, 1918.

Standard Pipe Thread. — The National standard pipe thread (also known as the American standard and as the American Briggs standard) establishes the outside diameter of the pipe, the diameters of external and internal pipe threads, the thread profile, the pitch, the effective thread length, the taper of the thread, and the gage dimensions. The standard pipe thread has an angle of 60 degrees, and the crest and root are truncated an amount equal to 0.033 × pitch of thread (see illustration). The thread depth equals 0.8 × pitch of thread, and the taper on the diameter is 1/16 inch per inch or 3/4 inch per foot. The dimensions of standard pipe threads are given in the table, pages 1154–1155. The pitch diameter at the end of the pipe thread and at the gaging notch of the plug gage, as well as the effective pipe thread length, are determined by the following formulas:



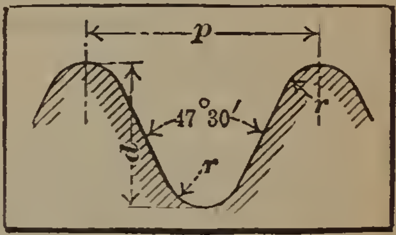
$$F = B - (0.05\,B + 1.1)\,P; \quad E = F + 0.0625 \times D; \quad C = (0.8 \times B + 6.8)\,P$$

in which F = pitch diameter at end; E = pitch diameter at gaging notch; B = outside diameter of pipe; D = normal engagement, by hand, between external and internal threads; C = effective length of external thread; P = pitch of thread.

The working gages for each size consist of one taper threaded plug gage and one taper threaded ring gage. The length of thread on the plug gage equals length C of the effective thread (see table) and the thickness of the ring gage equals length D of normal engagement, by hand, between external and internal threads. The crests of threads on ring and plug gages are truncated an amount equal to 0.1 × pitch.

In cutting Briggs pipe threads, the threading tool should be set with its center line at right angles to the axis of the piece to be threaded. When cutting British standard pipe threads (Whitworth form), the thread tool is set square with the side or conical surface of the work.

British Association Standard Thread (B.A.S.). — This standard is used abroad for very small screws. The form of the thread is shown in the accompanying engraving. The angle between the sides of the thread is 47 degrees 30 minutes. If p = pitch of thread, d = depth of thread and r = radius at top and bottom of thread, then:

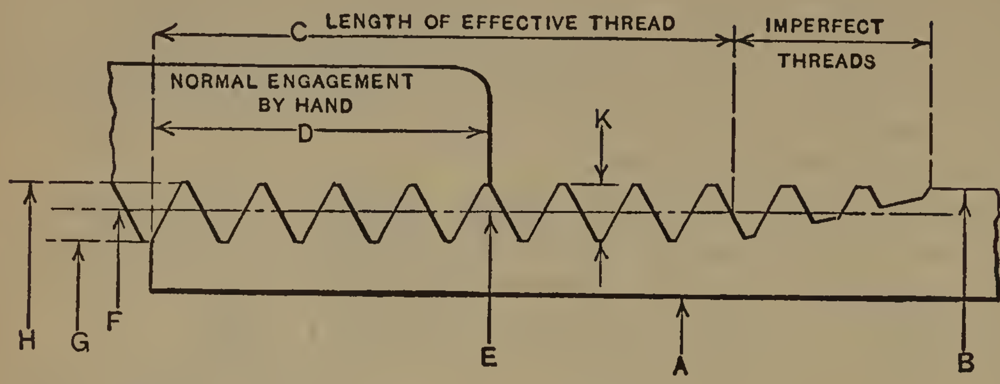


$$d = 0.6\,p, \quad r = \frac{2\,p}{11}$$

British Association Standard Thread

British Association Number	Diameter		Pitch		Depth of Thread	Radius	Double Depth of Thread
	Millimeters	Inches	Millimeters	Inches	Inches	Inches	Inches
0	6.0	0.2362	1.0	0.0394	0.0236	0.0072	0.0472
1	5.3	0.2087	0.90	0.0354	0.0212	0.0064	0.0425
2	4.7	0.1850	0.81	0.0319	0.0191	0.0058	0.0383
3	4.1	0.1614	0.73	0.0287	0.0172	0.0052	0.0345
4	3.6	0.1417	0.66	0.0260	0.0156	0.0047	0.0312
5	3.2	0.1260	0.59	0.0232	0.0139	0.0042	0.0279
6	2.8	0.1102	0.53	0.0209	0.0125	0.0038	0.0250
7	2.5	0.0984	0.48	0.0189	0.0113	0.0034	0.0227
8	2.2	0.0866	0.43	0.0169	0.0101	0.0031	0.0203
9	1.9	0.0748	0.39	0.0154	0.0092	0.0028	0.0184
10	1.7	0.0669	0.35	0.0138	0.0083	0.0025	0.0165
11	1.5	0.0591	0.31	0.0122	0.0073	0.0022	0.0146
12	1.3	0.0511	0.28	0.0110	0.0066	0.0020	0.0132
13	1.2	0.0472	0.25	0.0098	0.0059	0.0018	0.0118
14	1.0	0.0394	0.23	0.0091	0.0055	0.0016	0.0109
15	0.90	0.0354	0.21	0.0083	0.0050	0.0015	0.0099
16	0.79	0.0311	0.19	0.0075	0.0045	0.0014	0.0090
17	0.70	0.0276	0.17	0.0067	0.0040	0.0012	0.0080
18	0.62	0.0244	0.15	0.0059	0.0035	0.0011	0.0071
19	0.54	0.0213	0.14	0.0055	0.0033	0.0010	0.0066
20	0.48	0.0189	0.12	0.0047	0.0028	0.0009	0.0057
21	0.42	0.0165	0.11	0.0043	0.0026	0.0008	0.0052
22	0.37	0.0146	0.098	0.0039	0.0023	0.0007	0.0046
23	0.33	0.0130	0.089	0.0035	0.0021	0.0006	0.0042
24	0.29	0.0114	0.080	0.0031	0.0019	0.0006	0.0038
25	0.25	0.0098	0.072	0.0028	0.0017	0 0005	0.0034

National or American Briggs Standard Pipe Thread



For all dimensions see corresponding reference letters in table.

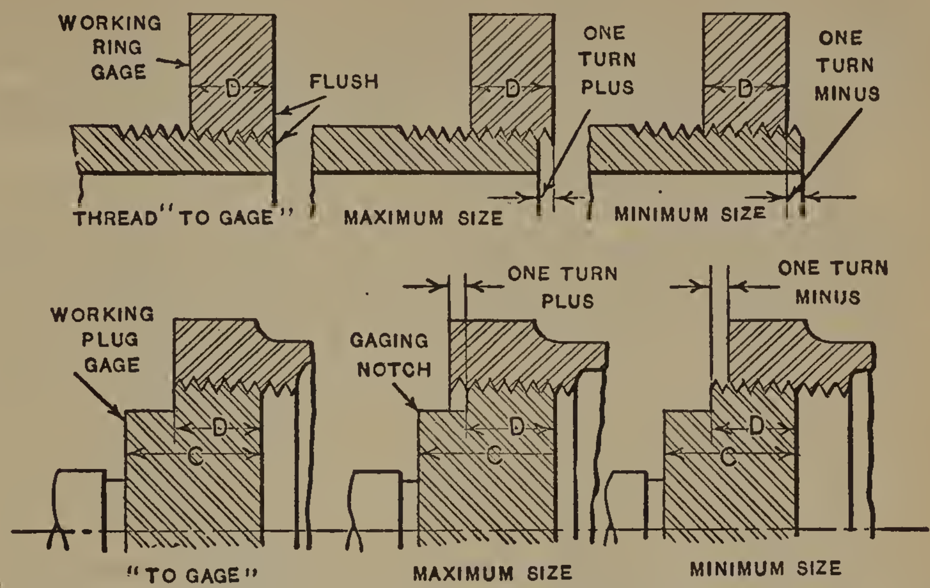
Angle between sides of thread is 60 degrees when measured in the axial plane and a line bisecting this angle is perpendicular to the axis of the pipe.

The thread depth is 0.8 × pitch of thread and the crest and root are truncated an amount equal to 0.033 × pitch. (See illustration, page 1152.)

All dimensions in inches.

Diameter of Pipe			No. of Threads per Inch	Length of Effective Thread	Length of Normal Engagement by Hand	Pitch Diameter at Gaging Notch	Pitch Diameter at Small End of Thread	Root Diameter at Small End of Thread
Nominal Inside	Actual Inside	Actual Outside						
	A	*B*		*C*	*D*	*E*	*F*	*G*
⅛	0.269	0.405	27	0.264	0.18	0.3748	0.3635	0.334
¼	0.364	0.540	18	0.402	0.20	0.4899	0.4774	0.433
⅜	0.493	0.675	18	0.408	0.24	0.6270	0.6120	0.568
½	0.622	0.840	14	0.534	0.32	0.7784	0.7584	0.701
¾	0.824	1.050	14	0.548	0.34	0.9889	0.9677	0.911
1	1.049	1.315	11½	0.683	0.40	1.2386	1.2136	1.144
1¼	1.380	1.660	11½	0.707	0.42	1.5834	1.5571	1.488
1½	1.610	1.900	11½	0.724	0.42	1.8223	1.7961	1.727
2	2.067	2.375	11½	0.757	0.44	2.2963	2.2690	2.199
2½	2.469	2.875	8	1.138	0.68	2.7622	2.7195	2.620
3	3.068	3.500	8	1.200	0.77	3.3885	3.3406	3.241
3½	3.548	4.000	8	1.250	0.82	3.8888	3.8375	3.738
4	4.026	4.500	8	1.300	0.84	4.3871	4.3344	4.234
4½	4.506	5.000	8	1.350	0.88	4.8859	4.8313	4.731
5	5.047	5.563	8	1.406	0.94	5.4493	5.3907	5.291
6	6.065	6.625	8	1.513	0.96	6.5060	6.4461	6.346
7	7.023	7.625	8	1.613	1.00	7.5023	7.4398	7.340
8	7.981	8.625	8	1.713	1.06	8.5000	8.4336	8.334
9	8.941	9.625	8	1.813	1.13	9.4980	9.4273	9.327
10	10.020	10.750	8	1.925	1.21	10.6209	10.5453	10.445
11	11.000	11.750	8	2.025	1.29	11.6194	11.5391	11.439
12	12.000	12.750	8	2.125	1.36	12.6178	12.5328	12.433

National or American Briggs Standard Pipe Thread (*Continued*)



For dimensions *C* and *D*, see corresponding reference letters in section of table on opposite page.

Nominal Inside Diameter	Outside Diam. at Small End of Thread	Reamer Diam. at Large End of Reamed Hole	Depth of Thread	Reference Gages		New Working Gages		
	H		*K*	Minimum Pitch Diam., Small End	Maximum Pitch Diam., Small End	Minimum Pitch Diam., Small End	Maximum Pitch Diam., Small End	Equivalent Longitudinal Variation
⅛	0.393	0.345	0.0296	0.3633	0.3637	0.3631	0.3639	0.0064
¼	0.522	0.445	0.0444	0.4772	0.4776	0.4770	0.4778	0.0070
⅜	0.656	0.583	0.0444	0.6118	0.6123	0.6115	0.6125	0.0077
½	0.816	0.721	0.0571	0.7582	0.7587	0.7579	0.7590	0.0083
¾	1.025	0.932	0.0571	0.9674	0.9680	0.9671	0.9682	0.0090
1	1.283	1.169	0.0696	1.2133	1.2139	1.2130	1.2142	0.0096
1¼	1.627	1.514	0.0696	1.5568	1.5575	1.5565	1.5578	0.0102
1½	1.866	1.753	0.0696	1.7958	1.7964	1.7954	1.7968	0.0109
2	2.339	2.227	0.0696	2.2687	2.2694	2.2683	2.2697	0.0115
2½	2.820	2.662	0.1000	2.7192	2.7200	2.7188	2.7203	0.0122
3	3.441	3.289	0.1000	3.3403	3.3410	3.3399	3.3414	0.0122
3½	3.938	3.789	0.1000	3.8371	3.8379	3.8367	3.8383	0.0131
4	4.434	4.287	0.1000	4.3340	4.3348	4.3335	4.3352	0.0138
4½	4.931	4.786	0.1000	4.8308	4.8317	4.8304	4.8322	0.0144
5	5.491	5.349	0.1000	5.3903	5.3912	5.3898	5.3917	0.0150
6	6.546	6.406	0.1000	6.4456	6.4466	6.4451	6.4471	0.0163
7	7.540	7.402	0.1000	7.4393	7.4404	7.4387	7.4409	0.0176
8	8.534	8.400	0.1000	8.4330	8.4342	8.4324	8.4348	0.0189
9	9.527	9.398	0.1000	9.4267	9.4280	9.4261	9.4286	0.0202
10	10.645	10.521	0.1000	10.5447	10.5460	10.5440	10.5466	0.0211
11	11.639	11.519	0.1000	....	....	....	....	....
12	12.633	12.518	0.1000	12.5321	12.5336	12.5313	12.5343	0.0237

British Standard Pipe Threads

The form of thread is that of the Whitworth system; the sides of the thread form an angle of 55 degrees with each other, and the top and bottom of the threads are rounded to a radius equal to 0.1373 × the pitch of the thread. For taper pipe threads the taper is 3/4 inch per foot, or 1/16 inch per inch, measured on the diameter. This system has been approved by the British Engineering Standards Association as the standard pipe thread system in Great Britain, and is known as the "British Standard Pipe Thread for Iron and Steel Pipes and Tubes." Complete data are given in the table below.

Nominal Inside Diameter	Approx. Outside Diameter of Pipe	No. of Threads per Inch	Depth of Thread	Diameter at Root of Thread	Length of Thread on Pipe End	Length of Thread in Coupler	Gage Diameter at Top of Thread	Distance of Gage Diam. from Pipe End
1/8	13/32	28	0.0230	0.337	3/8	3/4	0.383	5/32
1/4	17/32	19	0.0335	0.451	3/8	3/4	0.518	3/16
3/8	11/16	19	0.0335	0.589	1/2	1	0.656	1/4
1/2	27/32	14	0.0455	0.734	5/8	1 1/4	0.825	1/4
5/8	15/16	14	0.0455	0.811	5/8	1 1/4	0.902	1/4
3/4	1 1/16	14	0.0455	0.950	3/4	1 1/2	1.041	3/8
7/8	1 7/32	14	0.0455	1.098	3/4	1 1/2	1.189	3/8
1	1 11/32	11	0.0580	1.193	7/8	1 3/4	1.309	3/8
1 1/4	1 11/16	11	0.0580	1.534	1	2	1.650	1/2
1 1/2	1 29/32	11	0.0580	1.766	1	2	1.882	1/2
1 3/4	2 5/32	11	0.0580	2.000	1 1/8	2 1/4	2.116	5/8
2	2 3/8	11	0.0580	2.231	1 1/8	2 1/4	2.347	5/8
2 1/4	2 5/8	11	0.0580	2.471	1 1/4	2 1/2	2.587	11/16
2 1/2	3	11	0.0580	2.844	1 1/4	2 1/2	2.960	11/16
2 3/4	3 1/4	11	0.0580	3.094	1 3/8	2 3/4	3.210	13/16
3	3 1/2	11	0.0580	3.344	1 3/8	2 3/4	3.460	13/16
3 1/4	3 3/4	11	0.0580	3.584	1 1/2	3	3.700	7/8
3 1/2	4	11	0.0580	3.834	1 1/2	3	3.950	7/8
3 3/4	4 1/4	11	0.0580	4.084	1 1/2	3	4.200	7/8
4	4 1/2	11	0.0580	4.334	1 5/8	3 1/4	4.450	1
4 1/2	5	11	0.0580	4.834	1 5/8	3 1/4	4.950	1
5	5 1/2	11	0.0580	5.334	1 3/4	3 1/2	5.450	1 1/8
5 1/2	6	11	0.0580	5.834	1 7/8	3 3/4	5.950	1 1/4
6	6 1/2	11	0.0580	6.334	2	4	6.450	1 3/8
7	7 1/2	10	0.0640	7.322	2 1/8	4 1/4	7.450	1 3/8
8	8 1/2	10	0.0640	8.322	2 1/4	4 1/2	8.450	1 1/2
9	9 1/2	10	0.0640	9.322	2 1/4	4 1/2	9.450	1 1/2
10	10 1/2	10	0.0640	10.322	2 3/8	4 3/4	10.450	1 5/8
11	11 1/2	8	0.0800	11.290	2 1/2	5	11.450	1 5/8
12	12 1/2	8	0.0800	12.290	2 1/2	5	12.450	1 5/8
13	13 3/4	8	0.0800	13.520	2 5/8	5 1/4	13.680	1 5/8
14	14 3/4	8	0.0800	14.520	2 3/4	5 1/2	14.680	1 3/4
15	15 3/4	8	0.0800	15.520	2 3/4	5 1/2	15.680	1 3/4
16	16 3/4	8	0.0800	16.520	2 7/8	5 3/4	16.680	1 7/8
17	17 3/4	8	0.0800	17.520	3	6	17.680	2
18	18 3/4	8	0.0800	18.520	3	6	18.680	2

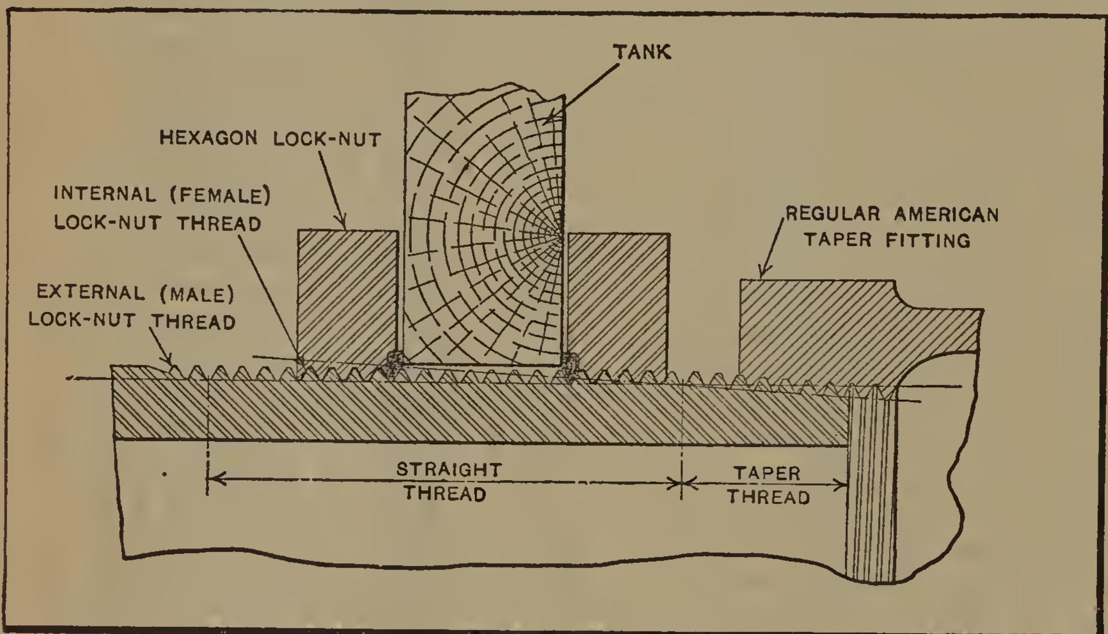
Whitworth Screw Threads for Hydraulic Iron Piping

Diameter of Piping, Internal	Diameter of Piping, External	Diameter at Bottom of Thread	Pressure in Pounds per Square Inch	No. of Threads per Inch
¼	⅝	0.5335	4000	14
	¾	0.6585	6000	
	⅞	0.7835	8000	
	1	0.9085	10000	
⅜	¾	0.6585	4000	14
	⅞	0.7835	6000	
	1	0.9085	8000	
	1⅛	1.0335	10000	
½	1	0.9085	4000	14
	1⅛	1.0335	6000	
	1¼	1.1335	8000	11
	1⅜	1.2585	10000	
⅝	1⅛	1.0335	4000	14
	1¼	1.1335	6000	11
	1⅜	1.2585	8000	
	1½	1.3835	10000	
¾	1¼	1.1335	4000	11
	1⅜	1.2585	6000	
	1½	1.3835	8000	
	1⅝	1.5085	10000	
⅞	1⅜	1.2585	4000	11
	1½	1.3835	6000	
	1⅝	1.5085	8000	
	1¾	1.6335	10000	
1	1½	1.3835	4000	11
	1⅝	1.5085	6000	
	1¾	1.6335	8000	
	1⅞	1.7585	10000	
1⅛	1⅝	1.5085	4000	11
	1¾	1.6335	6000	
1⅛	1⅞	1.7585	8000	11
	2	1.8835	10000	
1¼	1¾	1.6335	4000	11
	1⅞	1.7585	6000	
	2	1.8835	8000	
	2⅛	2.0085	10000	
1⅜	1⅞	1.7585	4000	11
	2	1.8835	6000	
	2⅛	2.0085	8000	
	2¼	2.1335	10000	
1½	2	1.8835	4000	11
	2⅛	2.0085	6000	
	2¼	2.1335	8000	
	2⅜	2.2585	10000	
	2½	2.3835	10000	
1⅝	2⅛	2.0085	4000	11
	2¼	2.1335	6000	
	2⅜	2.2585	8000	
	2½	2.3835	10000	
1¾	2¼	2.1335	3000	11
	2⅜	2.2585	4000	
	2½	2.3835	6000	
	2¾	2.6335	10000	
1⅞	2⅜	2.2585	3000	11
	2½	2.3835	4000	
	2¾	2.6335	8000	
2	2½	2.3835	3000	11
	2¾	2.6335	6000	
	3	2.8835	10000	

Straight Pipe Threads. — On certain classes of internal (female) fittings, straight pipe threads are used with entire satisfaction when the engaging male thread is tapered and to American or Briggs standard, as a "make up" non-leaking joint is obtained. The use of straight male threads under these conditions is not to be recommended, as an excessive amount of cement is necessary to prevent leaks. In fixture work, however, such a joint often is not necessary and is in many cases undesirable because of finish, so that in such cases straight male threads may be used to advantage. According to the specifications of the National Screw Thread Commission, straight threaded internal wrought-iron or wrought-steel couplings of the weight known as "standard" may be used with taper threaded pipe for ordinary pressures as they are sufficiently ductile to adjust themselves to the tapered external thread when the pipe and coupling are properly screwed together. For high pressures, only tapered external and internal threads should be used. Straight external (male) threads are approved only for special applications such as "long screws" and "tank nipples." Long screws are used to a limited extent, although this joint is not considered satisfactory when subjected to temperature or pressure. In this application the coupling has a straight thread and makes a joint with a standard taper pipe thread.

The straight pipe thread is the same as American or Briggs taper pipe thread in regard to pitch and depth of thread. The basic pitch diameter for straight pipe threads equals the pitch diameter at the gaging notch of the taper plug gage. These pitch diameters for various nominal pipe sizes are given in the accompanying table; they represent the pitch diameters of standard taper pipe threads measured at a distance from the end of the pipe equal to the dimensions given in column "*M*" of the preceding table for Briggs taper pipe threads. The straight pipe thread is gaged with a taper threaded plug gage and should gage flush at the face with the gaging notch, allowing a maximum variation of one turn plus or minus from the notch.

Lock-nut Threads. — Occasionally it is required to have a straight pipe thread of the largest diameter it is possible to get. These sizes have been standardized



and are known as the "maximum external and minimum internal lock-nut threads." (For dimensions see table.) The tank nipple shown in the accompanying illustration is an example of the application of this thread. After cutting the external lock-nut thread, a standard taper thread is cut on the end for receiving the taper fitting as the illustration shows.

National (American) Straight Pipe Threads — Internal

Nominal Pipe Size, Inches	Basic Pitch Diam., Inches	Nominal Pipe Size, Inches	Basic Pitch Diam., Inches	Nominal Pipe Size, Inches	Basic Pitch Diam., Inches	Nominal Pipe Size, Inches	Basic Pitch Diam., Inches
1/8	0.3748	1 1/4	1.5834	4	4.3871	9	9.4980
1/4	0.4899	1 1/2	1.8223	4 1/2	4.8859	10	10.6209
3/8	0.6270	2	2.2963	5	5.4493	11	11.6194
1/2	0.7784	2 1/2	2.7622	6	6.5060	12	12.6178
3/4	0.9889	3	3.3885	7	7.5023	14 O.D.	13.8726
1	1.2386	3 1/2	3.8888	8	8.5000	15 O.D.	14.8742

National (American) Lock-nut Threads

Nominal Size, Inches	Pitch Diameter: Max. External Lock-nut Thread	Pitch Diameter: Min. Internal Lock-nut Thread	Nominal Size, Inches	Pitch Diameter: Max. External Lock-nut Thread	Pitch Diameter: Min. Internal Lock-nut Thread	Nominal Size, Inches	Pitch Diameter: Max. External Lock-nut Thread	Pitch Diameter: Min. Internal Lock-nut Thread
1/8	0.3840	0.3863	1 1/2	1.8441	1.8495	5	5.4805	5.4887
1/4	0.5038	0.5072	2	2.3180	2.3234	6	6.5372	6.5450
3/8	0.6409	0.6444	2 1/2	2.7934	2.8012	7	7.5336	7.5414
1/2	0.7963	0.8007	3	3.4197	3.4276	8	8.5313	8.5391
3/4	1.0067	1.0112	3 1/2	3.9200	3.9279	9	9.5292	9.5370
1	1.2604	1.2658	4	4.4184	4.4262	10	10.6522	10.6600
1 1/4	1.6051	1.6105	4 1/2	4.9172	4.9250	12	12.6491	12.6569

Löwenherz Thread. — The Löwenherz thread has flats at the top and bottom the same as the U. S. standard form, but the angle is 53 degrees 8 minutes. The depth equals 0.75 × the pitch, and the width of the flats at the top and bottom is equal to 0.125 × the pitch. This screw thread is based on the metric system (see table) and is used extensively for the fine threads of measuring instruments, optical apparatus, etc., especially in Germany:

Löwenherz Thread

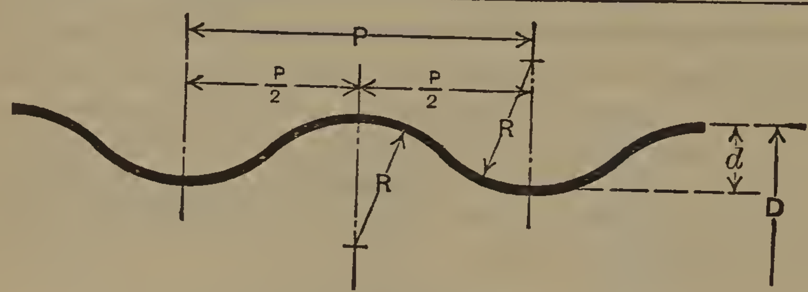
Diameter: Millimeters	Diameter: Inches	Pitch, Millimeters	Approximate No. of Threads per Inch	Diameter: Millimeters	Diameter: Inches	Pitch, Millimeters	Approximate No. of Threads per Inch
1.0	0.0394	0.25	101.6	9.0	0.3543	1.30	19.5
1.2	0.0472	0.25	101.6	10.0	0.3937	1.40	18.1
1.4	0.0551	0.30	84.7	12.0	0.4724	1.60	15.9
1.7	0.0669	0.35	72.6	14.0	0.5512	1.80	14.1
2.0	0.0787	0.40	63.5	16.0	0.6299	2.00	12.7
2.3	0.0905	0.40	63.5	18.0	0.7087	2.20	11.5
2.6	0.1024	0.45	56.4	20.0	0.7874	2.40	10.6
3.0	0.1181	0.50	50.8	22.0	0.8661	2.80	9.1
3.5	0.1378	0.60	42.3	24.0	0.9450	2.80	9.1
4.0	0.1575	0.70	36.3	26.0	1.0236	3.20	7.9
4.5	0.1772	0.75	33.9	28.0	1.1024	3.20	7.9
5.0	0.1968	0.80	31.7	30.0	1.1811	3.60	7.1
5.5	0.2165	0.90	28.2	32.0	1.2599	3.60	7.1
6.0	0.2362	1.00	25.4	36.0	1.4173	4.00	6.4
7.0	0.2756	1.10	23.1	40.0	1.5748	4.40	5.7
8.0	0.3150	1.20	21.1	......	......	......	......

Standard Threads for Lamp Base and Socket Shells. — The standard threads listed in the accompanying table "Standard Threads for Lamp Base and Socket Shells" are recommended by the American Society of Mechanical Engineers and by most of the large manufacturers of products requiring rolled threads on sheet metal shells or parts, such as lamp bases, fuse plugs, attachment plugs, etc.

This is known as the "American Standard." There are four sizes, designated as the "miniature size," the "candelabra size," the "medium size" and the "mogul size." The maximum and minimum dimensions given in the table represent the diameters at the top of the thread in each case.

It is recommended that for each size of lamp base shell, there should be two threaded ring gages to govern the diameter at the bottom of the thread and the form of the thread and in addition, two plain ring gages to govern the outside diameter at the top of the thread. The "go" gage for the top of the lamp base thread corresponds to the maximum diameter given in the table, and the "not go" gage to the minimum diameter. The diameters of the "go" and "not go" gages for the bottom of the thread may be obtained by subtracting twice the thread depth (d) from the diameters of the "go" and "not go" gages for the top of the thread.

Standard Threads for Lamp Base and Socket Shells



Name of Size	Threads per Inch	Radius R, Inch	Depth d, Inch	Lamp Base or Plug Diam. D		Socket Shell Diam. D	
				Max.	Min.	Max.	Min.
Miniature..........	14	0.021	0.020	0.375	0.370	0.3835	0.3775
Candelabra.........	10	0.03125	0.025	0.465	0.460	0.476	0.470
Medium...........	7	0.047	0.033	1.037	1.031	1.053	1.045
Mogul.............	4	0.0906	0.050	1.555	1.545	1.577	1.565

For each size of socket shell there should be two threaded plug gages to govern the diameter of the top of the thread inside, and the form of the thread; in addition there should be two plain plug gages to govern the diameter of the bottom of the thread inside. The "go" gage for the top of the thread corresponds to the minimum socket shell diameter given in the table and the "not go" gage corresponds to the maximum diameter. The diameters of the "go" and "not go" gages for the bottom of the thread are obtained by subtracting twice the thread depth (d) from the diameters of the "go" and "not go" gages for the top of the thread.

The Cadillac Screw Thread. — The Cadillac screw thread is used by the Cadillac Motor Car Co., Detroit, Mich. The thread angle is 60 degrees and the top is flat like the U. S. standard thread, but the bottom or root of the thread is a sharp V. Thus it is a cross between the U. S. standard thread and the sharp V-thread.

British Standard Fine Screw Thread (B. S. F.). — The form of this thread is the same as that of the Whitworth thread, but the number of threads per inch for a given diameter is greater than in the Whitworth standard system. The accompanying table gives the standard dimensions for the British Standard Fine Screw Threads as revised in 1918 by the British Engineering Standards Association.

British Standard Fine Screw Threads

Nominal Diameter of Screw, Inch	No. of Threads per Inch	Pitch of Thread, Inch	Bolts						Nuts					
			Full Diameter		Effective Diam.		Core Diameter		Full Diameter		Effective Diam.		Core Diameter	
			Max.	Min.	Max.	Min.	Max.	Min.	Min.	Max.	Min.	Max.	Min.	Max.
7/32	28	0.03571	0.2188	0.2131	0.1960	0.1922	0.1731	0.1665	0.2208	0.2284	0.1980	0.2018	0.1751	0.1808
1/4	26	0.03846	0.2500	0.2441	0.2254	0.2215	0.2007	0.1929	0.2520	0.2598	0.2274	0.2313	0.2027	0.2086
9/32	26	0.03846	0.2813	0.2754	0.2566	0.2527	0.2320	0.2242	0.2833	0.2911	0.2586	0.2625	0.2340	0.2399
5/16	22	0.04545	0.3125	0.3061	0.2834	0.2791	0.2543	0.2458	0.3145	0.3230	0.2854	0.2897	0.2563	0.2627
3/8	20	0.05000	0.3750	0.3683	0.3430	0.3385	0.3110	0.3020	0.3770	0.3860	0.3450	0.3495	0.3130	0.3197
7/16	18	0.05556	0.4375	0.4304	0.4019	0.3972	0.3664	0.3570	0.4395	0.4489	0.4039	0.4086	0.3684	0.3755
1/2	16	0.06250	0.5000	0.4925	0.4600	0.4550	0.4200	0.4100	0.5020	0.5120	0.4620	0.4670	0.4220	0.4295
9/16	16	0.06250	0.5625	0.5550	0.5225	0.5175	0.4825	0.4725	0.5645	0.5745	0.5245	0.5295	0.4845	0.4920
5/8	14	0.07143	0.6250	0.6170	0.5793	0.5740	0.5335	0.5228	0.6270	0.6377	0.5813	0.5866	0.5355	0.5435
11/16	14	0.07143	0.6875	0.6795	0.6418	0.6365	0.5960	0.5853	0.6895	0.7002	0.6438	0.6491	0.5980	0.6060
3/4	12	0.08333	0.7500	0.7413	0.6966	0.6908	0.6433	0.6318	0.7520	0.7635	0.6986	0.7044	0.6453	0.6540
13/16	12	0.08333	0.8125	0.8038	0.7591	0.7533	0.7058	0.6943	0.8145	0.8260	0.7611	0.7669	0.7078	0.7165
7/8	11	0.09091	0.8750	0.8660	0.8168	0.8108	0.7586	0.7465	0.8770	0.8891	0.8188	0.8248	0.7606	0.7696
1	10	0.10000	1.0000	0.9905	0.9360	0.9297	0.8719	0.8593	1.0020	1.0146	0.9380	0.9443	0.8739	0.8834
1 1/8	9	0.11111	1.1250	1.1150	1.0539	1.0472	0.9827	0.9694	1.1270	1.1403	1.0559	1.0626	0.9847	0.9947
1 1/4	9	0.11111	1.2500	1.2400	1.1789	1.1722	1.1077	1.0944	1.2520	1.2653	1.1809	1.1876	1.1097	1.1197
1 3/8	8	0.12500	1.3750	1.3644	1.2950	1.2879	1.2149	1.2008	1.3770	1.3911	1.2970	1.3041	1.2169	1.2275
1 1/2	8	0.12500	1.5000	1.4894	1.4200	1.4129	1.3399	1.3258	1.5020	1.5161	1.4220	1.4291	1.3419	1.3525
1 5/8	8	0.12500	1.6250	1.6144	1.5450	1.5379	1.4649	1.4508	1.6270	1.6411	1.5470	1.5541	1.4669	1.4775
1 3/4	7	0.14286	1.7500	1.7387	1.6585	1.6509	1.5670	1.5519	1.7520	1.7671	1.6605	1.6681	1.5690	1.5803
2	7	0.14286	2.0000	1.9887	1.9085	1.9009	1.8170	1.8019	2.0020	2.0171	1.9105	1.9181	1.8190	1.8303
2 1/4	6	0.16667	2.2500	2.2378	2.1433	2.1351	2.0366	2.0203	2.2520	2.2683	2.1453	2.1535	2.0386	2.0508
2 1/2	6	0.16667	2.5000	2.4878	2.3933	2.3851	2.2866	2.2703	2.5020	2.5183	2.3953	2.4035	2.2886	2.3008
2 3/4	6	0.16667	2.7500	2.7378	2.6433	2.6351	2.5366	2.5203	2.7520	2.7683	2.6453	2.6535	2.5386	2.5508
3	5	0.20000	3.0000	2.9866	2.8719	2.8630	2.7439	2.7260	3.0020	3.0199	2.8739	2.8828	2.7459	2.7593

Maximum and Minimum Diameters. — The table gives the maximum and minimum values for the full or outside diameter, the effective or pitch diameter, and the core or root diameter for bolts and nuts. The core diameter of a nut is measured between the crests of the thread and corresponds approximately to the core or root diameter of a bolt. The diameters are given to four decimal places. While it is recognized that this degree of accuracy is not required in general manufacturing, the table is also intended for use in gage-making. This particular table applies to ordinary bolts and nuts. As a close fit at the crest and root is not desirable, wider tolerances on the full and core diameters have been allowed.

Close Fits. — In addition to the screw thread dimensions listed in the table, there is another class of British Standard Fine Threads which are intended for close fits. These screw threads of the close-fit class are the same as those listed in the table, excepting that the tolerances are reduced one-half. The maximum diameters of the bolts and the minimum diameters of the nuts are the same as those given in the table. The minimum diameter of a bolt of the close-fit class can be obtained by subtracting from the maximum diameter given in the table, one-half the tolerance (one-half the difference between the maximum and minimum sizes given in the table).

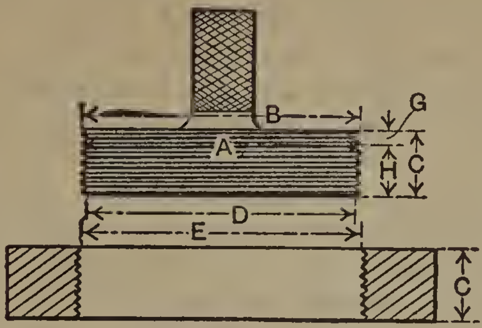
Tolerance Formulas. — An analysis of existing data and the examination of various screw threads showed that the quantity $0.01\sqrt{p}$, in which p is the pitch in inches, forms a convenient unit of tolerance; therefore the tolerances for different elements of the screw threads are made equal to some multiple of this quantity. For the ordinary class of fits represented by the accompanying table, the tolerance formulas for the bolts are as follows: Tolerance for full diameter $= 3 \times 0.01\sqrt{p}$; tolerance for effective diameter $= 2 \times 0.01\sqrt{p}$; tolerance for core diameter $= 4 \times 0.01\sqrt{p}$. The tolerance formulas for the nuts are as follows: Tolerance for full diameter $= 4 \times 0.01\sqrt{p}$; tolerance for effective diameter $= 2 \times 0.01\sqrt{p}$; tolerance for core diameter $= 3 \times 0.01\sqrt{p}$. The tolerance formulas for close fits are modified by reducing the first factor in the foregoing formulas one-half; thus the tolerance for the full diameter of a close-fitting bolt $= 1.5 \times 0.01\sqrt{p}$.

Watch Screw Threads

Waltham Watch Co.			Elgin National Watch Co.					
Diam. of Thread, Inch	Diam. of Thread, Millimeters	Threads per Inch	Diam., Inch	Diam., Millimeters	Threads per Inch	Diam., Inch	Diam., Millimeters	Threads per Inch
0.0591	1.50	110	0.0132	0.33	360	0.0428	1.07	120
0.0473	1.20	110	0.0148	0.37	320	0.0448	1.12	110
0.0433	1.10	120	0.0168	0.42	260	0.0468	1.17	110
0.0394	1.00	140	0.0208	0.52	220	0.0488	1.22	140
0.0366	0.93	160	0.0228	0.57	260	0.0488	1.22	200
0.0528	1.34	170	0.0248	0.62	220	0.0508	1.27	110*
0.0394	1.00	180	0.0268	0.67	180	0.0548	1.37	180
0.0327	0.83	180	0.0288	0.72	220	0.0608	1.52	110
0.0256	0.65	200	0.0308	0.77	180	0.0608	1.52	110*
0.0217	0.55	220	0.0308	0.77	220	0.0708	1.77	180*
0.0177	0.45	240	0.0368	0.92	140	0.0768	1.92	110*
0.0138	0.35	254	0.0368	0.92	220	0.0772	1.93	80*
.....	......	.....	0.0408	1.02	120*	0.0892	2.23	80*
.....	......	.....	0.0408	1.02	200	......	.....	.....

Note: Asterisk (*) indicates left-hand threads.

Dimensions of Standard Oil Well Casing Gages



The total taper is 3/8 inch per foot. The ring gage tapers for its whole length, and the plug gage on the outside for a distance *H* from the small end, the distance *G* being straight. The root of the thread tapers the whole length of the plug gage. The bottom of the thread in both plug and ring gage is sharp.

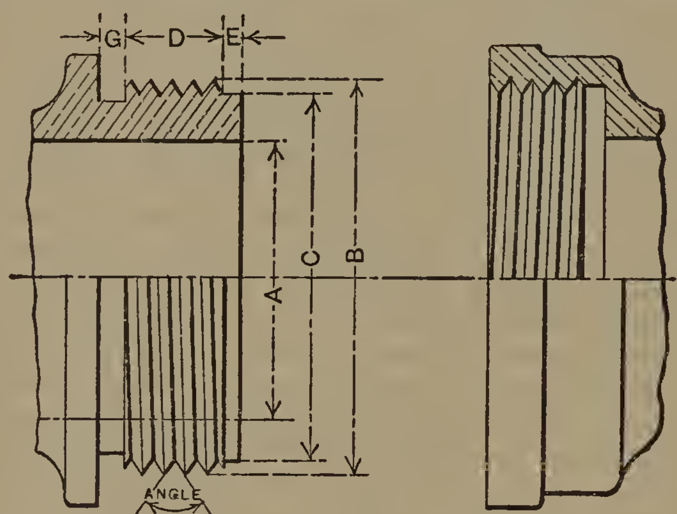
Nominal Size	No. of Threads per Inch	Diameter of Gage at Size Line	Diam. of Gage at Large End, if Turned Taper for Whole Length of Thread	Length of Threaded Portion of Gage	Diameter of Root of Thread at Small End	Diam. over Flats of Threads, Large End of Ring Gage	Flat on Top of Threads	Length of Straight Portion of Thread	Length of Taper Thread
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
2	14	2.250	2.254	0.968	2.105	2.140	0.0027	0.143	0.825
2¼	14	2.500	2.504	0.993	2.355	2.390	0.0027	0.143	0.850
2½	14	2.750	2.754	1.018	2.604	2.640	0.0027	0.143	0.875
2¾	14	3.000	3.004	1.043	2.853	2.890	0.0027	0.143	0.900
3	14	3.250	3.254	1.068	3.102	3.140	0.0027	0.143	0.925
3¼	14	3.500	3.504	1.093	3.351	3.390	0.0027	0.143	0.950
3½	14	3.750	3.754	1.118	3.601	3.640	0.0027	0.143	0.975
3¾	14	4.000	4.004	1.143	3.850	3.890	0.0027	0.143	1.000
4	14	4.250	4.254	1.168	4.099	4.140	0.0027	0.143	1.025
4¼	14	4.500	4.504	1.193	4.348	4.390	0.0027	0.143	1.050
4½	14	4.750	4.754	1.218	4.597	4.640	0.0027	0.143	1.075
4¾	14	5.000	5.004	1.243	4.847	4.890	0.0027	0.143	1.100
5	14	5.250	5.254	1.268	5.096	5.140	0.0027	0.143	1.125
5	11½	5.250	5.255	1.299	5.070	5.116	0.0033	0.174	1.125
5 3/16	14	5.500	5.504	1.293	5.345	5.390	0.0027	0.143	1.150
5 3/16	11½	5.500	5.505	1.324	5.319	5.366	0.0033	0.174	1.150
5⅝	14	6.000	6.004	1.343	5.844	5.890	0.0027	0.143	1.200
5⅝	11½	6.000	6.005	1.374	5.818	5.866	0.0033	0.174	1.200
6¼	14	6.625	6.629	1.405	6.467	6.515	0.0027	0.143	1.262
6¼	11½	6.625	6.630	1.436	6.441	6.491	0.0033	0.174	1.262
6⅝	14	7.000	7.004	1.443	6.840	6.890	0.0027	0.143	1.300
6⅝	11½	7.000	7.005	1.474	6.815	6.866	0.0033	0.174	1.300
7¼	14	7.625	7.629	1.505	7.464	7.515	0.0027	0.143	1.362
7¼	11½	7.625	7.630	1.536	7.438	7.491	0.0033	0.174	1.362
7⅝	11½	8.000	8.005	1.574	7.811	7.866	0.0033	0.174	1.400
8¼	11½	8.625	8.630	1.636	8.434	8.491	0.0033	0.174	1.462
8⅝	11½	9.000	9.005	1.674	8.808	8.866	0.0033	0.174	1.500
9⅝	11½	10.000	10.005	1.774	9.805	9.866	0.0033	0.174	1.600
10⅝	11½	11.000	11.005	1.874	10.802	10.866	0.0033	0.174	1.700
11⅝	11½	12.000	12.005	1.974	11.799	11.866	0.0033	0.174	1.800
12½	11½	13.000	13.005	2.074	12.796	12.866	0.0033	0.174	1.900
13½	11½	14.000	14.005	2.174	13.793	13.866	0.0033	0.174	2.000
14½	11½	15.000	15.005	2.274	14.790	14.866	0.0033	0.174	2.100
15½	11½	16.000	16.005	2.374	15.786	15.866	0.0033	0.174	2.200

U. S. Navy Standard Hose Couplings. — The accompanying table gives the dimensions for the standard hose couplings used by the Navy Department. The dimensions given were issued on September 1, 1916, and are the ones that are now in force. Complete dimensions and instructions for composition hose fittings, forged steel spanners, hose expanders and nozzles used by the Navy Department are given in the department's specifications 34F3a, which may be obtained upon request from the department.

The material from which composition hose fittings are made shall show on analysis, unless otherwise specified, as follows: Copper, not less than 83 per cent; tin, not less than 5 per cent; lead, not more than 3 per cent; zinc, not more than 7 per cent. As regards the outside of couplings, it is specified that the surface should be finished and polished, and the inside of couplings should be lathe finished, except at the point where the corrugations bind the hose.

When supplied to the government by a manufacturer, the couplings must be stamped with the contractor's name, the year and month of manufacture, and the size of hose, in letters ⅛ inch high.

Standard Hose Couplings — U. S. Navy Specifications



THREADS SHARP AT TOP AND BOTTOM

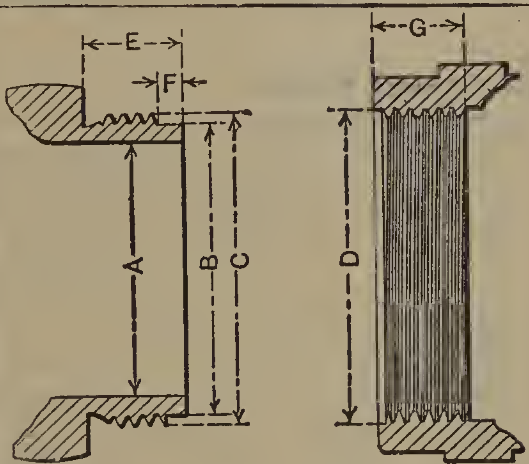
Size	Name	A	B	C	D	E	G	Angle, Degrees	No. of Threads per Inch
2½	Suction	2½	3 11/32	3 3/32	¾	⅛	⅛	60	7
2½	Fire	2½	3 11/32	3 3/32	¾	⅛	⅛	60	7
1½	Wash Deck	1½	2 3/32	1 15/16	½	1/16	⅛	60	10
1¼	Water Supply	1¼	2 3/32	1 15/16	½	1/16	⅛	60	10
¾	Steam	¾	1 17/64	1.115	½	1/16	⅛	60	11½

Standards for Hose Couplings. — Hose couplings for 2½-, 3-, 3½- and 4½-inch sizes have been standardized and adopted by the American Waterworks Association, the New England Waterworks Association, the National Firemen's Association, the National Fire Protection Association, etc. The 2½-inch hose coupling is the size most generally used by public fire departments. The 3- and 3½-inch sizes are used mainly for high pressure or fire-boat services and are not in general use.

For sizes under 2½ inches there is no universal standard; there are at least six different so-called "standards" used, known as follows: Eastern gage hose thread (used in the New England States); Pacific Coast hose thread, known also as the California standard hose thread (used on the Pacific Coast); Chicago hose thread (used in the Middle West); Pittsburg hose thread; Boston hose thread; and the iron pipe thread, which is the general standard for pipe threads.

The accompanying tables give a comparison of the dimensions of the outside diameter and the number of threads in the various standards, and also complete dimensions for the California standard hose and Chicago hose thread, as furnished by the Crane Co. of Chicago. In addition to these "standards," there is a great diversity of 2½-inch threads used by the fire departments of various cities. As regards the standards, there is no absolute agreement as to the dimensions, so that it is possible that some manufacturers deviate slightly from those given in the tables. It will be seen, for example, that the number of threads per inch as given in the comparative table (prepared by the aid of data furnished by the Elkhart Brass Mfg. Co., Elkhart, Ind.) for 1-inch Pacific Coast or California hose thread differs from the number given for the same thread in the table of California standard hose coupling thread. The "National Standard" for fire hose connections, given in the following table, conforms to the specifications of the National Screw Thread Commission.

Fire Hose Connections — National Standard



Form of Thread: 60°V with 0.010 inch cut off the top of thread and 0.010 inch left at the bottom of the groove on the 2½-, 3- and 3½-inch couplings, and 0.020 inch cut off the top and 0.020 inch left at the bottom on the 4½-inch coupling.

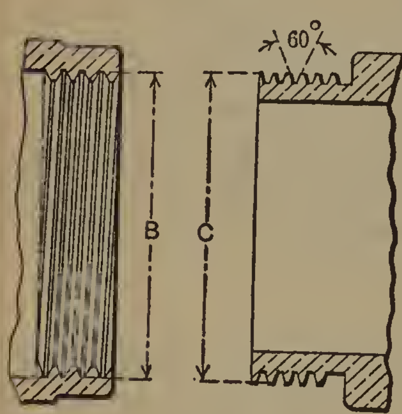
Adopted by the National Board of Fire Underwriters, American Waterworks Association, New England Waterworks Association, National Firemen's Association, National Fire Protection Association, etc.

Dimensions	2½-inch Size		3-inch Size		3½-inch Size		4½-inch Size	
	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters
A	2½	63.50	3	76.20	3½	88.90	4½	114.30
B	2.8715	72.94	3.3763	85.76	4.0013	101.63	5.397	137.08
C	3$\frac{1}{16}$	77.79	3⅝	92.07	4¼	107.95	5¾	146.05
D	3.0925	78.55	3.6550	92.84	4.280	108.71	5.80	147.32
E	1	25.40	1⅛	28.57	1⅛	28.57	1⅜	34.92
F	¼	6.35	¼	6.35	¼	6.35	¼	6.35
G	⅞	22.22	1	25.40	1	25.40	1¼	31.75
Threads per Inch	7½	......	6	......	6	......	4	......
Clearance between Male and Female Threads	0.030	0.76	0.030	0.76	0.030	0.76	0.050	1.27

Comparison of Hose Coupling Threads

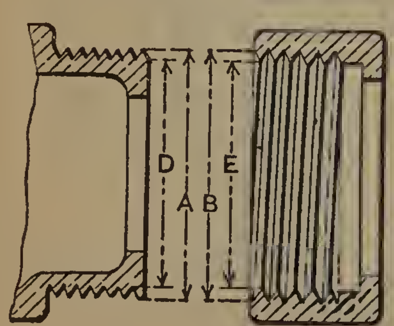
Nominal Size	Eastern Hose Thread		Pacific Coast Hose Thread		Pittsburg Hose Thread		Boston Hose Thread		National Std. Hose Thread		Iron Pipe Thread	
	Outside Diam.	No. of Threads per Inch	Outside Diam.	No. of Threads per Inch	Outside Diam.	No. of Threads per Inch	Outside Diam.	No. of Threads per Inch	Outside Diam.	No. of Threads per Inch	Outside Diam.	No. of Threads per Inch
3/4	1 1/16	11	1 1/16	11	1 3/16	11 1/2	1 1/16	11	...	...	1.050	14
1	1 5/16	11 1/2	1 5/16	11 1/2	...	...	1 15/32	11	...	...	1.315	11 1/2
1 1/4	1 23/32	11 1/2	1 57/64	11	...	...	...	...	...	...	1.660	11 1/2
1 1/2	1 31/32	11 1/2	2 9/64	11	...	...	...	...	...	...	1.900	11 1/2
2	2 35/64	8	2 19/32	10	...	...	...	...	...	...	2.375	11 1/2
2 1/2	3 1/32	7 1/2	3	7 1/2	...	...	...	...	3 1/16	7 1/2	2.875	8
3	...	...	...	...	...	...	...	...	3 5/8	6	3.500	8
3 1/2	...	...	...	...	...	...	...	...	4 1/4	6	4.000	8
4 1/2	...	...	...	...	...	...	...	...	5 3/4	4	5.000	8

California Standard Hose Coupling Thread



Nominal Size	B	C	No. of Threads per Inch	Clearance between Male and Female Threads
3/4	1.080	1.070	11	0.010
1	1.320	1.310	11	0.010
1 1/4	1.860	1.850	11	0.010
1 1/2	2.120	2.110	11	0.010
2	2.560	2.550	10	0.010
2 1/2	3.050	3.040	7 1/2	0.010

Chicago Standard Hose Coupling Thread



Nominal Size	A	B	D	E	No. of Threads per Inch
3/4	1.081	1.099	0.931	0.949	11 1/2
1	1.295	1.315	1.145	1.165	11 1/2
1 1/4	1.705	1.723	1.580	1.598	11 1/2
1 1/2	1.946	1.964	1.796	1.814	11 1/2
2	2.522	2.542	2.306	2.326	8
2 1/2	3.043	3.047	2.812	2.816	7

A. S. M. E. Standard for Machine Screw Threads. — The American Society of Mechanical Engineers has adopted a standard for machine screw threads, in which the basic form of thread is the same as that of the U. S. standard system, but in which certain definite limits are given both for screw and tap threads. The formulas from which these limits are calculated are as follows (T.P.I. = number of threads per inch):

Screws

Max. external diam. = basic external diam.
Max. pitch diam. = basic pitch diam.
Max. root diam. = basic root diam.

$$\text{Min. external diam.} = \text{basic external diam.} - \frac{0.336}{\text{T.P.I.} + 40}$$

$$\text{Min. pitch diam.} = \text{basic pitch diam.} - \frac{0.168}{\text{T.P.I.} + 40}$$

$$\text{Min. root diam.} = \text{basic root diam.} - \left[\frac{0.10825}{\text{T.P.I.}} + \frac{0.168}{\text{T.P.I.} + 40}\right]$$

Taps

$$\text{Max. external diam.} = \text{basic external diam.} + \frac{0.10825}{\text{T.P.I.}} + \frac{0.224}{\text{T.P.I.} + 40}$$

$$\text{Max. pitch diam.} = \text{basic pitch diam.} + \frac{0.224}{\text{T.P.I.} + 40}$$

$$\text{Max. root diam.} = \text{basic root diam.} + \frac{0.336}{\text{T.P.I.} + 40}$$

$$\text{Min. external diam.} = \text{basic external diam.} + \frac{0.112}{\text{T.P.I.} + 40}$$

$$\text{Min. pitch diam.} = \text{basic pitch diam.} + \frac{0.112}{\text{T.P.I.} + 40}$$

$$\text{Min. root diam.} = \text{basic root diam.} + \frac{0.112}{\text{T.P.I.} + 40}$$

Machine Screw Threads, Old Standard — Tap Sizes

Number	Diameter	Threads per Inch	Number	Diameter	Threads per Inch	Number	Diameter	Threads per Inch
1	0.071	64	8	0.166	32	16	0.272	18
1½	0.081	56	9	0.180	30	18	0.298	18
2	0.089	56	10	0.194	24	20	0.325	16
3	0.101	48	11	0.206	24	22	0.350	16
4	0.113	36	12	0.221	24	24	0.378	16
5	0.125	36	13	0.234	22	26	0.404	16
6	0.141	32	14	0.246	20	28	0.430	14
7	0.154	32	15	0.261	20	30	0.456	14

A. S. M. E. Standard and Special Machine Screws

Standard or Special	Number	Outside Diameter	Threads per Inch	Outside Diameter		Pitch Diameter		Root Diameter	
				Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
Standard Sizes	0	0.060	80	0.0572	0.060	0.0505	0.0519	0.0410	0.0438
	1	0.073	72	0.0700	0.073	0.0625	0.0640	0.0520	0.0550
	2	0.086	64	0.0828	0.086	0.0743	0.0759	0.0624	0.0657
	3	0.099	56	0.0955	0.099	0.0857	0.0874	0.0721	0.0758
	4	0.112	48	0.1082	0.112	0.0966	0.0985	0.0807	0.0849
	5	0.125	44	0.1210	0.125	0.1082	0.1102	0.0910	0.0955
	6	0.138	40	0.1338	0.138	0.1197	0.1218	0.1007	0.1055
	7	0.151	36	0.1466	0.151	0.1308	0.1330	0.1097	0.1149
	8	0.164	36	0.1596	0.164	0.1438	0.1460	0.1227	0.1279
	9	0.177	32	0.1723	0.177	0.1544	0.1567	0.1307	0.1364
	10	0.190	30	0.1852	0.190	0.1660	0.1684	0.1407	0.1467
	12	0.216	28	0.2111	0.216	0.1904	0.1928	0.1633	0.1696
	14	0.242	24	0.2368	0.242	0.2123	0.2149	0.1808	0.1879
	16	0.268	22	0.2626	0.268	0.2358	0.2385	0.2014	0.2090
	18	0.294	20	0.2884	0.294	0.2587	0.2615	0.2208	0.2290
	20	0.320	20	0.3144	0.320	0.2847	0.2875	0.2468	0.2550
	22	0.346	18	0.3402	0.346	0.3070	0.3099	0.2649	0.2738
	24	0.372	16	0.3660	0.372	0.3284	0.3314	0.2810	0.2908
	26	0.398	16	0.3920	0.398	0.3544	0.3574	0.3070	0.3168
	28	0.424	14	0.4178	0.424	0.3745	0.3776	0.3204	0.3312
	30	0.450	14	0.4438	0.450	0.4005	0.4036	0.3464	0.3572
Special Sizes	1	0.073	64	0.0698	0.073	0.0613	0.0629	0.0494	0.0527
	2	0.086	56	0.0825	0.086	0.0727	0.0744	0.0591	0.0628
	3	0.099	48	0.0952	0.099	0.0836	0.0855	0.0677	0.0719
	4	0.112	40	0.1078	0.112	0.0937	0.0958	0.0747	0.0795
	4	0.112	36	0.1076	0.112	0.0918	0.0940	0.0707	0.0759
	5	0.125	40	0.1208	0.125	0.1067	0.1088	0.0877	0.0925
	5	0.125	36	0.1206	0.125	0.1048	0.1070	0.0837	0.0889
	6	0.138	36	0.1336	0.138	0.1178	0.1200	0.0967	0.1019
	6	0.138	32	0.1333	0.138	0.1154	0.1177	0.0917	0.0974
	7	0.151	32	0.1463	0.151	0.1284	0.1307	0.1047	0.1104
	7	0.151	30	0.1462	0.151	0.1270	0.1294	0.1017	0.1077
	8	0.164	32	0.1593	0.164	0.1414	0.1437	0.1177	0.1234
	8	0.164	30	0.1592	0.164	0.1400	0.1424	0.1147	0.1207
	9	0.177	30	0.1722	0.177	0.1529	0.1553	0.1277	0.1337
	9	0.177	24	0.1718	0.177	0.1473	0.1499	0.1158	0.1229
	10	0.190	32	0.1853	0.190	0.1674	0.1697	0.1437	0.1494
	10	0.190	24	0.1848	0.190	0.1603	0.1629	0.1288	0.1359
	12	0.216	24	0.2108	0.216	0.1863	0.1889	0.1548	0.1619
	14	0.242	20	0.2364	0.242	0.2067	0.2095	0.1688	0.1770
	16	0.268	20	0.2624	0.268	0.2327	0.2355	0.1948	0.2030
	18	0.294	18	0.2882	0.294	0.2550	0.2579	0.2129	0.2218
	20	0.320	18	0.3142	0.320	0.2810	0.2839	0.2389	0.2478
	22	0.346	16	0.3400	0.346	0.3024	0.3054	0.2550	0.2648
	24	0.372	18	0.3662	0.372	0.3330	0.3359	0.2909	0.2998
	26	0.398	14	0.3918	0.398	0.3485	0.3516	0.2944	0.3052
	28	0.424	16	0.4180	0.424	0.3804	0.3834	0.3330	0.3428
	30	0.450	16	0.4440	0.450	0.4064	0.4094	0.3590	0.3688

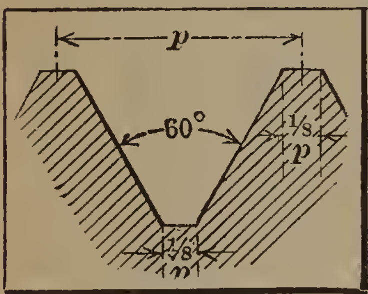
A. S. M. E. Standard and Special Machine Screw Taps

Standard or Special	Number	Outside Diameter	Threads per Inch	Outside Diameter Minimum	Outside Diameter Maximum	Pitch Diameter Minimum	Pitch Diameter Maximum	Root Diameter Minimum	Root Diameter Maximum
Standard Sizes	0	0.060	80	0.0609	0.0632	0.0528	0.0538	0.0447	0.0466
	1	0.073	72	0.0740	0.0765	0.0650	0.0660	0.0560	0.0580
	2	0.086	64	0.0871	0.0898	0.0770	0.0781	0.0668	0.0689
	3	0.099	56	0.1002	0.1033	0.0886	0.0897	0.0770	0.0793
	4	0.112	48	0.1133	0.1168	0.0998	0.1010	0.0862	0.0887
	5	0.125	44	0.1263	0.1301	0.1116	0.1129	0.0968	0.0995
	6	0.138	40	0.1394	0.1435	0.1232	0.1246	0.1069	0.1097
	7	0.151	36	0.1525	0.1569	0.1345	0.1359	0.1164	0.1193
	8	0.164	36	0.1655	0.1699	0.1475	0.1489	0.1294	0.1323
	9	0.177	32	0.1786	0.1835	0.1583	0.1598	0.1380	0.1411
	10	0.190	30	0.1916	0.1968	0.1700	0.1716	0.1483	0.1515
	12	0.216	28	0.2176	0.2232	0.1944	0.1961	0.1712	0.1745
	14	0.242	24	0.2438	0.2500	0.2167	0.2184	0.1896	0.1931
	16	0.268	22	0.2698	0.2765	0.2403	0.2421	0.2108	0.2144
	18	0.294	20	0.2959	0.3031	0.2634	0.2652	0.2309	0.2346
	20	0.320	20	0.3219	0.3291	0.2894	0.2912	0.2569	0.2606
	22	0.346	18	0.3479	0.3559	0.3118	0.3138	0.2757	0.2796
	24	0.372	16	0.3740	0.3828	0.3334	0.3354	0.2928	0.2968
	26	0.398	16	0.4000	0.4088	0.3594	0.3614	0.3188	0.3228
	28	0.424	14	0.4261	0.4359	0.3797	0.3818	0.3333	0.3374
	30	0.450	14	0.4521	0.4619	0.4057	0.4078	0.3593	0.3634
Special Sizes	1	0.073	64	0.0741	0.0768	0.0640	0.0651	0.0538	0.0559
	2	0.086	56	0.0872	0.0903	0.0756	0.0767	0.0640	0.0663
	3	0.099	48	0.1003	0.1038	0.0868	0.0880	0.0732	0.0757
	4	0.112	40	0.1134	0.1175	0.0972	0.0986	0.0809	0.0837
	4	0.112	36	0.1135	0.1179	0.0955	0.0969	0.0774	0.0803
	5	0.125	40	0.1264	0.1305	0.1102	0.1116	0.0939	0.0967
	5	0.125	36	0.1265	0.1309	0.1085	0.1099	0.0904	0.0933
	6	0.138	36	0.1395	0.1439	0.1215	0.1229	0.1034	0.1063
	6	0.138	32	0.1396	0.1445	0.1193	0.1208	0.0990	0.1021
	7	0.151	32	0.1526	0.1575	0.1323	0.1338	0.1120	0.1151
	7	0.151	30	0.1526	0.1578	0.1310	0.1326	0.1093	0.1125
	8	0.164	32	0.1656	0.1705	0.1453	0.1468	0.1250	0.1281
	8	0.164	30	0.1656	0.1708	0.1440	0.1456	0.1223	0.1255
	9	0.177	30	0.1786	0.1838	0.1569	0.1585	0.1353	0.1385
	9	0.177	24	0.1788	0.1850	0.1517	0.1534	0.1247	0.1282
	10	0.190	32	0.1916	0.1965	0.1713	0.1728	0.1510	0.1541
	10	0.190	24	0.1918	0.1980	0.1647	0.1664	0.1377	0.1412
	12	0.216	24	0.2178	0.2240	0.1907	0.1924	0.1637	0.1672
	14	0.242	20	0.2439	0.2511	0.2114	0.2132	0.1789	0.1826
	16	0 268	20	0.2699	0.2771	0.2374	0.2392	0.2049	0.2086
	18	0.294	18	0.2959	0.3039	0.2598	0.2618	0.2237	0.2276
	20	0.320	18	0.3219	0.3299	0.2858	0.2878	0.2497	0 2536
	22	0.346	16	0.3480	0.3568	0.3074	0.3094	0.2668	0.2708
	24	0.372	18	0.3739	0.3819	0 3378	0.3398	0.3017	0.3056
	26	0.398	14	0.4001	0.4099	0 3537	0.3558	0.3073	0.3114
	28	0.424	16	0.4260	0.4348	0.3854	0.3874	0.3448	0.3488
	30	0.450	16	0 4520	0.4608	0.4114	0.4134	0.3708	0.3748

Instrument Makers' System. — The standard screw system of the Royal Microscopical Society of London, England, also known as the "Society Thread," is employed for microscope objectives and the nose pieces of the microscope into which these objectives screw. The form of the thread is the standard Whitworth form. The number of threads per inch is 36. The dimensions of the thread are as follows:

Male thread, outside diam.,	max. 0.7982 inch,	min. 0.7952 inch;
root diam.,	max. 0.7626 inch,	min. 0.7596 inch;
Female thread, root of thread,	max. 0.7674 inch,	min. 0.7644 inch;
top of thread,	max. 0.8030 inch,	min. 0.8000 inch.

French and International System Standard Thread. — The form of the thread is the same as that of the U. S. standard thread system. The standard thread of the International system, denoted S. I., was adopted by an International Congress for the Unifying of Screw Threads, held in Zürich, 1898. This system conforms, in general, with the system earlier adopted in France, the French standard thread (S. F.), but there are some variations. In order to provide clearance at the bottom of the thread, the Congress specified that "the clearance at the bottom of the thread shall not exceed one-sixteenth of the height of the original triangle. The shape of the bottom of the thread, resulting from said clearance, is left to the manufacturers. However, the Congress recommends a rounded profile for said bottom."



Elements of the French and International System Standard Thread

Pitch, Mm.	Depth of Thread, Inches	Width of Flat, Inches	Double Depth of Thread, Inches	Pitch, Mm.	Depth of Thread, Inches	Width of Flat, Inches	Double Depth of Thread, Inches
8	0.2046	0.0394	0.4092	3	0.0767	0.0148	0.1534
7.75	0.1982	0.0382	0.3964	2.75	0.0703	0.0135	0.1406
7.5	0.1918	0.0369	0.3836	2.5	0.0639	0.0123	0.1279
7.25	0.1854	0.0357	0.3708	2.25	0.0575	0.0111	0.1151
7	0.1790	0.0344	0.3580	2	0.0511	0.0098	0.1023
6.75	0.1726	0.0332	0.3452	1.75	0.0448	0.0086	0.0895
6.5	0.1662	0.0320	0.3324	1.5	0.0384	0.0074	0.0767
6.25	0.1598	0.0308	0.3196	1.25	0.0320	0.0062	0.0639
6	0.1534	0.0295	0.3068	1	0.0256	0.0049	0.0511
5.75	0.1470	0.0283	0.2940	0.9	0.0230	0.0044	0.0460
5.5	0.1406	0.0271	0.2812	0.85	0.0217	0.0042	0.0435
5.25	0.1343	0.0259	0.2685	0.8	0.0205	0.0039	0.0409
5	0.1279	0.0246	0.2557	0.75	0.0192	0.0037	0.0384
4.75	0.1215	0.0234	0.2429	0.7	0.0179	0.0034	0.0358
4.5	0.1151	0.0221	0.2301	0.6	0.0153	0.0030	0.0307
4.25	0.1087	0.0209	0.2174	0.55	0.0141	0.0027	0.0281
4	0.1023	0.0197	0.2046	0.5	0.0128	0.0025	0.0256
3.75	0.0959	0.0185	0.1918	0.4	0.0102	0.0020	0.0205
3.5	0.0895	0.0172	0.1790	0.3	0.0077	0.0015	0.0153
3.25	0.0831	0.0160	0.1662	0.25	0.0064	0.0012	0.0128

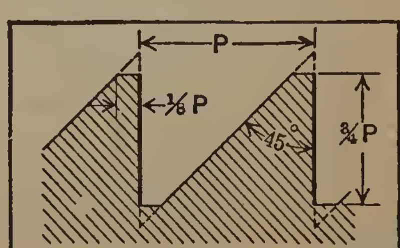
International Standard Thread

Diameter		Pitch, Mm.	Root Diameter		Diameter		Pitch, Mm.	Root Diameter	
Mm.	Inches		Mm.	Inches	Mm.	Inches		Mm.	Inches
1	0.0394	0.25	0.676	0.0266	24	0.9449	3.0	20.10	0.7915
2	0.0787	0.45	1.415	0.0557	27	1.0630	3.0	23.10	0.9095
3	0.1181	0.6	2.220	0.0874	30	1.1811	3.5	25.45	1.0020
4	0.1575	0.75	3.025	0.1191	33	1.2992	3.5	28.45	1.1201
5	0.1969	0.9	3.833	0.1509	36	1.4173	4.0	30.80	1.2126
6	0.2362	1.0	4.701	0.1851	39	1.5354	4.0	33.80	1.3307
7	0.2756	1.0	5.702	0.2245	42	1.6535	4.5	36.15	1.4232
8	0.3150	1.25	6.378	0.2511	45	1.7716	4.5	39.15	1.5413
9	0.3543	1.25	7.376	0.2904	48	1.8898	5.0	41.51	1.6343
10	0.3937	1.5	8.052	0.3170	52	2.0472	5.0	45.51	1.7918
11	0.4331	1.5	9.052	0.3564	56	2.2047	5.5	48.86	1.9235
12	0.4724	1.75	9.728	0.3830	60	2.3622	5.5	52.86	2.0810
14	0.5512	2.0	11.402	0.4489	64	2.5197	6.0	56.21	2.2130
16	0.6299	2.0	13.401	0.5276	68	2.6772	6.0	60.21	2.3705
18	0.7087	2.5	14.752	0.5808	72	2.8346	6.5	63.56	2.5023
20	0.7874	2.5	16.751	0.6595	76	2.9921	6.5	67.56	2.6598
22	0.8661	2.5	18.750	0.7382	80	3.1496	7.0	70.91	2.7918

French Standard Thread

Diameter		Pitch, Mm.	Root Diameter		Diameter		Pitch, Mm.	Root Diameter	
Mm.	Inches		Mm.	Inches	Mm.	Inches		Mm.	Inches
3	0.1181	0.5	2.35	0.0925	24	0.9449	3.0	20.10	0.7915
4	0.1575	0.75	3.03	0.1191	26	1.0236	3.0	22.10	0.8702
5	0.1969	0.75	4.03	0.1585	28	1.1024	3.0	24.10	0.9490
6	0.2362	1.0	4.70	0.1851	30	1.1811	3.5	25.45	1.0020
7	0.2756	1.0	5.70	0.2245	32	1.2598	3.5	27.45	1.0807
8	0.3150	1.0	6.70	0.2639	34	1.3386	3.5	29.45	1.1595
9	0.3543	1.0	7.70	0.3032	36	1.4173	4.0	30.80	1.2126
10	0.3937	1.5	8.05	0.3170	38	1.4961	4.0	32.80	1.2914
12	0.4724	1.5	10.05	0.3957	40	1.5748	4.0	34.80	1.3701
14	0.5512	2.0	11.40	0.4489	42	1.6535	4.5	36.15	1.4232
16	0.6299	2.0	13.40	0.5276	44	1.7323	4.5	38.15	1.5020
18	0.7087	2.5	14.75	0.5808	46	1.8110	4.5	40.15	1.5807
20	0.7874	2.5	16.75	0.6595	48	1.8898	5.0	41.51	1.6343
22	0.8661	2.5	18.75	0.7382	50	1.9685	5.0	43.51	1.7130

Buttress Threads. — This thread is not standardized but is usually made to the form shown in the illustration. The front face may be perpendicular to the axis of the screw or it may incline slightly (usually from 1 to 5 degrees). According to one rule, the pitch $P = 2 \times$ screw diameter $\div$ 15. The buttress form is adapted for axial loads in one direction only.



Pitch or Angle Diameters — Metric Pitches

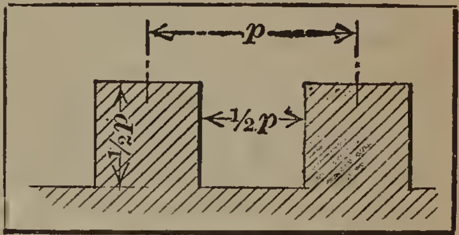
The table below gives the pitch diameters of metric pitch screw threads, as measured by thread micrometers. The outside diameter of the screw thread is given both in inches and millimeters, the pitch in millimeters only, and the pitch diameter in inches only. This arrangement is the most convenient for practical use.

Diameter		Pitch, Mm.	Pitch Diam., Inches	Diameter		Pitch, Mm.	Pitch Diam., Inches
Mm.	Inches			Mm.	Inches		
3	0.1181	0.50	0.1053	41	1.6142	4.00	1.5119
4	0.1575	0.75	0.1383	42	1.6535	4.50	1.5385
5	0.1969	0.75	0.1777	43	1.6929	4.50	1.5778
6	0.2362	1.00	0.2107	44	1.7323	4.50	1.6172
7	0.2756	1.00	0.2500	45	1.7717	4.50	1.6566
8	0.3150	1.25	0.2830	46	1.8110	4.50	1.6960
8	0.3150	1.00	0.2894	47	1.8504	4.50	1.7353
9	0.3543	1.25	0.3224	48	1.8898	5.00	1.7619
9	0.3543	1.00	0.3288	49	1.9291	5.00	1.8013
10	0.3937	1.50	0.3553	50	1.9685	5.00	1.8406
11	0.4331	1.50	0.3947	51	2.0079	5.00	1.8800
12	0.4724	1.75	0.4277	52	2.0472	5.00	1.9194
12	0.4724	1.50	0.4341	53	2.0866	5.00	1.9588
13	0.5118	1.75	0.4671	54	2.1260	5.00	1.9981
14	0.5512	2.00	0.5000	55	2.1653	5.00	2.0375
15	0.5906	2.00	0.5394	56	2.2047	5.50	2.0641
16	0.6299	2.00	0.5788	57	2.2441	5.50	2.1035
17	0.6693	2.00	0.6182	58	2.2835	5.50	2.1428
18	0.7087	2.50	0.6447	59	2.3228	5.50	2.1822
19	0.7480	2.50	0.6841	60	2.3622	5.50	2.2216
20	0.7874	2.50	0.7235	61	2.4016	5.50	2.2609
21	0.8268	2.50	0.7628	62	2.4409	5.50	2.3003
22	0.8661	2.50	0.8022	63	2.4803	5.50	2.3397
23	0.9055	2.50	0.8416	64	2.5197	6.00	2.3663
24	0.9449	3.00	0.8682	65	2.5591	6.00	2.4056
25	0.9843	3.00	0.9075	66	2.5984	6.00	2.4450
26	1.0236	3.00	0.9469	67	2.6378	6.00	2.4844
27	1.0630	3.00	0.9863	68	2.6772	6.00	2.5237
28	1.1024	3.00	1.0257	69	2.7165	6.00	2.5631
29	1.1417	3.00	1.0650	70	2.7559	6.00	2.6025
30	1.1811	3.50	1.0916	71	2.7953	6.00	2.6418
31	1.2205	3.50	1.1310	72	2.8346	6.50	2.6684
32	1.2598	3.50	1.1703	73	2.8740	6.50	2.7078
33	1.2992	3.50	1.2097	74	2.9134	6.50	2.7472
34	1.3386	3.50	1.2491	75	2.9528	6.50	2.7865
35	1.3780	3.50	1.2885	76	2.9921	6.50	2.8259
36	1.4173	4.00	1.3150	77	3.0315	6.50	2.8653
37	1.4567	4.00	1.3544	78	3.0709	6.50	2.9046
38	1.4961	4.00	1.3938	79	3.1102	6.50	2.9440
39	1.5354	4.00	1.4331	80	3.1496	7.00	2.9706
40	1.5748	4.00	1.4725	......	....	......	......

Whitworth Standard Thread System for Watch and Mathematical Instrument Makers

Diam., Inches	Threads per Inch	Diam., Inches	Threads per Inch	Diam., Inches	Threads per Inch	Diam., Inches	Threads per Inch	Diam., Inches	Threads per Inch
0.010	400	0.017	250	0.028	180	0.045	120	0.080	60
0.011	400	0.018	250	0.030	180	0.050	100	0.085	60
0.012	350	0.019	250	0.032	150	0.055	100	0.090	60
0.013	350	0.020	210	0.034	150	0.060	100	0.095	60
0.014	300	0.022	210	0.036	150	0.065	80	0.100	50
0.015	300	0.024	210	0.038	120	0.070	80	....	.....
0.016	300	0.026	180	0.040	120	0.075	80	....	......

Square Thread. — The sides of the square thread are parallel. The depth of the thread is equal to the width of space between the teeth; this space is, theoretically, equal to one-half of the pitch. It is necessary, in practice, however, to make the space in the nut a trifle wider than the thread, so as to permit of a sliding fit. The threads in the screws are made exactly according to the theoretical standard. The width of the point of the tool for cutting screws is, therefore, exactly one-half of the pitch, but the width of the point of the tool for cutting taps which are to be used for tapping nuts is slightly less than one-half the pitch, and the width of an inside thread tool for threading nuts is slightly more than one-half the pitch. The table below gives the width of tools for cutting taps, screws and nuts.



Tools for Square Thread

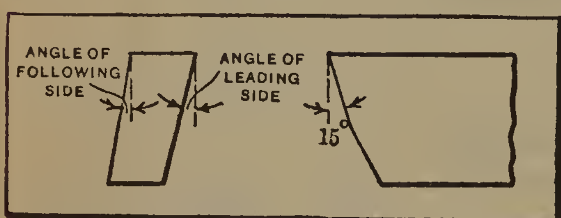
No. of Threads per Inch	Width of Point of Tool: For Taps	Width of Point of Tool: For Screws	Width of Point of Tool: For Inside Thread Tools for Nuts	No. of Threads per Inch	Width of Point of Tool: For Taps	Width of Point of Tool: For Screws	Width of Point of Tool: For Inside Thread Tools for Nuts
1	0.4965	0.5000	0.5035	8	0.0615	0.0625	0.0635
1⅓	0.3715	0.3750	0.3785	9	0.0546	0.0556	0.0566
1½	0.3303	0.3333	0.3363	10	0.0490	0.0500	0.0510
1¾	0.2827	0.2857	0.2887	11	0.0445	0.0455	0.0465
2	0.2475	0.2500	0.2525	12	0.0407	0.0417	0.0427
2½	0.1975	0.2000	0.2025	13	0.0375	0.0385	0.0395
3	0.1642	0.1667	0.1692	14	0.0352	0.0357	0.0362
3½	0.1409	0.1429	0.1449	15	0.0328	0.0333	0.0338
4	0.1235	0.1250	0.1265	16	0.0307	0.0312	0.0317
4½	0.1096	0.1111	0.1126	18	0.0273	0.0278	0.0283
5	0.0985	0.1000	0.1015	20	0.0245	0.0250	0.0255
5½	0.0894	0.0909	0.0924	22	0.0222	0.0227	0.0232
6	0.0818	0.0833	0.0848	24	0.0203	0.0208	0.0213
7	0.0699	0.0714	0.0729	......	......	......	....

Elements of the Square Thread

Threads per Inch	Depth of Thread	Double Depth of Thread	Threads per Inch	Depth of Thread	Double Depth of Thread	Threads per Inch	Depth of Thread	Double Depth of Thread
1	0.5000	1.0000	4½	0.1111	0.2222	12	0.0417	0.0833
1⅓	0.3750	0.7500	5	0.1000	0.2000	13	0.0385	0.0769
1½	0.3333	0.6667	5½	0.0909	0.1818	14	0.0357	0.0714
1¾	0.2857	0.5714	6	0.0833	0.1667	15	0.0333	0.0667
2	0.2500	0.5000	7	0.0714	0.1429	16	0.0312	0.0625
2½	0.2000	0.4000	8	0.0625	0.1250	18	0.0278	0.0556
3	0.1667	0.3333	9	0.0556	0.1111	20	0.0250	0.0500
3½	0.1429	0.2857	10	0.0500	0.1000	22	0.0227	0.0455
4	0.1250	0.2500	11	0.0455	0.0909	24	0.0208	0.0417

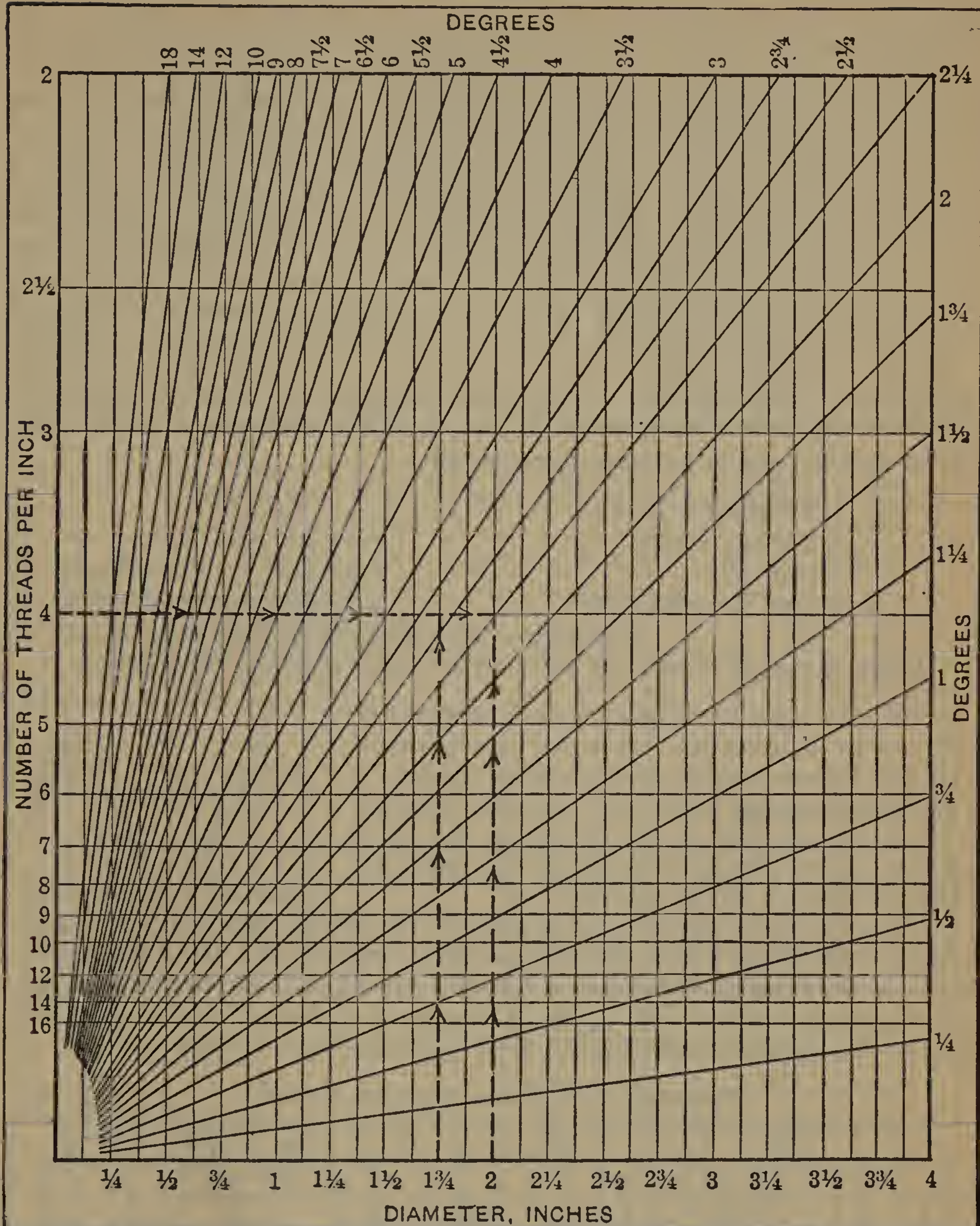
Diagram for Obtaining the Clearance Angles for Square Threading Tools. — The accompanying diagram makes it possible to find, without calculation, the clearance angles on the sides of threading tools for square threads. As indicated in the accompanying illustration, the angle on the "leading" side is slightly larger than the angle on the "following" side. This is due to the fact that the angle on the leading side is calculated to correspond to the root diameter of the thread, and the angle on the following side to the outside diameter. The end view in the illustration shows a tool for cutting a right-hand thread with the point towards the observer.

As an example of the use of the diagram, assume that a square thread screw, 2 inches in diameter, with four (single) threads per inch is to be cut. Find the required clearance angles for the threading tool. In this case, the root diameter equals 2 − ¼ = 1¾ inch. To find the clearance angle for the leading side of the threading tool, follow the vertical line from 1¾ inch diameter, as indicated on the scale at the bottom of the diagram, to the intersecting horizontal line from four threads per inch. From the intersection of these two lines, follow the nearest diagonal line, thus finding the angle to be 2½ degrees. To find the angle for the following side, follow the vertical line from 2 inches diameter to the intersecting horizontal line for 4 threads per inch. From the intersection, follow the nearest diagonal line as before. The angle on the following side is thus found to be 2¼ degrees. These angles are the theoretical clearance angles. For practical purposes, of course, slightly greater clearance is given to the tool, or, in other words, the angle of the leading side is made slightly more, and that of the following side slightly less, than the values found from the diagram.



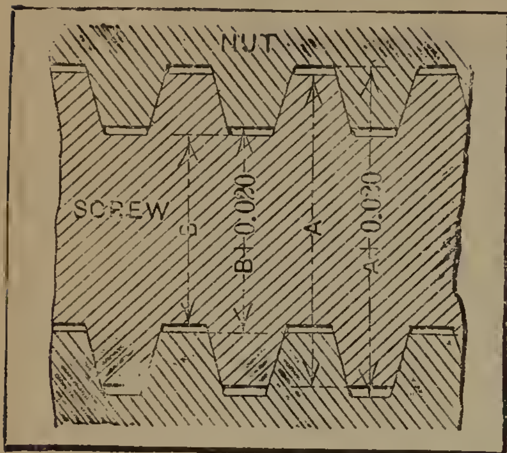
If the screw is multiple-threaded, the number of threads per inch located on the left-hand scale in the diagram will be the actual number of turns of one thread in one inch and not the number of grooves in the screw. For example, a screw with ¼-inch lead, ⅛-inch pitch, double thread, has four threads per inch, double. Assume that the outside diameter is 2 inches. The clearance angle for the following side would be located the same as in the example just given; but as the pitch of the screw is ⅛ inch, the root diameter would be 2 − ⅛ = 1⅞ inch, and this diameter should be used in finding the clearance angle for the leading side. The difference in the angle is very small and of no practical importance.

Diagram for Finding Clearance Angles for Square Thread Tools



Difficulties in Cutting Square Threads with Taps and Dies. — The square form of thread is usually made about twice as coarse in pitch as the V or U.S. standard threads. Partly for this reason and also because of the perpendicular walls of the thread, it is a troublesome thread to cut with taps and dies. Difficulties are also met with when more than one cut is made for producing the finished thread, owing to the succeeding taps or dies not having a lead exactly like the one of the partly cut thread, and hence the thread already formed is cut away. On account of these difficulties, the square thread, which was formerly used to a great extent on adjusting and power conveying screws, has of late been, to an ever increasing extent, replaced by screws provided with Acme threads, as these threads are much easier to produce accurately.

Acme Thread. — The angle between the sides of the Acme thread is 29 degrees. The depth of the thread is one-half the pitch plus 0.010 inch for clearance. The



screw is made of standard or nominal diameter, but the thread in the nut is made 0.020 inch over-size The relation between the diametral dimensions in the screw and nut are clearly shown in the illustration. If A is the diameter of the screw over the top of the thread, and B the diameter of the screw at the root, the corresponding dimensions in the nut are $A + 0.020$ inch and $B + 0.020$ inch. If p = pitch of thread, d = depth of thread, f = width of flat at top of thread, c = width of flat at root of thread, then the following formulas give the thread dimensions for the Acme screw thread as well as for taps for tapping nuts to fit Acme thread screws.

For screws:

$d = \frac{1}{2}\,p + 0.010$ inch;

$f = 0.3707\,p$;[1]

$c = 0.3707\,p - 0.0052$ inch.

For taps:

$d = \frac{1}{2}\,p + 0.020$ inch;

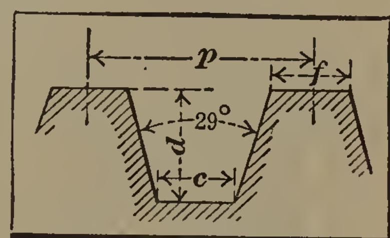
$f = 0.3707\,p - 0.0052$ inch;

$c = 0.3707\,p - 0.0052$ inch.

Diameter of tap = diameter of screw + 0.020 inch.

Diam. at root of thread (tap and screw) = diam. of screw − (p + 0.020).

For very heavy parts it may be advisable to allow the screw to bear at either the top or root of the thread to prevent it from wedging and binding in the nut.



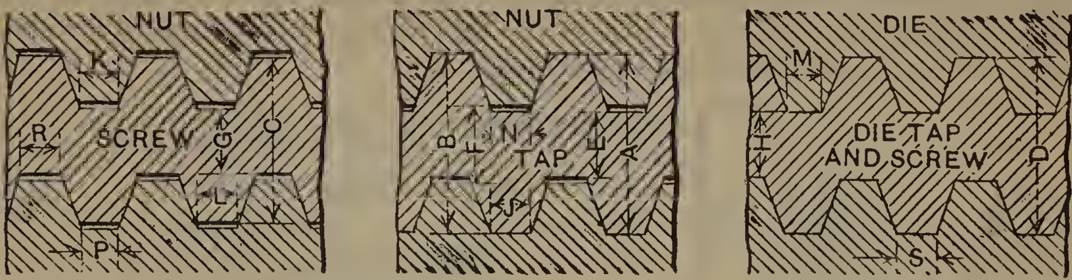
The accompanying tables give complete formulas of the Acme standard thread for screws, taps, nuts and dies, and the details of the thread for screws for various numbers of threads per inch.

The Acme thread is used to a great extent at the present time, and has in many instances replaced the square thread in machine construction. The advantages of the Acme thread are its strength and the ease with which it can be cut, compared with the square thread. This is due to the greater strength of the teeth in both taps and dies, as well as to the facility with which the chips free themselves.

Elements of the Acme Standard Thread

Threads per Inch	Depth of Thread	Width of Flat at Top of Thread	Width of Flat at Root of Thread	Double Depth of Thread	Threads per Inch	Depth of Thread	Width of Flat at Top of Thread	Width of Flat at Root of Thread	Double Depth of Thread
1	0.5100	0.3707	0.3655	1.0200	5	0.1100	0.0741	0.0689	0.2200
1½	0.3433	0.2471	0.2419	0.6867	5½	0.1009	0.0674	0.0622	0.2018
2	0.2600	0.1853	0.1801	0.5200	6	0.0933	0.0618	0.0566	0.1867
2½	0.2100	0.1483	0.1431	0.4200	7	0.0814	0.0530	0.0478	0.1629
3	0.1767	0.1236	0.1184	0.3533	8	0.0725	0.0463	0.0411	0.1450
3½	0.1529	0.1059	0.1007	0.3057	9	0.0656	0.0412	0.0360	0.1311
4	0.1350	0.0927	0.0875	0.2700	10	0.0600	0.0371	0.0319	0.1200
4½	0.1211	0.0824	0.0772	0.2422	12	0.0517	0.0309	0.0257	0.1033

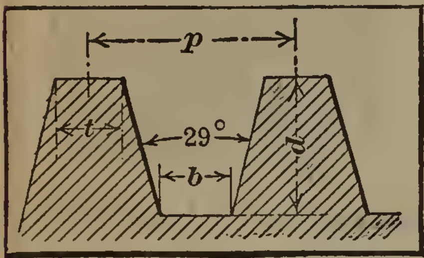
Table of Acme Thread Parts



In the formulas below, O=nominal size, or actual outside diameter of Acme thread screw; n = number of threads per inch.

Dimension Required	Class of Thread	Dimension	Formula	Example: Nominal Diam., 2 Inches; 4 Threads per Inch
Outside Diameter	Tap	A	$O + 0.020$ inch	$2.000 + 0.020 = 2.020$
	Nut	B	$O + 0.020$ inch	$2.000 + 0.020 = 2.020$
	Screw and Die Tap	C	O	2.000
	Die	D	O	2.000
Root Diameter	Tap	E	$O - \left(\frac{1}{n} + 0.020\right)$	$2.000 - (\frac{1}{4} + 0.020) = 1.730$
	Nut	F	$O - \frac{1}{n}$	$2.000 - 0.250 = 1.750$
	Screw and Die Tap	G	$O - \left(\frac{1}{n} + 0.020\right)$	$2.000 - (\frac{1}{4} + 0.020) = 1.730$
	Die	H	$O - \left(\frac{1}{n} + 0.020\right)$	$2.000 - (\frac{1}{4} + 0.020) = 1.730$
Width of Flat on Top of Thread	Tap	J	$\frac{0.3707}{n} - 0.0052$	$\frac{0.3707}{4} - 0.0052 = 0.0875$
	Nut	K	$\frac{0.3707}{n}$	$\frac{0.3707}{4} = 0.0927$
	Screw and Die Tap	L	$\frac{0.3707}{n}$	$\frac{0.3707}{4} = 0.0927$
	Die	M	$\frac{0.3707}{n} - 0.0052$	$\frac{0.3707}{4} - 0.0052 = 0.0875$
Width of Flat at Bottom of Thread	Tap	N	$\frac{0.3707}{n} - 0.0052$	$\frac{0.3707}{4} - 0.0052 = 0.0875$
	Nut	P	$\frac{0.3707}{n} - 0.0052$	$\frac{0.3707}{4} - 0.0052 = 0.0875$
	Screw and Die Tap	R	$\frac{0.3707}{n} - 0.0052$	$\frac{0.3707}{4} - 0.0052 = 0.0875$
	Die	S	$\frac{0.3707}{n}$	$\frac{0.3707}{4} = 0.0927$

Standard Worm Thread. — The standard worm thread resembles the Acme thread, in that the angle between the sides of the thread equals 29 degrees. The depth of the thread, and the width of the flat at top and bottom, differ. The formulas for the thread parts are as follows:



$$p = \text{pitch} = \frac{1}{\text{No. of threads per inch}}$$

$$d = \text{depth of thread} = 0.6866\,p = \frac{0.6866}{\text{No. of threads per inch}}$$

t = width at top of thread = $0.335\,p$;

b = width at bottom of thread = $0.310\,p$.

Elements of the Standard Worm Thread

Threads per Inch	Depth of Thread	Width of Flat at Top of Thread	Width of Flat at Bottom of Thread	Double Depth of Thread	Threads per Inch	Depth of Thread	Width of Flat at Top of Thread	Width of Flat at Bottom of Thread	Double Depth of Thread
1	0.6866	0.3350	0.3100	1.3732	5	0.1373	0.0670	0.0620	0.2746
1¼	0.5492	0.2680	0.2480	1.0984	6	0.1144	0.0558	0.0517	0.2289
1½	0.4577	0.2233	0.2066	0.9144	7	0.0981	0.0479	0.0443	0.1962
2	0.3433	0.1675	0.1550	0.6866	8	0.0858	0.0419	0.0388	0.1716
2½	0.2746	0.1340	0.1240	0.5492	9	0.0763	0.0372	0.0344	0.1526
3	0.2289	0.1117	0.1033	0.4577	10	0.0687	0.0335	0.0310	0.1373
3½	0.1962	0.0957	0.0886	0.3924	12	0.0572	0.0279	0.0258	0.1144
4	0.1716	0.0838	0.0775	0.3433	16	0.0429	0.0209	0.0194	0.0858
4½	0.1526	0.0744	0.0689	0.3052	20	0.0343	0.0167	0.0155	0.0687

Gas Fixture Threads. — Thin brass tubing is threaded with 27 threads per inch, irrespective of diameter. The so-called "ornament brass sizes" have 32 threads per inch. The standard sizes of the thread are 0.196 inch (large ornament brass size) and 0.148 inch (small ornament brass size).

Gas Fixture Threads

(Brass Pipe Sizes)

Nominal Size	Actual Diam. of Thread	Threads per Inch	Nominal Size	Actual Diam. of Thread	Threads per Inch	Nominal Size	Actual Diam. of Thread	Threads per Inch
0.148	0.148	32	⅜	0.390	27	¾	0.770	27
0.196	0.196	32	7/16	0.459	27	⅞	0.885	27
No. 4	0.246	27	½	0.515	27	1	1.006	27
¼	0.260	27	9/16	0.578	27	.....	.....	......
5/16	0.342	27	⅝	0.637	27	.....	.....	......

Cycle Engineers' Institute Standard Thread.— This thread is made with a 60-degree angle and rounded at top and bottom. The depth of the thread equals the pitch × 0.5327; the radius of the round at top and bottom equals ⅙ of the pitch.

Cycle Engineers' Institute Standard Thread

Diameter, Inches	No. of Threads per Inch	Diameter, Inches	No. of Threads per Inch	Diameter, Inches	No. of Threads per Inch	Diameter, Inches	No. of Threads per Inch
0.056	62	0.125	40	0.281	26	1.370	24
0.064	62	0.154	40	0.3125	26	*1.4375	24
0.072	62	0.175	32	0.375	26	1.500	24
0.080	62	0.1875	32	0.5625	20	......	......
0.092	56	0.250	26	1.000	26	......	......
0.104	44	0.266	26	*1.290	24	......	......

* For right-hand thread only

Wood Screw Thread, American Screw Co.'s Standard

No. of Screw	Diameter	Threads per Inch	No. of Screw	Diameter	Threads per Inch	No. of Screw	Diameter	Threads per Inch
0	0.058	32	11	0.203	12	22	0.347	7
1	0.071	28	12	0.216	11	23	0.361	7
2	0.084	26	13	0.229	11	24	0.374	7
3	0.097	24	14	0.242	10	25	0.387	7
4	0.110	22	15	0.255	10	26	0.400	6
5	0.124	20	16	0.268	9	27	0.413	6
6	0.137	18	17	0.282	9	28	0.426	6
7	0.150	16	18	0.295	8	29	0.439	6
8	0.163	15	19	0.308	8	30	0.453	6
9	0.176	14	20	0.321	8	.....	.....	......
10	0.189	13	21	0.334	8	.....	.....	......

Wood Screw Thread, Asa S. Cook Co.'s Standard

No. of Screw	Diameter	Threads per Inch	No. of Screw	Diameter	Threads per Inch	No. of Screw	Diameter	Threads per Inch
0	0.058	30	9	0.176	14	18	0.295	8
1	0.071	28	10	0.189	13	20	0.321	7.5
2	0.084	26	11	0.203	12.5	22	0.347	7.5
3	0.097	24	12	0.216	12	24	0.374	7
4	0.110	22	13	0.229	11	26	0.400	6.5
5	0.124	20	14	0.242	10	28	0.426	6.5
6	0.137	18	15	0.255	9.5	30	0.453	6
7	0.150	17	16	0.268	9	.....	.....	......
8	0.163	15	17	0.282	8.5	.....	.....	......

Lag Screw Thread Systems in Common Use

Diameter	Alternate Systems: Threads per Inch	Alternate Systems: Threads per Inch	Diameter	Alternate Systems: Threads per Inch	Alternate Systems: Threads per Inch	Diameter	Alternate Systems: Threads per Inch	Alternate Systems: Threads per Inch
1/4	10	10	1/2	6	6	3/4	4 1/2	5
5/16	9 1/2	9	9/16	5	6	7/8	4 1/2	4
3/8	7	8	5/8	5	5	1	3	4
7/16	7	7	11/16	4 1/2	5	....	......	......

Measuring Screw Threads

Pitch and Lead of Screw Threads. — The *pitch* of a screw thread is the distance from the center of one thread to the center of the next thread. This applies no matter whether the screw has a single, double, triple or quadruple thread. The *lead* of a screw thread is the distance the nut will move forward on the screw if it is turned around one full revolution. In a single-threaded screw, the pitch and lead are equal, because the nut would move forward the distance from one thread to the next, if turned around once. In a double-threaded screw, the nut will move forward two threads, or twice the pitch, so that in this case the lead equals twice the pitch. In a triple-threaded screw, the lead equals three times the pitch, and so on.

The word "pitch" is often, although improperly, used to denote the *number of threads per inch*. Screws are spoken of as having a 12-pitch thread, when twelve threads per inch is what is really meant. The number of threads per inch equals 1 divided by the pitch, or expressed as a formula:

$$\text{Number of threads per inch} = \frac{1}{\text{pitch}}$$

The pitch of a screw equals 1 divided by the number of threads per inch, or:

$$\text{Pitch} = \frac{1}{\text{number of threads per inch}}$$

If the number of threads per inch equals 16, the pitch = 1/16. If the pitch equals 0.05, the number of threads equals $1 \div 0.05 = 20$. If the pitch is 2/5 inch, the number of threads per inch equals $1 \div 2/5 = 2\frac{1}{2}$.

Confusion is often caused by the indefinite designation of multiple-thread screws (double, triple, quadruple, etc.). The expression, "four threads per inch, triple," for example, is not to be recommended. It means that the screw is cut with four triple threads or with twelve threads per inch, if the threads are counted by placing a scale alongside the screw. To cut this screw, the lathe would be geared to cut four threads per inch, but they would be cut only to the depth required for twelve threads per inch. The best expression, when a multiple-thread is to be cut, is to say, in this case, "1/4 inch lead, 1/12 inch pitch, triple thread." For single-threaded screws, only the number of threads per inch and the form of the thread are specified. The word "single" is not required.

Measuring Screw Thread Pitch Diameters by Thread Micrometers. — As the pitch or angle diameter of a tap or screw is the most important dimension, it is necessary that the pitch diameter of screw threads be measured, in addition to the outside diameter. One method of measuring in the angle of a thread is by

Pitch Diameters — Thread Micrometer Reading for United States Standard Screws

Diam. of Screw	Threads per Inch	Pitch Diam.	Diam. of Screw	Threads per Inch	Pitch Diam.	Diam. of Screw	Threads per Inch	Pitch Diam.
1/16	64	0.0524	15/16	9	0.8653	2	4 1/2	1.8557
3/32	50	0.0807	1	8	0.9188	2 1/8	4 1/2	1.9807
1/8	40	0.1088	1 1/16	7	0.9697	2 1/4	4 1/2	2.1057
5/32	36	0.1382	1 1/8	7	1.0322	2 3/8	4	2.2126
3/16	32	0.1672	1 3/16	7	1.0947	2 1/2	4	2.3376
7/32	28	0.1955	1 1/4	7	1.1572	2 5/8	4	2.4626
1/4	20	0.2175	1 5/16	6	1.2042	2 3/4	4	2.5876
5/16	18	0.2764	1 3/8	6	1.2667	2 7/8	3 1/2	2.6894
3/8	16	0.3344	1 7/16	6	1.3292	3	3 1/2	2.8144
7/16	14	0.3911	1 1/2	6	1.3917	3 1/8	3 1/2	2.9394
1/2	13	0.4500	1 9/16	5 1/2	1.4444	3 1/4	3 1/2	3.0644
9/16	12	0.5084	1 5/8	5 1/2	1.5069	3 3/8	3 1/4	3.1751
5/8	11	0.5660	1 11/16	5 1/2	1.5694	3 1/2	3 1/4	3.3001
11/16	11	0.6285	1 3/4	5	1.6201	3 5/8	3 1/4	3.4251
3/4	10	0.6850	1 13/16	5	1.6826	3 3/4	3	3.5335
13/16	10	0.7475	1 7/8	5	1.7451	3 7/8	3	3.6585
7/8	9	0.8028	1 15/16	5	1.8076	4	3	3.7835

Pitch Diameters — Thread Micrometer Reading for Standard Sharp V-thread Screws*

Diam. of Screw	Threads per Inch	Pitch Diam.	Diam. of Screw	Threads per Inch	Pitch Diam.	Diam. of Screw	Threads per Inch	Pitch Diam.
1/16	72	0.0505	15/16	9	0.8413	2	4 1/2	1.8075
3/32	56	0.0783	1	8	0.8917	2 1/8	4 1/2	1.9325
1/8	40	0.1033	1 1/16	8	0.9542	2 1/4	4 1/2	2.0575
5/32	32	0.1292	1 1/8	7	1.0013	2 3/8	4 1/2	2.1825
3/16	24	0.1514	1 3/16	7	1.0638	2 1/2	4	2.2835
7/32	24	0.1826	1 1/4	7	1.1263	2 5/8	4	2.4085
1/4	20	0.2067	1 5/16	7	1.1888	2 3/4	4	2.5335
5/16	18	0.2644	1 3/8	6	1.2307	2 7/8	4	2.6585
3/8	16	0.3209	1 7/16	6	1.2932	3	3 1/2	2.7526
7/16	14	0.3756	1 1/2	6	1.3557	3 1/8	3 1/2	2.8776
1/2	12	0.4278	1 9/16	6	1.4182	3 1/4	3 1/2	3.0026
9/16	12	0.4903	1 5/8	5	1.4518	3 3/8	3 1/4	3.1085
5/8	11	0.5463	1 11/16	5	1.5143	3 1/2	3 1/4	3.2335
11/16	11	0.6088	1 3/4	5	1.5768	3 5/8	3 1/4	3.3585
3/4	10	0.6634	1 13/16	5	1.6393	3 3/4	3	3.4613
13/16	10	0.7259	1 7/8	4 1/2	1.6825	3 7/8	3	3.5863
7/8	9	0.7788	1 15/16	4 1/2	1.7450	4	3	3.7113

* The figures are for the theoretical pitch diameter. If the sharp V-thread for practical purposes is provided with a flat on the top of the thread, the figures for the pitch diameter, as given, should be increased by an amount equal to width of flat × 1.732.

means of a special screw thread micrometer, as shown in the accompanying engraving, Fig. 1. The fixed anvil is V-shaped so as to fit over the thread, while the movable point is cone-shaped so as to enable it to enter the space between two threads, and at the same time be at liberty to revolve. The contact points are on the sides of the thread, as they necessarily must be in order that the pitch diameter may be determined. The cone-shaped point of the measuring screw is slightly rounded so that it will not bear in the bottom of the thread. There is also sufficient clearance at the bottom of the -shaped anvil to prevent it from bearing on the top of the thread. The movable point is adapted to measuring all pitches, but the fixed anvil is limited in its capacity. To cover the whole range of pitches, from the finest to the coarsest, a number of fixed anvils are, therefore, required.

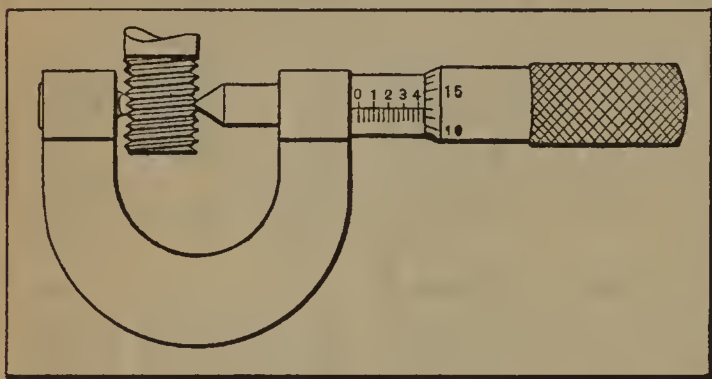


Fig. 1

To find the theoretical pitch diameter, which is measured by the micrometer, subtract the single depth of the thread from the standard outside diameter. The depth of the thread for U. S., V- and Whitworth standard threads is given in the section on screw thread systems. The accompanying tables give the pitch diameters for all standard U. S. and V-thread screws, that is, the reading of the standard thread micrometer if the thread on the screw or tap is correct.

Ball-point Micrometers. — If standard plug gages are available, it is not necessary to actually measure the pitch diameter, but merely to compare it with the standard gage. In this case, a ball-point micrometer, as shown in Fig. 2, may be employed. Two types of ball-point micrometers are ordinarily used. One is simply a regular plain micrometer with ball points made to slip over both measuring points. (See *B*, Fig. 2.) This makes a kind of combination plain and ball-point micrometer, the ball points being easily removed. These ball points, however, do not fit solidly on their seats, even if they are split, as shown, and are apt to cause errors in the measurements. The best, and, in the long run, the cheapest, method is to use a regular micrometer arranged as shown at *A*. Drill and ream out both the end of the measuring screw or spindle and the anvil, and fit ball points into them as shown. Care should be taken to have the ball point in the spindle run true. The holes in the micrometer spindle and anvil and the shanks on the points are tapered to insure a good fit. The hole *H* in spindle *G* is provided so that the ball point can be easily driven out when a change for a larger or smaller size of ball point is required.

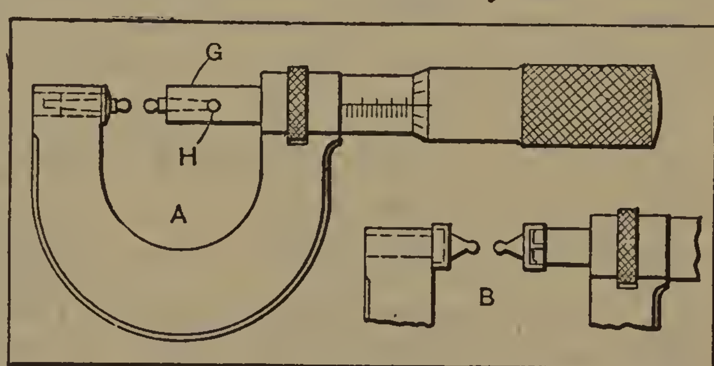
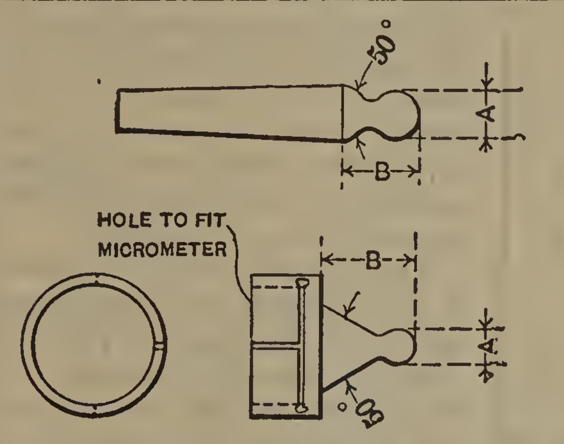


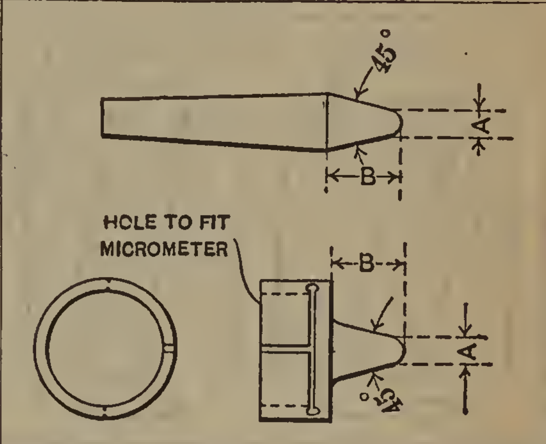
Fig. 2

A table is given with dimensions for ball points to be used for different numbers of threads per inch. These points will fit approximately halfway between the top and the root of the thread in a U. S. standard thread.

If one wishes to obtain the correct *angle* of a screw thread, as compared with that of a gage, this can be done with a ball-point micrometer by using different sizes of ball points, comparing the size first near the root of the thread, then (using a larger ball point) at about the point of the pitch diameter, and finally near the top of the thread (using in the latter case, of course, a much larger ball point). If these three measurements agree on the gage and the thread being measured, the thread angle is correct.

Table of Points for Ball-Point Micrometers





No. of Threads per Inch Calculated for	A	B	No. of Threads per Inch Used for	No. of Threads per Inch Calculated for	A	B	No. of Threads per Inch Used for
3½	0.164	9/32	3½	18	0.031	3/32	18–20
4	0.143	9/32	4	22	0.026	1/16	22
4½	0.127	9/32	4½	24	0.023	1/16	24–26
5	0.111	9/32	5	28	0.020	1/16	27–30
6	0.095	5/32	6	32	0.018	1/16	32–34
7	0.081	5/32	7	36	0.016	1/16	36–38
8	0.070	5/32	8–9	40	0.014	1/16	40–46
10	0.059	3/32	10–11	52	0.011	1/16	48–60
12	0.049	3/32	12–13	66	0.0086	1/16	62–72
14	0.042	3/32	14–15	....	....	....	....
16	0.036	3/32	16	....	....	....	....

Measuring Screw Threads by Three-wire Method. — The *effective* or *pitch diameter* of a screw thread may be measured very accurately by means of some form of micrometer and three wires of equal diameter. This method is extensively used in checking the accuracy of threaded plug gages and other precision screw threads. Two of the wires are placed in contact with the thread on one side and the third wire in a position diametrically opposite (as illustrated by the diagrams, Fig. 3, which represent a sharp V-thread and the U. S. standard thread) and the dimension over the wires is determined by means of a micrometer. An ordinary micrometer is commonly used but some form of "floating micrometer" is preferable, especially for measuring thread gages and other precision work. The floating micrometer is mounted upon a compound slide so that it can move freely in directions parallel or at right angles to the axis of the screw, which is held in a horizontal position between adjustable centers. With this arrangement the micrometer is held constantly at right angles to the axis of the screw so that only one wire on each side may be used instead of having two on one side and one on the other, as is necessary when using an ordinary micrometer. The accuracy of the pitch diameter may be determined provided the correct micrometer reading for wires of a given size is known. The micrometer reading for a U. S. standard thread can be determined by the following rule:

Micrometer Reading for U. S. Standard Thread. — Multiply the constant 1.5155 by the pitch of the thread (equals 1 ÷ number of threads per inch); subtract the product from the standard outside diameter of the screw and then add to the difference three times the diameter of the wires used; the result equals the correct micrometer reading which should be compared with the actual micrometer measurement over the wires.

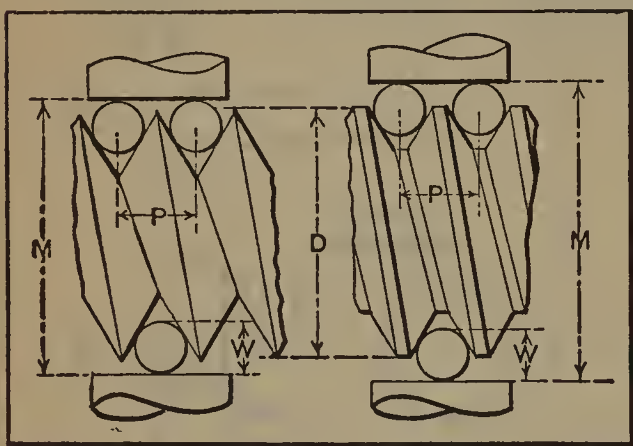


Fig. 3

The foregoing rule is expressed in the following as a formula, and additional formulas are given for different standard threads. In these formulas: M = the micrometer reading or measurement over the wires (see Fig. 3); D = the standard outside diameter of the screw; P = the pitch of the thread; and W = the diameter of the wires used.

U. S. Standard Thread:	$M = D - 1.5155 \times P + 3\,W.$
Whitworth Thread:	$M = D - 1.6008 \times P + 3.1657\,W.$
British Association:	$M = D - 1.7363 \times P + 3.4829\,W.$
Löwenherz:	$M = D - 1.75 \times P + 3.2359\,W.$
Sharp V-thread:	$M = D - 1.732 \times P + 3\,W.$

The formula given for the U. S. standard thread may also be used for the A.S.M.E. standard and the French and International standard, the pitch in millimeters, in the latter case, being changed to pitch in inches before using the formula.

Example: A screw 1½ inch in diameter and having 12 threads per inch of the U. S. standard form, is to be measured by the three-wire method; the wires are 0.070 inch in diameter. What is the correct micrometer reading?

$$1\tfrac{1}{2} - (1.5155 \times \tfrac{1}{12}) + (3 \times 0.070) = 1.5837 \text{ inch.}$$

If the micrometer reading is 1.591 inch instead, it indicates that the pitch diameter of the screw is too large. The amount of the error is the difference between the actual micrometer reading of the screw and the theoretical reading as found from the formula. In this case, then, 1.591 − 1.5837 = 0.0073 inch, which is the amount that the pitch diameter is too large. The outside diameter may be correct, or 1½ inch, but the flat on the top of the thread may be incorrect so as to account for the difference.

Tables are given herewith of the constants 1.5155 P, 1.732 P and 1.6008 P, as used in the formulas for U. S. standard, metric, V- and Whitworth threads, respectively. Tables are also given where the dimensions over the wires may be obtained directly for given diameters and pitches.

Limits for Diameters of Wires. — When wires are used in conjunction with a micrometer for measuring screw threads, the minimum wire diameter must be such that the wires extend beyond the top of the thread in order to prevent the micrometer from bearing on the threads instead of on the wires, and the maximum limit must be such that the wires bear upon the sides of the thread and not upon the corners or top edges. The following formulas for determining wire diameters do not give the extreme theoretical limits but the smallest and largest sizes which are practicable. The maximum and minimum diameters given in the accompanying table "Diameters of Wires for Measuring U. S. Standard and Whitworth Screw Threads" are based on these formulas:

Diameters of Wires for Measuring U. S. Standard and Whitworth Screw Threads

Threads per Inch	Pitch, Inch	Wire Diameters for U. S. Standard Threads			Wire Diameters for Whitworth Standard Threads		
		Max.	Min.	Pitch-line Contact	Max.	Min.	Pitch-line Contact
4	0.2500	0.2250	0.1400	0.1443	0.1900	0.1350	0.1409
4½	0.2222	0.2000	0.1244	0.1283	0.1689	0.1200	0.1253
5	0.2000	0.1800	0.1120	0.1155	0.1520	0.1080	0.1127
5½	0.1818	0.1636	0.1018	0.1050	0.1382	0.0982	0.1025
6	0.1667	0.1500	0.0933	0.0962	0.1267	0.0900	0.0939
7	0.1428	0.1286	0.0800	0.0825	0.1086	0.0771	0.0805
8	0.1250	0.1125	0.0700	0.0722	0.0950	0.0675	0.0705
9	0.1111	0.1000	0.0622	0.0641	0.0844	0.0600	0.0626
10	0.1000	0.0900	0.0560	0.0577	0.0760	0.0540	0.0564
11	0.0909	0.0818	0.0509	0.0525	0.0691	0.0491	0.0512
12	0.0833	0.0750	0.0467	0.0481	0.0633	0.0450	0.0470
13	0.0769	0.0692	0.0431	0.0444	0.0585	0.0415	0.0434
14	0.0714	0.0643	0.0400	0.0412	0.0543	0.0386	0.0403
16	0.0625	0.0562	0.0350	0.0361	0.0475	0.0337	0.0352
18	0.0555	0.0500	0.0311	0.0321	0.0422	0.0300	0.0313
20	0.0500	0.0450	0.0280	0.0289	0.0380	0.0270	0.0282
22	0.0454	0.0409	0.0254	0.0262	0.0345	0.0245	0.0256
24	0.0417	0.0375	0.0233	0.0240	0.0317	0.0225	0.0235
28	0.0357	0.0321	0.0200	0.0206	0.0271	0.0193	0.0201
32	0.0312	0.0281	0.0175	0.0180	0.0237	0.0169	0.0176
36	0.0278	0.0250	0.0156	0.0160	0.0211	0.0150	0.0156
40	0.0250	0.0225	0.0140	0.0144	0.0190	0.0135	0.0141

$$\text{U. S. Standard}\begin{cases}\text{Smallest wire diameter} = 0.56 \times \text{pitch}\\ \text{Largest wire diameter} = 0.90 \times \text{pitch}\\ \text{Diameter for pitch-line contact} = 0.57735 \times \text{pitch}\end{cases}$$

$$\text{Whitworth}\begin{cases}\text{Smallest wire diameter} = 0.54 \times \text{pitch}\\ \text{Largest wire diameter} = 0.76 \times \text{pitch}\\ \text{Diameter for pitch-line contact} = 0.56368 \times \text{pitch}\end{cases}$$

Testing Angle of Thread by Three-wire Method. — The error in the angle of a thread may be determined by using sets of wires of two diameters, the measurement over the two sets of wires being followed by calculations to determine the amount of error, assuming that the angle cannot be tested by comparison with a standard plug gage which is known to be correct. The diameter of the small wires for a U. S. standard thread is usually about 0.6 times the pitch and the diameter of the large wires, about 0.9 times the pitch. The total difference between the measurements over the large and small sets of wires is first determined. If the thread is a U. S. standard or any other form having an included angle of 60 degrees, the difference between the two measurements should equal three times the difference between the diameters of the wires used. Thus, if the wires are 0.116 and 0.076 inch in diameter, respectively, the difference equals 0.116 − 0.076 = 0.040 inch. Therefore the difference between the micrometer readings for a standard angle of 60 degrees equals 3 × 0.040 = 0.120 inch in this case. If the angle is incorrect, the amount of error may be determined by the following formula, which applies to any thread regardless of angle:

$$\sin a = \frac{A}{B - A}$$

In this formula,

A = difference in diameters of the large and small wires used;
B = total difference between the measurements over the large and small wires;
a = one-half the included thread angle.

Example: The diameter of the large wires used for testing the angle of a thread is 0.116 inch and of the small wires 0.076 inch. The measurement over the two sets of wires shows a total difference of 0.122 inch instead of the correct difference, 0.120 inch, for a standard angle of 60 degrees when using the sizes of wires mentioned. Therefore the amount of error is determined as follows:

$$\sin a = \frac{0.040}{0.122 - 0.040} = \frac{0.040}{0.082} = 0.4878$$

By referring to a table of sines it will be seen that this value (0.4878) is the sine of 29 degrees 12 minutes, approximately. Therefore the angle of the thread is 58 degrees, 24 minutes or 1 degree 36 minutes less than the standard angle.

Projection Method of Testing Screw Threads. — An effective method of testing screw threads is by means of an optical projection apparatus. An image of an external thread is projected directly upon a screen and is then compared with a suitable standard. When testing internal threads by this method, a small cast or impression of the thread is made, and the image of this cast is then projected upon the screen. The projection method shows errors in pitch, angle, and form of thread.

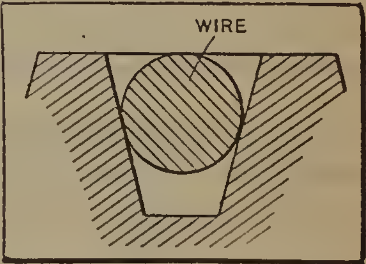
Constants Used for Measuring Pitch Diameters of Metric Screws by the Three-wire System

Pitch in Mm.	Pitch in Inches	Metric Thread, 1.5155 P	Pitch in Mm.	Pitch in Inches	Metric Thread, 1.5155 P	Pitch in Mm.	Pitch in Inches	Metric Thread, 1.5155 P
0.5	0.0197	0.0298	2.5	0.0984	0.1492	6.0	0.2362	0.3580
0.75	0.0295	0.0447	3.0	0.1181	0.1790	6.5	0.2559	0.3878
1.0	0.0394	0.0597	3.5	0.1378	0.2088	7.0	0.2756	0.4177
1.25	0.0492	0.0746	4.0	0.1575	0.2387	7.5	0.2953	0.4475
1.5	0.0590	0.0895	4.5	0.1772	0.2685	8.0	0.3150	0.4773
1.75	0.0689	0.1044	5.0	0.1969	0.2983	9.0	0.3543	0.5370
2.0	0.0787	0.1193	5.5	0.2165	0.3282	10.0	0.3937	0.5966

Values of Constants Used in Formulas for Measuring Pitch Diameters of Screws by the Three-wire System

No. of Threads per Inch	V-Thread, 1.732 *P*	U. S. Thread, 1.5155 *P*	Whitworth Thread, 1.6008 *P*	No. of Threads per Inch	V-Thread, 1.732 *P*	U. S. Thread, 1.5155 *P*	Whitworth Thread, 1.6008 *P*
2¼	0.7698	0.6736	0.7115	18	0.0962	0.0842	0.0889
2⅜	0.7293	0.6381	0.6740	20	0.0866	0.0758	0.0800
2½	0.6928	0.6062	0.6403	22	0.0787	0.0689	0.0728
2⅝	0.6598	0.5773	0.6098	24	0.0722	0.0631	0.0667
2¾	0.6298	0.5511	0.5821	26	0.0666	0.0583	0.0616
2⅞	0.6025	0.5271	0.5568	28	0.0619	0.0541	0.0572
3	0.5774	0.5052	0.5336	30	0.0577	0.0505	0.0534
3¼	0.5329	0.4663	0.4926	32	0.0541	0.0474	0.0500
3½	0.4949	0.4330	0.4574	34	0.0509	0.0446	0.0471
4	0.4330	0.3789	0.4002	36	0.0481	0.0421	0.0445
4½	0.3849	0.3368	0.3557	38	0.0456	0.0399	0.0421
5	0.3464	0.3031	0.3202	40	0.0433	0.0379	0.0400
5½	0.3149	0.2755	0.2911	42	0.0412	0.0361	0.0381
6	0.2887	0.2526	0.2668	44	0.0394	0.0344	0.0364
7	0.2474	0.2165	0.2287	46	0.0377	0.0329	0.0348
8	0.2165	0.1894	0.2001	48	0.0361	0.0316	0.0334
9	0.1925	0.1684	0.1779	50	0.0346	0.0303	0.0320
10	0.1732	0.1515	0.1601	52	0.0333	0.0291	0.0308
11	0.1575	0.1378	0.1455	56	0.0309	0.0271	0.0286
12	0.1443	0.1263	0.1334	60	0.0289	0.0253	0.0267
13	0.1332	0.1166	0.1231	64	0.0271	0.0237	0.0250
14	0.1237	0.1082	0.1143	68	0.0255	0.0223	0.0235
15	0.1155	0.1010	0.1067	72	0.0241	0.0210	0.0222
16	0.1083	0.0947	0.1001	80	0.0217	0.0189	0.0200

Gaging Acme Thread Screws with the Wire System. — In the table below, diameters of wires to be used when measuring the pitch diameter of Acme standard thread screws or taps are given. The diameters of the wires are so selected that when the thread is of correct shape and the pitch diameter correct, the wire will be flush with the tops of the threads in a tap, and project 0.010 inch above the tops of the threads on a screw. The formula for finding the diameter of wire for any number of threads per inch is:



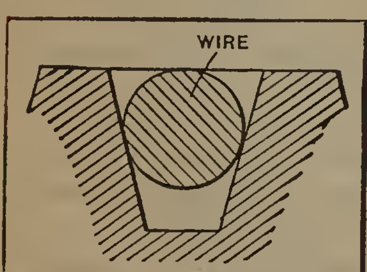
$$\text{Diam. of wire} = \left(\frac{0.6293}{\text{no. of threads}} + 0.0052\right) \times 0.7743.$$

Threads per Inch	Diam. of Wire	Threads per Inch	Diam. of Wire	Threads per Inch	Diam. of Wire	Threads per Inch	Diam. of Wire
1	0.4913	3	0.1664	5	0.1014	8	0.0649
1½	0.3288	3½	0.1432	5½	0.0926	9	0.0581
2	0.2476	4	0.1258	6	0.0852	10	0.0527
2½	0.1989	4½	0.1123	7	0.0736	12	0.0446

Table for Measuring V- and U. S. Standard Threads by the Three-wire System

Diam. of Screw	No. of Threads per Inch	Diam. of Wire used	Dimension over Wires, V-Thread	Dimension over Wires, U. S. Thread	Diam. of Screw	No. of Threads per Inch	Diam. of Wire used	Dimension over Wires, V-Thread	Dimension over Wires, U. S. Thread
1/4	18	0.035	0.2588	0.2708	7/8	8	0.090	0.9285	0.9556
1/4	20	0.035	0.2684	0.2792	7/8	9	0.090	0.9525	0.9766
1/4	22	0.035	0.2763	0.2861	7/8	10	0.090	0.9718	0.9935
1/4	24	0.035	0.2828	0.2919	15/16	8	0.090	0.9910	1.0181
5/16	18	0.035	0.3213	0.3333	15/16	9	0.090	1.0150	1.0391
5/16	20	0.035	0.3309	0.3417	1	8	0.090	1.0535	1.0806
5/16	22	0.035	0.3388	0.3486	1	9	0.090	1.0775	1.1016
5/16	24	0.035	0.3453	0.3544	1 1/8	7	0.090	1.1476	1.1785
3/8	16	0.040	0.3867	0.4003	1 1/4	7	0.090	1.2726	1.3035
3/8	18	0.040	0.3988	0.4108	1 3/8	6	0.150	1.5363	1.5724
3/8	20	0.040	0.4084	0.4192	1 1/2	6	0.150	1.6613	1.6974
7/16	14	0.050	0.4638	0.4793	1 5/8	5 1/2	0.150	1.7601	1.7995
7/16	16	0.050	0.4792	0.4928	1 3/4	5	0.150	1.8536	1.8969
1/2	12	0.050	0.5057	0.5237	1 7/8	5	0.150	1.9786	2.0219
1/2	13	0.050	0.5168	0.5334	2	4 1/2	0.150	2.0651	2.1132
1/2	14	0.050	0.5263	0.5418	2 1/4	4 1/2	0.150	2.3151	2.3632
9/16	12	0.050	0.5682	0.5862	2 1/2	4	0.150	2.5170	2.5711
9/16	14	0.050	0.5888	0.6043	2 3/4	4	0.150	2.7670	2.8211
5/8	10	0.070	0.6618	0.6835	3	3 1/2	0.200	3.1051	3.1670
5/8	11	0.070	0.6775	0.6972	3 1/4	3 1/2	0.200	3.3551	3.4170
5/8	12	0.070	0.6907	0.7087	3 1/2	3 1/4	0.250	3.7171	3.7837
11/16	10	0.070	0.7243	0.7460	3 3/4	3	0.250	3.9226	3.9948
11/16	11	0.070	0.7400	0.7597	4	3	0.250	4.1726	4.2448
3/4	10	0.070	0.7868	0.8085	4 1/4	2 7/8	0.250	4.3975	4.4729
3/4	11	0.070	0.8025	0.8222	4 1/2	2 3/4	0.250	4.6202	4.6989
3/4	12	0.070	0.8157	0.8337	4 3/4	2 5/8	0.250	4.8402	4.9227
13/16	9	0.070	0.8300	0.8541	5	2 1/2	0.250	5.0572	5.1438
13/16	10	0.070	0.8493	0.8710	....	....	.....	.....	.....

Gaging Standard Worm Threads by the Wire System. — The table on next page gives the diameter of wire to be used when measuring worm threads by the wire system. The diameter of the wire is so selected that it will be flush with the tops of the finished threads when it rests in the thread groove, provided the pitch diameter and form of thread are correct. The formula for finding the diameter of wire for any number of threads per inch is:



$$\text{Diam. of wire} = \frac{0.5149}{\text{no. of threads per inch}}$$

The wire system is especially useful for measuring Acme standard and worm threads, as special points and anvils are required if these threads are to be measured by regular thread micrometers.

Wire Sizes for Measuring Standard Worm Threads

Threads per Inch	Diam. of Wire	Threads per Inch	Diam. of Wire	Threads per Inch	Diam. of Wire	Threads per Inch	Diam. of Wire
½	1.0298	3	0.1716	6	0.0858	12	0.0429
1	0.5149	3½	0.1471	7	0.0735	14	0.0368
1½	0.3432	4	0.1287	8	0.0643	16	0.0322
2	0.2574	4½	0.1144	9	0.0572	18	0.0286
2½	0.2060	5	0.1030	10	0.0515	20	0.0257

Table for Measuring Whitworth Standard Threads by the Three-wire Method

Diam. of Thread	No. of Threads per Inch	Diam. of Wire used	Diam. Measured over Wires	Diam. of Thread	No. of Threads per Inch	Diam. of Wire used	Diam. Measured over Wires
⅛	40	0.018	0.1420	2¼	4	0.150	2.3247
3/16	24	0.030	0.2158	2⅜	4	0.150	2.4497
¼	20	0.035	0.2808	2½	4	0.150	2.5747
5/16	18	0.040	0.3502	2⅝	4	0.150	2.6997
⅜	16	0.040	0.4015	2¾	3½	0.200	2.9257
7/16	14	0.050	0.4815	2⅞	3½	0.200	3.0507
½	12	0.050	0.5249	3	3½	0.200	3.1757
9/16	12	0.050	0.5874	3⅛	3½	0.200	3.3007
⅝	11	0.070	0.7011	3¼	3¼	0.200	3.3905
11/16	11	0.070	0.7636	3⅜	3¼	0.200	3.5155
¾	10	0.070	0.8115	3½	3¼	0.200	3.6405
13/16	10	0.070	0.8740	3⅝	3¼	0.200	3.7655
⅞	9	0.070	0.9187	3¾	3	0.200	3.8495
15/16	9	0.070	0.9812	3⅞	3	0.200	3.9745
1	8	0.090	1.0848	4	3	0.200	4.0995
1 1/16	8	0.090	1.1473	4⅛	3	0.200	4.2245
1⅛	7	0.090	1.1812	4¼	2⅞	0.250	4.4846
1 3/16	7	0.090	1.2437	4⅜	2⅞	0.250	4.6096
1¼	7	0.090	1.3062	4½	2⅞	0.250	4.7346
1 5/16	7	0.090	1.3687	4⅝	2⅞	0.250	4.8596
1⅜	6	0.120	1.4881	4¾	2¾	0.250	4.9593
1 7/16	6	0.120	1.5506	4⅞	2¾	0.250	5.0843
1½	6	0.120	1.6131	5	2¾	0.250	5.2093
1 9/16	6	0.120	1.6756	5⅛	2¾	0.250	5.3343
1⅝	5	0.120	1.6847	5¼	2⅝	0.250	5.4316
1 11/16	5	0.120	1.7472	5⅜	2⅝	0.250	5.5566
1¾	5	0.120	1.8097	5½	2⅝	0.250	5.6816
1 13/16	5	0.120	1.8722	5⅝	2⅝	0.250	5.8066
1⅞	4½	0.150	1.9942	5¾	2½	0.250	5.9011
1 15/16	4½	0.150	2.0567	5⅞	2½	0.250	6.0261
2	4½	0.150	2.1192	6	2½	0.250	6.1511
2⅛	4½	0.150	2.2442	......	......	....	....

Measuring Flat or Radius at Point of Thread Tools. — To measure the width of the flat of a U. S. thread tool or the radius of a Whitworth thread tool, a micrometer arranged as shown in Fig. 4 may be used. The angle *ABC* of block *D* must, of course, be made to fit over the angle of the tool, and a different block must be used for the U. S. and the Whitworth thread tools, on account of the different

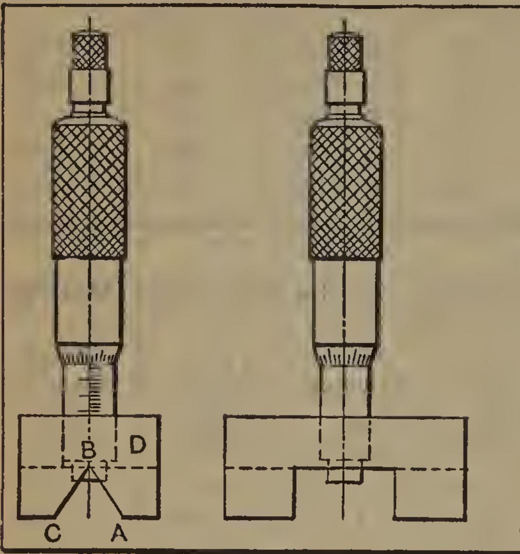


Fig. 4

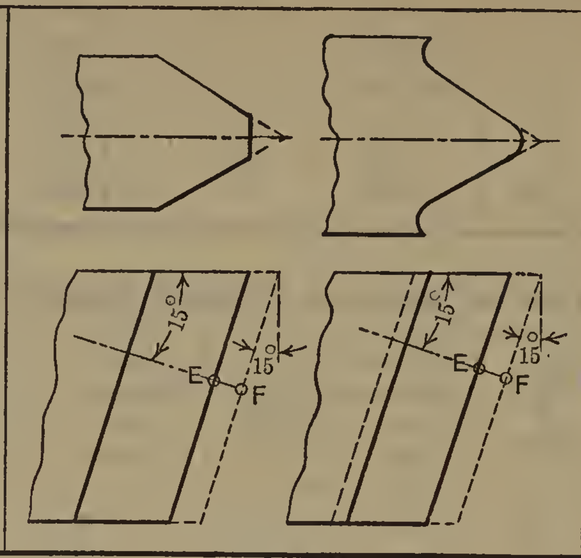


Fig. 5

angles of the threads. By this micrometer, the distance *EF*, Fig. 5, can be measured from which the exact width of flat or the correct radius can be determined. Tables are given herewith showing what the distance *EF* should be for thread tools for different numbers of threads per inch, for correct flats or radii.

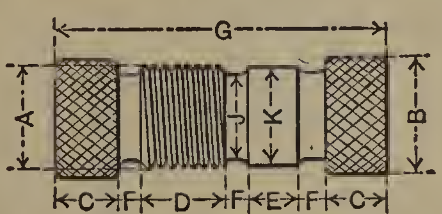
Table for Determining Flat or Radius at Point of Thread Tools
(U. S. and Whitworth Threads — 15-degree Clearance Angle)

No. of Threads per Inch	U. S. Thread, *EF* on Thread Tool, Fig. 5	Whitworth Thread, *EF* on Thread Tool, Fig. 5	No. of Threads per Inch	U. S. Thread, *EF* on Thread Tool, Fig. 5	Whitworth Thread, *EF* on Thread Tool, Fig. 5	No. of Threads per Inch	U. S. Thread, *EF* on Thread Tool, Fig. 5	Whitworth Thread, *EF* on Thread Tool, Fig. 5
2¼	0.0465	0.0687	9	0.0116	0.0172	34	0.0031	0.0045
2⅜	0.0440	0.0651	10	0.0105	0.0155	36	0.0029	0.0043
2½	0.0418	0.0619	11	0.0095	0.0141	38	0.0027	0.0041
2⅝	0.0398	0.0589	12	0.0087	0.0129	40	0.0026	0.0039
2¾	0.0380	0.0562	13	0.0080	0.0119	42	0.0025	0.0037
2⅞	0.0364	0.0538	14	0.0075	0.0110	44	0.0024	0.0035
3	0.0349	0.0515	15	0.0070	0.0103	46	0.0023	0.0034
3¼	0.0322	0.0476	16	0.0065	0.0097	48	0.0022	0.0032
3½	0.0299	0.0442	18	0.0058	0.0086	50	0.0021	0.0031
4	0.0261	0.0387	20	0.0052	0.0077	52	0.0020	0.0030
4½	0.0232	0.0344	22	0.0048	0.0070	56	0.0019	0.0028
5	0.0209	0.0309	24	0.0044	0.0064	60	0.0017	0.0026
5½	0.0190	0.0281	26	0.0040	0.0059	64	0.0016	0.0024
6	0.0174	0.0258	28	0.0037	0.0055	68	0.0015	0.0023
7	0.0149	0.0221	30	0.0035	0.0052	72	0.0015	0.0021
8	0.0131	0.0193	32	0.0033	0.0048	80	0.0013	0.0019

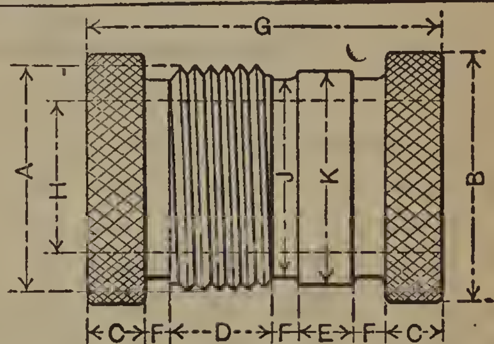
Table for Determining Radius at Point of British Association Standard Thread Tools — (15-degree Clearance Angle)

British Asso. No.	Dimension EF on Thread Tool, Fig. 5	British Asso. No.	Dimension EF on Thread Tool, Fig. 5	British Asso. No.	Dimension EF on Thread Tool, Fig. 5	British Asso. No.	Dimension EF on Thread Tool, Fig. 5
0	0.0102	7	0.0049	14	0.0023	21	0.0011
1	0.0092	8	0.0044	15	0.0021	22	0.0010
2	0.0083	9	0.0040	16	0.0019	23	0.0009
3	0.0075	10	0.0036	17	0.0017	24	0.0008
4	0.0068	11	0.0032	18	0.0015	25	0.0007
5	0.0060	12	0.0029	19	0.0014	......	......
6	0.0054	13	0.0025	20	0.0012	......	......

Reference Screw Thread Gages



A	B	C	D	E	F	G
5/32	1/4	1/2	1/4	3/16	3/32	1 19/32
3/16	5/16	1/2	5/16	1/4	3/32	1 23/32
1/4	3/8	1/2	3/8	1/4	3/32	1 25/32
5/16	7/16	1/2	7/16	1/4	1/8	2 1/16
3/8	1/2	1/2	1/2	5/16	1/8	2 3/16
7/16	9/16	1/2	1/2	5/16	1/8	2 3/16
1/2	5/8	1/2	9/16	3/8	3/16	2 1/2
9/16	11/16	1/2	9/16	3/8	3/16	2 1/2
5/8	3/4	1/2	9/16	3/8	3/16	2 1/2
3/4	7/8	1/2	5/8	7/16	3/16	2 5/8
7/8	1	1/2	5/8	7/16	1/4	2 13/16
1	1 1/8	1/2	5/8	7/16	1/4	2 13/16
1 1/8	1 1/4	1/2	5/8	7/16	1/4	2 13/16
1 1/4	1 3/8	1/2	3/4	1/2	1/4	3
1 3/8	1 1/2	1/2	3/4	1/2	1/4	3
1 1/2	1 5/8	1/2	3/4	1/2	1/4	3

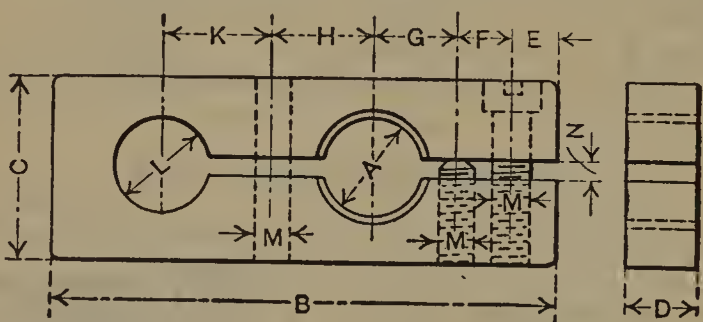


A	B	C	D	E	F	G	H
1 5/8	1 3/4	5/8	7/8	1/2	1/4	3 3/8	3/4
1 3/4	1 7/8	5/8	7/8	1/2	1/4	3 3/8	3/4
2	2 1/8	5/8	7/8	1/2	1/4	3 3/8	1
2 1/4	2 3/8	5/8	7/8	1/2	1/4	3 3/8	1
2 1/2	2 5/8	5/8	7/8	1/2	1/4	3 3/8	1 1/4
2 3/4	2 7/8	5/8	7/8	1/2	1/4	3 3/8	1 1/4
3	3 1/4	5/8	1	5/8	1/4	3 5/8	1 1/2
3 1/4	3 1/2	5/8	1	5/8	1/4	3 5/8	1 1/2
3 1/2	3 3/4	5/8	1	5/8	1/4	3 5/8	2
4	4 1/4	3/4	1	5/8	3/8	4 1/4	2
4 1/2	4 3/4	3/4	1	5/8	3/8	4 1/4	2 1/2
5	5 1/4	3/4	1 1/4	3/4	3/8	4 5/8	3
5 1/2	5 3/4	3/4	1 1/4	3/4	3/8	4 5/8	3 1/2
6	6 1/4	3/4	1 1/4	3/4	3/8	4 5/8	4
6 1/2	6 3/4	3/4	1 1/4	3/4	3/8	4 5/8	4 1/2
7	7 1/4	3/4	1 1/4	3/4	3/8	4 5/8	5

Reference Thread Gages. — The table of "Reference Screw Thread Gages" shows a type of gage used for reference purposes only. At each end of the gage a knurled collar is provided, larger in diameter than the thread. These collars prevent nuts from being screwed onto the threads and also prevent damage to the thread in handling and storage. The blank diameter at K gives the inside diameter of the corresponding ring thread gage. Up to 2 inches diameter, the following limits for thread gages are commercially obtainable: Pitch, + 0.0007 to − 0.0007 inch per inch of length; diameter of plug gage, 0.000 inch to − 0.0003 inch; inside diameter (smallest) of ring gage, 0.000 to + 0.0003 inch.

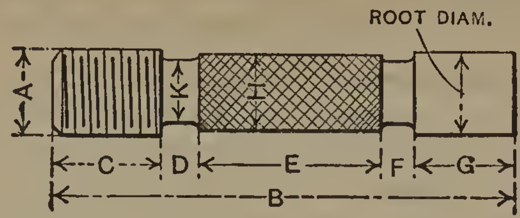
The material used for reference thread gages is commonly a good quality of mild steel, the blanks being annealed before and after rough turning. After the second annealing, they are again turned all over, rough threaded, and the collars knurled. They are then carbonized to a depth of from 1/32 to 1/16 inch, allowed to cool in the carbonizing box, and then reheated to about 1500 degrees F. and quenched in water at about 75 degrees F. They are then drawn to a bluish purple.

Dimensions of Thread Gages



A	B	C	D	E	F	G	H	K	L	M	N
1/4	2	3/4	5/16	3/16	1/4	1/4	1/4	5/8	3/8	1/8	1/16
5/16	2 1/8	13/16	5/16	3/16	1/4	9/32	9/32	11/16	3/8	1/8	1/16
3/8	2 5/16	7/8	3/8	3/16	1/4	11/32	11/32	3/4	3/8	5/32	1/16
7/16	2 7/16	15/16	7/16	3/16	1/4	3/8	3/8	13/16	3/8	5/32	1/16
1/2	2 3/4	1	1/2	1/4	5/16	7/16	7/16	7/8	7/16	3/16	1/16
9/16	2 15/16	1 1/8	1/2	1/4	5/16	15/32	15/32	15/16	7/16	3/16	1/16
5/8	3 1/16	1 3/16	9/16	1/4	5/16	1/2	1/2	1	7/16	3/16	1/16
11/16	3 1/8	1 1/4	9/16	1/4	5/16	17/32	17/32	1	7/16	3/16	1/16
3/4	3 1/4	1 3/8	9/16	1/4	5/16	9/16	9/16	1 1/16	7/16	3/16	1/16
13/16	3 3/8	1 7/16	9/16	1/4	5/16	5/8	5/8	1 1/16	7/16	3/16	1/16
7/8	3 11/16	1 9/16	5/8	5/16	3/8	11/16	11/16	1 1/8	1/2	1/4	3/32
15/16	3 3/4	1 5/8	5/8	5/16	3/8	23/32	23/32	1 1/8	1/2	1/4	3/32
1	3 7/8	1 3/4	5/8	5/16	3/8	3/4	3/4	1 3/16	1/2	1/4	3/32
1 1/8	4 1/8	1 7/8	11/16	5/16	3/8	13/16	13/16	1 1/4	1/2	1/4	3/32
1 1/4	4 5/16	2	11/16	5/16	3/8	7/8	7/8	1 5/16	1/2	1/4	3/32
1 3/8	4 9/16	2 1/8	3/4	3/8	3/8	15/16	15/16	1 3/8	9/16	1/4	1/8
1 1/2	4 3/4	2 5/16	3/4	3/8	3/8	1	1	1 3/8	9/16	1/4	1/8
1 5/8	5 1/8	2 1/2	13/16	3/8	3/8	1 1/8	1 1/8	1 1/2	9/16	1/4	1/8
1 3/4	5 1/4	2 5/8	13/16	3/8	3/8	1 3/16	1 3/16	1 1/2	9/16	1/4	1/8
1 7/8	5 3/8	2 3/4	7/8	3/8	3/8	1 1/4	1 1/4	1 1/2	9/16	1/4	1/8
2	5 1/2	3	7/8	3/8	3/8	1 5/16	1 5/16	1 1/2	9/16	1/4	1/8
2 1/4	6 1/8	3 1/4	15/16	7/16	7/16	1 1/2	1 1/2	1 5/8	5/8	5/16	1/8
2 1/2	6 1/2	3 1/2	15/16	7/16	7/16	1 5/8	1 5/8	1 5/8	5/8	5/16	1/8
2 3/4	6 3/4	3 7/8	1	7/16	7/16	1 3/4	1 3/4	1 5/8	5/8	5/16	1/8
3	7	4 1/4	1	7/16	7/16	1 7/8	1 7/8	1 5/8	5/8	5/16	1/8

Dimensions of Thread Gages



A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*
1/4	2 1/2	5/16	3/16	1 1/2	3/16	5/16	1/4	3/16
5/16	2 5/8	5/16	3/16	1 5/8	3/16	5/16	5/16	1/4
3/8	2 3/4	3/8	3/16	1 5/8	3/16	3/8	3/8	9/32
7/16	2 7/8	7/16	3/16	1 5/8	3/16	7/16	7/16	11/32
1/2	3	1/2	3/16	1 5/8	3/16	1/2	1/2	3/8
9/16	3 1/8	1/2	3/16	1 3/4	3/16	1/2	1/2	7/16
5/8	3 1/4	9/16	3/16	1 3/4	3/16	9/16	9/16	1/2
11/16	3 3/8	9/16	1/4	1 3/4	1/4	9/16	9/16	1/2
3/4	3 1/2	9/16	1/4	1 7/8	1/4	9/16	5/8	9/16
13/16	3 5/8	9/16	1/4	2	1/4	9/16	5/8	9/16
7/8	3 3/4	5/8	1/4	2	1/4	5/8	3/4	11/16
15/16	3 7/8	5/8	1/4	2 1/8	1/4	5/8	3/4	11/16
1	4	5/8	1/4	2 1/4	1/4	5/8	7/8	13/16
1 1/8	4 1/8	11/16	1/4	2 1/4	1/4	11/16	7/8	13/16
1 1/4	4 1/4	11/16	5/16	2 1/4	5/16	11/16	1	7/8
1 3/8	4 3/8	3/4	5/16	2 1/4	5/16	3/4	1	7/8
1 1/2	4 1/2	3/4	5/16	2 3/8	5/16	3/4	1 1/8	1
1 5/8	4 5/8	13/16	5/16	2 3/8	5/16	13/16	1 1/8	1
1 3/4	4 3/4	13/16	5/16	2 1/2	5/16	13/16	1 1/4	1 1/8
1 7/8	4 7/8	7/8	5/16	2 1/2	5/16	7/8	1 1/4	1 1/8
2	5	7/8	5/16	2 5/8	5/16	7/8	1 1/4	1 1/8
2 1/4	5 1/4	15/16	5/16	2 3/4	5/16	15/16	1 1/4	1 1/8
2 1/2	5 1/2	15/16	3/8	2 7/8	3/8	15/16	1 3/8	1 1/4
2 3/4	5 3/4	1	3/8	3	3/8	1	1 3/8	1 1/4
3	6	1	3/8	3 1/4	3/8	1	1 3/8	1 1/4

Grinding Gage Threads. — The threads of gages may be finished after hardening either by a grinding and lapping operation, or by lapping without grinding. It is more practicable to grind threads of medium or coarse pitch than those of fine pitches, because the small amount of wear at the edge of the grinding wheel may not be objectionable if the thread is of medium or coarse pitch, but the allowable wear diminishes with the pitch and it is difficult to keep the wheel to the right form in the case of fine threads. The grinding operation is intended to correct any errors due either to thread-cutting or to hardening, and the thread can be finished very close to size so that the lapping removes very little material. The allowance for grinding usually varies from 0.003 or 0.004 to 0.007 or 0.008 inch, depending upon the size of the gage. When threads are ground prior to the lapping, the lapping operation is simplified and the laps are not subject to so much wear as when they are relied upon to correct all the errors in the hardened gage. Aluminous abrasives such as aloxite or alundum are extensively used for thread gage grinding. The grain of the wheel should be fine enough to permit dressing the wheel to as sharp an edge as is required. The grain sizes vary from about 80 to 120 for the coarser threads up to 180 or 200 for the finer threads. The root diameter of U. S. standard gages is usually ground smaller than the theoretical size.

Allowance for Lapping Thread Gages. — The allowance for lapping should be reduced to a minimum by careful and accurate preliminary work, so that the lap merely corrects very slight errors. The lapping time is thereby reduced and more accurate gages are obtained. The lapping allowance may be reduced to a minimum by exercising care in the operations of thread-cutting, hardening, and also grinding, when the thread is ground prior to lapping. The allowance for lapping is ordinarily from 0.0002 to 0.0005 inch, although it may be 0.001 or more, the amount increasing with the size of the gage.

Some gage-makers prefer cast-iron laps, and others, laps made of soft steel. When lapping duplicate gages, it is good practice to time the lapping periods in order to determine the amount of material which is removed in a given time. When data of this kind are obtained, the gage can be reduced to size more easily and quickly, and without taking so many measurements.

TAPS AND THREADING DIES

Hand Taps. — These taps are usually made in sets of three, termed taper, plug and bottoming taps. The point of the taper tap is turned down to the diameter at the bottom of the thread for a length of about three or four threads, and then about six threads are chamfered or tapered until the full diameter of the tap is reached. On the plug tap, three threads are chamfered at the point. On the bottoming tap only about one thread is chamfered. The diameter of the straight portion of the thread of all the taps is the same in the regular type of hand tap. Taps are made in sets, however, where only the bottoming tap is of the full diameter, while the other taps gradually decrease in diameter, so as to distribute the work between the three taps. A simple rule used by one tap manufacturer for proportioning U. S. standard thread taps when made in this manner, is to make the diameter of the first tap in a set equal to the diameter of the finishing tap less the depth of the thread, and to make the diameter of the second tap equal to the diameter of the finishing tap less one-third of the depth of the thread. Frequently hand taps are not relieved or backed off in the threads, except on the chamfered or tapered portion, where the top of the thread should be slightly backed off. The diameter of the thread should, however, be made slightly smaller toward the shank than at the point. The amount of "back taper," as it is called, should be about 0.001 inch per inch.

Some manufacturers relieve the thread in the angle for the full length of the tap, leaving the top of the thread cylindrical, and other makers leave one-third of the width of the land of the thread concentric or unrelieved both on the top of the thread and in the angle. The remaining two-thirds of the land are relieved both on the top and in the angle of the thread. In this way, the advantages of a good support for the tap in the hole to be threaded, and of a free and easy cutting tap, due to the relief, are obtained.

The number of flutes should be as follows: For diameters from 1/4 to 1 3/4 inch, 4 flutes; 1 7/8 inch diameter and larger, 6 flutes. Some tap makers cut 8 flutes in taps larger than 3 inches in diameter. The dimensions of hand taps given in the accompanying table have been adopted by the Tap and Die Institute of America.

Dimensions of Squares on Tap Shanks. — The tap manufacturers have agreed upon a uniform standard for the squares on taps, and also a standard for the diameter of the shank. According to this standard, the diameter of the shank is equal to the diameter of the tap minus 1.6 times the standard pitch for a V-thread

Dimensions of Hand Taps — U. S. Standard Thread

Diam. of Tap, Inches	Length, Inches		Diam. of Shank, Inches	Size of Square, Inches	Diam. of Tap, Inches	Length, Inches		Diam. of Shank, Inches	Size of Square, Inches
	Thread	Over-all				Thread	Over-all		
1/4	1	2 1/2	0.2530	0.1897	1 1/4	2 9/16	5 3/4	1.0215	0.7661
17/64	1	2 1/2	0.2686	0.2015	1 9/32	2 9/16	5 3/4	1.0527	0.7895
9/32	1	2 1/2	0.2843	0.2132	1 5/16	2 9/16	5 3/4	1.0840	0.8130
19/64	1	2 1/2	0.2999	0.2249	1 3/8	3	6 1/16	1.1083	0.8312
5/16	1 1/8	2 23/32	0.3155	0.2366	1 13/32	3	6 1/16	1.1395	0.8546
21/64	1 1/8	2 23/32	0.3311	0.2483	1 7/16	3	6 1/16	1.1708	0.8781
11/32	1 1/8	2 23/32	0.3468	0.2601	1 1/2	3	6 3/8	1.2333	0.9250
23/64	1 1/8	2 23/32	0.3624	0.2718	1 17/32	3	6 3/8	1.2645	0.9484
3/8	1 1/4	2 15/16	0.3785	0.2839	1 5/8	3 3/16	6 11/16	1.3050	0.9787
25/64	1 1/4	2 15/16	0.3941	0.2956	1 21/32	3 3/16	6 11/16	1.3362	1.0021
13/32	1 1/4	2 15/16	0.4098	0.3074	1 3/4	3 3/16	7	1.4300	1.0725
....	....	....	.....	.....	1 25/32	3 3/16	7	1.4612	1.0959
3/8	1 1/4	2 15/16	0.2750	0.2062	1 7/8	3 9/16	7 5/16	1.5195	1.1396
25/64	1 1/4	2 15/16	0.2906	0.2179	1 29/32	3 9/16	7 5/16	1.5507	1.1630
13/32	1 1/4	2 15/16	0.3062	0.2296	2	3 9/16	7 5/8	1.6445	1.2334
27/64	1 1/4	2 15/16	0.3218	0.2414	2 1/32	3 9/16	7 5/8	1.6757	1.2568
7/16	1 7/16	3 5/32	0.3232	0.2424	2 1/8	3 9/16	8	1.7694	1.3271
29/64	1 7/16	3 5/32	0.3388	0.2541	2 5/32	3 9/16	8	1.8007	1.3505
15/32	1 7/16	3 5/32	0.3544	0.2658	2 1/4	3 9/16	8 1/4	1.8944	1.4208
31/64	1 7/16	3 5/32	0.3700	0.2775	2 9/32	3 9/16	8 1/4	1.9257	1.4443
1/2	1 21/32	3 3/8	0.3667	0.2750	2 3/8	4	8 1/2	2.0194	1.5146
33/64	1 21/32	3 3/8	0.3823	0.2867	2 13/32	4	8 1/2	2.0507	1.5380
17/32	1 21/32	3 3/8	0.3979	0.2984	2 1/2	4	8 3/4	2.1000	1.5750
35/64	1 21/32	3 3/8	0.4135	0.3101	2 17/32	4	8 3/4	2.1313	1.5984
9/16	1 21/32	3 19/32	0.4292	0.3219	2 5/8	4	8 3/4	2.2250	1.6687
37/64	1 21/32	3 19/32	0.4448	0.3336	2 21/32	4	8 3/4	2.2563	1.6922
19/32	1 21/32	3 19/32	0.4604	0.3453	2 3/4	4	9 1/4	2.3500	1.7625
39/64	1 21/32	3 19/32	0.4760	0.3570	2 25/32	4	9 1/4	2.3813	1.7859
5/8	1 13/16	3 13/16	0.4796	0.3597	2 7/8	4	9 1/4	2.4750	1.8562
41/64	1 13/16	3 13/16	0.4952	0.3714	2 29/32	4	9 1/4	2.5063	1.8797
21/32	1 13/16	3 13/16	0.5108	0.3831	3	4 9/16	9 3/4	2.5429	1.9072
11/16	1 13/16	4 1/32	0.5421	0.4066	3 1/32	4 9/16	9 3/4	2.5742	1.9306
23/32	1 13/16	4 1/32	0.5733	0.4300	3 1/8	4 9/16	9 3/4	2.6679	2.0009
3/4	2	4 1/4	0.5900	0.4425	3 5/32	4 9/16	9 3/4	2.6992	2.0244
25/32	2	4 1/4	0.6212	0.4659	3 1/4	4 9/16	10	2.7929	2.0946
13/16	2	4 15/32	0.6525	0.4894	3 9/32	4 9/16	10	2.8242	2.1181
27/32	2	4 15/32	0.6837	0.5128	3 3/8	4 9/16	10	2.8827	2.1620
7/8	2 7/32	4 11/16	0.6973	0.5230	3 13/32	4 9/16	10	2.9140	2.1855
29/32	2 7/32	4 11/16	0.7285	0.5464	3 1/2	4 15/16	10 1/4	3.0077	2.2557
15/16	2 7/32	4 29/32	0.7598	0.5698	3 17/32	4 15/16	10 1/4	3.0390	2.2792
31/32	2 7/32	4 29/32	0.7910	0.5932	3 5/8	4 15/16	10 1/4	3.1327	2.3495
1	2 1/2	5 1/8	0.8000	0.6000	3 21/32	4 15/16	10 1/4	3.1640	2.3730
1 1/32	2 1/2	5 1/8	0.8312	0.6234	3 3/4	5 5/16	10 1/2	3.2167	2.4125
1 1/16	2 1/2	5 1/8	0.8625	0.6469	3 25/32	5 5/16	10 1/2	3.2479	2.4359
1 1/8	2 9/16	5 7/16	0.8965	0.6724	3 7/8	5 5/16	10 1/2	3.3417	2.5062
1 5/32	2 9/16	5 7/16	0.9277	0.6958	3 29/32	5 5/16	10 1/2	3.3729	2.5297
1 3/16	2 9/16	5 7/16	0.9590	0.7192	4	5 5/16	10 3/4	3.4667	2.6000

tap or screw of the diameter of the tap. Hence, if a 1-inch tap were made with, say, twelve threads per inch, the diameter of shank would be still figured from the standard number of threads per inch of a 1-inch tap, which is eight threads, and the shanks and squares on all 1-inch taps, therefore, would be alike, irrespective of the number of threads per inch. It has been further agreed that the size of the square across flats should be made 3/4 times the diameter of the shank. The accompanying table "Dimensions of Hand Taps" gives the diameter of shank and size of square as figured from these formulas, excepting the first eleven diameters ranging from 1/4 to 13/32 inch, inclusive, which have larger shanks and squares.

Tolerances for Hand Taps. — The tolerances for the lengths, shank diameters, and sizes of wrench squares, vary for different sizes of hand taps as follows:

Length Over-all. — For tap diameters from 1/4 inch to 1 1/32 inch inclusive, there is a plus or minus tolerance of 1/32 inch for the over-all length; for diameters from 1 1/16 to 4 inches inclusive, there is a plus or minus tolerance of 1/16 inch.

Length of Thread. — The tolerance for the length of thread on taps of all sizes is plus or minus 3/64 inch.

Diameter of Shank. — For tap diameters up to 1 inch inclusive, the tolerance is +0 and −0.005 inch; for tap diameters from 1 1/32 to 2 inches inclusive, the tolerance is +0 and −0.007 inch; for tap diameters over 2 inches, the tolerance is +0 and −0.009 inch.

Size of Square. — For tap diameters up to 1/2 inch inclusive, the tolerance is +0 and −0.004 inch; for tap diameters from 33/64 to 1 inch inclusive, the tolerance is +0 and −0.006 inch; for tap diameters from 1 1/32 to 2 inches inclusive, the tolerance is +0 and −0.008 inch; for tap diameters from 2 1/32 to 4 inches inclusive, the tolerance is +0 and −0.010 inch.

These tolerances, which have been adopted by the Tap & Die Institute of America, apply to the hand taps listed in the table, "Dimensions of Hand Taps."

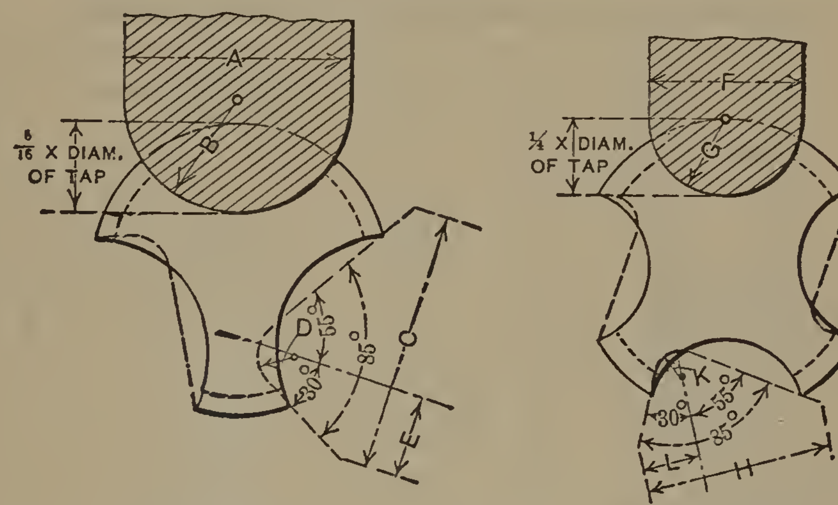
Adjustable Taps. — Many adjustable taps are now used, especially for accurate work. Some taps of this class are made of a solid piece of steel which is split and provided with means of expanding sufficiently to compensate for wear. Most of the larger adjustable taps have inserted blades or chasers which are rigidly held, but capable of radial adjustment. The use of taps of this general class enables standard sizes to be maintained readily. There is considerable variation in the design of these taps.

Advantages of Collapsing Taps. — Collapsing taps are similar in principle to self-opening dies, except that the action is reversed, the tap chasers moving inward to permit the rapid removal of the tap from the hole. This collapsing action may be due to the engagement of a collar gage-plate or lever on the tap with the surface of the work or with a fixed stop. With some designs, the collapsing action occurs only after the travel of the turret-slide is discontinued and is due to the relative motion between parts or sections of the tap itself. Collapsing taps not only reduce the time for the tapping period, but they avoid marring the thread.

Combination Taps and Dies. — Combination tools arranged for cutting an external thread and tapping a hole at the same time may be used to advantage in some cases. If the tap of a combination tool is used for cutting threads of different pitch, the difference in the rate at which they advance is compensated for by providing a floating movement for either the tap or the die. Taps combined with boring and facing cutters have proved very efficient on many kinds of work.

Tapping Square Threads. — If it is necessary to tap square threads, this should be done by using a set of taps that will form the thread by a progressive cutting action, the taps varying in size in order to distribute the work, especially for threads of comparatively coarse pitch. From three to five taps may be required in a set, depending upon the pitch. Each tap should have a pilot to steady it. The pilot of the first tap has a smooth cylindrical end from 0.003 to 0.005 inch smaller than the hole, and the pilots of following taps should have teeth and be of the same diameter as the body of the preceding tap. The teeth of different taps in a set should also increase in width progressively to distribute the work of cutting the thread groove.

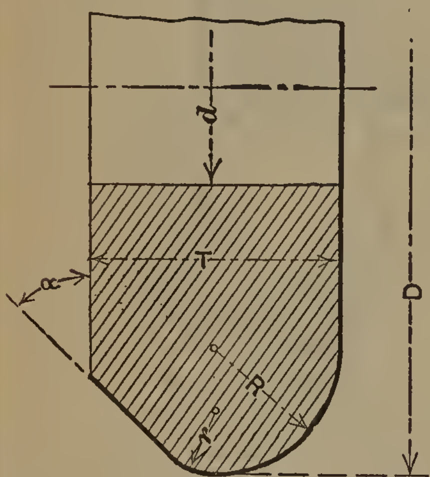
Double-angle and Convex Fluting Cutters for Taps



Diameter of Tap	Three Flutes					Four Flutes				
	Convex Fluting Cutter		Angular Fluting Cutter			Convex Fluting Cutter		Angular Fluting Cutter		
	Width A	Rad. B	Width C	Rad. D	From Side to Center of Arc, E	Width F	Rad. G	Width H	Rad. K	From Side to Center of Arc, L
1/8	3/32	3/64	3/8	1/64	1/8	1/16	1/32	3/8	1/64	1/8
1/4	3/16	3/32	3/8	1/32	1/8	1/8	1/16	3/8	1/32	1/8
1/2	3/8	3/16	1/2	1/16	5/32	1/4	1/8	1/2	3/64	5/32
3/4	9/16	9/32	3/4	3/32	7/32	3/8	3/16	5/8	1/16	7/32
1	3/4	3/8	7/8	1/8	5/16	1/2	1/4	3/4	3/32	1/4
1 1/4	15/16	15/32	1 1/4	5/32	3/8	5/8	5/16	7/8	1/8	5/16
1 1/2	1 1/8	9/16	1 1/4	3/16	7/16	3/4	3/8	1	1/8	3/8
1 3/4	1 5/16	21/32	1 3/4	7/32	9/16	7/8	7/16	1 1/8	5/32	3/8
2	1 1/2	3/4	1 3/4	1/4	9/16	1	1/2	1 1/4	3/16	7/16
2 1/4	1 11/16	27/32	2	9/32	11/16	1 1/8	9/16	1 3/8	7/32	7/16
2 1/2	1 7/8	15/16	2 1/4	5/16	3/4	1 1/4	5/8	1 5/8	1/4	1/2
2 3/4	2 1/16	1 1/32	2 3/8	11/32	3/4	1 3/8	11/16	1 3/4	9/32	5/8
3	2 1/4	1 1/8	2 5/8	3/8	7/8	1 1/2	3/4	1 7/8	5/16	5/8

Fluting Cutters. — Three kinds of fluting cutters are in use for fluting taps, general dimensions for which are given in the accompanying tables. The angular cutter with an 85-degree included angle, 30 degrees on one side and 55 degrees on the other, has, in the past, been used to a great extent, but does not produce a very good form of flute. The advantage of this cutter is that it is cheaper to use than formed cutters, and that one cutter can be used for a greater range of tap diameters. The convex cutter, dimensions for which are given in conjunction with the dimensions for the 85-degree cutter, is widely used and produces a tap of good cutting qualities. The cutter shown in the table "Dimensions of Improved Form of Tap Fluting Cutters" gives a flute of more correct form than either of the two already mentioned. When diameters of cutters and diameters of holes are given in the tables, they are intended for guidance only.

Dimensions of Improved Form of Tap Fluting Cutters



Number of Flutes, Angles and Diameters of Cutters

Size of Taps, Inches	No. of Flutes	Angle α, Degrees	Diam. of Cutter D	Diam. of Hole d
1/16 to 1/8	3	60	1 3/4	3/4
5/32 to 11/16	4	45	1 3/4	3/4
3/4 to 15/16	4	45	2 1/4	1
1 to 1 3/4	4	45	2 1/2	1
1 7/8 to 4	6	40	2 1/2	1

Diam. of Tap	Large Radius R	Small Radius r	Thickness T	Diam. of Tap	Large Radius R	Small Radius r	Thickness T
1/16	0.023	0.010	0.047	1 7/16	23/64	11/64	23/32
3/32	0.035	0.015	0.070	1 1/2	3/8	3/16	3/4
1/8	0.046	1/64	3/32	1 9/16	25/64	3/16	25/32
5/32	0.039	1/64	5/64	1 5/8	13/32	13/64	13/16
3/16	3/64	1/64	3/32	1 11/16	27/64	13/64	27/32
1/4	1/16	1/32	1/8	1 3/4	7/16	7/32	7/8
5/16	5/64	1/32	5/32	1 7/8	0.322	15/64	0.644
3/8	3/32	3/64	3/16	2	0.344	1/4	0.687
7/16	7/64	3/64	7/32	2 1/8	0.365	17/64	0.730
1/2	1/8	1/16	1/4	2 1/4	0.387	9/32	0.773
9/16	9/64	1/16	9/32	2 3/8	0.408	19/64	0.816
5/8	5/32	5/64	5/16	2 1/2	0.430	5/16	0.879
11/16	11/64	5/64	11/32	2 5/8	0.451	21/64	0.902
3/4	3/16	3/32	3/8	2 3/4	0.473	11/32	0.945
13/16	13/64	3/32	13/32	2 7/8	0.494	23/64	0.988
7/8	7/32	7/64	7/16	3	0.516	3/8	1.031
15/16	15/64	7/64	15/32	3 1/8	0.537	25/64	1.074
1	1/4	1/8	1/2	3 1/4	0.558	13/32	1.117
1 1/16	17/64	1/8	17/32	3 3/8	0.580	27/64	1.160
1 1/8	9/32	9/64	9/16	3 1/2	0.601	7/16	1.203
1 3/16	19/64	9/64	19/32	3 5/8	0.623	29/64	1.246
1 1/4	5/16	5/32	5/8	3 3/4	0.644	15/32	1.289
1 5/16	21/64	5/32	21/32	3 7/8	0.666	31/64	1.332
1 3/8	11/32	11/64	11/16	4	0.687	1/2	1.375

Commercial Tolerance for U. S. Standard Hand Taps and Pulley Taps

(Adopted by The Tap and Die Institute)

Nominal Size and Threads per Inch	Basic Size		Outside Diameter of Tap			Pitch Diameter of Tap		
	Outside Diam.	Pitch Diam.	Min. = Basic Plus	Max. = Basic Plus	Tolerance	Min. = Basic Plus	Max. = Basic Plus	Tolerance
¼–20	0.2500	0.2175	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
5/16–18	0.3125	0.2764	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
⅜–16	0.3750	0.3344	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
7/16–14	0.4375	0.3911	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
½–13	0.5000	0.4500	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
9/16–12	0.5625	0.5084	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
⅝–11	0.6250	0.5660	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
¾–10	0.7500	0.6851	0.0010	0.0050	0.0040	0.0005	0.0035	0.0030
⅞– 9	0.8750	0.8028	0.0010	0.0050	0.0040	0.0005	0.0035	0.0030
1 – 8	1.0000	0.9188	0.0010	0.0050	0.0040	0.0005	0.0035	0.0030
1⅛– 7	1.1250	1.0322	0.0015	0.0060	0.0045	0.0005	0.0040	0.0035
1¼– 7	1.2500	1.1572	0.0015	0.0060	0.0045	0.0005	0.0040	0.0035
1⅜– 6	1.3750	1.2668	0.0015	0.0060	0.0045	0.0005	0.0040	0.0035
1½– 6	1.5000	1.3918	0.0015	0.0060	0.0045	0.0005	0.0040	0.0035
1⅝– 5½	1.6250	1.5069	0.0015	0.0070	0.0055	0.0010	0.0050	0.0040
1¾– 5	1.7500	1.6201	0.0015	0.0070	0.0055	0.0010	0.0050	0.0040
1⅞– 5	1.8750	1.7451	0.0015	0.0070	0.0055	0.0010	0.0050	0.0040
2 – 4½	2.0000	1.8557	0.0015	0.0070	0.0055	0.0010	0.0050	0.0040
2⅛– 4½	2.1250	1.9807	0.0020	0.0080	0.0060	0.0010	0.0055	0.0045
2¼– 4½	2.2500	2.1057	0.0020	0.0080	0.0060	0.0010	0.0055	0.0045
2⅜– 4	2.3750	2.2126	0.0020	0.0080	0.0060	0.0010	0.0055	0.0045
2½– 4	2.5000	2.3376	0.0020	0.0080	0.0060	0.0010	0.0055	0.0045
2⅝– 4	2.6250	2.4626	0.0020	0.0090	0.0070	0.0010	0.0060	0.0050
2¾– 4	2.7500	2.5876	0.0020	0.0090	0.0070	0.0010	0.0060	0.0050
2⅞– 3½	2.8750	2.6894	0.0020	0.0090	0.0070	0.0010	0.0060	0.0050
3 – 3½	3.0000	2.8144	0.0020	0.0090	0.0070	0.0010	0.0060	0.0050

A maximum lead error of plus or minus 0.003 inch in one inch of thread is permitted. Taps with pitch coarser than S. A. E. Standard will take U. S. Standard tolerances. Those with pitch finer than S. A. E. Standard will take S. A. E. Standard tolerances. The dimensions given in this table apply to taps and not to tapped holes.

Commercial Tolerances for Taps. — The tolerances for taps, as given in the accompanying tables, were established by The Tap and Die Institute (which includes among its members the leading tap manufacturers of the United States), to meet commercial requirements in regard to both the manufacture and the use of different classes of taps. The average requirements in the trade were taken as the most desirable standard for commercial purposes. The tolerances given in these tables are based upon actual tap-making practice as determined by careful measurements of 12,600 hand taps supplied by representative tap manufacturers and ranging from ¼ inch up to 2 inches in diameter. It should be understood that the dimensions given in the tables apply to the taps themselves and not to the tapped holes. The maximum lead error permitted is plus or minus 0.003 inch in one inch of thread.

Commercial Tolerances for Machine Screw Taps — 1.

(Adopted by The Tap and Die Institute)

Screw No. and Threads per Inch	Basic Size		Outside Diam. of Tap			Pitch Diam. of Tap		
	Outside Diam.	Pitch Diam.	Min. = Basic Plus	Max. = Basic Plus	Tolerance	Min. = Basic Plus	Max. = Basic Plus	Tolerance
0–80	0.060	0.0519	0.0010	0.0025	0.0015	0.0005	0.0015	0.0010
1–56	0.073	0.0614	0.0010	0.0025	0.0015	0.0005	0.0015	0.0010
64	0.073	0.0629	0.0010	0.0025	0.0015	0.0005	0.0015	0.0010
72	0.073	0.0640	0.0010	0.0025	0.0015	0.0005	0.0015	0.0010
2–56	0.086	0.0744	0.0010	0.0025	0.0015	0.0005	0.0015	0.0010
64	0.086	0.0759	0.0010	0.0025	0.0015	0.0005	0.0015	0.0010
3–48	0.099	0.0855	0.0010	0.0025	0.0015	0.0005	0.0015	0.0010
56	0.099	0.0874	0.0010	0.0025	0.0015	0.0005	0.0015	0.0010
4–32	0.112	0.0917	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
36	0.112	0.0940	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
40	0.112	0.0958	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
48	0.112	0.0985	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
5–36	0.125	0.1070	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
40	0.125	0.1088	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
44	0.125	0.1102	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
6–32	0.138	0.1177	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
36	0.138	0.1200	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
40	0.138	0.1218	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
7–30	0.151	0.1294	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
32	0.151	0.1307	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
36	0.151	0.1330	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
8–30	0.164	0.1423	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
32	0.164	0.1437	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
36	0.164	0.1460	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
40	0.164	0.1478	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
9–24	0.177	0.1499	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
30	0.177	0.1553	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
32	0.177	0.1567	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
10–24	0.190	0.1629	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
28	0.190	0.1668	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
30	0.190	0.1684	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
32	0.190	0.1697	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
12–24	0.216	0.1889	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
28	0.216	0.1928	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
32	0.216	0.1957	0.0010	0.0030	0.0020	0.0005	0.0020	0.0015
14–20	0.242	0.2095	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
24	0.242	0.2149	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
16–18	0.268	0.2319	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
20	0.268	0.2355	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020

A maximum lead error of plus or minus 0.003 inch in one inch of thread is permitted. The dimensions given in this table apply to taps and not to tapped holes.

Commercial Tolerances for Machine Screw Taps — 2.

(Adopted by The Tap and Die Institute)

Screw No. and Threads per Inch	Basic Size		Outside Diam. of Tap			Pitch Diam. of Tap		
	Outside Diam.	Pitch Diam.	Min. = Basic Plus	Max. = Basic Plus	Toler-ance	Min. = Basic Plus	Max. = Basic Plus	Toler-ance
16–22	0.268	0.2385	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
18–18	0.294	0.2579	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
20	0.294	0.2615	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
20–16	0.320	0.2794	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
18	0.320	0.2839	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
20	0.320	0.2875	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
22–16	0.346	0.3054	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
18	0.346	0.3099	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
24–16	0.372	0.3314	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
18	0.372	0.3359	0.0010	0.0035	0.0025	0.0005	0.0025	0.0020
26–14	0.398	0.3516	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
16	0.398	0.3574	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
28–14	0.424	0.3776	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
16	0.424	0.3834	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
30–14	0.450	0.4036	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025
16	0.450	0.4094	0.0010	0.0040	0.0030	0.0005	0.0030	0.0025

A maximum lead error of plus or minus 0.003 inch in one inch of thread is permitted.

Commercial Tolerances for S. A. E. Standard Hand Taps

(Adopted by The Tap and Die Institute)

Nominal Size and Threads per Inch	Basic Size		Outside Diam. of Tap			Pitch Diam. of Tap		
	Outside Diam.	Pitch Diam.	Min. = Basic Plus	Max. = Basic Plus	Toler-ance	Min. = Basic Plus	Max. = Basic Plus	Toler-ance
1/4–28	0.2500	0.2268	0.0010	0.0035	0.0025	0.0005	0.0020	0.0015
5/16–24	0.3125	0.2854	0.0010	0.0035	0.0025	0.0005	0.0020	0.0015
3/8–24	0.3750	0.3479	0.0010	0.0035	0.0025	0.0005	0.0020	0.0015
7/16–20	0.4375	0.4050	0.0010	0.0040	0.0030	0.0005	0.0025	0.0020
1/2–20	0.5000	0.4675	0.0010	0.0040	0.0030	0.0005	0.0025	0.0020
9/16–18	0.5625	0.5264	0.0010	0.0040	0.0030	0.0005	0.0025	0.0020
5/8–18	0.6250	0.5889	0.0010	0.0040	0.0030	0.0005	0.0025	0.0020
3/4–16	0.7500	0.7094	0.0010	0.0050	0.0040	0.0005	0.0030	0.0025
7/8–14	0.8750	0.8286	0.0010	0.0050	0.0040	0.0005	0.0030	0.0025
1 –14	1.0000	0.9536	0.0010	0.0050	0.0040	0.0005	0.0030	0.0025
1 1/8–12	1.1250	1.0709	0.0015	0.0060	0.0045	0.0005	0.0035	0.0030
1 1/4–12	1.2500	1.1959	0.0015	0.0060	0.0045	0.0005	0.0035	0.0030
1 1/2–12	1.5000	1.4459	0.0015	0.0060	0.0045	0.0005	0.0035	0.0030

A maximum lead error of plus or minus 0.003 inch in one inch of thread is permitted.
The dimensions given in these tables apply to taps and not to tapped holes.

Commercial Tolerances for S. A. E. Standard Nut and Tapper Taps
(Adopted by The Tap and Die Institute)

Nominal Size and Threads per Inch	Basic Size		Outside Diam. of Tap			Pitch Diam. of Tap		
	Outside Diam.	Pitch Diam.	Min. = Basic Plus	Max. = Basic Plus	Toler-ance	Min. = Basic Plus	Max. = Basic Plus	Toler-ance
¼–28	0.2500	0.2268	0.0020	0.0050	0.0030	0.0010	0.0030	0.0020
⅜–24	0.3750	0.3479	0.0020	0.0050	0.0030	0.0010	0.0030	0.0020
½–20	0.5000	0.4675	0.0020	0.0060	0.0040	0.0010	0.0035	0.0025
⅝–18	0.6250	0.5889	0.0025	0.0065	0.0040	0.0015	0.0040	0.0025
¾–16	0.7500	0.7094	0.0025	0.0070	0.0045	0.0015	0.0045	0.0030
⅞–14	0.8750	0.8286	0.0025	0.0070	0.0045	0.0015	0.0045	0.0030
1 –14	1.0000	0.9536	0.0025	0.0070	0.0045	0.0015	0.0045	0.0030
1⅛–12	1.1250	1.0709	0.0030	0.0080	0.0050	0.0020	0.0055	0.0035
1¼–12	1.2500	1.1959	0.0030	0.0080	0.0050	0.0020	0.0055	0.0035
1½–12	1.5000	1.4459	0.0030	0.0080	0.0050	0.0020	0.0055	0.0035

Acme and Square-threaded Taps. — These taps are usually made in sets, three taps in a set undoubtedly being the most common. For very fine pitches, two taps in a set will be found sufficient, while as many as five taps in a set are used for coarse pitches. Tables are given herewith for proportioning both Acme and square-threaded taps when made in sets. One leading tap maker in cutting the threads of square-threaded taps makes them according to the following rules: The width of the groove between two threads is made equal to one-half the pitch of the thread, less 0.004 inch. This makes the width of the thread itself equal to one-half of the pitch, plus 0.004 inch. The depth of the thread is made equal to 0.45 times the pitch, plus 0.0025 inch. This latter rule produces a thread which for all the ordinarily used pitches for square-threaded taps has a depth less than the generally accepted standard depth, this latter depth being equal to one-half the pitch. The object of this shallow thread is to insure that if the hole to be threaded by the tap is not bored out so as to provide clearance at the bottom of the thread, the tap will cut its own clearance. The hole should, however, always be drilled out large enough so that the cutting of the clearance is not required of the tap.

Another maker follows under ordinary conditions the dimensions given in the accompanying tables, making the diameter at the end of the chamfer of the first tap equal to the root diameter of the thread, plus 0.010 inch. The diameter at the end of the chamfer of the second and third taps is made equal to the diameter of the straight portion of the next previous tap, minus 0.005 inch.

For Acme thread taps, this manufacturer makes the actual root diameter on the first tap 0.010 inch, and on the second tap 0.005 inch less than the standard root diameter. The finishing tap is made with standard root diameter, and a standard thread tool is used for all three taps in a set.

The table, "Dimensions of Acme Thread Taps in Sets of Three Taps" may be used for the length dimensions for Acme taps. The dimensions in this table apply to single-threaded taps. For multiple-threaded taps or taps with very coarse pitch, relative to the diameter, the length of the chamfered part of the thread may be increased. Square-threaded taps are made to the same table as Acme taps, with the exception of the figures in column K, which for square-threaded taps should be equal to the nominal diameter of the tap, no oversize allowance being customary in these taps. The first tap in a set of Acme taps (not square-threaded taps) should be turned taper in bottom of the thread for a distance of about one-quarter of the length of the threaded part. The taper should be so selected that the root diameter is about 1/32 inch smaller at the point than the proper root diameter of the tap. The first tap should preferably be provided with a short pilot at the point. For very coarse pitches, the first tap may be provided with spiral flutes at right angles to the angle of the thread. Acme and square-threaded taps should be relieved or backed off on the top of the thread of the chamfered portion on all of the taps in the set. When the taps are used as machine taps, rather than as hand taps, they should be relieved in the angle of the thread, as well as on the top, for the whole length of the chamfered portion. Acme taps should also always be relieved on the front side of the thread to within 1/32 inch of the cutting edge.

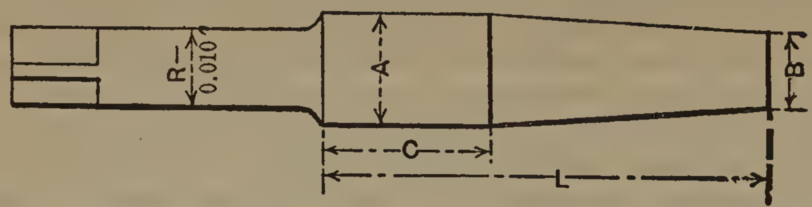
Table for Making Acme Thread Taps in Sets of Three Taps

No. of Threads per Inch	Amount in Inches to be Added to Root Diameter of Tap to Obtain Diameter of Straight Part of Thread of		No. of Threads per Inch	Amount in Inches to be Added to Root Diameter of Tap to Obtain Diameter of Straight Part of Thread of	
	1st Tap	2d Tap		1st Tap	2d Tap
1	0.468	0.832	5	0.108	0.192
1½	0.318	0.566	5½	0.100	0.178
2	0.243	0.432	6	0.093	0.166
2½	0.198	0.352	7	0.082	0.146
3	0.168	0.298	8	0.074	0.132
3½	0.147	0.261	9	0.068	0.121
4	0.130	0.232	10	0.063	0.112
4½	0.118	0.210	12	0.055	0.098

Table for Making Square-threaded Taps in Sets of Three Taps

No. of Threads per Inch	Amount in Inches to be Added to Root Diameter of Tap to Obtain Diameter of Straight Part of Thread of		No. of Threads per Inch	Amount in Inches to be Added to Root Diameter of Tap to Obtain Diameter of Straight Part of Thread of	
	1st Tap	2d Tap		1st Tap	2d Tap
1	0.410	0.800	5	0.082	0.160
1½	0.273	0.533	5½	0.075	0.146
2	0.205	0.400	6	0.068	0.133
2½	0.164	0.320	7	0.059	0.114
3	0.137	0.267	8	0.051	0.100
3½	0.117	0.229	9	0.046	0.089
4	0.102	0.200	10	0.041	0.080
4½	0.091	0.178	12	0.034	0.067

Proportions of Acme and Square-threaded Taps Made in Sets

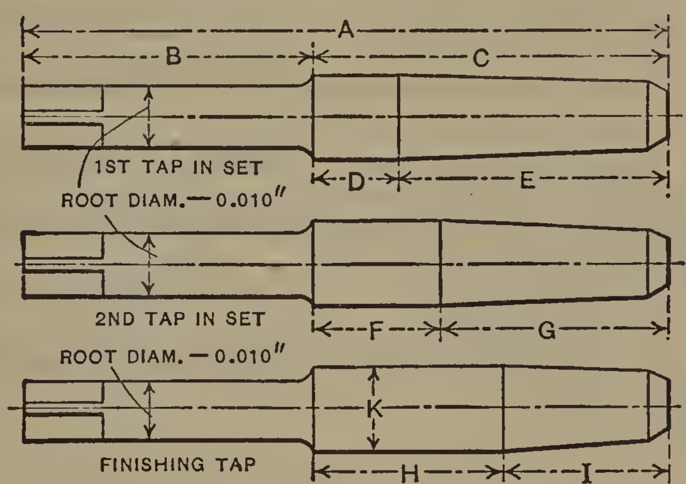


R = root diameter of thread. D = full diameter of tap.
T = double depth of full thread.

Kind of Tap	No. of Taps in Set	Order of Tap in Set	A	B	C
Acme Thread Taps	2	1st 2d	$R+0.65T$ D	$R+0.010$ A on 1st tap − 0.005	⅛ L to ⅙ L ¼ L to ⅓ L
	3	1st 2d 3d	$R+0.45T$ $R+0.80T$ D	$R+0.010$ A on 1st tap − 0.005 A on 2d tap − 0.005	⅛ L to ⅙ L ⅙ L to ¼ L ¼ L to ⅓ L
	4	1st 2d 3d 4th	$R+0.40T$ $R+0.70T$ $R+0.90T$ D	$R+0.010$ A on 1st tap − 0.005 A on 2d tap − 0.005 A on 3d tap − 0.005	⅛ L ⅙ L ⅕ L ¼ L to ⅓ L
	5	1st 2d 3d 4th 5th	$R+0.37T$ $R+0.63T$ $R+0.82T$ $R+0.94T$ D	$R+0.010$ A on 1st tap − 0.005 A on 2d tap − 0.005 A on 3d tap − 0.005 A on 4th tap − 0.005	⅛ L ⅙ L ⅕ L ⅕ L to ¼ L ¼ L to ⅓ L
Square-threaded Taps	2	1st 2d	$R+0.67T$ D	R A on 1st tap − 0.005	⅛ L to ⅙ L ¼ L to ⅓ L
	3	1st 2d 3d	$R+0.41T$ $R+0.80T$ D	R A on 1st tap − 0.005 A on 2d tap − 0.005	⅛ L to ⅙ L ⅙ L to ¼ L ¼ L to ⅓ L
	4	1st 2d 3d 4th	$R+0.32T$ $R+0.62T$ $R+0.90T$ D	R A on 1st tap − 0.005 A on 2d tap − 0.005 A on 3d tap − 0.005	⅛ L ⅙ L ⅕ L ¼ L to ⅓ L
	5	1st 2d 3d 4th 5th	$R+0.26T$ $R+0.50T$ $R+0.72T$ $R+0.92T$ D	R A on 1st tap − 0.005 A on 2d tap − 0.005 A on 3d tap − 0.005 A on 4th tap − 0.005	⅛ L ⅙ L ⅕ L ⅕ L to ¼ L ¼ L to ⅓ L

Steel for Taps. — The best steel to use for tapping cast iron and brass is one containing from two to three per cent tungsten, but otherwise having the same composition as an ordinary high-carbon steel, that is, with from about 1.15 to 1.25 per cent carbon. This steel, if uniform in its composition, will contract or shorten about 0.002 inch per inch in hardening, the same as most carbon steels. When hardening, it should be heated to about 1525 degrees F. The best steel for taps to be used on steel, as far as strength is concerned, is vanadium alloy steel containing from 0.25 up to 1 per cent of vanadium. The carbon content is the same as in regular carbon steels used for this purpose — from 1.15 to 1.25 per cent. The objection to this steel for taps is, however, that it is uncertain as regards its change in hardening. It is likely either to shorten or lengthen up to 0.002 inch per inch; but it is easily hardened and will stand a variation in the hardening heat of about 100 degrees F. without injury to the tap. Expensive special steels are obtainable in the market that show practically no change in either the lead or the diameter of the tap when hardened, but these are not used commercially on account of their high cost. Good grades of English and Swedish steels are very uniform and are

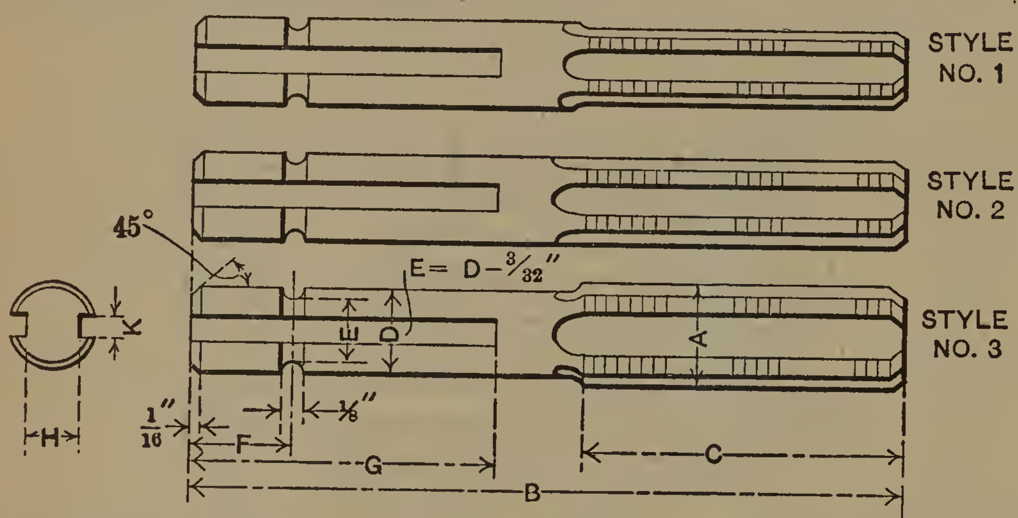
Dimensions of Acme Thread Taps in Sets of Three Taps



Nominal Diam.	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*K*
½	4¼	1⅞	2⅜	½	1⅞	⅝	1¾	⅞	1½	0.520
9/16	4⅞	2⅛	2¾	9/16	2 3/16	¾	2	1	1¾	0.582
⅝	5½	2⅜	3⅛	⅝	2½	⅞	2¼	1⅛	2	0.645
11/16	6	2½	3½	11/16	2 13/16	15/16	2 9/16	1¼	2¼	0.707
¾	6½	2 11/16	3 13/16	11/16	3⅛	1	2 13/16	1⅜	2 7/16	0.770
13/16	6⅞	2 13/16	4 1/16	¾	3 5/16	1 1/16	3	1 7/16	2⅝	0.832
⅞	7¼	3	4¼	¾	3½	1⅛	3⅛	1½	2¾	0.895
15/16	7 9/16	3⅛	4 7/16	13/16	3⅝	1 3/16	3¼	1 9/16	2⅞	0.957
1	7⅞	3¼	4⅝	13/16	3 13/16	1¼	3⅜	1⅝	3	1.020
1⅛	8½	3 9/16	4 15/16	⅞	4 1/16	1 5/16	3⅝	1¾	3 3/16	1.145
1¼	9	3¾	5¼	15/16	4 5/16	1⅜	3⅞	1⅞	3⅜	1.270
1⅜	9½	4	5½	1	4½	1 7/16	4 1/16	2	3½	1.395
1½	10	4¼	5¾	1	4¾	1½	4¼	2⅛	3⅝	1.520
1⅝	10½	4½	6	1	5	1½	4½	2⅛	3⅞	1.645
1¾	11	4¾	6¼	1 1/16	5 3/16	1 9/16	4 11/16	2¼	4	1.770
1⅞	11⅜	4⅞	6½	1 1/16	5 7/16	1 9/16	4 15/16	2¼	4¼	1.895
2	11¾	5	6¾	1⅛	5⅝	1⅝	5⅛	2⅜	4⅜	2.020
2¼	12½	5¼	7¼	1⅛	6⅛	1¾	5½	2½	4¾	2.270
2½	13¼	5½	7¾	1 3/16	6 9/16	1⅞	5⅞	2⅝	5⅛	2.520
2¾	14	5¾	8¼	1¼	7	2	6¼	2¾	5½	2.770
3	15	6¼	8¾	1¼	7½	2	6¾	3	5¾	3.020

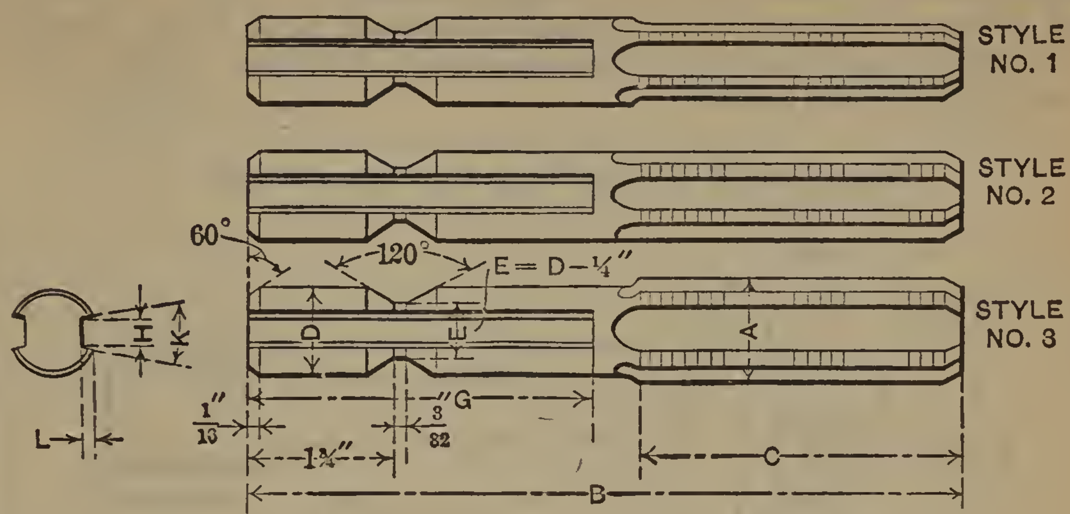
suitable for tap manufacture. They nearly always lengthen instead of shorten in hardening about 0.002 inch per inch. Some newer types of American steel show the same tendency. A high-speed steel suitable for taps should contain from 0.60 to 0.75 per cent carbon and from 15 to 20 per cent tungsten. This steel hardens at from 2100 to 2200 degrees F. The temper should be drawn at from 500 to (in some cases) 1000 degrees F. High-speed steel taps are especially good for automatic screw machine work, particularly when tapping brass or bronze. Under these conditions the production per tap may be increased from five to fifteen times as compared with that for taps made of ordinary carbon steel. One manufacturer, using small taps with rolled threads, finds that these will tap five times as many holes before losing their size as taps cut in the ordinary way.

Hand Taps for Nos. 1 and 2 Beaman & Smith Holders



No. of Holder	Diameter of Tap A	Total Length B	Length of Thread C	Diameter of Shank D	Center of Groove from End F	Length of Slot G	H Max.	H Min.	K Max.	K Min.	Style of Tap, Number
No. 1 Holder	1/4	2 3/4	1	3/8	5/8	1 1/16	0.24	0.22	0.12	0.11	1
	5/16	3	1 1/8	3/8	5/8	1 1/16	0.24	0.22	0.12	0.11	1
	3/8	3 1/4	1 1/4	3/8	5/8	1 1/16	0.24	0.22	0.12	0.11	2
	7/16	3 1/2	1 3/8	1/2	13/16	1 5/16	0.36	0.34	0.13	0.12	1
	1/2	3 3/4	1 1/2	1/2	13/16	1 5/16	0.36	0.34	0.13	0.12	2
	9/16	4	1 5/8	1/2	13/16	1 5/16	0.36	0.34	0.13	0.12	3
	5/8	4	1 5/8	1/2	13/16	1 5/16	0.36	0.34	0.13	0.12	3
No. 2 Holder	11/16	4 1/8	1 7/8	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	1
	3/4	4 7/16	2	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	2
	13/16	4 3/4	2 1/8	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	3
	7/8	5 1/16	2 1/4	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	3
	15/16	5 3/8	2 3/8	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	3
	1	5 11/16	2 1/2	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	3
	1 1/16	5 11/16	2 1/2	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	3
	1 1/8	6	2 5/8	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	3
	1 3/16	6	2 5/8	3/4	13/16	1 3/4	0.54	0.52	0.21	0.20	3

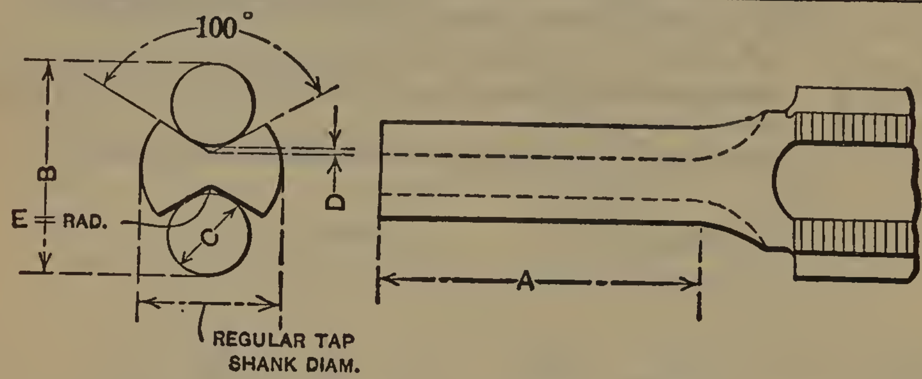
Hand Taps for Nos. 2½ and 3 Beaman & Smith Holders



No. of Holder	Diameter of Tap A	Total Length B	Length of Thread C	Diameter of Shank D	Length of Slot G	Width in Bottom of Slot H	Angle of Sides of Slot, Degrees K	Depth of Slot L	Style of Tap, No.
No. 2½ Holder	1	6¾	2½	1½	2¼	¼	20	3/16	1
	1 1/16	6¾	2½	1½	2¼	¼	20	3/16	1
	1⅛	6¾	2½	1½	2¼	¼	20	3/16	1
	1 3/16	6¾	2½	1½	2¼	¼	20	3/16	1
	1¼	7	2⅝	1½	2¼	¼	20	3/16	1
	1 5/16	7	2⅝	1½	2¼	¼	20	3/16	1
	1⅜	7¼	2¾	1½	2¼	¼	20	3/16	1
	1½	7½	2⅞	1½	2¼	¼	20	3/16	2
	1⅝	7¾	3	1½	2¼	¼	20	3/16	3
	1¾	8	3⅛	1½	2¼	¼	20	3/16	3
	1⅞	8¼	3¼	1½	2¼	¼	20	3/16	3
	2	8½	3⅜	1½	2¼	¼	20	3/16	3
No. 3 Holder	2⅛	9	3½	2	3	5/16	16	7/32	3
	2¼	9¼	3⅝	2	3	5/16	16	7/32	3
	2⅜	9½	3¾	2	3	5/16	16	7/32	3
	2½	9¾	3⅞	2	3	5/16	16	7/32	3

Special Tap Shanks. — Hand taps are frequently made with special shanks for holders or chucks such as the Beaman & Smith or the Graham. Dimensions for these shanks are given in the accompanying tables. Besides hand taps, regular Briggs' standard pipe taps are frequently made with shanks to fit the Beaman & Smith holders When this is the case, taps of ⅛ and ¼ inch nominal size generally fit the No. 1 holder; taps from ⅜ to ¾ inch nominal size are made to fit the No. 2 holder; taps from 1 to 1½ inch nominal size are made to fit the No. 2½ holder; and taps from 2 to 3 inches nominal size are made to fit the No. 3 holder. The tap shank should be made long enough to allow some clearance between the end of the slot cut in the shank and the threaded portion of the tap. The slot, the length of which is given as *G* in the tables of Beaman & Smith holders, should be flat or parallel in the bottom for a length equal to that dimension. Graham shanks are also frequently used for drills.

Dimensions of Tap Shanks to Fit Graham Chucks

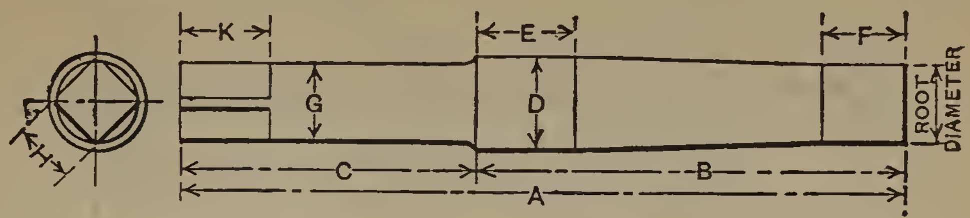


No. of Shank	Sizes of Hand Taps Used for, Inches	Sizes of Pipe Taps Used for, Nominal	*A*	*B*	*C*	*D*	*E*
1	¼ to 15/16	⅛ to ½	1⅝	0.710	¼	0.0206	1/64
2	1 to 2½	¾ to 3	2½	1.713	⅝	0.0402	1/32

Hob and Die Taps. — Hob taps are intended only for the final finishing or sizing of the thread in dies. Straight hob taps are made with the same dimensions as regular hand taps. (See table for hand taps.) They are not relieved either on the top or in the angle of the thread of the straight portion, but two or at most three threads are chamfered at the point of the tap, and these chamfered threads are relieved on the top the same as are hand taps. A taper hob should be relieved slightly both on the top and in the angle of the thread for its whole length. The number of flutes is greater than in hand taps and should be as follows: ¼–7/16 inch, six flutes; ½–⅞ inch, eight flutes; 15/16–1½ inch, ten flutes; 1⅝–2½ inches, twelve flutes; 2⅝–3 inches, fourteen flutes; 3¼ and larger, sixteen flutes. The flutes are cut with a single angle cutter with rounded point, the angle of the cutter being 50 degrees inclusive angle. Sellers hobs are a special kind of hob taps differing from the ordinary hob tap in that they are provided with a guide at the point of the thread. This guide or pilot is always hardened and ground.

Die taps, also known as long taper die taps, are used for cutting the thread in a die in a single operation from the blank, and are intended to be followed by a sizing hob tap. Die taps are similar to machine taps and made in almost exactly the same way. They are relieved both on the top and in the angle of the thread on the chamfered portion and are threaded with a taper in the bottom of the thread for a short distance from the point of the tap. On the end of the tap a straight pilot should be provided, although this is not always done in manufactured die taps. Die taps for pitches finer than 28 threads per inch are not relieved in the angle of the thread.

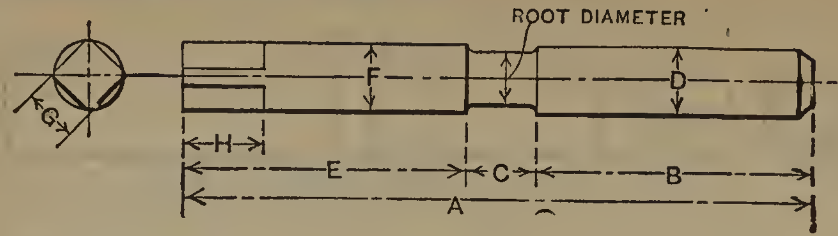
Dimensions of Taper Die Taps, U. S. Standard Thread



Diam. of Tap	Total Length	Length of Thread	Length of Shank	Length of Straight Thread	Length of Pilot	Diam. of Shank*	Size of Square	Length of Square	No. of Flutes
D	*A*	*B*	*C*	*E*	*F*	*G*	*H*	*K*	
1/4	5 3/16	2 13/16	2 3/8	1/4	3/8	0.170	0.127	9/16	5
5/16	5 1/2	3 1/16	2 7/16	5/16	7/16	0.224	0.168	5/8	5
3/8	5 7/8	3 5/16	2 9/16	3/8	1/2	0.275	0.206	11/16	5
7/16	6 1/4	3 5/8	2 5/8	7/16	9/16	0.323	0.242	11/16	5
1/2	6 5/8	3 7/8	2 3/4	1/2	5/8	0.367	0.275	3/4	5
9/16	6 15/16	4 1/8	2 13/16	9/16	5/8	0.429	0.322	13/16	5
5/8	7 5/16	4 3/8	2 15/16	5/8	11/16	0.480	0.360	13/16	5
11/16	7 11/16	4 11/16	3	11/16	11/16	0.542	0.407	7/8	6
3/4	8 1/16	4 15/16	3 1/8	3/4	3/4	0.590	0.442	7/8	6
13/16	8 3/8	5 3/16	3 3/16	13/16	3/4	0.652	0.489	15/16	6
7/8	8 3/4	5 7/16	3 5/16	7/8	13/16	0.697	0.523	1	6
15/16	9 1/8	5 3/4	3 3/8	15/16	13/16	0.760	0.570	1	6
1	9 1/2	6	3 1/2	1	7/8	0.800	0.600	1 1/16	6
1 1/8	10 3/16	6 1/2	3 11/16	1 1/8	15/16	0.896	0.672	1 1/8	6
1 1/4	10 15/16	7 1/16	3 7/8	1 1/4	1	1.021	0.766	1 3/16	7
1 3/8	11 5/8	7 9/16	4 1/16	1 3/8	1 1/16	1.108	0.831	1 5/16	7
1 1/2	12 3/8	8 1/8	4 1/4	1 1/2	1 1/8	1.233	0.925	1 3/8	7
1 5/8	13 1/16	8 5/8	4 7/16	1 5/8	1 1/8	1.305	0.979	1 7/16	7
1 3/4	13 13/16	9 3/16	4 5/8	1 3/4	1 3/16	1.430	1.072	1 1/2	8
1 7/8	14 1/2	9 11/16	4 13/16	1 7/8	1 1/4	1.519	1.140	1 5/8	8
2	15 1/4	10 1/4	5	2	1 5/16	1.644	1.233	1 11/16	8
2 1/8	15 15/16	10 3/4	5 3/16	2 1/8	1 5/16	1.769	1.327	1 3/4	8
2 1/4	16 11/16	11 5/16	5 3/8	2 1/4	1 3/8	1.894	1.421	1 13/16	9
2 3/8	17 3/8	11 13/16	5 9/16	2 3/8	1 7/16	2.019	1.515	1 15/16	9
2 1/2	18 1/8	12 3/8	5 3/4	2 1/2	1 1/2	2.100	1.575	2	9
2 5/8	18 9/16	12 5/8	5 15/16	2 5/8	1 1/2	2.225	1.669	2	9
2 3/4	19	12 7/8	6 1/8	2 3/4	1 9/16	2.350	1.762	2	10
2 7/8	19 7/16	13 1/8	6 5/16	2 7/8	1 5/8	2.475	1.856	2 1/16	10
3	19 7/8	13 3/8	6 1/2	3	1 5/8	2.543	1.907	2 1/16	10
3 1/4	20 3/4	13 7/8	6 7/8	3 1/4	1 11/16	2.793	2.095	2 1/16	10
3 1/2	21 5/8	14 3/8	7 1/4	3 1/2	1 3/4	3.008	2.256	2 1/8	10
3 3/4	22 1/2	14 7/8	7 5/8	3 3/4	1 13/16	3.217	2.412	2 1/8	10
4	23 3/8	15 3/8	8	4	1 7/8	3.467	2.600	2 3/16	10

* For sharp V-thread taps, make diameter of shank equal to root diameter of thread less 0.010 inch.

General Dimensions of Machine Screw Taps, A.S.M.E. St'd.

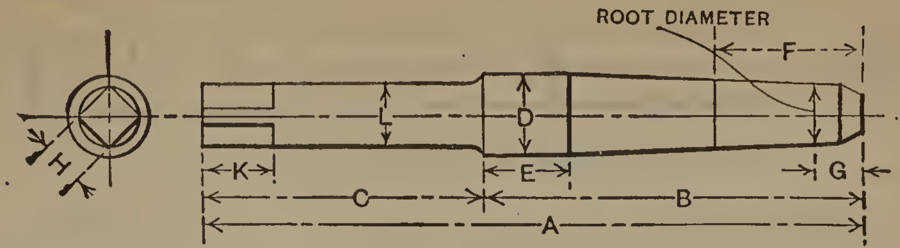


Number of flutes: Nos. 0 to 6, 3 flutes; No. 7 and larger sizes, 4 flutes.

No. of Tap	Diam. of Tap	St'd No. of Threads per In.	Total Length	Length of Thread	Length of Neck	Length of Shank	Diam. of Shank	Size of Square	Length of Square
	D		*A*	*B*	*C*	*E*	*F*	*G*	*H*
0	0.060	80	1 1/2	1/2	.....	1	0.125	0.093	3/16
1	0.073	72	1 5/8	9/16	.....	1 1/16	0.125	0.093	3/16
2	0.086	64	1 3/4	5/8	.....	1 1/8	0.125	0.093	3/16
3	0.099	56	1 13/16	11/16	.....	1 1/8	0.125	0.093	3/16
4	0.112	48	1 7/8	11/16	.....	1 3/16	0.125	0.093	3/16
5	0.125	44	1 15/16	3/4	.....	1 3/16	0.125	0.093	7/32
6	0.138	40	2	13/16	.....	1 3/16	0.138	0.104	7/32
7	0.151	36	2 1/16	13/16	.....	1 1/4	0.151	0.113	7/32
8	0.164	36	2 1/8	7/8	1/8	1 1/8	0.164	0.123	7/32
9	0.177	32	2 3/16	15/16	1/8	1 1/8	0.177	0.132	1/4
10	0.190	30	2 1/4	15/16	5/32	1 5/32	0.190	0.142	1/4
12	0.216	28	2 7/16	1 1/16	5/32	1 7/32	0.216	0.163	9/32
14	0.242	24	2 9/16	1 1/8	3/16	1 1/4	0.242	0.179	9/32
16	0.268	22	2 11/16	1 3/16	7/32	1 9/32	0.268	0.203	5/16
18	0.294	20	2 13/16	1 1/4	7/32	1 11/32	0.294	0.218	5/16
20	0.320	20	2 15/16	1 3/8	7/32	1 11/32	0.320	0.241	11/32
22	0.346	18	3 1/16	1 7/16	1/4	1 3/8	0.346	0.254	11/32
24	0.372	16	3 3/16	1 1/2	9/32	1 13/32	0.372	0.277	3/8
26	0.398	16	3 5/16	1 9/16	5/16	1 7/16	0.398	0.298	3/8
28	0.424	14	3 7/16	1 11/16	5/16	1 7/16	0.424	0.322	13/32
30	0.450	14	3 9/16	1 3/4	5/16	1 1/2	0.450	0.334	7/16

Tapper and Machine Nut Taps. — Tapper taps and machine nut taps are used for tapping nuts in tapping machines. The tapper tap is the simpler in design of the two and is not as well adapted for tapping nuts in materials of tough structure as is the machine nut tap. Tapper taps are relieved only on the top of the thread on the tapered portion. The main difference between tapper taps and machine nut taps is in the threading and relieving. While the tapper tap is threaded straight for the whole length of the threaded portion, the machine tap is threaded with a taper in the bottom of the thread for a distance *F* (see illustration with table, "Dimensions of Machine Nut Taps"). Machine nut taps are relieved on the top of the thread as well as in the angle of the thread for the whole of the chamfered part. At the point, there is a secondary chamfer, as shown. The diameter at the point of this second chamfer should be equal to the root diameter less the depth of the thread, or, in other words, equal to the full diameter of the tap minus three times the depth of the thread.

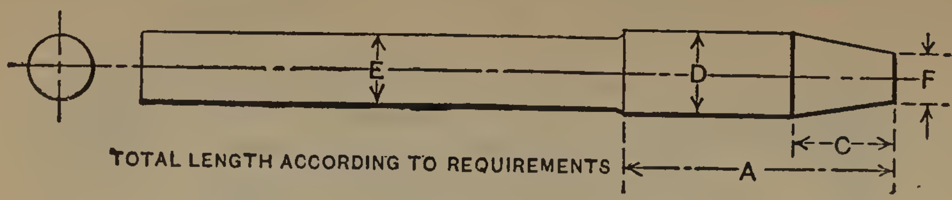
Dimensions of Machine Nut Taps, U. S. Standard Thread



Diam. of Tap	Total Length	Length of Thread	Length of Shank	Length of Full Thread	Length of Taper in Angle	Length below Root Diam.	Size of Square	Length of Square	Diam. of Shank*	Number of Flutes
D	*A*	*B*	*C*	*E*	*F*	*G*	*H*	*K*	*L*	
1/4	5	1 5/8	3 3/8	3/8	7/16	3/16	0.127	5/8	0.170	4
5/16	5 1/2	1 3/4	3 3/4	7/16	1/2	7/32	0.168	5/8	0.224	4
3/8	6	2 1/16	3 15/16	1/2	9/16	1/4	0.206	11/16	0.275	4
7/16	6 1/2	2 3/8	4 1/8	1/2	9/16	1/4	0.242	3/4	0.323	5
1/2	7	2 3/4	4 1/4	9/16	5/8	5/16	0.275	3/4	0.367	5
9/16	7 1/2	2 3/4	4 3/4	5/8	11/16	5/16	0.322	13/16	0.429	5
5/8	8	3	5	11/16	3/4	5/16	0.360	7/8	0.480	5
11/16	8 1/2	3	5 1/2	11/16	3/4	3/8	0.407	7/8	0.542	5
3/4	9	3 1/4	5 3/4	3/4	13/16	3/8	0.442	15/16	0.590	5
13/16	9 1/2	3 1/4	6 1/4	13/16	7/8	3/8	0.489	1	0.652	5
7/8	10	3 5/8	6 3/8	7/8	15/16	7/16	0.523	1	0.697	5
15/16	10 1/2	3 5/8	6 7/8	7/8	15/16	7/16	0.570	1 1/16	0.760	5
1	11	4 1/16	6 15/16	15/16	1	7/16	0.600	1 1/8	0.800	5
1 1/8	11 1/2	4 11/16	6 13/16	1 1/16	1 1/8	1/2	0.672	1 3/16	0.896	5
1 1/4	12	4 11/16	7 5/16	1 1/8	1 3/16	9/16	0.766	1 1/4	1.021	5
1 3/8	12 1/2	5 3/8	7 1/8	1 1/4	1 5/16	9/16	0.831	1 3/8	1.108	5
1 1/2	13	5 3/8	7 5/8	1 5/16	1 3/8	5/8	0.925	1 7/16	1.233	5
1 5/8	13 1/2	5 1/2	8	1 7/16	1 1/2	11/16	0.979	1 1/2	1.305	5
1 3/4	14	5 1/2	8 1/2	1 1/2	1 9/16	11/16	1.072	1 5/8	1.430	5
1 7/8	14 1/2	6 1/8	8 3/8	1 5/8	1 11/16	3/4	1.140	1 11/16	1.519	5
2	15	6 1/8	8 7/8	1 11/16	1 3/4	3/4	1.233	1 3/4	1.644	5
2 1/8	15 1/2	6 1/8	9 3/8	1 3/4	1 13/16	3/4	1.327	1 3/4	1.769	6
2 1/4	16	6 1/8	9 7/8	1 13/16	1 7/8	13/16	1.421	1 3/4	1.894	6
2 3/8	16 1/2	6 1/8	10 3/8	1 13/16	1 15/16	13/16	1.515	1 13/16	2.019	6
2 1/2	17	6 7/8	10 1/8	1 7/8	2	7/8	1.575	1 13/16	2.100	6
2 5/8	17 1/2	6 7/8	10 5/8	1 15/16	2 1/16	7/8	1.669	1 13/16	2.225	6
2 3/4	18	6 7/8	11 1/8	2	2 1/8	7/8	1.762	1 13/16	2.350	6
2 7/8	18 1/2	7 3/8	11 1/8	2	2 3/16	15/16	1.856	1 7/8	2.475	6
3	19	8 1/4	10 3/4	2 1/16	2 1/4	15/16	1.907	1 7/8	2.543	6
3 1/4	19 1/2	8 1/4	11 1/4	2 3/16	2 3/8	1	2.095	1 7/8	2.793	7
3 1/2	20	8 3/8	11 5/8	2 1/4	2 1/2	1 1/16	2.256	1 15/16	3.008	7
3 3/4	20 1/2	9	11 1/2	2 3/8	2 5/8	1 1/16	2.412	1 15/16	3.217	7
4	21	9	12	2 7/16	2 3/4	1 1/8	2.600	2	3.467	7

* For sharp V-thread taps make diameter of shank equal to root diameter of thread less 0.015 inch.

Dimensions of Tapper Taps



Number of flutes: 3/16–1½ inch, 4 flutes; larger sizes, 5 flutes.

Diam. of Tap	Length of Thread	Length of Taper	Diam. of Shank, E		Diam. of Point, F		Diam. of Tap	Length of Thread	Length of Taper	Diam. of Shank, E		Diam. of Point, F	
D	A	C	U.S. St'd	V. St'd	U. S. St'd	V. St'd	D	A	C	U. S. St'd	V. St'd	U. S. St'd	V. St'd
3/16	1½	9/16	0.14	0.11	0.140	0.110	7/8	3	1¼	0.72	0.67	0.722	0.674
¼	1¾	11/16	0.17	0.15	0.179	0.157	15/16	3¼	1⅜	0.78	0.73	0.783	0.735
5/16	2	¾	0.23	0.21	0.234	0.210	1	3¼	1⅜	0.82	0.77	0.828	0.774
⅜	2	¾	0.28	0.25	0.287	0.260	1⅛	3½	1½	0.92	0.86	0.928	0.867
7/16	2¼	⅞	0.33	0.30	0.338	0.306	1¼	3½	1½	1.04	0.98	1.053	0.992
½	2¼	⅞	0.39	0.34	0.393	0.349	1⅜	4	1⅝	1.14	1.07	1.147	1.074
9/16	2½	1	0.44	0.40	0.446	0.410	1½	4	1⅝	1.26	1.19	1.272	1.199
⅝	2½	1	0.49	0.45	0.499	0.460	1⅝	4½	1¾	1.37	1.26	1.376	1.266
11/16	2¾	1⅛	0.56	0.52	0.561	0.522	1¾	4½	1¾	1.47	1.38	1.476	1.390
¾	2¾	1⅛	0.61	0.56	0.611	0.568	1⅞	5	2	1.59	1.46	1.601	1.476
13/16	3	1¼	0.67	0.62	0.673	0.630	2	5	2	1.69	1.59	1.696	1.600

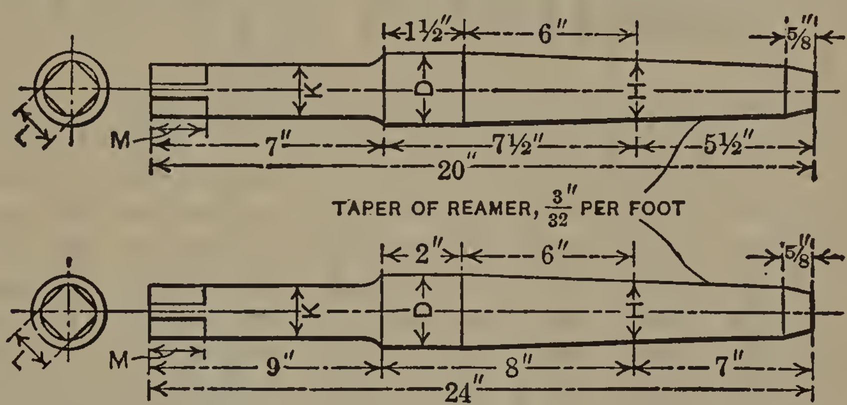
Commercial Tolerances for U. S. Standard Nut and Tapper Taps
(Adopted by The Tap and Die Institute)

Nominal Size and Threads per Inch	Basic Size		Outside Diam. of Tap			Pitch Diam. of Tap		
	Outside Diam.	Pitch Diam.	Min. = Basic Plus	Max. = Basic Plus	Toler-ance	Min. = Basic Plus	Max. = Basic Plus	Toler-ance
¼–20	0.2500	0.2175	0.0020	0.0050	0.0030	0.0015	0.0040	0.0025
⅜–16	0.3750	0.3344	0.0020	0.0050	0.0030	0.0015	0.0040	0.0025
½–13	0.5000	0.4500	0.0020	0.0060	0.0040	0.0015	0.0045	0.0030
¾–10	0.7500	0.6851	0.0025	0.0070	0.0045	0.0020	0.0055	0.0035
1 – 8	1.0000	0.9188	0.0025	0.0070	0.0045	0.0020	0.0055	0.0035
1¼– 7	1.2500	1.1572	0.0030	0.0080	0.0050	0.0025	0.0065	0.0040
1½– 6	1.5000	1.3918	0.0030	0.0080	0.0050	0.0025	0.0065	0.0040
1¾– 5	1.7500	1.6201	0.0030	0.0090	0.0060	0.0025	0.0070	0.0045
2 – 4½	2.0000	1.8557	0.0030	0.0090	0.0060	0.0025	0.0070	0.0045
2¼– 4½	2.2500	2.1057	0.0040	0.0110	0.0070	0.0030	0.0080	0.0050
2½– 4	2.5000	2.3376	0.0040	0.0110	0.0070	0.0030	0.0080	0.0050
2¾– 4	2.7500	2.5876	0.0040	0.0120	0.0080	0.0030	0.0085	0.0055
3 – 3½	3.0000	2.8144	0.0040	0.0120	0.0080	0.0030	0.0085	0.0055

A maximum lead error of plus or minus 0.003 inch in one inch of thread is permitted.

Staybolt Taps. — These are used extensively in locomotive boiler work. The ordinary or radial staybolt tap is shown in the table "Dimensions of Regular Staybolt Taps." The spindle staybolt tap, which has derived its name from the guiding spindle upon which the tap proper revolves, is shown in connection with the table of this class of taps. The point of the regular staybolt tap, from the end to dimension *H*, is a taper reamer for reaming the hole previous to tapping. The remainder of the tap is threaded, but part of the threaded portion is chamfered or tapered on the top of the thread, leaving only a short part of the full diameter *D* of the thread. The taper of the reamer is 3/32 inch per foot. The diameter at *H*

Dimensions of Regular Staybolt Taps



Thread: 12 sharp V-threads per inch; also commonly made with Whitworth thread. Number of flutes in all sizes: 5.

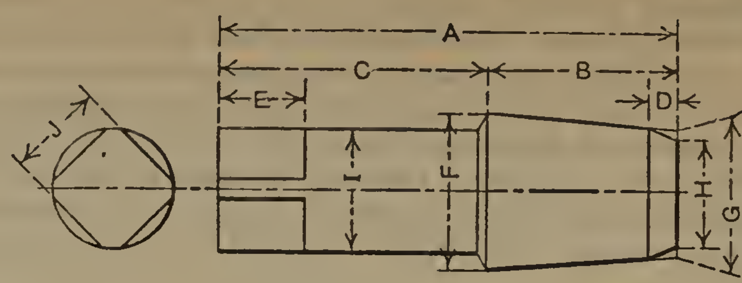
Diam. of Tap *D*	Root Diameter *H*	Diam. of Shank *K*	Size of Square *L*	Length of Square *M*	Diam. of Tap *D*	Root Diameter *H*	Diam. of Shank *K*	Size of Square *L*	Length of Square *M*
3/4	0.606	0.601	0.450	3/4	1 1/8	0.981	0.976	0.732	1
13/16	0.668	0.663	0.497	3/4	1 3/16	1.043	1.038	0.779	1
7/8	0.731	0.726	0.544	3/4	1 1/4	1.106	1.101	0.826	1 1/4
15/16	0.793	0.788	0.591	3/4	1 5/16	1.168	1.163	0.873	1 1/4
1	0.856	0.851	0.638	1	1 3/8	1.231	1.226	0.920	1 1/4
1 1/16	0.918	0.913	0.685	1	....	.....	.....	.....	....

equals the root diameter of the thread. Staybolts have in the past usually been made with 12 sharp V-threads per inch, and the accompanying table gives the root diameter *H* based on this practice. Of late, however, the Whitworth thread has been introduced for staybolts by the large locomotive builders. Staybolt taps, after hardening, should be between the nominal size and 0.002 inch above the nominal size in diameter for taps smaller than 1 inch, and not over 0.003 inch oversize for larger taps. The standard lengths in which the staybolt taps are made are 20 and 24 inches. However, staybolt taps must often be made to suit special conditions.

The spindle staybolt tap is fluted about halfway of the threaded portion. The remaining part of the thread acts as a guide and should be made about 0.010 inch smaller in pitch diameter than the cutting portion.

Dimensions of Briggs Standard Pipe Taps *

Taper, ¾ inch per foot.
Thread: Briggs standard pipe thread.



Nominal Size	No. of Threads per Inch	Total Length	Length of Thread	Length of Shank	Length of Chamfer	Length of Square	Diam. at Large End	Diam. at Small End before Chamfering	Diam. at Chamfered Point	Diam. of Shank	Size of Square	No. of Flutes
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	
⅛	27	2⅛	¾	1⅜	3/32	⅜	0.420	0.373	0.309	5/16	0.234	4
¼	18	2 7/16	1 1/16	1⅜	3/16	7/16	0.559	0.493	0.397	7/16	0.328	4
⅜	18	2 9/16	1 1/16	1½	3/16	½	0.694	0.628	0.532	9/16	0.421	4
½	14	3⅛	1⅜	1¾	7/32	⅝	0.865	0.779	0.656	11/16	0.515	4
¾	14	3¼	1⅜	1⅞	7/32	11/16	1.075	0.989	0.866	29/32	0.679	5
1	11½	3¾	1¾	2	¼	13/16	1.350	1.240	1.091	1⅛	0.843	5
1¼	11½	4	1¾	2¼	¼	15/16	1.693	1.584	1.434	1 5/16	0.984	6
1½	11½	4¼	1¾	2½	¼	1	1.932	1.822	1.672	1½	1.125	6
2	11½	4½	1¾	2¾	¼	1⅛	2.405	2.297	2.147	1⅞	1.406	7
2½	8	5½	2 9/16	2 15/16	⅜	1¼	2.921	2.761	2.545	2¼	1.687	8
3	8	6	2⅝	3⅜	⅜	1⅜	3.547	3.383	3.167	2⅝	1.968	9
3½	8	6½	2 11/16	3 13/16	⅜	1½	4.047	3.879	3.663	2 13/16	2.109	10
4	8	6¾	2¾	4	⅜	1⅝	4.547	4.375	4.159	3	2.250	11
4½	8	8	3⅞	4⅛	⅜	1 11/16	5.125	4.883	4.667	3 3/16	2.390	12
5	8	8¼	4	4¼	⅜	1¾	5.687	5.437	5.221	3⅜	2.530	14
6	8	8¾	4½	4¼	⅜	1⅞	6.766	6.485	6.269	3¾	2.812	14
7	8	9¼	4¾	4½	⅜	2	7.773	7.476	7.260	4⅛	3.094	16
8	8	9¼	4¾	4½	⅜	2⅛	8.773	8.476	8.260	4½	3.375	18
9	8	9¾	5	4¾	⅜	2¼	9.781	9.469	9.253	4⅞	3.650	18
10	8	9¾	5	4¾	⅜	2⅜	10.906	10.594	10.378	5¼	3.940	20

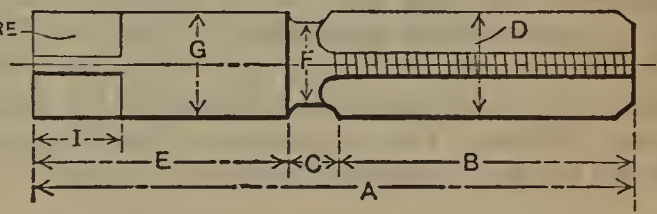
* Dimensions up to and including 4-inch nominal size are according to a standard adopted by the tap manufacturers in 1919; the shanks of taps made by some manufacturers are larger than the diameters listed, for nominal tap sizes up to ¾ inch inclusive. The dimensions for the sizes above 4 inches are based on average practice.

Briggs Standard Pipe Taps and Hobs. — All taper taps must be relieved both on the top and in the angle of the thread. The size of pipe taps is usually measured by a ring gage. The limits of accuracy are ordinarily determined by the amount that the end of the tap projects through or comes short of the face of the ring gage at the small end of the hole, the ring gage being made so that the end of the correct tap comes flush with the face of the gage. The tolerances adopted by The Tap and Die Institute are as follows: For sizes from ⅛ to 1 inch the end of the tap must not project through or come short of the ring-gage face more than 1/16 inch, for pipe sizes from 1 to 3 inches, 3/32 inch, and for sizes 3½ inches and larger, ⅛ inch.

English or Whitworth taper pipe taps are made to the same general length dimensions as are Briggs standard pipe taps, but the threaded portion should be made to conform to the standard for pipe threads adopted by the Engineering Standards Committee of Great Britain. The form of thread is the Whitworth and the number of threads per inch is according to the standard mentioned. A table of this standard is given in the section on "Screw Thread Systems."

Dimensions of Straight Pipe Taps

Thread: Briggs standard pipe thread.

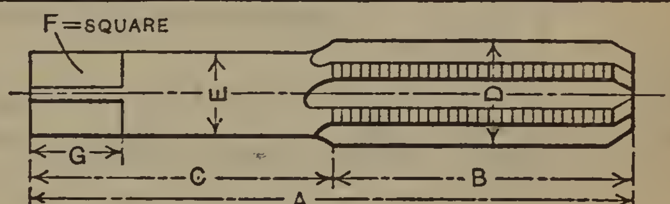


Nominal Size	Diam. of Tap* D	Total Length A	Length of Thread B	Length of Neck C	Length of Shank E	Diam. of Neck F	Diam. of Shank G	Size of Square H	Length of Square I	No. of Flutes
1/8	0.404	2 5/8	1	5/16	1 5/16	0.335	3/8	0.281	7/16	4
1/4	0.534	2 7/8	1 1/8	3/8	1 3/8	0.440	1/2	0.375	1/2	4
3/8	0.671	3 1/16	1 1/4	3/8	1 7/16	0.575	5/8	0.469	9/16	4
1/2	0.836	3 1/4	1 3/8	7/16	1 7/16	0.705	3/4	0.562	5/8	4
3/4	1.046	3 5/8	1 5/8	1/2	1 1/2	0.915	1	0.750	3/4	5

* Outside diameter at gaging notch of standard taper plug gage.

Dimensions of Straight Pipe Taps

Thread: Briggs standard pipe thread.



Nominal Size	Diam. of Tap* D	Total Length A	Length of Thread B	Length of Shank C	Diam. of Shank E	Size of Square F	Length of Square G	No. of Flutes
1	1.308	4	1 3/4	2 1/4	1 1/8	0.844	13/16	5
1 1/4	1.653	4 7/16	1 15/16	2 1/2	1 1/4	0.937	15/16	6
1 1/2	1.892	4 7/8	2 1/8	2 3/4	1 3/8	1.031	1	6
2	2.366	5 11/16	2 7/16	3 1/4	1 5/8	1.219	1 3/16	7
2 1/2	2.862	6 1/2	2 3/4	3 3/4	1 7/8	1.407	1 5/16	8
3	3.488	7 3/8	3 1/8	4 1/4	2 1/8	1.593	1 1/2	9
3 1/2	3.989	8 3/16	3 7/16	4 3/4	2 3/8	1.781	1 11/16	10
4	4.487	9	3 3/4	5 1/4	2 5/8	1.968	1 13/16	10

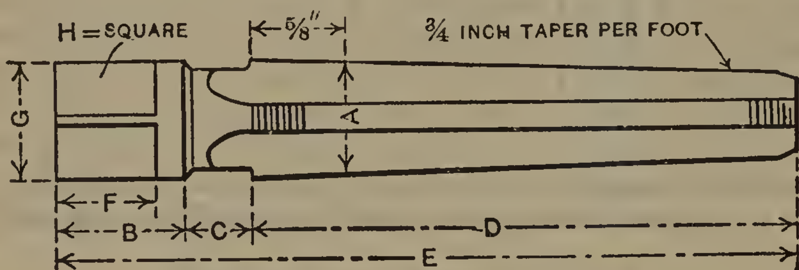
* Outside diameter at gaging notch of standard taper plug gage.

Pipe hobs are used for sizing pipe dies after the thread has been cut nearly to size either in a lathe or by a pipe tap. The dimensions given in the accompanying table are those generally used by tap manufacturers. The flutes are cut with a 50-degree single-angle cutter. Pipe hobs are relieved both in the angle and on the top of the thread.

Straight Pipe Taps. — These are only a variation of hand taps, having the same number of threads per inch as the corresponding sizes of taper pipe taps, and a diameter that is the same as the diameter at the gaging notch of a standard taper plug gage for Briggs pipe threads. English or Whitworth straight pipe taps have the Whitworth form of threads, and the diameter and number of threads per inch are according to the standard adopted by the Engineering Standards Committee of Great Britain. The dimensions for length, shank, square, etc., are made the same as for the Briggs standard straight pipe taps.

Pipe Taps and Drills Combined. — The threaded portion of these tools is an exact duplicate of the threaded part of a pipe tap. The drill part has two flutes like a twist drill and the point is ground to the same angle — 59 degrees with the center line.

Dimensions of Blacksmiths' Taper Taps

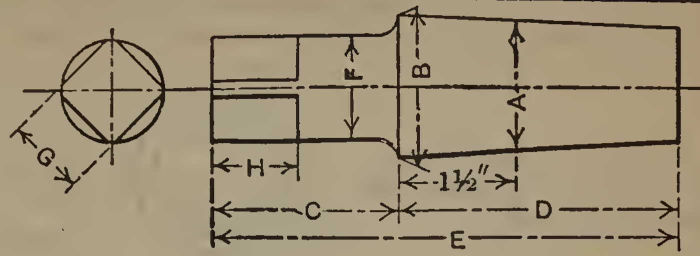


Taper, 3/4 inch per foot. Number of flutes in all sizes: 4.
Thread: Standard V-thread.

Nominal Diam. of Tap	No. of Threads per Inch	Length of Shank	Length of Neck	Length of Thread	Total Length	Length of Square	Diam. of Shank	Size of Square
A	V-thread	*B*	*C*	*D*	*E*	*F*	*G*	*H*
1/4	20	1/2	3/8	1 1/2	2 3/8	3/8	5/16	0.234
5/16	18	5/8	7/16	1 11/16	2 3/4	3/8	3/8	0.281
3/8	16	13/16	7/16	1 7/8	3 1/8	7/16	7/16	0.328
7/16	14	1	7/16	2 1/16	3 1/2	7/16	1/2	0.375
1/2	12	1 1/8	1/2	2 1/4	3 7/8	1/2	9/16	0.422
9/16	12	1 5/16	1/2	2 7/16	4 1/4	1/2	5/8	0.469
5/8	11	1 1/2	1/2	2 5/8	4 5/8	9/16	11/16	0.516
3/4	10	1 9/16	9/16	3	5 1/8	5/8	13/16	0.609
7/8	9	1 11/16	9/16	3 3/8	5 5/8	11/16	15/16	0.703
1	8	1 3/4	5/8	3 3/4	6 1/8	3/4	1 1/16	0.797
1 1/8	7	1 13/16	11/16	4 1/8	6 5/8	13/16	1 3/16	0.891
1 1/4	7	1 7/8	11/16	4 9/16	7 1/8	7/8	1 5/16	0.984
1 3/8	6	1 7/8	3/4	4 3/4	7 3/8	15/16	1 7/16	1.078
1 1/2	6	1 7/8	3/4	5	7 5/8	1	1 9/16	1.172

Dimensions of Pipe Hobs

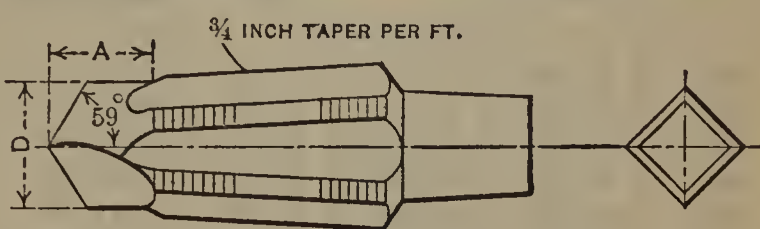
Taper, 3/4 inch per foot.
Thread: Briggs standard pipe thread.



Nominal Size	No. of Threads per Inch	Actual Size	Diam. at Large End	Length of Shank	Length of Thread	Length Overall	Diam. of Shank	Size of Square	Length of Square	No. of Flutes
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	
1/8	27	0.445	0.539	2	3 3/16	5 3/16	3/8	0.281	1/2	5
1/4	18	0.573	0.667	2	3 3/8	5 3/8	1/2	0.375	5/8	6
3/8	18	0.719	0.813	2 1/16	3 1/2	5 9/16	5/8	0.469	3/4	6
1/2	14	0.885	0.979	2 1/16	3 5/8	5 11/16	13/16	0.609	7/8	8
3/4	14	1.104	1.198	2 1/8	3 7/8	6	1 1/16	0.797	1	10
1	11 1/2	1.363	1.457	2 1/8	4	6 1/8	1 5/16	0.984	1 1/8	12
1 1/4	11 1/2	1.721	1.815	2 3/16	4 1/8	6 5/16	1 5/8	1.219	1 1/4	16
1 1/2	11 1/2	1.955	2.049	2 3/16	4 1/4	6 7/16	1 7/8	1.406	1 3/8	18
2	11 1/2	2.460	2.554	2 1/4	4 1/2	6 3/4	2 3/8	1.781	1 1/2	22
2 1/2	8	2.963	3.057	2 5/16	4 11/16	7	2 7/8	2.156	1 5/8	26
3	8	3.620	3.714	2 3/8	4 7/8	7 1/4	3 9/16	2.671	1 3/4	32
3 1/2	8	4.062	4.156	2 7/16	5 1/16	7 1/2	3 11/16	2.768	1 3/4	36
4	8	4.485	4.579	2 1/2	5 1/4	7 3/4	3 3/4	2.812	1 7/8	40
4 1/2	8	5.000	5.094	2 9/16	5 3/8	7 15/16	3 13/16	2.860	1 7/8	44
5	8	5.565	5.659	2 5/8	5 1/2	8 1/8	3 7/8	2.906	2	48
6	8	6.620	6.714	2 3/4	5 3/4	8 1/2	4	3.000	2	58

Length and Diameters of Drill Points in Combined Pipe Taps and Drills

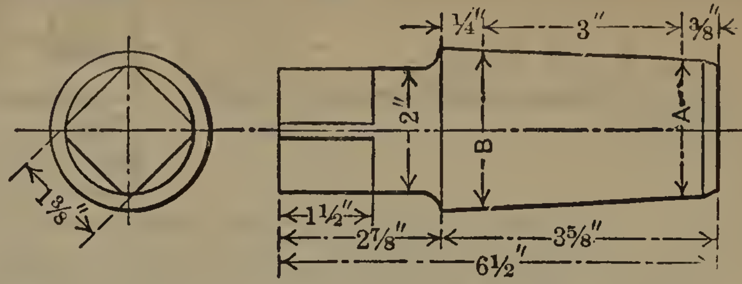
Tap part is the same as regular pipe taps.



Pipe Tap Size	Length of Drill Point	Diam. of Drill	Pipe Tap Size	Length of Drill Point	Diam. of Drill	Pipe Tap Size	Length of Drill Point	Diam. of Drill
	A	*D*		*A*	*D*		*A*	*D*
1/8	7/8	11/32	1/2	1 1/4	45/64	1 1/4	1 5/8	1 1/2
1/4	1	7/16	3/4	1 3/8	15/16	1 1/2	1 3/4	1 23/32
3/8	1 1/8	37/64	1	1 1/2	1 5/32	2	1 7/8	2 3/16

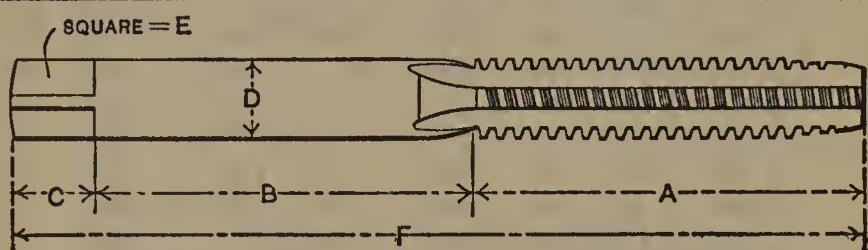
Dimensions of Mud or Wash-out Taps
(Also known as " Arch Pipe Taps ")

Taper, 1 1/4 inch per foot.
Thread: 12 sharp V-threads per inch.



No. of Tap	Diam. at Small End	Diam. at Large End	No. of Flutes	No. of Tap	Diam. at Small End	Diam. at Large End	No. of Flutes	No. of Tap	Diam. at Small End	Diam. at Large End	No. of Flutes
	A	*B*			*A*	*B*			*A*	*B*	
0	1 7/16	1 3/4	5	2	2 1/16	2 3/8	7	4	2 11/16	3	8
1	1 3/4	2 1/16	6	3	2 3/8	2 11/16	8	5	3	3 5/16	8

Dimensions of Stove-bolt Taps



Nominal Size	Actual Outside Diameter	Pitch Diameter	Root Diameter	Flat in Bottom of Thread Corresponding to	Flat on Top of Thread, Inches	Number of Threads per Inch
5/32	0.166	0.1500	0.134	20 U. S.	0.0110	28
3/16	0.197	0.1745	0.152	20 U. S.	0.0094	24
7/32	0.223	0.1965	0.170	18 U. S.	0.0079	22
1/4	0.257	0.2285	0.200	11 U. S.	0.0112	18
5/16	0.317	0.2855	0.254	10 U. S.	0.0067	18
3/8	0.354	0.3165	0.279	10 U. S.	0.0067	16

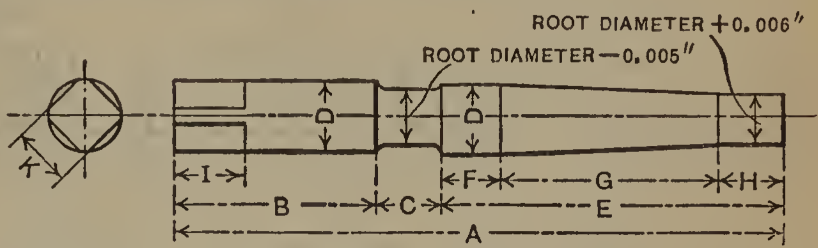
Nominal Size	*A*	*B*	*C*	*D*	*E*	*F*
5/32	7/8	1 5/32	7/32	0.166	1/8	2 1/4
3/16	15/16	1 3/16	1/4	0.197	9/64	2 3/8
7/32	15/16	1 1/4	1/4	0.223	11/64	2 7/16
1/4	1	1 7/32	9/32	0.257	3/16	2 1/2
5/16	1 1/8	1 5/16	5/16	0.317	15/64	2 3/4
3/8	1 1/4	1 5/16	3/8	0.354	9/32	2 15/16

Stove-bolt Taps. — The thread form is not the regular U. S. standard, but the thread is provided with a wider flat at the top and bottom than that corresponding to the number of threads per inch in the taps. The angle of the thread, however, is 60 degrees, and the flat at the bottom corresponds to that of the standard U. S. thread for pitches greater than the actual number of threads per inch in the tap, so that the taps can be cut with standard thread tools.

Straight Boiler Taps. — These are only a special class of hand taps. They have a long chamfer or taper on the top of the thread and a straight guide at the end or point. The chamfered portion is relieved on the top of the thread.

Dimensions of Straight Boiler Taps

Thread: 12 sharp V-threads per inch.



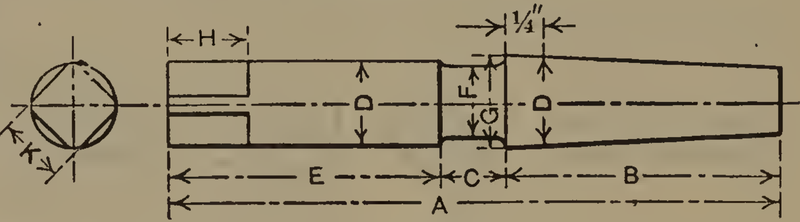
Diam. of Tap	Total Length	Length of Shank	Length of Neck	Length of Thread	Length of Full Thread	Length of Cham-fer	Length of Pilot	Length of Square	Size of Square	No. of Flutes
D	*A*	*B*	*C*	*E*	*F*	*G*	*H*	*I*	*K*	
1/2	4 1/4	1 3/4	1/2	2	1/2	1 1/8	3/8	1/2	0.375	4
9/16	4 5/8	1 13/16	1/2	2 5/16	9/16	1 3/8	3/8	9/16	0.422	4
5/8	5	1 15/16	1/2	2 9/16	5/8	1 1/2	7/16	5/8	0.469	4
11/16	5 1/4	1 15/16	9/16	2 3/4	11/16	1 5/8	7/16	11/16	0.516	4
3/4	5 1/2	2	9/16	2 15/16	3/4	1 11/16	1/2	3/4	0.562	4
13/16	5 3/4	2 1/16	9/16	3 1/8	13/16	1 13/16	1/2	13/16	0.609	4
7/8	6	2 3/16	9/16	3 1/4	7/8	1 7/8	1/2	7/8	0.656	4
15/16	6 1/4	2 3/16	5/8	3 7/16	15/16	2	1/2	15/16	0.703	4
1	6 1/2	2 3/16	5/8	3 11/16	1	2 1/8	9/16	1	0.750	4
1 1/16	6 3/4	2 5/16	5/8	3 13/16	1 1/16	2 3/16	9/16	1 1/16	0.797	4
1 1/8	6 7/8	2 5/16	5/8	3 15/16	1 1/8	2 3/16	5/8	1 1/8	0.844	4
1 3/16	7	2 5/16	11/16	4	1 3/16	2 3/16	5/8	1 3/16	0.891	4
1 1/4	7 1/8	2 5/16	11/16	4 1/8	1 1/4	2 3/16	11/16	1 1/4	0.937	4
1 5/16	7 1/4	2 5/16	11/16	4 1/4	1 5/16	2 1/4	11/16	1 5/16	0.984	4
1 3/8	7 3/8	2 5/16	11/16	4 3/8	1 3/8	2 5/16	11/16	1 3/8	1.031	4
1 7/16	7 1/2	2 5/16	11/16	4 1/2	1 7/16	2 5/16	3/4	1 7/16	1.078	4
1 1/2	7 5/8	2 3/8	3/4	4 1/2	1 7/16	2 5/16	3/4	1 1/2	1.125	4
1 5/8	7 3/4	2 3/8	3/4	4 5/8	1 9/16	2 5/16	3/4	1 1/2	1.219	4
1 3/4	7 7/8	2 3/8	13/16	4 11/16	1 5/8	2 5/16	3/4	1 1/2	1.312	6
1 7/8	8	2 3/8	13/16	4 13/16	1 3/4	2 5/16	3/4	1 1/2	1.406	6
2	8	2 3/8	13/16	4 13/16	1 3/4	2 5/16	3/4	1 1/2	1.500	6
2 1/8	8	2 3/8	13/16	4 13/16	1 3/4	2 5/16	3/4	1 5/8	1.594	6
2 1/4	8	2 3/8	13/16	4 13/16	1 3/4	2 5/16	3/4	1 5/8	1.687	6
2 3/8	8	2 3/8	13/16	4 13/16	1 3/4	2 5/16	3/4	1 5/8	1.781	6
2 1/2	8	2 3/8	13/16	4 13/16	1 3/4	2 5/16	3/4	1 5/8	1.875	6

Taper Boiler Taps and Patch-bolt Taps. — These are used in steam boiler work where a steam tight fit is required. The size line at which boiler taps are measured is located ¼ inch from the large end of the thread; the limits of error in the location of the size line may be made the same as for pipe taps. Patch-bolt taps are only a modified form of taper boiler taps, but are shorter. The size line is located ⅝ inch from the large end of the thread.

The taper of these taps is the same as that for regular Briggs standard pipe taps — three-quarters inch per foot. Both taper boiler taps and patch-bolt taps are provided with 12 sharp V-threads per inch for all sizes. The accompanying tables give dimensions for all commercial sizes.

Dimensions of Taper Boiler Taps

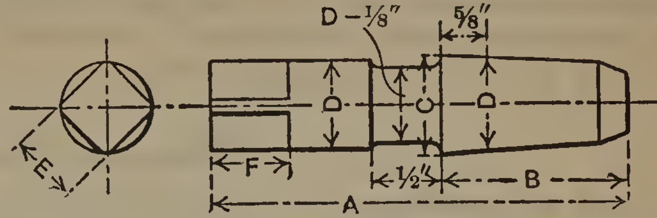
Taper, ¾ inch per foot.
Thread: 12 sharp V-threads per inch.



Diam. of Tap	Total Length	Length of Thread	Length of Neck	Length of Shank	Diam. of Neck	Diam. at Large End of Thread	Length of Square	Size of Square	No. of Flutes
D	*A*	*B*	*C*	*E*	*F*	*G*	*H*	*K*	
½	4¼	2¼	½	1½	11/32	0.516	½	0.375	4
9/16	4⅝	2⅜	½	1¾	13/32	0.578	9/16	0.422	4
⅝	5	2 7/16	½	2 1/16	15/32	0.641	⅝	0.469	4
11/16	5¼	2½	9/16	2 3/16	17/32	0.703	11/16	0.516	4
¾	5½	2 9/16	9/16	2⅜	19/32	0.766	¾	0.562	4
13/16	5¾	2⅝	9/16	2 9/16	21/32	0.828	13/16	0.609	4
⅞	6	2 11/16	9/16	2¾	23/32	0.891	⅞	0.656	4
15/16	6¼	2¾	⅝	2⅞	25/32	0.953	15/16	0.703	4
1	6½	2 13/16	⅝	3 1/16	27/32	1.016	1	0.750	4
1 1/16	6¾	2⅞	⅝	3¼	29/32	1.078	1 1/16	0.797	4
1⅛	6⅞	2 15/16	⅝	3 5/16	31/32	1.141	1⅛	0.844	4
1 3/16	7	2 15/16	11/16	3⅜	1 1/32	1.203	1 3/16	0.891	4
1¼	7⅛	3	11/16	3 7/16	1 3/32	1.266	1¼	0.937	4
1 5/16	7¼	3	11/16	3 9/16	1 5/32	1.328	1 5/16	0.984	4
1⅜	7⅜	3 1/16	11/16	3⅝	1 7/32	1.391	1⅜	1.031	4
1 7/16	7½	3 1/16	¾	3 11/16	1 9/32	1.453	1 7/16	1.078	4
1½	7⅝	3⅛	¾	3¾	1 11/32	1.516	1½	1.125	4
1⅝	7¾	3⅛	¾	3⅞	1 15/32	1.641	1½	1.219	5
1¾	7⅞	3 3/16	¾	3 15/16	1 19/32	1.766	1½	1.312	5
1⅞	8	3¼	13/16	3 15/16	1 23/32	1.891	1½	1.406	5
2	8	3¼	13/16	3 15/16	1 27/32	2.016	1⅝	1.500	5
2⅛	8	3¼	13/16	3 15/16	1 31/32	2.141	1⅝	1.594	6
2¼	8	3¼	13/16	3 15/16	2 3/32	2.266	1⅝	1.687	6
2⅜	8	3¼	13/16	3 15/16	2 7/32	2.391	1⅝	1.781	6
2½	8	3¼	13/16	3 15/16	2 11/32	2.516	1⅝	1.875	6

Dimensions of Patch-bolt Taps

Taper, ¾ inch per foot.
Number of flutes in all sizes: 4.
Thread: 12 sharp V-threads per inch.



Diam. of Tap	Total Length	Length of Thread	Diam. at Large End of Thread	Size of Square	Length of Square	Diam. of Tap	Total Length	Length of Thread	Diam. at Large End of Thread	Size of Square	Length of Square
D	*A*	*B*	*C*	*E*	*F*	*D*	*A*	*B*	*C*	*E*	*F*
1/2	3	1 1/4	0.539	0.375	1/2	1 1/16	3 9/16	1 3/8	1.102	0.797	3/4
9/16	3 1/16	1 5/16	0.602	0.422	1/2	1 1/8	3 5/8	1 3/8	1.164	0.844	13/16
5/8	3 1/8	1 5/16	0.664	0.469	9/16	1 3/16	3 11/16	1 7/16	1.227	0.891	13/16
11/16	3 3/16	1 5/16	0.727	0.516	9/16	1 1/4	3 3/4	1 7/16	1.289	0.937	7/8
3/4	3 1/4	1 5/16	0.789	0.562	5/8	1 5/16	3 13/16	1 7/16	1.352	0.984	7/8
13/16	3 5/16	1 5/16	0.852	0.609	5/8	1 3/8	3 7/8	1 7/16	1.414	1.031	15/16
7/8	3 3/8	1 3/8	0.914	0.656	11/16	1 7/16	3 15/16	1 7/16	1.477	1.078	15/16
15/16	3 7/16	1 3/8	0.977	0.703	11/16	1 1/2	4	1 7/16	1.539	1.125	1
1	3 1/2	1 3/8	1.039	0.750	3/4	...	...	...	....	....	....

Dimensions of Flat for Set-screws on Round Shanks

B	*A*	*C*	*B*	*A*	*C*	*B*	*A*	*C*
			9/16	0.211	0.021	1 3/4	0.656	0.064
			5/8	0.234	0.023	1 13/16	0.679	0.067
			11/16	0.258	0.025	1 7/8	0.703	0.069
			3/4	0.281	0.027	1 15/16	0.726	0.071
			13/16	0.304	0.030	2	0.750	0.073
			7/8	0.328	0.032	2 1/16	0.773	0.075
1/8	0.047	0.005	15/16	0.351	0.034	2 1/8	0.797	0.078
5/32	0.058	0.006	1	0.375	0.037	2 3/16	0.820	0.080
3/16	0.070	0.007	1 1/16	0.398	0.039	2 1/4	0.844	0.082
7/32	0.082	0.008	1 1/8	0.422	0.042	2 5/16	0.867	0.084
1/4	0.094	0.009	1 3/16	0.445	0.044	2 3/8	0.890	0.087
9/32	0.105	0.010	1 1/4	0.469	0.046	2 7/16	0.914	0.089
5/16	0.117	0.011	1 5/16	0.492	0.048	2 1/2	0.937	0.091
11/32	0.129	0.013	1 3/8	0.515	0.051	2 5/8	0.984	0.096
3/8	0.140	0.014	1 7/16	0.539	0.053	2 3/4	1.031	0.100
13/32	0.152	0.015	1 1/2	0.562	0.055	2 7/8	1.078	0.105
7/16	0.164	0.016	1 9/16	0.586	0.058	3	1.125	0.110
15/32	0.176	0.017	1 5/8	0.609	0.060	....	......	......
1/2	0.187	0.018	1 11/16	0.633	0.062	....	......	......

Width of Flat for Set-screws on Round Shanks. — The width of the flat for set-screws on round shanks of tools may not seem to be of great importance, but, nevertheless, it is advisable to have a standard for this. The accompanying table gives, therefore, the width of the flat A for a number of different diameters B figured from the formula $A = \frac{3}{8} B$. If the flat is milled to this width the appearance of the tool shanks will be satisfactory. The depth C will be required by the milling machine operator when milling the flats.

Threading Dies

Steel for Die Chasers. — Carbon steel chasers are generally preferred for dies which are intended for cutting smooth, accurate threads, although there is a difference of opinion regarding the relative merits of the use of carbon steel and high-speed steel for die chasers. High-speed steel is preferred when maximum production is the principal object, and carbon steel when finish is of especial importance. Semi-high-speed steel which has been used for many die chasers contains much less tungsten than high-speed steel turning tools. The National Acme Co. recommends the use of carbon steel chasers for cold-rolled screw stock, Bessemer, and open-hearth steel, 3- and 5-per cent nickel steel, malleable iron, brass, bronze, and similar alloys. For cutting threads in chrome-vanadium steel, tough alloy steels, cast iron, drop-forgings, and all heat-treated steels, either high-speed steel chasers or those made of semi-high-speed steel are recommended in preference to carbon steel.

Methods of Cutting Die-chaser Teeth. — The teeth of die chasers may be formed by three different methods. The first is by using a hob having helical teeth like the teeth of a tap. The second is by using a milling cutter which has annular rows of teeth that are perpendicular to the axis. This cutter is set to the helix angle of the screw thread when milling the chaser teeth, and it is fed across the end of the blank and forms a series of straight teeth. The third method is by using a milling cutter like the one just referred to. This cutter is set to the proper helix angle as in the previous case, and is then sunk into the end of the chaser blank, thus forming concave or circular teeth instead of straight teeth. Another form of milled chaser which has straight teeth is found in the Landis die-heads. In this case, the chasers are set tangentially to the work and the milled teeth, instead of being across the ends of the chasers, extend the full length of the chasers; the latter are sharpened by grinding on the ends.

Angle of Chamfer for Die Chasers. — The leading side or "throat" of a die is chamfered to provide a more gradual cutting action, unless it is necessary to cut a full thread close to a shoulder. The throat angle should preferably be such that the work of cutting a thread to the full depth will be distributed over at least two or three teeth on the leading side of the die. The chamfer, according to common practice, extends from the root or base of the most advanced tooth in a set of chasers back to the top of the third tooth, which may be slightly beveled. Each chaser should be ground to the same angle so that each throat will be the same distance from the die axis. The angles recommended for Hartness dies vary according to the pitch of the thread, the angle being 15 degrees relative to the axis of the die for threads varying from 4 to 5½ per inch, 20 degrees for threads varying from 6 to 8 per inch, and 25 degrees for nine or more threads per inch.

Relief of Die Chasers. — The throat or chamfered edge of each chaser should have clearance back of the cutting edge or in a circumferential direction. This clearance should be just enough to insure free cutting. If there is not enough relief, the cutting action will be either prevented entirely or retarded, and the die will not advance as fast as it should. On the contrary, excessive relief tends to increase the rate of advance, assuming that the die is self-leading. It is of especial importance that die chasers for brass have as little clearance as possible, because the throats of the chasers steady the die when starting a thread. The sides of the chaser teeth are sometimes relieved instead of the leading sides or corners being chamfered, when a thread must be cut close to a shoulder in brass. This relieving is done by using a brass lap which has an angle of about 50 degrees for a 60-degree chaser tooth.

Amount of Rake for Threading Dies. — The front face of each die chaser should lie in a plane intersecting the axis of a die that is used for cutting threads on parts of cast brass, cast iron, or brittle materials of a granular structure. For the more tenacious or tougher materials which are not brittle, such as wrought iron, steel, copper and yellow brass, the chasers should have positive rake, the cutting faces lying in planes that are in advance of the die axis. Most aluminum castings, on account of the zinc in their composition, cut very much like cast brass, and should preferably be threaded with dies having little or no rake. Many of the dies used for cutting threads in machine steel have the front faces of the chasers located ahead of the die axis a distance equal to about one-fifth of the die radius. When grinding the front faces of die chasers, care should be taken to maintain the rake angles. According to experiments of the National Tube Co., the rake angles of dies for pipe threading should vary from 15 to 25 degrees, the latter angle being suitable for threading open-hearth steel pipe.

Number of Chasers for Pipe Dies. — To obtain the best results with pipe dies, the number of chasers should vary according to the size of the die, four chasers being used for diameters up to 1¼ inch; six chasers for diameters of from 1½ to 4 inches; eight chasers for diameters of from 4½ to 8 inches; twelve chasers for diameters of from 9 to 12 inches; fourteen chasers for diameters of from 13 to 16 inches; and sixteen chasers for diameters of from 17 to 20 inches. This information is based on experiments of the National Tube Co.

Dies for Cutting Taper Threads. — While short taper threads are commonly cut with solid dies, the type of die to use for accurate work, particularly when the length of thread exceeds ordinary die widths, is one having chasers which taper to correspond to the taper on the work and are arranged to move outward radially as the die moves along. Such dies are of the self-opening type. The radial outward movement of the chasers is controlled by a taper plate which allows the cam or scroll ring of the die-head to turn slowly as the die advances. When the thread is finished, the chasers spring out rapidly to clear the work. In dies of this class, the taper plate for controlling the movement of the chasers serves about the same purpose as the adjustable slide or bar of a lathe taper attachment.

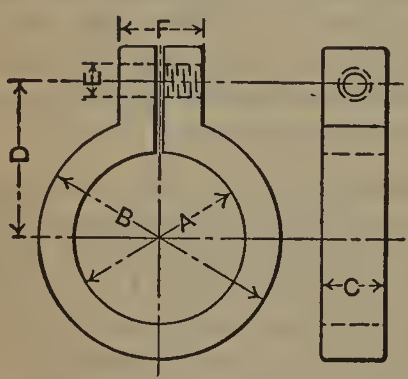
Dies for Cutting Square Threads. — If dies are to be used for cutting square threads, the sides of the teeth should be relieved to prevent the teeth from binding and breaking. If the die is to be self-leading (the feeding movement not being controlled by a lead-screw), this side relief or clearance should be very slight because, if there is too much relief on the sides of the chaser teeth, the die will not be supported properly and is liable to cut a thread that is quite incorrect in lead. The use of a lead-screw is preferable when cutting square threads with a die. The Acme thread may be cut readily with dies.

Positive Control of Die-feeding Movement. — While most dies are self-leading, it is sometimes advisable to control positively the longitudinal motion of the die relative to the work. This control may be utilized merely to start the die, or the arrangement may be such that the longitudinal motion of the die is controlled positively throughout the entire screw-cutting operation. This positive action may be derived from a lead-screw or from a cam, depending upon the type of machine. A lead-screw is sometimes applied to a threading machine of the bolt-cutter type, especially when cutting square threads, or special forms. For screw-cutting operations of this kind, if the die follows its own lead, the accumulated error is often considerable. It is essential that the pitch of the die teeth correspond to the leading movement obtained from the lead-screw. The general method of cutting threads on the automatic screw machine is to use a cam that starts a die on the work and then allows the turret-slide to lag behind somewhat so that the die can lead itself on. The die-holder is designed to allow the die to follow its own lead or move independently of the turret-slide.

Automatic or Self-opening Dies. — Dies which open automatically to permit their removal when the thread is finished, not only save time, but may prevent injury to the thread, which sometimes results when the chips wedge between the teeth of a non-opening die and the work. Self-opening dies differ both in regard to the mechanism for opening the die chasers automatically at the completion of a cut, and as to the method of resetting the chasers in the working or cutting position after the die has been removed. These dies, in general, are formed of two main sections, which have a certain relative motion for opening and closing the die. This motion may be parallel to the axis of the die, or it may be rotary. The radial movement of the chasers is derived from either cam surfaces or the conical surface of a slide in contact with the chasers. Dies of this class may be opened or tripped (1) by stopping the travel of the turret, (2) by the engagement of an outside tripping latch with a fixed stop, or (3) by the engagement of the end of the work with a tripping plate located in the center of the die back of the chasers. Most of the self-opening dies are of the non-revolving type, but some are designed for attachment to a revolving spindle.

Spring Screw Threading Dies. — These dies are usually tapped with a straight tap and hob, although this practice is somewhat objectionable on account of the fact that a slight inaccuracy is produced in the shape of the threads of the screw to be threaded when the prongs of the die are forced in by the adjustment of the clamp collar. It is, therefore, better to tap out this die from the back end with a tap that tapers an amount equal to the clearance required in the die when cutting.

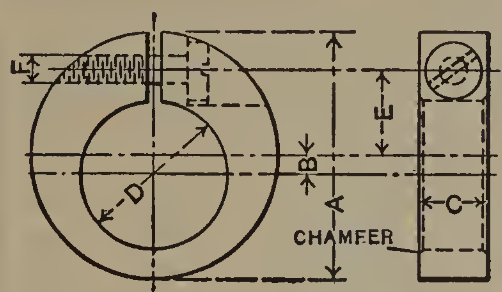
Clamp Collars for Spring Screw Threading Dies



A	*B*	*C*	*D*	*E*	*F*
½	¾	5/16	13/32	5/32	7/16
¾	1 1/16	3/8	9/16	3/16	½
1	1 3/8	7/16	23/32	¼	5/8
1 3/16	1 9/16	½	27/32	¼	11/16
1 ¼	1 11/16	½	29/32	5/16	11/16
1 3/8	1 13/16	9/16	31/32	5/16	¾
1 5/8	2 1/8	5/8	1 1/8	3/8	7/8
2	2 5/8	11/16	1 3/8	7/16	1
2 ½	3 ¼	13/16	1 5/8	½	1 3/16
3 ¼	4	15/16	2	½	1 ¼

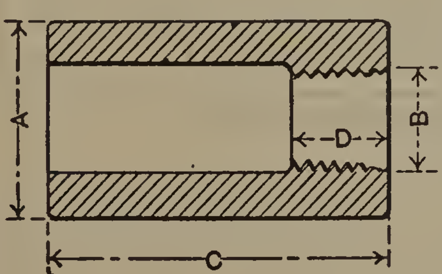
In that case, the die should be cut to the correct cutting size at the point, and not oversize, as is the case when it is cut with a straight tap. The amount of back taper may be made from about 0.005 to 0.010 inch per inch for iron and steel, and from 0.008 to 0.015 inch per inch for dies cutting brass, copper and metals of similar structure. Spring screw dies are generally made with four flutes, but for several reasons three flutes would be preferable. When four flutes are used, as a rule only two of the lands are cutting. If a die is made with three flutes, it should be fluted with a 60-degree angular cutter. If made with four flutes, the cutter should be a 48-, 45- or 40-degree angular cutter, according to the size of the die, the 48-degree cutter being used for the smallest dies and the 45-degree cutter for all ordinary sizes. Dies ½ inch in outside diameter or smaller are never made with more than three lands. If the die is not to cut close to a shoulder, about three threads should be chamfered off at the end. When dies are to cut close to a shoulder, not more than 1 or 1½ threads shou d be chamfered. The threads should be relieved on the chamfered part. It is common practice to make the length of the thread in a spring screw die about seven times the pitch. Tables are given herewith showing the general dimensions of spring screw threading dies as ordinarily manufactured, the length of the thread for various pitches, and the oversize required in taps for hobbing spring screw dies when these are cut with straight taps.

New Style Clamp Collars for Spring Screw Threading Dies



D	*A*	*B*	*C*	*E*	*F*
½	1	9/64	⅜	5/16	3/16
¾	1 5/16	9/64	7/16	13/32	7/32
1	1 11/16	11/64	½	9/16	¼
1 3/16	1⅞	5/32	9/16	21/32	¼
1¼	1⅞	⅛	9/16	23/32	¼
1⅜	2 3/16	5/32	⅝	¾	5/16
1⅝	2⅝	7/32	11/16	⅞	⅜
2	3⅛	9/32	13/16	1 1/16	⅜
2½	3¾	5/16	⅞	1 5/16	⅜
3¼	4½	5/16	15/16	1 11/16	⅜

Spring Screw Threading Dies



Outside Diam.	Diam. of Thread	Length of Die	Outside Diam.	Diam. of Thread	Length of Die
A	*B*	*C*	*A*	*B*	*C*
½	3/32–¼	1¼	1⅜	½– ¾	2½
¾	¼–⅜	1¾	1⅝	⅝–1	2½
1	5/16–½	2	2	¾–1¼	3
1 3/16	⅝–¾	2¼	2½	1 –1½	3½
1¼	⅜–¾	2½	3¼	1⅝–2⅛	4

Length of Thread for Different Pitches

No. of Threads per Inch	Length of Thread, *D*	No. of Threads per Inch	Length of Thread, *D*	No. of Threads per Inch	Length of Thread, *D*	No. of Threads per Inch	Length of Thread, *D*	No. of Threads per Inch	Length of Thread, *D*
40	3/16	24	5/16	14	½	10	¾	6	1 3/16
36	7/32	20	⅜	13	9/16	9	13/16	5½	1 5/16
32	¼	18	13/32	12	⅝	8	⅞	5	1 7/16
28	9/32	16	7/16	11	11/16	7	1	4½	1 9/16

Oversize of Taps for Hobbing Spring Screw Dies

No. of Threads per Inch	Oversize	No. of Threads per Inch	Oversize	No. of Threads per Inch	Oversize	No. of Threads per Inch	Oversize	No. of Threads per Inch	Oversize
4½	0.015	8	0.007	13	0.006	22	0.005	40	0.003
5	0.013	9	0.007	14	0.005	24	0.004	48	0.003
5½	0.012	10	0.006	16	0.005	28	0.004	56	0.003
6	0.010	11	0.006	18	0.005	32	0.004	64	0.002
7	0.008	12	0.006	20	0.005	36	0.004	72	0.002

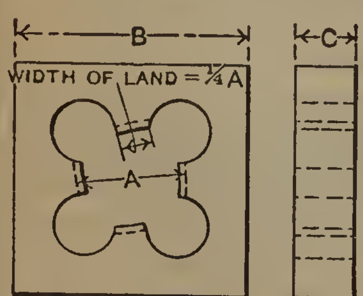
Solid Round Gas Fixture Dies

(For fixtures and thin brass tubing—60-degree V-thread with slight flat top and bottom)

Nominal Size	Diam. of Thread	No. of Threads per Inch	Outside Diam. of Die	Thickness of Die	Nominal Size	Diam. of Thread	No. of Threads per Inch	Outside Diam. of Die	Thickness of Die
0.148	0.148	32	⅝	¼	½	0.515	27	1 7/16	⅜
0.196	0.196	32	⅝	¼	9/16	0.578	27	1 7/16	⅜
No. 4	0.246	27	⅝	¼	⅝	0.637	27	1 7/16	⅜
¼	0.260	27	1	5/16	¾	0.770	27	2	½
5/16	0.342	27	1	5/16	⅞	0.885	27	2	½
⅜	0.390	27	1 7/16	⅜	1	1.006	27	2	½
7/16	0.459	27	1 7/16	⅜	....	....	.....	.....	.....
Straight Iron Pipe Dies*									
⅛	0.404	27	1 7/16	⅜	⅜	0.666	18	1 7/16	⅜
¼	0.531	18	1 7/16	⅜	½	0.821	14	2	½

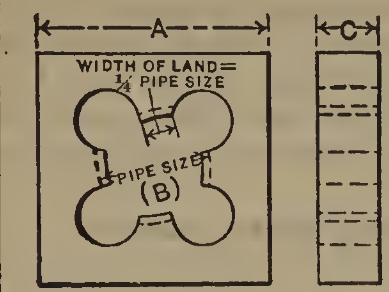
* Manufacturers of dies differ slightly as to this standard.

Solid Square Bolt Dies



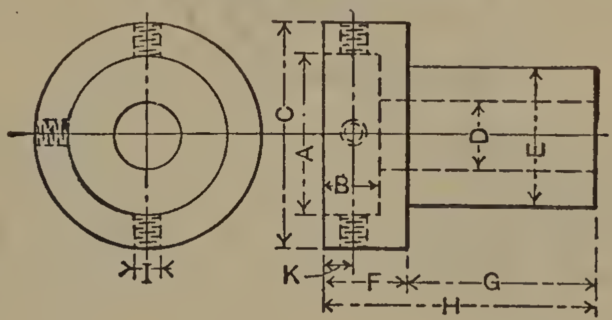
Diam. of Thread A	Size of Square B	Thickness C	Diam. of Thread A	Size of Square B	Thickness C
¼	2½	½	15/16	2½	¾
5/16	2½	½	1	2½	1
⅜	2½	½	1⅛	2½	1
7/16	2½	½	1¼	2½	1
½	2½	¾	1⅜	2½	1
9/16	2½	¾	1½	3	1
⅝	2½	¾	1⅝	3	1
11/16	2½	¾	1¾	3	1¼
¾	2½	¾	1⅞	3½	1½
13/16	2½	¾	2	3¾	2
⅞	2½	¾	.....	.....	.....

Solid Square Pipe Dies



Size of Square A	Pipe Size (Nominal) B	Thickness C	Size of Square A	Pipe Size (Nominal) B	Thickness C
2	⅛– ½	½	3⅞	1¼–2	⅞
2⅜	¼–1	¾	3⅞	1¼–2	1
2½	¼–1	¾	4	1¼–2	⅞
2⅞	¾–1¼	¾	4	1¼–2	1
3	¾–1¼	¾	5	2½–3	1¼

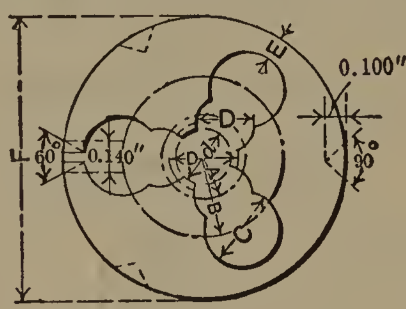
Lathe Die Holders



Diam. of Recess A	Depth of Recess B	Outside Diam. C	Diam. of Hole in Shank D	Diam. of Shank E	Length of Body F	Length of Shank G	Total Length H	Size of Screws I	Location of Screws K
0.632	¼	1	¼	⅝	⅜	¾	1⅛	5/32	0.135
0.821	¼	1¼	⅜	13/16	⅜	¾	1⅛	3/16	0.135
1.009	⅜	1½	9/16	1	9/16	1⅛	1 11/16	7/32	0.197
1.511	½	2 3/16	13/16	1⅜	¾	1½	2¼	¼	0.260
2.013	⅝	2⅞	1	1⅝	15/16	1⅞	2 13/16	5/16	0.322
2.515	11/16	3 9/16	1¼	2	1	2 1/16	3 1/16	⅜	0.354

Solid Dies. — A solid die, as a rule, is of square shape and is used principally for threading in bolt cutters and for pipe dies. In pipe dies the thread, of course, is tapered. A tapered die, in order to cut a thread smoothly and correctly, should be relieved in the angle of the thread, but as the difficulties of relieving an internal thread like that of a pipe die are very great, this is not ordinarily done and, therefore, pipe dies, as well as other taper dies, cannot be used for cutting the threads of taps, where the thread is required to be smooth and cut closely to the correct shape, but can be used only on pipes and similar soft metal where a perfect thread shape is not essential. Solid square dies are always provided with four lands, except when very large, when five lands are preferable. The width of the land should be about ¼ of the diameter of the thread to be cut, and the clearance holes are laid out so as to provide for this width of land. The center of the clearance holes should be located a trifle outside of the circle representing the outside of the screw to be cut. In very large dies it is not possible to make circular clearance holes, as these would then be of too large a diameter. In such cases, two clearance holes are drilled between each two of the lands and the metal between them removed. The chamfer on the top of the thread should extend to about three or four threads; the die should be relieved on the top of the chamfered threads. The outside size of the square of a solid die should, in general, not be less than twice the diameter of the thread to be cut.

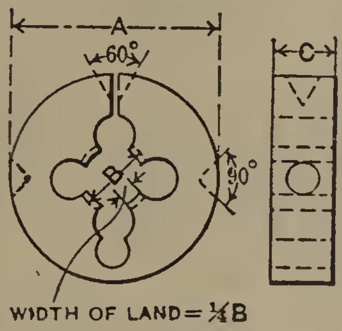
Adjustable Round Split Dies for A.S.M.E. Standard Machine Screws



No.	Diam. of Thread D	No. of Threads per Inch	Max. Root Diam. of Thread d	A	B	C	E	Diam. of Die Blank F	Thickness of Blank
0	0.060	80	0.0438	0.043	0.185	0.254	3/32	13/16	7/32
1	0.073	72	0.0550	0.053	0.192	0.241	3/32	13/16	7/32
2	0.086	64	0.0657	0.063	0.198	0.228	3/32	13/16	7/32
3	0.099	56	0.0758	0.072	0.205	0.216	3/32	13/16	7/32
4	0.112	48	0.0849	0.081	0.211	0.203	3/32	13/16	7/32
5	0.125	44	0.0955	0.091	0.217	0.190	3/32	13/16	7/32
6	0.138	40	0.1055	0.100	0.224	0.177	3/32	13/16	7/32
7	0.151	36	0.1149	0.110	0.230	0.165	3/32	13/16	7/32
8	0.164	36	0.1279	0.120	0.190	0.245	3/32	1	1/4
9	0.177	32	0.1364	0.129	0.196	0.233	3/32	1	1/4
10	0.190	30	0.1467	0.139	0.202	0.220	3/32	1	1/4
12	0.216	28	0.1696	0.158	0.215	0.194	3/32	1	1/4
14	0.242	24	0.1879	0.177	0.322	0.169	3/32	1	1/4
16	0.268	22	0.2090	0.196	0.335	0.143	3/32	1	1/4
18	0.294	20	0.2290	0.215	0.347	0.118	3/32	1	1/4
20	0.320	20	0.2550	0.235	.....	.....	....	1	1/4
22	0.346	18	0.2738	0.254	0.431	0.201	3/32	1 1/4	5/16
24	0.372	16	0.2908	0.272	0.448	0.166	3/32	1 1/4	5/16
26	0.398	16	0.3168	0.283	0.457	0.148	3/32	1 1/4	5/16
28	0.424	14	0.3312	0.290	0.464	0.135	3/32	1 1/4	5/16
30	0.450	14	0.3572	0.294	0.469	0.125	3/32	1 1/4	5/16

Round Split Adjustable Dies. — These dies have three lands for sizes up to and including 3/16 inch. For all larger sizes, four lands are used. When hardening these dies, draw to a blue back of the clearance holes in order to insure a good spring temper. About three threads should be chamfered and relieved on the top of the chamfer on the leading side of the die. When these dies are intended to be used in die stocks, they should be chamfered on both sides or ends, in order to permit the turning over of the die, and its cutting close to a shoulder. In such cases, the chamfer on the back side should be from 1 to 1½ threads.

Round Split Adjustable Dies



Outside Diam. A	Diameter of Thread B	Thickness C	Outside Diam. A	Diameter of Thread B	Thickness C
5/8	1/16–17/64	1/4	1½	1/4–5/8	1/2
13/16	1/16–5/16	1/4	2	3/8–7/8	5/8
1	3/16–1/2	3/8	2½	1/2–1¼	11/16

MILLING CUTTERS

Plain Milling Cutters. — Cutters with a width of face greater than four inches should preferably be made in two or more interlocking sections. Cutters larger than five inches in diameter should preferably be made with inserted teeth, although solid cutters are made as large as ten inches in diameter. Inserted blade cutters must have a smaller number of teeth than solid cutters. The pitch or spacing between the teeth should be about 1½ inch. This gives about twelve teeth for a 6-inch inserted blade cutter, and sixteen teeth for an 8-inch cutter. On larger

Number of Teeth in Plain Roughing Milling Cutters With Coarse Pitch

Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth
2	8	3	8	5	10	7	14	9	18
2¼	8	3½	9	5½	11	7½	14	9½	18
2½	8	4	9	6	12	8	16	10	20
2¾	8	4½	10	6½	12	8½	16	....	...

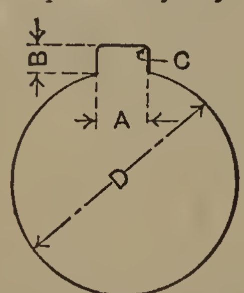
sizes, the spacing should be still coarser. For example, a 10-inch cutter may not have over 18 teeth. The thickness of an inserted steel blade should be 7/16 inch on cutters up to six inches in diameter, ½ inch for cutters up to 10 inches, and 9/16 inch for larger cutters. Two tables are given herewith showing numbers of teeth in solid plain milling cutters. In one table the number of teeth for the finer pitches is given, and in the other, the number of teeth in roughing milling cutters with the coarse pitch now used extensively. In end mills with coarse pitch teeth use 4 teeth in mills up to 1 inch; use 5 in 1¼-inch; 6 in 1½-inch; and 8 in 2-inch mills. When cutters are to be used exclusively on brass, the number of teeth may be made 25 per cent less than that given for the ordinary plain milling cutter. The teeth in plain milling cutters should be cut with a regular 60-degree angular cutter if the teeth are cut straight. If they are cut spiral, they should be cut either with 60-degree double-angle cutters (12 degrees on one side and 48 on the other) or with 65-degree, double-angle cutters (12 degrees on one side and 53 on the other). The angular cutter for the teeth should have a slightly rounded point so as not to produce a sharp corner in the bottom of the tooth being cut. The backing off of the teeth by grinding is made to an angle of about five degrees with the tangent to the cylindrical surface of the cutter. When the teeth are cut spiral, the lead should be so selected that the angle of spiral equals about fifteen degrees.

Plain and Side Milling Cutters

Diameter of Cutter	Plain Milling Cutter		Side Milling Cutter		Width of Land on Teeth	Diameter of Cutter	Plain Milling Cutter		Side Milling Cutter		Width of Land on Teeth
	Number of Teeth	Radius of Point of Fluting Cutter	Number of Teeth	Angle of Cutter for Side Teeth			Number of Teeth	Radius of Point of Fluting Cutter	Number of Teeth	Angle of Cutter for Side Teeth	
2	14	5/64	22	For wide cutters, use 70- or 75-degree angle cutter. For thin cutters, use 80-, and in extreme cases, 85-degree cutter.	1/32	5½	22	9/64	28	For wide cutters, use 70- or 75-degree angle cutter. For thin cutters, use 80-, and in extreme cases, 85-degree cutter.	3/64
2¼	14	5/64	22		1/32	6	24	5/32	30		1/16
2½	16	3/32	24		1/32	6½	24	5/32	30		1/16
2¾	16	3/32	24		1/32	7	26	3/16	30		1/16
3	18	7/64	24		1/32	7½	26	3/16	30		1/16
3½	18	7/64	24		3/64	8	28	3/16	30		5/64
4	20	1/8	26		3/64	8½	28	3/16	32		5/64
4½	20	1/8	26		3/64	9	30	3/16	32		5/64
5	22	9/64	28		3/64	10	30	3/16	32		5/64

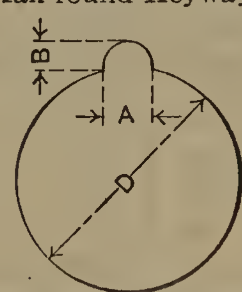
Standard Keyways for Milling Cutters

Square Keyway



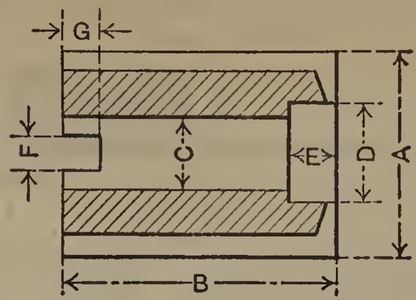
D = Diameter of Hole	A = Width of Key-way	B = Depth of Key-way	C = Radius of Corners
3/8 to 9/16 inch	3/32	3/64	0.020
5/8 to 7/8 inch	1/8	1/16	0.030
15/16 to 1⅛ inch	5/32	5/64	0.035
1 3/16 to 1⅜ inch	3/16	3/32	0.040
1 7/16 to 1¾ inch	¼	1/8	0.050
1 13/16 to 2 inch	5/16	5/32	0.060
2 1/16 to 2½ inch	3/8	3/16	0.060
2 9/16 to 3 inch	7/16	3/16	0.060

Half-round Keyway



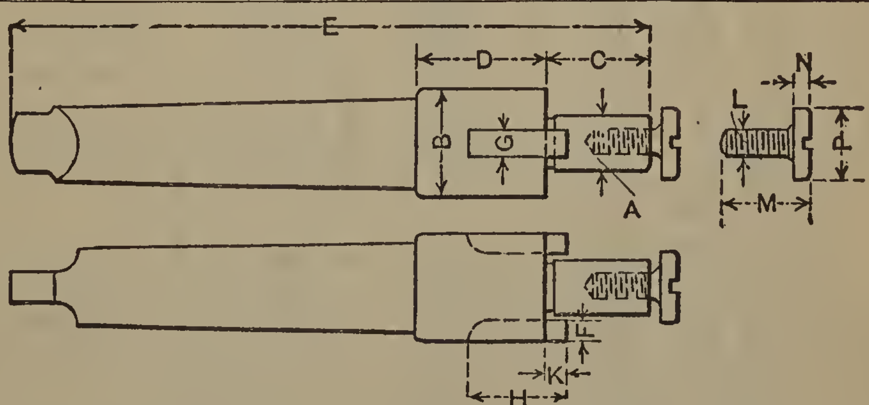
D = Diameter of Hole	A = Width of Key-way	B = Depth of Key-way
3/8 to 5/8 inch	1/8	1/16
11/16 to 13/16 inch	3/16	3/32
7/8 to 1 3/16 inch	¼	1/8
1¼ to 1 7/16 inch	5/16	5/32
1½ to 2 inch	3/8	3/16
2 1/16 to 2 7/16 inch	7/16	7/32
2½ to 3 inch	½	¼
....	......	......

Dimensions of Shell End Mills



Diam.	Total Length	Diam. of Hole	Diam. of Recess	Depth of Recess	Width of Key-way	Depth of Key-way	No. of Teeth	Angle of Cutter for Milling Teeth	
A	*B*	*C*	*D*	*E*	*F*	*G*		On Side	On End
1¼	1¼	½	¾	5/16	3/16	3/16	16	65°	60°
1 5/16	1¼	½	¾	5/16	3/16	3/16	16	65°	60°
1⅜	1¼	½	¾	5/16	3/16	3/16	16	65°	60°
1 7/16	1¼	½	¾	5/16	3/16	3/16	16	65°	60°
1½	1¾	⅝	⅞	5/16	3/16	3/16	16	65°	60°
1⅝	1¾	⅝	⅞	5/16	3/16	3/16	18	65°	60°
1¾	1¾	⅝	⅞	5/16	3/16	3/16	18	65°	60°
1⅞	1¾	⅝	⅞	5/16	3/16	3/16	18	65°	60°
2	1¾	⅝	⅞	5/16	3/16	3/16	18	65°	60°
2⅛	2¼	¾	1	7/16	¼	¼	18	65°	60°
2¼	2¼	¾	1	7/16	¼	¼	18	65°	60°
2⅜	2¼	¾	1	7/16	¼	¼	18	65°	60°
2½	2¼	¾	1	7/16	¼	¼	18	65°	60°
2⅝	2¼	1	1 5/16	½	5/16	5/16	20	65°	60°
2¾	2¼	1	1 5/16	½	5/16	5/16	20	65°	60°
2⅞	2¼	1	1 5/16	½	5/16	5/16	20	65°	60°
3	2¼	1	1 5/16	½	5/16	5/16	20	65°	60°

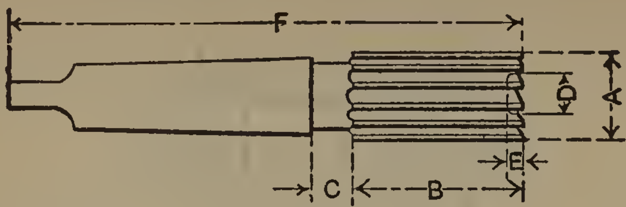
Dimensions of Arbors for Shell End Mills



Diam. of Arbor	Shell End Mill Sizes used for	Diam. of Body	Length of Arbor	Length of Body	Number of Morse Taper	Total Length	Depth of Key	Width of Key	Length of Key	Projection of Key	Diam. of Screw	No. of Threads per Inch of Screw	Length of Screw	Thickness of Head	Diam. of Head
*A**		*B*	*C*	*D*		*E*	*F*	*G*	*H*	*K*	*L*		*M*	*N*	*P*
½	1¼–1 7/16	1 3/16	⅞	2⅛	3	7	5/16	11/64	1¼	5/32	¼	24	⅝	7/32	11/16
⅝	1½–2	1⅜	1⅛	2⅛	3	7¼	⅜	11/64	1⅜	5/32	5/16	22	¾	3/16	13/16
¾	2⅛–2½	1¾	1 9/16	2 3/16	4	8⅞	7/16	15/64	1½	7/32	⅜	20	15/16	¼	15/16
1	2⅝–3	2	1 9/16	2 3/16	4	8⅞	7/16	19/64	1⅝	9/32	½	16	1⅛	5/16	1¼

* This dimension should be made from 0.00025 to 0.0005 inch under size.

Dimensions of End Mills



Diam.	Length of Cut	Length of Neck	Diam. of End Recess	Depth of Recess	Morse Taper Shank End Mills		Brown & Sharpe Taper Shank End Mills		Number of Flutes	Angle of Cutter for Milling Teeth	
A	*B*	*C*	*D*	*E*	No. of Morse Taper	Total Length, *F*	No. of B. & S. Taper	Total Length, *F*		On Side	On End
1/4	3/4	1/4	5/64	1/16	1	3 9/16	4	2 3/8	5	12°×83°	Filed to shape
1/4	3/4	5/16	5/64	1/16	....	....	5	2 15/16	5	"	
5/16	7/8	1/4	3/32	1/16	1	3 11/16	4	2 1/2	5	"	"
5/16	7/8	5/16	3/32	1/16	....	....	5	3 1/16	5	"	"
3/8	7/8	1/4	1/8	1/16	1	3 11/16	4	2 1/2	6	"	"
3/8	7/8	5/16	1/8	1/16	....	....	5	3 1/16	6	"	"
7/16	1	1/4	3/16	1/16	1	3 13/16	4	2 5/8	6	"	"
7/16	1	5/16	3/16	1/16	2	4 3/8	5	3 3/16	6	"	"
1/2	1 1/8	5/16	1/4	1/16	1	4	5	3 5/16	6	"	"
1/2	1 1/8	3/8	1/4	1/16	2	4 9/16	7	5 1/8	6	"	"
9/16	1 1/4	5/16	1/4	1/16	1	4 1/8	5	3 7/16	6	"	12°×63°
9/16	1 1/4	3/8	1/4	1/16	2	4 11/16	7	5 1/4	6	"	"
5/8	1 3/8	5/16	1/4	1/16	....	....	5	3 9/16	7	12°×78°	"
5/8	1 3/8	3/8	1/4	1/16	2	4 13/16	7	5 3/8	7	"	"
11/16	1 1/2	3/8	1/4	1/16	2	4 15/16	7	5 1/2	7	"	"
11/16	1 1/2	1/2	1/4	1/16	....	....	9	6 3/4	7	"	"
3/4	1 5/8	3/8	5/16	1/16	2	5 1/16	7	5 5/8	7	"	"
3/4	1 5/8	1/2	5/16	1/16	3	5 7/8	9	6 7/8	7	"	"
7/8	1 3/4	3/8	3/8	1/16	2	5 3/16	7	5 3/4	8	12°×63°	12°×68°
7/8	1 3/4	1/2	3/8	1/16	3	6	9	7	8	"	"
1	1 7/8	3/8	3/8	3/32	2	5 5/16	7	5 7/8	8	12°×68°	"
1	1 7/8	1/2	3/8	3/32	3	6 1/8	9	7 1/8	8	"	"
1 1/8	2	3/8	7/16	3/32	....	....	7	6	9	"	"
1 1/8	2	1/2	7/16	3/32	3	6 1/4	9	7 1/4	9	"	"
1 1/4	2	1/2	1/2	3/32	3	6 1/4	7	6 1/8	9	"	"
1 1/4	2	1/2	1/2	3/32	4	7 1/4	9	7 1/4	9	"	"
1 3/8	2 1/8	1/2	5/8	1/8	3	6 3/8	9	7 3/8	10	12°×63°	"
1 3/8	2 1/8	1/2	5/8	1/8	4	7 3/8	....	....	10	"	"
1 1/2	2 1/4	1/2	3/4	1/8	3	6 1/2	9	7 1/2	10	"	"
1 1/2	2 1/4	1/2	3/4	1/8	4	7 1/2	....	....	10	"	"
1 5/8	2 1/4	1/2	13/16	1/8	4	7 1/2	9	7 1/2	10	12°×68°	"
1 3/4	2 3/8	1/2	7/8	1/8	4	7 5/8	9	7 5/8	11	12°×63°	"
1 7/8	2 1/2	1/2	15/16	1/8	4	7 3/4	11	9 3/4	11	"	"
2	2 1/2	1/2	1	1/8	4	7 3/4	11	9 3/4	11	"	"

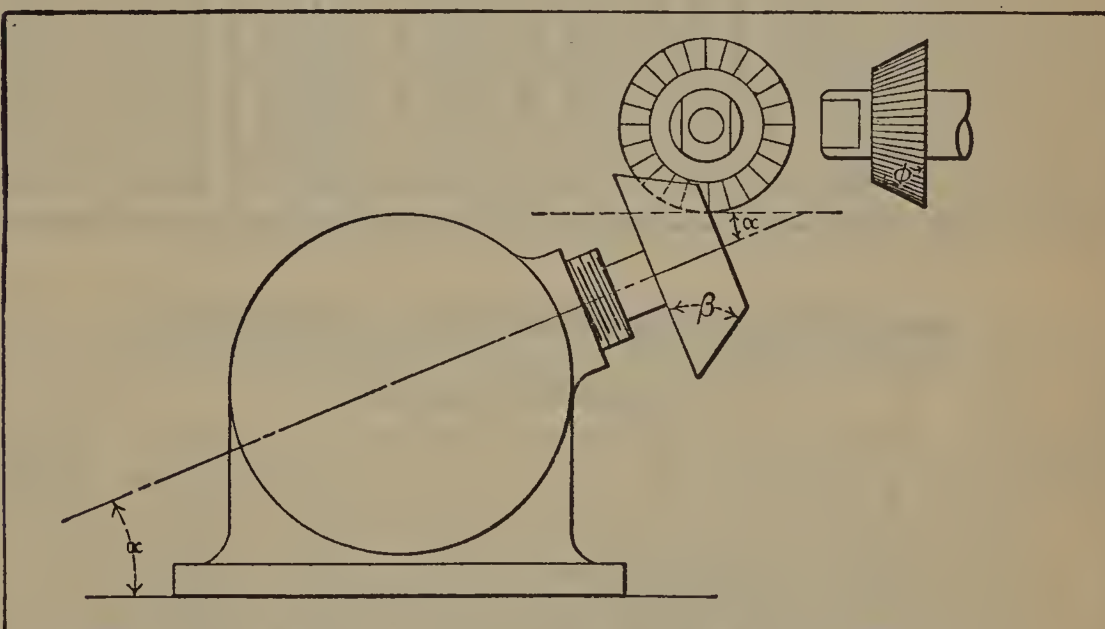
The angle of spiral in spiral cut end mills should be from 7 to 10 degrees.

Angular Milling Cutters. — These are usually made with an included angle between the teeth of 40, 50, 60, 70 or 80 degrees. They are used mainly for fluting milling cutters and end mills, and are ordinarily made in three sizes, 2½, 2¾ and

3 inches in diameter, ½ inch thick, with 1-inch hole in the two smaller sizes and 1¼-inch hole in the largest. The number of teeth is made 20, 22 and 24, respectively, for the three sizes.

Double-angle Cutters, such as are used for fluting spiral milling cutters, are usually made 2½, 2¾ and 3 inches in diameter with a width of ½ inch and a hole 1 inch in diameter in the two smaller sizes and 1¼ inch in the largest. The number of teeth is made 18, 20 and 22, respectively, for the three sizes. The teeth in these cutters are milled with angular cutters. For double-angle cutters with 12 degrees on one side and 40 or 48 degrees on the other, a 75-degree cutter is used for milling the teeth on the 12 degree side and a 60-degree cutter for the teeth on the 40 or 48 degree side.

Setting-angles for Milling End Mills, Angular Cutters and Taper Reamers. — In the calculation of setting-angles for the dividing head for milling angular cutters, end mills and taper reamers, the number of angles involved usually makes the calculation difficult and uncertain, and, in general, the settings can be obtained by the "cut-and-try" method in less time than it takes to compute them. It is,



however, preferable to have the drawings contain all the information relating to the work. The accompanying tables, therefore, give the angles required for setting the dividing head for a great number of combinations. The angle to which the dividing head must be set depends upon two factors; the number of the teeth in the mill to be cut and the angle of the cutter with which the teeth are cut or fluted. When the number of teeth in the cutter to be made and the angle of the cutter used for milling the teeth are given, the setting-angle of the dividing head is found in the body of the tables. For example, assume that twelve teeth are to be cut in the end of an end mill with a 60-degree cutter. By following the horizontal line from twelve teeth, we read in the column under 60 degrees that the dividing head should be set to an angle of 70 degrees 32 minutes. This is the angle α in the accompanying engraving. The angle of the blank in which the teeth are to be milled is designated β, and tables are given for every 5-degree variation in this angle. The angle of the fluting cutter is designated ϕ in the illustration, and the tables give data for fluting cutters with nine different angles.

Angles of Elevation for Milling Teeth in End Mills

No. of Teeth	Angle of Fluting Cutter							
	85	80	75	70	65	60	55	50
5	74° 23′	57° 8′	34° 27′	....	....	....	....	....
6	81 17	72 13	62 21	50° 55′	36° 8′	....	....	....
7	83 42	77 13	70 22	62 50	54 12	43° 36′	....	....
8	84 59	79 51	74 27	68 39	62 12	54 44	45° 33′	32° 57′
9	85 47	81 29	77 0	72 13	66 58	61 1	54 1	45 15
10	86 21	82 38	78 46	74 40	70 12	65 12	59 25	52 26
11	86 47	83 29	80 5	76 28	72 34	68 13	63 15	57 22
12	87 6	84 9	81 6	77 52	74 23	70 32	66 9	61 2
13	87 22	84 41	81 54	78 59	75 48	72 21	68 26	63 52
14	87 35	85 8	82 35	79 54	77 1	73 51	70 17	66 10
15	87 46	85 30	83 9	80 40	78 1	75 6	71 50	68 4
16	87 55	85 49	83 38	81 20	78 52	76 10	73 8	69 40
17	88 3	86 5	84 3	81 53	79 36	77 4	74 15	71 1
18	88 11	86 19	84 24	82 23	80 14	77 52	75 14	72 13
19	88 17	86 32	84 43	82 49	80 47	78 34	76 6	73 15
20	88 22	86 43	85 0	83 13	81 17	79 11	76 51	74 11
21	88 27	86 53	85 15	83 33	81 44	79 44	77 31	74 59
22	88 32	87 2	85 29	83 52	82 8	80 14	78 8	75 44
23	88 36	87 10	85 42	84 9	82 30	80 42	78 41	76 24
24	88 39	87 18	85 53	84 24	82 49	81 6	79 11	77 0

Angles of Elevation for Milling Teeth in 5-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	74° 12′	59° 11′	42° 43′	21° 41′	......	......	......	......	......
6	80 4	71 29	62 34	53 52	41° 41′	27° 22′	......	......	......
7	82 1	75 47	69 22	62 35	55 9	46 33	36° 12′	21° 36′	......
8	82 57	77 58	72 52	67 32	61 47	55 23	48 0	38 56	25° 40′
9	83 29	79 18	75 2	70 35	65 49	60 36	54 43	47 46	38 30
10	83 50	80 13	76 31	72 41	68 35	64 9	59 11	53 27	46 4
11	84 4	80 52	77 36	74 12	70 37	66 43	62 24	57 28	51 15
12	84 14	81 21	78 25	75 23	72 10	68 42	64 52	60 31	55 5
13	84 21	81 44	79 4	76 13	73 23	70 15	66 48	62 54	58 4
14	84 27	82 3	79 36	77 4	74 24	71 32	68 23	64 50	60 28
15	84 32	82 19	80 3	77 43	75 15	72 30	69 42	66 27	62 28
16	84 35	82 31	80 25	78 14	75 57	73 30	70 49	67 48	64 7
17	84 38	82 42	80 44	78 42	76 34	74 16	71 46	68 58	65 33
18	84 41	82 52	81 1	79 7	77 6	74 57	72 36	69 59	66 47
19	84 43	83 0	81 16	79 28	77 34	75 33	73 20	70 52	67 42
20	84 45	83 8	81 29	79 47	77 59	76 4	73 59	71 39	68 50
21	84 46	83 14	81 40	80 3	77 21	76 32	74 33	72 20	69 40
22	84 47	83 19	81 50	80 17	78 40	76 57	75 4	72 58	70 26
23	84 48	83 24	81 59	80 30	78 58	77 20	75 32	73 32	71 7
24	84 49	83 29	82 7	80 43	79 15	77 40	75 57	74 3	71 44

Angles of Elevation for Milling Teeth in 10-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	60° 16′	46° 45′	32° 9′	14° 31′	......	......	......	......	......
6	70 34	62 11	53 50	44 37	34° 5′	20° 57′	......	......	......
7	74 12	68 8	61 55	55 20	48 9	39 57	30° 2′	16° 32′	......
8	76 0	71 8	66 9	60 56	55 19	49 6	41 56	33 12	20° 39′
9	77 2	72 56	68 45	64 23	59 21	54 7	48 52	42 6	33 8
10	77 42	74 8	70 31	66 44	62 44	57 22	53 30	47 54	40 42
11	78 10	75 1	71 48	68 28	64 56	61 6	56 52	52 2	45 56
12	78 30	75 40	72 46	69 47	66 37	63 12	59 26	55 10	49 50
13	78 44	76 9	73 31	70 48	67 56	64 51	61 26	57 36	52 51
14	78 56	76 34	74 9	71 39	69 2	66 12	63 6	59 36	55 19
15	79 5	76 54	74 40	72 21	69 56	67 19	64 28	61 15	57 20
16	79 12	77 10	75 5	72 57	70 41	68 16	65 37	62 39	59 1
17	79 18	77 23	75 27	73 27	71 20	69 4	66 36	63 51	60 28
18	79 22	77 34	75 45	73 52	71 53	69 46	67 27	64 58	61 43
19	79 26	77 44	76 1	74 15	72 23	70 23	68 12	65 46	62 48
20	79 30	77 54	76 16	74 35	72 44	70 56	68 52	66 34	63 47
21	79 33	78 2	76 29	74 53	73 12	71 25	69 28	67 17	64 38
22	79 35	78 8	76 40	75 9	73 33	71 51	69 59	67 55	65 25
23	79 37	78 18	76 50	75 23	73 52	72 14	70 28	68 29	66 6
24	79 39	78 20	76 59	75 30	74 9	72 35	70 54	69 1	66 44

Angles of Elevation for Milling Teeth in 15-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	49° 4′	37° 3′	24° 52′	10° 32′	......	......	......	......	......
6	61 49	54 9	46 12	37 40	28° 4′	16° 26′	......	......	......
7	66 44	60 57	55 1	48 45	41 57	34 14	25° 2′	12° 57′	......
8	69 15	64 33	59 46	54 44	49 21	43 24	36 34	28 21	17° 34′
9	70 43	66 45	62 41	58 28	53 58	49 3	43 30	37 2	29 4
10	71 40	68 12	64 41	61 1	57 8	52 55	48 12	42 47	36 18
11	72 20	69 16	66 8	62 54	59 27	55 44	51 37	46 56	41 24
12	72 48	70 2	67 13	64 18	61 13	57 54	54 14	50 5	45 13
13	73 10	70 39	68 5	65 26	62 38	59 37	56 18	52 34	48 14
14	73 26	71 7	68 46	66 20	63 46	61 0	57 59	54 35	50 38
15	73 39	71 30	69 20	67 5	64 42	62 10	59 22	56 15	52 39
16	73 50	71 50	69 49	67 43	65 30	63 9	60 33	57 40	54 20
17	73 58	72 6	70 12	68 14	66 11	63 58	61 33	58 51	55 46
18	74 5	72 20	70 33	68 42	66 46	64 41	62 26	59 54	57 0
19	74 11	72 32	70 51	69 6	67 17	65 19	63 11	60 49	58 6
20	74 16	72 42	71 6	69 28	67 44	65 53	63 52	61 37	59 3
21	74 20	72 51	71 20	69 46	68 7	66 22	64 27	62 20	59 54
22	74 24	72 59	71 32	70 3	68 29	66 49	65 0	62 59	60 40
23	74 27	73 6	71 43	70 18	68 49	67 13	65 29	63 33	61 22
24	74 30	73 12	71 53	70 32	69 6	67 35	65 56	64 5	61 59

Angles of Elevation for Milling Teeth in 20-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	40° 20′	30° 4′	19° 46′	8° 4′	......	......	......	......	......
6	53 57	46 55	39 39	31 55	23° 18′	13° 11′	......	......	......
7	59 43	54 17	48 42	42 51	36 30	29 23	21° 1′	10° 23′	......
8	62 46	58 18	53 45	48 59	43 53	38 16	31 53	24 16	14° 31′
9	64 35	60 47	56 54	52 52	48 34	43 53	38 38	32 32	25 5
10	65 47	62 28	59 4	55 33	51 50	47 47	43 18	38 9	32 1
11	66 36	63 39	60 38	57 30	54 12	50 38	46 11	42 12	36 56
12	67 12	64 32	61 49	59 0	56 2	52 50	49 18	45 19	40 40
13	67 39	65 13	62 44	60 11	57 28	54 34	51 22	47 47	43 36
14	68 0	65 46	63 29	61 8	58 39	55 59	53 4	49 47	46 0
15	68 17	66 13	64 6	61 55	59 38	57 10	54 28	51 27	47 58
16	68 30	66 34	64 36	62 34	60 26	58 9	55 39	52 51	49 38
17	68 41	66 53	65 2	63 8	61 8	59 0	56 40	54 3	51 4
18	68 50	67 8	65 24	63 37	61 44	59 44	57 32	55 5	52 17
19	68 57	67 21	65 43	64 2	62 15	60 22	58 18	55 59	53 21
20	69 3	67 32	65 59	64 23	62 43	60 55	58 58	56 47	54 18
21	69 9	67 42	66 14	64 42	63 8	61 25	59 34	57 30	55 9
22	69 14	67 51	66 28	64 59	63 30	61 52	60 7	58 9	55 55
23	69 18	67 59	66 39	65 15	63 50	62 16	60 36	58 44	56 36
24	69 21	68 5	66 49	65 30	64 7	62 38	61 2	59 14	57 12

Angles of Elevation for Milling Teeth in 25-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	33° 32′	25° 0′	16° 5′	6° 27′	......	......	......	......	......
6	47 0	40 38	34 6	27 10	19° 33′	10° 48′	......	......	......
7	53 12	48 10	43 0	37 35	31 43	25 17	17° 44′	8° 31′	......
8	56 36	52 25	48 8	43 40	38 55	33 41	27 47	20 50	11° 33′
9	58 40	55 4	51 24	47 36	43 33	39 8	34 13	28 33	21 15
10	60 2	56 53	53 40	50 21	46 47	42 58	38 43	32 53	27 47
11	61 0	58 11	55 18	52 20	49 12	45 48	42 4	37 49	32 32
12	61 42	59 9	56 33	53 52	51 2	47 59	44 38	40 51	36 10
13	62 14	59 54	57 32	55 5	52 30	49 44	46 41	43 15	39 2
14	62 38	60 29	58 19	56 3	53 41	51 8	48 20	45 12	41 22
15	62 57	61 0	58 57	56 52	54 39	52 18	49 43	46 50	43 18
16	63 13	61 22	59 29	57 32	55 29	53 17	50 53	48 13	44 57
17	63 26	61 42	59 54	58 6	56 11	54 8	51 54	49 23	46 21
18	63 37	61 59	60 19	58 36	56 48	54 52	52 46	50 25	47 34
19	63 46	62 13	60 38	59 1	57 20	55 30	53 31	51 19	48 38
20	63 53	62 25	60 56	59 23	57 47	56 4	54 11	52 6	49 33
21	63 59	62 36	61 11	59 43	58 11	56 34	54 47	52 48	50 23
22	64 5	62 46	61 25	60 1	58 34	57 1	55 19	53 26	51 9
23	64 10	62 55	61 37	60 17	58 54	57 25	55 48	54 0	51 50
24	64 14	63 3	61 47	60 31	59 12	57 46	56 13	54 30	52 26

Angles of Elevation for Milling Teeth in 30-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	28° 9′	20° 51′	13° 17′	......	......	......	......	......	......
6	40 54	35 12	29 22	23° 13′	16° 32′	8° 59′	......	......	......
7	47 12	42 35	37 52	32 56	27 38	21 47	15° 6′	7° 5′	......
8	50 46	46 53	42 55	38 47	34 24	29 36	24 12	17 55	10° 14′
9	53 0	49 38	46 13	42 40	38 53	34 48	30 14	25 1	18 47
10	54 29	51 31	48 30	45 22	42 3	38 29	34 31	30 1	24 44
11	55 32	52 52	50 10	47 22	44 25	41 13	37 43	33 45	29 8
12	56 18	53 53	51 26	48 54	46 14	43 21	40 12	36 38	32 32
13	56 54	54 42	52 27	50 8	47 41	45 4	42 12	38 58	35 15
14	57 21	55 19	53 15	51 7	48 52	46 27	43 49	40 51	37 27
15	57 42	55 49	53 54	51 55	49 50	47 35	45 9	42 25	39 17
16	58 0	56 14	54 27	52 36	50 39	48 34	46 19	43 47	40 52
17	58 14	56 35	54 54	53 10	51 21	49 24	47 17	44 55	42 12
18	58 26	56 53	55 18	53 40	51 57	50 7	48 7	45 53	43 20
19	58 36	57 8	55 38	54 6	52 29	50 45	48 51	46 46	44 22
20	58 44	57 21	55 55	54 28	52 56	51 18	49 30	47 31	45 15
21	58 51	57 32	56 10	54 47	53 20	51 47	50 5	48 12	46 3
22	58 57	57 42	56 24	55 5	53 42	52 13	50 36	48 44	46 46
23	59 3	57 51	56 37	55 21	54 2	52 37	51 4	49 21	47 25
24	59 8	57 59	56 48	55 36	54 20	52 59	51 30	49 52	48 0

Angles of Elevation for Milling Teeth in 35-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	23° 49′	17° 35′	11° 10′	4° 22′	......	......	......	......	......
6	35 32	30 29	25 19	19 53	14° 3′	7° 1′	......	......	......
7	41 41	37 20	33 14	28 46	24 1	18 48	12° 54′	5° 58′	......
8	45 17	41 43	38 5	34 19	30 18	25 56	21 4	15 27	8° 41′
9	47 34	44 28	41 18	38 1	34 32	30 47	26 37	21 52	16 16
10	49 7	46 22	43 33	40 39	37 35	34 17	30 38	26 30	21 40
11	50 14	47 46	45 14	42 38	39 53	36 55	33 40	30 0	25 44
12	51 3	48 48	46 30	44 8	41 39	38 58	36 2	32 44	28 55
13	51 40	49 36	47 30	45 20	43 3	40 36	37 55	34 55	31 28
14	52 9	50 15	48 19	46 18	44 12	41 57	39 28	36 42	33 33
15	52 32	50 46	48 58	47 6	45 9	43 4	40 46	38 12	35 17
16	52 50	51 11	49 20	47 46	45 56	43 59	41 51	39 28	36 45
17	53 5	51 32	49 57	48 20	46 37	44 47	42 47	40 33	38 1
18	53 18	51 50	50 21	48 49	47 12	45 29	43 36	41 31	39 8
19	53 29	52 6	50 42	49 14	47 43	46 5	44 19	42 21	40 6
20	53 38	52 19	50 59	49 36	48 10	46 37	44 57	43 5	40 57
21	53 46	52 31	51 15	49 56	48 34	47 6	45 31	43 44	41 43
22	53 53	52 42	51 29	50 14	48 56	47 32	46 1	44 19	42 24
23	53 59	52 51	51 42	50 30	49 15	47 55	46 28	44 51	43 1
24	54 4	52 59	51 53	50 44	49 32	48 16	46 52	45 20	43 35

Angles of Elevation for Milling Teeth in 40-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	20° 13′	14° 53′	9° 24′	3° 39′	......	......	......	......	......
6	30 48	26 21	21 48	17 3	11° 58′	6° 22′	......	......	......
7	36 37	32 52	29 2	25 3	20 49	16 12	11° 1′	5° 2′	......
8	40 7	36 53	33 36	30 10	26 33	22 38	18 16	13 20	7° 23′
9	42 24	39 34	36 41	33 41	30 31	27 26	23 20	19 4	14 3
10	43 57	41 26	38 51	36 11	33 32	30 21	27 3	23 16	18 55
11	45 4	42 48	40 28	38 4	35 32	32 49	29 50	26 29	22 38
12	45 54	43 50	41 43	39 32	37 14	34 45	32 3	29 2	25 33
13	46 33	44 38	42 42	40 41	38 35	36 19	33 50	31 4	27 54
14	47 3	45 17	43 29	41 38	39 41	37 36	35 19	32 46	29 51
15	47 26	45 47	44 7	42 24	40 35	38 39	36 32	34 10	31 28
16	47 45	46 13	44 39	43 3	41 21	39 32	37 33	35 21	32 50
17	48 1	46 34	45 6	43 36	42 0	40 18	38 27	36 23	34 2
18	48 14	46 52	45 29	44 4	42 34	40 58	39 13	37 17	35 5
19	48 25	47 8	45 49	44 28	43 3	41 33	39 54	38 4	35 59
20	48 35	47 22	46 7	44 50	43 30	42 4	40 30	38 46	36 47
21	48 43	47 33	46 23	45 9	43 53	42 31	41 2	39 23	37 30
22	48 50	47 43	46 36	45 26	44 13	42 55	41 30	39 56	38 8
23	48 56	47 52	46 48	45 41	44 31	43 17	41 55	40 25	38 42
24	49 1	48 0	46 58	45 55	44 48	43 36	42 19	40 52	39 15

Angles of Elevation for Milling Teeth in 45-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	17° 10′	12° 36′	7° 57′	3° 5′	......	......	......	......	......
6	26 34	22 41	18 43	14 35	10° 11′	5° 23′	......	......	......
7	31 56	28 36	25 13	21 42	17 56	13 55	9° 24′	4° 15′	......
8	35 16	32 22	29 25	26 22	23 8	19 39	15 48	11 25	5° 58′
9	37 27	34 54	32 17	29 36	26 45	23 41	20 19	16 31	11 49
10	38 58	36 41	34 21	31 57	29 24	26 40	23 40	20 18	16 10
11	40 4	38 0	35 53	33 42	31 24	28 57	26 15	23 14	19 32
12	40 54	39 0	37 5	35 5	33 0	30 45	28 18	25 33	22 13
13	41 32	39 47	38 1	36 11	34 15	32 12	29 57	27 36	24 23
14	42 1	40 24	38 46	37 4	35 17	33 22	31 18	28 58	26 9
15	42 25	40 55	39 23	37 48	36 9	34 22	32 26	30 17	27 40
16	42 44	41 20	39 54	38 25	36 52	35 12	33 24	31 23	28 57
17	43 0	41 41	40 20	38 57	37 29	35 55	34 14	32 20	30 4
18	43 13	41 58	40 42	39 24	38 1	36 33	34 56	33 10	31 1
19	43 24	42 13	41 1	39 47	38 28	37 5	35 34	33 54	31 51
20	43 34	42 26	41 18	40 8	38 53	37 34	36 8	34 33	32 37
21	43 42	42 37	41 33	40 26	39 15	38 0	36 38	35 7	33 17
22	43 49	42 47	41 46	40 42	39 34	38 23	37 5	35 38	34 53
23	43 55	42 56	41 57	40 56	39 52	38 43	37 29	36 6	35 26
24	44 0	43 4	42 7	41 9	40 7	39 1	37 50	36 31	35 55

Angles of Elevation for Milling Teeth in 50-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	14° 32′	10° 39′	6° 42′	2° 33′	......	......	......	......	......
6	22 45	19 23	15 58	12 24	8° 38′	4° 32′	......	......	......
7	27 37	24 42	21 44	18 39	15 24	11 54	8° 1′	3° 36′	......
8	30 41	28 8	25 31	22 50	19 59	16 55	13 33	9 45	5° 20′
9	32 44	30 28	28 9	25 45	23 14	20 31	17 32	14 13	10 22
10	34 10	32 7	30 2	27 54	25 39	23 12	20 32	17 34	14 9
11	35 13	33 22	31 28	29 31	27 28	25 16	22 52	20 11	17 6
12	36 0	34 18	32 34	30 47	28 53	26 54	24 42	22 15	19 27
13	36 36	35 2	33 26	31 48	30 3	28 13	26 11	23 56	21 22
14	37 5	35 38	34 9	32 47	31 1	29 18	27 26	25 21	22 58
15	37 28	36 7	34 44	33 18	31 49	30 13	28 28	26 32	24 20
16	37 47	36 31	35 13	33 53	32 29	31 0	29 22	27 33	25 30
17	38 2	36 50	35 37	34 22	33 3	31 38	30 7	28 24	26 29
18	38 15	37 7	35 58	34 47	33 33	32 13	30 46	29 10	27 21
19	38 26	37 22	36 17	35 9	33 59	32 43	31 21	29 50	28 7
20	38 35	37 34	36 32	35 28	34 21	33 9	31 52	30 25	28 47
21	38 43	37 45	36 46	35 45	34 41	33 33	32 19	30 57	29 24
22	38 50	37 55	36 58	36 0	34 59	33 55	32 44	31 26	29 57
23	38 56	38 3	37 9	36 14	35 15	34 14	33 6	31 51	30 26
24	39 1	38 10	37 19	36 25	35 30	34 30	33 25	32 14	30 52

Angles of Elevation for Milling Teeth in 55-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	12° 13′	8° 57′	5° 37′	2° 10′	......	......	......	......	......
6	19 17	16 25	13 30	10 28	7° 15′	3° 48′	......	......	......
7	23 35	21 4	18 31	15 51	13 4	10 3	6° 44′	3° 1′	......
8	26 21	24 8	21 52	19 31	17 3	14 25	11 30	8 17	4° 17′
9	28 13	26 14	24 12	22 7	19 55	17 34	14 59	12 6	8 34
10	29 32	27 45	25 55	24 2	22 3	19 55	17 36	15 1	11 52
11	30 30	28 52	27 12	25 29	23 41	21 45	19 39	17 18	14 27
12	31 14	29 44	28 12	26 38	24 59	23 13	21 17	19 8	16 32
13	31 48	30 25	29 0	27 33	26 2	24 24	22 37	20 38	18 15
14	32 15	30 58	29 39	28 18	26 53	25 25	23 43	21 53	19 40
15	32 36	31 24	30 11	28 55	27 35	26 11	24 38	22 56	20 52
16	32 54	31 47	30 38	29 27	28 12	26 53	25 26	23 51	21 54
17	33 9	32 6	31 1	29 54	28 44	27 29	26 7	24 38	22 49
18	33 21	32 21	31 20	30 17	29 10	28 0	26 43	25 18	23 35
19	33 31	32 34	31 36	30 36	29 33	28 27	27 14	25 54	24 17
20	33 40	32 46	31 51	30 54	29 54	28 51	27 42	26 25	24 53
21	33 47	32 56	32 3	31 9	30 12	29 12	28 6	26 53	25 25
22	33 54	33 5	32 15	31 23	30 29	29 31	28 28	27 19	25 55
23	34 0	33 13	32 25	31 36	30 44	29 48	28 48	27 42	26 22
24	34 5	33 20	32 34	31 47	30 57	30 4	29 7	28 3	26 46

Angles of Elevation for Milling Teeth in 60-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	10° 7′	7° 25′	4° 39′	1° 47′	......	......	......	......	......
6	16 6	13 41	11 12	8 42	6° 2′	3° 9′	......	......	......
7	19 48	17 40	15 30	13 16	10 55	8 22	5° 36′	2° 30′	......
8	22 13	20 19	18 24	16 24	14 19	12 4	9 37	6 53	3° 44′
9	23 52	22 10	20 26	18 39	16 46	14 46	12 34	10 7	7 19
10	25 2	23 30	21 56	20 19	18 37	16 48	14 49	12 36	10 5
11	25 54	24 30	23 4	21 35	20 2	18 23	16 34	14 34	12 16
12	26 34	25 16	23 57	22 36	21 10	19 39	17 59	16 9	14 13
13	27 5	25 53	24 40	23 25	22 6	20 41	19 9	17 27	15 31
14	27 29	26 22	25 14	24 4	22 51	21 32	20 6	18 32	16 44
15	27 49	26 46	25 43	24 37	23 29	22 15	20 55	19 27	17 47
16	28 5	27 6	26 7	25 5	24 1	22 52	21 37	20 14	18 40
17	28 18	27 23	26 27	25 29	24 28	23 23	22 13	20 55	19 26
18	28 29	27 37	26 44	25 49	24 52	23 50	22 44	21 30	20 6
19	28 38	27 49	26 58	26 7	25 12	24 14	23 11	22 1	20 42
20	28 46	27 59	27 11	26 22	25 30	24 35	23 35	22 29	21 14
21	28 53	28 8	27 23	26 36	25 46	24 54	23 57	22 54	21 42
22	29 0	28 17	27 34	26 49	26 2	25 12	24 17	23 17	22 8
23	29 5	28 24	27 43	27 0	26 15	25 27	24 35	23 37	22 32
24	29 9	28 30	27 50	27 9	26 26	25 40	24 50	23 55	22 52

Angles of Elevation for Milling Teeth in 65-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	8° 12′	6° 0′	3° 46′	1° 27′	......	......	......	......	......
6	13 7	11 10	9 8	7 4	4° 53′	2° 33′	......	......	......
7	16 13	14 28	12 41	10 50	8 54	6 49	4° 33′	2° 1′	......
8	18 15	16 40	15 6	13 26	11 42	9 51	7 50	5 30	3° 1′
9	19 39	18 14	16 48	15 19	13 45	12 5	10 16	8 14	5 57
10	20 40	19 23	18 4	16 44	15 19	13 48	12 9	10 19	8 15
11	21 25	20 14	19 3	17 49	16 31	15 9	13 38	11 58	10 4
12	21 59	20 54	19 48	18 40	17 28	16 12	14 49	13 17	11 32
13	22 26	21 26	20 35	19 22	18 15	17 5	15 48	14 23	12 46
14	22 48	21 52	20 55	19 56	18 54	17 48	16 37	15 17	13 48
15	23 5	22 13	21 19	20 24	19 26	18 24	17 18	16 4	14 40
16	23 18	22 29	21 39	20 47	19 53	18 55	17 53	16 43	15 24
17	23 30	22 43	21 56	21 8	20 17	19 22	18 23	17 17	16 3
18	23 40	22 55	22 11	21 25	20 37	19 46	18 50	17 47	16 37
19	23 48	23 5	22 24	21 40	20 55	20 6	19 13	18 14	17 7
20	23 55	23 14	22 35	21 54	21 10	20 24	19 33	18 38	17 34
21	24 1	23 22	22 45	22 6	21 24	20 39	19 51	18 58	17 58
22	24 6	23 29	22 53	22 16	21 36	20 53	20 8	19 17	18 20
23	24 11	23 36	23 1	22 26	21 47	21 7	20 23	19 34	18 39
24	24 15	23 43	23 8	22 34	21 57	21 18	20 36	19 50	18 57

Angles of Elevation for Milling Teeth in 70-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	6° 25′	4° 42′	2° 57′	......	......	......	......	......	......
6	10 18	8 44	7 9	5° 32′	3° 48′	......	......	......	......
7	12 47	11 23	9 59	8 31	6 58	5° 21′	3° 33′	......	......
8	14 26	13 11	11 55	10 36	9 14	7 45	6 9	4° 23′	2° 21′
9	15 35	14 27	13 18	12 7	10 53	9 33	8 6	6 30	4 41
10	16 25	15 23	14 21	13 15	12 8	10 55	9 37	8 9	6 30
11	17 2	16 5	15 8	14 8	13 7	12 0	10 48	9 28	7 57
12	17 30	16 38	15 45	14 50	13 53	12 51	11 45	10 31	9 8
13	17 52	17 4	16 15	15 24	14 30	13 33	12 32	11 23	10 6
14	18 9	17 24	16 38	15 51	15 1	14 8	13 11	12 7	10 55
15	18 23	17 41	16 58	16 14	15 28	14 38	13 44	12 44	11 37
16	18 35	17 55	17 15	16 33	15 50	15 3	14 13	13 17	12 13
17	18 45	18 7	17 30	16 50	16 9	15 25	14 38	13 46	12 45
18	18 53	18 17	17 42	17 5	16 26	15 44	14 59	14 10	13 13
19	19 0	18 26	17 52	17 17	16 40	16 1	15 18	14 32	13 38
20	19 6	18 35	18 1	17 28	16 53	16 16	15 35	14 51	13 59
21	19 11	18 41	18 9	17 38	17 5	16 29	15 50	15 8	14 18
22	19 15	18 46	18 16	17 46	17 15	16 40	16 3	15 22	14 35
23	19 19	18 51	18 23	17 54	17 25	16 50	16 15	15 36	14 51
24	19 22	18 55	18 29	18 0	17 33	16 59	16 25	15 48	15 5

Angles of Elevation for Milling Teeth in 75-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	4° 44′	3° 28′	2° 10′	......	......	......	......	......	......
6	7 38	6 29	5 19	4° 6′	2° 50′	1° 29′	......	......	......
7	9 29	8 27	7 24	6 17	5 10	3 57	2° 38′	1° 10′	......
8	10 44	9 48	8 51	7 50	6 51	5 45	4 34	3 14	1° 45′
9	11 36	10 46	9 54	9 0	8 5	7 5	6 0	4 49	3 27
10	12 14	11 28	10 40	9 52	9 1	8 7	7 8	6 3	4 49
11	12 42	12 0	11 16	10 32	9 45	8 56	8 2	7 1	5 54
12	13 4	12 25	11 45	11 4	10 21	9 35	8 45	7 49	6 47
13	13 21	12 45	12 8	11 29	10 50	10 7	9 21	8 29	7 31
14	13 34	13 0	12 26	11 50	11 13	10 33	9 50	9 2	8 7
15	13 45	13 13	12 41	12 7	11 33	10 55	10 15	9 30	8 39
16	13 54	13 24	12 54	12 22	11 50	11 14	10 37	9 54	9 7
17	14 2	13 33	13 5	12 35	12 5	11 31	10 56	10 16	9 31
18	14 8	13 41	13 14	12 46	12 17	11 45	11 12	10 34	9 51
19	14 13	13 48	13 22	12 55	12 28	11 58	11 26	10 50	10 10
20	14 18	13 54	13 29	13 4	12 38	12 9	11 39	11 5	10 27
21	14 22	13 59	13 36	13 12	12 46	12 19	11 50	11 17	10 41
22	14 25	14 3	13 41	13 18	12 53	12 28	12 0	11 29	10 54
23	14 28	14 7	13 46	13 24	13 0	12 36	12 9	11 40	11 6
24	14 31	14 11	13 50	13 29	13 7	12 44	12 18	11 50	11 18

Angles of Elevation for Milling Teeth in 80-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	3° 7′	2° 17′	1° 26′	0° 43′	......	......	......	......	......
6	5 2	4 16	3 30	2 42	1° 52′	0° 58′	......	......	......
7	6 16	5 35	4 53	4 10	3 25	2 36	1° 45′	0° 46′	......
8	7 6	6 29	5 51	5 12	4 31	3 48	3 2	2 8	1° 8′
9	7 42	7 8	6 34	5 58	5 21	4 42	3 59	3 11	2 17
10	8 7	7 36	7 5	6 33	5 59	5 22	4 44	4 0	3 11
11	8 26	7 58	7 29	7 0	6 28	5 55	5 19	4 39	3 54
12	8 41	8 15	7 48	7 21	6 52	6 22	5 48	5 11	4 29
13	8 53	8 29	8 4	7 38	7 12	6 43	6 12	5 38	4 59
14	9 2	8 40	8 16	7 52	7 28	7 1	6 32	6 0	5 24
15	9 9	8 48	8 26	8 4	7 40	7 16	6 48	6 19	5 45
16	9 15	8 55	8 35	8 14	7 51	7 28	7 3	6 33	6 3
17	9 20	9 1	8 42	8 22	8 1	7 39	7 15	6 49	6 19
18	9 24	9 6	8 48	8 29	8 10	7 49	7 26	7 1	6 33
19	9 28	9 11	8 53	8 36	8 17	7 58	7 36	7 12	6 45
20	9 31	9 15	8 58	8 42	8 24	8 5	7 44	7 21	6 56
21	9 34	9 19	9 3	8 47	8 30	8 12	7 52	7 30	7 6
22	9 36	9 22	9 6	8 51	8 35	8 18	7 59	7 38	7 15
23	9 38	9 24	9 9	8 55	8 39	8 23	8 5	7 45	7 23
24	9 40	9 26	9 13	8 59	8 43	8 28	8 11	7 51	7 30

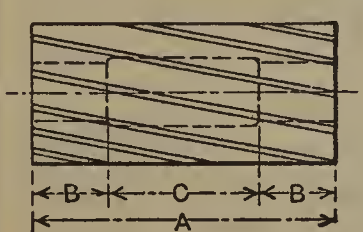
Angles of Elevation for Milling Teeth in 85-degree Blank

No. of Teeth	Angle of Fluting Cutter								
	90	85	80	75	70	65	60	55	50
5	1° 33′	1° 8′	......	......	......	......	......	......	......
6	2 30	2 7	1° 44′	1° 20′	0° 55′	......	......	......	......
7	3 7	2 46	2 26	2 4	1 42	1° 18′	0° 50′	......	......
8	3 32	3 13	2 55	2 35	2 15	1 53	1 29	1° 3′	0° 34′
9	3 50	3 33	3 16	2 58	2 40	2 20	1 59	1 35	1 8
10	4 3	3 48	3 32	3 16	2 59	2 41	2 21	1 59	1 35
11	4 13	3 59	3 44	3 30	3 14	2 57	2 39	2 19	1 57
12	4 20	4 7	3 53	3 40	3 25	3 10	2 53	2 35	2 15
13	4 26	4 14	4 1	3 48	3 35	3 21	3 6	2 48	2 30
14	4 30	4 19	4 7	3 55	3 43	3 29	3 15	2 59	2 42
15	4 34	4 23	4 12	4 1	3 50	3 37	3 24	3 9	2 52
16	4 37	4 27	4 17	4 6	3 56	3 44	3 30	3 17	3 1
17	4 40	4 30	4 21	4 11	4 1	3 50	3 37	3 24	3 9
18	4 42	4 33	4 24	4 15	4 5	3 55	3 43	3 30	3 16
19	4 44	4 35	4 27	4 18	4 9	3 59	3 48	3 36	3 22
20	4 46	4 37	4 29	4 21	4 12	4 3	3 52	3 41	3 28
21	4 47	4 39	4 31	4 23	4 15	4 6	3 56	3 45	3 33
22	4 48	4 41	4 33	4 25	4 18	4 9	3 59	3 49	3 37
23	4 49	4 42	4 35	4 27	4 20	4 12	4 2	3 53	3 41
24	4 50	4 43	4 36	4 29	4 22	4 14	4 5	3 56	3 45

Angles of Elevation of Dividing Head for Milling Teeth in Side Mills or End Mills with Large Number of Teeth

No. of Teeth	Angle of Fluting Cutter				
	45	50	60	70	80
25	75° 7′	77° 33′	81° 28′	84° 38′	87° 24′
26	75 44	78 4	81 49	84 51	87 30
27	76 17	78 32	82 8	85 3	87 36
28	76 49	78 58	82 26	85 14	87 42
29	77 17	79 21	82 42	85 24	87 46
30	77 44	79 43	82 57	85 34	87 51
32	78 32	80 23	83 24	85 51	87 59
34	79 14	80 59	83 48	86 6	88 7
36	79 51	81 29	84 9	86 19	88 13
38	80 24	81 58	84 29	86 31	88 19
40	80 53	82 22	84 45	86 42	88 24
42	81 20	82 44	85 0	86 51	88 29
44	81 44	83 4	85 14	87 0	88 33
46	82 7	83 23	85 27	87 8	88 37
48	82 26	83 39	85 38	87 16	88 40
50	82 45	83 55	85 49	87 22	88 43
52	83 3	84 9	85 59	87 28	88 46
54	83 17	84 22	86 8	87 34	88 49
56	83 31	84 34	86 16	87 39	88 52
58	83 46	84 46	86 24	87 44	88 54
60	83 58	84 57	86 31	87 49	88 56

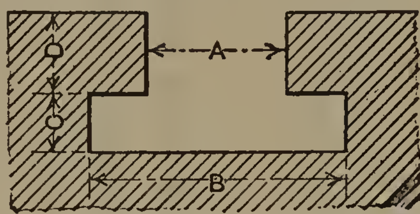
Length of Recess in Milling Cutters



Length of Cutter	Length of Bearing at Each End	Length of Recess
A	B	C
½	⅛	¼
⅝	3/16	¼
¾	3/16	⅜
⅞	¼	⅜
1	¼	½
1⅛	5/16	½
1¼	5/16	⅝
1⅜	⅜	⅝
1½	⅜	¾
1⅝	7/16	¾
1¾	7/16	⅞
1⅞	½	⅞
2	½	1
2¼	9/16	1⅛
2½	⅝	1¼
2¾	11/16	1⅜
3	¾	1½
3¼	13/16	1⅝
3½	⅞	1¾
3¾	⅞	2
4	15/16	2⅛
4¼	1 1/16	2⅛
4½	1⅛	2¼
4¾	1 3/16	2⅜
5	1 3/16	2⅝
5¼	1¼	2¾
5½	1 5/16	2⅞
5¾	1 7/16	2⅞
6	1½	3
......	......	.....

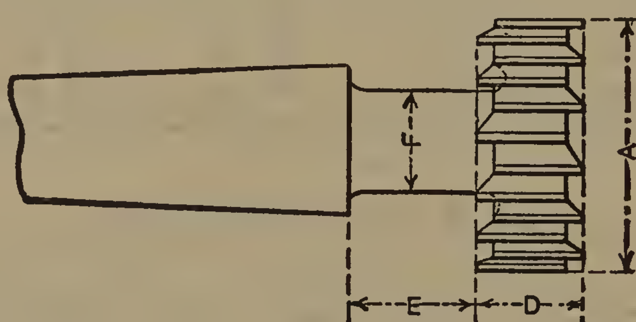
T-slot Cutters. — In order to permit the grinding of T-slot cutters without making the slot cut by them too small, they are originally made 1/32 inch larger in diameter and 1/64 inch greater in thickness than the nominal size, which is the minimum size of the T-slot. In order to provide a cutter that will cut more easily than would be the case if all the teeth were full, every other tooth is often cut away at the ends. It should be observed when doing this that when a tooth is cut off at the end face it should be left full at the back of the face, and *vice versa*. Some makers leave one tooth full at both ends to facilitate measuring the thickness of the cutter.

Standard Dimensions of T-slots



A	B	C	D	A	B	C	D	A	B	C	D
1/4	1/2	5/32	5/16	7/16	13/16	7/32	7/16	3/4	1 5/16	17/32	1
5/16	5/8	5/32	3/8	1/2	15/16	9/32	9/16	7/8	1 5/8	1 1/16	1 1/16
3/8	11/16	7/32	7/16	5/8	1 3/16	13/32	3/4	1	1 7/8	1 3/16	1 3/16

Dimensions of T-slot Cutters



Nominal Size of Cutter	Actual Size of Cutter	Nominal Thickness of Cutter	Actual Thickness of Cutter	Length of Neck	Diam. of Neck	No. of Morse Taper Shank	No. of B.&S. Taper Shank	No. of Teeth	Angle of Cutter for Milling Teeth on both Side and End
	A		D	E	F				
1/2	17/32	5/32	11/64	3/8	7/32	1	4, 5	8	75° single angle cutter for face, and 65° for end for all sizes.* (Rad. on point from 1/64 inch to 1/32 inch.)
5/8	21/32	5/32	11/64	7/16	9/32	1	5, 7	8	
11/16	23/32	7/32	15/64	1/2	11/32	2	5, 7	8	
13/16	27/32	7/32	15/64	1/2	3/8	2	7, 9	10	
15/16	31/32	9/32	19/64	5/8	7/16	2	7, 9	10	
1 3/16	1 7/32	13/32	27/64	13/16	17/32	3	9	10	
1 5/16	1 11/32	17/32	35/64	1 1/16	21/32	3	9	12	
1 5/8	1 21/32	11/16	45/64	1 1/8	25/32	4	9	12	
1 7/8	1 29/32	13/16	53/64	1 1/4	29/32	4	9	14	

* On narrow cutters, a 75-degree cutter is used for the end teeth.

Single Corner-rounding Cutters

Size of Radius A	Diam. of Cutter B	Width of Flange C	Total Width D	No. of Teeth	Size of Radius A	Diam. of Cutter B	Width of Flange C	Total Width D	No. of Teeth
1/16	2 1/4	5/32	7/32	16	1/2	3 1/4	3/8	7/8	10
3/32	2 1/4	3/16	9/32	16	9/16	3 1/2	13/32	31/32	10
1/8	2 1/4	3/16	5/16	16	5/8	3 1/2	7/16	1 1/16	8
5/32	2 1/2	7/32	3/8	12	11/16	3 3/4	15/32	1 5/32	8
3/16	2 1/2	7/32	13/32	12	3/4	3 3/4	1/2	1 1/4	8
7/32	2 1/2	1/4	15/32	12	13/16	4	1/2	1 5/16	8
1/4	2 1/2	1/4	1/2	12	7/8	4 1/4	9/16	1 7/16	8
5/16	2 3/4	9/32	19/32	10	15/16	4 1/4	9/16	1 1/2	8
3/8	3	5/16	11/16	10	1	4 1/2	5/8	1 5/8	8
7/16	3	11/32	25/32	10	.....	....	....	....	....

Double Corner-rounding Cutters

Size of Radius A	Diam. of Cutter B	Width of Flange C	Total Width D	No. of Teeth	Size of Radius A	Diam. of Cutter B	Width of Flange C	Total Width D	No. of Teeth
1/16	2 1/4	5/32	9/32	16	1/2	3 1/4	3/8	1 3/8	10
3/32	2 1/4	3/16	3/8	16	9/16	3 1/2	13/32	1 17/32	10
1/8	2 1/4	3/16	7/16	16	5/8	3 1/2	7/16	1 11/16	8
5/32	2 1/2	7/32	17/32	12	11/16	3 3/4	15/32	1 27/32	8
3/16	2 1/2	7/32	19/32	12	3/4	3 3/4	1/2	2	8
7/32	2 1/2	1/4	11/16	12	13/16	4	1/2	2 1/8	8
1/4	2 1/2	1/4	3/4	12	7/8	4 1/4	9/16	2 5/16	8
5/16	2 3/4	9/32	29/32	10	15/16	4 1/4	9/16	2 7/16	8
3/8	3	5/16	1 1/16	10	1	4 1/2	5/8	2 5/8	8
7/16	3	11/32	1 7/32	10	.....	.....	.....	....	....

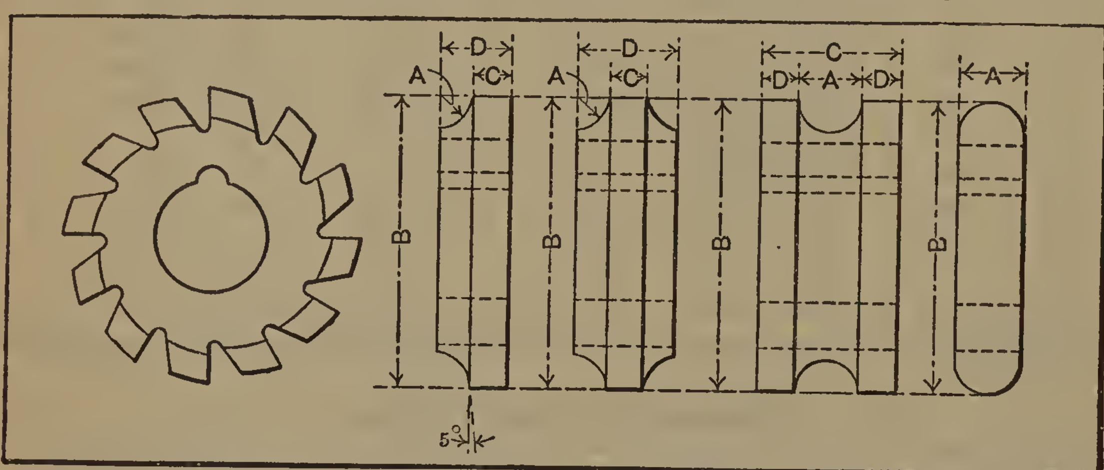
Dimensions of Convex Cutters

Diam. of Circle A	Diam. of Cutter B	Number of Teeth	Diam. of Circle A	Diam. of Cutter B	Number of Teeth	Diam. of Circle A	Diam. of Cutter B	Number of Teeth	Diam. of Circle A	Diam. of Cutter B	Number of Teeth
1/8	2 1/4	16	1/2	2 1/2	12	7/8	3	10	1 1/2	3 3/4	8
3/16	2 1/4	16	9/16	2 3/4	10	15/16	3 1/4	10	1 5/8	4	8
1/4	2 1/4	16	5/8	2 3/4	10	1	3 1/4	10	1 3/4	4 1/4	8
5/16	2 1/2	12	11/16	2 3/4	10	1 1/8	3 1/2	10	1 7/8	4 1/4	8
3/8	2 1/2	12	3/4	3	10	1 1/4	3 1/2	8	2	4 1/2	8
7/16	2 1/2	12	13/16	3	10	1 3/8	3 3/4	8	....	....	...

Dimensions of Concave Cutters

Diam. of Circle	Diam. of Cutter	Width of Cutter	Width of Flanges	No. of Teeth	Diam. of Circle	Diam. of Cutter	Width of Cutter	Width of Flanges	No. of Teeth
A	*B*	*C*	*D*		*A*	*B*	*C*	*D*	
1/8	2 1/4	7/16	5/32	16	7/8	3	1 7/16	9/32	10
3/16	2 1/4	1/2	5/32	16	15/16	3 1/4	1 9/16	5/16	10
1/4	2 1/4	9/16	5/32	16	1	3 1/4	1 5/8	5/16	10
5/16	2 1/2	11/16	3/16	12	1 1/8	3 1/2	1 13/16	11/32	10
3/8	2 1/2	3/4	3/16	12	1 1/4	3 1/2	1 15/16	11/32	8
7/16	2 1/2	7/8	7/32	12	1 3/8	3 3/4	2 1/8	3/8	8
1/2	2 1/2	15/16	7/32	12	1 1/2	3 3/4	2 5/16	13/32	8
9/16	2 3/4	1	7/32	10	1 5/8	4	2 1/2	7/16	8
5/8	2 3/4	1 1/8	1/4	10	1 3/4	4 1/4	2 5/8	7/16	8
11/16	2 3/4	1 3/16	1/4	10	1 7/8	4 1/4	2 13/16	15/32	8
3/4	3	1 1/4	1/4	10	2	4 1/2	3	1/2	8
13/16	3	1 3/8	9/32	10	....	....	.....	.....	.....

Formed Milling Cutters.—The spacing of the teeth in formed and eccentrically relieved cutters is much coarser than in ordinary milling cutters. The width of the space between the front of one tooth and the back of the next should be equal to about 1/2 of the width of the land. The grooves are milled with single-angle cutters varying from 30 to 45 degrees included angle. An angular cutter of 35 degrees included angle is satisfactory for most conditions. The most common of all formed cutters, outside of gear cutters, which form a class by themselves,



Notation used in Tables of Corner-rounding, Convex and Concave Cutters

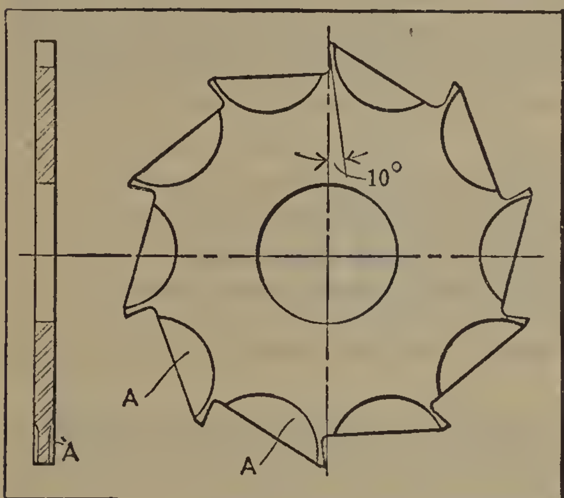
are concave, convex and corner-rounding cutters, the latter being made in two types, single or double. This latter cutter should not be made with the rounded part a full quarter of a circle but the shape should merge into a tangent 5 degrees to a line perpendicular to the axis of the cutter, as shown in the illustration.

Metal Slitting Cutters.—Thin cutters intended for cutting off or slitting purposes are termed metal slitting cutters. The sides of these cutters are ground to run true, but are made slightly thicker at the outside edge than at the hole or center, to provide clearance and prevent binding in the slot cut. A table is given showing the number of teeth used in these cutters for cutting steel. For brass and very deep slots the number of teeth should be only about two-thirds of the number of teeth given in the table. For light slotting, like screw slotting, etc., a cheaper grade of cutters with very fine teeth, not ground on the sides, is used. These are commonly termed screw slotting cutters. The number of teeth in these cutters for the most common diameters are as follows: 1¾ inch diameter, 52 teeth; 2 inches, 56 teeth; 2¼ inches, 60 teeth; 2½ inches, 64 teeth; 2¾ inches, 68 teeth; 3 inches, 72 teeth.

Number of Teeth in Metal Slitting Saws

(For Cutting Steel)

Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth	Diam. of Cutter	No. of Teeth
2½	30	3½	34	4½	38	5½	42	6½	46	7½	54
3	32	4	36	5	40	6	44	7	50	8	58



Saws for Copper. — Copper is one of the most difficult materials to cut with milling saws, and a special construction is necessary. One of the most successful types is shown in the accompanying engraving. The front of the tooth has a rake of 10 degrees and the metal is ground away on the sides between the teeth for clearance, as indicated at *A*. The number of teeth should be comparatively small. A pitch of about 1 inch, which would give 10 teeth in a 3-inch saw, gives good results.

Milling Cutters with Helical Teeth. — It is the general practice to cut the teeth straight on narrow cutters, up to about ¾ inch in width, and to cut the teeth helical or "spiral" on wider cutters. The angle between the helix and cutter axis should equal about 15 degrees. Cutters having helical teeth are generally used in preference to the type with straight or parallel teeth, especially for milling comparatively wide surfaces, because the former cut more smoothly. When the teeth are parallel to the axis, each tooth begins to cut along its entire width at the same time; hence, if a wide surface is being milled, a shock is produced as each tooth engages the metal. This difficulty is not experienced with helical teeth which, being at an angle, begin to cut at one side and continue across the work with a smooth shaving action. Helical cutters also require less power for driving and produce smoother surfaces.

Interlocking Cutters. — There are several methods of interlocking milling cutters which are made in sections. A simple method consists in milling a straight slot across the end of one cutter and providing the other cutter with a corresponding tongue fitting loosely in the slot. Ordinary side-mills are sometimes placed side by side with their hubs ground down so that the teeth of one of the cutters interlock into the spaces between the teeth of the other. Another method consists in cutting away two or more sectors on one end of each of the two cutters in such a manner that the remaining high sectors in one cutter fit loosely into the spaces cut away in the other section. The interlocking ends of some cutters are milled or planed off to produce an angular face, and a recess is provided in the cutter ends in which a washer is placed, against which the finished surfaces of the recess bear. Milling cutters are generally interlocked either (1) to maintain a standard width of cutter; (2) to enable the milling of two or more distinct widths with the same set of cutters; or (3) to enable long cutters to be made in sections, and still have a continuous tooth surface.

Cutter Grinding

Wheels for Sharpening Milling Cutters. — Grinding wheels for cutter sharpening should be of a medium-soft grade and not too fine — never finer than 60 grit. Fine wheels cut slowly and tend to burn the teeth. Wheels for sharpening cutters made of high-speed steels can be a little coarser than those used on carbon steel. If the wheel is too soft, it will wear rapidly, which makes it difficult to keep the cutter round while sharpening it. This difficulty can be overcome by using a wheel at least ¾-inch wide, instead of the ⅜- or ½-inch wheels commonly used. A wide soft wheel will last as long as a narrow one of harder grade, and being softer, it tends to eliminate the danger of burning. For sharpening ordinary milling cutters, a wide wheel will not be especially inconvenient, as generally there is plenty of room. The following "Aloxite" wheels have given good results for cutter sharpening operations:

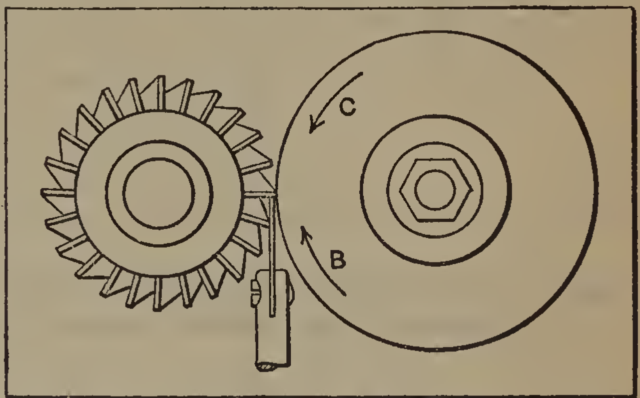
For carbon steel mills and cutters, 50 grit, O grade, D496 bond.
For high-speed steel mills and cutters, 40 grit, O grade, D496 bond.
For carbon-steel formed cutters, 50 grit, P grade, D495 bond.
For high-speed steel formed cutters, 40 grit, P grade, D495 bond.

To obtain the best results, these wheels should run at a surface speed of about 5,000 feet per minute.

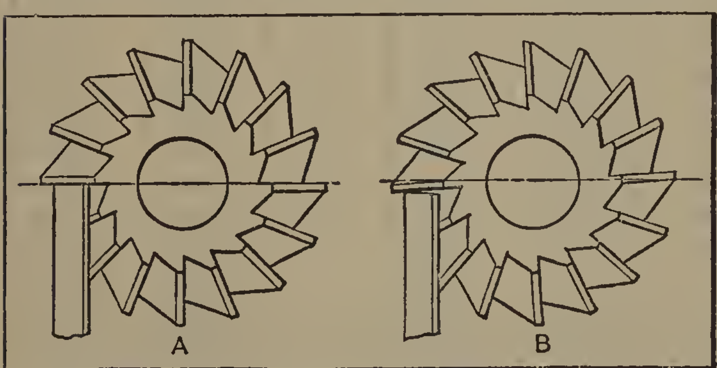
Form of Wheel for Cutter Grinding. — Ordinary milling cutters may be sharpened either by using the periphery of a disk wheel or the face of a cup-wheel. The latter grinds the lands of the teeth flat, whereas the periphery of a disk wheel leaves the teeth slightly concave. It has often been claimed that the cutter relieved with a disk wheel is inferior to the cutter relieved with the cup-wheel, owing to the smaller amount of metal directly back of the cutting edge. While the use of a cup-wheel theoretically possesses certain advantages over the disk type of wheel, actual tests indicate that longer life and higher rates of production are obtained with cutters relieved with the disk wheel. The conditions under which each cutter was tested were the same and the steel for the cutters was taken from the same bar. All cutters were given the same heat-treatment and one milling machine was used for all the tests. These tests proved that regardless of what combination of speeds and feeds was used, the result was practically always in favor of the cutter relieved with the disk wheel. In nearly all cases, from 20 to 30 per cent longer service, in inches milled, was obtained with cutters relieved with the disk wheel. The grinding wheels used, which proved to be of suitable grade and grain, were Norton 38–36–I silicate alundum for the disk wheel, and Norton 38–46–J alundum for the cup-wheel (Brown & Sharpe No. 50 shape). Great care was exercised in relieving the cutters so as not to burn the cutting edges.

Position of Tooth-rest on Cutter Grinder. — When grinding a cylindrical cutter having helical or "spiral" teeth, the tooth-rest should remain in a fixed position relative to the grinding wheel. The tooth being ground will then slide over the tooth-rest, thus causing the cutter to turn as it moves longitudinally, so that the edge of the helical tooth is ground to a uniform distance from the center, throughout its length. When grinding a straight-fluted cutter, it is also preferable to have the tooth-rest in a fixed position relative to the wheel, unless the cutter is quite narrow, because any warping of the cutter in hardening will result in inaccurate grinding, if the tooth-rest moves with the work. When the tooth-rest is fixed relative to the wheel, it should be somewhat wider than the wheel face so that the cutter will have a support before it reaches the wheel and also after it has been traversed past the wheel face. Narrow tooth-rests may be used when they are attached to the table, and remain fixed relative to the work.

Rotation of Wheel Relative to Cutter. — When grinding the teeth of cutters, reamers, etc., in a regular cutter grinding machine, the wheel can be revolved either against the cutting edge of the tooth as shown by arrow *B* (see illustration), or away from it, as indicated by arrow *C*. The cutter can be presented in either way, by swiveling the work around the stationary column of the machine, to either side of the wheel. By revolving the wheel against the tooth, as at *B*, a keen edge will be obtained without forming a burr, and there is also less danger of drawing the temper, thus enabling the grinding to be done more rapidly. Care must be taken, however, to hold the work securely against the tooth-rest as otherwise the wheel may draw the cutter away from the rest and score the tooth. There is also danger of the wheel being broken. Rotating the wheel as shown by arrow *C* is the safer method, as the wheel then holds the tooth against the rest. With the wheel reversed this way, a slight burr is left on the cutting edge, which should be removed with an oilstone.

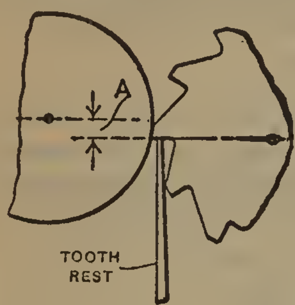


Sharpening Angular Cutters. — In sharpening cutters of this type, the tooth-rest should be set exactly on the center as indicated at *A* (see illustration), so that the cutter will be ground to the angle shown by the graduations on the machine. The angle for clearance, in this case, is obtained either by raising the wheel or lowering the swiveling head, depending upon the design of the machine. The tooth-rest should never be set below the center, as at *B*. Many machinists and toolmakers overlook this point, which is often the cause of angular cutters being inaccurate and unfit for close work.



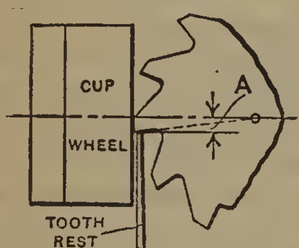
Clearance Angle for Teeth. — Milling cutters usually have from 5 to 7 degrees of clearance. The practice in some shops, where special roughing and finishing cutters are used, is to give the roughing cutters a clearance of 7 degrees and finishing cutters, 5 degrees. Excessive clearance causes chattering when milling, and the teeth become dull quickly. The clearance angle is regulated, when grinding, by setting the center of the grinding wheel slightly above the center of the cutter, or by adjusting the tooth-rest slightly below the center. (See the accompanying tables.) The tooth-rest should always bear against the tooth that is being sharpened, and for cutters having helical teeth, it is attached to the wheel head so as to remain in a fixed position relative to the wheel.

Distance to Set Center of Wheel Above the Cutter Center (Disk Wheel)



Diam. of Emery Wheel	*A* for 5 Deg. Clearance	*A* for 7 Deg. Clearance	Diam. of Emery Wheel	*A* for 5 Deg. Clearance	*A* for 7 Deg. Clearance
2	3/32	1/8	4 1/4	3/16	17/64
2 1/4	3/32	9/64	4 1/2	13/64	9/32
2 1/2	7/64	5/32	4 3/4	13/64	19/64
2 3/4	1/8	11/64	5	7/32	5/16
3	1/8	3/16	5 1/4	15/64	21/64
3 1/4	9/64	13/64	5 1/2	15/64	11/32
3 1/2	5/32	7/32	5 3/4	1/4	23/64
3 3/4	5/32	15/64	6	17/64	3/8
4	11/64	1/4	.....	.....	......

Table for Setting Tooth Rest Below Cutter Center to Obtain 5 and 7 Degrees Clearance When Grinding Milling Cutter Teeth With Cup Wheel



Diam. of Cutter	*A* for 5 Deg. Clearance	*A* for 7 Deg. Clearance
1/4	0.011	0.015
3/8	0.015	0.022
1/2	0.022	0.030
5/8	0.028	0.037

Diam. of Cutter	*A* for 5 Deg. Clearance	*A* for 7 Deg. Clearance	Diam. of Cutter	*A* for 5 Deg. Clearance	*A* for 7 Deg. Clearance
3/4	0.033	0.045	3 1/2	0.154	0.210
7/8	0.037	0.052	3 3/4	0.165	0.225
1	0.044	0.060	4	0.176	0.240
1 1/8	0.050	0.067	4 1/2	0.198	0.270
1 1/4	0.055	0.075	5	0.220	0.300
1 1/2	0.066	0.090	5 1/2	0.242	0.330
1 3/4	0.077	0.105	6	0.264	0.360
2	0.088	0.120	6 1/2	0.286	0.390
2 1/4	0.099	0.135	7	0.308	0.420
2 1/2	0.110	0.150	7 1/2	0.330	0.450
2 3/4	0.121	0.165	8	0.352	0.480
3	0.132	0.180	9	0.396	0.540
3 1/4	0.143	0.195	10	0.440	0.600

Sharpening Formed Cutters. — Formed cutters should be ground radially or so that the faces of the teeth lie in planes passing through the axis of the cutter. The teeth should also have the same height to insure each tooth doing an equal amount of work. When setting up the grinder for formed cutters, the grinding side of the wheel should run true and be in line with the centers. If the wheel is set off center and the teeth are not ground radially, the cutter will not mill the desired shape. To insure grinding all the teeth to the same height or so that all cutting edges are at the same radial distance, a dial gage is sometimes used, having a point which bears against the formed surface or back of each tooth being ground. The cutter is adjusted so that the gage shows the same reading for each tooth; hence all the faces are ground to the same relative distance from that part of the formed surface which comes in contact with the gage.

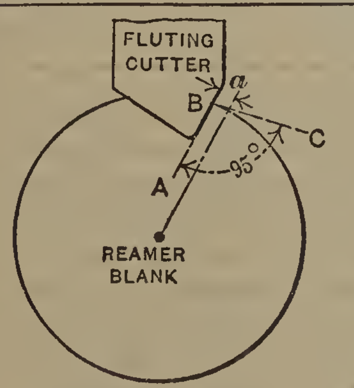
REAMERS

Hand Reamers. — Hand reamers should be provided with a short straight guide of length *G* (see illustration with table of hand reamers). In general, the diameter of this guide should be from 0.005 to 0.010 inch smaller than the standard size of the reamer for diameters up to 1 inch, and from 0.010 to 0.015 inch smaller, for diameters from 1 to 3 inches. At the upper end of the guide there is a small groove for clearance when grinding, and then a tapered portion extending from about ⅜ to ⅝ inch for the smaller and from ¾ to 1¼ inch for the larger sizes. Commercial hand reamers made for the market are not generally provided with this guide, although it is a very valuable feature. Reamers should be from 0.010 to 0.025 inch oversize before hardening, according to size and length of reamer, or in general, ½ inch size, 0.010 inch oversize; 1-inch, 0.015 inch; 2-inch, 0.020 inch. On shell reamers allow about 0.005 inch more than these figures on all sizes.

The diameter of the shank should be from 0.001 to 0.002 inch below the diameter of the reamer. That part of the shank which is squared should be turned smaller in diameter than the shank itself, so that, when applying a wrench, no burr may be raised which may mar the reamed hole if the reamer is passed clear through it.

When fluting reamers, the cutter is so set with relation to the center of the reamer blank that the tooth gets a slight negative rake; that is, the cutter should be set *ahead* of the center, as shown in the illustration accompanying the table giving the amount to set the cutter ahead of the radial line. The amount is so selected that a tangent to the circumference of the reamer at the cutting point makes an angle of approximately 95 degrees with the front face of the cutting edge.

Amount to Set Cutter Ahead of Radial Line to Obtain Negative Front Rake



Size of Reamer	Dimension *a*, Inches	Size of Reamer	Dimension *a*, Inches	Size of Reamer	Dimension *a*, Inches
¼	0.011	⅞	0.038	2	0.087
⅜	0.016	1	0.044	2¼	0.098
½	0.022	1¼	0.055	2½	0.109
⅝	0.027	1½	0.066	2¾	0.120
¾	0.033	1¾	0.076	3	0.131

When fluting reamers, it is necessary to "break up the flutes"; that is, to space the cutting edges unevenly around the reamer. The difference in spacing should be very slight and need not exceed two degrees one way or the other. The manner in which the breaking up of the flutes is usually done is to move the index head to which the reamer is fixed a certain amount more or less than would be the case if the spacing were regular. A table is given showing the amount of this additional movement of the index crank for reamers with different numbers of flutes. When a reamer is provided with helical flutes, the angle of spiral should be such that the cutting edges make an angle of about 10 or at most 15 degrees with the axis of the reamer.

The relief of the cutting edges should be comparatively slight. An eccentric relief, that is, one where the land back of the cutting edge is convex, rather than flat, is used by one or two manufacturers, and is preferable for finishing reamers, as the reamer will hold its size longer. When hand reamers are used merely for removing stock, or simply for enlarging holes, the flat relief is better, because the reamer has a keener cutting edge. The width of the land of the cutting edges should be about 1/32 inch for a 1/4-inch, 1/16 inch for a 1-inch, and 3/32 inch for a 3-inch reamer.

Irregular Spacing of Teeth in Reamers

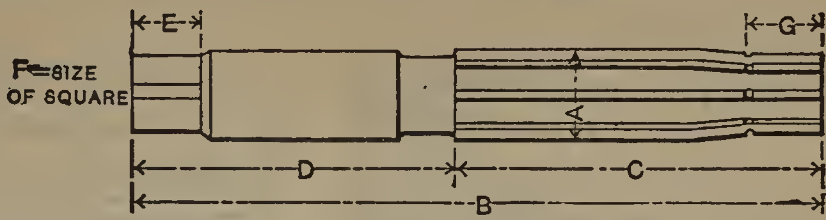
Number of flutes in reamer	4	6	8	10	12	14	16
Index circle to use	39	39	39	39	39	49	20
Before cutting	Move Spindle the Number of Holes below More or Less than for Regular Spacing						
2d flute	8 less	4 less	3 less	2 less	4 less	3 less	2 less
3d flute	4 more	5 more	5 more	3 more	4 more	2 more	2 more
4th flute	6 less	7 less	2 less	5 less	1 less	2 less	1 less
5th flute	.....	6 more	4 more	2 more	3 more	4 more	2 more
6th flute	.....	5 less	6 less	2 less	4 less	1 less	2 less
7th flute	.....	.....	2 more	3 more	4 more	3 more	1 more
8th flute	.....	.....	3 less	2 less	3 less	2 less	2 less
9th flute	.....	.....	.....	5 more	2 more	1 more	2 more
10th flute	.....	.....	.....	1 less	2 less	3 less	2 less
11th flute	.....	.....	.....	.....	3 more	3 more	1 more
12th flute	.....	.....	.....	.....	4 less	2 less	2 less
13th flute	.....	.....	.....	.....	.....	2 more	2 more
14th flute	.....	.....	.....	.....	.....	3 less	1 less
15th flute	.....	.....	.....	.....	.....	.....	2 more
16th flute	.....	.....	.....	.....	.....	.....	2 less

Threaded-end Hand Reamers. — Hand reamers are sometimes provided with a thread at the extreme point in order to give them a uniform feed when reaming. The diameter on the top of this thread at the point of the reamer is slightly smaller than the reamer itself, and the thread tapers upward until it reaches a dimension of from 0.003 to 0.008 inch, according to size, below the size of the reamer; at this point the thread stops and a short neck about 1/16 inch wide separates the threaded portion from the actual reamer which is provided with a short taper from 3/16 to 7/16 inch long up to where the standard diameter is reached. The length of the threaded portion and the number of threads per inch for reamers of this kind are given in the accompanying table. The thread employed is a sharp V-thread.

Dimensions for Threaded-end Hand Reamers

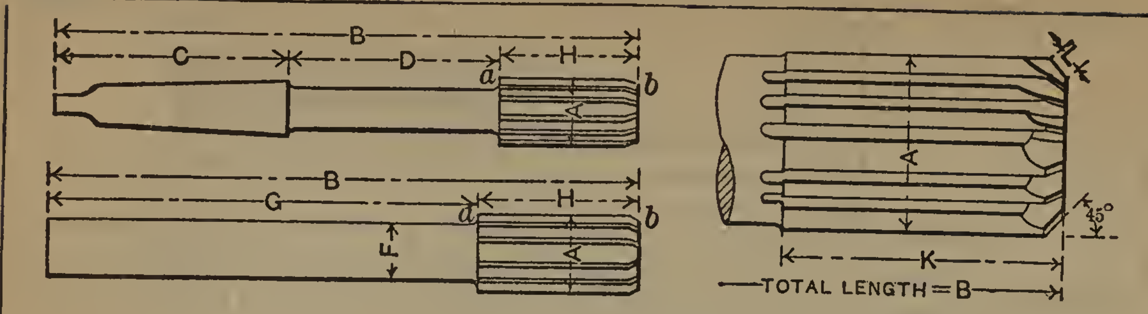
Sizes of Reamers	Length of Threaded Part	No. of Threads per Inch	Diam. of Thread at Point of Reamer	Sizes of Reamers	Length of Threaded Part	No. of Threads per Inch	Diam. of Thread at Point of Reamer
			Full diameter				Full diameter
1/8–5/16	3/8	32	−0.006	1 1/32 –1 1/2	9/16	18	−0.010
11/32–1/2	7/16	28	−0.006	1 17/32–2	9/16	18	−0.012
17/32–3/4	1/2	24	−0.008	2 1/32 –2 1/2	9/16	18	−0.015
25/32–1	9/16	18	−0.008	2 17/32–3	9/16	18	−0.020

Dimensions of Hand Reamers



Diameter	Total Length	Length of Flute	Length of Shank	Length of Squared Part	Size of Square	Length of Guide	No. of Flutes
A	B	C	D	E	F	G	
1/16	2 3/16	7/8	1 5/16	7/32	3/64	3/16	4
1/8	2 5/8	1 1/8	1 1/2	1/4	3/32	7/32	6
3/16	3 1/16	1 3/8	1 11/16	9/32	9/64	1/4	6
1/4	3 1/2	1 5/8	1 7/8	5/16	3/16	5/16	6
5/16	3 15/16	1 7/8	2 1/16	11/32	15/64	3/8	6
3/8	4 3/8	2 1/8	2 1/4	3/8	9/32	13/32	6
7/16	4 13/16	2 3/8	2 7/16	13/32	21/64	7/16	6
1/2	5 1/4	2 5/8	2 5/8	7/16	3/8	1/2	6
9/16	5 11/16	2 7/8	2 13/16	15/32	27/64	9/16	8
5/8	6 1/8	3 1/8	3	1/2	15/32	19/32	8
11/16	6 9/16	3 3/8	3 3/16	17/32	33/64	5/8	8
3/4	7	3 5/8	3 3/8	9/16	9/16	11/16	8
13/16	7 7/16	3 7/8	3 9/16	19/32	39/64	3/4	8
7/8	7 7/8	4 1/8	3 3/4	5/8	21/32	25/32	8
15/16	8 5/16	4 3/8	3 15/16	21/32	45/64	13/16	8
1	8 3/4	4 5/8	4 1/8	11/16	3/4	7/8	8
1 1/8	9 3/8	4 15/16	4 7/16	3/4	27/32	15/16	8
1 1/4	9 3/4	5 3/16	4 9/16	13/16	15/16	1	8
1 3/8	10 1/8	5 3/8	4 3/4	7/8	1 1/32	1 1/16	10
1 1/2	10 1/2	5 5/8	4 7/8	15/16	1 1/8	1 1/8	10
1 5/8	10 7/8	5 13/16	5 1/16	1	1 7/32	1 3/16	10
1 3/4	11 1/4	6 1/16	5 3/16	1 1/16	1 5/16	1 1/4	10
1 7/8	11 5/8	6 1/4	5 3/8	1 1/8	1 13/32	1 5/16	12
2	12	6 1/2	5 1/2	1 3/16	1 1/2	1 3/8	12
2 1/8	12 3/8	6 11/16	5 11/16	1 1/4	1 19/32	1 7/16	12
2 1/4	12 3/4	6 15/16	5 13/16	1 5/16	1 11/16	1 1/2	12
2 3/8	13 1/8	7 1/8	6	1 3/8	1 25/32	1 9/16	14
2 1/2	13 1/2	7 3/8	6 1/8	1 7/16	1 7/8	1 5/8	14
2 5/8	13 7/8	7 9/16	6 5/16	1 1/2	1 31/32	1 11/16	14
2 3/4	14 1/4	7 13/16	6 7/16	1 9/16	2 1/16	1 3/4	14
2 7/8	14 5/8	8	6 5/8	1 5/8	2 5/32	1 13/16	16
3	15	8 1/4	6 3/4	1 11/16	2 1/4	1 7/8	16

Dimensions of Fluted and Rose Chucking Reamers

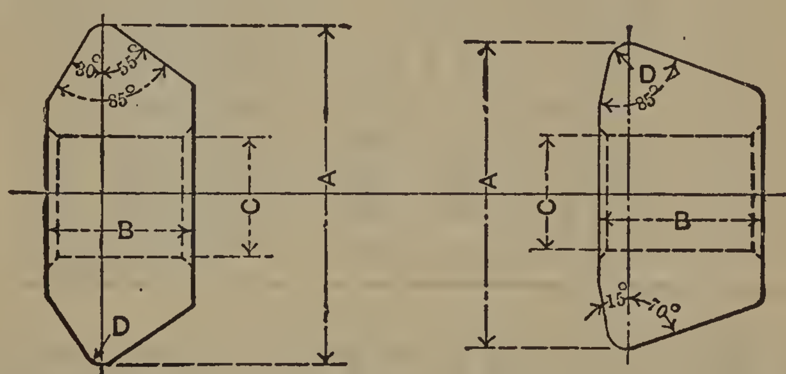


Diameter of Reamer	Total Length, Fluted and Rose Type	Morse Taper Shanks: Number of Shank	Morse Taper Shanks: Length of Shank	Morse Taper Shanks: Length of Neck, Fluted Type	Straight Shank: Diameter of Shank	Straight Shank: Length of Shank, Fluted Type	Fluted Type, Length of Flute	Rose Type: Length of Body	Rose Type: Cutting Edge	Number of Flutes or Cutting Edges
A	B		C	D	F	G	H	K	L	
1/4	6	1	2 9/16	2 7/16	3/16	5	1	1 1/2	1/32	6
5/16	6	1	2 9/16	2 7/16	1/4	5	1	1 1/2	1/32	6
3/8	7	1	2 9/16	3 5/16	5/16	5 7/8	1 1/8	1 3/4	1/32	6
7/16	7	1	2 9/16	3 5/16	3/8	5 7/8	1 1/8	1 3/4	3/64	6
1/2	8 1/8	1	2 9/16	4 5/16	7/16	6 7/8	1 1/4	2	3/64	6
9/16	8 1/8	1	2 9/16	4 5/16	1/2	6 7/8	1 1/4	2	3/64	8
5/8	9 1/8	2	3 1/16	4 11/16	9/16	7 3/4	1 3/8	2 1/4	3/64	8
11/16	9 1/8	2	3 1/16	4 11/16	9/16	7 3/4	1 3/8	2 1/4	1/16	8
3/4	9 5/8	2	3 1/16	5 1/16	5/8	8 1/8	1 1/2	2 1/2	1/16	8
7/8	10 1/4	2	3 1/16	5 7/16	3/4	8 1/2	1 3/4	2 5/8	1/16	8
1	10 3/4	3	3 3/4	5 1/8	7/8	8 7/8	1 7/8	2 3/4	5/64	8
1 1/8	11 1/4	3	3 3/4	5 1/2	1	9 1/4	2	2 7/8	5/64	10
1 1/4	11 3/4	4	4 3/4	4 7/8	1 1/16	9 5/8	2 1/8	3	3/32	10
1 3/8	12 1/4	4	4 3/4	5 1/4	1 3/16	10	2 1/4	3 1/4	3/32	10
1 1/2	12 3/4	4	4 3/4	5 5/8	1 1/4	10 3/8	2 3/8	3 1/2	7/64	10
1 5/8	13 1/4	4	4 3/4	6	1 5/16	10 3/4	2 1/2	3 3/4	7/64	12
1 3/4	13 3/4	5	6	5 1/8	1 3/8	11 1/8	2 5/8	4	1/8	12
1 7/8	14 1/4	5	6	5 1/2	1 7/16	11 1/2	2 3/4	4 1/4	1/8	12
2	14 1/4	5	6	5 1/2	1 1/2	11 1/2	2 3/4	4 1/4	1/8	12
2 1/8	14 3/4	5	6	5 7/8	1 9/16	11 7/8	2 7/8	4 1/2	5/32	14
2 1/4	15 1/4	5	6	6 1/4	1 5/8	12 1/4	3	4 1/2	5/32	14
2 3/8	15 1/4	5	6	6 1/4	1 11/16	12 1/4	3	4 3/4	5/32	14
2 1/2	15 1/2	5	6	6 3/8	1 11/16	12 3/8	3 1/8	4 3/4	3/16	14
2 3/4	15 3/4	5	6	6 1/2	1 3/4	12 1/2	3 1/4	5	3/16	16
3	16 1/4	5	6	6 3/4	1 3/4	12 3/4	3 1/2	5 1/4	7/32	16

Fluted Chucking Reamers. — These reamers are used in machines for enlarging holes and finishing them smooth and true to size. They are provided with either a straight or standard taper shank. They are not intended for removing a large amount of metal, 0.005 to 0.010 inch being all that should be required. The cutting edges are along the lines *ab* (see illustration given in connection with the table of dimensions of these reamers). At the front end there is a slight round, as shown at *b*. The diameter of the neck between the fluted part of the reamer and the taper shank should be about 1/32 inch smaller than either the diameter of the reamer or the diameter of the large end of the shank, depending upon which of these two diameters is the smaller. The object of this is to have the neck small enough in diameter so that the grinding wheel will clear that portion when the reamer part and the shank are ground.

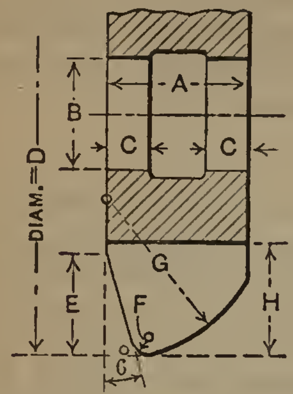
Rose Chucking Reamers. — These reamers are used for enlarging cored holes and are so constructed that they are able to remove a considerable amount of metal. As shown in the engraving with the table giving dimensions of these reamers, the cutting edges are on a 45-degree bevel on the end of the reamer. The cylindrical part of the reamer has no cutting edges, but merely grooves cut for the full length of the reamer body, providing a way for the chips to escape and a channel for lubricant to reach the cutting edges. There is no relief on the cylindrical surface of the body part, but it is slightly back-tapered so that the diameter at the point with the beveled cutting edges is slightly larger than the diameter further back. The back-taper should not exceed 0.001 inch per inch. This form of reamer usually

Fluting Cutters for Reamers



Diameter of Reamer	Diameter of Fluting Cutter A	Thickness of Fluting Cutter B	Diameter of Hole in Cutter C	Radius between Cutting Faces of Cutter D	Diameter of Reamer	Diameter of Fluting Cutter A	Thickness of Fluting Cutter B	Diameter of Hole in Cutter C	Radius between Cutting Faces of Cutter D
⅛	1¾	3/16	¾	sharp corner, no radius.	1	2¼	½	1	3/64
					1¼	2¼	9/16	1	1/16
3/16	1¾	3/16	¾	sharp corner, no radius.	1½	2¼	⅝	1	1/16
					1¾	2¼	⅝	1	5/64
¼	1¾	3/16	¾	1/64	2	2½	¾	1	5/64
⅜	2	¼	¾	1/64	2¼	2½	¾	1	5/64
½	2	5/16	¾	1/32	2½	2½	⅞	1	3/16
⅝	2	⅜	¾	1/32	2¾	2½	⅞	1	3/16
¾	2	7/16	¾	3/64	3	2½	1	1	3/16

Dimensions of Formed Reamer Fluting Cutters



The making and maintenance of cutters of the formed type involves greater expense than the use of the angular cutters of which dimensions are given on the previous page; but the form of flute produced by the formed type of cutter is preferred by many reamer users. The claims made for the formed type of flute are that the chips can be more readily removed from the reamer, and that the reamer has greater strength and is less likely to crack or spring out of shape in hardening.

Size of Reamers used for	Number of Teeth in Reamer	Diam. of Cutter	Width of Cutter	Diam. of Hole	Width of Bearing	Length of Bevel	Radius	Radius	Depth of Tooth	Number of Teeth in Cutter
		D	*A*	*B*	*C*	*E*	*F*	*G*	*H*	
1/8–3/16	6	1 3/4	3/16	7/8	....	0.125	0.016	7/32	0.21	14
1/4–5/16	6	1 3/4	1/4	7/8	....	0.152	0.022	9/32	0.25	13
3/8–7/16	6	1 7/8	3/8	7/8	1/8	0.178	0.029	1/2	0.28	12
1/2–11/16	6–8	2	7/16	7/8	1/8	0.205	0.036	9/16	0.30	12
3/4–1	8	2 1/8	1/2	7/8	5/32	0.232	0.042	11/16	0.32	12
1 1/16–1 1/2	10	2 1/4	9/16	7/8	5/32	0.258	0.049	3/4	0.38	11
1 9/16–2 1/8	12	2 3/8	5/8	7/8	3/16	0.285	0.056	27/32	0.40	11
2 1/4 –3	14	2 5/8	11/16	7/8	3/16	0.312	0.062	7/8	0.44	10

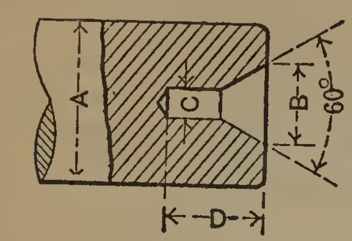
produces holes slightly larger than its size and it is, therefore, always made from 0.005 to 0.010 inch smaller than its nominal size, so that it may be followed by a fluted reamer for finishing. The grooves on the cylindrical portion are cut by a convex cutter having a width equal to from one-fifth to one-fourth the diameter of the rose reamer itself. The depth of the groove should be from one-eighth to one-sixth the diameter of the reamer. The teeth at the end of the reamer are milled with a 75-degree angular cutter; the width of the land of the cutting edge should be about one-fifth the distance from tooth to tooth. If an angular cutter is preferred to a convex cutter for milling the grooves on the cylindrical portion, because of the higher cutting speed possible when milling, an 80-degree angular cutter slightly rounded at the point may be used.

Cutters for Fluting Rose Chucking Reamers. — The cutters used for fluting rose chucking reamers on the end are 80-degree angular cutters for 1/4 and 5/16 inch diameter reamers; 75-degree angular cutters for 3/8 and 7/16 inch reamers; and 70-degree angular cutters for all larger sizes. The grooves on the cylindrical portion are milled with convex cutters of approximately the following sizes for given diameters of reamers: 5/32-inch convex cutter for 1/2-inch reamers; 5/16-inch cutter for 1-inch reamers; 3/8-inch cutter for 1 1/2-inch reamers; 13/32-inch cutters for 2-inch reamers; and 15/32-inch cutters for 2 1/2-inch reamers. The smaller sizes of reamers, from 1/4 to 3/8 inch in diameter, are often milled with regular double-angle reamer fluting cutters having a radius of 1/64 inch for 1/4-inch reamer, and 1/32 inch for 5/16- and 3/8-inch sizes.

Vertical Adjustment of Tooth-rest for Grinding Clearance on Reamers

Size of Reamer	Hand Reamer for Steel. Cutting Clearance Land 0.006 inch Wide		Hand Reamer for Cast Iron and Bronze. Cutting Clearance Land 0.025 inch Wide		Chucking Reamer for Cast Iron and Bronze. Cutting Clearance Land 0.025 inch Wide		Rose Chucking Reamers for Steel
	For Cutting Clearance	For Second Clearance	For Cutting Clearance	For Second Clearance	For Cutting Clearance	For Second Clearance	For Cutting Clearance on Angular Edge at End
½	0.012	0.052	0.032	0.072	0.040	0.080	0.080
⅝	0.012	0.062	0.032	0.072	0.040	0.090	0.090
¾	0.012	0.072	0.035	0.095	0.040	0.100	0.100
⅞	0.012	0.082	0.040	0.120	0.045	0.125	0.125
1	0.012	0.092	0.040	0.120	0.045	0.125	0.125
1⅛	0.012	0.102	0.040	0.120	0.045	0.125	0.125
1¼	0.012	0.112	0.045	0.145	0.050	0.160	0.160
1⅜	0.012	0.122	0.045	0.145	0.050	0.160	0.175
1½	0.012	0.132	0.048	0.168	0.055	0.175	0.175
1⅝	0.012	0.142	0.050	0.170	0.060	0.200	0.200
1¾	0.012	0.152	0.052	0.192	0.060	0.200	0.200
1⅞	0.012	0.162	0.056	0.196	0.060	0.200	0.200
2	0.012	0.172	0.056	0.216	0.064	0.224	0.225
2⅛	0.012	0.172	0.059	0.219	0.064	0.224	0.225
2¼	0.012	0.172	0.063	0.223	0.064	0.224	0.225
2⅜	0.012	0.172	0.063	0.223	0.068	0.228	0.230
2½	0.012	0.172	0.065	0.225	0.072	0.232	0.230
2⅝	0.012	0.172	0.065	0.225	0.075	0.235	0.235
2¾	0.012	0.172	0.065	0.225	0.077	0.237	0.240
2⅞	0.012	0.172	0.070	0.230	0.080	0.240	0.240
3	0.012	0.172	0.072	0.232	0.080	0.240	0.240
3⅛	0.012	0.172	0.075	0.235	0.083	0.240	0.240
3¼	0.012	0.172	0.078	0.238	0.083	0.243	0.245
3⅜	0.012	0.172	0.081	0.241	0.087	0.247	0.245
3½	0.012	0.172	0.084	0.244	0.090	0.250	0.250
3⅝	0.012	0.172	0.087	0.247	0.093	0.253	0.250
3¾	0.012	0.172	0.090	0.250	0.097	0.257	0.255
3⅞	0.012	0.172	0.093	0.253	0.100	0.260	0.255
4	0.012	0.172	0.096	0.256	0.104	0.264	0.260
4⅛	0.012	0.172	0.096	0.256	0.104	0.264	0.260
4¼	0.012	0.172	0.096	0.256	0.106	0.266	0.265
4⅜	0.012	0.172	0.096	0.256	0.108	0.268	0.265
4½	0.012	0.172	0.100	0.260	0.108	0.268	0.265
4⅝	0.012	0.172	0.100	0.260	0.110	0.270	0.270
4¾	0.012	0.172	0.104	0.264	0.114	0.274	0.275
4⅞	0.012	0.172	0.106	0.266	0.116	0.276	0.275
5	0.012	0.172	0.110	0.270	0.118	0.278	0.275

Dimensions of Centers for Reamers and Arbors



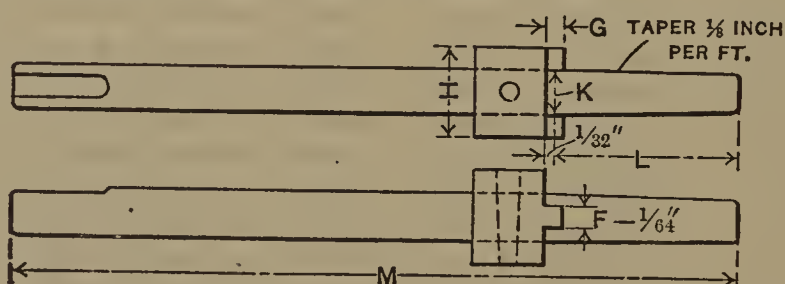
Diameter of Arbor A	Largest Diameter of Center B	Number of Drill C	Depth of Hole D
1/4	1/8	55	5/32
5/16	5/32	52	3/16
3/8	3/16	48	7/32
7/16	7/32	43	1/4
1/2	1/4	39	5/16
9/16	9/32	33	11/32
5/8	5/16	30	3/8
11/16	11/32	29	13/32

Diameter of Arbor A	Largest Diameter of Center B	Number of Drill C	Depth of Hole D
3/4	3/8	25	7/16
13/16	13/32	20	1/2
7/8	7/16	17	17/32
15/16	15/32	12	9/16
1	1/2	8	19/32
1 1/8	33/64	5	5/8
1 1/4	17/32	3	21/32
1 3/8	35/64	2	21/32
1 1/2	9/16	1	11/16
....	....	Letter	...
1 5/8	37/64	A	23/32
1 3/4	19/32	B	23/32
1 7/8	39/64	C	3/4
2	5/8	E	3/4
2 1/8	41/64	F	25/32
2 1/4	21/32	G	13/16
2 3/8	43/64	H	27/32

Diameter of Arbor A	Largest Diameter of Center B	Letter of Drill C	Depth of Hole D
2 1/2	11/16	J	27/32
2 5/8	45/64	K	7/8
2 3/4	23/32	L	29/32
2 7/8	47/64	M	29/32
3	3/4	N	15/16
3 1/8	49/64	N	31/32
3 1/4	25/32	O	31/32
3 3/8	51/64	O	1
3 1/2	13/16	P	1
3 5/8	53/64	Q	1 1/16
3 3/4	27/32	R	1 1/16
3 7/8	55/64	R	1 1/16
4	7/8	S	1 1/8
4 1/4	29/32	T	1 1/8
4 1/2	15/16	V	1 3/16
4 3/4	31/32	W	1 1/4
5	1	X	1 1/4

Dimensions of Shell Reamer Arbors

F, G and H are given in table on next page



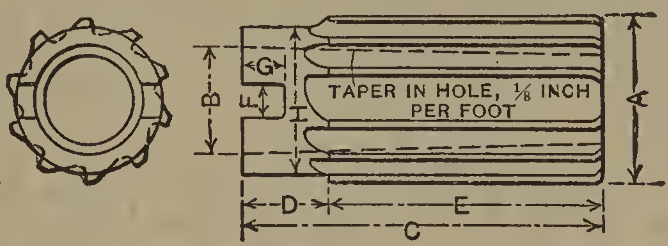
Diam. at Size Line K	Length from Size Line to End of Arbor L	Total Length M
1/8	1 1/2	6
3/16	1 3/4	7
1/4	2	8
3/8	2 1/4	9
1/2	2 1/2	9 1/2

Diam. at Size Line K	Length from Size Line to End of Arbor L	Total Length M
5/8	2 3/4	10
3/4	3	11
1	3 1/2	12
1 1/4	3 3/4	13
1 1/2	4	14

Diam. at Size Line K	Length from Size Line to End of Arbor L	Total Length M
1 3/4	4 1/2	15
2	5	16
2 1/4	5 1/2	17
2 1/2	6	18
.....	......	.....

Shell Reamers. — In order to save the material which goes into the shank, shell reamers having a hole through the center, by means of which they are mounted on arbors, are largely used. As one arbor can be used for several sizes of reamers, the saving is quite considerable. Arbors, as well as driving collars, for shell reamers should, however, preferably be made out of tool steel and the collars should be hardened. These reamers may be fluted either like fluted chucking reamers or like rose chucking reamers. The negative front rake on shell reamers should not be more than about 3 degrees.

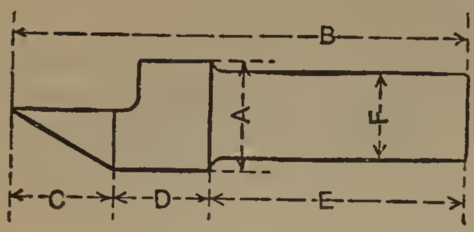
Dimensions of Shell Reamers

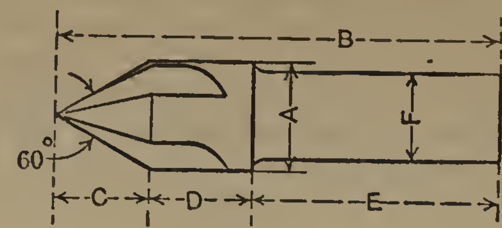


Diam. of Reamers	Diam. of Hole, Large End	Total Length	Length of Turned-down Portion	Length of Flutes	Width of Key-way	Depth of Key-way	Constant for finding Diam. H	No. of Flutes
A	*B*	*C*	*D*	*E*	*F*	*G*	*A–H*	
1/4 – 5/16	1/8	1 1/2	3/8	1 1/8	1/16	1/8	0.006	6
11/32 – 7/16	3/16	1 3/4	3/8	1 3/8	3/32	1/8	0.006	6
15/32 – 9/16	1/4	2	1/2	1 1/2	7/64	5/32	1/64	8
19/32 – 11/16	3/8	2 1/4	1/2	1 3/4	9/64	3/16	1/64	8
23/32 – 15/16	1/2	2 1/2	1/2	2	11/64	1/4	1/32	10
31/32 – 1 1/4	5/8	2 3/4	5/8	2 1/8	13/64	1/4	1/16	10
1 9/32 – 1 5/8	3/4	3	5/8	2 3/8	1/4	5/16	1/8	12
1 21/32 – 2	1	3 1/2	5/8	2 7/8	1/4	5/16	1/8	12
2 1/32 – 2 1/2	1 1/4	3 3/4	3/4	3	5/16	3/8	1/8	14
2 17/32 – 3	1 1/2	4	3/4	3 1/4	5/16	3/8	1/8	14
3 1/32 – 3 1/2	1 3/4	4 1/2	1	3 1/2	5/16	3/8	1/8	16
3 17/32 – 4	2	5	1	4	5/16	3/8	1/8	16
4 1/32 – 4 1/2	2 1/4	5 1/2	1	4 1/2	5/16	3/8	1/8	18
4 17/32 – 5	2 1/2	6	1	5	5/16	3/8	1/8	18

Center Reamers. — Center reamers are made in two different styles, as shown in the engraving with the table of these tools. The older style has only one cutting edge formed by cutting away the metal down to the center of the tool and relieving the beveled portion of the remaining half so that a cutting edge is produced. The second and later style is provided with four flutes or cuts. These are straight and the lands between them are relieved on the beveled part. The included angle of the point is, of course, the same as that used for lathe centers, or 60 degrees. Use regular side milling cutters for fluting center reamers. Use a 2½-inch cutter for a ¼-inch center reamer, and increase the cutter ¼ inch in diameter for each ⅛ inch increase in center reamer size. This gives a 4-inch cutter for a 1-inch center reamer.

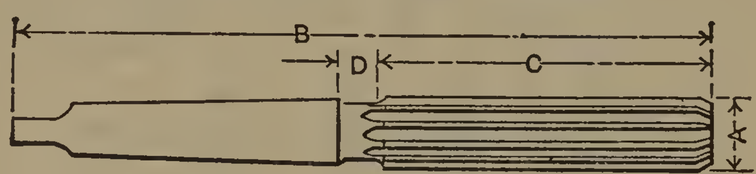
Dimensions of Center Reamers





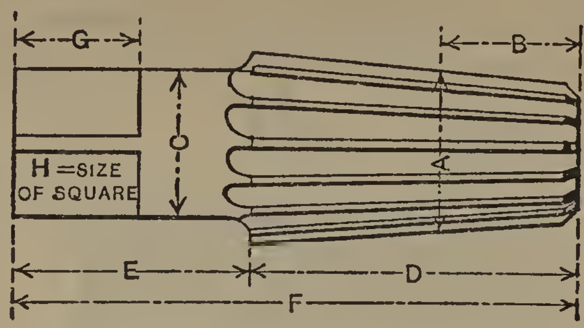
Full Diameter of Reamer	Total Length	Length of Beveled Portion, Approx.	Length of Straight Portion	Length of Shank	Diameter of Shank	Full Diameter of Reamer	Total Length	Length of Beveled Portion, Approx.	Length of Straight Portion	Length of Shank	Diameter of Shank
A	*B*	*C*	*D*	*E*	*F*	*A*	*B*	*C*	*D*	*E*	*F*
1/4	1 3/8	7/32	5/32	1	3/16	3/4	2 5/8	21/32	23/32	1 1/4	1/2
3/8	1 5/8	5/16	5/16	1	1/4	7/8	2 7/8	3/4	3/4	1 3/8	9/16
1/2	2	7/16	7/16	1 1/8	3/8	1	3 1/8	7/8	3/4	1 1/2	5/8
5/8	2 3/8	17/32	19/32	1 1/4	7/16	.....	....	......	....	....	....

Dimensions of Jobbers' Reamers



Diameter of Reamer	Total Length	Length of Flute	Length of Neck	No. of Morse Taper Shank	No. of Flutes	Diameter of Reamer	Total Length	Length of Flute	Length of Neck	No. of Morse Taper Shank	No. of Flutes
A	*B*	*C*	*D*			*A*	*B*	*C*	*D*		
1/4	5 3/16	2	5/8	1	6	1 3/8	12 13/16	6 5/16	1 3/4	4	10
5/16	5 1/2	2 1/4	11/16	1	6	1 1/2	13 1/8	6 1/2	1 7/8	4	10
3/8	5 13/16	2 1/2	3/4	1	6	1 5/8	13 1/8	6 1/2	1 7/8	4	10
7/16	6 1/8	2 3/4	13/16	1	6	1 3/4	14 11/16	6 3/4	1 15/16	5	10
1/2	6 7/16	3	7/8	1	6	1 7/8	15	7	2	5	12
9/16	6 3/4	3 1/4	15/16	1	8	2	15	7	2	5	12
5/8	7 9/16	3 1/2	1	2	8	2 1/8	15 1/2	7 1/4	2 1/4	5	12
11/16	8	3 7/8	1 1/16	2	8	2 1/4	15 1/2	7 1/4	2 1/4	5	12
3/4	8 3/8	4 3/16	1 1/8	2	8	2 3/8	16	7 1/2	2 1/2	5	14
7/8	9 3/16	4 7/8	1 1/4	2	8	2 1/2	16	7 1/2	2 1/2	5	14
1	10 3/8	5 5/16	1 5/16	3	8	2 3/4	16 1/2	7 3/4	2 3/4	5	14
1 1/8	10 7/8	5 13/16	1 5/16	3	8	3	17	8	3	5	16
1 1/4	12 9/16	6 1/8	1 11/16	4	8	....	....	....	....	...	...

Dimensions of Pipe Reamers



Taper, ¾ inch per foot.

Pipe Size	Diam. at Size Line	Distance from Size Line to Small End	Diam. of Shank	Length of Fluted Part	Length of Shank	Total Length	Length of Square	Size of Square	No. of Flutes
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	
⅛	0.345	25/64	11/32	1	1⅝	2⅝	½	¼	6
¼	0.445	9/16	7/16	1⅛	1¾	2⅞	9/16	5/16	6
⅜	0.582	9/16	9/16	1¼	1⅞	3⅛	⅝	7/16	6
½	0.721	¾	¾	1½	2	3½	11/16	9/16	8
¾	0.932	¾	15/16	1⅝	2¼	3⅞	¾	11/16	8
1	1.169	15/16	1⅛	1¾	2½	4¼	13/16	13/16	10
1¼	1.514	31/32	1 5/16	1⅞	2¾	4⅝	1	1	10
1½	1.753	1	1½	2	3	5	1⅛	1⅛	12
2	2.228	1	1⅞	2¼	3½	5¾	1⅜	1⅜	12
2½	2.662	1½	2¼	2⅞	4	6⅞	1 11/16	1 11/16	14
3	3.288	1 9/16	2⅝	3¼	4½	7¾	1 15/16	1 15/16	14
3½	3.789	1⅝	2 13/16	3⅝	4 9/16	8 3/16	2⅛	2⅛	16
4	4.287	1 11/16	3	3¾	4⅝	8⅜	2¼	2¼	18

Taper Reamers. — The most commonly used taper reamers are those used for standard taper pins and for the standard taper sockets — Morse, Brown & Sharpe and Jarno. Reamers for taper sockets are usually made in sets of two, a roughing reamer and a finishing reamer. The roughing reamer is generally made smaller at the small end than the finishing reamer and is provided with a groove cut like a thread along the cutting edges, as indicated in the illustration accompanying the tables for taper reamers for standard taper sockets. The purpose of this groove is to break up the chips. On ordinary right-hand reamers this groove is cut left-hand. The sides of the groove incline at an included angle of 29 degrees, making it possible to use the point of an Acme threading tool for cutting the groove.

In connection with these reamers, tables are also given of the Morse and Brown & Sharpe taper sockets. No table is given for the Jarno taper sockets, as this is based on such simple formulas that practically no calculations are required when the number of the taper is known. The taper per foot of all Jarno taper sizes is 0.600 inch on the diameter. The diameter at the large end of the taper is as many eighths, the diameter at the small end, as many tenths, and the length, as many half inches as are indicated by the number of the taper. For example, a No. 7 Jarno taper is 7/8 inch in diameter at the large end; 7/10, or 0.700 inch at the small end; and 7/2, or 3½ inches long. Expressing these rules as formulas: N = number of Jarno taper; D = diameter at large end; d = diameter at small end; L = length of taper.

$$D = \frac{N}{8} \qquad d = \frac{N}{10} \qquad L = \frac{N}{2}$$

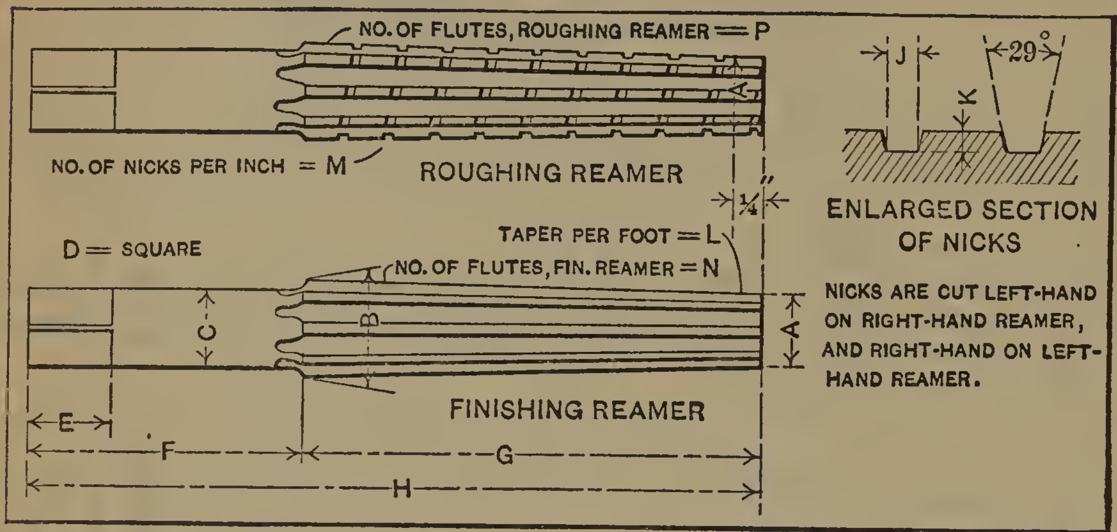


Illustration giving Notation used in Tables of Taper Reamers for Standard Sockets

Taper Reamers for Jarno Standard Taper Sockets

Number of Taper	Diameter at Small End	Diameter at Large End	Diameter of Shank	Size of Square	Length of Square	Length of Flute	Total Length	Width at Bottom of Groove	Depth of Groove	Taper per Foot	Number of Nicks per Inch	Number of Flutes, Fin. Reamer	No. of Flutes, Roughing Reamer
	A	*B*	*C*	*D*	*E*	*G*	*H*	*J*	*K*	*L*	*M*	*N*	*P*
1	0.100	0.144	1/8	3/32	3/16	7/8	1 3/4	1/32	0.025	0.600	3 1/2	6	4
2	0.200	0.269	15/64	11/64	9/32	1 3/8	2 5/8	1/32	0.025	0.600	3 1/2	6	4
3	0.300	0.400	3/8	9/32	3/8	2	3 1/2	1/32	0.025	0.600	3 1/2	6	4
4	0.400	0.531	15/32	23/64	1/2	2 5/8	4 3/8	3/64	0.025	0.600	3 1/2	6	4
5	0.500	0.659	19/32	29/64	5/8	3 3/16	5 1/8	3/64	0.025	0.600	3	6	4
6	0.600	0.787	11/16	33/64	5/8	3 3/4	5 7/8	1/16	1/32	0.600	3	8	5
7	0.700	0.916	13/16	39/64	3/4	4 5/16	6 5/8	1/16	1/32	0.600	3	8	5
8	0.800	1.044	15/16	45/64	7/8	4 7/8	7 3/8	5/64	1/32	0.600	3	8	5
9	0.900	1.169	1 1/16	51/64	7/8	5 3/8	8 1/8	5/64	1/32	0.600	3	8	5
10	1.000	1.297	1 1/8	27/32	1	5 15/16	8 7/8	3/32	3/64	0.600	2 1/2	8	6
11	1.100	1.422	1 3/16	57/64	1	6 7/16	9 1/2	3/32	3/64	0.600	2 1/2	8	6
12	1.200	1.550	1 5/16	63/64	1 1/16	7	10 1/8	3/32	3/64	0.600	2 1/2	8	6
13	1.300	1.675	1 7/16	1 5/64	1 1/16	7 1/2	10 3/4	7/64	3/64	0.600	2 1/2	10	6
14	1.400	1.800	1 1/2	1 1/8	1 1/8	8	11 3/8	7/64	3/64	0.600	2 1/2	10	6
15	1.500	1.928	1 5/8	1 7/32	1 1/8	8 9/16	12	7/64	3/64	0.600	2 1/2	10	6
16	1.600	2.053	1 3/4	1 5/16	1 1/8	9 1/16	12 5/8	7/64	3/64	0.600	2 1/2	10	6
17	1.700	2.181	1 13/16	1 23/64	1 1/4	9 5/8	13 3/8	7/64	3/64	0.600	2 1/2	10	6
18	1.800	2.306	1 7/8	1 13/32	1 1/4	10 1/8	14	1/8	1/16	0.600	2 1/2	12	6
19	1.900	2.431	1 15/16	1 29/64	1 1/4	10 5/8	14 5/8	1/8	1/16	0.600	2 1/2	12	6
20	2.000	2.556	2	1 1/2	1 3/8	11 1/8	15 1/4	1/8	1/16	0.600	2 1/2	12	6

Taper Reamers for Brown & Sharpe Standard Taper Sockets *

(See illustration on opposite page for notation used in table.)

Number of Taper	Diam. 1/16 Inch from Small End	Diameter at Large End	Total Length	Length of Flute	Diameter of Shank	Distance Across Flats	Distance Across Corners	Length of Square	Length of Shoulder for Square	Depth of Nicks	Width of Nicks	Number of Flutes
		B	*H*	*G*	*C*	*D*		*E*		*K*	*J*	
1	0.200	0.317	4 3/4	2 7/8	1/4	1/4	0.337	3/8	1/2	0.010	1/32	6
2	0.250	0.377	5 1/8	3 1/8	5/16	1/4	0.337	3/8	1/2	0.010	1/32	6
3	0.3125	0.450	5 1/2	3 3/8	3/8	5/16	0.409	7/16	5/8	1/64	1/32	6
4	0.350	0.501	5 7/8	3 11/16	7/16	3/8	0.499	1/2	5/8	1/64	1/32	6
5	0.450	0.614	6 3/8	4	9/16	7/16	0.589	1/2	11/16	1/64	1/32	8
6	0.500	0.680	6 7/8	4 3/8	5/8	1/2	0.679	9/16	3/4	1/64	1/32	8
7	0.600	0.800	7 1/2	4 7/8	3/4	5/8	0.803	11/16	15/16	1/32	1/16	8
8	0.750	0.976	8 1/8	5 1/2	7/8	11/16	0.924	3/4	1	1/32	1/16	8
9	0.900	1.152	8 7/8	6 1/8	1 1/16	7/8	1.194	7/8	1 1/8	1/32	1/16	8
10	1.0446	1.337	9 3/4	6 7/8	1 3/16	1	1.290	1	1 1/4	1/32	1/16	10
11	1.250	1.565	10 5/8	7 5/8	1.450	1 1/8	1.450	1 1/8	.....	1/32	1/16	10
12	1.500	1.841	11 3/8	8 1/4	1.450	1 1/8	1.450	1 1/8	.....	1/32	1/16	10
13	1.750	2.111	12	8 3/4	1.620	1 1/4	1.620	1 1/4	.....	1/32	1/16	10
14	2.000	2.382	12 1/2	9 1/4	1.620	1 1/4	1.620	1 1/4	.....	1/32	1/16	10
15	2.250	2.654	13 1/8	9 3/4	1.950	1 1/2	1.950	1 3/8	.....	1/32	1/16	10
16	2.500	2.924	13 1/2	10 1/4	1.950	1 1/2	1.950	1 3/8	.....	1/32	1/16	12
17	2.750	3.195	13 3/4	10 3/4	1.950	1 1/2	1.950	1 3/8	.....	1/32	1/16	12
18	3.000	3.466	14 1/4	11 1/4	1.950	1 1/2	1.950	1 3/8	.....	1/32	1/16	12

* Roughing and finishing reamers are the same except for the nicks in the teeth of roughing reamers; these grooves or nicks make three turns per inch and the depth and width are given in columns *K* and *J*.

The taper of both roughing and finishing reamers is 1/2 inch per foot except No. 10 which tapers 0.5161 inch per foot.

Taper Reamers for Morse Standard Taper Sockets

(See illustration on opposite page for notation used in table)

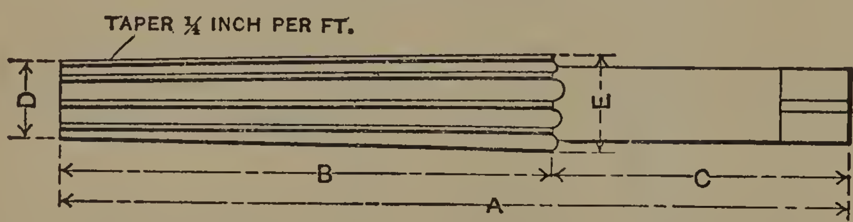
Number of Taper	Diameter at Small End	Diameter at Large End	Diameter of Shank	Size of Square	Length of Square	Length of Flute	Total Length	Width at Bottom of Groove	Depth of Groove	Taper per Foot	Number of Nicks per Inch	Number of Flutes, Fin. Reamer	No. of Flutes Roughing Reamer
	A	*B*	*C*	*D*	*E*	*G*	*H*	*J*	*K*	*L*	*M*	*N*	*P*
0	0.252	0.369	11/32	1/4	5/16	2 1/4	3 3/4	1/32	0.025	0.625	3 1/2	6	4
1	0.369	0.510	15/32	23/64	1/2	2 13/16	4 11/16	3/64	0.025	0.600	3 1/2	6	4
2	0.572	0.741	21/32	1/2	5/8	3 3/8	5 5/8	1/16	1/32	0.602	3	8	5
3	0.778	0.979	7/8	21/32	13/16	4	6 5/8	5/64	1/32	0.602	3	8	5
4	1.020	1.280	1 1/8	27/32	1	5	8	3/32	3/64	0.623	2 1/2	8	6
5	1.475	1.790	1 1/2	1 1/8	1 1/8	6	9 3/8	7/64	3/64	0.630	2 1/2	10	6
6	2.116	2.559	2	1 1/2	1 5/16	8 1/2	12 1/4	1/8	1/16	0.626	2 1/2	14	8
7	2.750	3.375	2 5/8	1 31/32	1 1/2	12	16 1/8	9/64	1/16	0.625	2 1/2	14	8

Taper Reamers for the American Taper Sockets

(See illustration on page 1262 for notation used in table)

Number of Taper	Diameter at Small End	Diameter at Large End	Diameter of Shank	Size of Square	Length of Square	Length of Flute	Total Length	Width at Bottom of Groove	Depth of Groove	Taper per Foot	Number of Nicks per Inch	Number of Flutes, Fin. Reamer	No. of Flutes, Roughing Reamer
	A	*B*	*C*	*D*	*E*	*G*	*H*	*J*	*K*	*L*	*M*	*N*	*P*
1	0.300	0.435	⅜	9/32	⅜	2¾	4¾	1/32	0.025	0.591	3½	6	4
2	0.416	0.574	½	⅜	½	3 1/16	5½	3/64	0.025	0.621	3½	6	4
3	0.586	0.750	11/16	½	⅝	3⅝	6¼	1/16	1/32	0.543	3	8	5
4	0.758	0.943	⅞	21/32	¾	3¾	6¾	5/64	1/32	0.592	3	8	5
5	1.033	1.286	1 3/16	⅞	1	4⅞	8¼	3/32	3/64	0.623	2½	8	6

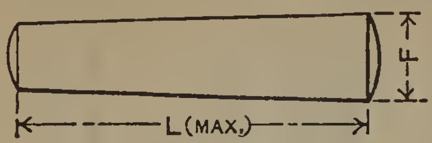
Standard Taper Pin Reamers*



No. of Taper Pin Reamer	Diameter at Large End of Reamer	Diameter at Small End of Reamer	Total Length of Reamer	Length of Cutting Edges	Length of Shank	No. of Flutes
	E	*D*	*A*	*B*	*C*	
00000	0.0984	0.075	1⅝	1⅛	½	4
0000	0.1140	0.088	1¾	1¼	½	4
000	0.1296	0.101	2	1⅜	⅝	4
00	0.1452	0.114	2¼	1½	¾	4
0	0.1608	0.127	2⅜	1⅝	¾	6
1	0.1824	0.146	2½	1¾	¾	6
2	0.2036	0.162	3	2	1	6
3	0.2298	0.183	3½	2¼	1¼	6
4	0.2600	0.208	4	2½	1½	6
5	0.3024	0.240	4½	3	1½	6
6	0.3544	0.279	5	3⅝	1⅜	6
7	0.4246	0.331	6	4½	1½	6
8	0.5072	0.398	6¾	5¼	1½	8
9	0.6094	0.482	8	6⅛	1⅞	8
10	0.7266	0.581	9	7	2	8
11	0.8776	0.706	11¼	8¼	3	8
12	1.050	0.842	13⅜	10	3⅜	......
13	1.2586	1.009	16	12	4	......
14	1.5412	1.250	18¼	14	4¼	......

* Adopted by manufacturers of reamers and taper pins.

Standard Taper Pins



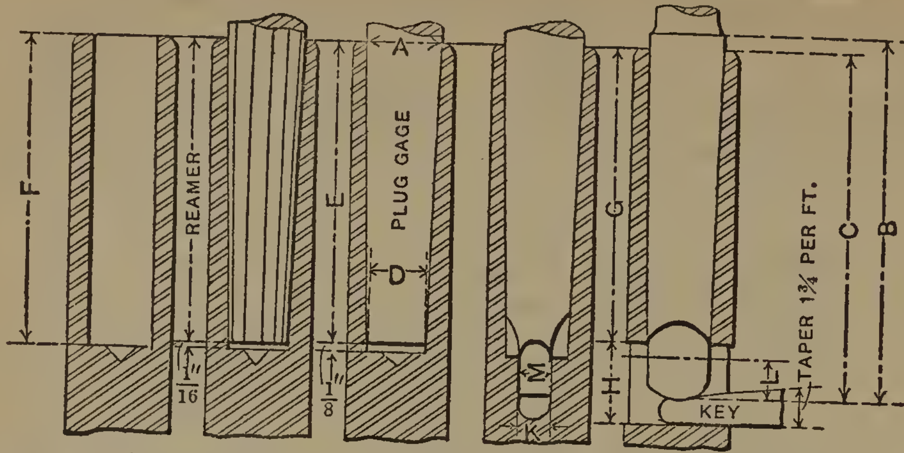
Taper, ¼ inch per foot.

No. of Taper Pin	Diam. *F* at Large End of Pin	Approx. Fractional Size *F*	Max. Length *L*
00000	0.094	3/32	¾
0000	0.109	7/64	⅞
000	0.125	⅛	1
00	0.141	9/64	1⅛
0	0.156	5/32	1¼
1	0.172	11/64	1¼
2	0.193	3/16	1½
3	0.219	7/32	1¾
4	0.250	¼	2
5	0.289	19/64	2¼
6	0.341	11/32	3
7	0.409	13/32	3¾
8	0.492	½	4½
9	0.591	19/32	5¼
10	0.706	23/32	6
11	0.860	55/64	7¼
12	1.032	1 1/32	9
13	1.241	1 15/64	11
14	1.523	1 33/64	13

Diameters at Small End of Standard Taper Pins

Size of Pin	0	1	2	3	4	5	6	7	8	9	10
Diam., Large End	0.156	0.172	0.193	0.219	0.250	0.289	0.341	0.409	0.492	0.591	0.706
Length in Inches	Diameter of Small End in Inches										
¾	0.140	0.156	0.177	0.203	0.235	0.273	0.325	0.393	0.476	0.575	0.690
1	0.135	0.151	0.172	0.198	0.230	0.268	0.320	0.388	0.471	0.570	0.685
1¼	0.130	0.146	0.167	0.192	0.224	0.263	0.315	0.382	0.466	0.565	0.680
1½	0.125	0.141	0.162	0.187	0.219	0.258	0.310	0.377	0.460	0.560	0.675
1¾	0.120	0.136	0.157	0.182	0.214	0.252	0.305	0.372	0.455	0.554	0.669
2	0.114	0.130	0.151	0.177	0.209	0.247	0.299	0.367	0.450	0.549	0.664
2¼	0.109	0.125	0.146	0.172	0.204	0.242	0.294	0.362	0.445	0.544	0.659
2½	0.104	0.120	0.141	0.166	0.198	0.237	0.289	0.356	0.440	0.539	0.654
2¾	0.099	0.115	0.136	0.161	0.193	0.232	0.284	0.351	0.434	0.534	0.649
3	0.094	0.110	0.131	0.156	0.188	0.227	0.279	0.346	0.429	0.528	0.643
3¼	...	...	...	0.151	0.182	0.221	0.273	0.340	0.424	0.523	0.638
3½	...	...	...	0.146	0.177	0.216	0.268	0.335	0.419	0.518	0.633
3¾	...	...	...	0.141	0.172	0.211	0.263	0.330	0.414	0.513	0.628
4	...	...	...	0.136	0.167	0.206	0.258	0.326	0.409	0.508	0.623
4¼	...	...	...	0.131	0.162	0.201	0.253	0.321	0.403	0.502	0.617
4½	...	...	...	0.125	0.156	0.195	0.247	0.315	0.398	0.497	0.612
5	...	...	...	...	0.146	0.185	0.237	0.305	0.389	0.487	0.602
5½	...	...	...	...	...	...	...	0.294	0.377	0.476	0.591
6	...	...	...	...	...	...	...	0.284	0.367	0.466	0.581

Brown & Sharpe Taper Shanks

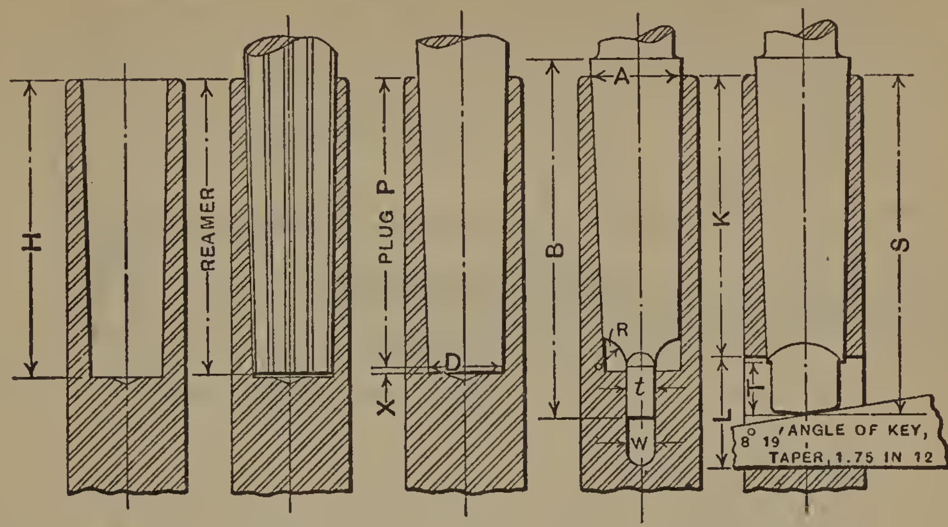


Taper per foot is ½ inch, except for No. 10 shank; where the taper is 0.5161 inch per foot.

Number of Taper	Diameter at End of Socket	Whole Length of Shank	Shank Depth	Diameter of Plug at Small End	Standard Plug Depth	Depth of Hole	End of Socket to Keyway	Length of Keyway	Width of Keyway	Length of Tongue	Thickness of Tongue
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	*L*	*M*
1	0.239	1 9/32	1 3/16	0.200	15/16	1 1/16	15/16	3/8	0.135	3/16	1/8
2	0.299	1 19/32	1 1/2	0.250	1 3/16	1 5/16	1 11/64	1/2	0.166	1/4	5/32
3	0.375	1 31/32	1 7/8	0.312	1 1/2	1 5/8	1 15/32	5/8	0.197	5/16	3/16
3	0.385	2 7/32	2 1/8	0.312	1 3/4	1 7/8	1 23/32	5/8	0.197	5/16	3/16
3	0.395	2 15/32	2 3/8	0.312	2	2 1/8	1 31/32	5/8	0.197	5/16	3/16
4	0.402	1 3/4	1 21/32	0.350	1 1/4	1 3/8	1 13/64	11/16	0.228	11/32	7/32
4	0.420	2 3/16	2 3/32	0.350	1 11/16	1 13/16	1 41/64	11/16	0.228	11/32	7/32
5	0.523	2 9/32	2 3/16	0.450	1 3/4	1 7/8	1 11/16	3/4	0.260	3/8	1/4
5	0.533	2 17/32	2 7/16	0.450	2	2 1/8	1 15/16	3/4	0.260	3/8	1/4
5	0.539	2 21/32	2 9/16	0.450	2 1/8	2 1/4	2 1/16	3/4	0.260	3/8	1/4
6	0.599	2 31/32	2 7/8	0.500	2 3/8	2 1/2	2 19/64	7/8	0.291	7/16	9/32
6	0.635	3 27/32	3 3/4	0.500	3 1/4	3 3/8	3 11/64	7/8	0.291	7/16	9/32
7 *	0.704	3 1/8	3 1/32	0.600	2 1/2	2 5/8	2 13/32	15/16	0.322	15/32	5/16
7 †	0.720	3 1/2	3 13/32	0.600	2 7/8	3	2 25/32	15/16	0.322	15/32	5/16
7	0.725	3 5/8	3 17/32	0.600	3	3 1/8	2 29/32	15/16	0.322	15/32	5/16
7 *	0.767	4 5/8	4 17/32	0.600	4	4 1/8	3 29/32	15/16	0.322	15/32	5/16
8	0.898	4 1/4	4 1/8	0.750	3 9/16	3 11/16	3 29/64	1	0.353	1/2	11/32
8	0.917	4 11/16	4 9/16	0.750	4	4 1/8	3 57/64	1	0.353	1/2	11/32
9	1.067	4 3/4	4 5/8	0.900	4	4 1/8	3 7/8	1 1/8	0.385	9/16	3/8
9	1.077	5	4 7/8	0.900	4 1/4	4 3/8	4 1/8	1 1/8	0.385	9/16	3/8
10	1.260	5 27/32	5 23/32	1.0446	5	5 1/8	4 27/32	1 5/16	0.447	21/32	7/16
10	1.289	6 17/32	6 13/32	1.0446	5 11/16	5 13/16	5 17/32	1 5/16	0.447	21/32	7/16
10	1.312	7 1/16	6 15/16	1.0446	6 7/32	6 11/32	6 1/16	1 5/16	0.447	21/32	7/16
11	1.498	6 25/32	6 21/32	1.250	5 15/16	6 1/16	5 25/32	1 5/16	0.447	21/32	7/16
11	1.531	7 19/32	7 15/32	1.250	6 3/4	6 7/8	6 19/32	1 5/16	0.447	21/32	7/16
12	1.797	8 1/16	7 15/16	1.500	7 1/8	7 1/4	6 15/16	1 1/2	0.510	3/4	1/2
13	2.073	8 11/16	8 9/16	1.750	7 3/4	7 7/8	7 9/16	1 1/2	0.510	3/4	1/2
14	2.344	9 9/32	9 5/32	2.000	8 1/4	8 3/8	8 1/32	1 11/16	0.572	27/32	9/16
15	2.615	9 25/32	9 21/32	2.250	8 3/4	8 7/8	8 17/32	1 11/16	0.572	27/32	9/16
16	2.885	10 3/8	10 1/4	2.500	9 1/4	9 3/8	9	1 7/8	0.635	15/16	5/8
17	3.156	.....	.....	2.750	9 3/4	9 7/8	....	....	.....	....	....
18	3.427	.....	.....	3.000	10 1/4	10 3/8	....	....	.....	....	....

* Obsolete. † Used only in the Brown & Sharpe Co.'s shops.

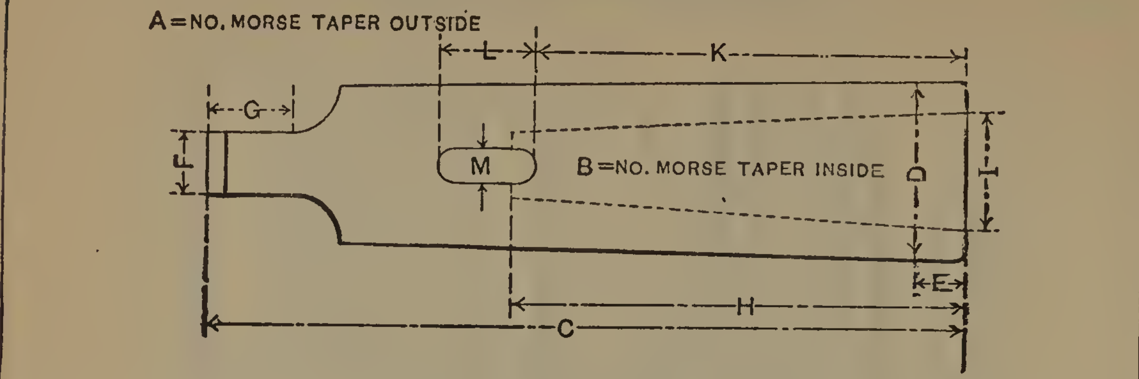
Morse Standard Taper Shanks



Number of Taper	Diam. of Plug at Small End	Diam. at End of Socket	Shank		Depth of Hole	Standard Plug Depth	Tongue		Keyway		End of Socket to Keyway	Taper per Foot
			Whole Length	Depth			Thickness	Length	Width	Length		
	D	*A*	*B*	*S*	*H*	*P*	*t*	*T*	*W*	*L*	*K*	
0	0.252	0.356	2 11/32	2 7/32	2 1/32	2	5/32	1/4	0.160	9/16	1 15/16	0.625
1	0.369	0.475	2 9/16	2 7/16	2 3/16	2 1/8	13/64	3/8	0.213	3/4	2 1/16	0.600
2	0.572	0.700	3 1/8	2 15/16	2 5/8	2 9/16	1/4	7/16	0.260	7/8	2 1/2	0.602
3	0.778	0.938	3 7/8	3 11/16	3 1/4	3 3/16	5/16	9/16	0.322	1 3/16	3 1/16	0.602
4	1.020	1.231	4 7/8	4 5/8	4 1/8	4 1/16	15/32	5/8	0.478	1 1/4	3 7/8	0.623
5	1.475	1.748	6 1/8	5 7/8	5 1/4	5 3/16	5/8	3/4	0.635	1 1/2	4 15/16	0.630
6	2.116	2.494	8 9/16	8 1/4	7 3/8	7 1/4	3/4	1 1/8	0.760	1 3/4	7	0.626
7	2.750	3.270	11 5/8	11 1/4	10 1/8	10	1 1/8	1 3/8	1.135	2 5/8	9 1/2	0.625
						Short Shanks						
0	0.271	0.356	1 31/32	1 27/32	1 21/32	1 5/8	0.188	1/4	0.193	5/8	1 17/32	0.625
1	0.388	0.475	2 3/16	2	1 13/16	1 3/4	0.251	5/16	0.260	13/16	1 21/32	0.600
2	0.600	0.700	2 9/16	2 3/8	2 1/16	2	0.376	7/16	0.385	1 3/16	1 27/32	0.602
3	0.816	0.938	3 1/8	2 15/16	2 1/2	2 7/16	0.501	9/16	0.512	1 5/16	2 7/32	0.602
4	1.062	1.231	4 1/16	3 13/16	3 5/16	3 1/4	0.626	5/8	0.637	1 1/2	2 31/32	0.623
5	1.532	1.748	5 1/16	4 13/16	4 3/16	4 1/8	1.001	3/4	1.012	2	3 21/32	0.630
6	2.201	2.494	7 1/16	6 3/4	5 3/4	5 5/8	1.251	1 1/8	1.263	2 3/4	5 1/16	0.626
7	2.857	3.270	9 11/16	9 5/16	8 1/16	7 15/16	1.627	1 1/2	1.639	3 5/8	7 1/8	0.625

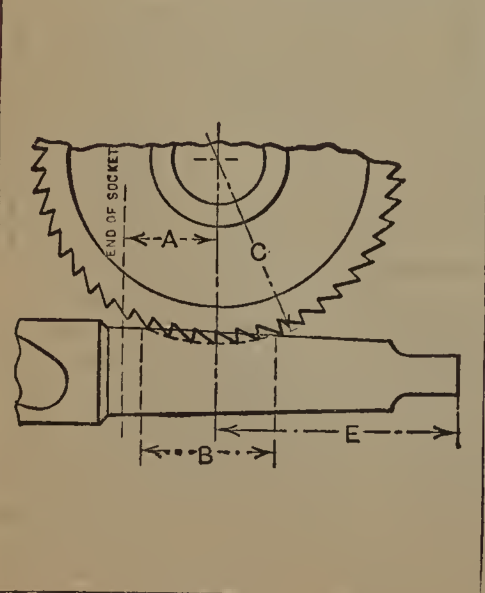
Dimension *X* (amount reamer projects through gage) equals 1/32 inch for taper Nos. 0 and 1; 3/64 inch for taper number 2; 1/16 for tapers 3 and 4; and 5/64 for tapers 5, 6 and 7. Dimensions (*t*) for short shanks are maximum; the minimum values are minus 0.002 inch for tapers up to No. 5 inclusive, minus 0.003 inch for No. 6 and minus 0.004 inch for No. 7. Keyway widths (*W*) for short shanks are minimum; the maximum values are plus 0.003 for taper Nos. 0, 1 and 2, plus 0.004 inch for Nos. 3, 4 and 5, and plus 0.005 inch for Nos. 6 and 7. Cutter radius *R* (see illustration) for long shanks varies as follows: taper No. 0, R = 5/32; No. 1, 3/16; No. 2, 1/4; No. 3, 9/32; No. 4, 5/16; No. 5, 3/8; No. 6, 1/2; No. 7, 3/4. Short shanks: taper No. 0, R = 3/16; No. 1, 1/4; No. 2, 9/32; No. 3, 5/16; No. 4, 3/8; No. 5, 1/2; No. 6, 5/8; No. 7, 3/4.

Dimensions of Morse Taper Sleeves



A	B	C	D	E	F	G	H	I	K	L	M
2	1	3 9/16	0.700	5/8	1/4	7/16	2 3/16	0.475	2 1/16	3/4	0.213
3	1	3 15/16	0.938	1/4	5/16	9/16	2 3/16	0.475	2 1/16	3/4	0.213
3	2	4 7/16	0.938	3/4	5/16	9/16	2 5/8	0.700	2 1/2	7/8	0.260
4	1	4 7/8	1.231	1/4	15/32	5/8	2 3/16	0.475	2 1/16	3/4	0.213
4	2	4 7/8	1.231	1/4	15/32	5/8	2 5/8	0.700	2 1/2	7/8	0.260
4	3	5 3/8	1.231	3/4	15/32	5/8	3 1/4	0.938	3 1/16	1 3/16	0.322
5	1	6 1/8	1.748	1/4	5/8	3/4	2 3/16	0.475	2 1/16	3/4	0.213
5	2	6 1/8	1.748	1/4	5/8	3/4	2 5/8	0.700	2 1/2	7/8	0.260
5	3	6 1/8	1.748	1/4	5/8	3/4	3 1/4	0.938	3 1/16	1 3/16	0.322
5	4	6 5/8	1.748	3/4	5/8	3/4	4 1/8	1.231	3 7/8	1 1/4	0.478
6	1	8 5/8	2.494	3/8	3/4	1 1/8	2 3/16	0.475	2 1/16	3/4	0.213
6	2	8 5/8	2.494	3/8	3/4	1 1/8	2 5/8	0.700	2 1/2	7/8	0.260
6	3	8 5/8	2.494	3/8	3/4	1 1/8	3 1/4	0.938	3 1/16	1 3/16	0.322
6	4	8 5/8	2.494	3/8	3/4	1 1/8	4 1/8	1.231	3 7/8	1 1/4	0.478
6	5	8 5/8	2.494	3/8	3/4	1 1/8	5 1/4	1.748	4 15/16	1 1/2	0.635
7	3	11 5/8	3.270	3/8	1 1/8	1 3/8	3 1/4	0.938	3 1/16	1 3/16	0.322
7	4	11 5/8	3.270	3/8	1 1/8	1 3/8	4 1/8	1.231	3 7/8	1 1/4	0.478
7	5	11 5/8	3.270	3/8	1 1/8	1 3/8	5 1/4	1.748	4 15/16	1 1/2	0.635
7	6	12 1/2	3.270	1 1/4	1 1/8	1 3/8	7 3/8	2.494	7	1 3/4	0.760

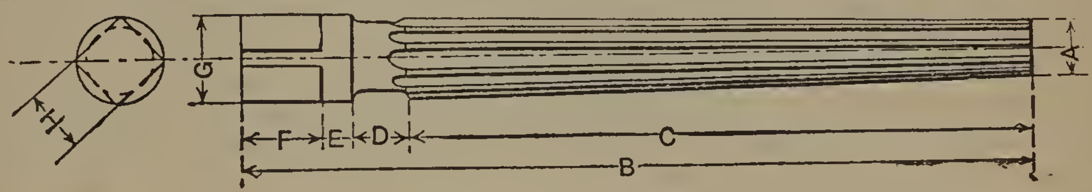
Taper Shanks for Cleveland Grip Socket



Number of Morse Taper Shank	End of Socket to Center of Cutter A	Length of Groove B	Diameter of Cutter C	Center of Cutter to End of Shank E	Thickness of Cutter
1	15/32	1 1/8	4	1 29/32	0.161
2	11/16	1 3/8	4 1/2	2 3/16	0.192
3	29/32	1 3/4	5	2 21/32	0.225
4	13/16	2	5 1/2	3 11/16	0.288
5	1 7/32	2 5/32	6	4 17/32	0.350

Locomotive Taper Reamers. — Taper reamers for locomotive work are generally made in two styles, with squared and with taper shanks. The commonly accepted standard taper for these reamers is 1/16 inch per foot. The fluted part of taper shank reamers is made the same as for those with squared shanks, the over-all length, of course, depending on the Morse taper shank used, which is the same for corresponding sizes as for chucking reamers. The size is measured at the extreme point.

Dimensions of Locomotive Taper Reamers with Squared Shank



Taper, 1/16 inch per foot.

Diam. at Small End of Reamer	Total Length	Length of Flutes	Length of Neck	Length of Collar	Length of Square	Diam. of Collar	Size of Square	No. of Flutes
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	
1/4	5 5/16	4	7/16	3/8	1/2	5/16	1/4	6
5/16	5 5/16	4	7/16	3/8	1/2	3/8	9/32	6
3/8	6 5/16	5	7/16	3/8	1/2	7/16	5/16	6
7/16	7 5/16	6	7/16	3/8	1/2	1/2	3/8	6
1/2	8 5/8	7	1/2	3/8	3/4	9/16	7/16	6
9/16	9 7/8	8	9/16	7/16	7/8	11/16	1/2	8
5/8	9 7/8	8	9/16	7/16	7/8	3/4	9/16	8
11/16	9 7/8	8	9/16	7/16	7/8	13/16	5/8	8
3/4	9 7/8	8	9/16	7/16	7/8	7/8	11/16	8
7/8	11 1/4	9	5/8	1/2	1 1/8	1	3/4	8
1	11 1/4	9	5/8	1/2	1 1/8	1 1/8	13/16	8
1 1/8	12 1/4	10	5/8	1/2	1 1/8	1 1/4	15/16	8
1 1/4	12 1/4	10	5/8	1/2	1 1/8	1 3/8	1	8
1 3/8	14 1/2	12	5/8	1/2	1 3/8	1 1/2	1 1/8	10
1 1/2	14 1/2	12	5/8	1/2	1 3/8	1 5/8	1 3/16	10
1 5/8	16 1/2	14	5/8	1/2	1 3/8	1 3/4	1 5/16	10
1 3/4	16 1/2	14	5/8	1/2	1 3/8	1 7/8	1 3/8	10
1 7/8	18 1/2	16	5/8	1/2	1 3/8	2	1 1/2	12
2	18 1/2	16	5/8	1/2	1 3/8	2 1/8	1 9/16	12

TWIST DRILLS AND COUNTERBORES

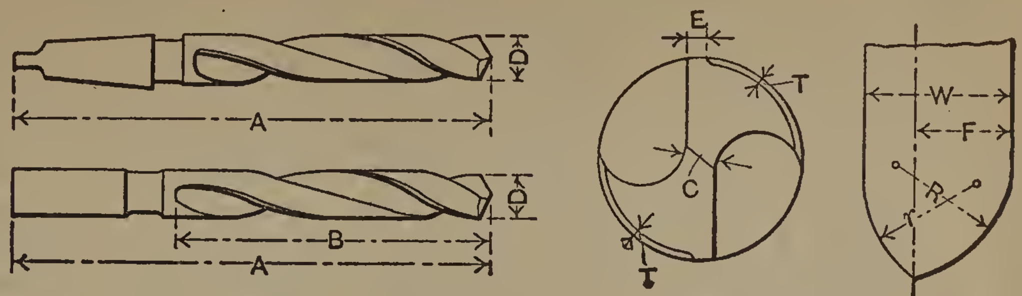
Twist drills 1/4 inch in diameter and larger are made with either straight or taper shanks. The taper of the shank is almost always the Morse standard. A short neck is provided between the grooved portion and the shank. Smaller sizes of drills are, in nearly all cases, made with straight shank only, and have no neck. The lead of the helix of the groove is either the same for the full length of the groove (constant angle drills), or the lead is gradually increased from the point towards the shank (increased twist drills). For constant-angle drills the lead of the helix is usually from 6.5 to 7 times the diameter of the drill. For increased-twist drills the lead at the point is 6 times the drill diameter, and increases at such a rate that the first turn of the helix is made in a distance equal to 7 times the drill diameter. The inclusive angle of the cutting point is 118 degrees; that is, the cutting edge makes an angle of 59 degrees with the center line of the drill. The thickness of the web or central portion between the bottom of the grooves usually increases from the point towards the shank. The increase is at the rate of 0.010 inch per inch for drills larger than ½ inch, and at the rate of 0.013 inch per inch for smaller sizes. The space between the grooves and the width of the grooves should be equal. All drills larger than No. 60 wire gage are backed off or relieved. Smaller sizes are usually not relieved. The cylindrical part or barrel of the drill is ground slightly tapered from the point towards the shank, the largest diameter being at the point. This taper is from 0.006 to 0.010 inch per foot. Straight shank drills, ⅜ inch in diameter and less are usually not ground after hardening. Larger sizes of straight shank drills and all taper shank drills are ground after hardening. From 0.010 to 0.015 inch should be allowed for grinding on carbon steel drills, and from 0.020 to 0.025 inch on high-speed steel drills. The temperatures to which drills are drawn after hardening are for carbon steel drills about as follows: Wire gage sizes Nos. 80–67, 480 degrees F.; Nos. 66–57, 470 degrees F.; Nos. 56–31, 460 degrees F.; Nos. 30–1, 450 degrees F.; 15/64–23/64 inch, 440 degrees F.; ⅜–47/64 inch, 435 degrees F.; ¾–1½ inch, 425 degrees F.; 1½–2 inches, 415 degrees F.; 2 inches and over, 410 degrees F. High-speed steel drills are drawn to from 500 to 600 degrees F. The accompanying tables give dimensions for twist drills and drill fluting cutters.

Dimensions of Twist Drills and Grooving Cutters — Wire Gage Sizes

See illustrations with table on opposite page.

Drill Gage No.	Diam. in Inches, *D*	Total Length, *A*	Length of Flute, *B*	Thickness of Web at Point, *C*	Width of Land, *E*	Total Clearance, $2T$	Cutter Dimensions			
							W	*r*	*R*	*F*
80	0.0135	¾	3/16	0.003	No clearance	No clearance	0.013	0.008	0.011	0.007
76	0.0200	1	¼	0.004			0.018	0.011	0.014	0.010
72	0.0250	1¼	7/16	0.005			0.022	0.015	0.017	0.012
68	0.0310	1 7/16	9/16	0.006			0.027	0.017	0.021	0.015
64	0.0360	1 9/16	⅝	0.007	0.015	0.003	0.030	0.019	0.025	0.018
60	0.0400	1 11/16	11/16	0.008	0.015	0.003	0.033	0.022	0.028	0.020
56	0.0465	1 13/16	25/32	0.009	0.015	0.0035	0.043	0.027	0.035	0.025
52	0.0635	1⅞	⅞	0.010	0.015	0.004	0.055	0.035	0.046	0.032
48	0.0760	2 1/16	1 1/16	0.012	0.020	0.0045	0.064	0.041	0.053	0.037
44	0.0860	2 3/16	1 3/16	0.014	0.020	0.0055	0.073	0.047	0.061	0.042
40	0.0980	2⅜	1 11/32	0.016	0.020	0.0065	0.081	0.052	0.068	0.047
36	0.1065	2 9/16	1½	0.017	0.025	0.007	0.088	0.055	0.073	0.051
32	0.1160	2 11/16	1⅝	0.019	0.025	0.008	0.101	0.065	0.085	0.059
28	0.1405	2⅞	1 13/16	0.021	0.025	0.009	0.116	0.075	0.097	0.068
24	0.1520	3 1/16	1 15/16	0.023	0.030	0.0095	0.124	0.081	0.104	0.073
20	0.1610	3 3/16	2 1/16	0.025	0.030	0.0105	0.133	0.087	0.112	0.079
16	0.1770	3⅜	2 3/16	0.027	0.030	0.0115	0.143	0.094	0.119	0.085
12	0.1890	3 9/16	2 5/16	0.029	0.030	0.0125	0.152	0.100	0.128	0.090
8	0.1990	3 11/16	2 15/32	0.031	0.035	0.0135	0.160	0.105	0.135	0.096
4	0.2090	3⅞	2 19/32	0.033	0.035	0.014	0.173	0.114	0.146	0.103
1	0.2280	4	2 21/32	0.035	0.035	0.015	0.187	0.123	0.157	0.111

Dimensions of Twist Drills and Grooving Cutters



Diam., *D*	Total Length, *A*	No. of M. T. Shank	Length of Flutes, Straight Shank, *B*	Thickness of Web at Point, *C*	Width of Land, *E*	Total Clearance, 2 *T*	Cutter Dimensions			
							W	*r*	*R*	*F*
1/4	6 1/8	1	4	0.038	0.040	0.017	0.200	0.131	0.168	0.119
5/16	6 3/8	1	4 1/16	0.045	0.050	0.021	0.250	0.164	0.211	0.148
3/8	6 3/4	1	4 1/4	0.051	0.055	0.025	0.300	0.197	0.253	0.178
7/16	7 1/4	1	4 5/8	0.058	0.060	0.029	0.350	0.230	0.295	0.208
1/2	7 3/4	1	5	0.066	0.060	0.033	0.400	0.262	0.337	0.238
5/8	8 3/4	2	5 3/4	0.082	0.065	0.041	0.500	0.328	0.422	0.297
3/4	9 3/4	2	6 3/8	0.098	0.075	0.049	0.600	0.394	0.506	0.356
7/8	10 1/2	2	7	0.114	0.085	0.053	0.700	0.459	0.590	0.416
1	11	3	7 3/16	0.128	0.090	0.057	0.800	0.525	0.675	0.475
1 1/8	11 3/4	3	7 7/8	0.140	0.095	0.061	0.900	0.591	0.759	0.534
1 1/4	12 1/2	3	8 1/2	0.152	0.100	0.065	1.000	0.656	0.844	0.594
1 3/8	14 1/2	4	9 1/2	0.164	0.105	0.067	1.100	0.722	0.928	0.653
1 1/2	15	4	9 7/8	0.176	0.110	0.069	1.200	0.788	1.012	0.713
1 3/4	16	4	10 1/2	0.200	0.120	0.073	1.400	0.919	1.181	0.831
2	16 1/2	4	11	0.224	0.130	0.075	1.600	1.050	1.350	0.950
2 1/4	17 1/2	5	10 1/4	0.244	0.135	0.075	1.800	1.181	1.518	1.069
2 1/2	19	5	11 1/2	0.260	0.140	0.075	2.000	1.313	1.687	1.187
2 3/4	20 1/2	5	12 3/4	0.276	0.150	0.080	2.200	1.444	1.856	1.306
3	22	5	14	0.292	0.160	0.085	2.400	1.575	2.025	1.425

Limits for Drill Rod. — The Navy Department specifies that drill rods may have a maximum variation of 0.0005 inch on sizes 7/16 inch diameter or less, and 0.001 inch on sizes larger than 7/16 inch. Variations of 0.003 inch are allowed on cold-rolled or cold-drawn machinery steel rods and bars up to and including 1 inch. Above 1 inch and including 2 1/2 inches, the allowable variation is 0.004 inch, and above 2 1/2 inches, 0.005 inch. A manufacturer of drill rods advises that all sizes of drill rods produced up to 1 1/2 inch round, are within 0.0005 inch above or below the specified size, and for all sizes less than 1 inch, the limit above or below will not exceed 0.00035 inch of the specified size. The General Electric specifications call for the following: Up to and including 0.3-inch diameter drill rods, over size 0.000, under size 1 per cent of the diameter, eccentricity 1/2 per cent of the diameter; above 0.3 inch and including 1 inch, over size 0.000, under size 0.003 inch, eccentricity 0.0015 inch; above 1 inch and including 2 1/2 inches, over size 0.000, under size 0.004 inch, eccentricity 0.002 inch; above 2 1/2 inches, over size 0.000, under size 0.005 inch, eccentricity 0.0025 inch. The Brown & Sharpe Mfg. Co. states that drill rod is accepted with a variation of 0.0005 inch large or small. In cold-rolled or drawn machinery steel, the limits vary from 0.001 to 0.005 inch, depending on where the stock is to be used.

Decimal Equivalents of Letter Size Drills

Letter	Size of Drill in Inches	Letter	Size of Drill in Inches	Letter	Size of Drill in Inches	Letter	Size of Drill in Inches
Z	0.413	S	0.348	L	0.290	E	0.250
Y	0.404	R	0.339	K	0.281	D	0.246
X	0.397	Q	0.332	J	0.277	C	0.242
W	0.386	P	0.323	I	0.272	B	0.238
V	0.377	O	0.316	H	0.266	A	0.234
U	0.368	N	0.302	G	0.261	......	......
T	0.358	M	0.295	F	0.257	......	......

Twist Drill and Steel Wire Gage
(Manufacturers' Standard)

No.	Size of Drill in Inches	No.	Size of Drill in Inches	No.	Size of Drill in Inches	No.	Size of Drill in Inches
1	0.2280	21	0.1590	41	0.0960	61	0.0390
2	0.2210	22	0.1570	42	0.0935	62	0.0380
3	0.2130	23	0.1540	43	0.0890	63	0.0370
4	0.2090	24	0.1520	44	0.0860	64	0.0360
5	0.2055	25	0.1495	45	0.0820	65	0.0350
6	0.2040	26	0.1470	46	0.0810	66	0.0330
7	0.2010	27	0.1440	47	0.0785	67	0.0320
8	0.1990	28	0.1405	48	0.0760	68	0.0310
9	0.1960	29	0.1360	49	0.0730	69	0.0292
10	0.1935	30	0.1285	50	0.0700	70	0.0280
11	0.1910	31	0.1200	51	0.0670	71	0.0260
12	0.1890	32	0.1160	52	0.0635	72	0.0250
13	0.1850	33	0.1130	53	0.0595	73	0.0240
14	0.1820	34	0.1110	54	0.0550	74	0.0225
15	0.1800	35	0.1100	55	0.0520	75	0.0210
16	0.1770	36	0.1065	56	0.0465	76	0.0200
17	0.1730	37	0.1040	57	0.0430	77	0.0180
18	0.1695	38	0.1015	58	0.0420	78	0.0160
19	0.1660	39	0.0995	59	0.0410	79	0.0145
20	0.1610	40	0.0980	60	0.0400	80	0.0135

Twist Drill and Steel Wire Gages. — The differences in the gages for small diameter twist drills and steel wire are a constant source of trouble to those who have to deal with drills and steel wire; and gage numbers mean nothing except if the name of the gage employed is specified. Some drill manufacturers are discouraging the use of gage sizes and are asking that the sizes of drills ordered be denoted in decimals of an inch. There are three well-known standards for twist drills and steel wire that are commonly used at the present time. These are the Stubs steel wire gage, the gage used by the Standard Tool Co. and the gage used by other leading manufacturers, such as the Morse Twist Drill & Machine Co., and Brown & Sharpe Mfg. Co. The latter has been termed, in the accompanying tables, "the manufacturers' standard."

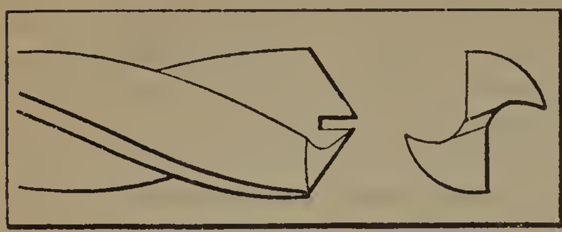
The Stubs steel wire gage is used for measuring steel wire and drill rod, but it is not used as much in this country at the present time as in the past. The gage used by the Standard Tool Co. was originally adopted for drill sizes in this country, but other manufacturers changed the numbers corresponding to certain sizes, while the Standard Tool Co. retained the original numbers, but interpolated half sizes in order to agree as to the actual diameters of drills furnished by other manufacturers. The Standard Tool Co.'s gage agrees with the "manufacturers' standard" for the sizes from No. 1 to No. 60, inclusive, but does not agree with the Stubs steel wire gage. From No. 61 to No. 80, inclusive, it agrees with the Stubs gage, half sizes being omitted. It also agrees with the "manufacturers' standard," as far as the diameters used are concerned, but the numbers corresponding to given diameters are different.

Table for Comparison of Twist Drill and Wire Gages

Gage Number	Stubs Steel Wire Gage Diam., Inches	Drill Manufacturers' Standard Diam., Inches	Gage Number	Stubs Steel Wire Gage Diam., Inches	Drill Manufacturers' Standard Diam., Inches	Stubs Steel Wire & Standard Tool Co.'s Drill Gage Nos.*	Manufacturers' Standard Drill Gage Nos.	Diam., Inches
1	0.227	0.2280	31	0.120	0.1200	60½	61	0.0390
2	0.219	0.2210	32	0.115	0.1160	61	62	0.0380
3	0.212	0.2130	33	0.112	0.1130	62	63	0.0370
4	0.207	0.2090	34	0.110	0.1110	63	64	0.0360
5	0.204	0.2055	35	0.108	0.1100	64	65	0.0350
6	0.201	0.2040	36	0.106	0.1065	65	66	0.0330
7	0.199	0.2010	37	0.103	0.1040	66	67	0.0320
8	0.197	0.1990	38	0.101	0.1015	67	68	0.0310
9	0.194	0.1960	39	0.099	0.0995	68	.....	0.0300
10	0.191	0.1935	40	0.097	0.0980	68½	69	0.0292
11	0.188	0.1910	41	0.095	0.0960	69	.....	0.0290
12	0.185	0.1890	42	0.092	0.0935	69½	70	0.0280
13	0.182	0.1850	43	0.088	0.0890	70	.....	0.0270
14	0.180	0.1820	44	0.085	0.0860	71	71	0.0260
15	0.178	0.1800	45	0.081	0.0820	71½	72	0.0250
16	0.175	0.1770	46	0.079	0.0810	72	73	0.0240
17	0.172	0.1730	47	0.077	0.0785	73	.....	0.0230
18	0.168	0.1695	48	0.075	0.0760	73½	74	0.0225
19	0.164	0.1660	49	0.072	0.0730	74	.....	0.0220
20	0.161	0.1610	50	0.069	0.0700	74½	75	0.0210
21	0.157	0.1590	51	0.066	0.0670	75	76	0.0200
22	0.155	0.1570	52	0.063	0.0635	76	77	0.0180
23	0.153	0.1540	53	0.058	0.0595	77	78	0.0160
24	0.151	0.1520	54	0.055	0.0550	78	.....	0.0150
25	0.148	0.1495	55	0.050	0.0520	78½	79	0.0145
26	0.146	0.1470	56	0.045	0.0465	79	.....	0.0140
27	0.143	0.1440	57	0.042	0.0430	79½	80	0.0135
28	0.139	0.1405	58	0.041	0.0420	80	.....	0.0130
29	0.134	0.1360	59	0.040	0.0410	......	.....	.....
30	0.127	0.1285	60	0.039	0.0400	......	.....	.....

* Half sizes are included only in Standard Tool Co.'s gage.

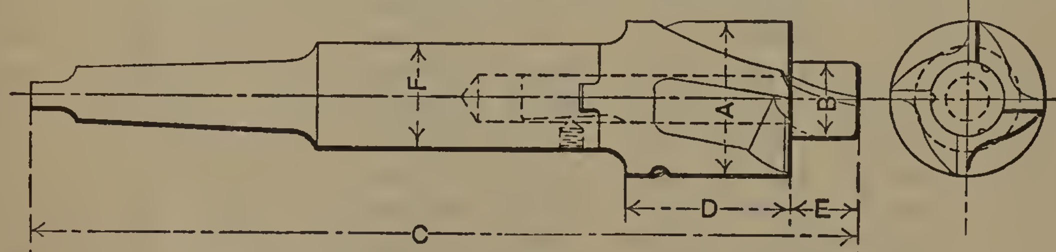
Drilling Marble. — The drilling of marble and similar materials, especially when the drill is fed by hand, is greatly facilitated by grinding or filing a narrow slot in the point of the drill, as shown by the illustration. This slot should be about ⅛ to ¼ inch deep, according to the size of the drill, and at an angle of a little less than 90 degrees with the cutting edges. If the slotting is carefully done, the drill will give good results.



Counterbores. — Counterbores for screw holes are generally made in sets. Each set contains three counterbores: one with the body of the size of the screw head and the pilot the size of the hole to admit the body of the screw; one with the body the size of the head of the screw and the pilot the size of the tap drill; and the third with the body the size of the body of the screw and the pilot the size of the tap drill. Counterbores are usually provided with four flutes cut on a right-hand spiral. The angle of the spiral is 15 degrees with the center line of the counterbore, which corresponds to a lead of the flute equal to about twelve times the diameter of the body of the counterbore. Counterbores for brass are fluted straight.

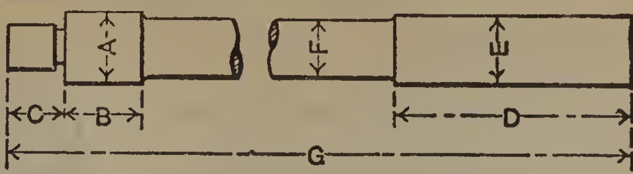
Small counterbores are often made with three flutes, but should then have the size plainly stamped on them before fluting, as they cannot afterwards be conveniently measured. The flutes should be deep enough to come below the surface of the pilot. The counterbore should be relieved on the end of the body only, and not on the cylindrical surface. To facilitate the relieving process, a small neck is turned between the guide and the body for clearance. The amount of clearance on the cutting edges is, for general work, from 4 to 5 degrees. The accompanying table gives dimensions for straight shank counterbores. The same dimensions, except for the shank part, may be used for Morse taper shank counterbores. The number of shank used for counterbores with bodies of different diameters is usually as follows: Up to ½ inch diameter body, No. 1 Morse taper shank; from 9/16 to ⅞ inch, No. 2; from 15/16 to 1⅜ inch, No. 3; from 1 7/16 to 2 inches, No. 4; from 2 1/16 to 3 inches, No. 5 shank.

Counterbores With Interchangeable Cutters and Guides



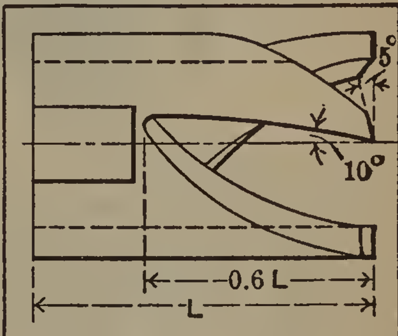
No. of Holder	No. of Morse Taper Shank	Range of Cutter Diameters, *A*	Range of Pilot Diameters, *B*	Total Length, *C*	Length of Cutter Body, *D*	Length of Pilot, *E*	Diam. of Shank, *F*
1	1 or 2	¾–1 1/16	½ – ¾	7¼	1	⅝	¾
2	2 or 3	1⅛–1 9/16	1 1/16 – 1⅛	9½	1⅜	⅞	1⅛
3	3 or 4	1⅝–2 1/16	⅞ – 1⅝	12½	1¾	1⅛	1⅝
4	4 or 5	2⅛–3½	1 – 2⅛	15	2¼	1⅜	2⅛

Dimensions of Counterbores



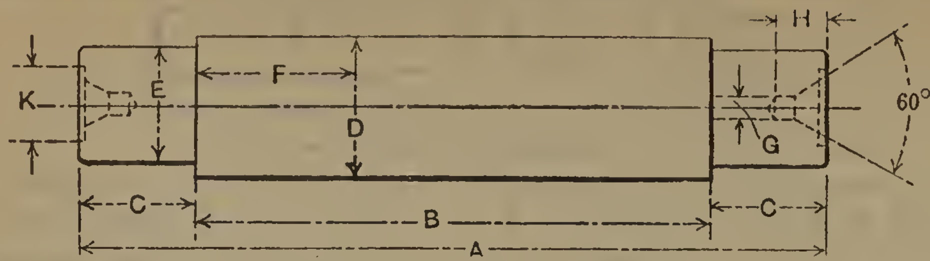
A	B	C	D	E	F	G	A	B	C	D	E	F	G
1/4	11/32	3/16	2 1/2	1/4	7/32	4 7/8	1 5/16	1 3/4	63/64	3 9/16	1 5/16	1 9/32	12 5/16
5/16	13/32	15/64	2 9/16	5/16	9/32	5 5/16	1 3/8	1 27/32	1 1/32	3 5/8	1 3/8	1 11/32	12 3/4
3/8	1/2	9/32	2 5/8	3/8	11/32	5 3/4	1 7/16	1 29/32	1 5/64	3 11/16	1 7/16	1 13/32	13 3/16
7/16	19/32	21/64	2 11/16	7/16	13/32	6 3/16	1 1/2	2	1 1/8	3 3/4	1 1/2	1 7/16	13 3/8
1/2	21/32	3/8	2 3/4	1/2	15/32	6 5/8	1 5/8	2 3/32	1 7/32	3 15/16	1 17/32	1 15/32	13 3/4
9/16	3/4	27/64	2 13/16	9/16	17/32	7 1/16	1 3/4	2 3/16	1 5/16	4 1/8	1 19/32	1 17/32	14 1/8
5/8	27/32	15/32	2 7/8	5/8	19/32	7 1/2	1 7/8	2 9/32	1 13/32	4 5/16	1 5/8	1 9/16	14 1/2
11/16	29/32	33/64	2 15/16	11/16	21/32	7 15/16	2	2 3/8	1 1/2	4 1/2	1 21/32	1 19/32	14 7/8
3/4	1	9/16	3	3/4	23/32	8 3/8	2 1/8	2 15/32	1 19/32	4 11/16	1 23/32	1 21/32	15 1/4
13/16	1 3/32	39/64	3 1/16	13/16	25/32	8 13/16	2 1/4	2 9/16	1 11/16	4 7/8	1 3/4	1 11/16	15 5/8
7/8	1 5/32	21/32	3 1/8	7/8	27/32	9 1/4	2 3/8	2 21/32	1 25/32	5 1/16	1 25/32	1 23/32	16
15/16	1 1/4	45/64	3 3/16	15/16	29/32	9 11/16	2 1/2	2 3/4	1 7/8	5 1/4	1 27/32	1 25/32	16 3/8
1	1 11/32	3/4	3 1/4	1	31/32	10 1/8	2 5/8	2 27/32	1 31/32	5 7/16	1 7/8	1 13/16	16 3/4
1 1/16	1 13/32	51/64	3 5/16	1 1/16	1 1/32	10 9/16	2 3/4	2 15/16	2 1/16	5 5/8	1 29/32	1 27/32	17 1/8
1 1/8	1 1/2	27/32	3 3/8	1 1/8	1 3/32	11	2 7/8	3 1/32	2 5/32	5 13/16	1 31/32	1 29/32	17 1/2
1 3/16	1 19/32	57/64	3 7/16	1 3/16	1 5/32	11 7/16	3	3 1/8	2 1/4	6	2	1 15/16	17 7/8
1 1/4	1 21/32	15/16	3 1/2	1 1/4	1 7/32	11 7/8	...	....	....	....	....	....	....

Hollow Mills. — A leading tool manufacturer's practice is as follows: The hole in hollow mills, from the cutting edges backwards, should be back tapered at the rate of 1/4 inch per foot for cutting steel and 3/8 inch per foot for cutting brass. Adjustable hollow mills are always provided with three flutes, cut straight, if the mill is to be used for brass, or cut on an angle not exceeding ten degrees, when the tool is used for steel. The cutters used for cutting the flutes are 55-degree, double-angle cutters, 12 degrees on one side and 43 degrees on the other. The land of a mill with only three flutes becomes too wide when milled with this cutter, so that it must be made narrower either by milling once more or by filing. The length of the fluted part should be 0.6 times the whole length of the mill. The outside diameter and the length of hollow mills are usually made to correspond to those of spring screw threading dies (see table of dimensions for these tools in the section on "Threading Dies").



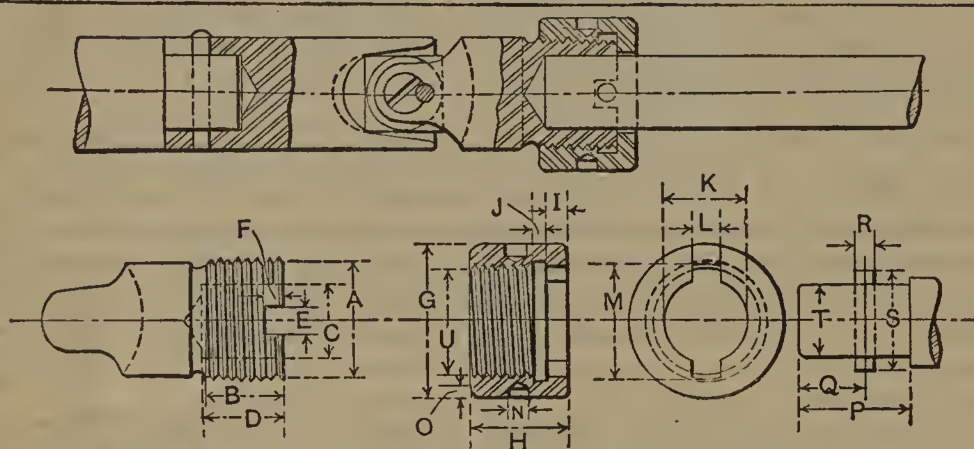
Lathe Arbors. — Arbors are usually tapered about 0.006 inch per foot. The diameter or nominal size *D* in the table is at a distance *F* from the small end. The diameter *G* of the drills for the centers conforms to Stub's steel wire gage. The "width of flat," listed in the last column, is for the driving dog. The centers of arbors intended for very heavy duty may be made somewhat larger than those given in the table. As to hardening, the practice at the present time, among manufacturers, is to harden arbors all over, but for extremely accurate work, an arbor having hardened ends and a soft body is generally considered superior, as there is less tendency of distortion from internal stresses. Hardened arbors should be "seasoned" before finish grinding to relieve internal stresses.

Proportions of Solid Lathe Arbors



D	*A*	*B*	*C*	*E*	*F*	*G*	*H*	*K*	Width of Flat
1/4	4	2 3/8	13/16	7/32	5/8	0.046	1/8	3/16	5/64
1/2	5	3 1/8	15/16	7/16	13/16	0.063	1/4	1/4	5/32
1	7	4 5/8	1 3/16	7/8	1 3/16	0.096	1/2	3/8	5/16
1 1/2	9	6 1/8	1 7/16	1 5/16	1 9/16	0.128	3/4	1/2	15/32
2	11	7 5/8	1 11/16	1 3/4	1 15/16	0.157	1	11/16	5/8
2 1/2	12 1/2	8 5/8	1 15/16	2 3/16	2 3/16	0.189	1 1/16	13/16	25/32

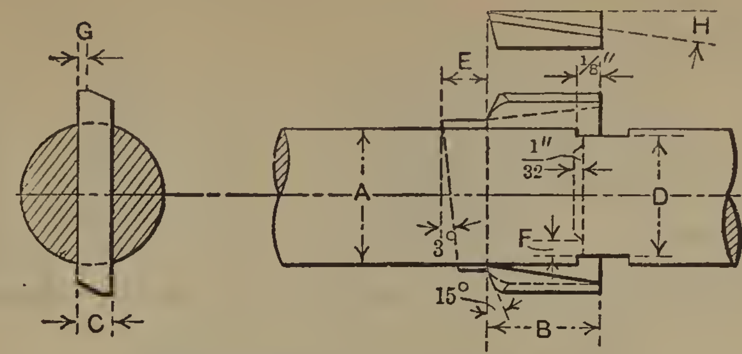
Boring-bar Couplings



A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*
3/4	1/2	1/2	7/16	3/16	5/32	1	5/8	1/8	3/32	17/32	3/16	11/16
1	1/2	5/8	9/16	1/4	7/32	1 3/8	5/8	5/32	3/32	21/32	1/4	15/16
1 1/4	5/8	3/4	5/8	5/16	9/32	1 5/8	13/16	3/16	1/8	25/32	5/16	1 5/32
1 1/2	13/16	1	7/8	7/16	13/32	2	1 1/16	1/4	1/8	1 1/32	7/16	1 13/32
2 1/2	7/8	1 1/2	1 3/8	9/16	17/32	3	1 1/8	1/4	1/8	1 17/32	9/16	2 3/8
3	7/8	1 7/8	1 3/4	11/16	21/32	3 1/2	1 1/4	1/4	1/8	1 29/32	11/16	2 7/8

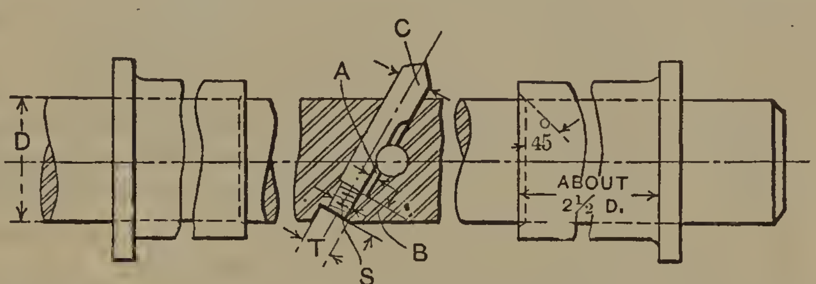
N	*O*	*P*	*Q*	*R*	*S*	*T*	*U*, Bore	Threads per Inch	Used for Bars
9/64	1/8	3/4	3/8	1/8	21/32	0.499	0.6610	16	3/8 to 1/2
11/64	5/32	7/8	15/32	3/16	7/8	0.624	0.9110	16	1/2 to 3/4
13/64	5/32	1 1/8	1/2	1/4	1 1/16	0.749	1.1615	12	3/4 to 1
15/64	7/32	1 3/8	11/16	3/8	1 5/16	0.999	1.4115	12	1 to 1 1/2
19/64	7/32	1 7/8	1 1/8	1/2	2 1/4	1.4985	2.4115	12	1 1/2 to 2
21/64	7/32	2 5/16	1 7/16	5/8	2 3/4	1.8735	2.9115	12	2 and over

Boring-bar Cutters — 1



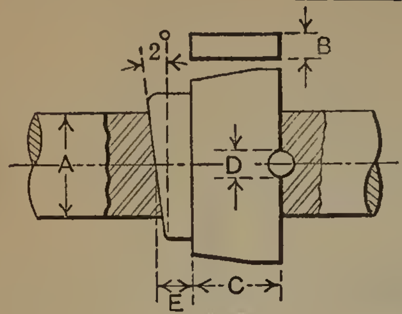
A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H
½	⅝	⅛	15/32	⅛	1/16	1/32	5°	2¼	1½	7/16	2 3/16	5/16	¼	1/16	8°
⅝	⅝	5/32	19/32	5/32	⅛	1/32	6	2½	1½	½	2 7/16	5/16	5/16	3/32	10
¾	¾	3/16	11/16	5/32	⅛	1/32	7	2¾	1½	½	2 11/16	5/16	5/16	3/32	10
⅞	13/16	7/32	13/16	¼	⅛	1/32	7	3	1½	½	2 15/16	7/16	⅜	⅛	10
1	⅞	¼	15/16	¼	5/32	1/32	8	3¼	1 11/16	⅝	3 5/32	7/16	½	⅛	10
1¼	1 3/16	5/16	1 3/16	¼	5/32	3/64	7	3½	1 11/16	⅝	3 13/32	7/16	½	⅛	10
1½	1¼	5/16	1 7/16	¼	3/16	3/64	7	4	2	¾	3 29/32	½	½	⅛	10
1¾	1½	⅜	1 11/16	5/16	3/16	3/64	7	4½	2	¾	4⅜	½	½	⅛	10
2	1½	7/16	1 15/16	5/16	¼	1/16	8	5	2	¾	4⅞	⅝	½	⅛	10

Boring-bar Cutters — 2



Diameter of Bar, *D*	Diameter of Cutter, *C*	Diameter of Pin	Depth of Flat, *A*	Diameter of Screw, *S*	Diameter *T* of Counterbore	Length *B* of Thread	Diameter of Bar, *D*	Diameter of Cutter, *C*	Diameter of Pin	Depth of Flat, *A*	Diameter of Screw, *S*	Diameter *T* of Counterbore	Length *B* of Thread
⅜	⅛	1/16	1/64	1/16	⅛	5/64	1½	⅜	9/32	3/64	5/16	⅜	⅜
7/16	⅛	3/32	1/64	1/16	⅛	5/64	1⅝	7/16	5/16	3/64	5/16	⅜	⅜
½	⅛	3/32	1/64	1/16	⅛	5/64	1¾	7/16	5/16	3/64	⅜	7/16	½
9/16	⅛	⅛	1/32	⅛	5/32	⅛	1⅞	½	11/32	1/16	⅜	7/16	½
⅝	3/16	⅛	1/32	⅛	5/32	⅛	2	½	⅜	1/16	7/16	½	9/16
11/16	3/16	5/32	1/32	⅛	3/16	5/32	2⅛	9/16	13/32	1/16	7/16	½	9/16
¾	3/16	5/32	1/32	⅛	3/16	5/32	2¼	9/16	13/32	1/16	½	9/16	⅝
13/16	¼	5/32	1/32	3/16	¼	3/16	2⅜	⅝	7/16	5/64	½	9/16	⅝
⅞	¼	5/32	1/32	3/16	¼	3/16	2½	⅝	15/32	5/64	9/16	⅝	11/16
15/16	¼	3/16	1/32	3/16	¼	¼	2⅝	11/16	½	5/64	9/16	⅝	11/16
1	¼	3/16	1/32	3/16	¼	¼	2¾	11/16	½	5/64	⅝	11/16	¾
1⅛	5/16	7/32	1/32	¼	5/16	5/16	2⅞	¾	17/32	3/32	⅝	¾	⅞
1¼	5/16	¼	3/64	¼	5/16	5/16	3	¾	9/16	3/32	⅝	¾	⅞
1⅜	⅜	¼	3/64	¼	5/16	5/16	...	...	...	...	...	...	...

Boring-bar Cutters — 3



A	B	C	D	E
¾ to 15/16	¼	¾	3/16	¼
1 to 1⅜	5/16	¾	3/16	¼
1½ to 2⅜	⅜	¾	¼	¼
2½ to 3	½	1	¼	5/16
3⅛ and larger	⅝	1¼	5/16	⅜

Boring Bars and Cutter Heads

Various methods of attaching cutters to boring-bars, and different designs of cutter-heads, are shown on the following pages. Many of the designs illustrated have been extensively used.

Fig. 1. A simple method of holding a flat cutter in a boring-bar. The cutter is held in a rectangular slot in the bar, by wedge *W*, tapered on one side to about 2 degrees; the cutter is centered by shoulders *S*.

Fig. 2. An excellent design in which the cutter is held and centered by a conical-ended screw *B*, which bears against a conical seat in the cutter. The screw end should have a taper of about 30 degrees included angle.

Fig. 3. Boring and facing cutter also held by a conical-ended screw which passes through a hole in the cutter. The hole is slightly offset so that the cutter will be forced back against its seat.

Fig. 4. Simple and inexpensive form of cutter made from drill rod and held by a taper pin which bears against a circular locating seat in the center.

Fig. 5. The cutter is secured by a fine threaded sleeve *B* and is centered by shoulders *S*. This form eliminates any tendency of springing the bar, and for that reason is sometimes preferred to wedges. The particular cutter shown in this bar is excellent for light finishing cuts. It has circular ends with slight clearance.

Fig. 6. A wedged cutter which is centered by pin *C*. The cutter is a plain rectangular piece of steel, except for the circular locating seat.

Fig. 7. The cutter is wedged by a piece of drill rod *W* having a tapering flat side, and is centered by the projecting shoulders *S*. The edge of the cutter against which the pin bears should be rounded, to locate the bearing in the center instead of on one corner. The cutter is solidly supported on the rear side.

Fig. 8. The clamping wedge is placed at right angles and engages a central notch in the cutter in order to bind and also center the latter. The objection to this design is that it requires two slots.

Fig. 9. Adjustable cutters made preferably of round steel flattened on one side for binding screws *B*. The adjusting screw *A* has a conical end which enters between the inner ends of the cutters.

Fig. 10. An inexpensive method of holding a cutter near the end of a bar.

Fig. 11. Boring and facing cutter clamped in its slot by an inclined screw *B*, which forces the flattened side of a hardened bushing against the cutter.

Fig. 12. Simple method of holding and adjusting single-ended cutter, adjustment being effected by screw *A*.

Fig. 13. A turret bar with tool held at an angle for boring blind holes, etc. Adjustments are effected by screw *A*.

Fig. 14. A modification of the type illustrated in Fig. 13.

Fig. 15. Cutter held by pin *B*, having a tapering flat side which bears against the flat side of the round cutter.

Types of Boring Bars

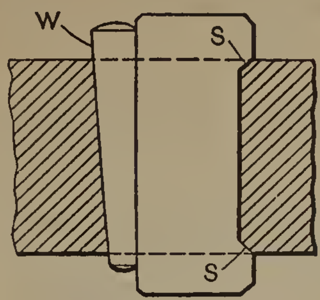


Fig.1

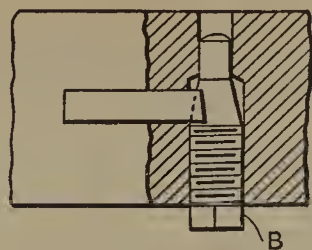


Fig.2

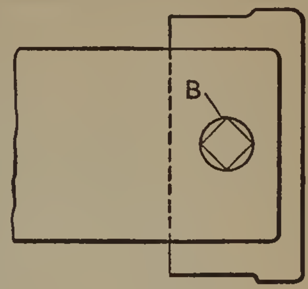


Fig.3

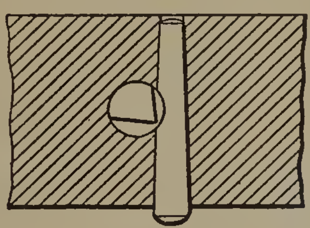


Fig.4

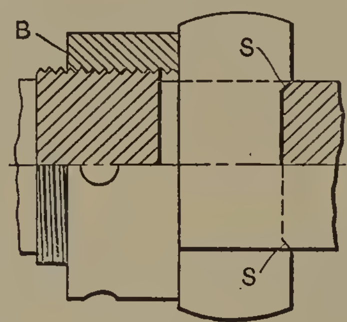


Fig.5

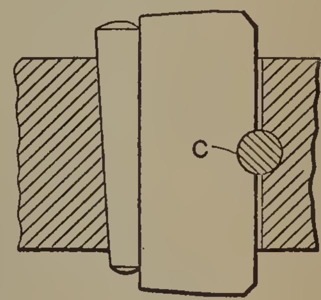


Fig.6

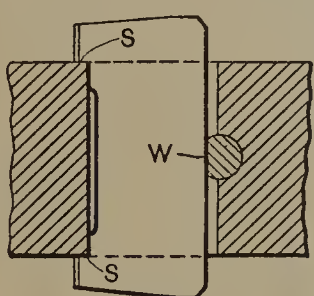


Fig.7

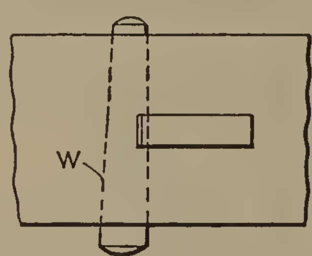


Fig.8

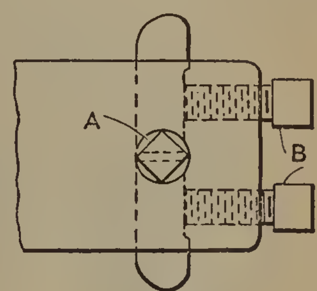


Fig.9

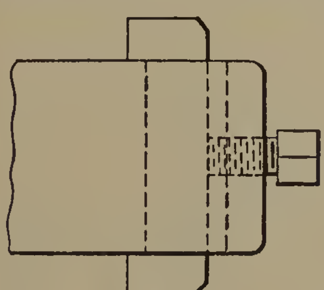


Fig.10

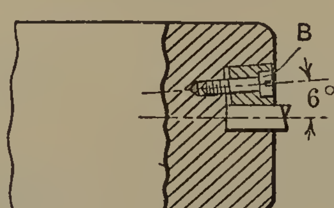


Fig.11

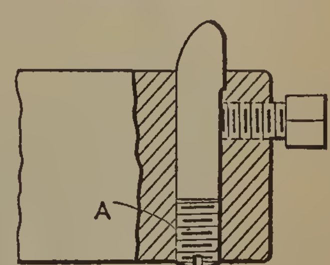


Fig.12

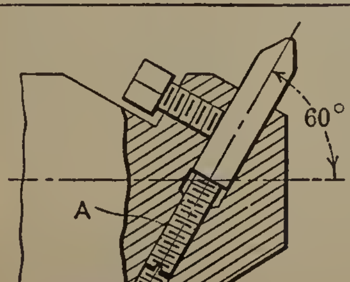


Fig.13

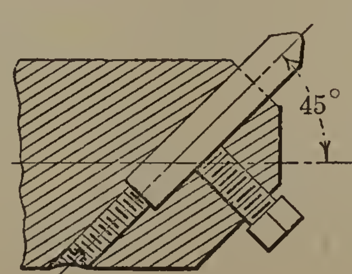


Fig.14

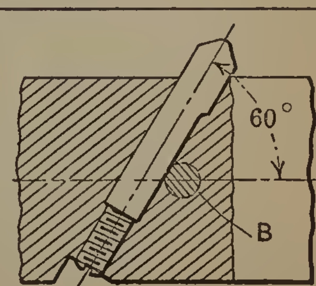


Fig.15

Fig. 16. Bar especially adapted for cast iron, having two cutters held by screws *B* and adjusted by hardened screws *C*, the heads of which fit into slots in the cutters.

Fig. 17. Boring head for steel with four cutters, clamped by screws *B* and adjusted by screws *C* located in ring *D*.

Fig. 18. Bar for brass and other light work, having two cutters which are adjusted by changing their lengthwise position in tapering slots, and are clamped by driving in taper pins *B*.

Fig. 19. Boring head for finishing cuts, having six tools which fit in tapering slots for adjustment. The cutters are held by screws *B* and clamps *D*.

Fig. 20. Boring head for finishing, having six cutters which are clamped by screwing in taper-headed screws *B*.

Fig. 21. Half-section of cutter-head such as is used in horizontal boring machines for holes of comparatively large diameter. The cutters are held at an angle of 45 degrees and are clamped by screws *B* and adjusted, within limits, by screws *A*.

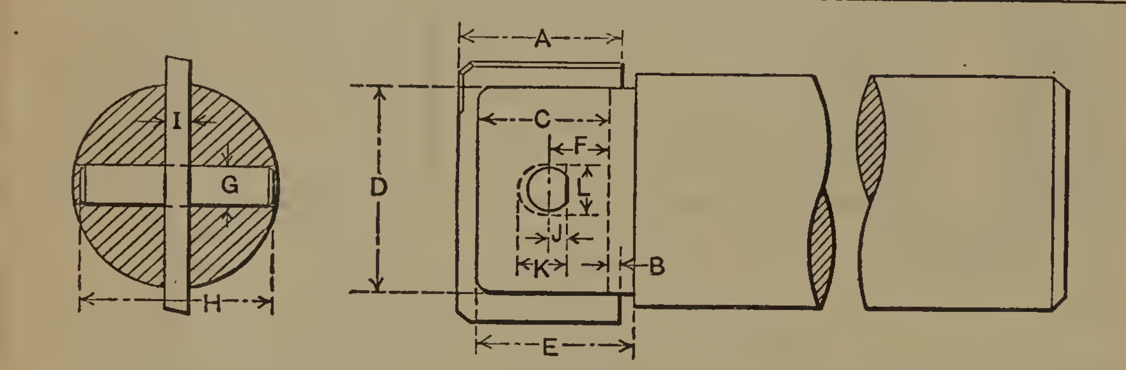
Fig. 22. Type of bar for light work, having two cutters which are held by flattened bushings and screws inserted at an angle, to secure a wedging effect.

Fig. 23. Half-section of cutter-head for comparatively large holes having means for adjusting the tools.

Fig. 24. Cutters for boring and facing, which are adjusted by conical-pointed screw *C* and held by bolts *B*, the heads of which enter grooves *D*.

Fig. 25. Half-section of boring head, the tools of which are inserted in slots or grooves in the face of the disk and are held by slotted clamping posts *B*.

Boring-bar Cutters — 4



Diam. of Bar	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
3	1 7/8	1/16	1 5/8	2 7/8	1 3/4	3/4	9/16	2 3/4	1/2	1/8	5/8	19/32
2 3/4	1 3/4	1/16	1 1/2	2 5/8	1 5/8	11/16	1/2	2 1/2	7/16	1/8	9/16	17/32
2 1/2	1 5/8	1/16	1 3/8	2 3/8	1 1/2	11/16	1/2	2 1/4	3/8	1/8	9/16	17/32
2 1/4	1 1/2	1/16	1 1/4	2 1/8	1 3/8	5/8	7/16	2	3/8	1/8	1/2	15/32
2	1 1/2	1/16	1 1/4	1 7/8	1 3/8	5/8	7/16	1 7/8	3/8	1/8	1/2	15/32
1 3/4	1 3/8	1/16	1 1/8	1 5/8	1 1/4	9/16	3/8	1 5/8	5/16	3/32	7/16	13/32
1 1/2	1 1/4	1/16	1 1/8	1 3/8	1 1/4	9/16	5/16	1 3/8	5/16	3/32	3/8	11/32
1 3/8	1 1/4	1/16	1 1/16	1 5/16	1 3/16	17/32	5/16	1 1/4	1/4	3/32	3/8	11/32
1 1/4	1 1/8	1/16	15/16	1 3/16	1 1/16	15/32	1/4	1 1/8	1/4	3/32	5/16	9/32
1 1/8	1	1/16	13/16	1 1/16	15/16	13/32	1/4	1	1/4	3/32	5/16	9/32
1	7/8	1/16	11/16	15/16	13/16	11/32	1/4	7/8	1/4	3/32	5/16	9/32

The bars and cutters listed above are for machining " blind " holes. The cutters are held in position by hardened parallel pins having flats tapering 3/8 inch per foot. A hole is drilled in the cutter, a little larger in diameter than the pin, and one side is squared and tapered to correspond with the taper of the pin.

Types of Boring Bars

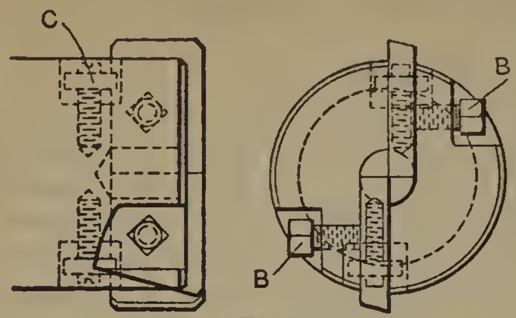


Fig.16

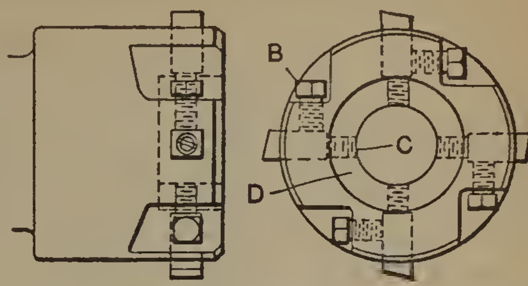


Fig.17

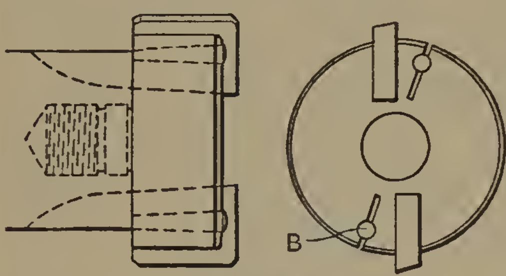


Fig.18

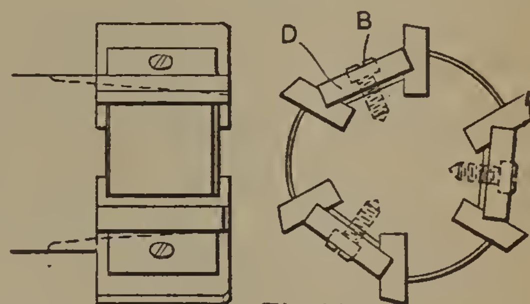


Fig.19

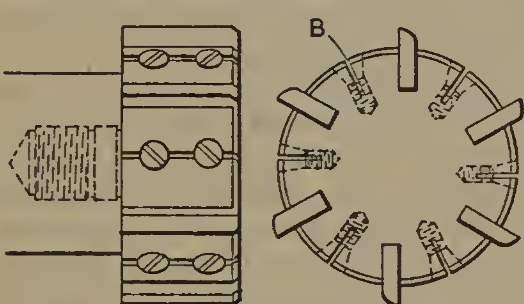


Fig.20

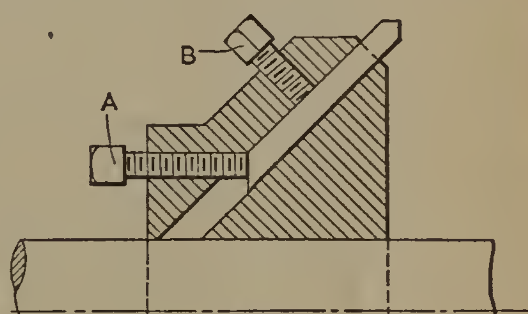


Fig.21

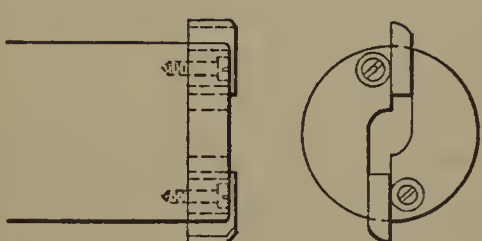


Fig.22

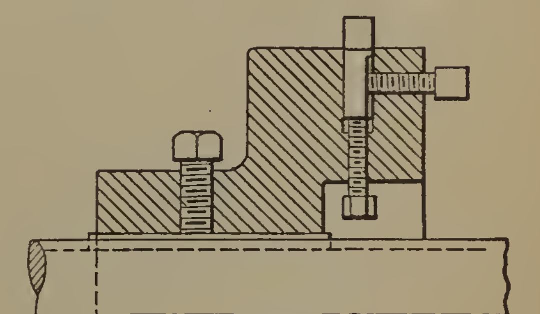


Fig.23

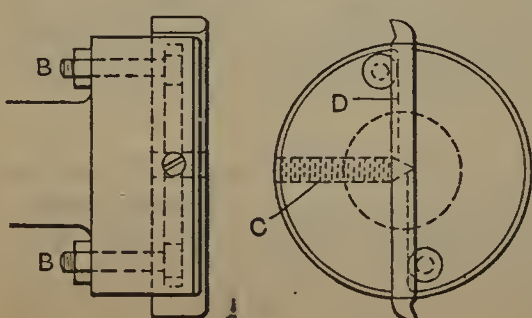


Fig.24

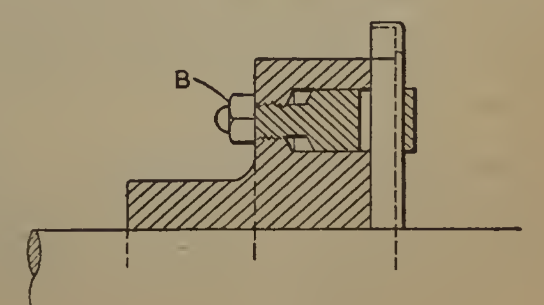


Fig.25

HEAT TREATMENT OF STEEL

Furnaces and Baths for Heating Steel

Steel Heating Furnaces. — The furnaces used for the hardening or tempering of steel are heated either by gas, oil, electricity or solid fuel. Furnaces using oil or gas are made in many different styles and sizes to suit various classes of work, but differ very little in their general arrangement. Crude oil and kerosene are commonly used in oil-heated furnaces. To insure an unvarying temperature, the air and fuel pressures should be uniform. Gas furnaces use either natural, artificial or producer gas. Some gas furnaces are equipped with an automatic apparatus which operates in conjunction with a pyrometer for controlling the temperature to within a few degrees of a given point. The air supply is generally obtained from a positive blower, although where a compressor is installed for operating pneumatic tools, the air is sometimes utilized for the furnaces by interposing reducing valves to diminish the pressure. Artificial gas is more expensive than oil, but is cleaner, and the installation of supply tanks, such as are required for oil, is avoided. Producer gas obtained from a separate plant is not economical, unless there is a considerable number of furnaces. When oxidization or the formation of scale is particularly objectionable, furnaces of the muffle type are often used. These furnaces contain a refractory retort in which the steel is placed, thus excluding the products of combustion. These muffles must be replaced quite frequently and more fuel is required than for an oven type of furnace.

Electrically-heated furnaces are generally considered very satisfactory for the heat-treatment of high-grade work, although the cost of electricity exceeds that of liquid or gaseous fuels. A type of electric furnace that is commonly used, derives its heat from a heavy, low-voltage current, which passes through electrodes to resistance elements in the heating chamber. This type of furnace gives a uniform heat and is adapted to accurate regulation. Electrically-heated furnaces are also used in conjunction with heating baths, the current being transmitted through a bath of metallic salts by two electrodes on opposite sides of the crucible. The conductivity of the salt is very small at normal temperatures, but at high temperatures, when the salt is in the molten condition, it offers a comparatively low resistance to the electric current, and, therefore, when the bath is hot, it forms an electric conductor, and each part of the bath produces its own heat.

Solid fuels, such as coke, coal, charcoal, etc., are also used, in many cases, for heating steel. A common type of solid-fuel furnace is equipped with a grate upon which the fuel is burned, and an arch above the grate which reflects the heat back to the plate that holds the steel to be heated. This plate should be so located that the flames will not come into direct contact with the steel and injure the finished surfaces. To prevent this, the steel is sometimes safeguarded by placing it inside of a clay or cast-iron retort, which is encircled by the flames. The solid-fuel type of furnace is inferior to other types, for most purposes, because it is almost impossible to maintain a uniform temperature, and the gases of combustion are liable to injure the steel.

Heating Steel in Liquid Baths. — The liquid baths commonly used for heating steel tools preparatory to hardening are molten lead, cyanide of potassium, barium chloride, a mixture of barium and potassium chloride and other metallic salts. The molten substance is retained in a crucible which is usually heated by gas or oil. The principal advantages of heating baths are as follows: No part of the work can be heated to a temperature above that of the bath; the temperature can be easily maintained at whatever degree has proved, in practice, to give the best results; the submerged steel can be heated uniformly, and the finished surfaces are protected against oxidization.

The Lead Bath. — The lead bath is extensively used, but is not adapted to the high temperatures required for hardening high-speed steel, as it begins to vaporize at about 1190 degrees F., and, if heated much above that point, rapidly volatilizes and gives off poisonous vapors; hence, lead furnaces should be equipped with hoods to carry away the fumes. Lead baths are especially adapted for heating small pieces which must be hardened in quantities. Gas is the most satisfactory fuel for heating the crucible. It is important to use pure lead that is free from sulphur. The work should be pre-heated before plunging it into the molten lead.

Cyanide of Potassium Bath. — Many steel hardeners prefer cyanide of potassium to lead, for heating steel cutting tools, dies, etc. When cyanide is used, the parts should be suspended from the side of the crucible by means of wires or wire cloth baskets, to prevent them from sinking to the bottom. Steel will not sink in a lead bath, as lead has a higher specific gravity than steel. Cyanide of potassium should be carefully used, as it is a violent poison. The fumes are very injurious, and the crucible should be enclosed with a hood connecting with a chimney or ventilating shaft. This bath is extensively used for hardening in gun shops, in order to harden parts and at the same time secure ornamental color effects.

Barium Chloride Bath. — As a temperature of about 2400 degrees F. can be obtained with this bath, it is used to some extent for heating high-speed steel. Owing to certain disadvantages, however, barium chloride has been discarded by many manufacturers. (See "Disadvantages of Barium Chloride Bath.") When barium chloride is used for the lower temperatures required for carbon steel, it is mixed with chloride of potassium. For temperatures between 1400 and 1650 degrees F., use three parts of barium chloride and two parts chloride of potassium. For higher temperatures, the amount of potassium chloride should be proportionately reduced, pure barium chloride being used for temperatures above 2000 degrees F. All steel should be pre-heated to 600 or 800 degrees F. before being immersed in the bath. Temperatures below 1075 degrees F. can be obtained by using equal parts of potassium nitrate and sodium nitrate. This mixture sets at 400 degrees F., and is used as a tempering bath.

Disadvantages of Barium Chloride Bath. — While barium chloride baths are still used to a considerable extent, both in this country and abroad, the results obtained in the heat-treatment of steel have not been as favorable as was at first expected. In fact, many former users of barium chloride have abandoned it in preference to other methods of heating. The principal difficulty has been that steel heated in barium chloride has a film of soft metal probably 0.003 to 0.006 inch deep. Tests made to determine the influence on the cutting qualities of tools showed conclusively that those heated in a barium chloride bath would not stand as high a cutting speed as steel heated in an oven furnace. When about 0.010 inch is ground from the cutting edges, the effect of heating in barium chloride is not apparent. The injurious effects are increased when using an electric hardening furnace with a barium-chloride bath. The objections mentioned in the foregoing apply only to the hardening of high-speed steel; the electric hardening furnace using a barium chloride bath has proved very satisfactory for ordinary carbon steel tools.

As mentioned in the preceding paragraph, the lower temperature required for carbon steel is obtained by adding a certain percentage of potassium chloride. The barium chloride bath has the advantage of protecting the heated steel, while it is being transferred to the quenching bath, by a thin coating which usually falls off when the steel is being quenched.

Baumé Gravity and Corresponding Specific Gravities, Weights per Gallon and Calorific Value of Fuel Oil

Kind of Oil	Baumé	Specific Gravity	Pounds per Gallon	Calculated B.T.U. per Pound	Calculated B.T.U. per Gallon
Mexico, California, Texas and Kansas Crude Fuel Oil	14	0.9722	8.10	18,810	152,361
	15	0.9655	8.05	18,850	151,743
	16	0.9589	7.99	18,890	150,931
	17	0.9523	7.94	18,930	150,304
	18	0.9459	7.88	18,970	149,484
	19	0.9395	7.83	19,010	148,848
	20	0.9333	7.78	19,050	148,209
	21	0.9271	7.73	19,090	147,506
	22	0.9210	7.68	19,130	146,918
	23	0.9150	7.63	19,170	146,267
	24	0.9090	7.58	19,210	145,612
	25	0.9032	7.54	19,250	145,145
	26	0.8974	7.49	19,290	144,482
	27	0.8917	7.44	19,330	143,815
Kansas, Indian Territory and Illinois Crudes, Penn. Fuel, California Refined Fuel Oil	28	0.8860	7.39	19,370	143,144
	29	0.8805	7.34	19,410	142,469
	30	0.8750	7.29	19,450	141,790
	31	0.8695	7.25	19,490	141,303
	32	0.8641	7.21	19,530	140,811
	33	0.8588	7.16	19,570	140,121
	34	0.8536	7.12	19,610	139,623
	35	0.8484	7.07	19,650	138,926
	36	0.8433	7.03	19,690	138,421
Ohio, Penn. and West Virginia Crude, California and Kansas Refined	37	0.8383	6.99	19,730	137,913
	38	0.8333	6.95	19,770	137,402
	39	0.8284	6.91	19,810	136,887
	40	0.8235	6.87	19,850	136,370
	41	0.8187	6.83	19,890	135,849
	42	0.8139	6.80	19,930	135,524
	43	0.8092	6.76	19,970	134,997
	44	0.8045	6.72	20,010	134,467
	45	0.8000	6.68	20,050	133,934
Kerosene and Gasoline	46	0.7954	6.64	20,090	133,398
	47	0.7909	6.60	20,130	132,858
	48	0.7865	6.57	20,170	132,517
	49	0.7821	6.53	20,210	131,971
	50	0.7777	6.49	20,250	131,423

Characteristics of Fuel Oils. — The calorific values in B.T.U. per pound of oil, as given in the table, "Baumé Gravity and Corresponding Specific Gravities, Weights per Gallon and Calorific Value of Fuel Oil," were determined by the formula: B.T.U. = 18650 + 40 (No. of Degrees Baumé − 10). Sixty-four samples of petroleum oils, ranging from heavy crude oil to gasoline, and representing the products of the principal oil fields of the United States, were examined for calorific power by combustion in oxygen in the Atwater Mahler bomb calorimeter with results ranging from 18,572 to 21,120 B.T.U. per pound. In general, the decrease in calorific power with increase in specific gravity was fairly regular, so that the relation between the two may be expressed, approximately, by means of a simple formula. When the calorific powers calculated from the densities by means of this formula were compared with those actually determined, it was found that in one-ninth of the cases the difference was greater and in eight-ninths it was less than one per cent; in only one-thirtieth was it greater than two per cent; and in no case was it as great as 3 per cent; hence, the calorific value of commercially pure petroleum oils can be determined from the density with sufficient accuracy for most practical purposes.

The heat value of oil is reduced by the presence of small percentages of water. Therefore, if the oil contains water, it should be passed through a filtering tank before going to the burners. In this filtering tank the water settles to the bottom and can be easily drawn off. The oil should be heated before going to the filtering tank, as the water in the oil is more easily separated out of hot oil than cold oil, first, because heated oil offers less resistance to freeing the water, and, second, because there is a greater expansion of oil than water due to the heat, and the water, therefore, has a relatively greater specific gravity.

Pyrometers

Pyrometers are of great value in connection with the heat-treatment of steel, as they make it possible to determine high temperatures accurately; moreover, the temperature, when heating for hardening, can be regulated to conform with the temperature that has given the best results in practice. There are several different types of pyrometers commonly used in industrial service, which may be classified according to the principle upon which they operate.

Thermo-electric Pyrometer. — In this type of pyrometer, temperature variations are determined by the measurement of an electric current generated by the action of heat on the junction of two dissimilar metals; that is, when one junction of the thermo-couple has a temperature different from the other, a current is developed and a meter indicates the temperature, the relation between the strength of current and the temperature being constant. The thermo-couple and the meter form the essential parts. The two dissimilar metals composing the thermo-couple are connected at one end, which is called the "hot end," and placed in the furnace or heated place, the temperature of which is required. Except at the hot end, the two wires or elements do not touch. The free ends, called the "cold end," are kept away from the heat. When the hot end is heated, the intensity of the current generated depends upon the difference between the temperature of the hot and cold ends. The meter is connected to the cold end and shows the value of the current in degrees Fahrenheit or Centigrade. Some pyrometers of this type may be used, intermittently, for temperatures up to 3000 degrees F.

Resistance Pyrometer. — The variation in electric conductivity due to changes in temperature is the principle upon which the resistance pyrometer is based. This type is very accurate for temperatures below 1600 degrees F., but should not be used continuously for higher temperatures. The maximum temperature is about 2200 degrees F. The thermo-electric type is preferable for indicating high-speed steel hardening temperatures, etc., because the resistance type will not stand exposure to intense heats, except for short periods.

Radiation Pyrometer. — This type measures radiated heat and is adapted for very high temperatures. The Féry radiation pyrometer is practically a reflecting telescope having a concave mirror which focuses the radiant heat of the object upon the "hot" junction of a small thermo-couple. There is a diaphragm for reducing the aperture when the instrument is pointed at a very hot object, in order to prevent over-heating the thermo-couple. With the Brown radiation pyrometer, the rays of heat from the furnace or molten metal which enter the pyrometer tube are reflected from a concave mirror onto a sensitive thermo-couple, and the temperature is indicated on a milli-voltmeter, graduated in temperature degrees, the same as a thermo-electric pyrometer. No part of the instrument is inserted in the high heat to be measured. If the temperature of a furnace is being measured, the tube is either held on a tripod or in the hand, and is pointed toward the door of the furnace. The temperature can then be read off on the indicator.

Optical Pyrometers. — There are several classes of optical pyrometers. The *Morse thermo-gage* indicates the temperature by heating the filament of an electric lamp to the same color as that of the incandescent body, the temperature of which is required. The small low-voltage lamp is placed inside a tube through which the heated object is observed. To determine the temperature, the current for the lamp is so regulated (by means of a rheostat) that the color of the lamp filament corresponds to that of the heated object which is observed through the instrument. The current then being consumed is indicated by a milli-ammeter, and the corresponding temperature is determined. This instrument is accurate to within 2 or 3 degrees C. When absorbent glasses are used to reduce the brilliancy of the heated part, the highest temperatures required for industrial work can be gaged. The *Mesure and Nouel optical pyrometer* is a very simple type, which, by means of prisms and reflectors, enables temperatures to be determined by utilizing the colored field produced by the polarization and refraction of light from the heated part. The accuracy of a reading depends upon the observer's judgment of relative colors and may vary 50 degrees C. (90 degrees F.) or more, at temperatures above 1000 degrees C. (1832 degrees F.). This type is adapted to the taking of frequent readings. With the *photometric type* (including the Wanner and Le Chatelier optical and Féry absorption pyrometers) there is an illuminated field, one-half of which receives light from the heated object, and the other half, from a standard source of light forming part of the instrument. With the Le Chatelier instrument, the amount of light admitted from the heated part is regulated by an adjustable diaphragm. When both halves are of the same intensity or brightness, the temperature is indicated by a scale on the diaphragm.

Judging Temperatures by Color. — The U. S. Bureau of Standards states that skilled observers may vary as much as 100 degrees F. in their estimation of relatively low temperatures by color; beyond 2200 degrees F. it is practically impossible to make estimations with any certainty.

Seger Temperature Cones. — The "sentinel" pyrometer or Seger temperature cones are in the form of triangular pyramids (about 3 inches high), composed of metallic and mineral substances which fuse at certain temperatures. They are made in series, each successive cone having a fusing temperature that differs slightly from the one above or below in the scale; that is, if the series were placed in a furnace and the temperature gradually raised, one cone after another would melt as its melting point was reached. These cones are sometimes used in pairs to determine the minimum and maximum temperatures for a given process, one cone being selected for the lowest and another for the highest temperature required. Tests have shown that this method for determining temperatures is very trustworthy within 35 degrees F.

Melting Temperatures of Seger Cones

No. of Cone	Melting Temp., Deg. F.	No. of Cone	Melting Temp., Deg. F.	No. of Cone	Melting Temp., Deg. F.	No. of Cone	Melting Temp., Deg. F.	No. of Cone	Melting Temp., Deg. F.
010	1743	01	2066	9	2390	18	2714	27	3038
09	1778	1	2102	10	2426	19	2750	28	3074
08	1814	2	2138	11	2462	20	2786	29	3110
07	1850	3	2174	12	2498	21	2822	30	3146
06	1886	4	2210	13	2534	22	2858	31	3182
05	1922	5	2246	14	2570	23	2894	32	3218
04	1958	6	2282	15	2606	24	2930	33	3254
03	1994	7	2318	16	2642	25	2966	....	....
02	2030	8	2354	17	2678	26	3002	....	....

Calibration of Pyrometers. — Pyrometers should occasionally be compared with a standard pyrometer or be calibrated in some other way. The following general instructions are given by the Hoskins Mfg. Co. The accuracy of both meter and thermo-couple should be checked. When checking the meter see that all connections are tight and that the protection tubes are sound. Set the zero of the check meter and the service meter to the same temperature and then take readings of both meters when they are alternately connected to any couple. If the instruments are both calibrated for the same external resistance, then only one set of leads is necessary, this set being connected first to one and then the other meter. If the meters are calibrated for different external resistances, then the individual leads of proper resistance must be used with each. In this method of checking, the check meter and the one being tested must be of the same kind; that is, both must be high or low resistance. When checking the thermo-couple, if only one thermo-couple is being used with the meter, see that the zero setting of the meter corresponds to the temperature of the "cold end" of the couple. If several couples are used with one meter, the zero setting should be in agreement with the average temperature of the cold ends of the several couples. Set the zero of the check meter in agreement with the cold-end temperature of the check couple; place check couple and service couple in same protection tube and compare the readings of the two meters.

If the meter operates with only one couple, then the *indicated* error is the *actual* error of the thermo-couple, assuming that the zero settings of both the check and the service meter are correct. If the service meter proved to be accurate, and it is serving more than one couple, then the difference between the readings of the two meters in this test is the combined error of the thermo-couple and the error due to the cold-end setting of the meter. To determine the portion of this due to the thermo-couple, note the *difference* in temperature between the zero setting of the service meter and the actual cold-end temperature of the particular couple being tested. Subtract this difference from the *indicated* error, as shown by the meter readings and the result is the error in the thermo-couple.

Calibrating by the Melting Point of Copper. — For calibrating pyrometers for temperatures above a red heat, the welded or "hot end" of the thermo-couple should be covered with a tight winding of No. 14 or 16 B. & S. gage, standard melting-point wire. The couple should then be inserted in a tube furnace with the welded end approximately in the center. The furnace should be of the required heat before inserting the couple, and should be kept at a temperature approximately 100 degrees F. higher than the melting point of the calibrating wire. The pointer of the meter will then move up the scale with a gradually decreasing speed until the calibrating wire begins to melt, when the pointer will come to rest. After the wire has melted, the pointer will again move upward. Pure copper wire, under oxidizing conditions, melts at 1083 degrees C. (1981 degrees F.), and pure zinc wire, at 419 degrees C. (786 degrees F.). In order to have a strictly oxidizing atmosphere, an open-end electric furnace should be used for calibrating. With this method of calibrating, care should be taken not to have the furnace temperature too far above the melting point of the calibrating wire, because the pointer will move so rapidly and the melting will be of such short duration that the temporary pause of the pointer may not be observed.

Calibrating by the Freezing Point of Melted Salts. — A very satisfactory way of calibrating pyrometers is by using the "freezing points" of melted salts. Pure common salt (NaCl) is melted in a pure graphite crucible. When the salt has been raised to a temperature of 100 to 200 degrees F. above its melting point, the bare welded end of the thermo-couple is inserted to a depth of 2 or 3 inches. The crucible is then removed from the furnace and allowed to cool. The pointer on the meter will drop gradually until the salt begins to freeze or solidify; then the pointer will stop until the salt is frozen. The freezing point of pure salt is taken at 800 degrees C. (1472 deg. F.). After calibrating and before being further used, the couple end should be washed in hot water to remove all traces of the salt, as otherwise the couple will deteriorate rapidly, especially when heated considerably above the melting point of salt in an open furnace. When calibrating pyrometers, care should be taken that the zero setting of the meter agrees with the cold end of the couple, which is always kept away from the heat and generally at the temperature of the outside air. The following table gives the latest available data by the Bureau of Standards on certain substances which may be used for calibrating pyrometers.

Water boils at	100 deg. C.	(212 deg. F.)
Tin freezes at	231.9 deg. C.	(449.4 deg. F.)
Zinc freezes at	419.4 deg. C.	(786.9 deg. F.)
Common salt freezes at	800 deg. C.	(1472 deg. F.)
Copper freezes at	1083 deg. C.	(1981.4 deg. F.)

Hardening

Critical Temperatures. — The "critical points" of carbon tool steel are the temperatures at which certain changes in the chemical composition of the steel take place, during both heating and cooling. Steel at normal temperatures has its carbon (which is the chief hardening element) in a certain form called *pearlite* carbon, and if the steel is heated to a certain temperature, a change occurs and the pearlite becomes *martensite* or hardening carbon. If the steel is allowed to cool slowly, the hardening carbon changes back to pearlite. The points at which these changes occur are the decalescence and recalescence or critical points, and the effect of these molecular changes is as follows: When a piece of steel is heated to a certain point, it continues to absorb heat without appreciably rising in temperature, although its immediate surroundings may be hotter than the steel. This is the *decalescence* point. Similarly, steel cooling slowly from a high heat will, at a certain temperature, actually increase in temperature, although its surroundings may be colder This takes place at the *recalescence* point. The recalescence point is lower than the decalescence point by anywhere from 85 to 215 degrees F., and the lower of these points does not manifest itself unless the higher one has first been fully passed. These critical points have a direct relation to the hardening of steel. Unless a temperature sufficient to reach the decalescence point is obtained, so that the pearlite carbon is changed into a hardening carbon, no hardening action can take place; and unless the steel is cooled suddenly before it reaches the recalescence point, thus preventing the changing back again from hardening to pearlite carbon, no hardening can take place. The critical points vary for different kinds of steel and must be determined by tests in each case. It is the variation in the critical points that makes it necessary to heat different steels to different temperatures when hardening.

Determining Hardening Temperatures. — The temperatures at which decalescence occurs vary with the amount of carbon in the steel, and are also higher for high-speed steel than for ordinary crucible steel. The decalescence point of any steel marks the correct hardening temperature, and the steel should be removed from the source of heat as soon as it has been heated uniformly to this temperature. Heating the piece slightly above this point may be desirable, either to insure the structural change being complete throughout, or to allow for any slight loss of heat which may occur in transferring the work from the furnace to the quenching bath. When steel is heated above the temperature of decalescence, it is non-magnetic. If steel is heated to a bright red, it will have no attraction for a magnet or magnetic needle, but at about a "cherry-red," it regains its magnetic property. This phenomenon is sometimes taken advantage of for determining the correct hardening temperature, and the use of a magnet is to be recommended if a pyrometer is not available. The only point requiring judgment is the length of time the steel should remain in the furnace after it has become non-magnetic, as the time varies with the size of the piece. When applying the magnetic needle test, be sure that the needle is not being attracted by the tongs.

The correct hardening temperature for any carbon steel can be determined accurately by the use of a pyrometer. A form of apparatus often used for testing specimens of steel consists of a small electric furnace in which to heat the specimen, and a special thermo-couple pyrometer (see "Pyrometers") for indicating the range of temperatures through which the steel passes. The pyrometer consists of a thermo-couple, connecting leads and an indicating meter. The thermo-couple is of small wire so as to respond readily to any slight temperature variation. When testing a piece of steel with this apparatus, the temperature indicated by the meter rises uniformly until the decalescence point is reached. At this temperature, the indicating pointer of the meter remains stationary, the added heat being consumed by internal changes. When these changes are completed, the temperature again rises, the length of the elapsed period depending upon the speed of heating. The temperature at which this pause in the motion of the indicating pointer occurs should be carefully noted. To obtain the lower critical point, the temperature is first raised about 100 degrees F. above the decalescence point; the steel is then removed from the furnace and is allowed to cool. The decrease of temperature is immediately shown by the fall of the meter pointer, and, at a temperature somewhat below the decalescence point, there is again a noticeable lag in the movement of the pointer. The temperature at which the movement ceases entirely is the recalescence point. Immediately following, there may occur a slight rising movement of the pointer. During these intervals of temperature lag, both during heating and cooling, there may occur a small fluctuation in the temperature; hence, a definite point in each of these intervals should be considered when a test is made, both critical temperatures being taken at the time the pointer first becomes stationary.

While it is possible to harden steel within a temperature range of about 200 degrees and obtain what might seem to be good results, the best results are obtained within a very narrow range of temperatures which are close to the decalescence point. The hardening temperature for both low tungsten and carbon steel can be located with accuracy, and the complete change from soft to hard occurs within about a range of 10 degrees F. or less. After the temperature has been increased more than from 35 to 55 degrees F. above the hardening point, the hardness of the steel is lessened by a higher temperature, provided the heating is sufficiently prolonged for the steel to be thoroughly heated.

Hardening or Quenching Baths. — When steel heated above the critical point is plunged into a cooling bath, the rapidity with which the heat is absorbed by the bath affects the degree of hardness; hence, baths of various kinds are used for different classes of work. Clear cold water is commonly employed, and brine is sometimes substituted to increase the degree of hardness. Sperm and lard oil baths are used for hardening springs, and raw linseed oil is excellent for cutters and other small tools. The effect of a bath upon steel depends upon its composition, temperature and volume. The bath should be amply large to dissipate the heat rapidly, and the temperature should be kept about constant, so that successive pieces will be cooled at the same rate. Greater hardness is obtained from quenching in salt brine, and less in oil, than is obtained by the use of water. This is due to the difference in the heat-dissipating qualities of these substances. When water is used, it should be "soft," as unsatisfactory results will be obtained with "hard" water. If thin pieces are plunged into brine, there is danger of cracking, owing to the suddenness of the cooling.

The temperature of the hardening bath has a great deal to do with the hardness obtained. In certain experiments a bar quenched at 41 degrees F. showed a scleroscopic hardness of 101. A piece from the same bar quenched at 75 degrees F. had a hardness of 96, while, when the temperature of the water was raised to 124 degrees F., the bar was decidedly soft, having a hardness of only 83. The higher the temperature of the quenching water, the more nearly does its effect approach that of oil, and if boiling water is used for quenching, it will have an effect even more gentle than that of oil; in fact, it would leave the steel nearly soft. With oil baths, the temperature changes have little effect upon the degree of hardness. Parts of irregular shape are sometimes quenched in a water bath that has been warmed somewhat to prevent sudden cooling and cracking. A water bath having one or two inches of oil on top is sometimes employed to advantage for tools made of high-carbon steel, as the oil through which the work first passes reduces the sudden action of the water.

Irregularly shaped parts should be immersed so that the heaviest or thickest section enters the bath first After immersion, the part to be hardened should be agitated in the bath; the agitation reduces the tendency of the formation of a vapor coating on certain surfaces, and a more uniform rate of cooling is obtained. The work should never be dropped to the bottom of the bath until quite cool. High-speed steel is cooled for hardening either by means of an air blast or an oil bath. Both fresh and salt water are also used, although, as a general rule, water should not be used for high-speed steel. Various oils, such as cotton-seed, linseed, lard, whale oil, kerosene, etc., are also employed; many prefer cotton-seed oil. Linseed has the objection of becoming gummy, and lard oil has a tendency to become rancid. Whale oil or fish oil give satisfactory results, but have offensive odors, although this can be overcome by the addition of about three per cent of heavy "tempering" oil.

A quenching solution of 3 per cent sulphuric acid and 97 per cent of water will make hardened carbon steel tools come out of the quenching bath bright and clean. This bath is sometimes used for drills and reamers which are not to be polished in the flutes after hardening. Another method of cleaning drills and similar tools after hardening is to pickle them in a solution of 1 part hydrochloric acid and 9 parts of water. Still another method is to use a heating bath consisting of 2 parts barium chloride and 3 parts potassium chloride. This method is satisfactory for reamers and tools which are not to be polished in the flutes after hardening.

Oil Quenching Baths. — Oil is used very extensively as a quenching medium as it gives the best proportion between hardness, toughness and warpage for standard steels. Special compounded oils of the soluble type are now used in many plants instead of such oils as fish oil, linseed oil, cotton-seed oil, etc. The soluble properties enable the oil to make an emulsion with water. A good quenching oil should possess a flash and fire point sufficiently high to be safe under the conditions used and 350 degrees F. should be about the minimum point. The specific heat of the oil regulates the hardness and toughness of the quenched steel, and the greater the specific heat, the harder the steel will be. Specific heats of quenching oils vary from 0.20 to 0.75, the specific heats of fish, animal and vegetable oils usually being from 0.2 to 0.4, and of soluble and mineral oils, from 0.5 to 0.7. The oil should not contain water, gum when used, have a disagreeable odor or become rancid. A

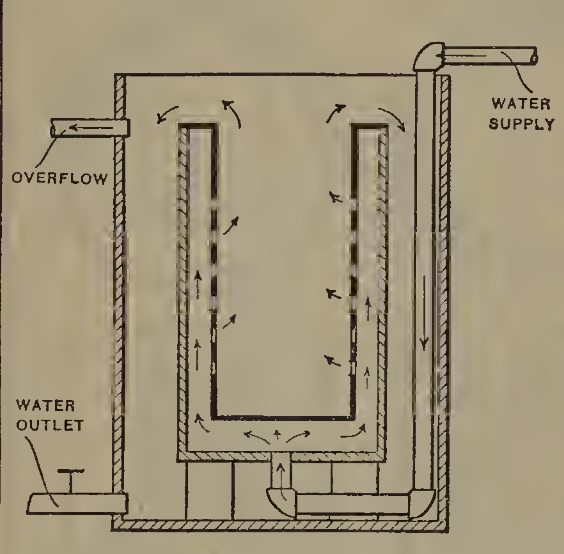


Fig. 1.

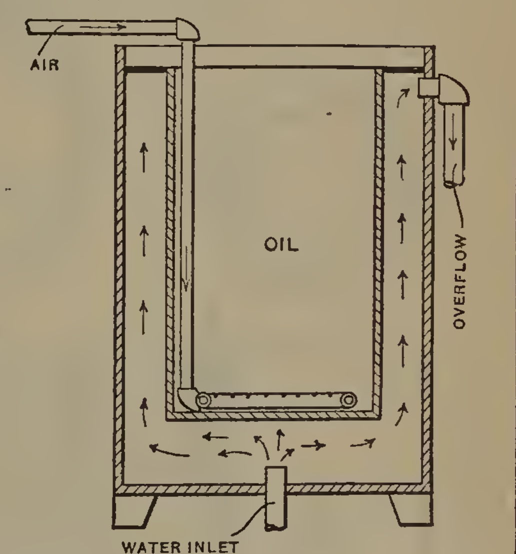


Fig. 2

great many concerns use paraffin and mineral oils for quenching, while a few use crude fuel oils. The quantity of steel that can be quenched per gallon of oil depends on the fluidity of the oil, or its draining qualities. The so-called "refrigerating qualities" are really the capacity of the oil to remove the heat from the steel at a fast rate and then radiate its own heat to the atmosphere.

Tanks for Quenching Baths. — The main point to be considered in a quenching bath is to keep it at a uniform temperature, so that successive pieces quenched will be subjected to the same heat. The next consideration is to keep the bath agitated, so that it will not be of different temperatures in different places; if thoroughly agitated and kept in motion, as is the case with the bath shown in Fig. 1, it is not even necessary to keep the pieces in motion in the bath, as steam will not be likely to form around the pieces quenched. Experience has proved that if a piece is held still in a thoroughly agitated bath, it will come out much straighter than if it has been moved around in an unagitated bath. This is an important consideration, especially when hardening long pieces. It is, besides, no easy matter to keep heavy and long pieces in motion unless it be done by mechanical means.

In Fig. 1 is shown a water or brine tank for quenching baths. Water is forced by a pump or other means through the supply pipe into the intermediate space between the outer and inner tank. From the intermediate space it is forced into the inner tank through holes as indicated. The water returns to the storage tank by overflowing from the inner tank into the outer one and then through the overflow pipe as indicated. In Fig. 3 is shown another water or brine tank of a more common type. In this case the water or brine is pumped from the storage tank and continuously returned to it. If the storage tank contains a large volume of water, there is no need of a special means for cooling. Otherwise, arrangements must be made for cooling the water after it has passed through the tank. The bath is agitated by the force with which the water is pumped into it. The holes at *A* are drilled at an angle, so as to throw the water toward the center of the tank. In Fig. 2 is shown an oil quenching tank in which water is circulated in an outer surrounding tank for keeping the oil bath cool. Air is forced into the oil bath to keep it agitated Fig. 6 shows a water and oil tank combined. The oil is kept cool by a coil passing through it in which water is circulated, which later passes into the water tank. The water and oil baths in this case are not agitated.

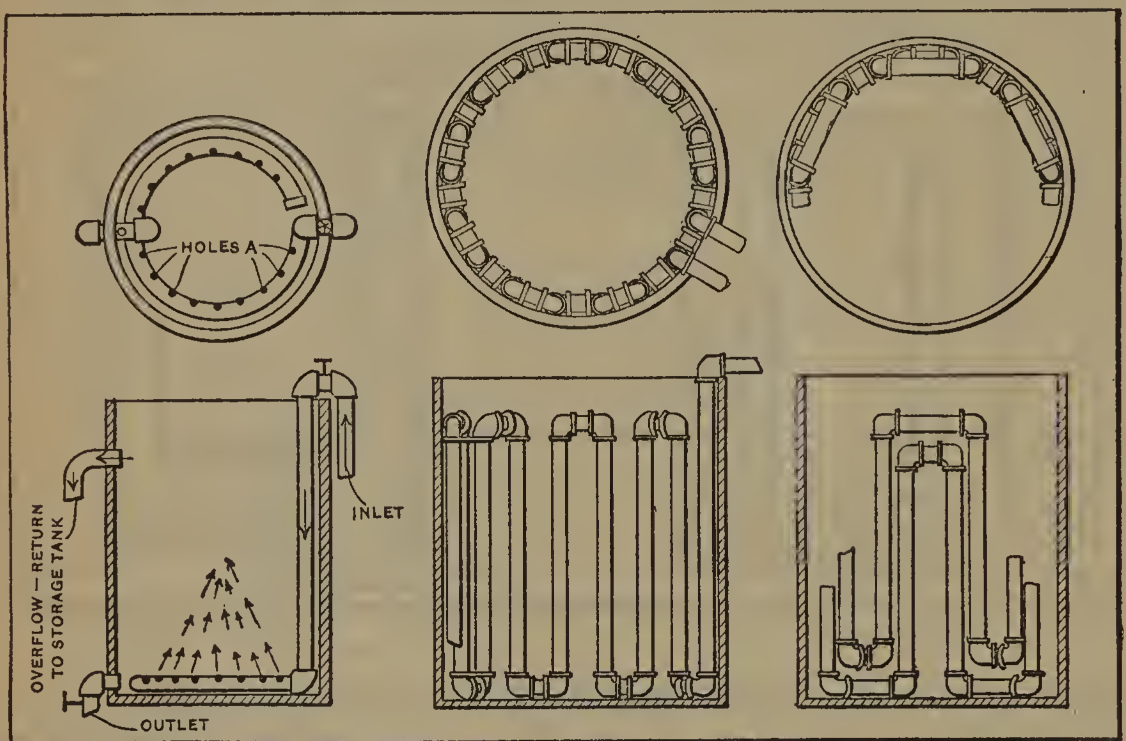


Fig. 3 Fig. 4 Fig. 5

Fig. 4 shows the ordinary type of quenching tank cooled by water forced through a coil of pipe. This can be used for either oil, water or brine. Fig. 5 shows a similar type of quenching tank, but with two coils of pipe. Water flows through one of these and steam through the other. By this means it is possible to keep the bath at a constant temperature.

Hardening High-speed Steel. — High-speed steel must be heated to a much higher temperature for hardening than carbon steel. A temperature of from 1400 to 1600 degrees F. is sufficient for carbon steel; high-speed steel requires from 1800 to 2200 degrees F. The usual method of hardening a high-speed steel tool, such as a turning or planing tool, is to heat the cutting end slowly to a temperature of about 1800 degrees F., and then more rapidly to about 2200 degrees F., or until the end is at a dazzling white heat and shows signs of melting down. The tool point is then cooled either by plunging it in a bath of oil (such as linseed or cotton-seed) or by placing the end in a blast of dry air. When an oil quenching bath is used, its temperature is varied from the room temperature to 350 degrees F., according to the steel used. The exact treatment varies for different steels and it is advisable to follow the directions given by the steel makers. High-speed steel parts that would be injured by a temperature high enough to melt the edges are hardened by heating slowly to as high a degree as possible and then cooling, as described. Formerly, the air blast was recommended by most steel makers, but oil is now extensively used. Care should be taken to quench the heated steel rapidly after removing from the source of heat. The barium-chloride bath has been used quite extensively for heating machine-finished, high-speed steel tools preparatory to hardening. The barium-chloride forms a thin coating on the steel, which is thus protected from oxidization while being transferred from the heating bath to the cooling bath. Tests have demonstrated, however, that barium-chloride baths have certain dis-

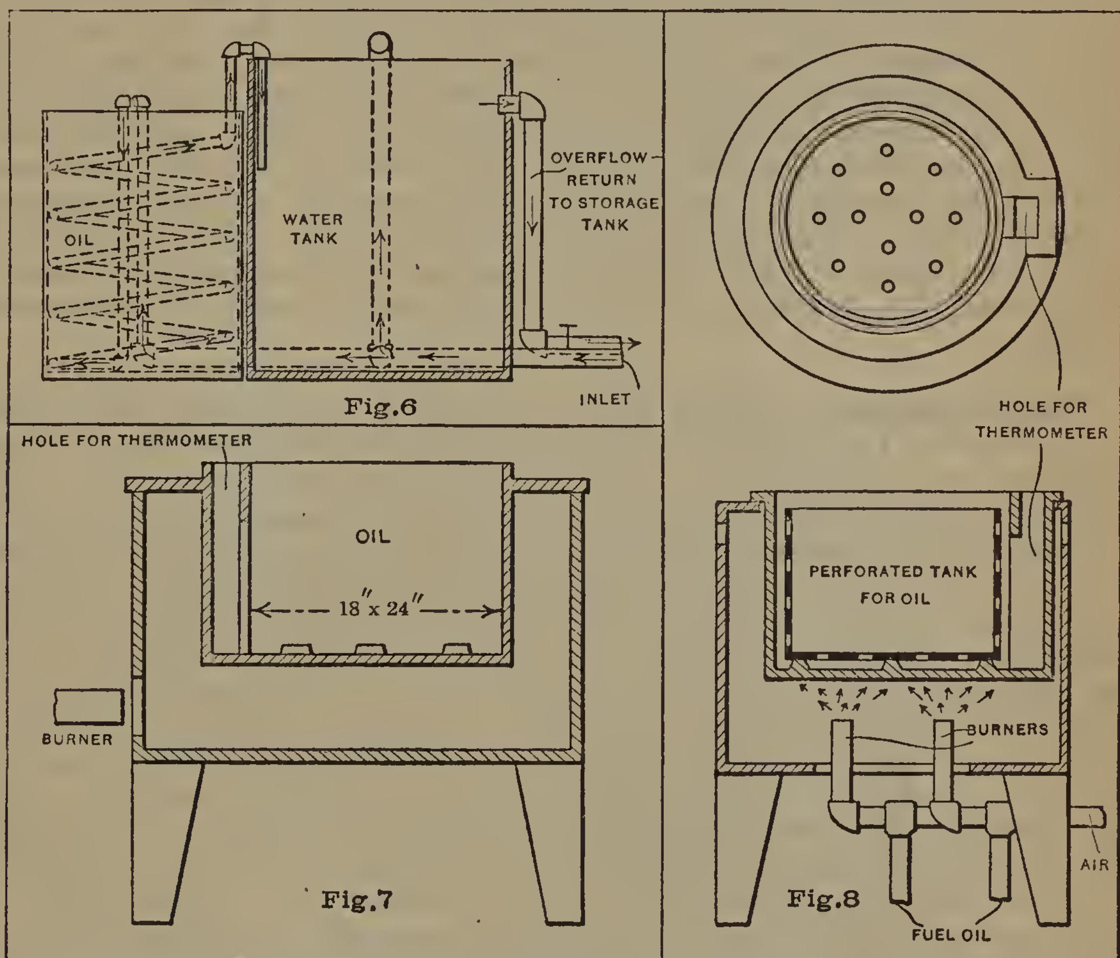


Fig.6 Fig.7 Fig.8

advantages for heating high-speed steel preparatory to hardening, because if the steel is heated to the required temperature, the surface of the tool is softened to some extent. These tests indicate that whenever this salt is used as a heating bath, the temperature should not be raised above 2050 degrees F. When about 0.010 inch is ground from the cutting edges of the tools, the influence of heating in barium chloride may be negligible. (See "Disadvantages of Barium-chloride Bath.")

Very satisfactory results in hardening high-speed steel tools, such as cutters, drills, etc., have been obtained by the following method: First pre-heat in an oven-type gas furnace to from 1300 to 1500 degrees F.; then transfer the steel to another gas furnace having a temperature varying from about 2000 to 2200 degrees F.; when the steel has attained this temperature, quench in a metallic salt bath having a temperature varying from 600 to 1200 degrees F., depending upon the kind of high-speed steel used. The piece to be hardened should be stirred vigorously in the bath until it has obtained the temperature of the bath; then it is cooled, preferably in the air, and requires no further tempering; or it may be put directly into the tempering oil, which should be at a temperature anywhere between 100 and 600 degrees F.

The tempering bath is then gradually raised to the heat required for tempering. The salt bath for quenching should consist of calcium chloride, sodium chloride and potassium ferro-cyanide, in proportions depending upon the required heat. Various kinds of steel require different temperatures for the metallic salt bath. After the temper of the tool has been drawn in the oil, the work is dipped in a tank of caustic soda, and then in hot water. This will remove all oil which might adhere to the tools, and is a method that applies to all tools after being tempered.

The Taylor-White Process. — This process of hardening high-speed steel is, in brief, as follows: The first method, commonly known as the "high-heat treatment," is effected by heating the tool slowly to 1500 degrees F., and then rapidly from that temperature to just below the melting point, after which the tool is quickly cooled below 1550 degrees. At this point, the cooling is continued either fast or slow to the temperature of the air. It is important to avoid any increase of temperature during the cooling period. The second or "low-heat treatment" consists in re-heating a tool which has had the high-heat treatment to a temperature somewhere between 700 and 1240 degrees F., preferably in a lead bath, for a period of five minutes. The tool is then cooled to the temperature of the air either rapidly or slowly.

Heat-Treatment of Spring Steel. — A number of experiments were made at the Baldwin Locomotive Works, to determine the effect of different heat-treatments on the transverse elastic limit and the modulus of elasticity of steels commonly used for locomotive springs. The points investigated were the effect of annealing, the comparative effect of quenching in water and oil, and the effect of re-heating the steel to various temperatures after complete cooling in water or oil. The steel used for the tests was basic open-hearth spring steel of the following composition: Carbon, 1.01 per cent; manganese, 0.38 per cent; phosphorus, 0.032 per cent; sulphur, 0.032 per cent, silicon, 0.13 per cent. This steel was found to reach its decalescence point at 1360 degrees F. Previous experiments had shown that, for annealing, it should be heated 40 or 50 degrees above this temperature, and for hardening, from 50 to 100 degrees above the point of decalescence. For the experiments, the following temperatures were used: For annealing, 1400 degrees F.; for quenching in oil, 1450 degrees F.; for quenching in water, 1425 degrees F. The results obtained are given in the table, "Results of Tests on Spring Steel."

As the table shows, the highest elastic limit obtainable, when heating to 1450 degrees and quenching in oil, was 187,400 pounds per square inch, which was obtained when the temper was not drawn after quenching. The higher the tempering temperature, the lower the elastic limit fell. When the steel was quenched at 1425 degrees F. in water, and the temper not drawn, it was brittle and broke when deflected 0.175 inch. Drawing the temper to 600 degrees F., after hardening in water, gave an elastic limit of 219,800 pounds. When the temper was drawn to 1050 degrees F., the elastic limit dropped to 180,700 pounds, but the test piece did not break at 1.1 inch deflection. The tests show that the modulus of elasticity is practically constant, and apparently independent of the heat-treatment. The conclusions are that steel of 1 per cent carbon, when quenched in cold water at its critical temperature or slightly above, is usually too hard and brittle for making springs or tools. The tests also show that the elastic limit of 1 per cent carbon steel can be made to vary from 78,500 to 240,800 pounds per square inch, by changes in the heat-treatment, and that very small changes in temperature when drawing the temper are sufficient to affect the elastic limit of the steel. Hence, to obtain good results, it is necessary to have means of heating the steel uniformly to the proper temperature, as well as cooling at the desired rate in a medium, the temperature and heat conductivity of which can be kept reasonably constant.

Results of Tests on Spring Steel

Hardened in Oil, Deg. F.	Hardened in Water, Deg. F.	Drawn to, Deg. F.	Diam. Test Piece	Elastic Limit	Modulus of Elasticity	Moment of Inertia	Breakage Deflection*
1450	......	560	1.000	137,500	28,700,000	0.04909	....
1450	......	500	1.000	160,400	27,150,000	0.04909	....
1450	......	400	0.991	177,600	29,080,000	0.04730	....
1450	......	not drawn	0.993	187,400	28,610,000	0.04772	....
....	1425	1050	0.997	180,700	28,070,000	0.04850	....
....	1425	900	0.998	233,900	28,860,000	0.04870	....
....	1425	750	0.994	240,800	29,220,000	0.04790	0.744
....	1425	600	0.991	219,800	30,420,000	0.04730	0.175
....	1425	not drawn	0.991	212,800	29,960,000	0.04730	0.175
Annealed in Lead at 1400 deg. F.			0.991	78,500	27,550,000	0.04730	....

* Seven test pieces did not break when deflected at the center 1.1 inch. The test pieces were placed on supports 12 inches apart and load was applied at center.

Local Hardening. — One method of hardening locally is to cover the part that is to remain soft with a thin metal shield, so that it prevents the surface from being suddenly cooled by the direct action of the cooling medium. The steam or vapor which forms beneath the cover prevents the cooling medium from entering until the work has cooled sufficiently to prevent hardening; hence, a rather loose-fitting shield is desirable. The shield should be made of sheet iron or steel of about No. 29 gage (0.014 inch), for ordinary work. It is composed of one or more pieces, depending upon the shape of the part, and, when several pieces are required, they can be bound together with wires or rivets. Of course, the surfaces to be hardened are left exposed. The heating should be done in a furnace or open-forge fire. A lead bath should not be used, because the hot lead beneath the shield will cause an explosion when the part is cooled. The quenching bath can be the same as when the shield is not used.

Local hardening is also effected by the application of a compound called "Enamelite" to the parts which are to remain soft. This compound, for tool steel, is in the form of a powder which is mixed with hot water to form a paste. It has the property of clinging to the steel and liberating hydrogen (the greatest known nonconductor) when the heated steel is plunged into water. This causes the steel to retain its heat long enough to escape the chill, so that it remains soft where the enamelite has been applied.

Defects in Hardening. — Uneven heating is the cause of most of the defects in hardening. Cracks of a circular form, from the corners or edges of a tool, indicate uneven heating in hardening. Cracks of a vertical nature and dark-colored fissures indicate that the steel has been burned and should be put on the scrap heap. Tools which have hard and soft places have been either unevenly heated, unevenly cooled, or "soaked," a term used to indicate prolonged heating. A tool not thoroughly moved about in the hardening fluid will show hard and soft places, and have a tendency to crack. Tools which are hardened by simply dropping them to the bottom of the tank, sometimes have soft places, owing to contact with the floor or sides of the tank. They should be thoroughly quenched before dropping. When a tool appears soft and will not harden, it probably has been decarbonized on the surface by too much heat or by soaking too long. The surface must be removed before the tool will harden properly. Tools are sometimes soft because the cooling bath is not large enough for the tools being hardened, and becomes too warm after a few pieces have been quenched.

Overheated Steel. — Overheated steel that is not actually burned can be *partly* restored by heating to the proper heat, and allowing it to cool slowly in hot ashes or sand; when cold, the steel is hardened again at the proper hardening heat. Tools treated in this way are not as good as when treated at the proper heat throughout, but they are partially restored, and if the overheating originally took place in forging, the risk of cracking in hardening will be lessened by adopting the process mentioned. Care should be taken that the tuyere of the forge is well covered when heating tool steel; a tool coming in direct contact with the air blast will become surface burned, show soft places in hardening and wear badly in use.

Scale on Hardened Steel. — The formation of scale on the surface of hardened steel is due to the contact of oxygen with the heated steel; hence, to prevent scale, the heated steel must not be exposed to the action of the air. When using an oven heating furnace, the flame should be so regulated that it is not visible in the heating chamber. The heated steel should be exposed to the air as little as possible, when transferring it from the furnace to the quenching bath. An old method of preventing scale and retaining a fine finish on dies used in jewelry manufacture, small taps, etc., is as follows: Fill the die impression with powdered boracic acid and place near the fire until the acid melts; then add a little more acid to insure covering all the surfaces. The die is then hardened in the usual way. If the boracic acid does not come entirely off in the quenching bath, immerse the work in boiling water. Dies hardened by this method are said to be as durable as those heated without the acid.

Tempering

Tempering by the Color Method. — Hardened steel can be tempered or made softer and less brittle by re-heating it to a certain temperature (depending upon the nature of the steel and its intended use), and then cooling. When steel is tempered by the color method, the temper is gaged by the colors formed on the surface as the heat increases. First the surface is brightened to reveal the color changes, and then the steel is heated either by placing it upon a piece of red-hot metal, a gas-heated plate or in any other available way. As the temper increases, various colors appear on the brightened surface. First there is a faint yellow which blends into straw, then light brown, dark brown, purple, blue and dark blue, with various intermediate shades. The temperatures corresponding to the different colors and shades are given in the table on temperatures and colors for tempering. Turning and planing tools, chisels, etc., are commonly tempered by first heating the cutting end to a cherry-red, and then quenching the part to be hardened. When the tool is removed from the bath, the heat remaining in the unquenched part raises the temperature of the cooled cutting end until the desired color (which will show on a brightened surface) is obtained, after which the entire tool is quenched. The foregoing methods are convenient, especially when only a few tools are to be treated, but the color method of gaging temperatures is not dependable, as the color is affected, to some extent, by the composition of the metal. The modern method of tempering, especially in quantity, is to heat the hardened parts to the required temperature in a bath of molten lead, heated oil or other liquids; the parts are then removed from the bath and quenched. The bath method makes it possible to heat the work uniformly, and to a given temperature within close limits.

High Temperatures judged by Color, and Colors for Tempering

Degrees Centigrade	Degrees Fahrenheit	High Temperatures judged by Color	Degrees Centigrade	Degrees Fahrenheit	Colors for Tempering
400	752	Red heat, visible in the dark	221.1	430	Very pale yellow
474	885	Red heat, visible in the twilight	226.7	440	Light yellow
525	975	Red heat, visible in the daylight	232.2	450	Pale straw-yellow
581	1077	Red heat, visible in the sunlight	237.8	460	Straw-yellow
700	1292	Dark red	243.3	470	Deep straw-yellow
800	1472	Dull cherry-red	248.9	480	Dark yellow
900	1652	Cherry-red	254.4	490	Yellow-brown
1000	1832	Bright cherry-red	260.0	500	Brown-yellow
1100	2012	Orange-red	265.6	510	Spotted red-brown
1200	2192	Orange-yellow	271.1	520	Brown-purple
1300	2372	Yellow-white	276.7	530	Light purple
1400	2552	White welding heat	282.2	540	Full purple
1500	2732	Brilliant white	287.8	550	Dark purple
1600	2912	Dazzling white (bluish-white)	293.3	560	Full blue
			298.9	570	Dark blue

Tempering in Oil. — Oil baths are extensively used for tempering tools (especially in quantity), the work being immersed in oil heated to the required temperature, which is indicated by a thermometer. It is important that the oil have a uniform temperature throughout and that the work be immersed long enough to acquire this temperature. Cold steel should not be plunged into a bath heated for tempering, owing to the danger of cracking it. The steel should either be pre-heated to about 300 degrees F., before placing it in the bath, or the latter should be at a comparatively low temperature before immersing the steel, and then be heated to the required degree. A temperature of from 650 to 700 degrees F. can be obtained with heavy tempering oils; for higher temperatures, a lead bath is generally used. A tempering oil which has given satisfactory results in practice has the following characteristics: Composition, mineral oil, 94 per cent; saponifiable oil, 6 per cent; specific gravity, 0.920; flash point, 550 degrees F.; fire test, 625 degrees F. The foregoing figures apply to new oil. When the oil has been used long enough to be rendered practically useless, an analysis shows the following changes: Composition, mineral oil, 30 per cent; saponifiable oil, 70 per cent; specific gravity, 0.950; flash point, 475 degrees F.; fire test, 550 degrees F. The great difference in the composition of the new and old oil is due to the loss of mineral oil, resulting from the high heats to which tempering oil is frequently or constantly subjected; hence, the durability of the tempering bath can be increased by occasionally adding new mineral oil.

Flash Point and Fire Test. — The distinction between the "flash point" and the "fire test" of an oil is as follows: The flash point is the temperature at which the amount of vapor given off is sufficient to form an inflammable or explosive mixture with the air over the surface of the oil, so that the gaseous mixture ignites and burns with a momentary flash when a flame is applied. As the temperature of the oil rises, more vapor is given off, and when the production of vapor is rapid enough to maintain a continuous flame, the oil takes fire and burns. The temperature at which this occurs is called the fire test, firing point or burning point of the oil.

Tempering in a Lead Bath. — The lead bath is commonly used for heating steel preparatory to tempering, as well as for hardening. The bath is first heated to the temperature at which the steel should be tempered; the pre-heated work

Temperatures of Lead Bath Alloys

Parts Lead	Parts Tin	Melting Temp., Deg. F.	Parts Lead	Parts Tin	Melting Temp., Deg. F.	Parts Lead	Parts Tin	Melting Temp., Deg. F.
200	8	560	39	8	510	19	8	460
100	8	550	33	8	500	17	8	450
75	8	540	28	8	490	16	8	440
60	8	530	24	8	480	15	8	430
48	8	520	21	8	470	14	8	420

is then placed in the bath long enough to acquire this temperature, after which it is removed and cooled. As the melting temperature of pure lead is 618 degrees F., tin is commonly added to it to lower the temperature sufficiently for tempering. Reductions in temperature can be obtained by varying the proportions of lead and tin, as shown by the table, "Temperatures of Lead Bath Alloys."

To Prevent Lead from Sticking to Steel. — To prevent hot lead from sticking to parts heated in it, mix common whiting with wood alcohol, and paint the part that is to be heated. Water can be used instead of alcohol, but in that case the paint must be thoroughly dry, as otherwise the moisture will cause the lead to "fly." Another method is to make a thick paste according to the following formula: Pulverized charred leather, 1 pound; fine wheat flour, 1½ pound; fine table salt, 2 pounds. Coat the tool with this paste and heat slowly until dry, then proceed to harden. Still another method is to heat the work to a blue color, or about 600 degrees F., and then dip it in a strong solution of salt water, prior to heating in the lead bath. The lead is sometimes removed from parts having fine projections or teeth, by using a stiff brush just before immersing in the cooling bath. This is necessary to prevent the formation of soft spots.

Pots for Lead Baths. — Melting pots for molten lead baths, etc., should, preferably, be made from seamless drawn steel rather than from cast iron. Experience has shown that the seamless pots will sometimes withstand six months' continuous service, whereas cast-iron pots will last, on an average, only a few days, under like conditions. Cast steel melting pots, if properly made, are as durable as those made of seamless drawn steel.

Tempering in Sand. — The sand bath is used for tempering certain classes of work. One method is to deposit the sand on an iron plate which is heated by suitable means as indicated in the accompanying illustration, Fig. 9. With this method of tempering, tools such as boiler punches, etc., can be given a varying temper by placing them endwise in the sand. As the temperature of the sand bath is higher toward the bottom, a tool can be so placed that the color of the lower end will be a deep dark blue when the middle portion is a very dark straw, and the working end or top a light straw color, the hardness gradually increasing from the bottom up. Tools to be heated by this method must be polished, as the temper is judged by the color. For tempering parts in quantity, sand tempering machines have been developed. One well-known design has a horizontal revolving cylinder containing rows of perforated pockets which become filled with sand when in the lower position and carry it upwards, thus sifting the heated sand in steady streams upon the work. The drum revolves at different rates of speed for different classes of

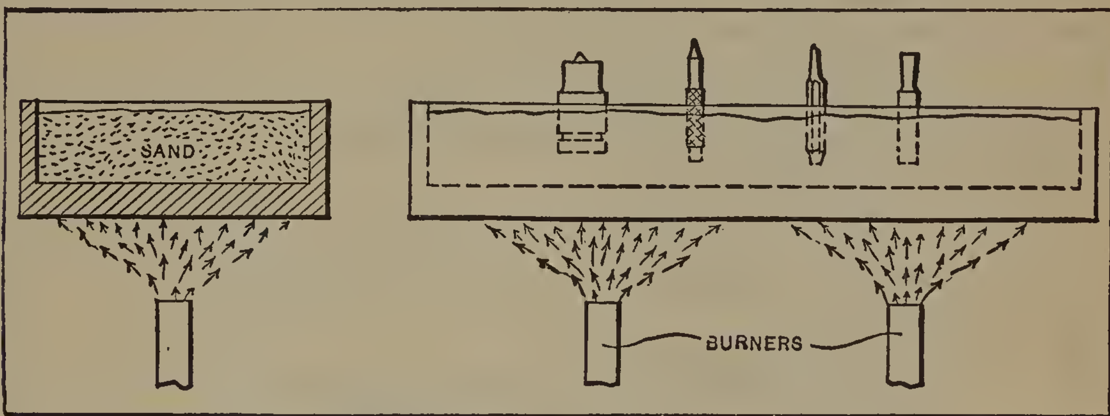


Fig. 9. Arrangement used for Sand Tempering

work, usually making from 3 to 10 revolutions per minute. The heat is supplied by a gas burner. The machine is equipped with a thermometer, which does not indicate the actual temperature of the sand, but a somewhat lower temperature than would be required for the same tempering color, under other conditions.

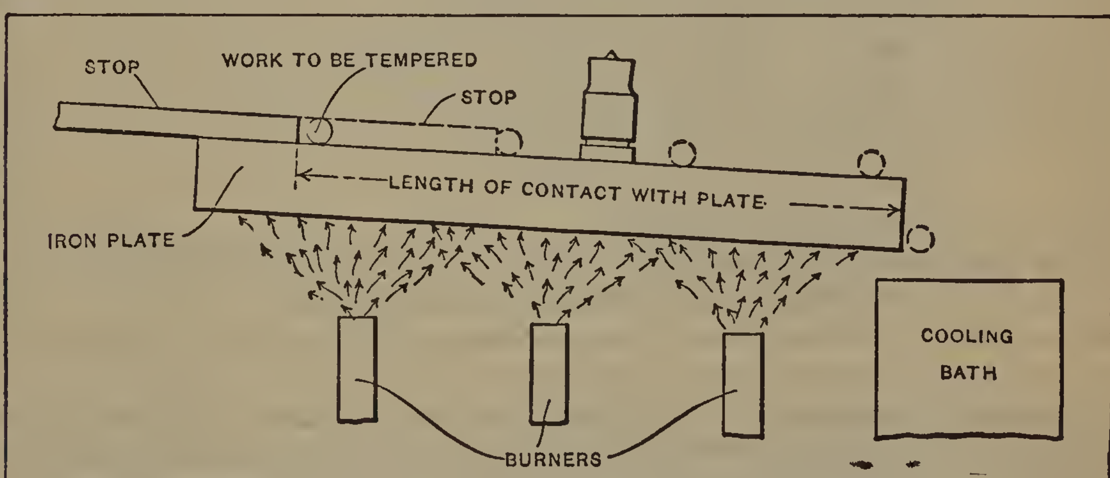


Fig. 10. Tempering by utilizing a Heated Inclined Plate on which the Objects roll down to the Cooling Bath

The thermometer reading, therefore, is relative and not a precise indication of the tempering temperature.

A plate arranged as shown in Fig. 10 will be found very convenient when drawing small, round pieces. The pieces are rolled on the inclined plate which is heated as indicated. The length of time the work is in contact with the plate can be regulated by adjusting the amount of the incline, as well as the location of the "stop." This arrangement can also be used for such work as punches, etc., in which case the plate, of course, should stand level and not in an inclined position.

Tempering Temperatures for Various Tools

Degrees F.	Class of Tool
495 to 500	Taps ½ inch or over, for use on automatic screw machines.
490 to 495	Taps ½ inch or over, for use on screw machines where they pass through the work.
495 to 500	Nut taps ½ inch and under.
515 to 520	Taps ¼ inch and under, for use on automatic screw machines.
525 to 530	Thread dies to cut thread close to shoulder.
500 to 510	Thread dies for general work.
495	Thread dies for tool steel or steel tube.
440 to 445	Circular thread chaser for use on lathes.
525 to 540	Dies for bolt threader threading to shoulder.
460 to 470	Thread rolling dies.
430 to 435	Hollow mills (solid type) for roughing on automatic screw machine work.
450 to 455	Hollow mills (solid type) for use on the drill press.
485	Knurls.
450	Twist drills for hard service.
450	Centering tools for automatic screw machine.
430	Forming tools for automatic screw machine.
430 to 435	Cut-off tools for automatic screw machine.
440 to 450	Profile cutters for milling machine.
430	Formed milling cutters.
435 to 440	Milling cutters.
430 to 440	Reamers.
460	Counterbores and countersinks.
440 to 450	Fly-cutters for use on the drill press.
480	Cutters for tube or pipe-cutting machine.
430 to 440	Dies for heading bicycle spokes.
430	Punches for heading bicycle spokes.
430	Backer blocks for spoke drawing dies.
400	Drawing dies for bicycle spokes.
800	Leaf or carriage springs.
460 and 520	Snaps for pneumatic hammers — harden full length, temper to 460 degrees, then bring point to 520 degrees.

Tempering Furnaces. — In tempering furnaces the only really important consideration is to insure that the furnace is so built as to heat the bath uniformly throughout. It is doubtful if there can be found a tempering furnace on the market that will fill this requirement entirely, although many give good results in general. It is never safe, however, to let any tools being tempered rest against the bottom or sides of the tank, as no matter how scientifically the furnace may be built these parts are, in most cases, hotter than the fluid itself. It is, of course, just as important not to let the thermometer rest against any of these parts in order to insure correct readings. After the pieces tempered are taken out of the oil bath, they should immediately be dipped in a tank of caustic soda (not registering over 8 or 9), and after that in a tank of hot water. This will remove all oil which might adhere to the tools.

Fig. 7, page 1293, shows an ordinary type of tempering furnace. In this the flame does not strike the walls of the tank directly. The tools to be tempered are laid in a basket which is immersed in the oil. In Fig. 8 is shown a tempering furnace in which means are provided for preventing the tools to be tempered from coming in contact with the walls or bottom of the furnace proper. The basket holding the tools is immersed in the inner perforated oil tank. This same arrangement can, of course, be applied to the furnace shown in Fig. 7.

In tempering, the best method is to immerse the pieces to be tempered in the oil before starting to heat the latter. They are then heated with the oil.

Tempering High-speed Steel. — Heavy high-speed steel tools having well-supported cutting edges (such as large planing or turning tools) are often used after hardening and grinding, without tempering. Tools that are comparatively weak should be toughened by tempering to suit the particular service required. The steel is generally heated in a bath of lead, oil, or salts. The tempering temperatures recommended by high-speed steel manufacturers usually vary from 400 to 1000 degrees F., so that definite information should be obtained from the maker of the particular steel to be used. One well-known manufacturer recommends re-heating hardened lathe tools to 1000 degrees F., and tools such as milling cutters, taps, dies, etc., to 500 or 650 degrees F. According to another manufacturer, it is desirable to temper most high-speed steel tools in order to make them more resistant to shocks, the drawing temperatures varying from 600 to 1100 degrees F. Still another steel maker advises tempering lathe and similar tools to 950 degrees F. Lower temperatures varying from 400 to 500 degrees F. are sometimes recommended for tools such as cutters, dies, reamers, etc.

Annealing

Annealing Steel. — The purpose of annealing is not only to soften steel for machining, but to remove all strains incident to rolling or hammering. A common method of annealing is to pack the steel in a cast-iron box containing some material, such as powdered charcoal, charred bone, charred leather, slaked lime, sand, fireclay, etc. The box and its contents are then heated in a furnace to the proper temperature, for a length of time depending upon the size of the steel. After heating, the box and its contents should be allowed to cool at a rate slow enough to prevent any hardening. It is essential, when annealing, to exclude the air as completely as possible while the steel is hot, to prevent the outside of the steel from becoming oxidized.

The temperature required for annealing should be slightly above the critical point, which varies for different steels. Low-carbon steel should be annealed at about 1650 degrees F., and high-carbon steel at between 1400 and 1500 degrees F. This temperature should be maintained just long enough to heat the entire piece evenly throughout. Care should be taken not to heat the steel much above the decalescence or hardening point. When steel is heated above this temperature, the grain assumes a definite size for that particular temperature, the coarseness increasing with an increase of temperature. Moreover, if steel that has been heated above the critical point is cooled slowly, the coarseness of the grain corresponds to the coarseness at the maximum temperature; hence, the grain of annealed steel is coarser, the higher the temperature to which it is heated above the critical point.

If only a small piece of steel or a single tool is to be annealed, this can be done by building up a firebrick box in an ordinary blacksmith's fire, placing the tool in it, covering over the top, then heating the whole, covering with coke and leaving it to cool over night. Another quick method is to heat the steel to a red heat, bury it in dry sand, sawdust, lime or hot ashes, and allow it to cool. Quick annealing can also be partially effected by heating the piece to a dull black-red and plunging it into hot water. This method is not to be recommended.

Annealing High-speed Steel. — The following method of annealing high-speed steel is recommended by one of the largest high-speed tool steel manufacturers in America, and corresponds in all important points to the practice of most other manufacturers: Use an iron box or pipe of sufficient size to allow at least one-half inch of packing between the pieces of steel to be annealed and the sides of the box or pipe. It is not necessary that each piece of steel be kept separate from every other piece, but only that the steel be prevented from touching the sides of the annealing pipe or box. Pack carefully with powdered charcoal, fine dry lime or mica (preferably charcoal), and cover with an air-tight cap, or lute with fireclay; heat slowly to a full red heat (about 1475 or 1500 degrees F.) and keep at this heat from 2 to 8 hours, depending upon the size of the pieces to be annealed. A piece 2 by 1 by 8 inches requires about three hours. Cool as slowly as possible, and do not expose to the air until cold. A good way is to allow the box or pipe to remain in the furnace until cold.

A series of experiments made to determine the proper temperature to which to heat high-speed steel for annealing, showed the following results: When the steel was heated to below 1250 degrees F. and slowly cooled, as in annealing, it retained the original hardness and brittleness imparted in forging. When heated to between 1250 and 1450 degrees F., the Brinnell test indicated that the steel was soft, but impact tests proved that the steel still retained its original brittleness. However, when heated to between 1475 and 1525 degrees F., the steel became very soft; it had a fine-grained fracture, and all of the initial brittleness had entirely disappeared. In carrying these tests further, to 1600, 1750, and 1850 degrees F., it was found that the steel became very soft, but there was a gradual increase in brittleness and in the size of the grain, until at 1850 degrees F. the steel again became as brittle as unannealed steel. Dried air-slaked lime was used as a packing medium in making these tests.

Casehardening

Casehardening is the process of hardening the surface of low-carbon steel or iron by carbonizing the surface. When parts must be casehardened in quantity, they should be packed in an iron box containing some carbonaceous material. The box and its contents are then heated for a certain length of time, depending upon the depth of hardened surface desired and the nature of the material. The heat for casehardening varies from 1600 to 1800 degrees F., the temperature being governed, to some extent, by the requirements. The absorption of carbon begins when the steel reaches about 1300 degrees F. At the end of the carbonizing period, the box is withdrawn from the furnace and is allowed to become quite cold. The articles are then placed in a muffle furnace and are re-heated to about 1470 degrees F., after which they are quenched in cold water, tepid water or oil, the bath depending upon the purpose for which the parts are to be used. For ordinary purposes, clear cold water is satisfactory. To produce a very hard surface, use salt water. When a hard surface is not as important as a tough core, use an oil bath. The practice of allowing the box and its contents to cool, and then re-heating prior to quenching, is based on the old rule of hardening on a rising heat. This method gives more satisfactory results than that of dumping the parts into a tank of cold water at the end of the carbonizing period.

Different Methods of Casehardening. — A committee of the American Society for Testing Materials recommended the following practice for casehardening carbon-steel parts. Four different conditions were considered, varying from the heat-treatment that would give the hardest surface and the least strength, to that which would give the greatest strength with the least hardness of surface: When a hard case is the only requirement and lack of toughness or even brittleness is unimportant, the articles may be quenched by emptying the contents of the casehardening boxes directly into cold water or oil. In this way both the core and the case are coarsely crystallized and the strength is reduced. If the articles are allowed to cool to a temperature slightly exceeding the critical range of the casehardening, usually from 800 to 825 degrees C. (1472 to 1517 degrees F.), and then quenched, the core and case still remain crystalline, but the danger of distortion or cracking in the quenching bath is reduced and the strength is somewhat increased. The next recommended method is to increase the toughness and strength of the article and refine the case. The articles are allowed to cool slowly in the carbonizing pot to a temperature of about 650 degrees C. (1200 degrees F.), are then re-heated to a temperature slightly exceeding the lower critical point of the case, which usually is from 775 to 825 degrees C. (1427 to 1517 degrees F.), and are then quenched in water or oil. They should be removed from the quenching bath before their temperature has fallen below 100 degrees C. (212 degrees F.). By allowing them to cool slowly to a temperature of about 650 degrees C. (1200 degrees F.) and then re-heating to a temperature of about 900 to 950 degrees C. (1652 to 1740 degrees F.), followed by quenching in oil, from which they are removed before they have dropped below a temperature of 100 degrees C. (212 degrees F.), then re-heating to about 800 degrees C. (1472 degrees F.) and again quenching in water or oil, both the case and the core will be thoroughly refined and their toughness greatly increased. In order to reduce the hardening stress created by quenching, the objects, as a final treatment, may be tempered by re-heating them to a temperature not exceeding 200 degrees C. (392 degrees F.).

Casehardening Packing Boxes. — Hardening boxes are made of either cast or wrought iron. The box should not be too large, as pieces in the center may then not become sufficiently carbonized. For small articles, a box measuring 12 by 10 by 8 inches is large enough, and for such parts as bicycle axles, pedal pins, etc., the maximum size should be about 18 by 12 by 11 inches. The boxes should have a plate lid which must be luted or sealed with clay after the contents are packed.

Carbonizing Materials. — The carbonizing materials in general use are charred leather, powdered bone, cyanide of potassium, wood and "animal" charcoal, prussiate of potash and other compositions consisting of mixtures of carbonaceous matter and certain cyanides or nitrates. For slight hardening, cyanides are often used. Charred leather gives good results, although poorly charred leather or that made from old boots, belting, etc., should not be used. A mixture preferred by some to charred leather consists of 60 parts of wood charcoal and 40 parts of barium carbonate. The casehardening compound employed at the Altoona Shops of the Pennsylvania Railroad is made in the following proportions: 11 pounds prussiate of potash; 30 pounds sal-soda; 20 pounds coarse salt; 6 bushels powdered charcoal (hickory preferred). These ingredients are mixed together, 30 quarts of water being added.

Steels for Casehardening. — The percentage of carbon in steels ordinarily used for parts to be casehardened varies, as a general rule, from 0.15 to 0.20 per cent. If the carbon exceeds 0.20 per cent, it tends to give a hard instead of a soft core. If the carbon content is too low, the steel may be difficult to machine; hence, steels containing as much as 0.20 to 0.25 per cent carbon are often used for casehardening. For general work, steel of the following composition will be found satisfactory: Carbon, 0.16 to 0.20 per cent; manganese, less than 0.35 per cent; silicon, not over 0.30 per cent. The sulphur and phosphorus should be as low as possible, not exceeding 0.1 per cent.

Degree and Depth of Hardened Surface. — The percentage of carbon contained in the casehardened surface should vary according to requirements. A high-carbon case containing 1.1 per cent carbon gives a very hard wearing surface suitable for work that must withstand a fairly constant pressure, as shafts running in bearings, etc., but for parts which must withstand repeated shocks, this amount of carbon would render them too brittle, and in such cases it is advisable not to exceed 0.90 to 1 per cent carbon. For most purposes, 0.90 per cent carbon is preferable. Recent investigations indicate that the percentage of carbon in the hardened crust varies with the depth of the latter; the deeper the penetration, the higher the carbon content. Crusts about 0.050 inch deep usually have from 0.85 to 0.90 per cent carbon on the surface. In many instances, a penetration of 0.040 inch is sufficient, but if the work is to be ground after casehardening, it is advisable to carbonize to a depth of about 1/16 inch. Too deep a carbonized case makes the work more brittle, partly because of the prolonged exposure to a high temperature and partly on account of the increase in the hardened section and the decrease in the softer and more ductile core; hence, parts to withstand bending stresses, like gear teeth, should not be carbonized too deeply. The penetration of the carbon increases with the temperature and with the time of exposure, but not in direct proportion to these two factors. Carbonization takes place rapidly until the crust is saturated with carbon, when there is a sudden diminution in the rate of carbonization, which varies according to the temperature.

Casehardening for Colors. — For hardening and at the same time coloring such parts as wrenches, etc., the following mixture may be used: Mix 10 parts of charred bone, 6 parts of wood charcoal, 4 parts of charred leather and 1 part of powdered cyanide. The leather should be black, crisp and well pulverized, and the four ingredients well mixed. The object in charring the bone and leather is to remove all grease. The parts to be colored must be well polished and should not be handled with greasy hands. To obtain satisfactory work, these rules must be observed. If the colors obtained are too gaudy, the cyanide may be omitted, and if there is still too much color, leave out the charcoal. The parts to be colored and hardened should be packed in a piece of common gas pipe having a closed end. Pipe is preferable because the pieces can be dumped into the cooling water with little or no exposure to the air. The open end of the pipe can be placed close to the surface of the water before the parts are removed, but with a box there would be more or less exposure. This class of work should be heated to a dark cherry-red and kept at that temperature for about four or five hours. If the temperature is too high, no colors will appear. The tank should be arranged with a compressed air pipe connecting with the water pipe at the bottom in such a way that a jet of air is forced upward, thus filling the tank with bubbles. There should also be a sieve or basket in the tank for receiving the work. After quenching, place the parts in boiling water for five minutes and then bury them in dry sawdust for half an hour. Another mixture recommended for coloring consists of 10 parts granulated bone, 2 parts bone black and 1 part granulated charred leather.

Gas Process of Casehardening. — Owing to the growth of the armor plate industry, efforts were made to find a better and cheaper method of carbonizing the plate. The first method employed was to place an armor plate in a pit and cover it with a layer of charcoal; then another plate was laid onto it, after which the pit was covered and heated to a temperature high enough to cause the steel to absorb the carbon from the charcoal. The next method tried was to force a current of carbonaceous gas between the two plates, instead of using charcoal. This caused the carbon to penetrate in less time and was found to be more economical. Later the plates were heated by electricity, and the use of electricity and carbonaceous gas gave a more uniform carbon penetration. This process was followed by the development of a muffle carbonizing furnace. In this the work is placed in a revolving retort through which is forced a current of carbonaceous gas. This retort serves as a muffle and is surrounded with the flames of the heating gases. It is claimed that in this furnace small pieces can be carbonized much more quickly and at about one-half the cost, as compared with packing in iron boxes and baking. Some of the gases that have been experimented with are methane, ethylene, illuminating gas, carbon-monoxide, carbon-dioxide, and gases made from petroleum, naphtha and gasoline. Carbon-monoxide was found superior to other gaseous materials, but, while it is capable of rapid penetration, there is an oxidizing effect that might spoil small parts which cannot be ground afterwards. To overcome these bad effects of carbon-monoxide, a new process has been developed in which the work is packed with wood charcoal in a cylinder, and, when heated to the carbonizing temperature, a current of carbon-dioxide is injected into the cylinder.

To Clean Work after Casehardening. — To clean work, especially if knurled, where dirt is likely to stick into crevices after casehardening, wash it in caustic soda (1 part soda to 10 parts water). In making this solution, the soda should be put into hot water gradually, and the mixture stirred until the soda is thoroughly dissolved. A still more effective method of cleaning is to dip the work into a mixture of 1 part sulphuric acid and 2 parts water. Leave the pieces in this mixture about three minutes; then wash them off immediately in a soda solution.

Pack-hardening. — Pack-hardening, as the term is generally understood, consists in treating steel (generally tool steel) with some carbonaceous material and quenching it in oil. The terms "pack-hardening" and "casehardening" are often used interchangeably and the two processes are similar. The surface of the steel is supplied with additional carbon by the use of some carbonaceous material that will not be injurious. To do this, the steel is packed in sealed iron boxes with the carbonizing material. Bone should not be used for pack-hardening tool steel, as it contains a high percentage of phosphorus, which tends to make the steel weak and brittle. For steel that is to contain not more than 1.25 per cent carbon, charred leather is recommended. For obtaining a higher carbon content, use charred hoofs or horns or a mixture of the two. The leather, hoofs or horns can be used repeatedly by adding a quantity of new material each time. A mixture of charred leather and charcoal is also used for pack-hardening. The work should be so packed that it does not come in contact with the box. First place a layer of carbonizing material in the bottom and then a layer of work, no two pieces touching each other. When treating gages, or parts that are likely to spring, they should be so packed that there will be little liability of springing when they are drawn up through the packing material. The parts should not be dumped into the quenching bath, as it is better to handle the pieces separately. It is a good plan to attach a piece of iron wire to each part to facilitate removal from the box. If there are several layers of work, the wires should be so arranged that the various layers can be taken out in the proper order, beginning with the top row. The temperature for pack-hardening should be as low as is consistent with the desired results, and should be uniform throughout the box. To gage the heat, holes may be drilled through the cover at the center so that test wires (say, 3/16 inch in diameter) can extend to the bottom of the box. When the latter has been in the fire long enough to heat the contents to about a dull red (as near as can be judged) a wire is withdrawn; if it is red hot begin timing the heat; if not, wait and draw another later, the test being continued until one is withdrawn that has the desired heat. The length of time necessary for heating depends upon the depth of hardening surface desired. For ordinary snap-gages, from one and one-half to two hours after the steel is red hot is sufficient. Ordinary work requires a temperature of about 1475 degrees F. Pack-hardening minimizes the danger of cracking and warping.

Casehardening Alloy Steels. — When nickel steels are heat-treated by casehardening, nickel seems to retard the process somewhat and the hardness of the "case" is somewhat lower than that obtainable in ordinary carbon steels. On the other hand, nickel tends to oppose the crystallization of the steel at high temperatures and to eliminate the consequent brittleness. With a 2 per cent nickel steel, the following temperatures are recommended: The steel should first be quenched from a temperature of 1830 degrees F. It is then given a second heating to 1380 degrees F., and is again quenched, after cooling to about 1290 degrees F. A single quenching from 1290 degrees F. gives the greatest hardness in the case, but not the greatest tenacity in the core. Quenching from 1380 degrees F. gives a somewhat higher tenacity but a slightly lower hardness in the case. A 6 per cent nickel steel should be quenched first from 1560 degrees F., and after re-heating, from 1245 degrees F. Since this high nickel percentage almost completely prevents the brittleness of the core, one quenching from about 1290 degrees F. is, in most cases, sufficient. Steels with from 1 to 1.2 per cent chromium are sometimes used when an especially hard case is required. This element aids the crystallization of the core and the double quenching is necessary. Chrome-nickel steels with a low chromium content require about the same heat treatment as pure nickel steels. A mixture of 60 parts wood charcoal and 40 parts of barium carbonate is recommended for carbonizing.

Casehardened Gears. — There are four general classes of steel used for casehardened gears, *viz.*, straight-carbon, nickel, chrome-vanadium, and chrome-nickel steel, and, in each of these classes, several modifications will be found in the market. On the whole, the steels containing chromium are preferable. Before being carbonized, the carbon content of each of the steels mentioned should be about 0.20 per cent; under no circumstances should it be more than 0.25 per cent, to avoid brittleness in the teeth. The carbon in the "case" should be increased to about 0.90 per cent, which can readily be done by using the proper carbonizing material and temperature for carbonizing. This temperature, in general, should be about 1600 to 1650 degrees F. for the classes of steel mentioned. Lower temperatures do not give sufficient depth of case, unless the heating operation is much prolonged Conversely, higher temperatures result in a case of excessive carbon content and a core of such large grain-size that it will not respond as readily to the subsequent heat-treatment The proper heat-treatment, after case-carbonizing, is very important. The work is first allowed to cool in the box after carbonizing. It is then re-heated to 1550 or 1625 degrees F. and quenched in a suitable medium to refine the core; next, it is re-heated to 1350 or 1425 degrees F. and is again quenched to harden the case; finally, it is drawn in oil to a temperature not over 400 degrees F., to further increase the strength and toughness of the material. The temperatures given are approximate, and more definite information concerning any particular steel should be obtained from the steel-maker. Casehardened gears have harder surfaces, as shown by the scleroscope test, than tempered gears.

Tempered Gears.— Unlike casehardened gears, tempered gears are of uniform carbon content, and when hardened have a uniform hardness throughout the tooth section. The steels used for tempered gears are of three general classes, *viz.*, silico-manganese, chrome-vanadium and chrome-nickel steel, the last named, in its different modifications, being the most generally used. The carbon content for the different classes varies from 0.40 to 0.60 per cent. The heat-treatment of all these steels consists simply in heating the gears slowly and uniformly to the hardening temperature, which is usually about 1500 degrees F., quenching in oil, and afterward drawing in an oil bath. Tempered alloy steel gears are preferred to casehardened gears for some purposes, especially where strength is the main consideration. They are particularly adapted for "clash gears."

Application and Heat Treatment of S. A. E. Carbon and Alloy Steels

The following data and information on various carbon and alloy steels is condensed from reports of the Iron and Steel Division of the Society of Automotive Engineers, Inc., as revised up to September, 1920. The steels referred to are intended primarily for use in automobile construction, but have proved of such value in other fields that they have been adopted by the Society of Automotive Engineers, Inc. (S. A. E.), for general use in aeronautic, marine, motor cycle, stationary engine and tractor industries. The accompanying tables give the compositions conforming to S. A. E. specifications as applied to various carbon and alloy steels. The notes and instructions given in the following, regarding physical characteristics, heat treatments, etc., are not to be considered as part of the S. A. E. specifications, but are added solely for the guidance of users of these steels and to assist buyers in selecting the proper steels for different purposes.

When referring to the tables, "Physical Properties of Heat-treated Carbon Steels" and "Physical Properties of Heat-treated Alloy Steels," the following points should be considered: (1) The figures given indicate what can be expected as the average product of a given composition when treated in the manner specified, and as applied to average sections prevailing in automobile work; (2) the values given are low enough so as to protect the makers of heat-treated stock and parts from unreasonable demands, the idea being to give values which coincide with the results obtained when stock of medium to high grade is purchased in the open market and treated by means of commercially efficient equipment controlled by commercially accurate instruments. For the sake of simplicity it was deemed advisable to adopt only average minimum values for tensile strength, elastic limit, reduction of area and elongation. These values are based upon the following considerations, the heat treatment being kept constant: The lowest tensile strength and elastic limit occur with steels at the bottom of a given range in carbon. The lowest reductions in area and elongations occur with steels at the top of a given range in carbon. True elastic limits are given, because these are constantly lower than corresponding yield points. The yield point is measured by the drop of the testing machine beam, and while this is the most readily and widely used measure of the so-called elastic limit, the results obtained by this method are generally from 5000 to 15,000 pounds higher than the true elastic limit when the latter property is not in excess of 125,000 pounds per square inch. The values given are very conservative and average results in practice will generally exceed appreciably the figures given, which serves to increase the factor of safety and protect both the engineer and the manufacturer.

S. A. E. Specification Numbers for Steels. — A numeral index system has been adopted by the Society of Automotive Engineers, Inc., for representing the different classes of steel included in the S. A. E. specifications. This system makes it possible to employ specification numerals on shop drawings and blueprints that are partially descriptive of the steel to which the numbers apply. The first figure of the number indicates the general class to which the steel belongs: thus, 1, indicates carbon steel; 2, nickel steel; 3, nickel-chromium steel; 5, chromium steel; 6, chromium-vanadium steel; 7, tungsten steel; and 9, silico-manganese steel. In the case of alloy steels, the second figure generally indicates the approximate percentage of the chief alloying element. The last two or three figures indicate the average carbon content in "points" or hundredths of one per cent. For example, specification No. 2512 indicates a nickel steel with approximately 5 per cent nickel, and 0.12 per cent carbon; and specification No. 71660 indicates a tungsten steel with about 16 per cent tungsten and 0.60 per cent carbon.

Heat Treatments for Carbon and Alloy Steels — 1

(Recommended for various steels conforming to S. A. E. specifications)

Heat Treatment A

After forging or machining:
1. Carbonize between 1600° F. and 1750° F. (1650°–1700° F. desired).
2. Cool slowly or quench.
3. Reheat to 1450°–1500° F. and quench.

Heat Treatment B

After forging or machining:
1. Carbonize between 1600° F. and 1750° F. (1650°–1700° F. desired).
2. Cool slowly in the carbonizing mixture.
3. Reheat to 1550°–1625° F.
4. Quench.
5. Reheat to 1400°–1450° F.
6. Quench.
7. Draw in hot oil varying from 300°–450° F., depending upon hardness desired.

Heat Treatment D

After forging or machining:
1. Heat to 1500°–1600° F.
2. Quench.
3. Reheat to 1450°–1500° F.
4. Quench.
5. Reheat to 600°–1200° F. and cool slowly.

Heat Treatment E

After forging or machining:
1. Heat to 1500°–1550° F.
2. Cool slowly.
3. Reheat to 1450°–1500° F.
4. Quench.
5. Reheat to 600°–1200° F. and cool slowly.

Heat Treatment F

After shaping or coiling:
1. Heat to 1425°–1475° F.
2. Quench in oil.
3. Reheat to 400°–900° F., according to temper desired, and cool slowly.

Heat Treatment G

After forging or machining:
1. Carbonize between 1600° F. and 1750° F. (1650°–1700° F. desired).
2. Cool slowly in the carbonizing material.
3. Reheat to 1500°–1550° F.
4. Quench.
5. Reheat to 1300°–1400° F.
6. Quench.
7. Reheat to 250°–500° F. (depending upon work) and cool slowly.

Heat Treatment H

After forging or machining:
1. Heat to 1500°–1600° F.
2. Quench.
3. Reheat to 600°–1200° F. and cool slowly.

Heat Treatment K

After forging or machining:
1. Heat to 1500°–1550° F.
2. Quench.
3. Reheat to 1300°–1400° F.
4. Quench.
5. Reheat to 600°–1200° F. and cool slowly.

Heat Treatment L

After forging or machining:
1. Carbonize at a temperature between 1600° F. and 1750° F. (1650°–1700° F. desired).
2. Cool slowly in the carbonizing mixture.
3. Reheat to 1400°–1500° F.
4. Quench.
5. Reheat to 1300°–1400° F.
6. Quench.
7. Reheat to 250°–500° F. and cool slowly.

Heat Treatment M

After forging or machining:
1. Heat to 1450°–1500° F.
2. Quench.
3. Reheat to 500°–1250° F. and cool slowly.

Heat Treatment P

After forging or machining:
1. Heat to 1450°–1500° F.
2. Quench.
3. Reheat to 1375°–1450° F.
4. Quench.
5. Reheat to 500°–1250° F. and cool slowly.

Heat Treatment Q

After forging:
1. Heat to 1475°–1525° F. (Hold at this temperature one-half hour to insure thorough heating.)
2. Cool slowly.
3. Machine.
4. Reheat to 1375°–1425° F.
5. Quench.
6. Reheat to 250°–550° F. and cool slowly.

Heat Treatments for Carbon and Alloy Steels — 2

(Recommended for Various Steels conforming to S. A. E. specifications)

Heat Treatment R

After forging:
1. Heat to 1500°–1550° F.
2. Quench in oil.
3. Reheat to 1200°–1300° F. (Hold at this temperature three hours.)
4. Cool slowly.
5. Machine.
6. Reheat to 1350°–1450° F.
7. Quench in oil.
8. Reheat to 250°–500° F. and cool slowly.

Heat Treatment S

After forging or machining:
1. Carbonize at a temperature between 1600° F. and 1750° F. (1650°–1700° F. desired).
2. Cool slowly in the carbonizing mixture.
3. Reheat to 1650°–1750° F.
4. Quench.
5. Reheat to 1475°–1550° F.
6. Quench.
7. Reheat to 250°–550° F. and cool slowly.

Heat Treatment T

After forging or machining:
1. Heat to 1650°–1750° F.
2. Quench.
3. Reheat to 500°–1300° F. and cool slowly.

Heat Treatment U

After forging:
1. Heat to 1525°–1600° F. (Hold for about one-half hour.)
2. Cool slowly.
3. Machine.
4. Reheat to 1650°–1700° F.
5. Quench.
6. Reheat to 350°–550° F. and cool slowly.

Heat Treatment V

After forging or machining:
1. Heat to 1650°–1750° F.
2. Quench.
3. Reheat to 400°–1200° F. and cool slowly.

Ten Per Cent Carbon Steel (Specification No. 1010). — This steel is usually known in the trade as soft, basic open-hearth steel. It is commonly used for seamless tubing, pressed steel frames, pressed steel brake drums, sheet steel brake bands, and a variety of other pressed steel parts. In a natural or annealed condition this steel has little tenacity, and should not be used where much strength is required. The quality is considerably improved by cold-drawing or cold-rolling, the yield point being raised by such mechanical working. When this steel, after such cold working, is heated as for bending, pressing, welding, etc., the yield point returns to that corresponding with the annealed steel. This is also true of all materials that are given a higher yield point by cold working. This steel has the following physical characteristics:

	Annealed	Cold-drawn
Yield point, pounds per square inch........	28,000 to 36,000	40,000 to 60.000
Reduction of area, per cent......................	65–55	55–45
Elongation in 2 inches, per cent	40–30	Unimportant

Heat Treatment. — The 0.10 per cent carbon steel in the natural or annealed state does not machine freely, and will tear badly in turning, threading, and broaching operations. Heat treatment is of little benefit although the steel is made somewhat tougher. If this steel is heated to 1500 degrees F., and quenched in oil or water, this will produce a little stiffness and put the steel in better condition for machining operations. No drawing is required. While this steel may be case-hardened, it is not as suitable as the 0.20 per cent carbon steel. For data on the composition, see the table "S. A. E. Specifications for Carbon Steels."

S. A. E. Specifications for Carbon Steels

Carbon, Per Cent		Manganese, Per Cent		Maximum Percentage		S. A. E. Specification Number
Desired	Min. and Max.	Desired	Min. and Max.	Phosphorus	Sulphur	
0.10	0.05–0.15	0.45	0.30–0.60	0.045	0.050	1010
0.20	0.15–0.25	0.45	0.30–0.60	0.045	0.050	1020
0.25	0.20–0.30	0.65	0.50–0.80	0.045	0.050	1025
0.35	0.30–0.40	0.65	0.50–0.80	0.045	0.050	1035
0.45	0.40–0.50	0.65	0.50–0.80	0.045	0.050	1045
0.95	0.90–1.05	0.35	0.25–0.50	0.040	0.050	1095

S. A. E. Specifications for Nickel and Nickel-Chromium Steels *

Carbon, Per Cent		Manganese, Per Cent		Nickel, Per Cent		Chromium, Per Cent		S. A. E. Specification No.
Desired	Min. and Max.	Desired	Min. and Max.	Desired	Min. and Max.	Desired	Min. and Max.	
Nickel Steels								
0.15	0.10–0.20	0.65	0.50–0.80	3.50	3.25–3.75	....	....	2315
0.20	0.15–0.25	0.65	0.50–0.80	3.50	3.25–3.75	....	....	2320
0.30	0.25–0.35	0.65	0.50–0.80	3.50	3.25–3.75	....	....	2330
0.35	0.30–0.40	0.65	0.50–0.80	3.50	3.25–3.75	....	....	2335
0.40	0.35–0.45	0.65	0.50–0.80	3.50	3.25–3.75	....	....	2340
0.45	0.40–0.50	0.65	0.50–0.80	3.50	3.25–3.75	....	....	2345
0.12	0.17	0.45	0.30–0.60	5.00	4.50–5.25	....	....	2512†
Nickel-Chromium Steels								
0.20	0.15–0.25	0.65	0.50–0.80	1.25	1.00–1.50	0.60	0.45–0.75	3120
0.25	0.20–0.30	0.65	0.50–0.80	1.25	1.00–1.50	0.60	0.45–0.75	3125
0.30	0.25–0.35	0.65	0.50–0.80	1.25	1.00–1.50	0.60	0.45–0.75	3130
0.35	0.30–0.40	0.65	0.50–0.80	1.25	1.00–1.50	0.60	0.45–0.75	3135
0.40	0.35–0.45	0.65	0.50–0.80	1.25	1.00–1.50	0.60	0.45–0.75	3140
0.20	0.15–0.25	0.45	0.30–0.60	1.75	1.50–2.00	1.10	0.90–1.25	3220
0.30	0.25–0.35	0.45	0.30–0.60	1.75	1.50–2.00	1.10	0.90–1.25	3230
0.40	0.35–0.45	0.45	0.30–0.60	1.75	1.50–2.00	1.10	0.90–1.25	3240
0.50	0.45–0.55	0.45	0.30–0.60	1.75	1.50–2.00	1.10	0.90–1.25	3250
0.15	0.10–0.20	0.60	0.45–0.75	3.00	2.75–3.25	0.80	0.60–0.95	3415
0.35	0.30–0.40	0.60	0.45–0.75	3.00	2.75–3.25	0.80	0.60–0.95	3435
0.50	0.45–0.55	0.60	0.45–0.75	3.00	2.75–3.25	0.80	0.60–0.95	3450
0.20	0.15–0.25	0.45	0.30–0.60	3.50	3.25–3.75	1.50	1.25–1.75	3320
0.30	0.25–0.35	0.45	0.30–0.60	3.50	3.25–3.75	1.50	1.25–1.75	3330
0.40	0.35–0.45	0.45	0.30–0.60	3.50	3.25–3.75	1.50	1.25–1.75	3340

* The phosphorus in the nickel and nickel-chromium steels must not exceed 0.040 per cent. The sulphur in nickel steels must not exceed 0.045 per cent. The sulphur in nickel-chromium steels up to and including specification No. 3140 must not exceed 0.045 per cent, and for all specification numbers above 3140, the sulphur must not exceed 0.040 per cent.

† When it is necessary to machine, after carburizing, nickel steel parts made according to specification No. 2512, the nickel content should be maintained as close to the lower limit as possible.

S. A. E. Specifications for Chromium and Chromium-Vanadium Steels *

Carbon, Per Cent		Manganese, Per Cent		Chromium, Per Cent		Vanadium, Per Cent		S. A. E. Specification No.
Desired	Min. and Max.	Desired	Min. and Max.	Desired	Min. and Max.	Desired	Min.	
Chromium Steels								
0.20	0.15–0.25	†	†	0.75	0.60–0.90	.....	....	5120
0.40	0.35–0.45	†	†	0.75	0.60–0.90	.....	....	5140
0.65	0.60–0.70	†	†	0.75	0.60–0.90	.....	....	5165
1.00	0.95–1.10	0.35	0.20–0.50	1.35	1.20–1.50	.....	....	52100
Chromium-Vanadium Steels								
0.20	0.15–0.25	0.65	0.50–0.80	0.95	0.80–1.10	0.18	0.15	6120
0.25	0.20–0.30	0.65	0.50–0.80	0.95	0.80–1.10	0.18	0.15	6125
0.30	0.25–0.35	0.65	0.50–0.80	0.95	0.80–1.10	0.18	0.15	6130
0.35	0.30–0.40	0.65	0.50–0.80	0.95	0.80–1.10	0.18	0.15	6135
0.40	0.35–0.45	0.65	0.50–0.80	0.95	0.80–1.10	0.18	0.15	6140
0.45	0.40–0.50	0.65	0.50–0.80	0.95	0.80–1.10	0.18	0.15	6145
0.50	0.45–0.55	0.65	0.50–0.80	0.95	0.80–1.10	0.18	0.15	6150
0.95	0.90–1.05	0.35	0.20–0.45	0.95	0.80–1.10	0.18	0.15	6195

* The phosphorus in chromium steels up to specification No. 5165 inclusive must not exceed 0.040 per cent; the maximum amount for No. 52100 is 0.030 per cent. The maximum sulphur content is 0.045 per cent except for steel No. 52100 which must not have over c.030 per cent sulphur. The maximum amount of both phosphorus and sulphur for all chromium-vanadium steels is 0.040 per cent, except No. 6195 which must not have over 0.03 per cent.

† Two types of steel are available in this class, one with manganese from 0.25 to 0.50 per cent (0.35 per cent desired) and silicon not over 0.20 per cent; the other with manganese from 0.60 to 0.80 per cent (0.70 per cent desired) and silicon from 0.15 to 0.50 per cent.

Twenty Per Cent Carbon Steel (Specification No. 1020). — This steel is known to the trade as 0.20 per cent carbon open-hearth steel and often as machine steel. It is intended primarily for casehardening, forges well, machines well, but should not be considered as screw machine stock. This steel may be used for a large variety of forged, machined, and casehardened automobile parts where strength is not paramount. Steel of this quality may also be drawn into tubes and rolled into cold-rolled forms, and it is better for frames than the 0.10 per cent carbon steel as it is stronger. For automobile parts this steel may be used interchangeably with the 0.10 per cent carbon steel as far as cold-pressed shapes are concerned, and it is only the most difficult cold-forming operations that will cause trouble from cracking.

The physical properties of this steel after heat treatment, and of others of higher carbon content, are given in the table, "Physical Properties of Heat-treated Carbon Steels." These values apply to ½- to 1½-inch round specimens which were heated from 15 to 30 minutes to the temperatures given in the table, quenched in oil, re-heated for 30 minutes and finally cooled in air. The physical properties given in the table referred to apply to three re-heating temperatures.

Heat Treatment. — Heat treatment of the 0.20 per cent carbon steel produces but little change so far as strength is concerned, but it does cause a desirable refine-

S. A. E. Specifications for Tungsten and Silico-Manganese Steels

Tungsten Steels

Carbon, Per Cent		Manganese, Max. Per Cent	Phosphorus, Max. Per Cent	Sulphur, Max. Per Cent	Chromium, Per Cent	Tungsten, Per Cent	S. A. E. Specification No.
Desired	Min. and Max.						
0.60	0.50–0.70	0.30	0.035	0.035	3.00–4.00	12.0–15.0	71,360
0.60	0.50–0.70	0.30	0.035	0.035	3.00–4.00	15.0–18.0	71,660
0.60	0.50–0.70	0.30	0.035	0.035	0.50–1.00	1.5– 2.00	7,260

Silico-Manganese Steels

Carbon, Per Cent		Manganese, Per Cent		Silicon, Per Cent		Phosphorus and Sulphur, Max.	S. A. E. Specification No.
Desired	Min. and Max.	Desired	Min. and Max.	Desired	Min. and Max.		
0.50	0.45–0.55	0.70	0.60–0.80	1.95	1.80–2.10	0.045	9250
0.60	0.55–0.65	0.60	0.50–0.70	1.65	1.50–1.80	0.045	9260

ment of the grain after forging, and materially increases the toughness. The machining qualities can often be improved by heat treatment *H* (information regarding the heat treatments indicated by different letters will be found on pages 1308 and 1309). Casehardening is the most important heat treatment for this quality of steel. The heat treatment depends upon the importance of the part and upon its shape and size. When parts are not intended to carry much load or withstand any shock, and the principal requirement is hardness, the simplest form of casehardening as obtained by heat treatment *A* will suffice. Screws and rod-end pins are examples of this class of work. For more important parts, such as gears, steering wheel pivot pins, cam rollers, push rods, and many similar automobile parts, which must not only be hard on the surface but possess strength, the desired treatment is one which first refines and strengthens the interior and uncarbonized metal. This is then followed by a treatment for refining the exterior or carbonized metal. Heat treatment *B* is employed. In the case of very important parts, the last drawing operation should be continued from one to three hours. The object of drawing is to relieve all internal strains produced by quenching, and decrease the hardness. The last drawing operation can be omitted with a large number of pieces. This steel when cold-rolled or cold-drawn will have a yield point of from 40,000 to 75,000 pounds per square inch for sections not over ½ inch round, or ¼ inch thick in the case of sheets or flat stock.

Hardness. — The various degrees of hardness of the 0.20 per cent carbon steel conforming to the different heat treatments listed in the table "Physical Properties of Heat-treated Carbon Steels" are as follows: For a re-heating temperature of 400 degrees F., 180 Brinell and 34 scleroscope; for a re-heating temperature of 900 degrees F., 140 Brinell and 32 scleroscope; for a re-heating temperature of 1400 degrees F., 100 Brinell and 30 scleroscope.

Twenty-five Per Cent Carbon Steel (Specification No. 1025). — This steel is used extensively for frames and for ordinary drop-forgings where moderate ductility is desired but great strength is not essential. It is not intended for casehardening, although by careful manipulation it may be so treated. This should be done in emergencies only, rather than as regular practice, always employing the double heat treatment followed by a drawing operation.

Heat Treatment. — Heat treatment has a moderate effect on the physical properties of the 0.25 per cent carbon steel, but this effect is not nearly so marked as in the case of the 0.35 per cent carbon steel. Heat treatment *H* or *D* may be employed, the former being simpler. The drawing operation must be varied to suit each individual case. For instance, if great toughness and little increase in strength are desired, the higher drawing temperatures (1100 or 1200 degrees F.) may be used; whereas, if considerable strength is desired and a little toughness, the lower temperatures are used. With some parts the drawing operation can be omitted entirely. The double heat treatment *D* gives better results than heat treatment *H*, a better refinement of grain being obtained.

Hardness. — The various degrees of hardness conforming to the heat treatments listed in the table "Physical Properties of Heat-treated Carbon Steels" are as follows: For a re-heating temperature of 400 degrees F., 215 Brinell and 37 scleroscope; for a re-heating temperature of 900 degrees F., 160 Brinell and 34 scleroscope; for a re-heating temperature of 1400 degrees F., 110 Brinell and 30 scleroscope.

Thirty-five Per Cent Carbon Steel (Specification No. 1035). — This steel is sometimes referred to in the trade as 0.35 per cent carbon machine steel. It is intended primarily for use as structural steel. It forges well, machines well, and responds to heat treatment as regards strength and toughness. It can be used for all forgings such as axles, driving shafts, steering pivots, and other structural parts. It is the best all-around structural steel for such use as its strength warrants.

Heat Treatment. — Heat treatment for toughening and increasing the strength is important with this steel. The heat treatment must be modified in accordance with the experience of each user and to suit the size of the work as well as the combination of strength and toughness desired. The steel should be heat treated whenever reliability is essential. Heat treatments *H*, *D*, or *E* may be employed. Machining may precede the heat treatment, depending somewhat upon the convenience and nature of the treatment. When heat treatment *E* is applied, machining may follow the second operation or the slow cooling after heating to from 1500 to 1550 degrees F. To obtain the most strength, a quenching medium like brine should be used. The yield point will then be correspondingly high, and the steel harder and more difficult to machine. If a moderately high yield point will suffice, oil may be used for quenching, and then machining may follow without any difficulty.

Hardness.—The various degrees of hardness conforming to the different heat treatments listed in the table "Physical Properties of Heat-treated Carbon Steels" are as follows: For a re-heating temperature of 400 degrees F., 260 Brinell and 42 scleroscope; for a re-heating temperature of 900 degrees F., 200 Brinell and 37 scleroscope; for a re-heating temperature of 1400 degrees F., 135 Brinell and 32 scleroscope.

Forty-five Per Cent Carbon Steel (Specification No. 1045). — This steel is ordinarily known in the trade as a 0.45 per cent carbon machine steel. It is a structural steel of greater strength than the 0.35 per cent carbon steel, but its uses are more limited and are confined in general to such parts as require a higher degree of strength and considerable toughness. With the proper heat treatment the fatigue-resisting or endurance qualities are very high — higher than in any of the steels previously mentioned. The 0.45 per cent carbon steel is commonly used for crankshafts, driving shafts, and propeller shafts. It has also been used for transmission gears, but is not quite hard enough without casehardening and is not tough enough with casehardening to produce safe gears. This steel should not be used for casehardened parts. When properly annealed it machines well, but not well enough for screw machine work.

Heat Treatment. — When 0.45 per cent carbon steel requires annealing, heat treatment *E* is suitable. Machining may follow operation 2, after the steel has been slowly cooled from a temperature ranging between 1500 to 1550 degrees F. Heat treatment *E* is especially adapted to crankshafts and similar parts. Heat treatment *H* is also commonly applied to this quality of steel.

Hardness. — The various degrees of hardness conforming to the different heat treatments listed in the table "Physical Properties of Heat-treated Carbon Steels" are as follows: For a re-heating temperature of 400 degrees F., 300 Brinell and 45 scleroscope; for a re-heating temperature of 900 degrees F., 230 Brinell and 40 scleroscope; for a re-heating temperature of 1400 degrees F., 160 Brinell and 35 scleroscope.

Ninety-five Per Cent Carbon Steel (Specification No. 1095). — This grade of steel is generally used for springs, and when properly heat treated very good results are possible. The physical characteristics of heat-treated spring steel are best determined by transverse tests, because steel as hard as tempered spring steel is difficult to hold in the jaws of a tensile testing machine. There is more or less slip and also side strains, all of which tends to produce misleading results.

Heat Treatment. — Heat treatment *F* is recommended. It should be understood that the higher the drawing temperature, the lower the yield point, but if the material is drawn to too low a temperature it will be brittle. A few practical trials will indicate the best temper for any given shape or size of spring. The elastic limit of steel subjected to heat treatment *F*, as determined by transverse tests, varies from 90,000 to 180,000 pounds per square inch.

Nickel Steel — 0.15 Per Cent Carbon (Specification No. 2315). — This steel, containing 0.15 per cent carbon and 3.5 per cent nickel, is suitable for carburizing purposes and will produce parts with exceedingly strong and tough cores combined with a high-carbon exterior. This steel may also be used for structural purposes, although it should not be selected for such a purpose when ordering materials, as much better results will be obtained with a nickel steel higher in carbon. The 0.15 per cent carbon nickel steel is intended for casehardened gears of transmission systems and for other casehardened parts requiring a very tough, strong steel with a hardened outer surface. The composition of this steel and of others having a higher carbon content, may be obtained from the accompanying table "S. A. E. Specifications for Nickel and Nickel-chromium Steels." When used for structural purposes, the physical characteristics will range about as given in the table "Physical Properties of Annealed and Heat-treated Alloy Steels," which also includes various other alloy steels referred to later.

Heat Treatment. — Alloy steels in general should be heat-treated and not be used in an annealed or natural condition, because the advantage of an annealed alloy steel as compared with a plain carbon steel is as a rule not in proportion with the increased cost. By means of heat treatment, however, there is a very marked improvement in physical characteristics

The method of casehardening nickel steel No. 2315 may be varied considerably. As a rule, those parts which are made of nickel steel require the best treatment for casehardening. Heat treatment *G* is recommended. The second quenching (operation 6) should be at the lowest temperature at which the material will harden, which is sometimes below 1300 degrees F. The final drawing (operation 7) may be omitted in some cases. Parts of intricate shape with abrupt changes of thickness, sharp corners, etc. (especially sliding gears), should always be drawn to relieve internal strains.

Nickel Steel — 0.20 Per Cent Carbon (Specification No. 2320). — This steel, containing 0.20 per cent carbon and 3.5 per cent nickel, may be used interchangeably with steel No. 2315. The former, although intended primarily for casehardening, may properly be used for structural parts, and when suitably heat treated, will give elastic limits somewhat higher than the nickel steel containing 0.15 per cent carbon.

Heat Treatment. — For casehardening heat treatment *G* is recommended, and for structural purposes heat treatment *H* or *K*. The quenching temperatures, as with other steels, may be modified to meet individual cases.

Hardness. — The various degrees of hardness conforming to the different heat treatments listed in the table "Physical Properties of Heat-treated Alloy Steels" are as follows: For a re-heating temperature of 400 degrees F., 375 Brinell and 55 scleroscope; for a re-heating temperature of 900 degrees F., 280 Brinell and 42 scleroscope; for a re-heating temperature of 1400 degrees F., 125 Brinell and 28 scleroscope.

Nickel Steel — 0.30 Per Cent Carbon (Specification No. 2330). — This steel containing 0.30 per cent carbon and 3.5 per cent nickel, is intended primarily for heat-treated structural parts when strength and toughness are desired, as in the case of axles, front wheel spindles, crankshafts, driving shafts and transmission shafts. The physical characteristics of this steel are practically the same as those of No. 2320, slight modifications in the heat treatment much more than offsetting the slight difference in carbon content.

Heat Treatment. — Heat treatment *H* may be employed, although a higher refinement may be obtained by heat treatment *K*. Wide variations of yield point or elastic limit are possible by the use of different quenching mediums (oil, water, or brine) and by varying the drawing temperatures from 500 up to 1200 degrees F.

Hardness. — The various degrees of hardness conforming to the different heat treatments listed in the table "Physical Properties of Heat-treated Alloy Steels" are as follows: For a re-heating temperature of 400 degrees F., Brinell 436, scleroscope 60; for a re-heating temperature 900 degrees F., Brinell 300, scleroscope 46; for a re-heating temperature 1400 degrees F., Brinell 150, scleroscope 30.

Nickel Steel — 0.35 Per Cent Carbon (Specification No. 2335). — The preceding remarks regarding nickel steel No. 2330 may also be applied to this steel which contains 3.5 per cent nickel and 0.35 per cent carbon. It will respond a little more sharply to heat treatment, and can be forced to higher elastic limits.

Nickel Steel — 0.40 Per Cent Carbon (Specification No. 2340). — This steel, containing 0.40 per cent carbon and 3.5 per cent nickel, is not used extensively. As the carbon content is higher than generally used, greater hardness is obtained by quenching and as this is accompanied by increased brittleness, the treatments must be modified to meet this condition. For example, the final quenching may be at a relatively low temperature and the final drawing temperature must be determined carefully in order to produce the desired toughness and other physical characteristics.

Nickel-chromium Steel — 0.20 Per Cent Carbon (Specification No. 3120). — By referring to the accompanying table, "S. A. E. Specifications for Nickel and Nickel-chromium Steels," it will be seen that there are five nickel-chromium steels (specifications Nos. 3120 to 3140 inclusive) which differ as to carbon content but have the same percentage of manganese, nickel and chromium. The nickel-chromium steel conforming to specification No. 3120 is intended primarily for casehardening, and it may also be used for structural parts if suitably heat-treated. This steel should not be used in a natural or untreated condition.

The four grades of steel conforming to specification numbers 3125, 3130, 3135 and 3140 are intended primarily for structural purposes, and are used in a heat-treated condition. Steels Nos. 3125 and 3130 may be used for casehardening.

Heat Treatment. — In general, the heat treatments and the properties resulting from them are much the same for nickel-chromium steels as for plain nickel steels, except that the effects of heat treatment are somewhat augmented by the chromium and increase with increasing amounts of nickel and chromium. Steel conforming to specification No. 3120 is casehardened by heat treatment *G*, and when used in structural parts is given heat treatment *H* or *D*. Heat treatment *H*, *D* or *E* is applied to steels Nos. 3125, 3130, 3135 and 3140.

Other Nickel-chromium Steels. — The important applications of the other nickel-chromium steels listed in the table "S. A. E. Specifications for Nickel and Nickel-chromium Steels" are as follows:

Specification No. 3220. — This steel is intended for casehardened parts, but when this grade of steel is required, very careful heat treatment is necessary, heat treatment *G* being recommended. This same steel may also be used for structural purposes, in which case it should receive heat treatment *H* or *D*.

Specification No. 3230. — This grade of nickel-chromium steel is intended for the most important structural parts and should be used only in a heat-treated condition. Heat treatment *H* or *D* is recommended.

Specification No. 3240. — This quality of steel is suitable for structural parts requiring unusual strength. Higher elastic limit is possible with a given heat treatment than in the case of a steel like No. 3230. The toughness will not be quite as great, but the steel is applicable where strength rather than toughness is the controlling factor. Heat treatment *H* or *D* is recommended.

Specification No. 3250. — This steel is intended for gears requiring extreme strength and hardness. Either heat treatment *M* or *Q* should be applied, *Q* giving the better results.

Specification No. 3415. — This steel is intended primarily for casehardening. It is considerably higher in nickel than the nickel-chromium steels previously referred to. Heat treatment *G* should be followed unless the steel is used for structural parts, when heat treatment *M* is applicable.

Specification No. 3435. — This steel is intended for very important structural parts such as crankshafts, axles, spindles, driving shafts and transmission shafts. Heat treatment *P* or *R* is recommended. This steel is not intended for casehardening.

Specification No. 3450. — This quality of steel may also be used for gears requiring extreme strength and hardness. Heat treatment *R* should be used, although heat treatment *P* is also applicable.

Specification No. 3320. — The remarks made in connection with No. 3220 apply to this steel also. There is no appreciable difference in the physical characteristics. Heat treatment *L* should be used for carburizing.

Specification No. 3330. — This steel, like 3230, is intended for very important structural parts. The high nickel and chromium contents make it exceedingly tough and strong when treated according to heat treatment *P* or *R*.

Specification No. 3340. — This steel is suitable for gears to be hardened without carburizing. The remarks made in connection with steel No. 3240 and 3250 apply. Heat treatment *P* or *R* should be used.

Chromium Steels. — Four grades of chromium steels are included in the accompanying table "S. A. E. Specifications for Chromium and Chromium-vanadium Steels." Chromium steel No. 5120 is a casehardening grade of much better quality than carbon steel and is similar in this respect to nickel steel No. 2320 and nickel-chromium steel No. 3120. Heat treatment *B* is recommended for steel No. 5120. Chromium steel No. 5140 is similar to nickel-chromium steel No. 3140. When given heat treatment *H* or *D*, it is suitable for high-duty shafting, etc. The drawing temperature should be moderately high, in order to secure a safe degree of toughness.

Physical Properties of Heat-treated Carbon Steels
(From Reports of Iron and Steel Division—Society of Automotive Engineers, Inc.)

Range of Carbon Content, Per Cent	Range of Manganese Content, Per Cent	Physical Properties — Average Minimum Values given *					
		Heating Temp., Deg. F.	Re-heating Temp., Deg. F.	Tensile Strength, Lbs. per Sq. In.	Elastic Limit, Lbs. per Sq. In.	Reduction of Area, Per Cent	Elongation in 2 Inches, Per Cent
0.15–0.25	0.30–0.60	1560–1580	400	80,000	50,000	60.0	20.0
			900	75,000	42,500	65.0	26.5
			1400	70,000	35,000	70.0	32.5
0.20–0.30	0.50–0.80	1540–1560	400	90,000	60,000	55.0	17.0
			900	82,500	50,000	61.0	23.5
			1400	75,000	40,000	67.5	30.0
0.30–0.40	0.50–0.80	1510–1530	400	105,000	75,000	42.5	15.0
			900	94,000	63,000	52.5	21.5
			1400	82,000	50,000	62.5	28.0
0.40–0.50	0.50–0.80	1490–1510	400	125,000	90,000	35.0	12.5
			900	110,000	75,000	45.0	17.5
			1400	95,000	60,000	55.0	22.5

* These values apply to round specimens varying from ½ to 1½ inch in diameter, which were heated from fifteen to thirty minutes, quenched in oil, re-heated for thirty minutes at the temperatures given in the table and finally cooled in air.

Chromium-vanadium Steels. — The specifications of eight different grades are given in the table "S. A. E. Specifications for Chromium and Chromium-vanadium Steels." The principal applications of these steels are as follows:

Specification No. 6120. — This quality of steel is intended primarily for casehardening. It is used for the most important parts, such as casehardened shafts, gears, etc. While this steel may be used in a heat-treated condition for structural purposes, some of the steels referred to in the following are preferable particularly where greater strength is desired. Heat treatment *S* is recommended for casehardening and heat treatment *T* for structural parts.

Specification No. 6125. — The difference between this steel and No. 6120 is very slight, and they may be used interchangeably for structural purposes. This steel may be casehardened, but it is not first choice for this purpose.

Specification No. 6130. — This steel can be used interchangeably with No. 6125 for structural purposes, but it should not be used for casehardening. When subjected to heat treatment *T*, it possesses a high degree of combined strength and toughness.

Specification No. 6135. — This specification provides an excellent quality of steel for structural parts that are to be heat-treated. The fatigue-resisting or endurance qualities of this steel are very good. Heat treatment *T* is recommended.

Specification No. 6140. — This is a very good quality of steel for use where a high degree of strength is desired in conjunction with considerable toughness. Its fatigue-resisting qualities are very high, and it is a first-class material for high-duty shafts. Heat treatment *T* is recommended.

Specification No. 6145. — This quality of steel contains sufficient carbon in combination with chromium and vanadium to harden to a considerable degree when quenched at a suitable temperature, and it may be used for gears and springs. For gears this steel should be annealed after forging and before machining, the annealing being done by following operations 1 and 2 of heat treatment *U*. For structural parts requiring great strength, heat treatment *T* is recommended.

Specification No. 6150. — The remarks regarding steel No. 6145 apply to this steel. It is suitable for springs, and when given the right heat treatment, very high elastic limits are obtained. For spring material heat treatment *U* is recommended, except that the last drawing (operation 6) temperature should be higher — probably from 700 to 1100 degrees F. — the temperature varying with the section of the material.

Silico-manganese Steels. — The two silico-manganese steels listed in the accompanying table of specifications have been standardized by usage principally as spring steel. No. 9260 is also used to some extent for gears. Neither steel is suitable for use without heat treatment. The two specifications are given to meet the requirements of, first, those manufacturers believing in relatively low-carbon and high-silicon steel, and those desiring higher carbon and lower silicon. When

Physical Properties of Heat-treated Alloy Steels

(From Reports of Iron and Steel Division — Society of Automotive Engineers, Inc.)

Range of Carbon Content, Per Cent	Range of Nickel Content, Per Cent	Physical Properties — Average Minimum Values given *						
		Heating Temp., Deg. F.	Re-heating Temp., Deg. F.	Tensile Strength, Lbs. per Sq. In.	Elastic Limit, Lbs. per Sq. In.	Reduction of Area, Per Cent	Elongation in 2 Inches, Per Cent
Nickel Steels							
			400	170,000	140,000	45.0	11.0
0.15–0.25	3.25–3.75	1510–1540	900	130,000	99,000	60.5	21.5
			1400	70,000	40,000	75.0	30.0
			400	220,000	190,000	35.0	10.0
0.25–0.35	3.25–3.75	1485–1515	900	140,000	115,000	54.0	16.0
			1400	80,000	50,000	70.0	25.0
			400	240,000	215,000	32.5	10.0
0.35–0.45	3.25–3.75	1435–1465	900	155,000	130,000	51.0	16.0
			1400	90,000	60,000	62.5	22.5
Nickel-Chromium Steels †							
			400	160,000	120,000	52.5	15.0
0.15–0.25	1.00–1.50	1585–1615	900	111,000	84,000	69.0	21.0
			1400	75,000	50,000	72.5	35.0
			400	190,000	155,000	37.5	10.0
0.25–0.35	1.00–1.50	1535–1565	900	134,000	102,000	63.0	17.5
			1400	80,000	70,000	70.0	30.0
			400	230,000	200,000	27.0	7.5
0.35–0.45	1.00–1.50	1485–1515	900	157,000	126,000	46.5	14.0
			1400	90,000	75,000	62.0	20.0

* These values apply to round specimens varying from ½ to 1½ inch in diameter, which were heated from fifteen to thirty minutes, quenched in oil, re-heated for thirty minutes at the temperatures given in the table and finally cooled in air.

† The chromium content for all the steels listed ranges from 0.45 to 0.75 per cent in the standard specifications, 0.60 per cent being desired.

Physical Properties of Annealed and Heat-treated Alloy Steels
(From Reports of Iron and Steel Division — Society of Automotive Engineers, Inc.)

S. A. E. Specification Number *	Annealed			Heat-treated			
	Yield Point, Lbs. per Sq. In.	Reduction of Area, Per Cent	Elongation in 2 in., Per Cent	Heat Treatment Letter †	Yield Point, Lbs. per Sq. In.	Reduction of Area, Per Cent	Elongation in 2 in., Per Cent
				Nickel Steels			
2315	35,000–45,000	65–45	35–25	*H* or *K*	40,000– 80,000	65–40	35–15
2320	40,000–50,000	65–40	30–20	*H* or *K*	50,000–125,000	65–40	25–10
2330	40,000–50,000	60–40	30–20	*H* or *K*	60,000–130,000	60–30	25–10
2335	45,000–55,000	55–35	25–15	*H* or *K*	65,000–160,000	55–25	25–10
2340	55,000–65,000	50–30	25–15	*H* or *K*	70,000–200,000	55–15	25– 5
				Nickel-Chromium Steels			
3120	30,000–40,000	55–40	35–25	*H* or *D*	40,000–100,000	65–40	25–15
3125	40,000–55,000	50–35	30–20	*H*, *D* or *E*	50,000–125,000	55–25	25–10
3130	40,000–55,000	50–35	30–20	*H*, *D* or *E*	50,000–125,000	55–25	25–10
3135	45,000–60,000	45–30	25–15	*H*, *D* or *E*	55,000–150,000	50–25	20– 5
3140	45,000–60,000	45–30	25–15	*H*, *D* or *E*	55,000–150,000	50–25	20– 5
3220	35,000–45,000	60–45	25–20	*H* or *D*	45,000–120,000	65–30	20– 5
3230	40,000–50,000	55–40	25–15	*H* or *D*	60,000–175,000	60–30	20– 5
3240	45,000–60,000	50–40	25–15	*H* or *D*	65,000–200,000	50–20	15– 2
3250	50,000–60,000	50–40	25–15	*M* or *Q*	150,000–250,000	25–15	15– 2
3415	35,000–45,000	60–45	25–20	*M*	40,000–100,000	65–30	20– 5
3435	45,000–55,000	55–40	25–15	*P* or *R*	60,000–175,000	60–30	20– 5
				Chromium-Vanadium Steels			
6120	40,000–50,000	65–50	30–20	*T*	55,000–100,000	65–45	25–10
6125	40,000–50,000	65–50	30–20	*T*	55,000–100,000	65–45	25–10
6130	45,000–55,000	60–50	25–20	*T*	60,000–150,000	55–25	15– 5
6135	45,000–55,000	60–50	25–20	*T*	60,000–150,000	55–25	15– 5
6140	50,000–60,000	55–45	25–15	*T*	65,000–175,000	50–15	15– 2
6145	55,000–65,000	55–40	25–15	*U*	150,000–200,000	25–10	10– 2
6150	60,000–70,000	50–35	20–15	*U*	150,000–225,000	35–15	10– 2
				Silico-Manganese Steels			
9250	55,000–65,000	45–30	25–20	*V*	60,000–180,000	40–10	20– 5
9260	55,000–65,000	45–30	25–20	*V*	60,000–180,000	40–10	20– 5

* The compositions represented by these specification numbers are given in the accompanying tables of S. A. E. specifications.

† The heat treatments represented by the different letters are given on pages 1308 and 1309.

heat-treated, the physical properties will not differ appreciably, although steel No. 9250 will probably be a little tougher. Heat treatment *V* is suitable for both gears and springs.

High-Chromium or "Stainless" Steel. — High-chromium, or what is called "stainless" steel contains from 11 to 14 per cent chromium. This steel was originally developed for cutlery manufacture but it is now used to a considerable extent for exhaust valves in airplane engines because of its resistance to oxidation or scaling at high temperatures. This steel should also prove very satisfactory for water-pump rods or shafting and other parts subject to objectionable corrosion. The composition is about as follows: Carbon 0.20 to 0.40 per cent; manganese not to exceed 0.50 per cent; phosphorus not to exceed 0.035 per cent; sulphur not to exceed 0.035 per cent; chromium 11.50 to 14.00 per cent; and silicon not to exceed 0.30 per cent.

Heat Treatment. — For forging, the steel should be heated slowly and forged at a temperature above 1750 degrees F., and preferably between 1800 and 2200 degrees F. If forged at temperatures between 1650 and 1750 degrees F., there is considerable danger of rupturing the steel because of its hardness at red heat. Owing to the air-hardening property of the steel, drop-forgings should be trimmed while hot. Thin forgings should be re-heated to redness before trimming, as otherwise they are liable to crack. Forgings will be hard if they are allowed to cool in air. This hardness varies over a range of from 250 to 500 Brinell, depending on the original forging temperature. Annealing can be done by heating to temperatures ranging from 1290 to 1380 degrees F. and cooling in air or quenching in water or oil. After this treatment the forgings will have a hardness of about 200 Brinell and a tensile strength of from 100,000 to 112,000 pounds per square inch. If softer forgings are desired they can be heated to a temperature of from 1560 to 1650 degrees F. and cooled very slowly. Although softer, the forgings will not machine as smoothly as when annealed at the lower temperature. The forgings can be hardened by cooling in still air or quenching in oil or water from a temperature between 1650 and 1750 degrees F.

Heat Treatment for Valves. — The usual heat treatment for valves is to quench in oil from 1650 degrees F. and temper or draw at 1100 to 1200 degrees F. According to one valve manufacturer valves of this steel are hardened by heating the previously annealed valves to 1650 degrees F. and cooling in still air. This treatment gives a scleroscope hardness of about 50.

Physical Properties. — The physical properties do not vary greatly when the carbon is within the range previously given, or when the steel is hardened and tempered in air, oil, or water. Valves have generally been made to the following specifications of physical properties: Yield point, 70,000 pounds per square inch; tensile strength, 90,000 pounds per square inch; elongation in two inches, 18 per

Comparison of Physical Properties for High-Chromium Steels of Different Carbon Content

Heat Treatments and Physical Properties	Composition		
	C 0.20 Mn 0.45 Cr 12.56	C 0.27 Mn 0.50 Cr 12.24	C 0.50 Cr 14.84
Quenched in oil from deg. F......	1,600	1,600	1,650
Tempered at deg. F.............	1,160	1,080	1,100
Yield point, lb. per sq. in........	78,300	75,000	91,616
Tensile strength, lb. per sq. in....	104,600	104,250	123,648
Elongation in 2 in., per cent.....	25.0	23.5	14.5
Reduction of area, per cent......	52.5	51.4	33.5

cent; and reduction of area, 50 per cent. This steel can be drawn into wire, rolled into sheets and strips, and drawn into seamless tubes.

Corrosion. — High-chromium steel, like any other steel when distorted by cold working, is more sensitive to corrosion and will rust. Rough cut surfaces will rust. Surfaces finished with a fine cut are less liable to rust. Ground and polished surfaces are practically immune to rust.

Cobaltcrom Steel. — This is a tungstenless alloy steel or high-speed steel which contains approximately 1.5 per cent carbon, 12.5 per cent chromium, and 3.5 per cent cobalt. Tools such as dies, milling cutters, etc., made from cobaltcrom steel can be cast to shape in suitable molds, the teeth of cutters being formed so that it is necessary only to grind them.

Before the blanks can be machined, they must be annealed; this operation is performed by pack-annealing at the temperature of 1800 degrees F., for a period of from three to six hours, according to the size of the castings being annealed. The following directions are given for the hardening of blanking and trimming dies, milling cutters, and similar tools made from cobaltcrom steel: Heat slowly in a hardening furnace to about 1830 degrees F., and hold the temperature at this point until the tools are thoroughly soaked. Then reduce the temperature about 50 degrees, withdraw the tools from the furnace, and allow them to cool in the atmosphere. As soon as the red color disappears from the cooling tool, place it in quenching oil until cold. The slight drop of 50 degrees in temperature while the tool is still in the hardening furnace is highly important in order to obtain proper results. The steel will be injured if the tool is heated above 1860 degrees F. In cooling milling cutters or other rotary tools, it is suggested that they be suspended on a wire to insure a uniform rate of cooling.

Tools that are subjected to shocks or vibration, such as pneumatic rivet sets, shear blades, etc., should be heated slowly to 1650 degrees F., after which the temperature should be reduced to about 1610 degrees F., at which point the tool should be removed from the furnace and permitted to cool in the atmosphere. There is no appreciable scaling present in the hardening of cobaltcrom steel tools.

General Properties of Alloy Steels. — Alloy or "special" steels are combinations of iron and carbon with some other element, such as nickel, chromium, tungsten, vanadium, manganese and molybdenum. All of these metals give certain distinct properties to the steel, but in all cases the principal quality is the increase in hardness and toughness.

Nickel steel usually contains from 3 to 3.5 per cent nickel (ordinarily not over 5 per cent), and from 0.20 to 0.40 per cent carbon. This steel is used for armor plate, ammunition, bridge construction, rails, etc. One of the reasons why nickel steel is adapted for armor plate is that it does not crack when perforated by a projectile. The Krupp steel used for armor plate contains approximately 3.5 per cent nickel, 1.5 per cent chromium and 0.25 per cent carbon. The advantages claimed for nickel steel for railroad rails are its increased resistance to abrasion and high elastic limit. On sharp curves, it has been estimated that a nickel steel rail will outlast four ordinary rails.

Chromium steel is well adapted for armor-piercing projectiles, owing to its hardness, toughness and stiffness, and is extensively used for this purpose. Chromium steel is also used in the construction of safes and for castings subjected to unusually severe stresses, such as those used in rock-crushing machinery, etc. The percentage of chromium used in chromium steels varies over quite a wide range in the low-chromium and high-chromium steels.

Tungsten steel is largely employed for high-speed metal cutting tools and magnet steels. It has also been used in the manufacture of armor plate and armor-piercing projectiles, in which case it is combined either with nickel or chromium or with both of these metals. The property that tungsten imparts to steel is that of hardening in the air, after heating to the required temperature. This steel usually contains from 5 to 15 per cent tungsten (although the percentage is sometimes as high as 24 per cent) and from 0.4 to 2 per cent carbon.

Vanadium steels ordinarily contain from 0.16 to 0.25 per cent vanadium. The effect of vanadium is to increase the tensile strength and elastic limit, and it gives the steel the valuable property of resisting, to an unusual degree, repeated stresses. Vanadium steel is especially adapted for springs, car axles, gears subjected to severe service, and for all parts which must withstand constant vibration and varying stresses.

Manganese steel (also known as Hadfield manganese steel) contains about 12 per cent manganese and from 0.8 to 1.25 per cent carbon. If there is only 1.5 per cent manganese, the steel is very brittle, and additional manganese increases this brittleness until the quantity has reached 4 to 5.5 per cent, when the steel can be pulverized under the hammer. With a further increase of manganese, the steel becomes ductile and very hard, these qualities being at their highest degree when the manganese content is 12 per cent. The ductility of the steel is brought out by sudden cooling, the process being opposite that employed for carbon steel.

Molybdenum steels have properties similar to tungsten steels, except that a smaller quantity of molybdenum than of tungsten is required to secure corresponding results.

Screw Stock. — The composition of ordinary screw stock should be, in general, about as follows: Carbon, from 0.08 to 0.20 per cent; manganese, 0.30 to 0.80 per cent; phosphorus, not to exceed 0.12 per cent; sulphur, 0.06 to 0.12 per cent. Screw stock is easily machined and cheap, but lacks strength and toughness and is not safe for vital parts. Screws made from hot-rolled bars of this material should be heat-treated and not used in an annealed condition. Screws made from cold-rolled bars are much stronger, but the best results, in either case, are obtained by the following heat-treatment: After machining, heat to 1500 degrees F.; quench; re-heat from 600 to 1300 degrees F., and cool slowly.

TESTING THE HARDNESS OF METALS

Different Methods of Hardness Testing. — There are four typical methods for testing the hardness of metals. These are the sclerometer method introduced by Turner in 1886; the scleroscope method recently invented by Shore; the indentation test adopted by Brinell about 1900; and the drill test introduced by Keep a few years earlier. The principles underlying each of the four methods are briefly described in the following.

Turner's Sclerometer. — In this form of test a weighted diamond point is drawn, once forward and once backward, over the smooth surface of the material to be tested. The hardness number is the weight in grams required to produce a standard scratch. The scratch selected is one which is just visible to the naked eye as a dark line on a bright reflecting surface. It is also the scratch which can just be felt with the edge of a quill when the latter is drawn over the smooth surface at right angles to a series of such scratches produced by regularly increasing weights.

Shore's Scleroscope. — In this instrument, a small cylinder of steel, with a hardened point, is allowed to fall upon the smooth surface of the metal to be tested, and the height of the rebound of the hammer is taken as the measure of hardness. The hammer weighs about 40 grains, the height of the rebound of hardened steel is in the neighborhood of 100 on the scale, or about 6¼ inches, while the total fall is about 10 inches or 254 millimeters.

Brinell's Test. — In this method, a hardened steel ball is pressed into the smooth surface of the metal so as to make an indentation of a size such as can be conveniently measured under the microscope. The spherical area of the indentation being calculated, and the pressure being known, the stress per unit of area when the ball comes to rest is calculated, and the hardness number obtained. Within certain limits, the value obtained is independent of the size of the ball and of the amount of pressure.

Keep's Test. — In this form of apparatus a standard steel drill is caused to make a definite number of revolutions, while it is pressed with standard force against the specimen to be tested. The hardness is automatically recorded on a diagram on which a dead soft material gives a horizontal line, while a material as hard as the drill itself gives a vertical line, intermediate hardness being represented by the corresponding angle between 0 and 90 degrees.

Hardness Scales Compared

Metal	Sclerometer	Scleroscope	Brinell Method *	Mohs's Scale for Minerals
Lead	1.0	1.0	1.0	Talc — 1
Tin	2.5	3.0	2.5	Gypsum — 2
Zinc	6.0	7.0	7.5	Calcite — 3
Copper, soft	8.0	8.0	....	Fluor Spar — 4
Copper, hard	....	12.0	12.0	Apatite — 5
Softest iron	15.0	....	14.5	Orthoclase — 6
Mild steel	21.0	22.0	16–24	Quartz — 7
Soft cast iron	21–24	24.0	24.0	Topaz — 8
Rail steel	24.0	27.0	26–35	Sapphire or Corundum — 9
Hard cast iron	36.0	40.0	35.0	
Hard white iron	72.0	70.0	75.0	
Hardened steel	....	95.0	93.0	Diamond — 10

* Actual numerals have been divided by 6 for purposes of comparison.

Comparison between Testing Methods. — Each form of test has its advantages and its limitations. The sclerometer is cheap, portable and easily applied, but it is not applicable to materials which do not possess a fairly smooth reflecting surface and the standard scratch is only definitely recognized after some experience. The Shore test is simple, rapid and definite for materials for which it is suited, but results obtained vary somewhat with the size and thickness of the sample. As a comparative measure of the hardness of material of the same quality and structure, however, it is quite accurate, but it is not reliable for comparing the hardness of two different metals. The Brinell test is especially useful for constructive materials. It is definite, and, with the new appliances recently brought out, easily applied. It cannot be applied, however, to very brittle materials, such as glass, nor is it satisfactory for use on hardened high-carbon steel. Keep's test is especially suited for castings of all kinds, as it records not only the surface hardness, but also the hardness of the whole thickness, and gives indications of blow-holes, hard streaks and spongy places. Obviously, it cannot be applied to materials too hard to be conveniently drilled by a hardened steel drill.

The accompanying table gives values obtained on the same materials by the scleroscope, sclerometer and the Brinell test, the figures being reduced to a common unit, assumed as 1 as a starting point; thus the actual Brinell numerals have been divided by 6, thereby reducing the hardness values for purposes of comparison.

Hardness Numerals — Brinell System

Steel ball of 10 millimeters diameter

Diameter of Impression, mm.	Hardness Numeral, Pressure in Kilograms		Diameter of Impression, mm.	Hardness Numeral, Pressure in Kilograms		Diameter of Impression, mm.	Hardness Numeral, Pressure in Kilograms		Diameter of Impression, mm.	Hardness Numeral, Pressure in Kilograms		Diameter of Impression, mm.	Hardness Numeral, Pressure in Kilograms	
	3000	500		3000	500		3000	500		3000	500		3000	500
2.00	946	158	3.00	418	70	4.00	228	38.0	5.00	143	23.8	6.00	95	15.9
2.05	898	150	3.05	402	67	4.05	223	37.0	5.05	140	23.3	6.05	94	15.6
2.10	857	143	3.10	387	65	4.10	217	36.0	5.10	137	22.8	6.10	92	15.3
2.15	817	136	3.15	375	63	4.15	212	35.0	5.15	134	22.3	6.15	90	15.1
2.20	782	130	3.20	364	61	4.20	207	34.5	5.20	131	21.8	6.20	89	14.8
2.25	744	124	3.25	351	59	4.25	202	33.6	5.25	128	21.5	6.25	87	14.5
2.30	713	119	3.30	340	57	4.30	196	32.6	5.30	126	21.0	6.30	86	14.3
2.35	683	114	3.35	332	55	4.35	192	32.0	5.35	124	20.6	6.35	84	14.0
2.40	652	109	3.40	321	54	4.40	187	31.2	5.40	121	20.1	6.40	82	13.8
2.45	627	105	3.45	311	52	4.45	183	30.4	5.45	118	19.7	6.45	81	13.5
2.50	600	100	3.50	302	50	4.50	179	29.7	5.50	116	19.3	6.50	80	13.3
2.55	578	96	3.55	293	49	4.55	174	29.1	5.55	114	19.0	6.55	79	13.1
2.60	555	93	3.60	286	48	4.60	170	28.4	5.60	112	18.6	6.60	77	12.8
2.65	532	89	3.65	277	46	4.65	166	27.8	5.65	109	18.2	6.65	76	12.6
2.70	512	86	3.70	269	45	4.70	163	27.2	5.70	107	17.8	6.70	74	12.4
2.75	495	83	3.75	262	44	4.75	159	26.5	5.75	105	17.5	6.75	73	12.2
2.80	477	80	3.80	255	43	4.80	156	25.9	5.80	103	17.2	6.80	71.5	11.9
2.85	460	77	3.85	248	41	4.85	153	25.4	5.85	101	16.9	6.85	70	11.7
2.90	444	74	3.90	241	40	4.90	149	24.9	5.90	99	16.6	6.90	69	11.5
2.95	430	73	3.95	235	39	4.95	146	24.4	5.95	97	16.2	6.95	68	11.3

Application of the Brinell Method. — The Brinell method, as mentioned, consists in partly forcing a hardened steel ball into the sample to be tested so as to effect a slight spherical impression. The diameter of the impression is measured and the surface of the spherical concavity calculated. The pressure required in kilograms for effecting the impression is divided by the area of the impression in square millimeters; the quotient is an expression of the hardness of the material tested and is called the *hardness numeral*. The standard diameter of the ball is 10 millimeters (0.3937 inch) and the pressure, 3000 kilograms (6614 pounds) in the case of iron and steel, while in the case of softer metals, a pressure of 500 kilograms (1102 pounds) is used. The diameter of the impression in the original instrument is measured by means of a microscope, after which the hardness numeral may be obtained without calculation directly from the table of "Hardness Numerals — Brinell System." Instruments have been constructed later so as to eliminate the need of the use of a microscope for measuring the diameter of the impression.

Relation between Hardness of Materials and Ultimate Strength. — A constant relationship exists between the hardness numeral as determined by the Brinell test and the ultimate strength of the material tested. The coefficients by which the hardness numerals must be multiplied to obtain the ultimate strength in kilograms per square millimeter may be determined by tests, and are constant for each class and kind of material, but they differ slightly for different materials and for materials treated in a different manner. The following coefficients are given for different grades of steel:

Steels, extra soft	$K = 0.360$
Steels, soft and semi-hard	$K = 0.355$
Steels, semi-hard	$K = 0.353$
Steels, hard	$K = 0.349$

It will be seen that these coefficients differ by but a slight amount for steel of different composition, and, as a general rule, the factor 0.355 may be used for all grades of steel.

Example: — Assume that a hardness test of structural steel (semi-hard) by the Brinell method gave an impression of 4.6 millimeters. The hardness numeral, from the table, would be 170, and the ultimate strength, $0.355 \times 170 = 60$ kilograms per square millimeter.

Accuracy of Brinell Hardness Test. — When commercial apparatus, as ordinarily used for making the Brinell test, is employed, and the test is carried out with ordinary care and precaution, it is reliable within an error of five Brinell units above or below the actual hardness. In other words, if the hardness of two pieces of metal is tested, and the difference on the Brinell scale is more than ten hardness units, it is certain that there is an absolute difference in the hardness of the pieces tested. With regard to the conditions under which the tests should be made, it may be stated that the pressure should be gradually applied for two minutes or more, and the pressure should be kept on the test piece for a period of at least five minutes.

Relation between Hardness and Wear of Steel. — There is no definite relation between hardness, as measured by the Brinell hardness testing method, and wear. While, in general, a high Brinell hardness number may be expected to indicate a metal which will give better wear, there are so many exceptions that this test for indicating wearing properties would be unreliable. As an example, Hadfield's manganese steel, which has a low Brinell hardness number, is one of the best steels as far as wear is concerned. The relation of either Brinell tests or ordinary wear tests to wear in actual practice is a subject which requires further investigation. Wear tests should be made along different lines, according to the actual uses to which the metal is to be put.

Scleroscope Hardness Scale*

Name of Metal	Annealed	Hammered
Lead (cast)	2–5	3–7
Babbitt metal	4–9	....
Gold	5	8½
Silver	6½	20–30
Brass (cast)	7–35	....
Pure tin (cast)	8	....
Brass (drawn)	10–15	24–25
Bismuth (cast)	9	....
Platinum	10	17
Copper (cast)	6	14–20
Zinc (cast)	8	20
Iron, pure	18	25–30
Mild steel, 0.15 per cent carbon	22	30–45
Nickel annode (cast)	31	55
Iron, gray (cast)	30–45	....
Iron, gray (chilled)	....	50–90
Steel, tool, 1 per cent carbon	30–35	40–50
Steel, tool, 1.65 per cent carbon	35–40	....
Vanadium steel	35–45	....
Chrome-nickel steel	47	....
Chrome-nickel steel (hardened)	....	60–95
Steel, high-speed (hardened)	....	70–105
Steel, carbon tool (hardened)	....	70–105

* The figures given are subject to variation, owing to the differences in composition of the metals tested.

PRINCIPLES OF IRON AND STEEL MANUFACTURE

Iron Ore. — Iron ore is an oxide of iron containing, ordinarily, from 35 to 65 per cent of iron, and, in addition, oxygen, phosphorus, sulphur, silica (sand) and other impurities. If the ore contains less than 40 per cent iron, it must first be concentrated, and if less than 25 per cent iron, it is not considered a commercial product, owing to the excessive cost of smelting. At the present time, the ores mined in this country average slightly over 50 per cent iron, although the "Lake" ores sometimes contain over 60 per cent.

Pig Iron. — When iron ore is charged in a blast furnace, mixed with limestone as a flux, and melted down with either charcoal, coke or anthracite coal as fuel, the resulting metal is what is commercially known as pig iron. It contains about 93 per cent of pure iron, from 3 to 5 per cent of carbon, and some silicon, phosphorus, sulphur, etc. Pig iron is used in foundries for the manufacture of iron castings by simply re-melting it in a cupola and without materially changing its chemical composition. Pig iron is classified: (1) according to the method of manufacture; (2) its intended use; (3) its composition. The methods of manufacture produce:

(1) coke pig iron, which is smelted with coke and always with a hot blast; (2) charcoal pig, which is smelted with charcoal and either a hot or cold blast; (3) anthracite pig, smelted with anthracite coal mixed with coke, using a hot blast. Classifications according to intended use are: (1) Bessemer pig, used for the Bessemer and acid open-hearth processes of making steel; (2) basic pig, used for the basic process; (3) malleable pig, used for malleable cast-iron castings; (4) foundry pig, used for foundry work; (5) forge pig, an inferior grade used for puddling and some classes of foundry work. The grading according to composition was formerly done by breaking the pig and examining the fracture; the modern method is by chemical analysis. In this country pig iron is usually sold in tons of 2240 pounds.

Classes of Steel. — The word steel is applied to many mixtures which differ greatly from each other in their chemical as well as physical qualities. The ingredient that exerts the most influence on steel is carbon. High-grade razor steel contains about 1.25 per cent of carbon, spring steel 1 per cent, steel rails from 0.50 to 0.75 per cent, and soft steel boiler plate may have as little as 0.062 per cent of carbon. Steel which is very low in carbon can easily be welded, but it cannot be hardened; when the carbon is above 0.33 per cent, welding is more difficult and can be done only by the use of borax or some other flux, or by the electric or thermit processes. Steel with carbon above 0.75 per cent can readily be hardened. In tool steel, other ingredients than carbon are sometimes used to influence its hardness, such as nickel, manganese, chromium, tungsten, etc., the last named playing an important part in so-called "high-speed steels," that is, tool steels that will cut metal at a high speed without losing their temper or hardness. Pig iron and cast iron contain about 4 per cent of carbon, and wrought iron only a trace of it, while steel is between these two extremes; hence, in the manufacture of steel it is important to get the right proportion of carbon. One method is to burn the carbon out of pig iron, as in the Bessemer and open-hearth processes, and the other method is to add carbon to wrought iron, as in the crucible process.

Puddling Process. — In the manufacture of wrought iron, pig iron is melted in a puddling furnace where most of the silicon, carbon, phosphorus, and other impurities are separated from the iron, forming the puddle cinder. Pig iron melts at about 2100 degrees F., steel at about 2500 degrees, and wrought iron at about 2800 degrees. The temperature in the puddling furnace is high enough to melt pig iron, but not high enough to keep wrought iron in a liquid state; therefore, as soon as the small particles of iron become purified, they partly congeal ("come to nature") forming a spongy mass in which small globules of iron are in a semi-plastic state. This mass is divided by the puddler into "puddle balls" or lumps of about 200 pounds each. These lumps are formed into elongated blooms in a rotary squeezer and, while still hot, are rolled out into "muck bars." These bars are cut into pieces of from 2 to 4 feet long and are stacked up in piles varying in weight from 100 to 2000 pounds. These piles are heated in a re-heating furnace, and when white hot, are taken to the rolls to be rolled out and welded together into iron bars, sheets, plates, or structural shapes. This material, after re-rolling, is known as refined iron, and if subjected to a second piling, heating and re-rolling, is called double-refined iron. Instead of starting the process with pig iron, as described, scrap iron can also be employed. "Common iron" is wrought iron made from coke pig, whereas finer grades or "charcoal iron" are made from charcoal pig.

Bessemer Process. — In the Bessemer process of making steel, the molten pig iron is put into a large pear-shaped vessel, called the "converter," and the impurities are oxidized and removed by blowing air up through the molten mass. The molten iron (from 10 to 15 tons at a time) is poured into the converter while the latter is in a horizontal position; then the compressed air is turned into the blast box at the bottom, as the converter rises to a vertical position. The air has sufficient pressure (about 20 pounds per square inch) to prevent the molten metal from entering the tuyeres. The air streams pass up through the molten metal burning out the carbon, silicon, etc. The changes which take place are indicated by the appearance of the flame. The 15 tons of molten pig iron contain nearly ¾ of a ton of carbon, and since this carbon is all burned out in less than ten minutes, the rapid rate of combustion greatly increases the heat of the metal. The flame is red at first and becomes brighter and brighter until it is white. A "blow" generally lasts about nine or ten minutes, when the sudden dropping of the flame gives notice that the carbon is all burned out. The metal in the converter is then practically liquid wrought iron. The converter is again turned to a horizontal position, the blast is shut off, and a certain amount of spiegeleisen or ferro-manganese is added in a liquid form to give the steel the proper amount of carbon and manganese to make it suitable for the purpose desired. The liquid steel is then poured out into "ingot molds," and the resulting ingots, while still hot, are rolled into blooms, billets, or rails without any additional re-heating, except a short period in the "soaking pits." If the pig iron used contains too high a percentage of silicon, too much heat will be generated and the charge is said to blow hot. On the other hand, if the percentage of silicon is too low, the charge is said to blow cold.

Open-hearth Process. — The open-hearth process, sometimes called "the Siemens-Martin process," is a method of producing steel by removing the impurities contained in a bath of iron lying on the hearth of a regenerative furnace, the hearth being open or exposed to the action of the flame. It is similar to the puddling process, but on a much larger scale. The furnaces generally have a capacity of from 40 to 50 tons of molten metal (in some exceptional cases as high as 200 tons); they are heated by gas made from bituminous coal (oil and natural gas have also been used). The gas and the air are heated to a high temperature (over 1000 degrees) before entering the combustion chamber, by passing through regenerative chambers. Owing to this preheating of the gas and air, a very high temperature can be maintained in the furnace so as to keep the iron liquid. The charge of molten metal has added to it a certain proportion of ore, iron scale or other oxides, the chemical reaction of which keeps the molten iron in a state of agitation. In the Bessemer process only pig iron is used, but in the open-hearth furnace it is practicable to use also scrap of wrought iron or steel, as the high temperature in the furnace will readily melt it. When the pig iron or scrap contains too much phosphorus, burnt lime is added to the charge; the resulting slag will absorb the phosphorus, thus taking it out of the metal. This dephosphorization by means of burnt lime is called the "basic process," in contra-distinction to the "acid process." The latter requires no lime but care must be taken that the metal charged is low in phosphorus. In this country, the basic process is at present used only in connection with open-hearth furnaces, while in Europe it is also used in many Bessemer plants producing the so-called "basic Bessemer steel."

Crucible or Tool Steel. — Crucible or tool steel (formerly called cast steel) is made by using high-grade, low-phosphorus wrought iron and adding carbon to it. The oldest method of adding carbon was the "cementation process," in which iron bars were packed in air-tight retorts with powdered charcoal between the bars. The filled retorts were put into a furnace where they were heated to a red heat for several days. The carbonized bars, commonly called "blister steel," were then cut into small pieces, re-melted in a crucible and poured into molds, forming small billets. The newer method is to put small pieces of wrought iron into an air-tight crucible containing the proper amount of powdered charcoal, and melt it down. The other ingredients, such as chromium, tungsten, etc., are also added in the crucible.

Percentage of Carbon in Carbon Steel Tools

Name of Tool — Machinists' Tools	Carbon, Per Cent	Name of Tool — Blacksmiths' Tools	Carbon, Per Cent
Turning and Planing Tools	1.15	Cold Chisel	0.75
Chipping Chisels	0.85	Hot Chisel	0.85
Saw Arbor	0.75	Hot Punch	0.85
Lathe Center	1.05	Flatter	0.85
Chuck Jaw	0.85	Anvil Facing	0.75
Milling Cutters	1.15	Hammer	0.75
Twist Drills	1.15	Miscellaneous	
Ordinary Files	1.25	Rivet Set	0.70
Machinists' Hammer	0.95	Roll Expander	1.05
Mandrel	1.05	Beading Tool	0.80
Pliers	0.75	Threading Dies	1.05
Reamer Blades	1.10	Wire Drawing Dies	1.40
Hand Reamers	1.05	Drop-forging Dies	0.70
Saw for Steel	1.60	Pipe Cutter	1.15
Screw Driver	0.65	Circular Saw	0.85
Taps	1.10	Band Saw	0.75
Vise Jaws	0.75	Ball Bearing Races	1.15
Wrenches	0.75	Crowbar	0.75

The above table is intended as an approximate guide in selecting steels for various purposes. Average figures are given; the percentage of carbon might vary 0.05 per cent either way, in most cases, without seriously affecting the quality of the tool.

Defects in Crucible Steel. — The most common defects in finished bars of tool steel are: 1. *Seams.* — These can generally be seen with the naked eye on the surface of the bar and run lengthwise. Tools made from steel containing seams are liable to open in forging or crack in hardening. 2. *Laps.* — These are caused either in rolling or drawing down under the hammer. They present an appearance similar to a seam, but are usually longer, and in the case of rolled bars frequently run the whole length of the bar. 3. *Pipes.* — These defects are caused by the contraction, when cooling, in the top half of the ingot. The lower half is usually sound, as its contraction, when cooling, is fed by the molten metal in the top half. The larger the ingot the more difficult this defect is to deal with. This defect causes tools to split from the center in hardening.

Electric Steel. — The most important uses of electric furnaces in steel plants and foundries are for making special alloy steels, tool steel, and for melting the steel used in making steel castings. Electric furnaces are also used for melting the ferro-alloys which are added to "special steels." The three general types of electric furnaces may be classified, electrically, as *arc* furnaces, *resistance* furnaces, and *induction* furnaces. The arc furnace is the important type in the steel industry. Various makes differ in regard to the arrangement and number of the electrodes and other details. The heat is derived in each case from electrical resistance, but the nature or form of the resistance varies. Most electrical furnaces are used to melt the cold charge, but larger sizes are often arranged to receive molten charges. The latter is known as the "hot metal" process. When electric furnaces receive molten charges, they may be used in conjunction with either Bessemer or open-hearth furnaces. Electric furnaces permit of very close control of the composition of steel, and alloy additions may be made in the furnace itself rather than in the ladle. It is practicable to use relatively cheap raw materials in the electric furnace. This furnace may be employed for recovering alloy scrap.

Malleable Iron Castings. — Malleable iron castings are produced by converting the combined carbon of white cast iron into an amorphous uncombined condition, by packing the castings with crushed slag, rolling mill scale, or a mixture of both, in malleable iron annealing pots and heating to a temperature of from 1350 to 1450 degrees F. Hematite ore is often used as a packing material, especially if the iron for the castings was melted by the cupola process. The maximum temperature of the furnace should not exceed 1700 degrees F. The desired heat is maintained from forty-eight to sixty hours, after which the oven is allowed to cool very slowly, at about 6 degrees per hour. This prolonged annealing process brings the iron carbide out of solution, giving the castings a steely nature.

Physical Properties of Malleable Iron. — Malleable, or annealed white cast iron, can be bent and twisted to some extent and is especially suitable for parts which are subjected to shock or strain. It is stronger than cast iron but weaker than cast steel and not suitable when high tensile strength is required. The composition of malleable iron recommended by the Society of Automotive Engineers is: Manganese, preferably 0.50 per cent, but varying from 0.30 to 0.70 per cent; silicon, 1 per cent maximum and 0.60 per cent desired; phosphorus, 0.20 per cent maximum, and 0.17 per cent desired; sulphur, 0.60 per cent maximum. The tensile strength of malleable iron varies from 35,000 to 55,000 pounds per square inch, with from 3 to 8 per cent elongation in 2 inches.

Steel Castings. — Steel castings are especially adapted for machine parts that must withstand shocks or heavy loads. They are stronger than either wrought iron, cast iron, or malleable iron and are very tough. The steel used for making steel castings may be produced either by the Bessemer, open-hearth, electric, or crucible processes. The raw materials used are steel scrap, pig iron, and iron ore, the materials and their proportions varying according to the process and the type of furnace used. Steel for comparatively small castings may be made by the Bessemer or crucible furnaces, whereas for large castings, the open-hearth furnace is preferable. The electric furnace is now used considerably, some of the larger sizes being employed in conjunction with open-hearth furnaces and Bessemer converters which partially refine the charge. Steel castings are used for such parts as cylinder covers, cross-heads, cross-head guides, bearing caps, bedplates, stern-posts for ships, rudder frames, gun mounts, locomotive side frames, etc. They are of special importance in ship construction and for various classes of railway equipment.

Annealing Steel Castings. — Steel castings are usually annealed by heating in a furnace to a temperature ordinarily between 1500 and 1600 degrees F., the temperature varying with the carbon content of the steel. Annealing temperatures, according to the American Society for Testing Materials, should be as follows:

Carbon, up to 0.16 per cent, 925 degrees C. (1697 degrees F.).
Carbon, from 0.16 to 0.34 per cent, 875 degrees C. (1609 degrees F.).
Carbon, from 0.35 to 0.54 per cent, 850 degrees C. (1562 degrees F.).
Carbon, from 0.55 to 0.79 per cent, 830 degrees C. (1526 degrees F.).

The castings should be heated slowly and uniformly to these temperatures, which may be 50 or, in special cases, 100 degrees C. higher than specified, if necessary to obtain the desired result. The castings should be heated to the maximum temperature long enough to insure refining the grain, and should then be cooled slowly and uniformly in the furnace, if the steel is to have the maximum softness. If a higher tensile strength and elastic limit are required, the rate of cooling may be accelerated by withdrawing the castings from the furnace and burying them in a bed of material that is a poor conductor of heat.

Strength of Steel Castings. — The tensile strength of steel castings usually varies from 60,000 to 80,000 pounds per square inch. The specifications prescribed by the American Society for Testing Materials include two general classes of steel castings, known, respectively, as "Class A" and "Class B." Class A represents castings for which no physical requirements are specified. The physical requirements for castings of Class B are divided into three grades, designated as hard, medium, and soft. The minimum requirements covering tensile properties are as follows:

Soft grade: Tensile strength, 60,000 pounds per square inch; yield point, 27,000 pounds per square inch; elongation in two inches, 22 per cent; reduction of area, 30 per cent.

Medium grade: Tensile strength, 70,000 pounds per square inch; yield point, 31,500 pounds per square inch; elongation in two inches, 18 per cent; reduction of area, 25 per cent.

Hard grade: Tensile strength, 80,000 pounds per square inch; yield point, 36,000 pounds per square inch; elongation in two inches, 15 per cent; reduction of area, 20 per cent.

Composition of Steel Castings. — The composition of steel castings recommended especially for automobile construction, but suitable for many other classes of work, is as follows: Carbon, preferably 0.35 per cent with variations of from 0.30 to 0.40 per cent; manganese, preferably 0.70 per cent and varying from 0.50 to 0.80 per cent; silicon, from 0.10 to 0.30 per cent; phosphorus, not over 0.05 per cent; sulphur, not over 0.05 per cent. These specifications have been adopted by the Society of Automotive Engineers. The elastic limit of an annealed casting of this composition is approximately 35,000 pounds per square inch.

The percentages of carbon, silicon, and manganese are sometimes varied according to the size of the castings or their intended use. The following data were obtained from a German source: For small machine parts, 0.50 per cent of carbon, 0.25 per cent of silicon, and 0.50 per cent of manganese; for large machine parts, from 0.10 to 0.40 per cent of carbon, from 0.20 to 0.40 per cent of silicon, and from 0.50 to 0.80 per cent of manganese; castings for ships, such as sternposts and rudder frames, from 0.20 to 0.40 per cent of carbon, 0.30 per cent of silicon, and 0.50 per cent of manganese. Hard castings for ore crushers, etc., from 0.80 to 1.00 per cent of carbon, from 0.20 to 0.40 per cent of silicon, and from 0.50 to 1.00 per cent of manganese.

It has been found that steel castings containing about 0.2 per cent of vanadium show a great increase in tensile strength and elastic limit. Such castings have been made having a tensile strength of 90,000 pounds per square inch. The average tensile strength, however, is about 80,000 pounds and the elastic limit, about 45,000 pounds. Tests have indicated that the quality of steel castings may also be greatly improved by the addition of titanium, which is said to improve the density, strength, toughness, and durability of the steel when properly applied.

Manganese Steel Castings. — The metal for manganese steel castings is refined in a Bessemer converter from which it is poured into a ladle in which the proper quantity of ferromanganese has been previously placed. Manganese steel castings are generally allowed to cool in the mold, and are then annealed for from 3 to 26 hours at temperatures ranging from 1800 to 2000 degrees F. Unannealed manganese steel castings are exceptionally brittle and almost glass hard. After the heat-treatment, they are tough and ductile with a tensile strength of about 90,000 pounds and an elastic limit of about 60,000 pounds per square inch.

Semi-steel Castings. — What is commonly known as "semi-steel" and less frequently as "toughened cast iron" is produced by adding soft steel or wrought-iron scrap to the charge in a cupola. The semi-steel castings obtained from this mixture are cast in the same manner as ordinary castings. The mixture or charge for making semi-steel castings usually contains about 20 per cent of steel scrap, although any amount up to about 70 per cent may be used. Semi-steel castings have less total carbon than ordinary cast iron, there seldom being more than 3 per cent. The fine grain of semi-steel is due to the low percentage and fineness of the graphitic carbon. Semi-steel is commonly used for large gears, for the tables, saddles, slides, etc., of machine tools, and for parts requiring a good appearance.

The Temper of Steel. — The term "temper" or "carbon-temper" is used by steel makers to designate the proportion of carbon in the steel. The temper marks or numbers are arbitrarily selected, their relation to the percentage of carbon varying with different makers. The following list of tempers and the purpose for which the various steels are adapted is given by Joseph T. Ryerson & Son:

Temper Number	Per Cent Carbon	Tools for which Steel is Adapted
7	0.65 to 0.75	Blacksmiths' hammers, table knives, dies for drop hammers, large hot forgings, flatters, fullers, track chisels and tools.
8	0.75 to 0.85	Large shear knives, punches, chisels, hammers, boilers, boiler makers' tools, lathe centers, etc.
9	0.85 to 0.95	Punches and dies, hand chisels, mining tools, shear blades, etc.
10	0.95 to 1.05	Drills, large milling cutters, axes, taps, reamers, bolt header dies and similar tools.
11	1.05 to 1.15	Granite chisels, milling cutters, taps, reamers, mill picks, threading dies, cups, cones, etc.
12	1.15 to 1.25	Milling cutters, small taps, threading dies, twist drills, forming and boring tools, mandrels, razors.
13	1.25 to 1.35	Inserted milling cutter teeth, lathe, planer and slotter tools, and tools requiring great hardness.
14	1.35 to 1.45	Cutting disks, granite lathe tools, paper knives, engravers' tools, roll corrugating and chilled roll turning tools.
15	1.45 to 1.55	Steel for turning chilled rolls, etc., requiring great hardness.

Iron and Steel Definitions. — At the Brussels Congress of the International Association for Testing Materials held in 1906, the following definitions of the most important forms of iron and steel were adopted:

Alloy cast irons. — Irons which owe their properties chiefly to the presence of an element other than carbon.

Alloy steels. — Steels which owe their properties chiefly to the presence of an element other than carbon.

Basic pig iron. — Pig iron containing so little silicon and sulphur that it is suited for easy conversion into steel by the basic open-hearth process (restricted to pig iron containing not more than one per cent of silicon).

Bessemer pig iron. — Iron which contains so little phosphorus and sulphur that it can be used for conversion into steel by the original or acid Bessemer process (restricted to pig iron containing not more than 0.10 per cent of phosphorus).

Bessemer steel. — Steel made by the Bessemer process, irrespective of carbon content.

Blister steel. — Steel made by carbonizing wrought iron by heating it in contact with carbonaceous matter.

Cast iron. — Iron containing so much carbon or its equivalent that it is not malleable at any temperature. It is recommended drawing the line between cast iron and steel at 2.20 per cent carbon.

Cast steel. — The same as crucible steel; obsolete and confusing.

Charcoal hearth cast iron. — Cast iron which has had its silicon and usually its phosphorus removed in the charcoal hearth, but still contains so much carbon as to be distinctly cast iron.

Converted steel. — The same as blister steel.

Crucible steel. — Steel made by the crucible process, irrespective of carbon content.

Gray pig iron and gray cast iron. — Pig iron and cast iron in the fracture of which the iron itself is nearly concealed by graphite, so that the fracture has the gray color of graphite.

Malleable castings. — Castings made from iron which when first made is in the condition of cast iron, and is made malleable by subsequent treatment without fusion.

Malleable pig iron. — An American trade name for the pig iron suitable for converting into malleable castings through the process of melting, treating when molten, casting in a brittle state, and then making malleable without re-melting.

Open-hearth steel. — Steel made by the open-hearth process irrespective of carbon content.

Pig iron. — Cast iron which has been cast into pigs direct from the blast furnace.

Puddled iron. — Wrought iron made by the puddling process.

Puddled steel. — Steel made by the puddling process, and necessarily slag-bearing.

Refined cast iron. — Cast iron which has had most of its silicon removed in the refinery furnace, but still contains so much carbon as to be distinctly cast iron.

Shear steel. — Steel, usually in the form of bars, made from blister steel by shearing it into short lengths, piling, and welding these by rolling or hammering them at a welding heat. If this process of shearing, etc., is repeated, the product is called "double-shear steel."

Steel. — Iron which is malleable at least in some one range of temperature, and, in addition, is either (*a*) cast into an initially malleable mass; or, (*b*) is capable of hardening greatly by sudden cooling; or, (*c*) is both so cast and so capable of hardening.

Steel castings. — Unforged and unrolled castings made of Bessemer, open-hearth, crucible or any other steel.

Washed metal. — Cast iron from which most of the silicon and phosphor have been removed by the Bell-Krupp process without removing much of the carbon; still contains enough carbon to be cast iron.

Weld iron. — The same as wrought iron; obsolete and needless.

White pig iron and white cast iron. — Pig iron and cast iron in the fracture of which little or no graphite is visible, so that their fracture is silvery and white.

Wrought iron. — Slag-bearing, malleable iron, which does not harden materially when suddenly cooled.

IRON CASTINGS

Causes of Blowholes. — Blowholes are the result of an outrush of gas from the core or mold materials, into the molten iron at the time of solidification. If the solidification has proceeded so far that the outrushing gas or steam cannot bubble through it and escape through the vents which should be provided for the purpose, it will be imprisoned in the casting, forming one or more holes according to the shape of the casting and the quantity of the escaping gas. These holes may not be apparent on the outside, and quite often occur in a location where they do no particular harm, but they are frequently located at some point where they are unsightly or greatly weaken the casting.

Moisture in the Sand. — The gases which cause blowholes may come from three sources: They may be, and generally are, caused by the generation of quantities of steam from the moisture contained in the molding sand by the heat of the iron. In the case of dry sand and loam castings, the quantity of steam generated is so insignificant, if the molds have been properly heated, that it gives no trouble. In the case of green sand castings, however, the moisture present, and, therefore, the steam generated, is quite large in amount, and special precautions have to be taken to prevent blowholes.

When the molten iron is poured into a green sand mold, all the moisture in the layer of sand immediately in contact with the iron will at once be transformed into steam. The depth of the sand layer so affected depends on the thickness and extent of the molten metal. The steam must either force its way through the molten iron in the form of a mass of bubbles, or else it must escape through the sand. To facilitate its escape, the mold is vented; that is, after the damp sand has been packed around the wooden pattern by ramming it closely into place, a wire is thrust repeatedly into the mold, making numerous passages for the escape of the steam and gas. Obviously, one of these vent-holes cannot extend to every point in the layer of sand adjacent to the casting, so it is necessary that most of the steam and gas should force its way for a short distance through the sand before it can reach a vent-hole. It can only do this when the sand is somewhat porous. If the sand is too tightly rammed, it will lack the necessary porosity, and even though it be unusually dry and the venting carefully done, the casting will be full of blowholes.

Decomposition of Binder in Cores. — The second cause of blowholes is the decomposition of the material (generally flour or molasses) used as a binder for the cores, and its escape in the form of gas into the iron at the time of pouring. It is impossible to prepare and bake a core in such a way that it will not give off large quantities of gas when the iron is poured; therefore, means must be provided for the escape of this gas. In order to do this, the cores are prepared with wax strips running through them. When the core is baked, the wax melts, leaving passages known as core vents for the escape of these gases. If the core is of such form and so set in the mold that the gases can escape from these vents in an upward or sidewise direction, and leave the mold without forcing their way through the molten iron, no blowholes will result.

Entrapping of Air. — A third source of blowholes is the entrapping of air in certain parts of the mold, and its mixing with the iron. This trouble is due to insufficient venting of the mold.

Dry Sand or Loam for Large Castings. — In the case of large and complicated castings, it is generally advisable to make dry sand or loam castings, in order to avoid, as far as possible, the chance of blowholes. When the mold is very large, it is difficult to vent it thoroughly, and when the work on it extends over a period of three or four weeks, it is impossible to keep the vents from filling up; hence the general use of dry sand work for large castings. Often, however, for the sake of economy, fairly large and complicated pieces must be undertaken in green sand, and it becomes a matter of importance that they be so designed that the molder will not be compelled to invite disaster by keeping his sand too wet, or ramming it too hard, and that there is no part of the mold which cannot be thoroughly vented.

Wetting and Ramming Green Sand Molds. — Up to a certain point, the wetter the sand is, the better it will stick, but it should not be any more wet than is necessary. The more tightly the sand is rammed, the better its particles will cohere, but tight ramming and wet sand, while they make a solid and easily handled mold, invariably produce blowholes. If a pattern be of complicated form or hard to draw, or if, when it is drawn, it leaves the sand in such a form that the mold will easily fall together if there be a little jarring, the molder will be compelled to wet the sand more and to ram it harder than usual. Small, deep openings, sharp fillets, and thin walls and partitions of sand are especially troublesome. Not only do they make it difficult to draw the pattern, and handle the mold, and so make excessive wetting and hard ramming imperative, but they form places in the mold which the venting wire is unlikely to reach. Hence, they are to be avoided when possible in any class of molding, and on no account should such work be permitted in the case of large green sand castings. When designing a casting to be made in green sand, the designer ought to know the position which it will occupy in the mold when poured. In general, the parts of a casting which lie in the bottom of the mold will be the soundest, and those parts which must be machined, or which require the greatest strength, should therefore occupy the bottom of the mold, if possible, when the casting is poured. To insure a sound casting, the sand in the lower parts of the mold must be comparatively dry and loosely rammed. This condition is, generally, not difficult to attain, since all the work on the sand is done with the pattern in place, and that part of the mold is not generally moved or handled after the support of the pattern has been withdrawn. In the lower part of the mold, the sand is generally supported at all points in a very thorough manner by the sand lying under it, and so hard ramming or wet sand is unnecessary. The upper part of the mold must of necessity be rammed harder than the lower part, since the sand is not supported from beneath but hangs from above.

Using both Green Sand and Dry Sand. — It often occurs that the larger part of a casting is of simple form and easy to mold. A certain part of it, however, may be of a form exceedingly difficult to mold, and therefore likely to give a great deal of trouble. It is not necessary that the whole casting be made in a dry sand mold, but a core-box may be made to take care of the difficult part of the work, even though the work would ordinarily be done without a core. It is just as easy, and often just as desirable, to cast the external face of a casting against a core as the internal face. While it may not pay to do this when only one casting is wanted it is often the cheapest method when a great many are required; it reduces to a minimum both the work of the molder and the chance of a spoiled casting. Forms can often be cast in this way which could not be cast by any other method.

Supporting the Cores. — In arranging the cores in a mold, it is always better, if possible, to support them at the top. The gases formed in the core being light, tend to rise, and if the core is supported at the bottom only, they tend to escape into the iron and bubble through it. If the gases can escape at the top, they will pass off without coming in contact with the iron. When it is impossible to support the cores at the top, they should be so arranged that the gases can pass off at the sides and escape from the mold without coming in contact with the iron.

Sponginess. — Sponginess is due to the formation of gas bubbles in iron at the instant of solidification. In all ordinary cases, this is due to an improper mixture of iron. However, if the casting is very thick at one place, but otherwise thin, it will be impossible to obtain a mixture which will have satisfactory properties for general work and not be spongy at the points of extraordinary thickness. It is an excellent rule to allow no part of a casting to be at a greater distance from a sand surface than 2½ inches. In case this rule is strictly adhered to, and the castings are of fairly uniform thickness, no trouble will be experienced from sponginess, except from the use of poor iron. When it is necessary to have a considerable quantity of metal at one place, and a greater thickness than 5 or 6 inches, it should either be at some point where the sponginess will do no harm or else provision be made to avoid it. The only practical method of doing this is to place a riser directly over the heavy section. When the metal is poured and the riser is full, a rod of wrought iron is inserted and worked up and down until the metal has almost solidified; the bubbles then have a better chance to escape, and the iron is left perfectly solid.

Shrink-holes. — A shrink-hole is a cavity caused by the shrinking away of the metal in cooling. Like sponginess, this defect is most likely to occur in those parts of a casting which are excessively thick. If practicable, avoid sudden changes in the thickness of a section. If an unusually thick section does not have to be machined, the difficulty may be overcome by placing in the mold at that point a piece of iron so that the casting will be caused to solidify at that point first, on account of the chilling effect of the cold iron. If the casting has to be machined, shrink-holes are best prevented (as described for the prevention of sponginess) by the use of a heavy riser, and the working of the iron with a rod when it is cooling.

The designer should avoid heavy sections whenever possible, because they are likely to produce these two serious faults: sponginess and shrink-holes. Heavy sections should be avoided especially in those parts which are to be machined or which are subjected to heavy stresses. If they cannot be avoided entirely, in such a case, they should be so arranged that risers may be placed immediately over them.

Scabbiness. — Although iron in the molten condition does not permeate the sand of a mold as water would, nevertheless, on account of its great weight, there is considerable erosive action on the mold. If the iron eats away fillets or partitions, or scours away patches of sand as it flows into the mold, the casting will not be of the proper form, but will have its angles partly filled up and unsightly protuberances upon its surfaces. Such imperfections are known as scabs. The sand so washed from its proper place may float on the iron and rise to the top of the mold, where it forms a dirty mixture, which, when cleaned away, leaves a rough depression in the surface of the casting, also known as a scab. The remedy for this trouble is to avoid, as far as possible, sharp fillets and thin tongues of sand projecting into the mold, and to so gate the casting that the current of iron, as it enters the mold, will spread out and not concentrate in any particular direction.

Sand-holes. — Sand-holes are almost invariably associated with scabbiness. If the sand which has been eroded by the entering current of iron does not rise immediately to the surface, the iron may partially solidify before it floats to the top. As a result, it will rise till it strikes a part of the iron which has so solidified, and remain there imprisoned within the body of the casting. Sand-holes generally occur in that part of a casting which lies near the top of the mold. Occasionally they form large cavities which seriously impair the strength of the casting, but more often they form very small holes which, being full of sand, injure the cutting edges of the tools and leave the finished surface pitted and unsightly.

Floating Cores. — Floating cores are often the cause of unsound castings. The buoyant effect of the molten iron on a core is equal to about three times the weight of the core, if the core is solid, and very much more than that if it is hollow. Large cores are generally built up about cast-iron skeletons known as core frames. These core frames are of about the same shape as the core and serve to support it and bind it together. A projecting piece of core having a volume of four cubic feet, for instance, will weigh approximately 500 pounds, and have a buoyant force of about 1500 pounds thrusting it upward when the mold is poured. If the core frame is not amply strong and stiff, this force will bend the projection upward or even break it off entirely; hence the necessity of making large cored cavities of such a form that the cores may be rigidly secured. One of the most difficult things to cast properly is a long iron pipe having a small diameter and thin walls. If such a pipe be cast in the usual position, that is, lying horizontally, there will be an upward thrust along the whole length of the core tending to bend it. On account of its slenderness, the central portions will be deflected upward, making the walls of the pipe thinner on one side than on the other. Often the deflection proves sufficient to thrust the core against the side of the mold, if it is long, or, in case the thickness of the wall is great in proportion to the length of the core, to break it off entirely. On this account, pipes and hollow columns of cast iron are often cast on end, thus avoiding any deflection of the core. The same principle may be applied to many other pieces by taking care to so design them that long and slender cores have a vertical position when the mold is poured.

Cold Shuts. — A cold shut is caused by the imperfect uniting of two or more streams of molten iron flowing together, which are too cold to coalesce. Such a fault often occurs on the upper side of a thin cylinder cast horizontally, when the iron is not sufficiently hot at the instant of pouring. It appears as a seam in the side of the cylinder, and it is very apparent that the metal has united imperfectly. Such a defect will cause the casting to split if subjected to any great stress, and it will leak under pressure. This imperfection is generally due to improper gating or thinness of metal. If the iron is obliged to flow in thin streams for long distances, it will be cooled very much, and probably the advancing face will be partially solidified. Consequently, when it meets a similar advancing face of metal which has been similarly cooled, there is small likelihood of their uniting properly. The arrangement of gates and risers is often of great importance in minimizing cold shuts, and if the casting is large, and at the same time has thin walls, the designer must see that the gates are so arranged that the iron can quickly fill up the mold. While the arrangement of the gating is generally determined by the molder, he may often be limited by the shape of the casting and obliged to place the gate at some point where the iron, in flowing in, must spread into a thin sheet, or pass for a considerable distance through a narrow passage. Under such circumstances, a cold shut is hardly to be avoided.

Shrinkage Strains. — When some sections of a casting are much larger in cross-section than others, shrinkage strains often result, due to unequal rates of cooling. The best way to eliminate them is to so arrange the thickness of the various parts that the entire casting will solidify at about the same time. If this is not feasible, the casting should be so designed that unequal contraction will not produce dangerous stresses at any point. To avoid unequal contraction and dangerous stresses, there should be no sharp corners and the various parts should be free to expand when necessary. For instance, a wheel or pulley with a solid rim is likely to have severe stresses set up within the arms by unequal cooling, but if the hub be divided through the center by means of a thin core (the halves being bolted subsequently) no shrinkage strains will occur, since the arms are free to expand or contract independently of the rim. Shrinkage strains often become so serious that it is necessary to make castings in two or more parts. Large jacketed cylinders for steam and gas engines are good examples.

Firebrick

Melting Points of Firebrick. — According to the United States Bureau of Standards, the melting points of firebricks are as follows: The most common firebrick will melt at a temperature ranging from 2830 to 3140 degrees F.; bauxite brick, from 2950 to 3245 degrees; silica brick, from 3090 to 3100 degrees; chromite brick, at 3720 degrees; and magnesia brick, at 4950 degrees. These melting points, which represent the lowest temperature at which a small piece of the brick could be distinctly seen to flow, were determined in an electric vacuum furnace, the temperature being measured with an optical pyrometer.

The heat conductivity of firebrick is six times as great at 3000 degrees F. as at 200 degrees F.

General Information about Firebrick. — To obtain the best results from brick work, observe the following precautions: All firebrick should be kept in a dry place, as moisture, especially in cold weather, will greatly injure any brick. Use good fireclay equal in refractory qualities to the brick itself. Apply thin with dipped joints and rub the brick to make a brick-to-brick joint. Warm up slowly to expel moisture. Remember that fireclay bricks contract, and silica, chromium and magnesia bricks expand under high temperatures. All furnaces in which silica brick is used should be heated and cooled slowly and uniformly. From 250 to 350 pounds of fireclay or silica cement are enough to lay up one thousand brick. Finely ground fireclay should be used for laying up fireclay brick, and silica cement for silica brick. For estimating on firebrick work, use the following figures:

One square foot of 4½-inch wall requires 7 bricks; 1 square foot of 9-inch wall requires 14 bricks; 1 square foot of 13½-inch wall requires 21 bricks; 1 cubic foot of brick work requires seventeen 9-inch straight bricks; 1 cubic foot of fireclay brick work weighs 150 pounds; 1 cubic foot of silica brick work weighs 130 pounds; 1000 bricks (closely stacked) occupy 56 cubic feet; 1000 bricks (loosely stacked) occupy 72 cubic feet. For estimating on red brick work, figure on 9 cubic feet of sand and three bushels of lime for laying 1000 bricks.

Cement for High Temperatures. — As a binder for a cement to stand high heat (1475 degrees F.), use sodium silicate (water glass) diluted with rain water until a specific gravity of 20 degrees on the Baumé hydrometer is obtained. Sand or quartz may be used in the cement to give the proper consistency.

Methods of Cleaning Castings

Pickling Solutions. — The pickling solutions used for removing scale from castings and forgings preparatory to milling or other machining operations are usually composed of dilute sulphuric acid or oil of vitriol, although hydrofluoric acid is sometimes used. A sulphuric acid bath is commonly made of one part acid and from four to ten parts water. The acid should be poured into the water while the latter is being stirred, but *the water should never be added to the acid*, as this may suddenly generate steam which may cause an explosion. Such an accident would be likely to throw the concentrated acid over the workman and cause serious burns. Sulphuric acid will not attack the sand or black oxide of iron which forms the scale upon castings, but it soaks through and attacks the iron beneath the scale and finally dissolves a sufficient amount to loosen the scale. Then the castings should be washed, preferably with hot water; if the castings are small, it is good practice after washing to immerse them in a soda solution to neutralize any acid that may remain.

A hydrofluoric acid pickling solution is preferable in some respects to the sulphuric acid solution, but it is more dangerous and must be handled carefully. Hydrofluoric acid is commonly sold in three grades, the first containing 30 per cent acid, the second 48 per cent, and the third 52 per cent, the balance of the solution being water. The 30 per cent solution is the one commonly used for pickling castings. The pickling bath should contain one gallon of the 30 per cent solution to from twenty to twenty-five gallons of water. If less water is used, the pickling can be done more rapidly, but by using slightly more water, the acid can be used for a longer period. Hydrofluoric acid attacks and dissolves the sand and the black oxide of iron. A bath can be used repeatedly by adding one-third the original quantity of acid before immersing a new lot of castings. Hydrofluoric acid cannot be kept in a crock, jug or glass receptacle and must be placed in a lead carboy, except when diluted, in which case it can be kept in wooden tubs or barrels. When handling concentrated hydrofluoric acid, a workman should always use rubber gloves, and if any acid comes into contact with the hands, it should be washed off at once with water and dilute ammonia.

Solutions for Cleaning Brass Castings. — A solution commonly used for pickling brass castings is made by mixing three parts of sulphuric acid and two parts of nitric acid, and adding to each quart of the mixture about a handful of common salt. This solution must be kept in an earthenware crock or in a vitrified receptacle, and the bath must be large enough to dip the castings into it. The following proportions are also used for cleaning brass castings: 1. Two parts nitric acid; one part sulphuric acid. 2. For a bright dip: One part nitric acid; one part sulphuric acid.

Recovering Copper from Dipping Acids. — Some copper is dissolved when dipping brass castings in acid to clean them. In order to recover this copper, run the acid liquids from the washing tanks into a large wooden vat containing milk of lime, and stir thoroughly. The lime neutralizes the acids and precipitates the sulphuric acid as sulphate of calcium. This acid is neutralized first, then the nitric acid is acted upon, and nitrate of calcium is produced. The clear liquid contains nitrate of calcium and nitrate of copper. Run this solution through any kind of scrap iron. The iron removes the copper from the solution, dissolving and taking the place of the copper. The copper settles at the bottom of the tank as a slush. At intervals, the copper may be taken out of the vat, dried and melted into ingots. All of the copper may be recovered if the supply of scrap iron is kept up.

Soda Cleaning Solution. — The solution used in soda kettles for removing oil or grease from machine parts should contain about one-half pound of sal-soda to each gallon of water. If old paint is to be removed, the solution should consist of about one-quarter pound of caustic soda to each gallon of water. As caustic soda is a strong alkali, care should be taken to prevent it from getting onto the hands. These solutions should be heated to the boiling point before immersing the parts to be cleaned; then the work will dry quickly after being removed, and will not rust. A wire basket or perforated bucket is convenient for washing small pieces. The time required for cleaning depends somewhat upon the nature of the grease and to what extent it has dried and hardened.

Dipping Solution to Preserve Iron Castings. — Mix 100 pounds of dry lamp black to a paste, with benzine. Then make a mixture of 5 gallons lamp black paste, 2 gallons turpentine japan, 1 gallon boiled oil, 1 to 2 gallons cheap varnish.

For very good work, add to the above 12½ pounds of refined lamp black ground in japan. Dilute the mixture with turpentine until it is as thin as water, and clean the castings before dipping.

The Sand Blast. — To obtain good results with a sand blast, it is necessary to provide an ample supply of air. The table "Cubic Feet of Free Air for Sand-blast Nozzles" gives the amount of air required for nozzles of different diameter and different air pressures. The pressure that should be used depends upon the nature of the work. For cleaning light castings, such as stove castings, etc., use from 5 to 10 pounds; for medium- and heavy-grade iron castings, 15 to 20 pounds; for steel castings, 30 to 75 pounds; for buildings and steel structures, 5 to 30 pounds, depending upon the height. The air piping connecting the receiver and the sand blast should not be less in diameter than the air connection of the sand blast; but if the distance between the receiver and the sand blast is over 75 feet, the piping should be larger than the air connection of the sand blast, to allow for the loss of pressure from friction. It is not that the sand blast will require all of this air, but the most satisfactory results are obtained by having a "backing of air" behind the jets. The piping should also be protected from condensation, if the lines are long, and moisture must be kept from the sand to insure proper working. The sand should be sharp, clean quartz or silica, sifted through a screen of the proper mesh and dried long enough beforehand to be cold when used. When using a ¼-inch nozzle, the sand should be passed through a No. 8 mesh screen. With a ½-inch nozzle, much coarser sand can be used. A coarse sand will be found effective for general work in the foundry, especially for cleaning steel castings.

Cubic Feet of Free Air for Sand-blast Nozzles

Nozzle Diameter, Inches	Air Gage Pressures, Pounds per Square Inch					
	5	10	15	20	25	30
	Cubic Feet of Free Air per Minute					
¼	14.4	21.8	26.7	30.8	34.5	40.0
⅜	34.6	49.0	60.0	69.0	77.0	90.0
½	61.6	87.0	107.0	123.0	138.0	161.0
⅝	96.5	136.0	167.0	193.0	216.0	252.0
¾	133.0	196.0	240.0	277.0	310.0	362.0
⅞	189.0	267.0	326.0	378.0	422.0	493.0

Pattern Materials — Shrinkage, Draft and Finish Allowances

Woods for Patterns. — Woods commonly used for patterns are white pine, mahogany, cherry, maple, birch, white wood and fir. For most patterns, white pine is considered superior because it is easily worked, readily takes glue and varnish, and is fairly durable. For medium- and small-sized patterns, especially if they are to be extensively used, a harder wood is preferable. Mahogany is often used for patterns of this class, although many prefer cherry. As mahogany has a close grain, it is not as susceptible to atmospheric changes as a wood of coarser grain. Mahogany is superior in this respect to cherry, but is more expensive. In selecting cherry, never use young timber. Maple and birch are employed quite extensively, especially for turned parts, as they take a good finish. White wood is sometimes substituted for pine, but it is inferior to the latter in being more susceptible to atmospheric changes.

Selection of Wood. — It is very important to select wood for patterns that is well seasoned; that is, it should either be kiln dried or kept one or two years before using, the time depending upon the size of the lumber. During the seasoning or drying process the moisture leaves the wood cells and the wood shrinks, the shrinkage being almost entirely across the grain rather than in a lengthwise direction. Naturally, after this change takes place, the wood is less liable to warp, although it will absorb moisture in damp weather. Patterns also tend to absorb moisture from the damp sand of molds, and to minimize troubles from this source they are covered with varnish. Green or water-soaked lumber should not be put in a drying room, because the ends will dry out faster than the rest of the log, thus causing cracks. In a log there is what is called the "sap wood" and the "heart wood." The outer layers form the sap wood which is not as firm as the heart wood and is more likely to warp; hence, it should be avoided, if possible.

Pattern Varnish. — Patterns intended for repeated use are varnished to protect them against moisture, especially when in the damp molding sand. The varnish used should dry quickly to give a smooth surface that readily draws from the sand. Yellow shellac varnish is generally used. It is made by dissolving gum shellac in grain alcohol. Wood alcohol is sometimes substituted, but is inferior. The color of the varnish is commonly changed for covering core prints, in order to readily distinguish the prints from the body of the pattern. Black shellac varnish (which is the color generally used) is made by the addition of lamp black. This should be of good quality and free from grit. Red varnish can be made by adding Chinese vermilion. All coloring powders should be well pulverized. At least three coats of varnish should be applied to patterns, the surfaces being rubbed down with sand paper after applying the preliminary coats, in order to obtain a smooth surface.

Glue for Patterns. — There are many qualities of glue both in the liquid, sheet and pulverized form. Animal glue in the sheet or flake form is generally used for pattern work. As a rule, the best quality is of amber color and the flakes are rather thin. Where glue is used in small quantities, the pulverized form has the advantage of being quickly prepared. Freshly made glue is the strongest and, if of good quality, can be drawn out into thin threads. Whenever practicable, glued joints should be reinforced by nails or screws. A joint to be glued should be accurately fitted because glue does not get a grip unless the parts are in close contact.

Before applying glue, clean the surfaces of sand paper dust or other foreign material, so that the glue can enter the pores of the wood. This is very important. If the end grain must be glued, first apply a sizing coat to fill the openings among the fibers. When the sizing coat is dry, apply a second coat to the surface and unite. If the preliminary coat is not applied, the open end grain is liable to absorb the glue so rapidly as to weaken the joint. The hot glue should be thin enough to spread easily. It can be thicker, however, for pine than for wood of closer grain, like mahogany, because (aside from the quality of the glue) the holding or binding property depends upon the extent to which the glue enters the pores of the wood. All glued joints should be firmly pressed together with clamps immediately after applying the glue. The latter should be given plenty of time in which to set; ten or twelve hours in a dry place should be sufficient.

Shrinkage Allowance. — The shrinkage allowances ordinarily made on patterns to compensate for the contraction of castings in cooling are as follows: Cast iron, $\frac{3}{32}$ to $\frac{1}{8}$ inch per foot; common brass, $\frac{3}{16}$ inch per foot; yellow brass, $\frac{7}{32}$ inch per foot; bronze, $\frac{5}{32}$ inch per foot; aluminum, $\frac{7}{32}$ to $\frac{1}{4}$ inch per foot; steel casting, $\frac{3}{16}$ inch per foot. The amount of shrinkage, in any case, depends to some extent upon the shape and size of the casting. A plain casting that is long in proportion to its width will contract differently from one that is more compact, even though both castings have the same weight and were cast from the same material. A heavy iron casting may shrink only 1/10 inch per foot or even less, whereas a lighter casting of the same material may shrink 1/8 inch per foot. A cylindrical or column-shaped casting will contract more in a lengthwise direction than radially. Hence, when making patterns for rather large castings of this kind, the allowance should be about 1/10 inch lengthwise and from 1/20 to 1/16 inch per foot radially. For pipes or other hollow castings, the lateral shrinkage is very much less than for solid castings or those having thick walls. The patternmakers' shrink rule has graduations which are longer than standard measurement, to give the allowance directly.

A general rule for columns of comparatively small diameter but great length, such as are used for building purposes, is to allow 1/8 inch per foot for shrinkage lengthwise and make no allowance on the diameter. The "one-tenth" shrinkage rule is the standard (for cast iron) in most machine pattern-shops. Although this is not the proper allowance for all forms of casting, the adoption of a standard eliminates the confusion that would follow the use of a number of rules for different classes of work. There can be no fixed rule governing shrinkage allowance, as it is largely a question of local conditions and practice.

Shrinkage of Castings

The usual allowance for each foot in length is as follows:			
In large cylinders	= 3/32 in.	In zinc	= 5/16 in.
In small cylinders	= 1/16 in.	In lead	= 5/16 in.
In beams and girders	= 1/10 in.	In tin	= 1/4 in.
In thick brass	= 5/32 in.	In copper	= 3/16 in.
In thin brass	= 3/16 in.	In bismuth	= 5/32 in.
In cast-iron pipe	= 1/8 in.	In malleable iron	= 1/8 in.
In steel	= 1/4 in.	In aluminum	= 1/5 in.

Draft for Patterns. — The draft or the amount of taper given to patterns to facilitate withdrawing them from the mold, depends somewhat upon the size and shape of the pattern. A general rule is to taper each side 1/8 inch for each foot of surface to be drawn. The average amount for small patterns is about 1/16 inch per foot, although in some cases it can be less, but, as a rule, there should be at least 1/32 inch draft. The draft slopes away from the pattern "face" which is usually uppermost in the mold when the pattern is drawn. Some patterns do not require draft because none of the surfaces are at right angles to the face. In some cases, very small patterns are made without draft.

Finish Allowance. — The amount added to a pattern to allow for machining the casting varies widely. It depends upon the method of machining, the size of the casting, and the importance of having a clean surface, free from flaws or defective spots. If castings are to be finished from the rough upon disk grinders, very little allowance is necessary; in fact, the molder may rap a pattern enough to allow for the finish. On small castings to be finished by milling or planing about 1/8 inch is usually allowed for the machining operation, whereas large castings for engine beds, flywheels, pump cylinders, etc., often have an allowance of 3/4 to 1 inch.

Metal Patterns. — Metal patterns are especially adapted to molding machine practice, owing to their durability and superiority in retaining the required shape. The original master pattern is generally made of wood, the casting obtained from the wood pattern being finished to make the metal pattern. The materials commonly used are brass, cast iron, aluminum and steel. Brass patterns should have a rather large percentage of tin, as this gives a good surface for the casting. Cast iron is generally used for patterns of large size, as it is cheaper than brass and more durable. Cast-iron patterns are largely used on molding machines. Aluminum patterns are light but they shrink considerably. White metal is sometimes used when it is necessary to avoid shrinkage. The gates for the mold may be cast or made of sheet brass. Some patterns are made of vulcanized rubber, especially for light match-board work.

Obtaining Weight of Casting from Pattern Weight. — To obtain the approximate weight of a casting, multiply the weight of the pattern by the factor given in the accompanying table. For example, if the weight of a white-pine pattern is 4 pounds, what is the weight of a solid cast-iron casting obtained from that pattern? Casting weight = 4 × 16 = 64 pounds. If the casting is cored, fill the core-boxes with dry sand, and multiply the weight of the sand by one of the following factors: For cast iron, 4; for brass, 4.65; for aluminum, 1.4. Then subtract the product of the sand weight and the factor just given from the weight of the solid casting, to obtain the weight of the cored casting. As the weight of wood varies considerably, the results obtained by the use of the table are only approximate, the factors being based on the average weight of the woods listed. For metal patterns, the results are more accurate.

Factors for Obtaining Weight of Casting from Pattern Weight

Pattern Material	Factors				
	Cast Iron	Aluminum	Copper	Zinc	Brass, 70% Copper, 30% Zinc
White pine	16.00	5.70	19.60	15.00	19.00
Mahogany, Honduras	12.00	4.50	14.70	11.50	14.00
Cherry	10.50	3.80	13.00	10.00	12.50
Cast iron	1.00	0.35	1.22	0.95	1.17
Aluminum	2.85	1.00	3.44	2.70	3.30

Branch Pipes for Exhausting Shavings from Wood Working Machines. — The sizes of branch pipes, given in the accompanying table, are correct for pipes not exceeding twenty feet in length. Where branch pipes contain a number of elbows, or exceed twenty feet in length, the area should be proportionately increased. Where the work is light and the branch pipes short, smaller connections than those given can sometimes be used. The area of the main duct should be equal to, or slightly larger than, the sum of the areas of the connecting branches. This proportion should be carefully maintained. If the main pipe is too small, the suction will be impaired, and if it is too large, the velocity of the air may be reduced to such an extent that the material being exhausted will settle in the bottom of the pipe, thereby reducing the area. If the main pipe is unusually long (exceeding 100 feet) the area should be increased from 10 to 20 per cent. Avoid abrupt turns in the piping, and never enter branch pipes at right angles to the main pipe, but always connect them at an angle of from 30 to 45 degrees. Branch pipes should never enter the main at the bottom but always at the side, and two pipes should not enter directly opposite each other.

Branch Pipes for Exhausting Shavings from Wood Working Machines

Type of Machine	Diam. of Branch Pipe, Inches	Type of Machine	Diam. of Branch Pipe, Inches
Saws:		Planer knives:	
Rip, Cut-off, Split, Swing, Bracket, Groove — 18 inches diam. or less	4	Length of knife, 24 inches	6
Rip, Cut-off, Split, Swing, Bracket, Groove — 18 to 24 inches diam.	5	Length of knife, 30 inches	7
Heavy cut-off, 24 to 42 inches diameter	6	Matcher heads, each	5
Band	4	Door tenoner	5
Band resaw, ¾ to 1 inch	5	Sash tenoner	5
Band resaw, 1½ to 2½ ins.	6	Sticker machines, each head	4
Planer knives:		Sand drum, 24 inches long	4
Length of knife, 5 inches	4	Sand drum, 30 inches long	5
Length of knife, 10 inches	5	Floor sweep-up	6
Length of knife, 14 inches	6	Heavy timber planer, each head	7
		Diagonal planer for doors	7
		Diagonal planer for doors, with sand drum	8

Speed of Circular Saws for Wood

Size of Saw, Ins.	Rev. per Min.	Size of Saw, Ins.	Rev. per Min.	Size of Saw, Ins.	Rev. per Min.	Size of Saw, Ins.	Rev. per Min.	Size of Saw, Ins.	Rev. per Min.
8	4500	20	1800	32	1125	44	840	56	650
10	3600	22	1650	34	1050	46	800	58	625
12	3000	24	1500	36	1000	48	750	60	600
14	2600	26	1400	38	950	50	725	64	550
16	2200	28	1300	40	900	52	700	68	525
18	2000	30	1200	42	870	54	675	72	500

Extrusion of Metals

The Extrusion Process. — The extrusion process is a method by means of which shapes of fairly plastic metals are produced by forcing the metal, which is usually heated, under high pressure through an aperture of the shape to be produced. In this manner, a continuous bar or pipe of the cross-section of the aperture or die is produced. Lead and tin can be extruded at comparatively low temperatures (250 degrees F.), while copper requires a temperature of about 1750 degrees F. The advantages of the extrusion process are that it permits parts of unusual cross-section to be produced cheaply. On account of the high pressure under which the metal is extruded, its structure becomes more compact and its strength is increased. The surfaces are smooth and free from flaws and other defects. The dimensions of the extruded shapes can be gaged with great accuracy, so that they can be used directly with no or very little additional finishing. Sometimes metals are extruded at atmospheric temperatures, in which case a higher pressure must be used, but the metal will be more condensed and the grain refined, adding to its strength, hardness and toughness. It requires, however, five times the pressure to extrude aluminum at 70 degrees F., as compared with the pressure required at 600 degrees F. The accompanying table shows the influence of the extrusion process on the properties of metals.

Influence of the Extrusion Process on the Properties of Metals

Metal	Cast		Extruded	
	Tensile Strength, Pounds per Square Inch	Elongation, Per Cent	Tensile Strength, Pounds per Square Inch	Elongation, Per Cent
Copper	28,500	35	34,000	38–40
Magnalium	43,000–64,000	5	53,000–71,000	10
Aluminum	14,000–17,000	3	33,000–38,000	4.3
Delta-metal	83,000	11	98,000	21.8
Durana-metal	58,000–74,000	35	60,000–81,000	38

Metals and Metal Alloys used in the Extrusion Processes. — Lead was the first metal used in the extrusion process. It is the most plastic of the metals. Other metals that have been used in extrusion are tin, copper, zinc, antimony, aluminum and bismuth. Better results are obtained, however, by using alloys; 20 per cent of zinc alloyed with 80 per cent of copper produces an exceedingly malleable alloy. One of the most common alloys used is composed of 55 per cent of copper and 45 per cent of zinc; brasses containing from 50 to 60 per cent copper and from 40 to 50 per cent zinc are also frequently used. Small quantities of iron add to the strength of the brasses and do not increase the dfficulty of extruding them; hence, Delta-metal, manganese-bronze and similar alloys can be used in extrusion processes. Bronzes with from 8 to 10 per cent of aluminum are easily extruded. Alloys containing 90 per cent copper and 10 per cent aluminum, or 85 per cent copper, 10 per cent zinc and 5 per cent aluminum are also used. With these aluminum bronzes a tensile strength of from 65,000 to 85,000 pounds per square inch is obtained, with an elastic limit of from 40,000 to 60,000 pounds, an elongation in 8 inches of about 18 per cent, and a reduction in area of about 20 per cent.

Pure zinc can be strengthened by an extrusion process conducted in the following way: The area of the die opening in relation to the area of the zinc billet to be extruded should be in the ratio of 1 to 15. The temperature of the metal should be kept between 85 and 180 degrees F., and the extruded metal should be submitted to a pressure of not less than 90,000 pounds per square inch. By this means, the coarse crystalline structure of ordinary zinc is transformed into a fine grain and the zinc assumes the properties of brass, having a tensile strength of 29,000 pounds per square inch and an elongation of from 26 to 70 per cent. Ordinarily, zinc has a tensile strength of only 5000 pounds per square inch and hardly any percentage of elongation.

Die-Casting

Die-casting is a method of producing finished castings by forcing molten metal into a suitable mold, which is arranged to open after the metal has solidified so that the casting can be removed. The die-casting process makes it possible to secure accuracy and uniformity in castings, and machining costs are either eliminated altogether or are greatly reduced. The greatest advantage of the die-casting process is due to the fact that parts are accurately and, usually, completely finished when taken from the dies. When the dies are properly made, castings may be accurate within 0.001 inch or even less and a limit of 0.002 and 0.003 inch can be maintained easily on many classes of work.

Die-castings are extensively used in the manufacture of such products as cash registers, meters, time-controlling devices, small housings, phonograph castings, and parts for a great variety of mechanisms. Lugs and gear teeth are cast in place and both external and internal screw threads can be cast. Holes can be formed within about 0.001 inch of size and the most accurate bearings require only a finish-reaming operation. Figures and letters may be cast sunken or in relief on wheels for counting or printing devices, and owing to numerous developments, many shapes which formerly were believed too intricate for die casting are now produced successfully by this process.

As to the limitations of the die-casting process it may be mentioned that the cost of dies is high, and, therefore, die-casting is applicable only when a large number of duplicate parts are required. The stronger and harder metals cannot be die-cast so that the process is not applicable for casting parts which must necessarily be made of iron or steel, although special alloys have been developed for die-casting which have considerable tensile and compressive strength.

Metals Used for Die-castings. — In the early development of the die-casting process, many alloys were experimented with and used, but the number of different compositions now employed has been reduced to a few standardized metals which may be divided into four general groups as follows: (1) the white-metal group; (2) the tin-base group; (3) the aluminum group; and (4) the brass and bronze group. In addition, lead-base alloys are used quite generally for bearing purposes.

The White-metal Alloys. — The white-metal group comprises the zinc-base alloys which frequently are also known as "white brasses." The metals in this group used for die-casting purposes contain zinc, tin, and copper. The zinc-base alloys are more easily handled in the die-casting process than any other alloys and are, therefore, most generally used. The tensile strength of the alloy is less than that of yellow brass, so that castings made from this alloy can be used to replace yellow-brass castings only when the tensile strength required does not exceed from 17,000 to 20,000 pounds per square inch. Zinc base alloys of this strength are obtainable when the metal is properly treated during the process of alloying or casting. Zinc-base castings present about the same wearing surface as cast iron or yellow brass and are used extensively for small electric motor frames, magneto castings, many automobile parts, and castings used in the electrical industries. The most commonly used composition for a zinc base alloy contains about 86 per cent of zinc, from 7 to 10 per cent of tin, from 4 to 7 per cent of copper, and possibly a slight trace of aluminum or bismuth in the metal. A higher percentage of tin will make the alloy very ductile, while a higher percentage of copper will produce greater hardness.

The Tin-base Alloys. — The tin-base alloys are used principally for bearings, and are similar to the so-called babbitt metals. The composition may vary considerably as in the case of babbitt metals some of which contain up to 90 per cent of tin, whereas other cheaper metals in the babbitt group contain a much smaller percentage of tin. In addition to being used for bearings, the tin-base alloys are also employed for parts that must be resistant to acids. The most acid-proof of the tin-base alloys are those which are high in tin, a favored composition being 90 per cent of tin, 4 per cent of copper, and 6 per cent of antimony, the copper and antimony increasing the hardness of the alloy.

Aluminum and Aluminum Alloys. — Pure aluminum can be die-cast on a commercial scale, but this is seldom done, because there is no particular commercial application for such castings. They are too soft for commercial use, and the standard aluminum alloys which contain a high percentage of aluminum alloyed with copper are so far superior in every respect to the pure aluminum that these alloys are nearly always used when a light casting is required having considerable strength. A common aluminum alloy consists of 92 per cent of aluminum and 8 per cent of copper. This alloy, when produced by die-casting, will have a tensile strength of 20,000 pounds per square inch, and is employed whenever a light strong alloy is required. The die-casting process increases the tensile strength of this alloy about 25 per cent. When gears are die-cast, this alloy is generally used, as it produces a good wearing surface.

Another alloy that is quite frequently used for automobile parts is a magnesium-aluminum alloy containing from 2 to 5 per cent of magnesium and from 3 to 7 per cent of copper, with the remainder aluminum. With proper treatment in melting and casting, it is possible to obtain a tensile strength of from 32,000 to 35,000 pounds per square inch with this alloy which is especially suitable for intricate castings, because it flows freely and will fill all cavities in the mold perfectly. Nickel is sometimes added to this alloy in quantities of from 1 to 3 per cent. The effect of the nickel is to produce greater hardness than is obtained by copper alone, and in addition, the casting will have a much brighter finish.

Brass and Bronze Die-castings. — Prior to 1913 all die-castings produced commercially were composed of either tin-, lead-, or zinc-base alloys, the last being used for about 90 per cent of the output. As the result of improvements, the die-casting of brass and bronze is now done on a commercial basis. The difficulties encountered when die-casting brass arise from the relatively high melting temperatures, and the excessive expansion and contraction of the casting and the resulting deterioration of the die which should produce at least from 10,000 to 12,000 castings in order to pay for itself. Very small holes cannot be cored or produced directly in a brass casting, but must usually be drilled afterward. The reason for this is that the pins of small diameter which are necessary to produce these small holes, tend to warp when subjected to the high temperatures which are required for keeping brass and bronze in a molten condition. The advantage of die-cast brass and bronze pieces over those cast in sand is that the surface of the casting is harder than a sand casting would be.

Air Pressures for Die-casting Machines. — The machines of the pneumatic type require air pressures for forcing the metal into the molds, ranging from 100 to 1500 pounds per square inch, according to the kind of metal being cast and the size and shape of the casting. The more intricate the shape of the casting being made, the higher must be the pressure, so that all the parts of the mold may be properly filled. The size and section thus determine entirely the pressure required. The lowest pressure is used on heavy large castings. On ordinary white-metal and aluminum castings this pressure varies from 100 to 500 pounds per square inch, with 300 pounds per square inch as an average for small castings. Bearing castings are made with high pressures in order to obtain a firm solid casting, although proper location of the gates and vents is also essential. The pressure used when casting bearings may vary from 500 to 1500 pounds per square inch. Brass and bronze are cast under comparatively low pressures, not generally exceeding 80 pounds per square inch.

Dies for Die-casting Machines. — The dies or molds of die-casting machines are generally made of steel although cast-iron and also non-metallic materials of a refractory nature have been used, the latter being intended especially for bronze or brass castings, which, owing to their comparatively high melting temperatures, would injure ordinary steel dies. The steel most generally used is a low-carbon steel. Chromium-vanadium and tungsten steels have been employed when dies must withstand relatively high temperatures.

The making of these dies requires considerable skill and experience. They must be so designed that the metal will rapidly flow to all parts of the impression and at the same time allow the air to escape through very small grooves cut into the parting of the die, as otherwise blow-holes or air-pockets will result. In order to secure solid castings, the gates and vents must be located with reference to the particular shape to be cast. Shrinkage is another important feature, especially on accurate work. The amount usually varies from 0.002 to 0.007 inch per inch, but to determine the exact shrinkage allowance for an alloy containing three or four metals is difficult except by experiment.

Die-casting Bearing Metals in Place. — Practically all the metals that are suitable for bearings can be conveniently die-cast in place. Automobile connecting-rods are an example of work to which this process has been applied successfully. In this case, after the bearings are cast in place, they are finished by reaming. The best metals for the bearings, and those that also can be die-cast most readily, are the babbitts containing about 85 per cent tin with the remainder copper and antimony. These metals should not contain over 9 per cent copper. The copper constitutes the hardening element in the bearing. A recommended composition for a high-class bearing metal is 85 per cent tin, 10 per cent antimony, and 5 per cent copper. The antimony may vary from 7 to 10 per cent and the copper from 5 to 8 per cent. As bearing metals with so high a percentage of tin are expensive, a number of bearing metals have been developed containing a high percentage of lead. One of these metals contains from 95 to 98 per cent lead which has been treated in the electric furnace. After die-casting, the metal becomes harder upon seasoning a few days. In die-casting bearings, the work is located from the bolt holes which are drilled previous to die-casting. It is important that the bolt holes be accurately drilled with relation to the remainder of the machined surfaces.

FORGE SHOP EQUIPMENT

Blacksmiths' Anvils. — The quality of an anvil can generally be judged by its ring, a good anvil giving out a clear, sharp sound when struck with a hammer. If soft or defective, the sound will be dull. A good anvil so mounted that it gives out a full volume of sound is easier to work upon than one having a dead ring. Anvils ordinarily vary in weight from 150 to 300 pounds. A mistake is often made in selecting anvils that are too light for the service required. A 300-pound anvil is suitable for almost any kind of machine blacksmithing, and, if of this weight or heavier, it will not move around while in use or need to be strapped to its block. The square hole in the face of an anvil for receiving the cutting and forming tools is called the "hardie hole," and the small round hole near it is called the "pritchel hole." Anvils are usually made with a wrought-iron body to which is welded a hardened steel face.

Setting the Anvil. — The height of an anvil should be such that when standing beside it the knuckles of the hands will just reach the top surface or face. A solid oak block set endwise in the ground is often used as a foundation, but a cast-iron mounting block is preferable as it can easily be moved. The casting should have a fairly broad base, and a pocket at the top for receiving the anvil; a flat block of wood is provided to act as a cushion. An anvil should not be strapped rigidly to its foundation, as this checks the vibration which tends to keep the face free from scales, and renders a high-grade wrought-iron anvil little better than one made of cast iron. When a wooden block is used under the anvil, it is necessary to drive in a few spikes to keep the anvil in place, but these should be so placed that they do not bear directly upon or bind against the corners.

Steam Hammer Rating. — The capacity of a steam hammer or its rating is the weight of the ram and its attached parts, such as the piston and rod. The steam pressure behind the piston is not considered as far as the rating is concerned. For example, a 1000-pound hammer has reciprocating parts of that weight. The steam pressures for operating hammers usually vary from 75 to 100 pounds per square inch.

Capacity of Steam Hammers. — The capacity of a steam hammer or the proper size to use for working iron and steel of a given cross-sectional area can be determined approximately by the following rule: Multiply the area of the largest cross-section to be worked by 80, if of steel, or 60, if of iron, and the product will be the required rating of the hammer in pounds. For example, the capacity of a hammer for working steel billets 5 inches square would be determined as follows: $5 \times 5 = 25$; and $25 \times 80 = 2000$, which is the rating of the hammer in pounds. A hammer rated according to this rule is an economical size to use, although it can, of course, be employed for heavier work.

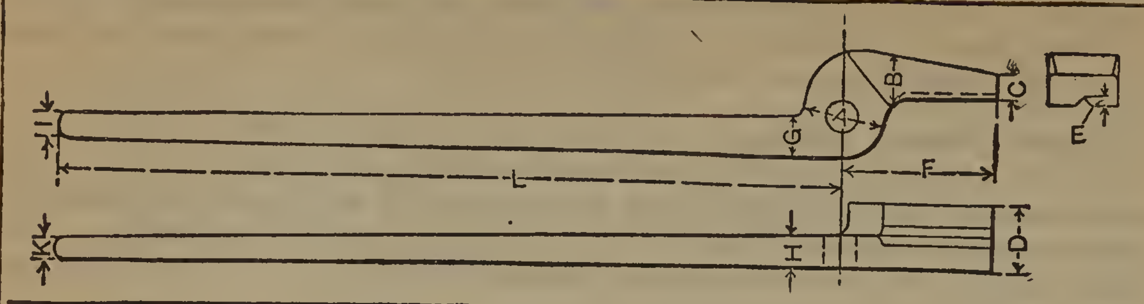
Power for Operating Steam Hammers. — The boiler horsepower for operating a steam hammer depends upon the service required and the number of hammers in use. Ordinarily, the boiler capacity can be less where there are a number of hammers, because all of the hammers are rarely, if ever, used at the same time; consequently, there is a reserve power; but with a single hammer, especially when in constant service, the boiler capacity should be proportionately greater. For average conditions, the boiler horsepower can be determined approximately by the following rule: Divide the rated capacity of the hammer in pounds by 100, and the quotient will be the boiler horsepower required for continuous operation. For example, if the hammer is rated at 2000 pounds, the boiler horsepower would equal $2000 \div 100 = 20$ H.P. This rule is also applicable in cases where the hammer is not used continually, by estimating the amount of idle time and making suitable allowance, but the boiler capacity must not be reduced to such an extent that there is a decided diminution in the pressure during the working period.

For foundations for steam hammers, see section on "Machinery and Hammer Foundations."

Board Drop-hammers. — This type of hammer is generally considered superior to the steam hammer for producing drop-forgings of small and medium size. When the work is heavy and requires a great deal of "breaking down" or drawing, or even when the forgings are light, but have thin sections that cool quickly, thus requiring sharp, rapid blows, the steam hammer will usually give better results than a "board drop." The capacity of most of the board drop-hammers in use varies from 800 to 1500 pounds; the steam hammers found in drop-forging plants usually range from 2000 to 5000 pounds capacity, for handling average work. It does not seem practicable to build board drops larger than 3000 pounds falling weight, and where the forgings are heavy enough to require a capacity over 1500 or 2000 pounds, steam hammers are usually preferred. The latter type is also preferred in some forge shops for all classes of work. It is generally conceded that the cost of operation and repairs is greater for steam hammers, but the latter has a greater output for a given capacity.

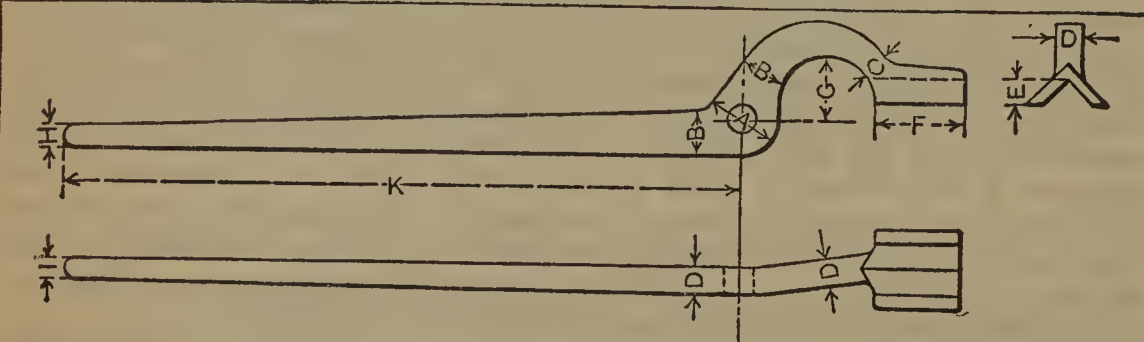
The power required for operating board drop-hammers varies considerably with the nature of the work. Very little power is required at the point of "pick up,"

Dimensions of Flat-jawed Tongs



Capacity, Inches	A	B	C	D	E	F	G	H	I	K	L	Rivet
0–1/8	5/8	3/8	1/4	5/8	1/16	2 1/8	1/2	5/16	5/16	1/4	14	1/4
3/16–5/16	3/4	7/16	5/16	5/8	1/16	2 1/4	9/16	5/16	5/16	1/4	15	1/4
3/8–7/16	7/8	1/2	5/16	3/4	1/16	2 1/2	5/8	3/8	3/8	5/16	16	5/16
1/2–5/8	1	9/16	3/8	7/8	3/32	2 3/4	11/16	7/16	3/8	5/16	18	3/8
3/4–7/8	1 1/8	5/8	3/8	1	5/32	3	3/4	1/2	7/16	3/8	20	7/16
1–1 1/8	1 1/4	11/16	7/16	1 1/8	3/16	3 1/4	13/16	9/16	1/2	7/16	22	1/2
1 1/4–1 3/8	1 3/8	3/4	1/2	1 1/8	1/4	3 1/2	7/8	9/16	1/2	7/16	24	9/16
1 1/2–1 5/8	1 1/2	3/4	1/2	1 1/4	3/8	3 3/4	1	5/8	5/8	1/2	26	5/8
1 3/4–1 7/8	1 5/8	13/16	9/16	1 3/8	7/16	4	1 1/16	11/16	5/8	1/2	28	5/8
2	1 3/4	7/8	5/8	1 1/2	7/16	4 1/4	1 1/8	3/4	11/16	1/2	30	11/16

Dimensions of Goose-neck Tongs



Capacity, Inches	A	B	C	D	E	F	G	H	I	K	Rivet
1/4–5/16	5/8	1/2	7/16	5/16	1/8	1	1/2	5/16	1/4	14	1/4
3/8–7/16	3/4	9/16	1/2	5/16	3/16	1 1/8	9/16	5/16	1/4	16	5/16
1/2–5/8	7/8	5/8	9/16	3/8	1/4	1 1/4	5/8	3/8	5/16	18	3/8
3/4–7/8	1	3/4	5/8	7/16	3/8	1 1/2	3/4	3/8	5/16	20	7/16
1–1 1/8	1 1/8	7/8	11/16	1/2	1/2	1 3/4	7/8	3/8	5/16	20	1/2
1 1/4–1 3/8	1 1/4	1	3/4	9/16	5/8	2	1	7/16	3/8	22	1/2
1 1/2–1 3/4	1 3/8	1 1/8	7/8	5/8	3/4	2 1/8	1 1/8	1/2	3/8	24	9/16
1 7/8–2 1/8	1 3/8	1 3/16	15/16	11/16	1	2 1/4	1 1/4	1/2	3/8	26	5/8
2 1/4–2 1/2	1 1/2	1 1/4	1	3/4	1 1/8	2 1/2	1 1/2	9/16	7/16	28	5/8
2 5/8–2 7/8	1 1/2	1 5/16	1 1/16	3/4	1 1/4	2 3/4	1 3/4	9/16	7/16	30	3/4
3–3 1/4	1 5/8	1 3/8	1 1/8	3/4	1 1/2	3	2	5/8	1/2	32	3/4
3 1/2–3 3/4	1 3/4	1 1/2	1 1/4	3/4	1 3/4	3 1/4	2 1/4	5/8	1/2	34	3/4
4–4 1/4	2	1 5/8	1 5/16	13/16	2	3 1/4	2 1/2	11/16	9/16	36	3/4
4 1/2–4 3/4	2 1/8	1 5/8	1 5/16	13/16	2 1/8	3 1/4	2 3/4	11/16	9/16	38	3/4
5	2 1/4	1 3/4	1 3/8	7/8	2 1/4	3 1/2	3 1/4	3/4	5/8	40	7/8

if the work is practically "die to die;" but when the work is soft and there is no rebound, a great deal more power is required, as the rolls have to pick up a "dead load" from rest and there is little kinetic energy in the driving pulleys. When there is a good rebound, with the knock-off properly timed, the board will be moving upward with considerable velocity when engaged by the rolls, and much less power is required. Seasoned maple boards have proved superior to any other kind for board drop-hammers. Paper fiber has been tried with fair results, but at present the cost of this material is too high.

For foundations for drop-hammers, see section on "Machinery and Hammer Foundations."

Forging Presses. — The power of forging presses for the average line of work is approximately as follows: For mild steel at a fair heat, a pressure of from 3 to 5 tons per square inch on the faces of the tools is generally sufficient, but when swages or dies are used, it may be necessary to double these pressures. For the very hardest steels, the pressure required may be as high as 10 or even 15 tons per square inch, but this is an exceptional case.

Capacity of Forging Presses

Capacity of Press, Tons	Maximum Diam. of Ingots, Inches	Capacity of Press, Tons	Maximum Diam. of Ingots, Inches
300	10	1500	36
500	14	2000	48
800	20	3000	60
1200	27	4000	72

For small forgings, including such parts as can be made from 8-inch square blooms or 12- by 6-inch flats, a press of 300 tons is sufficient, and for larger forgings, such as those used for heavy marine shafts and cranks, a 3000-ton press is generally considered sufficient and can readily handle a 60-inch ingot. The table above indicates, in a general way, the capacity of presses for handling ingots of various diameters.

A press of comparatively small capacity may, with suitable appliances, handle work that is really too heavy for it, but at some sacrifice of speed; for economical operation, there should be ample power. As is generally known, the forging press is superior to the steam hammer for comparatively large forgings, because the hammer tends to spread the surface metal without acting upon the center of the ingot to the required degree. With a press, the forging action goes right to the center of the ingot, as evidenced by the bulging that takes place at the sides, and if there is a cavity in the ingot, forging under the press closes it, whereas a hammer, by spreading the surface metal, may tend to enlarge it. As forgings diminish in size, the difference in favor of the press is less marked. Owing to the recent increase in the operating speed of forging presses, however, they now compete with power hammers in the forging of comparatively light work, and the range of presses has been greatly extended.

Air Pressures and Pipe Sizes for Forges. — Blacksmiths' forges require air pressures varying from 1½ to 6 ounces per square inch. Small forges with the blower close to them are adequately supplied with 1½ ounce pressure. If the blower is some distance away and a long discharge pipe with many bends leads to the forge, even though the latter be small, it may be necessary to carry 3 ounces pressure or more, to overcome the friction in the air ducts. Large forges usually require from 3 to 6 ounces pressure. The table, "Air Pressures and Pipe Sizes for Forges," gives the diameters of discharge mains for various tuyere sizes and numbers of forges.

Air Pressures and Pipe Sizes for Forges*

Diam. Forge Tuyere, Inches	Number of Forges Supplied by Blower									
	1	2	3	4	5	6	7	8	9	10
	Diameter Discharge Main at Blower, Inches									
3/4	1 1/2	1 1/2	2	2	2 1/2	2 1/2	3	3	3	3
1	1 1/2	2	2 1/2	3	3	3 1/2	3 1/2	4	4	4
1 1/4	2	2 1/2	3	3 1/2	4	4	4 1/2	5	5	5
1 1/2	2	3	3 1/2	4	4 1/2	5	6	6	6	6
1 3/4	2 1/2	3 1/2	4	4 1/2	5	6	6	7	7	7
2	3	4	4 1/2	5	6	7	7	8	8	8
2 1/4	3	4	5	6	7	7	8	9	9	9
2 1/2	3 1/2	5	6	7	8	8	9	9	10	10
2 3/4	4	5	6	7	8	9	10	10	11	11
3	4	6	7	8	9	10	11	11	12	12
3 1/2	4 1/2	7	8	9	10	11	12	13	14	14
4	6	8	9	11	12	13	14	15	16	17

* American Blower Co.

The Cold Swaging Process. — Cold swaging is a method of reducing or forming steel or other material while cold, by drawing to a point or reducing the diameter, as may be required. This is performed by a machine that causes the work to be struck a large number of successive blows by a pair of dies shaped to give the required form. This process is principally applied to the reduction of wires, rods and tubes, and is the only method by which rolled or plated stock can be reduced without destroying the plating or coating. For this reason, it is largely employed for jewelers' work. It is also extensively used for pointing rods or tubes which are to be drawn. The process is used in the manufacture of needles, bicycle spokes, button hooks, crochet needles, etc.

Forging Machines. — Some forging machines are intended especially for bolt and rivet heading, and others for more general work. The form or shape into which a part is forged is governed by dies of the required shape and also by a heading tool or plunger which bends or upsets the heated bar of metal and forces it into the die impression. The die may have a single impression, or two or three impressions may be required in order to forge the part by successive operations.

Dies for Bolt and Rivet Forging Machines. — Bolt and rivet dies used in forging machines are, as a rule, made from steel containing from 0.60 to 0.80 per cent carbon and are hardened and drawn. The heading tool, which must be tougher than the dies, is generally made from steel containing from 0.40 to 0.50 per cent carbon, and is drawn considerably more than the forming dies.

Dies and Tools Used in Hot-pressed Center-feed Nut Machines. — The dies used in hot-pressed center-feed nut machines are usually made from chilled iron castings, the dies being ground to size. It is claimed that dies made from this material will last fully eight times as long as those made from ordinary carbon steel, but as it is somewhat difficult to obtain the proper amount of chill, many manufacturers use a good grade of open-hearth crucible steel instead. A crucible steel which is found to give good results contains from 0.90 to 1.10 per cent carbon. In many cases, vanadium alloy steel is used for dies for nut forging machines. The composition of vanadium steel for dies varies. Two grades of vanadium tool steel are recommended for forging machine dies by the American Vanadium Co., of Pittsburgh, Pa. One is composed of carbon, 0.50 per cent; chromium, from 0.80 to 1.10 per cent; manganese, from 0.40 to 0.60 per cent; vanadium, not less than 0.16 per cent; silicon, not more than 0.20 per cent. The heat-treatment recommended for this steel is as follows: Heat to 1550 degrees F. and quench in oil; then re-heat to from 1425 to 1450 degrees F., and quench in water, submerging the face of the die only.

The second kind of vanadium tool steel recommended has the following analysis: Carbon, from 0.65 to 0.75 per cent; manganese, from 0.40 to 0.60 per cent; vanadium, not less than 0.16 per cent; silicon, not more than 0.20 per cent. The heat-treatment for this steel should be as follows: Heat to 1525 degrees F. and quench in water, with only the face of the die submerged. Ordinary carbon tool steel dies should be drawn to a light straw color.

Bulldozer Dies. — Many of the tools or dies used on bulldozers are made of cast iron, in order to reduce the cost, and those parts of the dies which are subjected to wear are faced with hardened steel plates which may readily be replaced, if necessary. Whenever hot punching or cutting is done, high-speed self-hardening steel should be used for the working members of the tool.

Helve Hammers. — Power hammers of the helve type are adapted especially for relatively light forging operations, particularly when a rapid succession of blows is required. Ordinary helve hammers are usually built in sizes ranging from 15 to 200 pounds, this rating being based upon the weight of the hammer head. Some "upright helve" hammers are made in sizes up to 500 pounds.

Vertical Power Hammers. — Vertical power hammers of the crank- and pneumatically-operated types are used for general forging operations, especially on the lighter classes of work. Power hammers of the vertical type usually range in size from 25 pounds up to 500 pounds.

Efficiency of Forging Hammers. — The Heim method for determining the efficiency of forging hammers is based on the results of numerous tests conducted by allowing an ordinary drop-hammer to fall a predetermined distance upon a pure lead cylinder, the height of which is 1.5 times its diameter. The diameters of the cylinders which have been adopted for use in testing various sizes of hammers

Table 1. Dimensions of Lead Plugs Used for Testing Various Sizes of Hammers

Falling Weight of Hammer		Diameter of Lead Cylinder *		Falling Weight of Hammer		Diameter of Lead Cylinder *	
Pounds	Kilograms	Inches	Millimeters	Pounds	Kilograms	Inches	Millimeters
66	30	1.18	30	330	150	2.36	60
110	50	1.38	35	506	230	2.76	70
165	75	1.57	40	770	350	3.15	80
220	100	1.97	50	1100	500	3.54	90

* Height equals 1.5 × diameter.

Table 2. Values of Factors Used in Calculating Power of Hammers

A	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
B	1.01	1.63	2.31	3.08	3.94	4.90	5.97	7.17	8.52	10.03	11.73

(with regard to their falling weight) are given in Table I. The following formula gives the number of foot-pounds of work done by one blow of the hammer:

Work $= 36.75\, D^3\, [8.85\, A + 13.12\, (A^2 + A^4)]$ foot-pounds, where D = diameter of lead cylinder; $A = (H - H_1) \div H$; H = original height of cylinder; H_1 = height of cylinder after being struck by the hammer.

If the expression inside the brackets in the formula is designated by B, the formula may be expressed in the following form:

$$\text{Work} = 36.75\, D^3 B$$

After the lead cylinder has been struck by the hammer, the value of A is calculated and the number of foot-pounds of work developed by the hammer is then obtained by taking the value of B from Table II and substituting in the formula.

Example: — Suppose a 100-kilogram (220-pound) hammer striking 180 blows per minute is allowed to strike a lead cylinder, the original dimensions of which are 50 millimeters (1.97 inch) in diameter by 75 millimeters (2.95 inches) high. After the blow has been struck, the resulting height of the cylinder is 48 millimeters (1.90 inch). From the preceding formula:

$$A = \frac{2.95 - 1.90}{2.95} = 0.35 \qquad B = 4.9, \text{ from Table II}$$

Substituting the values of D and B in the formula for the work done by one blow of the hammer:

$36.75 \times 1.97^3 \times 4.9 = 1376$ foot-pounds = work done by one blow.

$\dfrac{1376 \times 180}{60} = 4128$ foot-pounds = work done per second.

The maximum power required to drive the hammer is 10.3 horsepower. As one horsepower is equivalent to 550 foot-pounds of work per second, the amount of power consumed by the hammer per second is: $10.3 \times 550 = 5665$ foot-pounds. The efficiency of the hammer is found to be:

$$\text{Efficiency} = \frac{\text{useful work}}{\text{power supplied}} = \frac{4128}{5665} = 72 \text{ per cent.}$$

The Heim formula and method of testing may be applied to all types of hammers, but, when used on steam hammers, the test must be made while the hammer is running continuously and not when set to deliver a single blow.

Machinery and Hammer Foundations

The materials commonly used are concrete, stone, brick, and wood in conjunction with concrete for machines subjected to considerable vertical shock. The principal characteristics of these materials are briefly as follows: Concrete is an ideal foundation material, as it becomes practically one solid piece and is much cheaper than a masonry foundation. Stone, in addition to being strong and durable, has great vibration-absorbing power, but is quite costly. Brick is not so durable as stone, but is cheaper and available everywhere. In building a foundation, provision should be made for the foundation bolts, when these are necessary. Sometimes the bolts are set permanently in the foundation, or they may be placed in pipes and have pockets at the lower ends, thus permitting adjustment or removal, if necessary. The bolts are usually located in the proper position by making a wooden templet in which holes are bored to coincide with the holes in the machine base. The inclination of the sides of a foundation should vary from 1½ to 3 inches per foot from the vertical. The foundation pit should be excavated below the frost line of the locality.

Concrete Foundations. — The timber used for making the forms in which concrete foundations are molded should be about 1 inch thick, dressed on the inner side to give a smooth surface. The form should be braced externally about every 2½ feet, and internally about every foot in height. As the form is being filled, the inside braces can be removed. If pocket molds are used for the lower ends of the bolts, they should be soaked in water two or three hours before using, to prevent their swelling and sticking in the concrete. Do not use concrete that has been mixed over twenty minutes. Ram with rammers weighing about 1 pound per square inch of face area, the ramming being continued until water just shows at the

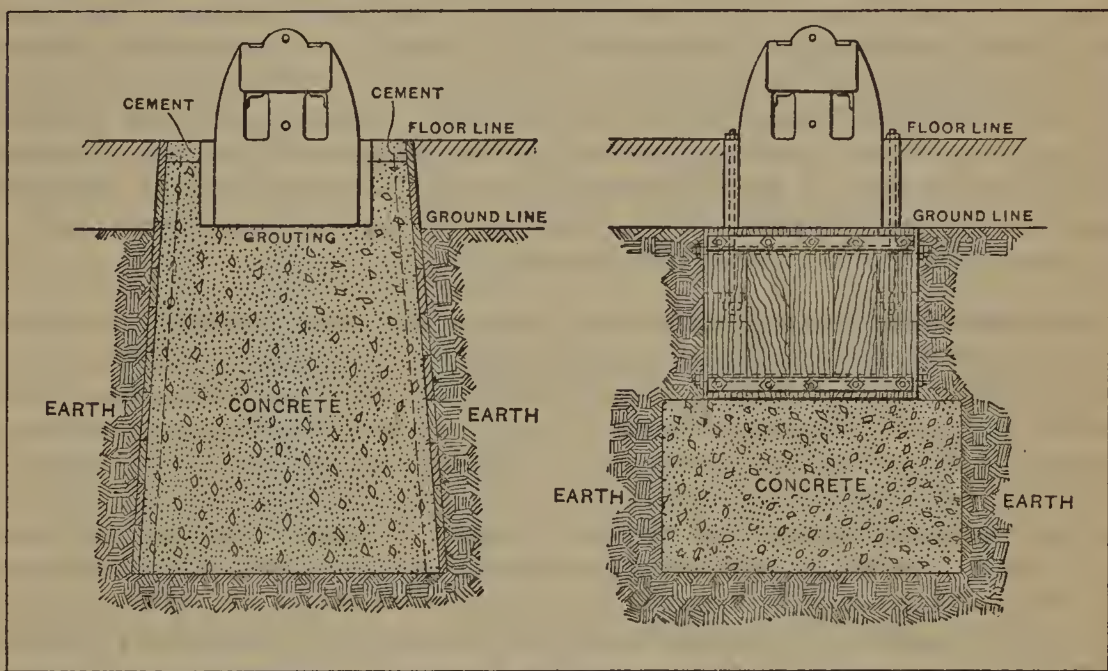


Fig. 1 Fig. 2

surface. Put down the concrete in layers about 6 inches thick and work it onto the form with a shovel, to obtain a smooth, even surface. The foundation may be partly filled with stones about the size of a man's head, placed approximately one foot apart and not less than one foot from the foundation surfaces. These stones should be wet before laying. If the work is stopped at night before completion, make grooves in the surface and when starting the next day, sprinkle and dust over with dry cement. As soon as the concrete has set, remove the form, as it is much easier to patch when the cement is somewhat "green." Foundations are sometimes "slushed" instead of being rammed. In this case, the concrete is mixed just wet enough so that it cannot be piled up. It is then dumped into the molds and worked in them to prevent air bubbles. The first method gives a more homogeneous structure as there is no chance for the broken stones to settle. When the machine is in position, the space around the foundation bolts may be filled with liquid cement, lead or melted sulphur.

Drop-hammer Foundations. — The following drop-hammer foundations are recommended by the E. W. Bliss Company:

Concrete Foundation: Excavate a hole from 10 to 14 feet deep and from 8 to 12 feet square; build up a block of concrete with tapering sides, as shown in Fig. 1, having a top about 6 to 12 inches wider, all around, than the base of the anvil. Place the anvil in position and wedge it level; then run a thin mixture of concrete under the anvil and allow it to set. Next move the wedges and build up a wall of concrete from 4 to 6 inches thick around the anvil. (See illustration.) This will make the use of bolts unnecessary and the anvil will set solid and will not be likely to shift. Solid concrete makes an excellent foundation that does not deteriorate, as is the case with timber when subjected to dampness from the earth or atmospheric moisture. Another advantage is that it is almost impervious to sparks or hot pieces of metal.

Timber-concrete Foundations: Excavate a hole somewhat larger than the anvil or base of the hammer. At the bottom lay a bed of concrete from 1 to 2 feet thick, as shown in Fig. 2. On this concrete bed place, endwise, Georgia pine timbers 12 by 12 inches by 6 to 8 feet long. These should be securely strapped together by steel bands on the outside fastened with through bolts. The timber base should preferably be a little larger than the anvil. To preserve the timbers coat them with oil of tar or creosote. The tops of the timbers should be adzed off evenly to obtain a level surface for the anvil. . Another method of making a foundation of this kind, for small and medium sized hammers, is to put the timbers upon a foot or more of gravel rammed down on a hard-pan bottom. When the timbers, which are also placed endwise and bolted, are in position, the space around the sides is filled with gravel tightly rammed.

Foundation for Steam Hammer. — To secure the greatest efficiency from steam power hammers, the foundations on which they are mounted must be solid. Concrete resting upon hard-pan has given better results than the combination of heavy wooden beams and concrete often used. When making solid concrete foundations, there should be several inches of cement placed over the concrete, and a cushion of wood, at least 3 inches thick, between the cement and base of the anvil, to give the necessary resiliency and prevent the concrete from being pulverized by the impact of the blows. In the front and rear of the hammer there should be openings down to the level of the anvil base, so that it can be leveled or adjusted by wedging and grouting with cement, in case it should sag or get out of alignment with the upper parts of the hammer. These openings can be covered with hatches set level with the floor.

Machine and Forge Shop Floor Materials

Machine shop floors are commonly made of wood or concrete. Probably there is no floor for the machine shop as good as one made of selected hard maple, properly laid and supported, as it wears smoothly and evenly. Concrete, however, has its advantages, the most important of which is its fire-resisting qualities. There are few objections to a wooden floor, and from the standpoint of health it is generally considered superior to concrete. Where there is much heat, or large quantities of moisture or chemicals in bulk, wooden floors should not be used. In certain classes of store-rooms, or where there is a likelihood of considerable moisture, as in wash rooms, concrete floors are considered superior to wood.

Concrete and Wooden Floors. — The following information on shop floors and their materials is abstracted from a paper by Mr. L. C. Wason read before the American Society of Mechanical Engineers. While the factor of cost is apt to be considered first, very often the maintenance and adaptability for the particular service required is of first importance. The initial cost of a granolithic floor surface is at no disadvantage compared with a wooden floor, as the cost of such a surface laid in the best manner is about equal to the cost of seven-eighths maple flooring. In addition, the granolithic surface is fire-proof and will not decay or disintegrate as the result of moisture, which is one of the weak points of the wooden floor. On the other hand, a wooden floor is more easily repaired than a granolithic surface. In making a comparison between wooden and granolithic floors, it is also necessary to consider the workmanship. With a maple top floor, the wearing quality depends comparatively little on the skill of the one who lays the floor, but with a granolithic finish, the work must be done carefully and intelligently. Among the objections to the granolithic surface, one of the most prominent is the bad effect of a concrete floor upon the health and comfort of the workmen. This is not due to the hardness of the floor, but rather to its heat conductivity. When a workman stands for hours on a concrete floor, the heat of the body is conducted to the floor quite rapidly, which tends to disarrange the circulation and cause physical ailments, such as rheumatism, etc. For men working steadily at machines, and usually in one position, this objectionable feature can be overcome by the use of insulating foot-boards or wooden gratings upon which to stand.

The dust produced by the wear of some granolithic surfaces has proved harmful to delicate machinery, whereas a wooden floor does not of itself produce a dust capable of any appreciable abrasive action. It is possible, however, by gluing battleship linoleum to concrete floors, to obtain many of the advantages of a wooden surface. Linoleum is also an effective insulation against the loss of bodily heat.

High resistance to wear and practically complete dustlessness can be secured in a granolithic surface if properly made. To secure a durable and practically dustless floor, proceed as follows: Do not use sand, as sand grains are quickly broken by abrasion and form dust. The granolithic finish should contain the highest possible proportion of tough stone aggregate. Use stone suitable for macadam road, and of a size that will pass through a half-inch round mesh screen, but use nothing smaller than that passed by a 20-mesh screen. Mix the concrete dry, and of a consistency for making blocks, so that considerable tamping will be required to bring enough water to the surface for troweling. Finally, do the troweling before the mortar sets. Prolonged troweling of a wet mixture brings to the top the "laitance" of the concrete, which is the part incapable of a true set. A top layer of laitance is therefore porous and wears down quickly. Even the fine particles of good cement should not be brought to the surface, as they form a layer which is weakly bonded to the rest of the concrete and wears away rapidly, appearing in the air as dust.

To Prevent Dust on Concrete Floors. — The Aberthaw Construction Co. of Boston, contracting engineers specializing in concrete, recommends the following method of curing a dusty concrete floor: Have the surface entirely dry; then paint it with a mixture of boiled linseed oil thinned with gasoline. Apply several coats, until the oil shows glossy on the top. The theory of this is that the linseed oil, having been boiled, has lost most of its volatile components and is practically permanent. The gasoline thins this down enough so that it will strike into the pores. A little experimenting will show the proper proportions. The thinner it is, the more coats will be required and the deeper it will strike in. A floor that is causing serious trouble from dust can often be cured with very little trouble and expense in this way.

Floors for Forge Shops. — There is considerable difference in opinion as to the best material for blacksmith shop flooring. Wood is too inflammable, bricks crack and break from the heat, cement or concrete has the same objectionable features, and asphalt is out of the question. Perhaps nothing is superior to or cheaper than dirt mixed with ashes. If kept moist by sprinkling at least once a day, it is more comfortable to stand upon than the other materials mentioned. It is easily repaired and leveled in case holes are worn in it, and is not affected by dropping heavy or hot pieces upon it. The space between the walls and forges, however, may be covered with concrete to facilitate the handling of such appliances as portable surface-plates and vises.

CEMENT AND CONCRETE

The cements used in concrete construction are classified as (1) Portland cements, (2) natural cements, and (3) Pozzuolanic (or Pozzuolan) cements; the last are also known as slag cements. These different classes are all hydraulic cements as they will set or harden under water. When the powdered cement is mixed with water to a plastic condition, the cement sets or solidifies as the result of chemical action. After the preliminary hardening or initial set, the cement slowly increases in strength, the increase extending over months or years.

Portland Cement. — The Portland and natural cements are the kinds most commonly used. The Portland cement should be used for all structures which must withstand heavy stresses, and for masonry that is either under water, exposed to water, or exposed to the weather. According to the specifications of the American Society for Testing Materials, the specific gravity of Portland cement must be not less than 3.1. If the tested cement is below this requirement, a second test should be made on a sample ignited at a low red heat. The ignited cement should not lose in weight more than four per cent. A satisfactory Portland cement must not develop initial set in less than 30 minutes; it must not develop hard set in less than 1 hour; but the time required for developing hard set must not exceed 10 hours. The minimum requirements for tensile strength in pounds, for briquettes one square inch in cross-section, should be as follows:

For cement 24 hours old in moist air, 175 pounds.

For cement 7 days old, one day in moist air and six days in water, 500 pounds.

For cement 28 days old, one day in moist air and 27 days in water, 600 pounds.

For one part of cement and three parts of standard Ottawa sand, 7 days old, one day in moist air and six days in water, 200 pounds.

For one part of cement and three parts of standard Ottawa sand, 28 days old, one day in moist air and 27 days in water, 275 pounds.

The cements must under no circumstances show a decrease in strength during the time periods specified.

Natural Cement. — Natural cement is used in mortar for ordinary brick work and stone masonry, street sub-pavements, as a backing or filling for massive concrete or stone masonry and for similar purposes. Natural cement does not develop its strength as quickly and is not as uniform in composition as Portland cement. It should not be used for columns, beams, floors or any structural members which must withstand considerable stress. Natural cement is also unsuitable for work that is exposed to water. Foundations which are subjected to moderate compressive stresses may be made of natural cement, which is also satisfactory for massive masonry where weight rather than strength is the essential feature.

The American Society for Testing Materials gives the following specifications for natural cement: An initial set must not develop in less than 10 minutes, and the hard set must not develop in less than 30 minutes, but must develop in less than three hours. The minimum requirements for tensile strength in pounds, for briquettes one inch in cross-section, are as follows:

For natural cement 24 hours old in moist air, 75 pounds.

For natural cement 7 days old, one day in moist air and six days in water, 150 pounds.

For natural cement 28 days old, one day in moist air and 27 days in water, 250 pounds.

For one part of cement and three parts of standard Ottawa sand, 7 days old, one day in moist air and six days in water, 50 pounds.

For one part of cement and three parts of standard Ottawa sand, 28 days old, one day in moist air and 27 days in water, 125 pounds.

Pozzuolanic or Slag Cement. — This cement is adapted for structures which are constantly exposed to fresh or salt water and for drains, sewers, foundation work underground, etc. It is not suitable where masonry is exposed to dry air for long periods. The Pozzuolanic cement sets slowly but its strength increases considerably with age. While this cement is relatively cheap, it is not as strong, uniform, or reliable as Portland and natural cements, and is not used extensively.

Concrete. — The principal ingredients of concrete are the matrix or mortar and the "coarse aggregate." The matrix consists of cement and sand mixed with water, and the coarse aggregate is usually broken stone or gravel. What is known as *rubble concrete* or cyclopean masonry contains large stones which are used for reducing the cost of massive dams and walls. These rubble stones may vary from a few per cent to over one-half the volume. When concrete without much strength but light in weight is required, cinders may be used. This cinder concrete is porous and is used for light floor construction or fire-proofing.

Concrete Mixtures. — In the mixing of concrete, it is desirable to use as little cement as is consistent with the required strength, because the cement is much more expensive than the other ingredients. The proportioning of the ingredients is usually by volume and mixtures are generally designated by giving the amount of each ingredient in a fixed order, as 1 : 2 : 5, the first figure indicating the amount of cement by volume, the second the amount of sand, and the third the amount of broken stone or gravel.

For ordinary machine foundations, retaining walls, bridge abutments, and piers exposed to the air, a 1 : 2½ : 5 concrete is satisfactory; and for ordinary foundations, heavy walls, etc., a lean mixture of 1 : 3 : 6 may be used. For reinforced floors, beams, columns, and arches, as well as for machine foundations which are subjected to vibration, a 1 : 2 : 4 concrete is generally used. This composition is also employed when concrete is used under water. For water tanks and similar structures subjected to considerable pressure and required to be water-tight, mixtures rich in cement and composed of either 1 : 1 : 2 or 1 : 1½ : 3 concrete are used. Portland cement should preferably be used in concrete construction.

Sand, Gravel and Stone for Concrete. — The sand used must be free from dust, loam, vegetable or other organic matter; it should pass, when dry, through a screen with holes of ¼-inch mesh. The gravel should consist of clean pebbles free from foreign matter, and should be of such coarseness that it will not pass through a screen of ¼-inch mesh. Gravel containing loam or clay should be washed by a hose before mixing. The broken stone should be of a hard and durable kind, such as granite or limestone. This stone should pass through a 2½-inch screen.

Amount of Water for Mixing Concrete. — The amount of water required to combine chemically with cement is about 16 per cent, by weight, but in mixing concrete a greater amount than this must be used, because of losses and the difficulty of uniformly distributing the water. In hot weather more water is required than in cool weather on account of the loss by evaporation. The same applies when absorbent sand is used, or when the concrete is not rammed tightly. An excess of water is not desirable, because this excess will flow away and carry some of the cement with it. The water must be free from oils, acids, and impurities that would prevent a proper chemical combination with the cement. It is important to mix the ingredients thoroughly. The cement, sand and stone should be mixed while dry, preferably by using a machine. Enough water should then be added to produce a mixture which will flow readily and fill different parts of the form.

Reinforced Concrete. — Concrete reinforced with steel is widely used, especially where the concrete must resist tensile as well as compressive stresses. This reinforcement may be in the form of round bars, twisted square bars, corrugated bars, expanded metal, steel mesh or wire fabric. The proportions for reinforced concrete structures are usually 1 : 2 : 4, or 1 barrel of Portland cement, 2 barrels of sand and 4 barrels of broken stone or gravel. The lateral spacing between reinforcement bars should be not less than three times the bar diameter from center to center, with a clear space between the bars of at least one inch. The distance from the side of a beam to the center of the nearest bar should be not less than two diameters.

Strength of Concrete. — The strength varies greatly depending upon the quality and proportions of the ingredients and the care in mixing and depositing in the forms. The compressive strength of concrete which, after having been mixed and laid, has set 28 days, varies from 1000 to 3300 pounds per square inch, according to the mixture used. If made in the proportion 1 : 3 : 6, using soft limestone and sandstone, a compressive strength of only 1000 pounds per square inch may be expected, whereas a mixture of 1 : 1 : 2, made with soft limestone and sandstone, will show a strength of 2200 pounds per square inch. A mixture of 1 : 3 : 6, made from granite or trap rock, will have a compressive strength of 1400 pounds per square inch, while a mixture of 1 : 1 : 2, made from granite or trap rock, will have a strength of 3300 pounds per square inch. Other mixtures will have values between those given. The richer in cement in proportion to sand, gravel, and stone, the stronger will be the concrete. The strongest concretes are also obtained by using granite or trap rock. A medium strength is obtained by using gravel, hard limestone, or hard sandstone, whereas the least strength is obtained by using soft limestone or sandstone. Concrete may also be mixed with cinders, but, in this case, very inferior strength is obtained; the richest mixtures will only give a strength of about 800 pounds per square inch.

Durability of Concrete in Sea Water. — Experiments have been made to determine the durability of different mixtures of concrete when exposed to sea water. It has been found that the mixtures that give the best results are those that are richest in cement. Mixtures of 1 : 1 : 2, for example, will give much better results than mixtures of 1 : 3 : 6. Also, very wet mixtures seem to give better results than those that are comparatively dry when deposited. It has also been found that, in order to insure the permanence of Portland cement concrete in sea water, the cement must contain as little lime and alumina as possible and must also be free from sulphates, and the proportion of sand and stones in the concrete must be such that the structure is practically non-porous. Natural cement should never be used for concrete exposed to sea water.

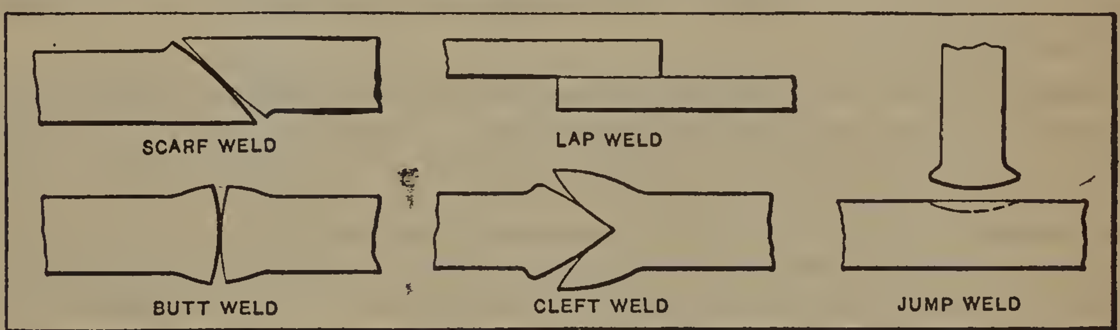
Waterproofing Concrete. — Several formulas for making concrete waterproof have been successfully used but some of them are too expensive for general application. One of the simplest, cheapest, and most effective is that developed by the U. S. Geological Survey. A heavy residual mineral oil of 0.93 specific gravity, mixed with Portland cement, makes it waterproof and does not weaken when the concrete consists of, say, cement, 1 part, sand, 3 parts, and oil, not more than 10 per cent, by weight, of the cement. Concrete mixed with oil requires about fifty per cent longer time to set hard, and the compressive strength is slightly decreased, but not seriously. The bond or grip of oil concrete on steel is much decreased when plain bars are used, but formed bars, wire mesh, or expanded metal act as effectively in it as in ordinary concrete.

Resistance to Acids and Oils. — Concrete of a good quality, that has thoroughly hardened, resists the action of acids and mineral oils as well as other building materials, but vegetable oils containing fatty acids produce injurious effects by combining with the lime in the cement and causing disintegration of the concrete.

WELDING METHODS

Classes of Welds. — Welds are classified according to the way the ends are formed prior to making the weld. The different welds ordinarily made in hand-forging practice are the scarf weld, butt weld, lap weld, cleft or split weld and jump weld. These welds are shown by the accompanying illustration. It will be seen that the surfaces, in most instances, are rounded or crowned. This is done so that when the heated ends are brought together they will unite first in the center. Any slag or dirt which may have adhered to the heated surfaces will then be forced out as the welding proceeds from the center outward. When making a lap weld, the hammering should begin at the center in order to work all the slag out, as the faces in this case are not rounded.

Welding Heat. — When two pieces of wrought iron or mild steel are heated until they become soft and plastic and will stick together when one is pressed or hammered against the other, they have reached what is commonly known as a welding heat. The quality of the weld depends largely upon the welding heat. If the ends to be heated are not hot enough, they will not stick together; inversely, if the work remains in the fire too long, it becomes overheated and burned, which greatly injures the metal. Iron which has been overheated has a rough, spongy appearance and



Different Kinds of Welds

is brittle. The danger of burning is increased when the air blast is too strong and the fire is oxidizing. It is important to heat the work slowly to secure a uniform temperature throughout the ends to be heated. With rapid heating, the outside may be raised to the welding temperature, while the interior is much below it; consequently, the weld will be defective.

Fire for Welding. — When heated iron comes into contact with the air it absorbs oxygen, thus forming a scale or oxide of iron on the surface, which prevents the formation of a good weld. A fire for heating parts to be welded should have a fairly thick bed between the tuyere and the work, so that the oxygen in the air blast will be consumed before it reaches the parts being heated. When there is only a thin bed of fuel beneath the work, or if too strong a blast is used, the excess of oxygen will pass through and oxidize the iron. The hotter the iron, the greater the formation of scale. The surface being heated can be given an additional protection by covering it with some substance that will exclude the air. (See "Fluxes for Welding.") Ordinarily, the air blast for a forge fire should have a pressure varying from 3 to 5 ounces per square inch. (See "Air Pressures and Pipe Sizes for Forges.")

Fluxes for Welding. — When iron is being heated preparatory to welding, the heated surfaces are oxidized to some extent or covered with oxide of iron, which forms a black scale when the hot iron comes into contact with the air. If this scale is not removed, it will cause a defective weld. Wrought iron can be heated to a high enough temperature to melt this oxide so that the latter is forced out from between the surfaces by the hammer blows; but when welding machine steel, and especially tool steel, a temperature high enough to melt the oxide would burn the steel, and it is necessary to use what is called a flux. This is a substance, such as sand or borax, having a melting temperature below the welding temperature of the work, and it is sprinkled upon the heated ends when they have reached about a yellow heat. The flux serves two purposes: It melts and covers the heated surfaces, thus protecting them from oxidation, and, when molten, aids in dissolving any oxide that may have formed, the oxide melting at a lower temperature when combined with the flux. Wrought iron can be welded in a clean, well-kept fire without using a flux of any kind, except when the material is very thin. The fluxes commonly used are fine clean sand and borax. When borax is used, it will give better results if burned. This can be done by heating it in a crucible until reduced to the liquid state. It should then be poured onto a flat surface to form a sheet; when cold, it can easily be broken up and pulverized. The borax powder can be used plain or it can be mixed with an equal quantity of fine clean sand and about 25 per cent iron (not steel) filings. For tool steel, a flux made of one part sal-ammoniac and twelve parts borax is recommended. When pieces are put together previous to welding, as in split welds, or when taking a second heat (usually termed a "wash"), a flux that will flow easily should be used. There are many welding compounds on the market, some of which are suited for one class of welding and some for another.

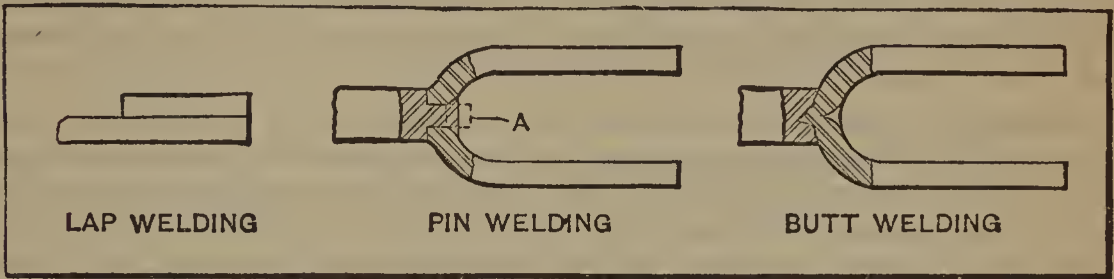
Fuels for Forge. — Coke, coal, charcoal, oil and gas are used as fuels for heating iron and steel preparatory to forging or welding. For general work, a coke fire is the best, although bituminous coal is extensively used. With anthracite coal, it is difficult to get a hot enough fire, especially on a small forge. Coke or bituminous coal should be low in sulphur, because sulphur makes the iron "hot short" or brittle while hot. Sulphur, lead, bronze or brass must not be in the fuel or fire to be used for heating iron or steel. A weld may be spoiled by throwing brass filings into a fire before heating the work.

Machine Welding. — There are three common types of welds that can be made satisfactorily in a forging machine, simple examples of which are shown in the accompanying illustration.

Lap-welding: This is one of the most successful methods that can be used in joining pieces together in a forging machine, whenever requirements will permit. There are several applications of this type of welding: Two pieces can be joined together (as shown in the illustration) or several pieces can be welded together in one block. Machine lap-welding is also employed for enlarging the diameter of a bar, this being accomplished by welding a U-shaped piece of rectangular stock to the end, and then upsetting the mass into the shape desired. An end plunger is used to upset the bar after the latter is securely held between the opposing faces of the gripping dies.

Pin-welding: In order to make a pin weld, the end of the bar is reduced and inserted in a hole in the part to which it is to be joined (see illustration). The reduced end is usually made from one-quarter to one-half the diameter of the original bar. The U-shaped piece, or other part which is to be joined to it, is generally made thicker where the weld is made, in order to strengthen the weld. The welding operation is effected by a plunger in the ram of the machine, which upsets the "pin" and at the same time forms the joint.

Butt-welding: This method of machine welding is not as common as the other two methods referred to, but is satisfactory when properly applied. To make a butt weld, it is not necessary to prepare the stock beforehand, although the pieces should have practically the desired shape. The weld is effected by a plunger having a pointed end which is forced through the forward member to be joined, thus closely pressing together the material and insuring a solid weld (see illustration). This method of welding is not considered as practicable as pin-welding, but when properly handled, it is satisfactory for many classes of work. Wrought iron is welded in a forging machine without using any flux, but the parts to be joined must be clean and free from scale. As a rule, compressed air is used to remove the



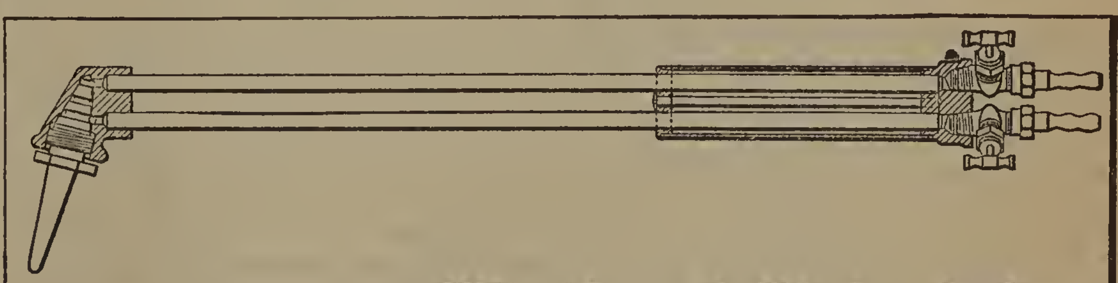
scale formed by oxidization. A small jet of air is directed against the work just before the machine is operated. For welding steel having a comparatively high carbon content, it is necessary to use a flux to make a satisfactory weld. (See "Fluxes for Welding.")

Autogenous or Gas Welding

Autogenous welding is usually defined as a process of uniting metals by fusing them together without compression or hammering and also without using a flux or adding new material to form the joint. When using a high-temperature gas flame, such as is obtained with an oxy-acetylene torch, additional material (in the form of a rod or wire) is ordinarily fused between the parts to be welded and a flux may also be used; hence, the term "autogenous welding" which has been commonly used to designate this method, is not strictly accurate and for this reason the expression "gas welding" is also used to indicate the welding of metals by means of a high-temperature gas flame.

Many different metals can be welded with an oxy-acetylene torch, although in the practical application of this method it is essential to distinguish between classes of work that can be welded on a commercial basis and work which should be done only to meet an emergency and which, under ordinary conditions, might be done more cheaply by other methods.

Welding Torch. — The intense local heat required for welding metal by the gas process is obtained by the combustion of a mixture of gases such as oxygen and



acetylene or oxygen and hydrogen. The arrangement of a Davis-Bournonville oxy-acetylene torch, suitable for medium and heavy welding, is shown by the accompanying illustration. This torch is a simple device, consisting of a handle, two needle valves, a tube for the oxygen and another tube for the acetylene, a head and a tip. The upper tube is connected with the oxygen supply and the lower one with the acetylene supply. The acetylene is admitted under lower pressure than the oxygen, and regulators on the storage tanks serve to control the working pressure of both gases. The mixing of the acetylene and oxygen gases takes place in the head of the torch, where the oxygen, flowing longitudinally, meets the cross currents of the acetylene gas flowing in through the holes in the sides. The diameter of the holes in the tip and the pressures of the respective gases determine the quality of the mixture. The diameters of the holes are graded and the tips are numbered to correspond. These tips are interchangeable, different sizes being required for various classes of work.

Adjusting the Torch. — Before using the welding torch, it is essential to regulate the acetylene and oxygen gas pressures with reference to the thickness of metal to be welded. The accompanying table (compiled by Davis-Bournonville Co.) gives the tip number, the range of metal thicknesses for which it is suited, the acetylene and oxygen pressures and the approximate gas consumption in cubic feet per hour. When using gases from acetylene and oxygen cylinders, it is customary to increase the working pressures slightly above those given in the table, when starting to weld, in order to compensate for the loss of pressure as the gases are used from the cylinders.

The following general instructions for regulating the pressures and adjusting the flame are given by the Davis-Bournonville Co.: First, identify the oxygen and acetylene hose and their respective valves and regulators. Next, open the oxygen needle valve in the torch handle; then, open the oxygen cylinder valve very slowly. Adjust the oxygen gas regulator to obtain a working pressure suitable for the thickness of metal (see table). Close the oxygen needle valve and open the acetylene needle valve. Then, open the acetylene cylinder valve slowly. Adjust the gas regulator to obtain the required working pressure, and light the gas quickly with a "spark lighter."

Obtaining a Neutral Flame. — The oxygen and acetylene must now be adjusted to obtain a *neutral flame*, which will neither oxidize (burn) the metal nor carbonize it. When a white cone appears at the tip and beyond the cone there is a white

Acetylene and Oxygen Pressures for Welding *

Tip No.	Thickness of Metal, Inches	Acetylene Pressure, Pounds	Oxygen Pressure, Pounds	Acetylene † Consumption Per Hour, Cu. Ft.	Oxygen † Consumption Per Hour, Cu. Ft.	Tip No.	Thickness of Metal, Inches	Acetylene Pressure, Pounds	Oxygen Pressure, Pounds	Acetylene † Consumption Per Hour, Cu. Ft.	Oxygen † Consumption Per Hour, Cu. Ft.
00	Very Light	1	1	0.6	0.8	6	5/16–3/8	6	12	25.0	28.5
0	Very Light	1	2	1.0	1.3	7	7/16–1/2	6	14	33.2	37.9
1	1/32–1/16	1	2	3.2	3.7	8	1/2–5/8	6	16	42.0	47.9
2	1/16–3/32	2	4	4.8	5.5	9	5/8–3/4	6	18	58.0	65.9
3	3/32–1/8	3	6	8.1	9.3	10	3/4–up	6	20	82.5	94.0
4	1/8–3/16	4	8	12.5	14.3	11	Extra Heavy	8	22	89.0	101.2
5	1/4–5/16	5	10	17.8	21.3	12	Extra Heavy	8	24	114.5	130.5

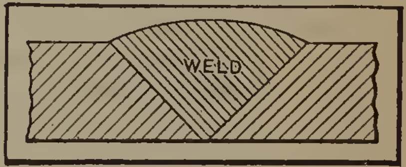
* Davis-Bournonville Style C welding torches with Style 99 and 100 tips.
† Gas consumption per hour is the maximum with torch burning continuously.
Operators frequently adjust the pressure regulators from one to two pounds above the figures given in the table to allow for gage variations and drop of pressure when the gases are supplied in cylinders.

envelope of flame with a long blue streamer flame, the combustion is unbalanced. There is insufficient oxygen and an excess of carbon in the flame. Now, if the oxygen needle valve is opened full, the white-hot cone at the tip will be very short and surrounded by a blue-white envelope. Combustion is now unbalanced because of the excess of oxygen, which combines with all of the acetylene gas available while the remainder tends to attack the metal. Combustion is accompanied by a roaring sound, and the flame is oxidizing. If the oxygen valve is closed slightly until combustion in the flame is balanced, two distinct cones will become visible. The one next to the tip is white hot and beyond is a long blue cone, which is also very hot. The sound differs from that of the carbonizing and oxidizing flames. This is the neutral flame which should be used.

Temperatures. — The temperature of the oxy-hydrogen flame is estimated to be about 4000 degrees F., and the temperature of the oxy-acetylene flame is estimated to be about 5400 degrees F. by some authorities and as high as 6300 degrees F. by others. The exact temperatures are not known. With the oxy-acetylene flame the number of British thermal units per cubic foot is approximately five times greater than is obtained with the oxy-hydrogen flame.

Size of Torch Tip to Use. — The proper size of tip to use for welding depends upon the thickness of metal and the rate at which the heat is dissipated. Sometimes the rate of conduction and radiation is affected by the location of the parts to be welded. The tip sizes given in the accompanying table represent conservative practice, and welders are advised to use as large a tip as possible without overheating the metal, in order to weld faster and use less gas. If the flame is too small for the thickness of the metal to be welded, the heat will be radiated too rapidly.

Preparing Joint for Welding. — When material is only about 1/16 to 1/8 inch thick, it can be welded successfully when the edges are square, but when thicker plates are to be welded, it is necessary to bevel the edges to obtain the best results, as otherwise the heat of the flame suitable for welding will not penetrate and cause perfect fusion beyond a depth of, say, from 1/8 to 3/16 inch; hence, when material is thicker than about 1/8 inch, the edges are beveled and the V-shaped channel thus formed is filled by using adding material. The edges of steel and iron plate should be beveled to an angle of 45 degrees, so that when the plates are placed together, the included angle of the V-shaped channel is about 90 degrees. (See accompanying illustration.) The angle may be smaller for brass and bronze when the work is done by experienced men. If the plates are quite thick, it is preferable to bevel and weld on both sides, the bevel extending into the center of the plate from each side. When it is necessary to weld thick material from one side only, it is advisable to reduce the included angle to about 60 degrees. Aluminum need not be beveled as much as steel, and if the plates are 1/4 inch thick or less, beveling is not necessary when a welding iron or spud is employed to break up the oxide, although experience is required to work successfully in this manner.



Provision for Expansion and Contraction. — As a general rule, 1 or 2 per cent of the length of the weld is allowed for expansion and contraction when welding two plates of steel together, especially if the weld is long. For example, if the joint is 12 inches long, the pieces should be laid so that they are 1/4 inch farther apart at one end than at the end where the welding is begun; thus, there is a tapering space between them. After the parts are "tacked" together and as the welding proceeds, the molten metal cools and contracts. This contraction causes the welded section to act like a hinge, and the unwelded ends draw closer together. It is often difficult to provide for expansion and contraction when welding castings.

Welding Rods or Adding Materials. — Mild steel should be welded with a welding rod or wire that is low in carbon. A welding rod having the composition of pure Swedish iron is preferable. The use of commercial steel wire is not recommended. Cast-iron welding rods should be made from a fine grade of cast iron, high in silicon and low in manganese and sulphur. The best rods are cast in metal molds, which insure density of metal and freedom from blow-holes and sand. Cast aluminum welding rods are used on aluminum castings and the drawn rods or wire are preferable for welding rolled aluminum sheets. Brass and bronze are welded with cast welding rods and drawn bronze rods. Brass wire is not suitable as it contains too much zinc which burns out, leaving the welding material defective. The diameter of the rod or wire should be chosen with reference to the thickness of the joint and the size of the tip and flame. A small wire should not be used for welding a heavy plate.

General Procedure when Welding. — To become proficient in the art of autogenous welding requires experience and practice, but a knowledge of some of the fundamental principles will enable the operator to make more rapid progress. It is advisable to begin by welding thin strips of iron or steel not over 1/8 inch in thickness. Such light metals can be welded without the addition of a filling-in material. The torch should be given a semicircular or zig-zag motion. This movement tends to blend the metal and reduces the liability of overheating. The beveled surfaces of thicker materials are heated by a semicircular movement of the flame, care being taken to melt them to a soft, plastic state without burning the metal. Wherever fusion occurs, new metal should be added from a "welding rod," the composition of which is suitable for the work in hand. The surface should be thoroughly fused before adding metal from the welding stick, and the latter should be held close to, or in contact with, the puddle of molten metal. The heat is then radiated from the puddle to the welding rod, whereas if the metal were allowed to drop through the flame, it might be burned to an injurious extent.

Pre-heating Castings. — As a general rule, it is necesary to pre-heat all cast-iron parts before welding, although some parts of very simple form may be welded without pre-heating. The source of heat for pre-heating depends upon local conditions and the character of work. Oil burners, charcoal fires, or pre-heating stoves using oil or gas are used. A hard-wood charcoal fire is sometimes used in preference to oil blow-torches. All pre-heated parts should be covered with asbestos paper as much as possible during pre-heating and welding, and the welded section should be immediately protected from radiation and rapid cooling after welding.

Welding Cast Iron. — It is highly important to pre-heat all cast iron before welding except possibly very simple parts, as explained in the preceding paragraph. The metal should be covered as soon as the weld is finished and be allowed to cool slowly. If the metal is more than 1/4 inch thick, the edges should be beveled at an angle of about 45 degrees on each side. For comparatively heavy welds, it is well to leave three small points of contact for aligning the broken parts in their original position. To make the weld, the flame should be passed for some distance around the fracture and then be directed on it until the metal is cherry-red. When this occurs, have an assistant throw on a little scaling powder, and when the metal begins to run, add cast iron from the cast-iron "welding stick," which should be of specially refined material. Powder should be added only when the metal does not flow well, as little as possible being used. Never attempt to re-weld pieces that have been previously welded or brazed, without first cutting away all of the old metal.

Welding Steel. — Steel less than 1/8 inch thick can be welded without the addition of any welding metal. If the thickness exceeds 1/8 inch, the edges should be beveled or chamfered. It is very important not to add the welding material until the edges are fused or molten at the place where the weld is being made. In no case should the flame be held at one point until a foam is produced, as this is an indication that the metal is being burned. Do not hold the flame steadily in the center of the weld, but give it a semicircular or zig-zag motion with an uplifting movement at each revolution, the object being to drive the molten metal toward the center of the weld. When welding a crack located in the middle of a heavy steel sheet, begin by chamfering the metal on each side of the fracture at an angle of 45 degrees, the slope extending to the bottom; then apply the welding torch to the sheet beyond the end of the crack, until there is sufficient expansion to open the crack perceptibly. The weld should then be made, and, as a rule, it will be found that the expansion will compensate for the contraction when cooling. No flux is required when welding low-carbon steel, because the oxide melts at a lower temperature than the fusing point of the metal. Flux may be used on the higher carbon steels to prevent oxidation.

Welding Aluminum. — Aluminum that is to be welded should be scraped and cleaned. The oxy-acetylene flame can be reduced or "softened" by using an excess of acetylene to a degree which will be indicated by the extension of the acetylene cone from 1 to 1½ inch beyond the white cone. This excess of acetylene does not injure aluminum but lowers the flame temperature which is desirable when welding aluminum. Before welding this metal, heat the entire piece in a charcoal fire or furnace to about 300 or 400 degrees below the melting point. Then cover it with asbestos or other material (leaving an opening where the weld is to be made), in order to keep the work hot until the weld is completed. When the weld is made it should be covered completely, as a protection against drafts, to insure slow cooling and prevent shrinkage cracks. Many aluminum parts can be welded without pre-heating, such as lugs or projecting pieces broken off completely. When a welding flame is applied to aluminum, it will be noticed that the metal does not run together. A flattened iron rod should be used to puddle the aluminum, enough to break the film of oxide, and this rod should be wiped frequently, so that it will not become coated. The rod should not be allowed to reach a red heat, thus causing oxide of iron to form on it, as this would cause a defective weld. A good aluminum flux will be found advantageous. The aluminum to be added should be in sticks of special composition, obtainable from the makers of welding apparatus. The quality of the welding metal has much to do with the quality of the weld.

Welding Brass. — In many cases it is preferable to braze rather than to weld brass parts. When the oxy-acetylene torch is used for brass, adjust the flame until there is a single cone, as for steel welding. Keep the point of the white flame slightly away from the weld, according to the thickness of the piece, so that the heat will not be sufficient to burn the copper in the brass or volatilize the zinc. If a white smoke appears, remove the flame, as this indicates excessive heat. A little borax should be used as a flux. For brass welding, it is advisable to use a tip about one size larger than for the same thickness of steel. As the weld is really cast brass, it will not have the strength of rolled sheet brass. Do not breathe the fumes while welding brass.

Welding Copper. — The purest copper wire obtainable should be used as a filler when welding copper and great care should be taken to prevent oxidation. Use the same kind of flame as for steel, but a much larger tip for corresponding dimensions, because of the great radiating property of copper. Pre-heating is necessary when a large piece of copper is to be welded, as otherwise so much heat from the torch will be dissipated by radiation that little will be left for fusing the metal. Copper will weld at about 1930 degrees F.; hence, the flame need not have so high a temperature as for steel and it must not be concentrated on so small a surface. On account of the radiation, however, the total quantity of heat must be greater. Welded copper has the strength of cast copper, but can be rendered more tenacious by hammering. The radiation of heat from copper can be considerably lessened by covering it with asbestos sheets while heating. To weld copper to steel, first raise the steel to a white heat (the welding point); then put the copper into contact with it and the two metals will fuse together. When the copper begins to flow, withdraw the flame slightly to prevent burning. Ordinary borax may be used as a flux for copper welding.

Welding High-speed Steel to Machine Steel. — Compression welding can be employed advantageously for welding small high-speed drills which require carbon steel shanks of special length. A jig made to hold the carbon steel shank and the high-speed steel drill in line should be employed, and the ends set about 1/8 inch apart. When the ends have been heated by the torch to the welding temperature, or until the ends are in a pasty condition, the drill is given a smart rap with a wooden mallet and driven against the steel shank, thus producing a butt or compression weld. This method is most successful for lengthening small twist drills with the oxy-acetylene torch.

Welding Cast Iron to Steel. — To weld cast iron to steel, cast-iron rods are used as welding material. The steel must be first heated to the melting point, as cast iron melts at a lower temperature. A very little scaling powder should be used. Pre-heating the cast iron and steel is generally recommended when the joint exceeds three or four inches in length, especially if adjacent parts are thin and likely to crack.

Welding Steel Castings. — As steel castings vary in carbon content, no fixed rule can be laid down for welding them; but, in general, they offer no serious difficulty to welding provided approved welding rods are used. This statement applies to castings whose carbon content is not more than 0.25 per cent. Steel castings containing carbon much in excess of 0.25 or 0.30 per cent may require the use of special welding rods such as those containing nickel. Nickel welding rods give good results on certain grades of steel castings, railway track steel, switch points and other steel products having comparatively high carbon content.

Welding Malleable Iron. — Welding of malleable iron with the oxy-acetylene torch is not recommended by the Davis-Bournonville Co., the reason being that the welded joint is necessarily weaker than the malleable iron due to the changes in physical structure produced by the high temperature. Brazing with an approved bronze rod is recommended. A brazed joint in malleable iron will generally be nearly, if not quite, as strong as the original material. Brazing can be effected at a temperature that causes little change in the malleable characteristics.

Welding Tool Steel. — Welding tool steel of the higher carbon contents, with the oxy-acetylene torch, is not generally recommended. The carbon content is changed and the weld produced may be valueless for tool purposes. However, a skilled welder can produce welds in high-carbon steel under proper conditions with fair success and if properly heat-treated they may give satisfaction under some conditions of use.

Welding Alloy Steels. — The alloy steels such as vanadium, chrome-nickel, etc., used in motor car construction may be welded successfully with adding material containing vanadium or nickel. The welds should be heat-treated afterwards in order to restore the joint portion to the physical condition of the part before welding.

Efficiency of Joints Welded by Oxy-Acetylene Process. — In order to determine the strength of joints in mild steel plates welded by the oxy-acetylene process, a series of tests was made at the Univerity of Illinois. These tests included (1) static load in tension, (2) repeated load (bending) and (3) impact in tension (in a drop-testing machine). The principal object of the tests was to compare the strength of the welded joints with the strength of the original plate. This ratio of strength is defined as the *efficiency of the joint*. The efficiency may be computed by either of two methods. The first method for obtaining the joint efficiency consists in comparing the strength of a test piece containing the welded joint, with the strength of a test piece of equal width cut from the same plate, no allowance being made for the additional thickness of the joint due to the addition of filler material. The second method for obtaining the *efficiency of the joint material* is based on the intensity of stress at the yield point or rupture both for the joint and the plate, as computed from the load and dimensions of the cross-section. The tests were made on joints welded by skilled workmen and in a shop especially fitted for oxy-acetylene welding. For joints made without subsequent treatment after welding, the joint efficiency for static tension was about 100 per cent for plates ½ inch thick or less; the efficiency decreased for thicker plates. For static tension tests the efficiency of the material in the joints may vary from 75 to 85 per cent, depending upon the skill of the operator. For repeated stress tests, the joint efficiency was about 100 per cent for plates ½ inch thick or less. Hammering or drawing the weld while hot, increases the strength. The impact tests showed that oxy-acetylene welded joints are decidedly weaker under shock than is the original material.

Cutting Metals with an Oxidizing Flame

The oxy-hydrogen and oxy-acetylene flames are especially adapted to cutting metals. When iron or steel is heated to a high temperature, it has a great affinity for oxygen and readily combines with it to form different oxides, which causes the metal to be disintegrated and burned with great rapidity. The metal cutting or burning torch operates on this principle. A torch tip is designed to pre-heat the metal, which is then burned or oxidized by a jet of pure oxygen. The kerf or path left by the flame is suggestive of a saw cut when the cutting torch has been properly adjusted and used. The traversing motion of the torch along the work may be controlled either by hand or mechanically.

The Cutting Torch. — The hand-operated gas-cutting torch is similar in its general appearance to the welding torch, but differs in the construction and method of control. The Davis-Bournonville cutting torch has three metal gas tubes, two being for oxygen and the third for acetylene, hydrogen or some other combustible gas. These three tubes are united in the head, where the gases required for the pre-heating flame are mixed the same as in a welding torch. Some cutting torches have a number of pre-heating flame ports surrounding the central oxygen port, so that a pre-heating flame will precede the oxygen regardless of the way in which the torch is moved. This arrangement has been used to advantage in mechanically guided torches. The rate of cutting varies with the thickness of the steel, the size of the tip and the oxygen pressure. The accompanying table shows the relation between the thickness of metal cut and the acetylene and oxygen pressures.

Adjustment and Use of Cutting Torch. — When using the cutting torch for the cutting of steel plate, the pre-heating flame first comes into contact with the edge of the plate and quickly raises it to a white hot temperature, and then the oxygen valve is opened by pulling a trigger on the torch and, as the pure oxygen comes into contact with the heated metal, the latter is burned or oxidized. The following general directions are given for adjusting and operating the Davis-Bournonville Style C cutting torch:

Place a tip on the torch which is suitable for the thickness of metal to be cut (see accompanying table), open the oxygen cylinder valve and adjust the regulator to the working pressure required for the thickness of metal. Next, close the oxygen needle valve in the torch and open the acetylene cylinder valve. Adjust the acetylene regulator to obtain the required working pressure, the acetylene needle valve being open. Ignite the acetylene gas, open the acetylene needle valve and adjust it to obtain a neutral or slightly oxidizing flame. Direct the flame against the edge of the plate until the metal has become white hot and then pull the trigger controlling the oxygen valve. It is important to hold the torch easily and cut as narrow a kerf as possible. The best results are secured by giving the torch a slightly zigzag motion across the cut, the movement being so short as to be barely perceptible.

Metals that can be Cut. — Metals such as wrought iron and steels of comparatively low carbon content can be cut readily with the cutting torch. High carbon steels may be cut successfully if pre-heated to a temperature that depends somewhat on the carbon content. The higher the carbon content, the greater the degree of pre-heating. A black heat is sufficient for ordinary tool steel, but a low red may be required for some of the alloy tool steels. Developments indicate that gray cast iron may eventually be cut with as smooth and narrow a kerf as mild steel, but at the present time cast iron cutting is a combination of cutting and melting. Brass and bronze plates have been cut by interposing them between steel plates.

Cutting Steel Castings. — When cutting steel castings, care should be taken to prevent burning pockets in the metal when the flame strikes a blow-hole. If a blow-hole is penetrated, the molten oxide will splash into the cavity and the flame will be diverted. The presence of the blow-hole is generally indicated by

Acetylene and Oxygen Pressures for Cutting Torches *

Tip No.	Thickness of Metal, Inches	Acetylene Pressure, Pounds	Oxygen Pressure, Pounds	Acetylene † Consumption Per Hour, Cu. Ft.	Oxygen † Consumption Per Hour, Cu. Ft.	Tip No.	Thickness of Metal, Inches	Acetylene Pressure, Pounds	Oxygen Pressure, Pounds	Acetylene † Consumption Per Hour, Cu. Ft.	Oxygen † Consumption Per Hour, Cu. Ft.
1	1/8	3	10	12.2	42	3	2	4	50	19.7	202
1	3/16	3	15	12.2	48	3	3	4	60	19.7	232
1	1/4	3	20	12.2	55	4	3	5	60	30.6	316
1	5/16	3	20	12.2	55	4	4	5	70	30.6	356
2	1/4	3	10	12.2	62	4	5	5	85	30.6	416
2	1/2	3	20	12.2	84	4	6	5	100	30.6	476
2	3/4	3	30	12.2	106	5	6	6	90	30.6	600
2	1	3	35	12.2	116	5	7	6	100	30.6	668
3	1	4	30	19.7	142	5	8	6	125	30.6	838
3	1 1/2	4	40	19.7	172	5	10	8	150	30.6	1008

* Davis-Bournonville Style C cutting torches with Style 12 tips.

† Gas consumption per hour is the maximum with torch burning continuously.

Operators frequently adjust the pressure regulators from one to two pounds above the figures given in the table to allow for gage variations and drop of pressure when the gases are supplied in cylinders.

excessive sparks. The operator should immediately move the torch back along the cut and direct it at an angle so as to strike the metal beneath the blow-hole and burn it away if possible beyond the cavity, when cutting in the normal position may be resumed.

Thickness of Metal that can be Cut. — The maximum thickness of metal that can be cut by these high-temperature flames depends largely upon the gases used and the pressure of the oxygen, which may be as high as 150 pounds per square inch; the thicker the metal the higher the pressure required. When using the oxy-acetylene flame, it might be practicable to cut iron or steel up to 12 or 14 inches in thickness, whereas, the oxy-hydrogen flame has been used to cut steel plates 24 inches thick. The oxy-hydrogen flame will cut thicker material principally because it is longer than the oxy-acetylene flame and can penetrate to the full depth of the cut, thus keeping all the oxide in a molten condition so that it can easily be blown out by the oxygen cutting jet. A mechanically guided torch will cut thick material more satisfactorily than a hand-guided torch, because the flame is directed straight into the cut and does not wabble, as it tends to do when the torch is held by hand. With any flame, the cut is less accurate and the kerf wider, as the thickness of the metal increases. When cutting light material, the kerf might be 1/16 inch wide, whereas, for heavy stock it might be 1/4 or 3/8 inch wide.

Application of Cutting and Welding Torches. — A few of the purposes for which cutting and welding torches are commonly used are as follows: For cutting steel wreckage, steel piling, steel beams in structural work, risers from steel castings, openings through steel plates, etc.; for welding seams, reclaiming cracked castings, filling blowholes in castings, adding metal to worn surfaces to secure the original thickness, welding piping without removal, filling holes that have been incorrectly located, replacing broken gear teeth by welding in new material, sealing riveted seams to secure tight joints without calking, etc.

High- and Low-pressure Torches. — The difference between high- and low-pressure welding and cutting torches, according to the generally accepted meaning of the term, is in the pressure of the acetylene. The first oxy-acetylene torches developed by Fouche were of the high-pressure type, using acetylene dissolved in acetone. Later, he developed a low-pressure torch, working on the injector principle, acetylene being drawn into the carburetor chamber where it mixed with the oxygen. The high pressures originally employed in the first torches could not be safely employed with the acetylene produced in generators, because the safe pressure of acetylene in volume should never exceed from 15 to 20 pounds per square inch, and the pressure is limited to 15 pounds per square inch by the Underwriters' Association in the United States; hence, the medium pressure which is in general use was developed. The proportion of oxygen to acetylene varies somewhat in the different torches. Usually from 1.04 to 1.12 times more oxygen is consumed than acetylene.

Welders and cutters should be provided with goggles or spectacles fitted with approved colored lenses that protect the eye from destructive light rays, flying sparks and globules of molten metal.

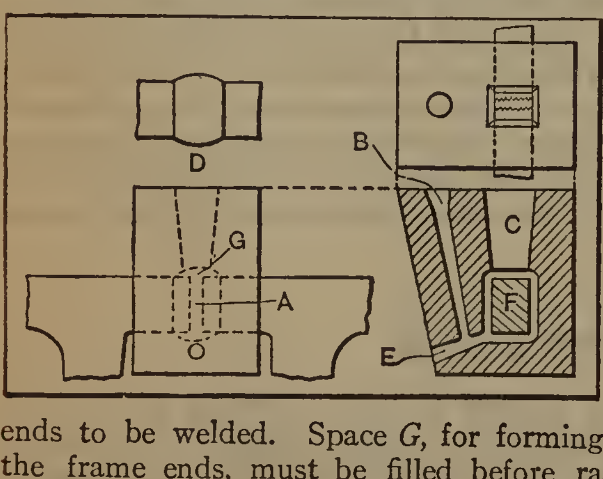
Welding with Thermit

Thermit Process. — This process of welding metals is effected by pouring superheated thermit steel around the parts to be united. Thermit is a mixture of finely divided aluminum and iron oxide. This mixture is placed in a crucible and the steel is produced by igniting the thermit in one spot by means of a special powder, which generates the intense heat necessary to start the chemical reaction. When the reaction is once started, it continues throughout the entire mass, the oxygen of the iron being taken up by the aluminum (which has a strong affinity for it), producing aluminum oxide (or slag) and superheated thermit steel. Ordinarily, the reaction requires from 35 seconds to one minute, depending upon the amount of thermit used. As soon as it ceases, the steel sinks to the bottom of the crucible and is tapped into a mold surrounding the parts to be welded. As the temperature of the steel is about 5400 degrees F., it fuses and amalgamates with the broken sections, thus forming a homogeneous weld.

It is necessary to pre-heat the sections to be welded before pouring, to prevent chilling the steel. The principal steps of the welding operation are, to clean the sections to be welded, remove enough metal at the fracture to provide for a free flow of thermit steel, align the broken members and surround them with a mold to retain the steel, pre-heat by means of a gasoline torch to prevent chilling the steel, ignite the thermit and tap the molten steel into the mold. This process is especially applicable to the welding of large sections. It has been extensively used for welding locomotive frames, broken motor casings, rudder- and stern-posts of ships, crank-shafts, spokes of driving wheels, connecting rods, and heavy repair work in general. One of the great advantages of the thermit process is that broken parts can usually be welded in place. For example, locomotive frames are welded by simply removing parts that would interfere with the application of a suitable mold. Thermit is also used for pipe welding, rail welding, and in foundry practice, to prevent the "piping" of ingots.

Preparation of Part to be Welded. — The first step in the operation of thermit welding is to clean the fractured parts and cut away enough metal to insure an unobstructed flow of the molten thermit. The oxy-acetylene or oxy-hydrogen cutting torch is very efficient for this operation. The amount that should be cut away depends upon the size of the work. Assuming that a locomotive frame is to be welded, the space should be about ¾ inch wide for a small frame, and 1 inch wide for a large frame. The frame sections are then jacked apart about ¼ inch to allow for contraction of the weld when cooling; trammel marks are scribed on each side of the fracture to show the normal length. If the weld is to be made on one member of a double-bar frame, the other parallel member should be heated with a torch to equalize the expansion in both sections and prevent unequal strains.

Mold for Thermit Welding. — The mold surrounding the fractured part should be so arranged that the molten thermit will run through a gate to the lowest part of the mold and rise through and around the parts to be welded into a large riser. The accompanying illustration shows a mold applied to a locomotive frame that is broken between the pedestals at *A*. The thermit steel is poured through gate *B*, and rises into space *C* after passing around and between the ends of frame *F*. The mold must allow for a reinforcing band or collar of thermit steel to be cast around the ends to be welded. Space *G*, for forming this collar, and the opening between the frame ends, must be filled before ramming up the mold. Yellow wax is ordinarily used for this purpose. The shape of this band or collar should be as indicated by the view of the completed weld at *D*. The thickest part is directly over the fracture and the band overlaps the edges of the fracture at least one inch.



For a frame of average size, the collars are made about 4 inches wide and 1 inch thick at the center, the thickness being increased for comparatively large sections. An opening is also made at *E* for pre-heating the ends to be welded.

Patterns for the riser, pouring and heating gates can be made of wood. The riser *C* should be quite large because the steel that first enters the mold is chilled somewhat by coming into contact with the metal, even when pre-heated. This chilling effect is overcome by using enough thermit steel to force the chilled portion up into the riser and replacing it by metal which has practically the full temperature received during reaction. The mold must be made of a refractory material, owing to the intense heat. The best material is made of one part firesand, one part fire-clay and one part ground firebrick, thoroughly mixed while dry and moistened just enough to pack well. If these ingredients cannot be obtained, one part fire-clay and one part clean, dry sand may be used. When the mold and box are filled and tamped, the wooden runner and riser patterns are withdrawn. The mold is then ready for the pre-heating and drying operation which causes the wax matrix to melt and run out.

Thermit Required for Welding. — The quantity of thermit required for making a weld can be determined from the cubic contents of the weld. Calculate the cubic contents of the weld and its reinforcement in cubic inches; double this amount to allow for filling the gate and riser, and multiply by 0.56 to get the number of pounds of thermit required. When wax is used for filling, the weight of the thermit can be determined as follows: Weigh the wax supply before and after filling the fracture. The difference in weight (in pounds), or the quantity used, multiplied by 32 will give the weight of thermit in pounds.

Thermit Additions. — When a quantity of more than 10 pounds of thermit is to be used, add 10 per cent of steel punchings (not over ½ inch in diameter) or steel scrap, free from grease, into the thermit powder. If the thermit exceeds 50 pounds, 15 per cent of small mild steel rivets may be mixed with it. One per cent (by weight) of pure manganese and 1 per cent of nickel-thermit should be added to increase the strength of the thermit steel.

Pre-heating — Making a Weld. — The ends to be welded should be red hot at the moment the thermit steel is tapped into the mold. This pre-heating is done, preferably, by a gasoline, compressed-air torch, and, as previously mentioned, it melts the wax matrix used for filling the fracture to form the pattern for the reinforcing band. When the ends have been heated red, quickly remove the torch and plug the pre-heating hole *E* with a dry sand core, backing it up with a few shovel-fuls of sand, well packed. The end of the cone-shaped crucible should be directly over the pouring gate and not more than 4 inches above it. To start the reaction, place one-half teaspoonful of ignition powder on top of the thermit and ignite with a storm match. It is important that sufficient time be allowed for the completion of the thermit reaction and for fusion of the steel punchings which have been mixed with the thermit. With charges containing from 30 to 50 pounds of thermit, the crucible should not be tapped in less than 35 seconds; with charges containing from 50 to 75 pounds, 40 seconds; 75 to 100 pounds, 50 seconds to one minute.

When welding a frame broken as shown in the illustration previously referred to, the screw jack used for forcing the pedestals apart should be turned back somewhat to release the pressure gradually as the weld cools. After pouring, the mold should remain in place as long as possible (preferably 10 or 12 hours) to anneal the steel in the weld, and, in any case, it should not be disturbed for at least two hours after pouring.

When welding a broken spoke in a driving wheel, or a similar part, it is necessary to pre-heat the adjacent spokes in order to prevent undue strains due to expansion and contraction. If a section of a spoke is broken out, it can be cast in, but if the space is over 6 inches long, it is better to insert a piece of steel and make a weld at each end Owing to the high temperature (5400 degrees F.) and the violent ebullition of thermit during reaction, the crucible must be lined with a very refractory material The crucibles used for this purpose have a sheet-iron shell and are lined with magnesia.

Filling Shrinkage Holes and Surface Flaws. — The filling of surface flaws in castings and forgings usually requires from 2 to 10 pounds of thermit. To make a weld of this kind, place an open mold around the part to be filled, large enough to overlap it about ½ inch. Clean the hole thoroughly and heat to a red heat by means of a strong blow-torch Use eighteen ounces of thermit for each cubic inch of space to be filled, but do not use less than two pounds for any one weld. Place a small amount of thermit in the crucible which, in this case, is of a small size for hand use. Ignite the thermit with ignition powder and as soon as it begins to burn, add the remainder, feeding it fast enough to keep the combustion going. When the reaction is completed, quickly pour the slag (which is about three-fourths of the total liquid) into dry sand; then pour the steel into the open mold and sprinkle loose thermit on top to prolong the reaction, as the casting, even when pre-heated, will have a chilling effect on the steel.

Composition of Thermit Steel. — An average analysis of thermit steel is as follows: Carbon, 0.05 to 0.10 per cent; manganese, 0.08 to 0.10 per cent; silicon, 0.09 to 0.20 per cent; sulphur, 0.03 to 0.04 per cent; phosphorus, 0.04 to 0.05 per cent; aluminum, 0.07 to 0.18 per cent. The tensile strength is about 65,000 pounds per square inch.

Electric Welding

There are two general methods of heating metals to a welding temperature by means of an electric current. One may be defined as electric resistance welding and the other as electric arc welding. These two main systems of electric welding include different processes which vary in regard to certain important details.

Thomson Process. — The Thomson or "incandescent" process of electric resistance welding consists of passing a large volume of electric current at a low voltage or pressure through the contacting surfaces of the metals to be welded. The electric resistance of the metals at the contacting surfaces is so great that they become heated immediately to a welding temperature. Pressure is then applied mechanically and the current turned off, thus producing a weld.

Electric welding machines for this process of welding are built in many different designs for handling various classes of work. They consist principally of a transformer for changing the current to low voltage and high amperage, clamping electrodes for holding the work and transmitting the current through it, mechanism for forcing the electrodes together, and apparatus for controlling the flow of the current and regulating its strength in accordance with the area of section being welded. The range of work that can be welded on different machines is determined by the area of the cross-section of metal at the point of weld and also by the kind of metal. Good conductors of electricity, such as copper, brass, etc., require a much larger flow of current and consequently a larger machine than corresponding sizes of welds in iron or steel.

Current Used. — A single-phase alternating current of any commercial frequency may be used for the resistance process. The voltage is usually 220 or 440 and the frequency, 60 cycles, although higher voltages and lower frequencies can be employed. Where there is a multi-phase circuit, the welding machine can be connected to one phase of the circuit. Direct current is not adapted to resistance welding, because there is no way of reducing the voltage without interposing resistance which wastes power. To obtain the low voltage for welding, a special transformer inside the machine reduces the 220 or 440 volts, as the case may be, down to from 3 to 5 volts.

Different Methods of Resistance Welding. — The general process of electric welding, by passing a current of large volume through the work, is commonly classified according to the way in which the parts are welded together. The names commonly applied to different methods are as follows:

Butt-welding, when pieces having practically the same cross-section are welded end to end; the pieces are held in alignment by suitable clamping electrodes, and the current passes from one part to the other until the metal at the ends (which is the point of greatest resistance) is fused; then the ends are forced together by mechanical means, thus welding them together.

Spot-welding is employed to join sheets of metal by fusing them at various points. One method of making a spot weld is to employ a welder equipped with pointed copper dies or electrodes between which the work is clamped at the point where the weld is to be made.

Point-welding is a special form of spot-welding and consists in raising one or more points or projections above the plane surfaces of the parts to be welded, prior to the welding operation. These points serve to concentrate or localize the current so that the metal is quickly fused; that is, the current divides itself among these points, and as a result of their resistance to its passage each becomes heated to the welding temperature; pressure is then applied by means of properly shaped electrodes, and all the welds are made simultaneously.

Ridge-welding consists in raising continuous ridges above the plane surfaces, which, when crossed by corresponding ridges on the other part to be welded, give the same practical results as the raised points with the advantage of greater ease in assembling the parts.

Tee-welding, as the name implies, is the welding of parts together in the form of a T.

Lap-welding is the forming of a joint by overlapping the two pieces of metal and squeezing or forcing them together while hot, either by passing them between rollers or by means of a press, thus leaving the welded joint practically of the same thickness as the stock. This method is limited in its application and is not as extensively used as the other methods referred to.

Welding Properties of Different Metals. — Various metals and alloys may be welded by the resistance process. Some combinations of different metals that have been successfully welded together by the Thomson electric welding process, are as follows: *Copper* to brass, silver, German silver, or gold; *brass* to iron, tin, mild steel, German silver, or platinum; *iron* to mild steel, tool steel, cast steel, alloy steel, cast brass, nickel, or German silver; *mild steel* to tool steel; *nickel steel* to machine steel; *tin* to zinc, brass or lead; *gold* to silver, German silver or platinum; *steel* to platinum; *silver* to platinum.

Cast iron cannot be commercially welded, as it is high in carbon and silicon and passes suddenly from a crystalline to a fluid condition when raised to a welding temperature. When welding iron and steel, it is necessary to keep the temperature below the melting point to avoid injuring the metal, and consequently considerable pressure is required to make a weld. High-carbon steel can be welded but must be annealed after welding to overcome the strains due to local heating. Good commercial results are difficult to obtain when there is 0.75 per cent carbon or more. Nickel steel welds readily.

Percussive Electric Welding. — Percussive electric welding differs from the Thomson process in that the heating of the parts to be welded is done instantly by the sudden discharge of a heavy electric current from a condenser at the same moment as the two parts are forced together with a rapid blow. Hence, the resistance to the sudden rush of current momentarily melts the portions of the work that are to be joined at the very moment when they are suddenly forced together, so that an intimate joint is formed. The process is applied mainly to the welding of wires of the same or dissimilar metals, and can also be used for the welding of a wire to an object of large dimensions.

Electric Arc Welding Processes. — When welding by the arc process one wire of an electric circuit is connected with the part to be welded and the other is attached to an electrode, so that the current passes across a short air gap between the work and the electrode, thus generating intense heat, the air being a poor conductor and offering a high resistance to the electricity. The heat of the arc is the hottest flame obtainable, the temperature varying between 3500 and 4000 degrees Centigrade (6332–7232 F.) according to estimates. By placing the electrode in contact with the metal and instantly withdrawing it a short distance, the arc is established. As the result of the heat thus produced, the metal may be entirely melted away or fused to another piece of metal, as desired. The electrode is the negative terminal and the metal to be welded is the positive terminal. Electric arc welding processes differ both in regard to the type of electrodes used and the kind of electric current. The Slavianoff and Bernardos processes are the ones in common use.

Slavianoff Process. — The electrode used with this process is usually of the same material as the metal to be welded instead of a carbon electrode being used. One reason for using a metallic electrode is to prevent hard welds which sometimes result with the Bernardos process owing principally to the transfer of carbon from the electrode to the weld. The metal is deposited directly from the electrode so that a separate rod of metal is not required as is the case when using a carbon electrode. The arc is short as compared with the Bernardos process, the length being only about 1/8 or 3/16 inch. (See also "Metallic and Carbon Electrodes.")

Bernardos Process. — The characteristic principle of the Bernardos process is that an electric arc is drawn between the metal to be welded, which forms one electrode for the circuit, and a carbon electrode manipulated by the workman. Experience has demonstrated that the metal to be welded should be the positive electrode, and the carbon, the negative electrode. If this order is reversed, a shorter arc is obtained, and when the flow of current is toward the metal, particles of carbon tend to enter the weld, thus making it hard and liable to crack.

Current for Arc Welding. — Some types of arc welding apparatus require a direct current and other types an alternating current. Opinions differ in regard to the relative merits of the direct and alternating current systems. A motor-generator set is used in connection with the direct current method, the motor being designed to operate on the available power circuit. If the current is alternating, it is transformed to direct by the motor-generator. According to the Westinghouse Electric & Mfg. Co., the voltage across the arc will never grealty exceed 22 volts for bare electrodes and 35 volts for coated electrodes in the case of the metallic electrode process, assuming that the proper length of arc is uniformly maintained on clean work. The arc length, however, will vary more or less owing to the physical impossibility of an operator holding the electrode at a uniform distance from the metal throughout the time required to make a weld. The amount of current depends upon the thickness of the plate to be welded, when its value is, say, 3/4 inch or less. The temperature of the plate and the type of joint also affect the amount of current used. When welding mild steel plates about 1/8 inch thick with direct current, from 50 to 85 amperes would ordinarily be required; whereas, for plates 3/4 inch thick, from 150 to 200 amperes would be needed, the electrode diameter in one case being 3/32 inch and in the other 3/16 inch. The current used with the carbon electrode process is usually between 300 and 450 amperes. For some special applications, 600 to 800 amperes may be required, especially if considerable speed is desired.

It is essential to maintain the proper arc length for the metallic electrode process. The operator should maintain as short and as uniform an arc as possible, and under normal conditions the arc length should be such that the arc voltage is between 18 and 22 volts, and does not exceed 25 volts.

Preparation of Work. — The surfaces to be welded should be free from dirt, grease, rust or other foreign matter. If two plates are to be welded together, the plates being in a common plane, the edge of each plate should be beveled to an angle of 45 degrees, thus forming a 90 degree channel into which the metal is deposited. Cracks in castings are also beveled prior to welding. Although it is not absolutely necessary to pre-heat cast iron before arc welding, this is done in some instances. Pre-heating medium and mild steel is not necessary and will only enable the operator to make a weld with a lower current value but at a sacrifice of welding speed, because the rate of depositing the molten metal is dependent upon the amount of current flowing.

Metallic and Carbon Electrodes. — Metallic electrodes are used for a large percentage of the work welded by the electric arc process. They may be of soft Norwegian or Swedish iron, or low-carbon steel. The mild steel electrodes contain about 0.18 per cent carbon, not over 0.5 per cent manganese and only a trace of phosphorus, sulphur and silicon. Such electrodes have been found generally satisfactory for welding low- and high-carbon as well as alloy steels. They have been used for cast iron and malleable iron, although more dependable results can be obtained by brazing, using a copper-aluminum-iron electrode and a suitable flux. The diameter of the electrode depends upon the current used and the class of work. The diameters ordinarily required are 1/8 inch, 5/32 inch, and 3/16 inch.

A coated electrode has a coating of some kind applied to its surface for the purpose of improving the metal in the weld by totally or partially excluding the atmosphere from the metal while in a molten condition as it passes through the arc and after it has been deposited. By employing such coatings, it has been possible to use special alloy steel electrodes such as manganese steel, nickel steel, vanadium steel and tungsten steel.

Carbon or graphite electrodes are used in welding thick plates or when it is desired to heat over large areas and add considerable metal. The carbon electrode is adapted for filling holes in large castings and for similar work, and it is especially suited to welding cast iron and non-ferrous metals and also for cutting or melting metals. The carbon pencil may vary from 1/4 inch to 1 1/2 inch in diameter.

Metals Welded by Electric Arc Process. — The metals which are best adapted to the arc welding process are wrought iron, low-carbon steels and steel castings. Steel that is high in carbon can be welded only with difficulty. While cast iron can be welded, greater skill is required and pre-heating is often desirable, especially for castings of intricate form. Brass or bronze having less than 3 per cent zinc and also copper can be successfully welded if the work is done carefully, but brass having a high percentage of zinc is liable to have a porous weld, as the zinc volatilizes at comparatively low temperatures. Aluminum is very difficult to weld because it oxidizes readily at a temperature below its melting point, while the fusing point of the oxide is considerably higher.

Strength of Joints Welded by Electric Arc Process. — An extensive series of tests to determine the strength of arc-welded joints in steel plates for ship construction (containing about 0.15 per cent carbon) showed the following results:

The ultimate strength of small welded specimens was over 100 per cent of the strength of the unwelded steel plate for thicknesses of ½ inch and averaged 90 per cent for plates ¾ and 1 inch thick. Up to the point of fracture, the extensions of the welded specimens are not sensibly different from those of similar unwelded material. At stresses greater than the elastic limit, the welded material is less ductile than mild steel and the ultimate elongation of a welded specimen, measured on a length of 8 inches, is only 10 per cent, as compared with 25 or 30 per cent for mild steel. Welded specimens are not capable of being bent without fracture over a given radius for more than 80 degrees with ¼-inch plate, and the bend is reduced to about 20 degrees in the case of 1-inch plate. Unwelded material under the same conditions can be bent to 180 degrees.

Butt welds have a tensile strength of from 90 to 95 per cent in welded plate. Lap welds with fillets on both edges have a tensile strength of from 70 to 80 per cent of the unwelded material. While the tensile strength of large butt welds is about the same as the unwelded material, it was thought that greater reliability of workmanship could be obtained with joints that are either lapped or strapped. It was found that the lap joint is about as strong as the riveted lap joint, and will withstand more trying conditions than the riveted lap joint.

Cutting Metals with Electric Arc. — The carbon electrode is used for such work as the cutting off of risers and sink heads in foundries, for cutting up scrap and similar work, when a smooth cut is not essential. The current value usually

Time Required for Cutting Steel and Cast Iron with Electric Arc *

Steel Plates				Square Cast-iron Blocks		Cast-iron of Circular Cross-section	
Thickness, Inches	Speed, Minutes Per Ft.	Current in Amperes	Kilowatt-Hours Per Ft.	Inches Square	Time, Minutes, Approx.	Diameter, Inches	Time, Minutes, Approx.
⅜	0.50	400	0.312	2	1	2	1
½	1.20	400	0.75	4	4	4	3½
⅝	2.14	400	1.34	6	11	6	7
¾	3.00	400	1.88	8	22	8	12½
1	3.75	600	3.50	10	38	10	23
1⅜	4.32	600	4.10	12	60	12	36
2	6.75	600	6.30	14	82	14	53
4	16.90	600	15.50	16	110	16	72
6	29.00	800	36.20	18	140	18	91
8	40.50	800	50.00	......	....	......	......
10	59.00	800	74.00	......	....	......	......
12	65.00	800	82.00	......	....	......	......

* Westinghouse Electric & Mfg. Co.

ranges from 350 to 800 amperes, depending upon the thickness of metal and the desired speed of cutting. A general idea of the time required for cutting both steel and cast iron may be obtained from the accompanying table, which is based on an extensive series of observations by the Westinghouse Electric & Mfg. Co.

Protection of the Operator when Welding. — Owing to the intense nature of the light and heat rays from the electric arc, the necessity for careful protection of the operator is very important. This is particularly so when using a carbon electrode, as the volume of light and heat is very great. For the protection of the body, ordinary clothing is sufficient; the hands and wrists should be covered with gauntlet gloves. For the protection of the head, the best device is a hood, consisting of a cylinder of micarta fiber, suitably shaped at the bottom so as to rest easily on the shoulders. The support in front for the window holds several thicknesses of glass a few inches from the operator's eyes. Hoods may also be arranged so that the glass window can be pushed up and back, so as to permit an unobstructed view of the work without removing the hood.

The really important part of the hood is the glass window. The arc is rich in ultra-violet rays, and it is against these rays that the eye must be protected. Recent scientific investigations of the subject seem to indicate that the best arrangement is to have three layers of glass, the outside thickness being of ordinary white glass, simply for protection against the heat of the arc. If broken, it can easily be replaced at slight expense. The second layer should be of a greenish or amber tint and should be chosen with regard to its opacity to the ultra-violet rays. The third layer should be of a neutral tint with sufficient density of color to reduce the brilliancy of the light of the arc to a comfortable degree. For short welds, a shield held in the hand may be used, this being constructed of sheet steel or aluminum. The hood is recommended for general use and must be used when filling in material is to be supplied to the weld.

Soldering and Brazing

Solders. — Solders for joining metallic surfaces or edges are almost invariably composed of an alloy of two or more metals. The solder used must have a lower melting point than the metals to be joined by it, but the fusing point should approach, as nearly as possible, that of the metals to be joined so that a more tenacious joint is effected. Solders may be divided into two general classes, hard and soft. The former fuses at a red heat; the latter, at a comparatively low temperature. These solders are also subdivided into a variety of classes such as brass, silver, gold, copper, tin, plumbers' solder and others — the name, in most cases, designating the application.

Soft Solders. — Soft solders consist chiefly of lead and tin, although other metals are occasionally added to lower the melting point. Lead-tin alloys melt at a lower temperature, with an increase in the percentage of tin, up to a certain point, but when the tin exceeds 67 per cent, the melting point rises gradually to the melting point of tin, as shown by the table "Melting Temperatures of Lead-tin Alloys." This table also gives the Brinell hardness test. The results show that the hardest alloy contains 66 per cent of tin and 34 per cent of lead. Soft solders are termed "common," "medium" and "fine," according to the tin content, those containing the most lead being the cheapest and having the highest melting temperatures.

Fine solder is largely used for soldering britannia metal, brass and tin-plate articles. It is also used for soldering cast iron, steel, copper and many alloys. The soft solder called "common" is used by plumbers for ordinary work; this solder contains two parts of lead to one part of tin. The best soft solders are made from pure lead and pure tin. Antimony is an objectionable impurity as it renders the solder less fluid when melted and tends to prevent perfect adhesion of the surfaces. Zinc also has an injurious effect on soft solder, causing it to flow sluggishly. Aluminum acts in a similar way. A small percentage of phosphorus renders soft solder very "lively"; that is, the solder has a tendency to run freely. Too much phosphorus is injurious, and if added to thin the solder it should be in the form of phosphor-tin. One or two ounces of five per cent phosphor-tin to one hundred pounds of solder is generally sufficient.

Melting Temperatures of Lead-tin Alloys

Percentage		Melting Temp., Deg. F.	Brinell Hardness Test	Percentage		Melting Temp., Deg. F.	Brinell Hardness Test
Tin	Lead			Tin	Lead		
0	100	618.8	3.9	60	40	368.6	14.6
10	90	577.4	10.1	66	34	356.0	16.7
20	80	532.4	12.16	70	30	365.0	15.8
30	70	491.0	14.5	80	20	388.4	15.2
40	60	446.0	15.8	90	10	419.0	13.3
50	50	401.0	15.0	100	0	450.0	4.1

Melting Temperatures of Copper-zinc Alloys

Percentage		Melting Temp., Deg. F.	Percentage		Melting Temp., Deg. F.	Percentage		Melting Temp., Deg. F.
Copper	Zinc		Copper	Zinc		Copper	Zinc	
100	0	1980	71	29	1746	41	59	1544
96	4	1967	66.4	33.6	1684	35	65	1501
86	14	1890	63	37	1666	33	67	1477
80	20	1846	60	40	1634	29	71	1467
76	24	1796	50	50	1616	24	76	1364
72	28	1756	48	52	1598	20	80	1301

Hard Soldering and Brazing. — Hard solder is used for joining such metals as copper, silver and gold, and alloys such as brass, German silver, gun metal, etc., which require a strong joint and often a solder the color of which is near that of the metal to be joined. The hard soldering of copper, iron, brass, etc., is generally known as brazing, and the solder as spelter. The operations of hard soldering and brazing are identical, and the two terms are often used interchangeably. According to common usage, however, there is the following distinction. Brazing is generally understood to mean the joining of metals by a film of brass, whereas hard soldering (which is the term used by jewelers) ordinarily means that "silver solder" is used as the uniting medium. (See "Silver Solders.") For hard soldering or brazing, a red heat is necessary, and borax is used as a flux to protect the metal from oxidation, and to dissolve the oxides formed. Heating cannot be done with a soldering iron, but should be effected by a blowpipe, blowtorch, gas forge or a coke or charcoal fire.

As a greater degree of heat is required to melt spelter than soft solder, brazed work will withstand more heat without breaking or weakening than parts which are soldered. The chief advantage of a brazed joint, however, lies in its superior strength. Before work is assembled for brazing, it should be carefully cleaned; the parts are then fastened together in the position they are to occupy when joined. Usually the pieces are secured by pinning, but sometimes wire, bolts or clamps are used for holding the parts together. If practicable, they should be secured in such a way that the work can be turned over during the process of brazing without disturbing the relation of the parts, thus affording a better chance to apply the flux and spelter.

Composition of Brazing Alloys

Percentage				Characteristics	Color
Copper	Zinc	Tin	Lead		
58	42	....	....	Very strong	Reddish-yellow
53	47	....	....	Strong	Reddish-yellow
48	52	....	....	Medium	Reddish-yellow
54.5	43.5	1.5	0.5	Medium	Reddish-yellow
34	66	....	....	Easily fusible	White
44	50	4	2	Easily fusible	Gray
55	26	15	4	White solder	White

Soft and Hard Solders for Various Metals

Metal to be Soldered	Flux	Soft Solder		
		Tin	Lead	Other Constituents*
Aluminum........	Stearin........................	70	....	Zn 25, Al 3, P-tin 2
Brass..........	Chloride of zinc, rosin, or chloride of ammonia..................	66	34	......
Gun metal		63	37	......
Copper.........		60	40	......
Lead............	Tallow or rosin.................	33	67	......
Block tin.........	Chloride of zinc................	99	1	......
Tinned steel......	Chloride of zinc or rosin.........	64	36	......
Galvanized steel...	Hydrochloric acid..............	58	42	......
Zinc............	Hydrochloric acid..............	55	45	......
Pewter...........	Gallipoli oil....................	25	25	Bi 50
Iron and steel.....	Chloride of ammonia............	50	50	......
Gold............	Chloride of zinc................	67	33	......
Silver............	Chloride of zinc................	67	33	......
Bismuth..........	Chloride of zinc................	33	33	Bi 34

* Zn = zinc. Al = aluminum. P-tin = phosphor-tin. Bi = bismuth.

Metal to be Soldered	Flux	Hard Solder			
		Copper	Zinc	Silver	Gold
Brass, soft........	Borax............	22	78	......	......
Brass, hard.......	Borax............	45	55	......	......
Copper...........	Borax............	50	50	......	......
Gold............	Borax............	22	......	11	67
Silver............	Borax............	20	10	70	......
Cast iron.........	Cuprous oxide.....	55	45	......	......
Iron and steel.....	Borax............	64	36	......	......

Fluxes for Soldering. — As two pieces to be soldered must be thoroughly alloyed with the material used as a solder, the temperature must be raised and maintained at such a point that inter-penetration can take place completely. It is necessary that the surfaces to be joined be perfectly clean, and means must be provided to prevent oxidation during soldering, oxides tending to prevent interfusion. This is accomplished by using a coating of some substance that melts at the fusing temperature of the solder, and thus excludes the air. The coating should have a solvent action on the oxide, thus keeping the metal clean and enabling the metal and solder to unite thoroughly. The fluxes generally used are rosin, sal-ammoniac, zinc chloride and borax. The flux is added first and the solder melted by means of a flame or soldering iron, the latter first having been smoothed with a file and then properly tinned. Rosin and chloride of zinc are fluxes commonly used for soft soldering tin (tinned iron) brass, etc. For hard soldering or brazing, use burnt or calcined borax, or boracic acid in powdered form. In all cases where zinc chloride is used as a flux, the article should be cleaned after soldering to prevent subsequent corrosion of the metal.

Alloys for Brazing Solders. — The alloys or "spelters" used for brazing are composed of copper-zinc alloys. The melting point of these alloys depends upon the percentage of zinc. As the proportion of zinc increases, the melting point is lowered, as shown by the table "Melting Temperatures of Copper-zinc Alloys." The fusing point of the spelter should be as close as possible to that of the article to be brazed, as a more tenacious joint is thereby secured. An easily fusible spelter may be made of two parts zinc to one part copper, but the joint will be weaker than when an alloy more difficult to fuse is employed. A spelter that is readily fused may be made of 44 per cent copper, 50 per cent zinc, 4 per cent tin and 2 per cent lead. Alloys containing much lead should be avoided, since lead does not transfuse with brass and thus decreases the strength of the joint. A hard solder for the richer alloys of copper and zinc may be produced from 53 parts copper and 47 parts zinc. Copper and iron have a much higher melting point than brass, thus allowing the use of a richer copper alloy. Tin is often added as one of the ingredients, but it should be sparingly used, because it increases the brittleness of the solder. The addition of tin to brass lightens the color, giving it a gray tint. A variety of different brazing alloys and their characteristics are given in the table "Composition of Brazing Alloys."

Silver Solders. — Silver solder is a hard solder containing silver, copper and zinc or brass. The composition of silver solders varies considerably according to the nature of the work. A silver solder extensively used by jewelers contains 70 parts silver and 30 parts copper. Silver coins can be used for small work. Silver soldering is employed for uniting comparatively small parts requiring a strong joint. The heating is usually effected by a blowpipe, and borax or powdered boracic acid may be used as a flux. The flux should be applied before heating, if possible.

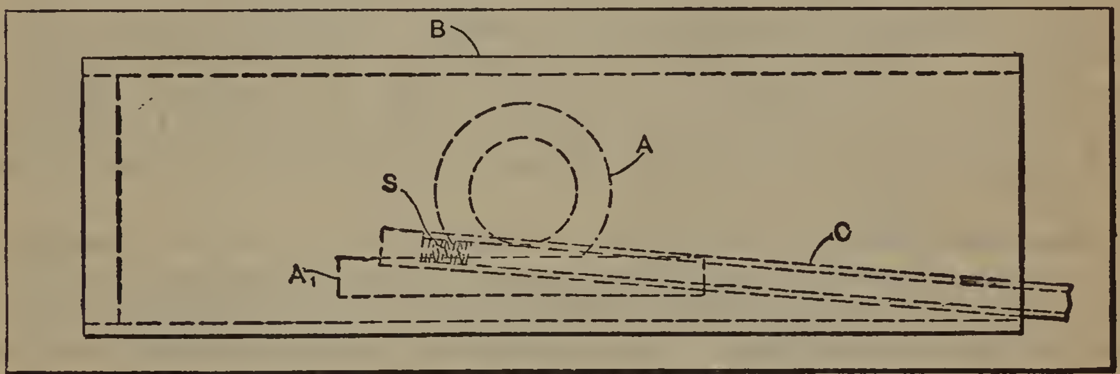
A hard solder of low fusing point, that is used extensively by one of the largest electrical companies, is composed of 34.36 per cent copper; 49.24 per cent silver; and 16.40 per cent zinc. Borax is used as a flux.

Solders for German silver are generally made of the same materials as those which compose the alloy to be soldered, but in such proportions that the melting point is lower. In some cases, silver solder is used for uniting German silver articles, and German silver solder is also used for soldering articles of iron and steel on account of its high melting point and tenacity. German silver solder is known under different names, as argentan, arguzoid, etc. It is rendered moderately fusible by an addition of zinc to the copper and nickel. If the solder is too brittle, this is an indication of too much zinc, which defect can be remedied by adding the requisite amount of copper and nickel. For soldering alloys composed of from 16 to 22 per cent nickel, the following proportions may be used as a solder: Copper 47 per cent; nickel 11 per cent; and zinc 42 per cent.

Dip Brazing. — The principal difference between "dip brazing" and ordinary brazing is that with the former method the work is immersed into the molten spelter until the parts are heated sufficiently to be united by it. This method is extensively employed in bicycle manufacture as it is more economical for duplicate work.

Brazing Band-saws. — Band-saws are joined by lap brazing, silver solder being generally used. The lap is formed by beveling the ends to a sharp edge for a length of about ⅜ inch for saws up to ¾ inch wide, and ½ inch for saws from ¾ to 1½ inch wide. After a flux (such as borax) has been applied to the joint, the ends are clamped together. The solder is sometimes melted into the joint from the edges, or it may be applied between the two surfaces. A convenient method is as follows: Pound a piece of silver, about the size of a dime, until quite thin; then clamp it between the surfaces to be brazed after applying damp, powdered borax. When the heat is applied and the silver melts, clamp the beveled ends of the saw tightly together. The heating should be localized as much as possible. For a flux, use lump borax and pulverize it as used.

Muffle Brazing. — This method of brazing differs from ordinary brazing in that the parts to be united are enclosed in a tube or muffle. This insures uniform heating, clean smooth surfaces, and is especially adapted to brazing alloys, the



melting temperatures of which are rather close to that of the spelter. An example of muffle brazing is shown by the illustration, *A* and A_1 being the parts to be brazed, and *B* the muffler, which is made of wrought-iron pipe closed at one end and having a cover for the opposite end. Parts *A* and A_1 are tied together with iron wire, the spelter is placed around the joint and the latter is wrapped with cloth to retain the spelter. A piece of ⅛-inch gas pipe *C* having a closed end, which is filled for about ½ inch with spelter *S*, is inserted so as to incline slightly, as shown. The inner end is located about opposite the joint to be brazed. The remaining space in tube *B* is filled with charcoal after which the cover, having an opening for pipe *C*, is placed on the end. The muffler is then inserted in the furnace and when the proper degree of heat is reached, the brass filings *S* will melt and run out of the small pipe. A short time is allowed to elapse after this occurs, to insure a good joint; the muffler is then removed and laid aside to cool. The old German locksmiths practiced muffle brazing to a considerable extent, but the mufflers they used were made of common clay.

Cleaning Surfaces to be Brazed. — Before brazing iron or steel, the surfaces should be thoroughly cleaned, either by filing, grinding or by the use of a sand blast. Brass or bronze parts can be cleaned by dipping in a solution of one-third nitric acid and two-thirds sulphuric acid. This same solution can be used to remove the scale after brazing.

Brazing Cast Iron. — First remove all dirt and grease from the pores of the metal by heating to a dull red. The surfaces should then be cleaned either with a sand blast or a wire brush. After cleaning, the parts must be fastened together in the position they are to occupy when brazed. The fastening may be effected by the use of screws, wires, bolts, clamps, etc. If practicable, the parts should be held in such a way that they can be turned over during the process of brazing without disturbing the alignment, in order to facilitate the application of flux and spelter. A good formula for cast-iron brazing is as follows: Boric acid, 16 ounces; chloride of potash (pulverized), 4 ounces; carbonate of iron, 3 ounces. This mixture should be kept dry, as moisture or long exposure to the air renders it less effective. The preparation is mixed with grain spelter immediately before using. Heat the work to a bright yellow color and apply the flux and spelter with an iron rod flattened at the end. After brazing cast iron, let it cool slowly, as sudden chilling may be injurious.

Navy Department Specifications for Brazing Metal. — Scrap must not be used, except such as may result from the process of manufacture of articles of similar composition. The chemical properties are to be as follows: Copper, 84 to 86 per cent; iron (maximum), 0.06 per cent; lead (maximum), 0.3 per cent; zinc, remainder. The color of a fracture section of test pieces, and the grain of the metal, must be uniform throughout. This material is suitable for the following purposes: All flanges for copper pipe and other fittings that are to be brazed.

Navy Department Specifications for Solder. — The solder should consist of practically equal amounts, by weight, of lead and tin, and be made from new tin — Straits, Malacca or Australian — and commercially pure new lead. Any bar selected at random from a delivery of solder must show on analysis: Total tin and lead, not less than 99.8 per cent; tin, between 49 and 51 per cent; antimony, not more than 0.10 per cent; zinc, none.

Aluminum Solder. — A thoroughly reliable aluminum solder that is used by a large electrical concern is composed of 75.5 parts (by weight) of tin; 18 parts of zinc; and 2.5 parts of aluminum. No flux is needed. It is advisable to slightly heat the parts to be soldered. Another aluminum solder is composed of 80 per cent tin and 20 per cent zinc, stearic acid being used as a flux.

Soldering Steel to Cast Iron. — To solder steel to cast iron, first copperize the cast-iron surface with vitriol. It is necessary that the cast-iron surface be thoroughly cleaned and polished before the vitriol is put onto it. After having applied the vitriol, allow the surface to dry thoroughly and then solder the steel to the copperized surface the same as in ordinary soldering.

Sweating. — When parts are soldered together by heating them sufficiently to melt the solder, instead of using a soldering iron, the operation is often known as sweating. Brass boxes for engine connecting-rods are usually sweated together prior to machining, in order to hold the two halves in alignment while finishing the sides and boring. The finished surfaces forming the joint between the brasses are first tinned or covered with solder. This is done by heating the brasses enough to melt the solder, then applying a flux (such as sal-ammoniac), and finally the solder. After tinning, the brasses are again heated if the solder has hardened; they are then put together and allowed to cool. The halves are separated after machining by heating them until the solder melts.

ETCHING AND ETCHING FLUIDS

Etching Fluids for Different Metals. — A common method of etching names or simple designs upon steel is to apply a thin, even coating of beeswax or some similar substance which will resist acid; then mark the required lines or letters in the wax with a sharp-pointed scriber, thus exposing the steel (where the wax has been removed by the scriber point) to the action of an acid, which is finally applied. To apply a very thin coating of beeswax, place the latter in a silk cloth, warm the piece to be etched, and rub the pad over it. Regular coach varnish is also used instead of wax, as a "resist."

An etching fluid ordinarily used for carbon steel consists of nitric acid, 1 part; water, 4 parts. It may be necessary to vary the amount of water, as the exact proportion depends upon the carbon in the steel and whether it is hard or soft. For hard steel, use nitric acid, 2 parts; acetic acid, 1 part. For high-speed steel, nickel or brass, use nitro-hydrochloric acid (nitric, 1 part; hydrochloric, 4 parts). For high-speed steel it is sometimes better to add a little more nitric acid. For etching bronze, use nitric acid, 100 parts; muriatic acid, 5 parts. For brass, nitric acid, 16 parts; water, 160 parts; dissolve 6 parts potassium chlorate in 100 parts of water; then mix the two solutions and apply.

A fluid which may be used either for producing a frosted effect or for deep etching (depending upon the time it is allowed to act) is composed of 1 ounce sulphate of copper (blue vitriol); ¼ ounce alum; ½ teaspoonful of salt; 1 gill of vinegar, and 20 drops of nitric acid. For aluminum, use a solution composed of alcohol, 4 ounces; acetic acid, 6 ounces; antimony chloride, 4 ounces; water, 40 ounces.

The National Twist Drill Co. employs the following method for etching on cutters and other tools: The steel is brushed with asphaltum varnish which is allowed to stand until it thickens and hardens to the right degree; then the desired inscription is pressed through the asphaltum with a rubber stamp and the etching fluid (nitro-hydrochloric acid or *aqua regia*) is applied with a medicine dropper. Practice and experience are required to judge just when the varnish has dried to the right consistency. A similar method, which has been successfully used for etching names on cutlery, is to coat the surface with gum guaiacum varnish. A rubber stamp having the name or design is then coated with a thin layer of potash solution. When this stamp is applied to the work, the varnish is "cut" by the potash wherever the coated stamp comes into contact with it; the surface is then brushed lightly with water to remove the loosened varnish and expose the lettering or design, which is then etched by applying dilute nitric acid. The rubber-stamp method is a very cheap and rapid process. One method of applying the potash is to press the stamp against a pad soaked with the solution. The action of etching fluids on steels varies somewhat according to the composition, high-carbon and alloy steels being acted upon more slowly than low-carbon steel or wrought iron.

Etching Brass Nameplates. — Etched brass nameplates having a black background are now often used in preference to cast plates, as they are less expensive. The etched plate is produced by coating a flat and polished sheet of brass with a thin layer of bichromated albumen, and exposing it to the light for a few minutes under a glass negative upon which are a number of the desired nameplate designs. (In order to prepare the bichromated albumen, mix together 10 parts of the white of egg with 30 parts of water. A second mixture is then made consisting of 2 parts of potassium bichromate and 58 parts of water. The first mixture composed of the white of egg and water, and the second mixture containing potassium bichromate and water, are next mixed together in a dark room. The bichromated albumen thus obtained should be kept and used in the dark.) When the brass plate is developed, this removes the albumen not exposed to the light (or that which has been protected by the black portions of the negative), and leaves the brass free to be etched. The etching solution will not attack the parts protected by the albumen or "resist."

The etching is done by a solution of perchlorate of iron, or by making the plate the anode in an acid-copper solution. When the plate has been etched to the required depth, it is washed. If the etched surface is tarnished, as it usually is after drying, a solution made of 2 parts of water and 1 part of muriatic acid should be spread over the surface to remove the stains and leave it clean and uniform. The plate should then be rinsed, but not dried. Then, without removing the resist, it is treated in some manner to produce a black background. When this has been done, the resist is removed and the sheet is cut up to form the individual name-plates, which are then lacquered.

Producing a Black Background. — The use of a black nickel deposit is the best method of producing a black background on etched brass name-plates. This solution does not affect any of the various kinds of resist used, and a large number of plates can be treated in a tank at one time. The black nickel bath is composed of water, 1 gallon; double-nickel salts, 8 ounces; ammonium sulpho-cyanate, 2 ounces; zinc sulphate, 1 ounce. This solution is used cold, with a weak current of about 1 volt. With a greater voltage, the deposit will be streaked and gray. As soon as the deposit is black, remove the plates, rinse, dry and cut to the desired size; then lacquer immediately in order to prevent the brownish discoloration which will otherwise form on the surface of the deposit. This solution can be used for brass, copper, bronze, etc.

Etching Ornamental Designs in Metal. — When metal plates having an ornamental design are required in small quantities, the etching process is sometimes used. The photographic method which is employed for nearly all intricate designs is as follows: The design is first drawn on white paper to any convenient scale, in black and white. A photographic negative is then made, or this may be procured from photo engravers who make a specialty of such work. The blacks and whites must be, respectively, opaque and transparent. This negative is used to print the design on the work to be etched, the metal, in order to take the design, being coated with a sensitized emulsion of bi-chromated albumen which has the property of remaining insoluble in water after exposure to the light. The portions corresponding to the opaque parts of the negative thus wash out in warm water, leaving the metal bare. Just prior to washing, however, the surface is coated with special lithographic ink, by means of a roller. The design is now on the metal, surrounded by a resist of a bi-chromated albumen base covered with a sticky ink. This resist is further reinforced by sprinkling the surface with dragon's blood. The latter is melted by heating and adheres to the resist, but forms a powder on the unprotected surface which can readily be blown off. This resist is effective, provided the etching is not done too deeply. For brass and copper, a strong solution of perchloride of iron is generally preferred as an etching fluid, as this does not attack the resist like strong acids, although its action is comparatively slow. Nitric acid may be used with proper resists. While etching is usually employed for cutting into the surface of the metal, the same process can be used for perforating the design in the plate.

COLORING METALS

General Requirements in the Coloring of Metal Surfaces. — Copper is more susceptible to coloring processes and materials than any of the other metals, and hence the alloys containing large percentages of copper are readily given various shades of yellow, brown, red, blue, purple and black. Alloys with smaller percentages of copper (or none at all) can be given various colors, but not as easily as if copper were the principal ingredient, and the higher the copper content, the more readily can the alloy be colored. The shades, and even the colors, can be altered by varying the density of the solution, its temperature and the length of time the object is immersed. They can also be altered by finishing the work in different ways. If a cotton buff is used, one shade will be produced; a scratch brush will produce another, etc. Thus to color work the same shade as that of a former lot, all the data in connection with these operations must be preserved so they can be repeated with exactness.

In making the solutions it should be remembered that a strong solution will produce the color quickly, and a weak solution more slowly. When a uniform coating can be produced, a strong solution is the best, owing to the time factor, but the most effective and lasting results are obtained with the weaker solutions; hence they are used for high-grade work. While these solutions are often used cold, there are many cases where better results can be obtained when they are heated.

Cleaning Metals for Coloring. — Metal surfaces to be colored chemically must first be thoroughly cleaned. To remove grease from small parts, dip in benzine, ether or some other solvent for the grease. Boil large pieces in a solution of one part caustic soda and ten parts water. For zinc, tin or britannia metal, do not use caustic soda, but a bath composed of one part carbonate of soda or potash and ten parts water. After boiling, wash in clean water. Do not touch the clean surfaces with the fingers, but handle the objects by the use of tongs or wires.

Pickling Solutions or Dips for Coloring. — The grease removal should be followed by chemical cleansing, which principally serves the purpose of removing the greenish or brownish films which form on copper, brass, bronze, etc. The composition of the bath or mixture for pickling varies for different metals. For copper and its alloys, a mixture of 100 parts concentrated sulphuric acid (66 degrees Baumé) and 75 parts nitric acid (40 degrees Baumé) is sometimes used. If the metal is to be given a luster instead of a mat or dull finish, add about 1 part common salt to 100 parts of the pickling solution, by weight. A better dip for a mat surface consists of 90 parts nitric acid (36 degrees Baumé), 45 parts concentrated sulphuric acid, 1 part salt, and from 1 to 5 parts of sulphate of zinc, by weight. The composition of copper-zinc alloys will produce different color tones in the same dip and will affect the results of chemical coloring. After pickling, washing in water is necessary.

Another good method of removing these films is to soak the work in a pickle composed of spent aquafortis until a black scale is formed, and then dip it for a few minutes into a solution of 64 parts water, 64 parts commercial sulphuric acid, 32 parts aquafortis, and 1 part hydrochloric acid. After that the work should be thoroughly rinsed several times with distilled water. If heating is not injurious, the work can be treated as follows: Heat to a dull red and plunge into dilute sulphuric acid; then soak the work in old aquafortis and rinse it thoroughly. The metal should be soaked until it has a uniform metallic appearance, and the bath should be large enough in volume to prevent an excessive increase in temperature. The best results are obtained with straw-colored aquafortis, as the white is too weak and the red too strong.

Coloring Brass. — Polished brass pieces can be given various shades from golden yellow to orange by immersing them for a certain length of time in a solution composed of 5 parts, by weight, of caustic soda, 50 parts water and 10 parts copper carbonate. When the desired shade is reached, the work must be well washed with water and dried in sawdust. Golden yellow may be produced as follows: Dissolve 100 grains lead acetate in 1 pint of water and add a solution of sodium hydrate until the precipitate which first forms is re-dissolved; then add 300 grains red potassium ferro-cyanide. With the solution at ordinary temperatures, the work will assume a golden yellow, but heating the solution darkens the color, until at 125 degrees F. it has changed to a brown.

To Produce a Rich Gold Color. — Brass can be given a rich gold color by boiling it in a solution composed of 2 parts, by weight, of saltpeter, 1 part common salt, 1 part alum, 24 parts water and 1 part hydrochloric acid. Another method is to apply a mixture of 3 parts alum, 6 parts saltpeter, 3 parts sulphate of zinc, and 3 parts common salt. After applying this mixture the work is heated over a hot plate until it becomes black, after which it is washed with water, rubbed with vinegar, and again washed and dried.

White Colors or Coatings. — The white color or coating that is given to such brass articles as pins hooks and eyes, buttons, etc., can be produced by dipping them in a solution made as follows: Dissolve 2 ounces fine-grain silver in nitric acid, then add 1 gallon distilled water, and put this into a strong solution of sodium chloride. The silver will precipitate in the form of chloride, and must be washed until all traces of the acid are removed. Testing the last rinse water with litmus paper will show when the acid has disappeared; then mix this chloride of silver with an equal amount of potassium bitartrate (cream of tartar), and add enough water to give it the consistency of cream. The work is then immersed in this solution and stirred around until properly coated, after which it is rinsed in hot water and dried in sawdust.

Silvering — A solution for silvering, that is applicable to such work as gage or clock dials, etc., can be made by grinding together in a mortar 1 ounce of very dry chloride of silver, 2 ounces cream of tartar, and 3 ounces common salt, then add enough water to obtain the desired consistency and rub it onto the work with a soft cloth. This will give brass or bronze surfaces a dead-white thin silver coating, but it will tarnish and wear if not given a coat of lacquer. The ordinary silver lacquers that can be applied cold are the best. Before adding the water, the mixture, as it leaves the mortar, can be kept a long time if put in very dark colored bottles, but if left in the light it will decompose.

To Produce Gray Colors. — A solution of 1 ounce of arsenic chloride in 1 pint of water will produce a gray color on brass, but if the work is left in this solution too long it will become black. The brass objects are left in the bath until they have assumed the correct shade, and are then washed in clean warm water, dried in sawdust and finally in warm air.

Blue and Violet Shades. — To give brass a blue color, dissolve 1 ounce of antimony chloride in 20 ounces of water, and add 3 ounces hydrochloric acid; then warm the work and immerse it in this solution until the desired blue is obtained. After that wash in clean water and dry in sawdust. A permanent and beautiful blue-black can be obtained by using just enough water to dissolve 2 ounces copper sulphate and then adding enough ammonia to neutralize and make it slightly alkaline. The work must be heated before immersion.

A beautiful violet color can be produced on polished brass with a mixture of two solutions: First, 4 ounces sodium hyposulphite is dissolved in 1 quart of water; then 1 ounce of sugar of lead is dissolved in another quart of water, and the two are well stirred together. By heating this to 175 degrees F. and immersing the work the correct length of time, a violet color is obtained. The work first turns a golden yellow, and then gradually turns to violet. If left a longer time, the violet will turn to blue and then to green. Thus, this same preparation can be used for all of these colors by correctly limiting the time that the work is immersed.

To Give Brass a Green Tint. — One solution that will produce the verde antique, or rust green, is composed of 3 ounces crystallized chloride of iron, 1 pound ammonium chloride, 8 ounces verdigris, 10 ounces common salt, 4 ounces potassium bitartrate and 1 gallon of water If the objects to be colored are large, the solution can be put on with a brush. Several applications may be required to give the desired depth of color. Small work should be immersed and the length of time it remains in the solution will govern the intensity of the color After immersion, stippling the surface with a soft round brush, dampened with the solution, will give it the variegated appearance of the naturally aged brass or bronze.

Blackening Brass. — There are many different processes and solutions for blackening brass Trioxide of arsenic, white arsenic or arsenious acid are different names for the chemical that is most commonly used. It is the cheapest chemical for producing black on brass, copper, nickel, German silver, etc., but has a tendency to fade, especially if not properly applied, although a coat of lacquer will preserve it a long time. A good black can be produced by immersing the work in a solution composed of 2 ounces white arsenic, 5 ounces cyanide of potassium, and 1 gallon of water. This should be boiled in an enamel or agate vessel, and used hot Another cheap solution is composed of 8 ounces of sugar of lead, 8 ounces hyposulphite of soda and 1 gallon of water. This must also be used hot and the work afterwards lacquered to prevent fading. When immersed, the brass first turns yellow, then blue and then black, the latter being a deposit of sulphide of lead.

Preservation of Color. — After a part has been given the desired color, it is usually washed in water and then dried with clean sawdust. The colored surfaces of alloys are commonly protected and preserved by coating with a colorless lacquer, such as japan lacquer. Small parts are coated by dipping, and large ones by rubbing the lacquer on. The lacquer is hard after drying, and insoluble in most fluids; hence, it can be washed without injury.

Niter Process of Bluing Steel. — The niter process of bluing iron and steel is as follows: The niter or nitrate of potash (often called saltpeter) is melted in an iron pot and heated to about 600 degrees F. The parts to be blued are cleaned and polished and then immersed in the molten niter until a uniform color of the desired shade has been obtained. This requires only a few seconds. The articles are then removed and allowed to cool, after which the adhering niter is washed off in water. Parts which will not warp may be immersed immediately after removing from the niter bath. After cleaning, dry in sawdust, and then apply some suitable oil, such as linseed, to prevent rusting. To secure uniform coloring, a pyrometer should be used to gage the temperature of the niter, because a higher heat than 600 degrees F. will produce a dark color, whereas a lower heat will give a lighter shade.

Bluing Steel by Heat-treatment. — Polished steel parts can be given a blue color by heating in hot sand, wood ashes or pulverized charcoal. Place the substance in an iron receptacle and stir constantly, while heating, in order to heat uniformly. Heat just hot enough to char a pine stick. The parts to be blued must be absolutely free from grease. They are placed in the heated substance until the desired color is obtained. Further coloring is then checked by immersing in oil. Small parts are sometimes heated by a Bunsen burner or by laying upon a heated plate. For a light blue color, heat in sand or wood ashes, and for a dark blue, use pulverized charcoal. The quality of the color depends largely upon the fineness of the finish. Still another method of coloring by heat is to immerse the parts in a molten bath of potassium nitrate and sodium nitrate. The coloring is then checked by plunging the work into boiling water.

Blue-black Finish. — To obtain a blue-black finish on small steel parts, use a mixture of 16 parts, by weight, of saltpeter and 2 parts of black oxide of manganese. This mixture is heated to a temperature of 750 degrees F. and the objects are immersed in it. The oxide of manganese is deposited on the work and must, therefore, be frequently replenished in the mixture.

Black Finish. — To obtain a black rust-protecting finish on hardened parts, temper, after hardening, in "heavy" cylinder oil; then immediately place the part with the oil on it in an oven having a temperature of from 300 to 350 degrees F. Remove the work in from 5 to 8 minutes, when the black finish is baked onto it.

Gun Metal Finish. — Several different chemical solutions have been used successfully for giving steel a gun metal finish or black color. Among these are the following: 1. Bismuth chloride, one part; copper chloride, one part; mercury chloride, two parts; hydrochloric acid, six parts; and water, fifty parts. 2. Ferric chloride, one part; alcohol, eight parts; and water, eight parts. 3. Copper sulphate, two parts; hydrochloric acid, three parts; nitric acid, seven parts; and perchloride of iron, eighty-eight parts. Other solutions have been prepared from nitric ether, nitric acid, copper sulphate, iron chloride, alcohol and water and from nitric acid, copper sulphate, iron chloride and water. The method of applying these and finishing the work is practically the same in all cases.

The surface is given a very thin coating with a soft brush or sponge that has been well squeezed, and is then allowed to dry. The work is then put in a closed retort to which steam is admitted and maintained at a temperature of about 100 degrees F., until the parts are covered with a slight rust. They are then boiled in clean water for about fifteen minutes and allowed to dry. A coating of black oxide will cover the surface, and this is scratch brushed. After brushing, the surface will show a grayish black. By repeating the sponging, steaming and brushing operations several times, a shiny black lasting surface will be obtained. For the best finishes, these operations are repeated as many as eight times.

Another process employs a solution of mercury chloride and ammonium chloride which is applied to the work three times and dried each time. A solution of copper sulphate, ferric chloride, nitric acid, alcohol and water is then applied three times and dried as before. A third solution of ferrous chloride, nitric acid and water is applied three times, and the work is boiled in clean water and dried each time. Finally, a solution of potassium chloride is applied and the work boiled and dried three times. The work is then scratch brushed and given a thin coating of oil. Ordnance for the French Government is treated in this way. The above methods are applicable to hardened and tempered steels, as 100 degrees F. does not affect the required temperature of the hardness. For steels that will stand 600 degrees temperature without losing the desired hardness, better and much cheaper methods have been devised.

The American Gas Furnace Co. has developed a process employing a furnace with a revolving retort. The work is charged in this, together with well-burnt bone. A chemical solution that gasifies when it enters the furnace is then injected into this retort while the work is heated to the proper temperature. This solution has been named "Carbonia." The color does not form a coating on the outside, as with the other processes, but a thin layer of the metal itself is turned to the proper color. By varying the temperature of the furnace, the time the work is in it, and the chemical, different colors can be produced from light straw to brown, blue, purple and black, or gun metal finish. Rough or sand-blasted surfaces will have a frosted appearance, while smooth polished surfaces will have a shiny brilliant appearance.

Browning Iron and Steel. — A good brown color can be obtained as follows: Coat the steel with ammonia and dry it in a warm place; then coat it with muriatic or nitric acid and dry it in a warm place; then place the steel in a solution of tannin or gallic acid and again dry it. The color can be deepened by placing the work near the fire, but it should be withdrawn the minute the desired shade is reached or it will turn black. The U. S. Government adopted the following formula for browning gun barrels: Alcohol, three ounces; tincture of iron, three ounces; corrosive sublimate, three ounces; sweet spirits of niter, three ounces; blue vitriol, two ounces; nitric acid, one and a half ounce; and warm water, two quarts. The solution is applied with a sponge, allowed to dry for twenty-four hours, and then the loose rust is removed by scratch brushing. A second coat is given in the same manner, after which the work is boiled in water and dried quickly. A thin coat of boiled linseed oil or lacquer is then put on to preserve the color.

To Produce a Bronze Color. — A bronze-like color can be produced by exposing iron or steel parts to the vapors of heated *aqua regia*, dipping them in melted vaseline, and then heating them until the vaseline begins to decompose, when it is wiped off with a soft cloth. Another method of producing this bronze-brown color is to slightly heat the work, evenly cover the surfaces with a paste of antimony chloride (known as "bronzing salt"), and let the object stand until the desired color is obtained. The paste can be made more active by adding a little nitric acid.

To Produce a Gray Color. — A gray color on steel can be obtained by immersing the work in a heated solution of ten grains of antimony chloride, ten grains of gallic acid, 400 grains of ferric chloride and five fluid ounces of water. The first color to appear is pale blue, and this passes through the darker blues to the purple, and, finally, to the gray. If immersed long enough, the metal will assume the gray color, but any of the intermediate colors may be produced. When used cold, this is also one of the bronzing solutions.

Mottled Coloring. — Mottled colors on steel can be produced by heating the objects to a good cherry-red for several minutes in cyanide of potassium, then pouring the cyanide off, and placing the receptacle containing the work back on the fire for five minutes. The contents are then quickly dumped into clean water. To heighten the colors, boil afterward in water and oil.

Another method for obtaining a fine mottled effect on steel is to first highly polish the object to be treated, after which it is very carefully cleaned from all grease in a hot soda solution. The object is then heated to a temperature of from 150 to 200 degrees by placing it, say, on the hot firebrick covering on the top of a furnace, and then putting it in a pot of heated cyanide of potassium and bringing it to a dark red heat. It is then dipped into clear water and vigorously moved about in the bath. Unless the work is moved about in the water, the mottled effect will not be obtained.

Coppering Solution. — A coppering solution for coating finished surfaces in order that lay-out lines may be more easily seen, is composed of the following ingredients: To 4 ounces of distilled water (or rain water) add all the copper sulphate (blue vitriol) it will dissolve; then add 10 drops of sulphuric acid. Test by applying to a piece of steel, and, if necessary, add four or five drops of acid. The surface to be coppered should be polished and free from grease. Apply the solution with clean waste, and, if a bright copper coating is not obtained, add a few more drops of the solution; then scour the surface with fine emery cloth, and apply rapidly a small quantity of fresh solution.

White Coatings for Laying Out Lines. — Powdered chalk or whiting mixed with alcohol is commonly used for coating finished metal surfaces preparatory to laying out lines for machining operations. Alcohol is preferable to water, because it will dry quicker and does not tend to rust the surface. This mixture can be applied with a brush and is more convenient than a coppering solution for general work. For many purposes, the surface can be coated satisfactorily by simply rubbing dry chalk over it.

MOTOR POWER FOR MACHINE TOOLS AND FORGING MACHINERY

The following tables of horsepower required for different types of machine tools and forging machinery are based upon information collected by the Westinghouse Electric & Mfg. Co., and embody average practice. The horsepower given may be decreased for very light work and must often be increased for heavy work. The type of motor to be used in each case is indicated by symbols *A*, *B*, *C*, etc. The meaning of these symbols is as follows:

A. Adjustable speed, shunt-wound, direct-current motor, wherever a number of speeds are essential.

B. Constant speed, shunt-wound, direct-current motor, when the required speeds are obtainable by a gear-box or cone pulley arrangement, or when only one speed is required.

C. Squirrel-cage induction motor, when direct current is not available; a gear-box or cone pulley arrangement must be used to obtain different speeds.

D. Constant speed, compound-wound, direct-current motor, when speeds are obtainable by a gear-box or cone pulley arrangement, or when only one speed is required.

E. Wound secondary or squirrel-cage induction motor with approximately 10 per cent slip, when direct current is not available.

F. Adjustable speed, compound-wound, direct-current motor.

G. Standard bending roll motor.

H. Standard machine tool traverse motor.

Motor Power for Machine Tools and Forging Machinery

Planers				Engine Lathes		
Reversing Motor, Type *A*			Non-reversing Motor, Type *C*, *D* or *F*	Type of Motor: *A*, *B* or *C*		
Planer Size, Inches	Horse-power	H.P., Cross-rail Motors	Horse-power	Swing of Lathe, Inches	Service and H.P. Average	Heavy
24 × 24	5	2¼	5	12	1	2
30 × 30	5 to 10	2¼	7½	14	1½	2– 3
36 × 36	10	2¼	10	16	2½	3– 5
42 × 42	10 to 20	3½	15	18	3½	5– 7½
48 × 48	20 to 25	3½	20	20	3½	5– 7½
54 × 54	25	3½	....	22	5	7½–10
56 × 56	....	....	25	24	7½	10–15
60 × 60	25 to 35	5	....	27	10	10–15
62 × 62	....	....	35	30	10	10–15
72 × 72	35	5	35	33	10	10–15
84 × 84	35 to 50	5	35 to 50	36	15	15–20
96 × 96	50	7½	50	42	20	25–30
120 × 120	50	7½	50	48	20	25–30
144 × 144	50	8½	....	54	25	30–35
168 × 168	50 to 75	10	....	60	25	30–35
192 × 168	75 to 100	10	....	72	25	30–35

Motor Power for Machine Tools and Forging Machinery

Special Lathes
Type of Motor: *A*, *B* or *C*

Wheel Lathes

Swing, Inches	Max. Distance between Faceplates, Inches	Horsepower	Tailstock Motor Horsepower
Driving Wheel Lathes			
51	108	15	8½
60	108	20	8½
69	108	20	8½
79	108	25	8½
84	108	25 to 50	8½
90	108	30 to 50	8½
100	108	50	8½
Car Wheel Lathes			
42	90	40	8½
46	94	50	8½
48	98	25	8½
50	98	50	8½

Axle Lathes

Swing over Tool-slide, Inches	Maximum Distance between Centers, Inches	Horsepower
10	90	15
12½	96	25
14	96	25

Buffing Lathes
Type of Motor: *B* or *C*

Wheel Diam., Inches	Horsepower *	Wheel Diam., Inches	Horsepower *
6	¼ to ½	12	2 to 3
10	1 to 2	14	3 to 5

* For brass tubing use about double the horsepower given above.

Bulldozers or Forming or Bending Machines
Type of Motor: *D* or *E*

Width, Inches	Head Movement, Inches	Horse power
29	14	5
34	16	7½
39	16	10
45	18	15
63	20	20

Shears
Type of Motor: *D* or *E*

Vertical Type

Soft Steel, Width, Inches	Thickness of Plate, Inches	Horsepower
30 to 42	1/32	¾ to 1
36 to 62	1/16	2 to 3
36 to 144	⅛	3 to 10
36 to 144	3/16	4 to 12
42 to 168	¼	6 to 20
54 to 126	⅜	15 to 20

Lever Type

Soft Steel, Square Bar, Size, Inches	Horsepower	Soft Steel, Square Bar, Size, Inches	Horsepower
¾	2 to 5	2½	10 to 20
1	3 to 5	2¾	15 to 20
1¼	5 to 7½	3	15 to 25
1½	5 to 7½	3¼	20 to 30
1¾	5 to 10	3½	20 to 40
2	7½ to 10	4	30 to 50
2¼	10 to 15	.....	......

Plate Shears

Size of Plate, Inches	Length of Stroke, Inches	Horsepower
⅜ × 24	3	10
1 × 24	3	15
2 × 14	4¼	30
1 × 42	4	20
1½ × 42	4½	60
1¼ × 54	6	75
1½ × 72	5½	100

Power Hacksaws
Type of Motor: *A*, *B* or *C*

Length of Blade, Inches	Stroke, Inches	Horsepower
14	6	½ to 1
16	6	½ to 1
18	6	¾ to 1½
20	6	¾ to 1½
24	6	1 to 2

Motor Power for Machine Tools and Forging Machinery

Power Hammers
Type of Motor: *D* or *E*

Weight, Pounds	Horsepower	Weight, Pounds	Horsepower
15	½ to ¾	200	5 to 10
25	¾ to 2	250	7½ to 15
50	1 to 3	300	7½ to 15
75	1½ to 3	400	10 to 15
100	2 to 5	500	10 to 15
125	2 to 5	700	15 to 25
150	2½ to 7½	1000	25 to 35

Drop-hammers require approximately 1 horsepower for every 100-pound weight of hammer head.

Keyseaters
Type of Motor: *A*, *B* or *C*

Size of Keyway in Steel, Inches	Horsepower	Size of Keyway in Steel, Inches	Horsepower
⅝	1 to 2	2	3 to 5
¾	1 to 2	2½	4 to 6
1	2 to 3	3½	5 to 7½
1¼	2 to 3	4	5 to 7½
1½	3 to 5	......	....

Vertical Turning and Boring Machines
Type of Motor: *A*, *B* or *C*

Swing, Inches	Horsepower	Swing, Feet	Horsepower
24	5	7	15 to 20
36	5 to 10	8⅓	20 to 25
42	7½ to 10	10	25 to 35
48	10 to 15	12	25 to 35
54	10 to 15	14	30 to 40
60	10 to 15	16	30 to 40
72	15 to 20	20	40 to 50

Horizontal Boring, Drilling and Milling Machines
Type of Motor: *A*, *B* or *C*

Spindle Diam., Inches	Horsepower	Spindle Diam., Inches	Horsepower
2½	5	6½	10 to 15
3½	5 to 7½	7	15 to 20
4½	7½ to 10	8	15 to 20
5	10 to 12½	9½	20 to 25

Cylinder Boring Machines
Type of Motor: *A*, *B* or *C*

Max. Diam., Inches	Horsepower	Max. Diam., Inches	Horsepower
20	7½	60	15 to 20
30	7½ to 10	70	15 to 20
40	10 to 15	80	15 to 20
50	10 to 15	100	20 to 25

Spur Gear Cutters
Type of Motor: *A*, *B* or *C*

Max. Gear Diameter, Inches	Max. Gear Width, Inches	Horsepower
36	9	2 to 3
48	10	3 to 5
30	12	5 to 7½
60	12	5 to 7½
72	14	7½ to 10
64	20	10 to 15

Rotary Planers
Type of Motor: *A*, *B* or *C*

Cutter Diam., Inches	Horsepower	Cutter Diam., Inches	Horsepower
25	5	70	25
30	7½	80	30
40	10	90	40
50	15	100	40
60	20	120	40

Heavy Forge Planers
Type of Motor: *B* or *C*

Size, Feet	Horsepower	
	Platen	Cross-rail
12 × 10	60	10
14 × 12	75	12

Hydraulic Wheel Presses
Type of Motor: *B* or *C*

Capacity, Tons	Horsepower	Capacity, Tons	Horsepower
100	3 to 3½	400	7½ to 10
200	5 to 7½	500	10 to 15
300	6 to 7½	600	12½ to 15

Motor Power for Machine Tools and Forging Machinery

Milling Machines
Type of Motor: *A*, *B* or *C*

Universal Milling Machines

Max. Feeding Movements, Inches: Lengthwise	Lateral	Vertical	Horsepower
22	8	18	3 to 5
28	10	18	5 to 7½
34	12	19	7½ to 10
42	14	20	10 to 15
50	14	20	15 to 20

Plain Milling Machines

Max. Feeding Movements, Inches: Lengthwise	Lateral	Vertical	Horsepower
22	8	19	3
28	10	19	5 to 7½
34	12	20	7½ to 10
42	14	20	10 to 15
50	14	21	15 to 20

Vertical Milling Machines

Max. Feeding Movements, Inches: Lengthwise	Lateral	Vertical	Horsepower
22	12	18	3 to 5
22	13	20	5 to 7½
34	14	22	7½ to 10
42	15	22	10 to 15
52	12	24	15 to 20

Vertical Slabbing Millers

Table Width, Inches	Max. Height, Inches	Horsepower
18	12	7½
24	22	10
30	26	10
36	27	15

Horizontal Slabbing Millers

Table Width, Inches	Max. Height, Inches	Horsepower
19	22	7½
23	25	10 to 15
27	25	15 to 20
30	37	20 to 25
42	50	35 to 50
50	57	40 to 60
60	72	75

Drilling Machines
Type of Motor: *A*, *B* or *C*

Single-spindle Sensitive Drills

Max. Drill Size, In.	Horsepower	Max. Drill Size, In.	Horsepower
3/16	¼	⅝	½ to 1
⅜	¼ to ½	⅞	1
½	½ to ¾	1⅛	1 to 1½

Upright Drilling Machines

Max. Work Diam., Inches	Horsepower
12 to 20	1 to 2
24 to 28	2 to 3
30 to 32	3 to 5
36 to 40	5 to 7½
50 to 60	7½ to 10

Radial Drilling Machines

Max. Drilling Radius, Feet	Service and Horsepower: Average	Heavy
2½	3	....
3	3	5
4	5	7½
5	6 to 7½	10
6	10	15 to 20
7	10	15 to 20
8	10	15 to 20

Multiple-spindle Drilling Machines

Max. Drill Diam. for Cast Iron, Inches	Maximum Number of Spindles	Horsepower
3/16 to ⅜	20– 8	5
¼ to ⅝	14– 4	7½ to 3½
¼ to ⅝	24–12	10 to 7½
¼ to 1	24– 6	10 to 7½
5/16 to 1	52–16	25 to 20
⅜ to ½	16–12	10 to 7½
⅜ to 1	20–10	15 to 10
⅝ to 1	24–12	20 to 15
⅝ to 1½	22– 8	20
¾ to 1	12– 8	20 to 10
¾ to 1½	22–10	40 to 25
1	12	15 to 20

Motor Power for Machine Tools and Forging Machinery

Cylindrical Grinding Machines
Type of Motor: *A*, *B* or *C*

Size of Wheel, Inches	Distance between Centers, Inches	Horsepower
10 × ¾	20 to 30	2 to 4
10 × 1½	20 to 30	2 to 4
12 × 1¼	32 to 66	5 to 8
12 × 1½	32 to 66	5 to 8
12 × 2½	32 to 96	10 to 12
14 × 1½	20 to 86	5 to 8
16 × 3	30 to 90	7½ to 10
18 × 2	27 to 120	7½ to 10
20 × 2	36 to 96	10 to 15
20 × 2½	39 to 123	12 to 15
24 × 2	96 to 168	15 to 20
24 × 3	98 to 172	25 to 35

Miscellaneous Grinders
Type of Motor: *B* or *C*

Type of Machine	Horsepower
Wet tool grinder..........	2 to 3
Swinging polishing machine	3
Angle cock grinder.........	3
Twist-drill grinder........	2
Automatic tool grinder.....	3 to 5

Cold Cutting-off Machines
Type of Motor: *A*, *B* or *C*

Saw Diameter, Inches	Max. Diameter, Inches, Soft Steel Round Stock	Horsepower
12	3	1 to 2
14	4	1 to 2
16	5½	1 to 2
18	5½	2 to 3
20	6	3 to 4
22	7	3 to 4
24	7½	3 to 5
26	7½	5 to 7½
30	9	5 to 7½
32	9½	7½ to 10
36	11½	10 to 15
40	13	10 to 15
42	14	15 to 20
48	15	15 to 25
50	17	15 to 25
54	18	20 to 25
58	19	20 to 25
62	21	20 to 25

Grinding Wheels
Type of Motor: *B* or *C*

Size of Wheel, Inches	Number of Wheels	Horsepower
6 × 1	2	1½
8 × 1	2	1½ to 2
10 × 2	2	2 to 3
12 × 2½	2	3 to 4
14 × 3	2	4 to 5
16 × 3	2	5 to 7½
20 × 3	2	7½ to 8
24 × 3½	2	8 to 10
26 × 4½	2	10 to 12
30 × 4½	2	10 to 15
36 × 4	2	10 to 15

Pipe Threading and Cutting-off Machines
Type of Motor: *A*, *B* or *C*

Size of Pipe, Inches	Horsepower	Size of Pipe, Inches	Horsepower
¼ to 2	1 to 2	2½ to 8	4 to 5
¼ to 3	2 to 3	2½ to 10	5 to 6
1 to 4	3 to 4	4 to 12	5 to 6
1¼ to 6	4 to 5	8 to 18	7½

Slotting Machines
Type of Motor: *A*, *B* or *C*

Max. Stroke, Inches	Horsepower	Max. Stroke, Inches	Horsepower
Crank Slotters			
6	1 to 2	22	10 to 15
8	2 to 3	24	10 to 15
10	3 to 5	26	10 to 15
12	3 to 5	28	10 to 15
14	5 to 7	30	10 to 20
16	5 to 7½	32	10 to 20
18	5 to 10	36	10 to 20
20	7½ to 15	.....	......
Geared Slotters			
24	10	60	12 to 15
30	10	68	15 to 20
36	10 to 12	72	20 to 25
40	10 to 15	92	20 to 25
50	12 to 15	.....	......

Motor Power for Machine Tools and Forging Machinery

Bolt Heading, Upsetting and Forging Machinery
Type of Motor: *D*, *E* or *F*

Size, Inches	Horsepower	Size, Inches	Horsepower
¾	5	3	20
1	7½	3½	25
1¼	7½	4	25
1½	10	5	35
2	15	6	40
2½	15	......	......

Bolt Cutters
Type of Motor: *A*, *B* or *C*

Type of Machine	Size, Inches	Horsepower
Single-spindle	¼ to 1	1½
Single-spindle	¼ to 1½	2
Single-spindle	½ to 2	3

Shapers
Type of Motor: *A*, *B* or *C*

Maximum Stroke, Inches	Horsepower	Maximum Stroke, Inches	Horsepower
Ordinary Type			
14	1 to 2	24	3 to 8
16	3 to 5	26	4 to 8
18	3 to 5	28	5 to 8
20	3 to 6	32	5 to 8
Traversing Head Type			
17	5	22	5 to 7½
18	5½	26	7½ to 10
20	5 to 7½	36	10

Punch Presses
Type of Motor: *A*, *C*, *D* or *E*

Soft Steel, Hole Diam., Inches	Thickness of Plate, Inches	Horsepower *
¼	¼	½ to 1
⅜	⅜	½ to 1½
½	½	¾ to 3
⅝	⅝	1½ to 2
¾	¾	1 to 5
⅞	⅞	1½ to 5
1	1	2 to 6
1¼	1	3 to 8
1½	1	7½
1¾	1	10
2	1	10
2¼	1⅛	10 to 15
2¼	1⅜	15 to 20
2½	1½	15 to 20
3	2	20 to 25
4	1½	25
6	1½	40

* The variations in horsepower are due entirely to the design of the press, especially with regard to flywheel and speed.

Bending and Straightening Rolls
Type of Motor: *E* or *G*

Soft Steel Capacity, Width, Feet	Thickness of Stock, Inches	Horsepower
5	⅜	5 to 7½
6	½	7½ to 10
8	⅝	10 to 15
10	¾	20 to 25
10	⅞	25 to 30
10	1	30 to 35
10	1⅛	35 to 40
10	1¼	35 to 40
10	1⅜	40 to 45
10	1½	45 to 50
10	1⅝	50 to 55

Application of Motors to Machine Tools

Bench and Speed Lathes. — Bench lathes may be either individually motor driven or driven from a countershaft mounted on the wall or bench which is in turn driven by a motor. Any kind except a series-wound or heavily compounded motor will be satisfactory. The object of the motor drive is to have the machine in the best possible location, without regard to the location of the line shafting. A number of these machines may be driven by a common line shaft, which in turn is driven by a motor. Speed lathes should be driven from a countershaft, or by a direct-connected motor. In the latter case a variable-speed motor is to be preferred, if direct current is available.

Engine Lathes. — This type of machine tool is very readily adapted to individual motor drive either adjustable or constant, depending upon whether the machine has a sufficient number of mechanical speed changes in the headstock. The adjustable-speed motor is preferable, as the speed changes can be secured at the carriage of the machine by means of an apron-operated controller, which gives any desired number of speeds. This arrangement is especially desirable on lathes having long beds, where considerable time would be lost if the operator had to return to the headstock, whenever speed changes were necessary.

Axle and Wheel Lathes. — It is of the greatest importance that axle lathes and car and driving wheel lathes should have the highest possible efficiency, and the most convenient location. These machines are mostly used in locomotive and car repair shops, where time saved does not mean merely the saving of wages, but each day gained means an added day in the earning capacity of the engine or car. These machines should be driven if possible by a direct-current adjustable-speed motor, as small variations in speed are often required due to the difference in hardness of the metal being turned.

Chucking Lathes. — This type of machine should ordinarily be driven by a constant-speed motor, but on the large sizes, adjustable-speed motors would enable the operator to obtain small graduations in speed which cannot be obtained by the mechanical speed changes.

Automatic Screw Machines. — Machines of this classification are almost universally equipped with constant-speed motors. Adjustable-speed motors could be used to advantage where arrangements are made to automatically change the motor speed as the work varies in diameter.

Sensitive and Upright Drills. — Constant-speed motors are used almost exclusively on this type of machine, as all the speeds necessary can usually be obtained by mechanical means.

Radial Drills. — These machines are usually driven by means of adjustable-speed motors due to convenience of obtaining speed changes at the head of the machines. Constant-speed motors can be used, however, and the speed changes obtained by means of a gear-box.

Boring Machines. — Where the work for boring machines is specialized and the machines perform only one operation a constant-speed motor can be used. Where the machine is used for a multiplicity of operations, such as drilling, boring, reaming and facing, an adjustable-speed motor is used. The range of speed of the motor should be as wide as possible, so that no gears need to be shifted for the entire set of operations on a single hole. Especially where a boring machine is used for facing, this variable speed will be found economical.

Grinders. — Constant-speed motors are universally used for driving grinders although a motor with a slight variation in speed to compensate for wheel wear is ideal.

Planers. — Adjustable-speed reversible motors are to be recommended for this type of machine tool. Motors for raising cross-rails should be a heavy compound-wound type. By using reversible motors in conjunction with dynamic braking, quick setting of work may be obtained, with a variety of cutting, and high return speeds.

Shapers and Slotters. — Adjustable-speed motors with dynamic braking should be used. The dynamic braking allows for stopping the machine at any point of its stroke and quick setting of work.

Milling Machines. — Constant-speed and also adjustable-speed motors are used for milling machines, depending upon the design of the driving mechanism or the provision for obtaining speed changes. The constant-speed drive through gear combinations which give the required speed changes, is extensively used. The driving shaft revolving at constant speed always delivers a uniform amount of power to the machine whereas, with a variable-speed type of drive, the speed and, consequently, the amount of power must be reduced in order to obtain a smaller number of revolutions per minute when using cutters of large diameter. With a constant-speed drive, the feeding mechanism may also be driven from the shaft which revolves at a uniform speed, so that the rate of feed is not affected by changes of the spindle speed as is the case when the feeding mechanism is driven from a shaft having variable speeds.

Punch Presses. — Constant-speed motors designed for dropping off in speed when the load is applied, and thereby using the energy stored in the flywheel for the actual shearing of the material, are most generally used. However, motors having enough speed adjustment to compensate for dullness of dies and hardness of material have been found to be of decided advantage for this class of machinery, especially in connection with roller feed presses. Drawing presses should be driven by adjustable speed motors when possible, as a decided advantage is gained when these machines are required to perform a variety of work.

Bending Rolls, Shears, etc. — This class of machinery, used largely for boiler, bridge, structural and ship building work, is generally placed in high shops and under cranes, and in locations and directions most convenient for the routing of the work. The shops in which it is placed are generally large and contain a relatively small amount of machinery, so that the amount of transmission gearing required is large in proportion to the amount of machinery. For this reason it is advisable in almost all cases to drive this class of machinery by an electric motor, which, of course, does not need to be of the variable-speed type.

Types of Motors for Different Requirements. — The general classification of direct-current motors suitable for individual machine tool drives and now included in the standard products of nearly all motor manufacturers is as follows:

Speed and Load Requirements	Type of Motor
Approximately constant speed, no load to full load	Shunt-commutating pole motor
Semi-constant speed, no load to full load	Compound motor
Adjustable speed, remaining approximately constant for one adjustment, no load to full load	Shunt-commutating pole motor with adjustable field resistance
Adjustable speed semi-constant for one adjustment, no load to full load	Compound motor, with adjustable shunt field resistance
Varying speed, varying with the load	Series motor Series-commutating pole motor

Estimating Horsepower to take a Given Cut on a Lathe using Round-nosed Type of Tool. — Under average shop conditions, the following is the horsepower required to remove one cubic inch per minute from various materials with a round-nosed lathe tool: Cast iron from 0.3 to 0.5 H.P.; cast steel from 1 to 1.8 H.P.; wrought iron and machine steel from 0.5 to 0.6 H.P.; steel 0.50 per cent carbon and over 1.00 to 1.25 H.P.; brass and similar alloys, from 0.2 to 0.25 H.P. The average horsepower for drilling may be estimated by doubling the horsepower values given for turning (per cubic inch of material removed).

Individual Motor Drive. — It is now generally agreed among manufacturers that individual motor drive for machine tools is much superior to group drive. The principal reasons are that greater flexibility may be obtained, belt troubles may be almost completely eliminated, better lighting conditions prevail as all overhead transmissions are unnecessary, speed controls for individual machines may be regulated to a nicer degree, interruptions are fewer as the shutting down of a motor affects only a particular machine, and transmission and friction power losses are greatly reduced. In fact, on a large class of machine tools, in order to procure maximum production, individual motor drive is essential.

Group Driving. — In the group drive, the motor drives a length of lineshafting from which, in turn, the machine tools are driven by belts. The best arrangement for group drive is to divide the machine shop into small units, having a motor for each department or each kind of machine. The lineshafting should be as short as possible and the motors placed in accessible positions. A small platform makes an excellent mounting for a motor. Motors suspended from the ceiling do not as a rule, receive the careful attention they need, and it is difficult to replace them when mounted in this way. The horsepower of the motor required for group driving is less than the sum of the horsepower required for each individual machine tool. The reason for this is that some of the machines are idle and others consume only a small amount of power at the time when the remaining machines absorb the maximum amount. Actual experience has shown that for group driving in an ordinary machine shop a motor of from ⅓ to ¾ the total horsepower of the motors required for the individual machines is sufficient.

The accompanying table "Power Requirements of Group-driven Machine Tools," gives data based on power consumption in an actual installation, as determined by tests. The table will be found useful when making estimates of power requirements.

Woodworking Machines. — In the woodworking department, the motors, in many cases, can be direct-connected to the machines, and in most cases high-speed motors can be used. If direct-current motors are used for this class of service they should be shunt-wound and entirely enclosed, the starting box also being enclosed in a metallic case lined with asbestos. If the short-circuit type induction motor is used for this class of service, it need not be dustproof, but the bearings should be dustproof and arrangements should be made to have the sawdust and shavings blown out of the motors at regular intervals; the motors of large sizes should also be provided with oil-immersed compensators. In general, motors in the woodworking department should have shaft extensions on both ends, for there are many cases where it is important that each machine be provided with its own blower for carrying away the sawdust. Where machines are equipped in this way with their own individual blowers, a great saving is effected, as the blower is in use only during the time that the machine is in service, whereas, if one common blower is used for the whole woodworking department, the load is practically the same whether a few or all the machines are in operation. Individual blowers are saving some woodworking departments a steady load of between 40 and 50 horsepower.

Power Requirements of Group-driven Machine Tools

Kind of Machine	Kind of Work	Per Cent of Machines Running	Floor Area for Machine and Operator in Square Feet	Total Average Power Per Machine in Watts *	Total Average Power Per Machine in Watts †	Friction and Line-shaft Load Per Machine in Watts †	Average Power Used in Doing Actual Work, in Watts *	Total Power Per Square Foot of Floor Area, in Watts
No. 2 Horizontal Rockford Boring Mills	Boring Bearings in Aluminum Cases	85	150	1620	1320	1100	300	8.8
No. 4 Cincinnati Millers	Light Milling on Aluminum	100	120	995	995	830	500	8.3
16-inch Lodge & Shipley Lathes	Turning Small Forgings	60	55	900	555	500	87	10.1
Double Disk Grinders; Double Buffers; Two Wheel Emery Stands	Grinding and Polishing	55	55	1800	1000	300	830	18.2
24-inch Bullard Vertical Lathes	Heavy Cuts on Cast Iron Fly-wheels	100	100	1350	1350	350	1000	13.5
24-inch Gould & Eberhardt Gear Cutters	Cutting Small Cast Iron Gears	100	65	333	333	250	83	5.1
Four Head Ingersoll Milling Machines	Making Four Cuts on Cast Iron Cylinders	100	300	3550	3550	2300	1250	11.8
Baker Single and Bausch Multi-Spindle Drills	Drilling and Tapping Cast Iron	40	70	1530	637	550	217	9.1
Heald Grinders, No. 60 Internal Grinders ‡	Cylinder Grinding	85	70	2830	2430	1860	500	34.7
No. 6 Whitney Hand Millers	Keyseating Small Cast Iron Gears	60	40	365	220	120	165	5.5
Landis No. 2 Grinders	Grinding Cam Shafts	80	90	1875	1500	1000	625	16.7
Norton 10 by 50-inch Grinders	Grinding Pistons and Small Forgings	70	100	2000	1400	1100	450	14 0
Jones & Lamson Flat Turret Lathes	Machining Small Forgings	85	65	675	560	200	375	8.6
Eight Spindle Cincinnati Gang Drills	Drilling and Reaming Connecting Rods (8 holes)	100	110	2840	2840	2000	840	25 8
Potter and Johnston Automatics	Turning Small Cast Iron Gears	100	75	690	690	440	250	9 2
1⅜-inch Gridley Automatics	Machining Cast Iron Pistons	100	200	1520	1520	1250	270	7.6
No. 4 Warner & Swasey Turret Lathe	Machining Small Forgings	65	55	560	360	310	70	6.5
24-inch Cincinnati Drill Presses	Small Drilling on Forgings	90	40	520	474	345	100	11.8

* Deducting Idle Machines. † Including Idle Machines. ‡ Exhaust Fan not considered.

DYNAMO AND MOTOR TROUBLES

Direct Current Dynamos and Motors

The various troubles to which direct current dynamos and motors are subject may be placed in six general classes. First, sparking of the brushes; second, heating of the parts; third, noises; fourth, variations in speed; fifth, miscellaneous derangements peculiar to motors as distinguished from dynamos; sixth, miscellaneous derangements peculiar to dynamos and generators as distinguished from motors. It is again possible to divide each of these major classes into minor ones. The sparking of the brushes, for instance, may be due, first, to faults of the brushes; second, to faults of the commutator; third, to excessive currents in the armature; fourth, to faults in the armature. Each of these divisions may be again subdivided and an appropriate individual remedy indicated.

To make this clearer, the arrangement in the present section has been adopted for stating precisely, and in a limited space, the troubles met with and the remedies to apply. As an illustration, suppose that the armature of a motor becomes dangerously hot after running for a time. The methodically arranged table or chart which follows is consulted and under the heading of "Heating of Parts" the sub-head "Armature" is found. There are seven different causes given here for heating of the armature. It may be due to overload of the motor, to a short circuit due to carbon dust, etc., on the commutator bars, or it may be caused by a broken circuit, a cross connection, moisture in the coils, eddy-currents in the core, or heat conveyed from a hot box or journals through the shaft. Each of these seven causes may be investigated in turn. For instance, it may be found that the armature core is warmer than the winding which surrounds it. If this is the case, the trouble is due to eddy-currents in the core, or to heat conducted through the shaft from a hot box. If the latter, the shaft will of course be hotter than the armature, and the bearings still hotter than the shaft. If the trouble is due to eddy-currents the armature will be found to be made of solid metal, or to be not sufficiently laminated. In either case the trouble is readily discovered.

Sparking at the Brushes

Faults of Brushes.

1. *Not set diametrically opposite.* — Should have been set properly at first, by counting bars, or by measurements on the commutator. Can be done if necessary while running; move rocker until brush on on-side sparks least; then adjust other rockers so they do not spark.

2. *Not set at neutral points.* — Move rocker back and forth slowly until sparking stops.

3. *Not properly trimmed.* — Brushes should be properly trimmed before starting by bending back and cutting off loose wires or ragged copper. If there are two or more brushes, one may be removed and retrimmed while running. Clean with benzine, soda or potash (alcohol or ether for carbon); then file or grind to standard jig and reset carefully. For instructions for setting see 1, 4, and 38.

4. *Not in line.* — Adjust each brush until bearing is on line and square on commutator bar, bearing evenly the whole width.

5. *Not in good contact.* — Clean commutator of oil and grit. See that brushes touch. Adjust tension screws and springs to secure light, firm, and even contact.

Faults of Commutator.

6–7. *Rough; worn in grooves or ridges; out of round.* — Grind with fine sand paper on curved block, and polish with crocus cloth. Never use emery in any form. If too bad to grind down, turn off true in a lathe or preferably on its own bearings, with a light tool and rest and a light cut, running slowly. Armature should have $\frac{1}{16}$ inch to $\frac{1}{8}$ inch end motion when running, to wear commutator evenly and smoothly. See also that foundation is level. If there is no end motion, file or turn ends of boxes or shoulders on shaft to provide end motion; then line up shaft and belt, so that there is no end thrust on shaft, but so that the armature plays freely endways when running.

8. *High bars.* — Set "high bar" down carefully with mallet or block of wood, then clamp end nuts tightly, or file, grind, or turn true. A high bar may cause singing. If so apply stearic acid (adamantine), candle, vaseline, or cylinder oil to commutator and wipe off; only a trace should be applied. Move brushes in and out of holder to get a firm, smooth, gentle pressure, free from hum or buzz.

9. *Low bars.* — Grind or turn commutator true to the surface of the low bars.

10. *Weak magnetic field.* — Broken circuit in field coils, or short circuit in field coils; repair if external, rewind if internal. Machine not properly wound or without proper amount of iron; no remedy but to rebuild it.

Excessive Current in Armature of Generator.

11. *Excessive load.* — Reduce number of lamps and load.

Ground and leak from short circuit on line. — Test out, locate, and repair.

Dead short circuit on line. — Dead short circuit will or should blow safety fuse. Shut down; locate fault and repair before starting again, and put in a new fuse.

Excessive Current in Armature of Motor.

11A. *Excessive voltage.* — Use proper current only, and with proper rheostat, controller, and switch.

Excessive amperes on constant current circuit. — See that controller, etc., are suitable with ample resistance.

Friction. — Reduce load on motor to its rated capacity or less. Clean with benzine. Bearings may be loose or worn out; perhaps new bearings are needed. For bearings out of line, see 30.

Too great load on pulley. — See that there is no undue friction anywhere.

Armature Faults.

12. *Short-circuited coils.* — (*a*). Remove copper dust, solder, or other metallic contact between commutator bars. (*b*). See that clamping rings are perfectly free, and insulated from commutator bars, and that there is no copper dust, carbonized oil, etc., to cause an electrical leak. (*c*). Test for cross connection or short circuit, and if such is found rewind armature to correct. (*d*). See that brush holders are perfectly insulated, with no copper dust, carbon dust, oil or dust, to cause an electrical leak.

13. *Broken coils.* — (*a*). Bridge the break temporarily by staggering the brushes until machine can be shut down (to save bad sparking) and then repair. (*b*). Shut down machine if possible, and repair loose or broken connection to commutator bar. (*c*). If coil is broken inside, rewinding is the only sure remedy. May be temporarily repaired by connecting to next coil, across mica. (*d*). Solder commutator lugs together, or put in a "jumper," and cut out, and leave open the broken coil. Be careful not to short circuit a good coil in doing this.

14. *Cross connections.* — Cross connections may have same effect as short circuit; treat as such, see 12. Each coil should test complete, with no cross and no ground.

Heating of Parts

Armature.

15. *Overloaded.* — Too many amperes, lights, or too much power being taken from machine. See 11A.

16. *Short circuit.* — Generally dirt, etc., at commutator bars. See 12.

17. *Broken circuit.* — Often caused by a loose or broken band. See 12, 13, and 14.

18. *Cross connection.* — Often caused by a loose coil abrading on another coil or core. See 12 and 14.

19. *Moisture in coils.* — Dry out by gentle heat; may be done by sending a small current through, or causing machine to generate a small current itself, by running slowly.

20. *Eddy currents in core.* — Iron of armature hotter than coils after a run: faulty construction. Core should be made of finely laminated insulated sheets. No remedy but to rebuild.

21. *Friction.* — Hot boxes or journals may effect armature. See 25 and 33.

Field Coils.

22. *Excessive current.* — When shunt wound decrease voltage at terminals by reducing speed; increase field resistance by winding on more wire, finer wire, or putting resistance in series with fields. When series wound, decrease current through fields by shunt, removing some of the field winding, or rewind with coarser wire. Excessive current may be caused by a short circuit, or by moisture in coils, producing a leakage. See 24.

23. *Eddy currents.* — Pole pieces hotter than coils after short run, due to faulty construction, or fluctuating current; if latter, regulate and steady current.

24. *Moisture in coils.* — Coils not dry show less than normal resistance; may cause short circuit or body contact to iron of dynamo. Dry out as in 19.

Bearings.

25. *Not sufficient or poor oil.* — See that plenty of good mineral oil, filtered clean, and free from grit, is fed to bearings; be careful that it does not get on commutator or brush holder, (See 12.) Cylinder oil or vaseline may be used if necessary to complete run, mixed with sulphur or white lead, or hydrate of potash. Then clean up and put in good order.

26. *Dirt or grit in bearings.* — (*a*). Wash out grit with oil while running, then clean up and put in order. Be careful not to flood the commutator and brush holder. (*b*). Remove caps and clean and polish journals and bearings perfectly; then replace. See that all parts are free and lubricate well. (*c*). When shut down, if hot, remove bearings and let them cool naturally; then clean, scrape and polish, assemble, seeing that all parts are free, and lubricate well.

27. *Rough journals or bearings.* — Smooth and polish in a lathe, removing all burrs, scratches, tool marks, etc., and re-babbitt old boxes or fit new ones.

28–29. *Journals too tight in bearings; bent shaft.* — Slacken cap bolts, put in liners and re-tighten till run is over; then scrape, ream, etc., as may be needed, bend or turn true in lathe or grind true. Possibly a new box or shaft will be needed.

30. *Bearings out of line.* — Loosen bearing bolts, line up and block until armature is in center of pole pieces, ream out dowel and bolt holes and secure in new position.

31. *End pressure of pulley hub or shaft collars.* — See that foundation is level and armature has free end motion. If there is no end motion, file or turn ends of boxes or shoulders on shaft to provide end motion. Then line up shaft and belt, so that there is no end thrust on shaft, but so that the armature plays freely endways when running.

32. *Belt too tight.* — (*a*). Reduce load so that belt may be loosened and yet not slip. Avoid vertical belts if possible. (*b*). Choose larger pulleys, wider and longer belts with slack side on top. Vibrating and flapping belts cause winking lamps.

33. *Armature out of center of pole pieces.* — (*a*). Bearings throwing armature out of center may be worn out and need replacing. (*b*). To repair, however,

center armature in polar space, and adjust bearings. Loosen bearing bolts, line up and block until armature is in center of pole pieces, ream out dowel and bolt holes and secure in new position. (*c*). File out polar space to give equal space all around. (*d*). Spring pole away from armature and secure in place; this may be difficult or impossible in large machines.

Noises

34. *Armature or pulley out of balance.* — Faulty construction; armature and pulley should have been balanced when made. May be helped by balancing on knife edges now.

35. *Armature strikes or rubs pole pieces.* — (*a*). Bend or press down any projecting wires, and secure with tie bands. (*b*). File out pole pieces where armature strikes. See also 30 and 33.

36. *Collars or shoulders on shaft strike or rub box.* — Bearings may be loose or worn out. Perhaps new bearings are needed. See also 30 and 31.

37. *Loose bolt connection or screws.* — See that all bolts and screws are tight, and examine daily to keep them so.

38. *Brushes sing or hiss.* — (*a*). Apply stearic acid (adamantine), candle, vaseline, or cylinder oil to commutator and wipe off; only a trace should be applied. (*b*). Move brushes in and out of holder to get a firm, smooth, gentle pressure, free from hum or buzz. See also 3, 8, and 9.

39. *Flapping of belt.* — Use an endless belt if possible; if a laced belt must be used, have square ends neatly laced.

40. *Slipping of belt from overload.* — Tighten belt or reduce load. See also 32.

41. *Humming of armature lugs or teeth.* — (*a*). Slope end of pole piece so that armature does not pass edges all at once. (*b*). Decrease magnetism of field, or increase magnetic capacity of tooth.

Variations in Speed

Runs Too Fast.

42. *Engine fails to regulate with varying load.* — Adjust governor of engine to regulate properly, from no load to full load.

43. *Series motor; too much current; runs away.* — Series motor on constant current: (*a*). Put in a shunt and regulate to proper current. (*b*). Use regulator or governor to control magnetism of field for varying load. Series motor on constant potential: (*a*) Insert resistance and reduce current. (*b*). Use a proper regulator or controlling switch. (*c*). Change to automatic speed regulating motor.

44. *Shunt motor; regulator not properly set.* — Adjust regulator to control motor.

Shunt motor; not proper current. — Use current of proper voltage and no other, with a proper rheostat.

Shunt motor; motor not properly proportioned. — Install better motor, one properly designed for the work.

Runs Too Slow.

45. *Engine fails to regulate.* — Adjust governor of engine to regulate properly, from no load to full load.

46. *Overload.* — Reduce number of lamps and load.

47. *Short circuit in armature.* — (*a*). Remove copper dust, solder or other metallic contact between commutator bars. (*b*). See that clamping rings are perfectly free, and insulated from commutator bars, and that there is no copper dust, carbonized oil, etc., to cause an electrical leak. (*c*). Test for cross connection or short circuit, and if such is found, rewind armature to correct. (*d*). See that brush holders are perfectly insulated, with no copper dust, carbon dust, oil or dust, to cause an electrical leak.

48. *Striking or rubbing of armature.* — (*a*). Bend or press down any projecting wires, and secure with tie bands. (*b*). File out pole pieces where armature strikes. See also 30 and 33.

49. *Friction.* — Clean with benzine. See also 25.

50. *Weak magnetic field.* — Broken circuit in field coils or short circuit in field coils: repair if external, rewind if internal. Machine not properly wound, or without proper amount of iron: no remedy but to rebuild it.

Motor

Stops or Fails to Start.

51–52. *Great overload; excessive friction.* — Open switch, find and repair trouble. Keep switch open and rheostat "off" to see if everything is in good order. With series motor no great harm will result from failing to start or stop. With shunt motor on constant potential circuit, fuse may blow or armature burn out. Reduce load on motor to its rated capacity or less. See that there is no undue friction or mechanical resistance anywhere. See also 25, 33, and 35.

53. *Circuit open; fuse melted or switch open.* — Find trouble. Put in fuse after opening switch. (If fuse is blown on account of dead short circuit, shut down, and locate and repair fault before starting again.)

Circuit open; broken wire or connection. — Open switch, find and repair trouble as instructed under 13.

Circuit open; brushes not in contact. — Open switch and adjust as stated under 5.

Circuit open; current fails or is shut off. — Open switch; return starting box lever to off position; wait for current.

54–55–56. *Short circuit of field, armature, or switch.* — Test for, and repair if possible. Examine insulation of binding posts and brush holders. Poor insulation, dirt, oil, and copper or carbon dust often result in a short circuit.

Runs Backwards.

57. *Wrong connections.* — Connect up correctly per diagram; if no diagram is at hand, reverse connections to brushes, or other connections, until direction of rotation is satisfactory.

Dynamo or Generator

Reversed Residual Magnetism.

58. *Reversed current through field coils.* — Use current from another machine or a battery through field in proper direction to correct fault. Test polarity with a compass.

Reversed connections. — If connections or windings are not known, try one way and test; if not correct, reverse connections, try again and test.

Earth's magnetism. — Connect up per diagram for desired rotation; see that connections to shunt and series coils are properly made.

Proximity of another dynamo. — Shift brushes until they operate better. See 1 and 2.

Brushes not in right position. — See 1 and 2.

Too Weak Residual Magnetism and Short Circuit.

59. *Too weak residual magnetism.* — Use current from another machine or a battery through field in proper direction to correct fault. Test polarity with a compass.

60. *Short circuit in machine.* — (*a*). Remove copper dust, solder or other metallic contact between commutator bars. (*b*). See that clamping rings are perfectly free, and insulated from commutator bars, and that there is no copper dust, carbonized oil, etc., to cause an electrical leak. (*c*). Test for cross connection or short circuit, and if such is found rewind armature to correct. (*d*). See that brush

holders are perfectly insulated, with no copper dust, carbon dust, oil or dust, to cause an electrical leak. See also 54–56.

61. *Short circuit in external circuit.* — A lamp socket, etc., may be short circuited or grounded, and prevent building up shunt or compound machines. Find and remedy before closing switch. See also 54–56.

62. *Field coils opposed to each other.* — Reverse connections of one of field coils and test. Find polarity with compass; if necessary, use current from another machine or a battery through field in proper direction to correct fault. Test polarity with a compass. Connect up per diagram for desired rotation, and see that connections to shunt and series coils are properly made. Try shifting brushes until they operate better. If necessary reverse connections and recharge in opposite directions.

Open Circuit.

63. *Broken wire.* — Search out and repair as stated in 13.

Faulty connections. — Search out and repair as stated in 37.

Brushes not in contact. — Search out and repair as stated in 5.

Safety fuses melted or broken. — Search out and repair as stated in 53.

External circuit open. — Search out and repair with dynamo switch open until repairs are completed.

Excessive Load or Resistance.

64. *Too great load on dynamo.* — (*a*). Reduce load to pilot lamp on shunt and incandescent machines; after voltage is obtained close switches in succession slowly, and regulate voltage. (*b*). Reduce number of lamps and load. (*c*). Bring up to voltage gradually with rheostat, and watch pilot lamp, regulating carefully.

65. *Too great resistance in field rheostat.* — Bring up to voltage gradually with rheostat, and watch pilot lamp; regulate carefully.

Alternating Current Machinery

In treating of the chief troubles of alternating current apparatus the following classes of machinery will be dealt with: 1. Alternators; 2. Synchronous Motors; 3. Induction Motors. Troubles due to mechanical causes, such as heating of bearings, etc., will not be considered, as the same remedies as are referred to in the previous section on direct current machinery apply to this class of apparatus also.

Alternators

Practically all alternator troubles may be sub-divided into the following: Failure to Generate; Excessive Heating; Poor Voltage Regulation; and Bad Parallel Operation.

Failure to Generate. — Failure to generate is very often due to exciter troubles, and it is therefore essential that exciter failures be analyzed in detail. Exciter troubles are considered in the previous section, "Direct Current Dynamos and Motors," under Dynamos. With the assumption that full exciter voltage is applied to the field terminals, the failure of an alternator to generate the normal terminal voltage at no load may be due to any of the following causes:

Armature winding open circuited. — Failure to generate from this cause can only occur with Y-connected machines, while for the delta-connected machines full voltage will be obtained across all terminals. The open circuited phase can be found by a magneto and bell, ringing from the neutral to each of the terminals.

Field winding open circuited. — This is the most general cause, and the break usually occurs in the connection between the coils.

Field coils short circuited. — Sometimes a short circuit may take place between adjacent field turns. This may be caused by mechanical injury, or by high induced voltages set up in the field winding, if the circuit is suddenly opened without being at field discharge resistance. A short circuit sooner or later results in a burnout. A resistance test should be taken of the field winding with the drop across each individual coil. A short circuited pole will show a reduced drop.

Reversal of individual field coils. — This may occur when reconnecting disassembled machines. It results in a reduced terminal voltage. Tests for determining if coils are reversed can readily be made by an ordinary compass.

Excessive Heating of Field Coils.

Generator overloaded. — No defect of machine. Relieve generator of part of the load before the consequent overheating reaches a dangerous point. The generator may also become overloaded due to incorrect instrument calibrations.

Low power factor of load. — If the power factor of the load is too low, the field distortion due to armature reaction will increase, thus weakening the resultant effective field. Increased excitation, followed by increased heating, is therefore required in order to maintain the voltage. Generator can only carry partial load when the power factor is lower than that for which the machine is designed.

Operation below normal speed. — In this case the field excitation must be increased to maintain the rated voltage, which may result in excessive heating of the field coils. Check speed.

Operation above rated voltage. — This may be due to incorrect voltmeter readings, or intentionally, in order to compensate for excessive voltage drop. It can only be obtained by increasing the field excitation and heating may result.

Short circuit in field winding. — This will render a portion of the field winding inoperative and a consequent increase in the exciting current is required to compensate for the now smaller number of effective field turns. It can best be located by resistance measurement of the field system. To guard against any danger of an internal breakdown of the field coils, field discharge switches with resistances should always be provided. Where this has not been done, turn in all resistance of both rheostats before opening the alternator field switch.

Excessive Heating of Armature Iron. — The causes which are responsible for excessive heating of the armature iron are generally the same as those causing overheating in the field coils, *i.e.*, overload, low power factor of load, slow speed, operation above rated voltage, and short circuit in field winding.

Excessive Heating Due to Defective Insulation between Stator Laminations. — This will result in energy losses with consequent heating due to hysteresis and eddy currents. Can only be remedied by rebuilding the core.

Excessive Heating of Armature Coils.

Generator overloaded. — May be due to incorrect meter readings, or may also be intentional.

Conduction of heat from armature iron. — Due to causes referred to under "Excessive Heating of Armature Iron."

Open circuit in one phase. — This occurs only with delta-wound armatures. The result is that the entire load is placed on the remaining two phases, thus overloading them and causing overheating. An open circuit can be tested out simply by a magneto and bell, or by applying to the armature winding a low voltage with an ammeter in the circuit, comparing the results for the different phases.

Grounding of one phase. —With Y-connected windings having a grounded neutral, a ground in any one phase will short circuit part of the winding. With delta or non-grounded Y-wound windings there must be two grounds in order to make a short circuit.

Short circuit in armature coils. — This will sooner or later result in the burning out of the short-circuited coils, due to excessive circulating currents therein. The overheating of certain coils can generally be detected by smoke issuing therefrom or by feeling the end connections with the hand. A definite method to detect a short-circuited phase is by resistance measurements. This is readily accomplished with Y-connection having grounded neutral and involves only a comparison of the resistances of the three phases, from the neutral to each terminal. With delta-connection or Y-connection with insulated neutral it becomes necessary to measure the resistance of two phases at a time and compare the results, which will then reveal which phase is short-circuited.

Collector rings. — Overheating of the collector rings is often due to excessive field excitation, the same as causes an overheating of the field coils. This cause can readily be detected by inserting an ammeter in the field connection and measuring the field current. Heating of collector rings may also be caused by excessive brush friction due to too high brush-tension. Incorrect brush alignment or dirty collector rings may also be the cause.

Poor Voltage Regulation.

Low power factor. — A poor regulation is often found to be caused by the fact that a highly inductive load is being placed on the machine, which has not been provided for in the design of the machine or guaranteed. If the load cannot be changed, the only remedies are to redesign the alternator or provide synchronous condensers which will supply the magnetizing current for the inductive load and thus raise the power factor of the load at the generator.

Alternator speed variations. — Variation in voltage may be caused by poor speed regulation of the prime movers, due to defective governors.

Exciter troubles. — A speed variation of the exciter sometimes causes voltage fluctuations, especially when the exciters are direct-connected to the main generators and thus subject to speed variations of the latter under different loads.

Automatic voltage regulators. — Almost all plants of any importance are nowadays provided with T. A. automatic voltage regulators, and, if so, moderate speed variations or load changes should not impair the regulation. Troubles may, however, occur with the regulator, and can be investigated as follows:

Should the regulator fail to build up its voltages, see that the reversing switches are thrown to the extreme position either up or down; see if the rheostat shunt circuit switches are closed on the exciters which it is intended to run; look for improper connections.

Should the voltage fall, examine the rheostat shunt circuit connections to see if they are not so connected as to short circuit the exciter field instead of its field rheostat.

If, after placing the regulator in service, the potential fluctuates to the extent of several volts, proceed as follows:

See if contact screws are loose. If so, they should be properly adjusted and set-screws securely tightened; observe both levers at the points where the core stems are attached, to see that there is no friction at these points.

The regulator should not be subjected to excessive vibrations, such as might be the case when it is mounted on iron brackets. If this is so, some rigid support should be provided to overcome the vibration.

The dashpot should be carefully inspected to see that it is actually full of oil to within ⅛ inch of the top; the dashpot should also be examined to see that it is securely attached to the supporting posts; the dashpot may be adjusted for too free a movement. This adjustment should be made as free as possible without causing pumping of the voltage at no-load.

Examine cores to see that they do not touch the inside of the magnet spools.

Carefully inspect all wiring, also look for flat spots on the commutators, loose brushes, or any other poor contact, that might cause an unsteady voltage.

If there is an error in the voltage from no-load to full-load without the compensating winding in circuit, it must be due to improper adjustment of the alternating current magnet core.

If, after the load has become steady in going from no-load to full-load, the main alternating current voltage has fallen off, the alternating current magnet core should be slightly lowered until the voltage is the same at full-load as at no-load. If the main alternating current voltage is too high, the alternating current core should be raised slightly to overcome the error and the voltage will be the same at full-load as at no-load.

If there is excessive arcing at the relay contacts, check the connections of the rheostat shunt circuit to see that they are properly made, and that the rheostat only is being short-circuited. The connections of the condensers should be checked.

Bad Parallel Operation.

Resonance. — The trouble mostly encountered in the parallel operation of engine-driven alternators consists in a tendency of the impressed oscillations or impulses to coincide with the natural periods or swing of the alternator, causing resonance. This trouble can often be overcome by the attachment of simple dashpots to the governors, which delay the governor sufficiently to prevent the troublesome periodic action and at the same time do not prevent sufficient promptness in governing.

It is often feasible to decrease the amplitude of the oscillations when the generators are rope- or belt-driven, by changing the length of the drive. The rotor displacement can be reduced or increased by providing a heavier or lighter flywheel. The natural period of swing of the alternator can be changed and resonance avoided by increasing or reducing the reactance of the machine.

With gas-engine-driven generators it is customary to place amortisseur windings in the pole faces to act as an additional drag on the engine to prevent displacement. Sudden tendencies to change in speed cause currents to be induced in the low resistance bars of this winding which produce a torque similar to that of a squirrel-cage induction motor and help out the flywheel to some extent.

Excessive cross currents. — Cross currents between alternators operating in parallel may be either wattless, or they may represent a transfer of energy. The former is caused by a difference in the excitation or voltage of the two machines, while the latter is caused by periodic oscillations of the generators, this being generally due to irregularities in the speed of the prime movers. They can therefore be reduced by improving the parallel operation as previously described.

Synchronous Motors

Excessive Heating. — Excessive heating in the different parts of a synchronous motor is generally caused by the same defects as for alternators and may be located and corrected in the same manner.

Power factor correction. — Excessive heating is very often caused when the field excitation of the motor is increased for low power factor correction, in order that the motor may draw a leading current and compensate for heavy inductive loads in other parts of the system. Such increased excitation will therefore increase the total armature current, slowly at first, but thereafter more rapidly, at low leading power factors. Over-excitation should therefore be kept within certain safe limits.

Open field circuit. — In the same manner as an increase in the field excitation causes the motor to draw a leading current from the line, a decrease in the normal excitation will cause the motor to draw a lagging current from the line, thus also increasing the total armature current and the heating. It becomes important therefore to have the field current permanently established.

Voltage too low. — A synchronous motor should be operated normally at its rated voltage, as a reduction in voltage means increased current with consequent increased heating of armature.

Frequency low. — Operation at normal frequency is desirable, as at a reduced frequency and normal voltage the iron losses are increased due to higher density, and consequently also the heating.

Wrong polarity. — Since the winding of a synchronous motor armature is in series all the way around the circumference and under all of the poles, except in exceedingly rare cases, the trouble from a reversed pole is not very serious. Everything operates fairly satisfactorily, the only trouble being that the fields require more current than they should, to make up for the pole that is opposing the other fields. If, therefore, excessive field current is required for the minimum input to the motor, try the polarity of all the spools, using a compass for the purpose.

Difficulties in Starting. — The majority of synchronous motors have their revolving fields provided with amortisseur windings, and are started by applying a reduced voltage to the armature winding. Failure to start and come up to synchronous speed may be due to faults of starting auxiliaries or of motor.

Open circuit. — Difficulty in starting may be caused by an open circuit in one of the lines to the motor. When this happens, a polyphase motor becomes single-phase and is not self starting. The motor will stand still and soon get hot.

Applied voltage too low. — The starting torque of a synchronous motor is proportional to the square of the applied voltage. When, therefore, a synchronous motor will not start, it may be due to the fact that the voltage on the line is pulled down below the value necessary for starting. If the motor is provided with a starting compensator it may be possible to re-connect the leads and thus obtain a higher terminal voltage. Care, however, should be taken that the starting current does not become too high.

Increased friction. — Difficulties in starting may be caused by an increase in the static friction, due to bearings being too tight, cutting of bearings, belt friction, etc. For very large motor-generator sets it sometimes becomes necessary to reduce the bearing friction by oil pressure.

Field excited. — If the field is on, most synchronous motors will not start at all.

Incorrect design of starting winding. — It sometimes happens that the manufacturer has not been properly informed as to the true operating conditions, and that the proper starting winding has not been furnished. A motor driving a generator will, for example, require a high starting torque compared to the pull-in torque, while for a motor driving a centrifugal pump the reverse is the case.

Synchronous Speed. — It is necessary to bring all synchronous motors up to 92 to 96 per cent of synchronism, depending somewhat on the flywheel effect of the load they are starting, before the field excitation can be put on. If the excitation is applied below this speed the motor fails to pull into synchronism and either shuts down entirely or runs along at a reduced speed, taking a heavy fluctuating current. In some cases additional torque near synchronism can be obtained by short circuiting the field winding through the field rheostat. This has the effect of reducing the resistance of the motor winding to some extent and causing the motor to have less slip with a given load. The gain from this source is small, however, in most cases, as the self-inductance of the field winding is so high as to allow very little current to flow, even if the field is dead short circuited, so that the total effective resistance of the rotor winding is not materially reduced. In some cases, where the torque is nearly sufficient, however, enough may be gained to take care of the conditions. If the field is short circuited before the motor is started, there will be a reduction in starting torque and an increase in current from the line, so that, if this method is resorted to, arrangements should be made to short circuit the field after the motor has come to constant speed.

It often occurs, when the motor has reached synchronous speed and the direct current excitation has not been previously applied, that when it is then excited by direct current, the poles will not be in the proper position relative to the stator conductors. If the direct current excitation is increased sufficiently, the rotor will "slip a pole," and then, the poles being in the proper relative position, the rotor will lock into synchronism. This method of forcing the rotor to "slip a pole" may cause excessive currents to flow in the armature conductors, as it tends to lessen the flux to the stator magnetizing currents, which generate the counter electro-motive forces. However, this condition may be prevented and the desired results obtained by either reversing the polarity of the applied excitation, by slightly loading the machine mechanically, or by disconnecting the motor from the supply. There is, of course, an even chance that the same polarity may be in the same relative position to the stator conductors.

Incorrect Connections. — Improper armature connections usually manifest themselves in a synchronous motor by unbalanced entering currents and by the fact that the starting torque is very much less than it should be, or perhaps negligible. The circuits should be traced out and the connections remade until the three entering currents in the case of three-phase, or the two entering currents in case of two-phase, agree approximately. It should be noted that these currents do not agree even with correct connection when the armature is standing still. The reading should be taken with the armature revolving slowly, mechanically, which, with the proper connection, should average up the entering currents.

Failure of Motor to Develop Full Load Torque. — The failure of a synchronous motor to develop full load torque after having attained synchronous speed is due entirely to failure of excitation.

Exciter failure. — The exciter should be inspected as to whether it is operating satisfactorily.

Open circuit. — This may be the case either in the leads between the exciter and the motor field, or in the connections between the field coils. The majority of the troubles of this nature are generally to be found in the coils themselves. In starting a synchronous motor, a transformer action exists between the armature winding and the field winding, the comparatively large number of turns on the field causing a high voltage to be induced in the field windings, resulting in a break down of the same. A field discharge switch and resistance should therefore always be provided with synchronous motors.

Motor Stops under Maximum Load. — The maximum torque of a synchronous motor is in proportion to the square of the voltage and over-excitation of the field will also increase it to some extent. If the motor, however, breaks down from excess of load it will not recover when the load is removed, as will the induction motor, but it is necessary to remove the field excitation and start up in the regular way.

Hunting. — When synchronous motors are operating in parallel under normal conditions, periodic variations in motive power or load may cause the machines to oscillate in speed. These oscillations, unless opposed in some manner, may become more and more aggravated, until the machines either "fall out of step" or serious line surges occur. One of the most valuable features of the amortisseur winding is its tendency to prevent these oscillations. When the machine is retarded or accelerated, the bars of the winding are cut by the flux. The cutting of the flux produces energy currents in the bars which tend to dampen out these oscillations. The energy that would otherwise be expended in retarding or accelerating the motor is thus absorbed and dissipated.

Induction Motors

Failure to Start.

No voltage. — No voltage at the motor terminals may be due to blown fuses or breaks in the connections to the motor. See also under "Defects in Starting Compensator."

Low voltage. — Low voltage is the most frequent external trouble causing a motor to refuse to start. Since the torque of an induction motor is proportional to the square of the applied voltage, a reduction in the applied voltage has a very great effect upon the starting torque. A low applied voltage may be due to several causes. The motor leads may be connected to a too low voltage tap on the compensator or the voltage drop in the supply circuit may be excessive. In testing for low voltage, measure the voltage at the motor terminals as soon as the motor is thrown on the line. Although the no-load voltage may be all right when the motor is thrown on the line, the heavy starting current taken may pull the voltage down too low, if there is not carrying capacity enough in the transformer windings and the line.

One phase of stator open circuited. — Reading of current can be obtained only in two legs of the three-phase leads if the motor is Y-wound and the motor will not start by itself. In the case of a three-phase delta-wound motor, an open circuit in one phase will result in an open circuited delta, excited with three-phase current. Such a winding will produce a rotating magnetic field and consequently develop a starting torque, although of smaller value than the normal three-phase torque. If, therefore, the normal starting torque is just sufficient to start the load, the motor will most likely refuse to start when suffering from this defect.

One phase of stator reversed. — The motor will have a greatly reduced torque and probably will not start with any load.

All phases of rotor open circuited. — Motor will not start, and the current taken is the exciting or no-load current. This trouble can be distinguished from blown fuses and open circuits in the stator and leads by connecting an ammeter in the line; if it shows current flowing in each of the phases, the trouble is in the rotor. If there were open circuits in the stator or in the leads, no current would flow in the affected lead.

One phase of rotor open circuited. — Running light, the current taken in the three lines will be practically balanced, but the motor in starting has a decided tendency to remain at half synchronous speed.

Defects in starting compensator. — When a motor will not start, having been connected to a compensator, the cause may be entirely in the compensator. This may have become open-circuited, due to a flash within. The compensator switch may have become deranged, so that it will not close, or a connection within the compensator may have become loosened. Possibly when a motor will not start when connected to a compensator just put into use, a secondary coil may be bucked against another secondary coil within the compensator, so that no voltage is produced by the compensator at the motor. This results in no particular heating which would account for the motor not starting. An ammeter in the motor leads would show the absence of current, or a voltmeter the absence of voltage.

Starting resistance too high. — A few tests may be made in short-circuiting some of the units in the starting resistance.

Controller troubles. — The motor will sometimes run one way and refuse to start in the other direction. The cause of this is that the contacts on the primary drum touch in one position, but fail to make contact when thrown to the other position. If the motor will not start in either direction and the fuses are all right, set the controller on the first notch and pull out the fuses one at a time, leaving the two fuses of one phase in each time. If the motor hums loudly on each phase, the primary connections are all right, and the trouble must be looked for either in the secondary drum contacts, the collector rings or the wiring.

Excessive load at starting. — This can only be remedied by removing some of the load. Excessive bearing friction or too tight belts may often be the cause.

Excessive Current at Starting.

Voltage too high. — It is often found that a motor is connected to a compensator in such a way that the applied voltage is too high, resulting in an excessive starting current. Compensators are usually supplied with various taps, and one should be chosen which produces the least disturbance on the line, giving at the same time the desired starting torque on the motor.

Wrong connections. — Sometimes a mistake in connection is made on the compensator so that full voltage is used at starting and the lower voltage after throwing over the switch. Thus the motor at starting takes excessive current.

Overheating and Failure to Carry Load.

Voltage too low. — Since the torque is proportional to the square of the voltage it can be seen that lowering the voltage has a positive effect upon the ability of the motor to carry load, and may be the cause of its stopping.

Overload. — The load put on the motor may be more than the designed maximum output. Excessive bearing friction or worn bearings may also be the cause for overloading. For some reason the bearings may have become worn, so that the air-gap at the bottom has become so reduced that the rotor commences to rub on the stator. The friction may then become so great that the load represented by it is more than the motor can carry, with the result that it shuts down.

Too frequent starting. — On account of the increased current required, in starting, a motor should not be started too often, as this may cause overheating.

Speed too low. — With variable speed motors it is often found that full load is carried on the motor at reduced speeds. This will cause overheating.

One phase of stator open circuited. — In this case the motor will continue to operate single phase, with a pronounced lack of power.

Short circuit in stator. — A short-circuited coil in the stator will cause it to buzz and hum, and if the motor is run for some time the coil will overheat and smoke.

Open circuit in Y-connected rotor. — This usually manifests itself by overheating, and if the motor is running under load, the speed will drop considerably.

Short circuit in rotor winding. — This will cause an overheating of the rotor; the short circuits are generally caused by winding being grounded in two places.

Unbalanced voltages. — The maximum output can be seriously affected by the fact that the voltages applied to the motor are not equal in amount.

Changed voltage. — If a standard induction motor is run on a changed voltage, the heating will vary in accordance with the losses. The copper losses being greater than the core loss results in increased heating at reduced voltage, when operating with the same horsepower output.

Variation of frequency. — An increase in the frequency results in a considerable reduction in the maximum load which an induction motor can carry.

Sparking. — Sparking of the collector ring brushes with phase-wound induction motors is generally due to a high current density therein, caused by an excessive overload. The collector rings become pitted, and the brushes wear away rapidly. Excessive brush pressure may also cause overheating.

Vibration. — Vibration may be due to mechanical unbalancing, but this occurs chiefly in high speed machines and is usually easily detected. Loose parts are also often responsible for this defect. When due to an unbalancing of magnetic pull caused by a combination of faulty air-gap and eccentricity of rotor circumference, it may in some cases be remedied by the adoption of a multiple-wound stator. It can always be improved by "truing up" the rotor.

MATERIALS

Melting Points. — The table, "Specific Gravity and Properties of Metals" gives the melting points of the more common commercial metals. The melting points of commercial metals vary, however, due to slight impurities, and should, therefore, be considered approximate only. The melting points of nearly all chemical elements in their pure state is given in the table " Melting Points of Chemical Elements." The data here given are accepted by the United States Bureau of Standards as the most carefully determined and most probably correct. In the table, the Fahrenheit temperatures are exact conversions from the Centigrade scale, but it should be understood that melting points above 2000 degrees F. are not known exactly to within 5 or 10 degrees. Impure metals usually have lower melting points than pure metals. The melting point of an alloy cannot always be directly inferred from the melting points of its component elements.

Melting Points of Chemical Elements

Chemical Element	Deg. Cent.	Deg. F.	Chemical Element	Deg. Cent.	Deg. F.
Aluminum *	658.7	1217.7	Molybdenum	2550	4620
Antimony *	630.0	1166.0	Neodymium	840?	1544
Argon	−188	−306	Neon	−253?	−423
Arsenic	850	1562	Nickel *	1452	2646
Barium	850	1562	Nitrogen	−210	−346
Beryllium	1280	2336	Osmium	2700?	4890
Bismuth	271	520	Oxygen	−218	−360
Boron	2200–2500?	4000–4500	Palladium *	1549	2820
Bromine	−7.3	18.9	Phosphorus	44	111
Cadmium *	320.9	609.6	Platinum *	1755	3191
Caesium	26	79	Potassium	62.3	144.1
Calcium	810	1490	Praseodymium	940	1724
Carbon	>3600	>6500	Radium	700	1292
Cerium	640	1184	Rhodium	1950	3542
Chlorine	−101.5	−150.7	Rubidium	38	100
Chromium	1615	2939	Ruthenium	2450?	4440
Cobalt	1480	2696	Samarium	1300–1400	2370–2550
Columbium	1700?	3090	Selenium	217–220	423–428
Copper *	1083.0	1981.4	Silicon	1420	2588
Fluorine	−223	−369	Silver *	960.5	1760.9
Gallium	30	86	Sodium	97.5	207.5
Germanium	958	1756	Sulphur	S_1 112.8	235.0
Gold *	1063.0	1945.5	Tantalum	2900	5250
Helium	<−271	<−456	Tellurium	452	846
Hydrogen	−259	−434	Thallium	302	576
Indium	155	311	Thorium	>1700	>3090
Iodine	113.5	236.3	Tin *	231.9	449.4
Iridium	2350?	4260	Titanium	1800	3272
Iron *	1530	2786	Tungsten *	3400	6152
Krypton	−169	−272	Uranium	<1850	<3360
Lanthanum	810?	1490	Vanadium	1720	3128
Lead *	327.4	621.3	Xenon	−140	−220
Lithium	186	367	Yttrium	1490	2714
Magnesium	651	1204	Zinc *	419.4	786.9
Manganese	1230	2245	Zirconium	1700?	3090
Mercury *	−38.87	−37.97	....	....	....

* The melting points of elements marked with asterisk (*) are used by the Bureau of Standards as standard temperatures for the calibration of thermometers and pyrometers. Table revised July, 1918.

Specific Gravity and Properties of Metals

Metal or Composition	Chemical Symbol	Specific Gravity	Weight per Cubic Inch, Pounds	Weight per Cubic Foot, Pounds	Melting Point, Deg. F.	Structure*	Linear Expansion per Unit Length per Deg. F.	Electric Conductivity; Silver = 100	Approx. Value per Pound, Dollars
Aluminum	Al	2.56	0.0924	159.7	1218	*M*	0.00001234	63.00	0.34
Antimony	Sb	6.71	0.2422	418.7	1166	*B*	0.00000627	3.59	0.23
Barium	Ba	3.75	0.1354	234.0	1562	*M*	....	30.61	512.00
Bismuth	Bi	9.80	0.3538	611.5	520	*B*	0.00000975	1.40	1.60
Boron	B	2.60	0.0939	162.2	4000–4500	*H*	....	....	270.00
Brass: 80 C., 20 Z.	....	8.60	0.3105	536.6					0.15
70 C., 30 Z.	....	8.40	0.3032	524.1					0.14
60 C., 40 Z.	....	8.36	0.3018	521.7	1700–1850	*M*	0.00000957	....	0.13
50 C., 50 Z.	....	8.20	0.2960	511.6					0.12
Bronze	....	8.85	0.3195	552.2	1675	*B*	0.00000986	....	0.20
Cadmium	Cd	8.60	0.3105	536.6	610	*M*	....	24.38	1.60
Calcium	Ca	1.57	0.0567	98.0	1490	*M*	....	21.77	1080.00
Chromium	Cr	6.50	0.2347	405.6	2939	*B*	....	16.00	0.80
Cobalt	Co	8.65	0.3123	539.8	2696	*M*	....	16.93	1.60
Copper	Cu	8.82	0.3184	550.4	1981	*M*	0.00000887	97.67	0.15
Gold	Au	19.32	0.6975	1205.6	1945	*M*	0.00000786	76.71	320.00
Iridium	Ir	22.42	0.8094	1399.0	4260	*M*	0.00000356	13.52	1500.00
Iron, cast	Fe	7.20	0.2600	449.2	2300	*B*	0.00000556	....	0.01
Iron, wrought	Fe	7.85	0.2834	489.8	2750	*M*	0.00000648	16.80	0.02
Lead	Pb	11.37	0.4105	709.5	621	*S*	0.00001571	8.42	0.06
Magnesium	Mg	1.74	0.0628	108.6	1204	*M*	....	39.44	1.50
Manganese	Mn	7.42	0.2679	463.0	2246	*B*	....	15.75	0.70
Mercury (60° F.)	Hg	13.58	0.4902	847.4	−38	*F*	....	1.75	0.55
Molybdenum	Mo	8.56	0.3090	534.2	4620	*B*	....	17.60	1.10
Nickel	Ni	8.80	0.3177	549.1	2646	*M*	0.00000695	12.89	0.45
Platinum, rolled	Pt	22.67	0.8184	1414.6					
Platinum, wire	Pt	21.04	0.7595	1312.9	3191	*M*	0.00000479	14.43	550.00
Potassium	K	0.87	0.0314	54.3	144	*S*	....	19.62	3.20
Silver	Ag	10.53	0.3802	657.1	1761	*M*	0.00001079	100.00	10.40
Sodium	Na	0.98	0.0354	61.1	207	*S*	....	31.98	3.20
Steel	Fe	7.80	0.2816	486.7	2500	*M*	0.00000636	12.00	....
Tellurium	Te	6.25	0.2256	390.0	846	*B*	....	0.001	80.00
Tin	Sn	7.29	0.2632	454.8	449	*M*	0.00001163	14.39	0.40
Titanium	Ti	3.54	0.1278	220.9	3272	*M*	....	13.73	800.00
Tungsten	W	18.77	0.6776	1171.2	6152	*B*	....	14.00	0.64
Vanadium	Va	5.50	0.1986	343.2	3128	*M*	....	4.95	2.75
Zinc, cast	Zn	6.86	0.2476	428.1		*B*			
Zinc, rolled	Zn	7.15	0.2581	446.1	787	*M*	0.00001407	29.57	0.06

* *B* = brittle; *F* = fluid; *H* = hard; *M* = malleable; *S* = soft.

Melting Points of Alloys of Low Fusing Point

Composition in Per Cent				Melting Point, Degrees F.	Composition in Per Cent			Melting Point, Degrees F.
Bismuth	Lead	Tin	Cadmium		Bismuth	Lead	Tin	
50.0	25.0	12.5	12.5	149	20.0	40.0	40.0	293
50.1	26.6	13.3	10.0	158	19.0	38.0	43.0	298
38.4	30.8	15.4	15.4	160	18.1	36.2	45.7	304
27.5	27.5	10.5	34.5	167	17.3	34.6	48.1	311
50.0	34.5	9.3	6.2	171	16.6	33.2	50.2	316
50.0	25.0	25.0	.....	187	16.0	36.0	48.0	311
50.0	31.2	18.8	.....	201	15.3	38.8	45.9	309
55.6	.....	33.3	11.1	203	14.8	40.2	45.0	307
50.0	.....	25.0	25.0	203	14.0	43.0	43.0	309
47.0	35.5	17.5	.....	208	13.7	44.8	41.5	320
42.1	42.1	15.8	.....	226	13.3	46.6	40.1	329
40.0	40.0	20.0	.....	235	12.8	49.0	38.2	342
36.5	36.5	27.0	.....	243	12.5	50.0	37.5	352
33.3	33.4	33.3	.....	253	11.7	46.8	41.5	333
30.8	38.4	30.8	.....	266	11.4	45.6	43.0	329
28.5	43.0	28.5	.....	270	11.2	44.4	44.4	320
25.0	50.0	25.0	.....	300	10.8	43.2	46.0	318
23.5	47.0	29.5	.....	304	10.5	42.0	47.5	320
22.2	44.4	33.4	.....	289	10.2	41.0	48.8	322
21.0	42.0	37.0	.....	289	10.0	40.0	50.0	324

Average Specific Gravity of Woods (Dry)

Wood	Sp. Gr.	Weight per Cubic Inch, Pounds	Weight per Cubic Foot, Pounds	Wood	Sp. Gr.	Weight per Cubic Inch, Pounds	Weight per Cubic Foot, Pounds
Apple......	0.77	0.028	48.1	Holly.........	0.76	0.027	47.5
Ash........	0.75	0.027	46.8	Lignum vitæ...	1.10	0.040	68.6
Bamboo....	0.35	0.013	21.8	Linden........	0.60	0.022	37.4
Beech......	0.75	0.027	46.8	Locust........	0.73	0.026	45.6
Birch......	0.64	0.023	40.0	Mahogany.....	0.90	0.032	56.2
Box........	1.04	0.038	64.9	Maple........	0.70	0.025	43.7
Cedar......	0.57	0.021	35.6	Mulberry......	0.75	0.027	46.8
Cherry.....	0.70	0.025	43.7	Oak, live......	1.15	0.042	71.7
Chestnut...	0.60	0.022	37.4	Oak, red......	0.75	0.027	46.8
Cork.......	0.24	0.009	15.0	Oak, white....	0.80	0.029	49.9
Cypress....	0.60	0.022	37.4	Pear..........	0.65	0.023	40.6
Ebony.....	1.25	0.045	78.0	Pine, white....	0.50	0.018	31.2
Elm.......	0.67	0.024	41.9	Pine, yellow...	0.65	0.023	40.6
Fir........	0.55	0.020	34.3	Poplar........	0.50	0.018	31.2
Hazel......	0.60	0.022	37.4	Spruce........	0.45	0.016	28.1
Hemlock...	0.40	0.014	25.0	Teak..........	0.85	0.031	53.0
Hickory....	0.80	0.029	49.9	Walnut.......	0.65	0.023	40.6

Specific gravity is a number indicating how many times a certain volume of a material is heavier than an equal volume of water. As the density of water differs slightly at different temperatures, it is the usual custom to make comparisons on the basis that the water has a temperature of 62 degrees F. The weight of one cubic inch of pure water at 62 degrees F. is 0.0361 pound. If the specific gravity of any material is known, the weight of a cubic inch of the material can, therefore, be found by multiplying its specific gravity by 0.0361.

Example: — The specific gravity of cast iron is 7.2. Find the weight of 5 cubic inches of cast iron.

$$7.2 \times 0.0361 \times 5 = 1.2996 \text{ pound.}$$

To find the weight per cubic foot of a material, the specific gravity of which is known, multiply the specific gravity by 62.355.

If the weight of a cubic inch of a material is known, the specific gravity is found by dividing the weight per cubic inch by 0.0361.

Example: — The weight of a cubic inch of gold is 0.697 pound. Find the specific gravity.

$$0.697 \div 0.0361 = 19.31.$$

If the weight per cubic foot of a material is known, the specific gravity is found by multiplying this weight by 0.01604.

Average Specific Gravity of Miscellaneous Substances

Substance	Sp. Gr.	Weight per Cubic Foot, Lbs.	Substance	Sp. Gr.	Weight per Cubic Foot, Lbs.
Asbestos	2.8	175	Gypsum	2.2	137
Asphaltum	1.4	87	Ice	0.9	56
Borax	1.75	109	Ivory	1.85	115
Brick, common	1.8	112	Limestone	2.6	163
Brick, fire	2.3	144	Marble	2.7	169
Brick, hard	2.0	125	Masonry	2.4	150
Brick, pressed	2.15	134	Mica	2.8	175
Brickwork, in mortar	1.6	100	Mortar	1.5	94
Brickwork, in cement	1.8	112	Phosphorus	1.8	112
Cement, Portland	3.1	194	Plaster of Paris	1.8	112
Chalk	2.6	163	Quartz	2.6	163
Charcoal	0.4	25	Salt, common	2.1	131
Coal, anthracite	1.5	94	Sand, dry	1.6	100
Coal, bituminous	1.27	79	Sand, wet	2.0	125
Concrete	2.2	137	Sandstone	2.3	144
Earth, loose	1.2	75	Slate	2.8	175
Earth, rammed	1.6	100	Soapstone	2.7	169
Emery	4.0	250	Soil, common black	2.0	125
Glass	2.6	163	Sulphur	2.0	125
Granite	2.65	166	Trap	3.0	187
Gravel	1.75	109	Tile	1.8	112

The weight per cubic foot is calculated on the basis of the specific gravity, and considers the material solidly packed. With many substances this is practically impossible, and a cubic foot of ordinary anthracite coal, for example, does not weigh more than from 55 to 65 pounds, due to the air spaces between the pieces of coal.

Specific Gravity of Liquids. — The specific gravity of liquids is the number which indicates how much a certain volume of the liquid weighs compared with an equal volume of water, the same as in the case of solid bodies. The density of liquids is also often expressed in degrees on the hydrometer, an instrument for determining the density of liquids, provided with graduations made to an arbitrary scale. The hydrometer consists of a glass tube with a bulb at one end containing air, and arranged with a weight at the bottom so as to float in an upright position in the liquid, the density of which is to be measured. The depth to which the hydrometer sinks in the liquid is read off on the graduated scale. The most commonly used hydrometer is the Baumé. The value of the degrees on the Baumé scale differs according to whether the liquid is heavier or lighter than water. The specific gravity for liquids heavier than water equals 145 ÷ (145 − degrees Baumé). For liquids lighter than water, the specific gravity equals 140 ÷ (130 + degrees Baumé).

Specific Gravity of Gases. — The specific gravity of gases is the number which indicates their weight in comparison with that of an equal volume of air. The specific gravity of air is 1, and the comparison is made at 32 degrees F.

Specific Gravity of Gases
(At 32 degrees F.)

Gas	Sp. Gr.	Gas	Sp. Gr.	Gas	Sp. Gr.
Air	1.000	Ether vapor	2.586	Marsh gas	0.555
Acetylene	0.920	Ethylene	0.967	Nitrogen	0.971
Alcohol vapor	1.601	Hydrofluoric acid	2.370	Nitric oxide	1.039
Ammonia	0.592	Hydrochloric acid	1.261	Nitrous oxide	1.527
Carbon dioxide	1.520	Hydrogen	0.069	Oxygen	1.106
Carbon monoxide	0.967	Illuminating gas	0.400	Sulphur dioxide	2.250
Chlorine	2.423	Mercury vapor	6.940	Water vapor	0.623

1 cubic foot of air at 32 degrees F. and atmospheric pressure weighs 0.0807 pound.

Average Weights and Volumes of Fuels

Anthracite coal, 1 cubic foot = 55 to 65 pounds.
1 ton (2240 pounds) = 34 to 41 cubic feet.
Bituminous coal, 1 cubic foot = 50 to 55 pounds.
1 ton (2240 pounds) = 41 to 45 cubic feet.
Charcoal, 1 cubic foot = 18 to 18.5 pounds.
1 ton (2240 pounds) = 120 to 124 cubic feet.
Coke, 1 cubic foot = 28 pounds.
1 ton (2240 pounds) = 80 cubic feet.

The average weight of a bushel of charcoal is 20 pounds; of a bushel of coke, 40 pounds; of a bushel of anthracite coal, 67 pounds; and of a bushel of bituminous coal, 60 pounds.

Weight of Wood. — The weight of seasoned wood per cord is approximately as follows, assuming about 70 cubic feet of *solid wood* per cord: Beech, 3300 pounds; chestnut, 2600 pounds; elm, 2900 pounds; maple, 3100 pounds; poplar, 2200 pounds; white pine, 2200 pounds; red oak, 3300 pounds; white oak, 3500 pounds.

Weight per Foot of Wood, Board Measure. — The following is the weight in pounds of various kinds of woods, commercially known as dry timber, per foot board measure: White oak, 4.16; white pine, 1.98; Douglas fir, 2.65; short-leaf yellow pine, 2.65; red pine, 2.60; hemlock, 2.08; spruce, 2.08; cypress, 2.39; cedar, 1.93; chestnut, 3.43; Georgia yellow pine, 3.17; California spruce, 2.08.

Specific Gravity of Liquids

Liquid	Sp. Gr.	Liquid	Sp. Gr.	Liquid	Sp. Gr.
Acetic acid	1.06	Fluoric acid	1.50	Petroleum oil	0.82
Alcohol, commerical	0.83	Gasoline	0.70	Phosphoric acid	1.78
Alcohol, pure	0.79	Kerosene	0.80	Rape oil	0.92
Ammonia	0.89	Linseed oil	0.94	Sulphuric acid	1.84
Benzine	0.69	Mineral oil	0.92	Tar	1.00
Bromine	2.97	Muriatic acid	1.20	Turpentine oil	0.87
Carbolic acid	0.96	Naphtha	0.76	Vinegar	1.08
Carbon disulphide	1.26	Nitric acid	1.22	Water	1.00
Cotton-seed oil	0.93	Olive oil	0.92	Water, sea	1.03
Ether, sulphuric	0.72	Palm oil	0.97	Whale oil	0.92

Degrees on Baumé's Hydrometer Converted into Specific Gravity

Deg. Baumé	Specific Gravity: Liquids Heavier than Water	Specific Gravity: Liquids Lighter than Water	Deg. Baumé	Specific Gravity: Liquids Heavier than Water	Specific Gravity: Liquids Lighter than Water	Deg. Baumé	Specific Gravity: Liquids Heavier than Water	Specific Gravity: Liquids Lighter than Water
0	1.000	......	27	1.229	0.892	54	1.593	0.761
1	1.007	......	28	1.239	0.886	55	1.611	0.757
2	1.014	......	29	1.250	0.881	56	1.629	0.753
3	1.021	......	30	1.261	0.875	57	1.648	0.749
4	1.028	......	31	1.272	0.870	58	1.667	0.745
5	1.036	......	32	1.283	0.864	59	1.686	0.741
6	1.043	......	33	1.295	0.859	60	1.706	0.737
7	1.051	......	34	1.306	0.854	61	1.726	0.733
8	1.058	......	35	1.318	0.849	62	1.747	0.729
9	1.066	......	36	1.330	0.843	63	1.768	0.725
10	1.074	1.000	37	1.343	0.838	64	1.790	0.721
11	1.082	0.993	38	1.355	0.833	65	1.813	0.718
12	1.090	0.986	39	1.368	0.828	66	1.836	0.714
13	1.099	0.979	40	1.381	0.824	67	1.859	0.710
14	1.107	0.972	41	1.394	0.819	68	1.883	0.707
15	1.115	0.966	42	1.408	0.814	69	1.908	0.704
16	1.124	0.959	43	1.422	0.809	70	1.933	0.700
17	1.133	0.952	44	1.436	0.805	71	1.959	0.696
18	1.142	0.946	45	1.450	0.800	72	1.986	0.693
19	1.151	0.940	46	1.465	0.796	73	2.014	0.689
20	1.160	0.933	47	1.480	0.791	74	2.042	0.686
21	1.169	0.927	48	1.495	0.787	75	2.071	0.683
22	1.179	0.921	49	1.510	0.782	76	2.101	0.679
23	1.189	0.915	50	1.526	0.778	77	2.132	0.676
24	1.198	0.909	51	1.542	0.773	78	2.164	0.673
25	1.208	0.903	52	1.559	0.769	79	2.197	0.669
26	1.219	0.897	53	1.576	0.765	80	2.230	0.666

Weight of Round Steel Bars per Running Inch

Diam. of Bar, Ins.	Weight per In., Lbs.	Diam. of Bar, Ins.	Weight per In., Lbs.	Diam. of Bar, Ins.	Weight per In., Lbs.	Diam. of Bar, Ins.	Weight per In., Lbs.	Diam. of Bar, Ins.	Weight per In., Lbs.
1/64	0.00005	2	0.89	5	5.56	8	14.3	14	43.6
1/32	0.00022	2 1/16	0.94	5 1/16	5.70	8 1/8	14.7	14 1/8	44.3
1/16	0.00087	2 1/8	1.00	5 1/8	5.84	8 1/4	15.1	14 1/4	45.1
3/32	0.0020	2 3/16	1.06	5 3/16	5.98	8 3/8	15.6	14 3/8	45.9
1/8	0.0035	2 1/4	1.13	5 1/4	6.13	8 1/2	16.1	14 1/2	46.7
5/32	0.0054	2 5/16	1.19	5 5/16	6.27	8 5/8	16.5	14 5/8	47.5
3/16	0.0078	2 3/8	1.25	5 3/8	6.42	8 3/4	17.0	14 3/4	48.4
7/32	0.0106	2 7/16	1.33	5 7/16	6.57	8 7/8	17.5	14 7/8	49.2
1/4	0.0139	2 1/2	1.39	5 1/2	6.72	9	18.0	15	50.0
9/32	0.0176	2 9/16	1.46	5 9/16	6.88	9 1/8	18.5	15 1/8	50.8
5/16	0.0217	2 5/8	1.53	5 5/8	7.03	9 1/4	19.0	15 1/4	51.7
11/32	0.0263	2 11/16	1.61	5 11/16	7.19	9 3/8	19.5	15 3/8	52.5
3/8	0.0313	2 3/4	1.68	5 3/4	7.35	9 1/2	20.1	15 1/2	53.4
13/32	0.0367	2 13/16	1.76	5 13/16	7.51	9 5/8	20.6	15 5/8	54.3
7/16	0.0425	2 7/8	1.84	5 7/8	7.67	9 3/4	21.1	15 3/4	55.1
15/32	0.0488	2 15/16	1.92	5 15/16	7.84	9 7/8	21.7	15 7/8	56.0
1/2	0.0556	3	2.00	6	8.00	10	22.2	16	56.9
17/32	0.0627	3 1/16	2.08	6 1/16	8.17	10 1/8	22.8	16 1/8	57.8
9/16	0.0703	3 1/8	2.17	6 1/8	8.34	10 1/4	23.4	16 1/4	58.7
19/32	0.0784	3 3/16	2.26	6 3/16	8.51	10 3/8	23.9	16 3/8	59.6
5/8	0.0868	3 1/4	2.35	6 1/4	8.68	10 1/2	24.5	16 1/2	60.5
21/32	0.0957	3 5/16	2.44	6 5/16	8.86	10 5/8	25.1	16 5/8	61.4
11/16	0.105	3 3/8	2.53	6 3/8	9.03	10 3/4	25.7	16 3/4	62.4
23/32	0.115	3 7/16	2.63	6 7/16	9.21	10 7/8	26.3	16 7/8	63.3
3/4	0.125	3 1/2	2.72	6 1/2	9.39	11	26.9	17	64.2
25/32	0.136	3 9/16	2.82	6 9/16	9.57	11 1/8	27.5	17 1/8	65.2
13/16	0.147	3 5/8	2.92	6 5/8	9.76	11 1/4	28.1	17 1/4	66.1
27/32	0.158	3 11/16	3.02	6 11/16	9.94	11 3/8	28.8	17 3/8	67.1
7/8	0.170	3 3/4	3.13	6 3/4	10.1	11 1/2	29.4	17 1/2	68.1
29/32	0.182	3 13/16	3.23	6 13/16	10.3	11 5/8	30.0	17 5/8	69.0
15/16	0.195	3 7/8	3.34	6 7/8	10.5	11 3/4	30.7	17 3/4	70.0
31/32	0.21	3 15/16	3.45	6 15/16	10.7	11 7/8	31.3	17 7/8	71.0
1	0.22	4	3.56	7	10.9	12	32.0	18	72.0
1 1/16	0.25	4 1/16	3.67	7 1/16	11.1	12 1/8	32.7	18 1/8	73.0
1 1/8	0.28	4 1/8	3.78	7 1/8	11.3	12 1/4	33.4	18 1/4	74.0
1 3/16	0.31	4 3/16	3 90	7 3/16	11.5	12 3/8	34.0	18 3/8	75.0
1 1/4	0.35	4 1/4	4.01	7 1/4	11.7	12 1/2	34.7	18 1/2	76.1
1 5/16	0.38	4 5/16	4.13	7 5/16	11.9	12 5/8	35.4	18 5/8	77.1
1 3/8	0 42	4 3/8	4.25	7 3/8	12.1	12 3/4	36.1	18 3/4	78 1
1 7/16	0.46	4 7/16	4.38	7 7/16	12.3	12 7/8	36.8	18 7/8	79.2
1 1/2	0.50	4 1/2	4 50	7 1/2	12.5	13	37.6	19	80.2
1 9/16	0.54	4 9/16	4 63	7 9/16	12.7	13 1/8	38.3	19 1/8	81.3
1 5/8	0.59	4 5/8	4.75	7 5/8	12.9	13 1/4	39.0	19 1/4	82.4
1 11/16	0.63	4 11/16	4.88	7 11/16	13.1	13 3/8	39.8	19 3/8	83.4
1 3/4	0.68	4 3/4	5.01	7 3/4	13.3	13 1/2	40.5	19 1/2	84.5
1 13/16	0.73	4 13/16	5.15	7 13/16	13.6	13 5/8	41.3	19 5/8	85.6
1 7/8	0.78	4 7/8	5.28	7 7/8	13.8	13 3/4	42.0	19 3/4	86.7
1 15/16	0.83	4 15/16	5.42	7 15/16	14.0	13 7/8	42.8	19 7/8	87.8

Weight of Round Steel Bars per Running Inch

Diam. of Bar, Ins.	Weight per In., Lbs.	Diam. of Bar, Ins.	Weight per In., Lbs.	Diam. of Bar, Ins.	Weight per In., Lbs.	Diam. of Bar, Ins.	Weight per In., Lbs.	Diam. of Bar, Ins.	Weight per In., Lbs.
20	88.9	26	150.3	32	227.6	38	321.0	44	430.3
20⅛	90.0	26⅛	151.7	32⅛	229.4	38⅛	323.1	44⅛	432.8
20¼	91.1	26¼	153.2	32¼	231.2	38¼	325.2	44¼	435.2
20⅜	92.3	26⅜	154.6	32⅜	233.0	38⅜	327.3	44⅜	437.7
20½	93.4	26½	156.1	32½	234.8	38½	329.5	44½	440.1
20⅝	94.6	26⅝	157.6	32⅝	236.6	38⅝	331.6	44⅝	442.6
20¾	95.7	26¾	159.0	32¾	238.4	38¾	333.8	44¾	445.1
20⅞	96.9	26⅞	160.5	32⅞	240.2	38⅞	335.9	44⅞	447.6
21	98.0	27	162.0	33	242.1	39	338.1	45	450.1
21⅛	99.2	27⅛	163.5	33⅛	243.9	39⅛	340.2	45⅛	452.6
21¼	100.4	27¼	165.1	33¼	245.7	39¼	342.4	45¼	455.1
21⅜	101.6	27⅜	166.6	33⅜	247.6	39⅜	344.6	45⅜	457.6
21½	102.7	27½	168.1	33½	249.4	39½	346.8	45½	460.2
21⅝	103.9	27⅝	169.6	33⅝	251.3	39⅝	349.0	45⅝	462.7
21¾	105.1	27¾	171.2	33¾	253.2	39¾	351.2	45¾	465.2
21⅞	106.4	27⅞	172.7	33⅞	255.1	39⅞	353.4	45⅞	467.8
22	107.6	28	174.3	34	256.9	40	355.6	46	470.3
22⅛	108.8	28⅛	175.8	34⅛	258.8	40⅛	357.9	46⅛	472.9
22¼	110.0	28¼	177.4	34¼	260.7	40¼	360.1	46¼	475.4
22⅜	111.3	28⅜	179.0	34⅜	262.6	40⅜	362.3	46⅜	478.0
22½	112.5	28½	180.5	34½	264.6	40½	364.6	46½	480.6
22⅝	113.8	28⅝	182.1	34⅝	266.5	40⅝	366.8	46⅝	483.2
22¾	115.0	28¾	183.7	34¾	268.4	40¾	369.1	46¾	485.8
22⅞	116.3	28⅞	185.3	34⅞	270.3	40⅞	371.4	46⅞	488.4
23	117.6	29	186.9	35	272.3	41	373.6	47	491.0
23⅛	118.9	29⅛	188.5	35⅛	274.2	41⅛	375.9	47⅛	493.6
23¼	120.1	29¼	190.2	35¼	276.2	41¼	378.2	47¼	496 2
23⅜	121.4	29⅜	191.8	35⅜	278.1	41⅜	380.5	47⅜	498.9
23½	122.7	29½	193.4	35½	280.1	41½	382.8	47½	501.5
23⅝	124.1	29⅝	195.1	35⅝	282.1	41⅝	385.1	47⅝	504.1
23¾	125.4	29¾	196.7	35¾	284.1	41¾	387.4	47¾	506.8
23⅞	126.7	29⅞	198.4	35⅞	286.1	41⅞	389.8	47⅞	509.4
24	128.0	30	200.0	36	288.1	42	392.1	48	512.1
24⅛	129.4	30⅛	201.7	36⅛	290.1	42⅛	394.4	48⅛	514.8
24¼	130.7	30¼	203.4	36¼	292.1	42¼	396.8	48¼	517.5
24⅜	132.1	30⅜	205.1	36⅜	294.1	42⅜	399.1	48⅜	520.1
24½	133.4	30½	206.8	36½	296.1	42½	401.5	48½	522.8
24⅝	134.8	30⅝	208.5	36⅝	298.1	42⅝	403.8	48⅝	525.5
24¾	136.2	30¾	210.2	36¾	300.2	42¾	406.2	48¾	528.2
24⅞	137.5	30⅞	211.9	36⅞	302.2	42⅞	408.6	48⅞	530.9
25	138.9	31	213.6	37	304.3	43	411.0	49	533.7
25⅛	140.3	31⅛	215.3	37⅛	306.3	43⅛	413.4	49⅛	536.4
25¼	141.7	31¼	217.1	37¼	308.4	43¼	415.8	49¼	539.1
25⅜	143.1	31⅜	218.8	37⅜	310.5	43⅜	418.2	49⅜	541.9
25½	144.5	31½	220.5	37½	312:6	43½	420.6	49½	544.6
25⅝	146.0	31⅝	222.3	37⅝	314.7	43⅝	423.0	49⅝	547.4
25¾	147.4	31¾	224.1	37¾	316.8	43¾	425.4	49¾	550.1
25⅞	148.8	31⅞	225.8	37⅞	318.9	43⅞	427.9	50	555.7

Weights of Square and Round Steel Bars in Pounds per Lineal Foot

Diam. of Round or Side of Square, Inches	Weight in Pounds		Diam. of Round or Side of Square, Inches	Weight in Pounds		Diam. of Round or Side of Square, Inches	Weight in Pounds	
	Square Bar	Round Bar		Square Bar	Round Bar		Square Bar	Round Bar
1/16	0.013	0.010	2 1/16	14.46	11.36	5	85.00	66.76
3/32	0.030	0.023	2 1/8	15.35	12.06	5 1/8	89.30	70.14
1/8	0.053	0.042	2 3/16	16.27	12.78	5 1/4	93.71	73.60
5/32	0.083	0.065	2 1/4	17.21	13.52	5 3/8	98.23	77.15
3/16	0.120	0.094	2 5/16	18.18	14.28	5 1/2	102.9	80.78
7/32	0.163	0.128	2 3/8	19.18	15.06	5 5/8	107.6	84.49
1/4	0.212	0.167	2 7/16	20.20	15.87	5 3/4	112.4	88.29
9/32	0.269	0.211	2 1/2	21.25	16.69	5 7/8	117.4	92.17
5/16	0.332	0.261	2 9/16	22.33	17.53	6	122.4	96.13
11/32	0.402	0.316	2 5/8	23.43	18.40	6 1/8	127.6	101.8
3/8	0.478	0.376	2 11/16	24.56	19.29	6 1/4	132.8	104.3
13/32	0.561	0.441	2 3/4	25.71	20.19	6 3/8	138.2	108.5
7/16	0.651	0.511	2 13/16	26.90	21.12	6 1/2	143.7	112.8
15/32	0.747	0.587	2 7/8	28.10	22.07	6 5/8	149.2	117.2
1/2	0.850	0.668	2 15/16	29.34	23.04	6 3/4	154.9	121.7
17/32	0.960	0.754	3	30.60	24.03	6 7/8	160.7	126.2
9/16	1.076	0.845	3 1/16	31.89	25.05	7	166.6	130.8
19/32	1.199	0.941	3 1/8	33.20	26.08	7 1/8	172.6	135.6
5/8	1.328	1.043	3 3/16	34.55	27.13	7 1/4	178.7	140.4
21/32	1.464	1.150	3 1/4	35.92	28.21	7 3/8	184.9	145.2
11/16	1.607	1.262	3 5/16	37.31	29.30	7 1/2	191.3	150.2
23/32	1.756	1.380	3 3/8	38.73	30.42	7 5/8	197.7	155.3
3/4	1.913	1.502	3 7/16	40.18	31.55	7 3/4	204.2	160.4
25/32	2.076	1.628	3 1/2	41.65	32.71	7 7/8	210.9	165.6
13/16	2.245	1.763	3 9/16	43.15	33.89	8	217.6	170.9
27/32	2.420	1.900	3 5/8	44.68	35.09	8 1/8	224.5	176.3
7/8	2.603	2.044	3 11/16	46.23	36.31	8 1/4	231.4	181.8
29/32	2.792	2.192	3 3/4	47.82	37.55	8 3/8	238.5	187.3
15/16	2.988	2.347	3 13/16	49.42	38.81	8 1/2	245.7	192.9
31/32	3.192	2.508	3 7/8	51.05	40.10	8 5/8	252.9	198.6
1	3.400	2.670	3 15/16	52.71	41.40	8 3/4	260.3	204.4
1 1/16	3.838	3.015	4	54.40	42.73	8 7/8	267.8	210.3
1 1/8	4.303	3.380	4 1/16	56.11	44.07	9	275.4	216.3
1 3/16	4.795	3.766	4 1/8	57.85	45.44	9 1/8	283.1	222.3
1 1/4	5.313	4.172	4 3/16	59.62	46.83	9 1/4	290.9	228.5
1 5/16	5.857	4.600	4 1/4	61.41	48.24	9 3/8	298.8	234.7
1 3/8	6.428	5.049	4 5/16	63.23	49.66	9 1/2	306.9	241.0
1 7/16	7.026	5.518	4 3/8	65.08	51.11	9 5/8	315.0	247.4
1 1/2	7.650	6.008	4 7/16	66.95	52.58	9 3/4	323.2	253.8
1 9/16	8.301	6.519	4 1/2	68.85	54.07	9 7/8	331.6	260.4
1 5/8	8.978	7.051	4 9/16	70.78	55.59	10	340.0	267.0
1 11/16	9.682	7.604	4 5/8	72.73	57.12	10 1/4	357.2	280.6
1 3/4	10.41	8.178	4 11/16	74.71	58.67	10 1/2	374.9	294.4
1 13/16	11.17	8.773	4 3/4	76.71	60.25	10 3/4	392.9	308.6
1 7/8	11.95	9.388	4 13/16	78.74	61.85	11	411.4	323.1
1 15/16	12.76	10.02	4 7/8	80.80	63.46	11 1/2	449.7	353.2
2	13.60	10.68	4 15/16	82.89	65.10	12	489.6	384.5

Weight in Pounds per Foot and per Inch of Hexagon Steel Bars

Width across Flats, Inches	Weight in Pounds: Per Running Foot	Weight in Pounds: Per Running Inch	Width across Flats, Inches	Weight in Pounds: Per Running Foot	Weight in Pounds: Per Running Inch	Width across Flats, Inches	Weight in Pounds: Per Running Foot	Weight in Pounds: Per Running Inch
1/4	0.195	0.016	3/4	1.66	0.138	1 9/16	7.17	0.597
9/32	0.23	0.019	13/16	1.91	0.159	1 5/8	7.76	0.647
5/16	0.29	0.024	7/8	2.25	0.187	1 11/16	8.37	0.697
11/32	0.36	0.030	29/32	2.40	0.200	1 3/4	9.00	0.750
3/8	0.43	0.036	15/16	2.58	0.215	1 7/8	10.32	0.860
13/32	0.50	0.042	1	2.94	0.245	2	11.78	0.982
7/16	0.56	0.047	1 1/16	3.33	0.277	2 1/8	13.30	1.108
15/32	0.64	0.053	1 3/32	3.52	0.293	2 1/4	14.91	1.242
1/2	0.73	0.061	1 1/8	3.73	0.311	2 3/8	16.61	1.384
17/32	0.82	0.068	1 3/16	4.15	0.346	2 1/2	18.40	1.533
9/16	0.93	0.077	1 1/4	4.60	0.383	2 5/8	20.29	1.691
19/32	1.10	0.092	1 5/16	5.07	0.422	2 3/4	22.27	1.856
5/8	1.15	0.096	1 3/8	5.57	0.464	3	26.50	2.208
11/16	1.40	0.117	1 7/16	6.07	0.506	3 1/4	31.10	2.592
23/32	1.52	0.127	1 1/2	6.62	0.552	3 1/2	36.07	3.006

Weights of Steel Plates. — For obtaining the weights of steel plates in pounds, the accompanying table may be used. First, multiply the length of the plate in inches by its breadth, and then this product by the constant C given in the table opposite the thickness of the plate.

Thickness	C	Thickness	C	Thickness	C	Thickness	C
3/16	0.053	3/8	0.105	9/16	0.158	3/4	0.210
1/4	0.070	7/16	0.123	5/8	0.175	7/8	0.245
5/16	0.087	1/2	0.140	11/16	0.193	1	0.280

Example: — Find the weight per square foot of area of steel plate, 7/16 inch thick.

$12 \times 12 \times 0.123 = 17.71$ pounds.

Weights of Hot-pressed Hexagon Nuts

Size of Bolt	Thickness of Nut	Width across Flats	Weight of 100 Nuts, Pounds	Size of Bolt	Thickness of Nut	Width across Flats	Weight of 100 Nuts, Pounds	Size of Bolt	Thickness of Nut	Width across Flats	Weight of 100 Nuts, Pounds
1/4	1/4	1/2	1.3	5/8	5/8	1 1/4	19.1	1 1/8	1 1/4	2	96
5/16	5/16	5/8	2.4	5/8	3/4	1 1/4	22.9	1 1/4	1 3/8	2 1/4	134
3/8	3/8	3/4	4.1	3/4	3/4	1 3/8	27.2	1 3/8	1 1/2	2 1/2	180
7/16	7/16	7/8	6.8	3/4	7/8	1 1/2	39.0	1 1/2	1 5/8	2 3/4	235
1/2	1/2	7/8	7.1	7/8	7/8	1 5/8	44.0	1 5/8	1 3/4	3	300
1/2	1/2	1	9.8	7/8	1	1 5/8	50.0	1 3/4	1 7/8	3 1/4	370
9/16	9/16	1 1/8	14.0	1	1	1 3/4	57.0	1 7/8	2	3 1/2	460
5/8	5/8	1 1/8	14.7	1	1 1/8	1 3/4	64.0	2	2	3 1/2	450

Weights of Flat Rolled Steel per Lineal Foot in Pounds

U.S. St'd Gage for Plate	Thickness in Inches	Width of Flat Steel, Inches						
		⅛	3/16	¼	5/16	⅜	7/16	½
0000000	0.5000	0.2126	0.3189	0.4252	0.5315	0.6378	0.7441	0.8504
000000	0.4688	0.1992	0.2988	0.3984	0.4980	0.5976	0.6972	0.7968
00000	0.4375	0.1860	0.2790	0.3720	0.4650	0.5580	0.6510	0.7440
0000	0.4063	0.1728	0.2592	0.3456	0.4320	0.5184	0.6048	0.6912
000	0.3750	0.1594	0.2391	0.3188	0.3985	0.4782	0.5579	0.6376
00	0.3438	0.1462	0.2193	0.2924	0.3655	0.4386	0.5117	0.5848
0	0.3125	0.1328	0.1992	0.2656	0.3320	0.3984	0.4648	0.5312
1	0.2813	0.1196	0.1794	0.2392	0.2990	0.3588	0.4186	0.4784
2	0.2656	0.1130	0.1695	0.2260	0.2825	0.3390	0.3955	0.4520
3	0.2500	0.1062	0.1593	0.2124	0.2655	0.3186	0.3717	0.4248
4	0.2344	0.0996	0.1494	0.1992	0.2490	0.2988	0.3486	0.3984
5	0.2188	0.0930	0.1395	0.1860	0.2325	0.2790	0.3255	0.3720
6	0.2031	0.0864	0.1296	0.1728	0.2160	0.2592	0.3024	0.3456
7	0.1875	0.0798	0.1197	0.1596	0.1995	0.2394	0.2793	0.3192
8	0.1719	0.0730	0.1095	0.1460	0.1825	0.2190	0.2555	0.2920
9	0.1563	0.0664	0.0996	0.1328	0.1660	0.1992	0.2324	0.2656
10	0.1406	0.0598	0.0897	0.1196	0.1495	0.1794	0.2093	0.2392
11	0.1250	0.0532	0.0798	0.1064	0.1330	0.1596	0.1862	0.2128
12	0.1094	0.0466	0.0699	0.0932	0.1165	0.1398	0.1631	0.1864
13	0.0938	0.0398	0.0597	0.0796	0.0995	0.1194	0.1393	0.1592
14	0.0781	0.0332	0.0498	0.0664	0.0830	0.0996	0.1162	0.1328
15	0.0703	0.0298	0.0447	0.0596	0.0745	0.0894	0.1043	0.1192
16	0.0625	0.0266	0.0399	0.0532	0.0665	0.0798	0.0931	0.1064
17	0.0563	0.0240	0.0360	0.0480	0.0600	0.0720	0.0840	0.0960
18	0.0500	0.0212	0.0318	0.0424	0.0530	0.0636	0.0742	0.0848
19	0.0438	0.0186	0.0279	0.0372	0.0465	0.0558	0.0651	0.0744
20	0.0375	0.0160	0.0240	0.0320	0.0400	0.0480	0.0560	0.0640
21	0.0344	0.0146	0.0219	0.0292	0.0365	0.0438	0.0511	0.0584
22	0.0313	0.0133	0.0201	0.0268	0.0335	0.0402	0.0469	0.0536
23	0.0281	0.0120	0.0180	0.0240	0.0300	0.0360	0.0420	0.0480
24	0.0250	0.0106	0.0159	0.0212	0.0265	0.0318	0.0371	0.0424
25	0.0219	0.0094	0.0141	0.0188	0.0235	0.0282	0.0329	0.0376
26	0.0188	0.0080	0.0120	0.0160	0.0200	0.0240	0.0280	0.0320
27	0.0172	0.0074	0.0111	0.0148	0.0185	0.0222	0.0259	0.0296
28	0.0156	0.0066	0.0099	0.0132	0.0165	0.0198	0.0231	0.0264
29	0.0141	0.0060	0.0090	0.0120	0.0150	0.0180	0.0210	0.0240
30	0.0125	0.0054	0.0081	0.0108	0.0135	0.0162	0.0189	0.0212
31	0.0109	0.0046	0.0069	0.0092	0.0115	0.0138	0.0161	0.0184
32	0.0102	0.0044	0.0066	0.0088	0.0110	0.0132	0.0154	0.0176
33	0.0094	0.0040	0.0060	0.0080	0.0100	0.0120	0.0140	0.0160
34	0.0086	0.0036	0.0054	0.0072	0.0090	0.0108	0.0126	0.0144
35	0.0078	0.0034	0.0051	0.0068	0.0085	0.0102	0.0119	0.0136
36	0.0070	0.0030	0.0045	0.0060	0.0075	0.0090	0.0105	0.0120
37	0.0066	0.0028	0.0042	0.0056	0.0070	0.0084	0.0098	0.0112
38	0.0063	0.0026	0.0039	0.0052	0.0065	0.0078	0.0091	0.0104

Weights of Flat Rolled Steel Per Lineal Foot in Pounds *(Continued)*

U. S. St'd Gage for Plate	Width of Flat Steel, Inches							
	9/16	5/8	11/16	3/4	13/16	7/8	1	2
0000000	0.9567	1.0630	1.1693	1.2756	1.3819	1.4882	1.7008	3.4016
000000	0.8964	0.9960	1.0956	1.1952	1.2948	1.3944	1.5936	3.1872
00000	0.8370	0.9300	1.0230	1.1160	1.2090	1.3020	1.4880	2.9760
0000	0.7776	0.8640	0.9500	1.0368	1.1232	1.2096	1.3824	2.7648
000	0.7173	0.7970	0.8767	0.9564	1.0361	1.1158	1.2752	2.5504
00	0.6579	0.7310	0.8041	0.8772	0.9505	1.0234	1.1696	2.3392
0	0.5976	0.6640	0.7300	0.7968	0.8632	0.9296	1.0624	2.1248
1	0.5382	0.5980	0.6578	0.7176	0.7774	0.8372	0.9568	1.9136
2	0.5085	0.5650	0.6215	0.6780	0.7345	0.7910	0.9040	1.8080
3	0.4779	0.5310	0.5841	0.6372	0.6903	0.7434	0.8496	1.6992
4	0.4482	0.4980	0.5478	0.5976	0.6474	0.6972	0.7968	1.5936
5	0.4185	0.4650	0.5115	0.5580	0.6045	0.6510	0.7440	1.4880
6	0.3888	0.4320	0.4752	0.5184	0.5616	0.6048	0.6912	1.3824
7	0.3591	0.3990	0.4389	0.4788	0.5187	0.5586	0.6384	1.2768
8	0.3285	0.3650	0.4015	0.4380	0.4745	0.5110	0.5840	1.1680
9	0.2988	0.3320	0.3652	0.3984	0.4316	0.4648	0.5312	1.0624
10	0.2691	0.2990	0.3289	0.3588	0.3887	0.4186	0.4784	0.9568
11	0.2394	0.2660	0.2926	0.3192	0.3458	0.3724	0.4256	0.8512
12	0.2097	0.2330	0.2563	0.2796	0.3029	0.3262	0.3728	0.7456
13	0.1791	0.1990	0.2189	0.2388	0.2587	0.2786	0.3184	0.6368
14	0.1494	0.1660	0.1826	0.1992	0.2158	0.2324	0.2656	0.5312
15	0.1341	0.1490	0.1639	0.1788	0.1937	0.2086	0.2384	0.4768
16	0.1197	0.1330	0.1463	0.1596	0.1729	0.1862	0.2128	0.4256
17	0.1080	0.1200	0.1320	0.1440	0.1560	0.1680	0.1920	0.3840
18	0.0954	0.1060	0.1166	0.1272	0.1378	0.1484	0.1696	0.3392
19	0.0837	0.0930	0.1023	0.1116	0.1209	0.1302	0.1488	0.2976
20	0.0720	0.0800	0.0880	0.0960	0.1040	0.1120	0.1280	0.2560
21	0.0657	0.0730	0.0803	0.0876	0.0949	0.1022	0.1168	0.2336
22	0.0603	0.0670	0.0737	0.0804	0.0871	0.0938	0.1072	0.2144
23	0.0540	0.0600	0.0660	0.0720	0.0780	0.0840	0.0960	0.1920
24	0.0477	0.0530	0.0583	0.0636	0.0689	0.0742	0.0848	0.1696
25	0.0423	0.0470	0.0517	0.0564	0.0611	0.0658	0.0752	0.1504
26	0.0360	0.0400	0.0440	0.0480	0.0520	0.0560	0.0640	0.1280
27	0.0333	0.0370	0.0407	0.0444	0.0481	0.0518	0.0592	0.1184
28	0.0297	0.0330	0.0363	0.0396	0.0429	0.0462	0.0528	0.1056
29	0.0270	0.0300	0.0330	0.0360	0.0390	0.0420	0.0480	0.0960
30	0.0243	0.0270	0.0297	0.0324	0.0351	0.0378	0.0432	0.0864
31	0.0207	0.0230	0.0253	0.0276	0.0299	0.0322	0.0368	0.0736
32	0.0198	0.0220	0.0242	0.0264	0.0286	0.0308	0.0352	0.0704
33	0.0180	0.0200	0.0220	0.0240	0.0260	0.0280	0.0320	0.0640
34	0.0162	0.0180	0.0198	0.0216	0.0234	0.0252	0.0288	0.0576
35	0.0153	0.0170	0.0187	0.0204	0.0221	0.0238	0.0272	0.0544
36	0.0135	0.0150	0.0165	0.0180	0.0195	0.0210	0.0240	0.0480
37	0.0126	0.0140	0.0154	0.0168	0.0182	0.0196	0.0224	0.0448
38	0.0117	0.0130	0.0143	0.0156	0.0169	0.0182	0.0208	0.0416

Weights of Flat Rolled Steel per Lineal Foot in Pounds (*Continued*)

U. S. St'd Gage for Plate	Width of Flat Steel, Inches							
	3	4	5	6	7	8	9	10
0000000	5.1024	6.8032	8.504	10.204	11.905	13.606	15.307	17.008
000000	4.7808	6.3744	7.968	9.561	11.155	12.748	14.342	15.936
00000	4.4640	5.9520	7.440	8.928	10.416	11.904	13.392	14.880
0000	4.1472	5.5296	6.912	8.294	9.676	11.059	12.441	13.824
000	3.8256	5.1008	6.376	7.651	8.926	10.201	11.476	12.752
00	3.5088	4.6784	5.848	7.017	8.187	9.356	10.526	11.696
0	3.1872	4.2496	5.312	6.374	7.436	8.499	9.561	10.624
1	2.8704	3.8272	4.784	5.740	6.697	7.654	8.611	9.568
2	2.7120	3.6160	4.520	5.424	6.328	7.232	8.136	9.040
3	2.5488	3.3984	4.248	5.097	5.947	6.796	7.646	8.496
4	2.3904	3.1872	3.984	4.780	5.577	6.374	7.171	7.968
5	2.2320	2.9760	3.720	4.464	5.208	5.952	6.696	7.440
6	2.0736	2.7648	3.456	4.147	4.838	5.529	6.220	6.912
7	1.9152	2 5536	3.192	3.830	4.468	5.107	5.745	6.384
8	1.7520	2.3360	2.920	3.504	4.088	4.672	5.256	5.840
9	1.5936	2.1248	2.656	3.187	3.718	4.249	4.780	5.312
10	1.4352	1.9136	2.392	2.870	3.348	3.827	4.305	4.784
11	1.2768	1.7024	2.128	2.553	2.979	3.404	3.830	4.256
12	1.1184	1.4912	1.864	2.236	2.609	2.982	3.355	3.728
13	0.9552	1.2736	1.592	1.910	2.228	2.547	2.865	3.184
14	0.7968	1.0624	1.328	1.593	1.859	2.124	2.390	2.656
15	0.7152	0.9536	1.192	1.430	1.668	1.907	2.145	2.384
16	0.6384	0.8512	1.064	1.276	1.489	1.702	1.915	2.128
17	0.5760	0.7680	0.960	1.152	1.344	1.536	1.728	1.920
18	0.5088	0.6786	0.848	1.027	1.187	1.357	1.526	1.696
19	0.4464	0.5952	0.744	0.892	1.041	1.190	1.319	1.488
20	0.3840	0.5120	0.640	0.768	0.896	1.024	1.152	1.280
21	0.3504	0.4672	0.584	0.700	0.817	0.934	1.051	1.168
22	0.3216	0.4288	0.536	0.643	0.750	0.857	0.964	1.072
23	0.2880	0.3840	0.480	0.576	0.672	0.768	0.864	0.960
24	0.2544	0.3392	0.424	0.508	0.593	0.678	0.763	0.848
25	0.2256	0.3008	0.376	0.451	0.526	0.601	0.676	0.752
26	0.1920	0.2560	0.320	0.384	0.448	0.512	0.596	0.640
27	0.1776	0.2368	0.296	0.355	0.414	0.473	0.532	0.592
28	0.1584	0.2112	0.264	0.316	0.369	0.422	0.475	0.528
29	0.1440	0.1920	0.240	0.288	0.336	0.384	0.432	0.480
30	0.1272	0.1768	0.216	0.259	0.302	0.353	0.388	0.432
31	0.1104	0.1472	0.184	0.220	0.257	0.297	0.331	0.368
32	0.1056	0.1408	0.176	0.211	0.246	0.281	0.316	0.352
33	0.0960	0.1280	0.160	0.192	0.224	0.256	0.288	0.320
34	0.0864	0.1152	0.144	0.172	0.201	0.230	0.259	0.288
35	0.0816	0.1088	0.136	0.163	0.190	0.217	0.244	0.272
36	0.0720	0.0960	0.120	0.148	0.168	0.192	0.216	0.240
37	0.0674	0.0896	0.112	0.134	0.156	0.179	0.201	0.224
38	0.0624	0.0832	0.104	0.124	0.145	0.166	0.187	0.208

Weights of Flat Rolled Steel per Lineal Foot in Pounds (*Continued*)

U. S. St'd Gage for Plate	Width of Flat Steel, Inches							
	11	12	13	14	15	16	18	20
0000000	18.708	20.409	22.110	23.811	25.512	27.212	30.614	34.016
000000	17.529	19.123	20.716	22.310	23.904	25.497	28.684	31.872
00000	16.368	17.856	19.344	20.832	22.320	23.808	26.784	29.760
0000	15.206	16.588	17.971	19.353	20.736	22.118	24.883	27.648
000	14.027	15.302	16.577	17.852	19.128	20.403	22.953	25.504
00	12.865	14.035	15.204	16.374	17.544	18.713	21.052	23.392
0	11.686	12.748	13.811	14.873	15.936	16.998	19.123	21.248
1	10.524	11.481	12.438	13.395	14.352	15.308	17.222	19.136
2	9.944	10.848	11.752	12.656	13.560	14.464	16.272	18.080
3	9.345	10.195	11.044	11.894	12.744	13.593	15.292	16.992
4	8.764	9.561	10.358	11.155	11.952	12.748	14.342	15.936
5	8.184	8.928	9.672	10.416	11.160	11.804	13.392	14.880
6	7.603	8.294	8.985	9.676	10.368	11.059	12.441	13.824
7	7.022	7.660	8.299	8.937	9.576	10.214	11.491	12.768
8	6.424	7.008	7.592	8.176	8.760	9.344	10.512	11.680
9	5.843	6.374	6.905	7.436	7.968	8.499	9.561	10.624
10	5.262	5.740	6.219	6.697	7.176	7.654	8.611	9.568
11	4.681	5.107	5.532	5.958	6.384	6.809	7.660	8.512
12	4.100	4.473	4.846	5.219	5.592	5.964	6.710	7.456
13	3.502	3.820	4.139	4.457	4.776	5.094	5.731	6.368
14	2.921	3.187	3.452	3.718	3.984	4.249	4.780	5.312
15	2.622	2.860	3.099	3.337	3.576	3.814	4.291	4.768
16	2.340	2.553	2.766	2.979	3.192	3.404	3.830	4.256
17	2.112	2.304	2.496	2.688	2.880	3.072	3.456	3.840
18	1.865	2.055	2.204	2.374	2.544	2.714	3.056	3.392
19	1.636	1.785	1.934	2.083	2.232	2.380	2.638	2.976
20	1.408	1.536	1.664	1.792	1.920	2.048	2.304	2.560
21	1.284	1.401	1.518	1.635	1.752	1.868	2.102	2.336
22	1.179	1.286	1.393	1.500	1.608	1.715	1.929	2.144
23	1.056	1.152	1.248	1.344	1.440	1.536	1.728	1.920
24	0.932	1.017	1.102	1.187	1.272	1.356	1.526	1.696
25	0.827	0.902	0.977	1.052	1.128	1.203	1.353	1.504
26	0.704	0.768	0.832	0.896	0.960	1.024	1.192	1.380
27	0.651	0.710	0.769	0.828	0.888	0.947	1.065	1.184
28	0.580	0.633	0.686	0.739	0.792	0.844	0.950	1.056
29	0.528	0.576	0.624	0.672	0.720	0.768	0.864	0.960
30	0.475	0.518	0.561	0.604	0.648	0.707	0.777	0.864
31	0.404	0.441	0.478	0.515	0.552	0.594	0.662	0.736
32	0.387	0.422	0.457	0.492	0.528	0.563	0.633	0.714
33	0.352	0.384	0.416	0.448	0.480	0.512	0.576	0.640
34	0.316	0.345	0.374	0.403	0.432	0.460	0.518	0.576
35	0.299	0.326	0.359	0.380	0.408	0.435	0.489	0.544
36	0.264	0.296	0.312	0.336	0.360	0.384	0.432	0.480
37	0.246	0.279	0.291	0.313	0.336	0.358	0.403	0.448
38	0.228	0.249	0.270	0.291	0.312	0.332	0.374	0.416

Weight of Flat Rolled Steel Bars in Pounds per Lineal Foot

(One cubic foot of rolled steel weighs 489.6 pounds.)

Thickness of Bar, Ins.	Width of Bar, Inches																	
	1/4	1/2	3/4	1	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3	3 1/4	3 1/2	3 3/4	4	4 1/4	4 1/2
1/16	0.053	0.106	0.159	0.21	0.27	0.32	0.37	0.42	0.48	0.53	0.58	0.64	0.69	0.74	0.80	0.85	0.90	0.96
1/8	0.106	0.212	0.319	0.42	0.53	0.64	0.74	0.85	0.96	1.06	1.17	1.27	1.38	1.49	1.59	1.70	1.81	1.91
3/16	0.159	0.319	0.478	0.64	0.80	0.96	1.12	1.28	1.43	1.59	1.75	1.91	2.07	2.23	2.39	2.55	2.71	2.87
1/4	0.213	0.425	0.638	0.85	1.06	1.28	1.49	1.70	1.91	2.13	2.34	2.55	2.76	2.98	3.19	3.40	3.61	3.83
5/16	0.266	0.531	0.797	1.06	1.33	1.59	1.86	2.13	2.39	2.66	2.92	3.19	3.45	3.72	3.98	4.25	4.52	4.78
3/8	0.319	0.638	0.956	1.28	1.59	1.91	2.23	2.55	2.87	3.19	3.51	3.83	4.14	4.46	4.78	5.10	5.42	5.74
7/16	0.372	0.744	1.12	1.49	1.86	2.23	2.60	2.98	3.35	3.72	4.09	4.46	4.83	5.21	5.58	5.95	6.32	6.69
1/2	0.425	0.850	1.28	1.70	2.13	2.55	2.98	3.40	3.83	4.25	4.68	5.10	5.53	5.95	6.38	6.80	7.22	7.65
9/16	0.478	0.956	1.43	1.91	2.39	2.87	3.35	3.83	4.30	4.78	5.26	5.74	6.22	6.69	7.17	7.65	8.13	8.61
5/8	0.531	1.06	1.59	2.13	2.66	3.19	3.72	4.25	4.78	5.31	5.84	6.38	6.91	7.44	7.97	8.50	9.03	9.56
11/16	0.584	1.17	1.75	2.34	2.92	3.51	4.09	4.68	5.26	5.84	6.43	7.01	7.60	8.18	8.77	9.35	9.93	10.52
3/4	0.638	1.28	1.91	2.55	3.19	3.83	4.46	5.10	5.74	6.38	7.01	7.65	8.29	8.93	9.56	10.20	10.84	11.48
13/16	0.691	1.38	2.07	2.76	3.45	4.14	4.83	5.53	6.22	6.91	7.60	8.29	8.98	9.67	10.36	11.05	11.74	12.43
7/8	0.744	1.49	2.23	2.98	3.72	4.46	5.21	5.95	6.69	7.44	8.18	8.93	9.67	10.41	11.16	11.90	12.64	13.39
15/16	0.797	1.59	2.39	3.19	3.98	4.78	5.58	6.38	7.17	7.97	8.77	9.56	10.36	11.16	11.95	12.75	13.55	14.34
1	0.850	1.70	2.55	3.40	4.25	5.10	5.95	6.80	7.65	8.50	9.35	10.20	11.05	11.90	12.75	13.60	14.45	15.30
1 1/16	0.903	1.81	2.71	3.61	4.52	5.42	6.32	7.23	8.13	9.03	9.93	10.84	11.74	12.64	13.55	14.45	15.35	16.26
1 1/8	0.956	1.91	2.87	3.83	4.78	5.74	6.69	7.65	8.61	9.56	10.52	11.48	12.43	13.39	14.34	15.30	16.26	17.21
1 3/16	1.01	2.02	3.03	4.04	5.05	6.06	7.07	8.08	9.08	10.09	11.10	12.11	13.12	14.13	15.14	16.15	17.16	18.17
1 1/4	1.06	2.12	3.19	4.25	5.31	6.38	7.44	8.50	9.56	10.63	11.69	12.75	13.81	14.88	15.94	17.00	18.06	19.13
1 5/16	1.12	2.23	3.34	4.46	5.58	6.69	7.81	8.93	10.04	11.16	12.27	13.39	14.50	15.62	16.73	17.85	18.97	20.08
1 3/8	1.17	2.34	3.50	4.68	5.84	7.01	8.18	9.35	10.52	11.69	12.86	14.03	15.19	16.36	17.53	18.70	19.87	21.04
1 7/16	1.22	2.44	3.66	4.89	6.11	7.33	8.55	9.78	11.00	12.22	13.44	14.66	15.88	17.11	18.33	19.55	20.77	21.99
1 1/2	1.27	2.55	3.82	5.10	6.38	7.65	8.93	10.20	11.48	12.75	14.03	15.30	16.58	17.85	19.13	20.40	21.68	22.95
1 9/16	1.33	2.66	3.98	5.31	6.64	7.97	9.30	10.63	11.95	13.28	14.61	15.92	17.27	18.59	19.92	21.25	22.58	23.91
1 5/8	1.38	2.76	4.14	5.53	6.91	8.29	9.67	11.05	12.43	13.81	15.19	16.58	17.96	19.34	20.72	22.10	23.48	24.86
1 11/16	1.43	2.87	4.30	5.74	7.17	8.61	10.04	11.48	12.91	14.34	15.78	17.21	18.65	20.08	21.52	22.95	24.38	25.82
1 3/4	1.49	2.97	4.46	5.95	7.44	8.93	10.41	11.90	13.39	14.88	16.36	17.85	19.34	20.83	22.31	23.80	25.29	26.78
1 13/16	1.54	3.08	4.62	6.16	7.70	9.24	10.78	12.33	13.87	15.41	16.95	18.49	20.03	21.57	23.11	24.65	26.19	27.73
1 7/8	1.59	3.19	4.78	6.38	7.97	9.56	11.16	12.75	14.34	15.94	17.53	19.13	20.72	22.31	23.91	25.50	27.09	28.69
1 15/16	1.65	3.29	4.94	6.59	8.23	9.88	11.53	13.18	14.82	16.47	18.12	19.76	21.41	23.06	24.70	26.35	28.00	29.64
2	1.70	3.40	5.10	6.80	8.50	10.20	11.90	13.60	15.30	17.00	18.70	20.40	22.10	23.80	25.50	27.20	28.90	30.60

Weight of Flat Rolled Steel Bars in Pounds per Lineal Foot

(One cubic foot of rolled steel weighs 489.6 pounds.)

Thickness of Bar, Ins.	Width of Bar, Inches																	
	4¾	5	5¼	5½	5¾	6	6½	7	7½	8	8½	9	9½	10	10½	11	11½	12
1/16	1.01	1.06	1.11	1.17	1.22	1.27	1.38	1.49	1.59	1.70	1.81	1.91	2.02	2.12	2.23	2.34	2.44	2.55
1/8	2.02	2.12	2.23	2.34	2.44	2.55	2.76	2.97	3.18	3.40	3.61	3.82	4.04	4.25	4.46	4.67	4.89	5.10
3/16	3.03	3.19	3.35	3.51	3.67	3.83	4.14	4.46	4.78	5.10	5.42	5.74	6.06	6.38	6.69	7.01	7.33	7.65
1/4	4.04	4.25	4.46	4.68	4.89	5.10	5.53	5.95	6.38	6.80	7.23	7.65	8.08	8.50	8.93	9.35	9.78	10.20
5/16	5.05	5.31	5.58	5.84	6.11	6.38	6.91	7.44	7.97	8.50	9.03	9.56	10.09	10.63	11.16	11.69	12.22	12.75
3/8	6.06	6.38	6.69	7.01	7.33	7.65	8.29	8.93	9.56	10.20	10.84	11.48	12.11	12.75	13.39	14.03	14.66	15.30
7/16	7.07	7.44	7.81	8.18	8.55	8.93	9.67	10.41	11.16	11.90	12.64	13.39	14.13	14.88	15.62	16.36	17.11	17.85
1/2	8.08	8.50	8.93	9.35	9.78	10.20	11.05	11.90	12.75	13.60	14.45	15.30	16.15	17.00	17.85	18.70	19.55	20.40
9/16	9.08	9.56	10.04	10.52	11.00	11.48	12.43	13.39	14.34	15.30	16.26	17.21	18.17	19.13	20.08	21.04	21.99	22.95
5/8	10.09	10.63	11.16	11.69	12.22	12.75	13.81	14.88	15.94	17.00	18.06	19.13	20.19	21.25	22.31	23.38	24.44	25.50
11/16	11.10	11.69	12.27	12.86	13.44	14.03	15.19	16.36	17.53	18.70	19.87	21.04	22.21	23.38	24.54	25.71	26.88	28.05
3/4	12.11	12.75	13.39	14.03	14.67	15.30	16.58	17.85	19.13	20.40	21.68	22.95	24.23	25.50	26.78	28.05	29.33	30.60
13/16	13.12	13.81	14.50	15.19	15.88	16.58	17.96	19.34	20.72	22.10	23.48	24.86	26.24	27.63	29.01	30.39	31.77	33.15
7/8	14.13	14.88	15.62	16.36	17.11	17.85	19.34	20.83	22.31	23.80	25.29	26.78	28.26	29.75	31.24	32.73	34.21	35.70
15/16	15.14	15.94	16.73	17.53	18.33	19.13	20.72	22.31	23.91	25.50	27.09	28.69	30.28	31.88	33.47	35.06	36.66	38.25
1	16.15	17.00	17.85	18.70	19.55	20.40	22.10	23.80	25.50	27.20	28.90	30.60	32.30	34.00	35.70	37.40	39.10	40.80
1 1/16	17.16	18.06	18.97	19.87	20.77	21.68	23.48	25.29	27.09	28.90	30.71	32.51	34.32	36.13	37.93	39.74	41.54	43.35
1 1/8	18.17	19.13	20.08	21.04	21.99	22.95	24.86	26.78	28.69	30.60	32.51	34.43	36.34	38.25	40.16	42.08	43.99	45.90
1 3/16	19.18	20.19	21.20	22.21	23.22	24.23	26.24	28.26	30.28	32.30	34.32	36.34	38.36	40.38	42.39	44.41	46.43	48.45
1 1/4	20.19	21.25	22.31	23.38	24.44	25.50	27.63	29.75	31.88	34.00	36.13	38.25	40.38	42.50	44.63	46.75	48.88	51.00
1 5/16	21.20	22.31	23.43	24.54	25.66	26.78	29.01	31.24	33.47	35.70	37.93	40.16	42.39	44.63	46.86	49.09	51.32	53.55
1 3/8	22.21	23.38	24.54	25.71	26.88	28.05	30.39	32.73	35.06	37.40	39.74	42.08	44.41	46.75	49.09	51.43	53.76	56.10
1 7/16	23.22	24.44	25.66	26.88	28.10	29.33	31.77	34.21	36.66	39.10	41.54	43.99	46.43	48.88	51.32	53.76	56.21	58.65
1 1/2	24.23	25.50	26.78	28.05	29.33	30.60	33.15	35.70	38.25	40.80	43.35	45.90	48.45	51.00	53.55	56.10	58.65	61.20
1 9/16	25.23	26.56	27.89	29.22	30.55	31.88	34.53	37.19	39.84	42.50	45.16	47.81	50.47	53.13	55.78	58.44	61.09	63.75
1 5/8	26.24	27.63	29.01	30.39	31.77	33.15	35.91	38.68	41.44	44.20	46.96	49.73	52.49	55.25	58.01	60.78	63.54	66.30
1 11/16	27.25	28.69	30.12	31.56	32.99	34.43	37.29	40.16	43.03	45.90	48.77	51.64	54.51	57.38	60.24	63.11	65.98	68.85
1 3/4	28.26	29.75	31.24	32.73	34.21	35.70	38.68	41.65	44.63	47.60	50.58	53.55	56.53	59.50	62.48	65.45	68.43	71.40
1 13/16	29.27	30.81	32.35	33.89	35.43	36.98	40.06	43.14	46.22	49.30	52.38	55.46	58.54	61.63	64.71	67.79	70.87	73.95
1 7/8	30.28	31.88	33.47	35.06	36.66	38.25	41.44	44.63	47.81	51.00	54.19	57.38	60.56	63.75	66.94	70.13	73.31	76.50
1 15/16	31.29	32.94	34.58	36.23	37.88	39.53	42.82	46.11	49.41	52.70	55.99	59.29	62.58	65.88	69.17	72.46	75.76	79.05
2	32.30	34.00	35.70	37.40	39.10	40.80	44.20	47.60	51.00	54.40	57.80	61.20	64.60	68.00	71.40	74.80	78.20	81.60

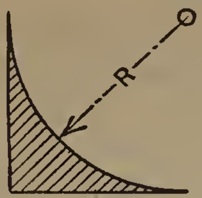
Weights of Sheets and Plates of Steel, Wrought Iron, Copper and Brass, in Pounds per Square Foot

Birmingham Gage						U. S. St'd Gage, 1893			
No. of Gage	Thickness, Inches	Weight per Square Foot				No. of Gage	Thickness, Inches (Approx.)	Weight per Square Foot	
		Steel	Iron	Copper	Brass			Iron	Steel
0000	0.454	18.52	18.16	20.57	19.43	0000000	0.500	20.00	20.40
000	0.425	17.34	17.00	19.25	18.19	000000	0.469	18.75	19.12
00	0.380	15.50	15.20	17.21	16.26	00000	0.437	17.50	17.85
0	0.340	13.87	13.60	15.40	14.55	0000	0.406	16.25	16.57
1	0.300	12.24	12.00	13.59	12.84	000	0.375	15.00	15.30
2	0.284	11.59	11.36	12.87	12.16	00	0.344	13.75	14.02
3	0.259	10.57	10.36	11.73	11.09	0	0.312	12.50	12.75
4	0.238	9.71	9.52	10.78	10.19	1	0.281	11.25	11.47
5	0.220	8.98	8.80	9.97	9.42	2	0.266	10.62	10.84
6	0.203	8.28	8.12	9.20	8.69	3	0.250	10.00	10.20
7	0.180	7.34	7.20	8.15	7.70	4	0.234	9.37	9.56
8	0.165	6.73	6.60	7.47	7.06	5	0.219	8.75	8.92
9	0.148	6.04	5.92	6.70	6.33	6	0.203	8.12	8.29
10	0.134	5.47	5.36	6.07	5.74	7	0.187	7.50	7.65
11	0.120	4.90	4.80	5.44	5.14	8	0.172	6.87	7.01
12	0.109	4.45	4.36	4.94	4.67	9	0.156	6.25	6.37
13	0.095	3.88	3.80	4.30	4.07	10	0.141	5.62	5.74
14	0.083	3.39	3.32	3.76	3.55	11	0.125	5.00	5.10
15	0.072	2.94	2.88	3.26	3.08	12	0.109	4.37	4.46
16	0.065	2.65	2.60	2.94	2.78	13	0.094	3.75	3.82
17	0.058	2.37	2.32	2.63	2.48	14	0.078	3.12	3.19
18	0.049	2.00	1.96	2.22	2.10	15	0.070	2.81	2.87
19	0.042	1.71	1.68	1.90	1.80	16	0.062	2.50	2.55
20	0.035	1.43	1.40	1.59	1.50	17	0.056	2.25	2.29
21	0.032	1.31	1.28	1.45	1.37	18	0.050	2.00	2.04
22	0.028	1.14	1.12	1.27	1.20	19	0.044	1.75	1.78
23	0.025	1.02	1.00	1.13	1.07	20	0.037	1.50	1.53
24	0.022	0.90	0.88	1.00	0.94	21	0.034	1.37	1.40
25	0.020	0.82	0.80	0.91	0.86	22	0.031	1.25	1.27
26	0.018	0.73	0.72	0.82	0.77	23	0.028	1.13	1.15
27	0.016	0.65	0.64	0.72	0.68	24	0.025	1.00	1.02
28	0.014	0.57	0.56	0.63	0.60	25	0.022	0.87	0.89
29	0.013	0.53	0.52	0.59	0.56	26	0.019	0.75	0.76
30	0.012	0.49	0.48	0.54	0.51	27	0.017	0.69	0.70
31	0.010	0.41	0.40	0.45	0.43	28	0.016	0.63	0.64
32	0.009	0.37	0.36	0.41	0.39	29	0.014	0.56	0.57
33	0.008	0.33	0.32	0.36	0.34	30	0.012	0.50	0.51
34	0.007	0.29	0.28	0.32	0.30	31	0.011	0.44	0.45
35	0.005	0.20	0.20	0.23	0.21	32	0.010	0.40	0.41
36	0.004	0.16	0.16	0.18	0.17	33	0.009	0.37	0.38
.....	.....	.....	.....	....	....	34	0.009	0.34	0.35
.....	.....	.....	.....	....	....	35	0.008	0.31	0.32
.....	.....	.....	.....	....	....	36	0.007	0.28	0.29
.....	.....	.....	.....	....	....	37	0.007	0.27	0.27
.....	.....	.....	.....	....	....	38	0.006	0.25	0.25

Weights of Sheets and Plates of Steel, Wrought Iron, Copper and Brass, in Pounds per Square Foot

American or Brown & Sharpe Gage						Standard Decimal Gage		
No. of Gage	Thickness, Inches	Weight per Square Foot				Thickness of Plate, Inches	Weight per Square Foot	
		Steel	Iron	Copper	Brass		Iron	Steel
0000	0.460	18.77	18.40	20.84	19.69	0.002	0.08	0.08
000	0.410	16.71	16.39	18.56	17.53	0.004	0.16	0.16
00	0.365	14.88	14.59	16.53	15.61	0.006	0.24	0.24
0	0.325	13.25	12.99	14.72	13.90	0.008	0.32	0.33
1	0.289	11.80	11.57	13.11	12.38	0.010	0.40	0.41
2	0.258	10.51	10.31	11.67	11.03	0.012	0.48	0.49
3	0.229	9.36	9.18	10.39	9.82	0.014	0.56	0.57
4	0.204	8.34	8.17	9.26	8.74	0.016	0.64	0.65
5	0.182	7.42	7.28	8.24	7.79	0.018	0.72	0.73
6	0.162	6.61	6.48	7.34	6.93	0.020	0.80	0.82
7	0.144	5.89	5.77	6.54	6.17	0.022	0.88	0.90
8	0.128	5.24	5.14	5.82	5.50	0.025	1.00	1.02
9	0.114	4.67	4.58	5.18	4.90	0.028	1.12	1.14
10	0.102	4.16	4.08	4.62	4.36	0.032	1.28	1.31
11	0.091	3.70	3.63	4.11	3.88	0.036	1.44	1.47
12	0.081	3.30	3.23	3.66	3.46	0.040	1.60	1.63
13	0.072	2.94	2.88	3.26	3.08	0.045	1.80	1.84
14	0.064	2.61	2.56	2.90	2.74	0.050	2.00	2.04
15	0.057	2.33	2.28	2.59	2.44	0.055	2.20	2.24
16	0.051	2.07	2.03	2.30	2.18	0.060	2.40	2.45
17	0.045	1.85	1.81	2.05	1.94	0.065	2.60	2.65
18	0.040	1.64	1.61	1.83	1.72	0.070	2.80	2.86
19	0.036	1.46	1.44	1.63	1.54	0.075	3.00	3.06
20	0.032	1.30	1.28	1.45	1.37	0.080	3.20	3.26
21	0.028	1.16	1.14	1.29	1.22	0.085	3.40	3.47
22	0.025	1.03	1.01	1.15	1.08	0.090	3.60	3.67
23	0.023	0.92	0.90	1.02	0.97	0.095	3.80	3.88
24	0.020	0.82	0.80	0.91	0.86	0.100	4.00	4.08
25	0.018	0.73	0.72	0.81	0.77	0.110	4.40	4.49
26	0.016	0.65	0.64	0.72	0.68	0.125	5.00	5.10
27	0.014	0.58	0.57	0.64	0.61	0.135	5.40	5.51
28	0.013	0.52	0.51	0.57	0.54	0.150	6.00	6.12
29	0.011	0.46	0.45	0.51	0.48	0.165	6.60	6.73
30	0.010	0.41	0.40	0.45	0.43	0.180	7.20	7.34
31	0.0089	0.36	0.36	0.40	0.38	0.200	8.00	8.16
32	0.0080	0.32	0.32	0.36	0.34	0.220	8.80	8.98
33	0.0071	0.29	0.28	0.32	0.30	0.240	9.60	9.79
34	0.0063	0.26	0.25	0.29	0.27	0.250	10.00	10.20
35	0.0056	0.23	0.22	0.25	0.24	....	.....	.....
36	0.0050	0.20	0.20	0.23	0.21	....	......	.....
37	0.0045	0.18	0.18	0.20	0.19	....	.....	.....
38	0.0040	0.16	0.16	0.18	0.17	....	.....	.....
39	0.0035	0.14	0.14	0.16	0.15	....	.....	.....
40	0.0031	0.13	0.13	0.14	0.13	....	.....	.....

Areas and Weights of Fillets of Steel, Cast Iron and Brass



Calculations are based on the following weights:
Steel ....489.6 pounds per cubic foot.
Cast iron.....450 pounds per cubic foot.
Cast brass....504 pounds per cubic foot.

Radius R, Inches	Area, Square Inches	Weight of Steel		Weight of Cast Iron		Weight of Cast Brass	
		Per Foot	Per Inch	Per Foot	Per Inch	Per Foot	Per Inch
¼	0.0134	0.0455	0.0038	0.0418	0.0035	0.0469	0.0040
5/16	0.0209	0.0712	0.0059	0.0655	0.0054	0.0733	0.0061
⅜	0.0302	0.1027	0.0085	0.0945	0.0078	0.1058	0.0088
7/16	0.0411	0.1397	0.0116	0.1285	0.0107	0.1439	0.0120
½	0.0536	0.1825	0.0152	0.1679	0.0140	0.1880	0.0157
9/16	0.0679	0.2310	0.0192	0.2125	0.0177	0.2380	0.0200
⅝	0.0834	0.2847	0.0237	0.2619	0.0218	0.2932	0.0244
11/16	0.1014	0.3447	0.0287	0.3171	0.0264	0.3550	0.0300
¾	0.1207	0.4105	0.0342	0.3777	0.0315	0.4228	0.0352
13/16	0.1416	0.4817	0.0401	0.4432	0.0369	0.4962	0.0414
⅞	0.1643	0.5580	0.0465	0.5134	0.0428	0.5747	0.0479
15/16	0.1886	0.6405	0.0534	0.5893	0.0491	0.6597	0.0550
1	0.2146	0.7300	0.0608	0.6716	0.0559	0.7519	0.0626
1⅛	0.2716	0.9250	0.0771	0.8510	0.0709	0.9527	0.0794
1¼	0.3353	1.140	0.0950	1.049	0.0874	1.174	0.0979
1⅜	0.4057	1.200	0.1000	1.104	0.0920	1.236	0.1030
1½	0.4828	1.642	0.1368	1.511	0.1259	1.691	0.1410
1⅝	0.5668	1.930	0.1608	1.776	0.1479	1.988	0.1657
1¾	0.6572	2.235	0.1862	2.056	0.1713	2.302	0.1920
1⅞	0.7545	2.565	0.2137	2.360	0.1970	2.642	0.2202
2	0.8585	2.917	0.2431	2.684	0.2237	3.005	0.2504
2⅛	0.9692	3.292	0.2743	3.029	0.2502	3.391	0.2826
2¼	1.086	3.695	0.3079	3.399	0.2832	3.806	0.3172
2⅜	1.210	4.115	0.3429	3.786	0.3155	4.238	0.3532
2½	1.341	4.560	0.3800	4.195	0.3496	4.697	0.3914
2⅝	1.478	5.030	0.4192	4.628	0.3857	5.181	0.4317
2¾	1.623	5.507	0.4589	5.066	0.4222	5.672	0.4727
2⅞	1.774	6.027	0.5022	5.545	0.4621	6.208	0.5017
3	1.931	6.565	0.5471	5.940	0.4950	6.762	0.5635
3⅛	2.096	7.125	0.5937	6.555	0.5462	7.339	0.6116
3¼	2.267	7.700	0.6417	7.084	0.5903	7.931	0.6609
3⅜	2.444	8.300	0.6917	7.636	0.6363	8.549	0.7124
3½	2.629	8.925	0.7438	8.211	0.6926	9.193	0.7661
3⅝	2.820	9.575	0.7979	8.809	0.7341	9.862	0.8220
3¾	3.018	10.27	0.8523	9.448	0.7873	10.58	0.8817
3⅞	3.222	10.97	0.9142	10.09	0.8408	11.30	0.9417
4	3.434	11.65	0.9709	10.72	0.8933	12.00	1.000
4¼	3.876	13.15	1.096	12.10	1.008	13.54	1.130
4½	4.346	14.77	1.231	13.59	1.132	15.21	1.270
4¾	4.842	16.45	1.371	15.13	1.261	16.94	1.412
5	5.365	18.25	1.521	16.79	1.400	18.80	1.570

Tin Plate Base Weight and Thickness

Base * Weight, Lbs., and Symbols	Weight per Sq. Ft.	Approx. Thickness, Inch	Base Weight, Lbs., and Symbols	Weight per Sq. Ft.	Approx. Thickness, Inch	Base Weight, Lbs., and Symbols	Weight per Sq. Ft.	Approx. Thickness, Inch
55	0.253	0.006	128–IXL	0.588	0.015	255–7 X	1.125	0.028
60	0.276	0.007	135–IX	0.620	0.015	268–D 4 X	1.230	0.031
65	0.298	0.007	139–DC	0.638	0.016	275–8 X	1.263	0.032
70	0.321	0.008	155–2 X	0.712	0.018	295–9 X	1.355	0.034
75	0.344	0.009	175–3 X	0.804	0.020	315–10 X	1.447	0.036
80	0.367	0.009	180–DX	0.827	0.021	335–11 X	1.539	0.038
85	0.390	0.010	195–4 X	0.895	0.022	355–12 X	1.631	0.041
90	0.413	0.010	210–D 2 X	0.964	0.024	375–13 X	1.722	0.043
95	0.436	0.011	215–5 X	0.988	0.025	395–14 X	1.814	0.045
100–ICL	0.459	0.011	235–6 X	1.08	0.027	415–15 X	1.906	0.048
107–IC	0.491	0.012	240–D 3 X	1.10	0.027	435–16 X	1.998	0.050

* Weight of standard "base box" containing 112 sheets, 14 × 20 inches.

Sheet Zinc Gage
(Matthiessen & Hegeler Zinc Co.)

Gage No.	Thickness, Inches	Gage No.	Thickness, Inches	Gage No.	Thickness, Inches	Gage No.	Thickness, Inches
1	0.002	8	0.016	15	0.040	22	0.090
2	0.004	9	0.018	16	0.045	23	0.100
3	0.006	10	0.020	17	0.050	24	0.125
4	0.008	11	0.024	18	0.055	25	0.250
5	0.010	12	0.028	19	0.060	26	0.375
6	0.012	13	0.032	20	0.070	27	0.500
7	0.014	14	0.036	21	0.080	28	1.000

American "Russia-Iron" Gage

Gage No.	Thickness, Ins.	Gage No.	Thickness, Ins.	Gage No.	Thickness, Ins.	Gage No.	Thickness, Ins.	Gage No.	Thickness, Ins.
7	0.015	9	0.017	11	0.020	13	0.024	15	0.027
8	0.016	10	0.018	12	0.021	14	0.025	16	0.030

Weight in Pounds per Square Foot of Zinc Plate

Gage No.	Weight in Pounds per Sq. Foot	Gage No.	Weight in Pounds per Sq. Foot	Gage No.	Weight in Pounds per Sq. Foot	Gage No.	Weight in Pounds per Sq. Foot
1	0.07	8	0.60	15	1.50	22	3.37
2	0.15	9	0.67	16	1.68	23	3.75
3	0.22	10	0.75	17	1.87	24	4.70
4	0.30	11	0.90	18	2.06	25	9.40
5	0.37	12	1.05	19	2.25	26	14.00
6	0.45	13	1.20	20	2.62	27	18.75
7	0.52	14	1.35	21	3.00	28	37.50

Table giving Number of Pieces in One Pound, when Weight of One Hundred Pieces is Known

Example: — 100 pieces weigh 11 pounds 5 ounces. From the table, there are then 8.84 pieces in one pound.

Ounces	Pounds 0	1	2	3	4	5	6	7	8
0	......	100.00	50.00	33.33	25.00	20.00	16.67	14.29	12.50
1	1600 00	94.12	48.48	32.65	24.61	19.75	16.49	14.16	12.40
2	800.00	88.88	47.06	32.00	24.24	19.51	16.33	14.03	12.31
3	533.33	84.21	45.71	31.37	23.88	19.27	16.16	13.91	12.21
4	400.00	80.00	44.44	30.77	23.52	19.05	16.00	13.79	12.12
5	320.00	76.19	43.24	30.19	23.19	18.82	15.84	13.67	12.03
6	266.66	72.73	42.11	29.63	22.86	18.60	15.69	13.56	11.94
7	228.57	69.57	41.03	29.09	22.53	18.39	15.53	13.44	11.85
8	200.00	66.67	40.00	28.57	22.22	18.18	15.38	13.33	11.76
9	177.78	64.00	39.02	28.07	21.92	17.98	15.24	13.22	11.68
10	160.00	61.54	38.09	27.58	21.62	17.78	15.09	13.11	11.59
11	145.45	59.26	37.21	27.12	21.33	17.58	14.95	13.01	11.51
12	133.33	57.14	36.36	26.67	21.05	17.39	14.81	12.90	11.43
13	123.08	55.17	35.56	26.23	20.78	17.20	14.68	12.80	11.35
14	114.29	53.33	34.78	25.81	20.51	17.02	14.54	12.70	11.27
15	106.67	51.61	34.04	25.40	20.25	16.84	14.41	12.60	11.19

Ounces	Pounds 9	10	11	12	13	14	15	16	17
0	11.11	10.00	9.09	8.33	7.69	7.14	6.66	6.25	5.88
1	11.03	9.94	9.04	8.29	7.65	7.11	6.64	6.23	5.86
2	10.96	9.88	8.99	8.25	7.62	7.08	6.61	6.20	5.84
3	10.89	9.82	8.94	8.20	7.58	7.04	6.59	6.17	5.82
4	10.81	9.76	8.89	8.16	7.54	7.01	6.56	6.15	5.80
5	10.74	9.69	8.84	8.12	7.51	6.98	6.53	6.13	5.78
6	10.67	9.64	8.79	8.08	7.47	6.95	6.50	6.11	5.76
7	10.59	9.58	8.74	8.04	7.44	6.92	6.47	6.08	5.74
8	10.53	9.52	8.69	8.00	7.41	6.89	6.45	6.06	5.72
9	10.46	9.47	8.65	7.96	7.37	6.86	6.43	6.04	5.70
10	10.39	9.41	8.60	7.92	7.34	6.84	6.40	6.01	5.68
11	10.32	9.36	8.56	7.88	7.31	6.81	6.37	5.98	5.66
12	10.25	9.30	8.51	7.84	7.27	6.78	6.35	5.96	5.64
13	10.19	9.25	8.46	7.80	7.24	6.75	6.32	5.94	5.62
14	10.13	9.19	8.42	7.76	7.21	6.72	6.30	5.92	5.60
15	10.06	9.14	8.38	7.73	7.17	6.69	6.27	5.90	5.58

Ounces	Pounds 18	19	20	21	22	23	24	25	26
0	5.56	5.26	5.00	4.76	4.54	4.35	4.16	4.00	3.84
1	5.54	5.24	4.98	4.74	4.53	4.34	4.15	3.99	3.83
2	5.52	5.23	4.96	4.73	4.52	4.33	4.14	3.98	3.82
3	5.50	5.21	4.95	4.71	4.51	4.32	4.13	3.97	3.81
4	5.48	5.19	4.94	4.70	4.50	4.30	4.12	3.96	3.81
5	5.46	5.18	4.92	4.69	4.49	4.29	4.11	3.95	3.80
6	5.44	5.16	4.90	4.68	4.48	4.28	4.10	3.94	3.79
7	5.42	5.14	4.89	4.66	4.46	4.27	4.09	3.93	3.78
8	5.40	5.12	4.88	4.65	4.44	4.25	4.08	3.92	3.77
9	5.38	5.11	4.86	4.64	4.43	4.24	4.07	3.91	3.76
10	5.36	5.09	4.84	4.63	4.42	4.23	4.06	3.90	3.75
11	5.34	5.08	4.83	4.61	4.41	4.22	4.05	3.89	3.74
12	5.32	5.07	4.82	4.60	4.39	4.21	4.04	3.88	3.73
13	5.31	5.05	4.81	4.59	4.38	4.19	4.03	3.87	3.73
14	5.30	5.04	4.79	4.57	4.37	4.18	4.02	3.86	3.72
15	5.28	5.02	4.78	4.56	4.36	4.17	4.01	3.85	3.71

COMPOSITION OF ALLOYS

Composition of Non-ferrous Alloys. — The Bureau of Steam Engineering of the U. S. Navy Department issued specifications in 1910 for materials to be supplied to the navy. The compositions of these materials are based on the most approved practice and on thorough experience with metals of this kind. The properties required in the most important of the non-ferrous casting materials are as follows:

Gun-bronze: The minimum tensile strength must be 30,000 pounds per square inch; the minimum yield-point, 15,000 pounds per square inch; and the elongation in 2 inches, 15 per cent.

Manganese-bronze: The minimum tensile strength must be 60,000 pounds per square inch; the yield-point, 30,000 pounds per square inch; and the elongation in 2 inches, 20 per cent.

Monel metal: The minimum tensile strength must be 65,000 pounds per square inch; the yield-point, 32,500 pounds per square inch; and the elongation in 2 inches, 25 per cent.

Phosphor-bronze: The minimum tensile strength must be 40,000 pounds per square inch; the yield-point 20,000 pounds per square inch; and the elongation in 2 inches, 20 per cent.

Specifications for Rolled Plates, Sheets, Shapes, etc. — The principal requirements of the more important non-ferrous materials used for rolled plates, sheets, shapes, etc., are as follows:

Copper: Ultimate tensile strength, 30,000 pounds per square inch; elongation in 2 inches, 25 per cent.

Muntz metal: Tensile strength, 40,000 pounds per square inch; elongation in 2 inches, 25 per cent.

Phosphor-bronze: Tensile strength, 50,000 pounds per square inch; elongation in 2 inches, 25 per cent.

Naval brass, 1 inch and below: Tensile strength, 62,000 pounds per square inch; elongation in 2 inches, 25 per cent. Above 1 inch: Tensile strength, 60,000 pounds per square inch; elongation in 2 inches, 28 per cent.

Manganese-bronze, 1 inch and below: Tensile strength, 72,000 pounds per square inch; elongation in 2 inches, 28 per cent. Above 1 inch: Tensile strength, 70,000 pounds per square inch; elongation in 2 inches, 30 per cent.

Monel metal, 1 inch and below: Tensile strength, 84,000 pounds per square inch; yield-point, 47,000 pounds per square inch; elongation in 2 inches, 25 per cent. Above 1 inch: Tensile strength, 80,000 pounds per square inch; yield-point, 45,000 pounds per square inch; elongation in 2 inches, 28 per cent.

In the case of Muntz metal, phosphor-bronze, naval brass and manganese-bronze, the yield-point should be one-half of the ultimate tensile strength specified.

White Metal. — Composition: 3.7 per cent of best refined copper; 88.8 per cent Banca tin; 7.5 per cent of regulus of antimony. (Well fluxed with borax and rosin in mixing.)

Turbine Materials. — Blading and binding strips may be made of the following compositions:

1. Monel metal.
2. Composition: 71–73 per cent copper; remainder zinc; lead not to exceed 0.2 per cent.
3. Cupro-nickel: 79–81 per cent copper; remainder nickel; iron not to exceed 0.75 per cent.
4. Composition: 53–56 per cent copper; 1.5 per cent nickel alloy; remainder zinc; lead not to exceed 0.5 per cent.
5. Phosphor-bronze.

U. S. Navy Specifications for Non-Ferrous Casting Metals

Letter	Name	Composition by Percentage					
		Copper	Tin	Zinc	Iron, Maximum	Lead, Maximum	Miscellaneous
B-c	Commercial brass	64–68	....	32–34	2.0	3.0	....
D-c	Muntz metal.....	59–62	....	39–41	....	0.6	....
F	Brazing metal....	84–86	....	Remainder	0.06	0.3	....
G	Gun bronze	87–89	9–11	1–3	0.06	0.2	....
H	Journal bronze...	82–84	12.5–14.5	2.5–4.5	0.06	1.0	....
M	Valve bronze.....	87	7	Remainder	0.06	1.0	....
Mn-c	Manganese-bronze*	57–60	0.75	37–40	1.0	....	Aluminum, 0.5; manganese, 0.3.
Mo-c	Monel metal......	Remainder	....	....	6.5	....	Aluminum, 0.5; nickel, 60 (min.).
N-c	Cast naval brass .	59–63	0.5–1.5	Remainder	0.06	0.6	....
P-c	Phosphor-bronze*	80–90	6–8	Remainder	0.06	0.2	Phosphorus, 0.3.
S-c	Screw pipe fittings, brass	77–80	4	13–19	0.1	3.0	....
Ni	Nickel...........	....	....	....	....	....	Nickel, 97 (min.).
Cu	Copper...........	99.8 (min.)	....	....	....	....	To be free from sulphur and other impurities and metals.
Zn	Zinc (rolled plates for boilers or slabs)	....	....	98½ (min.)	0.08	.....	....
Sn	Tin..............	....	99.6 (min.)	....	.....	....	....
Pb						Min.	....
	Lead (1)..........	....	....	....	....	99½	
	Lead (2)..........	....	....	....	....	97½	
Mo-c	Monel metal (ingots or shot)	....	....	....	3.0	....	Nickel, 60 per cent (min.); manganese, 2 per cent (min.) balance copper, with traces of other non-injurious ingredients.

* The figures given are approximate and are a guide as to the proportions of the elements except those for lead, aluminum and iron, which are maximum limits.

U. S. Navy Specifications for Non-Ferrous Rolled Metals

Letter	Name	Composition by Percentage					
		Copper	Tin	Zinc	Lead, Maximum	Iron, Maximum	Miscellaneous
A	Admiralty.......	70 (min.)	1 (min.)	Remainder	0.075	0.06	....
Be-r	Benedict nickel ..	84–86	....	....	....	....	Remainder nickel.
B-r	Sheet brass and piping	60–70	....	Remainder	0.5	....	....
B-r	Commerical brass rod	60–63	....	Remainder	3.0	....	....
Cu-r	Copper...........	99.5 (min.)	....	....	....	....	....
D-r	Muntz metal.....	59–62	....	Remainder	0.6	....	....
P-r	Phosphor-bronze*	85–95	5–10	4 (max.)	0.2	0.06	Phosphorus, 0.15.
Mn-r	Manganese-bronze*	57–60	0.5	37–40	....	1.0	Manganese, 0.30.
Mo-r	Monel metal.....	Remainder	....	....	....	3.5	Nickel, 60 (min.); aluminum, 0.5 (max.).
N-r	Rolled naval..... brass	59–63	0.5–1.5	Remainder	0.2	0.06	....

* The figures given are approximate and are a guide as to the proportions of the elements, except those for lead, aluminum and iron, which are maximum limits.

Composition of Miscellaneous Alloys

Alloys	Antimony	Bismuth	Copper	Iron	Lead	Nickel	Silver	Tin	Zinc
Brass, common yellow.....	...	...	61.6	....	2.9	....	....	0.2	35.3
Brass, to be rolled.........	...	...	32	....	....	....	....	1.5	10
Brass castings, common...	...	...	20	....	....	....	....	2.5	1.25
Gun metal................	...	...	8	....	....	....	....	1	....
Copper flanges............	...	...	9	....	....	....	....	0.26	1
Bronze statuary..........	...	...	91.4	....	1.37	....	....	1.7	5.53
German silver............	...	...	2	6.5	....	7.9	....	....	6.3
Britannia metal..........	50	25	....	....	....	....	....	25	....
Chinese white copper......	...	...	20.2	....	....	15.8	....	1.3	12.7
Pattern letters...........	15	15	....	....	70	....	....	....	....
Bell metal................	...	...	4	....	....	....	....	1	....
Chinese gongs............	...	...	40.5	....	....	....	....	9.2	....
White metal, ordinary.....	28.4	...	3.7	....	....	....	....	14.2	3.7
Spelter..................	...	...	1	....	....	....	....	....	1
Type metal...............	1	...	....	....	3–7	....	....	....	....

U. S. Navy Specifications for Non-Ferrous Metals

Letter	Name	Purposes for which suitable
B-c	Commercial brass .	Name and number plates; cases for instruments; oil cups; distributing boxes.
Cu-r	Copper...........	Copper pipe and tubing.
D-c	Muntz metal......	Where not subject to action of salt water.
F	Brazing metal.....	Brazing metal, and all flanges and fittings that are to be brazed.
G	Gun bronze.......	All composition valves 4 inches in diameter and above; expansion joints, flanged pipe fittings, gear wheels, bolts and nuts, miscellaneous brass castings, all parts where strength is required of brass castings or where subjected to salt water, and for all purposes where no other alloy is specified. Composition valves: Safety and relief, feed check and stop, surface blow, drain, air and water cocks, main stop, throttle, reducing, sea, safety sluice and manifolds at pumps. Condenser, distiller, feed-water heater, oil cooler. (Heads, shapes and water chests.) Pumps: Air-pump casing, valve seats, buckets, main circulating, water cylinders, valve boxes, water pistons, stuffing boxes, followers, glands, in general the water end of pumps complete except as specified. Stuffing boxes: Glands, bushings for iron or steel boxes. Blowers: Bearing boxes. Journal boxes: Distance pieces. Miscellaneous: Grease extractors; steam strainers separators, casing for stern tube and propeller shafts, propeller hub caps. Bearings: Main, stern tube, strut and spring. Spring bearings: Glands and baffles. Reciprocating engine: Intermediate and low pressure relief valves and casings, crosshead brasses, crank-pin brasses, eccentric straps and distance pieces.
H	Journal bronze....	Journal boxes, guide gibs, bushings, sleeves, slippers, etc. Reciprocating engine: Valve stem crosshead bottom brass; link block gibs, suspension link brasses.
M	Valve bronze......	All valves below 4 inches in diameter, for steam and general purposes, for which the material is not otherwise specified; hose couplings and fittings.
Mn-c	Manganese-bronze	Propeller hubs, blades, engine framing, and composition castings requiring great strength.

U. S. Navy Specifications for Non-Ferrous Metals

Letter	Name	Purposes for which suitable
Mo-c	Monel metal......	Same as for Mn-c, and pump liners, valve seats and castings requiring great strength, hardness and incorrodibility; shaft nuts and caps.
N-c	Cast naval brass ..	Valve handwheels, hand-rail fittings, ornamental and miscellaneous castings, and valves in water chests of condensers.
P-c	Phosphor-bronze..	Castings where strength and incorrodibility are required.
S-c	Screw pipe fittings.	For composition screwed fittings.
Ni	Nickel...........	For valve seats.
A	Admiralty metal ..	Condenser, distiller, feed-water heater and evaporator tubes.
B-n	Benedict metal....	Condenser, distiller, feed-water heater and evaporator tubes.
D-r	Muntz metal......	Bolts and nuts not subject to action of salt water.
P-r	Phosphor-bronze..	Pump rods, valve stems, valve springs, etc., exposed to salt water.
Mn-r	Manganese-bronze.	Rolled rounds, used principally for propeller blade bolts, air pump and condenser bolts, and parts requiring strength and incorrodibility.
Mo-r	Monel metal......	Rolled rounds, used principally for propeller blade bolts, air pump and condenser bolts, and parts requiring strength and incorrodibility, and pump rods.
N-r	Rolled naval brass.	Rolled rounds, used principally for propeller blade bolts, air pump condenser bolts, and parts requiring strength and incorrodibility, pump rods, tube sheets, supporting plates and shaft for valves in water heads for condensers.
B-r	Commercial rolled brass	Sheet brass: For liners, trim, etc. Brass pipe: Boiler dry pipe, hand rails. Distributing oil tubes and water pipes. Commercial brass rod for trim, and purposes where strength and incorrodibility are not required.
W	Anti-friction metal.	Lining bearings.

Calking Strips. — Composition: 62–63 per cent copper; remainder zinc; lead not to exceed 0.6 per cent.

Dummy and Gland Strips. — 1. Naval rolled brass. 2. Composition: 71–73 per cent copper; remainder zinc; lead not to exceed 0.2 per cent.

Thrust Rings. — Composition: 82–84 per cent copper; 12.5–14.5 per cent tin; 2.5–4.5 per cent lead.

Acid-resisting Alloy. — The following alloy is claimed to possess exceptional qualities with regard to its ability to resist the action of acids: Nickel, 66.6 per cent; chromium, 18 per cent; copper, 8.5 per cent; tungsten, 3.3 per cent; aluminum, 2 per cent; manganese, 1 per cent; titanium, 0.2 per cent; boron, 0.2 per cent and lithium, 0.2 per cent. This alloy is difficult to cast because it contracts considerably at the point of solidification. It can be drawn into wire and is easy to work.

Composition of Tin-Antimony-Copper Alloys*

Name of Metal	Tin, Per Cent	Antimony, Per Cent	Copper, Per Cent	Other Metals, Per Cent
English Britannia	94	5	1	....
Bearing	91	4.5	4.5	....
English Britannia, sheet	90.6	7.8	1.6	....
English Britannia, cast	90.6	9.2	0.2	....
Bearing	90	6	4	....
Bearing, Russian R.R.	90	8	2	....
English Britannia	90	6	2	Bismuth, 2
English Britannia	90	7	3	....
Bearing	89.3	8.9	1.8	....
Pewter	89.4	7	1.8	Lead, 1.8
Bearing	88.9	7.4	3.7	....
Bearing	88.8	7.5	3.7	....
Queen's metal	88.5	7.1	3.5	Zinc, 0.9
Queen's metal	88.5	7	3.5	Bismuth, 1
Bearing	87	7	6	....
English Britannia	85.5	9.7	1.8	Zinc, 3
Bearing, heavy	85	7.5	7.5	....
Jacoby metal	85	10	5	....
German Britannia	84	9	2	Zinc, 5
French, car bearings	83.3	11.2	5.5	....
Bearing	83.4	8.3	8.3	....
Bearing, German R.R.	83	11	6	....
Bearing, valve rods, etc	82	10	8	....
Bearing, French R.R.	82	12	6	....
Britannia, Baumgartel	81.9	16.3	1.8	....
Bearing, Swiss R.R.	80	10	10	....
Britannia, Ashberry	77.8	19.4	2.8	....
Britannia, Ashberry	77.8	19.4	....	Zinc, 2.8
Bearing, German	76	17	7	....
Bearing	73	18	9	....
Bearing	72	26	2	....
German Britannia	72	24	4	....
Bearing, Karmarsch	71.4	7.2	21.4	....
Bearing, valve packing	71	24	5	....
Bearing, Karmarsch	70.8	19.7	9.5	....
Minofor, Britannia	68.5	18.2	3.3	Zinc, 10
Bearing, G. W. R., England	67	11	22	....
Bearing, French R.R	67	22	11	....
Dewrance metal, locomotive	33.3	44.5	22.2	....

* From "Notes on the Metallography of Alloys," by William Campbell, presented at the Cleveland, 1912, meeting of the American Institute of Mining Engineers.

Composition of Lead-Tin-Antimony Alloys*

Name of Metal	Lead, Per Cent	Tin, Per Cent	Antimony, Per Cent	Other Metals, Per Cent
Electrotype metal	93	3	4	....
Bearing metal	86	1	13	....
Linotype metal	85	3	12	....
Bearing	83.3	8.3	8.4	....
Stereotype metal	82	6	12	....
Bearing	82	2	16	....
Stereotype	82	3.2	14.8	....
Bearing	80	10	10	....
Bearing, French R.R.	80	12	8	....
Bearing	80	5	15	....
Bearing, like Glyco, etc.	80.5	4.5	14.5	Arsenic, 0.5
Bearing, like Magnolia	78	6	16	....
Bearing, Magnolia and Tandem	77.3	5.9	16.8	....
Type metal	77.5	6.5	16	....
Bearing, anti-friction	77	10	12.5	Copper, 0.5
Bearing, like Coleco	77	8	14	Copper, 1.0
Bearing	76	7	17	....
Metallic packing, French R.R.	76	14	10	....
Bearing, American R.R.	73.5	8	18.5	....
Piston packing, French R.R.	73	12	15	....
Bearing, French R.R.	70	20	10	....
Stereotype, Mackenzie metal	70	13	17	....
Bearing, Paris-Lyon-Mediterranee R.R.	70	10	20	....
Type	70	10	18	Copper, 2
Bearing, American R.R.	68	21	11	....
Bearing, graphite metal	68	15	17	....
Stereotype	68	17	15	....
Type	63.2	12	24	Copper, 0.8
Bearing	62.5	26.2	10	Copper, 1.3
Bearing	62	28	10	....
Type	60.5	14.5	24.25	Copper, 0.75
Type	60	35	5	....
Bearing	60	20	20	....
Type, common	60	10	30	....
Solder	60	39	1	....
Type	55.5	40	4.5	....
Type, best	50	25	25	....
Bearing	48	40	10	Copper, 2
Bearing, American R.R., No. 2	46	36.5	16.5	Copper, 1
Hoyle's metal	42	46	12	....
Bearing, French R.R.	42	42	16	....
Bearing	40	45	15	....
Bearing, German	40	42	16	Copper, 2
Bearing, Italian R.R.	37	38	25	....
Stereotype	35	60	5	....
White metal	33	54	10.6	Copper, 2.4
Bearing	10	75	15	....
Bearing	10	75	12	Copper, 3
For small castings	5	75	20	....

* See note on preceding page for explanation.

Aluminum Alloys. — In order to increase the tensile strength and rigidity of aluminum and still retain the valuable property of lightness, aluminum has been alloyed with various materials such as copper, zinc, and magnesium. Plates and bars made from these alloys have ultimate tensile strengths varying from 40,000 to 50,000 pounds per square inch, an elastic limit of from 55 to 60 per cent of the tensile strength, an elongation of 20 per cent in 2 inches and a reduction in area of 25 per cent. Some of the special aluminum alloys on the market have ultimate tensile strengths which exceed considerably the figures given.

Aluminum-zinc Alloys. — The addition of zinc to aluminum alloys facilitates the production of good castings and effectively resists corrosion. The zinc content should not be less than 15 per cent and 20 or 25 per cent of zinc will increase the tensile strength. The aluminum-zinc alloys in general usually contain from 15 to about 33 per cent zinc. The addition of a small percentage of copper greatly increases the tensile strength. An aluminum-zinc alloy containing 25 per cent zinc is probably the most generally used. This alloy can be rolled into bars and drawn into wire, and when cast in sand molds has a tensile strength of about 27,000 pounds per square inch and a specific gravity of 3.4. An alloy known by the trade name "Alzene" contains two parts of aluminum to one of zinc. It is used for castings and the strength is equal to cast iron. This alloy is used for crank and transmission cases of automobile motors and for other parts requiring lightness.

Aluminum-brass. — This alloy is used when accurately sized castings are required. The alloy contains 3.1 per cent aluminum, 26.4 per cent zinc, and 70.5 per cent copper. The material cannot be worked easily if the aluminum exceeds 4 per cent. The tensile strength of this alloy is about 42,000 pounds per square inch and the elastic limit about 17,000 pounds per square inch.

Aluminum-bronze. — In aluminum-bronze, copper is the basic metal as in the case of aluminum-brass. The aluminum content varies from 5 to 11 per cent and the remainder of the alloy is principally copper. The tensile strength of aluminum-bronze varies in general from 40,000 to 60,000 pounds per square inch and may be much higher under favorable conditions. Tests made with aluminum-bronze containing exceptionally high-grade material, made from very pure metals, showed a tensile strength of 100,000 pounds per square inch for forged bars, when the aluminum content was 10 per cent. As the aluminum content was reduced the tensile strength diminished. About 9 or 10 per cent of aluminum is preferable.

Aluminum-copper Alloys. — Alloys of this class contain from 2 to 10 per cent of copper, the remainder being aluminum. Such alloys can be cast or rolled into bars. The tensile strength for a 2-per cent copper alloy is about 43,000 pounds per square inch and for an 8-per cent alloy about 56,000 pounds per square inch.

Aluminum-magnesium Alloys. — Alloys of the aluminum-magnesium class may be divided into two general groups, one of which contains aluminum as the basic metal whereas the other is composed chiefly of magnesium. One of the most important alloys of the first group is known under the trade name "Magnalium." This alloy usually contains from 1.6 to 2 per cent of magnesium and in addition small percentages of copper, nickel, tin and lead, the composition varying. This metal is lighter than aluminum castings or even lighter than pure aluminum, the specific gravity varying from 2.5 to 2.57. Magnalium can be forged and also drawn into fine wire as well as cast. The castings are more dense than ordinary aluminum castings. This metal has been used successfully for the cylinders of aeronautic and automobile motors as well as for many others parts requiring light weight. The tensile strength of magnalium castings is increased considerably by rapid cooling; thus the tensile strength of magnalium sand castings containing

2 per cent of magnesium is nearly 18,000 pounds per square inch. When castings containing the same amount of magnesium are chilled, the tensile strength is about 28,000 pounds per square inch and water-chilled castings with 2 per cent magnesium have a tensile strength of 40,000 pounds per square inch. By increasing the magnesium content to 10 per cent, the tensile strength of a chilled casting is about 33,000 pounds per square inch and of a water-chilled casting, about 61,000 pounds per square inch. The tensile strength of magnalium wire drawn from forged material was 53,000 pounds per square inch. The tensile strength of a drawn bar was 60,000 pounds and that of a tube 74,000 pounds per square inch.

Another aluminum alloy which has remarkable physical properties is known as "Duralumin." This alloy is a little heavier than pure aluminum and it is not suitable for castings but it can be drawn, rolled or forged. The tensile strength ranges from 30,000 to 88,000 pounds per square inch.

Alloys of the second group previously referred to contain from 80 to 95.5 per cent magnesium, the remainder of the composition being principally aluminum.

Copper-aluminum Alloys. — An alloy containing about 90 per cent of copper and 10 per cent of aluminum is remarkable for its high tensile strength, its resistance to corrosion, and its wearing qualities. The physical properties resemble those of 0.35 per cent carbon Bessemer steel, and are about as follows: Ultimate tensile strength, 70,000 pounds per square inch; elongation in two inches, 20 per cent; reduction in area, 21 per cent; specific gravity, 7.5. This bronze is about 10 per cent lighter than either yellow brass or manganese-bronze; 17 per cent lighter than phosphor-bronze; and 15 per cent lighter than red brass.

Copper-zinc-aluminum Alloys. — An important alloy in this group is composed of 56 per cent of copper, 43.5 per cent of zinc, 0.5 per cent of aluminum, with a very small percentage of manganese. This alloy is commonly known as "manganese-bronze." It is used for castings where great strength, toughness and resistance to corrosion are required. It is not a good bearing bronze. The ultimate tensile strength is about 60,000 pounds per square inch but may be as high as 100,000 pounds per square inch for rolled manganese bronze.

S. A. E. Standard Aluminum Alloys. — The Society of Automotive Engineers' standard specifications for three classes of aluminum alloys are as follows:

Specification No. 30. — This is one of the lightest of the aluminum alloys, possessing a high degree of strength and applicable where a tough light alloy is required in automobile construction. *Composition:* Aluminum, not less than 90 per cent; copper, 8.5 to 7 per cent. Total impurities should not exceed 1.7 per cent, of which not over 0.2 per cent shall be zinc. The only impurities allowed are carbon, silicon, iron, zinc, and manganese.

Specification No. 31. — This alloy possesses strength and closeness of grain and can be cast solid and free from blow-holes. It is a light metal, the specific gravity being approximately 3. *Composition:* Aluminum, not less than 80 per cent; zinc, not over 15 per cent; copper, between 2 and 3 per cent, and manganese, not over 0.4 per cent. The total impurities should not exceed 1.65 per cent, of which silicon should not exceed 0.5 per cent; iron should not exceed 1.0 per cent, and lead should not exceed 0.15 per cent.

Specification No. 32. — This alloy can be used where cheap castings are desired which will not be subjected to any great strains. It is used for flat plates, foot boards and running boards of automobiles, etc. *Composition:* Aluminum, 65 per cent; zinc, 35 per cent; total impurities not to exceed 1.65 per cent.

Phosphor-Bronze. — Phosphor-bronze is an alloy consisting of copper, tin, and phosphorus, with small percentages of zinc, iron and lead. For cast phosphor-bronze, the United States Navy specifications require a composition of from 80 to

90 per cent copper, from 6 to 8 per cent of tin, 0 3 per cent of phosphorus, not more than 0.06 per cent of iron, and not more than 0.2 per cent of lead, with the remainder zinc. For rolled metals, the same specifications call for from 85 to 90 per cent of copper, from 5 to 10 per cent of tin, 0.15 per cent of phosphorus, with a maximum of 0.06 per cent of iron, 0.2 per cent of lead, and 4 per cent of zinc. In many phosphor-bronzes, the phosphorus content may reach 2 per cent, but should not exceed this value.

Phosphor-bronze for worms and gears usually contains from 10 to 12 per cent of tin and from 0.7 to 1 per cent of phosphorus. If the wear is excessive, the phosphorus content may be increased to from 1 to 1.5 per cent. Phosphor-bronze resists corrosion to a considerable extent and is used for parts that are exposed to the action of salt water. The tensile strength varies from 20,000 to 30,000 pounds per square inch, with an elastic limit of from 10,000 to 15,000 pounds per square inch, and an elongation of from 2 to 6 per cent.

S. A. E. Standard Phosphor-Bronze. — This metal is similar to that specified by many railroads and it is an excellent composition where good anti-friction qualities are desired. It is adapted to heavy loads and severe usage and should be used only against hardened steel in automobile construction. *Composition:* Copper, 80 per cent; tin, 10 per cent; lead, 10 per cent; phosphorus, from 0.05 to 0.25 per cent.

S. A. E. Standard for Manganese Bronze Sheets and Rods. — Manganese bronze is one of the strongest non-ferrous alloys and it can readily be worked both hot and cold. The tensile strength is approximately 70,000 pounds per square inch, the elastic limit 30,000 to 35,000 pounds per square inch, and the elongations 25 to 30 per cent in 2 inches. Cold rolling or drawing increases the tensile strength and elastic limit, and reduces the elongation correspondingly. Manganese bronze is obtainable in cold-rolled sheets, strips and rods, cold-drawn rods, and hot-rolled sheets and rods. *Desired composition:* Copper, 56 to 60 per cent; tin, 0.5 to 1.5 per cent; iron, 0.5 to 1.5 per cent; manganese, 0.1 to 0.75 per cent; lead and impurities not over 0.25 per cent; zinc, remainder.

Sheet manganese bronze in cold-rolled strips, sheets, and rolls, is furnished either "annealed" or "hard-rolled."

S. A. E. Standard Cast Manganese Bronze. — Manganese bronze is understood to mean a metal consisting principally of copper and zinc in the approximate proportion of 60 to 40 with iron in small quantities and manganese in variable quantities. This bronze is of value for castings, where strength and toughness are required. The tensile strength should be about 60,000 pounds per square inch; the yield point one-half of this value; and elongation in 2 inches, 20 per cent.

S. A. E. Standard Gear Bronze. — This bronze is commonly known as "English gear bronze" and is used extensively both in Europe and in the United States. It is very serviceable for gears and worms and is adapted for severe duty and when quiet operation is essential. Sometimes this alloy is tempered with a ferrous hardener, quantities up to 4 per cent giving excellent results. *Composition:* Copper, 88 to 89 per cent; tin, 11 to 12 per cent, and phosphorus, 0.15 to 0.30 per cent.

S. A. E. Standard Hard Cast Bronze. — This bronze is similar to the U. S. government bronze "G." It is a general utility bronze, adapted for severe working conditions where heavy pressures and high speeds obtain. The tensile strength is approximately 35,000 pounds per square inch. *Composition:* Copper, from 87 to 88 per cent; tin, from 9.5 to 10.5 per cent, and zinc, from 1.5 to 2.5 per cent.

S. A. E. Standard Brass Casting Metals. — Specification No. 27 applies to "red brass," which is a high grade of composition metal that is excellent as a bearing metal where speed and pressure are not excessive. It is largely used for light castings and possesses good machining qualities. *Composition:* Copper, 85 per cent; tin, 5 per cent; lead, 5 per cent; zinc, 5 per cent. A tolerance of 1 per cent, plus or minus, is allowed. Impurities of over 0.25 per cent are not permitted.

Specification No. 28 is for a high grade of "yellow brass," which is tough and possesses good machining qualities. Its use is suggested in preference to ordinary commercial yellow brass castings, some of which contain from 1 to 3 per cent of iron, which is a very undesirable element as it is liable to cause blow-holes and hard spots. *Composition:* Copper, from 62 to 65 per cent; lead, from 2 to 4 per cent; zinc, from 31 to 36 per cent. Total impurities in excess of 0.5 per cent are not permitted.

Stellite. — Stellite is a non-ferrous alloy containing varying percentages of chromium and cobalt with a small percentage of molybdenum or tungsten, the latter two elements making the alloy especially suitable for the manufacture of high-speed lathe tools, or cutters. Stellite cannot be forged or machined except by grinding, but can be readily cast. Tools made from stellite retain their cutting quality even at red heat and can be used at a much higher peripheral speed than tools made from alloy steels.

When alloyed with tungsten, the metal becomes distinctly harder than when molybdenum is used. A content of 10 per cent of tungsten produces an excellent alloy for cold chisels and woodworking tools; 40 per cent of tungsten makes the metal suitable for turning cast iron. Stellite works best when operated at high speed and with a comparatively light feed. For some classes of work, stellite has its disadvantages. For instance, on soft iron or steel, where a large amount of metal must be removed in one cut, little, if any, advantage will be gained by using a stellite tool in preference to a good high-speed steel tool. The reason is that a high-speed steel tool will resist a much greater pressure than a stellite tool, and while it cannot be worked at as high a speed, it will remove a greater amount of metal in a given time, owing to its strength. However, there are many cases in which a stellite tool shows a marked increase in production over any other type of cutting tool.

Nichrome. — Nichrome is an alloy composed of nickel and chromium, which is practically non-corrosive and far superior to nickel in its ability to withstand high temperatures. Its melting point is about 1550 degrees C. (about 2800 degrees F.). Nichrome is not injured by oxidation of the exposed surface at high temperatures, and it is very strong even when heated red hot. The strength of a nichrome casting, when cold, varies from 45,000 to 50,000 pounds per square inch. At a temperature of 1800 degrees F., nichrome has a tensile strength of about 30,000 pounds per square inch, and it is tough and will bend considerably before breaking, even when heated red or white hot. This alloy has been used for some time in the manufacture of resistance coils for electrical heating apparatus and rheostats, and the physical properties which adapt it for use in electrical work have proved of practical value in various other ways.

Bakelite. — This is a synthetic organic substance resulting from the chemical condensation of phenol and formaldehyde. It may be used in either the liquid or solid form. As a liquid it is used for impregnating porous materials, for enameling under heat and pressure and as a binding agent for molded compounds. Solid bakelite is unaffected by water, steam, oils and almost all chemicals. The solid substance does not melt or soften at ordinary machine temperatures and is destroyed only at temperatures in the vicinity of 300 degrees C. It is easily and accurately molded and compares favorably with rubber and gutta-percha in every way except that it is not as flexible as those substances. It is used in electrical insulation, and in making imitation amber and many other products.

HEAT

Thermometer Scales. — There are three thermometer scales in general use: the Fahrenheit (F.), which is generally used in the English speaking countries; the Centigrade (C.) or Celsius, which is used in several continental countries and in scientific work; and the Réaumur (R.), which is used to some extent on the European continent, notably in Germany.

In the Fahrenheit thermometer, the freezing point of water is marked at 32 degrees on the scale and the boiling point, at atmospheric pressure, at 212 degrees. The distance between these two points is divided into 180 degrees. On the Centigrade scale, the freezing point of water is at 0 degrees and the boiling point at 100 degrees. On the Réaumur scale, the freezing point is at 0 degrees and the boiling point at 80 degrees. The following formulas may be used for converting temperatures given on any one of the scales to the other scales:

$$\text{Degrees Fahrenheit} = \frac{9 \times \text{degrees C.}}{5} + 32 = \frac{9 \times \text{degrees R.}}{4} + 32.$$

$$\text{Degrees Centigrade} = \frac{5 \times (\text{degrees F.} - 32)}{9} = \frac{5 \times \text{degrees R.}}{4}$$

$$\text{Degrees Réaumur} = \frac{4 \times \text{degrees C.}}{5} = \frac{4 \times (\text{degrees F.} - 32)}{9}$$

Tables are given herewith for converting degrees Centigrade into degrees Fahrenheit. The tables can, of course, be conveniently used in the reverse order. The table for "Conversion from Degrees Centigrade to Degrees Fahrenheit" covers the whole range of practically or scientifically obtained temperatures. As an example of the use of the table, it will be seen that 1040 degrees Centigrade equals 1904 degrees Fahrenheit, and that − 130 degrees Centigrade equals − 202 degrees Fahrenheit.

Absolute Temperature and Absolute Zero. — A point has been determined on the thermometer scale, by theoretical considerations, which is called the absolute zero and beyond which a further decrease in temperature is inconceivable. This point is located at − 273 degrees Centigrade or − 459.2 degrees F. A temperature reckoned from this point, instead of from the zero on the ordinary thermometers, is called absolute temperature. To find the absolute temperature when the temperature in degrees F. is known, add 459.2 to the number of degrees F. For example, find the absolute temperature of the freezing point of water (32 degrees F.).

$$459.2 + 32 = 491.2 \text{ absolute temperature Fahrenheit.}$$

Measures of the Quantity of Heat. — The unit of quantity of heat used in the English speaking countries is the British thermal unit (B.T.U.), which is the quantity of heat required to raise the temperature of one pound of pure water one degree F. The French thermal unit or *kilogram calorie*, is the quantity of heat required to raise the temperature of one kilogram of pure water one degree C. One kilogram calorie = 3.968 British thermal units = 1000 gram calories.

The number of foot-pounds of mechanical energy equivalent to one British thermal unit is called the *mechanical equivalent of heat*, and equals 778 foot-pounds. One foot-pound equals 0.001285 heat unit.

Comparison Between Degrees Centigrade and Degrees Fahrenheit

Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.
−40	−40.0	8	46.4	56	132.8	104	219.2	152	305.6	200	392.0
−39	−38.2	9	48.2	57	134.6	105	221.0	153	307.4	201	393.8
−38	−36.4	10	50.0	58	136.4	106	222.8	154	309.2	202	395.6
−37	−34.6	11	51.8	59	138.2	107	224.6	155	311.0	203	397.4
−36	−32.8	12	53.6	60	140.0	108	226.4	156	312.8	204	399.2
−35	−31.0	13	55.4	61	141.8	109	228.2	157	314.6	205	401.0
−34	−29.2	14	57.2	62	143.6	110	230.0	158	316.4	206	402.8
−33	−27.4	15	59.0	63	145.4	111	231.8	159	318.2	207	404.6
−32	−25.6	16	60.8	64	147.2	112	233.6	160	320.0	208	406.4
−31	−23.8	17	62.6	65	149.0	113	235.4	161	321.8	209	408.2
−30	−22.0	18	64.4	66	150.8	114	237.2	162	323.6	210	410.0
−29	−20.2	19	66.2	67	152.6	115	239.0	163	325.4	211	411.8
−28	−18.4	20	68.0	68	154.4	116	240.8	164	327.2	212	413.6
−27	−16.6	21	69.8	69	156.2	117	242.6	165	329.0	213	415.4
−26	−14.8	22	71.6	70	158.0	118	244.4	166	330.8	214	417.2
−25	−13.0	23	73.4	71	159.8	119	246.2	167	332.6	215	419.0
−24	−11.2	24	75.2	72	161.6	120	248.0	168	334.4	216	420.8
−23	− 9.4	25	77.0	73	163.4	121	249.8	169	336.2	217	422.6
−22	− 7.6	26	78.8	74	165.2	122	251.6	170	338.0	218	424.4
−21	− 5.8	27	80.6	75	167.0	123	253.4	171	339.8	219	426.2
−20	− 4.0	28	82.4	76	168.8	124	255.2	172	341.6	220	428.0
−19	− 2.2	29	84.2	77	170.6	125	257.0	173	343.4	221	429.8
−18	− 0.4	30	86.0	78	172.4	126	258.8	174	345.2	222	431.6
−17	+ 1.4	31	87.8	79	174.2	127	260.6	175	347.0	223	433.4
−16	3.2	32	89.6	80	176.0	128	262.4	176	348.8	224	435.2
−15	5.0	33	91.4	81	177.8	129	264.2	177	350.6	225	437.0
−14	6.8	34	93.2	82	179.6	130	266.0	178	352.4	226	438.8
−13	8.6	35	95.0	83	181.4	131	267.8	179	354.2	227	440.6
−12	10.4	36	96.8	84	183.2	132	269.6	180	356.0	228	442.4
−11	12.2	37	98.6	85	185.0	133	271.4	181	357.8	229	444.2
−10	14.0	38	100.4	86	186.8	134	273.2	182	359.6	230	446.0
− 9	15.8	39	102.2	87	188.6	135	275.0	183	361.4	231	447.8
− 8	17.6	40	104.0	88	190.4	136	276.8	184	363.2	232	449.6
− 7	19.4	41	105.8	89	192.2	137	278.6	185	365.0	233	451.4
− 6	21.2	42	107.6	90	194.0	138	280.4	186	366.8	234	453.2
− 5	23.0	43	109.4	91	195.8	139	282.2	187	368.6	235	455.0
− 4	24.8	44	111.2	92	197.6	140	284.0	188	370.4	236	456.8
− 3	26.6	45	113.0	93	199.4	141	285.8	189	372.2	237	458.6
− 2	28.4	46	114.8	94	201.2	142	287.6	190	374.0	238	460.4
− 1	30.2	47	116.6	95	203.0	143	289.4	191	375.8	239	462.2
0	32.0	48	118.4	96	204.8	144	291.2	192	377.6	240	464.0
+ 1	33.8	49	120.2	97	206.6	145	293.0	193	379.4	241	465.8
2	35.6	50	122.0	98	208.4	146	294.8	194	381.2	242	467.6
3	37.4	51	123.8	99	210.2	147	296.6	195	383.0	243	469.4
4	39.2	52	125.6	100	212.0	148	298.4	196	384.8	244	471.2
5	41.0	53	127.4	101	213.8	149	300.2	197	386.6	246	474.8
6	42.8	54	129.2	102	215.6	150	302.0	198	388.4	248	478.4
7	44.6	55	131.0	103	217.4	151	303.8	199	390.2	250	482.0

Table for Conversion from Degrees Centigrade to Degrees Fahrenheit

Degrees Centigrade	0	10	20	30	40	50	60	70	80	90
	Degrees Fahrenheit									
−200	−328	−346	−364	−382	−400	−418	−436	−454	.....	.....
−100	−148	−166	−184	−202	−220	−238	−256	−274	−292	−310
−0	+32	+14	−4	−22	−40	−58	−76	−94	−112	−130
0	32	50	68	86	104	122	140	158	176	194
100	212	230	248	266	284	302	320	338	356	374
200	392	410	428	446	464	482	500	518	536	554
300	572	590	608	626	644	662	680	698	716	734
400	752	770	788	806	824	842	860	878	896	914
500	932	950	968	986	1004	1022	1040	1058	1076	1094
600	1112	1130	1148	1166	1184	1202	1220	1238	1256	1274
700	1292	1310	1328	1346	1364	1382	1400	1418	1436	1454
800	1472	1490	1508	1526	1544	1562	1580	1598	1616	1634
900	1652	1670	1688	1706	1724	1742	1760	1778	1796	1814
1000	1832	1850	1868	1886	1904	1922	1940	1958	1976	1994
1100	2012	2030	2048	2066	2084	2102	2120	2138	2156	2174
1200	2192	2210	2228	2246	2264	2282	2300	2318	2336	2354
1300	2372	2390	2408	2426	2444	2462	2480	2498	2516	2534
1400	2552	2570	2588	2606	2624	2642	2660	2678	2696	2714
1500	2732	2750	2768	2786	2804	2822	2840	2858	2876	2894
1600	2912	2930	2948	2966	2984	3002	3020	3038	3056	3074
1700	3092	3110	3128	3146	3164	3182	3200	3218	3236	3254
1800	3272	3290	3308	3326	3344	3362	3380	3398	3416	3434
1900	3452	3470	3488	3506	3524	3542	3560	3578	3596	3614
2000	3632	3650	3668	3686	3704	3722	3740	3758	3776	3794
2100	3812	3830	3848	3866	3884	3902	3920	3938	3956	3974
2200	3992	4010	4028	4046	4064	4082	4100	4118	4136	4154
2300	4172	4190	4208	4226	4244	4262	4280	4298	4316	4334
2400	4352	4370	4388	4406	4424	4442	4460	4478	4496	4514
2500	4532	4550	4568	4586	4604	4622	4640	4658	4676	4694
2600	4712	4730	4748	4766	4784	4802	4820	4838	4856	4874
2700	4892	4910	4928	4946	4964	4982	5000	5018	5036	5054
2800	5072	5090	5108	5126	5144	5162	5180	5198	5216	5234
2900	5252	5270	5288	5306	5324	5342	5360	5378	5396	5414
3000	5432	5450	5468	5486	5504	5522	5540	5558	5576	5594
3100	5612	5630	5648	5666	5684	5702	5720	5738	5756	5774
3200	5792	5810	5828	5846	5864	5882	5900	5918	5936	5954
3300	5972	5990	6008	6026	6044	6062	6080	6098	6116	6134
3400	6152	6170	6188	6206	6224	6242	6260	6278	6296	6314
3500	6332	6350	6368	6386	6404	6422	6440	6458	6476	6494
3600	6512	6530	6548	6566	6584	6602	6620	6638	6656	6674
3700	6692	6710	6728	6746	6764	6782	6800	6818	6836	6854
3800	6872	6890	6908	6926	6944	6962	6980	6998	7016	7034
3900	7052	7070	7088	7106	7124	7142	7160	7178	7196	7214
4000	7232	7250	7268	7286	7304	7322	7340	7358	7376	7394

Coefficients of Heat Transmission

Heat transmitted, in British thermal units, per second, through metal 1 inch thick, per square inch of surface, for a temperature difference of 1° F.

Metal	B.T.U. per Second	Metal	B.T.U. per Second	Metal	B.T.U. per Second
Aluminum	0.00203	German silver	0.00050	Steel, soft	0.00062
Antimony	0.00022	Iron	0.00089	Silver	0.00610
Brass, yellow	0.00142	Lead	0.00045	Tin	0.00084
Brass, red	0.00157	Mercury	0.00011	Zinc	0.00170
Copper	0.00404	Steel, hard	0.00034	....	....

Coefficients of Heat Radiation

Heat radiated, in British thermal units, per square foot of surface per hour, for a temperature difference of 1° F.

Surface	B.T.U. per Hour	Surface	B.T.U. per Hour
Cast-iron, new	0.6480	Sawdust	0.7215
Cast-iron, rusted	0.6868	Sand, fine	0.7400
Copper, polished	0.0327	Silver, polished	0.0266
Glass	0.5948	Tin, polished	0.0439
Iron, ordinary	0.5662	Tinned iron, polished	0.0858
Iron, sheet-, polished	0.0920	Water	1.0853
Oil	1.4800	....	....

Freezing Mixtures

Mixture	Temperature Change, Degrees F. From	Temperature Change, Degrees F. To
Common salt ($NaCl$), 1 part; snow, 3 parts	32	±0
Common salt ($NaCl$), 1 part; snow, 1 part	32	−0.4
Calcium chloride ($CaCl_2$), 3 parts; snow, 2 parts	32	−27
Calcium chloride ($CaCl_2$), 2 parts; snow, 1 part	32	−44
Sal ammoniac (NH_4Cl), 5 parts; saltpeter (KNO_3), 5 parts; water, 16 parts	50	+10
Sal ammoniac (NH_4Cl), 1 part; saltpeter (KNO_3), 1 part; water, 1 part	46	−11
Ammonium nitrate (NH_4NO_3), 1 part; water, 1 part	50	+ 3
Potassium hydrate (KOH), 4 parts; snow, 3 parts	32	−35

Ignition Temperatures. — The following temperatures are required to ignite the different substances specified: Phosphorus, transparent, 120 degrees F.; bisulphide of carbon, 300 degrees F.; gun cotton, 430 degrees F.; nitro-glycerine, 490 degrees F.; phosphorus, amorphous, 500 degrees F.; rifle powder, 550 degrees F.; charcoal, 660 degrees F.; dry pine wood, 800 degrees F.; dry oak wood, 900 degrees F.

Latent Heat. — When a body changes from the solid to the liquid state or from the liquid to the gaseous state, a certain amount of heat is used to accomplish this change. This heat does not raise the temperature of the body and is called latent heat. When the body changes again from the gaseous to the liquid, or from the liquid to the solid state, this quantity of heat is given out by it. The *latent heat of fusion* is the heat supplied to a solid body at the melting point; this heat is absorbed by the body although its temperature remains nearly stationary during the whole operation of melting. The *latent heat of evaporation* is the heat that must be supplied to a liquid at the boiling point to transform the liquid into a vapor. The latent heat is generally given in British thermal units per pound. When it is said that the latent heat of evaporation of water is 966.6, this means that it takes 966.6 heat units to evaporate one pound of water after it has been raised to the boiling point, 212 degrees F.

Latent Heat of Fusion

Substance	B.T.U. per Pound	Substance	B.T.U. per Pound	Substance	B.T.U. per Pound
Bismuth..........	22.75	Paraffine.........	63.27	Sulphur........	16.86
Beeswax..........	76.14	Phosphorus......	9.06	Tin............	25.65
Cast iron, gray...	41.40	Lead.............	10.00	Zinc...........	50.63
Cast iron, white..	59.40	Silver	37.92	Ice.............	144.00

Latent Heat of Evaporation

Liquid	B.T.U. per Pound	Liquid	B.T.U. per Pound	Liquid	B.T.U. per Pound
Alcohol, ethyl....	371.0	Bisulphide of carbon.........	160.0	Sulphur dioxide	164.0
Alcohol, methyl..	481.0	Ether...........	162.8	Turpentine.....	133.0
Ammonia........	529.0			Water.........	966.6

Boiling Points of Various Substances at Atmospheric Pressure

Substance	Boiling Point, Degrees F.	Substance	Boiling Point, Degrees F.	Substance	Boiling Point, Degrees F.
Aniline...........	363	Chloroform.......	140	Saturated brine	226
Alcohol..........	173	Ether............	100	Sulphur........	833
Ammonia........	140	Linseed oil.......	597	Sulphuric acid..	590
Benzine..........	176	Mercury..........	676	Water, pure....	212
Bromine.........	145	Napthaline.......	428	Water, sea......	213.2
Carbon bisulphide.....	118	Nitric acid.......	248	Wood alcohol...	150
		Oil of turpentine..	315	....	......

Specific Heat. — The specific heat of a substance is the ratio of the heat required to raise the temperature of a certain weight of the given substance one degree F. to that required to raise the temperature of the same weight of water one degree. As the specific heat is not constant at all temperatures, it is generally assumed that it is determined by raising the temperature from 62 to 63 degrees F. For most substances, however, it is practically constant for temperatures up to 212 degrees F.

Average Specific Heats of Various Substances

Substance	Specific Heat	Substance	Specific Heat
Alcohol (absolute)	0.700	Kerosene	0.500
Alcohol (density 0.8)	0.622	Lead	0.031
Aluminum	0.214	Limestone	0.217
Antimony	0.051	Magnesia	0.222
Benzine	0.450	Marble	0.210
Brass	0.094	Masonry, brick	0.200
Brickwork	0.200	Mercury	0.033
Cadmium	0.057	Naphtha	0.310
Charcoal	0.200	Nickel	0.109
Chalk	0.215	Oil, machine	0.400
Coal	0.240	Oil, olive	0.350
Coke	0.203	Phosphorus	0.189
Copper, 32° to 212° F.	0.094	Platinum	0.032
Copper, 32° to 572° F.	0.101	Quartz	0.188
Corundum	0.198	Sand	0.195
Ether	0.503	Silica	0.191
Fusel oil	0.564	Silver	0.056
Glass	0.194	Soda	0.231
Gold	0.031	Steel, mild	0.116
Graphite	0.201	Steel, high carbon	0.117
Ice	0.504	Stone (generally)	0.200
Iron, cast	0.130	Sulphur	0.178
Iron, wrought, 32° to 212° F.	0.110	Sulphuric acid	0.330
32° to 392° F.	0.115	Tin	0.056
32° to 572° F.	0.122	Turpentine	0.472
32° to 662° F.	0.126	Water	1.000
Iron, at high temperatures:		Wood, fir	0.650
1382° to 1832° F.	0.213	Wood, oak	0.570
1750° to 1840° F.	0.218	Wood, pine	0.467
1920° to 2190° F.	0.199	Zinc	0.095

Specific Heat of Gases

Gas	Constant Pressure	Constant Volume	Gas	Constant Pressure	Constant Volume
Acetic acid	0.412	....	Chloroform	0.157	....
Air	0.238	0.168	Hydrogen	3.409	2.412
Alcohol	0.453	0.399	Nitrogen	0.244	0.173
Ammonia	0.508	0.299	Oxygen	0.217	0.155
Carbonic acid	0.217	0.171	Ethylene	0.404	0.332
Carbonic oxide	0.245	0.176	Steam	0.480	0.346
Chlorine	0.121	....	....	....	....

Heat Loss from Uncovered Steam Pipes. — The loss of heat from a bare steam or hot water pipe varies with the difference between the temperature inside the pipe and that of the surrounding air. The loss is 2.15 B.T.U. per hour, per square foot of pipe surface, per degree F. of temperature difference when the latter is 100 degrees; for a difference of 200 degrees, the loss is 2.66 B.T.U.; for 300 degrees, 3.26 B.T.U.; for 400 degrees, 4.03 B.T.U.; for 500 degrees, 5.18 B.T.U. Thus, if the pipe area is 1.18 square feet per foot of length, and the temperature difference 300 degrees F., the loss per hour per foot of length = 1.18 × 300 × 3.26 = 1154 B.T.U.

This loss is considerably reduced by covering the steam pipe with a material as non-conductive to heat as possible. The best insulating materials are those which hold air confined in minute cells. Incombustible mineral substances are to be preferred to combustible material. No covering should be less than one inch in thickness. Mineral wool, a fibrous material made from blast furnace slag, is the best non-combustible covering, but it is brittle and, therefore, liable to be reduced to a powder when subjected to vibration. The percentage of steam lost through a covering of mineral wool about 1¼ inch is about 1/10 of that lost from bare pipes. A heat insulation composed of a number of layers of asbestos paper in which are imbedded small pieces of sponge is also very effective. The amount of heat lost with the general commercial pipe coverings is from ⅛ to ⅙ of the amount that would be lost from bare pipes.

Relative Value of Heat-insulating Materials

(Hair- or wool-felt is assumed to have an insulating value=1.00)

Heat-insulating Material	Relative Value	Heat-insulating Material	Relative Value
Loose Wool	3.35	Loam, dry and open	0.55
Loose Lampblack	1.12	Chalk, ground, Spanish white	0.51
Geese Feathers	1.08	Coal Ashes	0.35–0.49
Felt, Hair or Wool	1.00	Gas-house Carbon	0.47
Carded Cotton	1.00	Asbestos Paper	0.47
Charcoal from Cork	0.87	Paste of Fossil Meal and Asbestos	0.47
Mineral Wool	0.68–0.83	Asbestos, fibrous	0.36
Fossil Meal	0.66–0.70	Plaster of Paris, dry	0.34
Straw Rope, wound spirally	0.77	Clay with vegetable fiber	0.34
Rice Chaff, loose	0.76	Anthracite Coal, powdered	0.29
Carbonate of Magnesia	0.67–0.76	Coke, in lumps	0.27
Charcoal from Wood	0.63–0.75	Air Space, undivided	0.14–0.22
Paper	0.50–0.74	Sand	0.17
Cork	0.71	Baked Clay, Brick	0.07
Sawdust	0.61–0.68	Glass	0.05
Paste of Fossil Meal and Hair	0.63	Stone	0.02
Wood Ashes	0.61		
Wood, cross grain	0.40–0.55		

Linear Expansion of Various Substances Between 32 and 212 Deg. Fahr.

(For linear expansion of metals see "Specific Gravity and Properties of Metals")
Expansion of volume = 3 × linear expansion.

Substance	Linear Expansion for 1 Deg. Fahr.	Substance	Linear Expansion for 1 Deg. Fahr.
Brick	0.0000030	Masonry, brick, from	0.0000026
Cement, Portland	0.0000060	to	0.0000050
Concrete	0.0000080	Plaster	0.0000092
Ebonite	0.0000428	Porcelain	0.0000020
Glass, thermometer	0.0000050	Quartz, from	0.0000043
Glass, hard	0.0000040	to	0.0000079
Granite	0.0000044	Slate	0.0000058
Marble, from	0.0000031	Sandstone	0.0000065
to	0.0000079	Wood (pine)	0.0000028

PROPERTIES, COMPRESSION AND FLOW OF AIR

Properties of Air. — Air is a mechanical mixture composed of 78 per cent, by volume, of nitrogen, 21 per cent of oxygen and 1 per cent of argon. The weight of pure air at 32 degrees F. and atmospheric pressure (29.92 inches of mercury or 14.70 pounds per square inch) is 0.08073 pound per cubic foot. The volume of a pound of air at the same temperature and pressure is 12.387 cubic feet. The weight of air at any other temperature or pressure is:

$$W = \frac{1.325 \times B}{T}$$

in which W = weight in pounds per cubic foot; B = height of barometric pressure in inches of mercury; T = absolute temperature Fahrenheit.

Volume and Weight of Air at Different Temperatures, at Atmospheric Pressure

Temperature, Degrees Fahr.	Volume of 1 Pound of Air in Cubic Feet	Weight per Cubic Foot, Pounds	Temperature, Degrees Fahr.	Volume of 1 Pound of Air in Cubic Feet	Weight per Cubic Foot, Pounds	Temperature, Degrees Fahr.	Volume of 1 Pound of Air in Cubic Feet	Weight per Cubic Foot, Pounds
0	11.57	0.0864	172	15.92	0.0628	800	31.75	0.0315
12	11.88	0.0842	182	16.18	0.0618	900	34.25	0.0292
22	12.14	0.0824	192	16.42	0.0609	1000	37.31	0.0268
32	12.39	0.0807	202	16.67	0.0600	1100	39.37	0.0254
42	12.64	0.0791	212	16.92	0.0591	1200	41.84	0.0239
52	12.89	0.0776	230	17.39	0.0575	1300	44.44	0.0225
62	13.14	0.0761	250	17.89	0.0559	1400	46.95	0 0213
72	13.39	0.0747	275	18.52	0.0540	1500	49.51	0.0202
82	13.64	0.0733	300	19.16	0.0522	1600	52.08	0.0192
92	13.89	0.0720	325	19.76	0.0506	1700	54.64	0.0183
102	14.14	0.0707	350	20.41	0.0490	1800	57.14	0.0175
112	14.41	0.0694	375	20.96	0.0477	2000	62.11	0 0161
122	14.66	0.0682	400	21.69	0.0461	2200	67.11	0.0149
132	14.90	0.0671	450	22.94	0.0436	2400	72.46	0.0138
142	15.17	0.0659	500	24.21	0.0413	2600	76.92	0.0130
152	15.41	0.0649	600	26.60	0.0376	2800	82.64	0.0121
162	15.67	0.0638	700	29.59	0.0338	3000	87.72	0.0114

The absolute zero from which all temperatures must be counted when dealing with the weight and volume of gases is assumed to be −459.2 degrees F. Hence, to obtain the absolute temperature T used in the formula above, add to the temperature observed on a regular Fahrenheit thermometer the value 459.2.

In obtaining the value of B, 1 inch of mercury at 32 degrees F. may be taken as equal to a pressure of 0.491 pound per square inch.

Example. — What would be the weight of a cubic foot of air at atmospheric pressure (29.92 inches of mercury) at 100 degrees F.?

$$W = \frac{1.325 \times 29.92}{100 + 459.2} = 0.0709 \text{ pound.}$$

Weight of Air at Different Pressures and Temperatures

Temp. of Air, Degrees Fahr.	Gage Pressure, Pounds														
	0	5	10	20	30	40	50	60	80	100	120	150	200	250	300
	Weight in Pounds per Cubic Foot														
−20	0.0900	0.1205	0.1515	0.2125	0.274	0.336	0.397	0.458	0.580	0.702	0.825	1.010	1.318	1.625	1.930
−10	0.0882	0.1184	0.1485	0.2090	0.268	0.328	0.388	0.448	0.567	0.687	0.807	0.989	1.288	1.588	1.890
0	0.0864	0.1160	0.1455	0.2040	0.263	0.321	0.380	0.438	0.555	0.672	0.790	0.968	1.260	1.553	1.850
10	0.0846	0.1136	0.1425	0.1995	0.257	0.314	0.372	0.429	0.543	0.658	0.774	0.947	1.233	1.520	1.810
20	0.0828	0.1112	0.1395	0.1955	0.252	0.307	0.364	0.420	0.533	0.645	0.757	0.927	1.208	1.489	1.770
30	0.0811	0.1088	0.1366	0.1916	0.246	0.301	0.357	0.412	0.522	0.632	0.742	0.908	1.184	1.460	1.735
40	0.0795	0.1067	0.1338	0.1876	0.241	0.295	0.350	0.404	0.511	0.619	0.727	0.890	1.161	1.431	1.701
50	0.0780	0.1045	0.1310	0.1839	0.237	0.290	0.343	0.396	0.501	0.607	0.713	0.873	1.139	1.403	1.668
60	0.0764	0.1025	0.1283	0.1803	0.232	0.284	0.336	0.388	0.493	0.596	0.700	0.856	1.116	1.376	1.636
80	0.0736	0.0988	0.1239	0.1738	0.224	0.274	0.324	0.374	0.473	0.572	0.673	0.824	1.074	1.325	1.573
100	0.0710	0.0954	0.1197	0.1676	0.215	0.264	0.312	0.360	0.455	0.551	0.648	0.794	1.035	1.276	1.517
120	0.0686	0.0921	0.1155	0.1618	0.208	0.255	0.302	0.348	0.440	0.533	0.626	0.767	1.001	1.234	1.465
140	0.0663	0.0889	0.1115	0.1565	0.201	0.246	0.291	0.336	0.426	0.516	0.606	0.742	0.968	1.194	1.416
150	0.0652	0.0874	0.1096	0.1541	0.198	0.242	0.286	0.331	0.419	0.508	0.596	0.730	0.953	1.175	1.392
175	0.0626	0.0840	0.1054	0.1482	0.191	0.233	0.275	0.318	0.403	0.488	0.573	0.701	0.914	1.128	1.337
200	0.0603	0.0809	0.1014	0.1427	0.184	0.225	0.265	0.305	0.388	0.470	0.552	0.674	0.879	1.084	1.287
225	0.0581	0.0779	0.0976	0.1373	0.177	0.216	0.255	0.295	0.374	0.452	0.531	0.649	0.846	1.043	1.240
250	0.0560	0.0751	0.0941	0.1323	0.170	0.208	0.247	0.284	0.360	0.436	0.513	0.627	0.817	1.007	1.197
275	0.0541	0.0726	0.0910	0.1278	0.164	0.201	0.233	0.274	0.348	0.421	0.494	0.605	0.789	0.972	1.155
300	0.0523	0.0707	0.0881	0.1237	0.159	0.194	0.230	0.265	0.336	0.407	0.478	0.585	0.762	0.940	1.118
350	0.0491	0.0658	0.0825	0.1160	0.149	0.183	0.216	0.249	0.316	0.382	0.449	0.549	0.715	0.883	1.048
400	0.0463	0.0621	0.0779	0.1090	0.140	0.172	0.203	0.235	0.297	0.360	0.423	0.517	0.674	0.831	0.987
450	0.0437	0.0586	0.0735	0.1033	0.133	0.163	0.192	0.222	0.281	0.340	0.399	0.488	0.637	0.786	0.934
500	0.0414	0.0555	0.0696	0.0978	0.126	0.154	0.182	0.210	0.266	0.322	0.379	0.463	0.604	0.746	0.885
550	0.0394	0.0528	0.0661	0.0930	0.120	0.146	0.173	0.200	0.253	0.306	0.359	0.440	0.573	0.749	0.841
600	0.0376	0.0504	0.0631	0.0885	0.114	0.139	0.165	0.190	0.241	0.292	0.343	0.419	0.547	0.675	0.801

Relation between Pressure, Temperature and Volume of Air. — This relationship is expressed by the formula:

$$\frac{P \times V}{T} = 53.3,$$

in which P = absolute pressure in pounds per square foot; V = volume in cubic feet of one pound of air at the given pressure and temperature; T = absolute temperature in degrees F.

Example. — What is the volume of one pound of air at a pressure of 24.7 pounds per square inch and at a temperature of 210 degrees F.?

$$\frac{24.7 \times 144 \times V}{210 + 459.2} = 53.3, \text{ or } V = \frac{53.3 \times 669.2}{24.7 \times 144} = 10.03 \text{ cubic feet.}$$

Relation Between Barometric Pressure, and Pressures in Pounds per Square Inch and Square Foot

Barometer, Inches	Pressure in Pounds per Square Inch	Pressure in Pounds per Square Foot	Barometer, Inches	Pressure in Pounds per Square Inch	Pressure in Pounds per Square Foot	Barometer, Inches	Pressure in Pounds per Square Inch	Pressure in Pounds per Square Foot
28.00	13.75	1980	29.25	14.36	2068	30.50	14.98	2156
28.25	13.87	1997	29.50	14.48	2086	30.75	15.10	2174
28.50	13.99	2015	29.75	14.61	2103	31.00	15.22	2192
28.75	14.12	2033	30.00	14.73	2121	31.25	15.34	2210
29.00	14.24	2050	30.25	14.85	2139	.....	.....	......

Expansion and Compression of Air. — The formula for the relationship of pressure, temperature and volume of air just given indicates that when the pressure remains constant the volume is directly proportional to the absolute temperature. If the temperature remains constant, the volume is inversely proportional to the absolute pressure. Theoretically, air (as well as other gases) can be expanded or compressed according to two different laws. *Adiabatic* expansion or compression takes place when the air is expanded or compressed without transmission of heat to or from it; as for example, if the air could be expanded or compressed in a cylinder of an absolutely non-conducting material. Let:

P_1 = initial absolute pressure in pounds per square foot;
V_1 = initial volume in cubic feet;
T_1 = initial absolute temperature in degrees F.;
P_2 = absolute pressure in pounds per square foot, after compression;
V_2 = volume in cubic feet, after compression;
T_2 = absolute temperature in degrees F., after compression.

Then:

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{0.71} \qquad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{1.41} \qquad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{0.41}$$

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{2.46} \qquad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{3.46} \qquad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{0.29}$$

These formulas are also applicable if all pressures are in pounds per square inch, or if all volumes are in cubic inches.

Isothermal expansion or compression takes place when the gas is expanded or compressed with an addition or transmission of sufficient heat to maintain a constant temperature. Let:

P_1 = initial pressure in pounds per square foot;
V_1 = initial volume in cubic feet;
P_2 = absolute pressure in pounds per square foot, after compression;
V_2 = volume in cubic feet, after compression;
C = constant depending on the temperature.

Then:

$$P_1 \times V_1 = P_2 \times V_2 = C.$$

For a temperature of 32 degrees F., constant C equals 26,200 foot-pounds, and for other temperatures it may be found from the formula, $C = 53.3\ T$, in which T is the absolute temperature which is maintained during the expansion or compression.

Example. — A volume of 165 cubic feet of air, at a pressure of 15 pounds per square inch, is compressed adiabatically to a pressure of 80 pounds per square inch. What will be the volume at this pressure?

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{0.71} = 165 \left(\frac{15}{80}\right)^{0.71} = 50 \text{ cubic feet, approx.}$$

Example. — The same volume of air is compressed isothermally from 15 to 80 pounds per square inch. What will be the volume after compression?

$$V_2 = \frac{P_1 \times V_1}{P_2} = \frac{15 \times 165}{80} = 31 \text{ cubic feet.}$$

Foot-pounds of Work Required in Compression of Air

Initial Pressure = 1 atmosphere = 14.7 pounds per square inch

Gage Pressure in Pounds per Square Inch	Foot-pounds Required per Cubic Foot of Air at Initial Pressure			Gage Pressure in Pounds per Square Inch	Foot-pounds Required per Cubic Foot of Air at Initial Pressure		
	Isothermal Compression	Adiabatic Compression	Actual Power Required		Isothermal Compression	Adiabatic Compression	Actual Power Required
5	619.6	649.5	637.5	55	3393.7	4188.9	3870.8
10	1098.2	1192.0	1154.6	60	3440.4	4422.8	4029.8
15	1488.3	1661.2	1592.0	65	3577.6	4645.4	4218.2
20	1817.7	2074.0	1971.4	70	3706.3	4859.6	4398.1
25	2102.6	2451.6	2312.0	75	3828.0	5063.9	4569.5
30	2353.6	2794.0	2617.8	80	3942.9	5259.7	4732.9
35	2578.0	3111.0	2897.8	85	4051.5	5450.0	4890.1
40	2780.8	3405.5	3155.6	90	4155.7	5633.1	5042.1
45	2966.0	3681.7	3395.4	95	4254.3	5819.3	5187.3
50	3136.2	3942.3	3619.8	100	4348.1	5981.2	5327.9

Work Required in Compression of Air. — The total work required for compression and expulsion of air, adiabatically compressed, is:

$$\text{Total work in foot-pounds} = 3.46\ P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{0.29} - 1\right]$$

in which P_1 = initial absolute pressure in pounds per square foot;
P_2 = absolute pressure in pounds per square foot, after compression;
V_1 = initial volume in cubic feet.

The total work required for isothermal compression is:

$$\text{Total work in foot-pounds} = P_1 V_1 \text{ hyp. log. } \frac{V_1}{V_2}$$

in which P_1, P_2 and V_1 denote the same quantities as in the previous equation, and V_2 = volume of air in cubic feet, after compression.

The work required to compress air isothermally, that is, when the heat of compression is removed as rapidly as produced, is considerably less than the work required for compressing air adiabatically, or when all the heat is retained. In actual practice, neither of these two theoretical extremes are obtainable, but the power required for air compression is about the medium between the powers that would be required for each. The accompanying table gives the average number of foot-pounds of work required to compress air.

Horsepower Required to Compress Air. — In the accompanying tables is given the horsepower required for compressing one cubic foot of free air per minute (isothermally and adiabatically) from atmospheric pressure (14.7 pounds per square inch) to various gage pressures, for one-, two- and three-stage compression. The formula for calculating the horsepower required to compress, adiabatically, a given volume of free air to a given pressure is:

$$\text{H. P.} = \frac{144\,NPVn}{33000\,(n-1)}\left[\left(\frac{P_2}{P}\right)^{\frac{n-1}{Nn}} - 1\right]$$

in which N = number of stages in which compression is accomplished;
P = atmospheric pressure in pounds per square inch;
P_2 = absolute terminal pressure in pounds per square inch;
V = volume of air, in cubic feet, compressed per minute, at atmospheric pressure;
n = exponent of the compression curve = 1.41 for adiabatic compression.

For different methods of compression and for one cubic foot of air per minute, this formula may be simplified as follows:

For one-stage compression: $\text{H. P.} = 0.015\,P\,(R^{0.29} - 1)$
For two-stage compression: $\text{H. P.} = 0.030\,P\,(R^{0.145} - 1)$
For three-stage compression: $\text{H. P.} = 0.045\,P\,(R^{0.0975} - 1)$
For four-stage compression: $\text{H. P.} = 0.060\,P\,(R^{0.0725} - 1)$

In these latter formulas $R = \frac{P_2}{P}$ = number of atmospheres to be compressed.

The formula for calculating the horsepower required to compress isothermally a given volume of free air to a given pressure is:

$$\text{H. P.} = \frac{144\,PV}{33000}\left(\text{hyp. log. } \frac{P_2}{P}\right)$$

Hyperbolic logarithms are obtained by multiplying common logarithms by 2.302585. See also the tables of hyperbolic logarithms, page 144.

Horsepower Required to Compress Air

Horsepower Required for Compressing One Cubic Foot of Free Air per Minute (Isothermally and Adiabatically) from Atmospheric Pressure (14.7 pounds per square inch) to Various Gage Pressures. — Single-stage Compression

(Initial Temperature of Air, 60° F. — Jacket-cooling not considered)

Gage Pressure, Pounds	Absolute Pressure, Pounds	Number of Atmospheres	Isothermal Compression		Adiabatic Compression			
			Mean Effective Pressure	Horsepower	Mean Effective Pressure, Theoretical	Mean Eff. Pressure plus 15 per cent Friction	Horsepower, Theoretical	Horsepower plus 15 per cent Friction
5	19.7	1.34	4.13	0.018	4.46	5.12	0.019	0.022
10	24.7	1.68	7.57	0.033	8.21	9.44	0.036	0.041
15	29.7	2.02	11.02	0.048	11.46	13.17	0.050	0.057
20	34.7	2.36	12.62	0.055	14.30	16.44	0.062	0.071
25	39.7	2.70	14.68	0.064	16.94	19.47	0.074	0.085
30	44.7	3.04	16.30	0.071	19.32	22.21	0.084	0.096
35	49.7	3.38	17.90	0.078	21.50	24.72	0.094	0.108
40	54.7	3.72	19.28	0.084	23.53	27.05	0.103	0.118
45	59.7	4.06	20.65	0.090	25.40	29.21	0.111	0.127
50	64.7	4.40	21.80	0.095	27.23	31.31	0.119	0.136
55	69.7	4.74	22.95	0.100	28.90	33.23	0.126	0.145
60	74.7	5.08	23.90	0.104	30.53	35.10	0.133	0.153
65	79.7	5.42	24.80	0.108	32.10	36.91	0.140	0.161
70	84.7	5.76	25.70	0.112	33.57	38.59	0.146	0.168
75	89.7	6.10	26.62	0.116	35.00	40.25	0.153	0.175
80	94.7	6.44	27.52	0.120	36.36	41.80	0.159	0.182
85	99.7	6.78	28.21	0.123	37.63	43.27	0.164	0.189
90	104.7	7.12	28.93	0.126	38.89	44.71	0.169	0.195
95	109.7	7.46	29.60	0.129	40.11	46.12	0.175	0.201
100	114.7	7.80	30.30	0.132	41.28	47.46	0.180	0.207
110	124.7	8.48	31.42	0.137	43.56	50.09	0.190	0.218
120	134.7	9.16	32.60	0.142	45.69	52.53	0.199	0.229
130	144.7	9.84	33.75	0.147	47.72	54.87	0.208	0.239
140	154.7	10.52	34.67	0.151	49.64	57.08	0.216	0.249
150	164.7	11.20	35.59	0.155	51.47	59.18	0.224	0.258
160	174.7	11.88	36.30	0.158	53.70	61.80	0.234	0.269
170	184.7	12.56	37.20	0.162	55.60	64.00	0.242	0.278
180	194.7	13.24	38.10	0.166	57.20	65.80	0.249	0.286
190	204.7	13.92	38.80	0.169	58.80	67.70	0.256	0.294
200	214.7	14.60	39.50	0.172	60.40	69.50	0.263	0.303

Horsepower Required to Compress Air

Horsepower Required for Compressing One Cubic Foot of Free Air per Minute (Isothermally and Adiabatically) from Atmospheric Pressure (14.7 pounds per square inch) to Various Gage Pressures. — Two-stage Compression

(Initial Temperature of Air, 60° F. — Jacket-cooling not considered)

Gage Pressure, Pounds	Absolute Pressure, Pounds	Number of Atmospheres	Correct Ratio of Cylinder Volumes	Intercooler Gage Pressure	Isothermal Compression		Adiabatic Compression				Percentage of Saving over One-stage Compression
					Mean Effective Pressure	Horsepower	Mean Eff. Pressure, Theoretical	Mean Eff. Pressure plus 15 per cent Friction	Horsepower, Theoretical	H.P. plus 15 per cent Friction	
50	64.7	4.40	2.10	16.2	21.80	0.095	24.30	27.90	0.106	0.123	10.9
60	74.7	5.08	2.25	18.4	23.90	0.104	27.20	31.30	0.118	0.136	11.3
70	84.7	5.76	2.40	20.6	25.70	0.112	29.31	33.71	0.128	0.147	12.3
80	94.7	6.44	2.54	22.7	27.52	0.120	31.44	36.15	0.137	0.158	13.8
90	104.7	7.12	2.67	24.5	28.93	0.126	33.37	38.36	0.145	0.167	14.2
100	114.7	7.80	2.79	26.3	30.30	0.132	35.20	40.48	0.153	0.176	15.0
110	124.7	8.48	2.91	28.1	31.42	0.137	36.82	42.34	0.161	0.185	15.2
120	134.7	9.16	3.03	29.8	32.60	0.142	38.44	44.20	0.168	0.193	15.6
130	144.7	9.84	3.14	31.5	33.75	0.147	39.86	45.83	0.174	0.200	16.3
140	154.7	10.52	3.24	32.9	34.67	0.151	41.28	47.47	0.180	0.207	16.7
150	164.7	11.20	3.35	34.5	35.59	0.155	42.60	48.99	0.186	0.214	16.9
160	174.7	11.88	3.45	36.1	36.30	0.158	43.82	50.39	0.191	0.219	18.4
170	184.7	12.56	3.54	37.3	37.20	0.162	44.93	51.66	0.196	0.225	19.0
180	194.7	13.24	3.64	38.8	38.10	0.166	46.05	52.95	0.201	0.231	19.3
190	204.7	13.92	3.73	40.1	38.80	0.169	47.16	54.22	0.206	0.236	19.5
200	214.7	14.60	3.82	41.4	39.50	0.172	48.18	55.39	0.210	0.241	20.1
210	224.7	15.28	3.91	42.8	40.10	0.174	49.35	56.70	0.216	0.247	.....
220	234.7	15.96	3.99	44.0	40.70	0.177	50.30	57.70	0.220	0.252	.....
230	244.7	16.64	4.08	45.3	41.30	0.180	51.30	59.10	0.224	0.257	.....
240	254.7	17.32	4.17	46.6	41.90	0.183	52.25	60.10	0.228	0.262	.....
250	264.7	18.00	4.24	47.6	42.70	0.186	52.84	60.76	0.230	0.264	.....
260	274.7	18.68	4.32	48.8	43.00	0.188	53.85	62.05	0.235	0.270	.....
270	284.7	19.36	4.40	50.0	43.50	0.190	54.60	62.90	0.238	0.274	.....
280	294.7	20.04	4.48	51.1	44.00	0.192	55.50	63.85	0.242	0.278	.....
290	304.7	20.72	4.55	52.2	44.50	0.194	56.20	64.75	0.246	0.282	.....
300	314.7	21.40	4.63	53.4	45.80	0.197	56.70	65.20	0.247	0.283	.....
350	364.7	24.80	4.98	58.5	47.30	0.206	60.15	69.16	0.262	0.301	.....
400	414.7	28.20	5.31	63.3	49.20	0.214	63.19	72.65	0.276	0.317	.....
450	464.7	31.60	5.61	67.8	51.20	0.223	65.93	75.81	0.287	0.329	.....
500	514.7	35.01	5.91	72.1	52.70	0.229	68.46	78.72	0.298	0.342	.....

Horsepower Required to Compress Air

Horsepower Required for Compressing One Cubic Foot of Free Air per Minute (Isothermally and Adiabatically) from Atmospheric Pressure (14.7 pounds per square inch) to Various Gage Pressures. — Three-stage Compression

(Initial Temperature of Air, 60° F. — Jacket-cooling not considered)

Gage Pressure, Pounds	Absolute Pressure, Pounds	Number of Atmospheres	Correct Ratio of Cylinder Volumes	Intercooler Gage Pressure, First and Second Stages	Isothermal Compression		Adiabatic Compression				Precentage of Saving over Two-stage Compression
					Mean Effective Pressure	Horsepower	Mean Eff. Pressure, Theoretical	Mean Eff. Pressure plus 15 per cent Friction	Horsepower, Theoretical	H.P. plus 15 per cent Friction	
100	114.7	7.8	1.98	14.4– 42.9	30.30	0.132	33.30	38.30	0.145	0.167	5.23
150	164.7	11.2	2.24	18.2– 59.0	35.59	0.155	40.30	46.50	0.175	0.202	5.92
200	214.7	14.6	2.44	21.2– 73.0	39.50	0.172	45.20	52.00	0.196	0.226	6.67
250	264.7	18.0	2.62	23.8– 86.1	42.70	0.186	49.20	56.60	0.214	0.246	6.96
300	314.7	21.4	2.78	26.1– 98.7	45.30	0.197	52.70	60.70	0.229	0.264	7.28
350	364.7	24.8	2.92	28.2–110.5	47.30	0.206	55.45	63.80	0.242	0.277	7.64
400	414.7	28.2	3.04	30.0–121.0	49.20	0.214	58.25	66.90	0.253	0.292	8.33
450	464.7	31.6	3.16	31.8–132.3	51.20	0.223	60.40	69.40	0.263	0.302	8.36
500	514.7	35.0	3.27	33.4–142.4	52.70	0.229	62.30	71.70	0.273	0.314	8.38
550	564.7	38.4	3.38	35.0–153.1	53.75	0.234	65.00	74.75	0.283	0.326	8.80
600	614.7	41.8	3.47	36.3–162.3	54.85	0.239	66.85	76.90	0.291	0.334	8.86
650	664.7	45.2	3.56	37.6–171.5	56.00	0.244	67.90	78.15	0.296	0.340	9.02
700	714.7	48.6	3.65	38.9–180.8	57.15	0.249	69.40	79.85	0.303	0.348	9.18
750	764.7	52.0	3.73	40.1–189.8	58.10	0.253	70.75	81.40	0.309	0.355	....
800	814.7	55.4	3.82	41.4–199.5	59.00	0.257	72.45	83.25	0.315	0.362	....
850	864.7	58.8	3.89	42.5–207.8	60.20	0.262	73.75	84.90	0.321	0.369	....
900	914.7	62.2	3.95	43.4–214.6	60.80	0.265	74.80	86.00	0.326	0.375	....
950	964.7	65.6	4.03	44.6–224.5	61.72	0.269	76.10	87.50	0.331	0.381	....
1000	1014.7	69.0	4.11	45.7–233.3	62.40	0.272	77.20	88.80	0.336	0.383	....
1050	1064.7	72.4	4.15	46.3–238.3	63.10	0.275	78.10	90.10	0.340	0.391	....
1100	1114.7	75.8	4.23	47.5–248.3	63.80	0.278	79.10	91.10	0.344	0.396	....
1150	1164.7	79.2	4.30	48.5–256.8	64.40	0.281	80.15	92.20	0.349	0.401	....
1200	1214.7	82.6	4.33	49.0–261.3	65.00	0.283	81.00	93.15	0.353	0.405	....
1250	1264.7	86.0	4.42	50.3–272.3	65.60	0.286	82.00	94.30	0.357	0.411	....
1300	1314.7	89.4	4.48	51.3–280.8	66.30	0.289	82.90	95.30	0.362	0.416	....
1350	1364.7	92.8	4.53	52.0–287.3	66.70	0.291	84.00	96.60	0.366	0.421	....
1400	1414.7	96.2	4.58	52.6–293.5	67.00	0.292	84.60	97.30	0.368	0.423	....
1450	1464.7	99.6	4.64	53.5–301.5	67.70	0.295	85.30	98.20	0.371	0.426	....
1500	1514.7	103.0	4.69	54.3–309.3	68.30	0.298	85.80	98.80	0.374	0.430	....
1550	1564.7	106.4	4.74	55.0–317.3	68.80	0.300	86.80	99.85	0.378	0.434	....
1600	1614.7	109.8	4.79	55.8–323.3	69.10	0.302	87.60	100.80	0.382	0.438	....

Flow of Air in Pipes. — The following formulas are used by the B. F. Sturtevant Co.:

$$v = \sqrt{\frac{25{,}000\, dp}{L}} \qquad p = \frac{Lv^2}{25{,}000\, d}$$

in which v = velocity of air in feet per second;
p = loss of pressure due to flow through the pipes in ounces per square inch;
d = inside diameter of pipe in inches;
L = length of pipe in feet.

The quantity of air discharged in cubic feet per second is the product of the velocity as obtained from the formula above and the area of the pipe in square feet. The horsepower required to drive air through a pipe equals the volume of air in cubic feet per second multiplied by the pressure in pounds per square foot, and this product divided by 550.

Volume of Air Transmitted, in Cubic Feet per Minute, Through Pipes

Velocity of Air in Feet per Second	Actual Inside Diameter of Pipe, Inches									
	1	2	3	4	6	8	10	12	16	24
1	0.33	1.31	2.95	5.2	11.8	20.9	32.7	47.1	83.8	188
2	0.65	2.62	5.89	10.5	23.6	41.9	65.4	94.2	167.5	377
3	0.98	3.93	8.84	15.7	35.3	62.8	98.2	141.4	251.3	565
4	1.31	5.24	11.78	20.9	47.1	83.8	131.0	188.0	335.0	754
5	1.64	6.55	14.7	26.2	59.0	104.0	163.0	235.0	419.0	942
6	1.96	7.85	17.7	31.4	70.7	125.0	196.0	283.0	502.0	1131
7	2.29	9.16	20.6	36.6	82.4	146.0	229.0	330.0	586.0	1319
8	2.62	10.50	23.5	41.9	94.0	167.0	262.0	377.0	670.0	1508
9	2.95	11.78	26.5	47.0	106.0	188.0	294.0	424.0	754.0	1696
10	3.27	13.1	29.4	52.0	118.0	209.0	327.0	471.0	838.0	1885
12	3.93	15.7	35.3	63.0	141.0	251.0	393.0	565.0	1005.0	2262
15	4.91	19.6	44.2	78.0	177.0	314.0	491.0	707.0	1256.0	2827
18	5.89	23.5	53.0	94.0	212.0	377.0	589.0	848.0	1508.0	3393
20	6.55	26.2	59.0	105.0	235.0	419.0	654.0	942.0	1675.0	3770
24	7.86	31.4	71.0	125.0	283.0	502.0	785.0	1131.0	2010.0	4524
25	8.18	32.7	73.0	131.0	294.0	523.0	818.0	1178.0	2094.0	4712
28	9.16	36.6	82.0	146.0	330.0	586.0	916.0	1319.0	2346.0	5278
30	9.80	39.3	88.0	157.0	353.0	628.0	982.0	1414.0	2513.0	5655

Flow of Compressed Air in Pipes. — When there is a comparatively small difference of pressure at the two ends of the pipe, the volume of flow in cubic feet per minute is found by the formula:

$$V = 58\sqrt{\frac{pd^5}{WL}}$$

in which V = volume of air in cubic feet per minute;
p = difference in pressure at the two ends of the pipe in pounds per square inch;
d = inside diameter of pipe in inches;
W = weight in pounds of one cubic foot of entering air;
L = length of pipe in feet.

Velocity of Escaping Compressed Air

Pressure Above Atmospheric Pressure: In Atmospheres	In Inches Mercury	In Lbs. per Sq. In.	Theoretical Velocity, Feet per Second	Pressure Above Atmospheric Pressure: In Atmospheres	In Inches Mercury	In Lbs. per Sq. In.	Theoretical Velocity, Feet per Second
0.010	0.30	0.147	94.4	0.680	20.4	10.0	780
0.066	2.10	1.00	246.0	0.809	24.3	12.0	855
0.100	3.00	1.47	299.0	1.0	30.0	14.7	946
0.136	4.08	2.00	348.0	2.0	60.0	29.4	1094
0.204	6.12	3.00	427.0	5.0	150.0	73.5	1219
0.272	8.16	4.00	493.0	10.0	300.0	147.0	1275
0.340	10.20	5.00	552.0	20.0	600.0	294.0	1304
0.408	12.24	6.00	604.0	40.0	1200.0	588.0	1323
0.500	15.00	7.35	673.0	100.0	3000.0	1470.0	1331
0.544	16.32	8.00	697.0	200.0	6000.0	2940.0	1334
0.611	18.34	9.00	741.0	......	......	......	....

The theoretical velocities in the table above must be reduced by multiplying by a "factor of discharge," which varies with the orifice and the pressure. The following factors are used for orifices in thin plate and short tubes.

Type of Orifice	Pressures in Atmospheres Above Atmospheric Pressure: 0.01	0.1	0.5	1	5	10	100
Orifice in thin plate........	0.65	0.64	0.57	0.54	0.45	0.44	0.42
Orifice in short tube......	0.83	0.82	0.71	0.67	0.53	0.51	0.49

Velocity of Air under Low Pressures. — The table "Velocity of Air under Low Pressures" gives the theoretical velocity for the discharge of air into the atmosphere. These theoretical velocities are modified by multiplying them by a factor varying with the form of the orifice. For an orifice with sharp edges in a thin plate, this factor equals 0.65. For a plate with the inside of the orifice rounded, the factor equals 0.70 to 0.75, and for a well-shaped nozzle, 0.93.

Velocity of Air Under Low Pressures

(Temperature 62° F.)

Gage Pressure, Ounces per Square Inch	Theoretical Velocity, Feet per Second	Gage Pressure, Ounces per Square Inch	Theoretical Velocity, Feet per Second	Gage Pressure, Ounces per Square Inch	Theoretical Velocity, Feet per Second	Gage Pressure, Ounces per Square Inch	Theoretical Velocity, Feet per Second	Gage Pressure, Ounces per Square Inch	Theoretical Velocity, Feet per Second
0.006	6.61	0.115	29.5	0.346	51.2	0.866	80.9	2.308	132.0
0.012	9.35	0.173	36.2	0.404	55.3	1.153	93.5	2.597	140.0
0.023	13.20	0.231	41.8	0.461	59.1	1.442	104.0	2.885	148.0
0.040	17.40	0.260	44.3	0.519	62.7	1.731	114.0	3.462	162.0
0.058	20.90	0.289	46.7	0.577	66.1	2.020	124.0	....	....

Loss of Pressure Due to Flow through Pipes. — The table "Loss of Pressure by Friction in Pipes" is based on data published by the B. F. Sturtevant Co., and gives the loss in pressure due to friction of air in pipes 100 feet long. For any other length the loss is proportional to the length.

Loss of Pressure by Friction in Pipes

Velocity, Feet per Minute	Diameter of Pipe in Inches											
	1	2	3	4	5	6	7	8	9	10	11	12
	Loss in Ounces per Square Inch per 100 Feet											
600	0.4	0.2	0.13	0.1	0.08	0.07	0.06	0.05	0.04	0.04	0.04	0.03
1200	1.6	0.8	0.53	0.4	0.32	0.27	0.23	0.20	0.18	0.16	0.14	0.13
1800	3.6	1.8	1.20	0.9	0.72	0.60	0.51	0.45	0.40	0.36	0.33	0.30
2400	6.4	3.2	2.13	1.6	1.28	1.07	0.91	0.80	0.71	0.64	0.58	0.53
3000	10.0	5.0	3.33	2.5	2.00	1.67	1.43	1.25	1.11	1.00	0.91	0.83
3600	14.4	7.2	4.80	3.6	2.88	2.40	2.06	1.80	1.60	1.44	1.31	1.20
4200	...	9.8	6.53	4.9	3.92	3.27	2.80	2.45	2.18	1.96	1.78	1.63
4800	...	12.8	8.53	6.4	5.12	4.27	3.66	3.20	2.84	2.56	2.33	2.13
6000	...	20.0	13.33	10.0	8.00	6.67	5.71	5.00	4.44	4.00	3.64	3.33

Velocity, Feet per Minute	Diameter of Pipe in Inches									
	14	16	20	24	28	32	36	40	44	48
	Loss in Ounces per Square Inch per 100 Feet									
600	0.029	0.025	0.02	0.017	0.014	0.012	0.011	0.01	0.009	0.008
1200	0.114	0.100	0.08	0.067	0.057	0.050	0.044	0.04	0.036	0.033
1800	0.257	0.225	0.18	0.150	0.129	0.112	0.100	0.09	0.082	0.075
2400	0.457	0.400	0.32	0.267	0.239	0.200	0.178	0.16	0.145	0.133
3000	0.714	0.625	0.50	0.417	0.357	0.312	0.278	0.25	0.227	0.208
3600	1.029	0.900	0.72	0.600	0.514	0.450	0.400	0.36	0.327	0.300
4200	1.400	1.225	0.98	0.817	0.700	0.612	0.544	0.49	0.445	0.408
4800	1.829	1.600	1.28	1.067	0.914	0.800	0.711	0.64	0.582	0.533
6000	2.857	2.500	2.00	1.667	1.429	1.250	1.111	1.00	0.909	0.833

Effect of Bends on the Flow of Air. — The formulas given for the flow of air through pipes relate to straight pipes only. The effect of a bend in a fitting or pipe varies to a great extent with the character of the bend. The resistance offered is least when the radius of the bend is equal to five times the radius of the pipe. The usual way for stating the resistance offered by a bend is in terms of the equivalent length of straight pipe which offers the same frictional resistance to the flow of air as does the bend. The following formula may be used:

$$L = 12.85\, l \left(\frac{r}{R}\right)^{0.83}$$

in which L = equivalent length of straight pipe in feet; R = mean radius of bend in feet or inches; r = inside radius of pipe in feet or inches; l = length of bend in feet measured along the center line.

The resistance of a bend, the radius of which is five times the radius of the pipe, is equal to the resistance of a straight length of pipe, 3⅓ times the length of the bend measured along the center line.

Effect of Fittings. — The reduction of pressure produced by elbows, tees and globe valves may also be stated in an equivalent length of straight pipe. The following table (National Tube Co.) gives the additional length required to equal the friction due to globe valves. For elbows and tees, take two-thirds of the length given in the table.

Diam. of Pipe,	1	1½	2	3	4	5	6	8	10	12	15	18	24
Additional Length, Feet	2	4	7	13	20	28	36	53	70	88	115	143	200

Loss of Pressure in Pounds per Square Inch of Air at 80 Pounds Gage Pressure in 1000 Feet of Pipe

(Ingersoll-Rand Co.)

Size of Pipe	Delivery in Cubic Feet of Compressed Air per Minute at 80 Pounds Gage									
	7.74	11.3	15.2	19.4	23.2	27.2	31.0	38.7	46.5	62.0
	Equivalent Delivery in Cubic Feet of Free Air per Minute									
	50	75	100	125	150	175	200	250	300	400
1	14.31	.....	.....	.....	.....	.....	.....	.....	.....	.....
1¼	3.96	8.46	15.31	.....	.....	.....	.....	.....	.....	.....
1½	1.53	3.26	5.92	9.64	13.79	.....	.....	.....	.....	.....
2	0.33	0.71	1.28	2.09	2.99	4.09	5.34	8.32	12.01	.....
2½	0.10	0.21	0.39	0.64	0.91	1.25	1.63	2.54	3.67	6.53
3	0.03	0.08	0.14	0.24	0.34	0.47	0.61	0.96	1.38	2.45
3½	0.01	0.03	0.06	0.11	0.15	0.21	0.27	0.43	0.62	1.11
4	.....	0.01	0.03	0.05	0.07	0.10	0.13	0.21	0.30	0.54
4½	.....	.....	0.02	0.03	0.04	0.06	0.07	0.12	0.17	0.30
5	.....	.....	0.01	0.01	0.02	0.03	0.04	0.07	0.09	0.17
6	.....	.....	.....	.....	0.01	0.01	0.01	0.02	0.03	0.06
7	.....	.....	.....	.....	.....	.....	.....	0.01	0.01	0.03
8	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.01

Size of Pipe	Delivery in Cubic Feet of Compressed Air per Minute at 80 Pounds Gage									
	77.4	92.9	124.0	152	232	310	387	465	620	774
	Equivalent Delivery in Cubic Feet of Free Air per Minute									
	500	600	800	1000	1500	2000	2500	3000	4000	5000
2½	10.81	.....	.....	.....	.....	.....	.....	.....	.....	.....
3	3.83	5.61	9.86	.....	.....	.....	.....	.....	.....	.....
3½	1.73	2.46	4.42	6.64	15.41	.....	.....	.....	.....	.....
4	0.85	1.22	2.18	3.29	7.62	13.62	.....	.....	.....	.....
4½	0.47	0.68	1.19	1.82	4.24	7.58	11.79	.....	.....	.....
5	0.27	0.39	0.69	1.04	2.43	4.32	6.88	9.72	.....	.....
6	0.10	0.15	0.27	0.40	0.95	1.69	2.64	3.79	6.78	10.55
7	0.05	0.06	0.12	0.18	0.43	0.77	1.19	1.73	3.07	4.79
8	0.02	0.03	0.06	0.09	0.22	0.39	0.60	0.87	1.55	2.46
9	0.01	0.02	0.03	0.05	0.12	0.21	0.33	0.48	0.85	1.33
10	.....	0.01	0.02	0.03	0.06	0.12	0.19	0.28	0.49	0.77
12	.....	.....	0.01	0.01	0.02	0.04	0.07	0.11	0.19	0.30
14	.....	.....	.....	.....	0.01	0.02	0.03	0.05	0.09	0.14
16	.....	.....	.....	.....	.....	0.01	0.01	0.02	0.04	0.07

Inside Diameter of Pipes, in Inches, Required to Transmit Air at Given Velocities

(Buffalo Forge Co.)

Cubic Feet of Air Transmitted per Minute	Velocity of Air, Feet per Minute											
	500	600	800	1000	1200	1500	1800	2000	2500	3000	3500	4000
	Diameter of Pipe, Inches											
200	9	8	7	7	6	6	6	6	6	6	6	6
400	13	11	10	9	8	8	7	7	6	6	6	6
600	15	14	12	11	10	9	8	8	7	7	6	6
800	18	16	14	13	12	10	9	9	8	8	7	7
1000	20	18	16	14	13	12	10	10	9	8	8	7
1200	21	20	17	15	14	13	11	11	10	9	9	8
1400	23	21	18	16	15	14	12	12	11	10	9	9
1600	25	23	20	18	16	15	13	13	11	11	10	9
1800	26	24	21	19	17	16	14	13	12	11	10	10
2000	28	25	22	20	18	16	15	14	13	12	11	10
2200	29	27	23	21	19	17	15	15	13	12	11	11
2400	30	28	24	21	20	18	16	15	14	13	12	11
2600	31	29	25	22	20	18	17	16	15	13	12	11
2800	33	30	26	23	21	19	18	16	15	14	13	12
3000	34	31	27	24	22	20	18	17	15	14	13	12
3200	34	32	28	25	23	20	19	18	15	15	13	13
3400	36	33	28	25	23	21	19	18	16	15	14	13
3600	37	34	29	26	24	21	20	19	16	15	14	13
3800	38	35	30	27	25	22	21	19	17	16	15	14
4000	39	35	31	28	25	22	21	20	18	16	15	14
4200	40	36	32	28	26	23	21	20	18	16	15	14
4400	41	37	32	29	26	24	22	21	18	17	16	15
4600	42	38	33	30	27	24	22	21	19	17	16	15
4800	42	39	34	30	28	25	22	21	19	18	16	15
5000	43	40	34	31	28	25	23	22	20	18	17	16
5200	44	40	35	31	29	25	24	22	20	18	17	16
5400	..	..	35	32	29	26	24	23	21	18	18	16
5600	..	..	36	33	30	27	24	23	21	19	18	17
5800	..	..	37	33	30	27	25	24	21	19	18	17
6000	..	..	38	34	31	28	25	24	21	20	18	17
6200	..	..	38	34	31	28	25	24	21	20	18	17
6400	..	..	39	35	32	28	26	25	22	20	19	18
6600	..	..	39	36	32	29	26	25	22	21	19	18
6800	..	..	40	36	33	29	27	25	23	21	19	18
7000	..	..	40	36	33	30	27	26	23	21	19	18
7200	..	..	41	37	34	30	28	26	23	21	20	19
7400	..	..	41	37	34	30	28	27	24	21	20	19
7600	..	..	42	38	34	31	28	27	24	22	20	19
7800	..	..	43	38	36	31	29	27	24	22	21	19
8000	..	..	43	39	36	32	29	28	25	22	21	20
8200	..	..	..	39	36	32	29	28	25	23	21	20
8400	..	..	..	40	36	33	30	28	25	23	21	20

Care of Pneumatic Hammers. — The life of pneumatic hammers can be greatly prolonged, and the necessity for making repairs largely eliminated, by keeping the working parts clean and well lubricated. It is a good plan to occasionally submerge the hammer in a bath of benzine for a few hours, and then blow it out under pressure to dislodge any foreign matter that may have entered. Before using the hammer, the working parts should be lubricated with a good quality of light machine oil. The source of pneumatic hammer troubles is sometimes in the pipe line. The moisture in the air rusts the pipes, and if a hammer is connected without first blowing out the pipes, the sediment is liable to enter the working parts and cause the valve or piston to stick. Rubber also deteriorates rapidly, and particles of the hose may blow into the valve box and interfere with the operation of the hammer. The use of a poor grade of heavy-bodied oil will also cause the ports to become clogged and render the hammer inoperative. In such cases, it is a good plan to clean the working parts by injecting a liberal quantity of benzine through the throttle handle, as this dislodges the foreign matter and "cuts" the thick oil, which can then be removed by blowing air through the hammer.

The use of strainers or filters attached to the hammer or placed in the supply pipe is to be recommended. Perhaps the most serious abuse to which riveting hammers are subjected is the use of short pistons. When the length of the piston is reduced, the hammer delivers a more rapid blow and, for a time, facilitates the work; hence workmen sometimes substitute short pistons to increase their output, especially when working on a piece-work basis. Short pistons are objectionable as they tend to crumble, and the broken parts will cut the inner casing of the cylinder. If the latter is not damaged beyond repair from this cause, it is soon cracked or the hammer is otherwise broken.

Pneumatic Hammer Capacity and Air Consumption *

Piston Diam., Inches	Stroke, Inches	Weight, Pounds	Cu. Ft. of Free Air per Min.	Blows per Minute	Size of Hose Required	Work Adapted for
			Riveting Hammers			
1 1/16	1	10 1/4	15	3100	1/2	Driving 1/4-inch Hot Rivets
1 1/16	2	11 1/4	15	2200	1/2	Driving 5/16-inch Hot Rivets
1 1/16	3	12 1/2	15	1600	1/2	Driving 3/8-inch Hot Rivets
1 1/16	4	15	15	1100	1/2	Driving 1/2-inch Hot Rivets
1 1/8	5	17	20	1000	3/4	Driving 3/4-inch Hot Rivets
1 1/16	6	19	21	900	3/4	Driving 7/8-inch Hot Rivets
1 1/16	8	21	22	770	3/4	Driving 1-inch Hot Rivets
1 1/16	9	22	23	700	3/4	Driving 1 1/8-inch Hot Rivets
1 1/8	9	22	25	700	3/4	Driving 1 1/4-inch Hot Rivets
1 1/16	9	42	22	800	3/4	Driving Stay-bolts — All Sizes
			Chipping and Calking Hammers			
3/4	1 5/16	5 1/2	7	5000	1/2	Very Light Chipping and Calking
15/16	1 1/4	7	10	4000	1/2	Very Light Chipping and Calking
1 1/16	1	10 1/4	15	3100	1/2	Very Light Chipping and Calking
1 1/16	2	11 1/4	15	2200	1/2	Medium Chipping, Calking and Beading
1 1/16	3	12 1/2	15	1600	1/2	General Chipping and Calking
1 1/16	4	15	15	1100	1/2	Heavy Chipping and Calking

* Independent Pneumatic Tool Co.

Pneumatic Hammer Capacity and Air Consumption *

Diam. of Piston, Inches	Length of Stroke, Inches	Blows per Minute	Cu. Ft. of Free Air per Minute †	Size of Hose Connection	Diam. of Piston, Inches	Length of Stroke, Inches	Capacity, Rivet Diam.	Blows per Minute	Cu. Ft. of Free Air per Minute	Size of Hose Connection
Chipping and Calking Hammers					Riveting Hammers					
3/4	1 3/8	2900	6	1/4	1 1/16	5	3/4	1000	25	3/8
15/16	1 1/4	2200	8	1/4	1 1/16	6	7/8	760	25	3/8
1 1/16	1 1/2	2800	12	1/4	1 1/16	8	1 1/8	700	25	3/8
1 1/8	1	3200	10	1/4	1 1/16	9	1 1/4	620	25	3/8
1 1/8	2	2800	12	1/4	1 1/16	6	7/8	760	25	3/8
1 1/8	3	2400	13	1/4	1 1/16	8	1 1/8	700	25	3/8
1 1/8	4	1600	14	1/4	1 1/16	9	1 1/4	620	25	3/8
1 1/8	1	3200	21	1/4	1 3/16	8	1 1/2	800	28	3/8
1 1/8	2	2800	23	1/4	....	....	....	....	....	....
1 1/8	4	1300	26	1/4	....	....	....	....	....	....
1 3/16	2	2800	10	1/4	....	....	....	....	....	....

* Chicago Pneumatic Tool Co. † Air consumption based upon an air pressure of 80 pounds per square inch.

Air Consumption of Pneumatic Drills. — A general idea of the number of cubic feet of free air per minute required for operating pneumatic drilling machines may be obtained from the following figures. The air consumption of drills made by different manufacturers is, of course, subject to some variation. These figures are based upon an initial air pressure of 80 pounds per square inch:

1. Cylinder diameter, 1 1/4 inch; stroke, 7/8 inch; size of hose connection, 1/4 inch; air consumption, 15 cubic feet per minute.

2. Cylinder diameter, 1 1/2 inch; stroke, 1 1/4 inch; size of hose connection, 1/2 inch; air consumption, from 18 to 20 cubic feet per minute.

3. Cylinder diameter, 1 11/16 inch; stroke, 1 5/8 inch; size of hose connection, 1/2 inch; air consumption, from 20 to 25 cubic feet per minute.

4. Cylinder diameter, 2 inches; stroke, 1 7/8 inch; size of hose connection, 1/2 inch; air consumption, from 30 to 35 cubic feet per minute.

Pneumatic Hoists. — The air-motor or pneumatic geared type of hoist is equipped with some form of air motor which drives the lifting drum through suitable reduction gearing. There are three general classes of air hoists of the cylinder type. With the *single-acting type*, compressed air is admitted to the lower or stuffing-box side of the piston only, and when lowering the hoist, this air is exhausted. This type is intended for ordinary work, especially where a delicate control is not necessary. The *air-balanced type* of hoist is so arranged that there is full air pressure on the stuffing-box side of the piston at all times. The load is hoisted by exhausting air from the space above the piston; the unbalanced area due to the space occupied by the piston-rod aids in forcing the piston downward. The advantage of this arrangement is accuracy of control. The *double-acting type* differs from the balanced type in that air may be admitted and exhausted from either side of the piston, so that the latter may be moved in either direction with equal power. Thus, with a balanced hoist, there is a constant pressure on one side of the piston and a variable pressure on the other, whereas, with a double-acting type, the pressure on either side of the piston may be varied in accordance with the amount of the load and the direction in which the force must be applied.

PRESSURES AND FLOW OF WATER

Water Pressures. — Water is composed of two gases, hydrogen and oxygen, in the ratio of two volumes of the former to one of the latter. Water boils under atmospheric pressure at 212 degrees F. and freezes at 32 degrees F. Its greatest density is at 39.1 degrees F., when it weighs 62.425 pounds per cubic foot. The pressure in pounds per square inch of water that is not moving, against the sides of any pipe, vessel, container or dam, is due solely to the "head," or height of the surface of the water above the point at which the pressure is considered. The pressure is equal to 0.433 pound per square inch for every foot of the head, at a temperature of 62 degrees F. For higher temperatures, the pressure slightly decreases in the proportion indicated by the table "Weight of Water per Cubic Foot at Different Temperatures." The pressure per square inch is equal in all directions, downwards, upwards and sideways. Water can be compressed only in a very slight degree, the compressibility being so slight that even at the depth of a mile, a cubic foot of water weighs only about one-half pound more than at the surface.

Pressure in Pounds per Square Inch for Different Heads of Water

Head, Feet	0	1	2	3	4	5	6	7	8	9
0	....	0.43	0.87	1.30	1.73	2.16	2.60	3.03	3.46	3.90
10	4.33	4.76	5.20	5.63	6.06	6.49	6.93	7.36	7.79	8.23
20	8.66	9.09	9.53	9.96	10.39	10.82	11.26	11.69	12.12	12.56
30	12.99	13.42	13.86	14.29	14.72	15.15	15.59	16.02	16.45	16.89
40	17.32	17.75	18.19	18.62	19.05	19.48	19.92	20.35	20.78	21.22
50	21.65	22.08	22.52	22.95	23.38	23.81	24.25	24.68	25.11	25.55
60	25.98	26.41	26.85	27.28	27.71	28.14	28.58	29.01	29.44	29.88
70	30.31	30.74	31.18	31.61	32.04	32.47	32.91	33.34	33.77	34.21
80	34.64	35.07	35.51	35.94	36.37	36.80	37.24	37.67	38.10	38.54
90	38.97	39.40	39.84	40.27	40.70	41.13	41.57	42.00	42.43	42.87

Heads of Water in Feet Corresponding to Certain Pressures in Pounds per Square Inch

Pressure, Lbs.	0	1	2	3	4	5	6	7	8	9
0	....	2.3	4.6	6.9	9.2	11.5	13.9	16.2	18.5	20.8
10	23.1	25.4	27.7	30.0	32.3	34.6	36.9	39.3	41.6	43.9
20	46.2	48.5	50.8	53.1	55.4	57.7	60.0	62.4	64.7	67.0
30	69.3	71.6	73.9	76.2	78.5	80.8	83.1	85.4	87.8	90.1
40	92.4	94.7	97.0	99.3	101.6	103.9	106.2	108.5	110.8	113.2
50	115.5	117.8	120.1	122.4	124.7	127.0	129.3	131.6	133.9	136.3
60	138.6	140.9	143.2	145.5	147.8	150.1	152.4	154.7	157.0	159.3
70	161.7	164.0	166.3	168.6	170.9	173.2	175.5	177.8	180.1	182.4
80	184.8	187.1	189.4	191.7	194.0	196.3	198.6	200.9	203.2	205.5
90	207.9	210.2	212.5	214.8	217.1	219.4	221.7	224.0	226.3	228.6

Comparison of Different Methods of Measuring Pressures

Ounces and Pounds per Square Inch, and Inches of Water and Mercury

Ounces per Square Inch	Pounds per Square Inch	Inches of Water	Inches of Mercury	Ounces per Square Inch	Pounds per Square Inch	Inches of Water	Inches of Mercury
0.25	0.016	0.433	0.0319	8	0.500	13.856	1.020
0.50	0.031	0.866	0.0638	9	0.562	15.588	1.148
1	0.062	1.732	0.1275	10	0.625	17.320	1.275
2	0.125	3.464	0.2551	11	0.687	19.052	1.403
3	0.187	5.196	0.3826	12	0.750	20.784	1.531
4	0.250	6.928	0.5102	13	0.812	22.516	1.658
5	0.312	8.660	0.6377	14	0.875	24.248	1.786
6	0.375	10.392	0.7653	15	0.937	25.980	1.913
7	0.437	12.124	0.8928	16	1.000	27.712	2.041

Pounds per Square Inch, Inches and Feet of Water and Inches of Mercury

Pounds per Square Inch	Inches of Water	Feet of Water	Inches of Mercury	Pounds per Square Inch	Inches of Water	Feet of Water	Inches of Mercury
1	27.71	2.31	2.041	14	387.97	32.33	28.57
2	55.42	4.62	4.081	14.7	407.37	33.95	30.00
3	83.14	6.93	6.122	15	415.68	34.64	30.61
4	110.85	9.24	8.163	16	443.40	36.95	32.65
5	138.56	11.55	10.20	17	471.11	39.26	34.69
6	166.27	13.86	12.24	18	498.82	41.57	36.73
7	193.99	16.17	14.28	19	526.53	43.88	38.77
8	221.70	18.47	16.33	20	554.25	46.19	40.81
9	249.41	20.78	18.37	21	581.96	48.50	42.85
10	277.12	23.09	20.41	22	609.67	50.81	44.89
11	304.84	25.40	22.45	23	637.38	53.12	46.94
12	332.55	27.71	24.49	24	665.10	55.42	48.98
13	360.26	30.02	26.53	25	692.81	57.73	51.02

Volume of Water at Different Temperatures

Degrees Fahr.	Volume	Degrees Fahr.	Volume	Degrees Fahr.	Volume	Degrees Fahr.	Volume
39.1	1.00000	86	1.00425	131	1.01423	176	1.02872
50	1.00025	95	1.00586	140	1.01678	185	1.03213
59	1.00083	104	1.00767	149	1.01951	194	1.03570
68	1.00171	113	1.00967	158	1.02241	203	1.03943
77	1.00286	122	1.01186	167	1.02548	212	1.04332

Weight of Water per Cubic Foot at Different Temperatures

Temperature, Degrees F.	Weight per Cubic Foot, Pounds	Temperature, Degrees F.	Weight per Cubic Foot, Pounds	Temperature, Degrees F.	Weight per Cubic Foot, Pounds	Temperature, Degrees F.	Weight per Cubic Foot, Pounds	Temperature, Degrees F.	Weight per Cubic Foot, Pounds	Temperature, Degrees F.	Weight per Cubic Foot, Pounds
32	62.42	130	61.56	220	59.63	320	56.66	420	52.6	520	47.6
40	62.42	140	61.37	230	59.37	330	56.30	430	52.2	530	47.0
50	62.41	150	61.18	240	59.11	340	55.94	440	51.7	540	46.3
60	62.37	160	60.98	250	58.83	350	55.57	450	51.2	550	45.6
70	62.31	170	60.77	260	58.55	360	55.18	460	50.7	560	44.9
80	62.23	180	60.55	270	58.26	370	54.78	470	50.2	570	44.1
90	62.13	190	60.32	280	57.96	380	54.36	480	49.7	580	43.3
100	62.02	200	60.12	290	57.65	390	53.94	490	49.2	590	42.6
110	61.89	210	59.88	300	57.33	400	53.50	500	48.7	600	41.8
120	61.74	212	59.83	310	57.00	410	53.00	510	48.1	...	...

Table of Horsepower Due to Certain Head of Water

The table gives the horsepower of 1 cubic foot of water per minute, and is based on an efficiency of 85 per cent.

Heads in Feet	Horse-power	Heads in Feet	Horse-power	Heads in Feet	Horse-power	Heads in Feet	Horse-power	Heads in Feet	Horse-power
1	0.0016	170	0.274	340	0.547	520	0.837	1250	2.012
10	0.0161	180	0.290	350	0.563	540	0.869	1300	2.093
20	0.0322	190	0.306	360	0.580	560	0.901	1350	2.173
30	0.0483	200	0.322	370	0.596	580	0.934	1400	2.254
40	0.0644	210	0.338	380	0.612	600	0.966	1450	2.334
50	0.0805	220	0.354	390	0.628	650	1.046	1500	2.415
60	0.0966	230	0.370	400	0.644	700	1.127	1550	2.495
70	0.1127	240	0.386	410	0.660	750	1.207	1600	2.576
80	0.1288	250	0.402	420	0.676	800	1.288	1650	2.656
90	0.1449	260	0.418	430	0.692	850	1.368	1700	2.737
100	0.1610	270	0.435	440	0.708	900	1.449	1750	2.818
110	0.1771	280	0.451	450	0.724	950	1.529	1800	2.898
120	0.1932	290	0.467	460	0.740	1000	1.610	1850	2.978
130	0.2093	300	0.483	470	0.757	1050	1.690	1900	3.059
140	0.2254	310	0.499	480	0.773	1100	1.771	1950	3.139
150	0.2415	320	0.515	490	0.789	1150	1.851	2000	3.220
160	0.2576	330	0.531	500	0.805	1200	1.932	2100	3.381

Flow of Water in Pipes. — The quantity of water that will be discharged through a pipe depends primarily on the head and also upon the diameter of the pipe, the character of the interior surface, and the number and shape of the bends. The head may be either the actual distance between the levels of the surface of water in a reservoir and the point of discharge, or it may be caused by mechanically applied pressure, as by pumping, in which case the head is calculated as the vertical distance corresponding to the pressure. One pound per square inch is equal to 2.309 feet head, or 1 foot head is equal to a pressure of 0.433 pound per square inch.

All formulas for finding the amount of water that will flow through a pipe in a given time are approximate. The formula below will give results within 5 or 10 per cent of actual results, if applied to pipe lines carefully laid and in a fair condition.

$$V = C\sqrt{\frac{hD}{L + 54D}}$$

in which V = approximate mean velocity in feet per second;
C = coefficient from the accompanying table;
D = diameter of pipe in feet;
h = total head in feet;
L = total length of pipe line in feet.

Values of Coefficient C

Diam. of Pipe		C	Diam. of Pipe		C	Diam. of Pipe		C
Feet	Inches		Feet	Inches		Feet	Inches	
0.1	1.2	23	0.8	9.6	46	3.5	42	64
0.2	2.4	30	0.9	10.8	47	4.0	48	66
0.3	3.6	34	1.0	12.0	48	5.0	60	68
0.4	4.8	37	1.5	18.0	53	6.0	72	70
0.5	6.0	39	2.0	24.0	57	7.0	84	72
0.6	7.2	42	2.5	30.0	60	8.0	96	74
0.7	8.4	44	3.0	36.0	62	10.0	120	77

Example. — A pipe line, 1 mile long, 12 inches in diameter, discharges water under a head of 100 feet. Find the velocity and quantity of discharge.

From the table, the coefficient C is found to be 48 for a pipe 1 foot in diameter, hence:

$$V = 48\sqrt{\frac{100 \times 1}{5280 + 54 \times 1}} = 6.57 \text{ feet per second.}$$

To find the discharge in cubic feet per second, multiply the velocity found by the area of cross-section of the pipe in square feet:

$$6.57 \times 0.7854 = 5.16 \text{ cubic feet per second.}$$

The loss of head due to a bend in the pipe is most frequently given in the equivalent length of straight pipe, which would cause the same loss in head as the bend. Experiments show that a right-angle bend should have a radius of about three times the diameter of the pipe. Assuming this curvature, then, if D is the diameter of the pipe in inches and L is the length of straight pipe in feet, which causes the same loss of head as the bend in the pipe, the following formula gives the equivalent length of straight pipe that should be added to compensate for a right-angle bend:

$$L = 4D \div 3.$$

Thus the loss of head due to a right-angle bend in a six-inch pipe would be equal to that in 8 feet of straight pipe. Experiments undertaken to determine the losses due to valves in pipe lines indicate that a fully open gate valve in a pipe causes a loss of head corresponding to that in a length of pipe equal to six diameters.

Flow of Water Through Nozzles in Cubic Feet per Second

Head in Feet, at Nozzle	Pressure, Pounds per Square Inch	Theoretical Velocity, Feet per Second	Diameter of Nozzle, Inches							
			1	1½	2	2½	3	3½	4	4½
5	2.17	17.93	0.10	0.22	0.39	0.61	0.88	1.20	1.56	2.04
10	4.33	25.36	0.14	0.31	0.55	0.86	1.24	1.69	2.21	2.87
20	8.66	35.86	0.19	0.44	0.78	1.22	1.76	2.39	3.13	4.07
30	12.99	43.92	0.24	0.54	0.96	1.50	2.16	2.93	3.83	4.98
40	17.32	50.72	0.28	0.62	1.10	1.73	2.49	3.39	4.43	5.75
50	21.65	56.71	0.31	0.70	1.24	1.93	2.78	3.79	4.95	6.43
60	25.99	62.12	0.34	0.76	1.35	2.12	3.05	4.15	5.42	7.04
70	30.32	67.10	0.37	0.82	1.46	2.29	3.29	4.48	5.86	7.61
80	34.65	71.73	0.39	0.88	1.56	2.44	3.52	4.79	6.26	8.13
90	38.98	76.08	0.42	0.94	1.66	2.59	3.73	5.08	6.64	8.63
100	43.31	80.20	0.44	0.99	1.75	2.73	3.94	5.38	7.00	9.09
120	51.97	87.88	0.49	1.08	1.87	3.00	4.31	5.87	7.67	9.96
140	60.63	94.89	0.52	1.17	2.07	3.23	4.66	6.35	8.28	10.76
160	69.29	101.45	0.56	1.25	2.21	3.46	4.98	6.78	8.86	11.50
180	77.96	107.59	0.59	1.32	2.34	3.67	5.28	7.19	9.39	12.20
200	86.62	113.41	0.62	1.39	2.47	3.87	5.57	7.57	9.90	12.86
250	108.50	126.80	0.70	1.56	2.76	4.32	6.22	8.47	11.07	14.38
300	130.20	138.91	0.76	1.71	3.03	4.74	6.82	9.27	12.13	15.75
350	151.90	150.04	0.82	1.84	3.27	5.12	7.37	10.02	13.10	17.01
400	173.60	160.40	0.88	1.97	3.50	5.47	7.87	10.71	14.00	18.19
450	195.30	170.12	0.93	2.09	3.71	5.80	8.35	11.36	14.85	19.39
500	216.00	179.33	0.99	2.21	3.91	6.11	8.80	11.98	15.65	20.34

Head in Feet, at Nozzle	Pressure, Pounds per Square Inch	Theoretical Velocity, Feet per Second	Diameter of Nozzle, Inches							
			5	6	7	8	9	10	11	12
5	2.17	17.93	2.44	3.52	4.81	6.3	7.9	9.8	12.8	14.1
10	4.33	25.36	3.46	4.98	6.78	8.8	11.2	13.8	16.7	19.9
20	8.66	35.86	4.88	7.04	9.58	12.5	15.8	19.6	23.7	28.2
30	12.99	43.92	5.99	8.62	11.74	15.3	19.4	23.9	29.0	34.5
40	17.32	50.72	6.92	9.96	13.56	17.7	22.4	27.7	33.5	39.8
50	21.65	56.71	7.73	11.13	15.16	19.8	25.0	30.9	37.4	44.5
60	25.99	62.12	8.44	12.19	16.60	21.7	27.4	33.9	41.0	48.8
70	30.32	67.10	9.15	13.17	17.93	23.4	29.6	36.6	44.3	52.7
80	34.65	71.73	9.78	14.08	19.17	25.0	31.7	39.1	47.3	56.4
90	38.98	76.08	10.38	14.93	20.35	26.6	33.6	41.5	50.2	59.7
100	43.31	80.20	10.94	15.74	21.44	28.0	35.4	43.7	52.9	63.0
120	51.97	87.88	11.99	17.25	23.49	30.7	38.8	47.9	58.0	69.0
140	60.63	94.89	12.94	18.63	25.36	33.1	41.9	51.7	62.6	74.5
160	69.29	101.45	13.84	19.91	27.12	35.4	44.8	55.3	67.0	79.7
180	77.96	107.59	14.67	21.12	28.76	37.6	47.5	58.7	71.0	84.5
200	86.62	113.41	15.47	22.26	30.31	39.6	50.1	61.8	74.8	89.1
250	108.50	126.80	17.29	24.86	33.89	44.3	56.0	69.2	83.7	99.6
300	130.20	138.91	18.90	27.27	37.13	48.5	61.4	75.8	91.7	109.1
350	151.90	150.04	20.46	29.45	40.10	52.4	66.3	81.8	99.0	117.8
400	173.60	160.40	21.88	31.49	42.87	56.0	70.9	87.5	105.9	126.0
450	195.30	170.12	23.20	33.39	45.26	59.4	75.2	92.8	112.2	133.6
500	216.00	179.33	24.46	35.20	47.93	62.6	79.2	97.8	118.4	140.8

Theoretical Velocity of Water Due to Head in Feet

Head in Feet	Theoretical Velocity, Feet per Second	Theoretical Velocity, Feet per Minute	Head in Feet	Theoretical Velocity, Feet per Second	Theoretical Velocity, Feet per Minute	Head in Feet	Theoretical Velocity, Feet per Second	Theoretical Velocity, Feet per Minute
1	8.02	481	48	55.60	3336	95	78.22	4693
2	11.34	682	49	56.17	3370	96	78.63	4718
3	13.90	834	50	56.74	3405	97	79.04	4742
4	16.05	963	51	57.31	3438	98	79.44	4767
5	17.94	1077	52	57.87	3472	99	79.85	4791
6	19.66	1179	53	58.42	3505	100	80.25	4815
7	21.23	1274	54	58.97	3538	105	82.23	4934
8	22.70	1362	55	59.51	3571	110	84.17	5050
9	24.07	1445	56	60.05	3603	115	86.06	5163
10	25.38	1523	57	60.59	3635	120	87.91	5274
11	26.61	1597	58	61.12	3667	125	89.72	5383
12	27.80	1668	59	61.64	3698	130	91.50	5490
13	28.93	1736	60	62.16	3730	135	93.24	5594
14	30.03	1802	61	62.68	3761	140	94.95	5697
15	31.08	1865	62	63.19	3791	145	96.63	5798
16	32.10	1926	63	63.70	3822	150	98.28	5897
17	33.09	1985	64	64.20	3852	155	99.91	5994
18	34.05	2043	65	64.70	3882	160	101.50	6090
19	34.98	2099	66	65.19	3912	165	103.08	6185
20	35.89	2153	67	65.69	3941	170	104.63	6278
21	36.77	2206	68	66.17	3970	175	106.16	6370
22	37.64	2258	69	66.66	4000	180	107.66	6460
23	38.49	2309	70	67.14	4028	185	109.15	6549
24	39.31	2359	71	67.62	4057	190	110.61	6637
25	40.12	2407	72	68.09	4086	195	112.06	6724
26	40.92	2455	73	68.56	4114	200	113.49	6809
27	41.70	2502	74	69.03	4142	205	114.90	6894
28	42.46	2548	75	69.50	4170	210	116.29	6978
29	43.21	2593	76	69.96	4198	215	117.66	7060
30	43.95	2637	77	70.42	4225	220	119.03	7142
31	44.68	2681	78	70.87	4252	225	120.00	7222
32	45.40	2724	79	71.33	4280	230	121.70	7302
33	46.10	2766	80	71.78	4307	235	123.02	7381
34	46.79	2783	81	72.22	4333	240	124.32	7459
35	47.48	2848	82	72.67	4360	245	125.60	7537
36	48.15	2889	83	73.11	4387	250	126.88	7613
37	48.81	2929	84	73.55	4413	255	128.15	7649
38	49.47	2968	85	73.99	4439	260	129.39	7764
39	50.12	3007	86	74.42	4465	270	131.86	7912
40	50.75	3045	87	74.85	4491	280	134.28	8057
41	51.38	3083	88	75.28	4517	290	136.66	8200
42	52.01	3120	89	75.71	4542	300	138.99	8340
43	52.62	3157	90	76.13	4568	310	141.29	8478
44	53.23	3194	91	76.55	4593	320	143.55	8613
45	53.83	3230	92	76.97	4618	330	145.78	8761
46	54.43	3266	93	77.39	4643	340	147.97	8878
47	55.02	3301	94	77.80	4668	350	150.13	9008

Loss of Head in Pipes by Friction

(Pelton Water-wheel Co.)

L = loss of head, in feet, for 100 feet length of pipe.
Q = quantity of water discharged per minute, in cubic feet.

Velocity, Feet per Second	Inside Diameter of Pipe in Inches											
	1		2		3		4		5		6	
	L	*Q*	*L*	*Q*	*L*	*Q*	*L*	*Q*	*L*	*Q*	*L*	*Q*
2.0	2.37	0.65	1.18	2.62	0.79	5.9	0.59	10.4	0.47	16.3	0.39	23.5
2.2	2.80	0.73	1.40	2.88	0.94	6.5	0.70	11.5	0.56	18.0	0.47	25.9
2.4	3.27	0.79	1.64	3.14	1.09	7.1	0.82	12.5	0.65	19.6	0.55	28.2
2.6	3.78	0.86	1.89	3.40	1.26	7.6	0.94	13.6	0.76	21.3	0.63	30.6
2.8	4.32	0.92	2.16	3.66	1.44	8.2	1.08	14.6	0.86	22.9	0.72	32.9
3.0	4.89	0.99	2.44	3.92	1.62	8.8	1.22	15.7	0.98	24.5	0.81	35.3
3.2	5.47	1.06	2.73	4.18	1.82	9.4	1.37	16.7	1.10	26.2	0.91	37.7
3.4	6.09	1.12	3.05	4.45	2.04	10.0	1.52	17.8	1.22	27.8	1.02	40.0
3.6	6.76	1.19	3.38	4.71	2.26	10.6	1.69	18.8	1.35	29.4	1.13	42.4
3.8	7.48	1.26	3.74	4.97	2.49	11.2	1.87	19.9	1.49	31.0	1.25	44.7
4.0	8.20	1.32	4.10	5.23	2.73	11.8	2.05	20.9	1.64	32.7	1.37	47.1
4.4	9.77	1.45	4.89	5.76	3.25	12.9	2.43	23.0	1.95	36.0	1.62	51.8
4.8	11.45	1.58	5.72	6.28	3.81	14.1	2.85	25.1	2.27	39.2	1.90	56.5
5.0	12.33	1.65	6.17	6.54	4.11	14.7	3.08	26.2	2.46	40.9	2.05	58.9
5.2	13.24	1.72	6.62	6.80	4.41	15.3	3.31	27.2	2.65	42.5	2.21	61.2
5.6	15.16	1.85	7.58	7.32	5.06	16.5	3.79	29.3	3.03	45.8	2.53	65.9
6.0	17.23	1.98	8.61	7.85	5.74	17.7	4.31	31.4	3.45	49.1	2.87	70.7
7.0	22.89	2.31	11.45	9.16	7.62	20.6	5.72	36.6	4.57	57.2	3.81	82.4

Velocity, Feet per Second	Inside Diameter of Pipe in Inches											
	7		8		9		10		12		14	
	L	*Q*	*L*	*Q*	*L*	*Q*	*L*	*Q*	*L*	*Q*	*L*	*Q*
2.0	0.34	32.0	0.30	41.9	0.26	53	0.24	65	0.20	94	0.17	128
2.2	0.40	35.3	0.35	46.1	0.31	58	0.28	72	0.23	103	0.20	141
2.4	0.47	38.5	0.41	50.2	0.36	64	0.33	78	0.27	113	0.23	154
2.6	0.54	41.7	0.47	54.4	0.42	69	0.38	85	0.31	122	0.27	167
2.8	0.62	44.9	0.54	58.6	0.48	74	0.43	92	0.36	132	0.31	179
3.0	0.70	48.1	0.61	62.8	0.54	79	0.49	98	0.41	141	0.35	192
3.2	0.78	51.3	0.69	67.0	0.61	85	0.55	105	0.46	151	0.39	205
3.4	0.87	54.5	0.76	71.2	0.68	90	0.61	111	0.51	160	0.44	218
3.6	0.97	57.7	0.85	75.4	0.75	95	0.68	118	0.57	169	0.48	231
3.8	1.07	60.9	0.94	79.6	0.83	101	0.75	124	0.62	179	0.53	243
4.0	1.17	64.1	1.03	83.7	0.91	106	0.82	131	0.68	188	0.59	256
4.4	1.39	70.5	1.22	92.1	1.09	116	0.98	144	0.81	207	0.70	282
4.8	1.63	76.9	1.43	100.0	1.27	127	1.14	157	0.95	226	0.82	308
5.0	1.76	80.2	1.54	105.0	1.37	132	1.23	163	1.03	235	0.88	321
5.2	1.89	83.3	1.65	109.0	1.47	138	1.32	170	1.10	245	0.95	333
5.6	2.17	89.8	1.89	117.0	1.68	148	1.51	183	1.26	264	1.08	359
6.0	2.46	96.2	2.15	125.0	1.92	159	1.71	196	1.43	283	1.23	385
7.0	3.26	112.0	2.85	146.0	2.52	185	2.28	229	1.91	330	1.63	449

Loss of Head in Pipes by Friction

(Pelton Water-wheel Co.)

L = loss of head, in feet, for 100 feet length of pipe.
Q = quantity of water discharged per minute, in cubic feet.

Velocity, Feet per Second	Inside Diameter of Pipe in Inches									
	15		16		18		20		24	
	L	Q	L	Q	L	Q	L	Q	L	Q
2.0	0.158	147	0.147	167	0.132	212	0.119	262	0.098	377
2.2	0.187	162	0.175	184	0.156	233	0.140	288	0.116	414
2.4	0.218	176	0.205	201	0.182	254	0.164	314	0.136	452
2.6	0.252	191	0.236	218	0.210	275	0.189	340	0.157	490
2.8	0.288	206	0.270	234	0.240	297	0.216	366	0.180	528
3.0	0.325	221	0.306	251	0.271	318	0.245	393	0.204	565
3.2	0.366	235	0.343	268	0.305	339	0.275	419	0.229	603
3.4	0.408	250	0.383	284	0.339	360	0.306	445	0.255	641
3.6	0.452	265	0.425	301	0.377	382	0.339	471	0.283	678
3.8	0.499	280	0.468	318	0.416	403	0.374	497	0.312	716
4.0	0.548	294	0.513	335	0.456	424	0.410	523	0.342	754
4.4	0.651	324	0.611	368	0.542	466	0.488	576	0.407	829
4.8	0.763	353	0.715	402	0.636	509	0.572	628	0.476	905
5.0	0.822	368	0.770	419	0.685	530	0.617	654	0.513	942
5.2	0.883	383	0.828	435	0.736	551	0.662	680	0.552	980
5.6	1.011	412	0.949	469	0.843	594	0.758	733	0.632	1055
6.0	1.148	442	1.076	502	0.957	636	0.861	785	0.717	1131
7.0	1.520	515	1.430	586	1.270	742	1.143	916	0.953	1319

Velocity, Feet per Second	Inside Diameter of Pipe in Inches									
	28		30		36		42		48	
	L	Q	L	Q	L	Q	L	Q	L	Q
2.0	0.084	513	0.079	589	0.066	848	0.057	1155	0.050	1508
2.2	0.099	564	0.093	648	0.078	933	0.067	1270	0.059	1658
2.4	0.116	616	0.109	707	0.091	1018	0.079	1385	0.069	1809
2.6	0.134	667	0.126	766	0.104	1100	0.090	1500	0.079	1960
2.8	0.153	718	0.144	824	0.119	1188	0.103	1617	0.090	2110
3.0	0.174	770	0.163	883	0.135	1273	0.117	1730	0.102	2260
3.2	0.195	821	0.182	942	0.152	1357	0.131	1845	0.115	2410
3.4	0.218	872	0.204	1001	0.169	1442	0.146	1961	0.128	2560
3.6	0.242	923	0.226	1060	0.188	1527	0.162	2079	0.142	2715
3.8	0.267	974	0.249	1119	0.207	1612	0.178	2190	0.156	2865
4.0	0.293	1026	0.273	1178	0.228	1697	0.195	2310	0.171	3016
4.4	0.348	1129	0.325	1296	0.271	1866	0.232	2540	0.203	3318
4.8	0.409	1231	0.381	1414	0.318	2036	0.270	2770	0.238	3619
5.0	0.440	1283	0.411	1472	0.342	2121	0.294	2885	0.256	3770
5.2	0.473	1334	0.441	1531	0.368	2206	0.317	3000	0.278	3920
5.6	0.542	1437	0.506	1649	0.421	2376	0.374	3230	0.319	4222
6.0	0.615	1539	0.574	1767	0.479	2545	0.408	3461	0.358	4524
7.0	0.817	1796	0.762	2061	0.636	2868	0.545	4030	0.476	5277

Capacity of Hydraulic Presses

Diam. of Ram, Inches	Area of Ram, Sq. Ins.	Pressure in Pounds per Square Inch on End of Ram										
		2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000
		Capacity of Hydraulic Press in Tons										
1	0.785	0.8	0.8	0.9	0.9	0.9	1.0	1.0	1.1	1.1	1.1	1.2
2	3.142	3.1	3.3	3.5	3.6	3.8	3.9	4.1	4.2	4.4	4.5	4.7
3	7.069	7.0	7.4	7.8	8.1	8.5	8.8	9.2	9.5	9.9	10.2	10.6
4	12.566	12.5	13	14.0	14.5	15.0	15.5	16.0	17.0	17.5	18.0	19
5	19.635	20	21	21.5	22.5	23.5	24.5	25.5	26.5	27.5	28.5	29
6	28.274	28	30	31	33	34	35	37	38	40	41	42
7	38.484	38	40	42	44	46	48	50	52	54	56	58
8	50.265	50	53	55	58	60	63	65	68	70	73	75
9	63.617	63	67	70	73	76	80	83	86	89	92	95
10	78.540	78	82	86	90	94	98	102	106	110	114	118
11	95.033	95	100	105	109	114	119	124	128	133	138	143
12	113.097	113	119	124	130	136	141	147	153	158	164	170
13	132.732	132	139	146	153	159	166	172	179	186	193	199
14	153.938	154	162	169	177	185	192	200	208	216	223	231
15	176.715	177	185	194	203	212	221	230	239	247	256	265
16	201.062	201	211	221	231	241	251	261	271	281	292	302
17	226.980	227	238	250	261	272	284	295	306	318	329	340
18	254.469	254	267	280	293	305	318	331	344	356	369	382
19	283.529	284	298	312	326	340	354	369	383	397	411	425
20	314.160	314	330	346	361	377	393	408	424	440	456	471
21	346.361	346	364	381	398	416	433	450	468	485	502	520
22	380.133	380	399	418	437	456	475	494	513	532	551	570
23	415.476	415	436	457	478	499	519	540	561	582	602	623
24	452.390	452	475	498	520	543	565	588	611	633	656	679
25	490.875	491	515	540	565	589	614	638	663	687	712	736
26	530.930	531	557	584	612	637	664	690	717	743	770	796
27	572.556	573	601	630	658	687	716	744	773	802	830	859
28	615.753	616	647	677	708	739	770	800	831	862	893	924
29	660.521	661	694	727	760	793	826	859	892	925	958	991
30	706.860	707	742	778	813	848	884	919	954	990	1025	1060

Rules and Formulas for Hydraulic Press Calculations. — To find the total pressure of a hydraulic press when the diameter of the ram in inches and the water pressure (gage pressure) in pounds per square inch are given, multiply the area of the cross-section of the ram by the pressure per square inch, and divide by 2000. The result is the capacity of the hydraulic press in tons. The same result may be obtained as follows: Multiply the square of the diameter of the ram by the pressure per square inch, and multiply this product by 0.00039. The result is the total pressure of the press in tons.

The pressure per square inch on the material under pressure in the press can be determined when the total pressure of the press and the area of the material under pressure are known. Multiply the total pressure of the press in tons by 2000, and divide the product by the area of the material to be pressed. The quotient is the pressure in pounds per square inch on the surface of the material.

Capacity of Hydraulic Presses

Diam. of Ram, Inches	Area of Ram, Sq. Ins.	Pressure in Pounds per Square Inch on End of Ram									
		3100	3200	3300	3400	3500	3600	3700	3800	3900	4000
		Capacity of Hydraulic Press in Tons									
1	0.785	1.2	1.3	1.3	1.3	1.4	1.4	1.4	1.5	1.5	1.6
2	3.142	4.9	5.0	5.2	5.3	5.5	5.7	5.8	6.0	6.1	6.3
3	7.069	10.9	11.3	11.7	12.0	12.4	12.7	13.1	13.4	13.8	14.1
4	12.566	19.5	20	20.5	21	22	22.5	23	24	24.5	25
5	19.635	30	31	32	33	34	35	36	37	38	39
6	28.274	44	45	47	48	49	51	52	54	55	56
7	38.484	60	62	64	66	67	69	71	73	75	77
8	50.265	78	80	83	85	88	90	93	95	98	100
9	63.617	99	102	105	108	111	115	118	121	124	127
10	78.540	122	126	130	134	137	141	145	149	153	157
11	95.033	147	152	157	162	166	171	176	181	185	190
12	113.097	175	181	187	192	198	204	209	215	221	226
13	132.732	206	212	219	226	232	238	245	252	259	265
14	153.938	239	246	254	262	269	277	285	293	300	308
15	176.715	274	283	292	300	309	318	327	336	345	353
16	201.062	312	322	332	342	352	362	372	382	392	402
17	226.980	352	363	375	386	397	409	420	431	443	454
18	254.469	394	407	420	433	445	458	471	483	496	509
19	283.529	439	454	468	482	496	510	525	539	553	567
20	314.160	487	503	518	534	550	566	581	597	613	628
21	346.361	537	554	571	589	606	623	641	658	675	693
22	380.133	589	608	627	646	665	684	703	722	741	760
23	415.476	644	665	686	706	727	748	769	789	810	831
24	452.390	701	724	746	769	792	814	837	860	882	905
25	490.875	761	785	810	834	859	884	908	933	957	982
26	530.930	823	850	876	903	929	956	982	1009	1035	1062
27	572.556	887	916	945	973	1002	1031	1059	1088	1116	1145
28	615.753	954	985	1016	1047	1078	1108	1139	1170	1201	1232
29	660.521	1024	1057	1090	1123	1156	1189	1222	1255	1288	1321
30	706.860	1096	1131	1166	1202	1237	1272	1308	1343	1378	1414

When a certain pressure per square inch on the material under pressure is required, the gage pressure of the press necessary to obtain this pressure may be calculated as follows: Multiply the area of the surface under pressure by the pressure per square inch desired on the material. Divide this product by 0.7854 times the square of the diameter of the ram. The quotient will be the desired gage pressure.

Expressing these rules as formulas, let D = diameter of ram in inches; P = water pressure in pounds per square inch (gage pressure); C = total pressure or capacity of press in tons; A = area of material to be pressed, in square inches; P_a = pressure in pounds per square inch on material under pressure; then:

$$C = 0.00039\, D^2 \times P; \quad P_a = \frac{2000\, C}{A}; \quad P = \frac{A \times P_a}{0.7854\, D^2}$$

PIPE AND PIPE FITTINGS

Pipe Connections. — Wrought-iron and steel pipe is usually connected by screwed or flanged joints. Pipe varying from ⅛ inch to 15 inches, inclusive, is regularly threaded on the ends and connected by threaded couplings. Sizes of 1¼ inch and larger are also frequently connected by drilled flanges bolted together,

Dimensions and Weight of Standard Pipe

(National Tube Co.)

Nominal Size	Diameters		Thickness	Weight per Foot, Pounds		Threads per Inch	Couplings		
	External	Internal		Plain Ends	Threads and Couplings		Diameter	Length	Weight, Pounds
⅛	0.405	0.269	0.068	0.244	0.245	27	0.562	⅞	0.029
¼	0.540	0.364	0.088	0.424	0.425	18	0.685	1	0.043
⅜	0.675	0.493	0.091	0.567	0.568	18	0.848	1⅛	0.070
½	0.840	0.622	0.109	0.850	0.852	14	1.024	1⅜	0.116
¾	1.050	0.824	0.113	1.130	1.134	14	1.281	1⅝	0.209
1	1.315	1.049	0.133	1.678	1.684	11½	1.576	1⅞	0.343
1¼	1.660	1.380	0.140	2.272	2.281	11½	1.950	2⅛	0.535
1½	1.900	1.610	0.145	2.717	2.731	11½	2.218	2⅜	0.743
2	2.375	2.067	0.154	3.652	3.678	11½	2.760	2⅝	1.208
2½	2.875	2.469	0.203	5.793	5.819	8	3.276	2⅞	1.720
3	3.500	3.068	0.216	7.575	7.616	8	3.948	3⅛	2.498
3½	4.000	3.548	0.226	9.109	9.202	8	4.591	3⅝	4.241
4	4.500	4.026	0.237	10.790	10.889	8	5.091	3⅝	4.741
4½	5.000	4.506	0.247	12.538	12.642	8	5.591	3⅝	5.241
5	5.563	5.047	0.258	14.617	14.810	8	6.296	4⅛	8.091
6	6.625	6.065	0.280	18.974	19.185	8	7.358	4⅛	9.554
7	7.625	7.023	0.301	23.544	23.769	8	8.358	4⅛	10.932
8	8.625	8.071	0.277	24.696	25.000	8	9.358	4⅝	13.905
8	8.625	7.981	0.322	28.554	28.809	8	9.358	4⅝	13.905
9	9.625	8.941	0.342	33.907	34.188	8	10.358	5⅛	17.236
10	10.750	10.192	0.279	31.201	32.000	8	11.721	6⅛	29.877
10	10.750	10.136	0.307	34.240	35.000	8	11.721	6⅛	29.877
10	10.750	10.020	0.365	40.483	41.132	8	11.721	6⅛	29.877
11	11.750	11.000	0.375	45.557	46.247	8	12.721	6⅛	32.550
12	12.750	12.090	0.330	43.773	45.000	8	13.958	6⅛	43.098
12	12.750	12.000	0.375	49.562	50.706	8	13.958	6⅛	43.098
13	14.000	13.250	0.375	54.568	55.824	8	15.208	6⅛	47.152
14	15.000	14.250	0.375	58.573	60.375	8	16.446	6⅛	59.493
15	16.000	15.250	0.375	62.579	64.500	8	17.446	6⅛	63.294

the joint being made by a gasket between the flange faces. The flanges are attached to the pipe in various ways. The most common method for sizes from 1¼ inch to 15 inches, inclusive, is to screw the flanges onto the pipe sections. For sizes larger than 6 inches, peened flanges are often used. The peened flange is shrunk onto the end of the pipe, and the latter is then peened over or expanded into a recess in the flange face, after which the flange is sometimes faced off in a lathe. Steel flanges are also welded to pipe, and loose flanges are used by flanging over the pipe ends. For water pipe which does not have to stand very high pressures, leaded joints are often used. The most common leaded joints are the Converse lock-joint and the Matheson joint. The Converse joint is made by means of a special cast-iron coupling or hub having an annular groove in each end and, in addition, two T-shaped grooves a short distance from the circular groove. The pipe has two holes punched a short distance from the end on opposite sides, into which rivets are driven. In making-up this joint, the heads of the rivets slip into the T-shaped slots of the hub and the pipe is turned slightly, thus holding the sections from pulling out of the hub endwise. The joint is then made tight by pouring lead into the annular grooves and calking. The Matheson joint is another type of leaded joint used for water or gas lines.

Dimensions and Weight of Extra Strong and Double Extra Strong Pipe

(National Tube Co.)

Nominal Size	Diameter, External	Diameter, Internal	Thickness	Wt. per Foot, Plain Ends, Pounds	Nominal Size	Diameter, External	Diameter, Internal	Thickness	Wt. per Foot, Plain Ends, Pounds
				Extra Strong Pipe — Black and Galvanized					
1/8	0.405	0.215	0.095	0.314	4½	5.000	4.290	0.355	17.611
¼	0.540	0.302	0.119	0.535	5	5.563	4.813	0.375	20.778
3/8	0.675	0.423	0.126	0.738	6	6.625	5.761	0.432	28.573
½	0.840	0.546	0.147	1.087	7	7.625	6.625	0.500	38.048
¾	1.050	0.742	0.154	1.473	8	8.625	7.625	0.500	43.388
1	1.315	0.957	0.179	2.171	9	9.625	8.625	0.500	48.728
1¼	1.660	1.278	0.191	2.996	10	10.750	9.750	0.500	54.735
1½	1.900	1.500	0.200	3.631	11	11.750	10.750	0.500	60.075
2	2.375	1.939	0.218	5.022	12	12.750	11.750	0.500	65.415
2½	2.875	2.323	0.276	7.661	13	14.000	13.000	0.500	72.091
3	3.500	2.900	0.300	10.252	14	15.000	14.000	0.500	77.431
3½	4.000	3.364	0.318	12.505	15	16.000	15.000	0.500	82.771
4	4.500	3.826	0.337	14.983	....	....	....	.....	.....
				Double Extra Strong Pipe — Black and Galvanized					
½	0.840	0.252	0.294	1.714	3½	4.000	2.728	0.636	22.850
¾	1.050	0.434	0.308	2.440	4	4.500	3.152	0.674	27.541
1	1.315	0.599	0.358	3.659	4½	5.000	3.580	0.710	32.530
1¼	1.660	0.896	0.382	5.214	5	5.563	4.063	0.750	38.552
1½	1.900	1.100	0.400	6.408	6	6.625	4.897	0.864	53.160
2	2.375	1.503	0.436	9.029	7	7.625	5.875	0.875	63.079
2½	2.875	1.771	0.552	13.695	8	8.625	6.875	0.875	72.424
3	3.500	2.300	0.600	18.583	....	....	....	.....	.....

Weight of Pipe per Foot. — The weight of pipe per linear foot can be determined by the following formula:

$$W = K\ (D^2 - d^2)$$

in which D = outside diameter; d = inside diameter; W = weight per running foot; K = 2.66 for wrought iron; K = 2.45 for cast iron; K = 2.82 for brass;

$K = 3.03$ for copper; $K = 3.86$ for lead. The constant for cast iron (2.45) is based on cast iron weighing 0.26 pound per cubic inch, or 450 pounds per cubic foot, and it is advisable to add 10 per cent to the figures obtained for cast iron from the formula, to allow for overweight in the castings.

Working Pressures. — Valves and fittings, when classified under the five general headings, — low pressure, standard, medium pressure, extra heavy and hydraulic, usually are designed for the following steam working pressures:

Low Pressure: — Suitable for pressures up to 25 or 50 pounds per square inch.

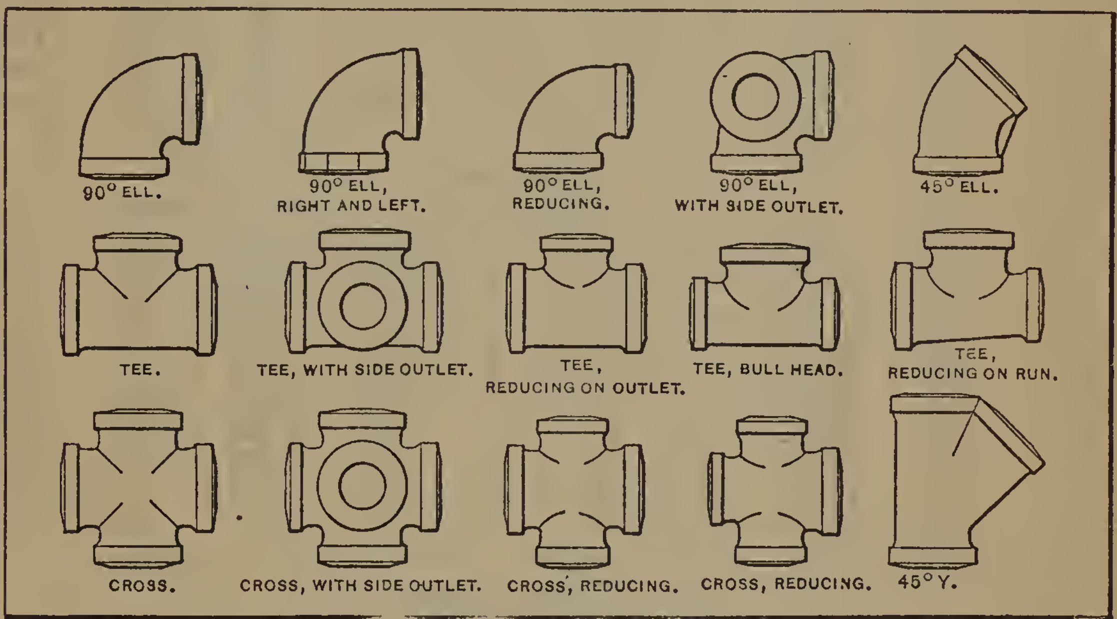
Standard: — Suitable for working pressures up to 125 pounds per square inch.

Medium Pressure: — Suitable for working pressures from 125 to 175 pounds per square inch.

Extra Heavy: — Suitable for working pressures from 175 to 250 pounds per square inch.

Hydraulic: — Water pressures from 800 to 3000 pounds per square inch.

According to the "American Standard" all standard weight fittings must be guaranteed for a working pressure of 125 pounds per square inch, and extra-heavy fittings for 250 pounds per square inch.



Different Types of Pipe Fittings

Bursting Pressure of Pipes. — The bursting pressure of pipes can be determined approximately by the following formula (Barlow's):

$$P = \frac{2\,T \times S}{O}$$

in which P = bursting pressure in pounds per square inch; T = thickness of wall, in inches; O = outside diameter of pipe, in inches; S = tensile strength of material, in pounds per square inch. The value of S as determined by actual bursting tests is 40,000 pounds for butt-welded steel pipe, and 50,000 pounds for lap-welded steel pipe. The accompanying table, "Bursting and Working Pressures for Pipes," is based on the foregoing formula, the accuracy of which has been tested by an exhaustive series of tests conducted by the National Tube Co. In these tests, all types of pipe and tubing were burst, and a number of different methods of plugging the ends were employed, to obtain results for different strains. These results were carefully checked with all available formulas and the Barlow formula came nearer to the experimental results than any of the others.

Bursting and Working Pressures for Pipes

Size of Pipe, Inches	Bursting Pressure, Pounds per Sq. In.	Working Pressures Factor of Safety 6	Working Pressures Factor of Safety 8	Working Pressures Factor of Safety 10	Size of Pipe, Inches	Bursting Pressure, Pounds per Sq. In.	Working Pressures Factor of Safety 6	Working Pressures Factor of Safety 8	Working Pressures Factor of Safety 10
Standard Pipe									
¼	13,032	2172	1629	1303	3½	5610	935	701	561
⅜	10,784	1797	1348	1078	4	5266	877	658	526
½	10,384	1731	1298	1038	4½	4940	823	618	494
¾	8,608	1434	1076	860	5	4630	772	579	463
1	8,088	1348	1011	808	6	4220	703	528	422
1¼	6,744	1124	843	674	7	3940	657	493	394
1½	6,104	1017	763	610	8	3730	622	466	373
2	5,184	864	648	518	9	3550	591	444	355
2½	5,648	941	706	564	10	3390	565	424	339
3	4,936	823	617	493	12	2940	490	368	294
Extra Strong									
¼	17,624	2937	2203	1762	3½	7950	1325	994	795
⅜	14,928	2488	1866	1492	4	7480	1246	935	748
½	14,000	2333	1750	1400	4½	7100	1183	887	710
¾	11,728	1954	1716	1172	5	6740	1123	842	674
1	10,888	1814	1611	1088	6	6550	1091	819	655
1¼	9,200	1533	1150	920	7	6520	1086	815	652
1½	8,416	1403	1052	841	8	5780	963	722	578
2	7,336	1223	917	733	9	5190	865	649	519
2½	7,680	1280	960	768	10	4650	775	581	465
3	6,856	1142	857	685	12	3920	653	490	392
Double Extra Strong									
½	28,000	4666	3500	2800	3½	15,900	2650	1987	1590
¾	23,464	3910	2933	2346	4	14,970	2495	1871	1497
1	21,776	3629	2722	2177	4½	14,200	2367	1775	1420
1¼	18,408	3068	2301	1840	5	13,480	2247	1685	1348
1½	16,840	2807	2105	1684	6	13,040	2173	1630	1304
2	15,360	2560	1920	1536	7	11,470	1912	1434	1147
2½	14,680	2447	1835	1468	8	10,140	1690	1267	1014
3	13,714	2285	1714	1371	......	......	......	......	......
Large O. D. Pipe — ⅜ inch Thick					**Large O. D. Pipe — ½ inch Thick**				
14	2680	447	335	268	14	3570	595	446	357
15	2500	417	313	250	15	3333	556	417	333
16	2340	390	293	234	16	3120	520	390	312
18	2080	347	260	208	18	2770	462	346	277
20	1870	312	234	187	20	2500	417	313	250
22	1700	283	213	170	22	2270	378	284	227
24	1560	260	195	156	24	2080	347	260	208

Transverse and Surface Areas of Standard and Extra Strong Pipe

Nominal Size of Pipe, Inches	Diameters, Inches		Transverse Areas, Sq. In.			Length of Pipe in Feet per Sq. Ft. of Surface Area		Length in Feet Containing One Cu. Ft.
	External	Internal	External	Internal	Metal	External	Internal	
Standard Wrought Pipe								
⅛	0.405	0.269	0.129	0.057	0.072	9.431	14.199	2533.775
¼	0.540	0.364	0.229	0.104	0.125	7.073	10.493	1383.789
⅜	0.675	0.493	0.358	0.191	0.167	5.658	7.747	754.360
½	0.840	0.622	0.554	0.304	0.250	4.547	6.141	473.906
¾	1.050	0.824	0.866	0.533	0.333	3.637	4.635	270.034
1	1.315	1.049	1.358	0.864	0.494	2.904	3.641	166.618
1¼	1.660	1.380	2.164	1.495	0.669	2.301	2.767	96.275
1½	1.900	1.610	2.835	2.036	0.799	2.010	2.372	70.733
2	2.375	2.067	4.430	3.355	1.075	1.608	1.847	42.913
2½	2.875	2.469	6.492	4.788	1.704	1.328	1.547	30.077
3	3.500	3.068	9.621	7.393	2.228	1.091	1.245	19.479
3½	4.000	3.548	12.566	9.886	2.680	0.954	1.076	14.565
4	4.500	4.026	15.904	12.730	3.174	0.848	0.948	11.312
4½	5.000	4.506	19.635	15.947	3.688	0.763	0.847	9.030
5	5.563	5.047	24.306	20.006	4.300	0.686	0.756	7.198
6	6.625	6.065	34.472	28.891	5.581	0.576	0.629	4.984
7	7.625	7.023	45.664	38.738	6.926	0.500	0.543	3.717
8	8.625	8.071	58.426	51.161	7.265	0.442	0.473	2.815
8	8.625	7.981	58.426	50.027	8.399	0.442	0.478	2.878
9	9.625	8.941	72.760	62.786	9.974	0.396	0.427	2.294
10	10.750	10.192	90.763	81.585	9.178	0.355	0.374	1.765
10	10.750	10.136	90.763	80.691	10.072	0.355	0.376	1.785
10	10.750	10.020	90.763	78.855	11.908	0.355	0.381	1.826
11	11.750	11.000	108.434	95.033	13.401	0.325	0.347	1.515
12	12.750	12.090	127.676	114.800	12.876	0.299	0.315	1.254
12	12.750	12.000	127.676	113.097	14.579	0.299	0.318	1.273
Extra Strong Wrought Pipe								
⅛	0.405	0.215	0.129	0.036	0.093	9.431	17.766	3966.392
¼	0.540	0.302	0.229	0.072	0.157	7.073	12.648	2010.290
⅜	0.675	0.423	0.358	0.141	0.217	5.658	9.030	1024.689
½	0.840	0.546	0.554	0.234	0.320	4.547	6.995	615.017
¾	1.050	0.742	0.866	0.433	0.433	3.637	5.147	333.016
1	1.315	0.957	1.358	0.719	0.639	2.904	3.991	200.193
1¼	1.660	1.278	2.164	1.283	0.881	2.301	2.988	112.256
1½	1.900	1.500	2.835	1.767	1.068	2.010	2.546	81.487
2	2.375	1.939	4.430	2.953	1.477	1.608	1.969	48.766
2½	2.875	2.323	6.492	4.238	2.254	1.328	1.644	33.976
3	3.500	2.900	9.621	6.605	3.016	1.091	1.317	21.801
3½	4.000	3.364	12.566	8.888	3.678	0.954	1.135	16.202
4	4.500	3.826	15.904	11.497	4.407	0.848	0.998	12.525
4½	5.000	4.290	19.635	14.455	5.180	0.763	0.890	9.962
5	5.563	4.813	24.306	18.194	6.112	0.686	0.793	7.915
6	6.625	5.761	34.472	26.067	8.405	0.576	0.663	5.524
7	7.625	6.625	45.664	34.472	11.192	0.500	0.576	4.177
8	8.625	7.625	58.426	45.663	12.763	0.442	0.500	3.154
9	9.625	8.625	72.760	58.426	14.334	0.396	0.442	2.464
10	10.750	9.750	90.763	74.662	16.101	0.355	0.391	1.929
11	11.750	10.750	108.434	90.763	17.671	0.325	0.355	1.587
12	12.750	11.750	127.676	108.434	19.242	0.299	0.325	1.328

Steel and Wrought-iron Pipe. — The term "wrought-iron pipe" is often used indiscriminately to designate all butt- or lap-welded pipe whether made from wrought iron or steel, but the term "wrought pipe" is preferable for designating either steel or wrought-iron pipe. A large percentage of the "wrought pipe" now used is made of steel. When wrought-iron pipe is desired the term "genuine wrought iron" or "guaranteed wrought iron" should be used, as otherwise the manufacturer will invariably supply steel pipe.

Formerly wrought iron was preferred for the best classes of work, but records of installations and tests have demonstrated that steel pipe is equal to wrought-iron pipe for general work and, according to some authorities, resists corrosion, in the average case, as well as wrought iron; the steel pipe is also cheaper than wrought iron, and according to one estimate 90 per cent of the wrought pipe made in the United States is of steel. The term "galvanized iron pipe" is applied to ordinary wrought pipe which has been galvanized.

Grades of Pipe. — Steel and wrought-iron pipe is usually graded as "standard," "extra heavy" or "extra strong," and "double extra heavy" or "double extra strong," the thickness and weight increasing progressively. Changes in thickness and weight of the various grades are made by varying the inside diameter only. The outside diameter remains constant so that any grade of pipe may be used with any grade of fitting, flange, coupling, or valve.

Standard weight pipe is commonly used for heating work, exhaust lines, and all pressures below 100 pounds per square inch; extra heavy pipe should be employed for pressures from 100 to 200 pounds per square inch, and where there is liable to be considerable corrosion. These pressures are far below the ultimate strength of the pipe. Special hydraulic pipe for service on lines requiring the highest possible grade of material and workmanship are bored from solid forgings and are made to order for pressures up to 10,000 pounds per square inch.

Making Screwed Joints Tight. — When making up screwed joints, the threads should be clean, and red or white lead, or some standard pipe joint cement or lubricant, should be applied to the threads in order to decrease the friction of the bearing surfaces of the threads; the joint should not be screwed up fast enough to produce excessive friction. Friction of the threads produces heat, thus causing the metal of the pipe to expand before the joint is properly made, with the result that, when the pipe cools again and contracts in the flange or fitting, the joint may be loose and cause leakage when the pressure is turned on the piping system.

Pipe Coverings. — Steam and feed-water pipes are protected with heat-insulating coverings in order to prevent loss of heat by radiation. Under ordinary conditions, about 3 British thermal units per square foot per hour, per degree difference in temperature, radiate from a bare steam pipe. Good commercial heat-insulating materials used for pipe covering will save from 75 to 85 per cent of this loss. Among the various materials used for covering pipe may be mentioned hair felt, cork, magnesia, and mineral wool. Asbestos is a very poor non-conductor of heat, but it may be used to advantage as a binder in other insulating substances. A common covering consists of 85 per cent carbonate of magnesia mixed with 15 per cent of asbestos. The covering should be at least 1 inch thick and preferably from 2 to 3 inches, depending on the size of the pipe. It is generally manufactured in sections molded in halves to fit the pipe. Valves and fittings may be covered with the same material in a plastic state. The covering is secured in place by means of heavy duck or canvas and bands made of brass or sheet iron placed at regular intervals along the pipes. Many commercial pipe coverings are made from two or more of these substances. Pipe laid in trenches may be insulated by the use of ashes, coke, loam, or charcoal.

Lap-welded and Seamless Boiler Tubes. — The following specifications for lap-welded and seamless boiler tubes have been approved by the Boiler Tube Manufacturers of America. Lap-welded tubes shall be made of open-hearth steel or knobbled hammered charcoal iron. Seamless tubes shall be made of open-hearth steel. The steel shall conform to the following requirements as to chemical composition: Carbon, from 0.08 to 0.18 per cent; manganese, from 0.30 to 0.50 per cent; phosphorus, not over 0.04 per cent; and sulphur, not over 0.045 per cent.

Thicknesses of Tubes for Water-tube Boilers.—According to the A. S. M. E. "Boiler Code," the minimum thicknesses of tubes used in water-tube boilers, measured by the Birmingham wire gage, for maximum allowable working pressures not exceeding 165 pounds per square inch, shall be as follows: Diameters less than 3 inches, No. 12 B.W.G.; 3 inches or over but less than 4 inches, No. 11 B.W.G.; 4 inches or over but less than 5 inches, No. 10 B.W.G.; 5 inches, No. 9 B.W.G. The foregoing gages shall be increased as follows for maximum allowable working pressures higher than 165 pounds per square inch: Over 165 pounds but not exceeding 235 pounds, 1 gage; over 235 pounds but not exceeding 285 pounds, 2 gages; over 285 pounds but not exceeding 400 pounds, 3 gages. Tubes over 4 inches in diameter shall not be used for maximum allowable working pressures above 285 pounds per square inch.

Thicknesses of Tubes for Fire-tube Boilers. — The minimum thicknesses of tubes used in fire-tube boilers, measured by the Birmingham wire gage, for maximum allowable working pressures not exceeding 175 pounds per square inch, shall be as follows: Diameters less than 2½ inches, No. 13 B.W.G.; 2½ inches or over but less than 3¼ inches, No. 12 B.W.G.; 3¼ inches or over but less than 4 inches, No. 11 B.W.G.; 4 inches or over but less than 5 inches, No. 10 B.W.G.; 5 inches, No. 9 B.W.G. For higher maximum allowable working pressures than previously given, the thicknesses shall be increased one gage.

Lap-welded Steel or Charcoal Iron Boiler Tubes

External Diameter, Inches	Standard Thickness		Nominal Weight per Foot — Pounds		External Diameter, Inches	Standard Thickness		Nominal Weight per Foot — Pounds	
	Birmingham Wire Gage	Inches	Standard Thickness	One Extra Wire Gage		Birmingham Wire Gage	Inches	Standard Thickness	One Extra Wire Gage
1¾	13	0.095	1.679	1.910	4½	10	0.134	6.248	6.879
2	13	0.095	1.932	2.201	5	9	0.148	7.669	8.520
2¼	13	0.095	2.186	2.492	6	8	0.165	10.282	11.188
2½	12	0.109	2.783	3.050	7	8	0.165	12.044	13.110
2¾	12	0.109	3.074	3.370	8	8	0.165	13.807	15.033
3	12	0.109	3.365	3.691	9	7	0.180	16.955	19.072
3¼	11	0.120	4.011	4.459	10	6	0.203	21.240	22.979
3½	11	0.120	4.331	4.817	11	5	0.220	25.329	27.355
3¾	11	0.120	4.652	5.175	12	......	0.229	28.788	31.188
4	10	0.134	5.532	6.088	13	4	0.238	32.439	35.243

Relative Strengths of Lap- and Butt-welded Pipe. — If seamless steel tubes are assumed to have a strength of 100 per cent, butt-welded steel pipe has a comparative strength of 73 per cent, and lap-welded steel pipe of 92 per cent. From this it will be seen that the strength of a butt-weld is only about 80 per cent of that of a lap-weld. The relative strengths of wrought iron and steel pipe are as follows: Butt-welded wrought-iron pipe has 70 per cent of the strength of similar butt-welded steel pipe, and lap-welded wrought-iron pipe has 57 per cent of the strength of similar lap-welded steel pipe.

Standards for Flanged Fittings. — The use of flanges and flanged fittings has increased rapidly of late years, and flanged joints are the standard form of connection for many classes of piping. For most purposes, the ordinary screw connection should not be used for pipe sizes over 6 inches, on account of the difficulty of making and breaking joints.

In the early days, each manufacturer made patterns for whatever sizes of flanged fittings were called for, according to his own personal ideas; hence, there was a great variation in the diameter and thickness of flanges as well as in the diameter of bolt circle, size and number of bolts, etc. On account of this, the American Society of Mechanical Engineers and the Master Steam & Hot Water Fitters Association, assisted by the manufacturers of fittings, adopted a standard which covered flange dimensions and bolting only, and which was known as the A. S. M. E. Standard of 1894. A few years later, the manufacturers realized that there was need for a standard of extra-heavy 250-pound flanges and bolting, and the Manufacturers' Standard of 1901 was adopted. In order to standardize the center-to-face and face-to-face dimensions of all flange fittings, the American Society of Mechanical Engineers and the Master Steam & Hot Water Fitters Association adopted another standard known as the 1912 U. S. Standard. This standard, however, differed, in some respects, from the dimensions which had been quite generally used by manufacturers, and at a meeting held in New York City in July, 1912, a standard known as the Manufacturers' Standard was adopted by the pipe fitting manufacturers. In order to avoid the confusion of having two standards in the field, an effort was made to bring about a compromise of the differences between these two standards, and this compromise, which became effective January 1, 1915, is known as the "American Standard." "The 1915 U. S. Standard," which name was adopted by the National Association of Master Steam and Hot Water Fitters, is the same as the American Standard.

American Standard Flanged Fittings. — The following general remarks relate to the American Standard and the 1915 U. S. Standard flanged fittings:

Standard and extra-heavy reducing elbows have the same center-to-face dimensions as regular elbows of largest straight size.

Standard and extra-heavy tees, crosses and laterals, reducing on run only, have the same dimensions face-to-face as largest straight size.

If flanged fittings for lower working pressure than 125 pounds are made, they must conform in all dimensions, except thickness of shell, to this standard, and must have the guaranteed working pressure cast on each fitting.

All standard weight fittings must be guaranteed for 125 pounds working pressure and extra-heavy fittings for 250 pounds working pressure, and each fitting must have some mark cast on it indicating maker and guaranteed working pressure.

All extra-heavy fittings and flanges must have a raised surface of 1/16 inch high inside of the bolt holes for gaskets; standard weight fittings and flanges must be plain-faced; the bolt holes must be 1/8 inch larger in diameter than the bolts; the bolt holes must straddle the center line.

The size of all fittings scheduled indicates the inside diameter of the ports, except for extra-heavy fittings 14 inches and larger, where the port diameter is 3/4 inch smaller than the nominal size.

Square head bolts with hexagonal nuts are recommended. For bolts 1 5/8 inch diameter and larger, studs with a nut on each end are satisfactory. Hexagonal nuts for pipe sizes 1 to 46 inches, on the 125-pound standard, and 1 to 16 inches on the 250-pound standard, can be conveniently pulled up with open wrenches of minimum design of heads. Hexagonal nuts for pipe sizes 48 to 100 inches on the 125-pound, and 18 to 48 inches on the 250-pound standards, can be conveniently pulled up with box wrenches.

Tees and crosses 16 inches and smaller, reducing on the outlet, have the same dimensions as straight sizes of the larger port. Sizes 18 inches and larger, reducing on the outlet, are made in two lengths depending upon the size of the outlet as given in the tables of dimensions. Laterals 16 inches and smaller, reducing on the branch, have the same dimensions as straight sizes of the larger port. Sizes 18 inches and larger, reducing on the branch, are made in two lengths depending upon the size of the branch as given in the tables of dimensions. The dimensions of reducing flanged fittings are always regulated by the reductions of the outlet or a branch. For fittings reducing on the run only, the long body pattern will always be used. Y's are special and are made to suit conditions. Double sweep tees are not made reducing on the run.

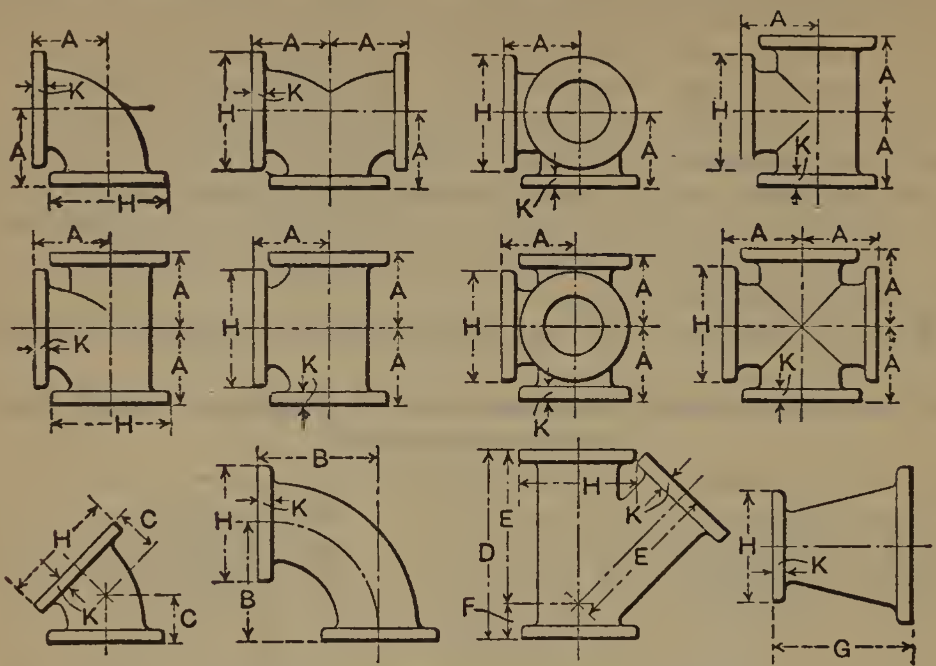
Steel flanges, fittings and valves are recommended for superheated steam.

Pipe Flanges for Working Pressures up to 125 Pounds per Square Inch — American Standard

Size	Diam. of Flange	Thickness of Flange	Diam. of Bolt Circle	No. of Bolts	Size of Bolts	Size	Diam. of Flange	Thickness of Flange	Diam. of Bolt Circle	No. of Bolts	Size of Bolts
1	4	7/16	3	4	7/16	42	53	2⅝	49½	36	1⅝
1¼	4½	½	3⅜	4	7/16	44	55¼	2⅝	51¾	40	1⅝
1½	5	9/16	3⅞	4	½	46	57¼	2 11/16	53¾	40	1⅝
2	6	⅝	4¾	4	⅝	48	59½	2¾	56	44	1⅝
2½	7	11/16	5½	4	⅝	50	61¾	2¾	58¼	44	1¾
3	7½	¾	6	4	⅝	52	64	2⅞	60½	44	1¾
3½	8½	13/16	7	4	⅝	54	66¼	3	62¾	44	1¾
4	9	15/16	7½	8	⅝	56	68¾	3	65	48	1¾
4½	9¼	15/16	7¾	8	¾	58	71	3⅛	67¼	48	1¾
5	10	15/16	8½	8	¾	60	73	3⅛	69¼	52	1¾
6	11	1	9½	8	¾	62	75¾	3¼	71¾	52	1⅞
7	12½	1 1/16	10¾	8	¾	64	78	3¼	74	52	1⅞
8	13½	1⅛	11¾	8	¾	66	80	3⅜	76	52	1⅞
9	15	1⅛	13¼	12	¾	68	82¼	3⅜	78¼	56	1⅞
10	16	1 3/16	14¼	12	⅞	70	84½	3½	80½	56	1⅞
12	19	1¼	17	12	⅞	72	86½	3½	82½	60	1⅞
14	21	1⅜	18¾	12	1	74	88½	3⅝	84½	60	1⅞
15	22¼	1⅜	20	16	1	76	90¾	3⅝	86½	60	1⅞
16	23½	1 7/16	21¼	16	1	78	93	3¾	88¾	60	2
18	25	1 9/16	22¾	16	1⅛	80	95¼	3¾	91	60	2
20	27½	1 11/16	25	20	1⅛	82	97½	3⅞	93¼	60	2
22	29½	1 13/16	27¼	20	1¼	84	99¾	3⅞	95½	64	2
24	32	1⅞	29½	20	1¼	86	102	4	97¾	64	2
26	34¼	2	31¾	24	1¼	88	104¼	4	100	68	2
28	36½	2 1/16	34	28	1¼	90	106½	4⅛	102¼	68	2⅛
30	38¾	2⅛	36	28	1⅜	92	108¾	4⅛	104½	68	2⅛
32	41¾	2¼	38½	28	1½	94	111	4¼	106¼	68	2⅛
34	43¾	2 5/16	40½	32	1½	96	113¼	4¼	108½	68	2¼
36	46	2⅜	42¾	32	1½	98	115½	4⅜	110¾	68	2¼
38	48¾	2⅜	45¼	32	1⅝	100	117¾	4⅜	113	68	2¼
40	50¾	2½	47¼	36	1⅝	....	....	...	....	....	..

Bolt holes are drilled ⅛ inch larger than nominal diameter of bolts.

Standard Flanged Fittings, Straight Sizes — American Standard



L = minimum metal thickness of body.

Size	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	*L*
1	3½	5	1¾	7½	5¾	1¾	....	4	7/16	7/16
1¼	3¾	5½	2	8	6¼	1¾	....	4½	½	7/16
1½	4	6	2¼	9	7	2	....	5	9/16	7/16
2	4½	6½	2½	10½	8	2½	....	6	⅝	7/16
2½	5	7	3	12	9½	2½	....	7	11/16	7/16
3	5½	7¾	3	13	10	3	6	7½	¾	7/16
3½	6	8½	3½	14½	11½	3	6½	8½	13/16	7/16
4	6½	9	4	15	12	3	7	9	15/16	½
4½	7	9½	4	15½	12½	3	7½	9¼	15/16	½
5	7½	10¼	4½	17	13½	3½	8	10	15/16	½
6	8	11½	5	18	14½	3½	9	11	1	9/16
7	8½	12¾	5½	20½	16½	4	10	12½	1 1/16	⅝
8	9	14	5½	22	17½	4½	11	13½	1⅛	⅝
9	10	15¼	6	24	19½	4½	11½	15	1⅛	11/16
10	11	16½	6½	25½	20½	5	12	16	1 3/16	¾
12	12	19	7½	30	24½	5½	14	19	1¼	13/16
14	14	21½	7½	33	27	6	16	21	1⅜	⅞
15	14½	22¾	8	34½	28½	6	17	22¼	1⅜	⅞
16	15	24	8	36½	30	6½	18	23½	1 7/16	1
18	16½	26½	8½	39	32	7	19	25	1 9/16	1 1/16
20	18	29	9½	43	35	8	20	27½	1 11/16	1⅛
22	20	31½	10	46	37½	8½	22	29½	1 13/16	1 3/16

Standard Flanged Fittings, Straight Sizes — American Standard

Size	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	*L*
24	22	34	11	49½	40½	9	24	32	1⅞	1¼
26	23	36½	13	53	44	9	26	34¼	2	1⁵⁄₁₆
28	24	39	14	56	46½	9½	28	36½	2¹⁄₁₆	1⅜
30	25	41½	15	59	49	10	30	38¾	2⅛	1⁷⁄₁₆
32	26	44	16	....	....	....	32	41¾	2¼	1½
34	27	46½	17	....	....	....	34	43¾	2⁵⁄₁₆	1⁹⁄₁₆
36	28	49	18	....	....	....	36	46	2⅜	1⅝
38	29	51½	19	....	....	....	38	48¾	2⅜	1¹¹⁄₁₆
40	30	54	20	....	....	....	40	50¾	2½	1¾
42	31	56½	21	....	....	....	42	53	2⅝	1¹³⁄₁₆
44	32	59	22	....	....	....	44	55¼	2⅝	1⅞
46	33	61½	23	....	....	....	46	57¼	2¹¹⁄₁₆	1¹⁵⁄₁₆
48	34	64	24	....	....	....	48	59½	2¾	2
50	35	66½	25	....	....	....	50	61¾	2¾	2¹⁄₁₆
52	37	69	26	....	....	....	52	64	2⅞	2⅛
54	39	71½	27	....	....	....	54	66¼	3	2³⁄₁₆
56	41	74	28	....	....	....	56	68¾	3	2¼
58	42	76½	29	....	....	....	58	71	3⅛	2⁵⁄₁₆
60	44	79	30	....	....	....	60	73	3⅛	2⁷⁄₁₆
62	45	81½	31	....	....	....	62	75¾	3¼	2½
64	47	84	32	....	....	....	64	78	3¼	2⁹⁄₁₆
66	48	86½	33	....	....	....	66	80	3⅜	2⅝
68	50	89	34	....	....	....	68	82¼	3⅜	2¹¹⁄₁₆
70	51	91½	35	....	....	....	70	84½	3½	2¾
72	53	94	36	....	....	....	72	86½	3½	2¹³⁄₁₆
74	54	96½	37	....	....	....	74	88½	3⅝	2⅞
76	56	99	38	....	....	....	76	90¾	3⅝	2¹⁵⁄₁₆
78	58	101½	39	....	....	....	78	93	3¾	3
80	59	104	40	....	....	....	80	95¼	3¾	3¹⁄₁₆
82	60	106½	41	....	....	....	82	97½	3⅞	3⅛
84	62	109	42	....	....	....	84	99¾	3⅞	3³⁄₁₆
86	63	111½	43	....	....	....	86	102	4	3¼
88	65	114	44	....	....	....	88	104¼	4	3⁵⁄₁₆
90	67	116½	45	....	....	....	90	106½	4⅛	3⅜
92	68	119	46	....	....	....	92	108¾	4⅛	3½
94	69	121½	47	....	....	....	94	111	4¼	3⁹⁄₁₆
96	71	124	48	....	....	....	96	113¼	4¼	3⅝
98	73	126½	49	....	....	....	98	115½	4⅜	3¹¹⁄₁₆
100	74	129	50	....	....	....	100	117¾	4⅜	3¾

Extra Heavy Flanged Fittings, Straight Sizes — American Standard

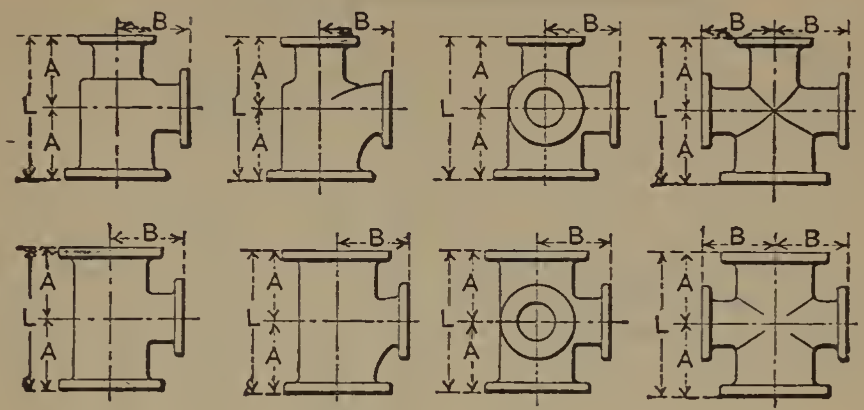
(See illustration on page 1488 for notation.)

Size	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	*L*
1	4	5	2	8½	6½	2	....	4½	11/16	½
1¼	4¼	5½	2½	9½	7¼	2¼	....	5	¾	½
1½	4½	6	2¾	11	8½	2½	....	6	13/16	½
2	5	6½	3	11½	9	2½	....	6½	⅞	½
2½	5½	7	3½	13	10½	2½	....	7½	1	9/16
3	6	7¾	3½	14	11	3	6	8¼	1⅛	9/16
3½	6½	8½	4	15½	12½	3	6½	9	1 3/16	9/16
4	7	9	4½	16½	13½	3	7	10	1¼	⅝
4½	7½	9½	4½	18	14½	3½	7½	10½	1 5/16	⅝
5	8	10¼	5	18½	15	3½	8	11	1⅜	11/16
6	8½	11½	5½	21½	17½	4	9	12½	1 7/16	¾
7	9	12¾	6	23½	19	4½	10	14	1½	13/16
8	10	14	6	25½	20½	5	11	15	1⅝	13/16
9	10½	15¼	6½	27½	22½	5	11½	16¼	1¾	⅞
10	11½	16½	7	29½	24	5½	12	17½	1⅞	15/16
12	13	19	8	33½	27½	6	14	20½	2	1
14	15	21½	8½	37½	31	6½	16	23	2⅛	1⅛
15	15½	22¾	9	39½	33	6½	17	24½	2 3/16	1 3/16
16	16½	24	9½	42	34½	7½	18	25½	2¼	1¼
18	18	26½	10	45½	37½	8	19	28	2⅜	1⅜
20	19½	29	10½	49	40½	8½	20	30½	2½	1½
22	20½	31½	11	53	43½	9½	22	33	2⅝	1 9/16
24	22½	34	12	57½	47½	10	24	36	2¾	1⅝
26	24	36½	13	....	....	....	26	38¼	2 13/16	1 13/16
28	26	39	14	....	....	....	28	40¾	2 15/16	1⅞
30	27½	41½	15	....	....	....	30	43	3	2
32	29	44	16	....	....	....	32	45¼	3⅛	2⅛
34	30½	46½	17	....	....	....	34	47½	3¼	2¼
36	32½	49	18	....	....	....	36	50	3⅜	2⅜
38	34	51½	19	....	....	....	38	52¼	3 7/16	2 7/16
40	35½	54	20	....	....	....	40	54½	3 9/16	2 9/16
42	37	56½	21	....	....	....	42	57	3 11/16	2 11/16
44	39	59	22	....	....	....	44	59¼	3¾	2 13/16
46	40½	61½	23	....	....	....	46	61½	3⅞	2⅞
48	42	64	24	....	....	....	48	65	4	3

Wrenches used in Screwing up Pipe Flanges. — The Crane Company has made a number of tests in order to determine the size of wrench that should be used in tightening up the nuts on the bolts in pipe flanges. The following lengths of wrenches are recommended when used by one man: For ¾-, ⅞- and 1-inch bolts, 16-inch wrench; for 1⅛-, 1¼- and 1⅜-inch bolts, 36-inch wrench; for 1½-, 1⅝- and 1¾-inch bolts, 60-inch wrench; for 2-inch bolts, 72-inch wrench.

Strength of Materials used for Flanges. — The Crane Company gives the following values relating to the strength per square inch of various materials used for pipe flanges: Ordinary grade cast iron, 14,000 pounds; high-grade gray cast iron, 22,500 pounds; ferro-steel, 33,500 pounds; malleable iron, 37,000 pounds; forged steel, 51,000 pounds; cast steel, 67,000 pounds.

Standard Flanged Fittings, Reducing Tees and Crosses — American Standard



Short Body Pattern

Size	Max. Size of Outlet*	*L*	*A*	*B*	Size	Max. Size of Outlet*	*L*	*A*	*B*
1					42	28	46	23	30
1¼					44	28	46	23	31
1½					46	30	48	24	33
2					48	32	52	26	34
2½					50	32	52	26	35
3	Sizes 16 inches and smaller have same center-to-face dimensions as straight size fittings.				52	34	54	27	36
3½					54	36	58	29	37
4					56	36	58	29	39
4½					58	38	62	31	40
5					60	40	66	33	41
6					62	40	66	33	42
7					64	42	68	34	44
8					66	44	70	35	45
9					68	44	70	35	46
10					70	46	74	37	47
12					72	48	80	40	48
14					74	48	80	40	49
15					76	50	84	42	50
16					78	52	86	43	52
18	12	26	13	15½	80	52	86	43	53
20	14	28	14	17	82	54	88	44	54
22	15	28	14	18	84	56	94	47	56
24	16	30	15	19	86	56	94	47	57
26	18	32	16	20	88	58	96	48	58
28	18	32	16	21	90	60	100	50	61
30	20	36	18	23	92	60	100	50	62
32	20	36	18	24	94	62	104	52	63
34	22	38	19	25	96	64	106	53	64
36	24	40	20	26	98	64	106	53	65
38	24	40	20	28	100	66	110	55	67
40	26	44	22	29	....	.....	....	....	....

* Long body patterns are used when outlets are larger than given in table. They have the same center-to-face dimensions as straight fittings.

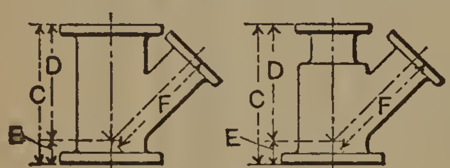
Extra Heavy Flanged Fittings, Reducing Tees and Crosses — American Standard

(See illustration on page 1491 for notation.)

Size	Max. Size of Outlet*	*L*	*A*	*B*	Size	Max. Size of Outlet*	*L*	*A*	*B*
1 to	Sizes 16 inches and smaller have same center-to-face dimensions as straight size fittings.				28	18	38	19	24
9					30	20	41	20½	25½
10					32	20	41	20½	26½
12					34	22	44	22	28
14					36	24	47	23½	29½
15					38	24	47	23½	30½
16					40	26	50	25	31½
18	12	28	14	17	42	28	53	26½	33½
20	14	31	15½	18½	44	28	53	26½	34½
22	15	33	16½	20	46	30	55	27½	35½
24	16	34	17	21½	48	32	58	29	37½
26	18	38	19	23	....	.....	....	....	....

* The dimensions in the table above are for short body patterns. Long body patterns are used when outlets are larger than given in table. They have the same center-to-face dimensions as straight fittings.

Standard Reducing Laterals — American Standard



Short Body Pattern

Sizes 16 inches and smaller have same center-to-face dimensions as straight size fittings.

Size	Max. Size of Branch*	*C*	*D*	*E*	*F*
18	9	26	25	1	27½
20	10	28	27	1	29½
22	10	29	28½	½	31½
24	12	32	31½	½	34½
26	12	35	35	0	38
28	14	37	37	0	40
30	15	39	39	0	42

* Long body patterns are used when outlets are larger than given in table. They have the same center-to-face dimensions as straight size fittings.

Extra Heavy Reducing Laterals — American Standard

(See illustration in table above for notation.)

	Size	Max. Size of Branch*	*C*	*D*	*E*	*F*
Sizes 16 inches and smaller have the same center-to-face dimensions as straight size fittings.	18	9	34	31	3	32½
	20	10	37	34	3	36
	22	10	40	37	3	39
	24	12	44	41	3	43

* The dimensions in the table above are for short body patterns. Long body patterns are used when outlets are larger than given in table. They have the same center-to-face dimensions as straight fittings.

Extra Heavy Pipe Flanges for Working Pressures up to 250 Pounds per Square Inch — American Standard

Size	Diam. of Flange	Thickness of Flange	Diam. of Bolt Circle	No. of Bolts	Size of Bolts	Size	Diam. of Flange	Thickness of Flange	Diam. of Bolt Circle	No. of Bolts	Size of Bolts
1	4½	1 1/16	3¼	4	½	16	25½	2¼	22½	20	1¼
1¼	5	¾	3¾	4	½	18	28	2⅜	24¾	24	1¼
1½	6	13/16	4½	4	⅝	20	30½	2½	27	24	1⅜
2	6½	⅞	5	4	⅝	22	33	2⅝	29¼	24	1½
2½	7½	1	5⅞	4	¾	24	36	2¾	32	24	1⅝
3	8¼	1⅛	6⅝	8	¾	26	38¼	2 13/16	34½	28	1⅝
3½	9	1 3/16	7¼	8	¾	28	40¾	2 15/16	37	28	1⅝
4	10	1¼	7⅞	8	¾	30	43	3	39¼	28	1¾
4½	10½	1 5/16	8½	8	¾	32	45¼	3⅛	41½	28	1⅞
5	11	1⅜	9¼	8	¾	34	47½	3¼	43½	28	1⅞
6	12½	1 7/16	10⅝	12	¾	36	50	3⅜	46	32	1⅞
7	14	1½	11⅞	12	⅞	38	52¼	3 7/16	48	32	1⅞
8	15	1⅝	13	12	⅞	40	54½	3 9/16	50¼	36	1⅞
9	16¼	1¾	14	12	1	42	57	3 11/16	52¾	36	1⅞
10	17½	1⅞	15¼	16	1	44	59¼	3¾	55	36	2
12	20½	2	17¾	16	1⅛	46	61½	3⅞	57¼	40	2
14	23	2⅛	20¼	20	1⅛	48	65	4	60¾	40	2
15	24½	2 3/16	21½	20	1¼	....	.....	.....	.....	....	....

Bolt holes are drilled ⅛ inch larger than nominal diameter of bolts.

Riveted Steel and Spiral Pipe. — Pipe made of sheet steel (1) may be formed of straight riveted sections; (2) it may have a spiral (helical) riveted seam; or (3) the joint of the spiral pipe may be formed by an interlocking seam. Pressed or forged steel flanges are either riveted or welded to the pipe ends. Pipes of this general class are used for low-pressure work in connection with exhaust steam mains, irrigation and dredging, hydraulic mining, exhaust systems, etc.

Approximate Bursting Pressures for Spiral Pipe

Inside Diam., Inches	Thickness, U. S. Gage	Bursting Pressure Lb., Sq. Ft.	Inside Diam., Inches	Thickness, U. S. Gage	Bursting Pressure Lb., Sq. Ft.	Inside Diam., Inches	Thickness, U. S. Gage	Bursting Pressure Lb., Sq. Ft.
Spiral Riveted Pipe — American Spiral Pipe Works								
4	16	1875	12	16	625	24	12	540
6	16	1250	14	14	670	28	10	605
8	16	935	16	14	585	32	10	525
10	16	750	20	14	470	40	10	420
Interlocking Seam — Standard Spiral Pipe Works								
4	16	1875	12	10	1400	26	10	635
	22	845		20	280		20	153
6	16	1250	14	10	1200	30	10	540
	22	470		20	240		18	180
8	16	935	18	10	930	34	10	475
	22	350		20	235		18	158
10	10	1510	22	10	760	40	10	410
	20	405		20	190		18	144

American Low-pressure Standard for End Flanges, Boltings and Body Thickness — for Working Pressure of 50 Pounds per Square Inch

Size	Size of Bolts	Body Thickness	Size	Size of Bolts	Body Thickness	Size	Size of Bolts	Body Thickness
12	3/4	5/8	42	1 1/8	1 1/4	74	1 1/4	2
14	3/4	11/16	44	1 1/8	1 5/16	76	1 1/4	2 1/16
15	3/4	11/16	46	1 1/8	1 3/8	78	1 3/8	2 1/8
16	3/4	3/4	48	1 1/8	1 7/16	80	1 3/8	2 1/8
18	7/8	3/4	50	1 1/8	1 1/2	82	1 3/8	2 3/16
20	7/8	13/16	52	1 1/8	1 1/2	84	1 3/8	2 1/4
22	7/8	7/8	54	1 1/8	1 9/16	86	1 3/8	2 1/4
24	7/8	7/8	56	1 1/4	1 5/8	88	1 3/8	2 5/16
26	1	15/16	58	1 1/4	1 5/8	90	1 3/8	2 3/8
28	1	1	60	1 1/4	1 11/16	92	1 3/8	2 3/8
30	1	1	62	1 1/4	1 3/4	94	1 3/8	2 7/16
32	1	1 1/16	64	1 1/4	1 3/4	96	1 3/8	2 1/2
34	1	1 1/8	66	1 1/4	1 13/16	98	1 3/8	2 1/2
36	1	1 1/8	68	1 1/4	1 7/8	100	1 3/8	2 9/16
38	1 1/8	1 3/16	70	1 1/4	1 15/16	....	......	......
40	1 1/8	1 1/4	72	1 1/4	1 15/16	....	......	......

For sizes 10 inches and smaller, use regular 125-pound American Standard flange dimensions.

For sizes 12 inches and larger, use 125-pound American Standard flange diameters, bolt circles, and number of bolts, using bolt diameters as shown above, thereby maintaining interchangeability with 125-pound American Standard flanges.

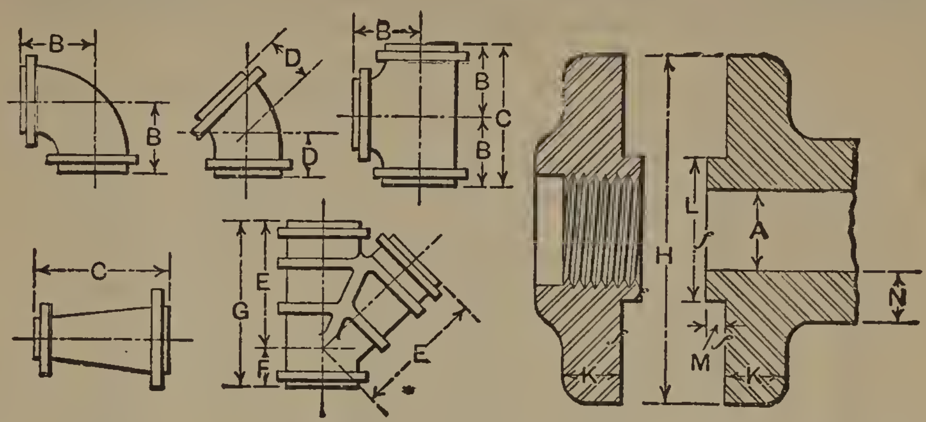
Screwed companion flanges should not be thinner than 125-pound American Standard thickness.

Working Pressures for Water and Steam. — The working pressures recommended by the manufacturers of fittings and valves are much higher for cold water, oil, etc., than for steam. According to the Crane Co., the following conservative rules may be used for determining approximately the relation between the working pressures for steam and water. Low pressure standard, and medium valves and fittings will withstand a water working pressure of 40 per cent greater than a steam working pressure on sizes 12 inches and smaller, and sizes 14 inches and larger will withstand 20 per cent greater water working pressure. A much greater range of pressures may be employed safely with comparatively small sizes of valves and fittings. The foregoing rule also applies to extra heavy globe valves, screwed fittings, and also extra heavy fittings larger than 8 inches.

Working pressures based on different factors of safety are given in the table "Bursting and Working Pressures for Pipes" on page 1482.

O. D. Pipe. — The abbreviated expression "O. D. pipe," which is found in manufacturers' catalogues, is applied to large wrought pipe, the nominal size of which is designated by the outside diameter instead of the inside diameter as in the case of smaller sizes. It is common practice to designate the nominal sizes of all pipes above 12 inches by giving the outside diameter, although this is not an invariable rule. The National Tube Co. designates all pipes above 15 inches inside diameter, by the outside diameter. The nominal sizes of boiler tubes also indicate the outside diameter.

800-Pound Hydraulic American Standard Flanged Fittings for Full-weight Wrought Pipe. Semi-steel and Cast Steel *



M = ¼ inch for all sizes of 800-, 1200- and 3000-pound fittings. All center-to-face and face-to-face dimensions include the raised face M.

Size, Inches	A	B	C	D	E	F	G	H
½	½	3¼	6½	2	5½	1¾	7¼	3½
¾	¾	3¾	7½	2¼	6	2	8	4
1	1	4¼	8½	2¼	6¾	2¼	9	4½
1¼	1¼	4½	9	2¾	7½	2½	10	5
1½	1½	4¾	9½	3	8¾	2¾	11½	6
2	2	5¾	11½	4¼	11	3½	14½	7
2½	2½	6½	13	4½	12½	3½	16	7½
3	3	7	14	5	13½	4	17½	8½
3½	3½	7½	15	5½	14½	4½	19	9½
4	4	8½	17	6	16½	4½	21	10¾
4½	4½	9	18	6½	17½	5½	23	11½
5	5	10	20	7	19½	6	25½	13
6	6	11	22	7½	21	6½	27½	14
7	7	12	24	8	22½	6½	29	15
8	8	13	26	8½	24½	7	31½	16½
9	9	14½	29	9	27½	7½	35	18½
10	10	15½	31	9½	29½	8	37½	20
12	12	16½	33	10	31½	8½	40	22

Size, Inches	K Semi-steel	K Cast Steel	L	N Semi-steel	N Cast Steel	Diam., Bolt Circle	No. of Bolts	Diam., Bolts
½	9/16	....	1⅜	7/16	....	2⅜	4	7/16
¾	⅝	....	1 11/16	7/16	....	2⅞	4	½
1	11/16	....	2	½	....	3¼	4	½
1¼	¾	....	2¼	½	....	3¾	4	½
1½	13/16	....	2⅞	½	....	4½	4	⅝
2	1¼	1	3⅝	9/16	½	5¼	4	¾
2½	1⅜	1⅛	4⅛	⅝	9/16	5⅞	8	¾
3	1½	1¼	5	⅝	9/16	6½	8	¾
3½	1⅝	1⅜	5½	¾	⅝	7½	8	⅞
4	1⅞	1½	6	⅞	11/16	8½	8	⅞
4½	2	1⅝	6½	1	¾	9¼	8	⅞
5	2⅛	1¾	7¼	1⅛	13/16	10½	8	1
6	2¼	1⅞	8⅜	1⅛	⅞	11½	12	1
7	2⅜	2	9⅜	1¼	1	12½	12	1
8	2½	2⅛	10⅝	1⅜	1 1/16	13¾	12	1⅛
9	2¾	2⅜	11⅝	1½	1⅛	15½	16	1⅛
10	2⅞	2½	12¾	1⅝	1¼	17	16	1¼
12	3	2⅝	15¼	1¾	1⅜	19¼	20	1¼

* These fittings are recommended for pump columns, oil-transmission lines, gas lines and other hydraulic service where shock is negligible for a maximum working pressure of 800 pounds per square inch, and a maximum temperature of 100 degrees F. Where subject to shock they are recommended for a maximum working pressure of 500 pounds. The diameter of port is nominal size. Reducing fittings carry same dimensions center to face as straight-size fittings corresponding to largest opening.

1200-Pound Hydraulic American Standard Flanged Fittings for Extra Strong Wrought Pipe. Semi-steel and Cast Steel *

(For notation see illustration accompanying table for 800-pound fittings.)

Size, Inches	A	B	C	D	E	F	G	H
½	½	3½	7	2½	7	2	9	4¼
¾	¾	4	8	3	8	2½	10½	4¾
1	15/16	4½	9	3½	9	2½	11½	5½
1¼	1¼	5	10	4	10	3	13	6
1½	1½	5¾	11½	4¼	11	3½	14½	6¾
2	1⅞	6½	13	4½	12½	3½	16	8
2½	2¼	7	14	5	13½	4	17½	8½
3	2¾	7½	15	5½	14½	4½	19	9½
3½	3¼	8½	17	6	16½	4½	21	10¾
4	3¾	9	18	6½	17½	5½	23	11½
4½	4¼	10	20	7	19½	6	25½	13
5	4¾	11	22	7½	21	6½	27½	14
6	5¾	12	24	8	22½	6½	29	15
7	6¾	13	26	8½	24½	7	31½	16½
8	7⅝	14½	29	9	27½	7½	35	18½
9	8¾	15½	31	9½	29½	8	37½	20
10	9¾	16½	33	10	31½	8½	40	21½
12	11¾	17½	35	11	34½	9	43½	24

Size, Inches	K Semi-steel	K Cast Steel	L	N Semi-steel	N Cast Steel	Diam., Bolt Circle	No. of Bolts	Diam., Bolts
½	¾	....	1⅜	½	....	3	4	½
¾	⅞	....	1 11/16	½	....	3⅜	4	½
1	1	....	2	9/16	....	4	4	⅝
1¼	1⅛	....	2¼	9/16	....	4⅜	4	⅝
1½	1¼	....	2⅞	⅝	....	4⅞	4	¾
2	1⅜	1⅛	3⅝	¾	⅝	6	4	⅞
2½	1½	1¼	4⅛	⅞	11/16	6½	8	⅞
3	1⅝	1⅜	5	1	13/16	7½	8	⅞
3½	1⅞	1½	5½	1 3/16	⅞	8½	8	1
4	2	1⅝	6	1⅜	1	9¼	8	1
4½	2⅛	1¾	6½	1½	1 1/16	10½	8	1⅛
5	2¼	1⅞	7¼	1⅝	1⅛	11½	8	1⅛
6	2⅜	2	8⅜	1¾	1¼	12½	12	1⅛
7	2½	2⅛	9⅜	1⅞	1⅜	13¾	12	1¼
8	2¾	2⅜	10⅝	2	1½	15½	12	1⅜
9	2⅞	2½	11⅝	2⅛	1⅝	17	16	1⅜
10	3	2⅝	12¾	2¼	1¾	18½	16	1⅜
12	3⅛	2¾	15¼	2½	1⅞	21	20	1⅜

* These fittings are recommended for pump columns, oil-transmission lines, gas lines and other hydraulic service where shock is negligible for a maximum working pressure of 1200 pounds per square inch, and a maximum temperature of 100 degrees F. Where subject to shock they are recommended for a maximum working pressure of 800 pounds.

The diameter of port is about the same as the inside diameter of extra strong pipe.

Reducing fittings carry the same dimensions center to face as straight-size fittings corresponding to largest opening.

Flanges may be attached to the pipe by any of the following methods: Screw flanges; lap flanges; shrunk, peened or riveted flanges; flanges welded to pipe.

Square-head bolts with hexagonal nuts are recommended. Hexagonal nuts on sizes 8 inches and smaller can be conveniently pulled up with open-end wrenches with minimum-design heads. Hexagonal nuts on sizes 9 inches and larger can be conveniently pulled up with box wrenches.

When flanges are screwed, shrunk, peened or riveted on the pipe it is recommended that the end of the pipe and flange be refaced.

3000-Pound Hydraulic American Standard Flanged Fittings for Double Extra Strong Wrought Pipe. Cast Steel*

(For notation see illustration accompanying table for 800-pound fittings.)

Size, In.	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*	*L*	*N*	Diam. Bolts
½	⅜	3½	7	2½	7	2	9	4¼	¾	⅞	7/16	½
¾	½	4	8	3	8	2½	10½	4¾	⅞	1 1/16	½	⅝
1	¾	4½	9	3½	9	2½	11½	5½	1	1 5/16	½	⅝
1¼	15/16	5	10	4	10	3	13	6	1⅛	1 11/16	9/16	¾
1½	1¼	5¾	11½	4¼	11	3½	14½	6¾	1¼	1 15/16	9/16	⅞
2	1½	6½	13	4½	12½	3½	16	8	1⅜	2⅜	⅝	⅞
2½	1⅞	7	14	5	13½	4	17½	8½	1½	2⅞	¾	⅞
3	2¼	7½	15	5½	14½	4½	19	9½	1⅝	3½	⅞	1
3½	2¾	8½	17	6	16½	4½	21	10¾	1⅞	4	1	1
4	3¼	9	18	6½	17½	5½	23	11½	2	4½	1 3/16	1⅛
4½	3¾	10	20	7	19½	6	25½	13	2⅛	5	1⅜	1¼
5	4¼	11	22	7½	21	6½	27½	14	2¼	5 9/16	1½	1¼
6	4¾	12	24	8	22½	6½	29	15	2⅜	6⅝	1⅝	1¼
7	5¾	13	26	8½	24½	7	31½	16½	2½	7⅝	1¾	1⅜
8	6¾	14½	29	9	27½	7½	35	18½	2¾	8⅝	1⅞	1½
9	7⅝	15½	31	9½	29½	8	37½	20	2⅞	9⅝	2	1½
10	8¾	16½	33	10	31½	8½	40	21½	3	10¾	2⅛	1½
12	10¾	17½	35	11	34½	9	43½	24	3⅛	12¾	2⅜	1½

* These fittings are recommended for hydraulic service where shock is negligible for a maximum working pressure of 3000 pounds per square inch, and a maximum temperature of 100 degrees F. For diameters of bolt circles and numbers of bolts, see table for 1200-pound fittings. Where subject to shock they are recommended for a maximum working pressure of 2000 pounds.

The diameter of port is about the same as the inside diameter of double extra strong pipe.

Reducing fittings carry same dimensions center to face as straight-size fittings corresponding to largest opening.

Flanges may be attached to the pipe by either of the following methods: Screw flanges; flanges welded to pipe. Screw flanges are furnished with plain face and are threaded with American Briggs Standard lock-nut threads. The pipe should be threaded with American Briggs Standard lock-nut threads, and the end of the pipe should be faced off square. The pipe should be screwed through the flange until the end projects about ¼ inch beyond the face of the flange and bears against the gasket. When flanges are welded on the pipe the end of the pipe should project through the flange and should be faced off square to form the raised face.

Square-head bolts with hexagonal nuts are recommended. Hexagonal nuts on sizes 5 inches and smaller can be conveniently pulled up with open-end wrenches with minimum-design heads. Hexagonal nuts on sizes 6 inches and larger can be conveniently pulled up with box wrenches.

Soft metallic gaskets at least 1/16 inch thick are recommended.

Hydraulic American Standard. — Three standards for hydraulic fittings have been adopted by the American Society of Mechanical Engineers. These are known, respectively, as the 800-pound, the 1200-pound, and the 3000-pound Hydraulic American Standard. The dimensions are given in the accompanying tables. All reducing fittings have the same center-to-face dimensions as straight size fittings corresponding to the largest openings. Flanges on fittings and valves and also all companion flanges except those for a lap joint, should have a ¼-inch raised face, unless otherwise specified, and all center-to-face and face-to-face dimensions are measured from this raised face. All bolt holes are ⅛ inch larger in diameter than the bolts and the holes straddle the center-line.

Dimensions of British Standard Pipe Flanges

For Steam Pressures up to 55 Lbs. per Sq. In. and Water Pressures up to 200 Lbs. per Sq. In.								For Steam Pressures up to 125 Lbs., 225 Lbs., and 325 Lbs., per Sq. In.											
Inside Diam. of Pipe	Diam. of Flange	Diam. of Bolt Circle	Number of Bolts	Diam. of Bolts *	Thickness of Flanges			Diam. of Flange	Diam. of Bolt Circle	Number of Bolts	Diam. of Bolts		Thickness of Flanges, Cast Iron, Steel or Iron Welded on			Thickness of Flanges, Steel (cast or riveted on) and Bronze		
					Cast Iron, Steel, or Iron Welded	Cast Steel, Bronze	Iron or Steel Stamped or Forged				125 Lbs. 225 Lbs.	325 Lbs.	125 Lbs.	225 Lbs.	325 Lbs.	125 Lbs.	225 Lbs.	325 Lbs.
½″	3¾″	2⅝″	4	½″	½″	⅜″	3/16″	3¾″	2⅝″	4	½″	½″	½″	½″	⅝″	⅜″	7/16″	⅝″
¾	4	2⅞	4	½	½	⅜	3/16	4	2⅞	4	½	½	½	½	⅝	⅜	7/16	⅝
1	4½	3¼	4	½	½	⅜	3/16	4¾	3 7/16	4	⅝	⅝	½	⅝	¾	⅜	½	1 1/16
1¼	4¾	3 7/16	4	½	⅝	½	¼	5¼	3⅞	4	⅝	⅝	⅝	⅝	¾	½	½	1 1/16
1½	5¼	3⅞	4	½	⅝	½	¼	5½	4⅛	4	⅝	⅝	⅝	¾	⅞	½	9/16	¾
2	6	4½	4	⅝	¾	9/16	5/16	6½	5	4	⅝	¾	¾	⅞	1	⅝	11/16	⅞
2½	6½	5	4	⅝	¾	9/16	5/16	7¼	5¾	8	⅝	¾	¾	⅞	1	⅝	11/16	⅞
3	7¼	5¾	4	⅝	¾	9/16	⅜	8	6½	8	⅝	¾	¾	1	1¼	⅝	¾	1
3½	8	6½	4	⅝	¾	9/16	⅜	8½	7	8	⅝	¾	⅞	1	1¼	¾	¾	1
4	8½	7	4	⅝	⅞	1 1/16	⅜	9	7½	8	⅝	¾	⅞	1⅛	1⅜	¾	⅞	1⅛
5	10	8¼	8	⅝	⅞	1 1/16	½	11	9¼	8	¾	⅞	1	1¼	1½	⅞	1	1¼
6	11	9¼	8	⅝	⅞	1 1/16	½	12	10¼	12	¾	⅞	1	1¼	1½	⅞	1	1¼
7	12	10¼	8	⅝	1	¾	½	13¼	11½	12	¾	⅞	1	1⅜	1⅝	⅞	1⅛	1⅜
8	13¼	11½	8	⅝	1	¾	½	14½	12¾	12	¾	⅞	1⅛	1⅜	1⅝	1	1⅛	1⅜
9	14½	12¾	8	⅝	1	¾	⅝	16	14	12	⅞	1	1⅛	1½	1¾	1	1¼	1½
10	16	14	8	¾	1	¾	⅝	17	15	12	⅞	1	1⅛	1½	1⅞	1	1¼	1⅝
12	18	16	12	¾	1⅛	⅞	⅝	19¼	17¼	16	⅞	1	1¼	1⅝	2	1⅛	1⅜	1¾
14	20¾	18½	12	⅞	1¼	1	¾	21¾	19½	16	1	1⅛	1⅜	1¾	2⅛	1¼	1½	1⅞
15	21¾	19½	12	⅞	1¼	1	¾	22¾	20½	16	1	1⅛	1⅜	1⅞	2⅛	1¼	1⅝	1⅞
16	22¾	20½	12	⅞	1¼	1	¾	24	21¾	20	1	1⅛	1⅜	1⅞	2¼	1¼	1⅝	2
18	25¼	23	12	⅞	1⅜	1⅛	⅞	26½	24	20	1⅛	1¼	1½	2	2⅜	1⅜	1¾	2⅛
20	27¾	25¼	16	⅞	1½	1¼	1	29	26½	24	1⅛	1¼	1⅝	2⅛	2½	1½	1⅞	2¼
21	29	26½	16	⅞	1½	1¼	1	30	27½	24	1⅛	1¼	1⅝	2¼	2⅝	1½	2	2⅜
24	32½	29¾	16	1	1⅝	1⅜	1⅛	33½	30¾	24	1¼	1⅜	1¾	2⅜	2¾	1⅝	2⅛	2½

* Holes for ½-inch a[illegible] inch larger, and for larger sizes of bolts, ⅛ inch larger than the bolt diameters.

Dimensions of British Standard Welded-on Flanges for Pipe Lines
(For working steam pressures up to 125 lbs., 225 lbs. and 325 lbs. per square inch)

Inside Diam. of Pipe	Diameter of Flange	Diameter of Bolt Circle	Number of Bolts	Diameter of Bolts †		Thickness of Steel or Iron Welded-on Flanges		
				125 Lbs. 225 Lbs.	325 Lbs.	125 Lbs.	225 Lbs.	325 Lbs.
2″	6″	4½″	4	⅝″	⅝″	½″	9/16″	¾″
2½	6½	5	4	⅝	¾	⅝	11/16	⅞
3	7¼	5¾	8	⅝	¾	⅝	11/16	⅞
3½	8	6½	8	⅝	¾	⅝	¾	1
4	8½	7	8	⅝	¾	¾	¾	1
*4½	9	7½	8	⅝	¾	¾	⅞	1⅛
5	10	8¼	8	¾	⅞	¾	⅞	1⅛
6	11	9¼	8	¾	⅞	⅞	1	1¼
7	12	10¼	12	¾	⅞	⅞	1	1¼
8	13¼	11½	12	¾	⅞	⅞	1⅛	1⅜
9	14½	12¾	12	¾	⅞	1	1⅛	1⅜
10	16	14	12	⅞	1	1	1¼	1½
*11	17	15	12	⅞	1	1	1¼	1⅝
12	18	16	16	⅞	1	1⅛	1⅜	1⅝
*13	19¼	17¼	16	⅞	1	1⅛	1⅜	1¾
14	20¾	18½	16	1	1⅛	1⅛	1½	1¾
15	21¾	19½	16	1	1⅛	1¼	1½	1⅞
16	22¾	20½	16	1	1⅛	1¼	1⅝	1⅞
*17	24	21¾	20	1	1⅛	1¼	1⅝	2
18	25¼	23	20	1	1⅛	1⅜	1¾	2⅛
*19	26½	24	20	1⅛	1¼	1⅜	1¾	2⅛
20	27¾	25¼	20	1⅛	1¼	1⅜	1⅞	2¼
21	29	26½	24	1⅛	1¼	1½	1⅞	2¼
*22	30	27½	24	1⅛	1¼	1½	2	2⅜
*23	31	28½	24	1⅛	1¼	1½	2	2⅜
24	32½	29¾	24	1¼	1⅜	1⅝	2⅛	2½

* These sizes not recommended for general use.

† The diameters of the holes for ½-in. and ⅝-in. bolts to be 1/16 in. larger than the diameters of the bolts, and for larger sizes of bolts, ⅛ in. Bolt-holes to be drilled off center lines.

British Standard Pipe Flanges. — The pipe flanges approved by the British Engineering Standards Association are divided into four classes as follows:

Low-pressure Standard for steam pressures up to 55 pounds and water pressures up to 200 pounds per square inch.

Intermediate-pressure Standard for steam pressures over 55 pounds, but not exceeding 125 pounds per square inch.

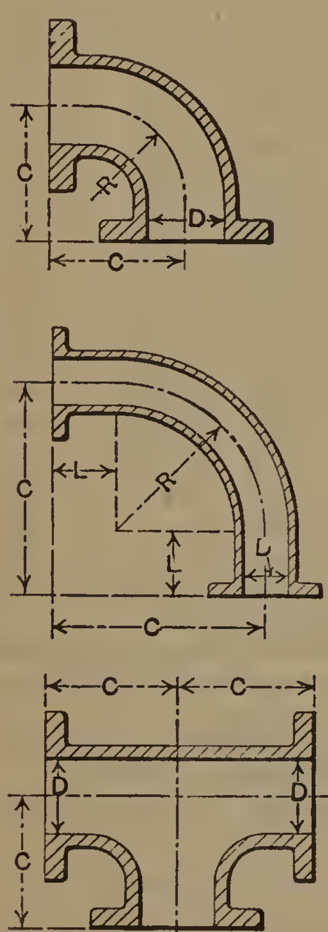
High-pressure Standard for steam pressures over 125 pounds but not exceeding 225 pounds per square inch.

Extra High-pressure Standard for steam pressures over 225 pounds but not exceeding 325 pounds per square inch.

The dimensions of flanges for these four standards are given in the accompanying table. That part of the table covering the low-pressure standard does not apply to boiler feed pipes or other water pipes subject to exceptional shocks. This table does not include flange dimensions for 1¾-inch, 4½-inch, 11-inch, 13-inch, 17-inch, 19-inch, 22-inch and 23-inch pipe sizes, as these sizes are not recommended for general use. The bolt holes should be so placed that the spaces between them are bisected by the main center lines.

British Standards for Flanged Bends and Tees. — The standards recommended by the British Engineering Standards Association for short bends and tees of cast metal and for long bends of wrought iron and steel are given in the accompanying table. For short bends and tees the dimensions from the center line to the face of the flange are equal to the pipe diameter plus three inches. The dimensions of long bends of wrought iron and steel were determined with reference to the requirements of manufacture and are given for pipe sizes up to and including 20 inches which is considered the maximum for bends of this type.

Dimensions of British Standard Short Bends, Long Bends, and Tees



Inside Diam. of Pipe, D	Short Bends and Tees †		Long Bends Wrought Iron and Steel		
	C	R	L	C	R
½″	3½″	2½″	2½″	4½″	2″
¾	3¾	2¾	2½	5	2½
1	4	2¾	3	6	3
1¼	4¼	3	3	6¾	3¾
1½	4½	3	3	7½	4½
*1¾	4¾	3¼	3½	8¾	5¼
2	5	3¼	3½	9½	6
2½	5½	3¾	4	11½	7½
3	6	4	4	13	9
3½	6½	4½	5	15½	10½
4	7	4¾	5	17	12
*4½	7½	5¼	6	19½	13½
5	8	5½	6	21	15
6	9	6½	7	25	18
7	10	7¼	7	31½	24½
8	11	8¼	8	36	28
9	12	9	8	39½	31½
10	13	10	9	49	40
*11	14	10¾	9	53	44
12	15	11¾	10	58	48
*13	16	12½	11	69½	58½
14	17	13½	11	74	63
15	18	14¼	12	79½	67½
16	19	15¼	13	93	80
*17	20	16	14	99	85
18	21	17	14	104	90
*19	22	17¾	15	119½	104½
20	23	18¾	16	126	110
21	24	19½	.....	.....	.....
*22	25	20½	.....	.....	.....
*23	26	21¼	.....	.....	.....
24	27	22¼	.....	.....	.....

* These sizes not recommended for general use.

† The dimensions of unequal tees having branches which do not vary in diameter more than 3 to 1, are the same as the dimensions given in the table for the largest branch diameter.

Cast-iron Pipe. — Cast-iron pipe is used instead of wrought pipe where the pipes must be placed under ground or submerged, and also for main steam pipes and branches which are subjected to acids. Cast-iron pipe is extensively employed for cold water on lines 4 inches in diameter and above. Commercial cast-iron pipe is unsuitable for lines subjected to expansion strains, contraction, and vibration unless the pipe is very heavy. It is not suitable for superheated steam or for temperatures above 575 degrees F. The cast-iron pipe used for underground work generally has the *bell-and-spigot* ends which are leaded and calked to secure a tight joint. Exposed cast-iron pipes usually have flanged ends.

Thickness Formula for Cast-iron Pipe. — The following formula for determining the thickness of cast-iron pipe was adopted by the American Water Works Association, and is in general use.

$$T = 0.25 + \frac{(P + X)\,R}{3300}$$

T = pipe thickness in inches; P = maximum static pressure in pounds per square inch for which the pipe is intended; X = allowance made for water ram; and R = inside radius in inches. For ordinary water-works conditions and pipe diameters from 42 to 60 inches, 70 pounds per square inch is a conservative value for X, but for smaller pipe, Brackett allows the following values:

Diameter of pipe, inches,	= 36	30	24	20	16	12	10 to 3
Values of X	= 75	80	85	90	100	110	120

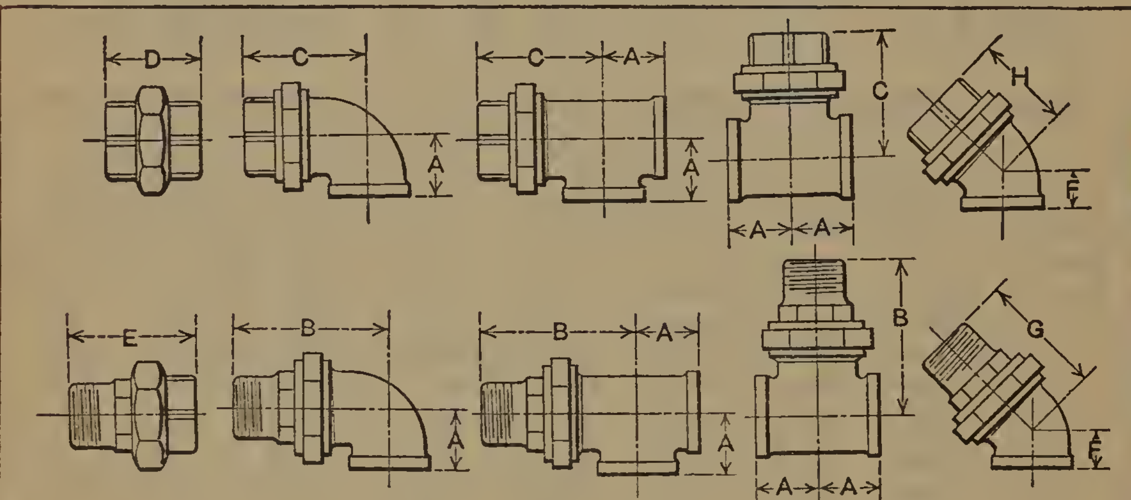
Standard Thickness and Weights of Cast-iron Pipe — Bell-and-spigot Joint

Nominal Inside Diam., In.	Class A 100-Feet Head 43 Lb. Pressure		Class B 200-Feet Head 86 Lb. Pressure		Class C 300-Feet Head 130 Lb. Pressure		Class D 400-Feet Head 173 Lb. Pressure	
	Thickness, Inches	Weight per Foot	Thickness, Inches	Weight per Foot	Thickness, Inches	Weight per Foot	Thickness, Inches	Weight per Foot
4	0.42	20.0	0.45	21.7	0.48	23.3	0.52	25.0
6	0.44	30.8	0.48	33.3	0.51	35.8	0.55	38.3
8	0.46	42.9	0.51	47.5	0.56	52.1	0.60	55.8
10	0.50	57.1	0.57	63.8	0.62	70.8	0.68	76.7
12	0.54	72.5	0.62	82.1	0.68	91.7	0.75	100.0
14	0.57	89.6	0.66	102.5	0.74	116.7	0.82	129.2
16	0.60	108.3	0.70	125.0	0.80	143.8	0.89	158.3
18	0.64	129.2	0.75	150.0	0.87	175.0	0.96	191.7
20	0.67	150.0	0.80	175.0	0.92	208.3	1.03	229.2
24	0.76	204.2	0.89	233.3	1.04	279.2	1.16	306.7
30	0.88	291.7	1.03	333.3	1.20	400.0	1.37	450.0
36	0.99	391.7	1.15	454.2	1.36	545.8	1.58	625.0
42	1.10	512.5	1.28	591.7	1.54	716.7	1.78	825.0
48	1.26	666.7	1.42	750.0	1.71	908.3	1.96	1050.0
54	1.35	800.0	1.55	933.3	1.90	1141.7	2.23	1341.7
60	1.39	916.7	1.67	1104.2	2.00	1341.7	2.38	1583.3
72	1.62	1283.4	1.95	1545.8	2.39	1904.2	.....	......
84	1.72	1633.4	2.22	2104.2	.....	......	.....	......

The approximate "laying length" for all diameters is 12 feet. The weights given include an allowance for the bell.

Screwed Pipe Fittings. — Screwed pipe fittings for use with "wrought pipe" are usually made of cast iron or malleable iron but they may also be made of either cast or forged steel for exceptionally high-pressure service. There are different weights of fittings for different weights of pipe, such as the standard cast or malleable iron, extra heavy, and hydraulic. The fittings may be plain or have banded or beaded ends. The plain type is generally used for low pressure gas and water lines, house plumbing, and railing work. The Crane Co. standard cast-iron flat-band fittings are recommended for steam working pressures up to 125 pounds per square inch and water working pressures up to 175 pounds. The standard malleable iron fittings are recommended for steam working pressures up to 150 pounds per square inch and the extra heavy malleable iron fittings for steam pressures up to 250 pounds. Extra heavy hydraulic malleable iron fittings are suitable for pressures ranging from 500 to 2000 pounds per square inch, depending upon the size of the fitting. Double extra heavy hydraulic forged steel screwed fittings may be obtained suitable for cold water or oil working pressures up to 6000 pounds per square inch, when used in hydraulic installations in which there is practically no shock. The use of screwed fittings in general is not recommended for sizes above six inches. When the size exceeds six inches it is preferable to employ fittings having flanges.

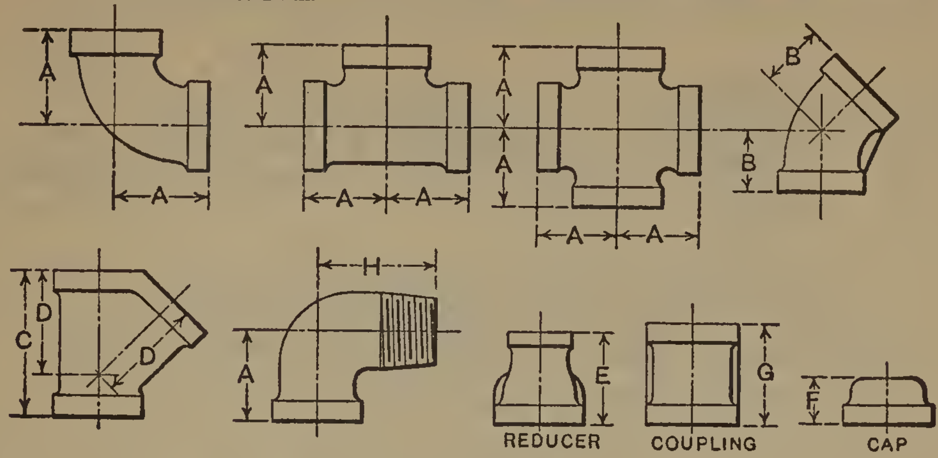
Malleable Iron Unions, Union Elbows and Union Tees (Crane Co.)



Size, Inches	*A*	*B*	*C*	*D* *	*D* *	*D* *	*E*	*F*	*G*	*H*
1/8	....	....	....	1 1/2	....	....	....	....	....	....
1/4	13/16	2 7/16	1 13/16	1 5/8	1 5/8	2 3/16	2 1/4	3/4	2 3/16	1 9/16
3/8	15/16	2 3/4	2 1/16	1 3/4	1 13/16	2 1/4	2 7/16	13/16	2 3/8	1 11/16
1/2	1 1/8	3 1/16	2 5/16	1 7/8	1 15/16	2 5/16	2 11/16	7/8	2 11/16	1 7/8
3/4	1 5/16	3 1/2	2 5/8	2 1/8	2 1/8	2 1/2	3 1/16	1	3	2 1/8
1	1 7/16	3 15/16	3	2 3/8	2 3/8	2 5/8	3 5/16	1 1/8	3 3/8	2 7/16
1 1/4	1 3/4	4 7/16	3 7/16	2 5/8	2 11/16	2 7/8	3 11/16	1 5/16	3 3/4	2 3/4
1 1/2	1 15/16	4 3/4	3 13/16	2 15/16	2 15/16	3 1/8	4	1 7/16	4	3 1/16
2	2 1/4	5 3/8	4 3/8	3 1/4	3 3/8	3 9/16	4 5/16	1 11/16	4 1/2	3 1/2
2 1/2	2 11/16	6	4 7/8	3 9/16	3 7/8	4 1/16	4 5/8	....	....	....
3	....	....	....	3 15/16	4 7/16	4 7/8	....	....	....	....
3 1/2	....	....	....	4 3/8	....	....	....	....	....	...
4	....	....	....	4 1/2	....	....	....	....	....	....

* The end-to-end dimensions for three different standards are given as there is no universal standard.

Standard Malleable Iron Screwed Fittings (Crane Co.)



Size, Inches	A	B	C	D	E	F	G	H
1/8	1 1/16	......	......	.....	.....	.....	.....	1 1/8
1/4	1 3/16	3/4	......	.....	1	5/8	1 1/16	1 5/16
3/8	1 5/16	1 3/16	......	.....	1 1/8	3/4	1 3/16	1 7/16
1/2	1 1/8	7/8	2 1/2	1 11/16	1 1/4	7/8	1 5/16	1 5/8
3/4	1 5/16	1	2 7/8	2	1 7/16	1 1/16	1 1/2	1 7/8
1	1 7/16	1 1/8	3 7/16	2 7/16	1 11/16	1 3/16	1 11/16	2 1/8
1 1/4	1 3/4	1 5/16	4 1/16	2 15/16	2 1/16	1 1/4	1 15/16	2 1/2
1 1/2	1 15/16	1 7/16	4 1/2	3 5/16	2 5/16	1 5/16	2 1/8	2 11/16
2	2 1/4	1 11/16	5 7/16	4 1/16	2 13/16	1 7/16	2 1/2	3 3/16
2 1/2	2 11/16	1 15/16	6 1/4	4 11/16	3 1/4	1 5/8	2 7/8	3 13/16
3	3 1/8	2 3/16	7 1/4	5 9/16	3 11/16	1 3/4	3 3/16	4 1/2
3 1/2	3 7/16	2 3/8	......	.....	4	1 15/16	.....	.....
4	3 3/4	2 5/8	8 7/8	6 15/16	4 3/8	2	.....	5 11/16
4 1/2	4 1/16	2 13/16	......	.....	.....	.....	.....	.....
5	4 7/16	3 1/16	......	.....	.....	2 5/16	.....	.....
6	5 1/8	3 7/16	......	.....	.....	2 9/16	.....	.....
7	5 13/16	3 7/8	......	.....	.....	.....	.....	.....
8	6 1/2	4 1/4	......	.....	.....	.....	.....	.....

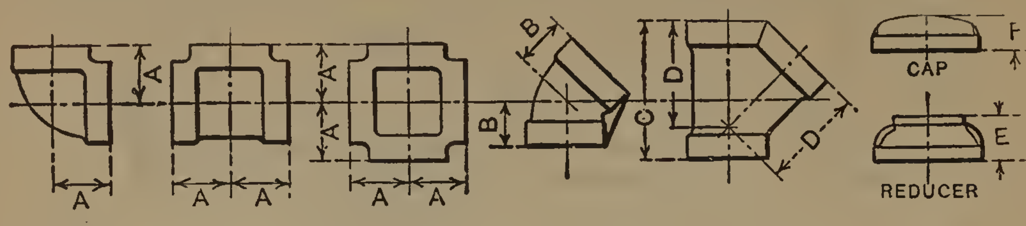
Standard Brass Screwed Fittings (Crane Co.)

(For notation, see illustration accompanying table, "Malleable Iron Screwed Fittings")

Size, Inches	A	B	C	D	E *	F	G	H
1/8	9/16	1/2	......	.....	.....	7/16	7/8	1
1/4	11/16	9/16	......	.....	15/16	9/16	15/16	1 1/8
3/8	3/4	5/8	......	.....	1 1/16	5/8	1 1/16	1 1/4
1/2	15/16	11/16	......	.....	1 1/8	11/16	1 1/8	1 7/16
3/4	1 1/8	13/16	2 7/8	2	1 5/16	3/4	1 5/16	1 5/8
1	1 5/16	15/16	3 7/16	2 7/16	1 1/2	7/8	1 1/2	1 7/8
1 1/4	1 9/16	1 1/8	4 1/16	2 15/16	1 11/16	1	1 11/16	2 3/16
1 1/2	1 3/4	1 1/4	4 1/2	3 5/16	1 7/8	1 1/8	1 7/8	2 7/16
2	2 3/16	1 7/16	5 7/16	4 1/16	2 3/16	1 5/16	2 3/16	2 7/8
2 1/2	2 9/16	1 11/16	6 1/4	4 11/16	3 1/4	1 1/2	2 9/16	.....
3	3 1/16	2	......	.....	3 11/16	1 3/4	3	.....
3 1/2	3 7/16	2 3/8	......	.....	.....	1 7/8	3 1/8	.....
4	3 3/4	2 1/2	......	.....	.....	2	3 3/8	.....

* For a reduction of one size only.

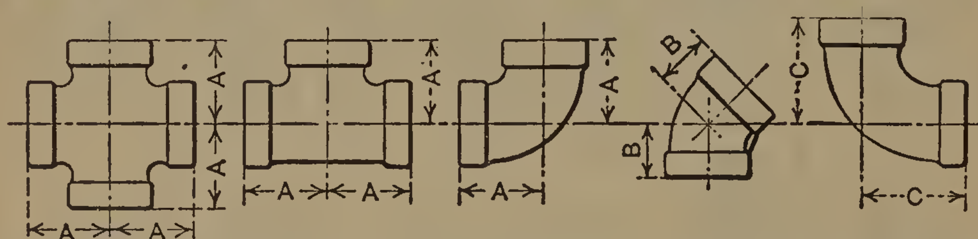
Standard and Extra Heavy Cast-iron Screwed Fittings (Crane Co.)



For steam pressures up to 125 pounds per square inch.

Size, Ins.	Dimensions in Inches				
	A	B	C	D	E
¼	1³⁄₁₆	¾	....	....	....
⅜	1⁵⁄₁₆	1³⁄₁₆	....	....	....
½	1⅛	⅞	2½	1⅞	....
¾	1⁵⁄₁₆	1	3	2¼	....
1	1⁷⁄₁₆	1⅛	3½	2¾	....
1¼	1¾	1⁵⁄₁₆	4¼	3¼	....
1½	1¹⁵⁄₁₆	1⁷⁄₁₆	4⅞	3¹³⁄₁₆	....
2	2¼	1¹¹⁄₁₆	5¾	4½	....
2½	2¹¹⁄₁₆	1¹⁵⁄₁₆	6¼	5³⁄₁₆	....
3	3⅛	2³⁄₁₆	7⅞	6⅛	2¹⁵⁄₁₆

Size, Ins.	Dimensions in Inches					
	A	B	C	D	E	F
3½	3⁷⁄₁₆	2⅜	8⅞	6⅞	3⅛	....
4	3¾	2⅝	9¾	7⅝	3⅜	2¹⁄₁₆
4½	4¹⁄₁₆	2¹³⁄₁₆	11⅝	9⅛	3⅝	2³⁄₁₆
5	4⁷⁄₁₆	3¹⁄₁₆	11⅝	9¼	3⅞	2⅜
6	5⅛	3⁷⁄₁₆	13⁷⁄₁₆	10¾	4⅜	2⅝
7	5¹³⁄₁₆	3⅞	15¼	12¼	4¹³⁄₁₆	2⅞
8	6½	4¼	16¹⁵⁄₁₆	13⅝	5¼	3⅛
9	7³⁄₁₆	4¹¹⁄₁₆	20¹¹⁄₁₆	16¾	5¹¹⁄₁₆	3⅜
10	7⅞	5³⁄₁₆	20¹¹⁄₁₆	16¾	6³⁄₁₆	3⅝
12	9¼	6	24⅛	19⅝	7⅛	4¼



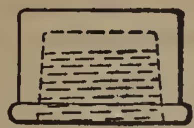
Extra-heavy cast-iron fittings — For pressures up to 250 pounds per square inch.

Size, Inches	Inches		Size, Inches	Inches		Size, Inches	Inches		Size, Inches	Inches	
	A	B		A	B		A	B		A	B
1	2	1⅜	2½	3½	2¼	4½	5½	3	8	9⅛	4¾
1¼	2¼	1½	3	4⅛	2½	5	6⅛	3⁵⁄₁₆	10	11⅜	4⅞
1½	2⁹⁄₁₆	1⅝	3½	4¹¹⁄₁₆	2⁹⁄₁₆	6	7¼	3¾	12	13⅜	5½
2	3	1¹⁵⁄₁₆	4	5⅛	2¾	7	8⅛	4	.....	.....	....

Extra-heavy hydraulic malleable-iron fittings.

Size, Inches	Inches			Size, Inches	Inches			Size, Inches	Inches		
	A	B	C		A	B	C		A	B	C
¼	1¹⁄₁₆	¾	....	1¼	2¼	1½	3	3½	4⅝	2⅝	6¼
⅜	1¼	⅞	....	1½	2½	1¹¹⁄₁₆	3½	4	5⅛	2¹³⁄₁₆	7
½	1½	1	....	2	3	2	4	4½	5⅝	....	7¾
¾	1¾	1⅛	....	2½	3½	2¼	4¾	5	6¼	....	8½
1	2	1⁵⁄₁₆	2½	3	4⅛	2½	5½	6	7¼	....	9½

Dimensions of Caps, Plugs, Bushings and Nuts

Size, Inside Cap and Nut; Outside Plug and Bushing	CAP 			PLUG 			BUSHING 			NUT 	
	Length	Diam. of Body	Diam. of Bead	Length of Thread	Length of Square	Side of Square	Length of Thread	Thickness of Hexagon	Across Flats	Thick-ness	Across Flats
⅛	....	....	....	⅜	¼	¼	...	...	...	...	...
¼	....	....	....	7/16	¼	⅜	...	...	...	¼	⅞
⅜	....	....	....	½	5/16	⅜	⅜	¼	¾	5/16	1
½	⅞	1 1/16	....	9/16	5/16	⅜	½	¼	15/16	⅜	1¼
¾	⅞	1¼	1½	⅝	⅜	7/16	½	¼	1⅛	⅜	1½
1	....	....	....	¾	7/16	11/16	½	⅜	1 7/16	7/16	1⅞
1¼	1¼	2	2¼	⅞	½	¾	¾	⅜	1¾	½	2⅛
1½	1⅜	2	2½	⅞	½	13/16	13/16	⅜	1⅞	½	2⅜
2	1½	2¾	3	1	9/16	1 1/16	⅞	⅜	2⅜	½	3
2½	1⅝	3¼	3½	1	⅝	1 3/16	15/16	7/16	3	¾	4⅛
3	1⅞	4	4¼	1⅛	⅝	1 5/16	1	½	3½	¾	4½
3½	2	4½	5	1¼	¾	1½	1¼	½	4½	1	5
4	2	5⅛	5⅝	1⅜	¾	1¾	1¼	½	5⅛	1	6¾
4½	2¼	6	6½	1⅜	13/16	2	1¼	9/16	5¼	1	7
5	2⅜	6½	7½	1½	13/16	2¼	1¼	9/16	6¼	1	7½
6	2½	7½	8½	1½	1	2½	1¼	¾	7½	1	8¼
7	3	9⅜	10⅜	1½	1	3	1½	1	8½	1	10
8	3	9¾	10¾	1½	1¼	3½	1½	1¼	9½	1	11¼
10	....	....	....	1⅝	1⅜	4⅜	1⅝	1¼	11	...	...
12	....	....	....	2	1½	5¼	...	...	...	...	...

Threads of caps, plugs and bushings conform to standard pipe threads of the same nominal size. See table, Briggs Pipe Thread and Gage Dimensions in section on Screw Thread systems. Lock-nuts have straight threads.

The Tensile Strength of Non-ferrous Piping. — The strength should be as follows: Brass piping 7000 pounds per square inch; copper piping, 6000 pounds per square inch; Benedict nickel piping, 14,000 pounds per square inch; and Monel metal piping, 20,000 pounds per square inch. Brass and copper pipe should not be tested, however, beyond 1000 pounds per square inch, and Benedict and Monel metal pipes should not be tested beyond 2000 pounds per square inch.

Experiments carried out with compressed seamless zinc tubes are said to have proved that in point of strength and elasticity they are approximately equal to copper and brass tubes. Owing to the fact that they are not chemically affected by naphtha or petroleum, they may be found useful in the petroleum refining industry.

Nipples and Couplings. — Nipples are short pieces of standard pipe threaded on each end. When the threads run together at the center, the term "close nipple" is used; if a small amount of unthreaded surface is left in the center, the name used is "short nipple." Longer nipples are classified as "long" and "extra long," the latter varying from 4 to 12 inches, the length increasing by even inches. Nipples are either threaded right-hand or right- and left-hand. Couplings are threaded internally for receiving the ends of pipes. They are also threaded either with right-hand or with right- and left-hand threads. "Wrought couplings" are commonly used for "wrought pipe." For dimensions see the accompanying table.

Unions. — Unions are generally classified under two headings, nut unions and flange unions. Nut unions are ordinarily used for 2-inch sizes and smaller, and flange unions for sizes larger than 2 inches. Nut unions are made of malleable iron, brass and malleable iron and all brass. The all malleable-iron union is the standard malleable union of the trade and requires a gasket. The brass and malleable-iron union (known as the "Kewanee") requires no gasket and is non-corrosive. The pipe or "thread end" having an external thread upon which the nut or ring screws is made of brass, and the other pipe end (called the bottom) and the nut or ring, are made of malleable iron. When the union is tightened, the harder iron makes a joint in the softer brass. All-brass unions have a circular or conical seat and no gaskets are required. Flange unions are made of cast iron or malleable iron in three weights, standard, extra heavy and hydraulic.

Dimensions of Couplings and Nipples

Couplings					Nipples						
Nom. Inside Diam. Pipe	Coupling			Threads per Inch	Pipe Size	Length in Inches					
	Inside Diam.	Outside Diam.	Length			Close	Short	Long			
1/8	11/32	19/32	13/16	27	1/8	3/4	1½	2	2½	3	3½
1/4	15/32	23/32	15/16	18	1/4	7/8	1½	2	2½	3	3½
3/8	37/64	27/32	1 1/16	18	3/8	1	1½	2	2½	3	3½
1/2	23/32	1	1 5/16	14	1/2	1⅛	1½	2	2½	3	3½
3/4	63/64	1 21/64	1 9/16	14	3/4	1⅜	2	2½	3	3½	4
1	1 11/64	1 9/16	1 13/16	11½	1	1½	2	2½	3	3½	4
1¼	1½	1 61/64	2⅛	11½	1¼	1⅝	2½	3	3½	4	4½
1½	1¾	2 7/32	2⅜	11½	1½	1¾	2½	3	3½	4	4½
2	2 7/32	2¾	2⅝	11½	2	1¾	2½	3	3½	4	4½
2½	2 21/32	3 9/32	2⅞	8	2½	2½	3	3½	4	4½	5
3	3¼	3 15/16	3⅛	8	3	2½	3	3½	4	4½	5
3½	3 25/32	4 7/16	3⅝	8	3½	2¾	4	4½	5	5½	6
4	4 17/64	5	3⅝	8	4	3	4	4½	5	5½	6
4½	4¾	5½	3⅝	8	4½	3	4	4½	5	5½	6
5	5 9/32	6 7/32	4⅛	8	5	3½	4½	5	5½	6	6½
6	6 11/32	7 5/16	4⅛	8	6	3½	4½	5	5½	6	6½
7	7⅜	8 5/16	4⅛	8	7	3½	5	...	...	...	6
8	8⅜	9 5/16	4⅝	8	8	3½	5	...	...	...	6
9	9 7/16	10⅜	5⅛	8	9	4	5	...	...	6	8
10	10 7/16	11 21/32	6⅛	8	10	4	5	...	...	6	8
11	11 15/32	12 21/32	6⅛	8	11	4	5	...	...	6	8
12	12 7/16	13⅞	6⅛	8	12	4	5	...	...	6	8
13	13 11/16	15 1/16	6⅛	8							
14	14 23/32	16⅜	6⅛	8							
15	15 11/16	17⅜	6⅛	8							

Extra-long nipples can be obtained in lengths up to 12 inches, varying by inches.

Pitch of Bolts for Water and Steam Joints

(U. S. Navy Standard)

Thickness of Flange for Through Bolts equals 1¼ to 1½ Bolt Diameters.
Thickness of Flange for Through Studs equals 1½ to 2 Stud Diameters.
Width of Flange equals 2¾ to 3½ Bolt or Stud Diameters.

Diameter of Bolt	Pressure in Pounds per Square Inch						Diameter of Bolt	Pressure in Pounds per Square Inch					
	0 to 50	50 to 90	90 to 125	125 to 150	150 to 175	175 to 200		0 to 50	50 to 90	90 to 125	125 to 150	150 to 175	175 to 200
½	3½	3	2⅝	2¼	2	1¾	1¼	8¾	7½	6½	5⅝	5	4⅜
⅝	4⅜	3¾	3¼	2¾	2½	2¼	1⅜	9⅝	8¼	7¼	6¼	5½	4⅞
¾	5¼	4½	4	3½	3	2¾	1½	10½	9	8	7	6	5¼
⅞	6⅛	5¼	4½	4	3½	3⅛	1⅝	11⅜	9¾	8½	7½	6½	5¾
1	7	6	5¼	4½	4	3½	1¾	12¼	10½	9	8	7	6¼
1⅛	7⅞	6¾	5⅞	5	4½	4	2	14	12	10½	9	8	7

Pitch of bolts is determined as follows: For pressures from 0 to 50 pounds, pitch = bolt diameter × 7; pressures from 50 to 90 pounds, pitch = bolt diameter × 6; from 90 to 125 pounds, pitch = bolt diameter × 5¼; from 125 to 150 pounds, pitch = bolt diameter × 4½; from 150 to 175 pounds, pitch = bolt diameter × 4; from 175 to 200 pounds, pitch = bolt diameter × 3½.

Linear Expansion of Steam Pipes*

(Increase of Length, in Inches, per 100 Feet)

Temp. Increase in Degrees Fahr.	Cast Iron	Wrought Iron	Steel	Brass and Copper	Temp. Increase in Degrees Fahr.	Cast Iron	Wrought Iron	Steel	Brass and Copper
50	0.36	0.40	0.38	0.57	450	3.89	4.28	4.08	6.18
100	0.72	0.79	0.76	1.14	475	4.20	4.62	4.41	6.68
125	0.88	0.97	0.92	1.40	500	4.45	4.90	4.67	7.06
150	1.10	1.21	1.15	1.75	525	4.75	5.22	4.99	7.55
175	1.28	1.41	1.34	2.04	550	5.05	5.55	5.30	8.03
200	1.50	1.65	1.57	2.38	575	5.36	5.90	5.63	8.52
225	1.70	1.87	1.78	2.70	600	5.70	6.26	5.98	9.06
250	1.90	2.09	1.99	3.02	625	6.05	6.65	6.35	9.62
275	2.15	2.36	2.26	3.42	650	6.40	7.05	6.71	10.18
300	2.35	2.58	2.47	3.74	675	6.78	7.46	7.12	10.78
325	2.60	2.86	2.73	4.13	700	7.15	7.86	7.50	11.37
350	2.80	3.08	2.94	4.45	725	7.58	8.33	7.96	12.06
375	3.15	3.46	3.31	5.01	750	7.96	8.75	8.36	12.66
400	3.30	3.63	3.46	5.24	775	8.42	9.26	8.84	13.38
425	3.68	4.05	3.86	5.85	800	8.87	9.76	9.31	14.10

* The expansion, for any length, between two temperatures, is found by dividing the difference in length at these temperatures by 100, and multiplying by length of pipe in feet.

Inside Diameter of Shelby Standard Cold-drawn Tubing

Figures in Body of Table give Inside Diameter in Inches

Outside Diameter, Inches	Thickness in Gage and Fractions of an Inch: 22 B.W.G.	20 B.W.G.	18 B.W.G.	1/16	3/32	1/8	5/32	3/16	7/32	1/4	5/16	3/8	1/2	5/8	3/4	7/8	1
1/2	0.444	0.430	0.402	0.375	0.3125	0.250	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
5/8	0.569	0.555	0.527	0.500	0.4375	0.375	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
3/4	0.694	0.680	0.652	0.625	0.5625	0.500	0.4375	0.375	.....	.....	.....	.....	.....	.....	.....	.....	.....
7/8	0.819	0.805	0.777	0.750	0.6875	0.625	0.5625	0.500	0.4375	.....	.....	.....	.....	.....	.....	.....	.....
1	0.944	0.930	0.902	0.875	0.8125	0.750	0.6875	0.625	0.5625	0.500	.....	.....	.....	.....	.....	.....	.....
1 1/8	1.069	1.055	1.027	1.000	0.9375	0.875	0.8125	0.750	0.6875	0.625	.....	.....	.....	.....	.....	.....	.....
1 1/4	1.194	1.180	1.152	1.125	1.0625	1.000	0.9375	0.875	0.8125	0.750	0.625	0.500	.....	.....	.....	.....	.....
1 3/8	......	1.305	1.277	1.250	1.1875	1.125	1.0625	1.000	0.9375	0.875	0.750	0.625	.....	.....	.....	.....	.....
1 1/2	......	1.430	1.402	1.375	1.3125	1.250	1.1875	1.125	1.0625	1.000	0.875	0.750	0.500	.....	.....	.....	.....
1 3/4	......	......	......	1.625	1.5625	1.500	1.4375	1.375	1.3125	1.250	1.125	1.000	0.750	.....	.....	.....	.....
2	......	......	......	1.875	1.8125	1.750	1.6875	1.625	1.5625	1.500	1.375	1.250	1.000	0.750	.....	.....	.....
2 1/4	......	......	......	2.125	2.0625	2.000	1.9375	1.875	1.8125	1.750	1.625	1.500	1.250	1.000	.....	.....	.....
2 1/2	......	......	......	2.375	2.3125	2.250	2.1875	2.125	2.0625	2.000	1.875	1.750	1.500	1.250	.....	.....	.....
2 3/4	......	......	......	2.625	2.5625	2.500	2.4375	2.375	2.3125	2.250	2.125	2.000	1.750	1.500	....	.....	.....
3	......	......	......	.....	2.8125	2.750	2.6875	2.625	2.5625	2.500	2.375	2.250	2.000	1.750	1.500	1.250	1.000
3 1/4	......	......	......	.....	3.0625	3.000	2.9375	2.875	2.8125	2.750	2.625	2.500	2.250	2.000	1.750	1.500	1.250
3 1/2	......	......	......	.....	3.3125	3.250	3.1875	3.125	3.0625	3.000	2.875	2.750	2.500	2.250	2.000	1.750	1.500
3 3/4	......	......	......	.....	.....	3.500	3.4375	3.375	3.3125	3.250	3.125	3.000	2.750	2.500	2.250	2.000	1.750
4	......	......	......	.....	.....	3.750	3.6875	3.625	3.5625	3.500	3.375	3.250	3.000	2.750	2.500	2.250	2.000
4 1/4	......	......	......	.....	.....	.....	3.9375	3.875	3.8125	3.750	3.625	3.500	3.250	3.000	2.750	2.500	2.250
4 1/2	......	......	......	.....	.....	.....	4.1875	4.125	4.0625	4.000	3.875	3.750	3.500	3.250	3.000	2.750	2.500
4 3/4	......	......	......	.....	.....	.....	4.4375	4.375	4.3125	4.250	4.125	4.000	3.750	3.500	3.250	3.000	2.750
5	......	......	......	.....	.....	.....	4.6875	4.625	4.5625	4.500	4.375	4.250	4.000	3.750	3.500	3.250	3.000
5 1/2	......	......	......	.....	.....	.....	5.1875	5.125	5.0625	5.000	4.875	4.750	4.500	4.250	4.000	3.750	3.500

Dimensions of Lead Pipe

Tensile strength of lead, 2240 pounds per square inch. Safe working pressures are based on a factor of safety of 5.

Inside Diameter, Inches	Outside Diameter, Inches	Thickness, Inches	Weight per Foot		Approximate Bursting Pressure in Pounds per Square Inch	Safe Working Pressure in Pounds per Square Inch	Commercial Designations	
			Lbs.	Oz.			Lead Pipe Manufacturers	Plumbing Supplies Catalogues*
3/8	0.74	0.183	1	12	2150	430	AAA	D. E. S.
3/8	0.72	0.173	1	8	2000	400	AA	E. S.
3/8	0.66	0.143	1	4	1650	330	A	Strong
3/8	0.63	0.128	1	0	1500	300	B	Medium
3/8	0.58	0.103	0	12	1250	250	C	Light
3/8	0.55	0.088	0	10	1050	210	D	E. L.
3/8	0.51	0.068	0	7	800	160	E	Aqueduct
7/16	0.66	0.111	1	0	1165	230	B	Medium
7/16	0.63	0.096	0	13	1000	200	C	Light
1/2	1.01	0.255	3	0	2000	400	AAA	D. E. S
1/2	0.87	0.185	2	0	1600	320	AA	E. S.
1/2	0.84	0.170	1	12	1500	300	A	Strong
1/2	0.76	0.130	1	4	1150	230	B	Medium
1/2	0.71	0.105	1	0	900	180	C	Light
1/2	0.67	0.085	0	12	700	140	D	E. L.
1/2	0.63	0.065	0	9	550	110	E	Aqueduct
5/8	1.13	0.253	3	8	1700	340	AAA	D. E. S.
5/8	1.05	0.213	2	12	1500	300	AA	E. S.
5/8	1.02	0.198	2	8	1300	260	A	Strong
5/8	0.96	0.167	2	0	1150	230	B	Medium
5/8	0.88	0.128	1	8	900	180	C	Light
5/8	0.80	0.088	1	0	600	120	D	E. L.
5/8	0.77	0.073	0	12	500	100	E	Aqueduct
3/4	1.31	0.280	4	12	1900	380	AAA	D. E. S.
3/4	1.21	0.230	3	8	1400	280	AA	E. S.
3/4	1.16	0.205	3	0	1150	230	A	Strong
3/4	1.07	0.160	2	4	950	190	B	Medium
3/4	1.01	0.130	1	12	750	150	C	Light
3/4	0.94	0.095	1	4	550	110	D	E. L.
3/4	0.91	0.080	1	0	450	90	E	Aqueduct
1	1.59	0.295	6	0	1300	260	AAA	D. E. S.
1	1.51	0.255	4	12	1100	220	AA	E. S.
1	1.42	0.210	4	0	900	180	A	Strong
1	1.36	0.180	3	4	775	155	B	Medium
1	1.28	0.140	2	8	600	120	C	Light
1	1.23	0.115	2	0	500	100	D	E. L.
1	1.17	0.085	1	8	400	80	E	Aqueduct

* D. E. S. = double extra strong; E. S. = extra strong; E. L. = extra light.

Dimensions of Lead Pipe (*Continued*)

Inside Diameter, Inches	Outside Diameter, Inches	Thickness, Inches	Weight per Foot		Approximate Bursting Pressure in Pounds per Square Inch	Safe Working Pressure in Pounds per Square Inch	Commercial Designations	
			Lbs.	Oz.			Lead Pipe Manufacturers	Plumbing Supplies Catalogues*
1¼	1.82	0.285	6	12	1000	200	AAA	D. E. S.
1¼	1.75	0.250	5	12	900	180	AA	E. S.
1¼	1.67	0.210	4	12	750	150	A	Strong
1¼	1.59	0.170	3	12	600	120	B	Medium
1¼	1.52	0.135	3	0	475	95	C	Light
1¼	1.50	0.125	2	8	450	90	D	E. L.
1¼	1.45	0.100	2	0	350	70	E	Aqueduct
1½	2.11	0.305	8	8	900	180	AAA	D. E. S.
1½	2.04	0.270	7	8	800	160	AA	E. S.
1½	1.96	0.230	6	8	700	140	A	Strong
1½	1.88	0.190	5	0	550	110	B	Medium
1½	1.82	0.160	4	4	475	95	C	Light
1½	1.78	0.140	3	8	400	80	D	E. L.
1½	1.75	0.125	3	0	350	70	E	Aqueduct
1¾	2.42	0.335	10	0	850	170	AAA	D. E. S.
1¾	2.26	0.255	8	8	650	130	AA	E. S.
1¾	2.21	0.230	7	0	600	120	A	Strong
1¾	2.15	0.200	6	0	500	100	B	Medium
1¾	2.09	0.170	5	0	450	90	C	Light
1¾	2.03	0.140	4	0	375	75	D	E. L.
2	2.59	0.295	11	12	650	130	AAA	D. E. S.
2	2.51	0.255	9	0	550	110	AA	E. S.
2	2.45	0.225	8	0	500	100	A	Strong
2	2.41	0.205	7	0	450	90	B	Medium
2	2.37	0.185	6	0	375	75	C	Light
2	2.26	0.130	4	12	275	55	D	E. L.

* D. E. S. = double extra strong; E. S. = extra strong; E. L. = extra light.

Brass and Copper Tubes. — Seamless drawn brass and copper tubes are made in sizes varying from ¼ to 8 or 10 inches in diameter. The sizes do not correspond with any universal standard, but usually increase by ⅛-inch increments up to 3 inches, and by ¼-inch increments for larger diameters. The nominal diameter of tubes may be either the inside or outside diameter, brass and copper tubes being made to conform with both methods of measurement. The term diameter, as applied to tubes, however, is generally understood to mean outside diameter. The thickness of the tube walls conforms to Stub's wire gage, and a given diameter (except small sizes below ¾ or 1 inch) can be obtained with any thickness of wall from a No. 1 gage (0.300 inch) to a No. 20 (0.035 inch) or No. 25 (0.020 inch) gage. Tubes ¼ inch in diameter are made in thicknesses up to about No. 13 gage (0.095 inch), and the range increases for larger sizes. Stub's gage is the standard for seamless drawn brass and copper tubing, but some special tubing is made to conform to the Brown & Sharpe gage. When ordering, state whether the diameter referred to is inside or outside.

Sizes and Weights in Pounds per Foot of Seamless Brass Tubes *

Outside Diam. of Tube, Inches	Thickness — Stub's or Birmingham Gage											
	3	4	5	6	7	8	9	10	11	12	13	14
	Decimal Equivalent of Gage Number, Inch											
	0.259	0.238	0.220	0.203	0.180	0.165	0.148	0.134	0.120	0.109	0.095	0.083
1/8	....	....	....	....	....	....	....	....	....	....	....	....
3/16	....	....	....	....	....	....	....	....	....	....	....	....
1/4	....	....	....	....	....	....	....	....	0.18	0.177	0.170	0.160
5/16	....	....	....	....	....	....	....	....	0.27	0.256	0.238	0.220
3/8	....	....	....	....	....	0.40	0.39	0.37	0.35	0.335	0.307	0.280
7/16	....	....	..	....	....	0.52	0.49	0.47	0.44	0.413	0.376	0.340
1/2	....	....	....	0.70	0.66	0.64	0.60	0.57	0.53	0.492	0.444	0.400
9/16	....	....	....	0.84	0.79	0.76	0.71	0.66	0.61	0.571	0.513	0.460
5/8	1.09	1.06	1.03	0.99	0.92	0.88	0.81	0.76	0.70	0.649	0.581	0.520
11/16	1.28	1.23	1.19	1.13	1.05	0.99	0.92	0.86	0.79	0.728	0.650	0.580
3/4	1.47	1.41	1.35	1.28	1.18	1.11	1.03	0.95	0.87	0.807	0.718	0.640
13/16	1.65	1.58	1.50	1.43	1.31	1.23	1.13	1.05	0.96	0.885	0.787	0.700
7/8	1.84	1.75	1.66	1.57	1.44	1.35	1.24	1.15	1.04	0.964	0.855	0.759
15/16	2.03	1.92	1.82	1.72	1.57	1.47	1.35	1.24	1.13	1.042	0.924	0.819
1	2.22	2.09	1.98	1.87	1.70	1.59	1.45	1.34	1.22	1.12	0.99	0.88
1 1/8	2.60	2.44	2.30	2.16	1.96	1.83	1.67	1.53	1.39	1.28	1.13	1.00
1 1/4	2.97	2.78	2.61	2.45	2.22	2.07	1.88	1.73	1.56	1.44	1.27	1.12
1 3/8	3.35	3.12	2.93	2.75	2.48	2.30	2.10	1.92	1.74	1.59	1.40	1.24
1 1/2	3.72	3.47	3.25	3.04	2.74	2.54	2.31	2.11	1.91	1.75	1.54	1.36
1 5/8	4.09	3.81	3.57	3.33	3.00	2.78	2.52	2.31	2.08	1.91	1.68	1.48
1 3/4	4.47	4.15	3.88	3.62	3.26	3.02	2.74	2.50	2.26	2.06	1.82	1.60
1 7/8	4.84	4.50	4.20	3.92	3.52	3.26	2.95	2.69	2.43	2.22	1.95	1.72
2	5.21	4.84	4.52	4.21	3.78	3.50	3.16	2.89	2.60	2.38	2.09	1.84
2 1/8	5.59	5.18	4.84	4.50	4.04	3.73	3.38	3.08	2.78	2.54	2.23	1.96
2 1/4	5.96	5.53	5.15	4.80	4.30	3.97	3.59	3.27	2.95	2.69	2.36	2.08
2 3/8	6.34	5.87	5.47	5.09	4.56	4.21	3.80	3.47	3.12	2.85	2.50	2.20
2 1/2	6.71	6.21	5.79	5.38	4.82	4.45	4.02	3.66	3.30	3.01	2.64	2.32
2 5/8	7.08	6.56	6.11	5.67	5.08	4.69	4.23	3.85	3.47	3.17	2.77	2.44
2 3/4	7.46	6.90	6.42	5.97	5.34	4.92	4.44	4.05	3.64	3.32	2.91	2.56
2 7/8	7.83	7.24	6.74	6.26	5.60	5.16	4.66	4.24	3.81	3.48	3.05	2.68
3	8.20	7.59	7.06	6.55	5.86	5.40	4.87	4.43	3.99	3.64	3.19	2.79
3 1/8	8.58	7.93	7.38	6.85	6.12	5.64	5.08	4.63	4.16	3.79	3.32	2.91
3 1/4	8.95	8.27	7.69	7.14	6.38	5.88	5.30	4.82	4.33	3.95	3.46	3.03
3 3/8	9.33	8.62	8.01	7.43	6.64	6.11	5.51	5.01	4.51	4.11	3.60	3.15
3 1/2	9.70	8.96	8.33	7.72	6.90	6.35	5.72	5.21	4.68	4.27	3.73	3.27
3 5/8	10.07	9.30	8.65	8.02	7.16	6.59	5.94	5.40	4.85	4.42	3.87	3.39
3 3/4	10.45	9.65	8.96	8.31	7.42	6.83	6.15	5.59	5.03	4.58	4.01	3.51
3 7/8	10.82	9.99	9.28	8.60	7.68	7.07	6.37	5.79	5.20	4.74	4.15	3.63

To determine weight per foot of a tube of a given *inside diameter*, add to weights in above list the weights in pounds per foot given below under corresponding gage numbers.

Gage No.	3	4	5	6	7	8	9	10	11	12	13	14
Weight Added	1.549	1.308	1.117	0.951	0.748	0.628	0.506	0.414	0.332	0.274	0.208	0.159

* Bridgeport Brass Co.

Sizes and Weights in Pounds per Foot of Seamless Brass Tubes

Outside Diam. of Tube, Inches	Thickness — Stub's or Birmingham Gage										
	15	16	17	18	19	20	21	22	23	24	25
	Decimal Equivalent of Gage Number, Inch										
	0.072	0.065	0.058	0.049	0.042	0.035	0.032	0.028	0.025	0.022	0.020
1/8	....	0.045	0.045	0.043	0.040	0.036	0.034	0.031	0.029	0.026	0.024
3/16	0.096	0.092	0.087	0.078	0.070	0.062	0.057	0.051	0.047	0.042	0.039
1/4	0.148	0.139	0.129	0.114	0.101	0.087	0.080	0.072	0.065	0.058	0.053
5/16	0.200	0.186	0.170	0.149	0.131	0.112	0.104	0.092	0.083	0.074	0.067
3/8	0.252	0.233	0.212	0.184	0.161	0.137	0.127	0.112	0.101	0.090	0.082
7/16	0.304	0.279	0.254	0.220	0.192	0.163	0.150	0.132	0.119	0.106	0.096
1/2	0.356	0.326	0.296	0.255	0.222	0.188	0.173	0.152	0.137	0.121	0.111
9/16	0.408	0.373	0.338	0.290	0.252	0.213	0.196	0.173	0.155	0.137	0.125
5/8	0.460	0.420	0.380	0.326	0.283	0.238	0.219	0.193	0.173	0.153	0.140
11/16	0.511	0.467	0.421	0.361	0.313	0.264	0.242	0.213	0.191	0.169	0.154
3/4	0.563	0.514	0.463	0.396	0.343	0.289	0.265	0.233	0.209	0.185	0.169
13/16	0.615	0.561	0.505	0.432	0.373	0.314	0.288	0.253	0.227	0.201	0.183
7/8	0.667	0.608	0.547	0.467	0.404	0.339	0.311	0.274	0.245	0.217	0.197
15/16	0.719	0.655	0.589	0.502	0.434	0.365	0.334	0.294	0.263	0.232	0.211
1	0.77	0.70	0.63	0.54	0.46	0.389	0.358	0.314	0.281	0.248	0.226
1 1/8	0.87	0.79	0.71	0.61	0.52	0.439	0.404	0.354	0.317	0.280	0.255
1 1/4	0.98	0.89	0.80	0.68	0.59	0.490	0.450	0.395	0.354	0.312	0.284
1 3/8	1.08	0.98	0.88	0.75	0.65	0.540	0.496	0.435	0.390	0.343	0.313
1 1/2	1.19	1.08	0.96	0.82	0.71	0.591	0.542	0.476	0.426	0.375	0.342
1 5/8	1.29	1.17	1.05	0.89	0.77	0.641	0.588	0.516	0.462	0.407	0.371
1 3/4	1.39	1.26	1.13	0.96	0.83	0.692	0.635	0.556	0.498	0.439	0.399
1 7/8	1.50	1.36	1.22	1.03	0.89	0.742	0.681	0.597	0.534	0.470	0.428
2	1.60	1.45	1.30	1.10	0.95	0.793	0.727	0.637	0.570	0.502	0.457
2 1/8	1.71	1.55	1.38	1.17	1.01	0.843	0.773	0.678	0.606	0.534	0.486
2 1/4	1.81	1.64	1.47	1.24	1.07	0.894	0.819	0.718	0.642	0.566	0.515
2 3/8	1.91	1.73	1.55	1.32	1.13	0.944	0.866	0.758	0.678	0.597	0.544
2 1/2	2.02	1.83	1.63	1.39	1.19	0.995	0.912	0.799	0.714	0.629	0.573
2 5/8	2.12	1.92	1.72	1.46	1.25	1.045	0.958	0.839	0.750	0.661	.....
2 3/4	2.23	2.01	1.80	1.53	1.31	1.096	1.004	0.880	0.786	0.693	.....
2 7/8	2.33	2.11	1.89	1.60	1.37	1.146	1.050	0.920	0.822	0.724	.....
3	2.43	2.20	1.97	1.67	1.43	1.197	1.096	0.960	0.859	0.756	.....
3 1/8	2.54	2.30	2.05	1.74	1.49	1.247	1.143	1.001	0.895	0.788	.....
3 1/4	2.64	2.39	2.14	1.81	1.55	1.298	1.189	1.041	0.931	0.820	.....
3 3/8	2.74	2.48	2.22	1.88	1.62	1.348	1.235	1.082	0.967	0.851	.....
3 1/2	2.85	2.58	2.30	1.95	1.68	1.399	1.281	1.122	1.003	0.883	.....
3 5/8	2.95	2.67	2.39	2.02	1.74	1.449	1.327	1.162	1.039	0.915	.....
3 3/4	3.06	2.76	2.47	2.09	1.80	1.50	1.373	1.203	1.075	0.946	.....
3 7/8	3.16	2.86	2.56	2.16	1.86	1.55	1.42	1.243	1.111	0.978	.....

To determine weight per foot of a tube of a given *inside diameter*, add to weights in above list the weights in pounds per foot given below under corresponding gage numbers.

Gage No.	15	16	17	18	19	20	21	22	23	24	25
Weight Added	0.120	0.097	0.078	0.055	0.041	0.028	0.024	0.018	0.014	0.011	0.009

Sizes and Weights in Pounds per Foot of Seamless Brass Tubes

Outside Diam. of Tube, Inches	Thickness — Stub's or Birmingham Gage									
	3	4	5	6	7	8	9	10	11	12
	Decimal Equivalent of Gage Number, Inch									
	0.259	0.238	0.220	0.203	0.180	0.165	0.148	0.134	0.120	0.109
4	11.19	10.33	9.60	8.90	7.94	7.31	6.58	5.98	5.37	4.89
4⅛	11.57	10.68	9.91	9.19	8.20	7.54	6.79	6.17	5.55	5.05
4¼	11.94	11.02	10.23	9.48	8.46	7.78	7.01	6.37	5.72	5.21
4⅜	12.32	11.36	10.55	9.77	8.72	8.02	7.22	6.56	5.89	5.37
4½	12.69	11.71	10.87	10.07	8.98	8.26	7.43	6.75	6.06	5.52
4⅝	13.06	12.05	11.18	10.36	9.24	8.50	7.65	6.94	6.24	5.68
4¾	13.44	12.39	11.50	10.65	9.50	8.73	7.86	7.14	6.41	5.84
4⅞	13.81	12.74	11.82	10.95	9.76	8.97	8.07	7.33	6.58	6.00
5	14.18	13.08	12.14	11.24	10.02	9.21	8.29	7.53	6.76	6.15
5⅛	14.56	13.42	12.45	11.53	10.28	9.45	8.50	7.72	6.93	6.31
5¼	14.93	13.77	12.77	11.82	10.53	9.69	8.71	7.91	7.10	6.47
5⅜	15.31	14.11	13.09	12.12	10.79	9.92	8.93	8.11	7.28	6.62
5½	15.68	14.45	13.41	12.41	11.05	10.16	9.14	8.30	7.45	6.78
5⅝	16.05	14.80	13.72	12.70	11.31	10.40	9.35	8.49	7.62	6.94
5¾	16.43	15.14	14.04	13.00	11.57	10.64	9.57	8.69	7.80	7.10
5⅞	16.80	15.48	14.36	13.29	11.83	10.88	9.78	8.88	7.97	7.25
6	17.17	15.83	14.67	13.58	12.09	11.12	9.99	9.07	8.14	7.41
6⅛	17.55	16.17	14.99	13.87	12.35	11.35	10.21	9.27	8.32	7.57
6¼	17.92	16.51	15.31	14.17	12.61	11.59	10.42	9.46	8.49	7.72
6⅜	18.30	16.86	15.63	14.46	12.87	11.83	10.64	9.65	8.66	7.88
6½	18.67	17.20	15.94	14.75	13.13	12.07	10.85	9.85	8.84	8.04
6⅝	19.04	17.54	16.26	15.05	13.39	12.31	11.06	10.04	9.01	8.20
6¾	19.42	17.89	16.58	15.34	13.65	12.54	11.28	10.23	9.18	8.35
6⅞	19.79	18.23	16.90	15.63	13.91	12.78	11.49	10.43	9.35	8.51
7	20.16	18.57	17.21	15.92	14.17	13.02	11.70	10.62	9.53	8.67
7⅛	20.54	18.92	17.53	16.22	14.43	13.26	11.92	10.81	9.70	8.83
7¼	20.91	19.26	17.85	16.51	14.69	13.50	12.13	11.01	9.87	8.98
7⅜	21.29	19.60	18.17	16.80	14.95	13.73	12.34	11.20	10.05	9.14
7½	21.66	19.95	18.48	17.10	15.21	13.97	12.56	11.39	10.22	9.30
7⅝	22.03	20.29	18.80	17.39	15.47	14.21	12.77	11.59	10.39	9.45
7¾	22.41	20.64	19.12	17.68	15.73	14.45	12.98	11.78	10.57	9.61
7⅞	22.78	20.98	19.44	17.98	15.99	14.69	13.20	11.97	10.74	9.77
8	23.15	21.32	19.75	18.27	16.25	14.93	13.41	12.17	10.91	9.93

To determine weight per foot of a tube of a given *inside diameter*, add to weights in above list the weights in pounds per foot given below under corresponding gage numbers.

Gage No.	3	4	5	6	7	8	9	10	11	12
Weight Added	1.549	1.308	1.117	0.951	0.748	0.628	0.506	0.414	0.332	0.274

Sizes and Weights in Pounds per Foot of Seamless Brass Tubes

Outside Diam. of Tube, Inches	Thickness — Stub's or Birmingham Gage											
	13	14	15	16	17	18	19	20	21	22	23	24
	Decimal Equivalent of Gage Number, Inch											
	0.095	0.083	0.072	0.065	0.058	0.049	0.042	0.035	0.032	0.028	0.025	0.022
4	4.28	3.75	3.26	2.95	2.64	2.23	1.92	1.601	1.466	1.284	1.147	1.010
4⅛	4.42	3.87	3.37	3.05	2.72	2.30	1.98	1.651	1.512	1.324	1.183	....
4¼	4.56	3.99	3.47	3.14	2.81	2.38	2.04	1.702	1.558	1.364	1.219	....
4⅜	4.69	4.11	3.58	3.23	2.89	2.45	2.10	1.752	1.604	1.405	1.255	....
4½	4.83	4.23	3.68	3.33	2.97	2.52	2.16	1.803	1.650	1.445	1.291	....
4⅝	4.97	4.35	3.78	3.42	3.06	2.59	2.22	1.853	1.697	1.486	...	....
4¾	5.11	4.47	3.89	3.52	3.14	2.66	2.28	1.904	1.743	1.526	...	....
4⅞	5.24	4.59	3.99	3.61	3.22	2.73	2.34	1.954	1.789	1.566	...	....
5	5.38	4.71	4.09	3.70	3.31	2.80	2.40	2.005	1.835	1.607	...	....
5⅛	5.52	4.83	4.20	3.79	3.39	2.87	2.46	2.055	1.881	....	...	....
5¼	5.65	4.95	4.30	3.89	3.48	2.94	2.52	2.106	1.928	....	...	....
5⅜	5.79	5.07	4.41	3.98	3.56	3.01	2.58	2.156	1.974	....	...	....
5½	5.93	5.19	4.51	4.08	3.64	3.08	2.65	2.207	2.02	....	...	....
5⅝	6.07	5.31	4.61	4.17	3.73	3.15	2.71	2.257	....	....	...	....
5¾	6.20	5.43	4.72	4.26	3.81	3.22	2.77	2.308	....	....	...	....
5⅞	6.34	5.55	4.82	4.36	3.89	3.29	2.83	2.358	....	....	...	....
6	6.48	5.67	4.93	4.45	3.98	3.37	2.89	2.409	....	....	...	....
6⅛	6.61	5.79	5.03	4.54	4.06	3.44	....	....	....	....	...	....
6¼	6.75	5.91	5.13	4.64	4.15	3.51	....	....	....	....	...	....
6⅜	6.89	6.03	5.24	4.73	4.23	3.58	....	....	....	....	...	....
6½	7.03	6.15	5.34	4.83	4.31	3.65	....	....	....	....	...	....
6⅝	7.16	6.27	5.45	4.92	4.40	3.72	....	....	....	....	...	....
6¾	7.30	6.39	5.55	5.01	4.48	3.79	....	....	....	....	...	....
6⅞	7.44	6.51	5.65	5.11	4.56	3.86	....	....	....	....	...	....
7	7.57	6.63	5.76	5.20	4.65	3.93	....	....	....	....	...	....
7⅛	7.71	6.75	5.86	5.29	....	....	....	...	....	....	...	....
7¼	7.85	6.87	5.96	5.39	....	....	....	....	....	....	...	....
7⅜	7.99	6.99	6.07	5.48	....	....	....	....	....	....	...	....
7½	8.12	7.11	6.17	5.58	....	....	....	....	....	....	...	....
7⅝	8.26	7.23	6.28	5.67	....	....	....	....	....	....	...	....
7¾	8.40	7.35	6.38	5.76	....	....	....	....	....	....	...	....
7⅞	8.53	7.47	6.48	5.86	....	....	....	....	....	....	...	....
8	8.67	7.58	6.59	5.95	....	....	....	....	....	....	...	....

To determine weight per foot of a tube of a given *inside diameter*, add to weights in above list the weights in pounds per foot given below under corresponding gage numbers.

Gage No.	13	14	15	16	17	18	19	20	21	22	23	24
Weight Added	0.208	0.159	0.120	0.097	0.078	0.055	0.041	0.028	0.024	0.018	0.014	0.011

Seamless Drawn Brass and Copper Pipe

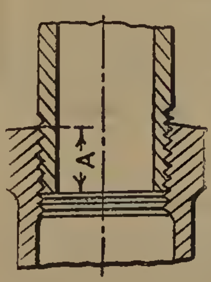
Made to correspond with iron pipe and to fit iron pipe fittings (American Tube Works).

Diameter			Approximate Weight per Foot, Pounds		Diameter			Approximate Weight per Foot, Pounds	
Iron Pipe Size	Approx. Outside Diam.	Exact Outside Diam.	Brass	Copper	Iron Pipe Size	Approx. Outside Diam.	Exact Outside Diam.	Brass	Copper
⅛	⅜	0.405	0.25	0.26	2½	2⅞	2.875	5.75	6.05
¼	9/16	0.540	0.43	0.45	3	3½	3.500	8.30	8.74
⅜	11/16	0.675	0.62	0.65	3½	4	4.000	10.90	11.47
½	13/16	0.840	0.90	0.95	4	4½	4.500	12.70	13.37
¾	1 1/16	1.050	1.25	1.32	4½	5	5.000	13.90	14.63
1	1 5/16	1.315	1.70	1.79	5	5 9/16	5.563	15.75	16.58
1¼	1⅝	1.660	2.50	2.63	6	6⅝	6.625	18.31	19.27
1½	1⅞	1.900	3.00	3.16	7	7⅝	7.625	23.73	24.98
2	2⅜	2.375	4.00	4.21	.....	.....	.....	....	....

Threading Pipe. — Clean, smooth pipe threads are essential to a good joint and depend largely upon the rake or lip angle and lead of the chasers, and the clearance, chip space and number of chasers in the die-head. The lip angle should vary from 15 to 25 degrees, depending upon the style and condition of the chasers and chaser holders. The chip space in front of the chasers should be large enough to allow room for accumulation of chips and at the same time provide means of

Length of Thread on Pipe Required to Make a Tight Joint

(Crane Co.)



Size of Pipe, Inches	Dimension A, Inches	Size of Pipe, Inches	Dimension A, Inches	Size of Pipe, Inches	Dimension A, Inches
⅛	¼	1½	⅝	5	1 3/16
¼	⅜	2	11/16	6	1¼
⅜	⅜	2½	15/16	7	1¼
½	½	3	1	8	1 5/16
¾	½	3½	1 1/16	9	1⅜
1	9/16	4	1 1/16	10	1½
1¼	⅝	4½	1⅛	12	1⅝

Dimensions do not allow for variation in tapping or threading.

lubricating the chasers. This is an important point, as insufficient chip space will cause the chips to clog and tear the threads. The lead of the chaser is the angle which is machined or ground on the leading or front side, to enable the die to start readily on the pipe, and also to distribute the work of cutting over a number of threads. To secure a good thread, the lead should cover the first three threads. As the heaviest cutting is done by this beveled part, it should have a slightly greater clearance angle than the rest of the threads on the chaser. When re-grinding chasers which have become dull on the lead, care should be taken to give each chaser the same length of lead, as otherwise the work will be unevenly distributed.

The number of chasers with which a die should be equipped depends upon the size of the die. The number recommended for different sizes is as follows:

Size of Die	Number of Chasers	Size of Die	Number of Chasers
Up to 1¼ inch	4	10 to 12 inches	12
1¼ to 4 inches	6	12 to 14 inches	14
4 to 7 inches	8	14 to 18 inches	16
7 to 10 inches	10	18 to 20 inches	18

Pipe threading dies should be lubricated with a good quality of lard oil or crude cotton-seed oil, the lubricant being used in liberal quantities.

Pipe Bending. — When it is necessary to bend pipes by hand without the use of special tools, this can be done by the following method: Completely fill the pipe with dry sand and block or cap the ends so that the filling will be retained. Heat the part to be bent; the heating should be restricted to this part, and overheating should be avoided. It is usually necessary to heat two or three times when making a bend of small radius, so that nothing is gained by heating to a high temperature. Clamp the pipe in a vise as close to the location of the bend as possible

Radii for Wrought Pipe Bends*

Pipe Size, Inches	Advisable Radius, Inches	Minimum Radius, Inches	Pipe Size, Inches	Advisable Radius, Inches	Minimum Radius, Inches	Pipe Size, Inches	Advisable Radius, Inches	Minimum Radius, Inches
2½	15	10	7	42	28	14	90	68
3	18	12	8	48	32	15	100	76
3½	21	14	9	54	36	18 O.D.	125	90
4	24	16	10	60	40	20 O.D.	150	120
4½	27	18	11	66	44	22 O.D.	165	132
5	30	20	12	72	48	24 O.D.	180	144
6	36	24	13	84	60	....	.....	......

* The radii given in above table apply to pipe of standard thickness. The minimum radii are as short as should be used to secure good results.

without gripping the heated part. Then cool the outside of the curve by water, so that the inside, being hot and plastic, is compressed as the bend is made; by forcing the bend to take place on the inside, the pipe walls are better supported by the filling material, because the cubic contents are slightly reduced by the compression; if the outside of the curve were allowed to stretch, the cubic contents would be slightly increased, which would result in a lack of support for the interior of the pipe. The use of water also plays an important part in the formation of the curve, the pipe being cooled wherever the required curvature has been obtained. The use of water for cooling the outside of the curve can usually be dispensed with when the radius exceeds 15 times the pipe diameter.

When bending butt-welded iron pipe, the seam should be, as a rule, on the side and not on the inside and outside of the curve, to lessen the danger of cracking. When heating must be avoided on account of the finish, or for other reasons, a filling is ordinarily used that can be melted at a low temperature, such as rosin for copper and brass, or a metal alloy having a low melting point, for small iron and steel pipes. Brass or copper pipe should be annealed before bending.

Relative Discharging Capacities of Pipes

Actual Internal Diam., Inches	0.269	0.364	0.493	0.622	0.824	1.049	1.380	1.610	2.067	2.469	3.068	3.548	4.026	4.506	5.047	6.065	7.023	7.981
Nom. Internal Diam.	**1/8**	**1/4**	**3/8**	**1/2**	**3/4**	**1**	**1 1/4**	**1 1/2**	**2**	**2 1/2**	**3**	**3 1/2**	**4**	**4 1/2**	**5**	**6**	**7**	**8**
1/4	2.1	1	....	....	....	....	....	....	....	....	...							
3/8	4.5	2.1	1	....	....	....	....	....	....	....	...							
1/2	8	3.8	1.8	1	....	....	....	....	....	....	...							
3/4	16	8	3.6	2	1	....	....	....	....	....	...							
1	30	14	6.6	3.7	1.8	1	....	....	....	....	...							
1 1/4	60	28	13	7	3.6	2	1	....	....	....	...							
1 1/2	88	41	19	11	5.3	2.9	1.5	1	....	....	...	...	...	...	...	...	...	...
2	164	77	36	20	10	5.5	2.7	1.9	1	....	...	...	...	...	...	...	...	...
2 1/2	255	120	56	31	16	8	4.3	2.9	1.6	1	...	...	...	...	...	...	...	...
3	439	206	97	54	27	15	7	5	2.7	1.7	1	...	...	...	...	...	...	...
3 1/2	632	297	139	78	38	21	11	7	3.9	2.5	1.4	1	...	...	...	...	...	...
4	867	407	191	107	53	29	15	10	5.3	3.4	2.0	1.4	1	...	...	...	...	...
4 1/2	1148	539	253	141	70	38	19	13	7	4.5	2.6	1.8	1.3	1	...	...	...	...
5	1525	716	335	188	93	51	26	17	9	6	3.5	2.4	1.8	1.3	1	...	...	...
6	2414	1133	531	297	147	80	40	28	15	9	5.5	3.8	2.8	2.1	1.6	1	...	...
7	3483	1635	766	428	212	116	58	40	21	14	8.0	5.5	4.0	3.0	2.3	1.4	1	...
8	4795	2251	1054	590	292	160	80	55	29	19	10.9	7.6	5.5	4.2	3.1	2.0	1.3	1
9	6369	2990	1401	783	388	212	107	73	39	25	14	10	7.3	5.5	4.2	2.6	1.8	1.3
10	8468	3976	1862	1042	516	282	142	97	52	33	19	13	10	7.4	5.6	3.5	2.4	1.8
11	10693	5020	2352	1315	651	356	179	122	65	42	24	17	12	9.3	7.0	4.4	3.0	2.2
12	13292	6240	2923	1635	809	443	223	152	81	52	30	21	15	12.0	8.7	5.5	3.8	2.8
13	17028	7994	3745	2094	1037	567	286	194	104	67	39	27	20	15.0	11.0	7.0	4.9	3.6
14	20425	9589	4492	2512	1244	680	343	233	125	80	46	32	24	18.0	13.0	8.5	5.9	4.3
15	24199	11361	5322	2976	1474	806	406	276	148	95	55	38	28	21.0	16.0	10.0	6.9	5.0

Figures in body of table represent number of small pipes having a discharging capacity equivalent to one large pipe of given diameter.

Definitions of Pipe Fittings

The following definitions for various pipe fittings are given by the National Tube Co.:

Armstrong Joint. — A two-bolt, flanged or lugged connection for high pressures. The ends of the pipes are peculiarly formed to properly hold a gutta-percha ring. It was originally made for cast-iron pipe. The two-bolt feature has much to commend it. There are various substitutes for this joint, many of which employ rubber in place of gutta-percha; others use more bolts in order to reduce the cost.

Bell and Spigot Joint. — (1) The usual term for the joint in cast-iron pipe. Each piece is made with an enlarged diameter or bell at one end into which the plain or spigot end of another piece is inserted when laying. The joint is then made tight by cement, oakum, lead, rubber or other suitable substance, which is driven in or calked into the bell and around the spigot. When a similar joint is made in wrought pipe by means of a cast bell (or hub), it is at times called hub and spigot joint (poor usage). Matheson joint is the name applied to a similar joint in wrought pipe which has the bell formed from the pipe. (2) Applied to fittings or valves, means that one end of the run is a "bell," and the other end is a "spigot," similar to those used on regular cast-iron pipe.

Bonnet. — (1) A cover used to guide and enclose the tail end of a valve spindle. (2) A cap over the end of a pipe (poor usage).

Branch. — The outlet or inlet of a fitting not in line with the run, but which may make any angle.

Branch Ell. — (1) Used to designate an elbow having a back outlet in line with one of the outlets of the "run." It is also called a heel outlet elbow. (2) Incorrectly used to designate side outlet or back outlet elbow.

Branch Pipe. — A very general term used to signify a pipe either cast or wrought, that is equipped with one or more branches. Such pipes are used so frequently that they have acquired common names such as tees, crosses, side or back outlet elbows, manifolds, double-branch elbows, etc. The term branch pipe is generally restricted to such as do not conform to usual dimensions.

Branch Tee (Header). — A tee having many side branches. (See Manifold.)

Bull Head Tee. — A tee the branch of which is larger than the run.

Bushing. — A pipe fitting for the purpose of connecting a pipe with a fitting of larger size, being a hollow plug with internal and external threads to suit the different diameters.

Card Weight Pipe. — A term used to designate standard or full weight pipe, which is the Briggs' standard thickness of pipe.

Close Nipple. — One the length of which is about twice the length of a standard pipe thread and is without any shoulder.

Coupling. — A threaded sleeve used to connect two pipes. Commercial couplings are threaded inside to suit the exterior thread of the pipe. The term coupling is occasionally used to mean any jointing device and may be applied to either straight or reducing sizes.

Cross. — A pipe fitting with four branches arranged in pairs, each pair on one axis, and the axes at right angles. When the outlets are otherwise arranged the fittings are branch pipes or specials.

Cross-over. — A small fitting with a double offset, or shaped like the letter *U* with the ends turned out. It is only made in small sizes and used to pass the flow of one pipe past another when the pipes are in the same plane.

Cross-over Tee. — A fitting made along lines similar to the cross-over, but having at one end two openings in a tee-head the plane of which is at right angles to the plane of the cross-over bend.

Cross Valve. — (1) A valve fitted on a transverse pipe so as to open communication at will between two parallel lines of piping. Much used in connection with oil and water pumping arrangements, especially on ship board. (2) Usually considered as an angle valve with a back outlet in the same plane as the other two openings.

Crotch. — A fitting that has the general shape of the letter *Y*. Caution should be exercised not to confuse the crotch and wye.

Double-branch Elbow. — A fitting that, in a manner, looks like a tee, or as if two elbows had been shaved and then placed together, forming a shape something like the letter *Y* or a crotch.

Double Sweep Tee. — A tee made with easy curves between body and branch, *i.e.*, the center of the curve between run and branch lies outside the body.

Drop Elbow. — A small sized ell that is frequently used where gas is put into a building. These fittings have wings cast on each side. The wings have small countersunk holes so that they may be fastened by wood screws to a ceiling or wall or framing timbers.

Drop Tee. — One having the same peculiar wings as the drop elbow.

Dry Joint. — One made without gasket or packing or smear of any kind, as a ground joint.

Elbow (Ell). — A fitting that makes an angle between adjacent pipes. The angle is always 90 degrees, unless another angle is stated. (See Branch, Service, and Union Ell.)

Extra Heavy. — When applied to pipe, means pipe thicker than standard pipe; when applied to valves and fittings, indicates goods suitable for a working pressure of 250 pounds per square inch.

Header. — A large pipe into which one set of boilers is connected by suitable nozzles or tees, or similar large pipes from which a number of smaller ones lead to consuming points. Headers are often used for other purposes — for heaters or in refrigeration work. Headers are essentially branch pipes with many outlets, which are usually parallel. Largely used for tubes of water-tube boilers.

Hydrostatic Joint. — Used in large water mains, in which sheet lead is forced tightly into the bell of a pipe by means of the hydrostatic pressure of a liquid.

Kewanee Union. — A patented pipe union having one pipe end of brass and the other of malleable iron, with a ring or nut of malleable iron, in which the arrangement and finish of the several parts is such as to provide a non-corrosive ball-and-socket joint at the junction of the pipe ends, and a non-corrosive connection between the ring and brass pipe end.

Lead Joint. — (1) Generally used to signify the connection between pipes which is made by pouring molten lead into the annular space between a bell and spigot, and then making the lead tight by calking. (2) Rarely used to mean the joint made by pressing the lead between adjacent pieces, as when a lead gasket is used between flanges.

Lead Wool. — A material used in place of molten lead for making pipe joints. It is lead fiber, about as coarse as fine excelsior, and when made in a strand, it can be calked into the joints, making them very solid.

Line Pipe. — Special brand of pipe that employs recessed and taper thread couplings, and usually greater length of thread than Briggs' standard. The pipe is also subjected to higher test.

Lip Union. — (1) A special form of union characterized by the lip that prevents the gasket from being squeezed into the pipe so as to obstruct the flow. (2) A ring union, unless flange is specified.

Manifold. — (1) A fitting with numerous branches used to convey fluids between a large pipe and several smaller pipes. (See Branch Tee.) (2) A header for a coil.

Matheson Joint. — A wrought pipe joint made by enlarging one end of the pipe to form a suitable lead recess, similar to the bell end of a cast-iron pipe, and which receives the male or spigot end of the next length. Practically the same style of a joint as used for cast-iron pipe.

Medium Pressure. — When applied to valves and fittings, means suitable for a working pressure of from 125 to 175 pounds per square inch.

Needle Valve. — A valve provided with a long tapering point in place of the ordinary valve disk. The tapering point permits fine graduation of the opening. At times called a needle point valve.

Nipple. — (1) A tubular pipe fitting usually threaded on both ends and under 12 inches in length. Pipe over 12 inches long is regarded as cut pipe. (See Close. Short, Shoulder and Space Nipple.)

Reducer. — (1) A fitting having a larger size at one end than at the other. Some have tried to establish the term "increaser" — thinking of direction of flow — but this has been due to a misunderstanding of the trade custom of always giving the largest size of run of a fitting first; hence, all fittings having more than one size are reducers. They are always threaded inside, unless specified flanged or for some special joint. (2) Threaded type, made with abrupt reduction. (3) Flanged pattern with taper body. (4) Flanged eccentric pattern with taper body, but flanges at 90 degrees to one side of body. (5) Misapplied at times, to a reducing coupling.

Run. — (1) A length of pipe that is made of more than one piece of pipe. (2) The portion of any fitting having its ends "in line" or nearly so, in contradistinction to the branch or side opening, as of a tee. The two main openings of an ell also indicate its run, and when there is a third opening on an ell, the fitting is a "side outlet" or "back outlet" elbow, except that when all three openings are in one plane and the back outlet is in line with one of the run openings, the fitting is a "heel outlet elbow" or a "single sweep tee" or sometimes a "branch tee."

Rust Joint. — Employed to secure rigid connection. The joint is made by packing an intervening space tightly with a stiff paste which oxidizes the iron, the whole rusting together and hardening into a solid mass. It generally cannot be separated except by destroying some of the pieces. One recipe is 80 pounds cast-iron borings or filings, 1 pound sal-ammoniac, 2 pounds flowers of sulphur, mixed to a paste with water.

Service Ell. — An elbow having an outside thread on one end. Also known as street ell.

Service Pipe. — A pipe connecting mains with a dwelling.

Service Tee. — A tee having inside thread on one end and on branch, but outside thread on other end of run. Also known as street tee.

Short Nipple. — One whose length is a little greater than that of two threaded lengths or somewhat longer than a close nipple. It always has some unthreaded portion between the two threads.

Shoulder Nipple. — A nipple of any length, which has a portion of pipe between two pipe threads. As generally used, however, it is a nipple about halfway between the length of a close nipple and a short nipple.

Space Nipple. — A nipple with a portion of pipe or shoulder between the two threads. It may be of any length long enough to allow a shoulder.

Standard Pressure. — A term applied to valves and fittings suitable for a working steam pressure of 125 pounds per square inch.

Tee. — A fitting, either cast or wrought, that has one side outlet at right angles to the run. A single outlet branch pipe. (See Branch, Bull Head, Cross-over, Double Sweep, Drop, Service and Union Tee.)

Union. — (1) The usual trade term for a device used to connect pipes. It commonly consists of three pieces which are, first, the thread end fitted with exterior and interior threads; second, the bottom end fitted with interior threads and a small exterior shoulder; and third, the ring which has an inside flange at one end while the other end has an inside thread like that on the exterior of the thread end. A gasket is placed between the thread and bottom ends, which are drawn together by the ring. Unions are very extensively used, because they permit of connections with little disturbance of the pipe positions.

Union Ell. — An ell with a male or female union at one end.

Union Joint. — A pipe coupling, usually threaded, which permits disconnection without disturbing other sections.

Union Tee. — A tee with male or female union at connection on one end of run.

Wiped Joint. — A lead joint in which the molten solder is poured upon the desired place, after scraping and fitting the parts together, and the joint is wiped up by hand with a moleskin or cloth pad while the metal is in a plastic condition.

Wye (Y). — A fitting either cast or wrought that has one side outlet at any angle other than 90 degrees. The angle is usually 45 degrees, unless another angle is specified. The fitting is usually indicated by the letter Y.

Lutes and Cements

Luting and cementing materials for various purposes in the laboratory and shops may be classified as follows: Water- and steam-proof; oil-proof; acid-proof; proof to hydrocarbon gases; chlorine-proof; elastic; general purposes; marine glue; gaskets; machinists'; leather (belting); crucible; iron; and stone.

Water-proof Compositions. — The asphalt fluid coatings for reservoir walls, concrete foundations, brick, wood, etc., are often of use to engineers. Asphalt only partly dissolves in petroleum naphtha, but when heated in a steam-jacketed kettle and not thinned out too much, a mixture of the two may be obtained in which the part of the asphalt not dissolved is held in suspension. Asphalt is entirely soluble in benzol or toluol, which are about the cheapest solvents for all the constituents of asphalt. Tar and pitch are sometimes used in this connection, but tar contains water, light oils and free carbon, and does not wear as well as good refined asphalt; pitch also contains free carbon, which is sometimes objectionable when it is thinned out with a solvent. Asphalt alone is somewhat pervious to water, but it can be improved in this respect by adding about one-fourth its weight of paraffin; it is also well to add a little boiled linseed oil. For thicker compositions, where body is required, asbestos, stone powder, cement, etc., may be added as fillers. Lutes of linseed oil thickened with clay, asbestos, red or white lead, etc., are waterproof if made thick enough. These are much used for steam joints. Flaxseed meal made into a paste with water is often serviceable, the oil contained serving as a binder as the water evaporates.

Oil-proof Cements. — The well-known "hektograph composition" is the most useful lute for small leaks, etc. It consists of the following ingredients: Good glue or gelatin, 2 parts; glycerin, 1 part; water, 7 parts. This preparation is applied warm and stiffens quickly on cooling. Another very useful composition is a stiff paste of molasses and flour. Another preparation, impervious to oil vapors, is the "flaxseed poultice," mentioned in the preceding paragraph, which is proof to oil vapors. One of the strongest cements, and one which is really oil-proof, waterproof and acid-proof, is a stiff paste of glycerin and litharge. These form a chemical combination which sets in a few minutes. If a little water is added, it sets more slowly, which is often an advantage. This cement is mixed when required for use. A mixture of plaster-of-paris and water is useful, and it is sometimes advantageous to mix asbestos, straw or hair with it. A solution of silicate of soda made into a stiff paste with carbonate of lime gets hard in six to eight hours.

Acid-proof Cements. — The asphalt compositions already mentioned, compositions of melted sulphur with fillers of stone powder, cement, sand, etc., may be used, and also the following, which withstands hydrochloric acid vapors: Rosin, 1 part; sulphur, 1 part; fireclay, 2 parts. The lute composed of boiled linseed oil and fireclay acts well with most acid vapors. The composition of glycerin and litharge previously referred to is useful in this connection, especially when made up according to the following formula: Litharge, 80 pounds; red lead, 8 pounds; "flock" asbestos, 10 pounds. It should be fed into a mixer, a little at a time, with small quantities of boiled oil (about six quarts of oil being used). Sockets in 3-inch pipes carrying nitric acid, calked with this preparation, showed no leaks in nine months.

A particularly useful cement for withstanding acid vapors, which is tough and elastic, is composed of crude rubber, cut fine, 1 part; linseed oil, boiled, 4 parts; fireclay, 6 parts. The rubber is dissolved in carbon disulphide to the consistency of molasses and is then mixed with the oil. Other acid-proof cements are as follows: "Black putty" made by carefully mixing equal portions of china-clay, gas-tar and linseed oil. The china-clay must be well dried by placing it over a boiler or by other means. Barytes cement is composed of pure, finely ground sulphate of barium, and is made into a putty with a solution of silicate of soda. This sets very hard when moderately heated, and is then proof against acids. The gravity of the silicate of soda should be between 1.2 and 1.4, 24 degrees to 42 degrees Baumé. If too thin, it does not hold; and when thicker than 1.4, it expands and breaks.

Gasket Compositions. — Almost any cementing substance may be used with rings of asbestos, etc., for gaskets, but some are especially adapted for the purpose. Asphalt, tar, petroleum residuum and soft or hard pitch are recommended. Silicate of soda is much used, and is sometimes advantageously mixed with casein, fine sand, clay, asbestos, carbonate of lime, caustic lime, magnesia, oxides of heavy metals, such as lead, zinc, iron and powdered barytes. A few mixtures that might be selected are: Silicate of soda and asbestos; silicate of soda, asbestos and slaked lime; silicate of soda and fine sand; silicate of soda and fireclay.

Machinists' Cements. — These are the well-known red and white leads. The red lead is often diluted with an equal bulk of silica or other inert substance to make it less powdery. The best way to do this is to add rubber or gutta-percha to the oil as follows: Linseed oil, 6 parts, by weight; rubber or gutta-percha, 1 part by weight. The rubber or gutta-percha is dissolved in sufficient carbon disulphide to give it the consistency of molasses, mixed with the oil, and left exposed to the air for about twenty-four hours. The red lead is then mixed to a putty. Oxide of iron makes a less brittle cement than red lead.

Leather Cements. — 1. Equal parts of good hide glue and American isinglass, softened in water for ten hours and then boiled with pure tannin until the whole mass is sticky. The surface of the joint should be roughened and the cement applied hot. 2. One pound of finely shredded gutta-percha digested over a water-bath with 10 pounds of benzol, until dissolved, and 12 pounds of linseed oil varnish stirred in. 3. Seven and one-half pounds of finely shredded india-rubber is completely dissolved in 10 pounds of carbon disulphide by treating while hot; 1 pound of shellac and 1 pound of turpentine are added, and the hot solution heated until the two latter ingredients are also dissolved.

Another leather cement is as follows: Gutta-percha, 8 ounces; pitch, 1 ounce; shellac, 1 ounce; sweet oil, 1 ounce. These are melted together. Still another is as follows: Fish glue is soaked in water twenty-four hours, allowed to drain for a like period, boiled well, and a previously melted mixture of 2 ounces of rosin and ½ ounce of boiled oil is added to every two pounds of glue solution.

Iron and Stone Cements. — When finely divided iron, such as filings or cast-iron borings that have been powdered, is mixed with an oxidizing agent, such as manganese dioxide, or a substance electro-negative to iron, such as sulphur, in a good conducting solution like salt or sal-ammoniac, galvanic action sets in very rapidly and the iron swells, by forming iron oxide, and cements the mass together. It is best diluted with Portland cement, the proportions being as follows: Iron filings, 40 parts; manganese dioxide or flowers of sulphur, 10 parts; sal-ammoniac, 1 part; Portland cement, 20 to 40 parts; water to form a paste. A hard stone-like composition is made as follows: Zinc oxide, 2 parts; zinc chloride, 1 part; water to make a paste. Magnesium oxide and chloride may also be used in like proportions. When used in considerable quantity, this cement is mixed with powdered stone, for reasons of economy, the proportions depending upon the character of the work.

Cement proof to Hydrocarbon Gases. — Compositions of plaster and cement, the former setting more quickly, are used; also compositions of casein, such as finely powdered casein, 2 parts; fresh slaked lime, 50 parts; fine sand, 50 parts. Water is added, when used, to form a thick mass. Various mixtures of silicate of soda are employed in which the thick silicate is absorbed in some inert material such as clay, sand or asbestos.

Cements proof to Chlorine. — The best and only reliable compositions are a few made with Portland cement, and the following is much used for electrolytic and chemical plants: Powdered glass, 1 part; Portland cement, 1 part; silicate of soda, 1 part; a small amount of powdered slate. This lute is said to stand acids and alkalies, as well as the influences of chlorine. Linseed oil made into a paste with fireclay serves for a time.

Elastic Cements. — The various cements containing rubber are elastic, if the rubber is in a predominating amount; many containing boiled linseed oil and the hektograph composition already mentioned are quite elastic. The rubber and linseed-oil cement, given in the paragraph headed "Acid-proof Cements," is very tough and useful for nearly all purposes except when oil vapors are to be confined. The most useful single rubber lute is probably the so-called Hart's india-rubber cement. Equal parts of raw linseed oil and pure masticated rubber are digested together by heating, and this mixture is made into a stiff putty with fine "paper stock" asbestos. It is more convenient, however, to dissolve the rubber first in carbon disulphide, and, after mixing the oil with it, to let the solvent evaporate spontaneously.

General Purposes. — Plaster-of-paris, especially when mixed with asbestos, straw, flush trimmings, hair, broken stone, etc., and used according to temperature, strain and other conditions, is one of the most useful preparations for general purposes. A putty of flour and molasses is a good composition to keep in a works ready for quick application when needed. It serves, for a time, almost any purpose at moderate temperatures. Casein compositions have great strength. The white of an egg made into a paste with slaked lime is strong and efficient, but must be used promptly on account of its quick setting qualities.

Marine Glue. — This can be purchased almost as cheaply as made. It consists of crude rubber, 1 part; shellac, 2 parts; pitch, 3 parts. The rubber must first be dissolved in carbon disulphide or turpentine before mixing with the heated combination of the other two ingredients.

Acid-proof Lining. — A lining for protecting tanks from the influence of acids is made from a mixture consisting of 75 parts (by weight), of pitch; 9 parts plaster-of-paris; 9 parts ochre; 15 parts beeswax; and 3 parts litharge. The tanks are covered on the inside with a thick coat of this mixture.

WEIGHTS AND MEASURES

Measures of Length

1 mile = 1760 yards = 5280 feet.
1 yard = 3 feet = 36 incnes.
1 foot = 12 inches.

The following measures of length are also used occasionally:

1 mil = 0.001 inch. 1 fathom = 2 yards = 6 feet.
1 rod = 5.5 yards = 16.5 feet. 1 hand = 4 inches. 1 span = 9 inches.

Surveyor's Measure

1 mile = 8 furlongs = 80 chains.
1 furlong = 10 chains = 220 yards.
1 chain = 4 rods = 22 yards = 66 feet = 100 links.
1 link = 7.92 inches.

Nautical Measure

1 league = 3 nautical miles.
1 nautical mile (knot) = 6080.26 feet = 1.1516 statute mile.

One degree at the equator = 60 nautical miles = 69.168 statute miles. 360 degrees = 21,600 nautical miles = 24,874.5 statute miles = circumference of earth at the equator.

Square Measure

1 square mile = 640 acres = 6400 square chains.
1 acre = 10 square chains = 4840 square yards = 43,560 square feet.
1 square chain = 16 square rods = 484 square yards = 4356 square feet.
1 square rod = 30.25 square yards = 272.25 square feet = 625 square links.
1 square yard = 9 square feet.
1 square foot = 144 square inches.

An acre is equal to a square, the side of which is 208.7 feet.

Measure used for Diameters and Areas of Electric Wires

1 circular inch = area of circle 1 inch in diameter = 0.7854 square inch.
1 circular inch = 1,000,000 circular mils.
1 square inch = 1.2732 circular inch = 1,273,239 circular mils.

A circular mil is the area of a circle 0.001 inch in diameter.

Cubic Measure

1 cubic yard = 27 cubic feet.
1 cubic foot = 1728 cubic inches.

The following measures are also used for wood and masonry:

1 cord of wood = 4 × 4 × 8 feet = 128 cubic feet.
1 perch of masonry = $16\frac{1}{2} \times 1\frac{1}{2} \times 1$ foot = $24\frac{3}{4}$ cubic feet.

Shipping Measure

For measuring entire internal capacity of a vessel:

1 register ton = 100 cubic feet.

For measurement of cargo:

1 U. S. shipping ton = 40 cubic feet = 32.143 U. S. bushels = 31.16 Imperial bushels.
British shipping ton = 42 cubic feet = 33.75 U. S. bushels = 32.72 Imperial bushels.

Dry Measure

1 bushel (U. S. or Winchester struck bushel) = 1.2445 cubic foot = 2150.42 cubic inches.
1 bushel = 4 pecks = 32 quarts = 64 pints.
1 peck = 8 quarts = 16 pints.
1 quart = 2 pints.
1 heaped bushel = $1\frac{1}{4}$ struck bushel.
1 cubic foot = 0.8036 struck bushel.
1 British Imperial bushel = 8 Imperial gallons = 1.2837 cubic foot = 2218.19 cubic inches.

Liquid Measure

1 U. S. gallon = 0.1337 cubic foot = 231 cubic inches = 4 quarts = 8 pints.
1 quart = 2 pints = 8 gills.
1 pint = 4 gills.
1 British Imperial gallon = 1.2003 U. S. gallon = 277.27 cubic inches.
1 cubic foot = 7.48 U. S. gallons.

Old Liquid Measure

1 tun = 2 pipes = 3 puncheons.
1 pipe or butt = 2 hogsheads = 4 barrels = 126 gallons.
1 puncheon = 2 tierces = 84 gallons.
1 hogshead = 2 barrels = 63 gallons.
1 tierce = 42 gallons.
1 barrel = $31\frac{1}{2}$ gallons.

Apothecaries' Fluid Measure

1 U. S. fluid ounce = 8 drachms = 1.805 cubic inch = $1/128$ U. S. gallon.
1 fluid drachm = 60 minims.
1 British fluid ounce = 1.732 cubic inch.

Measures of Weight

Avoirdupois or Commercial Weight

1 gross or long ton = 2240 pounds.
1 net or short ton = 2000 pounds.
1 pound = 16 ounces = 7000 grains.
1 ounce = 16 drachms = 437.5 grains.

The following measures for weight are now seldom used in the United States:
1 hundred-weight = 4 quarters = 112 pounds (1 gross or long ton = 20 hundred-weights); 1 quarter = 28 pounds; 1 stone = 14 pounds; 1 quintal = 100 pounds.

Troy Weight, used for Weighing Gold and Silver

1 pound = 12 ounces = 5760 grains.
1 ounce = 20 pennyweights = 480 grains.
1 pennyweight = 24 grains.
1 carat (used in weighing diamonds) = 3.086 grains.
1 grain Troy = 1 grain avoirdupois = 1 grain apothecaries' weight.

Apothecaries' Weight

1 pound = 12 ounces = 5760 grains.
1 ounce = 8 drachms = 480 grains.
1 drachm = 3 scruples = 60 grains.
1 scruple = 20 grains.

Measures of Pressure

1 pound per square inch = 144 pounds per square foot = 0.068 atmosphere = 2.042 inches of mercury at 62 degrees F. = 27.7 inches of water at 62 degrees F. = 2.31 feet of water at 62 degrees F.

1 atmosphere = 30 inches of mercury at 62 degrees F. = 14.7 pounds per square inch = 2116.3 pounds per square foot = 33.95 feet of water at 62 degrees F.

1 foot of water at 62 degrees F. = 62.355 pounds per square foot = 0.433 pound per square inch.

1 inch of mercury at 62 degrees F. = 1.132 foot of water = 13.58 inches of water = 0.491 pound per square inch.

Miscellaneous

1 great gross = 12 gross = 144 dozen.
1 gross = 12 dozen = 144 units.
1 dozen = 12 units.
1 score = 20 units.

1 quire = 24 sheets.
1 ream = 20 quires = 480 sheets.
1 ream printing paper = 500 sheets.

Decimal Equivalents of Fractions of an Inch

1/64	0.015 625	11/32	0.343 75	43/64	0.671 875
1/32	0.031 25	23/64	0.359 375	11/16	0.687 5
3/64	0.046 875	3/8	0.375	45/64	0.703 125
1/16	0.062 5	25/64	0.390 625	23/32	0.718 75
5/64	0.078 125	13/32	0.406 25	47/64	0.734 375
3/32	0.093 75	27/64	0.421 875	3/4	0.750
7/64	0.109 375	7/16	0.437 5	49/64	0.765 625
1/8	0.125	29/64	0.453 125	25/32	0.781 25
9/64	0.140 625	15/32	0.468 75	51/64	0.796 875
5/32	0.156 25	31/64	0.484 375	13/16	0.812 5
11/64	0.171 875	1/2	0.500	53/64	0.828 125
3/16	0.187 5	33/64	0.515 625	27/32	0.843 75
13/64	0.203 125	17/32	0.531 25	55/64	0.859 375
7/32	0.218 75	35/64	0.546 875	7/8	0.875
15/64	0.234 375	9/16	0.562 5	57/64	0.890 625
1/4	0.250	37/64	0.578 125	29/32	0.906 25
17/64	0.265 625	19/32	0.593 75	59/64	0.921 875
9/32	0.281 25	39/64	0.609 375	15/16	0.937 5
19/64	0.296 875	5/8	0.625	61/64	0.953 125
5/16	0.312 5	41/64	0.640 625	31/32	0.968 75
21/64	0.328 125	21/32	0.656 25	63/64	0.984 375

Table of Decimal Equivalents of a Foot Corresponding to Inches and Fractions of Inches. — Assume, for example, that it is required to find the equivalent of 6 7/32 inches in decimals of a foot. Locate 7/32 in the left-hand column and follow the horizontal line until the column headed "6" is reached. The figures 0.5182 read off in this column are the decimals of a foot corresponding to 6 7/32; in other words, 6 7/32 inches equals 0.5182 foot.

Inches into Decimals of a Foot

Inch	Inches: 0	1	2	3	4	5	6	7	8	9	10	11
	Decimals of a Foot											
....	....	0.0833	0.1667	0.2500	0.3333	0.4167	0.5000	0.5833	0.6667	0.7500	0.8333	0.9167
1/32	0.0026	0.0859	0.1693	0.2526	0.3359	0.4193	0.5026	0.5859	0.6693	0.7526	0.8359	0.9193
1/16	0.0052	0.0885	0.1719	0.2552	0.3385	0.4219	0.5052	0.5885	0.6719	0.7552	0.8385	0.9219
3/32	0.0078	0.0911	0.1745	0.2578	0.3411	0.4245	0.5078	0.5911	0.6745	0.7578	0.8411	0.9245
1/8	0.0104	0.0938	0.1771	0.2604	0.3438	0.4271	0.5104	0.5938	0.6771	0.7604	0.8438	0.9271
5/32	0.0130	0.0964	0.1797	0.2630	0.3464	0.4297	0.5130	0.5964	0.6797	0.7630	0.8464	0.9297
3/16	0.0156	0.0990	0.1823	0.2656	0.3490	0.4323	0.5156	0.5990	0.6823	0.7656	0.8490	0.9323
7/32	0.0182	0.1016	0.1849	0.2682	0.3516	0.4349	0.5182	0.6016	0.6849	0.7682	0.8516	0.9349
1/4	0.0208	0.1042	0.1875	0.2708	0.3542	0.4375	0.5208	0.6042	0.6875	0.7708	0.8542	0.9375
9/32	0.0234	0.1068	0.1901	0.2734	0.3568	0.4401	0.5234	0.6068	0.6901	0.7734	0.8568	0.9401
5/16	0.0260	0.1094	0.1927	0.2760	0.3594	0.4427	0.5260	0.6094	0.6927	0.7760	0.8594	0.9427
11/32	0.0286	0.1120	0.1953	0.2786	0.3620	0.4453	0.5286	0.6120	0.6953	0.7786	0.8620	0.9453
3/8	0.0313	0.1146	0.1979	0.2813	0.3646	0.4479	0.5313	0.6146	0.6979	0.7813	0.8646	0.9479
13/32	0.0339	0.1172	0.2005	0.2839	0.3672	0.4505	0.5339	0.6172	0.7005	0.7839	0.8672	0.9505
7/16	0.0365	0.1198	0.2031	0.2865	0.3698	0.4531	0.5365	0.6198	0.7031	0.7865	0.8698	0.9531
15/32	0.0391	0.1224	0.2057	0.2891	0.3724	0.4557	0.5391	0.6224	0.7057	0.7891	0.8724	0.9557
1/2	0.0417	0.1250	0.2083	0.2917	0.3750	0.4583	0.5417	0.6250	0.7083	0.7917	0.8750	0.9583
17/32	0.0443	0.1276	0.2109	0.2943	0.3776	0.4609	0.5443	0.6276	0.7109	0.7943	0.8776	0.9609
9/16	0 0469	0.1302	0.2135	0.2969	0.3802	0.4635	0.5469	0.6302	0.7135	0.7969	0.8802	0.9635
19/32	0.0495	0.1328	0.2161	0.2995	0.3828	0.4661	0.5495	0.6328	0.7161	0.7995	0.8828	0.9661
5/8	0.0521	0.1354	0.2188	0.3021	0.3854	0.4688	0.5521	0.6354	0.7188	0.8021	0.8854	0.9688
21/32	0.0547	0.1380	0.2214	0.3047	0.3880	0.4714	0.5547	0.6380	0.7214	0.8047	0.8880	0.9714
11/16	0.0573	0.1406	0.2240	0.3073	0.3906	0.4740	0.5573	0.6406	0.7240	0.8073	0.8906	0.9740
23/32	0.0599	0.1432	0.2266	0.3099	0.3932	0.4766	0.5599	0.6432	0.7266	0.8099	0.8932	0.9766
3/4	0.0625	0.1458	0.2292	0.3125	0.3958	0.4792	0.5625	0.6458	0.7292	0.8125	0.8958	0.9792
25/32	0.0651	0.1484	0.2318	0.3151	0.3984	0.4818	0.5651	0.6484	0.7318	0.8151	0.8984	0.9818
13/16	0.0677	0.1510	0.2344	0.3177	0.4010	0.4844	0.5677	0.6510	0.7344	0.8177	0.9010	0.9844
27/32	0.0703	0.1536	0.2370	0.3203	0.4036	0.4870	0.5703	0.6536	0.7370	0.8203	0.9036	0.9870
7/8	0.0729	0.1563	0.2396	0.3229	0.4063	0.4896	0.5729	0.6563	0.7396	0.8229	0.9063	0.9896
29/32	0.0755	0.1589	0.2422	0.3255	0.4089	0.4922	0.5755	0.6589	0.7422	0.8255	0.9089	0.9922
15/16	0.0781	0.1615	0.2448	0.3281	0.4115	0.4948	0.5781	0.6615	0.7448	0.8281	0.9115	0.9948
31/32	0.0807	0.1641	0.2474	0.3307	0.4141	0.4974	0.5807	0.6641	0.7474	0.8307	0.9141	0.9974

Decimal Equivalents of 6ths, 12ths, and 24ths of an Inch

1/24	0.041 667	9/24	0.375	17/24	0.708 333
1/12	0.083 333	5/12	0.416 667	9/12	0.75
3/24	0.125	11/24	0.458 333	19/24	0.791 667
1/6	0.166 667	3/6	0.5	5/6	0.833 333
5/24	0.208 333	13/24	0.541 667	21/24	0.875
3/12	0.25	7/12	0.583 333	11/12	0.916 667
7/24	0.291 667	15/24	0.625	23/24	0.958 333
2/6	0.333 333	4/6	0.666 667	......	....

Decimal Equivalents of 7ths, 14ths, and 28ths of an Inch

1/28	0.035 714	5/14	0.357 143	19/28	0.678 571
1/14	0.071 429	11/28	0.392 857	5/7	0.714 286
3/28	0.107 143	3/7	0.428 571	21/28	0.75
1/7	0.142 857	13/28	0.464 286	11/14	0.785 714
5/28	0.178 571	7/14	0.5	23/28	0.821 429
3/14	0.214 286	15/28	0.535 714	6/7	0.857 143
7/28	0.25	4/7	0.571 429	25/28	0.892 857
2/7	0.285 714	17/28	0.607 143	13/14	0.928 571
9/28	0.321 429	9/14	0.642 867	27/28	0.964 286

U. S. Gallons into Cubic Feet

Gallons	Cubic Feet	Gallons	Cubic Feet	Gallons	Cubic Feet	Gallons	Cubic Feet
1	0.134	20	2.674	300	40.10	4,000	534.72
2	0.267	30	4.010	400	53.47	5,000	668.40
3	0.401	40	5.347	500	66.84	6,000	802.08
4	0.535	50	6.684	600	80.21	7,000	935.76
5	0.668	60	8.021	700	93.58	8,000	1,069.44
6	0.802	70	9.358	800	106.94	9,000	1,203.12
7	0.936	80	10.694	900	120.31	10,000	1,336.81
8	1.069	90	12.031	1000	133.68	50,000	6,684.03
9	1.203	100	13.368	2000	267.36	100,000	13,368.06
10	1.337	200	26.736	3000	401.04	500,000	66,840.28

Cubic Feet into Gallons

(1 cubic foot = 7.4805 U. S. gallons; 1 gallon = 231 cubic inches = 0.13368 cubic foot.)

Cubic Feet	Gallons	Cubic Feet	Gallons	Cubic Feet	Gallons	Cubic Feet	Gallons
0.1	0.75	2	14.96	30	224.4	400	2,992.2
0.2	1.50	3	22.44	40	299.2	500	3,740.3
0.3	2.24	4	29.92	50	374.0	600	4,488.3
0.4	2.99	5	37.40	60	448.8	700	5,236.4
0.5	3.74	6	44.88	70	523.6	800	5,984.4
0.6	4.49	7	52.36	80	598.4	900	6,732.5
0.7	5.24	8	59.84	90	673.2	1,000	7,480.5
0.8	5.98	9	67.32	100	748.1	5,000	37,402.6
0.9	6.73	10	74.81	200	1496.1	10,000	74,805.2
1.0	7.48	20	149.61	300	2244.2	50,000	374,025.9

Contents in Cubic Feet and U. S. Gallons of Pipes and Cylinders One Foot in Length

Diam. in Inches	For 1 Foot in Length: Cubic Feet	For 1 Foot in Length: U. S. Gallons	Diam. in Inches	For 1 Foot in Length: Cubic Feet	For 1 Foot in Length: U. S. Gallons	Diam. in Inches	For 1 Foot in Length: Cubic Feet	For 1 Foot in Length: U. S. Gallons
¼	0.0003	0.0025	6¾	0.2485	1.859	19	1.969	14.73
5/16	0.0005	0.0040	7	0.2673	1.999	19½	2.074	15.51
⅜	0.0008	0.0057	7¼	0.2867	2.145	20	2.182	16.32
7/16	0.0010	0.0078	7½	0.3068	2.295	20½	2.292	17.15
½	0.0014	0.0102	7¾	0.3276	2.450	21	2.405	17.99
9/16	0.0017	0.0129	8	0.3491	2.611	21½	2.521	18.86
⅝	0.0021	0.0159	8¼	0.3712	2.777	22	2.640	19.75
11/16	0.0026	0.0193	8½	0.3941	2.948	22½	2.761	20.66
¾	0.0031	0.0230	8¾	0.4176	3.125	23	2.885	21.58
13/16	0.0036	0.0269	9	0.4418	3.305	23½	3.012	22.53
⅞	0.0042	0.0312	9¼	0.4667	3.491	24	3.142	23.50
15/16	0.0048	0.0359	9½	0.4922	3.682	25	3.409	25.50
1	0.0055	0.0408	9¾	0.5185	3.879	26	3.687	27.58
1¼	0.0085	0.0638	10	0.5454	4.080	27	3.976	29.74
1½	0.0123	0.0918	10¼	0.5730	4.286	28	4.276	31.99
1¾	0.0167	0.1249	10½	0.6013	4.498	29	4.587	34.31
2	0.0218	0.1632	10¾	0.6303	4.715	30	4.909	36.72
2¼	0.0276	0.2066	11	0.6600	4.937	31	5.241	39.21
2½	0.0341	0.2550	11¼	0.6903	5.164	32	5.585	41.78
2¾	0.0412	0.3085	11½	0.7213	5.396	33	5.940	44.43
3	0.0491	0.3672	11¾	0.7530	5.633	34	6.305	47.16
3¼	0.0576	0.4309	12	0.7854	5.875	35	6.681	49.98
3½	0.0668	0.4998	12½	0.8522	6.375	36	7.069	52.88
3¾	0.0767	0.5738	13	0.9218	6.895	37	7.467	55.86
4	0.0873	0.6528	13½	0.9940	7.436	38	7.876	58.92
4¼	0.0985	0.7369	14	1.069	7.997	39	8.296	62.06
4½	0.1104	0.8263	14½	1.147	8.578	40	8.727	65.28
4¾	0.1231	0.9206	15	1.227	9.180	41	9.168	68.58
5	0.1364	1.020	15½	1.310	9.801	42	9.621	71.97
5¼	0.1503	1.125	16	1.396	10.44	43	10.085	75.44
5½	0.1650	1.234	16½	1.485	11.11	44	10.559	78.99
5¾	0.1803	1.349	17	1.576	11.79	45	11.045	82.62
6	0.1963	1.469	17½	1.670	12.49	46	11.541	86.33
6¼	0.2131	1.594	18	1.767	13.22	47	12.048	90.13
6½	0.2304	1.724	18½	1.867	13.96	48	12.566	94.00

One cubic foot of water at 39.1 degrees F. weighs 62.4245 pounds.

One cubic foot of air at 32 degrees F., atmospheric pressure, weighs 0.08073 pound.

One pound of water at 39.1 degrees F. has a volume of 0.01602 cubic foot.

One pound of air at 32 degrees F., atmospheric pressure, has a volume of 12.387 cubic feet.

One gallon of water at 62 degrees F. weighs 8.336 pounds.

One pound of water at 62 degrees F. has a volume of 0.1199 U. S. gallon.

Contents of Circular Tanks in U. S. Gallons

Depth of Tank, Feet	Diameter of Tank, Feet: 5	6	7	8	9	10	11	12	13
	Contents of Tank, U. S. Gallons								
5	725	1060	1440	1875	2380	2,925	3,550	4,237	4,960
6	870	1270	1728	2250	2855	3,510	4,260	5,084	5,952
7	1015	1480	2016	2625	3330	4,095	4,970	5,931	6,944
8	1160	1690	2304	3000	3805	4,680	5,680	6,778	7,936
9	1305	1900	2592	3375	4280	5,265	6,390	7,625	8,928
10	1450	2110	2880	3750	4755	5,850	7,100	8,472	9,920
11	1595	2320	3168	4125	5230	6,435	7,810	9,319	10,912
12	1740	2530	3456	4500	5705	7,020	8,520	10,166	11,904
13	1885	2740	3744	4875	6180	7,605	9,230	11,013	12,896
14	2030	2950	4032	5250	6655	8,190	9,940	11,860	13,888
15	2175	3160	4320	5625	7130	8,775	10,650	12,707	14,880
16	2320	3370	4608	6000	7605	9,360	11,360	13,554	15,872
17	2465	3580	4896	6375	8080	9,945	12,070	14,401	16,864
18	2610	3790	5184	6750	8535	10,530	12,780	15,248	17,856
19	2755	4000	5472	7125	9010	11,115	13,490	16,095	18,848
20	2900	4210	5760	7500	9490	11,700	14,200	16,942	19,840

Depth of Tank, Feet	Diameter of Tank, Feet: 14	15	16	18	20	22	24	25
	Contents of Tank, U. S. Gallons							
5	5,765	6,698	7,520	9,516	11,750	14,215	16,918	18,358
6	6,918	8,038	9,024	11,419	14,100	17,059	20,302	22,030
7	8,071	9,378	10,528	13,322	16,450	19,902	23,680	25,701
8	9,224	10,718	12,032	15,225	18,800	22,745	27,070	29,372
9	10,377	12,058	13,536	17,128	21,150	25,588	30,454	33,043
10	11,530	13,398	15,040	19,031	23,500	28,431	33,838	36,714
11	12,683	14,738	16,544	20,934	25,850	31,274	37,222	40,385
12	13,836	16,078	18,048	22,837	28,200	34,117	40,606	44,056
13	14,989	17,418	19,552	24,740	30,550	36,960	43,990	47,727
14	16,142	18,758	21,056	26,643	32,900	39,803	47,374	51,398
15	17,295	20,098	22,260	28,546	35,250	42,646	50,758	55,069
16	18,448	21,438	24,064	30,449	37,600	45,489	54,142	58,740
17	19,601	22,778	25,568	32,352	39,950	48,332	57,520	62,411
18	20,754	24,118	27,072	34,255	42,300	51,175	60,910	66,082
19	21,907	25,458	28,576	36,158	44,650	54,018	64,294	69,753
20	23,060	26,798	30,080	38,062	47,000	56,861	67,678	73,424

A cylinder 7 inches in diameter and 6 inches high contains one gallon within 0.1 of a cubic inch.

The volume, in U. S. gallons, of a cylinder, equals the square of the diameter in inches × height of cylinder in inches × 0.0034.

Multiplication Table for Common Fractions and Whole Numbers

	1	2	3	4	5	6	7	8	9
1/64	0.0156	0.0312	0.0469	0.0625	0.0781	0.0937	0.1094	0.1250	0.1406
1/32	0.0312	0.0625	0.0937	0.1250	0.1562	0.1875	0.2187	0.2500	0.2812
3/64	0.0469	0.0937	0.1406	0.1875	0.2344	0.2812	0.3281	0.3750	0.4219
1/16	0.0625	0.1250	0.1875	0.2500	0.3125	0.3750	0.4375	0.5000	0.5625
5/64	0.0781	0.1562	0.2344	0.3125	0.3906	0.4687	0.5469	0.6250	0.7031
3/32	0.0937	0.1875	0.2812	0.3750	0.4687	0.5625	0.6562	0.7500	0.8437
7/64	0.1094	0.2187	0.3281	0.4375	0.5469	0.6562	0.7656	0.8750	0.9844
1/8	0.1250	0.2500	0.3750	0.5000	0.6250	0.7500	0.8750	1.0000	1.1250
9/64	0.1406	0.2812	0.4219	0.5625	0.7031	0.8437	0.9844	1.1250	1.2656
5/32	0.1562	0.3125	0.4687	0.6250	0.7812	0.9375	1.0937	1.2500	1.4062
11/64	0.1719	0.3437	0.5156	0.6875	0.8594	1.0312	1.2031	1.3750	1.5469
3/16	0.1875	0.3750	0.5625	0.7500	0.9375	1.1250	1.3125	1.5000	1.6875
13/64	0.2031	0.4062	0.6094	0.8125	1.0156	1.2187	1.4219	1.6250	1.8281
7/32	0.2187	0.4375	0.6562	0.8750	1.0937	1.3125	1.5312	1.7500	1.9687
15/64	0.2344	0.4687	0.7031	0.9375	1.1719	1.4062	1.6406	1.8750	2.1094
1/4	0.2500	0.5000	0.7500	1.0000	1.2500	1.5000	1.7500	2.0000	2.2500
17/64	0.2656	0.5312	0.7969	1.0625	1.3281	1.5937	1.8594	2.1250	2.3906
9/32	0.2812	0.5625	0.8437	1.1250	1.4062	1.6875	1.9687	2.2500	2.5312
19/64	0.2969	0.5937	0.8906	1.1875	1.4848	1.7812	2.0781	2.3750	2.6719
5/16	0.3125	0.6250	0.9375	1.2500	1.5625	1.8750	2.1875	2.5000	2.8125
21/64	0.3281	0.6562	0.9844	1.3125	1.6406	1.9687	2.2969	2.6250	2.9531
11/32	0.3437	0.6875	1.0312	1.3750	1.7187	2.0625	2.4062	2.7500	3.0937
23/64	0.3594	0.7187	1.0781	1.4375	1.7969	2.1562	2.5156	2.8750	3.2344
3/8	0.3750	0.7500	1.1250	1.5000	1.8750	2.2500	2.6250	3.0000	3.3750
25/64	0.3906	0.7812	1.1719	1.5625	1.9531	2.3437	2.7344	3.1250	3.5156
13/32	0.4062	0.8125	1.2187	1.6250	2.0312	2.4375	2.8437	3.2500	3.6562
27/64	0.4219	0.8437	1.2656	1.6875	2.1094	2.5312	2.9531	3.3750	3.7969
7/16	0.4375	0.8750	1.3125	1.7500	2.1875	2.6250	3.0625	3.5000	3.9375
29/64	0.4531	0.9062	1.3594	1.8125	2.2656	2.7187	3.1719	3.6250	4.0781
15/32	0.4687	0.9375	1.4062	1.8750	2.3437	2.8125	3.2812	3.7500	4.2187
31/64	0.4844	0.9687	1.4531	1.9375	2.4219	2.9062	3.3906	3.8750	4.3594
1/2	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000	4.0000	4.5000
33/64	0.5156	1.0312	1.5469	2.0625	2.5781	3.0937	3.6094	4.1250	4.6406
17/32	0.5312	1.0625	1.5937	2.1250	2.6562	3.1875	3.7187	4.2500	4.7812
35/64	0.5469	1.0937	1.6406	2.1875	2.7344	3.2812	3.8281	4.3750	4.9219
9/16	0.5625	1.1250	1.6875	2.2500	2.8125	3.3750	3.9375	4.5000	5.0625
37/64	0.5781	1.1562	1.7344	2.3125	2.8906	3.4687	4.0469	4.6250	5.2031
19/32	0.5937	1.1875	1.7812	2.3750	2.9687	3.5625	4.1562	4.7500	5.3437
39/64	0.6094	1.2187	1.8281	2.4375	3.0469	3.6562	4.2656	4.8750	5.4844
5/8	0.6250	1.2500	1.8750	2.5000	3.1250	3.7500	4.3750	5.0000	5.6250
41/64	0.6406	1.2812	1.9219	2.5625	3.2031	3.8437	4.4844	5.1250	5.7656
21/32	0.6562	1.3125	1.9687	2.6250	3.2812	3.9375	4.5937	5.2500	5.9062
43/64	0.6719	1.3437	2.0156	2.6875	3.3594	4.0312	4.7031	5.3750	6.0469
11/16	0.6875	1.3750	2.0625	2.7500	3.4375	4.1250	4.8125	5.5000	6.1875
45/64	0.7031	1.4062	2.1094	2.8125	3.5156	4.2187	4.9219	5.6250	6.3281
23/32	0.7187	1.4375	2.1562	2.8750	3.5937	4.3125	5.0312	5.7500	6.4687
47/64	0.7344	1.4687	2.2031	2.9375	3.6719	4.4062	5.1406	5.8750	6.6094
3/4	0.7500	1.5000	2.2500	3.0000	3.7500	4.5000	5.2500	6.0000	6.7500
49/64	0.7656	1.5312	2.2969	3.0625	3.8281	4.5937	5.3594	6.1250	6.8906
25/32	0.7812	1.5625	2.3437	3.1250	3.9062	4.6875	5.4687	6.2500	7.0312
51/64	0.7969	1.5937	2.3906	3.1875	3.9844	4.7812	5.5781	6.3750	7.1719
13/16	0.8125	1.6250	2.4375	3.2500	4.0625	4.8750	5.6875	6.5000	7.3125
53/64	0.8281	1.6562	2.4844	3.3125	4.1406	4.9687	5.7969	6.6250	7.4531
27/32	0.8437	1.6875	2.5312	3.3750	4.2187	5.0625	5.9062	6.7500	7.5937
55/64	0.8594	1.7187	2.5781	3.4375	4.2969	5.1562	6.0156	6.8750	7.7344
7/8	0.8750	1.7500	2.6250	3.5000	4.3750	5.2500	6.1250	7.0000	7.8750
57/64	0.8906	1.7812	2.6719	3.5625	4.4531	5.3437	6.2344	7.1250	8.0156
29/32	0.9062	1.8125	2.7187	3.6250	4.5312	5.4375	6.3437	7.2500	8.1562
59/64	0.9219	1.8437	2.7656	3.6875	4.6094	5.5312	6.4531	7.3750	8.2969
15/16	0.9375	1.8750	2.8125	3.7500	4.6875	5.6250	6.5625	7.5000	8.4375
61/64	0.9531	1.9062	2.8594	3.8125	4.7656	5.7187	6.6719	7.6250	8.5781
31/32	0.9687	1.9375	2.9062	3.8750	4.8437	5.8125	6.7812	7.7500	8.7187
63/64	0.9844	1.9687	2.9531	3.9375	4.9219	5.9062	6.8906	7.8750	8.8594

Multiplication Table for Common Fractions

	1/32	1/16	3/32	1/8	5/32	3/16	7/32	1/4
1/32	0.00098	0.00195	0.00293	0.00391	0.00488	0.00586	0.00684	0.00781
1/16	0.00195	0.00391	0.00586	0.00781	0.00977	0.01172	0.01367	0.01562
3/32	0.00293	0.00586	0.00879	0.01172	0.01465	0.01758	0.02051	0.02344
1/8	0.00391	0.00781	0.01172	0.01562	0.01953	0.02344	0.02734	0.03125
5/32	0.00488	0.00977	0.01465	0.01953	0.02441	0.02930	0.03418	0.03906
3/16	0.00586	0.01172	0.01758	0.02344	0.02930	0.03516	0.04102	0.04687
7/32	0.00684	0.01367	0.02051	0.02734	0.03418	0.04102	0.04785	0.05469
1/4	0.00781	0.01562	0.02344	0.03125	0.03906	0.04687	0.05469	0.06250
9/32	0.00879	0.01758	0.02637	0.03516	0.04394	0.05273	0.06152	0.07031
5/16	0.00977	0.01953	0.02930	0.03906	0.04883	0.05859	0.06836	0.07812
11/32	0.01074	0.02148	0.03223	0.04297	0.05371	0.06445	0.07519	0.08594
3/8	0.01172	0.02344	0.03516	0.04687	0.05859	0.07031	0.08203	0.09375
13/32	0.01269	0.02539	0.03809	0.05078	0.06348	0.07617	0.08887	0.10156
7/16	0.01367	0.02734	0.04102	0.05469	0.06836	0.08203	0.09570	0.10937
15/32	0.01465	0.02930	0.04394	0.05859	0.07324	0.08789	0.10254	0.11719
1/2	0.01562	0.03125	0.04687	0.06250	0.07812	0.09375	0.10937	0.12500
17/32	0.01660	0.03320	0.04980	0.06641	0.08301	0.09961	0.11621	0.13281
9/16	0.01758	0.03516	0.05273	0.07031	0.08789	0.10547	0.12305	0.14062
19/32	0.01855	0.03711	0.05566	0.07422	0.09277	0.11133	0.12988	0.14844
5/8	0.01953	0.03906	0.05859	0.07812	0.09766	0.11719	0.13672	0.15625
21/32	0.02051	0.04102	0.06152	0.08203	0.10254	0.12305	0.14355	0.16406
11/16	0.02148	0.04297	0.06445	0.08594	0.10742	0.12891	0.15039	0.17187
23/32	0.02246	0.04492	0.06738	0.08984	0.11230	0.13477	0.15723	0.17969
3/4	0.02344	0.04687	0.07031	0.09375	0.11719	0.14062	0.16406	0.18750
25/32	0.02441	0.04883	0.07324	0.09766	0.12207	0.14648	0.17090	0.19531
13/16	0.02539	0.05078	0.07617	0.10156	0.12695	0.15234	0.17773	0.20312
27/32	0.02637	0.05273	0.07910	0.10547	0.13184	0.15820	0.18457	0.21094
7/8	0.02734	0.05469	0.08203	0.10937	0.13672	0.16406	0.19141	0.21875
29/32	0.02832	0.05664	0.08496	0.11328	0.14160	0.16992	0.19824	0.22656
15/16	0.02930	0.05859	0.08789	0.11719	0.14648	0.17578	0.20508	0.23437
31/32	0.03027	0.06055	0.09082	0.12109	0.15137	0.18164	0.21191	0.24219
1	0.03125	0.06250	0.09375	0.12500	0.15625	0.18750	0.21875	0.25000

	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2
1/32	0.00879	0.00977	0.01074	0.01172	0.01269	0.01367	0.01465	0.01562
1/16	0.01758	0.01953	0.02148	0.02344	0.02539	0.02734	0.02930	0.03125
3/32	0.02637	0.02930	0.03223	0.03516	0.03809	0.04102	0.04394	0.04687
1/8	0.03516	0.03906	0.04297	0.04687	0.05078	0.05469	0.05859	0.06250
5/32	0.04394	0.04883	0.05371	0.05859	0.06348	0.06836	0.07324	0.07812
3/16	0.05273	0.05859	0.06445	0.07031	0.07617	0.08203	0.08789	0.09375
7/32	0.06152	0.06836	0.07519	0.08203	0.08887	0.09570	0.10254	0.10937
1/4	0.07031	0.07812	0.08594	0.09375	0.10156	0.10937	0.11719	0.12500
9/32	0.07910	0.08789	0.09668	0.10547	0.11426	0.12305	0.13184	0.14062
5/16	0.08789	0.09766	0.10742	0.11719	0.12695	0.13672	0.14648	0.15625
11/32	0.09668	0.10742	0.11816	0.12891	0.13965	0.15039	0.16113	0.17187
3/8	0.10547	0.11719	0.12891	0.14062	0.15234	0.16406	0.17578	0.18750
13/32	0.11426	0.12695	0.13965	0.15234	0.16504	0.17773	0.19043	0.20312
7/16	0.12305	0.13672	0.15039	0.16406	0.17773	0.19141	0.20508	0.21875
15/32	0.13184	0.14648	0.16113	0.17578	0.19043	0.20508	0.21973	0.23437
1/2	0.14062	0.15625	0.17187	0.18750	0.20312	0.21875	0.23437	0.25000
17/32	0.14941	0.16602	0.18262	0.19922	0.21582	0.23242	0.24902	0.26562
9/16	0.15820	0.17578	0.19336	0.21094	0.22852	0.24609	0.26367	0.28125
19/32	0.16699	0.18555	0.20410	0.22266	0.24121	0.25977	0.27832	0.29687
5/8	0.17578	0.19531	0.21484	0.23437	0.25391	0.27344	0.29297	0.31250
21/32	0.18457	0.20508	0.22559	0.24609	0.26660	0.28711	0.30762	0.32812
11/16	0.19336	0.21484	0.23633	0.25781	0.27930	0.30078	0.32227	0.34375
23/32	0.20215	0.22461	0.24707	0.26953	0.29199	0.31445	0.33691	0.35937
3/4	0.21094	0.23437	0.25781	0.28125	0.30469	0.32812	0.35156	0.37500
25/32	0.21973	0.24414	0.26855	0.29297	0.31738	0.34180	0.36621	0.39062
13/16	0.22852	0.25391	0.27930	0.30469	0.33008	0.35547	0.38086	0.40625
27/32	0.23730	0.26367	0.29004	0.31641	0.34277	0.36914	0.39551	0.42187
7/8	0.24609	0.27344	0.30078	0.32812	0.35547	0.38281	0.41016	0.43750
29/32	0.25488	0.28320	0.31152	0.33984	0.36816	0.39648	0.42480	0.45312
15/16	0.26367	0.29297	0.32227	0.35156	0.38086	0.41016	0.43945	0.46875
31/32	0.27246	0.30273	0.33301	0.36328	0.39355	0.42383	0.45410	0.48437
1	0.28125	0.31250	0.34375	0.37500	0.40625	0.43750	0.46875	0.50000

Multiplication Table for Common Fractions

	17/32	9/16	19/32	5/8	21/32	11/16	23/32	3/4
1/32	0.01660	0.01758	0.01855	0.01953	0.02051	0.02148	0.02246	0.02344
1/16	0.03320	0.03516	0.03711	0.03906	0.04102	0.04297	0.04492	0.04687
3/32	0.04980	0.05273	0.05566	0.05859	0.06152	0.06445	0.06738	0.07031
1/8	0.06641	0.07031	0.07422	0.07812	0.08203	0.08594	0.08984	0.09375
5/32	0.08301	0.08789	0.09277	0.09766	0.10254	0.10742	0.11230	0.11719
3/16	0.09961	0.10547	0.11133	0.11719	0.12305	0.12891	0.13477	0.14062
7/32	0.11621	0.12305	0.12988	0.13672	0.14355	0.15039	0.15723	0.16406
1/4	0.13281	0.14062	0.14844	0.15625	0.16406	0.17187	0.17969	0.18750
9/32	0.14941	0.15820	0.16699	0.17578	0.18457	0.19336	0.20215	0.21094
5/16	0.16602	0.17578	0.18555	0.19531	0.20508	0.21484	0.22461	0.23437
11/32	0.18262	0.19336	0.20410	0.21484	0.22559	0.23633	0.24707	0.25781
3/8	0.19922	0.21094	0.22266	0.23437	0.24609	0.25781	0.26953	0.28125
13/32	0.21582	0.22852	0.24121	0.25391	0.26660	0.27930	0.29199	0.30469
7/16	0.23242	0.24609	0.25977	0.27344	0.28711	0.30078	0.31445	0.32812
15/32	0.24902	0.26367	0.27832	0.29297	0.30762	0.32227	0.33691	0.35156
1/2	0.26562	0.28125	0.29687	0.31250	0.32812	0.34375	0.35937	0.37500
17/32	0.28223	0.29883	0.31543	0.33203	0.34863	0.36523	0.38184	0.39844
9/16	0.29883	0.31641	0.33398	0.35156	0.36914	0.38672	0.40430	0.42187
19/32	0.31543	0.33398	0.35254	0.37109	0.38965	0.40820	0.42676	0.44531
5/8	0.33203	0.35156	0.37109	0.39062	0.41016	0.42969	0.44922	0.46875
21/32	0.34863	0.36914	0.38965	0.41016	0.43066	0.45117	0.47168	0.49219
11/16	0.36523	0.38672	0.40820	0.42969	0.45117	0.47266	0.49414	0.51562
23/32	0.38184	0.40430	0.42676	0.44922	0.47168	0.49414	0.51660	0.53906
3/4	0.39844	0.42187	0.44531	0.46875	0.49219	0.51562	0.53906	0.56250
25/32	0.41504	0.43945	0.46387	0.48828	0.51269	0.53711	0.56152	0.58594
13/16	0.43164	0.45703	0.48242	0.50781	0.53320	0.55859	0.58398	0.60937
27/32	0.44824	0.47461	0.50098	0.52734	0.55371	0.58008	0.60644	0.63281
7/8	0.46484	0.49219	0.51953	0.54687	0.57422	0.60156	0.62891	0.65625
29/32	0.48144	0.50977	0.53809	0.56641	0.59473	0.62305	0.65137	0.67969
15/16	0.49805	0.52734	0.55664	0.58594	0.61523	0.64453	0.67383	0.70312
31/32	0.51465	0.54492	0.57519	0.60547	0.63574	0.66602	0.69629	0.72656
1	0.53125	0.56250	0.59375	0.62500	0.65625	0.68750	0.71875	0.75000

	25/32	13/16	27/32	7/8	29/32	15/16	31/32	1
1/32	0.02441	0.02539	0.02637	0.02734	0.02832	0.02930	0.03027	0.03125
1/16	0.04883	0.05078	0.05273	0.05469	0.05664	0.05859	0.06055	0.06250
3/32	0.07324	0.07617	0.07910	0.08203	0.08496	0.08789	0.09082	0.09375
1/8	0.09766	0.10156	0.10547	0.10937	0.11328	0.11719	0.12109	0.12500
5/32	0.12207	0.12695	0.13184	0.13672	0.14160	0.14648	0.15137	0.15625
3/16	0.14648	0.15234	0.15820	0.16406	0.16992	0.17578	0.18164	0.18750
7/32	0.17090	0.17773	0.18457	0.19141	0.19824	0.20508	0.21191	0.21875
1/4	0.19531	0.20312	0.21094	0.21875	0.22656	0.23437	0.24219	0.25000
9/32	0.21973	0.22852	0.23730	0.24609	0.25488	0.26367	0.27246	0.28125
5/16	0.24414	0.25391	0.26367	0.27344	0.28320	0.29297	0.30273	0.31250
11/32	0.26855	0.27930	0.29004	0.30078	0.31152	0.32227	0.33301	0.34375
3/8	0.29297	0.30469	0.31641	0.32812	0.33984	0.35156	0.36328	0.37500
13/32	0.31738	0.33008	0.34277	0.35547	0.36816	0.38086	0.39355	0.40625
7/16	0.34180	0.35547	0.36914	0.38281	0.39648	0.41016	0.42383	0.43750
15/32	0.36621	0.38086	0.39551	0.41016	0.42480	0.43945	0.45410	0.46875
1/2	0.39062	0.40625	0.42187	0.43750	0.45312	0.46875	0.48437	0.50000
17/32	0.41504	0.43164	0.44824	0.46484	0.48144	0.49805	0.51465	0.53125
9/16	0.43945	0.45703	0.47461	0.49219	0.50977	0.52734	0.54492	0.56250
19/32	0.46387	0.48242	0.50098	0.51953	0.53809	0.55664	0.57519	0.59375
5/8	0.48828	0.50781	0.52734	0.54687	0.56641	0.58594	0.60547	0.62500
21/32	0.51269	0.53320	0.55371	0.57422	0.59473	0.61523	0.63574	0.65625
11/16	0.53711	0.55859	0.58008	0.60156	0.62305	0.64453	0.66602	0.68750
23/32	0.56152	0.58398	0.60644	0.62891	0.65137	0.67383	0.69629	0.71875
3/4	0.58594	0.60937	0.63281	0.65625	0.67969	0.70312	0.72656	0.75000
25/32	0.61035	0.63477	0.65918	0.68359	0.70801	0.73242	0.75684	0.78125
13/16	0.63477	0.66016	0.68555	0.71094	0.73633	0.76172	0.78711	0.81250
27/32	0.65918	0.68555	0.71191	0.73828	0.76465	0.79102	0.81738	0.84375
7/8	0.68359	0.71094	0.73828	0.76562	0.79297	0.82031	0.84766	0.87500
29/32	0.70801	0.73633	0.76465	0.79297	0.82129	0.84961	0.87793	0.90625
15/16	0.73242	0.76172	0.79102	0.82031	0.84961	0.87891	0.90820	0.93750
31/32	0.75684	0.78711	0.81738	0.84766	0.87793	0.90820	0.93848	0.96875
1	0.78125	0.81250	0.84375	0.87500	0.90625	0.93750	0.96875	1.00000

Circular Mil Gage for Electrical Wires

Gage Number	Circular Mils	Diam. in Mils	Gage Number	Circular Mils	Diam. in Mils	Gage Number	Circular Mils	Diam. in Mils
3	3,000	54.78	65	65,000	254.96	170	170,000	412.32
5	5,000	70.72	70	70,000	264.58	180	180,000	424.27
8	8,000	89.45	75	75,000	273.87	190	190,000	435.89
12	12,000	109.55	80	80,000	282.85	200	200,000	447.22
15	15,000	122.48	85	85,000	291.55	220	220,000	469.05
20	20,000	141.43	90	90,000	300.00	240	240,000	489.90
25	25,000	158.12	95	95,000	308.23	260	260,000	509.91
30	30,000	173.21	100	100,000	316.23	280	280,000	529.16
35	35,000	187.09	110	110,000	331.67	300	300,000	547.73
40	40,000	200.00	120	120,000	346.42	320	320,000	565.69
45	45,000	212.14	130	130,000	360.56	340	340,000	583.10
50	50,000	223.61	140	140,000	374.17	360	360,000	600.00
55	55,000	234.53	150	150,000	387.30	.....	.....	......
60	60,000	244.95	160	160,000	400.00	.....	.....	......

METRIC SYSTEM OF MEASUREMENTS

In the metric system of measurements, the principal unit for length is the meter; the principal unit for capacity, the liter; and the principal unit for weight, the gram. The following prefixes are used for sub-divisions and multiples: milli = 1/1000; centi = 1/100; deci = 1/10; deca = 10; hecto = 100; kilo = 1000. In abbreviations, the sub-divisions are frequently used with a small letter and the multiples with a capital letter, although this practice is not universally followed everywhere where the metric system is used.

All the multiples and sub-divisions are not used commercially. Those ordinarily used for length are kilometer, meter, centimeter and millimeter; for capacity, square meter, square centimeter and square millimeter; for cubic measures, cubic meter, cubic decimeter (liter), cubic centimeter, and cubic millimeter. The most commonly used weights are the kilogram and gram. The metric system was legalized in the United States by an Act of Congress in 1866.

Measures of Length

10 millimeters (mm.)	= 1 centimeter (cm.).
10 centimeters	= 1 decimeter (dm.).
10 decimeters	= 1 meter (m.).
1000 meters	= 1 kilometer (Km.).

Square Measure

100 square millimeters (mm.^{2})	= 1 square centimeter (cm.^{2}).
100 square centimeters	= 1 square decimeter (dm.^{2}).
100 square decimeters	= 1 square meter (m.^{2}).

Surveyor's Square Measure

100 square meters (m.^{2})	= 1 are (ar.).
100 ares	= 1 hectare (har.).
100 hectares	= 1 square kilometer (Km.^{2}).

Cubic Measure

1000 cubic millimeters ($mm.^3$) = 1 cubic centimeter ($cm.^3$).
1000 cubic centimeters = 1 cubic decimeter ($dm.^3$).
1000 cubic decimeters = 1 cubic meter ($m.^3$).

Dry and Liquid Measure

10 milliliters (ml.) = 1 centiliter (cl.).
10 centiliters = 1 deciliter (dl.).
10 deciliters = 1 liter (l.).
100 liters = 1 hectoliter (Hl.).

1 liter = 1 cubic decimeter = the volume of 1 kilogram of pure water at a temperature of 39.2 degrees F.

Measures of Weight

10 milligrams (mg.) = 1 centigram (cg.).
10 centigrams = 1 decigram (dg.).
10 decigrams = 1 gram (g.).
10 grams = 1 decagram (Dg.).
10 decagrams = 1 hectogram (Hg.).
10 hectograms = 1 kilogram (Kg.).
1000 kilograms = 1 (metric) ton (T.).

Metric and English Conversion Table

Linear Measure

1 kilometer = 0.6214 mile.
1 meter = 39.37 inches. 3.2808 feet. 1.0936 yard.
1 centimeter = 0.3937 inch.
1 millimeter = 0.03937 inch.

1 mile = 1.609 kilometer.
1 yard = 0.9144 meter.
1 foot = 0.3048 meter.
1 foot = 304.8 millimeters.
1 inch = 2.54 centimeters.
1 inch = 25.4 millimeters.

Square Measure

1 square kilometer = 0.3861 square mile = 247.1 acres.
1 hectare = 2.471 acre = 107,640 square feet.
1 are = 0.0247 acre = 1076.4 square feet.
1 square meter = 10.764 square feet = 1.196 square yard.
1 square centimeter = 0.155 square inch.
1 square millimeter = 0.00155 square inch.

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1 square mile = 2.5899 square kilometers.
1 acre = 0.4047 hectare = 40.47 ares.
1 square yard = 0.836 square meter.
1 square foot = 0.0929 square meter = 929 square centimeters.
1 square inch = 6.452 square centimeters = 645.2 square millimeters.

Cubic Measure

1 cubic meter = 35.314 cubic feet = 1.308 cubic yard.
1 cubic meter = 264.2 U. S. gallons.
1 cubic centimeter = 0.061 cubic inch.
1 liter (cubic decimeter) = 0.0353 cubic foot = 61.023 cubic inches.
1 liter = 0.2642 U. S. gallon = 1.0567 U. S. quart.

1 cubic yard = 0.7645 cubic meter.
1 cubic foot = 0.02832 cubic meter = 28.317 liters.
1 cubic inch = 16.383 cubic centimeters.
1 U. S. gallon = 3.785 liters.
1 U. S. quart = 0.946 liter.

Weight

1 metric ton = 0.9842 ton (of 2240 pounds) = 2204.6 pounds.
1 kilogram = 2.2046 pounds = 35.274 ounces avoirdupois.
1 gram = 0.03215 ounce troy = 0.03527 ounce avoirdupois.
1 gram = 15.432 grains.

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1 ton (of 2240 pounds) = 1.016 metric ton = 1016 kilograms.
1 pound = 0.4536 kilogram = 453.6 grams.
1 ounce avoirdupois = 28.35 grams.
1 ounce troy = 31.103 grams.
1 grain = 0.0648 gram.

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1 kilogram per square millimeter = 1422.32 pounds per square inch.
1 kilogram per square centimeter = 14.223 pounds per square inch.
1 kilogram-meter = 7.233 foot-pounds.
1 pound per square inch = 0.0703 kilogram per square centimeter.
1 calorie (kilogram calorie) = 3.968 B.T.U. (British thermal unit).

The C.G.S. System of Measurement

The C.G.S. (centimeter-gram-second) system, frequently known as the absolute system of measurement, is based upon the length and weight units of the metric system, and the second as the time unit. In this system, the unit of distance is one centimeter, the unit of mass (or weight) is one gram, and the unit of time, one second. From these fundamental units are derived:

Unit of velocity = 1 centimeter in one second.
Acceleration due to gravity (at Paris) = 981 centimeters in one second.
Unit of force = 1 dyne = $1/981$ gram.
Unit of work = 1 erg = 1 dyne-centimeter.
Unit of power = 1 watt = 10,000,000 ergs per second.

The C.G.S. system of power measurements is becoming more and more used in the engineering field. It is used exclusively for electrical machines and apparatus on account of the simple relationship which exists between the various units. It is likely to be soon adopted in many other fields. The unit of work, erg, is so small that in practical work the joule is usually employed instead. One joule equals 10,000,000 ergs.

Standard of Length. — In 1866 the United States, by act of Congress, passed a law making legal the meter, the only measure of length that has been legalized by the United States Government. The United States yard is defined by the relation: 1 yard = $\frac{3600}{3937}$ meter. The legal equivalent of the meter for commercial purposes was fixed as 39.37 inches, by law, in July, 1866, and experience having shown that this value was exact within the error of observation, the United States

Office of Standard Weights and Measures was, in 1893, authorized to derive the yard from the meter by the use of this relation. The United States prototype meters Nos. 27 and 21 were received from the International Bureau of Weights and Measures in 1889. Meter No. 27, sealed in its metal case, is preserved in a fire-proof vault at the Bureau of Standards.

Comparisons made prior to 1893 indicated that the relation of the yard to the meter, fixed by the act of 1866, was by chance the exact relation between the international meter and the British imperial yard, within the error of observation. A subsequent comparison made between the standards just mentioned indicates that the legal relation adopted by Congress is in error 0.0001 inch; but, in view of the fact that certain comparisons made by the English Standards Office between the imperial yard and its authentic copies show variations as great if not greater than this, it cannot be said with certainty that there is a difference between the imperial yard of Great Britain and the United States yard derived from the meter. The bronze yard No. 11, which was an exact copy of the British imperial yard both in form and material, had shown changes when compared with the imperial yard in 1876 and 1888, which could not reasonably be said to be entirely due to changes in Bronze No. 11. On the other hand, the new meters represented the most advanced ideas of standards, and it therefore seemed that greater stability as well as higher accuracy would be secured by accepting the international meter as a fundamental standard of length.

Application of the Metric System. — In the practical application of the metric system in machine shop and drafting-room work, the part of the system with which the draftsman and machinist come into direct contact is the length measurements. The length units of the metric system that are most generally used in connection with any work relating to mechanical engineering are the meter, the centimeter, and the millimeter. The decimeter is not commonly used as a length measurement. On mechanical drawings all dimensions are generally given in millimeters, no matter how large they may be. In fact, dimensions of such machines as locomotives and large electrical apparatus are given exclusively in millimeters. This practice is adopted to avoid mistakes due to misplacing decimal points, or misreading dimensions if other units are used as well. When dimensions are given in millimeters, the majority can be given without resorting to decimal points, as a millimeter is only a trifle more than 1/32 inch. Only dimensions of precision need be given in decimals of a millimeter; such dimensions are generally given in hundredths of a millimeter — for example, 0.02 millimeter. As 0.01 millimeter is equal to 0.0004 inch, it is seldom that dimensions would be given with greater accuracy than to hundredths of a millimeter.

Drawings made to the metric system are not made to scales of 1/2, 1/4, 1/8, etc., as in the case of drawings made to the English system. If the object cannot be drawn full size, it is generally drawn one-fifth size, and, if this is too large, it is drawn one-tenth size. In exceptional cases, when very large objects are to be shown on a drawing, scales of one-twentieth, one-fiftieth, and one-one-hundredth may be used.

Tables of Metric Equivalents. — The following tables for converting millimeters to inches are based on the equivalent of the meter as legalized by the United States Government, according to which 1 meter = 39.37 inches and 1 inch = 25.4000508 + or practically 25.4 millimeters. The tables of equivalents for millimeters and inches published in previous editions of MACHINERY'S HANDBOOK are based on the metric equivalent of the British Standard yard, which differs from the United States legal standard by about 0.0001 inch. The actual error due to this difference is so slight as to be negligible in practically all cases.

Table for Converting Millimeters into Inches

Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches
1	0.0394	51	2.0079	101	3.9764	151	5.9449	201	7.9134
2	0.0787	52	2.0472	102	4.0157	152	5.9842	202	7.9527
3	0.1181	53	2.0866	103	4.0551	153	6.0236	203	7.9921
4	0.1575	54	2.1260	104	4.0945	154	6.0630	204	8.0315
5	0.1968	55	2.1653	105	4.1338	155	6.1023	205	8.0708
6	0.2362	56	2.2047	106	4.1732	156	6.1417	206	8.1102
7	0.2756	57	2.2441	107	4.2126	157	6.1811	207	8.1496
8	0.3150	58	2.2835	108	4.2520	158	6.2205	208	8.1890
9	0.3543	59	2.3228	109	4.2913	159	6.2598	209	8.2283
10	0.3937	60	2.3622	110	4.3307	160	6.2992	210	8.2677
11	0.4331	61	2.4016	111	4.3701	161	6.3386	211	8.3071
12	0.4724	62	2.4409	112	4.4094	162	6.3779	212	8.3464
13	0.5118	63	2.4803	113	4.4488	163	6.4173	213	8.3858
14	0.5512	64	2.5197	114	4.4882	164	6.4567	214	8.4252
15	0.5905	65	2.5590	115	4.5275	165	6.4960	215	8.4645
16	0.6299	66	2.5984	116	4.5669	166	6.5354	216	8.5039
17	0.6693	67	2.6378	117	4.6063	167	6.5748	217	8.5433
18	0.7087	68	2.6772	118	4.6457	168	6.6142	218	8.5827
19	0.7480	69	2.7165	119	4.6850	169	6.6535	219	8.6220
20	0.7874	70	2.7559	120	4.7244	170	6.6929	220	8.6614
21	0.8268	71	2.7953	121	4.7638	171	6.7323	221	8.7008
22	0.8661	72	2.8346	122	4.8031	172	6.7716	222	8.7401
23	0.9055	73	2.8740	123	4.8425	173	6.8110	223	8.7795
24	0.9449	74	2.9134	124	4.8819	174	6.8504	224	8.8189
25	0.9842	75	2.9527	125	4.9212	175	6.8897	225	8.8582
26	1.0236	76	2.9921	126	4.9606	176	6.9291	226	8.8976
27	1.0630	77	3.0315	127	5.0000	177	6.9685	227	8.9370
28	1.1024	78	3.0709	128	5.0394	178	7.0079	228	8.9764
29	1.1417	79	3.1102	129	5.0787	179	7.0472	229	9.0157
30	1.1811	80	3.1496	130	5.1181	180	7.0866	230	9.0551
31	1.2205	81	3.1890	131	5.1575	181	7.1260	231	9.0945
32	1.2598	82	3.2283	132	5.1968	182	7.1653	232	9.1338
33	1.2992	83	3.2677	133	5.2362	183	7.2047	233	9.1732
34	1.3386	84	3.3071	134	5.2756	184	7.2441	234	9.2126
35	1.3779	85	3.3464	135	5.3149	185	7.2834	235	9.2519
36	1.4173	86	3.3858	136	5.3543	186	7.3228	236	9.2913
37	1.4567	87	3.4252	137	5.3937	187	7.3622	237	9.3307
38	1.4961	88	3.4646	138	5.4331	188	7.4016	238	9.3701
39	1.5354	89	3.5039	139	5.4724	189	7.4409	239	9.4094
40	1.5748	90	3.5433	140	5.5118	190	7.4803	240	9.4488
41	1.6142	91	3.5827	141	5.5512	191	7.5197	241	9.4882
42	1.6535	92	3.6220	142	5.5905	192	7.5590	242	9.5275
43	1.6929	93	3.6614	143	5.6299	193	7.5984	243	9.5669
44	1.7323	94	3.7008	144	5.6693	194	7.6378	244	9.6063
45	1.7716	95	3.7401	145	5.7086	195	7.6771	245	9.6456
46	1.8110	96	3.7795	146	5.7480	196	7.7165	246	9.6850
47	1.8504	97	3.8189	147	5.7874	197	7.7559	247	9.7244
48	1.8898	98	3.8583	148	5.8268	198	7.7953	248	9.7638
49	1.9291	99	3.8976	149	5.8661	199	7.8346	249	9.8031
50	1.9685	100	3.9370	150	5.9055	200	7.8740	250	9.8425

Table for Converting Millimeters into Inches (Continued)

Milli-meters	Inches	Milli-meters	Inches	Milli-meters	Inches	Milli-meters	Inches	Milli-meters	Inches
251	9.8819	301	11.8504	351	13.8189	401	15.7874	451	17.7559
252	9.9212	302	11.8897	352	13.8582	402	15.8267	452	17.7952
253	9.9606	303	11.9291	353	13.8976	403	15.8661	453	17.8346
254	10.0000	304	11.9685	354	13.9370	404	15.9055	454	17.8740
255	10.0393	305	12.0078	355	13.9763	405	15.9448	455	17.9133
256	10.0787	306	12.0472	356	14.0157	406	15.9842	456	17.9527
257	10.1181	307	12.0866	357	14.0551	407	16.0236	457	17.9921
258	10.1575	308	12.1260	358	14.0945	408	16.0630	458	18.0315
259	10.1968	309	12.1653	359	14.1338	409	16.1023	459	18.0738
260	10.2362	310	12.2047	360	14.1732	410	16.1417	460	18.1102
261	10.2756	311	12.2441	361	14.2126	411	16.1811	461	18.1496
262	10.3149	312	12.2834	362	14.2519	412	16.2204	462	18.1889
263	10.3543	313	12.3228	363	14.2913	413	16.2598	463	18.2283
264	10.3937	314	12.3622	364	14.3307	414	16.2992	464	18.2677
265	10.4330	315	12.4015	365	14.3700	415	16.3385	465	18.3070
266	10.4724	316	12.4409	366	14.4094	416	16.3779	466	18.3464
267	10.5118	317	12.4803	367	14.4488	417	16.4173	467	18.3858
268	10.5512	318	12.5197	368	14.4882	418	16.4567	468	18.4252
269	10.5905	319	12.5590	369	14.5275	419	16.4960	469	18.4645
270	10.6299	320	12.5984	370	14.5669	420	16.5354	470	18.5039
271	10.6693	321	12.6378	371	14.6063	421	16.5748	471	18.5433
272	10.7086	322	12.6771	372	14.6456	422	16.6141	472	18.5826
273	10.7480	323	12.7165	373	14.6850	423	16.6535	473	18.6220
274	10.7874	324	12.7559	374	14.7244	424	16.6929	474	18.6614
275	10.8267	325	12.7952	375	14.7637	425	16.7322	475	18.7007
276	10.8661	326	12.8346	376	14.8031	426	16.7716	476	18.7401
277	10.9055	327	12.8740	377	14.8425	427	16.8110	477	18.7795
278	10.9449	328	12.9134	378	14.8819	428	16.8504	478	18.8189
279	10.9842	329	12.9527	379	14.9212	429	16.8897	479	18.8582
280	11.0236	330	12.9921	380	14.9606	430	16.9291	480	18.8976
281	11.0630	331	13.0315	381	15.0000	431	16.9685	481	18.9370
282	11.1023	332	13.0708	382	15.0393	432	17.0078	482	18.9763
283	11.1417	333	13.1102	383	15.0787	433	17.0472	483	19.0157
284	11.1811	334	13.1496	384	15.1181	434	17.0866	484	19.0551
285	11.2204	335	13.1889	385	15.1574	435	17.1259	485	19.0944
286	11.2598	336	13.2283	386	15.1968	436	17.1653	486	19.1338
287	11.2992	337	13.2677	387	15.2362	437	17.2047	487	19.1732
288	11.3386	338	13.3071	388	15.2756	438	17.2441	488	19.2126
289	11.3779	339	13.3464	389	15.3149	439	17.2834	489	19.2519
290	11.4173	340	13.3858	390	15.3543	440	17.3228	490	19.2913
291	11.4567	341	13.4252	391	15.3937	441	17.3622	491	19.3307
292	11.4960	342	13.4645	392	15.4330	442	17.4015	492	19.3700
293	11.5354	343	13.5039	393	15.4724	443	17.4409	493	19.4094
294	11.5748	344	13.5433	394	15.5118	444	17.4803	494	19.4488
295	11.6141	345	13.5826	395	15.5511	445	17.5196	495	19.4881
296	11.6535	346	13.6220	396	15.5905	446	17.5590	496	19.5275
297	11.6929	347	13.6614	397	15.6299	447	17.5984	497	19.5669
298	11.7323	348	13.7008	398	15.6693	448	17.6378	498	19.6063
299	11.7716	349	13.7401	399	15.7086	449	17.6771	499	19.6456
300	11.8110	350	13.7795	400	15.7480	450	17.7165	500	19.6850

Table for Converting Millimeters into Inches (Continued)

Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches
501	19.7244	551	21.6929	601	23.6614	651	25.6299	701	27.5984
502	19.7637	552	21.7322	602	23.7007	652	25.6692	702	27.6377
503	19.8031	553	21.7716	603	23.7401	653	25.7086	703	27.6771
504	19.8425	554	21.8110	604	23.7795	654	25.7480	704	27.7165
505	19.8818	555	21.8503	605	23.8188	655	25.7873	705	27.7558
506	19.9212	556	21.8897	606	23.8582	656	25.8267	706	27.7952
507	19.9606	557	21.9291	607	23.8976	657	25.8661	707	27.8346
508	20.0000	558	21.9685	608	23.9370	658	25.9055	708	27.8740
509	20.0393	559	22.0078	609	23.9763	659	25.9448	709	27.9133
510	20.0787	560	22.0472	610	24.0157	660	25.9842	710	27.9527
511	20.1181	561	22.0866	611	24.0551	661	26.0236	711	27.9921
512	20.1574	562	22.1259	612	24.0944	662	26.0629	712	28.0314
513	20.1968	563	22.1653	613	24.1338	663	26.1023	713	28.0708
514	20.2362	564	22.2047	614	24.1732	664	26.1417	714	28.1102
515	20.2755	565	22.2440	615	24.2125	665	26.1810	715	28.1495
516	20.3149	566	22.2834	616	24.2519	666	26.2204	716	28.1889
517	20.3543	567	22.3228	617	24.2913	667	26.2598	717	28.2283
518	20.3937	568	22.3622	618	24.3307	668	26.2992	718	28.2677
519	20.4330	569	22.4015	619	24.3700	669	26.3385	719	28.3070
520	20.4724	570	22.4409	620	24.4094	670	26.3779	720	28.3464
521	20.5118	571	22.4803	621	24.4488	671	26.4173	721	28.3858
522	20.5511	572	22.5196	622	24.4881	672	26.4566	722	28.4251
523	20.5905	573	22.5590	623	24.5275	673	26.4960	723	28.4645
524	20.6299	574	22.5984	624	24.5669	674	26.5354	724	28.5039
525	20.6692	575	22.6377	625	24.6062	675	26.5747	725	28.5432
526	20.7086	576	22.6771	626	24.6456	676	26.6141	726	28.5826
527	20.7480	577	22.7165	627	24.6850	677	26.6535	727	28.6220
528	20.7874	578	22.7559	628	24.7244	678	26.6929	728	28.6614
529	20.8267	579	22.7952	629	24.7637	679	26.7322	729	28.7007
530	20.8661	580	22.8346	630	24.8031	680	26.7716	730	28.7401
531	20.9055	581	22.8740	631	24.8425	681	26.8110	731	28.7795
532	20.9448	582	22.9133	632	24.8818	682	26.8503	732	28.8188
533	20.9842	583	22.9527	633	24.9212	683	26.8897	733	28.8582
534	21.0236	584	22.9921	634	24.9606	684	26.9291	734	28.8976
535	21.0629	585	23.0314	635	24.9999	685	26.9684	735	28.9369
536	21.1023	586	23.0708	636	25.0393	686	27.0078	736	28.9763
537	21.1417	587	23.1102	637	25.0787	687	27.0472	737	29.0157
538	21.1811	588	23.1496	638	25.1181	688	27.0866	738	29.0551
539	21.2204	589	23.1889	639	25.1574	689	27.1259	739	29.0944
540	21.2598	590	23.2283	640	25.1968	690	27.1653	740	29.1338
541	21.2992	591	23.2677	641	25.2362	691	27.2047	741	29.1732
542	21.3385	592	23.3070	642	25.2755	692	27.2440	742	29.2125
543	21.3779	593	23.3464	643	25.3149	693	27.2834	743	29.2519
544	21.4173	594	23.3858	644	25.3543	694	27.3228	744	29.2913
545	21.4566	595	23.4251	645	25.3936	695	27.3621	745	29.3307
546	21.4960	596	23.4645	646	25.4330	696	27.4015	746	29.3700
547	21.5354	597	23.5039	647	25.4724	697	27.4409	747	29.4094
548	21.5748	598	23.5433	648	25.5118	698	27.4803	748	29.4487
549	21.6141	599	23.5826	649	25.5511	699	27.5196	749	29.4881
550	21.6535	600	23.6220	650	25.5905	700	27.5590	750	29.5275

Table for Converting Millimeters into Inches (Continued)

Milli-meters	Inches	Milli-meters	Inches	Milli-meters	Inches	Milli-meters	Inches	Milli-meters	Inches
751	29.5669	801	31.5354	851	33.5039	901	35.4728	951	37.4409
752	29.6062	802	31.5747	852	33.5432	902	35.5117	952	37.4802
753	29.6456	803	31.6141	853	33.5826	903	35.5511	953	37.5196
754	29.6850	804	31.6535	854	33.6220	904	35.5905	954	37.5590
755	29.7243	805	31.6928	855	33.6613	905	35.6298	955	37.5983
756	29.7637	806	31.7322	856	33.7007	906	35.6692	956	37.6377
757	29.8031	807	31.7716	857	33.7401	907	35.7086	957	37.6771
758	29.8425	808	31.8110	858	33.7795	908	35.7480	958	37.7165
759	29.8818	809	31.8503	859	33.8188	909	35.7873	959	37.7558
760	29.9212	810	31.8897	860	33.8582	910	35.8267	960	37.7952
761	29.9606	811	31.9291	861	33.8976	911	35.8661	961	37.8346
762	29.9999	812	31.9684	862	33.9369	912	35.9054	962	37.8739
763	30.0393	813	32.0078	863	33.9763	913	35.9448	963	37.9133
764	30.0787	814	32.0472	864	34.0157	914	35.9842	964	37.9527
765	30.1180	815	32.0865	865	34.0550	915	36.0235	965	37.9920
766	30.1574	816	32.1259	866	34.0944	916	36.0629	966	38.0314
767	30.1968	817	32.1653	867	34.1338	917	36.1023	967	38.0708
768	30.2362	818	32.2047	868	34.1732	918	36.1417	968	38.1102
769	30.2755	819	32.2440	869	34.2125	919	36.1810	969	38.1495
770	30.3149	820	32.2834	870	34.2519	920	36.2204	970	38.1889
771	30.3543	821	32.3228	871	34.2913	921	36.2598	971	38.2283
772	30.3936	822	32.3621	872	34.3306	922	36.2991	972	38.2676
773	30.4330	823	32.4015	873	34.3700	923	36.3385	973	38.3070
774	30.4724	824	32.4409	874	34.4094	924	36.3779	974	38.3464
775	30.5117	825	32.4802	875	34.4487	925	36.4172	975	38.3857
776	30.5511	826	32.5196	876	34.4881	926	36.4566	976	38.4251
777	30.5905	827	32.5590	877	34.5275	927	36.4960	977	38.4645
778	30.6299	828	32.5984	878	34.5669	928	36.5354	978	38.5039
779	30.6692	829	32.6377	879	34.6062	929	36.5747	979	38.5432
780	30.7086	830	32.6771	880	34.6456	930	36.6141	980	38.5826
781	30.7480	831	32.7165	881	34.6850	931	36.6535	981	38.6220
782	30.7873	832	32.7558	882	34.7243	932	36.6928	982	38.6613
783	30.8267	833	32.7952	883	34.7637	933	36.7322	983	38.7007
784	30.8661	834	32.8346	884	34.8031	934	36.7716	984	38.7401
785	30.9054	835	32.8739	885	34.8424	935	36.8109	985	38.7794
786	30.9448	836	32.9133	886	34.8818	936	36.8503	986	38.8188
787	30.9842	837	32.9527	887	34.9212	937	36.8897	987	38.8582
788	31.0236	838	32.9921	888	34.9606	938	36.9291	988	38.8976
789	31.0629	839	33.0314	889	34.9999	939	36.9684	989	38.9369
790	31.1023	840	33.0708	890	35.0393	940	37.0078	990	38.9763
791	31.1417	841	33.1102	891	35.0787	941	37.0472	991	39.0157
792	31.1810	842	33.1495	892	35.1180	942	37.0865	992	39.0550
793	31.2204	843	33.1889	893	35.1574	943	37.1259	993	39.0944
794	31.2598	844	33.2283	894	35.1968	944	37.1653	994	39.1338
795	31.2991	845	33.2676	895	35.2361	945	37.2046	995	39.1731
796	31.3385	846	33.3070	896	35.2755	946	37.2440	996	39.2125
797	31.3779	847	33.3464	897	35.3149	947	37.2834	997	39.2519
798	31.4173	848	33.3858	898	35.3543	948	37.3228	998	39.2913
799	31.4566	849	33.4251	899	35.3936	949	37.3621	999	39.3306
800	31.4960	850	33.4645	900	35.4330	950	37.4015	1000	39.3700

Inches into Millimeters

Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters
1/64	0.3969	51/64	20.2406	2 5/32	54.7688	3 23/32	94.4564	5 9/32	134.144
1/32	0.7937	13/16	20.6375	2 3/16	55.5626	3 3/4	95.2502	5 5/16	134.938
3/64	1.1906	53/64	21.0344	2 7/32	56.3564	3 25/32	96.0439	5 11/32	135.732
1/16	1.5875	27/32	21.4312	2 1/4	57.1501	3 13/16	96.8377	5 3/8	136.525
5/64	1.9844	55/64	21.8281	2 9/32	57.9439	3 27/32	97.6314	5 13/32	137.319
3/32	2.3812	7/8	22.2250	2 5/16	58.7376	3 7/8	98.4252	5 7/16	138.113
7/64	2.7781	57/64	22.6219	2 11/32	59.5314	3 29/32	99.2189	5 15/32	138.907
1/8	3.1750	29/32	23.0187	2 3/8	60.3251	3 15/16	100.013	5 1/2	139.700
9/64	3.5719	59/64	23.4156	2 13/32	61.1189	3 31/32	100.806	5 17/32	140.494
5/32	3.9687	15/16	23.8125	2 7/16	61.9126	4	101.600	5 9/16	141.288
11/64	4.3656	61/64	24.2094	2 15/32	62.7064	4 1/32	102.394	5 19/32	142.082
3/16	4.7625	31/32	24.6062	2 1/2	63.5001	4 1/16	103.188	5 5/8	142.875
13/64	5.1594	63/64	25.0031	2 17/32	64.2939	4 3/32	103.981	5 21/32	143.669
7/32	5.5562	1	25.4001	2 9/16	65.0876	4 1/8	104.775	5 11/16	144.463
15/64	5.9531	1 1/32	26.1938	2 19/32	65.8814	4 5/32	105.569	5 23/32	145.257
1/4	6.3500	1 1/16	26.9876	2 5/8	66.6751	4 3/16	106.363	5 3/4	146.050
17/64	6.7469	1 3/32	27.7813	2 21/32	67.4689	4 7/32	107.156	5 25/32	146.844
9/32	7.1437	1 1/8	28.5751	2 11/16	68.2626	4 1/4	107.950	5 13/16	147.638
19/64	7.5406	1 5/32	29.3688	2 23/32	69.0564	4 9/32	108.744	5 27/32	148.432
5/16	7.9375	1 3/16	30.1626	2 3/4	69.8501	4 5/16	109.538	5 7/8	149.225
21/64	8.3344	1 7/32	30.9563	2 25/32	70.6439	4 11/32	110.331	5 29/32	150.019
11/32	8.7312	1 1/4	31.7501	2 13/16	71.4376	4 3/8	111.125	5 15/16	150.813
23/64	9.1281	1 9/32	32.5438	2 27/32	72.2314	4 13/32	111.919	5 31/32	151.607
3/8	9.5250	1 5/16	33.3376	2 7/8	73.0251	4 7/16	112.713	6	152.400
25/64	9.9219	1 11/32	34.1313	2 29/32	73.8189	4 15/32	113.506	6 1/16	153.988
13/32	10.3187	1 3/8	34.9251	2 15/16	74.6126	4 1/2	114.300	6 1/8	155.575
27/64	10.7156	1 13/32	35.7188	2 31/32	75.4064	4 17/32	115.094	6 3/16	157.163
7/16	11.1125	1 7/16	36.5126	3	76.2002	4 9/16	115.888	6 1/4	158.750
29/64	11.5094	1 15/32	37.3063	3 1/32	76.9939	4 19/32	116.681	6 5/16	160.338
15/32	11.9062	1 1/2	38.1001	3 1/16	77.7877	4 5/8	117.475	6 3/8	161.925
31/64	12.3031	1 17/32	38.8938	3 3/32	78.5814	4 21/32	118.269	6 7/16	163.513
1/2	12.7000	1 9/16	39.6876	3 1/8	79.3752	4 11/16	119.063	6 1/2	165.100
33/64	13.0969	1 19/32	40.4813	3 5/32	80.1689	4 23/32	119.856	6 9/16	166.688
17/32	13.4937	1 5/8	41.2751	3 3/16	80.9627	4 3/4	120.650	6 5/8	168.275
35/64	13.8906	1 21/32	42.0688	3 7/32	81.7564	4 25/32	121.444	6 11/16	169.863
9/16	14.2875	1 11/16	42.8626	3 1/4	82.5502	4 13/16	122.238	6 3/4	171.450
37/64	14.6844	1 23/32	43.6563	3 9/32	83.3439	4 27/32	123.031	6 13/16	173.038
19/32	15.0812	1 3/4	44.4501	3 5/16	84.1377	4 7/8	123.825	6 7/8	174.625
39/64	15.4781	1 25/32	45.2438	3 11/32	84.9314	4 29/32	124.619	6 15/16	176.213
5/8	15.8750	1 13/16	46.0376	3 3/8	85.7252	4 15/16	125.413	7	177.800
41/64	16.2719	1 27/32	46.8313	3 13/32	86.5189	4 31/32	126.206	7 1/16	179.388
21/32	16.6687	1 7/8	47.6251	3 7/16	87.3127	5	127.000	7 1/8	180.975
43/64	17.0656	1 29/32	48.4188	3 15/32	88.1064	5 1/32	127.794	7 3/16	182.563
11/16	17.4625	1 15/16	49.2126	3 1/2	88.9002	5 1/16	128.588	7 1/4	184.150
45/64	17.8594	1 31/32	50.0063	3 17/32	89.6939	5 3/32	129.382	7 5/16	185.738
23/32	18.2562	2	50.8001	3 9/16	90.4877	5 1/8	130.175	7 3/8	187.325
47/64	18.6531	2 1/32	51.5939	3 19/32	91.2814	5 5/32	130.969	7 7/16	188.913
3/4	19.0500	2 1/16	52.3876	3 5/8	92.0752	5 3/16	131.763	7 1/2	190.500
49/64	19.4469	2 3/32	53.1814	3 21/32	92.8689	5 7/32	132.557	7 9/16	192.088
25/32	19.8437	2 1/8	53.9751	3 11/16	93.6627	5 1/4	133.350	7 5/8	193.675

Feet and Inches into Millimeters

Inches	Millimeters	Inches	Millimeters	Ft.	In.	Millimeters	Ft.	In.	Millimeters	Feet	Millimeters
7 11/16	195.263	10 13/16	274.638	3	7	1092.20	7	9	2362.20	33	10,058.4
7 3/4	196.850	10 7/8	276.226	3	8	1117.60	7	10	2387.60	34	10,363.2
7 13/16	198.438	10 15/16	277.813	3	9	1143.00	7	11	2413.00	35	10,668.0
7 7/8	200.025	11	279.401	3	10	1168.40	8	0	2438.40	36	10,972.8
7 15/16	201.613	11 1/16	280.988	3	11	1193.80	8	1	2463.80	37	11,277.6
8	203.200	11 1/8	282.576	4	0	1219.20	8	2	2489.20	38	11,582.4
8 1/16	204.788	11 3/16	284.163	4	1	1244.60	8	3	2514.61	39	11,887.2
8 1/8	206.375	11 1/4	285.751	4	2	1270.00	8	4	2540.01	40	12,192.0
8 3/16	207.963	11 5/16	287.338	4	3	1295.40	8	5	2565.41	41	12,496.8
8 1/4	209.550	11 3/8	288.926	4	4	1320.80	8	6	2590.81	42	12,801.6
8 5/16	211.138	11 7/16	290.513	4	5	1346.20	8	7	2616.21	43	13,106.4
8 3/8	212.725	11 1/2	292.101	4	6	1371.60	8	8	2641.61	44	13,411.2
8 7/16	214.313	11 9/16	293.688	4	7	1397.00	8	9	2667.01	45	13,716.0
8 1/2	215.900	11 5/8	295.276	4	8	1422.40	8	10	2692.41	46	14,020.8
8 9/16	217.488	11 11/16	296.863	4	9	1447.80	8	11	2717.81	47	14,325.6
8 5/8	219.075	11 3/4	298.451	4	10	1473.20	9	0	2743.21	48	14,630.4
8 11/16	220.663	11 13/16	300.038	4	11	1498.60	9	1	2768.61	49	14,935.2
8 3/4	222.250	11 7/8	301.626	5	0	1524.00	9	2	2794.01	50	15,240.0
8 13/16	223.838	11 15/16	303.213	5	1	1549.40	9	3	2819.41	51	15,544.8
8 7/8	225.425	12	304.801	5	2	1574.80	9	4	2844.81	52	15,849.6
8 15/16	227.013	13	330.201	5	3	1600.20	9	5	2870.21	53	16,154.4
9	228.600	14	355.601	5	4	1625.60	9	6	2895.61	54	16,459.2
9 1/16	230.188	15	381.001	5	5	1651.00	9	7	2921.01	55	16,764.0
9 1/8	231.775	16	406.401	5	6	1676.40	9	8	2946.41	56	17,068.8
9 3/16	233.363	17	431.801	5	7	1701.80	9	9	2971.81	57	17,373.6
9 1/4	234.950	18	457.201	5	8	1727.20	9	10	2997.21	58	17,678.4
9 5/16	236.538	19	482.601	5	9	1752.60	9	11	3022.61	59	17,983.2
9 3/8	238.125	20	508.001	5	10	1778.00	10	0	3048.01	60	18,288.0
9 7/16	239.713	21	533.401	5	11	1803.40	11	0	3352.81	61	18,592.8
9 1/2	241.300	22	558.801	6	0	1828.80	12	0	3657.61	62	18,897.6
9 9/16	242.888	23	584.201	6	1	1854.20	13	0	3962.41	63	19,202.4
9 5/8	244.475	24	609.601	6	2	1879.60	14	0	4267.21	64	19,507.2
9 11/16	246.063	25	635.001	6	3	1905.00	15	0	4572.01	65	19,812.0
9 3/4	247.650	26	660.401	6	4	1930.40	16	0	4876.81	66	20,116.8
9 13/16	249.238	27	685.801	6	5	1955.80	17	0	5181.61	67	20,421.6
9 7/8	250.825	28	711.201	6	6	1981.20	18	0	5486.41	68	20,726.4
9 15/16	252.413	29	736.601	6	7	2006.60	19	0	5791.21	69	21,031.2
10	254.001	30	762.002	6	8	2032.00	20	0	6096.01	70	21,336.0
10 1/16	255.588	31	787.402	6	9	2057.40	21	0	6400.81	71	21,640.8
10 1/8	257.176	32	812.802	6	10	2082.80	22	0	6705.61	72	21,945.6
10 3/16	258.763	33	838.202	6	11	2108.20	23	0	7010.41	73	22,250.4
10 1/4	260.351	34	863.602	7	0	2133.60	24	0	7315.21	74	22,555.2
10 5/16	261.938	35	889.002	7	1	2159.00	25	0	7620.02	75	22,860.0
10 3/8	263.526	36	914.402	7	2	2184.40	26	0	7924.82	76	23,164.8
10 7/16	265.113	37	939.802	7	3	2209.80	27	0	8229.62	77	23,469.6
10 1/2	266.701	38	965.202	7	4	2235.20	28	0	8534.42	78	23,774.4
10 9/16	268.288	39	990.602	7	5	2260.60	29	0	8839.22	79	24,079.2
10 5/8	269.876	40	1016.00	7	6	2286.00	30	0	9144.02	80	24,384.0
10 11/16	271.463	41	1041.40	7	7	2311.40	31	0	9448.82	81	24,688.8
10 3/4	273.051	42	1066.80	7	8	2336.80	32	0	9753.62	82	24,993.6

Hundredths of a Millimeter into Inches

Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches
0.01	0.0004	0.21	0.0083	0.41	0.0161	0.61	0.0240	0.81	0.0319
0.02	0.0008	0.22	0.0087	0.42	0.0165	0.62	0.0244	0.82	0.0323
0.03	0.0012	0.23	0.0091	0.43	0.0169	0.63	0.0248	0.83	0.0327
0.04	0.0016	0.24	0.0094	0.44	0.0173	0.64	0.0252	0.84	0.0331
0.05	0.0020	0.25	0.0098	0.45	0.0177	0.65	0.0256	0.85	0.0335
0.06	0.0024	0.26	0.0102	0.46	0.0181	0.66	0.0260	0.86	0.0339
0.07	0.0028	0.27	0.0106	0.47	0.0185	0.67	0.0264	0.87	0.0343
0.08	0.0031	0.28	0.0110	0.48	0.0189	0.68	0.0268	0.88	0.0346
0.09	0.0035	0.29	0.0114	0.49	0.0193	0.69	0.0272	0.89	0.0350
0.10	0.0039	0.30	0.0118	0.50	0.0197	0.70	0.0276	0.90	0.0354
0.11	0.0043	0.31	0.0122	0.51	0.0201	0.71	0.0280	0.91	0.0358
0.12	0.0047	0.32	0.0126	0.52	0.0205	0.72	0.0283	0.92	0.0362
0.13	0.0051	0.33	0.0130	0.53	0.0209	0.73	0.0287	0.93	0.0366
0.14	0.0055	0.34	0.0134	0.54	0.0213	0.74	0.0291	0.94	0.0370
0.15	0.0059	0.35	0.0138	0.55	0.0217	0.75	0.0295	0.95	0.0374
0.16	0.0063	0.36	0.0142	0.56	0.0220	0.76	0.0299	0.96	0.0378
0.17	0.0067	0.37	0.0146	0.57	0.0224	0.77	0.0303	0.97	0.0382
0.18	0.0071	0.38	0.0150	0.58	0.0228	0.78	0.0307	0.98	0.0386
0.19	0.0075	0.39	0.0154	0.59	0.0232	0.79	0.0311	0.99	0.0390
0.20	0.0079	0.40	0.0157	0.60	0.0236	0.80	0.0315	1.00	0.0394

Decimals of an Inch into Millimeters

Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters	Inches	Millimeters
0.001	0.025	0.140	3.56	0.360	9.14	0.580	14.73	0.800	20.32
0.002	0.051	0.150	3.81	0.370	9.40	0.590	14.99	0.810	20.57
0.003	0.076	0.160	4.06	0.380	9.65	0.600	15.24	0.820	20.83
0.004	0.102	0.170	4.32	0.390	9.91	0.610	15.49	0.830	21.08
0.005	0.127	0.180	4.57	0.400	10.16	0.620	15.75	0.840	21.34
0.006	0.152	0.190	4.83	0.410	10.41	0.630	16.00	0.850	21.59
0.007	0.178	0.200	5.08	0.420	10.67	0.640	16.26	0.860	21.84
0.008	0.203	0.210	5.33	0.430	10.92	0.650	16.51	0.870	22.10
0.009	0.229	0.220	5.59	0.440	11.18	0.660	16.76	0.880	22.35
0.010	0.254	0.230	5.84	0.450	11.43	0.670	17.02	0.890	22.61
0.020	0.508	0.240	6.10	0.460	11.68	0.680	17.27	0.900	22.86
0.030	0.762	0.250	6.35	0.470	11.94	0.690	17.53	0.910	23.11
0.040	1.016	0.260	6.60	0.480	12.19	0.700	17.78	0.920	23.37
0.050	1.270	0.270	6.86	0.490	12.45	0.710	18.03	0.930	23.62
0.060	1.524	0.280	7.11	0.500	12.70	0.720	18.29	0.940	23.88
0.070	1.778	0.290	7.37	0.510	12.95	0.730	18.54	0.950	24.13
0.080	2.032	0.300	7.62	0.520	13.21	0.740	18.80	0.960	24.38
0.090	2.286	0.310	7.87	0.530	13.46	0.750	19.05	0.970	24.64
0.100	2.540	0.320	8.13	0.540	13.72	0.760	19.30	0.980	24.89
0.110	2.794	0.330	8.38	0.550	13.97	0.770	19.56	0.990	25.15
0.120	3.048	0.340	8.64	0.560	14.22	0.780	19.81	1.000	25.40
0.130	3.302	0.350	8.89	0.570	14.48	0.790	20.07	....	....

Example: — Find 0.856 inch in millimeters; 0.850 inch = 21.59 millimeters; 0.006 inch = 0.152 millimeter. Hence 21.59 + 0.152 = 21.742 millimeters = 0.856 inch.

Inches into Centimeters

Inches	0 Cm.	1 Cm.	2 Cm.	3 Cm.	4 Cm.	5 Cm.	6 Cm.	7 Cm.	8 Cm.	9 Cm.
0	......	2.54	5.08	7.62	10.16	12.70	15.24	17.78	20.32	22.86
10	25.40	27.94	30.48	33.02	35.56	38.10	40.64	43.18	45.72	48.26
20	50.80	53.34	55.88	58.42	60.96	63.50	66.04	68.58	71.12	73.66
30	76.20	78.74	81.28	83.82	86.36	88.90	91.44	93.98	96.52	99.06
40	101.60	104.14	106.68	109.22	111.76	114.30	116.84	119.38	121.92	124.46
50	127.00	129.54	132.08	134.62	137.16	139.70	142.24	144.78	147.32	149.86
60	152.40	154.94	157.48	160.02	162.56	165.10	167.64	170.18	172.72	175.26
70	177.80	180.34	182.88	185.42	187.96	190.50	193.04	195.58	198.12	200.66
80	203.20	205.74	208.28	210.82	213.36	215.90	218.44	220.98	223.52	226.06
90	228.60	231.14	233.68	236.22	238.76	241.30	243.84	246.38	248.92	251.46
100	254.00	256.54	259.08	261.62	264.16	266.70	269.24	271.78	274.32	276.86

Centimeters into Inches

Cm.	0 Inch	1 Inch	2 Inch	3 Inch	4 Inch	5 Inch	6 Inch	7 Inch	8 Inch	9 Inch
0	......	0.394	0.787	1.181	1.575	1.969	2.362	2.756	3.150	3.543
10	3.937	4.331	4.724	5.118	5.512	5.906	6.299	6.693	7.087	7.480
20	7.874	8.268	8.662	9.055	9.449	9.843	10.236	10.630	11.024	11.418
30	11.811	12.205	12.599	12.992	13.386	13.780	14.173	14.567	14.961	15.355
40	15.748	16.142	16.536	16.929	17.323	17.717	18.111	18.504	18.898	19.292
50	19.685	20.079	20.473	20.867	21.260	21.654	22.048	22.441	22.835	23.229
60	23.622	24.016	24.410	24.804	25.197	25.591	25.985	26.378	26.772	27.166
70	27.560	27.953	28.347	28.741	29.134	29.528	29.922	30.316	30.709	31.103
80	31.497	31.890	32.284	32.678	33.071	33.465	33.859	34.253	34.646	35.040
90	35.434	35.827	36.221	36.615	37.009	37.402	37.796	38.190	38.583	38.977
100	39.370	39.764	40.158	40.552	40.945	41.339	41.733	42.126	42.520	42.914

Feet into Meters

Feet	0 Meters	1 Meters	2 Meters	3 Meters	4 Meters	5 Meters	6 Meters	7 Meters	8 Meters	9 Meters
0	......	0.305	0.610	0.914	1.219	1.524	1.829	2.134	2.438	2.743
10	3.048	3.353	3.658	3.962	4.267	4.572	4.877	5.182	5.486	5.791
20	6.096	6.401	6.706	7.010	7.315	7.620	7.925	8.229	8.534	8.839
30	9.144	9.449	9.753	10.058	10.363	10.668	10.972	11.277	11.582	11.887
40	12.192	12.496	12.801	13.106	13.411	13.716	14.020	14.325	14.630	14.935
50	15.239	15.544	15.849	16.154	16.459	16.763	17.068	17.373	17.678	17.983
60	18.287	18.592	18.897	19.202	19.507	19.811	20.116	20.421	20.726	21.031
70	21.335	21.640	21.945	22.250	22.555	22.859	23.164	23.469	23.774	24.079
80	24.383	24.688	24.993	25.298	25.602	25.907	26.212	26.517	26.822	27.126
90	27.431	27.736	28.041	28.346	28.651	28.955	29.260	29.565	29.870	30.174
100	30.479	30.784	31.089	31.394	31.698	32.003	32.308	32.613	32.918	33.222

Meters into Feet

Meters	0 Feet	1 Feet	2 Feet	3 Feet	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet	9 Feet
0	.....	3.281	6.562	9.842	13.123	16.404	19.685	22.966	26.247	29.527
10	32.808	36.089	39.370	42.651	45.932	49.212	52.493	55.774	59.055	62.336
20	65.617	68.897	72.178	75.459	78.740	82.021	85.302	88.582	91.863	95.144
30	98.425	101.71	104.99	108.27	111.55	114.83	118.11	121.39	124.67	127.95
40	131.23	134.51	137.79	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50	164.04	167.32	170.60	173.88	177.16	180.45	183.73	187.01	190.29	193.57
60	196.85	200.13	203.41	206.69	209.97	213.25	216.53	219.82	223.10	226.38
70	229.66	232.94	236.22	239.50	242.78	246.06	249.34	252.62	255.90	259.19
80	262.47	265.75	269.03	272.31	275.59	278.87	282.15	285.43	288.71	291.99
90	295.27	298.56	301.84	305.12	308.40	311.68	314.96	318.24	321.52	324.80
100	328.08	331.36	334.64	337.93	341.21	344.49	347.77	351.05	354.33	357.61

Square Inches into Square Centimeters

Square Inches	0 Sq. Cm.	1 Sq. Cm.	2 Sq. Cm.	3 Sq. Cm.	4 Sq. Cm.	5 Sq. Cm.	6 Sq. Cm.	7 Sq. Cm.	8 Sq. Cm.	9 Sq. Cm.
0	.....	6.45	12.90	19.36	25.81	32.26	38.71	45.16	51.61	58.07
10	64.52	70.97	77.42	83.87	90.32	96.77	103.23	109.68	116.13	122.58
20	129.03	135.48	141.94	148.39	154.84	161.29	167.74	174.19	180.65	187.10
30	193.55	200.00	206.45	212.90	219.36	225.81	232.26	238.71	245.16	251.61
40	258.07	264.52	270 97	277.42	283.87	290.32	296.77	303.23	309.68	316.13
50	322.58	329.03	335.48	341.93	348.39	354.84	361.29	367.74	374.19	380.65
60	387.10	393.55	400.00	406.45	412.90	419.36	425.81	432.26	438.71	445.16
70	451.61	458.07	464.52	470.97	477.42	483.87	490.32	496.78	503.23	509.68
80	516.13	522.58	529.03	535.48	541.94	548.39	554.84	561.29	567.74	574.19
90	580.65	587.10	593.55	600.00	606.45	612.90	619.36	625.81	632.26	638.71
100	645.17	651.62	658.07	664.52	670.97	677.42	683.88	690.33	696.78	703.23

Square Centimeters into Square Inches

Square Cm.	0 Sq. In.	1 Sq. In.	2 Sq. In.	3 Sq. In.	4 Sq. In.	5 Sq. In.	6 Sq. In.	7 Sq. In.	8 Sq. In.	9 Sq. In.
0	......	0.155	0.310	0.465	0.620	0.775	0.930	1.085	1.240	1.395
10	1.550	1.705	1.860	2.015	2.170	2.325	2.480	2.635	2.790	2.945
20	3.100	3.255	3.410	3.565	3.720	3.875	4.030	4.185	4.340	4.495
30	4.650	4.805	4.960	5.115	5.270	5.425	5.580	5.735	5.890	6.045
40	6.200	6.355	6.510	6.665	6.820	6.975	7.130	7.285	7.440	7.595
50	7.750	7.905	8.060	8.215	8.370	8.525	8.680	8.835	8.990	9.145
60	9.300	9.455	9.610	9.765	9.920	10.075	10.230	10.385	10.540	10.695
70	10.850	11.005	11.160	11.315	11.470	11.625	11.780	11.935	12.090	12.245
80	12.400	12.555	12.710	12.865	13.020	13.175	13.330	13.485	13.640	13.795
90	13.950	14.105	14.260	14.415	14.570	14.725	14.880	15.035	15.190	15.345
100	15.500	15.655	15.810	15.965	16.120	16.275	16.430	16.585	16.740	16.895

Square Feet into Square Meters

Square Feet	0 Sq. Meters	1 Sq. Meters	2 Sq. Meters	3 Sq. Meters	4 Sq. Meters	5 Sq. Meters	6 Sq. Meters	7 Sq. Meters	8 Sq. Meters	9 Sq. Meters
0	......	0.0929	0.1858	0.2787	0.3716	0.4645	0.5574	0.6503	0.7432	0.8361
10	0.9290	1.0219	1.1148	1.2077	1.3006	1.3936	1.4865	1.5794	1.6723	1.7652
20	1.8581	1.9510	2.0439	2.1368	2.2297	2.3226	2.4155	2.5084	2.6013	2.6942
30	2.7871	2.8800	2.9729	3.0658	3.1587	3.2516	3.3445	3.4374	3.5303	3.6232
40	3.7161	3.8090	3.9019	3.9948	4.0878	4.1807	4.2736	4.3665	4.4594	4.5523
50	4.6452	4.7381	4.8310	4.9239	5.0168	5.1097	5.2026	5.2955	5.3884	5.4813
60	5.5742	5.6671	5.7600	5.8529	5.9458	6.0387	6.1316	6.2245	6.3174	6.4103
70	6.5032	6.5961	6, 6890	6.7819	6.8749	6.9678	7.0607	7.1536	7.2465	7.3394
80	7.4323	7.5252	7.6181	7.7110	7.8039	7.8968	7.9897	8.0826	8.1755	8.2684
90	8.3613	8.4542	8.5471	8.6400	8.7329	8.8258	8.9187	9.0116	9.1045	9.1974
100	9.2903	9.3832	9.4761	9.5690	9.6619	9.7548	9.8477	9.9406	10.0335	10.1264

Square Meters into Square Feet

Square Meters	0 Sq. Ft.	1 Sq. Ft.	2 Sq. Ft.	3 Sq. Ft.	4 Sq. Ft.	5 Sq. Ft.	6 Sq. Ft.	7 Sq. Ft.	8 Sq. Ft.	9 Sq. Ft.
0	......	10.76	21.53	32.29	43.06	53.82	64.58	75.35	86.11	96.88
10	107.64	118.40	129.17	139.93	150.69	161.46	172.22	182.99	193.75	204.51
20	215.28	226.04	236.81	247.57	258.33	269.10	279.86	290.62	301.39	312.15
30	322.92	333.68	344.44	355.21	365.97	376.74	387.50	398.26	409.03	419.79
40	430.55	441.32	452.08	462.85	473.61	484.37	495.14	505.90	516.67	527.43
50	538.19	548.96	559.72	570.48	581.25	592.01	602.78	613.54	624.30	735.07
60	645.83	656.60	667.36	678.12	688.89	699.65	710.42	721.18	731.94	642.71
70	753.47	764.23	775.00	785.76	796.53	807.29	818.05	828.82	839.58	850.35
80	861.11	871.87	882.64	893.40	904.16	914.93	925.69	936.46	947.22	957.98
90	968.75	979.51	990.28	1001.04	1011.80	1022.57	1033.33	1044.10	1054.86	1065.62
100	1076.39	1087.15	1097.92	1108.68	1119.44	1130.21	1140.97	1151.74	1162.50	1173.26

Cubic Inches into Cubic Centimeters

Cubic Inches	0 Cubic Cm.	1 Cubic Cm.	2 Cubic Cm.	3 Cubic Cm.	4 Cubic Cm.	5 Cubic Cm.	6 Cubic Cm.	7 Cubic Cm.	8 Cubic Cm.	9 Cubic Cm.
0	......	16.38	32.77	49.16	65.55	81.93	98.32	114.71	131.09	147.48
10	163.87	180.26	196.64	213.03	229.41	245.80	262.19	278.58	294.88	311.35
20	327.73	344.12	360.50	376.89	393.27	409.66	426.05	442.44	458.74	475.21
30	491.60	507.99	524.37	540.76	557.14	573.53	589.92	606.31	622.61	639.08
40	655.46	671.85	688.23	704.52	721.00	737.39	753.78	770.17	786.47	802.94
50	819.33	835.72	851.10	868.49	884.87	901.26	917.65	934.04	950.34	966.81
60	983.20	999.59	1016.0	1032.4	1048.7	1065.1	1081.5	1097.9	1114.2	1130.7
70	1147.1	1163.5	1179.9	1196.3	1212.6	1229.0	1245.4	1261.8	1278.1	1294.6
80	1310.9	1327.3	1343.7	1360.1	1376.4	1392.8	1409.2	1425.6	1441.9	1458.4
90	1474.8	1491.2	1507.6	1524.0	1540.3	1556.7	1573.1	1589.5	1605.8	1622.3
100	1638.7	1655.1	1671.5	1687.9	1704.2	1720.6	1737.0	1753.4	1769.7	1786.2

Cubic Centimeters into Cubic Inches

Cubic Cm.	0 Cubic Inches	1 Cubic Inches	2 Cubic Inches	3 Cubic Inches	4 Cubic Inches	5 Cubic Inches	6 Cubic Inches	7 Cubic Inches	8 Cubic Inches	9 Cubic Inches
0	......	0.0610	0.1221	0.1831	0.2441	0.3051	0.3661	0.4272	0.4882	0.5492
10	0.6102	0.6712	0.7323	0.7933	0.8543	0.9153	0.9763	1.0374	1.0984	1.1594
20	1.2205	1.2815	1.3426	1.4036	1.4646	1.5256	1.5866	1.6477	1.7087	1.7697
30	1.8308	1.8918	1.9529	2.0139	2.0749	2.1359	2.1969	2.2580	2.3190	2.3800
40	2.4410	2.5020	2.5631	2.6241	2.6851	2.7461	2.8071	2.8682	2.9292	2.9902
50	3.0513	3.1123	3.1734	3.2344	3.2954	3.3564	3.4174	3.4785	3.5395	3.6005
60	3.6615	3.7225	3.7836	3.8446	3.9056	3.9666	4.0276	4.0887	4.1497	4.2107
70	4.2718	4.3328	4.3939	4.4549	4.5159	4.5769	4.6379	4.6990	4.7600	4.8210
80	4.8820	4.9430	5.0041	5.0651	5.1261	5.1871	5.2481	5.3092	5.3702	5.4312
90	5.4923	5.5533	5.6144	5.6754	5.7364	5.7974	5.8584	5.9195	5.9805	6.0415
100	6.1025	6.1635	6.2246	6.2856	6.3466	6.4076	6.4686	6.5297	6.5907	6.6517

Cubic Feet into Cubic Meters

Cubic Feet	0 Cubic Meters	1 Cubic Meters	2 Cubic Meters	3 Cubic Meters	4 Cubic Meters	5 Cubic Meters	6 Cubic Meters	7 Cubic Meters	8 Cubic Meters	9 Cubic Meters
0	......	0.0283	0.0566	0.0850	0.1133	0.1416	0.1699	0.1982	0.2265	0.2549
10	0.2832	0.3115	0.3398	0.3681	0.3964	0.4248	0.4531	0.4814	0.5097	0.5380
20	0.5663	0.5947	0.6230	0.6513	0.6796	0.7079	0.7362	0.7646	0.7929	0.8212
30	0.8495	0.8778	0.9061	0.9345	0.9628	0.9911	1.0194	1.0477	1.0760	1.1044
40	1.1327	1.1610	1.1893	1.2176	1.2459	1.2743	1.3026	1.3309	1.3592	1.3875
50	1.4159	1.4442	1.4725	1.5008	1.6291	1.5574	1.5858	1.6141	1.6424	1.6707
60	1.6990	1.7273	1.7557	1.7840	1.8123	1.8406	1.8689	1.8972	1.9256	1.9539
70	1.9822	2.0105	2.0388	2.0671	2.0955	2.1238	2.1521	2.1804	2.2087	2.2370
80	2.2654	2.2937	2.3220	2.3503	2.3786	2.4069	2.4353	2.4636	2.4919	2.5202
90	2.5485	2.5768	2.6052	2.6335	2.6618	2.6901	2.7184	2.7468	2.7751	2.8034
100	2.8317	2.8600	2.8884	2.9167	2.9450	2.9733	3.0016	3.0300	3.0583	3.0866

Cubic Meters into Cubic Feet

Cubic Meters	0 Cubic Feet	1 Cubic Feet	2 Cubic Feet	3 Cubic Feet	4 Cubic Feet	5 Cubic Feet	6 Cubic Feet	7 Cubic Feet	8 Cubic Feet	9 Cubic Feet
0	......	35.3	70.6	105.9	141.3	176.6	211.9	247.2	282.5	317.8
10	353.1	388.5	423.8	459.1	494.4	529.7	565.0	600.3	635.7	671.0
20	706.3	741.6	776.9	812.2	847.5	882.9	918.2	953.5	988.8	1024.1
30	1059.4	1094.7	1130.1	1165.4	1200.7	1236.0	1271.3	1306.6	1341.9	1377.3
40	1412.6	1447.9	1483.2	1518.5	1553.8	1589.2	1624.5	1659.8	1695.1	1730.4
50	1765.7	1801.0	1836.4	1871.7	1907.0	1942.3	1977.6	2012.9	2048.2	2083.6
60	2118.9	2154.2	2189.5	2224.8	2260.1	2295.4	2230.8	2366.1	2401.4	2436.7
70	2472.0	2507.3	2542.6	2578.0	2613.3	2648.6	2683.9	2719.2	2754.5	2789.8
80	2825.2	2860.5	2895.8	2931.1	2966.4	3001.7	3037.0	3072.4	3107.7	3143.0
90	3178.3	3213.6	3248.9	3284.2	3319.6	3354.9	3390.2	3425.5	3460.8	3496.1
100	3531.4	3566.7	3602.0	3637.3	3672.7	3708.0	3743.3	3778.6	3813.9	3849.2

Cubic Feet into Liters (Cubic Decimeters)

Cubic Feet	0 Liters	1 Liters	2 Liters	3 Liters	4 Liters	5 Liters	6 Liters	7 Liters	8 Liters	9 Liters
0	......	28.32	56.63	84.95	113.26	141.58	169.89	198.21	226.53	254.84
10	283.16	311.47	339.79	368.11	396.42	424.74	453.06	481.37	509.69	538.00
20	566.32	594.64	622.95	651.27	679.58	707.90	736.22	764.53	792.85	821.16
30	849.48	877.80	906.11	934.43	962.74	991.06	1019.4	1047.7	1076.0	1104.3
40	1132.6	1160.8	1189.2	1217.5	1245.9	1274.2	1302.5	1330.8	1359.1	1387.4
50	1415.8	1444.0	1472.4	1500.7	1529.1	1557.4	1585.7	1614.0	1642.3	1670.6
60	1698.9	1727.2	1755.5	1783.8	1812.2	1840.5	1868.8	1897.1	1925.4	1953.7
70	1982.1	2010.3	2038.7	2067.0	2095.4	2123.7	2152.0	2180.3	2208.6	2236.9
80	2265.3	2293.5	2321.9	2350.2	2378.6	2406.9	2435.2	2463.5	2491.8	2520.1
90	2548.4	2576.6	2605.0	2633.3	2661.6	2690.0	2718.3	2746.6	2774.9	2803.2
100	2831.6	2859.8	2888.2	2916.5	2944.9	2973.2	3001.5	3029.8	3058.1	3086.4

Liters (Cubic Decimeters) into Cubic Feet

Liters	0 Cubic Feet	1 Cubic Feet	2 Cubic Feet	3 Cubic Feet	4 Cubic Feet	5 Cubic Feet	6 Cubic Feet	7 Cubic Feet	8 Cubic Feet	9 Cubic Feet
0	......	0.0353	0.0706	0.1059	0.1413	0.1766	0.2119	0.2472	0.2825	0.3178
10	0.3531	0.3884	0.4237	0.4590	0.4944	0.5297	0.5540	0.6003	0.6356	0.6709
20	0.7063	0.7416	0.7766	0.8122	0.8476	0.8829	0.9182	0.9535	0.9888	1.0241
30	1.0594	1.0947	1.1300	1.1653	1.2007	1.2360	1.2713	1.3066	1.3419	1.3772
40	1.4126	1.4479	1.4832	1.5185	1.5539	1.5892	1.6245	1.6608	1.6951	1.7304
50	1.7658	1.8011	1.8364	1.8717	1.9071	1.9424	1.9777	2.0130	2.0483	2.0836
60	2.1189	2.1542	2.1895	2.2248	2.2602	2.2955	2.3308	2.3661	2.4014	2.4367
70	2.4721	2.5074	2.5427	2.5780	2.6134	2.6487	2.6840	2.7193	2.7546	2.7899
80	2.8252	2.8605	2.8958	2.9311	2.9665	3.0018	3.0371	3.0724	3.1077	3.1430
90	3.1784	3.2137	3.2490	3.2843	3.3197	3.3550	3.3903	3.4256	3.4609	3.4962
100	3.5315	3.5668	3.6021	3.6374	3.6728	3.7081	3.7434	3.7787	3.8140	3.8493

U. S. Gallons into Liters

Gallons	0 Liters	1 Liters	2 Liters	3 Liters	4 Liters	5 Liters	6 Liters	7 Liters	8 Liters	9 Liters
0	.....	3.785	7.571	11.356	15.142	18.927	22.713	26.498	30.283	34.069
10	37.854	41.640	45.425	49.211	52.996	56.781	60.567	64.352	68.138	71.923
20	75.709	79.494	83.280	87.065	90.850	94.636	98.421	102.21	105.99	109.78
30	113.56	117.35	121.13	124.92	128.70	132.49	136.28	140.06	143.85	147.63
40	151.42	155.20	158.99	162.77	166.56	170.34	174.13	177.92	181.70	185.49
50	189.27	193.06	196.84	200.63	204.41	208.20	211.98	215.77	219.56	223.34
60	227.13	230.91	234.70	238.48	242.27	246.05	249.84	253.62	257.41	261.19
70	264.98	268.77	272.55	276.34	280.12	283.91	287.69	291.48	295.26	299.05
80	302.83	306.62	310.41	314.19	317.98	321.76	325.55	329.33	333.12	336.90
90	340.69	344.47	348.26	352.05	355.83	359.62	363.40	367.19	370.97	374.76
100	378.54	382.33	386.11	389.90	393.69	397.47	401.26	405.04	408.83	412.61

Liters into U. S. Gallons

Liters	0 Gallons	1 Gallons	2 Gallons	3 Gallons	4 Gallons	5 Gallons	6 Gallons	7 Gallons	8 Gallons	9 Gallons
0	.....	0.264	0.528	0.793	1.057	1.321	1.585	1.849	2.113	2.378
10	2.642	2.906	3.170	3.434	3.698	3.963	4.227	4.491	4.755	5.019
20	5.283	5.548	5.812	6.076	6.340	6.604	6.868	7.133	7.397	7.661
30	7.925	8.189	8.453	8.718	8.982	9.246	9.510	9.774	10.038	10.303
40	10.567	10.831	11.095	11.359	11.623	11.888	12.152	12.416	12.680	12.944
50	13.209	13.473	13.737	14.001	14.265	14.529	14.794	15.058	15.322	15.586
60	15.850	16.114	16.379	16.643	16.907	17.171	17.435	17.699	17.964	18.228
70	18.492	18.756	19.020	19.284	19.549	19.813	20.077	20.341	20.605	20.869
80	21.134	21.398	21.662	21.926	22.190	22.454	22.719	22.983	23.247	23.511
90	23.775	24.040	24.304	24.568	24.832	25.096	25.360	25.625	25.889	26.153
100	26.417	26.681	26.945	27.210	27.474	27.738	28.002	28.266	28.530	28.795

Pounds into Kilograms

Pounds	0 Kilograms	1 Kilograms	2 Kilograms	3 Kilograms	4 Kilograms	5 Kilograms	6 Kilograms	7 Kilograms	8 Kilograms	9 Kilograms
0	.....	0.454	0.907	1.361	1.814	2.268	2.722	3.175	3.629	4.082
10	4.536	4.990	5.443	5.897	6.350	6.804	7.257	7.711	8.165	8.618
20	9.072	9.525	9.979	10.433	10.886	11.340	11.793	12.247	12.701	13.154
30	13.608	14.061	14.515	14.969	15.422	15.876	16.329	16.783	17.237	17.690
40	18.144	18.597	19.051	19.504	19.958	20.412	20.865	21.319	21.772	22.226
50	22.680	23.133	23.587	24.040	24.494	24.948	25.401	25.855	26.308	26.762
60	27.216	27.669	28.123	28.576	29.030	29.484	29.937	30.391	30.844	31.298
70	31.751	32.205	32.659	33.112	33.566	34.019	34.473	34.927	35.380	35.834
80	36.287	36.741	37.195	37.648	38.102	38.555	39.009	39.463	39.916	40.370
90	40.823	41.277	41.730	42.184	42.638	43.091	43.545	43.998	44.453	44.906
100	45.359	45.813	46.266	46.720	47.174	47.627	48.081	48.534	48.988	49.442

Kilograms into Pounds

Kilograms	0 Lbs.	1 Lbs.	2 Lbs.	3 Lbs.	4 Lbs.	5 Lbs.	6 Lbs.	7 Lbs.	8 Lbs.	9 Lbs.
0	.....	2.205	4.409	6.614	8.818	11.023	13.228	15.432	17.637	19.842
10	22.046	24.251	26.455	28.660	30.865	33.069	35.274	37.479	39.683	41.888
20	44.092	46.297	48.502	50.706	52.911	55.116	57.320	59.525	61.729	63.934
30	66.139	68.343	70.548	72.752	74.957	77.162	79.366	81.571	83.776	85.980
40	88.185	90.389	92.594	94.799	97.003	99.208	101.41	103.62	105.82	108.03
50	110.23	112.44	114.64	116.84	119.05	121.25	123.46	125.66	127.87	130.07
60	132.28	134.48	136.69	138.89	141.10	143.30	145.51	147.71	149.91	152.12
70	154.32	156.53	158.73	160.94	163.14	165.35	167.55	169.76	171.96	174.17
80	176.37	178.57	180.78	182.98	185.19	187.39	189.60	191.80	194.01	196.21
90	198.42	200.62	202.83	205.03	207.23	209.44	211.64	213.85	216.05	218.26
100	220.46	222.67	224.87	227.08	229.28	231.49	233.69	235.89	238.10	240.30

Ounces Avoirdupois into Grams

Ounces	0 Grams	1 Grams	2 Grams	3 Grams	4 Grams	5 Grams	6 Grams	7 Grams	8 Grams	9 Grams
0	.....	28.35	56.70	85.05	113.39	141.74	170.09	198.44	226.79	255.14
10	283.48	311.83	340.18	368.52	396.87	425.22	453.57	481.92	510.27	538.62
20	566.97	595.32	623.67	652.01	680.36	708.71	737.06	765.41	793.76	822.11
30	850.46	878.81	907.16	935.50	963.85	992.20	1020.5	1048.9	1077.2	1105.6
40	1133.9	1162.2	1190.6	1218.90	1247.3	1275.6	1304.0	1332.3	1360.7	1389.0
50	1417.4	1445.7	1474.1	1502.4	1530.8	1559.1	1587.5	1615.8	1644.2	1672.5
60	1700.9	1729.2	1756.6	1785.9	1814.3	1842.9	1871.0	1899.3	1927.7	1956.0
70	1984.4	2012.7	2041.1	2079.4	2097.8	2126.1	2154.5	2182.8	2211.2	2239.5
80	2267.9	2296.2	2324.6	2352.9	2381.3	2409.6	2438.0	2466.3	2494.7	2523.0
90	2551.4	2579.7	2608.1	2636.4	2664.8	2693.1	2721.5	2739.8	2778.2	2806.5
100	2834.8	2863.1	2891.5	2919.8	2948.2	2976.5	3004.9	3033.2	3061.6	3089.9

Grams into Ounces Avoirdupois

Grams	0 Oz.	1 Oz.	2 Oz.	3 Oz.	4 Oz.	5 Oz.	6 Oz.	7 Oz.	8 Oz.	9 Oz.
0	.....	0.0353	0.0705	0.1058	0.1411	0.1768	0.2116	0.2469	0.2822	0.3175
10	0.3527	0.3880	0.4232	0.4585	0.4938	0.5295	0.5643	0.5996	0.6349	0.6702
20	0.7055	0.7408	0.7760	0.8113	0.8466	0.8823	0.9171	0.9524	0.9877	1.0230
30	1.0582	1.0935	1.1287	1.1640	1.1993	1.2350	1.2698	1.3051	1.3404	1.3757
40	1.4110	1.4463	1.4815	1.5168	1.5521	1.5878	1.6226	1.6579	1.6932	1.7285
50	1.7637	1.8040	1.8392	1.8745	1.9098	1.9455	1.9803	2.0156	2.0509	2.0862
60	2.1165	2.1518	2.1870	2.2223	2.2576	2.2933	2.3281	2.3634	2.3987	2.4340
70	2.4692	2.5045	2.5397	2.5750	2.6103	2.6460	2.6808	2.7161	2.7514	2.7867
80	2.8220	2.8573	2.8925	2.9278	2.9631	2.9988	3.0336	3.0689	3.1042	3.1395
90	3.1747	3.2100	3.2452	3.2805	3.3158	3.3515	3.3863	3.4216	3.4569	3.4922
100	3.5275	3.5628	3.5980	3.6333	3.6686	3.7043	3.7391	3.7744	3.8097	3.8450

Pounds per Square Inch into Kilograms per Square Centimeter

Pounds per Square Inch	0 Kg. per Sq. Cm.	1 Kg. per Sq. Cm.	2 Kg. per Sq. Cm.	3 Kg. per Sq. Cm.	4 Kg. per Sq. Cm.	5 Kg. per Sq. Cm.	6 Kg. per Sq. Cm.	7 Kg. per Sq. Cm.	8 Kg. per Sq. Cm.	9 Kg. per Sq. Cm.
0	.....	0.0703	0.1406	0.2109	0.2812	0.3515	0.4218	0.4921	0.5625	0.6328
10	0.7031	0.7734	0.8437	0.9140	0.9843	1.0546	1.1249	1.1952	1.2655	1.3358
20	1.4062	1.4765	1.5468	1.6171	1.6874	1.7577	1.8280	1.8983	1.9686	2.0389
30	2.1092	2.1795	2.2498	2.3202	2.3905	2.4608	2.5311	2.6014	2.6717	2.7420
40	2.8123	2.8826	2.9529	3.0232	3.0935	3.1639	3.2342	3.3045	3.3748	3.4451
50	3.5154	3.5857	3.6560	3.7263	3.7966	3.8669	3.9372	4.0075	4.0779	4.1482
60	4.2185	4.2888	4.3591	4.4294	4.4997	4.5700	4.6403	4.7106	4.7809	4.8512
70	4.9216	4.9919	5.0622	5.1325	5.2028	5.2731	5.3434	5.4137	5.4840	5.5543
80	5.6246	5.6949	5.7652	5.8356	5.9059	5.9762	6.0465	6.1168	6.1871	6.2574
90	6.3277	6.3980	6.4683	6.5386	6.6089	6.6793	6.7496	6.8199	6.8902	6.9605
100	7.0308	7.1011	7.1714	7.2417	7.3120	7.3823	7.4526	7.5229	7.5933	7.6636

Kilograms per Square Centimeter into Pounds per Square Inch

Kilograms per Sq. Cm.	0 Lbs. per Sq. In.	1 Lbs. per Sq. In.	2 Lbs. per Sq. In.	3 Lbs. per Sq. In.	4 Lbs. per Sq. In.	5 Lbs. per Sq. In.	6 Lbs. per Sq. In.	7 Lbs. per Sq. In.	8 Lbs. per Sq. In.	9 Lbs. per Sq. In.
0	.....	14.22	28.45	42.67	56.89	71.12	85.34	99.56	113.78	128.01
10	142.23	156.45	170.68	184.90	199.12	213.35	227.57	241.79	256.02	270.24
20	284.46	298.69	312.91	327.13	341.36	355.58	369.80	384.03	398.25	412.47
30	426.70	440.92	455.14	469.36	483.59	497.81	512.03	526.26	540.48	554.70
40	568.93	583.15	597.37	611.60	625.82	640.04	654.27	668.49	682.71	696.94
50	711.16	725.38	739.61	753.83	768.05	782.28	796.50	810.72	824.94	839.17
60	853.39	867.61	881.84	896.06	910.28	924.51	938.73	952.95	967.18	981.40
70	995.62	1009.8	1024.1	1038.3	1052.5	1066.7	1081.0	1095.2	1109.4	1123.6
80	1137.8	1152.1	1166.3	1180.5	1194.7	1209.0	1223.2	1237.4	1251.6	1265.9
90	1280.1	1294.3	1308.5	1322.7	1337.0	1351.2	1365.4	1379.6	1393.9	1408.1
100	1422.3	1436.5	1450.8	1465.0	1479.2	1493.4	1507.7	1521.9	1536.1	1550.3

Pounds per Square Foot into Kilograms per Square Meter

Pounds per Square Foot	0 Kg. per Sq. Meter	1 Kg. per Sq. Meter	2 Kg. per Sq. Meter	3 Kg. per Sq. Meter	4 Kg. per Sq. Meter	5 Kg. per Sq. Meter	6 Kg. per Sq. Meter	7 Kg. per Sq. Meter	8 Kg. per Sq. Meter	9 Kg. per Sq. Meter
0	.....	4.88	9.76	14.65	19.53	24.41	29.30	34.18	39.06	43.94
10	48.83	53.71	58.59	63.47	68.36	73.24	78.12	83.00	87.88	92.77
20	97.65	102.53	107.41	112.30	117.18	122.06	126.94	131.83	136.71	141.59
30	146.47	151.35	156.23	161.12	166.00	170.88	175.76	180.65	185.53	190.41
40	195.30	200.18	205.06	209.95	214.83	219.71	224.59	229.48	234.36	239.24
50	244.13	249.01	253.89	258.78	263.66	268.54	273.42	278.31	283.18	288.06
60	292.95	297.83	302.71	307.60	312.48	317.36	322.24	327.13	332.00	336.89
70	341.77	346.65	351.53	356.42	361.30	366.18	371.06	375.95	380.83	385.71
80	390.59	395.48	400.36	405.24	410.12	415.00	419.89	424.77	429.65	434.53
90	439.43	444.31	449.19	454.08	458.96	463.84	468.72	473.61	478.48	483.37
100	488.25	493.13	498.01	502.90	507.78	512.66	517.54	522.43	527.31	532.19

Kilograms per Square Meter into Pounds per Square Foot

Kilograms per Square Meter	0 Lbs. per Sq. Ft.	1 Lbs. per Sq. Ft.	2 Lbs. per Sq. Ft.	3 Lbs. per Sq. Ft.	4 Lbs. per Sq. Ft.	5 Lbs. per Sq. Ft.	6 Lbs. per Sq. Ft.	7 Lbs. per Sq. Ft.	8 Lbs. per Sq. Ft.	9 Lbs. per Sq. Ft.
0	.....	0.2048	0.4096	0.6144	0.8192	1.0240	1.2289	1.4337	1.6385	1.8433
10	2.0481	2.2529	2.4577	2.6625	2.8673	3.0721	3.2770	3.4818	3.6866	3.8914
20	4.0962	4.3010	4.5058	4.7106	4.9154	5.1202	5.3251	5.5299	5.7347	5.9395
30	6.1444	6.3492	6.5540	6.7588	6.9636	7.1684	7.3733	7.5781	7.7829	7.9877
40	8.1925	8.3973	8.6021	8.8069	9.0117	9.2165	9.4214	9.6262	9.8310	10.036
50	10.240	10.445	10.649	10.854	11.059	11.264	11.469	11.674	11.878	12.083
60	12.289	12.494	12.698	12.903	13.108	13.313	13.518	13.723	13.927	14.132
70	14.337	14.542	14.746	14.951	15.156	15.361	15.566	15.771	15.975	16.180
80	16.385	16.590	16.794	16.999	17.204	17.409	17.614	17.819	18.023	18.228
90	18.433	18.638	18.842	19.047	19.252	19.457	19.662	19.867	20.071	20.276
100	20.481	20.686	20.890	21.095	21.300	21.505	21.710	21.915	22.119	22.324

Foot-pounds into Meter-Kilograms

Foot-pounds	0 Meter-kilogr.	1 Meter-kilogr.	2 Meter-kilogr.	3 Meter-kilogr.	4 Meter-kilogr.	5 Meter-kilogr.	6 Meter-kilogr.	7 Meter-kilogr.	8 Meter-kilogr.	9 Meter-kilogr.
0	.....	0.138	0.276	0.415	0.553	0.691	0.829	0.967	1.106	1.244
10	1.382	1.520	1.658	1.796	1.934	2.073	2.211	2.349	2.487	2.625
20	2.764	2.902	3.040	3.178	3.316	3.455	3.593	3.731	3.869	4.007
30	4.146	4.284	4.422	4.560	4.698	4.837	4.975	5.113	5.251	5.389
40	5.528	5.666	5.804	5.942	6.080	6.219	6.357	6.495	6.633	6.771
50	6.910	7.048	7.186	7.324	7.462	7.601	7.739	7.877	8.015	8.153
60	8.292	8.430	8.568	8.706	8.844	8.983	9.121	9.259	9.397	9.535
70	9.674	9.812	9.950	10.088	10.227	10.365	10.503	10.641	10.779	10.918
80	11.056	11.194	11.332	11.470	11.609	11.747	11.885	12.023	12.161	12.300
90	12.438	12.576	12.714	12.855	12.991	13.129	13.267	13.405	13.544	13.682
100	13.820	13.958	14.096	14.235	14.373	14.511	14.649	14.787	14.925	14.064

Meter-Kilograms into Foot-pounds

Meter-kilograms	0 Foot-lbs.	1 Foot-lbs.	2 Foot-lbs.	3 Foot-lbs.	4 Foot-lbs.	5 Foot-lbs.	6 Foot-lbs.	7 Foot-lbs.	8 Foot-lbs.	9 Foot-lbs.
0	......	7.23	14.47	21.70	28.93	36.17	43.40	50.63	57.87	65.10
10	72.33	79.57	86.80	94.03	101.27	108.50	115.74	122.97	130.20	137.43
20	144.67	151.90	159.13	166.37	173.60	180.84	188.08	195.30	202.54	209.77
30	217.00	224.23	231.46	238.70	245.93	253.17	260.41	267.63	274.87	282.10
40	289.34	296.57	303.79	311.04	318.27	325.50	332.75	339.98	347.21	354.44
50	361.66	368.89	376.12	383.36	390.59	397.82	405.07	412.30	419.53	426.76
60	434.00	441.23	448.45	455.70	462.93	470.17	477.41	484.64	491.87	499.10
70	506.34	513.57	520.80	528.04	535.27	542.50	549.75	556.98	564.21	571.44
80	578.68	585.91	593.14	600.38	607.61	614.85	622.09	629.41	636.55	643.78
90	651.00	658.23	665.46	672.70	679.93	687.17	694.41	701.63	708.87	716.10
100	723.34	730.57	737.80	745.04	752.27	759.51	766.75	774.07	781.21	788.44

Foot-pounds into British Thermal Units

Foot-pounds	0 B.T.U.	1 B.T.U.	2 B.T.U.	3 B.T.U.	4 B.T.U.	5 B.T.U.	6 B.T.U.	7 B.T.U.	8 B.T.U.	9 B.T.U.
0	......	0.0013	0.0026	0.0039	0.0051	0.0064	0.0077	0.0090	0.0103	0.0116
10	0.0129	0.0141	0.0154	0.0167	0.0180	0.0193	0.0206	0.0218	0.0231	0.0244
20	0.0257	0.0270	0.0283	0.0296	0.0308	0.0321	0.0334	0.0347	0.0360	0.0373
30	0.0385	0.0398	0.0411	0.0424	0.0437	0.0450	0.0463	0.0475	0.0488	0.0501
40	0.0514	0.0527	0.0540	0.0552	0.0565	0.0578	0.0591	0.0604	0.0617	0.0630
50	0.0642	0.0655	0.0668	0.0681	0.0694	0.0707	0.0719	0.0732	0.0745	0.0758
60	0.0771	0.0784	0.0797	0.0809	0.0822	0.0835	0.0848	0.0861	0.0874	0.0886
70	0.0899	0.0913	0.0925	0.0938	0.0951	0.0964	0.0976	0.0989	0.1002	0.1015
80	0.1028	0.1040	0.1054	0.1066	0.1079	0.1092	0.1105	0.1118	0.1131	0.1143
90	0.1156	0.1169	0.1182	0.1195	0.1208	0.1221	0.1233	0.1246	0.1259	0.1272
100	0.1285	0.1298	0.1311	0.1324	0.1337	0.1350	0.1362	0.1375	0.1388	0.1401

British Thermal Units into Foot-pounds

B.T.U.	0 Foot-lbs.	1 Foot-lbs.	2 Foot-lbs.	3 Foot-lbs.	4 Foot-lbs.	5 Foot-lbs.	6 Foot-lbs.	7 Foot-lbs.	8 Foot-lbs.	9 Foot-lbs.
0	.....	778	1,557	2,335	3,114	3,892	4,670	5,449	6,227	7,006
10	7,784	8,562	9,341	10,119	10,897	11,676	12,454	13,233	14,011	14,789
20	15,568	16,346	17,125	17,903	18,681	19,460	20,238	21,017	21,795	22,573
30	23,352	24,130	24,909	25,687	26,465	27,244	28,022	28,800	29,579	30,357
40	31,136	31,914	32,692	33,471	34,249	35,028	35,806	36,584	37,363	38,141
50	38,920	39,698	40,476	41,255	42,033	42,811	43,590	44,368	45,147	45,925
60	46,703	47,482	48,260	49,039	49,817	50,595	51,374	52,152	52,931	53,709
70	54,487	55,266	56,044	56,823	57,601	58,379	59,158	59,936	60,714	61,493
80	62,271	63,050	63,828	64,606	65,385	66,163	66,942	67,720	68,498	69,277
90	70,055	70,834	71,612	72,390	73,169	73,947	74,726	75,504	76,282	77,061
100	77,839	78,618	79,396	80,174	80,953	81,731	82,510	83,288	84,066	84,845

Horsepower into Kilowatts

H.P.	0	1	2	3	4	5	6	7	8	9
	K.W.	K.W.	K.W.	K.W.	K.W.	K.W.	K.W.	K.W.	K.W.	K.W.
0	......	0.746	1.491	2.237	2.983	3.729	4.474	5.220	5.966	6.711
10	7.457	8.203	8.948	9.694	10.440	11.186	11.931	12.677	13.423	14.168
20	14.914	15.660	16.405	17.151	17.897	18.643	19.388	20.134	20.880	21.625
30	22.371	23.117	23.862	24.608	25.354	26.100	26.845	27.591	28.337	29.082
40	29.828	30.574	31.319	32.065	32.811	33.557	34.302	35.048	35.794	36.539
50	37.285	38.031	38.776	39.522	40.268	41.014	41.759	42.505	43.251	43.996
60	44.742	45.488	46.233	46.979	47.725	48.471	49.216	49.962	50.708	51.453
70	52.199	52.945	53.691	54.436	55.182	55.928	56.673	57.419	58.165	58.910
80	59.656	60.402	61.148	61.893	62.639	63.385	64.130	64.876	65.622	66.367
90	67.113	67.859	68.605	69.350	70.096	70.842	71.587	72.333	73.079	73.824
100	74.570	75.316	76.062	76.807	77.553	78.299	79.044	79.790	80.536	81.281

Kilowatts into Horsepower

K.W.	0	1	2	3	4	5	6	7	8	9
	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
0	......	1.341	2.682	4.023	5.364	6.705	8.046	9.387	10.728	12.069
10	13.410	14.751	16.092	17.433	18.774	20.115	21.456	22.797	24.138	25.479
20	26.820	28.161	29.502	30.843	32.184	33.525	34.866	36.208	37.549	38.890
30	40.231	41.572	42.913	44.254	45.595	46.936	48.277	49.618	50.959	52.300
40	53.641	54.982	56.323	57.664	59.005	60.346	61.687	63.028	64.369	65.710
50	67.051	68.392	69.733	71.074	72.415	73.756	75.097	76.438	77.779	79.120
60	80.461	81.802	83.143	84.484	85.825	87.166	88.507	89.848	91.189	92.530
70	93.871	95.212	96.553	97.894	99.235	100.58	101.92	103.26	104.60	105.94
80	107.28	108.62	109.96	111.30	112.65	113.99	115.33	116.67	118.01	119.35
90	120.69	122.03	123.37	124.71	126.06	127.40	128.74	130.08	131.42	132.76
100	134.10	135.44	136.78	138.12	139.47	140.81	142.15	143.49	144.83	146.17

Miles into Kilometers

Miles	0	1	2	3	4	5	6	7	8	9
	Km.	Km.	Km.	Km.	Km.	Km.	Km.	Km.	Km.	Km.
0	.....	1.609	3.219	4.828	6.437	8.047	9.656	11.265	12.875	14.484
10	16.093	17.703	19.312	20.921	22.531	24.140	25.750	27.359	28.968	30.578
20	32.187	33.796	35.406	37.015	38.624	40.234	41.843	43.452	45.062	46.671
30	48.280	49.890	51.499	53.108	54.718	56.327	57.936	59.546	61.155	62.764
40	64.374	65.983	67.593	69.202	70.811	72.421	74.030	75.639	77.249	78.858
50	80.467	82.077	83.686	85.295	86.905	88.514	90.123	91.733	93.342	94.951
60	96.561	98.170	99.779	101.39	103.00	104.61	106.22	107.83	109.44	111.04
70	112.65	114.26	115.87	117.48	119.09	120.70	122.31	123.92	125.53	127.14
80	128.75	130.36	131.97	133.58	135.19	136.79	138.40	140.01	141.62	143.23
90	144.84	146.45	148.06	149.67	151.28	152.89	154.50	156.11	157.72	159.33
100	160.93	162.54	164.15	165.76	167.37	168.98	170.59	172.20	173.81	175.42

Kilometers into Miles

Km.	0	1	2	3	4	5	6	7	8	9
	Miles	Miles	Miles	Miles	Miles	Miles	Miles	Miles	Miles	Miles
0	......	0.621	1.243	1.864	2.486	3.107	3.728	4.350	4.971	5.592
10	6.214	6.835	7.457	8.078	8.699	9.321	9.942	10.562	11.185	11.805
20	12.427	13.049	13.670	14.292	14.913	15.534	16.156	16.776	17.399	18.019
30	18.641	19.263	19.884	20.506	21.127	21.748	22.370	22.990	23.613	24.233
40	24.855	25.477	26.098	26.720	27.341	27.962	28.584	29.204	29.827	30.447
50	31.069	31.690	32.311	32.933	33.554	34.175	34.797	35.417	36.040	36.660
60	37.282	37.904	38.525	39.147	39.768	40.389	41.011	41.631	42.254	42.874
70	43.497	44.118	44.739	45.361	45.982	46.603	47.225	47.845	48.468	49.088
80	49.711	50.332	50.953	51.575	52.196	52.817	53.439	54.059	54.682	55.302
90	55.924	56.545	57.166	57.788	58.409	59.030	59.652	60.272	60.895	61.515
100	62.138	62.759	63.380	64.002	64.623	65.244	65.866	66.486	67.109	67.729

Power and Heat Equivalents

1 horsepower-hour = 0.746 kilowatt-hour = 1,980,000 foot-pounds = 2550 B.T.U. (British thermal units) = 2.64 pounds of water evaporated at 212° F. = 17 pounds of water raised from 62° to 212° F.

1 kilowatt-hour = 1000 watt-hours = 1.34 horsepower-hour = 2,653,200 foot-pounds = 3,600,000 joules = 3420 B.T.U. = 3.54 pounds of water evaporated at 212° F. = 22.8 pounds of water raised from 62° to 212° F.

1 horsepower = 746 watts = 0.746 kilowatt = 33,000 foot-pounds per minute = 550 foot-pounds per second = 2550 B.T.U. per hour = 42.5 B.T.U. per minute = 0.71 B.T.U. per second = 2.64 pounds of water evaporated per hour at 212° F.

1 kilowatt = 1000 watts = 1.34 horsepower = 2,653,200 foot-pounds per hour = 44,220 foot-pounds per minute = 737 foot-pounds per second = 3420 B.T.U. per hour = 57 B.T U. per minute = 0.95 B.T.U. per second = 3.54 pounds of water evaporated per hour at 212° F.

1 watt = 1 joule per second = 0.00134 horsepower = 0.001 kilowatt = 3.42 B.T.U. per hour = 44.22 foot-pounds per minute = 0.74 foot-pounds per second = 0.0035 pound of water evaporated per hour at 212° F.

1 B.T.U. (British thermal unit) = 1052 watt-seconds = 778 foot-pounds = 0.252 calorie = 0.000292 kilowatt-hour = 0.000391 horsepower-hour = 0.00104 pound of water evaporated at 212° F.

1 foot-pound = 1.36 joule = 0.000000377 kilowatt-hour = 0.00129 B.T.U. = 0.0000005 horsepower-hour.

1 joule = 1 watt-second = 0.000000278 kilowatt-hour = 0.00095 B.T.U. = 0.74 foot-pound.

Symbols of Chemical Elements

Chemical Element	Symbol	Chemical Element	Symbol	Chemical Element	Symbol
Aluminum........	Al	Indium...........	In	Ruthenium.....	Ru
Antimony.........	Sb	Iodine............	I	Samarium.......	Sm
Argon.............	A	Iridium...........	Ir	Scandium.......	Sc
Arsenic...........	As	Iron..............	Fe	Selenium.......	Se
Barium...........	Ba	Krypton..........	Kr	Silicon..........	Si
Beryllium.........	Be	Lanthanum.......	La	Silver...........	Ag
Bismuth..........	Bi	Lead..............	Pb	Sodium..........	Na
Boron.............	B	Lithium...........	Li	Strontium.......	Sr
Bromine..........	Br	Magnesium........	Mg	Sulphur..........	S
Cadmium.........	Cd	Manganese........	Mn	Tantalum.......	Ta
Caesium..........	Cs	Mercury..........	Hg	Tellurium.......	Te
Calcium..........	Ca	Molybdenum......	Mo	Terbium........	Tb
Carbon............	C	Neodymium......	Nd	Thallium........	Tl
Cerium............	Ce	Neon.............	Ne	Thorium........	Th
Chlorine..........	Cl	Nickel............	Ni	Thulium........	Tm
Chromium........	Cr	Niobium..........	Nb	Tin.............	Sn
Cobalt............	Co	Nitrogen..........	N	Titanium.......	Ti
Copper............	Cu	Osmium..........	Os	Tungsten.......	W
Erbium...........	Er	Oxygen...........	O	Uranium........	U
Fluorine..........	F	Palladium.........	Pd	Vanadium......	V
Gadolinium.......	Gd	Phosphorus.......	P	Xenon..........	Xe
Gallium...........	Ga	Platinum.........	Pt	Ytterbium......	Yb
Germanium.......	Ge	Potassium.........	K	Yttrium........	Y
Glucinum........	Gl	Praesodymium....	Pr	Zinc............	Zn
Gold..............	Au	Radium...........	Ra	Zirconium......	Zr
Helium...........	He	Rhodium..........	Rh		
Hydrogen.........	H	Rubidium........	Rb		

MANUFACTURING PLANT APPRAISAL

The method of appraising manufacturing plants depends somewhat upon the purpose of the appraisal. When making an appraisal for a prospective buyer, the appraiser will naturally decide upon a comparatively low range of values, knowing that the expert retained by the owners will decide upon the upper range, and *vice versa*. It may be impossible to determine absolutely the value of a machine, and the variation of a few dollars, one way or the other, might be equally justifiable under certain conditions. If the appraisal is made for the owners for purposes of taxation, accounting and insurance, it is well to take average values.

Terms used in Appraisal Work. — The meaning or definition of terms used in appraisal work and a brief explanation of the underlying theories are given in the following: *Unit Plant:* A unit portion of the plant equipment. *Replacement Value:* Actual cost of replacing a unit plant with one of the same type at prevalent market prices at time of appraisal. The replacement value, then, is the market price of the machine plus freight plus cost of installation In the case of large machine tools, the freight and installation items are large enough to be worth considering, especially when expensive foundations are necessary. When appraising small parts of machinery and small tools, these items are practically negligible when considering individual tools, but in appraising the contents of a tool-room, where large quantities of tools have been purchased in bulk, some allowance should be made for freight. *First Cost Installed:* The original cost of unit plant at market prices prevalent at the time of purchase of the plant under consideration plus freight and cost of installation. The "first cost installed" can often be obtained directly from the books of the business. *Scrap Value:* The actual cash return brought by the sale of materials (iron, copper, etc.) used in the construction of a machine or tool at current market prices, less cost of junking. The cost of junking will be high in the case of large and unwieldy machines, and in some cases will offset the return from sale of scrap, making the net scrap value zero or even a negative quantity. *Depreciation:* The lessening in value of unit plant due to (1) wear and tear; (2) age and deterioration; (3) inefficiency and inferiority in design. *Depreciable or Wearing Value:* The replacement value of plant less the scrap value. *Depreciated or Present Value:* Value of unit plant at the time of appraisal. Present value equals the replacement value less the accrued depreciation at time of appraisal.

Replacement Value. — The determination of the replacement value is simply a matter of applying unit prices to the machine or tool in question with proper additions for freight and installation. A careful check of unit prices is, of course, necessary to ascertain their accuracy at the time of the appraisal. The increased costs of material and labor have caused a marked rise in the prices of some of the larger machine tools within the past decade. In the case of small machine tools, prices, in many instances, have decreased within the past few years.

Present Value. — The computation of the theoretical present value can be most easily done by applying an annual rate of depreciation directly to the wearing value and subtracting the result from the replacement value. The wearing value equals the replacement value less the scrap value. The scrap value has a definite ratio to the replacement value for each type of plant. The reciprocal of the life of a machine or tool gives the rate per cent which, when applied to the wearing value and multiplied by the age in years, gives the accrued depreciation. This latter subtracted from the replacement value gives the present value. The present value can be obtained directly by applying the following formula: Present value = $\frac{a\,(l-f)+fb}{l}$, where a = replacement value; b = scrap value; f = age of machine:

l = life of machine. After the present value has been obtained by either of the methods outlined, allowance should be made for the actual condition of the machine or tool in question, as ascertained by careful inspection. For example, two drill presses of the same make and size, recently installed, are operating side by side. One is found to be in excellent condition while the other has a table badly mutilated by careless operators. Obviously, the accrued depreciation on the second should be greater and the present value less than on the first. It is in cases of this kind that the appraiser's judgment comes into play.

Average Life of Equipment. — The following list of average lives of various types of machines includes all three of the items given in the definition of depreciation, *i.e.* (1) wear and tear; (2) age and deterioration from natural causes; (3) inefficiency and inferiority in design. If item (3) were not considered, the averages would be considerably higher.

Large Machine Tools. — (Boring mills, planers, engine lathes, etc.) Estimated life, twenty-five years.

Small Machine Tools. — (Lathes, small drill presses, shapers, bench tools, etc.) Estimated life, twenty years.

Small Parts. — (Jigs, chucks, fixtures, etc.) Estimated life, fifteen years.

Small Tools. — (Reamers, boring bars, drills, etc.) Estimated life, ten years.

Miscellaneous. — (Closets, tool-stands, shop furniture, etc.) Estimated life, fifteen years.

Motors and Electrical Equipment. — Estimated life, twenty years.

Shafting. — Estimated life, fifteen years.

Belting. — Estimated life, ten years.

Depreciation of Machinery. — The amount to be deducted annually from the initial cost is approximately as follows:

Lathes and machine tools, first class	5 per cent
Engines, shafting, gearing and millwork	5 to 7 per cent
Lathes and machine tools, second class	10 per cent
Machinery in general	10 per cent
Boilers	7 to 9 per cent
Leather belting	10 to 12 per cent

Appraising Patterns and Drawings. — The proper method of appraising patterns and drawings is a subject regarding which there is considerable difference of opinion among appraisers. A detailed inspection of patterns is not feasible unless the manufacturer is willing to have the superintendent, or some official equally well informed, go over the stock with the appraiser and furnish information as to obsolete patterns, as well as those that are gradually becoming obsolete. It often happens that a company's sales contracts require them to furnish duplicate parts on demand. This necessitates the preservation of patterns for machines superseded by those of more recent design, in order to fill occasional orders. Obviously, the only way the appraiser can know of these cases is by the personal assistance of a representative of the company.

A satisfactory compromise may be effected by going over the pattern books with the superintendent and obtaining the original cost and date of manufacture, together with information as to obsolete and obsolescent patterns. The appraisal of drawings can be based on the assumption that each drawing is a necessary accessory of the pattern built from it. The value of the drawings then bears a definite ratio to the value of the corresponding patterns, with the exception of drawings relating to obsolete patterns. While from time to time it may be necessary to destroy obsolete patterns in order to avoid congestion in the pattern store-room, it is rarely advisable to destroy the corresponding drawing.

Depreciation Charges. — The depreciation in value of mechanical equipment takes place at a variable rate and is more or less compensated for by repairs and renewals, but an average can be determined by estimating the probable working life of the machine and then distributing over this period the difference between the cost of the machine and its scrap or junk value. The *replacement cost* of the machine includes the original price, the expense of installation, the cost of all repairs and renewals, and the expenditure for special tools and fixtures.

Replacements of expensive machinery, which is either worn out or obsolete, are generally provided for by setting aside annually a certain *depreciation charge*, thus creating a *depreciation fund* or reserve. The object of this plan is to avoid excessive operating charges during the years when replacements of expensive equipment are necessary. The depreciation fund may be invested in securities having a regular market value or it may be invested in the company's own business, preferably in such a way as to be available when needed.

Formulas for Determining Depreciation Charges. — If the value of a machine or other property is assumed to decrease at a uniform rate (which is the most common method), the annual depreciation in dollars, not considering interest on invested funds, may be found by subtracting the scrap value from the replacement cost, and dividing the difference by the estimated life of the machine in years. Thus, if D = annual depreciation in dollars; P = depreciation expressed on a percentage basis (decimal fraction); C = replacement cost; S = scrap value; n = number of years of useful life; then $D = \frac{C - S}{n}$. This is sometimes called the *straight line method*, because the depreciation charges are the same each year and if plotted would lie along a straight line; hence the name. When the depreciation charge is invested at a given percentage rate P, compounded annually, the depreciation charge may be determined by one of the following formulas: If the annual payments are made at the *end* of each year, then:

$$D = (C - S)\frac{P}{(1 + P)^n - 1}$$

If the annual payments are made at the *beginning* of each year, then:

$$D = (C - S)\frac{P}{(1 + P)^{n+1} - (1 + P)}$$

Example: The replacement cost of a machine is \$6000; the scrap value is \$100; and the useful life of the machine is estimated to be 20 years. Determine the annual depreciation charge, assuming that these sums are to be invested at the end of each year at six per cent compounded annually.

$$D = (6000 - 100)\frac{0.06}{(1 + 0.06)^{20} - 1} = \$160 \text{ approx.}$$

Diminishing Depreciation Charges. — When the depreciation charges are to be calculated on the decreasing value of equipment or its value at the beginning of each year, instead of taking a fixed replacement or original value as the basis, the following formulas may be used. The notation is similar to that previously given.

$$P = 1 - \sqrt[n]{\frac{S}{C}}$$

The depreciation charge at the end of a given number of years x is found by one of the following formulas:

$$D = PC\left(\frac{S}{C}\right)^{\frac{x-1}{n}} = PC\sqrt[n]{\left(\frac{S}{C}\right)^{x-1}}$$

PATENTS

Patentable Inventions. — Patents are issued by the United States Patent Office to any person who has invented or discovered any new and useful art, machine, method of manufacture, composition of matter, or any new and useful improvement along these lines. In order to obtain a patent for an invention, the latter must not have been known or used by others in this country previous to the time the invention was made by the person applying for the patent; nor must it have been described in any printed publication in this or any foreign country before the invention was made by the person applying for a patent in this country, or more than two years prior to the application for a patent. A patent cannot be granted if the article has been in public use or for sale in the United States for more than two years prior to the application for a patent.

A patent contains a grant to the patentee, his heirs or assigns, for a term of seventeen years, for the exclusive right to make, use or sell the invention or discovery throughout the United States. In case the inventor at the time of making his application believed himself to be the first inventor or discoverer, but it is subsequently found that the invention or discovery has been known or used in a foreign country before the time of his invention, he will not be refused a patent providing the article has not previously been patented or described in any printed publication. The application for a patent in this country must be filed within 12 months after an application for a patent may have been filed in a foreign country; otherwise, no patent will be granted in this country. In the case of ornamental or other designs, the foreign application must not be filed more than four months prior to the time when the application is made in this country.

Joint Inventions. — Joint inventors are entitled to a joint patent, but neither can claim one separately. Independent inventors of distinct and independent improvements in the same machine cannot obtain a joint patent for their separate inventions. If one person furnishes the capital and another makes the invention, they cannot make application as joint inventors. The inventor only can make such application; but they may become joint patentees by means of a deed of assignment.

Applications for Patents. — An inventor who wishes to apply for a patent, and is not familiar with the rules of patent practice, should apply to the Patent Office, Washington, D. C., for a copy of the "Rules of Practice," which will be sent upon request. It is also advisable that the services of a competent and duly registered patent attorney be secured, as the values of patents depend largely upon the preparation of the specifications and the claims. An inexperienced person will often prepare claims which cover only the particular design for the apparatus in which the invention at first may have been executed. The invention, however, may be much more fundamental in character, and the claims should cover all possible designs by means of which the same end may be obtained with the same fundamental principles of action of the device. The patent office, while it will not recommend any particular patent attorney or firm, advises applicants to avoid doing business with those attorneys who advertise the possession of unusual facilities for obtaining patents.

Applications for a patent must be made in writing to the Commissioner of Patents. The applicant must also file in the patent office a written description of the invention or discovery, in clear, concise and exact terms. In the case of a machine, it is necessary to particularly point out and distinctly claim the particular improvement or combination of which the inventor claims to be the discoverer. The specification and claim must be signed by the inventor. When the nature of the invention is such that drawings will make the description clearer, the applicant must furnish a drawing signed by the inventor or his attorney. If the patent office so requires, the applicant must also furnish a model of convenient size to exhibit advantageously the several parts of the invention, but a model should not be sent unless first called for by the patent office.

The applicant must make oath that he believes himself to be the original and first inventor of the invention or improvement for which he solicits a patent. He must also state of what country he is a citizen and where he resides, and whether he is the sole or joint inventor of the invention claimed. He must also state under oath that the invention has not been patented by himself or others with his knowledge or consent in this or any foreign country, and that an application for a patent has not been filed in any foreign country by himself or his legal representative more than 12 months prior to his application in this country (or four months in case of designs). If foreign applications have been filed, the country or countries where this has been done should be stated, with the date of application. This oath may be made before any person within the United States duly authorized by law to administer oaths, or, when the applicant resides in a foreign country, before any minister, chargé d'affaires, consul or commercial agent holding commission under the Government of the United States, or before any person having an official seal and authority to administer oath in a foreign country. In the latter case, the authority of such person must be proved by a certificate of a diplomatic or consular officer of the United States.

Fees. — Fees must be paid in advance, and are as follows: On filing each original application for a patent, $15. On issuing each original patent, $20. In design cases: For three years and six months, $10; for seven years, $15; for fourteen years, $30. On every application for the reissue of a patent, $30. On filing each disclaimer, $10. For certified copies of patents and other papers in manuscript, ten cents per hundred words and twenty-five cents for the certificate; for certified copies of printed patents, eighty-five cents. For uncertified copies of specifications and drawings of patents, ten cents each. For recording every assignment, agreement, power of attorney, or other paper, of three hundred words or under, $1; of over three hundred and under one thousand words, $2; for each additional thousand words, or fraction thereof, $1. For copies of drawings, the reasonable cost of making them. The Patent Office is prepared to furnish positive photographic copies of any drawing, foreign or domestic, in the possession of the office, in sizes and at rates as follows: Large size, 10 by 15 inches, twenty-five cents; medium size, 8 by 12½ inches, fifteen cents. Fee for examining and registering trade-marks, $10, which includes certificate. Stamps cannot be accepted by the Patent Office in payment of fees. Stamps and stamped envelopes should not be sent to the office for replies to letters, as stamps are not required on mail matter which is sent from the Patent Office. Mail sent to the Patent Office must, of course, be stamped.

Reissue of Patents. — A reissue of a patent is granted to the original patentee, or his legal representatives, if the original specification proves to have been defective or insufficient, or if the patentee has claimed for his invention or discovery more than he had a right to claim as new, so that the original patent is invalid, provided the error has arisen from ignorance or mistake and without any fraudulent or deceptive intention. A reissue application must be made and the specifications sworn to by the inventor, the same as in the case of the original application, providing the inventor is still living.

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